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The Automated Ordering of
Moiré Fringe Contours
Using Local Photometric Modelling
of Surface Shading

by

JAMES REGAN PEKELSKY, M.Sc.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University
April, 1988

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Using Local Photometric Modelling of Surface Shading"

submitted by JAMES REGAN PEKELSKY, M.Sc., in partial fulfilment
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Abstract

Moiré topography is an optical shape-encoding technique for superimposing a pattern of shadows (*moiré fringes*) on the image of an object such that the fringes are contours of elevation. The method ensures that fringes are properly formed at contours, but ordinarily does not label the *order* of the contours. The inability to determine from the fringes alone whether a contour lies above, at the same level, or below a neighbouring contour is known as the 'hill-or-valley dilemma', and has been cited as the principal weakness of moiré topography. Additional encoding can be introduced to resolve the ambiguity, and these methods are reviewed. However, a new method is reported here that uses information inherent to the ordinary moiré topogram. The method uses digital image processing techniques and can be fully-automated. It is based on the natural shading of the object which is visible through the moiré fringes. A photometric model, based on the reflectivity of the object and its orientation with respect to the light source and the observer, is used to predict the shading for the shape that corresponds to a possible ordering of neighbouring contours. From the possible contour-ordering cases, the Photometric Ordering Algorithm chooses the one which has an expected shading that is most consistent with the measured shading value. Design studies were conducted to learn about the shape and shading requirements for photometric moiré fringe ordering. Automated routines are described for discovering sequences of neighbouring contours to be ordered, for determining the list of possible orderings for sequences of contours, and for measuring the shading on the object in the presence of the moiré fringe pattern. Two programs are reported that use the routines to order a set of moiré fringe contours. The first was designed to demonstrate fully-automated ordering

of ordinary moiré fringes using a streamlined version of the Photometric Ordering Algorithm. The second was designed as an investigative tool to study the properties of the Algorithm, and has considerable flexibility in adjusting the parameters and conditions of the processing. Tests are reported that demonstrate that the method is effective for a variety of materials (painted wood, cardboard and live human skin) and topographic challenges that ranged from rolling hills to sharp ridges and saddle-points. Photometric ordering is shown to be a robust and efficient method for automated ordering of contours traced from ordinary moiré topograms.

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Ding Xinhong joined forces with me in 1985-86 as a Visiting Scientist on leave from the Nanjing Institute for Electronic Engineering, Peoples Republic of China. Mr. Ding wrote much of the code for the test programs and also designed the β -spline routine for drawing the contours in the digital display. Xinhong is a cheery spirit, a thoughtful mathematician and an eager, practical engineer. He is also a treasured friend.

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To Eleanor,
who suffered
the slings and arrows
of outrageous thesis.

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List of Notation

$\{\cdots\}$	lists of map attributes (M elements).....	100
$\langle\cdots\rangle$	lists of neighbourhood attributes (S elements).....	76
$[\cdot]$	lists of ordering-case attributes (N elements).....	102
a_i	additive distortion coefficient when measuring $\bar{p}'_i$	251
a	albedo of object in moiré topogram image.....	85
α	azimuth direction of surface normal $\mathbf{N}$	90
b_i	multiplicative distortion coefficient when measuring $\bar{p}'_i$	251
C	number of contour points in C_i	99
C_i	i th contour in the map.....	70
C'_A	vector tangent at $\mathcal{P}_A$ to gradient curve C_i	205
d	lamp height above camera in shadow-moiré instruments.....	70
D	facet-slope distribution in photometric model.....	87
Δ	moiré fringe interval.....	69
$\Delta_{i,j}$	relative moiré fringe number $(\phi_i - \phi_j)$	69
e	moiré fringe extrema (MFE) type.....	69
$\epsilon_j^{(k)}$	photometric analysis error for k th ordering case in region $\mathcal{R}_j$	94
ϵ'_j	photometric analysis error distorted by a biased shading estimator.....	251
ϵ_s	specular threshold.....	165
$\mathcal{E}$	map-list of fringe extrema types.....	100
ξ	parametric position of the middle point on a contour segment.....	236
f	specular fraction in photometric model.....	86
F_i	i th moiré fringe.....	72
$\mathcal{F}$	map-list of moiré fringes.....	72
F	Fresnel term in photometric model.....	86
g	linear distance along projection of $\mathcal{G}$ in $\hat{x}$ - $\hat{y}$ plane.....	207
$[g_i^-, g_i^+]$	shading-estimation integration interval.....	252
$\Delta g_{i,j}$	planimetric curve length of $\mathcal{G}$ in $\hat{x}$ - $\hat{y}$ plane $(g_j - g_i)$	227
γ	geometric scalar in the photometric model.....	86
$\mathcal{G}_j$	j th surface gradient curve.....	72
G	geometric attenuation factor in photometric model.....	87
G'_A	vector tangent at $\mathcal{P}_A$ to gradient curve $\mathcal{G}_j$	205
h_i	positive-valued elevation of the i th contour $(h_i = -z)$	396
H	direction of bisector to $\mathbf{V}$ and $\mathbf{L}$	86

$i_{\ell,p}$	element of image array	99
$[i_{\ell,p}]$	2-D array of elements in image I	99
I_0	nominal illumination intensity	85
I	array of image points	97
j	subscript for attributes of j th analysis region	110
(k)	superscript for attributes of k th ordering case	102
κ	extinction coefficient in photometric model	86
l	camera distance from grid	70
ℓ	line coordinate of image element	99
(ℓ, p)	image plane coordinates	99
λ_i	label in map-image for contour C_i	99
L	number of lines in image array I	99
(L_x, L_y, L_z)	components of $\mathbf{L}$	119
$\mathcal{L}$	map-list of contour labels	100
L	image element label field	100
$\mathbf{L}$	direction from the object point to illumination source	85
(m)	superscript for attributes of m th ordering case	106
m'	number of contour-pairs in a matched-mode sequence	214
m''	number of bright/dark contour-pairs in a mixed-mode sequence	214
μ_i	first moment M_i over the interval $[g_i^-, g_i^+]$	255
M	number of contours in map $\mathcal{M}$	72
M_i	moiré modulation function	252
$\overline{M}_i$	mean of the moiré modulation function over the interval $[g_i^-, g_i^+]$	255
$\mathcal{M}$	map-list of contours	72
n_i	i th 'known' in $\langle \Sigma \rangle$	78
η_i	normal to the gradient profile curve in the $\hat{g}$ - $\hat{z}$ plane	207
N	number of 'knowns' in $\langle \Sigma \rangle$	77
(N_x, N_y, N_z)	components of $\mathbf{N}$	120
N	number of ordering cases in $\langle \Sigma \rangle$	81
$\mathbf{N}$	direction of surface normal	85
$\omega_1, \dots, \omega_p$	photometric model parameters	116
Ω	contour ordering operator	78
p	pixel coordinate of image element	99
ϕ	absolute moiré fringe number	69
P	number of pixels per line in image array I	99
$\mathcal{P}$	shading-estimation patch in the image	250
$\mathcal{P}_j$	j th contour point	88
Φ	map-list of absolute fringe numbers	72
Φ_0	initial state of the absolute fringe numbers	100
Φ_q	state of the absolute fringe numbers after q th ordering step	100
$\langle \Phi \rangle^{(k)}$	sequence of ϕ 's for the k th ordering case	76
q	subscript for attributes of q th ordering step	100
Q	number of ordering steps in schedule V	79

r	distant from surface point to light source	85
ρ	reflectance (shading) value on the object	85
$\dot{\rho}$	slope of reflectance ($d\rho/dg$)	253
ρ_0	nominal reflectance in photometric model	85
ρ_d	diffuse reflectance component in photometric model	86
ρ_s	specular reflectance component in photometric model	86
$\bar{\rho}$	measured reflectance in image	91
$\hat{\rho}$	expected reflectance from photometric model	94
$\rho', \rho'_0, \bar{\rho}', \hat{\rho}'$	reflectances distorted by a biased shading estimator	251
R	number of analysis regions in $\langle \Sigma \rangle$	88
R	image element reflectance field	99
$\mathcal{R}_j$	j th analysis region	88
s	distance between grid strings	70
$\sigma_{i,j}$	separation of neighbours P_i, P_j measured along the gradient profile	112
S	number of contours in $\langle \Sigma \rangle$	76
$\{S\}$	data structure of regional attributes	112
$\langle \Sigma \rangle_q$	q th neighbourhood sequence of contours	76
t	parameter that measures the linear distance along C	236
τ	tilt direction of surface normal N	91
T	image-plane to grid-plane coordinate transformation	118
u_i	i th 'unknown' in $\langle \Sigma \rangle$	78
U	number of 'unknowns' in $\langle \Sigma \rangle$	77
V_q	q th schedule control vector	79
V	contour ordering schedule	79
V	direction from the object point to the viewer	85
w_i	width of the integration interval $[g_i^-, g_i^+]$	253
(x, y, z)	object-space coordinates	70
$(x_g, y_g, 0)$	grid-plane coordinates	118
$\hat{x}, \hat{y}, \hat{z}$	object-space basis vectors	70
X - Y plane	moiré grid plane	70
z_i	elevation of i th contour	69
Δz	contour elevation interval	69
Z	map-list of elevations	72

Chapter 1

Introduction to the Research

1.1 The Problem: Moiré Fringe Ordering

The topographic contour map is a well-known method for representing the shape of an object. In Figure 1.1 is a photograph of a person with a superimposed pattern of shadows, called *moiré fringes*, that are contours of elevation. The fringes are generated by an optical encoding technique called *moiré topography*, which ensures that fringes are properly formed at contours, but ordinarily does not label the *order* of the contours. One cannot determine from the fringes alone whether a contour lies above, at the same level, or below a neighbouring contour.

Sometimes the object is easily recognized, as in Figure 1.1, and its general form is known *a priori* so that the fringe ordering can be done by simple inspection. Sometimes, however, the shape of the object is ambiguous. Consider, for example, Figure 1.2, in which one can see from the fringes that the circular contours are on a sphere, and that the rectangular contours are on a truncated pyramid. Which of these objects is concave and which is convex? The problem is that ordinary moiré fringes are symmetric with the sign of the slope, so that 'uphill' and 'downhill' are encoded in the same way. This ambiguity is known in the literature as the 'hill-or-valley dilemma' and has been cited as the principal weakness of moiré topography.

There are ways of introducing additional encoding into the moiré topogram to

Figure 1.1: Moiré topogram from an orthopaedic clinic. The shadow pattern of moiré fringes on this patient's back represent 5 mm contours of elevation, parallel to the page. The symmetry of the fringe pattern on the right and left buttocks assures that the patient is aligned to the reference plane. The gross left-right asymmetry of the fringe pattern on the upper and middle sections of the back indicates a severe spinal deformity (scoliosis). The topogram was made using the NRC Moiré Contourgraph at the Ottawa Civic Hospital (Courtesy of G. W. D. Armstrong)

Figure 1.2: Demonstration of the 'hill-or-valley dilemma'. This is a moiré topogram of an engineered shape. The fringes are contours parallel to the page. The concentric-circular contours define a hemisphere. The concentric-rectangular contours define a truncated pyramid. One cannot tell from the fringes alone that the pyramid is concave and the sphere is convex. There might be other cues in the figure that suggest this interpretation is correct, due to shadows and shading, but in an effort to confuse those cues, the photo is shown here upside-down to reverse the sense of the illumination of the object.

break the ordering ambiguity. One can install fiducial devices which cast shadows with a characteristic geometry that are superimposed with the regular moiré pattern. Also, ingenious schemes for producing asymmetric fringes have been devised, so that the uphill side of each fringe differs from the downhill side. Another approach forms dynamic moiré fringes that can be made to move uphill, or downhill under phase-lock control, and thereby determine the sign of the local surface slope. These departures from ordinary moiré fringes are reviewed in detail in Chapter 2.

This thesis addresses the problem of ordering ordinary moiré fringes that have no additional encoding, and that have been recorded as a fixed pattern using inexpensive photography. There are many applications where moiré topography data is recorded in this fashion, and for various reasons the enhanced methods of Chapter 2 cannot be used. One particular application in biostereometrics, namely the measurement of spinal deformity, motivated this thesis (see Sec. 1.2). However, the results of this research have more general importance. Moiré topography is a versatile shape-measurement technique whose use ranges from monitoring the deformation of railway bridges to the microscopic wear in dental bridges. The reader is directed to Appendix B for a survey of the broad range of applications of moiré topography in engineering and medicine.

1.1.1 Photometric Ordering of Topographic Moiré Fringes

While investigating methods for automating the ordering of ordinary moiré topograms, the author discovered that there is already sufficient encoding in topographic moiré fringes to break the hill-and-valley ambiguity. Referring to Figure 1.1, one sees that the moiré topogram is just an ordinary photograph with a fringe pattern superimposed. The shading on the object is a slowly-varying envelope of reflected intensity, which depends on the orientation of the object with respect to the light source and the viewing position. The shading envelope is modulated by the moiré fringe pattern, which is a cyclic, symmetric function of elevation, independent of the object. The combined effect of the two functions is to darken the neighbouring fringe on the shady-side of each fringe, and to lighten the neighbouring fringe

on the bright-side. This natural shading asymmetry depends on the local shape of the object, which is exactly what is needed to decide on the fringe ordering (i.e., whether the neighbouring fringe is above, level with, or below a given fringe).

Photometric ordering refers to the fringe-ordering method invented by the author to capitalize on the topographic asymmetry of ordinary moiré fringes. In this method, each pair of moiré fringes is examined and the expected shading is computed for the test cases of a fringe being above, level with, or below its neighbour. The expected values are compared to the actual shading value, and the algorithm adopts the test case that agrees most consistently with the actual shading values at various points along the fringe contour. The technique is named for the *photometric model* used to predict the shading on the surface, which is a function of its position and orientation with respect to the light source and viewing position. The theory behind this method and its embodiment in the Photometric Ordering Algorithm are described in formal detail in Chapter 3.

1.1.2 Fringe Ordering Terminology

A working knowledge of moiré topography is assumed, and the author encourages the non-expert to read Appendix A for an introduction to the basic terminology, properties and formulas pertaining to moiré topography. Concepts specific to moiré fringe ordering are briefly introduced here, and are elaborated upon as needed throughout the text.

The fringe number ϕ refers to the cyclic nature of the moiré profile phase ψ_M

$$\phi = \psi_M / 2\pi \quad (1.1)$$

The bright and dark fringes occur along loci of local intensity extrema where the fringe number is an integer or odd half-integer, respectively:

$$\phi = \begin{cases} 0, 1, 2, \dots & \text{for bright fringes,} \\ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots & \text{for dark fringes.} \end{cases} \quad (1.2)$$

The term *fringe ordering* refers to the determination of ϕ for the discrete values given by Equation 1.2. The term *phase analysis* refers to the general case, in which

the determination of the moiré profile phase is at an arbitrary point in the pattern (i.e., not necessarily at a local intensity extremum).

The term *absolute ordering* refers to the determination of the fringe number ϕ for each and every contour, whereas *relative ordering* refers to the difference in fringe number $\Delta\phi$, between pairs of neighbouring contours, say A, B

$$\Delta\phi_{A,B} = \phi_A - \phi_B. \quad (1.3)$$

It is often adequate to know only the relative ordering of a set of contours. This allows the shape of the surface to be computed to the accuracy of the nominal contour interval, Δz . If the contour interval is significantly non-linear with respect to ϕ , and the highest accuracy shape determination is required, then it is necessary to determine the absolute fringe numbers so that the exact formula for the elevation z_ϕ can be applied.

There are useful properties exhibited by contours on topographically well-behaved surfaces that can aid in contour ordering. Armed with the tutorial on moiré topography in Appendix A and a few concepts, such as:

- contours never intersect;
- neighbouring contours differ in elevation by at most one interval;
- if one contour from a relatively-ordered set has known absolute ϕ , then every contour in the set can be (trivially) absolutely ordered;

the reader is adequately equipped to comprehend the fringe ordering methods reviewed in Chapter 2. The moiré fringe ordering problem is formally presented in Chapter 3.

1.2 Background to the Research

The results presented in this thesis represent research conducted by the author while with the Photogrammetric Research Section of the National Research Council of Canada (NRC), from the period 1980-87. The following paragraphs provide

some historical background on the motivation behind the author's research into the photometric ordering of topographic moiré fringes.

In the early 1970's, Gordon W. D. Armstrong, Head of Orthopaedic Surgery at the Ottawa Civic Hospital and Professor with the School of Medicine at the University of Ottawa, approached Theodor J. Blachut (Section Head) and Marius C. van Wijk, Senior Research Officers with the Photogrammetric Research Section at NRC, with a proposal to use photogrammetry (literally, *measurement with images*) to monitor surgical implantation procedures used for the correction of *scoliosis*, a type of spinal deformity.

Scoliosis is not as well-known to the public as it should be. A few facts are related here to underscore the importance of research in this field. The interested reader is advised to consult the excellent tutorial by Keim [1979] for more information. Scoliosis is a skeletal deformity that involves lateral curvature or longitudinal twisting of the spinal column. There is also *kyphosis*, a frontward curvature, and *lordosis*, a backward curvature of the spine. Scoliosis is very common, and severe cases are life-threatening: about one person in ten has a noticeable scoliosis, and about two in one hundred require aggressive treatment (*e.g.*, surgical implantation of Harrington steel-rods to straighten the spine). In some cases, scoliosis can be caused by congenital defects (*e.g.*, vertebral or rib fusion), neuromuscular disease (*e.g.*, cerebral palsy and muscular dystrophy), mesenchymal disorders (*e.g.*, rheumatoid arthritis), and a host of other maladies. However, adolescent idiopathic (genetic) scoliosis accounts for about 80% of all cases, and girls are affected seven times more often than boys. Dr. Keim writes

"It can be said that most children who develop idiopathic scoliosis are genetically programmed for the disease at the moment that the sperm and egg combine and a new life is formed. They are programmed, much as a computer might be, with a message to be printed out some 10 to 12 years later. The scoliotic genetic message will usually show itself during the juvenile or early adolescent years and will progress to a predetermined severity unless that course is interrupted by proper med-

ical treatment, usually spinal bracing with exercises, or surgery, or a combination of both—*never by exercises alone.*"

The urgent message from Dr. Keim, Dr. Armstrong and others, is the need for early detection of the disease so that its course *can* be interrupted. Experimental school-screening programs have been proven to be effective in identifying likely victims, but a safe, effective alternative to x-rays for detecting spinal deformity is still needed for mass screening.

The collaborative project between Dr. Armstrong and the NRC scientists to use photogrammetry for measuring the implantation of a modified Harrington rod was a success. In the autumn of 1974, Armstrong, Blachut and van Wijk presented a paper on their experiments at *ISP¹ Commission V Symposium Biostereometrics '74*, held in Washington, D.C. [Armstrong *et al.*, 1974]. These authors were considerably inspired by the content of another paper at that meeting, entitled simply "Moiré Topography" [Takasaki, 1974].

The simplicity and elegance of the shadow-moiré method for measuring surface relief, and its potential application in the detection and diagnosis of spinal deformity, spawned a flurry of activity in the Photogrammetric Research Section at NRC which continued for more than a decade. Soon after the 1974 ISP Symposium, a prototype instrument was fabricated in the NRC shops. Allen Way-Nee provided the essential photographic expertise for the design and testing of the prototype instruments. A pilot school-screening project for the early detection of scoliosis was reported in [Adair *et al.*, 1977].

At that time, Manfred Paulun commenced work on the design and development of the NRC Moiré Contourgraph, a translating-screen shadow-moiré instrument designed specifically for clinical assessment of human backs. Its design embodied both Takasaki's recommendations regarding moiré technique and NRC expertise in photogrammetric engineering. The Otal Precision Company² was licensed by NRC

¹The International Society for Photogrammetry, now called the International Society of Photogrammetry and Remote Sensing (ISPRS).

²Otal Precision Company, Ltd., 2730 Fenton Road, Ottawa, Canada. K1A 0R5

Figure 1.3: The NRC Moiré Contourgraph. Features include: an accurate 2 mm pitch string-grid; high-intensity, condensed illumination, precise grid translation during exposure; and a sturdy, rigid frame to accurately support the components. The contour interval is nominally 5 mm. The unit shown here has several modifications introduced by the author and M. Paulun, such as a new lamp housing that reduces stray light, a periscope that reflexes annotation into the topogram, and a motorized (instead of falling) screen translator.

to manufacture this device (Model MS-2040), and later a light-weight, stationary-screen folding model was introduced (Model SS-2040). The photogrammetric accuracy of the NRC designs was subsequently reported in [van Wijk, 1980 and 1981].

The development and promotion of a commercial shadow-moiré instrument was a key factor in NRC's strong commitment to automating the analysis of ordinary moiré topograms. Figure 1.3 shows an experimental version of the moving-screen NRC Moiré Contourgraph, used by the author to generate many of the topograms shown in this thesis. The inherent features of the commercial model of the instrument are still quite apparent in the figure. The unit has a cast aluminum base, and the shadow-grid can slide freely in-plane along stout, precision-ground, vertical shafts that are mounted rigidly to the base. Takasaki [1970] recommended translating the grid during exposure to eliminate the individual grid strings and their shadows, thereby improving the quality of the moiré fringes (see Fig. A.13 on p. 408). One of the design goals of the NRC instrument was to rigidly control the geometry of the moiré-forming elements. As can be seen in Figure 1.3, stiff steel struts radiate in triangles from the camera mount to the massive base, and another pair support the lamp unit, which had an additional strut to the top of the grid-guidance structure to prevent sway in that direction (the strut was not in place when the instrument was photographed). In this fashion, the camera and light sources can be assembled into their designed positions with respect to the grid: to within ± 1 mm, or about an order of magnitude better than required to restrict geometric errors to 'half a fringe' [van Wijk, 1981]. The assembled instrument is very stable, and can be freely rotated on wheels provided for orientation alignment about the vertical axis, by grasping the handgrip at the camera. Other aspects of the NRC Moiré Contourgraph design, such as the novel construction of the grid and the periscope for reflexing the annotation text into the topogram, can be found in [Pekelsky, 1987a]. Otherwise, critical features of the NRC instrumentation are described as needed in later chapters.

In terms of the author's role in this line of research, the stage had been set by 1980. In the autumn of that year, the author attended the first *International Sym-*

posium on Moiré Fringe Topography and Spinal Deformity, held at Basin Harbor, Vermont. About fifty specialists from seven countries gathered for the first time to discuss the state of the art [Moreland *et al.*, 1981b]. The prognosis was excellent, but guarded. Moiré topography instruments had been designed, built and commercialized by several companies. The moiré topograms they produced on 35-mm black-and-white film were accurate, inexpensive and archival. Various orthopaedic clinics were experimenting with home-built, purchased or borrowed units, and several school-screening pilot studies had been conducted. Hundreds, even thousands, of moiré topograms were being generated by eager orthopaedic scientists. The fringe patterns contained "all the topographic information" in a fashion easily interpreted by eye, but it was still not clear how surface measurements on the back related to skeletal deformity. The experts agreed that many measurements and studies were needed to establish a correlation. However, manual analysis of the patterns was extremely tedious, as it required tracing the fringe contours with coordinate-measuring instruments. The photogrammetrists knew of this tedium at the outset, but were caught up in the enthusiasm of the bio-engineers to apply this wonderful new diagnostic tool to back measurements. The users were already becoming frustrated, and they suspected that, despite its many advantages, moiré topography would be useless without automated analysis. The author agreed to study the problem of automating the analysis of moiré topograms.

The author's research had not progressed enough to present a paper at the second symposium held in Münster in 1982 [Drerup *et al.*, 1983a], but preliminary results on using shading to order moiré fringes were reported at the third meeting in Oxford in 1984 [Harris and Turner-Smith, 1986], and were well-received by the delegates [Pekelsky, 1986a]. By that time, NRC had established considerable leadership in this field and was asked to host the 1986 meeting.

In December of 1985, the author presented a paper at the SPIE³ Symposium *Biostereometrics '85*, held in Cannes [Pekelsky, 1986b]. This paper outlined the theory of photometric ordering, and reported on experimental results that demon-

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³The International Society for Optical Engineering

strated the principle. A shortened version was later published in *Optical Engineering* [Pekelsky, 1987b].

In 1985-86, the author was joined by Ding Xinhong, a visiting scientist on leave from the Nanjing Institute for Electronic Engineering, People's Republic of China. Together, they wrote the software to test the automation of the photometric ordering concept and conducted the experiments in fringe ordering that are documented in Chapters 8 and 9 in this thesis. The author presented a joint paper at the 1986 *SPIE Symposium on Optical Testing and Metrology* in Quebec City [Pekelsky and Ding, 1986], and Mr Ding presented a co-authored paper at the 1986 *ISPRS Commission V Symposium on Real-Time Photogrammetry*, held in Ottawa [Ding and Pekelsky, 1986].

During this period, the author was responsible for organizing the *Fourth International Symposium on Surface Topography and Spinal Deformity*, held at Mont Ste. Marie, Quebec in September, 1986. The meeting was very successful, attracting over seventy delegates from twelve countries, and forty-three papers were presented during the two-day symposium. The author presented a status report on moiré topogram production and analysis at NRC [Pekelsky, 1987a], and Mr Ding spoke on the photometric ordering technique [Ding and Pekelsky, 1987]. By that time, work on the thesis had started, and the next generation of software for photometric fringe ordering was in the planning stages.

Ironically, three weeks after the Mont Ste. Marie meeting, NRC announced expenditure cutbacks that resulted in the closing of the Photogrammetric Research Section on March 31, 1987. After several annual budget-tightenings, these new cuts forced every Division at NRC to terminate selected programs, in most cases irrespective of their scientific worthiness. Despite the outcry from our medical colleagues with regard to the importance of the moiré research (and, of course, from the photogrammetric community for the loss of the Section in general), the decision would not be reversed, and all the ongoing work had to be terminated by the end of the year. Time was short, and despite the aforementioned conference presentations, the full details on the photometric ordering method had not been

adequately documented. This thesis was the author's last chance to report on the theory, the software and the experiments, and every effort has been made to ensure its completeness. In this regard, the length of the document was not an issue.

In the few months remaining before the closing of the Section, the author co-edited the Proceedings for the Mont Ste. Marie meeting with colleagues from the University of Vermont [Stokes *et al.*, 1987], wrote with Marius van Wijk the chapter "Moiré Topography: Systems and Applications" for the new edition of the ASPRS⁴ Handbook of Non-Topographic Photogrammetry [Pekelsky and van Wijk, 1988], and started writing this document.

The Photogrammetric Research Section, one of the largest in the Division of Physics, had been in operation for thirty-six years. By April 1, 1987 its members were scattered, their developments were discarded, and the equipment was declared surplus.

1.3 Objectives of the Research

Automated analysis of moiré topograms is essential if this measuring technique is to be used in school-screening programs for the early detection of scoliosis. A city of one million with a screening program may have tens of thousands of topograms to be analyzed each year. The term *analysis* carries several meanings to the parties involved in this project, and it warrants clarification.

To the photogrammetrist, topogram analysis means generating the map from the photograph (*i.e.*, shape measurement). First, the contours are traced from the fringe pattern by measuring the coordinates of a sequence of points along the contours. Then, the contours are ordered by assigning an elevation to each contour. Finally, the extracted 3-D data can be transformed into one of several representations of the shape of the surface (*e.g.*, contours, profiles, grids). One popular representation is the Digital Elevation Model (DEM), an array of elevations that can be interpolated at regular planimetric positions from the contour points.

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⁴The American Society for Photogrammetry and Remote Sensing

Contour-to-DEM algorithms are well-known in photogrammetry, and are discussed in [Hierholzer *et al.*, 1983] and [Frobin and Hierholzer, 1986b].

To the medical specialist, topogram analysis implies making measurements that somehow give a diagnosis of the spinal deformity (*i.e.*, shape recognition and classification). This is a very difficult problem, even if the photogrammetry has been done. Progress is being made in this area, but the lack of a convenient means for extracting the map from the moiré topogram has frustrated the research to find statistically-significant diagnostic shape-parameters that correlate with spinal deformity.

The author's long-range objective was to develop the automated map-making tool needed by his medical colleagues. Digital image processing (DIP) techniques would be applied to digitized moiré topograms to extract the contours, order them, and generate the DEM. A working knowledge of DIP techniques is assumed in this document, and the non-expert may find [Pavlidis, 1982] and [Pratt, 1978] useful introductory texts. DIP methods have already been developed by others to automatically trace fringe patterns in images, and these are discussed in Appendix C. The one aspect of the map-making process that had not been solved was to automate the ordering of the moiré fringes. Inventing a DIP technique to do this is the primary objective of the thesis project.

Ordinary moiré topograms have grade-symmetric fringes with fixed encoding. They are the simplest to generate, require inexpensive apparatus, and can be recorded conveniently and reliably on inexpensive 35-mm black-and-white film. For these reasons, ordinary moiré topography is the preferred technology in school-screening programs for the early detection of spinal deformity, but it is a cost-effective choice only if the topograms can be automatically analyzed. Thus, the author was seeking an ordering method for *ordinary* moiré fringes that could be automated. This constraint meant that the ordering technique could not rely on the exotic grids, colour-coding, or dynamic fringe methods described in Chapter 2. A new method had to be invented and, if successful, it would significantly broaden the utility of ordinary moiré fringes in many applications where the need for automated

analysis had precluded their use.

What follows is a list of the objectives that have been met by the results presented in the thesis. Identified with each entry in the list is the chapter where the relevant material is reported. The objectives are:

1. To review the various methods that have been invented for fringe ordering in moiré topograms (Ch. 2).
2. To show that there is sufficient shading information in an ordinary moiré topogram to permit relative ordering of the contours, and that special additional encoding is not necessary (Ch. 4).
3. To invent an algorithm for the photometric ordering of ordinary topographic moiré fringes (Ch. 3).
4. To show that a simple shading model is adequate for ordering topographic moiré fringes on a broad variety of surface types (Ch. 4).
5. To examine and determine the shape and shading design requirements for photometric fringe ordering (Ch. 4).
6. To invent a method for discovering the neighbours of contours (i.e., the gradient neighbours) in a map, so that each pair of neighbouring contours can be ordered (Ch. 5).
7. To invent a method for measuring the local surface shape of the object from the moiré fringe pattern, since the photometric model needs the local shape to predict the shading there (Ch. 6).
8. To invent a method for measuring the local surface shading in the presence of the moiré fringe pattern, so that it can be compared to the expected shading value by the Photometric Ordering Algorithm (Ch. 7).
9. To write a computer program to demonstrate that the ordering process can be automated. This would show that the analysis of moiré topograms could be

fully automated, assuming the automated ordering process could be coupled with software for automated contour tracing (Ch. 8).

10. To write a computer program that permits the underlying properties of photometric ordering of moiré topograms to be investigated. (Ch. 9).
11. To conduct performance tests to show that the photometric ordering method is robust, fast and inexpensive (Ch. 8 and 9).
12. To establish a measure for the quality or confidence in the correctness of the ordering process (Ch. 8 and 9).
13. To propose further developments to this work (Ch. 10).

1.4 Organization of the Thesis

There is less text in this thesis than one might expect. Moiré topography is a very picture-oriented science, and the author has not hesitated to use a photograph or drawing to "save a thousand words" here and there. There are over two hundred figures, many of them full-page. Furthermore, the appendices are optional reading, and one of them contains over forty pages of program listings. Following this chapter, the remainder of the thesis is organized as follows.

In Chapter 2, various methods for ordering topography moiré fringes are reviewed. The methods are examined under three categories: ordinary moiré fringes, asymmetric moiré fringes, and dynamic moiré fringes. The chapter concludes with a discussion on the topography-dependent asymmetry of ordinary moiré fringes and the application of this property in the photometric ordering technique.

In Chapter 3, the theory of using photometric analysis of the shading in a moiré topogram to order the fringes is presented. The contour-ordering problem is defined, and a notational formalism is developed with the ultimate purpose of presenting the Photometric Ordering Algorithm. The chapter concludes with a discussion on the key assumptions, namely that sufficient photometric control exists in the topogram,

that the contours have already been traced, and that the shading on the object can be measured in the presence of the fringe pattern.

Chapter 4 reports on design studies conducted at the early stages of the project. The studies were intended to identify the crucial design issues, investigate appropriate shape and shading measurement methods, and specify control restrictions that must be imposed on the moiré topogram data. The moiré and DIP equipment and the test materials used in the research are introduced, and the measurement methods are described. The linearity of the reflectance measurements is examined, and an experiment to verify the accuracy of the shape determination is reported. Experiments in endogenous (*self*) shape determination are attempted, but the moiré instrument illumination is shown to have insufficient uniformity to support this approach to ordering. Propagated fringe ordering, using localized shading analysis based on a simple Lambertian photometric model, is adopted for the remainder of the project. Three issues in automating fringe ordering are identified: neighbour discovery, shape determination and shading determination. They are the subject of the next three chapters.

In Chapter 5, the problem of automating the identification of the neighbouring contours to a given contour is examined. It is assumed that the contours in the map have been already traced, but that the neighbour relationships are not known. Pairs of neighbouring contours must be automatically determined so that they can be ordered. The concept of the *gradient neighbour* is introduced, and the data structures used by the software are defined. A strategy for discovering the gradient-neighbour points to a given contour-point is presented, and its embodiment into software is described. The chapter concludes with remarks on the speed and robustness of the automated method.

Chapter 6 deals with determination of the shape of the object using the moiré fringe pattern. A connection is made between shape on the object (normals, tilts and azimuths) and shape information encoded in the moiré fringes in the topogram image. The uncertainty of the ordering amongst the members of a sequence of contours (called the *neighbourhood sequence*) results in a set of shape cases that

must be tested by the Photometric Ordering Algorithm. An algorithm is developed to generate the list of shape cases and to determine the local shape of the object at test points on the object for each case. The embodiment of this algorithm into software is described, and comments on the robustness and accuracy are given.

Chapter 7 examines the problem of making measurements of the shading on the object in the presence of the moiré fringe pattern. Averaging of the local data values is shown to be necessary, but the modulation of the shading envelope by the moiré fringes requires that the estimation patch be dynamically adjusted to the local fringe spacing in the image. Various patch geometries are examined. A circular patch is ultimately adopted, and its virtues and limitations are discussed, along with its software implementation. The chapter ends with a discussion of the performance of the automated shading estimator based on dynamic circular patches.

In Chapter 8, a program (Task JFO) is described that uses photometric analysis to automatically order a set of moiré fringe contours. The design philosophy and functionality of the program are explained, and its principal elements are described. Data files required by Task JFO are also specified. Results from experiments in contour ordering for two subjects (a painted wooden sphere and a live human back) are discussed in terms of the robustness of the software embodiment of the Photometric Ordering Algorithm and the efficiency of Task JFO's ordering scheduler. Included in this chapter is a description of the software engineering standards used by the author in this research.

In Chapter 9, a second program (Task MFO) is described that embodies the most general form of the Photometric Ordering Algorithm. Its purpose is to provide an investigative tool for studying the properties of the photometric ordering concept, and to this end, it offers much more flexibility than Task JFO, but with a lesser degree of automation. The design philosophy, functionality and special features that affect the execution of the algorithm are examined. The modules of Task MFO correspond to the block-diagrams presented in Chapter 3 that define the Photometric Ordering Algorithm. The overlay structure for the task is described, and the buffer restrictions that limit the complexity of the contour map that can

be processed by the program are also discussed. Experiments in ordering are reported that involve a variety of surface materials (grey cardboard, painted wood and live human skin) and a number of topological challenges that include sharp ridges and hilly terrain. The chapter concludes with a discussion of the performance of Task MFO, its usefulness as an analytic tool, and the prospects for fully automating its operation. The effect of *photometric mimics* on the robustness of the ordering process is also examined.

In Chapter 10, a summary of the conclusions from this project is given. Comments are made on the degree to which the stated objectives have been met, and recommendations are given for further work in this area.

Appendix A is a brief tutorial for the non-expert on the basic properties of topographic moiré fringes. Topics include some historical background, definitions and terminology, an analysis of the intensity distribution of the fringes, their topographic interpretation, fringe elevation formulas, fringe visibility and surface slope limits, and the effect of grid phase-shifting on the moiré fringe pattern.

Appendix B presents a survey of the applications of moiré topography in the fields of medicine and engineering. Citing over fifty articles, this appendix attempts to show by example the versatility of this measuring technique, and its imaginative application to very diverse tasks with a broad range of size, precision and accuracy requirements.

Appendix C reviews methods for digital elevation determination from moiré fringes. Three general approaches are examined: densitometric fringe tracking, densitometric fringe profiling, and phase-lock analysis. The problems with manual and automated methods are discussed, with numerous examples of actual systems. It is shown that for applications that can allow dynamic moiré fringes, the video-based phase-lock methods have been the most successful in fully-automated moiré topogram analysis.

Appendix D contains a listing of the program modules that comprise Task JFO, the program described in Chapter 8. The listing includes the command files to build the task, the Fortran subroutine-type files, and the Fortran include-type files.

Examples of the data files required by the task are also provided.

The Bibliography is a compilation of literature referenced in the chapters and the appendices, plus other publications related to moiré, shape-from-shading and image processing. A useful feature is the indexing of the thesis page(s) where each work has been cited.

Chapter 2

Methods for Ordering Moiré Fringes

2.1 Introduction

In this chapter, a number of fringe ordering methods are reviewed. These techniques span a broad range of complexity, performance and application. Some are widely used, while others remain academic exercises. Some are quite complex, requiring sophisticated (expensive) hardware and software; usually there is a client with a particular problem and a budget to motivate such developments. Automation is an essential feature for some users, whereas others cannot justify the associated cost. As usual, an engineering balance must be struck between the merits of each approach when considering an application, as there is no best overall choice.

The important merits and weaknesses of each method are included with the description of its principle, and examples of embodiment and application are given, but the author has avoided trying to rate the methods against one another. The reports in the literature are too uncorrelated, and subject to too many unstated parameters. Most of the cited articles convey the cost/performance of their technique in an implicit fashion (*e.g.*, hardware costs and processing speeds are inferred from equipment types and model numbers), or else with relative remarks like 'the grids were difficult to construct', or 'manual tracing of the fringes was very tedious'. It

was considered beyond the scope of this chapter to knit a comparison from these threads.

The chapter is organized into five sections as follows. This section concludes by classifying topographic moiré fringes into three groups: ordinary, asymmetric and dynamic. The next three sections examine the fringe ordering methods used for each class of fringes. The last section discusses the photometric ordering method as a technique that capitalizes upon the natural asymmetry in ordinary moiré fringes.

2.1.1 Moiré Fringe Profile Classes

Moiré fringes are cyclic functions along the gradient profile. The simplest moiré topographic arrangements produce profiles that are *even* functions, i.e., they are symmetric with respect to the sign of the gradient. Often they can be approximated as a sinusoidal distribution. Another characteristic of the most basic moiré arrangement is that the scene demodulation occurs prior to image recording, thus the moiré fringe pattern is fixed in the recorded image. The combination of *grade-symmetry* and *fixed-encoding* that characterizes ordinary moiré fringes is the root of the fringe ordering 'hill-or-valley' dilemma. Clever methods for overcoming this dilemma have been devised that depart in one way or another from the constraints of ordinary moiré fringes, and they form useful criteria for classifying moiré configurations:

- **Ordinary Moiré Fringes** are characterized by an even gradient profile function (*grade-symmetric*) and a fringe pattern that is fixed in the recorded image (*fixed-encoding*).
- **Asymmetric Moiré Fringes** are fixed-encoded moiré fringes designed to exhibit an asymmetric gradient profile distribution. Practiced techniques fall into two subclasses: those employing asymmetric intensity modulation and systems using asymmetric colour-coding (spectral modulation).
- **Dynamic Moiré Fringes**, in a broad sense, refers to those moiré techniques that rely upon comparing two or more shifted fringe patterns of the same object. Dynamic moiré is characterized by recording the distorted-grid image,

and forming the moiré fringe pattern(s) in a subsequent step by superimposing a suitable *analysis* or *demodulation grid*. By phase-shifting the demodulation grid, the fringes can be made to shift uphill or downhill (hence the fringes are *dynamic*, as opposed to *fixed-encoded*).

One of the advantages of moiré topography is that it encodes the 3-D information in a single image. Most of the dynamic techniques rely upon processing patterns generated from one record of the scene, although some require a sort of *calibration image* that can be used repeatedly. Thus, these methods still maintain the 'photographic economy' of moiré over stereo-photogrammetry in that respect.

Both of the above departures from the ordinary fringe case provide solutions to the fringe-ordering dilemma, but at the expense of spawning unique complications and costs.

2.2 Techniques for Ordinary Moiré Fringes

2.2.1 A priori Knowledge

A priori knowledge of the shape of the object will usually provide sufficient guidance to analyze the relative ordering of ordinary moiré fringes. Consider, for example, the fringe pattern presented in Figure 2.1. The shape is easily recognized, even though the object boundaries are incomplete and the natural shading on the surface is disturbed by the fringe pattern. Indeed, with little training, most observers are quick to use the fringe pattern to interpret first, the gross shape features (in Fig. 2.1, four 'hills' at the scapula and buttocks, and maybe two 'cylinders' forming the arms), and then, assess more subtle details (the left scapula hill is higher than the other).

Are there any regions in Figure 2.1 where the reader is not sure about ordering of the fringes? The closed-loop fringe at the 'small of the back' (along the spine, midway between the top hills and bottom hills) might be one, but an expert would know *a priori* that there is always a slight surface concavity there. The author

Figure 2.1: Moiré topogram of a human back, taken with the stationary-grid model of the NRC Contourgraph. The fringes are contours of elevation with respect to the grid plane (parallel to the plane of the printed image). The individual grid strings are clearly resolved in this image. The white horizontal and vertical fiducial strings lie in the grid plane, and intersect at the system coordinate origin, as does the optical axis of the camera lens. The black horizontal curve across the arms and back of the subject is the shadow of the horizontal fiducial string, and provides clues about the illumination (i.e., it is from above), and about the fringe ordering.

was told by an orthopaedic specialist¹ using moiré topography in a spinal clinic, that in every case, whether involving a normal or severely deformed back, there was never any problem in interpreting the fringe ordering. This remark holds for some classes of biomedical objects that tend to be smooth convex 'blobs' with contours running consistently downhill to the silhouette boundaries (see Fig. 2.2). For engineering applications, with a high likelihood of abrupt shape changes, or arbitrary convex/concave departures from flatness, *a priori* knowledge has limited utility.

A final consideration is the difficulty in automating human recognition skills. In some cases (such as inspection of identical manufactured objects), the expected shape can be expertly embodied into the analysis software, and the algorithm made robust enough to tolerate small departures. More flexible code is increasingly complex and expensive, and often it may be more cost-effective to alert a human helper, who can be highly adaptive in pattern recognition tasks. A common solution to general purpose fringe ordering is to have the human perform the 2-D pattern recognition (e.g., point to the hills and the valleys), and then let the software complete the bookkeeping task of sorting out the fringe ordering (see Fig. 2.3).

2.2.2 Fiducial Shadow Ordering

Moiré fringes are formed when the views of the grid lines and their shadows are made to interfere. Usually, the individual entities (lines and shadows) are not resolved, and measures may be taken to actually eliminate them from the final image. The fiducial shadow method makes one grid string and its shadow plainly visible in the final pattern. This additional encoding is carefully arranged to provide cues for fringe ordering.

Examples from the NRC Contourgraphs illustrate this method. Figure 2.4 depicts the shadow-moiré arrangement based on an upright grid (i.e., normal to the

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¹From a conversation with Dr. Richard E. Roger (Oxford Orthopaedic Engineering Centre), while at the First International Symposium on Moiré Topography and Spinal Deformity (Basin Harbor, 1980)

Figure 2.2: The surface gradient directions on a biomedical subject. Human torsos are sufficiently smooth and convex, so that most contours are crossed by gradient curves that run from the local peaks to the silhouette boundaries, and thus are easily ordered by inspection. Here, the single small-looped contour at the saddle-point (actually a depression at the small of the back) is the only contour left unresolved by the 'smooth-blob' ordering approach.

Figure 2.3: A method for interactive relative fringe ordering. An operator indicates the end points of slope lines with a light pen, where the order of point-indication signifies which end of the slope line has a higher fringe order. P and V are local peaks and valleys, respectively [Yatagai *et al.*, 1982a]

floor) with horizontal strings. The camera lens is aligned with its optical axis normal to the grid plane, and intersecting it at the coordinate origin. A distinguishable grid string is placed in the plane of the grid, parallel to the grid lines (at A in the figure), such that:

- the line can be identified in the image (even if the other lines cannot),
- its shadow on the object (at B in the figure) can be also identified in the image (even if the other line-shadows cannot).

One way to ensure that the fiducial string is resolved in the image is to make it much thicker than the other grid strings. On the NRC Contourgraphs, a white string is positioned over the gap between two black grid strings. Thus, a shadow of triple-width² is cast on the object (Fig. 2.5).

²On the translating-grid model, the horizontal fiducial is stretched between fixed struts, very close to the plane of moving grid strings; its shadow is only single-width, but the other string shadows are eliminated by the grid motion during exposure

Figure 2.4: Shadow-moiré geometry for the fiducial shadow ordering method. The camera lens axis is a horizontal line on the page, normal to the grid plane (shown in section, so that the grid strings are represented by a row of dots), and d , l and c are the distances from the lens to the lamp, grid and image, respectively. The fiducial string is the grid string where the lens axis intersects the grid. A is a point on the fiducial string, and A' is its image. B is the shadow of A cast on the object, and B' is its image. C is that point where ray BB' intersects the grid, so that C' (the image of C) coincides and occludes B' . Although the figure depicts the meridional case (i.e., rays AA' , AB and BB' in a plane containing the optic axis), the derived geometric relationships for fiducial shadow ordering hold for skew rays as well. The distance h_ϕ from B to the grid plane corresponds with the elevation of the ϕ th fringe.

Figure 2.5: A side view of the distorted-grid pattern on a human back. This oblique peek behind (rather than through) the grid of the NRC Contourgraph clearly shows how the triple-width fiducial shadow conforms to the surface relief. The lamp is at head-level, about 2 meters from the subject.

The topogram resulting from this arrangement was shown in Figure 2.1, and the white fiducial string and its dark, curved shadow can be clearly seen there. The horizontal fiducial string can be placed anywhere in the grid plane, but on the Contourgraph it passes through the camera lens axis, which simplifies the analysis. In particular, since the lamp is *above* the camera, the image of the horizontal fiducial shadow must always appear *below* that of the fiducial string. This is true for all objects placed behind the grid.

With knowledge of the apparatus geometry, the visible fiducial shadow can be used in several ways to determine the order of moiré fringes that it intersects, as depicted in Figure 2.6. Part (a) of this figure shows a portion of the topogram image in Figure 2.1, and in parts (b,c,d) the fiducial, its shadow, and the dark-fringe contours have been traced.

- **Grid Line Count Method:** If the grid lines are resolved in the image, then, as shown in part (b) of Figure 2.6, one can simply count the number of lines in the image between the fiducial and its shadow. More precisely, the number of grid-pitch intervals from the fiducial to its shadow matches the fringe number at the shadow. (This important property holds for fiducials placed *anywhere* on the shadow-moiré grid; see the derivation of Eq. A.17 on p. 391.) This method gives the absolute fringe number, but exact counting requires care and intimate knowledge of the moiré apparatus. For example, the NRC Contourgraph has a half grid-pitch interval bias on the separation between the fiducial and its shadow, which must be subtracted from the count. The shift occurs because the white fiducial string is actually suspended about 2 mm in *front* of a regular black string in the grid plane (as seen from the camera position), whereas the oblique illumination, from the lamp above the camera, is blocked by the fiducial string as if it were placed in the grid gap below it (i.e., half a grid-pitch interval lower). If the fiducial string was really placed in the indicated gap, it would look like just a bright gap. Instead, it blocks a black string and has bright gaps above and below it, so that its recognition is unmistakable in the image.

Figure 2.6: Fringe ordering methods using a fiducial shadow. (a) A magnified portion of the topogram in Figure 2.1, taken with the NRC Contourgraph. Visible are the black grid strings, the white fiducial strings, the dark, curved fiducial shadow, and the meandering moiré fringes. (b) The Grid Line Count Method. (A' and C' correspond to the points in Fig 2.4) (c) The Image Distance Method: (d) The Shadow Direction Method

- **Image Distance Method:** It may be more convenient just to measure the image distance y' from the fiducial string to its shadow, as shown in Figure 2.6c. The corresponding distance y in the grid plane (see Fig. 2.4) spans ϕ grid-pitch intervals of width s (i.e., $y = \phi s$), and if the image scale σ is known ($\sigma = \ell/c = y/y'$), then the order of the fringe intersected by the shadow is given by

$$\phi = \frac{y'\sigma}{s} \quad (2.1)$$

This method also yields the absolute fringe number, subject to the same cautions cited for the line counting method. However, it is useful even if the grid lines are not visible.

- **Shadow Direction Method:** One need only observe the direction of the fiducial shadow to decide upon the relative ordering of the fringes it intersects in sequence. Again, the key property is that the distance from the fiducial to where its shadow intersects a fringe increases with fringe order. Thus, as shown in Figure 2.6d, if the shadow goes up, across or down, when traversing from one fringe to the next, then the second fringe lies above, level with, or below the first fringe, respectively. This method gives only relative ordering, but does not require that the image scale be known, nor that the grid lines be visible (indeed, even the fiducial string is not directly needed in the image).

Fiducial shadow ordering is the only non-contact method of which the author is aware that allows the absolute fringe order to be determined (aside from targetting the object with calibrated control points). All other methods described in this chapter provide only relative ordering. Furthermore, the fiducial structure and its shadow are not restricted to lines, and targets such as dots, crosses or other machine-detectable symbols could be employed. These methods can be adapted to projection-moiré systems too (see App. A).

There are limitations to consider. The method requires features to be recognized within the context of the image, and although this is an easy task for human interpreters, it is difficult to automate. Furthermore, the fiducials and their shadows add clutter to the moiré pattern, which can hamper efforts to automate the

Figure 2.7: Portions of Figure 2.1 ordered by a single fiducial shadow. The fiducial shadow resolved the ordering of the 30 contours that span most of the back and arms of this subject. However, critical central areas missed are the single fringes on each scapula (zones A,B) and the small of the back (zone C). Peripheral zones have decreasing significance in this shape measurement application: the buttocks (zone D), the waist (zones E,F), the shoulders (zone G), and lower arms (zones H,I).

tracing of the moiré fringes. A more serious limitation is that fiducial methods are a strictly localized solution to the fringe ordering problem, and resolve only those fringes that intersect the fiducial shadow. Figure 2.7 shows that a single fiducial may solve nearly all of the fringe ordering in a topogram, but 'nearly all' is seldom enough.

2.2.3 Raster-Moiré

One approach to reduce the chance that the fiducial line shadow fails to intersect a fringe, is to use additional fiducial lines. In the application shown in Figure 2.7, two or

three more fiducials would provide better coverage, but some of the smaller contour loops would still be missed by such a sparse net. The term *raster-moiré* applies when *many* fiducials are embedded in the moiré pattern. The raster interval is determined by the tolerance to shape uncertainty. Depending upon the application, a small contour loop, missed by the raster shadows, may enclose an insignificant area, and failing to resolve the concavity/convexity of the surface there, may be acceptable.

To demonstrate this method, Takasaki and Ibrahim [1986] removed every m th string in a shadow-moiré grid to produce a dense raster-moiré pattern, which they reported as *two-frequency-moiré*. The example shown in Figure 2.8 sets $m = 20$, and is a rather busy, even confusing, pattern to analyze. In fact, a dense raster of fiducials adds so much clutter to the image, that it might be better not to form the moiré at all, but rather, to extract all the shape information directly from the distorted raster.³

2.2.4 Oblique or Tilted Moiré

On smooth, rotund objects, the ordering sequence is usually monotonic (i.e., it doesn't change direction), with possible exception at the closed loops about a local convexity/concavity, or in parallel contours traversing a horizontal ridge. One strategy is to abandon 'contour' planes, and select some other set of sheets that intersect the object with sufficient 'tilt' to avoid generating localized closed loops, and thus produce a set of quasi-straight curves that are then trivial to order. Dynamic fringe methods for achieving this will be discussed later, but a fixed-encoding method, termed *oblique moiré* or *tilted moiré*, has been used to demonstrate this principle [Takasaki, 1973].

Oblique moiré fringes result when the lamp and camera are at different distances from the grid. The geometry must be selected carefully, to at least shift the fringe-

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³This is the basis of another structured-light encoding technique for shape determination, aptly called *raster photogrammetry* and the reader is directed to a series of articles by Frobin and Hierholzer [1981, 1982a,b, 1983a,b, 1986a] that report on this technique.

Figure 2.8: A raster moiré topogram, created by removing every 20th string from the grid. Each bright stripe on the object, due to a missing string, is clearly visible in the moiré pattern and its position, where it intersects a moiré fringe in the topogram, could be used to determine the absolute elevation of the fringe [Takaşaki and Ibrahim, 1986]

Figure 2.9: Oblique moiré fringes on a mannequin, formed by placing the illumination and camera at different distances from the grid. In each photograph, the family of curved fringe sheets is seen to intersect a wall beside the mannequin, clearly illustrating that the fringes are not contours of constant elevation. In the right photo, the sheets intersect the torso so that no closed loops are formed, and the fringes simply increase in order going up the torso and over the shoulder [Takasaki, 1979a]

ordering problem if it can't be eliminated.⁴ Figure 2.9 shows two arrangements; the left example has simple ordering up the head of the mannequin, but still has closed loops on the torso, whereas the right topogram has problem fringes only on the face. The 'nearly-straight' curves on the torso in the right topogram are simpler to trace by hand or machine, but there is a much greater burden in computing the elevation of each point along the oblique fringe. Furthermore, visual interpretation of object shape is difficult compared to using contours of constant elevation.

⁴The First Law of Engineering reads "Trouble is conserved", thus an engineer can only move trouble, but never eliminate it!

Figure 2.10: Moiré fringe ordering using a phase-disjoint grid (a) The grid, with the top half shifted by $\Delta s \approx s/4$ with respect to the bottom half (b) The moiré fringe pattern resulting when a regular grid is superimposed on the disjoint grid. The effect is best seen by holding the page at a 'grazing' viewing angle in the fringe direction (this suppresses the fine-structure of the grids)

2.2.5 Phase-Disjoint Moiré

A fixed-fringe projection-moiré method that relied upon phase-shift properties was reported that used a *regionally phase-disjoint* projection grid [Benoit, *et al.*, 1975]. The technique requires making a Ronchi ruling in which the phase of the bars is periodically stepped by a fraction Δs of the bar spacing s . Benoit's grid appeared to have been sliced orthogonal to the bar direction in regular intervals, and then reassembled with slight, systematic misalignments. In Figure 2.10, part (a) shows a grid cut and reassembled parallel to the bars, and (b) shows the effect of superimposing a regular grid. Each fringe crossing the discontinuity suffers a slight, but distinct, phase shift with respect to the surface topography (say uphill); thus, the fringe ordering is indicated. If the shift is too much (i.e., $\Delta s \approx s/2$), then there is an ambiguity in linking fringes across the cut, hence defeating the purpose of the method.

This method achieves the same result as the fiducial shadow technique, in that

any intersected fringe is ordered. However, only relative ordering is possible, the disjoint grids are more difficult to make, and the method is restricted to projection-moiré (which uses separate encoding and demodulation grids). Furthermore, the disjoint fringes are more difficult to work with than those interrupted by fiducial shadows.

2.2.6 Object-Shifted Moiré

The position of the fringe contour planes is a function of the moiré apparatus, and not the object. One can think of the moiré process as immersing the object into the intersection volume, and fringes being localized where the fringe planes intersect the surface of the object. As the object is elevated (i.e., moved further from the base plane), the fringes shift downhill. Their corresponding shift in the topogram image can be used to determine the fringe ordering [Jaerisch and Makosch, 1973]. Figure 2.11 shows a simple example to illustrate this technique. Ordinary moiré topograms can only encode a single position of the object, thus a drawback of this method is that two images are required.

2.2.7 Stereo Moiré

One approach that literally doubles the image data bears mentioning. *Stereo moiré* [Takasaki, 1973], can be realized from a pair of moiré topograms taken from different camera positions, as in regular stereo-photogrammetry (Fig. 2.12). The key to this method is to record the *same* topographic fringe pattern from *different* perspectives, thereby introducing the horizontal parallax required for indirect stereoscopic vision (Fig. 2.13). This is the case if the camera positions are on a line parallel to the grid lines, as depicted in Figure 2.14.

The depth perception of the fringes makes their relative ordering immediately obvious when viewed stereoscopically. Clearly this method discards the single-image virtue of the moiré technique, and the remainder of this chapter will show that these measures are not necessary to determine relative fringe ordering.

Figure 2.11: Object-shifted moiré for fringe ordering. Four contours are shown for an object in its original position (labelled 1,2,3,4), and after being moved further from the moiré grid (labelled 1', 2',3',4'). The order of the contours can be interpreted as follows: since 2', 3' and 4' moved towards 1, 2' and 1 respectively, then (1 is above 2), (2 is above 3) and (1 is above 4); also, since 2' and 4' moved together, then (2 is level with 4)

Figure 2.12: Stereo moiré topograms of a living body. The grid had horizontal strings, allowing the same fringe pattern to be formed at horizontally-displaced camera positions. Note the shift in each view of the central white vertical fiducial string (in the grid plane) with respect to the navel of the subject. To view this figure stereoscopically, hold the page about 20" from your eyes and relax your attention. Now let the left eye focus on the center of left photo, while the right eye focuses on the center of the right photo. With practice, a 3-D image will appear 'between' the photos. Then, concentrate on the center image, which may be improved by slight rotational adjustments in the plane of the page. [Takasaki, 1979a]

Figure 2.13: Indirect stereoscopic observation of an object can be realized by providing the proper perspective image of the object for each eye. The images are arranged so that rays from each eye to a point A on the object intersect the images at the corresponding image of that point (A_1 and A_2 for the left and right eyes, respectively) [Hallert, 1960]

2.3 Techniques for Asymmetric Moiré Fringes

Departure from the constraints of ordinary moiré (fixed-encoding, grade-symmetric) has led to some truly ingenious methods of global fixed-encoding that capitalize upon the generation of grade-asymmetric fringes. These techniques fall into two classes of structured-light encoding: intensity modulation and spectral modulation.

2.3.1 Asymmetric Intensity Modulation by Grids

The simplest and most successful method of asymmetric encoding was invented by Windischbauer [1981]. He perfected an elegant grid for shadow-moiré that produces high-quality intensity-asymmetric fringes that vary {white, dark gray, light gray, black, ...} with increased elevation, thus making it trivial to determine the relative elevation of any fringe-pair. These *directional fringes* are very easily distinguished by eye from the topogram, as seen in Figure 2.15, and thus preserve the attractive ease of interpretation that characterizes the moiré method.

Figure 2.14: Instrument geometry for stereo moiré topography. For the cameras to record the same fringe pattern from different perspectives, they must be positioned on a line parallel to the grid lines. In the figure, the grid lines are horizontal and thus the cameras are placed on a parallel line at C_L and C_R . The change in perspective causes the vertical fiducial line V_g and point a_o in object space to have different positions in the left (V_L, a_L) and right (V_R, a_R) images, respectively. Note that the lamps may be distributed anywhere along the line L_1L_2 , which is also parallel to the grid lines.

Figure 2.15: Asymmetric moiré fringes from the Windischbauer grid. The intensity-asymmetric fringes vary from white to dark gray, to light gray, to black with increased elevation. This additional encoding makes it easy to determine the relative elevation of any fringe-pair. (Courtesy of G. Windischbauer)

Figure 2.16: String grid construction for shadow-moiré. A pair of C-clamps illustrates the principle of using the regular grooves of threaded rods to uniformly separate bars or strings stretched between them. In practice, the screws serve only as a bridge to establish the grid spacing, and a robust frame supports the screws and tensions the strings.

Windischbauer's technique is based upon a modified 'string grid'.⁵ A string grid is usually fabricated by stretching strings across a pair of threaded rods as depicted in Figure 2.16. In shadow-moiré, the strings cast shadows on the object when illuminated through the grid, and the strings also occlude parts of the object when viewed through the grid. The symmetric string grid is shown in cross-section in Figure 2.17, and since the illumination and viewing transmittances are symmetric functions, they convolve to form a moiré fringe intensity that is symmetric in elevation.

Novel to the Windischbauer grid are the *double-strings* stretched between a pair of 60°-vee threaded rods (Fig. 2.18), whose intensity modulation is asymmetric with the angle of incidence. The asymmetric fringe pattern results when the object is illuminated (source-encoded) through the grid at a different angle than that from which the camera records the demodulated moiré pattern.

Ibrahim [1982] studied the directionality of the Windischbauer grating (see

⁵Moiré grid construction is described in detail in [Pekelsky and van Wijk, 1988].

Figure 2.17: Moiré fringe intensity profile from a symmetric string grid. The transmittance of the grid is a symmetric function for both the illumination and view directions, and thus the moiré fringes are also symmetric.

Figure 2.18: The Windischbauer grid geometry. The sectional view through the grid shows how the pairs of strings are arranged to lie in the grooves of the threaded rod. The screw profile, its pitch, and the cross-section of the doubled strings establishes the asymmetric transmittance geometry of the grid. Since the illumination and viewing are from different directions, the respective transmittances are asymmetric functions, and so also is the intensity profile of the moiré fringes.

Figure 2.19: A comparison of asymmetric string-grids. Shown are the screw profiles and string arrangements to produce asymmetric intensity modulation for (a) Windischbauer's asymmetric vee-groove, (b) Ibrahim's skewed-groove, and (c) Takasaki and Ibrahim's double-threaded square groove

Fig. 2.19a), and proposed an alternate grid construction based upon a special skew-cut threaded rod whose angles are tuned to the illumination and observation directions (Fig. 2.19b). A third screw profile was reported based upon a double-pitch square-cut thread (Fig. 2.19c), and was predicted to produce directional fringes in the sequence {black, white, darkest gray, light gray, dark gray, lightest gray}, with the differences in the pairs of light and dark grays being a function of illumination angle in the shadow-moiré arrangement [Takasaki and Ibrahim, 1983]. In fact, the test fringes are rather symmetric to the eye, but these authors argued that densitometric analysis of the profile would disclose the directional characteristics.

Using a different approach, Chiang [1975b] reported a variation on *two-frequency moiré*, in which the spaces of a coarse Ronchi ruling are modulated by a finer Ronchi ruling for the purpose of producing intensity-asymmetric fringes. This method introduces a longer code than the binary {bright, dark, bright, ...} of ordinary moiré fringes, but is limited in the same way as Takasaki's *two-lamp moiré* method described below.

Figure 2.20: Two-lamp arrangement for asymmetric moiré fringes (a) Instrument geometry, with lamps at different heights d_n and d_m above the camera. (b) The doubly-modulated moiré fringe intensity function, with the further lamp causing a higher-frequency modulation of the low-frequency envelope due to the closer lamp. The {bright, dark, brighter, . . .} sequence is usefully asymmetric over a quarter-cycle of the envelope.

2.3.2 Asymmetric Intensity Modulation by Lights

Takasaki [1979a] suggested modulating a coarse moiré contour interval with a finer one, to extend the length of the binary moiré code. This is realized by using two light sources at different distances from the camera, as shown in Figure 2.20.

The double-modulation frequency depends on the relative lamp positions. The example in Figure 2.21 shows each coarse fringe modulated by about six fine fringes of varying grayness. Within a quarter cycle of the coarse fringe, the finer fringes can be ordered unambiguously, and this degree of intensity asymmetry may help to resolve some fringe ordering situations.

Figure 2.21: Doubly-modulated moiré fringes to aid in fringe order determination. By using two light sources at different distances from the camera, a fine and coarse fringe pattern are simultaneously formed. [Takasaki, 1979a]

2.3.3 Asymmetric Spectral Modulation by Grids

Colour has always been an attractive, if expensive, means of increasing the dimensionality of an image. Spatially-asymmetric spectral modulation to achieve *coloured fringes* that are tinged red on the uphill side and blue on the downhill side, was invented by Honda and his colleagues [Honda *et al.*, 1976]. They used three-colour Ronchi grids (sequences of red, blue and transparent bars) in a projection-moiré system, with the sequence reversed (blue, red, transparent) in the analysis grids. A similar coding, based upon projecting a {red, green, blue} Ronchi ruling was analyzed in [Eschbach and Bryngdahl, 1983]. The fabrication difficulty of these gratings, and the cost of colour photography to record the moiré patterns, has precluded the wide-spread use of this approach.

2.3.4 Asymmetric Spectral Modulation by Lights

Benoit *et al.* [1975] described a projection-moiré arrangement in which a double exposure is made using distinguishably-coloured illumination for each exposure (*e.g.*, red, and then blue). Between the exposures a small phase shift is introduced, resulting in a shifting of the fringe elevation. Thus, the fringes in the topogram are tinged red on the uphill side and blue on the downhill side, as in Honda's method. The advantage of Benoit's technique is that one need only install a colour filter on the camera of an otherwise conventional projection-moiré set-up. However, dynamic processes such as living subjects are too unstable to maintain alignment for the critical second exposure. Livnat *et al.* [1980] also invented *two-colour, double exposure moiré*, and showed that a white light exposure and a superimposed red filtered exposure produces a fringe that has a directional red-white-black asymmetry.

On a final note regarding colour: some of the dynamic methods described in the next section also use asymmetric colour to display the ordering of the fringes on a colour television *after* they have been ordered. The power of colour-coding to convey fringe ordering information to human interpreters can sometimes justify its cost.

2.4 Techniques for Dynamic Moiré Fringes

Grade-symmetric moiré has enjoyed the most success with the advent of dynamic encoding. The term *dynamic* refers to those moiré techniques that rely upon comparing two or more shifted fringe patterns of the same object. One of the advantages of moiré topography is that it encodes the 3-D information in a single image, and most of the dynamic techniques rely on processing patterns generated from one record of the scene. Some methods, however, require a sort of *calibration image* that can be used repeatedly, thus maintaining the photographic economy of moiré over stereo-photogrammetry.

Common to all the techniques described in this section are both the principle of *phase-shifting*, as described earlier, and the concept of the *source-encoded image*, or *distorted-grid image* as it is sometimes called. The distorted-grid image is produced by viewing the object from a different perspective than that from which the grid (or structured-light) is projected onto the object (as in Fig. 2.5). This is usually done by removing the analysis grid from a conventional projection-moiré arrangement.⁶

Dynamic moiré is characterized by forming the distorted-grid image, and producing the moiré fringe pattern in a subsequent step by superimposing a suitable analysis grid. By phase-shifting the analysis grid, the fringes are made to shift uphill or downhill (hence the fringes are *dynamic*, as opposed to *fixed-encoded*). Figure 2.22 illustrates this important technique.

Several manual methods have been described in the literature, and some have been developed into fully-automated fringe analysis systems. The dynamic moiré methods are discussed below under three main groupings: those using simple material grids, those with optical-interference virtual grids, and those based on detector-sampling grids.

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⁶In a projection-moiré system, a grid is optically projected onto the object (source-encoding), and the image is demodulated with a separate analysis grid in the image plane of the recording camera. Appendix A gives more details.

Figure 2.22: Dynamic moiré by analysis grid translation. (a) The distorted-grid image (b) The analysis grid. (c) and (d) show the superposition of the distorted-grid image and the analysis grid, at slightly different translations of the analysis grid, and with the appropriate phase shifts in the resulting moiré fringe pattern. The reader is encouraged to experiment with the mylar copy of (b) [in the pouch on the facing page] to best appreciate how small translations of the grid result in large phase changes in the moiré pattern. The distorted-grid image in (a) is from [Takasaki, 1981]

2.4.1 Dynamic Moiré using Material Grids

The term *material grid* refers to devices for blocking strips of a light beam with opaque material. String grids are common examples, as are grids formed on glass or plastic sheet by photography, lithography or other means. Various types of grid construction and their application to moiré topography are described in detail in [Pekelsky and van Wijk, 1988].

2.4.1.1 Object-and-Plate Translation

Miles and Speight [1975] reported a manual approach in which a distorted-grid image is made of the object, and then another is made of a flat screen or plate placed where the object was. Clearly, in a stable arrangement of camera and projector, the same flat screen photograph can be used to analyze a series of photographs of different objects. By superimposing the two images, moiré fringes are formed, and by translating the transparencies to introduce small calibrated phase-shifts, any point in the scene can be made to lie on a fringe, thus determining its elevation. This approach, based upon dynamic moiré using material grids, was a forerunner to a class of automated *phase-lock* techniques, and circumvented the fringe-ordering problem by reducing the moiré-phase to zero at each point via a calibrated phase-shift.

2.4.1.2 Object-and-Plate Rotation:

Livnat *et al.* [1980] also described an *object-and-plate* method, inspired in their case by the general moiré-difference scheme invented by Hovanesian and Hung [1971]. Livnat and his colleagues demonstrated a dynamic fringe ordering procedure in which the superimposed object and plate distorted-grid images are slightly *rotated* with respect to one another. Under *grid translation*, fringes tend to move up- or down-hill, so that, for example, a fringe looped around a hill becomes a smaller loop as it shifts towards the summit. Under the described *rotation*, however, the fringes tend to shift together, so that the common center of a series of concentric loops that characterize the summit of the hill shift from the summit in a prescribed

Figure 2.23: Dynamic moiré by analysis grid rotation. The distorted-grid image and analysis grid from Figure 2.22 are superimposed with a rotation. The reader can experiment with the effects of rotation using the mylar grid and part (a) of Figure 2.22.

direction, with less of a change in the size and shape of the fringes than with the previous method (Fig. 2.23). The shift for concave and convex fringes is in *opposite* directions, hence the application of this method to fringe ordering.

Roger *et al.* [1981] also used the object-and-plate rotation method, calling it *inclined-plane moiré*. As shown in Figure 2.24, they preferred to introduce significant tilt to the reference plane by rotating the analysis grid, with the intention of generating fringes that were trivial to order (there were no closed loops in the pattern, and the fringes were unambiguously monotonic-increasing in order). Livnat and Kafri [1983] described a *finite-fringe shadow-moiré* method that involved rotating the projection grid *in-plane* between two exposures, to finally produce fringes with no closed loops.

2.4.1.3 Illumination Direction Oscillation

Moore and Traux [1979] described an automated moiré system derived from phase-lock interferometry techniques. In shadow-moiré, the fringe elevations are governed by the geometric arrangement of the light source, the camera and the grid. For collimated illumination (infinitely distant source), it is the angle of incidence to the grid that affects the fringe formation. These authors arranged a telecentric optics such that the apparent direction to the light source was varied by an oscillating mirror. A laser provides a convenient point-source, but the method does not rely upon coherent illumination. A detector scans the object via a galvanometer-driven mirror. Rather than analyze the dynamic fringe pattern, the elevation of the object is adjusted on a computer-controlled stage so that the image of a selected fringe is centered on the detector. The detector raster-scans the view of the object, and the stage is raised or lowered as needed to keep the selected fringe tracking the profile of the object. By oscillating the light source direction $\pm 0.5^\circ$ at 2 kHz, Moore and Traux obtained the excellent signal-to-noise ratio that characterizes phase-lock techniques, and found they could track the surface profile to within $1/40$ of a fringe (about $10\times$ better than for static densitometric measurements). Clearly, the constant phase imposed at every point on the surface precludes the hill-or-valley issue.

2.4.1.4 Axial Grid Translation

Reid *et al.* [1984, 1985] reported on a system in which the shadow-moiré grid was mechanically moved along the optical axis (as opposed to in-plane translation) under phase-lock control. The phase-shifted images were captured on a CCD camera and the fringes were automatically ordered by software on a microcomputer. Their system is described in more detail in Appendix C.

2.4.2 Dynamic Moiré using Virtual Grids

Several techniques have been reported in the literature in which the patterns of one, or both, of the physical grids of a projection-moiré arrangement are produced

Figure 2.24: Dynamic inclined-plane fringes on a human back. A demodulation grid is superimposed on the distorted-grid image of a human back to produce moiré fringes. (a) The demodulation grid is aligned to produce contours parallel to the plane of the page. (b) The grid is rotated in-plane to produce inclined-plane moiré contours with no closed-loop fringes. The advantage of the latter pattern is that fringe ordering is trivial, being always monotonic-increasing [Roger *et al.*, 1981]

Figure 2.25: Off-line holographic phase-lock moiré analysis for the detection of spinal deformity (a) Conventional projection-moiré arrangement for recording the distorted-grid image (b) Optical configuration of the off-line phase-lock analysis system. [Chan *et al.*, 1981]

by interference effects. Coherent beams are made to interfere on the object such that the holographic fringe pattern (or *virtual grid*) encodes the object in the same fashion as a conventionally projected grid. The holographic approach has some distinct advantages that merit the use of expensive optics and lasers: very fine fringes (down to micrometer pitches) with superb depth of field may be created, and important to this discussion on fringe ordering is the ability to easily vary the grid pitch. Modifying the pitch causes the nominal elevation of the fringes to change, and the fringes can be made to huddle uphill or spread out downhill.

2.4.2.1 Holographic Phase-lock Moiré

Chan *et al.* [1981] reported a two-step process in which a distorted-grid image is photographed in the usual fashion, and the film is then placed in a phase-lock analysis system (Fig. 2.25). The virtual analysis grid is holographically formed by a Mach-Zehnder interferometer in which one of the folding mirrors is driven by a piezoelectric crystal to alter the path-length in one arm of the interferometer, thus causing the virtual grid to translate with respect to the film. The grid translation is phase-locked to the piezo-driving circuits, and profiles are tracked in the same

Figure 2.26: Schematic diagram of an off-line heterodyne optical-image processor for analysis of distorted-grid images. The interferometer uses polarized light to form the virtual analysis grid, and the electro-optic modulator oscillates the phase of the polarization, and hence the phase of the virtual grid illuminating the distorted-grid image in the photographic plate. The modulator and detector are phase-locked, so that the fringe ordering can be automatically determined as the moiré scene is scanned [Reid, 1983]

manner as in [Moore and Truax, 1979]. Perrin and Thomas [1979] also developed a holographic phase-lock system based on this method, but used a more sophisticated optical arrangement. They discussed the advantages and drawbacks of the method, and analyzed phase-lock and moiré photogrammetry in detail.

2.4.2.2 Heterodyne Moiré

Reid [1983] called his virtual-grid phase-lock approach *heterodyne moiré*, and reported on an analysis system for distorted-grid images that employs a virtual grid generated by interfering polarized laser beams, whose relative phase can be electronically modulated (Fig. 2.26). The demodulation grid is oscillated at 5 kHz, and a phase-locked detector is swept about the dynamic moiré image, automatically assessing the phase response (and hence the relative ordering) of each fringe.

2.4.3 Dynamic Moiré using Detector Grids

In the mid 1970's, a method based upon what could be called *detector grid moiré* was independently invented and developed by several separate research groups. Called *scanning moiré* by Idesawa *et al.* [1977], *electronic grating moiré* by Udea [1979], and later *video-electronic moiré* by Kugel and Lanzl [1979], it is perhaps the most successful scheme for automated moiré measurement of 3-D surfaces to date. Fundamental to the concept is the use of an electronically-scanned image detector (such as a TV camera) that can both scan the scene and impose a virtual analysis grid by modulating the image sampling. Thus, the grid is formed by *not sampling* where a physical analysis grid would have occluded the view of the distorted-grid image. There are variations on this theme, and the methods are grouped below, depending on whether the grid is formed by the basic raster, by sampling the raster, or by interpolation. For the interested reader, the generalized sampling conditions are formulated as a moiré extension to the Whittaker-Shannon sampling theorem in [Bell and Koliopoulos, 1984].

2.4.3.1 Detector Grid by Raster

The *video-electronic moiré* system developed by Kugel and Lanzl [1979] uses a TV camera to capture the distorted-grid image, and lets the inherent raster of the TV camera serve as the analysis grid (Fig. 2.27). This limits the opportunity to control the phase and tilt of the virtual grid (see Udea's method below). Clever manipulation of the two TV interlace-frames, however, allows them to cancel the carrier frequency, yielding the effect of virtual grid translation, and they could also colour-code the frames to produce asymmetrically-coloured (ordered) fringes on the display monitor in real-time. Solid-state CCD cameras have also been used for detector grid moiré based on the raster method [Nordbryhn, 1983].

Figure 2.27: Detector grid moiré by raster. The even and odd frames of interlaced TV scan alternate raster lines in the image, and thus serve as virtual analysis grids when viewing the distorted-grid image. By selecting one frame or the other, a stable moiré image is generated. Using video-mixing techniques, dynamic fringes with labelled ordering can be formed.

2.4.3.2 Detector Grid by Sampling

Udea's *electronic-grating moiré* system also used a computer-controlled TV camera to generate contours in real-time (i.e., video refresh rate) [Udea, 1979]. Unlike the previous method, however, the horizontal TV raster does not act as an analysis grid. Instead, the analysis grid is created perpendicular to the video raster using high-speed electronics to turn the camera gun off and on as it traverses each raster pass, and repeating this modulation scheme consistently for every line in the raster frame (Fig. 2.28). Udea uses a fast microprocessor and programmable counters to digitally adjust the pitch, phase and rotation of his electronic grid, and in this fashion has considerably more control over those parameters than that offered by the inherent raster grid of the camera.

Udea introduced several innovations that merit elaboration. He developed a simple digital filter to remove the carrier frequency and to generate smooth-edged binary fringes (Fig. 2.29), and he devised an electronic means for *virtual grid translation* to eliminate the second harmonic (alias) moiré pattern (Fig. 2.30), all at video frame rate. The electronic grid can be phase-shifted to determine the ordering of the fringes in the usual fashion, but Udea also devised a scheme to produce pseudo-

Figure 2.28: Detector grid moiré by sampling the raster. The analysis grid with a nominal pitch s is formed perpendicular to the raster by sampling at intervals longer than the raster pitch. The virtual grid can be given an offset s_0 and can be rotated by θ by introducing suitable delays into the initiation of sampling on each raster line. The moiré pattern is formed by viewing an appropriate distorted-grid image with the sampled raster.

coloured fringes by shifting the grid phase of successive frames by 0° , 120° and 240° , and applying the three resulting TV images to the respective red, green and blue guns of a colour monitor. The composite pattern produces coloured fringes that are sequenced {red, magenta, blue, cyan, green, yellow} with increasing elevation on the object (Fig. 2.31). This image can be viewed in real-time on a colour monitor and allows unambiguous ordering of the contours.

A forerunner to Udea's system, Hormière and Mathieu [1976] demonstrated the utility and control over the moiré geometry that was possible with vidicon cameras. Like Udea's system, the raster grid is controlled by special electronics that allow them to adjust the virtual period and phase of the grid. Rotation is optically introduced at the encoding projector. Their projection-moiré system is schematically shown in Figure 2.32. An important feature of this arrangement is the opto-electronic capability to freely orient the contour reference plane (ideal for determining fringe ordering), and Figure 2.33 shows some experimental results.

Figure 2.29: Electronic-grating moiré patterns. Shown are magnified portions of the moiré pattern to demonstrate the effect of a special digital filter. (a) The unfiltered pattern. (b) The pattern after filtering [Udea, 1979]

Figure 2.30: Virtual grid translation in electronic-grating moiré. (a) Digitally filtered output with stationary virtual grid. (b) Digitally filtered output with moving virtual grid, and reduced alias moiré effects. [Udea, 1979]

Figure 2.31: Real-time contour ordering by false-colour display (shown here in distinct gray-levels assigned to the indicated colours) (a) A curved surface (b) A human hand. [Udea, 1979]

Figure 2.32: Schematic of a projection-moiré system using a vidicon camera as a scanned grid to demodulate the distorted-grid view. In the projection system, the grid G could be rotated by θ_1 . The electronic device ED permitted the choice of virtual period p_1 and the phase ϕ [Hormière and Mathieu, 1976]

Figure 2.33: Experimental results of electronic moiré plane orientation using the system depicted in Figure 2.32. For images A, B and C, the moiré planes are made parallel to a given plane of the cube, whereas in D the moiré plane is adjusted normal to the viewing direction [Hormière and Mathieu, 1976]

Figure 2.34: Detector grid moiré by interpolating the raster. In detectors with spatially-fixed sensor sites (such as solid-state CCD cameras), the analysis grid can be formed by interpolating the values at virtual sensor positions. Like the sampled raster shown in Figure 2.28, an interpolated analysis grid with nominal pitch s can be formed perpendicular to the raster, with offset s_0 and rotation θ as shown

2.4.3.3 Detector Grid By Interpolation

Idesawa and his colleagues [1977] introduced *scanning moiré* as an off-line distorted-grid demodulation system that used a computer-controlled CRT to scan the photograph. Later, it was rendered as a near real-time method using a CCD linear-array scanner [Yatagai *et al.*, 1981b]. Unlike the vidicon camera used by Udea, the sampling positions in the CCD array are spatially fixed. To overcome this limitation, each line of pixels from the CCD scanner is resampled at different phases by using linear interpolation to estimate the intersensor image brightness (Fig. 2.34). By phase-shifting the resampling, three different phased moiré patterns could be recovered, from which the fringe-ordering along the profile could be automatically assessed.

