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TITLE OF THESIS/TITRE DE LA THÈSE "Analysis of the Flat Plate Structural Unit"

UNIVERSITY/UNIVERSITÉ Carleton University

DEGREE FOR WHICH THESIS WAS PRESENTED/
GRADE POUR LEQUEL CETTE THÈSE FUT PRÉSENTÉE M. Eng.

YEAR THIS DEGREE CONFERRED/ANNÉE D'OBTENTION DE CE DEGRÉ 1979

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ANALYSIS OF THE FLAT-PLATE

STRUCTURAL UNIT

by

Moustafa Fouad El-Kafrawy, B. Eng.

A THESIS SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
AND RESEARCH IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ENGINEERING

FACULTY OF ENGINEERING
CARLETON UNIVERSITY
OTTAWA, ONTARIO
JUNE 1979

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Studies and Research acceptance of the thesis
"Analysis of the Flat-Plate Structural Unit" submitted
by Moustafa Fouad El-Kafrawy, B. Eng., in partial
fulfillment of the requirements for the degree of
Master of Engineering.

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ABSTRACT

A finite element analysis of the elastic behaviour of rectangular flat plate slabs is presented.

The finite element model consists of an upper and lower column and a part of the slab bounded laterally between the centerlines of the four panels adjacent to the column. In the finite element analysis, the model is subdivided into triangular plate bending elements. The elements adjacent to the column are rigidly connected to the column element.

To include the column stiffness in the global stiffness matrix, a constraint method at the slab column interface is developed here. This allows the column rotation resulting from unbalanced loading conditions to be included. This technique allows investigation of the moment distribution in the slab in a realistic way and permits the calculation of the moments transmitted from the slab to the columns without introducing column finite elements or additional degrees of freedom beyond those of the slab.

To make full use of the core storage available, an out-of-core method is used in assembling the structural matrix and solving for the unknown displacements.

A comparison between the finite element results and the available mathematical solutions and the code methods of analysis is also presented in this study.

To my wife and my parents.

ACKNOWLEDGEMENTS

I wish to express deep gratitude to my supervisor and teacher, Professor G.A. Hartley, for his support and guidance throughout the course of this work. Professor Hartley devoted his time, maintained constant supervision and provided numerous suggestions.

My sincere thanks are due to Professor W.H. Bowes for his assistance and valuable suggestions in the finite element work of this study.

The research would not have been possible without the financial assistance of Carleton University, through the Carleton combined scholarship and teaching assistantship in the Department of Civil Engineering.

My education would never have been possible without the continued encouragement of my parents and my wife and the many personal sacrifices made by them. I shall always be grateful to them.

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NOMENCLATURE

[A]	square matrix comes from {p} evaluated at the nodes
a,b	slab panel dimensions
[B]	matrix of strain shape functions (derived from N)
{c}	vector of plate curvatures, $w_{,xx}$, $w_{,yy}$ and $w_{,xy}$
[C]	constraint (transformation) matrix
C_{TB}	carry over factor
D	flexural rigidity of the plate
[D]	element elastic property matrix
E	Young's modulus
{f ^(e) }	element force vector
{F}	global force vector
{F'}	global force vector after condensation
H	column height
H_c	column clear height
[I]	unit matrix
I_c	column moment of inertia
K	ratio of the column dimension to the slab dimension
[K]	global stiffness matrix of the plate
[K']	global stiffness matrix of the plate after condensation
K_c	column stiffness
K_{cx}	column stiffness about x-axis
K_{cy}	column stiffness about y-axis

K_{ec}	equivalent column stiffness
$[K_g]$	global stiffness matrix of the whole model
K_t	stiffness of attached torsional member
$\{m\}$	vector of the plate bending moments M_{xx} , M_{yy} and M_{xy}
M_o	total static moment
$[N]$	matrix of displacement shape functions
NBAND	maximum bandwidth
NCORE	available core storage
NDF	total number degrees of freedom
$\{p\}$	vector contains the assumed deflection functions
$\{P\}$	vector of applied surface traction
$\{q\}$	body force vector
q	uniform load intensities
Q	plate vertical shearing force
t	plate thickness
$[T]$	transformation (condensation) matrix
$\{U\}$	vector of displacements
U_s	strain energy
V	vertical shearing force along the plate edge
W_I	work done by the system before condensation
W_{II}	work done by the system after condensation
z	vertical distance measured from the plate neutral surface
w	plate (lateral) displacement
w'	modified plate displacement

$\{\Delta\}$	vector of nodal displacements
$\{\epsilon\}$	element strain vector
$\{\epsilon_0\}$	initial strain vector
$\{\sigma\}$	element stress vector
Π_p	total potential energy
ν	Poisson's ratio
$\{\alpha\}$	vector contains constant determined from the element geometry
α_1	ratio between the longitudinal beam stiffness and slab stiffness
β	bending moment factor
$\{\theta\}$	the column rotations (θ_x and θ_y) vector
$\{\delta\}$	vector contains the nodal parameters ω , ω_x and ω_y of the common nodes between the column and the plate
$\{\Delta'\}$	vector contains nodal parameters after condensation

Flat plate slab system: This thesis addresses, in the text and figures, the flat plate system, that is, two-way slabs with no supporting beams or column capitals or drop panels. The work can easily be extended to the flat slab system (with capitals or drop panels) since the column itself is reduced to a patch on the slab surface.

CHAPTER 1. INTRODUCTION

1.1 GENERAL

Reinforced concrete slabs are among the most widely used structural elements. In many structures, the slab, in addition to providing a versatile and economical method of supporting vertical loads and forming an integral portion of the structural frame to resist lateral forces, is also a floor/ceiling surface. The selection of the slab system is based on structural, architectural and economic factors.

Flat slabs and flat plate types are commonly used in hotels, offices, and residential buildings when the ratio of length to width of one slab panel is less than 2. This allows the slab to be supported in two directions making more efficient use of the material, in addition to obtaining a greater strength and stiffness for a given slab thickness.

Flat slabs and flat plates are classified as "two-way slabs without beams". They are reinforced in two or more directions, without beams or girders to transfer the load. Thus, the slab itself becomes a grid of two-way shallow beams supported directly on columns.

The flat plate type is used when the spans are not large and loads not particularly heavy. In flat slab floor constructions, "column capitals" or "drop panels" or both are used to increase the slab shear capacity and to reduce the stresses due to negative moment; thus increasing the slab's ability to carry heavier loads and have larger spans than in the case of flat plates.

Initially, flat slabs and flat plates were used chiefly for buildings like warehouses and factories which required large open floor-spaces. Their main advantage was that they permitted a given minimum clear height to be maintained with a reduced overall story height; though at the expense of calling for more material in the floors because of their reduced overall depth. Today, two further advantages have been gained which have made flat slabs and flat plates attractive also for offices and residential buildings. These are simplicity of formwork and the complete freedom with which internal partitions can be placed beneath an unbroken level soffit.

As flat slabs and flat plates have become extensively used, their correct design is of primary interest to ensure safety and, at the same time, to provide economical design. On the other hand, obtaining the exact stress distribution in these slabs is not an easy task due to the complexity of the plate behaviour and the large number of variables involved in the analysis. The work carried out in the course of this thesis is a stage in an attempt to establish the design of such a slab structure in a path toward a more theoretical analysis. This point is elaborated upon in the closing chapter of this thesis.

1.2 BACKGROUND

1.2.1 Development of slab analysis

The problem of the flat plate slab supported by rows of equidistant columns has been investigated by several authors using the classical theory of plates.

First attempts were made by A. Nádaï[1]* and B.G. Galerkin[2] who simplified the problem by assuming cross-sectional dimensions of the columns to be small and to contribute a negligible effect on deflection and moments at the center of the plate. Since such a solution assumes concentrated reactions acting at the points of support of the plate, infinitely large negative moments are calculated.

Other attempts were made by S. Woinowsky-Krieger[3], V. Lewe [4], and K. Frey [5] who assumed that the reactive forces were distributed uniformly over the cross-sectional area of the columns. The bending moments arising at the center of the supported area were finite, but the values obtained were still far from the exact solution especially when the column cross-sectional dimensions became large.

More recently, there have been efforts to investigate the problem with a more realistic approach considering the concrete slab rigidly connected to the column. A solution of the flat plate slab with square panels rigidly connected to circular column was given by F. Tölke[6].

In 1954, S. Woinowsky-Krieger[7] gave a solution, using the complex variable method, for infinite flat slab loaded uniformly and rigidly connected to square-shape columns or column capitals arranged to form the square panels.

The theory of plates usually requires the solution of partial differential equations with given boundary conditions. Only in the case of simple boundaries can these equations be treated in a rigorous manner; therefore, many authors turned to the method of finite differences

* Number in square parenthesis refers to the cited references on page 88.

in handling the problem[8,9,10], in principle, the method is applicable to any given boundary conditions. However, it becomes less appropriate if stress concentrations are present, as for example at the corners of square capitals rigidly connected with the slab.

After the finite element method had become an acceptable method of analysis, and with the rapid development of electronic computers, greater efforts have been made since the early 1960's to construct suitable plate bending elements. A considerable number of more or less refined solutions have been suggested by several authors[11]. With the help of these elements, the problem can be investigated by a more realistic approach.

In 1976, V. Walder[12] used a model for thin plates in bending, elastically supported with a constant reaction and compared the finite element results with the theory of plates solution by S. Woinowsky-Krieger, showing that the finite element method can be used with high accuracy. This work showed that the finite element method can be used to give a solution for flat plate slabs rigidly connected to rectangular columns.

1.2.2 Development of code design methods

Model studies of slabs supported directly on regularly-spaced columns began in the United States and Switzerland at the beginning of this century.

In the United States, Turner, one of the early advocates of a flat slab system, realized that in a slab supported at its four corners the principal bending moments would act radially outwards in all directions in the vicinity of the column head. He proposed distributing the reinforcement in four bands intersecting at 45° at each column head. He also used an enlarged column head and a square "drop panel" to distribute the column reaction and avoid excessive vertical shearing action[13,14].

The four-way reinforcement system was strongly criticized by Millart, who was responsible for the independent development of flat-slab systems in Switzerland[15]. He felt that it led to a concentration of bars at the column head and it was more efficient to run the reinforcement in the two directions of the column axes only. In 1905, Turner built the first such slab in Minneapolis but, since there was no rigorous mathematical basis for the design, a performance load test was required.

In 1908, the flat slab began being recognized as an acceptable floor system, but for many years, designers had difficulties with patent infringements. The ability of flat slab systems to carry load was substantiated by numerous tests of actual structures. The amount of reinforcement used had to later be increased to satisfy analysis by statics.

In 1914, Nichols [16] established a simple criteria

for the minimum moment that must be existing across critical sections to satisfy basic equilibrium conditions. This seemed contrary to the widely accepted understanding of the behaviour of flat slab systems. The difference between Nichols' theoretical requirements and the supposedly measured moments in the load tests were partially recognized by Westergaard and Slater in their paper[8]. Based on their work, a provision was adopted in the 1921 edition of the ACI Building Code that a reduction of moment coefficient from the statically required value of $0.125 \omega l^2$ to $0.09 \omega l^2$ may be made. This reduction was not regarded as a violation of statics but it was used as a way of permitting an increase in the allowable unit stress. In the 1963 ACI code, this coefficient was between 0.09 and 0.115. The 1971 code stipulates the use of the full statically required coefficient of 0.125.

In general, since the 1921 edition, there were no major changes until 1971 except for the introduction of the frame method in the 1941 edition.

To obtain reliable data of failure and to serve as the basis of evaluating several analytical investigations which were in progress, a series of slab tests was undertaken at the University of Illinois[17,18,19]. These tests were the basis of the 1971 edition of the ACI Building Code.

