**THE DESIGN OF A NOVEL ACCELEROMETER
FOR THE WEIGH-IN-MOTION APPLICATION**

By

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A Thesis submitted to the
School of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering

Ottawa Carleton Institute for Mechanical and Aerospace Engineering
Department of Mechanical and Aerospace Engineering
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ABSTRACT

The design of a novel accelerometer is presented, beginning with the definition of the need for such a device, and following through the detail design analysis, and resulting in the production of a prototype device. The application of the novel accelerometer is directed at the weigh-in-motion system. It is identified that such an application requires that the accelerometer have an especially low cross axis sensitivity, and the novel design attempts to meet this requirement. The thesis also explores the principles of precision instrument design by giving consideration to the influences of material properties and behaviour, as well as thermal sensitivities. The detail design analyses are comprehensive: covering geometrical optimization, electromagnetic optimization, dynamic feedback system analysis and compensation, heat transfer and, electronic analog circuit configuration. A set of performance tests are specified, to be used to evaluate the performance of the prototype device.

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This thesis is dedicated to anyone who has figured out that Design is at the top of the evolutionary chain of Engineering; not at the bottom.

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LIST OF SYMBOLS

The following list of symbols has been generalized in that subscripts have been omitted from symbols that have similar specific interpretations with several subscripts. Specific descriptions of all symbols, however, may be found in the text where they are first introduced. Where a symbol below has been indicated to have a vector interpretation, the symbol will appear with an over-arrow in the text when applicable.

α	1) angular acceleration 2) thermal coefficient of expansion 3) misalignment angle
β	misalignment angle
δ	1) spring deflection 2) misalignment angle
δx	variational quantity of x
Δx	change in x
ε	strain
γ	misalignment angle
θ	angle
μ	material relative magnetic permeability
μ_o	magnetic permeability of free space
ρ	material mass density
σ	component stress
σ_{el}	material elastic limit
τ	1) component shear stress 2) time constant
ω	1) circular frequency 2) angular speed
ψ	magnetic scalar potential
ζ	damping ratio
a	1) acceleration (and vector, where applicable) 2) diaphragm major diameter
A	1) amplitude of vibration 2) surface area 3) apparent input to accelerometer
A_g	air gap nominal area
b	1) Burger's vector 2) diaphragm minor diameter 3) damping coefficient
B	magnetic field strength
c	material heat capacity
C	capacitance

d	1) smallest displayed reading of a scale 2) distance 3) wire diameter
D	cylinder diameter
$D_c(s)$	lead compensator transmittance in Laplace domain
e	point voltage potential
E	material modulus of elasticity
F	force (and vector, where applicable)
g	gravitational field strength (and vector, where applicable)
G	material shear modulus
h	1) bulk heat convection coefficient 2) foil thickness
H	cylinder height
HA	hinge axis
H_m	magnetic field intensity
$H(s)$	Laplace domain transfer function
i	electrical current
IA	input axis
k	1) material thermal conductivity 2) spring stiffness
K	1) magnetic circuit loss factor 2) gain factor
K_{ix}	cross coupling of x axis to input axis
l	length
L	1) pinning point separation 2) length
L_{em}	electrical winding inductance
m	mass (both gravitational and inertial, which are assumed equal)
M	moment of force (and vector, where applicable)
n	number of parallel coil wires
N	number of turns in a coil
p	abscissa value of lead network pole
P	electrical power
PA	pendulous axis
q	heat energy flow rate
r	radial dimension
R	electrical resistance
R_g	air gap magnetic reluctance
s	Laplace domain variable
S_T	cross axis sensitivity
S_{ut}	material ultimate tensile strength
S_y	material yield strength
t	1) time 2) thickness
T	temperature

$T(s)$	Laplace domain closed loop transfer function
v	speed (and velocity vector, where applicable)
V	1) volume 2) electrical voltage potential
W	gravitational weight
x	displacement
$\dot{x}$	first time derivative of displacement (speed)
$\ddot{x}$	second time derivative of displacement (acceleration)
z	abscissa value of lead network zero

CHAPTER 1

INTRODUCTION

Canadian society has witnessed, on occasion, the subject of “garbage” brought to the forefront of our attention. It seems that barely a day goes by when one is not inundated with symbols of recycling, re-use and, reduction. And, it seems that barely a week goes by without yet another debate about this-or-that landfill site. In fact, the question: “What will we do with all of the commercial and residential refuse that we generate as a society?” is nagging most first world countries. Any useful technology that addresses the refuse processing task has a huge market potential throughout North America, Europe, and, Industrialized Asia. The processing of refuse implies, for the most part, collecting and landfilling it. Of course landfilling in Canada, with its huge land area, should hardly be an issue at all; however, our contemporary society has demanded that we seek “greener” alternatives whenever possible.

Refuse processing is a basic service that should not be abused or otherwise taken for granted. One method to ensure that a society respects the privilege of this service is to

operate it on a "user-pay" basis. The refuse collection industry is quickly moving in the direction whereby customers will pay, per unit mass, to process (i.e. landfill) their own refuse. The user-pay system has already been partially implemented on a non-certified basis in the commercial refuse collection industry. The term, "non-certified", implies that the collection company monitors the refuse weight with a machine not certified by Weights & Measures (Canada)[†]. The collector (e.g. Laidlaw) can then offer each client a contract renewal based on the knowledge of the average weight of refuse that the client produces. Should the client not wish to accept the contract offer, then the collector has no obligation to continue processing their refuse.

Such a system, as described above, is useful because the collection company must pay on a (certified) per-unit-mass basis to deposit the collected refuse at a landfill site and therefore, the value of the service provided to the client depends upon the mass of that client's refuse. And, in the last few years, a trend in the landfilling costs has become apparent, as illustrated with Figure 1.1.

Figure 1.1: Landfilling costs in Canada predicted to year 2000. Note that the rate of cost increase is growing rapidly, with the potential to explode, (as seen in 1994). Data from [1].

[†] Weights & Measures is the government agency responsible for such services as standardizing accuracy requirements for weigh scales.

It would appear from the figure that the landfilling costs are exponentially increasing. Therefore, common sense asserts that the collection industry adopts a certified per-unit-mass charging system, if it is to remain economically viable. Presently, there is no weigh-in-motion system that meets the Weights & Measures standards required to be certified as a "scale".

Of more interest to most readers is that a user-pay system will, eventually, also be required for residential refuse collection. In fact, despite the increasing level of municipal taxes, user-pay is probably inevitable. Fortunately, such a system is publicly fair, since the consumer personally makes the decisions that lead to how much he pays, vis-a-vis how much refuse he sets out at his curbside. Some municipalities have already addressed the issue with pilot projects. Some projects weigh refuse with non-certified scales and produce a mock invoice for each resident. Other projects require residents to purchase "tags" for each bag of refuse that they deposit on the curbside for collection.

Of course, there always is opposition to such user-pay projects, particularly in highly taxed regions, but economics dictate that user-pay be adopted. Ultimately, a certified, per-unit-mass user-pay system will be implemented by every major municipal centre, probably within the next 20 years. This is a direct consequence of the increasing landfilling costs, as illustrated by Figure 1.1.

1.1 WEIGH IN MOTION

One of the major obstacles to developing a user-pay system is the fact that the measurement of the weight of the customer's refuse must be incorporated into the regular collection cycle. It has been found that a commercial refuse collection cycle presently lasts about 20 seconds [1]. And, it has also been estimated that each one-second increase in the cycle time will result in an additional cost to the collection company of \$10,000 per vehicle per annum [1]. A similar situation exists for the residential refuse collection industry.

Apparently then, not only does the refuse weigh-scale for a user-pay system have to be mobile, but it must also be a "weigh-in-motion" (WIM) scale. By this, it is meant that the scale must operate as part of the cycle of depositing the refuse into the vehicle. And, the operation of the scale must be invisible to the collection workers. Any system that requires the collection worker to place the refuse on a static scale, wait for the scale to settle, and then proceed has been shown to be non-viable [1].

The scale must obtain the weight of the refuse, and its container, during the half-cycle represented by lifting the container and depositing the refuse into the vehicle. Then, the weight of the empty container is obtained by the scale during the latter half-cycle represented by lowering the container and detaching it from the vehicle. The weight of the refuse is calculated by finding the difference between these two measures. Canadian Weights & Measures has set out a specification on the accuracy required for certification for this type of scale. It is summarized as follows [2]:

$$\text{max error} = \text{greater of} \left[\begin{array}{l} \text{lesser of} \left[\begin{array}{l} 0.05\% \text{ max capacity} \\ \text{or } d \end{array} \right] \\ \text{or } 0.1125\% \text{ of load} + \frac{d}{2} \end{array} \right] \quad (1.1)$$

In this formula, d , is the smallest division recorded by the scale. For example, suppose the scale has a maximum capacity of 227kg (500lb_m), and has $d=0.1$ kg, then the following table may be drawn up:

Table 1.1: Weights & Measures error limits for a 227kg scale, $d=0.1$ kg. Note that the fractional error limits are challenging at best.

LOAD (kg)	ERROR LIMIT (kg)	FRACTIONAL ERROR (%)
50	0.110	0.220
100	0.165	0.165
150	0.220	0.142
200	0.275	0.138

On top of this, the certification procedure includes severe environmental testing, tilt testing, and even a test whereby the centre of mass of the refuse must shift within the container during the lifting cycle. To produce a certifiable WIM scale is, obviously, not a trivial task.

1.2 ACCELERATION AND WEIGH-IN-MOTION

In its simplest form, a WIM scale is represented by a free body diagram of the refuse mass. We treat the mass as a particle moving vertically upwards, as illustrated with Figure 1.2.

Figure 1.2: Free body diagram of refuse mass carried by WIM scale. The mass is being raised vertically by the scale. The symbols are defined in the text.

In the figure, the following terminology applies:

F = support force measured by the WIM scale;	m = actual mass of refuse;
a = vertical acceleration of refuse;	v = vertical velocity of refuse;
W = weight of refuse ($W=mg$);	g = gravitational field strength;

We see immediately that the support force, F , is given by:

$$F = m(a + g) \quad (1.2)$$

The force, F , is measurable by our WIM scale, and a value of "g" might be known to good accuracy. If we assume that the acceleration, a , is small (i.e., we don't attempt to measure it), then we calculate the apparent mass, m_{app} , of the refuse, to be:

$$m_{app} = \frac{F}{g} \quad (1.3)$$

Using equation (1.2), we see that:

$$m_{app} = m \left(\frac{a + g}{g} \right) \quad (1.4)$$

The relative error in our apparent mass is then:

$$\frac{m_{app} - m}{m} = \frac{a}{g} \quad (1.5)$$

Now, supposing $a=0.01g$, then we have 1% error in our scale measurement. Obviously, considering Table 1.1, this error is not acceptable.

We can conclude that a WIM scale needs some form of acceleration measurement if any accelerations exist in the motion. We will see later (Section 2.1.1) that, in fact, any WIM scale must be expected to experience significant accelerations.

In light of the data presented in Table 1.1, we would set an accuracy specification for a WIM scale of *at least* 0.1% in all environments. We also recognize that the error from a measurement system will arise from many different sources. As such, we would like to limit the contribution to error from any one source to be at least one order of magnitude less than the overall error specification. This implies that our ability to measure accelerations should not contribute to an error in measurement of the refuse mass of more than 0.01%.

1.3 ACCELEROMETERS

Since it has been concluded that measurement of acceleration is essential for a WIM scale, this section will present a survey of contemporary accelerometer technology. An accelerometer is a device that (somehow) measures the reaction force required to maintain the position of a mass (the "proof-mass") with respect to the body of the device. The device may, or may not, be accelerating in space along some axis. Figure 1.3 is a schematic of a generalized accelerometer.

Figure 1.3: Schematic diagram of a generalized accelerometer. The device consists of a body, proof-mass and reaction-force member.

One sees in Figure 1.3 that the proof-mass is supported by a beam cantilevered from the body. Now, suppose that the sensitive input axis (IA), of the accelerometer is aligned with the local vertical, and that the accelerometer is resting on a table (i.e. input acceleration $a=0$). In this case, the free body diagram of the proof-mass follows immediately as Figure 1.4:

Figure 1.4: Free body diagram of an accelerometer proof-mass. The body of the accelerometer is not accelerating.

The reaction force, F , supports the proof-mass, and fixes its position with respect to the body (actually, the proof-mass deflects a small amount). Therefore, in this case, F is non-zero. As a result, the device transduces the reaction force to an output signal, even though the acceleration is zero!

Alternatively, if the device is freely falling under the influence of gravity, no reaction force is required to maintain the position of the proof-mass with respect to the body. In this case, the output is zero even though the acceleration is non-zero! Thus, we see that an accelerometer is sensitive to gravitation fields as well as inertial acceleration. This is an important point to note for the WIM application: with the scale being mobile, we need to be able to detect the direction of the gravitational field strength vector with respect to the scale axis. This point will be elaborated upon in Section 2.1.2.

Having described a generalized accelerometer, the following sub-sections will briefly describe the principal types of accelerometers that have been developed, and have found contemporary application. These devices are:

- a) Strain gauge accelerometers
- b) Vibrating Beam Accelerometers (VBA)
- c) Force rebalanced accelerometers
- d) Pendulous Integrating Gyroscopic Accelerometers (PIGA)
- e) Electrostatic suspended accelerometers
- f) Piezoelectric accelerometers

1.3.1 Strain Gauge Accelerometers

A strain gauge accelerometer uses a strain gauge to transduce the reaction force that the body supplies to the proof mass. The common configuration is in the form of a

pendulous cantilever, very similar to that shown in Figure 1.3, and is elaborated upon by Figure 1.5.

Figure 1.5: Schematic diagram of a strain gauge accelerometer. The strain gauge sensitive elements are shown mounted on the cantilever member.

An input causes a *small* deflection of the cantilever, and consequentially, a reaction force. The reaction force is transduced to an output by the strain realized in the strain gauges mounted on the cantilever.

These devices are principally noted for their low cost, simplicity, and small size. Otherwise, they have poor sensitivity, high cross-axis sensitivity, poor mechanical and thermal stability, and moderate bandwidth.

One recent variation of this device is the silicon, micromachined accelerometer, as shown in Figure 1.6.

Figure 1.6: A silicon wafer etched with micro sized accelerometers. Reproduced from [5].

This device makes use of the piezo-resistive nature of silicon to transduce the strain into an output signal. Their exceptionally small size results in robustness, improved bandwidth, and low bulk cost. Presently, however, the accuracy and stability has usually proven to be very poor. They have found application as automobile air-bag sensors and in some high-g missiles.

1.3.2 Vibrating Beam Accelerometer (VBA)

VBA's are also pendulous cantilever devices. They use a quartz crystal to provide the reaction force as shown in Figure 1.7.

Figure 1.7: Schematic outline of a VBA. The device uses a stressed quartz crystal as the sensitive element.

The reaction force is transduced by measuring the frequency at which the quartz crystal may be driven into a mechanical resonance. Quartz is a piezoelectric material, so this transduction is rather straightforward. The crystals are dynamically balanced to provide low loss and superb resonant tuning.

The excellent mechanical properties of quartz give the VBA good stability, and the high stiffness of the quartz crystal ensures adequate bandwidth. Despite the open loop configuration, and the non-linear transduction process, the VBA has seen increasing application in inertial navigation units, both platform and strapdown. The device is also quite compact.

1.3.3 Force Rebalanced Accelerometers

Most accelerometers of this type have the pendulous cantilever configuration, but it is not intrinsic to the concept. The “force rebalance” principle implies the sensing of the deflection of the proof-mass and then applying a reaction force in order to restore the null position. The “rebalance” (i.e. “reaction”) force is usually provided

electromagnetically, so that a measurement of rebalance current is proportional to the reaction force.

The dominant design in this category is Allied Signal's (formerly Sundstrand), Q-flex as illustrated in Figure 1.8.

Figure 1.8: Schematic outline of a Q-flex force rebalanced accelerometer. Shown is the feedback loop from the pickoffs to the force coil. Reproduced from [5].

Obviously, this device is more complicated and expensive than the previous devices. It requires a rebalance loop that consists of torquer coils, magnetic field, pickoffs, and rebalance electronics. The advantages of minimizing the deflection include superior stability and reduced cross-axis sensitivity. The closed loop configuration allows for high bandwidth and the inclusion of electronic damping.

The Q-flex uses a quartz hinge in order to take advantage of this material's excellent mechanical properties and stability. Other designs have successfully employed metallic hinges [15 and 16]. These accelerometers are currently the principal designs used for inertial guidance engineering.

1.3.4 Pendulous Integrating Gyroscopic Accelerometer (PIGA)

The PIGA is a unique design that represents the state of the art of performance for platform inertial guidance systems. The PIGA is schematically outlined in Figure 1.9.

Figure 1.9: Schematic diagram of a PIGA. The diagram shows that the input axis is orthogonal to the spin axis of the momentum wheel. Reproduced from [5].

The PIGA operates with a rotor (wheel) that is spun up to generate a large angular momentum. The rotor is housed in a float whose mass centre is offset from its axis to effect a pendulous mass, as shown in Figure 1.9. An acceleration or gravity input gives rise to a torque about the float axis. The gyroscopic principle is to rotate the float about the precession axis to maintain the float in its null position. The rate of precession is then proportional to the input torque. By this fact, a velocity may be measured, and is proportional to the (acceleration) input, hence the integration concept. Going a step

further, the cumulative position of the float about the precession axis is proportional to the change in velocity along the input axis.

The fact that the sensor has produced an integration inherent to its operation makes the device attractive for inertial guidance systems. One other reason that the PIGA is very accurate is that the input and measurement axes are totally decoupled. Parasitic torques about the precession axis do not influence the accuracy at all. Such decoupling is a very desirable design principle, which is generally most feasible with some kind of gyroscopic element. Obviously, the complexity of the PIGA makes it very expensive. In fact, the principal application of the PIGA is found in intercontinental ballistic missiles (ICBMs).

1.3.5 Electrostatic Suspended Accelerometer

The electrostatic suspended accelerometer is also a force rebalanced device in essence, using electrostatic force to rebalance the proof-mass. What makes it unique, is that the body provides no mechanical support to the proof-mass. The arrangement is illustrated with Figure 1.10.

Figure 1.10: Schematic diagram of an electrostatic suspended accelerometer. It is seen that there is no mechanical contact between the proof-mass and the body.

The walls of the proof-mass and body serve, simultaneously, as electrostatic forcer plates, and as capacitive pickoffs. The device shown in Figure 1.10 is a three-axis sensitive device.

The absence of mechanical support gives this accelerometer unsurpassed stability, repeatability and threshold. The device is ideal for sensing very low inputs, but the dynamic range is somewhat limited. It is mainly an experimental device at present and is, therefore, very expensive.

1.3.6 Piezoelectric Accelerometers

The piezoelectric accelerometer is a familiar device, but it is more appropriately labelled as a "vibrometer". This is because its application is limited to sensing relatively high frequency, vibration induced accelerations. The device is illustrated with Figure 1.11.

Figure 1.11: Cut-away view of a typical piezoelectric accelerometer. The piezoelectric crystal is the sensitive element. Reproduced from [12].

The proof-mass is supported by piezoelectric crystals. These crystals generate a voltage across their faces when they support a time varying stress. This stress is, of course, manifested as a reaction force.

The high stiffness of the supporting crystals gives the device an outstanding bandwidth. However, the low frequency sensitivity and stability are poor, diminishing to nil at a dc input. For this reason, the piezoelectric vibrometer cannot sense steady gravitational inputs. The device is principally used to measure vibration levels.

1.3.7 Comparison of Accelerometers

This sub-section consists of the following table, (Table 1.2) that compares the main features of the accelerometer types described in this chapter. The reader may peruse the cited references for further details on the example devices. Also, some detail on other designs are described in many of the papers listed in the References.

TABLE 1.2: Summary of Accelerometer Types. Note that the given data is as presented by the manufacturer, and some definitions may not be consistent with ours. Therefore some data may seem to be unusually high or low.

TYPE/ MODEL	RANGE	ACCURACY	CROSS AXIS SENSITIVITY	BAND- WIDTH	SCALE FACTOR SENSITIVITY	COMMENTS
STRAIN GAUGE						
general	moderate	0.5%	poor	high	poor	[3] \$200-400
Entran EGAL-125-10	± 10 g	linearity $\pm 1\%$	3%	100 Hz	$\pm 2.5\%$ per 100°F	[4] piezo-resistive
Entran EGBL-125-10	± 10 g	linearity $\pm 2\%$	5%	100 Hz	N/A	[4] low cost version of above
VBA						
general	to ± 25 g	1 to 10 m/hr	moderate	high	10 ppm per °C	[5], [6]
Sundstrand Accelrex	± 5 g	linearity: 10 $\mu\text{g/g}^2$ SF: 100ppm	vibration rectification: 10 $\mu\text{g/g}^2$ max		temperature controlled	[7]
AdKEM Hi-G	1200g	linearity 1 $\mu\text{g/g}^2$	vibration rectification: 2 $\mu\text{g/g}^2$	2500 Hz	100 ppm/°C	[8]
FORCE REBALANCE						
general	moderate, to 500 g	0.1%, 1 m/hr, 0.5 $\mu\text{g/g}^2$	very good, vibration rectification: 0.5 $\mu\text{g/g}^2$ rms	medium, 200 Hz	very good	[3], [5], [6] >\$1000
Sundstrand QA-3000	± 25 g	linearity: 40 $\mu\text{g/g}^2$ threshold: <1 μg			100 ppm/°C temperature controlled	[9] best Q-flex >\$5000
Sundstrand QA-1200	± 25 g	linearity: 2-200 $\mu\text{g/g}^2$ repeatability: 0.001% FS threshold: 1 μg	axis alignment & transverse sensitivity: 0.002 g/g	300 Hz	180 ppm/°C temperature controlled	[10] Q-flex >\$2000
Sundstrand 303 GA	± 1 to ± 20 g	linearity: $\pm 0.05\%$ FS repeatability: 0.0005 g threshold: 5 μg	axis alignment & transverse sensitivity: 0.02 g/g	270 Hz	200 ppm/°C temperature controlled	[10] general purpose

Table 1.2: Continued

TYPE/ MODEL	RANGE	ACCURACY	CROSS AXIS SENSITIVITY	BAND- WIDTH	SCALE FACTOR SENSITIVITY	COMMENTS
PIGA						
general	to 50 g	0.01 m/hr, linearity: 0.05 ppm, resolution: 0.1 μ g	<1 mg/g ²	100 Hz		[5] >\$10000
ELECTRO- STATIC SUSPENDED						
CACTUS	± 20 g	linearity: 10 μ g/g ² resolution: 10 ⁻¹⁰ g	cross coupling: 10 μ g/g ²			[11] three axis design
PIEZO- ELECTRIC						
general	high	2-5%	very poor	very high	poor	[3] \$100 - 2000
Kistler 8604C50	± 50 g	linearity: $\pm 1\%$ threshold: 0.004 g rms	5%	10 kHz	-0.06% per °C	[12]
Kistler 8002	± 10000 g	linearity: $\pm 1\%$ threshold: 0.03 g rms	5%	7 kHz	-0.03% per °C	[12]

1.4 THESIS GOALS

Much of this chapter has been devoted to demonstrating the need for an accelerometer for the weigh-in-motion context. In order to more clearly define the need, we identify the following criteria which will be used to evaluate the potential for any device to find a role as a weigh-in-motion accelerometer (WIMA):

- performance;
- cost;
- robustness;
- size and weight, and;
- peripheral support requirements.

Each of these criteria is briefly elaborated upon in the following sub-sections.

1.4.1 Performance

The performance required for a WIMA will be specified in terms of:

- linearity and repeatability;
- cross axis sensitivity;
- thermal sensitivity, and;
- long term stability.

These terms will be examined thoroughly in subsequent chapters of this thesis. We will strive to quantify the performance of a prototype WIMA through testing.

1.4.2 Cost

A successful weigh-in-motion system will, eventually, see implementation throughout all industrialized nations. This implies that tens of thousands of vehicles will be equipped with WIM capability. As such, it is readily evident that the cost of the WIMA should be minimized. The world - wide outfitting cost of WIM vehicles is reduced by many thousands of dollars for each dollar reduction in WIMA cost.

1.4.3 Robustness

The implementation of WIM must fit invisibly into the current refuse collection practice. The environment for this practice is consistent with that of heavy industrial machinery. For this application, the WIMA must be a very durable device that requires no maintenance.

1.4.4 Size and Weight

As stated in the previous sub-section, the WIMA will find application in heavy machinery. Therefore, the WIMA need not be lightweight, nor particularly compact. Nevertheless, a small device will be unobtrusive, easy to locate, and, unlikely to suffer abuse. It is preferred that the WIMA should be as physically small as is practical.

1.4.5 Peripheral Support Requirements

The WIM system is to be mounted on standard urban road vehicles. This fact stipulates that the power consumption of the WIMA be compatible with the available support. High voltages, alternating currents, frequency generators, and, climate controls

are not desirable support requirements for the WIMA. Any requirement for special peripheral support must be considered as an increase in the total cost of the candidate device.

1.4.6 Statement of Thesis Objective

The objective of this thesis is to identify the requirements for a WIM accelerometer, principally on the basis of performance as outlined in Section 1.4.1. Using this information, and the other criteria as outlined in Sections 1.4.2 through 1.4.5, it will be determined if a suitable accelerometer is commercially available. This thesis will then propose and present a novel design of an accelerometer, suitable for WIM use. In order to assist in the synthesis of the design, some fundamental principles of instrument design will be established. The thesis will demonstrate a comprehensive exercise in mechanical design. The exercise will follow from the creative design stage, through to the detail design, prototyping, and, test planning stages.

CHAPTER 2

PRELIMINARY DESIGN CONSIDERATIONS

There is an unlimited variety of conceptual designs for an accelerometer. Indeed, six concepts, broadly classified on basic operation, were identified in Chapter 1, but many more concepts exist. For each class of basic operation, many physical arrangements can be envisioned. We observe that no mathematical analysis will lead directly to any particular configuration. Nor will it lead to any given geometry. These features are strictly the realm of creative design.

