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STRONG GROUND MOTIONS FOR BRIDGE DESIGN AND NON-LINEAR DYNAMIC RESPONSE ANALYSIS OF BRIDGES

by

Vietanh Phung

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements

for the degree of
Doctor of Philosophy

**Department of Civil and Environmental Engineering
Carleton University, Ottawa
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Submitted by

Vietanh Phung

In partial fulfillment of the requirements for
the degree of Doctor of Philosophy

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Dedicated to my parents

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Abstract

Recent major earthquakes have revealed some major problems in seismic analysis and design of bridge structures: (i) strong potential of directivity effects of ground motions are not explicitly considered in the design process, (ii) elastic analysis of bridge responses may not give sufficiently accurate results for safe design especially when large deformations can lead to significant damage, and (iii) uniform rigid support excitation does not adequately describe earthquake loadings on long-span bridges. In order to address some of these concerns and to advance the state of seismic bridge engineering, the objectives of this present study are: (i) to develop a methodology to characterize ground motions for seismic design of bridges, (ii) to develop non-linear time history analysis methods using Ritz vectors, (iii) to develop an accurate and efficient beam-column element model for bridge columns, and (iv) to study the effects of multi-support excitations on responses of long-span bridges.

The ground motion records of the 1999 Chi-Chi, Taiwan earthquake are used to develop a methodology to characterize ground motions in terms of site and directivity effects. A methodology is developed to estimate the site classification (rock vs. soil) of Taiwan stations. The method is based on the analysis of the shape of 5% damped pseudo-acceleration spectrum of horizontal ground motion component normalized with respect to PGA from 87 stations of known site conditions. The results obtained from the proposed methodology correlate well with the available known site information data. Developed methodology is applicable to estimate site classification at other regions. Directivity effects are quantified by the residuals or model error of the median ground motion

relationship expressed as a function of selected geometrical parameters related to the fault. The geometrical parameters include the fraction of the fault rupture that lies between the hypocenter and the site, and the angle between the fault plane and the path from the hypocenter to the site. Analysis indicates that directivity effects are significant for peak ground acceleration and pseudo-acceleration at all periods.

Advanced Ritz vector methods for inelastic response analysis of bridge structures are developed. The methods are then implemented in NEABS program. The Ritz algorithms are the Lanczos method and the WYD method, wherein both methods consider the spatial distribution of the earthquake loading. Two inelastic methods considered are pseudo-force method and tangent spectrum method. The pseudo-force method is efficient in terms of computation effort. This method provides an accurate numerical result if the non-linearities are not severe. It is found that the computational time to calculate the time history response of bridge systems using the derived Ritz vectors is approximately 25% to 75% of the time required by using direct step-by-step integration. The tangent spectrum method is more expensive in terms of computation effort. New set of base vectors is computed if second order norm of stiffness matrix is larger than a specified tolerance. A cut-off criterion based on the concept of effective mass participation factor is used to determine the number of derived Ritz vectors needed to be included in the time history analysis.

A 3D non-linear beam-column model is developed to account for accumulated damage in the bridge columns/piers experienced during earthquakes. The model implicitly includes the strength deterioration and stiffness degradation behaviour.

Response of bridges considering strength deterioration and stiffness degradation are significantly different from those obtained from elasto-plastic model.

Seismic response of long-span bridges to multi-support excitations is examined. It is found that for long-span bridges, multi-support excitations can induce a significantly different behaviour compared to those by uniform support excitations.

Nomenclature

$a_0, a_1, a_2, \text{ and } a_3$	Constants for yield surface for y-direction
A	Cross-sectional area, or Azimuth angle (radians)
$\mathbf{A}$	$\mathbf{A} = \mathbf{K}^{-1}\mathbf{M}$
$b_0, b_1, b_2, \text{ and } b_3$	Constants for yield surface for z-direction
b_i	Column i th of influence coefficient matrix $\mathbf{B}$
$\mathbf{B}$	(1) Distribution of load vector (2) Influence coefficient matrix
C	Softening coefficient
$\mathbf{C}$	(1) Damping matrix (2) Deterioration coefficient matrix
$C_1, C_2, C_3, C_4, \text{ and } C_5$	Coefficients of the ground motion relations
$C_i (i=1, 16)$	Constants for dynamic response analysis
$\mathbf{C}_L$	Damping matrix of linear members
$\mathbf{C}_N$	Damping matrix of non-linear members
$\mathbf{C}_{NL}$	Damping matrix of non-linear and linear members
d_b	Reinforcing bar diameter
$d\mathbf{r}$	Element displacement increment vector
$d\mathbf{s}$	Element force increment vector
D	(1) The shortest distance to the fault (km) (2) Damage index
$\mathbf{D}$	(1) Diagonal part of stiffness matrix (2) Damage matrix
E	Young's modulus
E_h	Dissipated energy
$\mathbf{f}(t)$	Applied dynamic load
$\mathbf{f}(s_k)$	Yield function
$\mathbf{H}$	Matrix relating damage and deformation
I	Inertia moment of a section
$\mathbf{I}$	Unit matrix
$\mathbf{K}$	Stiffness matrix
$\mathbf{K}^e$	Elastic stiffness matrix
$\mathbf{K}^{cd}$	Degradation stiffness matrix
$\mathbf{K}^{ep}$	Elasto-plastic stiffness matrix
$\mathbf{K}^{epd}$	Softening-elasto-plastic stiffness matrix
$\mathbf{K}_L$	Stiffness matrix of linear members
$\mathbf{K}_N$	Stiffness matrix of non-linear members
$\mathbf{K}_{NL}$	Stiffness matrix of linear and non-linear members
$\mathbf{K}_T$	Effective loading
l_p	Plastic hinge length
L	Length of beam-column element
$\mathbf{L}$	Lower triangular matrix ($\mathbf{LU}$ or Choleski factorization)
M_y	Yielding moment
M_{y0}	Yielding moment about y-axis

M_{yp}	Ultimate bending moment about y-axis
M_{yu}	Bending moment about y-axis
M_{z0}	Yielding moment about z-axis
M_{zp}	Ultimate bending moment about z-axis
M_{zu}	Bending moment about z-axis
$\mathbf{M}$	Mass matrix
P_t	Ultimate tensile axial force
p_u	Normalized axial force
P_0	Ultimate compressive axial force
P_u	Axial force
PGA	Peak Ground Acceleration
PSA	Pseudo-Spectral Acceleration
$\mathbf{Q}$	Restoring force
$\mathbf{q}$	Ritz vector
$\mathbf{r}$	Displacement vector
$\mathbf{r}$	Ritz vector
$\mathbf{s}$	Influence coefficient vector
S	Site condition parameter, $S = 1$ for soil, and $S = 0$ for rock
SR	Spectral Ratio
$S_1, S_2, S_3,$ and S_4	Site classification for hard rock, rock and stiff soil, medium stiff soil, and soft soil
T	Vibration period (sec)
T_g	Predominant period of ground motion (sec)
$\mathbf{T}_m$	Tridiagonal matrix
$\mathbf{u}$	Displacement vector
$\dot{\mathbf{u}}$	Velocity vector
$\ddot{\mathbf{u}}$	Acceleration vector
V_s	Shear wave velocity
$\mathbf{x}$	Ritz vector
Z	Eigenvalue of reduced system
α	$\mathbf{M}$ proportional coefficient
α_i	Amplitudes of q_i
β	$\mathbf{K}$ proportional coefficient
β_i	Amplitudes of q_{i-1}
γ_i	Amplitudes of q_{i-2}
ϕ_y	Curvature at yield
ϕ_r	Curvature beyond which deterioration begins
ϕ_{max}	Curvature at which moment capacity is zero
Φ	Mode shape matrix
λ	(1) Eigenvalue (2) Positive scalar of proportionality constant
ε	(1) Acceleration time history applied at supports (2) Error norm
ω	Natural frequency of vibration
Ψ	Mode shape vectors
ϕ	Eigenvector

δ	(1) Kronecker delta (2) Newmark integration coefficient (3) Displacement
σ	Newmark integration coefficient
ρ	Rayleigh ratio
ξ	Mass participation factor, or direction of plastic flow
η	Mass participation factor
θ_p	Plastic rotation angle (radians)
θ_r	Plastic rotation angle beyond which deterioration begins (radians)
$\theta_{\max}$	Maximum plastic rotation angle (radians)

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Chapter 1

Introduction

1.1 Seismic performance of bridges in recent major earthquakes

The occurrence of strong earthquakes in urban areas not only causes loss of lives and injuries, but also results in significant economic loss due to damage of infrastructure and disruption to business and other activities. A critical part of modern infrastructure of very significant economic importance is the highway system. A bridge is a key component of a highway network system, which supports crucial commercial and economic activities vital to the well-being of the society for three reasons: (i) it controls the capacity of the system; (ii) it is the highest cost per kilometer of the system; and (iii) if the bridge fails, the system fails (Barker et al., 1997). Every day, close to 4 billion vehicles cross bridges in the United States, and in Canada the equivalent traffic volume is about 40 million vehicle bridge crossings. The public expects these bridges to be safe and be able to accommodate this high volume of traffic. In the last 30 years, the United States has spent over \$55 billion in the Highway Bridge Replacement and Rehabilitation Program (HBRRP) by various levels of government to improve the condition of existing bridges in their highway networks (FHWA, 1999). The Federal Highway Administration (FHWA, 1999) has the mandate to ensure the safety and adequate performance of bridges and

mitigation of natural hazards including earthquakes to these important infrastructures. There are about 575,000 bridges in the United States, and Canada has about 10,000 bridges in the existing transportation networks. About 60 percent of these bridges were constructed before 1970 with little or no consideration given to seismic resistance in their design. Furthermore, the significant increase in traffic volume and loads in conjunction with deterioration of the conditions of bridge structures over the years have increased the vulnerability of these existing bridge structures to earthquake damage. Therefore, many existing bridges especially those constructed before 1970 need to be retrofitted to enhance their seismic performance during earthquakes to meet the current seismic design standards and codes.

During recent major earthquakes in California, Japan and Taiwan, a significant number of existing bridges suffered damage of varying degree from total collapse to major damage to their substructures and foundations. During the 1964 Prince William Sound, Alaska earthquake (1964), nearly every bridge along the then partially completed Cooper River Highway in Alaska was seriously damaged or destroyed. Seven years later during the San Fernando earthquake (1971), more than 60 bridges on the Golden State Freeway in California were damaged. Post-disaster investigations have concluded that bridge failures during the San Fernando earthquake were caused by excessive displacements between adjacent bridge frames at the expansion joints, which resulted in unseating failure of bridge spans at intermediate hinges. The earthquake costed the State of California approximately \$100 million in bridge repairs. In 1989, the Loma Prieta earthquake in California (1989) damaged more than 80 bridges and caused more than 40 deaths in bridge-related collapses. The cost of the earthquake damage to transportation

structures was \$1.8 billion, of which the damage to state-owned bridges was about \$300 million. In 1994, the Northridge earthquake in California (Moehle, 1995) caused 6 bridges to fail and 4 others were so severely damaged that they had to be replaced. The cause of failure was due primarily to poor detailings of the columns and joints. In 1995, the Hanshin/Awaji earthquake at Kobe and Awaji Island in Japan (Kawashima, 1995) caused intensive damage to highway and railway bridges. Post-earthquake investigations have found that the damage at 9 bridge sites was attributed to insufficient seat length for supporting bridge spans during the earthquake. In Taiwan, the 1999 Chi-Chi earthquake caused 12 bridge collapses and severely damaged many other bridges (Wallace, 2001). In many cases, the primary cause of collapse was again insufficient seat length for precast concrete girders supported on bent caps. Examples of this failure mode include the Shr-Wei bridge, the Ming-Ju bridge, the Wu-Shi bridge, the Bei-Feng bridge, the Chang-Geng bridge, and the Yi-Jiang bridge. Based on observations of bridge damage during recent major earthquakes, the unseating failure of bridge spans and poor detailings of bridge piers or columns are common causes of bridge collapse. The source of unseating failure can be traced back to underestimating the maximum deformation or displacement experienced by bridges and bridge components, which experienced significant inelastic actions during strong earthquakes. The underestimation of maximum deformation or displacement is due to a number of reasons. First, it is due to the lack of accurate analysis method to compute response of bridges. This can be corrected by using more accurate analysis methods such as non-linear time history response analysis methods. Second, it is due to the lack of accurate models to characterize the essential behaviour of bridge elements. Third, multi-support excitations can induce different behaviour to long-span

bridges. This was clearly observed in Loma Prieta (1989) and Kobe (1995) earthquakes. Therefore, the effects of multi-support excitations on the response of long-span bridges need to be considered. Fourth, effects of near-fault ground motions on response of bridges are not explicitly addressed. For bridge structures to be safe for the public, the above four mentioned issues need to be thoroughly studied.

1.2 Seismic design of bridge structures

Bridge design procedures are generally based on two requirements, namely safety and serviceability. Safety relates to extreme loadings such as earthquake and wind loadings, which have a very low probability of occurrence during the structure's life, and it is concerned with the collapse of the structure, major damage, and loss of life. Serviceability relates to medium to large loadings. For service loading, structures should remain operational. In accordance with the generally accepted design philosophy of bridges for seismic resistance, bridges are designed to resist small earthquakes with little damage, but may experience repairable damage during moderate earthquakes. During large earthquakes, bridges may suffer considerable and even non-repairable damage at critical locations, but should not collapse.

Prior to 1970, bridge structures were designed for strength by following the elastic design approach. The elastic design approach is still followed in some countries. In early seismic design of bridges, the concept or approach of strength-based design was adopted. The design earthquake force was determined from acceleration spectra of the design earthquake using the period of the design bridge, which was then applied as a uniform lateral load along the span of the bridge. In the strength-based design, a bridge

was designed to have sufficient strength to resist the earthquake inertial effect from horizontal ground shaking. Once the structure is proportioned, the stiffness properties are derived and serviceability constraints are checked. Iteration is usually necessary for convergence to an acceptable design. This approach is referred to as strength-based design since the elements are proportioned according to strength requirements. The strength-based design approach is appropriate when strength is the dominant design requirement. In the past, most structural design problems have fallen in this category. However, the effectiveness of the strength-based approach has become limited due to recent developments. The trend towards more flexible structures such as long-span bridges has resulted in more structural displacement under serviceability and safety loadings. Recent advances in material science and engineering have resulted in significant increase in the strength of traditional civil engineering materials. However, material stiffness has not increased at the same rate. This has led to the problem with satisfying the requirements on displacements. Furthermore, since major earthquakes are rare events within the life span of most bridges, it is uneconomical to design bridge structures to have sufficient elastic strength to resist the maximum design earthquake without any damage. Thus it is generally accepted to design bridges to have lower strength than that required to resist the earthquake actions in the elastic range; it is accepted that bridges may sustain some degree of damage during major earthquakes. The consequences of this design philosophy are that seismic deformations during strong earthquakes are seriously underestimated for bridges designed by the elastic strength-based design approach. In the elastic strength-based design, inelastic behaviour of bridge structures and the associated concepts of ductility are not considered. Major earthquakes

have shown that strength considerations are important; however, it is not sufficient to consider only strength. Earthquake damages are found to be more related to excessive displacement than strength.

In the 1970s, it was realized from lessons learned during major earthquakes that bridge structures designed based on elastic strength would survive shaking of greater intensity than that corresponding to the elastic ultimate strength. The concept of ductile response behaviour of the bridge system before the ultimate collapse state was recognized. This led to the development of the strength-based ductile design approach. Nowadays, most bridge design standards and codes around the world are based on the strength-based ductile design concept. In this approach, seismic elastic design strength of a bridge structure is determined from elastic response analysis using an elastic design spectrum, and then reduced by a force reduction factor to calculate inelastic demand, which is a function of ductility of the structural response at ultimate collapse state. The strength-based ductile design approach recognizes the high cost of designing structures to remain elastic during maximum design earthquake events, and takes advantage of the reserve ductility capacities of the structure to dissipate the seismic energy after its elastic strength is exceeded. It is thus expected that the strength-based ductile design approach leads to more economical design with acceptable seismic performance of structures. More recently, advances in seismic analysis and design of structures have shown that while the strength-based ductile design approach is a significant improvement from the elastic strength-based design, it may not lead to adequate estimation of the seismic demands, because it does not explicitly take into consideration the failure mechanisms of the structure.

Recognizing this shortcoming of the strength-based ductile design approach, the capacity-based design procedure has been developed (Paulay, 1992). The basic idea of the capacity design is to proportion and detail the structural elements or components of the structure based on a pre-selected specific failure mechanism of the structure, in accordance with the seismic performance of the systems. The failure mechanism is selected based on cumulative experiences of the seismic response characteristics and performance of that type of structure, or from more detailed non-linear inelastic analysis, such as quasi-static pushover or time history response analysis for more complex systems. In this approach, members expected to undergo plastic behaviour are designed with adequate detailings to ensure they have the reserve ductility capacity to sustain the large deformation demands on them during severe shaking of major earthquakes. Other members connecting to the inelastic components are designed to remain elastic throughout the earthquake to ensure that the assumed failure mechanism actually can occur. In the capacity-based design, the performance of the structure is controlled by structural elements or members with plastic behaviour that act as fuses, which limit the amount of force along the load path of the structures during its dynamic response. In bridge design, the most common capacity-based approach is to design the columns as ductile elements in which the load resistant capacities are limited by their plastic hinge moment capacities; the bridge girders are designed to remain elastic. To ensure the inelastic columns can sustain the large deformation and rotation demands imposed on them during strong earthquakes, the ductile behaviour of the columns is ensured by providing sufficient confinement to the column longitudinal reinforcements at the plastic hinge locations, which are generally at the column ends. The connecting superstructure

members and footings or other foundation components are designed to respond elastically to the maximum plastic force demands from the columns. Although the capacity design approach specifically defines the maximum seismic forces imposed on the design bridge and its components, there are still some limitations inherent to this approach. Non-linear behaviour of the structure is recognized in the design, but the dynamic response effects of the structure are still not adequately considered in some cases, resulting in the underestimation of column displacement demands.

In the latest developments of seismic design of structures, it is recognized that earthquake failure of a bridge is more related to excessive displacement rather than force-based design issues. Consequently, the concept of displacement-based design is incorporated into the capacity design concept to result in designs with more satisfactory performance during strong earthquakes. In the displacement-based design of bridges, the displacement capacity of the structure is typically computed from moment-curvature analysis of critical sections of the column with an assumed plastic hinge length.

Another problem in seismic design of bridges is that the design process does not adequately address the near-fault effects for bridges located near the fault ruptures; the seismic demands associated with large velocity pulses during near-fault ground motions are very high. As a result, many existing bridges susceptible to near-fault loading effects may not have the capacity to resist the seismic actions. To assess the seismic demands on bridges from near-fault ground motions, a non-linear time history response analysis may be required.

1.3 Seismic analysis of bridge structures

In order to determine the seismic demands of the analyzed bridge, a seismic analysis is required. Seismic analysis of bridges can be performed either by elastic methods or by inelastic methods. In the United States, the National Cooperative Highway Research Program (NCHRP) Project 12-49 Guidelines (1999) and in Canada, the Canadian Highway Bridge Design Code (2000) provide detailed instructions on four analysis methods for seismic design of bridges as follows:

- (1) Uniform load method
- (2) Single-mode spectral method
- (3) Multimode spectral method
- (4) Step-by-step time history method

The first three methods are often used for elastic analysis, whereas the step-by-step time history method can be used for either elastic or inelastic analysis. The choice of an analysis method is based on the complexity of the analyzed bridge. It is important that the seismic analysis of the bridges capture all of the essential behaviour of the bridge responses, such as yielding of ductile elements, the opening and closing of the expansion joints, strength and stiffness degradation behaviour, and the increase in the damping level when large permanent deformation occurs. Clearly, only inelastic analysis can accurately capture all these non-linear behaviour, however elastic analysis is still necessary and desirable and continues to be the most convenient and effective method for ordinary design because it can still be used to obtain useful estimates on the overall inelastic structural systems for design purposes. There are several reasons why the elastic analysis

is still used in seismic analysis of ordinary bridge structures, as opposed to the more realistic inelastic analysis:

- (1) It is relatively easy to model and analyze a bridge for linear response analysis,
- (2) The validity of the principle of superposition in elastic analysis allows the use of the response spectrum method to obtain estimates on the maximum response quantities such as maximum deck displacements and maximum column moments,
- (3) Analytical research results show that for typical bridges the inelastic behaviour can be approximately obtained by using elastic results, which may not be accurate but sufficiently so for design. For example, inelastic strength demands and displacement demands can be estimated from elastic analysis in conjunction with the force reduction factors.

However, post-earthquake investigations on bridge performance have shown that the use of elastic analysis results may not lead to conservative estimates of the inelastic behaviour and demands on the bridge for a safe design, especially when large deformations can result in significant damage or failure of the bridge. The use of the elastic analysis results in conjunction with the force reduction factor obtained from the analysis of equivalent single-degree-of-freedom systems does not accurately give the inelastic demands for a multi-degree-of-freedom system. The effects of closing and opening of the expansion joints is a non-linear behaviour that cannot be modeled in an elastic analysis. The effects of the changes to the damping characteristics and properties

on the responses as the structural system yields and undergoes permanent deformations during major earthquakes are not considered in elastic analysis. The strength and stiffness degradation behaviour are ignored in elastic analysis and design. Until recent years, elastic analysis has been sometimes still preferred because of the high demands on computing resources of inelastic analysis. With the latest developments in high speed computing, and non-linear structural modeling and analysis techniques, it is now possible to perform more accurate inelastic analysis of bridges on a routine basis. Non-linear time history analysis of bridge structures can be carried out to obtain more accurate assessment of the displacement demands of complex structures so that more meaningful capacity/demand ratios can be determined. This is especially necessary for complex and important bridges, or for bridges with damping devices, isolators, or other significant sources of inelastic behaviour (i.e., expansion joints, plastic hinges). Today's challenge is to design bridges for the variety of hazards that can occur during an earthquake. Particularly problematic are near-fault effects that may cause very large demands and challenge the traditional methods of seismic analysis and design of bridges (Yashinsky and Karshenas, 2003). Despite the advancements in computer resources, there are still limitations on accuracy in the modeling of the inelastic behaviour of bridge structures. Hence, there is still the need to develop efficient and accurate analysis procedures to evaluate the inelastic response of bridges.

Many existing structures were designed using outdated and/or inadequate design codes and standards and need to be considered for retrofit. Retrofit and rehabilitation of existing bridges require the use of accurate analysis such as non-linear inelastic analysis to determine the inelastic responses of the retrofitted bridge because the results have

significant impacts on the need, extent and approach of retrofit or rehabilitation, which have direct consequence on the economic costs. Non-linear inelastic analysis often reduces the cost of rehabilitation by minimizing the demand level of the retrofit.

During the last few decades, the development of high-speed digital computers and the Finite Element Method (FEM) have significantly expanded the range and complexity of structural problems that can be solved (Bathe, 1996). Improvements in accuracy of the FEM can be achieved by an increase in the number of degrees-of-freedom in the structural model. For large and complex bridge structures, the large number of degrees-of-freedom of the analyzed structure generally leads to large amounts of input data analysis, preparation and analysis results which are often difficult to interpret. Thus, effort is directed towards the development of more efficient non-linear structural modeling and analysis techniques, which can accurately compute the responses of bridge structures under severe dynamic loadings such as major earthquakes. The efficiency and accuracy of the analysis of large and complex structural systems has been the subject of a number of studies (Wilson et al., 1982 and Chang et al., 2001).

In the treatment of dynamic problems for large and complex structural systems, the reduction of computation effort in solving the dynamic equilibrium equations is important because the solution of the equations of motion must be performed repeatedly at different time steps to obtain the dynamic responses of the systems. There are two approaches to reduce the computational effort, namely:

- (1) Static condensation method
- (2) Rayleigh-Ritz method

The static condensation method is based on the assumption that the inertia effect may be neglected in some selected degrees-of-freedom of the structural finite element model, such as rotational degrees-of-freedom. The validity of this assumption depends on the characteristics of the loading patterns and dynamic behaviour of the structural systems. In the Rayleigh-Ritz method approach, the behaviour of the structural system is represented by a combination of assumed displacement patterns, and the amplitude of each is represented as a generalized coordinate or generalized degree-of-freedom of the system. Traditionally, the exact eigenvectors are used to reduce the size of structural model since experience has shown that the seismic responses of most bridges can be adequately represented by the dynamic responses of the first few vibration modes of the structure. The time history response of the structural system during earthquakes is computed by modal superposition. However, the solution of the eigenvalue problem to determine the vibration modes of large and complex structural systems is expensive. Another problem with the modal superposition method is that sometimes a large number of mode shapes is needed to obtain the dynamic responses of complex structures. Furthermore, the influence of load distribution is not reflected in the exact modes. Thus the use of a limited number of, or truncated series of, vibration modes to represent the dynamic responses of complex structures may result in missing mass or inadequate consideration of the inertia effects. In the Canadian Highway Bridge Design Code (2000), the seismic analysis model of the design bridge is required to capture at least 90% of the structural mass. In this consideration, some of the vibration modes are of no significance to the dynamic response of the structure if they are orthogonal to the load distribution acting on the structure. Therefore, this indicates that the spatial distribution of the load action should be

taken into consideration in the construction of the structural model for dynamic response analysis.

Research studies have shown that the direct engineering significance of vibrational frequencies and mode shapes of structures is of limited value (Wilson et al., 1982). Frequencies indicate possible resonant conditions, and mode shapes associated with low frequencies can indicate which regions of structure are most flexible. For most analyses, the only reason to use the exact eigenvectors is to reduce the size of the analysis model in the mode superposition method. In many cases, approximate values for frequencies and mode shapes can provide the same information. Lanczos (1950, 1952) had originally presented a procedure to compute the first few eigenvectors of a large sparse symmetric matrix. However, the procedure was not widely known to the engineering community because of the lack of computational stability. More recently an iterative procedure of the Lanczos approach was presented by Nour-Omid and Clough (1989). Wilson et al. (1982) independently presented a method for estimating a set of orthogonal vectors for direct superposition to obtain dynamic responses of structures. It has been found that the superposition of the special set of the derived Ritz vectors provides more accurate numerical results compared to those obtained from the superposition of exact modes. The Ritz vectors are constructed from the spatial distribution of the specified dynamic loads and are evaluated by a simple recurrence algorithm at a fraction of the computational effort required for calculation of exact mode shapes. Not only do the Ritz vector techniques significantly reduce computational time and problem size, the derived Ritz vector techniques highlight the fundamentals of the modal behaviour. The Ritz vector techniques present more physical meaning compared to

the finite element modeling, especially for large and complex bridge structures. Hence the use of derived Ritz vector techniques can lead to more direct engineering interpretation of the analysis results.

For linear structural systems, the superposition of derived Ritz vectors provides an efficient and accurate solution for time history response analysis (Wilson et al., 1982). The Ritz algorithms have been implemented in a computer program to evaluate the linear time history responses of bridge structures (Ruchala, 1997). For non-linear inelastic structural systems, the direct time step integration method of the full size coupled equations of motion is usually utilized. Recognizing the advantages of the Ritz vector algorithms, it is possible to develop a non-linear time history analysis procedure based on superposition of derived Ritz vectors. Two non-linear analysis procedures using superposition of derived Ritz vectors are presented in this study namely the pseudo-force method and the tangent spectrum method. The pseudo-force method uses the initial elastic properties of the structure throughout the analysis process and the non-linear effects are considered in the right hand side of the equations of motion. This method is efficient in computation time. However the loss of accuracy is significant when the extent of non-linearity in the behaviour of the gradually damaged structure become severe. The tangent spectrum method can give accurate numerical results with a sufficient number of derived Ritz vectors. However the computation cost is high because the properties of the structures are repeatedly updated and a new set of derived Ritz vectors is generated at many different steps during the responses to reflect the current damaged condition of the structure.

The rapid development of modern numerical analysis technique and high-speed digital computers has also provided structural engineers with powerful tools for a complete non-linear analysis of concrete structures. By using FEM and performing an incremental inelastic analysis, deformation and failure characteristics of concrete structures can be assessed with some degree of accuracy.

To date, there are a number of general-purpose non-linear analysis programs such as NEABS (Tseng and Penzien, 1973), ABAQUS (1998), ADINA (1995), and the DRAIN (Prakash, 1992) series of programs. However, most of these programs generally lack the features needed to make them useful for bridge analysis. They do not include elements, which are suitable for modeling bridge components, such as reinforced concrete bridge columns and expansion joints. It is fair to say that software specifically tailored for bridge engineering is needed, as special purpose stand alone programs and/or as additions to general purpose programs (Powell et al., 1992). The NEABS (Non-linear Dynamic Analysis of Bridge Systems) computer program is one of a few specialized programs for analyzing bridge structures subject to dynamic loads. The original version of the NEABS computer program was developed at the University of California, Berkeley by Tseng and Penzien (1973) for conducting detailed linear and non-linear dynamic analysis of bridge structures. However, this original program lacks some important features to make it a more useful tool for seismic analysis of bridges such as:

- (a) Frequency and mode shape analyses, which is used to estimate the maximum structural responses in the mode spectral methods

- (b) Non-linear beam-column model in the original release of the NEABS program does not have the modeling capabilities to characterize the strength and stiffness degradation behaviour of non-linear members.
- (c) Ground motion input is only modeled by uniform rigid support excitation.

In the present study, the NEABS program is further developed to perform frequency and modal analyses. The non-linear time history analysis procedures based on superposition of derived Ritz vectors by the pseudo-forced method and the tangent spectrum method are developed and implemented in the NEABS computer program. The strength and stiffness degradation of non-linear beam-column element stiffness are derived and used to compute non-linear time history responses of bridges. Finally, a module is developed for analysis of bridges subject to multiple-support excitations.

1.4 Characteristics of ground motions for bridge design

In earthquake resistant design of bridge structures, it is important to know the characteristics of the expected ground motions at the site because the responses of bridges are strongly dependent on the ground motion characteristics. In earthquake engineering, ground motion characteristics are often represented by ground motion relations, which are used to obtain estimates of force and/or displacement demands needed for structural design. The most commonly used ground motion intensity measure is the horizontal spectral pseudo-acceleration (PSA) at a specified damping level (usually 5% of the critical damping). A number of ground motion relations of this parameter have been proposed for various tectonic regimes: active region (Boore et al., 1997), subduction zone (Crouse, 1991), and intra-plate region (Atkinson, 2004). Ground motion relations

related to other intensity measures have also been proposed, including peak horizontal velocity, vertical spectral acceleration, duration-related parameters, and mean period. Ground motion relations are generally derived from or calibrated against, strong ground motion records. The ground motion relations specify the ground motion parameters (e.g., PSA) as functions of earthquake magnitude, distances to the fault, site conditions, and sometimes other parameters such as depth, focal mechanism, hanging wall and footwall effects, and possible directivity effects. The magnitude and distance parameters are especially important as ground motion amplitude generally increases with earthquake magnitude and attenuates with distance. The influence of local soil conditions on the characteristics of ground motions is another major factor in seismic design, particularly of bridges, which are often located in or near areas of soft river sediments. The significance of site effects has long been recognized by engineers and seismologists (Borcherdt, 1970 and Seed et al., 1970). It is known that soil sites tend to amplify ground motions more than rock sites, particularly at longer periods, and site amplification is a significant factor in determining damage potential of the ground motions. The devastating effect of site amplification was clearly demonstrated during the 1985 Mexico City earthquake, in which motions were amplified by an order of magnitude by the soft lakebed sediments of Mexico City (Singh et al., 1989). In general, the ground motion amplification factor of soil sites with respect to rock sites is between 1.5 and 3.0 on average (Takahashi et al., 2003). Therefore, in order to have a safe and efficient design it is important to know the site condition at the location of the engineered structure. Similarly, as ground motion characteristics are determined from ground motion relations,

the site conditions of recording stations are important to develop robust ground motion relations from empirical data.

In Taiwan, the 1999 Chi-Chi earthquake has increased the number of worldwide near-field strong motion recordings several fold (Lee et al., 1999). Prior to the 1999 Kocaeli, Turkey earthquake there were 8 near-fault recordings. The 1999 Kocaeli, Turkey earthquake added 5 recordings. The 1999 Chi-Chi, Taiwan earthquake added 68 recordings. However, the site conditions at many stations in Taiwan are unknown or missing. Hence, it is necessary to obtain the site condition of stations where strong-motions were recorded in order to properly separate source, path and site influences. A significant amount of effort on collecting geotechnical information for recording stations has been carried out in Taiwan (Kou, 1992-1994). The collection of such information by shear-wave velocity measurement is time consuming and expensive. Therefore, in order to obtain the required information in a timely manner, a number of alternatives for estimating the site effects can be applied, such as the methods utilizing the spectral H/V ratio of recorded ambient microtremor (Nakamura, 1989), the spectral ratio of horizontal to vertical components (Lermo and Chavez-Garcia, 1999), and the ratio of spectra on soil sites to those on nearby rock sites (Borcherdt, 1970). In the present study, two approaches are considered to estimate the site classification of the seismograph stations with unknown site conditions: (i) the horizontal-to-vertical spectral ratio technique; and (ii) the spectral shape technique. Both methods utilize the strong ground motions of the earthquake. The site classification methodology is developed for stations with known site condition and then applied to estimate site conditions of the remaining strong-motion stations in Taiwan.

Fault rupture mechanism is another factor affecting ground motion characteristics. Observations from reverse fault rupture events indicate that median ground motion amplitudes from these earthquakes are higher than those from strike-slip events (Somerville et al., 1997). Ground motion amplitudes of stations in the hanging wall side of thrust and normal faults are found to be higher than those in the footwall side because of shorter distance to the fault rupture and propagation effects (Brune et al., 2000).

Another phenomenon observed in recent major earthquakes to have significant influence on the damage potential of the ground motions is the directivity effects. Ground motion amplitudes are observed to be larger in the direction of wave propagation compared to those in other directions (Somerville et al., 1997), because the waves all arrive clustered within a short time window. Directivity effects often result in a single large velocity pulse at the beginning of the motion. Directivity effects have been observed during some recent major earthquakes, for examples the 1992 Landers earthquake in California, the 1994 Northridge earthquake in California, the 1995 Kobe earthquake in Japan, the 1999 Kocaeli earthquake in Turkey, and the 1999 Chi-Chi earthquake in Taiwan (Somerville, 2002). However, directivity effects are not presently considered in the design process. In order to better understand the behaviour of bridge structures during major earthquake events and mitigate the risks due to earthquakes, factors such as directivity that affect the accuracy of ground motion relations need to be thoroughly investigated.

One approach in the consideration of the directivity effects is to modify the ground motion relations using near-fault geometric parameters, such as percentage of fault rupturing toward sites, and site epicenter azimuth (Somerville et al., 1997).

Extensive evidence of larger slip distribution in the northern part of the epicentral region suggests strong directivity effects associated with the Chi-Chi earthquake (Ma et al., 2001). Many bridge and building structures located near the rupture fault and mainly in the northern part of the fault collapsed or were severely damaged (Wallace, 2001; and Dalguer et al., 2001). In order to relate the bridge damage pattern to the ground motion intensity in a more accurate manner, the directivity effects of Chi-Chi, Taiwan earthquake are evaluated and quantified. The ground motion relations for the Chi-Chi earthquake including directivity effects are developed for peak ground acceleration, and spectral acceleration at six vibration periods.

1.5 Objectives and scope

The large number of ground motions recorded during the Chi-Chi, Taiwan earthquake gives us a unique opportunity to evaluate and quantify these ground motions from a single large earthquake event. Post-earthquake investigations indicate that Chi-Chi earthquake caused 12 bridges to collapse and many others with different levels of damage (Wallace, 2001). Thus, the Chi-Chi ground motion records are ideal for the study of the strong ground motion characteristics and dynamic responses of bridges. The scope of the present study includes the five objectives as follows:

- (1) Develop a methodology to estimate site classification for the Taiwan stations using strong ground motion data from the 1999 Chi-Chi, Taiwan earthquake.
- (2) Evaluate and quantify the directivity effects of strong motions observed during the 1999 Chi-Chi, Taiwan earthquake.

- (3) Develop and implement two inelastic analysis procedures (the pseudo-force method and the tangent spectrum method) to compute time history response of bridge structures subject to strong ground motions based on direct superposition of derived Ritz vectors.
- (4) Develop and implement a non-linear beam-column model to consider the strength and stiffness degradation behaviour of non-linear beam-column elements.
- (5) Develop and implement a computational module for multiple-support seismic excitations analysis of bridge structures.

The present thesis is divided into 10 chapters, each largely self-contained as follows.

Chapter one gives an overview of developments of the seismic analysis and design for bridge structures. The problems in the current analysis and seismic design procedure for bridge structures are discussed. The need for an accurate analysis method such as the non-linear time history response analysis method is highlighted. The advantages of Ritz algorithms are reviewed. The use of Ritz reduction techniques to compute non-linear time history responses of bridge structures is considered. The NEABS program is briefly introduced. The strengths and weaknesses of the NEABS computer program are presented. The need of an accurate non-linear beam-column model to capture essential non-linear behaviour of bridges is highlighted. A modified non-linear beam-column model is briefly introduced. The need of multiple-support excitation analysis for long-span structures such as long bridges is addressed. The effects of local site condition on bridge responses are reviewed, which emphasizes to the importance of

knowing site condition in analysis and design of bridge structures. This leads to the need to estimate the site condition of Taiwan sites by using only ground motion data where the borehole and surface geology data are not available. The effects of ground motion directivity on bridge responses are also discussed. This suggests to study the directivity effects on the ground motion relations.

Chapter 2 proposes a methodology to estimate the site classification of strong-motion stations in Taiwan using ground motion data recorded in 1999 Chi-Chi, Taiwan earthquake. Reviews of available techniques (e.g., horizontal-to-vertical spectral ratio technique and spectral shape technique) for evaluating site responses are presented. The methodology to estimate the site classification of strong-motion stations is developed based on the information of ground motion recorded at 87 sites with known site condition. The developed methodology is then used to estimate site classification of the Taiwan stations.

Chapter 3 deals with the evaluation and quantification of the directivity effects observed during the 1999 Chi-Chi, Taiwan earthquake. The significance of parameters influencing directivity models is evaluated and quantified. The ground motion relations accounting for directivity effects for the 1999 Chi-Chi, Taiwan earthquake are developed.

Chapter 4 gives the basic concepts and properties of the eigensolution. Subspace iteration, Lanczos, and WYD methods for solving eigenproblem are introduced. The relationships of the Lanczos and WYD methods are discussed.

Chapter 5 presents the application of the derived Ritz vector techniques in computing linear time history response of bridge systems. Similarities between Lanczos

and WYD algorithms are presented. The improved WYD is presented, namely LWYD method. The cut-off criterion for the multi-directional earthquake loading is developed.

Chapter 6 presents the solution strategies to compute non-linear time history responses of bridges. A general procedure using direct integration method to compute non-linear time history responses of bridge is presented. The Ritz reduction techniques for non-linear time history response analysis of bridges are then introduced. Two inelastic analysis methods, namely pseudo-force method and tangent spectrum method are introduced.

Chapter 7 presents the developments of non-linear material model, which implicitly describes the strength deterioration and stiffness degradation behaviour of bridge columns/piers. Strength deterioration plastic stiffness matrix and elastic degrading stiffness matrix are derived.

Chapter 8 introduces the framework of the NEABS program and a number of newly developed modules and subroutines for non-linear time history analysis of bridge systems.

Chapter 9 presents numerical examples of non-linear time history response analysis of bridges. A multi-span bridge structure is used to illustrate the newly developed NEABS program. UHS compatible ground motion recordings are used in the analysis. Comparisons are conducted between results obtained from newly developed NEABS program and that from old version of the program. Furthermore, a subset of near-fault ground motion recordings obtained from Chi-Chi earthquake is selected to evaluate bridge response. Five ground motion recordings with distance to the fault less

than 15 km, represent for ground motions on different site condition (soil and rock), ground motion on different side of the fault (hangingwall and footwall), and ground motion on different directivity region (forward and backward).

Chapter 10 summaries the present study. Conclusions are presented. Directions for future studies are briefly introduced.

Finally, 11 appendices are presented.

Chapter 2

Methodology for Site Classification using Strong Ground Motion Data from 1999 Chi-Chi, Taiwan Earthquake

2.1 General

At 1:47 AM on 21 September 1999 an earthquake of magnitude $M=7.6$ (M is moment magnitude) occurred in central Taiwan. The epicenter was located at 23.78° N and 121.09° E near the town of Chi-Chi. After the earthquake, the Central Weather Bureau (i.e., Lee et al., 2001) successfully retrieved about 420 free-field records out of more than 650 free-field strong-motion stations. This set of records increases the number of near-field strong-motion records 10-fold in the existing worldwide dataset (Steward, 2003). Taiwan Strong Ground Motion Instrumentation Program was completed in 1995 however the site conditions of many of these strong-motion stations are not known or documented. In order to allow the proper use of this valuable set of strong-motion data to characterize source and propagation effects, it is important to obtain information on the site conditions of all the strong-motion stations in Taiwan. Kou (1992-1994) has classified strong-motion sites in the Taipei, Taoyuan, Hsinchum, Miaoli, and Chianan areas into four categories: S_1 (hard rock), S_2 (rock and hard stiff soil), which is also labeled as soft rock

in California, S_3 (medium stiff soil), and S_4 (soft soil), by using a combination of borehole and surface geology data. However, only 87 sites out of the 420 sites that recorded the Chi-Chi earthquake were classified (7 S_1 -sites, 33 S_2 -sites, 39 S_3 -sites, and 8 S_4 -sites). Because the classifications of these 87 sites are based on directly observed site parameters (from borehole and surface observations), these sites are considered as the ‘reference sites’ with known site conditions in the present investigation. The objective of this chapter is to present a methodology for estimating the site condition of the strong-motion stations with unknown site condition. The methodology is developed with the aid of the 87 reference sites based on the characteristics of ground motions recorded at these reference stations. Two techniques for classification of the sites are investigated, namely:

- The horizontal-to-vertical spectral ratio technique
- The spectral shape technique using the horizontal-component ground motions

The 420 free-field ground motion records of the Chi-Chi earthquake have three components (N-S, E-W, and vertical). The 5% damped pseudo-acceleration response spectra are computed for the 420 strong-motion records for natural periods up to 5 sec, after baseline correction. The response spectra are then smoothed by using a simple 9-point weighted triangular smoothing procedure. An average horizontal component of the ground motion for each record is obtained by averaging the natural logarithms of the ground motion amplitudes.

Seed et al. (1969, 1976) conducted some of the first quantitative studies of how the geological and local site conditions affect the response spectra. Following this pioneering study, the influences of the site conditions on the earthquake resistant design

of structures have been incorporated in the Uniform Building Code (UBC) (1997) and guidelines of the National Earthquake Hazards Reduction Program (NEHRP) (1997) through the use of simplified design spectra of different shapes for different types of sites. The site classification descriptions of UBC are summarized in Table 2.1. It is noted that the site classification in UBC is the same as that of NEHRP. The site classification is now well established in terms of shear-wave velocity. However, the practical application of using shear-wave velocity as a site classification criterion is limited due to the technical requirements and expense of obtaining shear-wave velocity measurements in the field. It is useful to develop a fast and simple methodology to estimate the site conditions based on the characteristics of the recorded ground motions. In the present study, a method is developed to classify the Taiwan strong-motion stations into different categories as defined in the design standards of UBC (1997), NEHRP (1997) and the Japan Design Specification for Highway Bridges (1996).

2.2 Site classification estimation techniques

The correlation between site amplification and surface geology has been the subject of many studies. One of the most popular yet controversial techniques to estimate site conditions is the horizontal-to-vertical spectral ratio technique (H/V technique) using the horizontal-to-vertical spectral ratio of earthquake records or microtremors. Nakamura (1989) used the horizontal-to-vertical component spectral ratios of the ambient seismic noise, dominated by the Rayleigh-waves to interpret micro-tremor measurements. Lermo and Chavez-Garcia (1994) were successful in using the H/V technique for site response evaluation of three cities in Mexico using earthquake records containing S-waves and

surface waves. Atkinson and Cassidy (2000) were also successful in estimating the soil response of the Fraser River Delta to the Duvall and Georgia Strait earthquake by using the H/V technique. The advantage of this technique is that it does not need a reference rock station; hence this technique is inexpensive and simple. The limitation of the technique is that it does not have a sound theoretical background (as discussed below), and the success of the technique has been mixed. Tsuboi et al. (2001) found that in general the spectral peaks of the H/V ratio and actual amplifications based on borehole-to-surface measurements between 1 Hz and 4 Hz agree well, but the agreement becomes worse for higher frequencies. Bonilla et al. (2002) have studied the borehole response at the Garner Valley Downhole Array, Southern California and found that the amplification obtained from the H/V spectral ratio is different from the one computed by the direct S-wave spectral ratio. Thus there are conflicting studies regarding the success of the H/V ratio as an estimator of site response.

Another method for site classification is based on the shape of response spectra obtained from earthquake records. The technique has its origins in the work of Borchardt (1970), who divided the spectrum obtained at a site of unknown site condition to that obtained at nearby reference site on competent bedrock. The statistical analysis performed by Seed et al. (1976) investigated the different shapes of normalized acceleration spectra for different site conditions. In the Japan Design Specification for Highway Bridges (1996), the site condition is classified based on the predominant period of the earthquake ground motion. In the following sections, we will evaluate the H/V spectral ratio method and a method based on spectral shape as site-class discriminants.

2.2.1 Horizontal-to-vertical spectral ratio technique

The reliability of the H/V technique in predicting the amplification effects of the surface soil layer is often questioned. This is because the basic principle of the H/V technique is largely qualitative. For a soft layer overlying a half-space, or for a velocity gradient, the horizontal component of the wave amplitude (S_H) is amplified by the soft soil layer as the wave propagates to the surface. There may also be amplification and attenuation of the vertical component, which tends to be dominated by S_v except at very high frequencies (i.e., Beresnev et al., 2002). However, the amplification of S_v is counterbalanced by the effects of refracting the ray path toward the vertical direction. Consequently, the net amplification effects of the vertical component are relatively small compared to those of the horizontal component, and may be considered negligible. Thus the H/V ratio is an estimate of site amplification, and may be useful in determining site condition. This view is supported by the studies by Nakamura (1989), Lermo and Chavez-Garcia (1994), and Atkinson and Cassidy (2000), but not supported by the study of Bonilla et al. (2002).

Here, amplitude and frequency characteristics of the horizontal-to-vertical spectral ratios are used to classify the site conditions of strong-motion stations in Taiwan. The H/V spectral ratio is computed for periods up to 2.5 sec for the 87 strong-motion stations where the site conditions are known. Figures 2.1 and 2.2 show the mean horizontal-to-vertical spectral ratio and its standard deviation (σ) for four site groups based on NEHRP site classes: $S_1 = A/B$ (7 strong ground motion records); $S_2 = C$ (33 strong ground motion records); $S_3 = D$ (39 strong ground motion records); and $S_4 = E/F$ (8 strong ground motion records). The peak of the mean H/V ratio increases in amplitude

and predominant period from rock sites to the medium-to-soft soil sites. Although the average H/V is different for each category plotted, there is considerable overlap of the H/V amongst classes when one considers the scatter as indicated by the $H/V \pm \sigma$. This scatter is an important constraint in developing a successful site class discriminant. It is observed that the peak H/V spectral ratios are 2.57, 2.56, 3.33, and 3.76 with predominant period at 0.55 sec, 0.4 sec, 0.75 sec, and 1.0 sec for NEHRP classes A/B, C, D, and E/F, respectively.

