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DETERMINATION OF STRESS INTENSITY FACTOR EQUATIONS FOR LIFE PREDICTION OF WELDS WITH SEMI-ELLIPTICAL WELD TOE CRACKS

© Tomas H. Lewinson

*A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the
Degree of Master of Engineering.*

**DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
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ABSTRACT

This report describes finite element analysis tests which were performed using the ASAS-H finite element program to determine the linear elastic stress intensity factors of 3-D semi-elliptical cracks at weld toes in "T" plates.

The substructuring capabilities of ASAS are applied to study the effect of crack geometry, load case and weld shape on the stress intensity factors for semi-circular and semi-elliptical weld toe cracks. The variables tested were:

- 1) crack geometry - $c/a=1.0, 2.0$ and 4.0 for $0.025 \leq a/t \leq 0.4$
- 2) load case - tension, 4-point bending, 3-point bending
- 3) weld shape - $70^\circ, 45^\circ$ and 30° welds

Empirical relations for the stress intensity factors have been found as functions of these variables for later use in a life determination program.

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NOMENCLATURE

a - crack depth into base

b - plate half width

c - half of crack length along surface

m - crack tip element length parameter

t - thickness of base plate

K - stress intensity factors

L - crack tip element length parameter

M_{SD} - stress intensity magnification factor

α - weld shape angle

ν - Poisson's Ratio

σ - stress

θ - angle to point on crack front from center of ellipse

Chapter 1

INTRODUCTION

1.1 Introduction

Along with increased efforts towards energy conservation worldwide, new and previously unfeasible oil sites are being sought out. Most of these new reserves lie at the bottom of major bodies of water. In order to reach these wells very sophisticated oil drilling platforms must be built. One of the many engineering problems associated with the design of these structures lies in the determination of the fatigue life of welds throughout. Due to the fact that these structures reside in the water, each wave motion actually acts as a loading cycle. The number of loading cycles, or the fatigue life of the welds, is of great importance to any 'water-based' drilling rigs due to possible catastrophic results if failure should occur. This paper will present formulas which can be used in the analysis of the fatigue life of these structures.

For many years now, techniques of fracture mechanics have been evolving due to the need to design structures and components which might develop cracks in service, using quantitative methods against the possibility of fracture.

Many structures suffer from such material defects as microporosity which can cause internal or embedded cracking. Other materials might contain surface cracks due to residual stresses or simply from local overloading near brittle areas such as welds, before they are even placed into service. Methods have to be determined for the quantitative evaluation of these cracks, or in other words can the part function with the crack without fear of fracture, or should it be replaced.

Offshore oil drilling platforms are usually constructed to stand on large tubular legs which extend down to the ocean floor. These legs are cross braced by other large tubes, which are fixed to the legs by welding. The toe of a weld is known to be a large stress concentration and usually causes the development of surface cracks at this point. These cracks are inevitable and sometimes occur before the platform is even placed into service. If the cracks do not occur at this stage due to thermal stressing at welding then the constant pounding of waves on the platform will almost surely force small surface flaws to form into cracks which would propagate with every wave load applied. Obviously it does not make sense for these platforms to be designed in such a way that parts would have to be replaced at the first sign of a crack. However, exactly how large a crack can be tolerated for a given material and a given weld geometry.

The current design practice for tubular frame structures is the S-N design curve approach (1),(2),(3),(4). Although this method has provided a successful means of design for past structures, there have been a number of criticisms directed towards this approach (5), the most outstanding is the fact that the S-N curve design method can only be applied to an undamaged structure, or in other words a crack free structure.

Linear elastic fracture mechanics (LEFM) offers an alternative method to the S-N curve method for estimating the life of a structure. This method, being fairly new, is not expected to replace the S-N curve method in the near future due to the abundance of data available for the S-N curve method and the relative lack of data for the LEFM method. However, LEFM does offer a method with which structures which contain flaws can be dealt with, and demonstrates the influence of a number of variables on fatigue crack propagation.

Fatigue damage in offshore platforms occurs at the welded joints at the intersection of the chord and brace in the weld toe region. Initial flaws or inclusions which are present at the weld toe propagate around the joint as more pronounced cracks until they reach critical size or penetrate the thickness of the structure. Since in tubular joints the crack propagates more rapidly along the weld compared to the growth in the depth direction, the investigation of this problem becomes a complex 3-dimensional analysis.

Through the study of stress fields at crack tips in elastic bodies it was found that one single parameter, the stress intensity factor 'K' could represent a particular structural geometry containing a crack at a given loading stress. Similarly the growth rate of a crack at a weld toe can be described using the stress intensity factor. In this approach the crack growth rate da/dN is related to the stress intensity range ΔK by the Paris Law (6)

$$da/dN = C\Delta K^m$$

where C and m are material constants. Using this technique it is possible to estimate the fatigue life of the structure once the solution for K as a function of crack shape is known. The value of K for complex geometries may be obtained using the finite element method (FEM).

Although the finite element method can in theory describe and analyze any shape of structure, the incredible number of nodes and elements which would be required to represent an actual weld toe cracked tubular joint makes the problem close to impossible to analyze properly due to computer cost and mesh construction difficulties. It is quite clear that actual experimental fatigue cycling tests of these massive tubular joints (some of which are 10m in diameter) would be just as difficult to perform as the FEM tests due to the enormity of the structure. Programs were therefore started by ECSC, UKOSRP, EMR and others to do testing on cracked 'T' plate specimens, since this was shown to be a fairly good comparison to the actual tubular joint. This can be gathered from FIGURE 1.1.

This report describes work which has been carried out on the calculation of stress intensity factors for 3-D semi-elliptical cracks at the toe of a weld in 'T' plate joints using the finite element method of analysis.

The purpose of these tests was to be able to derive an empirically based equation (or equations) for the stress intensity factors as functions of crack shape, load case, weld shape and crack position. The goal of this was to be able to use these equations in a crack growth prediction program.

1.2 Objectives

The objectives of this report were:

- 1) To investigate the effect of varying weld shape angles and load cases on the stress intensity factors for weld toe crack 'T' plate specimens using the finite element method.
- 2) To find empirical relations for the stress intensity factors as functions of the crack geometries, weld shape angles and load cases from the calculated results for use in a life prediction program.

The cases considered were:

- 1) The analysis of semi-elliptical cracks with aspect ratios of $c/a = 1.0, 2.0$ and 4.0 for ranges of a/t from 0.025 to 0.4 (FIGURE 1.2).
- 2) Investigation of the effect that weld shape has on the SIF. Each specimen was analyzed using a weld toe angle of $30^\circ, 45^\circ$, and 70° (FIGURE 1.3).
- 3) Each weld toe crack specimen was analyzed using load cases of tension, 4-point bending and 3-point bending.

FIGURE 1.1 - Example of tubular 'T' joint and plate 'T' joint model.

FIGURE 1.2 - Explanation of Crack Geometry Symbols

a) 70° Weld shape

b) 45° Weld shape

c) 30° Weld shape

FIGURE 1.3 - Weld geometries tested

Chapter 2

THEORY

2.1 Stress and Fatigue Analysis of Joints

Probably one of the most important aspects to consider in offshore structures, is the ability to withstand the time varying stresses caused by cyclic loading. Fatigue cracks are predominately due to the wave action which, over an anticipated 20-year life span, will subject the structure to approximately 10^8 cycles of stress variation. As a result of this wave action the Certifying Authorities specify requirements for fatigue analysis procedures which designers must meet in order to have their designs approved (1),(3),(4).

Offshore structures generally are of tubular construction. It is at the intersection of the chord and the brace where high local stresses adjacent to the connecting weld are induced. This location is commonly known as a hotspot (1) and it is at this location where fatigue damage will usually occur. It therefore follows that any fatigue analysis will be highly dependent upon the determination of the high local stress at this hotspot. Any deviation will result in a significant change in predicted fatigue life.

There has been much research into the assessment of fatigue in offshore structures in the past 10 years. A great deal of this research has been to develop an improved method of calculation and to date there has been considerable development. There has been extensive research programs in the United Kingdom (UKOSRP) and in Europe (ECSC) on the appropriate S-N curve to use and on the definition of stress and life. A lot of this work has been published in special conference proceedings (7), (8) and (9). Due to the vast amount of new information being produced by the UKOSRP and ECSC test programs revised fatigue design rules have been published by the UK

Department of Energy. These revisions are described in detail with supporting documentation in ref.(2).

In the new revised guidelines the fatigue design procedure is still based on the S-N approach. The difference from the old guidelines lies in the fact that a new curve (the T curve) has been proposed based on tests on tubular joints done by the above associations. Another major change is that the design stress to be used in a fatigue analysis of a tubular joint has been redefined as the hotspot stress. This ideal hot spot stress can be defined by linear extrapolation to the toe of the weld of the principal stresses at points A and B as shown in FIGURE 2.1, ref.(2). The new guidelines also define factors which must be applied to the basic S-N curve as they can influence the fatigue life of the structure. These variables are:

- i) thickness effect
- ii) environmental effects
- iii) stress concentration factors
- iv) post weld heat treatment (PWHT)
- v) weld improvement techniques

The basic S-N curve may be seen in FIGURE 2.2. The influence which the above variables have on the fatigue life of a structure will be discussed later in this report.

The hotspot stress at a tubular joint may be determined by mathematical or experimental modelling. However, due to the many different joint configurations in an offshore structure and the high cost of a finite element analysis or experimental tests, the designer must often resort to stress concentration parametric formulae. The three main collections of parametric formulae are those published by Kuang et al (10), Wordsworth and Smedley(11) and Gibstein (12). Unfortunately these formulae exist only for simple geometries and the tendency is usually to underestimate the actual stresses. Fisher (13) found that by comparing actual hotspot stresses measured on T and

X joints with predicted SCF's from parametric formulae, the formulae did indeed yield stresses which were lower than the actual values, (FIGURE 2.3). In fact, the formula underestimated the stresses by as much as 50%. This would result in an overestimate of the fatigue life by a factor of 8 (power of three), using the new S-N curve. This serves to show that great care must be taken even when simple joints are being considered.

2.2 Fracture Mechanics Analysis of Offshore Structures

The first S-N curves to be used in the design of offshore structures were developed from onshore welded fabrication design curves. They used a nominal member stress which allowed the fatigue life to be calculated very quickly. As development continued it became necessary to include in this design stress effects such as the weld toe stress concentration and the high local bending stresses produced by local bending of the chord and braces at the joint intersection. Once this was done the basic simplicity of the S-N curve was lost. Presently, due to long term tests of full scale steel joints, further tests are underway to produce S-N curves for the design of tubular joints which will include the effects of thickness, environment and post-weld heat treatment. The new DOE 'T' curve (FIGURE 2.4a) and the associated penalties and allowances on life is an example of this. In order to understand the effects of these variables on fatigue life many researchers are now turning to fracture mechanics concepts to explain the fatigue mechanisms and crack growth behavior of tubular joints.