This chapter will attempt to identify some fundamental principles that can be used to guide both the creative and detail design stages of the WIMA. However, it is first necessary to clearly set some kind of target specification that a WIMA needs to meet. Therefore, the topic of accuracy will be explored in detail.

2.1 ACCURACY

Returning to our generalized WIM scale of Figure 1.2, we repeat equation (1.2):

$$F = m(a + g) \quad (2.1)$$

It is required to determine the mass, m , from a measurement of the force, F . Suppose that the gravitational field strength, g , is well known. We see, immediately, that the variational relation for equation (2.1) is:

$$\frac{\delta F}{F} = \frac{\delta a}{a + g} \quad (2.2)$$

Assuming that our measurement of force is adequately precise, then F is a traceable measure of the mass, m . We identified previously, (Section 1.2), that the contribution to $\delta F/F$ (i.e. $\delta m/m$), of δa , should be no more than 0.01%. So, using this tolerance, and considering nominally, $a=0$, then equation (2.2) yields:

$$0.0001 = \frac{\delta a}{0 + g} \quad (2.3)$$

This equation may be solved to yield the tolerable acceleration error:

$$\delta a = 0.1mg = 0.000981 \text{ m/s}^2, \quad (2.4)$$

where we have introduced a conventional unit of $1g$ that equals 9.81 m/s^2 .

Equation (2.4) implies that the accuracy of our acceleration measurement must be of at least this order, $0.1mg$, even if our WIM machine is nominally a constant velocity device (i.e. $a=0$). In fact, we would suggest that an accuracy of $0.01mg=10\mu g$ is probably desirable for the WIMA.

We observe that we cannot increase our δa specification by setting the acceleration, a , of equation (2.2) to an arbitrarily large value. As a practical basis, $a \leq 0.1g$ for a WIM machine that will not self-destruct nor endanger the personal safety of its operator. The following sub-sections will expand upon this accuracy issue.

2.1.1 Acceleration Levels

In the previous section, we found that the allowable accelerations for a nominally constant speed WIM system are very small. In this sub-section we look at the issue more carefully in order to answer the question: "If a WIM machine was precisely constant speed, would we be able to avoid an acceleration measurement?". To answer this question, let us consider three separate issues. Firstly, we consider a WIM machine that has a constant speed, fixed-axis rotation, as described by Figure 2.1.

Figure 2.1: A constant speed rotating machine. The arm is rotating in the vertical plane.

Even with the relatively slow angular speed (approximately 90° in 4s), the normal acceleration of point P is:

$$a_n = 0.5m \cdot (0.4s^{-1})^2 = 0.08\pi^2 = 8.2mg \quad (2.5)$$

So, we see that accelerations up to two orders of magnitude greater than our critical δa , (of equation (2.4)) are readily involved with any rotating machinery.

Next, we consider the fact that the WIM scale is mobile, being mounted on a heavy truck. We know that the peak acceleration, a_{max} , in simple harmonic motion is given by:

$$a_{max} = A \cdot \omega^2 \quad (2.6)$$

where: A is the amplitude of the vibration; and, ω is the circular frequency. For a truck's idle-up engine speed of 1000rpm, we find the primary vibration frequency to be:

$$\omega = 105s^{-1} \quad (2.7)$$

Now, supposing that the amplitude of vibration is:

$$A = 1\mu m = 10^{-6}m \quad (2.8)$$

then we see from equation (2.6) that:

$$a_{\max} = 0.011g = 1.1mg \quad (2.9)$$

This acceleration is an order of magnitude above the critical δa , even for such a small vibration amplitude.

We note that the vibration frequency is approximately 16.7Hz. Hence, if data sampling took place at 10Hz, then the periodic vibration won't be "averaged" out; (a more rigorous assertion is that the sampling rate does not meet the Nyquist criterion). As a practical note, we recognize that 10Hz data sampling is not unrealistic for a mobile, real-time, data acquisition application.

Finally, we give consideration to the fact that the earth bound vehicle is not inertially fixed due, at the very least, to the earth's rotation. The angular rate of the earth is $7.2722 \times 10^{-5} \text{ s}^{-1}$, and, at the equator, the earth's radius is approximately 6000km. Hence the normal acceleration towards the earth's pole-axis is:

$$a_n = 0.03g = 3mg \quad (2.10)$$

Again, this value has an order of magnitude of significance on our critical δa . At the very least, this effect would need to be absorbed into our definition of the local value of g , for a WIM system.

The above issues demonstrate that it is imperative to include an accelerometer as part of a certifiable WIM system.

2.1.2 System Tilt

For the WIM context, it must be recognized that the mobile scale cannot be leveled with respect to the local direction of the gravitational field strength vector, $\vec{g}$.

We consider the generalized WIM scale, oriented at an angle, θ , to the direction of $\bar{g}$, as in Figure 2.2.

Figure 2.2: Free body diagram of WIM scale at an angle of tilt. The direction of motion is oriented at an angle, θ , with respect to the vertical.

It follows, immediately, that:

$$F = m(a + g \cos \theta) \quad (2.11)$$

Even for the constant speed case of $a=0$, it is apparent that an accelerometer is required in order to quantify the $g \cdot \cos \theta$ term of equation (2.11). If the tilt of the scale was neglected, the apparent mass would be given by:

$$m_{app} = \frac{F}{g} \quad (2.12)$$

But, the actual mass is given by:

$$m = \frac{F}{g \cos \theta} \quad (2.13)$$

Hence, the relative error in our measurement of mass is found to be:

$$\frac{m - m_{app}}{m} = 1 - \cos \theta \quad (2.14)$$

As per Section 1.2, we would limit the contribution to error due to tilt to be 0.01%.

Equation (2.14) then shows the maximum allowable tilt to be 0.81° . Unfortunately, it is not unreasonable to expect tilt angles of up to 15° in a mobile WIM application.

Therefore, we determine that it is necessary to account for the angle of tilt for a WIM system. Equation (2.11) demonstrates that the WIMA will also serve this purpose.

2.1.3 Cross - Axis Sensitivity

A linear accelerometer is supposed to sense inertial body forces along one axis, the input axis (IA). Unfortunately, all real devices are also sensitive to components of input directed along axes orthogonal to the IA. This sensitivity is called "cross-axis" or, "transverse", and we give it the symbol, S_T , which represents the maximum output reading of an accelerometer per unit input that is orthogonal to the IA. Figure 2.3 clarifies the situation.

Figure 2.3: Accelerometer subjected to a transverse input that is orthogonal to the input axis (IA).

The output registered by the device of Figure 2.3 due to the transverse acceleration, a_T , is interpreted as an apparent input along IA:

$$a_{app} = a_T \cdot S_T \quad (2.15)$$

If we assume that the transverse acceleration inputs are of the same order of magnitude as g , ie: $a_T \approx g$, and we require that the apparent acceleration be limited to $a_{app} = 0.1mg$ (as in Section 2.1), then Equation (2.15) gives:

$$S_T = \frac{0.0001g}{1g} = 0.0001\% \quad (2.16)$$

In reality, transverse accelerations can probably be limited to $a_T \approx 0.1g$, so that the cross-axis sensitivity requirement becomes somewhat less stringent:

$$S_T = 0.001\% \quad (2.17)$$

We now wish to examine the manner in which transverse sensitivity can be problematic when combined with system tilt. Recalling from Section 2.1.2, that a tilt angle of $\hat{\theta} = 0.81^\circ$ would give an unacceptable error in weight measurement if the angle was not known. Consider that the accelerometer was tilted at this critical angle:

Figure 2.4: Accelerometer tilted at the critical angle with respect to the vertical.

We see that the accelerometer will register an output corresponding to an apparent input of:

$$a_{app} = g \cos \hat{\theta} + S_T \cdot g \sin \hat{\theta} \quad (2.18)$$

And we ask: "For what value of S_T does this apparent input correspond to a pure input of g along the IA, as in Figure 2.5?"

Figure 2.5: Accelerometer subjected to a pure $1g$ input along the IA.

In Figure 2.5, the output of the device registers an apparent input of:

$$a_{app} = g \quad (2.19)$$

By equating equations (2.18) and (2.19), we deduce:

$$S_r = \frac{1 - \cos \hat{\theta}}{\sin \hat{\theta}} \quad (2.20)$$

Inserting the critical angle, $\hat{\theta} = 0.81^\circ$, into equation (2.20) we find:

$$S_r = 0.007 \text{ } \frac{\%}{^\circ} \quad (2.21)$$

What this implies is: any accelerometer with a cross-axis sensitivity poorer than that indicated by equation (2.21) cannot even detect a difference between the critical tilt angle, and no tilt at all!

2.1.4 Modelling the Accelerometer

Before proceeding, it is useful to review the transduction behaviour of an accelerometer, and how this behaviour leads to some specification of accuracy. In general, the input - output relationship of the accelerometer is modelled with a straight line. A detailed graph of a typical input - output relationship is shown in Figure 2.6.

Figure 2.6: Description of a general input - output relationship for an accelerometer.
The diagram is labelled to show the definition of the parameters used to describe the accuracy of an accelerometer.

The illustration is self-explanatory, so the reader is encouraged to examine it thoroughly. We see that the accelerometer's straight-line behaviour is principally identified by the scale factor (the slope of the best fit straight line), and the bias (the ordinate intercept).

On the other hand, the accuracy of the device is mainly a function of the maximum non-linearity, threshold, and resolution.

The true situation is found to be considerably more complicated in practice. Here, it is found that one primary measure of an instrument's accuracy is its *repeatability*. We define repeatability as the ability to produce the same output from repeated applications of an identical input. This concept inevitably leads to the need to specify the *stability* of an instrument. Principally, one is interested in the short-, and, long- term stabilities of bias and scale factor. These stabilities determine the steady state accuracy (short term stability), and, the need for recalibration or replacement (long term stability), of the sensor. It turns out that the scale factor and bias are also sensitive to temperature, so that we need to specify temperature stabilities also.

To summarize, the following parameters are fundamental to the specification of an accelerometer's behaviour:

- scale factor
- bias
- non-linearity
- threshold
- resolution
- scale factor stability (short and long term, and temperature)
- bias stability (short and long term, and temperature)

2.1.5 Modelling the Pendulous, Inertial Accelerometer

The pendulous accelerometer is the most common configuration used for inertial measurements. The VBA and Q-Flex types are the principal designs in use. The pendulous configuration has some distinct, intrinsic, sources of cross-axis sensitivity. The description of the first of these sources is aided with Figure 2.7.

Figure 2.7: Pendulous proof-mass of an accelerometer, showing the acceleration components along IA and PA. The HA is normal to the plane of the diagram, and runs through the root of the cantilever.

For simplicity, Figure 2.7 shows the application of D'Alembert's principle to a pendulous accelerometer subjected to an input acceleration, a_p , along the IA, and a simultaneous acceleration, a_i , along the orthogonal pendulous axis (PA). We see that because the pendulum deflects under the influence of ma_i , it becomes sensitive to ma_p , which tends to increase the deflection. A similar sensitivity appears along the second orthogonal axis parallel to the pendulum hinge (HA, the hinge axis), although, in this case, the effect is more non-linear since the transverse inertial force tends to twist the hinge.

It is apparent that this cross-axis sensitivity is not constant, but tends to increase with increasing input, a_p . For this reason, this sensitivity is called *cross-coupling*. Pendulous sensors are then modelled with a cross-coupling coefficient, K_{uc} , with units of $\%$. This coefficient represents the apparent output per unit transverse acceleration per unit input acceleration. For a WIM application, we have already noted (Section 2.1), that the input is on the order of $1g$, so that the equivalent cross-axis sensitivity is:

$$S_T = K_{uc} \times 1g \quad (2.22)$$

In force rebalance designs, the deflection of the pendulum under the influence of the input is reduced compared to open loop sensors (see Section 3.4.1). Therefore, the cross-coupling is also reduced. This is one of the main limiting factors of VBA's, which are strictly open loop devices.

There exists a second form of cross-coupling in pendulous accelerometers. This sensitivity occurs for periodic inputs, and is called "vibropendulosity". It can be shown,

[13], that the pendulum is acted upon by a steady state torque, proportional to the sine of twice the angle of incidence (see Figure 2.8), when subjected to a periodic acceleration (i.e. a vibration) with components along IA and PA.

Figure 2.8: Pendulous accelerometer subjected to a cross vibration. The input is periodic with a frequency of ω .

The effect is most severe when $\phi=45^\circ$. Obviously, the vibropendulous cross-coupling is also dependent upon the frequency of vibration, and can be quite severe near the natural frequency of the device.

One final effect that is problematic for pendulous accelerometers is their sensitivity to angular accelerations. Consider the accelerometer in fixed axis rotation about point O, as shown in Figure 2.9, at an instant when the angular velocity is zero.

Figure 2.9: Accelerometer in fixed axis rotation at an instant of $\omega=0$, with IA perpendicular to line OA. The angular acceleration is given by α .

We see that the accelerometer will register an acceleration of:

$$a_t = \alpha(r + d) \quad (2.23)$$

This is, in fact, the actual tangential acceleration of point "A", in the direction of IA. But, now consider the accelerometer oriented with its IA directed toward the centre, O:

Figure 2.10: Accelerometer in fixed rotation with IA parallel to line OA. The rotational speed is zero at this instant.

We see that the accelerometer will register an apparent input of :

$$a_i = d \cdot \alpha \quad (2.24)$$

This output exists even though there is, in fact, zero acceleration along IA. It is also noted that force rebalanced accelerometers are no less sensitive to this effect than are open loop sensors.

Since pendulous accelerometers are so widely used in inertial applications, the above effects, along with some others, have been accounted for into a standard model. The IEEE STD 337-1972 [5 and, 14] models the relationship between indicated acceleration and instrument output as:

$$A = \frac{E}{K_1} = K_0 + a_i + K_2 a_i^2 + K_3 a_i^3 + d_o a_p + K_{ip} a_i a_p - d_p a_o + K_{io} a_i a_o \quad (2.25)$$

where:

- A = applied acceleration or gravitational field strength
- E = output in instrument units (iu),
- K_1 = scale factor (iu/g),
- K_0 = bias (g),
- a_i = acceleration along IA (g),
- K_2 = non-linearity (g/g^2),
- K_3 = non-linearity (g/g^3),
- d_o = misalignment of IA to HA,
- a_p = acceleration along PA (g),
- K_{ip} = cross-coupling, (g/g^2),
- d_p = misalignment of IA to PA,
- a_o = acceleration along HA (g),
- K_{io} = cross-coupling, (g/g^2).

The additional moment about the HA due to angular acceleration and anisoinertia[†] errors is modelled by:

$$M = I_a \alpha_a + (I_p - I_a) \Omega_a \Omega_p \quad (2.26)$$

where:

I_a, I_o, I_p = moments of inertia about IA, HA, and, PA.

α_a = angular acceleration about HA.

Ω_a, Ω_p = angular velocities about IA and PA.

Inertial accelerometers are carefully tested individually to determine the coefficients of the model equations above. Inertial measurement systems can use the readings from three mutually orthogonal accelerometers to eliminate the cross-axis sensitivity errors, based on the model equations.

Although the IEEE model equation (2.25) is developed for pendulous accelerometers, its form is amenable to most other configurations also. We shall attempt to adhere to the form of this model equation when describing the behaviour of any particular accelerometer. We do, however, allow the use of a cross axis sensitivity coefficient, S_x , as long as we recognize the transformation from this to cross coupling, K_{ix} , by means of equation (2.22).

[†] One can see that the second term of the equation is a gyroscopic term that vanishes only if $I_p = I_a$, hence the name "anisoinertia".

2.2 THERMAL INFLUENCES ON INSTRUMENT DESIGN

Thermal influences are probably the most troublesome obstacles when designing a precision instrument. In fact, the changing thermal state of an accelerometer is the greatest contributor to short term scale factor and bias instability. Even the smallest temperature gradient can degrade the performance of an instrument. The problem is so severe that inertial grade instruments require very precise temperature control in order to meet their performance specifications.

We first note that, if the temperature field within the instrument and its surroundings changes, then the state of the device is fundamentally altered. The most important state changes are recognized to be:

- a) dimensional changes;
- b) stress changes; and,
- c) material property changes.

The following sub-sections will explore these thermal influences, and identify the implications that they hold for instrument design.

2.2.1 Thermal Material Properties

Before proceeding, it is necessary to define several material properties related to thermal effects. They are as follows:

a) *Thermal coefficient of expansion, α* . This parameter relates how the thermo-kinetic expansion of a material's atomic bonds is manifested, macroscopically, in the volume change of a "lump" of material.

b) *Thermal conductivity, k*. This parameter relates how readily a material conducts heat. Fourier's Law for a generalized linear coordinate direction, x , gives:

$$\frac{q}{A} = -k \frac{\partial T}{\partial x} \quad (2.27)$$

where: q is the rate of heat energy transfer, A is the area normal to direction x ; and, T is the temperature field. For a given, (instantaneous) temperature gradient, a material with large k will transfer heat more quickly than a material with low k . That is:

$$\frac{q_1}{q_2} = \frac{k_1}{k_2} \quad (2.28)$$

Or, if a certain, fixed, quantity of heat, q , must be dissipated, we see that the material with greater k will support a smaller temperature gradient. That is:

$$\frac{(\partial T / \partial x)_1}{(\partial T / \partial x)_2} = \frac{k_2}{k_1} \quad (2.29)$$

c) *Heat capacity, c*. This is a measure of how much heat energy is required to raise the temperature of a body by a certain amount. Consider a small, lumped mass at temperature, T_o , that is exposed to an environmental temperature, T_a , at time, $t=0$. The temperature, T , of the mass will generally behave transiently as follows:

$$T(t) = T_a - (T_a - T_o) \exp\left(-\frac{hA}{\rho V} t\right) \quad (2.30)$$

where:

- h = bulk convective heat transfer coefficient
- ρ = material density
- A = surface area of lumped mass
- V = lumped mass volume.

One can see that a material with a large heat capacity, c , will not change its temperature as rapidly, and will be less sensitive to thermal transients than a material with a low heat capacity. This is equivalent to saying that the mass has a large "thermal inertia". A large thermal inertia simply must absorb a relatively large quantity of heat energy in order to increase its temperature by any amount.

2.2.2 Dimensional Changes

All materials are sensitive to an atomic level process that manifests itself as thermal strain. The uniaxial thermal strain realized in an unconstrained object that is uniformly heated to a temperature, T_f , is given by:

$$\varepsilon_{th} = \alpha(T_f - T_i) = \alpha \cdot \Delta T \quad (2.31)$$

where the initial temperature is T_i . Schematically, we see the effect as illustrated with Figure 2.11.

Figure 2.11: Dimensional changes of an object subjected to a temperature increase.
The initial dimensions of the object are length, L , and thickness, t .

Obviously, an initial temperature, T_i , and a reference shape (i.e. L and t in the case illustrated) both must be defined in order to determine the shape of the object at the final temperature. In most cases, such an initial state is not easily defined. For example, the states at the time of manufacturing and at the time of assembly can be significantly different.

On the other hand, the final temperature may not be known at all, or, may lie somewhere between some upper and lower limiting temperatures. In this case, an infinite number of final configurations for the sensor are possible. In addition, the final temperature may be varying in time (i.e. not steady). Or, the sensor's temperature field may be non-uniform, and, varying in time as a result of the transfer of heat between it and the environment.

One possible solution to dimensional changes caused by thermal effects is to use a material with a zero coefficient of thermal expansion (i.e. $\alpha=0$). Some real materials (e.g. INVAR, Zerodur, and quartz) address this philosophy. These materials have a small value of α over a usable range of temperature. These materials tend to be expensive, and, have either moderate or unfavourable mechanical properties. As a result, their application is usually limited, and other solutions should be considered as a matter of course.

The instrument designer must try to select materials that minimize the *sensitivity* to thermal disturbances in order to ensure stability of the sensor. From our discussions of Section 2.2.1, we recognize that a small value of α is not the only desirable material property. We see, in fact, that a high value of c is desirable in an effort to meet the goal of thermal insensitivity. Rather than just consider each of these parameters separately, it has been suggested [17] that it is more appropriate to examine the material property grouping, $\alpha/c\rho$. Materials showing relatively low values of this grouping will demonstrate minimal dimensional sensitivity to thermal disturbances.

It is seen from equation (2.30) that a geometrical parameter also influences the thermal inertia of a machine. The volume-to-surface area ratio, V/A , should be kept large if the design is to be insensitive to fluctuating environmental temperatures. Conversely, a device that is required to dissipate heat rapidly should minimize the V/A ratio (with, for example, cooling fins). In this latter situation, a low value of the material property grouping, $\alpha/c\rho$ is desired, as evidenced by equations (2.28) and (2.30).

2.2.3 Stress Changes

In a situation where a machine component is constrained geometrically, we find that the thermal growth that would occur due to a temperature increase is prevented. This kind of constraint is possible because of the elasticity of real materials. As a

consequence, the constrained thermal strain results in a stressed state of the component. The performance of thermally stressed components of an instrument may be degraded. For example, if some safe stress limit (e.g. yield stress, elastic limit, or, buckling load) is exceeded, then the design is terminally flawed. In general, thermal stresses are best avoided in instrument design.

In order to minimize the thermal stress that can occur in a particular design, one may carefully consider the material selection for the sensitive components. Based on this criteria, we find that the material property groupings that should have low values are: αE , and, $\alpha E/k$ [17]. Alternatively, a design may be conceived where sensitive components are geometrically unconstrained, and are, therefore, free of stress.

It is, unfortunately, far more probable that the instrument will be in a state supporting temperature gradients, rather than having isolated components at uniform temperatures. The temperature gradient in a component can also result in a constraint-induced stress and strain. This is because the lower temperature region of the component will constrain the thermal growth of the higher temperature region. This kind of constraint leads to distortion, which is often a worse problem than the stress itself. We see the distortion effect illustrated in Figure 2.12. This effect gives rise to the desire for a low value of the material property grouping, α/k [17].

Figure 2.12: An object subjected to a temperature gradient. The object distorts because of the unequal thermal strain across the thickness.

We observe that an instrument that dissipates power (e.g. with strain gauges or a rebalance force) must, almost certainly, embody temperature gradients. If temperature

gradients cannot be eliminated, then we must design the geometry of the device to support the gradient in a benign manner. The most favourable configuration is such that the temperature gradient is symmetric about a line of geometric symmetry. If this is not feasible, then the next best situation is to align the gradient to be parallel to a line of geometric symmetry. The philosophy is illustrated with the following figures.

Figure 2.13: Philosophy of geometric design in context of temperature gradient. Arrows show direction of increasing temperature. a) Initial configuration. b) Temperature gradient symmetric about line of symmetry. c) Gradient parallel with line of symmetry. d) Gradient non-symmetric and non-parallel.

2.2.4 Material Property Changes

Measurement systems are frequently based on the elastic properties of materials. In the case of accelerometers, we have seen that the proof-mass is usually suspended by an elastic element. All of the devices discussed in Chapter 1 are of this configuration, except for the PIGA and electrostatic suspended designs. As such, the temperature dependence of the elastic modulus of a material can be problematic for precision measurement devices. There are materials that can be used to address this particular problem. ELINVAR is such a material which has a zero temperature coefficient of elastic modulus over a usable temperature range.

Another material property that is highly dependent upon temperature is electrical resistivity. And, therefore, any electro-mechanical transducer will experience some

physical change as the temperature is varied. In a similar fashion, the magnetomotive force provided by permanent magnet materials tends to decrease dramatically as the temperature is increased towards the Curie point. The implications for measurement, and sensor design, are serious indeed!

For small, localized components, such as many sensors are apt to be, temperature control may be a practical option. Such a solution is a bothersome, costly, and, energy consuming complexity, but can be very effective. Temperature control is not normally practical for large, mobile machines. For the WIMA, we have a strong preference for a device that is insensitive to thermal effects, but we reserve the use of temperature control, should it prove to be necessary.

2.2.5 Chart of Material Properties

The following chart summarizes some of the important material properties and groupings. The materials considered are candidates for instrument construction.

TABLE 2.1: Chart of material properties and groupings. Note that the last four columns (i.e. groupings) are normalized with respect to the first entry, CRS 1020. Data is from references [17], [18], [19], [20], [21], [22].

MATERIAL	E ksi (GPa)	S _y ksi (MPa)	σ _a ksi (MPa)	k W/m°C	ρ kg/m ³	c J/kg°C	α 10 ⁻⁶ °C ⁻¹	αE c.f. CRS	αE/k c.f. CRS	α/k c.f. CRS	α/cρ c.f. CRS
CRS 1020	30000 (205)	64 (437)		60	7900	420	11.7	1.0	1.0	1.0	1.0
4140	30000 (205)	105 (718)	30 (205)	60	7900	465	11.7	1.0	1.0	1.0	0.90
303 SS	28000 (193)	35 (239)		16.2	8000	500	17.2	1.38	5.1	5.4	1.2
Al 6061	10000 (70)	37 (253)		167	2700	896	24	0.70	0.25	0.74	2.81
Brass	16000 (110)	18 (123)		120	8500	375	20	0.92	0.45	0.85	1.78
INVAR	21800 (150)	40 (273)	20 (137)	13	8000	515	1.5	0.094	0.43	0.59	0.10
Quartz	10500 (72)	S _u =7 (48)	7 (48)	0.73	2200	870	0.5	0.015	1.2	3.5	0.074
Zerodur	13500 (93)	S _u =8.7 (59)	8.7 (59)	1.64	2500	821	0.007	0.0027	0.099	0.22	0.0097

2.3 HYSTERESIS AND INSTRUMENT DESIGN

In the previous section, we identified thermal effects to be a leading cause of short term instability. There are a number of other mechanisms that cause instability, and these are broadly classified as hysteresis. It is found that most materials exhibit anelastic and inelastic behaviour, as well as long term instability. Each of these effects appears as some form of hysteresis to the instrument designer. By hysteresis, we broadly imply that the present state of the material is dependent upon its state history.