Figure 2.3 shows the individual peak H/V and predominant period of the H/V spectra for S1 (A and B), S2 (C), S3 (D), and S4 (E and F) site classes. There are no pronounced differences in peak H/V and predominant period of H/V spectra between rock sites and soil sites, although the average H/V and the predominant period of average H/V spectra for soil sites (D, E and F) are larger than those for the rock sites (A, B and C).

Figures 2.4 to 2.6 show the individual H/V values for hard rock (A and B), soft rock (C), stiff soil (D) and medium-to-soft soil (E and F) for periods $T=0.1$ sec, 0.2 sec, 0.5 sec, 1.0 sec, 1.5 sec and 2.5 sec. For the period range from 0.1 to 2.5 sec, the H/V of soil sites and that of rock sites are overlapping. The degree of variability of H/V within each site class plays an important role in developing a site classification estimation scheme.

Some possible discriminants for site classification estimation based on the H/V technique have been examined; these included the use of peak H/V value, predominant period of H/V spectra, and values of H/V at specific periods. The use of the peak H/V

value and its period provides the most statistically significant discriminant among these alternatives. The trial site classification estimation method based on H/V works as follows. If the peak $H/V < 2$ then the site may be classified as rock, while if the peak $H/V \geq 5$ then the site may be classified as soil. For sites with $2 \leq H/V < 5$, the period at which the peak H/V occurs is used as a discriminant. If predominant period is larger than 0.6 sec, the site is classified as soil and if predominant period is smaller than 0.6 sec, the site is classified as rock. However, because of the considerable overlap of H/V between rock and soil sites, the accuracy of the method is only 61% (53 stations out of the 87 stations are classified correctly).

2.2.2 Response spectral shape technique

The use of response spectral shape as a characterization tool of soil response has its origins in the work of Borchardt (1970), who used the ratio of spectra on soil sites to those on nearby rock sites to estimate the response of the soil sites. The underlying assumption is that the rock sites are free of amplification, and thus the ratio of the soil to rock spectra provided the shape and amplitude of the site response. Seed et al. (1969, 1976) used normalized acceleration spectra to characterize the spectral shapes of several different site conditions. In the Japan Specification for Highway Bridges (1996), Part 5 Seismic Design, the values for predominant periods are approximately given as $T_g < 0.2$ sec for hard rock and rock sites (NEHRP site type A and B), $0.2 \text{ sec} \leq T_g < 0.6$ sec for rock and stiff soil sites (NEHRP site type C and D), and $T_g \geq 0.6$ sec for soft to medium soil sites (NEHRP site type E and F). Recently, Borchardt (1995) proposed a general methodology for developing estimates of site-dependent response spectra for design

based on the use of extensive borehole geologic, geotechnical and shear-wave velocity data. Thus the use of the response spectral shape, in various forms, has a long history as a site condition discriminant.

In this section, a method is developed to estimate the site condition for stations in Taiwan by using the spectral shape of the normalized acceleration spectra of the horizontal ground motion component. Normalized acceleration spectra are obtained as the response spectrum divided by the peak ground acceleration (PGA). Figures 2.7 and 2.8 show the spectral shapes for NEHRP site classes A/B, C, D and E/F, respectively, as obtained from the Chi-Chi data of the 87 sites with known site conditions.

The predominant periods of the normalized acceleration spectra for NEHRP site class A/B (S_1), C (S_2), D (S_3), and E/F (S_4) are approximately 0.3 sec, 0.35 sec, 0.75 sec and 1.0 sec, respectively. The normalized acceleration spectra for all site classes are very similar in the range of periods from 0.3 to 0.6 sec. In the range of periods from 0.6 sec to 1.0 sec, the normalized acceleration spectra for rock sites (A, B and C) decline quite dramatically (40% reduction in amplitude) with increasing period, while the amplitudes of the normalized acceleration spectra for soil sites (D, E and F) decrease more gently as period increases. This characteristic of the normalized acceleration spectra may be used as a discriminant for site classification. To utilize this observation, the shape of the normalized acceleration spectra is characterized by the shape ratio (SR), which is defined as the amplitude of the normalized acceleration spectra at T to that at its dominant (peak) period:

$$SR(T) = \frac{\frac{PSA(T)}{PGA}}{\frac{PSA(T_g)}{PGA}} \quad (2.1)$$

or

$$SR(T) = \frac{PSA(T)}{PSA(T_g)} \quad (2.2)$$

where T is period in seconds and T_g is predominant period in seconds.

The shape ratio SR at 1.0 sec and the predominant period of the spectra of the horizontal ground motion component are shown individually for each station in Figure 2.9. The mean of the SR at 1.0 sec and predominant period for soil sites are considerably larger than those for rock sites; however there is an overlap between soil sites and rock sites. The possibility that this overlap may be partly due to nonlinear soil response is investigated later.

Based just on the use of predominant period of the spectra shape, a simple classification procedure is developed as follows: if a site has a predominant period ≥ 0.6 sec it may be classified as soil (NEHRP site class D, E and F), while sites with predominant period < 0.6 sec may be classified as rock (NEHRP site class A, B and C). This technique accurately classifies 68 stations out of 87 stations (78% accuracy). As noted earlier, there is an overlap in predominant period of normalized acceleration spectra between rock and soil sites. There are a significant number of soil sites associated with large PGA that have a predominant period < 0.6 sec. At these sites, nonlinear soil response due to strong shaking may have some impact on the spectral shape of the

normalized acceleration spectra. This possibility is investigated in the next section, with an aim to improving the classification scheme to account for such effects.

2.3 Effects of ground motion intensity

Strong ground motion can cause soil to behave nonlinearly. Evidence of nonlinear behavior is usually observed when the PGA exceeds a threshold of 100 to 200 gal (Beresnev and Wen, 1996). Nonlinear soil effects include an increase in damping and reduction in resonant frequencies as ground motion intensity increases. Nonlinear soil effects during large earthquakes have been the subject of many studies since the pioneering work of Seed et al. (1969, 1976), in which soil nonlinearity was recognized to play an important role. However, for many years these effects were ignored because of the lack of data from strong-motion observations. The situation has changed due to evidence from recent observations from the 1985 Michoacan, Mexico (Singh et al., 1988), the 1989 Loma Prieta, (Chin and Aki, 1991), and the 1994 Northridge California, earthquakes (Beresnev et al., 1998). Other evidence of nonlinear site response during strong ground motion has been provided in many studies as reviewed by Beresnev et al. (1994). Yu et al. (1993) studied the effects of nonlinearity on Fourier and response spectra. Effects of nonlinearity on response spectra include: (i) at short periods, the response spectra at larger excitation level for the nonlinear case are always smaller than that in the linear case; and (ii) at long periods, there is almost no difference between the response spectra for linear and nonlinear cases. It is likely that the soil at some of the Taiwan sites behaved nonlinearly during the Chi-Chi earthquake and this may affect the

site classification results. In this section the relationship of the shape ratio, the H/V ratio and predominant period to the ground motion intensity is explored.

For soil sites with recorded $\text{PGA} \geq 100$ gal, nonlinear soil response may have occurred, which reduce the dominant period of the ground motion. Aki and Chin (1991) investigated the 1989 Loma Prieta earthquake and reported that the threshold PGA for nonlinear soil response is 0.1 g, while Wen et al. (1994, 1995) investigated the nonlinear site amplification at two downhole strong ground motion arrays in Taiwan and reported the nonlinear threshold as 0.15 g. It should also be noted that previous studies using the H/V ratio with micro tremor measurements by Nakamura (1989), aftershock measurement by Field and Jacobs (1995) or earthquake recordings by Lermo and Chavez-Garcia (1994), have been based on ground motions in the linear range.

From observations on relationship of predominant period with respect to PGA and results from the other studies on nonlinear soil effects, a PGA threshold of 120 gal for the onset of nonlinearity is adopted for the purposes of this present study. There are 98 stations out of 420 stations that recorded the Chi-Chi earthquake where the $\text{PGA} \geq 120$ gal, and which thus may have responded nonlinearly. In order to compare the spectral shape of high intensity ground motion and that of low intensity ground motion, the H/V spectra and the PSA/PGA spectra for rock and soil sites with $\text{PGA} \geq 120$ gal are plotted in Figure 2.10. Figure 2.10 also shows the spectra of the records having $\text{PGA} < 120$ gal for comparison. It is shown that the predominant period of normalized acceleration spectra for the soil sites is significantly reduced at high PGA levels. This reduction of predominant period would obscure the site classification scheme. These observations conflict with the findings by Yu et al. (1993) that nonlinear soil response increase

predominant period of ground motion. By examining the stations with high PGA, we found that those stations are located within 50 km away from the rupture fault. It suggests that the reduction of predominant period of ground motion on soil site is due to high frequency-content component of ground motion.

2.4 Effects of distance to rupture fault

To implicitly examine the effects of closest distance on predominant period, we plot the average spectral shape for rock and soil sites against closest distance. The average spectral shape for near-field ground motions (distance within 50 km) and far-field ground motions (distance from 50 km to 100 km) are shown in Figure 2.11. It is observed that for soil sites, predominant period is significantly reduced for stations close to the rupture fault indeed. This reduction of predominant period would obscure the classification estimation of rock and soil sites by using only predominant period. However, there is a considerable difference between the average spectral shape of rock and soil sites for period between 2 sec to 3 sec. This difference can be used to distinguish soil sites and rock sites for stations located within 50 km from the rupture fault.

2.5 Methodology for site classification of Taiwan stations

Based on the observations and statistical results presented in the previous sections, and considering nonlinear soil response, a site classification method is proposed as follows.

- (1) The stations are divided into two categories based on the closest distance to the rupture fault: stations with the closest distance ≥ 50 km and stations with the closest distance < 50 km.

- (2) The 5% damped pseudo-acceleration spectrum is computed using the average horizontal component of ground motion, and the spectra are normalized with respect to PGA.
- (3) For far-field stations located at distance of more than 50 km from the rupture fault, the predominant period of the spectra of the horizontal ground motion component is used as the discriminant for site classification. The site is classified as rock if predominant period < 0.6 sec. The site is classified as soil if predominant period ≥ 0.6 sec. (Note: if the spectrum has a plateau extending over a period range that exceeds 0.6 sec, it is classified as soil even if the period at which the maximum amplitude occurs is actually below 0.6 sec).
- (4) For near-field stations located within 50 km from the rupture fault, PSA (2.5 sec)/PGA is used as a discriminant. If $\text{PSA (2.5 sec)/PGA} \geq 0.5$ then the site is classified as soil. If $\text{PSA (2.5 sec)/PGA} < 0.5$ then the site is classified as rock.

By using the proposed site classification estimation methodology given above, 48 stations out of 54 stations (89%) with closest distance ≥ 50 km are accurately classified, and 27 stations out of 33 stations (82%) with closest distance < 50 km are accurately classified. Among the 53 sites classified as soil by this technique, one site is S_1 (A and B), 9 sites are S_2 (C), 36 sites are S_3 (D), and 8 sites are S_4 (E and F), according to Kou (1992-1994). Thus the proposed methodology has misclassified one A/B class and 9 C class sites as soil. Among the 34 sites classified as rock by this technique, 3 sites are S_3 (D), 25 sites are S_2 (C), and 6 sites are S_1 (A and B) (Kou, 1992-1994). Thus, the proposed methodology has misclassified 3 class D sites as rock sites. It is noted that site

classification is somewhat more accurate for sites of linear soil response than for those of nonlinear soil response.

The classification methodology based on spectral shape has a similar level of accuracy to that proposed by Lee et al. (2001) overall, but the classifications by the two methodologies can disagree. Lee et al. (2001) classified the sites into four different NEHRP classes (B, C, D, E) based primarily on interpretation of geologic maps and geomorphology data. The response spectral shape method and the horizontal-to-vertical spectral ratio were evaluated for each site by Lee et al. (2001), but no statistics on how these parameters were used as site discriminants was provided in their paper. About 77% of the sites classified by Lee et al. (2001) agree with site classifications obtained from the proposed methodology here. Among the 54 sites classified as soil here, 28 sites are classified as E class, 20 sites are classified as D class, and 4 sites are classified as C class by Lee et al. (2001), with 2 sites of no classification. Among the 33 sites classified as rock here, one site is classified as E class, 15 sites are classified as D class, 15 sites are classified as C class, and 2 sites are classified as B class by Lee et al. (2001). There is a considerable overlap between the D class sites of Lee et al. and the rock sites obtained from this study. Comparing the classifications of Lee et al. (2001) with those by Kou (1992-1994), the results by Lee et al. (2001) match the classifications by Kou (1992-1994) (i.e., A/B=S₁, C=S₂, D=S₃, E/F=S₄) in only 48% of the cases. However, the comparison shows that soil sites (S₃, S₄ = D, E and F) are correctly distinguished from rock sites (S₁, S₂ = A, B and C) by Lee et al. (2001) in 83% of the cases. The discrepancies in distinguishing between rock sites and soil sites involve sites classified as class D by Lee et al. (2001) that are actually S₂ (C) according to Kou (1992-1994). The

results (Figures 2.1 and 2.7) indicate that the amplification characteristics of the S_2 sites are very similar to those of the S_1 sites (rock), while Lee et al. (2001) classified many of the S_2 sites as soil. The rock classifications obtained from this study agree very closely with the known S_1 and S_2 classifications by Kou (1992-1994). Thus the proposed site classification methodology based on predominant period may be more diagnostic of rock sites than the classifications by Lee et al. (2001) based on map interpretation. It should be acknowledged that uncertainty in classification exists with both the proposed methodology and that of Lee et al. (2001). The differences between two classifications provide some measures of this uncertainty. Appendix 2.1 compares the site classifications from both this study and the Lee et al. (2001) study with the known conditions according to Kou (1992-1994).

We applied our classification method to the 333 strong-motion sites in Taiwan with unknown site conditions. Results are listed in Appendix 2.2. We classify 248 sites as soil. Of these, Lee et al. (2001) classified 15 sites as B class, 18 sites as C class, 132 sites as D class, 79 sites as E class and 4 sites with no classification. Approximately 13% of the sites that are classified as soil sites here are considered as rock sites (A, B and C) by Lee et al. (2001). There are 172 sites classified as rock by our method. Of these Lee et al. (2001) classified 35 sites as B class, 42 sites as C class, 69 sites as D class, 14 sites as E class and 12 sites with no classification. Approximately 48% of the sites classified as rock sites in this study are identified as soil sites (D, E and F) by Lee et al. (2001). Overall, the results from this study agree with those of Lee et al. (2001) at 71% of the sites. These discrepancies suggest there is still significant uncertainty in site classification for the strong-motion sites in Taiwan.

The results are also compared to the site classification obtained from recently measured shear-wave velocity. Comparisons indicate that 71% of our site classification agrees with shear-wave velocity results.

2.6 Conclusions

Site classification techniques are reviewed and applied to estimate the site condition for strong-motion sites in Taiwan that recorded the Chi-Chi earthquake. There is no evidence of nonlinear soil effects on the site classification. The effects of closest distance on spectral shape are more significant. For sites with the closest distance larger than 50 km, the predominant period can be used as a discriminant to distinguish soil sites from rock sites. The site is classified as rock if predominant period < 0.6 sec. The site is classified as soil if predominant period ≥ 0.6 sec. The comparison of site classification results with known soil conditions shows that 89% of these stations are classified correctly. For sites with the closest distance less than 50 km, the spectral shape at long periods needs also to be considered. The results show that 85% of these stations are classified correctly. It is therefore concluded that the proposed spectral shape technique is a good approach for site classification estimation of Taiwan stations.

Observed spectral shape characteristics depend not only on site conditions, but also on earthquake magnitude and distance. Thus discriminants of site condition based on shape should be re-examined using local earthquake data before application to other earthquake regions. However, the predominant period tends to be dominated by the influence of site amplification, as evidenced by results and observation reported extensively in the literature and practical applications of similar techniques (e.g., Japan

Specifications for Highway Bridges, 1996). Similar procedures may be developed to estimate the soil conditions of seismograph stations with unknown site conditions elsewhere, but would require statistical analyses to evaluate spectral shape characteristics in those regions.

Table 2.1 Site classification

Site class	Site class description by 1997 UBC Provisions ($\bar{V}_s$ is shear-wave velocity averaged over top 30 m)	Site class description in present study
A	Hard rock (typically eastern United States sites), $\bar{V}_s > 1500$ (m/s).	Rock
B	Rock, $\bar{V}_s$ is 760 to 1500 (m/s).	
C	Very dense soil and soft rock, $\bar{V}_s$ is 360 to 760 (m/s), Un-drained shear strength $S_u \geq 2000$ psf ($S_u \geq 100$ kPa) or $N \geq 50$ blows/ft (C class is typically called a rock site in California)	
D	Stiff soils, $\bar{V}_s$ is 180 to 360 (m/s), Stiff soil with un-drained shear strength $1000 \text{ psf} \leq S_u \leq 2000 \text{ psf}$ ($50 \text{ kPa} \leq S_u \leq 100 \text{ kPa}$), or $15 \leq N \leq 50$ blows/ft.	Soil
E	Soft soils, Profile with more than 10ft (3m) of soft clay defined as soil with plasticity index $PI > 20$, moisture content $w > 40\%$ and un-drained shear strength $S_u < 1000 \text{ psf}$ (50kPa), or $N < 15$ blows/ft.	
F	Soils requiring site-specific evaluations. (1) 1. Soil vulnerable to potential failure or collapse under seismic loading: e.g., liquefiable soils, quick and highly sensitive clays, collapsible weakly cemented soils. (2) 2. Peats and/or highly organic clays (10ft (3m) or thicker layer). (3) 3. Very high plasticity clays: (25ft (8m) or thicker layer with plasticity index > 75). (4) 4. Very thick soft/medium stiff clays: (120ft (36m) or thicker layer).

a) A and B (S_1) sites

b) C (S_2) sites

Figure 2.1 Average H/V spectral ratio for rock sites. Dotted lines indicate $\pm$ one standard deviation level.

a) D (S_3) sites

b) E and F (S_4) sites.

Figure 2.2 Average H/V spectral ratio for soil sites. Dotted lines indicate $\pm$ one standard deviation level.

a) Peak H/V

b) Predominant period of H/V

Figure 2.3 Peak H/V and predominant period of H/V spectra for A and B (S₁), C (S₂), D (S₃) and E and F (S₄) sites. Notations at each column give: site class, mean, standard deviation and number of records.

a) $T=0.1$ sec

b) $T=0.2$ sec

Figure 2.4 H/V spectral ratio for periods 0.1 and 0.2 sec. Notations at the top of each column give: site class, mean, standard deviation and number of records.

a) T=0.5 sec

b) T=1.0 sec

Figure 2.5 H/V spectral ratio for periods 0.5 and 1.0 sec. Notations at the top of each column give: site class, mean, standard deviation and number of records.

a) T=1.5 sec

b) T=2.5 sec

Figure 2.6 H/V spectral ratio for periods 1.5 and 2.5 sec. Notations at the top of each column give: site class, mean, standard deviation and number of records.

a) A and B (S_1) sites

b) C (S_2) sites

Figure 2.7 Average normalized spectra of horizontal strong-motion component for rock sites. Dotted lines indicate $\pm$ one standard deviation level.

a) D (S₃) sites

b) E and F (S₄) sites

Figure 2.8 Average normalized spectra of horizontal strong-motion component for soil sites. Dotted lines indicate $\pm$ one standard deviation level.

a) $\text{PSA}(1.0 \text{ sec})/\text{PSA}(T_g)$

b) Predominant period (T_g)

Figure 2.9 $\text{PSA}(1.0 \text{ sec})/\text{PSA}(T_g)$ and T_g of the spectra of horizontal ground motion component for A and B (S_1), C (S_2), D (S_3) and E and F (S_4) sites. Notations at each column give: site class, mean, standard deviation and number of records.

a) H/V spectra

b) Normalized acceleration spectra of horizontal ground motion component

Figure 2.10 Average H/V spectra and normalized horizontal ground motion component spectra (solid line for soil sites with $PGA < 120$ gal, dashed line for soil sites with $PGA \geq 120$ gal, dotted line for rock sites)

a) Closest distance within 50 km

b) Closest distance within 50 km and 100 km

Figure 2.11 Average normalized spectra of horizontal ground motion component (solid line for soil sites, dashed line for rock sites)

Chapter 3

Characterization of Directivity Effects Observed during 1999 Chi-Chi, Taiwan Earthquake

3.1 General

The wealth of data from the Chi-Chi earthquake provides a valuable opportunity to gain new insights into ground motion source, propagation and site effects. Studies of the characteristics of the Chi-Chi ground motions conducted to date have examined the effects of closest distance to the fault, site condition, and the role of thrust of the hanging wall (e.g., Loh et al., 2000; Chang et al., 2001; and Wang et al., 2002). The objective of this chapter is to evaluate and quantify the directivity effects in the ground motion records obtained from the 1999 Chi-Chi earthquake. The parameters evaluated are the horizontal-component peak ground acceleration (PGA) and the 5%-damped response spectral acceleration (PSA) at different periods.

Observations and inversions indicate that the Chi-Chi earthquake had a rupture length of about 100 km, a rupture width of about 48 km, strike of 3° , dip of 29° and rake of 66° (Ma et al., 2001). The pattern of surface deformation from the Chi-Chi earthquake varies along the Chelungpu fault, with progressively higher and wider fault scarps from the south to the north (Kelson et al., 2001). The southern portion of the fault showed

relatively constant rupture velocity with an average slip of about 1 m, whereas the northern portion of the fault showed significant variations in velocity and produced a larger amount of slip, up to 8 m (Ma et al., 2001). Of 12 bridges that collapsed during the Chi-Chi earthquake, 9 of them were in the northern part of the fault rupture (Wallace, 2001). These facts suggest that the ground motions in the northern part of the fault were stronger than those in the southern part of the fault, and that rupture propagation effects towards the north were significant.

It is widely recognized that ground motion amplitude is a function of earthquake magnitude and distance, site condition, faulting mechanism, hanging wall and footwall effects and possible directivity effects. The effects of earthquake magnitude, distance and site condition on ground motion amplitudes have been quantified in many empirical studies that have developed ground motion relations for various regions (e.g., see Abrahamson and Shedlock, 1997 and papers therein). However, it is only in recent years, with the collection of more extensive data sets, that it has become possible to empirically characterize the effects of faulting mechanism, hanging wall versus footwall, and directivity (e.g., Abrahamson and Somerville 1995; Somerville et al., 1995, 1997). In particular, it has been observed through recent earthquakes such as 1994 Northridge, 1995 Kobe and 1999 Chi-Chi, Taiwan earthquakes that the ground motion amplitudes are enhanced in the direction of fault rupture propagation. Directivity effects are known to result in ground motions having higher amplitudes and shorter durations in the forward direction of rupture propagation (i.e., when the rupture propagates toward the site), while those in the backward rupture direction have reduced amplitudes and longer durations. Ground motions in the direction perpendicular to rupture propagation are not affected.

There may also be significant differences between the strike-normal and strike-parallel components of horizontal ground motion amplitudes. For strike-slip faulting, the maximum of the radiation pattern for tangential S waves reinforces the directivity effects caused by rupture propagation toward the site, resulting in a large displacement pulse normal to the fault strike, as described by Somerville et al. (1997). For thrust earthquakes, the direction of the rupture propagation does not lead to reinforcement of the radiation pattern. Thus the strike-normal component is not necessarily larger than the strike-parallel component of horizontal ground motions. Preliminary analysis of the strike-normal to strike-parallel ratios for the Chi-Chi earthquake did not show any significant effect, so this ratio was not evaluated further in this study; this is in agreement with a study by Wang et al. (2002). The dimensions and the strike and dip angles of the fault, the direction of the slip (slip rake angle), and the hypocenter all play critical roles in determining the characteristics and amplitudes of near-fault ground motions. The role of these effects has been explored by Aagaard et al. (2002).

Characterization of directivity effects has been the subject of several studies. Research conducted by Somerville et al. (1995, 1997) to modify empirical strong ground motion attenuation relations to include the amplitude and duration effects of rupture directivity characterized the directivity effects for strike-slip and dip-slip faulting separately. Somerville et al. (1997) used ground motion data from 21 earthquakes, mostly in California, with magnitudes of 6 or larger to quantify average directivity effects in terms of ground motion amplitudes and durations, relative to standard ground motion relations for California. For dip-slip events, they examined just the directivity effects up-dip from the hypocenter, choosing not to model any along-strike component of the

propagation process for stations off the ends of the fault. The large dataset obtained from the Chi-Chi earthquake provides an opportunity to explore directivity observations in more detail for a large thrust earthquake, including effects both near and far from the fault.

3.2 Median ground motion relation for Chi-Chi earthquake

Ground motion relations predicting peak ground motions and response spectral ordinates as a function of magnitude and distance are a key input to seismic hazard analysis. In this investigation, a single earthquake is studied therefore an event-specific ground motion attenuation relation is derived rather than a general relation. However, this is a useful exercise because the nature of this dataset allows us to characterize features that cannot be readily explored in ground motion regressions for multi-event datasets. In particular, the regressions of the Chi-Chi earthquake data is used to generalize the effects of directivity on ground motion amplitudes. Such generalizations can then be used to predict how empirical regression equations for future earthquakes might be modified to better account for such effects.

The ground motion parameters evaluated are the horizontal-component peak ground acceleration (PGA) and the 5%-damped response spectral acceleration (PSA) at six different periods (0.2, 0.5, 1, 1.5, 2 and 5 sec). The average horizontal component is used in the regressions (where the average of the logs of the amplitudes is used). The ratio of the fault-normal to fault-parallel horizontal component was evaluated and found to be near unity, with the fault-normal component being on average just 10% larger than the fault-parallel component; thus only the average horizontal component needs to be

regressed. The ground motion relations without considering the directivity effects are determined. The simple least squares method is used to fit the observations to the form

$$\ln Y = C_1 + C_2 \ln d + C_3 d + C_4 S \quad (3.1)$$

where Y is the ground motion amplitude in cm/sec^2 (PGA or PSA(T)), d is the closest distance to the fault rupture surface, and S is a site category factor ($S=1$ for soil sites, $S=0$ for rock sites). Natural log units ($\ln$) are used for the ground motion variables. C_1 , C_2 , C_3 and C_4 are the coefficients to be determined.

The 5% damped pseudo-acceleration response spectra were computed for 420 strong-motion records for natural periods up to 5.0 sec. The computations were made using the whole time-history record, after baseline correction. The response spectra were smoothed using a simple 9-point weighted triangular smoothing technique. The site condition factor is either 1 for soil sites or 0 for rock sites. The site classifications for stations with unknown site conditions were estimated from the spectral shape technique as described in chapter 2. Among the 420 sites, 248 sites are classified as soil ($S=1$) and 172 sites are classified as rock ($S=0$).

Observed ground motion amplitudes and the attenuation curves determined by the regression for PGA and PSA at 4 periods (0.2, 0.5, 1.0 and 2.0 sec) are plotted against the closest distance to the fault in Figures 3.1 and 3.2. The site condition factor coefficient C_4 for PGA and PSA at low periods is negligible (not statistically significant) and is therefore dropped from the regression. For higher periods the site effects become significant (factor of 1.8) for PSA. The coefficients of this regression to obtain median

ground motion amplitudes for the horizontal component, without considering directivity effects, are given in Table 3.1.

3.3 Analysis of results to obtain directivity effects

The residuals of the regression to equation (3.1) are computed, where a residual is defined as the ratio of the observed ground motion value to the value predicted by equation (3.1) (with coefficients given in Table 3.1). The map view of the computed ground motion amplitude residuals is plotted in Figure 3.3 for PGA. It is observed that large positive residuals along the fault on the east side (hanging wall side), and to the north of the fault, with negative residuals to the south. Plots for spectral parameters show the same trends. This provides a clear evidence of directivity effects on ground motion amplitudes.

The apparent directivity effects in the ground motion amplitudes are quantified by relating the computed residuals to directivity parameters. Following the conventions developed by Somerville et al. (1997), it is expected that ground motion amplitude variations due to directivity effects will depend on two independent geometrical parameters: (i) the angle between the direction of wave propagation and the ray path of waves traveling from the fault to the site; and (ii) the fraction of the fault rupture surface that lies between the hypocenter and the site. The azimuth A and zenith Z angles and the fraction of the rupture fault surface that lies between the hypocenter and the site for along-strike and dip-slip components of rupture propagation are illustrated in Figure 3.4. It is noted that the directivity effects along-strike apply at all distances, while those up-dip apply only within the actual footprint of the fault rupture.

Somerville et al. (1997) used near-fault ground motion data from 21 strong earthquakes to study the directivity effects on the ground motion amplitudes and duration using the function $X \cos A$ for strike-slip faults, where X is length ratio and A is azimuth angle; and $Y \cos Z$ for dip-slip faults, where Y is width ratio and Z is zenith angle. The cosine function was chosen because it provides a smooth decay of the ground motion values with increasing angle. In their study of directivity effects for dip-slip events, Somerville et al. (1997) restricted their focus to sites near the fault, for which purely up-dip effects can be observed; they deliberately chose to exclude evaluation of the off-end effects of rupture propagation for dip-slip faults. In the present study, a variation of their approach is used to quantify the directivity effects of the Chi-Chi earthquake both station near the fault (up-dip direction) and stations further away (along strike direction). To achieve that objective, the effects of $X \cos A$ and $Y \cos Z$ on ground motion amplitudes is explored as shown in Figures 3.5 to 3.8. These effects should not be considered additive, because they have been examined separately (i.e., the residuals for the up-dip regression implicitly include any net along-strike effects). Thus they are alternative estimates of the magnitude of the potential directivity effects.

Figures 3.5 and 3.6 show the variations of ground motion residuals of PGA and PSA at 4 periods (0.2, 0.5, 1.0 and 2.0 sec) with respect to $X \cos A$ for both rock and soil sites. In Figures 3.5 and 3.6, note the increasing trend of the residuals as $X \cos A$ increases. Both rock and soil sites to the south of the epicenter (i.e., $X \cos A < 0$) have more negative residuals, while sites to the north of the epicenter (i.e., $X \cos A > 0$) have more positive residuals. This agrees with the map view of the residuals as shown in Figure 3.3. The residuals on Figures 3.5 and 3.6 were fit to

$$R = B_1 + B_2 X \cos A \quad (3.2)$$

where R is the natural logarithm of the residuals; thus for PGA, R is obtained from $R = \ln \text{PGA}(\text{observed}) - \ln \text{PGA}(\text{predicted})$, X is the length ratio along strike and A is the azimuth (Figure 3.4). B_1 and B_2 are the coefficients, given in Table 3.2 along with their standard errors.

Figures 3.7 and 3.8 present the residuals of PGA and PSA at 4 periods (0.2, 0.5, 1.0 and 2.0 sec), which examine up-dip directivity for stations within the overall fault footprint (within 50 km of the fault trace). Positive residuals are observed directly up-dip from the hypocenter (values of $Y \cos Z$ near 0) on the hanging wall. The data points on the left in Figures 3.7 and 3.8 (no apparent directivity effects) are from stations located further away from the epicenter (these are hanging-wall stations that would project onto the fault plane below the depth of the hypocenter). It is clear that there is a significant positive residual trend in $Y \cos Z$, but only for the hanging wall sites. The residuals of footwall sites cluster around zero, at a large value of $Y \cos Z$. In order to include the hanging wall effects for the dip-slip directivity, the residuals are fitted to

$$R = \text{HW} (B_3 + B_4 Y \cos Z) \quad (3.3)$$

where Y is the width ratio and Z is the zenith angle (Figure 3.4). $\text{HW} = 1$ for sites on the hanging wall and $\text{HW} = 0$ for sites on the footwall. The coefficients B_3 and B_4 and their standard errors are listed in Table 3.3.

Results from Tables 3.2 and 3.3 suggest that directivity both along-strike and up-dip have a significant effect on the ground motion amplitudes. The ground motion

amplitudes are approximately 1.5 times higher for sites in which rupture propagated strongly towards the site than those away from the site, along strike. Stations close to the fault up-dip from the hypocenter have strongly enhanced ground motion amplitudes, by a factor of about 2.7. These effects on amplitude are seen at all periods, and also for PGA.

3.4 Effects of ground motion intensity on the directivity models

The directivity effects have been evaluated and quantified using 420 ground motion recordings from the Chi-Chi earthquake. With the magnitude $M=7.6$, it is possible that nonlinear response has occurred at many soil sites during the Chi-Chi earthquake. Nonlinearity is known to change the characteristics of ground motions. In this section, the possible effects of ground motion intensity on the directivity models are examined.

Figures 3.9 and 3.10 search for possible nonlinear effects by plotting the ground motion residuals against the predicted PGA on rock given in equation (3.1) for each station. A decreasing trend of residuals with increasing PGA for the soil sites (relative to the residuals for the rock sites) would be expected if there was widespread nonlinear soil behavior. Such nonlinearity could potentially bias our results. From Figures 3.9 and 3.10, it appears that there are no residuals trends with increasing intensity level. To obtain nonlinear soil factor, the residuals on soil sites are regressed to fit the following equation:

$$NL = D_1 + D_2 \ln PGA(Rx) \quad (3.4)$$

where $PGA(Rx)$ is the expected PGA on rock based on equation (3.1), and D_1 and D_2 are the coefficients to be determined.

For PGA and all six spectral periods, it is noted that the coefficients D_1 and D_2 are not significantly different from zero. Thus there appears to be no significant nonlinear soil effects that could bias our results. The lack of soil nonlinearity may be due to the rather low PGA values for this earthquake. Typically, nonlinearity is observed for PGA values greater than about 100 to 200 cm/sec^2 (Beresnev and Wen, 1996).

3.5 Conclusions

Directivity effects observed during the 1999 Chi-Chi, Taiwan earthquake were evaluated and quantified by using 420 ground motion records. The dip-slip component of the fault rupture propagation has a more significant influence on the ground motion characteristics than the along-strike component. This is expected as the Chi-Chi earthquake was a thrust event. In the up-dip direction, ground motion amplitudes of the stations near the epicenter, towards which the rupture propagated, are enhanced by a factor of approximately 2.7. There is also a significant along-strike directivity effect. The amplitudes of PGA and PSA are approximately 1.5 times higher for sites in which rupture propagated strongly towards the site, along-strike. These results agree with those reported by Somerville et al. (1997) in terms of the general magnitude of the directivity effects to be expected. However, Somerville et al. (1997) found that directivity effects grow steadily with increasing period, while the Chi-Chi earthquake did not show this trend with period. Furthermore, even though the Chi-Chi earthquake was a thrust event, there is a significant apparent directivity effect along strike. Large ground motions were observed to the north, while weaker ground motions were observed to the south. More information on the soil conditions of the sites in these regions might shed further light on

whether there are other possible explanations for these observations. However, it appears likely that the greater strength of the rupture propagation towards the north was a significant factor influencing the observed ground motions.

Table 3.1 Coefficients of equation (3.1) for the Chi-Chi earthquake ground motions

Ground motion coefficients	PGA	PSA at 0.2 sec	PSA at 0.5 sec	PSA at 1.0 sec	PSA at 1.5 sec	PSA at 2.0 sec	PSA at 5.0 sec
C_1	6.092	6.283	6.315	6.088	5.715	5.443	4.818
C_2	-0.293	-0.253	-0.231	-0.368	-0.387	-0.401	-0.417
C_3	-0.011	-0.012	-0.011	-0.006	-0.005	-0.004	-0.005
C_4	0.0	0.0	0.0	0.405	0.504	0.597	0.579

Table 3.2 Coefficients of directivity effects model along-strike ($R=B_1+B_2X\cos A$)

Parameter	PGA	PSA at 0.2 sec	PSA at 0.5 sec	PSA at 1.0 sec	PSA at 1.5 sec	PSA at 2.0 sec	PSA at 5.0 sec
B_1 and error	-0.082 0.025	-0.062 0.024	-0.067 0.026	-0.077 0.028	-0.070 0.027	-0.063 0.027	-0.075 0.032
B_2 and error	0.464 0.059	0.346 0.058	0.381 0.062	0.433 0.066	0.397 0.064	0.354 0.066	0.426 0.078

Table 3.3 Coefficients of dip-slip directivity effects model ($R=HW (B_3+B_4Y\cos Z)$)

with $HW=1$ for the sites on the hanging wall.

Parameter	PGA	PSA at 0.2 sec	PSA at 0.5 sec	PSA at 1.0 sec	PSA at 1.5 sec	PSA at 2.0 sec	PSA at 5.0 sec
B_3 and error	0.594 0.116	0.724 0.116	0.518 0.132	0.637 0.207	0.544 0.179	0.275 0.176	-0.132 0.126
B_4 and error	1.048 0.305	1.301 0.307	1.094 0.347	1.134 0.546	1.248 0.471	1.224 0.463	1.109 0.332

a) PGA

b) PSA at 0.5 sec

Figure 3.1 PGA and PSA at 0.5 sec. amplitudes (symbols) and the regression curves for equation (3.1) (solid line for rock, dashed line for soil)

a) PSA at 1.0 sec

b) PSA at 2.0 sec

Figure 3.2 PSA at 1 and 2 sec. amplitudes (symbols) and the regression curves for equation (3.1) (solid line for rock, dashed line for soil)

Figure 3.3 Map view of the residuals in natural logarithm units (e.g., a residual R of 0.4 represents an amplitude that is 50% larger than predicted).

a) Along-strike (top view)

b) Up-dip (cross section)

Figure 3.4 Definition of rupture directivity parameters A and $X=S/L$ for along-strike (left) and Z and $Y=W/D$ for up-dip (right) components of propagation (from Somerville et al., 1997)

a) Residuals of PGA

b) Residuals of PSA at 0.5 sec

Figure 3.5 Variation of PGA and PSA at 0.5 sec. residuals with $X\cos A$. Residuals are in $\ln$ units. Best-fit lines (equation 3.2) are also shown.

a) Residuals of PSA at 1.0 sec

b) Residuals of PSA at 2.0 sec

Figure 3.6 Variation of PSA at 1 and 2 sec. residuals with $X\cos A$. Residuals are in natural logarithm units. Best-fit lines (equation 3.2) are also shown.

a) PGA

b) PSA at 0.5 sec

Figure 3.7 Variation of PGA and PSA at 0.5 sec. residuals with $Y\cos Z$. Residuals are in natural logarithm units. Filled symbols are sites on the hanging wall; open symbols are sites on the footwall. Circles (red) are rock sites while triangles (blue) are soil sites. The best-fit lines for the hanging-wall residuals are also shown.

a) PSA at 1.0 sec

b) PSA at 2.0 sec

Figure 3.8 Variation of PSA at 1 and 2 sec. residuals with $Y\cos Z$. Residuals are in ln units. Filled symbols are sites on the hanging wall; open symbols are sites on the footwall. Circles (red) are rock sites while triangles (blue) are soil sites. The best-fit lines for the hanging-wall residuals are also shown.

a) Residuals of PGA

b) Residuals of PSA at 0.5 sec

Figure 3.9 Effects of ground motion intensity (predicted PGA on rock) on PGA and PSA at 0.5 sec. residuals for rock sites (filled circles) and soil sites (open triangles).

a) Residuals of PSA at 1.0 sec

b) Residuals of PSA at 2.0 sec

Figure 3.10 Effects of ground motion intensity (predicted PGA on rock) on PSA at 1 and 2 sec. residuals for rock sites (filled circles) and soil sites (open triangles).

Chapter 4

Free Vibration Analysis

4.1 General

Dynamic analysis of structures by the mode superposition method requires the computation of the frequencies and mode shapes of structures. For linear dynamic response analysis, the computation of the frequencies and mode shapes of a structure is performed only once at the beginning of the analysis process. However, for non-linear dynamic response analysis, frequencies and mode shapes of a structure change during response and need to be computed again whenever there are changes in the configuration of the structure. The computation involved in determining the modal properties of a large and complex structure is time consuming, and this deficiency is further magnified in the calculation of the modal behaviour of structures in non-linear time history response analysis. Moreover, it has been recognized because of the changes in the modal behaviour in non-linear structures, direct engineering significance of frequencies and mode shapes of non-linear structures is limited. For a linear structure, frequencies indicate possible resonant conditions, and mode shapes associated with low frequencies can give indications which regions of the structure are most flexible. For most analyses, the advantage of using the exact eigenmodes is to reduce the size of the system by mode

superposition method. In many cases, the use of approximated frequencies and mode shapes can achieve similar reduction in the size of the problem and still retain the accuracy required for engineering purposes (Wilson, 1982).

For a complex structural system, obtaining solutions to the eigenvalue problem represents a major cost in the solution process. In this chapter, the definition and properties of the eigenproblem are reviewed, followed by discussions on the numerical methods based on the Rayleigh-Ritz approximation. The subspace iteration, the Lanczos and WYD methods for solving the eigenproblem are presented.

4.2 Eigenproblem formulation

After discretization, the equations of motion for the finite element model of a structural system are expressed as follows

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}(t) \quad (4.1)$$

where the vectors $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and $\mathbf{u}$ are the acceleration, velocity, and displacement vectors which describe the responses of the structure. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are respectively the mass, damping, and stiffness matrices, and $\mathbf{p}(t)$ is the vector of the time-dependent forcing function.

When the applied load vector in equation (4.1) is equal to zero, the equation describing the free vibration response of the system is written as follows

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \quad (4.2)$$

In which case the dynamic responses may be initiated when one or more degrees-of-freedom of the system are set in motion due to initial displacements and/or velocities. For the case damping is neglected, the finite element discretization of the structure leads to the general undamped free vibration equations of motion as follows

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \quad (4.3)$$

In deriving the solution of the free vibration response, the harmonic displacement response is assumed to be of the form

$$\mathbf{u}(t) = \boldsymbol{\phi} e^{i\omega t} \quad (4.4)$$

Substituting equation (4.4) into equation (4.3), it becomes

$$\mathbf{K}\boldsymbol{\phi} = \lambda \mathbf{M}\boldsymbol{\phi}, \quad \lambda = \omega^2 \quad (4.5)$$

where $\boldsymbol{\phi}$ is the eigenvector, λ is the corresponding eigenvalue, and ω is the natural frequency of vibration. Equation (4.5) is referred to as the general linear eigenvalue problem and it may also be expressed in another form as follows:

$$(\mathbf{A} - \lambda \mathbf{I})\boldsymbol{\phi} = \mathbf{0} \quad (4.6)$$

where $\mathbf{I}$ is an identity matrix of the same order of $\mathbf{A}$ and

$$\mathbf{A} = \mathbf{K}^{-1}\mathbf{M} \quad (4.7)$$

$$\nu = \frac{1}{\lambda} \quad (4.8)$$

Put in the form of equation (4.6), the free vibration eigenvalue problem is written as a standard symmetric eigenvalue problem $\mathbf{A}\phi = \lambda\phi$. The eigenvalues of the standard eigenproblem are the reciprocals of the eigenvalues of the generalized eigenproblem. The eigenvectors of both forms remain the same as ϕ .

4.3 Properties of eigensolution

Before the discussion on the numerical techniques for obtaining the eigensolution, it is useful to give an overview on the fundamental characteristics and properties of the eigensolution. In order to have a non-trivial solution, that is $\phi \neq 0$, the matrix $(\mathbf{K} - \lambda\mathbf{M})$ must be singular, i.e.

$$\det(\mathbf{K} - \lambda\mathbf{M}) = 0 \quad (4.9)$$

By expanding the determinant as represented by equation (4.9), the characteristic polynomial is obtained as follows

$$p(\lambda) = \det(\mathbf{K} - \lambda\mathbf{M}) \quad (4.10)$$

where the order of the polynomial is equal to the order of $\mathbf{K}$ and $\mathbf{M}$ matrices. There are n solution roots for equation (4.9), which give the eigenvalues of the eigenproblem. The eigenvalues are the squares of the natural frequencies. For each eigenvalue λ_i , a corresponding unique non-trivial eigenvector ϕ_i can be found by using equation (4.5) or equation (4.6) of which the magnitude may be arbitrarily specified. Thus, an eigenvector is defined only by its direction in the n -dimensional space. Furthermore, the eigenvectors have the orthogonal property with respect to both the mass matrix $\mathbf{M}$ and the stiffness

matrix $\mathbf{K}$. When appropriately normalized, the eigenvectors satisfy the following properties

$$\phi_i^T \mathbf{M} \phi_j = \delta_{ij} \quad (4.11)$$

$$\phi_i^T \mathbf{K} \phi_j = \lambda_i \delta_{ij} \quad (4.12)$$

where δ is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$, and $\delta_{ij} = 0$ for $i \neq j$). The eigenvectors can be grouped into a modal matrix Φ , where $\Phi = [\phi_1, \phi_2, \dots, \phi_n]$. The orthogonality property can then be expressed in the following form

$$\Phi^T \mathbf{M} \Phi = \mathbf{I} \quad (4.13)$$

$$\Phi^T \mathbf{K} \Phi = \Lambda \quad (4.14)$$

where Λ is the spectral matrix which is a diagonal matrix formed by the eigenvalues λ .

It is noted that the eigenvalues are all real values when both $\mathbf{K}$ and $\mathbf{M}$ matrices are symmetric. In most cases the matrices $\mathbf{K}$ and $\mathbf{M}$ are positive definite, so the eigenvalues are greater than zero. However, if the system is capable of rigid body motions (where the frequency of oscillation for the rigid body mode is zero), $\mathbf{K}$ will be positive semi-definite, and the eigenvalues will be greater than or equal to zero. In a case when the mass of the system is lumped at nodes, and $\mathbf{M}$ is a diagonal matrix with some zero diagonal elements, the corresponding eigenvalues of the zero elements are taken to be infinity. For the case of multiple roots for λ_i , where m is the multiplicity of the eigenvalue, the m eigenvectors associated with the eigenvalue λ_i are still linearly independent, and any linear combination of those vectors is also a valid mode shape for the eigenvalue λ_i .

4.4 The Rayleigh-Ritz approximation procedure

The Rayleigh-Ritz method was originally developed to reduce the size of a dynamic problem by which a partial solution of the sparse, large eigenvalue problem is obtained. The Rayleigh-Ritz method can be looked upon as the Ritz extension to the Rayleigh method. The essence of the Rayleigh's method is evident from the fact that the frequency of a single-degree-of-freedom system is defined as $\omega = \sqrt{\frac{k}{m}}$, where k and m are the system's mass and stiffness, respectively. For multi-degree-of-freedom system, the vibration frequency can be determined from this expression if k^* and m^* , represent the generalized stiffness and mass associated with a given assumed shape $\phi(x)$. The main idea of the Rayleigh method is based on the principle of conservation of energy (Clough and Penzien, 1993), which states that the total energy in a freely vibrating system must remain constant if there is no damping force to dissipate it. By equating the energy expression for the maximum strain energy to that for the maximum kinetic energy, a scalar ρ may be defined by the ratio as follows

$$\rho = \frac{\mathbf{u}^T \mathbf{K} \mathbf{u}}{\mathbf{u}^T \mathbf{M} \mathbf{u}} \quad (4.15)$$

where $\mathbf{K}$ and $\mathbf{M}$ represent the stiffness and mass matrices of order n , and $\mathbf{u}$ is an arbitrary vector also of the order n . The scalar ρ is known as the Rayleigh quotient. The Rayleigh quotient represents an upper bound estimate to the lowest eigenvalue or the fundamental frequency. Rayleigh method can provide an adequate approximation of the first mode of vibration to many structures. However, the dynamic analysis of a civil structure often

requires more than one mode to present dynamic response of the analyzed structure. Rayleigh-Ritz method is one of the most convenient methods to evaluate the first several modes of vibration. In the Rayleigh-Ritz method, a coordinate transformation is performed to reduce the number of degrees of freedom of the system, which reduces the computational effort required in dynamic response analysis. The basic assumption of the Ritz method is that the dynamic response may be represented by a set of space-dependent functions amplified by the time-dependent generalized coordinates (Clough and Penzien, 1993) as follows

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{Q}(\mathbf{x})\mathbf{z}(t) \quad (4.16)$$

where $\mathbf{Q}$ is referred here as the matrix of Ritz vectors, and $\mathbf{z}(t)$ is the vector of generalized coordinates. The size of $\mathbf{Q}$ matrix is determined by the total number of degrees-of-freedom n and the number of Ritz vectors q used in the transformation, where it is common that q is much smaller than n .

