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Subband Adaptive Filters

by

James Gregory Ryan

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
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Ottawa, Ontario, Canada, K1S 5B6

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the degree of Master of Engineering

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Abstract

This thesis studies subband adaptive filters. A review of previous research on this topic indicates that, although subband adaptive filters have been used in various applications, no thorough study of the effects of subbanding on system performance has been undertaken. To overcome this problem, computer experiments are performed using subband adaptive filters for both simulated and real conditions.

Adaptive noise cancelling experiments using simulated conditions demonstrate that subband adaptive filters can provide significant noise power reduction when the number of LMS taps per band is much less than the impulse response length of the unknown channel. In addition, subband adaptive filters provide higher initial convergence rates than time-domain LMS filters. Techniques for reducing the computational complexity of the adaptive filter also have a significant impact on performance.

These results suggest that subband adaptive filters are advantageously applied to problems where the input signals are coloured and the unknown channels are non-stationary. Experiments with primary and reference signals recorded in a moving automobile show that subband adaptive filters provide higher noise power reductions than TD LMS filters in this highly non-stationary environment.

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List of Abbreviations and Acronyms

ADF	Adaptive Digital Filter
ANC	Adaptive Noise Cancelling
AFB	Analysis Filterbank
dB	Decibel
DC	Direct Current (Frequency of zero in digital systems)
DFT	Discrete Fourier Transform
DSP	Digital Signal Processing
FB	Fullband
FD	Frequency Domain
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
IDFT	Inverse DFT
IFFT	Inverse FFT
IIR	Infinite Impulse Response
LMS	Least Mean Squares
MSE	Mean Squared Error
NLMS	Normalized LMS
QMF	Quadrature Mirror Filter
RAP	Row Action Projection
SB	Sub-band
SFB	Synthesis Filterbank
SSB	Single Sideband Modulation
TD	Time Domain

List of Mathematical Notation

α	Fraction of stability limit used as step size
$d(n)$	primary input signal or desired signal
$e(n)$	error signal
D	subband signal decimation factor
$f(n)$	synthesis filterbank lowpass prototype impulse response
$h(n)$	analysis filterbank lowpass prototype impulse response
$H(e^{j\omega})$	Fourier transform of signal $h(n)$
J_{ex}	excess mean squared error
J_{min}	minimum mean squared error
J_{tot}	total mean squared error
L	sparse LMS filter tap-spacing
λ_i	i^{th} eigenvalue of autocorrelation matrix
m	sample index for decimated time scale
M	number of LMS filter taps
μ	LMS algorithm step size parameter
n	sample index for non-decimated time scale
$\underline{p}$	cross-correlation vector
$r(k)$	autocorrelation function for a shift of k
$\mathbf{R}$	reference input signal autocorrelation matrix
σ_x^2	power or variance of signal $x(n)$
w_i	i^{th} adaptive filter tap weight
$\mathbf{w}_o$	optimum adaptive filter tap weight vector

ω	digital frequency
ω_k	center frequency of k^{th} band of filterbank
$\frac{\Delta\omega}{2}$	bandwidth of lowpass prototype filter, $h(n)$
$x(n)$	reference input signal
$X_k^{SSB}(m)$	k^{th} SSB subband output signal for input $x(n)$
$y(n)$	adaptive filter output signal
z^{-1}	unit sample delay operator
$\lfloor x \rfloor$	largest integer less than or equal to x

Chapter 1

Introduction

1.1 Description of Problem

With the advent of low-cost, high-performance digital signal processors, the application of adaptive filtering techniques to real-time signal processing problems is becoming more and more attractive. This is particularly true of adaptive filters based on the Least Mean Squares (LMS) algorithm.

The popularity of the LMS algorithm is due to its relatively simple implementation and, when applied to adapting the coefficients of a FIR filter, its guaranteed stability.

Although successfully applied to some problems [1, 2], the LMS algorithm has certain limitations. The most troublesome of these is the dependence of performance on the input signal characteristics and adaptive filter length. It is widely known that the LMS algorithm converges slowly for input signals with non-flat power spectra and for cases where long FIR filters are being adapted [3].

These limitations of the LMS algorithm have impeded its successful application in many areas involving acoustic signals such as adaptive noise cancelling for mobile communications. In these applications, the input signals rarely have flat power spectra and the length of the adaptive filters is usually quite long owing to the length of the impulse responses of acoustic channels.

Attempts to overcome the limitations of the LMS algorithm prompted researchers to investigate subband adaptive filters. In applications to adaptive noise cancelling in mobile communications, subband adaptive filters have been successful in eliminating the noise enhancement problem associated with LMS adaptive FIR filters [4–7].

Despite this successful application, no thorough study of the full effect of the subbanding operations on systems performance has been undertaken. Researchers studying the application of subband adaptive filters to acoustic echo cancellers have

attempted to provide some insight [8–10], however, several of the conclusions drawn seem counter-intuitive and require further investigation.

1.2 Thesis Objective

This thesis examines the use of subband adaptive filters to as a means to overcome some of the limitations of regular LMS adaptive filters. The focus is on digital signal processing (DSP) implementations of adaptive noise cancelling (ANC) in mobile communications using single reference inputs. The results can be extended to multi-reference ANC [7].

Specifically, this thesis examines the impact of the subbanding operation on the performance of the subband adaptive filters. Computer experiments of both simulated and real conditions are used to provide insight into the effects of the filterbank type, the number of bands and the various computational reduction techniques upon system performance.

The results obtained from the experiments with simulated input signals are used to explain the improvements noted in the experiments with real input signals. Based on these discussions, a class of applications which may benefit from the use of subband adaptive filters is identified.

1.3 Thesis Outline

The thesis is organized as follows. Chapter 2 provides a review of the relevant background material on the LMS algorithm and ANC and introduces Frequency Domain (FD) adaptive filters a class of adaptive algorithms which includes subband adaptive filters.

Chapter 3 begins with a summary of the important design characteristics of a filterbank for use in a subband adaptive filter. It then presents a subband adaptive filter structure for each of the two filterbank types discussed: modulated and non-modulated filterbanks. Included is a discussion of techniques for minimizing the computation rate required for implementing the adaptive portion of these algorithms.

Experimental results for both types of subband adaptive filters using simulated input signals are presented in Chapter 4 and using real input signals in Chapter 5. The main conclusions derived from this work and suggestions for further research are discussed in Chapter 6.

Chapter 2 Background

This chapter provides some background material for the discussion of subband adaptive filters to follow. The regular time-domain LMS algorithm is first introduced followed by a review of the relevant literature on adaptive noise cancelling (ANC) and frequency-domain (FD) adaptive filters. The chapter concludes with a thorough review of subband adaptive filters.

2.1 Time Domain LMS Algorithm

The LMS adaptive FIR filter is depicted in Figure 2.1. This implementation will be referred to as the time-domain (TD) LMS algorithm to distinguish it from the frequency-domain (FD) and subband algorithms to be introduced later.

Figure 2.1 Signal flow representation of the time-domain LMS algorithm.

The complete TD LMS algorithm can be summarized as follows (see Figure 2.1):

1. The filter output, $y(n)$, is the weighted sum of the output of a tapped delay line:

$$y(n) = \sum_{i=0}^{M-1} w_i x(n-i)$$

2. The error signal, $\epsilon(n)$, is the difference between the LMS filter output and the desired response, $d(n)$:

$$\epsilon(n) = d(n) - y(n)$$

3. The error signal is cross-correlated with the outputs of the tapped delay line and a correction is applied to each tap weight, $w_i(n)$. The size of the correction is controlled by the step size parameter, μ :

$$w_i(n+1) = w_i(n) + \mu x(n-i)\epsilon(n) \quad (2.3)$$

The algorithm is normally initialized with all $w_i(n) = 0$.

When applied to adapting the coefficients of an FIR filter, the LMS algorithm is iteratively seeking a solution to the normal equation [3]:

$$\mathbf{R}\mathbf{w}_o = \mathbf{p} \quad (2.4)$$

where $\mathbf{w}_o$ is the optimum Weiner filter coefficient vector, $\mathbf{R}$ is the input autocorrelation matrix and $\mathbf{p}$ is the cross-correlation vector between the desired response, $d(n)$, and the tap inputs, $x(n-i)$.

Since Equation 2.4 is a system of linear equations, some of the properties of linear equations apply. In general, there are three distinct cases [11]:

1. The matrix $\mathbf{R}$ is non-singular and well-conditioned. In this case a unique solution for $\mathbf{w}_o$ exists. A random input signal with a flat power spectrum produces this type of autocorrelation matrix.
2. The matrix $\mathbf{R}$ is non-singular but ill-conditioned. In this case a unique solution exists but it is extremely sensitive to any perturbations in the matrix, $\mathbf{R}$. Since the LMS algorithm uses instantaneous estimates of $\mathbf{R}$ and $\mathbf{p}$ in solving the normal equation, such perturbations will exist in $\mathbf{R}$ and will impede the progress of the algorithm in finding $\mathbf{w}_o$. An ill-conditioned autocorrelation matrix is caused by input signals with colored input spectra.

3. The matrix $\mathbf{R}$ is singular. When $\mathbf{R}$ is singular, the normal equation represents an over determined system. The solution for $\mathbf{w}_0$ will not be unique but the LMS algorithm will still converge to one solution, determined by initial conditions. A singular matrix is caused when the input signal consists of the sum of Q sinusoids and where this number is less than half the number of LMS taps, M [11].

The LMS algorithm, as presented above, minimizes the mean squared value of the error signal, $e(n)$. The steady-state value of the mean squared error produced by the LMS algorithm operating in a stationary environment is composed of two terms [3]: i) the theoretical minimum mean squared error, and, ii) an excess mean squared error due to the operation of the algorithm.

The theoretical minimum MSE, denoted J_{min} , is determined by the characteristics of the desired signal and by the capacity of the number of LMS taps to model the unknown system. The steady-state excess MSE, denoted J_{ex} , is determined by the selection of the step size parameter, μ . Under the assumptions of the independence theory, the theoretical value for J_{ex} is given by:

$$J_{ex} = \left(\frac{\mu \sum_{i=0}^{M-1} \lambda_i}{2 - \mu \sum_{i=0}^{M-1} \lambda_i} \right) J_{min} \quad (2.5)$$

where the λ_i are the eigenvalues of the autocorrelation matrix of the input signal, $x(n)$. The total MSE for a LMS adaptive filter is the sum of the minimum and excess mean squared errors:

$$J_{tot} = J_{min} \left[1 + \frac{\mu \sum_{i=0}^{M-1} \lambda_i}{2 - \mu \sum_{i=0}^{M-1} \lambda_i} \right] \quad (2.6)$$

As might be expected from viewing equation 2.6, the value assigned to the step size parameter, μ , has an important influence on the behaviour of the algorithm. A small value for μ produces a small total MSE but results in a slow adaptation of the

tap weights (see equation 2.3). Conversely, a large value for μ produces a larger total MSE but results in faster adaptation up to a point. The limit on the speed of adaptation is governed by the stability limit on μ . This limit states that to ensure the stability of the tap weight update, the step size must be chosen so that:

$$\mu < \frac{2}{\sum_{i=0}^{M-1} \lambda_i} = \frac{2}{M\sigma_r^2} \quad (2.7)$$

where σ_r^2 is the power of the input signal, $x(n)$. In some applications the step size is chosen as a fraction of this stability limit. That is:

$$\mu = \frac{2\alpha}{M\sigma_r^2} \quad (2.8)$$

where α is a number between 0 and 1.

In applications where the exact input power is not known or is highly variable, a more appropriate choice is to calculate the exact value for μ at each iteration. Equation 2.8 suggests the following form for the variable step size calculation:

$$\mu = \frac{2\alpha}{\epsilon + \sum_{i=0}^{M-1} x^2(n-i)} \quad (2.9)$$

where ϵ is a small positive constant used to prevent the step size from becoming too large and the $x(n-i)$ are the input signal samples stored in the LMS tapped delay line. This version of the LMS algorithm, with variable step size, is known as the normalized LMS algorithm [3, 12].

When the step size is calculated according to equation 2.9, an approximate expression for the steady-state MSE can be derived that depends only on the choice of α . This derivation is based on the fact that, for stationary input signals, the average value of μ will be:

$$\mu \approx \frac{2\alpha}{\sum_{i=0}^{M-1} \lambda_i} \quad (2.10)$$

Substituting this expression into equation 2.6 yields:

$$J_{tot} = J_{min} \left[1 + \frac{\alpha}{1 - \alpha} \right] \quad (2.11)$$

Two other factors which influence the behaviour of the LMS algorithm (including the normalized LMS algorithm) are the number of LMS FIR taps and the eigenvalues of the input autocorrelation matrix, λ_i .

Referring to equation 2.7, the maximum value of μ is inversely proportional to the number of tap weights, M . A larger value of M yields a smaller upper bound on μ and yields a slower upper bound on adaptation speed. Conversely, a smaller value of M yields a larger upper bound on μ and a higher upper bound on adaptation speed.

Equation 2.7 demonstrates the dependence of adaptation speed on the sum of the eigenvalues, λ_i . Transient analysis of the LMS algorithm shows that adaptation speed also depends on the distribution of the values of λ_i .

