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**AN IMPROVED METHOD FOR MODELLING
FULLY ROUGH TURBULENT BOUNDARY LAYER FLOWS**

by

Jean-François Gagné, B. Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirement for the degree of
Master of Engineering
in Aerospace Engineering

Ottawa Carleton Institute for Mechanical
and Aerospace Engineering

Department of Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario, Canada

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**"AN IMPROVED METHOD FOR MODELLING
FULLY ROUGH TURBULENT BOUNDARY LAYERS FLOWS"**

submitted by **Jean-François Gagné, B. Eng.**

in partial fulfilment of the requirement
for the degree of Master of Engineering

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Department of Mechanical and Aerospace Engineering

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CARLETON UNIVERSITY

Date 24 Sept./98

Abstract

A discrete element model has been implemented into an existing two-dimensional parabolic Navier-Stokes code for thin shear layers in order to model the effects of surface roughness on flow behaviour in boundary layers. The model modifies the equations of motion by including a form drag term in the momentum equation and accounting for the blockage effects of the roughness elements on the flow. Three turbulence models and a roughness element drag coefficient correlation were used for closure. The modified equations were derived in surface normal curvilinear co-ordinates and the effect of the roughness element blockage on the definition of flow parameters was assessed. It was found that the definitions of displacement and momentum thickness should be redefined to include blockage effects although this redefinition will not alter their values in a significant way. It was also shown that previous discrete element models use modified equations which are not consistent with the roughness blockage definitions. Validation of the implemented model was done using experiments involving roughness elements of two dimensional and three dimensional shape and agreement was found to be within acceptable error margins.

Original contributions in this thesis include :

- Derivation of the governing equations with improved definition of blockage factors in surface normal curvilinear co-ordinates ;
- Assessment of the effect of roughness blockage on flow parameters ;

- Implementation of the discrete element method in the thin shear layer computer code TSL ;
- Assessment of the performance of the discrete element method for roughness elements of two dimensional and three dimensional shape ;
- Assessment of the compatibility of the discrete element method in conjunction with the Baldwin-Lomax and $k-\omega$ turbulence models.

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Nomenclature

English Symbols

a	Cross-stream roughness dimension
a_i	Discretized standard equation coefficients
b	Stream-wise roughness dimension
d	Roughness diameter
h	Wall curvature parameter = $(1+y/R)$
k	Roughness height in Chapter 2 or kinetic energy of turbulence in Chapters 3 and 4
k_s	Equivalent sand roughness height
k_e	Turbulence kinetic energy at the free-stream edge
l	Cross-stream roughness separation distance
l	Mixing length
p_e	Pressure at the edge of the boundary layer
r	Radial co-ordinate
u	Stream-wise velocity component (usually denotes mean values; denotes instantaneous values in Section 3.2)
$\bar{u}$	Mean stream-wise velocity (in Section 3.2)
u'	Fluctuating turbulent velocity
u_e	Stream-wise velocity at the edge of the boundary layer
u^+	u/u_τ

u_τ	friction velocity = $(\tau_w/\rho)^{1/2}$
v	Stream-wise co-ordinate
x	Normal co-ordinate
y	Inner wall normal co-ordinate
y^+	yu_τ/ν
z	Cross-stream co-ordinate
A_x	Area open for flow in the stream-wise direction
A_y	Area open for flow in the normal direction
C	Constant in Dvorak's correlation for roughness effects (see Eq. 2.14)
C_D	Roughness element drag coefficient
C_f	Wall friction coefficient
C_P	Free-stream pressure coefficient
D	Roughness element drag
D	Diffusion conductance in the discretized general equation of Appendix C
F	Flow rate through a face of the control-volume of Figure C-1
G	Diffusive coefficient in the discretized general equation of Appendix C
H	Boundary layer shape factor = δ^*/θ
K	Smooth wall pressure gradient parameter, see Section 4.3.3
K_r	Rough wall pressure gradient parameter, see Section 4.3.3
L	Stream-wise roughness separation distance
L_{ref}	Reference length used for non-dimensionalisation

P	Peclet number, see Appendix C
R	Radius of curvature of the surface
Ri	Richardson number, see Section 3.3.2
Re _d	Reynolds number based on the roughness-element diameter
S	Curvature extra strain ratio, see Eqs. 3.41, 3.56 and 3.64
S _C	Constant source term
S _P	Variable source term coefficient
T	Temperature
U _{ref}	Reference velocity used for non-dimensionalisation
V	Velocity

Subscripts

inner	Inside the inner layer of the boundary layer
outer	Inside the outer layer of the boundary layer

Greek Symbols

α	Baldwin-Lomax-model correction factor
β	Clauser equilibrium parameter = $(\delta^*/\tau_w)(dp_e/dx)$
β_x	Stream-wise blockage factor
β_y	Normal blockage factor

δ	Boundary layer thickness
δ^*	Displacement thickness
ε	Dissipation rate of turbulence kinetic energy
ϕ	General flow parameter used in the standard equation of motion
κ	Von Karman constant ; $1/\kappa$ is the slope of the logarithmic velocity profile
λ	Roughness spacing parameter
μ	Fluid dynamic viscosity
θ	Angular co-ordinate
θ	Momentum thickness
ρ	Fluid density
τ	Shear stress
τ_t	Reynolds shear stress $= -\overline{\rho u'v'}$
τ_w	Shear stress at the wall
ν	Fluid kinematic viscosity
ν_e	Effective viscosity $= \nu + \nu_t$
ν_t	Turbulent eddy viscosity
ω	Specific dissipation rate of turbulence kinetic energy
ξ	Pressure gradient parameter $= (\delta/\tau_w)(dp_e/dx)$
ψ	Stretching parameter
Π	Coles wake parameter

Chapter 1

Introduction

The prediction of the skin friction of a surface immersed in a fluid flow has always been a major concern in the field of aerodynamics as the performance of aircraft and fluid machinery are strongly affected by skin friction. Accurate and reliable methods for predicting skin friction are thus of great importance. Surface roughness has an important influence on skin friction and prediction methods should be capable of including its effects. Many methods for modelling turbulent flow over smooth surfaces have been developed over the years but the effects of surface roughness have received relatively little attention and more research is required in this area. This thesis deals with a computational method that includes a model for predicting the effect of roughness on skin friction and flow development.

Most previous work has used a correlation approach to deal with surface roughness effects. A key difficulty with this approach is determining appropriate values of correlation factors for any particular roughness configuration. Another approach is the discrete element method in which the effects of roughness elements are represented directly by incorporating their aerodynamic drag and blockage into calculations of the near-wall flow.

The objectives of this thesis were to incorporate the discrete element approach into an existing computer code for flow over smooth surfaces, to verify the revised code and to validate its capability to accurately predict boundary layer development along surfaces covered with regular arrays of roughness elements. The existing code used was a modified version of the two-dimensional parabolic Navier-Stokes computer code for thin shear layers, called TSL, developed by Pajayakrit (1997).

This thesis is divided into 5 chapters, the first one being this introduction to the work at hand. In the second chapter, a review of the literature pertinent to the work is given, as well as some insight on how the governing equations are obtained and a description of the available experimental data cases with their limitations. Chapter 3 documents the steps involved in the implementation of the method into the existing code as well as a verification of this implementation. Chapter 4 presents comparisons between the results calculated with the discrete element approach and the experimental results and also discusses some possible reasons for discrepancies. The fifth chapter presents a discussion of the results and of their interpretation as well as conclusions which are drawn from this discussion and recommendations regarding the next steps to be taken.

Chapter 2

Literature Review

The ultimate objective of the present work is to improve prediction capabilities for flows that have rough boundary walls. This chapter reviews some aspects of the behaviour of such flows.

The modelling of the turbulence itself is very complex as the governing Navier-Stokes equations, though fundamental and rigorous, are non linear, non unique, complex and difficult to solve. Because of this, direct numerical solution of even the most simple of turbulent flows is prohibitively demanding on current computer resources (White, 1991). In order to solve this problem, most flow computations employ time averaged equations with semi-empirical turbulence models to achieve closure. Ideally, these semi-empirical turbulence models should rely as much as possible on sound physics pertaining to the mechanisms involved but as these mechanisms are not very well understood yet, most of the actual models are evaluated more on the basis of the accuracy of their obtained results than on the realism of their foundations. This is also true of the methods used to model the surface roughness. Some of the processes underlying those empirical relations were however deduced from physical principles and we will now review some of those principles.

2.1. Wall Bounded Flows

The phenomena to be modelled here are extremely complex and involve many characteristics such as viscous effects, non-linearity and turbulence. However, in flows with a high enough Reynolds number, the effects of viscosity are often confined to a very thin region called the shear layer or boundary layer. Outside this region, the flow can be computed using the inviscid flow theories by putting the properties calculated in the shear layer as boundary conditions for the inviscid calculations. It can therefore be seen that the viscous effects will also influence the inviscid region but through an indirect effect. In some flows however, such as fully developed pipe flow or flow in turbo-machinery passages, the viscous effects directly influence the majority or even all the flow field. In this case, one generally needs to solve the Navier-Stokes equations for the entire flow field. In either approach, accurate prediction of the behaviour of the flow near the wall is particularly important.

The flows which are of interest in the present work are those which are bounded by at least one solid surface. In order to solve the equations of motion which will be described in the next sections we need to have appropriate boundary conditions at the solid surface. White (1991) shows that for flows with relatively low Mach numbers, the flow can be considered to have no slip conditions at the solid surface. This implies that the velocity near the wall (V_w) is the same as that of the solid boundary or, in the case where the co-ordinate system is attached to the solid boundary, $V_w=0$. We also assume that there is no temperature jump at the wall/fluid interface so that $T_w=T_{\text{solid}}$.

Since in most engineering applications the Reynolds number is such that the flow is turbulent, there exists a region within the boundary layer in which the flow goes from zero to quite a substantial velocity in a very short distance. In this region very near the wall, called the viscous sub-layer, the boundary layer is mainly dominated by viscous (molecular) shear and the effects of turbulence are damped out. This region is characterised by relatively high shear stress and shear strain rates. Above the viscous sub-layer the flow is turbulent but is only directly dependent upon parameters which are affected by local conditions and can well be described by the "law of the wall". Together with the viscous sub-layer, this region of the flow is called the inner layer or law of the wall layer. Outside of the inner layer, turbulent mixing dominates and the wall merely acts as a source of retardation for the flow in a way that is strongly influenced by the pressure gradient. In solving the flows, the differential equations of motion can either be integrated right from the wall, or the "wall function" approach can be used. In the latter approach, law of the wall expressions are used for the near wall regions of the overall solutions. Regardless of the solution approach that is used, knowledge of the behaviour of the near wall flow is crucial. The direct effects of roughness are concentrated in the near wall region which is thus of particular interest in this thesis work.

2.2. The Law of the Wall for Smooth Walls

As was mentioned above, the boundary layer can be divided into inner and outer layers. In the inner layer, wall effects directly control the flow and in the outer layer,

mixing and pressure gradient effects dominate. There is also an overlapping zone which smoothly joins the two layers. All these sub-sections are depicted in Figure 2.1, where y^+ is a normalized normal co-ordinate (see Eq. 2.8) and δ is the boundary layer thickness.

Figure 2.1. Sub-layer distribution of the boundary layer.

2.2.1. Modelling Equations

Prandtl (1926) theorised that in the inner layer, the flow behaviour and so the velocity profile should be independent of free stream parameters and depend only upon wall shear stress, fluid properties and distance from the wall :

$$u_{inner} = f(\tau_w, \rho, \mu, y) \quad (2.1)$$

Von Karman (1930) proposed that in the outer layer, the viscous effects should be dominated by the momentum exchange due to turbulent eddy motion which causes a velocity defect, reducing the local velocity below the free stream velocity in a way which depends upon inertial effects such as free stream pressure gradient and boundary layer thickness.

$$u_{outer} = g(\tau_w, \rho, \delta, y, dp_e/dx) \quad (2.2)$$

Finally, in order to have a smooth and continuous velocity profile, there must be an overlapping region where both Eqs. (2.1) and (2.2) are valid.

$$u_{inner} = u_{outer} \quad (2.3)$$

From dimensional analysis, one can reduce Eqs. (2.1) and (2.2) to the form

$$\frac{\bar{u}_{inner}}{u_\tau} = f\left(\frac{yu_\tau}{\nu}\right) \quad \text{in the inner layer} \quad (2.4)$$

and
$$\frac{u_e - \bar{u}_{outer}}{u_\tau} = g\left(\frac{y}{\delta}, \xi\right) \quad \text{in the outer layer} \quad (2.5)$$

$$\text{where } u_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad \text{and} \quad \xi = \frac{\delta}{\tau_w} \frac{dp_e}{dx}$$

To satisfy Eq. (2.3), the functions f and g of Eqs. (2.4) and (2.5) must be logarithmic in the overlap zone which yields the familiar logarithmic law of the wall for smooth walls (White, 1991) :

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{yu_\tau}{\nu}\right) + B \quad (2.6)$$

where κ and B are near universal constants.

Although Nikuradse originally proposed values for κ and B of 0.4 and 5.5 respectively, White (1991) suggests using the more modern values of Coles and Hirst (1968) i.e.

$$\kappa = 0.41 \quad \text{and} \quad B = 5.0$$

Eq. (2.6) is valid in the inner layer all the way to very near the wall ($y \cdot u_\tau / \nu \approx 50$) where turbulence begins to be damped out; for $y \cdot u_\tau / \nu < 5$ the flow is dominated by viscous shear. In this region, the viscous sub-layer, the Reynolds stresses are negligible and the velocity profile becomes linear so that

$$\tau_w = \frac{\mu u}{y} \quad \text{or} \quad \frac{u}{u_\tau} = \frac{y u_\tau}{\nu} \quad (2.7)$$

Between the viscous sub-layer and the logarithmic region, i.e. for $5 < y \cdot u_\tau / \nu < 50$, there is a buffer layer where both the viscous and Reynolds shear stresses are of the same order of magnitude and where the velocity is neither linear nor logarithmic and cannot be described by any simple relation of $u = f(y)$. In order to simplify the expression of the velocity profile over the entire inner layer, Spalding (1961) proposed a single composite formula that fits the inner law data all the way from the wall to the point where the outer layer velocity begins to rise above the logarithmic curve :

$$y^+ = u^+ + e^{-\kappa B} \left[e^{\kappa u^+} - 1 - \kappa u^+ - \frac{(\kappa u^+)^2}{2} - \frac{(\kappa u^+)^3}{6} \right] \quad \text{where } y^+ = \frac{y \cdot u_\tau}{\nu} \quad (2.8)$$

and $u^+ = \frac{u}{u_\tau}$

White (1991) considers Eq. (2.8) to be completely accurate in the inner layer but cautions about its use in the outer layer as it becomes very sensitive to the pressure gradient parameter ξ of Eq. (2.5). In this region, the turbulence is characterised by large eddies, elongated in the main flow direction and the flow is quite similar to that observed in free shear layers such as jets and wakes. Clauser (1954) suggested that if the pressure gradient parameter ξ is constant, the outer layer will be in equilibrium and so its gross parameters can be scaled in terms of a single parameter, $\beta = (\delta^*/\tau_w)(dp_e/dx)$ which is preferred over $\xi = (\delta/\tau_w)(dp_e/dx)$.

Coles (1956) noted that the deviation of the outer layer velocity from the log-law, Eq. (2.6), has a wake like shape when viewed from the free stream. He therefore added this wake relation to the log-law to get an accurate approximation to the velocity distribution over both the overlap and the outer layers :

$$u^+ = \frac{1}{\kappa} \ln(y^+) + B + \frac{2\Pi}{\kappa} f\left(\frac{y}{\delta}\right) \quad (2.9)$$

Where Π is called *Coles wake parameter* and is related to Clauser's equilibrium parameter by the approximate correlation given by Das (1987) :

$$\beta \approx -0.4 + 0.76\Pi + 0.42\Pi^2 \quad (2.10)$$

The additive function $f(y/\delta)$ in Eq. (2.9) can be expressed by either of the two following curve fits :

$$f\left(\frac{y}{\delta}\right) \approx \sin^2\left(\frac{\pi}{2} \cdot \frac{y}{\delta}\right) \approx 3 \cdot \left(\frac{y}{\delta}\right)^2 - 2 \cdot \left(\frac{y}{\delta}\right)^3 \quad (2.11)$$

The last expression is somewhat easier to use in integral theories.

2.3. The Effects of Roughness on the Law of the Wall

Although wall roughness has little influence on laminar flows, even a small roughness will break up the thin viscous sub-layer and greatly increase the wall friction in turbulent flows. Since perfectly smooth surfaces are seldom encountered in engineering applications, it becomes necessary to be able to model the effect that roughness will have on the flow.