Thus, Ψ is the n by q matrix storing the Ritz base vectors. Applying the transformation of equation (4.16) to the equation of motion given in equation (4.1), it leads to

$$\mathbf{M}^* \ddot{\mathbf{z}}(t) + \mathbf{C}^* \dot{\mathbf{z}}(t) + \mathbf{K}^* \mathbf{z}(t) = \mathbf{p}^*(t) \quad (4.17)$$

where $\mathbf{M}^* = \mathbf{Q}^T \mathbf{M} \mathbf{Q}$, $\mathbf{C}^* = \mathbf{Q}^T \mathbf{C} \mathbf{Q}$, and $\mathbf{K}^* = \mathbf{Q}^T \mathbf{K} \mathbf{Q}$ are generalized mass, damping and stiffness matrix, respectively, and $\mathbf{p}^* = \mathbf{Q}^T \mathbf{p}$ is generalized force.

It is noted here that matrices $\mathbf{M}^*$, $\mathbf{C}^*$ and $\mathbf{K}^*$ are the projections of $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ onto the subspace spanned by the q Ritz vectors ψ_i , $i = 1, 2, \dots, q$. The eigenproblem for the reduced system is then expressed as follows

$$(\mathbf{K}^* - \Theta \mathbf{M}^*)\mathbf{Z} = 0 \quad (4.18)$$

where Θ is a diagonal matrix of the eigenvalues of the transformed eigenproblem, and $\mathbf{Z}$ is the matrix storing the $\mathbf{M}^*$ -orthonormal eigenvectors of the reduced system $\mathbf{z}_1, \dots, \mathbf{z}_q$. Approximations to the eigenvectors of the original problem are then obtained as follows

$$\Phi^* = \mathbf{Q}\mathbf{Z} \quad (4.19)$$

As noted by Bathe (1996), the eigenvalues calculated from the Ritz analysis are the upper bounds on the corresponding exact eigenvalues of the system. The quality of the approximate solution very much depends on the adequacy of the assumed set of vectors $\mathbf{Q}$. It is not so important that each Ritz vector is close to a required eigenvector, but rather that linear combinations of the Ritz vectors should produce good approximations of the eigenproblem $\mathbf{K}\Phi = \Lambda \mathbf{M}\Phi$.

4.5 Subspace iteration method

The subspace iteration method can be considered as a type of the Rayleigh-Ritz analysis procedure. This method is useful in determining the dominant eigenvalues and corresponding eigenvectors of a large sparse eigenproblem. It involves a simultaneous iteration process in which all the trial vectors are subjected to inverse iteration followed by orthogonalization procedures. The Gram-Schmidt orthogonalization is applied to prevent convergence of all the trial vectors to the most dominant eigenvector. The

orthogonalization procedure assures convergence to the first p eigenvalues and eigenvectors. The algorithm of the subspace iteration method is summarized in Table 4.1.

The subspace iteration method was originally developed by Bathe (1971). After each iteration, the matrices K_{k+1} and M_{k+1} tend to converge to a diagonal form and the generalized Jacobi method can be used as a efficient technique for the solution of the reduced eigenproblem. An important aspect of the subspace iteration method concerns the selection of the starting vectors and the number of the trial vectors that should be included in the iteration. It is generally observed that larger number of iteration vectors increases the rate of convergence. However, the disadvantage of this is that it increases the amount of computation in each iteration step. If p is the number of the required eigenvalues and q is the number of iteration vectors, it is found in practice that the method is most efficient when $q = \min \{2p, p+8\}$.

Furthermore, the proper choice of the starting vectors is a very important step in the subspace iteration method. If they are properly chosen, they can reduce the number of iterations and thus reduce the total cost of obtaining the eigensolution. The starting vectors should be chosen in a way that they excite the degrees-of-freedom with large mass and small stiffness. In general, the first Ritz vector in the first column of the matrix Q_1 can be the diagonal of M to ensure excitation of all the degrees-of-freedom of the system. For other columns of Q_1 except the last column, unit vectors may be selected with entries $+1$ at the degrees-of-freedom with the smallest ratio of k_{ii}/m_{ii} , and the last column is a random vector (Bathe, 1996). The Ritz eigenvalues Θ are upper bounds to the eigenvalues of the original eigenvalue problem, i.e.,

$$\lambda_j \leq \theta_j \quad (4.20)$$

for $j = 1, 2, \dots, p$ where $\theta_1 \leq \theta_2 \leq \dots \leq \theta_p$.

Higher Ritz eigenvalues are generally poorer approximates while the lower ones give better approximations and they often tend to converge first. The eigenvalues from the reduced eigenproblem are converged when

$$\frac{|\theta_i^{(k+1)} - \theta_i^{(k)}|}{\theta_i^{(k)}} \leq \text{tol} \quad \text{for } i = 1, 2, \dots, m \quad (4.21)$$

where $\theta_i^{(k)}$ is the i^{th} diagonal element of Θ_k at the k^{th} iteration and $m < p$.

Further modifications to the subspace iteration method have been introduced by Bathe and Ramaswamy (1980) to accelerate the computation. The basic steps in the original procedure are supplemented by a shifting technique, vector overrelaxation and the Lanczos method to generate a set of starting vectors. Finally, it is worthwhile to mention that the subspace iteration method is widely used by structural engineers and implemented in many computer programs for dynamic response analysis.

4.6 Ritz transformation method

Two Ritz transformation methods used to compute eigenvectors are presented, namely Lanczos method (1950, 1952) and the method developed by Wilson et al. (1982). The Lanczos method was primarily developed as an improvement to the Rayleigh-Ritz method in obtaining the eigensolution. In recent years, this technique has evolved to become one of the most effective procedures for determining a few eigenpairs of large

sparse problems (Sehmi, 1989). The main idea behind Lanczos method is to construct a set of M -orthonormal vectors by applying the Gram-Schmidt orthogonalization process to the Krylov sequence. This set of vectors, denoted by Q , transforms the generalized eigenproblem $K\phi = \lambda M\phi$ into the standard form of a tridiagonal matrix T

$$T = Q^T(MK^{-1}M)Q \quad (4.22)$$

The standard eigenproblem as given in equation (4.6) can be expressed in terms of T as follows

$$Ts = \frac{1}{\lambda} Is \quad (4.23)$$

where s is an eigenvector of equation (4.23). It is noted that the eigenvalues of T are the reciprocals of the eigenvalues of the problem $K\phi = \lambda M\phi$, and the eigenvectors are related as follows

$$\Phi = QS \quad (4.24)$$

Wilson et al. (1982) has developed a similar algorithm to the Lanczos algorithm. This set of vectors is named WYD after the researchers (Wilson, Yang, and Dickens). The only difference is that no iteration is required for the computation of WYD vectors, and the matrix T in equation (4.22) is not a tridiagonal matrix. The standard eigenproblem as given in equation (4.6) can be expressed as follows

$$Is = \frac{1}{\lambda} K^*s \quad (4.25)$$

in which $\mathbf{K}^* = \mathbf{Q}^T \mathbf{K} \mathbf{Q}$ and $\mathbf{I}$ is the unit matrix. The eigenvectors can be computed from equation (4.24).

Theoretically, only one eigenvector associated with a multi-eigenvalue can be calculated using the WYD or Lanczos method. Generally, if n exact eigenvectors are needed, approximately $2n$ Ritz vectors must be calculated. Although the lowest n vectors are almost identical to the eigenvectors, some other n Ritz vectors could differ from the exact eigenvectors.

Table 4.1 Subspace iteration procedure

1. Initiation

Form **K** and **M**

Select starting vectors **Q**₁

2. Iteration (for $k = 1, 2, \dots, \infty$)

2.1. Form $\mathbf{K}\bar{\mathbf{Q}}_{k+1} = \mathbf{M}\mathbf{Q}_k$ and solve for $\bar{\mathbf{Q}}_{k+1}$

2.2. Compute the projections of the matrices **K** and **M**

$$\mathbf{K}_{k+1} = \bar{\mathbf{Q}}_{k+1}^T \mathbf{K} \bar{\mathbf{Q}}_{k+1}$$

$$\mathbf{M}_{k+1} = \bar{\mathbf{Q}}_{k+1}^T \mathbf{M} \bar{\mathbf{Q}}_{k+1}$$

2.3. Solve for the reduced eigenproblem of order q by q

$$\mathbf{K}_{k+1} \mathbf{Z}_{k+1} = \mathbf{M}_{k+1} \mathbf{Z}_{k+1} \mathbf{\Lambda}_{k+1}$$

2.4. Find an improved approximation to the eigenvectors of original system.

$$\mathbf{Q}_{k+1} = \bar{\mathbf{Q}}_{k+1} \mathbf{Z}_{k+1}$$

2.5. Check the convergence, if convergence reached a required level of tolerance then stop, otherwise go to 2.1.

Chapter 5

Linear Time History Response Analysis of Bridges by Ritz Vectors

5.1 General

In the modeling of structural systems for linear dynamic response analysis, it is considered that the largest reduction in the size of the problem is achieved by using the exact eigenvectors of the system in the superposition method. However, the evaluations of the exact natural frequencies and eigenvectors require a large number of numerical operations. Furthermore, the direct engineering significance of this information may be limited. Frequency values indicate possible resonant conditions, and the mode shapes associated with low frequencies can indicate which regions of structure are the most flexible. In many cases approximate values can provide the same information. For most analyses, the only reason for the evaluation of the exact eigenvectors is their subsequent use to reduce the size of the system in the modal superposition analysis. Ritz and component mode synthesis methods have been used extensively for the dynamic response analysis of many practical structures. However, the major motivation for their use has been to avoid the large eigenvalue problem rather than to obtain more accurate results. A traditional approach in structural dynamic response analysis is to solve a generalized

eigenvalue problem by obtaining some lowest eigenvectors and then to expand the dynamic response into a series of these orthogonal vectors. The series is substituted into the dynamic response equilibrium equations and both sides of the equations are premultiplied by the transpose of the eigenvectors. If the damping is proportional, the equation of motion contains only diagonal matrices. After all degrees of freedom have been decoupled, a second order differential equation is formed with a lower order, which can be easily solved. This is the basic idea of the modal superposition method.

In this chapter, the development of the derived Ritz vectors (i.e., the Lanczos and the Wilson's Ritz vectors) is presented. The Lanczos method was originally developed to compute some low frequencies and mode shapes of a structural system (Lanczos, 1950). In practice, the algorithm is usually used to reduce the matrix to a tridiagonal form. The original Lanczos method is sensitive to the orthogonality requirement among the Lanczos vectors. Thus the original Lanczos method suffers from the problem of lack of computational stability, which led the method into a period of obscurity. Recently, with significant progress and improvement made in the numerical stability of the algorithm, the advantage of the Lanczos method has been better recognized and it has become a popular tool for solving eigenproblem of large sparse matrices. In the 1970s, the interest in the Lanczos method was revived because of advances in electronic computation for large and complex engineering problems, which often involves the solution of large sparse matrices. In the 1980s, a dynamic analysis procedure was independently developed by Wilson et al. (1982). The method was then named after its developers (Wilson, Yang, and Dickens) as the WYD method. This method does not require iteration to generate the Ritz vectors, and fewer vectors are needed than using exact eigenvectors

in the superposition analysis of dynamic responses. The WYD and Lanczos algorithms are very similar and a relation between the two algorithms can be obtained.

5.2 Lanczos algorithm

The Lanczos method, first introduced in 1950, is an efficient algorithm for extracting some frequencies and mode shapes of a structural system. It can be thought of as a means of constructing an orthogonal set of independent vectors known as Lanczos vectors, for use in the Rayleigh-Ritz procedure. It is shown here that the Lanczos vectors can be used to formulate an efficient means of dynamic response analysis without solving the vibration eigenproblem. These vectors do not have the full uncoupling property of the mode shapes, however they are much less expensive to generate. Moreover, they are derived from the applied load vector, and include the static displaced shape as the first vector of the series. Thus no 'static correction' is required no matter what the shape of the loading is or how few vectors are employed in the analysis. The Lanczos method is similar to the inverse iteration and the power method for calculating a single vibration mode shape and frequency of an eigenproblem. Given a pair of matrices $\mathbf{K}$ and $\mathbf{M}$ and a starting vector $\mathbf{r}$, these methods generate a sequence of vectors called the Krylov sequence as follows

$$[\mathbf{r}, \mathbf{K}^{-1}\mathbf{M}\mathbf{r}, (\mathbf{K}^{-1}\mathbf{M})^2\mathbf{r}, \dots, (\mathbf{K}^{-1}\mathbf{M})^j\mathbf{r}] \quad (5.1)$$

The sequence converges to the eigenvector corresponding to the smallest eigenvalue in magnitude of the eigenproblem $(\mathbf{K} - \lambda\mathbf{M})\mathbf{z} = 0$. The basic difference between the Lanczos method and other methods is that the information contained in each successive vector of

the Krylov sequence is employed in the dynamic response analysis, instead of using the final converged vector i.e., the Lanczos algorithm involves supplementing the Krylov sequence with a Gram-Schmidt orthogonalization process at each step; the result is a set of M -orthonormal vectors that is used to reduce the dimension of the dynamic equation set. In the form of the Lanczos algorithm described here, orthogonalization is needed to be applied only to the two preceding vectors, leading to a tridiagonal form of the dynamic equations that can be used with great advantage in either direct time step integration or in free vibration eigenproblem of the reduced system of equations of motion.

In the discussion of Lanczos algorithm, it is assumed that the first j Lanczos vectors, $(\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_j)$ have been found, and the analysis of the vector $\mathbf{q}_{j+1}$ is performed. These Lanczos vectors are normalized to satisfy the condition $\mathbf{q}_i^T \mathbf{M} \mathbf{q}_j = \delta_{ij}$, where δ_{ij} is the Kroneker delta ($\delta_{ij} = 1$ for $i = j$, and $\delta_{ij} = 0$ for $i \neq j$) so that the vectors are orthonormal with respect to the mass matrix. To calculate $\mathbf{q}_{j+1}$, a preliminary vector $\bar{\mathbf{r}}_j$ is first calculated from the previous vector, $\mathbf{q}_j$ as in the Krylov sequence

$$\bar{\mathbf{r}}_j = \mathbf{K}^{-1} \mathbf{M} \mathbf{q}_j \quad (5.2)$$

This preliminary vector $\bar{\mathbf{r}}_j$ contains components from each preceding vectors. Thus

$$\bar{\mathbf{r}}_j = \mathbf{r}_j + \alpha_j \mathbf{q}_j + \beta_j \mathbf{q}_{j-1} + \gamma_j \mathbf{q}_{j-2} + \dots \quad (5.3)$$

where $\mathbf{r}_j$ is the pure vector orthogonal to all previous Lanczos vectors, and $\alpha_j, \beta_j, \gamma_j \dots$ are the amplitudes of the previous vectors contained in $\bar{\mathbf{r}}_j$. These coefficients are evaluated

from the orthogonality of the Lanczos vectors. Thus if both sides of equation (5.3) are premultiplied by $\mathbf{q}_j^T \mathbf{M}$, one obtains

$$\mathbf{q}_j^T \mathbf{M} \bar{\mathbf{r}}_j = \mathbf{q}_j^T \mathbf{M} \mathbf{r}_j + \alpha_j \mathbf{q}_j^T \mathbf{M} \mathbf{q}_j + \beta_j \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-1} + \gamma_j \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-2} + \dots \quad (5.4)$$

Here the first term on the right hand side of equation (5.4) vanishes by definition and similarly all terms beyond the second term vanish due to $\mathbf{M}$ -orthogonality, which gives the amplitude of $\mathbf{q}_j$ along $\bar{\mathbf{r}}_j$ as follows

$$\alpha_j = \mathbf{q}_j^T \mathbf{M} \bar{\mathbf{r}}_j \quad (5.5)$$

Similarly, the amplitude of $\mathbf{q}_{j-1}$ contained in $\bar{\mathbf{r}}_j$ may be found by premultiplying equation (5.3) by $\mathbf{q}_{j-1}^T \mathbf{M}$. In this case all terms except the third term vanish due to $\mathbf{M}$ -orthogonality, which gives

$$\beta_j = \mathbf{q}_{j-1}^T \mathbf{M} \bar{\mathbf{r}}_j \quad (5.6)$$

Substituting equation (5.2) into equation (5.6) and applying transposition leads to

$$\beta_j = \mathbf{q}_{j-1}^T \mathbf{M} \mathbf{K}^{-1} \mathbf{M} \mathbf{q}_j = \bar{\mathbf{r}}_{j-1}^T \mathbf{M} \mathbf{q}_j = \mathbf{q}_j^T \mathbf{M} \bar{\mathbf{r}}_{j-1} \quad (5.7)$$

Expanding $\bar{\mathbf{r}}_{j-1}$ in terms of the purified vector, $\mathbf{r}_{j-1}$ and the preceding Lanczos vectors, equation (5.6) becomes

$$\beta_j = \mathbf{q}_j^T \mathbf{M} \mathbf{r}_{j-1} + \alpha_{j-1} \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-1} + \beta_{j-1} \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-2} + \gamma_{j-1} \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-3} + \dots \quad (5.8)$$

It is evident from equation (5.8) that all terms except the first term vanish on the right hand side. Now $\mathbf{q}_j$ is the vector obtained by normalizing $\mathbf{r}_{j-1}$ with β_j as the normalizing factor

$$\mathbf{q}_j = \frac{1}{\beta_j} \mathbf{r}_{j-1} \quad (5.9)$$

Then the value of β_j is given by equation (5.9) as follows

$$\beta_j = (\mathbf{r}_{j-1}^T \mathbf{M} \mathbf{r}_{j-1})^{\frac{1}{2}} \quad (5.10)$$

Following the same derivations, the amplitude of $\mathbf{q}_{j-2}$ contained in $\bar{\mathbf{r}}_j$ is found to be

$$\gamma_j = \mathbf{q}_{j-2}^T \mathbf{M} \bar{\mathbf{r}}_j \quad (5.11)$$

Following the procedure used to derive equation (5.10) leads to

$$\gamma_j = \mathbf{q}_j^T \mathbf{M} \mathbf{r}_{j-2} + \alpha_{j-2} \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-2} + \beta_{j-2} \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-3} + \gamma_{j-2} \mathbf{q}_j^T \mathbf{M} \mathbf{q}_{j-4} + \dots \quad (5.12)$$

But according to the normalizing relationship of equation (5.9), $\mathbf{r}_{j-2} = \beta_{j-1} \mathbf{q}_{j-1}$. Hence, when this is substituted into equation (5.12) it is clear that all terms on the right hand side of equation (5.12) vanish, the results is that $\gamma_j = 0$. A corresponding procedure can be used to demonstrate that all further terms in the expansion of $\bar{\mathbf{r}}_j$ vanish. In other words, the orthogonalization procedure used in generating each Lanczos vector needs be applied only to the previous two vectors. The Lanczos algorithm to derive the vector $\mathbf{q}_{j+1}$ is presented as shown in Table 5.1.

In theory, the starting vectors $\mathbf{q}_1$ of the Lanczos algorithm may be chosen arbitrarily. The algorithm described above can then be followed to generate as many vectors as needed.

The Lanczos algorithm is particularly advantageous for dynamic analysis of structures when the applied dynamic load is of the form

$$\mathbf{f} = \mathbf{b}\varepsilon(t) \quad (5.13)$$

where the spatial distribution of the load $\mathbf{b}$ remains constant and only its magnitude varies with time.

The starting vector in this case is taken in the direction of the static displacement. Thus

$$\mathbf{r}_0 = \beta_1 \mathbf{q}_1 = \mathbf{K}^{-1} \mathbf{b} \quad (5.14)$$

This choice gives the Lanczos vectors the important advantage noted before, that they automatically include the static displacement of the structure and thus it avoids the need of static correction to the analysis results. It is of interest to note that Wilson et al. (1982) adopted this starting vector for this reason. It is evident from equation (5.14) that if $\mathbf{b}$ is $\mathbf{M}$ -orthogonal to any vector then all the Lanczos vectors will be $\mathbf{M}$ -orthogonal to that vector.

An important type of dynamic loading of the form of equation (5.13) is earthquake excitation, where the effective loading results from accelerations applied at the supports. In this case, $\mathbf{b} = \mathbf{M}\mathbf{s}$, where $\mathbf{s}$ is the influence coefficient vector considering the displacements at the degrees-of-freedom of the structure due to a unit support displacement, and $\varepsilon(t)$ is the acceleration time history applied at the supports. The

Lanczos vectors then can be used in the Rayleigh-Ritz procedure to obtain approximates of the frequencies and mode shapes of the structural system. Substituting equation (5.2) into equation (5.3) and noting that the Lanczos vectors are made orthogonal to only two preceding vectors

$$\mathbf{r}_j = \mathbf{K}^{-1}\mathbf{M}\mathbf{q}_j - \alpha_j\mathbf{q}_j - \beta_j\mathbf{q}_{j-1} \quad (5.15)$$

From equation (5.9) $\mathbf{r}_j$ is written as follows

$$\mathbf{r}_j = \beta_{j+1}\mathbf{q}_{j+1} \quad (5.16)$$

Substituting equation (5.16) into equation (5.15) leads to

$$\mathbf{K}^{-1}\mathbf{M}\mathbf{q}_j = \alpha_j\mathbf{q}_j + \beta_j\mathbf{q}_{j-1} + \beta_{j+1}\mathbf{q}_{j+1} \quad (5.17)$$

Using equation (5.17) for $j = 1, \dots, m$

$$\mathbf{K}^{-1}\mathbf{M}[\mathbf{q}_1, \dots, \mathbf{q}_m] = [\mathbf{q}_1, \dots, \mathbf{q}_m] \begin{bmatrix} \alpha_1 & \beta_2 & & \\ \beta_2 & \alpha_2 & \beta_3 & \\ & & & \\ & & & \beta_m \\ & & \beta_m & \alpha_m \end{bmatrix} + [0, \dots, 0, \beta_{m+1}\mathbf{q}_{m+1}] \quad (5.18)$$

Let $\mathbf{T}_m$ be the tridiagonal matrix made up of the coefficient α_j and β_j

$$\mathbf{T}_m = \begin{bmatrix} \alpha_1 & \beta_2 & & \\ \beta_2 & \alpha_2 & \beta_3 & \\ & & & \\ & & & \beta_m \\ & & \beta_m & \alpha_m \end{bmatrix} \quad (5.19)$$

Premultiplying equation (5.18) by $\mathbf{Q}_m^T \mathbf{M}$, where $\mathbf{Q}_m^T$ is the column matrix of the generated Lanczos vectors $\mathbf{q}_1, \dots, \mathbf{q}_m$, the tridiagonal matrix $\mathbf{T}_m$ can be obtained as follows

$$\mathbf{T}_m = \mathbf{Q}_m^T \mathbf{M} \mathbf{K}^{-1} \mathbf{M} \mathbf{Q}_m \quad (5.20)$$

That is the projection of $\mathbf{M} \mathbf{K}^{-1} \mathbf{M}$ on the subspace $\mathbf{Q}_m$ is a tridiagonal matrix. The generalized eigenproblem is given in the form

$$\left(\mathbf{K}^{-1} \mathbf{M} - \frac{1}{\lambda} \mathbf{I} \right) \phi = 0 \quad (5.21)$$

In the formulation here, the pair $\{\theta_i^{(m)}, \mathbf{y}_i^{(m)}\}$ denotes a Ritz pair, which provides the best approximation to the eigenpair $\{\nu, \phi\}$. The relationship between λ and ν is given in equation (4.8). The Ritz vector $\mathbf{y}_i^{(m)}$ is expressed in terms of the Lanczos vectors $\mathbf{Q}_m$ and a coefficient vectors $\mathbf{s}_i^{(m)}$.

$$\mathbf{y}_i^{(m)} = \mathbf{Q}_m \mathbf{s}_i^{(m)}, i = 1, 2, \dots, m \quad (5.22)$$

Substituting equation (5.22) into equation (5.21) leads to

$$(\mathbf{K}^{-1} \mathbf{M} - \theta_i^{(m)} \mathbf{I}) \mathbf{Q}_m \mathbf{s}_i^{(m)} = 0 \quad (5.23)$$

Pre-multiplying by $\mathbf{Q}_m^T \mathbf{M}$ and noting orthonormal properties of the Lanczos vectors $\mathbf{Q}_m^T \mathbf{M} \mathbf{Q}_m = \mathbf{I}$, the eigenproblem of equation (5.23) is transformed to a tridiagonal form

$$\mathbf{T}_m \mathbf{s}_i^{(m)} - \theta_i^{(m)} \mathbf{s}_i^{(m)} = 0 \quad (5.24)$$

where $s_i^{(m)}$ is the eigenvector of the reduced tridiagonal system. As the number of Lanczos vectors increases, the eigenvalues of T_m , $\theta_i^{(m)}$ converge to the eigenvalues of the transformed problem represented by equation (5.20).

5.3 WYD algorithm

Another Ritz analysis approach for dynamic response analysis of structural systems has been independently developed to generate a set of Ritz vectors that also takes into consideration the spatial distribution of the applied load (Wilson et al., 1982). The vectors obtained by Wilson et al. in their method have been referred to as the load-dependent Ritz vectors. The Wilson's method is mathematically similar to the Lanczos algorithm. The vectors generated by both methods span the same Krylov subspace. In the original Wilson's method, each newly generated Wilson's Ritz vector is orthogonalized against all preceding Wilson's Ritz vectors, whereas in the Lanczos algorithm the orthogonalization procedure, which is based on the three term recurrence formula given in equation (5.17), affects only the two preceding Lanczos vectors.

The original Wilson algorithm, also called the WYD method named after the researchers Wilson, Yuan and Dickens, is based on the idea of selecting an orthonormalized Ritz basis in the Krylov space that accounts for the spatial distribution of the load. The method employs Gram-Schmidt orthogonalization to obtain a set of orthogonal vectors.

It is apparent that the selection of the Ritz vectors is a critical phase because they are directly used in the modal superposition analysis. The WYD vectors are generated from the following recurrence relationship

$$\mathbf{K}\mathbf{x}_i^* = \mathbf{M}\mathbf{x}_{i-1}^*, \text{ where } i = 2, \dots, n \quad (5.25)$$

The first vector $\mathbf{x}_1^*$ is obtained from the solutions of static equilibrium

$$\mathbf{K}\mathbf{x}_1^* = \mathbf{f} \quad (5.26)$$

The vectors are $\mathbf{M}$ -orthogonalized at each step by the use of the following procedure

$$\mathbf{x}_i^{**} = \mathbf{x}_i^* - \sum_{j=1}^{i-1} c_j \mathbf{x}_j \quad (5.27)$$

where

$$c_j = \mathbf{x}_j^T \mathbf{M} \mathbf{x}_i^* \quad (5.28)$$

Then it is $\mathbf{M}$ -normalized as follows

$$\mathbf{x}_i = \frac{\mathbf{x}_i^{**}}{(\mathbf{x}_i^{**T} \mathbf{M} \mathbf{x}_i^{**})^{1/2}} \quad (5.29)$$

The algorithm of the WYD method is shown in Table 5.2. The sequence of steps used to generate the Ritz vector has an interesting physical interpretation. It is noted that the first Wilson's Ritz vector represents a static response to the load vector $\mathbf{f}$ while ignoring the dynamic effect. In the second step, a load vector proportional to the dynamic inertia force resulting from the first Ritz vector of the form $\omega^2 \mathbf{M} \mathbf{x}_1$ is applied to generate the next Ritz

vector (i.e., equation (5.25)). The static solution at each step of the Wilson's algorithm neglects the dynamic effect, which is later used as the load vector in the next step for the generation of the next load dependent Ritz vector. As such, the generated vectors consist of vectors excited by the input loading. Thus, Wilson's Ritz vectors represent the major part of the response, whereas in the case of modal analysis using the exact modes, which are determined independent of the loading, there is no guarantee that the included modes actually participate in the dynamic response of the system. From previous studies and applications, the Gram-Schmidt orthogonalization procedure is sometimes known to be numerically unstable. In view of this, the original WYD algorithm has been modified by Leger (1987) with improvement on the numerical stability of the algorithm. The improvement at each step is achieved by using an improved static vector obtained from a one-step inverse iteration procedure instead of the previous Wilson's vector, x_{i-1} . The algorithm of this modified version of the Wilson's method referred to here as the LWYD algorithm is shown in Table 5.3. First, an initial vector u_1 is obtained from the static deflected shape. Using this static deflected shape as the initial vector in an one-step inverse iteration, a new Wilson's vector x_1 is then obtained. At each subsequent step, the initial vector u_1 is updated and all components of the previous vectors X in it are removed using the Gram-Schmidt orthogonalization procedure.

Leger (1987) suggests that by using the modified initial static deflected shape in each step, the rounding errors that may accumulate in the original algorithm are partially eliminated and a better control on the static correction is achieved. Theoretically, only one eigenvector associated with a multi-eigenvalue can be calculated using either the WYD or Lanczos method. Generally, if N exact eigenvectors are needed, approximately

$2N$ Ritz vectors are needed by the WYD method. Although the lowest N vectors are almost identical to the eigenvectors, some other N Ritz vectors could differ from the exact eigenvectors. In the following section, the solutions for approximate eigenvectors and eigenvalues are presented using the WYD and Lanczos vectors.

5.4 Ritz methods for linear time history response analysis

The Ritz vectors obtained by Lanczos and WYD algorithms can be used to obtain approximate modal properties or eigensolution to the eigenproblem of complex structures as presented in preceding two sections. In this section, the Ritz vectors are used to evaluate the dynamic responses of structural systems. Traditionally, the dynamic response of a linear multi-degree-of-freedom (MDOF) system can be obtained by the modal superposition method. The advantage of this method is that the size of the original problem is reduced as the dynamic response can be described adequately by the contributions from only the first few lower modes. The determination of the dynamic response of a structural system even though it may have hundreds of degrees-of-freedom is reduced to the solution of a set of only a few uncoupled modal equations by using the mode shapes to approximate the behaviour of the system. However, the direct engineering significance of the exact frequencies and mode shapes may be limited. For a large structural system, the need to obtain the eigenvectors for use in the modal superposition analysis may be inefficient. Furthermore, the determination and the use of the exact mode shapes do not take into account the spatial distribution of dynamic load. In comparison, the Ritz vectors generated by the Lanczos and WYD algorithms can essentially provide the same information with much less computation effort especially for

large complex structural systems. This is a major advantage of the Ritz vectors compared to the exact mode shapes used in the mode superposition method. The Rayleigh-Ritz vectors are required only to satisfy the geometric boundary conditions of the problem and be linear independent whereas the eigenmodes must satisfy not only the geometric boundary conditions but also more rigorous conditions in that they are also solutions to the eigenvalue problem of the structural system. Therefore, it is much less costly to generate a set of Rayleigh-Ritz vectors for the reduction of the system than the mode shapes. The disadvantage is that the resulting transformed matrices are no longer uncoupled and usually full. However, for a large complex system, the advantage of the Rayleigh-Ritz method usually more than compensates for the disadvantage. A special form of Rayleigh-Ritz procedure is the so called load-dependent Ritz vector algorithms, which transform the dynamic problem of a full size coupled equations of motion to a new system of tri-diagonal form of reduced size. The use of the load-dependent Ritz vectors as generalized coordinates in the modeling of the problem improves numerical efficiency and reduces the computational effort required in the analysis. Typically, fewer Ritz vectors are needed to describe the dynamic response of the system with the same or even higher level of accuracy in the results than required by the modal analysis method. This is in addition to the advantage that the need for the solution of the large eigenvalue problem is avoided.

5.4.1 Time history response analysis by modal superposition method

The equation (4.1) is rewritten as follows

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping and stiffness matrices respectively, $\mathbf{f}$ is the external load; $\mathbf{u}$ is the displacement vector.

The exact mode shapes are considered the special case of the Ritz vectors, thus a summary of modal analysis procedures presented below can be useful for later discussions of Ritz analysis. The standard mode superposition method is based on the transformation of the geometric displacement $\mathbf{u}(s,t)$ of the considered structural system to a generalized form

$$\mathbf{u}(s,t) = \Phi(s)\mathbf{z}(t) \quad (5.30)$$

In equation (5.30), the matrix Φ constitutes the set of mode shapes (eigenvectors), which serves to transform the generalized coordinate vector $\mathbf{z}(t)$ to the geometric displacement vector $\mathbf{u}(s,t)$ in the geometric coordinates. Substituting equation (5.30) into the matrix equation of motion for a structural system given in equation (4.1) yields to

$$\mathbf{M}\Phi\ddot{\mathbf{z}} + \mathbf{C}\Phi\dot{\mathbf{z}} + \mathbf{K}\Phi\mathbf{z} = \mathbf{p}(t) \quad (5.31)$$

Premultiplying both sides of equation (5.31) by Φ^T , the following equation is obtained

$$\mathbf{M}^*\ddot{\mathbf{z}} + \mathbf{C}^*\dot{\mathbf{z}} + \mathbf{K}^*\mathbf{z} = \mathbf{p}^* \quad (5.32)$$

where $\mathbf{M}^*$, $\mathbf{C}^*$, and $\mathbf{K}^*$ are the generalized mass, damping and stiffness matrices respectively, and $\mathbf{p}^*$ is the generalized external force vector, defined as follows

$$\begin{aligned}
\mathbf{M}^* &= \Phi^T \mathbf{M} \Phi \\
\mathbf{C}^* &= \Phi^T \mathbf{C} \Phi \\
\mathbf{K}^* &= \Phi^T \mathbf{K} \Phi \\
\mathbf{p}^* &= \Phi^T \mathbf{p}
\end{aligned}
\tag{5.33}$$

In general, although the generalized matrices in equation (5.33) remain symmetric, they are generally fully populated or tridiagonal. In contrast, if eigenvectors are used for Φ in the transformation of equation (5.33), the matrices $\mathbf{M}^*$, $\mathbf{K}^*$ will be reduced to a diagonal form. Special considerations are required for damped systems in the representation of the damping matrix $\mathbf{C}$. The definition of the damping matrix $\mathbf{C}$ usually presents some difficulties due to the complexity of the damping mechanisms involved in a structural system. To simplify the problem, assumptions are usually made concerning the damping mechanism in that the damping force is taken proportional to the velocity as in viscous damping. Moreover, the damping matrix is assumed to be proportional to $\mathbf{M}$ and $\mathbf{K}$ where $\mathbf{C}$ can be expressed as follows

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \tag{5.34}$$

The damping having the property of equation (5.34) is called Rayleigh damping after Lord Rayleigh, who first suggested its use or proportional damping. The constant coefficients α (sec^{-1}) and β (sec) are respectively the proportionality constants with $\mathbf{M}$ and $\mathbf{K}$ of the structural system.

In general, the time-varying load vector $\mathbf{p}(t)$ can be written as the product of a spatially distributed loading function $\mathbf{f}(s)$, which remains unchanged with time, and a time-varying scalar amplitude function $g(t)$ as follows

$$\mathbf{p}(s, t) = \mathbf{f}(s)g(t) \quad (5.35)$$

Using equation (5.35) for the load vector, equation (5.32) becomes

$$\mathbf{M}^* \ddot{\mathbf{z}} + \mathbf{C}^* \dot{\mathbf{z}} + \mathbf{K}^* \mathbf{z} = \mathbf{\Phi}^T \mathbf{f}(s)g(t) \quad (5.36)$$

After solving the above equation for the response of each modal coordinate, the response in the geometric coordinates can be obtained from equation (5.30). In modal response analysis, the number of mode shapes L is usually much less than the size of the problem (number of degrees of freedom). As a measure of the degree of participation of a particular mode in the dynamic response of a structural system, the modal participation factor can be used which is defined as follows

$$\Gamma_i = \frac{\phi_i^T \mathbf{f}(s)}{\phi_i^T \mathbf{M} \phi_i} \quad (5.37)$$

Alternatively, another important measure is an estimate of the error in the representation of the dynamic load. The spatial distribution of the dynamic load, $\mathbf{f}(s)$, can be expressed in terms of the mode shapes as follows

$$\mathbf{f} = \sum_{i=1}^N \phi_i \mathbf{f}^T \mathbf{M} \phi_i \quad (5.38)$$

with the mode shape ϕ_i normalized to unity with respect to the mass matrix.

In mode superposition analysis, a truncated series of L mode shapes is used with $L \ll N$. The higher order components of the load as presented by the higher modes

(from $L+1$ to N) are omitted. Therefore, an error vector in the load representation $\mathbf{e}_L$ exists, which is given as follows

$$\mathbf{e}_L = \mathbf{f} - \sum_{i=1}^L \phi_i \mathbf{f}^T \mathbf{M} \phi_i \quad (5.39)$$

Consequently, an error norm ε can be defined as follows

$$\varepsilon = \frac{|\mathbf{f}^T \mathbf{e}_L|}{\mathbf{f}^T \mathbf{f}} \quad (5.40)$$

which can be used as a cut-off criterion for determining the number of mode shapes that should be included in modal analysis. When the number of mode shapes L used in the modal analysis increases, ε_L becomes small and at some point the higher modes can be truncated.

The equations of motion of a MDOF system subject to earthquake loading can be expressed as follows

$$\mathbf{M}\ddot{\mathbf{u}}^t + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \quad (5.41)$$

where $\mathbf{u}^t$ represents the vector of the total displacement which is equal to the sum of the relative displacements $\mathbf{u}(t)$ and a rigid body displacements $\mathbf{u}_s(t)$ of the system

$$\mathbf{u}^t(t) = \mathbf{u}(t) + \mathbf{u}_s(t) \quad (5.42)$$

The rigid body displacements, also known as the quasi-static displacements, can be expressed in terms of an influence coefficient vector $\mathbf{r}$ which represents the

displacements of structure due to a unit displacement of the support, and the displacement of support $u_g(t)$ due to ground motion.

$$\mathbf{u}_s(t) = \mathbf{r}\mathbf{u}_g(t) \quad (5.43)$$

and

$$\mathbf{u}'(t) = \mathbf{u}(t) + \mathbf{r}\mathbf{u}_g(t) \quad (5.44)$$

Therefore, the equations of motion for a MDOF system have the form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\ddot{\mathbf{u}}_g = -\mathbf{M}\mathbf{r}\ddot{u}_g \quad (5.45)$$

The effective force vector as generated by the earthquake ground motion is

$$\mathbf{p}_{\text{eff}}(t) = -\mathbf{M}\mathbf{r}\ddot{u}_g(t) \quad (5.46)$$

Similar to the generalized loading case in equation (5.30), the modal participation factor for earthquake loading can be defined as follows

$$\Gamma_i = \frac{\phi_i^T \mathbf{M} \mathbf{r}}{\phi_i^T \mathbf{M} \phi_i} \quad (5.47)$$

Equations (5.39) and (5.40), which give the error estimates, are also valid for the case of earthquake loading in which the spatial load vector $\mathbf{f}(s)$ is expressed in terms of the structure mass matrix $\mathbf{M}$ and the distribution vector $\mathbf{r}$.

$$\mathbf{f}(s) = \mathbf{M} \mathbf{r} \quad (5.48)$$

The error norm in equation (5.40) for earthquake loading thus becomes

$$\varepsilon = \frac{\mathbf{r}^T \mathbf{M} \mathbf{r} - \sum_{i=1}^L (\phi_i^T \mathbf{M} \mathbf{r})^2}{\mathbf{r}^T \mathbf{M} \mathbf{r}} \quad (5.49)$$

When the mass matrix $\mathbf{M}$ is diagonal, the term $\mathbf{r}^T \mathbf{M} \mathbf{r}$ represents the total mass of the system that participates in the dynamic response due to the support motion. The term $\sum_{i=1}^L (\phi_i^T \mathbf{M} \mathbf{r})^2$ represents the part of the system mass considered in the modal analysis. The dynamic equations of motion of the system can be solved by a number of numerical schemes, including the Duhamel integral method and the direct time step integration procedure. In the direct time integration approach, the Newmark- β methods, which include the average and linear acceleration methods, have been widely used in time history response analysis. The accuracy of the Newmark- β methods depends on the size of the time step chosen for the numerical integration.

5.4.2 Load-dependent Ritz methods for linear time history response analysis

In the context of the present research, the mode superposition method can be looked upon as a special case of the Rayleigh-Ritz method in which the mode shapes are the Ritz vectors employed in the transformation of the original system. Within the context of the Rayleigh-Ritz method, it is not necessary to employ the restrictive eigenvectors, which lead to an uncoupled set of dynamic equations of motion to discretize the system. Numerical studies have shown that good approximation to the dynamic response and behaviour of the system can still be obtained by the use of less rigorous Ritz vectors for the transformation and reduction of the dynamic system.

The concept of the recently developed load-dependent Ritz vectors is presented here and two procedures, the Lanczos and Wilson's methods, for the generation of the special type of Ritz vectors are also discussed. Both, the Lanczos and Wilson's algorithms, are based on the idea of constructing a set of $\mathbf{M}$ -orthonormal vectors, which takes into consideration the static effect due to the spatial distribution of the dynamic loads. Premultiplying equation (4.1) by $\mathbf{MK}^{-1}$ leads to

$$\mathbf{MK}^{-1}\mathbf{M}\ddot{\mathbf{u}} + \mathbf{MK}^{-1}\mathbf{C}\dot{\mathbf{u}} + \mathbf{Mu} = \mathbf{MK}^{-1}\mathbf{p} \quad (5.50)$$

Substituting equation (4.16) into equation (5.50) one obtains

$$\mathbf{Q}^T\mathbf{MK}^{-1}\mathbf{MQ}\ddot{\mathbf{z}} + \mathbf{Q}^T\mathbf{MK}^{-1}\mathbf{CQ}\dot{\mathbf{z}} + \mathbf{Q}^T\mathbf{MQz} = \mathbf{Q}^T\mathbf{MK}^{-1}\mathbf{p} \quad (5.51)$$

Substituting equations (5.20) and (5.34) into equation (5.51) leads to

$$\mathbf{T}\ddot{\mathbf{z}} + [\alpha\mathbf{T} + \beta\mathbf{I}]\dot{\mathbf{z}} + \mathbf{z} = \mathbf{f} \quad (5.52)$$

in which

$$\mathbf{f} = \mathbf{Q}^T\mathbf{MK}^{-1}\mathbf{p} \quad (5.53)$$

Similarly, if the WYD vectors are used, the equations of motion are written as follows

$$\mathbf{I}\ddot{\mathbf{z}} + [\alpha\mathbf{I} + \beta\mathbf{K}^*]\dot{\mathbf{z}} + \mathbf{K}^*\mathbf{z} = \mathbf{Q}^T\mathbf{p} \quad (5.54)$$

5.4.3 Comparison of WYD and Lanczos methods

The fundamentals of the WYD method and the Lanczos method are similar. Both algorithms produce a set of orthogonal vectors, which are used to discretize the dynamic

behaviour of the system and to reduce the size of the problem. During the process of obtaining the Wilson's Ritz vectors or the Lanczos vectors, the Krylov sequence is orthogonalized and normalized by the Gram-Schmidt orthogonalization procedure. The procedures are non-iterative in nature. Consequently, both procedures are expected to be faster and more efficient when compared to the subspace iteration method. The first step of the Lanczos algorithm is exactly the same as that in the WYD method.

The differences between the two methods mainly occur in the fact that the generalized stiffness matrix $\mathbf{K}^*$ from the transformation by the Wilson's Ritz vectors is a full matrix in the projected subspace, whereas the Lanczos method leads to a tridiagonal system of equations of motion for the transformed eigenvalue problem. However, as it has been shown by Leger (1987) that the inverse of $\mathbf{K}^*$ is equal to the tridiagonal matrix $\mathbf{T}$ generated by the Lanczos algorithm. Moreover, the response solution in Lanczos coordinates is represented by equation (5.52) as follows

$$\mathbf{T}\ddot{\mathbf{z}} + [\alpha\mathbf{T} + \beta\mathbf{I}]\dot{\mathbf{z}} + \mathbf{z} = \mathbf{Q}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{p}$$

The equivalent equations of motion by WYD method is represented by equation (5.54) as follows

$$\mathbf{I}\ddot{\mathbf{z}} + [\alpha\mathbf{I} + \beta\mathbf{K}^*]\dot{\mathbf{z}} + \mathbf{K}^*\mathbf{z} = \mathbf{Q}^T\mathbf{p}$$

So that one approach is the inverse of the other. The Lanczos method is typically faster than the Wilson's method since the diagonalization of a full subspace matrix requires more time than that of a tridiagonal matrix (e.g., Yuan et al., 1988).

5.5 Cut-off criteria for multi-directional earthquake loading

The cut-off criteria for generating the derived Ritz vectors are based on two considerations: (a) representation of spatial distribution of load vector; and (b) representation of mass distribution. These two criteria are useful in the determination of the proper number of derived Ritz vectors that should be included in a dynamic response analysis to obtain the required accuracy in a truncated system.

For the present purpose, the spatially distributed load vector $\mathbf{f}(s)$ can be expressed as follows

$$\mathbf{f} = \sum_{i=1}^N \mathbf{x}_i \mathbf{f}^T \mathbf{M} \mathbf{x}_i \quad (5.55)$$

The error in the representation of the load when only L Wilson's Ritz vectors are used is given by

$$\mathbf{e}_L = \mathbf{f} - \mathbf{f}_L \quad (5.56)$$

where

$$\mathbf{f}_L = \sum_{i=1}^L \mathbf{x}_i \mathbf{f}^T \mathbf{M} \mathbf{x}_i \quad (5.57)$$

Finally, a scalar representation of the error, which can be treated as a cut-off criterion is defined as follows

$$\varepsilon_L = \frac{|\mathbf{f}^T \mathbf{e}_L|}{\mathbf{f}^T \mathbf{f}} \quad (5.58)$$

The second cut-off criterion based on the representation of the mass distribution can be formulated as follows

$$\xi_L = \frac{\sum_{i=1}^L (\bar{\mathbf{M}}^T \mathbf{x}_i)^2}{\sum_{i=1}^N \bar{\mathbf{M}}_i} \quad (5.59)$$

where $\bar{\mathbf{M}}$ is a vector whose i^{th} term is calculated as follows

$$\bar{\mathbf{M}}_i = \sum_{j=1}^N m_{ij} \quad (5.60)$$

where m_{ij} is the (i,j) element of matrix $\mathbf{M}$.

The two criteria discussed here can be applied simultaneously as either one of the two can be used as the cut-off criterion for Ritz algorithms. Xia and Humar (1992) have developed another algorithm similar to the Wilson's methods, which is based on the consideration of the frequency spectrum of the applied load. The method produces so called frequency dependent Ritz vectors for determining the response of a structural system subject to high frequency loading. This technique combines WYD method and LWYD method, with a shifting procedure, where the special error norms are developed to terminate the Ritz vector generation process when the required accuracy of the solution is achieved. As it was mentioned earlier, a commonly used criterion to measure the truncation error of the Rayleigh-Ritz analysis is the effective participating mass. Leger (1986, 1987) and Joo et al. (1988) have suggested that since the support movement is direction dependent, the criterion expressed in equation (5.59) should be applied in each

of the three directions separately in a general three-dimensional analysis. The influence vector $\mathbf{r}$ can be expressed in the Ritz vector subspace as follows

$$\mathbf{r} = \mathbf{X}\boldsymbol{\beta} \quad (5.61)$$

where $\boldsymbol{\beta}$ represents the generalized coordinates of the Ritz vector for $\mathbf{r}$.

