A DNS Study on the Forcing of 3D Instability Mechanisms of Laminar Planar Free-Shear Layers

by

Hristo Valtchanov
B.Eng. Aerospace Engineering

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Abstract

Direct numerical simulations were performed of a free-shear layer developing downstream from a flat no-slip splitter plate. The simulation results show that 3D streaky perturbations developing in the boundary layer on the high-speed side of the splitter-plate are convected downstream from the trailing edge and influence the development of the planar free-shear layer. Several other simulations are performed in which a spanwise-wavy shape is introduced to the trailing edge of the splitter plate to force the downstream development of the free-shear layer. Introduction of the spanwise-waviness to the trailing edge results in significantly earlier smaller-scale breakdown of the free-shear layer. The effect of varying the spanwise wavelength and height of the waviness on the downstream development of the free-shear layer is characterized. The events leading to smaller-scale breakdown of the free-shear layer are studied in each of the forced cases, and a general mechanism for smaller-scale breakdown is identified.

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Nomenclature

d	Local half-depth of free-shear layer computed as vorticity thickness (m)
h	Height of spanwise waviness of splitter-plate trailing edge (m)
L	Streamwise length of splitter plate (m)
$\dot{m}_i$	i^{th} component of mass flux vector, computed as $\rho u_i \Delta n_i$ (kg/s m ²)
n	Product of surface-normal vector with surface area (m ²)
P	Static pressure (Pa)
Q	Second invariant of the instantaneous velocity gradient tensor (1/s ²), computed as $Q = (\partial_i U_i \partial_j U_j - \partial_i U_j \partial_j U_i)$
q	Second invariant of the perturbation velocity gradient tensor (1/s ²), computed as $q = (\partial_i u_i \partial_j u_j - \partial_i u_j \partial_j u_i)$
$r, \tilde{r}$	non-normalized, and normalized residuals of a solution computed using ANSYS CFX [®] in Eqn. (3.16)
R	Velocity-difference ratio between high and low freestreams, computed as $R = \frac{\Delta U}{\bar{U}}$
Re_θ	Reynolds number based on momentum thickness on high-speed side of splitter plate, computed as $Re_\theta = \frac{U_2 \theta}{\nu}$
St	Strouhal number of 2D KH instability, computed as $St = f_{KH} \frac{2\theta}{U_1 + U_2}$
t	Time (s)
t_{KH}	Time period between consecutive Kelvin-Helmholtz billow sheddings (s)
U	Instantaneous velocity vector with components (U, V, W) in the x-, y-, and z-direction, respectively (m/s)
$\bar{U}$	Velocity vector corresponding to background flow-field as defined in Eqn. 4.1, with components $(\bar{U}(y, t), 0, 0)$ (m/s)

U_{KH}	Velocity vector corresponding to perturbation flow-field as defined in Eqn. 4.2, with components $(U_{KH}(x, y, t), V_{KH}(x, y, t), 0)$ (m/s)
$\bar{U}'$	Sum of $\bar{U}$ and U_{KH} velocity vectors (m/s)
u	Velocity vector corresponding to perturbation flow-field as defined in Eqn. 4.4, with components $(u(x, y, z, t), v(x, y, z, t), w(x, y, z, t))$ (m/s)
V_l	Velocity induced by Kelvin-Helmholtz billow at splitter-plate trailing edge (m/s)
U_c	Convective velocity of Kelvin-Helmholtz billows, computed as $U_c = \frac{(u_1 + u_2)}{2}$ (m/s)
U_1	Freestream velocity on the low-speed side of the splitter plate (m/s)
U_2	Freestream velocity on the high-speed side of the splitter plate (m/s)
u^*	Friction velocity in the boundary layer on the high-speed side of the splitter plate at the trailing edge, computed as $u^* = \sqrt{\frac{\tau_w}{\rho}}$ (m/s)
V	Volume (m ³)
x	displacement vector (m)
x	streamwise Cartesian coordinate (m)
y	wall-normal or cross-stream Cartesian coordinate (m)
z	spanwise Cartesian coordinate (m)

Greek letters

α_x	Azimuthal angle between peak and trough of waviness at splitter-plate trailing edge, computed as $\alpha_x = \tan^{-1}(2\pi \frac{h}{\Lambda_x})$
α_z	Spanwise angle between peak and trough of waviness at splitter-plate trailing edge
β	Blending factor used by ANSYS CFX [®] for convective-term discretization in Eqn. (3.9)

Γ_0	Circulation based on streamwise component of perturbation vorticity at splitter-plate trailing edge (m ² /s)
γ	Rotation rate (1/s)
$\Delta\phi$	Scaling quantity used by ANSYS CFX [®] to normalize r^n , which is based on global variations of a given generic variable.
ϵ	Strain rate (1/s)
η_i	Scaling quantity used by ANSYS CFX [®] to normalize r^n , based on the relative value of the finite volume at the i th node
θ	Momentum thickness, defined in Eqn. (4.5) (m)
Λ_z	Spanwise distance between two peaks of waviness at splitter-plate trailing edge (m)
λ_{KH}	Mean streamwise distance between two consecutive KH billows (m)
λ_z	Spanwise wavelength (m)
μ	Dynamic viscosity (Pa s)
ν	Kinematic viscosity (m ² /s)
ρ	Density (kg / m ³)
σ	Exponential perturbation growth rate, defined in Eqn. (4.7) for perturbation enstrophy (normalized by t_{KH})
τ_{ij}	ij th component of viscous stress tensor, computed as $\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ (N/m ²)
τ_w	Shear stress at wall (N/m ²)
ϕ	Generic flow quantity
ψ	Rayleigh's discriminant, defined in Eqn. (4.7)
Ω	Vorticity vector corresponding to instantaneous total velocity vector (1/s)
$\bar{\Omega}$	Vorticity vector corresponding to $\nabla \times \bar{\mathbf{U}}$ (1/s)
Ω_{KH}	Vorticity vector corresponding to $\nabla \times \mathbf{U}_{KH}$ (1/s)

$\bar{\boldsymbol{\Omega}}'$ Vorticity vector corresponding to $\nabla \times \bar{\boldsymbol{U}}'$ (1/s)

$\boldsymbol{\omega}$ Vorticity vector corresponding to $\nabla \times \boldsymbol{u}$ (1/s)

Cursive letters

$\mathcal{D}$ Spatially averaged viscous diffusion and dissipation of perturbation enstrophy, calculated as the 10th term in Eqn. (4.6) (1/s³)

d $\mathcal{D}$ normalized by spatially averaged perturbation enstrophy

$\mathcal{K}$ Kinetic energy (kg m²/s²)

$\mathcal{R}$ Ratio of $\mathcal{S}$ to $\mathcal{D}$

$\mathcal{S}$ Spatially averaged transport of perturbation enstrophy by inviscid mechanisms, sum of terms 1 through 6 in Eqn. (4.6) (1/s³)

s $\mathcal{S}$ normalized by spatially averaged perturbation enstrophy

Superscripts

⁺ Normalized by wall variables calculated on the high-speed side of the splitter-plate at the trailing edge, as is done for $\lambda_z^+ = \frac{\lambda_z u^*}{\nu}$

ⁿ Evaluated at nth iteration

⁰ Evaluated one time increment prior to ‘current time’

⁰⁰ Evaluated two time increments prior to ‘current time’

Subscripts

_{ip} Evaluated at integration point

_∞ Freestream conditions

Brackets

[.] Sparse Matrix variable

$\langle \cdot \rangle_x$ Variable inside of brackets is spatially averaged in subscript Cartesian spatial coordinate

Acronyms

NS Navier-Stokes

<i>TS</i>	Tollmien-Schlichting
<i>KH</i>	Kelvin-Helmholtz
<i>KH₀</i>	Upstream Kelvin-Helmholtz billow of a pair of consecutive billows
<i>KH₁</i>	Downstream Kelvin-Helmholtz billow of a pair of consecutive billows
<i>GGI</i>	General Grid Interface used in ANSYS CFX®
<i>PDE</i>	Partial differential equation
<i>CV</i>	A region enclosing two consecutive Kelvin-Helmholtz billows and convecting with U_C
<i>RMS</i>	Root-mean-squared
Key Terms	
Enstrophy	half of the square of vorticity magnitude, computed as $\frac{1}{2}\Omega^2$

1 Chapter: Introduction

1.1 Motivation

Free-shear layers are formed at the interface between two fluid-flow regions of differing freestream velocity. The free-shear layer represents the interface over which mass and momentum are exchanged between the streams on either side of the free-shear layer. A free-shear layer is pathologically unstable to perturbations at low Reynolds numbers, which leads to the development of self-sustaining turbulence that promotes mixing between the two freestreams. The study of turbulence characteristics of free-shear layers is therefore an important consideration in the modeling of a broad range of fluid-flow phenomena, such as ocean currents, astrophysical phenomena, heat exchangers, chemical reactors, aerodynamic wakes, and aerodynamic optimization of airfoils (Ho and Heurre, 1984). The last of these is a broad category in itself and the present study focuses on the optimization of airfoils in the low-pressure turbines of gas-turbine engines. To meet the ever-increasing demand for lighter, higher-performing aircraft engines, the aerodynamic loading on low-pressure turbine blades is being increased (Curtis 1997). An increase in aerodynamic loading requires airfoil configurations which have higher adverse streamwise pressure gradients on the suction-side surface of the airfoil, thus promoting laminar boundary layer separation. The separated laminar shear layer may be rapidly destabilized and transition into turbulence, causing the separated shear layer to reattach onto the airfoil. This phenomenon, resulting in the formation of a separation bubble, may occur under a wide variety of flow configurations. Figure 1.1 depicts such a separation bubble on a turbine blade in a gas turbine engine. In general, flow separation on an airfoil, unless it is accounted for in the aerodynamic design of the airfoil, may result in significant increase in viscous energy dissipation, and thus reduced aerodynamic efficiency.

Figure 1.1: a) Illustration of a Rolls-Royce Trent 1000 turbofan engine (Rolls-Royce, 2012). b) Diagram of a laminar separation bubble on a turbine blade (adapted from Brinkerhoff, 2010). c) Simplified geometry of a free-shear layer developing past a thin splitter plate, approximating a separated shear layer.

Flow development in a separation bubble is a highly complicated phenomenon, as it involves the amplification of disturbances via multiple instability modes. This makes it rather challenging to optimize airfoils over which separation bubbles occur. Recent efforts have been successful at modeling blade-surface boundary layers with separation bubbles, allowing for optimization of airfoil geometry and correspondingly turbine efficiency (Roberts and Yaras, 2005, McAuliffe and Yaras, 2007^{a,b}, Brinkerhoff and Yaras, 2011). To improve on such models, it is necessary to develop a better understanding of the process of laminar-to-turbulent transition in the separated shear layer. Such an understanding may be furthermore employed to devise means by which the process of laminar-to-turbulent transition may be manipulated to further improve the aerodynamic efficiency of airfoils. This can be accomplished more effectively through the study of a better-controlled flow environment. Such an idealized environment consists of a free-shear layer developing downstream from a

no-slip splitter plate. Once the instability mechanisms in a separated/free-shear layer are better understood, means of controlling these mechanisms may be sought. A recent study by Wiltse and Glezer (2011) effectively manipulated these mechanisms using spanwise arrays of streamwise-oriented counter-rotating vortices produced by a spanwise array of surface-film heaters mounted onto a splitter plate. A more practical alternative to producing a streamwise component of vorticity, is to introduce spanwise waviness to the shape of the splitter-plate trailing edge. This method of forcing will be the topic investigated, and it is the intention of the present study to characterize the parameters of the trailing-edge waviness that most affect the downstream development of the free-shear layer.

1.2 Objectives

This thesis investigates the three-dimensional (3D) secondary instability mechanisms that lead to smaller-scale breakdown (laminar-to-turbulent transition) of a planar, laminar free-shear layer. More specifically the present work studies how the 3D perturbations that develop in the boundary layer on the upstream splitter-plate interact with the secondary instability modes prevailing in a free-shear layer. Furthermore, the study investigates how the introduction of varying degrees of spanwise-waviness to the splitter-plate trailing edge affects the downstream development of a free-shear layer. The study is the first phase of a longer-term research effort, and as such concentrates on one Reynolds number in the presence of low environmental disturbances and with zero streamwise pressure gradient.

1.3 Approach

The non-linear dynamics of free-shear layers are investigated using fully-nonlinear, unsteady, 3D direct numerical simulations (DNS) of the Navier-Stokes equations that govern fluid flow, using the ANSYS CFX[®] commercial software. DNS is performed for a series of idealized cases, each comprising of a spatially and temporally growing planar free-shear layer

developing downstream from a planar no-slip splitter plate with a spanwise-wavy trailing edge shape. The spanwise waviness of the splitter-plate trailing edge was varied in height and spanwise wavelength with the intention of causing resonant and anti-resonant excitation of the fastest growing 3D (secondary) instability mode existing in the free-shear layer.

These idealized cases are meant to illustrate the effect of forced streamwise component of vorticity on the development of a free-shear layer. While the physical mechanisms studied in these cases are generally applicable to the laminar-to-turbulent transition of a laminar separation bubble, the idealized conditions under which the simulations are performed may differ substantially from those that are observed in the low-pressure turbines of a gas turbine engine. Some important considerations which will affect the development of 2D and 3D instability in the free-shear layer are: the effect of high-amplitude free-stream turbulence, unsteady inlet velocity and pressure conditions, the presence of a non-zero free-stream pressure gradient, the effect of free-shear layer proximity to a no-slip wall, the effect of thermophysical property gradients, compressibility effects, and the effect of a vibrating no-slip boundary condition. Although these effects are neglected in the present study in order to obtain a clear understanding of the complex mechanisms that govern 3D instability in free-shear layers, this study lays the ground work for a longer-term research effort also intended to elucidate the aforementioned effects.

2 Chapter: Literature Survey

This chapter reviews the published literature on the process of laminar-to-turbulent transition of planar boundary layers and free-shear layers in the presence of low environmental disturbance levels. Section 2.1 gives an overview of instability of attached laminar boundary layers, and Section 2.2 discusses instability of free-shear layers. In this review, particular attention is given to the development of 3D instability, and the manner in which it may be forced.

2.1 Instability of planar boundary layers

Subsections 2.1.1, 2.1.2, and 2.1.3 discuss the 2D primary instability mechanism, the 3D secondary instability mechanisms, and the laminar-to-turbulent transition of flat plate boundary layers, respectively. Section 2.1.4 reviews the effects of superposition of elongated streamwise-oriented vortices and steady velocity streaks on the primary and secondary instability mechanisms.

2.1.1 Primary instability of a boundary layer

Laminar-to-turbulent transition in a boundary layer is brought on by the selective amplification of disturbances by instability mechanisms. Using linear perturbation analysis, Tollmien and Schlichting (Schlichting, 1979) predicted that two-dimensional waves are first amplified downstream from the location where boundary layer Reynolds number reaches a critical value. This instability, called the primary Tollmien-Schlichting (TS) instability mode, is manifested as waves of streamwise-aggregated spanwise component of vorticity that propagate in the streamwise direction. Hot-wire anemometry experiments by Schubauer and Skramstad (1947) experimentally showed the existence of these TS waves. For further information regarding application of linear perturbation analysis, stability characteristics of a

variety of boundary layer velocity profiles, and the primary TS instability mode, the reader is referred to White (1991).

2.1.2 Secondary instability of a boundary layer

3D secondary instability is manifested in boundary layers as streamwise-oriented velocity streaks, formed through the spanwise variations in the streamwise component of velocity. Blackwelder and Eckelmann (1979) suggested that streaks coincide with streamwise-oriented counter-rotating vortices. They proposed that these streamwise-oriented vortices induce a wall-normal component of velocity that convects low-speed near-wall fluid upwards into high-speed regions and vice-versa, producing the streaks. The numerical experiments of Kim and Lim (2000) confirmed the role of wall-normal components of velocity and showed that it leads to the production of streaks. The presence of streaky perturbations coincides with the development of a spanwise waviness of the initially 2D TS waves. The TS waves are perturbed by the velocity streaks and are stretched in the streamwise direction by the shear of the undisturbed background flow field (Hon and Walker, 1987). The perturbed TS waves develop into vortices shaped like the Greek letter Λ , which are visualized in Figure 2.1 (Herbert, 1988).

Figure 2.1: Stretching of TS-waves into Λ -shaped vortices in a flat-plate boundary layer (Herbert, 1988).

Smith *et al.* (1991) asserted that these vortices develop into the coherent vortical structures known as ‘hairpin vortices,’ which are observed in fully turbulent boundary layers. The mechanisms by which this occurs are summarized in Figure 2.2.

Figure 2.2: Schematic illustration of a) perturbation of a spanwise-uniform vorticity line by streaky velocity perturbation (u), b) stretching of perturbation by the background shear into Λ -shaped vortices, lift up of their head by self-induced velocities, and c) lateral generation of Λ -shaped vortices. Adapted from Smith *et al.* (1991)

The legs of the developing Λ -shaped vortex induce a positive wall-normal velocity component between themselves, causing the head of the vortex to lift up away from the wall. Simultaneously, the legs of the Λ -shaped vortex induce a negative wall-normal component of velocity on the outboard sides of the vortex resulting in the lateral production of streaks that become unstable and in turn create more Λ -shaped vortices (Hon and Walker, 1987; Zhou *et al.* 1999). Adrian *et al.* (2000) suggested that this is the basis of an auto-regeneration cycle for hairpin vortices in turbulent boundary layers. Adrian *et al.* (2000, 2007) further found that wall-normal velocity component that is induced by the head of the hairpin vortex perturbs the flow in its wake and results in the production of offspring hairpin vortices. This consecutive reproduction of hairpin vortices results in a ‘wave-packet’ of several streamwise-aligned hairpin vortices. It is, however, noted that there are multiple other instability modes in the boundary layer that may result in the genesis of hairpin vortices, and that the auto-regeneration of turbulence structures in the boundary layer is still an active area of research.

Singer and Joslin (1994), Schroder and Kompenhans (2004), and more recently Brinkerhoff and Yaras (2012) provide a more in-depth analysis of the mechanisms by which hairpin vortices are created.

Streaks and hairpin vortices are ordered in the spanwise direction, and have been observed experimentally (Smith and Schwartz, 1983) and using DNS (Kim et al., 1987; Yu and Kawaguchi, 2003) to have a consistent average spanwise spacing of $\bar{\lambda}_z^+ = 100$ when normalized by wall units. Normalized spanwise spacing is defined as $\lambda_z^+ = \frac{\lambda_z u^*}{\nu}$, where λ_z is the dimensional streak spacing in the spanwise direction, $u^* = \sqrt{\frac{\tau_w}{\rho}}$ is the friction velocity, τ_w is the shear stress at the wall, ρ is the fluid density, and ν is the kinematic viscosity. Smith and Metzler (1983) observed experimentally that the characteristic spanwise spacing of streaks increases with wall-normal distance. Hon and Walker (1987) furthermore note that λ_z^+ decreases if background shear is locally increased.

Linear perturbation analysis has been utilized to accurately predict the spanwise spacing of streaks (Brooke and Hanratty, 1993, Schoppa and Hussain, 1997). Of particular note, Butler and Farrel (1993) used the Euler-Lagrange equation to formulate a variational calculus problem which searches for kinetic energy perturbations that exhibit maximum amplification. When this analysis, called optimum perturbation theory, was applied to a turbulent channel flow, it predicted that the most amplified perturbations have a spanwise spacing of $\lambda_z^+ = 110$, which increased with wall-normal distance in agreement with experiments.

Chernyshenko and Baig (2005) physically interpreted the results of optimum perturbation theory as the mechanism by which the background flow field selects the spanwise spacing of streaks. They concluded that streaks develop with a spanwise spacing

that achieves an optimum balance between viscous dissipation, which increases when the streaks are spaced further apart, and the wall-normal velocity component induced by the streamwise-oriented vortices, which decreases when the streaks are spaced further apart.

2.1.3 Breakdown into turbulence

The presence of a wave-packet of hairpin vortices destabilizes the surrounding regions, resulting in the reproductive spreading of turbulence structures (Adrian et al., 2007; Brinkerhoff and Yaras, 2012). This leads to the formation of a turbulent spot – an arrow-head shaped region of locally turbulent flow – which initiates the breakdown of the laminar boundary layer into turbulence. Smith *et al.* (1991), and Grek *et al.* (2000) proposed that turbulent spots are created via the self-sustaining streamwise and lateral reproduction of streaks, hairpin vortices, and their accompanying instability modes. Schroder and Kompenhans (2004), and Yaras (2007) found that fully turbulent boundary layers are formed by merging of numerous turbulent spots spreading laterally and in the streamwise direction.

2.1.4 Effect of superposition of steady velocity streaks or streamwise-oriented vortices on primary and secondary instability modes

Linear perturbation analysis performed by Nayfeh (1981) shed light on the effect of counter-rotating streamwise-oriented vortices superimposed on the boundary layer on the development of secondary instability. Nayfeh (1981) found that the presence of such vortices with a spanwise spacing equal to twice the spanwise wavelength of the secondary instability mode, doubled the growth rate of the secondary instability.

Wu and Choudhari (2003) performed linear perturbation analysis of a boundary layer in which steady velocity streaks were superimposed. They found that the presence of streaks with certain threshold amplitudes activate a local cross-flow-like inviscid instability mode at

the interfaces between high- and low-speed streaks that leads to rapid amplification of 3D perturbations. Cossu and Brandt (2004) found, however, that the superposition of lower-amplitude steady streaks in a boundary layer could stabilize the primary TS instability as a result of the shear between the high- and low-speed streaks. Increasing the amplitude of the steady streaks further stabilized the primary TS instability mode, until a threshold amplitude was exceeded which activated the cross-flow-like instability mode. Wu and Luo (2003) and Fransson *et al.* (2004) also found that imposition of spanwise-periodic steady streaks with spanwise spacing equal to that of the most amplified secondary instability mode increase the growth rate of the secondary instability mode. Liu *et al.* (2008) found the ability of steady streaks to alter secondary instability is further dependent on the width of the streaks imposed: wider streaks tend to promote instability by direct resonance with the most amplified secondary instability modes.