2.3 Fatigue Crack Growth in Weld Toes

Once a crack has been initiated at the toe of a weld the crack will exhibit growth or an elongation along a definite pathway. Various theories have been presented in the literature as to the direction in which a crack will grow. One such theory is that the crack will grow along a path perpendicular to the direction of maximum principal stress (50). In a tubular joint cracks commonly occur at weld toe defects at the intersection of the brace and the chord, thus resulting

in the use of the term 'weld toe cracks'. The rate of the crack growth or elongation, both on the surface and in the interior of the metal, is dependant on the following variables:

- i) the initial crack shape
- ii) the mean stress
- iii) the chance of multinyucleation and coalescence of defects
- iv) the type of loading
- v) the SCF at the weld toe

The fatigue crack propagation, taking into account the above variables, can be described by the Paris Law (6);

$$da/dN = C \Delta K^m$$

where:

da/dN = crack growth rate

C, m = material constants

ΔK = variation of stress intensity factor $= Y \Delta \sigma \sqrt{\pi a}$

The above equation may be integrated in order to obtain the predicted life of a structure.

$$\int_0^N dN = \frac{1}{\pi^{m/2} C \Delta \sigma^m} \int_{a_i}^{a_f} \frac{da}{Y^m a^{m/2}}$$

where:

a_i = initial crack length

a_f = final crack length

Although the Paris Law is widely used, it has certain limitations since it only applies over the central region of the fatigue crack growth curve (FIGURE 2.4b) and does not take into account such effects as mean stress, stress intensity threshold for fatigue crack growth (K_{th}) and fracture toughness (K_c). Various other equations have been published by numerous investigators to take into account these effects.

An improved version of the Paris equation which takes into account the stress ratio and K_c is one published by Forman et al (51).

$$da/dN = C \frac{\Delta K^m}{(1-R)K_c - \Delta K}$$

where $R = \frac{\sigma_{min}}{\sigma_{max}}$ and is the stress ratio

The original Paris equation was also modified to include the stress intensity threshold ΔK_{th} by Klesnil and Lukas (52) as follows:

$$da/dN = C(\Delta K^m - \Delta K_{th}^m)$$

Schulz (53) combined the previous equations to account for all of these effects in an equation which he claims fits the experimental data well:

$$da/dN = C \frac{(\Delta K^m - \Delta K_{th}^m)}{(1-R)K_c - \Delta K}$$

Hudak et al (54) describe the three regions (FIGURE 2.18) of fatigue crack growth in terms of the material's fatigue crack growth resistance (inverse of the growth rate). This equation can be expressed as follows:

$$\frac{1}{da/dN} = \frac{A_1}{\Delta K^{n_1}} + \frac{A_2}{\Delta K^{n_2}} + \frac{A_3}{[(1-R)K_c]^{n_3}}$$

where A_1 , n_1 , A_2 and n_2 are empirical constants. The above authors have shown that this expression gives reasonably good agreement between the predicted and measured lives of tubular joints tested in the air environment.

The fracture mechanics approach in the design of tubular joints has only been used in a minor role to date, and it is not likely to replace the S-N approach in the foreseeable future due to an abundance of data which has been accumulated for the S-N method. However, the designer can take advantage of the FM approach to gain insight into the fatigue crack growth mechanisms at joints so that variables such as geometry, loading, environment and fabrication procedures can be assessed in terms of importance. The FM method also provides the designer with a way of analyzing the importance of defects found during inspections and allows for the development of non-destructive evaluation systems, inspection periods and methods.

A great deal of further analytical and experimental development work is required before FM design reaches its full potential in the design of tubular joints, however references (5) and (14) describe in some detail many of the areas in which FM methodology can be used in the design, maintenance and repair of offshore structures.

Experimental and analytical investigations of the problem of fatigue crack growth around a tubular joint weld region are very costly and time consuming due to the extreme complexity of the problem. For this reason a great deal of the UKOSRP and ECSC programs are concerned with the analysis and testing of welded plate specimens, which it is believed can be related to the tubular joint structure (see FIGURE 1.1). This report will describe the procedures and results of testing various cracked welded plates.

2.4 Weld Toe Cracks

A crack occurring at the toe of a weld typically has an appearance such as that shown in FIGURE 2.3. There are three predominant mechanisms, acting either independently or in conjunction with one another, which cause these cracks to form. These mechanisms are listed below:

- 1) Stress concentration at the toe of a weld
- 2) Embrittlement due to microstructural transformations when the base metal surface is heated above the austenite region at welding and then is allowed to cool
- 3) Residual thermal stresses left in the base metal due to heating at the time of welding

Once one or more of the above factors has occurred in a manner so as to cause the initial formation of a weld toe crack, the tendency will be for the crack to grow in size. This growth occurs since each cyclic load which is placed on the crack front will tend to increase the crack size. As the crack grows larger the stress intensity at the crack tip increases and thus the crack will grow faster. This process continues until the stress intensity factor of the crack becomes greater than the critical toughness, K_{IC} of the material. At this point the part fractures.

2.4.1 Effect of Initial Crack Size

Cracks initiate at the toe of a welded joint as this is the area which is subject to large stress concentrations interacting in a highly complex manner. A crack will begin due to extremely high local stresses at the weld toe, combined with initial defects such as slag inclusions due to welding. These high local stresses will inevitably affect the fatigue strength of the joint. The predominant factors which are assumed to cause these high local stresses are as below:

- i) stress concentration due to weld shape
- ii) weld defects
- iii) residual thermal stresses

Of the above, a weld defect will cause, depending upon its severity, the highest stress concentrations and hence will greatly influence the fatigue strength of the weld.

In order to predict the fatigue life of a welded plate and tubular joints it is assumed that initial cracks the size of the weld toe defects exist and therefore the life is dominated by the crack propagation phase. However the question of the initial crack size, or weld toe defect, still remains to be determined. Many theories and sizes have been proposed to date, all utilizing the Linear Elastic Fracture Mechanics (LEFM) approach. Irvine (15) suggests that since the crack initiation will take place at the weld toe, within the small region of plasticity caused by the high local stresses, the LEFM conditions will not apply until the crack growth has progressed some distance. This assumption is based on the theory that the initial defect size is comparable to the size of the plastic zone. Engesvik (16) suggests that the typical grain size (0.01 mm) will be of the same order as the size of the plastic zone. He states further that the initial crack size, a_i , should be much greater than the plastic zone size, a value of about 0.1 mm. From these evaluations a reasonable lower limit on the size of the initial crack in order to apply the LEFM approach to life calculations is taken to be $a_i = 0.1 \text{ mm} \rightarrow 0.4 \text{ mm}$. This assumption is substantiated by metallurgical examinations which have shown the average depth of these weld toe defects to be 0.15 mm.

2.4.2 Thickness Effect

The new DOE guidelines impose an increasing penalty on life with increasing thickness for the fatigue design of welded joints. These thickness effect correction factors are of the form $(32/t)^{3/4}$ for tubular joints, in which the basic T curve applies for thicknesses of up to only 32 mm. This thickness effect penalty on the S-N curve is shown in FIGURE 2.4. A similar correction factor for non-tubular joints applies, and is of the form $(22/t)^{3/4}$, in which the basic T curve applies for thicknesses up to 22mm.

In order to analyze the fatigue strength of a weld it is first vital to investigate the effects of the plate thickness. Much research has been performed to date analyzing and experimenting on the thickness effects. Berge(17) examined the effect of increasing the plate thickness on the fatigue strength of transverse non-load carrying fillet welds in axial tension. He found that as the plate thickness was increased from 12.5 to 80.0 mm the fatigue strength of the joint was reduced by a factor of 0.2, for geometrically similar joints (FIGURE 2.6). Many other researchers have

published results all indicating similar findings as to the existence of a thickness effect. FIGURE 2.7 shows a comparison of these results by Johnston(18), Booth(19), Haibach(20), Wildschut(21), and Oliver(22).

The effects of varying the weld size and/or the attachment plate thickness were shown by Berge to be minor in comparison to the effect the thickness of the load-carrying plate had on the fatigue strength. Berge discovered that the weld size has a very minor influence on the fatigue strength. On the other hand, as the thickness of the attachment plate increased the fatigue strength decreased. However, although this relationship was found to exist, it is still relatively insignificant compared to the effect of the load-carrying plate thickness.

The need for an investigation into the thickness effect arose when it was noticed that small scale models of the joint had a considerably higher fatigue strength, proportionally, than did the actual full-size test specimens. This plate thickness effect can be easily explained with the aid of FIGURE 2.8. It can be seen that the butt welded plates have a geometrically similar stress distribution. However, it is important to notice that the stress gradient in the thin plate is steeper than that of the thicker plate. Thus, an initial crack in the thin plate will experience a lower stress than the same initial length crack would experience in the thicker plate. Hence, the thin plate will exhibit a lower crack growth rate than the thicker plate. The thinner joint will therefore have a longer life since the difference in the initial crack growth rate overcompensates the difference in crack length to cause failure.

This thickness effect can be predicted analytically since the stress gradient through the thickness enters into the calculation of the stress intensity factor K . This is accomplished by the use of the Paris crack growth law which was demonstrated by Gurney(23). He developed parametric equations for the SIF for different crack depths and weld shapes using the finite element method. With these he showed that the fatigue life does decrease with increasing thickness using calculations based on the Paris law.

2.4.3 Improvement of Weld Geometry and Treatment

A major goal in the study of welds is to improve the fatigue life. This can obviously be accomplished by reducing the stress concentration at the weld toe. In the previous sections it was shown that fatigue cracks tend to originate at the weld toe and are initiated at this location due to the small defects left at the toe during welding. To reduce this stress concentration would result in a longer fatigue life.

Investigators have determined that the best way to reduce the stress concentration is to control the weld geometry. It was shown by Gurney(23) that the fatigue strength of the joint could be increased by increasing the weld leg length. The notch root radius was found to be important according to Berge(17). He found that the weld geometry could be modelled using the parameters θ for the weld toe angle and ρ the notch root radius. The radius was found to be similar in size to the initial defects. Using this model Berge was able to produce a realistic simulation of the stress distributions at the weld toe which in turn produces a significant effect on the fatigue strength of the joint. Weld reinforcement was investigated by Mullen and Merwin(24) in order to see if a fatigue strength improvement would result. This process is a weld treatment method in which weld metal is laid down during or after the initial welding of the joint. The weld geometry is affected in so far as that by improving the weld angle the leg length is increased. They found that the fatigue life of the joint increased by a factor of 3-4.

Although the control of the weld geometry is an important engineering method of increasing the fatigue life of the joint the DOE guidelines do not consider this procedure as having a significant effect. The guidelines do, however, acknowledge that large improvements in the fatigue life can be obtained by local machining of the weld toe to remove toe defects. The reason for this is that by removing the defects, which are essentially small cracks, an initiation period will be introduced, thereby increasing the total life of the joint. The guidelines specify that in order for the defects to be removed the grinding must penetrate the surface of the plate as shown in FIGURE 2.9. If this procedure is followed the life of those joints tested in air will increase by a factor of 2. This improvement in the fatigue life has not as yet been demonstrated for unprotected joints tested in

sea water. The DOE guidelines have as their basis tests performed by Booth(25) and Gurney(26). Mullen and Merwin(24) conducted tests on 'T' specimens and found the life improvements to be in the order a factor of 3 - 11.