2.5 Photometric Ordering

The previous sections showed how one could introduce additional encoding to overcome the apparent limitation of ordinary moiré fringes for the purpose of determining their order. Elaborate schemes for generating asymmetric and dynamic fringes

have been invented to this end. There is, however, a natural asymmetry in ordinary moiré fringes which this author discovered could be used for ordering.

A moiré topogram can be interpreted as an ordinary photograph of the object with a shadow pattern superimposed. Thus, the natural *shading* of the object is *modulated* by the *moiré shadows*. The shading envelope is a slowly-varying function tracking the surface shape, whereas ordinary moiré fringes are even, cyclic functions of elevation. The combined effect is to darken the neighbouring fringe on the shady-side of each moiré fringe, hence the earlier remark that the fringes have a natural intensity asymmetry.

The next chapter is devoted to describing *photometric ordering*, a method invented by the author for ordering ordinary moiré fringes based upon the inherent shading on the surface of the object. The technique examines each pair of fringes and computes the expected shading for the cases of a fringe being above, at the same level, or below its neighbour fringe. These values are compared to the actual shading value (estimated in the presence of the fringes), and the most consistent match is adopted. The shading model (or *photometric model*) assumes that the object is a purely diffuse reflector (the simplest, and computationally most efficient model), and by performing the analysis at sufficient points about the contours, any localized departure from this assumption (i.e., a specular reflection or an image artifact) is rendered insignificant. In subsequent chapters, this 'shape-from-shading' technique is successfully demonstrated on surfaces with a variety of reflectance properties (e.g., painted wood, cardboard, live human skin) and with various topographic challenges (e.g., hills, saddle-points, sharp ridges).

Chapter 3

Photometric Ordering of Moiré Fringes

3.1 Introduction

This chapter examines the concept of using a photometric analysis of the shading in the moiré image to order a set of moiré fringe contours. The goal is to establish a photometric ordering algorithm which can be embodied into the test software described in later chapters. The remainder of the chapter is organized as follows.

The next section reiterates the contour ordering problem introduced in Chapter 1. The problem is defined for the general case, without reference to the particular technique used to order the contours. A notational formalism is developed with the ultimate purpose of presenting the ordering algorithm.

Photometric theory and the Photometric Ordering Operator are the subject of the third section. A brief review of 'shape-from-shading' research is given, and photometric models that link scene reflectance and surface geometry are described. The tasks of extracting the shading and shape characteristics from the moiré image are briefly sketched, and finally, the principle of applying a photometric analysis to order a sequence of contours is given.

The fourth section describes the algorithm developed by the author to photometrically order moiré fringe contours. The process description is structured as

modular, nested block-diagrams, and is designed to be efficiently embodied into automated software.

The last section discusses the key assumptions made in this research with regard to the photometric ordering of moiré fringe contours: that sufficient photometric control exists, that the contours have already been extracted, and that the shading can be extracted in the presence of the moiré fringe pattern.

3.2 The Contour Ordering Problem

The purpose of this section is to reiterate the contour ordering problem introduced in Chapter 1 in a generic fashion, without restricting the discussion to a specific method of ordering the contours. The special equivalence of moiré fringes and topographic contours is described, and terminology related to relative and absolute ordering is defined. Conditions for sufficiency and consistency in ordered contours are presented. The concepts of propagated and endogenous ordering are examined, the former being the basis for what is meant by 'photometric ordering' in this document. Propagated ordering uses sequences of neighbouring contours, and the section concludes with a look at the ordering schedule that controls the selection of neighbourhood sequences, and the ways in which a sequence of the contours may be ordered.

3.2.1 Moiré Fringes and Surface Contours

This investigation deals with shadow-moiré patterns in which the fringes represent topographic contours on the surface of the object. As described in Appendix A, this requires a particular arrangement of the shadow-moiré instrumentation, in which the light source and camera are in a plane parallel to the grid plane [Takasaki, 1970].

Given this special geometry, the moiré image intensity modulation is a cyclic, continuous function of the elevation above the base plane. As the elevation increases, the image intensity varies between maximum and minimum values, producing a sequence of bright and dark fringes. At any given elevation, the modulation is

characterized by the phase of the function.

The *absolute fringe number* of a moiré fringe (or simply 'fringe number' or 'phase number') will refer to the number of cycles counted from some reference elevation. In shadow-moiré instruments, the reference level is the base plane ($z = 0$). The symbol n or N is often used in the literature to denote the discrete fringe numbers, but the author uses ϕ as a reminder that the cyclic moiré *phase* is a continuous real-valued variable, which takes on simple values at the intensity extrema:

$$\phi = \begin{cases} 0, 1, 2, \dots & \text{for bright fringes,} \\ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots & \text{for dark fringes.} \end{cases} \quad (3.1)$$

The *moiré fringe extremum type* (MFE-type) of a contour is denoted by e , and refers to the type of fringe that was used to delineate it:

$$e = \begin{cases} 1 & \text{for bright fringes,} \\ 0 & \text{for dark fringes.} \end{cases} \quad (3.2)$$

A set of contours is called *matched-mode* if they are traced only from bright or dark fringes (i.e., all their e 's are the same), and they are called *mixed-mode* if they are traced from bright and dark fringes (i.e., $e_i \in \{0, 1\}$). The mode of a set of contours governs the possible ways they may be relatively ordered.

The *elevation* of a contour is its height z , with respect to a base plane (at $z = 0$). The term *contour interval* refers to the vertical separation between neighbouring contours that are at different elevations, and is denoted by Δz . In terms of phase numbers, the contour interval is denoted by Δ , and it is one moiré-function cycle for matched-mode contours and half a cycle for mixed-mode contours:

$$\Delta = \begin{cases} 1 & \text{for matched-mode contours,} \\ \frac{1}{2} & \text{for mixed-mode contours.} \end{cases} \quad (3.3)$$

The term *relative fringe number* will refer to the difference in phase number between adjacent contours, and is denoted by $\Delta_{i,j}$:

$$\Delta_{i,j} = \phi_i - \phi_j. \quad (3.4)$$

Figure 3.1: The shadow-moiré geometry. The grid lies in the X - Y plane of the coordinate system, with its strings parallel to the z -axis and spaced at a pitch s . The camera lens nodal point is in front of the grid, at $(0, 0, l)$, and the light source is above the camera, at $(0, d, l)$. The moiré-topogram image is formed in the U - V plane. The moiré fringe planes are parallel to the grid plane, and lie behind it. An object placed behind the grid is depicted as being intersected by the moiré fringe planes, forming contours on its surface.

For a pair of contours C_i and C_j , $\Delta_{i,j}$ depends on the mode of the contours

$$\Delta_{i,j} = \begin{cases} 0, \pm 1 & \text{if } e_i = e_j, \\ 0, \pm \frac{1}{2} & \text{if } e_i \neq e_j. \end{cases} \quad (3.5)$$

The functional relationship between the contour elevation and the fringe phase is established by the instrument geometry. Figure 3.1 shows the geometry of the shadow-moiré system used by the author. The grid lies in the X - Y plane of the coordinate system; its strings are parallel to the z -axis and have a pitch s . The camera lens nodal-point is at $(0, 0, l)$ and the lens axis is parallel to the z -direction. The light source is directly above the lens at $(0, d, l)$. The resulting series of alternating bright and moiré interference planes are at a distance from the grid plane (i.e., an elevation) given by

$$z = \frac{\phi l}{\phi - d/s}. \quad (3.6)$$

Note that fringes are formed for the volume behind the grid ($z < 0$) and that the choice of the coordinate system makes the contour elevations negative-valued

2

MicroD

Table 3.1: Fringe elevations for the NRC Moiré Contourgraph. Adapted from [Paulun, 1983].

ϕ_i	z_i (mm)	$\Delta z_{i,i-1}$ (mm)
0	0	—
1	-4.8	-4.80
2	-9.6	-4.83
3	-14.5	-4.86
4	-19.4	-4.88
5	-24.3	-4.91
6	-29.2	-4.94
7	-34.2	-4.97
8	-39.2	-5.00
9	-44.2	-5.03
10	-49.3	-5.06
$\vdots$	$\vdots$	$\vdots$

camera distance: $l = 1700$ mm
 lamp height: $d = 710$ mm
 grid pitch: $s = 2$ mm

for $\phi > 0$. If $\phi \in \{0, 1, 2, \dots\}$, then an inclined surface intersecting that moiré plane will produce a bright fringe in the topogram, whereas if $\phi \in \{\frac{1}{2}, \frac{3}{2}, \dots\}$, then a dark fringe results. Note that the surface *must* be tilted with respect to the $\hat{z}$ -axis for a contour to be defined there. Absolute and relative elevations for the first few bright fringes produced by the NRC Moiré Contourgraph are given in Table 3.1.

Throughout this document, the terms *fringe* and *contour* will be used interchangeably, with the distinction that the former is a curve in the image plane (be it the original photographic topogram or the display of a digital image analysis system) and the latter is a surface curve in object space. Thus, it is meaningful to refer to the fringe number ϕ of a contour.

The set of surface contours will be called a *map*. The set of contours will be assumed to be *complete* in that no contours are missing. More specifically, if a gradient curve G_k intersects a map contour C_i , then the next contour C_j that it intersects will also be a member of the map. The discussion will be further restricted to 'well-behaved' contours (continuous, non-intersecting lines).

Contours C_i and C_j on the object are *neighbours* if a surface gradient G_k intersects them without passing through another contour. If F_i and F_j are the corresponding neighbouring fringes in the topogram, then according to the axiomatic definition of the contour interval, their fringe numbers can differ by at most one interval. For matched-mode fringes

$$\phi_j = \begin{cases} \phi_i - 1, & \text{if } C_j \text{ is below } C_i, \\ \phi_i, & \text{if } C_j \text{ is level with } C_i, \\ \phi_i + 1, & \text{if } C_j \text{ is above } C_i, \end{cases} \quad (3.7)$$

and for mixed-mode fringes

$$\phi_j = \begin{cases} \phi_i - \frac{1}{2}, & \text{if } C_j \text{ is below } C_i, \\ \phi_i, & \text{if } C_j \text{ is level with } C_i, \\ \phi_i + \frac{1}{2}, & \text{if } C_j \text{ is above } C_i. \end{cases} \quad (3.8)$$

These expressions were derived from Equations 3.4 and 3.5.

3.2.2 Statement of the Ordering Problem

Consider a map consisting of a set of M contours

$$\mathcal{M} = \{C_1, C_2, \dots, C_M\} \quad (3.9)$$

having unknown elevations

$$\mathcal{Z} = \{z_1, z_2, \dots, z_M\}, \quad (3.10)$$

which must be determined. The corresponding set of M fringes

$$\mathcal{F} = \{F_1, F_2, \dots, F_M\} \quad (3.11)$$

have unknown fringe numbers

$$\Phi = \{\phi_1, \phi_2, \dots, \phi_M\}. \quad (3.12)$$

Clearly, if each of the absolute fringe numbers were determined, then the elevations could be computed using Equation 3.6, and the task would be complete. The

expressions 'solving the map', 'ordering the contours' and 'ordering the fringes' all refer to the execution of this task. A solution that results in the absolute fringe numbers and absolute elevations is aptly called an *absolute solution*.

In reviewing fringe ordering methods in Chapter 2, it was seen that only the fiducial-shadow methods could directly determine the absolute fringe number, and then only for a few select fringes.

The remaining ordering methods addressed all the fringes in the map, but they could only produce relative solutions. A *relative solution* is one where the relationship is determined only between neighbouring contours. This is referred to as 'relative fringe ordering', or just 'relative ordering'.

Sometimes a relative solution is adequate if the change in contour interval is negligible (see, for example, the Δz 's in Table 3.1). Note that, if the absolute fringe number ϕ is known for even one contour in a map that has been relatively ordered, then all the ϕ 's for the map can be found. Consider, as an example, the simple map $M = \{C_A, C_B, C_C, C_D, C_E\}$:

Relatively Ordered:		Absolutely Known:		Absolutely Ordered:
$\Delta_{A,B} = 1$	+	$\phi_C = 7$	$\Rightarrow$	$\phi_A = 9$ (from: $\phi_B + \Delta_{A,B}$)
$\Delta_{B,C} = 1$				$\phi_B = 8$ (from: $\phi_C + \Delta_{B,C}$)
$\Delta_{C,D} = 0$				$\phi_C = 7$ (given!)
$\Delta_{D,E} = -1$				$\phi_D = 7$ (from: $\phi_C - \Delta_{C,D}$)
				$\phi_E = 8$ (from: $\phi_D - \Delta_{D,E}$)

Thus, relative ordering can be the basis for an absolute solution, provided the absolute fringe number of at least one fringe can be established. There is a unique overhead to be borne by methods that rely on relative ordering, and that is the requirement to establish a list of the neighbour-pairs that span the map. It suffices here to say that generating the list of neighbours involves an exhaustive search of the image; the details of automatically conducting the search are the topic of Chapter 5.

Figure 3.2: A map to illustrate contour-neighbour sufficiency. Contours C_A , C_B and C_C are all neighbours, and it is sufficient to determine the relative ordering between only two of the three pairs.

3.2.3 Sufficiency and Consistency Considerations

Contour order relationships have *global validity* in that the elevation of each contour is constant throughout the map. With respect to relative ordering, this means that not every pair of contour neighbours need be scrutinized. Some contours, in a 'hilly' map may have several neighbours, but once a contour's elevation has been determined via its relationship to some neighbour contour, that elevation holds true everywhere on the contour, and it is unnecessary to re-establish that value by studying all the other pair possibilities.

Figure 3.2 illustrates this point with three contours that are mutual neighbours. Three contour-pairs can be formed: (C_A, C_B) , (C_B, C_C) and (C_A, C_C) . The sufficiency rule says that only two of the pairs need to be explicitly ordered to solve the map. The third pair can be resolved implicitly.

On the other hand, the global-validity property can be very useful to check for consistency in the solution. In the previous example, it was sufficient to determine $\Delta_{A,B}$ and $\Delta_{B,C}$ to solve the map, but if $\Delta_{A,C}$ is also evaluated, one can test for consistency via

$$\Delta_{A,C} \stackrel{?}{=} \Delta_{A,B} + \Delta_{B,C} . \quad (3.13)$$

This test expression is derived from Equation 3.4 as follows:

$$\Delta_{i,j} = \phi_i - \phi_j = \phi_i - \phi_k + \phi_k - \phi_j = \Delta_{i,k} + \Delta_{k,j} . \quad (3.14)$$

3.2.4 Propagated and Endogenous Ordering

Relative ordering establishes a relationship between pairs of neighbouring contours. In a previous example, it was shown how knowledge of the absolute fringe number of one contour could be *propagated* through a relatively-ordered map to determine the absolute phase of every fringe. In this sense, all absolute solutions are propagated from a 'seed' contour.

At a more primitive level, some relative-ordering methods must also propagate information from nearby contours to influence the ordering outcome. An example is the Smoothest-Profile Operator. It selects the smoothest profile through a set of contours, and thus needs to know how it has ordered the previous pair before it can proceed with the next. The Photometric Ordering Operator has similar requirements, and it works best if an absolute solution can be propagated while it processes each contour-pair. The Photometric Ordering Operator is described in detail later in this chapter.

In contrast to propagated solutions, contour ordering is *endogenous* if the relative ordering of a contour-pair can be determined without influence from other contours. The ordering solution can be thought of as 'growing or originating from within' the pair. Most of the phase-modifying ordering methods reviewed in Chapter 2 are of the endogenous variety. Experiments reported in the next chapter indicate that under certain conditions the Photometric Ordering Operator can be endogenous too.

The simple example in Figure 3.3 illustrates the difference between endogenous and propagated ordering. Two steps are required to order this map. At each step, there are three ordering cases to consider, with a possible surface shape associated with each case. An endogenous ordering operator could process the steps independently and in any sequence. The outcome of one step would not influence the analysis of another. On the other hand, an ordering operator requiring a propagated solution must proceed according to a prescribed sequence (e.g., Step #1, then Step #2) using the fact that one shape (say, No. 3) was selected in Step #1 to guide the choice of case in the next step.

Figure 3.3: Ordering the contours in a simple map. On the left is a map with three concentric contours. In the first ordering step, contours C_A and C_B are considered, and three shapes corresponding to the three ways those contours can be ordered have been drawn: a convex cone, a disk, and a concave cone. The second step poses the same choice of shapes, only now for contours C_B and C_C .

3.2.5 Propagated Ordering by Neighbourhood Sequences

Processing by propagated ordering requires that some contours with known fringe numbers (referred to as *knowns*) be used to determine the fringe number of contours with unknown fringe numbers (referred to as *unknowns*). The required contours can be arranged into a list of S contours called the *neighbourhood sequence*

$$\langle \Sigma \rangle_q = \langle C_1, C_2, \dots, C_S \rangle_q, \quad (3.15)$$

where the numbering $1, 2, \dots, S$ refers to the member-position locally within the sequence, and should not be confused with the member-position within the map-lists (as in Eq. 3.9). The lists of attributes related to the neighbourhood sequence will be enclosed in $\langle \dots \rangle$'s to help distinguish them from other lists.

The neighbourhood sequence is always specific to a particular processing step, and this will be emphasized when necessary with explicit subscripts for the q th step:

$$\text{the neighbourhood sequence: } \langle \Sigma \rangle_q = \langle C_1, C_2, \dots, C_S \rangle_q, \quad (3.16)$$

$$\text{the fringe numbers: } \langle \Phi \rangle_q = \langle \phi_1, \phi_2, \dots, \phi_S \rangle_q. \quad (3.17)$$

Figure 3.4: A sequence of gradient-neighbour contours. The contours in this simple map were assigned identification labels $A, B, C, \dots$ in the order in which they were discovered in a top-down horizontal raster search. G_1 is a surface gradient curve that cuts through some of the contours to form a sequence of neighbouring contours $\{C_D, C_B, C_E, C_F, C_G\}$.

An arbitrary sequence of contours is of little use, and the discussion will be restricted to sequences of contours that are *gradient neighbours*. A sequence of gradient-neighbour contours is formed in the order in which a specified surface gradient curve¹ intersects the contours. An example is shown in Figure 3.4. The appropriate gradient curve can be selected by a 'seed-point', usually on one of the contours in the desired neighbourhood sequence, and as the gradient curve is constructed in both directions (uphill/downhill) from the initiating contour point, other members of the neighbourhood sequence are 'discovered' as they are intersected. This fundamental process is referred to as *gradient neighbour discovery* and is pursued in detail in Chapter 5.

The neighbourhood sequence for the q th ordering step may be selected so that N knowns and U unknowns (i.e., contours with known and unknown ϕ , respectively)

— ¹The surface gradient curves are a family of loci orthogonal to the contour curves. Each gradient curve traces the local path of steepest descent on the surface.

can be partitioned

$$\langle \Phi \rangle_q = \langle \text{knowns} | \text{unknowns} \rangle_q, \quad (3.18)$$

$$= \langle \phi_{n_1}, \dots, \phi_{n_N} | \phi_{u_1}, \dots, \phi_{u_U} \rangle_q. \quad (3.19)$$

The *Ordering Operator* Ω is a process that converts the unknowns to knowns in a given sequence:

$$\Omega : \langle \text{knowns} | \text{unknowns} \rangle \Rightarrow \langle \text{knowns} | \rangle. \quad (3.20)$$

Thus the ordering process effectively 'moves the bar to the right end of the partitioned sequence' to indicate that all members of the sequence have known fringe numbers as an outcome of the process.

So far, the nature of the ordering process has not been specified. It could be based on *a priori* knowledge, or surface-smoothness constraints or, as in this thesis, photometric modelling.

The length of the sequence will depend upon the requirements of the ordering process. The shortest sequence would span a single neighbour-pair (e.g., $\langle C_a | C_b \rangle$), or for an endogenous ordering process $\langle | C_a, C_b \rangle$, whereas it will be shown that larger neighbourhoods, such as indicated in Equation 3.19, are required by the Photometric Ordering Operator.

3.2.6 The Ordering Schedule

Propagated ordering by neighbourhood sequences implies that the unknowns of a previous stage in the analysis are used as knowns in a subsequent step. For example, consider an ordering process which requires that two members of a sequence of three contours be knowns: $\langle C_1, C_2 | C_3 \rangle$, and furthermore that a simple sequence describes an example map shown in Figure 3.5: $\langle C_1, C_2 | C_3, C_4, C_5, \dots \rangle$. Then the following process steps could be used:

$$\begin{aligned} \text{Step 1: } \Omega : \langle C_1, C_2 | C_3 \rangle &\Rightarrow \langle C_1, C_2, C_3 | \rangle, \\ \text{Step 2: } \Omega : \langle C_2, C_3 | C_4 \rangle &\Rightarrow \langle C_2, C_3, C_4 | \rangle, \\ \text{Step 3: } \Omega : \langle C_3, C_4 | C_5 \rangle &\Rightarrow \langle C_3, C_4, C_5 | \rangle, \end{aligned} \quad (3.21)$$

Figure 3.5: A simple map to illustrate the Ordering Schedule concept. The map consists of a series of concentric contour loops $\mathcal{M} = \{C_1, C_2, C_3, C_4, C_5, \dots\}$. Only the first five loops are shown in the figure.

and so on. By writing a 'state diagram' for the map-list at the completion of each step (using "!" and "?" to denote known and unknown ϕ 's, respectively), one obtains an impression of the processing flowing (or propagating) through the map:

step	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	...
0	!	!	?	?	?	...
1	!	!	!	?	?	...
2	!	!	!	!	?	...
3	!	!	!	!	!	...
...	...	...	...	...	...	...

A series of process steps like this will be referred to as the *ordering schedule* to emphasize that a given step may not be executed before the previous steps in the schedule have been completed. In general, there are Q steps in the schedule V , and the parameters of each step are listed in each control vector V_q :

$$V = (V_1, V_2, \dots, V_q, \dots, V_Q). \quad (3.22)$$

The q th control vector V_q dictates which contours are to be analyzed at that step and how they are to be utilized.

Generating the schedule is rather straight-forward, but can be tedious. For an optimized schedule, an exhaustive search of the map may be conducted to form a list of all gradient-neighbour pairs, then this list can be organized into a schedule

Figure 3.6: A map with a branched ordering schedule. Since contour C_C has three neighbours, a graph of the map, and hence the ordering schedule, must be branched

of process steps with neighbourhood sequences that relate all unknowns in terms of knowns. Maps with several topographic singularities (such as hills, craters, saddle-points, etc.) are complicated by *branches* in the neighbour sequences. For example, the two-hilled map

$$\mathcal{M} = \{C_A, C_B, C_C, C_D, C_E, C_F, C_G, C_H\} \quad (3.23)$$

shown in Figure 3.6, has a schedule indicated by the following branched graph:²

$$\begin{array}{c}
 C_A \longrightarrow C_B \longrightarrow C_C \begin{array}{l} \nearrow C_E \longrightarrow C_F \\ \searrow C_G \longrightarrow C_H \end{array}
 \end{array} \quad (3.24)$$

Automating the generation of the schedule is desirable. Forming an efficient schedule efficiently is important; doing this optimally for the general case was considered beyond the requirements of this project. The author's interests lay with

²The graph of a map is drawn with contour labels at the nodes, and connecting lines indicating which nodes are neighbouring contours in the map. Not all neighbour relationships need to be indicated, and directed lines suggest the order of the steps in the processing schedule. An introduction to graph theory is given in [Pavlidis, 1982].

rather simple maps, and two approaches to scheduling were used.

One approach was crudely automated. The map-list (with knowns and unknowns tagged) was automatically scanned for unknowns with sufficient known gradient neighbours to permit analysis. When discovered, those unknowns were then photometrically analyzed and assigned fringe numbers, and then scanning of the list was resumed for any outstanding unknowns.

The other scheduling approach was sophisticated, but manual. The schedule was tightly controlled by a carefully contrived list of steps, which allowed considerable flexibility in studying the effects of the order in which the unknowns were resolved. These scheduling techniques are described in more detail in Chapters 8 and 9, where the computer programs for automated contour ordering are discussed.

3.2.7 Shape Cases for Neighbourhood Sequences

There is a definite neighbourhood sequence for each schedule step. Furthermore, there are a fixed number of ways N that a specified sequence of contours may be ordered. These ways are called *ordering cases*, and the Ordering Operator Ω must be able to pick the correct case from the possibilities. Since an ordered sequence of contours corresponds to a surface shape, each of the ordering cases is also a *shape case*.

The terms 'ordering case' and 'shape case' are used interchangeably, but with caution, since there is not a unique correspondence between them. There is an infinite number of objects with arbitrary shapes that degenerate to the same ordered sequence of contours, but most can be dismissed since they are physically unlikely. A residual degeneracy persists, however, even if only the smoothest shapes are admitted. Figure 3.3 shows an example of three shape cases for matched-mode pairs of contours. The true situation is more complex than depicted in that figure, since shape No. 2 (a flat disk) is impossible (surfaces must be inclined through a moiré irringe), and the ordering case of equal-height contours actually degenerates into two shape cases (a convex and a concave torus). This is a subtle, but important issue for ordering operators that pick shapes. The Photometric Ordering Operator

is in the shape-picking category, and the topic of shape degeneracy is pursued in Chapter 6.

A final remark is that longer neighbourhood sequences have more combinations of shapes to consider. The number N depends on the number of unknowns, the mode of the map, and the mode of each contour-pair in the sequence. For example, a sequence of three mixed-mode contours can be ordered in up to four ways, whereas three matched-mode contours can be ordered in nine ways. Formulas for counting the ways are developed in Chapter 6, along with methods for determining the shape corresponding to each case.

3.3 Photometric Analysis of Moiré Fringes

The purpose of this section is to describe a process, the Photometric Ordering Operator, that can pick the correct shape case by analyzing the local shading properties of the neighbourhood sequence. Following a brief review of the pioneering work in the 'shape-from-shading' field, the photometric models that link shape and shading are described. To use the models, the analysis region over which they are valid must be defined. The means for determining shape and shading estimates from the moiré fringe image are discussed, and the section concludes with a description of the Photometric Ordering Operator.

3.3.1 Photometric Analysis, Shading and Shadows

Photometric Analysis refers to the study of the shading distribution in a scene or image to infer various properties of the objects in the scene. In this document, the term will be restricted to relating the shape of the objects to their shading. In this context, the terms *shading analysis* and *photometric analysis* will be used interchangeably.

There is an important distinction between *shading* and *shadows*. The former term refers to brightness variations on the surface of the object (or in its image) due to surface reflectance properties, and its orientation with respect to the light source

and the viewer. The latter term refers to the effect of illumination obstructions, such as the pattern cast by the grid strings onto the object.

3.3.2 Shape from Shading

Man instinctively infers shape from shading cues. He constantly makes ongoing judgements about the shape of objects in his world by subconsciously analyzing the shaded scene he sees. His visual skills, in this respect, provide a highly-refined interface to the physical world [Pentland, 1983].

He can be misled by shading illusions. Artists can convey a convincing sense of three-dimensional depth on a flat canvas by using carefully varied shading in their pigments. A more subtle shape-from-shading trickery (by artists of a different school) involves the application of facial cosmetics to exaggerate cheek-bones or suppress large noses!

Considerable work has been done in the fields of machine vision and artificial intelligence to exploit the shading distribution in an image to deduce the shape of objects. Berthold K. P. Horn pioneered the concept of object-shape recovery from a densitometric analysis of an image [Horn, 1970, 1975 and 1977]. Horn showed that an array of local surface normals (a means of representing the shape of an object) could be computed from variations in the shading in a single image. Horn's analysis was based upon constraints regarding the surface continuity and reflectance, the image geometry and illumination, and the application of boundary conditions.

This technique was most successfully demonstrated with an iterative method that used occluding boundary information to propagate the solution along curves through the image [Ikeuchi and Horn, 1981]. There were severe limitations and difficulties that were never completely resolved for the general case: the initial set of object shapes was infinite, the aforementioned computations were extensive, and the result often reduced to several ambiguous shapes.

Several authors pursued methods that broke the shape ambiguity by imposing additional constraints. One tack was to use multiple scenes. Woodham [1981] introduced the concept of *photometric stereo*, whereby separate images were made

from a fixed object/camera placement using a different direction of illumination each time. A four-source arrangement was reported with a novel scheme devised to handle general surface texture and specularity [Coleman and Jain, 1981], and a three-source system was demonstrated to determine the shape of a variety of industrial work-pieces [Ray, 1982].

Other workers attempted more generalized approaches. Pentland [1982 and 1984] reported a method that avoided Horn's *a priori* scene assumptions (fixed light source position and constant surface albedo), and instead assumed local surface sphericity to determine the illuminant direction and the surface orientation at each image point. Smith [1983a] made no assumptions about the scene illumination and surface albedo, nor about the surface shape. His analysis required higher-order differential equations, and assumed perspective projection and some simple reflectance properties. However, the computational difficulties prevented a convergent solution from being realized.

In a critical review of the state of the art, Smith [1983b] cited difficulties and weaknesses inherent in the various 'shape-from-...' approaches, and remarked that, except for contrived situations involving trivial scenes, shape cannot be recovered from shading alone. He concluded, however, that shading could specify relative surface curvature and curvature changes.

In the case of the moiré fringe ordering task, selecting the relative curvature is all that is required of the shading analysis. That is, the shading information need only arbitrate in a rather limited choice of surface shapes. The surface shapes to be considered are determined, of course, from the moiré fringes, and not the shading.

The photometric analysis for moiré fringe ordering is based upon a reflectance model and image measurements that are similar to those used by Horn *et al.* In contrast, however, the contour-ordering task is much simpler than that of Horn's. There are only a few well-defined shapes to consider for each analysis neighbourhood. The calculations are few and not difficult. Furthermore, the shapes under consideration generally have such different expected shading values, that comparison with the measured value at an analysis point makes the choice clear.

Figure 3.7: The geometry for diffuse reflectance. The illumination direction is L , the surface normal is N , and the viewer's direction is V .

3.3.3 The Photometric Model

The photometric model is a mathematical expression that couples the observed shading at a point on an object with its local shape, subject to prescribed illumination and viewing conditions.

The simplest photometric situation is that of a single point-source of light [Pentland, 1984]. This is shown in Figure 3.7, where the geometry of the reflected light is governed by three vectors: the illumination direction L , the surface normal N , and the viewer's direction V . If one further assumes a Lambertian (i.e., purely diffuse) surface, then the reflectance function at any given point on the surface can be written as

$$\rho = a I_0 \frac{(N \cdot L)}{r^2}, \quad (3.25)$$

$$= \rho_0 \frac{(N \cdot L)}{r^2}, \quad (3.26)$$

where ρ is the reflectance value, a is the albedo of the surface, I_0 is the nominal intensity of the light source, r is the distance to the light source, and $(N \cdot L)$ is the scalar product. For simplicity, the nominal reflectance ρ_0 is used as a single parameter to replace $a I_0$ in the formula. This is a model for a purely-diffuse surface under finite-distance single-point illumination.

The simple Lambertian model has a convenient form, in that the reflectance

properties of the object (contained in ρ_0) can be separated from the shape factor γ . From Equation 3.26

$$\rho = \rho_0 \gamma, \quad (3.27)$$

where

$$\gamma = \frac{(\mathbf{N} \cdot \mathbf{L})}{r^2}, \quad (3.28)$$

and thus allows the shape and shading components of the model to be evaluated separately. Since the shape geometry of the Lambertian model is based upon a simple vector scalar-product, the parameter γ is called the *geometric scalar*.

Objects from the real world are never purely diffuse reflectors, but exhibit a specular component as well. Metals, glasses, ceramics and plastics all have 'highlights' that the Lambertian model fails to account for. From the computer graphics field, one finds several successful shading models used for simulating realistic images of objects. Phong [1975] devised a simple shading model that combines the diffuse and specular surface properties as separable components:

$$\rho = (1 - f)\rho_d + f\rho_s, \quad (3.29)$$

$$\rho_d = \rho_0 \frac{(\mathbf{N} \cdot \mathbf{L})}{r^2}, \quad (3.30)$$

$$\rho_s = \rho_0 (\mathbf{N} \cdot \mathbf{H})^\kappa, \quad (3.31)$$

where f is the specular fraction, κ is the extinction coefficient, and $\mathbf{H}$ is the bisector of $\mathbf{L}$ and $\mathbf{V}$, shown in Figure 3.8. Note that for this model, the reflectance properties of the object (ρ_0, f, κ) cannot be separated from the shape parameters, as is possible with the Lambertian model.

Cook and Torrance [1981] developed an expression similar to Phong's, but with a more elaborate specular component that accounted for its angular distribution in terms of the microfacets on the surface:

$$\rho_s = \frac{F}{\pi} \frac{D}{(\mathbf{N} \cdot \mathbf{L})} \frac{G}{(\mathbf{N} \cdot \mathbf{V})} \quad (3.32)$$

The Fresnel term F describes how light is reflected from each smooth microfacet, and is a function of the incident angle, the index of refraction of the material, and wavelength. Its evaluation involves solving the Fresnel equations for unpolarized

Figure 3.8: The photometric directions in the shadow-moiré arrangement. The object is illuminated and viewed through the grid. At each point on the object, four directions can be identified: N is the surface normal, L is the direction to the lamp, V is the direction to the viewer (camera), and H is the bisector of L and V .

light for each required angle of incidence. The geometrical attenuation factor G accounts for the shadowing and masking of one facet by another. The facet-slope distribution function D represents the fraction of facets that are oriented in the direction H . The interested reader is directed to the original article for the description of these functions, and for examples of some remarkably life-like synthetic images of objects made from visibly distinguishable materials (e.g., carbon, red rubber, lunar dust, copper, tin, stainless steel, etc.) In this respect, Cook and Torrence have produced a very complete photometric model that is strongly coupled to the physics of materials.

Ever more complex models could be cited to account for spatial and spectral light source distributions and other scene artifacts. However, the author found from trial and error that the more primitive models were adequate for photometric ordering. In Chapter 4, experiments with both the Lambertian model and Phong's diffuse/specular model are reported. It is shown that the simple Lambertian model (involving one vector scalar-product!) is effective if its application is restricted to sufficiently small analysis regions. For the remainder of this chapter, the discussion

will be restricted to the Lambertian model.

Before discussing how the photometric model is used to order contours, three concepts that influence that discussion will be examined: the valid range of the model, the extraction of shape estimates for the model, and the extraction of shading estimates for the model.

3.3.4 The Analysis Region for the Model

The nominal reflectance ρ_0 in the Lambertian photometric model is a constant value provided both the illumination and the surface reflectivity (albedo) of the object are constant. These conditions hold if the model is restricted to a sufficiently small region.

For photometric ordering of contours, the *photometric analysis region* $\mathcal{R}$ is defined to be an area large enough to span the neighbourhood sequence contours, and yet small enough to ensure that both the reflectance properties of the surface and the illumination remain sufficiently constant.

Each photometric analysis region $\mathcal{R}_i$ is formed about a *seed-point* $\mathcal{P}_i$ that lies on one of contours $\mathcal{C}_i$ in the neighbourhood sequence. As described earlier, the gradient curve $\mathcal{G}_i$ is constructed through $\mathcal{P}_i$ to discover the other analysis points in the region where it intersects the other contours in the neighbourhood sequence. The region $\mathcal{R}_i$ spans these points, and extends only far enough to permit the local shape and shading to be estimated.

Figure 3.9 shows a gradient neighbourhood with three contours. The middle contour $\mathcal{C}_0$ has supplied five seed-points. The local gradient curve has been constructed through each seed-point, and the points on the other two contours in the neighbourhood are indicated for each of the five analysis regions. A boundary has been drawn around each region to illustrate its limited domain.

Photometric Ordering could be performed using only a single region. Several, and even many regions about a neighbourhood sequence provide a more reliable basis for analysis. In general, there are R analysis regions in the neighbourhood. When there is more than one analysis region, the final ordering decision (i.e., choos-

Figure 3.9: The photometric analysis regions about a neighbourhood sequence. In this example, the neighbourhood sequence is $\langle C_B, C_C, C_D \rangle$, and five seed-points $P_1, \dots, P_5$ are taken along contour C_C . The gradient curves $G_1, \dots, G_5$ are constructed through the respective seed-points. The regions $R_1, \dots, R_5$ extend to enclose the area where each gradient curve intersects the other contours in the sequence, as shown.

ing the 'correct' ordering case) is based upon the average 'vote' by each region for each ordering case. This is important if the assumption of constant nominal reflectance breaks down at arbitrary places about the scene. An example of such a breakdown is provided if a strong highlight on the object occurs within a region, and a Lambertian model is being used to characterize the surface reflectance. Clearly, the diffuse model would give poor results under these specular conditions, whereas elsewhere on the object (away from the highlight), the model may work adequately and give the correct results.

3.3.5 Shape Determination for the Model

To use the model, the Photometric Ordering Operator must estimate the local shape about each analysis point within the region. More specifically, the geometric scalar γ must be evaluated for each ordering case. From Equation 3.28, this requires the surface normal N and the distance and direction to the light source (r and L , respectively) to be determined at each analysis point. The process is briefly described here, but the details and formulas are deferred to Section 3.4.9, starting on page 116.

Each ordering case temporarily fixes the phase number ϕ of all the contours in the sequence to a known value (to test that possible set of phase values). With the ϕ 's known for each contour, the elevation for each analysis point can be found (via Eq. 3.6) and the analysis point transformed from image-plane to object-space coordinates.

Once each analysis point in the sequence is projected to its object position, the distance r and direction L to the light source (with known position) is found with trivial vector algebra.

The surface normal N at each analysis point is determined from the shape of the contour and gradient curves through that point. The azimuth angle α is the planimetric normal of the contour measured from the $\hat{x}$ -axis, and can be evaluated by fitting a parametric curve through local points on the contour that are projected into object-space coordinates. Similarly, the profile of the surface gradient

curve can be curve-fitted to the sequence of gradient neighbour points projected into object-space coordinates, and the tilt angle τ of its normal from the vertical can be analytically extracted. The spherical coordinates $(1, \tau, \alpha)$ are then easily transformed into a Cartesian coordinate representation of N .

Note that each ordering case will alter the elevation assigned to the contours, and that this will have a drastic effect on the profile of the gradient curve (affecting τ), a slight effect on the position with respect to the light source (affecting r and V), but does not change the shape of the contour curves (keeping α constant). So, τ , r and L are said to be 'case-dependent' parameters, whereas α need only be calculated once for a point. These considerations will influence the order of operations in the Photometric Ordering Algorithm described in Section 3.3.7.

3.3.6 Shading Determination for the Model

The Photometric Ordering Operator must also measure the local shading $\bar{p}$ in the image about each analysis point within the region. The presence of the coarse pattern of the moiré fringes, and the fine image-structure due to the grid strings and their shadows, pose a fundamental complication to the measurement of the shading envelope. The basic considerations are briefly described here, with the details of this measurement task given in Chapter 7.

The shading measurement patch must be small enough to retain local shading characteristics, yet large enough to suppress noise and small artifacts, such as dust on the photographic image. This patch should not be confused with the photometric analysis region: the former is the immediate area about the analysis point, whereas the latter spans several analysis points and hence contains several such shading measurement patches. In a conventional (non-moiré) image, a constant patch size might be used. Indeed, the image might be digitized into a regular array of small picture elements (called *pixels*), which could be used individually to estimate the local shading about a point. If digitization noise is a problem, the shading measurement may average several local pixels.

Moiré topogram images formed with a stationary grid have coherent carrier-

noise³ that affects the shading measurement process. In the author's case, the analog moiré topogram images are digitized to a pixel size near that of the moiré grid pitch. The resulting pixel values represent the local *scene* (as opposed to *surface*) brightness in a particular direction to a point on the object. That view may be partially or wholly occluded by a moiré grid string, or it may be in the shadow of a grid string. Thus, the perceived surface shading is modulated by moiré shadows that range in spatial frequency from zero (broad fringes on level surfaces) to that of the grid strings.

Clearly, the shading measurement patch must average several pixels to smooth out the fine structure (i.e., the strings and their individual shadows). As the patch is increased to any significant size, however, the grosser moiré interference pattern imposes a gradual variation upon the values across the patch. Furthermore, the intensity distortion is not isotropic, but varies with the local orientation and separation of the fringes at each point. Thus, the measurement patch size must be dynamically adjusted to the fringe pattern. Discussion of which shape of patch does this perfectly is deferred to Chapter 7; it suffices here to state that a circular patch was ultimately adopted.

3.3.7 The Photometric Ordering Operator

The ordering schedule discussed earlier (p. 78) ensures that the neighbourhood spanned by each analysis region is partitioned into knowns and unknowns, with appropriate analysis points in *both* partitions. The sequence must be long enough, so that a sufficient number of points lie on each side of the partition to satisfy the previously-discussed measurement processes (shape and shading):

The Photometric Ordering Operator starts in the known partition of the sequence to determine the model parameters that characterize shading in the analysis region. Then, the Photometric Ordering Operator shifts to the unknown partition, uses the model to produce an expected shading value $\bar{p}$ for each of the trial ordering cases, and tests their consistency with the actual measured shading value $\bar{p}$.

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³See Appendix A for details on carrier-noise in moiré topograms.

The premise is that, on average, only one of the test cases will produce an expected value that is similar to the measured value. This is a reasonable expectation, since the various ordering cases usually create wildly-different surface normals,⁴ and only the correct one will (on average) reflect the appropriate amount of light to the camera. The term 'on average' implies this testing is performed for several regions about the neighbourhood, and that an averaging process is used to choose the most consistent case. At each region about a neighbourhood (Fig. 3.9), the object's orientation to the illumination can change dramatically, thereby altering the measured value of the shading even if the albedo of the surface is perfectly constant. The photometric model produces an expected shading value for the correct ordering case that should be *consistent* with the changes in orientation. For the other cases, the match may be good in some places, but on average it should be frustrated.

The Photometric Ordering Operator process for region $\mathcal{R}$, is formally described as follows.

- In the known partition of the region, use the fixed shading and shape properties to estimate the photometric model parameter(s),⁵ so that the model can be applied in the unknown partition of the region:

Step N.1: Measure the shading value $\bar{\rho}_n$

Step N.2: Measure the known shape γ_n .

Step N.3: Estimate the nominal reflectance ρ_0 for the region, using the Lambertian photometric model given in Equation 3.27:

$$\rho_0 = \frac{\bar{\rho}_n}{\gamma_n} \quad (3.33)$$

- In the unknown partition of the region, use the model to estimate the expected shading value for each of the test shape cases, and compute its consistency to the measured shading value at that point:

⁴If the reader has any doubt about the surface normal 'wildly changing direction' for each ordering case, imagine standing on a contour and having the terrain tip so that the next contour is above, then level with, then below you!

⁵The single-parameter Lambertian photometric model (Eq. 3.27) is used here as an example; other models may have several parameters that must be determined.

Step U.1: Measure the shading value $\bar{\rho}_u$.

Step U.2: Measure the test shape for each of N cases:

$$[\gamma_u] = \begin{bmatrix} \gamma_u^{(1)} \\ \vdots \\ \gamma_u^{(k)} \\ \vdots \\ \gamma_u^{(N)} \end{bmatrix} \quad (3.34)$$

Step U.3: Estimate the expected shading for each of N cases. For the k th ordering case, the Lambertian photometric model in Equation 3.27 becomes

$$\tilde{\rho}_u^{(k)} = \rho_0 \gamma_u^{(k)}, \quad (3.35)$$

which gives

$$[\tilde{\rho}_u] = \begin{bmatrix} \tilde{\rho}_u^{(1)} \\ \vdots \\ \tilde{\rho}_u^{(k)} \\ \vdots \\ \tilde{\rho}_u^{(N)} \end{bmatrix} \quad (3.36)$$

Step U.4: Compute the consistency between the measured and expected shading for each of the N ordering cases. A squared-error is used in this example as a measure of consistency:

$$\epsilon^{(k)} = (\gamma_u^{(k)} - \bar{\rho}_u)^2, \quad (3.37)$$

which results in N values for the j th region $\mathcal{R}_j$ (note the subscript ($\mathcal{R}_j$) has been introduced):

$$[\epsilon]_{\mathcal{R}_j} = \begin{bmatrix} \epsilon^{(1)} \\ \vdots \\ \epsilon^{(k)} \\ \vdots \\ \epsilon^{(N)} \end{bmatrix}_{\mathcal{R}_j} \quad (3.38)$$

The errors for each case can be summed over all of the R regions to obtain a total error for the neighbourhood $[\epsilon]_q$ (i.e., for the q th ordering step), where

$$\epsilon_q^{(k)} = \sum_{j=1}^R \epsilon_j^{(k)}. \quad (3.39)$$

Finally, the smallest value can be used to identify the most consistent ordering case, and the test values for the unknown ϕ 's of that case can be used to update the map-list of known phase numbers.

This concludes the description of the inner-workings of the Photometric Ordering Operator process at the regional level. In the next section, a formal algorithm for the overall process of ordering a map is given.

3.4 An Algorithm for Photometric Ordering

3.4.1 Introduction

The purpose of this section is to describe, in detail, the algorithm developed by the author to photometrically order a set of moiré fringe contours. The algorithm is presented in a structured, top-down formalism, with increasing detail given in deeper layers of the nested block-diagrams.

There may be better ways to nest and group the functions described here, but this was the design used by the author to develop the test software. Indeed, there is a one-to-one correspondence to the block-diagrams in this section and the program subroutine modules that comprise Task MFO (a program described in Ch. 9 that automatically orders moiré fringe contours using photometric analysis).

As a guide to the reader, Figure 3.10 lists the functions that characterize the algorithm. Each function is denoted by a three-letter acronym framed in a box (e.g., PMO), and the full name of the function is given beside its box in the figure. An attempt has been made in the figure to depict the hierarchy of the functions with horizontal indentation, and eight layers of nesting can be seen.

Figure 3.10: Nested functions for the Photometric Ordering Algorithm. The hierarchy of the functions is represented by the horizontal indentation. Note that for some functions, the vertical ordering of subfunctions has been shuffled for illustrative purposes.

Note that within some of the functions in Figure 3.10, the vertical listing of the subfunctions was shuffled to make the list easier to draw. The reader should refer to the specific block-diagram in the following figures to obtain the order in which subfunctions are executed.

The following conventions were used in the block-diagrams that describe the algorithm. Input data enters from the left, process control parameters enter from above, and output data exits to the right. Within the block, subprocessing proceeds in stages from left to right, such that all subprocesses to the left must be completed before the next subprocess can commence. Vertically-aligned subprocesses may be executed in any order (an opportunity for parallel processing). To reduce the mesh of paths, the external data is shown as 'sourceless' input to subprocesses which require it; explicit paths are drawn for all internal data. Accompanying each subprocess diagram is an icon of 'layered pages' to remind the reader of the relationship to the parent processes.

3.4.2 **PMO**: Photometric Map Ordering

Figure 3.11 shows the block-diagram for **PMO** (Photometric Map Ordering). It is the top-most process in the algorithm to photometrically order a set of moiré fringe contours. There are two subprocesses shown in the block-diagram:

POS	(Photometric Ordering Scheduler).
PNO	(Photometric Neighbourhood Ordering).

Before describing the functionality of **PMO**, the initial data structures will be defined.

The map $\mathcal{M}$ and the image $\mathcal{I}$ are the primary ingredients for the fringe ordering process. All other data structures are derived from them. They appear as input to many of the subprocesses in the figures that follow, and are always shown at the bottom of lists of input so that the 'novel' (internal) data can head the list in each figure.

Figure 3.11: Block-diagram for **PMO** Photometric Map Ordering

The map $\mathcal{M}$ is a list of M contours:

$$\mathcal{M} = \{C_1, C_2, \dots, C_M\}, \quad (3.40)$$

where each contour C_i is a list of contour-attributes plus a list of the C contour-points that define the contour curve in the map image:

$$C_i = (\lambda, e, \dots; P_1, P_2, \dots, P_C), \quad (3.41)$$

where λ is the contour's *map-label* (the code used to identify the contour in the map image), e is the contour's fringe extremum-type (bright=1, dark=0), and the ellipsis (...) imply there may be other topographic attributes, such as a list of the contour's neighbours. Associated with the j th point on each contour are its line and pixel coordinates in the image (ℓ, p) , and other possible topographic attributes specific to that point (such as the distance, direction and contour identity of the gradient-neighbour points on each side of the point):

$$P_j = (\ell, p, \dots), \quad (3.42)$$

Note that only the specifically-listed attributes are required as input to PMO. The other topographic attributes are generated by PMO as required, unless they are supplied by a prior process such as the contour extraction software. In that case, PMO could skip some of its internal processing.

The image data structure $\mathcal{I}$ is a two-dimensional array with L lines of P pixels, and is defined as

$$\mathcal{I} = [i_{\ell,p}] = \begin{bmatrix} i_{1,1} & i_{1,2} & \dots & i_{1,P} \\ i_{2,1} & i_{2,2} & \dots & i_{2,P} \\ \vdots & \vdots & \ddots & \vdots \\ i_{L,1} & i_{L,2} & \dots & i_{L,P} \end{bmatrix}, \quad (3.43)$$

where the image element i is made up of concatenated data fields:

$$i_{\ell,p} = (R, L)_{\ell,p}. \quad (3.44)$$

The reflectance field R is set to the measured reflectance value ρ in the image at that point:

$$R_{\ell,p} = \rho. \quad (3.45)$$

The label field L holds the map-labels that denote whether or not an element lies on a contour:

$$L_{\ell,p} = \begin{cases} \lambda_i & \text{if } (\ell,p) \in C_i, \\ 0 & \text{otherwise.} \end{cases} \quad (3.46)$$

It is convenient to organize the various contour attributes into lists that correspond to $\mathcal{M}$. Such lists will be enclosed by $\{\dots\}$'s, for example:

$$\text{the map list: } \mathcal{M} = \{C_1, C_2, \dots, C_M\}, \quad (3.47)$$

$$\text{the label list: } \mathcal{L} = \{\lambda_1, \lambda_2, \dots, \lambda_M\}, \quad (3.48)$$

$$\text{the extrema-type list: } \mathcal{E} = \{e_1, e_2, \dots, e_M\}. \quad (3.49)$$

One such list is particularly prominent in Figure 3.11, namely the list of moiré fringe numbers

$$\Phi = \{\phi_1, \phi_2, \dots, \phi_M\}. \quad (3.50)$$

In general, all the ϕ 's in the list are initially unknown. This initial state is denoted Φ_0 , and is shown as an input to $\boxed{\text{PMO}}$. At each stage in the processing, the output state of the previous step Φ_{q-1} is used as input to the q th execution of $\boxed{\text{PNO}}$ which produces Φ_q as its output state. At each step, one or more of the unknown ϕ 's are determined. There are Q steps ($Q \leq M$) required to determine all the unknowns, and Φ_Q is the final output of $\boxed{\text{PMO}}$, with all ϕ 's known and the map solved.

The overall process is controlled by the ordering schedule V , which is comprised of a list of Q control vectors

$$V = (V_1, V_2, \dots, V_q, \dots, V_Q). \quad (3.51)$$

As depicted in the block-diagram, $\boxed{\text{POS}}$ interprets V and sends the appropriate V_q to $\boxed{\text{PNO}}$, thereby spawning the execution of the q th ordering step.

For each step, the control vector dictates which contours are to be analyzed (i.e., the neighbourhood sequence), which are to be knowns/unknowns, which contours are to be updated (not necessarily all the unknowns), and which is to be the 'seed contour' for determining the analysis regions.

In addition, the control vector may pass parameters that adjust the behaviour of certain subprocesses, depending on the nature of the map. For example, the mode of the map (mixed-fringes or matched-fringes) will determine the ordering rules and how the image reflectance measurement patch is defined.

To simplify the notation in the block-diagrams, the control vector is not decomposed into specific components. It is always denoted in the figures as V_q for each process, with its role explained in the text. In this regard, the description of the algorithm has been kept as general as possible, and V_q shown as input to a subprocess is meant to indicate that a processing option could be selected at that point. Test software with most of the general control options was developed by the author (Task MFO), and is reported in Chapter 9.

Of course, many of the control options could be 'hard-wired' into the subprocesses (i.e., the control feature made implicit to the embodiment of the process). One scenario might require that the algorithm always assume a map of match-mode contours, that a neighbourhood sequence should always be three contours (with the first two known and the third an unknown), that the middle contour in the sequence is always the seed for generating the analysis regions, etc. The control vector for this 'optionless' algorithm would be much simpler than that for the general case. In the example just cited, the control vector need only supply the identity of the three contours at each step. Indeed, a program (Task JFO) was written to test this design, and it is described in Chapter 8.

It is possible that the schedule could be automatically generated within PMO, eliminating the explicit input V in the block-diagram. In this case, Figure 3.11 would show Φ_0 , $\mathcal{M}$ and $\mathcal{I}$ as input to POS, which would require access to this data to determine the schedule control vectors for each step. For example, the optionless Task JFO automatically generates its ordering schedule by searching about the seed-points on contours in the image to discover the members for a short neighbourhood sequence $\langle C_{n_1}, C_{n_2} | C_{u_1} \rangle$: when three such contours are found, POS submits their identities to PNO to determine the ϕ_{u_1} , and then POS repeats the process loop until no unknowns remain. The details of the schedule generation for Task JFO are

given in Chapter 8.

3.4.3 **PNO**: Photometric Neighbourhood Ordering

Figure 3.12 shows the block-diagram for **PNO** (Photometric Neighbourhood Ordering). At the q th ordering step, **PNO** is controlled by V_q to use $(\mathcal{M}, \mathcal{I})$ for changing the state of Φ_{q-1} to Φ_q . There are three subprocesses shown in the block-diagram:

OCG	(Ordering Case Generator).
CSU	(Case Select and Update).
PNA	(Photometric Neighbourhood Analysis).

OCG is given the specification of the q th neighbourhood sequence $\langle \Sigma \rangle_q$ by V_q , and uses the contour-ordering rules to generate a set of N ordering cases to be tested. Lists of attributes that correspond to the N ordering cases will be written vertically and enclosed by $[\cdot]$'s. For example, the output of **OCG** is the list of possible moiré fringe numbers for the neighbourhood sequence

$$[\langle \Phi \rangle]_q = \begin{bmatrix} \langle \Phi \rangle^{(1)} \\ \vdots \\ \langle \Phi \rangle^{(k)} \\ \vdots \\ \langle \Phi \rangle^{(N)} \end{bmatrix}_q \quad (3.52)$$

PNA uses the set of ordering cases $[\langle \Phi \rangle]_q$ and $(\mathcal{M}, \mathcal{I})$ to generate a set of photometric-consistency errors

$$[\epsilon]_q = \begin{bmatrix} \epsilon^{(1)} \\ \vdots \\ \epsilon^{(k)} \\ \vdots \\ \epsilon^{(N)} \end{bmatrix}_q \quad (3.53)$$

The error for the k th ordering case is a measure of how well the modelled shading for that ordering case matches the measured shading value. **CSU** uses the set of

Figure 3.12: Block-diagram for PNO Photometric Neighbourhood Ordering

errors $[\epsilon]_q$ to select the best ordering cases (i.e., the case that gave the photometric model estimates that were most consistent with the measured shading values). It then updates the previous state Φ_{q-1} to the new state Φ_q .

The internal details of **OCG**, **CSU** and **PNA** are described in separate sections below. **PNA** is explained last since it is the parent-process to the remainder of the algorithm.

3.4.4 **OCG**: Ordering Case Generator

The block-diagram for **OCG** is shown in Figure 3.13. For the q th ordering step, it uses the previous state $\langle \Phi \rangle_{q-1}$ and the map $\mathcal{M}$ to generate the set of ϕ -values to be tested for each of the N ordering cases. There are two subprocesses shown in the block-diagram:

NSE (Neighbourhood Sequence Encoder),
CRI (Case Rule Interpreter).

The control vector V_q specifies for **NSE** which contours in the map are to be used to form the q th neighbourhood sequence $\langle \Sigma \rangle_q$, and more specifically, which members are to be knowns and which are to be unknowns. **NSE** is called twice, first to encode the required known ϕ -values from Φ_{q-1} into $\langle \Phi \rangle_q$:

$$\begin{aligned} \Phi_{q-1} &= \{\phi_1, \phi_2, \dots, \phi_f, \dots, \phi_g, \dots, \phi_h, \dots, \phi_i, \dots, \phi_M\} \\ &\quad \Downarrow \quad \Downarrow \\ \langle \Phi \rangle_q &= \langle \phi_{n_1}, \dots, \phi_{n_N} \mid \phi_{u_1}, \dots, \phi_{u_U} \rangle_q \end{aligned} \quad (3.54)$$

and then to encode the fringe extremum-types from the map-list $\mathcal{E}$ (implied within $\mathcal{M}$ in the block-diagram) into the sequence $\langle \mathcal{E} \rangle_q$:

$$\begin{aligned} \mathcal{E} &= \{e_1, e_2, \dots, e_f, \dots, e_g, \dots, e_h, \dots, e_i, \dots, e_M\} \\ &\quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \\ \langle \mathcal{E} \rangle_q &= \langle e_{n_1}, \dots, e_{n_N} \mid e_{u_1}, \dots, e_{u_U} \rangle_q \end{aligned} \quad (3.55)$$

The two sequences $\langle \Phi \rangle_q$ and $\langle \mathcal{E} \rangle_q$, are used by **CRI** to form a set of ϕ 's for each of the N ordering cases. Embodied in **CRI** are the general contour ordering rules,

 Figure 3.13: Block-diagram for **OCG** Ordering Case Generator

and based on the mode of the map (mixed or matched, as instructed by V_q), the set $\langle\Phi\rangle_q$ is derived. For match-mode contours, the process would look like:

$$\begin{aligned}
 \text{case 1: } & [\phi_{u_1} = \phi_{n_N} + 1; \phi_{u_2} = \phi_{u_1} + 1; \dots; \phi_{u_U} = \phi_{u_{U-1}} + 1] \Rightarrow \langle\Phi\rangle^{(1)} \\
 \text{case 2: } & [\phi_{u_1} = \phi_{n_N} + 0; \phi_{u_2} = \phi_{u_1} + 1; \dots; \phi_{u_U} = \phi_{u_{U-1}} + 1] \Rightarrow \langle\Phi\rangle^{(2)} \\
 \text{case 3: } & [\phi_{u_1} = \phi_{n_N} + 0; \phi_{u_2} = \phi_{u_1} + 0; \dots; \phi_{u_U} = \phi_{u_{U-1}} + 1] \Rightarrow \langle\Phi\rangle^{(3)} \\
 & \vdots \\
 \text{case N: } & [\phi_{u_1} = \phi_{n_N} - 1; \phi_{u_2} = \phi_{u_1} - 1; \dots; \phi_{u_U} = \phi_{u_{U-1}} - 1] \Rightarrow \langle\Phi\rangle^{(N)}
 \end{aligned} \tag{3.56}$$

3.4.5 CSU: Case Select and Update

The block-diagram for CSU is shown in Figure 3.14. For the q th ordering step, it uses the set of photometric-consistency errors $[\epsilon]_q$ and the set of trial fringe numbers $\langle\Phi\rangle_q$ to update the ϕ -values of the previous state Φ_{q-1} to produce the new state Φ_q . There are two subprocesses shown in the block-diagram:

$$\begin{aligned}
 \text{MES} & \quad (\text{Minimum Error Selector}), \\
 \text{NSD} & \quad (\text{Neighbourhood Sequence Decoding}).
 \end{aligned}$$

MES scans the N error-values in $[\epsilon]_q$ looking for the minimum error, and thereby selects the m th ordering case. Thus, MES identifies the ordering case in $\langle\Phi\rangle_q$ that represents the shape that was most photometrically-consistent with the measured shading values.

NSD is essentially the reverse process to NSE, in this case mapping the determined ϕ -values from the selected m th ordering case in $\langle\Phi\rangle_q$ to update Φ_{q-1} to Φ_q :

$$\begin{aligned}
 \langle\Phi\rangle_q^{(m)} &= \langle\phi_{n_1}, \dots, \phi_{n_N} \mid \phi_{u_1}^{(m)}, \dots, \phi_{u_U}^{(m)}\rangle_q \\
 &\quad \downarrow \quad \quad \downarrow \\
 \Phi_q &= \{\phi_1, \phi_2, \dots, \phi_f, \dots, \phi_g, \dots, \phi_h, \dots, \phi_i, \dots, \phi_M\}.
 \end{aligned} \tag{3.57}$$

Note that V_q may specify that only some of the unknowns are to be assigned values by the q th decoding process. In this situation, the other unknowns will have been

Figure 3.14: Block-diagram for CSU Case Select and Update

scheduled to be converted to knowns at a later step, perhaps based on a sequence elsewhere in the map that has superior photometric data.

3.4.6 **PNA**: Photometric Neighbourhood Analysis

The block-diagram for **PNA** (Photometric Neighbourhood Analysis) is shown in Figure 3.15. For the q th ordering step, it uses the set of trial fringe numbers $\{\Phi\}_q$ and the map and image data $(\mathcal{M}, \mathcal{I})$ to photometrically analyse that portion of the image spanned by the q th neighbourhood sequence $(\Sigma)_q$. It produces a set of R photometric-consistency errors $[\epsilon]_q$, one for each of the possible ways of ordering the sequence of contours. There are two subprocesses shown in the block-diagram:

RSG (Region Seed Generator),
PRA (Photometric Regional Analysis).

RSG generates the R seed-points

$$P_{R_1}, P_{R_2}, \dots, P_{R_j}, \dots, P_{R_R} \quad (3.58)$$

for the R photometric analysis regions

$$R_1, R_2, \dots, R_j, \dots, R_R \quad (3.59)$$

that are distributed along the contours of the q th ordering neighbourhood.⁶ The control vector V_q specifies on which of the neighbourhood contours the seed-points are to lie. The list may be the contour list *per se* from within $\mathcal{M}$:

$$C_i = (\lambda, e, \dots; P_1, P_2, \dots, P_C)_i \quad (3.60)$$

or, V_q may specify a separate sub-list of contour points to be used.

PNA spawns a regional analysis process **PRA** for each of the R regions. Each **PRA** uses its designated seed-point to delineate an analysis region, and then tests

⁶To simplify the notation, the subscript ' q ' has been dropped from regional parameters (they bear a ' R_j ' for the j th region). Note, however, that the regional parameters are strictly unique to the q th ordering step.

Figure 3.15: Block-diagram for **PNA** Photometric Neighbourhood Analysis

the set of trial fringe numbers for the N ordering cases $\{\Phi\}_q$ by analyzing the map and image data $(\mathcal{M}, \mathcal{I})$. The j th **PNA** produces a set of N photometric-consistency errors $[\epsilon]_{\mathfrak{R}_j}$, one for each of the possible ways of ordering the sequence of contours. As shown in the block-diagram, the errors for each region are accumulated to give the total error for the neighbourhood $[\epsilon]_q$ (i.e., for the q th ordering step), where the total for the k th ordering case is given by

$$\epsilon_q^{(k)} = \sum_{j=1}^R \epsilon_j^{(k)} \quad (3.61)$$

The detailed functionality of the j th **PRA** is given in the following section.

3.4.7 **PRA**: Photometric Regional Analysis

The block-diagram for **PRA** (Photometric Regional Analysis) is shown in Figure 3.16. It uses the designated seed-point $\mathcal{P}_{\mathfrak{R}}$, to define its analysis region $\mathfrak{R}_j$, and then uses the map and image data $(\mathcal{M}, \mathcal{I})$ to photometrically analyze the set of trial fringe numbers $\{\Phi\}_q$. It produces a set of N regional photometric-consistency errors $[\epsilon]_{\mathfrak{R}_j}$, one for each of the possible ways of ordering the sequence of contours. There are three subprocesses shown in the block-diagram:

RND	(Regional Neighbourhood Discovery).
IRM	(Image Reflectance Measurement).
RCA	(Regional Case Analysis)

RND is responsible for discovering the regional analysis points that span the neighbourhood sequence. The identity of the S neighbourhood contours is provided by V_q , and searching the image about the seed-point $\mathcal{P}_{\mathfrak{R}}$, results in determining the distance, direction and coordinates of the S required points. As discussed earlier in this chapter, the topographic significance of these points is that they lie where the surface gradient curve drawn through the seed-point intersects each of the S contours in the neighbourhood sequence $\langle \Sigma \rangle_q$.

The important topographic attributes for photometric analysis are: the coordinates of the points, the planimetric azimuth direction of the contour at each point,

Figure 3.16: Block-diagram for PRA Photometric Regional Analysis

and the distance along the gradient to the neighbour point. These attributes can be organized into lists that correspond to the neighbourhood sequence:

$$\text{the line coordinates: } \langle \ell \rangle_{\mathbf{x}} = \langle \ell_1, \ell_2, \dots, \ell_S \rangle_{\mathbf{x}}, \quad (3.62)$$

$$\text{the pixel coordinates: } \langle p \rangle_{\mathbf{x}} = \langle p_1, p_2, \dots, p_S \rangle_{\mathbf{x}}, \quad (3.63)$$

$$\text{the azimuth directions: } \langle \alpha \rangle_{\mathbf{x}} = \langle \alpha_1, \alpha_2, \dots, \alpha_S \rangle_{\mathbf{x}}, \quad (3.64)$$

$$\text{the gradient separation: } \langle \sigma \rangle_{\mathbf{x}} = \langle \sigma_1, \sigma_2, \dots, \sigma_S \rangle_{\mathbf{x}}, \quad (3.65)$$

These lists of regional data are grouped into a data structure $\llbracket S \rrbracket_{\mathbf{x}}$, defined as

$$\llbracket S \rrbracket_{\mathbf{x}} = \llbracket \langle \ell \rangle_{\mathbf{x}}, \langle p \rangle_{\mathbf{x}}, \langle \alpha \rangle_{\mathbf{x}}, \langle \sigma \rangle_{\mathbf{x}} \rrbracket. \quad (3.66)$$

shown as the output of $\boxed{\text{RND}}$ and passed as internal data to the subprocesses $\boxed{\text{IRM}}$ and $\boxed{\text{RCA}}$.

$\boxed{\text{IRM}}$ measures the shading value in the image $\mathcal{I}$ at analysis points specified by V_q . Recall that the photometric analysis points are interior to the neighbourhood sequence (i.e., the end points of the sequence are not used, nor are necessarily all the other points). Thus, not every point in the sequence is used for the photometric analysis and, in fact, there may be only one point each amongst the knowns and unknowns where the measurement is conducted. The control vector identifies the measurement points by their position in the sequence list so that $\boxed{\text{IRM}}$ requires $\llbracket S \rrbracket_{\mathbf{x}}$ to get the image coordinates of the measurement points for each region.

$\boxed{\text{IRM}}$ measures the average reflectance in a small patch in the image $\mathcal{I}$, centered about the analysis point. As was discussed earlier in this chapter, the size and shape of the patch is very important if the shading is to be accurately gauged in the presence of the moiré fringe pattern. The coordinates of the neighbours on each side of the analysis point (provided by $\llbracket S \rrbracket_{\mathbf{x}}$) play a key role in determining the measurement patch dimensions and orientation. The measured reflectance value at each of the analysis points is denoted $\bar{\rho}$. To keep the notation simple, the output from $\boxed{\text{IRM}}$ is padded into a list $\langle \bar{\rho} \rangle_{\mathbf{x}}$, where it is understood that the value is only meaningful at the members that are designated as photometric analysis points.

$\boxed{\text{RCA}}$ uses the reflectance measurements $\langle \bar{\rho} \rangle_{\mathbf{x}}$ and the topographic data for the region $\llbracket S \rrbracket_{\mathbf{x}}$ to test the set of ϕ -values for the ordering cases $\llbracket \langle \Phi \rangle \rrbracket_q$. It produces a

measure of the photometric consistency for each ordering case on the basis of the measured and calculated properties of the regional data. $\boxed{\text{RCA}}$ is described in detail in the next section.

3.4.8 $\boxed{\text{RCA}}$: Regional Case Analysis

The block-diagram for $\boxed{\text{RCA}}$ (Regional Case Analysis) is shown in Figure 3.17. It embodies the principle of the Photometric Ordering Operator described in Section 3.3.7, starting on page 92. It uses the regional shading values $\langle \bar{\rho} \rangle_{\mathbf{x}}$, and the regional topographic attributes $\llbracket S \rrbracket_{\mathbf{x}}$, to test the set of ϕ -values for the N ordering cases $\llbracket \Phi \rrbracket_q$. It produces a set of photometric-consistency errors for the region, denoted by $\llbracket \epsilon \rrbracket_{\mathbf{x}}$.

Figure 3.17 shows that $\boxed{\text{RCA}}$ expands into a set of N independent subprocesses, one for each ordering case. The block-diagram for the k th Region Case Analysis $\boxed{\text{RCA}^{(k)}}$ is shown in Figure 3.18. Its output is a single value: the photometric-consistency errors for the region for the k th ordering case, denoted by $\epsilon_{\mathbf{x}}^{(k)}$. There are three subprocesses shown in the block-diagram:

$\boxed{\text{GSC}}$	(Geometric Scalar Calculation),
$\boxed{\text{MPE}}$	(Model Parameter Estimation),
$\boxed{\text{PCE}}$	(Photometric Consistency Error).

$\boxed{\text{GSC}}$ computes the geometric scalar γ for each of the analysis points designated by V_q . It uses the ϕ -values of a specific ordering case $\langle \Phi \rangle_q^{(k)}$ and the regional topographic attribute data $\llbracket S \rrbracket_{\mathbf{x}}$. Note that the variation in the ϕ -values for the unknowns can influence the value of γ for knowns as well as unknowns, and thus all the γ -values must be recomputed for each ordering case. $\boxed{\text{GSC}}$ produces values for the geometric scalar at the same points that $\boxed{\text{IRM}}$ measures the shading values and, like the shading values, the geometric scalars can be conveniently padded in a list $\langle \gamma \rangle_{\mathbf{x}}^{(k)}$. Furthermore, it is convenient to partition the list into separate lists, as shown in the block-diagram: $\langle \gamma |_{\mathbf{x}}^{(k)}$ for the analysis point(s) amid the knowns, and $| \gamma \rangle_{\mathbf{x}}^{(k)}$ for the analysis point(s) amid the unknowns. These partitioned entities (and

Figure 3.17: Block-diagram for $\boxed{\text{RCA}}$ Regional Case Analysis.

Figure 3.18: Block-diagram for $RCA^{(k)}$. Regional Case Analysis (k th)

their shading counterparts) are submitted to different subprocesses within $\boxed{\text{RCA}^{(k)}}$. The internal details of $\boxed{\text{GSC}}$ are presented in the next section.

$\boxed{\text{MPE}}$ estimates the photometric model parameter(s)⁷ $\omega_{\mathfrak{R}_j}^{(k)}$ at the known analysis points. Thus, it uses the partitions $\langle \bar{\rho} | \mathfrak{R}_j$ and $\langle \gamma | \mathfrak{R}_j^{(k)}$ to determine the reflectance characteristics of the region $\mathfrak{R}_j$. The control vector V_q specifies the analysis points to be used in this determination, and may also select an appropriate photometric model (i.e., Lambertian pure-diffuse, Phong's diffuse/specular, etc.) and, furthermore, may control the tolerances of the model-fitting technique.

$\boxed{\text{PCE}}$ uses the same photometric model as $\boxed{\text{MPE}}$. It is controlled by the model parameter(s) $\omega_{\mathfrak{R}_j}^{(k)}$ for the region, and uses the geometric scalars in the unknown partition $\langle \gamma | \mathfrak{R}_j^{(k)}$ to estimate the expected shading values $\langle \bar{\rho} | \mathfrak{R}_j^{(k)}$ for the shape represented by the k th ordering case. The expected values are compared to the measured values $\langle \bar{\rho} | \mathfrak{R}_j$, to generate the photometric-consistency error:

$$\epsilon_{\mathfrak{R}_j}^{(k)} = \sum_{u_a}^{\mathfrak{R}_j} (\bar{\rho}_{u_a}^{(k)} - \bar{\rho}_{u_a})^2, \quad (3.67)$$

where the sum is taken over the analysis points in the unknown partition of the region.

3.4.9 $\boxed{\text{GSC}}$: Geometric Scalar Calculation

The block-diagram for $\boxed{\text{GSC}}$ (Geometric Scalar Calculation) is shown in Figure 3.19. It uses the topographic attributes for the region $\llbracket S \rrbracket_{\mathfrak{R}_j}$ and the set of ϕ -values for the k th ordering case $\langle \Phi \rangle_q^{(k)}$. It produces the geometric scalars $\langle \gamma | \mathfrak{R}_j^{(k)}$ and $\langle \gamma | \mathfrak{R}_j$ for the analysis points in the known and unknown partitions, respectively. There are five subprocesses shown in the block-diagram:

$\boxed{\text{CEC}}$	(Contour Elevation Calculation),
$\boxed{\text{OCC}}$	(Object Coordinate Calculation),

⁷Some complex photometric models have several parameters to be determined, whereas the Lambertian model requires a single parameter. The possibility that $\omega_{\mathfrak{R}_j}^{(k)} = (\omega_1, \dots, \omega_2)_{\mathfrak{R}_j}^{(k)}$ is implicit in the notation.

 Figure 3.19: Block-diagram for $\boxed{\text{GSC}}$ Geometric Scalar Calculation.

IDC	(Illumination Direction Calculation),
SNC	(Surface Normal Calculation),
SPC	(Scalar Product Calculation).

Two additional data structures are shown as control input to the subprocesses of **GSC**. The first is denoted by **①** (Instrumentation Physical Dimensions), and contains the coordinates and critical dimensions of the components of the moiré fringe instrument. This file would include the position of the coordinate-system origin, the camera position $(0, 0, l)$, the light source position $(0, d, l)$, and the pitch of the grid s , for the purpose of calculating the elevation of a given fringe number. It might also include a calibrated table of elevations for each fringe. The second control data structure is denoted by **②** (Image-to-Object Transformation Coefficients), and contains the coefficients required to project image coordinates (ℓ, p) into the grid plane at $(x_g, y_g, 0)$. These two data structures have not been explicitly shown in the block-diagrams until this point, since only the subprocesses of **GSC** require them. They can be considered to be supplied by V_q .

CEC uses **①** to calculate (or look up) the elevations $\langle z \rangle^{(k)}$ that correspond to each of the ϕ -values for the k th ordering case $\langle \Phi \rangle_q^{(k)}$, using Equation 3.6:

$$z = \frac{\phi l}{\phi - d/s}.$$

OCC calculates the planimetric object-coordinates (i.e., their (x, y) -components, since the z -components have been computed by **CEC**) for the sequence of points in the region. The image coordinates for the sequence are provided in **[S]_q**. For the i th point, $(\ell, p)_i$ is first projected to the grid plane:

$$(x_g, y_g)_i = T(\ell, p)_i, \quad (3.68)$$

where T is the projection transformation, and the coefficients of the transformation are given in **②**. The transformation used by the author accounts for origin offset, rotation and differential scaling between the image-plane and grid-plane coordinate systems:

$$\begin{aligned} x_g &= a_0 + a_1 \ell_i + a_2 p_i, \\ y_g &= b_0 + b_1 \ell_i + b_2 p_i. \end{aligned} \quad (3.69)$$

More sophisticated rectification between the image and grid planes may be deemed necessary, and then standard methods of photogrammetry can be applied to model the projection distortion.

From the grid plane, the point is then projected to the required elevation at z_i using the instrument dimensions in ①:

$$x_i = x_g \left[1 + \frac{z_i}{l} \right], \quad (3.70)$$

$$y_i = y_g \left[1 + \frac{z_i}{l} \right], \quad (3.71)$$

where l is the distance from the camera to the grid plane.

The overall projection operation conducted for each P_i by CEC and OCC can be written as mapping a two-dimensional image position with a given fringe number, to a three-dimensional object-space position:

$$\mathcal{P}_i : (\ell, p, \phi)_i \longrightarrow (x, y, z)_i, \quad (3.72)$$

The output of OCC is shown in the block-diagram as $\langle (x, y) \rangle_{\mathcal{R}_j}^{(k)}$ to indicate that the planimetric coordinates for the sequence of points are both specific to the region $\mathcal{R}_j$ and are dependent upon the ϕ -values of the k th ordering case.

IDC computes the direction $\mathbf{L}$ and the distance r to the light source for the analysis points specified by V_q . It uses ① to obtain the lamp position in the instrument coordinate system (x_l, y_l, z_l) , and has as regional input the object coordinates $\langle (x, y, z) \rangle_{\mathcal{R}_j}^{(k)}$. For the i th analysis point P_i ,

$$\mathbf{L}_i = (L_{x_i}, L_{y_i}, L_{z_i}), \quad (3.73)$$

$$= \frac{(x_l - x_i, y_l - y_i, z_l - z_i)}{r_i}, \quad (3.74)$$

where

$$r_i = \sqrt{(x_l - x_i)^2 + (y_l - y_i)^2 + (z_l - z_i)^2}. \quad (3.75)$$

SNC computes the direction of the surface normal $\mathbf{N}$ for the regional analysis points in the sequence. This is a complicated process, and is fully described in Chapter 6. Briefly, however, it uses the planimetric azimuth angle α , determined

by **RND** from the contour curve through the i th point P_i , and it uses the sequence of neighbours at their trial elevations to determine the local tilt of the surface τ_i . Then, the normal is given as

$$\mathbf{N}_i = (N_{x_i}, N_{y_i}, N_{z_i}), \quad (3.76)$$

$$= (\sin \tau_i \cos \alpha_i, \sin \tau_i \sin \alpha_i, \cos \tau_i). \quad (3.77)$$

The required azimuth angles are in $[\mathbf{S}]_{\mathbf{x}_i}$. The sequence of object point coordinates $\langle (x, y, z) \rangle_{\mathbf{x}_i}^{(k)}$ is used for constructing the gradient profile curve, from which the surface tilt is determined. As with the other processes, the particular analysis points in the neighbourhood sequence are specified by V_q .

SPC calculates the scalar product γ_i for the i th analysis point P_i in the sequence. For the Lambertian photometric model represented by Equation 3.28, a simple vector scalar-product (normalized by the illumination inverse-square law) is computed:

$$\gamma_i = \frac{(\mathbf{L}_i \cdot \mathbf{N}_i)}{r_i^2}, \quad (3.78)$$

$$= \frac{(L_{x_i} N_{x_i} + L_{y_i} N_{y_i} + L_{z_i} N_{z_i})}{r_i^2}. \quad (3.79)$$

In **GSC**, after the object coordinates for all the regional points in the neighbourhood sequence have been calculated, the processing is shown as separate paths for the partition of knowns and the partition of unknowns. A single path, with partitioned outputs indicated at **SPC**, is also possible. The diagram was drawn with two paths to indicate at what stage in **GSC** an opportunity for parallel processing exists. A subtle point, explained in Chapter 6, is that one or more points from the other partition may be involved in the computation of the γ at a given analysis point, and thus *all* the points in the sequence should have their object coordinates evaluated before the γ 's in either partition are computed.

A final comment on the presentation of the algorithm reiterates what was said in the introduction. There are many different ways to depict, group and nest the functionality of the processes in the block-diagrams (some with undisputed improvements in efficiency and clarity!). As presented, however, this algorithm is embodied

in test software with the indicated data structures, and with essentially a one-to-one correspondence between the block-diagrams and the software modules.

3.5 Assumptions and Justifications

This section concludes the chapter with a brief discussion of some assumptions inherent to the Photometric Ordering Algorithm that was described in the previous section. A justification for accepting each assumption is also given.

3.5.1 Existence of Sufficient Control

The first assumption is that the data is collected in a controlled fashion, and that it is of a restricted form. In particular, the moiré topogram image is assumed to have been formed on a calibrated instrument, with established values for all the parameters required to relate the moiré fringe number with the elevation of the object from the instrument grid plane. It is implicit to the ordering algorithm that the fringes are true topographic contours. Fringes with other topographic interpretation may also be ordered by photometric analysis, but the algorithm would have to be modified to properly estimate the local shape (i.e., N) for each ordering case of the non-contour fringes. The algorithm further assumes a single point-source of illumination, but the model could easily include a summation over a distribution of light sources. Sufficient control must be imposed to ensure that the image scale, and orientation with respect to the grid plane are known.