Generally speaking, input signals with a large spread between the maximum and minimum eigenvalues will produce relatively slow adaptation of the tap weights whereas input signals where the maximum and minimum eigenvalues are approximately equal will produce relatively fast adaptation.

The physical interpretation of this is due to the relationship between the eigenvalues of the input autocorrelation matrix, R , and uniformly spaced samples of the power spectrum of the input signal [13]. As the order of the matrix, R , increases the eigenvalues approach the values of the power spectrum taken at uniform intervals along the frequency axis.

Thus, signals with a large spread between the maximum power and minimum power will cause slow adaptation whereas signals with maximum and minimum power relatively equal will converge more quickly.¹

The final important characteristic of the LMS and NLMS algorithms is the simplicity with which they can be implemented. The LMS algorithm requires the calculation of

¹ Sinusoidal inputs are an exception to this. For sinusoids, the signal spectrum is nonzero only at one frequency but the LMS algorithm converges very quickly. See [14-16] for more information on sinusoidal inputs.

an M tap FIR filter output and the computation of a new set of M tap weights for each output sample. The total number of calculations per sample is:

Multiplies	Adds	Divides
$2M$	$2M$	0

The added complexity of the NLMS algorithm is due to the calculation of the step size. As noted in [11], however, where sufficient memory storage is available to hold all the $x^2(n-i)$, the increase in the number of computations is limited to one multiply, one add, one subtraction and one divide. The total number of calculations per sample for the NLMS algorithm is:

Multiplies	Adds	Divides
$2M + 1$	$2M + 2$	1

Note that, in the above table and all subsequent tables of complexity, a subtraction operation is treated as equal to an addition.

The results on ANC to be presented in the next section deals with the form of the LMS algorithm presented above.

2.2 Adaptive Noise Cancelling

A large amount of research has been done on adaptive noise cancelling (ANC). This section focuses only on the research which has prompted this thesis. For a more complete review of ANC see [7].

The block diagram of a typical ANC application is shown in Figure 2.2. Basically, a desired signal, $d(n)$, is corrupted by a noise source, $x(n)$, that originates elsewhere and, thus, is modified by an unknown transmission channel. The goal is to recover the desired signal from the corrupted signal.

The primary input to the adaptive noise canceller is the corrupted signal. The reference input, usually obtained from a second transducer, is a measure of the reference signal, $x(n)$. The adaptive noise canceller attempts to remove the primary signal noise by estimating the modified noise signal and subtracting this estimate from the primary input.

Figure 2.2 An adaptive filter in a typical application to Adaptive Noise Cancelling (ANC)

Goubran studied the use of LMS adaptive filters for the cancellation of background acoustic noise in mobile telephones [4, 5]. The first attempt used a two microphone system: one primary microphone located near the driver of the vehicle measuring the speech signal and one reference microphone measuring the noise sources in the vehicle. The reference microphone was placed in various locations throughout the passenger compartment to evaluate the effects of the different noise sources. Simulations of full-band ANC systems using real measured car data showed overall noise power reductions; however, the noise power reduction was confined to low frequencies where the input

signal power was greatest. At higher frequencies, undesirable noise enhancement was observed. Similar findings were reported by other authors [17, 18].

Noting this uneven performance of the fullband ANC systems over the full frequency range, Goubran [5] suggested isolating those portions of the signal spectrum with good performance and performing the noise cancellation only in these regions. The proposed system, shown in Figure 2.3, uses a lowpass filter in the primary and reference inputs to the adaptive filter. The results presented show that this system provides noise cancellation in the lower frequency band without affecting the noise in the upper frequency band, thus providing a distinct advantage over fullband systems since no noise enhancement has been introduced by the canceller.

Figure 2.3 Lowpass adaptive noise canceller structure of Goubran [4].

Hébert [6] and Wallace [7, 19] extended the subband principle to cover the entire frequency range using two and three band systems, respectively. Separate LMS filters were applied to each band and the subband adaptive filter outputs were combined to yield a fullband output. This system is shown in Figure 2.4 for the two band case.

Figure 2.4 Two band adaptive noise canceller structure of Hébert [6].

The multiband systems proposed by Hébert and Wallace were successful at providing noise attenuation over the full signal spectrum but used an *a priori* knowledge of the input signal characteristics to choose the frequency placement of one or more fixed FIR filters. Also, insofar as its ability to reduce the computational complexity is concerned, the band limited nature of the subband inputs was not used advantageously.

2.3 Frequency Domain LMS Algorithm

The frequency domain adaptive filter has appeared in many forms since first proposed in [20]. It was not until [21], however, that the theory of block adaptive filtering was presented as a means of unifying all the frequency domain approaches.

The block adaptive filtering theory divides frequency domain algorithms into two groups: those that incorporate block processing of the input data and those that incorporate stream processing. This framework formed the basis of a recent tutorial paper by Shynk [22].

2.3.1 Block Frequency Domain Algorithms

All the algorithms in this category divide the input data into blocks of points for FFT processing. Output data is generated by an inverse block procedure such as the overlap-save or the overlap-add methods.

The basic idea of the block adaptive filter is to transform the FIR filtering and tap weight update of the LMS algorithm into the frequency domain. In the frequency domain, a time domain convolution can be implemented as a point by point multiplication. Whether the convolution so implemented is linear or circular will depend on the partitioning of the input data. Published algorithms on frequency domain adaptive filters have taken both approaches.

Where data partitioning is done properly, the frequency domain adaptive filters yield results identical to time domain adaptive filters.

2.3.1.1 Circular Convolution

The frequency domain algorithm proposed by Dentino et al. was a simple transformation of the entire LMS algorithm into the frequency domain, including error signal calculation [23]. The algorithm is shown in Figure 2.5.

The primary and reference input signals are divided into blocks of M points and transformed using M point FFTs. The complex LMS algorithm is then used to update the complex tap weights in the frequency domain, once per M point block. This structure was analyzed in [24] and [25].

Although the Dentino algorithm achieves a significant reduction in the number of operations required to implement a M point LMS filter, it ignores the distinction between circular and linear convolution. This is unacceptable for all but a few applications.

Figure 2.5 Frequency-domain adaptive filter of Dentino et al.[23].

2.3.1.2 Linear Convolution

Perhaps the earliest frequency domain adaptive filter was proposed by Walzman for use in an adaptive line enhancer (ALE) [20]. The motivation for this algorithm was to improve the ALE convergence speed for certain types of input signals while maintaining a low complexity. The structure of the algorithm is shown in Figure 2.6.

In this structure, the input data are partitioned into N point blocks where N is equal to twice the number of LMS filter taps, M . The additional $N - M$ points of the block are zero-filled. These data blocks are then transformed using N -point FFTs and the tap weight update is computed in the frequency domain.

In addition to the difference in data partitioning, the structure of Figure 2.6 also included a time-domain constraint on the tap weight vector. Since there are $2M$ FFT points used to implement an M tap LMS filter, an additional constraint must be placed upon the tap weight vector to ensure the tap weight solution represents a realizable M -tap FIR filter. The necessary operations are included in the shaded region of Figure 2.6. A similar constraint is included in all other frequency domain adaptive filters implementing

Figure 2.6 Frequency Domain Adaptive Filter proposed by Walzman and Maiwald [20, 26]. The portion of the diagram in the shaded box implements the time-domain constraint on the tap weight solution.

a linear convolution [27–33]. A recent paper on computational complexity for digital signal processor implementations has included studies of both types of algorithms [34].

An algorithm similar to Dentino's was proposed by Mansour and Grey [35]. This structure implements a linear convolution of an M -tap LMS filter with the input data without a time domain constraint on the tap weight solution. However, as pointed out by Clark, Mansour is simply trading the constraint on the tap weight vector for a constraint on the order of the unknown impulse response [28]. Thus, this algorithm only converges to the same solution as the time domain case when the order of the unknown system is less than M . That is, when the effects of the block processing of the FFTs can be neglected [25]. The algorithm is shown in Figure 2.7.

The main benefit of Mansour's algorithm was the use of a step size normalized to the expected power in each frequency bin to increase convergence speed.

Many frequency domain adaptive filter algorithms proposed since Walzman are

Figure 2.7 Frequency domain adaptive filter structure of Mansour and Grey [35].

variations of the same structure. For example, the system proposed by Ferrara, shown in Figure 2.8, uses a similar structure but incorporates the overlap save method for reconstructing the output sequence [27]. Note the differences in the data partitioning.

2.3.2 Stream Processing

The frequency domain algorithms in this category process the input data as a continuous stream. Techniques such as the transform domain adaptive filter, frequency sampling and subband adaptive filters are considered part of this group [22]. The subband adaptive filters, however, will be treated separately in this thesis.

The transform domain adaptive filter was first proposed by Narayan et al.[36, 37] and also studied by Lee et al.[38]. The basic structure is shown in Figure 2.9.

This adaptive filter utilizes a tapped delay line similar to the time domain LMS FIR. Instead of processing the tap outputs directly, however, they are transformed them using an orthogonal transform such as a DFT. A single, complex tap weight is then applied to the output of each transform point.

Figure 2.8 Frequency domain adaptive filter structure of Ferrara [27].

Figure 2.9 Frequency Domain LMS filter proposed by Narayan [36, 37]

While this structure promises an increase in convergence speed over the time domain LMS, there is no potential for reduced complexity since all processing is done at the original sampling rate.

A similar structure was proposed by Bitmead [39]. The difference is that the orthogonal transform is replaced by a bank of frequency sampling filters.

The last frequency domain method is somewhat of a hybrid scheme. It is called the Frequency Bin Adaptive filter and was proposed by Asharif et al. [40, 31]. In the frequency bin adaptive filter an FIR filter is placed in each FFT frequency bin. Due to this, shorter FFTs can be used since past FFT outputs are stored in the tapped delay line of the FIR filter.

Figure 2.10 Frequency Domain LMS filter proposed by Asharif et al.[40]

Although faster convergence is obtained, the initial formulation of this structure utilized circular convolution (as in the early frequency domain adaptive filters) and thus, did not converge to the same solution as the time domain filter. A later formulation of this system (incorporating the constraints of Ferrara) yields a system with slower convergence [31].

2.4 Subband Adaptive Filters

The block adaptive filter algorithms presented in the previous section all suffer from two distinct drawbacks. First, there is a large input to output delay associated with the

block processing. Recall that to implement an M tap LMS adaptive filter requires the computation of N point FFTs where $N > M$. This implies a delay of N samples which may be prohibitive in applications where M is large.

A second drawback is due to the FFT algorithm. Efficient schemes for computing the FFT exist only when the number of points is a highly composite number (e.g. a power of two). Thus the computational reductions promised by the various FD techniques may not be achievable if the desired number of taps is not close to a composite number.

FD methods based on stream processing overcome the delay problem, however, for algorithm dependent upon the FFT for computational advantages, the second drawback still exists. A technique which is capable of mitigating the delay problem and eliminating the dependence upon FFT processing constraints is the subband adaptive filter.

The advantages of the subband adaptive filter over FD adaptive filter is that the amount of signal delay is unrelated to the length of the adaptive filter. In addition, a complexity reduction is possible if multirate techniques are used in the subband signals.

The general structure of a subband adaptive filter is shown in Figure 2.11. This structure was first proposed by Kellerman for cancelling acoustic echoes in handsfree telephones [8, 9].

The primary and reference inputs are split into their various frequency components by analysis filterbanks AFB1 and AFB2. The subband error signals are then recombined by the synthesis filterbank, SFB, to form the fullband output.

As might be expected, the behaviour of the subband adaptive filter is highly dependent upon the characteristics of the AFB/SFB chosen.

The subband adaptive filter proposed by Kellerman is based on a polyphase DFT filterbank, a form of complex modulator filterbank. The back to back analysis/synthesis system is designed to give distortion free reconstruction of the input signal when the adaptive filters are disabled. The complex-valued subband signals are decimated by an integer factor, D , where D is less than K , the number of subbands. As discussed in [8, 9] choosing D less than K reduces the effect of aliasing for a given stopband attenuation.

Figure 2.11 Subband adaptive filter structure proposed by Kellerman [8, 9].

The adaptive filters in each subband operate at the reduced sampling thereby reducing the overall computation rate.

The system proposed by Kellerman provides faster convergence than TD LMS for certain types of input signals and has a lower computation rate when long LMS filters are used.

In his paper, Kellerman noted that as the number of subbands increased and the bandwidth decreased, the required number of LMS taps in the subbands increased. Similar findings have been reported by other authors [41].

The increase in the number of subband taps has been attributed to the bandlimiting effect of the analysis filters. That is, the narrow bandwidth of the analysis filterbank causes an increase in the length of the impulse response of the unknown channel. This behaviour is in apparent contradiction with the intuitive notion that as the bandwidth of a signal decreases, the number of taps necessary to provide adequate cancellation should also decrease. This point is explored further in subsequent chapters.

For applications where computational power is abundant, Gay and Mammone propose a method to improve convergence speed by using multiple iterations of the

adaptive algorithm (in this case the RAP algorithm) on past input data [42]. While this method does improve convergence speed, it is applying multiple repetitions of the adaptation algorithm and, thus, is restricted to applications where large amounts of processing power is available.

