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**PREDICTION OF OFF DESIGN PERFORMANCE OF TURBOJETS
AND TURBOFANS USING GAS DYNAMICS**

by

Mirza-Baig Faisal Sayeed

B.Sc.Eng., Aligarh M. University, INDIA, 1988

**A thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the degree of**

Master of Aerospace Engineering

**Ottawa Carleton Institute of
Mechanical and Aeronautical Engineering
Carleton University
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ISBN 0-315-60459-X

Acceptance Sheet

The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis; *Prediction of off design performance of turbojets and turbofans using gasdynamics*, submitted by Faisal Sayeed Mirza-Baig, in partial fulfillment of the requirements for the degree of Master of Aerospace Engineering.

Thesis Supervisor

Chairman

Department of Mechanical
& Aerospace Engineering

Abstract

The purpose of this thesis is to develop mathematical models which can be used to analyse the design and off-design performance of non-afterburning turbojets and twin or three spool non-afterburning turbofans with separate exhausts. The technique used is essentially based on mass flow and work compatibility between different components, reinforced with generalized turbine characteristic curves. Off design performance has been estimated by using gasdynamic properties of the exhaust nozzle, which regulates the aerothermodynamic behaviour of the upstream components. Approximate overall performance of a turbojet or turbofan at typical cruising conditions can be estimated, making the use of compressor and turbine maps redundant.

The computer program is validated by comparing it with the data available from three turbojets, with the best data available from a Rolls Royce Viper 20F-10. In the case of turbofans, the predicted results are compared with a twin spool Garrett TFE 731-2 and a three spool Rolls Royce RB211-22. To test the veracity of the methods developed, the program was again validated against a preliminary version of the PW300. Finally conclusions are drawn regarding the modelling procedure and what improvements could be made.

Dedicated

To all those who taught me

Acknowledgements

I wish to express my profound appreciation to my thesis supervisor Prof. H.I.H. Saravanamuttoo, for his scrupulous checking, invaluable suggestions and guidance throughout this research. I wish to thank Dr. Steen A. Sjolander for the suggestions offered during the course of my thesis work. Finally I would like to thank the Department of Mechanical and Aerospace Engineering for their financial support.

I would like to thank my father who helped me financially during my first year of study at Carleton and without whom I would never have been here on my own. Then I would like to thank all of my friends (Khoral, Ramesh, Raju etc etc) at Carleton University, who made the stay pleasant during all those long humdrum 'computer hacking' sessions.

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List of Symbols

A	- Exit area of exhaust nozzle
a	- Velocity of sound
C_p	- Specific heat at constant pressure
C_v	- Specific heat at constant volume
H.V.	- Heating value of the fuel
ECV	- Effective Calorific value of fuel
h	- Flight altitude
H	- Enthalpy
Ma	- Mach number
f	- fuel flow
F	- Specific Thrust
β	- bleed flow/core flow
m	- Mass flow
N	- Engine rotational speed
p	- Static pressure
P_t	- Total pressure
R	- Gas constant
T	- Static temperature
T_t	- Total temperature
T_g	- Gross thrust
T_n	- Net thrust
V	- Velocity
γ	- Specific heat ratio ($\gamma = C_p/C_v$)
θ	- Ratio of Ambient temperature to T_{OO} at Sea level
δ	- Ratio of Ambient pressure to P_{OO} at Sea level
ϵ_c	- Compressor pressure ratio
ϵ_f	- Fan pressure ratio

η_b	- Combustion efficiency
η_c	- Polytropic compressor efficiency
η_f	- Polytropic fan efficiency
η_n	- Isentropic nozzle efficiency
η_i	- Isentropic intake efficiency
η_t	- Polytropic turbine efficiency
η_m	- Mechanical efficiency
λ	- Bypass ratio (BPR)
ϕ	- Gross thrust parameter
ψ	- $\frac{T_{t5}/T_{t3}}{(T_{t5}/T_{t3})_{des}}$

Subscripts:

oo	-Sea level conditions
o	- Ambient atmosphere; free stream
isen	-Isentropic
a	- Air
c	- Cold flow
des,d	- Design
e	- Exit
f	- Fan
g	- Gas
h	- Hot flow
j	- Jet
crit	-Critical

Acronyms:

CDP - Compressor Delivery Pressure

BPR - Bypass ratio

FPR - Fan Pressure Ratio

OPR - Overall Pressure Ratio

LP - Low Pressure

IP - Intermediate Pressure

HP - High Pressure

LPC - Low Pressure Compressor

IPC - Intermediate Pressure Compressor

HPC - High Pressure Compressor

LPT - Low Pressure Turbine

IPT - Intermediate Pressure Turbine

HPT - High Pressure Turbine

sfc - Specific Fuel Consumption

shp -Shaft Horsepower

SLS - Sea Level Static

SOT - Stator Outlet Temperature

TET -Turbine Entry Temperature

ISA -International Standard Atmosphere

1.1 Objective

The objective of this research is to develop an engineering tool for analysing the off-design performance of turbojets or turbofans. Such a tool could be used to predict the overall performance parameters such as thrust and fuel flow at various flight conditions, this being the key information required for aircraft design studies.

Off-design performance of a gas turbine is of major interest, not only to the user but also to the manufacturer. The thrust or power developed by an engine is dependent on the mass of air entering the engine, and the energy imparted to it during the engine cycle; hence thrust is influenced by such variables as forward speed of the aircraft, altitude and climatic conditions.

With the help of modern test facilities nowadays it is possible to simulate atmospheric conditions at high altitudes, thus providing a means of assessing the performance capability of a turbojet or a turbofan engine in flight without the engine ever leaving the ground. This is important as the changes in ambient temperature and pressure encountered at high altitudes considerably influence the thrust of the engine and may cause operational problems.

The question arises as to why should a new mathematical model for off-design performance be needed, when a method based on component matching (Saravanan, 1972) is already available for both turbojets and turbofans. The answer is that the normal method requires compressor and turbine characteristics, which are usually unavailable. The analysis of the new model can be of use for preliminary performance calculations for parametric studies, particularly for aircraft

design purposes.

Early attempts to predict the off-design performance of turbojets and turbofans have been done by few organizations, the most notable being NASA, Royal Aircraft Establishment, aero-engine industries etc; they developed programs such as DYNGEN, SMOTE, GENENG, DIGISIM to name a few. The only problem was that all of them relied on compressor and turbine performance maps.

A major contribution made by Wittenberg (1976), basing his work on that of Saravanamuttoo (1972), showed that gasdynamic relationships can be used for matching of nozzles and turbines. The work of Wittenberg has been developed and applied to a range of turbojets and turbofans. Therefore the aim of this thesis is to develop a method based on Wittenberg's work, which can be used for a reasonable prediction of typical cruise performance variation having a knowledge of design point parameters.

The culmination of the work led to the development of computer programs to simulate different type of engines, with more stress given to the Viper 20F-10 turbojet, Garrett TFE 731-2, Rolls Royce RB211-22 and a preliminary version of a Pratt and Whitney Study engine, because published data available for these engines came from reliable sources.

1.2 Scope

In this thesis the analysis of turbojets is limited to simple single-spool turbojets, though with the addition of a simple subroutine afterburning could also be included. In the case of turbofans the analysis has been limited to non-afterburning, separate exhaust twin or three-spool turbofans. Some of the secondary effects have been also incorporated into the program, like the bleed flow simulation.

One of the main objectives of simulation is to improve detailed understanding of the influences on the complete system characteristics of the individual system

components. The computer is an indispensable tool in fulfilling this objective, though one must always take caution that the flexibility of the simulation model is maintained (Gould, 1970); this axiom has been carefully followed throughout the modelling.

The scope of the thesis is primarily divided into two areas :

- Design concepts
- Off-design analysis

The off-design concepts form the predominant part of the thesis. The design concept is critical because it determines the off-design performance .

1.2.1 Design Concepts

Before venturing into the area of modelling any engine, whether for design or off-design analysis, one must have some clear cut concepts about what should be achieved in the preliminary engine design specification. Some of the major basic requirements for any engine are

- 1) Thrust
- 2) Specific fuel consumption
- 3) Weight
- 4) Low Operating cost
- 5) High reliability

Out of these requirements, thrust and fuel flow occupy the paramount position and improvement in both is so crucial, that one tends to belittle the higher complexity and increased weight that are the side effects of this improvement.

The thermodynamic performance of an engine is determined essentially by two parameters: the cycle pressure ratio and the turbine entry temperature. These two parameters influence sfc, especially cycle pressure ratio (Morris, 1982). Figure A1 gives an idea about the increase of turbine entry temperature (TET) with growth

in technology. The size effect (or the power level) has also an indirect influence on fuel consumption, because it in turn influences the cycle pressure ratio and the turbine entry temperature. The component efficiencies have been assumed as per the state of art and their variation can be seen from Figure A2.

1.2.2 Off Design Concepts

The design point thermodynamic cycle analysis of an engine is simple and straightforward. On the other hand, when studying off-design performance, the design variables are already selected and the independent variables are basically the flight conditions and the throttle settings.

The off-design performance of any turbojet can be expressed as functional relationship between non-dimensional quantities. As shown by Cohen et al (1987) for a given engine of fixed geometry, burning a given fuel, the thrust and fuel flow are given as

$$F/\delta = f_1(N/\theta^{1/2}, Ma)$$

$$Q/(\delta\theta^{1/2}) = f_2(N\theta^{1/2}, Ma)$$

where N is the rotational speed of the engine, $\theta = T/T_{OO}$, $\delta = p/p_{OO}$ and Q is the mass of fuel burned per unit time. From the above two relations, specific fuel consumption (sfc) can be given as

$$sfc/\theta^{1/2} = f_3(N/\theta^{1/2}, Ma)$$

One can then find thrust and sfc for any specified combination of engine speed, flight Mach number and atmospheric temperature and pressure. The major problem is that the exact forms of f_1 and f_3 are not known, and they vary from engine to engine. The only option left is either to approach the problem empirically as shown by Toreenbeek (1973) or graphically cum analytically as shown by Wittenberg (1976).

Most often an engine is rated and guaranteed at full power under sea-level standard ambient conditions but thrust measurement in flight always has been a problem (Borger, 1981). The approximate performance analysis method as described in this thesis approaches the problem by quick 'back-of-the-envelope' approach, which gives approximately the power lapse rates with temperature, altitude and flight speed. This is very useful to a preliminary engine design analysis group.

**FIGURE A1 VARIATION OF TURBINE ENTRY TEMPERATURE
WITH TECHNOLOGY GROWTH
(REPRODUCED FROM NRC ACOP SYMPOSIUM BANFF, 1987)**

**FIGURE A2 A COMPREHENSIVE VIEW OF VARIATION OF
POLYTROPIC COMPRESSOR EFFICIENCY WITH
PRESSURE RATIO
(REPRODUCED FROM NRC ACOP SYMPOSIUM BANFF, 1987)**

2.1 Previous Work

Modern aircraft powerplants are very complex systems, and it has been recognized that computer simulation of powerplant behaviour plays an important role in their development. The prediction of steady state or transient performance consists of calculating gas turbine behaviour from a mathematical model of the thermodynamic processes involved.

If we consider the historical evolution of powerplant simulation, we will see that simulation has been extensively used in three primary areas. In the case of gas turbines simulation has been applied to steady-state prediction, transient performance prediction and transient simulation for controls system design. The mathematical models used for steady-state performance prediction can be very complex, depending on the accuracy of the information desired. The models are synthesised by integrating the component characteristics and thermodynamic relations that govern them. Usually an iterative procedure is required to solve the model. (Cottingham, 1974)

For off-design analysis, there are several methods available at the present time. The basic method is based on using a suitable matching procedure for engine components, but in this approach the use of performance maps of compressor(s) and turbine(s) is essential. These matching procedures are fully described in Saravanamuttoo (1972). Another method predicts off-design performance by using logarithmic differential equations to infer performance changes about a design point. The differential methods have been found quite helpful in predicting the performance

response to various cycle perturbations of small magnitude (Cockshutt, 1967).

Then there is an analytical cum empirical method as shown by Torenbeek (1973). He in 1968 and in 1973 tried to approach the problem of off-design performance of turbojets and turbofans by giving a series of analytical expressions, which depend to a large extent on empirical expressions and curves. The greatest drawback of the procedure was that it was based on the assumption that the engine operates under design conditions only. There was no explicit method for finding the bypass ratio, and so it was assumed to be constant throughout and equal to the value at the takeoff condition. Hence for the engines where the bypass ratio changed substantially the whole procedure failed.

The procedure also lacked in giving a sound method to calculate compressor pressure ratio ϵ_c or fan pressure ratio ϵ_f at any cruising altitude, and thus one was forced to assume $(\epsilon_c)_{cruise} = 1.05$ to 1.10 of $(\epsilon_c)_{takeoff}$. This made the whole procedure very approximate, and according to Torenbeek it was to be used where "errors of a few percent are acceptable".

Another approach uses gasdynamic relationships only for its matching procedures and does not require the compressor or turbine performance maps (Wittenberg, 1976). This approach by Wittenberg of Delft University is basically derived from the earlier work of Saravanamuttoo (1972).

Saravanamuttoo's work on matching procedures is given in References 26 and 27; but there is an extension of the idea, by bringing about the use of generalized turbine characteristics and gasdynamic relations, which together make performance maps of compressors redundant. If the compressor maps are available, they can be used to take into account the effect of changes of the compressor efficiency with the engine conditions.

The problem in this thesis deals with prediction of off-design performance of single spool turbojets, twin and three- spool turbofans without resorting to the use of fan, compressor or turbine detailed characteristic maps (which are usually

not available). All the earlier prediction programs (SMOTE, GENENG, DYN-GEN, DIGISIM etc) required the use of component performance maps; then came Wittenberg's method which provided realistic overall performance data at any specified cruise conditions knowing design point parameters, and so finally there was a method which made the use of component maps redundant.

2.2 Need for Simulation

The prime question to consider is the purpose for which the simulation is required. The most objective aim of simulation according to Barnes (1970) is "to derive statements about the properties of the system which may be read across to the real situation". According to Gould (1970) the main objective is to "improve engineer's understanding of the influences on the whole system characteristics of the individual system components". While according to Westbrook (1970) the most important objective is simply "to save money". With such diversifying definitions of simulation one finds oneself in a quandary, until and unless one keeps one's objective in clear sights.

In our case the main aim of the simulation is to provide overall performance data at any typical cruising conditions if we know the design point input parameters. The secondary benefits which can be accrued by this simulation is the optimisation of the independent design parameters. Though to a large extent, the choice of independent design variables is very much affected by the constraint of where we are going to install the gas turbine whether in a fighter aircraft or in a commercial aircraft, but still optimisation of design variables can be performed about a particular design point and their effects seen at the off-design conditions.

The next aspect is to pay proper emphasis on the modelling of each constituent. This is essential, because over-simplification or over-complication of any component affects the operational capabilities of other components. In this particular model the speed terms that occur both in compressor maps and in turbine maps

have been discarded. This modification in component modelling of compressors and turbines, makes the mathematical model very flexible to use and easy to modify, and flexibility of the simulation model is very important. The use of modular design i.e. separate subroutines for each component helps a lot, if later changes are to be done regarding modelling of a particular component.

2.3 Difficulties in Simulation

The basic reason why is it so difficult to predict the performance parameters, is because of the *unavailability of information*. Other factors include,

- Inherent complexity of the problem
- Abundance of independent input parameters, and the intricate way these factors affect each other further complicates the problem
- Secondary effects

2.3.1 Availability of Information

It is known that the accuracy of results is very much dependent on the availability and quality of information. Moreover it has been found that in practice, only very few design parameters are available from the general brochure of a gas turbine.

The design parameters that are supplied in the case of a turbojet, are mass flow rate of air, compressor pressure ratio, design point thrust and specific fuel consumption. In the case of turbofans, fan pressure ratio, overall pressure ratio and bypass ratio are also specified in addition to the above parameters. The turbine entry temperature at the design point may not be known to the user, nor is any information regarding the component efficiencies available. Secondary effect parameters such as bleed flow, accessory power at take-off, pressure losses in the

combustor and afterburner are also not known and have to be assumed by the user.

The effect of component efficiency variation can be seen from Figures 4.5 and 4.6 and also from Table 4. It can be seen that a 1% change in compressor efficiency brings as much as 3 to 4% change in the fuel flow and thrust at normal cruise conditions. Therefore the availability of information is very critical in predicting the design and off-design steady state performance of a gas turbine.

2.3.2 Interdependent Variable parameters

A major task facing a gas turbine modeller is the number of variable input and output parameters. According to Mattingly et al.,(1987), considering all the intermediate and supporting parameters, there are at least 35 such parameters, but out of these 7 parameters form the crux of the design and off-design analysis of an engine. Still the problem is not much simplified as can be seen by the problems faced by Torenbeek in his analytical approach.

According to Torenbeek (1973) the thrust lapse rate is given as

$$\frac{T}{T_{t_{oo}}} = \frac{\psi}{\psi_{t_{oo}}} \left(1 + \frac{\gamma - 1}{2} M_{oo}^2\right)^{1.5} \left[\frac{1}{\sqrt{\phi_{t_{oo}}}} + \frac{0.6}{\phi_{t_{oo}}^2} \left(1 + \frac{\gamma - 1}{2} M_{oo}^2\right)^{1.5} \left(\frac{\psi}{\psi_{t_{oo}}} + \frac{34.714 M_{oo}}{\psi_{t_{oo}}} \right) \right]$$

where ψ is the corrected specific thrust and $\psi_{t_{oo}}$ is the corrected specific thrust at SLS takeoff. Now the corrected specific thrust ψ is given as

$$\psi = 34.714 \left[\frac{1}{1 + \lambda} \sqrt{5\eta_n(1 + \eta_{nf}\lambda)} \sqrt{\left(G + 0.2 M_{oo}^2 \frac{\eta_d}{\eta_{nf}} \lambda \right)} - M_{oo} \right]$$

where G is the gas generator function.

To get thrust T at any altitude, one has to find out ψ which is a function of bypass ratio λ and gas generator function G . Now G in turn is itself a complicated function of compressor pressure ratio ϵ_c and turbine entry temperature (TET). Now ϵ_c or TET or λ at that cruising altitude are not known, and the

inter-dependence was the reason why analytical approach failed. So the enormity of the problem can be grasped, as one begins to realize the interdependence of various parameters on each other.

2.3.3 Secondary effects

Modelling must consider which secondary effects are to be modelled, and which are to be neglected. Until the sensitivity of the model to various simplifications is found out, it is better to at least simulate one secondary effect i.e the bleed flow. The reason is that this parameter has the maximum secondary effect on the performance of the aero-engine, and may lead to quite different results if it is neglected, especially in case of modern engines.

Main Features of the Program

3

3.1 Introduction

The purpose of modelling is to formulate the essential features of a real problem in simple mathematical form and to obtain practical results therefrom. The reduction of reality to a simple model, which can be treated mathematically, followed by a comparison of the predictions with observation or experiments, is the essence of mathematical modelling. Initially the model only provides partial explanations, so previously omitted aspects should also be included in order to make it more sophisticated. Mandl (1986). Mathematical modelling of the steady state performance of gas turbines is a well established discipline. Since in this thesis, the mathematical models for turbojets and turbofans are quite distinct, the author has explained them in detail in their respective Chapters 4 and 5. The main purpose of this chapter is to discuss the computer programs and the subroutines used. The program performs design and off-design calculations in accordance with the wishes of the user.

3.2 Program Hierarchy

The program has been written in modular form, with each module carrying out some specific operation. The program starts off by reading data either from some unformatted input file or by inputting it on the screen. Initially design point calculations are done and if the user is not satisfied with the output thrust and specific fuel consumption values, then the user can always change the component polytropic efficiencies or the isentropic efficiencies of the intake and the exhaust.

Because of lack of availability of data, the user has to spend some time on iterations in order to fix the design point. After getting satisfied with the design point performance values, the off-design analysis can be done. The operational hierarchy of the programs is shown in Figures 3.1 and 3.2 while the program module hierarchy is shown in Figures 3.3 and 3.4.

3.2.1 Software Specifications:

The program is written in FORTRAN 77 language, and extensive use is made of COMMON statements to transfer data between different subroutines. Broadly, the program contains a main program supported by nine subroutines for the turbojet program and by ten subroutines for the turbofan program.

3.3 Main Program

The main program is divided into four sections:

- 1) Data input
- 2) Design performance calculations
- 3) Off-design performance calculations
- 4) Data output

The data are read either from the screen or from the input file as explained before. From here the subroutine DESIGN is invoked to perform design point calculations with the help of CHOKED, UNCHOKED and ALTITU subroutines to supply other parameters. After this the user can proceed or iterate on the input parameters to get more accurate design point performance values. Once the design point has been fixed and before proceeding with the off design analysis, the user can vary compressor or fan efficiency.

Provision has been made for the user to input (if known) some values of compressor or fan polytropic efficiency and respective pressure ratios. A third order

polynomial is then fitted through these points, so that variable efficiency can be used as required.

The off-design performance calculations comprise the largest section of the main program. The techniques used in the off-design part of the main program have been explicitly described in Chapters 4 and 5. Finally all the output data that are generated, both from the on-design and off-design analysis, can be either viewed on the screen, or in a formatted output on a separate output file.

3.4 Module Structures

DESIGN module

This is the subroutine that forms the crux of the analysis as it performs design calculations. Once the necessary design variables have been input to the program, this subroutine performs component by component analysis until the exhaust nozzle is reached. This subroutine is different for a turbojet as compared to a turbofan, because of the different configuration of the two types. The bleed flow turbofan configuration is different from the ordinary turbofan configuration because of the mixing analysis involved between the bleed air and the air/fuel mixture.

COPR module

This is the subroutine that is used for getting the values of the specific heat at constant pressure C_p , enthalpy H , calorific value ECV and entropy function ψ at a particular temperature. This subroutine uses polynomial expressions for the above mentioned properties, and the polynomial coefficients as found in the appendix of Chappell & Cockshutt (1974).

ALTITU module

This is a subroutine which gives the values of the International Standard Atmosphere temperature and pressure, once the altitude has been specified. The

temperature and pressure obtained through this subroutine follow different physical laws above and below the 36,089 feet (tropopause). This subroutine is quite similar to the one used by Mattingly (1987).

LSTM module

This is the subroutine that finds the coefficients of the polynomial, once the order of the polynomial and the points have been supplied. This subroutine is extensively used when there is a change of compressor or fan efficiency with the compressor or fan pressure ratio. If the user knows the relationship between the two above mentioned parameters, then he can input a certain number of points and fit a polynomial between the two. This polynomial can be used to get fan or compressor efficiency at any compressor or fan pressure ratio.

UNCHOKED module

This subroutine is used only when the nozzle is unchoked. It gets as input the temperature and pressure at entry to the nozzle, and it gives as output specific thrust, gross thrust parameter, effective pressure and temperature, velocity and Mach number of the issuing jet. This subroutine in the case of turbofan program works for both the 'cold' and the 'hot' nozzle.

CHOKED module

This subroutine works only when the exhaust nozzle (whether cold or hot in case of a turbofan) is choked. The temperature and pressure at entry to the nozzle are supplied as input. The output of the subroutine is the same as that of the unchoked subroutine. Both the unchoked and choked subroutines are fitted with flags, which determine whether it is to act as a 'hot' or a 'cold' nozzle. The value of the flags is regulated by the main program.

COMPAR module

This subroutine works on a one-dimensional searching technique and is basically derived from the Bolzano bisection method. The purpose of the subroutine is to find the point of intersection between the 'turbine-nozzle matching curve' and the 'compressor curve'.

Between the two working points, it does a linear interpolation for the values of the compressor pressure ratio, mass flow parameter, bypass ratio and the fan pressure ratio until the exact intersection point is obtained. The intersection point gives the value of the overall pressure ratio, bypass ratio, fan pressure ratio and mass flow parameter from which other performance parameters can be calculated.

OZZCH module

This subroutine is designed to provide nozzle characteristics (both for 'cold' and 'hot' flow). For a certain range of pressure ratios across the nozzle, the subroutine gives the unchoked as well as choked non-dimensional mass flow parameter $m\sqrt{RT}/A_jP$. This non-dimensional mass flow term is multiplied by the $A_j/\sqrt{R}$ term, where A_j is the area of the nozzle. Eventually what one gets from the subroutine is the mass flow parameter $m\sqrt{T}/P$ for a range of nozzle pressure ratios.

CHONoz module

This subroutine is extensively used in the off-design analysis of turbojets. It works only for the case when the turbojet nozzle is choked (i.e during the usual cruise conditions). The purpose of this subroutine is to obtain the working point of the compressor, so it calculates mass flow at entry to the compressor and the compressor pressure ratio for any given turbine entry temperature (TET).

Another option is that compressor efficiency can be varied in the subroutine, if the engine shows significant variation of compressor efficiency.

TURBCH module

This subroutine is used to calculate turbine characteristics i.e. mass flow parameter $m\sqrt{T}/P$ for varying pressure ratio. In the case of unchoked turbojet nozzle, the subroutine works on the assumption that at design point the turbine is choked, because this assumption fixes the area of the turbine.

CUTT module

This subroutine is used in the off-design analysis of turbojets. It works for the case when the turbojet nozzle is unchoked. The purpose of this subroutine is to find the point of intersection of 'compressor curve' with the 'turbine/nozzle matching curve', for different turbine entry temperatures.

WUIGJ module

The purpose of this subroutine is to solve non-linear simultaneous equations in order to get the coefficients of the equation. This subroutine is used to get the coefficients of the equation that relates TET to mechanical speed N of the engine. Once the coefficients are known, then TET at any mechanical speed N can be determined.

FIGURE 3.1 OPERATIONAL HIERARCHY OF TURBOJET MODEL

FIGURE 3.2 OPERATIONAL HIERARCHY OF TURBOFAN MODEL

FIGURE 3.3 MODULE HIERARCHY FOR TURBOJET MODEL

FIGURE 3.4 MODULE HIERARCHY FOR TURBOFAN MODEL

4.1 Introduction

The calculation of design point performance of a simple single spool turbojet engine is quite easy. In these calculations, some independent design variables are taken such as compressor pressure ratio and turbine entry temperature, while component efficiencies and pressure losses are assumed as per the state of art. By varying the design parameters, it is possible to examine trends in engine performance and seek an optimum engine for a particular operation.

Once an engine cycle and its design conditions are selected, then off-design analysis is done to analyse the performance over its operating envelope. Off design cycle analysis is more critical than design cycle analysis because the user of a turbine is basically interested in finding out how the gas turbine will behave with changes in ambient temperature, throttle settings and flight conditions (if it is installed in an aircraft).

In this study, off design performances of engines has been estimated by adding some extra constraints to the design point analysis. The constraint is a choked exhaust nozzle, for estimating off-design cruise performance of a single spool turbojet. This constraint regulates the aerothermodynamic behaviour of components upstream of the nozzle.

Finally the off-design calculations obtained by the program are compared with the published test results of some existing engines.

4.2 Simplifying Assumptions

- 1) The gas properties i.e. specific heat of air C_{p_a} and γ_a for compressor, C_{p_g} and γ_g for turbine and nozzle etc are constant throughout the whole working range.
- 2) The turbine is choked over the operational working range, *especially at the design point*.
- 3) The polytropic efficiency of the turbine is independent of rotational speed and pressure ratio.
- 4) The mass of fuel added (f) is equal to air bled for disc cooling (β) etc in case of uncooled turbines.
- 5) The expansion in the exhaust nozzle(s) is assumed to be ideal, i.e. there is no under or overexpansion, hence the area of the fixed nozzle remains constant throughout.