In the author's case, all these conditions were attended to. The NRC Moiré Contourgraph is a stable, calibrated device. All the instrument parameters are known to sufficient accuracy ($< 1\%$). The geometry is carefully arranged to produce fringes that are true topographic contours. A compact line-source of illumination is used, and experiments described in the next chapter verify that it is sufficiently point-like. Calibrated fiducial targets in the corners of the grid plane, as well as a horizontal and vertical fiducial-string intersecting at the coordinate origin (in the center of the grid plane), were visible in the moiré image and ensured that

an accurate transformation from image to object coordinates could be performed reliably. Experiments reported in the next chapter verify the accuracy of coordinate determination for object points.

3.5.2 Existence of Extracted Contours

The Photometric Ordering Algorithm assumes that the contours have already been traced from the moiré topogram image, and that the only outstanding operation is to order them. It also assumes that at least one pair of neighbouring contours have known order.

The fringe-tracing problem has been adequately solved by other researchers for the broad class of images called *interferograms*. Many of the advances in this field come from the analysis of holographic fringe patterns (primarily for non-destructive testing), as well as from the wide application of in-plane and topographic moiré fringe patterns. Speckle interferometry, Schlieren interferometry, and even fingerprints are other popular subjects for fringe pattern analysis. Automated fringe-tracing techniques are reviewed in Appendix C.

For the development and testing of the Photometric Ordering Algorithm, the author manually traced contours from fringe patterns using interactive software. This method of contour extraction was considered acceptable, as relatively few sets of contours were required. The description of the interactive tracing software (Task JSP) is given in Section 5.3.3. The implementation of an automated fringe-tracing algorithm, to be coupled with the photometric ordering software, was considered to be a long-term goal of the moiré analysis research at the National Research Council of Canada.

3.5.3 Extractability of Shading in Moiré Topograms

The final assumption is that the shading on the object is extractable in the presence of the moiré fringe pattern. The problem of measuring the local surface reflectance has already been introduced earlier in this chapter. Only through experimentation

could the author verify whether or not adequate shading estimates could be made from the digitized moiré topograms. These experiments are reported in the next chapter, and based on the results of those tests, the problem and its solution are subsequently re-assessed, in Chapter 7.

Chapter 4

Design Studies

4.1 Introduction

This chapter reports on design studies conducted at an initial stage of the project. At the outset of this research, the ideas that led to the concept of photometric ordering of moiré fringe contours were based on several key assumptions. For example, it was assumed that one could measure the local surface shading of an object in the presence of the moiré pattern in the digitized image. It was also assumed that one could accurately determine the local surface shape for each trial ordering case by analysis of the shape of the moiré fringe pattern. Each of these assumptions is valid provided that the variables that modify these attributes (shape and shading) can be adequately controlled. As will be shown later in this chapter, some of the experimental variables (such as ambient illumination during topogram exposure) played an insignificant role, whereas other factors (such as the directional variation of the primary light source of the moiré instrument) became crucial design issues that had to be carefully considered. This chapter reports on some preliminary studies that were intended to:

- identify the crucial design issues.
- investigate appropriate shading and shape measurement methods.
- specify control restrictions that must be imposed on the data.

The experiments described in this chapter provided an initial look at the digital image data. The measurement methods were quite primitive, and the manual effort in interactively measuring a shading value or computing an object coordinate for a given pixel restricted the investigation to a few essential tests. Some of the specific questions to be addressed were:

- Is the digitized topogram reflectance sufficiently linear with respect to the local surface shading?
- How can the local surface shading be estimated in the presence of the moiré fringe pattern? What effect do the grid strings and their shadows have on the shading measurement?
- Can the object coordinates be calculated with sufficient accuracy from the digitized image?
- How complex a photometric model is required to represent real-life objects?
- Is the moiré instrument illumination sufficiently point-like for the photometric model, or must it be represented by a distribution of point sources?
- Is the surface reflectance and illumination uniform enough to permit endogenous ordering of contour fringes?

The studies reported in this chapter provided immediate answers to some of these questions, but some could not be completely answered until the techniques described in later chapters were developed. Despite this limitation, and most importantly, these initial design studies did provide the essential guidance that allowed the outstanding problems to be resolved, and ultimately led to the realization of software for the automated ordering of moiré fringe contours. The remainder of the chapter is organized as follows.

The next section reviews the equipment and materials used in the studies. The moiré instruments and topogram production are discussed, and construction details of the test objects are given. The section concludes with a look at the image digitization and digital image processing facilities used.

The third section briefly lists the measurement methods used in the studies. Procedures are described for selecting points in the digital image and for determining their image coordinates, the direction of the surface normal, the absolute moiré fringe number and the local surface shading value.

The fourth section describes tests to check the linearity of the reflectance measurements. Gray step tablets are calibrated and used to monitor the photographic process and the digitization of the topograms.

The fifth section describes an experiment to verify the accuracy of measuring shape from the moiré topogram. The coordinates of points along a spherical profile are recovered from digitized topogram image coordinates. The surface tilt at each point is also determined, and the accuracy of the results is analyzed.

In the sixth section, experiments into endogenous shape modelling are described. This method differs from the propagated ordering method used in the remainder of the thesis, in that no prior knowledge of the shape is required. The results are interesting: the method has potential, but the data are too noisy to pursue this approach.

In the seventh section, problems with the moiré instrument illumination are analyzed. The illumination from the lamp is not point-like, nor is it isotropic. Systematic departures are discussed in terms of modelling the effects of the lamp filament geometry and the condensor-optics of the lamp housing, but the disposable lamp proves to be the weak link in the calibrated system.

Experiments in propagated shape modelling are briefly reported in the following section. The results show that propagated fringe ordering is feasible on a variety of ~~surfaces~~, and that certain precautions are needed to handle surface specularities and illumination non-uniformities.

The chapter concludes with a review of the important results from the studies. The need for careful data acquisition is reiterated, and the Lambertian photometric model is adopted for the remainder of the thesis. The prospects for endogenous ordering are given a passing comment. Then the three main issues in automating propagated fringe ordering are listed: neighbour discovery, shape determination and

shading determination. These issues are examined in detail in Chapters 5, 6 and 7.

4.2 Equipment and Materials

4.2.1 Moiré Contourgraphs and Topograms

All the topograms used in this project were recorded using the NRC Moiré Contourgraphs. These instruments were described briefly in Chapter 1, and the translating-screen model was shown in Figure 1.3 on page 9. Some of the design details not mentioned previously proved to be quite important to shape and shading determination for photometric ordering of moiré fringes, and these instrumentation considerations will be elaborated upon at appropriate points in the remainder of this and subsequent chapters.

The Contourgraphs were originally designed to use a 35-mm camera with a bulk-film back and a power winder to record the topograms as photographic negatives for large numbers of subjects.¹ For in-house studies at NRC, it proved much more convenient to install a Polaroid camera on the moiré instruments. An adapter mount was machined to maintain the designed optical alignment: the camera lens nodal point is kept a certain distance from the center of the moiré grid, and its optical axis should intersect the center of the moiré grid (*i.e.*, where the horizontal and vertical fiducial strings intersect in the grid plane.) The adoption of Polaroid emulsions was a concern to the author, since the image density/exposure linearity (*i.e.*, the 'gamma' of the emulsion) was very sensitive to development time (a parameter often haphazardly controlled when several Polaroid exposures are being taken in succession). This issue is examined further in the reflectance linearity tests described later in the chapter.

The Contourgraphs were modified by the author to provide additional control over the shape and shading variables in the topogram in such a fashion that the

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¹This arrangement allowed up to several hundred school children to be efficiently screened in rapid succession for spinal deformity, and produced archive-quality photographic records for the medical examiners.

control information would be included with each topogram image. The concept of self-documenting photography has been long practised by the photogrammetric community, and most photogrammetric cameras (especially large-format aerial survey cameras) image a variety of important calibration parameters into the border of each frame. Figure 4.1 shows a typical moiré topogram from the enhanced instrument. The object commands the center of the image format, where lens distortion is minimal, and the annotation was reflexed into the unused border using a double periscope [Pekelsky, 1987a].

The annotation text is organized into two blocks. The upper block documents the characteristic moiré geometry for the instrument: in this case a grid pitch $s=2$ mm, a camera/grid distance $l=1700$ mm, and a lamp height $d=710$ mm. Also noted is that the white vertical and horizontal fiducial strings intersect on the camera axis to form the object coordinate system origin, and the nominal coordinates for the four fiducial targets in the grid plane are indicated. The lower text block displays the remaining information in three groups: choice of camera, lens, focus distance, aperture, shutter speed, and type of illumination; film type, film processing parameters, site and operator ID; subject ID, time and date. The author felt that not having to correlate written notes with photos merited this integration. Further enhancements could be made and, for example, the author considered including a bar-code for machine-readability of the parameters in the case of a fully-automated topogram analysis system [Pekelsky, 1986a]. The details of the text lettering system are given in [Pekelsky, 1987a].

With respect to shape-control, a fiducial target was installed in each corner of the moiré screen, as can be seen in the example topogram. Combined with the intersecting white horizontal and vertical fiducial strings, these targets with their calibrated object coordinates, are intended to provide the photogrammetric control required to transform digitized image coordinates into object coordinates. A test of the accuracy of object coordinate recovery is reported on later in the chapter.

With respect to shading-control, a pair of calibrated gray scales was mounted on the side supports of the moiré grid frame, as shown in the example topogram.

Figure 4.1: A topogram produced on the NRC Moiré Contourgraph, illustrating the principle of self-documenting photography. Control points give scale and orientation, gray scales calibrate the exposure and processing, and annotation specifies instrument geometry, photographic parameters, subject identification, date and time of day.

Combined with the white cardboard covering the side supports, these gray scales allow the linearity of the shading reflectance measurements to be monitored. The other components of the instrument are painted flat black, and special shrouds are installed on the instrument lamp housing to minimize stray light. Tests of reflectance linearity and the effect of ambient light are reported later in the chapter.

4.2.2 Test Objects

The NRC moiré instruments are designed to measure the shape of the human torso, and are engineered to provide a field of view and contour interval that is suited to that task. The human torso is a particularly difficult subject to analyze in terms of shape and shading parameters. The form of a live human subject is both topographically complex and very dynamic. Repositioning of live subjects is a long-standing problem in medical imaging research. The shading characteristics of living skin can also be highly variable, even on the same body. The reflectance properties of human skin are not well understood, and are subject to numerous parameters, such as racial pigmentation, perspiration, oils, wrinkles, tanning 'boundaries', diet, temperature, humidity, rashes, scars, age, sex, cosmetics, body hair, dander, freckles, time of day, month, year, etc. Despite these difficulties, it was considered essential that automated photometric ordering of moiré fringes work for human subjects (because of NRC's involvement with spinal deformity research), and many of the tests reported throughout this document use moiré topograms of live human subjects.

The other class of test material is comprised of inanimate objects, such as a painted wooden sphere, a cardboard sheet folded into the shape of an 'M', and the mannequin shown in Figure 4.1. There are several obvious advantages to this class of test objects, all relating to the issue of experimental control. Usually the surface of an inanimate test object can be treated to give a uniform reflectance (such as coating it with paint). Also, the shape can be restricted to a simple geometric form, as in the case of the wooden sphere described below. The third aspect is the ability to stably position the object for indefinite time intervals, which allows other parameters such as exposure time to be varied.

The details of each test object will be given at the appropriate points throughout the text, and as the painted wooden sphere dominates the initial tests reported in this chapter, its construction will be described here.

The sphere is part of the NRC Standard Object for Moiré Topography. By the early 1980's, several dozen laboratories around the world were conducting research into measuring spinal deformity using various types of moiré topography instruments. Some sort of standardization was needed to allow these researchers to exchange data and compare results. The NRC Standard Object is a test object designed to evaluate the photogrammetric properties of moiré instruments used for spinal research. It is comprised of simple geometric shapes (concave pyramidal sides and a convex spherical center) and displaces a volume comparable to the dimensions of the human torso (see Fig. 4.2). It was shaped from laminated mahogany on a numerically-controlled milling machine, which was also used to engrave an array of calibrated control points. The wood has been treated with stabilizing agents, and like the mannequin shown in Figure 4.1, it is painted to match Caucasian skin colour. Its construction and its use in the evaluation of several moiré instruments is reported in [Paulun, 1983].

A robust shipping case was also fabricated, and the Standard Object has been loaned to various institutes in Canada and the United States for the photogrammetric evaluation of moiré and other shape-measuring techniques. Figure 4.3 shows the encased Standard Object and its positioning stand, which is disassembled and stored in the lid of the shipping case [Pekelsky, 1987a].

Moiré topography instruments designed for spinal research should produce moiré fringes with good contrast over the depth of the Standard Object, and fringe loci that lie on true contour planes should produce straight parallel fringes on the pyramid and perfect concentric circles on the sphere. The array of calibrated control points allows the distortion in the recorded image to be assessed, and departures from the ideal fringe shapes to be measured. Thus, one can determine if a given instrument produces shape-encoding that is sufficiently accurate for the intended biomedical application. Figure 4.4 shows a topogram of the Standard Object taken

Figure 4.2: Design profiles of the NRC Standard Object. The interior of the model consists of a concave pyramid and a convex hemisphere. The stepped exterior is due to the laminations of mahogany used in its construction. The interior was shaped to an accuracy of ± 0.01 mm on an NC milling machine, which was also used to drill an array of control points on the interior surface.

Figure 4.3: The NRC Standard Object for Moiré Topography. The model is encased in a robust trunk for shipping, with a positioning stand that disassembles and stores in the lid of the trunk

with the stationary-screen version of the NRC Moiré Contourgraph.

As mentioned, the spherical center of the Standard Object (referred to simply as 'the painted sphere' throughout this document) was used in many of the tests described in this report because it is a simple and accurately-formed shape for analysis, it has uniform shading characteristics, and it was readily available.

4.2.3 Image Digitization

Careful consideration was given to digitizing the moiré topograms. Various devices such as television cameras and line scanners were available to this research, each having advantages and limitations. In these investigative studies, cost and convenience were put aside, and there remained two important considerations: the spatial quantization and the intensity quantization.

In the spatial domain, the accuracy of sampling a grid of points in the image establishes the photogrammetric limitations on recovering object coordinates from the row and column (line and pixel) number of an element in the digitized image. Thus, the sampling must be done with minimal distortion. Photodiode array cameras are generally better than vidicon cameras in this respect [El-Hakim *et al.*, 1988]. The spatial resolution is limited by the number of sample points in the camera format, and is balanced between the fineness of the smallest image structure to be resolved and the size of the image to be digitized. Moiré fringes have a spatial frequency that is bounded by the grid string pitch (in this case, 2 mm), and the topogram format spans 500 mm $\times$ 800 mm (or 250 'strings' $\times$ 400 'strings'). Thus to resolve fringes down to the grid structure, the Nyquist criterion requires 500-column $\times$ 800-row digitization of the topogram.² This resolution was beyond the limits of available solid-state cameras using two-dimensional CCD imaging array technology [Real and Fujimoto, 1988]. Thus it was decided to use line-scan cameras based on linear CCD arrays.

With regard to intensity quantization, the sufficiency of the 8-bit resolution

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²The Nyquist sampling frequency is twice the required cutoff frequency in the signal spectrum [Oppenheim and Schaffer, 1975].

Figure 4.4: Moiré topogram of the NRC Standard Object, taken with the stationary-screen version of the NRC Moiré Contourgraph. The moiré fringes show excellent contrast over the entire depth of the interior of the Object. Note the straightness of the fringes on the pyramidal sides and the circularity of the fringes on the sphere.

available on most image-digitizing devices was to be demonstrated by experiment. This dynamic range was a limitation over which the author had little control, although if it was found necessary, a flat-bed scanning microdensitometer with 12-bit analog-to-digital conversion (A/D) resolution was available at another institution. The linearity of the silicon photodiode response to exposure was not expected to be a problem, since this function is typically linear over six or seven orders of magnitude. The 2.4 orders of magnitude spanned by the 8-bit dynamic range of the A/D electronics could be optimized to lie within the linear region of the detector by adjusting the scanner scene illumination and with programmable pre-A/D amplification of the detected signal. The best signal-to-noise ratio is obtained with brightly-illuminated scenes whose brightest pixel results in a collected charge just below the saturation level of the detector element.

Image digitization for preliminary experiments reported in this chapter were conducted at DIPIX Systems, Ltd. (Ottawa), using their Eikonix 2048-element linear CCD camera to form $1K \times 1K$ digital images of the sample topograms. Eikonix line-scan cameras use a stepper motor to translate the sensor array over a series of rows, such that the sample points are spaced the same amount in both rows and columns. The Photogrammetric Research Section at NRC was eventually equipped with a $4K$ row $\times$ $4K$ column Eikonix CCD scanner (Model EC 78/99, shown in Fig. 4.5), capable of colour image digitization. This scanner was used for some of the experiments reported in subsequent chapters. Somewhat an overkill for the moiré research, the Eikonix EC 78/99 served other photogrammetric tasks for which it was considered only a *medium* resolution scanner (*high* resolution was deemed to be over 10K lines).

4.2.4 Image Analysis System

All the digital image analysis reported in this document was performed on an Ariens II digital image processing (DIP) system from DIPIX Systems. NRC purchased this medium-power, general-purpose DIP system in 1982 with minimal options, performance and cost, and over the next four years introduced a series of upgrades.

Figure 4.5: The Eikonix CCD Model 78/99 Image Digitizer. The scanner has a spatial resolution of 4K lines $\times$ 4K pixels, with 8-bit grey-level discrimination and filters for colour separation. The scanner head slides along the hollow vertical rail of a customized copy-stand, and is counter-weighted by a slug in the column (linked by a pulley/cable system) to allow easy adjustment of the vertical position of the head. The copy stand has illumination for transmissive and reflective imagery.

Figure 4.6: The Aries-II Digital Analysis System. The host computer ensemble is shown on the left, and on the right is the remote workstation.

In its present configuration, the host computer is a DEC PDP 11/73, running the RSX-11/M v3.2 operating system, with 1.3 MB RAM, a 300 MB hard disk, a tape drive, a console, and a laser printer. The host chassis ensemble is shown on the left in Figure 4.6, and the remote workstation, shown on the right in the figure, consists of a terminal, printer, colour monitor and graphics tablet.

The Aries-II image memory (2 MB RAM) can be dynamically configured into multiple images of specifiable dimension (lines, pixels and bit-depth), and is dual-ported so that the host and the display controller can access the images efficiently. The display controller is a DEC PDP 11/02, and through the workstation graphics tablet, the operator can invoke many built-in commands to modify the display on the high-resolution colour monitor, such as the selecting of images, zooming, roaming, annotating, etc.

The Aries-II system has many unique capabilities that served the author's needs in developing software for automated moiré fringe ordering. For experiments described in this chapter, the standard software utilities provided by DIPIX were used initially to manipulate and study the digitized moiré topograms. More in-depth

hardware considerations were required to program the automated contour ordering algorithm; details are deferred to the appropriate sections in later chapters.

4.3 Measurement Procedures

The design studies are based on rather crude measurements of the shape and shading attributes of digitized moiré topograms. The author avoided any special software development by using the DIPIX system utilities and the interactive display hardware to make the necessary measurements. There are several measurement operations of interest:

- Identify a point on a contour. This is a pattern recognition task that relies on the operator (the author) to pick representative points that lie on the center of a bright or dark moiré fringe. In some cases the operator selects the point by observing the fringe pattern on the display, and moves the display cursor onto that pixel. In other cases, the pixel reflectance values in the neighbourhood of the desired point are listed on the printer, and the values are manually examined to select the desired point.
- Determine the image coordinates of a point. In the case where the cursor is moved onto the point, the coordinates are read from the real-time display of the cursor position in the status block at the bottom of the image display screen. In the case where a printed pixel-dump is under examination, the line and pixel coordinates of a selected point can be determined from the printout.
- Measure the surface normal direction at a point. Perhaps the most crude, but effective method in the case of a displayed image point is to measure the azimuth angle α of the fringe with a protractor held up to the screen, and to compute the tilt angle $\tau = \tan^{-1}(\text{rise/run})$ by measuring the distance (in pixel-units) to the neighbouring fringe as the 'run', and using the moiré fringe contour interval as the 'rise'. In some experiments described below, the data points are selected where the surface normal assumes a simple evaluation.

such as along a spherical 'polar profile' ($\alpha = 0$), or in regular 15° intervals of α around a spherical 'zonal ribbon' with fixed τ .

- Measure the shading value at a point. This is the most difficult attribute to measure, especially if the individual strings and their shadows in a stationary-grid topogram are resolved in the digitized representation. Using a single pixel-value directly provides too noisy an estimate, and several pixels in the immediate vicinity must be examined to obtain an estimate that suppresses artifacts such as grid-string noise. One can manually examine printed listings of pixel values, or 'peek' at values in the image memory with an interactive display utility.
- Determine the absolute fringe value ϕ at a point. This attribute is determined easily by manually inspecting the fringe pattern to ascertain the relative fringe ordering, and then using the fiducial shadow methods described in Chapter 2 to assign the absolute order to the fringes. In the case of the painted sphere experiments, the relative ordering is a trivial matter.

4.4 Reflectance Measurement Linearity Test

The linearity of the digitized reflectance measurements, with respect to the actual shading on the object, was carefully examined. The photometric model is based on object shading, so if there is any significant departure from linearity, it will have to be compensated for in the model.

There are two processes to examine: 1) the photographic recording of the object shading distribution to produce the moiré topogram, and 2) the digitization of the photograph to produce an array of image reflectance measurements. The first process relies on photochemical detector sites in an emulsion, and produces a variation in the optical density of the emulsion that is nominally linear with respect to the exposure. The second process uses photoelectric conversion in an array of photodiodes, and produces an electrical charge at each photodiode that is a linear function of the exposure. In the first process, the exposure is due to the moiré instrument

illumination that reflects from the surface of the object (i.e., its shading), whereas in the second process, the exposure is due to the scanner instrument illumination that reflects from the surface of the topogram.

To test these two processes, a step tablet or gray scale with calibrated reflectances was substituted for the usual object in each case. The reflectances in the recorded 'images' (i.e., the photograph and the array of digitized numbers) could be compared to the calibrated values to check the linearity. The numerical accuracy of this method depended on the ability to measure the input reflectance for both processes (i.e., to calibrate the step tablet), and to measure the output reflectance for the photographic process (clearly, the image digitizer process inherently provided a numerical measure of the reflectance of the object it was scanning).

The step tablet image-method has the advantage that the exposure variation imposed by each step is recorded simultaneously for all the steps, so that if the overall incident illumination varies during the recording (very likely), all patches will be uniformly affected. Sequential viewing of the step tablet (i.e., photographing or digitizing one step at a time) requires a very stable light source.

4.4.1 Reflectance Gray Scales

Good-quality commercial gray scales were used. The large model (14 inches long) gray scale available from Eastman Kodak Co.³ is photographically printed on stabilized RC-paper by a sequence of accurately-stepped exposure intervals. Kodak claims careful control over the chromatic neutrality and uniformity of this product. The distribution of gray values over its twenty steps is depicted in Figure 4.7. The numbers on the tablet represent $10 \times$ the nominal optical reflectance density D for each step, relative to patch #0. The reflected illumination I_r from a surface with optical density D is

$$I_r = I_0 \cdot 10^{-D}, \quad (4.1)$$

where I_0 is the incident illumination. The Kodak tablet is printed with uniform steps in optical density (as opposed to uniform steps in reflectance) that range from

³Kodak Product No. Q-14: Color Separation Guides and Gray Scale (large).

Figure 4.7: Schematic of the gray scale used in the reflectance tests. The actual scale was produced by Eastman Kodak Co. on a 14-inch long strip of photographic paper. The numbers represent $10\times$ the nominal reflectance density.

$D = 0$ (100% reflectance) to $D = 1.9$ (1.3% reflectance).

Four step tablets were used, and they were calibrated using a MacBeth Transmission/Reflection Densitometer Model TR-524. This instrument is used in the NRC Sensitometry Calibration Facility as a secondary standard for routine calibration of sensitometry film strips (i.e., step tablets). In skilled hands and a stable environment, the MacBeth unit provides reliable measures of optical reflectance density to an accuracy of ± 0.01 over the range 0 to 4. Each Kodak gray scale was measured three times (over several hours), and the reading variation for each patch was bounded by ± 0.01 . All the calibrated steps were within ± 0.04 of the nominal density designated by Kodak, but in the tests that follow, the NRC calibrated values were used.

4.4.2 Photographic Linearity Tests

Photographic emulsions have a linear response only over a limited range of exposure. Figure 4.8 shows the characteristic curve published by Polaroid Corporation for the Type 667 positive-print film used in these experiments. This curve exhibits typical properties: it is nominally linear (with a slope, called the 'gamma of the film', of 1.40) over one order of magnitude in exposure (1 unit on the lower scale); for lower exposures the density is limited to a maximum value of 1.60; and for excessive exposures the emulsion saturates to a minimum density of 0.30. The curve as drawn is valid over a modest range of processing conditions, but extreme development times and temperatures will alter the image contrast (i.e., change gamma).

Figure 4.8: Characteristic curve for Polaroid Type 667 positive film. The emulsion has maximum and minimum densities of 1.60 and 0.30, respectively, and a gamma (slope in the linear region) of 1.40.

The overall goal of the exercise was to establish exposure conditions for the moiré instrument that would ensure that the fringe pattern and the shading on the object would be recorded within the linear region of the characteristic curve for Type 667 film. The initial challenge was to see how well the film could be characterized under ideal conditions. The calibration of the step tablets convinced the author that the MacBeth densitometer gave reliable, accurate measures of reflectance density. It remained to take photographs of the step tablets with different exposures and development conditions, and to measure the recorded density as a function of exposure.

The four step tablets were arranged in a montage as shown in Figure 4.9 so that each Polaroid photograph contained at least two patches of every nominal density on the step tablets, and that the patches were large enough for the MacBeth sampling probe to make accurate measurements. The arrangement also reserved space to document the experimental conditions within each photo.

A Polaroid 600SE camera with a Mamiya 150 mm lens (plus a #2 close-up lens) was focused at 2 meters from the montage of step tablets. Four 1000 watt quartz-halogen studio lamps (colour temperature 3200°K) were arranged at 45° incidence to the montage from opposite sides of the camera and at 3 meters distance. These were considered to be careful precautions to provide *very* uniform illumination for the

Figure 4.9: Four gray scales arranged to characterize Polaroid Type 667 film. The scales were mounted on cardboard, and staggered to best fill the film format with the entire range of the step tablets. This figure represents an actual Polaroid print from the test, and it self-documents the conditions of the exposure and development of the film.

Figure 4.10: Measured reflectance density for Polaroid Type 667 film as a function of relative exposure. The density values were measured from the photograph depicted in Figure 4.9 at each of the gray patches, and their previously calibrated values were used to determine the relative exposure at each patch. The shape of this curve agrees well with the manufacturer's data shown in Figure 4.8.

step tablets. If the lamps behaved as isotropic point-sources at this distance, then less than 0.5% variation in the illumination would be observed over the montage format, and this corresponded to density variations of less than 0.003.

Photographs were taken in a combination of exposures ($1/25$, $1/30$, $1/125$ and $1/250$ seconds at $F/32$) and development conditions (30, 60, 90 and 120 seconds at 21°C). The best contrast range (i.e., the most discernable gray levels in the photograph) was for $1/60$ second exposure with 60 second development. One of the three photographs taken at these conditions was measured with the MacBeth instrument, and the results are plotted in Figure 4.10. According to Equation 4.1, the calibrated reflectance density values for each patch in the montage give the relative exposure (assuming uniform I_0 due to the arrangement of four distant studio lamps).

Although the shape of this curve agrees well with Polaroid's specification (see Fig. 4.8), the data is much more scattered than expected (5% to 20%). Measurement uncertainty with the MacBeth densitometer is unlikely to be the source (recall the small 1% variations in the earlier calibration of the step tablets). No specification for the uniformity of the film is available, but this is not a likely explanation either. The author's suspicions lay with the so-called *uniform* illumination, and the next tests confirm this.

4.4.3 Digitizer Linearity Tests

The digitizer tests were done at DIPIX Systems on their machine. Digitizer illumination nonuniformity was a known problem at DIPIX. Conventional copy-stand illumination was used (see Fig. 4.5), with a 300 watt quartz-halogen flood-light on each side of the digitizer head. The lamps were mounted on fixed arms, and were aimed to be incident at 45° to the center of the scanner field of view. The distance from the bulb to the center of the copy material was only 80 cm, so point-source behaviour was compromised and factors such as distortions in the quartz lamp envelope could be significant. The lamp reflectors tended to concentrate the light in the forward direction, but it was not obvious if they contributed to the uniformity.

The visual impression was that the central 25 cm square portion of the copy field was quite uniformly illuminated, but tests from scanning a uniform gray card showed a distinct central hot-spot, with up to 50% falloff at the corners. Part of the corner-darkening was due to the $\cos^4$ falloff inherent to the lens used to image the scene onto the detector array, but in a macro lens (typically a 40° field of view) this accounts for only a 20% decrease.

DIPIX staff had tried mounting extra lamps on the copy stand arms to improve the uniformity, but the proximity of the hot lamps to the scanner head caused problems (one customer melted the plastic case of an early-model Eikonix camera!).

DIPIX engineers were developing software for their scanner that called for a 'white card scan' to be used as a normalization map for subsequent scans in order to compensate for variations in the illumination. Their experiments with this approach

Figure 4.11: A pair of gray scales arranged to test digitizer linearity. The labelling of the scales and their borders is referred to in subsequent figures

showed promise, and the author incorporated their ideas into his own tests.

A pair of tablets was arranged as shown in Figure 4.11 to test the linearity of the image digitizer. The tablets were placed lengthwise parallel to the line of the two lamps. The length of the gray scales was large compared to the format of the moiré topograms to be scanned by the digitizer (14 inches vs. 3 inches), and significant variations in the illumination were expected. The gray border on which Kodak prints the nominal density of each step was used to monitor the illumination variation along the step tablet. The border was measured with the MacBeth densitometer to have a density of $D = 0.59 \pm 0.01$, and although it was a lithographic halftone, its uniformity along the length of the step tablet proved to be excellent for this application.

The raw scanned image was 1728×1728 pixels, and each gray patch on the step tablets was about 115×80 pixels. A utility program on the Aries-II was used to define a 49×49 pixel sub-area image within each patch, and a statistics/histogramming program was used to find the mean and standard deviation of the reflectance in each patch.

A similar patch-statistics analysis was conducted along the gray scale borders in the image. Since the border had essentially uniform density, any variation in the measured reflectance could be attributed to nonuniform illumination. Figure 4.12 shows that variation to be almost 50 DU. Note that reflectance values from the digitizer are expressed in 'DU' (digitizer units, or A/D counts) and range from 0 to

Figure 4.12: Variation of the illumination for the scanner linearity tests. The graph shows the measured reflectance (in A/D counts) for the upper and lower borders of the gray scales in Figure 4.11 at various positions along the scales. Since the borders had a uniform optical density, the variation in their measured reflectance is due to the nonuniformity of the illumination during scanning.

255 DU for an 8-bit digitizer.

Figure 4.12 suggests that the illumination variation is a slowly-varying function, and that linear interpolation between the upper and lower borders (refer to Figure 4.11) could be used to estimate the correction to the illumination of the gray scale patches between the borders. Figure 4.13 shows a plot of the raw digitizer reflectance measurements from Scale 85-2, and of the adjusted values that result when illumination normalization is applied, as a function of the calibrated reflectance values for each gray step. (A similar plot results for Scale 85-1.)

In Figure 4.13, a straight line was fitted through the adjusted data (it was not extended through the lower values for clarity). Ideally, the line should pass through the coordinate origin (provided the dark current of the detector has been properly subtracted), and it should pass through A/D saturation at 100% input reflectance (provided the incident illumination has been properly adjusted). In this case, there is a residual offset at the origin, and the slope may not be optimal (there is no valid data at 100% reflectance to check this). Nonetheless, this figure demonstrates that

Figure 4.13: Digitizer reflectance measurement linearity test. The measured reflectance (in A/D counts) is plotted against the calibrated values for the step tablet patches in Scale 85-2. Shown are the raw digitizer values, and the adjusted values due to illumination normalization. The arrow indicates an inadmissible raw value that exceeded the range of the digitizer (the adjusted value for that point is also shown below it). A straight line was fitted through the adjusted data.

the digitizer has a suitably linear response function. These experiments also show that image digitization requires considerable skill in order to produce digital images that utilize the full dynamic range of the scanning instrument without saturation and with spatially uniform illumination characteristics.

In these linearity tests, illumination uniformity has been a significant consideration. Intuition can mislead, and what appeared as reasonable setups to produce visually-uniform illumination proved inadequate in both cases. The photographic linearity test was noisy because of lighting variations. The scanner linearity test required illumination normalization to demonstrate the straightness of its response function. Later in this chapter, when examining the experiments in endogenous shape modelling, it will be shown that the moiré instrument illumination was also nonuniform.

4.5 Shape Measurement Test

In this section, an experiment is described that tests the accuracy of measuring shape from the moiré topogram. The test examines two shape attributes that are important to using the photometric model (refer to Eq. 3.26 on p. 85). The first is the ability to transform from image coordinates to object coordinates. This means converting the line, pixel and fringe number (ℓ, p, ϕ) of a point in the digital image to the space coordinates (x, y, z) on the surface of the object. The coordinates of points on the object are required in the photometric model to compute the direction $\mathbf{L}$ to the illumination. The second attribute that is examined is the surface tilt τ taken with respect to the z -axis. The tilt must be combined with the azimuth angle α to fully describe the surface normal $\mathbf{N}$ for each of the fringe ordering trial cases.

The painted sphere is the test object used in this experiment. Its precisely-formed surface (125.0 mm radius) is ideal for testing shape measurement accuracy. The test points are selected along a longitudinal section (a 'polar profile') through the sphere, so that it is easy to check if the transformed points lie on a perfect circle of radius 125.0 mm and if their surface normals make the required tilt with respect

to the pole (i.e., the z-axis).

Figure 4.14 displays the digitized image of the painted sphere topogram shown previously in Figure 4.4. The horizontal polar profile is drawn through the center of the sphere (using the annotation feature of the Aries-II display), where in the figure we are looking *down* the 'north-pole' along the z-axis. Also indicated on the polar line are the positions where bright moiré fringes intersect the polar profile, as well as the approximate center of the sphere.

The display cursor could be used to select fringe intersection points along the polar profile, but the author found it was more instructive to examine a listing of the pixel reflectance values. A partial listing of those pixel values is given in Figure 4.15 and corresponds to the area in the small rectangle on the left side of Figure 4.14. The reader might well ponder: in which figure is it easier to select the fringe intersection points along the polar profile — the video analog display or the array of digital numbers? The bright and dark fringe intersections with the polar profile were manually identified from the pixel dump, and their image coordinates and reflectance values noted.

As a brief digression from the object-coordinate exercise, Figure 4.16 shows the reflectance values of the intersecting fringes plotted as a function of their pixel address. This is an important figure, as it gives the first numerical evidence of the asymmetry imparted to the moiré fringes by the shading envelope. For this topography, the uphill fringes are distinctly brighter than the downhill fringes. The photometric model can predict this behaviour and thus, it can be used to order the fringes.

Returning to coordinate measurement, the absolute moiré fringe value ϕ was determined for each fringe using the fiducial shadow method (Chapter 2), and the surface tilt was computed using the rise and run between fringes, as explained in Section 4.3. The image coordinates and fringe numbers were used to compute the object coordinates using the method and equations described in Section 3.4.9, starting on page 116.

Figure 4.14: Digitized topogram of painted sphere showing the polar profile line. This digital image was scanned from the topogram shown in Figure 4.4. Only the central portion is displayed here, recorded directly from the Aries-II screen with a Polaroid camera. The sphere is positioned approximately 4 mm below and 2 mm to the right of the object-space coordinate origin (i.e., the intersection of the white fiducial strings). The polar profile is annotated in the display as a horizontal white line through the center of the sphere. The rectangular box at the left end of the polar profile indicates where the 'pixel dump' in Figure 4.15 is taken from. Other image artifacts are also visible: note the distinct curved shadow of the horizontal fiducial string across the lower reaches of the sphere's surface, and the finer (but clearly-resolved) shadows of the individual grid strings.

	pixel address																							
line	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	
810	70	72	72	70	66	63	61	61	63	67	70	72	72	70	67	63	59	57	57	59	64	69	75	
811	64	66	66	64	61	57	55	55	57	60	64	66	67	65	62	58	54	51	51	53	57	63	69	
812	61	63	<u>63</u>	61	57	54	<u>52</u>	52	54	57	61	64	<u>64</u>	63	60	55	51	48	<u>48</u>	50	54	59	65	
813	60	62	62	60	56	53	51	50	53	56	60	63	64	63	59	55	51	48	47	49	53	58	64	
814	60	62	62	60	56	52	50	50	53	56	61	64	65	63	60	55	51	48	47	49	53	58	64	

Figure 4.15: Sample of the reflectance values along the polar profile, corresponding to the box indicated in Figure 4.14. The polar profile lies on line 812 in the digitized topogram, and values on the adjacent lines are listed to illustrate the variability in the raw digital image data (there are four vertical fringes intersecting this strip). This is only part of the 'pixel dump'; the polar profile spans about 300 pixels and is intersected by fringes about 30 times. The underscored reflectance values denote the brightness of the moiré fringe intersecting there, and o or • indicates whether it was a bright or dark fringe.

Figure 4.16: Reflectance values for fringes intersecting the polar profile. The modulation between bright and dark fringes is indicated by the straight lines connecting the plotted data points. The modulation across the top of the bright fringes is indicated by a gray curve, and is due to the shading variation along the surface of the sphere. The modulation across the bottom of the dark fringes (not indicated) is due to moiré fringe contrast degradation at the spatial cutoff frequency. The low-valued central 'bright fringe' is actually the vertical white fiducial string.

The results of coordinate and tilt computations for the twelve central bright fringe intersections are given in Table 4.1. Note that the y -values vary slightly from one side of the sphere to the other (this indicates the digitizer scan lines were not perfectly aligned to the y -axis in the topogram when scanning). When carefully plotted at full scale on 1-mm graph paper, the x - z values lay exactly (!) on a circle of radius 125.0 mm. This was convincing proof of the accuracy one could obtain with photogrammetric techniques to recover spatial coordinates from an image.

Using least-squares adjustment, a circle of radius 125.0 mm was fitted to the x - z values, and its center was found at ($x_c = 2.9, z_c = -105.2$), which agrees with the apparent position of the sphere in Figure 4.14. The radius error ϵ_{radius} from the circle center to each data point is given in the Table 4.1, and the reader may appreciate the tiny (submillimeter) residual errors in this exercise.⁴

The tilts of lines from the circle center through the data points were compared to the 'rise/run' tilts computed from the fringe pattern. The errors are shown in the last column of Table 4.1, and even this crude 'rise/run' method of estimating the surface tilt from the fringe pattern proved useful in estimating tilts to within a couple of degrees. This approach was considered adequate for the tests reported in this chapter, but more accurate methods are developed in Chapter 6.

The polar profile data points are used again in the next section. Although presented here as the basis of an accuracy report, this special set of data points served in the early stages of the software development as a necessary check that the proper transformation and modelling equations were encoded correctly and behaving predictably. All intermediate calculations were manually (tediously) verified, and it proved very useful to submit these 'well studied' data to each new phase of the software development.

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⁴These data lay near the paraxial region of the recording and scanning lenses, where image distortion is negligible. In the corners of the film format, one could expect distortions to introduce errors of 1-2 mm in object coordinate determination [van Wijk, 1980].

Table 4.1: Object coordinate and tilt estimation test for the polar profile.

pt. #	IMAGE SPACE			OBJECT SPACE				ERRORS	
	line #	pixel #	fringe ϕ	x (mm)	y (mm)	z (mm)	tilt (deg)	ϵ_{radius} (mm)	ϵ_{tilt} (deg)
1	812	744	21.5	-77.5	-3.8	-109.6	42.2	0.09	-2.1
2	812	754	20.5	-71.0	-3.8	-104.2	37.0	-0.15	-0.8
3	812	766	19.5	-63.2	-3.8	-98.8	34.6	-0.26	-2.8
4	812	781	18.5	-53.6	-3.8	-93.5	27.7	-0.18	-0.9
5	812	801	17.5	-40.8	-3.9	-88.1	21.1	0.01	-0.6
6	812	830	16.5	-22.6	-3.9	-82.9	11.1	0.07	0.7
7	812	909	16.5	26.9	-4.1	-82.9	11.1	0.37	0.0
8	812	940	17.5	46.4	-4.2	-88.1	22.0	0.08	-1.6
9	812	959	18.5	58.5	-4.3	-93.5	26.4	0.23	0.1
10	812	975	19.5	68.8	-4.3	-98.8	34.6	-0.15	-2.8
11	812	987	20.5	76.6	-4.4	-104.2	37.0	-0.03	-0.9
12	812	998	21.5	83.9	-4.4	-109.6	42.2	-0.30	-1.9
mean magnitude:								0.16	1.3

4.6 Experiments in Endogenous Shape Modelling

In this section, a brief summary is given on the experiments in endogenous shape modelling. These were important exploratory studies in a direction that was eventually abandoned in favour of the propagated ordering method described in the previous chapter. These experiments are described in more detail in [Pekelsky, 1986b] and [Pekelsky, 1987b].

In Chapter 3, endogenous ordering is described as a method that determines the relative ordering of the moiré fringes without prior knowledge of the shape of the object. This differs in an important way from propagated ordering, where a pair of contours with known ordering is required to 'seed' the ordering process.

Endogenous ordering of a neighbourhood sequence in which all the member contours are unknowns would involve the following steps:

- Form a list of all the possible orderings for the contours in the sequence. Each ordering represents a test shape for the object.
- For each test shape, use all points on all the members in the sequence to fit the

best photometric model through the data. For each test shape, the model is adjusted to the measured reflectances on the surface of the object, according to the local orientation of the surface with respect to the illumination source.

- Compare the reflectance predicted by the model to the measured reflectance over the test surface for each test shape. Choose the case with the most consistent results as being the correct shape.

The key to the approach is finding an adequate photometric model. The Lambertian model given in Equation 3.26 on page 85 assumes an isotropic point-source of illumination reflecting from a perfectly-uniform diffuse surface. Certainly real lights and real surfaces are more complex than this, but is a simple model adequate for this application? Is it useful to introduce more parameters to handle systematic departures? For example, Phong's diffuse/specular model (Eq. 3.31 on p. 86) admits a specular component, but it does not explicitly address the nonuniformities in the illumination and the surface. As it turns out, the nonuniformities in this moiré application are too large to be ignored and too random to model.

Two scenarios were analyzed. The first was the polar profile shown in Figure 4.14. The surface along a polar profile has continuously varying tilts at a fixed azimuth angle. The twelve bright fringe crossings listed in Table 4.1 formed the neighbourhood sequence, and although there are theoretically 3^{12} shape cases, only three surfaces were analyzed: a convex sphere, a plane and a concave sphere.

The second scenario was a 'zonal ribbon' taken about the third central bright fringe in Figure 4.14. A spherical zonal ribbon has fixed tilt and continuously varying azimuth. Twenty-four points spaced at regular azimuth intervals of 15° were used. The true tilt for the surface along the third bright fringe was 34.6° . An unknown zonal ribbon can only be one of three possible shapes: convex, plane and concave (like the shapes shown in Figure 3.3 on page 76).

The painted sphere has an obvious specular 'hot-spot' at the top of the most central bright fringe in Figure 4.14. In the preliminary experiments, Phong's 3-parameter specular/diffuse photometric model was adjusted to the polar profile and zonal-ribbon data, but in both cases the best fit resulted when the specular

Figure 4.17: Endogenous shape experiments with the polar profile. Models for a convex, plane and concave surface were fitted to the twelve bright fringes crossing the profile indicated in Figure 4.14, and the predicted reflectance is plotted for each, along with the measured values.

fraction vanished (i.e., the model reduced to the Lambertian formula). In fact, specularity is a localized effect on curved surfaces under point-illumination, and a short distance from the hot-spot the reflectance is essentially Lambertian.

The results for the two data sets are shown in Figures 4.17 and 4.18. These figures show the predicted reflectance values for each of the three test shapes (convex, plane and concave), along with the measured reflectance values. The desired outcome is that the measured values closely match the values from one (and only the correct one!) of the test shapes.

For the polar profile experiment (Fig. 4.17), the plane surface can be rejected, but there is not a convincing distinction between the convex and concave results (convex is the correct choice!). Part of the problem is the chance selection of this profile on the object: both a concave and convex sphere of that size and position with respect to the camera and light source would tend to reflect the same amount of light into the camera for each position along the profile. Another profile, such as one with vertical orientation, would show a stronger disparity between the two cases.

In the zonal ribbon results (Fig. 4.18), the disparity mentioned in the previous paragraph is quite evident (i.e., convex vs. concave at the top and bottom positions). The effect on the predicted reflectances due to the change in orientation of

Figure 4.18: Endogenous shape experiments with the zonal ribbon. Models for a convex, plane and concave surface were fitted to 24 points taken at 15° intervals around the third bright central fringe in Figure 4.14. The predicted reflectance for each surface is plotted, along with the measured values.

the ribbon with position along the fringe is quite dramatic (i.e., the three models have well-separated curves). The concave case is clearly not matched to the measured values. Although there is good discrimination between expected reflectances for the convex and plane shape cases, the measured values do not support one significantly better than the other.

Figure 4.18 suggests a reason for abandoning endogenous photometric ordering: the data was too noisy to make reliable choices. Endogenous ordering calls for fitting a model to data that is widely distributed over the format of the topogram, but random variations in the measured reflectances spoil the prospects for this approach. In the next section, experiments are reported that indicate the source of the variations and the hopelessness of controlling them.

4.7 Problems with Moiré Instrument Illumination

The NRC Moiré Contourgraph uses an extended light source to generate moiré fringes with maximum contrast.⁵ The lamp used in the Contourgraphs is shown in Figure 4.19.

The photometric model assumes an isotropic point-source of light. Recall Equation 3.25 on page 85

$$\rho = aI_0 \frac{(\mathbf{N} \cdot \mathbf{L})}{r^2},$$

where ρ is the reflectance value, a is the albedo of the surface, I_0 is the nominal intensity of the light source, r is the distance to the light source, and $(\mathbf{N} \cdot \mathbf{L})$ is the scalar-product of the surface normal and the direction to the light source.

The reflectance due to an extended light source can be modelled by partitioning the large source into an ensemble of n point-sources:

$$\rho = a \sum_{i=1}^n I_{0i} \frac{(\mathbf{N} \cdot \mathbf{L}_i)}{r_i^2}, \quad (4.2)$$

⁵Extended line-sources are permissible in moiré fringe topography [Pekelsky and van Wijk, 1988], and in fact, the translating-screen Contourgraph uses two of these lamps aligned horizontally above the camera to boost the illumination for reduced exposure times.

Figure 4.19: Tungsten-halogen lamp used in the NRC Moiré Contourgraphs. The filament coil is 6 mm in diameter and 22 mm long. This 1050 watt line-voltage lamp, ANSI type ELJ, has a colour temperature of 3200°K and produces a constant 30,000 lumens over a 75 hour expected lifetime. The quartz envelope is shown in its recommended horizontal orientation, and the small test should be aligned away from the optical path to the object.

where now the intensity, distance and direction of each point-source must be considered. For the coiled filament, the nominal intensity of each point-source can be treated as an average value

$$I_0 = I_0/n, \quad (4.3)$$

but if several independent sources are involved, each I_0 must be calibrated separately. The other factors to consider are the distance and direction to each source, and how the resulting illumination variation departs from that of a single point-source.

In the case of the coiled filament (6 mm diameter and 22 mm long), the effects are quite small. For objects placed 2 m from the lamp, the variation in distance to points along the filament with respect to the midpoint is only 0.002%. The directional variation is less than 0.3°, which affects the reflectance model by less than 0.3%.

If the naked filament could be used, then one could expect excellent point-like illumination. However, other artifacts complicate the situation. The lamp is mounted in a housing with condenser-optics that image the filament into the exit slit of the housing, which is set parallel to the grid strings of the moiré instrument. The housing is shown in Figure 4.20.

The photometric model assumes the filament is in a fixed position at $(0, d, l)$,

Figure 4.20: The lamp housing used on the NRC Moiré Contourgraphs. The cover has been removed to show the lamp/reflector unit (left) and the pair of cylindrical quartz lenses (center) for focusing the filament through the slit (right). The housing improves the contrast of the moiré fringes by absolutely limiting the thickness of the line-source of light to the width of the slit, and by eliminating stray illumination reflected from the walls, floor, etc. where the Contourgraph is used

just above the camera. Instead, there is a metal plate with a horizontal slit through which the light is projected (Fig. 4.20). From the object position, the filament always 'appears to be in the slit' when, in fact, it is positioned about 150 mm behind the slit, with a pair of cylindrical quartz lenses between the slit and the lamp. The $1/r^2$ intensity falloff across an illuminated object is not as pronounced when the filament is further away. Compared to the center of the moiré screen, the illumination at the corners is 2% brighter than if the filament were actually in the slit position.

The cylindrical lenses cause anisotropy in the illumination distribution. The cylinder axes are parallel to the horizontal grid strings. In the vertical direction, the filament image is stationary in the slit, but the nominal intensity has a $\cos^4$ falloff (as much as 20%, depending on the angle from the optical axis of the lamp housing, which intersects the center of the screen). In the horizontal direction, the nominal intensity is constant, but the horizontal position of the filament image in the slit shifts with the object-point position. Across the width of the screen (450 mm) the shift is 27 mm, which causes variations in the apparent direction to the illumination (a 1-2% effect).

All these effects could be modelled, and the author considered a series of refinements that would result in a stronger correlation between shape and shading. The moiré instrument geometry is very stable, so it was realistic to formulate a customized photometric model for each Contourgraph unit. However, the return on the effort in calibrating and parameterizing these phenomena was overshadowed by a discovery that led to the abandoning of the whole approach: the short-lived lamps had large, random, anisotropic light output! Goniometric measurements on seven lamps and housings confirmed the cylindrical lens behavior, but also showed that the quartz envelope of the lamp (especially the teat, see Fig. 4.19) imposed an individual 'signature' on the illumination propagating from the filament. The illumination modulation was typically a few percent, but one lamp showed a 20% dip in output over the center of the moiré screen (the teat was inadvertently pointed in the forward direction).

Modelling each lamp was considered impractical. The author concluded that moiré fringe ordering based on photometric modelling would have to operate in regions small enough for the illumination to be sufficiently isotropic and point-like.

4.8 Experiments in Propagated Shape Modelling.

The experiments in the previous sections showed that the photometric analysis region had to be small, but the question remained: how small? The author wanted experimental evidence that photometric ordering was possible with typical moiré topograms, before developing extensive software to embody the theoretical algorithm presented in Chapter 3.

The smallest useful region was merely a pair of image patches on adjacent fringes. The method described in Section 3.3.7 on page 92 was applied to a random selection of patch-pairs on test topograms. For each pair, one patch was selected as having a known shape (*i.e.*, orientation) and the local photometric model parameter was determined. Then the expected reflectance in the neighbouring patch was calculated for the three possible test shapes. The error between the calculated and measured values was used to form a probability that each shape was the correct shape.

Similar tests were done for the painted sphere, and some human-back topograms. The tediousness of the measuring and calculations restricted the experiment to about 20 patches. The results of this experiment have been published in [Pekelsky, 1986b] and [Pekelsky, 1987b], therefore the photographs of the test patches and the measurements will not be reproduced here.⁶ Briefly, however, the results can be summarized as follows:

- in general, the correct test shape for the unknown patch had the lowest reflectance estimation error (*i.e.*, the photometric ordering operator tended to choose the correct shape).
- in general, the process was independent of which patch in the pair was selected

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⁶Although these primitive results were exciting and significant evidence that the method worked, there are better results (based on thousands of patches) to support this in later chapters.

to have the known or unknown shape.

- the method worked for painted wood and live human skin.
- the method did not work well near surface specularities.
- the method worked well for both stationary and translated screen topograms (i.e., with grid strings resolved or smeared, respectively).

The experience of manually working the measurements and calculations for these tests was also an important outcome. The author gained an intimate insight into the subtleties of the problem. The design issues and constraints on the data are summarized in the next section.

4.9 Conclusions from the Design Studies

In the beginning of this chapter, it was stated that the purpose of these studies was threefold: to identify the design issues, to investigate shape and shading measurement methods, and to specify control restrictions on the data. Also, some specific questions were posed on the linearity of the measurements, the complexity of the required photometric model and prospects for endogenous ordering. The findings and conclusions are summarized in this section.

4.9.1 Restrictions on Topogram Data

The analysis in the previous sections showed that considerable care must be exercised in forming the moiré topogram if good data are to be obtained for subsequent analysis. This calls for good photogrammetric and photometric control.

Photogrammetric control refers to ensuring a stable, well-calibrated moiré geometry. The critical dimensions must be known to better than 0.05% if the object coordinates are to be recovered to 0.1%. Fiducial targets with known object coordinates are essential in this respect. The author confirmed that the NRC instruments have the required accuracy.

Photometric control refers to ensuring that adequately uniform illumination is used when photographically recording the topogram and when scanning the topogram to form the digitized image. Care must be taken to control the exposure in both of these operations to optimize the signal into the linear range of the detector. The author attempted to provide the best illumination conditions at every step. Lamps were carefully aligned to optimize uniformity. The author also verified the linearity of the detectors and ensured that he always worked within the linear range.

4.9.2 Choosing the Photometric Model

The Lambertian model was used exclusively after these experiments. An enormous advantage over other models is its simplicity. It involves only one parameter and this is easily determined from the closed-form expression in Equation 3.33 on page 93.

The outstanding concern was how well non-diffuse surfaces could be processed, especially near specularities. Surfaces with a specular component can be modelled as purely diffuse over a sufficiently small region. Near a specularity, the local model acts as if the nominal illumination has been increased. The propagated ordering method uses localized models for small regions, forming a unique, independent model for each region. Thus, even near a specularity, one may obtain reasonable performance. The tests show that patch-pairs define a small enough region for this principle to apply.

Another favourable aspect is that specularity can be considered rare in the sense that if data points are distributed uniformly on a curved surface, very few of them will fall in a specular region. Those that do can be ignored or excluded. The former tactic relies on average behaviour on the remainder of the surface to minimize specular effects in choosing the correct test shape. To exclude points, one can test for the specularity condition

$$\mathbf{N} \cdot \mathbf{H} < 1 - \epsilon_s, \quad (4.4)$$

where $\mathbf{N}$ is the surface normal, $\mathbf{H}$ bisects the directions to the illumination and viewer, and ϵ_s is the specular threshold. The threshold is determined empirically for each material. For example, calculations for the painted sphere show the hot-

spot is restricted to surface normals within 3° of $\mathbf{H}$, so $\epsilon_s = 0.002$ would be a suitable threshold for this material.

4.9.3 Prospects for Endogenous Ordering

Although endogenous ordering of moiré fringes was abandoned in this application because of illumination non-uniformities, the author feels this method has important potential. There may be other applications that use particularly uniform illumination to form the moiré fringe pattern. In this case, the advantage of shape determination without prior knowledge may be significant. There may also be ways to implement endogenous ordering in the presence of non-uniform illumination that the author has overlooked. An untried approach is suggested in Chapter 10.

4.9.4 Issues in Automated Fringe Ordering

The manual aspect of these tests made the author keenly aware of the need to automate the ordering process. He also learned to appreciate what aspects would be difficult, and which needed further investigation. There are three main topics to pursue in the chapters that follow:

- Neighbour Discovery. Human operators have no difficulty in recognizing which contour is the neighbour to a given point on another contour. A refinement to this problem is to select the nearest neighbour. During photometric ordering, this operation is performed at every analysis point in the topogram (hundreds, even thousands of points). Thus, it is essential that the software undertake this pattern-recognition task without operator intervention. This design issue is the subject of Chapter 5.
- Shape Determination. An algorithm to generate a list of the trial shapes for a specified sequence of contours is needed. Methods must be developed to measure the local shape from the fringe data. The tilt and azimuth must be calculated, and the object coordinates determined. This must be done for

every test shape at every analysis point. Thus, efficient fully-automated software is most important for the shape measurement elements of photometric ordering. The topic is pursued in detail in Chapter 6.

- **Shading Determination.** The digitized topogram can be very noisy in terms of measuring the reflectance. A robust method for obtaining a good estimate of the surface shading must be automated in software. It must work in the presence of the moiré fringes, and not be unduly sensitive to image artifacts or fringe shape. Chapter 7 is devoted to finding the solution to this problem

Figure 3.5: A simple map to illustrate the Ordering Schedule concept. The map consists of a series of concentric contour loops $\mathcal{M} = \{C_1, C_2, C_3, C_4, C_5, \dots\}$. Only the first five loops are shown in the figure.

and so on. By writing a 'state diagram' for the map-list at the completion of each step (using "!" and "?" to denote known and unknown ϕ 's, respectively), one obtains an impression of the processing flowing (or propagating) through the map:

step	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	...
0	!	!	?	?	?	...
1	!	!	!	?	?	...
2	!	!	!	!	?	...
3	!	!	!	!	!	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

A series of process steps like this will be referred to as the *ordering schedule* to emphasize that a given step may not be executed before the previous steps in the schedule have been completed. In general, there are Q steps in the schedule V , and the parameters of each step are listed in each control vector V_q :

$$V = (V_1, V_2, \dots, V_q, \dots, V_Q). \quad (3.22)$$

The q th control vector V_q dictates which contours are to be analyzed at that step and how they are to be utilized.

Generating the schedule is rather straight-forward, but can be tedious. For an optimized schedule, an exhaustive search of the map may be conducted to form a list of all gradient-neighbour pairs, then this list can be organized into a schedule

3

Micro-D

Chapter 5

Automated Gradient Neighbour Discovery

5.1 Introduction

The Photometric Ordering Algorithm described in Chapter 3 requires the identification of analysis points in the image that share certain shape and shading relationships. The sequence of points where a gradient curve intersects neighbouring contours has the required photometric significance. This chapter discusses the methods and problems of automating the discovery of points that are *gradient neighbours*.

The identification of gradient neighbours is a fundamental process in the photometric ordering of moiré fringes. The reliability of both the automated shape determination (Ch. 6), and the automated shading determination (Ch. 7) depends upon the robustness and accuracy of the search for analysis points.

The next section introduces the concept of *photometric significance*, which requires that analysis points for photometric ordering lie on contours and be close to one another. Gradient neighbours are shown to meet these requirements.

The subsequent section defines the scope and format of the data structures established by the author to support the photometric ordering analysis. The information is organized into two generic structures: images and lists. The characteristics of the digital image processing (DIP) hardware used by the author further influenced the

detail of the data structures, in particular regarding the storage, display and manipulation of the images. Initialization of the structures to furnish the data required by the Photometric Ordering Algorithm is also described.

The last section of this chapter reports on the tactics used to discover the gradient neighbours of a specified seed-point, and describes the embodiment of this non-interactive technique in automated software routines. The section concludes with some remarks on the execution speed and robustness of the programs.

5.2 Analysis Points with Photometric Significance

5.2.1 Criteria for Photometric Significance

The Photometric Ordering Operator links the shape of an object with the shading on its surface. Thus, the analysis must be conducted on sequences of points that have the following significant relationships:

- **Shape Significance.** The points must lie on a sequence of contours, with one point per contour. One or more of the contours in the sequence will be ordered by the analysis, and each possible ordering of the contours defines a unique shape for the surface.
- **Shading Significance.** For the points to share the same shading model parameters, they must be sufficiently close to one another such that the illumination and reflectance properties of the surface are constant.

Figure 5.1a illustrates that arbitrary contour neighbour points have the required shape significance, but may lack the shading coherence that is shared by the 'nearest' neighbours shown in part (b) of the figure. A secondary advantage of using nearest neighbours as opposed to arbitrary neighbours, is that the former provides a definitiveness of selection, which leads to a deterministic neighbour-search algorithm (i.e., the nearest neighbour is unique).

Figure 5.1: Analysis points for Photometric Ordering (a) Points A and B are on neighbouring contours C_1 and C_2 , respectively (Shape Significance), but may be too remote to share the same surface illumination and reflectance properties (b) Points A and B are now contour neighbours that are also nearest neighbours (Shading Significance) The curve G_1 passing through points A and B is a surface gradient curve

5.2.2 Gradient Neighbours as Significant Points

The term *nearest contour neighbour* is apt to be confusing (e.g., the nearest point on the next contour?, or the nearest point on the same contour?). A less ambiguous term that is topographically descriptive is the *gradient neighbour*. Gradient neighbours are those points where the surface gradient curve intersects a sequence of neighbouring contours.¹ In Figure 5.1b, the curve G_1 passing through points A and B is a surface gradient, and A and B are said to be gradient neighbours.

The surface gradient always intersects the contours orthogonally, and in most cases the shortest distance between two contours is along the gradient curve. By definition, gradient neighbours meet the requirements for shape significance, and in most cases they are close enough to ensure the necessary shading significance.

The gradient neighbour is the nearest neighbour as long as the surface of the object is sufficiently planar. This can be stated formally with reference to Figure 5.2. Consider a pair of contours C_1 and C_2 with A a point on contour C_1 . B is the point on C_2 where the gradient G_1 through A cuts C_2 , making B the gradient neighbour of

¹The term 'gradient contour neighbours' is perhaps more topographically descriptive, but it was considered too ponderous for frequent use.

Figure 5.2: Gradient neighbours as nearest neighbours. A is a point on contour C_1 . B is where the gradient G_j through A cuts contour C_2 , making B the gradient neighbour of A . (a) Perfectly planar surfaces have straight contours: the contour curvature at B has an infinite radius $r_B \gg |AB|$, and so B is also the 'nearest neighbour' to A . (b) Sufficiently planar surfaces have mildly curved contours: the curvature at B has a radius $r_B \geq |AB|$, and B is still the nearest neighbour to A . (c) Insufficiently planar surfaces have strongly curved contours: the curvature at B has a radius $r_B < |AB|$, and now B' , rather than B , is the nearest neighbour to A .

A . B is also the nearest neighbour of A if the contour curvature at B has a radius $r_B \geq |AB|$ (i.e., the distance from A to B).

Perfectly plane surfaces have contours that are straight, parallel lines, and in this case the gradient intersects the neighbour contour at the nearest point (see Fig. 5.2a). As the surface departs from planarity, the curvature of the contours decreases (Fig. 5.2b), but as long as its radius exceeds the distance between the contours, then gradient neighbours will be nearest neighbours. Figure 5.2c shows the case where the condition breaks down and a different point is a closer neighbour than the true gradient neighbour.

For photometric ordering of moiré fringes, the breakdown of the equivalence between gradient and nearest neighbours is not very serious for several reasons. First, it is a rare occurrence. For most of the potential analysis points along a pair of contours, there are usually very few places where the breakdown occurs.

Figure 5.3: Breakdown of the gradient neighbour condition on a ridge. Three contours C_1 , C_2 and C_3 traverse the ridge G_j is the gradient through the seed-point A , and B is the true gradient neighbour of A . Instead of finding B , the search routine discovers B' to be the nearest neighbour to A . However, B' lies on gradient curve G'_j that cuts C_1 at A' . Often A and A' are indistinguishable, so that B' can be considered an approximate gradient neighbour to A .

Second, it can be avoided. By comparing the distance to the discovered point with the radius of its contour there, one can omit problem-points from the analysis. Third, using the discovered nearest neighbour for photometric analysis is reasonable. This is because the nearest neighbour is, in fact, the gradient neighbour of a point sufficiently indistinguishable from the seed-point (Fig. 5.3).

5.2.3 Neighbours with Quantized Coordinates

The author has laboured to justify using the term *gradient neighbour* instead of *nearest neighbour*, but in practice the issue is academic. From the examples drawn in the figures supporting the preceding discussion, the reader might wrongly conclude that there are significant differences that warrant this care. In the next section, the digital representation of the contour curves used in the analysis is described in detail, but a few remarks are appropriate at this juncture.

Figure 5.4: Digital image representation of contour curves. Shown here are the same contours drawn in Figure 5.3, now delineated by a chain of square tiles. Each tile is determined to lie on the curve if the original continuous curve passed through that tile. Note the tiles that correspond to points A , A' and B' in the previous figure, and that in the digital image, point X is now the closest neighbour to A .

The continuous, analytic curves drawn in the preceding examples are misleading in the exactness of their shape and definition. The actual contour data consist of sparse *turning-points* or *break-points* in a piecewise-linear curve, so that there is considerable uncertainty in the interpolated shape of the surface. In the example of Figure 5.3, A and A' probably are not distinguishable, and B' (or indeed *any* point on C_2 near B) could be the gradient neighbour of A . The reader should appreciate that the curves are constructed with an artificial precision in the figures, whereas they really are quite crudely known in the moiré topogram image.

The vagueness of the contour delineation is further exaggerated when it is depicted in the quantized coordinates of the digital image. Figure 5.4 illustrates this with a spatially quantized version of the contours previously drawn in Figure 5.3. In this digital image, the contour curves are reconstructed as a ragged chain of the square tiles.² In drawing the digital contour curve, a tile is included in the chain if

² Tiles are reconstruction patches used to display the pixels of the digital image. Pixels are used.

the continuous version of the curve passes through it. Figure 5.4 also supports the earlier remark that points A and A' are likely to be indistinguishable, and that B is a valid candidate as gradient neighbour to A . Indeed, from the digital distortion of the curves, a new point X arises as the nearest neighbour to A . This departure from the niceties of the continuous domain typifies working with digital imagery. Coping with such problems is a recurring topic in the remainder of this and subsequent chapters.

5.3 Data Structures to Support the Analysis

The remainder of the thesis reports on specific techniques, algorithms and software developed to investigate the concept of photometric ordering of moiré fringe contours. In this section, the fundamental data structures used by the author to support the analysis are described.

5.3.1 Data Structure Requirements

5.3.1.1 Scope

Photometric analysis relates shape and shading information to determine the ordering of moiré fringe contours. There must be adequate data of both types to support the analysis.

The shading data consists of the measured brightness (sometimes called the *reflectance* or *gray-level*) at various points in the scene (*i.e.*, in the moiré topogram image). Preferably, the measurements are taken at regular intervals to sample the reflectance in a dense net or array of positions, and are stored as a two-dimensional array of shading values (*i.e.*, a digital image). Also required is a list of shading calibration data that establishes the illumination geometry of the moiré instrument.

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to define an attribute at *points* in the digitized image, such as the local brightness of the image. Pixels are spaced by the sampling interval, and tiles are centered on the pixel positions and butt one another. The author uses a square sampling interval to define the pixel positions, but other shapes are possible.

The shape data consists of the contour curves, traced out to form a topographic map. Like the shading data, the map can also be spatially quantized into a digital image, with the array values set to the local contour label code (e.g., set to "E" if contour C_E crosses the quantization interval and set to "0" if no contour crosses it). The data for an individual contour can also be represented as a list of the coordinates of those pixels that belong to the contour. The membership list is usually *ordered* (i.e., pixels are specified in a sequence that traces out the contour from beginning to end), and the list may be reduced to retain only the break-points in a piecewise-linear approximation to the curve. Added to the shape-data requirements is a list of the instrument geometry calibration data used to establish the scale and coordinate system of the map.

5.3.1.2 Image Data Format

The data is structured into two generic formats: images and lists. The following three image structures required for the analysis are shown in Figure 5.5.

- **The Shading Image** (Fig. 5.5a) is composed of measurements of the average brightness of each pixel, and is usually stored as positive integer values (8-bit values spanning a range of 0 to 255 is typical).
- **The Map Image** (Fig. 5.5b) is zero except where a pixel belongs to a contour, and then the pixel is assigned a value according to a label code (usually an integer, and in the author's case, 7-bit codes were used to identify up to 127 contours). Note that the bold labels on some pixels in the figure are for illustrative purposes only, and identify the break-points that define the piecewise-linear contours.
- **The Diagnosis Image** (Fig. 5.5c) is a 'scratch-pad' used in photometric ordering. It is usually zero unless a pixel has been tagged by the analysis process. The tag may be binary (0 or 1), or use several bits to represent a range of values used to discriminate between pixels to be included/excluded from various processing operations. In the example shown in Figure 5.5, the tags in

(pixel)	1	2	3	4	5	6	7	8	9	10	11	...
(line) 1	68	62	54	43	29	42	52	58	67	72	87	
2	79	69	67	51	42	35	47	59	62	75	79	
3	65	81	70	67	53	48	29	45	52	65	68	
4	59	65	75	72	61	52	44	31	43	39	43	
5	42	60	70	83	75	65	68	57	34	28	31	
6	37	53	63	79	82	73	79	66	47	52	43	
7	29	47	52	76	77	85	86	68	65	63	57	
8	26	27	43	51	72	84	97	79	76	70	72	
9	29	19	39	35	56	71	82	86	83	72	86	
10	41	36	32	27	37	56	62	76	63	57	49	
11	63	56	41	34	29	33	47	70	59	42	37	

(a)

(pixel)	1	2	3	4	5	6	7	8	9	10	11	...
(line) 1					G							
2						G						
3	W	W					G					
4			W					G				
5				W					G	G	G	
6					W							
7						W						
8	P						W					
9		P						W	W	W	W	
10			P	P								
11					P	P						

(b)

(pixel)	1	2	3	4	5	6	7	8	9	10	11	...
(line) 1												
2												
3												
4												
5												
6												
7						1						
8					1							
9				1								
10			1									
11												

(c)

Figure 5.5: The three image structures required for photometric ordering. (a) The Shading Image. (b) The Map Image. (c) The Diagnosis Image. The properties of each are described in the text.

part (c) are set to indicate which pixel-labels in part (b) were examined in a search operation, starting from the break-point at position (10,3) on contour C_P , and terminating with the gradient neighbour discovered on contour C_W .

It is important to stress that the three images should be registered to a common coordinate system so that the shading, map and diagnosis data can be correlated with minimal manipulation. In Figure 5.5, only the upper-left corner of a larger image is depicted, and individual pixels have coordinates (line,pixel).³ It should be appreciated that the column-index (pixel number) increases from left to right, but that the row-index (line number) increases from top to bottom. Transformations to Cartesian coordinates (x, y) must take this convention into account.

5.3.1.3 List Data Format

Associated with the image data structures are the lists. In this project, several lists were defined to serve different aspects of the processing. The Map List and the Contour List defined here are needed for the algorithms and software described in Chapters 5 to 7. Other list data, such as instrument calibration coefficients and processing schedules, are presented in Chapters 8 and 9.

- **The Map List** organizes the attribute data for all the contours in the map. It allows, for example, when searching about the image and discovering a labelled pixel, to efficiently examine the properties of the contour (such as its moiré fringe extrema-type (MFE-type), and whether it has known/unknown elevation), and to refer to the list of member pixels belonging to the contour. Figure 5.6 shows a Map List that corresponds to the image data in Figure 5.5.
- **The Contour List** is simple, but plays an important role. It lists the image coordinates of the pixels that are members of the contour. It may be a dense

³The author adopted the term *line and pixel coordinates* to conform to DIPIX software written for satellite image applications that traditionally prompt for the coordinates of a pixel point by its 'line and pixel number.' The term *row and column coordinates* avoids the ambiguity, but software conformity was deemed more important than clarity. The author will strive to embed adequate context to avoid any confusion between references to the value of a pixel and its position in the image.

Map List				
Map Name: BOY				
#	label	MFE	ϕ	C-List
1	G	dark	14.0	BOY 1
...	...	...	...	...
7	W	bright	13.5	BOY 7
...	...	...	...	...
12	P	dark	?	BOY 12

Figure 5.6: A Map List of contour attribute data. This map, named 'BOY', has twelve contours. The contour labels correspond to those in Figure 5.5. The moiré fringe extrema-type (MFE) specifies whether the contour was traced from a bright or dark fringe. The moiré fringe number ϕ is known for two of the contours, the rest must be ordered. The last entry is the name of the pixel-list file for each contour.

list comprised of all the labelled pixels in the Map Image that belong to a given contour. Usually it is a sparse list of the break-point pixels, so that not every labelled pixel in the Map Image needs to be listed.⁴ Figure 5.7 gives the structure of the Contour Lists named in Figure 5.6. Not indicated in this example is the fact that some lists may only have a few members while others may have hundreds, depending upon the complexity and extent of each contour.

In photometric ordering, the Contour List is accessed frequently to determine the local shape of the contour about some point on it. It is faster and more accurate to compute the shape from the local members read from the Contour List, than from contour pixels tracked in the Map Image. The photometric ordering process is not dependent upon whether full or break-point lists are used, as long as all the members of the contour can be identified. The author chose to design the algorithms to work with break-point lists because this is the usual form in which digitized topographic contours are recorded. Break-point representation can compress the size of the full list by one or two orders of magnitude.

⁴As will be seen, the break-points serve as seeds for piecewise-linear or spline reconstruction of the contour curve when creating the Map Image. Thus, the full list of members can be derived from the sparse list.

Contour List		Contour List		Contour List	
Name: BOY.1		Name: BOY.7		Name: BOY.12	
line	pixel	line	pixel	line	pixel
1	5	2	1	8	1
5	9	9	8	10	3
...	...	...	...	...	...
23	59	38	73	57	103
-1	-1	-1	-1	-1	-1

Figure 5.7: Contour Lists of member pixels, corresponding to those named in Figure 5.6. The naming convention links each contour (BOY *i*) to the map (BOY). In this example, the member list contains only the break-points of the piecewise-linear contour; they correspond to the curves drawn in Figure 5.5. The lists have lengths that depend on the complexity and extent of the individual contour, and a (-1, -1) code denotes the end of the list.

5.3.2 Hardware Considerations

So far, the data structures have been described generically, without reference to the hardware used to store and manipulate them. The author implemented photometric ordering software on a specific DIP system described in Chapter 4 (DIPIX Aries-II, with PIP 11/73 host computer and RSX-11/M v3.2 operating system), using many built-in utilities for image manipulation, file management and software development. All of the algorithms described in this report can be embodied on other machines. Some of the features of the author's equipment proved useful, but are not essential to the task.

The DIPIX system has a large video random-access memory (VRAM) for storing and displaying image data. Applications software (such as the photometric ordering tasks) process the contents of video memory while it is being displayed, thus the effect of a task is observed in real-time.⁵ The display has many features such as zoom, pan and colour selection, but particularly useful is the facility to create 'layered' images; and to control the layers (called *fields*) independently. This allows the three required images (Shading, Map and Diagnosis) to be superimposed (registered), so that by addressing a single *ixel*⁶ one can examine and modify the

⁵A *task* is a software program

⁶DIPIX coined the term *ixel* to mean 'image display pixel'. A pixel usually bears one image attribute (a shading value, a label, etc.), whereas the video-memory *ixel* is composed of compound

Figure 5.8: The three fields of the video-memory image. The feature field holds the 8-bit shading data, the theme field holds the 7-bit contour data, and the 1-bit cursor field is used as the diagnosis scratch-pad.

three attributes from the same memory location. Figure 5.8 shows the three fields of the DIPIX video memory: feature, theme and cursor, in which the shading, label and diagnosis data, respectively are installed.⁷

Unlike most commercial DIP systems, the size of the images and the depth of the fields in the DIPIX video memory are independently programmable. Up to eight concurrent images are allowed on the Aries-II system, and the dimension parameters for each is maintained in video memory by the DIPIX utilities: start line and pixel, number of lines and pixels, pixel depth (1, 2, 4, 8, 16 or 32 bits); one to three optional feature fields of selectable depth (1 to 8 bits), an optional theme fields for generating different components of the display (such as scenery with a coloured overlay of thematic information).

⁷The terms *feature* and *theme* are used in satellite image processing, for which the DIPIX system was originally intended. Feature and theme image files had a 'FF' or 'TT' embedded in their names, which the author had to adopt for his shading and label image files if the DIPIX utility software was to be easily used.

field up to 7 bits, and an optional 1-bit cursor field.

From this tremendous flexibility, the author used only the configuration shown in Figure 5.8, namely an 8-bit feature field for shading data, a 7-bit theme for contour labels, and a cursor bit for displayed diagnostics. The fields are assigned (i.e., shading $\Rightarrow$ feature field, etc.) to capitalize on the hardware functionality. The feature field (shading values) is displayed through colour look-up tables (CLUT's) to the red/green/blue guns of the colour monitor, and can be subjected to a battery of image enhancement hardware/software features available on the DIPIX system. Contour labels are ideally displayed from the theme field, since each theme (now, a label) is assigned a saturated colour from a palette (white, red, green, yellow, magenta, ...), and thus pixels on contours are displayed in that colour as an overlay to the shading background. The overall effect is to see the moiré topogram image with coloured contours boldly traced on it.

One can toggle the features and individual themes on/off if a particular situation calls for this. For example, it is convenient to monitor the gradient search operation with the shading field turned off (creating a black background), the seed-contour displayed in red, and all the other contours displayed in yellow. The search tracks are traced using the cursor bit (white), and special hardware features for this scratch-pad bit allow global erasure, manual drawing, and other operations. Figure 5.9 shows a few contours, plus some typical processing diagnostics traced into the cursor bit.

A further consideration for assigning certain data types to specific ixel fields is the way in which the DIPIX system stores image files on disk and loads them into video memory. Feature files are always arrays of 8-bit shading values, and can be loaded into an ixel feature field of 1 to 8 bits by a utility that masks off the unused least-significant bits. Themes are treated quite differently, as they are stored in files as 1-bit slices with up to eight themes per file. The image-load utility slices the requested theme from a file and writes its user-specified theme number into the ixel theme field. The cursor field is deemed to contain only disposable information, and no DIPIX utility exists to save this field on disk.

Figure 5.9: Contour images and processing diagnostics. Each contour has a unique theme number and display colour, which can be interactively toggled by the operator (or by an applications task). The 'web of lines' and the pair of 'snow-balls' are processing diagnostics traced into the cursor bit by software under test by the author. For this B&W image, the shading data was toggled off, and all the contours were set white, like the cursor.

The author uses a 16-bit ixel depth,⁸ and the bit assignment to words in video memory is shown in Figure 5.10. In Fortran, regular integer values are stored in two bytes (1 word = 2 bytes = 16 bits), thus the entire ixel can be efficiently fetched, altered and stuffed into video memory as an integer variable. Data values are packed and unpacked into the required field of the ixel using binary masking and shift operations available in Fortran.

A final hardware consideration is access to the video-memory image by the applications task. Digital images can require array storage that far exceeds the virtual address capability of the PDP-11 architecture.⁹ DIPIX allows an applications task to look into video memory through a 4K-word buffer or 'window' that is positioned in video memory by an address-offset register (AOR). A DIPIX utility returns the

⁸Some DIP systems support only 8-bit ixel depths. This limitation can be overcome by using shallow fields (say, 4-bit shading, 3-bit contour labels and a diagnostic bit), or by manipulating two 8-bit images (e.g., one for shading, and one for labels and the diagnostic bit).

⁹The PDP-11 family is limited to 1-word (16-bit) addressing of 64K words of virtual memory, and extended memory mapping is limited to 24-bit addressing of 16M words. The multi-spectral satellite images for which the DIPIX system is designed can be much larger than 16M words, therefore DIPIX uses a 32-bit address to locate the video-memory window.

Figure 5.10: The bit assignment for fields in the 16-bit ixel. The feature field is stored in the upper byte (bits 8 to 15), and contains the 8-bit shading data. The theme field spans bits 1 to 7, and can discriminate up to 127 unique contour labels. The cursor field is the lowest bit in the ixel.

starting address and ixel depth of the desired image in video memory to the user-task, which can then compute the AOR value to align the front of the window to a particular (line, pixel) in video memory. By making the window 4K words wide, a task can efficiently operate on many pixels before having to reposition the window. The working principle of the video-memory window is shown in Figure 5.11.

5.3.3 Creating the List Data

An assumption made at the outset of this research into contour ordering was that the contours had already been extracted by some other process, such as an automated fringe-tracing program. To test the concept of photometric ordering, access to contours that have been digitized from moiré topogram images is imperative.