The SBAF structures proposed by other authors can be categorized as attempting to alleviate one of the two main problems associated with these systems. These problems are reducing the computation rate and reducing the primary signal delay.

2.4.1 Reduced Complexity

In an attempt to further reduce the computational complexity, Gilloire [10] replaces the polyphase DFT filterbank used by Kellerman with the pseudo-QMF filterbanks of Rothweiler [43]. The use of pseudo-QMF filterbanks provides real subband signals and permits the use of the real LMS algorithm. A problem arises, however, since the subband signals are critically decimated. The adjacent band aliasing products, which would normally be cancelled on reconstruction, do not cancel due to the effects of the subband adaptive filters.

In a subsequent paper, Gilloire and Vetterli propose a method for cancelling these alias products by including adaptive cross-terms between adjacent subbands [44]. Two such cross-terms are illustrated in Figure 2.12.

The blocks denoted by F_i are fixed filters equal to the product of the transfer function of two adjacent bandpass filters. As a result, they pass only those frequencies common to two adjacent channels. The A_i are adaptive filters which operate on these reduced bandwidth signals and adaptively cancel the adjacent band aliasing.

This method, although successful at reducing aliasing, increased system complexity and slowed the system convergence speed [45].

Tzeng proposed the use of QMF band splitting filters for the analysis/synthesis filterbanks [41]. The filterbanks were of the type designed by Johnston [46]. However, since critical sub-sampling is used, aliasing distortion is present in the output.

Figure 2.12 Cross term cancellation structure proposed by Gilloire [44].

Slock recently proposed a method to reduce system complexity in subband adaptive filters by utilizing fractional decimation factors for the subband signals [47]. The fractional decimation is achieved by non-uniform sampling of the subband signals. Although complexity is reduced, the final levels of misadjustment are not as low as for the oversampled case due to the increased aliasing.

Other authors have attempted to keep complexity low while eliminating the aliasing problem. Two such attempts by Yasukawa and Zitzewitz use bandsplitting filters but maintain a small gap between the bands to avoid aliasing [48, 49]. Although successful, such a spectral gap is unacceptable in many applications.

Somayazulu et al. propose a scheme similar to Gilloire's for the elimination of interband aliasing when bandsplitting filterbanks are used [45]. The approach is to use an auxiliary subband which contains the complement of the signals in the main subbands. A short adaptive filter is used in this auxiliary band and is operated at the full system sampling rate.

A comparison is made between this method and that of Gilloire where cross-terms

are used to cancel interband aliasing. The computational complexity of the proposed scheme is approximately the same as that of Gilloire but promises faster adaptation.

2.4.2 Reduced Delay

The problem of reducing the input to output delay has also received some attention. Chen [50, 51] proposed a scheme whereby the analysis filterbank is totally eliminated from the primary signal path. See Figure 2.13.

Figure 2.13 No delay subband acoustic echo canceller proposed by Chen [50, 51].

This is compensated by another adaptive filter of the same length as the filterbank delay and operating at the full sampling rate. No convergence rate information is given but the extra adaptive filter required causes an increase in system complexity.

Amano et al. attempt to moderate the signal delay problem by providing two separate signal paths for the primary and reference signals [52]. See Figure 2.14.

The long delay path (switch position 2) is used when a near end signal is absent (normal adaptation). This position reduces the passband ripple effect on the subband echo canceller performance and permits the use of lower order filters.

Figure 2.14 Two path method proposed by Amano et al. [52].

When a local signal is detected, the switch is moved to the low delay path (position 1) and adaptation is frozen.

The problems with this approach are the signal distortion caused by the switching action and the poor echo canceller performance when a local signal is detected. Also, there is an added system complexity due to the extra synthesis filterbank required as compared to other subband schemes.

Chapter 3

Subband Adaptive Filter Structures

This chapter describes the two subband adaptive filter structures studied in this thesis: one based on modulated filterbanks and one based on non-modulated filterbanks. A discussion of the relevant issues in selecting and designing filterbanks suitable for subband adaptive filters is followed by a description of the structures employing each filterbank type. It is shown that the main difference between the two subband adaptive filters is the method used to minimize the computational complexity of the adaptive filters.

3.1 Filterbank Design Considerations

An important part of all subband LMS filtering schemes is the nature of the subband filterbanks used. This section discusses the relevant issues in selecting filterbanks suitable for a subband adaptive filter.

3.1.1 FIR or IIR

A fundamental decision in selecting a filterbank for a subband adaptive filter is the choice between an IIR or FIR implementation. As this discussion will show, there are several factors which favour the use of FIR filters for this purpose.

IIR filters are known to be more computationally efficient than FIR filters for a given amplitude response [53]; however, IIR filters lack the linear phase property of FIR filters. The linear phase characteristic is desirable for subband adaptive filters since it facilitates the alignment of delays between the signals in the various bands. In addition, FIR filter implementation becomes competitive with IIR when multirate processing techniques are employed [54].

Another factor to consider is the ease of design. As discussed in the next section, the back to back analysis/synthesis filterbanks should have a flat frequency response. This requirement will be seen to impose constraints on the frequency response of the

individual filters. These constraints are most easily satisfied using the window-based design techniques for FIR filters. This point is discussed in more detail in Appendix A.

3.1.2 Overall Frequency Response

The requirements on the back to back frequency response of the analysis/synthesis filterbanks vary depending on the structure of the subband adaptive filter.

For the structure proposed by Hébert [6] and Wallace [7] (see Figure 2.4), where the analysis/synthesis filterbanks are not inserted into the primary signal path, the requirements placed upon the frequency response are not stringent. This is shown by the results reported in [4, 6, 7, 55]. The only requirement under these conditions is that the filterbank contain no peaks or deep spectral gaps. Peaks would produce noise enhancement and deep gaps would prevent any noise cancellation at the frequencies of the gaps.

For the adaptive echo canceller proposed by Kellerman [8, 9] (see Figure 2.11), the analysis/synthesis filterbanks are inserted directly into the primary input signal path. In this case, it is essential that the back to back frequency response be flat.

3.1.3 Individual Frequency Response

The filterbank is required to separate portions of the input signal spectrum with highly disparate power levels. This is necessary to force the subband adaptive filters to operate on specific spectral regions. Accomplishing this task requires the use of filters with high stopband attenuation. The exact amount of stopband attenuation required depends on the input signal spectral dynamics and, thus, varies widely among different applications.

Achieving high stopband attenuation with real filters requires a trade-off between the amount of subband attenuation and the interband isolation (since the overall frequency response must be flat with no gaps). This trade-off is common in many filterbank designs and is usually satisfied by allowing the transition bands of adjacent channels to overlap. The overall flatness of the filterbank response is then ensured by careful design of the

individual filters (see Appendix A).

3.1.4 Distribution of Bands

Although some work on subband adaptive filters has included the effects of non-uniform distribution of bands [47], the choice is highly dependent upon the application. For the application considered herein, uniformly distributed bands have shown promising results from previous work [6, 7] and are convenient for filterbank design.

3.1.5 Multirate Processing

Since the outputs of the analysis filterbanks are band-limited, their sampling rates may, theoretically, be lowered. This is done to obtain reductions in the computation rate required to implement the filterbank and any other processing applied to the subband signals such as an adaptive filter or quantizer.

For the application to subband adaptive filtering, however, care must be taken to ensure that aliasing distortion is kept to a minimum.

In many applications it is sufficient to ensure that the synthesis filterbank output is free of aliasing distortion. Since the subband signals are not being significantly modified, a high level of aliasing can be tolerated. The aliasing in the subband signals is then cancelled at the filterbank output by careful design of the shapes of the individual filters. This is the approach taken in the design of Quadrature Mirror Filterbanks (QMF) and their relatives [46, 43, 56, 57].

In the application to subband adaptive filtering, however, the presence of aliasing in the subband signals must be eliminated. To see why, assume that a decimated signal, with aliasing, is the reference input to an adaptive filter and that a similarly decimated signal is input to the primary. The adaptive filter will attempt to remove those parts of the primary signal which are correlated with the reference. In the aliased spectral region, this requires that the adaptive filter transfer function attain two different values at the same frequency: one for the main component and one for the aliased component. Clearly, this is not possible and the adaptive filter will converge to a compromise, and

probably suboptimal, value in this region.

In addition, cancellation of the aliasing during reconstruction will be impossible since the adaptive filters in each subband are applying different modifications to the subband signals thereby destroying the necessary symmetry.

This sensitivity to aliasing distortion rules out many computationally efficient filterbank implementations based on aliasing cancellation [46, 43, 56, 57] without using a complicated adaptive filtering scheme such as those in [44, 45].

3.2 Structures Based on Non-modulated Filterbanks

3.2.1 Non-modulated Filterbanks

The frequency isolation requirements for a subband adaptive filterbank can be easily met using an arrangement of equispaced, uniform-bandwidth FIR filters. A suitable arrangement for a four-band system is shown in Figure 3.1. This filterbank consists of a lowpass filter, a highpass filter and two bandpass filters.

Figure 3.1 Individual filter responses of a typical four-band filterbank.

Such uniform bandwidth filterbanks produced using standard FIR design methods, Remez exchange algorithm, are linear, have well controlled amplitude responses and exactly linear phase. The design approach can also be modified to produce a filterbank

with an approximately flat overall frequency response. This design approach, however, becomes quite complicated for large numbers of bands.

A simpler approach is to base the filterbank design on modulated versions of a single, lowpass prototype filter and obtain the required bandpass coefficients from it. This design approach was used by Rothweiler and Chu in the design of pseudo-QMF filterbanks [43, 56].

The impulse responses for all channels of a pseudo-QMF filterbank are produced by modulating a lowpass prototype filter impulse response, $h(n)$. The bandpass impulse response for the k^{th} channel is obtained according to:

$$h_k(n) = \cos\left(\frac{2k-1}{4K}n\right)h(n) \quad (3.1)$$

The filterbank design problem is, thus, reduced to the problem of finding a suitable lowpass prototype filter, $h(n)$ which satisfies symmetry requirements similar to those of QMF filterbanks. This problem has been addressed differently by various authors, each producing acceptable solutions [46, 43, 56, 10]. For a more detailed discussion of this design problem, and yet another design approach, see Appendix A.

Using the design method of Appendix A, the lowpass prototype shown in Figure 3.2 was designed. The bandpass filter responses produced by this prototype are shown in Figure 3.3 and the overall filterbank frequency response in Figure 3.4.

This technique was also used to design filterbanks with 8, 16 and 32 bands. The results of the design process are summarized in Appendix A.

The main drawback of this type of non-modulated filterbank is that the subband signals cannot be decimated without introducing aliasing distortion.

Decimation of a signal by an integer factor, D , causes an inherent frequency translation of images of the signal spectrum by integer multiples of $2\pi/D$. To avoid aliasing, the k^{th} subband filter frequency responses should limit the signal spectrum to frequencies between:

$$\frac{k\pi}{D} < |\omega| < \frac{(k+1)\pi}{D} \quad (3.2)$$

Figure 3.2 Lowpass prototype filter for a four band BPF filterbank.

Figure 3.3 Bandpass filter frequency responses for a four band BPF filterbank.

Figure 3.4 Filterbank frequency response for a four band BPF filterbank.

That is, the filter transfer function should obey:

$$H(e^{j\omega}) = \begin{cases} 1 & \frac{k\pi}{D} < |\omega| < \frac{(k+1)\pi}{D} \\ 0 & \text{otherwise} \end{cases} \quad (3.3)$$

This constraint is referred to as the integer-band sampling constraint and applies to the decimation of bandpass signals [54].

This aliasing could be avoided by choosing a different decimation factor for each subband and/or maintaining highly oversampled subband signals, however, this approach eliminates the benefits of multirate processing and will not be considered further. Instead, the analysis filterbank output will be used at the full sampling rate requiring another technique to reduce the complexity of the subband adaptive filters.

Since the subband outputs are not decimated, the computationally efficient structures proposed in [43, 56] cannot be used. Instead, the bandpass filter coefficients will be used as regular FIR bandpass filters. This implementation will be referred to as the BPF filterbank in this thesis.

3.2.2 Adaptive Filter Structure #1

A non-modulated filterbank consists of a combination of linear time invariant lowpass, bandpass and highpass filters as described above. Placing an adaptive LMS FIR filter in each band results in the subband adaptive filter structure similar to that of Wallace [7] and Hébert [6] but with an arbitrary number of bands.

When the analysis filter outputs are not decimated, the adaptive filters in each band are excited by narrowband signals. As discussed earlier, a narrow band (coloured) input signal produces an ill-conditioned autocorrelation matrix and makes the tap weight solution very sensitive to small errors in estimating $\mathbf{R}$.

This sensitivity of the tap weights is manifested as a strong constraint on the frequency response of the adaptive filter in the passband of the analysis filters but a weak constraint in the stop band. That is, the frequency response in the stop band region can attain somewhat arbitrary values and still satisfy the normal equation within a small error tolerance.

Figure 3.5 Block diagram of Sparse LMS FIR filter.

As the bandwidth of the analysis filters is decreased, the input autocorrelation matrix becomes more ill-conditioned. The region of strong constraint on the frequency response becomes smaller and the weakly constrained regions becomes larger. It would seem reasonable, therefore, to expect that the number of taps required for an acceptable solution to the adaptive filtering problem would decrease as the analysis bandwidth decreases.