2.3.1 Anelastic Behaviour

It is found that the stress-strain behaviour of most materials is not a purely elastic mode. In fact, the stress-strain relationship tends to be dependent upon loading rate, not just the load. This fact accounts for inherent damping in materials. In some applications, high damping is desirable. For example, machine tool structures are often made of cast iron specially formulated to damp vibrations. For instruments, however, damping causes non-linearity, and (very-) short term drift.

A body of material under load is often modelled by including a dashpot-type damper with elastic elements as shown in Figure 2.14.

Figure 2.14: Tri-parameter model of an anelastic material. The model (right) shows the elastic and dissipative element properties of the real material (left).

The material model of Figure 2.14 is called the "tri-parameter" model [23]. The dashpot element is an energy dissipative device. Thus, the strain is dependent upon time, eventually reaching a steady state value. We see that this model has a unique steady state strain for every applied load (stress), and that the strain is fully recoverable. We find [17, 23 and, 24] that this kind of recoverable-strain damping is caused by thermoelasticity, grain boundary viscosity, and, interstitial re-ordering.

Fortunately, the damping in most materials is very small for low stresses. This also implies that the time to reach steady state is very short. Nevertheless, we must consider this behaviour when making a material selection for ultra-precision instruments. It is found [23] that quartz and single crystal silicon are nearly purely elastic. Hard, alloy steels are also found to have minimal damping [24].

2.3.2 Inelastic Behaviour

The yielding of many materials is a familiar phenomenon at high stress. For instrument design, however, we find that it is necessary to observe that many materials exhibit micro-yielding at surprisingly low stress. The critical microyield stress, τ , is given by:

$$\tau = \frac{Gb}{L} \quad (2.32)$$

where: G is the shear modulus; b is Burger's vector; and, L is the distance between pinning points. It is found, [25], that this stress is on the level of about 2000psi. Fortunately, this stress will not lead to measurable long-term creep.

However, there does exist an "elastic-limit" stress, σ_{el} , for most metals, that will lead to significant micro-creep. This elastic limit is often much lower than the yield stress, as is evidenced by the data included in Table 2.1. Micro-creep is an inelastic behaviour, and is not recoverable. For instrument design, we must maintain all stresses at a level much less than the elastic limit.

Inasmuch as low design stresses are necessary in instruments, the most problematic cause of microyield is the presence of residual stresses and stress concentrations. Eliminating these stress raisers is not so easily achieved as are low nominal stresses. The presence or absence of stress concentrations is the domain of geometric design. Symmetry can sometimes be of assistance in avoiding stress concentrations [24 and, 27].

It is found that tempering a material will relax residual stresses in the stock, and thereby improve stability. Unfortunately, when the stock is machined into a component, residual stresses may re-appear. It is suggested [26] that the component be machined in stages, alternating with stress relieving heat treatments. We recognize that welding, and the use of fasteners will lead to both residual stresses and stress concentrations. The use of high performance adhesives has been suggested [27] to avoid this problem.

Inelasticity is manifested as hysteresis principally as a consequence of the geometric alterations that occur under inelastic slip. That is to say that the micro-creep changes the load path in an instrument. On the other hand, cyclic modes of slip absorb energy. This energy not only increases the temperature of the material, but also interferes with the transduction process of an instrument. We recognize that bolted joints are one other potential source of inelastic slip. Shear loaded joints, in particular, are best avoided in instruments [24].

2.3.3 Long Term Material Instability

Even when stresses are nil, a material can still exhibit long term instability. In metals, this instability is primarily due to the metallurgy tending towards equilibrium. Once again, we find that quartz and single crystal silicon are very stable [6, 7, 17 and, 28]. On the other hand, poly-crystalline silicon is not particularly stable [28].

Non-equilibrium metallurgical states are not dimensionally stable because of phase transformations or precipitations that may occur over time. The various metallurgical phases of a particular alloy generally have different specific volumes, and this is the root of the dimensional instability. For example, martensitic steels can undergo a phase transformation, and are, therefore, not dimensionally stable despite their high strength. Similarly, the precipitation of solutes such as carbon in steel, and copper in aluminium alloys, is another source of instability. Materials for use in instruments should not be susceptible to these problems. As such, we find that thermal cycling can relax both modes. However, if a material is heat treated in this manner, one must ensure that its service temperature is well below the temperature that will enable the metallurgical transformation.

Another mode of instability occurs when residual stresses relax, ultimately leading to dimensional changes. This effect is familiar to machine tool manufacturers who age iron castings for several years before finishing the structure. In order to speed up the process, temperature is substituted for time to stress relieve a component.

In this sub-section, we also include a note regarding the use of polymers and composites. It is found [27] that such materials are particularly susceptible to absorption and desorption of chemicals. Therefore, not only may their material properties be altered but, in fact, the mass of a component may change. As such, these materials are rarely used in instrument design. Of course, when using metallic materials, one must take adequate steps to prevent oxidization, which has the same mass alteration effect.

2.4 DESIGN PRINCIPLES FOR THE WIMA

Up to this point, this chapter has identified the required accuracy of the WIMA, and has discussed the major causes of accuracy loss in instrument design. By comparing the analysis of Section 2.1.3 with Table 1.2, we see, immediately, that the most stringent requirement for the WIMA is the cross-axis sensitivity. In fact, only the better Q-flex models approach our specification for S_T . As noted previously (Section 2.1.5), inertial measurement systems make use of three orthogonal accelerometers in order to resolve the cross-axis difficulty. For a WIM system, we wish to avoid the expense and complexity involved with three high accuracy accelerometers. Therefore, we anticipate that a novel design for the WIMA is in order.

2.4.1 Axially Symmetric Principle

In our pursuit of a novel design, we first seek to better understand the causes of cross-axis sensitivity, and how it may be eliminated. After much careful consideration, we propose the following design principle:

If an accelerometer is to be a *truly* uniaxially sensitive device, then it *must* be axially symmetric about the sensitive axis, over a full 360°.

We consider this to be a novel concept since, to our knowledge, none of the literature has expounded the virtues of this principle. We do note that one paper [11] has suggested that a three-axis sensitive device should be, ideally, spherical.

The principle seems quite obvious once it is stated, but we note that this a common property of virtuous principles. Of course, it is not so obvious how the principle can be physically implemented. Nonetheless, we also seek some reassurance that an axially symmetric design has other benefits.

Firstly, we anticipate that the principles for superior thermal behaviour (Section 2.2.3) are readily incorporated into an axially symmetric design. That is, the principles of geometric symmetry, and, aligning thermal gradients about the line of symmetry should find easy application to an axisymmetric WIMA.

Secondly, we note that cylindrical components are, generally, the most economical to manufacture. And, finally, it is generally most feasible to eliminate stress concentrations in axially symmetric designs [27]. This last point is due to the rarity of abrupt directional changes on the surface of a symmetric object.

2.4.2 Force Rebalance Principle

We have identified (Section 2.1.5) that the force rebalanced, pendulous, accelerometer has low cross-axis sensitivity, as compared to open loop devices. In addition to this, we find that the rebalance scheme keeps the mobile proof-mass near to its null position. This fact reduces the stress in the flexure hinge, and, avoids non-linearities and temperature dependencies in the displacement behaviour of the spring. As a bonus, the rebalance scheme allows for electronic damping, and reduces the influence of the spring stiffness on the bandwidth of the device. The closed-loop stiffness of the system is mainly determined by the feedback gain as will be discussed in Section 3.4.

We recognize that all of these features are desirable for the WIMA. In particular, the low flexure stress should improve long term stability, and, electronic damping will improve dynamic response. At this point we conclude that the WIMA design should be of a force-rebalanced nature.

2.4.3 Material Selection

We see from Table 2.1 that Zerodur is a promising material from the thermal point of view. But, its poor machinability and low tensile strength and high cost allow us to reject it for a prototype WIMA. A similar sentiment applies to quartz. We then see that INVAR and aluminium and brass are candidate materials, though we are wary of the very large α of the latter two options.

The force rebalance scheme is anticipated to be of the electromagnetic configuration. We therefore see that INVAR is a good choice for structure that must act as a magnetic flux path. On the other hand, aluminium and brass may be appropriate for non-ferromagnetic components. We note, as a useful fact, that aluminium is considerably less dense than brass, and this may be a deciding factor between these two materials.

2.4.4 Rectilinear Motion

Section 2.1.5 demonstrated that the pendulous configuration for accelerometers has intrinsic cross-axis sensitivities. This can be linked to the fact that the proof-mass rotates about the fixed hinge axis. As such, any torque, including those arising from transverse and angular accelerations, tends to produce an output from the device. If the proof-mass can be arranged so that it moves in translation, then we anticipate that the intrinsic cross-axis sensitivity will be reduced.

One means of achieving near-rectilinear translation is by the use of parallel strip flexures. The concept is illustrated by Figure 2.15.

Figure 2.15: A proof-mass supported by parallel strip flexures, showing nearly rectilinear motion of the proof-mass as it displaces.

The parallel strip flexures of Figure 2.15 constrain the proof-mass to move in translation that is nearly rectilinear along IA. The strips are very flexible in the IA direction, but are several orders of magnitudes stiffer along the PA direction. Torques are supported by the strips, but the support is limited by the critical buckling load.

If we adopt the parallel strip flexure concept for the WIMA, and apply the axisymmetric principle (Section 2.4.1), then we synthesize the *parallel diaphragm* configuration:

Figure 2.16: Proof-mass supported axi-symmetrically by parallel diaphragm flexures. The proof-mass displaces along a precise rectilinear path.

As Figure 2.16 shows, the parallel diaphragm configuration constrains the proof mass to move in purely rectilinear translation along the IA. The diaphragms provide lateral support over a full 360° about IA. We see that, in fact, a PA cannot even be defined, so that the device is totally insensitive to angular accelerations. The critical buckling load of the diaphragms is much larger than that of the parallel strips since there is full circumferential support.

Transverse accelerations do not, inherently, lead to increased deflection of the above device. Hence, if we transduce displacement along IA, we get a measure of acceleration along IA only. This configuration is the most promising for WIMA application, and we choose to pursue it for detailed development.

2.4.5 Electromagnetic Forcer

The geometrical configuration of the WIMA was conceptualized in the previous section, and in Section 2.4.2, we adopted the force rebalance principle. For non-miniature devices, the most practical means of achieving force rebalance is to use an electromagnetic forcer, and we assume such an actuator for the WIMA.

An electromagnetic forcer requires a source of magnetic field, and this generally implies either:

- a) a field coil, or;
- b) permanent magnets.

Modern rare-earth permanent magnets (e.g. SmCo and NdFeB alloys) are now capable of very high field strengths, in conjunction with excellent stability. The newer NdFeB types are considerably less expensive than the SmCo composition, and are widely available. As such, we choose permanent magnets for the WIMA in order to avoid the additional complexity and power consumption of field coils.

The force, F , generated by the (average) radial magnetic field, B_{av} , on a cylindrical coil of radius, r , and N turns carrying current, i , is given by:

$$F = 2\pi N i r B_{av} \quad (2.33)$$

We assume that the field permeates the coil radially, and is produced by permanent magnets.

2.4.6 Initial Configuration and Preliminary Analysis

At this point, we observe that analysis cannot proceed until a detailed configuration is proposed through the creative design process. We can conceive of an unlimited number of configurations, but to maintain brevity, just one of our earlier designs will be presented here. It is displayed in Figure 2.17, and is fully labelled for the reader's information.

The very first calculation that we must examine, is an estimate of the force that can be reasonably realized by an electromagnetic forcer. The geometry shown in Figure 2.17 suggests a coil radius of approximately, $r=2.0\text{cm}$. We know, from experience, that the following values of parameters for equation (2.33) are typical:

$$N=2000 \quad i=0.01\text{A} \quad B_{av}=0.15\text{T}$$

We now find that the achievable force is:

$$F_{em} = 2\pi(2000)(0.01\text{A})(0.02\text{m})(0.15\text{T})\left(\frac{1\text{N/A}\cdot\text{m}}{1\text{T}}\right) = 0.38\text{N} \quad (2.34)$$

It is readily verified that the proof-mass shown in Figure 2.17 would weigh approximately 0.66N when made of brass. Apparently then, the proposed configuration is not feasible on this basis. We therefore express the following principle that should be adhered to in subsequent designs:

Figure 2.17: Proposed prototype WIMA for analysis of electromagnetic forcer feasibility. The proof-mass would weigh approximately 0.66N if made of brass.

The proof-mass should primarily consist of a coil form and windings. In this manner, the mass is almost entirely active, and there is little dead mass.

2.4.7 Pickoff Choice

For a force rebalance scheme, we must incorporate some means of sensing the displacement of the proof-mass. This sensor is commonly termed a "pickoff". Many commercial accelerometers use capacitive or optical pickoffs. We recognize that a capacitive pickoff must, generally, be integrated into the accelerometer structure with extremely tight tolerances. On the other hand, an optical pickoff design represents a complex diversion that we would rather avoid for the WIMA.

For the first WIMA, we choose to use miniature LVDT's as the pickoffs. The LVDT's are commercially produced as stand-alone units. This fact ensures that there will be very little interfacing effort required to incorporate them into the WIMA. We recognize that capacitive pickoffs would be less expensive for production versions, so we reserve the right to use them on future WIMA versions. As a design guideline, we state that the geometrical configuration of the WIMA should not preclude the use of capacitive pickoffs.

The pickoffs should be arranged in a "push - pull" configuration. This means that, as the proof-mass displaces, one pickoff senses an increasing displacement, and the other senses a decreasing displacement. This arrangement is desirable since thermal expansion of the proof-mass is rejected as "common-mode". Of course, there is also an improvement in the uncertainty of a measurement when taking more than one independent sample. On the other hand, this concept requires at least two pickoffs, and this is, obviously, more costly.

CHAPTER 3

DETAIL DESIGN ANALYSIS

In this chapter, we will present the final prototype design that has been developed for the WIMA. The design will be based upon the principles outlined in the previous chapter. As much as possible, the configuration of the device will be justified with analysis. The analysis will not simply be numerical, but will be conceptual, based upon good machine design practice. This is to say that our analysis will characterize the design in terms of the factors that influence performance of the WIMA.

3.1 PROOF MASS DIMENSIONS

The first issue to consider in detail, is the optimization of the dimensions of the proof-mass. We have already stated (Section 2.4.6) that the proof-mass should primarily be a coil form structure. As such, we propose a generalized proof-mass as illustrated with Figure 3.1.

Figure 3.1: Generalized proof-mass for the WIMA. The proof mass is modelled as a cylinder with wall thickness, t .

We wish to optimize the radius, length, and, thickness of the proof-mass towards increased performance of the WIMA. From Figure 3.1, we see that the weight of the proof-mass is given by:

$$W_{pm} = \pi \rho g \left((r_o + \frac{1}{2}t)^2 - (r_o - \frac{1}{2}t)^2 \right) l = 2\pi \rho g r_o t l \quad (3.1)$$

where: ρ is the material density; and, g is the gravitational field strength. Therefore, we see that, primarily:

$$W_{pm} \propto r l t \quad (3.2)$$

where r is some nominal radius, which is equivalent to r_o . We recall from equation (2.33) that the electromagnetic rebalance force is proportional as:

$$F_{em} \propto N i r B \quad (3.3)$$

Proceeding further, we find that N , the number of turns offorcer coil wire, is limited by geometric factors as shown in Figure 3.2.

Figure 3.2: Section of coil form showing dependence of N on d and l . The circles represent the turns of the coil wire.

Evidently, the number of turns of wire, of diameter, d , is proportional to:

$$N \propto \frac{l}{d} \quad (3.4)$$

On the other hand, the allowable operating current, i , of the coil is limited by the cross sectional area of the wire [29 and, 39], or:

$$i \propto d^2 \quad (3.5)$$

We will see in a later section (Section 3.3.1) that the magnetic field, B , is related to the dimensions of the air gap through which it passes. For an annular gap surrounding the generalized coil form of Figure 3.1, the field is related to the geometry as follows (where we recognize that the thickness of the air gap is essentially defined by the thickness of the proof mass):

$$B \propto \frac{1}{t} \quad (3.6)$$

To optimize performance, we insist that the ratio of rebalance force, F_{em} , to proof-mass weight, W_{pm} , be as large as possible. Combining the proportionalities (3.2) through (3.6), we realize:

$$\frac{F_{em}}{W_{pm}} \propto \frac{NirB}{rlt} \propto \frac{d}{t^2} \quad (3.7)$$

From this, we have determined that the thickness, t , of the coil form should be as small as is practical. The thickness of the coil form has a lower value that is limited by the need for it to be sufficiently rigid, and for it to be fastened to the flexure diaphragms. The coil wire should be as large a gauge as possible. But, we will see later that we place a limit on the practical current drawn from a power supply, and therefore, d also has an upper limit.

The length of the coil form, l , does not appear in the expression, equation (3.7), and, therefore, we reserve judgement on this parameter for further considerations. The radius of the coil form also does not appear in the expression, so we decide at this point to make it as small as is practical. This choice will ensure that the overall dimensions of the WIMA are not unnecessarily large.

In order to proceed with further analysis, it is necessary to propose a detailed design geometry for the WIMA at this point. Again, we look to the creative design process for this new geometry, and find a large number of concepts. In order to preserve continuity, we present only the final prototype design of the WIMA as given by Figure 3.3. Such a detailed design, obviously, does not follow immediately from the discussions up to this point. Rather, it represents the synthesis of all analysis and principles up to now, as well as much analysis yet to come. Many design iterations were required to develop the device illustrated in Figure 3.3, but our subsequent analyses will be specific to this final prototype design.

Figure 3.3: Final design configuration chosen for the WIMA prototype. Shown is an assembly layout of the device.

3.2 FLEXURE DIAPHRAGM DESIGN

The parallel diaphragm concept was introduced in Section 2.4.4, and incorporated into the prototype, Figure 3.3. We model the spring behaviour of each diaphragm as shown in Figure 3.4.

Figure 3.4: Spring model of diaphragm flexure. Shown are the principal dimensions, a , b , h , as well as the deflection, δ .

The deflection, δ , and maximum stress, σ , realized in the diaphragm are given by the following relations [30 and, 31]:

$$\delta = c' \frac{F a^2}{E h^3} \quad (3.8)$$

$$\sigma = c'' \frac{F}{h^2} \quad (3.9)$$

where: E is the elastic modulus of the material; and, c' , c'' are geometric scaling factors as shown:

Table 3.1: Coefficients for equations (3.8) and (3.9). Data is from references [30 and, 31]

a/b	1.25	1.5	2	3	4	5
c'	0.00129	0.0064	0.0237	0.062	0.092	0.114
c''	0.115	0.220	0.405	0.703	0.933	1.13

In order to make the equations (3.8) and (3.9) more manageable, we find (see Appendix A) that we can curve fit the data of Table 3.1 to obtain:

$$c' = 0.108975\sqrt{\gamma\%} - 0.12797 \quad (3.10)$$

$$c'' = 0.910257\sqrt{\gamma\%} - 0.89106 \quad (3.11)$$

We wish to try to optimize the geometrical parameters a , b , and h , of the diaphragm for the WIMA application. To this end, we establish the following design rules:

- a) The dimension, b , is equivalent to the proof-mass radius, r , and is, therefore, limited by the smallest practical radius (see Section 3.1)
- b) The maximum stress in the flexible element should be kept well below the elastic limit, (see Section 2.3.2). We arbitrarily require the stress to be less than 10% of the elastic limit.
- c) The stiffness of the spring, defined by the ratio, F/δ , should be low enough that there is measurable deflection of the proof-mass in the unpowered state. That is, the nominal deflection must be consistent with the range of the pickoffs (LVDT's). Hence, for given force, F , we see that the deflection, δ , should be as large as possible.

3.2.1 Deflection, Stress, and Geometry

For any given applied force, F , there will exist a unique stress, σ , and deflection, δ . The basic applied force to the diaphragms will be the weight of the proof-mass which was optimized in Section 3.1. So, we take the force to be equal to the proof-mass weight, and the dimension, b , to be equivalent to the radius, r , of the proof-mass. Therefore, from equation (3.2), and considering only those parameters that influence the flexure geometry, we find:

$$F \propto b \quad (3.12)$$

As such, we see from equation (3.8) that:

$$\delta \propto c' \frac{ba^2}{h^3} \quad (3.13)$$

Similarly, by considering a given force, F , and elastic modulus, E , we see from equation (3.9) that:

$$\sigma \propto c'' \frac{b}{h^2} \quad (3.14)$$

To meet our design rules b) and c) of Section 3.2, we wish to minimize the ratio of σ to δ . Hence, we combine equations (3.13) and (3.14) to find:

$$\frac{\sigma}{\delta} \propto \frac{c''}{c'} \frac{h}{a^2} \quad (3.15)$$

The approximations of equations of (3.10) and (3.11) show that c' and c'' are nearly proportional for larger values of $\%$, so we may reduce equation (3.15) to:

$$\frac{\sigma}{\delta} \propto \frac{h}{a^2} \quad (3.16)$$

We can now draw the following conclusions that will aid in the design:

- i) To minimize σ/δ , the diaphragm thickness, h , should be made small, and with a preference, the dimension, a , should be made large.
- ii) The dimension, b , has only a very small influence on σ/δ through the ratio c''/c' . Table 3.1 demonstrates that a smaller value of b reduces the ratio c''/c' slightly. Therefore, we have a weak preference for a small dimension, b .
- iii) If stress is problematic, then equation (3.14) shows that h should not be made too small. In other words, we use equation (3.16) to drive our design (as expressed by conclusion i)), but we use equation (3.14) to comply with our design rule b) of Section 3.2.

3.2.2 Torsional-Mode Behaviour

The diaphragm is flexible along its axis, but is very stiff in its plane. However, lateral moments can cause twisting of the spring, as shown in Figure 3.5.

Figure 3.5: Torsional spring model of the flexure diaphragm. Shown is the twist angle, θ , of the spring under application of the transverse moment, M .

The dimensions of the diaphragm are given by a , b , and, h , as in Figure 3.4. The angular stiffness is given by:

$$\frac{M}{\theta} = \frac{Eh^3}{c'''} \quad (3.17)$$

where: E is the elastic modulus; and, c''' is a geometric factor given by [32]:

Table 3.2: Geometric factor, c''' , for equation (3.17).

a/b	1.67	2	2.5	3.33
c'''	0.035	0.081	0.167	0.320

We can see (Appendix A) from the data of Table 3.2, that, to a degree of approximation:

$$c''' \propto \left(\frac{a}{b}\right)^3 \quad (3.18)$$

Therefore, we can interpret equation (3.17) to imply:

$$\frac{M}{\theta} \propto \frac{h^3 b^3}{a^3} \quad (3.19)$$

We want this torsional stiffness to be as high as possible to support the proof-mass against transverse loads. Therefore, the thickness of the diaphragm, h , should not be made unduly small, nor should the dimension, a , be made unduly large.

The dimension, b , should not be unduly small according to equation (3.19). This contradicts our previous assertion (Sections 3.1 and 3.2.1) that a small value is desired. But, we also recognize that a wider separation of the two parallel diaphragms will provide

superior torsional support of the proof-mass. We can use this fact to alleviate the contradiction. Hence, we set the diaphragms at the extreme ends of the proof-mass and maintain b at its smallest practical size.

3.2.3 Dimensions, Flexure Clamps, and Material Choice

As a further limitation on the major radius, a , of the diaphragm, we see that the over-all diameter of the WIMA is sized, principally, to enclose the diaphragms. A larger dimension, a , implies more bulk, and higher material costs for the WIMA. On a practical basis, the diameter of the WIMA should be limited to about 3 inches (75mm). Consequently, we limit the diameter of the diaphragm to be approximately $2a \approx 2$ inches (50mm). This will allow sufficient compliance along the axis as dictated by equation (3.16), while at the same time, maintain adequate torsional stiffness as described by equation (3.19).

The dimension, b , of the diaphragm (equivalent to the radius of the proof-mass) is the smallest practical in order to accommodate the magnets, and clamp the diaphragm securely as shown in Figure 3.3. We choose a clamped diaphragm configuration for the prototype WIMA, since it is simple to manufacture, and may be disassembled if necessary. On the other hand, we recall from Section 2.3.2 that bolted joints are a potential source of inelastic slip. The diaphragm is restrained by friction at its clamped edges so that microslip is possible. To alleviate the difficulty, we will use an adhesive to increase the effective coefficient of friction at the clamp. For future versions of the WIMA, we reserve the option of a more permanent fixing method for the flexure diaphragms.

From Section 2.2.2 we recall that a component with low ratio of volume to surface will be sensitive to thermal disturbances. For the diaphragms, this is indeed the situation, so we select INVAR as the flexure material. This material choice will help

prevent distortion of the diaphragms under thermal loads. INVAR has an elastic modulus only 70% as great as that of steel, which is helpful because it allows us to reduce the stiffness of the diaphragm without making the thickness, h , too small.

We see from Figure 3.3 that a certain length of proof-mass is dedicated to fixing the proof-mass to the diaphragms, and cannot be used to accommodate forcer windings. As such, we choose the length of the proof-mass to ensure that an adequate fraction of the length is "active" with forcer windings. The coil form must be non-ferromagnetic, and should be lightweight, as indicated in Section 3.1. Therefore, we choose 6061 aluminium as the material. To estimate the weight of the proof-mass, we first recognize that the forcer coil is made of copper wire, which is denser than aluminium. Using the dimensions suggested in Figure 3.3, we estimate the weight of the proof-mass to be:

$$W_{pm} = 0.060 lb_f = 0.27 N \quad (3.20)$$

3.2.4 Final Flexure Design

The points raised in the previous sub-sections are duly considered, and the following values of the diaphragm parameters are ultimately chosen:

Table 3.3: Final selection of flexure diaphragm parameters.

a inch (mm)	b inch (mm)	h inch (mm)	a/b	c' eq.(3.10)	c'' eq.(3.11)	E_{INVAR} ksi (GPa)
1.0 (25.4)	0.4 (10.2)	0.00394 (0.100)	2.5	0.044	0.548	21800 (150)

Equation (3.8) can be used to calculate the stiffness (spring rate) of each flexure diaphragm:

$$k_{st} = \frac{F}{\delta} = \frac{Eh^3}{c'a^2} = 302 \frac{lb_f}{in} = 529 \frac{N}{mm} \quad (3.21)$$

And, the maximum stress in a diaphragm under the influence of the weight of the proof-mass, considering that there are two supporting springs, is given by equations (3.9), (3.11), and, (3.20):

$$\sigma = \frac{1}{2} c^n \frac{W_{pm}}{h^2} = 11ksi = 75MPa \quad (3.22)$$

We are comfortable with this stress level since it is approximately 5% of the elastic limit of INVAR as shown in Table 2.1.