The direct effects of the roughness on the flow are concentrated in the wall region and will therefore only alter the velocity profile shape in this region. Hama (1955) theorised that this roughness effect could be taken into account in the law of the wall by adding an arbitrary intercept-shift function $\Delta u/u_\tau$ to the logarithmic velocity profile :

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{y \cdot u_\tau}{\nu} \right) + B - \frac{\Delta u}{u_\tau} \quad (2.12)$$

The first attempts to quantify this roughness effect were made by Nikuradse (1933) and Prandtl and Schlichting (1934) using standard grain roughness, requiring the surface to be covered with sand grains in a manner giving the highest sand density possible. This then enables the roughness to be described completely by the height of the sand, which is determined as the "size of mesh of the coarser of the two sieves through which the sand was sifted" (Prandtl, 1960), which corresponds to the maximum sand grain size. Nikuradse's (1933) data suggested that the effect of roughness should depend on wall variables only so that by dimensional analysis one could write:

$$\frac{\Delta u}{u_\tau} = f\left(\frac{k \cdot u_\tau}{\nu}\right) \quad (2.13)$$

where k is a typical length scale for the roughness elements. In Nikuradse's (1933) experiment, k was taken as the height of the sand grain as defined above and was thereafter labelled "standard sand-grain size" and often denoted as k_s . Eq. (2.13) was also confirmed by Hama (1955) for boundary layers as well as for pipe flow. Clauser (1956) suggested that the function f of Eq.(2.13) should be logarithmic for "fully rough" or "aerodynamically rough" surfaces and so the following form was adopted (Dvorak, 1969) :

$$\frac{\Delta u}{u_\tau} = \frac{1}{\kappa} \log\left(\frac{k \cdot u_\tau}{\nu}\right) + C[\lambda] \quad (2.14)$$

where C is a constant specified for any roughness geometry and depends upon λ , which is a roughness spacing parameter. The terms "fully rough" and "aerodynamically rough" are used to denote surfaces whose roughness is sufficiently great that velocity profiles and skin friction coefficient are independent of Reynolds number, that is independent of viscosity, ν . Note that when Eq. (2.14) is substituted into Eq. (2.12), the viscosity drops out and u/u_τ becomes a function of ku_τ/ν and λ only.

Schlichting (1936) proposed an alternative to eliminate the dependence on roughness type by introducing the concept of "equivalent sand" roughness. He defined this parameter as the size of sand as used in Nikuradse's (1933) experiment which would give the same resistance as that observed on a particular rough surface. He determined a

value of k_s for each of the surfaces in his experiment by evaluating C for each of these surfaces and comparing it to the standard sand value of -3 found by Nikuradse. By re-expressing Eq. (2.14) he then found a correlation between the ratio k_s/k and C . One of the basic assumptions behind his procedure is that all of the experimental data in his experiment were for measurements in fully rough flow. The argument of the function of Eq. (2.13), usually referred to as the roughness Reynolds number, has been used by many authors to characterise the roughness regime of the flow. For Nikuradse's (1933) experiment, Prandtl and Schlichting (1934) found the following regimes:

$$\frac{k \cdot u_\tau}{\nu} < 5$$

hydraulically smooth : The roughness has no apparent effect on the flow $\left(\frac{\Delta u}{u_\tau} \approx 0 \right)$.

$$5 \leq \frac{k \cdot u_\tau}{\nu} \leq 60$$

Transitionally rough : The roughness effect depends on the roughness Reynolds number ($C[\lambda]$ is a function of $\frac{k \cdot u_\tau}{\nu}$)

$$\frac{k \cdot u_\tau}{\nu} > 60$$

Fully rough: The roughness effect is independent of viscosity and consists mainly of form drag ($C[\lambda]$ is constant).

Although all three regimes have been identified by many authors, the above numerical values are valid only for the sand grain experiment of Nikuradse (1933) and values for other roughness types and distribution may vary somewhat from author to author. The differences between those values may even go as far as obtaining transitional

flow at ku/ν values as high as 200 (Chen and Roberson, 1974). In the same paper as he introduced the concept of equivalent sand-grain roughness, Schlichting (1936) proposed a concept that might give an explanation for this. He suggested that flow resistance of a rough surface be divided into two components: that due to form drag on the roughness elements and that due to the viscous shear on the smooth surface area between these elements. Therefore, when the spacing between roughness elements becomes sufficiently large, the form drag ceases to be the dominant source of friction and the viscous effects of the smooth surface can render a flow transitionally rough or even hydraulically smooth, even at roughness Reynolds numbers exceeding the standard sand fully rough values given above.

2.4. Approaches to Modelling the Roughness Effects

Two main approaches have been used to model the effects of surface roughness: the correlation approach (also called the classic equivalent sand-grain roughness approach) and the discrete element approach. The first method, originally proposed by Schlichting (1936), has been widely used due to its ease of implementation. However, the accuracy of the results that are obtained depends greatly upon the value chosen for the equivalent standard sand roughness, as explained above. To find appropriate relations between the original standard sand roughness used by Schlichting and the actual roughness geometry is a science of its own. It is for this reason that, apart from the following section, this thesis concerns itself mainly with the second approach, i.e. the

discrete element approach, as it was considered to be more in touch with the physics of the flow than the first.

2.4.1. The Correlation Approach

This approach consists of relating the roughness geometry of concern to a roughness geometry for which the effects on the flow are known. The first to propose this approach was Schlichting (1936) in his attempt to link the drag coefficient of roughened flat plates to the results of Nikuradse (1933). Following in his path, many other authors have since then tried to improve his correlations.

2.4.1.1. The Equivalent Sand-Grain Roughness Approach

As mentioned in section 2.3., Schlichting (1936) suggested a method to eliminate the dependence of the shift in velocity profile on roughness type by relating all the roughness geometries to the sand-grain roughness used by Nikuradse (1933). He proposed that when the resistance to the flow was independent of Reynolds number (fully rough regime), experiments finding the resistance of a surface roughened with the specific roughness type to the flow would, by comparison to the resistance of Nikuradse's (1933) roughness resistance, yield flow properties for any surface covered with that roughness type. His approach was already described in section 2.3.

2.4.1.2. The Dvorak Approach

Dvorak (1969) suggested an approach, which, in his view, was less limited in terms of range of validity of the skin friction relationship than those preceding it. He indeed argues that most available relations are limited to the use of sand roughness in the fully rough regime and to zero pressure gradients. He therefore proposed a method of calculation, stemming from the results of Bettermann (1966), which enabled the prediction of the turbulent boundary layer over rough surfaces, in pressure gradients, using a parameter which had been neglected in previous studies: the effect of roughness density. He also extended his correlation to the transitionally rough regime.

To formulate his calculation method, Dvorak (1969) started from Hama's (1955) expression for the skin friction law for rough surfaces in zero pressure gradient:

$$\frac{u_e}{u_\tau} = \left(\frac{\tau_w}{\rho u_e^2} \right)^{-1/2} = \left(\frac{2}{C_f} \right)^{1/2} = A \cdot \log_{10} \left(\frac{u \cdot \delta^+}{\nu} \right) + B - \frac{\Delta u}{u_\tau} \quad (2.15)$$

Using the definition of $\Delta u/u_\tau$ of Eq. (2.14) and data from Bettermann (1966) and from Schlichting (1960), he then suggested the following correlation for $C[\lambda]$ in the fully rough regime :

$$\begin{aligned} C[\lambda] &= 17.35 \cdot (1.625 \cdot \log_{10} \lambda - 1) & \text{for } \lambda \leq 5 \\ C[\lambda] &= -5.95 \cdot (1.103 \cdot \log_{10} \lambda - 1) & \text{for } \lambda > 5 \end{aligned} \quad (2.16)$$

The intercept of Eqs. (2.16) is at $\lambda=4.68$ and the correlation is shown in Fig. 2.2.

Dvorak then extended the use of Eq. (2.15) into the transitionally rough regime by using a

logarithmic polynomial to interpolate between the hydraulically smooth and the fully rough limits. That is:

$$\frac{\Delta u}{u_\tau} = C_0 + C_1 \cdot \ln\left(\frac{u_\tau \cdot k}{\nu}\right) + C_2 \cdot \ln^2\left(\frac{u_\tau \cdot k}{\nu}\right) + C_3 \cdot \ln^3\left(\frac{u_\tau \cdot k}{\nu}\right) \quad (2.17)$$

where the constants C_0 , C_1 , C_2 and C_3 are evaluated by finding the correct value of $\Delta u/u_\tau$ and its derivative for two values of $u_\tau k/\nu$ chosen as the upper limit for aerodynamically smooth flow and the lower limit of fully rough flow. He justified this approach on the basis of the measurements of Nikuradse (1933) and Hama (1954) and the calculations of Granville (1958).

Figure 2.2. Graphical representation of the value of the constant C as a function of the roughness density λ .

Dvorak (1969) also extended his method to flow with pressure gradients by adding another term to Eq. (2.15) in the form of $\Delta u_2/u_\tau$ as found by Arndt and Ippen (1967):

$$\begin{aligned} \frac{\Delta u_2}{u_\tau} &= 1.253 \cdot (G - 6.7) \quad \text{for } G \geq 6.7 \text{ (adverse pressure gradient)} \\ \text{or} \\ \frac{\Delta u_2}{u_\tau} &= 0.404 \cdot (G - 6.7) \quad \text{for } G < 6.7 \text{ (favorable pressure gradient)} \end{aligned} \quad (2.18)$$

Where G is Clauser's (1954, 1956) velocity profile shape and is given by :

$$G = \lambda \left(1 - \frac{1}{H} \right) \quad \text{where } H = \frac{\delta^*}{\theta} \quad \text{and } \lambda = \sqrt{\frac{2}{C_f}} \quad (2.19)$$

The value of G is related to the pressure gradient parameter, β , for equilibrium flows.

The skin friction law, Eq. (2.15), can therefore be rewritten as (using Bettermann's experimental values):

$$\left(\frac{2}{C_f} \right)^{1/2} = 5.6 \cdot \log_{10} \left(\frac{u \cdot \delta^*}{\nu} \right) + 4.9 - \frac{\Delta u_1}{u_\tau} + \frac{\Delta u_2}{u_\tau} \quad (2.20)$$

Where $\Delta u_1/u_\tau$ is evaluated from either of Eq. (2.16) or Eq. (2.17) and $\Delta u_2/u_\tau$ is evaluated from Eq. (2.18).

2.4.2. The Discrete Element Approach

2.4.2.1. Description of the method

In this approach, the effect of the presence of a collection of individual roughness elements on the flow is considered, generally by including a form-drag term in the momentum equation and accounting for the blockage effect of the roughness elements on the flow. As mentioned in section 2.3., Schlichting (1936) himself was the first to propose the principle that would become the foundation of this approach : that the flow resistance be divided into two separate components, mainly form drag on the roughness elements and viscous shear on the smooth surface between those elements. Subsequent workers took this idea, which Schlichting (1936) had only used in a brief, simple analysis, and pushed it a bit further by incorporating it in a full computational model. Finson and Clarke (1980) first did so by casting the governing boundary layer equations in a form to account for the blockage effects of the roughness elements. The form drag contribution of the individual elements is then described by adding a sink term in the momentum equation. Lin and Bywater (1980) decided to include modifications in their turbulent kinetic energy model equation; their modifications depended on k_s and included blockage effects in addition to including the different sinks and sources into the governing equations. Taylor et. al. (1985) have used the same approach as that of Finson et. al. (1980) but with a somewhat more thorough definition of the blockage factors. It will be shown in the next section however that their formulation of the governing equations is rather nebulous. Finally, Tarada (1987) has used the roughness model of Taylor et. al. (1985) with a modification of the k - ϵ model equation which accounts for the

roughness effects. The model used in the present study is a modified version of that of Taylor et. al. (1985), i.e. without the modifications to the turbulence models, as it was not clear a priori which approach would give the best results. It was therefore decided to keep the model to its simpler form, perhaps enabling later research to verify if including the roughness element effects in the turbulence models improves the accuracy of the present method.

2.4.2.2. Derivation of the New Modelling Equations

The modelling equations to be used here are the continuity equation as well as the stream-wise and normal momentum equations known as the Navier-Stokes equations. However, in order to evaluate the effect of the roughness elements, a blockage factor as well as a drag term must be introduced into these equations. We therefore offer here a derivation of the new equations for steady two-dimensional plane flow, in Cartesian coordinates.

For this derivation, we refer to Figure 2.3, which shows the control volume including an array of identical roughness elements of arbitrary shape used for the derivation of the governing equations. It is essential to notice from this figure that the areas of the control volume available for mass and momentum transport in the yz -plane (A_x) and the xz -plane (A_y), as well as the mass of fluid present in the control volume are decreased by the presence of the roughness elements. The areas on which shear stresses

Figure 2.3. Control volume with roughness elements.

and pressure forces act are affected in the same way. This blockage effect is taken into account by making use of the blockage factors β_x and β_y . These are defined as the fraction of area open for flow, through the yz and xz planes, respectively (Taylor et. al., 1985).

Note that in the most general case, these factors are functions of x and y . Since both the available area and volume are affected by these factors, it would seem appropriate to average these factors over the distances Δx and Δy . In the case of regular identical roughness elements, this implies the identity

$$\beta_x = \beta_y \quad (2.21)$$

This can be shown by noting from Figure 2.3 that, in the stream-wise direction, the area fraction available for flow around a single roughness element, at height y , is :

$$\beta_x = \frac{(l - \bar{a})\Delta y}{l\Delta y} = 1 - \frac{\bar{a}}{l} \quad (2.22)$$

where $\bar{a}$ is the average roughness width over the control volume which is obtained by evaluating the integral $\bar{a} = \frac{1}{L} \int_x^{x+L} a(x) dx$. For triangular elements as shown in

Figure 2.3., $\bar{a} = \frac{ab}{2L}$. Note that in general a and b vary with y so that $\bar{a}$ and β_x are

functions of y . Similarly, it can be seen that the available area fraction in the normal direction is :

$$\beta_y = \frac{L(l - \bar{a})}{Ll} = 1 - \frac{\bar{a}}{l} \quad (2.23)$$

which demonstrates the identity of Eq. (2.21) even for an arbitrary cross section as that of Figure 2.3.

This identity was recognised by Taylor et. al. (1985), but only for circular cross-sectioned elements. As shown above, the identity actually holds true for any regular array of identical roughness elements, provided that only solid blockage is important. There is however the possibility that the two blockage factors may be different when one considers the wakes that might originate from the presence of the roughness elements in the flow. For this reason , the following derivation has been done using β_x and β_y as distinct symbols to keep the equations as general as possible in case future research shows that the wakes of the roughness element do have an important influence. It must however be noted that Taylor et. al. (1985) suggest that the blockage factors are determined solely from the roughness element geometry, in which case the identity of Eq. (2.21) should apply.

2.4.2.2.1. The Continuity Equation

Taylor et. al. (1985) suggest using the following equation for the law of conservation of mass:

$$\frac{\bar{\partial}}{\partial x}(\rho\beta_x u) + \frac{\bar{\partial}}{\partial y}(\rho\beta_y v) = 0 \quad (2.24)$$

However, if one can state that the blockage factor is identical in both the x and the y directions, this equation merely reduces to the usual continuity equation :

$$\frac{\bar{\partial}}{\partial x}(\rho u) + \frac{\bar{\partial}}{\partial y}(\rho v) = 0 \quad (2.25)$$

2.4.2.2.2. The Stream-Wise Momentum Equation

Here we suppose that the mass of the flow can be calculated using either of the blockage factors. Taylor et. al. (1985) suggest using the following equation :

$$\begin{aligned} \beta_x \rho u \frac{\partial u}{\partial x} + \beta_y \rho v \frac{\partial u}{\partial y} = & -\frac{\partial}{\partial x}(\beta_x P) + \frac{\partial}{\partial y} \left[\beta_y \left(\mu \frac{\partial u}{\partial x} - \rho \overline{u'v'} \right) \right] \\ & - \frac{1}{2} \rho C_D d(y) \frac{u^2}{Ll} \left(1 + \frac{\overline{u'^2}}{u^2} \right) \end{aligned} \quad (2.26)$$

where $d(y)$, L and l are defined as in Figure 2.3. and C_D is the drag coefficient of the roughness elements.

It has come to the author's attention that the use of two different blockage factors in the left hand side of Eq. (2.26) seems inconsistent with the fact that both should stem from the same definition of the mass contained in the control volume. If we assume that the mass of moving fluid in the stream-wise direction is dictated by β_x , the Equation should therefore be :

$$\begin{aligned} \beta_x \rho u \frac{\partial u}{\partial x} + \beta_x \rho v \frac{\partial u}{\partial y} = & -\frac{\partial}{\partial x}(\beta_x P) + \frac{\partial}{\partial y} \left[\beta_y \left(\mu \frac{\partial u}{\partial x} - \rho \overline{u'v'} \right) \right] \\ & - \frac{1}{2} \rho C_D d(y) \frac{u^2}{Ll} \left(1 + \frac{\overline{u'^2}}{u^2} \right) \end{aligned} \quad (2.27)$$

Use of Eq. (2.27) will however bring problems when one wants to put it in the conservation form by using the formulation of Eq. (2.24) for the continuity equation as terms in β_x and β_y will not cancel each other out. When one uses either Eqs. (2.25) and (2.27) or Eqs. (2.24) and (2.26), this problem does not arise. In the computer code, it was

assumed that the identity of Eq. (2.21) does hold to put the equations in conservation form and Eqs. (2.25) and (2.27) were used.