Although the adaptive filter frequency response may not be zero for frequencies outside the passband, the effect on the fullband output signal is minimal for two reasons. First, there is little or no input signal energy to be passed to the output of the LMS filter. Secondly, any signal energy outside the analysis passband is attenuated by the synthesis filterbank which usually has a frequency response similar to the AFB.

This insensitivity of the fullband system output to the out-of-band response of the subband adaptive filters may be used to simplify the adaptive filter structure.

The approach is similar to the interpolating FIR filter designs of Neuvo et al. [58] where an FIR filter with sparsely located taps is followed by a lowpass interpolating filter to remove the unwanted frequency images. Making the non-zero taps adaptive results in the sparse LMS filter shown in Figure 3.5.

This adaptive filter was used by Abousaada to cancel the echo tail in a data communications problem [59]. The difference here is that a combination of lowpass, highpass and bandpass filters are used to select the frequency response images desired for each band. Figure 3.6 shows the sparse LMS adaptive filter concept extended to

the multiple subband case. This structure will be referred to as Structure #1 in the discussions to follow.

Structure #1 consists of K -band BPF filterbanks using sparse LMS filters with a tap spacing of L in each subband. Thus, for the sparse LMS filters to cover an impulse response length of M taps, there are $\lfloor \frac{M}{L} \rfloor + 1$ non-zero adaptive taps per band where $\lfloor x \rfloor$ indicates the largest integer less than or equal to x . The subband signals, as discussed previously, are not decimated.

Figure 3.6 Block diagram of subband adaptive filter structure #1 (based on non-modulated filterbank).

In Figure 3.6, since only one out of every L taps in the sparse adaptive filters is non-zero, the adaptive filter frequency responses are periodic with a period of $2\pi/L$. The synthesis filterbank selects the frequency response images in the desired spectral locations for the output.

Proper operation of this structure depends upon the correct selection of the sparse tap spacing, L . Since the sparse filter frequency response is periodic with a period of $2\pi/L$, L should be chosen so that the frequency response folding points, at intervals of π/L , are not near the analysis filter center frequencies (to avoid constraining the adaptive filter frequency response within the passband).

For K uniform bandwidth analysis filters, the center frequency of the k^{th} band is given by:

$$\omega_k = \frac{(2k-1)\pi}{2K} \quad \text{for } k = 1, 2, \dots, K$$

Thus, L should be chosen so that:

$$\frac{\pi}{L} \neq \frac{(2k-1)\pi}{2K} \quad \text{for } k = 1, 2, \dots, K$$

Since the computational complexity decreases with increasing L , as reported in [59], a somewhat conflicting requirement is that L be chosen as large as possible.

For all applications of Structure #1 studied in this thesis, the sparse tap spacing L , is equal to $K/2$. This produces folding frequencies outside the passbands of all analysis filters.

The computational complexity of the subband adaptive filter using a non-modulated filterbank is calculated based on the complexity of K separate sparse TD LMS filters. The length of the adaptive filters will be different, however, for different values of L .

For the subband adaptive filters with $L = 1M$, the complexity is simply K times the TD NLMS complexity. The total number of computations per output sample is:

Multiplies	Adds	Divides
$(2M+1)K$	$(2M+2)K$	K

For the subband adaptive filters with $L > 1$, the number of nonzero taps required to cover a time delay of M samples is $\lfloor \frac{M}{L} \rfloor + 1$ where $\lfloor x \rfloor$ indicates the largest integer

less than or equal to x . As stated above, all the systems using Structure #1 in this thesis have $L = \frac{K}{2}$ so the complexity of the subband adaptive filter taps is:

Multiplies	Adds	Divides
$[2(\lfloor \frac{M}{L} \rfloor + 1) + 1]K$	$[2(\lfloor \frac{M}{L} \rfloor + 1) + 2]K$	K

It should be noted that actual computational complexity figures for a real-time implementation of the above system must be architecture-specific. The numbers above are intended only as a guide for comparison of the given algorithms.

3.3 Structures Based on Modulated Filterbanks

3.3.1 Modulated Filterbanks

The most commonly used modulated filterbanks are forms of uniform FIR filterbanks [43, 56]. In this case, the bandpass filter outputs are shifted to a different frequency to enforce the constraint of equation 3.3. The structure of a modulated filterbank using complex modulation is shown in Figure 3.7.

In the analysis filter, the input signal is modulated by a complex exponential with a frequency

$$\omega_k = \frac{2\pi k}{K} \quad (3.6)$$

corresponding to the centre frequency of band k . The lowpass filter, $h(n)$, then removes all components outside a narrow band allowing the signal to be decimated by a factor D .

The synthesis filter reverses this process. The decimated signals are upsampled and interpolated by lowpass filter, $f(n)$, which is not necessarily the same as $h(n)$. The interpolated subband signals are modulated to reposition them at frequencies ω_k . The heavy lines in the diagram indicate complex signals.

Figure 3.7 Structure of one channel of a complex modulator filterbank: a) analysis filter, b) synthesis filter.

This type of filterbank was used by Kellerman for acoustic echo cancelling and uses the complex LMS algorithm to adaptively filter the complex-valued subband signals [9, 60].

A type of modulated filterbank which produces real subband signals can be obtained by changing the complex modulator in Figure 3.7 to a single-sideband (SSB) modulator. A typical SSB structure is shown in Figure 3.8 [54].

The operation is similar to the complex modulator filterbank except that the lowpass filtered signals are shifted in frequency by an amount equal to half the bandwidth of the lowpass filter. When the cosine and sine terms are added together, the result is a real-valued bandpass signal, $X_k^{SSB}(m)$, centered at a frequency of $\frac{\Delta\omega}{2}$, where $\frac{\Delta\omega}{2}$ is the bandwidth of the lowpass prototype, $h(n)$. Note that the indexes n and m refer to time at the high (non-decimated) and low (decimated) sampling rates, respectively.

In the SSB analysis filter, the input signal is modulated down to DC by cosine and sine signals at a frequency corresponding to the centre of each band. These modulated signals are then filtered by the same lowpass filter, $h(n)$, in each band and decimated by a factor, D . To obtain the real baseband, the lowpass signals are modulated again

Figure 3.8 Structure of one band of a typical SSB filterbank: a) analysis filter, b) synthesis filter.

by a frequency, $\frac{\Delta\omega}{2}$, equal to half the filter bandwidth, and the cosine and sine terms are added together.

The SSB synthesis filter reverses the analysis operations. The SSB baseband signal, $X_k^{SSB}(m)$, is first modulated to DC and lowpass filtered using $f(n)$, which is similar but not necessarily the same as $h(n)$. The lowpass signal is then repositioned to the correct frequency by the second modulating sequence.

Although sufficient for many applications, such as speech coding, the SSB structure shown in Figure 3.8 cannot be used for subband adaptive filtering. To see why, examine Figure 3.9 which shows the signal spectra in analysis band 2 of a four-band SSB filterbank without decimation.

The input signal spectrum is shown in Figure 3.9a) with numbers indicating the locations of the various bands of the filterbank.

Figure 3.9b) and Figure 3.9c) show the lowpass filtered cosine and sine terms, respectively. The two spectra shown in each figure correspond to the positive and negative frequency shifts associated with cosine and sine modulation. That is, since:

$$\cos(\omega n) = \frac{e^{j\omega n} + e^{-j\omega n}}{2} \quad (3.7)$$

and

$$\begin{aligned} \sin(\omega n) &= \frac{e^{j\omega n} - e^{-j\omega n}}{2j} \\ &= -j \frac{e^{j\omega n} - e^{-j\omega n}}{2} \end{aligned} \quad (3.8)$$

the input signal is shifted in both directions simultaneously. The additional 90° phase shift due to the complex operator on the sine terms is shown as a shaded region in the figures.

The SSB filterbank output signal is shown in Figure 3.9d). Note that the output spectrum contains overlapping frequency images in the regions near DC due to the positive and negative frequency shifts of the cosine and sine modulation. These images are due to the finite transition band of the lowpass filter and have the same effect upon the subband adaptive filters that aliasing does. Therefore, these images must be eliminated from the subband signals.

Figure 3.9 Signal spectra for band two of a typical four band SSB filterbank: a) input signal b) lowpass filtered cosine terms, c) lowpass filtered sine terms, and d) SSB modulated output signal. (Continued . . .)

Figure 3.9 Signal spectra for band two of a typical four band SSB filterbank: a) input signal b) lowpass filtered cosine terms, c) lowpass filtered sine terms, and d) SSB modulated output signal.

To overcome these frequency images, the SSB structure must be slightly modified. A suitable modification is to increase the second modulating frequency so that the spectral images do not overlap. The new structure is shown in Figure 3.10 and the resulting output signal spectra for band two is shown in Figure 3.11. Note that the centre of the spectrum is now located at $\Delta\omega$ due to the increased second modulation frequency and is free of imaging.

Figure 3.10 Structure of one band of a modified SSB filterbank: a) analysis filter, b) synthesis filter.

Figure 3.11 Output signal spectrum for band two of a four band modified structure #2.

Also note from Figure 3.11 that the resulting SSB signal is approximately a lowpass signal and, therefore, can be reduced in sampling rate without the aliasing problems associated with bandpass signals.

Four SSB modulated filterbanks with the structure shown in Figure 3.10 were

designed according to the method outlined in Appendix A. The results are summarized in Appendix A.

One other aspect of modulated filterbanks is that the impulse response of a given channel is shift variant. That is, the impulse response for each channel depends on the time shift of the input impulse. For example, Figure 3.12 shows two impulse responses for channel number one of the four band SSB filterbank designed according to Appendix A. The solid line is the response to an input at time zero, the dotted line is the response to an input at time seven corrected for the input shift. As seen in the figure, the two responses are significantly different. In fact, the amplitude of the difference exceeds the amplitude of the impulse response at some points. Figure 3.13 shows similar impulse responses but for the case when the subband signals are decimated.

This shift variant property of the modulated filterbank is due to action of the modulating sequences and, as might be expected, the shift variation is periodic. This fact was verified for each of the SSB designs used in this thesis. The periods for the four, eight, sixteen and thirty two band systems are summarized in Table 3.1.

These periods are directly related to the frequency of the second modulating sequence in the AFB, that is $\Delta\omega$. Since $\Delta\omega$ depends on the number of bands in the filterbank, the period of the impulse response variation will increase as the number of bands increases. Note, also, that the period of this time variation is unaffected by decimation.

As will be seen later, the effect of this time variation is to slow the adaptation of the subband adaptive filters.

Figure 3.12 Impulse response of channel one of an SSB filterbank: solid line, response to impulse at time zero; dotted line, response to impulse delayed by seven samples.

Figure 3.13 Decimated impulse response of channel one of an SSB filterbank: solid line, response to impulse at time zero; dotted line, response to impulse delayed by seven samples.

Number of Bands	Impulse Response Period	Second Modulating Frequency
4	16	$\frac{\pi}{4}$
8	32	$\frac{\pi}{8}$
16	64	$\frac{\pi}{16}$
32	128	$\frac{\pi}{32}$

Table 3.1 Periods of the impulse response variations in different SSB filterbanks.

3.3.2 Adaptive Filter Structure #2

A subband adaptive filter based on an SSB modulated filterbank is depicted in Figure 3.14. This will be referred to as Structure #2.

Structure #2 consists of K -band SSB filterbanks using independent TD LMS filters in each subband. The subband signals are decimated by a factor, D . Thus, for the subband LMS filters to cover an impulse response length of M taps, there are $\lfloor \frac{M}{D} \rfloor + 1$ adaptive taps per band where $\lfloor x \rfloor$ indicates the largest integer less than or equal to x . The subband adaptive filters do not employ sparse tap spacing.

As described in the previous section, the subband signals of an SSB filterbank are narrowband signals located at the same frequencies in all bands. It is this property that permits the use of decimation/interpolation to reduce the computation rate.

Note, however, that the decimated subband signals are no longer narrowband. This is due to the inherent frequency scaling of the decimation operation. This implies that using sparsely located taps cannot be combined with using decimation because the adaptive filter operating on the decimated signal is constrained at all frequencies.

The non-decimated subband signals are narrowband signals whose bandwidth is determined by the bandwidth of the AFB channels. When such a narrowband signal is reduced in sampling rate, in accordance with the constraints in equation 3.3, the resulting

Figure 3.14 Block diagram of subband adaptive filter structure #2 (based on SSB modulated filterbank).

signal appears as a wideband signal with reference to the lower sampling rate. In this case, however, the adaptive filter will be converging to the frequency translated transfer function of the unknown system due to the modulation of the input signals.

An adaptive filter which is operated at this lower sampling rate is excited by all frequencies and the adaptive filter frequency response will be constrained at all frequencies. It is not possible, therefore, to use the efficient sparse LMS tap scheme on decimated subband signals.

The computational complexity of the subband adaptive filter using a modulated filterbank is calculated based on the complexity of K separate TD LMS filters. The length of the adaptive filters will be different, however, for the decimated and non-decimated cases.