To create the required list data, the author wrote a simple program called Task JSP ("Store Points")¹⁰ to facilitate interactive contour tracing from displayed images. DIPIX utilities are used to create the image data structures in video memory and load the Shading Image into the feature field of the ixel. The main interaction in Task JSP calls for the operator to move the cursor to the next point on the contour and trigger the storage of the line and pixel coordinates in a file. The software draws a piecewise-linear contour in the cursor field as a reminder of which contours have been digitized. It also handles the file overhead automatically, producing a sequenced file name for each contour, opening the files, writing a header

¹⁰To distinguish his programs from the DIPIX utilities bearing two-letter names, the author usually put a 'J' before a pair of letters to form the task name.

Figure 5.11: Principle of a window from an application's task into video memory. The DPICOM is a 4K-word common block in the user task that is assigned by the task builder to a hard-wired location (XXX) in the task address space. The dual-port interface (DPI) polls address (XXX+4K-1) to read the AOR value at the bottom of the block, and maps the (4K-32)-word window into the video RAM address space according to the AOR value. The user task can manipulate the AOR registers to slide the window through video memory. The task then can directly read/write data to the windowed portion of video memory. Note that 32 words of the block are not available, 2 are for the AOR and 30 are reserved.

(with an option for comments by the operator), and closing the file at the end of each contour. A sample contour-list produced by Task JSP is shown in Figure 5.12. It took about half a minute to trace this contour (the rate-determining-step is definitely the operator's skill in moving the cursor in the display!).

The Map-List is created by manual editing of a file. Figure 5.13 is an example that has been transferred verbatim into this document from the data used to test Task MFO (see Ch. 9). The first record allows a task to skip over the records of the variable-length header, which permits the file to be flexibly self-documenting. From the file name in the first record, one infers: the disk directory is [200,200], that it is an index file (.IDX) for Task MFO (FO), that the image area name is ABC, and that this test-set of contours is called SOME. Note that the individual contour file-names (ABCiiii.SPF) listed in the index file also refer to the area name ABC, and to Task JSP that created them. Note that the comments in the file are for human consumption; they help to recall that these contours trace the bright and dark fringes about the scapula 'hills' on a human back. The corresponding Map Image for this list is shown in Figure 5.14.

5.3.4 Creating the Map Image

With the list data in hand and the Shading Image loaded into the feature field of the video-memory image, the only outstanding data structure to be created is the Map Image. The author specifies the functionality of Task JDC ("Draw Contours") as follows.

Through initial operator interaction, the task obtains the name of the Map File and of the image in video-memory, and verifies the existence of both. The Map List is opened, the header is skipped, and the contour records are processed in order. Each Contour List file is opened, and the coordinates of the break-points are read into a buffer. Task JDC then uses these break-points to draw the contour curve in the theme field of the video-memory image. The first version of Task JDC used piecewise-linear approximations when drawing, but the current version uses cubic Bessel spline interpolation [de Boor, 1978]. The operator may elect to control the

```

TOP OF FILE
1, 7 = # of header records
2, file name = ABCSPAA07.SPF
3,          CONTOUR 4.(W)
4,
5,
6,
7,-----end of header -----

LINE#  PIXEL#
540    818
613    809
652    791
670    757
672    708
652    670
605    632
567    613
540    606
491    608
464    618
448    638
438    663
405    718
403    743
429    777
459    801
503    816
540    818
-1     -1
END OF FILE

```

Figure 5.12: A Contour List produced by Task JSP. This is from an early software effort (with notably crude header information), but like all files created by the author's tasks, it allows a flexible number of header records that can be skipped by subsequent tasks. Note that the first and last points have the same coordinates, denoting a closed-loop contour. The code $(-1, -1)$ was used to mark the end of the list, rather than checking for EOF (end-of-file), so that subsequent tasks could be tricked to process partial contours from the list by manually inserting $(-1, -1)$ in the midst of the list.

TOP OF FILE

13= # of header records

File: [200,200]ABCFOSOME.IDX

This is a Master Index File. It contains a list of records, and each record contains delineation information for a single contour in the Map. The order of the records (1,2,3...) is just the Contour Number. Note that the Contour Label is the symbol used in the image overlay (On the ARIES: the 'theme number'), and that this may be a different value than the Contour Number. The Moire Fringe Extrema (MFE) type is 1 for a maxima (bright fringe), and 0 for a minima (dark fringe). The File Name specifies where the list of contour points can be found: these points are the 'turning points' for the piecewise-linear representation of the contour, and are used by Task JDC to display the contours on the VMA image for subsequent processing steps.

File Name	Number	Label	MFE	Comments
'ABCSPAA01.SPF'	1	1	1	top of the left scapula (hill)
'ABCSPAA02.SPF'	2	2	0	2nd down on left scapula
'ABCSPAA03.SPF'	3	3	1	3rd down on left scapula
'ABCSPAA04.SPF'	4	4	0	4th down on left scapula
'ABCSPAA05.SPF'	5	5	1	5th down on left scapula
'ABCSPAA06.SPF'	6	6	0	6th down on left scapula
'ABCSPAA07.SPF'	7	7	1	7th down on left scapula
'ABCSPAA08.SPF'	8	8	0	8th down on left scapula
'ABCSPAA09.SPF'	9	9	1	1st to loop around both hills
'ABCSPAA10.SPF'	10	10	0	2nd to loop around both hills
'ABCSPAA11.SPF'	11	11	1	top of the right scapula (hill)
'ABCSPAA12.SPF'	12	12	0	2nd down on right scapula
'ABCSPAA13.SPF'	13	13	1	3rd down on right scapula
'ABCSPAA14.SPF'	14	14	0	4th down on right scapula
'ABCSPAA15.SPF'	15	15	1	5th down on right scapula
'ABCSPAA16.SPF'	16	16	0	6th down on right scapula
'ABCSPAA17.SPF'	17	17	1	7th down on right scapula
'ABCSPAA18.SPF'	18	18	0	8th down on right scapula
'ABCSPAA19.SPF'	19	19	1	3rd to loop around both hills
'ABCSPAA20.SPF'	20	20	0	4th to loop around both hills

END OF FILE

Figure 5.13: A Map List: designed for Task MFO. The file is designed for human and machine consumption. It is self-documenting and has structured access to the data. The contents of the file are elaborated upon in the text.

Figure 5.14: The Map Image for the Map List shown in Figure 5.13. This overlay of twenty closed-loop contours was drawn in video memory by Task JDC, using the contour files named in the Map List. The contours span the moiré fringes on the upper portion of a human back, which was one of the test surfaces used in this research.

contour drawing parameters (display colour, theme number, or skip this contour), or may choose an automated loading of all contours in the list.

The smooth curves drawn by the spline interpolation algorithm are far superior to the piecewise-linear contours. They look better in the display, and behave better when subjected to analysis of their shape by the photometric ordering software. A brief description of the algorithm is merited.

Given N points $P_1, P_2, \dots, P_N$ in the x - y plane, where $P_i = (x_i, y_i)$, the curve produced by the cubic Bessel spline interpolation function has the following properties:

1. The curve passes through the N given points.
2. In each interval $[P_i, P_{i+1}]$, the curve is a cubic polynomial function.
3. The first derivative of this curve at $P_2, P_3, \dots, P_{N-1}$ is continuous. If $P_1 = P_N$ (a closed loop), then the first derivative is continuous at all points P_i for $1 \leq i \leq N$.

Let $P(x(t)), y(t)$ be the parametric equation representation of the curve, and let

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x, \quad (5.1)$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y, \quad (5.2)$$

where $0 \leq t \leq 1$, be the cubic polynomial functions in the interval $[P_i, P_{i+1}]$ (see Fig. 5.15). Then the coefficients a_x, b_x, c_x, d_x of $x(t)$ and the coefficients a_y, b_y, c_y, d_y of $y(t)$ can be determined as follows:

$$d_x = x_i, \quad (5.3)$$

$$c_x = \dot{x}_i, \quad (5.4)$$

$$b_x = 3x_{i+1} - 2c_x - 3d_x - \dot{x}_{i+1}, \quad (5.5)$$

$$a_x = x_{i+1} - b_x - c_x - d_x, \quad (5.6)$$

$$d_y = y_i, \quad (5.7)$$

$$c_y = \dot{y}_i, \quad (5.8)$$

Figure 5.15: The points on the spline interval used by Task JDC. For each interval $[P_i, P_{i+1}]$, exterior points P_{i-1} and P_{i+2} are required to control the continuity of the curve at the turning-points

$$b_y = 3y_{i+1} - 2c_y - 3d_y - \dot{y}_{i+1}, \quad (5.9)$$

$$a_y = y_{i+1} - b_y - c_y - d_y, \quad (5.10)$$

where the derivatives can be approximated by

$$\dot{x}_i = \frac{x_{i+1} - x_{i-1}}{2}, \quad (5.11)$$

$$\dot{x}_{i+1} = \frac{x_{i+2} - x_i}{2}, \quad (5.12)$$

$$\dot{y}_i = \frac{y_{i+1} - y_{i-1}}{2}, \quad (5.13)$$

$$\dot{y}_{i+1} = \frac{y_{i+2} - y_i}{2}. \quad (5.14)$$

$$(5.15)$$

Using these equations and by varying the parameter t from 0 to 1 in suitably small steps, the coordinates of all the interpolated points on the curve were computed, and then drawn into the Map Image by Task JDC. The Map Image in Figure 5.14 has only closed contour loops, but the algorithm can also handle the ends of broken contours, as shown in Figure 5.16.

Figure 5.16: Contours displayed in the Map Image by Task JDC. This object is a human back and illustrates the ability of Task JDC to draw both closed loops and broken contours using cubic spline interpolation.

5.4 Gradient Neighbour Discovery

This section studies the problem of discovering the gradient neighbour on each side of a point on a contour. The goal is to develop a program that can be given a seed-point and a search direction, and use the image structures to search for its gradient neighbours. The program should return the coordinates and identity of the discovered neighbours.

5.4.1 Search Strategy

A primitive search strategy is used to find 'well-behaved' gradient neighbours. The full meaning of this term will be made clearer, but, basically, neighbours are well-behaved if they are easily found by searching the Map Image in a predictable direction. If any problems arise, the search is called off. Refinements made to handle special cases are discussed at the end of the section.

Figure 5.17: The connectivity of pixels on a rectangular sampling grid. In a rectangular lattice, each pixel has eight neighbours as shown. (a) If pixels are only considered to be joined or touching at edges, then they are *four-connected*. (b) If the pixels touching at the corners are also allowed, then they are said to be *eight-connected*.

5.4.1.1 Assumptions

The following assumptions are applied to restrict the scope of the search and to ensure that sufficient data is available:

1. The calling program must supply both the seed-point from which to start the search, and the nominal search direction. Furthermore, the seed-point must lie on a contour, and the nominal search direction is assumed to be normal to the contour (i.e., the contour azimuth angle, α).¹¹
2. Contour curves must not be broken or have gaps such that the search path might miss the contour. They must be drawn in the Map Image as continuous strings of four-connected, or at least eight-connected pixels. Pratt [1978] defines a pixel on a rectangular sampling grid to be *four-connected* if its neighbours are considered to lie only above, below, or to the side, whereas it is *eight-connected* if the pixels on the diagonals are also admissible as touching neighbours (see Fig. 5.17).
3. There must be a closed boundary curve to restrict the search to the physical image. The boundary may be the edge of the image, but it is more efficient to terminate the search with a tighter boundary, such as the outer-most contour

¹¹The selection of seed-points and the calculation of the azimuth angle are the subject of the next chapter.

Figure 5.18: Search strategies to discover the intersection of a digital contour curve. The contour curve *A* in this example was drawn by Task JDC, and is a thin eight-connected curve. The search starting at *B* uses a thin eight-connected line to examine pixels; it fails to discover the contour. The thicker four-connected search vector launched from *C* will assuredly detect the contour as it crosses it.

in a series (e.g., Contour #20 in Fig. 5.13); then the search program must check for the label of the contour to be used as a boundary.

5.4.1.2 Method

The search method developed by the author is quite simple, but effective. Searching consists of examining the label field of each pixel along the line in the search direction, starting at the seed-pixel. Between contours, the label field of all the pixels that are not part of a contour is set to zero, so when a non-zero label is discovered, it is assumed that the neighbouring contour has been successfully intersected. Figure 5.18 illustrates the importance of using the thicker four-connected search vectors to ensure that an eight-connected contour curve is discovered when it is intersected.

In the first pass, a search is made in the azimuth direction. A valid 'hit' identifies the neighbouring contour and gives the coordinates of the intersected neighbour pixel. In most cases, this is the nearest neighbour. Non-valid hits (such as 'self-hits') are discussed later; they generate error codes to be returned to the calling program.

Figure 5.19: A bundle of search directions to find the closest neighbour to point P on contour C . In this example, the searches are conducted in directions $\pm 45^\circ$ about α in 15° steps to look for neighbours that are closer than the neighbour discovered in the α direction.

In a second phase, the search is repeated in a bundle of directions about the original direction to see if a closer neighbour exists. The bundle is adjustable in width and increment, and an example is shown in Figure 5.19 that spans $\pm 45^\circ$ in 15° steps. The potential refinement from the bundle search is beneficial for a small fraction of the seed-points ($< 10\%$). The problem-points always occur where the assumption of sufficient surface planarity is weak, as in Figure 5.3. No efficient method was developed for predicting whether or not the second phase will discover a closer neighbour than the first pass, so the bundle search is applied every time.

5.4.2 Software Implementation

Several versions of the gradient search algorithm were developed, either as separate subroutines, or embedded into the code of larger test modules. The central concepts are presented with much of the coding detail suppressed.

```

*-----*
|  SUBROUTINE SEARCH(,XXXXXX,<output>) |
*-----*

```

Figure 5.20: The limit point on the search path. The coordinates of point P_{E1} at the edge of the image can be computed for a given seed-point P_1 and search direction α_1 , provided the image boundaries are known. The values of (p_{MAX} , p_{MIN} , l_{MAX} , l_{MIN}) are stored in the Image Table, a common block containing the image format data.

The calling sequence¹² of the Fortran subroutine was changed to suit the task. For example, instead of being called twice, sometimes the subroutine was written to search on both sides of the contour to find the two gradient neighbours to a point. Another variation involves how the calling program passes the search direction: it may be in the form of the azimuth angle α , or it may supply the end-point of the search vector at the image boundary. The end-point concept is illustrated in Figure 5.20. Thus, the searching routine requires the following input information:

```

RXY(2,2)    ! search vector start and end points:
              ! (1,1) = start pixel
              ! (1,2) = end pixel
              ! (2,1) = start line
              ! (2,2) = end line
RAZMTH      ! azimuth angle of search vector
```

¹²The DIPIX software standards require that input and output arguments in a subroutine's calling sequence be separated by a dummy variable `XXXXXX`. See Chapter 8 for details on the coding standards used by the author.

The routine returns the following:

```

IERROR      ! error flag: +1 = success
              !          -1 = no contour found
              !          -2 = bad data: no label field
              !          -3 = bad data: vector is zero length
LINECO      ! neighbour line coordinate
PIXLCO      ! neighbour pixel coordinate
RUNVAL      ! distance to neighbour
BTHNUM      ! contour label of neighbour (theme number)

```

Various parameters are initialized, and in particular, SEARCH must construct the ixel mask used to test for non-zero labels in the theme field of the video-memory image. SEARCH has access to the Image Table, and can look-up the width of the theme field BFSP (in this case it is 7 bits), and whether or not the cursor field is present (in this case it is, and the theme mask must be left-shifted one bit):

```

      INTEGER*4 EPM                      ! mask (DIPIX reqd integer*4)
      DATA BMV/1,3,7,15,31,63,127/      ! theme field mask values
      :
      EPM = BMV(BFSP)                    ! mask = 000...000001111111
      IF (ICRS.EQ.'N') GOTO 25
      EPM = EPM*2                          ! mask = 000...000011111110
25    CONTINUE

```

To control the search positions in the video-memory image, the range of the loop counter and the incremental steps in x and y are computed. The x and y distance components to the end point are compared and the longer one is used to define the step size:

```

      SX = ABS(RXY(1,2)-RXY(1,1))        ! pixel distance to end point
      SY = ABS(RXY(2,2)-RXY(2,1))        ! line distance to end point
      IF(SY.GE.SX)
        THEN SDL = SY                     ! number of increments to use
        ELSE SDL = SX
      XNC = (RXY(1,2)-RXY(1,1))/SDL       ! x-increment
      YNC = (RXY(2,2)-RXY(2,1))/SDL       ! y-increment
      XS = RXY(1,1)                       ! initial search position (x)
      YS = RXY(1,2)                       ! initial search position (y)
      DO 80 K=1,SDL

```

```

XS = XS + XNC           ! increment position in x
YS = YS + YNC           ! increment position in y

```

A call is made to a routine that examines the theme field at position (XS,YS) in the video-memory image using the ixel mask EPM. The code shown here belies the fact that there are some integer $\leftrightarrow$ real conversions to attend to (omitted for brevity). The theme value is returned in IPV, with an error flag IER. If a non-zero theme is returned, then a neighbour has been discovered; otherwise a call is made to check as if only a vertical increment were used, and should that fail, then check as if only a lateral increment were used. In this way, a four-connect search vector is propagated:

```

CALL EXAM(XS,YS,EPM,XXXXXX,IER,IPV)      ! check (XS,YS)
IF(IPV.EQ.0)
  THEN GOTO 50
  ELSE GOTO 90
50 CALL EXAM(XS-XNC,YS,EPM,XXXXXX,IER,IPV) ! check 'up' only
IF(IPV.EQ.0)
  THEN GOTO 60
  ELSE XS = XS - XNC      ! retain this pixel coord value
  GOTO 90
60 CALL EXAM(XS,YS-YNC,EPM,XXXXXX,IER,IPV) ! check 'side' only
IF(IPV.EQ.0)
  THEN GOTO 80
  ELSE YS = YS - YNC      ! retain this line coord value
  GOTO 90

```

If the loop ends at the edge of the image without discovering a gradient neighbour, then the routine returns with the error flag suitably set:

```

80 CONTINUE
   IERROR = -1
   RETURN

```

Otherwise, the attributes of the discovered neighbour are loaded into the output arguments for a successful return to the calling program:

```

90 PIXLCO = XS
   LINECO = YS
   RUNVAL = SQRT((XS-RXY(1,1))**2+(YS-RXY(1,2))**2)
   BTHNUM = IPV
   IERROR = 1
   RETURN

```


5.4.3 Discussion of Implementation

5.4.3.1 Execution Speed

Not shown above is the code used to write diagnostics into the cursor field of the display while the task is running. Every examined pixel in the search is set 'white' in the cursor field, and the displayed effect is to trace the search vectors in the image in real-time. An example of the bundle of lines drawn for each search is shown in Figure 5.9 for a sequence of five seed-points along a gradient curve. In this example, the searching operation with displayed diagnostics took less than two seconds. The generation of diagnostic imagery in the display during searching operations is much faster than the operator can follow, thus there is an impression of fast execution, even though a complex map with hundreds or thousands of search operations takes several minutes to complete. This level of performance is quite adequate for the images tested in this research, and since the diagnostics are so useful in monitoring the progress of analysis about the image and for checking problem-points, it was decided not to suppress the diagnostics to speed up the processing.

5.4.3.2 Robustness

Nearly all searches are well-behaved, but there are a few contour geometries that are consistently troublesome. The most frequent is the case of the 'self-hit' and Figure 5.21 shows a typical example. The self-hit can occur in regions on the surface that are insufficiently planar, when a search vector misses the gradient neighbour contour completely, and continues on until it intersects its own contour. In the example in the figure, the bundle of search vectors does not consistently discover the proper neighbour. The SEARCH software simply selects the closest neighbour, even if it is a self-hit. It is the responsibility of the calling program to recognize problem-points returned from the gradient neighbour search.

Complex contour patterns can lead to 'mix-hits', in which candidates from more than one contour are discovered by the bundle search (see Fig. 5.22). As mentioned, the SEARCH software simply selects the closest, which is usually the optimal outcome.

Figure 5.21: A search that results in a 'self-hit'. Search vectors launched from seed-point A_i have intersected the proper neighbour contour at $B_{i,1}$ and $B_{i,2}$. Some have also intersected the seed-point's contour, causing 'self-hits' at $A_{i,3}$, $A_{i,4}$ and $A_{i,5}$.

Figure 5.22: A search that results in 'mix-hits'. In this example, the search conducted at seed-points A_j and A_k produces well-behaved neighbours. However, the search from seed-point A_i is characterized by 'mix-hits' in which more than one contour is intersected by the search vectors. In this case, candidate neighbours are discovered at $B_{i,1}$, $A_{i,2}$ (a self-hit), $C_{i,3}$, $C_{i,4}$ and $C_{i,5}$.

Careful monitoring of the display and printer diagnostics showed that the case of a self-hit or of the wrong mix-hit being selected as the closest neighbour in the bundle search was very rare. The simple selection criteria provide a robust filter, but it errs if the search bundle is too narrow to capture the proper gradient neighbour. This may happen for one or two points in a large map involving hundreds of analysis points. Making the bundle wider means either sacrificing the resolution of the bundle (a fixed number of search vectors to span a wider fan), or sacrificing efficiency by introducing more vectors. The author uses a fixed bundle, set empirically to give good performance ($\pm 40^\circ$, in 20° intervals). Image searching is a slow operation, and others may consider optimizing the efficiency of the filter by making the bundle width and step size dynamically adjustable.

Chapter 6

Automated Shape Determination

6.1 Introduction

An essential process in photometric ordering is the determination of the shape of the surface for all of the contour ordering possibilities that must be tested. This chapter discusses the methods and problems of automating this process.

In Chapter 3, the Photometric Ordering Operator Ω is described as choosing the correct ϕ -values from a set of possible elevations for the unknowns in a sequence of neighbouring contours. There are $\aleph$ ways of ordering the unknown contours and these ways are called *ordering cases*. The k th case is given by Equation 3.19 (p. 78) as

$$\langle \Phi \rangle^{(k)} = \langle \phi_{n_1}, \dots, \phi_{n_N} \mid \phi_{u_1}^{(k)}, \dots, \phi_{u_U}^{(k)} \rangle, \quad (6.1)$$

where only the unknowns change ϕ -value from case-to-case. One of the problems addressed in this chapter is to count the number of ordering cases $\aleph$ and determine the set of possible ϕ -values

$$[\langle \Phi \rangle] = \begin{bmatrix} \langle \Phi \rangle^{(1)} \\ \vdots \\ \langle \Phi \rangle^{(k)} \\ \vdots \\ \langle \Phi \rangle^{(\aleph)} \end{bmatrix}. \quad (6.2)$$

Figure 6.1: Three shape cases for a pair of contours. (a) The planimetric view, showing contours C_i and C_{i+1} cut by gradient curve G_j at P_A and P_B , respectively. (b) The gradient profile view for each shape case. In this example, P_B can be above, at the same height, or below P_A , and the gradient profile curves for the three shape cases are denoted $G_j^{(+)}$, $G_j^{(0)}$, and $G_j^{(-)}$, respectively. The surface normal $N_i^{(k)}$ at each P_i represents the local shape at those points, and it changes its vertical orientation (tilt) according to the shape case.

Forming the set of ϕ -values in Equation 6.2 corresponds to process **OCG** described in Section 3.4.3 (p. 102).

Each ordering case specifies a ϕ -value for every contour in the sequence. One can reconstruct a possible shape for an object that corresponds to a given ordering case. This shape is referred to as the *shape case* that corresponds to an *ordering case*.¹ In the discussion that follows, reference is also made to the *profile case*, which is the profile taken along a gradient curve on the object for a particular ordering case. To illustrate the relationship between ordering cases and profiles, Figure 6.1 shows the gradient profiles that correspond to three ordering cases for a pair of contours. Other examples are presented later in the chapter.

¹The correspondence between ordering cases and shape cases is discussed in Section 3.2.7 (p. 81).

The surface shape about a test point is represented by the local surface normal N , which is used in the photometric model to estimate the reflectance at that point. The surface normal is specified by its tilt τ from the vertical and by its azimuthal swing α about the vertical axis. To evaluate the surface normal at test points for each possible shape there are three sub-processes that must be automated:²

- determine the set of shape cases $\{\Phi\}$ for a sequence of contours;
- calculate the tilt $\tau^{(k)}$ for the k th shape case at the required test-points;
- calculate the azimuth α at the required test-points.

This chapter briefly reviews the connection between shape on the object (normals, tilts and azimuths) and shape information in the moiré fringe pattern. The shape degeneracy (i.e., the list of possible shapes) stems from the uncertainty of the contour ordering, and ordering rules are discussed that lead to an algorithm for automatically forming a list of shape cases for an arbitrary sequence of contours. Each shape case is characterized by a shape profile in the gradient direction, thus the surface tilt at each test point changes with each shape case. An algorithmic means is derived for extracting the surface tilt at selected points along the profile. In contrast, the azimuth angle is a planimetric constant for each point on the contour curve and it does not change its value with each shape case. Special considerations must be applied in automating the evaluation of the azimuth for arbitrary contour curves and an algorithm based on a parametric representation is presented. The processing steps and the equations for evaluating the surface normal are summarized and the chapter concludes with some remarks on the robustness, accuracy and compromises made by the software versions of the shape-determining subroutines.

6.2 Shape from Fringes

The shape of an object at a point on its surface can be specified by the surface normal N which is the vector perpendicular to the surface at that point (Fig. 6.2).

²In this discussion, *automated* refers to the development of deterministic closed-form algorithms or arithmetic expressions which are then encoded in non-interactive software

Figure 6.2: Surface normals depicting object shape. On this potato-like object, only a few points have been characterized with normal vectors.

The surface normal specifies only the orientation of an infinitesimal patch about the point and thus its usefulness as a shape descriptor is restricted.

The surface normal can be expressed in several ways. The Cartesian coordinate representation is very useful:

$$\mathbf{N} = (N_x, N_y, N_z), \quad (6.3)$$

where N_x , N_y and N_z are the components of the unit vector $\mathbf{N}$ along the $\hat{x}$, $\hat{y}$ and $\hat{z}$ directions, respectively. This representation is convenient when evaluating the photometric model, which involves a vector scalar-product with the illumination direction (i.e., $\mathbf{N} \cdot \mathbf{L}$ as in Eq. 3.25 on p. 85). The surface normal can also be specified in polar coordinate representation by its tilt τ from the vertical and its azimuthal swing α about the vertical direction measured from the $\hat{x}$ -axis (Fig. 6.3). One can convert from polar to Cartesian components via

$$N_x = \sin \tau \cos \alpha, \quad (6.4)$$

$$N_y = \sin \tau \sin \alpha, \quad (6.5)$$

$$N_z = \cos \tau. \quad (6.6)$$

Figure 6.3: Polar coordinate representation of the normal vector. The inclination or tilt τ is measured from the vertical direction $\hat{z}$. The azimuthal swing α is measured about the vertical direction from the $\hat{x}$ -axis. The length of the normal is always unity.

The polar representation of the surface normal is important in this discussion because the angles τ and α can be derived directly from the moiré topogram fringe pattern. Thus, one can obtain local shape on the surface of the object (in the form of N) from the fringes. The connection between the fringes and the surface normal is made through the contour and gradient curves. Consider Figure 6.4 which shows a cut-away version of the object shown previously in Figure 6.2. At each point P_A the surface normal is just the vector cross-product of the tangent vectors to the contour and gradient curves:

$$N_A = G'_A \times C'_A, \quad (6.7)$$

where G'_A and C'_A are the vector tangents at P_A to the gradient curve $\mathcal{G}$, and contour curve $\mathcal{C}$, respectively (Fig. 6.5).

Rather than apply the vector cross-product of Equation 6.7 directly, one can derive the tangential behaviour of the contour and gradient curves from their planimetric projection into the reference plane (i.e., the curves of the moiré fringe pattern). Figure 6.6 illustrates how the tilt τ is computed from the *run* between contour fringes (measured in the image plane) and the *rise* between them (deduced from

Figure 6.4: Surface normals from contours and gradient curves. A pair of contours C_1 and C_2 have been drawn on the potato-like object, and a section of the object has been removed by cutting along a pair of gradient curves G_1 and G_2 . The gradient curves intersect the contour curves at points P_A, P_B, P_C, P_D , and the corresponding normals N_A, N_B, N_C, N_D are formed by the vector cross-product between the tangent vectors to the contour and gradient curves at each point.

Figure 6.5: The surface normal as a vector product between G'_A and C'_A , the vector tangents at P_A to the gradient curve G_1 and contour curve C_1 , respectively.

the ϕ -values of the trial ordering):

$$\tan \tau = \frac{\text{rise}}{\text{run}} \quad (6.8)$$

The figure also indicates how the azimuth angle can be derived from the topogram image as the normal direction to the contour curve. Later in the chapter a formula is developed that gives the azimuth angle as a function of the coordinates of points that define the contour curve.

The preceding paragraph oversimplifies the calculation of the tilt and, in particular, it assumes the run between contours is a straight line. In general, the three-dimensional gradient profile traces a curved planimetric line in the $\hat{x}$ - $\hat{y}$ plane and, strictly speaking, the run must be measured along this path. The approach used by the author is to straighten-out this path so that the gradient profile flattens into a plane parallel to the vertical axis (Fig. 6.7). A plane-representation of the gradient profile simplifies the discussion of its possible shapes and it simplifies the computation of the normal to the profile. As shown in Figure 6.7, the parameter g measures the linear distance along the gradient curve $\mathcal{G}$, when it is projected into the $\hat{x}$ - $\hat{y}$ plane, and it can be used to flatten the gradient profile into the $\hat{z}$ - $\hat{y}$ plane. The normal to the plane-representation of the profile curve is denoted η_i to distinguish it from the object-space surface normal $\mathbf{N}_i$; both have the same tilt τ_i with respect to the $\hat{z}$ -axis. By transforming the gradient profile curve into a plane-representation, ordinary plane-geometry can be used to calculate the tilt at points along the profile. The appropriate formula is derived later in the chapter.

6.3 The List of Shape Cases

This section studies the problem of listing the $\aleph$ shape cases for a generalized sequence of contours. The goal is to develop a program that forms the set $[\langle \Phi \rangle]$ of all the contour ordering possibilities to be tested for a given sequence. The four rules for ordering moiré fringe contours are given, and examples are presented to illustrate their application. There are two independent aspects to the shape of an object at a

Figure 6.6: Surface normal tilt and azimuth from moiré fringes. In the upper part of the figure, a section of the object is drawn and the surface normal N at point P on the object is depicted by a large arrow. On the arrow, the tilt τ and azimuth α are indicated with respect to the Cartesian coordinate directions. The tilt of the normal is contained in the hatched *gradient profile section*, and can be calculated in this figure from the rise of two contour intervals, over the run spanning the two corresponding fringes in the planimetric view at the bottom of the figure (i.e., the moiré topogram in the image plane). The azimuthal swing α is also available in the image plane as the angle from the x -axis to the gradient line through the contour at the appropriate point.

Figure 6.7: Gradient profile curve projection into the $\hat{z}$ - $\hat{g}$ plane (a) The surface gradient $G_j(x, y)$ in the Cartesian object-coordinate space generally has a curved path when projected into the $\hat{x}$ - $\hat{y}$ plane. g is a parameter that measures the linear distance along the curve projected into the $\hat{x}$ - $\hat{y}$ plane. The surface normal on $G_j(x, y)$ at g_i is denoted by N_i , and it makes a tilt τ_i with respect to the $\hat{z}$ -axis. (b) The gradient profile has been projected into the $\hat{z}$ - $\hat{g}$ plane for plane-geometry analysis. The corresponding normal to the curve $G_j(g)$ at g_i is denoted η_i , and makes the same tilt τ_i with respect to the $\hat{z}$ -axis as did N_i .

point: the contour curvature specifies α and the gradient curvature specifies τ . The ordering rules only affect the gradient profile, and thus they control the computation of the tilt but not the azimuth. An algorithm for producing the list of profile cases for a generalized sequence is developed. The software code for implementing the profile-cases algorithm is sketched, and the section concludes with a discussion of the implementation used in the test programs.

6.3.1 Rules for Profiles

There are four rules that govern the profile cases. The first three rules can be applied to a sequence of contours to determine all the valid cases for the relative ordering of the members. Each case uniquely assigns a combination of relative elevations to the member contours, and this in turn allows a gradient profile curve to be drawn through the member contours. The fourth rule says that some of the profiles are physically unlikely.

- **Moiré Fringe Existence Rule:** The surface of the object *must* be inclined for a moiré fringe to be formed there.

Figure 6.8 illustrates this simple, but profound property. It is important to appreciate that it is *impossible* to have a profile that connects neighbour contours at the same level with a horizontal line (Fig. 6.9a). The profile *must* be a concave or convex curve (Fig. 6.9b) and yet be shallow enough so as not to intersect the next elevation interval (then a third moiré fringe would lie between the two neighbours, as in Fig. 6.9c).

- **Moiré Fringe Ridge Rule:** Two inclined surfaces meeting at a ridge parallel to the reference plane can produce a fringe pattern that is identical to a continuous inclined surface.

Figure 6.10 illustrates that, in general, it is *possible* for the surface inclination to abruptly change slope-sign *exactly* along a contour line, and that such shape cases must be admitted in the ordering process. Although such shapes may not

Figure 6.8: The Moiré Fringe Existence Rule says that the surface must be inclined to produce a moiré fringe. The upper part of the figure shows the profile of the object and the corresponding moiré topogram is shown below. (a) Points P_A , P_B and P_C lie on an inclined profile curve G_1 , and the corresponding moiré fringes are formed along F_A , F_B and F_C . (b) A level profile G_1 is drawn through point P_B and the corresponding moiré pattern is a 'fringe of infinite width' (i.e., no moiré fringe).

Figure 6.9: The required profile curvature for contours at the same elevation. Points P_A and P_B are on neighbouring contours at the same elevation, and N_A and N_B are their respective normals to the profile curve G_j drawn through the two points. (a) Illegal case: a horizontal profile violates the Moiré Fringe Existence Rule. (b) Valid cases: moderately convex/concave profiles ensure the surface is tilted at P_A and P_B so that separate moiré fringes are formed at those points. (c) Illegal cases: overly convex/concave profiles imply that a third (non-existent) moiré fringe lies between P_A and P_B .

Figure 6.10: The Moiré Fringe Ridge Rule says that two inclined surfaces meeting at a level ridge (a roof) can produce a fringe pattern that is identical to a continuous inclined surface (a) G_j is a roof profile with its peak at $\phi = 2.5$ such that it produces the same moiré fringe pattern (fringes F_A , F_B , F_C corresponding to profile points P_A , P_B , P_C) as for the continuous inclined plane in Figure 6.8. This holds for only one unique position, as seen in (b) with the roof peak at $\phi = 2.75$, the central dark fringe at $F_{B1,2}$ has broadened to split in the middle, and in (c) with the roof peak at $\phi = 3$, a bright fringe has formed in the middle of the pattern between the split halves F_{B1} and F_{B2} of the dark fringe.

seem likely and call for unique positioning with respect to the reference plane, these shapes should not be dismissed at the outset. For example, many engineered objects do have sharp edges and may be coincidentally or intentionally aligned to produce patterns that impersonate simpler objects.

• Moiré Neighbour Elevation Rule:³

1. For matched-mode contours: neighbours C_i , C_j always differ by at most one elevation interval ($\Delta_{i,j} = -1, 0, +1$).
2. For mixed-mode contours: neighbours C_i , C_j of the same MFE-type always have the same elevation ($\Delta_{i,j} = 0$), whereas ones of opposite MFE-type always differ by half an elevation interval ($\Delta_{i,j} = \pm \frac{1}{2}$).

³From Equation 3.5 on page 70

Figure 6.11: The Moiré Neighbour Elevation Rule for a sequence of five matched-mode fringes. Shown is the profile through a typical summit, and neighbour contours are seen to differ in elevation by an interval of +1, 0 or -1.

Figures 6.11 and 6.12 illustrate these rules with one of the smoother curves that can be drawn for each of the sequences.

The Moiré Neighbour Elevation Rule allows the number of possible shape cases N for a sequence of contours to be counted. The matched-mode rule says there are three cases for each of the m' pairs in a matched-mode sequence, and the mixed-mode rule says there are two cases for each of the m'' bright/dark pairs in a mixed mode sequence:

$$N = \begin{cases} 3^{m'} & \text{(matched-mode fringes)} \\ 2^{m''} & \text{(mixed-mode fringes)} \end{cases} \quad (6.9)$$

These formulas can be used to count the number of profile cases for some of the example sequences. Figure 6.11 shows a sequence of five matched-mode contours, and the four pairs of neighbours ($m' = 4$) give rise to $3^{m'} = 81$ shape cases to be analyzed. Figure 6.12 shows a sequence of nine mixed-mode contours, and there are seven pairs of neighbours with opposite MFE-type ($m'' = 7$) which results in $2^{m''} = 128$ valid profile curves. Both of these examples result in large numbers of possible profiles.

Figure 6.12: The Moiré Neighbour Elevation Rule for a sequence of nine mixed-mode fringes. Shown is a typical hill-and-valley profile. Contours of opposite MFE-type always differ in elevation by half an interval, whereas contours of the same MFE-type are always at the same elevation.

By breaking long sequences into a series of shorter sequences, the number of profiles to be tested is reduced. If the shortest sequence is used ($m = 1$ for two contours), then the first example can be broken up into four pair-sequences such that only $4 \cdot 3^1 = 12$ short curves need to be analyzed, and in the second example seven pair-sequences imply that only $7 \cdot 2^1 = 14$ short curves need to be analyzed. Fewer cases and short sequences lead to significant simplification of the software to perform photometric ordering (see Task JFO in Ch. 8), but longer sequences provide more points on the profile to give a better opportunity to reconstruct the true shape of the object and improve the shape-sensitivity of the photometric ordering process (see Task MFO in Ch. 9).

- **Surface Smoothness Rule:** a surface with a smooth profile is more likely than one with a disjoint gradient profile.

This rule paraphrases Occam's razor: if there is a simple explanation, then it is usually correct.⁴ Figure 6.13 shows an unlikely but valid shape case that must be

⁴The leading principle of the nominalism of William of Occam, that for the purposes of explanation things not known to exist should not, unless absolutely necessary, be postulated as existing [ref: The Shorter Oxford English Dictionary, 3rd ed., *razor*].

Figure 6.13: An unlikely groove-and-ridge profile curve for the sequence of five matched-mode fringe contours shown in Figure 6.11. The convex curve must rise to the mid-interval elevation in order to produce exactly two fringes (only one broad fringe is produced if the curve is too low, and a central third fringe appears if it is too high). Furthermore, the groove must fall and the roof ridge must peak exactly at the indicated elevations to produce a single fringe in each position.

precisely positioned in order to produce a fringe pattern that mimics the simpler profile shown for the same contours in Figure 6.11. One is tempted to eliminate such contrived cases, and rules could be formulated to predict and eliminate the unlikely profiles from the list of shape cases to be tested. This is not done here since an objective of this project is to study the strengths and weaknesses of photometric ordering in picking the correct shape based only on the shading considerations. The technique is shown to be more general if it does not rely on assumptions regarding the smoothness and position of the object.

6.3.2 Profile Cases Examples

To illustrate the profile case rules, a few simple examples are presented. The neighbour sequences are taken from Figure 6.14, where contours labelled $C_A, C_B, C_C, C_R, C_S, C_T$ are intersected by the gradient curve G_j at points $P_A, P_B, P_C, P_R, P_S, P_T$.

Figure 6.14: A contour map for profile cases examples in the text. Contours labelled $C_A, C_B, C_C, C_R, C_S, C_T$ are intersected in order by gradient curve G_1 at points $P_A, P_B, P_C, P_R, P_S, P_T$, respectively.

Example 6.1.

Let the contour set comprise bright fringes only, so that the matched-mode rule for the relative fringe number applies:

$$\Delta_{i,j}^{(k)} \in \{-1, 0, +1\}. \quad (6.10)$$

Consider a pair of neighbours, say C_A and C_B , and suppose the absolute fringe number of C_A is known to be $\phi_A = 14$. Then, the k th possible shape case is given by Equation 3.4 (p. 69) as

$$\phi_B^{(k)} = \phi_A + \Delta_{B,A}^{(k)}. \quad (6.11)$$

Table 6.1 lists the resulting ϕ -values for each of the three shape cases possible in this example.

Example 6.2.

Consider a neighbourhood sequence of three contours $\langle C_A, C_B, C_C \rangle$, and let them be drawn as if they were bright, dark and bright fringes, respectively. These mixed-mode contours all have opposite MFE-type neighbours, so the relative ordering rule

Table 6.1: The resulting ϕ -values for the three shape cases in Example 6.1

case	ϕ_A	ϕ_B
$k=1$	14	13
2	14	14
3	14	15

is

$$\Delta_{i,j}^{(k)} \in \{-\frac{1}{2}, +\frac{1}{2}\}. \quad (6.12)$$

Again, let the absolute fringe number of C_A be known to be $\phi_A = 14$. Then, the k th possible shape case for this example is given by Equation 3.4 as

$$\phi_B^{(k)} = \phi_A + \Delta_{B,A}^{(k)} \quad (6.13)$$

$$\phi_C^{(k)} = \phi_B^{(k)} + \Delta_{C,B}^{(k)} = \phi_A + \Delta_{B,A}^{(k)} + \Delta_{C,B}^{(k)} \quad (6.14)$$

Table 6.2 lists the resulting ϕ -values for each of the four shape cases possible in this example.

Table 6.2: The resulting ϕ -values for the four shape cases in Example 6.2

case	ϕ_A	ϕ_B	ϕ_C
$k=1$	14	13.5	13
2	14	13.5	14
3	14	14.5	14
4	14	14.5	15

Example 6.3.

In this final example, the entire neighbourhood sequence from Figure 6.14 is used: $\langle C_A, C_B, C_C, C_R, C_S, C_T \rangle$. As in the previous example, they are deemed a mixed-mode set of contours, but now one pair of neighbours (C_C, C_R) are of the same MFE-type (recall from Eq. 3.2 on p. 69 that $e = 1$ for bright fringes, $e = 0$ for dark

fringes). The initial conditions given in Table 6.3 show that contours C_A and C_B have their absolute fringe numbers specified *a priori*, and that the other neighbours are unknowns.

Table 6.3: The initial conditions for Example 6.3

map contour label	λ	A	B	C	R	S	T
MFE-type	e	1	0	1	1	0	1
absolute fringe number	ϕ	14	13.5	?	?	?	?

The k th possible shape case for this example is given by the following expressions:

$$\begin{aligned}
 \phi_A^{(k)} &= 14 \\
 \phi_B^{(k)} &= 13.5 \\
 \phi_C^{(k)} &= \phi_B^{(k)} + \Delta_{C,B}^{(k)} \quad \text{with } \Delta_{C,B}^{(k)} \in \{-\tfrac{1}{2}, +\tfrac{1}{2}\} \quad (e_B \neq e_C) \\
 \phi_R^{(k)} &= \phi_C^{(k)} + \Delta_{R,C}^{(k)} \quad \text{with } \Delta_{R,C}^{(k)} \in \{0\} \quad (e_C = e_R) \\
 \phi_S^{(k)} &= \phi_R^{(k)} + \Delta_{S,R}^{(k)} \quad \text{with } \Delta_{S,R}^{(k)} \in \{-\tfrac{1}{2}, +\tfrac{1}{2}\} \quad (e_R \neq e_S) \\
 \phi_T^{(k)} &= \phi_S^{(k)} + \Delta_{T,S}^{(k)} \quad \text{with } \Delta_{T,S}^{(k)} \in \{-\tfrac{1}{2}, +\tfrac{1}{2}\} \quad (e_S \neq e_T)
 \end{aligned}$$

Finally, Table 6.4 lists the resulting ϕ -values for each of the eight shape cases in this example. The gradient profile for two of the cases has been drawn in Figure 6.15. Note the ambiguity posed by Case 2 in this figure when trying to continue a smooth profile between points P_C and P_R . Strictly speaking, this *cusped* profile-case degenerates into two sub-cases that should be admitted into the photometric ordering analysis as distinguishable physical surfaces. Cusped profiles are discussed at the end of the chapter.

6.3.3 Algorithm Description

A pattern emerges from Table 6.4 that suggests how a general algorithm might generate a list of all valid shape cases for an arbitrary sequence of contours. Only the unknowns change value in the list, and all combinations of the valid relative fringe values for each neighbour-pair in the sequence constitute a complete list. One

Table 6.4: The resulting ϕ -values for the eight shape cases in Example 6.3

case	ϕ_A	ϕ_B	ϕ_C	ϕ_R	ϕ_S	ϕ_T
1	14	13.5	13	13	12.5	12
2	14	13.5	13	13	12.5	13
3	14	13.5	13	13	13.5	13
4	14	13.5	13	13	13.5	14
5	14	13.5	14	14	13.5	13
6	14	13.5	14	14	13.5	14
7	14	13.5	14	14	14.5	14
8	14	13.5	14	14	14.5	15

Figure 6.15: Profile curves for two cases from Example 6.3 (a) Shape Case 2 (b) Shape Case 4 The ambiguity in the curve for Case 2 is explained in the text

need only determine the valid relative values for each pair, and then loop through the combinations to construct the list.

The algorithm is described as follows. Using the notation introduced in Chapter 3, $\langle \Phi \rangle$ is a sequence of gradient neighbour contours that can be partitioned into sub-sequences of members with known and unknown ϕ -value (Eq. 3.54 on p. 104)

$$\langle \Phi \rangle = \langle \phi_{n_1}, \dots, \phi_{n_N} \mid \phi_{u_1}, \dots, \phi_{u_U} \rangle. \quad (6.15)$$

The desired outcome is to form a list of the N shape cases (Eq. 3.52, p. 102)

$$[\langle \Phi \rangle] = \begin{bmatrix} \langle \Phi \rangle^{(1)} \\ \vdots \\ \langle \Phi \rangle^{(k)} \\ \vdots \\ \langle \Phi \rangle^{(N)} \end{bmatrix}. \quad (6.16)$$

where the k th shape case is denoted

$$\langle \Phi \rangle^{(k)} = \langle \phi_{n_1}, \dots, \phi_{n_N} \mid \phi_{u_1}^{(k)}, \dots, \phi_{u_U}^{(k)} \rangle \quad (6.17)$$

to emphasize that only the unknowns change ϕ -value for each shape case.

The relative fringe number between the i th and $(i-1)$ th contour neighbours is given by Equation 3.4 (p. 69) as

$$\Delta_{i,i-1} = \phi_i - \phi_{i-1}, \quad (6.18)$$

which can take on different valid values that depend upon the contour ordering rules:

$$\Delta_{i,i-1} \in \begin{cases} \{-1, 0, +1\} & \text{if matched-mode contours,} \\ \{-\frac{1}{2}, +\frac{1}{2}\} & \text{if mixed-mode contours and } (e_i \neq e_{i-1}), \\ \{0\} & \text{if mixed-mode contours and } (e_i = e_{i-1}). \end{cases} \quad (6.19)$$

For the k th shape case, each $\Delta_{i,i-1}^{(k)}$ takes on a particular value. That is,

$$\Delta_{u_1, n_N}^{(k)}, \Delta_{u_2, u_1}^{(k)}, \dots, \Delta_{u_U, u_{U-1}}^{(k)} \quad (6.20)$$

all have a certain value selected from their lists of valid values, and all possible shape cases are generated by the valid combinations of the $\Delta_{i,i-1}^{(k)}$ values. The $\phi_i^{(k)}$ values for $i = u_1, \dots, u_L$ can be evaluated recursively using Equation 6.18, since ϕ_{n_N} is a known value:

$$\left. \begin{aligned} \phi_{u_1}^{(k)} &= \phi_{n_N} + \Delta_{u_1, n_N}^{(k)} \\ \phi_{u_2}^{(k)} &= \phi_{u_1}^{(k)} + \Delta_{u_2, u_1}^{(k)} \\ &\vdots \\ \phi_{u_L}^{(k)} &= \phi_{u_{L-1}}^{(k)} + \Delta_{u_L, u_{L-1}}^{(k)} \end{aligned} \right\} \quad (6.21)$$

or expanded in terms of the Δ 's as:

$$\left. \begin{aligned} \phi_{u_1}^{(k)} &= \phi_{n_N} + \Delta_{u_1, n_N}^{(k)} \\ \phi_{u_2}^{(k)} &= \phi_{n_N} + \Delta_{u_1, n_N}^{(k)} + \Delta_{u_2, u_1}^{(k)} \\ &\vdots \\ \phi_{u_L}^{(k)} &= \phi_{n_N} + \Delta_{u_1, n_N}^{(k)} + \Delta_{u_2, u_1}^{(k)} + \dots + \Delta_{u_L, u_{L-1}}^{(k)} \end{aligned} \right\} \quad (6.22)$$

The implementation of this algorithm in software is the subject of the next section

6.3.4 Software Implementation

There are several approaches to encoding the shape-case algorithm in software. The author uses nested DO-loops to cycle through the combinations of valid relative fringe values for each pair of contours in the neighbourhood sequence. The loop-limits are set according to the ordering rules by testing the MFE-type for each pair and knowing the mode. This information (mode and MFE's) is passed to the routine by the calling program, along with the ϕ -value of any knowns (at least the first member of the sequence must be a known). The routine returns a table of shape cases corresponding to $[\langle \Phi \rangle]$, the number of shape cases $\aleph$, and an error flag. The gist of the code is presented here, more in a form for understanding rather than directly copied from the source listings.

```

*-----*
| SUBROUTINE CASES(<input>,XXXXXX,<output>) |
*-----*

```

The routine requires at least the following input information:

```

MODE          ! contour set mode: 1=matched, 0=mixed
NUMS          ! no. of members in sequence (MAX=15)
MFE(15)       ! extrema types: 1=bright, 0=dark
STRPHI(15)    ! initial phi's known>0 ; unknown=-1

```

The routine returns the following:

```

SETPHI(32,15) ! up to 32 cases of phi values in table
NUMC          ! number of shape cases
IERR          ! error flag, set to 1 if all ok.

```

The error flag is assigned other values in the event of an error, and the calling program always tests the flag to take appropriate action. For example, invalid input data or too many shape cases for the table are flagged as an error. As will be seen below, it is useful to have both unique names and subscripts for some sets of variables such as loop-limits and loop-indices. This is done by the EQUIVALENCE-statement so that, for example, LL(2) reference the same variable as LL2:

```

EQUIVALENCE (DELT(1),DELT1), (DELT(2),DELT2), ...
EQUIVALENCE (LL(1),LL1), (LL(2),LL2), ...
EQUIVALENCE (UL(1),UL1), (UL(2),UL2), ...

```

Processing starts by finding the first unknown in the sequence. A loop is made through the initial ϕ -values, and when the first value below zero is discovered the pointer (U) is set to the first unknown:

```

      DO 50 U=1,NUMS
50    IF(STRPHI(U).LT.0) GOTO 60
51    ...           ! input error: no unknowns in sequence
      ::
60    IF(U.EQ.1) ... ! input error: at least one known req'd

```

Initialize pointers, counters:

```

NUMK = U-1          ! number of knowns
NUMU = NUMS - NUMK  ! number of unknowns
NUMC = 0            ! initialize case counter

```

Determine loop-limits: each pair of unknown contours has a relative fringe value $\text{DELT}(\mathbf{x}\mathbf{x})$ that will be varied by nested loops whose upper limits $\text{UL}(\mathbf{x}\mathbf{x})$ and lower limits $\text{LL}(\mathbf{x}\mathbf{x})$ must be set according to the contour ordering rules.

```

                IF(MODE.EQ.MATCH) THEN
                    DO 20 U=1,NUMU
                        LL(U) = -1 , UL(U) = 1          ! delta={-1,0,+1}
20                CONTINUE
                ELSE IF(MODE.EQ.MIX) THEN
                    DO 22 U=1,NUMU
                        IF(MFE(U+NUMK).EQ.MFE(NUMK+U-1))
                            THEN LL(U)=0    UL(U)=0    ! delta={0}
                            ELSE LL(U)=-.5  UL(U)=.5    ! delta={-.5,+.5}
22                CONTINUE
                ELSE ... ! input error: must be mix or match mode

```

Jump into the nested loops to the depth of the number of unknowns: The loop-limits and the number of unknowns determine N for the neighbourhood sequence. The number of cases is evaluated by counting the number of times the program cycles through the nested loops. This part of the program uses a computed-GOTO to neatly allow the nesting to be adjusted to the number of unknowns. Notice that the descending line numbering (allowable, and done here more for clarity) corresponds to the required depth of nesting.

```

                GOTO (1,2,3,4,5,6,7,8,9,10,11,12,13,14), NUMU
14    DO 800 DELT14 = LL14,UL14
13    DO 800 DELT13 = LL13,UL13
      :
3     DO 800 DELT3  = LL3,UL3
2     DO 800 DELT2 = LL2,UL2
1     DO 800 DELT1 = LL1,UL1

```

Compute the ϕ -values for a shape case: At the bottom of the 'nest' all the relative fringe values have been set for this shape case. To build $\langle \Phi \rangle^{(k)}$ according to Equation 6.17 first the knowns $\phi_{n_1}, \dots, \phi_{n_N}$ are loaded into the table. Then each unknown is evaluated by adjusting the previous member's known ϕ -value by the relative fringe number to produce the current member's ϕ -value as a known (Eq. 6.21).

Note that the member before the first unknown has a known value STRPHI(NUMK) (put into SETPHI(NUMC, NUMK) in the first loop) which serves as a seed to start the second loop.

```

      NUMC = NUMC + 1           ! count the case
      DO 500 K=1, NUMK         ! cycle through knowns
500   SETPHI(NUMC, K) = STRPHI(K)
      DO 600 U=1, NUMU         ! cycle through unknowns
600   SETPHI(NUMC, U+NUMK) = SETPHI(NUMC, NUMK+U-1) + DELT(U)
800   CONTINUE
      IERR=1                   ! success
      RETURN

```

6.3.5 Discussion of Implementation

Only the skeleton of SUBROUTINE CASES has been presented above. The details of variable-type declaration and error handling have been suppressed. A couple of FORTRAN-77 extensions have been carefully used to advantage, such as real-valued loop-indices and loop-limits.

Another implementation issue is variable buffer size. To keep the software flexible, it is a better practice to have the calling program establish the array dimensions, which requires that the subordinate module do some bookkeeping and limit-error checking. The simplistic code shown above defines array sizes locally, which restricts the shape case list to a 32-case $\times$ 15-member table of REAL*4 ϕ -values (requiring nearly 2 Kbytes of memory). The actual code requires that the calling program pass a one-dimensional buffer and the length of the neighbourhood sequence to CASES, which then maintains local pointers and offsets to process the buffer as a two-dimensional table. The workings of this aspect of the code have been suppressed above.

Two programs for automated photometric ordering (Tasks JFO and MFO) are described in Chapters 8 and 9. Task JFO is restricted to matched-mode pair analysis, and with only three trivial shape cases to consider each time, it does not use CASES. Task MFO is written for mixed-mode analysis of longer sequences (up to fifteen members), and uses a version of CASES without the mode-select option. The

mode-select was introduced by the author when preparing this document, as were a few other minor refinements that pertain to readability rather than functionality.

6.4 Surface Normal Tilt Determination

This section deals with the determination of the tilt of the surface at a point. The goal is to develop a closed-form expression for the tilt in terms of the coordinates of the point and its neighbours, subject to the elevations assigned by the ordering case.

The CASES algorithm specifies the vertical elevation as ϕ -values for the points in a sequence along the surface gradient. Combined with the planimetric (i.e., horizontal) separation of the points, one can reconstruct the gradient profile curve in the same way Figures 6.11, 6.12 and 6.13 are drawn. In this section, both piece-wise linear and higher-order functions are used to reconstruct the profile curve.

The surface profile tilt $\tau^{(k)}$ for the k th shape case is measured from the gradient profile at prescribed analysis positions. In the case of the piece-wise linear profile, the tilt is related to the slope of the straight line segments joining the analysis points, and has the familiar form

$$\tau = \tan^{-1} \left[\frac{\text{rise}}{\text{run}} \right] \quad (6.23)$$

In the case of continuous curves, the tilt is computed from the derivative with respect to the planimetric distance g along the curve:

$$\tau = \tan^{-1} \left[\frac{dz}{dg} \right] \quad (6.24)$$

There is no concern for modulo- π effects in evaluating the inverse trigonometric function since physical surface tilts are bounded by definition ($|\tau| \leq 90^\circ$). Indeed, in moiré topograms, fringe patterns are usually formed for physical surfaces tilted less than $\pm 60^\circ$ (see App. A).

6.4.1 Object-Space Rise and Run

To reconstruct the surface profile through a series of points, the rise and run between the points is computed in terms of object-space coordinates (i.e., at object scale). Here, the term *rise* is the change in elevation (Δz) between the points, and the *run* is the corresponding planimetric curve length (Δg).

One could use the nominal contour interval⁵ to represent the rise between any points on neighbouring contours. However, one must then monitor whether the interval is to be applied as a positive, zero or negative rise for each case, and more complex bookkeeping is required if mixed-mode (half-interval) contour sets are also handled. Furthermore, at large elevations the fixed nominal interval may be a poor approximation from which to compute tilts. Far simpler is to realize that photometric ordering requires the object coordinates of each analysis point to be explicitly determined anyway (to determine the illumination direction vector L), and so the exact vertical and lateral separation of each point are explicitly available for profile reconstruction. The equations for computing object-space coordinates from image coordinates are in Section 3.4.9, starting on page 116.

Consider a pair of points P_A and P_B with coordinates (x_A, y_A, z_A) and (x_B, y_B, z_B) , respectively. The rise between the points is the difference in their elevations:

$$\text{rise}_{A,B} = \Delta z_{A,B} = z_B - z_A. \quad (6.25)$$

Similarly, the run is the planimetric or lateral distance along the gradient curve between the two points, and can be approximated by their planimetric linear separation:

$$\text{run}_{A,B} = \Delta g_{A,B} \approx \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}. \quad (6.26)$$

6.4.2 Piece-Wise Linear Evaluation of Tilt

The simplest profile is drawn by joining the sequence of gradient neighbour points with straight lines to form a piece-wise linear profile, as shown in Figure 6.16. The

⁵On the NRC moiré instruments, $\Delta z \approx 5$ mm for matched-mode fringes

Figure 6.16: The tilt of a line between two points P_A and P_B projected into the $\hat{z}$ - $\hat{g}$ plane. The normal to the profile line segment joining P_A, P_B is $\eta_{A,B}$, and $\tau_{A,B}$ is its tilt from the vertical. The rise and run between the two points are denoted by $\Delta z_{A,B}$ and $\Delta g_{A,B}$, respectively.

tilt $\tau_{A,B}$ of the normal $\eta_{A,B}$ to the line segment spanning points P_A and P_B in the figure is trivial to compute in this case using Equation 6.23:

$$\tau_{A,B} = \tan^{-1} \left[\frac{\Delta z_{A,B}}{\Delta g_{A,B}} \right] \quad (6.27)$$

Photometric ordering compares the shading and geometry characteristics of neighbouring surface patches. An important distinction is made in the relationship between the position of the patches and the points on the contours of the neighbourhood sequence. The patches may be defined to lie between contour points (*mid-point modelling*) or else centered on the contour points (*on-point modelling*). The choice of definition has a strong influence on the form of the shape formulas developed in the remainder of this chapter and on the shading formulas developed in Chapter 7. The author tried both approaches: mid-point modelling is embodied in Task JFO (Ch. 8) and on-point modelling is embodied in Task MFO (Ch. 9).

Mid-point modelling is easily performed with piece-wise linear profiles. A minimum sequence of three contour points defines two profile line-segments with mid-points that have well-defined tilt values. Figure 6.17 illustrates mid-point modelling with piece-wise linear profiles. The astute reader may have noticed in Figure 6.17 that the example profile cannot correspond to an object that produces a moiré

Figure 6.17: Mid-point photometric modelling for a minimum sequence of three contour points P_A, P_B, P_C using piece-wise linear gradient profiles. The line joining P_A, P_B is bisected at P_R , and has a normal $\eta_{A,B}$, whereas the line joining P_B, P_C is bisected at P_S and has a normal $\eta_{B,C}$.

fringe pattern (surfaces must be inclined through contour points P_A and P_B for moiré fringes to be formed there). Nonetheless, whether a slight concave or convex profile curve is properly traced between these points, the tilt estimate at the midpoint will be very near vertical, matching the normal shown in the figure.

On-point modelling uses the contour points for analysis. In the strict sense, piece-wise linear profiles through these break-points have ill-conditioned normals at the break-points. One can define the tilt of the normal at a break-point to be a mean of the tilts to the lines on both sides, as illustrated in Figure 6.18. The simple mean gives a good approximation if P_A, P_B, P_C are evenly spaced:

$$\tau_B \approx \frac{\tau_{A,B} + \tau_{B,C}}{2} \quad (6.28)$$

If the middle point is much nearer to one end point, then one might consider weighting the mean by the relative length of the line segments from the middle point to each end (Fig. 6.18) so that the tilt of the shorter line-segment is favoured:

$$\tau_b \approx \frac{|P_BP_C| \cdot \tau_{A,B} + |P_AP_B| \cdot \tau_{B,C}}{|P_AP_B| + |P_BP_C|} \quad (6.29)$$

The author found that curve-fitting with a circular arc provides an elegant formula with this weighting property, as described in the next section.

Figure 6.18: Break-point tilt definition for on-point photometric modelling. At break-point P_B the tilt τ_B (depicted by the normal η_B) is given as an average of the tilt to lines joining P_A, P_B and P_B, P_C (depicted by $\eta_{A,B}$ and $\eta_{B,C}$, respectively) (a) Simple mean tilt (b) Weighted mean tilt, such that the tilt of the shorter segment is favoured.

6.4.3 Curve-Fit Evaluation of Tilt

Methods that fit high-order curves have the advantage that the Surface Smoothness Rule (p. 215) is inherently enforced, and that the profile will always be inclined ($\tau \neq 0$) as it passes through each point (Moiré Fringe Existence Rule, p. 210). Thus, the tilt is well-conditioned at the analysis points. This is desirable if the object is smooth, but profiles of real objects which have cusps like those shown in Figures 6.13 and 6.15 pose difficulties. Such objects are unlikely and handling cases with cusps was considered too complicated at this stage. Thus, a policy was adopted to treat all profiles as 'locally smooth' for the purpose of computing the tilt at a point. Figure 6.19 illustrates the use of local smoothness at points not at the cusps in parts (a) and (c), and also shows in (b) the problem of using a polynomial fit (order ≥ 3) that allows illegal inflection points.

For tilt calculation at a point, local smoothness is imposed only over the immediate neighbours. A circular arc is fitted through the point and its neighbour on each side, and the tilt through the point is taken as the inclination of the line joining the point and the circle center. Figure 6.20 shows the convex and concave cases.

Figure 6.19: Curve fitting profiles with potential cusps P_A, P_B, P_C, P_D are four contour points (a) To account for the observed fringes, a smooth curve through P_A, P_B, P_C imposes a convex cusp at P_C (b) A third-order polynomial produces a smooth curve through all four points, but introduces an illegal inflection point (arrow) which implies that a non-existent fringe lies there. (c) A smooth curve through the fringes at P_B, P_C, P_D imposes a concave cusp at P_B

Figure 6.20: The tilt of a circular arc through three points P_1, P_2, P_3 . The line joining the circle center P_C and P_2 is the normal η_2 , and has a tilt τ_2 with respect to the vertical $\hat{z}$. (a) The convex circular arc. (b) The concave circular arc

A formula was developed to express the tilt at the middle point on a circular arc through three neighbouring points $\mathcal{P}_A, \mathcal{P}_B, \mathcal{P}_C$ in terms of their relative lateral and vertical separations:

$$\tau_B = \tan^{-1} \left[\frac{(\Delta g_{A,B}^2 + \Delta z_{A,B}^2) \Delta z_{B,C} - (\Delta g_{B,C}^2 + \Delta z_{B,C}^2) \Delta z_{A,B}}{(\Delta g_{B,C}^2 + \Delta z_{B,C}^2) \Delta g_{A,B} - (\Delta g_{A,B}^2 + \Delta z_{A,B}^2) \Delta g_{B,C}} \right], \quad (6.30)$$

where

$$\Delta z_{A,B} = z_B - z_A, \quad (6.31)$$

$$\Delta z_{B,C} = z_C - z_B, \quad (6.32)$$

$$\Delta g_{A,B} \approx -\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}, \quad (6.33)$$

$$\Delta g_{B,C} \approx -\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}. \quad (6.34)$$

The formula is derived from first principles using the local plane coordinates shown in Figure 6.21. The neighbour points have coordinates $\mathcal{P}_1(u_1, v_1)$ and $\mathcal{P}_3(u_3, v_3)$ referenced to the middle point $\mathcal{P}_2(0, 0)$. Thus, v_1 and v_3 are the rises between the contours, and u_1 and u_3 are the corresponding runs. From the figure, it is clear that the normal to the curve at $\mathcal{P}_2$ has a tilt

$$\tau_2 = \tan^{-1} \left[\frac{u_c}{v_c} \right], \quad (6.35)$$

where (u_c, v_c) are the circle center coordinates. From the equation of the circle $r^2 = (u_1 - u_c)^2 + (v_1 - v_c)^2$, the equation at each point becomes:

$$r^2 = (u_1 - u_c)^2 + (v_1 - v_c)^2, \quad (6.36)$$

$$r^2 = u_c^2 + v_c^2, \quad (6.37)$$

$$r^2 = (u_3 - u_c)^2 + (v_3 - v_c)^2. \quad (6.38)$$

From Equations 6.36 and 6.37:

$$u_1^2 - 2u_1u_c + v_1^2 - 2v_1v_c = 0. \quad (6.39)$$

From Equations 6.37 and 6.38:

$$u_3^2 - 2u_3u_c + v_3^2 - 2v_3v_c = 0. \quad (6.40)$$

Figure 6.21: Local coordinate system for fitting a circle through three points. The origin is at $P_2(0,0)$. $P_1(u_1, v_1)$ and $P_3(u_3, v_3)$ are on opposite sides of the $\hat{z}$ -axis (i.e., $u_1 < 0$ and $u_3 > 0$). A circle drawn through P_1, P_2, P_3 has center $P_C(u_c, v_c)$ and radius r . The normal to the curve at P_2 has a tilt $\tau_2 = \tan^{-1}(u_c/v_c)$ with respect to the $\hat{z}$ -axis.

From Equations 6.39 and 6.40, one can solve for u_c :

$$(u_1^2 - 2u_1u_c + v_1^2)v_3 - (u_3^2 - 2u_3u_c + v_3^2)v_1 = 0, \quad (6.41)$$

which gives

$$u_c = \frac{(u_1^2 + v_1^2)v_3 - (u_3^2 + v_3^2)v_1}{2(u_1v_3 - u_3v_1)}. \quad (6.42)$$

Similarly, from Equations 6.39 and 6.40, one can solve for v_c :

$$(u_1^2 - 2v_1v_c + v_1^2)u_3 - (u_3^2 - 2v_3v_c + v_3^2)u_1 = 0, \quad (6.43)$$

which gives

$$v_c = \frac{(u_1^2 + v_1^2)u_3 - (u_3^2 + v_3^2)u_1}{-2(u_1v_3 - u_3v_1)}. \quad (6.44)$$

Equations 6.42 and 6.44 can be substituted into Equation 6.35 to give

$$\tau_2 = \tan^{-1} \left[\frac{(u_1^2 + v_1^2)v_3 - (u_3^2 + v_3^2)v_1}{(u_3^2 + v_3^2)u_1 - (u_1^2 + v_1^2)u_3} \right], \quad (6.45)$$

which is the required formula, namely Equation 6.30. (Q.E.D.)

Figure 6.22: The azimuth angle α_M to the curve through three points P_L, P_M, P_R is measured as a clockwise swing from the x -direction to the normal to the curve at the middle point P_M .

6.5 Surface Normal Azimuth Determination

This section deals with the determination of the surface normal azimuth angle at a point on the contour curve. The goal is to develop a closed-form expression for the azimuth in terms of the coordinates of the point and its neighbours.

For each point on a contour, the azimuth angle is a planimetric attribute that depends only on the shape of the contour curves in the moiré topogram image. Thus, it need only be calculated once, whereas the tilt at a point changes with the profile of each shape case. Computation of the azimuth angle involves reconstructing the contour curve through a sequence of three points in a plane, and determining the normal to the curve at the middle point. In many ways, this is like the tilt problem, but there are some important differences that call for a special approach.

Foremost is the fact that the curve for the azimuth angle is a topographic contour in the $\hat{x}$ - $\hat{y}$ reference plane. Unlike the tilt of a 'rolling terrain' whose angle is bounded by $\pm 90^\circ$, the azimuth can be in any direction in the $\hat{x}$ - $\hat{y}$ plane. Figure 6.22 shows that the azimuthal swing is measured clockwise from the $\hat{x}$ -direction to the normal to the curve at the middle point.

In general, contour segments plotted in the $\hat{x}$ - $\hat{y}$ plane are not single-valued functions of x (Fig. 6.23). Thus, the circular arc equation used for the tilt evaluation,

Figure 6.23: The functional nature of the contour curve precludes it, in general, from being a single-valued function of x .

(Eq. 6.30) cannot be easily adapted to compute azimuth angles. A parametric representation of the contour curves that resolves this difficulty is described later in the section.

6.5.1 Directionality Convention

There are two normals to a curve; they point in opposite directions. This is not a concern when measuring the surface tilt, as it is always taken in the positive vertical direction (topographic overhangs with negative tilts are not admissible shapes in moiré analysis). For the azimuth angle, a definition is required to impose a consistent choice of normal direction.

- **Definition:** The *direction of the normal vector* at a point P_M is taken at a right-angle *clockwise* to the direction of the curve passing through a sequence of points labelled P_L, P_M, P_R .

This means that with respect to the direction of the normal from P_M , P_R is to the right of P_M and P_L is to the left of P_M . This directionality convention can be seen in the figures throughout this section.

Figure 6.24: Parametric representation of the contour curve through points P_L, P_M, P_R , respectively (a) The contour curve $C(x, y)$ is drawn in the $\hat{x}$ - $\hat{y}$ plane. The parameter t is selected to take on values of $0, \xi, 1$ at points P_L, P_M, P_R , respectively (b) $C_x(t)$ is the x -component of the contour curve, drawn in the $\hat{x}$ - $\hat{t}$ parameter plane (c) $C_y(t)$ is the y -component of the contour curve, drawn in the $\hat{y}$ - $\hat{t}$ parameter plane. Both $C_x(t)$ and $C_y(t)$ are always single-valued functions of t

6.5.2 Parametric Curve-fitting Technique

In this section, a closed-form expression for the azimuth angle is derived in terms of the three points that define the curve. As mentioned, a parametric representation of the contour curve is useful for this purpose. Figure 6.24 illustrates how the parameter t serves to 'unwind' the contour curve $C(x, y)$. The parameter t is chosen to be linear with the distance along the curve, and is (arbitrarily) assigned convenient values at key points along the curve (e.g., $t_L = 0$ at point P_L and $t_R = 1$ at point P_R in Fig. 6.24). Since the exact position of P_M along the curve with respect to the other two points is not known, one sets $t_M = \xi$, where $t_L \leq t_M \leq t_R$ puts a bound on ξ . The x -values of $C(x, y)$ as t changes are mapped in the $\hat{x}$ - $\hat{t}$ parameter plane, and similarly the y -values of $C(x, y)$ as t changes are mapped in the $\hat{y}$ - $\hat{t}$ parameter plane. Regardless of the twisted nature of the contour curve $C(x, y)$ in the $\hat{x}$ - $\hat{y}$ plane, the parameter-plane curves $C_x(t)$ and $C_y(t)$ are always single-valued functions of the parameter.

The derivation of the formula for the azimuth angle is as follows. The parametric

curves are expanded as a second-order polynomial in t :

$$x = a_2 t^2 + a_1 t + a_0, \quad (6.46)$$

$$y = b_2 t^2 + b_1 t + b_0. \quad (6.47)$$

The parameter t has values assigned at each point in the sequence P_L, P_M, P_R :

$$t = \begin{cases} 0 & \text{at } (x_L, y_L), \\ \xi & \text{at } (x_M, y_M), \\ 1 & \text{at } (x_R, y_R), \end{cases} \quad (6.48)$$

where

$$\xi = \frac{\text{curve path distance from } P_L \text{ to } P_M}{\text{curve path distance from } P_M \text{ to } P_R} = \frac{\Delta t_{L,M}}{\Delta t_{M,R}}. \quad (6.49)$$

The parameter values can be substituted into the polynomials to solve for the coefficients. The x -expressions are:

$$x_L = a_0, \quad (6.50)$$

$$x_M = a_2 \xi^2 + a_1 \xi + a_0, \quad (6.51)$$

$$x_R = a_2 + a_1 + a_0, \quad (6.52)$$

which are trivially solved to give:

$$a_0 = x_L, \quad (6.53)$$

$$a_1 = \frac{\xi^2 x_R - x_M - (\xi^2 - 1)x_L}{\xi^2 - \xi}, \quad (6.54)$$

$$a_2 = \frac{\xi x_R - x_M - (\xi - 1)x_L}{\xi - \xi^2}. \quad (6.55)$$

Identical expressions result for b_2 , b_1 and b_0 in terms of the y -components. The azimuth angle θ of the tangent to the curve is given by the derivative:

$$\tan \theta = \frac{dy}{dx}. \quad (6.56)$$

The azimuth angle α of the normal to the curve is given by the negative inverse of the derivative (Fig. 6.25):

$$\tan \alpha = -\frac{dx}{dy} = -\frac{dx/dt}{dy/dt}. \quad (6.57)$$

Figure 6.25: The azimuth angle and the derivative of the contour curve. The azimuth angle of the normal is given by the negative inverse of the derivative of the contour curve.

The derivatives of the polynomials in Equations 6.46 and 6.47 are

$$dx/dt = 2a_2t + a_1, \quad (6.58)$$

$$dy/dt = 2b_2t + b_1. \quad (6.59)$$

These can be expanded at $t = \xi$ for a_2 and a_1 from Equations 6.54 and 6.55, and from the corresponding expressions for b_2 and b_1 , and substituted into Equation 6.57 to finally give the desired formula for the azimuth angle:

$$\alpha_M = \tan^{-1} \left[-\frac{(2\xi - 1)x_M + (\xi - 1)^2x_L - \xi^2x_R}{(2\xi - 1)y_M + (\xi - 1)^2y_L - \xi^2y_R} \right]. \quad (6.60)$$

All that remains is to establish an expression for ξ . It was considered impractical to evaluate the path lengths along the curve, so the chord length of each segment was used as reasonable estimate:

$$\xi = \frac{\Delta t_{L,M}}{\Delta t_{M,R}} \approx \sqrt{\frac{(x_M - x_L)^2 + (y_M - y_L)^2}{(x_M - x_R)^2 + (y_M - y_R)^2}}. \quad (6.61)$$

Equations 6.60 and 6.61 are given completely in terms of the coordinates of the three points P_L, P_M, P_R . (Q.E.D.)

Figure 6.26: Required neighbours to evaluate the surface normal at a point P_B . The gradient neighbours on each side of P_B are P_A and P_C , and are used to compute the tilt τ_B . The left and right contour neighbours, P_L and P_R , respectively, are needed to compute the azimuth α_B .

6.6 Processing Surface Normals

A brief summary is presented here to tie together the formulas used in evaluating the surface normal N at a point for each shape case. Recall that the k th case is characterized by a sequence of ϕ 's:

$$\langle \Phi \rangle^{(k)} = \langle \dots, \phi_A^{(k)}, \phi_B^{(k)}, \phi_C^{(k)} \dots \rangle \quad (6.62)$$

where points P_A, P_B, P_C are gradient neighbours in what may be a longer sequence. The objective is to compute the surface normal at P_B , and only its immediate gradient neighbours P_A and P_C play a role in the following equations. Also needed are P_L and P_R , the immediate left and right contour neighbours to P_B , as shown in Figure 6.26.

Each point in the moiré topogram image has digital line and pixel coordinates, denoted (ℓ, p) . Combined with a ϕ -value specified for the contour the point lies on, its object-space coordinates can be computed (the equations for the transformation

are given in Sec. 3.4.9, p. 116). For the k th shape case:

$$\begin{aligned}
 \mathcal{P}_A &: (\ell_A, p_A, \phi_A^{(k)}) \rightarrow (x_A, y_A, z_A)^{(k)}, \\
 \mathcal{P}_B &: (\ell_B, p_B, \phi_B^{(k)}) \rightarrow (x_B, y_B, z_B)^{(k)}, \\
 \mathcal{P}_C &: (\ell_C, p_C, \phi_C^{(k)}) \rightarrow (x_C, y_C, z_C)^{(k)}, \\
 \mathcal{P}_R &: (\ell_R, p_R, \phi_B^{(k)}) \rightarrow (x_R, y_R, z_B)^{(k)}, \\
 \mathcal{P}_L &: (\ell_L, p_L, \phi_B^{(k)}) \rightarrow (x_L, y_L, z_B)^{(k)},
 \end{aligned} \tag{6.63}$$

where $\mathcal{P}_L$ and $\mathcal{P}_R$ have the same ϕ -value as $\mathcal{P}_B$. From Equation 6.30, the tilt at $\mathcal{P}_B$ is

$$\tau_B^{(k)} = \tan^{-1} \left[\frac{(\Delta g_{A,B}^2 + \Delta z_{A,B}^2) \Delta z_{B,C} - (\Delta g_{B,C}^2 + \Delta z_{B,C}^2) \Delta z_{A,B}}{(\Delta g_{B,C}^2 + \Delta z_{B,C}^2) \Delta g_{A,B} - (\Delta g_{A,B}^2 + \Delta z_{A,B}^2) \Delta g_{B,C}} \right]^{(k)} \tag{6.64}$$

where

$$\Delta z_{A,B}^{(k)} = (z_B - z_A)^{(k)}, \tag{6.65}$$

$$\Delta z_{B,C}^{(k)} = (z_C - z_B)^{(k)}, \tag{6.66}$$

$$\Delta g_{A,B}^{(k)} \approx -\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}^{(k)}, \tag{6.67}$$

$$\Delta g_{B,C}^{(k)} \approx \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}^{(k)}. \tag{6.68}$$

The azimuth is given by Equations 6.60 and 6.61, and is constant for the shape cases (since it is the normal to the contour curve through $\mathcal{P}_B$, which does not change shape with the profile variations):

$$\alpha_B = \tan^{-1} \left[-\frac{(2\xi - 1)x_B + (\xi - 1)^2 x_L - \xi^2 x_R}{(2\xi - 1)y_B + (\xi - 1)^2 y_L - \xi^2 y_R} \right], \tag{6.69}$$

where

$$\xi = \sqrt{\frac{(x_B - x_L)^2 + (y_B - y_L)^2}{(x_B - x_R)^2 + (y_B - y_R)^2}}. \tag{6.70}$$

Finally, the surface normal at $\mathcal{P}_B$ for the k th case can be evaluated for $\tau_B^{(k)}$ and α_B using Equations 6.4 to 6.6:

$$N_x^{(k)} = \sin \tau_B^{(k)} \cos \alpha_B, \tag{6.71}$$

$$N_y^{(k)} = \sin \tau_B^{(k)} \sin \alpha_B, \tag{6.72}$$

$$N_z^{(k)} = \cos \tau_B^{(k)}. \tag{6.73}$$

6.7 Performance and Evaluation

6.7.1 Robustness

The formulas for angle calculations (tilt and azimuth) assume that the points that generate the appropriate curves are never coincident. In a continuous plane, the formulas are robust. However, in a plane with quantized coordinates, such as in the digital image, the angle-determining algorithms produce large changes in direction when the points on the curve are close together (*i.e.*, one, or a few pixels apart) and one of them is changed slightly. The software code guarded against computational blunders such as division by zero. Once the programs were debugged, no errors or input blunders were ever detected, and the angles produced were consistent and correct.

6.7.2 Angular Accuracy

The previous remark on the quality of the angle determinations is based upon empirical testing, such as determining angles from the fringe patterns of standard test-objects with accurately known geometry (Sec. 4.5, p. 150). An error analysis has not been performed to see what angular accuracy is required for photometric ordering. Instead, the best angular measures that can be extracted using the described methods are applied in a fail-or-succeed fashion. The determination of an error/performance trade-off analysis is recommended as an extension to this research.

6.7.3 Cusped Profiles

The angle-determining software has known compromises, particularly with respect to shape profiles that call for cusps to be formed at certain profile points. Strictly speaking, these shape cases should be split into two sub-cases with cusps (Fig. 6.15a). Instead, the author's software ignores the possibility of a cusp and always applies a local smooth circular arc through the profile point, making it convex or concave

according to the elevation of the neighbour points (Fig. 6.21).