4.3 Analysis of Design Point Performance

The design point analysis of a single spool turbojet can easily be done by a thermodynamic analysis of each component starting with diffuser, compressor, combustor, turbine and finally ending with the propelling nozzle. See Figure 4.1 for details.

- 1) Diffuser If we consider the diffuser, with an intake isentropic efficiency η_i then stagnation temperature $T_{t_2} = T_{t_0} = T_0(1 + (\gamma_a - 1)/2 * M_0^2)$, where T_0 is the ambient temperature. The stagnation pressure ratio $P_{t_2} = P_0(1 + \eta_i * (\gamma_a - 1)/2 * M_0^2)^{\gamma_a/(\gamma_a - 1)}$, where P_0 is the ambient pressure, η_i is the intake isentropic efficiency and since we are considering flight Mach number of 0.8, we can consider η_i to be independent of Mach number effects. (Cohen et al., 1987)

FIGURE 4.1 SCHEMATIC VIEW OF A SINGLE SPOOL TURBOJET, SHOWING STATION NUMBERING.

- 2) Compressor Across the compressor, the temperature rise is a function of compressor pressure ratio and polytropic efficiency, and is given as $T_{t3}/T_{t2} = \epsilon^{\gamma_a - 1/\eta_c \gamma_a}$.
- 3) Combustor If we assume constant combustion efficiency η_b then we have the equation $\eta_b * H.V. * m_f = H_4 - H_3$, from which we can find out the mass of fuel being added. Here H is the enthalpy of the working fluid at a particular station.
- 4) Turbine The pressure ratio across the turbine is dependent on the isentropic efficiency and temperature drop across it, and is given as $P_{t5}/P_{t4} = (T_{t5}/T_{t4})^{\gamma_g/\eta_t(\gamma_g - 1)}$, where T_{t4} is the turbine entry temperature. T_{t5} is obtained from the power equation, by which turbine produces power equal to that needed by the compressor. The power equation is $m_a C_{pa}(T_{t3} - T_{t2}) = \eta_m m_a C_{pg}(T_{t4} - T_{t5})$, where η_m is the mechanical efficiency of the spool.

- 5) Propelling Nozzle The pressure ratio P_{t_6}/P_O determines whether the nozzle is choking or not. If the nozzle is unchoked, then there is only momentum thrust across the nozzle, but if it is choked then pressure thrust will also be present.

4.3.1 Choked exhaust nozzle

($M_6=1$; $P_6 \geq P_O$) The velocity at exit from the nozzle is given as $V_6 = \sqrt{\gamma_g R T_6}$ while net specific thrust $F_s = (V_6 - V_O) + \frac{A_j}{m_g}(P_6 - P_O)$, where A_j is the jet area of the nozzle, m_g is the mass flow rate of gas across the nozzle and V_O is the aircraft velocity.

4.3.2 Unchoked exhaust nozzle

($M_6 \leq 1$; $P_6 = P_O$) The velocity at exit from the nozzle is given as $V_6 = \sqrt{2\eta_n C_{p_g} T_{t_6} (1 - (P_O/P_{t_6})^{\gamma_g-1/\gamma_g})}$, while net specific thrust $F_s = V_6 - V_O$. The total net thrust is calculated using net specific thrust and the mass flow rate of gas, while specific fuel consumption can be determined from the fuel flow and thrust.

There is a new parameter ϕ defined as gross thrust parameter which is given as $\phi = \frac{[(T_g/P_O A_6)+1]}{P_{t_2}/P_O}$ where gross thrust is given by $T_g = m_g V_6 + A_6(P_6 - P_O)$. On combining these two we get $\phi = \frac{P_6/P_O(\gamma_g M_6^2+1)}{P_{t_2}/P_O}$, so ϕ is a measure of pressure ratio at the exhaust nozzle to pressure ratio at the intake nozzle. If we estimate Mach number M_6 and if we know ϕ , ambient pressure P_O and exhaust nozzle area A_6 we can easily get gross thrust T_g .

4.4 Off Design Analysis

The off-design analysis revolves around two conditions

- 1) When aircraft is idling or descending (in both cases the turbine becomes unchoked).

- 2) Cruise flight (turbine is choked).

4.4.1 Turbine Characteristics

A map of a typical turbine characteristics can be seen from Figure A3.

FIGURE A3 TURBINE CHARACTERISTICS (Reproduced from Saravanamuttoo, H.I.H et al., 1987)

From this figure it can be seen that mass flow is nearly independent of speed for $P_{t3}/P_{t4} \geq 2.5$, where P_{t3}/P_{t4} is the pressure ratio across the turbine. The choking of the turbine may occur at the nozzle throats or at the rotor blade exits or outlet annulus, or it may be because of flow limitations at all these positions. When the turbine chokes because of choking of the nozzles (as is usually the case),

then the characteristics are as shown in Figure A3, i.e. $m\sqrt{T_t}/P_t$ is independent of $N/\sqrt{T_t}$ for large pressure ratios.

But on the other hand when the rotor outlet is choked, then $m\sqrt{T_t}/P_t$ would depend on the $N/\sqrt{T_t}$ for a given pressure ratio, because changes in rotor speed affect stagnation pressure and temperature drop across the rotor (Kerrebrock, 1977). On the other hand η_t does not vary rapidly with the off-design variations, hence for engine performance calculations, the turbine characteristics can be approximated by $m\sqrt{T_t}/P_t = \text{constant}$ and $\eta_t = \text{constant}$ for most of the cases. Furthermore, the approximation then yields little error because when the turbine is linked to the other components the whole operating range shown in Figure A3 is not used (Cohen et al, 1987).

4.4.2 Matching of turbine with exhaust nozzle

For the off-design analysis it is necessary to understand the matching procedures between the turbine and exhaust nozzle. In this study, the mass flow characteristics of turbines have been generalized to nozzle characteristics, i.e. *a single mass flow curve with a constant turbine efficiency has been assumed*. The effect of turbine rotational speed on mass flow and turbine efficiency, has been assumed to be of secondary nature because of the very restricted range of turbine when put in front of a nozzle (Cohen et al, 1987). Now to match the turbine mass flow and exhaust nozzle mass flow we have the relation,

$$\frac{m_g\sqrt{T_{t5}}}{P_{t5}} = \frac{m_g\sqrt{T_{t4}}}{P_{t4}} \frac{P_{t4}}{P_{t5}} \sqrt{\frac{T_{t5}}{T_{t4}}} \quad (1)$$

where $m_g\sqrt{T_t}/P_t$ comes from the non-dimensional mass flow term $m\sqrt{RT}/AP$ and if we assume fixed area of nozzle A_j , then we can get $m_g\sqrt{T_{t5}}/P_{t5}$. The area of the nozzle A_j is obtained when sizing of the engine is done at the design point. The temperature drop across the turbine is given by $T_{t5}/T_{t4} = (P_{t5}/P_{t4})^{\eta_t(\gamma_g-1)/\gamma_g}$,

hence equation (1) can be written as,

$$\frac{m_g \sqrt{T_{t5}}}{P_{t5}} = \frac{m_g \sqrt{T_{t4}}}{P_{t4}} \left(\frac{P_{t4}}{P_{t5}} \right)^{1 - \frac{\gamma_t(\gamma_g - 1)}{2\gamma_g}} \quad (2)$$

where $m_g \sqrt{T_{t4}}/P_{t4}$ is the turbine mass flow. Now, the mass flow across the nozzle is a function of pressure ratio across the nozzle, i.e. $m_g \sqrt{T_{t5}}/P_{t5} = f(P_{t5}/P_O)$, and hence using equation (2) we can find the turbine pressure ratio for any nozzle pressure ratio P_{t5}/P_O . A single curve representing the exhaust flow characteristics can readily be obtained by applying equation (2) to points on the single curve of the turbine flow characteristics. See Figure 4.2.

FIGURE 4.2 MATCHING OF TURBINE WITH THE EXHAUST NOZZLE.

4.4.3 Choked exhaust nozzle

When the propelling nozzle is choked, then it can be seen from Figure 4.2, there is a fixed turbine operating point, which is independent of nozzle pressure

ratio. Thus the maximum pressure ratio across the turbine is controlled by the choking of the exhaust nozzle. Due to the fixed operating point of the turbine, the temperature drop across the turbine T_{t_5}/T_{t_4} becomes fixed, and is the same for both design and off-design conditions. When we deal with the *compatibility of work between the various components*, we know that power supplied by the turbine to drive the compressor is given as,

$$m_a C_{p_a} (T_{t_3} - T_{t_2}) = \eta_m m_a (1 + f - \beta) C_{p_g} (T_{t_4} - T_{t_5}) \quad (3)$$

and it can be assumed that fuel flow $f = \beta$, the air bled for disc cooling for engine with uncooled blades. The compressor temperature ratio is given as $T_{t_3}/T_{t_2} = \epsilon^{(\gamma_a-1)/\eta_c \gamma_a}$ hence equation (3) can also be written as

$$m_a C_{p_a} T_{t_2} (T_{t_3}/T_{t_2} - 1) = \eta_m m_a C_{p_g} T_{t_4} (1 - T_{t_5}/T_{t_4})$$

With further modification equation (3) changes to

$$\frac{T_{t_3}}{T_{t_2}} - 1 = \eta_m \frac{C_{p_g}}{C_{p_a}} \frac{T_{t_4}}{T_{t_2}} \left(1 - \frac{T_{t_5}}{T_{t_4}}\right) \quad (4)$$

The compressor temperature ratio is given as $T_{t_3}/T_{t_2} = \epsilon^{(\gamma_a-1)/\eta_c \gamma_a}$ and since η_m , C_{p_g} and C_{p_a} are constant, equation (4) can be written as

$$\frac{T_{t_3}/T_{t_2} - 1}{(T_{t_3}/T_{t_2})_{des} - 1} = \frac{T_{t_4}/T_{t_2}}{(T_{t_4}/T_{t_2})_{des}}$$

because T_{t_5}/T_{t_4} is fixed, due to the fixed operating point of the turbine. The above equation can be further modified to give

$$\frac{\epsilon_c^{(\gamma_a-1)/\eta_c \gamma_a} - 1}{(\epsilon_c)^{(\gamma_a-1)/(\eta_c)_{des} \gamma_a} - 1} = \frac{T_{t_4}/T_{t_2}}{(T_{t_4}/T_{t_2})_{des}}$$

or compressor pressure ratio at new throttle setting (T_{t_4}/T_{t_2}) is given as

$$\epsilon_c = \left(1 + \frac{(T_{t_4}/T_{t_2})}{(T_{t_4}/T_{t_2})_{des}} \left((\epsilon_c)_{des}^{\gamma_a-1/(\eta_c)_{des} \gamma_a} - 1 \right) \right)^{\eta_c \gamma_a / \gamma_a - 1} \quad (5)$$

On satisfying the essential condition of *compatibility of flow between the turbine and compressor* we get for the compressor mass flow

$$\frac{m_a \sqrt{T_{t_2}}}{P_{t_2}} = \frac{m_a}{m_g} \frac{m_g \sqrt{T_{t_4}}}{P_{t_4}} \sqrt{\frac{T_{t_2}}{T_{t_4}}} \frac{P_{t_4}}{P_{t_3}} \frac{P_{t_3}}{P_{t_2}} \quad (6)$$

In this expression the turbine mass flow $m_g \sqrt{T_{t_4}}/P_{t_4}$ is determined by the turbine operating point and is the same for both design and off-design conditions. Moreover m_a/m_g and P_{t_4}/P_{t_3} (the combustor pressure loss) remain the same for both design and off-design conditions.

Hence equation (6) can be further modified to give

$$\frac{m_a \sqrt{T_{t_2}}}{P_{t_2}} = \left(\frac{m_a \sqrt{T_{t_2}}}{P_{t_2}} \right)_{des} \frac{(\sqrt{T_{t_2}/T_{t_4}})}{(\sqrt{T_{t_2}/T_{t_4}})_{des}} \frac{\epsilon_c}{(\epsilon_c)_{des}} \quad (7)$$

So by using equation (7), a new value of compressor mass flow can be estimated. Knowing compressor mass flow and pressure ratio, a unique working line on a compressor map can be obtained for different values of TET.

In equation (5) either one can work with constant compressor efficiency or by using a polynomial subroutine in the program find a new efficiency at that particular compressor ratio. This procedure gives more accuracy, especially in the case of high pressure ratio, variable stator engines. Once new values of ϵ_c , and $m_a \sqrt{T_{t_2}}/P_{t_2}$ are estimated, then other temperatures and pressures for different components can quite easily be calculated. This procedure gives new values of performance parameters such as specific thrust F_s , net thrust F , amount of fuel consumed m_f , gross thrust parameter ϕ , and specific fuel consumption .

4.4.4 Unchoked exhaust nozzle

The nozzle gets unchoked usually when the aircraft is landing or idling, also if the fuel is throttled too much the pressure ratio across the compressor goes down, and the nozzle becomes unchoked. From the power matching condition of the

compressor and turbine we know $m_a C_{p_a}(T_{t_3} - T_{t_2}) = \eta_m m_a (1 + f - \beta) C_{p_g}(T_{t_4} - T_{t_5})$ which can be further modified to give

$$T_{t_3}/T_{t_2} - 1 = \eta_m \frac{C_{p_g}}{C_{p_a}} \frac{T_{t_4}}{T_{t_2}} (1 - T_{t_5}/T_{t_4})$$

Now η_m , C_{p_g} and C_{p_a} are constant both at design and off-design conditions, hence the above equation can be modified to give equation (8).

$$\frac{(T_{t_3}/T_{t_2} - 1)}{(T_{t_3}/T_{t_2})_{des} - 1} = \frac{(T_{t_4}/T_{t_2})}{(T_{t_4}/T_{t_2})_{des}} \frac{1 - (T_{t_5}/T_{t_4})}{1 - (T_{t_5}/T_{t_4})_{des}} \quad (8)$$

Let the temperature ratio $\psi = T_{t_4}/T_{t_2}/(T_{t_4}/T_{t_2})_{des}$, hence equation (8) modifies to

$$\frac{1}{\psi} \frac{T_{t_3}/T_{t_2} - 1}{(T_{t_3}/T_{t_2})_{des} - 1} = \frac{1 - T_{t_5}/T_{t_4}}{1 - (T_{t_5}/T_{t_4})_{des}} \quad (9)$$

Now the left hand side of equation (9) is a function of overall pressure ratio P_{t_3}/P_O for a given ψ , M_O and intake efficiency η_i , while the right hand side is a function of nozzle pressure ratio P_{t_5}/P_O , which basically fixes the turbine pressure ratio P_{t_5}/P_{t_4} . From the turbine and nozzle matching conditions, we know that

$$\frac{m_g \sqrt{T_{t_5}}/P_{t_5}}{(m_g \sqrt{T_{t_5}}/P_{t_5})_{des}} = \frac{m_g \sqrt{T_{t_4}}/P_{t_4}}{(m_g \sqrt{T_{t_4}}/P_{t_4})_{des}} \left(\frac{P_{t_4}/P_{t_5}}{(P_{t_4}/P_{t_5})_{des}} \right)^{1 - \frac{\eta_t(\gamma_g - 1)}{2\gamma_g}} \quad (10)$$

Now we take a range of values of P_{t_5}/P_O and get $m_g \sqrt{T_{t_5}}/P_{t_5}$ terms by multiplying $m_g \sqrt{T_{t_5}} R / AP$ terms by $A_j / \sqrt{R}$. Similarly we get turbine characteristics by taking a range of values of P_{t_5}/P_{t_4} , and getting $m_g \sqrt{T_{t_4}}/P_{t_4}$ corresponding to each pressure ratio term. Then by using equation (10) we can get for each value of P_{t_5}/P_O , a definite value of P_{t_5}/P_{t_4} . Hence we can now get values of T_{t_5}/T_{t_4} and P_{t_3}/P_O because $P_{t_5}/P_O = (P_{t_5}/P_{t_4}) * (P_{t_4}/P_{t_3}) * (P_{t_3}/P_O)$. The curve between P_{t_3}/P_O and the right hand side of equation (9) is called a *turbine/nozzle characteristics curve*.

Then for that range of values of P_{t3}/P_O (as calculated earlier) one can get various values of ϵ_c , from equation (11)

$$\frac{P_{t3}}{P_O} = \left(1 + \eta_c \frac{(\gamma_a - 1)}{2} * M_O^2 \right)^{\gamma_a / \gamma_a - 1} (\epsilon_c) \quad (11)$$

Then from these values, different values of T_{t3}/T_{t2} can be found for a particular Mach number M_O and temperature ratio ψ . So a curve can be obtained between P_{t3}/P_O and the left hand side of the equation (9), and is called as the *compressor curve*. See Figure A4 for matching of compressor and turbine/nozzle characteristic curves. From the intersection point value of P_{t3}/P_O can be recorded for the case when the left hand side of equation (9) becomes equal to the right hand side. From this value of P_{t3}/P_O , values of compressor pressure ratio ϵ_c and exhaust conditions T_{t5} and P_{t5} can easily be found. Once we know these then the calculation of performance parameters like thrust, sfc etc is similar to the design point analysis.

FIGURE A4 MATCHING OF COMPRESSOR AND TURBINE/NOZZLE CHARACTERISTIC CURVES.

4.5 Comparison of Predictions

To check the validity of the computer program, the computer results must be compared with published data. Various types of turbojets are considered, the first one being a single spool low pressure turbojet J-35-A29, with design pressure ratio of 5.25 and design thrust of 5600 lbf at sea level static conditions.

Initially the program is supplied with design point compressor pressure ratio, air mass flow and turbine entry temperature. The turbine efficiency $\eta_t = 0.798$ is taken as constant, while compressor efficiency at design point is taken as $(\eta_c)_{des} = 0.838$ as deduced from published data.

Once the design thrust and sfc are known, then the program can be run either in choked nozzle mode or in unchoked nozzle mode. To get a compressor working line, we need to just vary T_{t_4}/T_{t_2} for a constant Mach number and get new values of compressor pressure ratio ϵ_c and compressor mass flow parameter $m_a \sqrt{T_{t_2}}/P_{t_2}$. Knowing these two, a unique compressor working line is drawn which is compared to manufacturer supplied data, as shown in Figure 4.3. Even when the program is run at constant compressor efficiency $\eta_c = (\eta_c)_{des}$, reasonably good values of ϵ_c and $m_a \sqrt{T_{t_2}}/P_{t_2}$ are obtained. In Figure 4.4 a graph of compressor pressure ratio and T_{t_4}/T_{t_2} is compared with published results (Walker, 1952).

In Table 1 experimental values for the J-35-A29 are shown. Using these values we get the polytropic efficiency of the compressor at different compressor speeds and pressure ratios. The predicted values are compared with the calculated values in Table 2. The two significant parameters compared are compressor pressure ratio and mass flow rate of air at different turbine entry temperatures. It can be seen from Table 2 that quite good agreement is obtained between the calculated values and the computer output.

A similar calculation was performed for the General Electric XJ-79-GE1 with

a design pressure ratio of 12.2 and design thrust of 9600 lbf at SLS conditions. In Figure 4.5 the compressor working line as obtained through computer program (when run at $\eta_c = (\eta_c)_{des} = 0.792$) is compared with published data. (Campbell, 1958).

The reason for the discrepancy between the two curves is because the XJ-79-GE1 is a high pressure ratio variable stator geometry engine so variation in compressor efficiency has to be taken in account. Same problem is encountered for the curve of compressor pressure ratio vs T_{t_4}/T_{t_2} as shown in Figure 4.6. This problem has been overcome in the computer program by curve fitting a (third order) polynomial, which gives polytropic compressor efficiency at a given pressure ratio. The program is capable of working both at constant polytropic compressor efficiency, as well as with high pressure ratio engines where variation in polytropic compressor efficiency is an important factor. On using variable efficiency, reasonably good predictions are obtained as shown in Figures 4.7 and 4.8. So there is a marked change brought about by efficiency variation.

In Table 3, comparison is made between the calculated and predicted characteristics of the XJ-79 GE1 at constant compressor efficiency. Importance is given to pressure ratio and mass flow terms, because these two fix the working line on the compressor map. In Table 4, again comparison is done between calculated and predicted characteristics of XJ-79 GE1, but now with variable compressor efficiency. It can be seen that predicted values (in Table 4) obtained with variable compressor efficiency are closer to calculated values, than the values (in Table 3) obtained with constant compressor efficiency.

In order to show how polytropic compressor efficiency varies with pressure ratio, the polytropic compressor efficiency was deduced from the published data and then plotted against compressor pressure ratio. It was found that a third order polynomial gives a very accurate fit, hence a third order polynomial subroutine was incorporated in the computer program. The shape of polytropic efficiency

variation with compressor pressure ratio can be seen in Figure 4.9 for XJ-79-GE-1 jet engine.

4.5.1 Simulation of Viper20 F-10

The Viper Mark 521 is a civil version of Viper 20 F-10 and at its design point of ISA +10° C, it develops 3000 lbf of SLS thrust, which results in an ISA SLS take-off rating of 3120 lbf. It is a single shaft turbojet with a seven-stage plus “zero” stage axial compressor driven by a single stage turbine. It also incorporates a fully automatic blow-off valve, and has improved thrust and higher turbine entry temperature when compared to older versions of Viper series (i.e 8 or 11).

The engine at sea level operates at maximum compressor speed of 13760 rpm, and at ISA SLS it is assumed there is no intake loss, ejector effect, air bleed or power take-off. The compressor operates at 97% of maximum engine speed at recommended maximum climb, and the speed drops to 94% at recommended maximum cruise conditions. At sea level we have maximum compressor speed conditions, but at any other altitude from 10,000 feet to 40,000 feet ceiling the compressor runs at 94% of maximum engine speed for cruise.

A detailed description of its design point characteristics is given below.

VIPER 20-F10

INPUT DATA:

Cycle parameters:

Turbine entry temperature (TET)	=	1145 K
Compressor pressure ratio (CPR)	=	5.15
Mass flow of air	=	22.32 kg/sec (49.2 lb/sec)
Polytropic compressor efficiency (AN.Viper, 1965)	=	0.89
Polytropic turbine efficiency (AN.Viper, 1965)	=	0.78
Combustion efficiency	=	0.99

Isentropic intake efficiency	=	1.00
Isentropic efficiency of propelling nozzle	=	0.99
Mechanical efficiency of spool	=	0.99
Combustion pressure loss	=	6% of CDP
Ma	=	0.0
Altitude	=	SLS (ISA+10)

OUTPUT:**Overall performance:**

Net thrust	=	13109 N (2948 lbf)
Fuel flow	=	1401 kg/hr (3088 lb/hr)
sfc	=	0.1069 kg/hr-N (1.048 lb/hr-lbf)

Component data:

Nozzle pressure ratio	=	1.985
Area of the propelling nozzle	=	134.2 in ²
Jet pipe temperature	=	962 K
Jet pipe pressure	=	2.01 bar

The purpose of this analysis, is to simulate the mathematical model with the experimental values which are obtained from the performance brochure (AN. Viper Mark 521, 1964). The main parameters we are trying to simulate are fuel flow and net thrust developed by the engine at various flight conditions. We can also easily find compressor pressure ratio, air flow, gross thrust parameter, exhaust gas parameter etc; but they are not of primary importance as compared to fuel flow and net thrust.

For the performance at sea level we get the values of N/N_D at different Mach numbers, and then for these values we find from reference 1¹ the values of turbine

¹ Bristol Siddeley P.S.198/S/2

entry temperature and gross thrust parameter ϕ , from which we can easily compute gross thrust by the relation $\phi = \frac{P_{O_1}}{P_{O_1}} (1 + \frac{X_G}{A P_{O_1}})$ where X_G is the gross thrust term.

The program is run at different altitudes and Mach numbers. When the program is run at sea level ISA conditions with varying Mach number, it requires turbine entry temperature as its input. The TET is read from reference (1) for different flight conditions, because there is a published curve available between TET and aerodynamic speed $N/N_D \sqrt{288.2/T_{t_1}}$. The behaviour of turbine entry temperature with the aerodynamic speed at various altitudes is shown in Tables 5,6,7,8 and 9. The program then gives all the performance parameters with net thrust and fuel flow of primary importance. When the aircraft is cruising at recommended cruise speed, at an engine power setting of $N/N_D = 0.94$, the turbine temperature drops quite drastically. This drop is due to the engine operating at cruise rating instead of take-off rating, and consequently at a lower inlet temperature. As the Mach number is changed at the same level of cruise engine speed, the TET varies in such a fashion that $m_g \sqrt{T_{t_4}}/P_{t_4}$ remains constant.

It is seen that for sea level and 10,000 ft altitude the TET remains nearly constant as Mach number is increased, but it goes down when Mach number is increased at 20,000, 30,000 or 40,000 ft altitude. This effect is tabulated in Table 10 and is also shown in Figure 4.14. The predicted values of net thrust and fuel flow as compared to published data can be seen in Figures 4.8, 4.9, 4.10, 4.11 and 4.12.

It is also seen that when the aircraft is above 30,000 ft altitude, the predicted fuel flow and net thrust are off by as much as 10% and 9% respectively at higher Mach numbers. In order to amend this feature, it was assumed that the compressor efficiency starts falling at high altitudes (due to low Reynolds number) especially at low Mach numbers. On inputting variable compressor efficiency good agreement is obtained, as compared to constant compressor efficiency cases.

The variation of performance parameters with efficiency variation can be seen

in Figures 4.11 and 4.12. In Figure 4.13 the deduced variation of compressor polytropic efficiency with compressor pressure ratio can be seen. The drop of compressor efficiency can be due to various reasons, out of which lower Reynolds number occupies a significant place. The published data as compared to predicted results can be seen in Tables 11, 12, 13, 14, 15, 16 and 17 for various altitudes and Mach numbers, but the variation of performance parameters with variation in compressor efficiency can be seen from Tables 14, 15, 16 and 17.

4.6 Correlation between TET and N

The only thing that the user has to input beside the flight condition (i.e. the altitude and the Mach number) is the turbine entry temperature. Now a user knows the TET at the design point and has no information about its variation with altitude, engine speed and Mach number. Moreover there is no data available to estimate turbine entry temperatures at different flight conditions or at different engine speeds. Hence we need a correlation between engine mechanical speed, inlet temperature and turbine entry temperature.

There can be no strict correlation between the engine speed and TET as it depends not only on the different configurations and class (thrust) of engines but also on their technological sophistication. This fact can be supplemented by the example that TET in the case of a PW J57 turbojet engine (Roy-Aikins, 1983) varies by 10-12 K for every 1% change in mechanical speed. In the case of Viper 20-F10 turbojet (AN. Approved performance data, 1964) it is found that TET varies by 12-50 K for every 1% change in mechanical speed depending on the flight conditions.

As the aircraft starts its climb, the engine speed is reduced from the take-off speed to climb speed, and finally settles down to cruise speed at normal cruising conditions. The takeoff engine speed is 100% of the mechanical engine speed, the climb speed around 97% of the engine speed and the cruise speed varies from 89%

to as high as 95%. From the literature it can be seen that TET is a function of the engine speed and for a turbojet it changes at approximately 2.5% for every 1% change in speed. This can be seen from Table 25 reproduced from Sobey & Suggs (1963).

TET is a function of both LP and HP spool speeds in the case of twin or three spool turbofans. Since the HP and LP speeds are aerodynamically coupled, one usually takes TET as a function of HP spool speed if the area of the exhaust nozzle(s) is fixed. Now one can see from Figure A5 how TET/T_{t1} behaves with the aerodynamic speed of the spool. Usually the relationship between the two is either quadratic or cubic, though it is linear in the speed range between 90% to 105% of the aerodynamic speed.