As can be seen from Eq. (2.27), an empirical model is needed to evaluate the drag coefficient, C_D . Taylor et. al. (1985) suggest using a model which is based on the local roughness element Reynolds number,

$$Re_d(y) = \frac{u(y)d(y)}{\nu} \quad (2.28)$$

which includes roughness element size and shape information through $d(y)$. Using data from Schlichting (1936) and the general shape of the drag coefficient versus Reynolds number curves for flow past transverse cylinders, Taylor et. al. (1985) suggest :

$$\begin{aligned} \log C_D &= -0.125 \log (Re_d) + 0.375 & (Re_d > 6 \times 10^4) \\ C_D &= 0.6 & (Re_d \leq 6 \times 10^4) \end{aligned} \quad (2.29)$$

The wall shear stress is then defined as the sum of the drag and the shear forces on the wall in the mean flow direction divided by the plan area of the wall. The corresponding skin friction coefficient is then :

$$C_f = \frac{\left(\beta_y\right)_w \mu \frac{\partial u}{\partial y} \Big|_w + \frac{1}{2} \frac{1}{Ll} \int_0^{\infty} (\rho d C_D u^2) dy}{\frac{1}{2} \rho_e u_e^2} \quad (2.30)$$

2.4.2.2.3. The Normal Momentum Equation

Since all the terms which would be affected by the stream-wise blockage factor are negligible in this equation, only the normal blockage factor remains and can therefore be neglected as with the continuity equation, yielding the familiar normal momentum equation :

$$0 = -\frac{\partial}{\partial y} \left(\frac{P}{\rho} + \overline{v^2} \right) \quad (2.31)$$

2.4.2.2.4. The Boundary Conditions

One of the advantages of the discrete element method over other roughness simulation methods is that no special boundary conditions are required. There is no need, as in many other methods, to define an effective wall location ($y = 0$) using an intercept of velocity profiles or any other method. The wall location is simply the smooth surface on which the roughness elements occur. An exception to this is the case of spheres packed in the most dense array as in the Stanford experiment (Healzer, 1974, Pimenta, 1975, and Coleman et. al., 1977) but this will be discussed in a later section. The boundary conditions are then simply that at $y = 0$, all velocities go to zero and as $y \rightarrow \infty$, $u \rightarrow u_c$.

2.4.2.3. Implications of the Discrete Element Approach on the Boundary Layer Characteristics

Characteristics

Since the governing equations are modified by the presence of the roughness elements, it becomes apparent that the integral flow parameters, which are defined from the integral control volume analysis of these equations, should be modified equally. This can be seen from the analysis made with the help of Figure 2.4. This analysis is similar to that given in White (1991).

Figure 2.4. Definition of the control volume for integral momentum analysis and flow characteristics evaluation.

2.4.2.3.1. Derivation of the New Displacement Thickness

By using the continuity equation in the form of Eq. (2.24), one can integrate it over the control volume of Figure 2.4., using the fact that no mass will pass through the top and bottom delimitations of the control volume as they are streamlines, to obtain :

$$\int_0^h (\rho \beta_x u_e) dy + \int_0^Y (\rho \beta_x u) dy = 0 \quad (2.32)$$

By assuming an incompressible flow (ρ is constant) one gets :

$$u_e \overline{\beta_x} h + \int_0^Y u \beta_x dy = 0 \quad \text{where} \quad \overline{\beta_x} = \int_0^{\delta} \beta_x dy \quad (2.33)$$

which can be rewritten as :

$$u_e \overline{\beta_x} h = \int_0^Y (u_e + u - u_e) \beta_x dy = \int_0^Y u_e \beta_x dy + \int_0^Y (u - u_e) \beta_x dy \quad (2.34)$$

or

$$u_e \overline{\beta_x} h = u_e Y \overline{\beta_x} + \int_0^Y (u - u_e) \beta_x dy \quad (2.35)$$

In Eq. (2.32) $\overline{\beta_x}$ can be seen as an average stream-wise blockage factor over the boundary layer thickness. Using the definition from Figure 2.4. of $\delta^* = Y - h$, this gives :

$$\delta^* = \int_0^Y \frac{\beta_x}{\overline{\beta_x}} \left(1 - \frac{u}{u_e} \right) dy \quad (2.36)$$

It can be seen that unless the blockage factor is constant through the boundary layer thickness, Eq. (2.36) is different from the usual definition of the displacement thickness.

2.4.2.3.2. Derivation of the New Momentum Thickness

This derivation is done by integrating the stream-wise momentum equation, Eq. (2.27), over the control volume of Figure 2.4., again using the fact that no mass will pass through the top and bottom streamlines of the control volume. The force term on the right hand side is all considered as drag and, for flow over a flat plate, the momentum thickness can be defined as the drag divided by ρu_e^2 . We can rewrite Eq. (2.27) as :

$$\int_0^y u(\rho \beta_x u) dy - \int_0^h u_e(\rho \beta_x u_e) dy = -D \quad (2.37)$$

where D represents all the tangential forces acting on the control volume but not the pressure gradient. By again assuming that ρ is constant one gets :

$$\frac{D}{\rho} = u_e^2 \overline{\beta_x} h - \int_0^y \beta_x u^2 dy \quad (2.38)$$

One then uses the definition from the previous section, (Eq. 2.33) :

$$h = \int_0^y \frac{\beta_x}{\overline{\beta_x}} \left(\frac{u}{u_e} \right) dy \quad (2.39)$$

to get :

$$\frac{D}{\rho} = \int_0^y \beta_x (u_e u) dy - \int_0^y \beta_x u^2 dy \quad (2.40)$$

And then by dividing Eq. (2.40) by u_e^2 the momentum thickness becomes :

$$\frac{D}{\rho u_e^2} = \theta = \int_0^{y \rightarrow \delta} \beta_x \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) dy \quad (2.41)$$

which again differs from the usual momentum thickness definition. The left hand side of this definition of the momentum thickness is only valid for a flat plate under no pressure gradient.

2.4.2.3.3. Sensitivity of the flow parameters to the modifications

Since the definition of the flow parameters have been modified to take account of the effect of the surface roughness, it was decided to verify how the new definitions would affect the value of the calculated parameters, compared with the value calculated using their definitions without modifications. Figure 2.5. shows the calculated results with and without modifications and clearly shows that the values are almost identical. However, in the present study, the definitions of the displacement and momentum thickness used were those without modifications as comparisons with experimental data were required. Since the values of θ and δ^* in the reports used were calculated with the unmodified definitions, this was a necessary choice. It is however suggested that in future research the new definitions be used, even though this does not affect the results in a significant way, as it is felt that this approach is more rigorous. In Figure 2.5., the BL, KE and KW abbreviations identify the results obtained with three different turbulence models, the Baldwin-Lomax model (BL), the k- ϵ model (KE) and the k- ω model (KW). Descriptions of these models are given in section 3.3. The experimental results of Figure 2.5. were obtained by Raupach et. al. (1986), using an array of thin metal strips extending fairly high into the boundary layer so that the blockage effect would be significant.

Figure 2.5. Comparison between results calculated with and without the modifications to the definitions of δ^* and θ .

2.5. Available Experimental Data

In order to evaluate the accuracy of a specific model, one must compare its results to actual measured data stemming from an experimental evaluation of a similar test case. Although the literature is filled with such experimental investigations for flow over smooth surfaces, the rough surface case has received comparatively low attention and test cases are much harder to find. Moreover, from the available test cases, only a parcel reveal themselves to be appropriate for the purposes of this study. This section reviews some of the test cases found and explains why they were retained or not.

2.5.1. Summary and Evaluation of Data

The test cases which are of interest for this study are those which concern two-dimensional boundary layers over rough surfaces. Most cases available treat the case of roughened pipes or channels and were therefore not suitable for this study.

2.5.1.1. Data for Verification of Roughness Model Implementation

These data were taken from Pajayakrit (1997) and include two analytical cases and three experimental studies. As these cases have been used by Pajayakrit (1997) to verify his computer code, they were considered the ideal cases to verify that the implementation of the roughness model did not alter its validity for smooth surfaces. Reasons for choosing these cases for validation are given in Pajayakrit (1997).

2.5.1.2. Data for Roughness Model Validation

The data available for rough surfaces is very limited compared to that for smooth surfaces. However, a respectable number of cases were found initially, which encouraged the author on the feasibility of the model validation. Sadly, many of the initial cases had to be discarded for various reasons. First, the discrete element method in its present form is limited to flows over regular roughness arrays. This ruled out the use of any surface roughened with random roughness elements, such as sand roughness experiments. Second, the computer code was designed to calculate flows in boundary layers, which disqualified all test cases for fully developed pipe and channel flows. Third, Taylor et. al. (1988) state that the discrete element method is restricted to roughness elements of three-dimensional shapes. Chapter 4 will show results that suggest the possibility of eliminating this restriction but for now, most test cases using roughness elements of two-dimensional shapes will be discarded. Finally, some experimental investigations were discarded on the basis that their reported data sets were either insufficient to start or compare a computer simulation or reported values were irrelevant to the flow parameters this study aimed at evaluating. Table 2.1 shows a summary of the rough wall test cases assessed for this study as well as whether they were accepted or rejected and reasons for their rejection.

AUTHORS	STATUS	REASONS
Nikuradse (1933)	Rejected	Random roughness element shape.
Schlichting (1936)	Rejected	Fully developed pipe flow.
Hama (1955)	Rejected	Two-dimensional roughness elements.
Sayre (1961)	Rejected	Irrelevant data reduction.
Perry and Joubert (1963)	Rejected	Two-dimensional roughness elements.
O'Loughlin and MacDonald (1964)	Rejected	Irrelevant data reduction.
Bettermann (1966)	Accepted	2-D limitation evaluation test case.
O'Loughlin and Annambhotla (1969)	Rejected	Insufficient available data.
Counihan (1971)	Rejected	Insufficient available data.
Wooding, Bradley and Marshall (1973)	Rejected	Insufficient available data.
Chen and Roberson (1974)	Rejected	Two-dimensional roughness elements.
Antonia and Wood (1975)	Rejected	Two-dimensional roughness elements.
Furuya, Miyata and Fulita (1976)	Rejected	Two-dimensional roughness elements.
Seginer, Mulhearn, Bradley and Finnigan (1976)	Rejected	Insufficient available data.
Boisvert, Gareem, Tsen and Vinh (1977)	Rejected	Two-dimensional roughness elements.
Coleman, Moffat and Kays (1977)	Accepted	
Gartshore and De Croos (1977)	Rejected	Insufficient available data.
Lee and Soliman (1977)	Rejected	Irrelevant data reduction.
Mulhearn and Finnigan (1978)	Rejected	Random roughness element shape.
Raupach, Thom and Edwards(1980)	Accepted	
Coleman, Hodge and Taylor (1984)	Rejected	Fully developed pipe flow.
Raupach, Coppin and Legg (1986)	Accepted	
Taylor, Scaggs and Coleman (1988)	Rejected	Fully developed pipe flow.
Scaggs, Taylor and Coleman (1988)	Rejected	Fully developed pipe flow.
Kind and Lawrysyn (1991)	Rejected	Random roughness element shape.
Hosni, Coleman and Taylor (1993)	Rejected	Insufficient available data.
Sullivan and Greely (1993)	Rejected	Insufficient available data.

Table 2.1. Summary of assessed experimental data.

Chapter 3

Implementation of the Discrete Element Method in a Two-Dimensional Parabolic Navier-Stokes Code for Thin Shear Layers

As was mentioned in the introduction, the validation of the discrete element method was done by modifying an existing two-dimensional parabolic Navier-Stokes computer code by introducing new subroutines to evaluate the effect of the presence of the roughness elements on the flow. This chapter describes the resulting code by stating the governing equations as well as describing the solving algorithm and the turbulence models used to achieve closure.

3.1. Basic Approach

3.1.1. Model Selection

Turbulence modelling can be seen as taking the boundary layer continuity and momentum equations as the three primary equations of motion and then adding a certain number of differential equations to evaluate the additional unknowns (White, 1991). The different models can be classified according to the number of additional differential equations they need to achieve closure as well as their potential range of applicability. From low end of applicability to high end we therefore have (White, 1991):

- Zero-equation models : the unknown Reynolds stresses in the momentum equation are directly modelled by introducing an eddy viscosity term which is evaluated algebraically from either a mixing length theory or any other algebraic relation such as the one proposed by Baldwin and Lomax (1978) ;
- One-equation models : an additional differential equation modelling the turbulent energy equation is used in conjunction with an algebraic correlation for the turbulence length scale ;
- Two-equation models : are similar to the previous method but add a second differential equation to model the rate of change of either the turbulence length scale, kinetic energy dissipation or vorticity fluctuations ;
- Reynolds stress models : directly compute the Reynolds stresses by either an algebraic stress model or from differential equations for the rate of change of each stress components ;
- Large-eddy simulation of turbulence : which model the small eddies but use direct numerical simulation for the larger eddies, making the method almost model free ;
- Direct numerical simulation of turbulence: which doesn't model anything at all but rather directly solves the entire set of equations.

In the present work, it was decided that the turbulence models to be used would be the same as those already implemented in the available computer code to reduce the workload. These are the Baldwin-Lomax model for zero-equation models and the k - ϵ and k - ω models for two equation models. This choice can easily be explained by noting that

one-equation models have been known to give results which are satisfactory but, apparently no better than those of the best zero-equation models, which are much easier to implement. Also, the large eddy and direct simulation methods are, with available computing resources, prohibitive in computing time. Finally, although a simplified version of the Reynolds stress models called the multiscale model was included in the computer code, it was decided that calculations of the roughness effects would not be done using that method since the cases to be investigated did not show any particular difficulty and that, for steady flows, it was not shown to be much more efficient than the two-equation models (Pajayakrit, 1997; Wilcox, 1988b). Additionally, it is considered to take about 50% more computation time (Wilcox, 1988b) and the implementation of the discrete element method in this type of modelling would have been more difficult due to the nature of its configuration in the original code.

3.1.2. Algorithm for Computation of Flow Development

Since the focus of this study was an assessment of the discrete element model for the evaluation of the roughness effect, the calculation domain would be limited to simple geometries. Flow separation and flow reversal not being of interest here, a relatively simple and fast algorithm for computing flow development was appropriate. Assuming that all the flows to be calculated could be assumed as two-dimensional thin shear layer flows with negligible axial diffusion, a parabolic, space-marching code already available (Pajayakrit, 1997) could therefore be used. That code would then be modified by the author to include subroutines enabling the calculation of the roughness effect by the

discrete element approach, therefore providing the author with some valuable experience in computational fluid dynamics.

3.2. Equations of Motion

3.2.1. Co-ordinate System

The body oriented curvilinear co-ordinate system is shown in Figure 3.1. The y-axis is aligned with the local normal and is directed away from the centre of curvature. The x-axis is perpendicular to the y-axis and is directed along the flow direction. The radius of curvature is positive for a convex surface.

3.2.2. Simplifying Assumptions

The flows in this study are assumed to be :

- incompressible, i.e. ρ is constant ;
- Steady, i.e. all time derivative are zero $\left(\frac{\partial}{\partial t} () = 0 \right)$
- Two-dimensional, i.e. $\bar{w} = 0$, $\frac{\partial}{\partial z} () = 0$; and
- thin, i.e. $\frac{\delta}{R} \ll 1$.

Figure 3.1. Body oriented curvilinear co-ordinate system.

3.2.3. System of Equations

As seen in section (2.4.2.2.), the blockage factors introduced by the roughness elements will modify the basic equations of motion. Since we want to be able to predict the flow on curved surfaces, we must rewrite the equations of section (2.4.2.2.) in surface-normal, curvilinear co-ordinates. To do so, we will start with the derivation of the equations in cylindrical co-ordinates and then transform them. We will start with the control volume of Figure 3.2 to derive each of the equations.

Figure 3.2. Body oriented curvilinear control volume with roughness elements used for the derivation of the governing equations.

3.2.3.1. The Continuity Equation

This equation states that all mass entering the control volume of Figure 3.2 must be accounted for by an equal amount of mass leaving it. In cylindrical co-ordinates, by taking into account the blockage factors defined in section 2.4.2.2., this can be written as :

$$\begin{aligned} & \left(u + \frac{\bar{\sigma}u}{\partial\theta} d\theta \right) \left(\beta_x + \frac{\partial\beta_x}{\partial\theta} d\theta \right) dr - u\beta_x dr + \\ & \left(v + \frac{\partial v}{\partial r} dr \right) \left(\beta_y + \frac{\partial\beta_y}{\partial r} dr \right) (r + dr) d\theta - v\beta_y r d\theta = 0 \end{aligned} \quad (3.1)$$

This can be rewritten after simplification as

$$\begin{aligned} & \left[\frac{\bar{\sigma}}{\partial\theta} (\beta_x u) + r \frac{\bar{\sigma}}{\partial r} (\beta_y v) + \beta_y v \right] dr d\theta - \left[\frac{\partial\beta_x}{\partial\theta} \frac{\bar{\sigma}u}{\partial\theta} \right] d\theta^2 dr \\ & + \left[r \left(\frac{\partial\beta_y}{\partial r} \frac{\partial v}{\partial r} \right) + r \frac{\partial}{\partial r} (\beta_y v) \right] dr^2 d\theta + \left(\frac{\partial\beta_y}{\partial r} \frac{\partial v}{\partial r} \right) dr^3 d\theta = 0 \end{aligned} \quad (3.2)$$

And by neglecting terms of third order or higher, one can divide Eq. (3.2) by $dr d\theta$ to get:

$$\frac{\bar{\sigma}}{\partial\theta} (\beta_x u) + \frac{\bar{\sigma}}{\partial r} (r\beta_y v) = 0 \quad (3.3)$$

To obtain the Reynolds averaged equation, which separates the mean and fluctuating parts of the properties, one assumes the following identities:

$$u = \bar{u} + u' \text{ and } v = \bar{v} + v' \quad (3.4)$$

and then averages the continuity equation over a sufficient lapse of time :

$$\overline{\frac{\partial}{\partial \theta}(\beta_x(\bar{u} + u'))} + \overline{\frac{\partial}{\partial r}(\beta_y r(\bar{v} + v'))} = 0 \quad (3.5)$$

which yields the resulting continuity equation for the mean part of the velocity :

$$\frac{\bar{\partial}}{\partial \theta}(\beta_x \bar{u}) + \frac{\bar{\partial}}{\partial r}(r \beta_y \bar{v}) = 0 \quad (3.6)$$

and subtracting Eq. (3.6) from Eq. (3.5) yields the resulting continuity equation for the fluctuating part of the velocity :

$$\frac{\bar{\partial}}{\partial \theta}(\beta_x u') + \frac{\bar{\partial}}{\partial r}(r \beta_y v') = 0 \quad (3.7)$$

Eq. (3.6) can then be rewritten in surface-normal, curvilinear co-ordinates (Figure 3.1) by noting that

$$\theta = \frac{x}{R}; d\theta = \frac{dx}{R}; r = R + y; dr = dy \text{ and } h = \left(1 + \frac{y}{R}\right) \quad (3.8)$$

which leads to

$$\frac{\bar{\partial}}{\partial x}(\beta_x \bar{u}) + \frac{\bar{\partial}}{\partial y}(\beta_y h \bar{v}) = 0 \quad (3.9)$$

It is interesting here to verify if the identity of Eq. (2.21) holds true for curvilinear co-ordinates too. As seen from Figure 3.2., if we assume that the average roughness width over the control volume which is obtained by evaluating the integral

$$\bar{a} = \frac{1}{\left(\frac{r+dr}{2}\right) d\theta} \int_{\theta-\frac{d\theta}{2}}^{\theta+\frac{d\theta}{2}} a(\theta) d\theta = \frac{2ab}{(r+dr)d\theta} \quad (3.10)$$

and that the angle $d\theta$ just encloses the roughness element spacing L , the area available for flow around a single roughness element in the stream-wise direction is :

$$\beta_x = \frac{l\Delta r - \left(\frac{2ab}{(r+dr)d\theta}\right)\Delta r}{l\Delta r} = 1 - \frac{2ab}{l(r+dr)d\theta} \quad (3.11)$$

Now, again from Figure 3.2. with the assumption that the angle $d\theta$ just encloses the roughness element spacing L , it can be seen that the available area in the normal direction is :

$$\beta_y = \frac{l\left(\frac{r+dr}{2}\right)d\theta - ab}{l\left(\frac{r+dr}{2}\right)d\theta} = 1 - \frac{2ab}{l(r+dr)d\theta} \quad (3.12)$$

which demonstrates the identity of Eq. (2.21) even in surface-normal curvilinear co-ordinates.