2.2 Instability of planar free-shear layers

Free-shear layers may be formed from the merging of two streams with differing velocities, or by boundary layer separation. Sections 2.2.1 through 2.2.4 discuss the ‘natural’ development of instability modes in a free-shear layer under low environmental perturbation conditions, and in far proximity from a wall. Section 2.2.6 provides an overview of literature focusing on manipulation of secondary instability modes in free-shear layers.

2.2.1 Primary instability of a planar free-shear layer

The free-shear layer between two parallel flows of differing freestream velocities has a characteristic hyperbolic-tangent cross-stream (y -direction) velocity profile with an inflection point at its center. Rayleigh’s inviscid instability criteria (White, 1991) dictate that a flow with an inflectional velocity profile is unstable. The 2D Kelvin-Helmholtz (KH) instability develops as a result of this instability. The development of the Kelvin-Helmholtz instability is illustrated by the analogy of a rolling-up vortex sheet, and is depicted in Figure 2.3.

Figure 2.3: Analogy of 2D KH instability using a perturbed vortex sheet. The small-amplitude perturbation in a) is aggregated in the streamwise direction causing the vortex sheet to ‘roll-up’ into

coherent 2D spanwise-oriented KH billows depicted at a later time in b) (Adapted from Drazin and Reid, 2004).

The thick wavy line in Figure 2.3a illustrates a sheet of spanwise (z) vorticity that is sinusoidally perturbed in the cross-stream (y) direction. Self-induced velocity of the vortex sheet causes the sinusoidal perturbation to increase in amplitude. Portions of the vortex sheet which are perturbed into the high-speed ($+U$) stream are convected downstream, whereas those that are perturbed into the low-speed ($-U$) stream, are convected in the upstream direction as shown in Figure 2.3a. These convective effects cause the sinusoidally perturbed vortex sheet to aggregate, or 'rollup', about its inflection points, as depicted in Figure 2.3b. The thicker portions of the line in Figure 2.3b indicate that spanwise vorticity is concentrated in the rolled-up vortex sheet. The regions where the vortex sheet is rolled up correspond to the 2D vortices called KH billows. In the region labelled as the braid, vorticity is depleted and the increasing KH-billow-induced velocity causes the vortex sheet to be stretched, indicating that this region is highly strained. The frequency of the 2D KH instability, f_{KH} , in a free-shear layer formed downstream from a splitter-plate is expressed as the Strouhal number, St . Strouhal number is given as $St = f_{KH} \frac{2\theta}{U_1 + U_2}$, where θ is the momentum thickness at the splitter-plate trailing edge, and U_1 and U_2 are the freestream velocities. The 2D KH instability is naturally most amplified in a range between $St_n = 0.016$, and 0.032 (Michalke and Herman, 1982). The KH billows convect with the mean of the two freestream velocities (Ho and Huerre, 1984) allowing conversion between, f_{KH} , and streamwise wavelength, λ_{KH} . Lastly, the mutually induced velocity of the KH billows results in the subharmonic pairing instability. KH billows that are perturbed into either the high- or low-speed streams are convected towards each other, until they meet and undergo a vortex merger interaction. This instability is called the subharmonic pairing instability of the primary KH instability, as it results in a doubling of wavelength and amplitude of the primary KH instability.

Figure 2.4 shows streamlines calculated from the local velocity field that develops subsequent to rollup of the KH billows. The spanwise component of vorticity is locally a minimum at the midpoint between the rolled up KH billows, which is labelled as the hyperbolic stagnation point in Figure 2.4. Likewise the elliptical stagnation point in the figure corresponds to the center of the KH-billow core, where spanwise component of vorticity is locally maximum. The velocity is zero at the ‘stagnation points’ in a reference frame travelling with the mean of the high- and low-speed freestream velocities.

Figure 2.4: Rolled up vortex sheet forming KH billows and elliptical and hyperbolic streamline patterns in core and braid regions, respectively.

Streamlines in the vicinity of the KH billows are elliptically shaped, whereas those in the braid region are hyperbolically shaped. The presence of elliptical or hyperbolic streamlines generally distinguishes what are labelled as the ‘core’ and ‘braid’ regions in the figure, respectively (Caulfield and Kerswell, 2000).

2.2.2 Secondary instability of a planar free-shear layer

Streamwise-oriented velocity streaks are present in the free-shear layer in the braid regions between consecutive KH billows and within the KH billows themselves. This is evident in the shadowgraphs taken by Konrad (1976) in the spanwise-plane view of a mixing layer shown in Figure 2.5a. Moore and Saffman (1975) concluded that the streak-structures in

the braid regions coincide with the presence of counter-rotating streamwise-oriented vortices. These authors also observed ribs on the KH billows with a mean spanwise spacing identical to that of the elongated streaks in the braid region. Wygnanski *et al.* (1979) reported, however, that streaks at the edges of the KH billows have differing spanwise spacings than those measured in the center of the KH billows. This indicated that there are several coexisting secondary instability modes in the free-shear layer. Breidenthal (1981) used a chemically-reacting dye visualization technique and obtained the instantaneous photographs of the mixing layer shown in Figure 2.5b. These photographs show that the KH-billow cores themselves develop a spanwise waviness.

Figure 2.5: a) Instantaneous spanwise-view shadowgraphs of Konrad (1976) and b) Instantaneous photograph of free-shear layer impregnated with chemically-reacting dye viewed from the wall-normal direction

Pierrehumbert and Widnall (1982) performed linear stability analysis of a streamwise-consecutive array of Stuart vortices – analytical solutions of an array of finite-vortex cores by Stuart (1967) – with vorticity distributions fitted to match an array of KH billows. They found the existence of a ‘translative’ instability involving growth of a spanwise-wavy perturbation to the center-line of the KH-billow core. They proposed that this waviness causes a spanwise-wavy strain field in the braid region which leads to the creation of streamwise-oriented vortices in the braid region.

An early study by Klaassen and Peltier (1985), however, noted that usage of Stuart vortices to model the KH instability is flawed because the resulting flow-field lacks the hyperbolic stagnation points in the braid regions. They performed linear perturbation analysis on a background flow which more realistically approximated the KH instability, and found that the hyperbolic stagnation point in the braid region is unstable in and of itself. This ‘hyperbolic instability’ leads to creation of streamwise-oriented vortices in the braid region independently of any spanwise-waviness of the KH-billow. Smyth and Peltier (1994), however, noted that the translative instability and the hyperbolic instability result from the stretching of a perturbation by the 2D strain field. These two instability modes are therefore manifestations of the same instability mode in different locations. As the characteristics of the background strain field in these respective regions differ, so do the spanwise wavelengths of the most amplified 3D perturbations and their respective growth rates. Caulfield and Kerswell (2000) found that in the linear perturbation regime, perturbation growth rate is limited from above by $\sigma = 2\sqrt{\epsilon^2 - \gamma^2}$, where ϵ represents the local strain-rate, and γ represents the local rotation rate. The core region is predominantly rotational, whereas the braid region is irrotational, resulting in the maximum growth rate in the braid region to be nearly four times greater than in the core region. Thus, the hyperbolic instability mode was found to be the

more-rapidly amplified instability mode. Aside from these instability modes which are generally the fastest growing, there are numerous other instability modes that exist in the free-shear layer. Sections 2.2.2.1 and 2.2.2.2 discuss the other instability modes that develop in the core and braid regions, respectively.

2.2.2.1 Secondary instability mechanisms in the core region

The elliptical streamline pattern found in the vicinity of the KH-billow core is susceptible to a large class of 3D instability modes, which are summarized in detail by Kerswell (2002). In the context of a free-shear layer, the most significant of these are: the translative instability, the helical pairing instability, and the bulging instability. The temporal evolution of these instability modes is illustrated schematically in Figure 2.6.

The translative instability observed by Pierrehumbert and Widnall (1982), consists of a spanwise-wavy perturbation to the center-line of the KH-billow core, as illustrated in Figure 2.6a. The perturbation is stretched by the background shear along the principle axis of strain, as illustrated in the figure. These perturbations warp the strain field of the streamwise adjacent KH billows, causing the perturbations in adjacent KH billows to be in phase with each other.

The helical pairing instability shown in Figure 2.6b is the 3D generalization of the 2D KH-billow pairing instability. A spanwise-wavy perturbation is imposed to the center-line of the KH-billow core in the wall-normal direction. Portions of the perturbation which protrude into the high-speed stream are convected in the downstream direction relative to those perturbed into the low-speed stream.

Figure 2.6: Temporal development of KH billows subject to a) translative instability b) helical pairing instability, and c) bulging instability, adapted from Schoppa *et al.* (1995).

The perturbed portion of the KH-billow core that is convected downstream induces velocity onto the downstream KH-billow core in the $-y$ direction, perturbing it into the low-speed stream. The portion of the downstream KH-billow core that is perturbed into the low-speed stream is then convected upstream in the reference frame moving at a speed equal to the average of the two freestream velocities. Similarly a portion of the KH-billow core convected upstream induces a $+y$ component of velocity on the upstream-adjacent KH-billow core. The KH-billow cores thus develop spanwise waviness that is out of phase in streamwise consecutive billows, as illustrated in Figure 2.6b. Pierrehumbert and Widnall (1982) suggested that because helical pairing type perturbations are out of phase in the streamwise direction, they interact destructively during KH-billow pairing. Schoppa *et al.* (1995) noted that, for a similar reason, helical pairing type perturbations also tend to interact destructively with the translative instability as well as suppressing perturbation growth in the braid region (Metcalf and Orszag, 1987).

The bulging instability was studied in depth by Schoppa *et al.* (1995). The temporal development of this instability is visualized in Figure 2.6. A local decrease in the radius of the KH-billow core results in locally increased rate of rotation relative to the unperturbed portions of the KH-billow core. As the perturbed, faster rotating portion of the KH-billow core rotates relative to the unperturbed region, the initially straight spanwise-oriented vorticity lines contained in the KH-billow core are twisted. The twisted vorticity lines have streamwise and wall-normal components of vorticity, that form vorticity rings around the KH-billow core. These vorticity rings induce circuits of flow that are illustrated by the dashed elliptically-shaped streamlines in Figure 2.6c. The induced flow circuits have a radially-outward component of velocity at the pinched portion of the KH-billow core that locally causes its expansion. As the pinched portion of the core is expanded, the relative rotation rate

of the pinched portion of the core is decreased and reverses in direction, causing the vorticity lines to twist in the opposite direction. As the vorticity lines twist in the opposite direction, the induced flow circuits also reverse in direction, and the core is contracted. The perturbation to the radius of the KH-billow core resonates, and the its amplitude initially grows linearly. Once a critical radial component of velocity is reached, however, the spanwise component of vorticity is ejected from the KH-billow core, breaking down the 2D coherence of the KH billow and leading to breakdown of the free-shear layer.

Figure 2.7 compares the variations of linear perturbation growth rate with spanwise wavenumber of the three core-centered instability modes obtained using linear perturbation analysis performed by Schoppa *et al.* (1995).

Figure 2.7: Variation of growth rate, σ_r , with spanwise wavenumber, β/α_i , predicted by linear perturbation analysis for the bulging (solid), helical pairing (dashed), and translative (dotted) instability modes (Schoppa *et al.*, 1995).

In the linear growth regime, the translative instability is the fastest growing instability mode, with greatest growth-rate at a spanwise wavelength that corresponds to $2.2d (\frac{1}{3} \lambda_{KH})$, where d is the local half-depth of the shear layer and is equal to vorticity thickness (Caulfield and Peltier, 2000). From Figure 2.7 it is evident that growth rate of the translative instability diminishes to zero in the 2D limit, and is relatively constant for perturbations with greater

spanwise wavenumbers (i.e. smaller spanwise wavelengths). Growth rate of the helical pairing instability is greatest at a spanwise wavenumber of zero, which corresponds to the 2D KH-billow pairing instability, and decreases with increasing wavenumber. For this reason, and the fact that the helical pairing instability interacts destructively with the translative instability, KH-billow pairing is generally 2D (Pierrehumbert and Widnall, 1982). Growth rate of the bulging instability in the linear growth regime is the least of the three instability modes. Schoppa *et al.* (1995), however, noted that the non-linear growth rate of the bulging instability is by far the greatest of the three instability modes. Finally, Schoppa *et al.* (1995) and Kerswell (2002) noted that these core-centered instability modes interact destructively with each other. It is therefore unlikely that multiple core-centered instability modes will appear simultaneously in a given case.

2.2.2.2 Secondary instability mechanisms in the braid region

Linear stability analysis performed by Klaassen and Peltier (1985) revealed that the hyperbolic stagnation point between consecutive KH billows is statically unstable to 3D kinetic energy perturbations. Friedlander and Vishik (1991) also noted that in general, the presence of a hyperbolic stagnation point is sufficient for instability. The linear stability analysis by Klaassen and Peltier (1989) showed that the perturbations with a spanwise wavelength of $\lambda_z = 0.8d$ are most amplified by the hyperbolic instability mode. Caulfield and Peltier (2000) found that the hyperbolic instability is inviscid in nature, and that perturbation growth is accomplished via stretching of perturbations by the background shear. Staquet (1995) and Caulfield and Kerswell (2000) further noted that strain rate is locally maximum at the hyperbolic stagnation point, and perturbation growth rate is correspondingly greatest there.

DNS by Caulfield and Peltier (2000) demonstrated that the hyperbolic instability mode leads to the formation of counter-rotating streamwise-oriented vortices in the braid region, with the maximum amplitude being centered at the hyperbolic stagnation point. A DNS study by McAuliffe and Yaras (2007) found similar streamwise-oriented vortical structures in the braid region between consecutive KH billows developing in the shear layer of a separation bubble. The presence of streamwise component of vorticity in the braid region was investigated experimentally by Bernal (1981) and Bernal and Roshko (1986). They likewise suggested that background shear in the braid region stretches a spanwise-oriented vortex-segment until it develops into the counter-rotating streamwise oriented vortices in the braid region. This is shown schematically in Figure 2.8. In their numerical analysis, Smyth and Peltier (1994) found that vortical perturbations that are stretched by the background shear experience growth along the principle axis of strain. They noted, however, that at the hyperbolic stagnation point itself, ‘tilting’ by background shear contributes more than actual stretching, which in fact has a small effect in its vicinity. These findings are in accordance with observations by Lasheras and Choi (1988), who performed experiments in which the principle axis of strain was skewed from the streamwise direction. They observed that the growth of vortical structures in the braid-region always occurs along the principle axes of strain, irrespective of the direction of the freestream velocities.

Figure 2.8: Stretching of a vorticity in the braid region by the strain field induced by KH billows. Adapted from Lasheras and Choi (1988).

Smyth and Peltier (1994) found that if the background flow field is rotating in a direction opposite to that of the KH billows, another instability mode dominates over the hyperbolic instability mode. This instability mode results from centrifugal instability of the concave elliptical streamlines outside of the KH billows (Potylitsin and Peltier, 1999; 2003), in a similar manner to the Görtler instability (White, 1991). The instability mode, called the ‘edge’ instability mode, results in elliptical rings of vorticity with axes coinciding with the elliptical streamlines surrounding the KH-billow core along which the centrifugal instability exists. The edge instability mode is in general faster-growing than the hyperbolic instability in the presence of weak background rotation, and is amplified over a broad range of spanwise wavelengths, with a peak wavelength of $\lambda_z = 0.55 d$. The edge instability mode is distinct in nature, however, from the bulging instability mode that similarly results in vorticity rings surrounding the KH billow, and which is also most amplified at a spanwise wavelength of $\lambda_z = 0.5 d$.

Potylitsin and Peltier (2003) performed DNS studies in which the hyperbolic and edge instability modes are present. Isosurfaces of vorticity magnitude are shown in Figure 2.9 to illustrate the general structure of these instability modes.

Figure 2.9: Isosurfaces of vorticity magnitude illustrating vortical structures created by the a) hyperbolic and b) edge instability modes. Blue- and green-coloured isosurfaces indicate positive and negative streamwise component of vorticity respectively. Adapted from Potylitsin and Peltier (2003).

Metcalfe and Orszag (1987), Smyth and Peltier (1994), Potylitsin and Peltier (1999), and Caulfield and Kerswell (2000) noted that inviscid growth rate and viscous dissipation rate of the 3D instability modes both increase with decreasing spanwise wavelength. They suggest that 3D secondary instability modes are most amplified at a spanwise wavelength that corresponds to an optimum balance between growth rate by inviscid mechanisms, and viscous dissipation. This idea corresponds to a similar idea proposed by Chernyshenko and Baig (2005) for the mechanism by which the background flow field in a boundary layer selects the spanwise spacing of streaks.

2.2.3 Phases of development of primary and secondary instability of a free-shear layer

The temporal development of the free-shear layer is subdivided into four sequential phases:

- 1) Onset of the primary 2D KH instability, during which 3D simulations are indistinguishable from 2D simulations. This phase is terminated by the saturation of 2D perturbation kinetic energy of the KH instability.
- 2) Commencement of secondary instability at the time of saturation of 2D kinetic energy of the KH instability, and subsequent amplification of 3D perturbation kinetic energy.
- 3) Reaching a critical magnitude of 3D perturbation kinetic energy necessary for 3D perturbations to affect the background flow, which is accompanied by a decline in the kinetic energy of the background flow field.
- 4) 3D perturbation kinetic energy reaches a maximum (saturates), indicating breakdown of the shear layer into disorderly smaller-scale structures accompanied by a sharp rise in viscous dissipation of kinetic energy.

Using DNS, Caulfield and Peltier (2000) studied the temporal evolution of a KH billow in a reference frame travelling with the convective velocity of its centroid. Their computational

domain encompassed the region between two consecutive hyperbolic stagnation points and used periodic streamwise and spanwise boundaries. In their analysis they used a spatial-averaging technique to separate the instantaneous velocity vector, $\mathbf{U}$, into its 2D background component, $\bar{\mathbf{U}}$, a component corresponding to the 2D KH instability, $\mathbf{U}_{KH}$, and a 3D perturbation component, $\mathbf{u}$. Figure 2.10 shows the time-variation of $\bar{\mathcal{K}}$, $\mathcal{K}_{KH}$, and $\mathcal{K}_{3D}$, which represent spatially averaged kinetic energy computed from the velocity vectors, $\bar{\mathbf{U}}$, $\mathbf{U}_{KH}$, and $\mathbf{u}$, respectively, commencing from the onset of phase (1). Kinetic energy in Figure 2.10 is normalized by the total kinetic energy at the onset of (1), $\mathcal{K}_0$; time, t , is normalized by d and $(U_2 - U_1)$. The time-variation of kinetic energy shown in Figure 2.10 illustrates the free-shear layer development phases described above, the onset of which are indicated in Figure 2.10.

Figure 2.10: Variation of kinetic energy contained in components of the flow-field associated with a) the background flow-field b) the 2D KH instability, and c) 3D secondary instabilities, using DNS of a temporally evolving free-shear layer. Adapted from Caulfield and Peltier (2000).

The simulation is initialized with a spanwise- and streamwise-uniform hyperbolic tangent velocity profile, which rolls up into a 2D KH billow located at the center of the domain. As the KH billow develops, the kinetic energy of the 2D KH flow field increases as is observed in Figure 2.10b, whereas the kinetic energy of the background flow-field is depleted, as is evident in Figure 2.10a. At $t = 35$ in Figure 2.10b, the kinetic energy of the 2D

KH instability reaches a maximum value, then declines and oscillates sinusoidally. The time at which the kinetic energy of the 2D KH instability is maximum is called '2D kinetic energy saturation', and is labelled as (2) in Figure 2.10. At this time the primary 2D KH instability ceases to grow, and the secondary pairing instability commences growth (Metcalf and Orszag, 1987). Caulfield and Peltier (2000) also note that at the time of kinetic energy saturation, the spanwise component of vorticity becomes minimal at the hyperbolic stagnation point. As this occurs, the hyperbolic instability becomes the dominant secondary instability mode and 3D perturbation growth rate increases substantially, as is observed in Figure 2.10c after (2).

Increase of 3D perturbation kinetic energy is linear until perturbation amplitude is sufficiently large to affect the background flow-field at $t = 110$, labelled (3) in Figure 2.10c. This is manifested as a greater rate of decline of kinetic energy in the background flow-field due to increased viscous dissipation, and is also accompanied by a decrease in background strain rate. 3D perturbation growth-rate accordingly declines after phase (3), which also corresponds to the time of laminar-to-turbulent transition. Saturation of 3D perturbation kinetic energy occurs at $t = 140$, which is labelled as (4) in Figure 2.10c. At this time the free-shear layer is fully turbulent.

2.2.4 Breakdown of a free-shear layer into smaller scales

There are numerous mechanisms that may lead to breakdown of a free-shear layer into smaller scales. For instance, Schoppa and Metcalfe (1995) suggested from the results of their non-linear modeling of the KH instability, that the interaction of the bulging instability and the KH-billow core results in the breakdown of the free-shear layer into smaller scales. Arendt *et al.* (1997) and Fritts *et al.* (1998) observed using DNS that the highly amplified helical pairing perturbations interacted to produce hairpin-like turbulence structures. A DNS

study performed by Alam and Sandham (2000) observed that Λ -shaped vortices were created from the extensively stretched streamwise-oriented vortices in the braid region.