Pre-multiplying both sides of the equation (5.61) by $\mathbf{x}_i^T \mathbf{M}$ and applying the mass orthonormality condition

$$\beta_i = \mathbf{x}_i^T \mathbf{M} \mathbf{r} \quad (5.62)$$

Now, the effective participating mass M_i^{eff} associated with the i th Ritz vector can be derived as follows

$$\mathbf{M}_i^{\text{eff}} = \mathbf{r}^T \mathbf{M} \mathbf{x}_i \mathbf{x}_i^T \mathbf{M} \mathbf{r} = \beta_i^2 \quad (5.63)$$

The mass participating factors for ground motion applied in one specific direction (when the first L Ritz vectors are used in the analysis) can be written as follows

$$\xi_L^1 = \frac{\sum_{i=1}^L (M_i^1)^{\text{eff}}}{\mathbf{r}_1^T \mathbf{M} \mathbf{r}_1}, \text{ where } (M_i^1)^{\text{eff}} = \mathbf{r}_1^T \mathbf{M} \mathbf{x}_i \mathbf{x}_i^T \mathbf{M} \mathbf{r}_1 = \beta_{1i}^2$$

$$\xi_L^2 = \frac{\sum_{i=1}^L (M_i^2)^{\text{eff}}}{\mathbf{r}_2^T \mathbf{M} \mathbf{r}_2}, \text{ where } (M_i^2)^{\text{eff}} = \mathbf{r}_2^T \mathbf{M} \mathbf{x}_i \mathbf{x}_i^T \mathbf{M} \mathbf{r}_2 = \beta_{2i}^2$$

$$\xi_L^3 = \frac{\sum_{i=1}^L (M_i^3)^{\text{eff}}}{\mathbf{r}_3^T \mathbf{M} \mathbf{r}_3}, \text{ where } (M_i^3)^{\text{eff}} = \mathbf{r}_3^T \mathbf{M} \mathbf{x}_i \mathbf{x}_i^T \mathbf{M} \mathbf{r}_3 = \beta_{3i}^2 \quad (5.64)$$

where $(M_i^1)^{\text{eff}}$, $(M_i^2)^{\text{eff}}$, and $(M_i^3)^{\text{eff}}$ are the effective participating masses associated with the i^{th} Ritz vector in the 1-, 2-, and 3-direction of ground excitation, respectively. The vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ are the influence vectors corresponding to the displacement at each active degree-of-freedom of the structure due to a unit base displacement in the 1-, 2-, and 3-direction of ground motion

The effectiveness of the truncated series of Ritz vectors in representing the dynamic load can be evaluated by consideration of the effective mass of the structure that participates in the dynamic response of the system. The analysis results are exact if the number of Ritz vectors used in the analysis account for all the mass of the structure that participates in the vibrational response of the system. However, accurate results can still be obtained even though the effective mass accounted for by the truncated series of Ritz vectors used in the analysis is less than the total participating mass. The typical range of effective mass that should be included in the analysis in order to guarantee high accuracy in the analysis results is 85%-95% of the total participating mass. For the derivation of the effective mass of a system subjected to multi-directional support excitation, the ground motion is assumed to occur in the 1- and 2-directions simultaneously. The total mass activated by the support movement $\mathbf{r}$ is expressed as follows

$$m_r = \mathbf{r}^T \mathbf{M} \mathbf{r} \quad (5.65)$$

The spatial distribution of the dynamic load is given as follows

$$\mathbf{f}(s) = \mathbf{f}_1(s) + \mathbf{f}_2(s) \quad (5.66)$$

In the case of support motion due to an earthquake, equation (5.66) becomes

$$\mathbf{f}(s) = \mathbf{M}\mathbf{r}_1 + \mathbf{M}\mathbf{r}_2 \quad (5.67)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are respectively the influence vectors corresponding to the displacement at each active degree-of-freedom of the structure due to a unit base displacement in the 1- and 2-directions. The total activated mass m_r thus has the form

$$m_r = \mathbf{r}^T \mathbf{M} \mathbf{r} = \mathbf{r}_1^T \mathbf{M} \mathbf{r}_1 + \mathbf{r}_2^T \mathbf{M} \mathbf{r}_2 + \mathbf{r}_1^T \mathbf{M} \mathbf{r}_2 + \mathbf{r}_2^T \mathbf{M} \mathbf{r}_1 \quad (5.68)$$

For a diagonal mass matrix $\mathbf{M}$, equation (5.68) has the form

$$m_r = \mathbf{r}_1^T \mathbf{M} \mathbf{r}_1 + \mathbf{r}_2^T \mathbf{M} \mathbf{r}_2 \quad (5.69)$$

The last two terms in equation (5.68) are equal to zero, whereas the first two represent the total activated masses in the 1- and 2-directions respectively. Now for a truncated vector basis containing L Ritz vectors, the effective participating mass associated with the i -th Ritz vector can be determined as follows

$$\begin{aligned} M_i^{\text{eff}} &= \mathbf{r}^T \mathbf{M} \mathbf{x}_i \mathbf{x}_i^T \mathbf{M} \mathbf{r} \\ &= (\mathbf{r}_1^T + \mathbf{r}_2^T) \mathbf{M} \mathbf{x}_i \mathbf{x}_i^T \mathbf{M} (\mathbf{r}_1 + \mathbf{r}_2) \\ &= (\beta_{1i} + \beta_{2i})(\beta_{1i} + \beta_{2i}) \\ &= \beta_{1i}^2 + \beta_{2i}^2 + \beta_{1i}\beta_{2i} + \beta_{2i}\beta_{1i} \end{aligned} \quad (5.70)$$

The term β_{1i}^2 represents the effective participating mass in the 1-direction due to ground motion in the 1-direction for the i^{th} Ritz vector. Similarly β_{2i}^2 is the effective participating

mass in the 2-direction due to ground motion in the 2-direction for the i^{th} Ritz vector. The two terms $\beta_{2i}\beta_{1i}$ and $\beta_{1i}\beta_{2i}$ represent, respectively, the effective participating mass in the 1-direction due to ground motion in the 2-direction and the effective participating mass in the 2-direction due to ground motion in the 1-direction for the i^{th} Ritz vector. These last two terms do not appear in equations (5.68) to (5.70) where the mass participating factors for two directions are formulated separately. Now, the combined mass participating factor for the activated mass by earthquake in two directions when the mass matrix is diagonal can be written as follows

$$\eta_L = \frac{\sum_{i=1}^L (\beta_{1i}^2 + \beta_{2i}^2 + 2\beta_{1i}\beta_{2i})}{\mathbf{r}_1^T \mathbf{M} \mathbf{r}_1 + \mathbf{r}_2^T \mathbf{M} \mathbf{r}_2} \quad (5.71)$$

Similarly, in a case of three directional ground motion, the mass participating factor can be formulated including the effective participating mass in one direction due to ground motion in another one. The revised form of the mass participating factor for multi-directional support motion can be defined as follows

$$\eta_L = \frac{\sum_{i=1}^L \left(\sum_{j=1}^M \sum_{k=1}^M \beta_{ji} \beta_{ki} \right)}{\sum_{j=1}^M \sum_{k=1}^M \mathbf{r}_j^T \mathbf{M} \mathbf{r}_k} \quad (5.72)$$

where M is a number of direction in which mass is activated by ground motion. The mass participation factor defined in equation (5.72) can be applied as a cut-off criterion for generating of a truncated basis composed of mode shapes or load-dependent Ritz vectors.

Table 5.1 Lanczos algorithm for generation of Ritz vectors

1. Initializing the vector $\mathbf{q}_0$

Set $\mathbf{q}_0 = \mathbf{0}$

2. Select a starting vector $\mathbf{r}_0$ and calculate

$$\beta_1 = (\mathbf{r}_0^T \mathbf{M} \mathbf{r}_0)^{1/2}$$

$$\mathbf{q}_1 = \frac{\mathbf{r}_0}{\beta_1}$$

3. For $j = 1, \dots, n$

$$\bar{\mathbf{r}}_j = \mathbf{K}^{-1} \mathbf{M} \mathbf{q}_j$$

$$\alpha_j = \mathbf{q}_j^T \mathbf{M} \bar{\mathbf{r}}_j$$

$$\mathbf{r}_j = \bar{\mathbf{r}}_j - \alpha_j \mathbf{q}_j - \beta_j \mathbf{q}_{j-1}$$

$$\beta_{j+1} = (\mathbf{r}_j^T \mathbf{M} \mathbf{r}_j)^{1/2}$$

$$\mathbf{q}_{j+1} = \frac{\mathbf{r}_j}{\beta_{j+1}}$$

Table 5.2 WYD algorithm for generation of Ritz vectors

1. Triangularize stiffness matrix

$$\mathbf{K} = \mathbf{LDL}^T$$

2. Solve for the first vector, $\mathbf{x}_1$

$$\mathbf{K}\mathbf{x}_1^* = \mathbf{f} \quad \text{solve for } \mathbf{x}_1^*$$

$$\mathbf{x}_1 = \frac{\mathbf{x}_1^*}{(\mathbf{x}_1^{*T} \mathbf{M} \mathbf{x}_1^*)^{1/2}} \quad \text{normalize with respect to } \mathbf{M} \text{ matrix}$$

3. Solve for additional vectors ($i = 2, 3, \dots, N$)

$$\mathbf{K}\mathbf{x}_i^* = \mathbf{M}\mathbf{x}_{i-1} \quad \text{solve for } \mathbf{x}_i^*$$

Compute for $j=1, 2, \dots, i-1$

$$c_j = \mathbf{x}_j^T \mathbf{M} \mathbf{x}_i^*$$

Make new vector orthogonal to previous vectors

$$\mathbf{x}_i^{**} = \mathbf{x}_i^* - \sum_{j=1}^{i-1} c_j \mathbf{x}_j$$

Normalize vector with respect to mass matrix

$$\mathbf{x}_i = \frac{\mathbf{x}_i^{**}}{(\mathbf{x}_i^{**T} \mathbf{M} \mathbf{x}_i^{**})^{1/2}}$$

4. Orthogonalized Ritz vectors

Form projective subspace

$$\mathbf{K}^* = \mathbf{X}^T \mathbf{K} \mathbf{X} = \text{full}$$

$$\mathbf{M}^* = \mathbf{X}^T \mathbf{M} \mathbf{X} = \mathbf{I}$$

Solve for $\mathbf{z}_i$ ($i = 1, 2, \dots, n$)

$$(\mathbf{K}^* - \omega_i^2 \mathbf{M}^*) \mathbf{z}_i = 0$$

Compute the final Ritz vectors

$$\mathbf{X}^0 = \mathbf{X} \mathbf{Z}, \text{ where } \mathbf{Z} = (\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n)$$

Table 5.3 L W Y D algorithm for generating of Ritz vectors

1. Given mass, stiffness matrices **M**, **K** and load vector **f**
2. Triangularize stiffness matrix

$$\mathbf{K} = \mathbf{LDL}^T$$

3. Solve for the first vector, $\mathbf{x}_1$

$$\mathbf{K}\mathbf{u}_1 = \mathbf{f} \quad \text{solve for } \mathbf{u}_1$$

$$\mathbf{K}\dot{\mathbf{x}}_1 = \mathbf{M}\mathbf{u}_1 \quad \text{solve for } \dot{\mathbf{x}}_1$$

$$\mathbf{x}_1 = \frac{\dot{\mathbf{x}}_1}{(\dot{\mathbf{x}}_1^T \mathbf{M} \dot{\mathbf{x}}_1)^{1/2}} \quad \mathbf{M}\text{-orthogonal vector } \mathbf{x}_1$$

4. Calculate additional vectors

For $i = 2, 3, \dots, L$

$$\mathbf{a}_{i-1} = \mathbf{x}_{i-1}^T \mathbf{M} \mathbf{u}_{i-1} \quad \text{update load vector}$$

$$\mathbf{u}_i = \mathbf{u}_{i-1} - \mathbf{a}_{i-1} \mathbf{x}_{i-1}$$

$$\mathbf{K}\dot{\mathbf{x}}_i = \mathbf{M}\mathbf{u}_i \quad \text{solve for } \dot{\mathbf{x}}_i$$

For $j = 1, \dots, i-1$

$$\mathbf{C}_j = \mathbf{x}_j^T \mathbf{M} \dot{\mathbf{x}}_i^{(0)}$$

$$\mathbf{x}_i^{*(j+1)} = \mathbf{x}_i^{*(j)} - \mathbf{C}_j \mathbf{x}_j \quad \mathbf{M}\text{-orthogonalize}$$

$$\mathbf{x}_i = \frac{\mathbf{x}_i^*}{(\mathbf{x}_i^{*T} \mathbf{M} \mathbf{x}_i^*)^{1/2}} \quad \mathbf{M}\text{-normalize}$$

5. Obtain reduced mass and stiffness matrices

$$\mathbf{M}^* = \mathbf{X}^T \mathbf{M} \mathbf{X}$$

$$\mathbf{K}^* = \mathbf{X}^T \mathbf{K} \mathbf{X}$$

6. Solve the eigenproblem of the reduced system

$$\mathbf{K}^* \mathbf{Z} = \mathbf{M}^* \mathbf{Z} \mathbf{\Lambda} \quad \text{where } \mathbf{\Lambda} \text{ is approximated eigenvalues}$$

7. Compute appropriate eigenvectors of the original system

$$\mathbf{X}^0 = \mathbf{X} \mathbf{Z}$$

Chapter 6

Non-linear Time History Response Analysis of Bridges by Ritz vectors

6.1 General

Non-linear time history response analysis is necessary to determine the behaviour of bridge structures that experience inelastic deformations or suffer significant damage during earthquakes, which can easily be the case when the bridge is located in a seismically active area especially at a near-fault site, and/or the bridge exhibits significant behaviour of non-linearity. The need to accurately assess the performance of bridge structures requires realistic material models and numerical procedures to compute accurate responses of bridges, which leads to further developments in non-linear time history response analysis. This chapter presents the developments of non-linear time history response analysis procedures for bridges by Ritz vectors. The next chapter presents the developments of a 3D beam-column model for accurate representation of the behaviour of this very important bridge component of bridge column contributing to the overall bridge structure behaviour.

The sources of non-linearity in structural systems are categorized into geometric non-linearity and material non-linearity. The geometric non-linearity arises from non-

linearity in the strain–displacement relations and leads to the need to formulate equilibrium conditions in the deformed configuration of the structure, i.e., the geometric non-linearity occurs as a result of large displacements. The material non-linearity occurs due to non-linearity in the stress–strain or member force–deformation relationships. Material non-linearity is often the cause of inelastic behaviour of civil engineering structures. A detailed presentation of geometric and material non-linear analysis is available from textbook by McGuire et al. (2000).

There are a number of general purpose computer programs developed to compute non-linear time history responses of structures such as ADINA (1995), ABAQUS (1998), DRAIN series (Prakash, 1992), and NEABS (1973), however, only the NEABS computer program has elements and features specifically developed to model the non-linear and inelastic behaviour of bridges (non-linear beam–column and expansion joint elements). Tseng and Penzien (1973) developed the original version of NEABS to compute non-linear time history responses of bridge structures subject to earthquake loadings. The program was later modified by Kawashima and Penzien (1976), Imbsen and Penzien (1978, 1986), Ruchala (1997) and Zhang (1999). The non-linear time history responses of bridges are often evaluated by direct integration of the equations of motion of the analyzed structure. An alternative is the modeling and analysis approach by the superposition method. The use of the superposition method is helpful for two reasons. First, it gives a better physical understanding of the structural behaviour. Second, it leads to a reduced system of lower order. The Ritz vectors can be considered as a special set of mode shapes which can be used effectively to compute time history responses of linear structures (Wilson, 1982). However, the applications of the Ritz vectors to non-linear

time history response analysis of bridge structures have not been thoroughly investigated. A detailed study of non-linear earthquake response analysis using Ritz vectors for building structures has been carried out by Leger et al. (1992). The objective of this chapter is to develop non-linear time history response analysis method for bridge structures using Ritz vectors. Two analysis methods are presented namely, the pseudo-force method and the tangent spectrum method.

6.2 Equations of motion

Equation of motion has been derived in previous studies (Tseng and Penzien, 1973; Imbsen and Penzien, 1986). Based on standard finite element discretization, the equation of motion for an N degree-of-freedom system may be formulated mathematically by considering equilibrium at the nodal degree-of-freedom (Clough and Penzien, 1993)

$$\mathbf{Q}^I(t) + \mathbf{Q}^D(t) + \mathbf{Q}^S(t) = \mathbf{F}(t) \quad (6.1)$$

where $\mathbf{F}(t)$ is the external applied dynamic nodal load vector at time t , and $\mathbf{Q}^I(t)$, $\mathbf{Q}^D(t)$ and $\mathbf{Q}^S(t)$ are the inertia force, damping force and internal restoring force vectors, respectively.

The mass property is generally assumed to be invariant with time, and thus the inertia force is proportional to the total (absolute) acceleration $\ddot{\mathbf{u}}^t(t)$. Thus

$$\mathbf{Q}^I(t) = \mathbf{M} \ddot{\mathbf{u}}^t(t) \quad (6.2)$$

where $\mathbf{M}$ is the constant mass matrix, and the superscript “ t ” indicates the total acceleration.

For practical applications in civil engineering, it is sufficient to assume Rayleigh-type viscous damping, where the damping force is proportional to the relative velocity $\dot{\mathbf{u}}(t)$. Thus

$$\mathbf{Q}^D(t) = \mathbf{C} \dot{\mathbf{u}}(t) \quad (6.3)$$

where $\mathbf{C}$ is the damping matrix, of which $\mathbf{C}$ is a function of the initial $\mathbf{K}$ and $\mathbf{M}$ matrices.

The internal restoring force $\mathbf{Q}^S(t)$ may be highly non-linear and dependent on the loading history. If the structure is linear, $\mathbf{Q}^S(t)$ may be expressed by

$$\mathbf{Q}^S(t) = \mathbf{K} \mathbf{u}(t) \quad (6.4)$$

where $\mathbf{K}$ is the constant stiffness matrix, and $\mathbf{u}(t)$ is the relative displacement. For inelastic behaviour, the non-linear restoring force vectors, $\mathbf{Q}^S(t)$, depends on the current deformation state of the structure and may be written as a function of $\mathbf{u}(t)$ as follows

$$\mathbf{Q}^S(t) = \mathbf{Q}^S(\mathbf{u}(t)) \quad (6.5)$$

If the external excitations do not come from supports, no distinction between the absolute and relative displacements is necessary. For earthquake type support excitations, it is computationally advantageous to define the relative displacement as the dynamic displacement response, expressed mathematically as follows

$$\mathbf{u}^t(t) = \mathbf{u}(t) + \mathbf{u}^{PS}(t) \quad (6.6)$$

The pseudo-static displacement component at instant t , $\mathbf{u}^{PS}(t)$ is the static response caused by the support movement, $\mathbf{u}_g(t)$

$$\mathbf{u}^{PS}(t) = \mathbf{B}\mathbf{u}_g(t) \quad (6.7)$$

where $\mathbf{B}$ is a constant influence coefficient matrix

Rigid support excitations (i.e., all the support points are rigidly connected to the ground and are prescribed by the same ground excitation vector, which consists of three translational components) can be expressed mathematically as follows

$$\mathbf{u}_g(t) = [u_{gx}(t) \ u_{gy}(t) \ u_{gz}(t)]^T$$

The corresponding influence coefficient matrix $\mathbf{B}$ is

$$\mathbf{B} = (\mathbf{b}_x \ \mathbf{b}_y \ \mathbf{b}_z)$$

where vectors $\mathbf{b}_i$ ($i=x, y, \text{ or } z$) have N components, each of which is equal to unity for the i th component and equal to zero for other components, where “ i ” is the direction of support excitation.

For differential support excitations (i.e., each support point moves independently), the ground excitation vector $\mathbf{u}_g(t)$ has N_F times 3 components, where N_F is the number of foundation support degrees-of-freedom and there are 3 independent translational directions to consider for the excitation. The corresponding influence coefficient matrix $\mathbf{B}$ can be determined by solving the static equilibrium equation

$$[\mathbf{K} \ \mathbf{K}_g] \begin{Bmatrix} \mathbf{u}^{PS} \\ \mathbf{u}_g \end{Bmatrix} = 0 \quad (6.8)$$

where $\mathbf{K}_g$ is the stiffness matrix coupling the active degrees-of-freedom with the support degrees-of-freedom. Substituting equation (6.7) in equation (6.8), matrix $\mathbf{B}$ is therefore obtained as follows

$$\mathbf{B} = -\mathbf{K}^{-1}\mathbf{K}_g$$

In both cases, the inertial force vector may be expressed as follows

$$\mathbf{Q}^I(t) = \mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{M}\mathbf{B}\ddot{\mathbf{u}}_g(t) \quad (6.9)$$

where $\mathbf{M}\mathbf{B}\ddot{\mathbf{u}}_g(t)$ is the equivalent dynamic loading due to ground acceleration excitations.

By substituting equations (6.2), (6.3) and (6.4) into equation (6.1), the governing equations of motion of a linear system are expressed as follows

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{R}(t) \quad (6.10a)$$

and for a non-linear system

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{Q}^S(\mathbf{u}(t)) = \mathbf{R}(t) \quad (6.10b)$$

where

$$\mathbf{R}(t) = \mathbf{F}(t) - \mathbf{M}\mathbf{B}\ddot{\mathbf{u}}_g(t) \quad (6.11)$$

For non-linear systems, the direct step-by-step integration method is generally adopted to calculate the dynamic responses. In the direct step-by-step integration method, the internal restoring force at time $t+\Delta t$ is linearized to facilitate numerical computation. By using the Taylor series expansion

$$\mathbf{Q}^S(\mathbf{u}_{n+1}) = \mathbf{Q}^S(\mathbf{u}_n) + \frac{\partial \mathbf{Q}^S}{\partial \mathbf{u}} \bigg|_{t_n} \Delta \mathbf{u}_{n+1} + \frac{\partial^2 \mathbf{Q}^S}{\partial \mathbf{u}^2} \bigg|_{t_n} \Delta \mathbf{u}_{n+1}^2 + \cdots + \frac{\partial^n \mathbf{Q}^S}{\partial \mathbf{u}^n} \bigg|_{t_n} \Delta \mathbf{u}_{n+1}^n$$

By ignoring the higher order terms, $\mathbf{Q}^S(\mathbf{u}(t_{n+1}))$ may be approximated as follows

$$\mathbf{Q}^S(\mathbf{u}_{n+1}) = \mathbf{Q}^S(\mathbf{u}_n) + \mathbf{K}_T(\mathbf{u}_n) \Delta \mathbf{u}_{n+1} \quad (6.12)$$

where $\mathbf{u}_{n+1} = \mathbf{u}(t_{n+1})$, $\Delta \mathbf{u}_{n+1} = \mathbf{u}(t_{n+1}) - \mathbf{u}(t_n)$ and the tangent stiffness matrix at $t = t_n$ is defined as follows

$$\mathbf{K}_T(\mathbf{u}_n) = \frac{\partial \mathbf{Q}^S(\mathbf{u}_n)}{\partial \mathbf{u}_n} \quad (6.13)$$

For computational purpose, a pseudo-non-linear restoring force vector, $\mathbf{Q}_n^{NS}$ may be defined as follows

$$\mathbf{Q}_n^{NS} = \mathbf{Q}^S(\mathbf{u}_n) - \mathbf{K} \mathbf{u}_n \quad (6.14)$$

where $\mathbf{K}$ is the initial linear elastic stiffness.

To avoid accumulating errors during numerical computations, it is desirable to compute $\mathbf{Q}_n^{NS}$ directly from the element deformation for the current time interval, as follows

$$\mathbf{Q}_n^{NS} = \sum_{j=1}^{NEL} \mathbf{q}_{j,n}^{NS} \quad (6.15)$$

where $\mathbf{q}_{j,n}^{NS}$ is the corresponding element pseudo-non-linear restoring force vector at t_n , and the summation is the typical finite element assembly of all non-linear elements (NEL is number of non-linear elements).

Using equations (6.12) and (6.14), the equilibrium equation at t_{n+1} may be expressed as follows

$$\mathbf{M}\ddot{\mathbf{u}}_{n+1} + \mathbf{C}\dot{\mathbf{u}}_{n+1} + \mathbf{K}_T(\mathbf{u}_n)\Delta\mathbf{u}_{n+1} = \mathbf{R}_{n+1} - \mathbf{K}\mathbf{u}_n - \mathbf{Q}_n^{NS} \quad (6.16)$$

where $\mathbf{M}$ is the constant mass matrix, independent of displacement parameters, $\mathbf{C}$ is the non-linear damping matrix, dependent on velocities and displacements, $\mathbf{K}_T$ is the instantaneous tangent stiffness matrix, dependent on displacements, $\mathbf{K}$ is the initial linear stiffness, $\mathbf{R}$ is the applied force vector and $\Delta\mathbf{u}_{n+1}$ is incremental displacement to be computed.

Solution techniques can be classified according to the manner in which the equations of motion are solved. There are two types of solution strategies based on the direct integration method and the superposition method. For a linear system, the dynamic responses may be solved by either the direct integration method or the superposition method. The superposition method is often used when the structural responses can be characterized by a small number of lower modes. The dynamic properties of the structural system are computed once at the beginning of the analysis process and can be used throughout the duration of the dynamic responses. For these reasons, the superposition method is more effective in solving linear elastic dynamic problems. For non-linear systems, the direct step-by-step integration method is often used to calculate

the dynamic responses of the systems. However, in many cases the superposition method can still be used to solve equations of motion for non-linear dynamic problems. In general, the solution procedure is based on an incremental step-by-step solution of the governing equation of motion. At each time step the incremental dynamic equilibrium equations are established and then solved using equilibrium iterations. This can be interpreted as the solution of an equivalent non-linear static problem at every time step. The solution techniques for solving equation (6.16) by the superposition method can be performed using instantaneous structural dynamic properties (tangent spectrum method) or with initial structural dynamic properties (pseudo-force method). It is recognized that the computation of the instantaneous structural dynamic properties at each time step is a costly procedure. If the extent of non-linearity in the behaviour of the structure is localized and limited in nature, structural dynamic properties may not change significantly from one time step to next and thus do not need to be calculated at each time step. The structural dynamic properties can be employed throughout the response calculation while the non-linearities in the behaviour are fully taken into account by the pseudo-forces placed on the right hand side of the equations of motion. The details of these two non-linear analysis procedures are presented in Section 6.5.

6.3 Non-linear dynamic analysis using step-by-step integration method

There are a number of direct step-by-step integration methods to solve the equations of motion. The most popular schemes are the Newmark Beta method, the Houbolt method, and the Wilson Theta method. The details of these methods for solving the increment form of the equilibrium equation in non-linear dynamics can be found in textbooks by

Clough and Penzien (1993); and Bathe (1996). Discussions of the computer implementation of the incremental integration at each time step for non-linear structures are summarized by Bathe and Cimento (1983). A review of the developments in the direct integration for non-linear structural dynamics is presented by Fellipa and Park (1985).

To carry out the step-by-step numerical integration of equation (6.16), it is necessary to introduce an approximate operator in the time domain to replace the differential operator. In the present analysis, the Newmark generalized acceleration method (Newmark, 1959) is adopted which assumes the following approximations for nodal velocities and displacements

$$\begin{aligned}\dot{\mathbf{u}}(t + \Delta t) &= \dot{\mathbf{u}}(t) + [(1 - \delta)\ddot{\mathbf{u}}(t) + \delta\ddot{\mathbf{u}}(t + \Delta t)] \\ \mathbf{u}(t + \Delta t) &= \mathbf{u}(t) + \dot{\mathbf{u}}(t)\Delta t + [(0.5 - \sigma)\ddot{\mathbf{u}}(t) + \sigma\ddot{\mathbf{u}}(t + \Delta t)]\end{aligned}\quad (6.17)$$

where parameters δ and σ can be chosen to give the required integration stability and accuracy. When $\delta=1/2$ and $\sigma=1/6$, the approximations correspond to the linear acceleration method, and when $\delta=1/2$ and $\sigma=1/4$, they correspond to the constant acceleration method. While the linear acceleration method is conditionally stable depending upon the magnitude of Δt i.e., $\frac{\Delta t}{T} \leq \frac{\sqrt{3}}{\pi} \approx 0.55$, where T is the vibration period of the highest mode which contribute significantly to the dynamic response of the structure, the constant acceleration method is unconditionally stable for any magnitude of Δt (Bathe and Wilson, 1975).

The approximations given by equation (6.17) can be expressed in the incremental form as follows

$$\begin{aligned}\ddot{\mathbf{u}}_{n+1} &= \ddot{\mathbf{u}}_n + C_1 \Delta \mathbf{u}_{n+1} - C_2 \dot{\mathbf{u}}_n - C_3 \ddot{\mathbf{u}}_n \\ \dot{\mathbf{u}}_{n+1} &= \dot{\mathbf{u}}_n + C_4 \Delta \mathbf{u}_{n+1} - C_5 \dot{\mathbf{u}}_n - C_6 \ddot{\mathbf{u}}_n\end{aligned}\quad (6.18)$$

where

$$\begin{aligned}C_1 &= \frac{1}{\sigma \Delta t^2}, & C_2 &= \frac{1}{\sigma \Delta t}, & C_3 &= \frac{1}{2\sigma}, \\ C_4 &= \frac{\delta}{\sigma \Delta t}, & C_5 &= \frac{\delta}{\sigma}, & C_6 &= \left(\frac{\delta}{2\sigma} - 1 \right) \Delta t\end{aligned}\quad (6.19)$$

Substituting equation (6.18) into equation (6.17), one obtains

$$\begin{aligned}(C_1 \mathbf{M} + C_4 \mathbf{C} + \mathbf{K}_T) \Delta \mathbf{u}_{n+1} &= \mathbf{R}_{n+1} - \mathbf{M} \ddot{\mathbf{u}}_n - \mathbf{C} \dot{\mathbf{u}}_n - \mathbf{K} \mathbf{u}_n - \mathbf{Q}_n^{\text{NS}} + \\ &\quad \mathbf{M}(C_2 \dot{\mathbf{u}}_n + C_3 \ddot{\mathbf{u}}_n) + \mathbf{C}(C_5 \dot{\mathbf{u}}_n + C_6 \ddot{\mathbf{u}}_n)\end{aligned}\quad (6.20)$$

Assuming a proportional damping as follows

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K}\quad (6.21)$$

and introducing the following constants

$$\begin{aligned}C_7 &= C_1 + \alpha C_4, & C_8 &= \beta C_4, & C_9 &= C_2 + \alpha C_5 - \alpha, \\ C_{10} &= 1 - C_3 - \alpha C_6, & C_{11} &= \beta(C_5 - 1), & C_{12} &= \beta C_6\end{aligned}$$

Equation (6.20) can be put into the form of equation (6.22) as follows

$$\bar{\mathbf{K}}_T \Delta \mathbf{u}_{n+1} = \Delta \bar{\mathbf{R}}_{n+1} \quad (6.22)$$

where

$$\bar{\mathbf{K}}_T = \mathbf{C}_7 \mathbf{M} + \mathbf{C}_8 \mathbf{K} + \mathbf{K}_T \quad (6.23)$$

$$\Delta \bar{\mathbf{R}}_{n+1} = \mathbf{R}_{n+1} - \mathbf{M}(\mathbf{C}_{10} \ddot{\mathbf{u}}_n - \mathbf{C}_9 \dot{\mathbf{u}}_n) - \mathbf{K}(\mathbf{u}_n - \mathbf{C}_{11} \dot{\mathbf{u}}_n - \mathbf{C}_{12} \ddot{\mathbf{u}}_n) - \mathbf{Q}_n^{\text{NS}} \quad (6.24)$$

and where matrix $\bar{\mathbf{K}}_T$ is the effective dynamic stiffness matrix and $\Delta \bar{\mathbf{R}}_{n+1}$ is the effective load vector. Equation (6.22) can be solved for $\Delta \mathbf{u}_{n+1}$ at each time instant $t_{n+1} = t_n + \Delta t$, and equations (6.18) can be used to compute $\Delta \dot{\mathbf{u}}_{n+1}$, and $\Delta \ddot{\mathbf{u}}_{n+1}$. The displacements, velocities, and accelerations at time step $t + \Delta t$ can then be determined from equation (6.25) as follows

$$\begin{aligned} \ddot{\mathbf{u}}_{n+1} &= \ddot{\mathbf{u}}_n + \mathbf{C}_1 \Delta \mathbf{u}_{n+1} - \mathbf{C}_2 \dot{\mathbf{u}}_n - \mathbf{C}_3 \ddot{\mathbf{u}}_n \\ \dot{\mathbf{u}}_{n+1} &= \dot{\mathbf{u}}_n + \mathbf{C}_4 \Delta \mathbf{u}_{n+1} - \mathbf{C}_5 \dot{\mathbf{u}}_n - \mathbf{C}_6 \ddot{\mathbf{u}}_n \\ \mathbf{u}_{n+1} &= \mathbf{u}_n + \Delta \mathbf{u}_{n+1} \end{aligned} \quad (6.25)$$

The displacement solution $\mathbf{u}_{n+1}$ can be used to calculate the internal force vector $\mathbf{S}_{n+1}$ and the new tangent stiffness matrix $\mathbf{k}_T^{\text{NEL}}$ for each non-linear element in the system. The new total tangent stiffness matrix $\mathbf{K}_T$ of the structure is then obtained by the standard assemblage procedure. It is important to note that when determining the element internal forces, the static forces existing in the element must always be included since the

calculation of element tangent stiffness depends upon the magnitudes of the element forces.

6.4 Equilibrium iteration

The equilibrium of the equations of motion is commonly obtained by an iterative procedure. The iteration can be carried out by the use of the modified Newton–Raphson method (Bathe, 1996 and Chopra, 2003). Equation (6.16) is derived based on approximation of the instantaneous stiffness relations between the incremental forces and the incremental displacements, which may lead to instability in the overall solution. A measurement of how well the dynamic equilibrium at time t_{n+1} is satisfied by the approximation solution of equation (6.16) may be expressed by the residual or unbalanced forces $\delta \mathbf{R}_{n+1}$. The accuracy of the solution using equation (6.16) may be assessed by comparing the Euclidean norm of the residual forces Δ_p with a specified tolerance Δ_{ps} , using the following relationship

$$\Delta_p = \|\delta \mathbf{R}_{n+1}\| \leq \Delta_{ps} \quad (6.26)$$

where

$$\delta \mathbf{R}_{n+1} = \mathbf{R}_{n+1} - \mathbf{M}\ddot{\mathbf{u}}_{n+1} - \mathbf{C}\dot{\mathbf{u}}_{n+1} - \mathbf{K}\mathbf{u}_{n+1} - \mathbf{Q}_{n+1}^{NS} \quad (6.27)$$

If the accuracy of the solution is not satisfactory, it may be improved by using a smaller time interval or by applying an equilibrium correction through an iteration process. The equation of motion for the i th equilibrium iteration at time t_{n+1} is expressed as follows

$$\delta \mathbf{R}_{n+1}^{(i)} = \mathbf{M}\delta \ddot{\mathbf{u}}_{n+1}^{(i)} + \mathbf{C}\delta \dot{\mathbf{u}}_{n+1}^{(i)} + \mathbf{K}_T^{(i)}\delta \mathbf{u}_{n+1}^{(i)} \quad (6.28)$$

where $\mathbf{M}$ and $\mathbf{C}$ are the constant mass and damping matrices, respectively; $\mathbf{K}_T^{(i)}$ is the tangent stiffness matrix at time t_{n+1} for the i th iteration; $\delta\ddot{\mathbf{u}}_{n+1}^{(i)}$, $\delta\dot{\mathbf{u}}_{n+1}^{(i)}$, and $\delta\mathbf{u}_{n+1}^{(i)}$ for the i th iteration are the corrective nodal accelerations, nodal velocity, and nodal displacements, respectively, defined as follows

$$\begin{aligned}\delta\ddot{\mathbf{u}}_{n+1}^{(i)} &= \ddot{\mathbf{u}}_{n+1}^{(i+1)} - \ddot{\mathbf{u}}_{n+1}^{(i)} \\ \delta\dot{\mathbf{u}}_{n+1}^{(i)} &= \dot{\mathbf{u}}_{n+1}^{(i+1)} - \dot{\mathbf{u}}_{n+1}^{(i)} \\ \delta\mathbf{u}_{n+1}^{(i)} &= \mathbf{u}_{n+1}^{(i+1)} - \mathbf{u}_{n+1}^{(i)}\end{aligned}\tag{6.29}$$

and where $\delta\mathbf{R}_{n+1}^{(i)}$ representing the nodal residual forces for the i th iteration is given by equation (6.28).

From the integration scheme of equation (6.28), the corrective accelerations and velocities of equation (6.29) can be written as follows

$$\begin{aligned}\delta\ddot{\mathbf{u}}_{n+1}^{(i)} &= \mathbf{C}_1\delta\mathbf{u}_{n+1}^{(i)} \\ \delta\dot{\mathbf{u}}_{n+1}^{(i)} &= \mathbf{C}_4\delta\mathbf{u}_{n+1}^{(i)}\end{aligned}\tag{6.30}$$

Substituting equations (6.29) and (6.30) into equation (6.28) leads to

$$\overline{\mathbf{K}}_T^{(i)}\delta\mathbf{u}_{n+1}^{(i)} = \delta\overline{\mathbf{R}}_{n+1}^{(i)}\tag{6.31}$$

where

$$\overline{\mathbf{K}}_T^{(i)} = \mathbf{K}_T^{(i)} + \mathbf{C}_7\mathbf{M} + \mathbf{C}_8\mathbf{K} \text{ and}$$

$$\delta \bar{\mathbf{R}}_{n+1}^{(i)} = \mathbf{R}_{n+1} - \mathbf{M}(\ddot{\mathbf{u}}_{n+1}^{(i)} + \alpha \dot{\mathbf{u}}_{n+1}^{(i)}) - \mathbf{K}(\mathbf{u}_{n+1}^{(i)} + \beta \dot{\mathbf{u}}_{n+1}^{(i)}) - \mathbf{Q}_{n+1}^{NS} \quad (6.32)$$

Solving equation (6.31) to obtain $\delta \mathbf{u}_{n+1}^{(i)}$; then the corrective displacements, velocities and accelerations can be calculated by using equation (6.29). The iteration is continued until the dynamic equilibrium of the motion is satisfied within the specified accuracy. The convergence of equilibrium iteration may be expressed by comparing the ratio of the Euclidean norm of the corrective displacement to that of total displacement Δ_u^i with a specified tolerance Δ_{us}^i in the form

$$\Delta_u^i = \frac{\|\delta \mathbf{u}_{n+1}^{(i)}\|}{\|\mathbf{u}_{n+1}^{(i)}\| + \|\mathbf{u}_{n+1}^{(i+1)}\|} \leq \Delta_{us}^i \quad (6.33)$$

and by comparing the Euclidean norm of the residual forces Δ_p^i with a specified tolerance Δ_{ps}^i in the form

$$\Delta_p^i = \|\delta \mathbf{R}_{n+1}^{(i)}\| \leq \Delta_{ps}^i \quad (6.34)$$

The iterative procedure using the Newton–Raphson method is described in Table 6.1 for the non-linear time history analysis problem. The step-by-step integration algorithm used herein for non-linear systems, which includes the equilibrium iteration, is summarized in Table 6.2.

6.5 Non-linear dynamic analysis using Ritz methods

The effectiveness of the superposition methods in non-linear dynamic problems depends on: (i) the number of required base vectors to characterize structural response; (ii) the efficiency of the algorithm used to calculate the initial vectors; and (iii) the frequency of

updating and recalculating the base vectors. The superposition methods in non-linear structural dynamics can be based either on the pseudo-force method or the tangent spectrum method. In the pseudo-force method, a single set of base vectors obtained from initial structural properties is used throughout the analysis. The pseudo-force method avoids the solution of the tangent eigenproblem at each time step during non-linear responses, and has been found to be competitive in terms of computational effort with the direct integration methods. In the tangent spectrum method, the discretizing base vectors are updated if necessary at each time step corresponding to the instantaneous system properties.

6.5.1 Pseudo-force method

The basis of the pseudo-force method is that the initial structural properties are used throughout the response solution and all the non-linear behaviour are grouped in the right side of the equations of motion with the external dynamic loads to form the pseudo-force vector. Since the order of the system is significantly reduced, the pseudo-force method provides a more effective computational solution compared to the direct integration method. However, the speed efficiency gained is offset by the disadvantage of significant loss of accuracy as non-linearities become severe. Therefore, this method should be considered only for structures with small amounts of non-linearity, such as those with limited non-linearity localized at specific regions of the structure.

Rearranging equation (6.16) as follows

$$\mathbf{M}\ddot{\mathbf{u}}_{n+1} + \mathbf{C}\dot{\mathbf{u}}_{n+1} + \mathbf{K}\Delta\mathbf{u}_{n+1} = \mathbf{R}_{n+1} - \mathbf{K}\mathbf{u}_n - (\mathbf{K}_T - \mathbf{K})\Delta\mathbf{u}_{n+1} - \mathbf{Q}_n^{\text{NS}} \quad (6.35)$$

Transforming the geometric displacement degree-of-freedom into the generalized coordinate by equation (5.29) as follows

$$\mathbf{u} = \Phi \mathbf{x}$$

Equations of motion in the generalized coordinates are expressed as follows

$$\mathbf{M}\Phi\ddot{\mathbf{x}}_{n+1} + \mathbf{C}\Phi\dot{\mathbf{x}}_{n+1} + \mathbf{K}\Phi\Delta\mathbf{x}_{n+1} = \mathbf{R}_{n+1} - \mathbf{K}\Phi\mathbf{x}_n - (\mathbf{K}_T - \mathbf{K})\Phi\Delta\mathbf{x}_{n+1} - \mathbf{Q}_n^{\text{NS}} \quad (6.36)$$

Pre-multiplying equation (6.36) by Φ^T leads to

$$\mathbf{M}^* \ddot{\mathbf{x}}_{n+1} + \mathbf{C}^* \dot{\mathbf{x}}_{n+1} + \mathbf{K}^* \Delta\mathbf{x}_{n+1} = \Phi^T (\mathbf{R}_{n+1} - \mathbf{K}\Phi\mathbf{x}_n - (\mathbf{K}_T - \mathbf{K})\Phi\Delta\mathbf{x}_{n+1} - \mathbf{Q}_n^{\text{NS}}) \quad (6.37)$$

where

$$\mathbf{M}^* = \Phi^T \mathbf{M} \Phi$$

$$\mathbf{C}^* = \Phi^T \mathbf{C} \Phi$$

$$\mathbf{K}^* = \Phi^T \mathbf{K} \Phi$$

Equation (6.37) is in a form similar to equation (6.16) but it has a lower order. The solution of equation (6.37) follows the same procedures as for equation (6.16) and thus is not repeated here. Equations (6.22) to (6.24) can be written in geometric coordinate as follows

$$\bar{\mathbf{K}}_T \Delta \mathbf{u}_{n+1} = \Delta \bar{\mathbf{R}}_{n+1}$$

where

$$\bar{\mathbf{K}}_T = \mathbf{C}_7 \mathbf{M} + \mathbf{C}_8 \mathbf{K} + \mathbf{K}$$

$$\Delta \bar{\mathbf{R}}_{n+1} = \mathbf{R}_{n+1} - \mathbf{M}(\mathbf{C}_{10} \ddot{\mathbf{u}}_n - \mathbf{C}_9 \dot{\mathbf{u}}_n) - \mathbf{K}(\mathbf{u}_n - \mathbf{C}_{11} \dot{\mathbf{u}}_n - \mathbf{C}_{12} \ddot{\mathbf{u}}_n) - \mathbf{Q}_n^{\text{NS}} - (\mathbf{K}_T - \mathbf{K}) \Delta \mathbf{u}_{n+1}$$

It is noted that the right hand side of equation (6.37) consists of the unknown $\Delta \mathbf{x}_{n+1}$ therefore, an iterative procedure is required. The unbalanced force is expressed as follows

$$\mathbf{f}_u = \Phi^T (\mathbf{K}_T - \mathbf{K}) \Delta \mathbf{x}_{n+1} \quad (6.38)$$

This unbalanced force is subtracted from the residual force presented in equation (6.32).

6.5.2 Tangent spectrum method

The pseudo-force method is more economical compared to the direct integration method because the amount of computations required for the generation of the base vectors plus the solution of the reduced system are typically less than solving the complete set of equations of motion directly. However, there is possible loss of accuracy with this solution approach if non-linearities in the structural responses become severe. When the base vectors are no longer representatives of the dynamic behaviour of the structure, the equilibrium iteration procedure may fail to converge. In this case, the tangent spectrum method using updated base vectors can be utilized. The effectiveness of the tangent spectrum method for non-linear dynamic analysis depends on: (i) the number of base vectors required to accurately simulate the response. It is the function of frequency content and spatial distribution of the loading versus the vibration characteristics of the systems; (ii) the efficiency of the algorithm used to calculate the base vectors; and (iii) the frequency of recalculating the base vectors.

It should be noted that the instantaneous frequencies of the system are continuously changed during the non-linear response. If the system stiffens during the response, the vibration frequencies of the structure generally increase. The number of base vectors needed to accurately describe linear behaviour of the system may be conservative to accurately describe non-linear behaviour of the system. However, in analysis of softening structures, the number of base vectors needed for non-linear response analysis may have to be larger than that for linear analysis.

Pre-multiplying both sides of equation (6.22) by Φ_n^T leads to

$$\Phi_n^T \bar{K}_T \Delta u_{n+1} = \Phi_n^T \Delta \bar{R}_{n+1} \quad (6.39)$$

where Φ_n^T is the transpose of the instantaneous base vectors at nth time step.

Substituting equation (5.29) into equation (6.39), the equation of motion in the generalized coordinates is obtained as follows

$$\Phi_n^T \bar{K}_T \Phi_n \Delta x_{n+1} = \Phi_n^T \Delta \bar{R}_{n+1} \quad (6.40)$$

where the effective stiffness and effective dynamic force can be written as follows

$$\bar{K}_T = C_7 M + C_8 K + K_T$$

$$\Delta \bar{R}_{n+1} = R_{n+1} - M(C_{10} \ddot{u}_n - C_9 \dot{u}_n) - K(u_n - C_{11} \dot{u}_n - C_{12} \ddot{u}_n) - Q_n^{NS}$$

Substituting above equations into equation (6.40) leads to

$$\begin{aligned} (C_7 \mathbf{M}_n^* + \mathbf{K}_n^* + \mathbf{K}_{n(T)}^*) \Delta \mathbf{x}_{n+1} = & \Phi_n^T \mathbf{R}_{n+1} - \Phi_n^T \mathbf{M} (C_{10} \ddot{\mathbf{u}}_n - C_9 \dot{\mathbf{u}}_n) - \\ & \Phi_n^T \mathbf{K} (\mathbf{u}_n - C_{11} \dot{\mathbf{u}}_n - C_{12} \ddot{\mathbf{u}}_n) - \Phi_n^T \mathbf{Q}_n^{NS} \end{aligned} \quad (6.41)$$

It is noted that on the left hand side of equation (6.41), only base vectors of the current state of the structure are involved however, on the right hand side of equation (6.41) both base vectors of the current state and those of the previous state of the structure are involved. The iteration equilibrium equation is the same as that described for equation (6.31) except of a lower order. However, it is important to point out that after equation (6.41) is used to compute the generalized displacement increment $\Delta \mathbf{x}_{n+1}$ the displacement increment $\Delta \mathbf{u}_{n+1}$ in the geometric coordinate is computed as follows

$$\Delta \mathbf{u}_{n+1} = \Phi_n \Delta \mathbf{x}_{n+1} \quad (6.42)$$

Equations (6.25) are then used to compute displacement, velocity and acceleration in the geometric coordinates. It is noted that since the Ritz vectors are frequently computed during the response, the number of converged Ritz vectors keeps changing, i.e., the number of required Ritz vectors may not be the number of converged Ritz vectors.