For structure #2 with $D = 1$ and M -tap subband LMS filters, the complexity is

simply K times the TD NLMS complexity. The total number of computations per output sample is:

Multiplies	Adds	Divides
$(2M + 1)K$	$(2M + 2)K$	K

For structure #2 with $D > 1$ the number of subband LMS taps required to cover an impulse response length of M samples is $\lfloor \frac{M}{D} \rfloor + 1$ where $\lfloor x \rfloor$ indicates the largest integer less than or equal to x . The total number of computations per output sample is:

Multiplies	Adds	Divides
$[2(\lfloor \frac{M}{D} \rfloor + 1) + 1] \frac{K}{D}$	$[2(\lfloor \frac{M}{D} \rfloor + 1) + 1] \frac{K}{D}$	$\frac{K}{D}$

Note that these figures are averages over D output samples due to the decimation.

As mentioned earlier, actual computational complexity figures for a real-time implementation of the above system must be architecture-specific. The numbers above are intended only as a guide for comparison of the given algorithms.

Chapter 4

Simulated Data Results

Other authors have presented results on the performance of subband adaptive filters. In many cases, however, the complexity of the real-life signals used has prevented a full analysis of system behaviour [6, 61, 7].

To overcome the difficulties of analyzing adaptive systems in very complex environments, a series of simulations with stationary input signals are performed. This chapter discusses the setup used and results obtained from these simulations.

4.1 Simulation Setup

The setup used to obtain the simulated data results is shown in Figure 4.1.

Figure 4.1 ANC simulation setup used to study subband adaptive filters.

The reference input to the adaptive noise canceller is either white or coloured noise. White noise is used to excite all frequencies of the systems under test to examine the optimal tap weight solutions for different parameter settings. Coloured noise is used to study convergence behaviour since most signals encountered in real applications are not spectrally flat.

A normally distributed, pseudorandom number generator with variance (i.e. noise power) equal to one is used as the white noise source. This produces an input autocorrelation matrix eigenvalue spread of one [3].

Coloured noise is obtained by passing white noise through a twelve pole IIR filter. The same IIR filter was used in [35] to compare the convergence rates of time-domain and frequency-domain LMS FIR filters. The power level of the white noise input to the IIR filter is adjusted so that the coloured noise power is also equal to one. The input signal thus obtained has an eigenvalue spread much greater than one [35].

The IIR filter coefficients are given in Table B.1 and the corresponding frequency response is shown in Figure 4.2. The noise equivalent bandwidth of this filter is 0.029, yielding a signal power reduction of -15.4 dB.

A fixed, FIR filter is used as the unknown channel modifying the reference input signal. A filter length of 32 taps is chosen so that the number of bands in the subband adaptive filter could be made equal to the number of LMS taps while maintaining a reasonable simulation execution time.

Except for ensuring that the frequency response is nonzero across the entire band, the transfer function of the unknown channel was chosen arbitrarily. This guarantees that the tap weights for the adaptive filter in each subband will not converge to a trivial solution.

The FIR filter coefficients for the unknown channel are shown in Table B.2 and the corresponding frequency response is shown in Figure 4.3. The noise equivalent bandwidth of this filter is 0.0512, yielding a signal power reduction of -12.9 dB.

The primary input to the adaptive noise canceller is the sum of the unknown channel output and a small amount of measurement noise. A normally distributed pseudo-random number generator, independent of the reference noise source, is used as the measurement noise. A power level of 10^{-6} fixes the noise floor of the system and, on plots of the time evolution of squared error, helps to illustrate the point at which convergence is completed.

Figure 4.2 Frequency response of IIR filter used to generate coloured noise [35].

Figure 4.3 Frequency response of FIR filter used as stationary channel.

Each of the two subband adaptive filter structures (#1 and #2) and the TD LMS adaptive filter are used in the block labelled Adaptive Noise Canceller. This permits the comparison of results for different algorithms.

The sparse tap spacing, L , for structure #1 and the decimation factor, D , for structure #2 are both equal to the number of subbands, K , divided by two. The normalized LMS algorithm is used to update all tap weights, see equation 2.9. The value of α was set to 0.5 in all simulations to achieve fast adaptation.

4.2 Varying the Number of Taps

For a regular TD LMS adaptive filter, the required number of taps is directly dependent upon the length of the unknown channel impulse response being modelled. Using fewer taps than required results in poor adaptive filter performance whereas using more taps than required slows adaptation and needlessly increases the computational complexity (see equation 2.3). Different behaviour has been reported, however, for subband adaptive filters.

Kellerman noticed that the required number of LMS taps per band for a subband adaptive filter based on a modulated filterbank seemed to increase as the bandwidth of the analysis filters decreased [8, 9]. Certain instances also required the insertion of a non-causal delay into the primary signal path. Other authors have reported similar findings [8, 9, 41].

The implications of increasing the number of LMS taps in each band as the analysis filter bandwidth decreases is that the subband adaptive filter impulse responses are covering a longer time span than a corresponding TD adaptive filter. The explanation offered by Kellerman is that the band-limiting applied to the input signals causes an apparent time stretching of the impulse response of the unknown channel. As the bandwidth decreases this time stretching increases.

While this argument seems plausible at first, it contradicts the ideas of LMS behaviour based on sinusoidal inputs, which are perfectly bandlimited [14, 15]. That is, an adaptive filter operating on sinusoidal inputs requires only two taps to completely

cancel the signal. Extending Kellerman's argument would require an infinite number of taps to do this. Clearly, this is not the case.

An analytic treatment of this problem is quite complicated, however, the simple experimental setup shown in Figure 4.1 can be used to examine the behaviour of subband adaptive filters as the analysis filter bandwidth decreases.

The idea is to simulate the two subband adaptive filter structures previously described using a fixed channel with a known impulse response length. All systems are trained using white noise and allowed to converge until the time evolution of the squared error ceases. A comparison of the performance for each system as the number of LMS taps is varied provides an indication of the required number of taps for a given structure.

Note that, for the structure #1 using $L > 1$ and for structure #2 using $D > 1$, the total delay compensation length of the adaptive filter will not be equal to the number of adaptive taps. That is, for an LMS filter with 3 taps and a sparse spacing of 2 samples, the total delay covered is actually 5 sample periods. Similar reasoning applies to the case of LMS filters operating on decimated signals.

The performance of the adaptive noise cancelling systems is measured using the total noise power reduction obtained. This quantity is given, in dB, by:

$$P_{red} = -10 \log_{10} \left(\frac{\sigma_e^2}{\sigma_y^2} \right) \quad (4.1)$$

The quantities σ_e^2 and σ_y^2 are estimated from the simulated data by the average power in 1024 points of the respective signals after convergence is complete. In the subband systems, the filterbank delay is compensated before calculating P_{red} .

For the TD LMS case, the theoretical maximum value of P_{red} can be estimated using equation 2.11.

Since the step size parameter, α , is equal to 0.5, the total mean squared error is

approximately equal to:

$$\begin{aligned} J_{tot} &= J_{min} \left[1 + \frac{\alpha}{1 - \alpha} \right] \\ &= J_{min} \left[1 + \frac{0.5}{1 - 0.5} \right] \\ &= 2J_{min} \end{aligned}$$

Thus, J_{tot} is equal to twice the value of J_{min} , resulting in a 3 dB increase in the minimum mean squared error using this value of α . For a properly sized LMS FIR filter, the value of J_{min} equals the measurement noise so that the value of J_{tot} is 2×10^{-6} .

Since the training signal is white noise with a power level of one, the primary signal power is simply equal to the noise equivalent bandwidth of the unknown channel filter, or 0.0512. This produces a power level reduction of 12.9 dB.

Thus, the theoretical maximum P_{red} for the time domain LMS adaptive noise canceller is:

$$\begin{aligned} P_{red} &= -10 \log_{10} \left(\frac{2 \times 10^{-6}}{0.0512} \right) \\ &= 44.1 \text{ dB} \end{aligned}$$

A theoretical value of P_{red} is significantly more difficult to calculate for the subband systems due to the interaction of the SFB frequency response and the power spectrum of the subband error signals. This problem is beyond the scope of this work.

4.2.1 Structure #1

Figure 4.4 shows the variation of noise power reduction as the delay compensation increases for TD LMS and subband LMS filters based on structure #1 for different numbers of bands. The subband sparse tap spacing factor, L , is equal to one in all cases.

When the delay compensation is less than 32, the length of the unknown channel impulse response, the subband systems provide a higher power reduction than the TD system. This is due to the fact that the subband systems are operating on narrowband input signals. With fewer taps, it is easier to provide a reasonable approximation of the required frequency response over a narrow band than over the full bandwidth.

Figure 4.4 Comparison of power reductions versus delay compensation for TD LMS and subband structure #1 ($L=1$): o = full band, x = 4 band, * = 16 band.

When the delay compensation equals 32, the TD system provides a level of power reduction consistent with the theoretical value calculated earlier. In addition, the subband adaptive filters provide the same, or slightly higher, power reductions. The improved performance of the subband adaptive filters is most likely due to the operation of the SFB. The operation of the adaptive filters produces some error signal energy outside the passband of the SFB due to the random input data. The SFB will remove all signal components outside the synthesis passband before reconstructing the fullband output signal, thereby causing an apparent power reduction in the output signal.

If the delay compensation is increased beyond 32, the performance provided by all adaptive filters remains relatively constant indicating that the optimum number of taps is 32. Thus it seems that, contrary to [8], the required number of subband taps is the same as the number of taps in the unknown channel impulse response, for this filterbank type. This is seen to be independent of the analysis filter bandwidth.

Figure 4.5 shows the variation of power reduction as the delay compensation is increased for subband structure #1 with $L = K/2$. Note that a sparse LMS filter covering a delay of M samples has only $\lfloor M/L \rfloor + 1$ nonzero taps and, as discussed in Chapter 3, requires fewer computations to implement.

The behavioural trends in Figure 4.5 are similar to those in Figure 4.4 except that the maximum level of power reduction achieved is significantly lower. The reason for this behaviour is that the sparse filters are unable to adequately estimate this particular unknown channel. In addition, the folding points of the sparse frequency responses do not all coincide with the stopband frequencies of the AFB. Since the frequency response of this unknown channel is asymmetric about these folding points, poor performance results.

Figure 4.6 summarizes the preceding discussion. It illustrates the behaviour of the maximum noise power reduction for various numbers of LMS taps per band as a function of the number of subbands. The case of one subband corresponds to the TD LMS filter.

Figure 4.5 Comparison of power reductions versus delay compensation for TD LMS and subband structure #1 ($L=K/2$): o = full band, x = 4 band, * = 16 band.

Figure 4.6 Noise power reduction versus delay compensation in structure #1 ($L=1$): o = 8 taps/band, x = 24 taps/band, * = 32 taps/band.

4.2.2 Structure #2

Figure 4.7 shows the variation of noise power reduction as the delay compensation increases for subband structure #2. The subband decimation factor, D , is equal to one.

When the delay compensation is less than 32, the subband systems provide a higher noise power reduction than the TD system. As in the case of structure #1, this is due to the narrowband input signals.

When the delay compensation equals 32, the noise power reduction provided by structure #2 is less than that provided by the TD filter. This suggests that more taps are required in the subband adaptive filters to model the unknown channel, as reported by Kellerman. Since this does not happen for structure #1, however, it is more likely due to the modulation of the input signals rather than the bandlimiting as suggested in [8, 9].

As the delay compensation is increased beyond 32, the performance of the four band system reaches a maximum and remains constant whereas the performance of the sixteen band system continues to increase. This suggests that the optimum number of bands is different in each case and that, in each case, it is higher than 32.

Figure 4.8 shows the noise power reduction obtained for structure #2 where the decimation factor, D , is equal to $K/2$. For an impulse response coverage of M samples, an LMS filter operating on signals decimated by a factor D needs only $\lfloor M/D \rfloor + 1$ nonzero taps and, as discussed in Chapter 3, requires fewer computations to implement. The performance trends are similar to those noted for the case $D = 1$ but the maximum noise power reductions obtained are somewhat different.

First, the performance for the same delay compensation is less for the decimated systems. This is due to the frequency scaling property of decimation. That is, decimating the subband signals spreads the signal energy over a wider frequency range, at the lower sampling rate. In this case, the subband adaptive filters are operating on a much wider band signal than the non-decimated systems and, thus, require a finer resolution in the frequency response to achieve a similar power reduction. This finer resolution requires more LMS taps.

Figure 4.7 Comparison of noise power reductions versus delay compensation for TD LMS and subband structure #2 ($D=1$): o = full band, x = 4 band, * = 16 band.

Figure 4.8 Comparison of noise power reductions versus delay compensation for TD LMS and subband structure #2 ($D=K/2$): o = full band, x = 4 band, * = 16 band.

Secondly, the maximum level of power reduction achievable with decimated systems is lower than that achievable with non-decimated systems since decimation prevents an exact duplication of the full-band impulse response. For example, consider an unknown channel consisting of a simple delay of seven samples. Assume that a subband adaptive filter is attempting to cancel a signal processed by this channel. If the subband signals are decimated, and if the phasing of the decimation is such that the seventh sample with respect to the reference input time origin is discarded, then the subband adaptive filter can only hope to find a compromise value for this seventh tap. Such a compromise value will not always provide perfect cancellation.

