An Algorithm for Preliminary Aeroelastic Analysis of Composite Wind Turbine Blades

by

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Abstract

Determining the structural response of composite horizontal axis wind turbine blades to wind loading is a challenging aeroelastic problem due to the coupling of unsteady aerodynamics and anisotropic structural dynamics. Methods such as Computational Fluid Dynamics (CFD) and Finite Element Analysis (FEA) are readily available through respected commercial software such as ANSYS, but are found to be unsuitable for the preliminary design phase in which many simulations are to be run, due to high computational time. A more efficient algorithm has been developed using a panel method for determining the unsteady aerodynamic blade loading, an anisotropic beam dynamics solver based on the Variational Asymptotic Beam Section (VABS) analysis tool developed by Khouli (2009) and the Ritz method, and the Blade Element Momentum Theory (BEMT).

The algorithm was implemented in MATLAB and validated by comparing the results with coupled two-way Fluid-Structure Interaction (FSI) simulations performed using ANSYS Workbench 14. Strong agreement was found between the algorithm results and the corresponding ANSYS simulations. Impressively, the algorithm achieved computational times of less than 2% of the ANSYS simulations. The algorithm is considered a success and has been found to be suitable for use in the preliminary design phase of horizontal axis wind turbines and other flexible, lightweight structures.

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Nomenclature

Items are listed by subsection, in the order that they appear. Duplicate items will appear in each table.

Chapter 2

2.1.1 Governing Equations of Fluid Dynamics

ρ	Fluid density
$\mathbf{V}$	Flow velocity
$\mathbf{T}$	Viscous stress tensor
$\mathbf{f}$	Body force vector
e	Internal energy per unit volume
q	Conductive heat flux
p	Pressure
W'	Work done per unit volume
Q'	Heat transfer per unit volume

2.1.2 Potential Flow

$\mathbf{V}$	Flow velocity
ϕ	Velocity potential
$\boldsymbol{\xi}$	Vorticity vector
ρ	Fluid density
p	Pressure

2.1.3 Extraction of the Pressure Field

Γ	Circulation
$\mathbf{V}$	Flow velocity
p	Pressure
ρ	Fluid density
V	Local flow speed
ϕ	Velocity potential

2.1.4 Velocity Potential Governing Equation

a	Local speed of sound
p	Pressure
ρ	Fluid density
ϕ	Velocity potential
V	Local flow speed
$\mathbf{V}$	Local flow velocity
Γ	Circulation

2.1.5 Small Disturbance Theory

u, v, w	Flow velocity x, y , and z - components
U	Free stream speed
U_x, U_y, U_z	Free stream velocity x, y , and z - components
$\mathbf{U}$	Free stream velocity
$\mathbf{V}$	Local flow velocity
ϕ	Velocity potential
ϕ_{FS}	Free stream velocity potential
ϕ'	Disturbance velocity potential
a	Local speed of sound
a_∞	Free stream speed of sound

2.1.6 2D Incompressible Unsteady Aerodynamics

ϕ_{FS}	Free stream velocity potential
U_x	Free stream speed, x - component
U_y	Free stream speed, y - component
U_z	Free stream speed, z - component
ϕ_S	Source velocity potential
Λ	Source strength
ϕ_V	Vortex velocity potential
Γ	Vortex strength

2.2 The Blade Element Momentum Theory

$\dot{m}$	Mass flow rate
P_{max}	Available wind power
U	Wind speed
ρ	Air density
A	Rotor disc area
c_P	Power coefficient
P	Developed power
c_T	Thrust coefficient
T	Developed thrust
c_Q	Torque coefficient
R	Rotor disc radius

2.2.1 Blade Element Theory

U_∞	Free stream wind speed
p_∞	Free stream pressure
U_D	Wind speed at rotor disc
P_D^+	Pressure directly in front of disc
P_D^-	Pressure directly behind disc
U_w	Wake velocity

2.2.1 Blade Element Momentum Theory

U_D	Axial wind speed at rotor disc
U	Free stream wind speed
a	Axial induction factor
$U_{t,D}$	Tangential wind speed at the rotor disc
Ω	Disc rotational speed
a'	Tangential induction factor
W	Relative wind speed at rotor disc
ϕ	Angle between relative wind vector and plane of rotation
a'	Tangential induction factor
T	Thrust
Q	Torque
ρ	Air density
B	Number of blades
c_l	2D lift coefficient
c_d	2D drag coefficient
c_x	Axial force coefficient
c_y	Tangential force coefficient
σ_r	Chord solidity

2.3.2 Overview of the Variational Asymptotic Method

ς	Beam slenderness ratio
a	Wavelength of beam deflection
ℓ	Axial induction factor
$\mathcal{F}$	Energy functional
Γ	Symmetric strain field
Υ	Matrix involving non-perturbation terms
z	Unknown perturbation of the 3D elastic field

2.3.3 Overview of Variational Asymptotic Method

h	Section thickness
$\nu_{1,2,3}$	Displacements along the u_1 - s - ξ coordinate axes
$\mathcal{U}$	3D strain energy functional

$D^{\alpha\beta\gamma\delta}$	Reduced 2D elastic modulus
$\epsilon_{\alpha\beta,\gamma\delta}$	Sectional strain
$\gamma_{\alpha\beta}, \rho_{\alpha\beta}$	Koiter and Sanders shell strain measures
$\bar{\mathcal{U}}$	Strain energy functional per unit length
z	Unknown perturbation of the 3D elastic field
u_i	Cartesian sectional displacement
$\mathcal{U}$	Strain energy functional
ρ_i	Bending strain measure

Chapter 3

3.1.1 Classical Thin Airfoil Theory and the Quasi-Steady Approach

c_l	2D lift coefficient
α	Angle of attack
z	Airfoil camberline curve
θ_0	Variable of integration
$c_{m,c/4}$	Pitching moment coefficient about quarter chord point
L	Lift per unit length
ρ	Free stream density
c	Chord length
$m_{c/4}$	Pitching moment about quarter chord point
U	Free stream air speed

3.1.2 Theodorsen's Unsteady Theory

k	Reduced frequency
ω	Circular frequency of pitch
c	Chord length
U	Free stream air speed
τ	Reduced time
c_l	2D lift coefficient

h	Airfoil plunge distance
α	Airfoil pitch
w	Downwash
ϕ'	Disturbance velocity potential

4.3.2 Flow Around a Vortex Panel

X_0	Panel start, X - coordinate
Z_0	Panel start, Z - coordinate
X_f	Panel end, X - coordinate
Z_f	Panel end, Z - coordinate
L	Panel length
γ	Vortex strength per unit length
ξ	Location on panel
ξ^*	Non-dimensional location on panel
γ_0	Constant component of panel strength
γ_1	Slope component of panel strength
$\mathbf{q}_{xz}$	Flow field in the $x - z$ coordinate system
χ	An influencing function in the $x - z$ coordinate system
v	An influencing function in the $x - z$ coordinate system
ψ	An influencing function in the $x - z$ coordinate system
ϕ	An influencing function in the $x - z$ coordinate system
θ	Rotation angle between the $x - z$ and $X - Z$ coordinate systems
X	An influencing function in the $X - Z$ coordinate system
Υ	An influencing function in the $X - Z$ coordinate system
Ψ	An influencing function in the $X - Z$ coordinate system
Φ	An influencing function in the $X - Z$ coordinate system

4.3.3 The Wake

α	Angle of attack
U	Wind speed
Γ_A	Circulation around the airfoil
$x^{(\alpha)}$	Position on wake line

$z^{(\alpha)}$	Coordinate normal to wake
γ_W	Vortex strength at a point on the wake
Γ_W	Circulation around the wake

4.3.4 Unsteady Kutta Condition

ϕ	Velocity potential
U	Wind speed
$x^{(\alpha)}$	Position on wake line
ϕ'	Disturbance potential
q	Local flow speed
p	Local pressure
p_∞	Free stream pressure
ρ	Air density
c_p	Pressure coefficient
ϕ'_∞	Disturbance potential value at infinity
$z^{(\alpha)}$	Coordinate normal to wake
c_{pa}	Pressure jump coefficient across wake
p_l	Local pressure just below the wake: $z^{(\alpha)} = 0^-$
p_u	Local pressure just above the wake: $z^{(\alpha)} = 0^+$
ϕ'_u	Local disturbance potential just above the wake: $z^{(\alpha)} = 0^+$
ϕ'_l	Local disturbance potential just below the wake: $z^{(\alpha)} = 0^-$
u'_u	Local disturbance velocity just above the wake: $z^{(\alpha)} = 0^+$
u'_l	Local disturbance velocity just below the wake: $z^{(\alpha)} = 0^-$
Γ_A	Circulation around the airfoil
γ_W	Vortex strength at a point on the wake
$B(s)$	Constant of integration
$\Gamma_{A,0}$	Circulation around the airfoil at $t = 0$
γ_{TE}	Vortex strength at the trailing edge

4.3.5 Non-circulatory Flow Contribution

X	Global X - coordinate
x_{EA}	Location along chord of elastic axis

e	Chordwise displacement
ξ	Position on chord line
ζ	Position perpendicular to chord line
φ	Twist angle
Z	Global Z - coordinate
h	Plunge displacement

4.4 The Unsteady Boundary Layer

u	Local flow speed, x - component
w	Local flow speed, z - component
ρ	Fluid density
p	Local pressure
ν	Kinematic viscosity
u_e	Local flow speed, x - component, at boundary layer edge
δ	Boundary layer thickness
δ^*	Displacement thickness
θ	Momentum thickness
C_f	Wall friction coefficient
τ_w	Wall shear stress
δ^k	Kinetic energy thickness
C_D	Viscous diffusion coefficient
η	Normalized coordinate perpendicular to wall, z/δ
μ	Boundary layer profile parameter
Re_θ	Reynolds number based on momentum thickness
Re_x	Reynolds number based on x

4.5.1 Airfoil Discretization and 4.5.2 Neumann Boundary Conditions

$\mathbf{q}_C$	Circulatory contribution to flow field
N	Number of airfoil panels
r	Number of wake panels

$\gamma_{0,j}$	Constant vortex strength component of j^{th} panel
$\gamma_{1,j}$	Slope vortex strength component of j^{th} panel
X	An influencing function in the $X - Z$ coordinate system
Υ	An influencing function in the $X - Z$ coordinate system
Ψ	An influencing function in the $X - Z$ coordinate system
Φ	An influencing function in the $X - Z$ coordinate system
ϕ	Velocity potential
n	Direction normal to airfoil surface
$\mathbf{q}_{NC}$	Non-circulatory contribution to flow field
$\bar{X}_i$	x - coordinate of midpoint of i^{th} panel
$\bar{Z}_i$	z - coordinate of midpoint of i^{th} panel
$\hat{\mathbf{n}}_i$	Unit normal vector at control point of i^{th} panel
θ_i	Orientation of i^{th} panel
$()_{ji}$	Influence of $()_j$ on the control point of i^{th} panel

4.5.3 Continuity Boundary Conditions

$\gamma_{0,j}$	Constant vortex strength component of j^{th} panel
$\gamma_{1,j}$	Slope vortex strength component of j^{th} panel
$\gamma_{0,j+1}$	Constant vortex strength component of $(j+1)^{\text{th}}$ panel
$\gamma_{0,N}$	Constant vortex strength component of N^{th} panel
$\gamma_{1,N}$	Slope vortex strength component of 1^{st} panel
N	Number of airfoil panels

4.5.4 The Unsteady Kutta Condition

$\gamma_{0,1}$	Constant vortex strength component of 1^{st} panel
$\gamma_{0,N+1}$	Constant vortex strength component of $(N+1)^{\text{th}}$ panel
U	Wind speed
Γ_A	Circulation around the airfoil
$\mathbf{q}_C$	Circulatory contribution to flow field
γ_c	Constant vortex strength superimposed over airfoil
N	Number of airfoil panels
r	Number of wake panels

L'	Lift per unit length
X	An influencing function in the $X - Z$ coordinate system
Ψ	An influencing function in the $X - Z$ coordinate system

4.5.5 Wake Propagation

N	Number of airfoil panels
r	Number of wake panels
α	Angle of attack
α_{max}	Maximum anticipated angle of attack
Γ_{max}	Maximum anticipated circulation around airfoil
L'	Lift per unit length
ρ	Fluid density
U	Wind speed
$c_{l\alpha}$	Lift curve slope
c	Chord length
q_{vortex}	Flow speed induced by point vortex
Γ	Strength of point vortex
d	Distance from point vortex
κ	q/U
Δt	Time step size
r_{max}	Maximum number of wake panels

4.5.6 The Kelvin Condition

Γ_A	Circulation around the airfoil
N	Number of airfoil panels
$\gamma_{0,j}$	Constant vortex strength component of j^{th} panel
$\gamma_{1,j}$	Slope vortex strength component of j^{th} panel
L_j	Length of j^{th} panel
r	Number of wake panels
$\Gamma_{A,0}$	Circulation around the airfoil at $t = 0$

4.5.7 Vortex Panel Method Solution

N	Number of airfoil panels
r	Number of wake panels
γ_C	Constant vortex strength
$\gamma_{C,W}$	Constant vortex strength
$[\mathcal{A}]$	Matrix of coefficients
$\{\gamma\}$	Solution vector
$\{\mathcal{B}\}$	Vector of coefficients

4.5.8 Determination of the Velocity Field

V_t	Tangential velocity
γ	Vortex strength

4.5.9 Determination of the Pressure Field

c_p	Pressure coefficient
p	Local pressure
p_∞	Free stream pressure
U	Wind speed
ϕ'	Disturbance velocity potential
$x^{(\alpha)}$	Position on wake
ϕ	Total velocity potential
ϕ_{FS}	Free stream velocity potential
ϕ_{TE}	Total velocity potential at the trailing edge
γ	Vortex strength
$\theta_{TE}(s)$	Angle from trailing edge to point s
γ_W	Wake vortex strength
Γ_W	Total circulation around the wake

4.5.10 Determination of the Lift and Pitching Moment

c_p	Pressure coefficient
-------	----------------------

c	Chord length
ρ	Air density
U	Wind speed
p	Local pressure
p_∞	Free stream pressure
U	Wind speed
c_l	2D lift coefficient
$c_{m_{c/4}}$	2D pitching moment coefficient about the quarter chord

4.6.1 Discretization of the Normal Modes and 4.6.2 Euler - Lagrange Formulation

u_1	Axial deformation of beam section
N_P	Number of assumed axial modes
p_j	Coefficient of j^{th} axial mode
P_j	j^{th} axial mode
y	Coordinate along beam span
θ	Twisting deformation of beam section
N_Q	Number of assumed twist modes
q_j	Coefficient of j^{th} twist mode
Θ_j	j^{th} twist mode
u_3	Bending deformation of beam section
N_R	Number of assumed bending modes for u_3
r_j	Coefficient of j^{th} bending mode for u_3
R_j	j^{th} bending mode
u_2	Bending deformation of beam section
N_S	Number of assumed bending modes
s_j	Coefficient of j^{th} bending mode for u_2
S_j	j^{th} bending mode for U_3
$[\bar{S}]$	Sectional stiffness matrix
F_1	Axial force acting on section
M_1	Torque acting on section
M_2	Bending moment acting on section
M_3	Bending moment acting on section

T	Kinetic energy
μ	Mass per unit length
I'	Rotational inertia per unit length
V	Strain energy
$\{\bar{\epsilon}\}$	Vector of deformations
$\bar{S}_{:j}$	j^{th} column of $[\bar{S}]$
W	Work done by nonconservative forces
b_i	Damping coefficient of i^{th} deformation
Q_i	Generalized force acting on i^{th} mode
$[M]$	Mass matrix
$[B]$	Damping matrix
$[K]$	Stiffness matrix
$\{c\}$	Vector of coefficients
$\{F\}$	Vector of generalized forces

4.6.3 Runge - Kutta Integration of the Equations of Motion

$\{\Xi\}_1$	Vector of coefficients
$\{c\}$	Vector of coefficients
$\{\Xi\}_2$	Vector of time derivatives of coefficients
$\{\Xi\}$	Total coefficient vector
$[M]$	Mass matrix
$\{F\}$	Vector of generalized forces
$[K]$	Stiffness matrix
$[B]$	Damping matrix
$[0]_{N \times N}$	$N \times N$ zero matrix
$[I]_{N \times N}$	$N \times N$ identity matrix
$[A]$	Matrix of coefficients
$\{g\}$	Vector of generalized forces
$[\tilde{\Xi}]$	Matrix containing eigenvectors of $[A]$ on the columns
$[\tilde{\Lambda}]$	Matrix containing eigenvalues of $[A]$ on the diagonal
$\{\varphi\}$	Vector of modal coordinates
$\{G\}$	Vector of modal generalized forces
$k_{1,2,3,4}$	Runge Kutta derivatives

Δt Time step size

Chapter 5

L	Lift per unit length
c_{l_α}	Lift curve slope
ρ	Air density
b	Half chord
U	Wind speed
h	Plunge distance
φ	Airfoil twist
a	Location of elastic axis
$C(k)$	Theodorsen's function
k	Reduced frequency
M	Pitching moment about quarter chord point
c	Chord length
α	Angle of attack
τ	Non-dimensional time
$[\bar{S}]$	Sectional stiffness matrix
E	Young's modulus
A	Sectional area
G	Shear modulus
J	Polar moment of area
I	Second moment of area
ω_n	Natural circular frequency
μ	Mass per unit length

Chapter 1

Introduction

Wind power has been harnessed for centuries. Early on, its primary use was through windmills for grinding grain and pumping water. Starting in the 19th century, wind turbines were developed to generate electricity. Wind turbines fell into disuse in the early 20th century, but eventually began increasing in popularity in the 1940s, and have since increased in use and power capacity. In efforts to continually increase the capacity for power generation, much research has been conducting in predicting the aerodynamic loading on wind turbine blades. Moreover, use of advanced composite materials in the design of rotor blades for horizontal axis wind turbines (HAWT) have been explored extensively. Through combined analysis of unsteady aerodynamic loading and the nonlinear behaviour of composite structures, it is possible to design wind turbines to be more efficient, generate more power, and reduce costs.

1.1 Background and Motivation

Historically, wind turbines have been over-designed using large safety factors due to the unknown nature of the aerodynamic loading [1]. As a result, older designs were produced far below optimum power output. Classical steady aerodynamic theory

allows for calculation of the 2D lift and pitching moment per unit span for thin airfoils using potential flow theory. In the 1930s, the unsteady aerodynamic loading on a 2D airfoil pitching and plunging in simple harmonic motion was determined by Theodorsen. This theory continually evolved over the next decades to include additional motion cases such as an impulsive change in angle of attack (Wagner), response to a sudden gust (Kussner), and the case of rotating wings (Loewy) [2].

1.1.1 Computational Aerodynamics

With increasing computer processing capacity throughout the latter half of the 20th century, numerical techniques have been implemented for computing more accurate flow characteristics around submersed bodies. Approaches range from those requiring high computational cost, such as Direct Numerical Simulation (DNS) of the Navier-Stokes equations and Computational Fluid Dynamics (CFD), to those requiring relatively little computing power, such as the Blade Element Theory (BET) and vortex panel methods.

1.1.2 Computational Structural Dynamics

In terms of blade structures, fiber reinforced composite materials have become increasingly popular for wing and blade construction over the past few decades. The result is lightweight but highly flexible wings and blades. When combined with anisotropic effects of composite materials, determining the structural properties and dynamic characteristics of such highly flexible structures becomes a challenging design and analysis problem. Again, with increasing computational capabilities come techniques such as Finite Element Analysis (FEA) which can be used to handle these challenges.

1.1.3 Aeroelasticity

When considering the dynamic behaviour of these flexible composite wings or rotor blades, one has to deal with a full aeroelastic problem that involves coupling unsteady aerodynamics and highly flexible anisotropic structural dynamics in order to determine the structural response. The response is very important for flexible turbine blades and other structures because induced vibrations have many different implications on the structural design. Performance parameters such as the generated power and torque will be subjected to fluctuations, and fluctuating stresses can have adverse effects such as material fatigue.

1.1.4 The Preliminary Design Phase

The goal of this work is to determine a method of solving the aerodynamic problem that is suitable for the preliminary design phase of wind turbines and other aerodynamically-excited structures where large numbers of simulations need to be performed to analyze the effects of various design parameters. Expensive techniques such as CFD and DNS, while capable of providing very accurate numbers, are unsuitable for performing large numbers of simulations due to the long computational times and extensive work required in setting up each simulation. The proposed method must be capable of rapid computations with relatively little set-up time, while maintaining a suitable degree of accuracy.

1.2 Approach

The Blade Element Theory (BET) is a popular choice in determining the aerodynamic loading of a slender structure due to its simplicity and fast computation times. The approach essentially involves discretization of the structure into a finite number of

spanwise sections, determining the 2D loading for each section, and integrating along the blade. Blade Element Momentum Theory (BEMT) is a popular extension of the BET for rotating blades that accounts for rotational effects through consideration of linear and angular momentum conservation through the rotor disc. Both methods often use 2D panel methods to rapidly determine the 2D aerodynamic loading for each section. In this work, a panel method will be developed to handle unsteady flow.