6.5.3 Criterion to update base vectors

In this study, a Euclidean norm of second order is used as a discriminant to update base vectors. Let $\mathbf{K}^N$ and $\mathbf{K}_T^N$ are the total elastic stiffness and tangent stiffness of non-linear elements respectively, that are obtained from assemblage procedure

$$\begin{aligned} \mathbf{K}^N &= \sum_{i=1}^{NEL} \mathbf{k}_i \\ \mathbf{K}_T^N &= \sum_{i=1}^{NEL} \mathbf{k}_{Ti} \end{aligned}$$

where $\mathbf{k}$ and $\mathbf{k}_T$ are the element elastic stiffness and tangent stiffness, respectively, and NEL is the number of non-linear elements. The change in stiffness is expressed as follows

$$\Delta \mathbf{K}^N = \mathbf{K}_T^N - \mathbf{K}^N = \sum_{i=1}^{NEL} (\mathbf{k}_{Ti} - \mathbf{k}_i) = \sum_{i=1}^{NEL} \Delta \mathbf{k}_i \quad (6.43)$$

Second order norm of $\Delta \mathbf{K}^N$ is

$$\|\Delta \mathbf{K}^N\| = \sqrt{\sum_{i=1}^{NEL} \Delta \mathbf{k}_i^2}$$

Second order norm of $\mathbf{K}^N$ is

$$\|\mathbf{K}^N\| = \sqrt{\sum_{i=1}^{NEL} \mathbf{k}_i^2}$$

The change of non-linear stiffness is evaluated as the ratio

$$\Delta = \frac{\|\Delta \mathbf{K}^N\|}{\|\mathbf{K}^N\|} = \frac{\sqrt{\sum_{i=1}^{NEL} \Delta \mathbf{k}_i^2}}{\sqrt{\sum_{i=1}^{NEL} \mathbf{k}_i^2}} \quad (6.44)$$

If Δ is less than the specified tolerance there is no need to compute new base vectors and vice versa.

6.6 Non-linear beam-column element

In assessing the non-linear time history responses of bridge structures, it is important to capture the essential behaviour of the non-linear structural members as accurately as possible. In seismic response of bridges, most of the non-linear behaviour and energy dissipation typically occur in bridge piers or columns, and usually near the ends of the columns where the moments are largest. The 3D beam-column element used in the

NEABS computer program was originally developed by Tseng and Penzien (1973) to model the non-linear behaviour of reinforced concrete bridge columns. The non-linear behaviour of reinforced concrete members is captured when the state of forces reaches the yield surface as defined by the biaxial interaction surface of two major bending moments and an axial force. The material behaviour of this beam-column element is assumed to be elasto-plastic. The modeling parameters of the elasto-plastic element in the NEABS computer program are as follows: (1) the definition of an element coordinate system and the corresponding deformation degrees-of-freedom to be characterized by elasto-plastic behaviour; (2) specification of an initial yield condition that defines the states of stress for which plastic flow is initiated; (3) definition of plastic flow for the post yield state of stress and strain; and (4) selection of hardening/softening rule that defines the modification of the yield condition after yielding.

It is accepted that there is an important role for non-linear time history response analysis of bridges subject to earthquake loading. For linear response analysis, only the initial stiffness of the structure needs to be modeled as it is unchanged throughout the entire duration of the dynamic responses. For non-linear analysis, however, properties such as strength deterioration and stiffness degradation also need to be considered and modeled. This section reviews the modeling of reinforced concrete structures, where the members resist axial force, bending, shear and possibly torsion. The emphasis is on how to capture the important aspects in the behaviour of beam and column members. The beam-column models can be based on: (1) lumped and distributed assumptions; and (2) force-displacement formulations. Cross section models are based on an analogy with plasticity theory. Recently, modeling approaches using fiber elements have been used by

many researchers (Filippou, 1992), which can also provide accurate results with reasonable computational effort.

Beams, columns and slender walls can be modeled using beam-column elements for which the effects of axial force, bending moment, shear force and possibly torsion are considered. In practice, it is common that most structural members are not subjected to significant torsional loading, and thus only the axial force and bending moment effects need to be considered in the model. Figure 6.1 shows the effects of axial force and bending moment on plastic zones. For uniform bending, the plastic zones extend along the full length and over part of the depth of the column members (Figure 6.1a). For non-uniform bending the plastic zones extend partially along the length, but are still symmetrical (Figure 6.1b). For combined axial force and non-uniform bending the plastic zones extend partially along the length and are unsymmetrical (Figure 6.1c). Moreover, the effects of cracks, softening/hardening and deterioration of strength can significantly influence the behaviour of reinforced concrete members. Therefore, the adoption of an accurate beam-column model that takes into account these effects in the analysis is required for accurate representation of the behaviour of reinforced concrete members under extreme loading conditions, such as major earthquakes.

In the development of a beam-column model, the plastic behaviour can be assumed to concentrate at the two ends of the member or distribute along the element length. In the lumped-plasticity model, the inelastic deformation is lumped in zero-length plastic hinges at the element ends. The concept of the zero-length plastic hinge implies infinite strains. In the distributed plasticity model, the inelastic behaviour is monitored at several discrete sections along the element length. In general, the distributed

plasticity models are more accurate than the lumped plasticity models. However, there are several advantages of using the equivalent plastic hinge model over the spread plasticity model. Firstly, it is computationally faster and requires less computer storage space. Secondly, additional effects of strain penetration into connecting members and tension shift from diagonal shear cracking can be simulated by simply increasing the equivalent plastic hinge length.

The inelastic behaviour of a hinge is dependent on the combined actions of bending moments and axial force in the member. The hinge and section stiffness of a beam-column member are defined by a 2 by 2 matrix representing the P-M interaction, and by a 3 by 3 matrix representing the P-M_y-M_z interaction. The well-known normality rule is assumed for plastic flow. The von Mises yield surface is used to develop the tangent stiffness after yield. Essentially, the von Mises yield criterion and the normal flow assumption define a rational theory for the interaction between stresses. However, there are a number of problems with this plastic model. The initial yield surface and the subsequent surfaces have identical shapes, and thus the stress-strain curve remains unchanged, ignoring the degradation effect from accumulated damage or plastic deformations sustained by the member. In spite of this limitation, this plastic model can capture some interaction effects and it can serve as a useful basic model. An advanced lumped-plasticity model capable of capturing the more complex behaviour is later developed in this study.

In the lumped plasticity approach, the length of the equivalent plastic hinge plays an important role in the overall member response. If the plastic hinge length is large relative to the length of the member, it implies the inelastic damage to the member is

spread over a large distance. For a given displacement, an increased plastic hinge length decreases the critical section curvature, resulting in reduced reinforcement and concrete strains and stresses. Since the member displacement capacity is typically related to the tensile strain capacity of the steel or the compressive strain capacity of the confined concrete, a larger plastic hinge length translates to an increased displacement capacity of the member. For a given displacement with a shorter plastic hinge length, the critical section curvature increases, with corresponding increases in fiber strains and stresses. For a cantilever column with the plastic hinge forming at the interface between the member and connecting footing or superstructure, the equivalent plastic hinge length used for non-linear analysis is suggested as follows (Priestley et al., 1996)

$$L_p = \begin{cases} 0.08L + 0.15f_{yc}d_{bl} \geq 0.3f_{yc}d_{bl} & (\text{for } f_{yc} \text{ in ksi}) \\ 0.08L + 0.022f_{yc}d_{bl} \geq 0.044f_{yc}d_{bl} & (\text{for } f_{yc} \text{ in MPa}) \\ 0.08L + 10d_{bl} \geq 20d_{bl} & (\text{for Grade 60 steel}) \end{cases} \quad (6.45)$$

where L_p is the plastic hinge length, L is the distance from the critical section of the plastic hinge to the point of contraflexure and d_{bl} is the diameter of the longitudinal reinforcement.

The first term in the plastic hinge length expression accounts for plastic curvatures along the member length and the tension shift effect due to diagonal shear cracking. The second term of the expression accounts for strain or yield penetration of the longitudinal reinforcement into the footing or connecting member. The minimum plastic hinge length indicated that strain penetration can occur on both sides of the critical section.

6.6.1 Yield function

In a one-dimensional stress space, a yield or proportional limit stress indicates when non-linear behaviour begins. However, when more than one stress component are present, different combinations of these forces may cause non-linear behaviour to occur. In a reinforced concrete column, different combinations of axial force and moments can cause non-linear behaviour at a section. A convenient mathematical representation to define these forces is the well-known interaction surface. Use of the interaction diagram to define this behaviour requires the following assumptions:

1. If the load combination falls inside the interaction surface, the behaviour is elastic;
2. If the load combination falls on the yield surface, the behaviour is elasto-plastic;
3. The load combination never falls outside the yield surface. Six generalized forces and displacements exist at any section of the element as shown in Figure 6.2.

Normal stresses are caused by the axial load P and the two bending moments M_y and M_z . The transverse shear stresses are caused by the torsion and the two shear forces Q_y and Q_z . The following assumptions are made in deriving the expression for the plastic hinge of a reinforced concrete column:

1. The generalized force-deformation relations of the element correspond to that of an elasto-plastic material having yield strength corresponding to the ultimate capacity of the member.

2. The member has ultimate shear strengths T_u , Q_{yu} , and Q_{zu} higher than the induced shear forces.
3. The ultimate axial force strength P_u and the ultimate bending strengths M_{yu} and M_{zu} are determined from the axial stress distribution present on the cross section under ultimate conditions and are independent of the shear stresses.
4. The interaction of P_u , M_{yu} , and M_{zu} can be represented by a three-dimensional generalized yield surface in the force space spanned by P , M_y , and M_z .
5. The element retains linear elastic behaviour between nodal points; however, elasto-plastic behaviour, as defined in the above assumptions, is permitted to occur over the cross section at each end for a distance approaching zero; i.e., the elasto-plastic behaviour is fully concentrated at the ends of the member.

The five assumptions listed above relate to material properties within the concentrated elasto-plastic hinges at the ends of the elements. Between these hinges, the material remains elastic. These assumptions are adopted to approximate the actual behaviour in which inelastic deformation may occur at any section along the column member. If a sufficient number of elements are used, a good approximation can be achieved.

The load-carrying capacity of a concrete column can be represented by a yield surface, as shown in Figure 6.3. The generalized yield surface included in the formulation by Tseng and Penzien (1973) follows a form similar to that proposed by Bresler (1960) for biaxial bending, and uses a cubic polynomial approximation for the axial force and bending interaction curves. Bresler's approach, known as the load contour method,

involves cutting a failure surface at a constant value of P_u to give a so-called “load contour” representing the interaction of M_{yu} and M_{zu} as shown in Figure 6.4. The equation of a load contour at a constant P may be expressed in non-dimensional form by

$$\left(\frac{M_{yu}}{M_{yp}}\right)^a + \left(\frac{M_{zu}}{M_{zp}}\right)^b = 1 \quad (6.46)$$

where M_{yu} and M_{zu} are the ultimate moments on the interaction surface for a fixed value of P_u , M_{yp} is the ultimate bending moment on the interaction surface for a fixed value of P_u for M_{zu} set equal to zero, M_{zp} is ultimate bending moment on the interaction surface for a fixed value of P_u for M_{yu} set equal to zero, and a and b are constants that depend on the dimension of the cross section, the steel reinforcement, the concrete strength, and the steel yield stresses. The load contours for constant P_u on yield surface are shown in Figure 6.4. Moments M_{yp} and M_{zp} can be approximated by the following equations

$$\begin{aligned} \left|\frac{M_{yp}}{M_{y0}}\right| &= 1.0 + a_1 \left(\frac{P_u}{P_0}\right) + a_2 \left(\frac{P_u}{P_0}\right)^2 + a_3 \left(\frac{P_u}{P_0}\right)^3 \\ \left|\frac{M_{zp}}{M_{z0}}\right| &= 1.0 + b_1 \left(\frac{P_u}{P_0}\right) + b_2 \left(\frac{P_u}{P_0}\right)^2 + b_3 \left(\frac{P_u}{P_0}\right)^3 \end{aligned} \quad (6.47)$$

$$-P_0 < P_u < P_t$$

where M_{y0} is the intercept on the M_y axis corresponding to zero axial load P and zero moment M_z ; M_{z0} is the intercept on the M_z axis corresponding to zero axial load P and zero moment M_y ; P_0 is the ultimate axial compression for the columns under conditions

of no bending; P_t is the ultimate axial tension for the columns under conditions of no bending; and $a_1, a_2, a_3, b_1, b_2,$ and b_3 are constants.

From equations (6.46) and (6.47) an approximate yield surface can be represented in the normalized form by

$$\left| \frac{m_{yu}}{1 + a_1 p_u + a_2 p_u^2 + a_3 p_u^3} \right|^a + \left| \frac{m_{zu}}{1 + b_1 p_u + b_2 p_u^2 + b_3 p_u^3} \right|^b = 1 \quad (6.48)$$

or symbolically as follows

$$f(p_u, m_{yu}, m_{zu}) = 1 \quad (6.49)$$

where

$$p_u = \frac{P_u}{P_0}; \quad m_{yu} = \frac{M_{yu}}{M_{y0}}; \text{ and } \quad m_{zu} = \frac{M_{zu}}{M_{z0}} \quad (6.50)$$

are the normalized resultant force components.

The six constants contained in the above equation are determined by using the methods previously described to fit smooth curves through the ultimate strength values obtained for a particular column. For typical bridge columns that have nearly oval-shaped cross-sections and are peripherally reinforced, an ellipse can be used to approximate the biaxial bending interaction curve. For this case, a value of 2 is used for both a , and b , leads to

$$\left| \frac{m_{yu}}{1 + a_1 p_u + a_2 p_u^2 + a_3 p_u^3} \right|^2 + \left| \frac{m_{zu}}{1 + b_1 p_u + b_2 p_u^2 + b_3 p_u^3} \right|^2 = 1 \quad (6.51)$$

In order to obtain six coefficients $a_1, a_2, a_3, b_1, b_2,$ and b_3 , a two dimensional moment-curvature analysis is required. A smooth yield surface of the cross section characterized by six coefficients is obtained by regression analysis. There are a number of computer programs to compute biaxial moment-curvature of a cross section, namely UCFIBER (Cross Section Analysis of Reinforced Concrete, Chadwell, 1998), and BIAX-2 (Analysis of Reinforced Concrete and Reinforced Masonry Sections, Wallace, 1992).

The force m_y, m_z and p in a normalized force space interact with one another to cause yielding in a given cross section. For a 3D reinforced concrete member, the yield function $\Phi(p, m_y, m_z)$ is a three-dimensional surface.

6.6.2 Flow rule

The yield surface defines the forces at which plastic flow is initiated. An additional rule governing the evolution of plastic flow must be postulated. It is customary to supplement the yield condition by a flow rule that defines the direction of plastic flow. The flow rule is deduced from experimental observations. From theoretical point of view, it is convenient to work with a rule that preserves the validity of the postulate of maximum plastic dissipation (von Mises, 1928).

Postulate of maximum plastic dissipation is described as follows. Letting r_p be a given plastic displacement and s to denote a force state, among all plastically admissible force state s , the power s/r_p is maximized by the actual force s if two conditions are satisfied: (i) the elastic domain is convex, and (ii) the direction of the plastic displacement rate is normal to the yield surface. These two requirements are referred as

convexity and normality. Illustrations of convexity and normality are presented in Figure 6.5.

As shown in Figure 6.6, the gradient ξ to the yield surface defines the direction of the plastic flow. The yield surface Φ translates, after some hardening has taken place, to a new position defined by the vector α . The yield function is expressed as follows

$$\Phi(p, m_y, m_z) = 0 \quad (6.52)$$

$$\text{where } p = \frac{P - \alpha_p}{P_0}, \quad m_y = \frac{M_y - \alpha_y}{M_{y0}}, \quad \text{and } m_z = \frac{M_z - \alpha_z}{M_{z0}}$$

yielding in matrix form

$$\begin{Bmatrix} P \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} pP_0 \\ m_y M_{y0} \\ m_z M_{z0} \end{Bmatrix} + \begin{Bmatrix} \alpha_p \\ \alpha_y \\ \alpha_z \end{Bmatrix}$$

and $S^T = \{P \ M_y \ M_z\}$ defines a point on the yield surface.

The origin of the yield surface, which is located at the coordinates α_p , α_y , and α_z , is the origin of the coordinate axes in the normalized force space shown in Figure 6.6.

The gradients in the non-normalized force space for the axial force and the two bending components, respectively, are as follows

$$\bar{\xi}_p = \frac{\partial \Phi}{\partial p} = \frac{\partial \Phi}{\partial P} \frac{\partial P}{\partial p} = P_0 \frac{\partial \Phi}{\partial P}$$

$$\bar{\xi}_y = \frac{\partial \Phi}{\partial m_y} = \frac{\partial \Phi}{\partial M_y} \frac{\partial M_y}{\partial m_y} = M_{y0} \frac{\partial \Phi}{\partial M_y} \quad (6.53)$$

$$\bar{\xi}_z = \frac{\partial \Phi}{\partial m_z} = \frac{\partial \Phi}{\partial M_z} \frac{\partial M_z}{\partial m_z} = M_{z0} \frac{\partial \Phi}{\partial M_z}$$

Thus

$$\frac{\partial \Phi}{\partial P} = \frac{\bar{\xi}_p}{P_0}, \quad \frac{\partial \Phi}{\partial M_y} = \frac{\bar{\xi}_y}{M_{y0}}, \quad \text{and} \quad \frac{\partial \Phi}{\partial M_z} = \frac{\bar{\xi}_z}{M_{z0}}$$

Putting in the matrix form

$$\xi = \begin{Bmatrix} \xi_p \\ \xi_y \\ \xi_z \end{Bmatrix} = \begin{Bmatrix} \partial \Phi / \partial P \\ \partial \Phi / \partial M_y \\ \partial \Phi / \partial M_z \end{Bmatrix} = \begin{Bmatrix} \bar{\xi}_p / P_0 \\ \bar{\xi}_y / M_{y0} \\ \bar{\xi}_z / M_{z0} \end{Bmatrix} \quad (6.54)$$

Letting the outward unit normal to the yield surface be denoted by n

$$n = \frac{\Phi_s}{(\Phi_s^T \Phi_s)^{1/2}} = \frac{1}{\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \quad (6.55)$$

6.6.3 Elasto-plastic tangent stiffness

In this section, the elasto-plastic tangent stiffness matrix k_t^p which relates the element force increments ds to the element nodal displacement dr for an elasto-plastic flexural column element yielding at one or both ends is derived using the generalized yield

function $f(p_u, m_{yu}, m_{zu})$ established previously and the associated flow rule for an elasto-plastic material. The desired relation is

$$ds = k_i^p dr \quad (6.56)$$

where

$$ds = \begin{Bmatrix} ds_i \\ ds_j \end{Bmatrix}, \text{ and } dr = \begin{Bmatrix} dr_i \\ dr_j \end{Bmatrix} \quad (6.57)$$

and $ds_k, dr_k, k = i, j$ are the vectors of element force increments and the corresponding element nodal displacement increments at node k .

By virtue of assumption (5) in Section 6.6.1 and the assumption that the element deformation increments can be additively decomposed into an elastic component and a plastic component, the element nodal displacement increment dr can be expressed as follows

$$dr = dr^e + dr^p \quad (6.58)$$

where

$$dr^e = \begin{Bmatrix} dr_i^e \\ dr_j^e \end{Bmatrix}, \text{ and } dr^p = \begin{Bmatrix} dr_i^p \\ dr_j^p \end{Bmatrix} \quad (6.59)$$

and where $dr_k^e, k = i, j$, represents the vectors of elastic nodal displacement increments at node k , and $dr_k^p, k = i, j$, represents the vectors of plastic displacement increments

resulting from concentrated plastic deformation at node k . The element forces are uniquely determined by the element elastic nodal displacements, i.e.

$$\mathbf{s} = \mathbf{k}^e \mathbf{r}^e \quad (6.60)$$

and therefore

$$d\mathbf{s} = \mathbf{k}^e d\mathbf{r}^e \quad (6.61)$$

where $\mathbf{k}^e$ is the elastic stiffness matrix of the element.

According to the associated flow rule for an elasto-plastic solid, the plastic deformation increment $d\mathbf{r}_k^p$ at a particular section k is governed by following equation,

$$d\mathbf{r}_k^p = \left[\frac{\partial f}{\partial \mathbf{s}_k} \right] d\lambda_k \quad (6.62)$$

where $d\lambda_k$ is a positive scalar proportionality constant which can be determined by the yield condition, i.e.

$$f(\mathbf{s}_k) = 1, df = \left[\frac{\partial f}{\partial \mathbf{s}_k} \right]^T d\mathbf{s}_k = 0 \quad (6.63)$$

where T indicates the transpose operation.

Note that if the section is not yielding, equation (6.62) still applies provided $\partial f / \partial \mathbf{s}_k$ is set equal to 0. Letting $k = i, j$, the flow rule can be expressed as follows

$$d\mathbf{r}^p = \begin{Bmatrix} d\mathbf{r}_i^p \\ d\mathbf{r}_j^p \end{Bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial s_i} & 0 \\ 0 & \frac{\partial f}{\partial s_j} \end{bmatrix} \begin{Bmatrix} d\lambda_i \\ d\lambda_j \end{Bmatrix} = \left[\frac{\partial f}{\partial s} \right] d\lambda \quad (6.64)$$

It is noted that vector $d\mathbf{r}^p$ has the size of 12 by 1, matrix $\left[\frac{\partial f}{\partial s} \right]$ has the size of 12 by 2 and

$d\lambda$ has the size of 2 by 1. The yield condition becomes

$$\mathbf{f} = \begin{Bmatrix} f(s_i) \\ f(s_j) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}, d\mathbf{f} = \begin{bmatrix} \frac{\partial f}{\partial s_i} & 0 \\ 0 & \frac{\partial f}{\partial s_j} \end{bmatrix}^T \begin{Bmatrix} ds_i \\ ds_j \end{Bmatrix} = \left[\frac{\partial f}{\partial s} \right]^T ds = 0 \quad (6.65)$$

Substituting equation (6.58) into equation (6.61) leads to

$$ds = \mathbf{k}^e d\mathbf{r}^e = \mathbf{k}^e (d\mathbf{r} - d\mathbf{r}^p) \quad (6.66)$$

Premultiplying both sides of the equation (6.66) by $\left[\frac{\partial f}{\partial s} \right]^T$, one obtains upon using equation (6.65) the relation

$$\left[\frac{\partial f}{\partial s} \right]^T ds = \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e (d\mathbf{r} - d\mathbf{r}^p) = 0 \quad (6.67)$$

Substituting equation (6.64) into equation (6.67) one obtains

$$\left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] d\lambda = \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e d\mathbf{r} \quad (6.68)$$

Solving for $d\lambda$ gives

$$d\lambda = \left\{ \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \right\}^{-1} \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e d\mathbf{r} \quad (6.69)$$

which after substituting into equation (6.64) gives plastic deformation as

$$d\mathbf{r}^p = \left[\frac{\partial f}{\partial s} \right] \left\{ \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \right\}^{-1} \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e d\mathbf{r} \quad (6.70)$$

Substituting equation (6.70) into equation (6.66) leads to

$$ds = (\mathbf{k}^e - \mathbf{k}_t^y) d\mathbf{r} = \mathbf{k}_t^{\text{ep}} d\mathbf{r} \quad (6.71)$$

where

$$\mathbf{k}_t^y = \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \left\{ \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \right\}^{-1} \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \quad (6.72)$$

and

$$\mathbf{k}_t^{\text{ep}} = \mathbf{k}^e - \mathbf{k}_t^y = \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \left\{ \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \right\}^{-1} \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \quad (6.73)$$

Physically, the matrix $\mathbf{k}_t^y$ represents the loss of elastic stiffness in the element due to yielding, and the elasto-plastic tangent stiffness matrix $\mathbf{k}_t^{\text{ep}}$ represents the reduced elastic stiffness of the element.

The evaluation of the elasto-plastic tangent stiffness matrix appearing in equation (6.73) is straight forward once the yield point representing the element force state on

yield surface is determined. However, in practice a step-by-step analysis procedure is usually required in which case the yield point determination is only approximate. Specifically, a point inside the yield surface can move outside the surface during a single time interval provided the element is in a yield state.

To alleviate this problem, it is necessary to employ an iteration procedure within each time interval. This procedure, however, requires a significant amount of computation effort. Therefore, in the present analysis, an approximate procedure that avoids iteration is adopted. This procedure is similar to the one considered by Chi and Powell (1973). Referring to Figure 6.7, assume that the column element at the beginning of a time interval Δt and at time t is in the elastic state as represented by point A inside the yield surface. Following the established procedures a vector of element nodal displacement increments $\Delta \mathbf{r}_t$ is determined and an apparent new element force vector $\mathbf{s}_{t+\Delta t}$ is then found using the relations,

$$\mathbf{s}_{t+\Delta t} = \mathbf{s}_t + \Delta \mathbf{s}_t^e = \mathbf{s}_t + \mathbf{k}^e \Delta \mathbf{r}_t \quad (6.74)$$

and yield function,

$$f(\mathbf{s}_{t+\Delta t}) > 1 \quad (6.75)$$

The method is used to determine yield point C proceeds by determining a scalar factor μ_1 , $0 \leq \mu_1 \leq 1$ such that $f(\mathbf{s}_t + \mu_1 \Delta \mathbf{s}_t^e) = 1$. One can then evaluate the elasto-plastic tangent stiffness matrix for point C. A new element force $\mathbf{s}_{t+\Delta t}$ as represented by point D is computed using the relation,

$$\bar{\mathbf{s}}_{t+\Delta t} = \mathbf{s}_t + \mu_1 \Delta \mathbf{s}_t^e + (1 - \mu_1) \mathbf{k}_t^{ep} \Delta \mathbf{r}_t \quad (6.76)$$

Usually, point D is also located outside the yield surface due to the local convex shape of the yield surface. A second scalar factor μ_2 , $0 \leq \mu_2 \leq 1$ is now introduced to bring point D back to point E on the yield surface along vector $\bar{s}_{t+\Delta t}$. The final element force vector is determined by,

$$s_{t+\Delta t} = \mu_2 \bar{s}_{t+\Delta t} \quad (6.77)$$

where μ_2 is determined so that $f(\mu_2) = 1$.

Finally, at the end of the time interval a new elasto–plastic tangent stiffness matrix is evaluated using the new yield force vector $s_{t+\Delta t}$ i.e., the vector to point E. This new tangent stiffness matrix is used as the element stiffness matrix during the next time interval.

Table 6.1 Equilibrium Iteration

For $i = 1, 2, \dots, n$

1. Compute the tangent stiffness and the residual force using equation (6.32)

$$\overline{\mathbf{K}}_T^{(i)} = \mathbf{K}_T^{(i)} + C_7 \mathbf{M} + C_8 \mathbf{K}$$

$$\delta \overline{\mathbf{R}}_{n+1}^{(i)} = \mathbf{R}_{n+1} - \mathbf{M}(\ddot{\mathbf{u}}_{n+1}^{(i)} + \alpha \dot{\mathbf{u}}_{n+1}^{(i)}) - \mathbf{K}(\mathbf{u}_{n+1}^{(i)} + \beta \dot{\mathbf{u}}_{n+1}^{(i)}) - \mathbf{Q}_{n+1}^{NS}$$

2. Compute the corrective displacement in the reduced coordinate using equations (6.31)

$$\mathbf{K}_T^{(i)} \delta \mathbf{u}_{n+1}^{(i)} = \delta \mathbf{R}_{n+1}^{(i)}$$

3. Compute the new displacement, velocity and acceleration using equation (6.29)

$$\mathbf{u}_{n+1}^{(i+1)} = \delta \mathbf{u}_{n+1}^{(i)} + \mathbf{u}_{n+1}^{(i)}$$

$$\dot{\mathbf{u}}_{n+1}^{(i+1)} = \delta \dot{\mathbf{u}}_{n+1}^{(i)} + \dot{\mathbf{u}}_{n+1}^{(i)}$$

$$\ddot{\mathbf{u}}_{n+1}^{(i+1)} = \delta \ddot{\mathbf{u}}_{n+1}^{(i)} + \ddot{\mathbf{u}}_{n+1}^{(i)}$$

Go to 1 if the residual force $\delta \overline{\mathbf{R}}$ is greater than the desired tolerance.

Table 6.2 Non-linear dynamic analysis framework by direct integration

1. Initial calculations
 - (a) Form initial stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$ of the system
 - (b) Solve for initial displacements and calculate element forces due to static loads
 - (c) Store dynamic load and ground acceleration time history
 - (d) Set initial conditions
 - (e) Calculate step-by-step integration constants C_i ($i=1$ to 12)
2. For each time increment Δt
 - (f) Form effective dynamic stiffness matrix $\bar{\mathbf{K}}_T$ and effective dynamic load vector $\bar{\mathbf{R}}_{n+1}$
 - (g) Solve for displacement increment $\Delta \mathbf{u}_{n+1}$
 - (h) Compute current accelerations, velocities and displacements
 - (i) Calculate current element forces, check for non-linear conditions and compute new element tangent stiffness matrix, then determine inelastic deformation vector and current inelastic element nodal restoring force
 - (j) Compute residual force vector $\delta \bar{\mathbf{R}}_{n+1}$ and check accuracy of the solution.
If calculated responses are sufficiently accurate, skip to (k), else go to (3)

(k) Repeat steps (f) through (j) for next time increment

3. Equilibrium iteration

(l) Form effective dynamic stiffness matrix $\bar{\mathbf{K}}_T^{(i)}$ and solve for corrective displacement vector $\delta \mathbf{u}_{n+1}^{(i)}$

(m) Compute i+1st corrected accelerations, velocities, and displacements.

(n) Calculated current element forces, check non-linearity conditions, and compute element tangent stiffness matrices; then determine inelastic deformation vectors and current element nodal restoring force vectors.

(p) Compute residual force vector $\delta \bar{\mathbf{R}}_{n+1}^{i+1}$ and check accuracy of solution. If corrected responses are sufficiently accurate, go back to (k); else repeat steps (l) to (p).

a) Uniform Bending

b) Non-uniform Bending

c) Bending and Axial Force

Figure 6.1 Plastic zones for different loading conditions (Powell, 1993)

Figure 6.2 Generalized forces in the beam-column element (Tseng et al., 1973)

Figure 6.3 Generalized yield surface for reinforced concrete columns (Tseng et al., 1973)

Figure 6.4 Load contours for column constant P_u on yield surface (Tseng et al., 1973)

a) Lack of normality

b) Lack of convexity

c) Normality and convexity

Figure 6.5 Normality and convexity (Bazănt et al., 2001)

Figure 6.6 Gradient to the yield surface in normalized force space (Tseng et al., 1973)

Figure 6.7 Determination of yield points (Tseng et al., 1973)

Chapter 7

Strength Deterioration and Stiffness Degradation Behaviour of Bridge Columns

7.1 General

The developments of derived Ritz vector methods for non-linear time history response analysis of bridges subject to earthquake loadings are presented in the previous chapter. The numerical results obtained from non-linear time history response analysis depend not only on the methods used to generate the discretizing Ritz vectors which are then used to formulate the equations of motion of the bridge system but also on the non-linear models used to characterize the non-linear behaviour of structural components or members of the bridge. The modeling assumptions and details of an analysis model have implications on the accuracy of the numerical results. In the original version of the NEABS program the non-linear behaviour of the beam-column elements is assumed to be elasto-plastic. However, experiments and observations from major earthquakes have indicated that reinforced concrete members often exhibit strength deterioration and stiffness degradation at large deformation. The decrease of flexural strength and stiffness of columns cause a redistribution of structural element forces, so that the seismic responses of the analyzed bridge can be significantly influenced. Therefore, there is the need for

advanced modeling schemes with improved non-linear modeling capacities. The objective of this chapter is to develop an accurate and effective non-linear model which accounts for strength deterioration and stiffness degradation behaviour of bridge columns during earthquakes. Cofer et al. (1999) have used the deformation level to quantify damage degree of bridge columns, however, it is believed that the damage of a member depends not only on deformation level but also depends on the number of cycles of loadings. Studies on inelastic damage analysis of reinforced concrete bridge columns have indicated that the Park and Ang index (Park and Ang, 1985) and the modified Park and Ang index (Kunnath et al., 1992) give the most reliable indication of structural failure (Williams et al., 1997; Hindi and Sexsmith, 2004).

The phenomenon of strength deterioration, which manifests as the negative slope of the load-deformation curve, is a material property and can be modeled by the plasticity theory. The strength deterioration behaviour occurs when deformation exceeds a certain deformation level. On the other hand, stiffness degradation behaviour is observed when the stiffness property decreases with increase in deformation upon repeated loading. Since the stiffness degradation is due mainly to micro-cracking or fracturing of material during response, it is not considered for modeling within the framework of plasticity theory. The derivation of the stiffness degradation formulation is based on the damage theory (Chen and Han, 1988). In this study, it is assumed that the strength deterioration behaviour depends on the ratio of maximum plastic deformation to the ultimate plastic deformation (deterioration slope remains constant), whereas stiffness degradation behaviour depends on the ratio of maximum plastic deformation to the ultimate plastic

deformation and the number of loading cycles in the loading history (degradation slope reduces).

7.2 Non-linear beam-column element with strength deterioration

Experimental studies and observations of damaged structures after major earthquakes have showed the phenomenon of strength deterioration of bridge columns has been observed and is significant, especially for bridges designed before 1970s. In order to accurately evaluate the earthquake responses of bridges, strength deterioration behaviour needs to be incorporated into the plasticity model employed to represent the behaviour and performance of bridges during major earthquakes. This is important for bridges designed before the 1970s because most of these bridge's columns or piers have inadequate confinement detailings and lap-slices at the connection zone between columns and footings. The hysteretic model adopted in the original version of the NEABS program is the elasto-plastic model, in which stiffness matrix derived from equation (6.73) does not include the strength deterioration behaviour in the pre-failure regime. In this chapter, the theoretical developments of the plastic stiffness accounting for strength deterioration of reinforced concrete members are presented. The moment-curvature curve of a member cross section considered in this study is shown in Figure 7.1.

7.2.1 Yield function with strength degradation behaviour

It is noted that the yield function mentioned in equation (6.52) is developed for elastic perfectly plastic material, where the yield stress level is independent of the degree of

plastification. This means that the yield surface is fixed, as shown in Figure 6.3. However, in reality, the yield surface may vary in size, shape, and/or position with increasing plastification. Furthermore, because the moment-curvature relation of a member section in flexure about y- and z- directions may not be the same, strength deterioration may thus initiate and proceed differently in the two separate y- and z- directions. To simplify the derivation of the plastic stiffness, it is assumed that the strength deterioration behaviour is isotropic, which thus implies that the subsequent yield surface is the result of uniform contraction of the original one, as shown in Figure 7.2. The cause of the yield surface contraction is that the yield stress level at a point decreases with increasing plastic deformation. To characterize the degree of strength deterioration, a strength deterioration coefficient C is introduced. Since strength deterioration behaviour occurs only when the deformation exceeds a certain deformation (often larger than yield deformation), strength degradation behaviour depends on the ratio of the maximum plastic deformation to the ultimate plastic deformation i.e., strength deterioration coefficient is obtained from following relation

$$C = \frac{\theta_{\max} - \theta_r}{\theta_u - \theta_r} \quad (7.1)$$

where $\theta_{\max}$ is the maximum rotation angle obtained during response, θ_r is the rotation angle beyond which strength deterioration begins, and θ_u is the ultimate rotation capacity of the section. It is important to note that the strength deterioration coefficient is not accumulative in nature. Total plastic rotation is calculated using the square root of summation of square rule as follows

$$\theta_{\max} = \sqrt{\theta_y^2 + \theta_z^2} \quad (7.2)$$

where θ_y and θ_z are the maximum plastic rotation in the y- and z- directions, respectively, during response, $\theta_{\max}$ is the total plastic rotation. The angles θ_r and θ_u may be calculated as follows

$$\theta_r = l_p (\phi_r - \phi_y) \quad (7.3)$$

$$\theta_u = l_p (\phi_u - \phi_y) \quad (7.4)$$

where l_p is the plastic hinge length, ϕ_r is the curvature beyond which strength degradation begins, ϕ_y is the yield curvature and ϕ_u is the curvature for which the maximum moment capability is equal to zero, theoretically.

The yield function that includes strength deterioration behaviour is defined as follows,

$$\left(\frac{M_{yu}}{M_{yp}} \right)^2 + \left(\frac{M_{zu}}{M_{zp}} \right)^2 = (1 - C)^2 \quad (7.5)$$

where C is the aforementioned strength deterioration coefficient and it is a function of plastic deformation ($0 \leq C \leq 1$). Because $(1 - C)^2$ is less than or equal to one, the yield surface shrinks. The yield function may be written in another form as follows:

$$f(s_k) = \kappa \quad (7.6)$$

where $f(s_k) = \left(\frac{M_{yu}}{M_{yp}}\right)^2 + \left(\frac{M_{zu}}{M_{zp}}\right)^2$, $\kappa = (1 - C)^2$ and s_k represents the generalized force vector, $k=i, j$.

The generalized force state is restricted to be either on or inside the yield surface. A generalized force state for which $f(s_k) = \kappa$ denotes a plastic state, whereas elastic behaviour is characterized by $f(s_k) < \kappa$. When $f(s_k) = \kappa$, there are two possibilities: (a) plastic loading from 1 to 3, or (b) elastic unloading from 1 to 2 (Figure 7.3). To use the stiffness corresponding correctly to the instant process of the element, the behaviour of the element needs to be determined. There can be difficulty in distinguishing between a reduction of generalized forces from strength degradation and one due to elastic unloading. Chen and Han (1988) have proposed procedures to distinguish between the two behaviour of elastic unloading and plastic loading. The change in incremental generalized forces, ds_k , which is calculated by using the elastic stiffness from equation (6.66), is substituted into equation (6.65) as follows

$$df = \left[\frac{\partial f}{\partial s_k} \right]^T ds_k$$

where $\left[\frac{\partial f}{\partial s_k} \right]^T$ can be obtained from the use of equation (6.51). The following loading criteria are then applied,

$$\begin{aligned} &\text{if } f(s_k) = \kappa \text{ and } df < 0, \text{ then elastic unloading occurs} \\ &\text{if } f(s_k) = \kappa \text{ and } df = 0, \text{ then neutral unloading on yield surface occurs} \\ &\text{if } f(s_k) = \kappa \text{ and } df > 0, \text{ then plastic loading occurs} \end{aligned} \tag{7.8}$$

7.2.2 Elasto-plastic stiffness matrix with strength deterioration

In Section 6.6.3, the derivation of the tangent stiffness for elasto-plastic flexural column elements has been presented. However, the derivation is obtained without consideration of the strength deterioration behaviour. In this section, the tangent stiffness for an elasto-plastic flexural column element is developed taking into account the strength degradation behaviour. After the yield function is rewritten as in equation (7.5), the next step is to form the incremental elastic-plastic-tangent stiffness matrix that includes the strength deterioration behaviour, $\mathbf{k}^{epd}$, using the flow rule (normality rule), which relates the element generalized force increments, ds , to the element nodal displacement increments, dr , for an elasto-plastic flexural beam-column element. In this approach, a plastic hinge is assumed to be developed at one or both ends of the beam-column element when the combination of generalized forces falls on the yield surface (Tseng et al., 1973). The yield surface is usually contracted proportionally to the increase in the plastic work increment dW^p (Kanchi, 1993). Figure 7.4 shows that

$$dW^p = M' d\theta^p \quad (7.9)$$

The yield function increment is written as follows

$$df = \frac{df}{dW^p} dW^p \quad (7.10)$$

Substituting equation (7.9) into equation (7.10) leads to

$$df = \frac{df}{M' d\theta^p} M' d\theta^p = \frac{\partial f}{\partial C} \frac{\partial C}{\partial \theta^p} d\theta^p = \frac{\partial f}{\partial C} dC \quad (7.11)$$

Substituting equation (7.11) into equation (7.7) leads to

$$df = \left[\frac{\partial f}{\partial s_k} \right]^T ds_k = -2(1 - C_k) dC_k, k=i, j \quad (7.12)$$

Equation (7.12) is similar to equation (6.63) except that the right hand side of equation (7.12) is a function of the strength deterioration coefficient C . Substituting equation (7.10) into equation (6.65) leads to

$$df = \begin{Bmatrix} df_i \\ df_j \end{Bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial s_i} & 0 \\ 0 & \frac{\partial f}{\partial s_j} \end{bmatrix}^T \begin{Bmatrix} ds_i \\ ds_j \end{Bmatrix} = \begin{bmatrix} -2(1 - C_i) & 0 \\ 0 & -2(1 - C_j) \end{bmatrix} \begin{Bmatrix} dC_i \\ dC_j \end{Bmatrix} \quad (7.13)$$

or

$$df = \left[\frac{\partial f}{\partial s} \right]^T ds = C dC \quad (7.14)$$

where

$$C = \begin{bmatrix} -2 \left(1 - \frac{\theta_{\max}^i - \theta_r^i}{\theta_u^i - \theta_r^i} \right) & 0 \\ 0 & -2 \left(1 - \frac{\theta_{\max}^j - \theta_r^j}{\theta_u^j - \theta_r^j} \right) \end{bmatrix} \quad (7.15)$$

Substituting equation (6.66) into equation (7.14) one obtains

$$df = \left[\frac{\partial f}{\partial s} \right]^T ds = \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e (d\mathbf{r} - d\mathbf{r}^p) = C dC \quad (7.16)$$

Substituting equation (6.64) into equation (7.16) and solving for $d\lambda$ leads to

$$d\lambda = \left(\left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \right)^{-1} \left\{ \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e d\mathbf{r} - \mathbf{C} d\mathbf{C} \right\} \quad (7.17)$$

Substituting equation (7.17) into equation (6.64) leads to the following relation

$$d\mathbf{r}^p = \left[\frac{\partial f}{\partial s} \right] \left(\left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial s} \right] \right)^{-1} \left\{ \left[\frac{\partial f}{\partial s} \right]^T \mathbf{k}^e d\mathbf{r} - \mathbf{C} d\mathbf{C} \right\} \quad (7.18)$$

In equation (7.18) the strength degradation coefficient matrix $\mathbf{C}$ has been defined according to equation (7.4), in which it is related to plastic deformation, therefore $d\mathbf{C}$ is the only unknown. It is assumed that $\theta_{\max}$ is computed as the square root of the summation of square of plastic rotations in both y- and z-directions as follows

$$\theta_{\max} = \sqrt{\theta_y^2 + \theta_z^2} \quad (7.2)$$

Substituting equation (7.2) into equation (7.1) one obtains

$$dC_k = \frac{2}{\theta_u^k} \frac{\theta_y^k d\theta_y^k + \theta_z^k d\theta_z^k}{\sqrt{(\theta_y^k)^2 + (\theta_z^k)^2}}, k = i, j \quad (7.19)$$

Using equation (7.19), $d\mathbf{C}$ is written as follows

$$d\mathbf{C} = \begin{Bmatrix} dC_i \\ dC_j \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & h_{1,5} & h_{1,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_{2,11} & h_{2,12} \end{bmatrix} d\mathbf{r}^p \quad (7.20)$$

or

$$d\mathbf{C} = \mathbf{H}d\mathbf{r}^p \quad (7.21)$$

where,

$$\begin{aligned} h_{1,5} &= \frac{2}{\theta_u^i} \frac{\theta_y^i}{\sqrt{(\theta_y^i)^2 + (\theta_z^i)^2}} \\ h_{1,6} &= \frac{2}{\theta_u^i} \frac{\theta_z^i}{\sqrt{(\theta_y^i)^2 + (\theta_z^i)^2}} \\ h_{2,11} &= \frac{2}{\theta_u^j} \frac{\theta_y^j}{\sqrt{(\theta_y^j)^2 + (\theta_z^j)^2}} \\ h_{2,12} &= \frac{2}{\theta_u^j} \frac{\theta_z^j}{\sqrt{(\theta_y^j)^2 + (\theta_z^j)^2}} \end{aligned} \quad (7.22)$$

Substituting equation (7.21) into equation (7.18) and solving for $d\mathbf{r}^p$, one obtains the equation of plastic displacement as follows

$$d\mathbf{r}^p = \left\{ \mathbf{I} + \left[\frac{\partial f}{\partial \mathbf{s}} \right] \left(\left[\frac{\partial f}{\partial \mathbf{s}} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial \mathbf{s}} \right] \right)^{-1} \mathbf{C} \mathbf{H} \right\}^{-1} \left[\frac{\partial f}{\partial \mathbf{s}} \right] \left(\left[\frac{\partial f}{\partial \mathbf{s}} \right]^T \mathbf{k}^e \left[\frac{\partial f}{\partial \mathbf{s}} \right] \right)^{-1} \left[\frac{\partial f}{\partial \mathbf{s}} \right]^T \mathbf{k}^e d\mathbf{r} \quad (7.23)$$

where $\mathbf{I}$ is a 12 by 12 unit matrix, $\mathbf{C}$ is strength deterioration coefficient matrix; $\mathbf{C}$ is a 2 by 2 matrix if there are damage at two ends; else $\mathbf{C}$ is scalar, $\mathbf{H}$ is a 2 by 12 coefficient matrix and $\partial f / \partial \mathbf{s}$ is a 12 by 2 matrix for 2-end yielded or a 12 by 1 matrix for 1-end

yielded. Substituting equation (7.23) into equation (6.66), the relationship between incremental generalized forces and incremental displacement is obtained as follows

$$ds = k_i^{epd} dr \quad (7.24)$$

where k_i^{epd} is expressed as follows

$$k^{epd} = k^e - k^e \left\{ I + \left[\frac{\partial f}{\partial s} \right] \left(\left[\frac{\partial f}{\partial s} \right]^T k^e \left[\frac{\partial f}{\partial s} \right] \right)^{-1} CH \right\}^{-1} \left[\frac{\partial f}{\partial s} \right] \left(\left[\frac{\partial f}{\partial s} \right]^T k^e \left[\frac{\partial f}{\partial s} \right] \right)^{-1} \left[\frac{\partial f}{\partial s} \right]^T k^e \quad (7.25)$$

and I is the 12 by 12 unit matrix.

Equations (7.23) and (7.24) are in the final form to compute incremental plastic deformation and incremental generalized forces in the strength degradation stage. However, from equation (7.21) due to difference in plastic deformation at 2 ends of beam-column element, it is noted that the new elasto-plastic stiffness matrix is asymmetric. The asymmetry arises due to the assumption of only flexural damage without considering damage due to buckling and shear. This causes problem with the standard storage scheme of NEABS program, which stores only the upper banded matrix. To alleviate this problem, a symmetric approximation to k_i^{epd} can be obtained by letting $H = 0$ (this is exactly the case of elasto-plastic behaviour). The symmetric stiffness matrix can then be used for solving the dynamic equilibrium equation at the current time step or iteration. The equilibrium unbalance caused by the use of symmetric stiffness matrix will be corrected by applying the equilibrium unbalance within the next time step

or iteration to restore equilibrium. Therefore, the derived asymmetrical $\mathbf{k}_t^{\text{epd}}$ is used to compute generalized force increments and convergence is readily obtained.