The new revised DOE guidelines now encourage weld improvement yet they do not allow any benefit to be claimed by this procedure. There are exceptions where the procedure may be used, but only in special circumstances such as in the final stages of design where a particular joint is found to be critical. Research is presently being performed in various offshore test programs in order to determine the effects of weld profile on fatigue life.

2.4.4 Environmental Effects on Weld Toe Cracks

After extensive testing carried out in the UKOSRP and the ECSC it was found that sea water had a detrimental effect on the fatigue life of joints. This effect could be minimized by the use of cathodic protection. For this reason the new DOE guidelines state that the relevant S-N curve should be reduced by a factor of 2 on life for unprotected joints and that there should be no assumed bend in the S-N curve at 10^7 cycles when considering small stresses. It is required that for protected joints the S-N curve in air should be used. It is also recommended that care should be taken so that the joints are not over-protected.

2.5 Finite Element Analysis

The finite element method was first designed for use in structural engineering. Within less than a decade the basic concept of this method was recognized as having a wide applicability and it was being used in many other areas. Subsequent development has been rapid, and techniques are now established within many scientific and engineering disciplines. Although there may be considerable diversity in the formulation of various methods, a finite element method can be distinguished by the following features:

- 1) the physical region of the problem is subdivided into subregions or finite elements.
- 2) At least one of the dependent variables is approximated in functional form over each element and hence over the entire domain. The parameters of the approximation subsequently become the unknowns of the problem.
- 3) Substitution of the approximations into the governing equations (or their equivalent) yields a set of equations in the unknown parameter and hence the approximate solution of the problem.

The actual finite element method formulation will not be discussed further in this report as a more thorough and comprehensive explanation can be found in (27), (28).

The dependent variables can either be displacements or force, defining either the displacement or force method of analysis. ASAS, the Finite Element package used for all of the tests conducted in this report uses the displacement method of analysis.

There are a great variety of 2-D and not so many 3-D elements. The displacement formulation usually has a polynomial form. Various orders of polynomials have been used ranging from linear to high orders of displacement variation.

In general the more elements that are used, the more accurate the results. Also the higher the order of the polynomial used, the higher the accuracy. The anticipated stress gradients in the structure will dictate the density of the element mesh required. In this respect crack tips are particularly crucial and a great deal of work has been focused on modelling the stress and displacement singularities which occur.

2.6 Stress intensity factors

In the theory of linear elastic fracture mechanics there are 3 basic modes of crack surface displacement which can cause crack growth. These are shown in FIGURE 2.10.

- I - The opening mode - the crack surfaces are pulled directly apart.

- II - The edge sliding mode - the crack surfaces move parallel to the crack front, and remain in the crack plane.
- III - The shear mode - the crack surfaces move parallel to the crack front and remain in the crack plane.

Together these three modes of fracture are sufficient to describe the most general case of crack surface displacement.

Usually in fracture mechanics only the macroscopic mode of crack growth is considered; 'shear fracture', which is crack growth in a 45° plane is a combination of both modes I and III, however it is widely accepted that treatment of this in calculations can be done as if it were Mode I. Generally, since a brittle crack always has a tendency to propagate in the direction which will minimize the shear loading, Mode I fracture is generally regarded as most important.

Associated with the 3 modes of fracture are 3 stress intensity factors K_I , K_{II} , K_{III} . The stress intensity factor may be described as the strength or amplitude of the inverse square root stress singularity at the crack tip. These are the results of altered stress flow patterns through the block. This can be represented easily in a 2-D drawing as in FIGURE 2.11.

It has been shown by Irwin (29) that for a circular embedded crack in a block in tension the stress flow distribution will be equal at any point on the crack tip, if the block is large and corner effects of the block can be neglected. This can be accomplished by making the block large enough so that it displays plane strain conditions along the entire crack front. Theoretical results for this are available and can be represented in the form of a simple equation.

$$K_I = \sigma \sqrt{\frac{4a}{\pi}}$$

In the case of a semi-circular surface crack, it is clear that the stress intensity at the free surface will not be the same as the stress intensity anywhere else along the crack due to the different boundary conditions. Also, while the deepest part of the crack displays plane strain conditions as

in the embedded crack, the crack front at the surface displays plane stress conditions. Since there cannot be a sudden transition from plane strain to plane stress, the values of K_I must vary around the crack. It is due to the complexity of this problem that much work has been done in obtaining K_I values for surface cracks.

2.7 Finite element applied to LEFM

2.7.1 Conventional Elements

In the initial development of the finite element method attempts at modelling cracks were done by use of great numbers of conventional elements, which are elements in which the formulation of the element did not take into account the stress singularity at the crack tip. Chan, Tuba, and Wilson (30) demonstrated that many hundreds of elements are required to achieve a 5% accuracy, even with the use of higher order polynomial elements. It was obvious that this approach had to be abandoned and an attempt would have to be made to find element formulations which take explicit account of the singularities.

2.7.2 Polynomial Singularity Function Elements

It is very desirable to make use of elementary polynomial displacement functions in order to maintain simple element geometry and integration of the stiffness parameters.

This approach was first used by Levy et al (31) to model the $1/r$ singularity found in elastic-plastic crack tip field. This was done by superimposing two adjacent nodes of a four-sided rectangular element, and reducing it to a triangle.

This was most important for further developments using isoparametric elements discussed in 2.7.3.

This approach has been intensely investigated by Tracey (32), Tracey et al (33), Blackburn (34) and others.

2.7.3 Isoparametric Elements

Another type of element which is used to model cracks and uses simple distorted polynomial shape functions to obtain the singularity are isoparametric elements. These elements exhibit complete interelement compatibility, contain correct rigid body or constant strain deformations, they can represent the $1/\sqrt{r}$ singularity which occurs in elastic crack problems and the $1/r$ singularity present in elasto-plastic crack tip fields.

Henshell and Shaw (35) were the first to recognize that by placing the 'mid-side' nodes at the quarter point in a quadratic isoparametric quadrilateral (or triangular) 2-D element, a singularity in strain would result at the nearest adjacent corner node. As required by the Westergaard solutions, the singularity turned out to be of the order $1/\sqrt{r}$. Thus by merely modifying the geometry of a standard isoparametric element, the elastic crack singularity could be modelled.

Barsoum (36), studied quadratic 2-D and 3-D isoparametric elements, and introduced the ideas of 'collapsing' the nodes along one edge, and placing the adjacent edge nodes at the quarter point (see FIGURES 2.12 and 2.13). The $1/\sqrt{r}$ singularity was also exhibited by these elements for elastic crack tip stress fields.

In a further paper Barsoum (37) demonstrated that the collapsed formulation of the 8 node triangle and the 20 node wedge contain the $1/\sqrt{r}$ singularity along any ray interior to the element and emanating from the crack tip nodes, whereas the 6 node triangle (2-D), 8 node quadrilateral (2-D), 15 node wedge (3-D), and 20 node brick (3-D) only displayed the $1/\sqrt{r}$ singularity on the boundaries and not in the interior of the elements.

Ingraffea et al (38) published an important paper where the quarter point wedge is applied to 3-D problems. The methodology of extracting SIF's as quadratic functions along the crack front is detailed. The optimum size of the crack tip elements and its effect on accuracy was studied. They recommended a value of $a/L=13.33$ (see FIGURE 4.10) for the tip element and found that as few as 4 elements along the crack face were adequate.

2.7.4 Stress and Displacement Extrapolation

This approach involves using the stresses and displacements obtained in the FEM numerical solution, in conjunction with the classical near field solution equations to extract the stress intensity factor. These equations can be evaluated along radial lines emanating from the crack at a series of nodal points. The results which were obtained from points close to the crack tip are discarded and upon extrapolation to $r=0$ a value of K is obtained.

Ingraffea et al (38) have used this approach in the determination of expressions for the three stress intensity factors K_I , K_{II} , K_{III} for use with collapsed isoparametric wedge elements (see FIGURE 2.14).

2.8 Numerical solutions of 2-D and 3-D crack problems

Prior to initiating any 3-D FE tests, it was believed important to determine the effects that various factors such as crack geometry, weld shape, and modelling procedures had on the stress intensity factors. Due to the high cost and labour involved in a 3-D analysis, these tests were conducted using 2-dimensional models.

For the first tests, a non-load carrying cruciform fillet weld was modelled. Since there are two planes of symmetry only one quarter of the joint was modelled. This joint was also analysed by Gurney and Johnston (55) who modelled the joint using a very large number of constant strain finite elements and by Smith (56) who represented the crack region of the joint by cubic isoparametric crack tip elements. The results for K_I produced by the ASAS program were within 6% of those published by Smith (56) which was deemed acceptable accuracy.

The next investigation was carried out on a T-specimen. Since this model had one vertical line of symmetry, only half of the specimen had to be modelled. This procedure however assumes that the specimen modelled has a crack at each weld toe. In most cases it is expected that a 'T' joint tested in an experimental program will have a toe crack at only one of the welds. This discrepancy was investigated by carrying out an analysis on a complete 'T' model with one unsymmetrical

crack. This analysis showed that there was no significant difference between the SIF's for a 'T' specimen with a single weld toe crack and one with symmetrical weld toe cracks. For this reason it was decided in the FE analysis to model the 2-D and 3-D 'T' joints with a vertical line of symmetry to reduce computing costs. From these 2-D investigations valuable experience was gained as to the mesh design and crack tip element size required to give results of adequate accuracy in the 3-D analyses. A more detailed discussion of the entire 2-D results is presented in reference (48).

At the present time two main numerical analysis tools exist for the determination of stress intensity factors for cracks. These are the finite element method (FEM) and the boundary integral method (BIE). Although very few solutions exist for the analysis of 3-D semi-elliptical weld toe cracks, a number of solutions are available for stress intensity factors for internally pressurised cylinders with elliptical cracks. Blackburn and Hellen (39) performed a finite element analysis of one quarter of such a cylinder using approximately 80, twenty-node quadratic brick elements and fifteen-node special singularity elements at the crack tip. The stress intensity factor was evaluated using the virtual crack extension method and accuracies claimed are within 1% of known solutions.

In another study Tan and Fenner (40) used the BIE method to obtain stress intensity factors for a similar pressurised cylinder. The structure was modelled using 82 eight-noded quadratic elements, with relatively small elements in the vicinity of the crack. The accuracy claimed for these tests was within 5%.

Labbens et al (57) conducted BIE solutions for thick-walled cylinders. Polynomial influence functions were calculated depending on the load and geometry of the cracked cylinder. The numerical values of the influence functions were used to calculate any stress intensity factor resulting from polynomial applied stresses. The accuracy claimed for these tests was within 5%.