In planar free-shear layers developing downstream from thin splitter plates, breakdown into disorderly smaller-scale structures has been experimentally observed when $R \frac{x}{\lambda_{KH}}$ is between 6 and 10, where R is the velocity-difference ratio, defined as $R = \frac{U_2 - U_1}{U_2 + U_1}$, x is the streamwise distance measured from the splitter-plate trailing edge, and U_2 and U_1 are the freestream velocities (Ho and Heurre, 1984). McAuliffe and Yaras (2007) observed using DNS that in a separation bubble, breakdown to smaller-scales occurs approximately one to two KH billow wavelengths downstream from the point of separation. They suggested that the streamwise-oriented vortices in the braid region are instrumental in causing breakdown to smaller scales. Brinkerhoff and Yaras (2011) further found using DNS that highly amplified secondary instabilities that are developed in the boundary layer upstream from the separation point interact with the KH instability to cause smaller-scale breakdown of the separated shear layer.

Using DNS and linear perturbation theory, Caulfield and Peltier (2000) and Peltier and Caulfield (2003) studied the role of the streamwise-oriented vortices in the braid region in the smaller-scale breakdown of a temporally evolving free-shear layer. They observed that the streamwise-oriented vortices originating in the braid region that are depicted in Figure 2.9, are stretched by the KH-billow-induced flow field outside of their native braid region, around the KH billows and towards the hyperbolic stagnation points of a neighboring braid region. Initially these vortices have small vortical amplitude, and as they infringe into a neighboring braid region they are deflected from the hyperbolic stagnation points and continue towards the next upstream/downstream braid region. The stretched streamwise oriented vortices are amplified by the stretching process and eventually acquire sufficiently

high vortical magnitude to penetrate into a hyperbolic stagnation point. As the infringing vortices penetrate into a braid region, they interact with the native braid vortices and are broken down into smaller-scale structures, in the vicinity of the hyperbolic stagnation point. A similar interaction was observed to cause breakdown to smaller scales in a separation bubble by Jones et al. (2008) using DNS.

2.2.5 Interaction of KH-billow pairing instability with secondary instability modes

Corcos and Lin (1983) and Metcalfe and Orszag (1987) showed that KH-billow pairing suppresses the growth of 3D perturbations with smaller spanwise wavelengths, whereas perturbations with greater spanwise wavelengths are less affected. Schoppa et al. (1995) observed using DNS, a similar attenuation of core-centered perturbation growth rate during KH-billow pairing, though they also found that the growth rate of the subharmonic of the bulging mode is in fact amplified. Caulfield and Peltier (2000) noted that, conversely, the presence of high-amplitude 3D perturbations in the braid region suppresses the development of the secondary KH pairing instability. A DNS study by Mashayek and Peltier (2011), however, found that if KH-billow pairing does occur in the presence of high-amplitude 3D perturbations, rapid smaller-scale breakdown ensues, particularly in the presence of the edge instability mode.

2.2.6 Forcing of a planar free-shear layer

Numerous studies have been conducted in which the primary 2D KH instability has been forced by various techniques, such as the vibrating ribbon and flapping trailing edge techniques (Wehrmann, 1967; Oster and Wygnanski, 1982; Ffowcs Williams and Möhring, 2000). Wiltse and Glezer (2004; 2011) provide reviews of recent efforts directed at controlling primary and secondary instability modes. A notable experimental study by Koochsfani and Dimotakis (1988) found that a flapping airfoil fixed to the trailing edge of a

flat splitter plate could be used to either enhance or completely suppress the dominant frequency of the KH instability.

Quasi-linear stability analysis of a forced free-shear layer was performed by Saxena and Liebovich (1998). Spanwise-waviness was introduced to a free-shear layer by superimposing a streamwise component of velocity that varies sinusoidally in the spanwise direction onto a hyperbolic tangent velocity profile, the combination of which comprised the steady background flow. The interaction between the spanwise-waviness of the shear layer and the perturbation velocity wave vector was modeled as a weakly-nonlinear wave interaction. They found that in general, growth rate of the 2D KH instability decreases as spanwise wavelength of the spanwise-varying component of velocity is decreased. They found, however, that while the growth rate of the 2D KH instability decreases, growth rate of KH billows with spanwise waviness is maximum when the wavelength of the spanwise-varying background velocity component is equal to the wavelength most amplified by the transverse instability. Their analysis considered only linear development of the KH instability, preventing analysis of the hyperbolic and edge instability modes that develop after the non-linear saturation of the 2D KH instability.

Forcing of a planar free-shear layer by introducing sinusoidal spanwise-waviness to the trailing edge of a thin splitter plate was studied experimentally by Lasheras and Choi (1988). The spanwise wavelength of the trailing-edge waviness was significantly larger than the most amplified hyperbolic instability mode. The authors found that the streamwise-oriented vortices introduced experience growth in the braid region while the 2D KH billows remained unperturbed. Spanwise waviness of the KH-billow core was observed subsequently, which had equal wavelength but opposite phase to the streamwise-oriented vortices.

Breakdown of the forced free-shear layer occurred significantly earlier than in the unforced case.

Nygaard and Glezer (1991) experimentally introduced counter-rotating streamwise-oriented vortices into a planar free-shear layer using a spanwise array of surface film heaters. They found that streamwise-oriented vortices could be forced in the braid region with a variety of spanwise spacings greater than λ_{KH} . The growth rate of the forced streamwise-oriented vortices did not diminish with increased spanwise spacing as expected. Consequently they were unable to identify a dominant 3D instability mode. Wiltse and Glezer (2004) performed similar experiments, and found that forced streamwise-oriented vortices with spanwise spacing equal to $\frac{1}{3}\lambda_{KH}$ excited the translative instability mode. Kit et al. (2007) also experimentally introduced streamwise-oriented vortices in a free-shear layer using a spanwise array of triangular chevrons protruding from the trailing edge of a splitter plate, with spanwise spacing of $0.435 \lambda_{KH}$. They found that both the translative and bulging instabilities are forced in different cases, but noted that streamwise-oriented vortices in the braid regions gradually decline. A follow-up study by Kit and Wyganski (2008) performed experiments and DNS studying a similarly forced free-shear layer using a spanwise-array of chevrons. They found that the bulging instability was predominantly forced if the chevrons are mounted onto a mechanism that flaps with frequency equal to f_{KH} . As in their previous study, they also found that streamwise oriented vortices in the braid region decay in magnitude over time.

Wiltse and Glezer (2011) used spanwise periodic arrays of surface film heaters, along with flow-measurement devices far downstream from the splitter-plate trailing edge to achieve closed-loop feedback control of secondary instability modes in a free-shear layer. They found that varying the spanwise spacing of small-amplitude forced streamwise-oriented vortices could force secondary instability modes into either a resonant or anti-resonant mode.

The resonant mode was able to enhance a particular secondary instability mode, producing organized flow-structures in the free-shear layer. The anti-resonant mode significantly attenuated the targeted instability mode such that the flow consists of 3D perturbations with multiple peaks at a variety of spanwise wavelengths. The configuration they used for feedback control was capable of significantly diminishing or enhancing turbulence kinetic energy and passive-scalar mixing in a free-shear layer.

3 Chapter: Numerical Method and Computational Domain

This chapter outlines the procedure used to perform the DNS computations. Section 3.1 describes the governing equations of fluid flow. Section 3.2 describes the numerical method by which the governing equations are discretized and solved in ANSYS CFX®, and the computing platform used. Section 3.3 describes the computational domain, spatial grid, convergence criteria and the duration of the simulations.

3.1 Governing equations of fluid flow

The differential governing equations describing conservation of mass, momentum and energy in a fluid medium, the Navier-Stokes (NS) system of equations, in which the gravitational effects are neglected, are expressed in their conservative forms in Eqns. (3.1) through (3.3) using index notation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho U_j) = 0 \quad (3.1)$$

$$\frac{\partial}{\partial t} (\rho U_i) + \frac{\partial}{\partial x_j} (\rho U_i U_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (3.2)$$

$$\frac{\partial}{\partial t} (\rho h_0) - \frac{\partial P}{\partial t} + \frac{\partial}{\partial x_j} (\rho U_j h_0) = \frac{\partial}{\partial x_j} \left(k \frac{\partial T}{\partial x_j} \right) + \frac{\partial}{\partial x_j} (U_i \tau_{ij}) \quad (3.3)$$

where,

ρ : density (kg/m³)

U_i : i^{th} Cartesian component of velocity (m/s)

P : pressure (Pa)

τ_{ij} : viscous stress tensor; in a Newtonian fluid, $\tau_{ij} = \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$ (Pa)

h_0 : total enthalpy (J/kg)

k : thermal conductivity (W/m K)

μ : dynamic Viscosity (kg/m s)

The fluid in the simulations was assumed to be Newtonian. Eqns. (3.1) through (3.3) are coupled with the equations of state for the working fluid. Air was used as the working fluid, and was modeled as an ideal gas.

3.2 Discretization of the governing equations of fluid flow

Direct numerical simulation (DNS) of the NS equations is performed using the ANSYS CFX[®] commercial CFD software. The methods by which these equations are discretized and solved in this software are illustrated here for their 2D incompressible formulation. Eqns. (3.1) through (3.3) are integrated over a small finite volume, $\mathcal{V}$. This integration is shown here for the mass and momentum conservation equations:

$$\int_{\mathcal{V}} \frac{\partial(\rho)}{\partial t} d\mathcal{V} + \int_{\mathcal{V}} \frac{\partial(\rho U_j)}{\partial x_j} d\mathcal{V} = 0 \quad (3.4)$$

$$\int_{\mathcal{V}} \frac{\partial(\rho U_i)}{\partial t} d\mathcal{V} + \int_{\mathcal{V}} \frac{\partial(\rho U_i U_j)}{\partial x_j} d\mathcal{V} = - \int_{\mathcal{V}} \frac{\partial P}{\partial x_i} d\mathcal{V} + \int_{\mathcal{V}} \frac{\partial}{\partial x_j} \left(\mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right) d\mathcal{V} \quad (3.5)$$

Volume integrals with spatial gradients are converted into surface integrals over the surface area, S , surrounding the finite volume using Gauss' divergence theorem:

$$\frac{\partial}{\partial t} \int_{\mathcal{V}} \rho d\mathcal{V} + \int_S \rho U_j dn_j = 0 \quad (3.6)$$

$$\frac{\partial}{\partial t} \int_{\mathcal{V}} \rho U_i d\mathcal{V} + \int_S \rho U_i U_j dn_j = - \int_S P dn_j + \int_S \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) dn_j \quad (3.7)$$

where n_j is the j^{th} Cartesian spatial component of the unit vector normal to the local surface and pointing outward from the volume in question. An example of the finite volumes used in ANSYS CFX[®] for 2D computational domains is illustrated in Figure 3.1.

Figure 3.1: Construction of a finite volume in CFX based on division of 2D space into elements. Adapted from ANSYS (2012).

The surface integrals in Eqns. (3.6) and (3.7) represent the summation of the mass and momentum fluxes across the boundaries surrounding the finite volume, which are represented as dashed lines in the figure. The surface integrals are discretized at integration points, denoted by ip , located on the boundaries of the finite volume, as shown in Figure 3.2.

Figure 3.2: An element containing integration points at the boundaries of the finite volume (ANSYS, 2012).

The surface integrals in Eqn. (3.6 and 3.7) are thus quantified by summing these mass and momentum fluxes over all of the integration points surrounding the finite volume:

$$\Delta V \left(\frac{\Delta(\rho)}{\Delta t} \right) + \sum_{ip} \dot{m}_{ip} = 0 \quad (3.8)$$

$$\Delta V \left(\frac{\Delta(\rho U_i)}{\Delta t} \right) + \sum_{ip} \dot{m}_{ip} (U_i)_{ip} = - \sum_{ip} (P \Delta n_j)_{ip} + \sum_{ip} \left(\mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \Delta n_j \right)_{ip} \quad (3.9)$$

where, ΔV corresponds to the value of the finite volume; Δn_j corresponds to the j^{th} component of the outward-pointing unit surface-normal vector at the ip^{th} integration point; and Δt corresponds to the time increment. The mass flux terms in Eqns. (3.8) and (3.9), corresponding to $\dot{m}_{ip} = (\rho U_j \Delta n_j)_{ip}$, are treated as coefficients when solving Eqn. (3.9) during the outer iteration loops discussed in Section 3.2.2. The mass-flux coefficients are estimated from a previous solution iteration or initial condition, and are iteratively solved within each outer iteration loop. Discretization and solution of the mass-flux terms uses the momentum-weighted interpolation scheme described by Majumdar (1988), which is fourth-order accurate and ensures adequate coupling of pressure and velocity.

3.2.1 Discretization of the advection, diffusion and unsteady terms

ANSYS CFX[®] uses a collocated grid technique, in which the solution variables are stored at the nodes, and the values and gradients at the integration points are interpolated from these values. This interpolation is accomplished by using tri-linear finite-element shape functions. More information regarding this interpolation method is found in the ANSYS CFX[®] solver theory guide (ANSYS, 2012). The advection term is evaluated at the integration points by estimating the value of a generic variable, ϕ_{ip} , from the value at an upstream node, ϕ_{up} , using the relation:

$$\phi_{ip} = \phi_{up} + \beta \frac{\partial \phi}{\partial x_i} \Delta x_i \quad (3.10)$$

where β is a blending factor that determines the spatial discretization scheme, and Δx_i is the i^{th} component of the displacement vector of the integration point relative to the upstream node. The i^{th} component of the gradient of the generic variable, $\partial \phi / \partial x_i$, is effectively second-order-accurate and is centered in space about the integration points. The value of β has a range between 0 and 1. A value of 0 results in the first-order accurate upwind discretization scheme, and a value of 1 results in the second-order-accurate central difference scheme. For all simulations conducted in the present study, $\beta = 1$. Further information regarding interpolation using shape functions is found in ANSYS (2012).

The diffusion terms are also treated using tri-linear shape functions. The spatial derivative of a generic variable is evaluated at an integration point using the relation:

$$\left. \frac{\partial \phi}{\partial x_i} \right|_{ip} = \sum_n \left. \frac{\partial N_n}{\partial x_i} \right|_{ip} \quad (3.11)$$

where, $\partial N_n / \partial x_i$ represents the derivative of the n^{th} shape function, N_n , with respect to the i^{th} Cartesian spatial direction. Summation over all the shape functions for an element yields the spatial derivative of ϕ at the integration point. The shape function derivatives are evaluated using the mapping procedure below:

$$\begin{bmatrix} \frac{\partial N_n}{\partial x} \\ \frac{\partial N_n}{\partial y} \\ \frac{\partial N_n}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial N_n}{\partial s} \\ \frac{\partial N_n}{\partial t} \\ \frac{\partial N_n}{\partial u} \end{bmatrix} \quad (3.12)$$

where, s , t , and u denote the local coordinate axes defined for the element.

The unsteady term in Eqn. (3.13) is discretized using the explicit second-order-accurate backwards Euler scheme:

$$\frac{\partial}{\partial t} \left(\int_V \rho \phi dV \right) \approx \Delta V \left(\frac{\Delta \rho \phi_i}{\Delta t} \right) = \Delta V \frac{1}{\Delta t} \left(\frac{3}{2} (\rho \phi_i) - 2 (\rho \phi_i)^{n-1} + \frac{1}{2} (\rho \phi_i)^{n-2} \right) \quad (3.13)$$

Discretization of the temporal gradient makes use of solution variables at two previous times, denoted by superscripts, $n - 1$ and $n - 2$, respectively.

3.2.2 The coupled solver

The above finite-volume discretization method is applied to all elements in the computational domain, which leads to the set of linearized equations summarized in Eqns. (3.14) through (3.16):

$$\sum_{nb,i} a_i^{nb} \phi_i^{nb} = \sum_{nb,i} b_i \quad (3.14)$$

$$a_i^{nb} = \begin{bmatrix} a_{puu} & a_{uv} & a_{uw} & a_{uw} \\ a_{vu} & a_{vv} & a_{vw} & a_{vw} \\ a_{vwu} & a_{wv} & a_{ww} & a_{ww} \\ a_{wpu} & a_{pv} & a_{pw} & a_{pw} \end{bmatrix}_i^{nb} \quad (3.15)$$

$$\phi_i^{nb} = \begin{bmatrix} p_i \\ u_i \\ v_i \\ w_i \end{bmatrix}_i^{nb}, \text{ and } b_i^{nb} = \begin{bmatrix} b_p \\ b_u \\ b_v \\ b_w \end{bmatrix}_i^{nb} \quad (3.16)$$

The coefficient matrix a_i^{nb} represents the relationship between the value of the solution vector, ϕ_i , at the i^{th} node to its neighbors, the nb^{th} nodes. The vector b_i on the right-hand side of the equation contains information regarding the boundary conditions and source terms. The above system of equations incorporates the spatially implicit discretization of the coupled Navier-Stokes equations. The linearized system of equations is rewritten in a general form using the coefficient matrix, $[A]$:

$$[A][\phi] = [b] \quad (3.17)$$

A correction term, $[\phi]'$, is added to a previous or initial solution, $[\phi]^n$, to obtain a corrected solution, $[\phi]^{n+1}$:

$$[\phi]^{n+1} = [\phi]^n + [\phi]' \quad (3.18)$$

Substitution of Eqn. (3.18) into Eqn. (3.17), yields:

$$[A] [\phi]' = [b] - [A][\phi]^n \quad (3.19)$$

The right-hand side of Eqn. (3.19) is equal to the error or residual, r^n , of the current solution iteration. Substituting r^n , Eqn. (3.19) becomes:

$$[A] [\phi]' = r^n \quad (3.20)$$

Eqn. (3.20) is solved in ANSYS CFX[®] using an incomplete lower upper (ILU) factorization method to obtain the correction term, $[\phi]'$, which is then used to obtain $[\phi]^{n+1}$. The solution is iteratively improved until r^n diminishes to a sufficiently small level. The residual is normalized at the i^{th} node by ANSYS CFX[®] using the relationship:

$$\tilde{r}_i^n = \frac{r_i^n}{\eta_i \Delta \phi} \quad (3.21)$$

where the coefficient η_i in Eqn. (3.21) represents a scaling quantity for the relative value of the finite volume at the node, and $\Delta \phi$ is represents a measure of the variation of ϕ over all elements.

The solution is also obtained on spatial grids with varying degrees of coarseness to accelerate the rate of convergence. ANSYS CFX[®] uses an algebraic multi-grid scheme (Raw, 1996), wherein adjacent groups of finite volumes on a computational grid are summed to form larger finite volumes and a coarser grid. The linearized system of equations corresponding to the refined grid is similarly summed over several adjacent elements,

yielding the discretized equations for the coarse grid. The exact procedure of coarsening and refining steps follows the additive correction technique of Hutchison and Raithby (1986). Additional information regarding this technique is available from the ANSYS CFX[®] solver theory guide (ANSYS, 2012).

3.2.3 Computing platform

The relatively large number of nodes of the simulations was managed by distributing the computational load over five four-core Intel[®] L5410 Xeon[™] processors and by making use of parallel computing techniques. The spatial grid was partitioned into several segments using the MeTiS partitioning method (Karypis and Kumar, 1995), each of which was solved on one processor core.

3.3 Computational domain

3.3.1 Computational domain and boundary conditions

The computational domain is illustrated in Figure 3.3. The domain incorporates three rectangular prism shaped regions: a region that contains a no-slip splitter plate, a region upstream from the splitter-plate leading edge, and a mixing region downstream from the splitter-plate trailing edge. The computational domain is further divided across the horizontal (xz) plane that bisects the domain ($y = 0$), for a total of six subregions.

The dimensions of the computational domain are indicated in the figure. The streamwise length of the splitter plate was established in conjunction with the inflow velocities to satisfy two conditions: 1) that viscous instability develop in the boundary layer on the high-speed side of the splitter plate, and 2) that the velocity-difference ratio, R , be similar to that observed in a laminar separation bubble (McAuliffe and Yaras, 2007). The height of the domain was sized such that the ceiling boundaries would be sufficiently

distanced from the free-shear layer to not affect the KH-billow shedding frequency, as specified by John (2012). The width of the domain was sized to be eight times greater than the anticipated most-amplified spanwise wavelength of the hyperbolic instability mode.

Figure 3.3: Schematic of the computational domain

The splitter plate was created by setting the bisecting xz plane between the upper and lower subregions in the splitter-plate region to be a no-slip boundary. The spanwise-wavy shape of the trailing edge was created by displacing the no-slip boundary in the y direction in a sinusoidal manner. The waviness of the splitter-plate trailing edge is characterized by its spanwise wavelength, Λ_z , and amplitude, h . h increases linearly with streamwise distance

from an initial value of zero, such that the peaks and troughs of the waviness each have an azimuthal angle of $\frac{1}{2}\alpha_x$ relative to the splitter plate.

Spatially-uniform and time-invariant streamwise (x) components of velocity of 11.7 m/s and 2.7 m/s were specified at the inflow boundaries on the lower and upper side of the splitter plate, respectively. The upper and lower (xz) boundaries were set to be free-slip surfaces, on which the shear stress was specified to be zero. The spanwise (xy) boundaries were set to be translationally periodic, which, sets primitive (non-gradient) variables at each xy location to be equal on both of the spanwise boundaries. The subregions are connected by a general grid interface (GGI), which conserves primitive variables across the interface. Zero averaged relative static pressure was specified at the outlet boundary.

3.3.2 Spatial grid

A structured grid consisting of hexahedral elements was mapped onto each subregion. A one-to-one node correspondence was ensured on either side of the mating interfaces between each subregion. The total grid is defined by 550, 182, and 138 nodes in the x, y, and z directions respectively. The variation of grid-node spacing in the x, y, and z directions is shown in wall units (denoted by a + superscript) in Figure 3.4. The values used for this normalization are evaluated at the splitter-plate trailing edge.