3.2.3.2. The Stream-wise Momentum Equation

This equation stems from Newton's second law that the mass of fluid included in the control volume of Figure 3.2 times its acceleration must be balanced by all the forces acting on the control volume. In cylindrical co-ordinates, by taking into account the blockage factors defined in section 2.4.2.2. and assuming that the mass of moving fluid in the stream-wise direction is dictated by β_x , this can be written as :

$$\begin{aligned}
& \left[\rho \beta_x \left(r + \frac{dr}{2} \right) d\theta dr \right] \left(r + \frac{dr}{2} \right) \left(\frac{u}{r} \frac{\partial u}{\partial \theta} + v \frac{\partial u}{\partial r} + \frac{uv}{r} \right) = \text{mass} \times \text{ang. accl'n.} = \\
& \left\{ - \left(p + \frac{p}{\partial \theta} d\theta \right) \left(r + \frac{dr}{2} \right) \left(\beta_x + \frac{\partial \beta_x}{\partial \theta} d\theta \right) dr + p \left(r + \frac{dr}{2} \right) \beta_x dr \right\} + \text{Pressure Moment} + \\
& \left\{ \left(\tau + \frac{\partial \tau}{\partial r} dr \right) (r + dr) \left(\beta_y + \frac{\partial \beta_y}{\partial r} dr \right) (r + dr) d\theta - \tau \beta_y r d\theta \right\} - \text{Shear Moments} - \\
& \left(r + \frac{dr}{2} \right) D \quad \text{Drag Moment}
\end{aligned} \tag{3.13}$$

This can be rewritten after simplification as

$$\begin{aligned}
& \left(\rho \beta_x \right) \left[r^2 dr d\theta + r dr^2 d\theta + \frac{dr^3 d\theta}{4} \right] \left(\frac{u}{r} \frac{\partial u}{\partial \theta} + v \frac{\partial u}{\partial r} + \frac{uv}{r} \right) = \\
& - \left\{ \frac{\partial(\beta_x p)}{\partial \theta} r dr d\theta + \frac{\partial(\beta_x p)}{\partial \theta} \frac{dr^2 d\theta}{2} + \left(\frac{\partial \beta_x}{\partial \theta} \frac{\partial p}{\partial \theta} \right) r dr d\theta^2 + \left(\frac{\partial \beta_x}{\partial \theta} \frac{\partial p}{\partial \theta} \right) \frac{dr^2 d\theta^2}{2} \right\} + \\
& \left\{ \left[2\beta_y \tau + \frac{\partial}{\partial r} (\beta_y \tau) r \right] r dr d\theta + \left[\beta_y \tau + 2r \frac{\partial}{\partial r} (\beta_y \tau) + r^2 \frac{\partial \beta_y}{\partial r} \frac{\partial \tau}{\partial r} \right] (dr^2 d\theta) + H.O.T. \right\} \\
& - \left(r + \frac{dr}{2} \right) D
\end{aligned} \tag{3.14}$$

where *H.O.T* stands for higher order terms.

By neglecting terms of third order or higher, one can divide Eq. (3.14) by $r^2 dr d\theta$ to get :

$$\rho \beta_x \left(\frac{u}{r} \frac{\partial u}{\partial \theta} + v \frac{\partial u}{\partial r} + \frac{uv}{r} \right) = - \frac{1}{r} \frac{\partial}{\partial \theta} (\beta_x p) + \frac{2\beta_y \tau}{r} + \frac{\partial}{\partial r} (\beta_y \tau) - \left(r + \frac{dr}{2} \right) \frac{D}{r^2 dr d\theta} \tag{3.15}$$

To obtain the Reynolds averaged equation, one assumes the same identities as in section 3.2.3.1. and adds the pressure identity :

$$u = \bar{u} + u' ; v = \bar{v} + v' \text{ and } p = \bar{p} + p' \quad (3.16)$$

and then averages the stream-wise momentum equation over a sufficient lapse of time :

$$\begin{aligned} \rho \beta_x \left[\frac{(\bar{u} + u')}{r} \frac{\partial(\bar{u} + u')}{\partial \theta} + (\bar{v} + v') \frac{\partial(\bar{u} + u')}{\partial r} + \frac{(\bar{u} + u')(\bar{v} + v')}{r} \right] = \\ - \frac{1}{r} \frac{\partial}{\partial \theta} \left[\beta_x (\bar{p} + p') \right] + \frac{2\beta_y \tau}{r} + \frac{\partial}{\partial r} (\beta_y \tau) - \left(r + \frac{dr}{2} \right) \frac{D}{r^2 dr d\theta} \end{aligned} \quad (3.17)$$

which, once averaged, yields the following :

$$\begin{aligned} \rho \beta_x \left[\frac{\bar{u}}{r} \frac{\partial \bar{u}}{\partial \theta} + 0 + 0 + \frac{\bar{u}'}{r} \frac{\partial \bar{u}'}{\partial \theta} + \bar{v} \frac{\partial \bar{u}}{\partial r} + 0 + 0 + \bar{v}' \frac{\partial \bar{u}'}{\partial r} + \frac{\bar{u} \bar{v}}{r} + 0 + 0 + \frac{\bar{u}' \bar{v}'}{r} \right] = \\ - \frac{1}{r} \frac{\partial}{\partial \theta} \left[\beta_x (\bar{p} + 0) \right] + \frac{2\beta_y \tau}{r} + \frac{\partial}{\partial r} (\beta_y \tau) - \left(r + \frac{dr}{2} \right) \frac{D}{r^2 dr d\theta} \end{aligned} \quad (3.18)$$

The first two fluctuating terms of Eq. (3.18) can be rewritten by noting that

$$\begin{aligned} \frac{u'}{r} \frac{\partial u'}{\partial \theta} + v' \frac{\partial u'}{\partial r} &= \frac{1}{r} \frac{\partial u'^2}{\partial \theta} - \frac{u'}{r} \frac{\partial u'}{\partial \theta} + \frac{\partial(u'v')}{\partial r} - u' \frac{\partial v'}{\partial r} = \frac{1}{r} \frac{\partial u'^2}{\partial \theta} + \frac{\partial(u'v')}{\partial r} - \frac{u'}{r} \left[\frac{\partial u'}{\partial \theta} - r \frac{\partial v'}{\partial r} \right] \\ &= \frac{1}{r} \frac{\partial u'^2}{\partial \theta} + \frac{\partial(u'v')}{\partial r} - \frac{u'}{r} \left\{ \frac{1}{\beta_x} \left[\frac{\partial(\beta_x u')}{\partial \theta} - u' \frac{\partial \beta_x}{\partial \theta} \right] + \frac{1}{\beta_y} \left[\frac{\partial(\beta_y r v')}{\partial r} - r v' \frac{\partial \beta_y}{\partial r} \right] - v' \right\} \\ &= \frac{1}{r} \frac{\partial u'^2}{\partial \theta} + \frac{\partial(u'v')}{\partial r} - \frac{u'}{r \beta_x} \left\{ \left[\frac{\partial(\beta_x u')}{\partial \theta} - \frac{\beta_x}{\beta_y} \frac{\partial(\beta_y r v')}{\partial r} \right] - \left[u' \frac{\partial \beta_x}{\partial \theta} + \frac{\beta_x}{\beta_y} r v' \frac{\partial \beta_y}{\partial r} \right] - \beta_x v' \right\} \end{aligned} \quad (3.19)$$

If we assume that $\beta_x = \beta_y$, the first term in the $\{\}$ brackets disappears from the continuity equation expressed in the form of Eq. (3.7). Eq. (3.19) can then be rewritten as :

$$\begin{aligned}
\frac{u'}{r} \frac{\partial u'}{\partial \theta} + v' \frac{\partial u'}{\partial r} &= \frac{1}{r} \frac{\partial u'^2}{\partial \theta} + \frac{\partial(u'v')}{\partial r} + \frac{u'v'}{r} + \frac{u'^2}{r} \frac{1}{\beta_x} \frac{\partial \beta_x}{\partial \theta} + \frac{u'v'}{\beta_x} \frac{\partial \beta_y}{\partial r} \\
&= \frac{1}{r\beta_x} \left[\beta_x \frac{\partial u'^2}{\partial \theta} + u'^2 \frac{\partial \beta_x}{\partial \theta} \right] + \frac{1}{\beta_x} \left[\frac{\beta_x}{\beta_y} \beta_y \frac{\partial u'v'}{\partial r} + u'v' \frac{\partial \beta_y}{\partial r} \right] + \frac{u'v'}{r}
\end{aligned} \tag{3.20}$$

Again assuming that $\beta_x = \beta_y$, we have :

$$\frac{u'}{r} \frac{\partial u'}{\partial \theta} + v' \frac{\partial u'}{\partial r} = \frac{1}{\beta_x} \left[\frac{1}{r} \frac{\partial}{\partial \theta} (\beta_x u'^2) + \frac{\partial}{\partial r} (\beta_y u'v') \right] + \frac{u'v'}{r} \tag{3.21}$$

which then enables the momentum equation to be rewritten as :

$$\begin{aligned}
\rho \beta_x \left[\frac{\bar{u}}{r} \frac{\partial \bar{u}}{\partial \theta} + \frac{\bar{v}}{r} \frac{\partial \bar{u}}{\partial r} + \frac{\bar{uv}}{r} + \frac{1}{\beta_x} \left[\frac{1}{r} \frac{\partial}{\partial \theta} (\beta_x u'^2) + \frac{\partial}{\partial r} (\beta_y u'v') \right] + \frac{\bar{u'v'}}{r} + \frac{\bar{u'v'}}{r} \right] = \\
-\frac{1}{r} \frac{\partial}{\partial \theta} [\beta_x \bar{p}] + \frac{2\beta_y \tau}{r} + \frac{\partial}{\partial r} (\beta_y \tau) - \left(r + \frac{dr}{2} \right) \frac{\bar{D}}{r^2 dr d\theta}
\end{aligned} \tag{3.22}$$

Rearranging to get only the mean velocity components on the left hand side :

$$\begin{aligned}
\rho \beta_x \left[\frac{\bar{u}}{r} \frac{\partial \bar{u}}{\partial \theta} + \frac{\bar{v}}{r} \frac{\partial \bar{u}}{\partial r} + \frac{\bar{uv}}{r} \right] &= -\frac{1}{r} \frac{\partial}{\partial \theta} \left[\beta_x (\bar{p} + \rho u'^2) \right] + \\
\frac{2}{r} [\beta_y \tau - \beta_x \rho \overline{u'v'}] &+ \frac{\partial}{\partial r} [\beta_y (\tau - \rho \overline{u'v'})] - \left(r + \frac{dr}{2} \right) \frac{\bar{D}}{r^2 dr d\theta}
\end{aligned} \tag{3.23}$$

And finally, by dividing both sides by $\rho \beta_x$ and by assuming that the density is constant and that here again, $\beta_x = \beta_y$, one gets :

$$\begin{aligned}
\frac{\bar{u}}{r} \frac{\partial \bar{u}}{\partial \theta} + \frac{\bar{v}}{r} \frac{\partial \bar{u}}{\partial r} + \frac{\bar{uv}}{r} &= -\frac{1}{\beta_x r} \frac{\partial}{\partial \theta} \left[\beta_x \left(\frac{\bar{p}}{\rho} + \overline{u'^2} \right) \right] + \frac{2}{r} \left[\beta_y \left(\frac{\tau}{\rho} - \overline{u'v'} \right) \right] + \\
+\frac{1}{\beta_x} \frac{\partial}{\partial r} \left[\beta_y \left(\frac{\tau}{\rho} - \overline{u'v'} \right) \right] &- \left(r + \frac{dr}{2} \right) \frac{\bar{D}}{\rho \beta_x r^2 dr d\theta}
\end{aligned} \tag{3.24}$$

It is now desired to put Eq. (3.24) in surface-normal, curvilinear co-ordinates (SNCC). We do so by again noting that

$$\theta = \frac{x}{R}; d\theta = \frac{dx}{R}; r = R + y; dr = dy \text{ and } h = \left(1 + \frac{y}{R}\right) \quad (3.25)$$

and assuming that if $dr \ll R$, then $\left(r + \frac{dr}{2}\right) = R + y + \frac{dy}{2} \approx Rh$ which leads to

$$\begin{aligned} \frac{-\partial \bar{u}}{\partial x} + h v \frac{-\partial \bar{u}}{\partial y} + \frac{\bar{uv}}{R} = & -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\bar{p}}{\rho} + \bar{u}^2 \right) \right] + \\ & \frac{2}{R} \left[\beta_y \left(\frac{\tau}{\rho} - \overline{u'v'} \right) \right] + \frac{h}{\beta_x} \frac{\partial}{\partial y} \left[\beta_y \left(\frac{\tau}{\rho} - \overline{u'v'} \right) \right] - \frac{\overline{D}}{\rho \beta_x dx dy} \end{aligned} \quad (3.26)$$

Now, assuming that the molecular shear force τ can be represented by $\tau = \nu \left(\frac{\partial \bar{u}}{\partial y} - \frac{\bar{u}}{hR} \right)$, and that the Boussinesq eddy viscosity assumption

$-\rho \overline{u'v'} = \tau_t = \nu_t \left(\frac{\partial \bar{u}}{\partial y} - \frac{\bar{u}}{hR} \right)$ holds, Eq. (3.26) can be rewritten as :

$$\begin{aligned} \frac{-\partial \bar{u}}{\partial x} + h v \frac{-\partial \bar{u}}{\partial y} + \frac{\bar{uv}}{R} = & -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\bar{p}}{\rho} + \bar{u}^2 \right) \right] - \\ & \left(\frac{(v + \nu_t)}{hR} + \frac{\partial \nu_t}{\partial y} \right) \frac{\bar{u}}{R} + \frac{1}{\beta_x} \frac{\partial}{\partial y} \left[\beta_y h (v + \nu_t) \frac{\partial \bar{u}}{\partial y} \right] - \frac{\overline{D}}{\rho \beta_x dx dy} \end{aligned} \quad (3.27)$$

By assuming the concept of an equivalent viscosity $\nu_e = \nu + \nu_t$ and by noting that $d\nu_e = d\nu_t$ this expression simplifies to :

$$\begin{aligned} \frac{-\partial \bar{u}}{\partial x} + h v \frac{-\partial \bar{u}}{\partial y} + \frac{\bar{uv}}{R} = & -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\bar{p}}{\rho} + \bar{u}^2 \right) \right] - \left(\frac{\nu_e}{hR} + \frac{\partial \nu_e}{\partial y} \right) \frac{\bar{u}}{R} \\ & + \frac{1}{\beta_x} \frac{\partial}{\partial y} \left[\beta_y h \nu_e \frac{\partial \bar{u}}{\partial y} \right] - \frac{\overline{D}}{\rho \beta_x dx dy} \end{aligned} \quad (3.28)$$

Finally, the last (drag) term on the right hand side is most conveniently handled by using a drag coefficient defined by (Taylor et. al., 1985) :

$$C_D = \frac{\overline{D}}{\frac{1}{2} \rho (\overline{u + u'})^2 d \cdot dy} \quad (3.29)$$

where $d \cdot dy$ is the projected area of the slice of a roughness element penetrating the CV.