Scmitt and Keim (41) also presented three-dimensional finite element solutions using twenty node isoparametric brick elements, but these elements were degenerated to wedges and the midside nodes moved to the quarter points in order to introduce the $1/\sqrt{r}$ singularity at the crack region. One quarter of the cylinder was modelled using 187 elements joined by 977 nodes and two different

loadcases of internal pressure and a temperature gradient applied. The accuracy was 6% for small cracks and 9% for the deeper cracks.

In 1979 Raju and Newman (42) published a report on stress intensity factors for a wide range of semi-elliptical cracks in finite thickness plates. These results were obtained by the finite element method, using meshes which were constructed using 3-D singularity isoparametric elements around the crack as shown in FIGURE 2.15. These elements were grouped together in an arrangement which permitted the use of 8 and in some cases 12 of these crack tip elements at any point along the crack (hoop direction). The number of degrees of freedom for most of the runs were over 4500, which can be considered an extremely refined mesh. In contrast the crack tip arrangement used for the ASAS finite element runs is shown in FIGURE 2.16.

In 1981 Newman and Raju (43) published a very important paper which contained an empirical relation for 3-D surface cracks in a tension block. These equations were based on their finite element results (shown in FIGURE 2.17) and were said to be accurate within $\pm 5\%$ of the FE results. The empirical results presented by Newman and Raju were used to non-dimensionalize the results obtained in the present work.

$a = 0.2 \sqrt{rt}$, but not smaller than 4mm.

FIGURE 2.1 -
DEFINITION OF REGIONS OF STRAIN LINEARITY FOR WELDED TUBULAR T JOINT (2)

FIGURE 2.2 - Design S-N curve for nodal joints (2)

FIGURE 2.3 COMPARISON OF PREDICTED AND EXPERIMENTAL SCFs (48)

FIGURE 2.4a-General form of S-N design curves for welded tubular joints in air (48)

FIGURE 2.4b - Schematic representation of three-component crack growth rate equation.

FIGURE 2.5 - Diagram of Weld Toe Crack

FIGURE 2.6 - EFFECT OF THICKNESS ON LIFE (48)

FIGURE 2.7 - Effect of thickness on life from various sources. (48)

FIGURE 2.8 -
 A MODEL FOR DESCRIBING THE THICKNESS EFFECT IN FATIGUE
 FAILURES DEVELOPING FROM THE WELD TOE (48)

FIGURE 2.9 -

Grinding a weld toe tangentially to the plate surface, as at A, will produce little improvement in strength. Grinding must extend below the plate surface, as at B, in order to remove toe defects. (48)

Basic modes of loading involving different crack surface displacements.

FIGURE 2.10 - Basic Modes of loading involving different Crack Surface Displacements

FIGURE 2.11- Stress flow pattern around a 2-D surface crack in a tension block

FIGURE 2.12 -
COLLAPSED ISOPARAMETRIC ELEMENT WITH MID-SIDE
NODES AT QUARTER POINTS

FIGURE 2.13 -
3-0 COLLAPSED FACE BRICK ELEMENT

$$K_I = \frac{E}{4(1-\nu^2)} \sqrt{\left(\frac{\pi}{2L_1}\right)} [2v_B - v_C + 2v_E - v_F + v_D - 2v_B + v_C - 2v_E + v_F - v_D \\ + \frac{1}{2}\eta(-4v_B + v_C + 4v_E - v_F + 4v_B - v_C - 4v_E + v_F) \\ + \frac{1}{2}\eta^2(v_F + v_C - 2v_D - v_F - v_C + 2v_D)]$$

$$K_{II} = \frac{E}{4(1-\nu^2)} \sqrt{\left(\frac{\pi}{2L_1}\right)} [2u_B - u_C + 2u_E - u_F + u_D - 2u_B + u_C - 2u_E + u_F - u_D \\ + \frac{1}{2}\eta(-4u_B + u_C + 4u_E - u_F + 4u_B - u_C - 4u_E + u_F) \\ + \frac{1}{2}\eta^2(u_F + u_C - 2u_D - u_F - u_C + 2u_D)]$$

and

$$K_{III} = \frac{E}{4(1+\nu)} \sqrt{\left(\frac{\pi}{2L_1}\right)} [2w_B - w_C + 2w_E - w_F + w_D - 2w_B + w_C - 2w_E + w_F - w_D \\ + \frac{1}{2}\eta(-4w_B + w_C + 4w_E - w_F + 4w_B - w_C - 4w_E + w_F) \\ + \frac{1}{2}\eta^2(w_F + w_C - 2w_D - w_F - w_C + 2w_D)]$$

FIGURE 2.14 - Arrangement of quarter-point wedge elements along segment of crack front with nodal lettering convention

FIGURE 2.15 - Crack Tip Arrangement Used By
Raju and Newman

FIGURE 2.16 - Crack Tip Arrangement Used for ASAS Runs

Surface crack in a finite plate.

$$K_I = (S_i + HS_s) \sqrt{\pi} \frac{a}{Q} F\left(\frac{a}{t}, \frac{a}{c}, \frac{c}{b}, \phi\right) \quad (1)$$

for $0 < a/c \leq 1.0$, $0 \leq a/t < 1.0$, $c/b < 0.5$ and $0 \leq \phi \leq \pi$. A useful approximation for Q ,

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65} \quad \left(\frac{a}{c} \leq 1\right) \quad (2)$$

$$F = \left[M_1 + M_2 \left(\frac{a}{t}\right)^2 + M_3 \left(\frac{a}{t}\right)^4 \right] f_{\phi} f_c \quad (3)$$

where

$$M_1 = 1.13 - 0.09 \left(\frac{a}{c}\right) \quad (4)$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + (a/c)} \quad (5)$$

$$M_3 = 0.5 - \frac{1.0}{0.65 + (a/c)} + 14 \left(1.0 - \frac{a}{c}\right)^{24} \quad (6)$$

FIGURE 2.17 - Raju and Newman Empirical Relation
for Stress Intensity Factors for a
Surface Crack in a Finite Plate (43)

$$g = 1 + \left[0.1 + 0.35 \left(\frac{a}{l} \right)^2 \right] (1 - \sin \phi)^2. \quad (7)$$

The function f_ϕ , an angular function from the embedded elliptical-crack solution

$$f_\phi = \left[\left(\frac{a}{c} \right)^2 \cos^2 \phi + \sin^2 \phi \right]^{1/4}. \quad (8)$$

The function f_w , a finite-width correction

$$f_w = \left[\sec \left(\frac{\pi c}{2b} \sqrt{\frac{a}{l}} \right) \right]^{1/2}. \quad (9)$$

The function H , developed herein also by curve fitting and engineering judgment, has the form

$$H = H_1 + (H_2 - H_1) \sin^p \phi \quad (10)$$

where

$$p = 0.2 + \frac{a}{c} + 0.6 \frac{a}{l} \quad (11)$$

$$H_1 = 1 - 0.34 \frac{a}{l} - 0.11 \frac{a}{c} \left(\frac{a}{l} \right) \quad (12)$$

$$H_2 = 1 + G_1 \left(\frac{a}{l} \right) + G_2 \left(\frac{a}{l} \right)^2 \quad (13)$$

In this equation for H_2 ,

$$G_1 = -1.22 - 0.12 \frac{a}{l} \quad (14)$$

FIGURE 2.17- (continued)

Chapter 3

ASAS-H FINITE ELEMENT SYSTEM

3.1 General Facilities of ASAS-H

The ASAS system incorporates facilities for linear static stress analysis, natural frequency analysis and heat conduction analysis. The program itself utilizes the finite element method and has at hand an extensive list of element types which will allow for the detailed analysis of almost any engineering structure. It is generally considered to be a very comprehensive all purpose finite element system. The many pre and post processors enable the user to select from a wide range of options. For example mesh generation to load case combination and stress contour plotting are just a few of the available selections. The graphics display system is an asset that cannot be emphasized enough as an aid to solving engineering problems. Also included are extensive data checking procedures for errors and consistency of input data.

The H version of ASAS (ASAS-H), which was used for all of the finite element analysis in this report is enhanced with multi-level substructuring. It also has an extended linear elastic fracture mechanics capability.

3.2 ASAS-H Fracture Mechanics Capabilities

The ASAS-H version contains several different crack tip elements for use with LEFM problems. Of these, only two have any relevance in this report. These elements are based on degenerate isoparametric elements described by Barsoum(36) and are as follows:

- 1) 2-D triangular membrane element with six nodes (CTM6) (FIGURE 2.12).
- 2) A wedge-shaped brick type element with 15 nodes (CB15) (FIGURE 2.13)

Both of the above are collapsed isoparametric elements. These elements do not yield a stress output because of the stress singularity at the crack tip and the rapid variation of the stresses across the element. The stress intensities can, however, be obtained from the displacements by using Ingraffea et al's (38) equations outlined in section 2.7.4

3.3 Multi-level Substructuring Capability of ASAS-H

The use of multi-level substructuring greatly extends the flexibility and power of the main program. A short description of this facility is included at this point in order to gain an appreciation of its applications. Should a detailed explanation be required reference (44) may be examined.

The use of substructuring, or component analysis, allows the user to create master components from finite elements of previously formed components. Once this is done the model of the total structure is formed from the component parts and solved for the displacements at the nodes, in a similar way to the assembly and solution of a single level finite element model. Displacements and stresses can then be recovered for any or all of the individual components from the ASAS backing files.

One great advantage to the use of substructuring is that it allows the ASAS system to rotate, translate and mirror any component of the structural model. In this way the symmetry of the structure may be utilized to its maximum extent. An example of this procedure taken from (44) is shown in FIGURE (3.1). This procedure can decrease the costs involved in an analysis considerably. A theoretical description of the process used in this component analysis is given in (45). The use of multi-level substructuring was used extensively in this project, particularly for the 3-D analysis.

STAGES IN THE CONSTRUCTION OF A MODEL FOR A
BRIDGE DECK

FIGURE 3.1
Multilevel Substructuring Analysis Using ASAS

Chapter 4

PRESENT WORK

4.1 Reasons For 3-D Analysis

The reason for performing a 3-D analysis on cracks is quite simple ; 3-dimensional objects very rarely have 2- dimensional cracks. Embedded cracks usually take the form of ellipses, surface cracks are generally of a semi-elliptical shape and corner cracks are typically in the shape of a quarter-ellipse. As the crack grows the shape of the ellipse will change until the part finally fractures. It is therefore obvious that a process such as this cannot be modelled by a purely 2-dimensional model. In fact it has been shown by Tan and Fenner (40) that a discrepancy of about 200% exists in the K_I value between the two models, with the 2-D model yielding the higher result (FIGURE 4.1). However, the 2-D K_I results for design purposes are on the conservative side as far as reliability goes. Due to the recent quest for a more accurate design technique, a more accurate stress intensity factor relation must be found. In this report a 3-D solution for K_I using the finite element method will be presented.

