MODELLING AIR FLOW IN DIE CASTING

by

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B. Eng.

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in partial fulfillment of the requirements for the degree of

Master of Applied Science

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the degree of

Master of Applied Science

A handwritten signature in black ink, appearing to read 'J. Golda', is written over a horizontal line. The signature is fluid and cursive.

Thesis Supervisor

A handwritten signature in black ink, appearing to read 'J. Red', is written over a horizontal line. The signature is more compact and less cursive than the one above.

Chair, Department of Mechanical and Aerospace Engineering

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Abstract

Porosity is a major issue in die casting products, it results from gas entrapment and shrinkage. It causes low mechanical properties of parts. Understanding the relationship among the process variables such as pressure, temperature and volume of the air trapped in the casting is very important for these reasons. It should help the designer to predict defects in the design stage and design dies that produce high quality casting.

In this thesis, computational mathematical models and algorithms describing air flow in die casting are developed based on compressible flow theory. Two projects, air exhaust and cavity air, which deal with compressible flow in the die casting process have been done based on their own mathematical models and algorithms.

As an alternative venting method, the influence of vacuum venting is also considered in this thesis.

The numerical results computed from the air exhaust and cavity air projects match well with theoretical expectation.

To

my wife Wei Wang

and

son Ruochuan Xie

Nomenclature

a	Speed of sound.
A	Surface area of the control volume.
c_p	Specific heat at constant pressure.
c_v	Specific heat at constant volume.
D	Hydraulic mean diameter.
e	Internal energy per unit mass.
E	Internal energy.
f	Friction factor.
F	Body force per unit mass.
g	Gravitational acceleration.
H	Enthalpy.
h	Specific enthalpy (enthalpy per unit mass).
k_s	Equivalent surface roughness for sand.
L	Length of duct.
m	Mass.
M	Mach number.
p	Static pressure.
p_b	Back pressure.
Q	Heat.
R	Characteristic gas constant.

R_e	Reynolds number.
S	Entropy.
s	Specific entropy (entropy per unit mass).
t	Time.
T	Absolute temperature.
u	Specific intrinsic internal energy (energy per unit mass).
v	Specific volume.
V	Velocity of the fluid.
$\mathcal{V}_1$	Volume of the control volume at time 1.
$\mathcal{V}_2$	Volume of the control volume at time 2.
$\mathcal{V}$	Volume of the control volume.
W	Work.
γ	Ratio of specific heat c_p to c_v .
μ	Dynamic viscosity.
ρ	Density.
ϵ	Relative surface roughness.
Ω	Control volume.
τ	Shear stress at wall.

Subscripts

0	Stagnation condition.
M	Condition at a Mach number M .

- 1 Condition at inlet.
- 2 Condition at outlet.

Superscripts

- * Condition at Mach number $M = 1$.
- Rate of change with time.
- Average value.

Contents

Acknowledgements	iii
Abstract	iv
Nomenclature	vi
Table of Contents	ix
List of Figures	xvii
1 Introduction to Die Casting	1
1.1 Metal-Shaping Technology	1
1.2 Different Casting Methods	2
1.3 Basic Die Casting Process	3
1.3.1 Gating Setup	7

1.3.2	Solidification	9
1.3.3	Lubrication	11
1.4	Die Design	12
1.5	Die Life and Quality Control	14
1.6	Casting Defects	17
1.7	Air Venting in Pressure Die Casting	19
1.8	Prior Work	20
1.9	Research Work	22
2	Basic Physical Laws and Equations of Compressible Flow	25
2.1	Introduction	25
2.2	The Compressible Fluid as a Continuum	26
2.3	Viscosity and Boundary Layer	27
2.4	Control Volume Approach	28
2.5	Conservation of Equations	28
2.5.1	Conservation of Mass: Continuity Equation	28
2.5.2	Newton's Second Law of Motion: Momentum Equation	30

2.5.3	The First Law of Thermodynamics: Conservation of Energy	31
2.5.4	Reversibility, Irreversibility, Adiabatic and Isotropic	31
2.5.5	The Second Law of Thermodynamics	32
2.6	Equation of State	33
2.7	Velocity of Sound	36
2.8	Subsonic and Supersonic Flow	37
2.9	Mach Number	38
2.10	The Stagnation and Total States	38
3	Isentropic Flow	40
3.1	Introduction	40
3.2	General Equations of Motion	41
3.3	Dependence of the Mach Number Through a Varying Channel	42
3.4	Isentropic Flow of a Perfect Gas	43
3.5	Choking in Isentropic Flow	44
4	Compressible Flow in a Duct with Friction	46

4.1	Introduction	46
4.2	Adiabatic Flow	47
4.3	Mach-Number Distribution Along a Duct	50
4.4	Variation of Pressure with Distance	53
4.5	Variation of Pressure and Mach Number Along a Duct vs. Inlet Static Pressure Increment	54
4.6	Choking due to Friction	56
5	Air Exhaust in Die Casting	57
5.1	Introduction	57
5.2	Modeling of Compressible Duct Flow in Die Casting	59
5.3	Algorithms	61
5.4	Test Problem From [3]	67
5.5	Numerical Results and Discussion	69
5.5.1	Computational Results of Pressure and Mach Number Varia- tion Along a Duct	70

5.5.2	Computational Results of Other Flow Properties Variation Along a Duct	72
5.6	Vacuum Venting in Die Casting Process	75
5.6.1	Introduction	75
5.6.2	The Model of Vacuum Venting	76
5.6.3	Simulation Results and Discussion of Vacuum Venting With the Change of Back Pressure	78
6	Cavity Air in Die Casting	85
6.1	Introduction	85
6.2	Modeling of Cavity Air in Die Casting	86
6.3	Algorithms	91
6.4	Results and Discussion	93
7	Conclusions	100

List of Figures

1.1	Schematic showing the principle components of a hot chamber die casting machine after Sully [23].	5
1.2	Schematic showing the principle components of a cold chamber die casting machine after Sully [23].	6
3.1	Effect of Mach number on property changes with area change in duct flow.	43
4.1	Control volume in a duct adapted from [3].	48
4.2	Distribution of static pressure and Mach number along a duct adapted from [3].	55
5.1	A schematic showing the shot sleeve, runner, gate, die cavity and air venting in pressure die casting after Bar-Meir et al [13].	58

5.2	A simplified model for air venting after Bar-Meir et al [13].	60
5.3	Flow chart of algorithm that distinguish choked and unchoked flow conditions.	62
5.4	Flow chart of algorithm in unchoked flow condition.	64
5.5	Flow chart of algorithm in choked flow condition.	66
5.6	Distribution of static pressure along a duct.	71
5.7	Distribution of Mach number along a duct.	71
5.8	Distribution of static temperature along a duct	73
5.9	Distribution of dynamic viscosity along a duct	73
5.10	Distribution of Reynolds number along a duct	74
5.11	Distribution of friction factor along a duct.	75
5.12	A schematic of an actual venting in pressure die casting arrangement after Bar-Meir al [26].	77
5.13	A simplified model for air vacuum venting after Bar-Meir et al [26]. .	78
5.14	Variations of outlet static pressure against the change of back pressure.	80
5.15	Variations of outlet static temperature against the change of back pres- sure.	81

5.16	Variations of outlet air density against the change of back pressure. .	82
5.17	Variations of outlet Reynolds number against the change of back pressure.	82
5.18	Variations of outlet Mach number against the change of back pressure.	83
5.19	Variations of velocity against the change of back pressure.	83
5.20	Variations of mass flow rate against the change of back pressure. . . .	84
6.1	A simplified model for air cavity in die casting.	87
6.2	A typical piston velocity diagram in die casting after Kaye and Street [26].	88
6.3	Flow chart of algorithm for cavity air state.	92
6.4	Variation of air mass in cavity with filling time.	94
6.5	Variation of stagnation pressure in cavity with filling time.	95
6.6	Variation of stagnation temperature in cavity with filling time.	96
6.7	Variation of density in cavity with filling time.	96
6.8	Variation of air mass flow rate in cavity with filling time.	97
6.9	Variation of cavity volume with filling time.	97
6.10	Variation of air density with the stagnation preassure in the cavity. .	98

6.11	Variation of air mass flow rate with stagnation pressure.	99
6.12	Variation of the air mass in cavity with cavity volume.	99

Chapter 1

Introduction to Die Casting

1.1 Metal-Shaping Technology

As we know, many useful engineering parts are made from metals and alloys by metal forming. Many ways of forming metals are available; they mainly include *casting, forging, welding, electroforming, powder metallurgy*, or a combination of these methods. In addition, whatever processes are used for a given part, machining (metal cutting) may be required to a greater or lesser degree.

Although it has some limitations and shortcoming such as part size limited, expensive patterns and equipment required, casting, as a method for shaping metals, is an efficient, economical process which, when used to its maximum potential, can replace assemblies of a variety of parts produced by various manufacturing processes

at significant saving in cost and labor.

1.2 Different Casting Methods

A summary of the various casting methods is as follows:

Sand Casting:

Sand casting is one of the older techniques. Classified according to the type of pattern used, it is (1) removable pattern, and (2) disposable pattern.

In the method employing a removable pattern, sand is packed around the pattern that is later withdrawn from the sand. The cavity produced is filled with molten metal to create the casting. Disposable patterns are made from polystyrene and molten metal is poured into the mold.

Die Casting:

Die casting is the most widely used of any of the permanent mold processes. Molten metal is forced into the metal mold die cavity at pressure from 0.7 to 70 MPa [16]. When the metal solidifies, the die halves open and the casting is ejected.

Centrifugal casting:

Centrifugal casting is the process of rotating a mold while the metal solidifies,

so as to utilize centrifugal force to position the metal in the mold. Greater detail on the casting is obtained, and the dense metal structure has superior physical properties. Castings of symmetrical shape lend themselves particularly to this method.

Precision or Investment Casting:

First make a wax or plastic replica of the piece to be cast. These replicas are surrounded with investment material, baked out, and metal is poured into the resultant cavity. Molds are broken to remove the castings.

Continuous Casting:

The process consists of continuously pouring molten metal into a mold, which has the facilities for rapidly chilling the metal to the point of solidification, and then withdrawing it from the mold.

The interested reader could refer to related foundry books for other casting techniques.

1.3 Basic Die Casting Process

Die casting is widely known to provide high volumes of metal castings in a short time span with tight tolerances. The source of the power for filling comes from the die casting machine that is hydraulically powered. However, the efficiency and quality of die cast parts are often taken for granted. It is a well-documented fact that

producing a defect free die casting is a very difficult task of engineering. In many engineering applications, one cannot change one parameter of the casting process without it somehow affecting another. Die casting is a cyclic process that involves several steps, each of which must be considered and analyzed carefully to ensure the quality of the casting. Generally, molten metal is injected into a die cavity at a high velocity and is allow to cool and solidify in the die by heat transfer to the die.

There are a variety of die casting processes. However they can be lumped into two main types of die casting techniques, the hot chamber and cold chamber process. The difference between the two is that in the hot chamber casting, the actuator that injects the metal into the die is in direct continuous contact with the molten metal, whereas the cold chamber process has the actuator in brief contact. However, the basic concepts and steps can be applied to both processes. In addition, instead of a shot sleeve used in a cold chamber casting, the hot chamber process uses a system of a gooseneck and a nozzle. The metal must be delivered to the shot sleeve, whereas the gooseneck is continuously filled with molten metal. The nozzle is analogous to the shot sleeve in the delivery process, but the nozzle is heated to prevent heat loss.

The Hot Chamber Process

The hot chamber process Figure 1.1 is employed for low melting points material such as zinc, lead, tin and magnesium. In this case, the exposure of the molten metal to phenomena like turbulence, oxidizing air, and heat loss during cooling

is greatly minimized due to system components being continuously in direct contact with the molten metal. However, it has one major drawback. Due to direct contact between the actuator and the molten metal for long time, this causes large thermal stresses on the machine and its components. It is not practical for higher melting point alloys.

Figure 1.1: Schematic showing the principle components of a hot chamber die casting machine after Sully [23].

The Cold Chamber Process

To overcome the above shortcoming, the cold chamber process Figure 1.2 is developed. In this process, the high melting point alloys such as aluminum, copper and even some ferrous can be used. The cold chamber process eliminates

the problem of having the hydraulic actuator in contact with the molten metal for long period time. The metal must be measured for each individual casting cycle and poured into the shot sleeve immediately before the cycle begins.

Figure 1.2: Schematic showing the principle components of a cold chamber die casting machine after Sully [23].

In both hot and cold chamber casting process, the entire casting cycle is broken down into an open and closed stage. The closed stage comes first. It is the stage where the die is closed and the metal injected, and allowed to solidify. The open stage is where the dies are opened, the solidified casting is ejected from the die, and the die is sprayed and allowed to cool, before it is closed and the entire casting cycle begins again.

The entire cycle can be divided into several crucial steps, that are discussed below.

1.3.1 Gating Setup

Gating involves the delivery of the molten metal from the shot sleeve to the die itself. Typical injection times are 0.2 sec [16]. This should be done as smoothly and as quickly as possible, which presents an obvious challenge in design. If turbulence is induced into the metal, harmful oxides and air impurities can become entrapped within the molten metal, lowering the quality of the finished casting.

After the liquid metal flows from the shot sleeve or nozzle, it flows into the sprues and runners. These have several purposes. First and foremost, they are designed to allow a smooth transition for the metal from the shot sleeve or nozzle. The sprue must also be designed to allow quick solidification inside of it, while maintaining the liquid state of the metal in the nozzle/shot sleeve, to allow a handling of the shot. The runner is the path the metal takes to the gate, the last section before injection into the die. This must also be able to be cooled quickly, and along with the sprue, are usually used to eject and handle the casting after solidification.