The static deflection of the proof-mass under the influence of gravity is easily calculated from equations (3.20) and (3.21):

$$\delta_{max} = \frac{W_{pm}}{2k_{s1}} = 0.001in = 0.025mm \quad (3.23)$$

Considering this deflection, we choose a pickoff LVDT that has the smallest available range of 0.005in (0.127mm) [33]. This complies with our design rule c) identified in Section 3.2. We expect the force rebalanced static deflection to be an order of magnitude smaller than the unpowered deflection expressed by equation (3.23), and this will help to avoid non-linearities in the spring stiffness that are caused by stretching (rather than simply bending) the flexure diaphragms at large deflections.

3.3 ELECTROMAGNETIC DESIGN

Central to the function of the WIMA, is the magnetic field that enables the generation of a rebalance force. We have already decided (Section 2.4.5) to make use of rare-earth permanent magnets to effect the magnetic field. Figure 3.3 shows that we have configured two magnets in a pole-opposed arrangement in order to establish a radial magnetic field passing through the forcer coil. This arrangement leads to an additional symmetry about the mid-plane of the WIMA. As discussed previously (Sections 2.2.3 and 2.3.2), any symmetry is generally helpful, so we are comfortable with this design.

3.3.1 Magnetic Circuit Design

A magnetic circuit generally has the function of establishing a magnetic field in an air gap. In the case of the WIMA, permanent magnets are the source of field intensity, and the air gap surrounds the forcer coils that have been wound onto the coil-form to comprise the proof-mass. The field is required to be radial since the Lorentz force on the forcer coils will be proportional to the cross product of the field and current vectors [34 and, 35].

The structure of the WIMA provides a magnetic flux return path and is, therefore, required to be a ferromagnetic material. From Section 2.4.3, we see that INVAR is a good material for this application, so we elect to use it. The ferromagnetic nature lowers the reluctance of the flux path, thereby providing a strong field in the gap. Apparently then, the gap width should be as narrow as possible. Figure 3.3 shows that the gap is chosen to clear the proof-mass by a small amount. The clearance is sufficient to allow for machining tolerances, and allow easy assembly.

We can show [34] that the field in an air gap, B_g , is approximated by:

$$B_g = \frac{1}{K} \frac{H_m l_m}{A_g R_g} \quad (3.24)$$

where: H_m is the field intensity within the magnet; l_m is the active length of the magnet; A_g is the average area of the gap which is:

$$A_g = 2\pi r l \quad (3.25)$$

where: r is the average gap radius; and, l is the axial length of the gap. Continuing with equation (3.24), we define K as a loss factor, and, R_g is the reluctance of the annular gap which is found [35] to be:

$$R_g = \frac{1}{2\pi l \mu_0} \ln \frac{r_2}{r_1} \quad (3.26)$$

where: r_1 and r_2 represent the inner and outer radial dimensions of the annular gap; and, μ_0 is the permeability of free space. The logarithm term can be approximated by:

$$\ln \left(\frac{r_2}{r_1} \right) = \ln \left(\frac{1 + l/2r}{1 - l/2r} \right) \approx \frac{l}{r} \quad (3.27)$$

where r is the average gap radius and, l is the gap width. Combining equations (3.24), through (3.27), we see that, primarily:

$$B_g \propto \frac{l_m}{l} \quad (3.28)$$

Therefore, we see that sufficient magnet length is desired, which is most feasible with a fairly long proof mass, as shown in Figure 3.3. The gap width, l , which is equivalent to the proof-mass thickness should be as small as possible. This is in accordance with our previous analysis of Section 3.1.

3.3.2 Field Calculation by Finite Differences

A reluctance calculation in the form of equation (3.24) can often be formulated to give a good estimate of the field strength in the gap of a magnetic circuit. Unfortunately, for the WIMA of Figure 3.3, the coefficient, K , is not easy to estimate. Therefore, we

need some means of calculating the average field strength in the gap for this complicated geometry.

We find [34 and, 35] that the magnetic field strength can be written as the gradient of a magnetic scalar potential, ψ :

$$\vec{B} = -\mu \vec{\nabla} \psi \quad (3.29)$$

where μ is the magnetic permeability of the material in which B exists. It turns out that this potential must satisfy Laplace's equation, in the absence of field currents.

$$\nabla^2 \psi \equiv 0 \quad (3.30)$$

Laplace's equation is readily solved by the method of finite differences, and we adopt this approach for calculating the radial field in the gap. We do this by writing a FORTRAN programme that is specific to the axi-symmetric configuration of our device. Actually, the programme is a modified version of a programme that was already in existence [36]. The modifications made consisted of converting the programme to cylindrical coordinates, and, specifying the geometry of the device.

Developing the programme, and using it to optimize the performance of the WIMA represented a significant portion of this work. However, it is not necessary to describe the details of the programme, nor the test results for every iteration of proposed WIMA geometry. Listings of the programme and its supporting programmes and input data are presented in Appendix B. Also, Table 3.4 is a summary of the important test results obtained with the programme.

The numerical analysis allows us to choose an optimum spacing between the two pole-opposed magnets. We choose a spacing that produces the most uniform field along the length of the gap. Also, we can optimize the length of the magnets to produce a strong field. Quantitatively, the programme provides the average radial field strength in the gap. For our final configuration (case 25 of Table 3.4) we see that:

$$B_{av} = 0.2T \quad (3.31)$$

Table 3.4 Summary of results obtained with finite difference program.

CASE	COMMENTS	RESULTS	Bav (T)
1	simple test case: magnet in free space	magnetic field qualitatively as expected	
2	test case: axial gap magnetic circuit; reluctance approximation yields $B_{av} = .715T$	result acceptable for quick estimate	.506
3	test case: radial gap magnetic circuit; reluctance approximation yields $B_{av} = .20T$	program diverges	
4	as per case 3; magnet interface slightly altered	program fails to converge	
5	central LVDT design WIMA; grid size 0.025in	adequate convergence	.153
6	as per case 5; grid size 0.0125in	program runs 15min	.154
7	as per case 5; grid size 0.00265in	program runs 4hrs; choose standard grid size 0.0125in	.153
8	as per case 6; more realistic geometry for end caps	no change in gap field; use this geometry as a basis	.154
9	as per case 8; double precision arithmetic	no change in output; use single precision as basis	.154
10	as per case 8; extra free space surrounding WIMA	no change in output	.154
11	as per case 8; lower permeability of INVAR specified	no change in output	.154
12	as per case 8; higher permeability of INVAR specified	no change in output	.154
13	as per case 8; coil form specified ferromagnetic	very weak field in gap	10^{-3}
14	as per case 8; slim magnet specified	field not maximum at centre plane	.144
15	rim LVDT design WIMA	gap field not altered	.154
16	as per case 15; low permeability of INVAR specified	no change in gap field	.154
17	as per case 15; wide magnet, small centre plane separation of magnets	high maximum field; rapid fall-off	.160
18	as per case 15; slim magnet	field not maximum at centre plane	.151
19	as per case 15; slightly slim magnet	smaller non-central peak	.151
20	as per case 15; slightly wide magnet	somewhat improved; use as basis	.157
21	as per case 20; grid spacing 0.00625in	2hrs to run; note $B_{av} = .223T$ over half gap length	.153
22	as per case 20; long magnet specified	$B_{av} = .230T$ over half gap length	.162
23	as per case 20; improved magnet design: mount with register	$B_{av} = .205$ over three quarter gap length	.173
24	new design with shorter coil form	improved average field	.197
25	as per case 24; slightly longer magnet	final geometry selected for prototype WIMA	.200

Figure 3.6: Radial component of magnetic field raised in air gap of WIMA, as calculated from the finite difference program (Case 25 in Table 3.4). Diagram shows the field magnitude from the centre plane of the forcer coil, out to the extreme end along the axial direction. The field, of course, is symmetrical about this centre plane.

This value is applicable to all of our electromagnetic force calculations. Figure 3.6 is a plot of the field along the gap, as obtained from the results of the finite difference calculation.

3.3.3 Field Calculation by Finite Elements

Although the finite difference programme appears to converge adequately for our proposed WIMA geometry, we see from Table 3.4 that convergence is unstable for some test geometries (cases 3 and 4). It was determined that there can be some numerical difficulties at the boundary between air and ferromagnetic structure. This is a result of the large difference in permeability in these materials that tends to magnify the error at the boundary, and this causes the non-convergence.

This observation leads us to seek some reassurance that our results for the WIMA magnetic field are accurate. Therefore, we choose to analyse the magnetic field with a well-tested commercial finite element software package. The software used is named "MagNET 5.0 ©", and its solution technique is based on a vector potential formulation. The software features automatic meshing and post-processing¹.

We implement the WIMA geometry on the finite element software and obtain the field solution. Hardcopy of the graphical solutions was not available, so we simply describe the key results here.

1. From the flux plot generated by MagNET © for the WIMA we observe that the field in the gap is quite radial, as desired.
2. A plot of the magnitude of the radial component of the field in the gap along its length was seen to be consistent with the results of the finite difference solution, Figure 3.6.
3. The average field from the finite element analysis is found to be:

¹ Facilities to use this software were graciously provided by Dr. Dawson of the Department of Electrical Engineering at Queen's University, Kingston, Ontario.

$$B_{\sigma} = 0.2T \quad (3.32)$$

The fact that the finite element solution is almost identical to the finite difference program, gives us the desired confidence in the results. We are able to proceed on this basis.

3.3.4 Electromagnetic Forcer Design

Now that the magnetic field in the gap has been established quantitatively, we are able to proceed with the design of the electromagnetic forcer. We must determine the optimum wire diameter and corresponding number of turns of wire required to generate adequate rebalance force. The normal operating force must rebalance the static weight of the proof-mass under the influence of gravity. It was identified in Section 3.2.3 that this weight is estimated to be 0.060lb_p (0.27N).

We first examine the options for winding the forcer coils. The coil must comprise a total length of wire, L , carrying current, i . The coil can be wound from a single strand of wire of length, L , and resistance, R , carrying current, i , as in Figure 3.7.

Figure 3.7: Model of single wire forcer coil of length L .

Ohm's law states that the voltage dropped over this wire is:

$$V = Ri \quad (3.33)$$

Alternatively, the coil can be wound with n parallel wires, each of length L/n , resistance R/n , and, each carrying current, i , as shown in Figure 3.8.

Figure 3.8: Model of multiple parallel wire forcer coil. Each wire has a length L/n .

As illustrated by Figure 3.8, the power supply must provide a total current, ni . In this case the voltage drop is:

$$V = \frac{1}{n} Ri \quad (3.34)$$

In either case of Figures 3.7 and 3.8, the power consumed is:

$$P = i^2 R \quad (3.35)$$

With the chosen geometry for the coil form as in Figure 3.3, we can estimate the maximum possible number of turns of wire that may be wound.

Figure 3.9: Schematic of wire lay on coil form. The coil form is shown in half section.

If the wires stack as shown in Figure 3.9, with the lowest packing factor, then the maximum number of turns, N_{max} , is evaluated by the formula:

$$N_{\max} = \frac{(.75in)(.0625in)}{d^2} = \frac{(159mm)(19.1mm)}{d^2} \quad (3.36)$$

We have previously found the electromagnetic force to be:

$$F_{em} = 2\pi N r_{av} i_{op} B_{av} \quad (3.37)$$

Equation (3.37) is subject to the following parameters:

$$\begin{aligned} F_{em} = W_{pm} &= 0.060 \text{ lbf} \quad (0.267 \text{ N}) & r_{av} &= 0.37 \text{ in} \quad (9.4 \text{ mm}) \\ B_{av} &= 0.2 \text{ T} & i_{op} &= \text{determined by wire diameter} \end{aligned}$$

The number of turns, N , required to realize the force is calculated from equation (3.37), by first assuming a wire diameter, d . The following table (Table 3.5) summarizes the possibilities. For each case in the table, the power consumed is 46mW.

The rebalance force is proportional to both the WIMA input and the forcer current. Therefore, we will use the current as a measure of the input acceleration. To measure the current, a ballast resistor, R_b , is required in series with the winding resistance. This ballast resistance will allow us to record a voltage as a traceable measure of the current. The total voltage drop over the device during normal operating conditions (i.e. 1g input) is:

$$V_{tot} = ni \times \left(\frac{R}{n^2} + R_b \right) \quad (3.38)$$

We note that the ballast resistor will be external to the WIMA structure, so that we need not be concerned with the heat that is dissipated by this component.

In order to make the best selection, we note the following criteria:

- 1) The WIMA should have a range of about $\pm 5g$ so that it does not overload during shock or heavy vibration. This implies that the peak current is $i_p = 5 \times ni$, and the peak voltage drop is $V_p = \pm 5 \times V_{tot}$
- 2) For a mobile application, the peak voltage drop is best limited to 12 volts (or ± 6 volts).

3) The maximum current drawn from the power supply should be limited to less than 0.5A.

4) The maximum number of parallel wires is 4, as any more would make assembly unwieldy.

5) The ratio $N/N_{\max}$ should be low since it allows more margin for increasing the capacity of the electromagnetic forcer. This is important since there may be additional losses in the magnetic flux path that are not accounted for in the calculation of the magnetic field. This ratio is seen to decrease with increasing wire gauge, which is in accordance with Section 3.1 where a preference for larger wire gauges was expressed. Larger wires also have a higher breaking strength, and are, thus, preferred for assembly.

6) We will see from Section 3.4.2 that the total resistance should not be unduly small. On the other hand, it is apparent that it should not be unduly large, or else the voltage drop will be unmanageable.

7) We will also see from Section 3.4.2 that there is a very strong preference for a small number of turns, N .

With the above considerations, we find that a good choice is:

gauge = 30
number of parallel wires $n = 1$
number of turns $N = 312$

winding resistance $R = 6.25\Omega$	operating current $i_{op} = 72.4 \text{ mA}$
ballast resistance $R_b = 10.25\Omega$	peak current $i_p = \pm 0.362 \text{ A}$
input impedance $R_{em} = 16.5\Omega$	operating voltage $V_{tot} = 1.19 \text{ volts}$
	peak voltage $V_p = \pm 6.0 \text{ volts}$

power consumption: winding resistance, $P_w = 33 \text{ mW}$
ballast resistance, $P_b = 53 \text{ mW}$

Table 3.5: Comparison of wire gauge, number of turns, configuration, current and voltage drop of various forcer coils. Wire data is from [38] and [39]. Diameter, d , is for heavy enamelled magnetwire.

wire gauge	d in (mm)	i_{op} mA	ρ_A Ω/ft	N	N_{max}	$N \div N_{max}$	n	R/n^2 Ω	ni mA	V Volts
28	.0148	162.0	.0649	140	210	.654	1	1.76	162.0	.285
	(0.376)						2	.440	324.0	.143
							4	.110	648.0	.071
30	.0120	101.8	.1032	222	326	.682	1	4.44	101.8	.452
	(0.305)						2	1.11	203.6	.226
							4	.278	407.2	.113
32	.0097	64.1	.1641	353	498	.708	1	11.2	64.1	.717
	(0.246)						2	2.81	128.2	.358
							4	.701	256.4	.179
34	.0077	40.3	.2609	561	790	.709	1	28.4	40.3	1.16
	(0.196)						2	7.09	80.6	.578
							4	1.77	161.2	.289
36	.0062	25.3	.4148	893	1219	.732	1	71.8	25.3	1.82
	(0.157)						2	17.9	50.6	.906
							4	4.49	101.2	.454
38	.0050	15.9	.6596	1421	1875	.756	1	181.6	15.9	2.89
	(0.127)						2	45.4	31.8	1.44
							4	11.35	63.6	.722
40	.0039	10.02	1.049	2255	3082	.732	1	458.3	10.02	4.59
	(0.099)						2	114.6	20.04	2.30
							4	28.64	40.08	1.15
42	.0031	6.302	1.668	3586	4878	.735	1	1159	6.302	7.30
	(0.079)						2	289.8	12.60	3.65
							4	72.42	25.21	1.83
44	.0026	3.964	2.652	5701	6934	.822	1	2929	3.964	11.61
	(0.066)						2	732.3	7.928	5.81
							4	183.1	15.86	2.90

3.4 DYNAMIC ANALYSIS AND DESIGN

Up to this point, our force-deflection analyses have considered only the static (i.e. steady state) case. Now, we must recognize that the WIMA is an electro-mechanical system that has a very important dynamic response. The fundamental mechanical structure of the WIMA is that of a mass, spring and damper as shown in Figure 3.10:

Figure 3.10: Mass, spring, damper model of the accelerometer. The displacement of the proof-mass from its static position is measured by x .

The force, F , in this figure represents the sum of the gravitational body force, the electromagnetic force, and, the inertial acceleration force (using D'Alembert's principle). The equation of motion for this system follows immediately:

$$m\ddot{x} = F - kx - b\dot{x} \quad (3.39)$$

The transfer function representing the mechanical accelerometer is now readily evaluated:

$$H(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \quad (3.40)$$

The mass and stiffness parameters are readily determined from equations (3.20) and (3.21).

$$m = \frac{W_{pm}}{g} = 0.0272 \text{ kg} \quad (3.41)$$

$$k = 2k_{st} = 10577 \text{ N/m} \quad (3.42)$$

We choose to use SI units for this analysis in order to be compatible with the units of the electromagnetic components in the Laplace domain.

The mechanical damping coefficient, b , is not so easily determined. We know that the mechanical damping will be small in the prototype WIMA (as it is due primarily to viscosity in the air gap), so we estimate that the damping ratio will be 0.1. That is:

$$\zeta \cong 0.1 \quad (3.43)$$

And, this allows us to estimate a value for the damping coefficient, b :

$$b = 2\zeta\sqrt{mk} = 3.392 \frac{Ns}{m} \quad (3.44)$$

For reference, we see that the natural frequency of the WIMA is:

$$\omega_n = \sqrt{k/m} = 624 s^{-1} = 99.2 Hz \quad (3.45)$$

It is instructive to evaluate the step response of the open loop accelerometer system. We can do this analytically, or by simulation on computer. We choose the latter option, and implement the block diagram shown in Figure 3.11 on Simulink®, and obtain the unit step response as shown in Figure 3.12. We see that the open loop static deflection of the proof-mass for a unit step input is:

$$\delta_{st} = \lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} s \frac{1N}{s} H(s) = \frac{1N}{k} = 9.5 \times 10^{-4} m \quad (3.46)$$

3.4.1 Force Rebalance Scheme, Less Inductance

Now, let us embed the mechanical accelerometer system into a feedback loop. This is done by amplifying the pickoff output in order to drive the electromagnetic rebalance forcer. Schematically, we represent the situation as in Figure 3.13.

Figure 3.11: Simulink ® block diagram of open loop WIMA accelerometer. The dynamics of the mechanical accelerometer are of the form of equation (3.40).

Figure 3.12: Unit step response of open loop WIMA accelerometer. The second order response is evident.

Figure 3.13: Proportional feedback scheme for WIMA. The displacement, x , gives rise to a pickoff signal, and results in a rebalance force, F_{cm} .

This figure models the forcer as a motor with a winding resistance. Although the winding inductance is significant, we choose to omit it here so that certain effects of the rebalance scheme will be more clear. We will include the inductance in the next subsection. The following relations apply to Figure 3.13:

$$V_{inh} = K_{inh}x \quad (3.47)$$

$$V_{amp} = -K_{amp}V_{inh} \quad (3.48)$$

$$i_f = \frac{1}{R_{cm}}(V_{amp} - K_{cm}\dot{x}) \quad (3.49)$$

$$F_{cm} = K_{cm}i_f \quad (3.50)$$

By combining the above relations, we see that:

$$F_{cm} = -\left(K_{inh}K_{amp}\frac{1}{R_{cm}}K_{cm}x + \frac{K_{cm}^2}{R_{cm}}\dot{x}\right) = -(K_px + \hat{b}\dot{x}) \quad (3.51)$$

In terms of a block diagram, we can represent the feedback system as in Figure 3.14.

Figure 3.14: Block diagram representation of WIMA feedback system. The displacement of the proof-mass is referenced to a null position.

We recognize that the disturbance input, F_{dist} , represents the sum of the gravitational and inertial body forces. The block diagram may be reduced by observing from Figure 3.14 that the reference position, x_{ref} is null. The reduced block diagram appears as illustrated with Figure 3.15.

Figure 3.15: Reduced block diagram of the WIMA feedback system. The null reference of Figure 3.14 has been eliminated.

And, we see that the transfer function of the reduced block diagram is given by:

$$T(s) = \frac{X(s)}{F_{dist}(s)} = \frac{1}{ms^2 + (b + \hat{b})s + (k + K_p)} \quad (3.52)$$

Equation (3.52) clearly demonstrates that the rebalance feedback has increased the accelerometer stiffness from k to $(k + K_p)$. With this increase in stiffness, we get improved bandwidth (i.e. increased natural frequency) and reduced static deflection of the proof-mass. The damping coefficient of the system has also increased because of the presense

of the additional dissipative element. In this case, the unit step deflection at steady state is found to be:

$$\delta_{cl} = \lim_{s \rightarrow 0} s \frac{1N}{s} T(s) = \frac{1N}{k + K_p} \quad (3.53)$$

The actual value of this deflection depends upon the choice of K_p . Suppose that we require the closed loop deflection, δ_{cl} , to be an order of magnitude less than the open loop deflection, δ_{ol} . That is:

$$\delta_{cl} = \frac{1}{10} \delta_{ol} \quad (3.54)$$

Therefore, by using equations (3.42), (3.46), (3.53) and, (3.54) we can define:

$$K_p = 9k = 95200 \text{ } \%/m \quad (3.55)$$

From Section 3.3.4, we have chosen a force impedance of 16.5Ω , driven by 1.19volts to give a force current of 0.072A. This current generates an electromagnetic force equal to the weight of the proof-mass. Hence, we see that the electromagnetic motor constant is:

$$K_{em} = \frac{W_{pm}}{i_{op}} = 3.686 \text{ } \%/A \quad (3.56)$$

The value of the additional damping term is now readily found to be:

$$\hat{b} = \frac{K_{em}^2}{R_{em}} = 0.8235 \frac{\text{N}}{m} \quad (3.57)$$

Again, we may examine the step response for the closed loop system by implementing the Simulink® block diagram shown in Figure 3.16. The step response from this simulation is illustrated by Figure 3.17. When we compare Figure 3.17 to 3.12, we see that the natural frequency is, indeed, increased by the closed loop rebalance scheme. However, the amount of oscillation evident in Figure 3.17 is somewhat disturbing. This prompts us to examine the root locus of equation (3.52) which is displayed in Figure 3.18.

Figure 3.16: Simulink ® block diagram of closed loop WIMA accelerometer.

Figure 3.17: Unit step response of closed loop WIMA accelerometer. The increased natural frequency of the rebalanced WIMA is evident.

Figure 3.18: Root Locus of closed loop WIMA accelerometer. The poles move parallel to the Imaginary axis. The system is always stable, but is lightly damped.

The root locus demonstrates that, as the gain, K_p , is increased, the natural frequency, ω_{ncl} of the closed loop system is raised. At the same time, the damping ratio, ζ_{cl} of the closed loop system is reduced, while the relative damping, $\zeta_{cl}\omega_{ncl}$, remains constant. For larger gains the static deflection is reduced, but the number of oscillation peaks increases. The settling time is slightly reduced compared to the open loop case since the relative damping is increased somewhat.

3.4.2 Modelling the Inductance

In the previous sub-section, we omitted the winding inductance in the dynamic model so that the effect of the feedback gain on the system would be revealed. We now include the inductance in our model to account for the additional dynamics that it introduces to the system. We see that Figure 3.19 models the inductance in series with the winding resistance.

Figure 3.19: Feedback scheme including model of winding inductance, L_{em} , in series with the winding resistance.

Relations (3.47), (3.48), and (3.50) still apply to Figure 3.19, but we must adjust equation (3.49) to give:

$$\frac{L_{em}}{R_{em}} \frac{di_f}{dt} + i_f = \frac{1}{R_{em}} (V_{amp} - K_{em} \dot{x}) \quad (3.58)$$

And, combining the given relations is most easily done in the Laplace domain to yield:

$$F_{em} = - \left(\frac{K_p + \hat{b}s}{1 + \tau s} \right) X \quad (3.59)$$

where:

$$\tau = \frac{L_{em}}{R_{em}} \quad (3.60)$$

Our block diagram now appears as in Figure 3.20

Figure 3.20: Block diagram of WIMA feedback system including the inductance pole.

Apparently, the inductance introduces an extra pole into the system, and we immediately desire to examine the root locus for Figure 3.20. But first, we need to calculate the inductance [37]:

$$L_{em} = \frac{\mu_o N^2 \pi r^2}{l} = 1.78mH \quad (3.61)$$

In this equation, N is the number of coil turns; r is the coil radius; and, l is the coil length. Actually, the inductance will be less than this due to flux leakage, but we use this conservative estimate. Now we may construct the root locus as in Figure 3.21.

The root locus demonstrates that the additional pole quickly pushes the other two poles into the unstable plane for even a moderate gain. Obviously, unstable dynamics are forbidden in the practical WIMA. As such, we observe that if τ (of equation (3.59)) is decreased, then the real axis pole of Figure 3.21 will move to the left, and the destabilizing effect is reduced. In fact, for a zero inductance, the pole is at minus infinity and the root locus is stable as in Figure 3.18. It is therefore desirable (see equation

Figure 3.21: Root locus of closed loop WIMA accelerometer, including the inductance pole.