7.2.3 Equilibrium equation

Rewriting equation (6.16)

$$\mathbf{M}\ddot{\mathbf{u}}_{n+1} + \mathbf{C}\dot{\mathbf{u}}_{n+1} + \mathbf{K}_T^S(\mathbf{u}_n)\Delta\mathbf{u}_{n+1} = \mathbf{R}_{n+1} - \mathbf{K}\mathbf{u}_n - \mathbf{Q}_n^{\text{NS}}$$

In the formulation here, the symmetric stiffness matrix $\mathbf{K}_T^S$ is used instead of the asymmetric stiffness matrix $\mathbf{K}_T^U$ for the solution of the displacement increment above. The discrepancy of this approximation is then rectified as unbalance force on the right hand side of equations of motion as discussed in the previous section. The equation of motion is written as follows

$$\mathbf{M}\ddot{\mathbf{u}}_{n+1} + \mathbf{C}\dot{\mathbf{u}}_{n+1} + \mathbf{K}_T^S(\mathbf{u}_n)\Delta\mathbf{u}_{n+1} + (\mathbf{K}_T^U(\mathbf{u}_n) - \mathbf{K}_T^S(\mathbf{u}_n))\Delta\mathbf{u}_{n+1} = \mathbf{R}_{n+1} - \mathbf{K}\mathbf{u}_n - \mathbf{Q}_n^{\text{NS}} \quad (7.26)$$

where $(\mathbf{K}_T^U(\mathbf{u}_n) - \mathbf{K}_T^S(\mathbf{u}_n))\Delta\mathbf{u}_{n+1}$ is the unbalanced force. This unbalanced force is then subtracted from the right hand side of equation (6.31), which leads to

$$\bar{\mathbf{K}}_T^{(i)}\delta\mathbf{u}_{n+1}^{(i)} = \delta\bar{\mathbf{R}}_{n+1}^{(i)} - (\mathbf{K}_T^U(\mathbf{u}_n) - \mathbf{K}_T^S(\mathbf{u}_n))\Delta\mathbf{u}_{n+1} \quad (7.27)$$

The equilibrium iteration follows the same as iteration procedure presented in Chapter 6 except that the unbalanced force is subtracted from the residual force.

7.3 Non-linear beam–column element with stiffness degradation

Stiffness degradation behaviour is primarily related to the accumulative effect of damage to material during response. For earthquake resistant design of typical reinforced concrete structures, the design philosophy of allowing some degree of damage to the structure but without collapse is adopted in order to achieve economical design with acceptable low risk to life safety for rare events such as major earthquakes. In order to accurately assess the performance of concrete structures subject to severe earthquake loadings, the concept of damage index is adopted (Park and Ang, 1985). The damage index may be defined locally for an individual member or globally for the entire structure. Generally, a reinforced concrete structure is damaged by a combination of stress reversals and high stress excursions. Therefore, the damage index is a function of maximum response as well as the effect of repeated cyclic loading. The global damage index may be calculated by taking a weighted average of the local indices of the entire structure, or by comparing the modal properties of the structure before and after the earthquake. To derive the stiffness matrix of a deteriorated structure, a damage index, D , is introduced. The damage index is defined such that a value of zero indicates no damage while a value of one means total damage. Failure of actual components is observed to occur at an intermediate value of damage, $D_{\max}$.

7.3.1 Park and Ang damage model

The Park and Ang damage index (1985) for a structural element is defined as follows,

$$D = \frac{\delta_{\max}}{\delta_u} + \frac{\beta}{F_y \delta_u} \int dE_h \quad (7.28)$$

where $\delta_{\max}$ is the maximum deformation caused by earthquake loading, δ_u is the ultimate deformation under monotonic loading, F_y is the yield strength, dE_h is the incremental absorbed hysteretic energy, and β is a non-negative parameter to account for cyclic load effect. The Park and Ang damage model accounts for damage due to maximum inelastic excursion and damage due to the history of deformations. The two contributing components to damage are linearly combined. During elastic response, the damage index is equal to zero.

For damage at the end section of an element, the following damage model modified from the original Park and Ang damage model has been introduced by Kunnath et al. (1992) as follows

$$D = \frac{\theta_{\max} - \theta_y}{\theta_u - \theta_y} + \frac{\beta}{M_y \theta_u} E_h \quad (7.29)$$

where $\theta_{\max}$ is the maximum rotation angle obtained during loading history, θ_y is the yield rotation angle when unloading, and θ_u is the ultimate rotation capacity of the section, E_h is the dissipated energy, and M_y is the yield moment.

7.3.2 Fatigue-based damage model

Since seismic loads induce several inelastic cycles at relatively large ductilities, the concept of using low-cycle fatigue theories to characterize damage of structural elements is more rational. A fatigue-based damage model has been introduced by Reinhorn and Valles (1995). The damage model is developed based on basic structural response considerations and a low-cycle fatigue rule. The damage index is defined as follows,

$$D = \frac{\theta_{\max} - \theta_y}{\theta_u - \theta_y} \left[\frac{1}{1 - \frac{E_h}{4(\theta_u - \theta_y)M_y}} \right] \quad (7.30)$$

where $\theta_{\max}$ is the maximum rotation obtained during loading history, θ_y is the yield rotation, θ_u is the ultimate rotation capacity of the section, E_h is the cumulative dissipated energy, and M_y is the yield moment.

Note that simplifying the fatigue based damage model for the case when ratio $\frac{\theta_{\max} - \theta_y}{\theta_u - \theta_y}$

is close to one, the equation (7.30) is equal to

$$D = \frac{\theta_{\max} - \theta_y}{\theta_u - \theta_y} + \frac{E_h}{4(\theta_u - \theta_y)M_y} \quad (7.31)$$

Equation (7.31) is equivalent to the Park and Ang damage model for the case $\beta=0.25$ and considering only the remaining rotation after subtracting out the part associated with yield deformation to damage due to cyclic load effect. Therefore, it can be considered that the Park and Ang damage model is similar to the fatigue-based damage model at maximum deformation close to the ultimate capacity of the element. It is noted that the displacement part of the damage index D is equivalent to the strength degradation coefficient C .

7.3.3 Cumulative dissipated energy

It is recognized that the damage index D is a function of the maximum rotation $\theta_{\max}$, the ultimate rotation θ_u , the yield rotation θ_y , the yield moment M_y , and the cumulative

dissipated energy E_h . Procedures to obtain θ_u , θ_y , and M_y are straight forward based on moment-curvature analysis. In the analysis, the maximum displacement $\theta_{\max}$ is obtained from keeping track the results of previous cycles of excursion in the response corresponding to each response direction. Procedures to compute the cumulative dissipated energy for the evaluation of damage index are presented next.

The input energy for an inelastic system is dissipated by both viscous damping and yielding behaviour. The various energy terms can be defined by integrating the equation of motion of an inelastic system, equation (5.45), as follows

$$\int_0^u m\ddot{u}(t)du + \int_0^u c\dot{u}(t)du + \int_0^u f_s(u, \dot{u})du = - \int_0^u m\ddot{u}_g(t)du \quad (7.32)$$

The right hand side of equation (7.32) is the total energy input into the structure due to earthquake excitation

$$E_i(t) = - \int_0^u m\ddot{u}_g(t)du \quad (7.33)$$

The first term on the left hand side of equation (7.32) is the kinetic energy of the mass associated with its motion relative to the ground

$$E_k(t) = \int_0^u m\ddot{u}(t)du = \int_0^u m\dot{u}(t)d\dot{u} = \frac{m\dot{u}^2}{2} \quad (7.34)$$

The second term on the left hand side of equation (7.32) is the energy dissipated by viscous damping

$$E_D(t) = \int_0^u c \dot{u}(t) du \quad (7.35)$$

The third term on the left hand side of equation (7.32) is the sum of the energy dissipated by yielding and the recoverable strain energy of the system. The recoverable strain energy of the system is expressed as follows

$$E_s(t) = \frac{[f_s(t)]^2}{2k} \quad (7.36)$$

where k is the initial stiffness of the elastic system. The hysteric energy dissipated by yielding is determined as follows

$$E_h(t) = \int_0^u f_s(u, \dot{u}) du - E_s(t) \quad (7.37)$$

where $f_s(u, \dot{u})$ is the inelastic resisting force, which is a function of displacement and velocity.

Dissipated energy expressed in equation (7.37) is a function of the total displacement. The cumulative dissipated energy can be represented as a function of the plastic displacement as follows

$$E_h(t) = \int_0^{u_p} f_s(u_p, \dot{u}_p) du_p \quad (7.38)$$

where $f_s(u_p, \dot{u}_p)$ is the inelastic resisting force and u_p is the plastic displacement. In the present study, the element plastic deformations are computed at each time step during

responses. These plastic deformations are then used to compute the dissipated energy using equation (7.38). Then the damage index is computed by using equation (7.31). More details of the derivation of the damage index adopted in this study are presented in Appendix G.

For a single degree of freedom elasto-plastic oscillator, the area enclosed by the hysteresis loop can be easily evaluated as follows

$$E_h = 4F_y (\delta_{\max} - \delta_y) \quad (7.39)$$

where F_y is the yield force, δ_y is the yield displacement and $\delta_{\max}$ is the maximum displacement response.

If moment-curvature relationship is used, the hysteresis dissipated energy can be written as follows

$$E_h = 4M_y (\theta_{\max} - \theta_y) \quad (7.40)$$

where M_y is the yield moment, θ_y is the yield rotation and $\theta_{\max}$ is the maximum rotation response. Substituting equation (7.40) into equation (7.31) leads to simplified equation for damage index for an elasto-plastic oscillator as follows

$$D = 2 \frac{\theta_{\max} - \theta_y}{\theta_u - \theta_y} \quad (7.41)$$

7.3.4 Elastic stiffness matrix with stiffness degradation behaviour

When subject to repeated cyclic loadings, reinforced concrete member's strength degrades. Therefore, in order to obtain more accurate inelastic behaviour of structures subject to earthquake loading, the stiffness degradation behaviour needs to be included in the non-linear beam-column model. It is assumed that the degradation of stiffness can only occur within the yield surface. If the generalized forces are on the yield surface, the elasto-plastic damage stiffness matrix $\mathbf{k}^{epd}$ derived in the previous section is used in the dynamic equilibrium equation for the current time step. The application of stiffness degradation is illustrated in the hysteretic model as shown in Figure 7.5.

The loading, unloading, and reloading behaviour are defined by the curve. The stiffness during unloading is the same as the initial elastic stiffness, but the stiffness during reloading is degraded. The basic non-linear behaviour of beam-column element are summarized in Table 7.1. Elastic behaviour consists of 2 loading types i.e., unloading and reloading. Unloading stiffness takes initial stiffness as given in equation (E.1) and the stiffness used for reloading takes into account degradation represented by the damage index. Reloading occurs when stress in both y- and z- directions change sign. A criterion is established to determine the starting point of a reloading phase by examining the change of sign of stress in either y- or z- directions. Inelastic behaviour is characterized by 3 loading types i.e., unloading, neutral and reloading. The elasto-plastic stiffness given in equation (6.73) or the damage tangent stiffness given in equation (7.25) is utilized depending on the maximum plastic rotation.

The beam-column element is a three dimensional element. During the elastic loading and reloading stage, the flexural capacities about the y- and x-axes are modeled as being fully continuous with no stiffness degradation. However, during the elastic reloading stage, the end conditions about the y- and z-axes are considered to be between fully continuous and fully hinged conditions, in which the degradation of stiffness occurs due to damage. Figure 7.6 shows a beam-column element that is fully restrained at both ends, which are denoted as ends i and j.

Let matrices $\mathbf{k}_{miy}^e$, $\mathbf{k}_{miz}^e$, $\mathbf{k}_{m jy}^e$, and $\mathbf{k}_{mjz}^e$ be the pinned end moment release stiffness matrices about the y- and z-axes at the i and j ends respectively. The difference of the two matrices $\mathbf{k}^e$ and $\mathbf{k}_{miy}^e$ is the elastic matrix for which the rotational stiffness about the y-axis at the i end is released. The matrices $\mathbf{k}^e$, $\mathbf{k}_{miy}^e$, $\mathbf{k}_{miz}^e$, $\mathbf{k}_{m jy}^e$, and $\mathbf{k}_{mjz}^e$ are shown in Appendix E. The detailed derivation of the release matrices is presented in Appendix F. One can write the damage stiffness matrix for the case of the i end damage as follows

$$\mathbf{k}^{ed} = \mathbf{k}^e - D_1 \mathbf{k}_{miy}^e - D_1 \mathbf{k}_{miz}^e \quad (7.42)$$

Similarly, one can write the damage stiffness matrix for the case of the j end damage as follows

$$\mathbf{k}^{ed} = \mathbf{k}^e - D_2 \mathbf{k}_{m jy}^e - D_2 \mathbf{k}_{mjz}^e \quad (7.43)$$

Extending this formulation to damage occurring simultaneously at both ends, the damage stiffness matrix can be expressed in the form as follows

$$\mathbf{k}^{ed} = \mathbf{k}^e - D_1 \mathbf{k}_{miy}^e - D_1 \mathbf{k}_{miz}^e - D_2 \mathbf{k}_{m jy}^e - D_2 \mathbf{k}_{mjz}^e \quad (7.44)$$

However, using the independent pinned end moment release stiffness matrices as defined in equations (7.42) and (7.43), the resulting damage stiffness matrix may overshoot the damage of the element because the progressive deterioration of the two ends of the beam-column element is path dependent and thus cannot be obtained from the superposition of the independent end released stiffness formulation at the two separate ends. This problem may be alleviated by using the matrix condensation technique. First, the beam-column element, as shown in Figure 7.6, is divided into two sub-elements with mid-node k in which only at the i and j ends are expected to sustain damage.

Therefore, the degradation stiffness for elements (ik) and (kj) may be expressed as follows

$$\mathbf{k}_1^{ed} = \mathbf{k}_1^e - D_1 \mathbf{k}_{miy}^e - D_1 \mathbf{k}_{miz}^e \quad (7.45)$$

$$\mathbf{k}_2^{ed} = \mathbf{k}_2^e - D_2 \mathbf{k}_{m jy}^e - D_2 \mathbf{k}_{mjz}^e \quad (7.46)$$

where $\mathbf{k}_1^{ed}$ and $\mathbf{k}_2^{ed}$ are degradation stiffness matrices for elements (ik) and (kj) , $\mathbf{k}_1^e$ and $\mathbf{k}_2^e$ are the usual elastic stiffness matrices for elements (ik) and (kj) and D_1 and D_2 are scalars represent for damage index of elements (ik) and (kj) .

Further, $\mathbf{k}_1^{ed}$ and $\mathbf{k}_2^{ed}$ may be written as follows

$$\mathbf{k}_1^{ed} = \begin{bmatrix} \mathbf{k}_{aa}^l & \mathbf{k}_{ab}^l \\ \mathbf{k}_{ba}^l & \mathbf{k}_{bb}^l \end{bmatrix} \quad (7.47)$$

and

$$\mathbf{k}_2^{ed} = \begin{bmatrix} \mathbf{k}_{aa}^2 & \mathbf{k}_{ab}^2 \\ \mathbf{k}_{ba}^2 & \mathbf{k}_{bb}^2 \end{bmatrix} \quad (7.48)$$

If $\mathbf{u}_i$, $\mathbf{u}_j$, and $\mathbf{u}_k$ are node displacement vectors at nodes i, j, and k, then the end displacements of elements (ik) and (kj) are related by the following equations

$$\begin{bmatrix} \mathbf{k}_{aa}^1 & 0 & \mathbf{k}_{ab}^1 \\ 0 & \mathbf{k}_{bb}^2 & \mathbf{k}_{ba}^2 \\ \mathbf{k}_{ba}^1 & \mathbf{k}_{ab}^2 & \mathbf{k}_{bb}^1 + \mathbf{k}_{aa}^2 \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \\ \mathbf{u}_k \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_i \\ \mathbf{f}_j \\ \mathbf{f}_k \end{Bmatrix} \quad (7.49)$$

where $\mathbf{f}_i$, $\mathbf{f}_j$, and $\mathbf{f}_k$ are force vectors at nodes i, j, and k, respectively.

Since node k is an inner node therefore

$$\mathbf{f}_k = 0 \quad (7.50)$$

Substituting equation (7.50) into equation (7.49) one obtains

$$\mathbf{k}_{pp} \mathbf{u}_p + \mathbf{k}_{ps} \mathbf{u}_s = \mathbf{f}_p \quad (7.51)$$

and

$$\mathbf{k}_{sp} \mathbf{u}_p + \mathbf{k}_{ss} \mathbf{u}_s = 0 \quad (7.52)$$

where

$$\mathbf{k}_{pp} = \begin{bmatrix} \mathbf{k}_{aa}^1 & 0 \\ 0 & \mathbf{k}_{bb}^2 \end{bmatrix}, \mathbf{k}_{ps} = \begin{Bmatrix} \mathbf{k}_{ab}^1 \\ \mathbf{k}_{ba}^2 \end{Bmatrix}, \mathbf{k}_{sp} = [\mathbf{k}_{ba}^1 \quad \mathbf{k}_{ab}^2], \mathbf{k}_{ss} = [\mathbf{k}_{bb}^1 + \mathbf{k}_{aa}^2]$$

$$\mathbf{u}_p = \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_j \end{Bmatrix}, \mathbf{u}_s = \{\mathbf{u}_k\} \text{ and } \mathbf{f}_p = \begin{Bmatrix} \mathbf{f}_i \\ \mathbf{f}_j \end{Bmatrix} \quad (7.53)$$

It is noted that we have partitioned the global equation to separate “primary” quantities (designated with a “p” subscript) from “secondary” quantities (designated with “s” subscript). Using equations (7.52) and (7.51), the inner node displacement $\mathbf{u}_s$ can be eliminated to solve for $\mathbf{u}_p$. Finally, the equation expressing end forces and displacements for nodes i and j becomes,

$$(\mathbf{k}_{pp} - \mathbf{k}_{ps}\mathbf{k}_{ss}^{-1}\mathbf{k}_{sp})\mathbf{u}_p = \mathbf{f}_p \quad (7.54)$$

where

$$\mathbf{k}^{ed} = \mathbf{k}_{pp} - \mathbf{k}_{ps}\mathbf{k}_{ss}^{-1}\mathbf{k}_{sp} \quad (7.55)$$

The $\mathbf{k}^{ed}$ matrix is thus the damage stiffness matrix of the 3D non-linear beam-column element.

The damage stiffness matrix obtained from equation (7.55) for the case of no damage at both ends is actually the elastic stiffness matrix. For simplification, consider the 4 by 4 stiffness matrix. The length of the half-element is L , and thus the length of the entire element is $2L$.

Elastic stiffness matrix of the entire element is as follows

$$\mathbf{K}^e = \frac{EI}{8L^3} \begin{bmatrix} 12 & 12L & -12 & 12L \\ 12L & 16L^2 & -12L & 8L^2 \\ -12 & -12L & 12 & -12L \\ 12L & 8L^2 & -12L & 16L^2 \end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 1.5 & 1.5L & -1.5 & 1.5L \\ 1.5L & 2L^2 & -1.5L & L^2 \\ -1.5 & -1.5L & 1.5 & -1.5L \\ 1.5L & L^2 & -1.5L & 2L^2 \end{bmatrix}$$

Elastic stiffness matrix of the half-element is

$$\mathbf{K} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Release stiffness matrix at the i-end is

$$\mathbf{K}_i = \frac{EI}{L^3} \begin{bmatrix} 9 & 6L & -9 & 3L \\ 6L & 4L^2 & -6L & 2L^2 \\ -9 & -6L & 9 & -3L \\ 3L & 2L^2 & -3L & L^2 \end{bmatrix}$$

Release stiffness matrix at the j-end is

$$\mathbf{K}_j = \frac{EI}{L^3} \begin{bmatrix} 9 & 3L & -9 & 6L \\ 3L & L^2 & -3L & 2L^2 \\ -9 & -3L & 9 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

a) In case of no damage

Damage stiffness matrix is computed by (7.55) as follows

$$\begin{aligned}
\mathbf{K}^{ed} &= \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & 0 & 0 \\ 6L & 4L^2 & 0 & 0 \\ 0 & 0 & 12 & -6L \\ 0 & 0 & -6L & 4L^2 \end{bmatrix} - \frac{EI}{L^3} \begin{bmatrix} -12 & 6L \\ -6L & 2L^2 \\ -12 & -6L \\ 6L & 2L^2 \end{bmatrix} \begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix}^{-1} \begin{bmatrix} -12 & -6L & -12 & 6L \\ 6L & 2L^2 & -6L & 2L^2 \end{bmatrix} \\
&= \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & 0 & 0 \\ 6L & 4L^2 & 0 & 0 \\ 0 & 0 & 12 & -6L \\ 0 & 0 & -6L & 4L^2 \end{bmatrix} - \frac{EI}{L^3} \begin{bmatrix} -12 & 6L \\ -6L & 2L^2 \\ -12 & -6L \\ 6L & 2L^2 \end{bmatrix} \begin{bmatrix} \frac{1}{24} & 0 \\ 0 & \frac{1}{8L^2} \end{bmatrix} \begin{bmatrix} -12 & -6L & -12 & 6L \\ 6L & 2L^2 & -6L & 2L^2 \end{bmatrix} \\
&= \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & 0 & 0 \\ 6L & 4L^2 & 0 & 0 \\ 0 & 0 & 12 & -6L \\ 0 & 0 & -6L & 4L^2 \end{bmatrix} - \frac{EI}{L^3} \begin{bmatrix} -0.5 & 0.75/L \\ -0.25L & 0.25 \\ -0.5 & -0.75/L \\ 0.25L & 0.25 \end{bmatrix} \begin{bmatrix} -12 & -6L & -12 & 6L \\ 6L & 2L^2 & -6L & 2L^2 \end{bmatrix} \\
&= \frac{EI}{L^3} \begin{bmatrix} 1.5 & 1.5L & -1.5 & 1.5L \\ 1.5L & 2L^2 & -1.5L & L^2 \\ -1.5 & -1.5L & 1.5 & -1.5L \\ 1.5L & L^2 & -1.5L & 2L^2 \end{bmatrix}
\end{aligned}$$

Then the damage stiffness in the case of no damage is the same as the elastic stiffness.

b) In case of the damage index equal to 0.7 at two ends

By repeating above procedure damage stiffness matrix can be obtained. Using equation

(7.44) the damage stiffness matrix is obtained as follows

$$\mathbf{K}^{ed} = \frac{EI}{L^3} \begin{bmatrix} -0.075 & -0.075L & 0.075 & -0.075L \\ -0.075L & 0.25L^2 & 0.075L & -0.4L^2 \\ 0.075 & 0.075L & -0.075 & 0.075L \\ -0.075L & -0.4L^2 & 0.075L & L^2 \end{bmatrix}$$

Using equation (7.55) the damage stiffness matrix is obtained as follows

$$\mathbf{K}^{ed} = \frac{EI}{L^3} \begin{bmatrix} 0.5455 & 0.5455L & -0.5455 & 0.5455L \\ 0.5455L & 0.8612L^2 & -0.5455L & 0.2297L^2 \\ -0.5455 & -0.5455L & 0.5455 & -0.5455L \\ 0.5455L & 0.2297L^2 & -0.5455L & 0.8612L^2 \end{bmatrix}$$

Equation (7.44) gives a stiffness matrix which is not positive definite. Therefore in the present study, damage matrix is obtained from equation (7.55).

Table 7.1 Basic processes in non-linear behaviour of beam-column element

Process	Elastic		Plastic		
	Loading	Unloading	Unloading	Neutral	Loading
$f(\sigma)^{(1)}$	negative	negative	zero	zero	zero
$f_{\sigma} : \mathbf{k}^e : \dot{\epsilon}$	arbitrary	arbitrary	negative	zero	positive
$\dot{f}_{\sigma}^{(2)}$	arbitrary	arbitrary	negative	zero	zero
$\dot{\lambda}^{(3)}$	zero	zero	zero	zero	positive
Stiffness	$\mathbf{k}^{ed}$	$\mathbf{k}^e$	$\mathbf{k}^{ed}$	$\mathbf{k}^{ed}$ or $\mathbf{k}^{ep}$ or $\mathbf{k}^{epd}$	$\mathbf{k}^{ep}$ or $\mathbf{k}^{epd}$

(1) Yield function

(2) Time derivative of yield function

(3) Rate of plastic multiplier

Figure 7.1 Idealized moment-curvature relation of a cross-section

Figure 7.2 Yield curve of isotropic material for a fixed axial load

Figure 7.3 Plastic loading in softening range (1-3) and elastic unloading (1-2)

Figure 7.4 Plastic work of moment through plastic rotation

Figure 7.5 Hysteretic model for beam-column element

Figure 7.6 Degrees of freedom of a three dimensional beam-column element

Chapter 8

Computer Implementations

8.1 General

Non-linear time history response analysis is often required to determine the realistic performance of bridges subject to earthquake loadings. The accuracy of the results depends on the selection of (i) analysis model; and (ii) numerical solution scheme. Chapters 6 and 7 describe the theoretical background for the developments of: (i) time history response analysis procedure for bridge structures by superposition of Ritz vectors (pseudo-force method and tangent spectrum method); (ii) 3-dimensional non-linear beam-column model accounting for strength deterioration and stiffness degradation behaviour of reinforced concrete members; and (iii) seismic responses of bridges to multiple-support excitations. Capabilities of computer programs for performing non-linear time history response analysis of bridge structures are still limited and not complete with respect to the abovementioned important aspects (Powell, 1993). However, many advances have been made on development of advanced numerical techniques and computer programs for bridge response analysis and performance evaluation. The computer implementation of the analysis model and numerical

techniques discussed in previous chapters, which comprehensively overcome the existing limitations mentioned above, are presented in this chapter.

Among the group of general-purpose computer programs for structural analysis, NEABS is one of the few computer programs specially developed for non-linear time history response analysis of bridge structures. The NEABS program was initially developed in 1973 by Tseng and Penzien to compute seismic response of bridges to earthquake loadings. Since then the NEABS program has been further developed by many researchers (Kawashima, 1976; Imbsen, 1976 and 1986; Liu, 1993; Ruchala, 1997; and Zhang, 1999). The existing version of NEABS program has basic non-linear models to capture non-linear behaviour of bridge elements such as 3D beam-column model and expansion joint model. In this study, the NEABS program is further improved by the addition of new analysis techniques and numerical models. The improvements to the NEABS program include the following advances:

- (1) Modeling of strength deterioration and stiffness degradation behaviour,
- (2) Modeling of effects of damage on structural behaviour,
- (3) Non-linear analysis by vector superposition method,
- (4) Consideration of multiple-support excitations, which is important for analysis of long span bridges.

The aim of this chapter is to present the development and implementation of the new features and their incorporation into the NEABS program. An overview of the original NEABS program's organization is presented first. The Ritz algorithms for generation of

the Ritz vectors are presented (Ruchala, 1997). The pseudo-force and tangent spectrum methods are then implemented for non-linear time history response analysis of bridge structures. A new non-linear beam-column model is implemented taking into account the effects of strength deterioration and stiffness degradation of bridge columns. Dynamic response analyses of bridges to multiple-support excitations are also presented.

8.2 NEABS program organization

The original version of NEABS program developed at the University of California, Berkeley by Tseng and Penzien (1973), has 60 main subroutines. The organization chart of NEABS program is shown in Figure 8.1. The organization of the NEABS program is summarized as follows,

- (1) Formulation of the static equilibrium equations of the bridge structural system (SETUP)
- (2) Solution for static dead load conditions (STATIC)
- (3) Generation of the dynamic load vector for a time step (LOADS)
- (4) Direct step-by-step time integration (INTGR)
- (5) Preparation of output results (OUTPUT).

8.3 Developments of NEABS program

New modules has been added to the NEABS program as follows

- (1) RITZ module: Ritz algorithms of Lanczos and load-dependent Ritz vectors have been implemented (Ruchala, 1997),
- (2) NRASR subroutine: a module of non-linear response analysis by superposition of Ritz vectors has been implemented for non-linear time history response analysis using the Ritz reduction method,
- (3) PBEAM subroutine: plastic beam element stiffness computation, and
- (4) INFLC subroutine: multiple-support seismic excitations analysis.

The following are descriptions of each developed subroutine. Supplement of the NEABS user's manual is provided in Appendix J.

8.3.1 RITZ subroutine

The RITZ module generates Ritz vectors to compute mode shapes and frequencies of the analyzed bridge. A flow chart of the RITZ module is presented in Figure 8.2. Three options have been implemented to perform the following tasks:

- (1) Compute exact mode shapes and frequencies by Lanczos algorithm with a combined partial and selective reorthogonalization procedure.
- (2) Compute approximate mode shapes and frequencies by Lanczos algorithm with a combined partial and selective reorthogonalization procedure.
- (3) Compute approximate mode shapes and frequencies by improved Wilson's algorithm.

Using the generated Ritz vectors, the equations of motion are formulated in the generalized coordinate system. Solution of the generalized coordinate system is obtained by step-by-step integration method. Solution in the geometric coordinate system is then obtained by transformations using the Ritz vectors. Time history response analysis using Ritz reduction methods is carried out by the INTGR subroutine. The new features introduced in time history response analysis of bridge systems are summarized as follows,

- (1) Modal superposition method using mode shapes generated by different algorithms
 - (a) Exact mode shapes by Sub-space iteration method
 - (b) Exact mode shapes by Lanczos method
 - (c) Approximate mode shapes by Lanczos method
 - (d) Approximate mode shapes by improved Wilson's method.
- (2) Direct step-step integration of reduced system obtained by using Ritz vectors generated by two algorithms
 - (a) Ritz vectors by Lanczos method
 - (b) Ritz vectors by improved Wilson's method.
- (3) Direct step-by-step integration of reduced system in a tridiagonal matrix T. Tri-diagonal system obtained using Ritz vectors by Lanczos method.

The efficiency and reliability of the reduction procedures are evaluated to ensure that the Ritz vectors used in the analysis can adequately describe the behaviour of the structure. A cut-off criterion based on the effective mass participation for multi-directional earthquake loading has been implemented to assess the effectiveness of the reduction procedures and to assess the rate of convergence. RITZ module is called from main NEABS program and from INTGR subroutine in case of tangent spectrum method.

8.3.2 NRASR subroutine

The NRASR subroutine performs time history analysis by direct superposition of Ritz vectors. Two analytical procedures are implemented namely,

- (1) Pseudo-force method
- (2) Tangent spectrum method

A flow chart of NRASR subroutine is shown in Figure 8.3.

8.3.3 PBEAM subroutine

The PBEAM subroutine computes the plastic stiffness accounting for strength deterioration and elastic damage stiffness of non-linear beam-column elements and determines which stiffness matrix is needed for the next time step. The old version of the NEABS program can handle only elastic stiffness and elasto-plastic stiffness and the plastic process is detected by the YIELD subroutine. The numerical procedures shown in Table 7.1 are carried out to select stiffness needs for next time step. By using equation (7.3) combined with recoverable displacement (commencing of degradation), plastic stiffness for the next time step is computed. A criterion is established to determine the

starting of reloading phase. By using equation (7.3) and procedure to keep track of the change of sign and magnitude of stresses, the elastic stiffness for next time step is computed. Figure 8.4 presents a flow chart of stiffness evaluation. The PBEAM subroutine is called from the NELSTF subroutine for each non-linear element.

The flow chart of basic processes of non-linear behaviour is shown in Figure 8.5. In this flow chart, f represents the internal force or moment. The magnitude and sign of f are needed to select stiffness for next time step. The magnitude of f is computed from

$$f = \sqrt{f_y^2 + f_z^2} \quad (8.1)$$

where f_y and f_z are forces or moments in the y - and the z - directions, respectively. The sign of f is assumed to be positive if both f_y and f_z are positive, else the sign of f is assumed to be negative. This is very rough assumption. It can be improved by considering the degradation in the y - and z - directions separately. The evolution of hysteretic model shown in Figure 7.5 is described as follows

- (1) O-1: hysteretic response starts from origin O and assumed to move towards point 1. Since the force increment is positive, this is loading process and hence the stiffness is k^{cd} . Note that no damage has been identified (structure responds elastically) therefore k^{cd} is the same as k^e .
- (2) 1-O: at point 1 response reverses in the opposite direction and moves towards origin O, hence the last unloading force (f_{ul}) is negative. During this unloading process the stiffness is k^e .
- (3) O-2: structure is reloaded. No damage has occurred, and k^{cd} is the same as k^e .

- (4) 2-3: nonlinearity occurs and the stiffness k^{cp} is used.
- (5) 3-4: strength degradation occurs and the stiffness k^{epd} is used.
- (6) 4-5: unloading occurs and the stiffness k^e is used.
- (7) 5-6: reloading occurs and stiffness degradation is allowed, the stiffness k^{cd} is used. Unloading and reloading can occur at any point during this reloading process.
- (8) 6-7: nonlinearity occurs and the stiffness k^{cp} is used.
- (9) 7-8: unloading occurs and the stiffness k^e is used.
- (10) 8-9: reloading occurs and the stiffness k^{cd} is used.

An alternative approach is to define loading or unloading stage based on only the load increment. If the load increment is positive, the next step is considered as loading or reloading and the stiffness k^{cd} is used. If the load increment is negative, the next step is considered as unloading and the stiffness k^e is used. The maximum loading force of each cycle is stored. The implementation of this approach is shown in Figure 8.6. Details of the computer implementation are as follows:

- (1) Elastic and release matrices of the half element are computed in NEWBM subroutine.
- (2) Damage index is computed in DINDEX subroutine.

(3) Damage stiffness is assembled in the `DSTIF` subroutine, which is called from `PBEAM` subroutine. This allows new stiffness (both damage and elasto-plastic) be transferred to `NELSTF` subroutine,

(4) Rotations are read from `team` subroutine. In this subroutine rotations are assigned to `ENPROP (N, I)` and then `ENPAR (I)` where `I` is from 12 to 18.

8.3.4 INFLC subroutine

The `INFLC` subroutine computes influence coefficient matrix **B** for multiple-support excitations. A column vector $\mathbf{b}_i$ of matrix **B** is the displacement vector of the analyzed bridge due to unit displacement at the *i*th support. Procedures for time history response analysis of bridge structure subject to time delay ground motions are implemented. A flow chart of `INFLC` subroutine is shown in Figure 8.7.

Table 8.1 Non-linear time history response analysis procedure by direct superposition of Ritz vectors

- (1) Generate Ritz vectors (Ruchala, 1997)
- (2) Compute projections of **K** and **M** matrices into **X** subspace

$$\mathbf{K}^* = \mathbf{X}^T \mathbf{K} \mathbf{X}$$

$$\mathbf{M}^* = \mathbf{X}^T \mathbf{M} \mathbf{X}$$

- (3) Step-by-step integration

Perform step-by-step integration of the reduced system and then superimpose the results to obtain response in geometric coordinates

- (4) Check for non-linearity

If nonlinearity occurs then continue with step 5. If not, go back to step 2 for new time step.

- (5) Instantaneous stiffness

Compute new elasto-plastic tangent stiffness using equation (6.73) or plastic degraded stiffness using equation (7.25).

- (6) Equilibrium equation

Compute residual force and iteration of equilibrium to correct the displacement, velocity and acceleration.

Figure 8.1 Main subroutine organization chart of NEABS program (Imbsen et al., 1991).

Figure 8.2 Flow chart of RITZ module

Figure 8.3 Flow chart of NRASR subroutine

Figure 8.4 Flow chart of PBEAM subroutine

Figure 8.5 Flow chart represents basic processes of non-linear behaviour (option 1)

Figure 8.6 Flow chart presents basic processes of non-linear behaviour (option 2)

Figure 8.7 Flow chart of INFLC subroutine

Chapter 9

Numerical Examples

9.1 General

In the previous chapter, computer implementations of new analysis and numerical modeling modules have been discussed. Computer implementations include: (1) two methods using Ritz vectors for non-linear time history response analysis of bridges; (2) a non-linear model accounts for strength deterioration and stiffness degradation behaviour of reinforced concrete beam-column elements; and (3) time history response analysis of bridges subject to multi-support seismic excitations. The capabilities of the developed analysis procedures and analytical model to provide accurate responses of bridges are demonstrated by numerical examples. Non-linear time history response analysis procedures generally include the following steps, namely establishing structural model of the analyzed bridge, computing linear and non-linear structural properties, selecting ground motion time series, performing analysis, and interpreting the results. The objectives of this chapter are: (1) to demonstrate the efficiency of the Ritz reduction methods for analysis of bridges; (2) to investigate the applicability of Ritz reduction methods for non-linear time history response analysis of bridges; (3) to study the effects of strength deterioration and stiffness degradation of non-linear beam-column elements

on the response of bridges; (4) to study the effects of multi-support seismic excitations on the response of long-span bridges; and (5) to evaluate the influence of site, hanging wall and footwall, and directivity to bridge response. A multi-span bridge is selected for a series of dynamic response analyses.

9.2 Time history response analysis of a multiple-span bridge

The bridge under consideration is represented in Figure 9.1. Non-linear responses of this bridge to multi-support excitations have been studied by Monti et al. (1996) except that the bridge deck is longitudinally hinged. It is a six-span continuous deck with five piers of the height of 10m and 10.5 m and 2.5m in diameter. The span length is 50m. The deck, longitudinally hinged to the piers and the abutments, has a dead load of 200kN/m. The piers are considered as fixed to the base supports. Table 9.1 lists the different types of time history response analysis conducted on the bridge. The results from the various Ritz reduction methods are compared to the exact solution obtained by the direct integration method. Letter L designates linear dynamic response analysis and letter N designates non-linear dynamic response analysis. Letters A, B, and C denote elastic, elastoplastic, and strength deterioration and stiffness degradation model, respectively. For the Ritz reduction method the number of Ritz vectors included in the analysis capture at least 90% of the participating mass (CHBDC, 2000). The accuracy and improvements of the developed non-linear model are examined. The effects of considering multiple-support excitations on time history response of the example bridge are studied.

9.2.1 Structural model

The lumped mass model used to idealize the bridge is shown in Figure 9.2a. The numbering for the nodes on the structure, and the reference nodes used to define the directions of the local axes of the elements are also shown in Figure 9.2a. The beam-column element is used to model the bridge's deck and columns. Elements are numbered as shown in Figure 9.2b. There are 27 nodal points and 26 beam-column elements. The cross-section properties of bridge piers and deck are shown in Table 9.2. The longitudinal reinforcement ratio is taken as 1%, which consists of 50 reinforcement bars of 35mm in diameter distributed evenly around the column perimeter (Figure 9.3). The concrete material properties of the column members are taken as Young's modulus $E_c=25,000,000$ kPa; Poisson's ratio $\nu=0.2$; strength $f_c=35,000$ kPa; ultimate strains $\epsilon_{cu}=0.006$; and the mass density $\rho=3,000$ kg/m³. For steel, Young modulus $E_s=200,000,000$ kPa, strength $f_y=300,000$ kPa; ultimate strains $\epsilon_{su}=0.1$. The cracked stiffness is obtained from moment-curvature analysis of the cross section of the bridge piers. A beam-column element located at the lower end of the columns is modeled as a non-linear beam-column element to allow yielding of the columns. The length of non-linear beam-column element is computed by using equation (6.45) as follows

$$L_p = 0.08L + 0.022f_y d_l = (0.08)(10) + (0.022)(440)(0.035) = 1.1\text{m}$$

The column is modeled by 4 beam-column elements, i.e., two 1.1m long elements at the two ends and two 3.9m long members at the center. Due to the bridge configurations, the plastic hinge is expected to form at the lower end of the column. The calculated ultimate strengths of the bridge column are computed by program BIAx (Wallace, 1993). For the

analyzed bridge the calculated ultimate strengths are taken as, P_0 172,000 kN, M_{y0} 16,500 kN-m, M_{z0} 16,500 kN-m, and P_1 22,000 kN. The moment-curvature relation of the pier member cross section is shown in Figure 9.4. Moment-curvature relation of the cross section of the bridge piers indicates that the yield curvature is 0.0015, the recoverable curvature is 0.005, and the ultimate curvature is 0.05, respectively. The yield surface shown in Figure 9.5 is plotted using the obtained coefficients defining the interaction of axial force and bending moment about the y-axis, a_0 is 1.0, a_1 is 6.58, a_2 is -8.24, and a_3 is 0.66, and by the coefficients defining the interaction of axial force and bending about the z-axis, b_0 is 1.0, b_1 is 6.58, b_2 is -8.24, and b_3 is 0.66.

9.2.2 Free vibration analysis

Free vibration analysis is performed by Ritz algorithms presented in Chapter 4. In order to obtain 90 percent of the mass participation, 10 derived Ritz vectors need to be included to obtain time history responses of the bridge. Complete response is obtained by 15 derived Ritz vectors. Tables 9.3 and 9.4 show 15 vibration periods of the bridge and effective mass participation factors of the mode shapes which contribute the most to the bridge response, respectively. Proportional damping coefficients can be obtained as follows

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2\xi}{\omega_m + \omega_n} \begin{Bmatrix} \omega_m \omega_n \\ 1 \end{Bmatrix} \quad (9.1)$$

where α , β are the mass proportional damping and stiffness proportional damping, respectively, ξ is damping ratio, and ω_m and ω_n are angular frequencies at modes m and n , respectively.

It is recommended that ω_m generally is taken as the fundamental frequency of the analyzed system and that ω_n is set among higher frequencies of the modes that contribute significantly to the dynamic response (Clough and Penzien, 1993). It is observed from Table 9.4 that the first mode and the fifth mode contribute the most to the bridge response and therefore the two frequencies corresponding to these two modes are used to compute proportional damping coefficients. For non-linear analysis damping coefficient ξ of 0.02 is used (Japan Design Specifications of Highway Bridges). From equation (9.1), Rayleigh damping coefficients are obtained as $\alpha=0.201$, and $\beta=0.0015$.

9.2.3 Earthquake ground motion time series

Dynamic response analyses of the example bridge have been conducted using simulated UHS compatible ground motions and records from the Chi-Chi earthquake. In the analyses using the UHS compatible ground motion records, the bridge is assumed to locate in Ottawa. In this case, earthquake excitations are applied in the longitudinal of the bridge. For the case using the Chi-Chi ground motion records, the bridge is assumed to locate near the Chelungpu fault ruptured during the Chi-Chi earthquake in Taiwan. In this case, three-component ground motion records are used. The objective of the first series of analysis is to illustrate the capabilities and validity of the non-linear analysis methods and structural models developed in this research presented in Chapters 6 and 7. The objective of the second series of analysis is to illustrate and understand the effects of site, hanging wall and footwall, and directivity on bridge responses and performance.

In the case of the analyzed bridge located in Ottawa, spectrum compatible ground motion time series (Atkinson and Beresnev, 1998) are used as input excitations in time

history response analysis of the bridge. The uniform hazard spectra for Ottawa for 2% probability of exceedence in 50 years are shown in Figure 9.6a for hard rock and firm soil site, respectively. It is noted that the current highway bridge codes (CHBDC, 2000) still requires 10% probability of exceedence in 50 years. However, the use of 2% probability of exceedence in 50 years has been suggested (Atkinson, 2004). The 2% probability of exceedence in 50 years has been used to study the inelastic seismic demand of buildings in Canada (Tremblay and Atkinson, 2001). It is expected that the next generation of bridge codes would require 2% probability of exceedence in 50 years. The compatible time history ground motion records are not ready to use because the spectra are generally different from the target uniform hazard spectra. The spectra of the compatible acceleration time histories need to be scaled up or down to match the target uniform hazard spectrum. It is suggested that scale factors are in the range from 0.5 to 2.0 (Atkinson, 1998). Let f be a vector of scale factors, $sa(t)$ be the pseudo-acceleration response, and $uhs(t)$ be coordinates of target uniform hazard spectrum. The error of scaled time history ground motion records compared to target spectrum is defined as total variance of two time series,

$$var_{i,j} = error_{i,j} = \sum_{t=a}^b [f_i sa_j(t) - uhs(t)]^2 \quad (9.2)$$

where “a” and “b” designates the period range where error is computed.

In this study, the target spectrum is matched in the 0-0.5s band by a moment magnitude M6 earthquake at short distance and matched in the 0.5-5s band by a moment magnitude M7 earthquake at further away distance. For M6 earthquake, $a=1$ and $b=50$ are selected,

and for M7.0 earthquake, $a=51$ and $b=500$ are selected. Two ground motion time series are selected corresponding to short- and long-period earthquake events. For a short-period event, spectrum compatible time series is characterized by earthquakes of M6 at distance of 50 km with a scale factor of 1.4, while for a long-period event, spectrum compatible time series is characterized by earthquakes of M7 at distance of 50 with a scale factor of 0.6. Since the fundamental period of the bridge is 0.926sec, the responses of the bridge will be dominant by the long-period events. The UHS ground motion recording for long-period earthquake events is used as input for analysis. Plot of ground motion time series for long-period events is shown in Figure 9.6b.

Response of bridges to multiple support excitations has been a subject of many studies (Bogdanoff, 1965; Hao, 1998; Zanardo, 2004). The multi-support excitation or non-synchronous input (due to non-correlated motions at supports) can cause responses different from those considering the uniform ground motion at each support (fully correlated ground motion at supports). In particular, multiple support excitations can induce differential movements of adjacent decks and may result in pounding between bridge decks if the design gap between decks is not enough to avoid such a collision and/or may cause the falling of bridge deck if the seating length is insufficient.

Closely spaced spatially varying ground motion records appropriate for use as input in multiple-support excitation analysis of bridge are rare because the distance between the supports of the structure under investigation are usually not equal to those between the seismograph stations of the earthquake station array. Therefore, simulation of the multiple support excitations, as a non-stationary random process, is needed. The spatial variation of seismic ground motions may be schematically thought of as the result

of three different phenomena: (1) the incoherence effect, resulting from reflections and refractions of waves through the soil during their propagation (this effect is referred to as geometric incoherence); (2) the so-called wave-passage effects, which leads to differences in the arrival times of seismic waves at different stations; and (3) the local effect, due to differences in local soil conditions under various stations.

For the purpose of demonstrating the effects on responses of long-span bridges due to multi-support excitations, the spectrum compatible ground motion time histories are modified to include the wave-passage effect and the soil condition effect. The waveform ground motion record is shifted along the time axis t -sec where t is the time delay for seismic wave to travel between two adjacent bridge supports. The time delay is defined as the time required that a shear wave travels from one pier to the adjacent one. The shear modulus G' is computed as follows

$$G' = \frac{E}{2(1 + \mu)} \quad (9.3)$$

where E is elastic modulus, μ is Poisson's ratio, the shear wave velocity V_s is computed as follows

$$V_s = \sqrt{\frac{G'}{\rho}} \quad (9.4)$$

where ρ is mass density of the rock foundation. The mass density of the rock foundation is computed as follows

$$\rho = \frac{\gamma_s}{g} \quad (9.5)$$

where γ_s is the unit weight, and g is gravity acceleration. Substituting equations (9.5) and (9.3) into equation (9.4), the shear wave velocity is obtained. The time delay from one pier to the next is now computed as follows

$$t = \frac{L}{V_s} \quad (9.6)$$

where t is time delay in ground motions, L is span length of the bridge, and V_s is shear wave velocity. For $V_s=500\text{m/sec}$, and $L=50\text{m}$ time delay is 0.1sec .

The site effect is characterized by defining different response spectra at different locations corresponding to their local soil conditions. The target UHS for soil site is defined as the product of the target UHS for rock site times soil amplification factors (Table 2, GSC, Open File 4459).