The gates are designed to deliver the fluid metal to the die cavity but since it is trimmed from the casting after solidification, it makes sense to make the cross section of the gate as small as possible. There is also a significant transition in cross sectional area between the runner and the gate, called the approach. However, the main consideration in the cross sectional area of the gate is not the trimming time needed, but the desired filling time and the metal flow rate. This is usually a

compromise because excessive velocities in the gate can cause die erosion.

Once the fluid metal is out of the gate, it enters the die cavity itself. Once there, the subject of venting must be addressed. Trapped gas in the die cavity can affect both the fluid flow and the quality of the metal cast. The vents must be positioned and sized so that the gas is vented effectively and the metal that follows solidifies before reaching the outside of the casting block. An alternative to venting is using a vacuum pump to pump out the air in the die cavity during the shot, and closing it in time to prevent metal from entering the vacuum system. However, this technology is relatively expensive, and as such is not widely used.

The final component in the metal flow is the overflow system. As the name suggests, this is where excess metal can flow out of the die cavity. These overflows can also act as a thermal balance because of their larger volume to surface area ratio, they can release more heat to the die where it most likely needs it, at the sections furthest away the gates.

The plunger is the device that gives the metal kinetic energy to flow into the casting, and travels in a pre-determined fashion. The pressure on it and velocity can be varied as it travels along its path. Thus, the pressure is held constant through the pre-fill and cavity filling stages while the cavity is filling, until it reaches a point called intensification. At this point, the plunger increases the pressure on the metal to force it into thinner sections of the die casting in question.

1.3.2 Solidification

Ideally, one wants the section nearest to the gate to cool last, so that metal can be fed into the die during the intensification stage of plunger movement. Solidification usually requires a considerable amount of undercooling. This means that the temperature of the liquid metal must be lowered below its solidification temperature. This is because the creation of a new surface requires a certain amount of energy before it can form. Since the mould wall is a pre-defined surface for solidification to begin, it lowers the degree of undercooling required. The amount it lowers the degree of undercooling required for solidification to occur depends on the temperature difference between the mould and the molten metal. Additionally, the latent heat of solidification is also released at the solid-liquid interface during solidification.

At the mould wall, chill crystals are nucleated at random orientations. At this newly formed solid-liquid interface, the temperature gradient is steep enough to cause instability. This instability causes crystals to form faster than from an otherwise stable interface, protruding into the increasingly supercooled liquid. These crystals, called columnar crystals, grow in preferred crystallographic directions, perpendicular to the mould wall, at this stage of solidification, the interface is no longer flat, but is a new dendritic. These dendrites can form secondary or even tertiary arms.

In alloys, solidification can occur in the center of the casting due to the combined effects of thermal and constitutional undercooling, rather than due to the growth of

chill crystals and columnar grains along the mould walls. These grains are called equiaxed grains, and randomly oriented. However, in most commercial castings, the structure is mainly dendritic [17], mainly due to constituents in the alloy acting as nuclei for solidification. The columnar crystals mentioned above, cease to grow once they encounter sufficient equiaxed crystals.

Thus for alloys, there can be three distinct zones of grain growth and development. They are, from the mould wall to the center, the chill zone, the columnar zone, and the equiaxed zone. The size of these zones can be affected by temperature of the cast metal and injection velocity, as well as the use of grain refiners. It has been found experimentally that with all other conditions being equal, the columnar zone increases in size as the pouring temperature is increased, and the equiaxed zone decreases as the pouring temperature decreases [17].

It should be noted that grain size has a profound effect on the material properties of the casting. Long solidification times promote the growth of large grains, and vice versa for short solidification times. Generally, the smaller the grain, the better the mechanical properties are, such as toughness, strength, and ductility. Thus, not only is it economically important to have minimum cooling times, it is also beneficial mechanically.

Once solidified and cooled sufficiently, the casting has to be removed from the die cavity. Ejection of the casting uses ejector pins. After ejection the preparation for

the next cycle starts. This step is followed by spray cooling and the application of lubricants that creates an insulating layer between the die surface and the casting. Finally the die is closed and it is ready for the next cycle.

1.3.3 Lubrication

Die face lubricants perform a parting function between cast metal and die surface to assist metal flow and casting release, reducing the tendency of die cast metals to weld to the die. Lubrication also plays a role in the surface finish of the die, and should not allow any excess gases to dissolve into the casting metal. As well, moving die parts such as slides, cores and ejector pins are lubricated to prevent seizure and wear. Lubricants are applied to the die face by several methods of spray systems, and should not create a large thermal shock to the hot die surface, and not create excessive gas, which may cause porosity.

Modern lubricants are dilutions of a paste within a type of solvent. They can be oil based, or more recently, water based. Water based lubricants tend to cool the die more than solvent-based lubricants due to their higher latent heat of vaporization content, and thus must be used with care to avoid excess thermal shock.

A common problem with lubrication is that the die is sometimes so hot that the lubricant actually evaporates before it contacts the die. This is common in water-based lubricants. This phenomenon is called the “leidenfrost vapor cushion” [16],

and to solve this problem, either the spray must increase in power or in the time of its application. Once the die temperature is reduced enough so that the lubricant actually comes into intimate contact with the die, it is said to have reached the wetting temperature.

Once applied, the lubricant must not run off the surface of the die. One advantage of a water based lubricant is that due to the evaporation of the water and the electric charges present in the constituents, one can get much thinner lubrication films than with oil based lubrication. The water-based lubricant leaves a film of concentrated lubricant on the die, which unlike solvents, does not spread easily.

1.4 Die Design

When designing a die, many factors must be taken into account. Die casting is such that no one variable dictates the quality of the finished product, and no one variable can be changed without affecting many other process variables. Thus, there are no single general empirical formulae that can be applied to any specific die. Each die casting is unique, with its own specific parameters, and thus, its own specific problems and inherent defects.

There are many theories for the proper design methodology of a die, and many are outlined by the North American Association of Die Casting (NAADC) and The

Society of Die Casting Engineers (SDCE). More often than not, it is a matter of experience and trial and error to make a certain die casting process run at its optimal operating capacity. Recently, with the aid of new finite element analysis techniques, this is becoming somewhat less of an art and more an exact science.

Before any of the gating calculations are made, the volume of the part, along with the gates and overflows, must be determined so that sufficient liquid metal is available to fill the casting. Once this has been completed, the designer must consider where to position the gates and runners to provide the required quality in the finished product.

The generally accepted procedure for regulating the actual flow of the metal should be such that the thinnest section of the die has sufficient time to fill. Thin sections are major problems in die casting, because of the heat content per unit of surface area is less than thicker sections. Thus, cooling here is quicker, and may be completed before the cavity is completely full. Higher injection temperature or die temperatures, or higher metal velocities can help in the filling of the thinner sections. Thus, if the thin sections should take care of the primary concern and are filled sufficiently, the thicker sections should take care of themselves. The injection temperature of the metal to be cast should be high enough so that the metal leaving the gate is in a totally liquid state, avoiding the semi-liquid slushy zone of alloys. Also, the die material itself should be sufficiently tough to withstand the stresses associated with die casting as well as provide sufficient cooling.

Good engineering judgment should be used when actually designing the part to be cast. One should consider the ease of ejection of the part and the accompanied runners, sprues and overflows once the casting has sufficiently solidified. Thus, ejection pins must be strategically located, and movement of the die in between open and closed stages must be precise. Also, the designer must avoid overly thick sections or stress concentrations such as sharp corners. These can cause excessive heat buildup, leading to problems such as soldering and die cracking.

1.5 Die Life and Quality Control

There are three main factors in die life. Firstly, the abrasive action of the injected metal causes die erosion, secondly, the chemical attack by the cast metal on the die during solidification, causing soldering, and finally, thermal fatigue, also called heat checking. The later factor, however, is usually a main factor with regards to die life. Low melting point alloys of lead and zinc have dies that last the longest, whereas ferrous alloys of steel have dies that can withstand only about 5000 shots before they need to be repaired or replaced [16]. Several measures can be taken to maximize die life.

Heat flow in the die is a very important aspect of quality control. Since die casting is a cyclic process, the temperatures in the die are not constant. Initially they fluctuate with each open and closing stage, increasing as more and more cycles

are applied. This fluctuation of temperatures occurs in a layer in the die called the thermal boundary layer (TBL), in which the temperature gradients are always changing. Eventually, the fluctuations of temperatures in the TBL reach a cyclic steady state, where the difference between minimum and temperatures stays constant. This condition is called a cyclic steady state, and usually occurs after approximately 300 casting [16].

Logically, the higher the casting temperature, the higher the thermal stresses induced in the die. Since the molten metal is completely encased by the die, the die acts as a massive heat sink for which much of the heat of the molten metal must pass through (spray removes some heat). The repeated violent cycles of heating and cooling, and consequent expansion and contraction of the die eventually leads to fine cracking, called craze cracking. The extent of this determines the surface quality of the casting, and thus the amount of machining required after the casting is completed. The die areas most quickly affected by craze cracking are the gate areas and cores which happen to be in direct line of the injected metal due to the high thermal gradient involved. It must also be noted that these areas are also very vulnerable to erosion as well. The issue of replacing or repairing the die due to craze cracking depends on one individual's own economic and quality standards, since replacing a die may cost millions of dollars.

The die should be as thermally balanced as possible, to prevent uneven temperature gradients, and with it, possible warping and distortion of the die. A number

of factors, mainly die geometry and the mass distribution of the casting, can cause uneven die temperature. Cooling lines within the die can control these temperatures. Also, the use of ample overflows can aid in the thermal balancing of the die.

However, allowing a die to cool too much in between castings can cause as much damage as overheating it, due to thermal fatigue. That being said, it should be ensured that the entire die, including cores and inserts are preheated to a constant temperature before injection, so as to not to induce uneven thermal gradients. Coolant flow should also be monitored closely; too high a flow can carry away too much heat, and can cause thermal shock to the die. Most cooling lines are made by drilling holes into the die, and running a cooling liquid, usually water, in them to carry the heat away. Often, since the water is not always pure, scale can build up and hinder in the removal of the heat. An alternative to water-cooling line is a heat pipe. A heat pipe is used in local hot spots within the die, and is usually a nickel-plated copper tube with a water-moistened wick lining the inside for capillary action. As one side of the pipe is heated, the water evaporates and travels to the other end of the pipe, where it condenses. It is then absorbed back into the wick material and carried back to the heat source by the capillary action. Conversely, cooling lines can be filled with hot oil or water to pre-heat the die before metal injection.

As well, local maximum and minimum temperatures in the die should be monitored. If a point in the die becomes too hot, hotter than the casting solidus temperature, then soldering will occur. Conversely, if a point in the die is too cold, especially

in a point where the metal is last filled, the metal will solidify too quickly, before the intensification stage, and may cause cold shuts or an incomplete fill. Thus, a suitable pre-injection die temperature should be suitably chosen and maintained.

The steel used to create the die must be sufficiently hard and strong to withstand the pressure and temperatures created by the die casting process, and must have a good finish to prevent premature crack initiation sites. More often than not, a heat treatment is used to prepare the die for the thermal punishment encountered during die casting. Stress concentrations such as sharp corners can also be a contributing factor to crack initiation sites, and local high temperatures, called hot spots. Also a low coefficient of thermal expansion will ensure that thermal strain is kept to a minimum. As well, a high thermal diffusivity constant will allow the heat to be quickly dissipated quickly, and hence cool the die faster.

1.6 Casting Defects

During solidification, many defects can occur in the die casting structure. Porosity is a common problem that occurs in die casting, and has many forms. Gas porosity occurs when gas or air is trapped in the solidifying metal, and can be caused by inadequate venting, poor metal flow during the casting, or excessive lubricant spray. Gas porosity can cause blisters to form if it is near the surface of the casting. Ideally, the intensification stage of the plunger movement should increase the cavity pressure

such that the remaining air is vented out, but often parts of the casting solidify too rapidly to benefit from this, as they trap the air.

Shrinkage during solidification is also a common problem, and can cause the formation of tiny holes within the casting. Shrinkage porosity can occur when there is insufficient feeding of metal during the intensification stage of the plunger movement, and can have an adverse effect on the tolerance of the casting, or in severe cases, on the eventual final shape. Solidification is usually very rapid in the die casting, so it is very difficult to feed the shrinkage. The shrink porosity can form along what is called the neutral thermal axis, which is the surface inside the casting where two solidification fronts from the die wall meet, and can locate hidden porosity [19].

When streams of metal solidify but do not fuse, it is called a cold shut. A cold shut can occur when solidification occurs too early in the mould, or the fluid does not mix properly. This can be caused by a gating design that causes bad flow, back pressure due to inadequate venting or excessively thin sections in the die.

If the die becomes locally overheated, soldering can occur between the die and the molten metal, and in extreme cases can erode the die itself. This is a common occurrence near the gate, due to induced turbulence of the high-speed metal flow. This can cause tears to occur in the casting during ejection, and can obviously have severe detrimental effects on the die itself. Adequate cooling of the die, and a careful gate design can prevent this.

During opening of the die after the casting has solidified, galling can occur. This defect causes visible marks to be created on the surface of the casting, and can be caused by ejecting the casting before adequate cooling, or poor die design. This can be easily corrected with a minor reconfiguration of the cavity.

Also, if the temperature of the casting is not highly uniform when it is ejected, defects such as warping and cracking may occur, obviously adversely affecting dimensional tolerances and surface quality. If during ejection the casting temperature is too high, the metal may be too soft to withstand the stresses induced by the process of ejection, and may cause tears or completely break the casting.

1.7 Air Venting in Pressure Die Casting

Porosity, a major problem in many die casting products, results from gas entrapment (gases entrapped in the liquid metal) and shrinkage (difference between liquid density and solid density). The porosity due to entrapped gases constitutes a large portion of the total porosity especially when the cast walls are very thin. The main causes for high porosity are insufficient vent area, lubricant evaporation (reaction processes), incorrect placement of the vents and mixing processes during the filling time.