4.2 Mesh Generation

The first 3-D finite element crack model was constructed without the use of a mesh generator. This task was found to be extremely tedious and error prone. Since it was known that a large number of crack geometries, and crack tip element geometries were to be tested, it was considered wise to write a mesh generation program so that the construction of the finite element meshes could be fully automated.

In order to ensure that the 3-dimensional ASAS crack tip element is not in error it is necessary to change a number of mesh parameters. By changing the number of crack tip elements around the crack, their size and the crack shape the accuracy of the crack tip element can be improved. A mesh generator for elliptical cracks, developed at Carleton University, produces input data in ASAS input format. This mesh generator has the following advantages:

- 1) Quick and easy production of 3-dimensional FE meshes
- 2) Error free data for ASAS
- 3) A regular mesh which produces a known progression into new meshes so that the accuracy and behaviour of the crack tip elements and the mesh refinement can be carried out in a systematic way.

This mesh generator will modify 3-dimensional FE crack meshes rapidly, without error and with a minimum of human effort. The only drawback is that the mesh produced is not optimized, which leads to increased solution times and hence costs. However, the advantages by far outweigh this disadvantage.

The mesh generator operates as follows:

- i) A two-dimensional crack mesh is input in ASAS format (FIGURE 4.2 shows the 2-D mesh for a surface crack in a tension block)
- ii) This mesh is then rotated around an elliptical path, of the required c/a ratio, in the Y-Z plane, to produce the 3-D mesh of the crack region (FIGURE 4.3)
- iii) The number of angles, (THETA), are chosen such that the length of each crack tip element along the crack front is equal and the number of elements along the crack front corresponds to the number of angles for which stress intensity factors are desired.

iv) The mesh generator can now output the element connectivity, nodal co-ordinates, nodal suppressions and nodal link data of the 3-D mesh in ASAS input format, in either polar or cartesian co-ordinates.

v) The mesh generator was also designed to output an additional information file (called the NODE file) which contained relevant crack tip element node numbers and their angular positions along the crack front and crack tip element lengths at these positions. This file was used in the post-ASAS Stress Intensity calculations, described in Section 4.4.

Using the mesh generator one can produce an accurate, error free mesh in approximately one hour. To produce a similar mesh manually would take over two weeks and would inevitably contain input errors

Before any actual attempt at modelling an elliptical weld toe crack with collapsed isoparametric brick elements could be made it was first considered necessary to explore the basic properties of the crack tip elements themselves. By performing this analysis beforehand, the ease with which acceptable results could be obtained was greater than if modelling was attempted without a general idea of the elements behavior. These results are presented in (46) and are reviewed in the following section.

4.3 Crack Tip Element Size and Mesh Refinement

Prior to initiating the weld toe crack FE runs, it was considered necessary to investigate how the size and length of the crack tip element affects the ASAS Finite Element Stress Intensity factor results.

For these tests a cube of side 2540mm subjected to a unit tensile stress in the Y direction was analyzed. The relevant dimensions of the cube are presented in FIGURE 4.4. Using these dimensions the cube approximates an infinite medium to 1% accuracy (ref. 47).

The actual F.E. structure for these tests is represented by the dashed lines in FIGURE 4.4. Using this structure and a well planned selection of link nodes the structure could be run as both an embedded crack and surface crack at little extra cost.

The investigation centered around 3 variables.

- 1) the number of elements required around the crack front
- 2) length of the element perpendicular to the crack front compared to the crack depth. (ie. a/L ratio, see FIGURE 4.5)
- 3) the length of the element perpendicular to the crack front compared to the length of the element along the crack front. (ie. m/L ratio, see FIGURE 4.5)

The embedded crack results were compared against Irwin's Exact solution as described in section 2.3. The surface crack was compared to the empirical equation obtained by Raju and Newman (43) as described in section 2.5.

The complete set of results is discussed fully in (46), however the major conclusion of the testing was that the best compromise between computing cost and accuracy was found for meshes with the following properties:

- 1) 4 crack tip elements around the crack
- 2) $m/L \simeq 2$
- 3) $a/L \simeq 5$

The tests performed were for c/a ratios of 1.0. Due to the fact that for the weld structures c/a ratios of 1.0, 2.0, and 4.0 had to be tested it was impossible to maintain both $a/L=5$ and $c/L=5$ for the elliptical crack shaped since the actual crack tip element size (L) was not permitted to vary along the crack front. The c/L ratio had to be kept constant for all three ellipse shapes (ie

1.0, 2.0, and 4.0) which had the same value of c so as to allow the linking of the weld structures to 3 different crack meshes. Therefore for all of the crack structures built c/L was set at 10, thus making the crack tip element ratios as follows:

- 1) for $c/a = 1.0$; $m/L = 3.9$; $a/L = 10.0$
- 2) for $c/a = 2.0$; $m/L = 3.0$; $a/L = 5.0$
- 3) for $c/a = 4.0$, $m/L = 2.7$, $a/L = 2.5$

All of the meshes created had the same number of elements and nodes independent of the crack shape. These were:

- 1) Crack Structure (SECA) , #nodes = 990 ; #elements = 112
- 2) Back Structure (SECC) ; #nodes = 486 , #elements = 52
- 3) Weld Structure (WELD1), #nodes = 330 , #elements = 34

See FIGURE 4.6 for a description of the section names.

4.4 Modelling of Weld Toe Cracks with ASAS

To produce the 3-dimensional FE model of the cracked 'T' specimen joint both the mesh generator and the multi-level substructuring capabilities of the ASAS FE program was extensively used. It was only necessary to consider 1/4 of the joint due to symmetry. This joint was modelled as follows (see FIGURE 4.6):

- i) The mesh for the substructure SECA was produced using the mesh generator from a 2-dimensional mesh of the X-Z plane (see FIGURE 4.2)

- ii) SECA was then mirrored about the plane of the crack to form substructure SECB
- iii) SECC and the weld structures (WELD1, WELD2 or WELD3) were then linked on to the crack structure (SECA and SECB) in the GLOBAL file. (ie. links subelements together to form global structure)

The mesh generator will also produce the correct linking nodes, linking sequence and boundary conditions for these model components. The linking sequences are shown in FIGURE 4.7. FIGURE 4.8 shows the manually produced substructures of WELD1, WELD2 and WELD3 representing the 30,45 and 70 (DEGREE) weld shapes

4.4.1 Weld Geometries and Load Cases Tested

By applying a series of loading and boundary conditions the three different load cases can be analyzed for each crack shape at the weld toe. These three load cases are:

- 1) Axial Tension - non-load carrying weld
- 2) 4-point bending - non load carrying weld
- 3) 3-point Bending - load carrying weld

This procedure enables nine different load-weld combinations to be analyzed for each crack shape, resulting in a dramatic cost reduction for each case considered. These nine cases are as below:

- 1) 70 weld - Tension
- 2) 70 weld - 4-point Bending
- 3) 70 weld - 3-point Bending
- 4) 45 weld - Tension
- 5) 45 weld - 4-point Bending

- 6) 45 weld - 3-point Bending
- 7) 30 weld - Tension
- 8) 30 weld - 4-point Bending
- 9) 30 weld - 3-point Bending

A structure was produced with no weld substructures in order to check the accuracy of the results and the modelling procedure. This case is equivalent to a semi-circular surface crack in a plate and thus the resulting K_I values could be compared against the Raju and Newman empirical equations for K_I . The two cases tested were:

- 10) No weld - Tension
- 11) No weld - 4-point Bending

4.4.2 Crack Geometries Tested

TABLE 4.1 indicates the 3-dimensional weld toe crack geometries which are being considered. FIGURE 4.9 gives the physical dimensions of the model. Again, as stated previously, it was only necessary to model 1/4 of the of the 'T' specimen due to symmetry and the FE input for the substructure analysis was produced as described in section 4.3. In each FE model there were four crack tip elements along the crack front and three different load cases were analyzed for each crack shape and weld shape considered. The axial stress, at the weld toe was set to a unit value by adjusting the loads applied (see FIGURE 4.10). All together 110 cases of crack shape, weld shape and loading were considered, of which 20 cases were tested under no weld conditions. A total of 10 different crack shape geometries were tested.

4.5 Analysis of Results

Upon completion of an entire cracked weld analysis the results were processed using the equation shown in FIGURE 2.14 to obtain the K_I values around the crack.

Manual processing of the K_I 's proved very slow and tedious since the nodal displacements required for the analysis resided in the ASAS RECOVERY file which was well over 6000 lines long. Also since each crack structure had 11 RECOVERY outputs associated with it, (see SECTION 4.4.1), and a total of 10 crack shapes were to be tested this would have involved a great deal of time. For this reason it was felt that the processing must be more automated.

A program was written which would read in the required displacements from the recovery file, and read in required data from the NODE file which was created by the mesh generator (ie. crack tip node numbers, angular position along the crack front, and crack tip element length). The program would calculate the stress intensities around the crack front from the Recovery file data and also the Raju and Newman stress intensities for a surface crack. The output was presented in tabular form, and is shown in the sample output of TABLE 4.2. The program could also create files necessary for a plotting routine which was used extensively.

4.6 Results

The 3-D cracked weld toe cases considered in this report are shown in TABLE 4.1 as discussed previously.

The resulting stress intensities found were non-dimensionalized with respect to an embedded circular crack and are shown in graphical form in FIGURES 4.11 to 4.50. In all these curves $\theta = 0^\circ$ is the deepest point of the crack and $\theta = 90^\circ$ is the surface of the crack (ie. weld toe). The surface point has been multiplied by a surface correction factor of $(1 - \nu^2)$ in order to obtain the result as plane stress as compared to all the other points around the crack front which are plane strain.

It should be noted that if a more precise account of the results is required, all of the data can be obtained in tabular form as in TABLE 4.2 from Prof. R. Bell, Department of Mechanical and Aeronautical Engineering, Carleton University.

FIGURE 4.51 shows a comparison of the ASAS 2-D results with the 3-D results obtained for a cracked weld toe.

Also included are FIGURES 4.52 to 4.61 which are reduced versions of FIGURES 4.11 to 4.50 and are given so that a comparison of the effects of weld geometries on the SIF's can be made for any given crack geometry.

The SIF's for the points ($\theta = 0^\circ$, and $\theta = 90^\circ$) are presented in TABLES 4.3 to 4.14, normalized with respect to $K_{I, \text{Rajak \& Newman}}$ for a surface crack of the same geometric dimensions

FIGURES 4.62-4.73 show the empirically based curves which were found to describe the $K'_{I\theta}$ as a function of crack position, load case, weld shape and crack geometry. These equations are presented in TABLES 4.15 and 4.16.