The anisotropic structural deformations will be handled by using the Variational Asymptotic Beam Section (VABS) tool developed by Khouli [3] to compute the stiffness of a 2D thin-walled cross section, and using the Ritz method to determine the equations of motion. This step only needs to be performed once at the beginning of the simulation, and then a simple integration scheme can be used to determine the structural deformations at each time step, assuming the deflections are small.

1.3 Algorithm Verification

The algorithm was tested by simulating several design cases alongside coupled CFD and FEA simulations using ANSYS Workbench 14. The simulations were compared for agreement of results and contrasted in terms of total computation time. A wind tunnel experiment using a scaled blade model was also performed as part of an ongoing effort by El Sherif (2014) to develop an experimental benchmark by which aeroelastic response algorithms can be evaluated, and an attempt was made to reproduce some of the wind tunnel results to demonstrate the capabilities of the proposed algorithm in predicting the response of a real blade.

1.4 Thesis Outline

This section aims to clarify the organization of the thesis, and should be referred to periodically.

Chapter 2 provides the background information required for the construction of the algorithm components. It contains a review of unsteady aerodynamics, Blade Element Momentum Theory (BEMT) for 3D blade loading, and an introduction to the Variational Asymptotic Method (VAM) which is the foundation of the selected beam stiffness analysis tool developed by Khouli (2009) [3].

Chapter 3 contains a review of the literature and outlines the theories currently available for use in the development of an aeroelastic response algorithm.

In Chapter 4, the sub-algorithms are developed based on the principles provided in Chapter 2, and the functionality is documented.

The results of the algorithm verification are presented in Chapter 5, and the conclusions and contributions are summarized in Chapter 6.

Chapter 2

Background

The aeroelastic analysis of a finite blade involves contributions from aerodynamics, structural mechanics, and dynamics as illustrated by the Collar Diagram shown in Fig. 2.1. The prediction of the motion of a structure interacting with a fluid is complicated since the external loading is coupled with the structural deformation. Before the mathematical model of a flexible rotating blade is developed, necessary theory from the fields of fluid dynamics and structural mechanics must be discussed.

Figure 2.1: Aeroelasticity collar diagram. [4]

2.1 Simplified Unsteady Aerodynamic Model

To determine the external loading on a body immersed in a fluid, the surrounding flow must be analyzed. For the topic at hand, the flow will be assumed to constitute a continuum. That is, the mean distance between fluid particles is assumed small compared to the particle size such that the state of the fluid may be described by continuous distributions of intensive properties. The number of state variables required to describe the flow depends on the assumptions made in the model.

2.1.1 Governing Equations of Fluid Dynamics

All real flows must satisfy the Navier-Stokes equations, which together comprise statements ensuring the conservation of mass and momentum within the fluid. These statements are respectively presented as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (2.1)$$

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}, \quad (2.2)$$

where ρ is the fluid density, $\mathbf{V}$ is the velocity field, p is the pressure field, $\mathbf{T}$ is the viscous stress tensor, $\mathbf{f}$ is the body force vector, and the substantial derivative operator $\frac{D}{Dt}$ is defined as:

$$\frac{D}{Dt} () = \frac{\partial}{\partial t} () + \mathbf{V} \cdot \nabla (). \quad (2.3)$$

A model for the fluid is needed to relate the stress tensor to the flow properties. As well, the body forces may depend on some factor such as an external gravitational field. For a Newtonian fluid, the entries of the stress tensor are linear functions of the velocity field. The body forces will be assumed negligible for the remainder of this

work. For a Newtonian fluid, the Navier-Stokes equations introduce three unknowns - the scalar density and pressure fields, and the vector velocity field. For compressible flows, the density ρ varies and the energy equation is required as well:

$$\rho \frac{D}{Dt} \left(e + \frac{V^2}{2} \right) = \rho \dot{q} - \nabla \cdot (p \mathbf{V}) + \rho (\mathbf{f} \cdot \mathbf{V}) + \left(\dot{W}' + \dot{Q}' \right)_{viscous}. \quad (2.4)$$

The energy equation states that the rate of change of energy of the system is balanced by: power accompanying conductive heat transfer, work done due to fluid motion, work done by body forces, and energy loss due to viscous forces. The energy equation introduces a new unknown - the internal energy, bringing the number of unknowns to four. To close the system of unknowns, thermodynamic equations must be invoked to relate the internal energy to temperature (a new unknown), and then an equation of state must be invoked.

2.1.2 Potential Flow

In the goal of simplifying the model, the required number of governing equations can be reduced with appropriate assumptions. In particular, if the flow is assumed incompressible (constant density), then the Navier-Stokes equations are sufficient to solve for the velocity and pressure fields and the required number of governing equations is two. It will also be shown in this chapter that compressible flows can be treated by linearizing the governing equations under certain conditions. The Navier-Stokes equations constitute a system of nonlinear partial differential equations which, in general, are very difficult if not impossible to solve. However, it is known from vector calculus that a conservative vector field must be expressible as the gradient of a scalar field. If the velocity field is conservative, then it is guaranteed that:

$$\mathbf{V} = \nabla\phi, \quad (2.5)$$

where ϕ is called is potential of the velocity field. A sufficient condition for the velocity field to be conservative is a vanishing curl:

$$\text{curl } \mathbf{V} = \nabla \times \mathbf{V} = \boldsymbol{\xi} = \mathbf{0}. \quad (2.6)$$

Such a velocity field is said to be irrotational. That is, all fluid elements have no angular velocity about their mass centers. In the absence of viscous forces, no mechanism exists to change the angular momentum of a fluid element. $\boldsymbol{\xi}$ is the vorticity and represents a metric of rotationality. As long as no angular momentum is brought into the flow from an external source, an inviscid flow must be irrotational. For the remainder of the document, the terms irrotational and inviscid will be used interchangeably, however, the reader should be aware of the difference between the two.

The velocity potential ϕ in Equation 2.5 represents a single unknown from which the vector velocity field can be extracted. A flow for which Equation 2.5 holds is called a potential flow. Due to the simplicity of analyzing potential flows, it is advantageous to divide a given flow into viscous and inviscid regions. In many cases, the viscous regions will be confined to thin shear layers along boundary surfaces and to the wake trailing behind an immersed body.

A governing equation for the velocity potential can be derived by combining Equation 2.6 with the Navier-Stokes Equations 2.1 and 2.2. In the absence of viscous and body forces, the Navier-Stokes equations reduce to the Euler equations of conservation of mass and momentum:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (2.7)$$

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p. \quad (2.8)$$

Substitution of Equation 2.5 into the Euler continuity equation (2.7) yields a governing PDE for ϕ . First, Equation 2.7 is expanded:

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla \rho = 0. \quad (2.9)$$

Taking the divergence of Equation 2.5:

$$\nabla \cdot \mathbf{V} = \nabla^2 \phi. \quad (2.10)$$

Equation 2.9 can now be expressed as:

$$\frac{\partial \rho}{\partial t} + \rho \nabla^2 \phi + \nabla \phi \cdot \nabla \rho = 0. \quad (2.11)$$

Equation 2.11 ensures conservation of mass. However, the momentum and energy equations need to be invoked to deal with the pressure and density fields. If the flow is incompressible, $\dot{\rho}$ and $\nabla \rho$ vanish, yielding Laplace's equation.

$$\nabla^2 \phi = 0. \quad (2.12)$$

The incompressible equation can be solved for the velocity potential outright and thus the assumption of incompressibility is extremely valuable where simplicity is concerned.

2.1.3 Extraction of the Pressure Field

With a PDE in place for the velocity potential, the momentum and energy equations are invoked to determine the pressure and density fields. Before proceeding, it is important to define the *circulation* Γ on any closed contour C as

$$\Gamma = \oint_C \mathbf{V} \cdot d\mathbf{s}, \quad (2.13)$$

The circulation is an invaluable tool in analyzing unsteady flow. To illustrate this, let us take the substantial derivative of the circulation 2.13:

$$\frac{D\Gamma}{Dt} = \oint \frac{D\mathbf{V}}{Dt} \cdot d\mathbf{s}. \quad (2.14)$$

Calling upon the Euler momentum equation, 2.8:

$$\frac{D\Gamma}{Dt} = \oint -\frac{\nabla p}{\rho} \cdot d\mathbf{s} = -\oint \frac{dp}{\rho}. \quad (2.15)$$

Equation 2.15 is Kelvin's theorem. For incompressible flow, or for barotropic flow in which the pressure is strictly a function of density (prevalent in most aerodynamics problems), Kelvin's theorem reduces to

$$\frac{D\Gamma}{Dt} = 0. \quad (2.16)$$

A physical interpretation of the circulation is obtained by applying Stokes' Theorem to Equation 2.13:

$$\Gamma = \oiint \nabla \times \mathbf{V} \cdot d\mathbf{A} = \oiint \boldsymbol{\xi} \cdot d\mathbf{A}. \quad (2.17)$$

Thus, the circulation around a closed path is a measure of the total vorticity enclosed

within. To invoke the Euler momentum equation, it is expanded to give

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p. \quad (2.18)$$

Recognizing that

$$\mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{2} \nabla (V^2) - \mathbf{V} \times (\nabla \times \mathbf{V}),$$

and since $\nabla \times \mathbf{V} = \mathbf{0}$ for a potential flow, Equation 2.18 becomes

$$\frac{\partial \mathbf{V}}{\partial t} + \frac{1}{2} \nabla (V^2) + \frac{\nabla p}{\rho} = 0. \quad (2.19)$$

Again, assuming barotropic flow, and inserting Equation 2.5, one obtains

$$\frac{\partial \nabla \phi}{\partial t} + \frac{1}{2} \nabla (V^2) + \nabla \int \frac{dp}{\rho} = 0.$$

Thus,

$$\nabla \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} V^2 + \int \frac{dp}{\rho} \right) = 0. \quad (2.20)$$

The quantity inside the gradient operator must be at most a function of time:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} V^2 + \int \frac{dp}{\rho} = F(t). \quad (2.21)$$

If the function is arbitrarily set to 0, *Kelvin's Equation* is obtained:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} V^2 + \int \frac{dp}{\rho} = 0. \quad (2.22)$$

Through Kelvin's Equation, the pressure field can be extracted once the velocity potential is known. If the flow is incompressible, then the result is the *Unsteady*

Bernoulli Equation:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}V^2 + \frac{p}{\rho} = 0. \quad (2.23)$$

Note that for steady flows, the Bernoulli Equation is recovered.

2.1.4 Velocity Potential Governing Equation

At this point, a PDE for the velocity potential has been developed (Equation 2.11), and the pressure field has been solved in the form of Kelvin's Equation (Equation 2.22). The density is yet to be determined, and so an additional equation is invoked to close the system. Air is assumed to be a perfect gas, and hence, for a barotropic flow,

$$a^2 = \frac{dp}{d\rho} = \frac{\partial p}{\partial \rho}, \quad (2.24)$$

where a is the local speed of sound. Differentiating the Kelvin Equation 2.22 with respect to time:

$$\frac{1}{\rho} \frac{\partial p}{\partial t} = -\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{2} \frac{\partial V^2}{\partial t}.$$

Making use of Equation 2.24,

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} = -\frac{1}{a^2} \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{1}{2} \frac{\partial V^2}{\partial t} \right).$$

.

Substituting from Equation 2.11

$$-\frac{1}{\rho} (\rho \nabla^2 \phi + \nabla \phi \cdot \nabla \rho) = -\frac{1}{a^2} \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{1}{2} \frac{\partial V^2}{\partial t} \right).$$

With $\nabla p = a^2 \nabla \rho$ and $\mathbf{V} = \nabla \phi$,

$$-\nabla^2 \phi - \frac{1}{a^2} \mathbf{V} \cdot \frac{\nabla p}{\rho} = -\frac{1}{a^2} \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{1}{2} \frac{\partial V^2}{\partial t} \right).$$

Finally, $\frac{\nabla p}{\rho}$ is given in Equation 2.19, so that one has:

$$\nabla^2 \phi - \frac{1}{a^2} \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{1}{2} \frac{\partial V^2}{\partial t} + \mathbf{V} \cdot \left(\frac{\partial \mathbf{V}}{\partial t} + \frac{1}{2} \nabla (V^2) \right) \right) = 0.$$

Simplifying, the resulting PDE is:

$$\nabla^2 \phi - \frac{1}{a^2} \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial V^2}{\partial t} + \mathbf{V} \cdot \frac{1}{2} \nabla (V^2) \right) = 0. \quad (2.25)$$

Equation 2.25 again reduces to Laplace's equation for incompressible flow. In the next section, it will be linearized for suitable use with compressible flows.

2.1.5 Small Disturbance Theory

The full compressible PDE 2.25 poses difficulties in its implementation due to its nonlinear nature. It is therefore desirable to linearize Equation 2.25 before use in solving the flow field. To this end, small disturbance theory can be used. Assuming that the presence of a body immersed in a flow causes the velocity to experience deviations with small magnitude relative to the free stream velocity U , that is,

$$\begin{aligned} u &\ll U, \\ v &\ll U, \\ w &\ll U. \end{aligned} \quad (2.26)$$

where u , v , and w are the orthogonal velocity components of $\mathbf{V}$. Then the V^2 term

can be effectively linearized as follows:

$$\begin{aligned} V^2 &= (U_x + u)^2 + (U_y + v)^2 + (U_z + w)^2, \\ &\approx U^2 + 2\mathbf{U} \cdot \mathbf{V}. \end{aligned} \quad (2.27)$$

Based on the form of Equation 2.27, it seems appropriate to separate the free stream velocity potential and the disturbance velocity potential from each other:

$$\phi = \phi_{FS} + \phi'. \quad (2.28)$$

The free stream velocity potential is given by:

$$\phi_{FS} = U_x x + U_y y + U_z z. \quad (2.29)$$

Substitution of Equations 2.29 and 2.27 in Equation 2.25 yields

$$\begin{aligned} \nabla^2 (\phi_{FS} + \phi') - \frac{1}{a^2} \left(\frac{\partial^2}{\partial t^2} (\phi_{FS} + \phi') + \frac{\partial}{\partial t} [U^2 + 2\mathbf{U} \cdot \mathbf{V}] + \mathbf{V} \cdot \frac{1}{2} \nabla [U^2 + 2\mathbf{U} \cdot \mathbf{V}] \right) \\ = 0. \end{aligned} \quad (2.30)$$

Given that the free stream velocity remains constant, the derivatives of ϕ_{FS} and U^2 vanish. The remaining terms can be evaluated as follows:

$$\begin{aligned} \frac{\partial}{\partial t} [2\mathbf{U} \cdot \mathbf{V}] &= 2\mathbf{U} \cdot \frac{\partial \nabla \phi}{\partial t}, \\ \mathbf{V} \cdot \frac{1}{2} \nabla [U^2 + 2\mathbf{U} \cdot \mathbf{V}] &= U^2 \nabla^2 \phi. \end{aligned}$$

Substituting back into Equation 2.30 yields

$$\nabla^2 \phi' - \frac{1}{a^2} \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right)^2 \phi' = 0 \quad (2.31)$$

The final step in linearizing the governing PDE requires the speed of sound term to be dealt with. As a heuristic argument, it seems plausible that the local speed of sound should not deviate too far from the free stream speed of sound a_∞ , and thus a may be replaced with the constant a_∞ . To verify this, it can be shown by using the energy equation that $\frac{a'}{a_\infty}$ varies with $\frac{1}{a_\infty^2}$. Thus, the full compressible, linearized PDE for the disturbance velocity potential is given as

$$\nabla^2 \phi' - \frac{1}{a_\infty^2} \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right)^2 \phi' = 0. \quad (2.32)$$

After the disturbance potential is found, it can be added to the free stream potential by use of Equation 2.28 to form the total potential. Again, for incompressible flow, Laplace's equation is recovered.

2.1.6 2D Incompressible Unsteady Aerodynamics

To solve a 2D, unsteady, immersed body problem, Equation 2.32 along with the appropriate boundary conditions and initial condition may be implemented. In the case of a 2D airfoil, the required boundary conditions are that the flow does not penetrate the surface body, Kelvin's theorem is upheld, the Kutta Condition is upheld (to be discussed), and the disturbance potential must vanish at infinity. A computational approach will involve discretizing the PDE and solving numerically.

In the case of incompressible flow, the problem reduces to solving Laplace's equation. Again, a full analytical solution is impractical in most cases, but with Laplace's equation it is possible to “build” a solution out of elementary solutions. Some examples of

elementary solutions for incompressible flow are outlined in the following subsections.

2.1.6.1 Uniform Flow

The velocity potential for uniform flow is a linear function of the rectangular spatial coordinates and hence satisfies Laplace's equation. A uniform flow is thus an elementary solution. The velocity potential is given as

$$\phi_{FS} = U_x x + U_y y + U_z z. \quad (2.33)$$

This flow is characterized by an infinite set of parallel streamlines with uniform flow speed.

2.1.6.2 Source and Sink Flow

In seeking elementary solutions, it is helpful to switch to polar coordinates. Two obvious, nontrivial solutions are given by:

$$\phi_S = \frac{\Lambda}{2\pi} \ln r, \quad (2.34)$$

$$\phi_V = \frac{\Gamma}{2\pi} \theta, \quad (2.35)$$

where the constants Λ and Γ represent the strengths of the solutions. Equation 2.34 gives the velocity potential for source or sink flow. Equation 2.35 gives the velocity potential for vortex flow and will be discussed in the next section.

Source flow is visualized by streamlines radiating from the origin at $r = 0$. The flow direction is radial and the velocity varies inversely with r , hence preserving mass flow. Λ represents the strength of the flow. For positive values of Λ , the flow will emanate from the origin (source) and for negative values of Λ , the flow will retract

into the origin (sink).

An important feature of the source/sink is that the circulation on any contour surrounding the origin must be 0 since there is no circumferential component of velocity. Hence, sources and sinks do not provide vorticity to the flow, which is required for the generation of lift according the Kutta-Joukowski theorem.

2.1.6.3 Irrotational Vortex Flow

Equation 2.35 gives the velocity potential for irrotational vortex flow. In contrast to the source flow, vortex motion consists of pure tangential velocity and no radial velocity. In this case, there must exist a non-zero circulation around a contour surrounding the origin. At first thought, this is counterintuitive since the existence of the velocity potential guarantees that the flow is irrotational. The confusion is relieved when one notes that there is a singularity in the flow velocity at $r = 0$ and hence the vorticity must be concentrated there. Application of Equation 2.13 around a circular contour yields insight here:

$$\oint_C \mathbf{V} \cdot d\mathbf{s} = \int_0^{2\pi} \frac{\Gamma R d\theta}{2\pi R} = \Gamma$$

Hence, the strength of the vortex represents the finite circulation associated with it. By the Kutta-Joukowski theorem, the irrotational vortex flow is a lifting flow, and is thus invaluable in the prediction of lift using potential flow theory.

2.2 The Blade Element Momentum Theory

To analyze the performance of a horizontal axis wind turbine, the developed torque and power must be determined. The developed power is typically nondimensionalized

by observing that the power available in the wind is given by (noting that $\dot{m} = \rho AU$):

$$P_{max} = \frac{1}{2}\dot{m}U^2 = \frac{1}{2}\rho AU^3 \quad (2.36)$$

and then defining the power coefficient c_P as the ratio of power developed to available power:

$$c_P = \frac{P}{P_{max}} = \frac{P}{\frac{1}{2}\rho AU^3} \quad (2.37)$$

The thrust and torque coefficients, c_T and c_Q , are defined in a similar manner:

$$c_T = \frac{T}{\frac{1}{2}\rho AU^2} \quad (2.38)$$

$$c_Q = \frac{T}{\frac{1}{2}\rho AU^2 R} \quad (2.39)$$

Determining these coefficients involves knowledge of the aerodynamic loading on the turbine blades. The BEMT can be used to find the loading.

2.2.1 Blade Element Theory

The simplest approach in determining the aerodynamic loading is the Blade Element Theory (BET). Each turbine blade is discretized into a finite number of spanwise sections, and a 2D airfoil analysis is performed for each section to determine the lift, moment, and drag. The loading is then integrated across the blade. The drawback to the BET is that the incoming flow velocity cannot be fully determined, as the motion of the rotor disc influences the flow around the turbine. One approach to figure out the incoming flow is to use Momentum Theory. The combination of both theories yields the BEMT.

Figure 2.2: Streamtube expanding through the rotor disc. [1].