FIGURE A5 CORRELATION BETWEEN TET AND LP SPOOL AERODYNAMIC SPEED FOR LM-1600. (REPRODUCED FROM ZHU PING)

On assuming a quadratic relationship between TET/T_{t1} and aerodynamic speed $N/\sqrt{T_{t1}}$ we get,

$$TET/T_{t1} = A + B(N/\sqrt{T_{t1}}) + C(N/\sqrt{T_{t1}})^2$$

or on further modification,

$$TET = A(T_{t1}) + BN\sqrt{T_{t1}} + CN^2 \quad (A)$$

Now, at takeoff rating ($N=1$), the values of TET and T_{t2} at SLS can be assumed in equation (A). Then one can put for any value of Mach number at sea level another value of TET and T_{t2} in equation (A). The value of TET at any other Mach number is very close to the value of TET at SLS condition, the reason being the small decrease in aerodynamic speed being compensated by increase in T_{t2} as Mach number is increased at sea level.

Since equation (A) involves three unknowns, three simultaneous equations are needed. To get a third equation we can take TET, mechanical speed N and T_{t2} at cruise conditions. Usually at cruise rating, N varies from 0.90 to 0.96 of the maximum spool speed.

Now on putting the third value of TET (cruise) and T_{t2} three simultaneous equations can be obtained. If the user does not know the cruise TET, the best solution is to input TET and T_{t2} at takeoff ratings at a Mach number different from the one taken earlier, and so get the three simultaneous equations.

But for a twin-spool or a three-spool engine, the inner HP spool or the HP and IP spool in case of a three spool turbofan, are isolated from the external conditions by the LP spool. According to Torenbeek (1976) the variation of TET from takeoff to cruise ratings for low bypass or turbojet engines is of the order of 150 to 200 K, but for medium to high bypass ratio turbofans it is only of the order of 50 to 100 K for the same class of engines. The actual data showing the variation of TET at takeoff to cruise can be seen from Table 27 and Table 28 reproduced

from Borger (1981). According to Borger it is seen that for turbojets the variation of TET from takeoff to cruise ratings is around 150 K, while for turbofans it is from 100 K to 170 K. The TET (takeoff to cruise) variation for different engines can be seen graphically in Figure 4.17. The same conclusions were obtained in this thesis when comparison was made for TET variation, between single spool turbojet Viper 20-F10 and twin spool turbofan TFE 731-2. So for a twin or three-spool one can assume with confidence that HP spool mechanical speed is the main factor that influences the TET, while LP spool mechanical speed is the main factor that influences the mass flow of the engine. (Young P.,1966).

4.7 Conclusions

The reasonable predictions given by the program, prove the validity of the major assumption that the constraint of a choked exhaust nozzle dictates the aerothermodynamic behaviour of all upstream components. Moreover for the unchoked nozzle case, one can get reasonably accurate fuel consumption, when the aircraft is either idling or taxiing at the runway or waiting for take-off clearance which nowadays can be a substantial time.

The agreement is quite good in the region where Mach number is between 0.3-0.8 for all the altitude conditions. Only when the altitude exceeds 30,000 feet do we need to take compressor efficiency variation into account for better simulation, because in single spool engines there is more variation of compressor efficiency as compared to a twin spool engine.

Besides the usual approach some new aspects have been added to the computer program. One is the way of obtaining temperature, pressure and density at a particular altitude, just by specifying the altitude of the aircraft. This is accomplished in a subroutine which transfers ISA data to the main program up to an altitude of 100,000 feet. This subroutine is based on a similar kind of a subroutine given by Mattingly et al.,(1986).

In the computer program, fuel flow is calculated as given in Gas Turbine Thermodynamic Tables (Chappell & Cockshutt, 1974) i.e it just requires the use of combustor entry and exit temperatures.

FIGURE 4.3 PREDICTED AND PUBLISHED WORKING LINE OF J35-A-29

FIGURE 4.4 PREDICTED AND PUBLISHED CHARACTERISTICS OF J35-A29

FIGURE 4.5 PREDICTED AND PUBLISHED WORKING LINE OF XJ79-GE1

FIGURE 4.6 PREDICTED AND PUBLISHED CHARACTERISTICS OF XJ-79-GE1

FIGURE 4.7 VARIATION OF COMPRESSOR POLYTROPIC EFFICIENCY

FIGURE 4.8. PREDICTED AND PUBLISHED PERFORMANCE DATA FOR VIPER20-F10.

FIGURE 4.9 PREDICTED AND PUBLISHED PERFORMANCE DATA FOR VIPER20-F10

FIGURE 4.10. PREDICTED AND PUBLISHED PERFORMANCE OF VIPER 20-F10

FIGURE 4.11 PREDICTED AND PUBLISHED PERFORMANCE DATA FOR VIPER20-F10

FIGURE 4.12 PREDICTED AND PUBLISHED PERFORMANCE DATA FOR VIPER20-F10

FIGURE 4.13 PREDICTED POLYTROPIC EFFICIENCY VARIATION

FIGURE 4.14 VARIATION OF TURBINE ENTRY TEMPERATURE AT DIFFERENT FLIGHT CONDITIONS.

FIGURE 4.15 COMPARISON OF TET AT TAKE OFF TO TET AT CRUISE FOR VARIOUS ENGINES

5.1 Introduction

The turbofan is probably the most common derivative of the gas turbine for aircraft propulsion. It is a 'bypass' engine, where part of the air is compressed fully and passed into the combustion chamber, while the remainder is compressed to a lower extent and ducted around the hot section. The 'hot' flow may either be mixed with 'cold flow' or both may be exhausted separately into the atmosphere. In both cases the result is reduced overall jet velocity, giving better propulsive efficiency at subsonic speeds, lower noise levels and improved specific fuel consumption, features which are preferred both by military and civil aircraft manufacturers.

Turbofans significantly differ from turbojets in two basic areas:

- 1) Air flow is higher for a given thrust.
- 2) sfc is lower at typical subsonic cruise conditions.

The higher airflow results in a lower thrust per kg of air-flow and hence in lower jet velocity, which in turn means higher propulsive efficiency for the turbofan as compared to a turbojet. The lower sfc is the prime factor that has caused the turbofans to replace the turbojet in subsonic commercial aircraft. (Rolls-Royce, 1986)

5.2 Turbofan Configurations

The cycle parameters for a turbofan have a profound effect on the mechanical design of the engine. This is because variation in bypass ratio and overall pres-

sure ratio implies variation in component diameters and rotational speeds (Saranamuttoo, 1987). For moderate bypass and overall pressure ratios the simplest arrangement is as shown in Figure A6. The HP compressor is usually driven by a single or two stage turbine, while the LP turbine drives the fan (or LP compressor). For high bypass ratio and high overall pressure ratio engines, one has to go for multi-spool turbofans with the fan followed by booster stages (or LP compressor) driven by an LP turbine, followed by a HP compressor driven by an HP turbine; in the case of the three-spool turbofan, the core is a twin-spool gas generator.

5.3 Design Point Calculations

In this study we will deal first with how to establish design point performance of the turbofan, and then deal with more complicated aspects of off-design performance predictions. Here, since we are dealing with twin spool turbofans we have to take into account two nozzles, the ‘hot nozzle’ and the ‘cold nozzle’. Now, at any off-design condition, the nozzles maybe unchoked or choked, or one choked with the other unchoked depending on the altitude and Mach number of the aircraft.

The various combinations of the two nozzles, make the picture more complex when compared to a single spool turbojet. Therefore a model has to be devised which is based on the assumption that the nozzles maybe unchoked or choked. Since in this study we are more concerned about off-design cruise performance, we will concentrate more on the choked analysis of the nozzle.

5.3.1 Design Point Characteristics

The design point is characterized by the following data:

- 1) Flight condition (altitude and Mach Number).
- 2) Fan pressure ratio $\epsilon_f = P_{t3}/P_{t2}$.

- 3) Bypass ratio $\lambda = m_c/m_h$.
- 4) Overall pressure ratio P_{t_4}/P_{t_1} .
- 5) Turbine entry temperature T_{t_5} .
- 6) Air flow m_a .
- 7) Component (polytropic) efficiencies of fan, LP compressor, HP compressor and turbines.
- 8) Losses such as pressure loss in combustion chamber, isentropic efficiency of the intake and the propelling nozzles.

5.4 Simplifying Assumptions

- 1) The polytropic efficiency of the turbine is assumed to be constant for all operating conditions.
- 2) The low pressure and high pressure turbines have equal polytropic efficiencies.
- 3) The polytropic fan (or compressor) efficiency is mainly dependent on rotational speed.
- 4) The expansion in the exhaust nozzle(s) is assumed to be ideal, i.e. there is no under or overexpansion, hence the area of fixed nozzle remains constant throughout.
- 5) The LP and HP compressors are assumed to have equal polytropic efficiencies though $\eta_c \neq \eta_t$.
- 6) The mass of fuel added ($m_h \cdot f$) is taken equal to the amount of air bleed ($m_h \cdot \beta$) for disc cooling of engine with uncooled blades.
- 7) The air bleed for cooling the turbines is accounted for separately.

FIGURE A6 SCHEMATIC VIEW OF A TWIN SPOOL TURBOFAN

5.5 Design Analysis of Turbofans

The design point analysis of turbofans is in fact a combination of individual component design analysis. The aerothermodynamic processes that occur in various components can be broadly classified as:

5.5.1 Intake

The basic purpose of a diffuser is to increase the static pressure of the free stream fluid. In this component the ambient air at temperature T_O and pressure P_O is brought to stagnation pressure P_{tO} and stagnation temperature T_{tO} under the effect of ram compression. So

$$T_{t2} = T_{tO} = T_O \left(1 + \frac{(\gamma_a - 1)}{2} * M_O^2 \right) \quad (5.1)$$

while

$$P_{t2} = P_O \left(1 + \eta_i \frac{(\gamma_a - 1)}{2} * M_O^2 \right)^{\gamma_a / \gamma_a - 1} \quad (5.2)$$

5.5.2 Fan

The air then passes through a fan which is used to raise the pressure of the incoming air to a suitable level. The fan is a key component of high bypass turbofan engines for jet airliners. A substantial portion of air entering the fan bypasses the gas generator of the engine and the fan acts like a very efficient ducted propeller to provide a significant proportion of the total thrust.

The fan pressure ratio is dependent on the bypass ratio of the engine, so the number of stages in a fan is a function of the bypass ratio. Usually for civil or commercial aircraft a single stage fan is used for reasons of weight, noise, bearing design etc.

Now the pressure ratio of the cold nozzle can be written as :

$$\frac{P_{t3}}{P_O} = \frac{P_{t3}}{P_{t2}} \frac{P_{t2}}{P_{tO}} \frac{P_{tO}}{P_O} = \epsilon_f \left(1 + \eta_i \frac{(\gamma_a - 1)}{2} * M_O^2 \right)^{\gamma_a / \gamma_a - 1} \quad (5.3)$$

while the temperature ratio across the fan is given by

$$T_{t3} = T_O \left(1 + \frac{(\gamma_a - 1)}{2} * M_O^2 \right) (\epsilon_f)^{\frac{\gamma_a - 1}{\eta_f \gamma_a}} \quad (5.4)$$

Depending on the pressure ratio that exists across the fan nozzle, the nozzle may be unchoked or choked. The exhaust velocity for the unchoked nozzle is given as

$$V_f = \sqrt{2\eta_n C_p T_{t2} \left(1 - (P_O/P_{t2})^{\frac{\gamma_g - 1}{\gamma_g}} \right)} \quad (5.5)$$

Hence the net cold nozzle thrust is given as

$$T_{ncold} = m_c (V_f - V_O) \quad (5.6)$$

where V_O is the aircraft velocity. For the choked cold nozzle the critical temperature is given as $T_c = \frac{2}{\gamma_a + 1} T_{t3}$ while critical pressure is given as $P_c = P_{t3} \left(1 - \frac{1}{\eta_n} \left(\frac{\gamma_a - 1}{\gamma_a + 1} \right) \right)^{\gamma_a / \gamma_a - 1}$. Therefore we get the exhaust velocity as

$$V_f = \sqrt{\gamma_a R_a T_c} = \sqrt{\frac{2\gamma_a R_a}{\gamma_a + 1} T_{t3}} \quad (5.7)$$

The net cold thrust will be given as $T_{ncold} = m_c (V_f - V_O) + A_j (P_c - P_O)$ where A_j is the fixed area of the fan nozzle. It can be seen that for the case of an unchoked nozzle the exhaust velocity is a function of pressure ratio across the fan nozzle.

Thus the momentum thrust part of the total fan thrust is a function of the fan pressure ratio. For flight conditions at low Mach numbers, especially at lower altitudes, the fan thrust (mostly momentum thrust) is the predominant part of the total thrust and is a very much a function of fan pressure ratio. Hence the choice of fan pressure ratio, is a major design variable for fixing the working envelope of the aircraft.

5.5.3 HP and LP spools

There are many different types of engine configurations, but usually engines have the following configuration as described below. The LP spool consists of

LP turbine driving the fan and the LP compressor (or the booster stages). Most likely the LP turbines are two or three stage axial flow turbines which powers three or four stages of axial flow compressor and a fan through a planetary reduction gearbox .

The HP spool usually has a single stage axial turbine driving a centrifugal compressor or several stages of axial compressors. The HP spool is aerodynamically coupled to the LP spool, though mechanically they are on two separate concentric shafts.

The power equation of the HP spool can be written as

$$m_h C_{pa}(T_{t4} - T_{t3}) = \eta_m m_h (1 + f - \beta) C_{pg}(T_{t5} - T_{t6}) \quad (5.8)$$

Here to simplify things we assume that the bleed air ($m_h * \beta$ for disc cooling) from the HP compressor is equal to the mass of fuel ($m_h * f$) added in the combustor. Air bled for cooling the turbines is to be separately accounted for, as it changes the power equation as well as mass matching equation to some extent as shown in Chapter 6.

The power equation for the LP spool can be written as

$$(m_c + m_h) C_{pa}(T_{t3} - T_{t2}) = \eta_m m_h C_{pg}(T_{t6} - T_{t7}) \quad (5.9)$$

where again we are making the assumption that the amount of air bled is equal to the mass of the fuel added, without taking cooling air for the turbine(s) into account. The term η_m is the mechanical efficiency of the LP spool and can be assumed equal to HP spool mechanical efficiency. Now adding equations (5.8) and (5.9) we get equation (5.10) as

$$m_c C_{pa} T_{t3} + m_h C_{pa} T_{t4} - m_c C_{pa} T_{t2} - m_h C_{pa} T_{t2} = \eta_m m_h C_{pg}(T_{t5} - T_{t7}) \quad (5.10)$$

On further modification one gets

$$m_c C_{pa}(T_{t3} - T_{t2}) + m_h C_{pa}(T_{t4} - T_{t2}) = \eta_m m_h C_{pg}(T_{t5} - T_{t7}) \quad (5.11a)$$

Finally dividing by the $(m_h C_{pa})$ terms throughout one gets

$$\lambda \left(\frac{T_{t3}}{T_{t2}} - 1 \right) + \left(\frac{T_{t4}}{T_{t2}} - 1 \right) = \eta_m \frac{C_{pg}}{C_{pa}} \frac{T_{t5}}{T_{t2}} \left(1 - \frac{T_{t7}}{T_{t5}} \right) \quad (5.11b)$$

Now T_{t3}/T_{t2} is the temperature ratio across the fan and is given as

$$\frac{T_{t3}}{T_{t2}} = (\epsilon_f)^{\frac{\gamma_a - 1}{\eta_f \gamma_a}} \quad (5.12)$$

while temperature ratio T_{t4}/T_{t2} is given as

$$\frac{T_{t4}}{T_{t2}} = (\epsilon_f)^{\frac{\gamma_a - 1}{\eta_f \gamma_a}} * (\epsilon_{HPC})^{\frac{\gamma_a - 1}{\eta_c \gamma_a}} \quad (5.13)$$

In equation (5.11b) the temperature ratio T_{t5}/T_{t2} can easily be found as T_{t5} is the known turbine inlet temperature, while T_{t2} is fixed by the flight conditions, so the temperature T_{t7} at the entrance of hot nozzle can be found. Moreover, the total pressure P_{t7} at the entrance of hot nozzle can be found as

$$\frac{P_{t7}}{P_{t5}} = \left(\frac{T_{t7}}{T_{t5}} \right)^{\frac{\gamma_g}{\eta_t(\gamma_g - 1)}} \quad (5.14)$$

which is based on the assumption that both turbines have equal polytropic efficiencies, and these efficiencies remain constant at all off-design conditions. Another point to be considered is that, due to the assumption of LP turbine driving fan only (instead of fan+LP compressor as it happens in reality), we may get LP turbine unchoked earlier because of the lower pressure drop across it. The reason for making this assumption is that we don't know the split of pressure rise between LP and HP compressors separately, and so have to combine them together and assume that HP turbine is driving the combined compressor configuration. Hence one will get inter-compressor or inter-turbine pressures and temperatures different from the actual values.

5.5.4 Hot Nozzle Flow

The pressure rise across the hot nozzle is given as

$$\frac{P_{t7}}{P_O} = \frac{P_{t7}}{P_{t5}} \frac{P_{t5}}{P_{t4}} \frac{P_{t4}}{P_{t3}} \frac{P_{t3}}{P_{t2}} \frac{P_{t2}}{P_O} \quad (5.15)$$

From the above equation we can see that the pressure ratio across the hot nozzle is a function of $\frac{P_{t4}}{P_{t3}} \frac{P_{t1}}{P_{t2}} \frac{P_{t2}}{P_O}$ or in other words is a function of overall compressor pressure ratio and ram compression ratio. So at the high Mach numbers and at higher altitudes we have higher ratio of hot thrust to cold thrust than at lower altitudes and at low Mach numbers.

Now for the unchoked hot nozzle the exhaust velocity is given by

$$V_e = \sqrt{2C_{pg}\eta_n T_{t7} \left(1 - \left(\frac{P_O}{P_{t7}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right)} \quad (5.16)$$

and the net hot thrust becomes equal to

$$T_{n_{hot}} = m_h(V_e - V_O) \quad (5.17)$$

When the hot nozzle is choked as it is during typical cruise conditions, the exhaust velocity is given as $V_e = \sqrt{\gamma_g R T_c}$ where $T_c = \frac{2}{\gamma_g+1} T_{t7}$. The effective pressure ratio across the nozzle is given as

$$\frac{P_e}{P_O} = \frac{P_{t7}}{P_O} \left(1 - \frac{1}{\eta_n} \frac{\gamma_g - 1}{\gamma_g + 1}\right)^{\frac{\gamma_g}{\gamma_g-1}} \quad (5.18)$$

Hence the net hot thrust is given as

$$T_{n_{hot}} = m_h(V_e - V_O) + A_j(P_e - P_O) \quad (5.19)$$

5.6 Off Design Performance

For a turbojet the off-design performance is quite easy to estimate because there is only one exhaust nozzle which can be either choked or unchoked. But a turbofan engine is more complicated because it has two nozzles- one for the cold flow and the other for the hot flow; depending on the throttle settings and the flight conditions, the nozzles may be unchoked or choked. The method to estimate the off-design performance has thus to be based on the assumption that the nozzles may be either choked or unchoked.

The off-design performance calculations in the case of turbofans is based on the assumption that *the low pressure turbine (LPT) is always choked*. The matching of turbines and a choked exhaust nozzle is shown in Figure A7. The assumption of choked LPT fixes the operating point of the high pressure turbine (HPT). Moreover it simplifies the analysis of turbofan performance, as now the behaviour of the high pressure spool can be estimated in the same way as for a single spool turbojet engine with a choked nozzle flow.

This means that the working line on the high pressure compressor map is fixed and is given by the relationship

$$\epsilon_{HPC} = f(m_h \sqrt{T_{t3}}/P_{t3}, T_{t5}/T_{t3})$$

The power matching equation of high pressure compressor and high pressure turbine is given as (assuming fuel added is equal to the bled air)

$$m_h C_{p_a} T_{t3} (T_{t4}/T_{t3} - 1) = \eta_m m_h C_{p_g} T_{t5} (T_{t6}/T_{t5} - 1) \quad (5.20)$$

Now it can be further modified as

$$(T_{t4}/T_{t3} - 1) = \eta_m \frac{C_{p_g}}{C_{p_a}} \frac{T_{t5}}{T_{t3}} (T_{t6}/T_{t5} - 1) \quad (5.21)$$

Since the HPT operating point is fixed, the value of T_{t6}/T_{t5} remains the same at design and off-design conditions. Assuming C_{p_g}/C_{p_a} and η_m are constant, one can get from equation (5.21), with respect to the design point,

$$\frac{T_{t4}/T_{t3} - 1}{(T_{t4}/T_{t3})_{des} - 1} = \frac{T_{t5}/T_{t3}}{(T_{t5}/T_{t3})_{des}} \quad (5.22)$$

But $T_{t4}/T_{t3} = (\epsilon_{HPC})^{\gamma_a - 1/\eta_c \gamma_a}$, so one can write

$$\frac{(\epsilon_{HPC})^{\gamma_a - 1/\eta_c \gamma_a} - 1}{(\epsilon_{HPC})_{des}^{\gamma_a - 1/\eta_c \gamma_a} - 1} = \frac{T_{t5}/T_{t3}}{(T_{t5}/T_{t3})_{des}} \quad (5.23)$$

or on further modification one gets

$$\epsilon_{HPC} = \left(1 + \frac{T_{t5}/T_{t3}}{(T_{t5}/T_{t3})_{des}} \left((\epsilon_{HPC})_{des}^{\gamma_a - 1/\eta_c \gamma_a} - 1 \right) \right)^{\eta_c \gamma_a / \gamma_a - 1} \quad (5.24)$$

FIGURE A7 MATCHING OF CHOKED TURBINES WITH THE CHOKED EXHAUST NOZZLE OF A TWIN SPOOL TURBOFAN.

From the mass flow matching conditions, one can get

$$\frac{m_h \sqrt{T_{t_3}}}{P_{t_3}} = \frac{m_h \sqrt{T_{t_5}}}{P_{t_5}} \sqrt{\frac{T_{t_3}}{T_{t_5}}} \frac{P_{t_5}}{P_{t_4}} \frac{P_{t_4}}{P_{t_3}} \quad (5.25)$$

which is based on the assumption that the fuel added is equal to the air bled for cooling of disc and bearings. Now the parameter $m_h \sqrt{T_{t_5}}/P_{t_5}$ is constant due to the fixed operating point of the HP turbine. The term P_{t_5}/P_{t_4} is the combustor pressure loss and can be assumed constant at design and off-design conditions. So on referencing the equation (5.25) one can get

$$\frac{m_h \sqrt{T_{t_3}}}{P_{t_3}} = \left(\frac{m_h \sqrt{T_{t_3}}}{P_{t_3}} \right)_{des} \sqrt{\frac{(T_{t_5}/T_{t_3})_{des}}{T_{t_5}/T_{t_3}}} \frac{\epsilon_{HPC}}{(\epsilon_{HPC})_{des}} \quad (5.26)$$

Having analysed the HP spool, we can now consider the power balance for the low-pressure spool which is given below

$$(m_c + m_h) C_{p_a} (T_{t_3} - T_{t_2}) = \eta_m m_h C_{p_g} (T_{t_6} - T_{t_7}) \quad (5.27)$$

The assumption that the amount of bled air is equal to the mass of fuel added is implicit in equation (5.27) On further modification we get

$$(\lambda + 1)(1 - T_{t_2}/T_{t_3}) = \eta_m \frac{C_{p_g}}{C_{p_a}} \frac{T_{t_5}}{T_{t_3}} \frac{T_{t_6}}{T_{t_5}} (1 - T_{t_7}/T_{t_6}) \quad (5.27b)$$

But the temperature ratio T_{t_6}/T_{t_5} is fixed, due to the fixed working point of HPT. From equation (5.27b) we can get

$$\left(\frac{\lambda + 1}{\lambda_{des} + 1} \right) \frac{(1 - T_{t_2}/T_{t_3})}{(1 - (T_{t_2}/T_{t_3})_{des})} = \frac{T_{t_5}/T_{t_3}}{(T_{t_5}/T_{t_3})_{des}} \frac{(1 - T_{t_7}/T_{t_6})}{(1 - (T_{t_7}/T_{t_6})_{des})} \quad (5.27c)$$

Let $\psi = \frac{T_{t_5}/T_{t_3}}{(T_{t_5}/T_{t_3})_{des}}$, hence (5.27c) can be further modified to

$$\frac{1}{\psi} \left(\frac{\lambda + 1}{\lambda_{des} + 1} \right) \frac{(1 - T_{t_2}/T_{t_3})}{1 - (T_{t_2}/T_{t_3})_{des}} = \frac{(1 - T_{t_7}/T_{t_6})}{1 - (T_{t_7}/T_{t_6})_{des}} \quad (5.28)$$

Now the left hand side of the equation (5.28) is a unique function of cold nozzle pressure ratio P_{t_3}/P_O for a given flight Mach number M_O and ψ . Similarly the

right hand side of the equation (5.28) is a function of hot nozzle pressure ratio P_{t7}/P_O . By estimating both the left hand side and the right hand side of (5.28) as functions of the same variable P_{t4}/P_O one can get the equilibrium point of the LP spool.

5.6.1 Programming aspect

The basic requirement is estimation of the performance of an aircraft at varying flight Mach numbers and altitudes. For a given flight Mach number and turbine entry temperature T_{t5} , a range of P_{t3}/P_O is assumed which gives a range of fan pressure ratios ϵ_f because

$$\frac{P_{t3}}{P_O} = \frac{P_{t3}}{P_{t2}} \frac{P_{t2}}{P_{tO}} \frac{P_{tO}}{P_O} = \epsilon_f \left(1 + \eta_c \frac{(\gamma_a - 1)}{2} * M_O^2 \right)^{\gamma_a / \gamma_a - 1} \quad (5.29)$$

Now from the known turbine inlet temperature T_{t5} and T_{t2} one can get a range of T_{t5}/T_{t3} because

$$\frac{T_{t5}}{T_{t2}} = \frac{T_{t5}}{T_{t3}} \frac{T_{t3}}{T_{t2}} = \frac{T_{t5}}{T_{t3}} * \epsilon_f^{\frac{(\gamma_a - 1)}{\eta_f \gamma_a}} \quad (5.30)$$

Knowing T_{t5}/T_{t3} it is easy to get a range of ϵ_{HPC} and $m_h \sqrt{T_{t3}}/P_{t3}$ from equations (5.24) and (5.26). Hence the operating point on the HPC working line now varies with P_{t3}/P_O .

For the same range of P_{t3}/P_O one can get a range of $m_c \sqrt{T_{t3}}/P_{t3}$ from the cold nozzle characteristics because the cold nozzle exhaust area A_c is assumed to be fixed. Knowing $m_c \sqrt{T_{t3}}/P_{t3}$ and $m_h \sqrt{T_{t3}}/P_{t3}$, a range of bypass ratios is calculated as $\lambda = \frac{m_c \sqrt{T_{t3}}/P_{t3}}{m_h \sqrt{T_{t3}}/P_{t3}}$. A range of T_{t3}/T_{t2} is obtained from various values of ϵ_f that have been calculated previously, because $T_{t3}/T_{t2} = \epsilon_f^{\gamma_a - 1 / \eta_f \gamma_a}$. Moreover a range of overall pressure ratios P_{t4}/P_O is obtained because $P_{t4}/P_O = \epsilon_{HPC} P_{t3}/P_O$. Hence all the terms of the left hand side of equation (5.28) can be calculated as functions of P_{t4}/P_O and a curve drawn between the left hand side of equation (5.28) vs P_{t4}/P_O for a given flight Mach number, altitude and turbine entry temperature T_{t5} .