In the second case, a subset of five near-fault ground motion recordings is selected to evaluate the seismic performance of the bridge and to examine the site effect, directivity effect, and hanging wall and footwall effect on the bridge response. Details of the ground motion recordings are shown in Table 9.5. Figures 9.7 to 9.11 show 3-component of 5 near-fault ground motion recordings. The ground motion recordings have the distance to the fault less than 15 km. Three ground motion recordings (TCU078, CHY028, and NSY) are in the footwall side, where no directivity effect was observed. It is expected that site effect is dominant for stations in the footwall side i.e., soil site ground motion recordings cause higher seismic demands than those on rock site. Two

ground motion recordings (TCU074 and TCU110) are in the hanging wall side; one is in the forward directivity region and the other is in the backward directivity region. It is expected that ground motion recordings in the forward directivity region cause higher seismic demands than those in the backward directivity region. Detailed bridge response results are presented in Section 9.2.8.

9.2.4 Linear time history response analysis

Linear time history response analyses of the bridge are carried out by direct integration method and Ritz methods. The bridge is analyzed using the spectrum compatible ground motion time series (Atkinson and Beresnev, 1998). Table 9.4 shows that to obtain 90 percent of mass participation factor, 10 Ritz vectors are included in the analysis. The maximum force and moment at the base of columns and the maximum displacement of the deck computed by direct integration method and Ritz methods are presented in Table 9.6. Computational time to perform linear time history response analysis using Ritz vectors is generally less than computational time required by direct integration method. Time history responses of deck acceleration, deck displacement, moment and shear force at base of columns are shown in Figures 9.12 and 9.13.

9.2.5 Non-linear time history response analysis

Non-linear time history response analyses of the bridge are performed by direct integration and Ritz reduction methods, namely pseudo-force method and tangent spectrum method. Numerical results obtained from two methods are compared to the exact solution. Table 9.7 shows the results obtained from non-linear dynamic response

analyses. Numerical results obtained from non-linear dynamic response analysis indicate that the computed deck displacement is significantly higher than that determined from linear analysis (Figure 9.14b). The moments at the pier base obtained by non-linear dynamic response analysis are less than those obtained by the linear dynamic response analysis (Figure 9.15b).

The numerical results obtained from pseudo-force method are poorly correlated to the exact solution (Figures 9.16 and 9.17). Non-linearity can only be partly presented by pseudo-force. Further analysis indicates that this method can be used for non-linear time history analysis in case of localized non-linearity at limited locations. Solution by pseudo-force method is stable because only base vectors corresponding to the initial stage of the bridge is utilized.

From analysis by tangent spectrum method, it is observed that the vibration periods change as the structure suffers inelastic damage during the time history in the non-linear dynamic analysis. The predominant period corresponding to the elastic state of the bridge is 0.92sec and the largest predominant period corresponding to the inelastic state of the bridge is 2.12sec. Time history responses of deck acceleration, deck displacement, shear force and moment at base of column are shown in Figures 9.18 and 9.19. The solution of the tangent spectrum method (using 30 vectors) compares well with the exact solution of the direct integration method. It is found that solution by tangent spectrum method is less stable due to the requirement of repeated computation of new base vectors. Figures 9.20 and 9.21 compare the bridge response by tangent spectrum method using different number of derived Ritz vectors. It is observed that the number of

Ritz vectors needed in non-linear time history analysis is larger than that needed for linear time history analysis.

9.2.6 Applications of developed non-linear beam-column model

The effects of strength deterioration and stiffness degradation behaviour on the bridge response are demonstrated. Non-linear time history response analysis of the bridge with and without considering strength deterioration and stiffness degradation are performed. Figures 9.22 to 9.23 show the non-linear time history response of the bridge from the non-linear analysis considering strength deterioration and stiffness degradation behaviour of non-linear elements. Table 9.7 shows the maximum response of the bridge using the developed model. The evolution of damage index of non-linear elements is shown in Figure 9.24. Figure 9.25 shows the contribution of deformation and dissipated energy to the damage index. It is shown that for 5 columns, the contribution of deformation to the damage index is dominant. The contribution of dissipated energy to damage index is about 10%.

9.2.7 Response of bridge to multi-support seismic excitations

Responses of the bridge to multiple-support excitation are evaluated. Multi-support seismic excitations include the wave-passage and site effects. The responses of bridge to multi-support excitations are computed and compared to the responses of bridge to uniform support excitation. The maximum response results are shown in Tables 9.6 and 9.7 for linear and non-linear analysis, respectively. Figures 9.26 and 9.27 compare time history responses of the bridge to multi-support excitations for the case of the identical soil condition on hard rock site (case 1) and uniform support excitation. In this case,

responses of the bridge are slightly less than those by uniform support excitation except for bridge deck acceleration. Figures 9.28 and 9.29 compare the responses of the bridge to multi-span support excitations for the case of the hard rock condition at the two end piers and the firm soil condition at the other piers (case 2) and uniform support excitation. In this case, responses of the bridge are significantly larger than those by uniform support excitation. Non-linear responses of the bridge to multi-support excitations for the case of different site condition at supports are significantly larger than those by uniform support excitation (Figures 9.30 and 9.31).

9.2.8 Response of bridge to Chi-Chi earthquake ground motions

Responses of the bridge to Chi-Chi earthquake ground motion recordings are computed. The selected Chi-Chi earthquake ground motion recordings are listed in Table 9.5. A summary of the bridge responses at pier 1 of the example bridge is shown in Table 9.8. For stations on the footwall side (TCU078 and CHY028) both on soil site, there is no clear indication of directivity effect that can be observed from comparison of bridge responses. However, the site effect is clearly shown as the responses of the bridge due to ground motion recording on the soil site TCU078 is larger than those due to ground motion recording on the rock site NSY. On the other hand, directivity effect is clearly observed on the hanging wall side. Figures 9.32 to 9.33 indicate that ground motion recording in the forward directivity region (TCU078) induces larger bridge responses than that in the backward directivity region (TCU110). This agrees with the observations of directivity effects presented in Chapter 3.

Table 9.1 Type of time history response analysis

Type of analysis	Analysis method	Analysis model
Linear analysis	Ritz vector L1	Elastic A
	Direct integration (exact) L2	Elastic A
	Multi-support excitations L3 (1) for identical site condition (2) for different site condition	Elastic A
Non-linear analysis	Pseudo-force N1	Elastoplastic B
	Tangent spectrum N2 (1) 10 Ritz vectors (2) 20 Ritz vectors (3) 30 Ritz vectors	Elastoplastic B
	Direct integration (exact) N3	Elastoplastic B
		Strength deterioration and stiffness degradation C
	Multi-support excitations N4 (1) for identical site condition (2) for different site condition	Elastoplastic B

Table 9.2 Properties of cross sections

Element	Area (m ²)	Shear area in y-axis (m ²)	Shear area in z-axis (m ²)	Moment of inertia about x- axis (m ⁴)	Moment of inertia about y- axis (m ⁴)	Moment of inertia about z- axis (m ⁴)
Deck	100	0	0	100	100	100
Column	4.91	0	0	0.767	0.767	1.534

Table 9.3 Vibration properties of the bridge

Mode No.	Frequency (rad/s)	Period (s)	Modal mass participation factor		
			X-dir	Y-dir	Z-dir
1	6.79E+00	9.26E-01	2.47E+03	-1.32E+01	1.89E-13
2	1.05E+01	5.96E-01	-2.12E-13	-1.13E-14	1.91E+03
3	1.23E+01	5.09E-01	-7.33E-13	1.60E-14	-1.05E+03
4	1.60E+01	3.94E-01	-4.84E-13	1.73E-14	4.53E+02
5	1.94E+01	3.24E-01	-3.17E+01	-1.03E+03	1.66E-14
6	2.03E+01	3.09E-01	-2.67E-11	5.80E-13	2.87E+02
7	2.45E+01	2.56E-01	-6.76E-13	-6.02E-14	-2.28E+02
8	1.10E+02	5.72E-02	-4.31E-04	1.93E+03	-1.08E-09
9	1.10E+02	5.69E-02	1.60E-03	-1.92E+02	-4.00E-10
10	1.12E+02	5.60E-02	2.28E-03	-5.38E+02	8.47E-11
11	1.15E+02	5.48E-02	2.40E+00	-2.52E-01	-1.70E-14
12	1.16E+02	5.43E-02	-1.20E-05	5.86E+01	-9.16E-10
13	1.57E+02	4.01E-02	4.58E+01	3.00E+00	1.54E-15
14	2.22E+02	2.84E-02	1.18E-01	-1.01E-01	-1.08E-11
15	3.84E+02	1.64E-02	3.30E-01	2.79E-02	-1.58E-08

Table 9.4 Modal mass participation factor

X-dir			Y-dir			Z-dir		
Mode number	Modal %	Total %	Mode number	Modal %	Total %	Mode number	Modal %	Total %
1	99.95	99.95	8	72.75	72.75	2	71.76	71.76
13	0.03	99.98	5	20.78	93.53	3	21.57	93.33
5	0.02	100	10	5.67	99.21	4	4.03	97.36
11	0	100	9	0.72	99.93	6	1.62	98.97
15	0	100	12	0.07	100	7	1.02	100

Table 9.5 Chi-Chi earthquake ground motion recordings for bridge analysis

No.	Station code	Distance (km)	Coordinate (degree)		PGA (cm/sec ²)			Region	Site
			Lat.	Lon.	V $\delta(V)$	NS $\delta(NS)$	EW $\delta(EW)$		
1	TCU078	8.27	23.8123	120.8455	171 +0.21	302 +0.45	440 +0.94	FW-FD	S
2	CHY028	8.67	23.6327	120.6052	336 +1.43	750 +2.66	624 +1.79	FW-BD	S
3	NSY	9.08	24.4162	120.7607	85 -0.39	116 -0.48	119 -0.48	FW-FD	R
4	TCU074	13.75	23.9622	120.9618	270 +1.52	368 +1.15	586 +2.25	HW-FD	S
5	TCU110	14.16	23.9617	120.5698	116 +0.10	188 +0.12	178 +0.00	HW-BD	S

Notes

V is the vertical component of ground motion, whose ground motion relation can be expressed as $\ln V = 5.97829 - 0.41717 \ln(d) - 0.01192d - 0.04436S$

NS is the north south component of ground motion, whose ground motion relation can be expressed as $\ln NS = 6.10804 - 0.27214 \ln(d) - 0.01134d - 0.09648S$

EW is the east west component of ground motion, whose ground motion relation can be expressed as $\ln EW = 6.30325 - 0.34768 \ln(d) - 0.01005d - 0.05643S$

$\delta(V)$, $\delta(NS)$, and $\delta(EW)$ are number of standard deviation for V, NS, and EW, respectively

FW is footwall

HW is hanging wall

FD is forward directivity region

BD is backward directivity region

S is soil site

R is rock site

Table 9.6 Linear time history response analysis of bridge subject to UHS ground motion recording

Maximum response	Column number	Linear dynamic response analysis			
		L1A	L2A	L3A1	L3A2
Acceleration at the top of the column (m/sec ²)	1	1.822	1.822	5.973	7.384
	2	1.822	1.822	5.962	7.372
	3	1.822	1.822	5.536	6.947
	4	1.822	1.822	5.259	6.671
	5	1.823	1.823	5.243	6.657
Displacement at the top of the column (cm)	1	3.950	3.950	2.620	5.032
	2	3.949	3.949	2.621	5.032
	3	3.948	3.948	2.622	5.032
	4	3.949	3.949	2.623	5.033
	5	3.950	3.950	2.624	5.035
Shear force at the base of the column (kN)	1	2042	2042	1354	2598
	2	2272	2272	1507	2894
	3	2272	2272	1508	2864
	4	2272	2272	1508	2895
	5	2272	2272	1509	2896
Moment at the base of the column (kN-m)	1	20470	20470	13540	26000
	2	22720	22720	15070	28940
	3	22720	22720	15080	28940
	4	22720	22720	15080	28950
	5	22720	22720	15090	28960

Table 9.7 Non-linear time history response analysis of bridge subject to UHS ground motion recording

Maximum response	Column number	Non-linear dynamic response analysis					
		N1B	N2B2	N2B3	N3B	N3C	N4B2
Acceleration at the top of the column (m/sec ²)	1	1.825	1.561	1.621	1.625	1.588	6.735
	2	1.824	1.556	1.632	1.638	1.588	6.735
	3	1.823	1.554	1.634	1.631	1.585	6.299
	4	1.823	1.559	1.624	1.621	1.586	6.023
	5	1.823	1.567	1.644	1.630	1.590	6.007
Displacement at the top of the column (cm)	1	3.957	4.966	5.693	5.576	5.689	8.405
	2	3.955	4.966	5.693	5.576	5.689	8.404
	3	3.955	4.966	5.693	5.576	5.689	8.405
	4	3.955	4.966	5.694	5.576	5.689	8.407
	5	3.956	4.967	5.694	5.576	5.690	8.410
Shear force at the base of the column (kN)	1	3572	2551	2268	2173	2715	2891
	2	3816	2629	4436	4693	3315	3458
	3	3636	2845	3592	3320	3047	3099
	4	3592	2696	3359	3586	3050	3998
	5	3604	2064	2826	3256	3302	4150
Moment at the base of the column (kN-m)	1	19830	17220	16970	16960	16890	17310
	2	19820	17520	17870	17780	17370	17900
	3	19600	17420	17440	17420	17110	17730
	4	19600	17520	17720	17640	17180	17570
	5	19650	17390	17710	17480	17230	18250
Plastic rotation at the base of column (rad)	1	0.0005	0.001	0.0019	0.0018	0.0018	0.0048
	2	0.0003	0.002	0.0034	0.0032	0.0027	0.0052
	3	0.0004	0.0025	0.0034	0.0030	0.0036	0.0040
	4	0.0003	0.0022	0.0032	0.0031	0.0036	0.0064
	5	0.0003	0.0019	0.0030	0.0030	0.0037	0.0069

Table 9.8 Non-linear time history response analysis of bridge subject to Chi-Chi earthquake ground motion recordings

Maximum response	Axis	Chi-Chi ground motion recordings				
		TCU078	CHY028	NSY	TCU074	TCU110
Acceleration at the top of the column (m/sec ²)	X-	5.886	13.39	1.686	21.72	4.306
	Y-	3.091	7.067	1.272	5.889	2.497
	Z-	5.920	1.444	2.199	10.86	2.811
Displacement at the top of the column (cm)	X-	12.96	29.57	6.486	57.02	18.55
	Y-	4.59	2.994	0.386	5.874	4.236
	Z-	3.79	9.168	2.040	7.658	2.344
Force at the base of the column (kN)	x-	4798	2306	1649	6288	3601
	y-	6406	2626	2634	6498	4458
	z-	6032	1999	1860	6971	5287
Moment at the base of the column (kN-m)	x-	13570	19420	11340	15410	10720
	y-	19010	22200	16880	20150	18320
	z-	18230	26050	15640	17820	16740
Plastic rotation at the base of the column (rad)	x-	-	-	-	-	-
	y-	0.01177	0.00648	0.00264	0.01465	0.01339
	z-	0.00299	0.00241	0.00154	0.00186	0.00157

Notes

(X, Y, Z) designates global axis directions

(x, y, z) designates local member axis directions

a) Plan

b) Elevation

Figure 9.1 Multiple span bridge structure

a) Joint numbering

b) Element numbering

Figure 9.2 Structural idealization

Figure 9.3 Cross section of bridge column

Figure 9.4 Moment-curvature of cross section of bridge column

Figure 9.5 Moment-axial load interaction curve of column cross-section

a) Uniform hazard spectra for Ottawa

b) Long-period event M7 at a distance of 50 km

Figure 9.6 UHS compatible ground motion time series

a) Vertical component

b) NS component

c) EW component

Figure 9.7 Chi-Chi ground motion recordings at station NSY

a) Vertical component

b) NS component

c) EW component

Figure 9.8 Chi-Chi ground motion recordings at station TCU078

a) Vertical component

b) NS component

c) EW component

Figure 9.9 Chi-Chi ground motion recordings at station TCU110

a) Vertical component

b) NS component

c) EW component

Figure 9.10 Chi-Chi ground motion recordings at station CHY028

a) Vertical component

b) NS component

c) EW component

Figure 9.11 Chi-Chi ground motion recordings at station TCU074

a) Acceleration of deck

b) Displacement of deck

Figure 9.12 Comparison of bridge deck response between direct integration method (L2A) and Ritz method (L1A) in linear dynamic response analysis

a) Shear force at base

b) Moment at base

Figure 9.13 Comparison of force at base response between direct integration method (L2A) and Ritz method (L1A) in linear dynamic response analysis

a) Acceleration of deck

b) Displacement of deck

Figure 9.14 Comparison of bridge deck response between linear analysis (L2A) and non-linear analysis (N3B)

a) Shear force at base

b) Moment at base

Figure 9.15 Comparison of force at base response between linear analysis (L2A) and non-linear analysis (N2B)

a) Acceleration of deck

b) Displacement of deck

Figure 9.16 Comparison of bridge deck response between pseudo-force method (N1B) and direct integration method (N3B)

a) Shear force and plastic rotation relation

b) Moment at base

Figure 9.17 Comparison of force at base response between pseudo-force method (N1B) and direct integration method (N3B)

a) Acceleration of deck

b) Displacement of deck

Figure 9.18 Comparison of bridge deck response between tangent spectrum method (N2B3) and direct integration method (N3B)

a) Shear force and plastic rotation relation

b) Moment at base

Figure 9.19 Comparison of force at base response between tangent spectrum method (N2B3) and direct integration method (N3B)

a) Acceleration of deck

b) Displacement of deck

Figure 9.20 Comparison of bridge deck response by tangent spectrum method using different number of derived Ritz vectors

a) Shear force and plastic rotation relation

b) Moment at base

Figure 9.21 Comparison of force at base response by tangent spectrum method using different number of derived Ritz vectors

a) Acceleration of deck

b) Displacement of deck

Figure 9.22 Comparison of bridge deck response between elastoplastic model (N3B) and developed model (N3C)

a) Shear force and plastic rotation relation

b) Moment at base

Figure 9.23 Comparison of force at base response between elastoplastic model (N3B) and developed model (N3C)

Figure 9.24 Damage index at non-linear elements

Figure 9.25 Contribution of maximum rotation (shaded area) and dissipated energy (empty area) to the damage of beam-column elements

a) Acceleration of deck

b) Displacement of deck

Figure 9.26 Comparison of bridge deck response between uniform support excitation (L2A) and multi-support excitations (L3A1) in linear dynamic response analysis

a) Shear force at base

b) Moment at base

Figure 9.27 Comparison of force at base response between uniform support excitation (L2A) and multi-support excitations (L3A1) in linear dynamic response analysis

a) Acceleration of deck

b) Displacement of deck

Figure 9.28 Comparison of bridge deck response between uniform support excitation (L2A) and multi-support excitations (L3A2) in linear dynamic response analysis

a) Shear force at base

b) Moment at base

Figure 9.29 Comparison of force at base response between uniform support excitation (L2A) and multi-support excitations (L3A2) in linear dynamic response analysis

a) Acceleration of deck

b) Displacement of deck

Figure 9.30 Comparison of bridge deck response between uniform support excitation (N3B) and multi-support excitations (N4B2) in non-linear dynamic response analysis

a) Shear force at base

b) Moment at base

Figure 9.31 Comparison of force at base response between uniform support excitation (N3B) and multi-support excitations (N4B2) in non-linear dynamic response analysis

a) Acceleration of deck

b) Displacement of deck

Figure 9.32 Effects of directivity on bridge deck response

a) Shear force at base

b) Moment at base

Figure 9.33 Effects of directivity on force at base response

Chapter 10

Summary, Conclusions, and Recommendations

10.1 Summary

The present thesis is divided into ten chapters each largely self-contained. Chapter one presented a general introduction about the current state of seismic resistant analysis and design of bridge structures. The current issues of engineering seismology and earthquake engineering were summarized as follows. Firstly, there is a need to estimate the site classification at seismograph stations where borehole information is not available. An example of this was the lack of site classification of strong ground motion stations of SMART-1 array in Taiwan. This led to the need for the development of an accurate and efficient method to estimate site classification for SMART-1 stations. Secondly, the directivity effects have been observed during recent major earthquakes. However, the directivity effects have not been studied using a large set of ground motion data from a single earthquake event due to the lack of recorded near-fault ground motions. The Chi-Chi, Taiwan earthquake provided a large number of near-fault ground motion data, which can be used to study the directivity effects in more detail. Thirdly, solution techniques for non-linear time history response analysis of bridge structures are often limited to the direct integration method. The reduction technique has been mostly used for linear elastic

systems. The applications of the reduction techniques to non-linear inelastic systems need to be investigated. Fourthly, 3-dimensional non-linear modeling of reinforced concrete bridge piers/columns is typically limited to the elastoplastic model. Recognizing the limitations of the elastoplastic model used for reinforced concrete members, it is necessary to develop an accurate and effective model to characterize essential non-linear behaviour of bridge piers/columns. Fifthly, multiple-support excitations can cause long-span bridges to respond in a different fashion than the response obtained from uniform support excitation. However, computer programs to perform such an analysis are limited. The remaining chapters of this study aimed to discuss the aforementioned issues in more detail and to present solution for these issues. Chapter two presented a methodology to estimate site classification of strong-motion stations at SMART-1 array in Taiwan using free-field ground motion data of the Chi-Chi, Taiwan earthquake. Chapter three evaluates and quantifies the directivity effects observed during the Chi-Chi, Taiwan earthquake. A directivity model is developed for the Chi-Chi, Taiwan earthquake. Chapter four presents a general solution for eigenproblem using standard methods and using Ritz methods. This chapter describes the fundamentals of Ritz algorithms which were applied to solve the time history response of linear and non-linear systems. Chapter five presents the solution techniques for time history response of linear systems by superposition of derived Ritz vectors. Chapter six extends the solution techniques presented in Chapter five to non-linear systems. Two methods, namely pseudo-force method and tangent spectrum method are employed to compute time history response of non-linear bridge systems. A method to compute time history response of long-span bridges subject to multiple-support seismic excitations is presented. Chapter seven presents the development of a non-linear

model for bridge columns/piers, which accounts for strength deterioration and stiffness degradation of bridge columns/piers. Both the maximum displacement and the dissipated energy are used to assess the damage level of bridge piers/columns. Chapter eight describes the computer implementation of solution techniques developed in Chapter six and non-linear beam-column model developed in Chapter seven. Chapter nine presents numerical examples for dynamic response analysis of a multi-span bridge.

10.2 Conclusions

Conclusions are presented in the order of the chapters as follows.

- (1) The site classification technique based on the 5% damped normalized acceleration spectra shape can be used to estimate the site conditions. When appropriately used, this technique can provide more than 80% of accuracy in estimating site classification. The site classification technique presented here applies to sites in Taiwan. Strictly speaking, the technique has only been demonstrated to apply to the Chi-Chi, Taiwan earthquake. Observed spectral shape characteristics depend on site conditions, earthquake magnitude and distance, and thus discriminants of site condition based on spectral shape should generally be re-evaluated before application to other earthquake conditions. However, the predominant period tends to be dominated by the influence of site amplification, as evidenced by the extensive literature and practical applications that reference similar techniques (e.g., Japan Specifications for Highway Bridges, 1996). Thus the technique may be generally applicable for sites in Taiwan. Similar procedures may be implemented to estimate the soil conditions of seismograph stations with

unknown site conditions elsewhere, but would require statistical analyses to evaluate spectral shape characteristics in those regions.

- (2) Horizontal-to-vertical spectral ratio technique did not provide a good estimation. It is believed that the fundamental assumption of the method, that the vertical component of ground motion does not amplify, is not always correct.
- (3) The non-linear soil response does not affect the site classification scheme since non-linearity would increase the predominant period of ground motion. The closest distance to fault plays an important role in estimating the site classification of seismograph stations in strong ground motion array such as SMART-1 array.
- (4) Evidence of significant directivity effects during Chi-Chi, Taiwan earthquake indicates that the directivity effects are important and need to be considered in the design of bridges. The dip-slip component of the fault rupture propagation has a more significant influence on the ground motion characteristics than the along-strike component. This is expected as the Chi-Chi, Taiwan earthquake was a thrust event. In the up-dip direction, ground motion amplitudes of the stations near the epicenter, towards which the rupture propagated, are enhanced by a factor of approximately 2.7. There is also a significant along-strike directivity effect. The amplitudes of PGA and PSA are approximately 1.5 times higher for sites in which rupture propagated strongly towards the site, along-strike.
- (5) The role of an efficient and accurate analysis method to capture essential behaviour of bridges is vital as today's new challenge is towards the strong ground motion at near-fault sites. Non-linear time history analysis can capture

essential behaviour of bridges and therefore should be used in seismic analysis of bridges. Ritz reduction methods can be used to reduce the order of equations of motion.

- (6) Pseudo-force method is efficient in terms of computation effort because only Ritz vectors corresponding to the initial elastic state of the structure are used. This method can provide accurate numerical results if the non-linearity is limited.
- (7) The tangent spectrum method is more expensive in terms of computation effort because new Ritz vectors are required when there is a change in the basic properties of the analyzed structure. The efficiency of this method depends on the type of analyzed bridge and the earthquake loadings. This method is applicable if the dynamic properties such as frequencies and/or mode shapes of the structure during response are of interest. This method can be compared to the direct integration method in terms of accuracy. The direct integration method does not provide information about mode shapes and frequencies during response of the analyzed structure however, this method is less expensive than the tangent spectrum method. Furthermore, it should be noted that the tangent spectrum method is not as stable as the pseudo-force method. The change of stiffness of non-linear beam-column elements causes the change in structure's stiffness and a new set of base vectors. It is not always possible to obtain enough base vectors to capture required effective mass factor during the response. If the number of converged base vectors is not enough the solution becomes instable.

- (8) The use of each method is primarily based on experience of a structural engineer and demand of the analysis. Comparisons between methods are provided to help to select a suitable method for analysis: (1) in terms of computational effort, the pseudo-force method is most efficient and the tangent spectrum method is most expensive; (2) in terms of accuracy, the direct integration method and the tangent spectral method should give similar results provided enough number of Ritz vectors is included. The pseudo-force method does not provide accurate results when non-linearity becomes severe and/or structure has many sources of non-linearity; (3) in terms of information on vibration properties during response, only the tangent spectrum method provides information on structural vibration.
- (9) Ritz methods can be used in non-linear dynamic response analysis of bridges if the converged Ritz vectors capture required effective mass factor. Plastic deformations and residual displacements are sensitive to the number of converged Ritz vectors.
- (10) Effects of strength deterioration and stiffness degradation behaviour on the dynamic response of bridges are studied. Displacement response of bridges could be significantly higher with the consideration of strength deterioration and stiffness degradation behaviour when non-linearity is severe. In particular, larger maximum displacements are observed for bridge deck, and less residual displacements are observed at the top of bridge piers/columns. Plastic rotation is also significantly larger when strength deterioration and stiffness degradation are considered.

- (11) Damage index of structural members are computed from linear combination of the maximum displacement and the dissipated energy. The contribution of the dissipated energy to damage index is not negligible and should be considered in the damage index computation.
- (12) Response of the bridge to multiple support excitations in the case of identical site condition (only wave-passage effect is considered) is slightly different from that to the uniform support excitation. However, in the case of different site condition at the location of supports (both wave-passage effect and site effects are considered) the bridge response is significantly different from that to the uniform support excitation.

10.3 Recommendations

During the course of this study, it is recognized that there are some areas that need to be further studied.

- (1) Response of bridges, especially of long-span bridges can be strongly affected by the non-synchronous seismic input ground motions. To obtain more accurate results of bridge response, the spatial variation of support ground motions should be considered. It is desirable to study the spatial variation of ground motions and its effects on the response of long-span bridges.
- (2) The damage stiffness matrix is derived by considering only the flexural behaviour of beam-column elements. However, bridge piers/columns may be damaged due to shear or shear and flexure combined. To obtain more accurate behaviour of

bridge piers/columns, it is desirable to consider shear damage of piers/columns. The damage in shear can be derived using a similar procedure as presented in Chapter 7.

- (3) The near-fault ground motions are different from the ordinary ground motions in that they often contain strong coherent dynamic long period pulses and permanent ground displacements. Many bridge structures have collapsed or been severely damaged during recent major earthquakes, which concentrated near or along the rupture faults. This suggests the pronounced near-fault effects of ground motions on the bridge responses. The dynamic response of bridge structures subject to the near-fault ground motions need to be studied in more details.
- (4) Characteristics of multi-component ground motions are primarily based on the results of the study by Penzien et al. (1975) where a limited number of ground motion records were used. It is desirable to study the characteristics of strong ground motion records obtained from the Chi-Chi, Taiwan earthquake to further investigate the effects of directivity on the properties of multi-component ground motions.
- (5) The stability of tangent spectrum method needs to be studied in more detail. Since the tangent spectrum method can provide information on the vibration properties of the structure, it is desirable to assess the damage of bridge structures using vibration properties. It is recognized that the non-linear behaviour is limited to specified locations, therefore, it may be possible to use static condensation technique to separate the stiffness of non-linear elements from the stiffness of

linear elements, so that only a limited number of vectors need to be regenerated due to the change of stiffness of non-linear elements.

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Appendix A

Site classification for 87 stations with known soil conditions

Stn. Name	Estimated site by			
	Kou	Lee et al.	RSS	HVSR
TCU047	S ₁	C	R	S
TCU046	S ₁	B	R	R
TCU045	S ₁	C	R	R
TCU042	S ₃	D	R	R
TCU040	S ₃	E	S	S
TCU039	S ₂	C	S	S
TCU038	S ₃	D	S	R
TCU036	S ₂	D	S	S
TCU034	S ₁	C	R	R
TCU033	S ₃	D	S	S
TCU031	S ₂	D	S	S
TCU029	S ₁	C	S	R
TCU026	S ₂	D	S	S
TCU025	S ₁	B	R	R
TCU018	S ₂	C	R	S
TCU017	S ₂	?	S	S
TCU015	S ₁	C	R	S
TCU014	S ₂	D	R	S
TCU011	S ₃	D	S	S
TCU010	S ₂	C	S	S
TCU009	S ₂	D	S	S
TCU008	S ₂	C	S	S
TCU007	S ₂	D	R	S
TCU006	S ₃	D	S	S
TCU003	S ₃	D	S	S
TAP028	S ₃	D	S	S
TAP026	S ₃	E	S	S
TAP024	S ₃	D	S	R
TAP021	S ₃	E	S	S
TAP020	S ₃	E	S	S
TAP017	S ₃	E	S	S
TAP014	S ₃	E	S	S
TAP013	S ₃	E	S	S
TAP012	S ₃	E	S	S
TAP010	S ₄	E	S	S
TAP008	S ₃	E	S	S
TAP007	S ₃	E	S	S
TAP006	S ₃	E	S	S
TAP005	S ₄	E	S	S
TAP003	S ₄	E	S	S
CHY100	S ₃	D	S	R
CHY099	S ₃	D	S	S
CHY096	S ₃	D	S	S
CHY090	S ₃	E	S	R
CHY088	S ₂	D	R	S
CHY087	S ₂	C	R	R

Stn. Name	Estimated site by			
	Kou	Lee et al.	RSS	HVSR
CHY081	S ₂	C	R	S
CHY079	S ₂	C	R	R
CHY078	S ₃	D	S	S
CHY074	S ₂	C	R	S
CHY071	S ₄	E	S	S
CHY070	S ₃	D	S	S
CHY069	S ₃	E	S	R
CHY067	S ₃	E	S	S
CHY066	S ₃	E	S	S
CHY065	S ₂	D	R	S
CHY063	S ₂	D	R	R
CHY062	S ₂	D	R	R
CHY061	S ₂	C	R	R
CHY060	S ₃	E	S	S
CHY059	S ₄	E	S	S
CHY058	S ₃	D	S	S
CHY057	S ₂	C	R	R
CHY055	S ₃	E	S	R
CHY054	S ₃	E	R	S
CHY052	S ₂	C	R	R
CHY050	S ₂	C	R	R
CHY047	S ₃	D	S	R
CHY046	S ₂	C	R	S
CHY044	S ₄	E	S	S
CHY042	S ₂	?	R	S
CHY041	S ₂	D	R	S
CHY039	S ₃	E	S	S
CHY036	S ₃	D	R	S
CHY035	S ₂	D	R	S
CHY034	S ₂	D	R	S
CHY023	S ₃	D	S	S
CHY022	S ₂	C	R	R
CHY019	S ₂	D	R	R
CHY017	S ₃	E	S	S
CHY016	S ₃	E	S	R
CHY015	S ₄	D	S	R
CHY014	S ₂	D	R	R
CHY012	S ₄	E	S	S
CHY010	S ₂	D	R	R
CHY008	S ₃	E	S	R
CHY006	S ₂	D	R	S

RSS is Response Spectral Shape.

HVSR is Horizontal to Vertical Spectral Ratio.

Appendix B

Site classification for 420 sites in Taiwan

Sites classified as soil (248 sites)	Sites classified as rock (172 sites)
HK; CHY002; CHY008; CHY012; CHY015; CHY016; CHY017; CHY023; CHY024; CHY025; CHY026; CHY032; CHY033; CHY039; CHY042; CHY044; CHY047; CHY055; CHY058; CHY059; CHY060; CHY066; CHY067; CHY069; CHY070; CHY071; CHY076; CHY078; CHY090; CHY092; CHY096; CHY099; CHY100; CHY101; CHY104; CHY116; HEN; HSN; HWA003; HWA007; HWA011; HWA012; HWA013; HWA014; HWA015; HWA016; HWA017; HWA019; HWA028; HWA029; HWA031; HWA036; HWA041; HWA054; ILA002; ILA003; ILA004; ILA005; ILA006; ILA007; ILA012; ILA013; ILA016; ILA021; ILA024; ILA027; ILA030; ILA032; ILA035; ILA036; ILA037; ILA039; ILA041; ILA042; ILA044; ILA046; ILA048; ILA049; ILA051; ILA054; ILA055; ILA056; ILA061; ILA064; KAU003; KAU006; KAU007; KAU008; KAU010; KAU011; KAU012; KAU015; KAU018; KAU020; KAU022; KAU030; KAU032; KAU033; KAU034; KAU037; KAU038; KAU039; KAU043; KAU044; KAU046; KAU048; KAU051; KAU052; KAU057; KAU058; KAU062; KAU063; KAU064; KAU066; KAU073; KAU074; KAU075; KAU081; KAU082; KAU083; KAU086; KAU087; KAU088; SGL; TAI1; TAP; TAP003; TAP005; TAP006; TAP007; TAP008; TAP010; TAP012; TAP013; TAP014; TAP017; TAP020; TAP021; TAP024; TAP026; TAP028; TAP032; TAP036; TAP041; TAP042; TAP046; TAP049; TAP060; TAP065; TAP066; TAP069; TAP075; TAP077; TAP078; TAP079; TAP081; TAP083; TAP084; TAP087; TAP090; TAP094; TAP095; TAP097; TAP098; TAP100; TAW; TCU003; TCU006; TCU007; TCU008; TCU009; TCU010; TCU011; TCU015; TCU017; TCU031; TCU033; TCU036; TCU038; TCU039; TCU040; TCU048; TCU050; TCU051; TCU052; TCU054; TCU055; TCU056; TCU059; TCU060; TCU061; TCU063; TCU064; TCU067; TCU068; TCU070; TCU075; TCU081; TCU082; TCU092; TCU094; TCU096; TCU101; TCU102; TCU103; TCU106; TCU107; TCU109; TCU110; TCU111; TCU112; TCU116; TCU117; TCU118; TCU120; TCU123; TCU128; TCU136; TCU141; TCU147; TTN; TTN001; TTN003; TTN004; TTN005; TTN006; TTN007; TTN008; TTN009; TTN010; TTN012; TTN013; TTN014; TTN015; TTN016; TTN022; TTN023; TTN027; TTN032; TTN033; TTN036; TTN042; TTN045; TTN047; TTN048; W GK; W SF; W TC.	ALS; CHY004; CHY006; CHY010; CHY014; CHY019; CHY022; CHY027; CHY028; CHY029; CHY034; CHY035; CHY036; CHY041; CHY046; CHY050; CHY052; CHY054; CHY057; CHY061; CHY062; CHY063; CHY065; CHY074; CHY079; CHY080; CHY081; CHY082; CHY086; CHY087; CHY088; CHY093; CHY094; CHY102; CHY107; CHY109; CHY110; ENA; ESL; HWA; HWA002; HWA005; HWA006; HWA009; HWA020; HWA022; HWA023; HWA024; HWA025; HWA026; HWA027; HWA030; HWA032; HWA033; HWA034; HWA035; HWA037; HWA038; HWA039; HWA043; HWA044; HWA045; HWA046; HWA048; HWA049; HWA050; HWA051; HWA053; HWA055; HWA056; HWA057; HWA058; HWA059; HWA060; HWA2; ILA001; ILA008; ILA010; ILA014; ILA015; ILA031; ILA043; ILA050; ILA052; ILA059; ILA062; ILA063; ILA066; ILA067; KAU001; KAU040; KAU042; KAU047; KAU050; KAU054; KAU069; KAU077; KAU078; KAU085; NCU; NSK; NST; NSY; PNG; SSD; STY; TAP034; TAP035; TAP043; TAP047; TAP051; TAP052; TAP053; TAP059; TAP067; TAP072; TAP086; TAP103; TCU; TCU014; TCU018; TCU025; TCU026; TCU029; TCU034; TCU042; TCU045; TCU046; TCU047; TCU049; TCU053; TCU057; TCU065; TCU071; TCU072; TCU074; TCU076; TCU078; TCU079; TCU083; TCU084; TCU085; TCU087; TCU089; TCU095; TCU098; TCU100; TCU104; TCU105; TCU113; TCU115; TCU119; TCU122; TCU129; TCU138; TCU140; TCU145; TTN002; TTN018; TTN020; TTN024; TTN025; TTN026; TTN028; TTN031; TTN040; TTN041; TTN044; TTN046; TTN050; TTN051; WNT.

Appendix C

Baseline Correction Procedure

It is often that the recorded strong motions are contaminated by noise and instrumental error. The baseline of ground motions is offset from zero-line, and therefore ground motions need to be corrected. This process of detrend a given ground motion to zero-line is called the baseline correction. Baseline correction can be carried out by two simple approaches: (i) remove average trace, and/or (ii) detrend whole trace. These procedures are used in the strong-motion processing subroutine (smp.f). It is reported for Chi-Chi earthquake data the baseline correction procedure does not significantly influence the response spectra of importance to engineering application (Boore, 1999; Boore, 2001; and Boore and Bommer, 2005).

C.1 Remove average trace

Baseline correction can be carried out by reducing time series by its mean value to satisfy the assumption of zero mean process (stationary). The average of time series is computed as follows,

$$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n} \quad (C.1)$$

The whole time series is recomputed by removing average value of time series

$$Y_i = Y_i - \bar{Y} \quad (C.2)$$

C.2 Detrend whole trace

This procedure is based on the theory of regression analysis (Draper and Smith, 1998).
Supposed we have available n sets of observations $(X_1, Y_1), (X_2, Y_2), \dots$, and (X_n, Y_n) .

The line passing through these points can be written as

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \text{ for } i=1, 2, \dots, n \quad (C.3)$$

The summation of squares of deviations from the true line is

$$S = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i)^2 \quad (C.4)$$

We shall determine values of b_0 and b_1 for β_0 and β_1 in equation (C.4) so that value of S obtained from equation (C.4) is minimized. We can determine values of b_0 and b_1 by differentiating equation (C.4) first with respect to β_0 and then with respect to β_1 and setting the results equal to zero.

$$\begin{aligned} \frac{\partial S}{\partial \beta_0} &= -2 \sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_i) \\ \frac{\partial S}{\partial \beta_1} &= -2 \sum_{i=1}^n X_i (Y_i - \beta_0 - \beta_1 X_i) \end{aligned} \quad (C.5)$$

so that the estimate of b_0 and b_1 are obtained by substituting (b_0, b_1) for (β_0, β_1)

$$\begin{aligned} \sum_{i=1}^n (Y_i - b_0 - b_1 X_i) &= 0 \\ \sum_{i=1}^n X_i (Y_i - b_0 - b_1 X_i) &= 0 \end{aligned} \quad (C.6)$$

From equation (C.6) we have

$$\sum_{i=1}^n Y_i - nb_0 - b_1 \sum_{i=1}^n X_i = 0$$

$$\sum_{i=1}^n X_i Y_i - b_0 \sum_{i=1}^n X_i - b_1 \sum_{i=1}^n X_i^2 = 0 \quad (C.7)$$

or

$$b_0 n + b_1 \sum_{i=1}^n X_i = \sum_{i=1}^n Y_i$$

$$b_0 \sum_{i=1}^n X_i + b_1 \sum_{i=1}^n X_i^2 = \sum_{i=1}^n X_i Y_i \quad (C.8)$$

Solution for equation (C.8) is

$$b_1 = \frac{\sum_{i=1}^n X_i Y_i - \frac{\left[\left(\sum_{i=1}^n X_i \right) \left(\sum_{i=1}^n Y_i \right) \right]}{n}}{\sum_{i=1}^n X_i^2 - \frac{\left(\sum_{i=1}^n X_i \right)^2}{n}}$$

$$b_0 = \bar{Y} - b_1 \bar{X} \quad (C.9)$$

where

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \text{ and } \bar{Y} = \frac{\sum_{i=1}^n Y_i}{n} \quad (C.10)$$

C.3 Response of an oscillator to a step in acceleration

This section is to illustrate that baseline correction procedure does not influence response spectra of engineers' interest. The materials presented herein are taken from the paper by

Boore (1999). The displacement response of an oscillator due to an impulse in acceleration equal to $a_{bl}\delta(t)$ is

$$u(t) = -a_{bl}\exp(-\omega_0\eta t)\frac{\sin\omega_\eta t}{\omega_\eta} \quad (C.11)$$

where $\omega_\eta = \omega_0\sqrt{1-\eta^2}$ and $\omega_0 = \frac{2\pi}{T}$ is radial frequency corresponding to the undamped oscillator period T and η is the factional damping of the oscillator. From (C.11), the response to a step in acceleration of height a_{bl} can be obtained by integration giving

$$u(t) = -\left(\frac{a_{bl}}{\omega_0^2}\right)\left\{1 - \exp(-\omega_0\eta t)\left[\frac{\eta}{\sqrt{1-\eta^2}}\sin\omega_\eta t + \cos\omega_\eta t\right]\right\} \quad (C.12)$$

The response spectrum is given by the maximum of this response. The maximum occurs at the time when the derivative of equation (C.12) equals zero. Because the derivative of equation (C.12) is equation (C.11), the time for a maximum of the response to a step in acceleration is

$$t_{\max} = \frac{\pi}{\omega_\eta} \quad (C.13)$$

and the peak response $|u_{\max}|$ is obtained by substituting equation (C.13) into equation (C.12) giving

$$|u_{\max}| = \left(\frac{a_{bl}}{\omega_0^2}\right)\left[1 + \exp\left(\frac{-\eta\pi}{\sqrt{1-\eta^2}}\right)\right] \quad (C.14a)$$

in term of radial frequency and

$$|u_{\max}| = a_{bl} T^2 \frac{1 + \exp\left(\frac{-\eta\pi}{\sqrt{1-\eta^2}}\right)}{4\pi^2} \quad (C.14b)$$

in terms of period.

In many cases the offsets a_{bl} are smaller than 1 cm/sec^2 . With $a_{bl}=1 \text{ cm/sec}^2$ and $\eta=0.05$,

$$|u_{\max}| = 0.047T^2 \quad (C.15)$$

Because of the strong dependence on period, the effect of a step in acceleration will be much more important at long periods than at short periods. For a period of 1 sec, the response is only 0.05 cm, whereas at 20 sec the response is 19 cm.

C.4 Integrated velocity and displacement of strong earthquake ground motion

Materials presented in this appendix are from Newmark (1959), and Berg and Housener (1961). Strong earthquake ground motion usually recorded by accelerometers since the ground acceleration is the most pertinent quantity for engineering purposes. However in some engineering purposes the knowledge of ground displacement during strong earthquakes is important. Ground displacement is measured by displacement meters. The difficulty involved in installing and maintaining a network of displacement meters having period sufficiently long to record accurately strong ground motions are truly formidable. Although ground velocities and displacements are not of as much interest to engineers as ground acceleration there is still interest in knowing the magnitudes involved and, therefore, in the past there have been attempts at integration of the accelerograms to obtain velocities and displacements (Housener, 1947). Since the baseline is not recorded on the accelerographs, it is necessary to establish a baseline for the purpose of

integration. The major difficulty has been determining the true axis (zero acceleration) of the accelerograms. As ground motion velocity must diminish to zero at the end of the earthquake, a criterion is established to approximate true axis of accelerograms. The integrations are then performed by Newmark method (1959). The final velocity and displacement is

$$\begin{aligned}\dot{v}_1 &= \dot{v}_0 + \int_0^h \ddot{v}(\tau) d\tau \\ v_1 &= v_0 + \int_0^h \dot{v}(\tau) d\tau\end{aligned}\tag{C.16}$$

In the Newmark formulation, equation (C.16) is expressed as follows,

$$\begin{aligned}\dot{v}_1 &= \dot{v}_0 + (1-\gamma)h\ddot{v}_0 + \gamma h\ddot{v}_1 \\ v_1 &= v_0 + h\dot{v}_0 + \left(\frac{1}{2}-\beta\right)h^2\ddot{v}_0 + \beta h^2\ddot{v}_1\end{aligned}\tag{C.17}$$

For $\gamma=1/2$ and $\beta=1/4$ the Newmark method is referred as the constant average acceleration method, for $\gamma=1/2$ and $\beta=1/6$ it is referred as the linear acceleration method

Figures C.1 and C.2 show the final acceleration, velocity, and displacement for constant acceleration and linear varying acceleration methods. For constant acceleration, velocity and displacement can be determined from,

$$\begin{aligned}\dot{v}_1 &= \dot{v}_0 + \frac{h}{2}(\ddot{v}_0 + \ddot{v}_1) \\ v_1 &= v_0 + \dot{v}_0 h + \frac{h^2}{4}(\ddot{v}_0 + \ddot{v}_1)\end{aligned}\tag{C.18}$$

For linear varying acceleration, velocity and displacement can be determined from,

$$\begin{aligned}\dot{v}_1 &= \dot{v}_0 + \frac{h}{2}(\ddot{v}_0 + \ddot{v}_1) \\ v_1 &= v_0 + \dot{v}_0 h + \frac{h^2}{6}(2\ddot{v}_0 + \ddot{v}_1)\end{aligned}\tag{C.19}$$

a) Acceleration (constant)

$$\ddot{v}_{av} = \frac{1}{2}(\ddot{v}_0 + \ddot{v}_1)$$

b) Velocity (linear)

$$\dot{v}(\tau) = \dot{v}_0 + \frac{\tau}{2}(\ddot{v}_0 + \ddot{v}_1)$$

c) Displacement (quadratic)

$$v(\tau) = v_0 + \dot{v}_0\tau + \frac{\tau^2}{4}(\ddot{v}_0 + \ddot{v}_1)$$

Figure C.1 Constant average acceleration

a) Acceleration (linear)

$$\ddot{v}_{av} = \frac{1}{2}(\ddot{v}_0 + \ddot{v}_1)$$

b) Velocity (quadratic)

$$\dot{v}(\tau) = \dot{v}_0 + \frac{\tau}{2}(\ddot{v}_0 + \ddot{v}_1)$$

c) Displacement (cubic)

$$v(\tau) = v_0 + \dot{v}_0\tau + \frac{\tau^2}{4}(\ddot{v}_0 + \ddot{v}_1)$$

Figure C.2 Linear varying acceleration

Appendix D

Newmark Beta Integration Method

D.1 Equation of motion

In this present study, dynamic responses of a single-degree-of-freedom (SDOF) system are computed by Newmark beta method (1969). In this appendix, the author would like to present the algorithm of direct time step integration method to compute dynamic responses of a SDOF system. The initial conditions are obtained and the requirement for lowest period is emphasized. The author found it is very important to satisfy the requirement to obtain stable numerical solution.