TABLE 4.1 - WELD TOE CRACK GEOMETRIES

c/a	a/t
1.0	0.05
	0.1
	0.2
	0.4
2.0	0.025
	0.05
	0.1
	0.2
4.0	0.025
	0.05

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		tension= 1.1565 4-point= 1.3152 3-point= 1.3245	tension= 0.9955 4-point= 1.0215 3-point= 1.0208	tension= 0.9171 4-point= 0.8546 3-point= 0.8434	tension= 0.9280 4-point= 0.8022 3-point= 0.7965
2.0	tension= 1.3336 4-point= 1.5979 3-point= 1.6292	tension= 1.1149 4-point= 1.2632 3-point= 1.2827	tension= 0.9711 4-point= 1.0222 3-point= 1.0281	tension= 0.9368 4-point= 0.9234 3-point= 0.9245	
4.0	tension= 1.3542 4-point= 1.6208 3-point= 1.6581	tension= 1.1379 4-point= 1.2900 3-point= 1.3069			

TABLE 4.3 - Table of $K_{I, \text{weld}}$ for $\text{THETA} = 0^\circ$, for weld shape angle of 70°
 $K_{I, \text{Raju}}$

a/t c/a	0.025	0.050	0.100	0.200	0.400
1.0		tension=1.1936 4-point=1.3152 3-point=1.3461	tension=1.0204 4-point=1.0563 3-point=1.0578	tension=0.9299 4-point=0.8897 3-point=0.8836	tension=0.9399 4-point=0.8288 3-point=0.8237
2.0	tension=1.3646 4-point=1.5699 3-point=1.5992	tension=1.1420 4-point=1.2735 3-point=1.2869	tension=0.9904 4-point=1.0486 3-point=1.0568	tension=0.9475 4-point=0.9359 3-point=0.9355	
4.0	tension=1.3827 4-point=1.5889 3-point=1.6130	tension=1.1675 4-point=1.2950 3-point=1.3138			

TABLE 4.4 - Table of $K_{I,weld}$ for $\theta = 0^\circ$, for weld shane angle of 45°
 $K_{I,Raju}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		tension=1.2010 4-point=1.2953 3-point=1.3110	tension=1.0400 4-point=1.0714 3-point=1.0774	tension=0.9482 4-point=0.9173 3-point=0.9143	tension=0.9486 4-point=0.8462 3-point=0.8416
2.0	tension=1.3569 4-point=1.4858 3-point=1.5039	tension=1.1517 4-point=1.2405 3-point=1.2532	tension=1.0106 4-point=1.0619 3-point=1.0686	tension=0.9555 4-point=0.9464 3-point=0.9457	
4.0	tension=1.3708 4-point=1.4785 3-point=1.5079	tension=1.1770 4-point=1.2685 3-point=1.2838			

TABLE 4.5- Table of $K_{I,weld}$ for $\theta = 0^\circ$, for weld shape angle of 30°
 $K_{I,Raju}$

$\frac{a}{t}$ $\frac{c}{a}$	0.025	0.050	0.100	0.200	0.400
1.0		70° weld= 1.1565 45° weld= 1.1936 30° weld= 1.2010	70° weld= 0.9955 45° weld= 1.0204 30° weld= 1.0400	70° weld= 0.9171 45° weld= 0.9299 30° weld= 0.9483	70° weld= 0.9280 45° weld= 0.9399 30° weld= 0.9486
2.0	70° weld= 1.3336 45° weld= 1.3646 30° weld= 1.3569	70° weld= 1.1149 45° weld= 1.1420 30° weld= 1.1517	70° weld= 0.9711 45° weld= 0.9904 30° weld= 1.0106	70° weld= 0.9368 45° weld= 0.9475 30° weld= 0.9555	
4.0	70° weld= 1.3542 45° weld= 1.3827 30° weld= 1.3708	70° weld= 1.1379 45° weld= 1.1675 30° weld= 1.1770			

TABLE 4.6- Table of $K_{I, \text{weld}}$ for $\text{THETA} = 0^\circ$, for load case of TENSION
 $K_{I, \text{Raju}}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		70° weld= 1.3152 45° weld= 1.3152 30° weld= 1.2953	70° weld= 1.0215 45° weld= 1.0563 30° weld= 1.0714	70° weld= 0.8546 45° weld= 0.8897 30° weld= 0.9173	70° weld= 0.8022 45° weld= 0.8288 30° weld= 0.8462
2.0	70° weld= 1.5979 45° weld= 1.5699 30° weld= 1.4858	70° weld= 1.2632 45° weld= 1.2735 30° weld= 1.2405	70° weld= 1.0222 45° weld= 1.0486 30° weld= 1.0619	70° weld= 0.9234 45° weld= 0.9359 30° weld= 0.9464	
4.0	70° weld= 1.6208 45° weld= 1.5889 30° weld= 1.4785	70° weld= 1.2900 45° weld= 1.2950 30° weld= 1.2685			

TABLE 4.7- Table of $K_{I, \text{weld}}$ for $\text{THETA} = 0^\circ$, for load case of 4-POINT BENDING.
 $K_{I, \text{Raju}}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		70° weld= 1.3245 45° weld= 1.3461 30° weld= 1.3110	70° weld= 1.0208 45° weld= 1.0578 30° weld= 1.0774	70° weld= 0.8434 45° weld= 0.8836 30° weld= 0.9143	70° weld= 0.7965 45° weld= 0.8288 30° weld= 0.8416
2.0	70° weld= 1.6292 45° weld= 1.5992 30° weld= 1.5039	70° weld= 1.2827 45° weld= 1.2869 30° weld= 1.2532	70° weld= 1.0281 45° weld= 1.0566 30° weld= 1.0686	70° weld= 0.9245 45° weld= 0.9355 30° weld= 0.9454	
4.0	70° weld= 1.6581 45° weld= 1.6130 30° weld= 1.5079	70° weld= 1.3069 45° weld= 1.3138 30° weld= 1.2838			

TABLE 4.8 - Table of $K_{I, \text{weld}}$ for $\text{THETA} = 0^\circ$, for load case of 3-POINT BENDING
 $\frac{K_{I, \text{weld}}}{K_{I, \text{Ray}}}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		tension=1.7618 4-point=2.2726 3-point=2.3329	tension=1.3075 4-point=1.6255 3-point=1.6637	tension=1.1334 4-point=1.4044 3-point=1.4303	tension=0.9807 4-point=1.0761 3-point=1.0785
2.0	tension=2.1722 4-point=2.8352 3-point=2.9135	tension=1.6813 4-point=2.1470 3-point=2.2145	tension=1.3468 4-point=1.6709 3-point=1.7111	tension=1.0662 4-point=1.1251 3-point=1.1332	
4.0	tension=2.2560 4-point=2.9212 3-point=3.0268	tension=1.7561 4-point=2.2280 3-point=2.2939			

TABLE 4.9 - Table of $K_{I, \text{weld}}$ for $\text{THETA} = 90^\circ$, for weld shape angle of 70°
 $K_{I, \text{Raju}}$

a/t c/a	0.025	0.050	0.100	0.200	0.400
1.0		tension=1.8017 4-point=2.1127 3-point=2.1581	tension=1.3571 4-point=1.5803 3-point=1.6115	tension=1.1904 4-point=1.4036 3-point=1.4270	tension=1.0215 4-point=1.0900 3-point=1.0900
2.0	tension=2.1586 4-point=2.5375 3-point=2.6019	tension=1.7175 4-point=2.0085 3-point=2.0606	tension=1.4051 4-point=1.6347 3-point=1.6666	tension=1.1119 4-point=1.1315 3-point=1.1328	
4.0	tension=2.2088 4-point=2.6000 3-point=2.6569	tension=1.7815 4-point=2.0723 3-point=2.1133			

TABLE 4.10 - Table of $K_{I, \text{weld}}$ for $\text{THETA} = 90^\circ$, for weld shape angle of 45°
 $K_{I, \text{Raju}}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		tension=1.6699 4-point=1.8588 3-point=1.8958	tension=1.3355 4-point=1.4832 3-point=1.5008	tension=1.1904 4-point=1.3509 3-point=1.3670	tension=1.0404 4-point=1.1140 3-point=1.1160
2.0	tension=1.9591 4-point=2.1619 3-point=2.1999	tension=1.6270 4-point=1.7892 3-point=1.8234	tension=1.3793 4-point=1.5297 3-point=1.5506	tension=1.1329 4-point=1.1590 3-point=1.1618	
4.0	tension=1.9966 4-point=2.2028 3-point=2.2384	tension=1.6797 4-point=1.8587 3-point=1.8892			

TABLE 4.11- Table of $K_{I,weld}$ for $\theta = 90^\circ$, for weld shape angle of 30°
 $\frac{K_{I,Raju}}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0		70° weld= 1.7618 45° weld= 1.8017 30° weld= 1.6699	70° weld= 1.3075 45° weld= 1.3571 30° weld= 1.3355	70° weld= 1.1334 45° weld= 1.1904 30° weld= 1.1904	70° weld= 0.9802 45° weld= 1.0215 30° weld= 1.0404
2.0	70° weld= 2.1722 45° weld= 2.1586 30° weld= 1.9591	70° weld= 1.6813 45° weld= 1.7175 30° weld= 1.6270	70° weld= 1.3468 45° weld= 1.4051 30° weld= 1.3793	70° weld= 1.0662 45° weld= 1.1119 30° weld= 1.1329	
4.0	70° weld= 2.2560 45° weld= 2.2088 30° weld= 1.9966	70° weld= 1.7561 45° weld= 1.7815 30° weld= 1.6797			

TABLE 4.12- Table of $K_{I, \text{weld}}$ for $\text{THETA} = 90^\circ$, for load case of TENSION
 $K_{I, \text{Raju}}$

$\frac{a/t}{c/a}$	0.025	0.050	0.100	0.200	0.400
1.0	70° weld= 2.2726 45° weld= 2.1127 30° weld= 1.8588	70° weld= 2.2726 45° weld= 2.1127 30° weld= 1.8588	70° weld= 1.6255 45° weld= 1.5803 30° weld= 1.4832	70° weld= 1.4044 45° weld= 1.4036 30° weld= 1.3509	70° weld= 1.0761 45° weld= 1.0900 30° weld= 1.1140
2.0	70° weld= 2.8352 45° weld= 2.5375 30° weld= 2.1619	70° weld= 2.1470 45° weld= 2.0085 30° weld= 1.7892	70° weld= 1.6709 45° weld= 1.6347 30° weld= 1.5297	70° weld= 1.1251 45° weld= 1.1315 30° weld= 1.1590	
3.0	70° weld= 2.9212 45° weld= 2.6000 30° weld= 2.2028	70° weld= 2.2280 45° weld= 2.0723 30° weld= 1.8583			