(3.60)) that the winding inductance, L_{em} , be made as low as possible, whereas the resistance, R_{em} , should not be unduly small. From equation (3.61), we see that there is a very strong incentive to minimize the number of turns, N , as alluded to in Section 3.3.4.

3.4.3 Lead Compensation

We have identified that a small inductance will minimize the destabilizing effect on the system. Nevertheless, even for the zero inductance case, the large overshoot and oscillation of the proof-mass evidenced by Figure 3.17 is not desirable. The long settling time (0.08sec) is also of concern. We therefore seek to increase the relative damping of the WIMA feedback system. The most effective means of doing this is to include a lead network compensator into the system. This is illustrated schematically by Figure 3.22.

Figure 3.22: Feedback arrangement including a lead compensation network. The network is modelled as a transmittance, D_c .

Relations, (3.47), (3.48), and, (3.50) still apply to Figure 3.22, but we now add the following relations:

$$V_c = D_c V_{amp} \quad (3.62)$$

$$\frac{L_{em}}{R_{em}} \frac{di_f}{dt} + i_f = \frac{1}{R_{em}} (V_c - K_{em} \dot{x}) \quad (3.63)$$

and, we find:

$$F_{em} = - \left(\frac{K_p D_c(s) + \hat{b}s}{1 + \tau s} \right) X \quad (3.64)$$

In the Laplace domain, the lead compensator has a transmittance:

$$D_c(s) = \frac{s+z}{s+p} \quad , \quad z < p \quad (3.65)$$

Normally, we place the zero in the s -plane somewhat to the left of the real part of the open loop poles, and the pole is placed sufficiently far away from the zero [40 and, 41].

As such, after some iteration, we choose a lead compensation transmittance:

$$D_c(s) = \frac{s+1000}{s+8000} \quad (3.66)$$

The accelerometer system now has a block diagram as illustrated by Figure 3.23.

Figure 3.23: Block diagram of WIMA system including lead compensation pole pair.

It is instructive to examine the root locus of this compensated system which is shown by Figure 3.24. We see that the effect of the compensator is to draw the poles away from the imaginary axis, thereby increasing the relative damping. The shape of the root locus suggests that the accuracy of our estimate of the open loop damping ratio, ζ , is not critical. That is, no matter how small ζ actually is, the lead compensator will be effective in drawing the poles away from the imaginary axis.

Unfortunately, the compensator also attenuates the signal, so that we require a greater gain from the amplifier. For the same static deflection as the uncompensated feedback system (Section 3.4.1) we find that:

Figure 3.24: Root locus of closed loop WIMA accelerometer including the inductance and lead compensation poles.

$$K_p = \frac{1}{D_c} \times 95200 \frac{\text{V}}{\text{m}} = 761600 \frac{\text{V}}{\text{m}} \quad (3.67)$$

Using this gain, we may implement a Simulink® block diagram, as shown in Figure 3.25 and, we obtain the step response as illustrated with Figure 3.26.

This step response is much more well behaved than the uncompensated case (Figure 3.17), as there is no oscillation nor overshoot. We also see that the settling time is now much improved over the uncompensated system. In fact, the settling time has been reduced by at least a factor of 10 which is quite satisfying. Essentially, for the gain we have chosen, the system is now dominated by the single closed loop pole on the real axis. This accounts for the appearance of the response which is a damped second order response superimposed upon a first order response. Therefore, the compensator has fulfilled the required task, and it is to be included into the WIMA system.

3.4.4 Gain Determination

In this sub-section, the overall amplifier gain will be determined. We need consider only the steady state term for this analysis. First, we observe from the LVDT supplier data [33] that, for a recommended excitation of 4 volts, the scale factor of the LVDT is:

$$K_{int} = \frac{1mV}{0.001in} = 1V/in \quad (3.68)$$

So, from equations (3.51), (3.67), and (3.68) we find the required amplifier gain.

$$K_{amp} = K_p \div \left(K_{int} \frac{1}{R_{cm}} K_{cm} \right) = 86600 \text{ } \nu/\text{ } \quad (3.69)$$

Of course, the implementation of the practical electronic feedback part of the WIMA system is much more complex than the simple schematic diagram depicted in Figure 3.22. The design of the electronics is a topic that merits a section of its own, and

Figure 3.25: Simulink ® block diagram of closed loop WIMA accelerometer, including inductance and lead compensation poles.

Figure 3.26: Unit step response of closed loop WIMA accelerometer, including the effects of the inductance and lead compensation poles. The lower curve is plotted on a time axis that has been expanded by a factor of 10.

will follow after a brief investigation of the thermal behaviour of the system that we have developed thus far.

3.5 HEAT TRANSFER

The basic description of the proposed WIMA is now almost totally complete. Since the power consumption has been calculated (Section 3.3.4) it is now possible to estimate the normal operating temperature field in the device.

We observe that the forcer coil is, essentially, a heat source. Fortunately, this heat source is symmetrically distributed about the axis and plane of geometric symmetry. This is exactly the kind of arrangement that is desired, as was expounded in Section 2.2.3. The coil form is made of aluminium, which has a very high coefficient of thermal expansion, α . Therefore, it is especially necessary that the thermal loading be symmetric.

We also note that the proof-mass is well isolated from the external environment. This isolation is provided by the heavy walls of the WIMA body and the air gap. As such, it will not be subjected to external thermal disturbances. On the other hand, it is required that there be adequate heat transfer from the forcer coil to the body. Otherwise, there may be gross expansion of the proof-mass under both normal operating, and, transient conditions.

3.5.1 Outer Wall Temperature

First, we examine the convection of heat from the accelerometer body to the surrounding environment. We treat the WIMA as a vertical cylinder, transferring a steady amount of heat, q , by free convection to the surrounding air. The situation is illustrated with Figure 3.27.

Figure 3.27: Model of heat transfer from WIMA to environment. The WIMA is modelled as a cylinder with surface temperature, T_o , interacting with environment at temperature, T_∞ .

The heat flow is known from Section 3.3.4 to be:

$$q = 0.033W \quad (3.70)$$

and, we wish to find the temperature difference:

$$\Delta T = T_o - T_\infty \quad (3.71)$$

The heat transfer relation is well known to be given by:

$$q = hA(T_o - T_\infty) \quad (3.72)$$

where A is the heat transfer area. The convective heat transfer coefficient, h , depends on the geometry, and can be estimated [43] by:

$$h = \left(1.42 \frac{W}{m^2 \cdot ^\circ C} \right) \left(\frac{T_o - T_\infty}{H} \frac{m}{^\circ C} \right)^{1/4} \quad (3.73)$$

Considering only the circumferential area, we see that:

$$A = \pi DH = 0.01811 m^2 \quad (3.74)$$

By simultaneously solving equations (3.70) through (3.74), we determine:

$$T_o - T_\infty = 0.7^\circ C \quad (3.75)$$

This implies that the exterior body will settle to a temperature approximately $0.7^\circ C$ above the ambient temperature. Such a temperature difference is quite satisfactory for the WIMA instrument.

3.5.2 Inner Wall Temperature

Moving inwards, we seek to estimate the temperature of the body at its inside surface near the proof-mass. For this calculation, we can represent the body of the WIMA as a thick walled cylinder.

Figure 3.28: Model of heat conduction through the heavy walls of the WIMA. The WIMA is modelled as a thick-walled cylinder.

For such a situation, Fourier's law of conduction can be integrated to give [43]:

$$T_i - T_o = \frac{q \ln \frac{D_o}{D_i}}{2\pi k H} \quad (3.76)$$

where: k is the conductivity of the body material. We have chosen INVAR as the body material, which is found to have a conductivity of 13W/m°C, as seen from Table 2.1.

Evaluating equation (3.76) results in:

$$T_i - T_o = 0.02^\circ C \quad (3.77)$$

This demonstrates that there will not be a large temperature difference across the wall of the WIMA body. Of course, this implies that the body will remain largely unstressed, thermally.

3.5.3 Temperature of Proof-Mass

Again moving inwards, we examine the temperature rise of the proof-mass. The proof-mass is adjacent to the inner wall, separated by a thin layer of air. We model the situation with Figure 3.29.

Figure 3.29: Model of heat transfer from forcer coil to WIMA body. The forcer coil is modelled as a cylinder at surface temperature, T_{pm} , separated by a thin layer of air from a surface at temperature, T_i .

We recognize that the proof-mass is a heat source at a temperature, T_{pm} . It is transferring heat across an annular gap of width:

$$\delta = \frac{1}{2}(D_i - D_{pm}) \quad (3.78)$$

to the WIMA body's inner wall which is at a temperature, T_i .

Actually, the proof-mass is attached to the diaphragms which act as cooling fins. We neglect this heat sink in order to gain a conservative (i.e. high) estimate of the temperature rise of the proof-mass. Also, we neglect to include the heat transfer to the inner core (magnets) of the WIMA for the same reason.

For such a narrow gap, it is found [43] that the heat transfer mechanism is largely film conduction rather than convection. In this case, the temperature difference is given by:

$$(T_{pm} - T_i) = \frac{q \ln D_i / D_{pm}}{2\pi k H} \quad (3.79)$$

For this equation, the conductivity, k , is that of air:

$$k = 0.02624 \text{ W/m}^\circ\text{C} \quad (3.80)$$

so that we find from equation (3.79) that:

$$(T_{pm} - T_i) = 0.5^\circ\text{C} \quad (3.81)$$

Considering also the heat transfer that we have neglected, we expect the actual temperature rise of the proof-mass to be less than one half of the value indicated by equation (3.81). Regardless, we see that the heat transfer from the proof-mass is adequate to prevent it from reaching an unacceptably high temperature.

3.6 ELECTRONIC MODULES DESIGN

In order to physically realize the feedback branch of the WIMA system, we need to transport the signals from the LVDT pickoffs to the forcer coil as a useful source of power. The signal from the LVDT's, however, cannot be used directly since it has neither sufficient power nor the necessary direct current form. As such, we must feed the pickoff signal through a series of electronic *conditioning* modules to convert it to a useful input to the forcer coil.

We choose analog electronic circuit configurations rather than digital processing for the prototype WIMA in order to preserve simplicity and continuity from the analog pickoffs to the analog forcer coil. However, we recognize that for advanced versions of the WIMA, a partly digital control circuit may be desirable since it would be relatively insensitive to thermal sensitivities, and allow temperature mapped compensation of the whole system. Also, a generic programmable processor would be significantly more economical than a special purpose analog integrated circuit.

In this section, the design of each of the electronic modules will be described in terms of its function and the arrangement of the electronic components used to make the module. We make extensive use of the simple operational amplifier integrated circuit (IC) in our modules, and the reader is assumed to be familiar with its basic function and properties.

3.6.1 Differential Input Isolation Amplifier

The output signal of the LVDT pickoff has a differential, AC voltage form. The first module that is needed to receive this signal is a differential input amplifier. This device will accept the differential (i.e., no reference) signal as an input, and provide a referenced-to-ground (i.e. single wire) output. The differential amplifier will pass the

differential voltage between the two signal wires, and reject any common mode voltage present.

When designing this module, we find that it can also provide two other functions simultaneously. The first additional function is that of a high-impedance isolator. This feature prevents the rebalance circuit from attempting to draw current (and therefore, power) from the LVDT's. Like most sensors, the LVDT's are not intended to deliver current, and they will display their most stable and linear behaviour if their output is connected to a high-impedance isolator.

The second additional function is that of providing gain to the signal. The output voltage of the LVDT is very small, and we need a large overall amplifier gain, as outlined in Section 3.4.4. If we introduce gain to the signal at the earliest point in the path, then the signal to noise ratio will be optimized. We also find that the common mode rejection ratio (CMRR) of our module is increased with increasing gain, and this provides another reason to include gain at this stage. The appropriate module is illustrated with the following schematic diagram:

Figure 3.30: Differential isolation amplifier schematic diagram. The differential inputs are e_1 and e_2 . The output is e_o .

The input-output relation for the module is given as follows:

$$e_o = -\frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) (e_1 - e_2) \quad (3.82)$$

3.6.2 Push-Pull Summing Amplifier

From Figure 3.3, it is seen that we have chosen to use 4 LVDTs as pickoffs to sense the displacement of the proof-mass. As previously noted in Section 2.4.7, the use of multiple, independent pickoffs will provide a more accurate measure of displacement, as well as allow the rejection of common-mode growth of the proof-mass. This second benefit is realized if the pickoffs are configured in a push-pull arrangement as we have designed the WIMA (i.e., two LVDTs on one end of the proof-mass, and two on the other end).

The output of each LVDT is connected to the input of an individual differential isolation module as described above. It is necessary to combine the four signals coming from the four isolation amplifiers into a single signal representing the displacement of the proof mass. We obtain the benefit of a more accurate measure by simply adding the signals, thereby effectively averaging the four signals. In fact, we obtain an additional benefit of realizing a gain of 4 by performing this sum without dividing by 4 to calculate the arithmetic average. The push-pull feature is incorporated by reversing the polarity of two of the LVDT output signals that feed the isolation amplifiers (i.e., e_1 and e_2 in Figure 3.30). The configuration of a summing amplifier is shown in the following illustration.

Figure 3.31: Schematic diagram of summation amplifier. The inputs are e_1 , e_2 , e_3 , and e_4 . The output is e_o .

The input-output relation of the above device is simply:

$$e_o = -(e_1 + e_2 + e_3 + e_4) \quad (3.83)$$

3.6.3 Rectifier

The LVDT output is in the form of an AC voltage that varies in amplitude as the LVDT core is displaced. It is necessary to demodulate this signal into a DC voltage that varies about zero volts as the proof mass is displaced. The first stage in this demodulation is a half-wave rectification of the AC signal. It is not practical to perform this rectification directly on the signal with a common Wheatstone bridge of diodes because diodes require relatively large currents to operate, and, introduce non-linearities to the signals that they transmit.

Instead, we may pass the WIMA signal and its inverted complement through a pair of CMOS switches that are triggered by a pair of in-phase square waves. The signals that pass through the switches are then re-combined at a hardwired “OR” junction. The square waves are generated by passing the LVDT excitation signal and its inverted complement through a pair of zero-crossing comparators. The LVDT input excitation is used as a signal to the square wave generator because its voltage level is relatively high compared to the output signal, and it is also constant in amplitude. The final arrangement is shown in the diagram below.

Figure 3.32: Schematic diagram of the rectifier circuit. The form of the signals is shown at each stage.

3.6.4 Primary Phase Shifter

In the previously described module, we had assumed that the primary excitation signal was in phase with the LVDT output signal. In practice, this assumption is false because the LVDT sensor shows a phase lag on the output. In order for the rectifier to work properly, we must trigger the CMOS switch with a signal that is in phase with the WIMA signal. We can realize the appropriate trigger signal if we pass the primary excitation signal through a phase-shift filter prior to introducing it to the square wave generator. We simply adjust the amount of phase shift until the trigger signal is in phase with the WIMA signal. The particular configuration of a phase-shift filter chosen for the WIMA is illustrated below.

Figure 3.33: Schematic diagram of phase-shift filter. The input signal is e_i and the output signal, e_o , may be brought into phase with the input by adjusting the potentiometer, R . Adapted from [44].

In the above circuit, the module is tuned to the input signal frequency, ω , by the following relation:

$$\omega = \frac{1}{R_2 C} \quad (3.84)$$

3.6.5 Demodulation Filter

The half-wave rectified WIMA signal coming from the rectifier module can be reduced into a DC voltage level by removing all of the Fourier series harmonics. We may achieve this demodulation by passing the signal through a low-pass filter. For the WIMA system, we choose a second order Sallen and Key configuration, which is illustrated in the following diagram.

Figure 3.34: Second order low pass demodulator filter. The input signal is e_i and the output signal is e_o . Adapted from [44].

The break frequency of the above filter is given by:

$$\omega_o = \frac{1}{R_2 C} \quad (3.85)$$

3.6.6 Level Adjust Module

It is recognized that even when there is no deflection of the WIMA proof mass, the LVDT's will not have a null output. This is due to the fact that it is not possible to adjust the position of each LVDT to precisely its null position when assembling the WIMA. In fact, even if the LVDT's were at their null position, they would still show a bias offset voltage signal. It is necessary to remove the static DC offset of the WIMA signal so that there is no current passed through the WIMA coil when it is at null. It is a simple matter to accomplish this compensation by using a summing amplifier and

including a constant voltage as one input. In actual fact, the constant voltage is adjustable in level by means of a trim pot as shown in the schematic of the level adjust circuit.

Figure 3.35: Level adjust module. The WIMA signal is represented by e_i and the output is e_o . The pot is connected between the rail voltages.

3.6.7 Lead Network

The lead compensator may be realized with an RC network [42] as shown in Figure 3.36.

Figure 3.36: RC network realization of a lead compensator. The input, V_{amp} , and the output, V_c , correspond to Figure 3.22.

We find that the transmittance of the above circuit is:

$$D_c(s) = \frac{s + \frac{1}{R_1 C}}{s + \frac{1}{R_1 C} + \frac{1}{R_2 C}} \quad (3.86)$$

Suppose that we arbitrarily choose a $0.1\mu\text{F}$ capacitor for the circuit. In this case, the resistances required to satisfy the compensator design of equation (3.66) follow immediately:

$$R_1 = \frac{1}{0.1\mu\text{F}} \frac{1\text{sec}}{1000} = 10\text{k}\Omega \quad (3.87)$$

and,

$$R_2 = \frac{1}{0.1\mu\text{F}} \frac{1\text{sec}}{8000 - 1000} = 1.43\text{k}\Omega \quad (3.88)$$

3.6.8 Voltage Gain Amplifier

The remaining voltage gain that is required to bring the WIMA signal up to the designed level can be easily realized by cascading as many simple feedback amplifiers as is necessary. The basic circuit for a voltage gain amplifier is given in the following figure.

Figure 3.37: Schematic diagram of a feedback voltage amplifier. The input is e_i and the output is e_o .

The gain relationship of this configuration is:

$$e_o = \frac{R_f}{R_i} e_i \quad (3.89)$$

3.6.9: Power Amplifier

In the previously described voltage gain amplifier, the current that can be delivered is minimal due to the limited continuous power output of the operational

amplifier integrated circuit. It is necessary to provide ample current gain to the signal in order to drive the WIMA forcer coil. This is done conveniently with the following transistor circuit.

Figure 3.38: Schematic diagram of power amplifier module. The voltage input is e_i , which is identically realized over the load resistance, R_{load} , with the transistors delivering the current.

In this module, a pair of complementary NPN and PNP transistors deliver the current from the power supply rails (at voltages $+V$ and $-V$) to the load resistance, R_{load} . In the WIMA system, the load resistance is the series combination of the ballast resistance and the forcer coil resistance. The gain relationships of the power amplifier circuit are given by:

$$e_o = -e_i \quad (3.90)$$

$$i_o = -\frac{R_1}{R_{load}} i_i \quad (3.91)$$

CHAPTER 4

PROTOTYPING AND TEST METHODS

The previous chapter presented the design analysis of a prototype WIMA, addressing both the mechanical and electronic parts. In this chapter, the actual production of the prototype device will be explored. This exercise includes the production and assembly of the machine elements that comprise the mechanical part of the WIMA system as well as the assembly of the components that make up the electronic part of the system. Each of these parts must be tested for functionality (and, in practice, this always requires a considerable effort in troubleshooting) individually before they are combined to synthesize the WIMA system. The device will then be subjected to some preliminary testing which essentially verifies the functionality of the combined mechanical and electronic parts.

This chapter will also include a description of the required tests that would result in the calibration of the WIMA and provide some sensitivity and stability data. Although the WIMA is not a pendulous device, we choose to use the IEEE model equation (equation (2.25)) as a skeleton for describing the behaviour of the device. This first phase

of testing will prepare the device for the second phase which will determine the cross axis and thermal sensitivities of the prototype WIMA. This second phase is also described in this chapter. The resources (both equipment and time) are simply not available for these described tests to be carried out as part of this work. However, developing the prototype WIMA to the functional stage leaves only the most easily tractable part of the development (the testing) to be executed. Having defined the required set of tests, all that remains is to procure the resources and perform the tests.

The future tests will determine the performance of the design concept as it relates to WIM application. As we have already identified in Section 2.4, the cross-axis sensitivity requirement for the WIMA is anticipated to be the most difficult to achieve. As such, the future test program will include the major task of quantifying this behaviour in the prototype WIMA. This test will determine if the proposed design has the potential to successfully fulfill a role as a WIMA.

It is to be noted that a critical stage in this design has been achieved by manufacturing and assembling the actual WIMA prototype. The mechanical parts have been machined and assembled, and the electronic component modules have been wired individually and together as a complete circuit. The WIMA prototype has been physically realized by combining the mechanical and electronic parts into a real hardware system. The present state of the WIMA prototype, as of this writing, is that of a functioning device that operates as intended, and is prepared to undergo performance testing.

4.1 PROTOTYPE PRODUCTION

We recognize that the prototype WIMA is not suitable for mass production in its current form. The prototype has the task of verifying the design concept and assessing its suitability for WIM use. As such, the prototype WIMA need not necessarily conform to every optimum design principle. Of course, the design concept must not preclude future improvements.

4.1.1 Material Choice

We have identified (Section 3.2.3) that INVAR is the material of choice for the WIMA body, and it is specified on the drawings in Appendix C. For this thesis exercise, however, we find it to be necessary to relax this requirement due to budgetary constraints. It turns out that INVAR has only 25% machinability, as compared to cold rolled steel (CRS). This thesis project could not afford the expense required to produce INVAR parts.

The decision is then made to produce the very first WIMA prototype largely out of CRS. This greatly reduces the costs of both the material stock and the machining. The flexure diaphragms are still made of INVAR foil. Of course, the original specification for INVAR was made on the basis of thermal material behaviour and stability. With this in mind, we prescribe that the testing of the CRS WIMA should be done under carefully temperature controlled conditions. By controlling the temperature, we remove the thermal instability factor. This should allow us to assess the critical performance of the design concept.

Additionally, we recognize that the magnetic permeability of INVAR is much greater than that of CRS. As such, we must realize that the use of CRS could adversely affect the performance of the electromagnetic forcer, as compared to its design intent. If the achievable magnetic field strength permeating the forcer coil is substantially less than

the designed field strength, then we may have to accept an increased steady state deflection of the proof mass while the coils pass the stated operating current. Again, this sub-optimization of material selection will not preventing us from assessing the basic function of the prototype WIMA.

4.1.2 Detail Drawings

This section simply makes reference to Appendix C. In this appendix are full detail drawings of the proposed prototype WIMA. The drawings are consistent with Figure 3.3. The specified tolerances are determined by the need for the components to align axially with great precision. The geometric tolerancing is based on ANSI STD Y14.5M-1982.

We wish to emphasize that the proposed WIMA is a prototype, and as such, is designed with a certain philosophy. Primarily, we recognize that the device is readily disassembled. This will allow future modifications if they are deemed necessary. The clearances are such that assembly and disassembly is not awkward. Unfortunately, the extensive use of non-permanent fasteners (i.e. machine screws) dramatically increases the number of parts, the machining required, and, the overall cost. For a prototype, we accept the high cost in order to allow us to evaluate the design concept. Furthermore, we have carefully designed the device such that future production versions will be much less expensive. This point will be expanded upon in Chapter 5.

In order to make the first prototype more affordable, the surface finish specifications called out on the detail drawings were relaxed somewhat in production. This is another sub-optimization that may affect the performance of the prototype, but we anticipate the this degradation will not be unmanageable.

4.1.3 Rebalance Electronics

In Section 3.4 we carefully designed the feedback control strategy for the WIMA, while in Section 3.6, we described the details of each module of the rebalance electronics. In this section, the assembly of electronic hardware is described with Figure 4.1. A large amount of effort is required to thoroughly troubleshoot the electronic network, but it is not necessary to describe that effort in detail. However, a number of points are listed here that were used as guidelines for constructing the practical circuit.

1. Metal-film precision resistors are used rather than carbon resistors principally for reason of superior stability.
2. Low leakage polystyrene capacitors are used in all filter elements (including the compensation network) so that the real filter breakpoints will match the calculated breakpoints and be stable.
3. Buffer capacitors are positioned near the power supply points of all IC elements to protect the elements from any voltage droops that may occur at the rails.
4. The input impedance of each element is sufficiently high so that there is minimal current drawn from the preceding element.
5. When the TL071 (or TL074 package) op-amp is used, we find it to be necessary to include a trim pot to null the offset voltage if it is comparable in magnitude to the output signal. Where a low offset voltage is essential, the OP-07 op-amp may be used.
6. The behaviour of all electronics is very sensitive to thermal disturbances. No attempt is made, at this point, to compensate the electronics for this sensitivity. Hence, we must be cautious when using the circuit by allowing it to reach a steady state temperature, and taking measures to isolate it from the external thermal environment.
7. The feedback gain of any one voltage amplifier should be limited to 10 in order to preserve linearity.

Figure 4.1: Circuit diagram of the electronic rebalance part of the WIMA system.

4.2 ACCELEROMETER TEST APPARATUS

The testing of the WIMA that comprises the determination of misalignments, bias, scale factor, resolution, and the investigation of stability can be performed in a static mode with a dividing head. The determination of the cross axis sensitivity requires a (non-static) centrifuge apparatus. This section will briefly describe these apparatuses, beginning with the static mode equipment.

4.2.1 Static Mode Apparatus

The static tests first require the use of a massive surface plate. The surface plate is usually made of granite, and has a smooth surface that is flat to better than 3 arcseconds (8.3×10^{-3} degrees or 1.45×10^{-5} radian) and must be well isolated from floor borne vibrations [45]. By using precision bubble levels, the basic true plane of the table may be horizontally levelled with respect to the local gravitational field strength vector to better than 3 arcseconds.