In engineering applications, the most widely used casting method opens the vent to the atmosphere. It is referred to herein as air venting. When the results for air

venting are poor other methods such as vacuum venting, Pore Free Technique (in zinc and aluminum casting) or squeeze casting are used.

The best ventilation is achieved for a die with a large vent area. However, to minimize the secondary machining such as trimming and to insure freezing of the liquid metal in the mold, the gates and vents have to be very narrow. In addition, higher injection pressure and temperature are hopefully achieved to help in filling of the thinner section. A typical size of the vents and gate thickness is in the range of 1 to 2 mm. The conflicting requirements on the vent area suggest an analysis of the air venting itself.

1.8 Prior Work

Ali Mahallati, a previous member of the research team, did valuable work for the air exhaust project. In his study, he used the following criterion, “If the duct is choked, the duct outlet stagnation pressure and temperature are higher than the atmospheric pressure and temperature. If the duct is not choked, the outlet stagnation pressure and temperature are equal to the atmospheric conditions” [27].

In Ali Mahallati’s algorithm, the mass flow rate is determined by finding the pressure value that gives the correct resistance coefficient f for both choked and unchoked flow. He used a Newton Raphson method. Even with special precautions,

the Newton Raphson algorithm often failed to converge.

In the new algorithm developed in this thesis, the distinction between of choked and unchoked cases is based on both the Mach number of unity and static pressure equal to back pressure at the outlet. The algorithm for choked and unchoked cases is described in detail chapter 4.

For unchoked flow, the mass flow rate is calculated based on assuming an initial inlet Mach number, and then determining the outlet pressure. The initial Mach number is varied until the outlet pressure matches the back pressure. For choked flow, an iterative approach is used again but iterating on the value of friction factor f . The solution is the value at which the correction $|df| < tolerance$. At the same time, a check is always made to be sure that outlet static pressure equals or exceeds back pressure. This algorithm has proved to be very robust and fast.

Professor Ali Nouri, Sharif University, Iran, worked on this problem as a Visiting Professor. However, he used another algorithm which I was not able to implement successfully.

Most of theoretical equations that were developed in this thesis for the air exhaust model were taken from Roberson & Crowe [3]. The strategy follows that of Roberson & Crowe. However, Roberson & Crowe do not present an algorithm and to author's knowledge, the numerical algorithm presented in this thesis that is based on the theory and strategy in Roberson & Crowe, is novel. Also numerical solutions for each

formula and differential equation have been tested and compared with the theoretical solution.

1.9 Research Work

Air trapped in the casting increases porosity. Porosity is a major issue in die casting. It causes common defects such as leakage of pressure tight casting and low mechanical properties. Gas trapped in die cast parts comes from different sources. Among them are air occupying die cavity and shot sleeve prior to die filling, and lubricant decomposition resulting from contact with the hot metal. It is desirable to extract an adequate amount of air to reduce porosity to a level under a specified limit. Vents are used to allow extraction of gas from the cavity during filling. Understanding the relationship among the process variables such as pressure, temperature and volume of the air trapped in the casting is very important for these reasons. It also should help the designer to predict defects in the design stage and design dies that produce high quality casting.

In this thesis, computational mathematical models and algorithms describing air flow in die casting are developed based on compressible flow theory. Two projects dealt with compressible flow in die casting process have been done based on their own mathematical models. They are the air exhaust project and the cavity air project.

Air Exhaust Project

Air flow is initiated by pressure build-up in the die or by application of vacuum. Higher pressure differential between the die cavity and the environment increases air mass flow rate, dependent on the geometry of the vents when the Mach number at the vent outlet equals one. This relationship is described in a quasi-steady state analysis. This process is called air exhaust. It deals with the mass flow rate of the air from one cavity with a given stagnation pressure and temperature to another cavity with a given stagnation pressure and temperature. The air flows through a duct of known cross-section, length and surface roughness. The mass flow rate and the outlet pressure and temperature of the air have been computed based on a mathematical model and algorithms.

Cavity Air Project

As the plunger moves forward to fill the die cavities with molten metal, it also compresses the air in the die which increases both air pressure and mass flow rate out through the vents. For a cavity with a known volume, stagnation temperature and pressure at time zero or an initial time say t_0 , the volume of the cavity is known as a function of time and the mass leak rate from the cavity is known as a function of time. The stagnation pressure and temperature (and hence also density and total mass of air in the cavity) have been computed based on a computational model and algorithms in this thesis.

It should be noted that the cavity air project uses the mass flow rate or leak rate of air computed by the air exhaust project, while the inlet stagnation pressure and temperature in air exhaust project are calculated by the cavity air project. The parameters such as back pressure from air exhaust and pressure and temperature of the cavity air are very important high level design parameters of die casting. Also these parameters can be used as boundary conditions to implement finite element analysis, such as temperature distribution and thermal stress analysis and so on.

Chapter 2

Basic Physical Laws and Equations of Compressible Flow

2.1 Introduction

Compressible fluid mechanics is a study of flow in which significant density variations occur throughout the fluid. With Mach numbers greater than about 0.3 and non-negligible density changes, the equations of incompressible fluid mechanics must be supplemented with those of thermodynamics. Four basic laws can be readily applied in compressible fluids. These fundamental laws upon which all the analyses presented in this thesis depend directly or indirectly are

1. Conservation of mass.
2. Conservation of linear & angular momentum.
3. Conservation of energy.
4. The second laws of thermodynamics.

In any flow analysis some information about the properties of the fluid must be known. So an equation of state of the substance is involved. The additional complexities introduced by compressible flow require that approximations be made in order to simplify the problems so that satisfactory engineering answers can be obtained.

The basic physical laws and equations of compressible flow are usually treated in thermodynamics and fluid mechanics. Therefore only a briefly review of this material is presented in this chapter.

2.2 The Compressible Fluid as a Continuum

In order to simplify the analysis of gas flows, the gas is approximated as a continuous substance, with only the average effects of all the molecules in a finite region of the gas being considered. This continuum assumption means that its variation in properties is so smooth that the differential calculus can be used to analyze the compressible

fluid.

2.3 Viscosity and Boundary Layer

In a continuum, when a viscous fluid flows over a fixed surface, layers of the fluid next to the surface are held back by the viscous forces and stick to the surface, in other words, the velocity component of the fluid at the fixed wall is zero, both normal and tangential to the wall surface. For most fluids, and certainly for gases, this viscosity is so small that the viscous effects are confined to a very thin layer in the vicinity of surface called the boundary layer. Outside the boundary layer, the fluid can be analyzed with inviscid theory.

Considering the flow of a gas through an internal passage, the boundary layer is usually thin enough that it can be assumed that the pressure gradient in the direction normal to the wall surface can be neglected. Thus the pressure distribution on an internal passage, even though a boundary layer exists, can be calculated using the simpler inviscid equations.

2.4 Control Volume Approach

Three basic approaches are available in the analysis of arbitrary flow problems, they are: control volume, differential and experimental analysis. Because control volume analysis overcomes the shortcomings and limitations of differential and experimental analysis in the light of numerical calculation and economic factor respectively, all physical equations derived in this thesis are based on control volume analysis approach.

2.5 Conservation of Equations

For a compressible flow, density becomes an additional variable; also significant variations in fluid temperature may occur as a result of density or pressure changes. There are total four possible unknowns: pressure p , temperature T , density ρ , and flow velocity V . Thus four equations are required for the solution of a problem in compressible gas dynamics: the conservation of mass, the conservation of momentum, the conservation of energy and a thermodynamic equation of state.

2.5.1 Conservation of Mass: Continuity Equation

Considering a generalized fixed control volume Ω embedded in with an arbitrary flow field, for a given differential area dA of surface, it will have a different velocity $\mathbf{V}$

making a different angle θ with the local normal to dA . Let $\mathbf{n}$ is defined as the *outward* normal unit vector everywhere on the control surface, then $\mathbf{V} \cdot \mathbf{n} = V_n > 0$ for outflow and $V_n < 0$ for inflow. The rate of increase of mass within the control volume must equal the rate of mass influx into the control volume, we have

$$\int_{\Omega} \frac{\partial \rho}{\partial t} d\Omega + \int_{\partial\Omega} \rho(\mathbf{V} \cdot \mathbf{n}) dA = 0 \quad (2.1)$$

For a control volume that has only a finite number of one-dimensional inlets and outlets, we get:

$$\int_{\Omega} \frac{\partial \rho}{\partial t} d\Omega + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0 \quad (2.2)$$

If the flow within the control volume is steady, then $\partial \rho / \partial t = 0$, and equation (2.1) and equation (2.2) reduces to equation (2.3) and equation (2.4) respectively. They are:

$$\int_{\partial\Omega} \rho(\mathbf{V} \cdot \mathbf{n}) dA = 0 \quad (2.3)$$

$$\sum_i (\rho_i A_i V_i)_{out} = \sum_i (\rho_i A_i V_i)_{in} \quad (2.4)$$

This law indicates that mass can neither be created nor destroyed. The mass of a control volume remains constant at steady state for a system.

2.5.2 Newton's Second Law of Motion: Momentum Equation

Applying Newton's second law to a small control volume in Cartesian coordinates, we get:

$$\frac{d}{dt}(m\mathbf{V})_{\text{sys}} = \sum \mathbf{F} = \frac{\partial}{\partial t} \left(\int_{\Omega} \mathbf{V} \rho d\Omega \right) + \int_{\partial\Omega} \mathbf{V} \rho (\mathbf{V} \cdot \mathbf{n}) dA \quad (2.5)$$

The left-hand side represents the resultant of all forces acting on the control volume. $\mathbf{F}$ may involve pressure forces, viscous forces, gravity, magnetic forces, electric forces, surface tension, and so on.

The right-hand side represents the rate of increase of linear momentum within the control volume added to the net rate of efflux of linear momentum from the control volume.

If the control volume has only one-dimensional inlets and outlets, equation (2.5) reduces to

$$\sum \mathbf{F} = \frac{\partial}{\partial t} \left(\int_{\Omega} \mathbf{V} \rho d\Omega \right) + \sum_i (\dot{m}_i \mathbf{V}_i)_{\text{out}} - \sum_i (\dot{m}_i \mathbf{V}_i)_{\text{in}} \quad (2.6)$$

Similarly, for steady flow,

$$\frac{\partial}{\partial t} \left(\int_{\Omega} \mathbf{V} \rho d\Omega \right) = 0 \quad (2.7)$$

so equation (2.5) and equation (2.6) simplifies to equation (2.8) and (2.9) as follows:

$$\sum \mathbf{F} = \int_{\partial\Omega} \mathbf{V} \rho (\mathbf{V} \cdot \mathbf{n}) dA \quad (2.8)$$

$$\sum \mathbf{F} = \sum_i (\dot{m}_i \mathbf{V}_i)_{out} - \sum_i (\dot{m}_i \mathbf{V}_i)_{in} \quad (2.9)$$

2.5.3 The First Law of Thermodynamics: Conservation of Energy

Let total energy of the fixed mass system as E , such as the internal energy, kinetic energy, potential energy of the system and other forms of storable energy; the heat added to the system as Q ; work done by the system as W . For a system consisting of a fixed mass of particles, the conservation of energy can be expressed as

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \quad (2.10)$$

Let e equal the energy per unit mass, then, the conservation of energy can be written as

$$\frac{dE}{dt} = \frac{\partial}{\partial t} \left(\int_{\Omega} e \rho d\Omega \right) + \int_{\partial\Omega} e \rho (\mathbf{V} \cdot \mathbf{n}) dA \quad (2.11)$$

If the system can be assumed to possess only internal, kinetic, and potential energies, then $e = h + \frac{1}{2}V^2 + gz$, where h is specific enthalpy.

2.5.4 Reversibility, Irreversibility, Adiabatic and Isotropic

A process is reversible if the system can be restored to its initial state and leaves no change in either system or surroundings.

A process is irreversible if the system and all parts of its surroundings cannot be restored exactly to their respective initial states after the process has occurred [25].

A process is irreversible if it has dissipation. Friction and fracture are examples of processes with dissipation.

When a system undergoes a process while enclosed by an insulated wall, i.e., there is no flow of heat in or out of the system, then that process is adiabatic. During an adiabatic process, entropy can increase or remains constant. A process that is adiabatic and involves no change in entropy is called isentropic, i.e., a process is isentropic if it is both adiabatic and reversible.

2.5.5 The Second Law of Thermodynamics

The second law of thermodynamics enable us to define ideal processes and hence to specify the degree of imperfection of actual processes. The thermodynamics property derivable from the law is entropy S , which is defined for a system under a reversible process by

$$dS = \frac{dQ}{T} \quad (2.12)$$

for an irreversible process,

$$dS > \frac{dQ}{T} \quad (2.13)$$

Examples of irreversible processes are friction, heat transfer, sudden expansion and mixing of different gases and so on. For a fluid flowing through the control surface, the pressure loss due to irreversibilities is the same as the pressure loss due to friction.

A useful thermodynamic equation for a pure substance, derivable from the first and second laws, is

$$Tds = dh - vdp \quad (2.14)$$

where s represents entropy per unit mass. This equation contains only thermodynamic properties, it is independent of the path of a process. It can be integrated between given end states to determine the entropy change regardless of whether the thermodynamic process involved is reversible or irreversible and whether the process takes place in a closed container or in steady flow.

2.6 Equation of State

An equation of state for a pure substance is a relation between pressure, density and temperature for that substance.

Ideal gases represent the limit in which the volume of atoms is small relative to the volume of the gas. In other words, in an ideal gas the average distance between molecules is large relative to the molecular diameter. As the volume of the gas decreases relative to the volume of atoms, equations of state for real gases tend to

deviate from the equations of state for ideal gases. Examples of equations of state for real gases include van der Waals equation and virial equations.