1.3. OBJECT AND SCOPE

In the design of flat plates or flat slabs, the whole structure may be considered as a set of frames each consisting of the columns and the floor slab bounded laterally between the centerlines of the two panels adjacent to the line of columns (Fig. 1.1). The analysis of these frames should give special consideration to the stiffness characteristics of the connections between columns and slabs in the frame and the appropriate boundary conditions on the slab edges (at the panel centerlines).

The main purpose of this study is to develop a finite element model consisting of an upper and lower column and a part of the slab bounded laterally between the centerlines of the four panels adjacent to the column to represent the typical frame connection (Fig. 1.2). The purpose of this model is not to design real building structures. The purpose is to investigate the distribution of moments in the slab system, within the elastic range, as various parameters in the system are changed.

The application of the finite element to the present study does not introduce any new fundamental principles for the finite element method but does involve the use of the existing principles in a manner not previously reported. In particular, to the author's knowledge, the column head slab interface has not been modelled by a compatibility stiffness approach as described in this thesis.

Studying the behaviour of such a connection would help to establish some fundamental aspects which may be useful towards the analysis of the whole frame and other problems of this type.

This thesis addresses mainly the flat plate slab system (i.e. no column capitals or drop panels). However, since the columns are basically reduced to a patch on the surface in the constraint method developed here, the extension to flat slabs is a minor one.

The object of this study can be summarized as follows:

1. Develop a finite element program for a numerical model of linear-elastic flat plate slab system.
2. Investigate the distribution of moments in the system as various parameters are changed, and compare the results with the Code methods of analysis.
3. Verify the finite element solution with the available analytical solutions.
4. Put these procedures into a form in which the work can be carried forward into a new phase, as described in the closing chapter of this thesis.

FIG. 1.1. Slab-Column Plane Frame.

FIG. 1:2. Finite Element Model.

CHAPTER 2. FINITE ELEMENT ANALYSIS

2.1 GENERAL

The finite element method is a general numerical stress analysis procedure based upon the concept of approximating any continuous field by a discrete model ~~that~~ is composed of a set of piecewise "shape functions" which are continuous over a finite number of subdomains called elements. The most popular formulation of the finite element equation starts by assuming a displacement field and then minimizes the potential energy to obtain the nodal values of the displacements. Once the displacements are known, one can solve for the strains and stresses.

The shape functions determine the displacements at every point in the element in terms of nodal displacements. This is expressed as:

$$\{U\} = [N]\{\Delta\} \quad (2.1)$$

where $[N]$ is the matrix of shape functions and $\{\Delta\}$ is the vector of nodal displacements.

The general strain-displacement relationship is given by:

$$\{\epsilon\} = [B]\{\Delta\} \quad (2.2)$$

where $\{\epsilon\}$ is the vector of element strains and $[B]$ is derived by performing the differentiation of corresponding elements of $[N]$. The generalized Hooke's law for linear materials gives:

$$\{\sigma\} = [D]\{\epsilon - \epsilon_0\} \quad (2.3)$$

where $\{\epsilon\}$ is the total strain and $\{\epsilon_0\}$ is an initial strain. $[D]$ is

symmetric matrix which contains the elastic constants of the material.

The strain energy for a differential element of volume dV is given by:

$$dU_s = \left[\frac{1}{2} \{\epsilon\}^T \{\sigma\} - \frac{1}{2} \{\epsilon_o\}^T \{\sigma\} \right] dV \quad (2.4)$$

Thus, the total strain energy is obtained by integrating over the volume:

$$U_s = \int_V \frac{1}{2} (\{\epsilon\}^T \{\sigma\} - \{\epsilon_o\}^T \{\sigma\}) dV \quad (2.5)$$

Since the total potential energy (Π_p) is equal to the difference between the total strain energy and the work done by the body forces and the external loads. Thus, the total potential energy of the elastic body may be given as:

$$\Pi_p = U_s - \int_V \{U\}^T \{q\} dV - \int_S \{U\}^T \{P\} ds \quad (2.6)$$

where $\{q\}$ is the vector of body force components and $\{P\}$ is the vector of applied surface tractions.

Substituting Eqns. (2.1), (2.2), (2.3), (2.5) into Eqn. (2.6), and realizing that the region is subdivided into a number of elements, the total potential energy may be written as:

$$\begin{aligned} \Pi_p = & \sum_{e=1}^E \left[\int_{V(e)} \frac{1}{2} \{\Delta\}^T [B^{(e)}] [D^{(e)}] [B^{(e)}] \{\Delta\} dV - \int_{V(e)} \{\Delta\}^T [B^{(e)}] [D^{(e)}] \{\epsilon_o^{(e)}\} dV \right. \\ & - \int_{V(e)} \{\epsilon_o^{(e)}\} [D^{(e)}] \{\epsilon_o^{(e)}\} ds - \int_{V(e)} \{\Delta\}^T [N^{(e)}] \{q^{(e)}\} dV \\ & \left. - \int_{S(e)} \{\Delta\}^T [N^{(e)}] \{P^{(e)}\} ds \right] \quad (2.7) \end{aligned}$$

To minimize Π_p , we differentiate with respect to $\{\Delta\}$ and set it equal to zero.

$$\frac{\partial \Pi_p}{\partial \{\Delta\}} = \sum_{e=1}^E \left[\int_{V(e)} [B^{(e)}]^T [D^{(e)}] [B^{(e)}] dV \{\Delta\} - \int_{V(e)} [B^{(e)}]^T [D^{(e)}] \{\epsilon_0^{(e)}\} dV \right. \\ \left. - \int_{V(e)} [N^{(e)}]^T \{q^{(e)}\} dV - \int_{S(e)} [N^{(e)}]^T \{p^{(e)}\} ds \right] = 0 \quad (2.8)$$

This formula can be written as:

$$\sum_{e=1}^E [K^{(e)}] \{\Delta\} - \sum_{e=1}^E \{f^{(e)}\} = 0 \quad (2.9)$$

where $[K^{(e)}]$ is the element stiffness matrix and $\{f^{(e)}\}$ is the element force vector, which are defined by Eqn. (2.8).

Having established the elements stiffness matrices and force vectors, they are assembled in the global stiffness matrix $[K]$ and global force vector $\{f\}$. This is expressed as:

$$[K] = \sum_{e=1}^E [K^{(e)}] \quad (2.10)$$

$$\{F\} = \sum_{e=1}^E \{f^{(e)}\} \quad (2.11)$$

The general relationship between $[K]$ and $\{F\}$ is:

$$[K] \{\Delta\} = \{F\} \quad (2.12)$$

This set of linear equations is then solved for displacements $\{\Delta\}$.

2.2 PLATE BENDING ELEMENT

The general approach to the element stiffness analysis for plate bending elements is based on the classical thin plate theory

(Kirchoff theory), where the straining in the plate is completely defined by the lateral deflection w , hence w and its derivatives (up to a chosen order) are taken as degrees of freedom.

The deflection w is defined by assuming shape functions:

$$\{w\} = \{p\}^T \{\alpha\} \quad (2.13)$$

where $\{p\}$ is a vector which contains the assumed deflection functions, and $\{\alpha\}$ is a vector which contains constants determined from the element geometry. Applying Eqn. (2.13) at the nodal points:

$$\{\Delta\} = [A]\{\alpha\} \quad (2.14)$$

where $[A]$ is a square matrix and comes from $\{p\}$ evaluated at the nodes. Substituting Eqn. (2.14) into Eqn. (2.13), the deflection can be expressed as

$$\{w\} = \{p\}^T [A]^{-1} \{\Delta\}$$

which can be written as

$$\{w\} = [N]\{\Delta\} \quad (2.15)$$

In the thin plate theory, it is assumed that the strains and stresses vary linearly from the neutral surface which is assumed to be equidistant from the upper and lower plate surfaces provided that the deflections of the plate are small in comparison to the thickness t . The strains can be expressed as:

$$\epsilon_{xx} = -w_{,xx} \cdot z$$

$$\epsilon_{yy} = -\omega_{,yy} \cdot z \quad (2.16)$$

$$\epsilon_{xy} = -\omega_{,xy} \cdot z$$

where z is the distance measured from the neutral surface. The relationships between the stresses and moments per unit length are given by:

$$\begin{aligned} \sigma_{xx} &= \frac{12 M_{xx}}{t^3} \cdot z \\ \sigma_{yy} &= \frac{12 M_{yy}}{t^3} \cdot z \\ \sigma_{xy} &= \frac{12 M_{xy}}{t^3} \cdot z \end{aligned} \quad (2.17)$$

Dealing with these strains and stresses in the determination of strain energy would require integration through the plate thickness. It is more convenient to deal with curvatures and moments. Once the curvatures and moments are known, the stresses and strains at any point can be found from Eqns. (2.16) and (2.17).

Considering an element of the plate, as shown in Fig. 2.1, the strain energy is expressed as:

$$dU_s = -\frac{1}{2} M_x \omega_{,xx} - \frac{1}{2} M_y \omega_{,yy} - \frac{1}{2} M_{xy} \omega_{,xy} - \frac{1}{2} M_{yx} \omega_{,xy}$$

since

$$M_{xy} = M_{yx}$$

$$dU_s = -\frac{1}{2} M_x \omega_{,xx} - \frac{1}{2} M_y \omega_{,yy} - M_{xy} \omega_{,xy}$$

The strain energy of the entire plate can be written as:

$$U_s = - \frac{1}{2} \int_A \{c\}^T \{m\} dA \quad (2.18)$$

where $\{c\}$ is the vector of curvatures

$$\{c\} = \begin{bmatrix} \omega_{,xx} \\ \omega_{,yy} \\ 2\omega_{,xy} \end{bmatrix} \quad (2.19)$$

and $\{m\}$ is the vector of bending moments

$$\{m\} = \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} \quad (2.20)$$

In the case of an elastic isotropic material, the bending moments are related to the curvatures as follows:

$$\begin{aligned} M_x &= -D(\omega_{,xx} + \nu \omega_{,yy}) \\ M_y &= -D(\omega_{,yy} + \nu \omega_{,xx}) \\ M_{xy} &= -D(1-\nu)\omega_{,xy} \end{aligned} \quad (2.21)$$

where

$$D = \frac{Et^3}{12(1-\nu)^2}$$

Eqn. (2.21) may be written

$$\{m\} = -[D]\{C\} \quad (2.22)$$

where

$$[D] = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix},$$

Substituting Eqn. (2.22) into Eqn. (2.18), one finds

$$U_s = \int_A \frac{1}{2} \{C\}^T [D] \{C\} dA \quad (2.23)$$

Using Eqn. (2.15), the curvature-displacement relationship can be expressed as:

$$\{C\} = [B]\{\Delta\} \quad (2.24)$$

where $[B]$ is derived by performing the differentiation of corresponding elements of $[N]$.

Substituting Eqn. (2.24) into Eqn. (2.23), one finds

$$U_s = \int_A \frac{1}{2} \{\Delta\}^T [B]^T [D] [B] dA \{\Delta\} \quad (2.25)$$

following the same procedure as in Pages 12 and 13.

The element stiffness matrix is given by:

$$[K] = \int_A [B]^T [D] [B] dA \quad (2.26)$$

2.3 FINITE ELEMENT MODEL

In order to study the elastic behaviour of a flat plate slabs system, a finite element model, introduced in Chapter 1 and illustrated in Fig. 1.2, is considered. The model consists of two different types of finite elements, plate bending elements representing the slab and column elements. In deriving the stiffness of the plate, it is divided into triangular elements each with 18 degrees of freedom. This element was first introduced by K. Bell in 1969[11]. It is also described in Bowes and Russell[21], and a program by Bowes at Carleton University was modified and used in the course of the present work.