For the LPT and the hot exhaust nozzle, the mass flow matching equation is given as

$$\frac{m_h \sqrt{T_{t7}}}{P_{t7}} = \frac{m_h \sqrt{T_{t6}}}{P_{t6}} \frac{P_{t6}}{P_{t7}} \sqrt{\frac{T_{t7}}{T_{t6}}} \quad (5.31)$$

Since the LPT is assumed to be choked throughout the design and off-design conditions, the mass flow parameter $\frac{m_h \sqrt{T_{t6}}}{P_{t6}}$ is constant. Hence

$$\frac{m_h \sqrt{T_{t7}}}{P_{t7}} = \left(\frac{m_h \sqrt{T_{t7}}}{P_{t7}} \right)_{des} \left(\frac{P_{t6}/P_{t7}}{(P_{t6}/P_{t7})_{des}} \right)^{1 - \frac{\gamma_t(\gamma_g - 1)}{2\gamma_g}} \quad (5.32)$$

because

$$T_{t7}/T_{t6} = (P_{t7}/P_{t6})^{(\gamma_g - 1)\eta_t/\gamma_g} \quad (5.33)$$

On assuming a range of P_{t7}/P_O we can get a range of $m_h \sqrt{T_{t7}}/P_{t7}$ from the hot nozzle characteristics assuming the hot nozzle exit area A_h is fixed. Then from equation (5.31) it is easy to get a range of LP turbine pressure ratios P_{t7}/P_{t6} . From (5.33) we can get a range of T_{t7}/T_{t6} values. Moreover a range of overall pressure ratios P_{t4}/P_O is obtained as

$$\frac{P_{t4}}{P_O} = \frac{P_{t7}}{P_O} \frac{P_{t6}}{P_{t7}} \frac{P_{t5}}{P_{t6}} \frac{P_{t4}}{P_{t5}}$$

where P_{t6}/P_{t5} is fixed pressure ratio of HPT while P_{t5}/P_{t4} is the assumed combustor pressure loss (remains constant throughout). Hence all the terms of the right hand side of the equation (5.28) are known, and a curve between the right hand side and P_{t4}/P_O can be drawn. It is important to note that the right hand side of equation (5.28) is independent of ψ , Mach number and altitude. The equilibrium condition of the low pressure spool is determined by the intersection of the two curves representing the left hand and right hand sides of equation (5.28) as functions of P_{t4}/P_O .

At the equilibrium point, the corresponding values of *compressor pressure ratio*, *fan pressure ratio*, *bypass ratio* and $m_h \sqrt{T_{t3}}/P_{t3}$ are selected. From the fan

pressure ratio the values of T_{t_3} and P_{t_3} are easily determined. Knowing the compressor pressure ratio, it is easy to get the values of T_{t_4} and P_{t_4} , as T_{t_3} and P_{t_3} are already known. Since the turbine inlet temperature is specified beforehand, the fuel flow can also be determined. Since the values of $m_h \sqrt{T_{t_3}}/P_{t_3}$, T_{t_3} and P_{t_3} are known, it is easy to get the value of hot (core) mass flow as

$$m_h = \frac{m_h \sqrt{T_{t_3}}}{P_{t_3}} \frac{P_{t_3}}{\sqrt{T_{t_3}}}$$

Knowing the value of m_h , the mass flow of cold air can also be found as $m_c = m_h \lambda$.

All these parameters can then be substituted in the power equation of the HP spool,

$$m_h C_{p_a} T_{t_3} (T_{t_4}/T_{t_3} - 1) = \eta_m m_h C_{p_g} T_{t_5} (T_{t_6}/T_{t_5} - 1)$$

to get T_{t_6} . Knowing T_{t_6}/T_{t_5} we can get the pressure ratio across the HP turbine. Again if the known parameters are substituted in the LP spool power equation,

$$(m_c + m_h) C_{p_a} (T_{t_3} - T_{t_2}) = \eta_m m_h C_{p_g} (T_{t_6} - T_{t_7})$$

then T_{t_7} can be found as it is the only unknown. Hence the pressure ratio across the LP turbine can also be calculated, and if one knows P_{t_7} then the pressure ratio across the hot nozzle can easily be calculated.

Since P_{t_3} and P_{t_7} are known, the thrusts for the hot and cold nozzle can be calculated as shown in the design point analysis. Thus all the important performance parameters such as fuel flow, thrust and specific fuel consumption can be calculated by the procedure described.

5.7 Verification of the Mathematical Model

Garrett TFE 731-2

The Garrett TFE 731-2 is a twin spool turbofan and at its design point (ISA SLS) it develops a thrust of 3500 lbf at takeoff conditions, with an sfc of 0.491

lb/hr-lb. It is a medium bypass ratio, two-spool turbofan engine, incorporating a single-stage gear-driven fan. The low pressure (LP) spool, consisting of a three-stage axial turbine and a four-stage axial compressor on a single shaft, drives the fan through a planetary reduction gearbox. The HP spool is aerodynamically coupled to the LP spool, and consists of a single-stage centrifugal compressor driven by a single-stage axial turbine.

Design point data are given below.

GARRETT TFE 731-2

INPUT DATA:

Cycle parameters:

Turbine inlet temperature (TET)	=	1260 K
Overall pressure ratio (OPR)	=	15.09
Bypass ratio	=	2.67
Fan pressure ratio	=	1.54
Mass flow of air	=	51.27 kg/sec (113 lb/sec)
Assumed component efficiencies		
Polytropic efficiency of fan	=	0.86
Polytropic efficiency of compressor	=	0.85
Polytropic efficiency of turbine	=	0.85
Combustion efficiency	=	0.98
Isentropic intake efficiency	=	0.99
Isentropic efficiency of propelling nozzle	=	0.98
Mechanical efficiency (both spools)	=	0.98
Combustion pressure loss	=	5% of CDP
Flight parameters		
Ma	=	0.0
Altitude	=	ISA SLS

OUTPUTS:**Overall performance:**

Total net thrust	=	15.63 kN (3514 lbf)
Fuel flow	=	782.67 kg/hr (1725 lb/hr)
sfc	=	0.0506 kg/hr-N (0.491 lb/hr-lbf)
Fan net thrust	=	5.34 kN (1202 lbf)
Hot nozzle net thrust	=	10.28 kN (2312 lbf)
Fan nozzle pressure ratio	=	1.54 (Unchoked)
Hot nozzle pressure ratio	=	1.42 (Unchoked)
Hot nozzle area	=	112.11 in ²
Cold nozzle area	=	172.83 in ²
Cold mass flow	=	82.21 lb/sec
Hot mass flow	=	30.79 lb/sec

An important thing to be considered is that the efficiencies (whether polytropic or isentropic) of the components are to be selected such that at the design point the correct design thrust, fuel flow and exit nozzle areas can be matched. It is necessary to iterate until a satisfactory solution is obtained.

When the aircraft is operating at maximum cruise conditions, one can vary the flight Mach number at a particular altitude and predict values of net thrust and fuel flow from the developed computer program. Figures 5.1, 5.2, 5.3, 5.4, 5.5 and 5.6 show the predicted values of thrust and fuel flow at different altitudes as compared with the published data. It can be seen that even when the component efficiencies (polytropic) are assumed constant throughout, there is a good relationship between the predicted and published data.

The numerical comparison of the predicted and published values of thrust, fuel

flow and specific fuel consumption can be seen in Tables 18, 19, 20, 21, 22 and 23. Meanwhile an interesting feature can be seen in Figures 5.7, 5.8, 5.9, 5.10, 5.11 and 5.12 where we can see that the fan thrust is initially about 60% of the total net thrust at sea level, but it decreases and becomes lower than 40% of total net thrust at higher altitudes. It can also be seen that the net hot nozzle thrust is nearly independent of the Mach number, while the net fan thrust decreases as the Mach number is increased. In other words it means that the effect of Mach number is being borne by the LP spool or the HP spool is shielded by the LP spool from the downstream conditions.

Figures 5.13 and 5.15 show the effect of Mach number and altitude on the fan pressure ratio and bypass ratio. From Figure 5.13 it can be seen that the fan pressure ratio increases at higher altitudes but decreases with increasing Mach number. From Figure 5.15 it can be seen that the bypass ratio decreases appreciably with increase in altitude, and although it tends to increase with increasing Mach number, its percentage increase is not rapid enough, and the net result is lower fan thrust at higher altitudes.

An important point noticed is that bypass ratio increases as the thrust is cut back. The reason why bypass ratio increases can be found out by simple logical reasoning as shown. Bypass ratio is given as m_c/m_h , which can be written in terms of exit nozzle mass flow parameters as

$$\frac{m_c}{m_h} = \left(\frac{m_c \sqrt{T_{t2}}}{A_c P_{t2}} \right) \left(\frac{A_h \sqrt{P_{t7}}}{m_h T_{t7}} \right) \frac{A_c}{A_h} \frac{P_{t2}}{P_{t7}} \sqrt{\frac{T_{t7}}{T_{t2}}}$$

Now in the cruise condition the nozzles are choked and so $\frac{m_c \sqrt{T_{t2}}}{A_c P_{t2}} = 0.396$ and $\frac{m_h \sqrt{T_{t7}}}{A_h P_{t7}} = 0.389$. So bypass ratio now depends only on

$$\frac{m_c}{m_h} \simeq 1 * (A_c/A_h) * \frac{P_{t2}}{P_{t7}} \sqrt{\frac{T_{t7}}{T_{t2}}}$$

As the engine is throttled back there is more decrease in P_{t7} than in any other

parameter, which pushes up the term $\frac{P_{t2}}{P_{t7}}$ and so the bypass ratio rises as thrust is cut back in cruise. See Table 26 for the variation in values as thrust is throttled.

In Figure 5.14 compressor pressure ratio behaves in a similar manner as fan pressure ratio, the only difference being a less rapid decrease of pressure ratio in compressor as compared to a fan. Figure 5.16 shows the variation of turbine inlet temperature (HP turbine) with altitude and Mach number. There is a linear relationship between the TET and the Mach number as can be seen from Figure 5.16.

5.8 Conclusions

The agreement is quite satisfactory in the region where Mach number is between 0.3-0.5 for all the altitude conditions, but this region is not of much concern. It can be seen that the agreement is very good for typical cruise conditions of Ma 0.6-0.8, altitude 30,000-50,000 feet although we have assumed constant fan and compressor efficiencies throughout.

FIGURE 5.1 PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 5.2 COMPARISON OF PREDICTED AND PUBLISHED DATA.

FIGURE 5.3 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 5.4 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

FIGURE 5.5 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 5.6 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 5.7 PREDICTED FAN AND HOT NOZZLE THRUST

FIGURE 5.8 PREDICTED FAN AND HOT NOZZLE THRUST WITH MACH NUMBER.

FIGURE 5.9 PREDICTED FAN AND HOT NOZZLE THRUST

FIGURE 5.10 PREDICTED FAN AND HOT NOZZLE THRUST

FIGURE 5.11 PREDICTED FAN AND HOT NOZZLE THRUST

FIGURE 5.12 PREDICTED FAN AND HOT NOZZLE THRUST

FIGURE 5.13 PREDICTED FAN PRESSURE RATIO.

FIGURE 5.14 VARIATION OF PREDICTED COMPRESSOR PRESSURE RATIO (HP & LP)

FIGURE 5.15 PREDICTED BYPASS RATIO

FIGURE 5.16 PREDICTED TURBINE ENTRY TEMPERATURE

Three Spool Turbofans

6

6.1 Introduction

The three-spool turbofan is basically an extension of the twin-spool, and it came into existence because of the need for both high overall pressure ratios and bypass ratios. In three-spool engine the pressure rise can be split between HP and IP compressors, driven by separate turbines, which can be run at different speeds giving maximum efficiency for each compressor. It was a concept that was introduced by Rolls-Royce, basically to avoid the problem of high pressure ratio across a single compressor, which leads to surge and mechanical complexities.

The three-spool configuration is schematically shown in Figure A8. The IP and HP spools can be considered as a single spool in the design and off-design conditions, if the polytropic efficiencies of the compressors and turbines are assumed to be equal. Hence a three-spool turbofan becomes mechanically equivalent to a twin spool turbofan using this assumption.

The added assumption that the LP turbine is choked, makes the three-spool case exactly similar to a twin-spool engine. The design point calculations are nearly the same as those for the twin spool, with the addition (if one wants) of splitting the IP and HP spool separately. So one can effectively use the same program for three-spool and twin-spool engines, with the only difference that the inter-turbine and inter-compressor temperatures and pressures calculated will be totally different from the actual values.

FIGURE A8 SCHEMATIC VIEW OF A THREE-SPOOL TURBOFAN

6.2 Comparison of Rolls-Royce RB211-22

The designation RB211 applies to a family of advanced technology three-shaft turbofans of high bypass ratio and high pressure ratios, with thrusts ranging from 166 kN (37,400 lbf) to 249 kN (56,000 lbf). Among the advantages offered by a three-shaft layout are its achievement of a high pressure ratio with fewer compressor and turbine stages while maintaining excellent handling. The need for variable compressor stator mechanisms can be minimised and the rotating assemblies can be made relatively short and rigid while preserving light construction. (Jane's All the World Aircraft, 1987-88)

The RB211-22B fitted to the L-1011-1 and -100 Tristar is flat rated at 187 kN (42,000 lbf) to 28.9°C. At the cruise condition (35,000 ft and Ma=0.8), the RB211-22B produces 43.1 kN (9,700 lbf) thrust and gives a specific fuel consumption (sfc)=0.618 lb/hr-lbf.

A detailed description of the design point characteristics of the RB211-22 is given below.

RB211-22

INPUT DATA:

Cycle parameters

Turbine entry temperature (TET)	=	1398 K
Overall pressure ratio (OPR)	=	24.50
Bypass ratio	=	4.77
Fan pressure ratio	=	1.48
Mass flow of air	=	594.37 kg/s (1310 lb/s)
Polytropic efficiency of fan (AN. RB211-22, 1968)	=	0.81
Polytropic efficiency of compressor (AN. RB211-22, 1968)	=	0.93

Polytropic efficiency of turbine (AN. RB211-22, 1968)	=	0.875
Combustion efficiency	=	0.99
Isentropic intake efficiency	=	1.00
Isentropic efficiency of propelling nozzle	=	1.00
Mechanical efficiency of spool	=	0.99
Combustion pressure loss	=	5% of CDP
Flight parameters		
Ma	=	0.0
Altitude	=	Sea level ISA

OUTPUT:**Overall performance:**

Total gross thrust	=	176.5 kN (39700 lbf)
Fuel flow	=	6633 kg/hr (14619 lb/hr)
sfc (take off)	=	0.0375 kg/hr-N (0.368 lb/hr-lbf)
Fan gross thrust	=	130454 N (29329 lbf)
Hot nozzle gross thrust	=	46079 N (10359 lbf)
Fan nozzle pressure ratio	=	1.48 (Unchoked)
Hot nozzle pressure ratio	=	1.60 (Unchoked)
Hot nozzle area	=	673 sq.in
Cold nozzle area	=	2404 in ²
Cold mass flow	=	1083 lb/sec
Hot mass flow	=	227 lb/sec

6.3 Performance at Sea Level Conditions

First the sea level static performance at various engine speeds will be considered. Table No. 24 shows the values as reproduced from Wittenberg (1976). The design point is one with the engine rpm set for maximum continuous operation

and is shown in italics. From Table 24, it can be determined that the polytropic efficiencies of IP and HP compressor are $\eta_{c_{tot}} = 0.93$ and it has been assumed that this efficiency remains constant at all off-design conditions.

The working line of the IP and HP compressor spool is established from the values of hot mass flow parameter $\frac{m_h \sqrt{T_{t3}/T_{OQ}}}{P_{t3}/P_{OQ}}$ and compressor pressure ratio ϵ_c computed from the program. These values are then compared from the published values taken from the Table 24, and a graphical representation can be seen in Figure 6.1. Though there are differences in $\eta_{c_{tot}}$ in both the cases, the calculations show a good agreement with the published data.

From the program we can obtain total mass flow parameter $\frac{m_a \sqrt{T_{t3}/T_{OQ}}}{P_{t3}/P_{OQ}}$ and also get the values of fan pressure ratio ϵ_f and bypass ratio λ . These values are then compared with the published data, and the graphical representation can be seen in Figures 6.2 and 6.3.

The fan pressure ratio simulation in Figure 6.3 is good but the bypass ratio does not match well with the published data. The causes for this discrepancy can be attributed to various reasons, the main being the nozzle area difference (around 7% for hot nozzle and 3.5% for cold nozzle) between the actual published values and those calculated by the program at the design point for SLS condition.

Figure 6.4 shows a very good simulation of the predicted net thrust and the hot nozzle thrust with the published data. It can be seen that as the mass flow increases, the fan thrust increases much faster than the hot nozzle thrust.

From Figure 6.5 and Figure 6.6 it can be seen that the predicted jet pipe temperatures T_{t7} and hot nozzle pressure ratios P_{t7}/P_{tO} match quite well with the published data at cruising values of thrust, but start showing a marked difference at values of thrust dropping down to 50%. Since the value of P_{t7}/P_{tO} dictates the hot nozzle thrust, one can intuitively guess that the value of hot thrust at part loads will be significantly different from the published values. Since the fan nozzle thrust is not at all influenced, the total thrust at part loads may or may not be

totally different from the actual values.

From Figure 6.7 it can be seen that the predicted fuel/air ratio deviates not more than 10% from the actual published values. This small deviation may be due to slightly different core flow or slightly different compressor delivery temperature T_{t4} .

6.4 Performance at Cruise Conditions

The aircraft is assumed flying at 30,000 feet and at Mach no.=0.8. At this flight condition, the program was run for various turbine inlet temperatures and the predicted values compared with the published data. In Figure 6.8 comparison is made between the predicted fan pressure ratio and the published data. It can be seen that the deviation is of the order of 10%, which is reasonable taking into account all the assumptions made for the off-design analysis. The predicted bypass ratio does not match well with the published values though it follows the trend, as can be seen from Figure 6.9.

From Figure 6.10 it can be seen that the predicted net thrust diverges from the published values only when thrust is less than 50% of the maximum cruise thrust. Both Figures 6.11 and 6.12 show that gross thrust developed by cold and hot nozzles matches 'in toto' with the published values. From Figure 6.13 one can see the invariance of the working line of IP+HP spool as it remains nearly the same to that found at ISA SLS. The reason for this is that the LP turbine always remains choked, so the IP+HP spool is shielded from the effects of varying Mach number as well as from the changing conditions downstream of LP turbine.

A parametric analysis was done to see the effects of Mach number on bypass ratio, sfc and overall efficiency. It can be seen from Figure 6.14 that the sfc goes down initially as the thrust is increased, but the ram drag also increases and so we have an increase in sfc as thrust is increased further. From Figure 6.15 it can be seen that the bypass ratio becomes nearly independent of Mach number as thrust

level is increased, and finally all the three curves merge together. The significance of this is that to produce the same thrust, a higher turbine inlet temperature is required for aircrafts operating at higher Mach numbers.

6.5 Simulation of Bleed Flow in Turbofans

6.5.1 Introduction

Nowadays the thrust of the aeroengine industry is to increase the power/weight ratio and to decrease the specific fuel consumption of engines. This requires changes to be made in various areas ranging from materials and internal aerodynamics to the thermodynamics of the engine. The trend of major thermodynamic parameters from the 50's to the 80's, shows a consistent steady growth of overall pressure ratio and turbine entry temperature (TET).

As the turbine entry temperature increases, the need for better heat resistant materials for the turbine blades also increases. Another thing that begins to occupy a paramount position as the turbine inlet temperature exceeds 1250-1300 K is the need of cooling air for the turbine (especially the first stage). This cooling air is bled from the HP compressor to the HP turbine (single or two stages as the case may be) and sometimes to the first stage of the LP turbine.

Because of the complexity of the cooling air outlets and inlets, it is clear that we have to make a certain number of assumptions in order to create a model of cooling phenomena in the engine. Some of these assumptions may appear questionable, but one should always bear in mind that we are trying to investigate a complicated phenomena, in the sense that it varies from engine to engine configuration and is also very much dependent on the current technology.

Figure A9 shows that the percentage of the cooling air with respect to TET has varied considerably from the 60's to the 80's (Sjolander et al., 1988). Another thing to be taken into account is that the mathematical model of the turbofan

(with the cooling flow model incorporated) is a generalized one, and so the cooling flow model may suit one configuration of engine but not match with a different configuration of turbofan.

FIGURE A9 HIGH PRESSURE TURBINE BLADE COOLING FLOW TRENDS.

(REPRODUCED FROM SJOLANDER ET AL.)

Since it is very difficult to obtain performance data which shows the effect of bleed on the performance of the engine, the author has tried to perform a parametric analysis on RB211-22 with and without bleed. In this analysis only the crucial parameters affected by bleed are taken into account.

6.5.2 Cooling

In modern aeroengines the need for air cooling the combustor and the turbine has become an important consideration. Cooling air is used to control the temperature of the turbine shafts and discs, control the thermal fatigue and uncontrolled expansion and contraction rates. Flow is controlled by interstage seals and, on completion of the cooling function, the air is expelled into the main gas stream. (Rolls-Royce, 1986).

According to Wilde (1977) the loss in efficiency is 0.79 % for every 1% of rotor cooling flow in a moderately loaded turbine operating at 1500 K.

6.5.3 Basic Assumptions

The cooling flow model needs several assumptions; some of these have been made in order to reduce the complexity level of the model.

- 1) The total air bleed from the HP compressor is composed of two major components:
 - The cooling air for the turbine(s).
 - The air bleed for cabin pressurization, pneumatic actuation of accessories etc.
- 2) The cooling air is a certain fixed percentage of the core flow, both for design and off-design analysis.
- 3) Air is bled from the exit of the HP compressor.
- 4) The air bleed for cabin pressurization, accessories etc is equal to the amount of fuel added in the combustor.
- 5) The cooling air is split between the HP turbine stages (in case of RB211-22 there is only one stage) and the IP or LP turbine stages as required. (Wilde, 1977)
- 6) The bleed air mixes with the core flow in the stator of the HP turbine.
- 7) The bleed air enthalpy contributes fully to the fuel/air mixture enthalpy i.e the mixing in the stator is ideal.
- 8) Throughout the model, the values of C_{p_g} , C_{p_a} and γ have been assumed to be constant.

6.5.4 Modelling of Cooling Flow

Searching the literature to find the effect of cooling flow on the performance

parameters of the engine, reveals that much has not been published in this area. According to Neitzel and Hemsworth (1966) a correction for cooling can be made merely by substituting an equivalent value of TET which is about 30 K below the actual TET value of the cooled turbine. This is a very simplified approach, so it is discarded without further consideration.

Broadly the procedure that has been followed to simulate bleed flow is that due to mixing of the cooling flow (lower enthalpy) with the core flow in the stator, the stator outlet temperature (SOT) is reduced which leads to a different working point of the HP+IP spool. For this setting of the bleed flow, LP spool equilibrium is obtained, after which calculations are done to get other performance parameters.

A simple way to approach the problem in the case of the RB211-22 is by modelling the HP+IP together and assuming that 70% of the bleed flow mixes with the core flow at the stator entry of the HP turbine, while the rest of the bleed flow goes to the stator of the IP turbine.

6.5.4.1 Mass flow matching

Taking the above assumption into account we can write

$$\frac{m_h \sqrt{T_{t3}}}{P_{t3}} = \left(\frac{(m_h - 0.3bl) \sqrt{T_{t5}}}{P_{t5}} \right) \sqrt{\frac{T_{t3}}{T_{t5}}} \frac{P_{t5}}{P_{t3}} \frac{m_h}{m_h(1 - 0.3bl)} \quad (6.1)$$

where bl is the ratio of bleed flow to core flow. The quantity $\frac{(m_h - 0.3bl) \sqrt{T_{t5}}}{P_{t5}}$ is a fixed quantity due to the choked HP turbine. Therefore we get an expression for the non dimensional mass flow

$$\frac{m_h \sqrt{T_{t3}}}{P_{t3}} = \left(\frac{m_h \sqrt{T_{t3}}}{P_{t3}} \right)_{des} \frac{\frac{1}{(1-0.3bl)}}{\frac{1}{(1-0.3bl)}_{des}} \sqrt{\frac{(T_{t5}/T_{t3})_{des}}{T_{t5}/T_{t3}}} \frac{\epsilon_{HPC}}{(\epsilon_{HPC})_{des}} \quad (6.2)$$

which is based on the assumption that at design point 70% of bleed flow goes to HP turbine and 30% to IP turbine, i.e. the same as at off-design conditions.

6.5.4.2 Power spool matching

If we assume that not all of the bleed flow is going to do useful work in the HP+IP turbine, then the power matching equation changes to

$$m_h C_{pa} T_{t3} (T_{t4}/T_{t3} - 1) = \eta_m (m_h - 0.3bl) C_{pg} T_{t5} (1 - T_{t6}/T_{t5})$$

on assuming that 70% of the bleed flow does useful work in the HP+IP turbine. On further modification we get

$$\frac{(T_{t4}/T_{t3} - 1)}{(T_{t4}/T_{t3})_{des} - 1} = \frac{(T_{t5}/T_{t3})}{(T_{t5}/T_{t3})_{des}} \frac{m_h(1 - 0.3bl)}{(m_h(1 - 0.3bl))_{des}}$$

which is again based on the assumption that 70% of bleed flow does useful work in HP+IP turbine at design point.

So the HP+IP spool setting is fixed with the only difference being that the parameter (SOT) is now taken into account. Hence the matching of the HP+IP spool gives new expressions for compressor pressure ratio and mass flow which are similar to equations (5.24) and (5.26) provided the above mentioned assumptions hold true.

6.5.4.3 LP spool matching

For LP spool power matching we get

$$m_a C_{pa} (T_{t3} - T_{t2}) = \eta_m m_h C_{pg} (T_{t6} - T_{t7})$$

because the bleed flow gets mixed with the core flow in the LP turbine and so there is no bleed effect in LP turbine. On referencing with respect to design point we get

$$\frac{1}{\psi} \left(\frac{\lambda + 1}{\lambda_{des} + 1} \right) \frac{(1 - T_{t2}/T_{t3})}{1 - (T_{t2}/T_{t3})_{des}} = \frac{(1 - T_{t7}/T_{t6})}{1 - (T_{t7}/T_{t6})_{des}} \quad (6.3)$$

Hence the LP spool power matching yields a similar expression as equation (5.28).

6.5.5 Parametric Analysis of RB211-22

The program was run at different settings of bleed flow at take off condition, and it was found that with increasing bleed flow there is a corresponding increase in HP+IP turbine stator areas, LP turbine stator areas and exhaust hot nozzle area. There was no change in cold nozzle area with variation in bleed flow. The hot nozzle area variation can be seen from Figure 6.16 with different design points at which the nozzles are to be sized. Usually bleed flow for cooling of turbines is less than 5% and hence the increase in area is less than 20% from the reference nozzle area at zero bleed. Once the areas of the exhaust nozzles are fixed at SLS takeoff conditions, then with bleed flow remaining constant the program is run at typical cruise conditions (30,000 ft, Ma=0.8), and the performance parameters obtained.

Since we are trying to find the effect of bleed on the performance parameters, the TET at cruise conditions was held constant for all the cases. The factor that is affected most is the hot nozzle thrust, because with the bleed flow there is a higher pressure drop across the LP turbine, leading to a decrease in pressure ratio across the hot nozzle. The decrease of hot nozzle thrust causes consequent decrease of total net thrust as can be seen from Figure 6.17. The variation of cold thrust is insignificant i.e. there is a 3% decrease in cold thrust when bleed (first stage rotor cooling) is 5% of the total core flow.