The number of roughness elements per unit area of the xz plane being $1/(Ll)$, Eq. (3.28)

can be rewritten as :

$$\begin{aligned} -\frac{\partial \overline{u}}{\partial x} + h \overline{v} \frac{\partial \overline{u}}{\partial y} + \frac{\overline{uv}}{R} = & -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\overline{p}}{\rho} + \overline{u^2} \right) \right] - \left(\frac{v_e}{hR} + \frac{\partial v_e}{\partial y} \right) \frac{\overline{u}}{R} \\ & + \frac{1}{\beta_x} \frac{\partial}{\partial y} \left[\beta_x h v_e \frac{\partial \overline{u}}{\partial y} \right] - \frac{1}{2\beta_x} C_D \frac{d}{Ll} \overline{u^2} \left(1 + \frac{\overline{u^2}}{u^2} \right) \end{aligned} \quad (3.30)$$

which is the stream- wise momentum equation used in the formulation of the computer code used for this study.

3.2.3.3. The Normal Momentum Equation

As was mentioned in Section 2.4.2.2.3., the normal momentum equation is not affected by the roughness elements and can therefore be written in SNCC as (Bradshaw, 1973) :

$$\frac{\overline{u^2}}{hR} = \frac{\partial}{\partial y} \left(\frac{\overline{p}}{\rho} + \overline{v^2} \right) \quad (3.31)$$

where $h = 1 + \frac{y}{R}$

3.2.4. Non-Dimensionalisation

The equations of motion, Eqs (3.8), (3.30) and (3.31) are non-dimensionalised by introducing the following non-dimensional variables :

$$\begin{aligned}
 x^* &= \frac{x}{L_{ref}} ; & y^* &= \frac{y}{L_{ref}} ; & R^* &= \frac{R}{L_{ref}} ; \\
 d^* &= \frac{d}{L_{ref}} ; & L^* &= \frac{L}{L_{ref}} ; & l^* &= \frac{l}{L_{ref}} ; \\
 \bar{u}^* &= \frac{\bar{u}}{U_{ref}} ; & \bar{v}^* &= \frac{\bar{v}}{U_{ref}} ; & & \\
 \overline{u^2}^* &= \frac{\overline{u^2}}{U_{ref}^2} ; & \overline{v^2}^* &= \frac{\overline{v^2}}{U_{ref}^2} ; & \tau^* &= \frac{\tau}{U_{ref}^2} ; \\
 \overline{C_p} &= \frac{\bar{p} - p_{ref}}{\frac{1}{2} \rho U_{ref}^2} ; & Re &= \frac{L_{ref} U_{ref}}{\nu} ; & & \\
 v_i^* &= \frac{v_i}{L_{ref} U_{ref}} ; & v_e^* &= \frac{v + v_i}{L_{ref} U_{ref}} ; & &
 \end{aligned}$$

where L_{ref} is a reference length, U_{ref} is a reference velocity, p_{ref} is a reference static pressure, A_p is the projected area of the slice of a roughness element penetrating the CV, $A_p = d \cdot \delta y$ and Re is the Reynolds number. Substituting the non-dimensional variables in the equations of motion yields :

Continuity :

$$\frac{\partial}{\partial x^*} (\beta_x \bar{u}^*) + \frac{\partial}{\partial y^*} (\beta_y \bar{v}^*) = 0 \quad (3.32)$$

Stream-Wise Momentum :

$$\begin{aligned} \overline{u} \frac{\partial \overline{u}}{\partial x^*} + h \overline{v} \frac{\partial \overline{u}}{\partial y^*} + \frac{\overline{u} \overline{v}}{R^*} = & -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\overline{C_p}}{2} + \overline{u^{*2}} \right) \right] - \left(\frac{\overline{v_c^*}}{h R^*} + \frac{\partial \overline{v_c^*}}{\partial y^*} \right) \frac{\overline{u}}{R^*} \\ & + \frac{1}{\beta_x} \frac{\partial}{\partial y^*} \left[\beta_y h \overline{v_c^*} \frac{\partial \overline{u}}{\partial y^*} \right] - \frac{h}{2 \beta_x} \overline{C_D} \frac{d^*}{L^* l^*} \overline{u}^{-2} \left(1 + \frac{\overline{u^{*2}}}{\overline{u}^2} \right) \end{aligned} \quad (3.33)$$

Normal Momentum :

$$\frac{\overline{u^{*2}}}{h R^*} = \frac{\partial}{\partial y^*} \left(\frac{\overline{C_p}}{2} + \overline{v^{*2}} \right) \quad (3.34)$$

3.3. Selected Turbulence Models

As mentioned in section 2.4.2., one of the advantages of the discrete element approach is that since the physical effect of the roughness elements are included explicitly in the equations of motion, the turbulence modelling is done in the same way as would be for smooth surface flows. This section reviews the turbulence models used in this study. This description can also be found in Pajayakrit (1997).

3.3.1. Baldwin-Lomax (BL)

This is a two layer algebraic (zero equation) eddy viscosity model presented by Baldwin and Lomax (1978) in which μ_t is given by :

$$\mu_t^* = \begin{cases} (\mu_t^*)_{inner}, & y^* \leq y_{cross}^* \\ (\mu_t^*)_{outer}, & y^* > y_{cross}^* \end{cases} \quad (3.35)$$

where $(\mu_t^*)_inner$ is the inner layer eddy viscosity,
 $(\mu_t^*)_outer$ is the outer layer eddy viscosity,
 y_{cross}^* is the minimum y^* where $(\mu_t^*)_inner = (\mu_t^*)_outer$

The Prandtl-Van Driest formulation is used to calculate the inner eddy viscosity,

$$(\mu_t^*)_inner = \rho l^{*2} |\omega^*| \quad (3.36)$$

where ω^* , the vorticity, is given for two-dimensional, thin layer flow by

$$\omega^* = \frac{\partial \bar{u}^*}{\partial y^*} + \frac{\bar{u}^*}{hR^*} \quad (3.37)$$

The mixing length, l^* , contains the wall damping, as well as curvature correction factors,

$$l^* = l_0^* F_F \quad (3.38)$$

the wall damping effect being given by

$$l_0^* = \kappa y^+ \left[1 - \exp\left(-\frac{y^+}{A^+}\right) \right] \quad (3.39)$$

and the curvature effect term being suggested by Shrewsbury (1989), who applied the correction factors as suggested by Bradshaw (1973) :

$$F_F = 1 - \alpha S \quad (3.40)$$

where S is the ratio of the "extra" strain, due to curvature, to the primary strain, or

$$S = \frac{\frac{\bar{u}^*}{hR^*}}{\frac{\partial \bar{u}^*}{\partial y^*}} \quad (3.41)$$

The value of the constant α is found by trial and error and may vary from flow to flow.

The outer eddy viscosity is calculated from :

$$(\mu_t^*)_{outer} = KC_{C_p} \rho F_{WAKE} F_{KLEB}, \quad (3.42)$$

$$F_{KLEB} = \left[1 + 55 \left(C_{KLEB} \frac{y^*}{y_{max}^*} \right)^6 \right]^{-1}, \text{ and} \quad (3.43)$$

$$F_{WAKE} = \text{Min} \left(y_{max}^* F_{max}, \frac{C_{WAKE} y_{max}^* \bar{u}_{DIFF}^2}{F_{max}} \right). \quad (3.44)$$

In the previous equations, $\bar{u}_{DIFF}^*$ is the difference between the maximum and minimum total non-dimensional mean velocity in the velocity profile at any specific x-position. For two-dimensional flows where no region of reverse flow is present, such as those studied in this thesis,

$$\bar{u}_{DIFF}^* = \left(\sqrt{\bar{u}^{*2} + \bar{v}^{*2}} \right)_{max}. \quad (3.45)$$

The y_{max} and F_{max} are evaluated from the following function at the point in the profile where it is a maximum :

$$F(y^*) = y^* |\omega^*| \left[1 - \exp \left(-\frac{y^*}{A^+} \right) \right] \quad (3.46)$$

The values of the above model constants are taken as :

$$\begin{array}{lll} \kappa = 0.4 ; & K = 0.0168 ; & A^+ = 26 ; \\ C_{Cp} = 1.6 ; & C_{WAKE} = 1.0 ; & C_{KLEB} = 0.3. \end{array}$$

3.3.2. Dash k-ε (KE)

This model consists of two layers, a mixing length model in the near wall region and a standard high Reynolds number k-ε model everywhere else. The equations in two-dimensional, body-oriented, curvilinear co-ordinates of the turbulent kinetic energy and the turbulence dissipation rate are, respectively :

$$\bar{u} \frac{\partial k^*}{\partial x^*} + h\bar{v} \frac{\partial k^*}{\partial y^*} = \frac{\partial}{\partial y^*} \left[\frac{h\bar{v}_\epsilon^*}{\sigma_k} \frac{\partial k^*}{\partial y^*} \right] + h(P - \epsilon^*) \quad (3.47)$$

$$\bar{u} \frac{\partial \epsilon^*}{\partial x^*} + h\bar{v} \frac{\partial \epsilon^*}{\partial y^*} = \frac{\partial}{\partial y^*} \left[\frac{h\bar{v}_\epsilon^*}{\sigma_\epsilon} \frac{\partial \epsilon^*}{\partial y^*} \right] + h\frac{\epsilon^*}{k^*} (C_1 P - C_2 \epsilon^*) \quad (3.48)$$

where

$$P = u_i^* \left(\frac{\partial \bar{u}^*}{\partial y^*} + \frac{\bar{u}^*}{hR^*} \right)^2, \quad (3.49)$$

$$u_i^* = C_\mu \frac{k^{*2}}{\epsilon^*} \quad (3.50)$$

and the non-dimensional k^* and ϵ^* are defined as

$$k^* = \frac{k}{U_{ref}^2} \text{ and } \epsilon^* = \frac{\epsilon L_{ref}}{U_{ref}^3} \quad (3.51)$$

The values of the model constants are :

$$\sigma_K = 1.0 ; \quad \sigma_\epsilon = 1.3 ; \quad C_1 = 1.43 (1 + C_C Ri) ;$$

$$C_2 = 1.92 ; \quad C_\mu = 0.09 ; \quad C_C = 0.16 ;$$

and Ri is a Richardson number which is defined by Launder, Pridden and Sharma (1977)

as

$$Ri = \frac{\bar{u}}{R} \left(\frac{k}{\varepsilon} \right)^2 \frac{\partial \bar{u}}{\partial y}$$

In the near wall region, the eddy viscosity is calculated using a mixing length model :

$$v_t^* = l^{*2} \left| \frac{\partial \bar{u}^*}{\partial y^*} \right| \quad (3.52)$$

where, as in the Baldwin-Lomax model, the mixing length, l^* , contains the wall damping, as well as curvature correction factors,

$$l^* = l_0^* F_F \quad (3.53)$$

the wall damping effect being given by

$$l_0^* = \kappa y^+ \left[1 - \exp \left(-\frac{y^+}{A^+} \right) \right] \quad (3.54)$$

and the curvature effect term being applied as follows :

$$F_F = \frac{1 - \alpha S}{1 - S} \quad (3.55)$$

where S is again the ratio of the "extra" strain, due to curvature, to the primary strain, or

$$S = \frac{\frac{\bar{u}^*}{hR^*}}{\frac{\partial \bar{u}^*}{\partial y^*}} \quad (3.56)$$

The value of the constant α was found by Dash et. al. (1983) to have little effect unless the wall curvature was large. They nevertheless suggested values for α in the range of 5-10.

The matching point between the near wall and the outer regions was set to occur at $y^+ \sim 50$. Dash et. al. (1983) reported that this value did not affect model prediction as long as it remained in the log-law region ($20 < y^+ < 100$). At this matching point, the value of the turbulent viscosity, ν_t , calculated from both the mixing length and the k- ϵ models are equal and the production and dissipation term in the k equation, Eq. (3.47) are in equilibrium. This yields boundary conditions for k and ϵ at the matching point which set these values at :

$$k_{I^*} = \frac{l_{I^*}^2 \left| \frac{\partial \bar{u}}{\partial y} \right|_{I^*}^2}{C_\mu^{1/2}} \text{ and} \quad (3.57)$$

$$\epsilon_{I^*} = \frac{C_\mu^{3/4} k_{I^*}^{3/2}}{l_{I^*}} \quad (3.58)$$

where the subscript I^* indicates values at the matching point. These values are therefore the wall boundary values of k and ϵ and the k- ϵ model equations are then solved only at the nodes above I^* .

The boundary conditions at the edge of the boundary layer are set by solving a reduced form of the k and ϵ equations. At the edge, it is assumed that $\frac{\partial}{\partial y^*}(\) = 0$ and that the production terms are also zero. Under these assumptions, the k and ϵ equations reduce to :

$$\bar{u}_\epsilon \frac{\partial k_\epsilon^*}{\partial x^*} = -h_\epsilon \epsilon_\epsilon^* \quad (3.59)$$

$$\bar{u}_e \frac{\partial \varepsilon_e^*}{\partial x^*} = -C_2 h_e \frac{\varepsilon_e^{*2}}{k_e^*} \quad (3.60)$$

where the subscript e denotes values at the boundary layer edge. k_e^* and ε_e^* can be solved using Runge-Kutta integration, as the hierarchy diagram of Figure 3.3 indicates.

3.3.2. Wilcox k- ω (KW)

This model was presented by Wilcox (1988a). Its main advantage over the other methods is that it can be integrated right from the wall, without the use of any damping terms. The equations in two-dimensional, body-oriented, curvilinear co-ordinates of the turbulent mixing energy and the specific dissipation rate are, respectively (Wilcox, 1993):

$$\bar{u} \frac{\partial k^*}{\partial x^*} + h v \frac{\partial k^*}{\partial y^*} + C_k v_i \frac{\bar{u}}{R^*} \frac{\partial \bar{u}}{\partial y^*} = \frac{\partial}{\partial y^*} \left[h \left(\frac{1}{\text{Re}} + \sigma^* v_i \right) \frac{\partial k^*}{\partial y^*} \right] + v_i h S^2 - \beta^* h \omega^* k^* \quad (3.61)$$

$$\text{and} \quad \bar{u} \frac{\partial \omega^*}{\partial x^*} + h v \frac{\partial \omega^*}{\partial y^*} = \frac{\partial}{\partial y^*} \left[h \left(\frac{1}{\text{Re}} + \sigma \omega_i^* \right) \frac{\partial \omega^*}{\partial y^*} \right] + \alpha h S^2 - \beta^* h \omega^{*2} \quad (3.62)$$

where the third term on the left hand side of the turbulent mixing energy, Eq. (3.61), is the curvature correction term suggested by Wilcox and Chambers (1977), and

$$\text{the eddy viscosity, } \nu_i^* = \frac{k^*}{\omega^*}, \quad (3.63)$$

$$\text{and the strain rate, } S = \frac{\partial \bar{u}^*}{\partial y^*} - \frac{\bar{u}^*}{h R^*} \quad (3.64)$$

and the non-dimensional k^* and ω^* are defined as

$$k^* = \frac{k}{U_{ref}^2} \text{ and } \omega^* = \frac{\omega L_{ref}}{U_{ref}} \quad (3.65)$$

The values of the model constants are :

$$\begin{aligned} \alpha &= 5/9 ; & \beta &= 3/40 ; & \beta^* &= 9/100 ; \\ \sigma &= 1/2 ; & \sigma^* &= 1/2 ; & C_K &= 9/2 ; \end{aligned}$$

The boundary conditions at the edge of the boundary layer are set in the same way as for the k- ϵ model, i.e. assuming that all $\frac{\partial}{\partial y}(\)$ and the production terms are zero at the edge. Under these assumptions, the k and ω equations reduce to :

$$\overline{u}_\epsilon \frac{\partial k_\epsilon^*}{\partial x^*} = -h_\epsilon \beta^* \omega_\epsilon^* k_\epsilon^* \quad (3.66)$$

$$\overline{u}_\epsilon \frac{\partial \omega_\epsilon^*}{\partial x^*} = -h_\epsilon \beta \omega_\epsilon^{*2} \quad (3.67)$$

Solution of these two equations therefore yields boundary conditions at the shear layer edge while the wall conditions are :

$$k_w = 0, \text{ and} \quad (3.68)$$

$$\omega_w = \frac{u_\tau^2}{\nu} \left(\frac{50}{k_R^+} \right)^2 = \left(\frac{\partial \overline{u}}{\partial x} \right)_w \left(\frac{50}{k_R^+} \right)^2. \quad (3.69)$$

the last condition having been suggested by Wilcox (1993). k_R^+ is the non-dimensional wall roughness. Wilcox also states that to apply the k- ω equations right up to the wall, care must be taken to ensure that the first grid point next to the wall be close enough to the wall so that $y^+ < 1$.

3.4. Flow Computation Algorithm

The calculations of this study were made using a FORTRAN two-dimensional parabolic Navier-Stokes computer code for thin shear layers called TSL and developed by Pajayakrit (1997). This existing code was modified to implement a model for surface roughness effects. The modified TSL program is available from the department of Mechanical and Aerospace Engineering of Carleton University upon request. A brief description of its features is given in this section. A description of the code is also available in Pajayakrit (1997).

3.4.1. General Description of the Algorithm

Figure 3.3. shows the hierarchy diagram of the modified computer code. On this diagram, subroutines which have not been modified, or have only slightly been modified, are shown as simple boxes. Subroutines that underwent serious modifications are shown as bold boxes while newly created subroutines, developed purely for the calculations of the roughness effect modelling, are shown as shadowed boxes. The main program, TSL.FOR, calls all other subroutines and checks for convergence through the use of special functions. An example of the input files needed to start the calculations is given in Appendix B, as well as some notes on these input files, on the output given by the code and some general instructions on how to use the code. Electronic files containing the modified program, some example input files and the technical notes will be available upon request.