For all gases, the compressibility factor Z is defined as $Z=pv/RT$. In another form, the compressibility factor Z is expressed in inverse powers of specific volume as $Z = 1 + B(T)/v + C(T)/v^2 + D(T)/v^3 + \dots$

In the limiting case where the gas molecules are assumed not to interact in any way, the coefficients B , C , D , etc. vanish and the equation reduces to $Z = pv/RT = 1$, or $p v = RT$. In the case ($Z=1$), $p v = RT$ is referred as the ideal gas equation of state.

All gases at sufficiently high temperatures and sufficiently low pressure (relative to their critical point) are in good agreement with the ideal-gas equation of state.

A die casting is produced by injecting a molten aluminum-silicon alloy at about 640 C into a die made from H13 tool steel operating with a die cavity temperature of about 300 C. The 20 Kg of aluminum is injected in about 30 ms at pressure as high as 700 bars.

Let us assume the pressure is 700 bars and temperature is 1000 K. For air, the critical pressure p_c and critical temperature T_c are 133 bars and 37.7 K respectively in Table A-1 [25]. The reduced pressure p_r and reduced temperature T_r can be calculated as $p_r = p/p_c = 700/133 = 5.263$ and $T_r = T/T_c = 1000/37.7 = 26.52$. With these values for the reduced pressure and reduced temperature, the value of Z obtained

from Figure A-2 [25] is approximately 1.005. For a fixed volume and temperature of 1000 K, the error in pressure is about 0.5% in an ideal gas relative to a real gas. So it is reasonable to assume air as an ideal gas in die casting process.

For the perfect gases, it is expressed as

$$p = \rho RT \quad (2.15)$$

where R is a gas constant. For air, $R = 287 \text{ m}^2/\text{s}^2\text{K}$.

The perfect gas law, simple as it is, yields uncomplicated expressions for the various thermodynamic properties and can be applied over a wide range of pressures and temperatures with a high degree of accuracy.

For a perfect gas, the internal energy is a function of temperature only:

$$u = u(T) \quad (2.16)$$

$$du = c_v \cdot dT \quad (2.17)$$

where c_v is specific heat,

$$c_v = \left(\frac{du}{dT} \right)_v \quad (2.18)$$

In a like manner specific enthalpy h and c_p of a perfect gas also vary only with temperature

$$dh = du + RdT \quad (2.19)$$

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p \quad (2.20)$$

The ratio of specific heats of a perfect gas is an important dimensionless parameter in compressible-flow analysis

$$\gamma = \frac{c_p}{c_v} \quad (2.21)$$

As a first approximation in airflow analysis we commonly take c_p , c_v , and γ to be constant, $\gamma_{\text{air}} \approx 1.4$.

2.7 Velocity of Sound

The sound speed is the rate of propagation of a pressure pulse of infinitesimal strength through a still fluid. It is defined as

$$a^2 = \frac{\partial p}{\partial \rho} \quad (2.22)$$

The evaluation of the derivative requires that the process must be adiabatic because there are no temperature gradients except inside the wave itself. For vanishing-strength sound waves we therefore have an infinitesimal adiabatic or isentropic process. The correct statement for the sound speed is

$$a = \left(\frac{\partial p}{\partial \rho} \right)_s^{1/2} = \left(\gamma \frac{\partial p}{\partial \rho} \right)_T^{1/2} \quad (2.23)$$

For a perfect gas, the velocity of sound can be expressed as the square root of the temperature:

$$a = \sqrt{\gamma RT} \quad (2.24)$$

2.8 Subsonic and Supersonic Flow

As a body moves through a stationary fluid, waves are emitted from each point on the body and travel outward at the velocity of sound. In an incompressible fluid, the velocity of sound is infinite, so an entire body of fluid is able to sense instantaneously the motion of an object passing through it. In a compressible fluid, the velocity of sound has a finite value.

If a body travels through a compressible fluid at a velocity less than that of sound, i.e. subsonic flow, waves emitted by the body are able to move ahead of the body and signal the fluid to adjust to the oncoming disturbance. In this case, the fluid is able to adjust gradually to a moving object, and smooth, continuous streamline patterns result.

If a body travels at a velocity greater than that of sound, i.e. supersonic flow, the waves are not able to signal the fluid ahead of the body. In this case, the adjustment of the flow is not gradual but takes place entirely in the shock wave itself. Shock waves result in discontinuous changes in fluid properties.

Shock waves can exist only in supersonic flow.

2.9 Mach Number

There are large differences in flow patterns with compressible flows. General behavior of the flow depends on whether the fluid velocity is greater or less than the local velocity of sound. The criterion for the type of flow is Mach number M , defined by

$$M = \frac{V}{a} \quad (2.25)$$

Mach number is an extremely important parameter in the study of compressible fluid flow. In the development of the equations of motion of a compressible fluid, much of the analysis will be expressed in terms of Mach number.

2.10 The Stagnation and Total States

In literature dealing with compressible flows, one often finds reference to “stagnation” condition — that is, “stagnation temperature” and “stagnation pressure”. Stagnation refers to the conditions that exist at a point in the flow where the velocity is zero, regardless of whether or not the zero velocity has been achieved by an adiabatic, or reversible process.

But “total” conditions, such as “total temperature” and “total pressure”, refer

to the conditions that the flow velocity is brought to zero in a reversible, adiabatic process.

In most cases, however, the difference between stagnation and total conditions are negligibly small.

Chapter 3

Isentropic Flow

3.1 Introduction

If the friction and heat transfer are negligible, then the flow may be considered reversible and adiabatic and hence isentropic. Variations in fluid properties are brought about by the area change. One-dimensional, steady flow of a perfect gas is assumed in order to reduce the equation to a workable form. For gas flow, changes in potential energy and gravitational forces are usually neglected. The analysis for this case gives the ideal conditions and can be used as a standard for comparison of the actual flows.

3.2 General Equations of Motion

By using the basic equations derived in chapter 2, for one-dimensional steady flow through a varying area channel, the general behavior of isentropic flow can be presented as follows:

The continuity equation (2.1) gives

$$\int_{\partial\Omega} \rho (\mathbf{V} \cdot \mathbf{n}) dA = 0 \quad (3.1)$$

For the one-dimensional flow,

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad (3.2)$$

The momentum equation (2.5) with $dV/dt = 0$, gives

$$\sum F = \int_{\partial\Omega} \mathbf{V} \rho (\mathbf{V} \cdot \mathbf{n}) dA \quad (3.3)$$

Simplifying yields

$$dp + \rho V dV = 0 \quad (3.4)$$

The energy equation (2.11) gives

$$dh + d\frac{V^2}{2} = 0 \quad (3.5)$$

3.3 Dependence of the Mach Number Through a Varying Channel

Consider the duct of varying area shown in table 3.1. It is assumed that the flow is isentropic and that the flow properties at each section are uniform. Since the mass flow is constant along the duct, combining the continuity and momentum equations derived in section 3.2, one can get

$$\frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{A} \frac{dA}{dx} + \frac{1}{V} \frac{dV}{dx} = 0 \quad (3.6)$$

The flow is assumed to be inviscid, so Euler's equation for steady flow is applicable:

$$\rho V \frac{dV}{dx} + \frac{dp}{dx} = 0 \quad (3.7)$$

Making use of $a^2 = \partial p / \partial \rho$ to the speed of sound in an isentropic flow, gives

$$\frac{1}{V} \frac{dV}{dx} = \frac{(1/A)(dA/dx)}{M^2 - 1} \quad (3.8)$$

Inspection of this equation, without actually solving it, reveals a fascinating aspect of compressible flow: property changes are of opposite sign for subsonic and supersonic flow because of the term $M^2 - 1$. The combination results of area change and Mach number are summarized in table 3.1.

<i>Duct geometry</i>	dA/dx	<i>Subsonic $M < 1$</i>	<i>Supersonic $M > 1$</i>
> 0	$dV < 0$ $dp > 0$ Subsonic diffuser	$dV > 0$ $dp < 0$ Supersonic nozzle
< 0	$dV > 0$ $dp < 0$ Subsonic nozzle	$dV < 0$ $dp > 0$ Supersonic diffuser

Figure 3.1: Effect of Mach number on property changes with area change in duct flow.

3.4 Isentropic Flow of a Perfect Gas

Some important variations of the fluid properties for isentropic flow of a perfect gas are given as follow in terms of Mach number M and stagnation states.

Pressure:

$$\frac{p_0}{p} = \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{1}{2}(\gamma - 1)M^2 \right]^{\frac{\gamma}{\gamma-1}} \quad (3.9)$$

Temperature:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2, \quad M = \frac{V}{a} \quad (3.10)$$

Density :

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T} \right)^{\frac{1}{\gamma-1}} = \left[1 + \frac{1}{2}(\gamma - 1)M^2 \right]^{\frac{1}{\gamma-1}} \quad (3.11)$$

For compressible flow, another convenient reference state is at Mach number unity. These sonic, or critical, properties are denoted by asterisks: p^* , T^* and ρ^* . There are certain ratios of the stagnation properties as given by equation (3.9), equation (3.10) and equation (3.11) when $M = 1$; for perfect gas, $\gamma = 1.4$

$$\begin{aligned} \frac{p^*}{p_0} &= \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}} = 0.5283 \\ \frac{T^*}{T_0} &= \frac{2}{\gamma + 1} = 0.8333 \\ \frac{\rho^*}{\rho_0} &= \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma-1}} = 0.6339 \end{aligned} \quad (3.12)$$

It should be noted that all critical properties are constant in all isentropic flow; in adiabatic non-isentropic flow, the p^* and ρ^* may vary. The usefulness of these critical values will become clearer as we study compressible duct flow with friction later in next chapter.

3.5 Choking in Isentropic Flow

For given stagnation conditions, the maximum possible mass flow passes through a duct when its throat is at the critical or sonic condition. The duct is said to be

choked and carry no additional mass flow unless the throat is widened. If the throat is constricted further with no other changes, the mass flow through the duct must decrease.

Chapter 4

Compressible Flow in a Duct with Friction

4.1 Introduction

The analysis of compressible flow in a duct is somewhat more difficult than the flow of liquid through a duct because of the dependence of density and pressure on temperature. The problem is also complicated by the fact that wall friction and heat transfer cannot be simply combined into a single head-loss parameter because of their distinct effect on the Mach number distribution along the duct. In real engineering problems, however, frictional forces are present and may have a decisive effect on the resultant flow characteristics. Naturally, the inclusion of friction terms in the equations of mo-

tion makes the resultant analysis for more complex. This chapter is concerned with compressible flow with friction in constant-area, insulated ducts, which eliminate the effects of the area change and heat addition, the theory closely follows [3]. In a practical sense, these restrictions limit the applicability of the resultant analysis; however, certain problems such as flow in short ducts can be handled very well with a compressible flow. For these reasons, to study the effect of the friction on compressible flow in ducts, certain basic assumptions will be placed on the flow as follows:

1. Steady one-dimensional adiabatic flow.
2. Perfect gas with constant specific heats.
3. Constant-area straight duct.
4. Negligible shaft-work and potential-energy changes.
5. Wall shear stress correlated by a Darcy friction factor.

In effect, we are treating a Moody-type pipe-friction problem but with large changes in kinetic energy, enthalpy and pressure in the flow.

4.2 Adiabatic Flow

Consider the elemental duct control volume of area A and length dx in Figure 4.1

Figure 4.1: Control volume in a duct adapted from [3].

The area is constant, but other flow properties (p, ρ, T, h, V) may vary with x . Application of the three conservation laws to this control volume gives three differential equations:

Conservation-of-mass:

$$\rho V = \text{constant} \quad (4.1)$$

which, expressed in differential form, becomes,

$$\frac{dV}{V} + \frac{d\rho}{\rho} = 0 \quad (4.2)$$

Conservation-of-energy:

For adiabatic flow,

$$h + \frac{V^2}{2} = \text{constant} \quad (4.3)$$

Using the relationships between enthalpy and temperature for an ideal gas, $h = c_p \cdot T$ and $c_p = \frac{\gamma R}{\gamma - 1}$, the above energy equation in differential form as follows:

$$\frac{\gamma R dT}{\gamma - 1} + V dV = 0 \quad (4.4)$$

Conservation-of-momentum:

By applying the momentum equation to a control volume of length Δx contained in a duct, as shown in Figure 4.1. Equating the forces acting on the system to the net efflux of momentum from the control volume result in

$$A[p - (p + \Delta p)] - \tau C \Delta x = \rho V A(-V + V + \Delta V) \quad (4.5)$$

Where C is the wetted perimeter of the duct, τ is the shear stress at the wall.

Introducing the Darcy friction factor f for τ ,

$$\tau = \frac{f \rho V^2}{8} \quad (4.6)$$

and simplifying, the momentum equation in differential form as follows:

$$\rho V dV + dp + \frac{f \rho V^2 dx}{2D} = 0 \quad (4.7)$$

where D is the duct diameter.

It should be noted that for noncircular ducts D is replaced by the hydraulic diameter D_h as equation $D_h = \frac{4 \cdot \text{area}}{\text{wetted-perimeter}}$.

4.3 Mach-Number Distribution Along a Duct

By combining the foregoing conservation equations with the equation of gas state, equation (2.15), one gets a statement for the Mach number distribution along a duct.