In this finite element, the displacement function selected is a complete 5th degree polynomial which contains 21 terms. Thus, the displacement w at any point of the element is expressed as:

$$\begin{aligned} w = & \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y \\ & + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^4 + \alpha_{12} x^3 y + \alpha_{13} x^2 y^2 + \alpha_{14} xy^3 + \alpha_{15} x^4 y + \alpha_{16} x^5 \\ & + \alpha_{17} x^4 y + \alpha_{18} x^3 y^2 + \alpha_{19} x^2 y^3 + \alpha_{20} xy^4 + \alpha_{21} y^5 \end{aligned} \quad (2.27)$$

This can be written as:

$$\{w\} = \{p\}^T \{\alpha\} \quad (2.13)$$

The deflection w and its derivatives $w_{,x}$ and $w_{,y}$ and the three curvatures $w_{,xx}$, $w_{,xy}$ and $w_{,yy}$ are introduced as degrees of freedom at each of the three corners, in addition to the edge normal slope $w_{,n}$ at each of the three mid-span nodes (Fig. 2.2). Afterwards, the mid-side

nodes are eliminated by assuming that the normal derivative on each side is a cubic.

The assumed displacement function, together with the selected nodal parameters, provides for both deflection and slope compatibility also the curvatures and consequently the internal moments are continuous between elements at the corner nodes if the thickness is continuous.

The element stiffness matrix and the consistent load vectors involve the evaluation of integrals of the form:

$$P_{rs} = \int_A x^r y^s dA$$

The integration is performed by using integration formulae for a plane triangle[11].

The column is taken from the centerline of the top floor to the centerline of the bottom floor slab. The portion of the column between the upper slab face and the lower slab face is assumed to have an infinite stiffness (Fig. 2.3).

After the global stiffness matrix of the plate has been assembled and the column rotational stiffness has been calculated, these should be added to form the stiffness matrix of the complete model. The column degrees of freedom θ_x and θ_y are in terms of the point of intersection between the column centerline and the plate middle surface, and the plate degrees of freedom refer to the nodal points at the plate and the column interface. Therefore, assumptions must be considered in order to obtain a relationship between the column degrees of freedom θ_x and θ_y .

and the degrees of freedom of the nodal points along the column and the slab interface. The assumptions are:

1. The column cross-section remains plane after bending.
2. The slab is rigidly connected to the column.
3. No axial deformations occur in the column.
4. The column is fixed at its remote ends.

According to the first and third assumptions, when the column starts to rotate due to unbalanced loading of the slabs, the middle point in the column cross-section will have no vertical displacement. Meanwhile, the column cross-section will rotate with the same angle of rotation as its centerline and remains plane (Fig. 2.4). This requires that the vertical displacement w at every point of the column cross-section be expressed as:

$$w = \theta_x \cdot x + \theta_y \cdot y \quad (2.28)$$

where θ_x and θ_y are the column rotations in the x and y directions respectively, and x and y are the Cartesian coordinates of the point measured from the column center.

On the other hand, in order to provide both displacement continuity and a complete fixation between the plate and the column, the element nodes lying on the column boundary should have compatible deflection and rotation.

This means that all of the degrees of freedom in w , w_x and w_y of these nodal points are not independent and they are functions of

the column rotations, θ_x and θ_y . This can be expressed as:

$$\{\delta\} = [T]\{\theta\} \quad (2.29)$$

where

$\{\theta\}$ is a vector containing the column rotation,

$[T]$ is the transformation (condensation) matrix, and

$\{\delta\}$ is a vector containing the nodal parameters ω , ω_x and ω_y of the common nodes between the column and the plate.

Generally, the relationship between the displacements of the whole model before and after condensation can be written as:

$$\begin{bmatrix} \Delta_1 \\ - \\ \delta \\ - \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ - & - & - \\ 0 & T & 0 \\ - & - & - \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \Delta_1 \\ - \\ \theta \\ - \\ \Delta_2 \end{bmatrix} \quad (2.30)$$

where $[I]$ is a unit matrix.

This can also be written in the form:

$$\{\Delta\} = [C]\{\Delta'\}$$

where

$\{\Delta\}$ is a vector containing the nodal parameters before condensation (transformation)

$\{\Delta'\}$ is a vector containing the nodal parameters after condensation (transformation)

$[C]$ is the constraint (transformation) matrix.

To obtain the relationship between the stiffness matrix and the force vector of the finite element model before and after transformation, a virtual displacement is applied.

The work done by the first system of nodal displacements is:

$$W_I = \{d\Delta\}^T \{F\}$$

where $\{F\}$ is the force vector of the first system.

The work done by the new system of nodal displacements is:

$$W_{II} = \{d\Delta'\}^T \{F'\}$$

where $\{F'\}$ is the force vector of the new system.

Equating the work done by the two systems, we get:

$$\{d\Delta'\}^T \{F'\} = \{d\Delta\}^T \{F\}$$

$$\{d\Delta'\}^T [K'] \{\Delta'\} = \{d\Delta\}^T [K] \{\Delta\}$$

Substituting Eqn. (2.30), we get:

$$\{d\Delta'\}^T [K'] \{\Delta'\} = [[C]\{d\Delta'\}]^T [K] [C] \{\Delta'\}$$

$$\{d\Delta'\}^T [K'] = [[C]\{d\Delta'\}]^T [K] [C]$$

$$\{d\Delta'\}^T [K'] = \{d\Delta'\}^T [C]^T [K] [C]$$

$$[K'] = [C]^T [K] [C] \quad (2.31)$$

Eqn. (2.31) states the relationship between the stiffness matrix after condensation $[K']$ and before condensation $[K]$.

The general relationship between the nodal forces and the displacements in the new set of equations is:

$$\{F'\} = [K']\{\Delta'\}$$

$$\{F'\} = [C]^T[K][C]\{\Delta'\}$$

$$\{F'\} = [C]^T[K]\{\Delta\}$$

$$\{F'\} = [C]^T\{F\} \quad (2.32)$$

Eqn. (2.32) states the relationship between the consistent load vector after condensation $\{F'\}$ and before condensation $\{F\}$. After the global stiffness of the plate has been assembled and modified, the column rotational stiffness should be added in its proper locations in the plate stiffness matrix forming the stiffness matrix of the complete model.

The general relationship between the forces and displacements is:

$$\begin{bmatrix} F' \end{bmatrix} = \begin{bmatrix} K' \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \theta_x \\ \theta_y \\ \Delta_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ & K_{cx} & 0 \\ 0 & 0 & K_{cy} \\ 0 & & 0 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \theta_x \\ \theta_y \\ \Delta_2 \end{bmatrix} \quad (2.33)$$

where K_{cx} and K_{cy} are the column rotational stiffnesses about the x and y axes. This can be expressed as:

$$\{F'\} = [K_g]\{\Delta'\}$$

where $[K_g]$ is the stiffness matrix of the whole model.

The use of the constraint patch method in combining the plate bending stiffness with the column stiffness has some advantages. For example:

1. It eliminates the need to model the column with new elements and therefore no additional degrees of freedom have to be considered.
2. It eliminates some degrees of freedom giving a modified matrix with smaller size than the original one.
3. It avoids a difficult problem of forming a column element into some plate bending elements with different characteristics.
4. Since the columns are basically reduced to a patch on the surface, the extension of the solution to flat slab systems is basically the same.

Generally, it can be stated that the constraint path method provides a simple and convenient solution to the problem of combining the column and slab stiffnesses.

2.4 ACCURACY OF THE SOLUTION

One of the important requirements of the finite element method is that its solutions converge to the exact solution of the problem as the mesh division is progressively subdivided. Increasing the mesh division is accompanied by:

1. a steep rise in computer running time
2. a steep rise in storage demands.

Thus, it is important to find a low number of elements giving a solution with sufficient accuracy.

A finite element solution to the problem of the flat plate with infinite extension and uniformly loaded square panels supported by square columns was made here using different mesh sizes. Because of the double symmetry of the plate, only one quarter of it was considered in the analysis (Fig. 2.5).

Table 2.1 contains the values of the bending moments in various points of the slab obtained by both the finite element and S. Woinowsky-Krieger solution by the classical theory of plates. From the table, it is clear that the difference ~~of~~ the results obtained by the two methods decreases by increasing the number of elements, and good results can be achieved by subdividing the plate to 48 elements.

Curves 2.6 and 2.7 show the distribution of the bending moments M_x and M_y , along the centerline of the plate and the line connecting the columns, obtained by the two methods. It can be concluded that there is only a small difference between the results obtained by the two methods. Because of the nature of the bending moment, using less elements results in obtaining an insufficient number of points necessary to draw it; whereas the larger the number of points the more accurate the moment curve becomes.

More accurate results for the bending moments near the stress concentration at the column corner can be obtained by subdividing the slab into smaller elements in the column area. Since the main objective is to study the flexibility of the column slab joint not the stresses in the column area, the selected grid was considered to be adequate enough for the purpose of the study.

2.5 ASSEMBLY AND SOLUTION OF LARGE SYSTEM OF SYMMETRIC LINEAR EQUATIONS

To obtain high accuracy of results, it is required as previously discussed to have at least 48 elements in each quarter making a total of 192 elements with 142 nodal points (Fig. 2.8). Since each nodal point has 6 degrees of freedom, the total number of degrees of freedom (NDF) becomes large. Meanwhile, the constraint method discussed before increases the bandwidth (NBAND). For example, in Fig. 2.8, the bandwidth before condensation can be expressed as:

$$\text{NBAND} = 6(m-L+1)$$

After condensing the degrees of freedom, the bandwidth becomes:

$$\text{NBAND} = 6(m-K+1) - 22$$

where the number 22 represents the reduction in the degrees of freedom.

To overcome this problem, a special way of numbering the nodal points is required to control the bandwidth; however, the bandwidth is still quite large. This leads to the problem of having a stiffness matrix (NDFXNBAND) too large to be entirely stored in the core.

It was decided to use out-of-core method[20] in the assembly of the stiffness matrix and in solving for the unknown displacements. In this method, coefficients not immediately in demand can be stored temporarily in secondary storage. To reduce the time consumption in reading and writing from the magnetic storage, every six equations in the stiffness matrix representing the degrees of freedom of one node are

transferred as a single record. Before the assembly of each element stiffness matrix in the global stiffness matrix, a check must be made to make sure that all the equations belonging to the element connecting nodes are in core. If these equations were not in the core at that time, equations not immediately in demand can be transferred to the discs and the required equations would be transferred back.

To solve the large system of linear equations, the Gaussian elimination method is used. During the Gaussian elimination procedure, the first six equations representing the first node contribute to the alteration of only the coefficients in the next (NBAND-6) equations (Fig. 2.9). Having eliminated these six unknowns, they are no longer needed during the forward elimination and as long as an up-to-date version exists in disc storage, the core location for these equations can be overwritten whenever necessary.

The out-of-core method reduces the number of coefficients required to be stored in core at one time to (NBANDxNBAND). The maximum bandwidth that can be used depends on the core storage, as follows:

$$\text{Maximum NBAND} = \sqrt{\text{NCORE}}$$

where NCORE is the available core storage.

While there is a maximum limit for the bandwidth, there is no limitation on the number of equations to be stored (NDF).

Because of the time consumed in reading and writing from the magnetic storage, the computer running time is increased especially when the number of equations becomes very large.

2.6 BOUNDARY CONDITIONS

In the finite element analysis, the slab edge conditions must be incorporated. In flat plate slabs with a large number of bays, the boundary conditions along the edges of a typical slab panel (not close to the floor boundary) depend on the applied pattern of loading. Fig. 2.10 shows the boundary conditions for various types of supports.