With regard to the tests done in this project, ignoring the cusps is not as serious a weakness as one might think. Only Task MFO uses the curve-fitting method (Task JFO uses piece-wise linear profiles which do not have this problem), and since Task MFO uses mixed-mode contour sets with non-overlapping analysis patches, the potential cusps will always lie between the analysis points. Thus, in a sense, cusps are implicitly admitted to the profile but explicit sub-cases are not necessary. Should Task MFO be modified to accommodate matched-mode contours with overlapping analysis patches, then some thought may have to be given to accounting for cusps. However, real objects with cusped profiles are rare and, furthermore, they must be positioned in a very special way in order to produce a moiré fringe pattern that might be confused with the pattern of a simpler object (Fig. 6.13). The concept of cusped profiles is presented here for academic completeness, but they have negligible practical consequence.

Chapter 7

Automated Shading Determination

7.1 Introduction

An essential process in photometric ordering is estimating the surface reflectance or shading at various points on the object from measurements on the moiré topogram. The shading on the object is recorded in the moiré topogram just as in any regular photographic image: the reflectance of the emulsion varies from black to white in accordance with the amount of light reflected from the object to the camera. However, as discussed at length in Chapter 4, the shading envelope is modulated by the cyclic moiré fringe pattern, and this complicates the process of estimating the shading at selected points on the object.

Figure 7.1 illustrates some of the considerations of the shading measurement process. In part (a) of the figure, a gradient curve G , is shown intersecting a sequence of contours that have been traced from moiré fringes. Part (b) shows the ideal shading envelope and the moiré modulation along G , that occurs when the moiré grid and its shadows are perfectly formed (triangular modulation) and perfectly reflected (unity contrast). The objective is to estimate the value of the shading envelope at selected points in the moiré fringe pattern. Part (c) of Figure 7.1

Figure 7.1: Shading estimation in the presence of moiré fringes (a) The map image, with a sequence of contours C_A, C_B, C_C, C_D, C_E intersected by a gradient curve G_j through contour points P_A, P_B, P_C, P_D, P_E . The objective is to estimate the shading, for example, about point P_B (b) The theoretical reflectance profile $\rho(g)$ taken along the gradient curve G_j , showing the ideal shading envelope and perfect moiré fringe modulation (c) The measured reflectance profile $\bar{\rho}(g)$ showing the degradation of the moiré fringe contrast. Stray reflections, grid strings that are not 'black', object reflectance effects, fuzzy grid shadows and other effects typically combine to spoil the modulation

represents typical moiré fringe modulation of the shading envelope, and illustrates some of the complications that must be attended to. Most obvious is the noise in the moiré modulation function; averaging over a *patch* on the image¹ is needed to suppress the moiré-carrier which may dominate the modulation function. Also noticeable is the degradation of the contrast and the departure from the perfect-triangular modulation function; detector nonlinearity and optical imperfections in the object and moiré instrument are the controlling factors. A more subtle problem in measuring the shading envelope is that the spacing between the moiré fringes is not constant; care must be taken not to bias the shading estimate, as is the case if a constant-sized image patch is used when averaging to suppress noise.

In this chapter, the task of automating the estimation of the local surface shading is examined. The shading measurements are estimated from the digitized moiré topogram image of the object. Averaging of the local data values within a small image-patch centered on selected points is shown to be necessary, and this in itself is not so difficult to automate. However, the variable spacing of the moiré fringes causes a fixed-size patch to introduce a variable bias to the shading estimate, and the need to dynamically adjust the shading estimator to the fringe spacing is analyzed. The attributes and weaknesses of various patch geometries are examined. Polygon patches are shown to be useful in maintaining a consistent bias in the estimator, but they are computationally tedious to determine. A circular patch is ultimately adopted, and its virtues and limitations are discussed along with its software implementation. The chapter ends with a discussion of the performance of the automated shading estimator based on dynamic circular patches.

7.2 Shading and Shadows

A moiré topogram of an object is a composite of shading and shadows, and it is important to distinguish between them. The term *shading* will always refer to the reflective properties of the object and their sensitivity to the viewing and illu-

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¹The shading measurement patch and its use by the Photometric Ordering Operator is described in Sections 3.3.6 and 3.3.7, starting on page 91.

mination orientation. Thus the shading is the variation in the surface brightness witnessed under 'regular illumination' (i.e., in the absence of the moiré grid and its effects). The term *shadows* will always refer to obstruction of the illumination, such as in the case of the moiré grid strings casting a pattern of shadows on the object.

7.3. Reflectance Data

The reflectance data used in the photometric analysis is corrupted and distorted by several noise factors that must be taken into consideration. Assumptions based upon the ideal moiré fringe pattern are valid if the carrier spacing (grid lines and shadows) and the image-digitization interval (pixel size) are fine compared to the spacing of the fringes. Figure 7.2 attempts to depict the departure that real moiré topogram images make from the ideal model. In all three parts of the figure, the moiré fringes are broad, straight, vertical bands. Part (a) shows synthetic ideal fringes, with perfect triangular modulation and grid lines and shadows that are too fine to be resolved. Part (b) has the same synthetic perfect fringes as (a), but the carrier noise dominates the images and the fringes are much more difficult to visualize. Part (c) is a magnified view taken from an actual moiré topogram (the grid lines are horizontal and defocused, whereas their shadows are clearly seen as ragged, diagonal lines) and the contrast of the moiré fringes is very poor.

The magnified view in Figure 7.2c is an extreme example of noisy image conditions. There are many figures in this document showing topograms of real objects with clearly-visible moiré fringes which appear free of such noise. However, at some location on most topograms, the moiré fringe spacing approaches the carrier spacing, resulting in reduced contrast and the possibility of alias fringes (see Sec. A.9, p. 405). Furthermore, if the carrier structure is resolved in the topogram, then the shading estimation method must accommodate this coherent pattern-noise. Figures A.13, A.15 and A.17 (starting on p. 408) show good examples of the carrier noise that typifies stationary-grid moiré instruments.

Image digitization introduces still more problems with the reflectance data. As

Figure 7.2: Comparison between ideal, perfect and actual moiré fringe patterns. In all three cases, the fringes are vertical and straight. (a) Ideal moiré fringes have a triangular modulation with unity contrast, and are formed by grid and shadow lines that are below the viewing resolution limit. (b) The moiré fringes from perfect 'black' grid and shadow lines ($\rho = 0$) that are perfectly formed and resolved have an average unity contrast, but the lines introduce a coarse carrier structure (coherent noise) that must be compensated for by local processing. (c) The actual grid and shadow lines are not truly black ($\rho \approx 3\%$), are fuzzy and ill-focused, and result in reduced contrast and compromised extraction of the background shading envelope.

a rule of thumb, one can usually resolve fringes in the topogram that are separated by several grid-pitches. Digitization to resolve these fringes sets the pixel size to be near the grid pitch, and alternate rows of pixels may lie nearly on, or off, the grid lines in the topogram image. Close inspection of the digital image so that individual pixels are resolved, reveals a rough and ragged appearance of the moiré fringes. An example of the digital representation of a magnified patch spanning a couple of fringes is shown in Figure 7.3. Thus the digital reflectance data in these studies is very noisy, but not from the customary sources. Quantization noise from the A/D converter is restricted to the least-significant bit, and is negligible in comparison to the wide fluctuations from pixel-to-pixel due to the moiré noise. It is obvious that the local shading estimate has to be extracted from many neighbouring pixels to average out the carrier noise.

7.4 Shading Estimation in the Presence of Moiré Fringes

A simple average over a fixed-size patch in a moiré pattern produces inconsistent shading estimates, even if it is centered on a fringe each time. The reason is that the moiré fringes modulate the shading values and their varied spacing causes the phase of the modulation to vary over a fixed-size patch. Since the fringes have already been traced as contours (an assumption made at the outset of this research), the moiré fringe phase could be interpolated everywhere in the topogram to demodulate the image. In principle, it is possible to eliminate the fringe pattern this way and regenerate the smooth envelope of the absolute shading values. Then fixed-size patches (indeed, single pixels!) could be used to measure the absolute shading value anywhere in the image. In practice, fringe pattern demodulation is a very difficult problem, but this approach might have been necessary if the photometric ordering method relied on absolute shading values. Fortunately, it does not.

(a)

60	38	10	25	52	77	100	84	65	48	20	35	57	77	100	74
25	18	0	0	25	50	80	62	35	23	0	0	15	40	70	57
67	45	18	38	56	73	96	90	62	35	8	29	51	73	100	100
50	23	0	0	18	36	53	60	40	18	0	1	28	46	63	65
74	50	25	13	21	67	90	97	79	60	35	23	24	67	85	87
35	20	3	0	13	39	66	77	45	25	3	0	3	29	56	72
80	68	37	28	45	60	82	100	75	59	27	18	40	60	87	100
55	35	9	0	3	23	40	70	45	30	9	0	13	33	50	75
88	65	38	11	27	55	78	100	83	75	48	21	32	55	71	90
41	36	15	0	0	28	53	71	51	41	15	0	0	16	43	66
95	74	60	37	32	49	71	94	90	64	50	27	27	49	76	100
77	50	23	0	0	11	27	58	67	45	23	5	4	21	37	60

(b)

Figure 7.3: Magnified digital image patch spanning a couple of typical moiré fringes (a) A gray-scale display of the digital values assigned to each pixel, giving a visual impression of the spatial relationship and the noisiness of the pattern (b) The actual pixel values, through which the software must draw the contour curves, and then extract an estimate of the shading on the surface of the object. It is not obvious which filament of pixels should define a contour, nor what the shading estimate should be about a selected pixel.

7.4.1 Justification for Using a Biased Shading Estimator

The photometric ordering process described in Section 3.3.7 (p. 92) does not require the absolute shading, since it involves a relative comparison of the model-dependent behaviour between neighbouring surface patches. The absoluteness of the shading values can be folded into the photometric model parameter and shared by both patches. Provided the shading estimate has a consistent distortion, the distorted model should provide the same ordering outcome as the model based upon absolute shading values. This can be seen from the following analysis.

The photometric model introduced in Chapter 3 expresses the reflectance ρ_i in terms of the nominal reflectance ρ_0 and a geometric factor γ_i . From Equation 3.27 (p. 86) the Lambertian model is

$$\rho_i = \rho_0 \gamma_i \quad (7.1)$$

Consider the case where the true reflectance can be measured without distortion. Two neighbouring patches P_1 and P_2 , with unbiased measured reflectance values $\bar{\rho}_1$ and $\bar{\rho}_2$, respectively, are assumed to have the same model parameter ρ_0 . P_1 has geometry γ_1 , and the Photometric Ordering Operator tests if P_2 has geometry γ_2 that is consistent with the shading model by computing the normalized absolute error ϵ between the expected and measured shading values:

$$\epsilon_2 = \frac{|\bar{\rho}_2 - \bar{\rho}_2|}{\bar{\rho}_2} \quad (7.2)$$

The model parameter is solved at P_1 using Equation 7.1. ✓

$$\rho_0 = \bar{\rho}_1 / \gamma_1 \quad (7.3)$$

The expected reflectance at P_2 is

$$\bar{\rho}_2 = \rho_0 \gamma_2 = \bar{\rho}_1 \gamma_2 / \gamma_1 \quad (7.4)$$

Thus the photometric analysis error due to an unbiased shading estimate is

$$\epsilon_2 = \frac{|\bar{\rho}_1 \gamma_2 / \gamma_1 - \bar{\rho}_2|}{\bar{\rho}_2} \quad (7.5)$$

Now consider the case where the measured reflectance ρ'_i has a patch-dependent distortion with respect to the true reflectance there:

$$\rho'_i = a_i + b_i \rho_i, \quad (7.6)$$

where a_i and b_i are the additive and multiplicative coefficients of distortion for the i th patch. Now patches P_1 and P_2 , with biased measured reflectance values $\bar{\rho}'_1$ and $\bar{\rho}'_2$, respectively, are assumed to have the same distorted model parameter ρ'_0 . As before, P_1 has geometry γ_1 , and the Photometric Ordering Operator tests if P_2 has geometry γ_2 that is consistent with the shading model by computing the distorted normalized absolute error ϵ' between the expected and measured shading values:

$$\epsilon'_2 = \frac{|\bar{\rho}'_2 - \bar{\rho}'_1|}{\bar{\rho}'_1} \quad (7.7)$$

The distorted model parameter is solved at P_1 using Equation 7.1:

$$\rho'_0 = \bar{\rho}'_1 / \gamma_1 = (a_1 + b_1 \bar{\rho}_1) / \gamma_1. \quad (7.8)$$

The distorted expected reflectance at P_2 is

$$\bar{\rho}'_2 = \rho'_0 \gamma_2 = (a_1 + b_1 \bar{\rho}_1) \gamma_2 / \gamma_1. \quad (7.9)$$

So the photometric analysis error due to the biased shading estimator is

$$\epsilon'_2 = \frac{|(a_1 + b_1 \bar{\rho}_1) \gamma_2 / \gamma_1 - (a_2 + b_2 \bar{\rho}_2)|}{(a_2 + b_2 \bar{\rho}_2)} \quad (7.10)$$

The photometric ordering outcome is based on the value of the total photometric analysis error for each ordering case, summed over all the regions for the neighbourhood (Eq. 3.39, p. 95). Thus, if the photometric analysis errors are the same ($\epsilon'_i \doteq \epsilon_i$), then the biased estimator will produce ordering results that are identical to an unbiased shading estimator. Equations 7.5 and 7.10 are identical if $a_1 = a_2 = 0$ and $b_1 = b_2 = b$. Thus, if the shading estimator introduces a consistent distortion of the form

$$\bar{\rho}'_i = b \bar{\rho}_i, \quad (7.11)$$

then the distorted model should provide the same ordering outcome as the model based upon absolute shading values.

This result can be extended to the squared-error given in Equation 3.37 (p. 94) as

$$\epsilon = (\tilde{\rho}_i - \bar{\rho}_i)^2, \quad (7.12)$$

which is the form used in Tasks JFO and MFO. In this case, if the shading estimator has a constant multiplicative bias as indicated in Equation 7.11, then

$$\epsilon'_i = b^2 \epsilon_i, \quad (7.13)$$

and the error for each ordering case in Equation 3.38 (p. 94) has a common factor

$$[\epsilon']_{\mathbf{x}_j} = [b^2 \epsilon]_{\mathbf{x}_j} \begin{bmatrix} b^2 \epsilon^{(1)} \\ \vdots \\ b^2 \epsilon^{(k)} \\ \vdots \\ b^2 \epsilon^{(N)} \end{bmatrix}_{\mathbf{x}_j} \quad (7.14)$$

The constant b^2 -factor scales all of the photometric analysis errors consistently. Despite the b^2 -factor, the biased and unbiased shading estimators pick the same case.

7.4.2 1-D Shading Envelope Estimation

In this section, the constraints to ensure that the estimator has a consistent bias are analyzed for the 1-D case. Figure 7.4 shows a pair of points on a gradient curve that have the same moiré phase value. Here they are chosen to lie at intensity maxima (bright fringes), but intensity minima (dark fringes) or other convenient phase values could have been used without any loss of generality.

The ideal reflectance profile $\rho(g)$ along the gradient curve $\mathcal{G}(g)$ is drawn in Figure 7.5, and the points g_1 and g_2 are shown with the integration interval $[g_1^-, g_1^+]$ used to estimate the shading value. The interval represents a constant moiré phase interval of the cyclic moiré modulation function $M_i(g)$, but in the spatial domain (as drawn in the figure) it will have a different length about each point as the fringes

Figure 7.4: Contours used in the 1-D shading estimation analysis. An alternating sequence of bright and dark contour fringes is intersected by the gradient curve G_1 . g_1 and g_2 are two points on the gradient curve with the same moiré phase (shown here at fringe intensity maxima).

have varying spacing. The function $\rho(g)$ may be nonlinear, but over the interval $[g_i^-, g_i^+]$ is assumed to be sufficiently linear such that

$$\rho(g) \approx [\rho(g_i) + \dot{\rho}_i(g - g_i)] \quad \forall g_i \in [g_i^-, g_i^+], \quad (7.15)$$

where $\dot{\rho}_i$ is the slope of $\rho(g)$ through g_i (i.e., $\dot{\rho} = d\rho/dg$).

Figure 7.6 shows the moiré modulation function $M_0(g)$ at $g = 0$ is bounded by the interval $[g_0^-, g_0^+]$. About other points g_i , the modulation is a scaled, shifted version of the function at the origin:

$$M_i(g) = M_0\left(\frac{w_0}{w_i}(g - g_i)\right), \quad (7.16)$$

where the width of the interval w is defined as

$$w_i = g_i^+ - g_i^-, \quad (7.17)$$

and the interval bounds are chosen about g_i such that

$$M_i(g_i^-) = M_0(g_0^-), \quad (7.18)$$

$$M_i(g_i^+) = M_0(g_0^+). \quad (7.19)$$

Figure 7.5: Reflectance profile used in 1-D shading estimation analysis. $\rho(g)$ is the smooth reflectance function along the gradient, and ρ_1 and ρ_2 are its values at g_1 and g_2 , respectively. Spanning each point is a moiré phase interval $[g_i^-, g_i^+]$ over which the integration for producing the shading estimate is taken.

Figure 7.6: The moiré modulation function used for the 1-D analysis. It is considered non-zero only in the analysis interval, although in general, it is periodic. It may be nonlinear, and depending upon the choice of analysis interval, it may be uneven about the origin of the analysis patch.

The local mean modulation-weighted shading reflectance $\bar{\rho}_i$ is given by integrating over the interval and normalizing:

$$\bar{\rho}_i = \frac{1}{w_i} \int_{g_i^-}^{g_i^+} M_i(g) \rho(g) dg. \quad (7.20)$$

This expands from Equation 7.15 to:

$$\bar{\rho}_i = \frac{1}{w_i} \int_{g_i^-}^{g_i^+} M_i(g) [\rho(g_i) + \dot{\rho}_i(g_i - g)] dg, \quad (7.21)$$

$$= \rho_i(g) \frac{1}{w_i} \int_{g_i^-}^{g_i^+} M_i(g) dg + \dot{\rho}_i \frac{1}{w_i} \int_{g_i^-}^{g_i^+} M_i(g)(g_i - g) dg, \quad (7.22)$$

$$= \rho_i(g) \bar{M}_i + \dot{\rho}_i \mu_i, \quad (7.23)$$

where $\bar{M}_i$ is the mean of the modulation over the interval:

$$\bar{M}_i = \frac{1}{w_i} \int_{g_i^-}^{g_i^+} M_i(g) dg = \frac{1}{w_0} \int_{g_0^-}^{g_0^+} M_0(g) dg = \bar{M}_0, \quad (7.24)$$

and μ_i is its first moment:

$$\mu_i = \frac{1}{w_i} \int_{g_i^-}^{g_i^+} M_i(g)(g_i - g) dg = \frac{1}{w_0} \int_{g_0^-}^{g_0^+} M_0(g)(g_i - g) dg = \mu_0. \quad (7.25)$$

The first term of Equation 7.23 can be compared to Equation 7.11, where $\bar{M}$ corresponds to the multiplicative bias in that equation, and is a constant if the estimation integral is taken over a constant moiré phase interval in a consistent fashion for each point g_i . Note that the first moment μ_i is always zero if the selected interval makes the modulation an even function about the point g_i , and thus the second term in Equation 7.23 can be made to vanish (or at least be kept negligible by careful selection of the shading averaging interval).

The 1-D analysis shows that it is important to estimate the local shading along a moiré phase interval that is consistent from patch-to-patch, so that the estimate has a consistent bias at each point. This suggests that the sample integration should be taken along the gradient curve in intervals that are bounded by characteristic cyclic properties of the fringes, rather than a fixed spatial distance. For example, the shading at a bright fringe could be estimated along an interval spanning the dark fringe on each side. Such a scheme also ensures that the moiré modulation in the interval is an even function (so that $\mu_i = 0$).

7.5 2-D Patch Considerations

A better estimate of the shading results if the integration is taken over a 2-D patch in the moiré image. The patch must be designed so that it spans the desired moiré phase interval in the gradient direction and has sufficient width in the contour direction to suppress the noise. Its shape is not arbitrary, but rather should perfectly bound a series of 1-D gradient intervals. The series of gradient intervals might be of uniform phase interval (as in the quadrilateral patch described below), but 'tapered' intervals (as in the circular patch described later) may also be used. The magnitude of the bias is not important, as long as it is a consistent factor for all shading measurements.

The adopted patch shape and size must be adjusted consistently in accordance with the local contour pattern so that samples (pixels) are always taken in the same fashion along the gradient and contour directions. Since the patch size is varied, care must be exercised to normalize the shading estimate by counting the number of samples contained in the patch. Small patches occur between contours near the digital spatial resolution limit, and empty patches can be avoided if the estimator always includes the samples on the patch boundary.

Adjusting the patch boundary to the local contour and gradient curves requires that the shading estimator have access to portions of the map data (*i.e.*, contour image and member lists) as well as the shading data (*i.e.*, reflectance image). Some methods of defining the boundary require that only the coordinates of the central point and key neighbours be supplied as seed-points from which the boundary is geometrically grown, whereas other methods require searching the map data to delineate segments of the local contour and gradient curves. Some of the seed-points on the critical curves that are used to define patch boundaries are shown in Figure 7.7.

Figure 7.7: Seed-points for generating shading estimation boundary about P_B . C_{i-1} , C_i and C_{i+1} are a sequence of contours, with gradient G_j intersecting them at P_A , P_B and P_C , respectively. P_E and P_F are contour-points on each side of P_B .

7.6 Polygon Patches

The *polygon patch* is a good approximation to the ideal patch containing a consistent series of gradient intervals. Its boundary is defined by seed-points in both the gradient and contour directions, and thus it conforms to the spacing variation in the contour pattern in an optimal fashion.

Integrating the interior of a polygon of arbitrary complexity is trivial if the boundary can be drawn (by setting a unique boundary-bit of the map image) as a closed, continuous 8-connected string of points. A first-in/first-out buffer is initialized with the coordinates of an interior point, whose reflectance is summed and boundary-bit set. Then, the coordinates of interior points are pulled in turn from the buffer, and the boundary-bit of each 4-neighbour is examined: if set, the neighbour is discarded; but if clear, its reflectance is summed, its boundary-bit is set and its coordinates put in the buffer. The number of interior points can be counted so that the summed reflectance can be normalized with respect to the area of the polygon. If the boundary is to be included in the integration (desirable if the patches tend to get small), a separate bit must be set to distinguish between processed

Figure 7.8: A map of a hill tiled with moiré cells. Contours C_A , C_B , C_C encircle the hill, and gradients G_a , G_b , G_c , G_d , G_e radiate from the summit. Each moiré cell is bounded by pairs of contours (i.e., moiré fringes) and by pairs of gradient curves. One moiré cell with vertices P_1 , P_2 , P_3 , P_4 has been shaded for illustrative purposes.

interior and boundary points.

7.6.1 The Moiré Cell

Figure 7.8 shows a simple map of a hill with closed contour loops, and a series of gradients radiating from its summit. The two sets of curves (contours and gradients) partition the map into a set of patches (one with vertices labelled P_1 , P_2 , P_3 , P_4 is shaded in Fig. 7.8). These patches are termed *moiré cells* since they fundamentally conform to the moiré fringe pattern: one pair of boundary curves (the contours) are loci of constant moiré phase, while the other pair of boundaries (the gradients) are loci of maximal change in moiré phase. Bounded by the true contour and gradient curves, the moiré cell provides an ideal shading estimation patch.

The primary difficulty with using the moiré cell as a shading estimation patch is the need to search for the boundary vertices. The generation of the moiré cell from the seed-points available to the shading estimator is depicted in Figure 7.9. The two contour arcs through P_A and P_C are known from the map image. However, the task

Figure 7.9: Seed-points for delineating the moiré cell centered at P_B . Contour curves C_{i-1} , C_i and C_{i+1} exist in the map image, seed-points P_A , P_C , P_E , P_F are given, and gradient G_j is known. The gradient G_E through P_E is drawn by searching the map image for intersections with contour C_{i-1} at vertex P_X and with contour C_{i+1} at vertex P_Y . The gradient G_F through P_F is similarly discovered to establish vertices P_U and P_V .

of discovering the gradient curves through P_E and P_F is tedious (see Ch. 5), and requires searching the map image from point P_E (in Fig. 7.9) to find the gradient neighbours P_X and P_Y , and from P_F to find P_U and P_V .

7.6.2 Polygon Approximations

Another disadvantage of using the moiré cell to define the shading measurement patch is that the generation of the boundary curves through the vertices is computationally expensive. Straight-line polygons are a more cost-effective approximation. The resulting distortion in the boundary is a moot point when one considers the crudeness of its representation in a spatially-quantized digital image. For moiré cells at moderate resolution, the arcs in question often span areas less than 10×10 pixel elements, and then it makes no sense to distinguish between a gentle arc or its chord when both are identically delineated by a 'staircase' line.

Further refinements, such as weighting the samples along the boundary to emulate sub-pixel accuracy, were not explored. This was deemed a concern only for

Figure 7.10: Mid-sized contour-centered moiré cell, centered about point P_B . P_E , P_A , P_F and P_C are available boundary points defined previously. P_1 , P_2 , P_3 and P_4 are the respective midpoints to P_B and points P_E , P_A , P_F and P_C . Corner points P_5 and P_8 are the midpoint gradient neighbours of P_1 (found by searching the map image), and similarly P_6 and P_7 are discovered by searching from P_3 . The mid-size moiré cell is thus bounded by P_1 , P_5 , P_2 , P_6 , P_3 , P_7 , P_4 , P_8 .

very small patches (i.e., ones with mostly boundary points), and since small patches were usually compared to likewise-small patches, consistent (unweighted) behaviour was expected to keep photometric comparisons in balance.

A final consideration is one of patch overlap. Referring to Figure 7.9, the moiré cell about P_B spans three contours. If the photometric evaluation compares the shading about immediate neighbours (say points P_B and P_C in that figure), then a smaller patch must be designed to avoid correlation of the shading characteristic assigned to each point. One approach for a mid-size patch is suggested in Figure 7.10, and requires the same searching overhead as for the full-sized moiré cell, but also involves computing eight midpoint coordinates. Each midpoint has the mean value coordinates of the appropriate endpoints, so this is a small addition to the cost of the gradient neighbour search operation.

Figure 7.11: The polygon star shading estimation patch formed about point P_B . Midpoints P_1 , P_2 , P_3 , P_4 are formed between P_B and the points P_E , P_A , P_F and P_C , and the overlapping quadrilaterals P_1 , P_A , P_3 , P_C and P_2 , P_F , P_4 , P_E enclose the shaded interior patch used to estimate the shading about P_B .

7.6.3 Polygon Star

A different approach to forming a polygon boundary was devised that avoided further gradient neighbour searches and yielded mid-sized patches. The formation of the *polygon star* is shown in Figure 7.11. Recall that points P_A , P_B , P_C , P_E , P_F are supplied to the shading estimator, and that the midpoints P_1 , P_2 , P_3 , P_4 are trivial to compute. The patch boundary is the interior intersection of the two quadrilaterals shown in the figure, and is rapidly traced using straight lines. In the figure, the interior of the patch is hatched with a series of gradient curve intervals to illustrate the balance of samples taken on each side of P_B 's contour curve.

Software was written (Task JEE) to estimate the local shading using the polygon star patch and the polygon-interior integrator described earlier. The subroutine calls for the five seed-points (P_A, P_B, P_C, P_E, P_F), draws the star in the boundary-bit of the map image, and then integrates the interior shading values. The normalized value is returned to the calling program, along with the patch size (number of pixels)

and an error code ($ok=1$).

One of the strengths of the star-shaped patch is its natural adaptation to meandering contours with drastically changing curvature (Fig. 7.12). Chapter 5 discussed how the contour curves are generated from a sparse string of break-points. In Task JEE, contour points P_E , P_B , P_F (see Fig. 7.11) are just sequences of break-points read from the contour-list file, and thus the polygon star automatically adjusts its shape to correspond to the contour curvature. Of course, this adaptability has a cost: the list and image data-structures must be searched to find the five seed-points required by the algorithm.

A weakness of all these boundary-drawing algorithms is that the drawing operation is rather slow. Furthermore, the boundary-bit remains set after the operation, and may interfere with the generation and scanning of neighbouring patches. Clearing the boundary-bit after each patch takes significant time.

7.7 Circular Patches

In this section, circular patches for shading estimation are examined. Circular boundaries have several advantages over their polygon counterparts. They have constant shape and are insensitive to orientation. Only two diametric points are required to define the circular boundary. The circular boundary has a simple generating equation, and need not be drawn to confine an interior scan which can be rasterized with calculated endpoints. Thus, scanning of the patch interior is more efficient and the setting/testing/clearing of boundary bits is avoided. There are weaknesses too and these are described below — but the circular patch was ultimately adopted as the best method.

7.7.1 Mid-point Circular Patch

The simplicity of the circular patch permitted easy adaptation to different photometric ordering schemes. For example, Task JFO (Ch. 8) considers the critical surface patch to always lie between a pair of contour fringes (mid-point photomet-

Figure 7.12: The polygon star as it adapts to changing contour curvature and orientation. The hatching of each estimation patch in the gradient direction suggests how well the star maintains a consistent series of moiré phase intervals, balanced about the contour 'spine' of the patch as its shape and orientation at each location is dynamically adjusted.

Figure 7.13: The mid-point circular patch is defined by contour points P_A and P_B on its diameter. Mid-point P_M is its calculated center and the shading estimate is assigned to P_M .

ric modelling), as shown in Figure 7.13. Only two contour points (P_A, P_B) have to be supplied; the mid-point P_M and the patch boundary are trivially calculated. Task JFO always uses bright or dark fringes (matched-mode), thus the moiré phase interval subtended by a contour pair is always an even function (ensuring consistent shading estimation bias).

7.7.2 Contour-point Circular Patch

Task MFO (Ch. 9) always uses three contours to define the circular patch (on-point photometric modelling), as shown in Figure 7.14. The exterior points (P_A, P_C) define the circle, and the interior point P_B is assumed to nominally lie at the center of the patch, so that the shading estimate is attributed to P_B .

Task MFO permits assessment of both moiré fringe contour modes: sets of bright fringes or of dark fringes (matched-mode), or of both bright and dark fringes (mixed-mode). By requiring three contour points, irrespective of the mode, the moiré phase interval is consistently an even function (again, maintaining a consistent bias on the shading estimator). To allow adjacent contours to be photometrically assessed

4

MicroD

Figure 7.14: The contour-point circular patch is defined by contour points P_A and P_C on its diameter. Contour-point P_B is nominally at the patch center in this figure, and the shading estimate extracted from the patch is assigned to P_B .

without patch overlap, a mid-size contour-point circular patch could be generated as shown in Figure 7.15.

7.7.3 Estimation Distortion by Circular Patches

The simplicity of the circular patch is also its weakness. An obvious difficulty is that contours are rarely equally spaced in the map-plane (i.e., the planimetric separation, as opposed to implied elevation intervals). Thus the circle center does not lie on the central contour of a contour trio. Figure 7.16 illustrates the difficulty of resolving the different distances to the neighbours with semi-circular patches. The method suggested in this figure is sensitive to orientation and, furthermore, the series of gradient-curve phase-intervals is not likely to be consistent from patch to patch. This implies the shading estimator bias will fluctuate, thereby spoiling the basis for photometric ordering.

Several corrective strategies were initially explored, but all lead to computational burdens without a demonstrated need. Therefore, instead of compensating for the distortion, the problem has been ignored at this stage. Simple circles as shown

Figure 7.15: A mid-size contour-point circular patch is defined by the midpoints from P_B to each of P_A and P_C on the neighbouring contours. Contour-point P_B is nominally at the patch center in this figure, and the shading estimate extracted from the patch is assigned to P_B . Similar patches could be formed at P_A and P_C without overlap.

Figure 7.16: Two semi-circular patches of different radii. The attempt to deal with neighbour contours at different distances results in an inconsistent distortion of the shading estimate at P_B .

Figure 7.17: Contour-point circular patch for differently spaced contours. Contour points P_A and P_C diametrically define the patch with circle center P_M at their midpoint. Contour point P_B may be a negligible distance from P_M .

Figure 7.17 are used and despite the compromise, the photometric ordering based on simple circular patches works very well with all the test imagery used by the author (see Chs. 8 and 9). The author suspects that several factors combined to favour success in this respect. Smooth objects tend to have fringes that change their planimetric spacing gradually, and the shading changes are, of course, correlated to these gradual shape changes. Thus, circular patches spanning opposite sides of a contour are likely to have the same degree of phase-sampling distortion, thereby ensuring that the bias factor in the shading estimation will be sufficiently constant.

7.8 Implementing the Circular Patch

In this section, some of the programming considerations of invoking a shading estimator based on circular patches are described.

7.8.1 Calling Sequence

Much of the previous discussion dwells on the potential distortion to estimates of the shading in the presence of the moiré fringes. If the photometric ordering

method depended upon rigorous measurements of the shading, then it would be too cumbersome to be practical. The author elected to ignore the pitfalls to see if a simple solution would suffice. In fact, the shading-estimator program described below works very well. The subroutine has a simple calling sequence: the input is the coordinates (X1,Y1) and (X2,Y2) of the two diametric points, and the output is the shading estimate REFL and a standard error flag (always set to 1 if processing is successful).

7.8.2 Circle Center and Radius

The computation of the circle parameters is trivial:

```
XC = (X1+X2)/2           ! x-coord of circle center
YC = (Y1+Y2)/2           ! y-coord of circle center
RR = ((X1-X2)**2+(Y1-Y2)**2)/4 ! circle diameter squared
R = SQRT(RR)              ! radius of circle
```

7.8.3 Pixel Reflectance Access

Access to the reflectance image data in video memory is through utility routines (DIPIX library modules). As described previously (Ch. 5), the attributes of each pixel are stored in user-defined fields spanning 16-bits. In these tests, the upper byte (8-bits) contains the shading values. The lower byte contains the contour label number if the pixel belongs to a contour (otherwise it is zero), plus an annotation-bit used to draw patch boundaries. The utility uses a bit-mask to extract the requested field.

7.8.4 Integration Schemes

As mentioned earlier, the polygon patches involve drawing the boundary and 'nibbling' out the interior. Circular patches avoid the need to draw the boundary since a raster scan of nested DO-loops can visit each pixel in the patch, with the start/stop limits of each scan line (i.e., the boundary pixels) explicitly computed as needed.

7.8.4.1 Trivial Limits

This method restricts the scan to the interior of a circumscribed square (see Fig. 7.18), thus the scan limits are easy to compute and are fixed for the patch. Each interior pixel was tested in turn, and only those within the radius distance from the center contribute to the shading estimate.

```

X1 = XC - R    X2 = XC + R    ! set column limits
Y1 = YC - R    Y2 = YC + R    ! set row limits
SUM = 0        NUM = 0        ! clear counters
DO 10 Yj = Y1,Y2                ! scan the rows
DO 10 Xi = X1,X2                ! scan the columns
RijsQ = (Xi-XC)*(Xi-XC) + (Yj-YC)*(Yj-YC) ! distance**2 to C
IF (RijsQ .GT. RR) GOTO 10      ! skip if too far
SUM = SUM + REF(Xi,Yj)         ! ...else sum the reflectance
NUM = NUM + 1                  ! and count the pixel
10 CONTINUE
REFL = SUM/NUM                  ! normalize shading estimate.

```

7.8.4.2 Solved Limits

The equation of the circular boundary can be solved for each row to compute the scan start/stop columns (see Fig. 7.19). This approach eliminates the need to test each pixel, but uses a (slow) math function call to SQRT for every row.

```

Y1 = YC - R    Y2 = YC + R    ! set row limits
SUM = 0        NUM = 0        ! clear counters
DO 10 Yj = Y1,Y2                ! scan the rows
XOF = SQRT(RR - (Yj-YC)*(Yj-YC)) ! compute the offset
X1 = XC - XOF    X2 = XC + XOF    ! set column limits
DO 10 Xi = X1,X2                ! scan the columns
SUM = SUM + REF(Xi,Yj)         ! ...sum the reflectance
NUM = NUM + 1                  ! and count the pixels
10 CONTINUE
REFL = SUM/NUM                  ! normalize shading estimate.

```

Figure 7.18: Trivial square-limits for scanning a circular interior. P_A and P_B are the diametric seed-points that define the circle. P_C is the center and R is the radius of the circle. The sides, top and bottom of the circumscribed square scanning patch are a distance R from P_C . The distance $r_{i,j}$ of each point from P_C is tested, and if not greater than R (so as to include boundary pixels), then that point's reflectance is included in the reflectance estimate.

Figure 7.19: Computed limits for scanning a circular boundary P_A and P_B are the usual seed-points, and P_C is at the circle center. $Y1$ and $Y2$ are the computed upper/lower limits for scanning the rows in the circle. For each row Y_j , the start/stop columns $X1_j$ and $X2_j$ are computed as described in the text.

7.8.4.3 Tabular Limits

A given patch size may be used hundreds of times in the photometric analysis of a moiré topogram. To further improve the efficiency, one might consider generating tables of boundary-offsets for the rows of various-sized circles. Selection of the correct table for a required circle could be made by a pointer computed from the seed-points coordinates. Then values to set scan limits could be read by keying the scan-row loop-index into the appropriate array of offset values. With this method, only one square-root is calculated per patch instead of n for an n -row patch, but the program requires more memory to store the tables.

7.8.5 Diagnostics and Error Checks

The subroutine code checks that the boundary always stays within the image, and that the number of pixels in the estimate is non-zero (prior to normalization). To provide visual indication of the processing, the annotation-bit is set in the image

display at each interior pixel as its reflectance value is included into the sum. This appears on the screen as a white 'snow-ball' that grows in a top-down raster scan pattern (Fig. 5.9, p. 182) and is quite useful for spotting where the circular patch fails to span the moiré gradient interval as intended. Note that for circular patches, the annotation-bit plays no role in defining the boundary nor in confining the scan of the patch interior (not the case with the polygon patches discussed previously), and so old 'snow-balls' can be left on the screen while each new patch is analyzed.

7.9 Performance

The shading estimation software used by the author has been designed for understanding rather than ultimate execution speed. The design of the algorithm prevents arithmetic errors from occurring, however, passing bad seed-points produces incongruous shading estimates for patches that are obviously inappropriate to the local fringes.

7.9.1 Computational Efficiency

In this study, more attention has been spent on diagnostics and error-free code development than on schemes for saving machine-cycles. It was essential that the code work correctly and that it provide enough processing parameters to verify the computations. It was sufficient that the execution was fast enough so that software development was not hampered. For example, on the DIPIX Aries-II machine a typical patch 10-20 pixels in diameter takes 1-2 seconds to process, including the painting of the 'snow-ball' on the display. With 10-30 patches per contour, a dozen contour-pairs can be ordered in about five minutes. This performance was deemed acceptable and the shading-estimator code is used in Tasks JFO and MFO without refinements (such as upgrading from trivial-limits code) and with the display diagnostics left switched-on.

7.9.2 Estimation Robustness

Over the course of testing several topogram images, thousands of patches have been analyzed and no algorithmic arithmetic errors have occurred. By ensuring that the boundary is always included, even the smallest patch spanning the closest fringes (such as the case of contours on adjacent pixels) has a non-zero membership, and so normalization never causes an error. By using double-precision arithmetic, the integration sum accommodates the largest patch (the full 10^6 pixel image of 8-bit shading values) without overflow errors.

The shading estimator accepts the supplied seed-points without prejudice and constructs a circular patch across them, irrespective of that circle's relationship to the underlying fringe pattern. Figure 7.12 shows how well the polygon star adapts to contours with sharp curvature. The same curves are shown in Figure 7.20, this time with circular patches. The circle has problems tracking the 'spine' of the ideal gradient patch, and the consistency of this sampling scheme certainly breaks down when the seed-points specify a circle with curvature that is not small when compared to the curvature of the contours. One could test for this condition or search along the boundary to discover undesirable intersections with contour curves, and thus eliminate such patches from the photometric ordering process. Neither Task JFO nor MFO undertake to filter such distorted sampling patches from the photometric analysis; the occurrence of bad patches was found to be statistically insignificant for the test images used by the author. This topic is discussed further in the next two chapters.

Figure 7.20: The circular patch as it adapts to changing contour curvature and orientation. The hatching of each estimation patch in the gradient direction suggests how well the circle maintains a consistent series of moiré phase intervals, balanced about the contour 'spine' of the patch as its size at each location is dynamically adjusted.

Chapter 8

Task JFO: Automated Fringe Ordering

8.1 Introduction

Two programs have been written to test the Photometric Ordering Algorithm developed in Chapter 3. The first program (Task JFO) is designed to demonstrate fully-automated ordering of moiré fringe contours, and uses streamlined code, a simple Lambertian model and a minimum length of neighbourhood contour sequences. The second program (Task MFO) is designed as an investigative tool, and allows considerable flexibility in adjusting the parameters and conditions of ordering tests. The purpose of this chapter is to describe Task JFO (Task MFO is reported in Ch. 9).

The remainder of the chapter is organized as follows. The next section gives an overview of Task JFO, with emphasis on the design philosophy behind this program, and on the functionality features incorporated into it. The third section deals with the software implementation of the task. Following a brief discussion of the software coding principles employed, the principle elements of the program are described. The data files required by the task are specified, and the program modules are related to the processes of the Photometric Ordering Algorithm. The fourth section reports on some contour-ordering experiments performed with Task

JFO. Solved maps are presented for two experimental subjects: a painted wooden sphere and a live human back. The last section discusses the experimental results, and in particular, comments on the robustness of photometric ordering as embodied in this program, and on the efficiency of Task JFO's ordering scheduler. Suggestions for improvements are given.

8.2 Overview of Task JFO

In this section, an overview of Task JFO is given. The philosophy behind the design of the task is explained, and the restrictions to its functionality are described. The significance of these restrictions in simplifying the embodiment of the Photometric Ordering Algorithm into the program is also examined in this section.

8.2.1 Design Philosophy

Task JFO was written while the Photometric Ordering Algorithm, described in Section 3.4 (p. 95), was still in the conceptual stages. The author designed the algorithm with considerable flexibility in its functionality (such as handling arbitrary-length neighbourhood sequences and allowing both mixed and matched modes of moiré fringe contours). The plan was to embody the algorithm into a flexible program (Task MFO, reported in the next chapter) that could test the various aspects of the algorithm. Task JFO was intended as a 'quick-test' of a streamlined version of the algorithm, to verify that the primary ideas behind propagated photometric ordering were sound.

The primitive functions, such as gradient neighbour discovery, geometric scalar evaluation, and image shading measurement, had been programmed and tested separately. These functions are described in detail in Chapters 5, 6 and 7, respectively. Task JFO was to tie these elements together with a simple control structure that could automatically process a set of moiré fringe contours.

For Task JFO, the emphasis was on getting the program running with minimal refinement. This meant ignoring sometimes obvious opportunities to 'save com-

puter cycles' or 'save memory'. It was felt that one could return later to make improvements. The program coding standards, however, were not compromised in this exercise.

8.2.2 Functionality Features

Task JFO features three restrictions to its functionality that sets it apart from Task MFO (which is not restricted in these ways):

- Fixed Contour Mode;
- Fixed Sequence Length;
- Fixed Schedule.

The decision to fix these variables in the Photometric Ordering Algorithm leads to considerable simplifications over the general case. The ramifications are described in the sections that follow.

8.2.3 Matched-Mode Contours for Task JFO

Task JFO can only handle matched-mode contours. This means that the set of contours in the map have to be traced from either bright moiré fringes only, or else dark moiré fringes only. Mixed-mode contours (traced from both bright and dark fringes) are not accommodated by the program. Indeed, Task JFO ignores any mode information (i.e., the value of e_i for each contour) that may reside in data files designed for Task MFO, and only the matched-mode ordering rules are encoded in Task JFO.

Recall that the contour mode determines the possible ordering cases for a sequence of contours, and from Equation 3.7 (p. 72) the three ordering cases to consider for a pair of contours C_n and C_u , are given by:

$$\begin{aligned}\phi_u^{(+)} &= \phi_n + 1, & \text{if } C_u \text{ is above } C_n, \\ \phi_u^{(0)} &= \phi_n, & \text{if } C_u \text{ is level with } C_n, \\ \phi_u^{(-)} &= \phi_n - 1, & \text{if } C_u \text{ is below } C_n.\end{aligned}\tag{8.1}$$

Note the switch from the numeric superscripts ($k = 1, 2, \dots, N$) of the general case, to the symbolic superscripts (+), (0) and (-) to denote these three ordering cases.

8.2.4 Minimum-Length Neighbourhood Sequences for Task JFO

Task JFO limits the neighbourhood sequence to three contours, and these must be partitioned as:

$$\langle C_m, C_n | C_u \rangle_q, \quad (8.2)$$

that is, the first two contours are knowns and the third is the unknown. The subscript 'q' denotes the qth ordering step in the schedule. Three contours is the minimum length for a neighbourhood sequence that can be used for propagated photometric ordering. This is because there must be two patches in an analysis region: one patch must have fixed geometry and measurable shading to estimate the photometric model parameter for the analysis region (and the patch between knowns C_m and C_n has this property); the second patch must allow trial geometries and have measurable shading (and the patch between known C_n and unknown C_u has that property).

Since there is always just one unknown in the sequence for Task JFO, then there are always only three ordering cases to consider. Fixing the number of ordering cases to a small number simplifies many of the data structures used in the task, and likewise simplifies the processing steps that manipulate these data values. For example, the list of trial fringe values $\llbracket \Phi \rrbracket_q$ for the qth ordering step is trivially given by Equations 8.1 and 3.56 (p. 106):

$$\llbracket \Phi \rrbracket_q = \begin{bmatrix} \langle \Phi \rangle^{(+)} \\ \langle \Phi \rangle^{(0)} \\ \langle \Phi \rangle^{(-)} \end{bmatrix}_q = \begin{bmatrix} \langle \phi_m, \phi_n | \phi_u^{(+)} \rangle \\ \langle \phi_m, \phi_n | \phi_u^{(0)} \rangle \\ \langle \phi_m, \phi_n | \phi_u^{(-)} \rangle \end{bmatrix}_q \quad (8.3)$$

In Task JFO, the C_n points along central contour C_n of the neighbourhood are the seed-points

$$P_1, P_2, P_3, \dots, P_{C_n} \quad (8.4)$$

Figure 8.1: The photometric analysis region for Task JFO P_j is the seed-point on contour C_n , and G_j is the surface gradient curve through P_j that intersects the neighbouring contours C_m and C_u at $P_{j,m}$ and $P_{j,u}$, respectively. P_{M_n} is the center of the mid-point circular patch spanning the known contour-pair (C_m, C_n) , and P_{M_u} is the center of the mid-point circular patch spanning the known/unknown contour pair (C_n, C_u) . The region R_j is shown as a boundary encircling the two analysis patches.

for the $R = C_n$ photometric analysis regions

$$R_1, R_2, R_3, \dots, R_R \quad (8.5)$$

that span the neighbourhood.

Restricting the neighbourhood to three contours fixes the analysis patches to be centered on the mid-points between each contour-pair, and this, in turn, designates the domain of the photometric analysis region R_j for the j th point P_j . The definition of the mid-point circular analysis patch was shown in Figure 7.13 on page 264. The geometry defining the centers of the two analysis patches in R_j for Task JFO is depicted in Figure 8.1.

With only three ordering cases at each analysis region, a simpler version of the photometric ordering process outlined in Section 3.3.7 (p. 92) can be used. For the j th region R_j along the central contour C_n , Task JFO first measures the shading $\bar{\rho}_n$ and the geometric scalar γ_n at the mid-point P_{M_n} between the knowns C_m and C_n , and then estimates the photometric model parameter ρ_0 for the region. It then

measures the shading $\bar{\rho}_u$ at the mid-point $\mathcal{P}_{M_u}$ between the known $\mathcal{C}_n$ and unknown $\mathcal{C}_u$, and then calculates the three geometric scalars there, one for each ordering case:

$$[\gamma_u] = \begin{bmatrix} \gamma_u^{(+)} \\ \gamma_u^{(0)} \\ \gamma_u^{(-)} \end{bmatrix} \quad (8.6)$$

Task JFO then estimates the expected shading for the three cases. For the k th ordering case, the Lambertian photometric model in Equation 3.27 (p. 86) becomes:

$$\bar{\rho}_u^{(k)} = \rho_0 \gamma_u^{(k)}, \quad (8.7)$$

which gives:

$$[\tilde{\rho}_u] = \begin{bmatrix} \tilde{\rho}_u^{(+)} \\ \tilde{\rho}_u^{(0)} \\ \tilde{\rho}_u^{(-)} \end{bmatrix} \quad (8.8)$$

Task JFO finally computes the consistency between the measured and expected shading for each of the three ordering cases, using the squared-difference as a measure of consistency:

$$\epsilon^{(k)} = (\tilde{\rho}_u^{(k)} - \bar{\rho}_u)^2, \quad (8.9)$$

which results in three values for the j th region $\mathcal{R}_j$ about contour $\mathcal{C}_n$:

$$[\epsilon]_{\mathcal{R}_j} = \begin{bmatrix} \epsilon^{(+)} \\ \epsilon^{(0)} \\ \epsilon^{(-)} \end{bmatrix}_{\mathcal{R}_j} \quad (8.10)$$

The way in which Task JFO makes use of the three consistency errors from the regional analysis is described in the next section.

8.2.5 Ordering Schedule for Task JFO

Task JFO has a built-in ordering scheduler which can only select the ordering steps in a fixed sequence that depends upon the sequence of the contours in the map-list.

Task JFO uses a 'pushbroom' approach to derive the schedule. A sweep is made from beginning-to-end through the map-list of ϕ 's:

$$\Phi = \{\phi_1, \phi_2, \dots, \phi_i, \dots, \phi_M\}, \quad (8.11)$$

where the ϕ -values of all the unknowns have been initialized to a convenient negative value, and the ϕ -values of the knowns are always non-negative:

$$\phi_i = \begin{cases} -1 & \text{for 'unknowns',} \\ \geq 0 & \text{for 'knowns'.} \end{cases} \quad (8.12)$$

The sweep through the list is interrupted each time a known contour C_n is found in the map-list. The known contour C_n has a list of contour-points

$$C_n = (\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \dots, \mathcal{P}_j, \dots, \mathcal{P}_C)_n, \quad (8.13)$$

and each point is processed in turn as a seed-point for a potential photometric analysis region along the contour.

The gradient neighbour on each side of the j th point $\mathcal{P}_j$ (denoted $\mathcal{P}_{j,m}$ and $\mathcal{P}_{j,u}$ in Fig. 8.1) is discovered by searching the image. If one-and-only-one of the discovered neighbours is from an unknown contour, then the region $\mathcal{R}_j$ about $(\mathcal{P}_{j,m}, \mathcal{P}_j, \mathcal{P}_{j,u})$ will be photometrically analyzed as described in the previous section; otherwise, the point is skipped.

For every contour in the map, Task JFO maintains four accumulators:

$$\text{the 'above' consistency error: } \{\epsilon^{(+)}\} = \{\epsilon_1^{(+)}, \epsilon_2^{(+)}, \dots, \epsilon_M^{(+)}\}, \quad (8.14)$$

$$\text{the 'level-with' consistency error: } \{\epsilon^{(0)}\} = \{\epsilon_1^{(0)}, \epsilon_2^{(0)}, \dots, \epsilon_M^{(0)}\}, \quad (8.15)$$

$$\text{the 'below' consistency error: } \{\epsilon^{(-)}\} = \{\epsilon_1^{(-)}, \epsilon_2^{(-)}, \dots, \epsilon_M^{(-)}\}, \quad (8.16)$$

$$\text{the number of regions: } \{R\} = \{R_1, R_2, \dots, R_M\}. \quad (8.17)$$

The consistency errors for the unknown contour in each analysis region are summed into that contour's accumulators, and its region-count is incremented. In this way, if more than one unknown contour is discovered while analyzing the $R = C_n$ regions about the known contour C_n , then the consistency errors for each unknown can be monitored as separate totals.

When all the points along the contour have been processed, Task JFO examines the accumulators. It considers only the unknown contour with the greatest number of regions associated with it *for this step* (i.e., the unknown contour that has the greatest number of analysis regions with the seed contour C_n). For that unknown contour C_u , Task JFO then chooses the shape case with the smallest consistency error, and then updates ϕ_u according to Equation 8.1.

In its sweep through the map-list, Task JFO may have to pass over some, or even many, known contours that have both neighbours as unknowns. If at least one unknown is resolved in the sweep, then the next sweep will address at least one of the previously skipped contour sequences. A 'lucky' arrangement of the contour list can result in Task JFO ordering the map in one sweep; a random arrangement will involve several sweeps, or in the worst case, one sweep will be required for each unknown. As written, Task JFO pays a severe penalty in this respect: in looking for any outstanding unknowns, it searches about every point along every known contour, even if previous searches have shown that those contours' neighbours have already been ordered. This obvious inefficiency in Task JFO's 'pushbroom' ordering schedule will be discussed further at the end of the chapter.

8.3 Software Implementation of Task JFO

In this section, the software implementation of Task JFO is described. The discussion starts by specifying the software engineering standard adopted by the author for developing the programs for this research project. Then the data files required by Task JFO are described and the program modules that comprise the task are reviewed. The section concludes with some comments on debugging the program, and on verifying the correctness of the computations and processing. The source-code listings for Task JFO and its related files (≈ 3000 lines) are given in Appendix D.

8.3.1 Software Engineering Principles

Although Task JFO was written and tested in a short length of time (a few weeks), an effort was made to consistently adhere to the software engineering principles used for all program development throughout this project. It was the author's experience that well-structured, error-free, self-documenting code was efficiently produced by these methods. Even complex programs were easy to write, debug and modify.

The author adopted the software standard used by DIPIX Systems Ltd. (Ottawa) to ensure compatibility with their system software and to maintain a channel for dialog with their engineers should problems arise. Most engineers can find fault with this software standard, but most will agree that software consistency is a sufficiently redeeming virtue for any reasonable standard.

DIPIX uses DEC (Digital Equipment Corporation) PDP-11 computers in their Aries-II image analysis systems, and the preferred language for *applications tasks* (such as Tasks JFO and MFO written for this project) is PDP-11 FORTRAN-77. DIPIX provides libraries with a wide assortment of primitive functions for image file management and image processing, and these modules are written in MACRO-11 assembly language when optimized processing is critical.

Listings of the source code for Task JFO are given in Appendix D, and browsing through these listings will give the reader an impression of the strengths and weaknesses of the coding standards. The following principles guided the development of all the programs written for the moiré fringe ordering project.¹

8.3.1.1 Names of Variables

All variables obey a type and usage naming convention. The first character of the name specifies the data type of a variable, according to the following statements that appear in every module:

```
IMPLICIT BYTE(B), COMPLEX(C), DOUBLE PRECISION(D), INTEGER*4(E)
IMPLICIT LOGICAL*1(A), INTEGER*2(F-Q), REAL(R-Z)
```

---

¹Naturally, there are some *intentional* exceptions to the rules!

All variables from the subroutine parameter list are six characters in length, all common variables are four characters in length, and all variables local to a module are one, two or three characters in length. These conventions allow one to determine the type and usage of any variable simply by examining its name.

8.3.1.2 Subroutine Calling Sequences

Parameters in subroutine calling sequences (i.e., the argument list) are always grouped depending on how they are modified by the subroutine. The three groups are: <input>, for variables whose initial values are supplied by the calling routine and are never modified by the subroutine; <output>, for variables that must be set by the subroutine and whose initial value does not affect the results of the subroutine process; <input/output>, for variables that may or may not be modified, and whose initial value is required by the subroutine. The groups are separated by dummy variables XXXXXX and YYYYYY, as in this example:

```
CALL AMFOPR(<input>,XXXXXX,<output>,YYYYYY,<input/output>)
```

Many of the DIPIX modules have a dozen (some have several dozen!) variables in their parameter list, and the sorting and separation into three lists helps to avoid the 'hardest' error to find in FORTRAN, namely, the corruption of values that can result from mismatched or missing subroutine parameters.

8.3.1.3 Pseudo Code

The process steps for each module are written as a sequence of pseudo code comment statements that give the step number, describe the process for that step, and indicate the step number of the next step if it doesn't immediately follow. Simple processing branches on error tests are indicated by <error>/<no-error>, as in the first comment statement in the following program segment from module AMFOMA.FTN.

```
C 75. CHECK THE IDENTIFIER. CHARACTERS MUST BE
C      'A',..., 'Z' OR '0',..., '9'. OTHERWISE REPORT
```

70/80


```

C      ERROR MESSAGE AND RE-INPUT FILE IDENTIFIER.
C
75     ISP=75
        CALL AMFOCH(BFR(6),4,0,XXXXXX,IER)
        IF (IER.EQ.1) GOTO 80
        CALL XOMESG(IER,17)
        GOTO 70

```

The processing for each pseudo code step is kept to a few lines of real code, such as a subroutine call and the error flag testing, or initializing some parameters. If an error condition is to be tested, the first statement is always labelled as the step number, and the step counter (ISP) is set equal to the step number. In this way, a branch to the error log can report at what step the problem originated.²

8.3.1.4 Error Reporting

The first output parameter of every subroutine is the error flag IERROR. The calling routine will call it IER, since it is a local variable there. The error flag is always set to '+1' if the process completes without error. On return, the calling process always checks this parameter, and reports any error condition with a call to the DIPIX error log routine XOERRM. For example, from AMFORN.FTN the code reads:

```

C 2000. ERROR REPORTING.                                     999
C
2000     MSG=20
        CALL XOERRM('AMFORN',ISP,IER,MSG)
        GOTO 999

```

The error log lists the module name (AMFORN), the step number (ISP), the returned error code (IER), and an error message that is retrieved by number (MSG) from the message file for the task, e.g.,

```
*** ERROR *** in AMFORN at step 15 (IER = -2) No themes in image!
```

²In this respect, Task JFO is not a rigorous example for illustrating the software standards: the task was hastily written, and Mr. Ding was not very enthusiastic about documentation, especially in updating the pseudo code as the real code was modified. Task MFO has better documentation, but at ≈ 6500 lines, it was too long to list. As is often the case for software projects, there was no time later to groom the documentation.

After logging the error, control returns to the calling routine, which reports at which step in its code the error occurred, and so on. The result is a series of one-line reports which may end in the task exiting (a fatal error), or continuing if the error can be processed. When writing the modules for a task, the error reporting code is automatically included in the skeleton (or template) of each module, so its form is always the same.³ Consistent error reporting makes debugging much easier.

8.3.1.5 Common Blocks

A well-known practice, but worth restating, is the concept of declaring common blocks in include-type files. Then modules that require access to the parameters in a common block will only require the appropriate `INCLUDE` statement in the declaration portion of the module. For example, the logical names for I/O devices are maintained in a file called `AMFOLU.INC`, and all modules that perform I/O would 'include' that file with their other declaration statements. Similarly, the moiré instrument parameters, the image transformation coefficients, and the display image parameters are stored in common blocks that are declared via include-type files. This allows a common block structure to be efficiently modified by editing one file instead of many.

8.3.1.6 I/O Device References

All input/output devices are referenced by a four-character logical name of the form `Lxxx`, such as `LTER` for the operator's terminal, and `LIDX` for the contour map index file. Every type of file (index, contour points, transform coefficients, etc.) used by the task has a logical name. The names are assigned logical unit numbers (LUN's) in the initialization module for the common block (`AMFOLU.FTN`), and the command file to build the task (`JFO.BLD`) specifies the assignment of the system's physical devices to the LUN's. This allows I/O to be conveniently redirected, so that for example, the error log can be sent to a terminal screen, or the line printer, or a file.

---

³In practice, however, one is tempted to streamline code, sometimes inviting disaster. In some modules in Task JFO for example, where 'nothing can go wrong' [sic], the error reporting section has been deleted; this is contrary to the standard.

8.3.1.7 Program Module Names

Program module names are always six characters in length and are of the form `ttttmm`. The first field `tttt` denotes the task, as in `AMFO` for 'automated moiré fringe ordering'. The second field `mm` denotes the module, as in `MA` for 'main' and `EX` for 'examine the theme value'. At DIPIX the emphasis is on using module names to identify the tasks for which the modules are written, rather than as mnemonic devices to describe the function of the module.

In a similar manner, DIPIX collects common modules for the ARIES-II system into suites of modules that are functionally related. The modules are named `Xsiiii`, where `X` means the module can be called from any applications task, `s` denotes the suite, and `iiii` is a module identifier assigned by the programmer. For example, the `XO`-suite is for operator interaction routines (such as `XOMESG` for messages on the console, or `XOERRM` for error logging), and the `XI`-suite is for image display routines (such as `XICREA` for creating an image in the display, or `XIREPO` for reporting the display image parameters). There are several hundred common modules organized into about twenty common suites.

8.3.1.8 File Names

Strict file-naming conventions are imposed to organize the numerous list and image files that relate to the same image area. Image files have nine-character names of the form `aaattiiii`, where `aaa` is the three-character area name, `tt` is the file type (feature or theme), and `iiii` is a user-assigned identifier. Files for the same area can be conveniently loaded into the display (for example, the red, green and blue components of a colour image, plus a graphics overlay) by specifying the area name, and then each of the four-character identifiers. The common-suite modules, and the applications tasks, rely on adherence to the naming convention to correctly build the file names and find them on the disk (as a check, an error will occur if the image information in the header of a file is not consistent with the display image parameters).

List files are managed by a common-suite interface to DEC system software, and

they have nine-character file names and three-character file extensions of the form `aaattiiii.ttF`. The fields `aaa` and `iiii` play the same role as for image files, but `tt` now denotes the task that created the file (e.g., for Task JFO, `tt = F0`). The DIPIX standard calls for the extension `ttF` to redundantly denote 'Task `tt` File', but the author relaxes this rule when writing an application task that requires several different types of list files in one task. For example, Task JFO expects the moiré instrument parameter file to end in `.XYZ`, and the image transformation coefficients file to end in `.CEF`.

Having remarked on relaxing the rules in a few cases, it is appropriate to conclude this discussion on software standards with this comment: for modest software projects such as the author's, coding standards provide a useful framework, and they should be upheld except where they seriously obstruct efficiency or clarity.

8.3.2 Required Data Files

Task JFO requires access to several types of data files. The list data falls into four categories:

- Moiré Instrument Parameters, required to define the physical geometry of the shadow moiré instrument used to form the moiré topogram image. The same file can be used for all topogram images produced on a given instrument. This file corresponds to the data structure ① in the Photometric Ordering Algorithm (refer to Fig. 3.19, p. 117).
- Image Transformation Coefficients, required to transform digitized image coordinates into the moiré grid-plane of the object-coordinate system. Usually a unique file is required for each digitized image. This is because the author did not employ an accurate means of positioning topogram image records (photographic prints) when digitizing. This file corresponds to the data structure ② in the Photometric Ordering Algorithm (refer to Fig. 3.19, p. 117).
- Moiré Image Map List, also called the 'map index file', contains the list of contours that comprise the map. In general, this file corresponds to the data

structure $\mathcal{M}$ in the Photometric Ordering Algorithm (refer to Fig. 3.11, p. 98). In the case of Task JFO, each contour-entry into the list consists of the entry (contour) number, the name of the contour-point list file, whether the contour is a broken ('B') or closed ('C') loop, and the theme (map-label) assigned to the contour in the displayed image.

- Moiré Image Contour-Point Lists, namely, the list of points that define the contour curve in the display image. There must be one of these files for each of the contours in the Map List

Examples of the data files used by Task JFO are given at the end of Appendix D. These are the actual data files used by Task JFO in one of the contour-ordering experiments described later in this chapter. The instrument parameters file was called MOIREINST.XYZ. The digitized topogram image was named BSPFFR100, so that the transformation coefficients were kept in a file named BSPFOR100.CEF. Several experimental sets of contours were traced from the painted sphere in this image; the set reported in the experimental results was called BBBB, and the map index file called BSPDCBBBB.OUT was automatically produced for Task JFO by Task JDC when it drew the contours in the display image to prepare the image data structure for photometric analysis. The contour-points are listed in files BSPSP0001.SPF to BSPSP0010.SPF, and were made by Task JSP (see Sec. 5.3.3, p. 183).

8.3.3 Task JFO Program Modules

The hierarchy chart for the modules in Task JFO is shown in Figure 8.2. Some of the standard calls to system modules, such as error logging and messages to the console, are indicated by superscripts (e.g., AMFOAZ^[a] means that AMFOAZ called XODETA and XOATTA). The short functional description given in the figure for each module will be elaborated upon below.

AMFOMA ^[a,e,m]	-system initialization and operator interaction
XOINIT	-initiate OIS
XBGVID	-attach VMA to this task
XXVLNK	-link VMA to DPI-common
XIREPO	-get image display parameters
AMFOCH ^[m]	-check file name validity
AMFOPR ^[a,e,m,f]	-process the contours in the map list
AMFOAZ ^[a]	-compute the azimuth at a given point
AMFORN ^[a,e]	-search for the neighbours to a point
AMFOEX ^[a]	-examine the VMA contents at a pixel
AMFONL ^[a]	-compute the geometric scalar at a point
AMFORV ^[a,e]	-measure the reflectance value in the image
XBRVID	-release VMA
XOEXIT	-return to OIS

Notation   [a] -calls to XODETA and XOATTA (permits direct I/O to console)
               [e] -calls to XOERRM (error log routine)
               [m] -calls to XOMESG (operator I/O)
               [f] -calls to ERRSYS (FORTRAN I/O error routine)

Figure 8.2: Hierarchy chart for Task JFO. To simplify the list, standard calls to system modules are indicated by superscripts, as noted

8.3.3.1 Module AMFOMA

AMFOMA is the main module for Task JFO, and deals primarily with system initialization, operator interaction, and task termination. The task runs under the DIPIX operating system OIS, and every applications task must first call XOINIT to initialize the OIS environment. OIS attaches to the operator console to maintain rigid control over all I/O. For slipping casual messages to the operator console, one trick is to call XODETA, dump the message, and then call XOATTA. This trick was used extensively when debugging the program.

OIS tasks that use the video memory array (VMA) must attach to this device, and XBGVID attends to the system details of this request. At the end of the session, a call to XBRVID releases the VMA for use by other tasks. Tasks that make direct access to the VMA must call XXVLNK to establish the mapping to the DPI common block (see Fig. 5.11, p. 184).

Task JFO assumes that the image data structure has already been created. AMFOMA prompts the operator for the image number, and calls XIREPO to read all the image display parameters into the common block /XZ1IPR/. It then interacts with the operator (using OIS's formal mode of console I/O, namely XOMESG) to be sure that the correct image is displayed, that the transformation coefficient file exists for it, and that the map-list file identifier has been properly specified. Calls to AMFOCH are made to check that valid characters are typed by the operator. If all is in order, AMFOMA calls AMFOPR to process this map (i.e., to photometrically-order the list of contours).

When AMFOPR returns, the operator is asked if any more map-lists are to be processed, and if not, a tidy exit is made to OIS via XOEXIT.

8.3.3.2 Module AMFOCH

Subroutine AMFOCH is a routine used in Task JFO and other tasks to check that the operator entered valid characters when prompted for input from the console.

8.3.3.3 Module AMFOPR

This processing subroutine is the workhorse of the task. It opens the specified files and reads the moiré instrument parameters and the image transformation coefficients into a set of buffers. It then opens the map-list file, and reads the contour identification data into memory. All the files are closed after reading. It prompts for two 'knowns' (i.e., the fringe numbers for a pair of neighbours), and then starts the pushbroom scheduler (described in Sec. 8.2.5), by considering each contour in the map-list one at a time.

First it clears the consistency-error accumulators. Then, it opens the contour's point-list file, and reads the digital image coordinates of the points into a buffer. AMFOPR then performs Task JFO's version of the photometric regional analysis about each point, as described in Section 8.2.4. It calls AMFOAZ to calculate the azimuth angle at the point, and calls AMFORN twice to discover the identity and distance to the gradient neighbour on each side of the point. If one-and-only-one neighbour is an unknown, then processing of the region proceeds; otherwise AMFOPR skips to the next point (i.e., the next region) in the contour-list.

To process the region, AMFOPR first calculates the photometric model parameter for the known geometry by calling AMFONL to compute the geometric scalar, and AMFORV to measure the image reflectance value. Then it calls AMFORV to measure the image reflectance value about the trial point between the known and unknown contour. The geometric scalar for the three trial geometries (Eq. 8.6) is obtained by successive calls to AMFONL, the three expected reflectance estimates are formed, and their differences with respect to the measured reflectance are squared and accumulated in the error buffers (Sec. 8.2.5).

When all the regions for a contour have been processed, AMFOPR examines the accumulators and picks the ordering case with the smallest error, for the most frequently encountered unknown neighbour to the seed contour. It updates the map-list fringe number for the picked contour, and then cycles to the next seed contour in the map-list.

8.3.3.4 Module AMFOAZ

Subroutine AMFOAZ computes the azimuth angle at a point on a contour, using the methods and equations described in Section 6.5 (p. 234). The calling routine supplies three points on the curve, and AMFOAZ fits a quadratic polynomial through the points and then computes the azimuth angle to the desired point by taking the first derivative of the curve there.

8.3.3.5 Module AMFORN

Subroutine AMFORN searches the image about the seed-point to discover the identity and distance to the gradient neighbour. The details of the search strategy and software implementation are described in Section 5.4 (p. 191). The identity of contour points (i.e., the contour label) is stored in the theme field of the display image, and AMFORN makes repeated calls to AMFOEX to examine the theme value of neighbouring pixels.

8.3.3.6 Module AMFOEX

This module is a low-level process that examines the theme field of a specified image element in the display memory. Using the (line, pixel) coordinates of the element, AMFOEX computes the Address Offset Register (AOR) value (refer to Fig. 5.11, p. 184) and extracts the theme value using a bit-mask. The value is returned to the calling routine.

8.3.3.7 Module AMFONL

Subroutine AMFONL computes the geometric scalar as described in the block diagram for GSC in Figure 3.19 (p. 117). It uses the specified image coordinates and fringe number to transform the point into object space. From there, the illumination direction $\mathbf{L}$ and the surface normal direction $\mathbf{N}$ are computed using the equations given in Section 3.4.9 (p. 116).

8.3.3.8 Module AMFORV

Subroutine AMFORV measures the reflectance value in a circular patch in the display image using the methods and software implementation described in Section 7.8 (p. 267).

8.3.4 Debugging and Computation Verification

Extensive use is made of the debug-option for the FORTRAN-77 compiler. Embedded in every module are sections of code which dump the values of important parameters at critical stages of the processing. As modules were tested and verified to reliably give the correct results, they could be recompiled with the debug switch turned off. Statements with a 'D' in the first column are affected by the switch: when turned on, these statements are compiled as normal code, and when turned off they are treated as comment-statements and ignored.

Verifying the computations is a tedious job. For example, the reflectance measuring algorithm has been checked by manually verifying that every pixel included in the patch is correctly admitted, that its value is correctly extracted from the display ixel, and that the arithmetic sum and average is calculated correctly.

Similarly, the computation of the object-space coordinates from the image transformation, the evaluation of the illumination direction cosines, and the arduous calculation (by curve-fitting) of the tilt, the azimuth, and their transformation to the surface normal direction cosines, and finally the working of the photometric model, have *all* been verified by dumping the intermediate values to the printer, and checking each result with a hand-calculator. The photogrammetry aspect of the computations has been shown in Chapter 4 to be very accurate, as absolute object-space coordinates on the calibrated NRC Standard Object could be recovered from its digitized topogram image to an accuracy of $\pm 1/4$ mm. It was beyond the means of the project to establish a similar verification of the absolute photometric measurements, and this was deemed unnecessary anyway, as only relative photometric measurements are required for the analysis.

8.4 Experiments with Task JFO

In this section, some of the experimental results obtained with Task JFO are reported. As mentioned at the beginning of this chapter, Task JFO was written to find out if the author had somehow overlooked an important detail in the basic design of the Photometric Ordering Algorithm. The program was to be an interim development tool, leading up to Task MFO.

Task JFO tended to be used repeatedly with the same pair of test patterns: a set of contours traced from a topogram of a painted wooden sphere, and another set of contours traced from the topogram of the back of a live subject (one of my colleagues!).

The same two patterns were processed by Task JFO many times to discover and isolate new bugs as the modules were gradually groomed into error-free code.

8.4.1 The Painted Sphere

The 'painted sphere' was a favourite test object in this study. It has a stable, simple geometry (a convex sphere of 125.00 mm radius), and predictable photometric properties (a uniform coating of semi-gloss paint). It is part of the NRC Standard Object, described in Chapter 4. The object was mechanically sculpted from a laminated wooden form, and has an overall shape and depth-of-field that pose a challenge to moiré instruments intended for measuring human backs in medical applications. The surface is painted to visually match a Caucasian skin colour.

Figure 8.3 is an enlargement of the original 4"×5" Polaroid print of the moiré topogram for the painted sphere data. The photographic parameters for recording the topogram were as follows. The camera was a Polaroid Model 600SE, with a Mamiya $f=150$ mm lens, set at an aperture of $F/11$ and focussed at 2 meters (about 30 cm behind the grid plane). The shutter speed was 1/30 second, and a single 1000 watt quartz-halogen lamp was used for illumination. The film was Polaroid Type 667, with an effective speed of 3000 ASA, and the development was for 60 seconds at 21°C. With these parameters, the curved shadows of the grid

Figure 8.3: Moiré topogram of the NRC Standard Object. The individual strings of the stationary grid are clearly visible on the original 4" x 5" Polaroid print

strings on the object are clearly resolved in the original print, while the grid strings *per se* are just out of focus, although they obviously contribute to the production of the high-contrast moiré fringes.

Figure 8.4 shows how the digitized topogram appears in the display of the image analysis system. This figure is reproduced from a Polaroid print made with a camera positioned in front of the display screen. The contrast has been stretched in this figure (using the display system utilities) for illustrative purposes: the fringe ordering software uses the raw values of the digitized image intensities. The digitizing scale was such that approximately three pixels span the moiré grid string interval (2 mm in object-space), and the curved grid-string shadows can still be resolved in the digital image display. The white fiducial strings and the dark shadow of the horizontal fiducial string are also clearly visible in the figure, and they allow the absolute fringe number to be independently determined for any fringe the dark shadow crosses (recall that Task JFO needs a pair of knowns to start its processing). The reader may also be able to see the bright specular 'highlight' at the top of the central bright fringe. The illumination is from above and in front of the sphere, and the resulting highlight on the painted surface can cause problems for a purely-diffuse photometric model.

Figure 8.5 shows the displayed image of the painted sphere with the contour data set superimposed by Task JDC (Sec. 5.3.4, p. 185). The actual data files used to trace the contours in the terre field of the image are listed at the end of Appendix D. They include the map-list file and the nine contour-point files. The contrast has not been stretched in this figure, as the data is ready to be processed by Task JFO.

Table 8.1 shows the results of Task JFO processing the painted sphere contours. In this test, the program is given initial values for ϕ_1 and ϕ_2 (the most central contours in Fig. 8.5), and Task JFO orders the remaining contours in seven steps. For each step, the table shows the neighbourhood contour sequence that is used (always only one unknown), the relative error sum for each of the three ordering cases, and the fringe number assigned to the unknown.

Figure 8.4: Display of the digitized NRC Standard Object moiré topogram. The contrast has been enhanced for illustrative purposes.

Figure 8.5: Test contours traced onto the painted sphere display image by Task JDC. The contrast has not been stretched for this figure. The contours have been traced along the central nine dark fringes, and appear as an overlay of fine white lines

Table 8.1: Experimental results for the painted sphere using Task JFO

process step no.	neighbourhood contour sequence $\langle \phi_m, \phi_n \phi_u \rangle$	relative error sum for each ordering case			fringe number assignment
		$\epsilon_u^{(+)}$	$\epsilon_u^{(0)}$	$\epsilon_u^{(-)}$	
0	$\phi_1 = 16.5$ $\phi_2 = 17.5$				
1	$\langle \phi_1, \phi_2 \phi_3 \rangle$	2.2	2.0	1.0	18.5)
2	$\langle \phi_2, \phi_3 \phi_4 \rangle$	5.6	2.0	1.0	19.5)
3	$\langle \phi_3, \phi_4 \phi_5 \rangle$	10.6	6.0	1.0	20.5)
4	$\langle \phi_4, \phi_5 \phi_6 \rangle$	30.1	19.7	1.0	21.5)
5	$\langle \phi_5, \phi_6 \phi_7 \rangle$	11.5	7.4	1.0	22.5)
6	$\langle \phi_6, \phi_7 \phi_8 \rangle$	50.6	56.3	1.0	23.5)
7	$\langle \phi_7, \phi_8 \phi_9 \rangle$	29.9	30.6	1.0	24.5)

The relative error is formed by dividing the smallest error into the other two larger errors. This normalizes the error with respect to the number of regions, and allows the performance of the photometric model to be more readily compared from step to step. Two features of the results in Table 8.1 stand out. One is that the ordering is correctly determined at every step. This is evident from the progression of increasing fringe numbers assigned in the last column of the table, as one would expect for the concentric contours on a sphere. The second feature of this test is the strength of the discrimination against the wrong choices. The larger the relative error for the other cases compared to the selected case, the stronger is the discrimination between them. Near the center of the sphere, the discrimination is poorest because of the specular effects that degrade the performance of the Lambertian photometric model, but the performance steadily improves as the analysis moves away from the specular hotspot on the painted surface.

Table 8.2 shows the state diagram for the map-list. Task JFO's pushbroom scheduler requires only one sweep through the map-list, since each known that it visits has been resolved in the immediately previous step. It is not chance: the list has been arranged to be so. In other tests, Task JFO was given different initial

Table 8.2: Task JFO processing schedule for the painted sphere

fringe	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9
step 0	!	!	?	?	?	?	?	?	?
1	!	!	*						
2		!	!	*					
3			!	!	*				
4				!	!	*			
5					!	!	*		
6						!	!	*	
7							!	!	*

! = known   ? = unknown   * = assigned

fringes to work with (such as ϕ_8 and ϕ_9), and although the ordering results were the same, it took several sweeps to solve the map. Schedule efficiency is discussed further at the end of the chapter.

Figure 8.6 shows a perspective view of the ordered contours from the painted-sphere map. This figure was photographed from the image analysis display. The perspective scene was created by Task VW,⁴ and was originally viewed in glorious colour (each elevation can be displayed in a different colour). Task VW provides a useful pictorial verification that the ordering software has produced the correct results.

8.4.2 The Human Back

Another important class of test objects for the biomedical moiré research at NRC involves live human subjects. In particular, the research into spinal deformity motivated perfecting the moiré technique to record the shape of the human back.

Figure 8.7 shows an enlargement of a topogram of the back of a volunteer, taken under the same photographic conditions as the painted sphere in Figure 8.3. For

⁴Task VW was designed and written by Mr. Ding just prior to completing his stay at NRC. Unfortunately, there was only enough time to apply this task to a few of the contour ordering test results.

Figure 8.6: A perspective view of the ordered contours for the painted sphere map. This image has been created by Task VW using the system display, and provides a pictorial check that Task JFO has correctly ordered the contours of the convex sphere.

Figure 8.7: Moiré topogram of the back of a live human subject. The individual strings of the stationary grid are clearly visible on the original 4" x 5" Polaroid print.

objects covered in 'live skin' (as opposed to semi-gloss paint), it was found that reducing the development time to 30 seconds provides a better contrast range in the fringe pattern.

Figure 8.8 shows the display of the upper portion of the digitized topogram. The author was limited to obtaining hardcopy of the processed digital images by directly photographing the system video display. Although the entire digitized topogram can be stored in the video memory, the image is too large to be displayed at full resolution. The display provides only a partial view of the stored image, and one has to interactively 'roam' the display window to view other parts of the image. It is possible to load a decimated version of the image, such that it fits into the display window, but it was not considered desirable to process decimated images. The hardware does not allow one to 'zoom back' for a decimated view of the image in memory. This makes photographing the processing results for large images rather awkward. For the tests described here, the author restricted the domain of the processing to whatever portion of the test image that would fit into the display window at full resolution. Naturally, the processing by Task JFO of the image in memory is only restricted by the size of video memory and not the size of the display window.

Figure 8.9 shows a set of contours that have been traced from the dark moiré fringes on the left and right scapula of the test subject. The contours in this figure have been reconstructed by Task JDC using a map-list index file and the individual contour-point list-files produced by Task JSP. There are ten contours in this map, and the topography is characterized by two hills (the left and right scapulas) on a larger hill.