Upon the surface plate rests a machinist's dividing head. The carriage of the dividing head can rotate about its axis through a full 360° . The angle of the carriage can be adjusted and measured with an accuracy of at least 3 arcseconds. The axis of the dividing head carriage is assured to be horizontal with respect to the surface plate. The device to be tested (i.e. the WIMA) can then be mounted in a sensor block, which in turn is fixed to the carriage. This arrangement allows us to rotate the sensor about an axis orthogonal to the gravity vector. The situation is schematically illustrated with Figure 4.2. Note that the sensor block is thermally isolated from the dividing head with quartz spacers.

Figure 4.2: Schematic of accelerometer test apparatus. The front and right side views are shown, and the diagram is fully labelled.

Although the dividing head axis is orthogonal to the gravity vector, this tells us little about the position of the WIMA. We can define a reference position, however. We do this by mounting a mirror on a flat surface of the sensor block as shown in Figure 4.2. By using an optical autocollimation technique, we can ensure that the reference face is precisely vertical. Hence the normal vector to the reference face, $\vec{F}$, is horizontal with respect to $\vec{g}$. We record the dividing head angle at this reference position.

4.2.2 Centrifuge Apparatus

For the determination of the cross axis sensitivity of the WIMA, we make use of a centrifuge apparatus that can turn the device about an axis at a constant speed. We use the centrifuge to apply a relatively large acceleration along the non-sensitive axis of the WIMA, without exceeding the input limits along the sensitive axis. We use the same sensor block that was used in the static tests, thereby making use of the knowledge gained regarding the mis-alignment angles. A schematic of the centrifuge apparatus is given in Figure 4.3 below.

Figure 4.3: Schematic diagram of centrifuge apparatus. The front and top views of the centrifuge table are shown.

We can see from the diagram that the cross axis acceleration is nominally given by:

$$a_r = r\omega^2 \quad (4.1)$$

and that the input along the sensitive axis is nominally equal to the gravitational field strength. The actual components along the axis depend on how well leveled is the centrifuge table, and on the misalignment angles which are measured with the static apparatus. As a final note, we see that either a system of slip rings or telemetry is required to read the output signal of the WIMA as it spins on the centrifuge table.

4.3 MISALIGNMENT MEASUREMENT

When the vector $\vec{F}$ of Figure 4.2 is horizontal, then the sensitive input axis (IA) of the WIMA is nominally vertical. In practice, however, the IA is not quite vertical, and as demonstrated in Section 2.1.2, this fact masks the true input to the accelerometer. As a result, it is imperative that the misalignment angles are determined in the course of calibration. We will first examine the definition of the misalignment angles.

4.3.1 Definition of Accelerometer Misalignment Angles

Figure 4.4 shows a reference coordinate system X-Y-Z. In this system, the acceleration components a_x , a_y , and a_z are considered to be known. Nominally, we would like the accelerometer's sensitive input axis to be aligned with the Z direction. In practice, the accelerometer is not perfectly aligned in this manner due to machining and assembly tolerances. So we consider a second set of coordinate axes, x-y-z that are fixed to the WIMA with the z direction aligned with the true IA. Since the WIMA is axisymmetric, it is not important how we define the orientation of the x and y axes, as long as they are mutually orthogonal to z.

Figure 4.4: Axes definition. The IA of the WIMA is aligned with the z axis. The acceleration components are known along the X-Y-Z axes.

We can define the misalignment of the WIMA by considering a rotation δ_y about the Y axis to the $x'-y'-z'$ system, followed by a rotation δ_x about the x' axis to give the final accelerometer axes, $x-y-z$.

If z is the IA, then the total input to the WIMA by the acceleration components along X, Y, and Z is:

$$A = a_z \cos \delta_y \cos \delta_x + a_x \sin \delta_y \cos \delta_x - a_y \sin \delta_x \quad (4.2)$$

For small misalignment angles, we may simplify equation (4.2) to give:

$$A = a_z + a_x \delta_y - a_y \delta_x \quad (4.3)$$

When we compare equation (4.3) to the IEEE model (equation (2.25)), we see that δ_y and δ_x are equivalent to the misalignments of IA to HA and PA (i.e. d_o and d_p) if a_z , a_x , and a_y are equivalent to a_i , a_p , and a_o respectively.

We must be careful to distinguish between these described misalignments and any intrinsic cross-axis sensitivity, S_T , of the WIMA. At this stage of testing we assume that the value of S_T is nil in order to measure the misalignment angles. At a more advanced stage of testing, these measured misalignments will allow the determination of the true value of S_T using centrifuge tests.

4.3.2 Measuring Misalignment with the Dividing Head

When the WIMA sensor block is mounted on the dividing head carriage, the IA is not perfectly vertical even if $\vec{F}$ (see Figure 4.2) is horizontal. We can illustrate the misalignment with Figure 4.5.

Figure 4.5: Definition of IA misalignment angles. The Z axis is vertical, and the Y axis is aligned with the dividing head rotational axis.

In Figure 4.5, the Y axis corresponds to the dividing head axis, and the X axis is parallel to $\vec{F}$. We see that the initial misalignment of the IA is defined by the angles β and δ .

The component of $\vec{g}$ along the IA is readily seen to be given by:

$$A = g \cos \beta \cos \delta \quad (4.4)$$

As the dividing head carriage is rotated about Y, we see that the misalignment angle δ remains constant. Now, suppose that we denote the dividing head angle to be α , which is zero when $\vec{F}$ (i.e. X) is horizontal. We then see that the IA leads α by the misalignment angle β so that the input to the WIMA is given by:

$$A = g \cos \delta \cos(\beta + \alpha) = g' \cos(\beta + \alpha) \quad (4.5)$$

The relation above may then be plotted, as in Figure 4.6.

Figure 4.6: Gravitational input along the IA of the WIMA with variation of dividing head angle, α . The amplitude, g' , corresponds to equation (4.5).

Now suppose that we rotate the WIMA sensor block by exactly 90° about an axis parallel to Z. In terms of Figure 4.5, this is equivalent to making axis X to be the dividing head axis. Obviously then, the misalignment angles of interest are now ϕ and γ , such that the input to the WIMA is given by:

$$A = g \cos \gamma \cos(\alpha - \phi) = g'' \cos(\alpha - \phi) \quad (4.6)$$

Therefore, in a manner analogous to Figure 4.6, we can extract the angle ϕ .

It now remains to determine the other two angles, δ and γ , in order to define the direction of IA. We do this by writing the following relations directly from Figure 4.4:

$$\cos \delta \sin \beta = \sin \gamma \quad (4.7)$$

$$\cos \gamma \sin \phi = \sin \delta \quad (4.8)$$

$$\cos \delta \cos \beta = \cos \gamma \cos \phi \quad (4.9)$$

Now, by taking the ratios of both sides of equations (4.7) and (4.9), we find that:

$$\tan \gamma = \cos \phi \tan \beta \quad (4.10)$$

And, similarly from equations (4.8) and (4.9):

$$\tan \delta = \cos \beta \tan \phi \quad (4.11)$$

Thus, now that the angles are determined, we can relate them to the required data. By comparing Figure 4.5 to Figure 4.4, we immediately see that:

$$\delta_y = \beta \quad (4.12)$$

and:

$$\delta_x = -\delta \quad (4.13)$$

As such, the determination of the misalignment angles is complete. The actual extraction of the misalignment angles from measurement data will follow from the next section.

4.4 MEASUREMENT OF SCALE FACTOR AND BIAS

If we recall Figure 2.6, we recognize that the output of an accelerometer will show a bias. That is, there will be a voltage output even when the input is zero. As we rotate the dividing head carriage, we expect the WIMA to show an output voltage even when the IA is horizontal; (we are still neglecting cross axis sensitivity at this point). Hence, if we use the IEEE notation of equation (2.25), then the voltage output is expected to follow the relation:

$$E = K_1(g \cos \delta \cos(\beta + \alpha)) + K_1 K_0 = K_1 g' \cos(\beta + \alpha) + K_1 K_0 \quad (4.14)$$

where: E is the WIMA output (volts); K_1 is the scale factor (volts/g); and, K_0 is the bias (g). The output may be plotted as a function of angle α , as illustrated with Figure 4.7.

Figure 4.7: Schematic representation of WIMA output signal over the dividing head travel, including the Bias entity.

We see that we must use this curve to obtain the angle β for the analysis of Section 4.3.2. At the same time, we record the quantities, $(K_1 g')$ and $(K_1 K_0)$. Then, once we have determined the misalignments, β and δ , we can find the scale factor and bias terms as follows:

$$K_1 = \frac{(K_1 g')}{g \cos \delta} \quad (4.15)$$

$$K_0 = \frac{(K_1 K_0)}{K_1} \quad (4.16)$$

In fact, we obtain the measured quantities using a “multi-positional test” technique. This is best described by first re-writing equation (4.14) in an alternate form.

$$E = K_1 g' (\cos \beta \cos \alpha + \sin \beta \sin \alpha) + K_1 K_o \quad (4.17)$$

By setting the position of the dividing head carriage to a number of discrete positions, we can produce sufficient equations to solve for all of the unknowns. As a minimum, we set the dividing head carriage to the following angles:

$$\alpha = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \quad (4.18)$$

and we obtain the following four equations:

$$E_1 = K_1 g' \cos \beta + K_1 K_o \quad (4.19)$$

$$E_2 = -K_1 g' \sin \beta + K_1 K_o \quad (4.20)$$

$$E_3 = -K_1 g' \cos \beta + K_1 K_o \quad (4.21)$$

$$E_4 = K_1 g' \sin \beta + K_1 K_o \quad (4.22)$$

And, by combining these equations in an appropriate manner, we find the desired quantities as shown below.

$$\beta = \arctan \left(\frac{E_4 - E_2}{E_1 - E_3} \right) \quad (4.23)$$

$$K_1 g' = \frac{1}{2} \sqrt{(E_1 - E_3)^2 + (E_4 - E_2)^2} \quad (4.24)$$

$$K_1 K_o = \frac{1}{4} (E_1 + E_2 + E_3 + E_4) \quad (4.25)$$

4.5 MEASUREMENT OF RESOLUTION

The previous tests have established the physical parameters of misalignment, bias, and scale factor. We now wish to examine a fundamental measure of the performance of an accelerometer, its “resolution”. From Figure 2.6, we see that resolution basically defines the minimum input change that yields a measurable change in sensor output. The obvious procedure for determining resolution is to apply decreasingly small discrete changes of input until no change of output is observed.

We must be careful about how we arrange our test setup. From equation (4.5) we see that the sensitivity of input to dividing head angle, α , is given by:

$$\left| \frac{\delta A}{\delta \alpha} \right| = g' \sin(\beta + \alpha) \quad (4.26)$$

This sensitivity is smallest when $\alpha = -\beta$. This means that for a given change in input, the greatest angle change is required when the IA is in the vertical plane. This is an important observation because we want to ensure that our ability to measure angle does not limit our ability to measure the resolution of the WIMA. Hence, when measuring resolution, we do not want to apply a small absolute input (as would be the case if the IA were nominally horizontal). Instead, we align the IA with the vertical plane so that the nominal input is $1g$, and apply a small *change* to the input.

In Section 2.1, the resolution specification for the WIMA was determined to be at least $0.1mg$. We can easily calculate that an angle change of 0.81° (approximately 3000 arcseconds or 50 arcminutes) is required to change the input by $0.1mg$ when the IA is nominally vertical. Since the dividing head has an angle positioning accuracy of 3 arcseconds, then we are comfortable with the capability of the test apparatus.

To test the WIMA’s resolution, we first set it to be nominally vertical (ie. IA is in the vertical plane when $\alpha = -\beta$) and record the output. The WIMA is then tilted by a small angle (say 400 arc minutes) and the output is recorded again. Following this, the WIMA

is reset to its vertical position. The WIMA is subsequently tilted by an angle that is one half of the previous tilt angle. We repeat this process, recording the output at each step, until there is no discernible change in output. In this manner, we determine the resolution of the WIMA, and at the same time, examine the repeatability of the device.

A final test related to resolution is an examination of hysteresis. We may repeatedly apply and remove an input change of 0.1mg (50 arcminutes) and record the output. This test will reveal whether or not the WIMA has any hysteresis behaviour at this level of input. This test should also be performed for gross input changes (on the order of 1g). This second hysteresis test will give an indication of the repeatability of the device after the application of large mechanical stresses.

4.6 MEASUREMENT OF STABILITY

In the previous section, the resolution tests included test for repeatability and hysteresis. As discussed in Section 2.1.4, these tests are an indication of short term stability. For this work, we wish to extend the scope of stability testing somewhat further. We do this by leaving the WIMA subjected to a constant input for an extended period of time (48 hours).

During this time, we record the output of the WIMA so that we may examine any drift. We must also record temperature since we have chosen to control the WIMA temperature for this work, and the electronic feedback part of the system is uncompensated for temperature variation. After the 48 hour period, we can repeat the bias and scale factor test to find if there has been any drift in these quantities.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This thesis has presented a concise overview of the design of a precise measurement instrument that has a specific application. We have seen that such an undertaking involves a very broad scope of engineering. Indeed, we see that our socio-economic considerations identified the potential for a WIM system. By following this up with some fundamental engineering analysis, we determined that an accelerometer would be required for a certifiable WIM system. Our next course was to thoroughly examine the contemporary accelerometer technology. Again, using engineering principles, we were able to set a specification for the WIMA device, and fully understand the behaviour and limitations of acceleration measuring instruments.

In order to expand our knowledge of the causes of the limitations of instruments, we explored the science of material behaviour. In particular, we examined thermal and hysteresis behaviour on a level that is not normally required for gross machine design. The instrument designer must be aware of the most minutely tangible material behaviour and its consequences within a measurement device. Armed with this knowledge, we

were able to tackle the creative design task in a well informed state. Our key result was the statement of the axially symmetric principle for single axis accelerometers.

Once our creative design phase had generated a WIMA concept that was promising on the basis of our instrument design knowledge, we proceed to do the detail analysis. A conscious effort was made to avoid accepting any design feature without some justification, or careful consideration of its potential consequences. Frequently, our most useful design tool was parametric analysis, which allows us to choose a geometry that tends towards optimum performance.

The scope of this work is very diverse, as we discovered during our attempts to calculate the magnetic field. Not only did we need a general understanding of electromagnetic physics, but we had to apply it to a computer assisted numerical computation. In this task, we again attempted to optimize the design of the magnetic circuit and electromagnetic forcer coil in a systematic manner.

The work took on a completely different face with the analysis of the dynamic WIMA system. We modelled the dynamic system and identified the weakness in its response and stability. Consequently, we draw on our knowledge of feedback control to effectively compensate the system and obtain a satisfactory response. In order to physically implement the feedback loop, a collection of electronic modules were synthesized. This required the application of analog IC engineering principles.

Having recognized early on that thermal influences on materials can be problematic for instrument design, we attempted to conceive of a device that has minimal thermal sensitivity. Our final analysis in this respect was a set of elementary heat transfer calculations to estimate the temperature field within the WIMA. Finally, a set of engineering production drawings are generated with due consideration of machining and assembly limitations. Using this set of drawings, the production of a prototype WIMA was supervised from material stock, to assembly, to preparation for testing.

Lastly, we tie together the entire work with the specification of a set of fundamental tests and calibrations. These tests will ultimately identify any shortcomings in the design. The next stage would be to use the test results as a platform for designing a production version of the WIMA. Of course, the WIMA is but one component of a successful WIM system, and the design of such a system is an even more major undertaking.

5.1 CONCLUSIONS

The overview of the work presented above gives an indication of the scope of the project. It is desired to now present some fundamental conclusions that may be drawn from the work. These conclusions are applicable to any future investigations into the WIM and WIMA technology.

- a) An accelerometer is required for any certifiable WIM system, whether the mechanism is constant speed or not.
- b) For a WIM system using only one accelerometer, the principal limitation of existing accelerometer technology is the cross axis sensitivity.
- c) Thermal and hysteretic material behaviour continues to be problematic for instrument design. However, by carefully selecting materials and inducing symmetry in design and minimizing stress concentrations and residual stresses, the instrument can be de-sensitized to these influences.
- d) For an accelerometer to be a truly uniaxially sensitive device, it must be axially symmetric about the sensitive axis over a full 360°.
- e) The WIMA prototype presented in this work incorporates most of the principles for sensor design that are described herewithin. By assembling the mechanical and electronic parts of the WIMA that are described in Chapters 3 and 4, the prototype device is found to function as designed. The device is now prepared to undergo the test program described in Chapter 4.

5.2 RECOMMENDATIONS FOR FUTURE EFFORT

The scope of this work has been very broad. As such, it is necessary to limit the duration, and this means that the final phase of testing is left to future effort. In particular, the WIMA prototype needs to be carefully tested for cross axis sensitivity. This sort of testing is most practical with a centrifuge apparatus. The lack of availability of this equipment set a limit on the scope of testing. The centrifuge tests are the very next phase that is to be addressed after the basic static tests in the pursuit of verifying the design concept.

As alluded to in Section 2.1.4, a critical subset of specifications for an accelerometer is comprised of the short- and long-term and temperature stabilities. This set of tests must also be addressed in the future of the WIMA development. Of course, the temperature sensitivity of the first prototype WIMA is anticipated to be large since a non-optimum material (ie. CRS) was used for construction. Nevertheless, a temperature sensitivity test could be useful for qualitatively assessing the behaviour of the design.

Finally, as mentioned in Section 4.1, the WIMA design concept is such that it is readily modified to a more favourable configuration, in terms of both economy and performance. We again stress that the prototype WIMA was designed so that it could be disassembled to allow for modifications or maintenance. These features are not necessary for a production version.

In keeping with the principles of design for manufacture and assembly (DFMA), the production version of the WIMA need not have any threaded fasteners. This fact alone can reduce the material and production cost of the WIMA by at least an order of magnitude as compared to the prototype. A schematic assembly drawing of a proposed production version of the WIMA has been prepared, and is illustrated as Figure 5.1. The device is readily seen to be similar in concept to Figure 3.3, but we may briefly describe the improvements as follows.

- 1 END CASE
- 2 CENTRE CASE
- 3 CLAMPING DIAPHRAGM
- 4 CLAMPING
- 5 MAGNET
- 6 CAPACITIVE PICKOFF

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Figure 5.1: Advanced proposed WIMA design. Note the efficient use of material, and the absence of threaded fasteners.

1) The production WIMA uses capacitive pickoffs instead of LVDTs. This eliminates the expense of LVDTs, while providing improved accuracy of displacement measurement. This will improve the resolution of the device. Also, no LVDT clamps are required, which implies a large reduction in machining and part count.

2) The production WIMA is shorter than the prototype. This is mainly due to the elimination of the LVDTs. As a result, less material and machining is required for the end caps. As Figure 5.1 reveals, the efficiency of material usage is improved, with a large reduction in wasted material from the end caps. For a WIMA made from INVAR, this implies a large savings in material cost.

3) There are no threaded fasteners in the production WIMA. Every piece is press fit or bonded (eg. the capacitive pickoff plates) to its adjacent component. This reduces both the part count and machining time by at least an order of magnitude. Full axial symmetry is also realized without the fasteners.

4) The device is hermetically sealed by electron-beam welding the end caps to the centre case. The electrical leads can pass through a Bendix type terminal located on one end cap. The interior of the device can be evacuated of air and filled with a dry Nobel gas (eg. Argon or Helium). This will reduce instability and electrical conductor degradation due to moisture. Also, the hermetic sealing eliminates changes in the buoyant force on the proof mass due to atmospheric pressure changes.

5) The flexure diaphragms are fixed to the proof mass by means of a snap fit, perhaps assisted by an adhesive or laser weld. This eliminates many threaded fasteners, and avoids any microslip that may occur with a clamped flexure. Machining and assembly costs are greatly reduced by this feature. Also, a smaller fraction of the proof mass is dead (ie. not active with forcer windings), so the design is more efficient.

6) The air gap of the magnetic circuit is somewhat narrower. The positively located assembly method allows the clearances to be smaller. As a result the

magnetic field in the gap is stronger, and the design is more efficient. The small clearance has the added benefit of increased mechanical damping. This helps to stabilize the device in its unpowered state.

7) The coil form could be made of Zerodur to improve its thermal stability and reduce the weight of the proof mass.

8) Finally, it is anticipated that the rebalance electronics required for the second generation WIMA could be largely handled by a programmable digital processor. As discussed earlier, the digital processing of the signal would allow for a temperature mapped compensation of the system as a whole. Such a processor would be economically developed by simply programming a generic device.

The above list makes it readily apparent that much improvement of the WIMA design is easily achieved for production versions. The identification of these improvements was considered throughout the design of the WIMA. After the fundamental tests are performed on the prototype WIMA, the next task would be to implement the above improvements into a second generation WIMA prototype. At this stage, it would then be feasible to fully assess the suitability of the design in terms of its stabilities, especially thermal. We see that the second generation prototype could be made economically from INVAR, so that the thermal stability is anticipated to be excellent.

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APPENDIX A

CURVE FIT SOLUTIONS FOR FLEXURE DIAPHRAGM DESIGN

The data of Table 3.1 may be fitted to an approximating function in order to make clear the relationship that is described by the data. This exercise leads to the equations (3.10) and (3.11). The development of these equations is described in this appendix.

Let us first consider the c' data, and its corresponding approximating equation, (3.10). We assume a form of the curve-fit equation to be:

$$c' = k_1 \sqrt{a/b} + k_2 \quad (\text{A.1})$$

In order to make the remaining analysis less cumbersome, we will operate on an equation of the form:

$$y = k_1 \sqrt{x} + k_2 \quad (\text{A.2})$$

Choosing a least-squares technique, and representing the tabulated data as y_i and x_i , we define the residual to be:

$$R = \sum_i (y_i - k_1 \sqrt{x_i} - k_2)^2 \quad (\text{A.3})$$

We then set the partial derivatives to zero:

$$\frac{\partial R}{\partial k_1} = 2 \sum_i (y_i - k_1 \sqrt{x_i} - k_2) (-\sqrt{x_i}) = 0 \quad (\text{A.4})$$

$$\frac{\partial R}{\partial k_2} = 2 \sum_i (y_i - k_1 \sqrt{x_i} - k_2) (-1) = 0 \quad (\text{A.5})$$

And we solve the equation pair (A.4) and (A.5) to obtain the simultaneous minimum:

$$k_1 = \frac{\sum y_i \sqrt{x_i} - \frac{1}{n} \sum y_i \sum \sqrt{x_i}}{\sum x_i - \frac{1}{n} (\sum \sqrt{x_i})^2} \quad (\text{A.6})$$

$$k_2 = \frac{1}{n} (\sum y_i - k_1 \sum \sqrt{x_i}) \quad (\text{A.7})$$

where it is implied that the summations are over i , and where n is the total number of data points.

We assume the same form of approximating equation for the c'' data of Table 3.1 (see equation (3.11)). After performing the calculations of (A.6) and (A.7) we obtain the desired coefficients as shown in the table below:

Table A.1.1: Coefficients for equations (3.10) and (3.11). Note that the curve fit for the c' data has omitted the point of $a/b=1.25$ because a better fit is obtained in this manner.

ITEM	k_1	k_2
c'	0.108975	-0.12797
c''	0.910257	-0.89106

The data of Table 3.1, and the corresponding approximations of equations (3.10) and (3.11) are plotted in Figures A.1 and A.2. From these figures, it is apparent that the fit of the curves to the data is acceptable for the purposes intended.

Figure A.1: Plot of curve fit equation (3.10) and c' data of Table 3.1. Note that the point, $a/b=1.25$ is omitted from the curve fit solution. Note that the spacing of the abscissa is non linear.

Figure A.2: Plot of curve fit equation (3.11) and c'' data of Table 3.1. Note that the spacing of the abscissa is non linear.

The data of Table 3.2 may also be fit to an approximating curve to reveal the nature of the functional relationship. We assume in this case, an approximate power function:

$$c''' = k_3 \left(\frac{a}{b}\right)^3 \quad (\text{A.8})$$

And, again, to make the analysis less cumbersome, we operate on the following equation:

$$y = k_3 x^3 \quad (\text{A.9})$$

The residual is defined to be:

$$R = \sum (y_i - k_3 x_i^3)^2 \quad (\text{A.10})$$

We then set the partial derivative to zero to obtain the minimum:

$$\frac{\partial R}{\partial k_3} = 2 \sum (y_i - k_3 x_i^3) \cdot (-x_i^3) = 0 \quad (\text{A.11})$$

or:

$$k_3 = \frac{\sum y_i x_i^3}{\sum x_i^6} \quad (\text{A.12})$$

When we perform the above calculation on the data of Table 3.2 we find:

$$k_3 = 0.0089 \quad (\text{A.13})$$

The data of Table 3.2 and the relation of equation (A.8), using the coefficient above, is plotted in Figure A.3. From this figure, we see that the fit of the curve is acceptable for the purpose of the analysis of Section 3.2.2.

Figure A.3: Plot of curve fit equation, $c''' = 0.0089\left(\frac{a}{b}\right)^3$, and data of Table 3.2.
Note that the spacing of the abscissa is non linear.

APPENDIX B

FINITE DIFFERENCE PROGRAMME AND SUPPORTING INFORMATION

A listing of the FORTRAN programme, "FICYL.F", used to calculate the magnetic field supported within the WIMA device is presented in this appendix. In addition to this programme, there are some important supporting documentations presented. This information is summarized as follows:

- A sketch of the finite difference grid used to represent the WIMA geometry within the program
- A data initialization file that contains the grid positions of all of the WIMA geometry boundaries defined in the grid sketch. The file also includes the material and grid size information required by FICYL.F.
- A second FORTRAN programme "INTFILE.F" that uses the data initialization file to define the material and boundary type of all grid points. The initial value of every point is also set by this programme. This information is stored in a database file that is subsequently operated on by the finite difference programme, "FICYL.F".

Figure A.4: Grid representation of WIMA geometry. One quarter of the WIMA is modelled since symmetry can be invoked to reduce the scope of the problem to be solved. The boundaries, corners and T-section points are labelled.