The code uses the control volume method described in Patankar (1980) to discretize the system of equations. When using this method, the location of the control volume faces can be specified in either of two ways: the faces can be located midway between the grid points or the grid points can be placed at the centre of the control volumes. For convenience of calculating the diffusion coefficient, the first practice was chosen. The grid generation was made by using a geometrical grid point distribution in the y direction, that is

$$\delta y_{i+1} = (1 + \psi) \delta y_i ,$$

where δy is the spacing between adjacent grid points and ψ is the stretching parameter, generally set between 0.05 and 0.1. Care was also taken to keep the first point next to the wall ($i=2$) at $y_2^+ < 1$. A visual marker was included to warn the user when this value was not respected so that the initial value of y_2 could be changed. The last grid point ($i=n$) was kept at $y_n/\delta > 1.75$.

Since the method of Patankar (1980) uses one common form of equation to discretize the system of equations, a common solver was used, enabling the use of analytical solutions to validate the calculated results. The stream-wise momentum and turbulence model equations could be all recast in a standard form as :

$$\frac{\partial}{\partial x^*} (\bar{u}^* \phi^*) + \frac{\partial}{\partial y^*} \left(h \bar{v}^* \phi^* - G \frac{\partial \phi^*}{\partial y^*} \right) + \frac{\bar{u}^* \bar{v}^*}{R^*} = S_p \phi^* + S_c \quad (3.70)$$

where G is the diffusive coefficient, and S_p and S_c represent the source terms which include the production and dissipation terms.

The discretized equation for the standard equation (3.70) is

$$a_P \phi_P^* = a_W \phi_W^* + a_N \phi_N^* + a_S \phi_S^* + b \quad (3.71)$$

where a_P , a_N , a_S and b are coefficients calculated from the convection, diffusion and source terms. ϕ_P^* , ϕ_W^* , ϕ_N^* and ϕ_S^* , are the unknown scalar variables ($\bar{u}^*$, k^* , ϵ^* , ω^* , etc.) and the subscripts P, W, N and S respectively refer to the node of calculation and its western, northern and southern neighbours, as labelled in Figure C-1 of Appendix C. The detailed derivation of equations (3.70) and (3.71) and the formulae for calculating a_P , a_N , a_S and b are also provided in Appendix C.

Figure 3.3 Hierarchy diagram of the modified TSL program.

3.4.2. Modifications to Account for Roughness

The modifications needed to implement the discrete element method were quite simple. Basically, two more subroutines were developed and some of the original subroutines had to be modified. The first subroutine developed (BLOCAGE.FOR) extracts the information from the input file which describes the roughness geometry of the surface to be studied. It then calculates the blockage factors at the present x-position for all y-positions as well as the roughness diameter used to calculate the drag coefficient in Eq. 2.28. The second subroutine (DRAG.FOR) uses the velocity profile as well as the information of the first subroutine (BLOCAGE.FOR) to calculate the drag of the roughness elements.

Some of the existing subroutines also had to be modified. The first of these is DEFI.FOR which opens the main input file, TSL.DAT, and reads the basic information to calculate the flow (see Appendix B). In this file, a line was added to input the roughness information. DEFI.FOR therefore had to be modified to read that line. Modifications also had to be made to the subroutine solving the stream-wise momentum equation (SMOM.FOR) to include the effect of the drag coefficient (see Eq. 3.33). The common solver therefore had to be modified to take these modifications into account which forced some of the other subroutines to be modified but, as these modifications were really minor, this was not shown in Figure 3.3. Finally, the subroutine PROP.FOR which calculates the flow properties such as C_f , δ , δ^* and θ was modified to take into account the new definition of C_f as given by Eq. 2.27. However, as explained in Section 2.4.2.3.,

the original definitions of the momentum and displacement thickness were kept to be consistent with the experimental data.

3.5. Verification of Overall Code

The basic code used in this study having previously been verified (Pajayakrit, 1997), the purpose of this section is to verify that the modifications introduced into the code to incorporate the effect of surface roughness did not introduce implementation errors. This was done using the same set of test cases used for the original verification and making sure that the revised code gave results which agreed with those given by the original code.

3.5.1. Verification with Analytical Cases

These sets of data were initially used to validate the common solver of the flow. Only two analytical cases were re-tested in this study as the results were very conclusive and only these two cases were pertinent to the study.

3.5.1.1. Blasius Flow

The most basic flow was the laminar flow over a flat plate at zero pressure gradient for which Blasius gave the analytical solution. As can be seen from Figure 3.4, the agreement between the new solution and the solution of the unmodified code is extremely good, with the discrepancies averaging 0.02%. This difference is so small that it does not show up on Figure 3.4. The results also compare very well with the exact

analytical solution, indicating that the common solver was not affected by the modifications to the code.

Figure 3.4 Comparison of calculated results with Blasius solution

3.5.1.2. Falkner-Skan Flow

The next test case was the laminar flow over a flat plate under an adverse pressure gradient known as the Falkner-Skan flow. This case was thought to be very important as one of the key feature of the present study was to calculate the flow over rough surfaces under various pressure gradients. The comparison between the results calculated both with and without the modifications and the Falkner-Skan solution is shown in Figure 3.5.

The calculations were carried out for the case of Reynolds number, $\frac{U_{\infty} L}{\nu} = 100$, and $m = -0.08257$, m being an indication of the change of the free stream velocity according to $U_{\infty} \propto x^m$, which in turn gives the pressure gradient. The Falkner-Skan velocity distribution and values of the integral parameters were obtained from White (1991). This particular case is the most severe adverse pressure gradient just short of separation for which the analytical solutions are provided. Figure 3.5 again shows that the computer solutions with and without the modifications are virtually identical therefore showing that the modifications do not affect the behaviour of the solver as far as pressure gradients are concerned. It can also be seen from Figure 3.5 that both solutions agree very well with the exact analytical solution.

Figure 3.5 Comparison of calculated results with Falkner-Skan solution

3.5.2. Preliminary Test of Unmodified Turbulence Models

Now that the solver has been shown to be unaffected by the modifications brought to the TSL code, it is desired to verify that the turbulence models are also still valid. The method used here is the same as the one in the previous section, mainly that cases having already been tested by the unmodified version of the TSL code were tested again with the modified version of the code. The agreement between both sets of results is then analysed to verify that the modifications did not alter the validity of the turbulence models implemented in the code. The different test cases have all been tested using the three turbulence models stated in section 3.3. and results showing the comparison between modified and unmodified code calculations are given for each model.

3.5.2.1. Turbulent Boundary Layer over a Smooth Flat Plate

This test was done for the case of turbulent flow over a smooth flat plate under zero pressure gradient with a unit Reynolds number of 296,000 per ft. As can be seen from Figure 3.6, the results obtained with both the modified and the unmodified code are once more almost identical, with an average difference of around 0.11% which is again too small to show on the comparison graph. It can also be seen that the results stemming from the different turbulence models are quite in agreement with one another. It can also be seen that the friction coefficient, C_f has an unrealistic value at the initial stream-wise position. This is due to the lack of precise information on the initial conditions to be specified to start the problem. This is not critical as the calculations readjust themselves

as they proceed downstream to obtain more realistic C_f values. Finally, it has been argued by Pajayakrit (1997) that these results were reasonably comparable to experimental data.

Figure 3.6 Comparison of calculated results with experimental data for a turbulent boundary layer over a smooth flat plate.

3.5.2.2. Samuel and Joubert Flow

The next test case is a turbulent boundary layer over a smooth flat plate under increasingly adverse pressure gradient. The test conditions and the experimental results were provided by Samuel and Joubert (1974). The calculations started at $x=1.1$ m., using the experimental data at nearest stream-wise positions as starting profiles. The pressure coefficient, C_p , at the wall was integrated from the experimental data for dC_p/dx . The calculated results from all turbulence models with and without modifications are compared with the experimental data in Figure 3.7. The results are again nearly identical for the original and the modified codes. As for the different turbulence models, Pajayakrit (1997) showed that although the BL model predicted slightly more accurate C_f values than the others, as can be seen from Figure 3.7, most results were very similar and all models significantly under-predicted values for k and $\overline{u^2}$.

3.5.2.3. Curved Boundary Layer over a Smooth Plate

This test case involves a turbulent boundary layer over a smooth convex wall under zero pressure gradient. The input and solution data come from one of the Stanford II (Kline, Cantwell and Lilley, 1982) test cases titled Flow 0233. The test geometry consists of a straight wall entry section, followed by a 90° turn curved section, then a straight wall recovery section. The origin of the co-ordinate system starts at the junction of the entry and the curved sections. The input data used were the first available experimental profiles given at $x = -0.3$ m. From the experimental data for the C_p , it was shown that the pressure remained essentially constant throughout all three sections,

Figure 3.7 Comparison of calculated results with experimental data for a turbulent boundary layer over a smooth flat plate under increasingly adverse pressure gradient.

whence the assumption of zero pressure gradient. Results from this test case, as displayed in Figure 3.8, once again show that the modifications to the computer code do not affect the output. The calculated results for all turbulence models can be seen to be almost identical whether the modified or the original version of the code was used. Pajayakrit (1997) notes that certain models, especially the KW model, can be very sensitive to the value of ω at the free stream edge, ω_e . However, he has shown that this sensitivity limits itself to the tendency of the k profile to "blow-up" near the free stream edge. This effect was not important for the present study as the concern was primarily on general flow parameters such as friction coefficient and boundary layer thickness. A sensitivity analysis was nonetheless done to verify the effect of varying the free stream turbulence intensity on the overall results and a discussion of this analysis is given in section 4.1.1.

Figure 3.8 Comparison of calculated results with experimental data

for a curved turbulent boundary layer over a smooth plate.

Chapter 4

Validation of the Discrete Element Approach for Rough Walls

This chapter shows a comparison between the available experimental data for rough walls, discussed in section 2.5., and the results calculated with the modified version of the TSL computer code.

4.1. Starting Profiles

As experimental data are never complete enough to start a numerical calculation without additional assumptions, the TSL program offers five schemes to make these assumptions, implemented in eight different input file formats. A detailed explanation of these schemes is given in Pajayakrit (1997). In the present study, only one of these schemes was used as the data available for the testing was very limited.

4.1.1. Profiles Used

The method used was to input an initial velocity versus y-position profile and to assume a step profile for the turbulence kinetic energy k :

$$k = \begin{cases} k_h, & y \leq \delta \\ k_l, & y > \delta \end{cases} \quad (4.1)$$

Values used for k_h and k_e and their importance is discussed in the next section. The turbulence length scale was then assumed as $l = \min(0.41y, 0.09\delta)$, and ω and ε were calculated from

$$\begin{aligned}\omega &= \frac{\sqrt{k}}{l}, \quad \text{and} \\ \varepsilon &= 0.09 \frac{k^{3/2}}{l}.\end{aligned}\tag{4.2}$$

The velocity profiles were generally taken as the first available experimental profiles although sometimes a velocity profile had to be assumed and the results were verified to assess their sensitivity to the starting profiles.

4.1.2. Effect of Starting Profile on Calculated Results

As discussed in the previous section and in section 3.5.3.3., the values inputted to the computer code in order to start the calculations may have a significant impact on the calculated results. It was especially shown that the KW model is very sensitive to the value of the free stream turbulence kinetic energy given in Eq. (4.1). In the cases where the experimental value was given, that was the value used. However, sometimes the value had to be assumed and so a sensitivity test was conducted. Figure 4.1 shows the results for the case of surface roughness consisting of 1.27 mm. diameter spheres packed in the most dense array under a mild pressure gradient, obtained with different values of the free stream turbulence intensity, including the experimental value. It can be seen from that figure that unless the assumed turbulence intensity is 100 times the actual values, the

results are not affected in a significant way. It can therefore be presumed that the assumed value is within this limit and does not influence the results in a significant way.

Figure 4.1 Comparison of calculated results for a rough surface in pressure gradient with different values of the free stream turbulence intensity.

4.2. Two-Dimensional Roughness Elements

Although Taylor et. al. (1985) suggest that the discrete element method is suitable only for three-dimensional roughness elements, it was decided to verify the limits of its use for two-dimensional roughness elements.

The test case used was the plate # 1 of Bettermann (1966) which consisted of an incompressible flow with no pressure gradient over a copper plate with two-dimensional square roughness. The roughness elements were spaced 2.65 roughness heights apart and started 0.17 m. aft of the leading edge. The available data in the Bettermann paper (1966) consisted of velocity profiles at various x stations, while profiles for $C_f/2$, θ/C and H were available in Dvorak's (1969) paper. For these calculations, the blockage factors were assumed to hold up to the actual wall location, as is customary for three-dimensional roughness elements, instead of using an equivalent wall location due to the total flow blockage of the two-dimensional elements. This was however not judged critical because the calculation was really just a test to see how the method would perform for two-dimensional roughness elements. The results are shown in Figure 4.2 and can be seen as being in very good agreement with the experimental results as far as skin friction is concerned. The shape factor seems to have been under-predicted by about 10% while the momentum thickness seems to have been underestimated by a factor of two for reasons which are not clear to the author at present. It was wondered if the values reported by Dvorak (1969) might not have been using a value for the reference length scale of $c=1$ instead of the reported value of $c=2$. In any case, these results were considered very

Figure 4.2 Comparison of calculated results and experimental data for Bettermann's flat plate under zero pressure gradient with two-dimensional roughness elements.

encouraging since the method had not been explicitly implemented for two-dimensional roughness elements.

4.3. Three-Dimensional Roughness Elements

Many cases were tested in this category although not all had sufficient data to come up with results that could be well trusted. Some of the better cases are assessed in the following sections.

4.3.1. Thin Vertical Strips

Since the discrete element method had been proven to give good results with roughness elements of circular cross-section (Taylor et. al., 1985), this case was very appealing to see how it would perform for elements with high frontal area but low thickness. The data used was that of Raupach, Coppin and Legg (1986), which was designed to model a plant canopy. The 3.0 m. long rough surface consists of an array of vertical aluminium strips, 10 mm. wide, 1 mm. thick and 60 mm. tall, arranged in a regular diamond pattern with 60 mm. cross-stream and 44 mm. stream-wise spacing. This roughness array was preceded and followed by two sections of rock like roughness to enhance flow development. This was irrelevant to the computer simulation as the developed velocity profile just upstream of the regular array was used for Initialisation. The flexible roof of the wind tunnel working section was adjusted to give zero stream-wise pressure gradient. The origin of the co-ordinate system was the leading edge of the regular roughness array.

The only available data for this experiment was the velocity profiles at various x -positions. The corresponding calculated profiles are plotted with these experimental points in Figure 4.3. From that figure, it can be seen that all the turbulence models give results which agree well with the measured profiles.

Figure 4.3 Comparison of calculated results and experimental data for the array of thin metal strips roughness elements of Raupach et. al. (1986) under zero pressure gradient.

4.3.2. Vertical Cylinders

The data used for this test case was that of Raupach, Thom and Edwards (1980), which was also designed to model a plant canopy. Although the roughness disposition was very well defined, very limited data was available to verify the accuracy of the computer code. The only available data were velocity profiles and values of C_f , δ^* and θ , which were only given at one stream-wise position, that is at $x = 2588$ mm. from the leading edge. The results are nevertheless shown here as this was an interesting case for it tested the effect of having different densities of roughness elements. The roughness consisted of small vertical cylinders, 6mm. in diameter and 6 mm. tall. The element configuration was changed four times, going from a diamond arrangement to a square arrangement and back to the diamond arrangement, in order to double the roughness density each time. Along with one test run for a smooth version of the test plate, this gave six velocity profiles as shown in Figure 4.4.

In order to start the problem, as no upstream velocity profile was given, it was decided to test different initial profiles along the smooth plate to come up with the calculated profile that would come closest to profile A of Figure 4.4. It was argued that since the leading edge of the test plate was free-standing in the flow, the flow over the first part of the plate should be laminar. A Blasius velocity profile with a δ value of 0.327 mm. was tried as the initial profile at a stream-wise position of 5 mm. aft of the leading edge. Since no means to evaluate the location of transition is included in the TSL code, the Blasius profile was used in conjunction with turbulence models from the leading

edge. Even though this seemed an unusual procedure at first, the resulting velocity profile obtained at $x = 2588$ mm. was shown to be in very good agreement with that found by Raupach et. al. (1980), as seen when comparing the different velocity profiles of Figure 4.5. with velocity profile A in Figure 4.4. In Figures 4.5. and 4.6., straight lines with the same slope u_τ/k as in Figure 4.4. have been superimposed on the profiles for comparison. Other initial stream-wise positions of the Blasius profile as well as different profiles were also tried without any improvement to the accuracy of the final velocity profile. For this reason, the original Blasius profile (at $x_i = 5$ mm.) was kept as the initial profile for all the test cases. The resulting velocity profiles for the cases with different roughness densities can be seen to be in good agreement with the experimental measurements of Raupach et. al. as can be seen from Figure 4.6. Finally, values for C_f , δ^* and θ were given at only one location so that good comparison of the evolution of those parameters along the plate was not possible. Table 4.1. however shows a comparison between those parameters as calculated by the code and the reported values and it can be seen that the results fall within an acceptable error margin for cases B, C and D. The discrepancy between calculated and experimental results is, however, rather large for the high density cases E and F.