Grid-node spacing was refined in the region of the boundary layer and free-shear layer where 2D and 3D instabilities were expected to develop. Correspondingly, nodes were closely spaced in the streamwise direction between $x/L = -0.333$ and $x/L = 0$, which corresponds to the region where $Re_\theta > 160$ on the high-speed side of the splitter-plate, indicating viscous instability. Grid-node spacing in the streamwise direction was further decreased near the splitter-plate trailing edge (at $x = 0$), in order to adequately resolve the

interaction of 3D instabilities prevailing in the boundary layer on the high-speed side of the splitter plate with the free-shear layer downstream from the trailing edge.

Figure 3.4: Variation of grid-node spacing in wall-units in the a) x direction, b) y direction, and c) z direction.

Downstream from the trailing edge, nodes were uniformly spaced in the streamwise direction up to $x = 7 \lambda_{KH}$, where, based on published studies, the free-shear layer was expected to have broken down into smaller-scales. Grid-node spacing was slowly coarsened downstream from this location in order to increase the distance of the outflow boundary from the region of interest, minimizing any artificial effect on the development of 3D instability.

In the spanwise direction, 100 nodes were uniformly distributed in the center of the domain within a spanwise length equal to four spanwise wavelengths of the anticipated most amplified hyperbolic instability mode. Spanwise grid-node spacing was gradually increased on either side of this region up to the side boundaries of the computational domain.

The y distribution of wall-normal grid-node spacing is symmetric about the bisecting xz plane at $y = 0$. This distribution was varied with streamwise distance downstream from the trailing edge to accommodate the spatial development of the free-shear layer. The local distribution of wall-normal grid-node spacing was linearly interpolated between the distributions at $x = 0$ and $x = 7 \lambda_{KH}$, shown in Figure 3.4b. This streamwise variation of wall-normal grid-node spacing ensured that upstream from the trailing edge, at least 30 nodes were contained within $y = 2.6 \theta$, where θ is the local momentum thickness. Downstream from the trailing edge 60 nodes were contained within $y = \pm 2.6 \theta$ in the free-shear layer. In regions of non-uniform grid-node spacing, spacing was not permitted to increase by more than 10% from one pair of nodes to the next in any spatial direction.

Table 3.1 compares the grid-node spacing of the refined region of interest used in the present simulations with those used in published DNS studies.

Table 3.1: Comparison of grid-node spacing of present and published DNS studies of free-shear layers

Study	Δx^+	Δy_{wall}^+	Δz^+
Present study (in refined region)	11	0.50	4.8
McAuliffe and Yaras (2007)	24	0.50	24
Na and Moin (1998)	18	0.11	24
Alam and Sandham (2000)	14	0.50	11
Caulfield and Peltier (2000)	6.68	10 (uniform in y)	9.5

The first three studies referred to in Table 3.1 are DNS studies of separation bubbles, and the last is a DNS study of a temporally evolving free-shear layer. The grid-node spacing used in the present simulations is generally as, or more refined than in the DNS studies listed in Table 3.1.

Lastly, the time increment used for the present simulations was set to $\Delta t = 1.5(10^{-6})$ s, which was determined to be adequate to resolve the development of 3D secondary instability based on the simulations performed by McAuliffe and Yaras (2007).

3.3.3 Grid convergence analysis

The 2D and 3D DNS studies of John (2012) were used to establish the streamwise and wall-normal distributions of grid-node spacing in the refined region of the present simulations. Spanwise grid-node spacing was iteratively decreased until the spatially- and temporally-averaged RMS value of the streamwise component of perturbation velocity at the splitter-plate trailing edge was grid independent.

3.3.4 Simulation initialization, convergence criteria and duration

The computational domains on the high- and low-speed sides of the splitter plate were uniformly initialized using the velocities specified at their respective inflow boundaries. Pressure and temperature were initialized uniformly throughout the computational domain at standard atmospheric pressure and temperature.

The simulations were advanced in time using a constant time increment, until certain time-varying parameters were considered statistically steady. During each time increment, the solution was converged such that the global maximum value of the normalized residual, $\tilde{r}_i^n$, was set to be 0.005, resulting in RMS residuals on the order of 10^{-7} indicating adequate convergence.

Whether the simulations were statistically steady was determined by monitoring the pressure and wall-normal component of vorticity at three points in space with spatial coordinates of (0,0,0), (0, 0.16 L, 0), (0, 0.33 L, 0) and (0, 0.5 L, 0). Several dominant fundamental frequencies were observed in the temporal variation of vorticity at these

monitoring points, with periods ranging from $1000\Delta t$ to $4\Delta t$. Statistical steadiness was considered to be achieved when the power spectral density at the frequencies corresponding to time-periods of $10 t_{KH}$ ($640 \Delta t$) and, t_{KH} ($64 \Delta t$), varied by less than 5% of their mean value in time. Statistical convergence was achieved following a period of $4000\Delta t$ after commencement of the simulations with the stated initial conditions. After this time, data was extracted every time step over a period of $1000\Delta t$.

4 Chapter: Results and Discussion

This chapter presents and discusses the results of the DNS studies performed. The chapter begins with a brief overview of the test cases performed and a description of the data reduction procedures in Section 4.1. Section 4.2 discusses the development of the free-shear layer downstream from a flat no-slip splitter plate. Section 4.3 discusses the development of a free-shear layer downstream from a flat no-slip splitter plate with a spanwise-wavy trailing edge with emphasis on the manner in which streamwise-oriented vortical structures result in the breakdown of the free-shear layer into smaller-scale structures. Lastly, Section 4.4 compares the temporal growth of 2D and 3D perturbations among all cases in the context of the parameters of the waviness of the trailing edge.

4.1 Test matrix and data reduction procedures

4.1.1 Overview of test cases

The planar free-shear layer is forced by introducing spanwise periodic waviness to the splitter-plate trailing edge, which reorients the spanwise vorticity developed in the boundary layers on either side of the splitter plate into the streamwise and wall-normal directions. The free-shear layer is forced in this manner to determine how the spanwise wavelength, Λ_z , and the wall-normal (y-direction) amplitude, h , of the waviness affect the development of a free-shear layer. The parameters characterizing the sinusoidal waviness were defined in Figure 3.3.

Along with an unforced case in which the splitter plate is flat, three forced cases were undertaken in which Λ_z and h are varied. Table 4.1 summarizes these test cases. Re_θ at the splitter-plate trailing edge, also given in Table 4.1, is sufficiently high to allow viscous instability to develop in the boundary layer on the high-speed side of the splitter plate. As

previously, the superscript $^+$ denotes normalization by wall variables, and R denotes the velocity difference ratio. In this case, $\Lambda_z^+ = \frac{\Lambda_z u^*}{\nu}$, where ν is the kinematic viscosity, and u^* is the friction velocity computed using shear stress at the wall, τ_w , at the trailing edge on the high-speed side of the splitter-plate.

Table 4.1: Summary of test cases and parameters of waviness of splitter-plate trailing edge varied

Case	h/θ	Λ_z^+	Γ_0	α_z	α_x	$Re_{\theta_{x=0}}$	R
0 (unforced)	0	-	-	-	0°	327	1.88
1 (forced)	2.6	75	0.90	76°	8°	327	1.88
2 (forced)	1.5	100	0.56	55°	8°	327	1.88
3 (forced)	2.6	100	0.89	68°	8°	327	1.88

The circulation, Γ_0 , indicating the magnitude of the streamwise-component of perturbation vorticity, ω_x , introduced into the free-shear layer is primarily affected by Λ_z^+ and h . Γ_0 was calculated by integrating ω_x in a yz plane at the trailing edge between the upper and lower free-slip boundaries, and between consecutive peaks and troughs of trailing-edge waviness. The values of Γ_0 are averaged over the spanwise extent of the domain, and over a time period of $7t_{KH}$. The wall-normal amplitude of the sinusoidal trailing-edge waviness in Case 1 is equal to a multiple of θ at the splitter-plate trailing edge in the unforced case (Case 0). The spanwise wavelength of the trailing-edge waviness in Case 1, $\Lambda_z^+=75$, is between the spanwise spacing of the most amplified streaky structure observed in the unforced case, $\lambda_z^+ = 100$, and its subharmonic. This case is intended to force the free-shear layer at a ‘detuned’ mode from the fastest-growing 3D instability mode. Case 3 has similar h and Γ_0 values to Case 1, indicating similar initial strength of forcing, but in which Λ_z^+ is equal to $\lambda_z^+ = 100$. In Case 2, h is decreased from that in Cases 1 and 3, but Λ_z^+ is equal to that in Case 3.

4.1.2 Data processing

To facilitate observation of the flow physics on the basis of the computed results, the computational data are post-processed to yield the perturbation components of velocity and vorticity, which are deemed to be most effective in highlighting certain flow features. The perturbation velocity vector, $\mathbf{u}$, is computed through the following steps:

$$\bar{\mathbf{U}}(y, t) = \langle \mathbf{U} \rangle_{xz} \quad (4.1)$$

$$\mathbf{U}_{KH}(x, y, t) = \langle \mathbf{U} - \bar{\mathbf{U}} \rangle_z \quad (4.2)$$

$$\bar{\mathbf{U}}'(x, y, t) = \bar{\mathbf{U}} + \mathbf{U}_{KH} \quad (4.3)$$

$$\mathbf{u}(x, y, z, t) = \mathbf{U} - \mathbf{U}_{KH} - \bar{\mathbf{U}} \quad (4.4)$$

In Eqn. (4.1), spatial averaging of the instantaneous velocity vector, $\mathbf{U}$, in the x and z directions, denoted by $\langle \mathbf{U} \rangle_{xz}$, yields the "background" velocity vector, $\bar{\mathbf{U}}$. In Eqn. (4.2), spanwise averaging of $\mathbf{U} - \bar{\mathbf{U}}$, denoted by $\langle \mathbf{U} - \bar{\mathbf{U}} \rangle_z$, yields the perturbation velocity vector, $\mathbf{U}_{KH}$, that is related to two-dimensional flow features driven by the Kelvin-Helmholtz (KH) instability mechanism. The summation of the background velocity vector with the KH perturbation velocity vector in Eqn. (4.3) yields the total 2D background velocity vector, $\bar{\mathbf{U}}'$. In Eqn. (4.4), subtracting the background velocity vector ($\bar{\mathbf{U}}$) and the KH perturbation velocity vector ($\mathbf{U}_{KH}$) from the local instantaneous velocity vector ($\mathbf{U}$) yields the perturbation velocity vector, $\mathbf{u}$, that accounts for all 3D perturbations of the velocity-field. Taking the curl of the velocity vectors $\mathbf{U}$, $\bar{\mathbf{U}}$, $\mathbf{U}_{KH}$, $\bar{\mathbf{U}}'$ and $\mathbf{u}$, respectively produces the vorticity vectors $\boldsymbol{\Omega}_0$, $\bar{\boldsymbol{\Omega}}$, $\boldsymbol{\Omega}_{KH}$, $\bar{\boldsymbol{\Omega}}'$ and $\boldsymbol{\omega}$. Furthermore, ∇^2 is the Laplace operator; and, subscripts x , y and z denote components of the parameter in question in the respective Cartesian coordinate direction. Finally, unless otherwise specified, nondimensionalization of all flow parameters is based on

the momentum thickness of the wake at the splitter-plate trailing edge, θ , and freestream velocity difference ($U_2 - U_1$). θ is computed as:

$$\theta = \int_{-\infty}^{+\infty} \frac{U(y) - U_1}{U_2 - U_1} \left(1 - \frac{U(y) - U_1}{U_2 - U_1} \right) dy \quad (4.5)$$

where $U(y)$ is the streamwise component of instantaneous velocity, and U_2 and U_1 are the freestream velocities on the high- and low-speed sides of the splitter plate, respectively.

4.2 Analysis of unforced case

4.2.1 Rollup of the free-shear layer into KH billows

The sequence of events resulting in the rolled up shear layer forming discrete Kelvin-Helmholtz (KH) billows is visualized in Figure 4.1. Isosurfaces of spanwise component of vorticity in the figure illustrate the rollup of the free-shear layer into discrete two-dimensional KH billows, which are connected by thin braids of spanwise-oriented vorticity. The isosurface value of $\Omega_z = 1$ was arbitrarily selected and matches the value of vorticity within the boundary layer at the wall-normal distance equal to 2.6θ on the high-speed side of the splitter-plate trailing edge. The 2D KH instability may be regarded to grow either spatially or temporally as the shear layer exhibits a high degree of temporal periodicity over the time, t_{KH} , it takes a KH billow to traverse the distance between two successive KH billows, λ_{KH} . The datums at $x/\lambda_{KH} = 0$ and $t/t_{KH} = 0$ indicate the streamwise location of the splitter-plate trailing edge, and the time at which a point travelling with the convective velocity of the KH billows and subsequently coincides with the centroid of an arbitrary KH billow is located at $x/\lambda_{KH} = 0$. Spatial distance in the wall-normal and spanwise directions is expressed in wall units with reference to friction velocity calculated on the high-speed side of the splitter-plate trailing edge in the unforced case, while streamwise distance in all cases is normalized by λ_{KH} .

Subsequent normalization denoted by “+” indicates normalization by wall-variables calculated as such in the unforced case.

The time $t/t_{KH} = 0.260$ corresponds to an instance in time just after the linear stage of instability development. Deviation from linear development occurs shortly downstream from the trailing edge, at about $x/\lambda_{KH} = 0.15$, and is characterized by sinusoidal wall-normal perturbation of the shear layer in time. This development may be interpreted as the commencement of the roll-up phase and is characterized by the local aggregation of spanwise-oriented vorticity in the streamwise direction. By $t/t_{KH} = 0.500$, the aggregated spanwise-oriented vorticity forms coherent KH-billows. Velocities induced by the KH billow result in the gradual convection of spanwise-oriented vorticity from the braid region into the KH-billow core between $t/t_{KH} = 0.500$ and 0.750 . Concentration of spanwise-oriented vorticity in the KH-billow core continues until at $t/t_{KH} = 0.750$ the spatially averaged kinetic energy in the KH-billow core reaches local maximum in time, henceforth referred to as kinetic energy saturation. In Figure 4.1 linear perturbation of the shear layer upstream from the newly created KH billow has commenced at $t/t_{kh} = 1$. After $t/t_{KH} = 1$, the spatially-averaged kinetic energy and enstrophy in the rolled up KH billow stop increasing, signifying the end of the development phase of the KH billow. The temporal trends observed for spanwise-averaged kinetic energy and enstrophy further downstream are discussed in Section 4.4.

The time-averaged Strouhal number for the present case is $St = 0.0365$, wherein KH billow shedding frequency is normalized by the average of the freestream velocities on the high- and low-speed sides of the splitter plate, and the momentum thickness, calculated at the splitter-plate trailing edge. This Strouhal number value compares well with experimentally measured values of $St = 0.032$, published by Michalke and Herman (1982).

Figure 4.1: Isosurfaces of spanwise component of instantaneous vorticity, $\Omega_z = -1$, at different instances in time over one KH billow shedding period. Solid-black arrows at $t/t_{KH} = 0.260$ schematically illustrate the profile of streamwise component of velocity.

4.2.2 Development of 3D instability of the free-shear layer

The evolution of the free-shear layer upstream of $x/\lambda_{KH} = 6$ is dominated by the 2D KH flow-field, as upstream of this location the spatially-averaged 3D perturbation kinetic energy in the shear layer is six orders of magnitude smaller than the spatially-averaged kinetic energy in the 2D KH flow-field. Caulfield and Peltier (2000) observed that

development of 3D instability commences at the time of kinetic energy saturation in the core of the KH billows. In the present results, this saturation is first reached at $x/\lambda_{KH} = 0.75$. However, three-dimensionality of the flow-field is observed far upstream from the splitter-plate trailing edge. This is due to a key difference between the present simulations and other free-shear layer DNS studies that utilize streamwise periodic boundary conditions, such as those performed by Clarkson and McMurtry (1990) and Caulfield and Peltier (2000, 2002), in that the free-shear layer in the present case is the downstream extension of boundary layers formed over the no-slip surface of the splitter plate. As could be expected, flow features that develop upstream of the splitter-plate trailing edge due to flow instability are convected into the free-shear layer.

There is evidence of viscous instability development in the boundary layer on the high-speed side of the splitter plate, leading to the formation of Tollmien-Schlichting (TS) waves and the accompanying spanwise-periodic variations in the streamwise velocity component. This spanwise variation in the streamwise velocity component of the boundary layer may be observed through the distribution of the wall-normal vorticity component, ω_y , or through the power spectral density (PSD) of the spanwise distribution of streamwise component of perturbation velocity, u . Figure 4.2 shows several of the latter distributions sampled along spanwise lines that intersect $y^+ = 0$ and $x/\lambda_{KH} = 0.00, 1.00, 2.00$ and 3.00 . The sampling locations are selected to coincide with the midpoints between consecutive KH billows at $t/t_{KH} = 0.50$, and were illustrated by dashed lines in Figure 4.1.

While the initially dominant streaky structure in the spanwise distribution of u with a spanwise spacing of $\lambda_z^+ = 200 \pm 10$ continues to be visible in the braid region as far downstream as $x/\lambda_{KH} = 7$, another spanwise-periodic flow phenomenon at a dominant wavelength of $\lambda_z^+ = 90 \pm 10$ is observed to develop at a greater growth rate commencing at

$x/\lambda_{KH} = 1.5$, dominating the PSD of the spanwise distribution of u by $x/\lambda_{KH} = 3$. The spanwise wavelength of the most amplified velocity-streak structure in the boundary layer was observed to be $\lambda_z^+ \approx 100$ in DNS studies performed by Yu and Kawaguchi (2003) and Kim et al (1987); and observed experimentally by Smith and Schwartz (1983).

Figure 4.2: Streamwise variation of the power spectral density of spanwise distributions of streamwise component of perturbation velocity, u .

The presence of the streaky structure with a larger spanwise spacing of $\lambda_z^+ = 200 \pm 10$ in the boundary layer on the high-speed side of the splitter plate is due to the by-product of a forcing effect in the DNS simulation affected by the increase of grid-node spacing towards the spanwise boundaries of the computational domain (depicted in Figure 4.1). The increase in spanwise grid-node spacing has the effect of locally increasing truncation error in a time-averaged sense, which is manifested as a corresponding increase in numerical dissipation. This coarsening of spanwise grid-node spacing in a spanwise periodic manner has the effect of imposing a spanwise periodic modulation in the effective viscosity distribution, and thus causing kinetic energy production via viscous instability to likewise vary periodically in the spanwise direction, with an apparent wavelength of $\lambda_z^+ = 200$. This notion

was verified by performing a DNS of a flat plate boundary layer with streamwise and wall-normal grid-node spacing and boundary conditions identical to those imposed on the computational domain on the high-speed side of the splitter plate, but for which the spanwise grid-node spacing was uniformly distributed. In this validation case, there was a notable absence of a spanwise-periodic perturbation in u with spanwise spacing of $\lambda_z^+ = 200$ at the splitter-plate trailing edge amidst the presence of each of the other spanwise periodic perturbations in u which were observed in the present case. Despite the unintended presence of numerically-driven streaky structures with spanwise spacing of $\lambda_z^+ = 200$, the downstream shear layer development is observed to be dominated by the physical streaky structures of $\lambda_z^+ = 100$ spacing, as the discussion below illustrates.

Further light is shed on the interaction of these spanwise periodic perturbations with the downstream free-shear layer in Figure 4.3 through isosurfaces of the y -component of perturbation vorticity, ω_y . Streaky structures developing in the xy plane with a consistent spacing of $\lambda_z^+ = 90 \pm 10$ are noted to dominate the flow field. These flow structures are consistent with the flow development attributed to the hyperbolic instability mode existing in the braid regions as described in Section 2.2. This flow-development in the braid region consists of 's'-shaped vortices oriented in the xy plane and centered at the hyperbolic stagnation point with a dominant spanwise spacing of $\lambda_z^+ = 93$. These streamwise-oriented vortical structures in the braid region are henceforth called 'primary' braid vortices. The ω_y distribution in Figure 4.3 is consistent with the presence of such streamwise-oriented braid vortices. However, the fact that the spanwise distribution in ω_y remains consistent as far upstream as the splitter trailing edge, i.e. well upstream of the flow region where braid regions with their local stability characteristics develop between KH billows, suggests that the three-dimensionality of the free-shear layer is affected by the streaky structures in the

upstream boundary layer as well as by the instabilities in the braid regions of the free-shear layer flow field.

Figure 4.3: Isosurfaces of ω_y highlighting streaky structures in the free-shear layer.

The variation of the perturbation enstrophy transport terms corresponding to the mechanisms of stretching, reorientation, convection and, viscous diffusion and dissipation are plotted along the center-line axis of a primary braid vortex at $t/t_{KH} = 2.25$. Smyth and Peltier (1994) show that the growth of vortical disturbances in the braid region reflects a complex combination of mechanisms: stretching of a vorticity perturbation by background strain, reorientation of background vorticity by perturbation shear, and tilting of the

perturbation by the background shear. The interaction of these mechanisms is presently studied through the perturbation enstrophy, which is defined as one-half of the square of the perturbation vorticity magnitude ($\frac{1}{2}\omega^2$). The perturbation enstrophy transport equation is presented in Eqn. (4.6). The derivation of this equation is presented in Appendix A.1.

$$\begin{aligned}
 & \text{(1), (2), (3): Effect of stretching by background strain in xy plane and in z-direction} \quad \text{(4): Effect of tilting by background shear in the xy plane} \\
 & \frac{\partial}{\partial t} \left(\frac{1}{2} \omega^2 \right) = \left(\omega_x^2 \left(\frac{\partial \bar{u}'}{\partial x} \right) + \omega_y^2 \left(\frac{\partial \bar{v}'}{\partial y} \right) + \omega_z \Omega_z \left(\frac{\partial w}{\partial z} \right) \right) + \omega_x \omega_y \left(\frac{\partial \bar{u}'}{\partial y} + \frac{\partial \bar{v}'}{\partial x} \right) \\
 & + \omega_x \Omega_z \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) + \omega_y \Omega_z \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) - \left(U \frac{\partial}{\partial x} \left(\frac{\omega^2}{2} \right) + V \frac{\partial}{\partial y} \left(\frac{\omega^2}{2} \right) + w \frac{\partial}{\partial z} \left(\frac{\omega^2}{2} \right) \right) + \frac{1}{Re_\theta} (\omega) \cdot \nabla^2 (\omega) \\
 & \text{(5), (6): Effect of reorientation of background z-vorticity by perturbation shear in the xz and yz planes} \quad \text{(7), (8), (9): Total instantaneous convective transport of perturbation enstrophy} \quad \text{(10) Sum of viscous dissipation and diffusion of perturbation enstrophy}
 \end{aligned} \tag{4.6}$$

The first three terms on the right-hand side of Eqn. (4.6) are related to the mechanism of stretching; the fourth term is related to the mechanism of tilting of the perturbation vorticity by background shear; the fifth and sixth terms account for reorientation of spanwise-component of background vorticity by the shear produced by the perturbation flow-field; the seventh, eighth and ninth terms account for the total instantaneous convection of perturbation enstrophy; and the tenth term represents the sum of viscous diffusion and dissipation of enstrophy. A positive value for the sum of these terms reflects perturbation enstrophy aggregation at the corresponding spatial location and vice-versa.