Moreover it can be seen from Figure 6.17 that for every 1% increase of bleed flow there is around 3.2% decrease of net thrust. A similar expression relating bleed flow to percent thrust loss is given by Raymer (1989) which states that

$$\text{percent thrust loss} = C_{\text{bleed}} \left(\frac{\text{bleed mass flow}}{\text{engine core flow}} \right)$$

where the value of C_{bleed} is approximately equal to 2. So the engine bleed air exacts a thrust penalty that is more-than-proportional to the percent of the total engine mass flow extracted as bleed air.

There is a decrease of fuel flow as the bleed flow increases for constant TET as can be seen from Figure 6.18, but the rate of change of fuel flow is not high enough to counter the higher rate of decrease of net thrust and so there is an increase of sfc. The variation of sfc with bleed flow can be seen in Figure 6.19, where it is almost linear which agrees well with the similar results (see Figure A10) from Toreenbeek (1976).

It is found that with increasing bleed flow the SOT drops which leads to lower values of HP+IP compressor pressure ratio and core flow. The variation of HP+IP spool characteristics due to increase in bleed flow about a particular working point (30,000 ft, $Ma=0.8$ & $TET=1270$ K) can be seen in Figure 6.20. The variation of all the relevant parameters with increase in bleed flow can be seen from Table 29, where the base is "zero" blade cooling.

FIGURE 6.1 COMPARISON OF PREDICTED AND PUBLISHED WORKING LINE OF IP+HP SPOOL

FIGURE 6.2 COMPARISON OF PREDICTED AND PUBLISHED BYPASS RATIO

FIGURE 6.3 COMPARISON OF PREDICTED AND PUBLISHED FAN PRESSURE RATIO

FIGURE 6.4 COMPARISON OF PREDICTED NET THRUST AND HOT NOZZLE THRUST WITH PUBLISHED DATA

FIGURE 6.6 COMPARISON OF PREDICTED HOT NOZZLE PRESSURE RATIO WITH THE PUBLISHED DATA

FIGURE 6.5 COMPARISON OF PREDICTED LP EXIT TEMPERATURE WITH THE PUBLISHED DATA

FIGURE 6.7 COMPARISON OF PREDICTED FUEL/AIR RATIO WITH PUBLISHED DATA

FIGURE 6.8 COMPARISON OF PREDICTED AND PUBLISHED FAN PRESSURE RATIO

FIGURE 6.9 COMPARISON OF PREDICTED BYPASS RATIO
WITH THE PUBLISHED DATA

FIGURE 6.10 COMPARISON OF PREDICTED NET THRUST WITH PUBLISHED DATA

FIGURE 6.11 COMPARISON OF PREDICTED GROSS COLD THRUST WITH PUBLISHED DATA

FIGURE 6.12 COMPARISON OF PREDICTED HOT NOZZLE GROSS THRUST WITH PUBLISHED DATA

FIGURE 6.13 COMPARISON OF PREDICTED WORKING LINE OF IP+HP SPOOL WITH PUBLISHED DATA

FIGURE 6.14 PREDICTED SPECIFIC FUEL CONSUMPTION WITH THRUST AT DIFFERENT MACH NUMBERS

FIGURE 6.15 PREDICTED BYPASS RATIO WITH NET THRUST
AT DIFFERENT MACH NUMBERS

FIGURE 6.16 EFFECT OF BLEED FLOW ON HOT NOZZLE AREA FOR DIFFERENT DESIGN POINTS

FIGURE 6.17 PREDICTED EFFECT OF BLEED ON NET THRUST AND HOT THRUST

FIGURE 6.18 PREDICTED EFFECT OF BLEED ON FUEL FLOW

FIGURE 6.19 PREDICTED EFFECT OF BLEED ON SFC

FIGURE A10 EFFECT OF BLEED TAKEOFF ON SFC
(REPRODUCED FROM TORENBECK)

FIGURE 6.20 PREDICTED EFFECT OF BLEED ON HP+IP COMPRESSOR CHARACTERISTICS

Further Validation of The Program 7

7.1 Introduction

The purpose of this chapter is to validate the program against any engine chosen randomly. Data were obtained from Pratt & Whitney Canada for a study engine which was a predecessor of the PW300. Enough data were available to show overall performance predictions over a wide range of flight conditions. The objectives were to ascertain the design point of the engine and then to predict the off-design performance without modifying any of the input parameters, in order to test the validity of the methods developed in the thesis.

7.2 Comparison of PWX

The PW study engine is an advanced technology high bypass turbofan engine in the 5000 lbf thrust class, which has been optimised for low specific fuel consumption. This Pratt & Whitney study engine will henceforth be referred to as PWX. The engine is flat rated (thermodynamically) at 23.2 kN (5214 lbf.) upto 30° C at SLS conditions. At the cruise condition (40,000 ft and $Ma = 0.8$), the study engine produces 4.72 kN (1060 lbf) uninstalled thrust with a specific fuel consumption (sfc) of $= 0.698 \text{ lb/hr-lbf}$.

The design point characteristics are given below.

PWX

INPUT DATA:

Cycle parameters

Turbine entry temperature (TET)	=	1475 K
Overall pressure ratio (OPR)	=	22.70
Bypass ratio	=	4.3
Fan pressure ratio	=	1.746
Mass flow of air	=	73.503 kg/sec (162 lb/sec)

Assumed Component efficiencies

Polytropic efficiency of fan	=	0.88
Polytropic efficiency of compressor	=	0.87
Polytropic efficiency of turbine	=	0.87
Combustion efficiency	=	0.99
Isentropic intake efficiency	=	1.00
Isentropic efficiency of propelling nozzle	=	0.98
Mechanical efficiency of spools	=	0.98
Combustion pressure loss	=	5% of CDP

Flight parameters

Ma	=	0.0
Altitude	=	Sea level
Pressure and Temperature	=	1.0132 bar and 15° C

OUTPUT:**Overall performance:**

Total net thrust	=	23.15 kN (5204 lbf)
Fuel flow	=	991.4 kg/hr (2185 lb/hr)
sfc (Take off)	=	0.04264 kg/hr-N (0.418 lb/hr-lbf)
Fan nozzle pressure ratio	=	1.75 (Unchoked)
Hot nozzle pressure ratio	=	1.27 (Unchoked)
Hot nozzle area	=	141 in ²
Cold nozzle area	=	241 in ²

Cold mass flow	=	131.43 lb/sec
Hot mass flow	=	30.57 lb/sec

7.3 Performance at Different Flight Conditions

First the program was run at ISA SL/3 conditions and the values of component efficiencies iterated until a satisfactory solution was obtained. The fixing of SLS thrust and fuel flow fixes the size of the exhaust nozzles. The program was then run at varying Mach numbers at sea level to get net thrust, fuel flow and other performance parameters. It was noticed that the value of TET does not change much as the Mach number is increased because there is little change in aerodynamic speed at sea level conditions. Figure 7.1 shows the predicted and published values of net thrust and fuel flow at take off conditions.

The program was then run at maximum cruise conditions with varying Mach number at a particular altitude. The predicted values of net thrust and fuel flow were then compared with published data (Reference 21) as shown in Figures 7.2, 7.3, 7.4 and 7.5. It can be seen that even when the component efficiencies are assumed constant, there is a good relationship between the predicted and published data. Only in the case of flight at ISA 40,000 ft the compressor polytropic efficiency had to be changed from 0.87 to 0.85, and this is quite reasonable because of lower Reynolds number at higher altitudes.

An interesting feature can be seen in Figures 7.6 and 7.7 where the ratio of cold nozzle net thrust to hot nozzle net thrust changes from nearly 3.1 to 2.6 as altitude increases. Hence at lower altitudes the fan produces nearly 80% of the total thrust but at higher altitudes (or at cruising altitudes) it comes down to nearly 75% of the total thrust. It can also be seen that as the Mach number increases the non dimensional mass flow $\frac{m_a \sqrt{\theta}}{\delta}$ goes down with a consequent decrease of cold net thrust.

Another thing that is noticed is that the hot nozzle thrust is nearly independent of Mach number, while the fan thrust decreases considerably as Mach number is increased. Since similar behaviour was observed with the Garrett 731-2, it can be broadly summarised that in case of twin spool or three spool turbofans the increase of Mach number is borne only by the outer LP spool, and the HP or the HP+IP spool is shielded by the LP spool from downstream conditions.

Figure 7.8 shows the effect of Mach number on the bypass ratio, where it can be seen that at 40,000 ft the predicted bypass ratios are higher than at 30,000 ft. This is caused by the decrease of compressor efficiency, which is equivalent to throttling of the HP spool which leads to an increase of bypass ratio as shown for the case of Garrett 731-2 in Chapter 5. The maximum change in bypass ratio that occurs with reference to the sea level value is only 7% .

Figures 7.9 and 7.10 show that as the Mach number is increased the fan or core pressure ratios come down because of decrease in aerodynamic speed with increasing fan or compressor entry temperature. The maximum increase in fan pressure ratio with reference to sea level fan pressure ratio is around 17% , while maximum change in core pressure ratio is around 11% with respect to sea level core pressure ratio of 13:1. The increases in fan or core pressure ratio are caused by the increase of aerodynamic speed that occurs as the altitude is increased.

Figure 7.11 shows the variation of predicted TET as Mach number is changed at a particular altitude. It shows that at lower altitudes the change in aerodynamic speed is not much as Mach number is increased, hence TET/T_{t1} variation is small, leading to a small increase of TET. At higher altitudes, there is a big change in aerodynamic speed which leads to more rapid rise of TET with Mach number. This is because at higher altitudes ambient temperature T_a goes down, and since $T_{t1} = T_a(1 + \frac{\gamma-1}{2}(Ma)^2)$ it causes T_{t1} to go down which in turn leads to a rapid rise of aerodynamic speed $N/\sqrt{T_{t1}}$.

7.4 Conclusions

The excellent agreement obtained with the published values assuming only state of the art component efficiencies proves the veracity and potency of the method. Above 30,000 ft, variation of compressor efficiency is to be taken into account for better simulation. Moreover the change in TET from $TET_{T.O.}$ to TET_{cruise} is around 100 K, which is consistent for this (5000 lbf) class of engines. (Personal conversation of Prof. Saravanamuttoo with B.J. Palfreeman of Pratt & Whitney Canada). This is also in trend with Garrett 731-2 which has a Δ TET variation of 70 K because it is a smaller (3500 lbf thrust) engine.

It was determined that TET variation with Mach number correlated best when the mechanical speed of the engine was taken as 94% of the maximum engine speed. Though there is no evidence to support this finding, but usually 93 - 94% is the HP spool mechanical speed of modern turbofans at cruise.

FIGURE 7.1 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 7.2 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 7.3 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 7.4 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 7.5 COMPARISON OF PREDICTED AND PUBLISHED PERFORMANCE DATA

FIGURE 7.6 PREDICTED FAN THRUST AND HOT NOZZLE THRUST WITH MACH NUMBER

FIGURE 7.7 PREDICTED FAN THRUST AND HOT NOZZLE THRUST WITH MACH NUMBER

FIGURE 7.8 PREDICTED BYPASS RATIO

FIGURE 7.9 PREDICTED FAN PRESSURE RATIO

FIGURE 7.10 PREDICTED COMPRESSOR PRESSURE RATIO

FIGURE 7.11 PREDICTED T.T. VARIATION WITH MACH NUMBER

Conclusions and Recommendations 8

8.1 Conclusions

- 1) For a single spool turbojet simple analytical expressions have been developed to establish the working line of the compressor and the off-design performance of the engine when the exhaust nozzle is choked. When the exhaust nozzle is unchoked, the procedure becomes slightly more complicated as one has to resort to generalized turbine characteristics and find the intersection point between the turbine/nozzle characteristics curve and the compressor curve.
- 2) The predicted results obtained for three turbojet engines agree well with the published values. In order to resolve the discrepancy between the predicted and published data, slight adjustments can be made in compressor efficiency especially at higher altitudes.
- 3) The off-design performance of twin-spool turbofans can be estimated with the assumption that the low pressure turbine is choked; this condition fixes the working line of the inner HP spool.
- 4) The performance calculations for the Garrett TFE 731-2 and PW Study engine agree quite well with the published data, even when the compressor efficiency is assumed constant throughout.
- 5) It was seen that the net hot nozzle thrust for Garrett 731-2 and PW Study engine is nearly independent of the Mach number, while the net fan thrust decreases as the Mach number is increased. In other words it means that the effect of the Mach number is being borne by

the LP spool only, as the HP spool is shielded by the LP spool from the downstream conditions. The performance of three spool turbofan engines can be calculated by the same method as the twin spool type by combining the IP and HP spool together into a single combined (IP+HP) spool.

- 6) The simulation as performed on RB211-22 showed good agreement with the published data. The values at sea level static conditions show good agreement down to 50% of the maximum thrust, after that the agreement is not reasonable. For the flight condition at a typical Mach number and altitude (30000 ft, $Ma=0.8$) the simulation is quite good, even at low values of thrust.
- 7) Certain assumptions like bleed being equal to fuel flow, C_p and C_v remaining constant throughout, cause considerable errors in individual component thermodynamic analysis. But it has been found empirically that several of these effects cancel each other out when the overall engine performance is considered. For conventional engine configurations the results are found out to be reasonably accurate, proving the validity of the above assumption.

8.2 Recommendations for Future Work

The model that has been made can be improved upon by incorporating mixing of hot and cold flow in the mixer. The need for it arises because nowadays quite a lot of mixed flow turbofans are being used in the market. The mixing brings about a theoretical thrust gain and also helps the fan running line to operate in a more efficient operating region. (Williams, 1984) The added constraint of fan and core streams having nearly same static pressure in the mixer, complicates the model, but it can be a fruitful exercise.

Then attention can be given to the concept of variable cycle engines, by incorporating a simple modification of variable exhaust nozzles in the mathematical model. The task of varying the area of the exhaust nozzles is quite simple, but incorporating variable area stators can be slightly daunting.

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TABLES

Table 1. Experimental data of J35-A29 jet engine.(Reproduced
from Wittenberg)

run no	Compressor			Combustion chamber				Turbine and nozzle		
	$\frac{n}{\sqrt{T_{t2}/T_0}}$	T_{t2} R°	ϵ_c	T_{t3} R°	$\frac{m_a \sqrt{T_{t3}/T_0}}{P_{t2}/P_0}$ (lb/sec)	T_{t4} R	P_{t4}/P_{t3}	P_{t4}/P_{t5}	η_t	P_{t4}/P_0
1 †	5646	516	2.015	679	53.9	869	0.9477	2.496	0.813	1.758
2	5744	523	2.137	697	56.1	923	0.9478	2.516	0.81	1.847
3	6935	516	2.974	749	67.0	1280	0.9581	2.521	0.80 0	2.563
4	7200	522	4.209	844	80.5	1790	0.9592	2.502	0.79	3.642
5	7856	518	5.083	899	89.3	2060	0.9638	2.505	0.795	4.516
6	8030	517	5.247	910	90.7	2130	0.9637	2.502	0.798	4.5638

†Design point

Table 2. Comparison of predicted data with published performance of J35-A29 engine

Calculated Data					Simulated output	
T_{t_4}/T_{t_2}	ψ	η_c	$\epsilon_c \ominus$	$m_a\sqrt{\theta}/\delta \ominus$	$\epsilon_c \diamond$	$m_a\sqrt{\theta}/\delta \diamond$
1.684	0.409	0.729	2.015	24.13	2.214	27.12
1.765	0.428	0.756	2.137	25.45	2.288	27.39
2.481	0.620	0.836	2.974	30.40	3.029	30.55
3.429	0.832	0.855	4.209	36.52	4.225	36.26
3.977	0.965	0.843	5.083	40.52	5.036	40.15
4.120 †	1.000	0.838	5.247	41.15	5.247	41.15

† Design point

$$\ominus \eta_c = \eta_{c_{des}}$$

$$\diamond \eta_c = \eta_{c_{var}}$$

Table 3. Comparison of predicted data with published performance of XJ79 GE-1 engine at constant compressor efficiency

Calculated Data					Simulated output	
T_{t_4}/T_{t_2}	ψ	η_c	ϵ_c	$m_a\sqrt{\theta}/\delta$	$\epsilon_c \diamond$	$m_a\sqrt{\theta}/\delta \diamond$
4.703 †	1.000	0.792	13.68	84.86	13.68	84.86
4.454	0.947	0.812	13.28	84.33	12.50	79.66
4.306	0.916	0.818	12.90	83.77	11.83	76.67
3.928	0.835	0.847	11.93	80.83	10.20	69.24
3.608	0.767	0.853	10.37	75.60	8.94	63.38
3.296	0.701	0.853	8.72	65.00	7.32	58.01

† Design point

$$\diamond \eta_c = \eta_{c_{des}}$$

Table 4. Comparison of predicted data with published performance of XJ79 GE-1 engine at variable compressor efficiency

Calculated Data					Simulated output	
T_{t_4}/T_{t_2}	ψ	η_c	ϵ_c	$m_a\sqrt{\theta}/\delta$	$\epsilon_c \diamond$	$m_a\sqrt{\theta}/\delta \diamond$
4.703 †	1.000	0.792	13.68	84.86	13.67	84.82
4.454	0.947	0.812	13.28	84.33	13.25	84.61
4.306	0.916	0.818	12.90	83.77	12.88	83.84
3.928	0.835	0.847	11.93	80.83	12.20	83.00
3.608	0.767	0.853	10.37	75.60	10.60	73.34
3.296	0.701	0.853	8.72	65.00	9.21	67.87

† Design point

$\diamond \eta_c = \eta_{cvar}$

Table 5. Variation of turbine inlet temperature with aerodynamic speed at ISA sea level

Mechanical speed $N/N_D = 1.0$							
MACH NO.	0.1	0.2	0.3	0.4	0.5	0.6	0.7
P_{t_1} bar	1.007	1.028	1.065	1.117	1.186	1.276	1.387
T_{t_1} K	288.8	290.5	293.4	297.4	302.6	309.0	316.4
$N/N_a\sqrt{288.2/T_{t_1}}$	.998	.996	.990	.985	.975	.965	.954
TIT K	1147	1145	1144	1145	1147	1143	1145

Table 6. Variation of turbine inlet temperature with aerodynamic speed at ISA 10,000 ft

Mechanical speed $N/N_D = .94$							
<i>MACH NO.</i>	0.1	0.2	0.3	0.4	0.5	0.6	0.7
P_{t_1} bar	0.692	0.707	0.733	0.768	0.816	0.877	0.954
T_{t_1} K	268.9	270.5	273.2	276.9	281.8	297.7	294.6
$N/N_d\sqrt{288.2/T_{t_1}}$	0.973	0.970	0.965	0.959	0.951	0.940	0.930
<i>TIT K</i>	1014	1012	1011	1010	1015	1016	1016

Table 7. Variation of turbine inlet temperature with aerodynamic speed at ISA 20,000 ft

Mechanical speed $N/N_D = .94$							
<i>MACH NO.</i>	0.2	0.3	0.4	0.5	0.6	0.7	0.8
P_{t_1} bar	0.473	0.490	0.514	0.546	0.586	0.638	0.700
T_{t_1} K	250.6	253.	256.5	261.0	266.5	272.9	280.4
$N/N_d\sqrt{288.2/T_{t_1}}$	1.007	1.003	0.996	0.987	0.978	0.965	0.953
<i>TIT K</i>	1023	1021	1019	1018	1018	1013	1012

Table 8. Variation of turbine inlet temperature with aerodynamic speed at ISA 30,000 ft

Mechanical speed $N/N_D = .94$							
<i>MACH NO.</i>	0.2	0.3	0.4	0.5	0.6	0.7	0.8
P_{t_1} bar	0.305	0.316	0.332	0.352	0.379	0.412	0.452
T_{t_1} K	230.6	232.9	236.5	240.2	245.2	251.2	258.0
$N/N_d\sqrt{288.2/T_{t_1}}$	1.051	1.046	1.039	1.030	1.020	1.007	.993
<i>TIT K</i>	1045	1042	1042	1036	1035	1033	1024

Table 9. Variation of turbine inlet temperature with aerodynamic speed at ISA 40,000 ft

Mechanical speed $N/N_D = .94$							
<i>MACH NO.</i>	0.2	0.3	0.4	0.5	0.6	0.7	0.8
P_{t_1} bar	0.191	0.198	0.207	0.220	0.237	0.258	0.283
T_{t_1} K	218.4	220.6	223.6	227.5	232.3	237.9	244.4
$N/N_d\sqrt{288.2/T_{t_1}}$	1.08	1.074	1.067	1.058	1.047	1.034	1.021
<i>TIT K</i>	1077	1074	1071	1070	1064	1059	1052

Table 10 . Variation of turbine inlet temperature with altitude and
Mach number for VIPER 20 F-10.

		TURBINE INLET TEMPERATURE K									
MACH NUMBER	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8			
SEA LEVEL	1147	1145	1144	1145	1148	1143	1145	—			
10,000 FT	1014	1012	1011	1010	1015	1016	1016	—			
20,000 FT	—	1023	1021	1019	1018	1018	1013	1012			
30,000 FT	—	1045	1042	1042	1036	1035	1033	1024			
40,000 FT	—	1077	1074	1071	1070	1064	1059	1052			

Table 11. Comparison of predicted performance parameters with published data of VIPER 20 F-10 at ISA sea level.

SIMULATED RESULTS																	
PUBLISHED DATA																	
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8					
NET THRUST lbf	2802	2792	2758	2808	2903	---	2819	2775	2781	2794	2827	---					
FUEL FLOW lbs/hr	3291	3429	3572	3708	3918	---	3379	3462	3595	3692	3870	---					
CPR	5.15	5.10	5.02	4.93	4.82	---	5.26	5.17	5.07	4.92	4.78	---					
MASS FLOW lbs/hr	24.20	25.0	26.0	27.3	28.9	---	24.26	25.0	26.04	27.2	28.7	---					
†S.F.C lbs/hr-lbf	1.175	1.228	1.283	1.336	1.350	---	1.199	1.248	1.293	1.333	1.369	---					

†Based on net thrust

Table 12. Comparison of predicted performance parameters with published data of VIPER 20 F-10 at ISA 10,000 ft.

PUBLISHED DATA																		SIMULATED RESULTS									
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8									
NET THRUST lbf	1732	1715	1703	1728	1764	—	1727	1693	1692	1706	1746	—	1727	1693	1692	1706	1746	—									
FUEL FLOW lbs/hr	1925	1995	2060	2144	2254	—	1952	1995	2067	2151	2261	—	1952	1995	2067	2151	2261	—									
CPR	4.91	4.87	4.77	4.67	4.58	—	4.94	4.85	4.76	4.64	4.53	—	4.94	4.85	4.76	4.64	4.53	—									
MASS FLOW lbs/hr	16.72	17.3	17.95	18.84	19.87	—	16.64	17.15	17.86	18.74	19.84	—	16.64	17.15	17.86	18.74	19.84	—									
†S.F.C lbs/hr-lbf	1.111	1.163	1.210	1.241	1.281	—	1.130	1.178	1.220	1.261	1.295	—	1.130	1.178	1.220	1.261	1.295	—									

†Based on net thrust

Table 13. Comparison of predicted performance parameters with
published data of VIPER 20 F-10 at ISA 20,000 ft.

MACH NO.	PUBLISHED DATA								SIMULATED RESULTS							
	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6
NET THRUST lbf	1326	1330	1341	1370	1393	1436	1395	1372	1364	1367	1374	1412	1395	1372	1364	1367
FUEL FLOW lbs/hr	1432	1478	1540	1614	1678	1778	1509	1543	1587	1639	1690	1778	1509	1543	1587	1639
CPR	5.29	5.22	5.15	5.05	4.87	4.74	5.48	5.38	5.25	5.10	4.92	4.77	5.48	5.38	5.25	5.10
MASS FLOW lbs/hr	12.07	12.48	13.00	13.64	14.44	15.41	12.32	12.69	13.17	13.77	14.48	15.40	12.32	12.69	13.17	13.77
†S.F.C lbs/hr-lbf	1.08	1.111	1.148	1.178	1.205	1.238	1.082	1.125	1.164	1.20	1.230	1.260	1.082	1.125	1.164	1.20

†Based on net thrust

Table 14. Comparison of predicted performance parameters with published data of VIPER 20 F-10 at ISA 30,000 ft at constant compressor efficiency.

PUBLISHED DATA													SIMULATED RESULTS						
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8							
NET THRUST lbf	991	1004	1009	1033	1057	1107	1124	1115	1109	1111	1117	1129							
FUEL FLOW lbs/hr	1034	1085	1121	1182	1234	1299	1171	1219	1236	1273	1314	1357							
CPR	5.71	5.65	5.55	5.45	5.32	5.18	6.26	6.20	6.00	5.81	5.59	5.36							
MASS FLOW lbs/hr	8.37	8.71	9.11	9.62	10.20	10.89	9.04	9.36	9.65	10.07	10.56	11.15							
†S.F.C lbs/hr-lbf	1.043	1.081	1.111	1.144	1.167	1.173	1.042	1.081	1.114	1.146	1.176	1.202							

†Based on net thrust

Table 15. Comparison of predicted performance parameters with published data of VIPER 20 F-10 at ISA 30,000 ft at variable compressor efficiency.

PUBLISHED DATA													SIMULATED RESULTS												
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8													
NET THRUST lbf	991	1004	1009	1033	1057	1107	1028	1044	1040	1058	1079	1107													
FUEL FLOW lbs/hr	1034	1085	1121	1182	1234	1299	1095	1151	1179	1229	1280	1338													
CPR	5.71	5.65	5.55	5.45	5.32	5.18	5.76	5.77	5.65	5.55	5.41	5.26													
MASS FLOW lbs/hr	8.37	8.71	9.11	9.62	10.20	10.89	8.32	8.71	9.09	9.62	10.22	10.94													
†S.F.C lbs/hr-lbf	1.043	1.081	1.111	1.144	1.167	1.173	1.065	1.102	1.134	1.162	1.186	1.204													

†Based on net thrust

Table 16. Comparison of predicted performance parameters with published data of VIPER 20 F-10 at ISA 40,000 ft at constant compressor efficiency.

	PUBLISHED DATA						SIMULATED RESULTS					
	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8
MACH NO.												
NET THRUST lbf	690	691	697	715	734	760	854	836	830	838	837	846
FUEL FLOW lbs/hr	727	738	774	806	827	894	879	889	908	943	964	996
CPR	6.00	5.97	5.83	5.72	5.60	5.45	7.12	6.93	6.73	6.54	6.25	5.98
MASS FLOW lbs/hr	5.47	5.66	5.92	6.27	6.69	7.16	6.30	6.45	6.66	6.96	7.27	7.66
†S.F.C lbs/hr-lbf	1.054	1.068	1.116	1.127	1.127	1.176	1.029	1.063	1.094	1.125	1.152	1.177

†Based on net thrust

Table 17 .Comparison of predicted performance parameters with published data of VIPER 20 F-10 at ISA 40,000 ft at variable compressor efficiency.

	PUBLISHED DATA						SIMULATED RESULTS						
	MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8
NET THRUST lbf		690	691	697	715	734	760	731	726	732	750	759	760
FUEL FLOW lbs/hr		727	738	774	806	827	894	782	798	826	866	895	954
CPR		6.00	5.97	5.83	5.72	5.60	5.45	6.17	6.08	5.98	5.89	5.70	5.51
MASS FLOW lbs/hr		5.47	5.66	5.92	6.27	6.69	7.16	5.46	5.66	5.92	6.26	6.62	7.14
†S.F.C lbs/hr-lbf		1.054	1.068	1.116	1.127	1.129	1.176	1.07	1.100	1.128	1.155	1.179	1.180

†Based on net thrust

Table 18 . Comparison of predicted performance parameters with published data of GARRETT TFE 731-2 at ISA Sealevel.