Dividing each term in equation (4.7) by the pressure p , and realizing that

$$\frac{p}{\rho} = RT = \frac{a^2}{\gamma}$$

we obtain:

$$\gamma M^2 \frac{dV}{V} + \frac{dp}{p} + \frac{\gamma f M^2 dx}{2D} = 0 \quad (4.8)$$

The equation of state can be written in differential form:

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

Using the continuity equation, equation (4.2), to replace ρ by V and the energy equation, equation (4.4), to replace T by V yields

$$\frac{dp}{p} = -\frac{dV}{V} - (\gamma - 1) M^2 \frac{dV}{V} \quad (4.9)$$

which when substituted in the momentum equation, equation (4.8), results in

$$(M^2 - 1) \frac{dV}{V} + \frac{\gamma f M^2 dx}{2D} = 0 \quad (4.10)$$

Note that the Mach number is also defined as

$$M = \frac{V}{(\gamma RT)^{1/2}} \quad (4.11)$$

which can be written in differential form as follows:

$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2} \frac{dT}{T} \quad (4.12)$$

Again, using equation (4.4) to eliminate T yields

$$\frac{dM}{M} = \frac{dV}{V} \left[1 + \frac{(\gamma - 1)M^2}{2} \right] \quad (4.13)$$

Using this equation to eliminate V in equation (4.10) results in the following differential equation for the Mach number and distance:

$$\frac{(1 - M^2)dM}{M^3\{1 + [(\gamma - 1)/2]M^2\}} = \frac{\gamma f dx}{2D} \quad (4.14)$$

The resistance coefficient f can be related to the local Reynolds number and relative roughness of the duct from, say, the Moody chart. In practice, however, a constant average $\bar{f}$ is always assumed, i.e. that it does not change along the duct. Since the Reynolds number $R_e = \frac{\rho V D}{\mu}$, the continuity equation requires that the mass velocity ρV be constant along a duct transporting a compressible fluid because the area A is constant. Reynolds number R_e only changes due to the variation of μ with T . This is small for gases. Furthermore, in turbulent duct flow f is a weak function of Reynolds number R_e . Thus little error is incurred by using an appropriate mean value of f . Thus it is reasonable to assume when integrating equation (4.14) that f is a constant and equal to the average value, $\bar{f}$, in the duct.

The left-hand side of the equation (4.14) can be presented as a sum of partial

fractions to facilitate integration:

$$\left(\frac{1}{M^3} - \frac{\gamma+1}{2M} + \frac{(\gamma+1)(\gamma-1)M}{4\{1 + [(\gamma-1)/2]M^2\}} \right) dM = \frac{\gamma \bar{f} dx}{2D} \quad (4.15)$$

Integrating each side gives

$$\frac{-1}{2M^2} - \frac{\gamma+1}{2} \ln M + \frac{\gamma+1}{4} \ln \left(1 + \frac{\gamma-1}{2} M^2 \right) = \frac{\gamma \bar{f} x}{2D} + C \quad (4.16)$$

where C is the integrating constant. It is convenient to evaluate C by defining x^* as the distance corresponding to a Mach number of unity.

$$C = \frac{-\gamma \bar{f} x^*}{2D} - \frac{1}{2} + \frac{\gamma+1}{4} \ln \left(\frac{\gamma+1}{2} \right) \quad (4.17)$$

Substituting this statement for C into equation (4.16) results in

$$\frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} \ln \left[\frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2} \right] = \frac{\bar{f}(x^* - x_M)}{D} \quad (4.18)$$

where x_M is the distance corresponding to a Mach number M .

Thus we see that it is relatively easy to solve for distance along a duct once the Mach number is known. However, it is difficult to solve for the change in Mach number given a distance along the duct. For this reason, the solution method is recommended in this thesis by using an iterative approach.

4.4 Variation of Pressure with Distance

The differential equation for pressure as a function of velocity and Mach number is equation (4.9),

$$\frac{dp}{p} = \frac{-dV}{V} [1 + (\gamma - 1)M^2] \quad (4.19)$$

Using equation (4.13) we can get a differential statement for pressure as a function solely of Mach number:

$$\frac{dp}{p} = \frac{-dM}{M} \left\{ \frac{1 + (\gamma - 1)M^2}{1 + [(\gamma - 1)/2]M^2} \right\} \quad (4.20)$$

Carrying out the division of the factors contained in the brackets in equation (4.20) and dividing by Mach number M , we arrive at

$$\frac{dp}{p} = \left\{ -\frac{1}{M} - \frac{[(\gamma - 1)/2]M}{1 + [(\gamma - 1)/2]M^2} \right\} dM \quad (4.21)$$

Integrating each side, one gets

$$\ln p = -\ln M - \frac{1}{2} \ln \left(1 + \frac{\gamma - 1}{2} M^2 \right) + C \quad (4.22)$$

the constant of integration C can be evaluated by setting $p = p^*$ at $M = 1$, so we have

$$\ln p^* = -\frac{1}{2} \ln \left(\frac{\gamma + 1}{2} \right) + C \quad (4.23)$$

which, when substituted back into equation 4.22, gives

$$\frac{p_M}{p^*} = \frac{1}{M} \left[\frac{\gamma + 1}{2 + (\gamma - 1)M^2} \right]^{1/2} \quad (4.24)$$

where p_M is the pressure at a Mach number M .

4.5 Variation of Pressure and Mach Number Along a Duct vs. Inlet Static Pressure Increment

From equation (4.20) one can conclude that $dp/dM < 0$. This means that pressure decreases with increasing Mach number and increases with decreasing Mach number. Thus for subsonic flow in a duct, the pressure decreases with distance, the negative pressure gradient providing the force to overcome the wall shear force and accelerate the fluid.

Based on the compressible flow theory, let us consider how the pressure and Mach number vary along a duct that discharges to the atmosphere as the upstream static pressure is increased. Considering the qualitative distribution of static pressure and Mach number along the duct shown in Figure 4.2, the duct discharges to a pressure p_b . Case *A* through *D* represent a continuous increase in the upstream static pressure.

Case A: The static pressure is uniform along the duct and equal to p_b . Thus there is no flow in the duct, and the Mach number is everywhere zero.

Case B: The static pressure uniformly decreases to p_b , while the Mach number increases along the duct.

Case C: The static pressure decrease more rapidly and cause the flow to accelerate to a Mach number of unity at the exit. In this case $p_b = p^*$.

Figure 4.2: Distribution of static pressure and Mach number along a duct adapted from [3].

Case D: The static pressure distribution is nearly identical to that in case *C* but shifted to a higher value. Thus the duct discharges at a pressure higher than p_b , and pressure equilibration is achieved by a series of expansion waves. The Mach number distribution differs little from that for case *C*, the flow continuing to discharge at sonic speed.

4.6 Choking due to Friction

It should be noted that for adiabatic frictional flow in a constant-area duct, no matter what the inlet Mach number M_1 is, the flow downstream tends toward the sonic point. There is a certain duct length L^* for which the exit mach number will be exactly unity. If the actual duct length L is greater than the predicted “maximum” length L^* , then the flow conditions must change.

From equation (4.14), one also can find that if the flow is subsonic, then $dM/dx > 0$ and the Mach number increases with distance along the duct. Conversely, if the flow is supersonic, the $dM/dx < 0$ and the Mach number decrease along the duct. Thus the effect of the wall friction is always to cause the Mach number to approach unity, it is impossible for the Mach number of a compressible flow in a duct to change from subsonic to supersonic. Consequently, the maximum Mach number that an initial subsonic flow can attain is unity, and this can be reached only at the exit end of the duct.

Shock waves, of course, can occur in the duct to change an initially supersonic flow to a subsonic flow.

Chapter 5

Air Exhaust in Die Casting

5.1 Introduction

The air inside the die cavity must be removed during the filling and before solidification of the molten metal in order to avoid casting defects. To vent air in the die cavity, thin ducts, called air exhaust lines, are provided between the mating die halves. Exhaust lines are typically constant cross section rectangular ducts with 1 or 2 mm in height, 25 mm in width and roughly 20 cm in length. In pressure die casting, the molten metal is pushed into the gating system from outside the die by the plunger in the shot sleeve. Figure 5.1 shows the fluid flow system used in a die casting process to inject the molten metal poured into the shot sleeve into the die cavity.

Figure 5.1: A schematic showing the shot sleeve, runner, gate, die cavity and air venting in pressure die casting after Bar-Meir et al [13].

When the molten metal is injected into the shot sleeve, it does not fill the sleeve completely. The shot sleeve is usually less than half full of metal. Initially, the plunger moves slowly to vent the air in the sleeve as much as possible. During the fast shot, the remaining air gets pushed into the die cavity. The air compression in the die cavity could be significant. Moreover, this high pressure air may cause pores.

Gas flow in micro-channels [8] is an emerging technology for cooling of electronic circuits, small high frequency fluidic control systems, infrared detectors and diode lasers. However, gas flow in exhaust lines for die castings has received very little attention. To my knowledge, only Bennett and Karni [14] recognized the importance of such analysis. Therefore, air exhaust with adiabatic frictional duct flow in die

casting has been studied based on compressible flow theory in this chapter. Results from this analysis are used to relate volume of cavity air to stagnation pressure and temperature in chapter 6.

5.2 Modeling of Compressible Duct Flow in Die Casting

In die casting, since the molten metal filling period is very short, a fraction of a second, we assume that the air at a stagnation pressure of p_0 flows adiabatically through a duct. Adiabatic conditions prevail if the duct is short and the flow so rapid that heat interaction is negligible. Even though adiabatic conditions may exist, the entropy of the system still increases because of the friction with the viscous shear of molecules of the gas. The irreversibility associated with friction causes a decrease in the stagnation pressure p_0 , and this in turn affects flow properties.

Consider one-dimensional duct flow for analysis of gas flow through the exhaust line, Figure 5.2. The air within a piston-cylinder arrangement is subjected to the force imposed by a piston, where the piston pressure is the pressure of the molten metal. In summary, besides the above assumptions mentioned, a quasi-steady analysis of the flow properties such as mass flow rate and pressure distribution along the duct is calculated under the following assumptions:

Figure 5.2: A simplified model for air venting after Bar-Meir et al [13].

1. The air is modeled as a perfect gas with constant specific heats across the duct.
2. Steady one-dimensional adiabatic flow.
3. The duct length/width and length/height ratios are high and the duct area is constant. Therefore, the fluid flow variables are functions of x -direction only.
4. The pressure, temperature and density are assumed uniform within any cross-section of the duct.
5. Negligible shaft-work and potential-energy changes.
6. Wall shear stress is correlated by a Darcy friction factor.

5.3 Algorithms

Chapter 4.5 explained how the pressure and Mach number vary along a duct that discharges to the atmosphere as inlet static pressure is increased based on compressible flow theory. The four situations, case *A* through *D*, are also presented in figure 4.2 showing a continuous increase in the inlet static pressure.

The flow is calculated based on adiabatic flow for two different cases, choked conditions and unchoked conditions. That means the algorithms used to solve this problem depends on whether the flow exits sonically (case *D*) or subsonically (case *B*). If the flow exits subsonically, the mass flow rate must be such that the exit pressure is equal to the atmospheric pressure (back pressure p_b). On the other hand, if the flow exits sonically, the mass flow rate is determined by finding the pressure level that gives the correct resistance coefficient f in the duct.

In order to establish which approach to use, we first determine the inlet pressure corresponding to case *C* describing in chapter 4.5, where the flow exits sonically at atmospheric pressure (back pressure p_b), i.e. back pressure is exactly equal to pressure p^* ($M = 1$). If the inlet pressure is bigger than that corresponding to case *C*, then case *D* (sonic flow) should correspond to the problem, otherwise, case *B* (subsonic flow) should be adopted to solve the problem.

The flow chart of this algorithm is presented by Figure 5.3 as follows:

Figure 5.3: Flow chart of algorithm that distinguish choked and unchoked flow conditions.

For the case B (subsonic flow) and case D (sonic flow), the iterative approach is used based on compressible flow theory and their own situation.

For unchoked flow (case B), one shall assume an initial inlet Mach number, calculate the exit Mach number, and then determine the exit pressure. The initial inlet Mach number is varied until the exit pressure matches the atmosphere pressure (back pressure p_b).

The flow chart of algorithm for unchoked flow is presented by Figure 5.4 as follows:

Figure 5.4: Flow chart of algorithm in unchoked flow condition.

For choked flow (case D), an iterative approach is again used but now assuming average friction coefficient f . One begins by assuming an average friction coefficient f , calculating the inlet Mach number and Reynolds number, then finding a new f , which is used as the assumed value for the next iteration. The solution is the value at which f no longer changes. In the numerical method, the change in f is less than specified tolerance. A check is always made to be sure that exit pressure (at Mach number equal to one) exceeds backpressure p_b .

The flow chart of algorithm for choked flow is presented by Figure 5.5 as follows:

Figure 5.5: Flow chart of algorithm in choked flow condition.

5.4 Test Problem From [3]

In order to further verify the validations of mathematical model and algorithm developed in this thesis, a comparison of results calculated with the code developed in this section is made with existing data in [3].

The case used to compare is as follows: A duct is 8 m in length and 3 cm in diameter with relative roughness 0.00005 brass tube. The stagnation temperature of air flow in the tube is 300 K. The tube discharges to atmosphere, where the back pressure is 100 KPa. We need to calculate the mass flow rate in the tube (1) when the inlet static pressure is 120 KPa and (2) when it is 400K Pa.

When inlet static pressure is 120 KPa, flow is unchoked. The results from Roberson & Crowe [3] are compared in table 5.1.

$p_1 = 120$ (kPa)	M_1	M_2	p_2 (kPa)	T_2 (K)	f	ρ_2 (kg/s)	a_2 (m/s)	$\dot{m}$ (kg/s)
A	0.195	0.23	100	298	0.019	1.4	346	0.067
B	0.20	0.234	100	297.7	0.0186	1.376	345.79	0.0668

Table 5.1: Comparison of results between Roberson & Crowe and algorithm developed in unchoked condition.

When inlet static pressure is 400 KPa, flow is choked. The results from Roberson & Crowe [3] are compared in table 5.2.

$p_1 = 400$ (kPa)	M_1	M_2	p_2 (kPa)	T_2 (K)	f	ρ_2 (kg/s)	a_2 (m/s)	$\dot{m}$ (kg/s)
A	0.35	1	127	293	0.0135	4.77	342	0.403
B	0.352	0.9999	129.02	292.74	0.0137	4.76	342.9	0.406

Table 5.2: Comparison of results between Roberson & Crowe and algorithm developed in choked condition.

A is data from Roberson & Crowe, page 509 [3].

B is data computed by the code developed in this thesis.