If the edge of the plate is simply supported, the deflection, the slope and the curvature along this edge must be zero. At the same time, this edge can rotate freely with respect to the edge line, i.e. there are no bending moments along this edge. Since the bending moment is a function of the curvature in two orthogonal directions and the curvature along the edge is zero, therefore the curvature along the line normal to the edge is also zero.

Assuming the edge of the plate is simply supported and parallel to the y-axis, the following boundary conditions should be applied:

$$\omega = 0$$

$$\omega_{,y} = 0$$

$$\omega_{,xx} = 0$$

$$\omega_{,yy} = 0$$

If the edge of the plate is built in, the deflection, the slope and the curvature of the plate along this edge must be zero. The tangent plane to the deflected middle surface along this edge coincides with the initial position of the middle plane of the plate.

Assuming the edge of the plate is built in and parallel to the y-axis, the analytical expressions of the boundary conditions in this case are:

$$\omega = 0$$

$$\omega_{,x} = 0$$

$$\omega_{,y} = 0$$

$$\omega_{,xy} = 0$$

$$\omega_{,yy} = 0$$

A free edge support condition involves zero normal moment to the plate edge and zero vertical shear along the plate edge.

Assuming the free edge of the plate is parallel to the y-axis two conditions can be written:

$$1. \quad M_x = 0 \quad \text{or} \quad \omega_{,xx} + \nu \omega_{,yy} = 0$$

$$2. \quad V_x = 0 \quad \text{or} \quad Q_x + M_{xy,y} = 0$$

The first condition can be treated by transforming the nodal parameters to a modified set of nodal parameters as the moments can be expressed in terms of the curvatures. The relationship between the two sets of nodal parameters is:

$$\begin{bmatrix} \omega \\ \omega_{,x} \\ \omega_{,y} \\ \omega_{,xx} \\ \omega_{,xy} \\ \omega_{,yy} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{1-\nu^2} & 0 & \frac{-\nu}{1-\nu^2} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{-\nu}{1-\nu^2} & 0 & \frac{1}{1-\nu^2} \end{bmatrix} \begin{bmatrix} \omega \\ \omega_{,x} \\ \omega_{,y} \\ \omega_{,xx} + \nu \omega_{,yy} \\ \omega_{,xy} \\ \omega_{,yy} + \nu \omega_{,xx} \end{bmatrix} \quad (2.34)$$

The second condition which involves zero vertical shear cannot be prescribed since it is not possible to express the vertical shear in terms of the nodal parameters.

In the case of plate with infinite extension (large number of bays), to study the distribution of moments in a typical panel, the boundary conditions along the model edges which lie on the centerlines of four adjacent panels, as previously described, may be prescribed for axes of symmetry or anti-symmetry depending on the loading conditions. Different loading patterns can be obtained as a combination of the symmetrical and anti-symmetrical cases.

If the edge of the plate lies on the axis of symmetry, the slope normal to the line of symmetry and the warping of the plate along the edge must be zero. If the edge of the plate lies on the axis of anti-symmetry, the deflection, the slope along the edge, the curvature along the edge and the line normal to the edge must be zero.

The analytical expressions of the boundary conditions in these two cases (assuming the edge of the plate is parallel to the y-axis) are:

1. Symmetrical case:

$$\omega_{,x} = 0$$

$$\omega_{,xy} = 0$$

2. Anti-symmetrical case:

$$\omega = 0$$

$$\omega_{,y} = 0$$

$$\omega_{,xx} = 0$$

$$\omega_{,yy} = 0$$

FIG. 2.1. Slab Moments.

FIG. 2.2. Nodal Point System and Nodal Parameters for 18 d.o.f. Triangular Bending Element.

(a) Slab System

(b) Column Stiffness

FIG. 2.3. Column Element.

16-element

30-element

48-element

FIG. 2.5. Use of Symmetry in Finite Element Grids.

FIG. 2.6. Bending Moment Along Line Connecting the Two Columns ($y=a/2$) for $\nu=0.2$ and $c/a=0.2$ ($M=\beta q a^2$).

FIG. 2.7. Bending Moment Along the Centerline of the Plate ($y=0$) for $\nu=0.2$ and $c/a=0.2$.
 $M=\beta qa^2$

FIG. 2.8. Finite Element Grid.

FIG. 2.9. Out-of-Core Solution of Linear Equations.

FIG. 2.10. Slab Edge Boundary Conditions.

TABLE 2.1. Bending Moments ($M=\beta qa^2$) of a Uniformly Loaded Square Panel of Flat Plate
where $\nu=0.2$ and $c/a=0.2$.

β	Location	Theory of Plate	16 Element		30 Element		48 Element	
			β	Error	β	Error	β	Error
$M_x=M_y$	$x=0, y=0$	0.0262	0.0270	3.05%	0.0250	4.58%	0.025765	1.66%
M_x	$x=a/2, y=0$	-0.0142	-0.0123	13.38%	-0.0128	9.86%	-0.013020	8.30%
M_y	$x=a/2, y=0$	0.0348	0.0330	5.17%	0.0337	3.16%	0.03425	1.58%
M_x	$x=a/2-c/2, y=a/2$	-0.0626	-0.078	24.60%	-0.0710	13.42%	-0.06075	2.95%

CHAPTER 3. ANALYSIS OF RESULTS

3.1 INTRODUCTION

The distribution of moments in reinforced concrete flat plate slabs depends not only on the applied loading pattern but also on the slab geometry. The change in the slab geometry alters the stiffness characteristics of the slab.

The factors affecting the distribution of moments are:

1. the column's cross-sectional dimension;
2. the ratio of the long span to the short span of the slab panel;
3. slab thickness;
4. column height; and
5. column shape.

To study the effect of these parameters on the distribution of the moments, the flat plate floor with infinite extension (large number of bays in both directions) and loaded with two different loading patterns is considered.

3.2 UNIFORMLY LOADED FLAT PLATE WITH INFINITE EXTENSION

Considering a uniformly loaded reinforced concrete flat plate slab with rectangular columns arranged to form equal rectangular slab panels (Fig. 3.1a), it can be concluded from the symmetry of the structure that the bending moment in all panels not close to the floor boundary is the same. Therefore, the problem can be limited to the bending of one

panel only and the finite element model represents the bending of all the panels not close to the floor boundary.

From the symmetry of the floor geometry, the edges of the model lie on axes of symmetry (Fig. 3.1b) and the nodal displacements at the model boundary can be prescribed as discussed in Chapter 2 giving the required boundary conditions. Making use of the double symmetry about the x-axis and y-axis of the model, the problem can be simplified by considering only one-quarter of the plate. Also, the deflected shape of the plate will be symmetrical and therefore the column rotation about the x-axis and the y-axis is equal to zero and consequently no moments are induced in the column. The Poisson's ratio of the plate was taken as 0.2 in the analysis, which is a good average value for the reinforcement and the concrete[7,24].

Since there is no bending in the columns, the factors affecting the distribution of the moments in the slab are reduced to:

1. the column cross-sectional dimension;
2. the ratio of the long span to the short span of the slab panel (a/b); and
3. the column shape.

3.2.1 The column cross-sectional dimension

To illustrate the effect of the column dimensions on the moments in the slab, a set of curves (Figs. 3.2, 3.3, 3.4 and 3.5) were drawn to show the bending moment at various points in the slab for different values of K which is the ratio of the column dimension to the slab

dimension ($K=c/a=d/b$). One can observe that decreasing the column dimensions increases the moment in the slab, especially the negative moment at or near the column edge. For example, the negative moment at the column edge for $K=0.05$ is double that for $K=0.15$ (Fig. 3.5). The transverse distribution of the longitudinal moment changes also with the change in the ratio K . When K increases, the distribution becomes more uniformly distributed across the slab width. (See Appendix A for the transverse distribution of longitudinal moments.)

3.2.2 The ratio of the long span to the short span (a/b)

The curves in Figs. 3.2, 3.3, 3.4 and 3.5 and Table 3.1 show the moments obtained for different values of the a/b ratio. It may be observed that increasing the ratio a/b increases the positive moment in the center of the slab panel while the positive moment at the line connecting the column decreases; also, the negative moment in the column neighbourhood decreases while the negative moment at half the distance between the columns increases. It can be concluded that increasing the a/b ratio gives more uniform moment distribution in the transverse direction (see Appendix A for the transverse distribution of longitudinal moments). From the curves, one may also observe that the effect of a/b ratio on the slab moment decreases with the increase of K . Table 3.1 shows that the rate of change in the moments resulting from the change in the a/b ratio decreases with the increase of a/b .

In the shorter direction, increasing the a/b ratio results in a rapid decrease in the moments especially for small values of K .

3.2.3 Column shape

Using circular columns instead of square columns to support the uniformly loaded plate with square panels (Fig. 3.6) changes the distribution of moments in the slab. The solution of this problem by the theory of thin plates was first introduced by F. Tölke in 1934 as previously stated in Chapter 1. Table 3.2 shows the values obtained by F. Tolke and the finite element solution for $D/a=0.2$ where D is the column diameter and a is the panel length. From the table, it is clear that the difference in the results obtained by the methods is small.

To illustrate the effect of the column shape on the distribution of moments, Table 3.3 gives the moments in uniformly loaded flat plate slabs with square panels supported by circular columns for different D/a ratios and the corresponding moments using square columns having the same area. It can be concluded that the moments in slabs with circular columns are higher than those obtained using square columns of the same area. This difference is small for small columns but increases with the increase in the column dimensions. Circular columns give rise to a uniform distribution of clamping moments while square columns produce clamping moments at the column corners higher than those at the column sides. This difference increases with the increase in the column dimensions.

3.3 FLAT PLATE WITH INFINITE EXTENSION AND UNIFORMLY LOADED WITH DIFFERENT LOAD INTENSITIES

Consider a reinforced concrete flat plate slab with rectangular panels and loaded with different load intensities as shown in Fig. 3.7.

Due to the difference in the load intensities in two successive panels, the moments in the slab system become different on either side of the supporting columns. These differences in moments are transferred to the columns. The value of these moments depends on:

1. The column stiffness which depends on:
 - a) the column cross-sectional dimension (K)
 - b) the column height (H)
 - c) the column shape.
2. The slab stiffness which depends on:
 - a) the slab panel dimension (a and b)
 - b) the slab thickness (t).

The finite element analysis of this problem cannot be made straightforward because the boundary conditions at the plate edges are not defined. The problem can be solved by combining two loading patterns; the first is symmetrical and the second is anti-symmetrical (Fig. 3.8). In the two problems, the slab edges can be prescribed as axes of symmetry giving the required boundary conditions.

The first loading pattern is the same as the problem previously discussed and the results obtained can be used in this problem. In the second problem, the x -axis is an axis of symmetry and the y -axis is an axis of anti-symmetry; therefore, only one-quarter of the plate is considered in the analysis. Because of the symmetry about the x -axis, the rotation of the column around the x -axis is zero. The column is free to rotate around the y -axis and only one-quarter of the column stiffness

should be considered.

In deriving the column rotational stiffness, the slab thicknesses on the top and bottom are considered to be equal. The column stiffness characteristics and the analogous column sections are shown in Fig. 3.9.