The terms associated with the stretching mechanism are positive if strain rate is positive, such that vorticity is aggregated about any point. This is illustrated in the case of an axisymmetric vortex tube under positive axial strain in an incompressible flow-field. A round (Burgers) vortex under positive axial strain has negative radial strain in a divergence free

flow-field. The radial strain concentrates vorticity towards the center of the vortex core (Lin and Corcos, 1984).

In Eqn. (4.6) the tilting mechanism describes the re-orientation of wall-normal perturbation vorticity component into the streamwise direction by background shear, and conversely the reorientation of the streamwise-oriented perturbation vorticity component into the wall-normal direction in ‘positive feedback’ (Smyth and Peltier, 1994). The tilting mechanism is ubiquitously present at a hyperbolic stagnation point (Sipp and Jacquin, 1999; and Smyth and Peltier, 1994). The reorientation mechanism is similar in principle, but involves reorientation of spanwise vorticity present in the background flow-field in a 3D manner. The tilting and reorientation mechanisms are henceforth simply referred to as reorientation to describe both of these effects.

The contribution of some of these mechanisms to the growth of perturbation enstrophy in the braid region between the KH billows may be observed at different locations in a primary braid vortex, illustrated in Figure 4.4. In Figure 4.4a, the braid vortex is visualized using the spanwise averaged second variant of the perturbation velocity gradient tensor, q . q is frequently used to visualize vortical structures in turbulent flows, and corresponds to $q = \frac{1}{4}(|\boldsymbol{\omega}|^2 - \epsilon^2)$, where the square of the strain rate, ϵ , is equal to the dissipation rate of perturbation kinetic energy (Martin et al., 1998). Enstrophy transport terms in Eqn. (4.6) are averaged in x- and z-directions over a distance of $\Delta y^+ = 10$ and $\Delta z^+ = 10$ centered about the center-line of each braid vortex, as indicated in Figure 4.4a. The streamwise distributions of the transport terms averaged over the seven braid vortices present between $x/\lambda_{KH} = 1.5$ and 2.75 are presented in Figure 4.4b. The summation of the fourth, fifth, and sixth terms in Eqn. (4.6) are labeled as “Reorientation” in the figure; and the summation of terms (1) through (10) in Eqn. (4.6) are likewise labeled as ‘Summation’.

Figure 4.4: Instantaneous variation of perturbation enstrophy transport terms of Eqn. (4.6) along the centerline of a primary braid vortex, at $t/t_{KH} = 2.30$.

At the hyperbolic stagnation point identified in Figure 4.4, strain and shear are both positive. Smyth and Peltier (1994) note that the strain rate that causes stretching is comparatively low at the hyperbolic stagnation point, leading the tilting mechanism to be predominantly responsible for the growth of vortical perturbations there. They likewise observed the stretching terms in Eqn. (4.6) to be positive at locations along the centerline of the braid vortex other than the hyperbolic stagnation point, thus causing positive enstrophy transport at the peripheries of the developing primary braid vortex due to this mechanism. In the present case, these observations are in general confirmed downstream from the location at which kinetic energy has saturated in the KH-billow core, which is reflected in the

distributions of the perturbation enstrophy transport terms observed in Figure 4.4. Instability by the reorientation mechanism accounts for the majority of positive enstrophy transport observed near the hyperbolic stagnation point, which is in effect the ‘source’ of positive enstrophy transport in the braid region. The stretching mechanism contributes very little at the hyperbolic stagnation point, but is the dominant mechanism at the extremities of the primary braid vortex, helping to offset the negative transport caused by the reorientation mechanism on enstrophy at $x/\lambda_{KH} = 2.65$ in Figure 4.4b.

Viscous diffusion and dissipation of perturbation enstrophy scales on the magnitude of the gradient and curvature of perturbation enstrophy, respectively, and have a comparatively small effect along much of the primary braid vortex, as noted in Figure 4.4b. Viscous effects increase with increased vorticity curvature approaching the periphery of the KH-billow core and cause positive perturbation enstrophy transport between $x/\lambda_{KH} = 2.55$ and 2.75, as is evident in Figure 4.4.

Convective transport of perturbation enstrophy, which is dominated by the KH-billow-induced velocity field, redistributes perturbation enstrophy from the hyperbolic stagnation point to the peripheries of the braid vortex, where the mechanism of stretching is in effect.

The mean values of the enstrophy transport terms shown in Figure 4.4b, evaluated along the centerline axis of the primary braid vortex and are: 4.62×10^{-6} , 1.63×10^{-7} , and -2.60×10^{-7} for the sum of the stretching and reorientation, convection, and viscous diffusion and dissipation terms, respectively. This indicates firstly that, stretching and reorientation are the dominant mechanisms responsible for growth in the braid vortices. Secondly, the convective transport of perturbation enstrophy contributes the least of all transport mechanisms to the growth of the braid vortex.

The instability mechanisms thus cause the formation of braid vortices in the free-shear layer about the hyperbolic stagnation points. The spatial orientation and position (topology) of the braid-vortices remains in a steady state in the reference frame travelling with the convective speed of the KH billows, and does not change when the KH billows are in the region between $x/\lambda_{KH} = 1.5$ and $x/\lambda_{KH} = 6$. The vortical topology of the free-shear layer during its steady state is illustrated in Figure 4.5 as isosurfaces of q .

Figure 4.5: Isosurfaces of $q = 0.25$, coloured by ω_x to indicate rotational sense, at $t/t_{KH} = 4.25$.

The isosurfaces are coloured by streamwise-component of perturbation vorticity, ω_x , indicating their rotational sense. The braid vortices develop with a spanwise spacing that corresponds to the streaky structures in u observed in Figure 4.3. At $x/\lambda_{KH} = 3.75$ in Figure 4.5, a primary braid vortex appears to approach, and infringe upon, the upstream adjacent braid region. Though the isosurface shown in Figure 4.5 encloses both the local braid structure and the infringing vorticity, these vortices are not in contact. In the present instance

the braid vortices approach the upstream adjacent braid-region as a consequence of being convected along the elliptical (or hyperbolic) streamlines perturbed by the KH-billow-induced velocity. The implications of this event to smaller-scale breakdown of the free-shear layer are discussed in Section 4.3.

4.3 Analysis of forced cases

4.3.1 Methodology for analysis of time-varying quantities

A general representation of the vortical structures that develop in the forced cases is visualized in Figure 4.6 using isosurfaces of the second invariant of the instantaneous velocity gradient tensor, Q . The development of the free-shear layer in the forced cases encompasses four visible phases numbered 1 through 4 in Figure 4.6. These phases correspond to the development phases summarized in Section 2.2.3. Due to inadequate grid-node resolution of the region downstream from $x/\lambda_{KH} = 7$, the fully turbulent phase is not analyzed in any of the present cases.

As in the unforced case discussed in Section 4.2, all forced cases exhibit a high degree of temporal periodicity of velocity and vorticity over a period of time equal to t_{KH} , until the smaller-scale breakdown of the free-shear layer. Correspondingly the vortical structures in the braid region of the KH billows may be analyzed as if they were developing temporally in a reference frame that is stationary with respect to the KH billows, thereby simplifying the results. The moving rectangular shaded region labelled CV in Figure 4.6 contains a consecutive pair of KH billows, labelled KH_0 and KH_1 , and has spatial dimensions as indicated in the figure.

Figure 4.6: Overview of free-shear layer development in Case 2, visualized as isosurfaces of $Q = 0.01$.

CV translates with constant velocity, U_c , equal to the mean of the convective speeds of KH_0 and KH_1 , and is thus stationary with respect to KH_0 and KH_1 . $t/t_{KH} = 0.0$ indicates the time when the upstream edge of CV coincides with the splitter-plate trailing edge. All variables that are henceforth indicated to be spatially averaged are calculated by performing a spatial average over the volume enclosed by CV at the indicated instance in time.

Of the parameters which define the spanwise waviness of the splitter-plate trailing edge, the circulation of the streamwise-oriented vortices at the trailing edge, Γ_0 , and the spanwise wavelength of trailing-edge waviness, Λ_z , are most characteristic of the resulting dynamics of the shear-layer. Section 4.3.2 discusses the manner in which vorticity is introduced into the free-shear layer by the spanwise-wavy splitter-plate trailing edge.

4.3.2 Generation of 3D vortical structures in the free-shear layer by the spanwise-wavy splitter-plate trailing edge

Figure 4.7 illustrates how the waviness of the splitter-plate trailing edge introduces a streamwise component of vorticity into the free-shear layer. The solid-gray arrows upstream of the trailing edge represent the velocity profile of the undisturbed boundary layer. Initially, the vorticity in the undisturbed boundary layer — represented in Figure 4.7 by the dashed-gray vorticity line labelled T_0 — is oriented primarily in the spanwise direction. As the flow approaches the splitter-plate trailing edge, the fluid aligned with the peaks of the wavy trailing edge is accelerated, causing the vorticity line in that region to be stretched in the downstream direction. The fluid aligned with the troughs is decelerated causing the vorticity line to be stretched in the upstream direction. Such spanwise-periodic stretching of the vorticity line labelled T_0 generates the wavy vorticity line labelled T_1 in Figure 4.7, and correspondingly a streamwise component of vorticity is introduced within the boundary layer. The stretching of T_1 — and hence the production of streamwise component of vorticity — increases until it reaches the splitter-plate trailing edge. Downstream of the trailing edge, the spanwise component of vorticity associated with the splitter plate boundary layer rolls-up into KH billows, and their induced velocity field causes the streamwise vorticity developed upstream to be wrapped around the KH billow, as shown by the vorticity line labelled T_2 . The circulation of this reoriented streamwise component of vorticity at the splitter-plate trailing edge is related to h and α_x (Greitzer et al, 2004), where α_x is half the angle at which the waviness is formed starting from the flat portion of the splitter plate, and was defined in Section 3.3.1.

Figure 4.7: Schematic illustration of generation of streamwise vorticity by the spanwise wavy trailing edge. Velocity vectors (solid) and vorticity lines (dashed) correspond to the velocity and vorticity in the boundary layer (gray) and KH-billow-induced effects (blue).

Downstream from the splitter-plate trailing edge, the developing KH billow induces velocity, V_l , at the trailing edge in the wall-normal direction, indicated by solid-blue arrows in Figure 4.7. The induced wall-normal velocity has a component that is tangential to the wall of the waviness, shown in Plane A, which experiences shear at the wall. The shear imposed on this induced component of velocity results in a velocity profile at the trailing edge in the yz plane – illustrated as solid-blue arrows in Figure 4.7 – which has a streamwise component of vorticity, represented as a dashed-blue vorticity line, labelled S_1 . The magnitude of the streamwise component of vorticity created by this induced effect is related to the component of the wall-normal induced velocity that is parallel to the wall at the trailing edge, $V_l \sin(\alpha_z)$. V_l is related to the circulation contained in the KH-billow core and the distance between its centroid and the trailing edge. Correspondingly, the induced streamwise component of vorticity at the trailing edge varies in time, and is likewise periodic over the time period t_{KH} .

As S_1 is convected downstream from the splitter-plate trailing edge, and a new KH billow is rolled up behind it, peak vorticity magnitude on S_1 occurs at the midpoint between two consecutive KH billows. Furthermore, as S_1 and T_2 are counter-rotating with respect to each other, vorticity cancellation occurs in regions where these structures are in close proximity. The magnitude of streamwise-oriented vorticity component of S_1 varies with V_1 and is greatest in the braid region, such that more of the vorticity of T_2 is cancelled in the braid region. Thus two distinct counter-rotating vortical structures result: the induced streamwise component of vorticity, S_1 , and the reoriented streamwise component of vorticity, T_2 . These structures have peak vorticity in the braid region and on the periphery of the KH billow, respectively. The vortices created by the induced effects are correspondingly henceforth called ‘forced braid vortices’.

Figure 4.8 shows the rollup of the forced free-shear layer into KH billows and the vortical structures created by the two aforementioned forcing effects, as isosurfaces of Q for all cases. Streamwise oriented isosurfaces of Q in the braid region of Case 1 at $t/t_{KH} = 0.95$, correspond to the forced braid vortices, and are labelled accordingly. It is evident that the isosurfaces corresponding to these structures are largest in the braid region in Case 1, smaller in the braid region in Case 3, and are not at all visible in Case 2. This suggests that the relative strength of these vortices in their respective cases, and therefore the degree of forcing in the braid region in these cases, increases with the value of α_z . The reoriented streamwise component of vorticity is observed in each case in Figure 4.8, as rib-like protrusions on the isosurfaces of Q corresponding to KH_0 . Given the relatively small values of Γ_θ of each forced case in comparison to the circulation of the KH-billow core in the unforced case ($\Gamma_{KH} = 19.1$ at the time of kinetic energy saturation), the background and KH velocity fields of the forced cases are initially unchanged by the presence of these ribs compared to the unforced case.

Figure 4.8: Isosurfaces of $Q = 0.01$ illustrating rollup of KH_0 in the CV for all forced cases. The splitter-plate trailing edge is located at $x/\lambda_{KH} = 0.0$.

The 2D elliptical (and hyperbolic) streamline pattern, calculated from $\bar{\mathbf{U}}'$ (calculated as $\bar{\mathbf{U}}' = \langle \mathbf{U} \rangle_{xz} + \langle \mathbf{U} - \bar{\mathbf{U}} \rangle_z$) in a reference frame travelling with CV, indicated for Case 2 in Figure 4.8, is identical for all cases at $t/t_{KH} = 0.95$. These streamlines define the paths along which vorticity is convected by the KH-billow-induced velocity. Likewise, 3D vortical perturbations grow along the principle axis of strain (Ashurst and Meiburg, 1988, Lasheras and Choi, 1988). The curves labelled ‘strain-field lines’ are tangent to the instantaneous spatially-varying principle axis (principle eigenvector) of the strain tensor of the vector $\bar{\mathbf{U}}'$, and indicate the direction of growth of 3D vortical perturbations. The regions where these strain-field lines are converging, at the lower and upper extremities of the KH billows, are labelled as ‘nodal-points’ in the background strain field.

The reoriented vorticity at the periphery of the KH billows (the ‘ribs’) convect around the KH-billow core along the elliptical streamlines. At $t/t_{KH} = 1.40$, the reoriented vorticity that was observed to be initially on the downstream side of the KH billow at $t/t_{KH} = 0.95$ in the figure, reaches the lower nodal point on KH_0 . Between $t/t_{KH} = 0.95$ and 1.85, vorticity which is aligned with the strain-field lines experiences growth. Isosurfaces of Q in Figure 4.8, corresponding to the ‘ribs’ on the KH billows, grow in the downstream direction along the strain-field lines, outwardly from the lower nodal points of strain. It will subsequently be evident that vortical perturbations similarly grow outwardly in the upstream direction from the upper nodal point of each KH billow. This growth into the braid region gives the appearance that vortices are ‘ejected’ from the KH billow: the resulting vortical structures are henceforth referred to as ‘core-ejected’ vortices, and are labelled as such in Figure 4.8. Furthermore, growth of reoriented vorticity located on KH_0 into the braid region is observed in all three forced cases to commence at identical times in Figure 4.8, suggesting that the time

of incidence of the ejection event is primarily dependent on the characteristics of the KH-billow cores, which are initially the same for all cases.

The core-ejected and forced braid vortices do not commence significant growth until these respective structures occupy the braid region, and kinetic energy has saturated in the KH-billow cores. The development of the vortical structures in the braid region, from $t/t_{KH} = 1.85$ until breakdown to smaller-scales in Case 1, Case 2 and Case 3, is discussed in Sections 4.3.3, 4.3.4 and 4.3.5, respectively.

4.3.3 Development of 3D vortical structures and mechanism of breakdown to smaller-scales in Case 1

Two consecutive KH billows, KH_0 and KH_1 , are arbitrarily selected in Case 1 and followed in the CV as they convect downstream, in the manner described in Section 4.3.1. Figure 4.9 shows yz plots in which isocontours of q are plotted as black-coloured contour lines. The spatial distributions of the terms in the perturbation enstrophy transport equation, given in Eqn. (4.6) in Section 4.2.2, are spanwise-averaged and also shown in the yz plots in the figure as blue- and red-coloured flood-plots. The flood-plots in Figure 5a corresponds to the sum of terms 1 through 6 of Eqn. (4.6) representing ‘stretching and reorientation’; and those in Figure 5b correspond to term 10 of Eqn. (4.6) representing the sum of viscous diffusion and dissipation as in Figure 4.4. The isocontours of q and the quantities shown as flood-plots are normalized by the value of $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ corresponding to the relevant instance in time.

Figure 4.9: Development of braid region in Case 1. Black-coloured isocontours indicate $\langle q \rangle_z / \langle \frac{1}{2} \omega^2 \rangle_{xyz}$ at values of 0.01 (dotted), 0.1 (solid-light), and 1.0 (solid-heavy) at $t/t_{KH} = 1.6$, $t/t_{KH} = 2.2$, and $t/t_{KH} = 7.0$, respectively. Flood-plots show perturbation enstrophy transport by mechanisms of a) stretching and reorientation, and b) viscous diffusion and dissipation. $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ used for normalization has the values of 0.0030, 0.0042, and 0.0040 at $t/t_{KH} = 1.6$, 2.2 and 7.0, respectively.

In Figure 4.9a, perturbation enstrophy due to stretching and reorientation is greatest at the hyperbolic stagnation point, and decreases with distance from it. This corresponds to the distribution observed in Section 4.2.2 for the unforced case, and suggests that the forced braid vortex is amplified by the hyperbolic instability that exists in the braid region. Localized regions of negative perturbation enstrophy transport are observed upstream from the lower nodal point of strain on KH_0 and downstream from the upper nodal point of strain on KH_1 . This negative transport may be explained by noting that in Figure 4.8 the strain-field lines upstream from the lower nodal point are perpendicular to the axis of the core-ejected vortex beneath KH_0 . This indicates that the perturbation vorticity vector and principle axis of strain are locally mutually orthogonal in those regions, which results in negative perturbation enstrophy transport via the stretching and reorientation mechanisms (Smyth and Peltier,

1994). Negative perturbation transport due to stretching and reorientation is observed at identical locations along the primary braid vortex in the unforced case, and in the remaining two forced cases.

Enstrophy transport due to viscous diffusion and dissipation are observed in Figure 4.9b to be generally negative along the axes of the vortices in the braid-region, indicating that viscous dissipation (which is negative definite) is more likely to be prevalent than viscous diffusion. This result is consistent with such observations for the unforced case.

At $t/t_{KH} = 1.6$, the forced braid vortex occupies the hyperbolic stagnation point in the braid region, and core-ejected vortices are located along the peripheries of the KH billows; these structures are labelled accordingly in Figure 4.9. The presence of a larger isocontour of $\langle q \rangle_z / \langle \frac{1}{2} \omega^2 \rangle_{xyz} = 1.0$ inside of the forced braid vortex suggests that this vortex is significantly stronger than the core-ejected vortices. By $t/t_{KH} = 2.2$, however, the core-ejected vortex beneath KH_0 has grown into the braid region, and its peak vortical magnitude exceeds that of the forced braid vortex, suggesting that growth rate at the periphery of the KH-billow core is greater than in the braid region. This is an interesting result, as it was expected that the hyperbolic instability mode, centered in the braid region, would be the most amplified 3D instability mode. The stability analysis and DNS of Potylitsin and Peltier (1999, 2003) shows that while the hyperbolic instability is most amplified at $\lambda_z^+ \cong 93$, the most amplified spanwise wave-number of the edge mode at the peripheries of the KH-billow core is near $\lambda_z^+ \cong 58$ (Smyth and Peltier, 1994). To determine whether the centrifugal edge instability mode is indeed present, Rayleigh's discriminant is calculated as per Eqn. (4.7) (Afanasyev and Peltier, 1998).

$$\psi(r) = \frac{1}{r^3} \frac{d}{dr} (r^2 \bar{\Omega}')^2 \quad (4.7)$$

In Eqn. (4.7), r is the radial coordinate in a cylindrical coordinate system with the origin defined at the elliptical stagnation point within KH_0 . It is found that $\psi(r)$ is positive along the elliptical streamlines at the periphery of the KH billow in all cases, indicating centrifugal instability in these regions. This is interesting as Potylitsin and Peltier (1999, 2003) find that the centrifugal edge instability mode exists primarily when the free-shear layer is in a rotating reference frame. The present DNS simulation, however, is computed in a non-rotating stationary reference frame, suggesting that the edge instability mode may also exist in the absence of background rotation. In the present case, the spanwise wavelength of forcing, $\Lambda_z^+ = 75$, is between the most amplified wavelengths of the hyperbolic and edge instability modes. The forced braid vortex, and core-ejected vortices are thus amplified by the hyperbolic and edge instability modes simultaneously. Of these, the growth rate of the edge-mode is greater in the present case. Correspondingly, by $t/t_{KH} = 2.2$ in Figure 4.9a, maximum growth has shifted away from the hyperbolic stagnation point, and towards the peripheries of the KH-billow core.