BROCHURE DATA †											SIMULATED RESULTS				
MACH NO.	0	0.1	0.2	0.3	0.4	0.5	0	0.1	0.2	0.3	0.4	0.5	0.5	0.5	0.5
NET THRUST lbf	3514	3180	2890	2630	2440	2300	3514	3104	2820	2593	2391	2237	2391	2391	2237
FUEL FLOW lbs/hr	1700	1735	1750	1770	1790	1795	1724	1724	1739	1764	1786	1823	1786	1786	1823
S.F.C lbs/hr-lbf	0.486	0.546	0.606	0.673	0.734	0.780	0.491	0.555	0.617	0.681	0.747	0.815	0.747	0.747	0.815

†Based on Takeoff speed

Table 19 . Comparison of predicted performance parameters with published data of GARRETT TFE 731-2 at ISA 10,000 ft.

BROCHURE DATA †											SIMULATED RESULTS				
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.8	0.8
NET THRUST lbf	2020	1875	1760	1640	1570	—	1900	1809	1715	1642	1583	—	1583	1583	—
FUEL FLOW lbs/hr	1210	1245	1280	1330	1370	—	1225	1277	1316	1360	1405	—	1405	1405	—
S.F.C lbs/hr-lbf	0.600	0.664	0.727	0.811	0.873	—	0.645	0.706	0.766	0.829	0.887	—	0.887	0.887	—

†Based on Cruise speed

Table 20. Comparison of predicted performance parameters with published data of GARRETT TFE 731-2 at ISA 20,000 ft.

BROCHURE DATA †													SIMULATED RESULTS					
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8						
NET THRUST lbf	1730	1630	1550	1485	1410	1350	1571	1490	1422	1369	1342	1325						
FUEL FLOW lbs/hr	970	997	1035	1070	1109	1167	971	1001	1031	1063	1110	1160						
S.F.C lbs/hr-lbf	0.561	0.612	0.668	0.720	0.786	0.864	0.618	0.672	0.725	0.777	0.827	0.876						

†Based on Cruise speed

Table 21. Comparison of predicted performance parameters with published data of GARRETT TFE 731-2 at ISA 30,000 ft.

BROCHURE DATA †											SIMULATED RESULTS							
MACH NO.	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8						
NET THRUST lbf	1220	1180	1140	1100	1080	1060	1262	1204	1160	1132	1100	1097						
FUEL FLOW lbs/hr	730	765	780	810	850	875	754	775	800	830	857	902						
S.F.C lbs/hr-lbf	0.598	0.648	0.684	0.736	0.787	0.825	0.597	0.644	0.690	0.733	0.779	0.822						

†Based on Cruise speed

Table 22. Comparison of predicted performance parameters with published data of GARRETT TFE 731-2 at ISA 40,000 ft.

MACH NO.	BROCHURE DATA †							SIMULATED RESULTS						
	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8		
NET THRUST lbf	850	795	770	765	763	755	856	815	789	778	768	763		
FUEL FLOW lbs/hr	490	510	545	580	605	615	497	509	526	552	577	605		
S.F.C lbs/hr-lbf	0.576	0.641	0.708	0.758	0.793	0.816	0.581	0.624	0.666	0.709	0.751	0.792		

†Based on Cruise speed

Table 23. Comparison of predicted performance parameters with published data of GARRETT TFE 731-2 at ISA 50,000 ft.

MACH NO.	BROCHURE DATA †							SIMULATED RESULTS						
	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8		
NET THRUST lbf	505	495	485	475	465	460	503	487	474	466	463	465		
FUEL FLOW lbs/hr	280	295	305	325	340	361	290	302	315	330	347	367		
S.F.C lbs/hr-lbf	0.555	0.596	0.629	0.684	0.731	0.784	0.576	0.621	0.665	0.708	0.749	0.791		

†Based on Cruise speed

Table 24. Data of RB211-22 at sea level static conditions.(Reproduced from Wittenberg)

Fan IP+HPC Hot nozzle Engine											
Data point	m_f (lb/sec)	λ	ϵ_f	η_f †	ϵ_{tot}	$\eta_{c,tot}$	T_h (lb)	T_{t_7} K	P_{t_7}/P_O	thrust (lb)	fuel flow (lb/sec)
1 †	1083	4.77	1.48	.81	16.51	0.93	10205	776	1.57	39551	1310
2	1050	4.83	1.45	0.82	15.94	0.93	9286	756	1.51	36878	1268
3	973	4.93	1.38	0.78	14.75	0.93	7441	715	1.40	31221	1170
4	878	5.02	1.31	0.78	13.27	0.92	5627	678	1.31	25117	1053
5	778	5.07	1.24	0.74	11.82	0.92	4135	648	1.21	19638	932
6	658	5.03	1.18	0.71	9.71	0.92	2655	615	1.12	14028	789

†Design point

‡derived from experimental data

Table 25. Guaranteed performance for a turbojet (Reproduced from Sobey & Suggs)

Turbojet Engine			
Condition	Engine speed	Thrust	Turbine Temperature †
Military †	100%	100%	100%
Normal §	97%	83%	92%
90% normal ¶	95%	73%	86%
75% normal	92%	62%	80%

†Maximum temperature

‡corresponds to takeoff

§corresponds to climb speed

¶corresponds to cruise speed

Table 26. Variation of bypass ratio as thrust is throttled.

Garrett 731-2 at 30000 ft, Ma=0.8						
TET K	T _{t2} K	P _{t2} Bar	T _{t7} K	P _{t7} Bar	$\frac{P_{t2}}{P_{t7}} \sqrt{\frac{T_{t7}}{T_{t2}}}$	BPR
1270	305	.732	789	.910	1.29	2.41
1200	301	.703	745	.810	1.36	2.53
1130	300	.674	702	.720	1.43	2.67
1050	292	.645	645	630	1.53	2.85

Table 27. Comparison of performance parameters for various types of aero-engines.

PERFORMANCE PARAMETERS					
TYPE	TURBOJET †	TURBOJET †	TURBOJET †	TURBOFAN †	TURBOFAN †
MODEL	P & W JT3C-6	P & W JT4A-12	P & W JT3D-3B	P & W JT8D-17R	
AIRCRAFT	B707-121	B707-321	DC8-55	DC9, B727-200	
THRUST-TO ◊ lbs	13500 ‡	17500	18000	16400	
AIRFLOW-TO lb/sec	175	256	452	328	
BYPASS RATIO -TO	—	—	1.37	0.98	
F.P.R-TO	—	—	1.74	1.99	
O.P.R-TO	12.5	12.5	13.6	18.5	
TET-TO K	1133	1206	1211	1378	
CRUISE SFC ◊ lb/hr-lbs	0.93	0.93	0.80	0.83 ◊	
CRUISE THRUST ◊ lbs	4080	5730	5200	5100 ◊	
CRUISE TET K	1042	1047	1042	1242	

†Twin spool

◊ 0.85M, 30000 ft

◊ 0.80M, 35,000 ft

‡Wet

◊ SLS, Ideal nozzle

Table 28. Comparison of performance parameters for various types of aero-engines.

TYPE	PERFORMANCE PARAMETERS			
	TURBOFAN †	TURBOFAN †	TURBOFAN †	TURBOFAN ⊖
MODEL	P & W JT9D-7A	P & W JT9D-7Q	GE CF6-50A	RB211-524B3
AIRCRAFT	B747	B747-200F	DC10-30	L1011-500
THRUST-TO ∅ lbs	46150	53000	49000	48000
AIRFLOW-TO lb/sec	1540	1640	1440	1506
BYPASS RATIO -TO	5.1	4.9	4.4	4.47
F.P.R-TO	1.52	1.52	1.69	1.66
O.P.R-TO	22.5	24.5	28.4	28.1
TET-TO K	1470	1547	1503	1567
CRUISE SFC ∅ lb/hr-lbs	0.695 ∅	0.67	0.67	0.65
CRUISE THRUST ∅ lbs	9890 ∅	11180	10600	10640
CRUISE TET K	1361	1394	1377	1417

†Twin spool

∅ 0.85M, 30000 ft

∅ 0.80M, 35,000 ft

†Wet

∅ SLS, Ideal nozzle

Table 29. Parametric Estimation of bleed flow on RB211-22

BLEED FLOW VARIATION						
BLEED FLOW %	0	1	2	3	4	5
NET THRUST (-) %	base	3.82	7.30	11.03	14.75	18.66
FUEL FLOW (-) %	base	1.60	3.08	4.60	6.07	7.53
HOT THRUST (-) %	base	6.49	15.57	23.71	33.41	43.57
SFC (+) %	base	2.34	4.52	7.32	10.13	13.71
HP+IP CPR (-) %	base	0.82	2.18	3.35	4.65	6.00
CORE FLOW (-) %	base	1.19	2.18	3.37	4.17	5.16

APPENDICES

Appendix - A

1. Propelling Nozzles

The propelling nozzles play an important role in this analysis, primarily because they control the aerothermodynamic behaviour of the upstream components. The nozzles control the off-design performance of the gas turbine because of their regulating position as a back pressure controller for the whole engine. Thus, the nozzle serves not only as a back pressure control but also acts as an acceleration device converting thermal energy into kinetic energy.

It can be shown that for an isentropic expansion the thrust produced is a maximum when complete expansion to ambient pressure occurs in the nozzle. For values of P_n/P_0 about 3 however, the thrust produced by a convergent nozzle is greater than that produced by a convergent-divergent nozzle. For subsonic aircraft a convergent propelling nozzle is always used (Cohen et al., 1987).

1.1 Nozzle Functions

Engine Back Pressure Control: The throat area of the nozzle is one of the main means available to control the thrust and fuel consumption characteristics of an existing engine. For a single spool engine, if we increase the area of the nozzle, we can move the operating line on the compressor map away from the stall or surge line. Opening the nozzle throat area to its maximum value reduces the back pressure on the turbine and increases its expansion ratio.

1.2 Characteristics of Convergent Nozzles

Let the index 1 denote the nozzle entry conditions and the index 2 the exit conditions at the nozzle throat. From the energy conservation law for adiabatic

flow it follows that total or stagnation enthalpy remains constant:

$$C_p T_{t_1} = C_p T_{t_2} = C_p T_2 + \frac{V_2^2}{2} \quad (1)$$

Then for the isentropic conditions it follows :

$$\frac{(T_2)_{isen}}{T_{t_1}} = \left(\frac{P_2}{P_{t_1}} \right)^{\gamma-1/\gamma} \quad (2)$$

Now the isentropic efficiency for the nozzle is given by :

$$\eta_n = \frac{T_{t_1} - T_2}{T_{t_1} - (T_2)_{isen}} \quad (3)$$

Using equations (1),(2) and (3) we get:

$$V_2 = \sqrt{2C_p(T_{t_1} - T_2)} = \sqrt{2C_p\eta_n T_{t_1} \left(1 - (P_2/P_{t_1})^{\gamma-1/\gamma} \right)} \quad (4)$$

$$T_2 = T_{t_1} (1 - \eta_n (1 - (P_2/P_{t_1})^{\gamma-1/\gamma})) = T_{t_1} \left(1 - \eta_n (1 - (P_2/P_{t_1})^{\gamma-1/\gamma}) \right) \quad (5)$$

The total pressure ratio can be calculated from (4) and (5) as:

$$P_{t_2}/P_{t_1} = \frac{P_2}{P_{t_1}} \frac{1}{\left(1 - \eta_n (1 - (P_2/P_{t_1})^{\gamma-1/\gamma}) \right)^{\gamma/\gamma-1}} \quad (6)$$

1.3 Choking Conditions

The energy conservation law can also be expressed as

$$T_{t_1} = T_{t_2} = T_2 \left(1 + \frac{(\gamma-1)}{2} M_2^2 \right) \quad (7)$$

For the choked nozzle : $M_c = 1$ so $T_{t_1}/T_2 = \frac{(\gamma+1)}{2}$

The corresponding critical pressure ratio follows from:

$$\eta_n = \frac{1 - T_2/T_{t_1}}{1 - (P_2/P_{t_1})_{crit}^{\gamma-1/\gamma}}$$

Thus

$$(P_{t1}/P_2)_{crit} = \frac{1}{\left(1 - \frac{1}{\eta_n} \frac{(\gamma-1)}{(\gamma+1)}\right)^{\gamma/\gamma-1}} \quad (8)$$

Knowing the $(P_{t1}/P_2)_{crit}$ one can easily find out the corresponding total pressure loss from the equation (6). (Shapiro A., 1953) The generalized nozzle characteristics for a hot nozzle at efficiencies of 1.0 and .95 can be seen from Figure A11.

1.4 Mass Flow

The dimensionless mass-flow parameter at the inlet using total temperature and total pressure is given as:

$$\frac{m\sqrt{RT_{t1}}}{P_{t1}A_2} = \frac{(\frac{P_2}{RT_2}V_2A_2)\sqrt{RT_{t1}}}{P_{t1}A_2} = \frac{P_2}{P_{t1}} \frac{V_2}{\sqrt{RT_{t1}}} \frac{T_{t1}}{T_2} \quad (9)$$

Substituting equations (4) and (5) in the equation (9) one finds :

$$\frac{m\sqrt{RT_{t1}}}{P_{t1}A_2} = \frac{P_2}{P_{t1}} \sqrt{\frac{2\gamma\eta}{\gamma-1} \left(1 - \left(\frac{P_2}{P_{t1}}\right)^{\gamma-1/\gamma}\right)} * \frac{1}{\left(1 - \eta_n \left(1 - \frac{P_2}{P_{t1}}\right)^{\gamma-1/\gamma}\right)} \quad (10)$$

For the unchoked nozzle ($M_2 \leq 1$) the pressure P_2 is equal to the ambient pressure P_O and one gets a unique relationship between mass flow parameter $\frac{m\sqrt{RT_{t1}}}{P_{t1}A_2}$ and the nozzle pressure ratio P_{t1}/P_O (Wittenberg H., 1976)

For the choked nozzle flow ($M_2 = 1$) the pressure ratio is given by equation (8) and the mass flow parameter is independent of the pressure ratio P_{t1}/P_2 . It is then given as shown below:

$$\left(\frac{m\sqrt{RT_{t1}}}{P_{t1}A_2}\right)_{crit} = \left(1 - \frac{1}{\eta_n} \frac{(\gamma-1)}{(\gamma+1)}\right)^{\gamma/\gamma-1} * \sqrt{\gamma(\gamma+1)/2} \quad (11)$$

FIGURE A11 NOZZLE CHARACTERISTICS

Appendix - B

1. Generalized Engine Configuration

Upto now we have been dealing with either single spool turbojets or twin or three spool turbofans each requiring a different procedure to tackle the problem. In this part of the thesis we will try to generalize the case, so that the method is valid for any turbofan configuration. Basically we can hypothesize that by specifying OPR, FPR, BPR, TET and the component efficiencies we can do the overall performance irrespective of how the engine is configured. Let Figure A12 be any generalized engine where we have combined the compressors and the fan together as a single compressor driven by a single turbine, which again is the combination of all the turbines. Hence we have reduced twin or three spool configurations to a single spool configuration, so that the analysis becomes valid for any turbofan configuration.

2. Mathematical Modelling

The analysis of this generalized engine can be done by using the power and mass flow matching spool calculations. If we start with the power matching calculations we get the relationship

$$m_h C_{p_g}(T_{t_5} - T_{t_6}) = m_a C_{p_a}(T_{t_3} - T_{t_2}) + m_h C_{p_a}(T_{t_4} - T_{t_3})$$

On further modification we get

$$T_{t_5}(1 - \frac{T_{t_6}}{T_{t_5}}) = (\lambda + 1) \frac{C_{p_a}}{C_{p_g}}(\frac{T_{t_3}}{T_{t_2}} - 1)T_{t_2} + \frac{C_{p_a}}{C_{p_g}}(\frac{T_{t_4}}{T_{t_3}} - 1)\frac{T_{t_3}}{T_{t_2}}T_{t_2}$$

which can be extended further to give

$$\frac{T_{t5}}{T_{t2}} \left(1 - \frac{T_{t6}}{T_{t5}}\right) = (\lambda + 1) \frac{C_{pa}}{C_{pg}} \left(\frac{T_{t3}}{T_{t2}} - 1\right) + \frac{C_{pa}}{C_{pg}} \left(\frac{T_{t4}}{T_{t3}} - 1\right) \frac{T_{t3}}{T_{t2}} \quad (A)$$

Now for a particular flight condition T_{t2} is fixed by the Mach number and altitude, while the user specifies the turbine entry temperature T_{t5} .

That leaves four unknowns (λ , T_{t6}/T_{t5} , T_{t4}/T_{t3} , T_{t3}/T_{t2}) to be solved in equation (A). In order to find the bypass ratio λ , we should know the corrected hot mass flow $m_h \sqrt{T_{t3}}/P_{t3}$. Now from the mass flow matching relations across the two sides of the combustion chamber we get

$$\frac{m_h \sqrt{T_{t4}}}{P_{t4}} = \frac{m_h \sqrt{T_{t3}}}{P_{t3}} \sqrt{\frac{T_{t4}}{T_{t3}}} \frac{P_{t3}}{P_{t4}}$$

and

$$\frac{m_h \sqrt{T_{t4}}}{P_{t4}} = \frac{m_h \sqrt{T_{t5}}}{P_{t5}} \sqrt{\frac{T_{t4}}{T_{t5}}} \frac{P_{t5}}{P_{t4}}$$

Equating the above two equations, we get

$$\frac{m_h \sqrt{T_{t5}}}{P_{t5}} \sqrt{\frac{T_{t4}}{T_{t5}}} \frac{P_{t5}}{P_{t4}} = \frac{m_h \sqrt{T_{t3}}}{P_{t3}} \sqrt{\frac{T_{t4}}{T_{t3}}} \frac{P_{t3}}{P_{t4}}$$

Now the term $\frac{m_h \sqrt{T_{t5}}}{P_{t5}}$ is fixed because the turbine is choked for all the operating range, while the combustor pressure loss P_{t5}/P_{t4} is assumed constant for any off design condition. Hence we can modify the above equation to get

$$\frac{m_h \sqrt{T_{t3}}}{P_{t3}} = K_1 K_2 \sqrt{\frac{T_{t3}}{T_{t5}}} \frac{P_{t4}}{P_{t3}} \quad (B)$$

So in equation (B) we have two unknowns, T_{t3} and compressor pressure ratio P_{t4}/P_{t3} . To solve for equation (B) we take a range of cold nozzle pressure ratios P_{t3}/P_O . From these we get a range of P_{t3}/P_{t2} because $P_{t3}/P_O = P_{t3}/P_{t2} * P_{t2}/P_O$ where P_{t2}/P_O is the ram pressure ratio. Knowing P_{t3}/P_{t2} we get a range of temperature ratios T_{t3}/T_{t2} , and since we know T_{t2} (for any flight condition) we can get T_{t3} . This can be put in equation (B), leaving P_{t4}/P_{t3} as the only unknown.

Now we proceed towards the hot nozzle. Here we take a range of hot nozzle pressure ratios P_{t6}/P_O . Knowing P_{t6}/P_O we get a range of $m_h \sqrt{T_{t6}}/P_{t6}$ using analytical equations for the nozzle. On applying mass flow matching between the nozzle and turbine we get

$$\frac{m_h \sqrt{T_{t6}}}{P_{t6}} = \frac{m_h \sqrt{T_{t5}}}{P_{t5}} \sqrt{\frac{T_{t6}}{T_{t5}}} \frac{P_{t5}}{P_{t6}}$$

The temperature ratio across the turbine is given by $T_{t6}/T_{t5} = (P_{t6}/P_{t5})^{\frac{\eta_t(\gamma_g-1)}{\gamma_g}}$

On substituting the above relation we get

$$\frac{m_h \sqrt{T_{t6}}}{P_{t6}} = \frac{m_h \sqrt{T_{t5}}}{P_{t5}} \left(\frac{P_{t5}}{P_{t6}} \right)^{1 - \frac{\eta_t(\gamma_g-1)}{2\gamma_g}}$$

Since the turbine is going to be choked over most of the operating conditions the term $m_h \sqrt{T_{t5}}/P_{t5}$ is a fixed quantity. Hence from the above relationship turbine pressure ratios P_{t5}/P_{t6} and temperature ratios T_{t5}/T_{t6} can be evaluated for every hot nozzle pressure ratio P_{t6}/P_O .

The overall pressure ratio P_{t4}/P_O is given as

$$\frac{P_{t4}}{P_O} = \frac{P_{t6}}{P_O} \frac{P_{t5}}{P_{t6}} \frac{P_{t4}}{P_{t5}}$$

Hence different values of P_{t4}/P_O can be calculated for a range of P_{t6}/P_O .

The overall pressure ratio P_{t4}/P_O is related to the cold nozzle pressure ratios P_{t3}/P_O by the relation

$$\frac{P_{t4}}{P_O} = \frac{P_{t4}}{P_{t3}} \frac{P_{t3}}{P_O}$$

So we can get a range of compressor pressure ratios P_{t4}/P_{t3} for a range of cold pressure ratios. These compressor pressure ratios can be put in equation (B) to calculate hot mass flow parameter $m_h \sqrt{T_{t3}}/P_{t3}$. Once we know the hot mass flow $m_h \sqrt{T_{t3}}/P_{t3}$ and cold mass flow parameters $m_c \sqrt{T_{t3}}/P_{t3}$ it is easy to get the bypass ratio λ because $\lambda = \left(\frac{m_c \sqrt{T_{t3}}/P_{t3}}{m_h \sqrt{T_{t3}}/P_{t3}} \right)$. Moreover compressor temperature ratio T_{t4}/T_{t3} can be also calculated for every P_{t4}/P_{t3} because $T_{t4}/T_{t3} = (P_{t4}/P_{t3})^{\frac{\gamma_a}{\eta_c(\gamma_a-1)}}$.

Hence we have calculated all the four unknowns λ , T_{t_6}/T_{t_5} , T_{t_3}/T_{t_2} and T_{t_4}/T_{t_3} . So all the parameters are known in equation (A) for a range of cold and hot nozzle pressure ratios P_{t_3}/P_O and P_{t_6}/P_O and when the left hand side becomes equal to the right hand side of equation (A), we stop the iterations as this is the equilibrium point.

This procedure was later programmed by the author and then validated against a twin spool turbofan. The results obtained from this procedure and those obtained from the normal method showed differences especially regarding the HP compressor pressure ratio. Regarding the overall performance i.e. net thrust and fuel flow the difference from the published data was of the order of 7 to 10 %.

FIGURE A12 SCHEMATIC VIEW OF A GENERALIZED TURBOFAN ENGINE

APPENDIX C

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C *****
C
C PROGRAM TSTF.FTN (TWIN OR THREE SPOOL TURBOFAN)
C
C THIS PROGRAM PERFORMS ON-DESIGN AND OFF-DESIGN PERFORMANCE ANALYSIS OF
C A TWIN SPOOL OR THREE SPOOL TURBOFAN WITHOUT BLEED FLOW ANALYSIS.
C
C PROGRAMMER: MIRZA FAISAL BAIG
C
C DATE: DECEMBER 19, 1989.
C SUBSTANTIALLY MODIFIED ON JAN 12, 1990.
C
C CENTER: CASCADE, CARLETON UNIVERSITY
C *****
C
C LAM=BYPASS RATIO
C MTO5=MASS FLOW PARAMETER AT ENTRY TO HPT
C MTO6=MASS FLOW PARAMETER AT ENTRY TO LPT
C FPRA=FAN PRESSURE RATIO
C ZP=TEMPERATURE RATIO
C MOF=MACH NUMBER
C MTO6Y=OFF-DESIGN MASS FLOW PARAMETER AT LPT
C MTO7Y=OFF-DESIGN MASS FLOW PARAMETER AT EXIT OF LPT
C
C IMPLICIT REAL*8(A-H,O-Z)
C REAL LAM,MA,MO,MC,MH,MH3,MH31,LAMA,MHNA,MCNA,MCNAF,MFN,MTO6
C REAL LAMF,MOF,MC1,MH1,MA1,LAMX,MACH,MHX,LAMY,MHHX,MA11,MTO5
C REAL MTO6Y,LAMME,MTO7Y,LAPUT,MAHUT
C DIMENSION PRNH(110),MHNA(110),MCNA(110),PRNHO(110)
C DIMENSION LAMA(110),FPRA(110),RR(110),PR4(110),HLEFT(110)
C DIMENSION TO3S(110),PRR4(110),HWRITE(110),TR7(110),PR7(110)
C DIMENSION TO3X(110),HLEFTX(110),RRX(110),LAMX(110),MHX(110)
C DIMENSION CPRX(110),ZP(110),TO53X(110),PRX4(110),FBC(2,7),COEP(5)
C DIMENSION POLM(1,3),APPU(3,4)
C COMMON/PARS/CMPL,HP,HG,HA,HD,ETAM,ETAJ,ETAI,HAA
C COMMON/CONS/CPA,CPG,R,GAMA,GAMG
C CHARACTER *6 VALL
C CPA=1005.
C CPG=1148.
C R=287.
C GAMA=1.4
C GAMG=1.333
C M1=4
C M2=5
C WRITE(*,*)'NAME THE FILE PLEASE, EITHER BEGIN2 OR JT2'
C READ(*,123)VALL
123 FORMAT(A6)
C OPEN(UNIT=5,FILE=VALL)
C REWIND(5)
C
C READ DATA FROM INPUT FILE
C
C WRITE(*,*)'INPUT THE ALTITUDE IN (FEET) & INPUT THE MACH NO.'
C READ(5,*)ALTI,MO
C WRITE(*,*)'GIVE VALUE OF TOTAL AIR MASS FLOW IN kg/s'
C READ(5,*)MA
C WRITE(*,*)'INPUT THE TOTAL P.R.'
C READ(5,*)TPR
C WRITE(*,*)'INPUT THE FAN P.R.AND TURBINE INLET TEMP IN KEL'
C READ(5,*)FPR,TIT
C WRITE(*,*)'GIVE THE BYPASS RATIO'
C READ(5,*)LAM
C WRITE(*,*)'GIVE POLYTROPIC EFFICIENCY OF FAN,COMPRESSOR & TURBINE'
C READ(5,*)EFAN,ETAC,ETAT
C WRITE(*,*)'GIVE MECHANICAL EFFICIENCY OF SPOOL & COMBUSTION EFFI'
C WRITE(*,*)'CIENCY'
C READ(5,*)ETAM,ETAB
C WRITE(*,*)'GIVE INTAKE & PROPELLING NOZZLES EFFICIENCY'
C READ(5,*)ETAI,ETAJ
C WRITE(*,*)'GIVE COMBUSTION PRESSURE LOSS IN DECIMAL FORM'

```

```

      READ(5,*)CMPL
      WRITE(*,*)'GIVE THE BLEED FLOW AS RATIO OF TOTAL INFLOW'
      READ(5,*)BL
      CLOSE(5)

C
C   CONSTANT PARAMETERS
C

      HG=((GAMG-1.)/GAMG)*ETAT
      HA=(GAMA-1.)/(GAMA*ETAC)
      HAA=(GAMA-1.)/(GAMA*EFAN)
      HD=GAMA/(GAMA-1.)
      HP=GAMG/(GAMG-1.)