The properties of the analogous section are:

$$A = \frac{1}{EI_c} \cdot H_c = \frac{H_c}{EI_c}$$

$$I = \frac{\left(\frac{1}{EI_c}\right)H_c^3}{12} = \frac{H_c^3}{12EI_c}$$

The rotational stiffness of the column is:

$$\begin{aligned} K_c &= \frac{1}{\left(\frac{H_c}{EI_c}\right)} + \frac{\left(\frac{H_c}{2} + \frac{t}{2}\right)^2}{\left(\frac{H_c^3}{12EI_c}\right)} \\ &= \frac{EI_c}{H_c} \left[4 + 6\left(\frac{t}{H_c}\right) + 3\left(\frac{t}{H_c}\right)^2 \right] \end{aligned} \quad (3.1)$$

The carryover factor is:

$$\begin{aligned} C_{TB} &= \frac{\frac{1}{\left(\frac{H_c}{EI_c}\right)} - \frac{\left(\frac{H_c}{2} + \frac{t}{2}\right)^2}{\left(\frac{H_c^3}{12EI_c}\right)}}{\frac{1}{\left(\frac{H_c}{EI_c}\right)} + \frac{\left(\frac{H_c}{2} + \frac{t}{2}\right)^2}{\left(\frac{H_c^3}{12EI_c}\right)}} = \frac{2 + 6\left(\frac{t}{H_c}\right) + 3\left(\frac{t}{H_c}\right)^2}{4 + 6\left(\frac{t}{H_c}\right) + 3\left(\frac{t}{H_c}\right)^2} \end{aligned} \quad (3.2)$$

where

t = slab thickness

H_c = column clear height.

The relationship between the rotations and deflection of the nodes attached to the column and the column rotation is:

$$\begin{bmatrix} \omega_i \\ \omega_{,x_i} \\ \omega_{,y_i} \\ \omega_j \\ \omega_{,x_j} \\ \omega_{,y_j} \\ \omega_k \\ \omega_{,x_k} \\ \omega_{,y_k} \end{bmatrix} = \begin{bmatrix} -c & -d \\ 1 & 0 \\ 0 & 1 \\ -c & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -d \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} \quad (3.3)$$

where

c = half the column cross-sectional dimension parallel to the x-axis

d = half the column cross-sectional dimension parallel to the y-axis

θ_x = the column rotation about the y-axis

θ_y = the column rotation about the x-axis

Tables 3.4, 3.5, 3.6 and 3.7 show the moments induced in rectangular columns due to the difference in the load intensity in two successive slab panels for different values of K , a/b , H and t . From the tables, it can be concluded that the change in the column cross-sectional dimension has the largest effect on the column moment while no significant effect is obtained by changing the slab thickness and the column height.

The results obtained from the analysis show that the change in the slab thickness and the column height has also no significant effect on the slab moments; hence the factors that have a big influence on the slab moments are the column cross-sectional dimensions, the a/b ratio and the column shape.

FIG. 3.1a. Uniformly Loaded Flat Plate Slab.

FIG. 3.1b. Finite Element Model.

FIG. 3.2. Bending Moment at Point A ($M_x = \beta q a^2$).

FIG. 3.3. Bending Moment at Point B ($M_x = \beta q a^2$).

FIG. 3.4. Bending Moment at Point C ($M_x = \beta q a^2$).

FIG. 3.5. Bending Moment at Point D ($M_x = \beta q a^2$).

(a) Typical Slab Panel

(b) Finite Element Grid

Fig. 3.6. Flat Plate with Square Panels and Supported by Circular Columns.

FIG. 3.7. Flat Plate-Slab with Different Load Intensities.

Fig. 3.8/ Finite Element Idealizations.

FIG. 3.9. Column Stiffness Characteristic.

TABLE 3.1. Bending Moments ($M=\beta qa^2$) of a Uniformly Loaded Rectangular Panel of Flat Plate,
 $\nu=0.2$, $K=0.1$.

β	Location	$a/b=1.0$	$a/b=1.2$	$a/b=1.4$	$a/b=1.6$	$a/b=1.8$	$a/b=2.0$
M_x	$x=0, y=0$	0.03024	0.03250	0.03406	0.03515	0.03572	0.03600
M_x	$x=a/2, y=0$	-0.01664	-0.02420	-0.02922	-0.03252	-0.03450	-0.03550
M_x	$x=0, y=b/2$	0.04438	0.04175	0.04000	0.03877	0.03796	0.03737
M_x	$x=a/2, y=b/2$	-0.12897	-0.11950	-0.11373	-0.11035	-0.10879	-0.10687
M_y	$x=0, y=0$	0.03024	0.01972	0.01415	0.01133	0.00950	0.00856
M_y	$x=0, y=b/2$	-0.01664	-0.00535	0.00045	0.00350	0.00516	0.00607
M_y	$x=a/2, y=0$	0.04438	0.03275	0.02470	0.01875	0.01412	0.01040
M_y	$x=a/2, y=b/2$	-0.12897	-0.09150	-0.06440	-0.04238	-0.02392	-0.00869

TABLE 3.2. Bending Moment ($M=\beta qa^2$) of Flat Plate Supported by Circular Columns when $\nu=0.2$ and $D/a=0.2$.

β	Location	Theory of Plate	Finite Element	
			β	Error
$M_x = M_y$	$x=0, y=0$	0.0292	0.02825	3.25%
M_y	$x=a/2, y=0$	0.0399	0.03975	0.37%
M_x	$x=a/2, y=0$	-0.0161	-0.01605	0.31%
M_r	$r=D/2$	-0.1682	-0.15800	6.06%

TABLE 3.3. Bending Moment ($M = \beta q a^2$) of a Uniformly Loaded Square Panel when $\nu = 0.2$.

β	Location	Circular Column ($D/a = 0.05$)	Equivalent Square Column	Difference	Circular Column ($D/a = 0.1$)	Equivalent Square Column	Difference	Circular Column ($D/a = 0.2$)	Equivalent Square Column	Difference
$M_x = M_y$	$x=0, y=0$	0.0326	0.03220	1.22%	0.0315	0.03070	2.6%	0.02825	0.02690	4.79%
M_y	$x=a/2, y=0$	0.0502	0.04882	2.75%	0.0474	0.04538	4.26%	0.03975	0.03675	7.55%
M_x	$x=a/2, y=0$	-0.0182	-0.01790	1.65%	-0.0175	-0.01700	2.85%	-0.01605	-0.01400	12.77%
M_x	$x=a/2-c/2, y=a/2$		-0.21000			-0.14100			-0.07300	
M_x	$x=a/2-c/2, y=a/2-c/2$		-0.26400			-0.20600			-0.12000	
M_T	$r=D/2$	-0.2400			-0.2185			-0.15800		

* c is the cross-sectional dimension of the equivalent square column.

TABLE 3.4. Bending Moment ($M=\beta qba^2$) in Columns for $a/b=1$, $a/H=1.5$,
 $a/t=35.3$ and $\nu=0.2$.

K	β
0	0
0.025	0.002127
0.035	0.007020
0.050	0.017100
0.075	0.029250
0.100	0.034830
0.125	0.038310
0.150	0.040350
0.175	0.041970
0.200	0.043760

TABLE 3.5. Bending Moment ($M=\beta \cdot \Delta q b a^2$) in Columns for $K=0.1$,
 $a/H=1.5$, $a/t=35.3$ and $\nu=0.2$.

a/b	
1.0	0.3483
1.1	0.3575
1.2	0.3650
1.3	0.3704
1.4	0.3776
1.5	0.3813
1.6	0.3855
1.7	0.3899
1.8	0.3925
1.9	0.3950
2.0	0.3968

TABLE 3.6. Bending Moment ($M=\beta \Delta q b a^2$) in Columns for $K=0.1$, $a/b=1$, $a/t=35.3$, and $\nu=0.2$.

a/H	β
1.30	0.03449
1.50	0.03483
1.75	0.03518
2.00	0.03541
2.40	0.03574

TABLE 3.7. Bending Moment ($M=\beta \Delta q b a^2$) in Columns for $K=0.1$, $a/b=1$, $a/H=1.5$ and $\nu=0.2$.

a/t	β
28	0.03300
30	0.03362
32	0.03415
34	0.03460
36	0.03496
38	0.03527
40	0.03550

CHAPTER 4. DESIGN METHODS OF SLABS

4.1 INTRODUCTION

This chapter describes two design methods recommended by the code and compares the results obtained by these methods with the finite element results.

4.2 CODE METHODS

For the design of two-way slabs, CSA Standard CAN3-A23.3-M77* allows the use of any design method provided the requirements of statics, compatibility and strength are satisfied. The two methods specified by the Code reduce the three-dimensional slab problem to a series of two-dimensional problems in two orthogonal directions. Each direction involves the consideration of rigid frames consisting of the columns above and below the floor, and the floor system bounded laterally between the centerlines of the two panels adjacent to the line of columns.

The portion of the floor considered in the analysis (slab strip) is formed by the "column strip" and the "middle strip". The width of a column strip is defined to be $1/4$ the shorter span length of a panel on each side of the column centerline. The width of a middle strip is the slab strip width minus the column strip width (Fig. 4.1).

* CSA Standard CAN3-A23.3-M77 "Code for the Design of Concrete Structures for Buildings".

The two methods recommended by the Code are:

1. Direct design method and
2. Equivalent frame method.

4.2.1 Direct design method

The direct design method is basically a fairly heavily parameterized tabulation scheme for picking off the distribution of moments directly, without any need for detailed analysis. It is restricted by limitations on continuity, dimensions and live-load to dead-load ratios to ensure regular stress patterns in the slab system.

The distribution of the moments in the longitudinal direction is found by proportioning the total static design moments according to a set of coefficients prescribed by the Code.

The longitudinal moments, obtained from the previous step, are distributed over the entire width of the slab strip. Each of these moments is to be divided to the column strip and the two half middle strips in accordance with another set of coefficients provided by the Code.

4.2.2 Equivalent frame method

The equivalent frame does involve a structural analysis and therefore is capable of providing a more realistic distribution of moments. It differs from the direct design method only in the way in which the longitudinal moments along the spans of the equivalent frame are obtained. In the equivalent frame method, the elastic analysis of

the equivalent rigid frame is made and the maximum positive moments within the span and negative moments at the supports are determined.

The equivalent frame consists of slab-beam members and equivalent column members. Equivalent columns are further divided into actual column members and an attached torsional member (Fig. 4.2).

The slab-beam member is formed by a slab strip taken longitudinally in the direction of the analysis. The width of a slab-beam is defined as the distance between the centerlines of an adjacent panel. The actual column members are taken from the centerline of the top floor slab to the centerline of the bottom floor slab. The portion of the column below the underside of the slab above and the face of the slab below will have a finite stiffness. Variations in cross-section, possibly due to column capital, should be considered in the stiffness computations.

Torsional members are introduced to estimate the actual moment distribution between the adjacent slab panels in a three-dimensional slab system.

The flexibility of the equivalent column member is defined by:

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} \quad (4.1)$$

This expression can be inverted to give the stiffness of the equivalent column:

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t} \quad (4.2)$$

where

K_{ec} is the stiffness of the equivalent column member

K_c is the stiffness of the actual column member

and K_t is the stiffness of the attached torsional member.

Due to lateral loads, unequal gravity loads or unsymmetrical spans, moments in the slab system may be different on either side of the supporting columns. These differences in moments are referred to as "unbalanced moments" and are transferred to columns. These moments are obtained directly from the frame analysis.

4.3. ANALYSIS OF RESULTS

To compare the results of the finite element analysis with the Code design methods, the two problems previously discussed in Chapter 3 were considered again using the Code design methods. The details of the required calculations are presented in Appendices B and C.

4.3.1 The column moment

The curves (4.3, 4.4) and tables (4.1, 4.2) show the bending moments transmitted to the columns by the equivalent frame and the finite element for different column dimensions, a/b ratios, column heights, and slab thicknesses.

It can be observed that:

1. The curves obtained by the two methods have the same shape.
2. Increasing the column dimensions increases the column moments.

3. The column height and the slab thickness have a small effect on the column moments.
4. The moments calculated by the finite element analysis are higher than those obtained by the equivalent frame method.

This difference in the moments obtained from the finite element method and the equivalent frame method is due to the different way of modelling the slab column joint in the two methods. The equivalent frame method incorporates a so-called equivalent column connected to the slab.