Figure 4.9 summarizes the preceding discussion. It illustrates the behaviour of the maximum noise power reduction for various numbers of LMS taps per band as a function of the number of subbands for structure #2. The case of one subband corresponds to the TD LMS filter. Comparing this plot with that shown in Figure 4.6 illustrates the main differences between the performance of structures #1 and #2.

The results presented in this section show the maximum noise power reductions obtained after the adaptive filters are allowed to fully converge. As the detailed results in Appendix B show, however, this can take a significantly different amount of time for each type of system. This particular point is discussed more fully in the next section.

4.3 Convergence Speed

A distinct advantage of adaptive filters over fixed filters is the adaptive filter's ability to change with time. This is particularly important in applications where the desired transfer function or the input signal characteristics are non-stationary [7, 55].

Ideally, the adaptive filter should be capable of making an accurate estimate of the desired transfer function in a time period that is short compared to the rate of change of the desired system.

This section discusses the convergence speed for subband adaptive filters under various controlled conditions. Other authors have studied the convergence behaviour of the time domain LMS algorithm based on a group of assumptions concerning the

Figure 4.9 Noise power reduction versus number of bands in structure #2 (D=1): o = 8 taps/band, x = 24 taps/band, * = 32 taps/band.

statistics of the input data. This collection of assumptions is commonly referred to as the independence theory [3]. The results of these studies indicates that the convergence behaviour of the LMS algorithm can be described as a sum of decaying exponentials. The starting magnitude and the rate of decay is determined by the eigenvalues of the input signal autocorrelation matrix.

These results, though useful for certain applications, accurately describe only the average convergence behaviour of the LMS algorithm and only when the step size parameter is very small. Since the step size used in the simulations in this thesis is large to provide fast adaptation, the theoretical convergence results do not provide accurate predictions and will not be used.

Convergence rate studies for subband adaptive filters have only been presented for special test signals [10, 8, 9]. As a result, general observations and inferences are difficult to make.

To overcome this problem, the results presented in this section are based on computer experiments under controlled conditions. Ensemble averaged time evolution of squared error is shown for time domain and subband systems for both white and coloured noise.

The question as to the most fair comparison between different subband systems and a time domain system is a very complicated one and deserves some attention.

Several authors have compared TD and SB systems on the basis of computational complexity [8, 9]. Doing so, however, relies on the presence of a detailed computational analysis of the filterbank used as this contributes to the overall system complexity. It also does not permit the comparison of different filterbank schemes. Arguments may also be presented that this particular comparison is unfair because the total number of adaptive taps may be different in each application.

Comparisons based on equal numbers of adaptive taps are also possible [10]. Although this approach may seem to provide a fair assessment of different schemes, the delay coverage of the subband systems is very small when the number of subbands is large. Also, as the number of subbands approaches and exceeds the number of TD taps, it presents a problem of how to deal with fractional taps values.

The comparison to be studied in this work is based on the amount of impulse response delay coverage. This makes the determination of the required number of taps for time domain and subband systems easier, even when decimation and sparsely located taps are used.

From the existing literature on the LMS algorithm, the effects of the parameters M and α are known. A longer LMS filter, large M , tends to converge more slowly whereas a larger value of α , up to the stability limit, speeds convergence. In the results of this section, the effects of these two parameters is not discussed.

The parameter variations that are studied in this section are the numbers of subbands and the effects of using sparsely spaced taps and decimation on the speed of convergence.

The following results are obtained for adaptive filter delay lengths of 32 taps and step size parameter, α , equal to 0.5 for fast adaptation. If longer LMS filters are used or a smaller step size is chosen, the convergence speed improvements of the subband adaptive filters over the TD adaptive filter is more pronounced [62].

Convergence studies of various adaptive filters often use a plot of the time evolution of averaged squared error to illustrate convergence of the algorithm. This type of curve, often called a learning curve, will be used in this thesis.

The learning curves in this thesis are obtained by averaging the squared error at each iteration over 10 independent simulation runs.

Although insufficient for complete convergence of all adaptive filters studied in this thesis, the number of input data points was limited to a maximum of 12,000 to maintain reasonable simulation execution times. This number appears sufficient to illustrate the important trends.

4.3.1 Convergence with White Noise

White, or spectrally flat, noise is the most favourable input to a TD LMS adaptive filter. The equal energy at all frequencies produces uniform convergence of all modes of the adaptive system and provides fullband identification of the unknown channel. In

addition, the simple statistical description of a white noise signal facilitates analysis of adaptive system behaviour.

This section presents the results of subband adaptive filter simulations where the reference input signal is white noise. In all cases the performance of the subband adaptive filter is compared to the performance of a TD LMS filter with an equal impulse response length.

4.3.1.1 Structure #1

Figure 4.10 shows the learning curve of a 32 tap, TD LMS filter. As seen, the average squared error reaches the theoretical minimum value, 3 dB above the noise floor of 10^{-6} , after approximately 300 iterations. During the transient period the squared error decays at a relatively constant rate of 0.2 dB/iteration. This behaviour is typical of a TD LMS filter trained with white noise.

The learning curve of two, eight-band subband adaptive filters based on structure #1 is shown in Figure 4.11. The curve labelled A is for a subband adaptive filter with $L = 1$. The curve labelled B is for a subband adaptive filter with $L = K/2$.

For the adaptive filter represented by curve A, full convergence requires slightly more than 12,000 iterations. The adaptive filter represented by curve B reaches full convergence much sooner, fewer than 2000 iterations, but converges to a much higher steady-state error.

During the transient period, the squared error does not decay at a constant rate as for the TD LMS filter. Instead, the initially high convergence rate slows as the subband adaptive filters approach the steady-state condition. A detailed illustration of the early convergence of the TD LMS and subband adaptive filters is shown in Figure 4.12.

As Figure 4.12 shows, the initial convergence rates for both subband adaptive filters is higher than the TD adaptive filter even though the TD adaptive filter reaches steady-state first. The result is that the subband adaptive filters provide a lower squared error output than the TD LMS filter for a brief period, approximately 100 iterations.

Figure 4.10 Learning curve for 32 tap, TD LMS adaptive filter trained with white noise ($\alpha = 0.5$).

**Figure 4.11 Learning curve for 8 band structure
#1 trained with white noise: A) $L=1$; B) $L=K/2$.**

Figure 4.12 Comparison of initial convergence rates for TD, and 8 band structure #1 trained with white noise: A) TD adaptive filter; B) structure #1 ($L=1$); C) structure #1 ($L=K/2$).

Since the input signals to the adaptive filters are stationary, the convergence behaviour is relatively easy to explain. As noted in [3], the convergence of the different modes of the LMS algorithm depend on the eigenvalues of the input autocorrelation matrix. For white noise inputs all eigenvalues are equal so all modes converge at the same rate producing a learning curve similar to Figure 4.10. For a coloured noise input, such as produced by the AFB of a subband adaptive filter, the eigenvalues are not equal and all modes converge at different rates. This results in a learning curve with a changing slope as the adaptation proceeds, similar to Figure 4.11.

The subband adaptive filter with sparse taps maintains the same high initial convergence rate because the characteristics of the subband signal have not been modified and the sparse adaptive filters have fewer taps to update. This, in general, produces faster convergence.

The effect of the number of subbands on the initial convergence speed is illustrated in Figure 4.13. Learning curves for four and eight band structure #1 systems and a TD LMS filter are compared for the first 500 iterations. As the number of subbands increases from four to eight, the convergence rate also increases. This trend is also observed for higher numbers of bands as well.

4.3.1.2 Structure #2

Slightly different results are observed for a subband adaptive filter based on structure #2. The learning curve for such an adaptive filter with eight bands is shown in Figure 4.14. In Figure 4.14, curve A represents the performance with non-decimated, $D = 1$, subband signals and curve B represents the performance with decimated subband signals, $D = K/2$.

As seen in Figure 4.14, the subband adaptive filter using non-decimated signals approaches a squared error level approximately 15 dB higher than the corresponding structure #1 adaptive filter ($L = 1$) for the same number of iterations. In addition, full convergence takes significantly longer as indicated by examining the detailed simulation results of Appendix B. The structure #2 subband adaptive filter using decimated signals

Figure 4.13 Comparison of initial convergence rates for TD, and structure #1 trained with white noise: A) TD adaptive filter; B) 4 band structure #1 ($L=1$); C) 8 band structure #1 ($L=1$).

**Figure 4.14 Learning curve for 8 band structure
#2 trained with white noise: A) $D=1$; B) $D=K/2$.**

reaches steady-state after approximately 2000 iterations but the level of squared error is even higher than for the non-decimated case.

The behaviour of structure #2 for non-decimated and decimated signals during the transient period is illustrated in Figure 4.15 which shows the first 500 iterations of the TD and subband adaptive filters.

Similar to the results of structure #1, the SSB-based adaptive filter using non-decimated signals (i.e. $D = 1$) has an initial convergence rate much higher than the TD LMS. When $D = K/2$, however, the structure #2 adaptive filter does not maintain this advantage.

Most of the discussion regarding the behaviour of the structure #1 adaptive filters applies to structure #2 as well. There are, however, two additional observations.

First, the non-decimated structure #2 adaptive filters takes much longer to converge than the corresponding structure #1 adaptive filters even though the analysis filter frequency responses are very similar. To understand why, recall from the filterbank discussion in Chapter 3 that the impulse response of the SSB modulated filterbank is periodically time varying, producing subband signals with periodically time varying properties. The adaptive filters in the subbands will attempt to average these time varying properties but will require more iterations to do so. As the period of the time variation increases, by an increasing number of bands, the adaptive filters will require more iterations to average the subband signal properties producing slower convergence.

The second point about structure #2 subband adaptive filters is the slower convergence speed with decimated signals. This is due to the fact that the decimated systems have fewer tap weight updates per unit time and to the effect of the decimation on the input signals. This last point is discussed further below.

If a bandlimited signal, $x(n)$, is decimated by a factor D , then the spectrum of the decimated signal is a scaled version of the original signal spectrum [54]. That is:

$$\begin{aligned} x(n) &\longleftrightarrow X(e^{j\omega}) \\ x(mD) &\longleftrightarrow X(e^{j\omega'}) = X(e^{j\frac{\omega}{D}}) \end{aligned} \quad (4.4)$$

Figure 4.15 Comparison of initial convergence rates for TD, and 8 band structure #2 trained with white noise: A) TD adaptive filter; B) structure #2 ($D=1$); C) structure #2 ($D=K/2$).

The effect of decimation, therefore, is to change a narrowband signal at the higher sampling rate into a wideband signal at the lower sampling rate with a corresponding change in the eigenvalues of the autocorrelation matrix. Thus, the convergence behaviour of the subband adaptive filter will be affected and, in the cases presented above, it is an adverse effect.

The effect of the number of subbands on the initial convergence rate for the non-decimated structure #2 adaptive filters is illustrated in Figure 4.16. Similar to structure #1 (Figure 4.13), as the number of subbands increases, the initial convergence rate increases.

When the subband signals are decimated, the initial convergence rate is not high, as already noted. For higher numbers of subbands, this is also the case. In fact, all structure #2 adaptive filters using decimated subband signals have approximately the same convergence rates. The reason for this is that, for a white noise input, all the decimated subband signals have a similar power spectrum and, thus, a similar eigenvalue spread.

The preceding discussion regarding subband adaptive filter performance applies equally to subband adaptive filters with different numbers of bands. Appendix B contains the learning curves of subband adaptive filters, for both structure #1 and #2, with four, sixteen and thirty-two bands.

4.3.2 Convergence with Coloured Noise

Very few signals encountered in real applications of adaptive filters have the spectrally flat characteristic of white noise. As mentioned in Chapter 2, however, signals with non-flat power spectra cause slow convergence of TD LMS adaptive filters. The results presented in this section deal with this type of input signal often referred to as coloured input signals.

4.3.2.1 Structure #1

Figure 4.17 shows the learning curve of 32 tap TD LMS filter trained with coloured noise, created using the IIR filter shown in Figure 4.2. As expected, the behaviour

Figure 4.16 Comparison of initial convergence rates for TD, and structure #2 trained with white noise: A) TD adaptive filter; B) 4 band structure #2 ($D=1$); C) 8 band structure #2 ($D=1$).

is significantly different from that observed for white noise training. Full convergence requires approximately 10,000 iterations and the squared error decay rate is not constant. Instead, the rate of decay of squared error is initially very high and slows as the adaptive filter approaches steady-state.

The learning curve of a similarly trained structure #1 subband adaptive filter is shown in Figure 4.18. In this case, the convergence behaviour is very similar to the convergence behaviour of the same system for white noise input, Figure 4.11. That is, a high initial convergence rate and full convergence only after 12,000 iterations.

The reason for this behaviour is that the analysis filters, similar to those depicted in Figure 3.3, have very high stopband attenuation and narrow transition bandwidths. Since the stopband attenuation is much higher than the input signal power spectrum differences, there is little impact upon adaptation.

The most significant difference with the convergence results for coloured noise data is seen by comparing TD and subband systems. Figure 4.19 shows the first 500 iterations of a TD LMS filter, a four band and an eight band structure #1 adaptive filter.

From this figure it is clear that the subband adaptive filter convergence exceeds that of the TD LMS for a much longer period than for a white noise input. It also illustrates that the length of this period increases as the number of subbands increases.