      CALL ALTITU(ALT1,TOD,PO,RHO)
      WRITE(*,1)TOD,PO,RHO
1   FORMAT('TO=',F6.2,3X,'K',/, 'PO=',F6.4,3X,'BAR',/, 'RHO=',F6.4)
      WRITE(*,*)'IS TEMP.GREATER THAN ISA? FEED EXTRA TEMP(-90 TO 90 C)'
      READ(*,*)EXT
      TO=TOD+EXT
      WRITE(*,2)TO
2   FORMAT('THE NEW TEMP. IS =',F6.2,3X,'K')
      VEL=MO*SQRT(GAMA*R*TO)
      VELL=VEL*3.6
      WRITE(*,3)VELL
3   FORMAT('VELOCITY OF AIRCRAFT=',F6.2,2X,'KM/HR')

C
C
      CALL DESIGN(MO,PO,TO,MA,BL,CPR,TIT,TO2,PO2,C,PO3,TO3,RS,ETAT,
      STO4,PO4,PO5,TO5,D,E,HH,G,ETAC,EFAN,PO6,TO6,TO7,PO7,FPR,TPR,LAM)

C
      FNOPR=PO3/PO
      CFNOPR=1./(1.-(1./ETAJ)*((GAMA-1.)/(GAMA+1.)))*HD
      WRITE(*,4)FNOPR,CFNOPR
4   FORMAT('AT FAN NOZZLE PRESSURE RATIOS ARE',/, 'FNOPR=',
      S F7.4,/, 'CFNOPR=', F7.4)
      HNOPR=PO7/PO
      CHNOPR=1./(1.-(1./ETAJ)*((GAMG-1.)/(GAMG+1.)))*HP
      WRITE(*,5)HNOPR,CHNOPR
5   FORMAT('AT HOT NOZZLE PRESSURE RATIOS ARE',/, 'HNOPR=',
      S F7.4,/, 'CHNOPR=', F7.4)

C
      B=TIT/TO3
      WRITE(*,*)'B=',B
      CALL COPR(TO4,CP4,THCP4,H4,THH4,PSI4,THPSI4,ECV4)
      CALL COPR(TIT,CP5,THCP5,H5,THH5,PSI5,THPSI5,ECV5)
      MC=(MA-BL)*(LAM/(LAM+1.))
      MH=(MA-BL)-MC
      WRITE(*,12)MC,MH
12  FORMAT('MASS OF COLD AIR=',F6.2,'KG/S',/, 'MASS OF HOT AIR=',F6.2,
1   'KG/S')
      PAC=MH*SQRT(TO3)/PO3
      WRITE(*,23)PAC
23  FORMAT('NON-DIMENSIONALISED HOT FLOW AT HPC=',F7.2)
      PACK=MH*SQRT(TO7)/PO7
      WRITE(*,25)PACK
25  FORMAT('NON-DIMENSIONALISED HOT FLOW AFTER LPT=',F7.2)
      MTO5=(MH-BL)*SQRT(TIT)/PO5
      MTO6=(MH-BL)*SQRT(TO6)/PO6
      WRITE(*,233)MTO5,MTO6
233 FORMAT('MTO5=',F7.2,'MTO6=',F7.2)

C
      F=(H5-H4)/(ETAB*ECV5)
      FU=(H5-H4)/(ETAB*ECV5)*(MH)*3600
      FM=(MH)/(ETAB*42801.)*(CPG*TIT-CPA*TO4)*3.6
      WRITE(*,*)'FUEL/AIR RATIO=',F
      WRITE(*,11)FU
11  FORMAT('FUEL FLOW=',F8.2,5X,'KG/HR')
      FUUU=FU*2.204
      WRITE(*,*)'FUUU=',FUUU
      IF(FNOPR.GE.CFNOPR)THEN

C
C   TO FIND THE THRUST OF THE INDIVIDUAL NOZZLES

```

```

C      K=1
      WRITE(*,*) '***** FAN NOZZLE IS CHOKING*****'
C
      CALL CHOKED(CFNOPR,PO,FPR,TO2,TIT,VEL,PO3,TO3,TF,PF,VELF,FS,PHI,
1 K,PO2,MC,MH)
      THO=FS*MC
      WRITE(*,13)THO
13     FORMAT('FAN NOZZLE NET THRUST=',F12.3,'N')
      ELSE
      K=1
      WRITE(*,*) '***** FAN NOZZLE IS UNCHOKED*****'
C
      CALL UNCHOKED(TO,PO,TO3,F,VEL,PO2,FNOPR,TF,PF,VELF,FS,PHI,K,
1 SPM)
      TH1=FS*MC
      WRITE(*,14)TH1
14     FORMAT('FAN NOZZLE NET THRUST=',F12.3,'N')
      ENDIF
      IF (HNOPR.GE.CHNOPR) THEN
      K=2
      WRITE(*,*) '***** HOT NOZZLE IS CHOKING*****'
C
      CALL CHOKED(CHNOPR,PO,CPR,TO2,TIT,VEL,PO7,TO7,TH,PH,VELH,FS,PHI,
1 K,PO2,MC,MH)
      TH2=FS*MH
      WRITE(*,15)TH2
15     FORMAT('HOT NOZZLE NET THRUST=',F12.3,'N')
      ELSE
      WRITE(*,*) '*****HOT NOZZLE IS UNCHOKED*****'
C
      K=2
      CALL UNCHOKED(TO,PO,TO7,F,VEL,PO2,HNOPR,TH,PH,VELH,FS,PHI,K,
1 SPM)
      TH3=FS*MH
      WRITE(*,16)TH3
16     FORMAT('HOT NOZZLE NET THRUST=',F12.3,'N')
      ENDIF
      THRU=THO+TH1+TH2+TH3
      WRITE(*,17)THRU
17     FORMAT('NET THRUST IS=',F12.3,'N')
      THRUUP=THRU/4.448
      WRITE(*,*) 'THRUUP=',THRUUP
C
      SFC=FU/(THRU)
      SS1=FM/THRU
      WRITE(*,*)SS1
      WRITE(*,18)SFC
18     FORMAT('SFC=',F7.5,3X,'KG/HR-N')
      SFFF=SFC*9.804
      WRITE(*,*) 'SFFF=',SFFF
C
C      FIXING THE AREA OF THE NOZZLES
C
      IF (HNOPR.GE.CHNOPR) THEN
      DIEN=3975.3
      AREE=MH*SQRT(TO7)/(PO7*DIE!!)
      WRITE(*,19)AREE
19     FORMAT('AREA OF HOT NOZZLE=',F6.4,'sq.m')
      ARAA=AREE*1549.44
      WRITE(*,20)ARAA
20     FORMAT('AREA OF HOT NOZZLE=',F6.2,'sq.in')
      ELSE
      DIEN1=(PH/PO7)*SQRT(8*(1.-(PH/PO7)**(1./HP)))*1./(1.-(1.~
S(PH/PO7)**(1./HP)))*5903.2
      WRITE(*,*)DIEN1
      BAL=MH*SQRT(TO7)/PO7
      WRITE(*,*)BAL
      AREE=MH*SQRT(TO7)/(PO7*DIEN1)
      WRITE(*,119)AREE
119    FORMAT('AREA OF HOT NOZZLE=',F6.4,'sq.m')
      ARAA=AREE*1549.44
      WRITE(*,120)ARAA
120    FORMAT('AREA OF HOT NOZZLE=',F6.2,'sq.in')
      ENDIF

```

```

C      IF(NOPR.GE.CFNOPR) THEN
C        DIEN2=4041.8
C        AREE1=MC*SQRT(TO3)/(PO3*DIEN2)
C        WRITE(*,21)AREE1
21      FORMAT('AREA OF COLD NOZZLE=',F6.4,'sq.m')
C        ARAA1=AREE1*1549.44
C        WRITE(*,22)ARAA1
22      FORMAT('AREA OF COLD NOZZLE=',F6.2,'sq.in')
C        ELSE
C        DIEN3=(PF/PO3)*SQRT(7*(1-(PF/PO3)**(1./HD)))*1./(1.-(1.-
C        S(PF/PO3)**(1./HD)))*5903.2
C        WRITE(*,*)DIEN3
C        BALL=MC*SQRT(TO3)/PO3
C        WRITE(*,*)BALL
C        AREE1=MC*SQRT(TO3)/(PO3*DIEN3)
C        WRITE(*,121)AREE1
121      FORMAT('AREA OF COLD NOZZLE=',F6.4,'sq.m')
C        ARAA1=AREE1*1549.44
C        WRITE(*,122)ARAA1
122      FORMAT('AREA OF COLD NOZZLE=',F8.2,'sq.in')
C        ENDIF
C
C      *****
C      OFF-DESIGN ANALYSIS BEGINS
C      *****
C
C      TO TAKE THE VARIABLE POLYTROPIC COMPRESSOR OR FAN
C      EFFICIENCY INTO ACCOUNT BY FITTING A THIRD OR FOURTH
C      ORDER POLYNOMIAL TO IT.
C
C      *****
C
24      WRITE(*,*)'WANT TO QUIT. PUT 1 FOR Y:,0 FOR NO'
C      READ(*,*)SUBM
C      IF(SUBM.EQ.1) GOTO 50
C      IF(SUBM.EQ.0)CONTINUE
C      WRITE(*,*)'WANT TO RUN PROGRAM AT CONSTANT COMPRESSOR EFFI'
C      WRITE(*,*)'OR WANT VARIABLE EFFICIENCY? PUT 1 FOR Y:0 FOR NO'
C      READ(*,*)CHO
C      IF(CHO.EQ.0)THEN
C        WRITE(*,*)'YOU MUST KNOW AT LEAST FIVE COMP.PR.AND CORRESPONDING'
C        WRITE(*,*)'EFFICIENCY POINTS.'
C        WRITE(*,*)'IF YOU DO NOT KNOW PUT: 1'
C        READ(*,*)DON
C        IF(DON.EQ.1)GOTO 24
C        WRITE(*,*)'WRITE NO. OF POINTS YOU WILL INPUT {5,6,7...}'
C        READ(*,*)N
C        WRITE(*,*)'START INPUTTING POINTS'
C        READ(*,*)((PBC(J,I),I=1,N),J=1,2)
C        CALL LSTM(N,M1,M2,PBC,COEP,S,FAL)
C        WRITE(*,*)COEP
C        ENDIF
C
C      *****
C
C      THIS PART OF MAIN PROGRAM IS TO PERFORM OFF-DESIGN CALCULATIONS,
C      FOR THE CASE WHEN THE TURBINE INLET TEMPERATURE IS VARIED, WITH THE
C      VARYING FLIGHT CONDITIONS OF THE AIRCRAFT.
C
C      *****
C
C      CORRELATION BETWEEN THE MECHANICAL SPEED AND TET.
C
C      ANN1=CRUISE ENGINE SPEED
C      ANN2=ROOT OF ANN1
C      TITI=TET AT ANY TAKEOFF OR CRUISE SPEED
C      TITLI=TET AT ANY TAKEOFF OR CRUISE SPEED
C      ANNA1=CRUISE SPEED AT WHICH TET IS TO BE DETERMINED
C      ANNA2=ROOT OF THE CRUISE SPEED AT WHICH TET IS TO BE DETERMINED
C      APPU=ARRAY OF TEMPERATURES ENTERED
C      TITI=COMPUTED TET AT THAT FLIGHT CONDITION

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52  CONTINUE
    WRITE(*,*) 'GIVE TIT, TO, TITI, TO2, TITLI, TO3'
    READ(*,*) TIT, TO, TITI, TO2, TITLI, TO3
    WRITE(*,*) 'PLEASE INPUT ANN1, ANN2'
    READ(*,*) ANN1, ANN2
    APPU(1,4)=TIT
    APPU(2,4)=TITI
    APPU(3,4)=TITLI
    APPU(1,1)=TO
    APPU(1,2)=SQRT(APPU(1,1))
    APPU(1,3)=1.
    APPU(2,1)=TO2
    APPU(2,2)=SQRT(APPU(2,1))
    APPU(2,3)=1.
    APPU(3,1)=TO3
    APPU(3,2)=SQRT(APPU(3,1))*ANN1
    APPU(3,3)=1.*ANN2
    WRITE(*,*) 'PLEASE INPUT ANNA1, ANNA2'
    READ(*,*) ANNA1, ANNA2
    CALL WUIGJ(APPU, 3, 4, DETA)
    WRITE(*,*) APPU(1,4), APPU(2,4), APPU(3,4)
    POLM(1,1)=APPU(1,4)
    POLM(1,2)=APPU(2,4)*ANNA1
    POLM(1,3)=APPU(3,4)*ANNA2
    WRITE(*,*) POLM(1,1), POLM(1,2), POLM(1,3)

C
C  INPUT THE OFF-DESIGN FLIGHT CONDITION
C

    WRITE(*,*) 'PLEASE GIVE ALT & MACH. NO.'
    READ(*,*) ALTIP, MACH
    CALL ALTITU(ALTIP, TOXD, POX, RHOX)
    WRITE(*,53) TOXD, POX, RHOX
53  FORMAT('TO=', F6.2, 3X, 'K', /, 'PO=', F6.4, 3X, 'BAR',
1  /, 'RHO=', F6.4, 3X, 'kg/cu.m')
    WRITE(*,*) 'IS TEMP. GREATER THAN ISA. FEED EXTRA TEMP(0 TO 50)'
    READ(*,*) EXTX
    TOX=TOXD+EXTX
    WRITE(*,54) TOX
54  FORMAT('THE NEW TEMP.IS TO=', F6.2, 3X, 'K')
    VELX=MACH*SQRT(GAMA*R*TOX)
    TO2X=TOX*(1.+(GAMA-1.)/2*MACH**2)
    PO2X=POX*(1.+ETAI*(GAMA-1.)/2*MACH**2)**HD
    WRITE(*,55) TO2X, PO2X
55  FORMAT('TO2=', F6.2, 3X, 'K', /, 'PO2=', F6.4, 3X, 'BAR')
    VELFX=VELX*3.6
    WRITE(*,56) VELFX
56  FORMAT('VELOCITY OF AIRCRAFT=', F6.2, 2X, 'KM/HR')

C
C  TO OBTAIN NEW TET AT NEW FLIGHT CONDITION
C

    TITL=POLM(1,1)*TO2X+POLM(1,2)*SQRT(to2x)+POLM(1,3)
    WRITE(*,*) 'TITL=', TITL

C
C  FOR VARIABLE COMPRESSOR OR FAN EFFICIENCY
C

    WRITE(*,*) 'WANT TO VARY HP COMPRESSOR EFFICIENCY IF YES PUT 1'
    READ(*,*) CHO
    IF(CHO.EQ.1) THEN
        WRITE(*,*) 'INPUT HP COMPRESSOR EFFICIENCY'
        READ(*,*) ECFE
    ENDIF
    WRITE(*,*) 'WANT TO VARY FAN EFFICIENCY IF YES PUT 1'
    READ(*,*) CHRO
    IF(CHRO.EQ.1) THEN
        WRITE(*,*) 'INPUT NEW FAN EFFICIENCY'
        READ(*,*) EFFAN
        HAAA=(GAMA-1.)/(GAMA*EFFAN)
    ENDIF

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```

WRITE(*,*)'WANT TO CONTINUE WITH SAME VALUE OR WANT'
WRITE(*,*)'TO CHANGE IT SLIGHTLY.IF YES OR NO ENTER'
WRITE(*,*)'NEW VALUE OF TIT'
WRITE(*,*)TITX
READ(*,*)TITX
AX=TITX/TO2X

C
C
C
TO GET THE LHS OF THE POWER MATCHING EQUILIBRIUM EQUATION

J=1
IF(J.EQ.1) THEN
CALL OZZCH(MCNA,MHNA,PRNHO,AREE1,J,PRNCC,PRNHC)
ENDIF
DO 57 I=1,79
FPRA(I)=PRNHO(I+1)/(ETAI*(1.+(GAMA-1.)/2.*MACH**2)**HD)
IF(CHRO.EQ.1) THEN
TO53X(I)=(TITX/TO2X)/(FPRA(I)**HAAA)
ELSE
TO53X(I)=(TITX/TO2X)/(FPRA(I)**HAA)
ENDIF
ZP(I)=TO53X(I)/B

IF(CHO.EQ.1) THEN
CPRX(I)=(1.+(ZP(I))*(CPR** (1./(HD*ETAC))-1.))** (ECFF*HD)
ELSE
CPRX(I)=(1.+(ZP(I))*(CPR** (1./(HD*ETAC))-1.))** (ETAC*HD)
ENDIF
MHX(I)=PAC*SQRT(1./ZP(I))*(CPRX(I)/CPR)
60 FORMAT('NON-DIMENSIONAL HOT FLOW AT HPC=',F7.2)
LAMX(I)=MCNA(I+1)/MHX(I)
IF(CHRO.EQ.1) THEN
TO3X(I)=TO2X*FPRA(I)**HAAA
ELSE
TO3X(I)=TO2X*FPRA(I)**HAA
ENDIF
RRX(I)=TO3X(I)/TO2X
PRX4(I)=PRNHO(I+1)*CPRX(I)
HLEFTX(I)=(1./ZP(I))*(1./(LAM+1.))*(1./(1.-(1./RS)))*(LAMX(I)+1.)*
& (1.-(RRX(I)**(-1)))
57 CONTINUE

C
C
C
TO GET THE RHS OF THE POWER MATCHING EQUILIBRIUM EQUATION

J=2
IF(J.EQ.2) THEN
CALL OZZCH(MCNA,MHNA,PRNH,AREE,J,PRNCC,PRNHC)
ENDIF
DO 61 J=1,79
PR6=(MHNA(J+1)/PACK)** (1./(1.-HG/2.))*HH
PR7(J)=1./PR6
TR7(J)=PR7(J)**HG
PRR4(J)=PRNH(J+1)*(1./PR7(J))*(1./(1.-CMPL))*D
HRITE(J)=(1.-TR7(J))/(1.-(1./G))
C
61 WRITE(*,*)PRX4(J),PRR4(J),HLEFTX(J),HRITE(J)
CONTINUE

C
C
C
TO FIND THE INTERSECTION POINT OF THE COMPRESSOR AND THE
TURBINE/NOZZLE CHARACTERISTIC CURVE

CALL COMPAR(HRITE,HLEFTX,PRX4,PRR4,HIH,PIH,FLIP,I,CPRX,MHX,
1 LAMX,FPRA,CAPUT,FAPUT,MAHUT,LAPUT)

WRITE(*,*)'FLIP=',FLIP
PR7Y=PR7(I)
TR7Y=TR7(I)

C
C
C
TO GET THE NEW COMPRESSOR AND FAN PRESSURE RATIO

CPRY=CAPUT

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FPRF=FAPUT
C
FLLIP=FLIP*POX
WRITE(*,*)'FLLIP=',FLLIP
PO3Y=FLLIP/CPRY
WRITE(*,62)PO3Y
62 FORMAT('PRESSURE AT EXIT FROM FAN=',F6.3,2X,'BAR')
C
C TO GET THE NEW MASS FLOW PARAMETER AND BYPASS RATIO
C
MHHX=MAHUT
LAMY=LAPUT
C
WRITE(*,*)'MHHX=',MHHX
GOG=(1.+(GAMA-1.)/2.*MACH**2)**HD
FPRT=PO3Y/PO2X
WRITE(*,29)FPRF
WRITE(*,*)FPRT
29 FORMAT('NEW FAN PRESSURE RATIO =',F6.3)
IF(CHRO.EQ.1)THEN
TO3Y=TO2X*FPRF**HAAJ.
ELSE
TO3Y=TO2X*FPRF**HAA
ENDIF
TO23Y=TO3Y-TO2X
WRITE(*,28)TO3Y
28 FORMAT('TEMP. AT EXIT FROM FAN=',F6.2,'K')

TR7Y=1.-HIH*(1.-(1./G))
PR7Y=TR7Y** (1./HG)
PO7YY=FLLIP*PR7Y*(1.-CMPL)/D
IF((PO3Y/POX).LE.PRNCC)THEN
MFN=(POX/PO3Y)*SQRT(2.*HD*ETAJ*(1.-(POX/PO3Y)
& ** (1./HD))) *1./ (1.-ETAJ*(1.-(POX/PO3Y)** (1./HD)))
MCNAF=MFN*(AREE1/16.94)*100000.
ELSE
MFN=(1.-((GAMA-1.)/(GAMA+1.))*(1./ETAJ))**HD
& *SQRT(GAMA*(GAMA+1.)/2.)
MCNAF=(AREE1/16.94)*MFN*100000.
ENDIF
GP=(1.+(GAMA-1.)/2.*MACH**2)
GPY=(TITX/TO3Y)
IF(CHRO.EQ.1)THEN
TO7YY=TR7Y*(1./E)*GPY*(FPRF**HAAA)*GP*TOX
ELSE
TO7YY=TR7Y*(1./E)*GPY*(FPRF**HAA)*GP*TOX
ENDIF
WRITE(*,*)TO7YY,PO7YY
LAMME=MCNAF/MHHX
WRITE(*,34)LAMY,LAMME
34 FORMAT('THE NEW BYPASS RATIO=',F6.3,2X,'LAMME=',F6.3)
WRITE(*,32)CPRY
32 FORMAT('THE NEW HPC PRESSURE RATIO=',F6.3)
PO4Y=PO3Y*CPRY
WRITE(*,30)PO4Y
30 FORMAT('PRESS. AT EXIT FROM HPC=',F5.2,'BAR')
IF(CHO.EQ.1)THEN

C
C ANALYSIS AFTER THE HP COMPRESSOR
C

TO4Y=TO3Y*(CPRY)**((GAMA-1.)/(GAMA*ECFF))
ELSE
TO4Y=TO3Y*(CPRY)**HA
ENDIF
WRITE(*,31)TO4Y
31 FORMAT('TEMP. AT EXIT FROM HPC=',F6.2,'K')
TO34Y=TO4Y-TO3Y
WRITE(*,35)TITX
35 FORMAT('THE NEW H.P. TURBINE INLET TEMP.=',F7.2,'K')
PO5Y=PO4Y*(1.-CMPL)
WRITE(*,33)PO5Y
33 FORMAT('PRESSURE AT ENTRY TO HPT=',F5.2,'BAR')
TO56Y=CPA*TO34Y/(ETAM*CPG)

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TO6Y=TITX-TO56Y
PO6Y=PO5Y*(TO6Y/TITX)**(1./HG)
WRITE(*,37)PO6Y,TO6Y
DFY=PO5Y/PO6Y
EFY=TITX/TO6Y
WRITE(*,70)EFY
70  FORMAT('THE TEMP. RATIO ACROSS H.P.TURBINE=',F6.3)
    WRITE(*,36)DFY
36  FORMAT('THE PR.ACROSS H.P.TURBINE =',F6.4)
37  FORMAT('AT EXIT FROM H.P.TURBINE',/, 'PO6=',F5.2,3X,'BAR',
    S /, 'TO6=',F7.2,3X,'K')
    TO67Y=CPA*(TO23Y)*(LAMY+1.)/(CPG*ETAM)
    TO7Y=TO6Y-TO67Y
    PO7Y=PO6Y*(TO7Y/TO6Y)**(1./HG)
    WRITE(*,38)TO7Y,PO7Y
38  FORMAT('AT EXIT FROM L.P.TURBINE',/, 'TO7=',F7.2,3X,'K',
    S /, 'PO7=',F5.2,3X,'BAR')
    HHFY=PO6Y/PO7Y
    GGY=TO6Y/TO7Y
    WRITE(*,71)GGY
71  FORMAT('THE TEMP. RATIO ACROSS L.P. TURBINE=',F6.3)
    WRITE(*,39)HHFY
39  FORMAT('THE PR.ACROSS L.P.TURBINE =',F6.4)
    TUOPR=1./(1.-(1./ETAT)*((GAMG-1.)/(GAMG+1.)))**HP
    WRITE(*,900)TUOPR
900  FORMAT('CRITICAL LP TURBINE P.R.=',F6.3)
    MH1=MHHX*PO3Y/SQRT(TO3Y)
    MC1=LAMY*MH1
    MA1=MC1+MH1
    MA11=MA1*2.204
    WRITE(*,40)MH1,MC1,MA1,MA11
40  FORMAT('NEW HOT MASS FLOW=',F6.2,/, 'NEW COLD AIR MASS=',F6.2,/,
    S 'TOTAL AIR MASS FLOW=',F6.2,/, 'MA11=',F6.2)
    TITLY=(MTO5*PO5Y/MH1)**2
    MTO6Y=MH1*SQRT(TO6Y)/PO6Y
    WRITE(*,235)TITLY,MTO6Y
235  FORMAT('TITLY=',F7.2, 'MTO6Y=',F7.2)
    FNOPR=PO3Y/POX
    CFNOPR=1./(1.-(1./ETAJ)*((GAMA-1.)/(GAMA+1.)))**HD
    WRITE(*,41)FNOPR,CFNOPR
41  FORMAT('AT FAN NOZZLE PRESSURE RATIOS ARE',/, 'FNOPR=',
    S F7.4,/, 'CFNOPR=', F7.4)
    MTO7Y=MH1*SQRT(TO7Y)/PO7Y
    WRITE(*,236)MTO7Y
236  FORMAT('MTO7Y=',F7.2)
    PRLP=(MTO7Y/MTO6Y)**(1./(1-HG/2.))
    WRITE(*,237)PRLP
237  FORMAT('PRLP=',F5.3)
    HNOPR=PO7Y/POX
C
C  TO GET THE MASS FLOW ACROSS THE HOT NOZZLE
C
    IF((PO7Y/POX).LE.PRnHC)THEN
        MFN=(POX/PO7Y)*SQRT(2.*HP*ETAJ*(1.-(POX/PO7Y)
        & *(1./HP)))**1./(1.-ETAJ*(1.-(POX/PO7Y)**(1./Hp)))
        MCNAF=MFN*(AREE/16.94)*100000.
        ELSE
        MFN=(1.-((GAMG-1.)/(GAMG+1.))*(1./ETAJ)))**HP
        & *SQRT(GAMG*(GAMG+1.)/2.)
        MCNAF=(AREE/16.94)*MFN*100000.
        WRITE(*,238)MCNAF
238  FORMAT('MTO7Y=',F7.2)
        ENDF
        CHNOPR=1./(1.-(1./ETAJ)*((GAMG-1.)/(GAMG+1.)))**HP
        WRITE(*,42)HNOPR,CHNOPR
42  FORMAT('AT HOT NOZZLE PRESSURE RATIOS ARE',/, 'HNOPR=',
    S F7.4,/, 'CHNOPR=', F7.4)
        CALL COPR(TO4Y,CP4,THCP4,H4,THH4,PSI4,THPSI4,ECV4)
        CALL COPR(TITX,CP5,THCP5,H5,THH5,PSI5,THPSI5,ECV5)
        F=(H5-H4)/(ETAB*ECV5)
        FU=(H5-H4)/(ETAB*ECV5)*(MH1)*3600
        FM=(MH1)/(ETAB*42800.)*(CPG*TITX-CPA*TO4Y)*3.6
        WRITE(*,544)f
544  FORMAT('FUEL AIR RATIO=',F7.5)
        FUL=FU*2.204

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43  WRITE(*,43)FU,FUL
    FORMAT('FUEL FLOW=',F8.2,2X,'KG/HR',5X,'FUL=',f8.2,2X,'LB/HR')

C
C  TO GET THE THRUSTS FOR CHOKED OR UNCHOKED NOZZLES
C

    THFO=0.
    THF1=0.
    THF2=0.
    THF3=0.
    IF (FNOPR.GE.CFNOPR) THEN
      K=1
      WRITE(*,*)'***** FAN NOZZLE IS CHOKING*****'
      WRITE(*,*)THFO
      CALL CHOKED(CFNOPR,POX,FPRF,TO2X,TITX,VELX,PO3Y,
1    TO3Y,TF,PF,VELQ,FSQ,PHI,K,PO2X,MCI,MH1)
      THFO=FSQ*MCI
      THFOO=THFO/4.448
      WRITE(*,44)THFO,THFOO
44  FORMAT('FAN NOZZLE NETTHRUST CHOKED=',F12.3,'N',5X,'THFOO=',f12.3)
      ELSE
      K=1
      WRITE(*,*)'***** FAN NOZZLE IS UNCHOKED*****'