From the above comparison it is clear that the results of the algorithm developed in the thesis compare well with the results published by Roberson and Crowe [3]. However, the thesis results appear to have higher precision.

5.5 Numerical Results and Discussion

In this section, the variations of a variety of properties of air exhaust, such as pressure and Mach number, along a duct that discharges to the atmosphere are tested as the upstream inlet static pressure is increased based on the computational model and algorithms developed.

To illustrate the above procedure and algorithms, for tests we choose the inlet static pressure and other test settings as follows:

For inlet static pressure, corresponding with case A to case D, we choose inlet static pressure as follows:

For case A: $p_1 = p_b = 100$ kPa.

For case B: $p_1 = 180$ kPa.

For case C: $p_1 = 273.55437347718625$ kPa. ($M=1$)

For case D: $p_1 = 310$ kPa.

For other settings, the initial conditions are as follows:

$\gamma = 1.4$. (Ratio of specific heat.)

$T_{01} = 300$ K. (Initial stagnation temperature.)

$p_b = 100$ kPa. (Back pressure.)

$M_0=0.31$. (Initial Mach number assumed.)

$dM=0.6$. (A factor used to compute an increment in the initial Mach number assumed.)

$k_s=0.0000015$. (Equivalent surface roughness for sand.)

$H = 0.002$ m. (Height of a duct.)

$W = 0.025$ m. (Width of a duct.)

$L = 0.2$ m. (Length of a duct.)

$D = \frac{2HW}{H+W}$ m. (Hydraulic diameter.)

$\epsilon = \frac{k_s}{D}$. (Relative surface roughness.)

5.5.1 Computational Results of Pressure and Mach Number Variation Along a Duct

Figure 5.6 and Figure 5.7 show the distribution of static pressure and Mach number along a duct when inlet static pressure is increased.

Figure 5.6: Distribution of static pressure along a duct.

Figure 5.7: Distribution of Mach number along a duct.

From the test results, one can find the test results match the theory prediction very well. For subsonical flow (case B), the exit pressure is equal to back pressure, while the Mach number increase along the duct. For sonic flow (case D), the static pressure distribution is nearly identical to that in Mach number of unity (case C) but shifted to a higher value. Thus the pipe discharges at a pressure higher than back pressure p_b , and pressure equilibration is achieved by a series of expansion waves. The Mach number distribution differs little from that for a Mach number of unity (case C). The flow continuing to discharge at sonic speed.

5.5.2 Computational Results of Other Flow Properties Variation Along a Duct

Figure 5.8 and Figure 5.9 illustrate the variations of static temperature and dynamic viscosity along a duct. It should be noted that the dynamic viscosity only changes due to the variation of temperature, but the gradient of variation of dynamic viscosity is not big, the maximum of change of the dynamic viscosity is only $2e-6$. This is roughly a 10% change.

Figure 5.8: Distribution of static temperature along a duct

Figure 5.9: Distribution of dynamic viscosity along a duct

Figure 5.10 illustrates the variations of Reynolds number along a duct. Considering the Reynolds number Re only changes due to the variation of dynamic viscosity with temperature for a constant area duct, it makes sense that the Reynolds number remains approximately constant along a duct.

Figure 5.10: Distribution of Reynolds number along a duct

Figure 5.11 illustrates the variation of friction factor along a duct. Just as we assumed before, although the friction factor f is related to the local Reynolds number and relative roughness of the duct, say the Moody chart, a constant average $\bar{f}$ is always assumed in practice, i.e. that it does not change along the duct.

Figure 5.11: Distribution of friction factor along a duct.

5.6 Vacuum Venting in Die Casting Process

5.6.1 Introduction

The mechanical and thermal properties of parts produced in pressure die casting are often compromised by high porosity. Gas/air porosity contributes a significant part of the total porosity. The most widely used casting method opens the vent to the atmosphere. It is referred to herein as air venting. When the results for air venting are poor other methods such as vacuum venting are used. Vacuum venting, a possible solution to reduce or eliminate this porosity, is applied and extract gas before it has the opportunity to mix with the liquid metal.

One of the differences between vacuum venting and air venting occurs at the start up time. For vacuum venting, a choking condition is established almost instantaneously, while in the air venting case the flow is not necessarily choked until late in the filling process. Once the flow is choked there is no difference in calculating the flow for the two cases.

The present work considers the influence of vacuum venting. It supposes the air from one cavity with a given stagnation pressure and temperature to another possibly under vacuum cavity with a given stagnation pressure and temperature where it could have pressure in the range of 1 atm to 0.1 atm.

5.6.2 The Model of Vacuum Venting

Traditionally, a large and expensive pump would be required to maintain the vacuum over the injection period. On the other hand, there is a long period of time between injections in which there is no need to apply a vacuum. This non-continuous demand suggests a system in which the pump is not directly connected to the die cavity. This arrangement provides an efficient pump use and a smaller pump is needed. A vacuum tank is maintained at almost constant pressure during the process. Figure 5.12 shows a schematic of a commercial die casting system with a vacuum tank. The following model is proposed to simplify the process. The unfilled shot sleeve, runner and die cavity are combined and called the cylinder. A piston moving the known velocity

pushes the liquid metal and both of them propel the air through a long duct. An equivalent long straight duct can be represented as the “zigzag” duct. The liquid metal can be considered part of the piston Figure 5.13.

Figure 5.12: A schematic of an actual venting in pressure die casting arrangement after Bar-Meir al [26].

It should be noted the mathematical model and algorithm for vacuum venting are based on that of air exhaust project.

Figure 5.13: A simplified model for air vacuum venting after Bar-Meir et al [26].

5.6.3 Simulation Results and Discussion of Vacuum Venting With the Change of Back Pressure

In this section, the variations of a variety of properties of air exhaust, such as pressure, temperature and Mach number, are tested with the change of back pressure p_b from 1 atm to 0.1 atm as the upstream inlet static pressure and temperature are constant.

To illustrate and simulate the above vacuum venting procedure and process in die casting, for tests we choose the following the inlet static pressure and other test settings:

$\gamma = 1.4$. (Ratio of specific heat.)

$T_{01} = 300 \text{ K}$. (Initial stagnation temperature.)

$p_{01}=150$ kPa. (Inlet stagnation pressure.)

$M_0=0.31$. (Initial Mach number assumed.)

$dM=0.6$. (A factor used to compute an increment in the initial Mach number assumed.)

$k_s=0.0000015$. (Equivalent surface roughness for sand.)

$H = 0.002$ m. (Height of a duct.)

$W = 0.025$ m. (Width of a duct.)

$L = 0.2$ m. (Length of a duct.)

$D = \frac{2HW}{H+W}$ m. (Hydraulic diameter.)

$\epsilon = \frac{k_s}{D}$. (Relative surface roughness.)

Figure 5.14 and Figure 5.15 exhibit the variations of outlet static pressure and temperature with the change of back pressure p_b from 1 atm to 0.1 atm. The flow is calculated based on adiabatic flow for two different cases, choked conditions and unchoked conditions. Because we assume the upstream inlet static pressure and temperature are constant, the outlet static pressure and temperature are maintained constant in the choked region at $p_b \leq 52000$ Pa, but one should note that the outlet static pressure shifts to a higher value than back pressure. It should be also noted

Figure 5.14: Variations of outlet static pressure against the change of back pressure.

that the outlet static pressure changes linearly with back pressure in unchoked region, and the outlet static pressure is exactly equal to back pressure p_b .

Figure 5.16 shows the change of outlet density against back pressure.

Figure 5.17 and Figure 5.18 describe the variations of the outlet Reynolds number and Mach number with the changes of back pressure. Just as we expect, the Mach number and Reynolds number remain constant in choked region and Mach number is exactly equal to unity.

Figure 5.19 describes the variations of velocity with the change of back pressure. Air exhaust velocity increases when back pressure decreases.

Finally, Figure 5.20 describes the variation of the mass flow rate with the change of

Figure 5.15: Variations of outlet static temperature against the change of back pressure.

back pressure. The mass flow rate reaches a maximum and remain constant in choked region. In unchoked region, the mass flow rate increases when the back pressure decreases.

It should make sense that the cavity air exhaust benefits from vacuum venting.

Figure 5.16: Variations of outlet air density against the change of back pressure.

Figure 5.17: Variations of outlet Reynolds number against the change of back pressure.

Figure 5.18: Variations of outlet Mach number against the change of back pressure.

Figure 5.19: Variations of velocity against the change of back pressure.

Figure 5.20: Variations of mass flow rate against the change of back pressure.

Chapter 6

Cavity Air in Die Casting

6.1 Introduction

When the plunger moves forward to fill the die cavities with molten metal, it also compresses the air in the die cavity itself. As we mentioned before, the cavity air project uses the mass flow rate or leak rate of air computed by the air exhaust project to calculate how much air is left when the cavity is filling by the molten metal, while the mass leak rate through the exhaust line depends on the stagnation pressure and temperature of the cavity air as well as the inlet Mach number and geometry of the exhaust line.

In this chapter, the mathematical model and algorithms are developed to determine the rate of change of mass and stagnation pressure and temperature of air in

die cavity which are connected to a piston cylinder as well as to a narrow air exhaust line.

The parameters such as stagnation pressure and temperature of the die cavity air are very important high level design parameters of die casting. These parameters can be used as boundary conditions for finite element analysis, such as temperature distribution and thermal stress analysis and so on.

For a cavity with a known volume at time zero or an initial time say t_0 with known stagnation temperature and pressure, the volume of the cavity and the mass leak rate from the cavity is known as a function of time. The stagnation pressure and temperature (and hence also density and total mass of air in the cavity) have been computed based on a computational model and algorithms in this chapter.

6.2 Modeling of Cavity Air in Die Casting

In die casting, air is trapped in the die cavity when the flow of the molten metal is poured in the mold by a moving plunger. A simplified model of cavity air in die casting is presented in Figure 6.1.

The molten metal moves with a known velocity. For each time step of the filling process analysis, the liquid velocity varies linearly. In the actual process, the velocity of the plunger that forces liquid metal into the die is not constant but increases rapidly

Figure 6.1: A simplified model for air cavity in die casting.

near the end of the piston stroke. Yet, during most of the piston stroke (=95 percent or more) the speed is almost constant [26]. Because the maximum speed of liquid metal is less than 60 m/s which is about Mach number equals to 0.2, the acoustic waves in the air cavity can be neglected. This means that the flow of air in the cavity, not in the air exhaust, can be treated as incompressible. Because the inertial forces in the air are small, air cavities can be treated quasi-statically. Figure 6.2 presents a typical velocity diagram after Kaye and Street (1982). The initial velocity is chosen to try to minimize air entrainment. The final increased velocity and increased pressure reduces the volume of the air that is already entrapped by compression.

For die cavity air, the following assumptions are made:

1. The air is modeled as a perfect gas with constant specific heats.
2. The stagnation pressure p_0 and temperature T_0 in the die cavity are uniform at any instant of time, i.e., there are no spatial gradients.

Figure 6.2: A typical piston velocity diagram in die casting after Kaye and Street [26].

3. Air in the die cavity is not heated from the molten metal poured.
4. The air compression in the die cavity is isentropic.
5. Negligible shaft-work and potential-energy changes.

A given die cavity is filled with air with volume $\mathcal{V}(t)$ as a known function of time. For each time step, $[t_1, t_2]$, the initial known data is volume $\mathcal{V}_1$, $\mathcal{V}_2$, air mass m_1 , stagnation pressure p_{01} and temperature T_{01} . We must calculate for left air mass m_2 , stagnation pressure p_{02} and temperature T_{02} at time t_2 .

Considering perfect air as the working fluid, the conservation of the mass in the

die cavity is:

$$\frac{dm}{dt} + \dot{m}_0(t) = 0 \quad (6.1)$$

The conservation of the energy in the die cavity is:

$$\dot{W} + \dot{H}_{out} + \dot{U} = \dot{H}_{in} + \dot{Q} \quad (6.2)$$

$\dot{H}_{in} = h(t)\dot{m}(t)$ is the rate change of the enthalpy due to gas flowing into the die cavity at time t . h is the specific enthalpy at time t . It is a surface integral. In this application, because no extra air flows into the die cavity, this term is zero.

$\dot{Q}(t)$ is the rate of heat flow into the die cavity at time t . Based on the above model assumption (3) and (4), neglected the energy lost due the gas escaping through the air exhaust, in this application, this term is also zero.

$\dot{W}(t)=p\dot{\mathcal{V}}$ is the work done on the air in the die cavity by external agents at time t .

$\dot{\mathcal{V}}$ is the rate of change of the air volume in the die cavity at time t .

$\dot{H}_{out}(t)$ is the rate of change of the enthalpy due to gas flowing out the die cavity at constant pressure through air exhaust at time t .

$\dot{U}(t)$ is the rate of change of the internal energy of air at constant volume in the die cavity at time t .