Vortical structures inside of the KH-billow cores in Figure 4.9 are generally at least one order of magnitude weaker than vortices in the braid region, and appear to grow at a slower rate. These structures do not appear to play a significant role in any forced case.

Both the core-ejected vortices and forced braid vortex are convected by the KH-billow-induced velocities, over KH_1 in the downstream direction, and beneath KH_0 in the upstream direction, as indicated by the black arrows in Figure 4.9b. These vortices traverse the upstream and downstream boundaries of CV, and are convected into the neighboring braid region. The vortices beneath KH_1 and above KH_0 are likewise convected into CV, towards the hyperbolic stagnation point, such that they infringe into the local braid region; this is circled in Figure 4.9a at $t/t_{KH} = 2.2$. Breakdown to smaller-scales in Case 1 is instigated

by the contact of the infringing vortices with the local forced braid vortex. The indicated smaller-scale structures in Figure 4.9a at $t/t_{KH} = 7.0$ appear in the region where the infringement occurred. This indicates that between $t/t_{KH} = 2.2$ and $t/t_{KH} = 7.0$, the forced braid vortex is broken down by its interaction with the infringing vortices. The precise time of commencement of breakdown to smaller-scales in the forced braid vortex is $t/t_{KH} = 3.8$. As the core-ejected vortices have substantially greater vortical magnitude than the forced braid vortex, they act as barriers between the smaller-scale structures and the KH-billow core. Correspondingly the smaller-scale vortical structures observed in Figure 4.9 at $t/t_{KH} = 7.0$ are localized to the region indicated, and the core-ejected vortices and surrounding structures remain coherent.

4.3.4 Development of 3D vortical structures and mechanism of breakdown to smaller scales in Case 2

The development of vortical structures in the braid region for Case 2 is shown in Figure 4.10 at $t/t_{KH} = 1.6, 3.4$, and 4.3 , in an identical manner as was done in Figure 4.9 for Case 1. At $t/t_{KH} = 1.6$, the core-ejected vortices are located at identical spatial locations as in Case 1, whereas the magnitude of the forced braid vortex is less than $\langle q \rangle_z / \langle \frac{1}{2} \omega^2 \rangle_{xyz} = 0.01$, and is therefore not present. Given the absence of the forced braid vortex, positive transport due to the stretching and reorientation mechanisms in Figure 4.10a is most prevalent around the periphery of the KH-billow core. The spatial distribution of perturbation enstrophy transport for the mechanism of viscous diffusion and dissipation is similar to those of Case 1, and the unforced case. Viscous diffusion and dissipation are negative along the axis of all vortical structures in the braid region.

The absence of the forced braid vortex simplifies the vortical dynamics in the braid region in comparison to Case 1. The upstream and downstream core-ejected vortices grow in the braid region towards the hyperbolic stagnation point. By $t/t_{KH} = 2.9$, these vortices form a unified streamwise-oriented vortex that occupies the braid region, which is visible at $t/t_{KH} = 3.4$ in Figure 4.10. This structure resembles the braid vortices in the unforced case and thus is henceforth simply referred to as the ‘braid vortex’. The fact that the core-ejected vortices merge in the braid region suggests that hyperbolic instability exists, and that the braid vortex is amplified by both the edge and hyperbolic instability modes simultaneously.

Figure 4.10: Development of a braid region in Case 2. Black-coloured isocontours indicate $\langle q \rangle_z / \langle \frac{1}{2} \omega^2 \rangle_{xyz}$ at values of 0.01 (dotted), 0.1 (solid-light), and 1.0 (solid-heavy) at $t/t_{KH} = 1.6$, $t/t_{KH} = 3.4$, and 4.3, respectively. Flood-plots show perturbation enstrophy transport by mechanisms of a) stretching and reorientation, and b) viscous diffusion and dissipation. $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ used for normalization has the values of 0.0026, 0.021, and 0.0325 at $t/t_{KH} = 1.6$, 3.4 and 4.3, respectively.

As in Case 1, the braid vortex in Case 2 is convected underneath KH_0 into the neighboring upstream braid region, and over KH_1 into neighboring downstream braid region.

At approximately $t/t_{KH} = 2.2$, at the same time as was observed in Case 1 and in the unforced case, the braid vortex originating downstream from KH_I is convected into the braid region in the CV, and comes into contact with the local braid vortex near the hyperbolic stagnation point. The consequence of this contact is circled and labelled as ‘braid vortex splitting’ in Figure 4.10b at $t/t_{KH} = 3.4$. The circled region shows a vortical structure separating from the braid vortex near the hyperbolic stagnation point. At $t/t_{KH} = 4.3$, the instability growing out of the braid vortex has increased in size and propagates upstream. Disorderly smaller-scale vortical structures, observed beneath KH_I at $t/t_{KH} = 4.3$ in Figure 4.10, commence precisely at $t/t_{KH} = 3.9$; this time is taken as the commencement of breakdown to smaller-scales in the CV.

The contact of the braid vortex at the hyperbolic stagnation point with a braid vortex convected to this location from the downstream-neighboring braid region appears to be related to breakdown to smaller-scales of the free-shear layer. The dynamics preceding this instability are observed using 3D isosurfaces of q , which are shown for $q/\langle \frac{1}{2} \omega^2 \rangle_{xyz} = 0.1$, at $t/t_{KH} = 2.1$ and 2.7 in Figure 4.11. Isosurfaces of q are also shown at $t/t_{KH} = 3.4$ and 4.3 in Figure 4.12, to illustrate the dynamics following this instability that lead to small-scale breakdown. Isosurfaces of q are coloured by ω_x to indicate the rotational sense of the vortical structures. The hyperbolic stagnation point is intersected by yz planes at each instance in time, labelled A and B in Figure 4.11, and C and D in Figure 4.12. Flood-plots are shown in the yz planes to indicate spatial distribution of $q/\langle \frac{1}{2} \omega^2 \rangle_{xyz}$. The scale of the flood-plot legend varies exponentially between $q/\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ values of 0.1 and 10.

Figure 4.11: Isosurfaces of $q/(\frac{1}{2}\omega^2)_{xyz} = 0.1$ in the CV for Case 2, prior to and after 'braid vortex splitting'. Distribution of $q/(\frac{1}{2}\omega^2)_{xyz}$ is shown using grayscale flood plots in Planes A and B. $(\frac{1}{2}\omega^2)_{xyz}$ used for normalization has the values of 0.0039 and 0.010 at $t/t_{KH} = 2.1$ and 2.7 respectively.

At $t/t_{KH} = 2.1$ in Figure 4.12, it is observed that the upstream end of the braid vortex from the braid region downstream of KH_1 has entered into the vicinity of the hyperbolic stagnation point. The spatial distribution of q shown in Plane A illustrates that the local braid vortex is substantially stronger than the upstream end of the infringing vortex.

Figure 4.12: Isosurfaces of $q/(\frac{1}{2}\omega^2)_{xyz} = 0.1$ in the CV for Case 2, prior to and after breakdown to smaller-scales. Distribution of $q/(\frac{1}{2}\omega^2)_{xyz}$ is shown using grayscale flood plots in Planes C and D. $(\frac{1}{2}\omega^2)_{xyz}$ used for normalization has the values of 0.024 and 0.032 at $t/t_{KH} = 3.4$ and 4.3 respectively.

The curved arrows indicate rotational sense of the vortices; the straight, solid arrows indicate the velocity vector at the centroids of the corresponding vortices; and the dashed curved arrows indicate streamlines calculated from the perturbation velocity, $\mathbf{u}$, and indicate the trajectory of the infringing vortices. These streamlines show that the stronger local braid

vortices induce substantial velocity component onto the infringing ends of the downstream braid vortices, causing them to be convected upwards.

In plane B, the cores of the infringing vortices, are located at the same y^+ location as the local braid vortices. Velocity induced by the infringing vorticity causes the cores of the local braid vortices to be squeezed together, reducing their spacing from $\Delta z^+ = 90$ to 75. Due to the spanwise displacement of the local braid vortices, however, the distribution of $q/\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ is now asymmetrical. Vorticity which is located further from the spanwise displaced local braid vortex core is induced upwards, and is rolled up into the upwards moving infringing vortex. The infringing vortex cores are observed in Plane C to have separated from the local braid vortices, but now contain substantially greater values of $q/\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ that are comparable to that of the remaining local braid vortices. These separated vortices correspond to the vortical structures observed near the hyperbolic stagnation point in at $t/t_{KH} = 3.4$ Figure 4.10, which give the appearance that the braid vortex is splitting; hence the terminology ‘braid vortex splitting’ is used to describe this interaction.

In Plane C, the like-rotating separated vortices and remaining local braid vortices induce velocities onto each other in opposing directions, causing these vortices to become disorderly in Plane D. The separated vortices are convected and experience growth in the direction indicated in Figure 4.12, until they eventually enter into close proximity of the KH-billow cores. The separated vortices penetrate into KH_1 at $t/t_{KH} = 3.4$, and into KH_0 at $t/t_{KH} = 4.3$, as indicated in Figure 4.12. Smaller-scale structures are observed subsequent to this penetration in the vicinity of KH_1 , at $t/t_{KH} = 4.3$ in Figure 4.10 and Figure 4.12.

The braid vortex splitting interaction was initiated in this case when an upstream braid vortex infringed into its neighboring upstream braid region. Braid vortices were also noted to

infringe into upstream-neighboring braid regions in Case 1 and in the unforced case. Breakdown of the braid vortex followed soon after this infringement in Case 1 and in the present case. A similar interaction was also observed using DNS by Caulfield and Peltier (2000), and Sandberg and Sandham (2008), and was suggested to result in breakdown to smaller-scales in the braid region (rather than near the core as in Case 2). Infringement of vorticity from a downstream braid region occurred in the unforced case and in Cases 1 and 2 at exactly $t/t_{KH} = 2.2$; braid vortex splitting for both Cases 1 and 2 at $t/t_{KH} = 2.7$; and smaller-scale structures were observed on both cases by $t/t_{KH} = 3.4$. The occurrence of the infringement event in three distinct cases suggests that it is a natural consequence of the presence of streamwise oriented vortical structures at the hyperbolic stagnation points created by the 2D KH instability. Due to the streamwise periodicity of the KH-billow-induced flow-field, vortical perturbations developing in a hyperbolic stagnation point are convected towards other hyperbolic stagnation points. The fact that all instances of the infringement event occurred at the same time furthermore suggests that the infringement event is dependent on the large-eddy turnover time of the KH billow, $d/(U_2 - U_1)$, where d is the half-depth of the free-shear layer (and approximately equal to the diameter of the KH billow). It was observed that in the unforced case, however, where $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ is six orders of magnitude less than in the forced cases (shown in Section 4.4), braid vortex splitting did not occur, and the infringing vortex was deflected by the KH-billow-induced flow field. Caulfield and Peltier (2000) also found this in their DNS of an unforced free-shear layer, and noted that breakdown to smaller-scales later after substantial 3D perturbation growth.

Disorderly smaller-scale vortical structures were first observed in the braid region at similar times (approximately $t/t_{KH} = 3.9$) in Case 1 and Case 2, despite these cases having substantially different initial values of perturbation magnitude and perturbation growth rate

(discussed in Section 4.4). This indicates that in addition to the infringement event, there exists a critical perturbation magnitude at which the braid vortex splitting mechanism occurs. It is furthermore noted that this mechanism is primarily governed by inviscid vortex mechanics. It is presently suggested that this critical vortical magnitude is related to the requirement that the local braid vortices have sufficient circulation to locally perturb the 2D hyperbolic streamline pattern – which defines the trajectory of the infringing vortex – to cause redirection of the infringing braid vortex into the braid region.

The rapid breakdown to smaller-scales may be hampered by numerous factors, such as the presence of competing 3D instability modes in the braid region, as was observed Case 1. Nevertheless, in a free-shear layer unstable to the 2D KH instability, this interaction represents a viable and perhaps predictable path to breakdown to smaller-scales. There are of course also other paths to breakdown in the free-shear layer, another of which is discussed in Section 4.3.5 for Case 3.

4.3.5 Development of 3D vortical structures and mechanism of breakdown to smaller scales in Case 3

The development of vortical structures in Case 3 is shown in Figure 4.13 in an identical manner and at identical instances in time as in Figure 4.10 for Case 2. In general the spatial distributions of the perturbation enstrophy transport terms of stretching and reorientation, and viscous diffusion and dissipation shown in the flood-plots in Figure 4.13b at $t/t_{KH} = 1.6$ are similar to those observed for Case 2.

Figure 4.13: Development of a braid region in Case 3. Black-coloured isocontours indicate $\langle q \rangle_z / \langle \frac{1}{2} \omega^2 \rangle_{xyz}$ at values of 0.01 (dotted), 0.1 (solid-light), and 1.0 (solid-heavy) at $t/t_{KH} = 1.6$, $t/t_{KH} = 3.4$, and 4.3, respectively. Flood-plots show perturbation enstrophy transport by mechanisms of a) stretching and reorientation, and b) viscous diffusion and dissipation. $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ used for normalization has the values of 0.0025, 0.021, and 0.039 at $t/t_{KH} = 1.6$, 3.4 and 4.3, respectively.

Unlike in Case 2, however, the forced braid vortex has significant magnitude, though it is less than that of the core-ejected vortices. Furthermore, unlike in Case 1, the core-ejected vortices are able to displace the weaker forced braid vortex, as they grow towards the braid region. At $t/t_{KH} = 3.4$, the forced braid vortex, indicated in Figure 4.13b, lies below the core-ejected vortices which have merged. Figure 4.14 shows 3D isosurfaces of q in the braid region, at instances in time before and after displacement of the forced braid vortex. As the core-ejected vortices grow towards the hyperbolic stagnation point, they induced velocity in the $+y$ direction onto the forced braid vortices, which are indicated using black arrows in the figure. The forced braid vortices move downwards in between the core-ejected vortices in an interaction similar to the braid vortex splitting. Velocity induced by the forced braid vortices

onto the core-ejected vortices during their interaction, likewise causes the core-ejected vortices to be displaced in the spanwise direction.

Figure 4.14: Isosurfaces of $q/(\frac{1}{2}\omega^2)_{xyz} = 0.1$ at instances in time before and after displacement of the forced braid vortex.

In Figure 4.14 at $t/t_{KH} = 2.7$ core-ejected vortices which induce velocity between themselves in the upwards (-y) direction, are squeezed together. The narrower gap between these vortices suggests that the infringing vortices from the downstream braid region would be unable to pass through them as they did in Case 2. Thus the braid vortex splitting mechanism is suppressed. The infringing vortices from the downstream braid region, are repelled, and are instead wrapped around the upstream end of KH_1 . The portions of the braid vortices that are wrapped around the KH-billow core are referred to as ‘edge vortices.’ At $t/t_{KH} = 3.4$ in Figure 4.13a, intense growth due to stretching and reorientation is observed to be located within the edge vortices, as indicated in Figure 4.13a at $t/t_{KH} = 3.4$. Maximum perturbation magnitude in the braid region correspondingly also occurs in this region; maximum magnitude is labeled in Figure 4.13a. Following this intense growth, disorderly smaller-scale structures are observed in the indicated

region near KH_1 at $t/t_{KH} = 4.3$ in Figure 4.13a. Figure 4.15 and Figure 4.16 show isosurfaces of q in Case 3 in an identical manner as was done in Figures 8 and 9 for Case 2. yz planes intersect the strong edge vortices, and are labelled A and B, in Figure 4.15, and C and D in Figure 4.16.

Figure 4.15: Isosurfaces of $q/(\frac{1}{2}\omega^2)_{xyz} = 0.1$ in the CV for Case 3, prior to and after vortical collapse. The distribution of $q/(\frac{1}{2}\omega^2)_{xyz}$ is shown using grayscale flood-plots in Planes A and B. $(\frac{1}{2}\omega^2)_{xyz}$ used for normalization has the values 0.010 and 0.021 at $t/t_{KH} = 2.7$ and 3.4, respectively.

Figure 4.16: Isosurfaces of $q/(\frac{1}{2}\omega^2)_{xyz} = 0.1$ in the CV for Case 3, following vortical collapse and during smaller-scale breakdown. Distribution of $q/(\frac{1}{2}\omega^2)_{xyz}$ is shown using grayscale flood-plots in Planes C and D. $(\frac{1}{2}\omega^2)_{xyz}$ used for normalization has the values 0.025 and 0.039 at $t/t_{KH} = 3.6$ and 4.3 respectively.

In a round, axially-elongated vortex, net transport of enstrophy occurring radially inwards from the vortex core generally constitutes vortical collapse (Lin and Corcos, 1984),

wherein the vortex decreases in size and becomes rapidly intensified in magnitude. Convection of enstrophy by self-induced velocity and positive axial strain tend to concentrate enstrophy radially inwards, whereas viscous diffusion transports enstrophy in the radially outward direction. The criterion for vortical collapse due to self-induced effects and axial strain is met in the region labelled as 'edge vortices' in plane A. Vortical collapse is observed when the edge vortices indicated in Plane A, are smaller in size and their maximum value of $q/(\frac{1}{2}\omega^2)_{xyz}$ exceeds 10 in Plane B. Subsequent to this collapse, the vortices surrounding the collapsed vortex undergo breakdown, as suggested by their spatial distribution of q in Plane B. One such vortex that is breaking down is indicated in Plane B in Figure 4.15, and appears to be separating into two distinct vortices.

In Plane C in Figure 4.16, the collapsed vortex induces velocity of high magnitude, indicated by the thick black arrow, onto the vortex that is breaking down. This velocity is induced in a different direction from the trajectory of the collapsing vortex (indicated by the thick-dashed arrow), and corresponds to the direction in which the vortex splits. This suggests that breakdown in the indicated vortices in Planes A and B occurs due to strong convective effects induced by the collapsed edge vortices. Mutually-induced convective effects also occur in other neighboring vortices causing their respective breakdown. These broken down vortices, shown in Plane C, destabilize neighboring vortices, resulting in the general disorderliness observed in Plane D. Disorderly smaller-scale structures are observed subsequently in the vicinity of the broken-down vortices near KH_1 , as isosurfaces of $q/(\frac{1}{2}\omega^2)_{xyz} = 0.1$ at $t/t_{KH} = 4.3$ in Figure 4.16. Breakdown of the free-shear layer is thus concluded to be caused by mutually induced vortical breakdown in the vicinity of the KH-billow core.

4.4 Time-variation of spatially-averaged kinetic energy and enstrophy in the 2D and 3D perturbation flow fields

This section discusses the time variation of spatially averaged enstrophy and kinetic energy calculated from $\mathbf{u}$, and $\mathbf{U}_{KH}$, which represent the spatially and temporally varying 3D perturbation and 2D KH velocity fields, as defined in Section 4.1.2. The temporal variation of spatially averaged kinetic energy, $\langle \frac{1}{2} \mathbf{U}_{KH}^2 \rangle_{xyz}$, and enstrophy, $\langle \frac{1}{2} \mathbf{\Omega}_{KH}^2 \rangle_{xyz}$, in the KH flow-field in the CV is presented in Figure 4.17, starting from the time of kinetic energy saturation in the unforced case, $t/t_{KH} = 0.5$. The times at which smaller-scale structures are observed in the CV are indicated in, and are: $t/t_{KH} = 3.8$ for Case 1, $t/t_{KH} = 3.9$ for Case 2 and $t/t_{KH} = 4.3$ for Case 3.

Figure 4.17: Time variation of (a) enstrophy in the KH flow-field, $\langle \frac{1}{2} \mathbf{\Omega}_{KH}^2 \rangle_{xyz}$, and (b) kinetic energy in the KH flow-field, $\langle \frac{1}{2} \mathbf{U}_{KH}^2 \rangle_{xyz}$.

The trends observed for $\langle \frac{1}{2} \mathbf{U}_{KH}^2 \rangle_{xyz}$, and $\langle \frac{1}{2} \mathbf{\Omega}_{KH}^2 \rangle_{xyz}$ in Figure 4.17 are similar among all cases until approximately $t/t_{KH} = 2.0$. This initial trend, which is continued in the unforced case until $t/t_{KH} = 4.5$, constitutes a general decline of kinetic energy and enstrophy in the KH

flow-field subsequent to kinetic energy saturation, characteristic of phase (2) in Section 2.2.3. Kinetic energy is observed to increase in the unforced case after $t/t_{KH} = 4.5$. This increase corresponds to events related to the KH pairing instability, which occurs at approximately $t/t_{KH} = 6.0$. In all forced cases, however, the KH pairing instability is suppressed by the presence of high-magnitude 3D perturbations in the braid region, also noted by Pierrehumbert et al., (1982), and Metcalfe and Orszag (1987). In general, the symmetry between kinetic energy and enstrophy is quite complex (Hamman et al., 2008) and beyond the scope of this research; thus for brevity the discussion will focus on the trends observed in enstrophy. In Figure 4.17, enstrophy in the KH flow-field increases in Cases 2 and 3, after $t/t_{KH} = 2.0$, whereas in Case 1 and in the unforced case it continues to decline with time.