MODEL	PLATE	B	C	D	E	F
Array Type		Diamond	Square	Diamond	Square	Diamond
Nearest neighbour separation D (mm.)		56.6	40	28.3	20	14.1
Inverse concentration $1/\lambda$		89	44	22	11	5.6
BL	δ^* (mm)	15.23	18.12	26.16	30.34	39.65
	θ (mm)	10.34	12.16	15.99	18.41	22.13
	u_τ (m/s)	0.885	0.883	1.065	1.06	1.191
KE	δ^* (mm)	16.22	19.86	30.17	35.49	43.85
	θ (mm)	11.52	13.96	19.35	22.49	25.49
	u_τ (m/s)	0.97	0.986	1.232	1.234	1.237
KW	δ^* (mm)	15.49	18.88	28.49	33.81	26.68
	θ (mm)	10.78	12.95	17.74	20.68	25.69
	u_τ (m/s)	0.945	0.952	1.187	1.186	1.379
Experimental Results	δ^* (mm)	16.1	21.4	23	25.7	27.4
	θ (mm)	10.8	13.6	14.1	14.8	15.6
	u_τ (m/s)	0.843	0.938	1	1.025	1.068

Table 4.1. Comparison between flow parameters as calculated by the code and experimental data of Raupach et. al. (1980).

Figure 4.4 Velocity profiles at $x=2588$ mm., as given by Raupach et. al. (1980)

Initial Blasius profile at
 $x_i=5$ mm.
($\delta_i = 0.327$ mm.)

Initial Blasius profile at
 $x_i=50$ mm.
($\delta_i = 1.034$ mm.)

Initial Thompson profile at
 $x_i=100$ mm.
($\delta_i = 2.946$ mm.)

Figure 4.5 Comparison of the velocity profile for the smooth plate case investigated by Raupach et. al. (1980) as calculated by the TSL program with different initial profiles.

Figure 4.6 (a)

For legend, see page 85

Plate B

Plate C

Plate D

Plate E

Plate F

Figure 4.6 (b)

For legend, see page 85

Plate B

Plate C

Plate D

Plate E

Plate F

Figure 4.6 Calculated velocity profiles for the different roughness densities in the test case of Raupach et. al. (1980).

[Calculations made using a) BL model, b) KE model, c) KW model].

4.3.3. Spheres Packed in the Most Dense Array

This case is the one for which the most complete data is available. It is the case, thoroughly studied in the Stanford programme, of a porous flat plate composed of uniform, 1.27 mm. diameter spheres packed in the most dense array. The free stream velocity at the inlet section was nominally 26.8 m/s and the turbulent boundary layer was in a fully rough state for all cases reported. This particular configuration was studied by Healzer (1974), Pimenta (1975) and Coleman et. al. (1977). The data presented here is that available in Coleman et. al. (1977) and Taylor et. al. (1985), as the Ph.D. reports (Healzer, 1974 and Pimenta, 1975) were not accessible for full data examination.

The most interesting particularity about the study of this surface is that Coleman et. al. (1977) have included the effect of the pressure gradient on the boundary layer development. For this reason, this section is divided into three sub-sections dealing respectively with zero pressure gradient flow, equilibrium flow with pressure gradient and non-equilibrium flow. The different pressure gradients were identified using a pressure gradient parameter for fully rough flows defined by Coleman et. al. (1977) as

$K_r = \frac{r}{U_e} \frac{dU_e}{dx}$. This is analogous to the acceleration parameter used for smooth wall

boundary layers $K = \frac{v}{U_e^2} \frac{dU_e}{dx}$.

4.3.3.1. Zero Pressure Gradient ($K_r = 0$)

This test was important because it was used to evaluate the apparent wall location which is one of the difficulties associated with this type of surface. Taylor et. al. (1985) suggest that the apparent wall location should be situated 0.2 diameters below the crests of the roughness elements. However, tests were conducted using the different turbulence models to see which wall location would give the best agreement between calculated results and those obtained experimentally. Figure 4.7 shows the results of those tests and it can be seen that the optimum wall location is not the same for all turbulence models. The BL and KE models seem to give the best overall results for a y_0 location situated 0.4 D below the crests of the roughness elements. For the KW model, the best wall origin location is hard to identify as putting y_0 at 0.2 D below the elements crests gives better C_f values but using $y_0=0.25D$ below the crests gives better values of momentum thickness and velocity profiles (see Figure 4.7). Since the skin friction coefficient is the parameter which generally is of greatest interest, it was decided to keep the value of y_0 at 0.2 D below the element crests as this was also the value suggested by Taylor et. al. (1985). Figure 4.7 also shows that the velocity profiles found were in good agreement with experimental results but mainly only in the outer region. The experimental values for the C_f values are taken from Figure 7 of Taylor et. al. (1985) and those for the momentum thickness, θ , were taken from Figure 2 of Coleman et .al. (1977).

Figure 4.7 (a) Comparison of flow parameters calculated with the BL model and measured flow parameters for the different apparent wall locations in the Stanford test case with no pressure gradient (velocity profiles taken at $x/L_{ref} = 15$).

Figure 4.7 (b) Comparison of flow parameters calculated with the KE model and measured flow parameters for the different apparent wall locations in the Stanford test case with no pressure gradient (velocity profiles taken at $x/L_{ref} = 15$).

Figure 4.7 (c) Comparison of flow parameters calculated with the KW model and measured flow parameters for the different apparent wall locations in the Stanford test case with no pressure gradient (velocity profiles taken at $x/L_{ref} = 15$).

4.3.3.1. With Pressure Gradient, Equilibrium Flows

The data sets for these test cases were given by Coleman et. al. (1977) in a very comprehensive format. Graphs of the experimental values of C_f , H , θ and the pressure gradient distribution were available and only the initial velocity profile had to be assumed. This was done by using the same velocity profile as that used for the case with zero pressure gradient. Two other types of profiles were tested, including a uniform velocity profile at the leading edge of the plate, with not much difference between the final results and so the original profile was assumed as satisfactory.

Two experimental cases given in Coleman et. al. (1977) were used, one with a mild adverse pressure gradient ($K_r = 0.15 \times 10^{-3}$) and one with a severe adverse pressure gradient ($K_r = 0.29 \times 10^{-3}$). The results given by the computer code are compared with the experimental data in Figures 4.9 and 4.10 for the mild and the severe adverse pressure gradients respectively. As can be seen, all turbulence models give good predictions for the skin friction coefficient under both mild and severe gradients. However, the momentum thickness θ obtained from the calculations seems to be in good agreement with a value of twice the reported experimental value. This was a very surprising effect and could not be explained by the author from theory or calculated results. However, a check was made to see if experimental and calculated results were consistent with results obtained using the momentum-integral equation. This was done using a spreadsheet and using inputs at every stream-wise position to calculate the momentum thickness at the next stream-wise position. The inputs used were either the reported values for the

experimental check or the values calculated by the code for the TSL check. A third check was made using twice the reported θ values in the momentum-integral calculations, to support the hypothesis that these would be the real experimental values. As shown in Figure 4.8, the calculated results do follow the integral approach results but the reported experimental values do not. The third check seems to indicate that the reported values of C_f , δ^* and u_e are in better agreement with values of twice the reported θ values. It was wondered if, since the experimental results are reported in the form of θ/r where r is the roughness element radius, an error involving use of the roughness element diameter instead of the radius could have taken place when the Coleman et. al. (1977) figures were made. This is the only explanation that the author has been able to find and the experimental θ values were therefore considered as twice the values reported. With this assumption, the calculated results can be considered in very good agreement with the experimental results. Finally, the calculated value of shape factor, H , from figures 4.9 and 4.10 can be seen to vary a little more from model to model than the other parameters but can also be considered to be in good agreement with the experimental values reported.

Figure 4.8 Verification of the calculated and reported momentum thickness versus x/L_{ref} that expected from the momentum integral approach.

Figure 4.9 Comparison of calculated and measured flow parameters for the case of flow under a mild adverse pressure gradient ($K_r = 0.15 \times 10^{-3}$) of the Stanford experiment.

Figure 4.10 Comparison of calculated and measured flow parameters for the case of flow under a severe adverse pressure gradient ($K_r = 0.29 \times 10^{-3}$) of the Stanford experiment.

4.3.3.1. With Pressure Gradient, Non-Equilibrium Flow

This case was very interesting as it enabled the verification of the discrete element model for a case of non-equilibrium flow, which is the most common case in real applications. Note that both previous cases also had regions of non-equilibrium flow and the method still proved itself reliable. Since the pressure gradient parameter K_r is only valid for equilibrium flows, the parameter used to quantify the pressure variation of the non-equilibrium case is the acceleration parameter used for smooth wall boundary layers,

$$K = \frac{v}{U_e^2} \frac{dU_e}{dx}. \text{ The case which was tested was that for which the value of } K \text{ was set as } K$$

$= 0.29 \times 10^{-6}$. The results are shown in Figure 4.11 and can be seen to have the same kind of agreement as for the two previous cases. The method therefore seems to be reliable whether the flow is in equilibrium or not. It can be again seen that the calculated results follow a trend which doubles the trend reported for the momentum thickness. This again supports the hypothesis that some error in reporting the value of θ might have occurred, probably by using the diameter instead of the radius in the non-dimensionalisation process.

Figure 4.11 Comparison of calculated and measured flow parameters for the case of non-equilibrium flow under a severe adverse pressure gradient ($K = 0.29 \times 10^{-6}$) of the Stanford experiment.

Chapter 5

Conclusions and Recommendations

5.1. Conclusions

The results obtained in the preceding chapters were all compared to experimental results taken from the available literature. However, complete data records for all of the experiments were not available and so some assumptions had to be made to generate the necessary information in order to be able to conduct the validations. These assumptions were made either from available information from other studies, from interpretation of graphical results or from common sense deductions. In light of this, the generally small discrepancies between calculated and measured flow properties can be considered as acceptable to within experimental and judgmental errors. For this reason, it is considered that the results shown in the previous chapters can be trusted to represent well the behaviour of the TSL algorithm.

From the results of the previous chapters, it can be concluded that the objectives of this study have been met :

1. A discrete element method designed to model the effects of surface roughness on flow parameters has been implemented into an existing two-dimensional parabolic Navier-

Stokes computer code without modifying the reliability of the code for smooth surfaces.

2. The governing equations for the discrete element method have been derived in surface-normal, curvilinear co-ordinates and their effect on the calculation of flow parameters have been assessed.
3. The results of the implemented method have been shown to give results which are in good agreement with experimental measurements for rough wall flows with and without pressure gradients which may or may not be under equilibrium conditions. This conclusion applies only to regular arrays of three-dimensional roughness elements. Some results for two-dimensional roughness elements show encouraging trends which suggest that the method could be extended to such cases.
4. All three tested turbulence models were shown to be compatible with the implemented discrete element model. For the different test cases, all models showed agreement which was within acceptable error margins. Although some models performed better for certain cases, it was not found that any model showed decisively better overall performance.
5. The discrete element model has been shown to be a viable resource for the modelling of turbulent flow properties over rough surfaces in cases where a two-dimensional parabolic Navier-Stokes marching code can be used.

5.2. Recommendations

Despite the apparent success of this study in modelling the effects of surface roughness on turbulent flow properties, sufficient data showing these effects as measured experimentally is still lacking. As discussed in Section 5.1., the foregoing conclusions are dependent upon the validity of the assumptions made to fill the lack of information available for turbulent boundary layers over rough surfaces. Extensive studies giving exhaustive data measurements for many different roughness geometries and densities should be conducted to ensure that all possible effects are included in the models and that these models can predict flows for a wider variety of surface roughness. Specifically, the experimental efforts should lie in the following domain:

1. Experimental re-evaluation of the results for roughness elements similar to those of Schlichting (1936) with full details of velocity profiles, kinetic energy distribution and dissipation, stress deviation and integral parameters.
2. Fully detailed evaluation of surfaces roughened with different roughness geometries, including two-dimensional roughness elements, and evaluation of effects of different roughness parameters on the validity of the equivalent sand-grain approach.
3. Detailed evaluation of surfaces roughened with randomly distributed roughness elements and with different roughness spacing or density.
4. Study of the effect of the wakes of roughness elements on the blockage factor of these elements.

5. Extensive study to accurately evaluate the drag coefficient of different roughness arrangements.

In addition, numerical improvements might be achieved by including the blockage effects as well as other roughness elements source or sink terms in the turbulence modelling equations. A study to assess how to include these effects in the method as well as whether it is an improvement or not would therefore be beneficial.

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Appendix A

Modified TSL Program

Program listing available in electronic format on request.

Appendix B

Program Notes and Example of Input Files

These notes and the example input files are also available in electronic format upon
request

Notes Regarding Using the TSL program

This write-up is contained in the word95 format file "TSLnotes.doc" or in text format in the "Readme3.txt" file.

Required Drivers

TSL program requires Watfor Graphics Kernel System (WGKS) and access to logical drives e: and f:. Optionally GRAPHICS.COM is also needed if direct graphic screen dump to printer is desired. File STANKE.BAT in the \bats\ directory shows how to load the drivers and make drives e: and f: available. Drives e: and f: are used to hold the output files from TSL. I normally put these files into a "Results" directory but any directory will do just as well as long as it is well specified. File STANKE.BAT also shows how the TSL.DAT file pertinent to this specific test case (Stanford rough plate, k-epsilon model) is imported from its respectful directory into the main TSL directory.

Input Files

The main input file is TSL.DAT which contains references to other input files, as well as values of various parameters. TSL program also reads several other model constant files. These are files with the extension *.mc. For example, KE.MC is the model constant file for the k-epsilon model. Files that look like *_std.mc are not accessed by TSL program. They contain standard values of the model constants and can be used when you need to do a "reset to default".

The format of the main input file TSL.DAT is as follows.

Line 1: Title, a string that describes the flow being calculated.

Line 2: Model, Reynolds no., Flow ID, Discretization Scheme;

Model, integer specifying the turbulence model, 0=laminar, 1=Baldwin-Lomax,
2=k-eps, 3=k-omeg, 4=multiscale,

Reynolds no., real value of $U_{ref} * L_{ref} / \nu$.

Flow ID, integer indicating some special flows, 0=general boundary layers,

10=general wall jets, see Appendix A for other values,

Discretization Scheme, integer specifying how the convection and diffusion are discretized, 0=power-law, 1=upwind.

Line 3: xi, dxi, xf, dxs;

xi, initial value of streamwise position, x/L_{ref} ,

dxi, initial value of streamwise position, dx/L_{ref} ,

xf, final value of streamwise position, x/L_{ref} ,

dxs, streamwise step size in multiple of delta for boundary layers and yhalf for wall jets.

Line 4: dyli, ymaxi, xx, stretching factor, ytop;

dyli, initial first delta y next to the wall,

ymaxi, initial maximum y/L_{ref} ,

xx, not used at present,

stretching factor, the ratio of $dy(i)/dy(i-1)$ for geometrical grid distribution,

ytop, value of y/L_{ref} at the top of the control volume used in calculating global momentum conservation. This value is optional and does not affect calculated results.

Line 5: omegu, omegv, omegnu, omegz, omegrs;

omegu, relaxation factor for u,

omegv, relaxation factor for v,

omegnu, relaxation factor for eddy viscosity,

omegz, relaxation factor for epsilon (k-eps model) or omega (k-omeg model),

omegars, relaxation factor for Reynolds stresses (multiscale model).

Line 6: IPscheme, P1, P2, P3, IPfile;

IPscheme, integer specifying how P1, P2, and P3 are to be interpreted, see details in Appendix B,

P1, P2, P3, parameters of the initial profiles, see Appendix B,

IPfile, initial profile filename, see Appendix B for file format.

Line 7: Wall curvature file, Wall static pressure file, Screen plot format file;

Wall curvature file (*KW), filename of the file describing Lref/R as a function of streamwise position, x/Lref in the format x/Lref, Lref/R;

Wall static pressure file, filename of the file describing wall static pressure coefficient, $C_p = (P - P_{ref}) / (1/2 * \rho * U_{ref}^2)$ in format x/Lref, Cp;

Screen plot format file, filename of the file describing what variables to display during a run and where they are displayed in following format.

Line 1: ymin, ymax

Line 2 and up: window, variable, vmin, vmax;

window, integer indicating where the variable will be displayed, available windows are from 1 to 15,

variable, a string identifying variable name to be

displayed, see file PROF.FOR for all available

variable names,

vmin, vmax, minimum and maximum values of the

variable in the plot window.

Line 8: Roughness model, Wall roughness file, roughness gradient factor;

Roughness Model : Integer specifying the physical aspect of the roughness

elements(0=circular cross section elements, 1=2-D roughness elements,

2=Not used at present, 3=THIN STRIPS, 4=spheres)

Wall roughness file : filename of the file describing wall roughness geometry.

The format is : XR(I),LX(I),LZ(I),D0(I),D1(I),D2(I),D3(I),

D4(I),YTOP(I)

XR : value of streamwise position, x/Lref, at which the roughness parameters are known;

LX : value of streamwise distance between two adjacent rows of roughness elements, Lx/Lref;

LZ : value of cross-stream distance between two adjacent rows of roughness elements, Lz/Lref;

D0,D1,D2,D3,D4 : values used in calculating the roughness geometry (varies with roughness model, see BLOCAGE.FOR)

YTOP : roughness height, k/Lref.

Roughness gradient factor, MR : Integer specifying the way the roughness geometry changes over the surface

(MR=0:ABRUPT CHANGE IN ROUGHNESS AT NEW STATION, MR=1:LINEAR CHANGE IN ROUGHNESS BETWEEN STATIONS)

Line 9 and up: x, Solution file, variable name, Print flag;

x, value of streamwise position, x/Lref;

Solution file, filename of the file containing solution to be displayed for comparison with the calculated results in format x,v or y,v;

Variable name, a string identifying the variable in the solution file, see file PROF.FOR for available variable names;

Print flag, a logical parameter specifying whether calculated profiles will be printed out at this x position.