C
      WRITE(*,*)THF1
      CALL UNCHOKED(TOX,POX,TO3Y,F,VELX,PO2X,FNOPR,TF,PF,VELQ,FSQ,PHI,K,
1    SPM)
      THF1=FSQ*MCI
      thf11=thf1/4.448
      WRITE(*,45)THF1,thf11
45  FORMAT('FANNOZZLE NETTHRUST UNCHOKE=',F12.3,'N',5X,'THF11=',f12.3)
      ENDIF
      IF (HNOPR.GE.CHNOPR) THEN
      K=2
      WRITE(*,*)'***** HOT NOZZLE IS CHOKING*****'

C
      CALL CHOKED(CHNOPR,POX,CPRY,TO2X,TITX,VELX,PO7Y,TO7Y,
1    TH,PH,VELH,FSH,PHI,K,PO2X,MCI,MH1)
      THF2=FSH*MH1
      THF22=THF2/4.448
      WRITE(*,46)THF2,THF22
46  FORMAT('HOT NOZZLE NETTHRUST CHOKED=',F12.3,5X,'N','THF22=',F12.3)
      ELSE
      WRITE(*,*)'*****HOT NOZZLE IS UNCHOKED*****'

C
      K=2
      CALL UNCHOKED(TOX,POX,TO7Y,F,VELX,PO2X,HNOPR,
1    TH,PH,VELH,FSH,PHI,K,SPM)
      THF3=FSH*MH1
      THF33=THF3/4.448
      WRITE(*,47)THF3,THF33
47  FORMAT('HOTNOZZLE NETTHRUST UNCHOKE=',F12.3,5X,'N','THF33=',F12.3)
      ENDIF
      THRUS=THFO+THF1+THF2+THF3
      THRUSS=THRUS/4.448
      WRITE(*,48)THRUS,THRUSS
48  FORMAT('NET THRUST IS=',F12.3,5X,'N','THRUSS=',f12.3)
      SFC=FU/(THRUS)
      SS1=FUL/THRUSS
      WRITE(*,49)SFC,SS1
49  FORMAT('SFC=',F7.5,3X,'KG/HR-N',5X,'SS1=',f7.5)
      GOTO 24

C
50  STOP
    END

C
C  *****
C  THIS SUBROUTINE IS TO PERFORM ON-DESIGN CALCULATIONS. IT RETURNS THE
C  THRUST,S.F.C,AND GROSS THRUST PARAMETER, TEMPS.& PRESSURES AT
C  VARIOUS POINTS ARE ALSO COMPUTED.
C  *****

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SUBROUTINE DESIGN(MO,PO,TO,MA,BL,CPR,TIT,TO2,PO2,C,PO3,TO3,RS,
1ETAT,TO4,PO4,PO5,TO5,D,E,HH,G,ETAC,EFAN,PO6,TO6,TO7,PO7,FPR,TPR,
2 LAM)
C
C D=PRESSURE RATIO ACROSS HP TURBINE
C HH=PRESSURE RATIO ACROSS LP TURBINE
C E=TEMPERATURE RATIO ACROSS HP TURBINE
C
IMPLICIT REAL*8(A-H,O-Z)
REAL MA,MO,LAM,MC,MH
COMMON/CONS/CPA,CPG,R,GAMA,GAMG
COMMON/PARS/CMPL,HP,HG,HA,HD,ETAM,ETAJ,ETAI,HAA

TO2=TO*(1.+(GAMA-1.)/2*MO**2)
PO2=PO*(1.+etai*(GAMA-1.)/2*MO**2)**HD
WRITE(*,1)TO2,PO2
1 FORMAT('AT EXIT FROM DIFFUSER',/, 'TO2=',F6.2,/, 'PO2=',F6.4)
C=MA*SQRT(TO2)/(PO2)
WRITE(*,2)C
2 FORMAT('THE VALUE OF NON-DIMENSIONAL MASS=',F7.2)
C
*****FAN FLOW*****
PO3=FPR*PO2
TO23=TO2*((FPR)**HAA-1.)
TO3=TO2+TO23
WRITE(*,3)TO3,PO3
3 FORMAT('AT EXIT FROM FAN',/, 'TO3=',F7.3,3X,'K',
1 /, 'PO3=',F5.2,3X,'BAR')
RS=TO3/TO2
CPR=TPR/FPR
TO4=TO3*(CPR)**HA
TO34=TO4-TO3
PO4=PO3*CPR
WRITE(*,4)TO4,PO4
4 FORMAT('AT EXIT FROM H.P.C',/, 'TO4=',F7.2,3X,'K',
1 /, 'PO4=',F5.2,3X,'BAR')
PO5=PO4*(1.-CMPL)
WRITE(*,5)TIT,PO5
5 FORMAT('AT ENTRY TO H.P.TURBINE',/, 'TIT=',F7.2,3X,'K',
1 /, 'PO5=',F5.2,3X,'BAR')
TO56=CPA*TO34/(ETAM*CPG)
TO6=TIT-TO56
PO6=PO5*(TO6/TIT)**(1./HG)
WRITE(*,7)PO6,TO6
C
C PARAMETERS THAT REMAIN CONSTANT FOR BOTH DESIGN AND OFF-DESIGN
C CONDITIONS, IF THE HP TURBINE REMAINS CHOKED THROUGHOUT.
C
D=PO5/PO6
E=TIT/TO6
WRITE(*,6)D,E
6 FORMAT('THE PR.ACROSS H.P.TURBINE =',F6.4,/, 'TEMP.
1 RATIO ACROSS H.P.TURBINE=',F6.4)
7 FORMAT('AT EXIT FROM H.P.TURBINE',/, 'PO6=',F5.2,3X,'BAR',
1 /, 'TO6=',F7.2,3X,'K')
C
TO67=CPA*(TO23)*(LAM+1.)/(CPG*ETAM)
TO7=TO6-TO67
PO7=PO6*(TO7/TO6)**(1./HG)
WRITE(*,8)TO7,PO7
8 FORMAT('AT EXIT FROM L.P.TURBINE',/, 'TO7=',F7.2,3X,'K',
1 /, 'PO7=',F5.2,3X,'BAR')
HH=PO6/PO7
G=TO6/TO7
WRITE(*,9)HH,G
9 FORMAT('THE PR.ACROSS L.P.TURBINE =',F6.4,/, 'TEMP.
1 RATIO ACROSS L.P.TURBINE=',F6.4)
TUNOPR=1./(1.-(1./ETAT)*((GAMG-1.)/(GAMG+1.))**HP)
WRITE(*,10)TUNOPR
10 FORMAT('CRITICAL LP TURBINE P.R.=',F6.3)
RETURN
END

```

```

C *****
C
C THIS SUBROUTINE IS TO PROVIDE NOZZLE CHARACTERISTICS, i.e. IT GIVES MASS
C FLOW PARAMETER VARIATION AT DIFFERENT PRESSURE RATIOS (BEYOND NOZZLE
C GETTING CHOKED). IT IS ESSENTIAL THAT ONE MUST ITERATE TO GET EXACT FIXED
C AREA OF THE NOZZLE BEFORE RUNNING THE PROGRAM.
C *****
C
C SUBROUTINE OZZCH(MCNA,MHNA,PRNH,ARE,K,PRNCC,
1 PRNHC)
C
C MCNA=COLD MASS FLOW PARAMETER
C MHNA=HOT MASS FLOW PARAMETER
C PRNH=PRESSURE RATIO
C
C IMPLICIT REAL*8(A-H,O-Z)
C REAL MHN,MHNA,MA,MP,MCN,MCNA
C DIMENSION PRNH(110),MHNA(110),MCNA(110)
C COMMON/PARS/CMPL,HP,HG,HA,HD,ETAM,ETAJ,ETA,HLA
C COMMON/CONS/CPA,CPG,R,GAMA,GAMG
C
C IF(K.EQ.1) THEN
C PRNCC=1./ (1.-(GAMA-1.)/(GAMA+1.)/ETAJ)**HD
C WRITE(*,2) PRNCC
2 FORMAT('CRITICAL PR. RATIO OF COLD N.=' ,F5.3)
C ELSE
C PRNHC=1./ (1.-(GAMG-1.)/(GAMG+1.)/ETAJ)**HP
C WRITE(*,3) PRNHC
3 FORMAT('CRITICAL PR. RATIO=' ,F5.3)
C ENDIF
C IF(K.EQ.1) THEN
C DO 4 N=1,INT((PRNCC-1.)*20)
C PRNH(1)=1.0
C PRNH(N+1)=PRNH(1)+(N/20.)
C MCN=(1./PRNH(N+1))*SQRT(2.*HD*ETAJ*(1.-(1./PRNH(N+1))
C & *(1./HD)))*1./ (1.-ETAJ*(1.-(1./PRNH(N+1))*(1./HD)))
C WRITE(*,*) PRNH(N+1),MCN
C MCNA(N+1)=MCN*(ARE/16.94)*100000.
C WRITE(*,*) PRNH(N+1),MCN
4 CONTINUE
C ELSE
C DO 5 N=1,INT((PRNHC-1.)*20)
C PRNH(1)=1.
C PRNH(N+1)=PRNH(1)+(N/20.)
C MHN=(1./PRNH(N+1))*SQRT(2.*HP*ETAJ*(1.-(1./PRNH(N+1))
C & *(1./HP)))*1./ (1.-ETAJ*(1.-(1./PRNH(N+1))*(1./HP)))
C MHNA(N+1)=MHN*(ARE/16.94)*100000.
5 CONTINUE
C ENDIF
C
C AFTER CRITICAL PRESSURE RATIOS BEYOND WHICH NOZZLE GETS CHOKED
C
C IF(K.EQ.1) THEN
C DO 6 L=INT((PRNCC-.9)*19),110
C PRNH(1)=.05
C PRNH(L+1)=PRNH(1)+(L/10.)
C MCN=(1.-((GAMA-1.)/(GAMA+1.))*(1./ETAJ))**HD
C & *SQRT(GAMA*(GAMA+1.)/2.)
C MCNA(L+1)=(ARE/16.94)*MCN*100000.
6 CONTINUE
C ELSE
C DO 7 L=INT((PRNHC-.9)*19),110
C PRNH(1)=.05
C PRNH(L+1)=PRNH(1)+(L/10.)
C MHN=(1.-((GAMG-1.)/(GAMG+1.))*(1./ETAJ))**HP
C & *SQRT(GAMG*(GAMG+1.)/2.)
C MHNA(L+1)=(ARE/16.94)*MHN*100000.
7 CONTINUE
C ENDIF
C RETURN
C END

```

3

of/de

3

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

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C *****
C
C THIS SUBROUTINE IS TO FIND OUT THE POINT OF INTERSECTION
C BETWEEN THE TURBINE-NOZZLE MATCHING CURVE AND THE COMPRESSOR
C CURVE. THE INTERSECTION POINT GIVES THE VALUE OF P04/PO, MASS FLOW
C PARAMETER, BPR, FPR AND CPR FROM WHICH OTHER PERFORMANCE PARAMETERS
C CAN BE CALCULATED.
C
C THE MATHEMATICAL TECHNIQUE USED IS CALLED AS ONE DIMENSIONAL
C SEARCHING TECHNIQUE AND IS ESSENTIALLY BASED ON BOLZANO'S
C BISECTION METHOD.
C *****
C
C SUBROUTINE COMPAR(HRITE,HLEFT,PR4,PRR4,HIH,PIH,FLIP,I,CPRX,MHX,
1 LAMX,FPRA,CAPUT,FAPUT,MAHUT,LAPUT)
C
C HRITE=RHS OF THE EQUILIBRIUM EQUATION
C HLEFT=LHS OF THE EQUILIBRIUM EQUATION
C PR4=OVERALL PRESSURE RATIO
C
C IMPLICIT REAL*8(A-H,O-Z)
C REAL MHX,MHXX,LAMX,LAMMYE,LAPUT,MAHUT
C COMMON/PARS/CMPL,HP,HG,HA,HD,ETAM,ETAJ,ETAI,HAA
C COMMON/CONS/CPA,CPG,R,GAMA,GAMG
C DIMENSION HRITE(110),HLEFT(110),PR4(110),PRR4(110)
C DIMENSION PP4P(55),HRITEP(55),HLEFTP(55),CPRX(110),MHX(110)
C DIMENSION CPRX(55),MHXX(55),LAMX(110),LAMYYE(55),FPRA(110)
C DIMENSION FPRAE(55)
C
C DO 1 I=1,109
C TAB=HRITE(I+1)
C PAB=HLEFT(I+1)
C IF(PAB.LE.TAB) THEN
C DDA=ABS(TAB-PAB)
C IF(DDA.LT..00005) THEN
C WRITE(*,*) 'DIFFICULTY IN SUBROUTINE COMPAR'
C GOTO 3
C ENDIF
C ELSE
C DO 2 K=1,50
C PP4=PR4(I)
C PP4P(K)=PP4+K/25.
C
C INTERPOLATING BETWEEN THE TWO POINTS TO GET THE EXACT
C VALUES OF MASS FLOW, BPR, FPR AND CPR.
C
C HRITEP(K)=HRITE(I)+(HRITE(I+1)-HRITE(I))/(PR4(I+1)-PR4(I))
1 * (PP4P(K)-PR4(I))
C HLEFTP(K)=HLEFT(I)+(HLEFT(I+1)-HLEFT(I))/(PR4(I+1)-PR4(I))
1 * (PP4P(K)-PR4(I))
C CPRXX(K)=CPRX(I)+(CPRX(I+1)-CPRX(I))/(PR4(I+1)-PR4(I))
1 * (PP4P(K)-PR4(I))
C MHXX(K)=MHX(I)+(MHX(I+1)-MHX(I))/(PR4(I+1)-PR4(I))
1 * (PP4P(K)-PR4(I))
C LAMMYE(K)=LAMX(I)+(LAMX(I+1)-LAMX(I))/(PR4(I+1)-PR4(I))
1 * (PP4P(K)-PR4(I))
C FPRAE(K)=FPRA(I)+(FPRA(I+1)-FPRA(I))/(PR4(I+1)-PR4(I))
1 * (PP4P(K)-PR4(I))
C TABO=HRITEP(K)
C PABO=HLEFTP(K)
C DADU=ABS(TABO-PABO)
C IF(DADU.LE..005) THEN
C FLIP=PP4P(K)
C HIH=HRITEP(K)
C PIH=HLEFTP(K)
C CAPUT=CPRXX(K)
C MAHUT=MHXX(K)
C FAPUT=FPRAE(K)
C LAPUT=LAMYYE(K)
C GOTO 3

```

```

      ENDIF
2     CONTINUE
      ENDIF
1     CONTINUE
3     CONTINUE
      RETURN
      END

C     *****
C
C     THIS SUBROUTINE WORKS ONLY WHEN THE NOZZLE IS CHOKED, IT GETS
C     AS INPUT TEMP. & PRESSURE AT ENTRY TO THE NOZZLE, & IT GIVES
C     AS OUTPUT SPECIFIC THRUST, GROSS THRUST PARAMETER & THE VELO-
C     CITY OF THE JET.
C     *****
C
C     SUBROUTINE CHOKED (CRNOPR, PD, CR, T2, TT, VL, PED, TED, TP, PP,
1     VELP, FFS, PHI, K, P2F, MC, MH)
C
C     FFS=SPECIFIC THRUST
C     PP=CRITICAL PRESSURE AT NOZZLE THROAT
C     VELP=VELOCITY OF THE JET
C
C     IMPLICIT REAL*8 (A-H, O-Z)
C     COMMON/ PARS/ CMPL, HP, HG, HA, HD, ETAM, ETAJ, ETAI, HAA
C     COMMON/ CONS/ CPA, CPG, R, GAMA, GAMG

      IF (K.EQ.1) THEN
        TP=2./ (GAMA+1.) *TED
      ELSE
        TP=2./ (GAMG+1.) *TED
      ENDIF
      PP=PED* (1./CRNOPR)
      RHOP=PP/ (R*TP)
      IF (K.EQ.1) THEN
        VELP=SQRT (GAMA*R*TP)
      ELSE
        VELP=SQRT (GAMG*R*TP)
      ENDIF
      IF (K.EQ.1) THEN
        WRITE (*, 1) TP, PP, VELP
1     FORMAT (' IN NOZZLE AT CRITICAL FLOW', /, 'TF=', F7.2, 3X, 'K',
2     /, 'PF=', F6.3, 3X, 'BAR', /, 'VELF=', F7.2, 3X, 'm/s')
      ELSE
        WRITE (*, 2) TP, PP, VELP
2     FORMAT (' IN NOZZLE AT CRITICAL FLOW', /, 'T.HOT=', F7.2, 3X, 'K',
3     /, 'P.HOT=', F6.3, 3X, 'BAR', /, 'VEL.HOT=', F7.2, 3X, 'm/s')
      ENDIF
      ARM=1./ (RHOP*VELP)
      FFS= (VELP-VL)+ARM* (PP-PD)
      WRITE (*, 4) FFS
4     FORMAT (' THE SPECIFIC THRUST IS', /, 'FS=', F8.3, 3X, 'N/(KG/S)')
      IF (K.EQ.1) THEN
        PHI=PP/P2F* (GAMA+1.)
      ELSE
        PHI=PP/P2F* (GAMG+1.)
      ENDIF
      PHI= (1.- (CPA*T2)/ (ETAM*CPG*TT)* (CR**HA-1.))** (1/HG)* (GAMG+1.)
      & *CR* (1.-CMPL)* (1.- (GAMG-1.)/ ((GAMG+1.)*ETAJ))**HP
      WRITE (*, 3) PHI
3     FORMAT (' PHI=', F7.3)
      RETURN
      END

C     *****
C
C     THIS SUBROUTINE WORKS ONLY WHEN THE NOZZLE IS UNCHOKED, IT
C     GETS AS INPUT TEMP. & PRESSURE AT ENTRY TO THE NOZZLE, & IT
C     GIVES AS OUTPUT SPECIFIC THRUST, GROSS THRUST PARAMETER & THE
C     VELOCITY OF THE JET.

```

```

C
C *****

SUBROUTINE UNCHOKED(TD,PD,TED,FD,VL,P2D,FNPR,TP,PP,VELZ,FZS,PHI,K,
1 SPM)
C
C FZS=SPECIFIC THRUST
C VELZ=COMPUTED VELOCITY OF JET
C FNPR=PRESSURE RATIO ACROSS THE NOZZLE
C
C IMPLICIT REAL*8(A-H,O-Z)
COMMON/PARS/CMPL,HP,HG,HA,HD,ETAM,ETAJ,ETAI,HAA
COMMON/CONS/CPA,CPG,R,GAMA,GAMG

PP=PD
IF(K.EQ.1)THEN
TOS=ETAJ*TED*(1.-(1./FNPR)**(1/HD))
ELSE
TOS=ETAJ*TED*(1.-(1./FNPR)**(1/HP))
ENDIF
TP=TED-TOS
IF(K.EQ.1)THEN
VELZ=SQRT(2.*CPA*(TOS))
VELM=SQRT(GAMA*R*TP)
WRITE(*,*)VELM
ELSE
VELZ=SQRT(2.*CPG*(TOS))
VELM=SQRT(GAMG*R*TP)
WRITE(*,*)VELM
ENDIF
IF(K.EQ.1)THEN
WRITE(*,1)TP,PP,VELZ
1 FORMAT('AT UNCHOKED NOZZLE',/, 'TF=',F7.2,3X,'K',/,
1 'PF=',F6.3,3X,'BAR',/, 'VELF=',F7.2,2X,'m/s')
ELSE
WRITE(*,2)TP,PP,VELZ
2 FORMAT('AT UNCHOKED NOZZLE',/, 'T.HOT=',F7.2,3X,'K',/,
1 'P.HOT=',F6.3,3X,'BAR',/, 'VEL.HOT=',F7.2,2X,'m/s')
ENDIF
WRITE(*,*)'VL=',VL
FZS=VELZ-VL
WRITE(*,3)FZS
3 FORMAT('THE SPECIFIC THRUST IS',/, 'FS=',F8.3,3X,'N/(KG/S)')
IF(K.EQ.1)THEN
YM=(VELZ**2)/(GAMA*R*TP)
ELSE
YM=(VELZ**2)/(GAMG*R*TP)
ENDIF
SPM=VELZ/VELM
WRITE(*,*)'SPM=',SPM
IF(K.EQ.1)THEN
PHI=(GAMA*YM+1.)/(P2D/PD)
ELSE
PHI=(GAMG*YM+1.)/(P2D/PD)
ENDIF
WRITE(*,4)PHI
4 FORMAT('PHI=',F7.3)
RETURN
END

C *****
C
C THIS SUBROUTINE IS NOTHING BUT A POLYNOMIAL CURVE FITTING PROGRAM, HERE
C USED TO FIT A THIRD ORDER POLYNOMIAL BETWEEN (1-TIT/TO5)/(1-(TIT/TO5)DES)
C AND OVERALL PRESSURE RATIO PO3/PO.
C *****
C
C SUBROUTINE LSTM(N,M1,M2,X,A,S,FAIL)
C

```



```

C      N= NO. OF POINTS TO BE INPUT
C      M1=ORDER OF POLYNOMIAL +1
C      M2=ORDER OF POLYNOMIAL +2
C      X= INPUT POINTS
C      A= COMPUTED COEFFICIENTS OF POLYNOMIAL
C
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER N,M1,M2
      DIMENSION X(2,N),A(M1),S(M1,M2)
      M=M1-1
      DO 5 I=1,M1
      DO 5 J=1,M2
      S(I,J)=0.0
5      CONTINUE
      S(1,1)=N
      DO 6 J=1,N
      S(1,M2)=S(1,M2)+X(2,J)
6      CONTINUE
      DO 7 I=2,M1
      I1=I-1
      MI=M1+I-2
      DO 7 J=1,N
      S(I,1)=S(I,1)+X(1,J)**I1
      S(M1,I)=S(M1,I)+X(1,J)**MI
      S(I,M2)=S(I,M2)+X(1,J)**I1*X(2,J)
7      CONTINUE
      DO 8 J=2,M
      DO 8 I=J,M
      S(I,J)=S(I+1,J-1)
8      CONTINUE
      DO 9 I=1,M
      I1=I+1
      DO 9 J=I1,M1
      S(I,J)=S(J,I)
9      CONTINUE
      CALL GS(M1,S,A,M2,FAIL)
      RETURN
      END
      SUBROUTINE GS(N,A,X,N1,FAIL)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER N,N1
      DIMENSION A(N,N1),X(N)
      DO 11 K=1,N
      DO 6 I=K,N
      IF (A(I,K).EQ.0.0) GOTO 6
      IO=I
      GOTO 7
6      CONTINUE
      FAIL=-1.
      RETURN
7      IF (IO.EQ.K) GOTO 9
      DO 8 J=K,N1
      T=A(K,J)
      A(K,J)=A(IO,J)
8      A(IO,J)=T
9      K1=K+1
      DO 10 J=K1,N1
      A(K,J)=A(K,J)/A(K,K)
      DO 11 I=K1,N
      DO 11 J=K1,N1
      A(I,J)=A(I,J)-A(I,K)*A(K,J)
11     CONTINUE
      X(N)=A(N,N1)
      NI=N-1
      DO 12 K=1,N1
      K1=NI-K+1
      X(K1)=A(K1,N1)
      K2=K1+1
      DO 12 J=K2,N
      X(K1)=X(K1)-A(K1,J)*X(J)
12     CONTINUE
      FAIL=0
      RETURN
      END

```

```

C *****
C
C THIS SUBROUTINE FITS A POLYNOMIAL, WHEN IT GETS X-AXIS POINT
C AND THE COEFFICIENTS FOR THE POLYNOMIAL. THUS IN OUTPUT IT
C GIVES THE VALUE OF Y-AXIS POINT.
C *****
C
SUBROUTINE FIT(X,A,Y,M1)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(M1)
Y=0.0
DO 25 I=1,M1
Y=Y+A(I)*X**(I-1)
25 CONTINUE
RETURN
END

C *****
C
C THIS SUBROUTINE (WORKS ON STANDARD I.S.A.)
C CALCULATES STANDARD ATMOSPHERIC PROPERTIES FOR INPUT ALTITUDE
C *****
C
SUBROUTINE ALTITU(HEIGHT,TD,PD,RHD)
C
C TD=TEMPERATURE
C PD=PRESSURE
C RHD=DENSITY
C
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER JALT,KALT
DIMENSION T(101),P(101)
DATA T/288.2,286.2,284.2,282.2,280.2,278.3,276.3,274.3,272.3,
1 270.3,268.3,266.3,264.3,262.4,260.4,258.4,256.4,254.4,252.5,
2 250.6,248.6,246.6,244.7,242.7,240.7,238.7,236.7,234.7,232.7,
3 230.8,228.8,226.9,224.9,222.9,220.9,218.9,216.9,46*216.7,
4 217.3,218.2,219.,220.,220.9,221.8,222.7,223.6,224.5,225.4,
5 226.3,227.2,228.1,229.0,229.9,230.8,231.8,232.7/
DATA P/1.01325,.9772,.9422,.9081,.8752,.8431,.8121,.7819,.7527,
1 .7244,.6969,.6704,.6445,.6134,.5954,.5721,.5494,.5275,.5064,
2 .4858,.4660,.4468,.4283,.4104,.3932,.3765,.3604,.3448,.3298,
3 .3154,.3015,.2880,.2751,.2626,.2506,.2391,.2279,.2173,.2071,
4 .1974,.1882,.1794,.1711,.1631,.1554,.1482,.1413,.1346,.1284,
5 .1224,.1166,.1112,.1059,.1011,.09632,.09184,.08756,.08343,
6 .07956,.07584,.07233,.06895,.0657,.06267,.0597,.05695,.05426,
7 .05178,.04936,.04702,.04481,.04274,.04075,.03889,.03702,.0353,
8 .03371,.03213,.03061,.02917,.02785,.0265,.0253,.02413,.02303,
9 .0219,.02096,.02,.01910,.0182,.01737,.01662,.01586,.01517,
1 .01447,.01386,.01324,.01262,.012,.01158,.01103/
IF (HEIGHT.LT.0.) HEIGHT=0
IF (HEIGHT.GE.100000.) GOTO 10
JALT=INT(HEIGHT/1000.)+1
KALT=JALT+1
FAC=HEIGHT/1000.-FLOAT(JALT-1)
TD=FAC*(T(KALT)-T(JALT))+T(JALT)
PD=FAC*(P(KALT)-P(JALT))+P(JALT)
RHD=PD*100000./(TD*287.)
RETURN
10 WRITE(*,11)
11 FORMAT(9('**'),'MAX. ALTITUDE IS 100,000 FEET',9('**'))
HEIGHT=34000
PD=.01103
TD=232.7
RHD=.01652
RETURN
END

```

```
30 DETA=0.0
   RETURN
20 DETA=DETA*A(K,K)
   DO 40 J=1,NCOL
   IF (J-K) 50,40,50
50 A(K,J)=A(K,J)/A(K,K)
40 CONTINUE
   A(K,K)=1./A(K,K)
   DO 10 I=1,N
   IF (I-K) 60,10,60
60 DO 70 J=1,NCOL
   IF (J-K) 80,70,80
80 A(I,J)=A(I,J)-A(I,K)*A(K,J)
70 CONTINUE
   A(I,K)=-A(I,K)*A(K,K)
10 CONTINUE
   RETURN
   END
```

```
C *****
C *****
```

END

1 2 0 4 9 1

FIN