Figure 4.18 illustrates isocontours of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_z$, before and after this buildup of enstrophy, which show that the buildup of enstrophy in Case 2 is localized to the KH-billow cores. The mechanism for this is related to the fact that the streamwise-oriented vortical structures straddle the KH billows, as illustrated in the figure. The streamwise vortices lying above the KH-billow core induce high-momentum fluid from the high-speed stream, between themselves, and directly into the KH-billow cores. As the high-momentum fluid is entrained into the core, the local velocity magnitude within the core increases causing an increase in the rotation rate of the KH billow core, hence causing an increase in enstrophy. Low-momentum fluid is likewise exchanged in this manner in between the braid vortices underneath the KH-billow core, decreasing enstrophy in the core. Perturbation magnitude (and thus induced velocity) was noted to be greater in the braid vortices near the high-speed stream in Sections 4.3.3 through 4.3.5, however, thus a net increase in kinetic energy and enstrophy in the KH-billow core is observed. Conversely, the lack of kinetic energy buildup in Case 1 is due to the presence of the forced braid vortices on top of the core ejected-vortices, which are counter

rotating with respect to each other. The y-component of velocity induced by the core-ejected vortices interacts destructively with that of the forced braid vortices, resulting in less net transfer of momentum from the high- and low-speed streams into the KH-billow core.

Figure 4.18: Isocontours of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_z$ for values of 1, 5, 10, and 15 ($\times 10^{-4}$), illustrating build-up of enstrophy in the KH-billow cores, in a xy plane in the CV for Case 2 at a) $t/t_{KH} = 2.0$ and b) $t/t_{KH} = 3.0$.

$\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ begins to decrease for Case 2 at $t/t_{KH} = 3.6$, following the penetration of vortical structures into KH_1 , described in Section 4.3.4. This rapid decline of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ suggests that rapid breakdown of the KH-billow core is occurring. A decline in $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ similarly begins for Case 3 at $t/t_{KH} = 3.6$, precisely following vortical collapse of the edge vortices discussed in Section 4.3.5. The increase and decline of spatially averaged enstrophy in the KH-billow cores is indicative of the change in the strength of the background strain field that drives perturbation growth. The variation of enstrophy in the KH-billow cores will coincide with changes in perturbation growth rate.

The time variation of the spatially averaged perturbation enstrophy, $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$, and kinetic energy, $\langle \frac{1}{2} u^2 \rangle_{xyz}$, in the CV are presented in Figure 4.19 for all cases, and in Figure 4.20 for only the forced cases.

Figure 4.19: Time variation of spatially-averaged perturbation a) enstrophy, $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$, and b) kinetic energy, $\langle \frac{1}{2} u^2 \rangle_{xyz}$ in the CV.

In Figure 4.19, the trends observed for the time variation of $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ are generally very similar to those of $\langle \frac{1}{2} u^2 \rangle_{xyz}$ in all cases prior to breakdown to smaller-scales. These quantities generally increase in all cases, and correspond to the mean perturbation enstrophy and kinetic energy that are contained in (and induced by) the dominant streamwise-oriented vortical structures.

Growth of perturbation enstrophy and kinetic energy in the unforced case commences after the time of kinetic energy saturation in the KH-bilow core, at $t/t_{KH} = 0.75$. Both perturbation enstrophy and kinetic energy in the unforced case undergo linear growth until $t/t_{KH} = 6.0$. At this time, KH-bilow pairing occurs, suppressing perturbation growth in the CV. The suppression of perturbation growth by KH-bilow pairing has been noted to occur by Pierrehumbert et al. (1984), and Smyth and Peltier (1994) and many others and is adequately explained in the literature. Initial perturbation enstrophy and kinetic energy in the unforced

case is six orders of magnitude less than in the forced cases, and in fact never exceeds the initial perturbation magnitude of the forced cases. Correspondingly, the unforced case does not experience breakdown to smaller-scales in the present analysis.

The time variation of perturbation enstrophy and kinetic energy in the forced cases are more clearly observed in Figure 4.20.

Figure 4.20: Time variation of spatially-averaged perturbation a) enstrophy, $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$, and b) kinetic energy, $\langle \frac{1}{2} u^2 \rangle_{xyz}$, of the forced cases.

As in the unforced case, growth of perturbation enstrophy and kinetic energy begins for all forced cases at approximately $t/t_{KH} = 0.75$. The time variation of perturbation enstrophy and kinetic energy observed in Figure 4.20 are similar for Cases 2 and 3, and distinct from Case 1. After $t/t_{KH} = 0.75$, perturbations in Cases 2 and 3 increase non-linearly until approximately $t/t_{KH} = 1.0$. Between $t/t_{KH} = 1.0$ and 3.5 , perturbation enstrophy and kinetic energy in Cases 2 and 3 experience linear growth. After $t/t_{KH} = 3.5$, perturbation growth rate begins to decline in Cases 2 and 3, following breakdown to smaller-scales. Case 1 similarly experiences a period of linear growth, though the growth rate does not decrease significantly due to the persistent coherence of the core-ejected vortices following breakdown of the forced braid vortex.

The definition of perturbation growth rate used in the present study is similar to that used by Metcalfe and Orszag (1987) and Caulfield and Peltier (2000). Instantaneous growth rate of perturbation enstrophy, $\sigma_{\frac{1}{2}\omega^2}$, is calculated as per Eqn. (4.8); growth rate of perturbation kinetic energy, $\sigma_{\frac{1}{2}u^2}$, is similarly calculated.

$$\sigma_{\frac{1}{2}\omega^2} = \frac{d}{dt} \ln \left(\left\langle \frac{1}{2} \omega^2 \right\rangle_{xyz} \right) \quad (4.8)$$

Instantaneous perturbation growth rates for all four cases are given in Table 4.2. Growth rates are averaged during the period of linear growth in each case (indicated in Figure 4.20 for Case 3). The times at which this period begins and ends are denoted by T_1 and T_2 , respectively.

Table 4.2: Perturbation growth rates during the linear growth phase for all cases

Case	$\sigma_{\frac{1}{2}u^2}$	$\sigma_{\frac{1}{2}\omega^2}$	T_1	T_2
0	2.4	2.4	1.0	6.0
1	0.34	0.40	1.5	3.5
2	1.5	1.6	1.5	3.5
3	1.4	1.5	1.5	3.5

From Table 4.2, time-averaged perturbation growth rate is by far greatest in the unforced case, next greatest for Cases 2 and 3, and least in Case 1. The perturbation growth rates and trends of $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ in time in Cases 2 and 3 are very similar. This relates to the values of h and Λ_z^+ : Cases 2 and 3 had identical Λ_z^+ , but differing h , from which it is evident that h does not significantly affect growth rate during the linear growth phase. Likewise, Γ_0 is very similar between Cases 1 and 3, though their respective growth rates differ substantially. This suggests that the smaller value of Λ_z^+ in Case 1 resulted in the lesser growth rate of Case 1 compared to Cases 2 and 3. Interestingly, growth rate is observed to be greater in the

unforced case, despite that Λ_z^t of Cases 2 and 3 were set to the spanwise spacing of the most amplified velocity streak structure in the unforced case.

Further light is shed on the differing values of $\sigma_{\frac{1}{2}}\omega^2$ observed amongst the cases by analysis of the time variation of the individual terms of the perturbation enstrophy transport equation, given in Eqn. (4.6), which are shown in Figure 4.21 for all cases. The summation of terms (1) through (6) in Eqn. (4.6), is plotted and labelled in the figure as ‘Stretching and reorientation,’ and is denoted by $\mathcal{S}$. Likewise, terms (7) through (9) of Eqn. (4.6) describe the 3D convective transport of enstrophy from the background and KH flow-fields; these terms are plotted and labelled in Figure 4.21 as ‘Convection,’ and denoted by $\mathcal{C}$. The tenth term in Eqn. (4.6) is plotted as an absolute value and labelled in the figure as ‘Viscous diffusion and dissipation,’ and is denoted by $\mathcal{D}$. The summation of the above terms yields the total transport of perturbation enstrophy in the CV, which is plotted in the figure and is labelled accordingly.

In Figure 4.21 it is observed that convection of perturbation enstrophy contributes the least to total enstrophy transport in all cases. This is because the summation of the convective perturbation enstrophy transport terms is predominantly equal to the amount of enstrophy entering and leaving CV, which is a small value since the braid vortices are relatively stationary with respect to CV. Positive transport due to the stretching and reorientation mechanisms was shown to be the principal contributor to the growth of perturbation enstrophy in the braid region in the unforced case, in Section 4.2.2, and appears to also be true in the forced cases. The trends for enstrophy transport by the stretching and reorientation mechanisms, and viscous diffusion and dissipation mechanisms are very similar, but appear to be vertically offset from each other in the figure for all cases. Furthermore, the time-variation of the perturbation enstrophy transport terms are similar to those observed for

perturbation enstrophy in Figure 4.19 and Figure 4.20, which suggests that perturbation enstrophy transport is in general proportional to perturbation enstrophy.

Figure 4.21: Time variation of spatially-averaged perturbation enstrophy transport terms by the mechanisms of convection (dashed), stretching and reorientation (small-dotted), viscous diffusion (large-dotted), and the summation of these terms (solid) in the CV for all cases.

This is in general not true given the non-linear nature of the governing equations. Further light is shed on this phenomenon by considering the general stretching and reorientation term:

$$\omega_i \omega_j \frac{\partial \bar{U}'_j}{\partial x_i} \quad (4.9)$$

which is derived in Appendix A.2. Eqn. (4.9) represents the product dot product of two tensors: the covariant vorticity tensor, $\omega_i \omega_j$, and the background strain tensor, $\partial \bar{U}_j / \partial x_i$.

Proportionality of perturbation enstrophy transport to perturbation enstrophy may occur under

two conditions. Firstly, the background strain tensor, $\partial U_j / \partial x_i$, must be steady, or more generally the direction of the principle axis of strain must be steady. In the latter case, strain rate may vary in time, but all terms of the strain tensor are uniformly scaled as the strain tensor changes in time. A similar condition may be applied to the vorticity vector: if the spatial orientation of the vorticity vector at each point is relatively steady, each term of the covariant vorticity product, $(\omega_i \omega_j)$ is uniformly scaled as it increases in time. Combined with an unchanging background strain tensor this yields direct proportionality of the stretching and reorientation transport term to the square of vorticity magnitude, or enstrophy. In general, after kinetic energy saturation in the KH-billow core, the spatial orientation of the principle axis of strain (exemplified by the strain field lines shown in Figure 4.8) does not change (Leblanc and Godeferd, 1999), and correspondingly as perturbation growth occurs along the principle axis of strain, the spatial orientation of the vorticity vector is also steady (Smyth and Peltier, 1994). Smyth and Peltier (1994) note that in general, unsteadiness of the background flow field results in decreased perturbation growth rate. Similarly, if the spatial distribution of vorticity remains relatively constant, the gradient of vorticity and correspondingly viscous dissipation increase proportionally to perturbation enstrophy.

Noting this proportionality, the spatially averaged transport terms, $\mathcal{S}$ and $\mathcal{D}$, are normalized by $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$, and the resulting quantities are represented by the smaller case cursive letters, $\mathcal{s}$ and $\mathcal{d}$, respectively. If the convective transport term, $\mathcal{C}$, is neglected as a source of perturbation growth, total spatially averaged perturbation enstrophy transport is:

$$\left\langle \frac{\partial}{\partial t} \left(\frac{\omega^2}{2} \right) \right\rangle_{xyz} = \mathcal{s}(t) \left\langle \frac{\omega^2}{2} \right\rangle_{xyz} + \mathcal{d}(t) \left\langle \frac{\omega^2}{2} \right\rangle_{xyz} \quad (4.10)$$

Using Eqn. (4.10), a relationship can be derived (see Appendix A.2) for spatially-averaged perturbation growth rate, $\sigma_{\frac{1}{2}\omega^2}$, from the normalized perturbation enstrophy transport terms, and is given as:

$$\sigma_{\frac{1}{2}\omega^2} = \mathcal{S}(t) \left(\frac{\mathcal{R}(t) - 1}{\mathcal{R}(t)} \right) \quad (4.11)$$

where, $\mathcal{R}(t) = -\mathcal{S}(t)/\mathcal{D}(t)$ is the ratio of spatially averaged inviscid growth to spatially averaged viscous diffusion and dissipation of perturbation enstrophy. From Eqn. (4.11), it is evident that the values of $\mathcal{R}$ and $\mathcal{S}$ specify the growth rate. The observed differences in $\sigma_{\frac{1}{2}\omega^2}$ for all cases may hence be interpreted in the context of these parameters, with $\mathcal{S}$ representing the inviscid growth rate due to stretching and reorientation, and $\left(\frac{\mathcal{R}(t)-1}{\mathcal{R}(t)} \right)$ represents an efficiency factor that incorporates viscous dissipation of perturbation enstrophy relative to growth.

The vertical distance between a point on the curves $\mathcal{S}$ and $\mathcal{D}$ in Figure 4.21 at a given instance in time, is indicative of $\mathcal{R}$. It is qualitatively observed in Figure 4.21b that Case 1 generally has the lowest value of $\mathcal{R}$. The time variation of $\mathcal{R}$ and $\mathcal{S}$ are shown in Figure 4.22 for all cases.

Figure 4.22: Time-variation of a) ratio of perturbation enstrophy transport due to stretching and reorientation to viscous diffusion and dissipation, $\mathcal{R}$, and b) inviscid growth rate, $\mathcal{s}$.

There are two distinct trends for $\mathcal{s}$ which are observed in Figure 4.22b, those for Case 1 and the unforced case, and those for Cases 2 and 3. In Cases 2 and 3, $\mathcal{s}$ is greater, and increases at times corresponding to the buildup of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ observed for these cases in Figure 4.17. The increase in $\mathcal{s}$ is particularly greatest in Case 3, and a sharp increase in $\mathcal{s}$ coincides with the time when the edge vortex was wrapped around KH_1 . The increase in $\mathcal{s}$ thus appears to be more pronounced when high-amplitude streamwise-oriented vortical structures are situated in close proximity to the KH-billow core. $\mathcal{s}$ in Cases 2 and 3 begins to decline precisely at the time of the decline of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ observed in Figure 4.17 for these cases. This is as expected, as the growth rate of 3D perturbations is facilitated by the KH-billow induced strain field and declines with it. Conversely, the temporal variation of $\mathcal{s}$ in Case 1 and in the unforced case, which is quite similar prior to breakdown of the forced braid vortices in Case 1 at $t/t_{KH} = 3.0$, is lower than in Cases 2 and 3 and reflects their similar temporal variation of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ observed in Figure 4.17. Notably, in these cases a buildup of $\langle \frac{1}{2} \Omega_{KH}^2 \rangle_{xyz}$ was not observed in Figure 4.17. Since it is well established that growth of 3D perturbations is facilitated by KH-billow-induced strain, it follows that in Cases 2 and 3 there is positive feedback between the magnitude of the background strain and the magnitude of the streamwise-oriented vortices, which induce growth of the KH billows. In light of these observations, it is concluded that the greater growth rates of Cases 2 and 3 compared to Case 1, are partially due to the positive feedback of 3D perturbations with the KH-billow core. Moreover, although in general the fastest growing linear instability mode in the free-shear layer is indeed the hyperbolic instability – as indicated by the dominance of braid vortices in Case 1 and in the unforced case – the present data suggests that forcing of the edge instability

mode at the periphery of the KH-billow core may produce significantly greater growth rates due to these non-linear positive feedback effects.

The inviscid perturbation growth rate, $\mathcal{s}$, is modified by the viscous perturbation growth efficiency, $\left(\frac{\mathcal{R}(t)-1}{\mathcal{R}(t)}\right)$, to determine overall perturbation growth rate; greater value of $\mathcal{R}$ indicates greater perturbation growth efficiency. In general it is observed from Figure 4.22a that $\mathcal{R}$ is greatest in the unforced case, very similar between Cases 2 and 3, and least in Case 1. It is evident from Figure 4.22a that perturbation growth rate in Cases 2 and 3 is less than in the unforced case, despite greater values of $\mathcal{s}$, due to their lesser values of $\mathcal{R}$. Case 1 has lowest values of $\mathcal{R}$, and despite having a similar initial value of $\mathcal{s}$ as in the unforced case, has the least perturbation growth rate. The lowest value of $\mathcal{R}$ being for Case 1 suggests that the smaller value of Λ_z^+ in that case causes a greater degree of viscous diffusion and dissipation. This is because the streamwise vorticity introduced at the splitter-plate trailing edge was concentrated into a narrower spanwise spacing in Case 1. More concentrated vorticity expectedly has greater vorticity gradients, and correspondingly greater viscous dissipation. Furthermore, in Case 1 it is noted that there were a greater number of vortical structures in the braid region in close wall-normal proximity to each-other, which by similar reasoning also contribute to the increased viscous dissipation. It is thus presently suggested that the value of $\mathcal{R}$ is generally related to the spacing of the streamwise-oriented vortical structures.

The time variation of spatially-averaged mean spanwise wavelength, $\langle \lambda_z^+ \rangle_{xyf_z}$ (note that the subscript f denotes averaging in the frequency domain), of streaky perturbations in the CV in the unforced case is calculated as per Eqn. (4.11), and is shown for the unforced case in Figure 4.23.

$$\frac{1}{\langle \lambda_z^+ \rangle_{xyf_z}} = \frac{\int_{CV} \left(\frac{1}{\lambda_z^+} u(x, y, f_z, t) \right) dx dy df_z}{\int_{CV} u u(x, y, f_z, t) dx dy df_z} \quad (4.12)$$

$u(x, y, f_z, t)$ in Eqn. (4.11), corresponds to the time-varying PSD of streamwise component of velocity, u , computed in the spanwise direction at all x and y locations in the CV, and f_z corresponds to spanwise wavenumber. Furthermore, the integral with subscript CV denotes integration over the region enclosed by the CV.

Figure 4.23: Time-variation of spatially-averaged spanwise wavelength of u , $\langle \lambda_z^+ \rangle_{xyf_z}$, in the unforced case.

In Figure 4.23, $\langle \lambda_z^+ \rangle_{xyf_z}$ is initially large due to the presence of a wider-spaced streaky structure with $\lambda_z^+ = 200$, which was developed in the upstream boundary layer on the high-speed side of the splitter plate, discussed in Section 4.2.2. $\langle \lambda_z^+ \rangle_{xyf_z}$ rapidly declines, and by $t/t_{KH} = 1.5$ converges to the wavelength of the fastest growing 3D instability mode in the free-shear layer, $\lambda_z^+ \cong 93$. The time of this initial decline in $\langle \lambda_z^+ \rangle_{xyf_z}$ in Figure 4.23 corresponds to the sharp increase in $\mathcal{R}$ observed in Figure 4.22a. After $t/t_{KH} = 1.5$ in Figure 4.23, $\langle \lambda_z^+ \rangle_{xyf_z}$ begins increasing in correspondence to the temporally increasing depth of the shear layer (Smyth and Peltier, 1994). At this time, $\mathcal{R}$ also increases and generally follows the trend of $\langle \lambda_z^+ \rangle_{xyf_z}$.

$\mathcal{R}$ is observed to decrease when $\langle \lambda_z^+ \rangle_{xyf_z}$ deviates from that of the fastest growing 3D instability mode. This was observed in the unforced case when $\mathcal{R}$ was quite low due to the value of $\langle \lambda_z^+ \rangle_{xyf_z} = 200$, and in Case 1 which generally had a lower value of $\mathcal{R}$ and in which $\Lambda_z^+ = 75$. Furthermore, the increase of both $\mathcal{R}$ and $\langle \lambda_z^+ \rangle_{xyf_z}$ after $t/t_{KH} = 1.5$ in Figure 4.23 suggests that $\langle \lambda_z^+ \rangle_{xyf_z}$ varied in the unforced case such that $\mathcal{R}$ is maximized, thus also maximizing $\sigma_{\frac{1}{2}\omega^2}$. The optimizing behavior of 3D perturbations is explained as follows. Greater perturbation growth efficiency corresponds to greater perturbation growth rate; perturbations with the greatest growth rate will grow to dominate the flow field, and thus overall perturbation growth efficiency will tend to increase in the flow-field, until breakdown to smaller scales. This increase in efficiency was notably observed to occur in the DNS studies by Caulfield and Peltier (2000, 2003).

In this context the unforced case exhibits the greatest value of $\mathcal{R}$ due to this optimizing effect. Conversely in the forced cases, streamwise component of vorticity is imposed with a particular spanwise spacing, hence their value of $\mathcal{R}$ remains relatively constant in Figure 4.22 until breakdown to smaller-scales. Furthermore, introducing streamwise component of vorticity in a spanwise-periodic manner in the free-shear layer with a Λ_z^+ value that deviates from the spanwise wavelength of the fastest growing 3D instability mode, results in a lesser value of $\mathcal{R}$, and possibly lesser perturbation growth rate. Finally, although streamwise component of vorticity was introduced in Cases 2 and 3 at a Λ_z^+ value equal to the spanwise wavelength of the fastest growing 3D instability mode in the free-shear layer at the splitter-plate trailing edge, the perturbation growth rates in these cases were observed to be lesser than in the unforced case due to a lesser overall value of $\mathcal{R}$.

5 Chapter: Conclusions and Future work

A free-shear layer developing past a no-slip splitter-plate was forced by introducing a spanwise-wavy shape to the splitter-plate trailing edge. Four DNS cases were performed: a flat plate case, a case where spanwise wavelength of the waviness was set equal to the spanwise wavelength of the most amplified secondary instability mode, and two cases where the wavelength and height of the waviness were independently varied. It was found that in the absence of forcing and significant environmental disturbances, 3D streaky perturbations developed in the boundary layer on the high-speed side of the splitter plate were convected downstream from the trailing edge of the splitter plate, and into the free-shear layer. These perturbations were then amplified by the secondary instability mechanisms inherent to the free-shear layer and formed streamwise-oriented vortices in the braid region with spanwise spacing characteristic of vortices generated by the hyperbolic instability mode in the braid region.