4.3.2.2 Structure #2

Similar results are also observed for the subband adaptive filters based on structure #2. The learning curve of an eight band structure #2 subband adaptive filter is shown in Figure 4.20. As for the structure #1 adaptive filter, this system is not adversely affected by the coloured input signal.

The initial convergence of four-band and eight-band structure #2 subband adaptive filters is compared to that of a TD LMS filter in Figure 4.21. Again, the trends are similar to that observed for structure #1: high initial convergence rates for longer periods as the number of subbands increases.

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

Figure 4.17 Learning curve for a 32 tap TD LMS algorithm trained with coloured noise ($\alpha = 0.5$).

**Figure 4.18 Learning curve for 8 band structure
#1 trained with coloured noise: A) $L=1$; B) $L=K/2$.**

Figure 4.19 Comparison of initial convergence rates for TD, and structure #1 trained with coloured noise: A) TD adaptive filter; B) 4 band structure #1 ($L=1$); C) 8 band structure #1 ($L=1$).

**Figure 4.20 Learning curve for 8 band structure #2
trained with coloured noise: A) $D=1$; B) $D=K/2$.**

The effect of decimating the subband signals is the same as for white noise input; all adaptive filters using decimated signals and trained with coloured noise have similar convergence rates, regardless of the number of subbands.

4.4 Summary of Observations

The experimental results from varying the number of taps show that subband adaptive filters can provide significant noise power reduction even when the unknown channel is under-modelled. This was contrasted to an under-modelled TD LMS filter whose performance was significantly degraded.

The reason for the improved performance of the subband adaptive filters is that the adaptive filters in the subband are operating on an isolated frequency range. It is easier for the adaptive filters to provide a reasonable estimate of the correct frequency response over a narrow band than over the full band. Since the SFB eliminate the effects of the adaptive filter response outside the passband, the error signal remains small.

The implication of this result is that a subband adaptive filter can be expected to perform better than a TD LMS filter in situations where the unknown channel impulse response is under-modelled by the adaptive filter. In most real applications, the actual impulse response length of the unknown channel is very long and the adaptive filter used to compensate it is short owing to practical limitations or adaptation speed considerations. In these circumstances, a subband adaptive filter would be expected to perform better.

Another important point illustrated by the simulated data results is that for the same input conditions, subband adaptive filters using modulated filterbanks require more LMS taps in each subband than subband adaptive filters using non-modulated filterbanks. Since the frequency responses of the two analysis filterbanks used in this thesis are very similar, the reason for this effect is the modulation of the input signals. That is, for a modulated filterbank, all the output signals are located at the same frequencies to permit multirate processing. As a result, the unknown channel transfer function values at the original signal frequencies are shifted into this region. This implies that the

Figure 4.21 Comparison of initial convergence rates for TD, and structure #2 trained with coloured noise: A) TD adaptive filter; B) 4 band structure #2 ($D=1$); C) 8 band structure #2 ($D=1$).

impulse response of the unknown channel will **not**, in general, be the optimum tap weight solution for the subband adaptive filters.

From the experiments concerning convergence speed, it is seen that subband adaptive filters have a high initial rate of convergence as compared to TD LMS filters with the same delay compensation (even if the total number of adaptive taps is higher). This behaviour is observed for both white and coloured noise, however, the effect is enhanced for coloured noise inputs. For applications where tracking a non-stationary unknown channel is necessary this faster convergence will be an obvious benefit.

In terms of the effects of the type of filterbank, the SSB modulated filterbanks were seen to produce slower converging subband signals (compare Figures 4.11 and 4.14). This can be attributed to the time variation of the impulse response of the analysis filters, as discussed in Chapter 3 . The subband adaptive filters must average the effects of the time variation causing a slowing of convergence.

Finally, the techniques available for minimizing the computational complexity of the adaptive filters was shown to have an impact of performance. The use of sparse taps maintained a high initial rate of convergence but at the expense of steady-state error. On the other hand, using decimated subband signals allowed lower steady-state error, but at the expense of adaptation speed. For applications where convergence speed is important, the sparse tap method is preferred.

Chapter 5

Real Data Results

This chapter presents some experimental results for the subband adaptive filter structures previously described applied to real signals. The observations from the simulated data cases are used to explain the results obtained.

5.1 Setup

The real data used in the experiments of this section was gathered by Hébert for the analysis of background acoustic noise in mobile communications [6].

A primary microphone was located near the driver's position in the vehicle to measure the desired speech signal. Three other reference microphones were placed in various locations throughout the vehicle to measure the different noise sources. One reference microphone was placed in the engine compartment, one under the dashboard and one near the floor. Digital recordings were made from all four microphones simultaneously as the vehicle was operated under various driving conditions.

A more detailed description of the measurement setup and recording technique can be found in [6] and [7].

Analysis of the recorded signals indicate that the noise spectra are highly coloured [6]. In addition, a high degree of non-stationarity has been reported [7].

5.2 Results

Of the many driving conditions for which recordings were made, the simulations performed in this section focus on those cases which benefit from single reference ANC techniques. Several such cases were tested and reported in [7].

In addition, the effects of parameter variations was extensively studied in [7]. As a result, the experiments described in this section assume certain parameters which provide the best performance as indicated in [7]. The parameters of main interest are:

1. The adaptive filters all have 100 taps per band: using a higher number of taps has been shown to be ineffective [7].
2. Cases are selected for which no primary signal delay compensation is required.
3. The recorded data is the telephony bandlimited version created by Wallace [7].

To facilitate comparison of the simulation results with those reported by Wallace, the performance measure used in the car data simulations is the amount of noise spectrum reduction. This data is calculated by performing a spectral estimate on the primary input and error signal output of the adaptive filters.

The power spectra of the input and output signals are estimated using the Welch method [53]. A block of data 512 points long is divided into sections of 128 points overlapping successive sections by 64 points. The 128 point sections are transformed into the frequency domain using an FFT and the FFTs from each section are averaged to yield the spectral estimates. The Hanning window is used in the FFT/IFFT calculations. The noise power reduction is obtained at the FFT points which fall within the telephony bandwidth as defined in [7]. Minimum, maximum and mean power reductions for each simulated case are calculated. The total noise power reductions are also included for comparison with results presented earlier in this thesis.

5.2.1 Structure #1

The input and output power spectra of an adaptive noise canceller using a 100 tap TD LMS are shown in Figure 5.1. Although the low frequency noise is attenuated, noise enhancement is actually observed in the higher frequencies.

The input and output power spectra of an adaptive noise canceller using a sixteen band structure #1 subband adaptive filter are shown in Figure 5.2. For the subband adaptive filter, the noise enhancement problem has been eliminated. In fact, substantial noise reduction has been achieved.

Figures 5.3 and 5.4 summarize the results obtained using the recorded car noise data in simulations of structure #1 subband adaptive filters for various numbers of bands. The results for a 100 tap TD LMS filter are shown as the single band case in the figure.

Figure 5.1 Input and output power spectra from a TD LMS adaptive noise canceller operating on car data: A) input; B) output.

Figure 5.2 Input and output power spectra from a sixteen-band structure #1 subband adaptive noise canceller: A) input; B) output.

The noise enhancement phenomenon is illustrated in the plot by a negative value for the minimum noise power reduction.

As seen in Figure 5.3, the minimum, mean and maximum noise reductions all increase as the number of subbands increases. This trend is also noted in Figure 5.4 for the sparsely spaced adaptive filters, except that levels of noise reduction are somewhat reduced.

Based on the observations made for the stationary data cases, two factors contribute to the enhanced performance of the subband adaptive filters.

First, as the number of subbands increases, the initial convergence rates of structure #1 adaptive filters increases, with little effect due to the use of sparse taps. Since the car noise data have been demonstrated to be coloured and highly non-stationary, the faster convergence is a distinct benefit.

Second, it was shown that the subband adaptive filter are capable of providing higher noise power reduction than an equivalent TD LMS when the number of LMS taps used is less than the length of the impulse response being modelled. For the acoustic signal paths inside an automobile, the impulse response will be much longer than 100 samples (assuming an 8 kHz sampling rate) due to the physical dimensions and the speed of sound in air. In fact, in a similar application of ANC, Boll has reported impulse responses of up to 1500 samples [63].

5.2.2 Structure #2

Figures 5.5 and 5.6 summarize the results obtained using the recorded car noise data in simulations of structure #2 subband adaptive filters for various numbers of bands. Similar to structure #1, the minimum, mean and maximum noise reductions all increase as the number of subbands increases for adaptive filters without decimation (i.e. for $D = 1$). For the decimated signals, however, the performance is significantly degraded.

The reason for the poor performance of the decimated systems, is due to their slower convergence as noted in the stationary data cases. Since the convergence is slow, the adaptive filters are unable to track the changing signals and the noise cannot be cancelled.

Figure 5.3 Noise spectrum reductions for Structure #1 adaptive noise canceller ($M=100$, $L=1$): o = minimum, x = mean, * = maximum.

Figure 5.4 Noise spectrum reductions for Structure #1 adaptive noise canceller ($M=100$, $L=K/2$): o = minimum, x = mean, * = maximum.

Figure 5.5 Noise spectrum reductions for Structure #2 adaptive noise canceller ($M=100$, $D=1$): o = minimum, x = mean, * = maximum.

Figure 5.6 Noise spectrum reductions for Structure #2 adaptive noise canceller ($M=100$, $D=K/2$): o = minimum, x = mean, * = maximum.

5.3 Summary of Observations

The experiments using real car data demonstrated that subband adaptive filters provide higher noise attenuation than TD LMS filters without the noise enhancement phenomenon. The reasons for this are twofold:

1. The subband adaptive filters are capable of faster adaptation than TD LMS filters. This effect is enhanced for coloured input signals such as those recorded in a moving automobile. The fact that structure #2 using decimation did not perform as well supports this since it was demonstrated earlier that decimation slows convergence.
2. Subband adaptive filters are capable of significant noise cancellation when the unknown channel is under-modelled. Since the impulse responses of signal paths inside the passenger compartment of an automobile are expected to be longer than the length of the adaptive filters used (i.e. longer than 100 taps) the setup described herein represents such an under-modelled system.

Chapter 6

Conclusion

6.1 Summary

There are several important findings presented in this thesis. The list below is a summary of these items. For a complete discussion, please refer to the summary at the end of each chapter.

Subband adaptive filters have the following advantages over TD LMS adaptive filters:

1. Subband adaptive filters are capable of eliminating the noise enhancement phenomenon associated with TD LMS filters applied to ANC [4, 17]. The benefits from subband processing increase with the number of bands.
2. Subband adaptive filters have initial convergence rates higher than corresponding TD LMS filters for both white and coloured signal inputs. They are, thus, beneficial in applications where the tracking of highly non-stationary systems is required, particularly when the input signals are coloured.
3. When the unknown channel impulse response is under-modelled, subband adaptive filters still provide a significant amount of noise power reduction in applications such as ANC. In a similar situation, the performance of a TD LMS filter is significantly degraded. This is important in acoustic applications where the actual impulse response length is unknown and, generally, quite long.
4. Since the subband signals are bandlimited, the computation rate required for implementation of the adaptive filters can be reduced by utilizing multirate processing or sparse LMS FIR taps.

In light of these benefits of subband adaptive filters, it is expected that such systems will provide enhanced performance over TD LMS filters in applications involving coloured input signals and non-stationary unknown systems.

There are also several important implications concerning the type and design of the analysis filterbank. These can be summarized as follows:

1. The use of modulated filterbanks changes the optimum tap weight solution for the subband adaptive filters due to the modulation of the input signals. For the cases studied in this thesis, the result is an increase in the number of adaptive taps required to achieve the same performance as a TD LMS filter.
2. The use of modulated filterbanks causes a slowdown in the convergence of the subband adaptive filters compared to systems using non-modulated filterbanks. This is due to the time variation of the analysis filterbank impulse response.
3. Techniques for reducing the computational complexity of the adaptive filters impact performance of the systems. Sparsely located taps maintain the convergence rate advantage but degrade steady-state performance. Multirate processing maintains low steady-state error but at the expense of convergence speed.

6.2 Areas for Further Research

As a result of the investigations reported in this thesis, the following areas are suggested for further research:

1. Methods of subband processing other than uniform bandwidth FIR filters may lead to more efficient or effective subband adaptive filters. Specifically, the area of constant-Q filters or wavelet transforms are potential candidates [64].
2. The problem of time-varying impulse responses of modulated filterbanks impedes adaptive filter convergence. Research is needed to determine if some other scheme can be used to exploit this behaviour (e.g. cyclostationary techniques [65]) to improve convergence speed.
3. Investigation of computationally efficient filterbanks schemes which satisfy the subband adaptive filter requirements outlined in Chapter 3. This would help to reduce the overall system complexity.

Appendix A Filterbank Design

For both types of filterbanks in Chapter 3, the design procedure is reduced to a process of finding a suitable lowpass prototype filter, $h(n)$. This appendix outlines the design approach used in this thesis.