2.2.2 Blade Element Momentum Theory (BEMT)

The momentum theory is used to account for the induced flow around the rotor disc due to the presence of the rotating blades. Fig. 2.2 indicates a streamtube expanding through the disc. It can be seen that the axial velocity decreases from its upstream value U to a value of U_D at the disc, and further decreases to U_W far downstream. The axial induction factor a is defined such that

$$U_D = U (1 - a). \quad (2.40)$$

Taking into account the rotation of the wake, the tangential induction factor a' can also be defined so that the induced tangential velocity $U_{t,D}$ at the rotor disc, assuming rotational speed Ω , is given by

$$U_{t,D}(r) = \Omega r (1 + a'), \quad (2.41)$$

as illustrated in Fig. 2.3. The resultant incoming velocity W can therefore be expressed in terms of the induction factors a and a' :

(a) Velocities

(b) Forces

Figure 2.3: Induced velocity and force components on a blade element. [1].

$$W(r) = \sqrt{U^2(1-a)^2 + r^2\Omega^2(1+a')^2} \quad (2.42)$$

W acts at an angle ϕ to the plane of rotation such that:

$$\sin \phi = \frac{U(1-a)}{W}, \quad (2.43)$$

$$\cos \phi = \frac{r\Omega(1+a')}{W}. \quad (2.44)$$

The incremental thrust and torque acting on a spanwise blade element are given by, respectively,

$$\delta T = \delta L \cos \phi + \delta D \sin \phi = \frac{1}{2} \rho W^2 B c (c_l \cos \phi + c_d \sin \phi) \delta r, \quad (2.45)$$

$$\delta Q = (\delta L \sin \phi - \delta D \cos \phi) r = \frac{1}{2} \rho W^2 B c r (c_l \sin \phi - c_d \cos \phi) \delta r \quad (2.46)$$

where L is the lift, D is the drag, B is the number of blades, and c is the chord length. To take into account the rotation of the wake, the vortex cylinder model is used in which a cylinder of air extending behind the rotor is induced into rotational motion. It can be shown by applying conservation of momentum in a control volume enclosing the streamtube that

$$\delta T = 2\pi r \delta r \rho U (1 - a) 2aU, \quad (2.47)$$

$$\delta Q = 2\pi r \delta r \rho U (1 - a) 2a' r^2 \Omega. \quad (2.48)$$

Equating Equations 2.45 and 2.46 with Equations 2.47 and 2.48, and performing some algebra,

$$\frac{a}{1 - a} = \frac{\sigma_r}{4 \sin^2 \phi} c_x, \quad (2.49)$$

$$\frac{a'}{1 + a'} = \frac{\sigma_r}{4 \sin \phi \cos \phi} c_y, \quad (2.50)$$

where,

$$c_x = c_l \cos \phi + c_d \sin \phi, \quad (2.51)$$

$$c_y = c_l \sin \phi - c_d \cos \phi, \quad (2.52)$$

and the chord solidarity σ_r is given by

$$\sigma_r(r) = \frac{Bc}{2\pi r}. \quad (2.53)$$

Equations 2.49 and 2.50 can be solved numerical and yield the induction factors a

and a' as functions of r . The lift and drag coefficients c_l and c_d depend on the choice of airfoil and flow conditions, and a technique for finding them will be developed in Chapter 4. The performance coefficients c_P , c_T , and c_Q can be determined after integrating Equations 2.47 and 2.48 along the blades. In the absence of rotation, the resultant incoming velocity is simply given by $W = U$ and the loading can be found by a simple integration of Equations 2.45 and 2.46.

2.3 Sectional Analysis of Thin-walled Composite Beams Using the Variational Asymptotic Method

For the structural dynamics solver, it is required to determine the distribution of stiffness along the beam. For this work, the Variational Asymptotic Beam Section (VABS) analysis tool developed by Khouli (2009) [3] was selected based on its capacity for handling composite materials with the possibility of including the effects of piezoelectric actuation (smart materials). The basis of this tool is the Variational Asymptotic Method (VAM) of Berdichevsky [3]. A very brief overview of this theory is presented here, although this section may be omitted without loss of continuity.

2.3.1 Motivation

In the past several decades, the use of composite materials in various applications has increased significantly [5]. Composite materials tend to be good candidates for lightweight structures such as aircraft wings and turbine blades due to their high strength at low weights. Popular choices for horizontal axis wind turbines include carbon fibre reinforced polymers which offer very good strength-to-weight ratios, as

well as other ideal characteristics such as ease of manufacture, ability to form strong joints, and smooth surface finish.

Unfortunately, the ability to manufacture composite structural components has improved more rapidly than the ability to analyze their behaviour under load [5]. Composite materials are anisotropic and hence simple 1-D analyses of rotors as beam elements are not appropriate to accurately predict their deformation. Finite Element Analysis (FEA) programs currently serve as accurate and popular tools in composite structural analysis, but as with CFD, they are costly and labour-intensive and thus are not suitable for use in the preliminary design phase. Less-costly beam programs are available, but are unable to accurately model the elastic couplings that exist in nonlinear structures that may be used to the advantage of the designer.

One method that has demonstrated good accuracy for nonlinear beam analysis is the Variational Asymptotic Beam Section (VABS) method. The basis of this method is the Variational Asymptotic Method, in which the general 3D nonlinear elastic beam problem is split into a 2D linear cross-sectional analysis and a nonlinear 1D beam analysis. The VABS method holds great promise for meeting the industry's requirements for an efficient, reliable analysis tool for composite beams [5].

2.3.2 Overview of the Variational Asymptotic Method

As mentioned previously, the VABS method involves the dissection of a fully-nonlinear 3D beam problem into a 2D linear cross-sectional analysis and a 1D nonlinear beam analysis. The Variational Asymptotic Method allows for the replacement of a full 3D structural model with a reduced-order model by expanding the energy functional as an asymptotic series in terms of certain small characteristic

Figure 2.4: Overview of the VABS algorithm. [6].

parameters of the structure [5]. In such an expansion, certain terms may afterwards be discarded in the energy functional, equivalent to dropping higher order terms when solving differential equations by asymptotic methods. Determining which terms to drop in the PDEs would otherwise be very difficult, if not impossible [3]. The general algorithm is depicted in Figure 2.4. In this section, the basic algorithm for performing the cross-sectional analysis is outlined.

The 3D elastic energy functional of a rotor blade may be expanded in terms of the ratio of the characteristic dimension of the cross-section, a , to the spanwise elastic deformation wavelength, ℓ , i.e.

$$\varsigma \equiv \frac{a}{\ell}. \quad (2.54)$$

Let the energy functional be written as

$$\mathcal{F}(\Gamma, \varsigma) = \mathcal{E}_1(\Upsilon, z_1) + {}^0\mathcal{E}_\varsigma(\Upsilon, z_1, \varsigma), \quad (2.55)$$

where Γ is a 6×1 column matrix representing a 3×3 symmetric strain field, $z_1 = [w_{11} \ w_{12} \ w_{13}]$ is a matrix representing some unknown perturbation of the 3D elastic field (the in/out of plane warping functions), and Υ is a matrix containing all terms not involving the unknown perturbations. For the 2D cross-sectional analysis, the displacement field then represents the in/out-of-plane warping functions to a first correction. Note that the energy functional is split into two parts. The first, $\mathcal{E}_1(\Upsilon, z_1)$, contains all terms of order ς^0 , while the second, ${}^0\mathcal{E}_\varsigma(\Upsilon, z_1, \varsigma)$ contains all terms of order ς^1 and higher. The left superscript on the second part indicates the level of construction of the displacement field. The cross-sectional analysis is an iterative process and new levels will be added with subsequent iterations.

The displacement field z_1 is approximated to a first correction by neglecting high-order powers of ς in the energy functional, and then minimizing the functional with respect to z_1 . The resulting solution is not unique due to the possibility of 4 rigid body modes: axial translation, two directions of in-plane translation, and in-plane rotation. These modes are eliminated by imposing 4 constraints on z_1 :

$$\int_A \bar{C}_n(z_1) dA = 0, \quad n = 1, 2, 3, 4 \quad (2.56)$$

In general, a closed-form solution for z_1 is not possible, and the problem is discretized over the cross-section and solved along with the constraints using the finite element method. The order of ς terms in z_1 is not known until the solution is found.

The displacement field z_1 will vary over the cross-section:

$$z_1 = f_1(\Upsilon, \zeta_2, \zeta_3) \quad (2.57)$$

where ζ_2 and ζ_3 are the coordinate measures along the perpendicular axes defining the cross-sectional plane. To find the next asymptotically-correct approximation of the displacement field, a new perturbation z_2 is defined such that:

$$z_2 = z_1 - f_1(\Upsilon, \zeta_2, \zeta_3) \quad (2.58)$$

This new perturbation is substituted back into the original energy function to obtain:

$$\mathcal{F}(\Gamma, \varsigma) = \mathcal{F}_1(\Gamma) + \mathcal{E}_2(\Upsilon, z_2) + {}^1\mathcal{E}_\varsigma(\Upsilon, z_2, \varsigma), \quad (2.59)$$

Once again, the higher order powers of ς are neglected, and the functional is minimized with respect to z_2 subjected to the same constraints as given in Equation 2.56. The solution procedure is again followed, and the new perturbation z_2 is obtained:

$$z_2 = f_2(\Upsilon, \zeta_2, \zeta_3) \quad (2.60)$$

The process is repeated until the new perturbation yields no terms in the energy functional that are of the highest order yielded by the previous perturbation [3]. The energy functional after the k^{th} perturbation is written as:

$$\mathcal{F}(\Gamma, \varsigma) = \mathcal{F}_1(\Gamma) + \mathcal{F}_2(\Gamma) + \dots + \mathcal{F}_k(\Gamma) + \mathcal{E}_{k+1}(\Upsilon, z_{k+1}) + {}^k\mathcal{E}_\varsigma(\Upsilon, z_{k+1}, \varsigma) \quad (2.61)$$

At this point, the energy functional has been expanded as an asymptotic series in terms of ς and can thus be written as

$$\mathcal{F}(\Gamma, \varsigma) = \mathcal{O}(\varsigma^0) + \mathcal{O}(\varsigma^1) + \mathcal{O}(\varsigma^2) + \dots + \mathcal{O}(\varsigma^k) \quad (2.62)$$

Figure 2.5: A composite thin-walled single-cell beam section. [7].

Accordingly, the 3D elastic energy has been effectively collapsed into 1D along the span of the beam. Assuming the theory correctly accounts for all terms up to the order $\mathcal{O}(\varsigma^k)$, the theory is said to be asymptotically correct to the order k [3].

2.3.3 Application of VABS to Composite Thin-walled Beams

This section will briefly outline the process of determining the closed-form 4×4 stiffness matrix of closed, thin-walled, single-cell composite beam sections. The theory can be easily adapted to open sections and multiple-cell sections with the effects of piezoelectric actuation (smart materials) included [3, 7].

For the development of the algorithm, the curvilinear and Cartesian coordinate systems shown in Figure 2.5 are used. The strain energy per unit volume, $\mathcal{U}$, is given by

$$2\mathcal{U} = D^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \quad (2.63)$$

where $D^{\alpha\beta\gamma\delta}$ is the reduced two-dimensional elastic modulus [8], and the Greek subscripts run from 1-2. The strain tensor can be expressed in terms of the generalized Koiter and Sanders shell strain measures, $\gamma_{\alpha\beta}$ and $\rho_{\alpha\beta}$ [7, 9]:

$$\epsilon_{\alpha\beta} = \gamma_{\alpha\beta} + \xi \rho_{\alpha\beta} \quad (2.64)$$

To ensure consistency with the strategy of collapsing the 3D strain energy into 1D along the span, the strain energy per unit length, $\bar{\mathcal{U}}$, is determined by integrating Equation 2.67 across the section thickness and around its contour:

$$2\mathcal{U} = \oint_C \int_{-h/2}^{h/2} D^{\alpha\beta\gamma\delta} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} d\xi ds \quad (2.65)$$

Next, an asymptotic expansion of the strain energy functional is sought to an order of $\mathcal{O}(\epsilon^2)$, where $\epsilon \ll 1$ is another small parameter which is the maximum magnitude of strain measures of the classical deformation modes [7]. To this end, two successive perturbations $\hat{w}_i$ and w_i of the displacement field are required. The Latin subscript may take on values 1-3. The generalized shell strains up to $\mathcal{O}(\epsilon^2)$ are given in terms of the unknown perturbations, section geometry, and deformation modes as: [7]:

$$\begin{bmatrix} \bar{\gamma}_{11} \\ 2\bar{\gamma}_{12} \\ \bar{\gamma}_{22} \\ h\bar{\rho}_{11} \\ h\bar{\rho}_{12} \\ h\bar{\rho}_{22} \end{bmatrix} = \begin{bmatrix} u'_1 \\ r_n\theta' + w_{1,2} \\ w_{2,2} + \frac{w_3}{R} \\ 0 \\ 0 \\ f \left[\left(\hat{w}_{3,2} - \frac{\hat{w}_2}{R} \right)_{,2} \right] \end{bmatrix} \quad (2.66)$$

where differentiation with respect to the curvilinear coordinate system is denoted with a subscripted comma followed by the level of differentiation, and differentiation with respect to the axial coordinate x_1 is denoted by a prime. The variable R represents the radius of curvature of the shell, u_i represents the axial extension and two bending deflections, respectively, θ represents the axial twist, and r_n is the component of r along $\mathbf{n}$ [7].

Recalling that the Euler-Lagrange minimization of the energy functional will result in redundancy due to the four rigid body degrees of freedom, it is apparent that four constraints must be imposed. This is accomplished through the application of four Lagrange multipliers, $\lambda_1 \dots \lambda_4$ to the expression for the energy functional. Applying Equation 2.66 as well, the expression for the energy functional reads as

$$2\mathcal{U} = \oint_C \left(\bar{\psi}^T Q \bar{\psi} + 2\bar{\phi}^T S \bar{\psi} + \bar{\phi}^T P \bar{\phi} + 2\bar{\phi}^T \hat{T} \bar{\lambda} + 2\bar{\epsilon}^T E^T \bar{\lambda} \right) ds \quad (2.67)$$

The matrices Q , S , and P contain the material properties related to the laminate composite layout along the contour [7]:

$$\begin{aligned}
Q &= \begin{bmatrix} E_e^{1111} & hE_{eb}^{1111} & 2hE_{eb}^{1112} \\ hE_{eb}^{1111} & h^2E_b^{1111} & 2h^2E_b^{1112} \\ 2hE_{eb}^{1112} & 2h^2E_b^{1112} & 4h^2E_b^{1212} \end{bmatrix} \\
S &= h \begin{bmatrix} E_e^{1111} & hE_{eb}^{1112} & 2hE_{eb}^{1212} \\ E_e^{1122} & hE_{eb}^{1122} & 2hE_{eb}^{1222} \\ hE_{eb}^{1122} & h^2E_b^{1122} & 2h^2E_b^{1222} \end{bmatrix} \\
P &= h \begin{bmatrix} E_e^{1212} & E_e^{1222} & hE_{eb}^{1222} \\ E_e^{1222} & E_e^{2222} & hE_{eb}^{2222} \\ hE_{eb}^{1222} & hE_{eb}^{2222} & h^2E_b^{2222} \end{bmatrix}
\end{aligned}$$

where

$$\left\{ E_e^{\alpha\beta\gamma\delta}, E_{eb}^{\alpha\beta\gamma\delta}, E_b^{\alpha\beta\gamma\delta} \right\} = \frac{1}{h} \int_{-h/2}^{h/2} D^{\alpha\beta\gamma\delta} \left\{ 1, \frac{\xi}{h}, \left(\frac{\xi}{h} \right)^2 \right\} ds.$$

The matrices E^T , $\hat{T}$, and $\bar{\lambda}$ are given by:

$$E^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -r_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & x_2 & x_3 & 1 \end{bmatrix}$$

$$\bar{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}.$$

The column matrix $\bar{\epsilon}$ contains the deformation modes:

$$\bar{\epsilon} = \begin{bmatrix} u'_1 \\ \theta' \\ -u''_3 \\ u''_2 \end{bmatrix}.$$

Minimizing the energy functional with respect to z_2 yields

$$\left(2ST\bar{\epsilon} + 2P\bar{\phi} + 2\hat{T}\bar{\lambda} \right) \delta\bar{\phi} = 0 \quad (2.68)$$

where

$$T = \begin{bmatrix} 1 & 0 & x_3 & -x_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2.69)$$

From Equation 2.68, $\bar{\phi}$ is obtained:

$$\bar{\phi} = -P^{-1}ST\bar{\epsilon} - P^{-1}\hat{T}\bar{\lambda}. \quad (2.70)$$

The Lagrange multipliers are determined by substituting $\bar{\phi}$ into the following 4 constraints which ensure that the shell bending strain measure ρ_{22} and the three orthogonal Cartesian displacements u_i are single-valued functions [3]:

$$\oint h\rho_{22}ds = 0. \quad (2.71)$$

$$\oint u_{1,2}ds = 0. \quad (2.72)$$

$$\oint u_{2,2}ds = 0. \quad (2.73)$$

$$\oint u_{2,3}ds = 0. \quad (2.74)$$

The solution is then

$$\bar{\lambda} = F^{-1}J\bar{\epsilon} \quad (2.75)$$

where

$$F = - \oint_C \begin{bmatrix} P_{11}^{-1} & x_2 P_{13}^{-1} & x_3 P_{13}^{-1} & P_{13}^{-1} \\ x_2 P_{13}^{-1} & x_2^2 P_{33}^{-1} & x_2 x_3 P_{33}^{-1} & x_2 P_{33}^{-1} \\ x_3 P_{13}^{-1} & x_2 x_3 P_{33}^{-1} & x_3^2 P_{33}^{-1} & x_3 P_{33}^{-1} \\ P_{13}^{-1} & x_2 P_{33}^{-1} & x_3 P_{33}^{-1} & P_{33}^{-1} \end{bmatrix} ds,$$

$$J = \oint_C \begin{bmatrix} c_1 & r_n & x_3 c_1 & -x_2 c_1 \\ x_2 c_3 & 0 & x_2 x_3 c_3 & -x_2^2 c_3 \\ x_3 c_3 & 0 & x_3^2 c_3 & -x_2 x_3 c_3 \\ c_3 & 0 & x_3 c_3 & -x_2 c_3 \end{bmatrix} ds,$$

and

$$c = P^{-1}S.$$

Substitution of Equation 2.75 back into Equation 2.70 gives the solution for $\bar{\phi}$:

$$\bar{\phi} = \left(-cT - P^{-1}\hat{T}F^{-1}J \right) \bar{\epsilon}. \quad (2.76)$$

Substitution of Equations 2.76 and 2.75 back into Equation 2.67 and noting that $\bar{\psi} = T\bar{\epsilon}$ gives [7]:

$$2\mathcal{U} = \oint_C [\bar{\epsilon}^T (T^T Q T - Y^T P Y + 2E^T F^{-1}J) \bar{\epsilon}] ds, \quad (2.77)$$

where

$$Y = -cT - P^{-1}\hat{T}F^{-1}J. \quad (2.78)$$

From Equation 2.77, the 4×4 stiffness matrix $\bar{S}$ is then extracted by inspection, and the beam deformations are then related to the external loading through

$$\begin{bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = [\bar{S}] \begin{bmatrix} u'_1 \\ \theta' \\ -u''_3 \\ u''_2 \end{bmatrix}. \quad (2.79)$$

Chapter 3

Modern Aeroelastic Response

Methodology

As a multidisciplinary field, aeroelasticity allows for a wide array of approaches to solving many types of problems. For each stage of the development of the proposed algorithm, there are many options available to choose from. This chapter serves to provide some background on the techniques available and can help justify the methods selected.

3.1 Unsteady Aerodynamics

At the heart of many algorithms for solving unsteady aerodynamic loading is the need to compute the loading for an aerodynamic body undergoing motion in an immersed flow. There is a very wide selection of theories and techniques available that can be used and adapted to the problem at hand. In this section, a selection of these approaches is summarized.

3.1.1 Classical Thin Airfoil Theory and the Quasi-steady Approach

The loading of a thin, cambered airfoil in a steady flow is given by a nice closed-form solution and has been known since the 19th century. The results are derived in many texts on the subject of introductory aerodynamics, such as Anderson's text [10]. In essence, a vortex sheet of varying strength $\gamma(s)$ is distributed over the camber line of the 2D airfoil, and the strength distribution is adjusted such that the camber line becomes a streamline of the flow and the vortex strength is 0 at the trailing edge to ensure no velocity jump. The result is the well-known 2D lift and pitching moment expressions:

$$c_l = 2\pi \left[\alpha + \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} (\cos \theta_0 - 1) d\theta_0 \right] \quad (3.1)$$

and

$$c_{m,c/4} = \frac{1}{2} \int_0^\pi \frac{dz}{dx} (\cos 2\theta_0 - \cos \theta_0) d\theta_0, \quad (3.2)$$

where

$$x = \frac{c}{2} (1 - \cos \theta_0), \quad (3.3)$$

and where $z(x)$ is the equation of the camber line, c is the chord length, α is the angle of attack, and c_l and $c_{m,c/4}$ are the lift and pitching moment coefficients given as

$$c_l = \frac{L}{\frac{1}{2}\rho U^2 c} \quad (3.4)$$

and

$$c_{m,c/4} = \frac{m_{c/4}}{\frac{1}{2}\rho U^2 c^2}. \quad (3.5)$$

The derivation is performed using steady potential flow theory and thus is unsuitable for unsteady flow. However, for slower motions, it can be used as a simple approximation and the results are dubbed “quasi - steady”. The quasi-steady approach provides dubious results even at moderate frequencies of motion.

3.1.2 Theodorsen’s Unsteady Theory

In the 1930s, Theodorsen developed an unsteady potential flow model for a flat plate pitching and plunging in simple harmonic motion [2, 4]. He modelled the wake as a line of distributed vortices emanating from the trailing edge and included the induced downwash into his model [11]. Linearizing the governing equation of potential flow using the small disturbance theory, Theodorsen used a clever conformal mapping to solve the resulting linear problem (Laplace’s equation). The derivation is included in Appendix A. The problem can also be solved in integral equation form as shown in Gulcat’s text [12].