In the cases where the free-shear layer was forced, streamwise-component of vorticity was introduced by the waviness of the splitter-plate trailing edge, preferentially at the periphery of the KH-billow core or in the braid region depending on the value of α_z at the splitter-plate trailing edge. Vorticity introduced near the KH-billow core was simultaneously amplified by a centrifugal ‘edge’ instability mode and by the hyperbolic instability mode, and formed coherent braid vortices as in the unforced case. The presence of vorticity near the KH-billow core in Cases 2 and 3 facilitated buildup of enstrophy and kinetic energy in the KH billow that caused a non-linear feedback relationship between growth of the braid vortices and KH billows. The inviscid component of perturbation growth rate in these cases was substantially higher than in Case 1 and in the unforced case, in which vorticity predominantly existed in the braid region.

Varying the height of the trailing edge waviness affected the initial magnitude of the streamwise-oriented vortices generated at the trailing edge, but did not significantly affect their growth rate. Perturbation growth rate appeared to be dependent on the spanwise spacing between the streamwise-oriented vortices. 3D perturbations with spanwise spacing that differed from the most amplified spanwise wavelength predicted by linear stability theory exhibited significantly increased viscous dissipation relative to growth. Perturbation growth rate in the unforced case was thus greater than in any of the forced cases due minimal viscous dissipation relative to inviscid growth.

The greater initial perturbation magnitude of the forced cases resulted in their earlier breakdown to smaller scales. The mechanisms leading to smaller-scale breakdown were studied in all forced cases. In two cases, smaller-scale breakdown occurred due to infringement of braid vortices into adjacent braid regions causing a braid vortex splitting interaction that destabilized the shear layer. It was suggested in the present study that the incident of the infringement of vorticity from neighboring braid regions was ubiquitous to the KH billow wave train. It was furthermore suggested that the time scale of this event was directly related to the large-eddy turnover time of the KH billow. Lastly, it was also found that smaller-scale breakdown of the shear-layer occurred at similar times in two forced cases, despite significantly different perturbation growth rates and magnitudes, which indicated the existence of a critical vortical magnitude required for smaller-scale breakdown by the infringement mechanism.

In light of the above conclusions, the objectives of the present study were achieved. The contributions of this study to the understanding of forced free-shear layers are summarized as follows. Firstly, building on the work presented in the literature, the braid-vortex splitting interaction was studied in detail and was found to be a ubiquitous instability

in the KH billow wave-train. Secondly, the effect of the spanwise wavelength, Λ_z , and amplitude, h , of a spanwise-wavy splitter-plate trailing edge on the growth rate of the 3D perturbations introduced into the wake of the splitter-plate were characterized. An optimum value of Λ_z was identified, which yielded maximum perturbation growth rate and hence the shortest mixing length. Finally, a representation was derived for the 3D perturbation growth rate with respect to the inviscid perturbation enstrophy transport terms and a viscous perturbation growth efficiency factor. The inviscid growth rate, and perturbation growth efficiency were found to be dependent on the location of perturbations within the shear layer, and on their spanwise spacing, respectively.

The present study was somewhat limited in scope, however. Future work may focus on more concretely establishing the relationship between the perturbation growth efficiency and the spanwise wavelength of the trailing-edge waviness, investigating the effect of wall proximity on secondary instability modes, investigating the possibility of a critical vortical magnitude for breakdown to smaller-scales due to the braid-vortex splitting interaction, and investigating the effect of thermo-physical property gradients on the secondary instability modes.

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Appendices

Appendix A

A.1 Derivation of perturbation enstrophy transport equation, Eqn. (4.6)

Derivations are performed using vector notation. Perturbation enstrophy transport is derived from the incompressible governing equation for linear momentum, expressed as:

$$\frac{\partial(\mathbf{U})}{\partial t} + (\mathbf{U} \cdot \nabla)(\mathbf{U}) = -\frac{1}{\rho} \nabla P + \frac{\mu}{\rho} \nabla^2 \mathbf{U} \quad (\text{A.1})$$

The advective derivative, $(\mathbf{v} \cdot \nabla)$, is expanded with the vector calculus identity:

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) \quad (\text{A.2})$$

Where, $\mathbf{A}$ and $\mathbf{B}$ are arbitrary 3D vector fields. Eqn. (A.1) becomes Eqn. (A.3).

$$\left(\frac{\partial(\mathbf{U})}{\partial t} + \frac{\nabla(\mathbf{U} \cdot \mathbf{U})}{2} - \mathbf{U} \times (\nabla \times \mathbf{U}) = -\frac{1}{\rho} \nabla P + \frac{\mu}{\rho} \nabla^2 \mathbf{U} \right) \quad (\text{A.3})$$

To obtain the vorticity transport equation, the curl operator is applied to Eqn. (A.3):

$$\nabla \times \left(\frac{\partial(\mathbf{U})}{\partial t} + \frac{\nabla(\mathbf{U} \cdot \mathbf{U})}{2} - \mathbf{U} \times (\nabla \times \mathbf{U}) = -\frac{1}{\rho} \nabla P + \frac{\mu}{\rho} \nabla^2 \mathbf{U} \right) \quad (\text{A.4})$$

This yields:

$$\begin{aligned} \left(\frac{\partial(\nabla \times \mathbf{U})}{\partial t} + \nabla \times \left(\frac{\nabla(\mathbf{U} \cdot \mathbf{U})}{2} \right) - \nabla \times (\mathbf{U} \times (\nabla \times \mathbf{U})) \right. \\ \left. = -\nabla \times \left(\frac{1}{\rho} \nabla P \right) + \frac{\mu}{\rho} \nabla \times (\nabla^2 \mathbf{U}) \right) \end{aligned} \quad (\text{A.5})$$

wherein, $\nabla \cdot (\nabla \times \mathbf{U}) = 0$, such that:

$$\frac{\partial(\nabla \times \mathbf{U})}{\partial t} - \nabla \times (\mathbf{U} \times (\nabla \times \mathbf{U})) = \frac{\mu}{\rho} \nabla^2 (\nabla \times \mathbf{U}) \quad (\text{A.6})$$

Vorticity, defined as $\boldsymbol{\Omega} = \nabla \times \mathbf{U}$, is substituted into Eqn. (A.6), yielding:

$$\frac{\partial(\boldsymbol{\Omega})}{\partial t} - \nabla \times (\mathbf{U} \times (\boldsymbol{\Omega})) = \frac{\mu}{\rho} \nabla^2 (\boldsymbol{\Omega}) \quad (\text{A.7})$$

Using Eqn. (A.8), the convective terms of the vorticity transport equation are obtained.

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} \quad (\text{A.8})$$

Eqn. (A.7) thus becomes:

$$\frac{\partial(\boldsymbol{\Omega})}{\partial t} - \mathbf{v}(\nabla \cdot \boldsymbol{\Omega}) + \boldsymbol{\Omega}(\nabla \cdot \mathbf{U}) - (\boldsymbol{\Omega} \cdot \nabla)\mathbf{U} + (\mathbf{U} \cdot \nabla)\boldsymbol{\Omega} = \frac{\mu}{\rho} \nabla^2 (\boldsymbol{\Omega}) \quad (\text{A.9})$$

Due to incompressibility, $\nabla \cdot \mathbf{v} = 0$.

$$\frac{\partial(\boldsymbol{\Omega})}{\partial t} + (\mathbf{U} \cdot \nabla)\boldsymbol{\Omega} = (\boldsymbol{\Omega} \cdot \nabla)\mathbf{U} + \frac{\mu}{\rho} \nabla^2 (\boldsymbol{\Omega}) \quad (\text{A.10})$$

To obtain the total instantaneous enstrophy transport equation, the dot product of vorticity transport with $\boldsymbol{\Omega}$ is evaluated.

$$\boldsymbol{\Omega} \cdot \left(\frac{D(\boldsymbol{\Omega})}{Dt} \right) = (\boldsymbol{\Omega} \cdot \nabla)\mathbf{U} + \frac{\mu}{\rho} \nabla^2 (\boldsymbol{\Omega}) \quad (\text{A.11})$$

This yields:

$$\frac{D}{Dt} \left(\frac{1}{2} \boldsymbol{\Omega}^2 \right) = \boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \cdot \nabla)\mathbf{U} + \frac{\mu}{\rho} \boldsymbol{\Omega} \cdot \nabla^2 (\boldsymbol{\Omega}) \quad (\text{A.12})$$

To obtain the perturbation enstrophy transport equation, the vorticity and velocity vectors are decomposed into their 2D background and 3D perturbation components, $\boldsymbol{\Omega} = (\bar{\boldsymbol{\Omega}}' + \boldsymbol{\omega})$ and $\mathbf{U} = (\bar{\mathbf{U}}' + \mathbf{u})$. These are substituted into the vorticity transport equation. This

makes the assumption that the 2D background flow field, represented by $\bar{\mathbf{U}}'$, is unaltered by the 3D perturbation flow field, $\mathbf{u}$. The vorticity transport equation for the 2D background flow-field is:

$$\frac{D}{Dt}(\bar{\boldsymbol{\Omega}}') = (\bar{\boldsymbol{\Omega}}' \cdot \nabla)\bar{\mathbf{U}}' + \frac{\mu}{\rho}\nabla^2(\bar{\boldsymbol{\Omega}}') \quad (\text{A.13})$$

The perturbation vorticity transport equation is therefore equal to the vorticity transport terms containing perturbation quantities, which are predominantly 3D.

$$\frac{D}{Dt}\boldsymbol{\omega} = (\bar{\boldsymbol{\Omega}}' \cdot \nabla)\mathbf{u} + (\boldsymbol{\omega} \cdot \nabla)\bar{\mathbf{U}}' + (\boldsymbol{\omega} \cdot \nabla)\mathbf{u} + \frac{\mu}{\rho}\nabla^2(\boldsymbol{\omega}) \quad (\text{A.14})$$

The perturbation flow field is restricted from causing transport into the background flow field, which is not the case in fully turbulent flows. As the background flow-field is defined as the spanwise averaged portion of the flow, the present assumption is valid so long as the spanwise averaged portion of Eqn. (A.14) remains small in comparison to its 3D components. Prior to fully turbulent flow, the spanwise averaged 2D component of Eqn. (A.14) is small compared to the 3D component, and thus for the present study this is an adequate approximation. If this formulation of the perturbation enstrophy transport equations is solved numerically, for example using Floquet Analysis, there is only a one-way coupling between the 2D background flow field and the 3D flow-field, and feedback effects between these terms are not manifested. In the present study these terms are evaluated at each instance in time from DNS data, and so the background flow field does in fact change, although Eqn. (A.13) does not completely describe its development. Should it be desirable to reinstate this coupling for modeling purposes, the spanwise-averaged component of Eqn. (A.14) must be added to Eqn. (A.13) as a source term.

Substituting $\boldsymbol{\omega} = (\bar{\boldsymbol{\Omega}}' + \boldsymbol{\omega})$ and $\mathbf{v} = (\bar{\mathbf{U}}' + \mathbf{u})$ into Eqn. (A.12), yields:

$$\begin{aligned} \frac{1}{2} \frac{D}{Dt} (\bar{\Omega}' + \omega)^2 & \quad (A.15) \\ &= (\bar{\Omega}' + \omega) \cdot ((\bar{\Omega}' + \omega) \cdot \nabla) (\bar{U}' + u) + \frac{\mu}{\rho} (\bar{\Omega}' + \omega) \cdot \nabla^2 (\bar{\Omega}' + \omega) \end{aligned}$$

Like kinetic energy, however, enstrophy is reference frame dependent. Thus $\frac{1}{2} (\bar{\Omega}' + \omega)^2$ has component $\bar{\Omega}' \cdot \omega$ which represents the change in enstrophy resulting from a change in reference frame from an a stationary one to one rotating with $\bar{\Omega}'$. The transport of this enstrophy component may be rewritten in terms of ω and $\bar{\Omega}'$. The transport of this mixed term is expanded using the multivariate product rule:

$$\frac{D}{Dt} (\bar{\Omega}' \cdot \omega) = \bar{\Omega}' \cdot \frac{D}{Dt} \omega + \omega \cdot \frac{D}{Dt} \bar{\Omega}' \quad (A.16)$$

The terms on the RHS of Eqn. (A.16) are obtained by substituting the background and perturbation vorticity transport equations, Eqns. (A.13) and (A.14), yielding:

$$\bar{\Omega}' \cdot \frac{D}{Dt} \omega = \bar{\Omega}' \cdot (\bar{\Omega}' \cdot \nabla) u + \bar{\Omega}' \cdot (\omega \cdot \nabla) \bar{U}' + \bar{\Omega}' \cdot (\omega \cdot \nabla) u + \frac{\mu}{\rho} \bar{\Omega}' \cdot \nabla^2 (\omega) \quad (A.17)$$

$$\omega \cdot \frac{D}{Dt} \bar{\Omega}' = \omega \cdot (\bar{\Omega}' \cdot \nabla) \bar{U}' + \frac{\mu}{\rho} \omega \cdot \nabla^2 (\bar{\Omega}') \quad (A.18)$$

Likewise, as it was assumed that the 2D background flow field was unaffected by the perturbation flow field, transport of background enstrophy is written as:

$$\frac{1}{2} \frac{D}{Dt} \bar{\Omega}'^2 = \bar{\Omega}' \cdot (\bar{\Omega}' \cdot \nabla) (\bar{U}') + \frac{\mu}{\rho} \bar{\Omega}' \cdot \nabla^2 (\bar{\Omega}') \quad (A.19)$$

Eqns. (A.17), (A.18), and (A.19) are subtracted from Eqn. (A.15) to yield the perturbation enstrophy transport equation:

$$\frac{1}{2} \frac{D}{Dt} \omega^2 = \omega \cdot (\bar{\Omega}' \cdot \nabla) (u) + \omega \cdot (\omega \cdot \nabla) (\bar{U}') + \omega \cdot (\omega \cdot \nabla) (u) + \frac{\mu}{\rho} \omega \cdot \nabla^2 (\omega) \quad (A.20a)$$

Eqn. (A.20a) is rewritten in index notation below.

$$\frac{1}{2} \frac{D}{Dt} (\omega_i^2) = \omega_i \bar{\Omega}'_j \partial_i u_j + \omega_i \omega_j \partial_i \bar{U}'_j + \omega_i \omega_j \partial_i u_j + \frac{\mu}{\rho} \omega_i \partial_j^2 \omega_i \quad (\text{A.20b})$$

Where, ∂_i , represents the spatial derivative in i^{th} Cartesian spatial direction. It is noted that the advective velocities found in the expanded advective derivative, $\frac{1}{2} \frac{D}{Dt} (\omega_i^2)$, remain as the instantaneous components of velocity, $(u_j + \bar{U}'_j)$. This can be shown to be true when performing the above derivation with the expanded advective derivative. Eqn. (A.20b) corresponds precisely to the form of the perturbation enstrophy transport equation found in Kundu and Cohen (2004) with the exception that the advective transport terms are not assumed to be small-valued.

The left hand side of Eqn. (A.20) is thus expressed as:

$$LHS = \frac{1}{2} \left(\frac{\partial \omega^2}{\partial t} + (\mathbf{U}) \cdot \nabla (\omega^2) \right) \quad (\text{A.21})$$

In the present cases, $\bar{\mathbf{U}}'$ only has non-zero components in the xy plane.

$$\bar{\mathbf{U}}' = \begin{bmatrix} \bar{U}'_x \\ \bar{U}'_y \\ 0 \end{bmatrix}, \quad \nabla \bar{\mathbf{U}}' = \begin{bmatrix} \frac{\partial \bar{U}'_x}{\partial x} & \frac{\partial \bar{U}'_x}{\partial y} & 0 \\ \frac{\partial \bar{U}'_y}{\partial x} & \frac{\partial \bar{U}'_y}{\partial y} & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \bar{\mathbf{\Omega}}' = \begin{bmatrix} 0 \\ 0 \\ \bar{\Omega}'_z \end{bmatrix}$$

Furthermore, when nondimensionalized by the time and length scales described in Section 4.1, Eqn. (A.20b) is rewritten as A.22.

$$\begin{aligned} \frac{1}{2} \frac{\partial}{\partial t} (\omega^2) = & \left(\omega_x^2 \left(\frac{\partial \bar{U}'_x}{\partial x} \right) + \omega_y^2 \left(\frac{\partial \bar{V}'_y}{\partial y} \right) + \omega_z \Omega_z \left(\frac{\partial \omega}{\partial z} \right) \right) + \omega_x \omega_y \left(\frac{\partial \bar{U}'_y}{\partial y} + \frac{\partial \bar{V}'_x}{\partial x} \right) \\ & + \omega_x \Omega_z \left(\frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z} \right) + \omega_y \Omega_z \left(\frac{\partial v}{\partial z} + \frac{\partial \omega}{\partial y} \right) - \frac{1}{2} \left((U) \frac{\partial}{\partial x} \left(\frac{\omega^2}{2} \right) + V \frac{\partial}{\partial y} \left(\frac{\omega^2}{2} \right) + w \frac{\partial}{\partial z} \left(\frac{\omega^2}{2} \right) \right) \\ & + \frac{1}{Re_\theta} (\omega) \cdot \nabla^2 (\omega) \end{aligned} \quad (\text{A.22})$$

Lastly, the viscous term may be decomposed using the product rule, into its viscous diffusion and viscous dissipation components:

$$\frac{1}{Re_\theta} (\boldsymbol{\omega}) \cdot \nabla^2 (\boldsymbol{\omega}) = \frac{1}{Re_\theta} \left(\frac{1}{2} \nabla^2 (\boldsymbol{\omega}^2) - (\nabla \boldsymbol{\omega})^2 \right) \quad (\text{A.23})$$

A.2 Derivation of perturbation growth rate relation n presented in Eqn. (4.11)

$$\sigma_{\frac{1}{2}\omega^2} = \frac{d}{dt} \ln \left(\langle \frac{1}{2} \omega^2 \rangle_{xyz} \right) \quad (\text{A.24})$$

$$\sigma_{\frac{1}{2}\omega^2} = \frac{1}{\langle \frac{1}{2} \omega^2 \rangle_{xyz}} \frac{d}{dt} \langle \frac{1}{2} \omega^2 \rangle_{xyz} \quad (\text{A.25})$$

As $\langle \frac{1}{2} \omega^2 \rangle_{xyz}$ is evaluated over the region enclosed by CV, and because the average operator, and the (partial) differential operator are both linear,

$$\frac{d}{dt} \langle \frac{1}{2} \omega^2 \rangle_{xyz} = \langle \frac{\partial}{\partial t} \frac{1}{2} \omega^2 \rangle_{xyz} \quad (\text{A.26})$$

Neglecting the convective terms from Eqn. A.22 and substituting it into A.26:

$$\begin{aligned} \langle \frac{1}{2} \frac{\partial}{\partial t} (\omega^2) \rangle_{xyz} = & \left\langle \left(\omega_x^2 \left(\frac{\partial \bar{U}'}{\partial x} \right) + \omega_y^2 \left(\frac{\partial \bar{V}'}{\partial y} \right) + \omega_z \bar{\Omega}'_z \left(\frac{\partial w}{\partial z} \right) \right) + \omega_x \omega_y \left(\frac{\partial \bar{U}'}{\partial y} + \frac{\partial \bar{V}'}{\partial x} \right) + \right. \\ & \left. \omega_x \bar{\Omega}'_z \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) + \omega_y \bar{\Omega}'_z \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right\rangle_{xyz} + \langle \frac{1}{2 Re_\theta} (\boldsymbol{\omega}) \cdot \nabla^2 (\boldsymbol{\omega}) \rangle_{xyz} \end{aligned} \quad (\text{A.27})$$

The inviscid and viscous transport terms are expressed in terms of $\mathcal{S}$ and $\mathcal{D}$.

$$\begin{aligned} \mathcal{S}(t) = & \left\langle \left(\omega_x^2 \left(\frac{\partial \bar{U}'}{\partial x} \right) + \omega_y^2 \left(\frac{\partial \bar{V}'}{\partial y} \right) + \omega_z \bar{\Omega}'_z \left(\frac{\partial w}{\partial z} \right) \right) + \omega_x \omega_y \left(\frac{\partial \bar{U}'}{\partial y} + \frac{\partial \bar{V}'}{\partial x} \right) \right. \\ & \left. + \omega_x \bar{\Omega}'_z \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) + \omega_y \bar{\Omega}'_z \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right\rangle_{xyz} \end{aligned} \quad (\text{A.28})$$

$$\mathcal{D}(t) = \langle \frac{1}{2 Re_\theta} (\boldsymbol{\omega}) \cdot \nabla^2 (\boldsymbol{\omega}) \rangle_{xyz} \quad (\text{A.29})$$

$$\left\langle \frac{\partial}{\partial t} \left(\frac{\omega^2}{2} \right) \right\rangle_{xyz} = \mathcal{S}(t) + \mathcal{D}(t) \quad (\text{A.30})$$

Perturbation growth rate is thus expressed in terms of the perturbation enstrophy transport terms.

$$\sigma_{\frac{1}{2}\omega^2} = \frac{1}{\langle \frac{1}{2}\omega^2 \rangle_{xyz}} (\mathcal{S}(t) + \mathcal{D}(t)) \quad (\text{A.31})$$

Next, the ratio of viscous diffusion and dissipation, $\mathcal{R}$, is defined:

$$\mathcal{R} = -\frac{\mathcal{S}(t)}{\mathcal{D}(t)} \quad (\text{A.32})$$

Eqn. (A.31) is recast in terms of the inviscid growth rate, $\mathcal{s}$, and an efficiency factor in Eqn. (A.34).

$$\sigma_{\frac{1}{2}\omega^2} = \frac{\mathcal{S}(t)}{\langle \frac{1}{2}\omega^2 \rangle_{xyz}} \left(\frac{\mathcal{R}(t)-1}{\mathcal{R}(t)} \right) \quad (\text{A.33})$$

$$\sigma_{\frac{1}{2}\omega^2} = \mathcal{s}(t) \left(\frac{\mathcal{R}(t)-1}{\mathcal{R}(t)} \right) \quad (\text{A.34})$$