The data file shown below is the initialization file that describes the grid positions of the boundaries, and the problem constant data.

```
1 12
2 36
3 19
4 39
5 22
6 22
7 52
8 19
9 92
10 112
11 24
12 42
13 47
14 34
15 36
16 37
17 82
18 47
19 92
20 112
21 0
22 0
23 0
24 0
32 32 0.31750E-03 0.10000E-04
0.10800E+01 0.80000E+06 0.31416E-02
```

The FORTRAN programme presented here, "INTFILE.F", uses the initialization data to generate the grid point database that is operated on by the finite difference programme.

```

C      PROGRAM INTFILE
C
C      IMPLICIT NONE
C
C      INTEGER N,M,I,J,AIR,IRON,MAGNET,TYPE
C      INTEGER K,KK,B(30)
C      INTEGER GAPI,GAPI
C      REAL GRID,CONVRG,BR,HC,MUFE
C      CHARACTER JLINE(170)
C      DATA AIR/1/, IRON/2/, MAGNET/3/
C
C      OPEN(UNIT=12,FILE='GRID.IN',STATUS='OLD')
C      DO K=1,24
C        READ(12,FMT='(2X,I2,1X,I3)')KK,B(KK)
C        WRITE(6,*)KK,B(KK)
C      ENDDO
C      READ(12,FMT='(2I5,2E15.5)')GAPI,GAPI,GRID,CONVRG
C      READ(12,FMT='(3E15.5)')BR,HC,MUFE
C      CLOSE(12)
C      N=180
C      M=170
C      OPEN(UNIT=10,FILE='GRID.TST',STATUS='NEW')
C      OPEN(UNIT=13,FILE='GRID.OUT',STATUS='NEW')
C      WRITE(UNIT=13,FMT='(2I5,2E15.5)')GAPI,GAPI,GRID,CONVRG
C      WRITE(UNIT=13,FMT='(3E15.5)')BR,HC,MUFE
C      WRITE(UNIT=6,FMT='(2I5,2E15.5)')GAPI,GAPI,GRID,CONVRG
C      WRITE(UNIT=6,FMT='(3E15.5)')BR,HC,MUFE
C      DO I=1,N
C        DO J=1,M
C          IF (J.LT.B(1)) THEN
C            IF (J.LT.B(15)) THEN
C              TYPE = AIR
C            ELSEIF (J.EQ.B(15)) THEN
C              TYPE = -25
C            ELSEIF (J.LT.B(20)) THEN
C              TYPE = IRON
C            ELSEIF (J.EQ.B(20)) THEN
C              TYPE = -30
C            ELSE
C              TYPE = AIR
C            ENDIF
C          ELSEIF (J.EQ.B(1)) THEN
C            IF (J.LT.B(5)) THEN
C              TYPE = -11
C            ELSEIF (J.EQ.B(5)) THEN
C              TYPE = -44
C            ELSEIF (J.LT.B(15)) THEN
C              TYPE = AIR

```

```

ELSEIF (J.EQ.B(15)) THEN
  TYPE = -25
ELSEIF (J.LT.B(20)) THEN
  TYPE = IRON
ELSEIF (J.EQ.B(20)) THEN
  TYPE = -30
ELSE
  TYPE = AIR
ENDIF
ELSEIF (J.LT.B(2)) THEN
  IF (J.LT.B(5)) THEN
    TYPE = MAGNET
  ELSEIF (J.EQ.B(5)) THEN
    TYPE = -15
  ELSEIF (J.LT.B(15)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(15)) THEN
    TYPE = -25
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (J.EQ.B(2)) THEN
  IF (J.LT.B(3)) THEN
    TYPE = MAGNET
  ELSEIF (J.EQ.B(3)) THEN
    TYPE = -41
  ELSEIF (J.LT.B(5)) THEN
    TYPE = -12
  ELSEIF (J.EQ.B(5)) THEN
    TYPE = -1
  ELSEIF (J.LT.B(15)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(15)) THEN
    TYPE = -25
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (J.LT.B(16)) THEN
  IF (J.LT.B(3)) THEN
    TYPE = MAGNET
  ELSEIF (J.EQ.B(3)) THEN
    TYPE = -13
  ELSEIF (J.LT.B(6)) THEN
    TYPE = IRON

```

```

ELSEIF (J.EQ.B(6)) THEN
  TYPE = -16
ELSEIF (J.LT.B(15)) THEN
  TYPE = AIR
ELSEIF (J.EQ.B(15)) THEN
  TYPE = -25
ELSEIF (J.LT.B(20)) THEN
  TYPE = IRON
ELSEIF (J.EQ.B(20)) THEN
  TYPE = -30
ELSE
  TYPE = AIR
ENDIF
ELSEIF (J.EQ.B(16)) THEN
  IF (J.LT.B(3)) THEN
    TYPE = MAGNET
  ELSEIF (J.EQ.B(3)) THEN
    TYPE = -13
  ELSEIF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(15)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(15)) THEN
    TYPE = -51
  ELSEIF (J.LT.B(17)) THEN
    TYPE = -26
  ELSEIF (J.EQ.B(17)) THEN
    TYPE = -52
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (J.LT.B(4)) THEN
  IF (J.LT.B(3)) THEN
    TYPE = MAGNET
  ELSEIF (J.EQ.B(3)) THEN
    TYPE = -13
  ELSEIF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(17)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(17)) THEN
    TYPE = -27
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON

```

```

ELSEIF (J.EQ.B(20)) THEN
  TYPE = -30
ELSE
  TYPE = AIR
ENDIF
ELSEIF (I.EQ.B(4)) THEN
  IF (J.LT.B(3)) THEN
    TYPE = -14
  ELSEIF (J.EQ.B(3)) THEN
    TYPE = -42
  ELSEIF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(17)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(17)) THEN
    TYPE = -27
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (I.LT.B(12)) THEN
  IF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(17)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(17)) THEN
    TYPE = -27
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (I.EQ.B(12)) THEN
  IF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(11)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(11)) THEN
    TYPE = -47
  ELSEIF (J.LT.B(14)) THEN
    TYPE = -22

```

```

ELSEIF (J.EQ.B(14)) THEN
  TYPE = -48
ELSEIF (J.LT.B(17)) THEN
  TYPE = AIR
ELSEIF (J.EQ.B(17)) THEN
  TYPE = -27
ELSEIF (J.LT.B(20)) THEN
  TYPE = IRON
ELSEIF (J.EQ.B(20)) THEN
  TYPE = -30
ELSE
  TYPE = AIR
ENDIF
ELSEIF (J.LT.B(13)) THEN
  IF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(11)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(11)) THEN
    TYPE = -21
  ELSEIF (J.LT.B(14)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(14)) THEN
    TYPE = -24
  ELSEIF (J.LT.B(17)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(17)) THEN
    TYPE = -27
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (J.EQ.B(13)) THEN
  IF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(11)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(11)) THEN
    TYPE = -46
  ELSEIF (J.LT.B(14)) THEN
    TYPE = -23
  ELSEIF (J.EQ.B(14)) THEN
    TYPE = -49
  ELSEIF (J.LT.B(17)) THEN
    TYPE = AIR

```

```

ELSEIF (J.EQ.B(17)) THEN
  TYPE = -53
ELSEIF (J.LT.B(19)) THEN
  TYPE = -28
ELSEIF (J.EQ.B(19)) THEN
  TYPE = -54
ELSEIF (J.LT.B(20)) THEN
  TYPE = IRON
ELSEIF (J.EQ.B(20)) THEN
  TYPE = -30
ELSE
  TYPE = AIR
ENDIF
ELSEIF (J.LT.B(7)) THEN
  IF (J.LT.B(6)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -16
  ELSEIF (J.LT.B(19)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(19)) THEN
    TYPE = -29
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (J.EQ.B(7)) THEN
  IF (J.LT.B(8)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(8)) THEN
    TYPE = -43
  ELSEIF (J.LT.B(6)) THEN
    TYPE = -17
  ELSEIF (J.EQ.B(6)) THEN
    TYPE = -45
  ELSEIF (J.LT.B(19)) THEN
    TYPE = AIR
  ELSEIF (J.EQ.B(19)) THEN
    TYPE = -29
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (J.LT.B(9)) THEN
  IF (J.LT.B(8)) THEN
    TYPE = IRON

```

```

ELSEIF (J.EQ.B(8)) THEN
  TYPE = -18
ELSEIF (J.LT.B(19)) THEN
  TYPE = AIR
ELSEIF (J.EQ.B(19)) THEN
  TYPE = -29
ELSEIF (J.LT.B(20)) THEN
  TYPE = IRON
ELSEIF (J.EQ.B(20)) THEN
  TYPE = -30
ELSE
  TYPE = AIR
ENDIF
ELSEIF (I.EQ.B(9)) THEN
  IF (J.LT.B(8)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(8)) THEN
    TYPE = -50
  ELSEIF (J.LT.B(19)) THEN
    TYPE = -19
  ELSEIF (J.EQ.B(19)) THEN
    TYPE = -55
  ELSEIF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (I.LT.B(10)) THEN
  IF (J.LT.B(20)) THEN
    TYPE = IRON
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -30
  ELSE
    TYPE = AIR
  ENDIF
ELSEIF (I.EQ.B(10)) THEN
  IF (J.LT.B(20)) THEN
    TYPE = -20
  ELSEIF (J.EQ.B(20)) THEN
    TYPE = -56
  ELSE
    TYPE = AIR
  ENDIF
ELSE
  TYPE = AIR
ENDIF

```

```

C
13  FORMAT(3I5,3E15.5)
C

```

```

  IF (TYPE.EQ.1) THEN

```



```

        JLINE(J)='- '
        WRITE(13,13)I,J,1,0,1,0,0,0,0
    ELSEIF (TYPE.EQ.2) THEN
        JLINE(J)='- '
        WRITE(13,13)I,J,2,0,0,0,1,0,0
    ELSEIF (TYPE.EQ.3) THEN
        JLINE(J)='-+'
        WRITE(13,13)I,J,3,0,0,0,0,0,9
    ELSEIF ((TYPE.LT.0).AND.(TYPE.GT.-11)) THEN
        JLINE(J)='-T'
        WRITE(13,13)I,J,TYPE,0,0,0,0,0,0
    ELSEIF ((TYPE.LT.-10).AND.(TYPE.GT.-41)) THEN
        JLINE(J)='-B'
        WRITE(13,13)I,J,TYPE,0,0,0,0,0,0
    ELSEIF ((TYPE.LT.-40).AND.(TYPE.GT.-70)) THEN
        JLINE(J)='-C'
        WRITE(13,13)I,J,TYPE,0,0,0,0,0,0
    ENDIF
C
    ENDDO
    WRITE(10,FMT='(2X,170A1)')JLINE
C
    ENDDO
    CLOSE(10)
    CLOSE(13)
C
    STOP
    END

```

The FORTRAN programme presented below, "FICYL.F", calculates, by means of a finite-difference technique, in cylindrical coordinates, the magnetic potential field supported in the WIMA prototype. The radial component of the magnetic field along the mid-cylinder of the air gap is also calculated.

```

C FICYL.F    MAGNETIC FIELD CALCULATION    ACCELEROMETER
C          BY FINITE DIFFERENCES
C
C THIS PROGRAM HAS BEEN ADAPTED FROM FIELDS.FOR, PROPERTY OF ANCON
C SPACE TECHNOLOGY CORP.. THE PROGRAM WAS MODIFIED TO SOLVE THE
C DIFFERENCE EQUATIONS IN CYLINDRICAL COORDINATES. ALSO, ADDITIONAL
C T-SECTION BOUNDARY SUBROUTINES HAVE BEEN ADDED, AND THE CALLS TO
C THE BOUNDARY SUBROUTINES HAVE BEEN MODIFIED TO MATCH THE GEOMETRY
C OF THE ACCELEROMETER
C
C   THE FOLLOWING ARRAYS NEED TO BE RE-DIMENSIONED IF
C   N, M, AND L ARE CHANGED. IT WILL ALSO BE NECESSARY
C   TO CHANGE A FORMAT STATEMENT IN SUBROUTINE STORE
C
C IMPLICIT NONE
C
C REAL PHI(180,170,3), MU(3), TMP(3), ROW(180), B(2)
C INTEGER TYPE(180,170)
C
C REAL BIGPHI, BR, CNVRG, GRID, HCOERC, MAXR, PI, FJUNK
C REAL ALPHA
C INTEGER I, J, K, L, M, N, GAPI, GAPJ, ITER, STDIN, STDOUT
C INTEGER AIR, IRON, MAGNET
C DATA PI/3.1415926/, STDIN/5/, STDOUT/6/
C DATA AIR/1/, IRON/2/, MAGNET/3/
C DATA N/180/, M/170/, L/3/
C
C ALPHA = 1.1
C MU(AIR) = 4.0 * PI * 1.0E-07
C
C ...INITIALIZE ARRAYS AND READ INPUT DATA FILE
C CALL INIT(N,M,L,PHI,TYPE,GAPI,GAPJ,GRID,CNVRG,
C & BR,HCOERC,MU,TMP)
C
C ...MAIN ITERATION LOOP BASED ON TEST OF MAXIMUM RELATIVE
C ...RESIDUAL VERSUS THE CONVERGENCE CRITERIA
C ITER = 0
C MAXR = 1.0E+6
C BIGPHI = 1.0E-6
C WHILE (MAXR/BIGPHI .GT. CNVRG)
C 150 CONTINUE
C     ITER = ITER + 1
C     CALL CALC(N,M,L,PHI,TYPE,MU,MAXR,ALPHA,GRID,BR)
C     CALL LARGE(N,M,L,PHI,BIGPHI)
C ...STORE SOME INTERMEDIATE RESULTS

```

```

        IF (MOD(ITER, 500) .EQ. 0 ) THEN
            CALL SAVE(N,M,L,PHI,TYPE,BIGPHI,MAXR,ITER,ROW,STDOUT,
                GAPI,GAPI,GRID,CNVRG,BR,HCOERC,MU)
        END IF
C    ...WRITE TO THE SCREEN EACH ITERATION
        WRITE(STDOUT,FMT="(2X,I4)")ITER
C    END WHILE
        IF (MAXR/BIGPHI .GT. CNVRG) GO TO 150
C
C    ...SAVE THE FIELD IN THE AIR GAP TO A FILE
        CALL GAPFLD(N,M,L,PHI,MU,GAPI,GAPI,GRID,B)
C
C    ...SAVE THE FINAL RESULTS
        CALL SAVE(N,M,L,PHI,TYPE,BIGPHI,MAXR,ITER,ROW,STDOUT,
            GAPI,GAPI,GRID,CNVRG,BR,HCOERC,MU)
        STOP
        END
C
C
        SUBROUTINE INIT(N,M,L,PHI,TYPE,GAPI,GAPI,GRID,CNVRG,
            &    BR,HCOERC,MU,TMP)
C
        IMPLICIT NONE
C
        INTEGER N, M, L, I, J, K, TYPE(N,M), GAPI, GAPI
        REAL PHI(N,M,L), GRID, CNVRG, BR, HCOERC, MU(L), TMP(L)
        INTEGER STDOUT, STDIN, INFILE, IOS, AIR, IRON, MAGNET, TYP
        CHARACTER*14 NAME
        DATA STDOUT/6/, STDIN/5/, INFILE/8/
        DATA AIR/1/, IRON/2/, MAGNET/3/
        1 FORMAT(1X, 'INPUT DATA FILE NAME? ')
        2 FORMAT(A14)
        3 FORMAT(2I5, 2E15.5)
        4 FORMAT(3E15.5)
        5 FORMAT(3I5, 3E15.5)
C
C    ...START WITH AIR ONLY
        DO 120 I = 1, N
            DO 110 J = 1, M
                TYPE(I,J) = AIR
                DO 100 K = 1, L
                    PHI(I, J, K) = 0.0
                100 CONTINUE
            110 CONTINUE
        120 CONTINUE
C
C    ...GET NAME OF THE THE INPUT DATA FILE
        WRITE(STDOUT, 1)
        READ(STDIN, 2) NAME
C    ...OPEN AND READ THE INPUT DATA
        OPEN(UNIT = INFILE, FILE = NAME, IOSTAT = IOS)
        IF (IOS .NE. 0) CALL ERROR(STDOUT, 1, 0)

```

```

C
  READ(INFILE, 3) GAPI, GAPI, GRID, CNVRG
  READ(INFILE, 4) BR, HCOERC, MU(IRON)
  MU(MAGNET) = BR/HCOERC
C
  READ(INFILE, 5) I, J, TYP, TMP(AIR), TMP(IRON), TMP(MAGNET)
130 CONTINUE
    TYPE(I,J) = TYP
    DO 140 K = 1, L
      PHI(I,J,K) = TMP(K)
140  CONTINUE
    READ(INFILE,5) I, J, TYP, TMP(AIR), TMP(IRON), TMP(MAGNET)
    IF (I .GT. 0) GO TO 130
C
  CLOSE(INFILE)
  RETURN
  END
C
C
  SUBROUTINE CALC(N,M,L,PHI,TYPE,MU,MAXR,ALPHA,GRID,BR)
C
  IMPLICIT NONE
C
  INTEGER N,M,L
  REAL PHI(N,M,L), MU(L), MAXR, ALPHA, GRID, BR
  INTEGER TYPE(N,M)
  INTEGER AIR, IRON, MAGNET
  INTEGER TOP, BOTTOM, LEFT, RIGHT, IN, OUT
  INTEGER BND(30)
  INTEGER KRNR(30)
  INTEGER TEE(10)
  INTEGER P
  INTEGER ANCHOR
  INTEGER I, J, K, NM1, NM2, MM1, MM2, STDOUT
  REAL R,RADIUS
C
  DATA AIR/1/, IRON/2/, MAGNET/3/
C
  DATA ANCHOR/-99/, STDOUT/6/
C
  DO P = 1,10
    TEE(P)=-1*P
  ENDDO
C
  DO P=1,30
    BND(P)=-1*P-10
    KRNR(P)=-1*P-40
  ENDDO
C
  NM1 = N - 1
  NM2 = N - 2
  MM1 = M - 1

```

```

MM2 = M - 2
MAXR = 0.0
C
C ...INDUCE SYMMETRY ALONG AXIAL CENTRE LINE (J=2)
DO 90 I = 1, N
    DO 80 K = AIR, MAGNET
        PHI(I,1,K) = PHI(I,3,K)
80 CONTINUE
90 CONTINUE
C
C ...INDUCE SYMMETRY ALONG MID PLANE LINE (I=2)
DO 110 J = 1, M
    DO 100 K = AIR, MAGNET
        PHI(1,J,K) = PHI(3,J,K)
100 CONTINUE
110 CONTINUE
C
C ...SET DERIVATIVES OF POTENTIAL TO ZERO AT OUTER BOUNDARIES
DO 120 I = 1, N
    PHI(I,M,AIR) = PHI(I,MM2,AIR)
120 CONTINUE
DO 130 J = 1, M
    PHI(N,J,AIR) = PHI(NM2,J,AIR)
130 CONTINUE
C
C ...SWEEP THROUGH FROM TOP TO BOTTOM
DO 150 J = MM1, 2, -1
C
C ... CALCULATE RADIUS
RADIUS = (J-2)*GRID
C
C ... SWEEP THROUGH LEFT TO RIGHT
DO 140 I = 2, NM1
C
    IF (TYPE(I,J) .EQ. AIR) THEN
C
C ... SET PHI INDEX TO AIR AND CALCULATE RESIDUAL/4
K = AIR
IF (J.EQ.2) THEN
    R=0.25*(PHI(I-1,J,K)+PHI(I+1,J,K)
& +2*PHI(I,J+1,K)+2*PHI(I,J-1,K)-6*PHI(I,J,K))
ELSE
    R=0.25*(PHI(I-1,J,K)+PHI(I+1,J,K)
& +(1+0.5*GRID/RADIUS)*PHI(I,J+1,K)
& +(1-0.5*GRID/RADIUS)*PHI(I,J-1,K))
& -PHI(I,J,K)
ENDIF
C
C ... CALCULATE NEW VALUE AND MAX RESIDUAL
PHI(I,J,K) = PHI(I,J,K) + ALPHA * R
MAXR = AMAX1(ABS(R),MAXR)
C

```

```

C      ELSE IF (TYPE(I,J) .EQ. IRON) THEN
C      SET PHI INDEX TO IRON
      K = IRON
      IF (J.EQ.2) THEN
        R=0.25*(PHI(I-1,J,K)+PHI(I+1,J,K)
&      +2*PHI(I,J+1,K)+2*PHI(I,J-1,K)-6*PHI(I,J,K))
      ELSE
        R=0.25*(PHI(I-1,J,K)+PHI(I+1,J,K)
&      +(1+0.5*GRID/RADIUS)*PHI(I,J+1,K)
&      +(1-0.5*GRID/RADIUS)*PHI(I,J-1,K))
&      -PHI(I,J,K)
      ENDIF
C
      PHI(I,J,K) = PHI(I,J,K) + ALPHA * R
      MAXR = AMAX1(ABS(R),MAXR)

      ELSE IF (TYPE(I,J) .EQ. MAGNET) THEN
C      ... SET PHI INDEX TO MAGNET
      K = MAGNET
      IF (J.EQ.2) THEN
        R=0.25*(PHI(I-1,J,K)+PHI(I+1,J,K)
&      +2*PHI(I,J+1,K)+2*PHI(I,J-1,K)-6*PHI(I,J,K))
      ELSE
        R=0.25*(PHI(I-1,J,K)+PHI(I+1,J,K)
&      +(1+0.5*GRID/RADIUS)*PHI(I,J+1,K)
&      +(1-0.5*GRID/RADIUS)*PHI(I,J-1,K))
&      -PHI(I,J,K)
      ENDIF
C
      PHI(I,J,K) = PHI(I,J,K) + ALPHA * R
      MAXR = AMAX1(ABS(R),MAXR)
C
C MATERIAL BOUNDARIES
      ELSE IF (TYPE(I,J) .EQ. BND(1)) THEN
        LEFT = AIR
        RIGHT = MAGNET
        CALL VRTCL(I,J,LEFT,RIGHT,GRID,-1*BR,N,M,L,PHI,MU)
      ELSE IF (TYPE(I,J) .EQ. BND(2)) THEN
        LEFT = MAGNET
        RIGHT = IRON
        CALL VRTCL(I,J,LEFT,RIGHT,GRID,BR,N,M,L,PHI,MU)
      ELSE IF (TYPE(I,J) .EQ. BND(3)) THEN
        TOP = IRON
        BOTTOM = MAGNET
        CALL HRZNTL(I,J,TOP,BOTTOM,GRID,0.0,N,M,L,PHI,MU)
      ELSE IF (TYPE(I,J) .EQ. BND(4)) THEN
        LEFT = MAGNET
        RIGHT = IRON
        CALL VRTCL(I,J,LEFT,RIGHT,GRID,BR,N,M,L,PHI,MU)
      ELSE IF (TYPE(I,J) .EQ. BND(5)) THEN

```

```

      TOP = AIR
      BOTTOM = MAGNET
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(6)) THEN
      TOP = AIR
      BOTTOM = IRON
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(7)) THEN
      LEFT = IRON
      RIGHT = AIR
      CALL VRTCL(I,J, LEFT, RIGHT, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(8)) THEN
      TOP = AIR
      BOTTOM = IRON
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(9)) THEN
      LEFT = AIR
      RIGHT = IRON
      CALL VRTCL(I,J, LEFT, RIGHT, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(10)) THEN
      LEFT = IRON
      RIGHT = AIR
      CALL VRTCL(I,J, LEFT, RIGHT, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(11)) THEN
      TOP = IRON
      BOTTOM = AIR
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(12)) THEN
      LEFT = AIR
      RIGHT = IRON
      CALL VRTCL(I,J, LEFT, RIGHT, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(13)) THEN
      LEFT = IRON
      RIGHT = AIR
      CALL VRTCL(I,J, LEFT, RIGHT, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(14)) THEN
      TOP = AIR
      BOTTOM = IRON
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(15)) THEN
      TOP = IRON
      BOTTOM = AIR
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(16)) THEN
      LEFT = IRON
      RIGHT = AIR
      CALL VRTCL(I,J, LEFT, RIGHT, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(17)) THEN
      TOP = IRON
      BOTTOM = AIR
      CALL HRZNTL(I,J, TOP, BOTTOM, GRID, 0.0, N, M, L, PHI, MU)
ELSE IF (TYPE(I,J) .EQ. BND(18)) THEN

```

```

LEFT = IRON
RIGHT = AIR
CALL VRTCL(I,J,LEFT,RIGHT,GRID,0.0,N,M,L,PHI,MU)
ELSE IF (TYPE(I,J) .EQ. BND(19)) THEN
TOP = IRON
BOTTOM = AIR
CALL HRZNTL(I,J,TOP,BOTTOM,GRID,0.0,N,M,L,PHI,MU)
ELSE IF (TYPE(I,J) .EQ. BND(20)) THEN
TOP = AIR
BOTTOM = IRON
CALL HRZNTL(I,J,TOP,BOTTOM,GRID,0.0,N,M,L,PHI,MU)
ELSE IF (TYPE(I,J) .EQ. BND(25)) THEN
TOP = AIR
BOTTOM = AIR
ELSE IF (TYPE(I,J) .EQ. BND(26)) THEN
LEFT = AIR
RIGHT = IRON
ELSE IF (TYPE(I,J) .EQ. BND(27)) THEN
LEFT = AIR
RIGHT = IRON
ELSE IF (TYPE(I,J) .EQ. BND(28)) THEN
TOP = AIR
BOTTOM = AIR
ELSE IF (TYPE(I,J) .EQ. BND(29)) THEN
LEFT = AIR
RIGHT = AIR
ELSE IF (TYPE(I,J) .EQ. BND(30)) THEN
TOP = AIR
BOTTOM = AIR