Theodorsen’s results are simple in form but valid only for thin, uncambered airfoils (with a flap) exhibiting small-amplitude simple harmonic motion in pitch and plunge. If the pitching frequency ω is non-dimensionalized into the reduced frequency k by

$$k = \frac{\omega c}{2U}, \quad (3.6)$$

Theodorsen showed that the quasi-steady lift and moment could be modified by a function $C(k)$ that depends solely on the reduced frequency. The function is complex-valued, and hence affects both the magnitude and phase of the lift and moment, as

would be expected for a truly unsteady flow.

3.1.3 Wagner's Impulse Angle Response

Wagner determined the response of an airfoil to an impulsive change in pitch angle [2, 4]. The quasi-steady approach can be modified by use of Wagner's function $\Phi(\tau)$, which depends only on the reduced time τ , defined by

$$\tau = \frac{2Ut}{c}. \quad (3.7)$$

Whereas the quasi-steady approach predicts an instantaneous jump in lift, the Wagner function modification causes the lift to jump initially by only half the final value and then approach steady state asymptotically. Garrick [2] eventually showed the relationship between Wagner's function and Theodorsen's function, with the two essentially being related through the Fourier transform. In fact, the problem of loading on a flat plate in general pitching motion could be determined through convolution using Wagner's response to a step input. Indeed, Gulcat [12] gives the response of a 2D airfoil in general motion through the Duhamel integrals:

$$c_l(\tau) = \frac{\pi c}{2U^2} (\ddot{h} + U\dot{\alpha}) - \frac{2\pi}{U} \left[w(3c/4, \tau)/2 + \int_0^\tau w(3c/4, \tau) \phi'(\tau - \xi) d\xi \right], \quad (3.8)$$

and

$$c_m(\tau) = -\frac{\pi c}{4U^2} \left(\frac{c}{8} \ddot{\alpha} + U \dot{\alpha} \right) - \frac{\pi}{U} \left[w(3c/4, \tau)/2 + \int_0^\tau w(3c/4, \tau) \phi'(\tau - \xi) d\xi \right]. \quad (3.9)$$

The integrals are written in terms of the downwash w and the disturbance potential

ϕ' evaluated at the three-quarter chord point. Although the format is convenient, it is only applicable to thin, uncambered airfoils and incompressible flow.

3.1.4 Extensions of Theodorsen's Theory

Theodorsen's work was extended to cover other specific motion cases [2]. Kussner determined the lift response to a gust, and Loewy determined the airfoil loading for a rotating propeller blade. These approaches each cover different specific cases, but are limited to the motion of thin, uncambered airfoils.

3.1.5 Thick airfoils

Theodorsen's theory eventually evolved to cover the motion of 2D airfoils of any given shape. One solution was provided in 1953 by Woods [13], in which some viscous effects have been semi-empirically included. Woods writes that the presence of viscous flow modifies the inviscid prediction in three ways:

1. The location of the rear stagnation point may deviate from the trailing edge.
2. The velocity distribution of the mean steady flow is disturbed, particularly near the trailing edge.
3. It contributes to the aerodynamic damping, even in still air.

Woods' theory incorporates the first two viscous effects. The results for the lift and pitching moment deviate from the flat plate results in that: (i) the reduced frequency k is replaced by αk , where α is the angle of attack, (ii) the force and moment derivatives are multiplied by factors of α , and (iii) the reduction of the wake velocity due to the thickness of the airfoil changes the flat plate derivatives with increasing effect for higher values of ω .

3.2 2D Unsteady Boundary Layers

The methods outlined in the previous section are mostly based on potential flow theory and hence are incapable of predicting the drag on an airfoil. In some cases the drag may be an important feature of the flow, particularly in blades with low bending stiffness in the chordwise direction and in drag-driven rotors such as those of vertical axis wind turbines. This section outlines some of the options available for drag prediction.

3.2.1 Steady Boundary Layers

Steady boundary layers can be analyzed easily in the case of laminar flow. For turbulent boundary layers, things are somewhat more complicated, but many approximations are available.

The simplest solutions for both laminar and turbulent boundary layers are the Blasius solutions [10] which give the boundary layer thickness and skin friction coefficients as functions of the downstream distance on a flat plate at zero angle of attack, assuming zero streamwise pressure gradient. The Blasius solutions are simply obtained by solving the Navier-Stokes equations with small quantities neglected.

In the case of non-zero streamwise pressure gradients, the Navier-Stokes equations can be combined and integrated to produce momentum integral equations. The momentum integral equations essentially transform the steady Navier-Stokes equations into ordinary differential equations whose unknowns are strictly a function of the streamwise position. These differential equations represent under-determined

systems and hence additional closure relations are required to obtain a full solution [14]. Many different approaches are yielded depending on the strategy used to close the system. For example, the Polhausen method closes the system by using a fourth-order polynomial approximation for the velocity profile, while Thwaites used approximating equations from empirical data to close the system [12].

There does exist an “unsteady” Thwaite’s method, but the approach is in fact quasi-steady and thus not suitable for most applications.

3.2.2 Unsteady Momentum Integral Methods

The unsteady Navier-Stokes equations can be combined and integrated over the boundary layer thickness to provide unsteady momentum integral equations. Bram Van Es [14] summarizes the modern approaches very well in his thesis. The majority of approaches use two particular momentum integral equations: the Von Karman momentum integral and the kinetic energy momentum integral, involving 5 unknown functions of the streamwise distance. The system can be closed by using a 1-parameter velocity profile such as Tani’s profile [14] for laminar flows and the momentum integral equations reduce to a closed hyperbolic system of two partial differential equations that can be solved using a finite volume method.

Currently, unsteady turbulent boundary layers are very difficult to analyze and a wide array of empirical closure relations are available for very specific cases [14].

3.3 Computational Fluid Dynamics and Direct Numerical Simulation

Computational Fluid Dynamics (CFD) and Direct Numerical Simulation (DNS) involve numerically solving the Navier-Stokes equations, often coupled with the energy equation, thermodynamic models, and turbulence models to obtain accurate solutions to complex problems. These methods are extremely popular and useful due to the ability to solve very complex flow problems. The drawbacks to these methods are the high computational times and the difficulties involved in setting up the problem, particularly in generating the mesh for the finite element method. Many modern software packages are available such as ANSYS CFX and Fluent that are capable of performing a very wide array of analyses.

3.4 2D and 3D Panel Methods

For the purposes of this work, the CFD and DNS approaches take too long and are thus unsuitable for use in the preliminary design phase of horizontal axis wind turbines and other structures. Fortunately, there are other methods available for analyzing complex flows based on the potential flow theory.

Panel methods are a popular choice in analyzing flows around complex geometry [10]. The general idea is to discretize immersed body surfaces into flat panels over which singularity flows such as the vortex, source, or doublet are distributed with varying strengths. The strengths are modelled using appropriate shape functions over the panel surfaces, and the governing coefficients are determined by setting up a system of equations resulting from the application of appropriate boundary conditions. Panel methods are much more efficient in terms of processing time than

CFD or DNS, although they do not allow quite the same level of problem complexity.

3.5 Blade Element Momentum Theory

The Blade Element Theory (BET) and Blade Element Momentum Theory (BEMT) allow for the analysis of slender wings and blades using only 2D aerodynamic analyses performed at a finite number of spanwise locations [1]. The spanwise loading is then integrated over the entire blade to determine the total loading.

The BET is very useful in reducing the problem of complexity, and the cost of accurately analyzing 3D flow effects such as spanwise flow. These methods can be augmented using empirical corrections to account for features such as spanwise flow and tip losses [15]. Combined with 2D panel methods, the blade loading can be determined very rapidly.

3.6 Dynamics of Composite Structures

The proposed algorithm will make use of the work of Khouli [3, 7] in determining the structural response of a composite turbine blade. Khouli used the Variational Asymptotic Method of Yu, Hodges, Volovoi, and Cesnik [6] to determine the stiffness distribution of an anisotropic beam and developed a nonlinear 1D kinematics model for the structural deformations. For the sake of reducing computational time, the method of Ritz [4] can be used to derive linear equations of motion for the structural deformations of the beam.

Chapter 4

Development of an Aeroelastic Algorithm

The approaches for determining blade structural response as presented in Chapter 3 are found to be unsuitable for the preliminary design phase. Methods such as Computational Fluid Dynamics (CFD) or Direct Numerical Simulation (DNS) are too computationally expensive and time-consuming, while algorithms such as the steady Blade Element Momentum Theory (BEMT) and classical flat plate solutions are too simplistic and cannot accurately predict aerodynamic loading on thick airfoils at moderate vibrational frequencies. Hence, a new algorithm is needed that is more rapid than CFD and DNS, but also more accurate than classical or steady aerodynamic theories.

4.1 Problem Statement and Algorithm Requirements

The goal of the algorithm is to determine the unsteady aerodynamic loading of an anisotropic, arbitrarily-supported blade immersed in a steady, uniform flow. The algorithm is to be implemented in MATLAB and will meet the following requirements:

1. The algorithm shall allow for the analysis of anisotropic blades.

2. The algorithm shall allow for the analysis of blades of varying geometry and material properties along the span.
3. The algorithm shall allow for blades with any arrangement of simple and fixed supports.
4. The algorithm shall account for the unsteady wake trailing behind the blade.
5. The algorithm shall account for bending, twisting, and extensional displacement of the blade.
6. The resulting software shall be shown to provide a blade loading history that is consistent with results from CFD analyses using a respected finite element package.
7. The resulting software shall be validated through wind tunnel testing of a scale model.
8. The resulting software shall be shown to run a minimum of 100 times faster than a CFD analysis undertaken on the same computer hardware.

4.2 Breakdown of the Aeroelastic Response Algorithm

The algorithm consists of three sub-algorithms operating in tandem as shown in Figure 4.1. The sub-algorithms include:

1. A cross-sectional stiffness analysis tool.
2. An unsteady aerodynamic loading solver.

3. A beam dynamics solver.

For the simulations performed in this work, the VABS cross-sectional analysis tool of Khouli (2009) was used to generate the cross-sectional stiffness matrices [3]. The theory behind this tool was briefly summarized in Section 2.3. In Section 4.3, the Vortex Panel Method (VPM) Solver is developed, which allows for the prediction of unsteady aerodynamic loading on a 2D blade section. Section 4.6 outlines the construction of the Beam Kinematics Solver, which computes the incremental blade deformation due to the output of the VPM Solver at each time step.

Figure 4.1: Feedback loop involving the three sub-algorithms.

4.3 The 2D Unsteady Vortex Panel Method

As the BEMT involves the determination of the 2D aerodynamic coefficients for each spanwise element, the algorithm needs to be capable of determining the 2D lift and pitching moment of an airfoil. The steady panel, method as described in Chapter 3, is a fast and accurate choice, but it must be modified to be used in the unsteady conditions resulting from blade vibration.

4.3.1 Discretization of the Airfoil Geometry

The Vortex Panel Method (VPM) allows the problem of flow around an immersed body to be discretized by dividing the body surface into a finite number of straight vortex sheets, as shown in Figure 4.2. The required tangency of the flow with the vortex panels can then be applied a finite number of times, once for each panel. The wake can also be modelled using vortex panels, though the Neumann boundary conditions are not necessary.

Figure 4.2: Discretization of airfoil geometry for the vortex panel method.

4.3.2 Flow Around a Vortex Panel

Consider a vortex panel that extends in the global $X - Z$ coordinate system from (X_0, Z_0) to (X_f, Z_f) as shown in Figure 4.3. In the local $x - z$ coordinate system, the panel extends from $(0, 0)$ to $(L, 0)$. The vortex strength γ varies along the panel. The induced velocity at a point (x, z) due to an infinitesimal element of strength $\gamma(\xi) d\xi$ from the panel is found by taking the gradient of Equation 2.35:

Figure 4.3: Induced velocity around a vortex panel.

$$d\mathbf{q}_{xz}(\xi^*) = \left(\frac{\gamma(\xi^*) d\xi^*}{2\pi(1 + \xi^{*2})} \right) \mathbf{i} + \left(\frac{\gamma(\xi^*) \xi^* d\xi^*}{2\pi(1 + \xi^{*2})} \right) \mathbf{k}, \quad (4.1)$$

where,

$$\xi^* = \frac{\xi - x}{z}. \quad (4.2)$$

The function $\gamma(\xi)$ may be chosen arbitrarily. The constant function is the simplest approach, but tends to yield oscillating solutions as continuity of vortex strength at the panel nodes cannot be enforced [10]. Linear and quadratic functions give better results. Choosing a linear vortex strength variation along the panel results in

$$\gamma(\xi) = \gamma_0 + \gamma_1 \frac{\xi}{L}. \quad (4.3)$$

Equation 4.1 may now be integrated over the panel to give the resulting flow field:

$$\mathbf{q}_{xz} = \left(\frac{\gamma_0}{2\pi} \chi + \frac{\gamma_1}{2\pi} v \right) \mathbf{i} + \left(\frac{\gamma_0}{2\pi} \psi + \frac{\gamma_1}{2\pi} \phi \right) \mathbf{k}, \quad (4.4)$$

where,

$$\chi(x, z; L) = \arctan\left(\frac{L-x}{z}\right) + \arctan\left(\frac{x}{z}\right), \quad (4.5)$$

$$\psi(x, z; L) = \ln \sqrt{\frac{(L-x)^2 + z^2}{x^2 + z^2}}, \quad (4.6)$$

$$v(x, z; L) = \frac{z}{L}\psi + \frac{x}{L}\chi, \quad (4.7)$$

$$\phi(x, z; L) = 1 - \frac{z}{L}\chi + \frac{x}{L}\psi. \quad (4.8)$$

Finally, transforming back to the global $X - Z$ coordinate system through the use of the translational and rotational transformations

$$\begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \cos \theta_j & -\sin \theta_j \\ \sin \theta_j & \cos \theta_j \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}, \quad (4.9)$$

the induced velocity field is given as (in terms of unit vectors $\mathbf{I}$ and $\mathbf{K}$ in the $X - Z$ coordinate system):

$$\mathbf{q}_{XZ} = \left(\frac{\gamma_0}{2\pi}X + \frac{\gamma_1}{2\pi}\Upsilon\right)\mathbf{I} + \left(\frac{\gamma_0}{2\pi}\Psi + \frac{\gamma_1}{2\pi}\Phi\right)\mathbf{K}. \quad (4.10)$$

Thus, the induced velocity field is linear with respect to the panel strength coefficients, γ_{0i} and γ_{1i} . The influencing functions X , Υ , Ψ , and Φ can be evaluated for a given panel given its location and orientation at any desired point in space.

4.3.3 The Wake

Theodorsen accurately modelled the unsteady, viscous wake trailing behind an airfoil as a 1-dimensional vortex sheet. The method can be adapted to the VPM by discretizing the wake into a finite series of vortex panels, as shown in Figure 4.4. A new

coordinate system, $x^{(\alpha)} - z^{(\alpha)}$, is attached to the wake as shown for use in analyzing its effect on the flow. After determining the appropriate strengths, the effect of the wake on the total velocity field can be calculated through use of Equation 4.10.

Figure 4.4: Propagation of vortex panels into the wake.

4.3.4 Unsteady Kutta Condition

Application of the Neumann boundary condition is not sufficient to produce a unique solution to Laplace's Equation for the velocity potential. The solution is fixed by application of the Kutta Condition, which states that the pressure must be continuous across the wake. To write this condition in terms of the velocity potential, it is advantageous to make use of the small disturbance theory. Let the free stream potential be separated from the rest of the flow such that

$$\phi = Ux^{(\alpha)} + \phi', \quad (4.11)$$

where ϕ' is the disturbance potential, representing the effect of the airfoil on the free stream flow. By virtue of Equation 2.5, the magnitude of the velocity is written as:

$$q = |\nabla\phi| = \left(U + \frac{\partial\phi'}{\partial x^{(\alpha)}} \right)^2 + \left(\frac{\partial\phi'}{\partial z^{(\alpha)}} \right)^2. \quad (4.12)$$

Noting that the disturbance velocities $\frac{\partial \phi'}{\partial x^{(\alpha)}, z^{(\alpha)}}$ are generally much smaller than the free stream velocity, the expression can be expanded and such quantities may be neglected, leading to:

$$q \approx U^2 + 2U \frac{\partial \phi'}{\partial x^{(\alpha)}}. \quad (4.13)$$

Setting $f(t) = \frac{\partial \phi'_\infty}{\partial t} + \frac{1}{2}U^2 + \frac{p_\infty}{\rho}$ in the unsteady Bernoulli equation (Equation 2.23), substituting in Equation 4.13 for q , and rearranging the terms results in

$$\frac{p - p_\infty}{\frac{1}{2}\rho U^2} = c_p = \frac{2}{U^2} \frac{\partial}{\partial t} (\phi'_\infty - \phi') + \frac{2}{U} \left(\frac{\partial \phi'_\infty}{\partial x^{(\alpha)}} - \frac{\partial \phi'}{\partial x^{(\alpha)}} \right). \quad (4.14)$$

The pressure coefficient, c_p , relates the local pressure and velocity at a point $(x^{(\alpha)}, z^{(\alpha)})$ to the free stream values. Similarly, a new pressure coefficient can be defined that relates the pressures and velocities just above (subscript u) and just below (subscript l) the wake [12]:

$$\frac{p_l - p_u}{\frac{1}{2}\rho U^2} = c_{p_a} = \frac{2}{U^2} \frac{\partial}{\partial t} (\phi'_u - \phi'_l) + \frac{2}{U} \frac{\partial}{\partial x^{(\alpha)}} (\phi'_u - \phi'_l). \quad (4.15)$$

The Kutta Condition may now be expressed as:

$$c_{p_a} (x^{(\alpha)} > 0) = 0. \quad (4.16)$$

Writing the disturbance potentials as integrals of the disturbance velocities:

$$\phi'_u - \phi'_l = \int_{-\infty}^{x^{(\alpha)}} (u'_u - u'_l) dx^{(\alpha)}. \quad (4.17)$$

The portion of the integral from $-\infty$ to 0 along the $x^{(\alpha)}$ axis encompasses the entire airfoil. As the velocity jump equals the vortex strength, this part of the integral must give the circulation over the airfoil, Γ_A . The velocity jump over the wake must give

the local wake vortex strength:

$$\phi'_u - \phi'_l = \Gamma_A + \int_0^{x^{(\alpha)}} \gamma_W dx^{(\alpha)}. \quad (4.18)$$

This in conjunction with Equation 4.15 may be substituted into the unsteady Kutta condition:

$$\dot{\Gamma}_A + \frac{\partial}{\partial t} \int_0^{x^{(\alpha)}} \gamma_W (x^{(\alpha)}, t) dx^{(\alpha)} + U \gamma_W (x^{(\alpha)}, t) = 0. \quad (4.19)$$

To determine the strength of the wake, the following procedure from Gulcat's text [12] can be used. First, the Laplace transform of Equation 4.19 is taken:

$$s\bar{\Gamma}_A - \Gamma_{A,0} + s \int_0^{x^{(\alpha)}} \bar{\gamma}_W (x^{(\alpha)}, t) dx^{(\alpha)} + U \bar{\gamma}_W (x^{(\alpha)}, t) = 0 \quad (4.20)$$

The result is then differentiated with respect to $x^{(\alpha)}$:

$$s\bar{\gamma}_W + U \frac{\partial \bar{\gamma}_W}{\partial x^{(\alpha)}} = 0. \quad (4.21)$$

Equation 4.21 is a first order linear ODE with the solution

$$\bar{\gamma}_W (x^{(\alpha)}, s) = B(s) \exp \left(-\frac{sx^{(\alpha)}}{U} \right). \quad (4.22)$$

The function $B(s)$ is determined by setting $x^{(\alpha)} = 0$ in Equations 4.20 and 4.22 and comparing. The Kutta condition now becomes

$$\bar{\gamma}_W (x^{(\alpha)}, s) = -\frac{s\bar{\Gamma}_A - \Gamma_{A,0}}{U} \exp \left(-\frac{sx^{(\alpha)}}{U} \right). \quad (4.23)$$

Upon applying the inverse transform, the Kutta Condition in the time domain is recovered as:

$$\gamma_W(x^{(\alpha)}, t) = -\frac{1}{U} \dot{\Gamma}_A \left(t - \frac{x^{(\alpha)}}{U} \right). \quad (4.24)$$

This is a remarkable result, as it indicates that a snapshot of the rate of change of circulation around the airfoil from every point in time is built into the wake at some point downstream. For this reason, it is said that the wake has memory of the time history of the airfoil motion. Also worth noting is that as a result of the unsteady Kutta condition,

$$\gamma_W(x^{(\alpha)} = 0, t) = \gamma_{TE}(t) = -\frac{1}{U} \dot{\Gamma}_A(t). \quad (4.25)$$

For a steady flow, the circulation remains constant around the airfoil, and Equation 4.25 reduces to the steady Kutta condition given by $\gamma_{TE} = 0$.