TABLE 4.13- Table of $K_{I, \text{weld}}$ for $\text{THETA} = 90^\circ$, for load case of 4-POINT BENDING
 $K_{I, \text{Raju}}$

a/t c/a	0.025	0.050	0.100	0.200	0.400
1.0		70° weld= 2.3329 45° weld= 2.1581 30° weld= 1.8958	70° weld= 1.6637 45° weld= 1.6115 30° weld= 1.5008	70° weld= 1.4303 45° weld= 1.4270 30° weld= 1.3670	70° weld= 1.0785 45° weld= 1.0902 30° weld= 1.1160
2.0	70° weld= 2.9135 45° weld= 2.6019 30° weld= 2.1999	70° weld= 2.2145 45° weld= 2.0606 30° weld= 1.8234	70° weld= 1.7111 45° weld= 1.6666 30° weld= 1.5506	70° weld= 1.1332 45° weld= 1.1328 30° weld= 1.1618	
4.0	70° weld= 3.0268 45° weld= 2.6569 30° weld= 2.2384	70° weld= 2.2939 45° weld= 2.1133 30° weld= 1.8892			

TABLE 4.14 - Table of $K_{I, \text{weld}}$ for $\text{THETA} = 90^\circ$, for load case of 3-POINT BENDING
 $K_{I, \text{Raju}}$

WELD SHAPE ANGLE	LOAD CASE	EQUATION
70	TENSION	$M_{3D} = 0.8521 + 0.01394X - 0.00005781X^2$
70	4-POINT	$M_{3D} = 0.6888 + 0.03373X - 0.0002884X^2$
70	3-POINT	$M_{3D} = 0.6763 + 0.03524X - 0.0002967X^2$
45	TENSION	$M_{3D} = 0.8550 + 0.01594X - 0.00008981X^2$
45	4-POINT	$M_{3D} = 0.7212 + 0.03303X - 0.0003046X^2$
45	3-POINT	$M_{3D} = 0.7056 + 0.03536X - 0.0003359X^2$
30	TENSION	$M_{3D} = 0.8667 + 0.01641X - 0.0001126X^2$
30	4-POINT	$M_{3D} = 0.7531 + 0.03102X - 0.0003280X^2$
30	3-POINT	$M_{3D} = 0.7439 + 0.03246X - 0.0003437X^2$

Where : $X = (a/t)^{-1} (c/a)^{.05} + 1$

$$KI_{\text{weld}} = M_{3D} \cdot KI_{\text{Raju, Newman}}$$

TABLE 4.15 - Equations for the calculation of KI_{weld} , for $\text{THETA} = 0^\circ$

WELD SHAPE ANGLE	LOAD CASE	EQUATION
70	TENSION	$M_{3D} = 0.6200 + 0.1290X - 0.002003X^2$
70	4-POINT	$M_{3D} = 0.4883 + 0.2131X - 0.004203X^2$
70	3-POINT	$M_{3D} = 0.4612 + 0.2253X - 0.004505X^2$
45	TENSION	$M_{3D} = 0.6509 + 0.1362X - 0.002656X^2$
45	4-POINT	$M_{3D} = 0.5778 + 0.1912X - 0.004322X^2$
45	3-POINT	$M_{3D} = 0.5528 + 0.2018X - 0.004654X^2$
30	TENSION	$M_{3D} = 0.7388 + 0.1155X - 0.002477X^2$
30	4-POINT	$M_{3D} = 0.7362 + 0.1440X - 0.003448X^2$
30	3-POINT	$M_{3D} = 0.7174 + 0.1521X - 0.003731X^2$

Where : $X = (a/t)^{-0.75}(c/a)^{.05} + (c/a)^{-7}$

$KI_{weld} = M_{3D} \cdot KI_{Raju, Newman}$

TABLE 4.16- Equations for the calculation of KI_{weld} , for $\theta = 90^\circ$

Stress intensity factors for a semi-elliptical surface crack in a pressurized cylinder, at the point of deepest penetration, with $b/a = 0.8$

FIGURE 4.1 - Comparison of 2-D stress intensity
Against 3-D Stress Intensity
(Fig. taken from 40)

FIGURE 4.2 - Example of 2-D crack mesh used as input for mesh generator

FIGURE 4.3 - Example of 3-D crack mesh created by mesh generator from the 2-D mesh shown in FIGURE 4.2. ($c/a=4.0$; $a/t=0.1$)

FIGURE 4 4 - TENSION BLOCK GEOMETRY

FIGURE 4.5 - DEFINITION OF M/L RATIO

FIGURE 4.6 - Method of Substructuring used for modelling weld toe crack problem.

1. Link Z-Y-X
2. Link Y-X
3. Link X-Y
4. Link X-Y-Z
5. Suppress Z
6. Suppress Y

Back Mesh

1. Link X-Y
2. Link X-Y-Z
3. Link Z-Y-X
4. Link X-Y
5. Link X-Y-Z
6. Link Z-Y-X
7. Link Y-X
8. Link X-Y
9. Link X-Y-Z
10. Suppress Z

Crack Mesh

FIGURE 4.7 - Linking Sequence used for weld toe crack problems.

FIGURE 4.8 - Example of 45° weld mesh used

FIGURE 4.9 - Dimensions used for weld toe crack problems

FIGURE 4.10 - Load cases considered for weld toe crack problem.

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FIGURE 4.11 - SIF curves for no weld, $c/a=1.0$, $a/t=0.05$

FIGURE 4.12 - SIF curves for no weld, $c/a=1.0$, $a/t=0.1$

FIGURE 4.13 - SIF curves for no weld, $c/a=1.0$, $a/t=0.2$

FIGURE 4.14 - SIF curves for no weld, $c/a=1.0$, $a/t=0.4$

FIGURE 4.15 - SIF curves for no weld, $c/a=2.0$, $a/t=0.025$

FIGURE 4.16 - SIF curves for no weld, $c/a=2.0$, $a/t=0.05$

FIGURE 4.17 - SIF curves for no weld, $c/a=2.0$, $a/t=0.1$

FIGURE 4.18 - SIF curves for no weld, $c/a=2.0$, $a/t=0.2$

FIGURE 4.19 - SIF curves for no weld, $c/a=4.0$, $a/t=0.025$

FIGURE 4.20 - SIF curves for no weld, $c/a=4.0$, $a/t=0.05$

FIGURE 4.21 - SIF curves for 70° weld, $c/a=1.0$, $a/t=0.05$

FIGURE 4.22 - SIF curves for 45° weld, $c/a=1.0$, $a/t=0.05$

FIGURE 4.23 - SIF curves for 30° weld, $c/a=1.0$, $a/t=0.05$

FIGURE 4.24 - SIF curves for 70° weld, $c/a=1.0$, $a/t=0.1$

FIGURE 4.25 - SIF curves for 45° weld, $c/a=1.0$, $a/t=0.1$

FIGURE 4.26 - SIF curves for 30° weld, $c/a=1.0$, $a/t=0.1$

FIGURE 4.27 - SIF curves for 70° weld, $c/a=1.0$, $a/t=0.2$

FIGURE 4.28 - SIF curves for 45° weld, $c/a=1.0$, $a/t=0.2$

FIGURE 4.29 - SIF curves for 30° weld, $c/a=1.0$, $a/t=0.2$

FIGURE 4.30 - SIF curves for 70° weld, $c/a=1.0$, $a/t=0.4$

FIGURE 4.31 - SIF curves for 45° weld, $c/a=1.0$, $a/t=0.4$

FIGURE 4.32 - SIF curves for 30° weld, $c/a=1.0$, $a/t=0.4$

FIGURE 4.33 - SIF curves for 70° weld, $c/a=2.0$, $a/t=0.025$

FIGURE 4.34 - SIF curves for 45° weld, $c/a=2.0$, $a/t=0.025$

FIGURE 4.35 - SIF curves for 30° weld, $c/a=2.0$, $a/t=0.025$

FIGURE 4.36 - SIF curves for 70° weld, $c/a=2.0$, $a/t=0.05$

FIGURE 4.37 - SIF curves for 45° weld, $c/a=2.0$, $a/t=0.05$

FIGURE 4.38 - SIF curves for 30° weld, $c/a=2.0$, $a/t=0.05$

FIGURE 4.39 - SIF curves for 70° weld, $c/a=2.0$, $a/t=0.1$

FIGURE 4.40 - SIF curves for 45° weld, $c/a=2.0$, $a/t=0.1$

FIGURE 4.41 - SIF curves for 30° weld, $c/a=2.0$, $a/t=0.1$

FIGURE 4.42 - SIF curves for 70° weld, $c/a=2.0$, $a/t=0.2$

FIGURE 4.43 - SIF curves for 45° weld, $c/a=2.0$, $a/t=0.2$

FIGURE 4.44 - SIF curves for 30° weld, $c/a=2.0$, $a/t=0.2$

FIGURE 4.45 - SIF curves for 70° weld, $c/a=4.0$, $a/t=0.025$

FIGURE 4.46 - SIF curves for 45° weld, $c/a=4.0$, $a/t=0.025$

FIGURE 4.47 - SIF curves for 30° weld, $c/a=4.0$, $a/t=0.025$

FIGURE 4.48 - SIF curves for 70° weld, $c/a=4.0$, $a/t=0.05$

FIGURE 4.49 - SIF curves for 45° weld, $c/a=4.0$, $a/t=0.05$

FIGURE 4.50 - SIF curves for 30° weld, $c/a=4.0$, $a/t=0.05$

3-POINT BENDING : 70 DEG. WELD

FIGURE 4.51 - Comparison of 2-D and 3-D weld toe crack results for 3-point bending with a 70° weld.

FIGURE 4.52 - SIF curves for $c/a=1.0$, $a/t=0.05$

FIGURE 4.53 - SIF curves for $c/a=1.0$, $a/t=0.1$

FIGURE 4.54 - SIF curves for $c/a=1.0, a/t=0.2$

FIGURE 4.55 - SIF curves for $c/a=1.0$, $a/t=0.4$

FIGURE 4.56 - SIF curves for $c/a=2.0$, $a/t=0.025$

FIGURE 4.57 - SIF curves for $c/a=2.0$, $a/t=0.05$

FIGURE 4.58 - SIF curves for $c/a=2.0$, $a/t=0.1$

FIGURE 4.59 - SIF curves for $c/a=2.0$, $a/t=0.2$

FIGURE 4.60 - SIF curves for $c/a=4.0$, $a/t=0.025$

FIGURE 4.61 - SIF curves for $c/a=4.0$, $a/t=0.05$

FIGURE 4.62 - Crack geometry equation vs. Magnification factor (M_{3D}) for 70° weld at $\text{THETA}=0^\circ$.

FIGURE 4.63 - Crack geometry equation vs. Magnification factor (M_{3D}) for
45° weld at $\theta = 0^\circ$.

FIGURE 4.64 - Crack geometry equation vs. Magnification factor (M_{3D}) for
30° weld at $\text{THETA}=0^\circ$.

FIGURE 4.65 - Crack geometry equation vs. Magnification factor (M_{3D}) for
TENSION load case at $\theta = 0^\circ$.

FIGURE 4.66 - Crack geometry equation vs. Magnification factor (M_{3D}) for 4-POINT bending load case at $\theta = 0^\circ$.

FIGURE 4.67 - Crack geometry equation vs. Magnification factor (M_{3D}) for 3-POINT bending load case at $\theta = 0^\circ$.