4.3.2 The slab moment

The longitudinal moments obtained from the frame analysis in the Code design methods are divided between the column strip and the middle strip. In each strip, the moments are assumed to be uniformly distributed over the entire strip width; therefore, only four values for the moments in each slab panel are obtained.

The curves in Figs. 4.5, 4.6, 4.7 and 4.8 show a comparison between the results of the different methods for the longitudinal bending moments taken at sections (1-1), (2-2), (3-3) and point 4 in Fig. 4.1.

Since the moments obtained from the finite element method vary across both the column and middle strip widths, the maximum and the minimum values of these moments are presented in the curves.

The curves in Fig. 4.5 show that the positive moment of the middle strip calculated by the Code methods is lower than the moments obtained from the finite element analysis. The difference in the results increases with the increase of the ratio a/b and the decrease in the column dimensions.

Fig. 4.6 shows that the positive moment of the column strip is higher for the Code methods than for the finite element method especially when the a/b ratio increases.

Fig. 4.7 shows that the direct design method gives larger negative middle strip moments than the equivalent frame, and the difference increases with the increase of K , e.g. when $K=0.25$, the difference is about 30%.

The finite element method gives small values for the moments at large values of K , but with the decrease in the value of K , the maximum moment increases rapidly giving higher moments than those obtained by the Code design methods.

The negative moments at the slab column interface (Fig. 4.8) obtained from the finite element method are high in comparison with the two other methods especially at small values of K .

In general, it may be observed that for the direct design method, no difference in the moments results with the change in a/b ratio, while in the equivalent frame method, a small difference in the moment is observed.

FIG. 4.1. Definition of Column Strip and Middle Strip.

FIG. 4.2. Model of Slab-Column Joint.

FIG. 4.3. Bending Moment in Column for $a/b=1$, $a/H=1.5$, $a/t=35.3$
 $(M=\beta \Delta q b a^2)$.

FIG. 4.4. Bending Moment in Column for $K=0.1$, $a/H=1.5$, $a/t=35.3$ ($M=\beta\Delta qba^2$).

Fig. 4.6. Bending Moment of Column Strip at Line (2-2) (Fig. 4.1)
($M=Bqa^2$).

FIG. 4.7. Bending Moment of Middle Strip at Line (3-3) (Fig. 4.1) ($M = \beta q a^2$).

FIG. 4.8. Bending Moment M_x at Point 4 (Fig. 4.1) ($M_x = Bqa^2$).

TABLE 4.1. Calculation of Bending Moment ($M=\beta \Delta q b a^2$) in Columns for $K=0.1$, $a/b=1$, $a/t=35.3$, and $v=0.2$.

a/H	β	
	Finite Element	Equivalent Frame
1.30	0.03449	0.02665
1.50	0.03483	0.02695
1.75	0.03518	0.02700
2.00	0.03541	0.02720
2.40	0.03574	0.02736

TABLE 4.2. Bending Moment ($M=\beta \Delta q b a^2$) in Columns for $K=0.1$, $a/b=1$, $a/H=1.5$, and $v=0.2$.

a/t	β	
	Finite Element	Equivalent Frame
28	0.03300	0.02528
30	0.03362	0.02585
32	0.03415	0.02629
34	0.03460	0.02663
36	0.03496	0.02697
38	0.03527	0.02722
40	0.03550	0.02745

CHAPTER 5. CONCLUSIONS AND FUTURE WORK

5.1 CONCLUSIONS

On the basis of the results obtained in this study, the following general conclusions are reached:

1. The constraint method, developed in this study to include the column rotation, offers an alternative approach for incorporating column stiffness into a finite element grid of plate bending elements.
2. The special purpose program developed in the course of this work is a tool for investigating the behaviour of a slab-column system as parameters are varied.
3. In flat plate slabs with rectangular columns, moments of very high values occur at the column corners. In reinforced concrete floors, these high negative moments are limited by the cracking of concrete in tension and local yielding of steel reinforcement. These cracks also reduce the moments transmitted to the columns.
4. Changing the ratio a/b results in changing the distribution of the longitudinal moments in the slab calculated by the finite element method, while no significant change in the moments calculated from the code design method is observed.
5. The ratio K (column to slab dimension) is the most effective factor that affects the slab and column moments. The slab

thickness and the column height have the least significant effect. It may be concluded that increasing the K ratio reduces the slab moments and at the same time increase the column moment.

6. Using circular instead of square columns increases the slab moments, as shown by Table 3.3, and gives more uniform distribution of clamping moments. The difference between the moments increases with the increase in the column dimensions; therefore, the code provision of replacing the circular columns with square columns having the same area underestimates the slab moments, especially when the column dimensions are large.
7. In the finite element method, the transverse distribution of the longitudinal moments in the slab changes with the change in a/b ratio and the column dimensions. In the code design methods, the factors for distributing the longitudinal moments between the column strip and the middle strip depend on the ratio between the longitudinal beam stiffness and slab stiffness (α_1). This ratio is always equal to zero in the flat plate slabs since no beams are used. Therefore, the distributing factors are always the same, and there is no change in the transverse distribution of the moments with the change in a/b ratio and the column dimensions.

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8. The constraint method, developed in this study, was used to investigate the distribution of moments in flat plate slabs (i.e. no drop panels or column heads). However, since the columns are basically reduced to a patch on the surface, the extension to flat slabs is basically a minor one.
9. All of the results obtained are valid for a "reversed flat slab" i.e. for a mat foundation carrying a system of equidistant columns in rigid connections with the slab, provided the soil pressure is distributed uniformly.

5.2 FUTURE WORK AND DEVELOPMENT

The finite element analysis of this thesis has been directed toward the study of the distribution of moment in the columns and slabs of a typical flat plate slab panel. In the analysis, the displacements along the edges of the finite element model should be prescribed to give the necessary boundary conditions of the problem. The limited nature of the boundary conditions, which can be applied, limits the generality of the solution which can be obtained. Thus, only flat plate slabs with infinite extensions (large number of bays) under the application of symmetrical or anti-symmetrical loading could be studied. Any repeated pattern of loading can also be presented by combining a symmetrical and anti-symmetrical loading. The tools developed are appropriate to parametric study and are not design tools.

In future research, more general problems can be studied by modifying and expanding the existing method to include the whole floor (Fig. 5.1), thus allowing the study of the distribution of moments in all columns and slab panels without having a problem with the boundary conditions. With this modification, any pattern of loading can be studied. The distribution of moments in the panels at or near the floor boundary can be found in this way. Such a model could be formed by putting together several of the structural units developed in the course of this thesis.

The floor system can be idealized as column and slab beam elements connected together with a rotational spring (Fig. 5.2). Results obtained from the previous solution can be used to investigate the spring coefficient at the joint in the idealized system. Once these coefficients are determined, the analysis could be carried out by any frame analysis method such as the stiffness method or moment distribution. For example, a number of programs are available for three-dimension analysis of frames. By inserting a torsional beam (out-of-plane) between the columns in one plane and beams in a parallel plane, these coefficients could be incorporated as the torsional stiffness as shown in Fig. 5.3. In the frame analysis P-delta effects can be included to give more accurate results especially when the bending moments due to lateral horizontal forces (wind loads or earthquakes) are required.

As previously calculated in the reinforced concrete slabs, the high negative moment occurring at the column corners is limited by the cracking of the concrete and yielding of the steel reinforcement. This results in a change in the slab mechanism accompanied by a re-distribution in the slab moments. The inelastic analysis of the selected model can help to study the problem in a more realistic approach as far as the ultimate capacity of the reinforced concrete is concerned.

FIG. 5.1. Equivalent Frame.

Actual Two-Dimensional Slab-Column Frame

Idealized Beam-Column Frame

FIG. 5.2.

FIG. 5.3. Three-Dimensional Slab-Column Frame.

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APPENDIX A. TRANSVERSE DISTRIBUTION OF LONGITUDINAL MOMENTS

This appendix contains the curves of the distribution of the longitudinal moment across the width of the slab panel at the middle of the span and at the column edge. The curves are drawn to show the change in the distribution of the moments with the change in a/b ratio and the column dimensions.

The first two curves show the distribution of the moments for $a/b=1.0$ which is the minimum value of a/b and $a/b=2.0$ which is the upper limit value. The other two curves show the distribution in a square panel ($a/b=1.0$) for $K=0.05$ and $K=0.25$.

From the curves, it may be concluded that increasing either the a/b ratio or the column dimension results in a more uniform distribution of longitudinal moments in the transverse direction.

FIG. A.1. Distribution of Longitudinal Moment M_x Along the Line A-B for $K=0.1$, $\nu=0.2$ ($M_x = \beta q a^2$).

FIG. A.2. Distribution of Longitudinal Moment M_x Along the Line C-D for $K=0.1$, $\nu=0.2$ ($M_x = B q a^2$).

FIG. A.3. Distribution of Longitudinal Moment M_x Along the Line 'A-'B for $a/b=1$, $\nu=0.2$ ($M_x = \beta q a^2$).

FIG. A.4. Distribution of Longitudinal Moment M_x Along the Line $C'D'$ for $a/b=1$, $\nu=0.2$ ($M_x = \beta q a^2$).

APPENDIX B. UNIFORMLY LOADED FLAT PLATE WITH INFINITE EXTENSION

The purpose of this appendix is to describe the solution of the problem of flat plate with infinite extension and uniformly loaded by the two methods given by the code, the direct design method and the equivalent frame method.

Design Data

Considering the flat plate slab shown by Fig. B.1, the basic geometry of the slab is:

Panel length = a

Panel width = b

Column cross-sectional dimension = cxd

Uniform load intensity = q

and $c/a=K$ $d/b=K$ $a/b=r$

Direct Design Method

1. The major direction

Total static moment

$$M_o = \frac{qb(a-c)^2}{8} \quad (11.4.2.2)$$

$$M_o = \frac{qb(a-aK)^2}{8} = 0.125(1-K)^2 qba^2$$

according to clause (11.4.3.3)

$$M_{(-ve)} = 0.65M_o = 0.08125(1-K)^2 qba^2$$

$$M_{(+ve)} = 0.35M_o = 0.04375(1-K)^2 qba^2$$

Assuming that

$$\beta_a = \left(\frac{D.L}{L.L}\right)_{\text{service}} > 2.0$$

the effect of patterns can be ignored (clause 11.4.6.1).

Design strip moments are assigned to the column strip and the middle strip. According to A23.3M clause 11.4.4, the percentage of longitudinal moment in column strip depends on α_1 (the longitudinal beam stiffness to slab stiffness) and β_t (the torsional rigidity of the edge beam section).

Since there are no beams in the case of flat plate slabs, α_1 and β_t are equal to zero.