Equation of motion is written as follows

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = p(t) \quad (D.1)$$

where m , c and k are mass, damping and stiffness of the system.

Newmark (1959) assumed the relation of velocity and displacement as follows

$$\dot{u}_1 = \dot{u}_0 + (1 - \gamma)\Delta t \dot{u}_0 + \gamma \Delta t \ddot{u}_1 \quad (D.2)$$

$$u_1 = u_0 + h \dot{u}_0 + \left(\frac{1}{2} - \beta \right) \Delta t^2 \ddot{u}_0 + \beta \Delta t^2 \ddot{u}_1 \quad (D.3)$$

From equation (D.3) one obtains

$$\ddot{u}_1 = \frac{1}{\beta \Delta t^2} (u_1 - u_0) - \frac{1}{\beta \Delta t} \dot{u}_0 - \frac{0.5 - \beta}{\beta} \ddot{u}_0 \quad (D.4)$$

Substituting equation (D.4) into equation (D.2) leads to

$$\dot{u}_1 = \dot{u}_0 + (1-\gamma)\Delta t \ddot{u}_0 + \gamma \Delta t \left[\frac{1}{\beta \Delta t^2} (u_1 - u_0) - \frac{1}{\beta \Delta t} \dot{u}_0 - \left(\frac{1}{2\beta} - 1 \right) \ddot{u}_0 \right] \quad (D.5a)$$

Rearranging equation (D.5a) as follows

$$\dot{u}_1 = \frac{\gamma}{\beta \Delta t} (u_1 - u_0) + \left(1 - \frac{\gamma}{\beta} \right) \dot{u}_0 + \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_0 \quad (D.5b)$$

Substituting equation (D.4) and (D.5b) into equation (D.1) one obtains

$$\begin{aligned} & m \left[\frac{1}{\beta \Delta t^2} (u_1 - u_0) - \frac{1}{\beta \Delta t} \dot{u}_0 - \frac{0.5 - \beta}{\beta} \ddot{u}_0 \right] + \\ & c \left[\frac{\gamma}{\beta \Delta t} (u_1 - u_0) + \left(1 - \frac{\gamma}{\beta} \right) \dot{u}_0 + \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_0 \right] + k u_1 = p(t) \end{aligned} \quad (D.6a)$$

Expanding equation (D.6a) leads to

$$\begin{aligned} & \left(\frac{m}{\beta \Delta t^2} + \frac{c\gamma}{\beta \Delta t} + k \right) u_1 = p(t) + m \left(\frac{1}{\beta \Delta t} \dot{u}_0 + \frac{0.5 - \beta}{\beta} \ddot{u}_0 + \frac{1}{\beta \Delta t^2} u_0 \right) + \\ & c \left(\frac{\gamma}{\beta \Delta t} u_0 - \left(1 - \frac{\gamma}{\beta} \right) \dot{u}_0 - \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_0 \right) \end{aligned} \quad (D.6b)$$

Let call

$$a_0 = \frac{1}{\beta \Delta t} \dot{u}_0 + \frac{0.5 - \beta}{\beta} \ddot{u}_0 + \frac{1}{\beta \Delta t^2} u_0$$

$$b_0 = \frac{\gamma}{\beta \Delta t} u_0 - \left(1 - \frac{\gamma}{\beta} \right) \dot{u}_0 - \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_0$$

Equation (D.6) becomes

$$\bar{k} u_1 = p(t) + m a_0 + c b_0 \quad (D.7)$$

D.2 Initial conditions

Using Duhamel integration displacement can be expressed as follows

$$u(t) = \frac{1}{m\omega_D} \int_0^t p(\tau) \sin\omega_D(t-\tau) e^{-\xi\omega(t-\tau)} d\tau \quad (D.8)$$

Let call

$$h(t-\tau) = \frac{1}{m\omega_D} \sin\omega_D(t-\tau) e^{-\xi\omega(t-\tau)} \quad (D.9)$$

$$u(t) = \int_0^t p(\tau) h(t-\tau) d\tau \quad (D.10)$$

First and second derivatives of $u(t)$ are as follows

$$\dot{u}(t) = \int_0^t p(\tau) \dot{h}(t-\tau) d\tau \quad (D.11)$$

$$\ddot{u}(t) = \int_0^t p(\tau) \ddot{h}(t-\tau) d\tau \quad (D.12)$$

Substituting equation (D.9) into equations (D.11) and (D.12) one obtains

$$\dot{h}(t-\tau) = \frac{1}{m} e^{-\xi\omega(t-\tau)} \left[\cos\omega_D(t-\tau) - \frac{\xi}{\sqrt{1-\xi^2}} \sin\omega_D(t-\tau) \right] \quad (D.13)$$

$$\begin{aligned} \ddot{h}(t-\tau) = & \frac{1}{m} (-\xi\omega) e^{-\xi\omega(t-\tau)} \left[\cos\omega_D(t-\tau) - \frac{\xi}{\sqrt{1-\xi^2}} \sin\omega_D(t-\tau) \right] + \\ & \frac{1}{m} e^{-\xi\omega(t-\tau)} \left[-\omega_D \sin\omega_D(t-\tau) - \frac{\xi}{\sqrt{1-\xi^2}} \omega_D \cos\omega_D(t-\tau) \right] \end{aligned} \quad (D.14)$$

Substituting $t=0$, $\tau=0$ into expression $h(t-\tau)$, $\dot{h}(t-\tau)$, and $\ddot{h}(t-\tau)$ leads to

$$h(0) = 0$$

$$\dot{h}(0) = \frac{1}{m}$$

$$\ddot{h}(0) = -\frac{2\omega}{m}\xi$$

Substituting these into equations (D.10), (D.11) and (D.12) one obtains

$$u(0) = 0 \tag{D.15a}$$

$$\dot{u}(0) = \frac{p(0)dt}{m} \tag{D.15b}$$

$$u(0) = \frac{p(0)dt}{m}(-2\omega\xi) \tag{D.15c}$$

For earthquake ground motions

$$u(0) = 0 \tag{D.16a}$$

$$\dot{u}(0) = -\ddot{x}_g(0)dt \tag{D.16b}$$

$$\ddot{u}(0) = 2\ddot{x}_g(0)dt\omega\xi \tag{D.16c}$$

If we use m , c and k instead of ω and ξ we have

$$\omega = \frac{2\pi}{T}$$

$$k = \omega^2 m$$

$$\xi = \frac{c}{2m\omega}$$

Rewriting equation (D.16) leads to

$$u(0) = 0 \quad (\text{D.17a})$$

$$\dot{u}(0) = -\ddot{x}_g(0)dt \quad (\text{D.17b})$$

$$\ddot{u}(0) = 2\ddot{x}_g(0)\frac{cdt}{m} \quad (\text{D.17c})$$

Appendix E

Elastic and release stiffness matrices for beam-column element

E.1 Elastic stiffness matrix

$$k^c = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3(1+\phi_y)} & 0 & 0 & 0 & 0 & 0 & \frac{12EI_z}{L^3(1+\phi_y)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12EI_y}{L^3(1+\phi_z)} & 0 & 0 & 0 & 0 & 0 & \frac{12EI_y}{L^3(1+\phi_z)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 \\ 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} & 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} & 0 \\ -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-12EI_z}{L^3(1+\phi_y)} & 0 & 0 & 0 & \frac{-6EI_z}{L^2(1+\phi_y)} & 0 & \frac{12EI_z}{L^3(1+\phi_y)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-12EI_y}{L^3(1+\phi_z)} & 0 & \frac{6EI_y}{L^2(1+\phi_z)} & 0 & 0 & 0 & \frac{12EI_y}{L^3(1+\phi_z)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 \\ 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 \\ 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} & 0 & \frac{-6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} \end{bmatrix}$$

Sym

where

A is cross sectional area of beam-column element,

E is Young modulus,

G is shear modulus,

I_y is bending inertia about the local y -axis,

I_z is bending inertia about the local z -axis,

$$\phi_y = \frac{12EI_z}{GA_{sy}L^2}$$

$$\phi_z = \frac{12EI_y}{GA_{sz}L^2},$$

A_{sy} is shear area in the local y -axis,

A_{sz} is shear area in the local z -axis.

E.2 Release stiffness matrices

$$k_{miy}^e = \begin{bmatrix} 0 & & & & & & & & & & \\ 0 & 0 & & & & & & & & & \\ 0 & 0 & \frac{36EI_y}{L^3(1+\phi_z)(4+\phi_z)} & & & & & & & & \\ 0 & 0 & 0 & 0 & & & & & & & \\ 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & \frac{-36EI_y}{L^3(1+\phi_z)(4+\phi_z)} & 0 & \frac{6EI_y}{L^2(1+\phi_z)} & 0 & 0 & 0 & \frac{36EI_y}{L^3(1+\phi_z)(4+\phi_z)} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & \frac{-6(2-\phi_z)EI_y}{L^2(1+\phi_z)(4+\phi_z)} & 0 & \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 & 0 & \frac{6(2-\phi_z)EI_y}{L^2(1+\phi_z)(4+\phi_z)} & 0 & \frac{(2-\phi_z)^2EI_y}{L(1+\phi_z)(4+\phi_z)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Sym}$$

$$k_{miz}^c = \begin{bmatrix} 0 & & & & & & & & \\ 0 & \frac{36EI_z}{L^3(1+\phi_y)(4+\phi_y)} & & & & & & & \\ 0 & 0 & 0 & & & & & & \\ 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & 0 & & & & \\ & & & & & \text{Sym} & & & \\ 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & \\ 0 & \frac{-36EI_z}{L^3(1+\phi_y)(4+\phi_y)} & 0 & 0 & 0 & \frac{-6EI_z}{L^2(1+\phi_y)} & 0 & \frac{36EI_z}{L^3(1+\phi_y)(4+\phi_y)} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & \frac{6(2-\phi_y)EI_z}{L^2(1+\phi_y)(4+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} & 0 & \frac{-6(2-\phi_y)EI_z}{L^2(1+\phi_y)(4+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)^2 EI_z}{L(1+\phi_y)(4+\phi_y)} \end{bmatrix}$$

$$k_{mij}^c = \begin{bmatrix} 0 & & & & & & & & \\ 0 & 0 & & & & & & & \\ 0 & 0 & \frac{36EI_y}{L^3(1+\phi_z)(4+\phi_z)} & & & & & & \\ 0 & 0 & 0 & 0 & & & & & \\ & & & & \text{Sym} & & & & \\ 0 & 0 & \frac{-6(2-\phi_z)EI_y}{L^2(1+\phi_z)(4+\phi_z)} & 0 & \frac{(2-\phi_z)^2 EI_y}{L(1+\phi_z)(4+\phi_z)} & & & & \\ 0 & 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\ 0 & 0 & \frac{-36EI_y}{L^3(1+\phi_z)(4+\phi_z)} & 0 & \frac{6(2-\phi_z)EI_y}{L^2(1+\phi_z)(4+\phi_z)} & 0 & 0 & 0 & \frac{36EI_y}{L^3(1+\phi_z)(4+\phi_z)} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 & 0 & \frac{6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_{miz}^e = \begin{bmatrix} 0 & & & & & & & \\ 0 & \frac{36EI_z}{L^3(1+\phi_y)(4+\phi_y)} & & & & & & \\ 0 & 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & 0 & 0 & & & \\ 0 & \frac{6(2-\phi_y)EI_z}{L^2(1+\phi_y)(4+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)^2 EI_z}{L(1+\phi_y)(4+\phi_y)} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & \frac{-36EI_z}{L^3(1+\phi_y)(4+\phi_y)} & 0 & 0 & 0 & \frac{-6(2-\phi_y)EI_z}{L^2(1+\phi_y)(4+\phi_y)} & 0 & \frac{36EI_z}{L^3(1+\phi_y)(4+\phi_y)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} & 0 & \frac{-6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} \end{bmatrix} \quad \text{Sym}$$

The element of release matrices can be represented by the corresponding element of the elastic matrix. The elements of the k_{miz}^e , which are different from corresponding elements of elastic stiffness matrix can be expressed as follows

$$k_{miz}^e(3,3) = \frac{36EI_y}{L^3(1+\Phi_z)(4+\Phi_z)} = \frac{3}{(4+\Phi_z)} \frac{12EI_y}{L^3(1+\Phi_z)} = \frac{0.75}{(1+0.5shfz)} k^e(3,3)$$

$$k_{miz}^e(11,3) = \frac{-6(2-\Phi_z)EI_y}{L^2(1+\Phi_z)(4+\Phi_z)} = \frac{(2-\Phi_z)}{(4+\Phi_z)} \frac{-6EI_y}{L^2(1+\Phi_z)} = \frac{1-shfz}{2+shfz} k^e(5,3)$$

$$k_{miz}^e(11,11) = \frac{(2-\Phi_z)^2 EI_y}{L(1+\Phi_z)(4+\Phi_z)} = \frac{(2-\Phi_z)}{(4+\Phi_z)} \frac{(2-\Phi_z)EI_y}{L(1+\Phi_z)} = \frac{1-shfz}{2+shfz} k^e(11,5)$$

The elements of the k_{miz}^e , which are different from corresponding elements of elastic stiffness matrix can be expressed as follows

$$k_{miz}^e(2,2) = \frac{36EI_z}{L^3(1+\Phi_y)(4+\Phi_y)} = \frac{3}{(4+\Phi_y)} \frac{12EI_z}{L^3(1+\Phi_y)} = \frac{0.75}{(1+0.5shfy)} k^e(2,2)$$

$$k_{miz}^e(12,2) = \frac{6(2-\Phi_y)EI_z}{L^2(1+\Phi_y)(4+\Phi_y)} = \frac{(2-\Phi_y)}{(4+\Phi_y)} \frac{6EI_z}{L^2(1+\Phi_y)} = \frac{1-shfy}{2+shfy} k^e(6,2)$$

$$k_{miz}^e(12,12) = \frac{(2-\Phi_y)^2 EI_z}{L(1+\Phi_y)(4+\Phi_y)} = \frac{(2-\Phi_y)}{(4+\Phi_y)} \frac{(2-\Phi_y)EI_z}{L(1+\Phi_y)} = \frac{1-shfy}{2+shfy} k^e(12,6)$$

Appendix F

Derivation of release stiffness matrices

The procedure to establish elastic stiffness for 2-end fixed member is well understood. Release stiffness matrices can be derived from elastic stiffness with 2-ends restrained by static condensation method. Consider the following static equation of equilibrium

$$\begin{Bmatrix} \mathbf{F}_p \\ \mathbf{F}_s \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_{pp} & \mathbf{K}_{ps} \\ \mathbf{K}_{sp} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_p \\ \mathbf{u}_s \end{Bmatrix} \quad (\text{F.1})$$

It is noted that we have partitioned the global equation to separate “primary” quantities (designated with a “p” subscript) from “secondary” quantities (designated with “s” subscript). What we plan to do is to eliminate the secondary displacements from equation (F.1). The secondary displacements will be those associated with the equation of condition introduced by the presence of the hinge.

Expanding equation (F.1) we have

$$\mathbf{F}_p = \mathbf{K}_{pp}\mathbf{u}_p + \mathbf{K}_{ps}\mathbf{u}_s \quad (\text{F.2})$$

$$\mathbf{F}_s = \mathbf{K}_{sp}\mathbf{u}_p + \mathbf{K}_{ss}\mathbf{u}_s \quad (\text{F.3})$$

We next solve equation (F.3) for $\mathbf{u}_s$ giving

$$\mathbf{u}_s = \mathbf{K}_{ss}^{-1}(\mathbf{F}_s - \mathbf{K}_{sp}\mathbf{u}_p) \quad (\text{F.4})$$

Substituting equation (F.4) into equation (F.2) we have

$$\mathbf{F}_p = \mathbf{K}_{pp} \mathbf{u}_p + \mathbf{K}_{ps} \mathbf{K}_{ss}^{-1} (\mathbf{F}_s - \mathbf{K}_{sp} \mathbf{u}_p) \quad (\text{F.5})$$

Rearranging equation (F.5) we have

$$\mathbf{F}_p - \mathbf{K}_{ps} \mathbf{K}_{ss}^{-1} \mathbf{F}_s = (\mathbf{K}_{pp} - \mathbf{K}_{ps} \mathbf{K}_{ss}^{-1} \mathbf{K}_{sp}) \mathbf{u}_p \quad (\text{F.6})$$

Solving equation (F.6) we obtain $\mathbf{u}_p$. Matrix $\mathbf{K}_{pp} - \mathbf{K}_{ps} \mathbf{K}_{ss}^{-1} \mathbf{K}_{sp}$ is the condensed matrix after releasing an end rotation, and matrix $\mathbf{K}_{ps} \mathbf{K}_{ss}^{-1} \mathbf{K}_{sp}$ is the release matrix aforementioned in Chapter 7.

The release matrix $\mathbf{k}_{miy}^e$ can be obtained by using the column and row corresponding to the degree-of-freedom about y-axis at i end as follows

$$\mathbf{k}_{miy}^e = \begin{bmatrix} 0 \\ 0 \\ -\frac{6EI_y}{L^2(1+\phi_z)} \\ 0 \\ \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} \\ 0 \\ 0 \\ 0 \\ \frac{6EI_y}{L^2(1+\phi_z)} \\ 0 \\ \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} \\ 0 \end{bmatrix} \frac{L(1+\phi_z)}{(4+\phi_z)EI_y} \begin{bmatrix} -\frac{6EI_y}{L^2(1+\phi_z)} & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & \frac{6EI_y}{L^2(1+\phi_z)} & \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The release matrix $\mathbf{k}_{miz}^e$ can be obtained by using the column and row corresponding to the degree-of-freedom about z-axis at i end as follows

$$k_{miz}^e = \begin{bmatrix} 0 \\ \frac{6EI_z}{L^2(1+\phi_y)} \\ 0 \\ 0 \\ 0 \\ \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} \\ 0 \\ -\frac{6EI_z}{L^2(1+\phi_y)} \\ 0 \\ 0 \\ 0 \\ \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} \\ 0 \end{bmatrix} \frac{L(1+\phi_y)}{(4+\phi_y)EI_z} \begin{bmatrix} 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} & 0 & \frac{-6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} \end{bmatrix}$$

The release matrix k_{mji}^e can be obtained by using the column and row corresponding to the degree-of-freedom about y-axis at j end as follows

$$k_{mji}^e = \begin{bmatrix} 0 \\ 0 \\ -\frac{6EI_y}{L^2(1+\phi_z)} \\ 0 \\ \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{6EI_y}{L^2(1+\phi_z)} \\ 0 \\ \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} \\ 0 \end{bmatrix} \frac{L(1+\phi_z)}{(4+\phi_z)EI_y} \begin{bmatrix} 0 & 0 & \frac{-6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(2-\phi_z)EI_y}{L(1+\phi_z)} & 0 & 0 & 0 & \frac{6EI_y}{L^2(1+\phi_z)} & 0 & \frac{(4+\phi_z)EI_y}{L(1+\phi_z)} & 0 \end{bmatrix}$$

The release matrix k_{mjz}^e can be obtained by using the column and row corresponding to the degree-of-freedom about z-axis at j end as follows

$$k_{mjz}^e = \begin{bmatrix} 0 \\ \frac{6EI_z}{L^2(1+\phi_y)} \\ 0 \\ 0 \\ 0 \\ \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} \\ 0 \\ -\frac{6EI_z}{L^2(1+\phi_y)} \\ 0 \\ 0 \\ 0 \\ \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} \end{bmatrix} \frac{L(1+\phi_y)}{(4+\phi_y)EI_z} \begin{bmatrix} 0 & \frac{6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(2-\phi_y)EI_z}{L(1+\phi_y)} & 0 & \frac{-6EI_z}{L^2(1+\phi_y)} & 0 & 0 & 0 & \frac{(4+\phi_y)EI_z}{L(1+\phi_y)} \end{bmatrix}$$

Appendix G

Hysteretic dissipated energy in bridge columns

It is noted that the Park and Ang damage index, and the fatigue-based damage index, were developed for 1-D modeling. For 2-D modeling, the assessment of damage index is more complicated. The use of deformation in the 2-D damage index model was discussed in Chapter 7. This appendix presents the formulation of hysteretic dissipated energy and maximum energy used in 3-D damage index model computation.

G.1 Hysteretic dissipated energy

Total dissipated energy is energy accumulated during response of element. Total dissipated energy is computed by

$$E_h = \sum_{i=1}^n (E_h)_i \quad (G.1)$$

At each time step, the dissipated energy is computed by dot product

$$(E_h)_i = M_i \cdot \Delta \theta_i \quad (G.2)$$

Figure G.1 shows the deformations and loadings in two directions of a non-linear bridge column element subject to earthquake loadings. Let θ_{py} , θ_{pz} , M_{py} and M_{pz} are deformation (rotation) and loading (moment) in the beam-column element. Expanding equation (G.2) leads to

$$(E_h)_i = (M_x)_i (\Delta \theta_x)_i + (M_y)_i (\Delta \theta_y)_i \quad (G.3)$$

Figure G.1 Deformations and loadings in non-linear beam-column element

G.2 Maximum energy

The maximum energy that a section absorbs is equal to the area of moment-curvature envelope. Simply, the maximum energy of an elasto-plastic system is written as follows

$$E_u = 4M_y(\theta_u - \theta_y) \quad (G.4)$$

where M_y is the yield moment, θ_y and θ_u is the yield and ultimate plastic rotation angles. Notice that M_y is a function of axial force, therefore E_u is a function of axial force. The damage index can be written in the form. For moment-curvature relation of Figure 7.1, the maximum energy is

$$E_u = 4 \left(M_y(\theta_c - \theta_y) + \frac{M_y(\theta_u - \theta_c)}{2} \right) \quad (G.5)$$

or,

$$E_u = 4M_y \left(\theta_c - \theta_y + \frac{\theta_u - \theta_c}{2} \right) = 4M_y \left(\frac{\theta_u}{2} + \frac{\theta_c}{2} - \theta_y \right) \quad (G.6)$$

If plastic rotation angles are used

$$E_u = 4M_y \left(\frac{\theta_u}{2} + \frac{\theta_c}{2} \right) = 2M_y (\theta_u + \theta_c) \quad (G.7)$$

Equations (G.5) to (G.7) are derived for 1-D element. From equation (G.3) it is found that the total maximum energy for 2-D element is

$$E_u = 4M_y (\theta_u + \theta_c - 2\theta_y) \quad (G.8)$$

Park and Ang damage model

$$D = \frac{\delta_{\max}}{\delta_u} + \frac{\sum dE_{hi}}{E_u} \quad (G.9)$$

Fatigue based damage model

$$D = \frac{\theta_{\max} - \theta_y}{\theta_u - \theta_y} + \frac{\sum dE_h}{E_u} \quad (G.10)$$

Appendix H

Influence Coefficient Matrix

In most of analytical procedures the structure is connected to the ground undergo identical motion that is prescribed. The assumption of rigid support excitation does not always give conservative results. The effects of multiple-support excitation may be significant in the case of long-span structures and cause structures to perform in undesirable fashion. This section describes formulation of equations of motion to allow for different-possibly even multicomponent-prescribed motions at the various supports.

Theoretically, the influence coefficient matrix **B** for identical ground motion is N by 3 (where N is number of DOFs and 3 is number of translational excitations in x-, y- and z-directions) matrix of one in direction of excitation and the coefficients in other directions are zero, for example, the influence coefficient matrix **B** for the case of rigid support motion applied in the x-direction

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

In case of multiple-support excitation the influence coefficient matrix **B** no longer includes only ones and zeros. In this present study, it is assumed the ground excitation is applied in only one translational direction i.e., no multi-directional ground excitation is considered. The displacement vector now contains two parts: (1) $\mathbf{u}^t$ includes N DOFs of the superstructure, where the superscript “t” denotes that these are total displacements;

and (2) $\mathbf{u}_g$ includes the N_g components of support displacements. The equation of dynamic equilibrium for all the DOFs is written in the partitioned form as follows

$$\begin{bmatrix} \mathbf{m} & \mathbf{m}_g \\ \mathbf{m}_g^T & \mathbf{m}_{gg} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}^t \\ \ddot{\mathbf{u}}_g \end{Bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{c}_g \\ \mathbf{c}_g^T & \mathbf{c}_{gg} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}^t \\ \dot{\mathbf{u}}_g \end{Bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k}_g \\ \mathbf{k}_g^T & \mathbf{k}_{gg} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^t \\ \mathbf{u}_g \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p}_g(t) \end{Bmatrix} \quad (\text{H.1})$$

Notice that no external forces are applied along the superstructure DOFs. In equation (H.1) the mass, damping, and stiffness matrices can be determined from the properties of the analyzed structure, while the support motions $\mathbf{u}_g(t)$, $\dot{\mathbf{u}}_g(t)$, and $\ddot{\mathbf{u}}_g(t)$ must be specified. It is desired to determine the displacement $\mathbf{u}^t$ in the superstructure DOFs and the support forces $\mathbf{p}_g$. Displacements $\mathbf{u}^t$, and $\mathbf{u}_g$ can be separated into two parts

$$\begin{Bmatrix} \mathbf{u}^t \\ \mathbf{u}_g \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}^s \\ \mathbf{u}_g \end{Bmatrix} + \begin{Bmatrix} \mathbf{u} \\ \mathbf{0} \end{Bmatrix} \quad (\text{H.2})$$

In this equation $\mathbf{u}^s$ is the vector of structural displacements due to static application of the prescribed support displacements $\mathbf{u}_g$ at each time instant. The two are related by

$$\begin{bmatrix} \mathbf{k} & \mathbf{k}_g \\ \mathbf{k}_g^T & \mathbf{k}_{gg} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^s \\ \mathbf{u}_g \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p}_g^s \end{Bmatrix} \quad (\text{H.3})$$

where $\mathbf{p}_g^s$ are the support force necessary to statically impose displacements $\mathbf{u}_g$ that vary with time; obviously $\mathbf{u}^s$ varies with time and is therefore known as the vector of quasi-static displacements. The remainder $\mathbf{u}$ of the structural displacements is known as dynamic displacements because a dynamic analysis is required to evaluate them.

Write the first of the two partitioned equations (H.1)

$$\mathbf{m}\ddot{\mathbf{u}}^t + \mathbf{m}_g \ddot{\mathbf{u}}_g + \mathbf{c}\dot{\mathbf{u}}^t + \mathbf{c}_g \dot{\mathbf{u}}_g + \mathbf{k}\mathbf{u}^t + \mathbf{k}_g \mathbf{u}_g = \mathbf{0} \quad (\text{H.4})$$

Substituting equation (H.2) and transfer all terms involving $\mathbf{u}_g$ and $\mathbf{u}^s$ to the right side leads to

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}_{\text{eff}}(t) \quad (\text{H.5})$$

where the vector of effective earthquake load is

$$\mathbf{p}_{\text{eff}}(t) = -(\mathbf{m}\ddot{\mathbf{u}}^s + \mathbf{m}_g \ddot{\mathbf{u}}_g) - (\mathbf{c}\dot{\mathbf{u}}^s + \mathbf{c}_g \dot{\mathbf{u}}_g) - (\mathbf{k}\mathbf{u}^s + \mathbf{k}_g \mathbf{u}_g) \quad (\text{H.6})$$

From equation (H.3) one obtained

$$\mathbf{k}\mathbf{u}^s + \mathbf{k}_g \mathbf{u}_g = \mathbf{0} \quad (\text{H.7})$$

Equation (H.7) also allows us to express the quasi-static displacement $\mathbf{u}^s$ in terms of the specified support displacements $\mathbf{u}_g$

$$\begin{aligned} \mathbf{u}^s &= \mathbf{B}\mathbf{u}_g \\ \mathbf{B} &= -\mathbf{k}^{-1}\mathbf{k}_g \end{aligned} \quad (\text{H.8})$$

where $\mathbf{B}$ is influence coefficient matrix because it describes the influence of support displacements on the structural displacements. Quasi-static displacement can be expressed in a different form

$$\mathbf{u}^s = \sum_{n=1}^{3N_g} \mathbf{B}_n \mathbf{u}_{gn}(t) \quad (\text{H.9})$$

Vector $\mathbf{B}_n$ is actually the influence vector associated with the support displacement at nth support $\mathbf{u}_{gn}$. Substituting equations (H.7) and (H.8) in equation (H.6) gives

$$\mathbf{p}_{\text{eff}}(t) = -(\mathbf{m}\mathbf{B} + \mathbf{m}_g)\ddot{\mathbf{u}}_g(t) - (\mathbf{c}\mathbf{B} - \mathbf{c}_g)\dot{\mathbf{u}}_g(t) \quad (\text{H.10})$$

In equation (H.10) the damping term is usually small relative to the inertia term and therefore can be dropped. For structures with mass idealized as lumped at the DOF, the mass matrix is diagonal, implying that $\mathbf{m}_g$ is null matrix and equation (H.10) can be reduced to

$$\mathbf{p}_{\text{eff}}(t) = -\mathbf{m}\mathbf{B}\ddot{\mathbf{u}}_g(t) \quad (\text{H.11})$$

or in the matrix form

$$\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{Bmatrix} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{21} & \cdots & b_{1ng} \\ b_{21} & b_{22} & \cdots & b_{2ng} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nng} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \vdots \\ \ddot{u}_{ng} \end{Bmatrix} \quad (\text{H.12})$$

Notice that $\mathbf{B}$ is N by N_g matrix, $\ddot{\mathbf{u}}_g(t)$ N_g vector, and $\mathbf{m}$ is N by N matrix.

The problem with previous approach is the computation of the inverse stiffness matrix $\mathbf{k}$ in equation (H.8). Another approach can also be developed to compute influence matrix $\mathbf{B}$ without obtaining inverse stiffness matrix $\mathbf{k}$ (Kassimali, 1999). This approach requires the calculation of the actions at the ends of members in the restrained structure due to the displacements of the support. These fixed-end actions are treated the same manner as those due to loads. The advantage of treating support displacements in this member-oriented fashion is that the analysis is cast into the same format as that for other

secondary effects. For each member that is attached to the supports that undergo displacement, the fixed-end force vector F_s due to support displacements u_g can be computed as follows

$$F_s = k u_g \quad (H.13)$$

The nodal displacements u due to support displacement u_g is obtained from solving equation

$$-F_s = K u \quad (H.14)$$

Notice that u is actually the n th vector of the influence coefficient matrix B . The INFLC subroutine to compute influence coefficient matrix B can be summarized as follows

```

      subroutine inflc (b,mband,neq,numel,numnp)
c*****
c***  subroutine for calculating the influence coefficient
c***  matrix b used in multiple support excitation
c***  variable declaration *****
c      ne, number of prescribed support excitations
c      n, support identification number
c      r, displacement at the nth support
c      tl, displacement matrix
c      nod, support at which excitation is imposed
c      nj, direction code
c      ubl, displacement vector
c      s, element stiffness matrix
c      f, force vector
c*****
      implicit double precision (a-h, o-z)
      common/imse/ ims,nex
      common/emti/ mtype,lm(24),nd,ns,lms(12),inpar(7),nlm(24,2),
      .           ijkl(124)
      common/emtx/ s(24,24),pf(24,4),xm(24),sa(12,24),tt(12,4),
      .           pbl(24),ubl(24),sbl(12),qqqq(981)
c
      dimension a(neq,mband),b(neq,nex),rr(6),tl(numnp,6),f(neq),
      .           nbmax(neq)
c
      do 5 i=1,neq

```

```

        do 5 j=1,nex
5 b(i,j)=0.
c
        do 110 in=1,nex
        do 70 n=1,neq
70 f(n)=0.
c
        do 80 n=1,numnp
        do 80 j=1,6
80 tl(n,j)=0.
c
        read (5,1000) nsi,(rr(j), j=1,6)
c
        do 200 j=1,6
        tl(nsi,j)=rr(j)
200 continue
c
c      set-up displacement vector
c
        rewind 2
        rewind 15
        do 300 n=1,numel
        read (2) lm,nd,ns,s,pf,xm,nlm
        call zeror(ubl,24)
        do 400 j=1,nd
        nod=nlm(j,1)
        nj=nlm(j,2)
        ubl(j)=tl(nod,nj)
400 continue
        write (15) nd,ubl,lm,s
300 continue
c
c      compute fixed-end force vector
c
        rewind 2
        rewind 15
        do 500 n=1,numel
        read (15) nd,ubl,lm,s
        do 600 k=1,nd
        ii=lm(k)
        if (ii.le.0.or.ii.gt.neq) go to 600
        do 700 j=1,nd
        f(ii)=f(ii)-s(k,j)*ubl(j)
700 continue
600 continue
500 continue
c
c      solve for i-th vector of influence coefficient matrix B
c
        rewind 4
        read (4) ((a(i,j),i=1,neq),j=1,mband)
        call cosol (f,a,nbmax,neq,mband)

```

```

        do 800 j=1,neq
          b(j,in)=f(j)
800    continue
c
      110 continue
c
    1000 format (i5,6f5.0)
      end

```

After the influence coefficient matrix **B** is computed, the effective dynamic force vector **P_{eff}** is obtained by using equation (H.11). The solution procedure for dynamic equation of motion is the same for the case of the rigid support excitation. The code to compute the effective dynamic force vector in LOADF subroutine is developed as follows

```

c
  subroutine loadf (b,z,p,neq,nfn,nt)
c
  implicit double precision (a-h, o-z)
  dimension b(neq,nex),xm(neq),p(neq),pp(neq,nt),z(nt,nfn)
c
  common/misc/ mfn,ngm,mmm(3),igm(3),rrr(3),fgm(3)
  common/imse/ ims,nex
c
  form dynamic load vector at each time step
c
  rewind 9
  read (9) xm
  rewind 12
c
  do 20 i=1,neq
    do 20 j=1,nt
      xx=0.
      do 10 k=1,neq
        do 10 l=1,nfn
          if (i.ne.k) xx=xx+0.
          if (i.eq.k) xx=xx+xm(i)*b(k,l)*z(j,l)
10      continue
20      pp(i,j)=xx
c
      do 30 j=1,nt
        do 40 i=1,neq
40      p(i)=-pp(i,j)
        write (12) (p(i), i=1,neq)
30      continue
c
  return
  end

```

Sample input

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.3 \\ 0.2 & 0.1 \end{bmatrix} \text{ and } UG = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Resulting output

$$P = \begin{bmatrix} 1.2 \\ 3.3 \\ 1 \end{bmatrix}$$

Notice that multiple-support excitation is considered in the computation process of effective dynamic load, i.e., the left side of the equation of motion remains intact and only its right side needs to be modified. NEABS program can perform dynamic analysis for multiple-component excitation but not for multiple-support excitation. NFN (number of time functions) therefore has value in the range from 1 to 3, i.e., $igm = [1 \ 2 \ 3]$ and NE (number of support excitations) is equal to number of supports of the structure.

Effective dynamic load is computed through `loads` subroutine following three steps: (1) compute `neq` by `nfn r` matrix and `neq`-vector `mass` through `loadr` subroutine; (2) read time histories `nt` by `nfn z` matrix through `inthis` subroutine; and (3) compute dynamic effective load `neq` by `nt p` matrix through `loadv` subroutine. It is noted that `neq`-vector `mass` is obtained from $6 \times \text{numnp}$ `ID` array (6 is the number of DOFs per node), i.e., let the `ID` array is,

$$\mathbf{ID} = \begin{bmatrix} 0 & 0 & 0 & 1 & 3 & 5 & 7 & 9 & 11 \\ 0 & 0 & 0 & 2 & 4 & 6 & 8 & 10 & 12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence the neq-vector **mass** is,

$$\mathbf{mass} = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12]$$

The above procedure is applied for the case of multiple-component excitation. For the case of the multiple-support excitation, INFLC subroutine is required to compute influence coefficient matrix **B** and some modifications to the sub- subroutines are necessary.

Appendix I

Inverse of a Matrix

The inverse of a given matrix is sometimes required, i.e., the inverse of $k_{\beta\beta}$ in equation (6-117). For the case of lower order matrix (i.e., $n=2$ or 3), the inverse of a matrix can be obtained directly from formulae. For higher order of matrix the procedure for calculating inverse of a given matrix A is obtained from a solving a system of linear equations (Conte and Boor, 1980). The j th column $A^{-1}i_j$ of the inverse matrix A^{-1} is the solution of the linear system,

$$Ax = i_j \quad (I.1)$$

The matrix A is written in the form,

$$A = LU \quad (I.2)$$

where L is a lower triangular matrix and U is upper triangular matrix.

Substituting equation (I.2) into equation (I.1) leads to,

$$LUx = i_j \quad (I.3)$$

Equation (I.1) can be divided into two triangular systems,

$$Ly = i_j \quad (I.4)$$

and

$$Ux = y \quad (I.5)$$

Thus, equation (I.4) can be solved for y by forward substitution, and equation (I.5) can be solved for x by back substitution. Hence, the solution of equation (I.1) can be obtained by two steps: (1) matrix factorization (Gauss elimination); and (2) solve each of the n systems (Forward- and back-substitution).

I.1 Gauss elimination

Let W is an n by $(n+1)$ working array or matrix which contains initially the coefficients and right side of the n equations $Ax = b$. Whenever some unknown is eliminated from an equation, the changed coefficients and right side for this equation are calculated and stored in the working array W in place of the previous coefficients and right side. For reasons to be made clear below, we store the multiplier $m = a_{ik}^{(k-1)}/a_{kk}^{(k-1)}$ (used to eliminate x_k from the i th equation) in w_{ik} in place of the number $a_{ik}^{(k-1)}$ since the latter is zero anyway. We also record the row interchanges made with the aid of an integer array p . Gauss elimination procedure is summarized as follows

- (a) Initialize the n -vector p to have $p_i = i, i = 1, \dots, n$
- (b) For $k = 1, \dots, n-1$ find (the smallest) $i \geq k$ such that $w_{ik} \neq 0$. If no such i exists, signal that A is not invertible and stop. Otherwise, exchange the contents of p_k and p_i , and of rows k and i of W and continue.
- (c) For $i = k+1, \dots, n$ compute

$$m = w_{ik} = w_{ik}/w_{kk} \quad (I.6)$$

(d) For $j = k+1, \dots, n+1$ compute

$$w_{ik} = w_{ik} - mw_{kj} \quad (I.7)$$

If $w_{nn} = 0$, signal that **A** is not invertible and stop

I.2 Forward and back-substitution

The solution of the equation (I.1) by forward- and back-substitution is summarized as follows. Given a final contents of the first n columns of the working array **W** and the n -vector **p**, the solution is obtained

(a) For $k = 1, \dots, n$ compute

$$y_k = b_{p_k} - \sum_{j=1}^{k-1} w_{kj} y_j \quad (I.8)$$

(b) For $k = n, n-1, \dots, 1$ compute

$$x_k = \frac{\left(y_k - \sum_{j=k+1}^n w_{kj} x_j \right)}{w_{kk}} \quad (I.9)$$

The vector $x = \{x_i\}$ now contains the solution of $Ax = b$.

A code is developed to compute the inverse of a given matrix. Example of the input and output for calculating the inverse of a matrix is described as follows.

Sample input

2 1 3
0 1 2
0 0 3

Resulting output

THE COMPUTED INVERSE IS

ROW 1 $1/2$ $-1/2$ $-1/6$
ROW 2 0 1 $-2/3$
ROW 3 0 0 $1/3$

Appendix J

NEABS User's Guide Supplement

J.1 Ritz methods

Control card

A line is added for to specify the Ritz method. Format and meaning of the parameters are as follows

Columns

1-10	METHD	Type of Ritz method
		1: Subspace method
		2: Lanczos method
		3: Load dependent method
11-20	MNOR	Type of analysis
		≥ 2 : Modal analysis
		< 2 : Time history response analysis

Ritz method parameters

Lanczos method (4 lines)

Line 1

Columns

1-5	NROOT	Number of vectors required
6-10	NC	Number of trial iteration vectors used

Line 2	Column	11-15	IFSS	Flag for Sturm sequence check
				0: No Sturm sequence check
				1: Sturm sequence check
		16-20	IVM	Type of starting vector
				1: static displacement vector
				2: vector of mass
				3: arbitrary vector
Line 3	Column	1-5	ENDL	Lowest wanted eigenvalue
		6-10	ENDR	Highest wanted eigenvalue
Line 4	Column	1-5	IVEC	Iteration vector type (default is 0)
Load dependent method (3 lines)	Columns	1-5	MPF (1)	Mass participation factor in X-dir
		6-10	MPF (2)	Mass participation factor in Y-dir
		11-15	MPF (3)	Mass participation factor in Z-dir
Line 1	Columns			
		1-5	NC	Number of trial iteration vectors used

6-10	NSR	Flag for static residual
		0: Last vector is static residual
		1: No static residual in basic

Line 2

Column

1-5	IVEC	Iteration vector type (default is 0)
-----	------	--------------------------------------

Line 3

Column

1-5	MPF (1)	Mass participation factor in X-dir
6-10	MPF (2)	Mass participation factor in Y-dir
11-15	MPF (3)	Mass participation factor in Z-dir

Subspace method (2 lines)

Line 1

Column

1-5	NROOT	Number of modes required
6-10	NITEM	Number of maximum iteration vectors
11-15	NC	Number of trial iteration vectors used
16-20	KKK	Type of starting vectors
		1: Standard procedure
		2: Lanczos algorithm
21-25	IFSS	Flag for Sturm sequence check
		0: No Sturm sequence check
		1: Sturm sequence check

26-30	IFPR	Flag for printing during iteration
		0: No printing
		1: Printing

Line 2

Column

1-5	IVEC	Iteration vector type (default is 0)
-----	------	--------------------------------------

Example of input parameters

Lanczos method

2	1		
20	40	1	1
0.0	10000.0		
0.0			
90.0	90.0	90.0	

Load independent method

33	1		
40	1		
0.0			
90.0	90.0	90.0	

Subspace method

1	1				
20	40	40	1	1	0
0.0					

J.2 Non-linear beam-column element

A new line is added to non-linear property line. The format and meaning of the parameters for this line are as follows:

Column

1-10	ENPROP (N, 13)	Critical plastic rotation angle at I-end
11-20	ENPROP (N, 14)	Maximum plastic rotation angle at I-end
21-30	ENPROP (N, 15)	Critical plastic rotation angle at J-end
31-40	ENPROP (N, 16)	Maximum plastic rotation angle at J-end

J.3 Damage index model parameters

Two lines are supplemented to the damage index model. The format and meaning of these lines are as follows:

Line 1

Columns

1-10	DIML	Damage index model
11-20	ACDI	Accumulative damage index

Line 2

Columns

1-10	ALPHA	Weighted coefficient for displacement
------	-------	---------------------------------------

11-20

BETA

Weighted coefficient for energy

J.4 Step-by-step time history static dynamic analysis

The input ground excitations are expanded to include multiple-support seismic excitations. A number of lines are supplemented to the dynamic analysis. The format and meaning of these lines are as follows:

Dynamic analysis control data line (3F10.0, I5)

Columns

1-10	DT	Time interval for integration in seconds
11-20	ALFA	The damping factor proportional to mass
21-30	BETA	The damping factor proportional to stiffness
31-35	INTG	Integration method indicator

Time increment control data line (I5, F10.0)

Columns

1-5	NSDIV	Number of subdivision
6-15	RTOLS	Relative tolerance of subdivision

Equilibrium iteration control data line (2I5, 2F10.0)

Columns

1-5	MAXIT	Maximum number of iterations
6-10	ITTYT	Iteration type
11-20	RTOLI	Relative tolerance to use for iteration
21-30	RTOLU	Relative tolerance of convergence

Input ground motion line (3I5, 3F10.0)

Columns

1-5	IGM(1)	Function number in X-direction
6-10	IGM(2)	Function number in Y-direction
11-15	IGM(3)	Function number in Z-direction
16-25	FGM(1)	Scale factor in X-direction
26-35	FGM(2)	Scale factor in Y-direction
36-45	FGM(3)	Scale factor in Z-direction

Dynamic load input data line (3I5, F10.0)

Columns

1-5	N	Nodal point number where load is applied
6-10	J	Displacement component number at which load is applied
11-15	K	Time function number
16-25	P	Scalar multiplier for scaling the time function

Prescribed dynamic displacement input data line (3I5, F10.0)

Columns

1-5	N	Nodal point number where displacement is applied
6-10	J	Displacement component number at which displacement is imposed
11-15	K	Time function number
16-25	P	Scalar multiplier for scaling the time

function

Multi-support seismic excitation data lines

Line 1 (2I5)

Columns

1-5	IMS	Multi-support excitation code (default=1)
6-10	NEX	Number of support excitation

This is followed by a blank line (default)

Line 2 (I5, 3I5)

Columns

1-5	NS	Support identification number
6-10	R(1)	Translational ground excitation in x-dir
11-15	R(2)	Translational ground excitation in y-dir
16-20	R(3)	Translational ground excitation in z-dir

Supply as many lines as required (NEX)

Time function data lines

Line 1

Columns

1-72	TITLE(12)	Title
------	-----------	-------

Line 2

Columns

1-5	NDT	Total number of digitized points
-----	-----	----------------------------------

6-15	SFTR	Scaling factor
------	------	----------------

16-25	TINC	Time interval of digitization
-------	------	-------------------------------

26-35	ZCOR	Zero correction
-------	------	-----------------

Line 3 (8F9.0)

Supply as many lines as required (NFN or NEX).

J.5 Program control (7I5)

Columns

1-5	N	Number of nodal points in the system, including all
-----	---	---

the auxiliary points needed for defining the element configuration in the structure

6-10	NELTYP	Number of element types
11-15	NUMEL	Total number of elements (linear and non-linear)
16-20	NFN	Number of input time functions
21-25	NGM	Ground motion input code
		-7: Dynamic analysis by superposition method
		-1: Frequency analysis
		0: Static analysis
		7: Dynamic analysis by direct integration method
		5: Multiple ground motion input
26-30	NT	Number of time steps to be computed
31-35	NOT	Number of time interval between two consecutive output points of response time history

Ten Principles

The following ten principles are taken from Abeles (1981) and applicable to any endeavor that the engineer is called upon to undertake (Nawy, 1996):

1. You cannot have everything. (Each solution has advantages and disadvantages that have to be tallied and trade off against each other.)
2. You cannot have something for nothing. (One has to pay in one way or another for something which is offered as a "free gift" into the bargain, notwithstanding a solution's being optimal for the problem.)
3. It is never too late (e.g., to alter a design, to strengthen a structure before it collapses, or to adjust or even change principles previously employed in the light of increasing knowledge and experience).
4. There is no progress without considered risk. (While it is important to ensure sufficient safety, overconservatism can never lead to an understanding of novel structures.)
5. The proof of the pudding is in the eating. (This is in direct connection with the previous principle indicating the necessity of tests.)
6. Simplicity is always an advantage, but beware of oversimplification. (The latter may lead to theoretical calculations which are not always correct in practice, or to a failure to cover all conditions.)
7. Do not generalize, but rather qualify the specific circumstances. (Serious misunderstandings may be caused by unreserved generalizations.)
8. The important question is how good, not how cheap an item is. (A cheap price given by an inexperienced contractor usually results in bad work; similarly, cheap, unproved appliances may have to be replaced.)
9. We live and learn. (It is always possible to increase one's knowledge and experience.)
10. There is nothing completely new. (Nothing is achieved instantaneously, but only by step-by-step development.)