In terms of air mass flow rate and mass of the die cavity at state 1 and 2, the

conservation of energy equation (6.2) becomes

$$\frac{1}{\Delta t} \int_{\mathcal{V}_1}^{\mathcal{V}_2} p_0 dV + \dot{m}_0 h_0 + \frac{m_2 u_2 - m_1 u_1}{\Delta t} = 0 \quad (6.3)$$

Introducing equation (6.1) into equation (6.3), the result will be:

$$\frac{1}{\Delta t} \int_{\mathcal{V}_1}^{\mathcal{V}_2} p_0 dV + \dot{m}_0 c_p \frac{T_{02} + T_{01}}{2} + \frac{1}{\Delta t} [(m_1 - \dot{m}_0 \Delta t) T_{02} - m_1 T_{01}] c_v = 0 \quad (6.4)$$

It should be noted that the air mass flow rate $\dot{m}$ is defined to be positive for mass flowing out of the cavity. Recognizing that p_{01} and $\mathcal{V}_1$ are fixed known data for this time step, we use $p_0 \mathcal{V}^\gamma = p_{01} \mathcal{V}_1^\gamma$ to substitute for p_0 and integrate to obtain,

$$\frac{p_{01} \mathcal{V}_1^\gamma}{1 - \gamma} (\mathcal{V}_2^{1-\gamma} - \mathcal{V}_1^{1-\gamma}) + \dot{m}_0 c_p \Delta t \frac{T_{02} + T_{01}}{2} + [(m_1 - \dot{m}_0 \Delta t) T_{02} - m_1 T_{01}] c_v = 0 \quad (6.5)$$

Divide the above equation by c_v and use the equation $\gamma = c_p/c_v$. Define $\mathcal{A}$ which is a function only of known data for this time step as:

$$\mathcal{A} = \frac{1}{c_v} \frac{p_{01} \mathcal{V}_1^\gamma}{1 - \gamma} (\mathcal{V}_2^{1-\gamma} - \mathcal{V}_1^{1-\gamma}) \quad (6.6)$$

Rearrange the above equation

$$\mathcal{A} + \left[m_1 + \left(\frac{\gamma}{2} - 1 \right) \dot{m}_0 \Delta t \right] T_{02} - (m_1 - \frac{\gamma}{2} \dot{m}_0 \Delta t) T_{01} = 0 \quad (6.7)$$

Define $\mathcal{B}$ and $\mathcal{C}$ that are function only of known data for this time step,

$$\mathcal{B} = m_1 + \left(\frac{\gamma}{2} - 1 \right) \dot{m}_0 \Delta t \quad (6.8)$$

$$\mathcal{C} = m_1 - \frac{\gamma}{2} \dot{m}_0 \Delta t \quad (6.9)$$

Finally, equation (6.7) becomes

$$\mathcal{A} + \mathcal{B}T_{02} - \mathcal{C}T_{01} = 0 \quad (6.10)$$

Solve for T_{02}

$$T_{02} = \frac{\mathcal{C}T_{01} - \mathcal{A}}{\mathcal{B}} \quad (6.11)$$

Using the perfect gas law for the air at state 1 and 2 respectively, we will have

$$\begin{aligned} p_{01}\mathcal{V}_1 &= m_1RT_{01} \\ p_{02}\mathcal{V}_2 &= (m_1 - \dot{m}_0\Delta t)RT_{02} \end{aligned} \quad (6.12)$$

Solve for p_{02}

$$p_{02} = \frac{(m_1 - \dot{m}_0\Delta t)RT_{02}}{\mathcal{V}_2} \quad (6.13)$$

Solve for m_2

$$m_2 = m_1 - \dot{m}_0\Delta t \quad (6.14)$$

6.3 Algorithms

For each time step, $[t_1 \ t_2]$, the flow chart of the algorithm for choked flow is presented by Figure 6.3 as follows:

Figure 6.3: Flow chart of algorithm for cavity air state.

6.4 Results and Discussion

To illustrate the above procedure and algorithms, for tests we choose all test setting as follows: Initial conditions:

$\gamma = 1.4$. (Ratio of specific heat.)

$T_{01}=400$ K. (Initial stagnation temperature.)

$p_{01}=150$ kPa. (Initial stagnation pressure.)

$p_b=100$ kPa. (Back pressure.)

$M_0=0.31$. (Initial Mach number assumed.)

$dM=0.6$. (A factor used to compute an increment in the initial Mach number assumed.)

$k_s=0.0000015$. (Equivalent sand roughness.)

$dt=0.05$ (s). (Time increment.)

$H = 0.002$ m. (Height of a duct.)

$W = 0.025$ m. (Width of a duct.)

$L = 0.2$ m. (length of a duct.)

$D = \frac{2HW}{H+W}$ m. (Hydraulic diameter.)

$$\epsilon = \frac{k_s}{D}. \text{ (Relative surface roughness.)}$$

$$\mathcal{V}_1 = 1.0 \text{ m}^3. \text{ (Initial cavity volume.)}$$

$$\mathcal{V}_2(t) = \mathcal{V}_1 - k * t$$

where k is a scalar. In an actual die casting, the cavity volume would be determined by an integration of the volume of a connected VOF (volume of fluid) set.

Figure 6.4 describes the air mass remaining in a cavity as a function of the time.

The air mass left in the cavity decreases as the air exhaust time increases.

Figure 6.4: Variation of air mass in cavity with filling time.

Figure 6.5 shows the stagnation pressure varying with filling time. The stagnation pressure increases with molten metal filling the cavity. It should be noted the pressure

increases sharply at the end of filling.

Figure 6.5: Variation of stagnation pressure in cavity with filling time.

Figure 6.6 and 6.7 describes the variations of stagnation temperature and air density in cavity with filling time. It shows the stagnation temperature and density also increases when cavity volume decreases.

Figure 6.8 shows the air mass leakage flow rate increases with filling time increasing. Considering the stagnation pressure also increases with the filling time increasing, it should make sense that the air mass leakage flow rate should increase with stagnation pressure increasing.

Figure 6.9 describes the change of cavity volume with filling time. For test we assumed the velocity of molten metal filling cavity is constant, and choose to set

Figure 6.6: Variation of stagnation temperature in cavity with filling time.

Figure 6.7: Variation of density in cavity with filling time.

Figure 6.8: Variation of air mass flow rate in cavity with filling time.

$$\mathcal{V}_2(t) = \mathcal{V}_1 - 0.1 * t.$$

Figure 6.9: Variation of cavity volume with filling time.

Figure 6.10 and Figure 6.11 describe the change of air density and leakage flow rate with stagnation pressure.

Figure 6.10: Variation of air density with the stagnation pressure in the cavity.

Figure 6.12 illustrates the the residual air mass fraction with the cavity volume vary.

Figure 6.11: Variation of air mass flow rate with stagnation pressure.

Figure 6.12: Variation of the air mass in cavity with cavity volume.

Chapter 7

Conclusions

In this thesis, more accurate mathematical models and algorithms describing air exhaust and cavity air in die-casting manufacturing process are developed based on compressible flow theory and engineering thermodynamics. The numerical results from the air exhaust and cavity air projects match well with theoretical expectations. The results can be used to predict the quality and behavior of practical die-casting manufacture process in the design stage, allow modification and improvements before the design reaches the production stage.

In the air exhaust project, static pressure and Mach number along a duct that discharges to the atmosphere are tested as upstream inlet static pressure is increased. From the numerical results, one finds that the test results match the theoretical prediction very well. For subsonic flow, the exit pressure is equal to the back pressure,

and the Mach number increase along the duct. For sonic flow, the static pressure distribution is nearly identical to that with exit Mach number of unity but shifts to a higher value. Thus the duct discharges as a pressure higher than back pressure p_b , and pressure equilibration is achieved by a series of expansion waves. The Mach number distribution differs little from a Mach number of unity, continuing to discharge at sonic speed. Meanwhile, variations of the other flow properties of air exhausts, such as static temperature, dynamic viscosity and Reynolds number, along a duct that discharges to the atmosphere are tested as the upstream inlet static pressure is increased based on the computational model and algorithms developed.

In the cavity air project, high level design parameters, such as stagnation pressure and temperature (and hence also density and total mass of air) in die cavity, are tested based on mathematical models and algorithms. The rate and tendency of change of these design parameters in die cavity are described. From simulation results, one finds that the stagnation pressure and temperature in the cavity increase sharply as filling time increases. All these parameters can be used as initial conditions and boundary conditions for finite element analysis, such as temperature distribution and thermal stress analysis, in order to improve production quality and reduce scrap rates.

The present work also considers the influence of vacuum venting. All test results can be used as a valuable reference by designer in vacuum venting manufacturing process.

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Appendix

A complete listing of the code for the Air exhaust and Cavity Air projects is given in this appendix. The code was written in the TCL language. This is a scripting language that is very convenient for rapid prototyping software.

```
#!/mrco/release/bin/MRCOShell4
```

```
proc FrictionDistance {M1} {
```

```
    global k
```

```
    set Y [expr ( (1.0 - $M1*$M1)/($k*$M1*$M1) + (($k  
+ 1.0)/(2.0*$k)) * (log((( $k + 1.0)*$M1*$M1)/(2.0 +  
($k - 1.0)*$M1*$M1))))]
```

```
    return $Y
```

```
}
```

```
proc FrictionFactor {Re} {
```

```
    global Roughness
```

```
    if {$Re <= 2200} {
```

```
set f [expr 64.0/$Re]
```

```
    } else {
```

```
set X1 6.0
```

```
set dX1 [expr(1.14-2.0*log10($Roughness+9.35*$X1/($Re))- $X1)]
```

```

set A [expr abs($dX1/$X1)]

while {$A > 0.000001} {

    set dX1 [expr(1.14-2.0*log10($Roughness+9.35*$X1/($Re))-X1)]

    set X1  [expr ($X1+$dX1)]

    set A    [expr abs($dX1/$X1)]

}

set f [expr (1.0/($X1*$X1))]

#puts "f $f"

}

return $f

}

proc DensityGas {P T} {

    global R

    set Density [expr ($P/($R*$T))]

    return $Density

}

proc ReynoldNo {V Density u} {

    global D

    set Re [expr (($Density*$V*$D)/$u)]

    return $Re

}

```

```

proc SoundSpeed {T} {

    global R k

    set Ce [expr (sqrt($k*$R*$T))]

    return $Ce

}

proc MassFlowRate {M C Density} {

    global XArea

    set mdot [expr ($M*$C*$Density*$XArea)]

    return $mdot

}

proc PressureRatio {M_1 M_0} {

    global k

    set P_M1 [expr((1.0/$M_1)*sqrt(($k+1.0)/(2.0+($k-1)*$M_1*$M_1)))]
    set P_M0 [expr((1.0/$M_0)*sqrt(($k+1.0)/(2.0+($k-1)*$M_0*$M_0)))]
    set Ratio [expr ($P_M1/$P_M0)]

    return $Ratio

}

proc PressureRatio_new {M_out M_in} {

    set Ratio [expr($M_in/$M_out)/(pow(1+0.2*$M_in*$M_in,
3)*pow(1+0.2*$M_out*$M_out, 0.5))]

    return $Ratio

}

```

```

}

proc PressureRatio1 {M_1} {

    global k

    set Ratio [expr((1/$M_1)*sqrt(($k+1)/(2+($k-1)*$M_1*$M_1)))]

    return $Ratio

}

proc InletStaticPressure {M_in M_e} {

    global Pb k

    set In_Ratio [expr((1.0/$M_in)*sqrt(($k+1)/(2+($k-1)*$M_in*$M_in)))]

    set Exit_Ratio[expr((1.0/$M_e)*sqrt(($k+1)/(2+($k-1)*$M_e*$M_e)))]

    set Ratio [expr ($Pb*$In_Ratio/$Exit_Ratio)]

    return $Ratio

}

#proc Inverse_FrictionDistance { FD_0 k M}

#M is a trial value or guess of Mach No.

#If M1 = 0.3 gives no trouble, then leave the code as is.

proc Reverse_FrictionDistance {FD_0} {

    global k

    set M1 0.31

    set A [expr

        ($FD_0+(1/$k)-(($k+1.0)/(2.0*$k))*log((($k+1.0)*$M1*$M1)

```

```

/(2.0+($k-1.0)*$M1*$M1)))]]

set dM1 [expr (1.0/(2.0*$k*$A*$M1) -$M1/2.0)]

set B [expr abs($dM1/$M1)]

while {$B > 0.0000001 } {

set A [expr

($FD_0+(1.0/$k)-(($k+1.0)/(2.0*$k))*log((($k+1.0)*$M1*$M1)

/(2.0+($k-1)*$M1*$M1)))]]

set dM1 [expr (1.0/(2.0*$k*$A*$M1) -$M1/2.0)]

set M1 [expr ($M1+$dM1)]

set B [expr abs($dM1/$M1)]

}

return $M1

}

proc StaticTemp {T0 M} {

global k

set T [expr ($T0/(1.0+$M*$M*($k-1.0)/2.0))]

return $T

}

proc Viscosity {T} {

set u [expr(1.4805e-14*$T*$T*$T-4.2616e-11*$T*$T+6.85e-8*$T+1.36055e-6)]

return $u

```

```

}

proc StaicPressureMP0 {P0 M} {

    global k

    set kf [expr ($k-1.0)/2]

    set AkM [expr (1.0+$kf*$M*$M)]

    set P_static [expr ($P0/(pow($AkM,$kf)))]

    return $P_static

}

proc NonChokedFlow {M P0 T0} {

    puts "\n\tNonChokedFlow\n"

    global L D

    puts "M $M"

    set T_static [StaticTemp $T0 $M]

    puts "Tstatic=$T_static"

    set P_static [StaicPressureMP0 $P0 $M]

    puts "P_static=$P_static"

    set C_M [SoundSpeed $T_static]

    #puts "C_M $C_M"

    set Density_Static [DensityGas $P_static $T_static]

    #puts "Density_Static $Density_Static"

    set V [expr ($M*$C_M)]

```



```

puts "V $V"

set u [Viscosity $T_static]

puts "u $u"

set Re [ReynoldNo $V $Density_Static $u]

puts "Re $Re"

set f_Re [FrictionFactor $Re]

puts "f_Re $f_Re, f_Re/D [expr $f_Re/$D]"

set FD_0 [FrictionDistance $M]

puts "FD_0 $FD_0"

set xxx [expr ($FD_0*$D/$f_Re)]

puts "xxx=$xxx"

set FD_1 [expr ($FD_0-$L*$f_Re/$D)]

puts "FD_1 $FD_1"

if {$FD_1 < 0} {
return [list -1 -1 -1 -1 -1]

}

set M_e [Reverse_FrictionDistance $FD_1]

puts "M_e $M_e"

set Te [StaticTemp $T0 $M_e]

puts "Te $Te"

set PressureRatio [PressureRatio $M_e $M]