Output Files

- ERRMSG.TXT : information on x positions where there were difficulties(see RECOV3.FOR).
- CONV.TRA : the residual errors after convergence at each streamwise step(see IPOUT.FOR).
- GLOB.TRA : Global streamwise angular momentum conservation(see GLOB.FOR).
- STRIP.TRA : Stripwise streamwise angular momentum conservation(see STRIP.FOR).
- OUTX.CAL : Integral and profile parameters as functions of x/L_{ref} , the format is
1. x/L_{ref}
 2. $C_f = \tau_{uw}/(1/2 \cdot \rho \cdot U_e^2)$ for b.l. or $\tau_{uw}/(1/2 \cdot \rho \cdot U_m^2)$ for w.j.
 3. δ^*/L_{ref} = displacement thickness
 4. θ/L_{ref} = momentum thickness
 5. y_{half}/L_{ref} = wall jet half width
 6. y_m/L_{ref} = wall jet y at U_{max}
 7. U_m/U_{ref} = wall jet maximum velocity
 8. δ_{99}/L_{ref} = shear layer thickness
 9. U_{pw}/U_{ref} = potential wall velocity (or U_e for plane flow)
 10. $U_x/U_{ref} = (U_m - U_{pw})/U_{ref}$
 11. $y_{t0}/L_{ref} = y$ @ zero shear stress point for wall jets
 12. L_{ref}/R = wall curvature
 13. C_{pw} = wall static pressure coefficient
 14. $y_{2+} = y_+$ of the first grid point next to the wall
 15. kR_+ = non-dim surface roughness for k- ω and multiscale models.
- OUTn.CAL : Calculated profiles at the streamwise positions specified in line 8 and above of the file TSL.DAT. OUT1.CAL corresponds to the first x position, OUT2.CAL to the second x and so on. The format is
1. y/L_{ref} ,
 2. u/U_{ref} ,
 3. v/U_{ref} ,
 4. C_p ,
 5. $\nu_{eff}/(U_{ref} \cdot L_{ref})$, the sum of eddy and molecular viscosity,
 6. k/U_{ref}^2 ,
 7. $\epsilon \cdot L_{ref}/U_{ref}^3$ or $\omega \cdot L_{ref}/U_{ref}$,
 8. $(k-\epsilon)/k$,
 9. τ/U_{ref}^2 ,
 10. S_x/U_{ref}^2 ,
 11. S_y/U_{ref}^2 ,
 12. $du/dy \cdot L_{ref}/U_{ref}$.
- OUTn.CSC : Scaled profiles at the streamwise positions specified in line 8 and above of the file TSL.DAT. The local length scale, L, and velocity, U, for scaling is δ^* and U_{pw} for boundary layers, and y_{half} and U_x for wall jets. The file format is
1. y/L ,
 2. u/U ,
 3. v/U ,
 4. k/U^2 ,
 5. $\epsilon \cdot L/U^3$ or $\omega \cdot L/U^2$,
 6. $(k-\epsilon)/k$,
 7. τ/U^2 ,
 8. S_x/U^2 ,
 9. S_y/U^2 ,
 10. $d(u/U)/d(y/L)$
 11. y_+ ,
 12. u_+ .
- OUTn.CM1 : Selected profiles just upstream of those in OUTn.CAL (see SOLU.FOR).

Disk Organization

There are 2 main directories :

- bats : contains the batch files required to run the different cases ;
- tsl : contains all fortran source codes and all test cases as well as the results directory.

The tsl directory is split in 5 sub directories :

- bl : contains all the smooth boundary layer cases computed by Palanunt Pajayakrit :
 - Flat plate zero press. grad. flow in dir. "tfp"
 - Samuel and Joubert flow in dir. "f0141"
 - Curved b.l. in dir. "f0233"
- lam : contains all the laminar boundary layer cases computed by Palanunt Pajayakrit :
 - Blasius flow in dir. "blasius"
 - Falkner-Skan flow with $m=-0.08257$ in dir. "fs08"
- rough : contains all the rough boundary layer cases computed in this study :
 - 2-Dimensionnal roughness flow in dir. "btrmn"
 - Thin metal strips roughness elements in dir. "Raup86"
 - Low aspect-ratio cylindrical roughness elements in dir. "Raup80"
 - High aspect-ratio cylindrical roughness elements in dir. "thom71"
 - Spheres packed in the most dense array in dir. "Stanford"
- Results : Contains the output files from the last test case ran ;
- Watgksli : contains the library files to run Watcom GKS.

Each test case data directory contains subdir. "bl", "ke" and "kw" which hold the TSL.DAT files for the respective models.

Running TSL

The following steps run the Tsl program from the Watfor87 executable window :

- 1) Set default drive to a: (or the directory where the content of the tsl disk is copied to) ;
- 2) In the "bats" directory, type the name of the batch file pertaining to the case to be tested ;
- 3) When prompted, type y to delete results from previous runs ;
- 4) In the Watfor87 window, Type TSL.

Appendix A : Flow ID

Flow ID identifies some special flows which require particular non-dimensional parameters or extra input. These flows were not used in the simulation of any rough surfaces but can still be treated with the TSL program. These flows are as follows :

Flow ID	Description	Notes
0	General b.l.	
1	Blasius	
2	Falkner-Skan	*.CPW file format : $U_e/U_x, 9dC_p/dx)_i$ $(dC_p/dx)_i = \text{initial prers. grad.} \approx -(U_e/U_x)^{**2}$
3	Laminar b.l. over a cylinder	
10	Wall jets in still air	
11	Curved wall jet over log-spiral	*.KW file format $y_{1/2} / R, L_{ref}/R$
20	Wall jets in moving stream	
21	Self-similar plane w.j.	*.CPW file format : $U_e/U_x, 9dC_p/dx)_i$ $(dC_p/dx)_i = \text{initial prers. grad.} \approx -(U_e/U_x)^{**2}$

Appendix B : Input Parameters

IPscheme	P1	P2	P3	IPfile format	Description
0				y, u	Laminar flow
1	Kh	Ke	Ee or We**	y, u	Assuming a step profile for k., Kh = k value in the shear layer, Ke = k value outside of the layer.
2	same as 3.				
3		Cf	Ke	Ee or We**	y, u Assuming a profile of shear stress, then calculate k and eps or omeg from eddy viscosity. Ke=value of k at the last grid point.
4		Ke*	Ee or We**	y, u, k, l	Given values of velo, k, and length scale profiles.
5		Ke*	Ee or We**	y, u, k, tau, Sx, Sy	All given values are in same file.
6	Ue*	Ke*	Ee or We**	'var', 'filename'	Starting profiles are in different files. The IPfile lists each that the profile is filename
variable available and the of the file that contains it. The format of each file is y, v.					
7	Ue*	Ke*	Ee or We*	OUTn.CAL	Using the calculated profiles from previous run as the starting profiles.
8	Ue*	Ke*	Ee or We*	OUTn.CSC	Using the scaled profiles from previous run as the starting profiles.

Notes

- * : If the specified value is >0, then the last grid point assumes that value.
If the specified value is <= 0, then the last grid point value is interpolated from the initial profile.
- ** : If the specified value is >0, then the last grid point assumes that value.
If the specified value is 0, then the last grid point value is calculated from the relation involving k and delta.
If the specified value is <0, then the last grid point value is calculated from the relation involving the eddy viscosity.

Examples of Input Files

TSL.DAT

```

STANFORD VERIFICATION; KE MODEL'
2, 154829.6, 0, 0
2.45, .05, 22.05, 0.5
0.00006, 2.45, .5, 1.10, .45
.5, 1., 1., 0.5, 1.
1, 0.004, 1.e-4, 0., ROUGH\STANFORD\INIT.u0'
FLAT.KW', CPW0.CPW', ROUGH\STANFORD\bNfp.pf'
4, 'STANFORD.rgh', 1
15., 'N/A', '', .TRUE.
25., ROUGH\STANFORD\THETA', THETA', .FALSE.
25., ROUGH\STANFORD\CF', CF', .FALSE.

```

TFP.PF

```

0., 6.
10, 'UBL', 0., 1.
13, 'THETA', 0., .015
14, 'DELS', 0., .01

```

STANFORD.RGH

```

0.00  0.01245  0.01245  0.01220  0.01245  0.00249  0.00000  0.00000  0.00249
23.92 0.01245  0.01245  0.01220  0.01245  0.00249  0.00000  0.00000  0.00249

```

FLAT.KW

```

0., 0.
1000., 0.

```

CPW0.CPW

```

-100., 0.
0., 0.
100., 0.
1000., 0.

```

THETA.SOL

5.402392976	0.017931527
9.326236295	0.026847283
12.45393749	0.038710615
15.24043492	0.049509543
18.08379965	0.057271622
21.04089896	0.06404524

CF.SOL

5.402392976	0.005470588
9.326236295	0.005088235
12.45393749	0.004764706
15.24043492	0.004558824
18.08379965	0.004441176
21.04089896	0.004352941

INIT.U0

0.	0.
0.000779118	0.159489809
0.000881127	0.176069112
0.001014161	0.195016887
0.001209046	0.218701606
0.001441381	0.242386324
0.001718362	0.266071043
0.002048568	0.289755762
0.002442229	0.31344048
0.002911536	0.337125199
0.003471028	0.360809917
0.004138033	0.384494636
0.004933213	0.408179354
0.005881197	0.431864073
0.00701135	0.455548792
0.008358676	0.47923351
0.00996491	0.502918229
0.011879804	0.526602947
0.01416267	0.550287666
0.016884221	0.594035412
0.020848944	0.625605053
0.025744658	0.663488622
0.030691845	0.703476834
0.037238525	0.741360403
0.045982811	0.791871829
0.053863902	0.838173969
0.063095749	0.888685395
0.073909862	0.926568964
0.083586763	0.949720034
0.097912873	0.972871104
0.108803113	0.97708039
0.125230474	0.97708039

Appendix C

Derivation of the Discretized Equations of Motion

The stream-wise momentum equation (2.26 or 3.33), The k and ϵ equations for the Dash k - ϵ model (equations 3.47 and 3.48) and the k and ω equations for the Wilcox k - ω model (equations 3.61 and 3.62) can be cast in the following standard form :

$$\frac{\partial}{\partial x^*}(\bar{u}^* \phi^*) + \frac{\partial}{\partial y^*} \left(h \bar{v}^* \phi^* - G \frac{\partial \phi^*}{\partial y^*} \right) = S_p \phi^* + S_c \quad (C-1)$$

the momentum equation (3.33) will be used to demonstrate the steps involved in putting it in a standard form. It must first be put in conservative form by adding to it the conservation equation (3.32) multiplied by $\bar{u}^*$. This will yield the following equation :

$$\begin{aligned} \frac{\partial}{\partial x^*}(\bar{u}^{*2}) + \frac{\partial}{\partial y^*}(\bar{h} \bar{v}^* \bar{u}^*) + \frac{\bar{u}^* \bar{v}^*}{R^*} = & -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\bar{C}_p}{2} + \bar{u}^{*2} \right) \right] - \left(\frac{\bar{v}_e^*}{h R^*} + \frac{\partial \bar{v}_e^*}{\partial y^*} \right) \frac{\bar{u}^*}{R^*} \\ & + \frac{1}{\beta_x} \frac{\partial}{\partial y^*} \left[\beta_y h \bar{v}_e^* \frac{\partial \bar{u}^*}{\partial y^*} \right] - \frac{h}{2 \beta_x} \bar{C}_D \frac{d^*}{L^* l^*} \bar{u}^{*2} \left(1 + \frac{\bar{u}^{*2}}{\bar{u}^*} \right) \end{aligned} \quad (C-2)$$

which can be recast in the form

$$\begin{aligned} \frac{\partial}{\partial x^*}(\bar{u}^{*2}) + \frac{\partial}{\partial y^*} \left(h \bar{v}^* \bar{u}^* - \frac{\beta_y}{\beta_x} h \bar{v}_e^* \frac{\partial \bar{u}^*}{\partial y^*} \right) = & \\ -\frac{1}{R^*} \left(\bar{v} + \frac{\bar{v}_e^*}{h R^*} + \frac{\partial \bar{v}_e^*}{\partial y^*} + \frac{h R}{2 \beta_x} \bar{C}_D \frac{d^*}{L^* l^*} \bar{u}^{*2} \left(1 + \frac{\bar{u}^{*2}}{\bar{u}^*} \right) \right) \bar{u}^* - \frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\bar{C}_p}{2} + \bar{u}^{*2} \right) \right] \end{aligned} \quad (C-1)$$

by assuming that β_x varies slowly in the y direction.

Comparing Equation (C-3) with the standard form, Eq. (C-1), one obtains :

$$\phi^* = \bar{u}^*, \quad (C-4)$$

$$G = \frac{\beta_y}{\beta_x} h v_e^*, \quad (C-4)$$

$$S_p = -\frac{1}{R^*} \left(v + \frac{v_e^*}{h R^*} + \frac{\partial v_e^*}{\partial y^*} + \frac{h R}{2 \beta_x} \overline{C_D} \frac{d^*}{L^*} \overline{u}^2 \left(1 + \frac{\overline{u^{2*}}}{\overline{u}^2} \right) \right) \text{ and} \quad (C-4)$$

$$S_c = -\frac{1}{\beta_x} \frac{\partial}{\partial x} \left[\beta_x \left(\frac{\overline{C_p}}{2} + \overline{u^{2*}} \right) \right] \quad (C-5)$$

This satisfies Basic rule number 3 of Patankar (1980) which states that to avoid physically unrealistic values during iteration, S_p must always be less than or equal to zero.

With these definitions, the standard equation (C-1) can be discretized by integrating over the control volume of Figure C-1.

$$\oint_{C.S.} \left[\frac{\partial}{\partial x^*} (\bar{u}^* \phi^*) + \frac{\partial}{\partial y^*} \left(h \bar{v}^* \phi^* - G \frac{\partial \phi^*}{\partial y^*} \right) \right] dA = \oint_{C.V.} (S_p \phi^* + S_c) dV \quad (C-6)$$

where C.S. stands for control surface and C.V. stands for control volume. For the very small two dimensional control volume of Figure (C-1), Eq. (C-6) can be approximated by

$$\bar{u}^* \phi^* \Delta A \Big|_w^e + \left(h \bar{v}^* \phi^* - G \frac{\partial \phi^*}{\partial y^*} \right) \Delta A \Big|_s^n = (S_p \phi^* + S_c) \Delta V \quad (C-7)$$

ΔA being the area of a surface of the control surface, ΔV the volume of the control volume and e, w, s and n are the four control surfaces as labelled in Figure C-1.

Patankar (1980) has shown that the coefficients in the discretized version of the standard equation (Equation 3.71) depend on the mass flow through the control surfaces F , the diffusive conductance across the control surfaces, D , and the volume of the control volume, ΔV , which are determined as follows

$$\begin{aligned} F_e &= \bar{u}_e \Delta y, \\ F_w &= \bar{u}_w \Delta y, \\ F_n &= \bar{u}_n \Delta x, \\ F_s &= \bar{u}_s \Delta x, \\ D_e &= 0, \\ D_w &= 0, \\ D_n &= G_n \frac{\Delta A_n}{\delta y_n} = G_n \frac{h_n \Delta x}{\delta y_n}, \\ D_s &= G_s \frac{\Delta A_s}{\delta y_s} = G_s \frac{h_s \Delta x}{\delta y_s}, \\ \Delta V &= \frac{h_n + h_s}{2} \Delta x \Delta y. \end{aligned}$$

The locations of the control surfaces are placed at the midpoint between the nodes simply for the convenience of calculating the coefficients. The results are such that

$$\begin{aligned} \bar{u}_e &= \frac{\bar{u}_P + \bar{u}_E}{2}, & \bar{u}_w &= \frac{\bar{u}_W + \bar{u}_P}{2}, \\ \bar{v}_n &= \frac{\bar{v}_P + \bar{v}_N}{2}, & \bar{v}_s &= \frac{\bar{v}_S + \bar{v}_P}{2}, \\ h_n &= 1 + \frac{y_P + y_N}{2R}, & h_s &= 1 + \frac{y_S + y_P}{2R}, \end{aligned}$$

$$G_n = \frac{2G_N G_P}{G_N + G_P} \quad \text{and} \quad G_s = \frac{2G_S G_P}{G_S + G_P}.$$

The final discretized equation is

$$a_P \phi_P^* = a_E \phi_E^* + a_W \phi_W^* + a_N \phi_N^* + a_S \phi_S^* + b$$

where $a_E = D_e A(|P_e|) + \| -F_e, 0 \| = 0,$

$$a_W = D_w A(|P_w|) + \| F_w, 0 \| = F_w,$$

$$a_N = D_n A(|P_n|) + \| -F_n, 0 \|,$$

$$a_S = D_s A(|P_s|) + \| F_s, 0 \|,$$

$$b = S_C \Delta V \quad \text{and}$$

$$a_P = a_E + a_W + a_N + a_S - S_P \Delta V$$

and where the symbol $\| , \|$ represents the maximum function. The Peclet numbers, P_i are defined as

$$P_i = \frac{F_i}{D_i}.$$

Patankar (1980) discusses several possible forms of the function $A(|P|)$ and recommends the power law scheme given by

$$A(|P|) = \| 0, (1 - 0.1|P|)^5 \|$$

as the most accurate scheme. However, Pajayakrit (1997) found that for some flows, the Power Law scheme produced discontinuous solutions near the free stream edge. For such cases, the Upwind scheme, given by $A(|P|)=1$, was used instead.

IMAGE EVALUATION TEST TARGET (QA-3)

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