4.3.5 Non-circulatory Flow Contribution

Due to the motion of the airfoil, each point on the surface sees an additional component of flow velocity that must be taken into account. Assuming the airfoil is rigid and exhibits planar motion in the $X - Z$ global coordinate system, 3 generalized coordinates must be assigned to capture the effects. Yet another coordinate system, the $\xi - \zeta$ coordinate system, is attached to the elastic axis (shear centre) as shown in Figure 4.5. The airfoil may experience changes in pitch, φ , vertical plunge, h , and heave, e . From Figure 4.5, the $X - Z$ coordinates can be related to the $\xi - \zeta$ coordinates using the transformation

$$X = x_{EA} + e + \xi \cos \varphi + \zeta \sin \varphi \quad (4.26)$$

$$Z = h - \xi \sin \varphi + \zeta \cos \varphi \quad (4.27)$$

The velocity of a point (X, Z) can then be computed by differentiating Equations 4.26 and 4.27:

$$\dot{X} = \dot{e} - (\xi \sin \varphi - \zeta \cos \varphi) \dot{\varphi} \quad (4.28)$$

$$\dot{Z} = \dot{h} - (\xi \cos \varphi + \zeta \sin \varphi) \dot{\varphi} \quad (4.29)$$

with ξ and ζ computed from Equations 4.26 and 4.27. Now, the global frame can be affixed to the airfoil, with the velocities of each point augmented by $-\dot{X}$ and $-\dot{Z}$ as defined by Equations 4.28 and 4.29.

The discussion up to this point covers all material needed to develop an algorithm for determining the unsteady, inviscid flow field around an airfoil for small angles of attack. This is sufficient for computation of the lift and pitching moment, but the drag is not computed as a result of the inviscid nature of the potential flow solution. Considering that the drag is generally much smaller than the lift, and that turbine blades tend to have high bending stiffnesses in the chordwise direction, the contribution of drag force to the motion of the structure will be deemed insignificant and no computational resources will be spent trying to predict the drag.

Figure 4.5: 2D airfoil motion with associated degrees of freedom.

4.4 The Unsteady Boundary Layer

The proposed algorithm may eventually find use in different applications where the drag force plays a significant role. Taking this into consideration, some groundwork is provided here to allow the user to implement a boundary layer solver. This section may be omitted without loss of continuity. Although Prediction of the unsteady boundary layer is a particularly difficult problem even from the standpoint of modern methods in fluid dynamics. The governing Partial Differential Equation (PDE) of potential flow theory is linear if the flow is incompressible. Even if the flow is compressible, under suitable conditions, the PDE can be linearized very easily [12, 14]. This is not so for the unsteady boundary layer equations, which are developed from the Navier-Stokes equations. Assuming incompressible flow, and assuming large curvature of the airfoil with respect to the thickness of the boundary layer, the Navier-Stokes equations reduce to give [12]:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (4.30)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2}, \quad (4.31)$$

$$\frac{\partial p}{\partial z} = 0, \quad (4.32)$$

with the convention given as in Figure 4.6. The first equation is the continuity equation. The second is the momentum equation. The third indicates that the pressure field remains unchanged through the boundary layer, and hence the drag force can be computed independently of the inviscid solution.

Figure 4.6: The boundary layer over a flat surface.

As currently posed, the boundary layer equations are said to be in field form. Van Es [14] points out that the boundary layer equations are more efficiently solved in the integral form. Integral forms of the boundary layer equations are created by integrating families of momentum equations, such as

$$[continuity] \times (u^{n+1} - u_e^{n+1}) + [momentum] \times (n+1) u^n, \quad (4.33)$$

where each value of $n \geq 0$ yields a different momentum equation. Integrating over the thickness from 0 to δ yields a momentum integral equation. Setting $n = 0$ yields the Unsteady Von Karman integral boundary layer equation [14]:

$$\frac{1}{u_e^2} \frac{\partial (u_e \delta^*)}{\partial t} + \frac{\partial \theta}{\partial x} + \left(2 + \frac{\delta^*}{\theta}\right) \frac{\theta}{u_e} \frac{\partial u_e}{\partial x} = \frac{C_f}{2}, \quad (4.34)$$

with three unknowns. The displacement thickness, momentum thickness, and wall

friction coefficient, are given respectively as:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dz, \quad (4.35)$$

$$\theta = \int_0^\delta \frac{u}{u_e} \left(1 - \frac{u}{u_e}\right) dz, \quad (4.36)$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_e^2}. \quad (4.37)$$

Since there are two more unknowns than there are equations, closure relations are needed to close the system. There are many modern methods available for obtaining closure relations which usually involve either an assumed velocity profile or use of empirical relationships. An example of the former approach is the Method of Hayasi [14]. First, returning to Equation 4.33 and letting $n = 1$ yields the Kinetic Energy Integral Equation [14]:

$$\frac{1}{u_e} \frac{\partial(\theta + \delta^*)}{\partial t} + 2 \frac{\theta}{u_e^2} \frac{\partial u_e}{\partial t} + \frac{1}{u_e^3} \frac{\partial(u_e^3 \delta^k)}{\partial x} = C_D \quad (4.38)$$

with 2 additional unknowns: the viscous diffusion coefficient and kinetic energy thickness given as, respectively,

$$C_D = \frac{2}{\rho u_e^3} \int_0^\delta \tau \frac{\partial u}{\partial z} dz, \quad (4.39)$$

$$\delta^k = \int_0^\delta \frac{u}{u_e} \left(1 - \frac{u^2}{u_e^2}\right) dz, \quad (4.40)$$

where τ is the local shear stress. Next, a single-parameter boundary layer profile $u = u(z; \mu)$ is chosen such that all 5 unknowns, δ^* , θ , C_f , C_D , and δ^k can be expressed as functions of δ and one parameter μ alone. Substitution into the Von Karman and Kinetic Energy Integral Equations yields a hyperbolic system of 2 PDEs that may

be solved using a finite volume approach. For laminar boundary layers, a single-parameter profile such as Tani's profile may be used [14]. Tani's profile is given as:

$$\frac{u}{u_e} = (6 - 8\eta + 3\eta^2) \eta^2 + \mu\eta(1 - \eta)^3 \quad (4.41)$$

with

$$\eta = \frac{z}{\delta} \quad (4.42)$$

For turbulent boundary layers, the power law can be used for a basic fit [14, 16]:

$$\frac{u}{u_e} = \eta^\mu \quad (4.43)$$

The transition from a laminar to a turbulent boundary layer depends on the Reynold number. One possible criterion is Michel's criterion [14]:

$$Re_\theta = 1.174 \left(1 + \frac{22,400}{Re_x} \right) Re_x^{0.46} \quad (4.44)$$

with Re_θ using the momentum thickness as the reference length, and Re_x using the x -coordinate leads to

$$Re_\theta = \frac{u_e \theta}{\nu} \quad (4.45)$$

$$Re_x = \frac{u_e x}{\nu} \quad (4.46)$$

Finally, with 1-parameter velocity profiles selected for all positions on the airfoil, the momentum integral equation unknowns can be expressed in terms of the profile parameter μ and the boundary layer thickness δ using Equations 4.35, 4.36, 4.37, 4.39, and 4.40. Substitution into the unsteady momentum integrals, Equations 4.34

and 4.38, yields a hyperbolic system of PDEs in δ and μ . The solution can then be carried out using a finite volume scheme.

The computation of additional unsteady boundary layer features, such as flow separation, are beyond the scope of this work. It should be noted, however, that phenomena such as dynamic stall are important in the analysis of turbine performance.

4.5 Implementation of the Unsteady 2D Vortex Panel Method

Based on the Potential Flow Theory and introduction to the Vortex Panel Method (VPM) as covered in Chapter 2, an algorithm for the determination of the lift and drag on a 2D airfoil can be developed.

The inviscid solution may be computed under the assumptions of a 2D, irrotational, incompressible flow. This allows for the use of Potential Flow Theory and the Unsteady Bernoulli Equation, Equation 2.23. The airfoil is assumed rigid, with 3 DOF for its motion as shown in Figure 4.5. It is also assumed that at the beginning of the simulation, steady flow conditions are present.

4.5.1 Airfoil Discretization

The airfoil geometry is discretized into N linear vortex panels, as shown in Figure 4.2. The panels are connected at the nodes, and the control points are at the centre. Initially, the wake is assumed to have zero strength due to the pre-existing steady flow. With each time step, the airfoil is allowed to move to a new configuration. The

change in circulation associated with the motion is balanced with the vorticity shed into the wake by means of a new vortex panel emanating from the trailing edge. The progression is shown in Figure 4.4. Thus, after the r^{th} time step, the total number of vortex panels is $N + r$. The circulatory flow velocity at the point (X, Z) is the sum of contributions from all the panels:

$$\mathbf{q}_C = \sum_{j=1}^{N+r} \left[\left(\frac{\gamma_{0,j}}{2\pi} X + \frac{\gamma_{1,j}}{2\pi} \Upsilon \right) \mathbf{I} + \left(\frac{\gamma_{0,j}}{2\pi} \Psi + \frac{\gamma_{1,j}}{2\pi} \Phi \right) \mathbf{K} \right]. \quad (4.47)$$

With two unknown panel strength parameters per panel, $\gamma_{0,j}$ and $\gamma_{1,j}$, the total number of unknowns is $2N + 2r$. The non-circulatory flow is entirely known at each time step as it depends only on the free-stream velocity and the airfoil motion. A suitable set of equations must now be determined.

4.5.2 Neumann Boundary Conditions

The “non-penetration” boundary conditions are expressed in Neumann form in terms of the velocity potential as:

$$\frac{\partial \phi}{\partial n} = 0, \quad (4.48)$$

where n is a direction normal to the surface. The Neumann conditions must be satisfied at a finite number of control points along the airfoil. These control points are taken to be the panel centres. In terms of the velocity field:

$$[\mathbf{q}_C(\bar{X}_i, \bar{Z}_i) + \mathbf{q}_{NC}] \cdot \hat{\mathbf{n}}_i = 0, \quad (4.49)$$

where $\bar{X}_i, \bar{Z}_i$ are the coordinates of the control point of the i^{th} panel, and $\hat{\mathbf{n}}_i$ is the normal vector at that location. The normal unit vectors are formulated from the

panel angles, θ_j :

$$\hat{\mathbf{n}}_i = -\sin \theta_i \mathbf{I} + \cos \theta_i \mathbf{K} \quad (4.50)$$

Applying this at each of the panel control points yields N equations given by:

$$\begin{aligned} \sum_{j=1}^{N+r} \left[-\left(\frac{\gamma_{0,j}}{2\pi} X_{ji} + \frac{\gamma_{1,j}}{2\pi} \Upsilon_{ji} \right) \sin \theta_i + \left(\frac{\gamma_{0,j}}{2\pi} \Psi_{ji} + \frac{\gamma_{1,j}}{2\pi} \Phi_{ji} \right) \cos \theta_i \right] \\ = -\mathbf{q}_{NC} \cdot \hat{\mathbf{n}}_i, \quad i = 1 \dots N \end{aligned} \quad (4.51)$$

where the notation $()_{ji}$ indicates that the influence of panel j on control point i is computed. Condensing the notation,

$$\sum_{j=1}^{N+r} (q_{0,ji} \gamma_{0,j} + q_{1,ji} \gamma_{1,j}) = b_i, \quad i = 1 \dots N \quad (4.52)$$

Hence, application of the Neumann boundary conditions gives N linear equations in the primary unknowns, $\gamma_{0,j}$ and $\gamma_{1,j}$.

4.5.3 Continuity Boundary Conditions

To ensure a continuous distribution of vorticity around the airfoil, it is required that two connecting panels share a common vortex strength at their node. Thus,

$$\gamma_{0,j} + \gamma_{1,j} = \gamma_{0,j+1}, \quad j = 1 \dots N-1 \quad (4.53)$$

$$\gamma_{0,N} + \gamma_{1,N} = \gamma_{0,1} \quad (4.54)$$

This yields another N equations, bringing the total to $2N$.

4.5.4 The Unsteady Kutta Condition

The formulation of the Kutta Condition, Equation 4.24, shows that for each point on the wake, the local vortex strength is proportional to the time rate of change of circulation over the airfoil at the instant the point was shed. Thus, the Kutta condition is enforced by requiring that

$$\gamma_{0,1} = \gamma_{0,N+1} = -\frac{1}{U}\dot{\Gamma}_A \quad (4.55)$$

at each time step. At this point, the number of equations used to close the system of $2N$ panel strengths is $2N + 1$, resulting in an overdetermined system. One approach to fixing this problem is to overwrite one of the previous equations. However, the numerical solution will differ depending on which equation is overwritten. Instead, the approach used here is to introduce an additional unknown through a set of vortex panels of constant strength γ_c over the airfoil. Doing so does not affect vortex strength continuity, nor does the addition of a constant affect the Neumann boundary conditions. Through use of Equation 4.47, the velocity induced by the constant strength sheet is:

$$\mathbf{q}_C = \frac{\gamma_c}{2\pi} \sum_{j=1}^{N+r} [X\mathbf{I} + \Psi\mathbf{K}] \quad (4.56)$$

Thus, the terms $\frac{\gamma_c}{2\pi}X_{ji}$ and $\frac{\gamma_c}{2\pi}\Psi_{ji}$ simply need to be added to the inner brackets, respectively, in Equation 4.51.

4.5.5 Wake Propagation

After the progression of each time step, the wake panels from the previous time step propagate downstream to make room for the new panel. Since $2r - 2$ of the

wake panel strength constants are known from the previous time step, $2r - 2$ simple equations can be introduced to specify these values. Additionally, the new wake panel must continuously join the vortex strength at the trailing edge to the nodal vortex strength of the previous wake panel. Hence, 2 additional simple equations are supplied, yielding $2N + 2r + 1$ total equations.

As the number of wake panels increases the system size, it is desirable to keep the number of panels to a minimum. The wake panels should be discarded after propagating a certain distance downstream, as their effect on the velocity field surrounding the airfoil becomes negligible with increasing distance. To determine an effective cutoff criterion, it can be assumed that the strongest pulse of vorticity that could be shed into the wake would come from a large step change in pitch from $\alpha = 0$ to $\alpha = \alpha_{max}$. The Kutta-Joukowski theorem gives the change in circulation, $\Delta\Gamma_{max}$, associated with this pulse as

$$\Delta\Gamma_{max} = \frac{\Delta L}{\rho U}, \quad (4.57)$$

where ΔL represents the change in lift during the pulse. This result is valid for steady flow, but can be used for the order of magnitude approximation. Noting that from simple airfoil theory, $\Delta L = \frac{1}{2}\rho U^2 c c_{l_\alpha} \alpha_{max}$:

$$\Delta\Gamma_{max} = \frac{\frac{1}{2}\rho U^2 c_{l_\alpha} \alpha_{max} c}{\rho U} = \frac{1}{2} U c_{l_\alpha} \alpha_{max} c, \quad (4.58)$$

where α_{max} is the maximum allowable angle of attack. The magnitude of the induced velocity far from a point vortex can be found by taking the magnitude of the gradient of Equation 2.35 to get:

$$q_{vortex} = \frac{\Gamma}{2\pi d} \quad (4.59)$$

where d is the distance from the vortex. After r equal timesteps (also the number of wake panels), the last wake panel will have propagated a distance of approximately $rU\Delta t$, where Δt is the time step size. The ratio of induced velocity to free stream velocity is:

$$\frac{q}{U} = \kappa = \frac{\frac{1}{2}U c_{l\alpha} \alpha_{max} c}{2\pi r U^2 \Delta t} \quad (4.60)$$

Solving for r , the maximum number of wake panels is found to be

$$r_{max} = \frac{c_{l\alpha} \alpha_{max} c}{4\pi U \kappa \Delta t} \quad (4.61)$$

The values of α_{max} and κ can be chosen arbitrarily. Suggested values are 5-10° and 0.001 to 0.01, respectively.

4.5.6 The Kelvin Condition

The Kelvin condition requires the vorticity to be conserved during each time step. Integrating over the airfoil panels, the circulation around the airfoil at any time step is given by

$$\Gamma_A = \sum_{j=1}^N \left(\gamma_{0,j} + \frac{1}{2} \gamma_{1,j} \right) L_j \quad (4.62)$$

and defined similarly for the wake. Invoking Kelvin's Theorem, Equation 2.16, and knowing the total circulation $\Gamma_{A,0}$ from the first time step, the Kelvin condition states that

$$\sum_{j=1}^{N+r} \left(\gamma_{0,j} + \frac{1}{2} \gamma_{1,j} \right) L_j = \Gamma_{A,0} \quad (4.63)$$

With the addition of one more equation, the system has again become overdetermined. An additional unknown may be introduced through the placement of another constant strength set of vortex panels around the airfoil with strength $\gamma_{C,W}$, as described in Section 4.5.4.

4.5.7 Vortex Panel Method Solution

The unknowns to be determined are the vortex panel strength parameters, $\gamma_{0,j}$ and $\gamma_{1,j}$ for $j = 1 \dots 2N + 2r$ and the two constant panel strengths, γ_C and $\gamma_{C,W}$. Thus, the total number of unknowns is $2N + 2r + 2$. The system of equations to be solved is linear and takes the form:

$$[\mathcal{A}] \{\gamma\} = \{\mathcal{B}\}. \quad (4.64)$$

Table 4.1 summarizes the equations derived in this chapter that need to be solved to obtain the vorticity distribution around the airfoil.

4.5.8 Determination of the Velocity Field

Since the distributed panel strength represents the jump in tangential velocity across a vortex sheet, and the velocity is zero inside the airfoil, the vortex strength represents the tangential velocity V_t at the surface of the airfoil. Then, the tangential velocity varies as a function of s , the distance traversed in a counterclockwise fashion around the airfoil from the trailing edge as shown in Figure 4.2, according to:

Table 4.1: Summary of equations to solve for unsteady vortex panel method.

Condition	Reference	Number of Equations
Airfoil vortex continuity	Equations 4.53 and 4.54	N
Neumann (tangency) conditions	Equation 4.52	N
Unsteady Kutta condition	Equation 4.55	1
The Kelvin condition	Equation 4.63	1
Previous wake panels	N/A	$2(r - 1)$
New wake panel	N/A	2
Total:		$2N + 2r + 2$

$$V_t(s) = \gamma(s). \quad (4.65)$$

The location of the leading edge stagnation point can then be found by interpolating to find the point where $\gamma(s) = 0$.

4.5.9 Determination of the Pressure Field

After the panel strengths are determined, the velocity potential can be evaluated at each panel control point. Recalling the definition of the pressure coefficient from Equation 4.14 and noting that far from the airfoil, the disturbance potential vanishes:

$$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2} = -\frac{2}{U^2} \frac{\partial \phi'}{\partial t} - \frac{2}{U} \frac{\partial \phi'}{\partial x^{(\alpha)}}. \quad (4.66)$$

Also recalling from Equation 2.28 that $\phi = \phi_{FS} + \phi'$, and noting that ϕ_{FS} is constant as it represents the free stream:

$$c_p = -\frac{2}{U^2} \frac{\partial \phi}{\partial t} - \frac{2}{U} \frac{\partial \phi}{\partial x^{(\alpha)}}. \quad (4.67)$$

For each time step, the velocity potential must be evaluated at each panel control point and then the pressure field can be extracted. Making use of the velocity potential equation for a vortex (Equation 2.35), the velocity potential as a function of s is given as:

$$\phi(s) = \phi_{TE} + \frac{1}{2\pi} \int_0^s \gamma(s) \theta_{TE}(s) ds, \quad (4.68)$$

where $\theta_{TE}(s)$ is the angle from the trailing edge to point $(X(s), Z(s))$. The potential at the trailing edge where $s = 0$ does not include the effects from the airfoil panels, but must include the effects from the wake. Because the wake extends far downstream from the trailing edge, it will be assumed that the angle from all points on the wake to the trailing edge is approximately π . Again invoking the velocity potential equation for a vortex (Equation 2.35), the potential at the trailing edge is given by:

$$\phi_{TE} = \frac{1}{2\pi} \int_0^\infty \gamma_W(x^{(\alpha)}) \pi ds. \quad (4.69)$$

The constant π can be taken out and the remaining integral represents the total circulation over the wake. Hence, the velocity potential at the trailing edge is:

$$\phi_{TE} = \frac{1}{2} \Gamma_W. \quad (4.70)$$

Finally, the velocity potential at the point $(X(s), Z(s))$ is given as:

$$\phi(s) = \frac{1}{2} \Gamma_W + \frac{1}{2\pi} \int_0^s \gamma(s) \arctan \left(\frac{Z(s) - Z(0)}{X(s) - X(0)} \right) ds. \quad (4.71)$$

The spatial derivatives of ϕ are computed at each control point using a central finite differencing scheme, and the time derivatives are computed from the values at the previous time steps using a backwards finite differencing scheme. Then, the pressure

is calculated at each control point using Equation 4.67.