Column strip moments:

$$M_{(-ve)} = 0.75 \times 0.08125(1-K)^2 qba^2 = 0.0609375(1-K)^2 qba^2$$

$$M_{(+ve)} = 0.60 \times 0.04375(1-K)^2 qba^2 = 0.02625(1-K)^2 qba^2$$

These moments are distributed over the column strip width which is equal to $b/2$. Therefore,

$$M_{(-ve)}/\text{unit width} = \frac{0.0609375(1-K)^2 qba^2}{b/2} = 0.121875(1-K)^2 qa^2$$

$$M_{(+ve)}/\text{unit width} = \frac{0.02625(1-K)^2 qba^2}{b/2} = 0.0525(1-K)^2 qa^2$$

Middle strip moments:

$$M_{(-ve)} = 0.25 \times 0.08125(1-K)^2 qba^2 = 0.023125(1-K)^2 qba^2$$

$$M_{(+ve)} = 0.40 \times 0.04375(1-K)^2 qba^2 = 0.0175(1-K)^2 qba^2$$

These moments are distributed over the middle strip width which is equal to $b/2$.

$$M_{(-ve)}/\text{unit width} = \frac{0.023125(1-K)^2 qba^2}{b/2} = 0.04625(1-K)^2 qa^2$$

$$M_{(+ve)}/\text{unit width} = \frac{0.0175(1-K)^2 qba^2}{b/2} = 0.035(1-K)^2 qa^2$$

2. The minor direction

$$M_o = \frac{qa(b-d)^2}{8} = \frac{qa(b-Kb)^2}{8} = 0.125(1-K)^2 qab^2$$

$$M_{(-ve)} = 0.65M_o = 0.08125(1-K)^2 qab^2$$

$$M_{(+ve)} = 0.35M_o = 0.04375(1-K)^2 qab^2$$

Middle strip moments:

$$M_{(-ve)} = 0.25 \times 0.08125(1-K)^2 qab^2 = 0.023125(1-K)^2 qab^2$$

$$M_{(+ve)} = 0.4 \times 0.04375(1-K)^2 qab^2 = 0.0175(1-K)^2 qab^2$$

$$\text{The strip width} = a - \frac{b}{2} = a - \frac{a}{2r}$$

Therefore,

$$M_{(-ve)}/\text{unit width} = \frac{0.023125(1-K)^2 qa \left(\frac{a}{r}\right)^2}{a - \frac{a}{2r}} = 0.04625 \frac{(1-K)^2}{r(2r-1)} qa^2$$

$$M_{(+ve)}/\text{unit width} = \frac{0.0175(1-K)^2 qa \left(\frac{a}{r}\right)^2}{a - \frac{a}{2r}} = 0.035 \frac{(1-K)^2}{r(2r-1)} qa^2$$

Column strip moments:

$$M_{(-ve)} = 0.75 \times 0.08125(1-K)^2 qab^2 = 0.0609375(1-K)^2 qab^2$$

$$M_{(+ve)} = 0.6 \times 0.04375(1-K)^2 qab^2 = 0.02625(1-K)^2 qab^2$$

The strip width = $\frac{b}{2}$

Therefore

$$M_{(-ve)}/\text{unit width} = \frac{0.0609375(1-K)^2 qab(a/r)}{b/2} = 0.121875 \frac{(1-K)^2}{r} qa^2$$

$$M_{(+ve)}/\text{unit width} = \frac{0.02625(1-K)^2 qab(a/r)}{b/2} = 0.0525 \frac{(1-K)^2}{r} qa^2$$

Equivalent Frame Method

1. The major direction

Since all the panels are equal and have the same uniform load (q) and all the columns have the same dimension. The fixed end moments will be the same and there is no need to calculate the moment distribution constants.

$$\text{Fixed end moments } (M_f) = m qa^2 b \quad (11.5.1)$$

where m varies with the change of K .

The negative moment at the column face ($M_{(-ve)}$) is:

$$= m q ba^2 - \frac{1}{2} q(ab-cd) \frac{c}{2} + q(b-d) \frac{c}{2} \cdot \frac{c}{4}$$

Therefore

$$M_{(-ve)} = [m - \frac{1}{8}(2K-K^2-rK^3)]qba^2$$

$$\begin{aligned} M_{(+ve)} &= \frac{1}{2} q(ab-c^2) \frac{a}{2} - \frac{1}{2} q(ab-c^2) \frac{a}{4} - m q a^2 b \\ &= [\frac{1}{8} (1-rK^2) - m] qa^2 b \end{aligned}$$

since the slab within the limitations of the direct design method.

Therefore, the design moments are reduced such that the total static design moment M_o is maintained (A23.3M, clause 11.5.5).

Correction factor (C.F.)

$$= \frac{M_o}{M_{(+ve)} + M_{(-ve)}} = \frac{0.125(1-K)^2}{0.125(1-K^2) - 0.125(2K-K^2-rK^3)}$$

Therefore

$$CF = \frac{(1-K)^2}{1-2K+(1-r)K^2+rK^3}$$

Therefore

$$M_{(+ve)} = CF[0.125(1-rK^2) - m]qa^2b$$

Distributing the moments between the column and middle strip as direct design moment, we get:

Middle strip moments:

$$M_{(-ve)}/\text{unit width} = 0.5[m - 0.125(2K-K^2-rK^3)]qa^2$$

$$M_{(+ve)}/\text{unit width} = 0.8 \times CF[0.125(1-rK^2) - m]qa^2$$

Column strip moments:

$$M_{(-ve)}/\text{unit width} = 1.5[m - 0.125(2K - K^2 - rK^3)]qa^2$$

$$M_{(+ve)}/\text{unit width} = 1.2 \times CF[0.125(1 - rK^2) - m]qa^2$$

2. The minor direction

Similarly, as the major direction. The negative moment at the column face ($M_{(-ve)}$)

$$= m q ab^2 - \frac{1}{2} q(ab-d^2) \frac{d}{2} + q(a-c) \frac{d}{2} \frac{d}{4}$$

Therefore

$$M_{(-ve)} = [m - 0.125(2K - K^2 - \frac{1}{r} K^3)]qab^2$$

$$\begin{aligned} M_{(+ve)} &= \frac{1}{2} q(ab-d^2) \frac{b}{2} - \frac{1}{2} q(ab-d^2) \frac{b}{4} - m q ab^2 \\ &= [0.125(1 - \frac{1}{r} K^2) - m]qab^2 \end{aligned}$$

$$CF = \frac{(1-K)^2}{1 - 2K + K^2 - \frac{K^2}{r} + \frac{K^2}{r}}$$

$$M_{(+ve)} = CF[0.125(1 - \frac{1}{r} K^2) - m]qab^2$$

Middle strip moments:

$$M_{(-ve)}/\text{unit width} = 0.5[m - 0.125(2K - K^2 - \frac{1}{r} K^3)] \frac{1}{r(2r-1)} qa^2$$

$$M_{(+ve)}/\text{unit width} = \frac{0.8 \times CF}{r(2r-1)} \times [0.125(1 - \frac{1}{r} K^2) - m]qa^2$$

Column strip moments:

$$M_{(-ve)}/\text{unit width} = \frac{1.5}{r} [m - 0.125(2K - K^2 - \frac{1}{r} K^3)] qa^2$$

$$M_{(+ve)}/\text{unit width} = \frac{1.2}{r} \times CF[0.125(1 - \frac{1}{r} K^2) - m] qa^2$$

FIG. B.1. Uniformly Loaded Flat Plate with Infinite Extension.

APPENDIX C. FLAT PLATE WITH INFINITE EXTENSION AND UNIFORMLY LOADED
WITH DIFFERENT LOAD INTENSITIES

Design Data

Panel dimension $= a \times b$

Column dimension $= c \times d$

Column height $= H$

Slab thickness $= t$

$K = c/a$ and $k = d/b$

The load intensities in two successive panels are q_1 and q_2 as shown in Fig. C.1.

Example

Assume:

$$a \times b = 6.00 \times 6.00 \text{ m}$$

$$c \times d = 0.60 \times 0.60 \text{ m}$$

$$H = 4.00 \text{ m}$$

$$t = 0.17 \text{ m}$$

$$q_1 = 10 \text{ KN/m}^2 = 1.0 \text{ N/cm}^2$$

$$q_2 = 12 \text{ KN/m}^2 = 1.2 \text{ N/cm}^2$$

Slab stiffness (K_s) (Clause 11.5.1(d))

Column dimension $c = d = 600 \text{ mm}$

$$K = \frac{c}{a} = 0.1$$

$$K = \frac{d}{b} = 0.1$$

Therefore

$$K = 4.18$$

$$C = 0.51$$

$$m = 0.0847$$

$$I_s = 6000(170)^3/12 = 2456.5 \times 10^6 \text{ mm}^4$$

$$K_s = K E_{cs} I_s / \ell_1 = \frac{4.18 \times 2456.5 \times 10^6 E_{cs}}{6000} = 1711.3616 \times 10^3 E_{cs}$$

Column stiffness (K_c): (Clause 11.5.1(e))

$$\frac{H}{H_c} = \frac{4000}{3830} = 1.0471204$$

$$K = 4.46$$

$$C = 0.55$$

$$I_c = \frac{(600)^4}{12} = 1.08 \times 10^{10} \text{ mm}^4$$

$$K_c = K E_{cc} I_c / H = \frac{4.46 \times 1.08 \times 10^{10} E_{cc}}{4000} = 12042 \times 10^3 E_{cc}$$

Torsional member stiffness (K_t): (Clause 11.5.1(e))

$$x = 170 \text{ mm}$$

$$y = 600 \text{ mm}$$

$$c = (1 - 0.63 \frac{x}{y}) \frac{x^3 y}{3} = 807205.98$$

Therefore,

$$\frac{d}{b} = 0.1$$

$$r_1 = 12.3$$

$$K_t = \frac{r_1 c E_{cs}}{\ell_2} = \frac{12.3 \times 807.205.98 \times E_{cs}}{6000} = 1654.7721 \times 10^3 E_{cs}$$

Equivalent column stiffness (K_{ec}): (Clause 11.5.1(e))

$$K_{ec} = \frac{K_c \times K_t}{K_c + K_t}$$

Therefore, for $E_{cc} = E_{cs}$

$$\begin{aligned} K_{ec} &= \frac{(2 \times 12042)(2 \times 1654.7721) \times 10^6 E_{cs}}{(2 \times 12042) + (2 \times 1654.7721) \times 10^3} \\ &= 2909.7 \times 10^3 E_{cs} \end{aligned}$$

Distribution factor

$$DF = \frac{1171.3616 \times 10^3}{(2 \times 1171.3616 + 2909.7) \times 10^3} = 0.27$$

Fixed end moment

$$\begin{aligned} FEM_1 &= m q_1 \ell_1^2 = 0.0847 \times (1 \times 600) \times (600)^2 = 182.952 \times 10^5 \text{ N.cm} \\ &= 182.952 \text{ KN.m} \end{aligned}$$

$$\begin{aligned} FEM_2 &= m q_2 \ell_1^2 = 0.0847 \times (1.2 \times 600) \times (600)^2 = 219.5424 \times 10^5 \text{ N.cm} \\ &= 219.5424 \text{ KN.m} \end{aligned}$$

← 0.51 →		← 0.51 →		← 0.51 →		← 0.51 →	
0.27 C 0.27		0.27 A 0.27		0.27 B 0.27			
-182.952	+219.5424	-219.5424	+182.952	-182.952	+219.5424	F.E.M.	
- 9.88	- 9.88	+ 9.88	+ 9.88	- 9.88	- 9.88	Bal.M	
+ 5.04	+ 5.04	- 5.04	- 5.04	+ 5.04	+ 5.04	C.O.M.	
- 2.72	- 2.72	+ 2.72	+ 2.72	- 2.72	- 2.72	Bal. M	
+ 1.39	+ 1.39	- 1.39	- 1.39	+ 1.39	+ 1.39	C.O.M.	
- 0.75	- 0.75	+ 0.75	+ 0.75	- 0.75	- 0.75	Bal. M	
+ 0.38	+ 0.38	- 0.38	- 0.38	+ 0.38	+ 0.38	C.O.M.	
- 0.20	- 0.20	+ 0.20	+ 0.20	- 0.20	- 0.20	Bal. M	
+ 0.105	+ 0.105	- 0.105	- 0.105	+ 0.105	+ 0.105	C.O.M.	
- 0.0567	- 0.0567	+ 0.0567	+ 0.0567	- 0.0567	- 0.0567	Bal. M	
-189.6437	+212.8507	-212.8507	+189.6437	-189.6437	+212.8507	Final M	

Therefore,

$$\text{the column moment} = \frac{212.8507 - 189.6437}{2} = 11.6035 \text{ KN.m}$$

Slab moment

Span (A-B)