FIGURE 4.68 - Crack geometry equation vs. Magnification factor (M_{3D}) for
70° weld at $\text{THETA}=90^\circ$.

FIGURE 4.69 - Crack geometry equation vs. Magnification factor (M_{3D}) for
45° weld at $\theta = 90^\circ$.

FIGURE 4.70 - Crack geometry equation vs. Magnification factor (M_{3D}) for
30° weld at $\theta = 90^\circ$.

FIGURE 4.71 - Crack geometry equation vs. Magnification factor (M_{3D}) for
TENSION load case at $\theta = 90^\circ$.

FIGURE 4.72 - Crack geometry equation vs. Magnification factor (M_{3D}) for 4-POINT bending load case at $\theta = 90^\circ$.

FIGURE 4.73 - Crack geometry equation vs. Magnification factor (M_{3D}) for 3-POINT bending load case at $\theta = 90^\circ$.

Chapter 5

DISCUSSION

5.1 Comparison of surface crack results

As was discussed earlier, each of the crack shapes was tested not only as a weld toe crack, but also as a surface crack in a plate so that the mesh accuracy could be determined by comparing it against the empirical equation of Raju and Newman (43). These results are shown in FIGURES 4.11 to 4.20. As can be seen the two results are in excellent correlation with each other, with the maximum error between any two points being less than 8%, and an average error of less than 3%. Since Raju and Newman's equation is claimed to be accurate to within 5% of the finite element results, the errors which we obtained are acceptable.

5.2 Comparison of 3-D results against 2-D results

The curves of FIGURE 4.51 show a sample comparison of the 3-D results for the deepest point of the crack (ie. $\theta = 0^\circ$) for $c/a=1.0, 2.0$ and 4.0 , in 3-point bending for a 70° weld, compared against the same structure tested in a 2-D analysis using the ASAS system. These 2-D results are fully discussed in Ref. (48). It can be expected that the 2-D results would correspond to a c/a ratio of infinity. As can be seen from the curves, the 3-D results do tend towards the 2-D results as c/a is increased. This graph also shows the necessity of doing 3-D analysis as compared to 2-D, since for the c/a cracks of 1.0 a discrepancy of almost 400% exists for the deepest cracks, with the 2-D results being on the conservative side of life expectancy.

5.3 Discussion of results

FIGURES 4.52 to 4.61 show how the variation of weld angle changes the stress intensity around the crack. As expected the SIF decreases with decreasing weld angle. Also the deeper the crack the less effect the weld shape has on the SIF. As an example FIGURES 4.52 and 4.55 show the increase in SIF between 30° and 70° weld shapes for $c/a=1.0$ is 24% at $a/t=0.05$, and a 6% decrease at $c/a=1.0$, $a/t=0.4$. As a matter of fact it was found that for this crack shape, the 70° weld results in a SIF about 2% less than the no-weld case, however this is most likely due to finite element analysis errors.

Another point worth noting is that for the shallow crack depths, the bending cases yielded higher results than the tension case at the deepest point of the crack, however as expected, as the crack depth increased, the tension case results became greater than the bending.

It is also worth noting that for all of the crack shapes tested except two, the SIF at the surface was larger than that at the deepest point. The only exceptions to this were for $c/a=4.0$ and $a/t=.025$ and $.05$ which were almost constant for the 45° weld and actually lower for the 30° weld, while the 70° weld still yielded higher results at the surface. The reason for this can be explained quite simply. At the deepest point the SIF is high due to the facts presented in FIGURE 4.51, that the larger the c/a ratio the higher the SIF at the deepest point. That along with the fact that for such a shallow crack the bending stress seen by the crack is not very much different at the surface as compared to the deepest point. The third point is that as was mentioned earlier the SIF at the surface points was multiplied by $(1 - \nu^2)$ where $\nu = 0.3$ in order to obtain plane stress conditions. Actually at these depths it is quite possible that the entire crack displays a mixture of plane stress and plane strain characteristics.

It should however be kept in mind that the results obtained from any finite element analysis for the point where the crack cuts the surface (ie. $\theta = 90^\circ$) are unreliable (49) and that these points should be treated with caution.

5.4 Equation for life determination

As was explained earlier, for it to be possible to make a life prediction of a 3-D growing crack using LEFM one would require information on how the stress intensity varies at the surface and at the deepest point of the crack depending on the crack depth and aspect ratio. Therefore a set of equations had to be found which would describe K_I for each of the 9 weld-load cases (3 loads, 3 welds) both at $\theta = 0^\circ$ and $\theta = 90^\circ$.

Since the addition of a weld to an uncracked stressed plate simply acts as a stress concentration factor on the nominal stress applied, it was believed that the stress intensity factor of a cracked plate would be multiplied by a similar stress intensity magnification factor (M_{3D}) when a weld was added to the structure. For this reason it was decided to find an equation for a stress intensity magnification factor, which could be multiplied by the Raju and Newman K_I equation for a surface crack in order to obtain the cracked weld toe stress intensity factors as shown below.

$$K_{I,weld} = M_{3D} \times K_{I,Raju \& Newman}$$

TABLES 4.3 to 4.14 show the M'_{3D} s for the 3 welds and 3 load cases for $\theta = 0^\circ$ and $\theta = 90^\circ$. It should be noted that the data in TABLES 4.3 to 4.8 is identical to that of TABLE 4.7 to 4.12, they are only presented in different formats for convenience of comparison.

From TABLES 4.3, 4.4 and 4.5 it can be seen that at the deepest point ($\theta = 0^\circ$), the M_{3D} is really only a function of the a/t ratio and only partly the weld shape angle, while the c/a ratio appears to have almost no significant effect at all. It is interesting to note that TABLE 4.6 shows that for $\theta = 0^\circ$ and the tension load case the 30° weld shape angle actually causes a higher M_{3D} than either the 45° or the 70° weld for all of the crack shapes tested except for the shallowest cracks of $a/t = .025$ where the 45° weld gives the highest value however it can be said that for this load case and crack position the weld shape only makes a 5% difference on the M_{3D} between the 70° and the 30° weld.

TABLES 4.7 and 4.8 show that for very shallow cracks in bending the weld shape has a larger effect than in tension, but the effect is only a 10% reduction in M_{3D} between the 70° and the 30° welds, for deeper welds the situation reverses again as in the tension case with the 30° weld yielding slightly higher results.

TABLES 4.9 to 4.11 show the M_{3D} for the crack surface ($\theta = 90^\circ$). From this data it is quite obvious that 3-point bending causes the highest M_{3D} of all the load cases, however it is only about 3% higher than the 4-point bending M_{3D} . It can also be seen that at the surface the shallow cracks experience an extremely high M_{3D} , actually 3 times as high as the deep cracks with the 70° weld. It should be remembered however that these surface results should be treated with caution as was explained previously.

TABLE 4.12 shows that for the surface of a crack in tension, the weld shape has very little effect causing a maximum difference of only 13% in M_{3D} between weld shapes for the shallowest cracks, with the steepest weld causing the highest M_{3D} . For cracks of $a/t \geq 0.1$ the 30° weld yielded slightly higher M_{3D} than the 45° or 70° welds.

From TABLES 4.13 and 4.14 it is evident that the weld shape has a very large effect on M_{3D} for shallow cracks in either 4-point or 3-point bending with the 30° weld giving a 26% reduction in M_{3D} over the 70° weld. As before the 30° weld showed a greater M_{3D} than either the 45° or 70° weld for the deepest crack.

The points of these tables were fitted along curves which could be used to determine the K_I at the $\theta = 0^\circ$ or $\theta = 90^\circ$ point of the crack if the c/a and a/t ratios were known. These equations are given in TABLE 4.15 for $\theta = 0^\circ$, and in TABLE 4.16 for $\theta = 90^\circ$. To use these equations one must follow these steps:

- 1) Enter the c/a and a/t ratios into the 'X' equation given in TABLES 4.15 or 4.16 depending on the θ angle desired.
- 2) select appropriate M_{3D} equation from Table 4.13 or 4.14 depending on θ angle, load case and weld shape angle desired.

3) input 'X' value into M_{3D} equation. The M_{3D} value is the stress intensity magnification factor.

4) Multiply the M_{3D} value by the value obtained from the Raju and Newman empirical relation for a surface crack from FIGURE 2.17. This result is the stress intensity value for the weld toe crack.

The M_{3D} equations have been plotted as a function of the 'X' equations in FIGURES 4.62 to 4.73. All of the equations for M_{3D} at $\theta = 0^\circ$ have a maximum error from the actual data points of less than $\pm 5\%$, and have an average error of less than $\pm 1.4\%$, while the equations for M_{3D} at $\theta = 90^\circ$ have a maximum error of less than $\pm 6\%$ and an average error of $\pm 2.2\%$.

The above stress intensity equations have been entered into a life prediction program, however results are not ready for presentation in this report.

Chapter 6

CONCLUSIONS

The conclusions which can be deduced from this report are:

- 1) K_I values for various c/a , a/t , weld toe crack geometries were calculated using the finite element method for various weld shape angles and load cases.
- 2) The SIF's calculated from the 2-D finite element analysis were up to 400% higher than the results obtained from the 3-D analysis.
- 3) The results for $K_{I, \text{ weld}}$ were expressed as a magnification factor (M_{3D}) times the Raju and Newman empirical results for a surface crack in a plate.
- 4) The equations fitted to the calculated results for K_I had a maximum error of less than 6% and an average error of only 2.2% from the actual values.
- 5) For shallow crack depths ($a/t \leq 0.1$), bending yields a higher M_{3D} for all points around the crack.
- 6) The weld shape angle does not have a large influence on the M_{3D} at the deepest point of the crack (ie. $\theta = 0^\circ$).
- 7) The weld shape angle only affects the M_{3D} for shallow cracks ($a/t \leq 0.1$) at the crack surface (ie. $\theta = 90^\circ$).
- 8) For all cracks at the deepest point ($\theta = 0^\circ$), the M_{3D} is only a function of the a/t ratio, and not of the c/a ratio or the weld shape angle.

- 9) For shallow cracks in tension ($a/t \leq 0.05$), the weld shape angle causes a maximum decrease of 13% in the M_{3D} between the 70° and 30° welds at the crack surface, (i.e. $\theta = 90^\circ$).
- 10) For shallow cracks in bending ($a/t \leq 0.1$), the weld shape angle causes a maximum decrease of 26% in the M_{3D} between the 70° and 30° welds at the crack surface (i.e. $\theta = 90^\circ$).
- 11) Multi level substructuring proved to be an important tool in the large scale finite element model of the weld toe crack.
- 12) The 3-dimensional ASAS-H finite elements proved to be quite accurate in the testing of semi-elliptical crack problems.
- 13) Mesh generation was found to be an extremely useful tool to the point that it can almost be considered mandatory for the analysis of this size of problem.
- 14) The empirical equations found for K_I could be incorporated in a life prediction program.

Chapter 7

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FIGURE 4.14 - SIF curves for no weld, $c/a=1.0$, $a/t=0.4$