4.5.10 Determination of the Lift and Pitching Moment

The last step of the vortex panel method algorithm is to compute the 2D lift and pitching moment about the quarter-chord point. From Equation 4.66,

$$\frac{c_p}{c} = \frac{1}{\frac{1}{2}\rho U^2 c} (p - p_\infty). \quad (4.72)$$

Integrating over the airfoil and noting that negative gage pressure (suction) generates positive lift on the top surface of the airfoil, the lift coefficient, c_l , is given by:

$$c_l = \frac{1}{\frac{1}{2}\rho U^2 c} \left[\int_c^0 (p_u - p_\infty) dx - \int_0^c (p_l - p_\infty) dx \right], \quad (4.73)$$

where the subscripts u and l denote the top and bottom of airfoil, respectively. Reversing the sense of the first integral and applying Equation 4.66, one arrives at

$$c_l = -\frac{1}{c} \int_0^c (c_{p_u} + c_{p_l}) dx. \quad (4.74)$$

Similarly, the pitching moment, $c_{m_{c/4}}$ (positive when pitching nose up) can be computed by taking the moments of the pressure forces about the quarter chord point:

$$c_{m_{c/4}} = \frac{1}{\frac{1}{2}\rho U^2 c^2} \left[\int_c^0 (p_u - p_\infty) \left(\frac{c}{4} - x \right) dx - \int_0^c (p_l - p_\infty) \left(\frac{c}{4} - x \right) dx \right]. \quad (4.75)$$

Again reversing the sense of the first integral and applying Equation 4.66:

$$c_{m_{c/4}} = -\frac{1}{c^2} \int_0^c (c_{p_u} + c_{p_l}) \left(\frac{c}{4} - x \right) dx. \quad (4.76)$$

The 2D aerodynamic loading obtained here is per unit of span, and the net loading will be determined upon integration over the blade.

4.6 The Beam Kinematics Solver

In developing a beam kinematics solver for use in the preliminary design phase of wind turbines, it is in the best interest of the developer to sacrifice some accuracy to decrease the computation time. Full numerical solution of the equations of motion for an anisotropic beam generally involves a computationally intensive method that involves a numerical integration of the equations of motion. A quicker approach is to assume some mode shapes for the beam and attempt to minimize the resulting energy functional in some way. To this end, the Ritz method may be used to develop a linear ODE governing the motion of the beam. Assuming a set of mode shapes for the lateral, torsional, and axial deflection of the blade and incorporating these into the energy functional, the Euler - Lagrange equations may be used to assemble the equations of motion. The structural solver then only requires a simple integration scheme such as the second or fourth order Runge - Kutta methods.

4.6.1 Discretization of the Normal Modes

Rather than segmenting the beam into spanwise elements, the mode shapes can be discretized using a series of basis functions. The beam deflections are assumed to be given by

$$u_1(y, t) = \sum_{j=1}^{N_P} p_j(t) P_j(y), \quad (4.77)$$

$$\theta(y, t) = \sum_{j=1}^{N_Q} q_j(t) \Theta_j(y), \quad (4.78)$$

$$u_3(y, t) = \sum_{j=1}^{N_R} r_j(t) R_j(y), \quad (4.79)$$

$$u_2(y, t) = \sum_{j=1}^{N_S} s_j(t) S_j(y), \quad (4.80)$$

where the functions denoted by capital letters denote the spatial basis functions for the deflections as defined in Chapter 2, and shown again in Figure 4.7. The basis functions are required to satisfy the geometric boundary conditions, but not the natural boundary conditions. The external forces and deflections are related through the stiffness matrix $[\bar{S}]$ as derived in Chapter 2 by:

Figure 4.7: Deflections of the beam cross section [7].

$$\begin{bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = [\bar{S}] \begin{bmatrix} u'_1 \\ \theta' \\ -u''_3 \\ u''_2 \end{bmatrix}. \quad (4.81)$$

4.6.2 Euler - Lagrange Formulation

To assemble the energy functional, the kinetic energy is first determined:

$$T = \frac{1}{2} \int_0^L \mu \left(\left[\frac{\partial u_1}{\partial t} \right]^2 + \left[\frac{\partial u_2}{\partial t} \right]^2 + \left[\frac{\partial u_3}{\partial t} \right]^2 \right) dy + \frac{1}{2} \int_0^L I' \left[\frac{\partial \theta}{\partial t} \right]^2 dy, \quad (4.82)$$

where μ is the linear mass density and I' is the linear rotational inertia density.

Substituting the assumed modes from Equations 4.77 to 4.80 yields

$$T = \frac{1}{2} \int_0^L \mu \left(\left[\sum_{j=1}^{N_P} \dot{p}_j P_j \right]^2 + \left[\sum_{j=1}^{N_R} \dot{r}_j R_j \right]^2 + \left[\sum_{j=1}^{N_S} \dot{s}_j S_j \right]^2 \right) dy + \frac{1}{2} \int_0^L I' \left[\sum_{j=1}^{N_Q} \dot{q}_j \Theta_j \right]^2 dy, \quad (4.83)$$

where the dots represent time derivatives and the primes represent spatial derivatives.

Since the spatial basis functions will be assumed beforehand, the time-dependent coefficients will be taken as generalized coordinates. Taking partial derivatives with respect to the generalized momenta, $\dot{p}_i$, $\dot{q}_i$, $\dot{r}_i$, and $\dot{s}_i$, results in

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_i} \right) = \frac{d}{dt} \int_0^L \mu P_i \sum_{j=1}^{N_P} \dot{p}_j P_j dy = \sum_{j=1}^{N_P} \ddot{p}_j \int_0^L \mu P_i P_j dy \quad (4.84)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) = \frac{d}{dt} \int_0^L I' P_i \sum_{j=1}^{N_Q} \dot{q}_j \Theta_j dy = \sum_{j=1}^{N_Q} \ddot{q}_j \int_0^L I' \Theta_i \Theta_j dy \quad (4.85)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{r}_i} \right) = \frac{d}{dt} \int_0^L \mu R_i \sum_{j=1}^{N_R} \dot{r}_j R_j dy = \sum_{j=1}^{N_R} \ddot{r}_j \int_0^L \mu R_i R_j dy \quad (4.86)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{s}_i} \right) = \frac{d}{dt} \int_0^L \mu S_i \sum_{j=1}^{N_S} \dot{s}_j S_j dy = \sum_{j=1}^{N_S} \ddot{s}_j \int_0^L \mu S_i S_j dy \quad (4.87)$$

Next, the elastic potential energy is determined. From Chapter 2, the elastic energy functional is given as

$$V = \frac{1}{2} \int_0^L \{\bar{\epsilon}\}^T [\bar{S}] \{\bar{\epsilon}\} dy \quad (4.88)$$

Again, substituting the assumed modes from Equations 4.77 to 4.80 yields

$$V = \frac{1}{2} \int_0^L \left\{ \begin{array}{c} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ - \sum_{j=1}^{N_R} r_j R''_j \\ \sum_{j=1}^{N_S} s_j S''_j \end{array} \right\}^T [\bar{S}] \left\{ \begin{array}{c} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ - \sum_{j=1}^{N_R} r_j R''_j \\ \sum_{j=1}^{N_S} s_j S''_j \end{array} \right\} dy \quad (4.89)$$

Next, the partial derivatives of the strain energy are taken with respect to the generalized coordinates. This is aided by noting that, if a scalar ν is written as $\nu = \mathbf{x}^T \mathbf{A} \mathbf{x}$,

$$\frac{\partial \nu}{\partial \mathbf{x}} = \mathbf{x}^T (\mathbf{A} + \mathbf{A}^T). \quad (4.90)$$

The result is:

$$\frac{\partial V}{\partial p_i} = \frac{\partial V}{\partial \{\bar{\epsilon}\}} \frac{\partial \{\bar{\epsilon}\}}{\partial p_i} = \frac{1}{2} \int_0^L \left\{ \begin{array}{c} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ - \sum_{j=1}^{N_R} r_j R''_j \\ \sum_{j=1}^{N_S} s_j S''_j \end{array} \right\}^T \left([\bar{S}] + [\bar{S}]^T \right) \left\{ \begin{array}{c} \frac{\partial}{\partial p_i} \sum_{j=1}^{N_P} p_j P'_j \\ \frac{\partial}{\partial p_i} \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ - \frac{\partial}{\partial p_i} \sum_{j=1}^{N_R} r_j R''_j \\ \frac{\partial}{\partial p_i} \sum_{j=1}^{N_S} s_j S''_j \end{array} \right\} dy \quad (4.91)$$

The stiffness matrix is symmetric, and hence $[\bar{S}] + [\bar{S}]^T = 2 [\bar{S}]$:

$$\frac{\partial V}{\partial p_i} = \int_0^L \left\{ \begin{array}{c} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ - \sum_{j=1}^{N_R} r_j R''_j \\ \sum_{j=1}^{N_S} s_j S''_j \end{array} \right\}^T [\bar{S}] \left\{ \begin{array}{c} P'_i \\ 0 \\ 0 \\ 0 \end{array} \right\} dy. \quad (4.92)$$

The result of $[\bar{S}] \left\{ \begin{array}{c} P'_i \\ 0 \\ 0 \\ 0 \end{array} \right\}$ is the multiplication of the first column of $[\bar{S}]$, $[\bar{S}_{:1}]$, by P'_i :

$$\frac{\partial V}{\partial p_i} = \int_0^L \left\{ \begin{array}{c} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ - \sum_{j=1}^{N_R} r_j R''_j \\ \sum_{j=1}^{N_S} s_j S''_j \end{array} \right\}^T [\bar{S}_{:1}] P'_i dy. \quad (4.93)$$

Continuing in a likewise fashion with the remaining generalized coordinates:

$$\begin{aligned} \frac{\partial V}{\partial p_i} = & \sum_{j=1}^{N_P} p_j \int_0^L \bar{S}_{11} P'_i P'_j dy + \sum_{j=1}^{N_Q} q_j \int_0^L \bar{S}_{21} P'_i \Theta'_j dy + \sum_{j=1}^{N_R} r_j \int_0^L -\bar{S}_{31} P'_i R''_j dy \\ & + \sum_{j=1}^{N_S} s_j \int_0^L \bar{S}_{41} P'_i S''_j dy \end{aligned} \quad (4.94)$$

$$\begin{aligned} \frac{\partial V}{\partial q_i} = & \sum_{j=1}^{N_P} p_j \int_0^L \bar{S}_{12} \Theta'_i P'_j dy + \sum_{j=1}^{N_Q} q_j \int_0^L \bar{S}_{22} \Theta'_i \Theta'_j dy + \sum_{j=1}^{N_R} r_j \int_0^L -\bar{S}_{32} \Theta'_i R''_j dy \\ & + \sum_{j=1}^{N_S} s_j \int_0^L \bar{S}_{42} \Theta'_i S''_j dy \end{aligned} \quad (4.95)$$

$$\begin{aligned} \frac{\partial V}{\partial r_i} = & \sum_{j=1}^{N_P} p_j \int_0^L -\bar{S}_{13} R''_i P'_j dy + \sum_{j=1}^{N_Q} q_j \int_0^L -\bar{S}_{23} R''_i \Theta'_j dy + \sum_{j=1}^{N_R} r_j \int_0^L \bar{S}_{33} R''_i R''_j dy \\ & + \sum_{j=1}^{N_S} s_j \int_0^L -\bar{S}_{43} R''_i S''_j dy \end{aligned} \quad (4.96)$$

$$\begin{aligned} \frac{\partial V}{\partial s_i} = & \sum_{j=1}^{N_P} p_j \int_0^L \bar{S}_{14} S''_i P'_j dy + \sum_{j=1}^{N_Q} q_j \int_0^L \bar{S}_{24} S''_i \Theta'_j dy + \sum_{j=1}^{N_R} r_j \int_0^L -\bar{S}_{34} S''_i R''_j dy \\ & + \sum_{j=1}^{N_S} s_j \int_0^L \bar{S}_{44} S''_i S''_j dy. \end{aligned} \quad (4.97)$$

Finally, the generalized forces are considered. The nonconservative forces acting on the beam are split into the external loading and the damping forces, and the work done by these forces is determined (recognizing that the support reactions do no

work):

$$W = \int_0^L \begin{Bmatrix} F_1 \\ M_1 \\ F_3 \\ F_2 \end{Bmatrix} \begin{Bmatrix} u'_1 \\ \theta' \\ u'_3 \\ u'_2 \end{Bmatrix}^T dy - \int_0^L \begin{Bmatrix} b_1 \dot{u}_1 \\ b_\theta \dot{\theta} \\ b_3 \dot{u}_3 \\ b_2 \dot{u}_2 \end{Bmatrix} \begin{Bmatrix} u'_1 \\ \theta' \\ u'_3 \\ u'_2 \end{Bmatrix}^T dy, \quad (4.98)$$

Substituting from Equations 4.77 to 4.80 yields

$$W = \int_0^L \begin{Bmatrix} F_1 \\ M_1 \\ F_3 \\ F_2 \end{Bmatrix} \begin{Bmatrix} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ -\sum_{j=1}^{N_R} r_j R'_j \\ \sum_{j=1}^{N_S} s_j S'_j \end{Bmatrix}^T dy - \int_0^L \begin{Bmatrix} b_1 \sum_{j=1}^{N_P} \dot{p}_j P_j \\ b_\theta \sum_{j=1}^{N_Q} \dot{q}_j \Theta P_j \\ -b_3 \sum_{j=1}^{N_R} \dot{r}_j R_j \\ b_2 \sum_{j=1}^{N_S} \dot{s}_j S_j \end{Bmatrix} \begin{Bmatrix} \sum_{j=1}^{N_P} p_j P'_j \\ \sum_{j=1}^{N_Q} q_j \Theta P'_j \\ -\sum_{j=1}^{N_R} r_j R'_j \\ \sum_{j=1}^{N_S} s_j S'_j \end{Bmatrix}^T dy, \quad (4.99)$$

where the damping coefficients corresponding to the cross-sectional deflections are to be determined experimentally. The generalized forces associated with the i^{th} generalized coordinates are then

$$Q_{p_i} = \frac{\partial W}{\partial p_i} = \int_0^L F_1 P'_i dy - \sum_{j=1}^{N_P} \dot{p}_j \int_0^L b_1 P'_i P_j dy \quad (4.100)$$

$$Q_{q_i} = \frac{\partial W}{\partial q_i} = \int_0^L M_1 \Theta'_i dy - \sum_{j=1}^{N_Q} \dot{q}_j \int_0^L b_\theta \Theta'_i \Theta_j dy \quad (4.101)$$

$$Q_{r_i} = \frac{\partial W}{\partial r_i} = \int_0^L F_3 R'_i dy - \sum_{j=1}^{N_R} \dot{r}_j \int_0^L b_3 R'_i R_j dy \quad (4.102)$$

$$Q_{s_i} = \frac{\partial W}{\partial s_i} = \int_0^L F_2 S'_i dy - \sum_{j=1}^{N_S} \dot{s}_j \int_0^L b_2 S'_i S_j dy \quad (4.103)$$

With all the partial derivatives determined, the Euler - Lagrange equations may be applied:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_i} \right) - \frac{\partial T}{\partial p_i} + \frac{\partial V}{\partial p_i} = Q_{p_i} \quad (4.104)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_{q_i} \quad (4.105)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{r}_i} \right) - \frac{\partial T}{\partial r_i} + \frac{\partial V}{\partial r_i} = Q_{r_i} \quad (4.106)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{s}_i} \right) - \frac{\partial T}{\partial s_i} + \frac{\partial V}{\partial s_i} = Q_{s_i} \quad (4.107)$$

Each term is linear with respect to the generalized coordinates and their time derivatives. Thus, Equations 4.104 through 4.107 constitute a linear system of ODEs of the form

$$[M] \{\ddot{c}\} + [B] \{\dot{c}\} + [K] \{c\} = \{F\}, \quad (4.108)$$

where the vector $\{c\}$ contains all the generalized coordinates and the mass, damping, and stiffness matrices contain the integral coefficients obtained in this section. The desired system of ODEs can now be coupled with a Runge -Kutta integration scheme to complete the beam kinematics algorithm.

4.6.3 Runge - Kutta Integration of the Equations of Motion

Before integrating the equations of motion, the system of N second order equations is reduced to a system of $2N$ first order equations by letting

$$\{\Xi\}_1 = \{c\}, \quad (4.109)$$

$$\{\Xi\}_2 = \{\dot{c}\}. \quad (4.110)$$

Then,

$$\{\dot{\Xi}\}_1 = \{\Xi\}_2, \quad (4.111)$$

$$\{\dot{\Xi}\}_2 = [M]^{-1} (\{F\} - [K] \{\Xi\}_1 - [B] \{\Xi\}_2). \quad (4.112)$$

This is collected in matrix form:

$$\begin{Bmatrix} \{\dot{\Xi}\}_1 \\ \{\dot{\Xi}\}_2 \end{Bmatrix} = \begin{bmatrix} [0]_{N \times N} & [I]_{N \times N} \\ -[M]^{-1} [K] & -[M]^{-1} [B] \end{bmatrix} \begin{Bmatrix} \{\Xi\}_1 \\ \{\Xi\}_2 \end{Bmatrix} + \begin{Bmatrix} \{0\} \\ [M]^{-1} \{F\} \end{Bmatrix}. \quad (4.113)$$

More compactly,

$$\{\dot{\Xi}\} = [A] \{\Xi\} + \{g\}. \quad (4.114)$$

The natural frequencies and modes of the system can now be determined from the eigenvalues and eigenvectors of $[A]$. In preparation for the Runge - Kutta integration,

the system is decoupled by noting that

$$[\tilde{\Xi}]^{-1} [A] [\tilde{\Xi}] = [\Lambda], \quad (4.115)$$

where $[\tilde{\Xi}]$ is a matrix whose columns contain the eigenvectors of $[A]$, and $[\Lambda]$ is a matrix with the eigenvalues of $[A]$ on the diagonal and zeros everywhere else. Making a change of variables such that

$$\{\Xi\} = [\tilde{\Xi}] \{\varphi\}, \quad (4.116)$$

and substituting into Equation 4.114:

$$[\tilde{\Xi}] \{\dot{\varphi}\} = [A] [\tilde{\Xi}] \{\varphi\} + \{g\}. \quad (4.117)$$

Multiplying through by $[\tilde{\Xi}]^{-1}$ and making use of Equation 4.115:

$$\{\dot{\varphi}\} = [\Lambda] \{\varphi\} + \{G\} = f(\{\varphi\}, t), \quad (4.118)$$

with $\{G\} = [\tilde{\Xi}]^{-1} \{g\}$. The equations of motion are hence decoupled and may be integrated using a Runge - Kutta integration scheme. The fourth order scheme is a popular choice in which the derivatives at the current time step,

$$\begin{Bmatrix} \{k_1\} \\ \{k_2\} \\ \{k_3\} \\ \{k_4\} \end{Bmatrix} = \begin{Bmatrix} f(\{\varphi\}, t) \\ f\left(\{\varphi\} + \frac{\{k_1\}\Delta t}{2}, t + \frac{\Delta t}{2}\right) \\ f\left(\{\varphi\} + \frac{\{k_2\}\Delta t}{2}, t + \frac{\Delta t}{2}\right) \\ f(\{\varphi\} + \{k_3\}\Delta t, t + \Delta t) \end{Bmatrix}, \quad (4.119)$$

are computed, and the value of $\{\varphi\}$ at the next time step is approximated by a

weighted average of the derivatives:

$$\{\varphi\}(t + \Delta t) = \{\varphi\}(t) + \frac{\{k_1\} + 2\{k_2\} + 2\{k_3\} + \{k_4\}}{6} \Delta t + O(\Delta t^5). \quad (4.120)$$

The solution is completed with a transformation back to the original variables through Equation 4.116.

4.7 Fluid/Structure Coupling

The unsteady panel method and beam kinematics solver algorithms may now be combined to predict the aeroelastic response of a flexible composite blade. The algorithm flow chart is shown in Figure 4.8. The following sections explain the algorithm in further detail.

4.7.1 User Inputs

The user must define the blade sections, mode shape basis functions, and external flow conditions. These will be used to initialize the algorithm parameters and perform the initial modal analysis.

4.7.1.1 Definition of Blade Sections

The blade is defined by a number of spanwise elements. For each element, the following must be provided:

1. Airfoil geometry. Arrays of x and z coordinates shall be provided that define the section geometry. The airfoil chord shall coincide with the x - axis.

2. Stiffness matrix. A 4×4 stiffness matrix $[\bar{S}]$ is required such that

$$\begin{Bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = [\bar{S}] \begin{Bmatrix} u'_1 \\ \theta' \\ -u''_3 \\ u''_2 \end{Bmatrix}$$

For this work, the stiffness matrices are determined using the Variational Asymptotic Beam Section (VABS) tool developed by Khouli in 2009 [3].

3. Mass and inertia per unit length, μ and I' . These properties are required to generate the coefficients for the mass, stiffness, and damping matrices.
4. Rigid angle of attack, α_0 .
5. Damping coefficient values. The values b_1 , b_θ , b_2 , b_3 used above are somewhat arbitrary and may, at best, be estimated experimentally. However, if Rayleigh damping coefficients α and β are known or can be determined for a given material candidate, then the damping matrix in Equation 4.108 will be given as:

$$[B] = \alpha [M] + \beta [K]. \quad (4.121)$$

6. (Optional) Lift curve slope, c_{l_α} , if a quasi-steady simulation is desired.

The number of spanwise elements is up to the user and will affect the computation time.