```

C

C ... CORNERS

```

ELSE IF (TYPE(I,J) .EQ. KRNR(1)) THEN
IN = IRON
OUT = MAGNET
CALL CORNR1(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(2)) THEN
IN = MAGNET
OUT = IRON
CALL CORNR3(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(3)) THEN
IN = AIR
OUT = IRON
CALL CORNR1(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(4)) THEN
IN = MAGNET
OUT = AIR
CALL CORNR4(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(5)) THEN
IN = IRON
OUT = AIR
CALL CORNR3(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(6)) THEN

```



```

      IN = IRON
      OUT = AIR
      CALL CORNR2(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(7)) THEN
      IN = IRON
      OUT = AIR
      CALL CORNR1(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(8)) THEN
      IN = IRON
      OUT = AIR
      CALL CORNR4(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(9)) THEN
      IN = IRON
      OUT = AIR
      CALL CORNR3(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(10)) THEN
      IN = AIR
      OUT = IRON
      CALL CORNR2(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(11)) THEN
      IN = IRON
      OUT = AIR
      CALL CORNR2(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(12)) THEN
      IN = AIR
      OUT = IRON
      CALL CORNR4(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(13)) THEN
      IN = IRON
      OUT = AIR
      CALL CORNR2(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(14)) THEN
      IN = AIR
      OUT = IRON
      CALL CORNR4(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(15)) THEN
      IN = AIR
      OUT = IRON
      CALL CORNR3(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(16)) THEN
      IN = IRON
      OUT = AIR
      CALL CORNR3(I,J,IN,OUT,1.0,N,M,L,PHI,MU,MAXR,GRID)
ELSE IF (TYPE(I,J) .EQ. KRNR(17)) THEN
      IN = AIR
      OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(18)) THEN
      IN = AIR
      OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(19)) THEN
      IN = AIR
      OUT = AIR

```

```

ELSE IF (TYPE(I,J) .EQ. KRNR(20)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(21)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(22)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(23)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(24)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(25)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(26)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(27)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(28)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(29)) THEN
  IN = AIR
  OUT = AIR
ELSE IF (TYPE(I,J) .EQ. KRNR(30)) THEN
  IN = AIR
  OUT = AIR
C
C .. T SECTIONS
  ELSE IF (TYPE(I,J) .EQ. TEE(1)) THEN
    CALL TSECT1(I,J,GRID,BR,N,M,L,PHI,MU)
  ELSE IF (TYPE(I,J) .EQ. TEE(2)) THEN
    CALL TSECT2(I,J,GRID,BR,N,M,L,PHI,MU)
C
  ELSE IF (TYPE(I,J) .EQ. ANCHOR) THEN
    PHI(I,J,MAGNET) = 0.0
  ELSE
    CALL ERROR(STDOUT,3, TYPE(I,J))
  END IF
140 CONTINUE
150 CONTINUE
RETURN
END
C
C
SUBROUTINE LARGE(N,M,L,PHI,BIGPHI)

```

```

C
IMPLICIT NONE
C
INTEGER N, M, L, I, J, K, MM1
REAL PHI(N,M,L), BIGPHI
BIGPHI = 0.0
MM1 = M - 1
DO 120 J = MM1, 2, -1
  DO 110 I = 2, N
    DO 100 K = 1, L
      BIGPHI = AMAX1(BIGPHI, ABS(PHI(I,J,K)))
100  CONTINUE
110  CONTINUE
120 CONTINUE
RETURN
END
C
C
SUBROUTINE SAVE(N,M,L,PHI,TYPE,BIGPHI,MAXR,ITER,ROW,STDOUT,
.      GAPI,GAPI,GRID,CNVRG,BR,HCOERC,MU)
C
IMPLICIT NONE
C
INTEGER N,M,L,TYPE(N,M),ITER,STDOUT,GAPI,GAPI
REAL PHI(N,M,L),BIGPHI,MAXR,ROW(N),GRID,CNVRG,BR,HCOERC,MU(L)
CHARACTER*14 NAME
INTEGER K, AIR, IRON, MAGNET, KOUT
DATA AIR/1/, IRON/2/, MAGNET/3/, KOUT/9/
NAME = 'AIR'
K = AIR
CALL STORE(N,M,L,K,PHI,BIGPHI,MAXR,ITER,KOUT,ROW,NAME,STDOUT)
NAME = 'IRON'
K = IRON
CALL STORE(N,M,L,K,PHI,BIGPHI,MAXR,ITER,KOUT,ROW,NAME,STDOUT)
NAME = 'MAGNET'
K = MAGNET
CALL STORE(N,M,L,K,PHI,BIGPHI,MAXR,ITER,KOUT,ROW,NAME,STDOUT)
C ...SAVE A COMPLETE DATA FILE TO RESTART IF NECESSARY
C ...OR TO CALCULATE VARIOUS FIELD QUANTITIES.
NAME = 'PHI.INT'
CALL INTRMD(N,M,L,PHI,TYPE,KOUT,NAME,GAPI,GAPI,
&      GRID,CNVRG,BR,HCOERC,MU)
RETURN
END
C
C
SUBROUTINE STORE(N,M,L,K,PHI,BIGPHI,MAXR,ITER,KOUT,ROW,
.      NAME,STDOUT)
C
IMPLICIT NONE
C
INTEGER N, M, L, K, ITER, KOUT, IOS, STDOUT, MM1

```

```

INTEGER LJ
REAL PHI(N,M,L), BIGPHI, MAXR, ROW(N)
CHARACTER*14 NAME
CHARACTER*24 FNAME
1 FORMAT('MAX RESIDUAL = ', E12.3, ' LARGEST PHI = ', E12.3)
2 FORMAT(A14, ' POTENTIAL, ITERATION ', I5)
C ...CHANGE FOLLOWING FORMAT IF N IS CHANGED
3 FORMAT(I3, 1X, 180(F6.4,1X))
C ...USE FOLLOWING FOR WATFOR77, NEXT ONE FOR RM/FORTRAN
C FNAME = '(TEXT:210)//NAME
FNAME = NAME
MM1 = M - 1
OPEN (UNIT= KOUT, FILE = FNAME, IOSTAT = IOS)
IF (IOS .NE. 0) CALL ERROR(STDOUT,4, 0)
WRITE(KOUT,1) MAXR, BIGPHI
WRITE(KOUT,2) NAME, ITER
DO 110 J = MM1, 2, -1
    DO 100 I = 2, N
        ROW(I) = PHI(I,J,K) / BIGPHI
100 CONTINUE
    WRITE(KOUT, 3) J, (ROW(I), I = 2, N)
110 CONTINUE
CLOSE(KOUT)
RETURN
END
C
C
SUBROUTINE INTRMD(N,M,L,PHI,TYPE,KOUT,NAME,GAPI,GAPJ,
& GRID,CNVRG,BR,HCOERC,MU)
C
C IMPLICIT NONE
C
INTEGER N, M, L, KOUT, GAPI, GAPJ
REAL PHI(N,M,L), GRID, CNVRG, BR, HCOERC, MU(L)
INTEGER TYPE(N,M)
CHARACTER*14 NAME
INTEGER I, J, K, JUNK, IOS, AIR, IRON, MAGNET, STDOUT
REAL FJUNK
DATA AIR/1/, IRON/2/, MAGNET/3/, JUNK/-999/, FJUNK/0.0/
DATA STDOUT/6/
1 FORMAT(2I5, 2E15.5)
2 FORMAT(3E15.5)
3 FORMAT(3I5, 3E15.5)
OPEN(UNIT = KOUT, FILE = NAME, IOSTAT = IOS)
IF (IOS .NE. 0) CALL ERROR(STDOUT, 9, 0)
WRITE(KOUT,1) GAPI, GAPJ, GRID, CNVRG
WRITE(KOUT,2) BR, HCOERC, MU(IRON)
DO 110 I = 1, N
    DO 100 J = 1, M
        WRITE(KOUT, 3) I, J, TYPE(I,J), PHI(I,J,AIR),
        PHI(I,J,IRON), PHI(I,J,MAGNET)
100 CONTINUE

```

```

110 CONTINUE
  WRITE(KOUT, 3) JUNK, JUNK, JUNK, FJUNK, FJUNK, FJUNK
  CLOSE(KOUT)
  RETURN
  END
C
C
SUBROUTINE GAPFLD(N,M,L,PHI,MU,GAPI,GAPJ,GRID,B)
C
  IMPLICIT NONE
C
  INTEGER N, M, L, GAPI, GAPJ
  REAL PHI(N,M,L), MU(L), GRID, B(GAPI)
  INTEGER AIR, STDOUT, KOUT, IOS, I
  DATA AIR/1/, KOUT/9/, STDOUT/6/
C
  I FORMAT(2X, E15.5)
  OPEN(UNIT = KOUT, FILE = 'B.GAP', IOSTAT = IOS)
  IF (IOS .NE. 0) CALL ERROR(STDOUT, 7, 0)
C
  WRITE(KOUT,FMT='(2X,2I5)')GAPI,GAPJ
  DO I = 2, GAPI
    B(I) = MU(AIR)*(PHI(I,GAPJ+1,AIR)-PHI(I,GAPJ-1,AIR))
    &      /(2.0*GRID)
    WRITE(KOUT,I)B(I)
  ENDDO
C
  CLOSE(KOUT)
  RETURN
  END
C
C
C
C
SUBROUTINE HRZNTL(IJ, TOP, BOTTOM, GRID, BR, N, M, L, P, I, MU)
C
  IMPLICIT NONE
C
  INTEGER I, J, L, M, N, TOP, BOTTOM, X, Y
  REAL X3, X4, X5, Y2, Y3, Y5
  REAL PHI(N,M,L), MU(L), GRID, BR
  X = BOTTOM
  Y = TOP
  X3 = PHI(I-1,J,X)
  X4 = PHI(I,J-1,X)
  Y2 = PHI(I,J+1,Y)
  Y3 = PHI(I-1,J,Y)
  X5 = (-GRID*BR + MU(X)*X4 + MU(Y)*(Y2 + X3 - Y3))
  . / (MU(X) + MU(Y))
  Y5 = Y2 - (MU(X)*(X5-X4) + GRID*BR) / MU(Y)
  PHI(I,J,X) = X5
  PHI(I,J,Y) = Y5

```

```

      RETURN
      END
C
C
      SUBROUTINE VRTCL(I,J,LEFT,RIGHT,GRID,BR,N,M,L,PHI,MU)
C
      IMPLICIT NONE
C
      INTEGER I, J, L, M, N, LEFT, RIGHT, X, Y
      REAL X3, X4, X5, Y2, Y3, Y5
      REAL PHI(N,M,L), MU(L), GRID, BR
      X = LEFT
      Y = RIGHT
      X3 = PHI(I,J+1,X)
      X4 = PHI(I-1,J,X)
      Y2 = PHI(I+1,J,Y)
      Y3 = PHI(I,J+1,Y)
      X5 = (-GRID*BR + MU(X)*X4 + MU(Y)*(Y2 + X3 - Y3))
      . / (MU(X) + MU(Y))
      Y5 = Y2 - (MU(X)*(X5-X4) + GRID*BR) / MU(Y)
      PHI(I,J,X) = X5
      PHI(I,J,Y) = Y5
      RETURN
      END
C
C
      SUBROUTINE ERROR(STDOUT,K,IDAT)
C
      IMPLICIT NONE
C
      INTEGER K, STDOUT, IDAT
      IF (K .EQ. 1) WRITE(STDOUT,1)
      IF (K .EQ. 2) WRITE(STDOUT,2)
      IF (K .EQ. 3) WRITE(STDOUT,3) IDAT
      IF (K .EQ. 4) WRITE(STDOUT,4)
      IF (K .EQ. 5) WRITE(STDOUT,5)
      IF (K .EQ. 6) WRITE(STDOUT,6)
      IF (K .EQ. 7) WRITE(STDOUT,7)
      IF (K .EQ. 8) WRITE(STDOUT,8)
      IF (K .EQ. 9) WRITE(STDOUT,9)
      IF (K .EQ. 10) WRITE(STDOUT,10)
      IF (K .LT. 1 .OR. K .GT. 10) WRITE(STDOUT,99)
      1 FORMAT(1X, 'ERROR OPENING INPUT FILE')
      2 FORMAT(1X, 'ERROR CLOSING INPUT FILE')
      3 FORMAT(1X, 'UNDEFINED TYPE OF POINT', I5)
      4 FORMAT(1X, 'ERROR OPENING AIR OUTPUT FILE')
      5 FORMAT(1X, 'ERROR OPENING IRON OUTPUT FILE')
      6 FORMAT(1X, 'ERROR OPENING MAGNET OUTPUT FILE')
      7 FORMAT(1X, 'ERROR OPENING GAP FIELD OUTPUT FILE')
      8 FORMAT(1X, 'ERROR OPENING MAGNET FIELD OUTPUT FILE')
      9 FORMAT(1X, 'ERROR OPENING INTERMEDIATE OUTPUT FILE')
      10 FORMAT(1X, 'ERROR CLOSING AN OUTPUT FILE')

```

```

99 FORMAT(1X, 'ERROR IN CALL TO ERROR')
STOP
END
C
C
SUBROUTINE CORNR1(I,J,IN,OUT,ALPHA,N,M,L,PHI,MU,MAXR,GRID)
C
IMPLICIT NONE
C
INTEGER I, J, KI, KO, L, M, N, IN, OUT
REAL PHI(N,M,L), MU(L), ALPHA, MAXR, R, RADIUS, GRID
C
RADIUS=(J-2)*GRID
C
KO = OUT
R=0.25*(PHI(I-1,J,KO)+PHI(I+1,J,KO)
& +(1.0+0.5*GRID/RADIUS)*PHI(I,J+1,KO)
& +(1.0-0.5*GRID/RADIUS)*PHI(I,J-1,KO))
& -PHI(I,J,KO)
PHI(I,J,KO) = PHI(I,J,KO) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
C
KI = IN
R = 0.25*(2.0*(PHI(I,J+1,KI)+PHI(I+1,J,KI))
& +(0.5*GRID/RADIUS-1.0)*PHI(I,J+1,KO)
& +(1.0-0.5*GRID/RADIUS)*PHI(I,J-1,KO)
& -PHI(I+1,J,KO) + PHI(I-1,J,KO))- PHI(I,J,KI)
PHI(I,J,KI) = PHI(I,J,KI) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
RETURN
END
C
C
SUBROUTINE CORNR2(I,J,IN,OUT,ALPHA,N,M,L,PHI,MU,MAXR,GRID)
C
IMPLICIT NONE
C
INTEGER I, J, L, M, N, KI, KO, IN, OUT
REAL PHI(N,M,L), MU(L), ALPHA, MAXR, R, RADIUS, GRID
C
RADIUS=(J-2)*GRID
C
KO = OUT
R=0.25*(PHI(I-1,J,KO)+PHI(I+1,J,KO)
& +(1.0+0.5*GRID/RADIUS)*PHI(I,J+1,KO)
& +(1.0-0.5*GRID/RADIUS)*PHI(I,J-1,KO))
& -PHI(I,J,KO)
PHI(I,J,KO) = PHI(I,J,KO) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
C
KI = IN
R = 0.25*(2.0*(PHI(I-1,J,KI)+PHI(I,J+1,KI))

```

```

& +(0.5*GRID/RADIUS-1.0)*PHI(I,J+1,KO)
& +(1.0-0.5*GRID/RADIUS)*PHI(I,J-1,KO)
& -PHI(I-1,J,KO) + PHI(I+1,J,KO))- PHI(I,J,KI)
PHI(I,J,KI) = PHI(I,J,KI) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
RETURN
END
C
C
SUBROUTINE CORNR3(I,J,IN,OUT,ALPHA,N,M,L,PHI,MU,MAXR,GRID)
C
IMPLICIT NONE
C
INTEGER I, J, L, M, N, KI, KO, IN, OUT
REAL PHI(N,M,L), MU(L), ALPHA, MAXR, R, RADIUS, GRID
C
RADIUS=(J-2)*GRID
C
KO = OUT
R = 0.25*(PHI(I-1,J,KO) + PHI(I+1,J,KO)
& +(1.0+0.5*GRID/RADIUS)*PHI(I,J+1,KO)
& +(1.0-0.5*GRID/RADIUS)*PHI(I,J-1,KO))
& - PHI(I,J,KO)
PHI(I,J,KO) = PHI(I,J,KO) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
C
KI = IN
R = 0.25*(2.0*(PHI(I-1,J,KI)+PHI(I,J-1,KI))
& +(1.0+0.5*GRID/RADIUS)*PHI(I,J+1,KO)
& -(1.0+0.5*GRID/RADIUS)*PHI(I,J-1,KO)
& -PHI(I-1,J,KO) + PHI(I+1,J,KO))- PHI(I,J,KI)
PHI(I,J,KI) = PHI(I,J,KI) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
RETURN
END
C
C
SUBROUTINE CORNR4(I,J,IN,OUT,ALPHA,N,M,L,PHI,MU,MAXR,GRID)
C
IMPLICIT NONE
C
INTEGER I, J, L, M, N, KI, KO, IN, OUT
REAL PHI(N,M,L), MU(L), ALPHA, MAXR, R, RADIUS, GRID
C
RADIUS=(J-2)*GRID
C
KO = OUT
R = 0.25*(PHI(I-1,J,KO) + PHI(I+1,J,KO)
& +(1.0+0.5*GRID/RADIUS)*PHI(I,J+1,KO)
& +(1.0-0.5*GRID/RADIUS)*PHI(I,J-1,KO))
& - PHI(I,J,KO)
PHI(I,J,KO) = PHI(I,J,KO) + ALPHA * R

```



```

MAXR = AMAX1(MAXR,ABS(R))
C
KI = IN
R = 0.25*(2.0*(PHI(I,J-1,KI)+PHI(I+1,J,KI))
& +(1.0+0.5*GRID/RADIUS)*PHI(I,J+1,KO)
& -(1.0+0.5*GRID/RADIUS)*PHI(I,J-1,KO)
& -PHI(I+1,J,KO) + PHI(I-1,J,KO))- PHI(I,J,KI)
PHI(I,J,KI) = PHI(I,J,KI) + ALPHA * R
MAXR = AMAX1(MAXR,ABS(R))
RETURN
END
C
C
SUBROUTINE TSECT1(I,J,GRID,BR,N,M,L,PHI,MU)
C
IMPLICIT NONE
C
INTEGER I, J, L, M, N, R1, R2, R3, AIR, IRON, MAGNET
REAL GRID, BR
REAL PHI(N,M,L), MU(L)
DATA AIR/1/, IRON/2/, MAGNET/3/
R1 = AIR
R2 = MAGNET
R3 = IRON
PHI(I,J,R2) = PHI(I-1,J,R2) - MU(R3)*(PHI(I-1,J,R1)
& -PHI(I+1,J,R1)+GRID*BR/MU(R3))/(MU(R2)+MU(R3))
PHI(I,J,R3) = PHI(I+1,J,R3) + MU(R2)*(PHI(I-1,J,R2)
& -PHI(I,J,R2))/MU(R3) - GRID*BR/MU(R3)
PHI(I,J,R1) = PHI(I,J+1,R1)
& -0.5*(MU(R2)*(PHI(I,J,R2)-PHI(I,J-1,R2))
& + MU(R3)*(PHI(I,J,R3)-PHI(I,J-1,R3)))/MU(R1)
RETURN
END
C
C
SUBROUTINE TSECT2(I,J,GRID,BR,N,M,L,PHI,MU)
C
IMPLICIT NONE
C
INTEGER I, J, L, M, N, R1, R2, R3, AIR, IRON, MAGNET
REAL PHI(N,M,L), MU(L), BR, GRID
DATA AIR/1/, IRON/2/, MAGNET/3/
R1 = AIR
R2 = AIR
R3 = AIR
RETURN
END

```

APPENDIX C

DETAIL DRAWINGS OF PROTOTYPE WIMA COMPONENTS

This appendix contains the detail drawings used to manufacture the prototype WIMA device. The first drawing is an assembly layout which shows clearly the arrangement of all of the components.

 CARLTON UNIVERSITY FACULTY OF ENGINEERING	DESIGNED BY	S.R.	COIL FORM	DATE	D-03
	QUANTITY	QTY: 1	MTL. AL. 6081-76	SCALE	4:1

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DATE	04/05/95	DATE	04/05/95	DATE	04/05/95
	QTY	4	QTY	4	QTY	4
	S.R.		S.R.		S.R.	
	ACCEL. MAGNET		ACCEL. MAGNET		ACCEL. MAGNET	
			D-05

INCHES
BREAK ALL EDGES

NOTE: TWO PIECES SHOULD BE
MADE TOGETHER AND MARKED
AS A SET UPON SEPARATION
ONE SET SHOWN

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DESIGNED BY S.R. DATE 19/03/95	LVDI CLAMP MATERIAL 211	D-06

INCHES
BREAK ALL EDGES

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY S.R. QTY: 4	TITLE LVDT CORE SPACER	DATE 28/03/85	SCALE 20:1	SHEET NO. D-07
	TITLE: INVAR 36

INCHES
BREAK ALL EDGES

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY: S.R. QTY: 4 DATE: 22/03/95	OVERLOAD BOSS MATL: INVAR 36 QTY: 20	D-08

NOTCH
0.015" ± 0.002 DP
1 PLACES
20 SP

DETAIL 1

INCHES
DE-BURR COMPLETELY

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY: S.R. QTY: 5 DATE: 23/06/95	DIAPHRAGM FOIL .0039 INVAR FOIL 2.1	D-09

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY 3 R. QTY: 2 DATE 23/06/35	PART NO. OUTER CLAMP MATERIAL: INVAR 36 SCALE 2:1	REV. D:10

DETAIL 2

NOTCH
 .02 R, .012 DP
 4 PLACES
 EQ. SP

INCHES
 BREAK ALL EDGES

CARLETON UNIVERSITY FACULTY OF ENGINEERING	S R QTY 2 DT 30/04/85	INNER CLAMP MATL INVAR QTY 2	D-11

INCHES
BREAK ALL EDGES

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY: S.R. CHECKED BY: Q.T.V. DATE: 22/09/95	ASSEMBLY MANUFACTURE MATERIAL: ALUM SCALE: 2:1	7-01

Technical drawing of a sensor block, showing front and side views with dimensions and specifications.

Front View (Top):

- Overall width: 1790 / 1780
- Overall height: 3210 / 3200
- Left side features a vertical slot with a width of 17.230.
- Right side features a circular feature with a diameter of 3300 DIA and a hole of 2.750 DIA.
- Bottom features a horizontal slot with a width of 17.230.

Side View (Bottom):

- Overall width: 1790 / 1780
- Overall height: 3210 / 3200
- Left side features a vertical slot with a width of 17.230.
- Right side features a circular feature with a diameter of 3300 DIA and a hole of 2.750 DIA.
- Bottom features a horizontal slot with a width of 17.230.

Specifications:

- 1.84 UNF - 35.00P 2 HOLES 10.50P 2 FACES
- 10.12 UNF - 50.00P 2 HOLES 10.50P 2 FACES
- 3.300 DIA 3.258 DIA
- 2.750 DIA
- 1.84 UNF - 35.00P 2 HOLES 10.50P 2 FACES
- 10.12 UNF - 50.00P 2 HOLES 10.50P 2 FACES

Material and Drawing Information:

- SENOR BLOCK
- MATL 8061 ALUM
- DATE 2/03/85
- CARLETON UNIVERSITY
- FACULTY OF ENGINEERING
- 7-02

Ø .014 THRU
4 PLACES

DRILL THRU 1/2
BORE .20 DP FOR
10-32 UNF SHCS - 4 PLACES

INCHES
BREAK ALL EDGES

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DESIGNED BY	S. RICHE	ADAPTER PLATE	REV	T-03
	QTY	1	MAT'L: 6061 ALUM	DATE	1-1
	DATE	04/09/55

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY: S. RICHE QTY: 1 DATE: 04/08/85	PART: 1.1 MATERIAL: CLR POLYCARBONATE FINISH: 7.01
---	--	--

INCHES
BREAK ALL EDGES

CARLETON UNIVERSITY FACULTY OF ENGINEERING	DRAWN BY S. RICHE	CHECKED BY S. BLOCK / INSERT	DATE 05/09/95	SCALE 1:1	PART NO. 7-05
	QTY: 1	MAT'L: 8061 ALUM

SPECIFICATIONS

MODEL	UNITED STATES MILITARY SPECIFICATION (MIL-STD-883C)	CASE DIA (INCH)	CASE LENGTH (INCH)	CORE DIA (INCH)	CORE LENGTH (INCH)	LINEARITY (%)	SENSITIVITY AT 100 Hz (mV/g)	100 Hz FREQUENCY (Hz)	LOAD (ohms)	MAX. WIND VOLTAGE (V)	TEMP. RATIO (°C)	OUTPUT (mV/g)	WIND INPUT (mV/g)	OUTPUT (mV/g)	REMARKS
DTS-1	20.005	.437	.125	.125	.125	.25	0.25	7500	5	10	85	310	400	400	250
DTS-8	20.050	.437	.125	.125	.125	.25	1.25	14000	100	1.25	85	240	400	400	240
DTS-9	20.100	.437	.125	.125	.125	.25	5.7	14000	50	0.30	85	240	400	400	240

OPERATING TEMPERATURE: -55°F TO +220°F

CASE: 8000

SHIELDING: ELECTROSTATIC AND MAGNETIC

NOTE: LINEAR RANGE FROM 10.20" TO 25.00" AVAILABLE WITH INTEGRAL ELECTRONICS

"100 HZ FREQUENCY" THE 100 HZ INPUT WITH THE INPUT AMPLIFIER BETWEEN THE INPUT AND OUTPUT TERMINALS IS ZERO. AT THIS FREQUENCY THE EFFECTS OF TEMPERATURE CHANGE ON THE INPUT AND OUTPUT ON THE OUTPUT ARE MINIMAL.

DTS MODELS 1, 8 AND 9
LINEAR VARIABLE
DIFFERENTIAL TRANSFORMERS
PICKERING & CO. INC.

MODEL DTS-1
DTS 1, 8, 8.5, 9