Table 8.3 shows the experimental results for ordering this map. In this test, Task JFO is given initial values for ϕ_1 and ϕ_2 (the contours at the summit of the left hill), and it orders the remaining contours in eight steps. For each step, the table gives the neighbourhood contour sequence, the relative error sum for each ordering case, and the fringe number assigned to the unknown.

The human back provides a more difficult ordering challenge than the map of

Figure 8.8: Display of the digitized human-back moiré topogram. The contrast has been enhanced for illustrative purposes.

Figure 8.9: Test contours traced onto the human back display image. The contrast has not been stretched for this figure. The contours have been traced along ten dark fringes that define the left and right scapula 'hills', and appear as an overlay of fine white lines drawn by Task JDC.

Table 8.3: Experimental results for the human back using Task JFO

process step no.	neighbourhood contour sequence $\langle \phi_m, \phi_n \phi_u \rangle$	relative error sum for each ordering case			fringe number assignment
		$\epsilon_u^{(+)}$	$\epsilon_u^{(0)}$	$\epsilon_u^{(-)}$	
0	$\phi_1 = 16.5$ $\phi_2 = 17.5$				
1	$\langle \phi_1, \phi_2 \phi_3 \rangle$	19.1	12.8	1.0	18.5)
2	$\langle \phi_2, \phi_3 \phi_4 \rangle$	11.1	5.9	1.0	19.5)
3	$\langle \phi_3, \phi_4 \phi_5 \rangle$	20.3	12.8	1.0	20.5)
4	$\langle \phi_4, \phi_5 \phi_{10} \rangle$	20.1	13.4	1.0	21.5)
5	$\langle \phi_{10}, \phi_5 \phi_9 \rangle$	1.0	2.1	2.4	19.5)
6	$\langle \phi_5, \phi_9 \phi_8 \rangle$	1.0	2.2	2.3	18.5)
7	$\langle \phi_9, \phi_8 \phi_7 \rangle$	1.0	3.3	3.1	17.5)
8	$\langle \phi_8, \phi_7 \phi_6 \rangle$	1.0	12.1	17.4	16.5)

the painted sphere. The presence of two hills means that the ordering schedule must branch, and it may even have to reverse direction in the map-list. Referring to Figure 8.9, one can see from Table 8.3 that the ordering progresses down the left scapula hill (ϕ_1, ϕ_2, ϕ_3 and ϕ_4), then down the shoulders (ϕ_5, ϕ_{10}), and then works up the right scapula hill (ϕ_9, ϕ_8, ϕ_7 and ϕ_6). Table 8.4 shows the evolution of the state diagram for the map-list at each step. The disruption in the flow through the list due to the branch in the schedule is quite clear in this table. As Task JFO backs-up the right hill, it has to make one sweep through the map list for each of the last four ordering steps.

Figure 8.10 shows a perspective plot of a larger set of contours from the same test subject, also ordered by Task JFO. As before, this type of view of the ordering results provides a quick visual check that the ordering has been done correctly.

Table 8.4: Task JFO processing schedule for the human back

fringe.	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}
step 0,	!	!	?	?	?	?	?	?	?	?
1	!	!	*							
2		!	!	*						
3			!	!	*					
4				!	!					*
5					!				*	!
6					!			*	!	
7							*	!	!	
8						*	!	!		

! = known

? = unknown

* = assigned

Figure 8.10: A perspective view of the ordered contours for the human back map. This image was created by Task VW using the system display, and provides a pictorial check that Task JFO has correctly ordered the contours of this map. Note that a few extra contours are shown in this figure (one more at the top of the shoulders, and five more in the lower back).

8.5 Discussion

8.5.1 General Comments

Task JFO has all the elements required for automated photometric ordering of a set of moiré fringe contours. The two sets of test contours described here present Task JFO with several interesting photometric ordering challenges. In obtaining the results presented in this chapter, it has been shown that photometric ordering can be realized with a simple, streamlined program (Task JFO) that can handle a variety of surface reflectance types (painted wood, live human skin) and hilly topographies with branched ordering schedules. Furthermore, the analysis has been based on a simple Lambertian photometric model and the minimum-length of neighbourhood contour sequence (three neighbours, with one an unknown).

This level of performance meets the requirements of the biomedical researchers looking for ways to automate the analysis of moiré topograms for spinal deformity research. With minimal modification, Task JFO can be adapted to other image processing systems, and used by these researchers. For this reason, the full listings for Task JFO's program modules are provided in Appendix D.⁵

8.5.2 Robustness of Photometric Ordering

One of the most important issues in using photometric analysis to order the moiré fringe contours is the choice of the photometric model. A simple model, such as the purely-diffuse Lambertian model used in Task JFO, has significant computational advantages over models that account for more realistic surface reflectance properties. The author wondered how robust the ordering method would be if it were based on a purely-diffuse model and applied to an object with pronounced specular components. Hope lay in the fact that specular effects are rare on most surfaces, and that they are highly localized.

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⁵A floppy-disk with the source code for all the tasks is available from the author. It includes some suggestions on how to modify the DIPIX-specific portions of the code for other image processing systems.

The painted sphere is a good candidate to test this inquiry. The strong highlight at the top of the sphere is known to produce the wrong value for the expected diffuse reflectance. As compensation for this sensitivity of the model, the specular aspect of the semi-gloss painted surface is localized to a small region where the geometry supports specular reflection. The other analysis regions along the contours have geometries bearing a diffuse characteristic that are well represented by the Lambertian model. It follows that if there is a sufficient number of well-behaved analysis regions along the contour, then the effect of one or two poorly-modelled regions will not be likely to sway the outcome. It is expected that the discrimination will be degraded, but not demolished.

This expectation is supported by the test results. The ordering-case discrimination for the central contours in the painted sphere results is good (the worst case has a value of 2.0), and it improves dramatically as one moves to contours that are distant from the specularity.

Rather than rely on having sufficient well-behaved regions, one can test for and skip the poorly-behaved regions. It is quite simple to test for the problem points, and Task JFO could be modified to include a test to skip those points. One need only test the bisector $\mathbf{H}$ to the illumination direction $\mathbf{L}$ and the viewing direction $\mathbf{V}$. If $\mathbf{H}$ significantly aligns with the surface normal $\mathbf{N}$ (i.e., as $\mathbf{H} \cdot \mathbf{N} \rightarrow 1$), then a specular component may be a problem. The significance of the alignment must be determined, but for most glossy non-metals the specular behaviour is confined to a solid-angle cone subtending a fraction of one degree in the visual field, which corresponds to a swing of few degrees between $\mathbf{H}$ and $\mathbf{N}$. Thus, one could reject points based on the specularity condition given in Equation 4.4 (p. 165).

8.5.3 Schedule Efficiency

Another obvious place to improve Task JFO is in the execution of the ordering schedule. There is no need to repeat the sweeps through the map-list in search of neighbourhood sequences with the one-and-only-one unknown. On the first sweep, Task JFO could fill a table with all the neighbourhood sequences it finds. Some may

be processed immediately, but the identity of the contours in the sequences that are skipped on the first pass should not be discarded. The neighbour information required for scheduling could be retrieved from the table, rather than searching the image each time to see if a contour has any neighbours that need ordering. After the first-pass search, a clever scheduler could use the table of neighbours and the map-list of ϕ -values to optimally schedule the ordering steps for a map with a minimum of image searching operations.

Chapter 9

Task MFO: Automated Fringe Ordering

9.1 Introduction

Task MFO was designed to embody the most general form of the Photometric Ordering Algorithm. This chapter describes the design and development of Task MFO, and reports on some experiments into the automated ordering of moiré fringe contours using this program. The remainder of the chapter is organized as follows.

The next section gives an overview of Task MFO, and the discussion focusses on the design philosophy behind the task and on its functionality. The various features of the task and how they affect the embodiment of the Photometric Ordering Algorithm are also described.

The third section deals with the software implementation of Task MFO. First, the data files required by the task are specified. Then the program modules are described, along with their correspondence to the block-diagrams that define the Photometric Ordering Algorithm in Chapter 3. Finally, the overlay structure for Task MFO is described, and the buffer restrictions that limit the complexity of the contour map that can be processed by the task are also discussed.

The fourth section reports on some experiments into contour ordering using Task MFO. The experiments involve a variety of surface materials (grey cardboard,

painted wood and live human skin), and a number of topological challenges that include sharp ridges and hilly terrain.

The last section presents a discussion on the performance of Task MFO. The test results are analyzed in terms of Task MFO's usefulness as an analytic tool for investigating photometric ordering. The effect of 'photometric mimics' on the robustness of the ordering process is examined. Finally, the prospects for automating the generation of the schedule file are discussed, along with other possible extensions to the program.

9.2 Overview of Task MFO

In this section, an overview of Task MFO is given. The design philosophy behind the task is explained, and the scope of the functionality of the program is described. The degree to which the various functionality features embody the general form of the Photometric Ordering Algorithm is also discussed.

9.2.1 Design Philosophy

Task MFO is designed to embody the most general form of the Photometric Ordering Algorithm, as it was described in Chapter 3. Unlike Task JFO, which has several restrictions that warrant a separate description of that task's mode of operation, Task MFO has an exact correspondence with the general prescription given in Chapter 3 for the propagated ordering of moiré fringe contours by photometric analysis. The details of the prescription will not be reiterated here. The reader may find it convenient to refer to Section 3.4 (starting on p. 95), as frequent reference will be made to essential data structures, processes and control elements that are described there.

The general form of the Photometric Ordering Algorithm allows for considerable flexibility in its function. For example, the neighbourhood sequences can be of arbitrary length for each ordering step (provided they are greater than two contours), and maps can be traced from mixed- or matched-mode moiré fringes. The general

algorithm does not restrict the analysis to a specific photometric model. A more subtle point, explained in the sections that follow, is that for the q th ordering step, not all the unknowns in the neighbourhood sequence are required to be assigned a fringe number at that step.

In the previous chapter, Task JFO was described as using a streamlined, simplistic version of the Photometric Ordering Algorithm. It proved to be a very successful program for the automated ordering of moiré fringe contours, yet had several restrictions that limited the scope of its application. Task MFO is intended to explore those restrictions, to probe into the algorithm in some practical situations, and allow the author to draw some conclusions about the design of a future generation of automated ordering software. A number of features were implemented primarily to serve investigative exercises.

9.2.2 Functionality Features

Task MFO has several features incorporated into its software that facilitate displaying the strengths, and exposing the deficiencies, of the Photometric Ordering Algorithm. There are five features that characterize the flexibility of Task MFO:

- Mixed or Matched Contour Modes;
- Flexible-Length Neighbourhood Sequences;
- Specifiable Roles for Sequence Members;
- Specifiable Regional Seed-points;
- Externally Prescribed Schedule.

The decision to allow flexibility in these aspects of the algorithm led to significant complications in designing the software. Before examining the software design, the investigative benefits of implementing these features are briefly discussed in the sections that follow.

9.2.3 General Contour Modes for Task MFO

Task MFO is designed to work with mixed- or matched-mode contours. It maintains a list of moiré fringe extrema-type for the contours, and uses the codes ($e = 1$ for bright fringes, $e = 0$ for dark fringes) to select the appropriate ordering rules when generating the trial ordering cases. This allows experiments with a variety of contour types (i.e., traced from all bright, all dark, or mixed fringes) to test the statement made in Section 7.7.2 (p. 264), that the shading estimator used by Task MFO would have a consistent bias irrespective of the contour mode.

9.2.4 Flexible-Length Neighbourhood Sequences for Task MFO

Task MFO allows the length of neighbourhood sequence to be adjusted for each ordering step in the schedule. The maximum length is limited to 15 by the current software buffer sizes. This allows the neighbourhood to stretch over more contours than the minimum three, so that one can test the algorithm over extended analysis regions. It also allows more than one analysis patch in both the known and unknown partitions of the neighbourhood.

9.2.5 Specifiable Roles for Members in Task MFO's Neighbourhood Sequences

Task MFO allows the members of the sequence to play different roles for each ordering step. An important role to be specified is the seed-point member from which the gradient curve is generated to define the neighbouring analysis points. Also specifiable are the members on which the photometric analysis patches are to be centered, and Task MFO maintains separate analysis-point lists for the members that are knowns and unknowns. Figure 9.1 shows an example of a sequence with 7 members. The third member provides the seed-points, and analysis patches are specified to be centered on members 2, 4 and 6. In this example the patches about

Figure 9.1: The photometric analysis region used by Task MFO can have a varied length. In this example, the neighbourhood sequence has 7 members $\{C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{23}, C_{24}\}$. The third member is specified as the seed-point contour, so for this analysis region the gradient curve G_j has been constructed from seed-point P_j on C_{14} , to intersect the neighbouring contours at the indicated points. Analysis patches have been specified to be centered on members 2, 4 and 6, and were drawn using the definition of the contour-point circular patch (refer to Figure 7.14). Finally, the photometric analysis region R_j is drawn as a boundary encircling the analysis patches.

members 2 and 4 might be on contours with known ordering, and the patch at member 6 might be subject to trial orderings.

Finally, Task MFO also allows the user to specify which of the unknowns are to be assigned ϕ -values as an outcome of a given ordering step. It may be desirable *not* to fix the most-exterior member of the neighbourhood, but rather to fix that contour in a later ordering step (when it is more interior to the sequence).¹ In Figure 9.1, it might be the case that members 5, 6 and 7 are unknowns, but that only members 5 and 6 are to be assigned ϕ -values in that ordering step.

For the rest of this chapter, members in the neighbourhood sequence that are to

¹The trial ϕ -value of the unknown contour at the end of the sequence may have had little influence on the photometric analysis. In general, the central contours control the fundamental shape of the gradient profile, with the end contour providing some fine-tuning of the surface normal at an interior analysis patch.

be photometric analysis points will be denoted with an underscore (i.e., $\underline{\phi}$'s), and unknowns that are not to be assigned ϕ -values at a given step will be enclosed with parentheses [i.e., (ϕ) 's].

9.2.6 Specifiable Regional Seed-points for Task MFO

Task MFO allows an arbitrary list of points on the seed-contour to be specified as the seed-points. Recall that Task JFO used the contour-list *per se*, so that *all* the sparse break-points that defined the contour curve in the map image had to be considered by that program as seeds for photometric analysis regions. Task MFO allows the user to provide a subset of the break-points, or even to use a denser set of points. This is useful for confining the analysis to a particular area in the map image, and for forcing the ordering outcome to be based on one or more specifiable analysis regions.

9.2.7 External Scheduler for Task MFO

To ensure that the user has total control over the ordering process parameters, Task MFO requires that an external schedule be provided. Task MFO expects the schedule information to be organized in a particular way (the format is described in detail in the next section). The required information consists of the initial ϕ -values to be assigned to any knowns, and then a series of ordering 'control vectors' with the specifications for each ordering step. The specifications include: the length of the neighbourhood sequence, the identity of the members (the map-label of the contours), the seed-point member, the members that define the known/unknown analysis points, the unknown members that are to be assigned ϕ -values at this step, and the name of list of seed-points. In this regard, the schedule required as input by Task MFO corresponds exactly to the ordering schedule V depicted as input to **PMO** in Figure 3.11 (p. 98). As currently written, the program has no internal mechanism for generating the scheduling steps, as is the case with Task JFO.

TOP OF FILE

---

6 = # of header records File: [200,200]ABCFOTRYG.TOP

This is a TOP CONTROL FILE for Task MFO. It supplies the names of all the files required by the task to execute a photometric ordering of the moire fringes.

'ABCFOAAAA.XYZ' = Topogram Coordinate Transformation Coefficients File
 'OTALSS.XYZ' = Moire Instrument Parameters File
 'ABCFOTRYG.SCH' = Fringe Ordering Schedule File
 'ABCFO SOME IDX' = Index File of Contours in the Map for BSPFFR100

END OF FILE

Figure 9.2: The top control file used by Task MFO

9.3 Software Implementation of Task MFO

This section deals with the software implementation of Task MFO. The data files required by the task are specified, then the program modules that comprise the task are reviewed. The task is too large to run directly on the system, and the required overlay structure for Task MFO is briefly described. In the last section, the restrictions imposed on the size of the processing buffers are discussed in terms of the complexity of the map that Task MFO can analyze.

9.3.1 Required Data Files

Task MFO requires access to several types of data files. The list data falls into the following categories:

- Top Control File, specifies the names of the data files used by Task MFO. This file is just a convenience to the investigator, who may design experimental runs that try various combinations of data files, and assign a 'name' to each test combination (*e.g.*, "BEND" or "TRYG"). Task MFO prompts for the name, and then reads the appropriate Top Control File to obtain the required data file names for that test configuration. Figure 9.2 shows an example of a top control file used in one of the experiments reported later in the chapter.
- Schedule File, contains two lists, and is unique to Task MFO. The first list

assigns ϕ -values to the initial knowns in the map. The second list provides values for the components of the control vector for each ordering step in the schedule, and includes: the step number; the length of the neighbourhood sequence and the identity of its members; the member on which the seed-points lie; lists of members that are to be used as knowns, as unknowns, and those to be assigned ϕ -values; and the file name of the list of seed-points. Figure 9.3 shows an actual schedule file used in one of the experiments reported later in this chapter. Note the verbose header in the example, which identifies the variable names used by Task MFO for each component in the control vector.

- Moiré Image Map List, also called the 'Master Index File', lists the contours that comprise the map. This index file differed from the one used by Task JFO by including the moiré fringe extrema-type for each contour. An example of a Master Index File used for an experiment discussed later in the chapter was shown in Figure 5.13 on page 187.
- Moiré Image Seed-Point Lists, are the source of seed-points for the analysis regions, and might be the same contour-point files used by Task JDC to draw the contours, or else separate files especially prepared for Task MFO containing subsets of seed-points. One list is required for each of the Q ordering steps.
- Moiré Instrument Parameters File, required to define the physical geometry of the moiré instrument used to form the moiré topogram image. This file is identical to the one used by Task JFO (Sec. 8.3.2 on p. 288). It corresponds to the data structure ① in the Photometric Ordering Algorithm (refer to Fig. 3.19 on p. 117).
- Image Transformation Coefficients File, required to transform digitized image coordinates into the moiré grid-plane of the object coordinate system. Again, this file is identical to the one used by Task JFO, and it corresponds to the data structure ② in Figure 3.19 (p. 117).

TOP OF FILE

30 = # rec.

File: [200,200]ABCFOTRYG.SCH

4 = # of records in second header.

This is a Master Fringe Ordering Schedule File for Task MFO. It contains two lists of records. The first list names contours (by number) whose moire fringe value is known...these values provide the necessary starting point for 'Propagated Shape Photometric Ordering' (PSPD). The end of this list is marked with zero-data.

The second list schedules the fringe ordering operation steps: the n-th record contains the Control Vector required for the n-th fringe-ordering step. The end of this list is the 'end of file'. The Control Vector has the following components:

NUMSTP step number
 NUMREG number of member contours in NEISEQ
 NEISEQ sequence of neighbourhood members (as member's contour-number)
 MEMSED neighbourhood member that is the seed contour
 NUMOFV number of contours in LISTSV
 LISTSV neighbourhood members with known fringe number
 NUMOFU number of contours in LISTSU
 LISTSU neighbourhood members with unknown fringe number
 NUMOFF number of contours in LISTSF
 LISTSF neighbourhood members whose fringe number will be determined this step
 BFMSD file name of list of neighbourhood seed-points.

 First List: Contours with Known Moire Fringe Numbers.

Cnt	Fring#	Comments:
1	15.0	top of left hill...it is a maxima (bright fringe)
2	15.5	
0	0.0	(denotes end of list)

Second List: Schedule of Fringe Ordering Operation Steps.

#	*+NEISEQ	MSD	*+LISTSV	*+LISTSU	*+LISTSF	BFMSD
1	5 1,2,3,4,5	4	1 2	1 4	2 3,4	'ABCSPAA04.SPF'
2	5 2,3,4,5,6	4	1 2	1 4	2 4,5	'ABCSPAA05.SPF'
3	5 4,5,6,7,8	4	1 2	1 4	2 4,5	'ABCSPAA07.SPF'
4	5 6,7,8,9,10	4	1 2	1 4	2 4,5	'ABCSPAA09.SPF'
5	5 10,9,18,17,16	4	1 2	1 4	3 3,4,5	'ABCSPAA17.SPF'
6	5 18,17,16,15,14	4	1 2	1 4	2 4,5	'ABCSPAA15.SPF'
7	5 16,15,14,13,12	4	1 2	1 4	2 4,5	'ABCSPAA13.SPF'
8	5 14,13,12,11,11	4	1 2	1 4	1 4	'ABCSPAA11.SPF'

END OF FILE

Figure 9.3: The Schedule File used by Task MFO

9.3.2 Task MFO Program Modules

The hierarchy chart for the modules in Task MFO is shown in Figure 9.4, and uses the same format as was used for Task JFO's chart in the previous chapter. As before, some of the standard calls to system modules, such as error logging and messages to the console, are indicated by superscripts (e.g., FOAZMT^[a] means that FOAZMT called XODETA and XOATTA). The short functional description given in the figure for each module will be elaborated upon below.

The modular structure of Task MFO has an exact correspondence to an early version of the Photometric Ordering Algorithm. Since its conception, the description of the algorithm has been modified to improve its clarity, and the version presented in Chapter 3 has a slightly different grouping and nesting of sub-processes than the original. The overall functionality is the same, but in one portion, a single software module (FOPROC) still spans several function-blocks of the algorithm, and in another portion of the program, modules are nested several levels to embody RND, which is shown as a single function block in the current description of the algorithm. Despite these exceptions, most of the functions of the Photometric Ordering Algorithm still have a one-to-one correspondence to the modules of Task MFO, and with a minor programming effort, the correspondence could be restored if it was deemed necessary. As will be seen in the sections that follow, the correspondence with the block-diagrams of the algorithm commences with module FOPROC and its subordinates.

9.3.2.1 Module FOMAIN

FOMAIN is the main module for Task MFO. It deals with the task initialization, operator interaction to specify the processing parameters, and task termination.

The details of the initialization routines called by this module were given in the previous chapter for Task JFO. X0INIT is called to initialize the DIPIX operating system (OIS) environment. XBGVID attaches the video memory array (VMA) to the task. XXVLNK establishes the mapping to DPI-common for direct access to the VMA.

Notation: [a] -calls to IODETA and IOATTA (permits direct I/O to console)
 [e] -calls to IOERRM (error log routine)
 [m] -calls to IONESQ (operator I/O)
 [f] -calls to KRRTS (FORTRAN I/O error routine)

Figure 9.4: Hierarchy chart for Task MFO. To simplify the list, standard calls to system modules are indicated by superscripts, as noted. The corresponding sub-processes of the Photometric Ordering Algorithm are listed in 's on the right.

FOGETS prompts the operator to indicate which image in VMA contains the required image data structure for Task MFO, and then calls XIREPO to read the appropriate image display parameters into the common block /XZ1IPR/. The operator is prompted to confirm that the correct image has been selected (VMA can store up to 8 independent images at once), and is then solicited for the 4-character file identifier of the top control file. FOCHAK ensures the operator specifies a valid file name. FOGETS opens the specified top control file, and returns to FOMAIN the names of the data files required to process the map.

FOMAIN then calls three file-reading routines: FOGETT reads the digitized image transformation coefficients into a common block, FOGETI reads the moiré instrument parameters into another common block, and FOGETF reads the map index file data into a set of buffers. Then FOPROC is called to process the map (this module is described in detail below), and after FOPROC has returned, FOMAIN calls XBRVID to release VMA, and finally calls XOEXIT to make the standard tidy exit to OIS.

9.3.2.2 Module FOPROC

FOPROC is the workhorse of Task MFO, as it oversees calling all the high-level functions of the Photometric Ordering Algorithm. Indeed, it spans several of the block diagrams of the Photometric Ordering Algorithm: **PMO** (see Fig. 3.11 on p. 98), **PNO** (see Fig. 3.12 on p. 103), and **PNA** (see Fig. 3.15 on p. 109).

In the **PMO** portion of FOPROC, FOSHIN is called to open the schedule file, and read any initial ϕ -values from the first list in the schedule file, and thereby establish the initial state of the map $(\Phi)_0$. Then FOVECT is called to perform the function depicted by **POS** in Figure 3.11, namely reading the control vector V_q for each ordering step in the schedule. The map-label for each member in the neighbourhood sequence is read into a buffer, and lists are also formed of which members are to be knowns, which are to be unknowns, and which are to be altered by the ordering step. The file name of the seed-point list is also read from the control vector.

In the **PNO** portion of FOPROC, the sub-functions of **OCG** (see Fig. 3.13 on p. 105) correspond directly with the next two modules of the hierarchy chart in

Figure 9.4. FOPREP sets up parameter buffers for the members of neighbourhood sequence for a given ordering step. The MFE-type and fringe number are extracted from the map-lists and put into neighbourhood-lists $\langle \phi \rangle_q$ and $\langle e \rangle_q$, respectively, in a fashion that corresponds with [NSE]. FOCASE embodies the process [CRI], and produces the set of trial fringe numbers $\{\Phi\}_q$ using the fringe ordering case rules for the proper contour mode.

In the [PNA] portion of FOPROC, the initialization for processing each neighbourhood sequence is completed by clearing the photometric-consistency error accumulators and the region-counter. Then FOPROC spawns a [PRA] (photometric regional analysis) for each point it reads from the seed-point file (an operation corresponding to [RSG]) by looping until the seed-points have been exhausted. The function of each [PRA] (see Fig. 3.15 on p. 111), is realized by a pair of subroutines to FOPROC, namely FOGRA and FORKAS, which are described in separate sections below.

To complete the [PNO] portion of the algorithm, FOPROC finally calls FOPICK to pick the most photometrically-consistent trial case, and to update the ϕ -list. Module FOPICK directly corresponds to the original function [CSU] (which was later decomposed into two logical sub-functions, as shown in Fig. 3.14 on p. 107). The brief code required to implement [MES] and [NSD] of that figure allows the two sub-functions to remain merged as a single subroutine in Task MFO. Besides choosing the best case and updating the ϕ -list, FOPICK also displays a short report on the operator's console for each ordering step. An example of the report is given with the experimental results (Fig. 9.9), but briefly it includes: the number of seed-points, the number of successfully-analyzed regions, and the total consistency-error $\epsilon_q^{(k)}$ and ϕ -values for each of the N trial ordering cases. The best case (i.e., the m th case, having smallest total error $\epsilon_q^{(m)}$) is indicated, and so is the next closest case. Also computed is the contrast κ between the best case and the other cases:

$$\kappa = \frac{\epsilon_q^{(k)} - \epsilon_q^{(m)}}{\epsilon_q^{(k)} + \epsilon_q^{(m)}} \quad (k \neq m), \quad (9.1)$$

as well as the average contrast for the $(N - 1)$ rejected cases. The contrast ranges between 1 and 0, and gives a good measure as to how well the photometric analysis

has distinguished between the m th case and the other cases. Perfect contrast ($\kappa = 1$) requires perfect photometric consistency for the m th case (i.e., $\epsilon_q^{(m)} = 0$), which is unlikely. As will be shown in the experimental results, high contrasts ($> 50\%$) are easily obtained on some test surfaces, with lower contrasts down to 5% still being quite convincing (i.e., that a distinct choice of shape can be made).

9.3.2.3 Module FOGRAD

Subroutine FOGRAD embodies function `RND` in Fig. 3.16 (p. 111). Its purpose is to fill `[S]k`, the gradient-neighbour attribute table that is required for each analysis region. The contents of the table were defined on page 112, but briefly they include for each member of the neighbourhood: the member number; its contour number (position in the map-index list); its map-label (in the image data structure); its line and pixel coordinates in the image; the azimuth direction and distance to its gradient neighbour.

The line and pixel coordinates, and the direction and distance to the gradient neighbour must be discovered by searching the image, as was described in Chapter 5. FOGRAD starts at the specified seed-point (normally in the midst of the neighbourhood sequence), and searches in one direction (called the 'forward' direction) to find the gradient neighbour on the next contour in the sequence, and then searches from that point to find the next gradient neighbour, and so on, until it has discovered all the gradient neighbours in forward direction in the sequence. FOGRAD then returns to the seed-point, and searches in the 'backward' direction along the sequence, discovering the gradient neighbours in turn, so that finally all the required regional attributes for each member of the neighbourhood sequence are determined.

The search function depicted simply by `RND` in Figure 3.16 (p. 111), turns out to be rather lengthy to encode in software (~ 2000 lines in 5 subroutines). Module FOGRAD organizes the forward and backward searches, pushes discovered attributes into the appropriate positions in the table, handles special cases and errors, and provides detailed diagnostics to monitor this deceptively complex task. Although all the actual searching of the image is performed by calling FOAZCL, module FOGRAD

is still over 600 lines in length.

As mentioned, subroutine FOAZCL oversees the image search for gradient neighbours. The programming techniques were described in Section 5.4.2 (p. 194), and the details will not be elaborated upon here. Briefly, however, FOAZCL uses the contour-point file to establish the contour-points on each side of the seed-point (i.e., forming three points on the contour curve) and then calls FOAZMT to compute the azimuth direction using polynomial curve-fitting techniques. FOAZCL then loops through a bundle of search directions about the azimuth direction, making a call to FOSEAR to search the image in each trial direction for the nearest intersection of a contour. FOSEAR calls a utility routine FOEXAM to examine the theme field (i.e., the map-label value) of each image element on the search path, to discover the event of intersecting a contour. FOAZCL keeps track of which search direction resulted in the closest neighbour being discovered, and passes the attributes of that discovered neighbour back to FOGRAD.

9.3.2.4 Module FORKAS

Module FORKAS invokes the remainder of the function of each **PRA** (see Fig. 3.16 on p. 111). In the **IRM** portion of the module, it makes calls to FOREFL to measure the image reflectance about each of the photometric analysis points. In the **RCA** portion of the code, FORKAS sets up a loop counter to call FOPMRA to compute the consistency-error for each trial ordering case.

9.3.2.5 Module FOPMRA

Module FOPMRA corresponds exactly to **RCA^(*)**, and oversees the functions depicted in Figure 3.18 on page 115. It realizes **GSC** by calling FOGEOM to get the geometric scalars for the analysis members of the region. It performs **MPE** by calling FOPARA to establish the photometric model parameters using the 'known' members. It executes **PCE** by calling FOPMER to compute the photometric consistency-error due to the trial ordering of the 'unknowns'.

The functionality of FOGEOM is described in more detail below. The calculations

performed by FOPARA and FOPMER are done according to the prescription given in Section 3.4.8 (p. 113), and are not elaborated upon further.

9.3.2.6 Module FOGIOM

Module FOGIOM has an exact correspondence to **GSC**, shown in Figure 3.19 on page 117. The routines called by FOGIOM use the equations and concepts described in Section 3.4.9, starting on page 116. In the first part of the module, FOGIOM performs function **CEC** by repeated calling of FOELEV to compute the elevation z of each point in the region, and then executes **OCC** by repeated calling of FOCORR to compute the (x, y) coordinates for each of the points.

FOGIOM then derives the scalar product for the 'unknown' analysis points. It executes **IDC** by calling FOLAMP to compute the direction to the lamp for each point, and then performs **SNC** by calling FOOIRIE to compute the orientation of the surface normal. Finally, it invokes **SPC** by calling FOPROD to compute the scalar product of the two vectors for each point. FOGIOM repeats this sequence of operations for the list of 'known' analysis points, and then passes the two lists of scalar products back to FOPMRA.

9.3.3 Task MFO Overlay Structure

Task MFO is too large² to run on the PDP-11 system using directly-linked object-code modules, and it is necessary to use overlay techniques when building the task image to reduce its apparent size. The restriction to task size is imposed by the system architecture. The primary addressing mechanism of the PDP-11 is the 16-bit computer word (1 word = 2 bytes = 16 bits). The maximum physical address space that the PDP-11 can reference at any one time is a function of the length of this word. Thus, the size of a non-privileged task is restricted to $2^{16} = 64K$ bytes on an unmapped system.³ Another constraint arises from reserved memory

²Task MFO has ~6500 lines of code, compared to ~3000 lines for Task JFO, and also has a large buffer-space allocated for maintaining attributes of variable-length neighbourhood sequences.

³Privileged tasks can reference beyond 64K bytes, and memory-mapped systems can also extend the size of a task's work space.

segments. The top 8K bytes of memory are reserved for the UNIBUS addresses that correspond to peripheral device registers (the I/O page), and the low 8K-byte segment is occupied by the Executive. A third segment is reserved for the 8K-byte DPI-window required by DIPIX tasks that make direct access to the video memory array (refer to Fig. 5.11 on p. 184). Of the 64K bytes, this leaves 40K bytes in which to build the task.

Overlaying a task involves partitioning the code into sections that can be swapped in and out of the working space as required. Code sections may be swapped from disk (resulting in significant delays for each swap) or from program memory (resulting in negligible delays). Subroutine modules, singly or in groups, provide a natural partitioning of the code. Swapping is permitted on the basis that the parent of the swapped routines must remain resident in the work space. Thus, for example, FOGIOM must remain in the work space, while FOELEV, FOCOR, FOLAMP, FOORIE and FOPROD are each swapped in and out as FOGIOM calls them in turn. The descriptor for such an overlay scheme (called a 'leg' or 'factor') may be written as:

```
LEGGED:  FOGIOM-(FOELEV,FOCOR,FOLAMP,FOORIE,FOPROD)
```

where LEGGED is the name assigned to the factor, and the punctuation syntax identifies which modules must be co-resident and which are to be swapped.

Figure 9.5 shows the complete overlay descriptor submitted to the PDP-11 RSX-11/M Task Builder to construct Task MFO. An Overlay Descriptor Language (ODL) line takes the form:

```
label: directive argument-list ;comment
```

and the three directives used in the figure are: .ROOT, which defines the structure of the overlaid task; .FCTR, which defines a factor in the structure, and permits complex overlay trees to be described; and .END, which terminates the overlay descriptor. Names are assigned only to the factors, and 'DR2:' refers to the disk where the modules reside. The hierarchy chart for the task in Figure 9.4 can guide the initial choice of the factors, but the size of each compiled module plays an

```

-----
NAME          MFO.ODL
PURPOSE       Overlay Descriptor for TASK MFO
AUTHOR        J.R. Pekelsky
DATE          April 22, 1986
FIRST USED ON ARIES-II Image Analysis System for
              automated moire fringe ordering
-----

THE MAIN ROOT

.ROOT  MAIN-LIB-*(LEGGTS,LEGGTT,LEGGTI,LEGGTF,LEGPRC)

MAIN   .FCTR  DR2:FOMAIN
LIB:   .FCTR  DR0:[100,377]COMMONR/LB
LEGGTS .FCTR  DR2:FOGETS-LIB-FOCHAK
LEGGTT .FCTR  DR2:FOGETT-LIB
LEGGTI .FCTR  DR2:FOGETI-LIB
LEGGTF .FCTR  DR2:FOGETF-LIB
LEGPRC .FCTR  DR2:FOPROC-LIB-(FOSHIN,FOVECT,FOPREP,FOCASE,LEGRAD,LEGRKS,FOPICK)
LEGRAD .FCTR  DR2:FOGRAD-(LIB,FOGREX,LEGAZL)
LEGAZL .FCTR  DR2:FOAZCL-(LIB,FOAZNT,FOSEAR-FOEXAM)
LEGRKS .FCTR  DR2:FORKAS-(LIB,FOREFL,LEGPME)
LEGPME .FCTR  DR2:FOPMRA-(LEGGED,FOPARA,FOPMER)
LEGGED .FCTR  DR2:FOGEOM-(FOELEV,FOCOOR,FOLAMP,FOORIE,FOPROD)
-----

END

```

Figure 9.5: The overlay descriptor file for Task MFO

essential role, and ultimately one must juggle the factorization of the task in a trial-and-error fashion. This was the author's first experience with overlays, and it took several attempts before the task was small enough to run.

9.3.4 Task MFO Map-Complexity Limitations

Task MFO has a number of buffers whose sizes set the limits on the complexity of the maps that may be analyzed by the task. Making the buffers arbitrarily large extends Task MFO's capabilities, but also increases the task size. A practical balance must be established, especially if the task must be overlaid in order to build it.

In the author's case, a reasonable level of overlay effort (i.e., no drastic partitioning of modules as originally designed, into smaller modules for the sake of overlaying) led to modest buffers that allowed: up to 52 contours in the map; up to 256 turning-points in a contour file; up to 15 members in the neighbourhood

sequence; and up to 32 trial ordering cases per regional analysis. As will be shown below, these so-called 'modest' buffers required over 5700 bytes of task space.

An effort was made to group the declarations that would affect the buffer limitations, so that they could be readily adjusted. Here is the section from FOMAIN where all the parameters associated with each member of the map are declared:

```

C----- Map Info from Index File -----
CHARACTER*14 BFN(52)  ! contour file name
C DIMENSION KON(52)   ! contour number
  DIMENSION MLA(52)   ! contour label
  DIMENSION MFE(52)   ! contour moire fringe extrema type
C----- Map Info from Schedule File -----
  DIMENSION ZFR(52)   ! contour moire fringe values
  DIMENSION BSY(52)   ! "!" = KNOWN / "?" = UNKNOWN FRINGE

data MXM/52/          ! maximum number of contours in map

```

Studying the data types shows that 25 bytes are required for each contour in the map list. Thus $MXM \times 25$ bytes must be reserved (in this case $25 \times 52 = 1300$ bytes).

The number of turning-points in a contour is limited by the buffers in FOAZCL for storing the line and pixel coordinates (the entire file is read into memory by the current design), and at 2 bytes per integer coordinate value, then 4 bytes are required per point. The current 256-point limit requires that 1K bytes must be reserved.

The length of the neighbourhood sequence and the number of ordering cases are loosely related. Generally, the longer the sequence, the more unknowns there are, and the number trial cases increases with the number of unknowns in a fashion that depends on the mode of the contour set. Most of the affected variables are declared in FQPRQC:

```

DIMENSION NEI(15)      ! neighbourhood sequence
DIMENSION LV(15)       ! list of 'knowns' members
DIMENSION LU(15)       ! list of 'unknowns' members
DIMENSION LF(15)       ! list of 'modifiables' members
DIMENSION ZF(15)       ! fringe values for N-members
DIMENSION MF(15)       ! moire fringe extrema type for N-members
DIMENSION LIN(15)      ! line coordinates for members of the region
DIMENSION PIX(15)      ! pixel coordinates for members of the region
DIMENSION SEP(15)      ! curve lengths separating members of the region
DIMENSION TAZ(15)      ! azimuth angles for members of the region

```

DIMENSION SET(480) ! Set of 0-cases: (MXK x MXR) array fringe value

DIMENSION TER(32) ! total PMA-error for each 0-case

C

DATA MXK/32/ ! maximum number of 0-cases that can be handled

DATA MXR/15/ ! maximum number of contours spanning the region

Note that MXK and MXR are set to the maximum values for the number of ordering cases and the length of the neighbourhood sequence, respectively, and that subroutines in Task MFO check that these limits are never exceeded. A few other subroutines also declare some parameters that relate to these limits:

C-----{from FORKAS}

DIMENSION RPH(15) ! neighbourhood member's fringe value

DIMENSION VRE(15) ! knowns member's reflectances

DIMENSION URE(15) ! unknowns member's reflectances

DIMENSION RER(32) ! case-wise PMA-errors

C-----{from FOPICK}

DIMENSION MXK(32) ! array of equally smallest cases

DIMENSION TRA(32) ! array of contrasts

C-----{from FOGDOM}

DIMENSION ICO(15) ! I-member's x coordinate

DIMENSION YCO(15) ! I-member's y coordinate

DIMENSION ZCO(15) ! I-member's z coordinate

DIMENSION RUN(15) ! object space distance between members

C-----{from FOCASE}

dimension xf(15) ! case-wise fringe numbers

dimension xll(15) ! case loop lower limit

dimension xul(15) ! case loop upper limit

dimension pro(15) ! pre-contour propagator

Inspection of the data types shows that: 8 integer and 13 real variables (68 bytes) are reserved for each of the MXR neighbourhood sequence members; 1 integer and 3 real variables (14 bytes) are reserved for each of the MXK ordering cases; and a real variable (4 bytes) is reserved for each entry in the (MXK x MXR) array of trial fringe values in SET. The total reserved space is

$$68(MXR) + 14(MXK) + 4(MXR)(MXK)$$

and with the current values of MXR=15 and MXK=32, this calls for 3420 bytes of the task space.

9.4 Experiments with Task MFO

This section reports on some experiments into contour ordering using Task MFO. In the previous chapter, ordering experiments with Task JFO were described for two test objects: the painted wooden sphere and a live human back. Those experiments are repeated here with Task MFO, with greater control over the ordering process parameters. In addition, a third test object is introduced: a corrugated cardboard "M" that presents an interesting new topographic challenge to the photometric ordering algorithm.

9.4.1 The Corrugated Cardboard

The corrugated cardboard test object was designed to present a difficult challenge to the photometric ordering algorithm, at least from the human interpreter's point of view. With minimal training, most people can learn to order moiré fringe contours by a quick inspection of the topogram. The human visual process is very complex, and many factors participate in the shape recognition task. Certainly surface shading plays a role, but intuitive guesses based on familiar shapes are potent cues for the decisions. This is particularly true for objects with smooth curved surfaces, such as the human back. However, even objects with sharp edges, such as the ridge at a fold in a sheet, are usually at an arbitrary alignment with respect to the reference plane, and cues such as the contours 'wrapping around the corner' will provide sufficient information to convey the shape even in the absence of surface shading.⁴

The corrugated cardboard object was designed to generate a set of contours that were evenly spaced, parallel lines, with no cues to suggest that sharp discontinuities (corrugation ridges) were present *except* for the variation in shading as the ridge was crossed. In principle, the variation in shading could be reduced to be imperceptible to the human observer, so that without the aid of machine-based photometric analysis, the human observer could not detect that the surface had corrugations. In the test presented here, the shading was kept obvious to provide visual verification

⁴Indeed, the modern engineer is particularly sensitized to recognizing the shape of a object from a 'wire frame' contour/profile representation that is typical of computer-aided design (CAD) systems

Figure 9.6: The cardboard "M" test object. The cardboard was creased along scores cut with a razor blade, and then it was stapled to a piece of plywood so that the corrugations were inclined 15° to the plywood. When photographed and digitized, the "M" was oriented at 45° with respect to the horizontal axis of the display. Shown in this figure are the labels {5,7,9} assigned to the moiré contours about the upper ridge used in the test.

of Task MFO's choice of shape.

Figure 9.6 shows a schematic of the cardboard test object. Compared to the elaborate fabrication of the NRC Standard Object (the painted sphere, described earlier), the cardboard test object is a rather primitive construction. The corrugated shape was formed by scoring the stiff (1.25 mm thick) cardboard with a razor blade along the ridge and groove lines so that the bends would make sharp creases. The nominal 15° inclination of the corrugated planes was established by stapling the cardboard along the central groove to a plywood sheet, and then arching the corrugation on each side to the desired inclination and stapling the tab at the end of the cardboard to the plywood as well. The gray side of the cardboard was faced outwards, to provide a reflectance more similar to that of the other test objects (painted wood, live human skin) than if the white surface were used.

Figure 9.7 shows the moiré topogram of the corrugated cardboard object. The topogram has been oriented in the figure with the same nominal 90° clockwise rotation that was used when digitizing the topogram (i.e., the figure orientation

Figure 9.7: Moiré topogram of the corrugated cardboard object. The topogram is shown rotated $\sim 90^\circ$ to the orientation from which it was digitized for photometric analysis. The cardboard test object was carefully positioned just behind the grid strings so that: 1) the surface shading varied with the tilt of the surface normal; 2) the moiré contours were parallel to the cardboard ridges; 3) a dark contour fringe was formed on each ridge.

matches the digital display of the topogram). The illumination is from the left, and the shading variation as each corrugation is crossed should be obvious to the reader. Another absolute cue to the shape comes from the rotation from vertical of the shadow of the vertical white fiducial string. Notice that the convex corrugation almost touches the grid plane at the intersection of the two white fiducial strings. Alignment of the object behind the moiré grid was difficult. The goal was to keep the ridges at $\sim 45^\circ$ to the grid strings, and to move the plywood sheet into a plane parallel to the grid plane so that the moiré fringes were evenly spaced straight lines and so that a dark fringe was formed on each ridge. This was realized on the upper-right ridge better than on the lower-left ridge.

Figure 9.8 shows the same area of the digital display under different viewing conditions. The photograph in part (a) has the contrast stretched to show the shading and shadow detail in the image. Just visible in the figure are the fine black vertical lines of the grid string shadows. Task MFO's shading measurement routine must deal with this form of structured noise, and its presence complicates the measurement process. Another artifact visible in the photograph is the modulation (due to overexposure) in the apparent thickness of the white horizontal fiducial string as it crosses bright and dark moiré fringes.

The photograph in part (b) of Figure 9.8 has the image contrast reduced to emphasize the display diagnostics produced by Task MFO. The diagnostic overlay was traced while Task MFO analyzed a neighbourhood that crosses the upper ridge on the cardboard test object. In the figure, 13 contours are traced (only the odd ones are labelled), and C_7 is the dark-fringe contour that lies on the ridge. The neighbourhood used in this test was $\langle \phi_5, \phi_6, \phi_7 | \phi_8, \phi_9 \rangle$, so that it was assumed that the contours leading up to the top of the ridge were of known ϕ -value, and the goal was to see if Task MFO could detect the change in the sign of the slope as the ridge was crossed. One can see in the figure a boundary curve B used to enclose the contours. This ensured that 'run-away' searches for gradient neighbours initiated from broken contours would be confined. The diagnostics show that this precaution was necessary as the two seed-points at each end of the ridge launched search vectors

(a)

(b)

Figure 9.8: Photometric analysis of a cardboard ridge. The two photographs show the same area of the digital display, at the same scale. (a) An enhanced (contrast stretched) version to illustrate the detail in the image. Clearly visible are several artifacts that survived digitization, such as the grid string shadows that appear as fine black vertical lines. (b) The display diagnostics produced by Task MFO for the analysis regions in the neighbourhood $(\phi_5, \phi_6, \phi_7 | \phi_8, \phi_9)$ that crosses the ridge at contour C_7 . Of the 13 seed-points along the ridge, only 9 resulted in a successful photometric analysis of the ordering cases.

```

[FOGETS.90] The following files are specified:

      Top Specification File = MSBFOTRYA.TOP
      Coordinate Transformation File = MMHCOEF.XYZ
      Moire Instrument Parameter File = OTALSS.XYZ
      Analysis Schedule File = MSBFOTRYA.SCH
      Topogram Contour Index File = MSBFOBEND.IDX

<fogett.36> Display-to-Object rotation = -89.66 degrees

#####fovect.90 reports:[]
Step 1: Seed contour is member #4 of Neighbourhood : 5 6 7 8 9
      Analysis contours with known fringe value: 6
      Analysis contours with unknown fringe value: 8
      Contours whose fringe value will change: 8 9
      File name of seed-points: MSBSPTT08.SPF
#####

REPORT FOR THIS NEIGHBOURHOOD ----- [FOPICK 35]

      13 = number of seed-points read from file.
      9 = number of successfully analyzed regions.

      case PHA-error contrast Case-wise Fringe Values

      1 4459.32 0.445 1.5 1.0 0.5 0.0 -0.5
      2 2020.79 0.083 <---closest 1.5 1.0 0.5 1.0 0.5
      3 2376.99 0.162 1.5 1.0 0.5 0.0 0.5
      4 1712.70 <---minimum 1.5 1.0 0.5 1.0 1.5
      ##### (!) (!) (!) (?) (?)
      (average: 0.230) (cntr> 5 6 7 8 9

      ..update from [FOPICK 55] (!=known & ?=unknown)

      Contour: 1 2 3 4 5 6 7 8 9 10 11 12 13 14
      Fringe: ? ? ? ? ! ! ! ! ? ? ? ? ?

```

Figure 9.9: Task MFO diagnostics for the corrugated cardboard test

that missed their targets (i.e., the neighbouring contours). The display diagnostics show that for the other 9 interior seed-points on the ridge, the photometric analysis for each region proceeded normally (i.e., localized bundles of search vectors, and a pair of small 'snow-balls' from measuring the reflectance at each analysis point in the region).

Figure 9.9 shows part of the analysis diagnostics that Task MFO sends to the operator console. This is from a minimal listing, with all the optional diagnostics switched off. Shown is a report from FOGETS as to the files specified for this test. Note that FOGETT reports the calculated rotation from display coordinates to object-space coordinates. FOVECT reports the parameters for each scheduling step (i.e., the

neighbourhood details), and for this test there was only one step in the schedule.

When all the seed-points have been tried, FOPICK lists some results while it updates the ϕ -values. It reports in this example, that of the 13 seed-points, only 9 were successfully analyzed (concurring with the display diagnostics in Fig. 9.8). It also lists for each ordering case the total photometric analysis (PMA) consistency-error, the contrast in the errors, and the actual ϕ -values assigned to each fringe for that trial ordering. The case with the minimum error is indicated (and is used to update the ϕ -values), and the next closest ordering case is also indicated. Below the table of values for the cases is the average contrast, a reminder as to the identity of the members of that neighbourhood (their map-labels), and whether they were knowns (!) or unknowns (?) during that step. FOPICK also dumps a 2-line chart of the map-list to indicate which contours are knowns/unknowns after that ordering step.

To aid in the interpretation of the ordering results, Figure 9.10 shows the gradient profile 'curve' (truly a piece-wise linear curve for this test object) that corresponds to each of the four trial orderings for the unknowns in the test neighbourhood ($\phi_5, \phi_6, \phi_7 \mid \phi_8, \phi_9$). Case #4 is the correct shape for the ridge. As mentioned earlier, the underlining of members in the neighbourhood sequence (ϕ_6 and ϕ_8 in this example) signifies that those members were used as the photometric analysis points in each region. In the figure, the labels of the knowns are punctuated with a '!' and the unknowns with a '?'. The profile curves are plotted against the actual contour elevation levels, and the small numbers for the ϕ -values indicate how near to the grid ($\phi_{\text{GRID}} = 0$) the cardboard object was positioned for this test ($\phi_7 = 0.5$ for Case #4 means that ridge was only 2.5 mm from the grid).

In Figure 9.10, small vectors are drawn to indicate the tilt of the surface normal at the analysis points for each ordering case. Note the wild variation of the surface normal from case-to-case for the unknown contour #8. This illustrates an earlier remark that the set of shapes under consideration generally have quite different expected shading values, and that only the correct shape will (on average) have expected shading values that are consistent with the measured shading value.

Figure 9.10: Profiles for the cardboard ridge trial ordering cases. There are four ways of ordering the unknowns in neighbourhood $(\phi_5, \phi_6, \phi_7 | \phi_8, \phi_9)$, and each way results in a gradient profile as depicted. Small vectors represent the surface normal at the two analysis points (one each on contours C_6 and C_8) for each case. The relative consistency-error ϵ_r (with respect to Case #4, the best case), and the error-contrast κ are given for each case. Notice that Case #1 introduces a non-physical profile ($\phi_9 = -0.5$ is before the grid).

Also shown for the k th case in Figure 9.10 is the *relative* consistency-error $\epsilon_r^{(k)}$, taken with respect to the total consistency-error of the m th (best) case:

$$\epsilon_r^{(k)} = \frac{\epsilon^{(k)}}{\epsilon^{(m)}}. \quad (9.2)$$

The error-contrast κ defined in Equation 9.1 (p. 324) is listed too. These measures indicate that Case #4 was strongly distinguished from the other cases by the photometric analysis, and represents the most likely ordering of the unknown contours.

9.4.2 The Painted Sphere

It has been mentioned several times that the painted sphere was a favourite test object in these studies. It provided a simple calibrated geometry, with shaded surfaces that were oriented in every direction, and it was a stable, fixed subject that facilitated repeatable trials. The photographing and subsequent digitizing of the sphere's moiré topogram image for analysis was described in Chapter 8 when discussing the experimental results for Task JFO. Recall that Figure 8.3 (p. 296) showed the moiré topogram of the sphere, and Figure 8.4 (p. 298) showed the enhanced display of the digitized topogram.

Figure 9.11 shows the test contours used by Task MFO for the ordering experiment on the painted sphere. There are 18 contours traced in this image, and for the human observer, the odd contours from 1 to 13 are visibly identified with annotation. When Task MFO analyzes the image, the contours are left as closed loops (i.e., not broken with annotation), as the software maintains the identity of every display pixel in its label field. Contour #1 is at the center of the sphere (closest to the grid plane), and the other contours are progressively further from the plane, with corresponding increasing ϕ -values. The actual ϕ -values are given in Table 9.1.

The schedule designed to analyze this set of contours had seven ordering steps. Figure 9.12 gives part of the analysis diagnostics for Step #4 that Task MFO sent to the operator console. When compared to the diagnostics for the cardboard test object (Fig. 9.9), one notes in this example that there are more seed-points, that

Figure 9.11: Test contours used by Task MFO for the painted sphere experiment. The contrast of the display image has not been stretched for this figure, so that the overlay of contour lines and labels are more clearly visible against the moiré fringe pattern. The annotation on the odd contours from 1 to 13 is for illustrative purposes only (Task MFO maintains the identification of every contour pixel in the label field of the pixel). Note that even with reduced contrast the specular highlight on the painted surface at the top of contour #2 is quite apparent

Table 9.1: Actual moiré fringe values for the painted sphere topogram in Figure 9.11

$\phi_1 = 15.5$	$\phi_7 = 18.5$	$\phi_{13} = 21.5$
$\phi_2 = 16.0$	$\phi_8 = 19.0$	$\phi_{14} = 22.0$
$\phi_3 = 16.5$	$\phi_9 = 19.5$	$\phi_{15} = 22.5$
$\phi_4 = 17.0$	$\phi_{10} = 20.0$	$\phi_{16} = 23.0$
$\phi_5 = 17.5$	$\phi_{11} = 20.5$	$\phi_{17} = 23.5$
$\phi_6 = 18.0$	$\phi_{12} = 21.0$	$\phi_{18} = 24.0$

```

#####[foveat 90 reports ]
Step 4: Seed contour is member 84 of Neighbourhood : 7 8 9 10 11
      Analysis contours with known fringe value:      8
      Analysis contours with unknown fringe value:    10
      Contours whose fringe value will change:        10 11
      File name of seed-points:                        BSPSPCC05.SPF
#####

REPORT FOR THIS NEIGHBOURHOOD ----- [FOPICK 35]

40 = number of seed-points read from file
40 = number of successfully analyzed regions.

case PHA-error contrast Case-wise Fringe Values
1 29345.64 0.927 18.5 19.0 19.5 19.0 18.5
2 18516.61 0.887 18.5 19.0 19.5 20.0 19.5
3 15402.28 0.866 <---closest 18.5 19.0 19.5 19.0 19.5
4 1108.55 <---minimum 18.5 19.0 19.5 20.0 20.5
      ##### (!) (!) (!) (?) (?)
      (average: 0.893) (cntr> 7 8 9 10 11

...update from [FOPICK.55] (!=known & ?=unknown)
Contour: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Fringe:  ! ! ! ! ! ! ! ! ! ! ? ? ? ? ? ? ?

```

Figure 9.12: Task MFO console diagnostics from the painted sphere test

Table 9.2: Experimental results for the painted sphere using Task MFO.

process step no.	neighbourhood contour sequence $(\phi_{n_1} \dots \phi_{n_N} \phi_{u_1} \dots \phi_{u_U})$	average contrast κ_{av}	best case fringe numbers ϕ 's	worst contrast κ_{min}	number of cases
0	$\phi_1 = 15.5$ $\phi_2 = 16.0$ $\phi_3 = 16.5$				
1	$(\phi_1, \phi_2, \phi_3 \phi_4, \phi_5)$	0.28	17.0, 17.5)	0.25	4
2	$(\phi_3, \phi_4, \phi_5 \phi_6, \phi_7)$	0.56	18.0, 18.5)	0.44	4
3	$(\phi_5, \phi_6, \phi_7 \phi_8, \phi_9)$	0.77	19.0, 19.5)	0.73	4
4	$(\phi_7, \phi_8, \phi_9 \phi_{10}, \phi_{11})$	0.89	20.0, 20.5)	0.87	4
5	$(\phi_9, \phi_{10}, \phi_{11} \phi_{12}, \phi_{13})$	0.79	21.0, 21.5)	0.75	4
6	$(\phi_{11}, \phi_{12}, \phi_{13} \phi_{14}, \phi_{15})$	0.92	22.0, 22.5)	0.91	4
7	$(\phi_{13}, \phi_{14}, \phi_{15} \phi_{16}, \phi_{17})$	0.93	23.0, 23.5)	0.92	4

higher error-contrasts are produced, and that the fringe values are larger. Closed-loop contours in these studies typically have several dozen or more seed-points to define them, which leads to a wide variety of positions and orientations about the surface of the test object. This variety influences the photometric error-contrast, as only the correct ordering case will produce consistent shading estimates at every position and orientation. Finally, the larger ϕ -values (e.g., 10 to 30) are typical for most test situations, as the object is usually kept well away from touching the delicate grid strings.

Table 9.2 gives the experimental results obtained with Task MFO for the painted sphere. In this test, the program was given initial values for ϕ_1 , ϕ_2 and ϕ_3 , (the most central contours in Fig. 9.11). For each step, the table shows the neighbourhood contour sequence that was used, the average error-contrast κ_{av} , the ϕ -values for the best ordering case, the error-contrast κ_{min} of the worst case (i.e., the rejected ordering case with the smallest error), and the number of ordering cases tested.

The rejected case with the smallest photometric-consistency error is termed the 'worst case', since its contrast κ_{min} is a measure of how well separated the selected

Table 9.3: Task MFO processing schedule for the painted sphere

fringe:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
MFE-type.	•	o	•	o	•	o	•	o	•	o	•	o	•	o	•	o	•
step 0.	!	!	!	?	?	?	?	?	?	?	?	?	?	?	?	?	?
1.	!	!	!	*	*												
2.			!	!	*	*											
3.				!	!	!	!	*	*								
4.							!	!	*	*							
5.									!	!	!	*	*				
6.											!	!	!	*	*		
7.													!	!	!	*	*

o = bright   • = dark   ! = known   ? = unknown   * = assigned

case was from the nearest rejected case. It represented the 'nearness' of a false-hit (i.e., choosing the wrong case).

There were no false-hits in this test. At every ordering step, the best case selected by Task MFO corresponded to the physically correct ϕ -values. For every ordering step, the contrast was convincingly strong, and it improved steadily as the analysis regions moved away from the specular behavior near the center of the sphere.

Table 9.3 shows the evolution of the state diagram for the map-list as Task MFO executed the schedule steps. From this table, one can appreciate the simple 'flow' of the ordering process from one end of the map-list to the other. The sphere possesses a topologically-simple set of contours (concentric circles), and the ordering schedule designed for this test was appropriately simple.

9.4.3 The Human Back

As mentioned in the previous chapter, the human back is a significant challenge to automated moiré topogram analysis. Good moiré fringe contrast is difficult to obtain on living skin. Patient motion during exposure blurs the fringes, and the

Table 9.4: Actual moiré fringe values for the human back topogram in Figure 9.13

$\phi_1=15.0$	$\phi_5=17.0$	$\phi_9=19.0$	$\phi_{13}=16.0$	$\phi_{17}=18.0$
$\phi_2=15.5$	$\phi_6=17.5$	$\phi_{10}=19.5$	$\phi_{14}=16.5$	$\phi_{18}=18.5$
$\phi_3=16.0$	$\phi_7=18.0$	$\phi_{11}=15.0$	$\phi_{15}=17.0$	$\phi_{19}=20.0$
$\phi_4=16.5$	$\phi_8=18.5$	$\phi_{12}=15.5$	$\phi_{16}=17.5$	$\phi_{20}=20.5$

contours can form quite a complex pattern of 'hilly terrain' to analyze.

The same digitized moiré topogram of a human back that was used in the tests for Task JFO, was also used to test Task MFO. Recall that the moiré topogram was shown in Figure 8.7 (p. 303), and that the display of the digitized topogram was shown in Figure 8.8 (p. 305). A more detailed set of contours was traced from this image for the testing of Task MFO, and they are shown in Fig. 9.13.

- There are 20 contours traced from both bright and dark moiré fringes in this image, and each is visibly identified with annotation in the figure. Contours #1 to #8 are labelled in order of descending elevation on the left scapula (i.e., increasing ϕ -values), contours #11 to #18 descend on the right scapula, and contours #9, #10, #19 and #20 surround both scapula hills, and continue the descent from the posterior to the anterior surface of the torso. The actual ϕ -values of these contours are given in Table 9.4.

An interesting aspect of this map is the topography, comprising two hills and a saddle-point. Several contour-ordering experiments were conducted with Task MFO using this set of contours. The first test schedule supplied some initial contour values at the top of the left scapula, and in 8 processing steps, Task MFO first ordered the remaining contours going down the left scapula, it then ordered those contours looping around both scapula, and finally it ordered the contours going up to the top of the right scapula. The branched flow of processing the map-list using this schedule is given in Table 9.5.

The specification of the scheduled neighbourhood sequences and the results at each step are summarized in Table 9.6. The table shows that initial values

Figure 9.13: Test contours used by Task MFO for the human back experiment. The contrast of the display image has not been stretched for this figure, so that the overlay of contour lines and labels are more clearly visible. There are 20 contours traced from both bright and dark moiré fringes, and in this photograph taken from the display, each contour is annotated with its label number. Contours #1 to #8 are labelled in descending order on the left scapula, contours #11 to #18 are labelled in descending order on the right scapula, and contours #9, #10, #19 and #20 loop around both scapula hills, and continue the descent from the posterior to the anterior surface of the torso.

Table 9.5: Task MFO processing schedule for the human back

fringe:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
MFE-type	o	•	o	•	o	•	o	•	o	•	o	•	o	•	o	•	o	•
step 0	!	!	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
1	!	!	*	*														
2		!	!	!	*	*												
3				!	!	!	*	*										
4						!	!	!	*	*								
5									!	!						*	*	*
6														*	*	!	!	!
7												*	*	!	!	!		
8										*	!	!	!					

o = bright   • = dark   ! = known   ? = unknown   * = assigned

Table 9.6: Experimental results for the human back using Task MFO

process step no.	neighbourhood contour sequence $\langle \phi_{n_1} \dots \phi_{n_N} \mid \phi_{u_1} \dots \phi_{u_U} \rangle$	average contrast κ_{av}	best case fringe numbers ϕ 's	worst contrast κ_{min}	number of cases
0	$\phi_1 = 15.0$ $\phi_2 = 15.5$				
1	$\langle \phi_1, \phi_2 \mid \phi_3, \phi_4, (\phi_5) \rangle$	0.62	$\mid 16.0, 16.5, 17.0 \rangle$	0.39	8
2	$\langle \phi_2, \phi_3, \phi_4 \mid \phi_5, \phi_6 \rangle$	0.86	$\mid 17.0, 17.5 \rangle$	0.84	4
3	$\langle \phi_4, \phi_5, \phi_6 \mid \phi_7, \phi_8 \rangle$	0.58	$\mid 18.0, 18.5 \rangle$	0.45	4
4	$\langle \phi_6, \phi_7, \phi_8 \mid \phi_9, \phi_{10} \rangle$	0.74	$\mid 19.0, 19.5 \rangle$	0.68	4
5	$\langle \phi_{10}, \phi_9 \mid \phi_{18}, \phi_{17}, \phi_{16} \rangle$	0.74	$\mid 18.5, 18.0, 17.5 \rangle$	0.42	8
6	$\langle \phi_{18}, \phi_{17}, \phi_{16} \mid \phi_{15}, \phi_{14} \rangle$	0.59	$\mid 17.0, 16.5 \rangle$	0.57	4
7	$\langle \phi_{16}, \phi_{15}, \phi_{14} \mid \phi_{13}, \phi_{12} \rangle$	0.81	$\mid 16.0, 15.5 \rangle$	0.77	4
8	$\langle \phi_{14}, \phi_{13}, \phi_{12} \mid \phi_{11}, \phi_{11} \rangle$	0.30	$\mid 15.0, 15.0 \rangle$	0.30	2

were given only for ϕ_1 and ϕ_2 , and that various lengths of contour sequences were used at each step. Each sequence is partitioned as usual into (knowns | unknowns), and the underscored members (ϕ 's) in each partition denote which members were used as photometric analysis points (i.e., where the shading measurements and estimates were formed). Of course, all the members play a role in each trial ordering case by defining the shape of the gradient profile from which the direction of the surface normal is derived.

Note that in Step #1, contour #5 was admitted to sequence of unknowns, but it was not assigned a fringe value until Step #2 (this was more a demonstration of the flexibility of the program than an indication that an unreliable assignment might have been made in Step #1.) Unknown contours like #5 that are not assigned fringe values at an ordering step are denoted by enclosing them in parentheses (ϕ)'s in the neighbourhood sequence.

For each step, Table 9.6 also gives the average error-contrast κ_{av} , the ϕ -values for the best ordering case, the error-contrast κ_{min} of the worst case, and the number of ordering cases tested.

Note in this experiment the number of ordering cases varied from 2 to 8. In Step #8 there were only two trial cases to consider since the two unknowns in the sequence were on the the same closed-loop contour (the gradient profile was drawn right across contour #11, cutting it twice) so that 'both' unknowns were forced to always have the same trial ϕ -value. Thus, there was only one ϕ -value involved to determine the number of trial ordering cases, whereas both unknown members were required to determine the shape of the gradient profile.

In this experiment, Task MFO selected the physically-correct ϕ -values (listed in Table 9.4) at every ordering step in the schedule. From the worst case contrasts in Table 9.6, one can conclude there was good discrimination against a false-hit (i.e., choosing the wrong ordering case). Notice that the contrasts degrade slightly about the tops of the scapula hills, suggesting that the simple diffuse photometric model used by Task MFO is sensitive to the specular component of human skin (the highlights on skin sometimes can be more pronounced than with a painted surface,

especially if the skin is oily).

9.4.4 The Bend Test Revisited

The most interesting challenge to automated ordering of moiré fringe contours is the detection of slope-sign reversal (i.e., the ability, when going uphill or downhill, to sense a change of slope direction). The experiment with the cardboard test object, reported in a previous section, addressed this issue, and showed that Task MFO could detect slope-sign reversal without any cues other than the variation in surface shading as the ridge was crossed. The cardboard test object was not the only item subjected to this type of experiment with Task MFO. Indeed, the first test was with the human back image from the previous section, and the initial test was a failure! The report follows below.

The human back topogram (see Fig. 8.7, p. 303) has an opportunity for a slope-sign reversal test (henceforth, simply 'bend test') between the two scapula 'hills'. The plan was to come down the left scapula with knowns, and cross the bend to the right scapula where all contours were to be unknowns. The correct outcome would be to start going 'up' the right scapula. Would Task MFO detect the change in slope sign, or would it keep going 'down'?

The parameters and diagnostics for the first bend test are shown in Figure 9.14. The neighbourhood sequence in this test is

$$\langle \phi_6, \phi_7, \phi_8 \mid \phi_{18}, \phi_{17}, \phi_{16} \rangle, \quad (9.3)$$

and five photometric analysis regions can be discerned in part (a) of the figure. In part (b) of the figure, the results for the 4 trial ordering cases are shown. From the list of actual fringe values in Table 9.4, one sees that the correct choice is Case #1 in Figure 9.14, but that Task MFO has wrongly selected Case #3.

The reason for Task MFO's confusion is shown in Figure 9.15. In this enhanced view of the test area, the white vertical fiducial string of the moiré instrument can be seen superimposed between contours #17 and #18. As indicated by the underscored members of the neighbourhood sequence in Equation 9.3, the shading measurements

```

#####[fovect.90 reports:]
Step 1: Seed contour is member #5 of Neighbourhood : 6 7 8 18 17 16
Analysis contours with known fringe value: 7
Analysis contours with unknown fringe value: 17
Contours whose fringe value will change: 18 17 16
File name of seed-points: ABCF00017.SPF
#####
REPORT FOR THIS NEIGHBOURHOOD ----- [FOPICK.35]

5 = number of seed-points read from file.
5 = number of successfully analyzed regions.

case PHA-error contrast Case-wise Fringe Values

1 594.97 0.638 17.5 18.0 18.5 18.5 18.0 17.5
2 193.96 0.041 <---closest 17.5 18.0 18.5 18.5 19.0 18.5
3 178.58 <---minimum 17.5 18.0 18.5 18.5 18.0 18.5
4 636.27 0.562 17.5 18.0 18.5 18.5 19.0 19.5
      mmmm      (!) (!) (!) (?) (?) (?)
      (average: 0.380) (cntr> 6 7 8 18 17 16

...update from [FOPICK.55] (!=known & ?=unknown)
Contour: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Fringe: ? ? ? ? ? ! ! ! ? ? ? ? ? ? ? ! ! ! ? ?

```

Figure 9.14: Task MFO bend analysis about contour #17 on the human back. (a) shows the display diagnostics from the photometric analysis at each of the five regions that traverse the bend. (b) shows the diagnostic summary from the operator console for this test.

Figure 9.15: Enhanced display of the bend-test area on the human back. In this photograph taken from the display, the contours have been traced with black lines and are annotated with black numerals so that they are visible against the bright moiré fringes. An important artifact in this image is the white vertical fiducial string that passes between contours #17 and #18. Attempts to measure the surface shading using patches that overlap this bright artifact will have erroneous results.

in the known partition were made along contour #7, and along contour #17 in the unknown partition. When Task MFO attempted to measure the shading on the surface of the object at points along contour #17, the bright image artifact caused erroneous values to be used (i.e., values that were artificially too bright compared to the expected value for the correct orientation of the object).

The flexibility of Task MFO allowed a different neighbourhood sequence to be used in a second bend test for this image:

$$\langle \phi_5, \phi_6, \phi_7, \phi_8 \mid \phi_{18}, \phi_{17}, \phi_{16}, \phi_{15} \rangle, \quad (9.4)$$

and the analysis points for the unknown partition were chosen to lie on contour #16, so as to avoid the problem of the systematic artifact. The display diagnostics for the new sequence are shown in part (a) of Figure 9.16, and one can see the sampling 'snowballs' that characterize the shading measurement process are now centered along contours #6 and #16. The report for the analysis is shown in part (b) of the figure, and of the 8 ordering cases associated with the neighbourhood sequence defined in Equation 9.4, Case #1 is still the correct case, and is the one selected by Task MFO. The average contrast is strong, indicating that the photometric ordering algorithm can indeed detect slope-sign reversal.

The astute reader will have noticed that the success of the second test is one based on 'the thinnest of margins': the contrast of the closest rejected case (the worst case!) was only 0.001 (i.e., negligible). The implication is that Cases #1 and #2 cannot be distinguished by photometric analysis. They are photometrically equivalent, and one could say that Case #2 is a 'photometric mimic' of Case #1. An explanation for this equivalence is found by examining the gradient profile curves associated with each of the trial fringe values for the two cases.

The gradient profiles are shown in Figure 9.17, and it is obvious how the two ordering cases can represent unique profile shapes with the same surface normal direction at the analysis points. Clearly, Case #2 represents an unlikely shape (at least, for a human back), and it could be rejected on grounds other than photometric (e.g., smoothness and arbitrary-position criteria).⁵ The problem of photometric

⁵The profile shown in Figure 9.17b has a *very* unlikely shape and is *not* arbitrarily positioned:

(a)

```

#####[fovect.90 reports.]
Step 1: Seed contour is member 86 of Neighbourhood : 5 6 7 8 18 17 16 15
Analysis contours with known fringe value: 6
Analysis contours with unknown fringe value: 16
Contours whose fringe value will change: 18 17 16 15
File name of seed-points: ABCF00017.SPF
#####

REPORT FOR THIS NEIGHBOURHOOD -----[FOPICK.35]

5 = number of seed-points read from file.
5 = number of successfully analyzed regions.

case PHA-error contrast Case-wise Fringe Values

1 11.83 <---minimum 17.0 17.5 18.0 18.5 18.5 18.0 17.5 17.0
2 11.85 0.001 <---closest 17.0 17.5 18.0 18.5 18.5 19.0 18.5 18.0
3 157.76 0.861 17.0 17.5 18.0 18.5 18.5 18.0 18.5 18.0
4 145.86 0.850 17.0 17.5 18.0 18.5 18.5 19.0 19.5 19.0
5 174.90 0.873 17.0 17.5 18.0 18.5 18.5 18.0 17.5 18.0
6 162.16 0.864 17.0 17.5 18.0 18.5 18.5 19.0 18.5 19.0
7 18.42 0.218 17.0 17.5 18.0 18.5 18.5 18.0 18.5 19.0
8 14.76 0.110 17.0 17.5 18.0 18.5 18.5 19.0 19.5 20.0
          =====
          (average: 0.540) (cntr> 5 6 7 8 18 17 16 15
                                (!) (!) (!) (!) (?) (?) (?) (?)

...update from [FOPICK.55] (!=known & ?=unknown)
Contour: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Fringe: ? ? ? ? ! ! ! ! ? ? ? ? ? ? ! ! ! ! ? ?

```

(b)

Figure 9.16: Task MFO bend analysis about contour #16 on the human back (a) shows the display diagnostics from the photometric analysis at each of the five regions that traverse the bend. (b) shows the diagnostic summary from the operator console for this test.

Figure 9.17: Photometrically-similar profiles from the Task MFO bend analysis. (a) and (b) are the gradient profiles drawn through the trial fringe values for ordering cases #1 and #2, respectively, in Figure 9.16. Although the two profiles have distinct differences, they each produce nearly-identical values for the surface normal at the analysis points on contours #6 and #16, and thus are photometrically equivalent.

mimics is linked to the number of unknowns in the neighbourhood sequence, and will be discussed further in the conclusions. Briefly, however, one can say that short sequences will avoid the problem.

9.5 Discussion

This section presents a discussion on the performance of Task MFO. The test results are reviewed in terms of Task MFO's usefulness as an analytic tool for investigating photometric ordering. The effect of image artifacts and 'photometric mimics' on the robustness of the ordering process is examined. Finally, the prospects for automating the generation of Task MFO's schedule file are discussed, along with other possible extensions to the program.

9.5.1 General Comments

Task MFO proved to be very useful in probing the nature of the photometric ordering algorithm when applied to a variety of surface topographies that ranged from sharp ridges to hilly terrain, and with surface reflectances that included painted wood, gray cardboard and live human skin. Its conceptual design and its implementation in software allow most of the parameters of the algorithm to be independently studied by virtue of its externally specified ordering schedule. By permitting the use of mixed or matched contour modes, flexible-length neighbourhood sequences, specifiable roles for sequence members and specifiable regional seed-points, Task MFO can be used to order an entire map-list of contours, or to carefully examine the photometric region about a single seed-point. A good example of its flexibility for isolating and analyzing problem situations was the series of bend tests reported in the previous section.

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the dip between contours #8 and #18 must fall exactly halfway between elevation 18.5 and 19.0, and the sharp ridges shown at contours #18 and #17 must each fall exactly at elevations 18.5 and 19.0 respectively, otherwise different moiré fringe patterns will be generated by the proposed profile.

9.5.2 Robustness of Photometric Ordering

In general, the results of the experiments with Task MFO support the claim that photometric ordering of moiré fringe contours is robust. The overall high contrast values in the experimental results for each of the test objects illustrates this point. The claim of robustness is qualified by the following remarks.

First, for photometric ordering to be reliable, a good distribution of seed-points about the contours must be used to minimize the disruption to the model by localized image artifacts (*e.g.*, specular reflectance components, obstruction of the surface by fiducial strings, etc.). As implemented here, contour ordering involves picking one case from a few choices in an unconditional fashion. A decision may be weakened by the reduced error-contrast that results from one or more regions with spoiled models, but once the choice is made, the next step in the schedule is executed independently of whether the choice was based on high or low error-contrasts. Averaging the consistency-error over many seed-points reduces the inconsistent influence a region with a spoiled photometric model can have on selecting the correct case. An example with an extended image artifact that spoiled the modelling for all the seed-points was the experiment shown in Figure 9.14.

A second consideration to the robustness of the method is the concept of the 'photometric mimics'. Photometric mimics are trial ordering cases that have distinctly different gradient profiles, but lead to the same surface normals at the selected analysis points, so that they cannot be distinguished by photometric modelling computations. The concept of the mimic arose from the experiments with Task MFO (recall Fig. 9.17), and it is one that is characteristic of long neighbourhood sequences. Photometric mimics arise when there are 'many' unknowns in the sequence, and thus there are many trial ordering cases (with their associated gradient profiles) to consider. With so many degrees of freedom to define the shape of the profile, it is likely that some of the cases will have photometrically-similar profile shapes about the selected analysis points. The fewer the unknowns, the less likely mimics will be a problem. As was shown with Task JFO in the previous chapter, photometric ordering can be performed very effectively with short sequences

involving a single unknown each time (thereby precluding the possibility of a mimic).

A third consideration to the robustness of the method is the use of the Lambertian (purely-diffuse) reflectance model to analyze physical objects with marked specular characteristics. The Lambertian model is computationally trivial (a vector scalar-product), which is a strong merit regarding program efficiency. Compared to the complexity of other models that account for specular reflectances, the simplicity of the Lambertian model allows some latitude in a computational budget for testing to ensure its validity at each surface point. Specularity is a systematic artifact, and one that can be economically (*i.e.*, easily) tested for. As mentioned previously in Section 4.9.2 (p. 165), a surface point has potential specularity when the bisector H of the illumination direction L and the viewing direction V aligns with the local surface normal N . One can simply test

$$0 \leq N \cdot H < \epsilon_s \leq 1, \quad (9.5)$$

where ϵ_s is the threshold beyond which the surface point is too specular, and regions exceeding the threshold can be excluded from the accumulated consistency-errors of the remaining (sufficiently-nonspecular) regions. Exclusion of specular points would improve the contrast. The value of ϵ_s depends on the glossiness of the surface, and the author found that essentially all of the specular points on the test objects used in this study would have been excluded by setting $\epsilon_s = 0.95$ (this corresponds to the bisector aligning to within 18° of the surface normal). In the tests reported here, only a few points on a few contours would have to be excluded to realize improved contrasts. Task MFO did not exclude specular points, and relied on averaging over many nonspecular regions to suppress specularity effects.

9.5.3 Schedule Considerations

Manually producing a schedule to control the execution of Task MFO allowed full control over the ordering process, as noted above. Picking the parameters for each step was not difficult, but it was tedious. It required recognizing sequences of neighbours with partitioned knowns and unknowns, and arranging the steps in the

schedule so that all the required knowns at any given step had been assigned ϕ -values at previous steps. This same process had been crudely automated within Task JFO, so the mechanisms for automatically generating the schedule had been developed and demonstrated there. Indeed, the real challenge with Task MFO's schedule was to manually type the long string of parameter values for each control vector into the schedule file without syntax errors. Automation of the process was under development when work on this project was terminated.

9.5.4 Extensions to Task MFO

The few experiments presented in this chapter were completed in the short time between the completion of Task MFO and the termination of support for this project. The tests demonstrate some of the features of Task MFO, but the potential for continued research with this program and with Task JFO is enormous.

It is not likely that the investigative features of Task MFO would have been incorporated into production software. The verbose diagnostics and flexibility in execution have a high processing overhead⁶ that is not justified in the routine moiré contour ordering that might typify a spinal analysis clinic, or an industrial quality-control station.

Nonetheless, some extensions to Task MFO were planned. As mentioned, the schedule file generation was to be automated. The program had provisions to handle most of the parameters of the general algorithm, although some sections of the code were not fully developed. In particular, the author wanted to allow for a specular photometric model option, so that metallic and glossy-plastic objects could be investigated. Another planned extension would have allowed for optional moiré instrument geometries, such as multiple-lamp illuminators or projection-moiré instruments.

The subject of extensions to this research is pursued in detail in the next chapter, but a brief comment on program development is relevant here. Other refinements to

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⁶Despite this overhead, Task MFO ordered maps almost as fast as its streamlined cousin, Task JFO (e.g., a few minutes for the human back map shown in Fig. 8.7).

Tasks MFO and JFO could be implemented, but these were inherently exploratory programs with limited scope for modification. Instead, the author planned a third generation program that was to be fast, robust and efficient, with a design based on the experiences with the forerunner programs. The new task would have the simplicity of Task JFO and some of the flexibility of Task MFO. It would tend to use short sequences with one unknown contour to avoid mimics, it would eliminate specular points to improve the robustness, and it would generate its own efficient schedule.