4.7.1.2 Assumed Mode Shapes

For each of the four cross-sectional deflections, u_1 , θ , u_2 , and u_3 , a finite number of mode shapes must be defined that satisfy the natural boundary conditions as shown in Figure 4.9. For example, consider a cantilevered blade, fixed at $y = 0$. A very simple set of modes that satisfy the geometric boundary conditions are:

$$P_1(y) = y, \quad (4.122)$$

$$\Theta(y) = y, \quad (4.123)$$

$$R_1(y) = y^2, \quad (4.124)$$

$$R_2(y) = y^3, \quad (4.125)$$

$$S_1(y) = y^2, \quad (4.126)$$

$$S_2(y) = y^3, \quad (4.127)$$

Alternative options may include Fourier series terms such as $\sin\left(\frac{(2n-1)\pi y}{2L}\right)$, which more closely resemble cantilevered beam vibration modes and thus may yield more accurate natural frequencies with fewer assumed modes. In general, increasing the number of basis functions results in higher frequency modes being uncovered, and hence will increase the required number of time steps. For this reason, it is desirable to use as few modes as possible. As a rule of thumb, the author suggests that for

each type of mode (bending, twisting, and extension), the number of assumed modes should exceed the number of desired modes by one.

4.7.1.3 Simulation Parameters and External Flow Parameters

The simulation parameters determine how rapidly and how accurately the code will run. These parameters are summarized in Table 4.2.

Table 4.2: Description of algorithm simulation and flow parameters.

Parameter	Description
Time step resolution	Number of time steps per highest frequency period
Number of spanwise elements	Number of locations where load is calculated
Option: quasi-steady	Yes: panel method replaced by simple lift theory
Option: boundary layer solver	Yes: use a boundary layer solver
Angle of attack	Global blade angle of attack
Flow speed	External free stream flow velocity
Density	Density of incoming air
Viscosity	Flow viscosity

4.7.2 Initial Modal Analysis

After the program is initialized, the initial modal analysis is performed by computing the eigenvalues and eigenvectors of the $[A]$ matrix from Equation 4.114. The highest frequency will be used to determine the time step size, with the corresponding period being divided using the specified time step resolution. The fundamental frequency can be used to determine the frequency of computation of the blade loading, if desired.

To compute the coefficients of the mass, stiffness, and damping matrices, the beam section properties and the assumed mode shapes will be integrated along the span as demonstrated earlier.

4.7.3 Generation of 2D Panels

At a finite number of spanwise locations, the cross-sectional geometry will be discretized into a number of vortex panels as specified by the user. The surface geometry can be interpolated from any number of supplied data points using MATLAB's spline function and the panel nodes can be determined either by dividing the surface into equal or unequal sections. An array will store, for each section, the coordinates of the panel nodes and control points.

4.7.4 2D Panel Method and Boundary Layer Solver

For each spanwise section, the algorithm as described in Section 4.3 will be executed to determine the 2D loading. During the initial time step, the number of vortex panels in the wake is zero and a steady state solution will be obtained. During subsequent time steps, an array containing past wake data for all sections will be used to populate the coefficient matrix. The number of previous wake panels to be kept is determined from Equation 4.61.

4.7.5 BEMT Integration

At each time step, the current positions, velocities, and accelerations of each spanwise section are supplied from the Beam Kinematics Solver, or are known from initial conditions at the first time step. The incoming wind speed and angle of attack for each section are computed appropriately in the case of blade rotation, in Section 2.2.2

With the unsteady 2D aerodynamic loading determined for each spanwise section, the total loading is determined by integration along the span.

4.7.6 Time Step Marching

If the end of the simulation is not yet reached, the Beam Kinematics Solver is used to compute the sectional positions, velocities, and accelerations for the next time step. These kinematic quantities are used by the unsteady panel method to compute the noncirculatory component of the flow field. This calculation sequence is repeated until the desired simulation is achieved.

4.8 Implementation Using MATLAB

The aeroelastic response algorithm was implemented using MATLAB. The Beam Kinematics Solver and the Unsteady Panel Method codes were written solely by the author, and may be run separately or in coupled mode for a full aeroelastic analysis. The Variational Asymptotic Beam Section (VABS) tool was written by Khouli in 2009, also in MATLAB [3].

Figure 4.8: Aeroelastic response algorithm flow chart.

Figure 4.9: Assumed mode shapes for cantilevered blade.

Chapter 5

Validation of the Aeroelastic Algorithm

In this chapter the speed and accuracy of the aeroelastic response algorithm are assessed through comparison of results with two-way coupled fluid-structure interaction simulations performed using the ANSYS Fluent and Transient Structural solvers. A scaled wind tunnel experiment was performed by El Sherif in 2014 and the results are also presented here [17].

5.1 Validation of the 2D Unsteady Panel Method

At the heart of the algorithm lies the 2D Unsteady Panel Method which computes the lift and moment per unit length acting on a spanwise blade section. Hence, the accuracy of this component is crucial to the algorithm's success. The panel method was validated using 3 sources:

1. Theodorsen's solution for an airfoil pitching and plunging in simple harmonic motion.
2. Wagner's solution for a step change in angle of attack.
3. A 2D transient CFD analysis of an airfoil in pitching and plunging motion using

ANSYS Fluent 14.0.

The results of these comparisons are presented and discussed in this section.

5.1.1 Simulation Hardware

The simulations were performed on a lab PC running Microsoft Windows 7. The hardware specifications are given in Table 5.1.

Table 5.1: Description of PC hardware used for simulation.

Processor	Intel i5-750 Quad core, 2.66 GHz
Installed memory	4 GB @ 1066 MHz
Hard drive	500GB SATA 2
Operating system	Windows 7 Professional

5.1.2 Comparison with Theodorsen's Solution

Theodorsen derived the lift and moment on an airfoil in pitching and plunging motion as [4]

$$L = \frac{1}{2}c_{l_\alpha}\rho b^2 \left[\ddot{h} + U\dot{\varphi} - ba\ddot{\varphi} \right] + \frac{1}{2}c_{l_\alpha}\rho b C(k) \left[U\dot{h} + U^2\varphi + Ub \left(\frac{1}{2} - a \right) \dot{\varphi} \right] \quad (5.1)$$

and

$$M = \frac{1}{2}c_{l_\alpha}\rho b^2 \left[ba\ddot{h} - Ub \left(\frac{1}{2} - a \right) \dot{\varphi} - b^2 \left(\frac{1}{8} + a \right) \ddot{\varphi} \right] + \frac{1}{2}c_{l_\alpha}\rho b^2 \left(a + \frac{1}{2} \right) C(k) \left[U\dot{h} + U^2\varphi + Ub \left(\frac{1}{2} - a \right) \dot{\varphi} \right], \quad (5.2)$$

where φ and h are the pitch and plunge coordinates, respectively, as shown in Figure 5.1, and a and b are geometric parameters as shown in Figure 5.2.

Figure 5.1: 2D motion of a blade section.

Figure 5.2: 2D airfoil geometry, modified from Wright [4].

Theodorsen's lift and moment from Equations 5.1 and 5.2 are valid for thin, uncambered airfoils experiencing oscillatory motion according to

$$\varphi = \varphi_0 e^{j\omega_\varphi t} \quad (5.3)$$

and

$$h = h_0 e^{j\omega_h t}. \quad (5.4)$$

Theodorsen's function $C(k)$ is given as

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + jH_0^{(2)}(k)}, \quad (5.5)$$

where $H_1^{(2)}$ and $H_0^{(2)}$ are first and second order Hankel functions of the second kind, respectively. The reduced frequency k is defined as

$$k = \frac{\omega c}{2U}, \quad (5.6)$$

where ω is the circular frequency of the motion and U is the freestream wind velocity. The first components of the lift and moment Equations (5.1 and 5.2) represent the noncirculatory contributions to 2D loading due to the acceleration of the airfoil in the surrounding flow. The second components represent the circulatory contributions due to the distribution of vorticity in the boundary layer and in the wake. At low frequencies of oscillation, the airfoil experiences relatively small accelerations and the circulatory contributions dominate the loading. At higher frequencies ($k > 1.0$), the non-circulatory contributions begin to dominate the loading. The effect of Theodorsen's function is to introduce a magnitude and phase change to the loading, as shown in Figure 5.3.

Figure 5.3: Effect of pitching frequency on unsteady aerodynamic loading [4].

5.1.2.1 Validation Using a Thin, Uncambered Airfoil

The 2D Unsteady Panel Method was run for two airfoils experiencing oscillatory pitching motion. The first was a thin, uncambered airfoil with profile boundaries defined by

$$z = \pm 0.05\sqrt{x} \left(1 - \frac{x}{c}\right). \quad (5.7)$$

The airfoil is shown in Fig. 5.4. Simulation parameters and run times are given in Table 5.2. The simulations were completed over 4 oscillations at a range of reduced frequencies as described in Table 5.2.

Figure 5.4: A thin, uncambered airfoil suitable for validation using Theodorsen's lift model.

The lift coefficients were computed with the 2D Unsteady Panel Method solver and using Theodorsen's classical solution. The values are compared in Figures 5.5 through 5.10 along with the quasi-steady approximations to demonstrate the effect of pitching oscillations on the lift.

Table 5.2: Simulation parameters for 2D Unsteady Panel Method simulations for pitching thin, uncambered airfoil.

Parameter		Value
Default pitch, α_0		5.0°
Number of airfoil panels, N_p		60
Number of wake panels, N_p		34
Number of time steps, N_t		200
Air density, ρ		1.225 kg/m^3
Chord, c		1.0 m
Angular frequency, ω	Reduced frequency, k	Required wind speed, U
12.0 rad/s	0.05	120 m/s
12.0 rad/s	0.10	60 m/s
12.0 rad/s	0.20	30 m/s
12.0 rad/s	0.50	12 m/s
12.0 rad/s	1.00	6 m/s
12.0 rad/s	2.00	3 m/s
Simulation time (Unsteady VPM)		10 s
Simulation time (ANSYS)		12 min

Figure 5.5: 2D Unsteady Panel Method results for pitching oscillations, $k = 0.05$.

Figure 5.6: 2D Unsteady Panel Method results for pitching oscillations, $k = 0.10$.

Figure 5.7: 2D Unsteady Panel Method results for pitching oscillations, $k = 0.20$.

Figure 5.8: 2D Unsteady Panel Method results for pitching oscillations, $k = 0.50$.

Figure 5.9: 2D Unsteady Panel Method results for pitching oscillations, $k = 1.00$.

Figure 5.10: 2D Unsteady Panel Method results for pitching oscillations, $k = 2.00$.

Figures 5.5 through 5.10 demonstrate a strong agreement (within 5%) between the 2D Unsteady Panel Method and the Theodorsen lift. The comparisons with the quasi-steady solution also illustrate the expected true lift behaviour as illustrated by Figure 5.2. Notice the significant increase in lift occurring at the higher reduced frequencies, as predicted by Equation 5.1.

5.1.2.2 Validation Using a Thick, Cambered Airfoil

To show that the 2D Unsteady Panel Method algorithm can produce results with comparable accuracy to those of CFD simulations, ANSYS Fluent was used to determine the lift of a thick airfoil in the time domain. The airfoil selected was the S809 developed by National Renewable Energy Laboratory (NREL) [18], as shown in Fig. 5.11. This airfoil was chosen as it is a candidate for use in wind turbine applications, exhibiting geometric characteristics such as high thickness ratio and unique curvature that distinguishes the lifting behaviour from that of a flat plate.

Figure 5.11: S809 airfoil geometry, NREL [18].

The simulations were again performed for a range of reduced frequencies. The 2D Panel Method simulation parameters are given in Table 5.3.

Table 5.3: Simulation parameters for 2D Unsteady Panel Method simulations for pitching S809 airfoil.

Parameter		Value
Default pitch, α_0		5.0°
Number of airfoil panels, N_p		80
Number of wakel panels, N_p		36
Number of time steps, N_t		200
Air density, ρ		1.225 kg/m^3
Chord, c		1.0 m
Angular frequency, ω	Reduced frequency, k	Required wind speed, U
12.0 rad/s	0.20	30 m/s
12.0 rad/s	0.50	12 m/s
12.0 rad/s	1.00	6 m/s
Simulation time (Unsteady VPM)		10 s
Simulation time (ANSYS)		12 min

For the CFD simulation, a mesh was first developed in ANSYS ICEM CFD, illustrated in Fig. 5.12. The mesh area around the airfoil extends 20 m downstream and upstream of the airfoil and 10 m in width to ensure that the velocity profile is uniform upstream of the rotor. Triangular elements were used to allow for mesh deformation due to airfoil motion during the CFD trials. This allowed for various motions such as pitching, plunging, and heaving. The motion of the airfoil was implemented through a User-Defined Function (UDF) written in C and compiled in ANSYS Fluent. Simulation parameters for the CFD analyses are given in Table 5.4.

Figure 5.12: ANSYS Fluent deformable mesh surrounding the S809 airfoil.

Table 5.4: Simulation parameters for CFD pitching oscillation simulations.

Parameter	Value
Chord, c	1.0 m
Air density, ρ	1.2 kg/m ³
CFD Simulation Parameters	
Number of elements	20,000
Run completion, max	480 s
Number of time steps	200

The results of the CFD simulations are shown in Figures 5.13 through 5.15. Good agreement is still achieved between the 2D Panel Method results and the Theodorsen model, but discrepancies are now observable due to thickness and camber effects associated with the S809 airfoil. Good agreement is achieved between the Panel Method and the ANSYS simulations, with maximum discrepancies not exceeding 5% in any trial, except near the beginning of the simulation due to differing initial conditions in the solvers.

Figure 5.13: 2D Unsteady Panel Method and ANSYS results for pitching oscillations, $k = 0.20$.

Figure 5.14: 2D Unsteady Panel Method and ANSYS results for pitching oscillations, $k = 0.50$.

Figure 5.15: 2D Unsteady Panel Method and ANSYS results for pitching oscillations, $k = 1.00$.

5.1.2.3 Validation for Simultaneous Pitching and Plunging

Next, a set of simulations were conducted with the S809 airfoil pitching and plunging. Again, ANSYS Fluent was used to produce the CFD results. The simulation

parameters are shown in Table 5.5. The results are given in Figures 5.16 through 5.18. Again, strong agreement was found with the ANSYS results. The solutions differ slightly near $t = 0$ due to the quasi-steady initialization of the panel method algorithm.

Table 5.5: Simulation parameters for 2D Unsteady Panel Method simulations for the S809 airfoil in pitching and plunging motion.

Parameter		Value
Default pitch, α_0		5.0°
Default plunge distance, h_0		0.1 m
Number of airfoil panels, N_p		60
Number of wake panels, N_r		34
Number of time steps, N_t		200
Air density, ρ		1.225 kg/m ³
Chord, c		1.0 m
Pitching ω (k)	Plunging ω (k)	Wind speed, U
12.0 rad/s (0.40)	12.0 rad/s (0.40)	15 m/s
12.0 rad/s (0.40)	30.0 rad/s (1.00)	15 m/s
30.0 rad/s (1.00)	12.0 rad/s (0.40)	15 m/s
Simulation time (Unsteady VPM)		10 s
Simulation time (ANSYS)		12 min

Figure 5.16: S809 airfoil in pitch ($k = 0.40$) and plunge ($k = 0.40$).

Figure 5.17: S809 airfoil in pitch ($k = 0.40$) and plunge ($k = 1.00$).

Figure 5.18: S809 airfoil in pitch ($k = 1.00$) and plunge ($k = 0.40$).

5.1.3 Comparison with Wagner's Solution

Wagner considered the time-dependent lift on a thin, symmetrical airfoil experiencing a step increase in angle of attack [4]. Consider Figure 5.19. Before $t = 0$ the airfoil rests at a constant angle of attack α_0 and produces constant lift L . At $t = 0$, the airfoil instantaneously experiences a jump in angle of attack of magnitude $\Delta\alpha = \frac{\alpha_0}{2}$. The quasi-steady approach suggests that the lift will instantaneously increase by 50%. The full unsteady solution shows that the lift makes a jump of only half the difference, and then asymptotically approaches steady-state.

Figure 5.19: Effect of sudden change in angle of attack on the lift [4].

The increase in lift is modelled by appending the quasi-steady solution with Wagner's function:

$$\Delta L = \frac{1}{2} \rho U^2 c c_{l_\alpha} \Delta \alpha \Phi(\tau) \quad (5.8)$$

where τ is the non-dimensional time, defined as

$$\tau = \frac{2U}{c} t \quad (5.9)$$

and Wagner's function is approximated as

$$\Phi(\tau) = \frac{\tau + 2}{\tau + 4}, \quad \tau > 0 \quad (5.10)$$

As Wagner's solution gives the response of the lift to a step change in pitch, the response to a continuous pitching motion could be calculated using a convolution integral.

For comparison with Wagner's solution, two simulation trials were carried out, as described in Tables 5.6 and 5.7.

Table 5.6: Simulation of Wagner problem, Trial # 1.

Parameter	Value
Default pitch, φ_0	0.0 °
Final pitch, $\varphi_0 + \Delta\varphi$	8.0 °
Simulation time	3.0 s
Transition time, t_j	1.5 s
Number of airfoil panels, N_p	60
Number of wake panels, N_r	28
Number of time steps, N_t	200
Air density, ρ	1.225 kg/m ³
Chord, c	1.0 m
Simulation time (Unsteady VPM)	10 s
Simulation time (ANSYS)	10 min

The simulations were performed using the 2D Unsteady Panel Method as well as with ANSYS Fluent. To avoid singularities and large values of the time derivatives, the step change in angle of attack was approximated using the arctangent function:

$$\varphi(t) = \varphi_0 + \Delta\varphi \frac{1}{\pi} \left[\arctan(75(t - t_j)) + \frac{\pi}{2} \right] \quad (5.11)$$

The results of the simulations are shown in Figures 5.20 and 5.21.

Table 5.7: Simulation of Wagner problem, Trial # 2.

Parameter	Value
Default pitch, φ_0	0.0°
Final pitch, $\varphi_0 + \Delta\varphi$	5.0°
Simulation time	1.5 s
Transition time, t_j	0.75 s
Number of airfoil panels, N_p	60
Number of wake panels, N_r	28
Number of time steps, N_t	200
Air density, ρ	1.225 kg/m^3
Chord, c	1.0 m
Simulation time (Unsteady VPM)	10 s
Simulation time (ANSYS)	10 min

Figure 5.20: Simulation results for Wagner problem, Trial # 1.

Figure 5.21: Simulation results for Wagner problem, Trial #21.

The results again show good agreement between the 2D Panel Method, ANSYS Fluent, and the classical solution, with maximum discrepancies under 10%. The 2D Panel Method is seen to slightly over-predict the lift compared to the Wagner solution due to the influence of thickness and camber on the lift. ANSYS Fluent is again seen to slightly under-predict the lift due to allowance of more complex features of the flow and mesh sensitivity. It is noted that the numerical solutions experience localized oscillations in the vicinity of the pitch jump. This occurs because of the very high derivative values at this point, although the stable solution quickly corrects itself and converges on the correct path. The lift is also noted in these cases to begin rising preemptively, due strictly to the arctangent approximation of the jump.

5.1.4 Simulation of a 2D Airfoil in General Motion

The S809 airfoil was subjected to 3 trials consisting of general pitching and plunging motions defined respectively according to:

$$\varphi(t) = 0.20e^{-10(t-1.0)^2} \sin(\pi t) \quad (5.12)$$

$$h(t) = -0.20e^{-20(t-1.25)^2} \quad (5.13)$$

The motions are plotted in Fig. 5.22. The first and second trials consisted of pure pitching and pure plunging, respectively, according to Equations 5.12 and 5.13. The third trial was a combination of the two motions.

Figure 5.22: Imposed pitching and plunging motions for the S809 airfoil.

The simulation parameters are given in Table 5.8 and the results are shown in Figures 5.23 through 5.25.

Table 5.8: Simulation parameters for the general motion trials.

Parameter	Value
Simulation time	2.0 s
Number of airfoil panels, N_p	60
Number of wake panels, N_r	28
Number of time steps, N_t	200
Air density, ρ	1.225 kg/m ³
Air speed, U	20 m/s
Chord, c	1.0 m
Simulation time (Unsteady VPM)	10 s
Simulation time (ANSYS)	12 min

Figure 5.23: General motion of the S809 airfoil, pitching only.

Figure 5.24: General motion of the S809 airfoil, plunging only.

Figure 5.25: General motion of the S809 airfoil, pitching and plunging.

Very good agreement is achieved between the two codes. The discrepancies observed near $t = 0$ are again explained by considering the differing initial conditions: the 2D Panel Method code assumes steady state conditions at the initial time, whereas ANSYS Fluent begins with no boundary layer developed over the airfoil.

5.2 Validation of the Beam Kinematics Solver

The validation process was divided into examining the accuracy of the modal analysis part of the algorithm and examining the accuracy of the forced response solver. The output of the kinematics tool was compared with classical beam vibration solutions, as well as with modal and transient structural analyses performed in ANSYS Workbench 14.

5.2.1 Accuracy of the Modal Analysis Feature

The quality of the modal analysis tool was assessed by comparison of results with classical solutions and with finite element structural analyses performed with ANSYS Workbench.