Negative moment at the column face

$$= 189.6437 - \frac{1}{2} \times 10(6 \times 6 - 0.6 \times 0.6) \times \frac{0.6}{2} + 10(6-0.6) \times \frac{0.6}{2} \times \frac{0.6}{4}$$

Therefore,

$$M_{(-ve)} = 189.6437 - 53.46 + 2.43 = 138.6137 \text{ KN.m}$$

Maximum positive moment

$$= \frac{1}{2} \times 10(6 \times 6 - 0.6 \times 0.6) \times \frac{6}{2} - \frac{1}{2} \times 10(6 \times 6 - 0.6 \times 0.6) \times \frac{6}{4} - 189.6437$$

Therefore,

$$M_{(+ve)} = 267.3 - 189.6437 = 77.6563 \text{ KN.m}$$

Simple span moment

$$M_o = \frac{10 \times 6(6-0.6)^2}{8} = 218.7 \text{ KN.m}$$

Correction factor

$$= \frac{218.7}{138.6137 + 77.6563} = 1.011236$$

Therefore,

$$M_{(+ve)} = 1.011236 \times 77.6563 = 78.5288 \text{ KN.m}$$

Span (C-A)

$$M_{(-ve)} = 212.8507 - \frac{1}{2} \times 12(6 \times 6 - 0.6 \times 0.6) \times \frac{0.6}{2} + 12(6-0.6) \times \frac{0.6}{2} \times \frac{0.6}{4} = 151.6147$$

$$M_{(+ve)} = \frac{1}{2} \times 12(6 \times 6 - 0.6 \times 0.6) \times \frac{6}{2} - \frac{1}{2} \times 12(6 \times 6 - 0.6 \times 0.6) \times \frac{6}{4} - 212.8507 = 107.9 \text{ KN.m}$$

$$M_o = \frac{12 \times 6(6 - 0.6)^2}{8} = 262.44 \text{ KN.m}$$

$$\text{Correction factor} = \frac{262.44}{151.6147 + 107.9} = 1.011236$$

Therefore,

$$M_{(+ve)} = 1.011236 \times 107.9 = 109.11 \text{ KN.m}$$

Design moments for column and middle strip:

The negative and positive moments are distributed to the column and middle strips on the basis of the proportions specified in Clause 11.5.3.

Bending Moment in Columns in Flat Plate Slabs with Square Panels.

$t = 17 \text{ cm}$ $H = 400 \text{ cm}$ $a=b=600 \text{ cm}$

$q_1 = 1 \text{ N/cm}^2$ $q_2 = 1.2 \text{ N/cm}^2$

K	K_s/E	K_c/E	K_t/E	K_{ec}/E	DF	M_{COL} (KN.m)	β^*
0	1637.667	0	0	0	0.5	0	0
0.025	1650.0	47.039	117.0	67.1	0.49	0.712	0.00165
0.030	1659.8	97.54	196.2	130.3	0.4811	1.308	0.00303
0.050	1674.5	752.625	557.481	640.52	0.4197	5.03	0.01164
0.075	1683.726	3810.159	1081.5	1684.78	0.3332	9.14	0.02115
0.100	1711.3616	12047.0	1654.7721	2909.8	0.2702	11.61	0.02687
0.125	1756.9	29399.3	2149.16	4005.5	0.2336	13.3	0.03078
0.150	1802.456	60962.6	3140.0	5972.38	0.1882	14.33	0.03317
0.175	1867.45	112940.6	4289.3	8264.72	0.1556	15.45	0.03576
0.200	1932.0	192672.0	5160.6	10052.0	0.1388	16.2	0.0375

$$* \beta = M/\Delta q \text{ ba}^2$$

where

$$\Delta q = q_2 - q_1$$

Bending Moment in Columns of Flat Plate Slabs with
Rectangle Panels.

$$t = 17 \text{ cm} \quad H = 400 \text{ cm} \quad K = 0.1 \quad a = 600 \text{ cm}$$

$$q_1 = 1 \text{ N/cm}^2 \quad q_2 = 1.2 \text{ N/cm}^2$$

a/b	K_s/E	K_c/E	K_t/E	K_{ec}/E	DF	M_{COL} (KN.m)	β^*
1.0	1711.3616	1204.7	1654.7721	2909.8	0.2702	11.61	0.02687
1.1	1555.78	10947.27	1820.25	3121.478	0.2496	11.16	0.02583
1.2	1426.	10035.	1985.72	3315.4	0.2312	10.7	0.02476
1.3	1316.43	9263.07	2151.2	3499.8	0.2146	10.3	0.02384
1.4	1222.4	8601.43	2316.68	3650.	0.2	9.83	0.02275
1.5	1140.9	8028.	2482.158	3791.9	0.1878	9.43	0.02182
1.6	1069.6	7526.25	2647.63	3917.24	0.1766	8.97	0.02076
1.7	1006.68	7083.5	2813.11	4026.968	0.1666	8.59	0.01988
1.8	950.75	6690.	2978.59	4121.96	0.1578	8.26	0.01912
1.9	900.71	6337.9	3144.06	4203.08	0.15	7.958	0.01842
2.0	855.68	6021.0	3309.544	4271.3	0.143	7.668	0.01775

$$* \beta = M/\Delta q \text{ } ba^2$$

where

$$\Delta q = q_2 - q_1$$

Bending Moment in Columns in Flat Plate Slabs with
Square Panels

$$t = 17 \text{ cm} \quad K = 0.1 \quad a = b = 600 \text{ cm}$$

$$q_1 = 1 \text{ N/cm}^2 \quad q_2 = 1.2 \text{ N/cm}^2$$

H	K_s/E	K_c/E	K_t/E	K_{ec}/E	DF	M_{COL} (KN.m)	β^*
250	1711.3616	20643.9	1654.7721	3063.9	0.2638	11.82	0.02736
300	1711.3616	16681.798	1654.7721	3010.8	0.266	11.75	0.0272
350	1711.3616	13991.75	1654.7721	2959.5	0.2681	11.68	0.02703
400	1711.3616	12047.0	1654.7721	2909.8	0.2702	11.61	0.02687
450	1711.3616	10568.3	1654.7721	2861.5	0.2723	11.54	0.02671

$$* \beta = M/\Delta q \text{ ba}^2$$

where

$$\Delta q = q_2 - q_1$$

Bending Moment in Columns in Flat Plate Slabs with
Square Panels

$H = 400 \text{ cm}$ $K = 0.1$ $a = b = 600 \text{ cm}$

$q_1 = 1 \text{ N/cm}^2$

$q_2 = 1.2 \text{ N/cm}^2$

t (cm)	K_s	K_c	K_t	K_{ec}	DF	M_{COL} (KN.m)	β^*
15	1175.6248	11880.796	1165.8	2123.27	0.2627	11.86	0.02745
16	1426.7731	11967.772	1397.2	2502.3	0.2664	11.74	0.02717
17	1711.3616	12047.0	1654.7721	2909.8	0.2702	11.61	0.02687
18	2031.4798	12114.497	1939.20	3343.2	0.2743	11.47	0.02655
19	2389.2181	12193.792	2251.16	3800.6	0.2785	11.32	0.0262
20	2786.6664	12273.799	2591.2	4279.0	0.2828	11.17	0.02585
21	3225.9146	12354.525	2959.77	4775.48	0.2873	11.01	0.02548

$$* \beta = M / \Delta q \text{ ba}^2$$

where

$$\Delta q = q_2 - q_1$$

FIG. C.1. Flat Plate with Infinite Extension and Uniformly Loaded with Different Load Intensities.

FIG. C.3. Bending Moment Along the Centerline of the Panel.

APPENDIX D. PROGRAM DESCRIPTION

Introduction

The Fortran program to be described is an application of the finite element technique to the problem of linear elastic analysis of flat plate floor systems. The program is modified from W.H. Bowes program PB11B[21]. The main features of the program are:

- a) Automatic mesh generation of the plate bending elements
- b) Deriving the slab stiffness using 18 degrees of freedom triangular plate bending elements.
- c) Condensing the degrees of freedom of the nodes attached to the column and adding the column rotational stiffness
- d) A solver of a system of a large linear symmetrical equation by out-of-core method
- e) Calculating the moments in the slab and the column.

Description of the Program

The following flow chart presents in more detail the process of the program and the subroutines used in it and their sequence.

- Print title. Read and print case title.
- Read and print; Young's modulus and Poisson's ratio.
- Read and print; Plate dimensions.
- Read and print; column dimensions.
- Calculate the column rotational stiffness. CCRS

- Fill in the relationship between moments and curvatures [D].
- Fill in the transformation matrix for the nodes with modified curvatures [T].
- Generate coordinates of nodes and element data.
- Read pressure on the plate. GCEL1
- Print coordinates of nodes and pressure.
- Fill zeros in the global stiffness matrix [S] and global force vector [F].
- Do for all elements.
- Determine coordinate of nodes referred to the center of the element.
- IF the new element is similar to the previous one.
- Determine [A], {CO}, {Si}, {FL}
- INVERT [A]
- Eliminate the mid side nodes $[G] = ([A^{-1}] \begin{bmatrix} I \\ H \end{bmatrix})$ EVO2B
- Evaluate integrals of polynomial terms over element [P]. TI01B
- Calculate element stiffness matrix
- $$[E] = [G]^T \int_V [B]^T [D] [B] [G] dV$$
 ES18B
- Calculate consistent load vector {FE}. CLV
- Add the element stiffness to the global stiffness matrix [S]. ASS
- Add the element consistent load vector to the global force vector [F]. ASS2
- Read node number at which curvatures are to be modified.

Modify [S].	MOD
Modify [F].	MOD2
Read known forces.	KF02B
Read known displacements.	KU01B
Operate P and I operation.	PI
Modify [F] and [S] for column rotations.	MOD3
Add the column rotational stiffness to [S].	
Solve for unknown displacements [U].	SOLVE
Transform [U] for original degrees of freedom.	RU01
Calculate and print column rotations and moments.	
Transform [U] for original curvatures.	RU02B
Print displacements.	
Calculate and print moments in the plate.	MP
STOP.	

Data Input

The input data required to use the program can be classified to three groups:

1. Slab and column elastic properties and dimensions, according to this set of data a mesh of triangular plate bending elements is generated; the element stiffness is calculated and assembled in the global stiffness matrix in addition to the calculation of the column rotational stiffness.
2. Load data; according to this, the consistent load vector is calculated.

3. The nodes with modified curvatures and the known displacements, to specify the boundary conditions of the problem.

The following table describes in more detail the data cards required and their format and sequence.

CARD	CARD COLUMN	FORMAT	VARIABLE	VARIABLE DESCRIPTION
1		20A4	NAME	JOB identifications.
2	1-15 15-22	F15.5 F7.4	YM GNU	Young Modulus Poisson's ratio
3	1-10 11-20 21-30	F10.5 F10.5 F10.5	AA BB TH	Slab panel length Slab panel width Slab thickness
4	1-10 11-20 21-30	F10.5 F10.5 F10.5	CC DD CL	Column cross-sectional dimensions Column height
5	1-10 11-20 21-30 31-40	F10.5 F10.5 F10.5 F10.5	P(1) P(2) P(3) P(4)	Pressure in the 1st quarter Pressure in the 2nd quarter Pressure in the 3rd quarter Pressure in the 4th quarter
	1-5	I5	IM	No. of nodes having modified curvature
				Blank card indicates end of nodes having modified curvature
	1-5 6-15	I5 F10.5	I FT	The number of nodes having non-zero forces. Force acting at the node.
				Blank card indicates the end of the
	1-5 6-15	I5 F10.5	I U	Node number Known displacement
				Blank card indicates end of data

The Program Listing

The program listing is at Carleton University, Computer Library.

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