Chapter 10

Discussion and Recommendations

10.1 Introduction

This chapter summarizes the results presented in the thesis and makes recommendations for future work. The thesis is organized such that the detailed discussion for particular findings is within each chapter, usually as its conclusion. The goal here is to reflect on the overall significance of the results with respect to the objectives listed in Chapter 1.

10.2 Summary of the Results

The natural shading of the object due to point-illumination in a moiré topogram has been noticed by others, and some have even proposed lighting methods that eliminate the shading. It occurred to the author, however, that this inherent shading might be useful in ordering the contours, since the human observer uses shading as a cue to guess the shape of an object. The author studied various methods invented by others to order moiré fringe contours (1,¹ Ch. 2), and was impressed by the ingenious shadow-grids that produced fringes with asymmetric intensity profiles. The author came to realize that the shading on the object rendered even ordinary moiré fringes

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¹Underlined numerals refer to the objectives of the thesis listed in Chapter 1.

with an asymmetric intensity profile. He hypothesized that the shape associated with each possible ordering of the contours would have an expected shading value at each point, but that only one would consistently match the measured shading at that point. This became the basis of *photometric ordering*, which takes its name from the *photometric model*, used to relate the reflectance or shading of an object with its position and orientation with respect to a light source and the viewing position.

In the initial stages of the research, tests were done to confirm that there was a relationship between shape and shading in the moiré topogram (2, Ch. 4). The author was convinced that the shading could be automatically measured, and that the measurement of the shape associated with each test-ordering of a sequence of contours could also be automated. Here lay the prospects and the challenge for automated ordering of ordinary moiré fringes.

The Photometric Ordering Algorithm was developed to cast these ideas into a formal statement of the solution (3, Ch. 3), and although it specified the required calculations, it did not suggest how the measurement of attributes within the topogram image were to be taken. Digital image processing techniques were tried, and further design studies satisfied the author that shading on the object could be measured 'between the fringes', that shape on the object could be measured from the curvature and separation of the fringes in the topogram, and finally that a simple Lambertian photometric model could be used to relate the two. The model worked well for a broad range of materials, such as gray cardboard, painted wood and live human skin (4, Ch. 4).

Design studies were conducted to learn about the shape and shading requirements for photometric fringe ordering (5, Ch. 4). The role of the illumination in recording the topogram and in scanning (digitizing) the image was carefully investigated. Reasonable care was needed to work within the linear-response range of the photographic emulsion and the detector in the image digitizer. Also checked was the accuracy of shape measurement from the fringe pattern. A calibrated moiré instrument geometry, and in particular, calibrated fiducial targets for image scaling,

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MicroD

provided the photogrammetric control required to accurately recover the encoded shape from the moiré fringe pattern.

Three sub-tasks were identified that required automating, and they were treated as separable problems with interdependent data. The solutions to these problems would provide the core processes upon which automated photometric ordering could be based. The first to be solved was 'gradient neighbour discovery' (6, Ch. 5), and it required searching the digitized topogram image for points that were on the neighbouring contours to a given point. An added constraint was that the points also had to lie on the same surface gradient line, hence they were *gradient neighbours*. The second task was to measure the local shape at a test point by analyzing the curves in the fringe pattern (7, Ch. 6). An algorithm was invented to generate the list of possible orderings of unknown contours about the point, and to use the curvature and separation of the local contour curves to reconstruct the surface shape for each test case. The third task required measuring the shading on the object at a test point in the presence of the moiré fringe pattern (8, Ch. 7). The individual pixel intensity values were too noisy, therefore averaging over a sampling patch was necessary. However, the variable spacing of the fringes required the sampling patch to be dynamically adjusted to fit between the fringes, and an algorithm was developed that used circular patches with the correct properties. For all three sub-tasks, the invented algorithms worked with, or generated, specified data structures that were compatible with the overall task (fully-automated moiré topogram analysis) and with the DIP hardware used by the author. In all three cases, the methods and the data structures developed by the author could be transported to other DIP systems.

Two major computer programs were written. In both programs, the test shapes represented possible orderings of unknown contours, and the shape with the smallest photometric error was selected as being the 'correct' ordering. The absolute error, normalized by the number of test points along the contour, could be used as a measure of the quality of the modelling process (12). The relative error between the test cases provided a measure of the confidence in the correctness of the selected case over the other cases (12).

The first program (Task JFO) was designed to demonstrate fully-automated ordering of ordinary moiré fringes (2, Ch. 8). This program embodies in streamlined code all the elements of the Photometric Ordering Algorithm, using a simple Lambertian photometric model and the minimum length of neighbourhood contour sequence (three neighbours, with one unknown).

The second-generation program (Task MFO) was designed as an investigative tool to study the properties of the Photometric Ordering Algorithm (10, Ch. 9). It has considerable flexibility in adjusting the parameters and conditions of the test runs, and allows one to vary the way and the sequence in which a set of contours was processed. This allows the strengths and weaknesses of the photometric method and its embodiment in software to be scrutinized, so that optimizations can be discovered and introduced into third-generation software.

In both programs, tests were performed to demonstrate that the photometric ordering method was robust and provided good discrimination between the correct ordering and the other test cases (11). The algorithms were invoked in efficient code, but the diagnostics I/O in both programs slowed down the processing. Most of the processing involved manipulating the data in the digital image memory (searching for gradient neighbours and measuring the shading in the sampling patch), with very few computations needed to evaluate the photometric model (one vector inner-product) and to determine the error between the expected and measured shading value. Much could have been done to speed up the execution, but there was no opportunity to conduct such tests.

The *photometric mimic* is an artifact discovered from the tests in Chapter 9. Its discovery was due to Task MFO's ability to order long sequences of neighbouring contours. The mimic is a test shape that is physically unlikely, but is photometrically indistinguishable from the correct shape. Small errors in the shading comparison between the two cases may lead the Photometric Ordering Algorithm to choose the mimic over the correct shape. Other tests can be used to disqualify the mimic, such as on the grounds of its unlikely shape. To avoid mimics, one should use neighbourhood sequences with minimum length (*i.e.*, three members), as does

Task JFO.

One outstanding weakness of the Photometric Ordering Algorithm is that it requires a pair of contours with known order to start the process. The author tried *endogenous* shape determination (Ch. 4), but the experiments failed because the moiré-instrument illumination was not sufficiently isotropic to support the photometric model over a large enough analysis region (5, Ch. 4). At that point, the author was still trying to prove whether or not photometric ordering was even feasible, and the fact that 'seed' contours were needed to start the photometric ordering process was dismissed as a minor criticism. If necessary, it was reasonable to use the Fiducial Shadow Method (Ch. 2) to provide the seeds, and the author was prepared to use DIP techniques to eventually automate this pattern recognition task. However, the author presents a new idea in the next section that may overcome this weakness without resorting to detecting fiducial shadows in the image.

One outstanding strength of the Photometric Ordering Algorithm is that it does not require that the contours be complete or unbroken. Sometimes, traced contours cannot be closed because they run off the edge of the field of view, or there are topographic discontinuities, such as skin folds on a human subject. Automated fringe tracing often produces maps with broken contours, and a separate expert process must attempt to repair the breaks. The tracing algorithm may fail because of artifacts in the digital images (such as reflectance changes due to surface defects), or it may become confused by abrupt changes in the expected direction of the fringe curve. The photometric ordering method can be made to tolerate broken contours by performing the analysis only where the contours are locally continuous. It would be relatively simple to build safe-guards into the software to sense when a search for a gradient neighbour has 'wandered too far', that is, the search path has passed through a local break in the neighbouring contour. The strength of the photometric ordering method is that it can be applied at as many points along the contour as needed to establish confidence in the ordering, and that only the most favourable points need to be included. This means that points near discovered breaks can be excluded (if the break prevents the gradient neighbour point from being properly

identified). It also means that an *expert* photometric algorithm could elect to use only those points that provide the highest expected discrimination between the test cases, and that are away from localized problems such as contour breaks and regions of high specularity. This would ensure the photometric ordering process was optimally robust. Furthermore, after ordering all the long contour segments, one could use the photometric method in a second pass to order any small fragments of contours that span two breaks, and thus help the contour-repair algorithm with its task of linking together fragments of the same contour.

10.3 Recommendations for Future Development

With regard to the theory of photometric ordering, one concept that warrants further investigation is *endogenous ordering* (Chs. 3 and 4). Recall that endogenous shape determination for a sequence of unknown contours does not depend on knowing the ordering of other contours. In its current form, the Photometric Ordering Algorithm is a propagated solution, and requires a known pair of contours to start the ordering process. The earlier comment was that the Fiducial Shadow Method (Ch. 2) could provide the seed contours. It would be better if the photometric ordering method could be rendered completely independent of any extra encoding, such as a fiducial shadow. The author thinks this is possible, and recommends the following approach.

Consider a sequence three neighbouring contours $\{C_A, C_B, C_C\}$ with unknown ordering. Let pair $\{C_A, C_B\}$ serve as the knowns (even if it is a guess) and pair $\{C_B, C_C\}$ be the unknowns in the usual photometric ordering method. Usually, when given $\{C_A, C_B\}$, then the ordering of $\{C_B, C_C\}$ is chosen as the shape with the most consistent shading correlation. The consistency is measured as the error between the expected and measured shading value for various test points along the contours. Since $\{C_A, C_B\}$ is not known, then all three possible orderings of $\{C_A, C_B\}$ must be tried. For each guess, there is a choice for $\{C_B, C_C\}$ that is the most photometrically consistent. One of them has the lowest overall error, and is most likely to be the

correct ordering for $\{C_A, C_B, C_C\}$. If the confidence in this outcome is high, then propagated ordering can proceed from these ordered contours. If not, then other combinations of contours can be tested. A low confidence may occur if the selected contours happen to generate a photometric mimic.

With regard to mimics, one has to remember that the photometric method relies on the contours looping about the topogram image so that different points along the contour take on very different orientations with respect to the light and to the observer. In this way, the shapes corresponding to two trial orderings may coincidentally reflect the same amount of light to the observer at one position along the contour (i.e., they are photometrically indistinguishable there), but it is unlikely to happen everywhere along the contour. Therefore, the distinguishability between test cases may be poor at one point, but on average, only the correct ordering will give the most consistent correlation with the measured shading values everywhere along the contour. Usually, a mimic will confuse the algorithm only at one orientation, and 'around the bend of the contour', the situation is resolved.

If the author had been permitted to continue with this research, the next phase (13) would have been to merge the best parts of Tasks JEO and MFO into a fast, robust, efficient third-generation program. The new program would have the simplicity of Task JFO and some of the flexibility of Task MEO. It would tend to use short sequences with one unknown contour to avoid mimics, it would eliminate specular points to improve robustness, and it would generate its own efficient schedule. The discussions at the end of Chapters 8 and 9 give details on avoiding specular points and photometric mimics to improve the robustness of the ordering, and the new program would incorporate those thoughts.

Task JFO has an automated, but inefficient scheduler. It exhaustively re-scans the map-list of contours, looking for any neighbourhood sequences with one-and-only-one unknown. The map-list does not provide the identity of neighbours to a contour, so the program must make a tedious search for neighbours by roaming about the topogram image. Image searches are slow operations, and Task JFO repeats searches needlessly because it discards all neighbours it finds that must be

ordered at a later stage.

The new program, however, could fill a table with all the neighbourhood sequences it finds on the first sweep through the map-list. This table of contour relationships could be used by the scheduler, in conjunction with the map-list of ϕ -values, to optimally schedule the ordering steps for a map with a minimum of image searching operations. The table could retain not only the identity of the contours, but also the coordinates of the discovered gradient neighbour points along the contours; this would ensure all searching was done only once, but it would place heavier demands on the program memory requirements. Task MFO's manual scheduler also works in this manner, in that the operator manually plans the schedule so that searching is done only once, and only as the contours are being ordered.

The author did his work using an expensive multi-workstation DIP facility, but he would recommend that others consider implementing the software on smaller machines that support the necessary DIP peripherals, such as the IBM PC/AT or IBM PS/2 computers. The minimum hardware configuration to equip a self-contained workstation for moiré topogram analysis would consist of a personal computer plus a video frame-grabber board to capture, process and display the digital image, with a solid-state video camera for digitizing topograms, and a monitor to display the digital images. For the monitoring and diagnosis of spinal deformity in a clinic that processes a few dozen patients per day, this configuration suits the budget and staff resources available at a typical hospital or private clinic. For municipal, provincial or national school-screening programs for the early detection of scoliosis (Ch. 1), a central processing facility with DIP systems that can handle a large volume of imagery would be more cost-effective.

10.4 Concluding Remarks

Moiré topography is a versatile optical method for measuring the shape of objects. Topographic contours based on ordinary moiré fringes can be easily and inexpensively produced with great accuracy, but in the past have suffered from a

'hill-or-valley' ambiguity that hampers their ordering. The photometric ordering method described in this thesis provides a means for automating the assignment of elevations to ordinary moiré fringe contours. This new method, when combined with automated fringe tracing techniques, forms the basis for fully-automated shape measurement of objects from images encoded with ordinary moiré fringes that are contours of elevation.

Appendix A

Properties of Topographic Moiré Fringes

This appendix presents enough detail on the basic properties of topographic moiré fringes so that a non-expert can grasp the essential concepts presented in the thesis. The topics include some historical background, definitions and terminology, an analysis of the intensity distribution of the fringes, their topographic interpretation, fringe elevation and interval formulas, fringe visibility and surface slope limits, and the effect of grid phase-shifting on the moiré fringe pattern. A more complete tutorial on moiré topography is in [Pekelsky and van Wijk, 1988].

A.1 The Moiré Phenomenon

When two similar patterns of opaque lines on separate sheets of transparent material are superimposed, dark fringes may be observed when the sheets are viewed against a light background. The dark portion of the fringes is caused where the opaque area of one pattern overlaps the transparent or opaque area of the other and thus blocks out the light. The bright zones between the dark fringes result when the transparent parts of the two patterns overlap each other. Figure A.1 shows a simple example of two superimposed line patterns and the interference or *moiré fringes* that result. Small differences in the pitch and the orientation of the two line patterns

Figure A.1; Moiré fringes obtained by superimposing two patterns. In this example, rather coarse line patterns have been selected to make clearly visible the intersection of the discrete lines in each pattern. The moiré fringes are the zig-zag 'bands' running vertically in the figure; for finer line patterns the fringes appear more continuous.

will affect the shape and spacing of the fringes [Bryngdahl, 1974 and 1975]. The interference fringes can be analyzed to derive information on the differences between the two superimposed line patterns, or the differences in their relative position and orientation.

A.1.1 Origins of the Terminology

The word *moiré* means *a watered or wavy appearance* [Gove, 1976]. The term evolved from the archaic French *mouaire* (c.1650), which itself is believed to be originally derived from the English *mohair*, the wavy fleece of the Angora goat [Robert, 1970]. The ancient Chinese developed a technique for pressing two layers of silk together to form wavy patterns on the fabric, and the popularity of *moiré silk* imported from the Orient by European spice and silk traders probably accounts for the adoption of the term into English usage.

As an optical phenomenon, moiré fringes were noted in the scientific literature more than 100 years ago, but the early authors referred to them as "watery or wavy patterns". Indeed, the earliest scientific application of moiré phenomena of which this author is aware, was described by Lord Rayleigh [1874], who suggested testing the quality of replicated diffraction gratings by superimposing a pair of gratings to

produce "bar patterns" whose geometry may be analyzed in terms of the geometry of the gratings.

At the turn of the century, many scientists were analyzing optical fringe patterns arising from a variety of effects, and introduced such terms as Brewster bands, Haidinger rings, Newton rings, or just "interference fringes" into their writings. Barus [1900] typified the situation:

"It has often been a matter of surprise to me that the shadow bands observed, for instance, on looking through one distant picket fence at another, are so seldom referred to in the literature of physics..."

and for want of a better term, he himself referred to the "well-known waterlines" in describing a moiré phenomenon. Moiré fringes are commonplace in an industrialized society, where structures with regular lines are preferred over random constructions (e.g., brick walls versus straw huts).

It seems Mulot [1925] introduced the word *moiré* for the first time as a scientific metaphor in a French article (concerning stress analysis of loading of mica sheets), that spoke of "des franges de moiré", or literally *watery fringes*, which survived translation into English in *Science Abstracts* as *Application of the "Moiré" to the Study of Mica Deformations*. The emphasis by the translator suggests the term was alien; the capitalization was perhaps unfortunate, as many today still think Moiré is a surname and continue to capitalize it. The confusion can go full circle, as French authors may read of the "moiré method" in English journals, and then use a literal translation in their papers; for example "*Une 'méthode des franges' en temps réel* ...." was one recent title [Dessus et al., 1975].

A.1.2 Early Research on Moiré Phenomena

Special credit must be given to Roger [1980], who prepared an exhaustive review of the scientific and engineering applications of moiré techniques, with over 200 references spanning the period from Lord Rayleigh to the late 1970's. The author's own literature search uncovered only a few early works not included in Roger's

bibliography. The next few paragraphs can only briefly mention the fields of activity detailed in the sixty pages of Roger's review.

As mentioned in the previous section, at the turn of this century, scientists were studying various phenomena that produced optical fringe patterns. A few papers were directed to moiré effects over the years, but in the 1960's, moiré patterns became fashionable for the modelling of beat phenomena that occur when wave-fronts interfere. Analogies were drawn between moiré patterns formed by certain grids and the solutions to problems in hydrodynamics, electrostatics, crystallography and other fields [Oster and Nishijima, 1963]. Even a hobby-kit (c.1964) for studying the science of moiré patterns was available from Edmund Scientific Co. (Cat.#9068), but it was out of production when the author inquired in 1980. For a well-researched bibliography, see [Oster, 1965].

From the pre-1920's to the present day, moiré effects were studied in the printing industry as troublesome artifacts that arise from the use of half-tone screens [Lees, 1919]. With the introduction of multiple screens for colour separation, the artifacts are more difficult to control [Tollenaar, 1945]. In the mid-1930's, the interaction of patterns reproduced on television screens with the raster lines was an emerging engineering issue [Mertz and Gray, 1934]. Anti-aliasing is still an important area of research in display engineering.

After 1945, rapid progress was made in the application of moiré techniques in materials research to the measurement of stress and strain. The basic method used to evaluate *in-plane* processes is quite simple: a grid (called the *specimen grid*) is printed on the surface of the test piece, which is then subject to loading. The loaded object deforms, and so does the specimen grid. When the specimen grid is optically or photographically superimposed with a *reference grid*, moiré fringes are formed, which can be interpreted in terms of the deformation. A detailed review of applications of this moiré technique in the field of engineering metrology is in [Theocaris, 1962]. An extension to this approach, called *reflection moiré*, has been used to study the deformation of mirrored surfaces. Instead of a material grid formed on the surface, the specimen grid is a relief-distorted reflection of the refer-

ence grid, and the superposition of the grid and its distorted reflection creates moiré patterns that allow the deformation to be measured [Chiang, 1978]. In the last ten years, moiré methods in materials research have developed into very sophisticated and sensitive investigative tools, and the reader is directed to [Reid, 1984] for a recent review of topics that include the determination of horizontality, angularity, deformation, vibration, refractive index, etc.

Moiré encoders for machine tool control and other position-measurement tasks became practical in the 1950's. The theory of moiré encoders had been known for some time, but cost-effective methods for making high-quality high-resolution grids (called *encoder scales*) were needed [Burch, 1963]. Most modern numerically-controlled machine tools rely on linear and rotary moiré encoders to monitor tool and workpiece positions.

The author agrees with Roger, that Mulot [1925] was probably the first to intentionally create moiré patterns for scientific measurement purposes. His application concerned the measurement of out-of-plane deflections of thin sheets of mica under load. The surface to be investigated was placed under a grid, and moiré fringes were formed by the interference of this grid and its shadow, cast on the surface by a collimated light. It was shown that the fringes represented planes parallel to the grid and could be used to calculate the deflections.

Mulot's work was overlooked, and his moiré method for measuring surface relief was independently invented by five other groups in the period 1948-1966. Developments in this period introduced several important methods for measuring surface curvature, slope and flatness, but they required collimated illumination and fine grids, and so had shallow working-depths and lens-aperture-limited fields of view. The breakthrough in moiré topography came with the *shadow-moiré method* described by Pirroda [1969] that allowed a point light-source to cast shadows of a large coarse grid, with the object viewed through the grid from a position directly in front of the object. This method was also invented by Takasaki [1970], Meadows *et al.* [1970] and Dykes [1971], and is the subject of the remainder of this appendix.

Figure A.2: The basic arrangement for shadow-moiré topography. A light source at point L illuminates an object positioned under the grid, such that shadows of the grid lines are cast on the object. Moiré fringes are formed when the distorted-grid shadow pattern on the object is viewed through the grid from point E . When the illumination and viewing points are at the same distance from the grid, the fringes lie in planes parallel to the grid plane. Note that for finite l , the fringe planes are not equally spaced [Takasaki, 1979b]

A.2 Moiré Topography

The application of moiré techniques to the measurement and display of an object's three-dimensional form is referred to as *moiré topography*. Two techniques are commonly applied:

- **Shadow-moiré Topography:** In this method, a grid is placed close to the surface to be evaluated, as can be seen in Figure A.2. A shadow of the grid is cast on the object by a light source. The shadow pattern is distorted as a result of the three-dimensional shape of the object. Moiré fringes are generated when the shadow pattern is observed or recorded through the same grid.

Figure A.3: The basic arrangement for projection-moiré topography. Grid G_1 is projected through L_1 to form a distorted-grid shadow pattern on the object, which is projected back through L_2 onto grid G_2 , where it interferes to form the moiré fringe pattern. When grids G_1, G_2 with identical pitch are coplanar, and are projected by lenses L_1, L_2 of matched focal length and with nodal points in a plane parallel to the grid plane, then the moiré fringes lie in planes parallel to the reference (image) plane of the grids. Note that for finite l , the fringe planes are not equally spaced [Takasaki, 1979b]

- **Projection-moiré Topography:** In this method, the shadow pattern of a grid on the object is obtained by projecting the image of a grid onto the object, as shown in Figure A.3. The distorted-grid image is photographed together with a separate grid and the moiré pattern is formed in the image plane of the recording camera as the result of the superposition of the two grid images.

The formation of *contours of elevation* by arranging the light source and viewing point to be the same distance from the grid is a key feature of moiré topography, as it leads to an encoding of the 3-D information that is particularly easy to interpret.

A.2.1 Three-Dimensional Information from a Single Image

The geometry of both the shadow- and projection-moiré techniques is closely related to stereo-photogrammetry in the sense that the image contains three-dimensional

information defined by the projective geometry of two projection centers. In the case of the shadow-moiré technique, these projection centers are the light source and the camera lens or the observer's eye (L and E , respectively in Figure A.2). For the projection-moiré technique they are represented by the lens of the projector, used for projecting the grid on the object, and the camera lens (L_1 and L_2 , respectively in Figure A.3). What sets moiré topography apart from the usual stereo methods is the special coding with shadows that allows the moiré topogram to contain the perspective information for *both* projection centers in a single image.

Moiré fringes encode the perspective geometry using structured light. The distance in depth between the fringes depends only on the relative position of the light source, the camera, the grid. These positions determine the photogrammetric *base-to-height ratio*, which affects the resolution and the accuracy of the depth information derived from moiré images. Also, since the moiré image is a central-perspective representation of the object, then a different scale factor applies to each fringe, depending upon its distance from the camera. Furthermore, the interval between successive fringes is not constant, but increases with the distance from the camera. Corrections for these geometric properties are relatively easy to apply, and formulas for these relationships are derived later in this appendix.

A.2.2 The Moiré Processes and Grids

The moiré geometries shown in Figures A.2 and A.3 can be interpreted as conventional close-range stereo-photogrammetry arrangements, with a pair of separated projection centers p_1 (the light source) and p_2 (the viewing point) [Perrin and Thomas, 1979]. A feature of the moiré topography process is that the parallax information for recovering the third dimension is encoded into a single 2-D image. The encoded image is produced through p_2 by virtue of the object-distorted bundle of structured light originating at p_1 . A camera is usually positioned at p_2 to record the encoded image, as shown in the figures.

For the purposes of the discussion in this section, it is useful to think of topographic moiré fringes as being formed by a pair of processes:

- **Source Encoding Process:** A source of illumination at p_1 is spatially modulated, and bathes the object with a bundle of structured rays that encode the object with shadows that characterize the perspective geometry of the source and the object.
- **Scene Demodulation Process:** The encoded scene is viewed from a different perspective at p_2 , and is spatially demodulated to extract the moiré fringe pattern.

The spatial modulation/demodulation is achieved with a variety of techniques and structures described below, but loosely speaking, they are embodied as *grids*. There are three conceptual 'grids' that enter the discussion, and they are identified by their role in the moiré processes:

- **Encoding Grid:** that which imparts spatial structuring (usually a pattern of bright and dark stripes) to the object illumination.
- **Distorted Grid:** that pattern of bright and dark stripes on the surface of the object which appears distorted by the surface relief when viewed from a different perspective than that of the illumination source.
- **Demodulation Grid:** that which spatially demodulates the view of the distorted-grid scene such that moiré fringes are formed.

These definitions for the processes and the grids are cast in terms of geometric optics and communications theory. Both of these theories are useful for modelling the moiré processes, as each has insight to contribute. For example, Perrin and Thomas [1979] refer to the encoding process as introducing a carrier frequency whose phase is modulated by the relief of the objects in the scene.

Clearly the encoding grid's role is to produce the appropriate distorted grid on the object. The most practical encoding arises from projecting a regular series of stripes of light onto the object, and physical embodiments for achieving this will be explored below.

Often overlooked is the concept that the encoded scene viewed from p_2 has the appearance of that scene under conventional illumination with a superimposed

Figure A.4: The distorted-grid pattern formed on a live human body. The fine line shadow pattern is resolved as a modulation of the natural shading of the surface. The translucence and specularity of living skin pose noticeable problems in maintaining the contrast of the fine grid shadows [Takasaki, 1981]

shadow-pattern arising from the structured light at p_1 , as shown in Figure A.4. Indeed, the object *per se* is usually an 'unspoken entity' and only the shadow-pattern, distorted by the surface relief of the object, is acknowledged. Several authors have coined the term *distorted-grid image* to describe the pattern as seen from p_2 , and this image is the essential input for the moiré-forming scene-demodulation process.

Nonetheless, this term belies the full information borne by the scene, namely the shading envelope defined by light reflected from the object. The natural appearance of the object, with its usual shading characteristics, renders the distorted-grid view and the subsequent moiré topogram, particularly easy to interpret with respect to object recognition. This is due to the presence of the encoding shadow-pattern, which is encoded at a higher spatial frequency than other structured-light techniques (e.g., raster stereo-photogrammetry, scanning profilers, etc.). Moiré topography

imposes a more continuous encoding of the parallax, and thus illuminates the object more completely.

The moiré fringes are formed when the distorted grid is viewed through the demodulation grid, which functions as a second modulator or sampler that closely matches the encoding grid. As a physical interpretation, Oster [1963] described moiré fringe formation as an optical analog computer process whose output is the fringe pattern. Several authors have found geometrical ray-tracing to provide a satisfying model to explain fringe localization. In the more popular communications formalism, Fourier analysis showed the fringes as resulting from low-pass spatial filtering to eliminate the source encoding carrier frequency [Perrin and Thomas, 1979]. These, and other models have been reviewed in [Roger, 1980], but common to them all are the two processes (source encoding and scene demodulation), and the three grids (encoding, distorted and demodulating).

A.3 Terminology and Classification

A.3.1 Generic Systems

There are two basic arrangements, and although they have been mentioned several times, their key attributes are defined here in terms of the grid roles:

- **Shadow-Moiré Systems** are characterized by a single material grid that serves the role of both the encoding and the demodulation grids. The grid is located close enough to the object to cast sharp shadows on it, and it must be large enough to simultaneously encode and provide sufficient aperture through which to view the scene.
- **Projection-Moiré Systems** are characterized by separate, physically remote encoding and demodulation grids that are imaged onto the object by projection optics. The grids may be small material gratings, conventionally imaged, or one or both of the grids may be emulated by one of the embodiments discussed in the next section.

A.3.2 Grid Embodiments

The term *grid* refers to a means of imposing a spatially-periodic structure to an optical channel (such as the view of a scene), to produce *intensity modulation* by the material grid or its emulator. The term *material* means that the grid is composed of structured matter (i.e., one can actually touch the grid *per se*). A distinction is made between the modulating action of such a material grid and that which may be emulated by a *virtual grid*, which is physically realized by nongrid-like structures. Despite the fineness of some grids used for moiré topography, the term *grating* is avoided unless coherence and diffraction effects are central to the technique.

As mentioned, the most practical encoding arises from projecting a regular series of straight stripes of light onto the objects in the scene. Several methods for achieving this are in practice, and they form popular adjectives in the literature for classifying the various moiré schemes: shadow, projection, holographic, scanning, etc. These terms are only defined here, along with some examples:¹

- **Shadow Grids** are material grids that are placed physically near the object, such that the intensity modulation of the illumination maintains sufficient spatial definition. Examples of embodiment are: frames supporting a series of closely-spaced stretched strings or wires, or transparent substrates (such as glass or plastic sheet) with a series of printed opaque bars. Examples of application are: the (large) grid of the generic shadow-moiré systems, and (smaller) photographic transparency grids that can be used to demodulate distorted-grid photographic records by physically sandwiching them.
- **Projected Grids** are material grids that are imaged by projection optics onto the object, again such that the modulation of the illumination maintains sufficient definition. Usually these grids can be distinguished from grids for shadow applications, as they are relatively small and finely pitched, and one relies upon high-quality projection optics to magnify their subtended image

¹The detailed considerations for the fabrication and application of grids for moiré topography are found in [Pekelsky and van Wijk, 1988].

area. Exceptions to this rule arise in shadow-moiré arrangements for small objects such as coins and ear-drums. The grids themselves may be embodied as a bar pattern exposed in photographic emulsion or etched in a metallic coating on glass plates. The inherent grid-structure of some solid-state detectors, such as CCD imagers, also falls into this class of grid. Matched copies of such grids are often employed as encoding and demodulation grids in projection-moiré systems.

- **Transform Grids** are virtual grids that are the Fourier transform of a grid structure in the spatial object-domain. They may be embodied as appropriate apertures in an opaque plate (thereby filtering out the unwanted frequencies) and may be employed in systems that provide access to the spatial frequency-domain. This technique has been used, for example, in a coherent analysis system for demodulating the distorted-grid image.
- **Coherence Grids** are virtual grids formed by coherently interfering beams of light. They are embodied in a variety of optical arrangements (interferometers) that usually involve a laser light source and beam expansion, splitting and combining optics. Examples of applications include source encoding in projection-moiré, and distorted-grid image demodulation in off-line analysis. Coherence grids are not to be confused with the material *holographic grids*, created by exposing photoplates in the interfering beams of a coherence grid.
- **Scanned Grids** are virtual grids formed over a period of time by sweeping an illumination spot across the grid-defining raster. Typical embodiments are cathode ray tube scanners and laser scanners. These grids have been applied as source encoders in projection-moiré systems and distorted-grid image demodulation in off-line analysis.

A.3.3 Modulation Scheme

The terms introduced in this section address the issue of whether a given method forms the moiré fringes optically or by some non-optical technique after the image

has been converted at the detector into some other representation (e.g., electrical signals).

- **Modulated Illumination** refers to those processes that introduce a modulating structure into the optical *illumination* channel without terminating that channel. Further optical processing is essential to the moiré process in as much as a view of the distorted-grid scene must be available for the demodulation process.
- **Modulated View** refers to those processes that introduce a modulating structure into the optical *viewing* channel without terminating the channel. Thus further optical processing is possible, and an optical image with demodulated moiré fringes could be formed on a detector such as the eye retina, a photographic emulsion, or the photosensors of a CCD array.
- **Modulated Detector** refers to those processes that introduce *primarily for the purpose of demodulating the moiré fringes* a spatial modulation at the detector, either simultaneously or after the optical signal has been converted into another representation. An example of post-conversion modulation is the raster-scan by the electron beam of a vidicon camera that modulates by sampling the spatially-continuous image formed on the phosphor of the distorted-grid view. A second important example is the aforementioned grid-like structure of the CCD photosensor sites, where clearly the spatial modulation occurs simultaneously with the photo-electric signal conversion.

A.3.4 Fringe Terminology

Moiré fringes are band-like shadow patterns formed along loci of constant parametric value. In the case of topographic encoding, the fringes are most commonly arranged to represent contours of elevation with respect to some *base plane*. These two sentences innocently use the term *fringe* to describe very different concepts: the former speaks about the 2-D cyclic or periodic nature of the shadow pattern, whereas the latter refers to the loci of local extrema in the encoding functions,

i.e., *extrema* curves that delineate the center of the bright, or alternately, the dark bands. Strictly speaking, the fringes are bands and the contours are curves. Typically, there is no problem in using the term *fringes* to describe both broad shadows and fine curves, as most authors keep these concepts clear by the context of the discussion.

This appendix dwells on the 2-D nature of the fringe pattern. The *fringe direction* is simply the direction of the tangent to the extrema curve. The *fringe profile* is the phase and amplitude distribution of the encoding function orthogonal to the local fringe direction. Unless otherwise specified, fringes are assumed to be contours of elevation. Such contours can define profiles that lie along the *gradient lines* of the surface, and the term *gradient profile* denotes this special case.

Moiré fringes are cyclic functions along the gradient profile. The simplest moiré topographic arrangements produce profiles that are *even* functions, i.e., they are symmetric with respect to the sign of the gradient. Often they can be approximated as a sinusoidal distribution. Another characteristic of the most basic moiré arrangements is that the scene demodulation occurs prior to image recording, and the moiré fringe pattern is thus 'frozen' in the recorded image. The combination of *grade-symmetry* and *fixed-encoding* that characterizes ordinary moiré fringes is the root of the fringe-ordering 'hill-or-valley' dilemma, which is the research topic addressed by this thesis. Clever methods for overcoming this dilemma have been devised that depart from the constraints of ordinary moiré fringes, such as *asymmetric moiré fringes* and *dynamic moiré fringes*. The complete definitions of these terms and their application to the fringe ordering problem are presented in Chapter 2.

A.3.5 Demodulation Schemes

Since the processes of source encoding and scene demodulation are separable, moiré fringes may be formed at the moment of image recording, or after the scene is recorded. Several classes of technique are characterized by different demodulation epochs in the temporal domain, and the following terminology will appear in later discussions:

- **On-line Demodulation** refers to processes that form the moiré fringes prior to, or at the moment of image recording. These are fixed-encoding systems if photographs are produced, but some video-based multi-frame systems use on-line demodulation in a dynamic-encoding configuration.
- **Off-line Demodulation** refers to processes that form the moiré fringes from a record of the image. These are always dynamic fringe systems.

In terms of the spatial domain, demodulation may be invoked in different fashions as well:

- **Global Demodulation** refers to demodulating the entire view to produce at once the full moiré pattern. This is the customary output of most on-line systems, an example being the moiré topogram as photographed in a conventional shadow-system.
- **Local Demodulation** refers to demodulating only a small patch of the distorted-grid image at a time, usually to process the phase information of this patch, and then to proceed to the next patch, and so on until the entire scene is analyzed. This scheme is embodied in the phase-lock analysis techniques described in detail in Section A.10.2.

A.4 Moiré Fringe Intensity Distribution

In this section, an expression is derived for the distribution of intensities in the fringe pattern, in terms of the geometric properties of the shadow-moiré arrangement. The analysis is restricted to a physical generalization of the author's apparatus. Results pertaining to other geometries are cited for comparison.

This thesis is concerned with topographic applications of moiré patterns, but the reader should appreciate from the forgoing introduction, that moiré effects permeate a much more general class of situations, namely those involving periodic structures. The moiré pattern may be the desired outcome, but often it is a bothersome artifact. Thus, it is not surprising that many authors have laboured over suitable models to

describe the moiré fringe formation process. It is beyond the scope of this document to review various methods, and the interested reader is directed to [Roger, 1980] where these approaches are discussed under five categories:

- simple geometrical analysis;
- indicial or parametric representation of the grids;
- communications theory, and the use of transmittance functions;
- representation of the periodic structures in Fourier or reciprocal space;
- full optical theory, accounting for diffraction effects.

Most of these approaches confine the analysis to the superposition *per se* of two similar in-plane patterns to produce a moiré effect. Although this is valid for topographic moiré as well, it does little to convey just how the patterns relate to the surface relief, which is perhaps the most important feature in the analysis of moiré topography. The so-called 'communications model', with some geometric ray tracing to tie in the projected grid-distortion, has been the most successful in this respect, and the discussion that follows is based largely on the development by Takasaki [1979a]. Also useful are the general expressions for the case of rotated and translated grids developed using Fourier theory in [Buitrago and Durelli, 1978], and the reader is directed to [Chiang, 1975a] for an alternate derivation based on a geometrical interpretation.

A.4.1 Moiré Fringes in Shadow-Moiré Systems

Figure A.5 shows a typical shadow-moiré arrangement. A grid G lies in the x - y plane of a 3-D Cartesian coordinate system, such that the grid lines are parallel to the x -axis, and are regularly spaced at a pitch s . Thus, at any point $P(x, y, z)$ on the grid, the relative phase of the grid ν is a function only of the y -coordinate and the pitch of the grid

$$\nu(x, y, z) = 2\pi(y/s). \quad (\text{A.1})$$

Figure A.5: Local shadow-moiré pattern analysis. A grid G with pitch s , lies in the x - y plane of a Cartesian coordinate system. The grid lines are parallel to the x -axis, and y_0 is the initial displacement of the grid with respect to the origin. A point light source is at $p_1 = (x_1, d, l_1)$ and an observer is at $p_2 = (0, 0, l_2)$. An object is placed under the grid, and rays from p_1 and p_2 , that meet at a point P on the object, intersect the grid plane at points Q and R . Grid patch G_Q is p_1 -projected onto the object to form a relief-distorted image G'_Q about P . This pattern is p_2 -projected back to the grid plane to form G''_Q about R . The superposition of G'_Q and G''_Q describes the moiré pattern seen from p_2 in the direction of P .

For convenience, the functional notation will be suppressed (i.e., simply $\nu = 2\pi y/s$), unless otherwise ambiguous, and then subscripts may indicate the intent (i.e., the phase at P is ν_P). The grid G has a transmittance

$$T_G(x, y, 0) = \frac{1}{2}[1 + \cos(\nu + \nu_0)] \quad (\text{A.2})$$

where the initial phase

$$\nu_0 = 2\pi(y_0/s) \quad (\text{A.3})$$

is given by the initial displacement y_0 of the grid with respect to the origin.

The point light source is placed at projection center p_1 at a finite elevation l_1 above the grid plane, a distance d in the y -direction and x_1 in the x -direction. The observer or camera is placed at projection center p_2 on the z -axis at a finite elevation l_2 above the coordinate system origin. Thus, p_1 and p_2 have coordinates (x_1, d, l_1) and $(0, 0, l_2)$, respectively. It will be shown that the fringe pattern is independent of x_1 (i.e., the position of the light source along the grid line direction).

An object is placed under the grid, as shown in the figure, and this discussion is concerned with the *local* moiré pattern observed from p_2 about a point P on the object. Rays from p_1 and p_2 that meet at P intersect the grid plane at points Q and R . When restricted to the grid plane ($z = 0$), one may conveniently switch to plane coordinates with no loss of generality; thus, points Q and R have plane coordinates (x_Q, y_Q) and (x_R, y_R) , respectively.

Irrespective of the particular roles of source and detector assigned to the projection centers, one may speak of geometrically projecting patches of grid G about Q and R onto the object at P to form relief-distorted grid patterns G'_Q and G'_R , respectively, where the prime symbol denotes perspective distortion of the grid due to projection. One could then analyze the moiré formed by those superimposed patterns.

It is a more convenient frame of reference, however, to consider the following. First, project via p_1 the patch of grid about Q onto the surface at P forming G'_Q (the distorted-grid patch on the object in Figure A.5). Then, project via p_2 that

shadow pattern to R as G''_Q (the rotated, compressed patch² on the grid in the figure). Thus, the moiré pattern is formed by interfering the doubly-projected (and distorted) grid patch originating at Q , with the original grid patch at R . Clearly, it is the surface relief of the object that imparts a perspective distortion of the pattern with each of the projections, and an attempt has been made to depict this effect in the figure.

From Equation A.2, the transmittance of the grid through Q is

$$T_G(x_Q, y_Q) = \frac{1}{2}[1 + \cos(\nu_Q + \nu_0)] \quad (\text{A.4})$$

where $\nu_Q = 2\pi(y_Q/s)$, as before.

A key assumption is that the object is sufficiently smooth: that is, projections of the grid G from p_1 and p_2 must produce patterns on the surface that are very finely spaced compared to the local surface curvature. Only then can the doubly-projected shadow of the grid patch G''_Q around R be approximated as a straight-line plane grid with pitch s'' and direction θ with respect to the original grid, and have the same absolute phase relationship (ν''_0) to point R as did the original grid G to point Q . The values of s'' and θ depend upon the orientation of the object at P , whereas the phase ν''_0 depends only upon the position of P with respect to the projection centers.

When these conditions are upheld, the intensity of the doubly-projected shadow pattern around point R is given by the effective transmittance

$$T_{G''_Q}(x, y) = \frac{1}{2}[1 + \cos(\nu'' + \nu''_0)] \quad (\text{A.5})$$

$$\nu'' = 2\pi[(x - x_R) \sin \theta + (y - y_R) \cos \theta]/s'' \quad (\text{A.6})$$

$$\nu''_0 = (\nu_Q + \nu_0) \quad (\text{A.7})$$

A very important property is that ν'' vanishes at point R , and Equation A.5 simplifies to an expression that is *object-independent*

$$T_{G''_Q}(x_R, y_R) = \frac{1}{2}[1 + \cos(\nu_Q + \nu_0)] \quad (\text{A.8})$$

²The double-prime denotes perspective distortion due to a double-projection.

The above equations are exact relationships only if the object is a true plane. Otherwise, small perturbations must be introduced into the assumptions of phase preservation under double-projection, and in particular Equation A.8 expands with terms that depend upon the surface relief. For most practical applications, the grid spacing is sufficiently fine compared to the surface curvature, and the approximate solutions presented here are adequate for analysis of the resulting moiré patterns.

It is assumed that the light source at p_1 has a nominal intensity I_0 , and that the pattern of shadows about P , on the surface of the object, is ideally reflected to p_2 . Thus, from p_2 , the intensity of the interfering patterns in the neighbourhood of R is observed to be

$$I_R = T_{GR} \times T_{GQ}'' \times I_0, \quad (\text{A.9})$$

which expands from Equations A.4 and A.5 to

$$I_R(x, y) = \frac{I_0}{4} \left[1 + \cos(\nu + \nu_0) + \cos(\nu'' + \nu_0'') + \cos(\nu'' - \nu + \nu_Q) + \cos(\nu'' + \nu + \nu_Q + 2\nu_0) \right]. \quad (\text{A.10})$$

At R , $\nu'' = 0$, and using Equation A.7, the distribution of intensities simplifies to

$$I(x_R, y_R) = \frac{I_0}{4} \left[1 + \cos(\nu_R + \nu_0) + \cos(\nu_Q + \nu_0) + \cos(\nu_Q - \nu_R) + \cos(\nu_R + \nu_Q + 2\nu_0) \right]. \quad (\text{A.11})$$

The terms of this expression depend only upon the grid phase at R and Q (i.e., the points where rays extended from the projection centers p_1, p_2 to the point P on the object intersect the grid). The geometric relationship between the projection centers and the grid is fixed in the apparatus (with $l_1, l_2 < \infty$ and $s, d \neq 0$). Thus, it is the variation in the position of P , as it is moved about the surface of the object, that determines different positions for R and Q , and accounts for the variation in the intensity terms.

The terms in Equation A.11 have a simple physical interpretation. Generally, all but the third cosine term are of a high spatial frequency, namely of the order of that of the grid. Indeed, the first cosine term represents the grid *per se*, the second cosine term represents the projected shadow of the grid, and the fourth cosine term represents the sum of the grid and the shadow frequencies.

The intensity of the moiré fringe pattern is determined by third cosine term

$$I_M(x_R, y_R) \propto \cos(\nu_Q - \nu_R). \quad (\text{A.12})$$

It is characterized by the low-frequency phase variation (ψ_M) arising from the difference between the spatial frequencies of the grid and its shadow

$$\psi_M = (\nu_Q - \nu_R). \quad (\text{A.13})$$

A very useful property is that only the moiré term in Equation A.11 is independent of the initial phase of the grid ν_0 , making the moiré pattern stationary against translations of the grid. Grid shifting is discussed at the end of this appendix.

A.4.2 Fringe Order and the Intensity Distribution

The *fringe order* ϕ associated with point P on the object, due to the moiré pattern intensity seen from the camera, is defined by

$$\phi = \psi_M / 2\pi. \quad (\text{A.14})$$

For shadow-moiré systems, ϕ is restricted to non-negative, real numbers. This means the intensity variation is continuous with distance from the grid, and is only physically valid on the 'shadow-side' of the grid ($\phi_{\text{grid}} = 0$). In projection-moiré systems, the reference plane is virtual where $\phi = 0$, and ϕ can be negative.

The term 'fringe order' implies discrete values, and yet moiré fringes are broad, fuzzy bands across which the fringe order varies.³ However, the bright fringes become brightest and the dark fringes darkest where the fringe phase becomes exactly

³The symbol n or N is often used in the literature to denote the discrete fringe numbers, but the author uses ϕ as a reminder that the cyclic moiré phase is a continuous real-valued variable

even and odd multiples of π , respectively. The loci of these extrema (termed *fringe skeletons*) are easy to recognize within the intensity pattern, and it has become customary to loosely refer to these thin curves as simply fringes, contours or skeletons. The fringe order assigned to those curves is given in integer and odd half-integer multiples

$$\phi = \begin{cases} 0, 1, 2, \dots & \text{for bright fringes,} \\ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots & \text{for dark fringes.} \end{cases} \quad (\text{A.15})$$

A useful property for determining the order of a fringe is that the number of grid-lines between points Q and R in Figure A.5 matches the order of the fringe they form. This follows from Equations A.1 and A.13, where

$$\psi_M = 2\pi(y_Q - y_R)/s, \quad (\text{A.16})$$

and from the definition of ϕ (Eq. A.14)

$$y_Q - y_R = \phi s. \quad (\text{A.17})$$

This property is exploited in the Fiducial Shadow Ordering Method discussed in Chapter 2.

A.4.3 Moiré Fringes in Projection-Moiré Systems

The analysis of fringe formation in projection-moiré configurations proceeds along similar arguments as presented for the shadow-moiré system, with the expected complications. There are now independent grids to be considered, which are located *above* the projection centers. The properties of the projection optics, such as focal lengths, magnification ratios and position of the nodal points of the lenses, play a significant role in the modelling of fringe formation. The interested reader is directed to [Idesawa *et al.*, 1977] for the general development of nodal-ray formulas, and to [Takeda, 1982] for an analysis based on principal-ray optics.

A.5 Topographic Fringe Loci

A.5.1 Fringes as Contours of Elevation

Special arrangements of the grid(s) and projection centers discussed in the previous section lead to practical simplifications of the fringe patterns. Most significant by far is the case of the lamp and camera placed at the same distance from the shadow-moiré grid (i.e., $l_1 = l_2$). In this case, the fringe planes are parallel to the grid (reference) plane and enjoy a simple topographic interpretation: they are true contours of elevation with respect to the reference plane. This is shown schematically in Figure A.2, where the family of fringe planes (numbered $\phi = 1, 2, \dots$) can be seen as parallel to the grid plane. Most users strive for this condition in their arrangements, and the NRC Moiré Contourgraphs used by the author have been designed with great care to establish this special geometry.

A.5.2 Fringes from Curved Sheets

In general, however, the fringes represent the intersection loci of a series of curved sheets intersecting the surface of the object [Frobin and Hierholzer, 1982b]. This is depicted schematically in Figure A.6, where $p_1 = L$ and $p_2 = E$ are at different distances from the grid, and the resulting moiré fringes represent a family of curved sheets, asymptotic to the grid plane. This effect was elegantly recorded by Takasaki in the photographs shown in Figure 2.9 on page 36.

A.5.3 Fringes from Surface Differences

Hovanesian and Hung [1971] described a general projection-moiré technique which they called the *moiré contour-difference* method, which produces fringes of constant elevation-difference between two surfaces. The method is quite simple: first an object is illuminated by the encoding grid and the distorted grid view is recorded on film; then a second object is exchanged for the first and a second exposure is made of the new distorted-grid image. A double-exposure can collect both views,

Figure A.6: Shadow-moiré geometry for curved surface fringes. When the illumination L and viewing E are not at the same distance from the grid, the fringes lie on a family of curved sheets, asymptotic to the grid plane [Takasaki, 1979b]

or else separate photographic images can be sandwiched later.

Their analysis shows that the resulting fringes are indeed contour-differences, and this paper inspired several important developments. First, as they demonstrated in an example, is the prospect of quality control in manufacturing, where the initial object is a 'perfect' sample, and subsequent samples are examined by differencing to detect surface defects.

Perhaps more exploited is the concept that, if one of the objects (called the *reference object*) is a flat plate, then the fringes are true contours of elevation with respect to the plate. This solves a scaling problem with off-line demodulation schemes; namely, it produces a suitable demodulation grid that is properly scaled to the distorted-grid image [Miles and Speight, 1975]. Furthermore, each view of the object and plate undergoes the same optical distortion of the imaging system, thereby reducing the distortion of the final moiré pattern. This *object-and-plate* technique is also useful for relative fringe ordering (see Ch. 2).

A.5.4 Other Topographic Fringes

There have been numerous schemes reported that generate moiré fringes to encode other topographic information, and a few are named here for the interested reader to pursue. Hovanesian and Hung [1971] described a *contour-sum* arrangement with fringes that sum the elevation between two surfaces. Kao and Chiang [1982] reviewed a family of techniques to produce fringes of surface slope and curvature, and cited over fifty references in this field. Kafri and Livnat [1983] reported a method for moiré contour mapping the second and third shape derivatives of surfaces using a *shearing moiré* technique.

A.6 Fringe Contour Formulas

In this section, formulas for the elevation and spacing of moiré fringe contours are derived for the case of the separated lamp and camera ($d \neq 0$), at a finite distance from the grid plane ($l_1, l_2 < \infty, s \neq 0$). Again this corresponds to the author's equipment. Henceforth, when referring to the distance of a fringe to the reference plane, or to an adjacent fringe, the terms *fringe elevation* and *fringe interval* will be restricted to the special case of moiré fringes that are true contours of elevation.

A.6.1 Shadow-Moiré Fringe Contour Elevation

The derivation of a formula for the fringe elevation in shadow-moiré systems comes from inspecting the phase ψ_M of the moiré intensity when written in terms of coordinate components, as in Equation A.16. The following analysis expresses components y_R, y_Q in terms of the geometry of the projectors and surface point, and leads to the desired fringe elevation formula.

Figure A.7 shows the geometry of the essential points, redrawn from Figure A.5 and relabelled with general coordinates for this analysis, subject to two physical constraints. First, the two separated projection centers must be at a finite distance on the opposite side of the grid plane to the point on the object surface, so that

Figure A.7: Points for vector analysis of contour fringe interval, redrawn from Figure A.5. The two projection centers are now labelled $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, with arbitrary positions above the x - y plane with respect to the origin, but not colinear in the x -direction. The point on the surface is now $P_3(x_3, y_3, z_3)$, with arbitrary position below the x - y plane. $Q(x_Q, y_Q, z_Q)$ and $R(x_R, y_R, z_R)$ are points where lines P_1P_3 and P_2P_3 intersect the x - y plane, respectively.

shadows are cast on the object. Second, the projections centers must not be on a line that has a component parallel to the grid lines, so that the fringes have a finite elevation. The two projection centers are now labelled $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, and the point on the surface is now $P_3(x_3, y_3, z_3)$. The equation of line P_1P_3 is

$$(x, y, z) = (x_1, y_1, z_1) + \alpha_{1,3}(x_3 - x_1, y_3 - y_1, z_3 - z_1), \quad (\text{A.18})$$

and it intersects the x - y plane at $Q(x_Q, y_Q, z_Q)$. Since $z_Q = 0$, then Equation A.18 gives

$$\alpha_{1,3} = z_1 / (z_1 - z_3), \quad (\text{A.19})$$

and this can be substituted back into Equation A.18 to solve for the y -component at Q

$$\begin{aligned} y_Q &= y_1 + \alpha_{1,3}(y_3 - y_1) \\ &= \frac{y_1(z_1 - z_3) + z_1(y_3 - y_1)}{(z_1 - z_3)} \\ &= \frac{z_1 y_3 - y_1 z_3}{(z_1 - z_3)} \end{aligned} \quad (\text{A.20})$$

Similarly, the equation of line P_2P_3 is

$$(x, y, z) = (x_2, y_2, z_2) + \alpha_{2,3}(x_3 - x_2, y_3 - y_2, z_3 - z_2), \quad (\text{A.21})$$

and it intersects the x - y plane at $R(x_R, y_R, z_R)$. Again, since $z_R = 0$, then Equation A.21 gives

$$\alpha_{2,3} = z_2/(z_2 - z_3), \quad (\text{A.22})$$

and when substituted back into Equation A.21, the y -component at R is

$$\begin{aligned} y_R &= y_2 + \alpha_{2,3}(y_3 - y_2) \\ &= \frac{y_2(z_2 - z_3) + z_2(y_3 - y_2)}{(z_2 - z_3)} \\ &= \frac{z_2 y_3 - y_2 z_3}{(z_2 - z_3)} \end{aligned} \quad (\text{A.23})$$

The two components (Eqs. A.20 and A.23) can now be substituted into Equation A.16 to obtain a general expression for the moiré phase

$$\begin{aligned} \psi_M &= 2\pi(y_Q - y_R)/s \\ &= 2\pi \left(\frac{z_1 y_3 - y_1 z_3}{s(z_1 - z_3)} - \frac{z_2 y_3 - y_2 z_3}{s(z_2 - z_3)} \right) \\ &= 2\pi \left(\frac{z_3 y_3(z_2 - z_1)}{s(z_1 - z_3)(z_2 - z_3)} - \frac{z_3 y_1}{s(z_1 - z_3)} + \frac{z_3 y_2}{s(z_2 - z_3)} \right) \end{aligned} \quad (\text{A.24})$$

When the points P_1, P_2, P_3 assume their positions as shown in Figure A.5 (namely: $y_1 = d, y_2 = 0, z_1 = l_1, z_2 = l_2$, and $z_3 = -h$ with h positive in the $-\hat{z}$ direction), then Equation A.24 becomes

$$\psi_M = 2\pi \left(\frac{hy(l_2 - l_1)}{s(l_1 + h)(l_2 + h)} + \frac{hd}{s(l_2 + h)} \right), \quad (\text{A.25})$$

where, reiterating their physical significance, l_1, l_2 are the projection-center elevations, d is their separation in the y -direction, and h is the elevation of the point on the object below the grid, which has a pitch of s .

In general, $l_1 \neq l_2$, and the loci of constant moiré phase are curved sheets due to the y -dependence in the first term of Equation A.25 for point $P(x, y, -h)$ on the surface of the object.

A.6.1.1 Contours with Non-Linear Elevation

If the projection centers are placed at the same elevation, the loci become planes:

$$\boxed{\text{Condition for Contours: } l_1 = l_2 = l} \quad (\text{A.26})$$

and Equation A.25 simplifies to

$$\psi_M = 2\pi \frac{hd}{s(l+h)} \quad (\text{A.27})$$

which is now a function of only h for a given l and d . Thus, the constant moiré phase surface corresponds to a constant elevation plane in the object space.

By combining Equations A.14 and A.27, one obtains the well-known expression for the ϕ th moiré fringe contour elevation:

$$h_\phi = \frac{\phi l}{d/s - \phi} \quad (\text{A.28})$$

This formula is valid for shadow-moiré systems in which the lamp and camera are at the same distance l from the grid (with pitch s) and are separated by a distance d orthogonal to the grid lines. Note that contour elevation is non-linear in ϕ ; in fact, the contour spacing *increases* with distance from the grid. Formulas for the contour interval are given in Section A.6.2.

A.6.1.2 Contours with Linear Elevation

In some applications, the slight non-linearity of Equation A.28 can be ignored.⁴ If the lamp and camera separation d is much larger than the grid pitch s ,⁵ then the following linear relation may be used as a good approximation for the elevation of the ϕ th fringe

$$h_\phi \approx \frac{\phi l s}{d}, \quad (d/s \gg \phi) \quad (\text{A.29})$$

⁴For example, the NRC Moiré Contourgraph is said to have a "nominal contour interval of 5 mm", with less than 0.6% spacing non-linearity for fringes with $\phi < 30$; this is negligible for some orthopaedic applications, such as detecting spinal deformity.

⁵On the NRC apparatus, $d = 710$ mm and $s = 2$ mm.

Contours with perfect uniform spacing result when the lamp and camera are at an infinite distance from the grid. This can be realized by using telecentric optics [Marasco, 1975; Doty, 1975], and then the elevation of the ϕ th contour is given by

$$h_{\phi} = \frac{\phi s}{\tan \alpha + \tan \beta} \quad (\text{A.30})$$

where α and β are the angles of incidence to the grid by the illumination and observation beams, respectively.

A.6.2 Shadow-Moiré Contour Interval Formulas

Generally, one uses the appropriate elevation formula to establish look-up tables that cover a working range of ϕ for a given apparatus geometry. In designing the apparatus, it is useful to monitor the contour spacing as a function of the system parameters, particularly if it is non-linear in ϕ . The contour interval formula is the most direct analytical tool.

The contour interval Δh_{ϕ} is defined as the distance between the ϕ th and $(\phi-1)$ th contours

$$\Delta h_{\phi} = h_{\phi} - h_{\phi-1} \quad (\text{A.31})$$

For non-uniform spaced shadow-moiré contours, an exact expression can be developed from Equation A.28 as

$$\begin{aligned} \Delta h_{\phi} &= \frac{\phi l}{d/s - \phi} - \frac{(\phi-1)l}{d/s - (\phi-1)} \\ &= \frac{l}{(d/s) + (2\phi-1) + (\phi^2 - \phi)(d/s)^{-1}} \end{aligned} \quad (\text{A.32})$$

For nearly-uniform spaced contours due to widely-separated lamp and camera, the contour interval is given by Equation A.29 as

$$\Delta h_{\phi} \approx \frac{ls}{d}, \quad (d/s \gg \phi) \quad (\text{A.33})$$

The telecentric-projection case results in contours with a uniform interval, as given by Equation A.30

$$\Delta h_{\phi} = \frac{s}{\tan \alpha + \tan \beta} \quad (\text{A.34})$$

A.6.3 Projection-Moiré Elevation Formulas

Figure A.3 shows the geometry of a practical projection-moiré system intended to produce fringes that are true contours of elevation. The lenses and grids are identical for each projector, and arranged such that the grids are coplanar and the nodal points of the lenses are in a plane parallel to the grid plane. For this arrangement, the elevation of the ϕ th fringe is given by Takeda [1982] as

$$h_{\phi} = \frac{\phi s l (l - f)}{d f - \phi s (l - f)} \quad (\text{A.35})$$

where, as shown in Figure A.3, h_{ϕ} is measured from the plane conjugate to the plane of the grids G_1 and G_2 (i.e., their image plane), s is the pitch of the grids, f the focal length of the lenses, d the separation of the nodal points of the lenses, and l is the distance from the nodal points to the reference plane. Equation A.35 can be rewritten as

$$h_{\phi} = \frac{\phi l}{(d/sm) - \phi} \quad (\text{A.36})$$

where the magnification m is the relative size of the grid-images formed in the reference plane. This form can be compared to Equation A.28 for the shadow-moiré situation, and suggests that the lens and grid of each arm of the projection arrangement need only match to the extent that the independently-magnified grid-images formed in the reference plane be the same for each arm. Naturally, the nodal points for dissimilar projectors would have to be shifted to accommodate the difference in working distance at different magnifications.

Equation A.35 is widely used in practice, although its derivation is based upon nodal-ray assumptions that can introduce large errors (10%) when the object-distance approaches the focal length of the projectors. Takeda [1982] analyzed this failing, and developed corrections based upon principal-ray considerations.

A.7 Moiré Fringe Localization

In general, topographic moiré fringes are not formed on the surface of the object, nor simply in the grid plane as discussed in the previous section. The eye of the observer

Figure A.8: Demonstration of moiré fringe localization. The lens aperture was set to give a shallow depth of focus, and photographs were taken of the shadow-moiré contours with lens focused (a) on the object, (b) well in front of the object, and (c) well behind the object. The visibility is quite different for each set of fringes [Yoshizawa and Tashiro, 1982]

dynamically focuses on the fringes, tracking the optimal focal depth as the spatial localization varies over the encoded scene. Yoshizawa and Tashiro [1982] suggested this may be the reason that moiré fringe patterns that appear sharp to the eye are sometimes quite fuzzy when photographed (Fig. A.8). They introduced a theory based upon geometric obstruction to analyze fringe localization, but the results were highly dependent upon the complex optical and geometric characteristics of each system, and precluded a simple formula. They recommended that the object be placed as close to the shadow-grid or the sharpest image of the projection-grid as possible, and that the lens apertures be kept as small as possible to maximize the depth of field.

One method that does generate topographic moiré fringes directly on the object has been called *additive projection-moiré* by Takasaki [1982]. In this case, two grids are projected from different angles onto the test object using separate projectors (see Fig. A.9). The highly-localized moiré fringes are formed as a result of the non-linear response of the observer's eye, or of the photographic emulsion. Although the

Figure A.9: An arrangement to produce additive moiré fringes. Each projector forms a distorted-grid shadow pattern on the object, which is superimposed on the surface *per se*, and thus, the shape of the resulting moiré fringes is independent of the viewing position [Idesawa *et al.*, 1977]

fringe visibility is rather low, this method has the advantage that several observers may view the same pattern *in situ*, independent of their position with respect to the projectors.

A.8 Moiré Fringe Visibility

Visibility refers to several processes that combine to affect the distinguishability of the fringes. Such issues as coherent noise at the carrier frequency, contrast of the fringes and sharpness of their profile are discussed here.

A.8.1 Coherent Noise

As in communications theory for amplitude-modulation (AM) transmitter/receiver models, the moiré grid-lines can be considered as the *carrier function* that modulates and demodulates the shading *envelope function* of the object. When the individual grid lines or their shadows on the object are resolved in the final moiré image, the carrier must be considered as a coherent source of noise. There are two cases

Figure A.10: The coherent noise in an ideal moiré fringe pattern. A magnified portion of the moiré fringe pattern is shown, formed from the superposition of an ideal bar grid and its ideal, rotated shadow. The vertical light and dark fringes are centered alternately about the white 'columns of diamonds' and the black 'zig-zags', respectively. Shown below is a schematic representation of the moiré fringe mean intensity profile, in this case the envelope is a triangular function with transmittance bounded at $T_p = 0.5$ [Yokozeki, 1982]

in which unfiltered noise is troublesome: microscopic examination of fringes, and fringes formed on steep slopes. The former refers to the task of tracking along a moiré fringe in the image, where the smooth curve depicting the ideal center-ridge of a fringe is desired. What actually is observed is the fringe envelope modulated by structured noise. This can be seen in Figure A.10, where a bright moiré fringe lies along each vertical column of white 'diamonds', and a dark fringe centers on each black 'zig-zag stripe' running between each bright fringe. Clearly, the carrier noise is so dominant in this figure that it is difficult to say where the transition from a bright to a dark fringe occurs. While our superb cortical image processing system has little trouble in tracking the center of the fringes in this example, it is a problem to automate such tracking by machines. The subject of fringe tracking is examined in Appendix C.

The problem with carrier noise also manifests itself on steep surfaces, where the shadow lines have drastically changed in pitch and/or orientation with respect to the grid lines. When resolved by the optics, the coherent noise becomes a troublesome,

regular artifact, especially where it begins to approach the local spatial frequency of the moiré signal. Alias moiré patterns, running orthogonal to the true fringe pattern, tend to confuse the fringe pattern, and reduce the visibility of the fringes.

Carrier noise is generally of higher frequency than that of the moiré signal, and a slightly defocused lens that fails to resolve the carrier may effectively low-pass filter this noise. Yoshino [1982] described a Fourier optical filtering technique that selectively removes the grid noise. Ideally, the method calls for placing a slit aperture in the iris plane (i.e., the Fourier spatial-frequency domain) of the lens of the recording camera, although Yoshino reported good results by simply placing the filter in front of the lens to avoid dismantling it. The long axis of the slit is made parallel to the grid lines, and the width of the slit is chosen to filter-out spatial frequencies at, or above, the fundamental frequency of the grid. The length of the slit aperture can be proportionally increased to maintain constant pupil area with regard to exposure.

Another approach to the problem is to suppress carrier noise by integration. One may sum with an open camera shutter the 'many-shifted-views' of the grid translation techniques described in Section A.10.1. Or, in the case of electronic imaging, it may be possible to arrange for 180° phase-shifted distorted-grid images that can be electronically summed to exactly cancel the carrier noise [Kugel and Lanzl, 1979]. Udea [1979] developed a digital filter for his electronic-grating moiré method based on this principle, and examples of both filtered and unfiltered images from his system are shown in Figure 2.29 on page 62.

A.8.2 Contrast

Contrast is generally defined as the ratio of the difference over the sum of the maximum and minimum signals. In the case of image intensities, one writes

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (\text{A.37})$$

Moiré fringe contrast is a complex topic. It is dependent upon the reflectance and shape properties of the object, and upon the design of the moiré instrument com-

ponents. Illumination, grid design, imaging optics, and image detector sensitivity and linearity, are all important factors. Moiré fringe contrast is measured *across* the fringes (*i.e.*, orthogonal to the fringe direction, which in the case of fringes that are contours, is in the direction of the surface gradient).

Loosely speaking, moiré fringe contrast is maximized if the grids and their shadows are *perfectly* resolved, and if the shadows are *perfectly* black and the light stripes are *perfectly* reflected at the surface of the object. Figure A.10 shows a simple pattern and illustrates an important point in discussing the contrast of moiré fringes: the maximum and minimum intensity signals are averages taken along the fringes to smear any carrier noise, rather than simply using local absolute intensities. Thus, the transmitted intensity profile drawn in Figure A.10 is a triangular function ranging between 0 and 0.5.

In this ideal example, the contrast is perfect ($C = 1$), since even though the bright fringes are only middle-gray on average, the black areas are truly zero. The contrast is degraded if the grid lines or their shadows become fuzzy or no longer appear perfectly black. Illumination from the light source that is reflected directly from the grid lines to the camera (in a shadow-moiré arrangement), and ambient light spilling onto the object that spoils the quality of the grid shadows, significantly degrade the fringe contrast available in the scene [Takasaki, 1981]. Naturally, the act of recording the scene, for example on photographic emulsions, will affect the contrast in accordance with the response function of the detector. Janssens *et al.* [1985] described a theoretical study on the effect of extended light sources and finite-aperture recording systems on the visibility of moiré fringes. They included experimental evidence for the predicted degradation in visibility.

A.8.3 Sharpness

Sharpness refers to the narrowing of the bright/dark fringe boundary about the center of a fringe. This is useful, as it reduces the uncertainty in tracking the center of the sharpened fringe. Fringe sharpening may be invoked by altering the transmission profile of the grids. Binary grids have a square-wave transmittance,

Figure A.11: Demonstration of moiré fringe sharpening. The binary grids have bar-to-space ratios of 10:1 and 1:10 (low and high transmittance, respectively), and have been superimposed in combinations of (a) high+high, (b) high+low and (c) low+low. The latter combinations produce noticeably sharper dark or sharper light fringes. [Yokozeki, 1982]

and are composed of alternating opaque bars and transparent spaces. Binary grids with bars and spaces of equal width are called *Ronchi rulings* and are the most commonly used profiles. A model for fringe sharpening based upon grids that depart from the Ronchi profile was discussed in [Yokozeki, 1982]. Figure A.11 shows some examples of moiré fringe sharpening based on this principle, where the bar-to-space ratios were combinations of 10:1 and 1:10. The use of two-colour grids to improve fringe sharpening was discussed in [Subramanian and Selvan, 1983].

A.9 Surface Slope Limits and Alias Moiré Fringes

Large surface inclinations tend to broaden the grid shadow pattern on the surface, lowering its effective spatial frequency. Takasaki [1979a], and other authors, pointed out that this low end of the grid-shadow frequency spectrum tends to overlap the high moiré frequencies that characterize these surface inclinations, and gives rise to alias moiré patterns. Figure A.12 shows the alias moiré that typifies adequately-illuminated surfaces with steep inclination. Halioua *et al.* [1983] commented that with stationary grids, alias moiré was noticeable on surfaces with as little as 30°

Figure A.12: Moiré contours on a full-size mannequin. $s = 1$ mm, $l = 200$ cm, $d = 100$ cm, $\Delta h = 2$ mm. The vertical distance of the white fiducial lines was 100 mm. The vertical-string grid was stationary during exposure. Note the alias moiré fringes on the left and right cheeks [Takasaki, 1979a]

inclination. Perrin and Thomas [1979] discussed slope limits from the theoretical aspects of their phase-modulated carrier frequency analysis. The equations indicated that the upper bound is about 60° of inclination for a tilted plate, and is less for more general shapes.

The above comments pertain to stationary grids. In the following section, grid translation techniques are described that eliminate the alias term and increase the surface slope limits. In their experiments with translated projection grids, Halioua *et al.* [1983], could produce high-definition fringes on surfaces inclined up to 85° . Such fringes were actually *finer* than the grid line shadows, and also typify the extended range of slope limits due to translating shadow-moiré grids.

A.10 Grid Phase Shifting

A.10.1 Grid Shifting for Pattern Noise Reduction

As mentioned in the development of Equation A.11, the cosine term representing the moiré signal is independent of the initial phase ν_0 of the shadow-moiré grid, whereas the other three terms retained this dependence. This subtle property was noted by Takasaki [1970], who appreciated the profound impact on the quality of moiré images by translating the grid of a shadow-moiré system in its plane orthogonal to the grid line direction *during* the exposure. If the grid is continuously translated at least $s/2$ (half a grid pitch) during the exposure, then the individual grid lines, their shadows, and the grid/shadow sum-term are effectively eliminated as an integrated blurr, and only the stationary moiré pattern is resolved. Allen and Meadows [1971] analyzed the case of summing four discrete phase-shifts to eliminate the alias moiré terms, as well as the case of continuous translation. An example of the benefits offered by shadow-grid translation is shown in Figure A.13.

The translation speed must be tailored to the required exposure. For example, a 1/10 second exposure when using a grid with 2 mm pitch requires translation velocities of at least 1 cm per second to traverse one grid line during the exposure. Takasaki [1982] recommended that up to ten grid lines be traversed to improve the

Figure A.13: Demonstration of the grid translation method. The moiré patterns were produced by a stationary grid (left) and a grid which was translated during the exposure (right). The object was a mannequin, and the photographs were produced by placing the camera 200 mm from the grid of the NRC Moiré Contourgraph.

accuracy of the moiré pattern by averaging out the pitch errors of individual lines.

A variation on grid translation to smooth alias fringes in a shadow-moiré system was discovered by Chu and Block [1982]. Their moiré apparatus had four legs that extended from the camera housing to hold the grid at a calibrated distance (Fig. A.14). The legs could be adjusted to render the grid plane parallel to the camera housing, and these authors induced the fringe-averaging effect of a translated shadow-grid by *tapping* the side of the suspended grid during exposure to cause small in-plane vibrations or oscillations (Fig. A.15).

These smoothing methods are not restricted to shadow-moiré systems. Takasaki remarked that the same result could be invoked in a projection-moiré system if the two grids were translated simultaneously (Fig. A.16). Such a system with mechanically-coupled encoding and demodulation grids was reported by Halioua *et al.* [1982], with excellent results, as shown in Figure A.17. Udea's video-based projection-moiré system [1979] is a good example of virtual grid translation that can be realized with electronic signal processing (see Fig. 2.30 on p. 62). However,

Figure A.14: A shadow-moiré system with an oscillating grid that was designed for radiation therapy planning. C = camera (127 mm focal-length lens with Polaroid back), G = grid, $25 \times 25 \text{ cm}^2$ with 4.8 mm pitch, attached to a platform with four retractable legs for height adjustment, and L = two 15 watt light sources. The rigidly-suspended grid could be made to oscillate side-to-side by tapping it [Chu and Bloch, 1982]

Figure A.15: Demonstration of the grid oscillation method. These moiré contour patterns are of a breast phantom used in radiation therapy planning: (a) with a stationary shadow-grid, and (b) with the rigidly-suspended grid set into in-plane oscillation by hand. [Chu and Bloch, 1982]

Figure A.16: Schematic of translated-grid projection-moiré system. In this arrangement, the two grids were translated simultaneously in-plane to eliminate the alias moiré fringes [Halioua *et al.*: 1983]

despite the obvious visual improvement in the fringe patterns, smoothing is not a wide-spread practice with projection-moiré systems. Generally, it is costly to engineer temporal control over the grids. Systems that do have this control usually employ it for automated phase-shift fringe ordering, as described below. In most applications, smooth patterns are not as important as automated pattern analysis.

A.10.2 Grid Shifting for Phase-Lock Analysis

In projection-moiré systems, the fringe elevation on the surface of the object depends upon the relative phase between the encoding and the analysis grids. This dependence was not shown explicitly in Equation A.35, but for completeness its properties will be mentioned here. One may distinguish between two types of phase-shifting in projection-moiré systems:

- **Demodulation-Grid Translation:** By translating the demodulation grid relative to the distorted-grid image, the fringes can be made to 'move' uphill or downhill in accordance with the phase change.

(a)

Figure A.17: Mechanically-linked grid translation in a double-projection system. (a) Schematic of the double-projection arrangement. Comparative results, showing (b) the moving-grid pattern and (c) the stationary-grid pattern [Halioua *et al.*, 1983]

- **Demodulation-Grid Rotation:** The rotated straight-line grid has the same effect as demodulating the scene with the distorted-grid image of an inclined plane. As described in the contour-difference method in Section A.5, this results in fringes that represent the difference between the generating surfaces. By rotating the grid slightly, the fringe planes 'tip', causing the fringes to shift accordingly uphill or downhill as they are re-oriented.

Thus, by shifting the relative grid phase by a known translation or rotation, the fringes can be made to undergo a prescribed shift in elevation. This is termed *phase-lock analysis*, and knowing that all the fringes will shift, say, uphill, it becomes trivial to determine the relative ordering of the fringes.

Phase-lock analysis is also possible with shadow-moiré systems. Reid *et al.* [1984, 1985] invented a method in which the shadow-moiré grid was mechanically moved along the optical axis (as opposed to in-plane translation). This amounts to varying l in Equation A.28, and causes the fringe phase to shift as in the projection-moiré methods.

The principles of manual and automated phase-lock techniques for fringe ordering are described in more detail in Chapter 2. Systems that embody phase-lock methods for digital elevation determination from moiré fringes are described in Appendix C.

Appendix B

Applications of Moiré Topography

Moiré topography is a photogrammetric technique, whereby shape information is derived from images of the object. In many applications, the images may be recorded on film or loaded into the memory of a computer. Working with recorded images presents some distinct advantages to the user: dynamic or hazardous objects can be remotely assessed; additional measurements can be taken from the image at any time; and 2-D information can be extracted, yielding 3-D shape if the images are suitably encoded. Moiré topography has some additional advantages over other photogrammetric methods (*e.g.*, stereo-photogrammetry) in that it uses simple equipment, requires only a single image, encodes 3-D shape information directly as visible contours in the image, and is sensitive over a broad range of sizes (micrometer to decimeter contour intervals). It is not surprising that moiré topography has been widely applied, particularly in the fields of medicine and engineering. A brief overview is presented here to provide the reader with an impression of the potential of this shape-measurement technique in those fields.

B.1 Medical Applications

As a whole-body, 3-D surface-measuring technique, moiré topography has been especially welcomed by the medical profession. Its non-invasive illumination (compared to x-rays) and directness of shape representation (contour fringes) generated

considerable excitement amongst biomedical engineers. It was, in particular, the late Hiroshi Takasaki (*d.*1985) who pioneered the application of the moiré principle to the measurement of the human body form [Takasaki, 1970], and who, over the years, made important contributions to perfect the photographic technique for producing high quality moiré fringes on living subjects [Takasaki, 1973, 1979a, 1979b, 1981 and 1982].

As a result of work by Takasaki and his colleagues, the moiré technique has been accepted in Japan for use in school screening programs for the early detection of scoliosis [Ohtsuka *et al.*, 1981; Harada *et al.*, 1981 and 1986]. School screening projects have also been conducted in other countries; examples include work in Canada [Adair *et al.*, 1977], Sweden [Willner, 1979 and 1983], Italy [Frontino *et al.*, 1983] and Austria [Neugebauer and Windischbauer, 1987].

The interest in applying the technique to the measurement of human body forms has been most active in the detection and diagnosis of spinal deformity. Since 1980, four successful international symposia on this subject have been organized in North America and Europe [Moreland *et al.*, 1981a; Drerup *et al.*, 1983a; Harris and Turner-Smith, 1986 and Stokes *et al.*, 1987]. An extensive bibliography on moiré fringe topography and spinal deformity appeared in [Moreland *et al.*, 1981a], with most of the 256 citations covering the 1970-1980 period.

Scoliosis, kyphosis and lordosis are skeletal deformities, whereas moiré topography measures the shape of exterior surfaces. Work is being done at different institutes to establish the degree of correlation between back surface measurements from moiré topograms and radiographic measurements of the spine [Shinoto *et al.*, 1981; Stokes *et al.*, 1983; Willner and Willner, 1982], as well as to compare other surface topography methods in this application [Neugebauer and Windischbauer, 1986]. In addition, associated asymmetries in scoliosis have been investigated by moiré topography, such as facial asymmetry [Mehta, 1981] and anterior chest wall deformities [Shochat and Csongradi, 1983].

Although the human back is a complex, variable shape, scoliosis screening is concerned primarily with detecting asymmetry about the spinal column, and thus

Figure B.1: Moiré contours for monitoring body changes during pregnancy. The potential for recording the complete form of both breasts in a single frontal topogram may be realized by raising the arms over the head, as demonstrated by the subject in the left photograph [Tympanidis and Karras, 1984]

has good prospects for eventual automation. At present, interactive systems are used for detailed analysis of moiré images in diagnosis, with the fringe order being assigned by the operator [Yatagai *et al.*, 1983; Suzuki *et al.*, 1986; Ong *et al.*, 1987]. Another potential field of application for these semi-automatic techniques is dynamic study, such as gait analysis [Lewis *et al.*, 1981], torso expansion/contraction during respiration [Suzuki, N. *et al.*, 1981b], dynamic posture studies [Asazuma *et al.*, 1986; Swain *et al.*, 1986], and the effect of leg-length inequalities on spinal curvature [Swain *et al.*, 1987].

Other applications involving the analysis of human body forms include the determination of volume and surface area of human body parts during pregnancy [Karras and Tympanidis, 1983; Karras, 1984; Tympanidis and Karras, 1984], and morphometric studies of the feet in clinical diagnosis provided by moiré images of the sole and lateral views of the foot [Soares, 1983; Soares *et al.*, 1983]. An example from a study of the variation in breast geometry during pregnancy is shown in Figure B.1.

Moiré has also been used in the field of reconstruction of human anatomical parts, such as leg and foot prostheses [Duncan, 1986; Duncan and Mair, 1983; Saunders, 1981 and 1983]. A moiré device was fitted on a commercial simulator to obtain patient contours for radiation therapy planning and verification of patient position [Chu and Bloch, 1982]. In a different field, moiré topography was used to determine the corneal configuration of the human eye [Chander *et al.*, 1976].

The special considerations when designing moiré topography apparatus for the evaluation of the human body are treated in detail in [Pekelsky and van Wijk, 1988], but some brief comments follow. Torso and limb measurements generally require fringe intervals of 5 mm, which involve a shadow-grid pitch in the order of a millimeter. At this level of sensitivity, physical limitations, such as the finite size of the light source, can normally be overcome by simple precautions (such as the use of a condenser lens in the illumination system to make the lamp sufficiently 'point-like'). In certain cases, the optical properties of the skin have been found to limit the image quality of the moiré fringes. The contrast between light and shadow projected on the skin is spoiled by short-wavelength scattering and reflection from deeper skin layers, which become particularly disturbing with finer moiré patterns. The use of blue-filtered light has been suggested to improve the fringe contrast [Drerup, 1978; Takasaki, 1973].