A.1 Design Procedure

The frequency response of this lowpass prototype must satisfy certain symmetry requirements. These symmetry requirements state that the filters of adjacent bands must be symmetric about the point of overlap so that the overall filterbank response is flat. In a more exact statement, the requirement on the frequency response, $H(e^{j\omega})$, of $h(n)$ is:

$$\begin{aligned} H(e^{j\omega}) &= 1 & \omega &= 0 \\ H(e^{j\omega}) &= H(e^{j\frac{\pi}{2K}-\omega}) & 0 < \omega &\leq \frac{\pi}{2K} \\ H(e^{j\omega}) &= 0 & \omega &> \frac{\pi}{K} \end{aligned} \quad (\text{A.1})$$

The use of a standard algorithm, such as the Remez exchange, for designing $h(n)$ is not very convenient as the transfer function must be specified at many points for an acceptable design. Instead, the filter coefficients, $h(n)$, have been found by other authors through various techniques such as parameter optimization [46, 43], gradient search [56] and polynomial factorization [66]. However, since the filterbank requirements for this application are not stringent, a simple iterative design procedure is used and is found to provide adequate results.

It is well known that the window method of designing FIR filters possesses certain symmetry properties [67]. This fact suggests that a simple iterative approach based on the window design procedure can be used to determine a suitable $h(n)$. The method is outlined in Figure A.1.

The design is initiated by estimating the cutoff frequency of the desired lowpass prototype and designing a filter using a window method. An initial value of $\pi/2K$ is a reasonable starting point.

Figure A.1 Iterative window-based design procedure.

This lowpass prototype is then used to calculate the overall filterbank frequency response. If this response is adequate, the design cycle terminates and the estimated lowpass prototype is a suitable choice. If the response is inadequate, the estimated corner frequency is increased or decreased, as appropriate and the cycle is repeated. For the filterbanks designed in this thesis, an acceptable lowpass prototype (less than 0.1 dB of ripple) was typically obtained in fewer than ten iterations.

A.2 Results

Figures A.6 through A.17 show the lowpass prototypes designed using the above technique for 4, 8, 16 and 32 band systems. For maximum stopband attenuation, the Blackman window is used in all the following designs. The overall filterbank responses for these prototypes is shown in Figures A.2 to A.13.

Figure A.2 Overall frequency response of a four-band SSB filterbank.

Figure A.3 Overall frequency response of an eight-band SSB filterbank.

Figure A.4 Overall frequency response of a sixteen-band SSB filterbank.

Figure A.5 Overall frequency response of a thirty-two band SSB filterbank.

Figure A.6 Frequency Response of lowpass prototype filter used in the SSB filterbank of Figure A.2.

Figure A.7 Frequency Response of lowpass prototype filter used in the SSB filterbank of Figure A.3.

Figure A.8 Frequency Response of lowpass prototype filter used in the SSB filterbank of Figure A.4.

Figure A.9 Frequency Response of lowpass prototype filter used in the SSB filterbank of Figure A.5.

Figure A.10 Overall frequency response of a four-band BPF filterbank.

Figure A.11 Overall frequency response of an eight-band BPF filterbank.

Figure A.12 Overall frequency response of a sixteen-band BPF filterbank.

Figure A.13 Overall frequency response of a thirty-two band BPF filterbank.

Figure A.14 Frequency Response of lowpass prototype filter used in the BPF filterbank of Figure 3.4.

Figure A.15 Frequency Response of lowpass prototype filter used in the BPF filterbank of Figure A.11.

Figure A.16 Frequency Response of lowpass prototype filter used in the BPF filterbank of Figure A.12.

Figure A.17 Frequency Response of lowpass prototype filter used in the BPF filterbank of Figure A.13.

Appendix B Detailed Simulation Data

This Appendix contains detailed simulation results from the simulations described in Chapter 4.

B.1 Filter Coefficients

Tables B.1 and B.2 contain the filter coefficients used for the IIR coloured noise filter and the FIR unknown channel filter, respectively. Although there are 32 FIR coefficients, only the first 16 are shown since the last 16 are equal (linear phase property).

Feed Back	Feed Forward
1.0	0.0594
-1.4603	0
1.2664	0
-0.8505	0
0.6295	0
-0.4975	0
0.2737	0
-0.1682	0
0.2579	0
-0.2384	0
0.5081	0
-0.3794	0
0.2043	0

Table B.1 IIR filter coefficients.

Feed Forward 0 to 16
-0.0760
0.0051
-0.0053
0.0057
-0.0060
0.0065
-0.0071
0.0078
-0.0088
0.0101
-0.0118
0.0143
-0.0183
0.0255
-0.0425
0.1275

Table B.2 Unknown FIR channel filter coefficients.

B.2 Number of LMS Taps

The following tables contain the values of power reduction obtained from simulations and used to generate the graphs of Chapter 4.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	-3.1	8
16	16	-0.24	8
24	24	5.9	8
32	32	43.4	8
40	40	43.5	8
48	48	43.5	8
56	56	43.6	8
64	64	43.6	8

Table B.3 Power reductions for adaptive noise canceller based on TD LMS filter trained with white noise.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	2.3	10
16	16	10.7	10
24	24	12.8	10
32	32	39.3	20
40	40	47.1	50
48	48	46.0	50
56	56	46.2	50
64	64	46.0	50

Table B.4 Power reductions for adaptive noise canceller based on four band structure #2 trained with white noise: D=1.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	7.1	12
16	16	13.8	12
24	24	20.6	50
32	32	41.0	50
40	40	43.7	50
48	48	44.5	50
56	56	44.0	50
64	64	44.0	50

Table B.5 Power reductions for adaptive noise canceller based on eight band structure #2 trained with white noise: D=1.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	16.0	12
16	16	21.6	12
24	24	26.0	12
32	32	33.5	50
40	40	37.3	50
48	48	30.2	50
56	56	41.6	50
64	64	43.3	50

Table B.6 Power reductions for adaptive noise canceller based on sixteen band structure #2 trained with white noise: D=1.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	23.2	12
16	16	25.7	12
24	24	30.1	12
32	32	33.0	50
40	40	36.6	50
48	48	39.6	50
56	56	40.1	50
64	64	42.5	50

Table B.7 Power reductions for adaptive noise canceller based on thirty-two band structure #2 trained with white noise: $D=1$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	5	0.4	12
16	9	8.1	50
24	13	10.2	50
32	17	41.4	50
40	21	45.1	50
48	25	44.4	50
56	29	45.1	50
64	33	44.6	50

Table B.8 Power reductions for adaptive noise canceller based on four band structure #2 trained with white noise: $D=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	3	2.9	12
16	5	7.0	50
24	7	10.2	50
32	9	14.6	50
40	11	19.4	50
48	13	27.6	50
56	15	38.5	50
64	17	38.4	50

Table B.9 Power reductions for adaptive noise canceller based on eight band structure #2 trained with white noise: $D=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	2	2.0	12
16	3	6.3	40
24	4	14.8	40
32	5	19.8	40
40	6	27.3	40
48	7	27.4	40
56	8	31.4	40
64	9	31.6	40

Table B.10 Power reductions for adaptive noise canceller based on sixteen band structure #2 trained with white noise: $D=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8			
16	2	8.0	12
24			
32	3	15.9	12
40			
48	4	21.4	12
56			
64	5	24.9	12

Table B.11 Power reductions for adaptive noise canceller based on thirty-two band structure #2 trained with white noise: $D=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	1.4	12
16	16	9.1	50
24	24	10.9	50
32	32	47.2	50
40	40	46.6	50
48	48	45.6	50
56	56	46.0	50
64	64	46.2	50

Table B.12 Power reductions for adaptive noise canceller based on four band structure #1 trained with white noise: $L=1$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	7.2	12
16	16	10.7	12
24	24	17.9	12
32	32	48.3	50
40	40	49.4	50
48	48	48.7	50
56	56	47.1	50
64	64	48.1	50

Table B.13 Power reductions for adaptive noise canceller based on eight band structure #1 trained with white noise: $L=1$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	18.2	12
16	16	20.3	12
24	24	27.6	12
32	32	50.2	50
40	40	49.4	50
48	48	50.1	50
56	56	50.4	50
64	64	48.5	50

Table B.14 Power reductions for adaptive noise canceller based on sixteen band structure #1 trained with white noise: $L=1$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	8	31.3	12
16	16	35.6	12
24	24	42.3	12
32	32	55.4	30
40	40	52.9	30
48	48	50.1	30
56	56	50.3	30
64	64	50.7	12

Table B.15 Power reductions for adaptive noise canceller based on thirty-two band structure #1 trained with white noise: $L=1$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	5	1.7	50
16	9	8.5	50
24	13	10.6	50
32	17	20.5	50
40	21	20.0	12
48	25	20.8	12
56	29	21.2	12
64	33	21.6	12

Table B.16 Power reductions for adaptive noise canceller based on four band structure #1 trained with white noise: $L=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	3	4.9	12
16	5	10.4	12
24	7	16.5	12
32	9	18.9	12
40	11	18.2	12
48	13	18.5	12
56	15	19.7	12
64	17	20.3	12

Table B.17 Power reductions for adaptive noise canceller based on eight band structure #1 trained with white noise: $L=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8	2	9.2	12
16	3	14.5	12
24	4	19.6	12
32	5	19.7	12
40	6	20.6	12
48	7	20.9	12
56	8	20.3	12
64	9	20.5	12

Table B.18 Power reductions for adaptive noise canceller based on sixteen band structure #1 trained with white noise: $L=K/2$.

Time Span	Number of LMS Taps	Power Reduction (dB)	Number of Iterations (thousands)
8			
16	2	14.7	12
24			
32	3	14.4	12
40			
48	4	15.0	12
56			
64	5	14.3	12

Table B.19 Power reductions for adaptive noise canceller based on thirty-two band structure #1 trained with white noise: $L=K/2$.

B.3 Learning Curves

The only learning curves presented in the body of the thesis are for eight band SSB and BPF systems. This Appendix contains learning curves for the other subband systems tested.

B.3.1 White Noise Training

This section contains the learning curves for subband adaptive filters trained with white noise inputs. Figures B.1 to B.3 show the learning curves for subband adaptive filters based on structure #2 and Figures B.4 to B.6 show the learning curves for subband adaptive filters based on structure #1.

Figure B.1 Four band structure #2 trained with white noise: A) $D=1$, B) $D=K/2$.

Figure B.2 Sixteen band structure #2 trained with white noise: A) $D=1$, B) $D=K/2$.

**Figure B.3 Thirty-two band structure #2
trained with white noise: A) $D=1$, B) $D=K/2$.**

Figure B.4 Four band structure #1 trained with white noise: A) $L=1$, B) $L=K/2$.

Figure B.5 Sixteen band structure #1 trained with white noise: A) $L=1$, B) $L=K/2$.

Figure B.6 Thirty-two band structure #1 trained with white noise: A) $L=1$, B) $L=K/2$.

B.3.2 Coloured Noise Training

This section contains the learning curves for subband adaptive filters trained with coloured noise inputs. Figures B.7 to B.9 show the learning curves for subband adaptive filters based on structure #2s and Figures B.10 to B.12 show the learning curves for subband adaptive filters based on structure #1.

Figure B.7 Four band structure #2 trained with coloured noise: A) $D=1$, B) $D=K/2$.

Figure B.8 Sixteen band structure #2 trained with coloured noise: A) $D=1$, B) $D=K/2$.

Figure B.9 Thirty-two band structure #2 trained with coloured noise: A) $D=1$, B) $D=K/2$.

Figure B.10 Four band structure #1 trained with coloured noise: A) $L=1$, B) $L=K/2$.

Figure B.11 Sixteen band structure #1 trained with coloured noise: A) $L=1$, B) $L=K/2$.

Figure B.12 Thirty-two band structure #1 trained with coloured noise: A) $L=1$, B) $L=K/2$.

B.4 Real Data

The following tables contain the values of power reduction obtained from simulations and used to generate the graphs of Chapter 5.

Number of Bands	Noise Power Reduction (dB)			
	Minimum	Mean	Maximum	Total
4	-1.5	7.9	30.2	7.1
8	0.6	8.5	29.9	9.0
16	2.6	11.3	30.0	12.2
32	3.4	12.9	33.2	9.8

**Table B.20 Power reductions obtained from structure #1
subband adaptive filters operating on car data: $L=1$.**

Number of Bands	Noise Power Reduction (dB)			
	Minimum	Mean	Maximum	Total
4	0.8	7.9	30.4	7.9
8	-4.4	8.1	30.8	9.0
16	0.3	8.3	30.2	9.8
32	-0.1	8.7	27.6	10.4

**Table B.21 Power reductions obtained from structure #1
subband adaptive filters operating on car data: $L=K/2$.**

Number of Bands	Noise Power Reduction (dB)			
	Minimum	Mean	Maximum	Total
4	-1.5	7.9	30.2	7.1
8	0.6	8.5	29.9	9.0
16	2.6	11.3	30.0	12.2
32	3.4	12.9	33.2	9.8

**Table B.22 Power reductions obtained from structure #2
subband adaptive filters operating on car data: $D=1$.**

Number of Bands	Noise Power Reduction (dB)			
	Minimum	Mean	Maximum	Total
4	-3.0	3.8	24.4	3.9
8	-8.8	1.6	17.8	3.3
16	-3.1	0.7	10.5	0.9
32	-7.4	-0.1	13.0	5.2

**Table B.23 Power reductions obtained from structure #2
subband adaptive filters operating on car data: $D=K/2$.**

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