5.2.1.1 Modal Analysis of an Aluminum Tube

The modal analysis of an isotropic, uniform beam of annular cross section was used as a starting point based on the simplicity of the analytical solution. The stiffness matrix of such a bar is given by

$$[\bar{S}] = \begin{bmatrix} EA & 0 & 0 & 0 \\ 0 & GJ & 0 & 0 \\ 0 & 0 & EI_{zz} & 0 \\ 0 & 0 & 0 & EI_{xx} \end{bmatrix} \quad (5.14)$$

Hence, the extensional, torsional, and bending deflections are all uncoupled and the undamped natural frequencies ω_n for the cantilever configuration are well known and given by:

$$\omega_{n,e} = \frac{2n-1}{2L} \pi \sqrt{\frac{E}{\rho}}, \quad (5.15)$$

$$\omega_{n,t} = \frac{2n-1}{2L} \pi \sqrt{\frac{G}{\rho}}, \quad (5.16)$$

$$\omega_{n,bxx} = \alpha_n^2 \sqrt{\frac{EI_{xx}}{\mu L^4}}, \quad (5.17)$$

$$\omega_{n,bzz} = \alpha_n^2 \sqrt{\frac{EI_{zz}}{\mu L^4}}, \quad (5.18)$$

with

$$\alpha_n = \{1.875, 4.694, 7.885\}. \quad (5.19)$$

where the subscripts e , t , bxx , and bzz represent extensional, torsional, and bending modes about the x and z axes respectively. The geometric parameters for the test case are given in Table 5.9.

Table 5.9: Geometric parameters for clamped aluminum tube.

Parameter	Value
Diameter, d	50 mm
Thickness, t	2 mm
Length, L	500 mm
Density, ρ	2500 kg/m ³
Young's modulus, E	70×10^9 N/m ²
Shear modulus, G	26×10^9 N/m ²

For use in the Beam Kinematics solver, the basis functions were chosen as those specified in Equations 4.122 through 4.127. For the ANSYS Workbench simulation, the first 6 natural frequencies were requested. The results are shown in Table 5.10.

Table 5.10: Natural frequencies for clamped aluminum bar.

Mode		$f_n = \frac{\omega_n}{2\pi}$, Hz		
Description	Classical	ANSYS	Beam Kinematics	
1st Bending, xx	210	202	210	
1st Bending, zz	210	202	210	
2nd Bending, xx	1313	1258	1326	
2nd Bending, zz	1313	1258	1326	
1st Torsional	1612	1596	1780	
1st Extensional	2646	2757	2917	

A strong agreement is achieved for the pair of fundamental frequencies, which are expected to dominate the response of the turbine blades since the aerodynamic excitation is coupled with the elasticity of the structure. Good agreement is also achieved with the second bending modes, which would be sufficient for an aeroelastic analysis as it is clear in this case that the motion would be dominated by bending. Acceptable accuracy is achieved for the torsional and extensional modes. This could be improved by implementing higher order basis functions, but at the cost of introduction of higher frequencies into the model which would involve more time steps.

5.2.1.2 Modal Analysis of the S809 wind tunnel blade model

The modal analysis was repeated using the wind tunnel blade model. The specimen consists of a straight, uniform hollow blade with the S809 airfoil as the cross section.

The geometric parameters are given in Table 5.11.

Table 5.11: Properties of the S809 blade model.

Parameter	Value
Chord, c	127 mm
Length, L	539 mm
Density, ρ	930 kg/m ³
Young's modulus, E	1700×10^6 N/m ²
Poisson's ratio, ν	0.40

For use in the Beam Kinematics solver, the basis functions were chosen as those specified in Equations 4.122 to 4.127. For the ANSYS Workbench simulation, the first four natural frequencies were requested. The results are shown in Table 5.12. Due to the asymmetrical nature of the cross section, no classical solutions are available.

Table 5.12: Natural frequencies of the S809 specimen.

Mode		$f_n = \frac{\omega_n}{2\pi}, Hz$
Number	ANSYS	Beam Kinematics
1	16.4	23.1
2	30.8	50.6
3	91.2	110.6
4	138.0	228.0

In this case, a sharper discrepancy was found between simulation trials. However, what is most important is that the fundamental frequency is correctly matched, and

the agreement in this regard is only moderate. The main reason for this discrepancy comes from the differing ways the fixed support is modelled in each case. In ANSYS, the entire surface at $y = 0$ is constrained from motion, and thus restricts the blade motion in the vicinity of the support. In the Beam Kinematics solution, the supported end is fixed at a single point due to the 1D nature of the VABS analysis. The true beam behaviour is likely somewhere in between these two extremes.

5.2.2 Accuracy of the Forced Response Feature

With the modal analyses performed, the forced response of the two test structures was examined using a sinusoidally-varying load in ANSYS Workbench and with the Beam Kinematics Solver.

5.2.2.1 Forced Response of an Aluminum Tube

The simplest case to analyze is the response of the cylindrical tube to a constant transverse point load at the tip of 1 kN. According to simple Euler beam theory, the steady state tip deflection is given by

$$\hat{\delta} = \frac{PL^3}{3EI} = 6.1\text{mm}.$$

First the cylinder response was analyzed for a sinusoidally varying tip load of 1 kN. For the Beam Kinematics solver, a tight Gaussian distribution was used to model the tip load:

$$F_3(y) = 1000 \cdot \frac{2}{\sigma\sqrt{2\pi}} e^{-\frac{(y-L)^2}{2\sigma^2}} \cos(2\pi ft), \quad (5.20)$$

where $\sigma = L/200$ was chosen to concentrate the load at the tip. The integral of Equation 5.20 over the span yields the net load of 1 kN. The undamped responses

are shown in Figures 5.26 through 5.28, for these different forcing frequencies.

Figure 5.26: Forced cylinder response at 100 Hz, undamped.

Figure 5.27: Forced cylinder response at 250 Hz, undamped.

Figure 5.28: Forced cylinder response at 500 Hz, undamped.

Based on the strong agreement between fundamental frequencies between the Beam Kinematics Solver and ANSYS, good agreement is seen between the forced beam responses.

5.2.2.2 Forced Response of the S809 Wind Tunnel Blade Model

The forced response of the S809 blade specimen was determined according to the varying pressure loading:

$$p(y, t) = 25 \cos(2\pi ft). \quad (5.21)$$

To compare solvers, the simulation was performed with forcing frequencies of $f = 0.6f_{n,1}$ and $f = 1.2f_{n,1}$, where $f_{n,1}$ represents the fundamental natural frequency. The resulting tip deflections are plotted in Figures 5.29 and 5.30.

Figure 5.29: Forced S809 blade response at $f = 0.6f_{n,1}$, undamped.

Figure 5.30: Forced S809 blade response at $f = 1.2f_{n,1}$, undamped.

In spite of the noticeable mismatch of frequencies as noted during the modal analysis simulations, the motion is dominated by the fundamental frequency and the forced responses match fairly well. The tip deflection is under-predicted by the Beam Kinematics solver because it solves for a stiffer beam.

5.3 Validation of the 3D Unsteady Blade Element Momentum Algorithm with a Two-way Fluid/Structure Interaction CFD Simulation

At this point it looks promising that the Unsteady Aeroelastic Response algorithm can serve as a rapid tool for analyzing the dynamics of wind turbine blades and other structures. The next step is to demonstrate that useful 3D results can be generated in a fraction of the time taken to perform a similar analysis in ANSYS.

5.3.1 Two-way Coupled Fluid/Structure Interaction (FSI) with ANSYS Workbench

ANSYS Workbench is capable of coupling the Fluent CFD solver with its Transient Structural solver to predict the aeroelastic response of structures immersed in a flow, as shown in Figure 5.31. The two solvers are linked to share the same time stepping scheme, and information is passed between the two at each time step. The unsteady pressure field from the CFD solver is passed to the corresponding immersed surfaces and the finite element structural solver then computes the displacement field, passes it back to the CFD solver, deforms the mesh accordingly, and the process is repeated.

Figure 5.31: System coupling of transient structural and fluid dynamics solvers in ANSYS Workbench.

The two-way FSI technique can be difficult to set up without sufficient training and experience with the solver. Fluid meshes need to be quite well defined to ensure success when deforming at each time step. The convergence of the CFD solver at each time step may also be difficult to achieve depending on the flow situation. The fluid volume requires a large buffer zone between the walls and the immersed body to prevent wall effects from affecting the solution. Hence, the process generally requires large numbers of elements and will take a long time to finish for a large number of time steps.

The 2D Unsteady Panel method on the other hand requires only about 50 panels and is robust in the sense that convergence is guaranteed for each time step. The Beam Kinematics Solver also avoids convergence issues and its speed depends only on the desired number of time steps and choice of basis functions.

5.3.2 Comparison of Results for Two-way FSI and Unsteady Aeroelastic Response Algorithm

To validate the accuracy of the Aeroelastic Response algorithm, three 3D coupled FSI simulations were performed in ANSYS Workbench. The fluid mesh around the airfoil was designed such that an element-dense box enclosed the expected zone of deformation of the blade, and a dense strip of elements extended downstream of the trailing edge in anticipation of the wake. The rest of the enclosure was more sparsely populated with elements to reduce the simulation time. Sample meshes are shown in Figures 5.32 and 5.33

Figure 5.32: ANSYS Fluent mesh around blade at 0 degree angle of attack.

Figure 5.33: ANSYS Fluent mesh around blade at 5 degree angle of attack.

Tables 5.13, 5.14, and 5.15 show the simulation parameters for the FSI simulations. The results are given in Figures 5.34, 5.35, and 5.36.

Figure 5.34: 3D coupled FSI results for $U = 15$ m/s at a 0 degree angle of attack, Trial # 1.

Table 5.13: Two-way FSI simulation Trial # 1.

Parameter	Value
Chord, c	127 mm
Length, L	539 mm
Section	S809
Material	PA2200
Wind speed, U	15.0 m/s
Angle of attack, α_0	0.0°
Heat transfer model	None
Turbulence model	None
Time step size	0.001 s
Requested number of time steps	200
Time steps before failure	52
Number of elements	279,000
Algorithm simulation time	33 s
ANSYS simulation time	5 hr, 5 min

Figure 5.35: 3D coupled FSI results for $U = 17.3$ m/s at a 5 degree angle of attack, Trial # 2.

Table 5.14: Two-way FSI simulation Trial # 2.

Parameter	Value
Chord, c	127 mm
Length, L	539 mm
Section	S809
Material	PA2200
Wind speed, U	17.3 m/s
Angle of attack, α_0	5.0°
Heat transfer model	None
Turbulence model	None
Time step size	0.001 s
Requested number of time steps	100
Time steps before failure	53
Number of elements	237,000
Algorithm simulation time	34 s
ANSYS simulation time	4 hr, 14 min

Figure 5.36: 3D coupled FSI results for $U = 19.7$ m/s at a 5 degree angle of attack, Trial # 3.

Table 5.15: Two-way FSI simulation Trial # 3.

Parameter	Value
Chord, c	127 mm
Length, L	539 mm
Section	S809
Material	PA2200
Wind speed, U	19.7 m/s
Angle of attack, α_0	5.0 °
Heat transfer model	None
Turbulence model	None
Time step size	0.001 s
Requested number of time steps	100
Time steps before failure	52
Number of elements	237,000
Algorithm simulation time	33 s
ANSYS simulation time	4 hr, 7 min

Unfortunately, despite best efforts, the ANSYS simulations all failed after about a half cycle of oscillation due to errors in deforming the mesh further. Even so, it is compelling from the results that the Unsteady Aeroelastic Response algorithm is capable of generating similar results to commercial a CFD code. Better yet is the fact that the Aeroelastic algorithm is capable of generating similar results in less than 1% of the time.

5.4 Validation of 3D Unsteady Blade Element Momentum Algorithm with a Scaled Wind Tunnel Experiment

The work of El Sherif (2014) involved designing and running a scaled wind tunnel experiment that serves as a benchmark for aeroelastic response algorithm testing [17]. As part of the work, 6 experimental trials were conducted to measure the lift reactions on a flexible S809 blade model, involving two different wind speeds and three different angles of attack. An attempt to duplicate the experimental results was conducted and the results are shown in this section.

5.4.1 The Test Model

Testing was performed using a reduced-scale blade which was cantilevered in a low speed wind tunnel. The blade was designed with a uniform cross section based on the S809 airfoil, and is shown schematically in Figure 5.37. The blade was fabricated by 3D printing from ShapeWays using the PA2200 flexible laser sintered nylon plastic. The blade was manufactured in two pieces, with a removable strip on the upper surface to allow for the installation of 5 strain gauges internally. A 3D CAD model of the blade is shown in Figure 5.37.

Figure 5.37: 3D CAD model of the wind tunnel blade specimen [17].

The straight portion of the specimen matches the model used to perform the modal analysis in the previous section. Relevant dimensions are found in Tables 5.13 through 5.15. Below the hollow bending portion is a tapered section ending in a flange designed to mate with the load cell connector. This portion of the blade is solid to discourage bending deformations and extend the cantilevered support into the wind tunnel test section.

Figures 5.38 and 5.39 show the load cell coupling at the base of the taper, and a top-down view of the blade is shown in Figure 5.40.

Figure 5.38: Bottom view of the load cell coupling.

Figure 5.39: Top view of the load cell coupling.

Figure 5.40: Top view of the S809 blade profile.

5.4.2 Wind Tunnel Experimental Setup and Simulations

The wind tunnel experiment was performed on November 12th, 2014 in the low-speed closed-loop wind tunnel at Carleton University, ME3224. The wind tunnel allows for removable test cells that can be swapped out of the test section. The blade and load cell were protruded through the side of the test section and mounted to an external rotating plexi-glass plate, which was clamped to the outside of the test section using a fabricated wooden support structure. The setup is shown in Figures 5.41 and 5.42.

Figure 5.41: The blade inserted in the test section with rotating blade visible on right.

Figure 5.42: Implementation of the test section clamping system.

The experiment was performed in 6 trials, with 2 different wind speeds and three different angles of attack. The trials are detailed in Table 5.16.

Table 5.16: Description of wind tunnel test trials.

Trial	Angle of attack, degrees	Wind speed, m/s
#1	+10	17.3
#2	+10	19.7
#3	0	17.3
#4	0	19.7
#5	-10	17.3
#6	-10	19.7

The load cell data was recorded using LabView at a sampling rate of 1 kHz to capture motion from both of the anticipated bending modes. The resulting data files were exported to MATLAB to perform Fourier transforms for comparison with the output from the Aeroelastic Blade Response Algorithm. The results are given in the next section.

5.4.3 Experimental Results

For each trial, the total lift was measured as a function of time. The responses were imported into MATLAB and the Fast Fourier Transform (FFT) was used to generate the spectra shown in Figures 5.43 to 5.48.

Figure 5.43: Comparison of lift results for 10 degree angle of attack, 17.3 m/s wind speed.

Figure 5.44: Comparison of lift results for 10 degree angle of attack, 19.7 m/s wind speed.

Figure 5.45: Comparison of lift results for 0 degree angle of attack, 17.3 m/s wind speed.

Figure 5.46: Comparison of lift results for 0 degree angle of attack, 19.7 m/s wind speed.

Figure 5.47: Comparison of lift results for -10 degree angle of attack, 17.3 m/s wind speed.

Figure 5.48: Comparison of lift results for -10 degree angle of attack, 19.7 m/s wind speed.

The peaks generated by the Aeroelastic Response Algorithm do appear to have corresponding peaks from the wind tunnel data and hence the algorithm is capable of predicting some features of the true response. In each case, the predicted peaks are at higher frequencies than their counterparts from the wind tunnel experiment. It should be noted that from the modal analyses, ANSYS predicted lower natural frequencies than the Beam Kinematics Solver, which would partially explain why the experimental peaks are at lower frequencies. This can also be partially explained by the fact that damping was not taken into consideration during use of the Beam Kinematics Solver, whereas the experimental peak frequencies should be lowered by the presence of damping. The wind tunnel data also contains many other peaks that were not accounted for by the aeroelastic algorithm. At this time, it cannot be said with certainty what the source of these peaks is, but it seems reasonable that some of them are due to deflections in the blade supporting structure.

5.5 Summary of Aeroelastic Response Algorithm Performance

In each simulation trial, the Aeroelastic Response Algorithm was shown to provide results in agreement with those from coupled two-way FSI simulations running in ANSYS Workbench 14. The simulation times were found to be significantly lower when using the Aeroelastic Response Algorithm, as summarized in Table 5.17. In the table, t_{ARA} denotes the simulation time of the Aeroelastic Response Algorithm, and t_{FSI} denotes the simulation time of the ANSYS FSI simulation.

Table 5.17: Summary of simulation times for computational trials.

Simulation	Type	t_{ARA}	t_{FSI}	t_{ARA}/t_{FSI}
Pitching flat plate, $k = 0.05$	2D	10 s	12 min	1.4%
Pitching flat plate, $k = 0.10$	2D	10 s	12 min	1.4%
Pitching flat plate, $k = 0.20$	2D	10 s	12 min	1.4%
Pitching flat plate, $k = 0.50$	2D	10 s	12 min	1.4%
Pitching flat plate, $k = 1.00$	2D	10 s	12 min	1.4%
Pitching flat plate, $k = 2.00$	2D	10 s	12 min	1.4%
Pitching S809, $k = 0.20$	2D	10 s	12 min	1.4%
Pitching S809, $k = 0.50$	2D	10 s	12 min	1.4%
Pitching S809, $k = 1.00$	2D	10 s	12 min	1.4%
S809 Pitch and plunge, Trial # 1	2D	10 s	12 min	1.4%
S809 Pitch and plunge, Trial # 1	2D	10 s	12 min	1.4%
S809 Pitch and plunge, Trial # 1	2D	10 s	12 min	1.4%
S809 Wagner step, Trial # 1	2D	10 s	10 min	1.7%
S809 Wagner step, Trial # 2	2D	10 s	10 min	1.7%
S809 General motion, Trial # 1	2D	10 s	10 min	1.7%
S809 General motion, Trial # 2	2D	10 s	10 min	1.7%
S809 General motion, Trial # 3	2D	10 s	10 min	1.7%
S809 Two-way FSI, Trial # 1	3D	33 s	305 min	0.18%
S809 Two-way FSI, Trial # 2	3D	34 s	254 min	0.22%
S809 Two-way FSI, Trial # 3	3D	33 s	247 min	0.22%

Chapter 6

Conclusions and Future Work

Based on the results from the previous chapter, the Unsteady Aeroelastic Response algorithm has been shown to make predictions that are in agreement with the results from coupled CFD and FEA analyses performed in ANSYS Workbench. Furthermore, the computation time has been shown to be very rapid, and well under 2% of the time taken by ANSYS. The algorithm generally produced results that were in good agreement with those obtained from ANSYS. The Aeroelastic Response Algorithm was also successful in predicting some features of the lift response obtained during a wind tunnel experiment performed by El Sherif, 2014 [17].

6.1 Significance of the Work

The value of this algorithm lies in its ability to perform 3D analyses in less than 2% of the time taken by a respected commercial finite element package such as ANSYS. Considering that using ANSYS it took in excess of four hours to generate a half-cycle worth of data for the blade, it could take days to produce enough data to generate a response spectrum for a single simulation, which is unsuitable for performing large numbers of simulations for the preliminary design phase. With the Aeroelastic

Response Algorithm, response data can be generated in a few minutes, allowing for dozens of simulations to be run in a single day. Furthermore, the time taken to perform large numbers of trials is reduced even further given the simplicity of setting up the simulations. The completion times given in Table 5.17 do not take into account the effort taken in generating the mesh in ANSYS and the number of failed attempts in trying to get a simulation to complete successfully.

6.2 Future Work

Given the results, this work serves as a good starting point in terms of being able to quickly generate large amounts of data that agree with results from a respected finite element solver. The features of this algorithm are somewhat limited, however, and future revisions could be made to implement some of the following extensions:

- Refine accuracy by implementing a deformable wake. For the Unsteady Vortex Panel Method, it was assumed that the wake panels propagate along a straight line emanating from the trailing edge. In real flows, the trailing vortices induce velocities on each other and hence the wake deviates from a straight line beyond the trailing edge [10]. The effect on the airfoil loading is more pronounced at higher reduced frequencies and could be implemented with relative ease.
- Implement a boundary layer solver. The groundwork has been laid out to set up a boundary layer solver. This would allow for the prediction of drag, which is important for lightweight structures with low bending stiffnesses about a plane perpendicular to the chord line.
- Add the ability to predict additional features of the flow. The ability to predict the flow at separation and during stall would allow for the analysis of blades at

higher angles of attack.

- Implement turbulence models. Turbulence plays a large role in the aerodynamics of real turbine blades and the ability to predict these effects would greatly improve the utility of this algorithm.

Of course, the addition of more complex features would have an adverse effect on the computational time. Still, it would be advantageous for certain design concepts to give the user control over these aspects of the simulation.

6.3 Final Words

The development of this algorithm has been a very humbling experience given the incredibly complex nature of fluid flows and their interaction with flexible structures. The work undertaken here only scratches the surface of what can be accomplished in this field, and the capabilities of more complex software such as ANSYS are to be admired. Even though the prediction of unsteady flow is very difficult, the reward is great as there are so many applications that benefit mankind. It has been a pleasure to take part in this work.

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