```

```

puts "PressureRatio=$PressureRatio"

set P_e      [expr ($P0*$PressureRatio)]

puts "P0/P_e = [expr $P0/$P_e]"

puts "P_e = $P_e"

set Density_e [DensityGas $P_e $Te]

#set M_0 $M_1 to interpolate for next iteration.

set M_inlet $M

set M_exit $M_e

return [list $P_e $M_exit $Te $Re $Density_e]
}

proc AB {P0 T0 f_0} {

    global D L

    puts "f_0 $f_0"

    set FD_0      [expr ($f_0* $L/$D)]

    puts "FD_0 $FD_0"

    set M_0      [Reverse_FrictionDistance $FD_0]

    puts "M_0 $M_0"

    set FD_1      [expr ($FD_0-$L*$f_0/$D)]

    puts "FD_1 $FD_1"

    set M_1      [Reverse_FrictionDistance $FD_1]

    puts "M_1 $M_1"

```

```

set Ratio [PressureRatio $M_1 $M_0 ]

puts "Ratio $Ratio"

set P_e [expr ($Ratio*$P0)]

puts "P_e $P_e"

set Tstatic [StaticTemp $T0 $M_0]

set T_e [StaticTemp $T0 $M_1]

puts "Tstatic $Tstatic"

set Density [DensityGas $P0 $Tstatic]

puts "Density $Density"

set Density_e [DensityGas $P_e $T_e]

set C [SoundSpeed $Tstatic]

puts "C $C"

set V0 [expr $M_0*$C]

puts "V0 $V0"

set mu [Viscosity $Tstatic]

set Re [ReynoldNo $V0 $Density $mu]

puts "Re $Re"

set f_1 [FrictionFactor $Re]

puts "f_1 $f_1"

return [list $f_1 $M_0 $M_1 $P_e $T_e $C $Density $Re $Density_e]
}

```

```

proc ChokedFlow {P0 T0} {
    global iterationLimit

    puts "\n\tChokedFlow\n"

    set f1 0.0

    set f2 0.02

    set cnt 1

    while {(abs($f1 - $f2)) > 0.000001 && $cnt <=$iterationLimit} {
set f1 $f2

foreach {f2 Mi Me Pe Te C Density Re Density_e} [AB $P0 $T0 $f1] {}

puts ""

incr cnt

    }

    if {$cnt >= $iterationLimit} {

puts "\ncnt out of limit! Quit!\n"

return [list -1 -1 -1 -1 -1]

    }

    set mdot [MassFlowRate $Mi $C $Density]

    puts ""

    #puts "mdot=$mdot"

    return [list $mdot $Mi $Me $Pe $Re $Te $Density $Density_e ]

}

```

```

proc IsFlowChoked {T0 P0 } {

    global k R D Roughness L Pb

    set Te [expr ($T0*2/($k+1))]

    set Ce [SoundSpeed $Te]

    set Density [DensityGas $Pb $Te]

    set mu [Viscosity $Te]

    set Re [ReynoldNo $Ce $Density $mu]

    set f [FrictionFactor $Re]

    set FD_0 [expr ($f*$L/$D)]

    set M_1 [Reverse_FrictionDistance $FD_0]

    set Ratio [PressureRatio1 $M_1]

    set P_static [expr $Ratio*$Pb]

    puts "P_static $P_static"

    set P_in_static [StaicPressureMPO $P0 $M_1]

    if {$P_in_static < $P_static} {

return 0

    } else {

return 1

    }

}

#####

```

```

#set filePlotMDOTvsPb [open "PlotMDOTvsPb.dat"

#set filePlotMevsPb [open "PlotMevsPb.dat" w]

#set fileCheckAli [open "CheckAli.dat" w]

#set filePlotPevsPb [open "PlotPevsPb.dat" w]

#set filePlotTevsPb [open "PlotTevsPb.dat" w]

#set filePlotDensityvsP0 [open "PlotDensityvsP0.dat" w]

#foreach P0 {1e6 1e7 1e8 1e9}

#for {set P0 120000} {$P0 < 500000} {incr P0 2000} {}

#for {set i 10} {$i<120} {incr i} {}

#set Pb [expr 120000-$i*1000]

#for {set Pb 100} {$Pb < 120000} {incr Pb 10000} {}

#foreach P0 {105000 110000 120000 140000 160000 180000
200000 250000 300000}

proc AirExhaust {P1 T1 M0} {

    # puts "\nStart AirExhaust P1 = $P1, T1 = $T1, M0= $M0\n"

    global k R D Roughness L tol iterationLimit XArea dM0 Pb

    set P0 $P1

    set T0 $T1

    if [IsFlowChoked $T0 $P0] {

set flagChoked "Choked-1"

foreach {mdot Mi Me Pe Re Te Density Density_e }

```

```

[ChokedFlow $P0 $T0 ] {}

} else {

#We guess two values of inlet velocity M1A and M1B.

set Mi $M0

set M1A $M0

set dM $dM0

#puts "Mi=$Mi"

#foreach {PeA n n n n} [NonChokedFlow $M1A $P0 $T0] {}

#puts "case 1"

foreach {PeA Me Te Re Density_e} [NonChokedFlow $M1A $P0 $T0] {}

#puts "Me=$Me"

#break

# find a proper initial guess M1B

set counter 1

set Pe $PeA

while {[expr abs($Pe - $Pb)/$Pb] > $tol && $counter<= $iterationLimit} {

    #puts "\n find iter $counter \n"

    if {$Pe == -1} {

set dM [expr $dM/2]

#puts "dM $dM"

set Mi [expr $Mi - $dM]

```

```

#puts "case 2"

foreach {Pe Me Te Re Density_e} [NonChokedFlow $Mi $PO $TO] {}

#puts "Me = $Me"

#puts "Me=$Me"

#break

    } elseif {$Pe < $Pb} {

break

set dM [expr $dM/2]

#puts "dM $dM"

set Mi [expr $Mi - $dM]

#puts "case 3"

foreach {Pe Me Te Re Density_e} [NonChokedFlow $Mi $PO $TO] {}

puts "Me = $Me"

    } else {

set Mi [expr $Mi + $dM]

#puts "case 4"

foreach {Pe Me Te Re Density_e} [NonChokedFlow $Mi $PO $TO] {}

#puts "Me = $Me"

    }

    incr counter

}

```



```

if {$counter >= $iterationLimit} {

    # choked flow

    set flagChoked "Choked-2"

    foreach {mdot Mi Me Pe Re Te Density Density_e} [ChokedFlow $P0 $T0 ] {}

} else {

    set flagChoked "NonChoked"

    #set M1B [expr $Mi+0.1]

    set M1B $Mi

    #set PeB [expr ($Pe+1)]

    set PeB $Pe

    set dPRel 1

    set dMiRel 1

    while {$dPRel > $tol && $dMiRel > $tol} {

#puts "\niter $counter \n"

#puts "detM=$M1A-$M1B"

set m [expr (($PeA - $PeB)/($M1A-$M1B))]

set b [expr ($PeA-$Pb - $m*$M1A)]

set Mi [expr ((-$b)/$m)]

#puts "m=$m"

foreach {Pe Me Te Re Density_e} [NonChokedFlow $Mi $P0 $T0] {}

#puts "Me = $Me"

```

```

#puts "M1A $M1A, M1B $M1B, Mi $Mi"

#puts "PeA $PeA, PeB $PeB, Pe $Pe"

set dMiRel [expr abs($Mi - $M1A)/$M1A]

if {[expr ($Pe - $Pb)*($PeA - $Pb)] < 0} {

    set M1B $Mi

    set PeB $Pe

} else {

    set M1A $Mi

    set PeA $Pe

}

set dPRel [expr abs($Pe - $Pb)/$Pb]

incr counter

}

set Density0 [DensityGas $P0 $T0]

set C0 [SoundSpeed $T0]

set mdot [MassFlowRate $Mi $C0 $Density0]

set Me $Me

#puts "case1"

#puts "Me=$Me"

#puts $filePlotMeVsP0 "$P0 $Me"

#puts $filePlotP0vsPe "$P0 $Pe"

```

```

}

}

#puts $filePlotMDOTvsPb "$Pb $mdot $flagChoked"

#puts $filePlotMevsPb "$Pb $Me"

#puts $filePlotPevsPb "$Pb $Pe"

#puts $filePlotTevsPb "$Pb $Te"

#puts $filePlotDensityvsP0 "$P0 $Density_e"

#puts "\nmdot=$mdot"

#puts "\nMe=$Me"

return [list $mdot $Me]

#puts "DONE!"

}

#}

*****

#The following data define the problem will not
change in the problem.

set tcl_precision 17

set tol 0.0000000001

set iterationLimit 100

set k 1.4

set R 287.0

```

```

set dt    0.1

set P1    150000.0

set T1    400.0

set Cv    [expr ($R/($k-1))]

set Cp    [expr ($k*$R/($k-1))]

set k     [expr ($Cp/$Cv)]

set M0    0.31

set dM0   0.6

set Pb    100000.0

set Ks    0.0000015

if 0 {

    #D is diameter of a round pipe but must be computed for a
    rectangular pipe.

    set D    0.03

    set L    8.0

    set Roughness [expr ($Ks/$D)]

    set XArea [expr (0.785*$D*$D)]

    set P0 $P1

    set T0 $T1

} else {

    # Ali's test

```

```

    set H 0.002

    set W 0.025

    set D [expr 2*$H*$W/($H + $W)]

    set L 0.2

    set Roughness [expr ($Ks/$D)]

    set XArea [expr $W*$H]

    set P0 $P1

    set T0 $T1

}

if 0 {

    set P0 120000.0

    set file [open "plot.dat" w]

    for {set i 100} {$i < 290} {incr i} {

set Mi [expr $i*0.001]

puts "Mi $Mi"

puts "case 6"

foreach {Pe Me Te Re Density_e} [NonChokedFlow $Mi $P0 $T0] {}

puts "Me = $Me"

puts $file "$Mi $Pe"

    }

    close $file

```

```

}

*****

# the follow part is for Air Cavity

#set PlotMdot_1vst2 [open "PlotMdot_1vst2.dat" w]

#set filePlotP1vst2      [open "PlotP1vst2.dat" w]

set filePlotMdot_1vsts  [open "MassFlowRatevsTime.dat" w]

set filePlotT1vsts      [open "StagTempvsTime.dat" w]

set filePlotm1vsts      [open "AirMassvsTime.dat" w]

set filePlotV1vsts      [open "CavityVolvsTime.dat" w]

set filePlotP1vsts      [open "StagPresvsTime.dat" w]

set filePlotDensityvsts [open "AirDensityvsTime.dat" w]

set filePlotMdot_1vsP1  [open "MassFlowRatevsStagPres.dat" w]

set filePlotm1vsV1      [open "AirMassvsCavityVol.dat" w]

set filePlotDensityvsP1 [open "AirDensityvsStagPres.dat" w]

#set filePlotconstvsts  [open "Plotconstvsts.dat" w]

set dt    0.05

set V1    1.0

set t1    0.0

set counter 1

set m1 [expr ($P1*$V1/($R*$T1))]

for {set ts 0} {$ts < 20 } {incr ts} {

```

```

puts "\n find time_step $ts \n"

#set t [expr 0.999999*$ts]

#set t1 [expr ($ts*28*1.7*$dt)]

#set t2 [expr ($t1+2000*$dt)]

set V2 [expr (1.0 - $dt*$ts)]

#set V2 $V1

#puts "t1=$t1"

#puts "t2=$t2"

set Density [expr ($P1/($R*$T1))]

foreach {Mdot_1 Me} [AirExhaust $P1 $T1 $M0] {}

#set Mdot_1 0.0

#puts "M0=$M0"

#puts "M_e=$M_e"

#set m1 [expr ($P1*$V1/($R*$T1))]

set A [expr((1.0/$Cv)*($P1*(pow($V1,$k)))/(1-$k)

*(pow($V2,(1-$k))-pow($V1,(1-$k))))]

puts "A=$A"

set B [expr ($m1+(0.5*$k-1)*$Mdot_1*$dt)]

set C [expr ($m1-0.5*$k*$Mdot_1*$dt)]

set T1 [expr (($C*$T1-$A)/$B)]

set P1 [expr (((m1-$Mdot_1*$dt)*$R*$T1)/$V2)]

```

```

#set Density [expr ($P1/($R*$T1))]

set const [expr $P1*(pow($V2,1.4))]

set m1 [expr ($m1-$Mdot_1*$ts*$dt)]

#puts ""

#puts "const=$const"

puts "t1=$t1"

#puts "t2=$t2"

puts "P1=$P1"

puts "V2=$V2"

puts "T1=$T1"

puts "Mdot_1=$Mdot_1"

#set t1 $t2

set V1 $V2

puts "ts $ts"

#puts $filePlotMdot_1vst2 "$t2 $Mdot_1"

#puts $filePlotP1vst2 "$t2 $P1"

puts $filePlotMdot_1vsts "$ts $Mdot_1"

puts $filePlotT1vsts "$ts $T1"

puts $filePlotm1vsts "$ts $m1"

puts $filePlotV1vsts "$ts $V1"

puts $filePlotP1vsts "$ts $P1"

```



```

    puts $filePlotDensityvsts "$ts $Density"

    puts $filePlotMdot_1vsP1 "$P1 $Mdot_1"

    puts $filePlotm1vsV1 "$V1 $m1"

    puts $filePlotDensityvsP1 "$P1 $Density"

    #puts $filePlotconstvsts "$ts $const"

    set MO $Me

    #puts "MO_1=$MO"

    #break

    incr counter
}

#close $filePlotMdot_1vst2

#close $filePlotP1vst2

close $filePlotMdot_1vsts

close $filePlotT1vsts

close $filePlotm1vsts

close $filePlotV1vsts

close $filePlotP1vsts

close $filePlotDensityvsts

close $filePlotDensityvsP1

close $filePlotMdot_1vsP1

close $filePlotm1vsV1

```

```
#close $filePlotconstvsts
```

```
puts "DONE!"
```