Normally, the above applications are based on the use of incoherent light sources. A biomedical application of a different accuracy requirement is the real-time contouring of tooth imprints for the measurement of wear in dental restorations. In this application, the moiré fringes were formed by the interference of two coherent laser beams. A 5 μm contour interval was obtained by the technique [Jongsma *et al.*, 1985 and 1986].

Biomedical applications have not been restricted to human subjects, and the practical problems of monitoring domestic livestock have been reported in [Miles and Speight, 1975] and [Windischbauer and Schedle, 1983].

B.2 Engineering Applications

The engineering application of moiré fringe patterns is probably best known in the field of stress analysis. These patterns are not restricted to topographic contours, but may be designed to depict in-plane deformations as well. Moiré methods for non-destructive testing were reviewed by Chiang [1978].

Topographic moiré contours can be produced with simple equipment and ordinary incandescent illumination, which is particularly useful if the apparatus must be scaled to handle large engineering structures, such as the 3.6 m $\times$ 1.9 m brick arch described in [Burch and Forno, 1982]. Using incoherent illumination, both shadow- and projection-moiré topography are widely used in engineering where the accuracy requirements call for contour intervals ranging from tens of centimeters down to approximately 0.1 mm. A typical example is shown in Figure B.2, where projection-moiré has been used to map the shape of a three-lobed cam, with a contour interval of 1 mm [Reid, 1984].

Members of the aeronautical industry were among the first to use moiré topography. Their interest lay with the measurement of out-of-plane deformations of large structural members of aircraft fuselages. Moiré topography is particularly suited for on-line inspection of mechanical parts [Dessus *et al.*, 1975]. In this case, a *demodulation grid* may be created by photographing the image of a grid projected onto a master object, and subsequently be used in the moiré analysis of a test object, with small differences or defects creating characteristic moiré patterns. Yatagai and Ide-sawa [1977] described a technique for computer-generating the distorted-grid image of a 'perfect' part to serve as a demodulation grid for the inspection of real parts.

As in the medical field, the scope of applications has been further broadened by the introduction of video techniques. These extend the use of moiré topography into areas such as on-line dimensional control in computer-aided manufacturing (CAM). The use of video techniques, particularly those using solid-state cameras with their high geometric stability, allows one to obtain geometric information on the object in real-time, without the need for an intermediate photographic stage.

Figure B.2: Topographic moiré fringes on a three-lobed cam, obtained by the projection-moiré technique, and with a contour interval of approximately 1 mm. (Courtesy of G. T. Reid, National Engineering Laboratory, Glasgow, UK)

Video techniques also extend the use of moiré methods to industrial robotics, where structured-light is often used in machine vision for automated recognition and positioning of objects and tooling in CAM operations [Lamy *et al.*, 1983; Hovanesan and Hung, 1983].

The moiré technique has also been used in conjunction with high-speed photography to study dynamic phenomena, such as propagation shockwaves and vibrations, often in real-time [Dessus *et al.*, 1975; Hovanesan and Hung, 1971]. A wide range of applications of the shadow- and projection-moiré techniques are reviewed in [Pirodda, 1982].

The use of shadow- and projection-moiré techniques in conjunction with conventional illumination and imaging systems is limited to cases where the pitch of the grid can be sufficiently coarse to ensure the necessary depth of field. Higher resolutions are possible when a coherent light source is used, as this allows the generation of extremely fine grids over extended working depths. An example is *moiré interferometry*, where a fine grid is formed on the surface of the specimen and viewed together with a superimposed demodulation grid, which itself is formed by two interfering beams of coherent laser light. This virtual demodulation grid interacts with the distorted grid to form an interference pattern directly on the object, which can then be recorded by a camera [Post, 1985].

Moiré interferometry has also been used in conjunction with the scanning electron microscope (SEM). In this case, the distorted grid is recorded in a photosensitive coating on the surface of the object as it is illuminated by a pair of interfering laser beams. By imaging the object in a SEM, and providing a demodulation grid by suitable selection of the scan raster of the microscope, moiré fringes can be created when the scan raster is aligned with the grid on the object. The technique was used to measure small, deeply-curved surfaces, such as the conical Rockwell diamond indenter shown in Figure B.3, with a contour interval of $4.2 \mu\text{m}$ [Robinson, 1981].

Additional metrological applications of moiré topography, including measurements of deformations, vibrations and displacements, and determination of refractive index and refractive index gradients, were recently reviewed in [Reid, 1984].

Figure B.3: Scanning electron micrograph showing moiré fringes on a Rockwell indenter, at a contour interval of approximately $4.2 \mu\text{m}$ (Courtesy of D. W. Robinson, National Physical Laboratory, Teddington, UK).

Appendix C

Digital Elevation Determination from Moiré Fringes

C.1 Introduction

One of the key advantages of moiré topography is that the 3-D shape of an object is encoded as a visible contour pattern in a single image of the object. Coupled with the simplicity of the apparatus required to generate the fringe patterns, the moiré method has been widely applied in the fields of medicine and engineering (see App. B).

In some cases, the goal of the user can be met by examining the fringe pattern to detect critical shapes in the fringe curves themselves. These pattern recognition techniques use a variety of criteria, such as the *symmetry* of the fringe pattern about some natural axis of the object (e.g., about the spinal column in a human back), or the *frequency* of the fringes, which increases in areas of high curvature (e.g., at a bulge in a defective pressure vessel), or by localized *signatures* in the fringe pattern (e.g., a small 'bulls-eye' of fringes may occur at a dent in an auto panel). The methods and equipment developed for moiré fringe pattern interpretation were recently reviewed in [Pekelsky and van Wijk, 1988].

In this appendix, the task of extracting 3-D numerical coordinate information is briefly examined. The emphasis is on the various methods currently practiced in

Figure C.1: Fringe detection by densitometric scanning (a) The fringe pattern of a small region of an investment casting, with pattern noise clearly visible. (b) A microdensitometer trace, which has a false fringe peak (arrow). [Reid, 1983]

decoding the distribution of shadow intensities in the fringe pattern into a distribution of elevations that represent the shape of the object. When interpolated into a regular array, such a distribution is called a Digital Elevation Model or DEM.

There are several approaches to decoding the moiré pattern into an array of heights. The densitometric methods tend to be off-line techniques that work with recorded fixed-pattern images. They analyze the intensity values in the image, using the fact that the bright and dark fringes are on loci of constant moiré fringe phase, and have locally high and low intensities, respectively. Fringe *tracking* attempts to directly trace the contour, whereas fringe *profiling* tries to find fringe intersections in a set of densitometric profiles scanned across the image. Trackers may need operator intervention to find the beginning of a fringe, and then may fail to track in the presence of artifacts such as object image detail, image noise or poor fringe contrast. Profilers are also affected by noise (see Fig. C.1), and may have trouble connecting fringes between profiles. Both methods may collapse in the presence of surface discontinuities, such as shadowed skin folds on the human body.

Phase-lock analysis overcomes many of these weaknesses because dynamic fringe encoding allows 'several looks' to resolve problems. These methods tend to be used

in on-line systems, or else off-line with the distorted-grid image. Instead of moving about the frozen image, phase-lock methods analyze the fringes as they are driven past each point in the image, thereby assessing the moiré-phase (i.e., the fractional ϕ th fringe order) of each point. The moiré fringe interval equation can then be used to compute the elevations. The principle of moiré phase-lock analysis is described in Section A.10.2, and its application to fringe ordering is reviewed in Chapter 2.

Examples of systems that embody phase-lock, fringe tracking and fringe profiling are given below. Moiré fringes have much in common with holographic fringes, and there is considerable overlap in the methods of evaluation of each species of the generic class of *interference fringes*. The subject of automatic fringe analysis was recently reviewed in [Reid, 1986], and although the 87 referenced articles were predominantly concerned with holographic applications, much of the commentary had relevance to moiré fringe analysis.

C.2 Densitometric Fringe Tracking

C.2.1 Manual Contour Digitization

In the field of spinal deformity research, Windischbauer [1981b] has promoted the DEM for representing the body surface to free the practitioner from the problems of orientation that plague asymmetry judgements based on pattern interpretation methods. It was argued that 3-D shape analysis could be better executed on a numerical surface representation, where rigid-body rotations and translations could be mathematically removed [Neugebauer, 1983]. He used a desktop PC, programmed in BASIC, and facilitated image I/O with a graphics pad and plotter. On a typical topogram of a human back, manual digitizing of 600–1000 points was done in 30–45 minutes [Windischbauer and Neugebauer, 1986]. Using combinations of linear and spline interpolation, orthogonal-projection maps like that shown in Figure C.2 were produced.

Figure C.2: Topographic contours of a human body. (a) Finite-perspective moiré fringes produced using the Windischbauer shadow-moiré apparatus. (b) Computer-corrected orthogonal-projection map, with contour curves fitted by spline interpolation through manually-digitized points from the moiré topogram. [Windischbauer, 1981b]

Figure C.3: Tracking moiré fringes of a cat eardrum. (a) The digital image, with computer-generated 'seeds' to start fringe tracking. (b) Scheme for selecting pixel neighbours to consider in the forward direction of the trace. (c) The first three fringes traced by the computer, with an error near the end of the third fringe [Funnell, 1981]

C.2.2 Automated Fringe Tracking

In some applications, the moiré pattern is relatively simple, and simple approaches to fringe tracking can be used. An example is the moiré topogram of a cat eardrum replica, shown in Figure C.3. Funnell [1981] manually indicated the center of the pattern of concentric circles, and the software then searched along a densitometric profile in vertical direction in the image. Alternating maximum and minimum values were 'discovered' at intersections with bright and dark fringes, and these were used as starting points from which subsequent tracking was initiated for each fringe (Fig. C.3a). Funnell used a simple nearest-neighbour criterion to move to the next pixel when tracking along a fringe. The choice of neighbours was limited to those within $\pm 90^\circ$ of the local forward tracking direction, as shown in part (b) of the figure, and the brightest or darkest of those pixel neighbours was selected, depending on whether the fringe being traced was bright or dark, respectively. Part (c) of the figure displays an error in which the tracking side-stepped to, and continued along, a neighbouring fringe.

Figure C.4: A noisy holographic fringe pattern in which expert software has traced a contour line by tracking a fringe [Robinson, 1983]

C.2.3 Tracking Noisy Fringes

Figure C.4 is a good example of a noisy holographic fringe pattern that has been analyzed by a densitometric fringe tracking method. This example was taken from [Robinson, 1983], in which software to cope with noise in a variety of pattern types was described. Usually, advantage is taken of *a priori* knowledge of the fringe pattern, especially if the ideal form of the curves has a simple mathematical expression. Candidates for this approach are the concentric circles and nearly straight lines that often occur in quality control situations. Thus, one may coax the tracking software through the noise, placing higher confidence in searching further in the forward direction, and taking into account the likely changes in direction. The strategy used for the pattern in Figure C.4 is shown in Figure C.5, and involves global hunts to find a good starting point, local tracking in several directions to pick up the best starting direction, and limited tracking at several radii to select a suitable curvature.

The less that is known about the fringe shape, the more expert the strategy

Figure C.5: Processing sequence for tracing the contours of the concentric-circular fringes in the noisy pattern shown in Figure C.4 [Robinson, 1983]

Figure C.6: A 5×5 non-linear directional fringe peak detector. (a) The pixel filter coefficient matrix (b) The detection directions [Yatagai *et al.*, 1982a]

must be. Yatagai *et al.* [1982a] described a technique that works well on moiré fringe patterns with an arbitrary shape. They first preprocess the digital image, using 3×3 and 5×5 neighbourhood operators to locally smooth the intensity and eliminate speckle, and apply a histogram modification method to enhance the fringe contrast. Then, a sparse 5×5 non-linear directional fringe peak detector is passed over the image, to produce a new 1-bit image for each of the four directions shown in Figure C.6.

For each direction, the grayness transition that characterizes crossing a fringe peak is tested in the 8-bit input image. If the directional transition is above a selected threshold, then that pixel is set in the corresponding 1-bit filter image. The four 1-bit images are then logically summed, to produce a 1-bit *fringe skeleton* image. The skeleton pixels are set to one if two adjacent directions are detected at a pixel, otherwise the pixel is set to zero. An example of this filtering operation is shown in Figure C.7, which also includes further processing to thin the filtered skeletons to single-pixel-width filaments. Yatagai *et al.* discussed thinning algorithms, and strategies to handle broken contours when finally tracking the thinned skeletons. Their invention of the described peak-detection filter is an important contribution to densitometric fringe analysis, and their method has been adopted by other researchers [Ong *et al.*, 1987].

C.2.4 Ordering Tracked Fringes

The relative ordering of tracked fringes is the central topic of this thesis, and a variety of methods is reviewed in Chapter 2. Only a few of the relative fringe ordering methods listed in Chapter 2 have been applied to automated densitometric analysis. In some cases, the fringes may be quasi-parallel lines that never form closed loops and are always of monotonic-increasing order (as in the oblique-moiré and tilted-moiré patterns described in Sect. 2.2.4), and then tracking and subsequent ordering may be automatically performed as reported in [Birch *et al.*, 1983]. For a specific class of objects and limited variation of the fringe pattern, one can embody in the software the *a priori* or expected ordering of the fringes [Yatagai *et al.*, 1982a]. A

Figure C.7: Skeletoning and thinning of interferometric fringes. (a) Original interferometric fringe pattern. Skeletons of the original fringes, in the case of detecting peaks in (b) one or more, and (c) two or more of the four directions, respectively. (d) Thinned skeletons of (c), with the area outside the boundary removed [Yatagai *et al.*, 1982a]

more expert system may allow greater variation in the fringe pattern, and even cope with sparse or vague *a priori* knowledge. This is unsatisfactory for a general-purpose fringe analyzer, however, and most systems rely upon the operator to intervene (see Fig. 2.3 on p. 26).

As an aside: for fully-automated densitometric analysis, the photometric ordering technique described in this thesis has solved many of the shortcomings of the previous methods. The form embodied in the test software requires one pair of contours somewhere in the image to have a known relationship, after which, all others are ordered by photometric analysis. The author believes that expert software can be developed to start the process with an unordered pair, by considering the three ordering possibilities, and selecting the most photometrically-consistent possibility to seed the rest of the ordering process. This idea is developed in Chapter 10.

C.3 Densitometric Fringe Profiling

C.3.1 Sparse Profiles

In many applications, the moiré fringes are rather smooth curves that meander about the image with few abrupt changes in direction. This means that one could reconstruct the contour curves from a sparse sampling of points along the curves, using for example, spline interpolation functions. Here *sparse* means significantly fewer points ($< 10\%$) than the immediately-adjacent points that define a digital image contour. One practical approach to sampling the contour pattern is to generate a coarse raster to regularly section the image. Patterns treated in this fashion are reduced to a sequence of 1-D fringe detection and identification tasks, which represent a significant reduction in processing compared to full scene 2-D analysis.

The RIFRAN interactive fringe analyzer is a good example of a system based upon sparse profiles, [Yatagai and Idesawa, 1982; Yatagai *et al.*, 1982b]. The topogram is digitized by a TV camera and stored in the frame memory. The digital image is continuously displayed on a monitor, with various graphical information overlaid for the benefit of the operator (Fig. C.8).

Figure C.8: Display of the RIFRAN interactive fringe analyzer. Superimposed on the fringe pattern to be analyzed is a series of ten horizontal section lines through the image. At the sides of the image are labelled command areas that the operator can activate with a light pen [Yatagai *et al.*, 1982b]

In particular, the selectable horizontal profile lines that sectioned the pattern are overlaid (spaced in the figure about 40–50 pixels apart). The analysis sequence is shown in Figure C.9. The operator uses a light pen to indicate the photogrammetric control in the image, to establish the vertical analysis-line position, and the horizontal sectioning interval. Peak detection is then automatic, and the operator is allowed to edit the results. At each profile section, the operator is solicited to indicate the relative ordering of the fringes, as shown in Figure C.10a, and this aspect relies upon the operator's familiarity with the object. In a subsequent step, the relative fringe ordering between adjacent profiles is adjusted by the operator, providing the linkage between the scans (Fig. C.10b). The surface can be reconstructed with a denser net of values using spline interpolation, as shown in Figure C.11.

Figure C.9: Processing sequence for the RIFRAN interactive fringe analyzer [Yatagai et al., 1982b]

Figure C.10: Fringe order determination in the RIFRAN system. (a) The display of ordering by the software after the operator has indicated the change of slope-sign points along the horizontal section (b) The ordering between nearby peaks detected along the vertical section (L_1) and each horizontal section is provided by the operator [Yatagai *et al.*, 1982b]

Figure C.11: Perspective plot of a reconstructed human back. Solid lines show the detailed structure of a 3-D model which was produced by interpolating the sparse profiles (broken lines) analyzed using the RIFRAN system [Yatagai *et al.*, 1982b]

Figure C.12: Densitometric analysis by fringe profiling. The pattern is a time-average interferogram and shows a resonant mode of a turbine fan blade. The computer automatically identified the fringe maxima. [Robinson and Williams, 1986]

C.3.2 Ordering Profiled Fringes

Earlier remarks on ordering tracked fringes also apply to profiled fringes, with the added complication of defining the interprofile fringe continuity. The RIFRAN solution requires a high degree of operator intervention in this respect, perhaps because the profiles are too sparse to rely upon automated ordering based upon trends in neighbouring profiles. Robinson and Williams [1985] revealed their automated profiling strategy: detected fringes are assigned a relative fringe order number by incrementing a counter every time a fringe is crossed, and when a turning point (slope-sign reversal) is detected the count direction is reversed. An example from their work with holographic non-destructive testing is shown in Figure C.12.

Turning points are detected by looking for characteristic clues, such as changes in the sign of the fringe angle, peaks in the derivative of the local fringe spacing, and positive identification of the connection of two fringe peaks by a continuous fringe. In some cases, routines for tracking along a fringe to confirm its relative order number are necessary. The author concurs with the comment by Robinson and Williams that expert software can go a long way in ordering densitometrically-traced fringes, but phase-lock analysis avoids the whole issue of fringe ordering.

C.4 Phase-Lock Analysis

C.4.1 Optical Phase-Lock Methods

The science of optical phase-lock analysis was considerably advanced in the mid-1970's at the University of Rochester. Moore and his colleagues perfected A-C interferometry, and subsequently moved the technology into the moiré arena. The systems reported in [Moore and Truax, 1979] and [Chan *et al.*, 1981] were prototypes, applied in real-time to small items (using 200 μm contours), and off-line to distorted-grid images of large items (using 2.5 mm contours), respectively. They demonstrate the potential of phase-locking an optically-modulated grid. These systems were described earlier in Chapter 2.

C.4.2 Mechanical Phase-Lock Methods

More recently, Reid *et al.* [1984, 1985] reported on the Auto-MATE (Automatic Moiré Analysis for Topographic Evaluation) moiré system. The schematic of the projection version is depicted in Figure C.13, and the prototype shadow-moiré version is shown in Figure C.14. In both versions, the grid is mechanically moved under phase-lock control.

Phase-shifted images, like those shown in Figure C.15, are captured by the charge-couple device (CCD) camera and stored in memory. The resultant series-of intensities at each pixel is processed to recover the moiré phase, which determines the local elevation. Details can be found in the referenced articles. The big advantage cited by Reid is that neither operator intervention nor *a priori* knowledge of the object is required, so the system is *fully* automatic. A further advantage of processing a whole digital image in this manner is that the planimetric resolution is usually sufficient, and interpolation is not required to densify the DEM.

The system is microcomputer-based, and it takes about 90 seconds to measure and analyze 28,000 points on a turbine fan blade with 0.5 mm contour intervals. Reid estimates the precision to be 1/180 of a contour. The accuracy is only

Figure C.13: Phase-lock moiré analysis using the Auto-MATE projection-moiré system. The projection grid is mechanically moved while a series of phase-shifted images are put into computer memory. [Reid *et al.*, 1984]

Figure C.14: Real-time fringe analysis with a personal computer. Shown here is the Auto-MATE shadow-moiré system, which employs phase-measuring techniques to automatically evaluate the moiré pattern. The test object is a small turbine fan blade. (Courtesy of G. T. Reid, National Engineering Laboratory, Glasgow, UK)

Figure C.15: Phase-shifted moiré fringe contour images of a fan blade. The photographs were taken at three different positions of the Auto-MATE projection grid [Reid *et al.*, 1985]

1/10 of a contour because of assumptions made on the geometry [Reid *et al.*, 1986]. This matches the accuracy recently reported for densitometric fringe analysis in [Ennos *et al.*, 1985]. Perspective plots of the fan blade from Figure C.15 and of an automobile door panel are shown in Figure C.16.

C.4.3 Electronic Phase-Lock Methods

In this section the detector-grid moiré systems introduced in Chapter 2 are briefly discussed in terms of their attributes for phase-lock analysis.

The schematic diagram for the *electronic grating moiré* system [Udea, 1979 and 1980], is shown in Figure C.17a. As mentioned in Section 2.4.3.2, Udea's system produced three phase-shifted binary patterns. When assigned to different colour-guns on a CRT display, the additive colours produced an asymmetric colouring of the fringes for human interpretation. Udea also developed dedicated hardware

Figure C.16: Perspective plots of objects analyzed by Auto-MATE. (a) A large automobile door panel (b) A small turbine fan blade [Reid *et al.*, 1985]

Figure C.17: Electronic-grating moiré system. (a) General layout of the phase-shift logic hardware. (b) Detail of the Section Generator hardware. [Udea, 1979]

Figure C.18: Examples of real-time section evaluation and display by the electronic-gating moiré method (a) A profile in the horizontal direction (b) A profile in the vertical direction. [Udea, 1979]

(Fig. C.17b) that could assign a numerical level to each of the three images, which, when summed, produced a unique code sequence. The logic circuits could automatically track the code along a selected vertical or horizontal section to produce in real-time the object elevation profile. Examples of this elegant technique are shown in Figure C.18, and it is reasonable to assume that after nearly a decade of progress in electronics, the same sectioning computations can now be applied to entire images in real-time at comparable costs.

The other important technique in this class is the *scanning moiré* method, pioneered by Idesawa and his co-workers [1977] as an off-line phase-lock analysis system. Later, it was rendered as a near real-time method using a modified FM-40 projection-moiré camera [Yatagai *et al.*, 1981b]. The optical arrangement is depicted in Figure C.19a. In part (b) of the figure, the distorted-grid image detected by the CCD scanner is shown. Each line of pixels from the CCD scanner is numerically resampled at different phases, using linear interpolation to estimate intersensor image brightness values. As described previously (Ch. 2), this resampling demodulates the distorted-grid image to form the moiré fringe pattern, a line at a time. By phase-shifting the resampling, three different-phased patterns can be recovered, from which the fringe-ordering along the profile is automatically deter-

Figure C.19: Real-time scanning moiré with a modified FM-40 projection-moiré camera. (a) Schematic of the camera modifications, showing the scanner that replaces the standard 35 mm film recorder (b) The distorted-grid image produced by the CCD scanner [Yatagai *et al.*, 1981b]

mined. In this fashion, like all moiré phase-lock methods, the relative elevations of the entire image can be automatically analyzed. Yatagai *et al.* do not cite the processing performance specifications, but they mention that their system was invoked on a DEC LSI 11/02 with only 64 kB of memory. A modern arrangement could probably analyze TV-frame images in real-time. Currently, video systems cannot offer the resolution and accuracy of film-based photogrammetry. For many applications, however, such as on-line machine vision for automated manufacturing, the real-time advantages of video systems make them important alternatives [El-Hakim *et al.*, 1988].

Appendix D

Program Listings for Task JFO

D.1 Introduction

This appendix contains a listing of the program modules that comprise Task JFO. The appendix is organized into four additional sections. The second section lists the command files that build the task. The third section lists the FORTRAN subroutine-type files, and the fourth section lists the FORTRAN include-type files. The last section contains examples of the data files used by the task. A local table of contents is provided here for convenience.

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D.2 Command Files to Build Task JFO

D.2.1 JFO.BLD

COMMAND FILE TO BUILD TASK JFO ON ARIES-II SYSTEM

JFO.TSK/FP/CP,JFO MAP/-SP=

AMFOMA
AMFOPR
AMFOAZ
AMFORN
AMFOEX
AMFORV
AMFONL
AMFOCH
AMFOLU

DR0:[100,377]COMMON/LB

LUN 8 PURPOSE

LUN 1 AR: CONFIGURATION FILE
LUN 2 IM: IMAGE DISK
LUN 3 DR2: INDEX FILE
LUN 4 DR2: CONTOUR FILE
LUN 5 DR2: COEFFICIENT FILE
LUN 6 DR2: MOIRE PARAMETER FILE


```

LUN 7  OC  TERMINAL
LUN 8  TT1  LINE PRINTER
LUN 9  HL:  DYNAMICALLY ASSIGNED

```

```

UNITS=9
ASG=AR:1,IM:2,DR2:3:4 5 6,OC:7,TT1:8,HL:9
VWDWS=1
VSECT=DPICOM 120000.20000

```

```

LIBR=F77RES RO
UIC=[200,200]
TASK=JFO
/

```

D.2.2 JFO.CMP

```

COMMAND FILE TO COMPILE ALL MODULES FOR
TASK 'JFO' ON ARIES-II SYSTEM

```

```

COMPILE ALL MODULES

```

```

F77 AMFOMA,AMFOMA=AMFOMA
F77 AMFOPR,AMFOPR=AMFOPR
F77 AMFOAZ,AMFOAZ=AMFOAZ
F77 AMFORN,AMFORN=AMFORN
F77 AMFOEX,AMFOEX=AMFOEX
F77 AMFORV,AMFORV=AMFORV
F77 AMFONL,AMFONL=AMFONL
F77 AMFOCH,AMFOCH=AMFOCH
F77 AMFOLU,AMFOLU=AMFOLU

```

D.2.3 JFO.DOC

```

COMMAND FILE TO DOC ALL MODULES FOR
TASK 'JFO'

```

```

DOC ALL MODULES

```

```

DOC JFO DOC
DOC JFO TOP
DOC JFO BLD
DOC JFO CMP
DOC AMFOMA.FTH
DOC AMFOPR.FTH
DOC AMFOAZ.FTH
DOC AMFORN.FTH
DOC AMFOEX.FTH
DOC AMFORV.FTH
DOC AMFONL.FTH
DOC AMFOCH.FTH

```

DOC AMFOLU.FTH
 DOC AMFOTC.INC
 DOC AMFOMR.INC
 DOC AMFOLU.INC

D.2.4 JFO.TOP

```

TOP LEVEL COMMAND FILE FOR TASK JFO

ASK $DOC DO YOU WISH TO DOC ALL THE FILES
    IFF $DOC .GOTO 5
        DOC ALL FILES
    @JFO.DOC
    .5:
    ASK $CON DO YOU WISH TO COMPILE ALL THE FILES
    IFF $CON .GOTO 10
        MODULE COMPILE
    @JFO.CMP
    .10:
    ASK $TKB DO YOU WISH TO BUILD THE TASK
    IFF $TKB .GOTO 15
        TASK BUILD
    TKB @JFO.BLD
    .15:
    ASK $INS DO YOU WISH TO INSTAL THIS TASK
    IFF $INS .GOTO 20
        INSTAL TASK
    DR1
    UIC 100 377
    SRD JFO.TSK/DE
    PIP /MV=DR2:[171,110]JFO.TSK
    DR2
    UIC 171,110
    .20:
    
```

D.3 FORTRAN Subroutine-Type Files for Task JFO

D.3.1 AMFOAZ.FTN

```

C+
C      ADDRESS      DR2:[171,110]
C      NAME         AMFOAZ.FTN
C      PURPOSE      CALCULATE THE AZIMUTH OF A NORMAL
C                   VECTOR AT A SPECIFIED POINT
C      LANGUAGE     FORTRAN
C      AUTHOR       DING XINHONG
C      DATE         3-DEC- 85
C      FIRST USED ON ARIES-II IMAGE ANALYSIS SYSTEM
C
C-----
C
C      DESCRIPTION
C
C      THIS IS A SUBROUTINE FOR TASK 'JFO'. IT CALCULATES THE AZIMUTH
C      OF A NORMAL VECTOR AT A SPECIFIED POINT OF A CONTOUR. TO CALCULATE
C      THE AZIMUTH OF A NORMAL VECTOR, THIS SUBROUTINE NEEDS 4 POINTS:
C      GIVEN POINT ITSELF, LEFT POINT, RIGHT POINT AND THE MIDDLE POINT.
C
C-----
C
C      VARIABLE DEFINITION SECTION
C
C      INPUT  VARIABLES
C
C      IPXLEF  PIXEL VALUE FOR LEFT POINT
C      ILXLEF  LINE VALUE FOR LEFT POINT
C      IPXMID  PIXEL VALUE FOR MIDDLE POINT
C      ILXMID  LINE VALUE FOR MIDDLE POINT
C      IPXRIG  PIXEL VALUE FOR RIGHT POINT
C      ILXRIG  LINE VALUE FOR RIGHT POINT
C
C      OUTPUT VARIABLES
C
C      IERROR  ERROR FLAG
C      RVECT1  (2,2)NORMAL VECTOR
C      RVECT2  (2,2)OPPOSITE OF NORMAL VECTOR
C      RAZMT1  AZIMUTH OF NORMAL VECTOR
C      RAZMT2  AZIMUTH OF OPPOSITE NORMAL VECTOR
C
C      LOCAL VARIABLES
C
C      IPL     PIXEL # OF LEFT POINT
C      ILL     LINE # OF LEFT POINT
C      IPM     PIXEL # OF MIDDLE POINT
C      ILM     LINE # OF MIDDLE POINT
C      IPR     PIXEL # OF RIGHT POINT
C      ILR     LINE # OF RIGHT POINT
C      RL1     VECTOR LENGTH
C      RL2     VECTOR LENGTH
C      R       RATIO OF TWO VECTOR LENGTH
C      IP1     PIXEL # OF END POINT OF VECTOR 1
C      IL1     LINE # OF END POINT OF VECTOR 1
C      IP2     PIXEL # OF END POINT OF VECTOR 2
C      IL2     LINE # OF END POINT OF VECTOR 1
C      RMP     REAL VERSION OF IPM

```

```

C      RML      REAL VERSION OF ILM
C      RRP      REAL VERSION OF IP2
C      REL      REAL VERSION OF IL2
C      RAZ      atan()
C      RC       COEFFICIENT FROM RADIUS TO ANGLE
C
C-----

C      DECLARATIONS
C
C      SUBROUTINE AMFOAZ(ILLILL,IPPPPP,ILILEF,IPXLEF,
1          ILIMID,IPXIMID,ILIRIG,IPXIRIG,
2          BFLAGE,XXXXXX,IERROR,RVECT1,
3          RVECT2,RAZNT1,RAZNT2)
C
C      IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C      IMPLICIT LOGICAL*1(A),INTEGER*2(F-Q),REAL(R-Z)
C
C      DIMENSION RVECT1(2,2),RVECT2(2,2),RA(4),RB(4)
C
C      INCLUDE 'AMFOLU.INC'
C
C-----

C      PROCESS
C
C      10.  ASSUME NO ERROR.
C
C      IERROR=1
C
C      15.  TRANSFER INPUT PARAMETERS TO LOCAL REAL VARIABLES.      20
C
C      RPL=IPXLEF
C      RLL=ILILEF
C      RPH=IPXIMID
C      RLM=ILIMID
C      RPR=IPXIRIG
C      RLR=ILIRIG
C      PIX=IPPPPP
C      LIN=ILLILL
C
C      20.  CALCULATE THE COEFFICIENTS OF QUADRATIC POLYNOMIAL.
C
C      RA(3)=RPL
C      RA(2)=0.5*(4*RPH-RPR-3*RPL)
C      RA(1)=RPH-RA(2)-RA(3)
C
C      RB(3)=RLL
C      RB(2)=0.5*(4*RLM-RLR-3*RLL)
C      RB(1)=RLM-RB(2)-RB(3)
C
C      25.  CALCULATE Y VALUE CORRESPONDING TO GIVEN PIXEL AND LINE.
C
C      YD=RPR-RPL
C      YD=RLR-RLL
C      IF (ABS(YD).GE.ABS(YD)) GOTO 26
C      IF (ABS(YD).GE.ABS(XD)) GOTO 27
C
C      26  T=(-RA(3)-PIX)/RA(2)
C      IF (RA(1).EQ.0) GOTO 30
C
C      T1=(-RA(2)+SQRT(RA(2)*RA(2)-4*RA(1)*(RA(3)-PIX)))/(2*RA(1))
C      T2=(-RA(2)-SQRT(RA(2)*RA(2)-4*RA(1)*(RA(3)-PIX)))/(2*RA(1))

```

```

IF ((BFLAG.EQ.'H').AND.(T1.GT.0.1).AND.(T1.LT.1.9))T=T1
IF ((BFLAG.EQ.'H').AND.(T2.GT.0.1).AND.(T2.LT.1.9))T=T2
IF ((BFLAG.EQ.'L').AND.(T1.GT.0.4).AND.(T1.LT.1.0))T=T1
IF ((BFLAG.EQ.'L').AND.(T2.GT.0.4).AND.(T2.LT.1.0))T=T2
IF ((BFLAG.EQ.'R').AND.(T1.GT.1.0).AND.(T1.LT.1.6))T=T1
IF ((BFLAG.EQ.'R').AND.(T2.GT.1.0).AND.(T2.LT.1.6))T=T2

```

```

GOTO 30

```

```

27  T=(-RB(3)-LIN)/RB(2)
    IF (RB(1).EQ.0) GOTO 30
    T1=(-RB(2)+SQRT(RB(2)*RB(2)-4*RB(1)*(RB(3)-LIN)))/(2*RB(1))
    T2=(-RB(2)-SQRT(RB(2)*RB(2)-4*RB(1)*(RB(3)-LIN)))/(2*RB(1))
    IF ((BFLAG.EQ.'H').AND.(T1.GT.0.1).AND.(T1.LT.1.9))T=T1
    IF ((BFLAG.EQ.'H').AND.(T2.GT.0.1).AND.(T2.LT.1.9))T=T2
    IF ((BFLAG.EQ.'L').AND.(T1.GT.0.4).AND.(T1.LT.1.0))T=T1
    IF ((BFLAG.EQ.'L').AND.(T2.GT.0.4).AND.(T2.LT.1.0))T=T2
    IF ((BFLAG.EQ.'R').AND.(T1.GT.1.0).AND.(T1.LT.1.6))T=T1
    IF ((BFLAG.EQ.'R').AND.(T2.GT.1.0).AND.(T2.LT.1.6))T=T2

```

```

C
C 30.  CALCULATE NORMAL VECTOR AT GIVEN POINT.
C

```

```

30  ISP=30
    RC=180.0/3.1416

```

```

D    CALL XODETA(0)
D    WRITE(LTER,4002) T1,T2,T
D4002 FORMAT(1X,'T1,T2,T',5X,3(F10.2,2X))
D    CALL XOATTA(0)

```

```

    XN=-2*RB(1)+T*RB(2)
    YN= 2*RA(1)+T*RA(2)

```

```

    RX1=PIX+XN
    RY1=LIN+YN

```

```

    RAZ1=180
    IF ((2*RA(1)+T*RA(2)).EQ.0) GOTO 31
    RAZ1=ATAN((2*RB(1)+T*RB(2))/(2*RA(1)+T*RA(2)))*RC+90

```

```

31  RX2=PIX-XN
    RY2=LIN-YN

```

```

    RAZ2=RAZ1+180

```

```

D    CALL XODETA(0)
D    WRITE(LTER,4004) RAZ1,RAZ2
D4004 FORMAT(1X,'RAZ1,RAZ2',5X,2(F12.2,2X))
D    CALL XOATTA(0)

```

```

C
C 35.  SET OUTPUT PARAMETERS.
C

```

```

    RVECT1(1,1)=PIX
    RVECT1(2,1)=LIN
    RVECT1(1,2)=RX1
    RVECT1(2,2)=RY1
    RVECT2(1,1)=PIX
    RVECT2(2,1)=LIN
    RVECT2(1,2)=RX2
    RVECT2(2,2)=RY2

```

```

      RAZNT1=180-RAZ1
      RAZNT2=RAZNT1+180
C
C 40.  OUTPUT THE RESULTS TO TERMINAL.
C
D      CALL IOGETA(0)
D      WRITE(LTER,4006) RVECT1(1,1),RVECT1(2,1),RVECT1(1,2),RVECT1(2,2)
D      WRITE(LTER,4008) RVECT2(1,1),RVECT2(2,1),RVECT2(1,2),RVECT2(2,2)
D      WRITE(LTER,4010) RAZNT1,RAZNT2
D      CALL IOATEA(0)
C
C 999  RETURN
C
999  RETURN
C
C-----
C      FORMAT STATEMENTS
C
4006  FORMAT(1X,'RVECT1(1,1)=' ,F8.2,/,
1      1X,'RVECT1(2,1)=' ,F8.2,/,
2      1X,'RVECT1(1,2)=' ,F8.2,/,
3      1X,'RVECT1(2,2)=' ,F8.2)
4008  FORMAT(1X,'RVECT2(1,1)=' ,F8.2,/,
1      1X,'RVECT2(2,1)=' ,F8.2,/,
2      1X,'RVECT2(1,2)=' ,F8.2,/,
3      1X,'RVECT2(2,2)=' ,F8.2)
4010  FORMAT(1X,'RAZNT1=' ,F10.2,/,
1      1X,'RAZNT2=' ,F10.2)
C
C-----
      END

```

D.3.2 AMFOCH.FTN

```

C+
C      ADDRESS      DR2:[171,110]
C      NAME         AMFOCH.FTN
C      PURPOSE      CHECK THE INPUT CHARACTERS.
C      LANGUAGE     FORTRAN
C      AUTHOR       DING YINHONG
C      DATE         3-DEC-85
C      FIRST USED ON ARIES II IMAGE ANALYSIS SYSTEM FOR
C                   AUTOMATED NOISE FRINGE ORDERING
C
C-----
C      DESCRIPTION
C
C      THIS SUBROUTINE CHECKS THE INPUT CHARACTERS WHICH MUST BE A,B,...,Z,
C      0,1,...,9, OR ' '. OTHERWISE SET ERROR FLAG AND RETURNS.
C
C      ERROR MESSAGES:
C
C          IERROR=1      NO ERROR
C          IERROR=-1     CHARACTERS NOT IN RANGE
C
C-----
C      VARIABLE DEFINITION SECTION
C

```

```

C          INPUT  VARIABLES
C
C          BCHBUF  : INPUT CHARACTER BUFFER
C          NUMCHA  : NUMBER OF CHARACTERS
C          BDOCPT  : ' ' POSITION
C
C          OUTPUT VARIABLES
C
C          IERROR  : ERROR FLAG
C
C          LOCAL VARIABLES
C
C          BCH      : CORRECT CHARACTER SET
C
C-----
C
C          DECLARATIONS
C
C          SUBROUTINE ANFOCH (BCHBUF,NUMCHA,BDOCPT,IXIIII,IERROR)
C
C          IMPLICIT BYTE(B),COMPLEX(UMPLICIT LOGICAL*1(A),INTEGER(F-Q),REAL(R-Z)
C
C          DIMENSION BCHBUF(14),BCH(36)
C
C          DATA BCH/'A','B','C','D','E','F','G','H','I','J','K','L','M',
C 1          'N','O','P','Q','R','S','T','U','V','W','X','Y','Z',
C 2          '0','1','2','3','4','5','6','7','8','9',' ','O/
C
C-----
C
C          PROCESS
C
C 10. CHECK FILE NAME.
C
C          IF (BDOCPT.EQ.0) GOTO 10
C          IF (BCHBUF(BDOCPT).NE.BCH(37)) GOTO 15
C          BCHBUF(BDOCPT)='A'
C 10 DO 12 I=1,NUMCHA
C          DO 11 J=1,36
C          IF (BCHBUF(I).EQ.BCH(J)) GOTO 12
C 11 CONTINUE
C          GOTO 15
C 12 CONTINUE
C          IERROR=1
C          GOTO 999
C
C 15. FILE NAME IS INCORRECT.
C
C 15 CALL IONESG(IER,21)
C          IERROR=-1
C          GOTO 999
C
C 999. RETURN.
C
C 999 RETURN
C
C-----
C
C          FORMAT STATEMENTS
C
C          NONE
C
C-----
C          END

```

D.3.3 AMFOEX.FTN

```

C+
C   ADDRESS      DR2:[171,110]
C   NAME         AMFOEX.FTN
C   PURPOSE      EXAMINE THE THEME VALUE AT A SPECIFIED VMA POSITION.
C   LANGUAGE     FORTRAN
C   AUTHOR       DING XIHONG
C   DATE         3-DEC-85
C   FIRST USED ON ARIES II IMAGE ANALYSIS SYSTEM
C
C-

```

DESCRIPTION

THIS SUBROUTINE EXAMINES THE THEME VALUE AT A SPECIFIED VMA POSITION.

ERROR MESSAGES:

```

C   IERROR=1      NO ERROR
C   IERROR=-1     IXL BITS NO CORRECT

```

VARIABLE DEFINITION SECTION

INPUT VARIABLES

```

C   IPIXCO  PIXEL COORDINATE
C   ILINCO  LINE COORDINATE
C   IXLDEP  IXL DEPTH
C   EPMASK  FIELD MASK (THE CORR. BITS WILL BE EXAMINED)

```

OUTPUT VARIABLES

```

C   IERROR  ERROR FLAG
C   EVALUE  PIXEL VALUE

```

COMMON VARIABLES

```

C   ESTL  START LINE OF DISK IMAGE
C   ESTP  START PIXEL OF DISK IMAGE
C   IVPF  (4096) BUFFER FOR MAPPING VMA
C   IAOB  (2) ADDRESS OFFSET REGISTER

```

LOCAL VARIABLES

```

C   RAD  ADDRESS OFFSET
C   IA   EQUIVALENCED TO RAD
C   EJ   BYTE NO. OF CURRENT PIXEL
C   EK   SHIFT FACTOR FOR CURRENT PIXEL
C   EL   BYTE VALUE
C   EM   SHIFTED AND. SELECTED THE FIELD TO BE EXAMINED
C   III  DUMMY TO ALLOW BYTE ASSIGNMENTS
C   BBB  EQUIVALENCED TO III
C   K1   A VALUE THAT WILL BE PUT INTO CURSOR BIT

```

DECLARATIONS

```

C   SUBROUTINE AMFOEX(IPIXCO,ILINCO,IXLDEP,EPMASK,BBFLAG,
1      XXXXX,IERROR,IVALUE)

```


APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

452

```

C      IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C      IMPLICIT LOGICAL*1(A),INTEGER*2(F-Q),REAL(R-Z)
C
C      DIMENSION BVBF(8128),IA(2),BBB(2),IM(2),IV(2)
C
C      INCLUDE 'AMFOLU.INC'
C
C      COMMON /DPICOM/ IVBF(4064),IIII(30),IAOR(2)
C      COMMON /IZIIPR/ BINB, BXLD, BARE(4), RDEC,
1      ISTL, ISTD, IDIL, IDIP,
2      ESAD, BWFE, BETH, BFSP(8,10),
3      IDEC, JJJJ, FWPL, ICBS, RASP,
4      EFLC, EFPC, ELLC, ELPC, NBYT, BTBT, BNTF,
5      BRED, BGRW, BBLU,
6      BBB1, BTCD(128), BBB2, BCSP, IYAR, IXAR, ITAT,
7      ESTL, ESTP, EBYT, BUNI, BLOC(11)
C
C      EQUIVALENCE (EAD,IA(1)),(IVBF(1),BVBF(1)),(BBB(1),III)
C
C-----
C
C      PROCESS
C
C      10. ASSUME NO ERROR. 15
C      10. IERROR=1
C
C      20. CALCULATE ADDRESS OFFSET. 25
C
C      20. IPI=IPIXCO-ESTP+1
C      ELI=ILINCO
C      ENL=ELI-ESTL
C      EAD=ESAD+(ENL*FWPL*2)
C      IAOR(1)=IA(1)
C      IAOR(2)=IA(2)
C
C      25. PROCESS CORRESPONDING IXEL DEPTH. 999/40/45/50
C
C      4 BIT IXEL DEPTH 40
C      8 BIT IXEL DEPTH 45
C      16 BIT IXEL DEPTH 50
C
C      IF (IXLDEP.EQ.4) GOTO 40
C      IF (IXLDEP.EQ.8) GOTO 45
C      IF (IXLDEP.EQ.16) GOTO 50
C      IERROR=-1
C      GOTO 999
C
C      40. EXAMINE 4 BIT IXEL. 55
C
C      40. EJ = (IPI+1)/2
C      EK = 4*LAND(IPI+1,1)
C      EL = BVBF(EJ)
C      EM = ISHFT(EPMASK,EK)
C      EXI = LAND(EL,EM)
C      IVALUE = BBB(1)
C      IF (EK.EQ.0) GOTO 41
C      IVALUE = BBB(1)/16
C      41. E1=ISHFT(1,EK)
C      IF (BBFLAQ.EQ.'N') III=IOR(EL,E1)
C      BVBF(EJ)=BBB(1)
C      GO TO 55
C
C

```

```

C 45. EXAMINE 8 BIT IXEL.                                     55
C
45      KL = BVBF(IPX)
        III = IAND(KL,EPMASK)
        IVALUE = BBB(1)
        BVBF(IPX)=BBB(1)+1
        IF (BBFLAG.NE.'C') GOTO 55
        BVBF(IPX)=BVBF(IPX).AND.(-2)
        GOTO 55
C
C 50 EXAMINE 16 BIT IXEL.
C
50      KL=IVBF(IPX)
        IVALUE=IAND(KL,EPMASK)
51      IVBF(IPX)=IOR(IVBF(IPX),1)
        IF (BBFLAG.NE.'C') GOTO 55
        IVBF(IPX)=IAND(IVBF(IPX),-2)
        GOTO 55
C
C 55. DISPLAY VMA VALUES ON TERMINAL.                         999
C
55      CONTINUE
D       CALL IODETA(0)
D       WRITE(LTER,5001) IVALUE
D       CALL IOATTA(0)
C
C 999. RETURN.
C
999     RETURN
C
C-----
C
C   FORMAT STATEMENTS
C
5000    FORMAT(1X,'IPX=',I4,/,
1       1X,'ILINCO=',I6,/,
2       1X,'IXLDEP=',I3,/,
3       1X,'EPMASK=',I8)
5001    FORMAT(1X,'IVALUE=',I8)
C
C-----
C
C   END

```

D.3.4 AMFOLU.FTN

```

C+
C
C   NAME          AMFOLU.FTN
C   PURPOSE       DEFINE ALL LOGICAL UNITS USED IN THE TASK
C   LANGUAGE      FORTRAN
C   AUTHOR        DING XINHONG
C   DATE          10-MARCH-86
C   FIRST USED ON  ARIES II IMAGE ANALYSIS SYSTEM FOR THE
C                  AUTOMATED MOIRE FRINGE ORDERING TASK
C
C-----
C
C   DESCRIPTION
C
C   TO ASSIGN VALUE TO THE COMMON BLOCK /AMFOAINS THE
C   LOGICAL UNIT NAMES REQUIRED BY THIS TASK. 4
C

```

```

C          LCFG  CONFIGURATION FILE
C          LVMA  VMA DISK
C          LIDX  INDEX FILE
C          LCON  CONTOUR POINT FILE
C          LCEF  TRANSFORMATION COEFFICIENT FILE
C          LMOR  NOIRE INSTRUMENT PARAMETER FILE
C          LTER  LOGICAL UNIT FOR TERMINAL
C          LPRT  LOGICAL UNIT FOR LINE PRINTER
C
C-----

```

```

C  .  DECLARATIONS
C

```

```

C  BLOCKDATA AMFOLU
C

```

```

C  IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C  IMPLICIT LOGICAL*1(A),INTEGER*2(F-Q),REAL(R-Z)
C

```

```

C  INCLUDE 'AMFOLU.INC'
C

```

```

C  DATA  LCFG/1/,
C         1  LVMA/2/,
C         2  LIDX/3/,
C         3  LCON/4/,
C         4  LCEF/5/,
C         5  LMOR/6/,
C         6  LTER/7/,
C         7  LPRT/8/
C
C-----

```

```

C  END
C-----

```

D.3.5 AMFOMA.FTN

```

C+
C  ADDRESS      DR2:[171,110]
C  NAME         AMFOMA.FTN
C  PURPOSE      MAIN PROGRAM FOR TASK 'JFO'
C  LANGUAGE     FORTRAN
C  AUTHOR       DING XIFENG
C  DATE         10-MARCH-86
C  FIRST USED ON ARIES-II IMAGE ANALYSIS SYSTEM FOR
C                AUTOMATED NOIRE FRINGE ORDERING
C
C-----

```

```

C  DESCRIPTION
C

```

```

C  INPUT FILES:
C

```

```

C  (1).  AAADCIIII.OUT      CONTOUR INDEX FILE
C  (2).  AAAFOIIII.CEF      COEFFICIENT FILE
C  (3).  AAAFOIIII.MOR      NOIRE PARAMETER FILE
C

```

```

C  OUTPUT FILES:
C

```

```

C  (1).  AAAFOIIII.FRI      FRINGE ORDERING FILE
C  (2).  AAAFOIIII.DIA      DIAGNOSTIC FILE
C

```

```

C  PROCESSING PROCEDURES:
C

```

```

C      (1).  ENTER IMAGE NUMBER
C      (2).  ENTER INDEX FILE NAME
C      (3).  ENTER INITIAL DATA
C      (4).  PROCESS
C      (5).  OUTPUT ORDERING RESULTS
C

```

VARIABLE DEFINITION SECTION

COMMON VARIABLES

```

C      IWIN   (4064) A WINDOW TO MAP VNA DATA
C      IALO   LOWER PART OF ADDRESS OFFSET FOR IWIN
C      IAH1   HIGHER PART OF ADDRESS OFFSET FOR IWIN
C      BINH   IMAGE NUMBER
C      BILD   IXEL DEPTH MINUS 1 (15,7,3,1)
C      BARE   (4) AREA NAME
C      ISTL   START LINE IN CCT COORDINATES
C      ISTP   START PIXEL IN CCT COORDINATES
C      EDIL   NUMBER OF DISPLAY LINES
C      EDIP   NUMBER OF DISPLAY PIXEL
C      BDFE   NUMBER OF FEATURES
C      BNTH   NUMBER OF THEMES, ASSUMING PACKED FORMAT
C      BFSP   (8,10) FILE SPECIFICATION
C      EFLC   CCT LINE NUMBER OF 1ST LINE DISC IMAGE (WHICH COULD
C             DIFFER FROM THE FIRST DISC LINE TO BE MAPPED TO VNA)
C      EFPC   CCT PIXEL NUMBER OF 1ST PIXEL DISC IMAGE (WHICH COULD
C             DIFFER FROM THE FIRST DISC PIXEL TO BE MAPPED TO VNA)
C      ELLC   CCT LINE NUMBER OF LAST LINE OF DISC IMAGE
C      ELPC   CCT PIXEL NUMBER OF LAST PIXEL OF DISC IMAGE
C      BTHT   THEME TYPE (PACKED/BIT SPECIFIC)
C      BNTF   NUMBER OF THEME FILES
C      BYCO(128) THEME COLOUR TABLE. INITIALLY VALUES 0 TO 63

```

LOCAL VARIABLES

```

C      BBB    (26) DUMMY
C      BID    (14) CONTOUR INDEX FILE
C      BCF    (14) TRANSFORMATION COEFFICIENT FILE
C      BMR    (14) NOISE PARAMETER FILE
C      BFR    (14) FRINGE ORDERING RESULT FILE
C      BYE    'Y' OR 'N' RESPONSE
C      IER    ERROR FLAG
C      IIN    IMAGE NUMBER
C      ISP    STEP NUMBER
C      IVN    VNA NUMBER
C      MSG    MESSAGE NUMBER

```

DECLARATIONS

```

C      IMPLICIT BYTE(B), COMPLEX(C), DOUBLE PRECISION(D), INTEGER*4(E)
C      IMPLICIT LOGICAL*1(A), INTEGER(F-Q), REAL(R-Z)

```

```

C      DIMENSION BBB(26), BID(14), BCF(14), BMR(14), BFR(14), BB(256)

```

```

C      INCLUDE 'AMFOLU.INC'

```

```

C      COMMON /DPICOM/ IWIN(4064), IIII(80), IALO, IAH1
C      COMMON /XZ1IPR/ BINH, BILD, BARE(4), RDEC, ISTL, ISTP, EDIL, EDIP,
1      ESAD, BDFE, BNTH, BFSP(8,10), IDEC, JJJJ, NVPL, ICAS,
2      RASP, EFLC, EFPC, ELLC, ELPC, NBYT, BTHT, BNTF, BRED,

```

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

456

3 BGRN, BBLU, BBB1, BTCO(128), BBB2, BCSP, IYAR, IIAR,
4 ITAT, ESTL, ESTP, EBYT, BUNI, BLOC(11)

DATA BID/'0','0','0','D','C','0','0','0','0','0','0','0','U','T','0/
DATA BCF/'0','0','0','F','0','0','0','0','0','0','0','0','C','E','F','0/
DATA BMR/'0','0','I','R','E','I','H','S','T','0','0','0','X','Y','Z','0/
DATA BFR/'0','0','0','F','0','0','0','0','0','0','0','0','F','R','I','0/

C

C-----

C PROCESS

C

C 10 INITIALIZE OIS. 2000/15

C

10 ISP=10
CALL X0INIT(XXXXXX,IER,BBB)
IF (IER.NE.1) GOTO 2000

C

C 15 DISPLAY TASK HEADER MESSAGES 2000/999/20

C

15 ISP=15
CALL X0HESG(IER,1)
IF (IER.EQ.-273) GOTO 999
IF (IER.NE.1) GOTO 2000

C 20 ATTACH VMA 2000/25

C

20 ISP=20
CALL XBGVID(LCFG,'Y',XXXXXX,IER,IVN)
IF (IER.NE.1) GOTO 2000

C

C 25 LINK VMA TO COMMON BLOCK 'DPICOM' 2000/30

C

25 ISP=25
CALL XXVLNK(IVIN,IVN,LCFG,XXXXXX,IER)
IF (IER.NE.1) GOTO 2000

C 30 INPUT IMAGE NUMBER. 2000/999/35

C

IMN=0
30 ISP=30
IMN=IMN+1
CALL X0HESG(IER,2+"4000,1,IMN)
IF (IER.EQ.-273) GOTO 999
IF (IER.NE.1) GOTO 2000

C

C 35 GET IMAGE PARAMETERS. IF IMAGE DOES NOT 2000/30/40
EXIST, REPORT ERROR MESSAGE AND RE-INPUT IMAGE NUMBER.

C

35 ISP=35
CALL XIREPO(IVN,IMN,XXXXXX,IER,BINN)
IF (IER.NE.-300) GOTO 3
CALL X0HESG(IER,3)
GOTO 30
35 IF (IER.NE.1) GOTO 2000

C 40 DISPLAY IMAGE PARAMETERS. 45

C

40 ISP=40
CALL X0DETA(0)
WRITE(LTER,5000) BINN,(BARE(I), I=1,3),ESTL,ESTP
WRITE(LTER,5001) NDIL,NDIP,BILD,BIFE,BNTH,BTET,BNTF
WRITE(LTERPC,ELLC,ELPC)

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

457

```

      CALL IOATTA(0)
C
C 45.  ASK OPERATOR IF THIS IS THE DESIRED IMAGE.                2000/999/30/50
C      'Y' --- CONTINUE.
C      'N' --- RE-PROMPT FOR IMAGE NUMBER
C      'Z' --- EXIT.
C
45      ISP=45
      BYE='Y'
      CALL IONESG(IER,4+"4000,1,BYE)
      IF (IER.EQ.-273) GOTO 999.
      IF (IER.NE.1) GOTO 2000
      IF (BYE.EQ.'Y') GOTO 50
      IF (BYE.EQ.'N') GOTO 30
      GOTO 45

C
C 50  WOULD YOU LIKE TO PROCESS THIS IMAGE?                    2000/999/90/55
C      'Y' --- CONTINUE
C      'N' --- GOTO 90
C      'Z' --- EXIT
C
50      ISP=50
      BYE='Y'
      CALL IONESG(IER,5+"4000,1,BYE)
      IF (IER.EQ.-273) GOTO 999
      IF (IER.NE.1) GOTO 2000
      IF (BYE.EQ.'Y') GOTO 55
      IF (BYE.EQ.'N') GOTO 90
      GOTO 50

C 55.  CHECK IMAGE THEME BITS.                                  2000/999/60
C
55      ISP=55
      IF (BETH.NE.0) GOTO 60
      CALL IONESG(IER,6)
      GOTO 90

C 60.  SET AREA NAME FOR ALL FILE AND INPUT                    2000/999/65
C      ITS FILE IDENTIFIER(4-CHAR).
C
60      ISP=60
      DO 61 I=1,3
      BID(I) = BARE(I)
      BCF(I) = BARE(I)
61      CONTINUE
      CALL IONESG(IER,7+"4000,4,BID(6))
      IF (IER.EQ.-273) GOTO 999
      IF (IER.NE.1) GOTO 2000

C 65.  CHECK THE IDENTIFIER. CHARACTERS MUST BE                60/70
C      'A',..., 'Z' OR 'O',..., '9' OTHERWISE REPORT
C      ERROR MESSAGE AND RE-INPUT FILE IDENTIFIER.
C
65      ISP=65
      CALL ANPOCH(BID(6),4,0,XXXXXX,IER)
      IF (IER.EQ.1) GOTO 70
      CALL IONESG(IER,17)
      GOTO 60

C 70.  SET AREA NAME FOR FRINGE ORDERING FILE AND INPUT
      2000/999/75
C      ITS FILE IDENTIFIER(4-CHAR). DEFAULT
C      WILL BE THE SAME IDENTIFIER AS INDEX FILE.

```

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

458

```

C
70      ISP=70
        DO 71 I=1,3
          BFR(I)=BALE(I)
71      CONTINUE
        DO 72 I=6,9
          BFR(I)=BID(I)
72      CONTINUE
        CALL IOMESG(IER,8+"4000,4,BFR(6))
        IF (IER.EQ.-273) GOTO 999
        IF (IER.NE.1) GOTO 2000

C 75     CHECK THE IDENTIFIER. CHARACTERS MUST BE
C        'A',...,'Z' OR '0',...,'9' OTHERWISE REPORT
C        ERROR MESSAGE AND RE-INPUT FILE IDENTIFIER
C
75      ISP=75
        CALL AMPQCH(BFR(6),4,0,XXXXXX,IER)
        IF (IER.EQ.1) GOTO 80
        CALL IOMESG(IER,17)
        GOTO 70

C 80     CALL A SUBROUTINE TO PROCESS THIS IMAGE
C
80      ISP=80
        DO 82 I=6,9
          BCF(I)=BID(I)
82      CONTINUE
        CALL AMPOPR(BID,BCF,BMR,BFR,XXXXXX,IER)
        IF (IER.EQ.-273) GOTO 999
        IF (IER.NE.-2) GOTO 81
        CALL IOMESG(IER,18)
        GOTO 85
81      IF (IER.EQ.-1) GOTO 85
        IF (IER.NE.1) GOTO 2000

C 85     ASK IF THERE ARE MORE INDEX FILE
C
85      ISP=85
        BYE='N'
        CALL IOMESG(IER,15+"4000,1,BYE)
        IF (IER.EQ.-273) GOTO 999
        IF (IER.NE.1) GOTO 2000
        IF (BYE.EQ.'Y') GOTO 60
        IF (BYE.EQ.'N') GOTO 90
        GOTO 85

C 90     ASK IF THERE ARE MORE IMAGES
C
90      ISP=90
        BYE='N'
        CALL IOMESG(IER,16+"4000,1,BYE)
        IF (IER.EQ.-273) GOTO 999
        IF (IER.NE.1) GOTO 2000
        IF (BYE.EQ.'Y') GOTO 30
        IF (BYE.EQ.'N') GOTO 95
        GOTO 90

C 95     RELEASES VMA.
C
95      ISP=95
        CALL IBRVID(IVN,XXXXXX,IER)
        IF (IER.NE.1) GOTO 2000
C

```

70/80

2000/999/85

2000/999/85/60/90

2000/999/90/30/95

2000/999

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

468

```

131      IF (RFR(BTH2) NE 0) GOTO 132
        LIN=LIN1
        PIX=PIX1
        LLL=LIN2
        PPP=PIX2
        RAZ=RAL
        BTM=BTH1
        BTH=BTH2
        GOTO 133

132      LIN=LIN2
        PIX=PIX2
        LLL=LIN1
        PPP=PIX1
        RAZ=RAL
        BTM=BTH2
        BTH=BTH1
        GOTO 133

133      IF (RAZ LT 180) RAZ=RAZ-180
        IF (RAZ GT 180) RAZ=MOD(RAZ,180 0)

D        CALL XODETA(0)
D        WRITE (LTER,4040) BTM,RAZ,RFR(BTM),RFR(M)
D4040    FORMAT(1X,'BTM,RAZ,RFR(BTM),RFR(M)',2X,I4,2X,F12 4,2(F8 2))
D        CALL XODATA(0)

        IF (RFR(BTM) GT RFR(M)) RIA=RAZ+180
        IF (RFR(BTM) LE RFR(M)) RIA=RAZ
        CALL AMFOWL(LIN,PIX,IML,IMP,RIA,RFR(BTM),RFR(M),XXXXXX,IER,RML)

C
C 135    ESTIMATE REFLECTANCE VALUE AT POINT
C        N=(1/2(LIN+IML),1/2(PIX+IMP))
C
135      ISP=135
        CALL AMFORV(LIN,PIX,IML,IMP,XXXXXX,IER,RV,NUM)

C
C 140    CALCULATE MODEL PARAMETER AT POINT
C        N=(1/2(LIN+IML),1/2(PIX+IMP))
C
140      ISP=140
        RO=RV/RML

D        CALL XODETA(0)
D        L=0.5*(LIN+IML)
D        P=0.5*(PIX+IMP)
D        WRITE (LTER,4014) L,P,RV,RIA,RML,RO
D4014    FORMAT(1X,'L,P,RV,RIA,RML,RO',
D          1      2X,I4,F8 4,F8 2,F12 8,2X,F14 4)
D        CALL XODATA(0)

C
C 145    ESTIMATE NEXT FRINGE NUMBER
C
145      ISP=145
        CALL AMFORV(IML,IMP,LLL,PPP,XXXXXX,IER,RN,NN)

        RA1=RAZ
        CALL AMFOWL(IML,IMP,LLL,PPP,RA1,RFR(M),
D          1      RFR(M)+1,XXXXXX,IER,RML1)
        RE1=(RN-RO+RML1)*(RN-RO+RML1)

```



```

C 999 EXIT.
C
999 CALL IOEXIT
C
C 2000 ERROR REPORTING 999
C
2000 MSG=20
      CALL IOERRM('AMFOMA',ISP,IER,MSG)
      GOTO 999
C
C-----

C      FORMAT STATEMENTS
C
5000 FORMAT(1X,'IMAGE NUMBER = ',I4,/,
1      1X,'AREA NAME = ',3A1,/,
2      1X,'START LINE FOR DISPLAY = ',I6,/,
3      1X,'START PIXEL FOR DISPLAY = ',I6)
5001 FORMAT(1X,'NUMBER OF DISPLAY LINES = ',I6,/,
1      1X,'NUMBER OF DISPLY PIXELS = ',I6,/,
2      1X,'PIXEL DEPTH = ',I4,/,
3      1X,'NUMBER OF FEATURES = ',I4,/,
4      1X,'NUMBER OF THEMES = ',I4,/,
5      1X,'THEME TYPE = ',I4,/,
6      1X,'NUMBER OF THEME FILES = ',I4)
5002 FORMAT(1X,'FIRST LINE=',I6,/,
1      1X,'FIRST PIXEL=',I6,/,
2      1X,'LAST LINE=',I6,/,
3      1X,'LAST PIXEL=',I6,/)
C
C-----

C      END

```

D.3.6 AMFONL.FTN

```

C+
C      ADDRESS      DR2: [171,110]
C      NAME         AMFONL.FTN
C      PURPOSE      TO COMPUTE INNER PRODUCT BETWEEN
C      =            SURFACE NORMAL AND LIGHT VECTOR
C      LANGUAGE     FORTRAN
C      AUTHOR       DING XINHONG
C      DATE         10-MARCH-86
C      FIRST USED ON ARIES-II SYSTEM FOR
C                  MOIRE FRINGE ANALYSIS
C
C-----

C      DESCRIPTION
C
C      THIS SUBROUTINE COMPUTES THE INNER PRODUCT BETWEEN THE SURFACE
C      NORMAL AND THE SURFACE POINT TO LIGHT VECTOR
C
C-----

C      VARIABLES
C
C      INPUT  VARIABLES
C
C      LINEST  START LINE
C      PIXEST  START PIXEL

```

```

C          LINEED END LINE
C          PIXEED END PIEL
C          RAZMTH AZIMUTH, IN ANGLE
C          FRING1 FRINGE 1
C          FRING2 FRINGE 2
C
C          OUTPUT VARIABLES
C
C          IERROR ERROR FLAG
C          RPRODT INNER PRODUCT (M,L)
C
C          LOCAL VARIABLES
C
C-----

C          DECLARATIONS
C
C          SUBROUTINE AMFONL(LINEST,PIXEST,LINEED,PIXEED,RAZMTH,
1              RRING1,RRING2,XXXXXX,IERROR,RPRODT)
C
C          IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C          IMPLICIT LOGICAL*1(A),INTEGER(F-Q),REAL(R-Z)
C
C          DIMENSION
1              X(3),Y(3),Z(3),S(3),SMX(4,3),
2              ZK(3),SCA(3),VECTOR(2,2),ZFRING(2)
C
C          INCLUDE 'AMFOTC.INC'
C          INCLUDE 'AMFOMR.INC'
C          INCLUDE 'AMFOLU.INC'
C
C-----

C          PROCESS
C
C 10 SET ERROR FLAG
C
C          IER=1
C
C 15 GIVEN (1)TWO POINT COORDINATES IN IMAGE PLANE,
C          (2)THE MOIRE INSTRUMENT PARAMETERS AND
C          (3)THE FRINGE NUMBERS OF THE POINTS, WE CAN
C          OBTAIN TWO CORRESPONDING POINTS ON THE
C          SURFACE OF THE OBJECT
C
C          ZFRING(1)=RRING1
C          ZFRING(2)=RRING2
C          VECTOR(1,1)=PIXEST
C          VECTOR(2,1)=LINEST
C          VECTOR(1,2)=PIXEED
C          VECTOR(2,2)=LINEED
C
C          DO 17 I=1,2
C
C          ZK(I)=-ZFRING(I)*RZMP(3)/(RYMP(4)/RYMP(2)-ZFRING(I))
C          SCA(I)=1-ZK(I)/RZMP(3)
C
C          X(I)=(TRA0+TRA1*VECTOR(1,I)+TRA2*VECTOR(2,I))*SCA(I)
C          Y(I)=(TRB0+TRB1*VECTOR(1,I)+TRB2*VECTOR(2,I))*SCA(I)
C          Z(I)=ZK(I)
C
17 CONTINUE

```

```

C
C 30  CALCULATE TILT.
C
      XD=X(2)-X(1)
      YD=Y(2)-Y(1)
      ZD=Z(2)-Z(1)
      ZZ=SQRT(XD*XD+YD*YD+ZD*ZD)
      RTL=ACOS(ZD/ZZ)
      RTL=RTL*(180/3.1416)-90
      RTL2=RTL
      RTL=ABS(RTL)
      RTL3=RTL
      TIL=RTL

D      CALL XODETA(0)
D      WRITE(LTER,8006) RTL1,RTL2,RTL3
D8006  FORMAT(1X,'RTL1,RTL2,RTL3',3F10.5)
D      WRITE(LTER,4006) RTL
D4006  FORMAT(1X,'TILT',5X,F8.2)
D      CALL XOATTA(0)

C
C 35  OBTAIN UNIT NORMAL VECTOR
C
      RC=3.1416/180.0
      RAZ=RAZMTH*RC
      RTL=RTL*RC
      RN1=COS(RAZ)*SIN(RTL)
      RN2=SIN(RAZ)*SIN(RTL)
      RN3=COS(RTL)

C
C 40  OBTAIN UNIT ILLUMINATION VECTOR
C
      XM=(X(1)+X(2))/2.0
      YM=(Y(1)+Y(2))/2.0
      ZM=(Z(1)+Z(2))/2.0

      RL1=RIMP(4)-XM
      RL2=RYMP(4)-YM
      RL3=RZMP(4)-ZM
      RVL=SQRT(RL1*RL1+RL2*RL2+RL3*RL3)
      RL1=RL1/RVL
      RL2=RL2/RVL
      RL3=RL3/RVL

C
C 45  CALCULATE INNER PRODUCT
C
      RVL=RN1*RL1+RN2*RL2+RN3*RL3
      RPRODT=RVL

C
C 50.  DISPLAY OUTPUT PARAMETERS
C
D      CALL XODETA(0)
D      WRITE(LTER,400 TIL,RN1,RN2,RN3,RL1,RL2,RL3,RVL
D4008  FORMAT(1X,'AMPONL 50 TL,W,L',1X,F6.2,1X,3(F8.4),
D      1      1X,3(F8.4),1X,'RVL',F8.2)
D      CALL XOATTA(0)

C
C 999  RETURN

```

```

C
999  I=IER
      RETURN
C
C-----
C
C      FORMAT STATEMENTS
C
C      NONE
C
C-----
C
      END

```

D.3.7 ANFOPR.FTN

```

C+
C      ADDRESS      DR2:[171,110]
C      NAME         ANFOPR.FTN
C      PURPOSE      PREPARE DATA FOR SUBSEQUENT SUBROUTINES
C      LANGUAGE     FORTRAN
C      AUTHOR       DING XINHONG
C      DATE         10-MARCH-86
C      FIRST USED ON ARIES-II IMAGE ANALYSIS SYSTEM FOR
C                   AUTOMATED MOIRE FRINGE ORDERING
C
C-----
C
C      DESCRIPTION
C
C      THIS SUBROUTINE PERFORMS THE FOLLOWING FUNCTIONS
C      (1).  OBTAIN THE IMAGE PLANE TO GRID PLANE
C            TRANSFORMATION COEFFICIENTS.
C      (2).  OBTAIN THE MOIRE INSTRUMENT PARAMETERS.
C      (3).  CALCULATE THE SCALE FACTOR AND THE AMOUNT OF ROTATION
C      (4).  OPEN CONTOUR INDEX FILE AND READ IN ALL THE INFORMATION
C      (5).  INPUT TWO INITIAL FRINGE NUMBER.
C      (6).  COMPUTE THE TRUE IMAGE BRIGHTNESS VALUE BASED ON KNOWN
C            CONTOUR ORDERING RELATIONSHIP.
C      (7).  COMPUTE THE ESTIMATED IMAGE BRIGHTNESS VALUE FOR EACH OF
C            THREE POSSIBLE ORIENTATION CASES.
C      (8).  SELECT THE CASE THAT WILL PRODUCE THE LEAST TOTAL ERROR
C            BETWEEN THE TRUE IMAGE BRIGHTNESS VALUE AND THE ESTIMATED
C            IMAGE BRIGHTNESS VALUE.
C      (9).  SEND THE RESULTS TO THE TERMINAL.
C
C-----
C
C      VARIABLE DEFINITION SECTION
C
C      INPUT VARIABLES
C
C            BINDEX (14)INDEX FILE NAME
C            BCOEPT (14)TRANSFORMATION COEFFICIENT FILE NAME
C            BMOIRE (14)MOIRE INSTRUMENT PARAMETER FILE NAME
C            BFRING (14)FRINGE ORDERING FILE NAME
C
C      OUTPUT VARIABLES
C
C            IERROR  ERROR FLAG

```

C LOCAL VARIABLES

```

C      NHR      NUMBER OF HEADER RECORDS
C      RTC(12)   TRANSFORMATION COEFFICIENTS
C      RXMP(10)  NOISE INSTRUMENT PARAMETERS
C      RYMP(10)  NOISE INSTRUMENT PARAMETERS
C      RZMP(10)  NOISE INSTRUMENT PARAMETERS
C      SCAL      SCAL FACTOR
C      AZMO      ROTATION ANGLE
C      BCF(14,20) CONTOUR FILE NAMES
C      BTY(20)   CONTOUR TYPE
C      BTN(20)   THEME NUMBER
C      NCF      NUMBER OF CONTOUR FILES
C      RFR(20)   RESULTED FRINGE NUMBER BUFFER
C      ILO(256)  LINE DATA
C      IPO(256)  PIXEL DATA
C      IPT      NUMBER OF POINTS
C      RT1      TOTAL ERROR FOR + CASE
C      RTO      TOTAL ERROR FOR 0 CASE
C      RT2      TOTAL ERROR FOR - CASE
C      ILL      LEFT POINT LINE
C      ILP      LEFT POINT PIXEL
C      IML      MIDDLE POINT LINE
C      IMP      MIDDLE POINT PIXEL
C      IRL      RIGHT POINT LINE
C      IRP      RIGHT POINT PIXEL
C      RV1(2,2)  VECTOR 1
C      RV2(2,2)  VECTOR 2
C      RAL      AZIMUTH 1
C      RAR      AZIMUTH 2
C      LIN1      LINE 1
C      PIX1      PIXEL 1
C      RRV1      ROW VALUE 1
C      BTH1      THEME # 1
C      LIN2      LINE 2
C      PIX2      PIXEL 2
C      RRV2      ROW VALUE 2
C      BTH2      THEME #2
C      ILN(256)  LINE DATA
C      IPI(256)  PIXEL DATA
C      NTH(20)   THEME FLAG
C      RTE1(20)  TOTAL ERROR FOR + CASE
C      RTE0(20)  TOTAL ERROR FOR 0 CASE
C      RTE2(20)  TOTAL ERROR FOR - CASE
C      ISP      STEP NUMBER
C      IER      ERROR FLAG
C      MSG      MESSAGE NUMBER

```

C -----
C DECLARATIONS

```

C      SUBROUTINE AMFOPR(BINDEX,BCOEFT,BMOIRE,BFRING,XXXXX,IERROR)
C
C      IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C      IMPLICIT LOGICAL*1(A),INTEGER(F-Q),REAL(R-Z)
C
C      DIMENSION BINDEX(14),BCOEFT(14),BMOIRE(14),BFRING(14),
1      ILN(256),IPI(256),RV1(2,2),RV2(2,2),BCF(14,20),
2      BTY(20),BTN(20),ILO(256),IPO(256),RTC(12),
3      RFR(20),RT1(10),RTO(10),RT2(10),NTH(20),
4      RTE1(20),RTE0(20),RTE2(20)

```

C

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

464

```

INCLUDE 'AMFUTC.INC'
INCLUDE 'AMFOMR.INC'
INCLUDE 'AMFOLU.INC'

COMMON /DPICOM/ IWIN(4064), IIII(80), IALD, IAH1
COMMON /XZIIPR/ BINN, BXLD, BARE(4), RDEC,
1      ISTL, ISTD, NDIL, NDIP,
2      ESAD, BNFE, BNTH, BFSP(8,10),
3      IDEC, JJJJ, NWPL, ICRS, RASP,
4      EFLC, EFPC, ELIC, ELRC, EBYT, BNTH, BNTH,
5      BNED, BGRN, BBLU,
6      BBB1, BTCO(128), BBB2, BCSP, IYAR, IXAR, ITAT,
7      ESTL, ESTP, EBYT, BUNI, BLOC(11)

C
EQUIVALENCE (TRA0,RTC(1)),(TRA1,RTC(2)),(TRA2,RTC(3)),
1      (TRB0,RTC(4)),(TRB1,RTC(5)),(TRB2,RTC(6)),
2      (TRC0,RTC(7)),(TRC1,RTC(8)),(TRC2,RTC(9)),
3      (TRD0,RTC(10)),(TRD1,RTC(11)),(TRD2,RTC(12))

C
C-----
C      PROCESS
C
C 10. SET NO ERROR CODE.
C
10      IER=1
C
C 15 OPEN IMAGE PLANE/GRID PLANE TRANSFORMATION COEFFICIENT FILE
15      ISP=15
      OPEN(UNIT=LCEF,NAME=BCOEFT,TYPE='OLD',READONLY,ERR=2005)
C
C 20 SKIP HEADER RECORDS.
C
20      ISP=20
      READ(LCEF,*,END=2000,ERR=2005) NNN,NHR
      DO 21 I=1,NHR-1
      READ(LCEF,*,END=2000,ERR=2005)
21      CONTINUE
C
C 25 READ IN ALL TRANSFORMATION PARAMETERS
C
25      ISP=25
      DO 26 I=1,4
      I1=(I-1)*3+1
      I2=I1+2
      DO 27 J=I1,I2
      READ(LCEF,*,ERR=2005) RTC(J)
27      CONTINUE
      READ(LCEF,*,ERR=2005)
26      CONTINUE
C
C 30 CLOSE THIS FILE.
C
30      ISP=30
      CLOSE(UNIT=LCEF,ERR=2005)
C
C 35 OPEN MOIRE INSTRUMENT PARAMETER FILE.
C

```

```
35   ISP=35
      OPEN(UNIT=LCEF,NAME=BMOIRE,TYPE='OLD',READONLY,ERR=2005)

C
C 40.  SKIP HEADER RECORDS.
C
40   ISP=40
      READ(LCEF,*,END=2000,ERR=2005) NHR,NHR
      DO 41 I=1,NHR-1
41   READ(LCEF,*,END=2000,ERR=2005)

C
C 45  READ IN ALL XYZ-COORDINATES OF MOIRE INSTRUMENT
C
45   ISP=45
      DO 46 I=1,10
      READ(LCEF,*,ERR=2005) RIMP(I),RYMP(I),RZMP(I)
46   CONTINUE

C
C 50  CLOSE THIS FILE
C
50   ISP=50
      CLOSE(UNIT=LCEF,ERR=2005)

C
C 55  CALCULATE SCALE FACTOR AND AMOUNT OF ROTATION
C
55   ISP=55
      SCAL=SQRT((TRA0**2+TRB0**2)/(TRC0**2+TRD0**2))
      AZMO=0

C
C 60  OPEN THE INDEX FILE
C
60   ISP=60
      OPEN(UNIT=LIDI,NAME=BINDEX,TYPE='OLD',READONLY,ERR=2005)

C
C 65  SKIP HEADER RECORDS
C
65   ISP=65
      READ(LIDI,*,END=2000,ERR=2005) NHR
      DO 67 I=1,NHR-1
67   READ(LIDI,5004,END=2000,ERR=2005)

C
C 70  READ IN ALL CONTOUR FILE NAME
C      AND CORRESPONDING THEME NUMBER
C
70   ISP=70
      DO 71 J=1,20
      READ(LIDI,5004,END=75,ERR=2005) BBB,(BCF(I,J),I=1,13),BTY(J),BTM(J)
71   CONTINUE

C
C 75  CLOSE CONTOUR INDEX FILE.
C
75   ISP=75
      NCF=J-1
      CLOSE(UNIT=LIDI,ERR=2005)

C
C 76  CLEAR FRINGE * BUFFER
```

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

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```
C
      DO 76 I=1,NCF
      RFR(I)=0
76    CONTINUE

C
C 77.  INPUT TWO INITIAL FRINGE NUMBERS.
C
77    ISP=77
      CALL XOMESG(IER,21*'4000,4,RFR(1),22*'4000,4,RFR(2))
      IF (IER EQ.-273) GOTO 999
      IF (IER.NE 1) GOTO 2000

C
C 78.  FOR EACH CONTOUR DO:
C
78    DO 80 M=2,NCF-1

C      CLEAR TOTAL ERROR BUFFERS

      DO 79 I=1,NCF
      RT1(I)=0
      RTO(I)=0
      RT2(I)=0
      RTH(I)=0
79    CONTINUE

A.  C      IF THE FRINGE # OF THIS CONTOUR IS UNKNOWN, TRY NEXT CONTOUR
      IF (RFR(M) EQ.0) GOTO 80

C
C 85  OPEN THE CONTOUR
C
85    ISP=85
      OPEN(UNIT=LCON,NAME=BCF(1,M),TYPE='OLD',READONLY,ERR=2005)

C
C 90  SKIP HEADER RECORDS
C
90    ISP=90
      READ(LCON,*,END=2000,ERR=2005) NHH,NHR
      DO 91 I=1,NHR+1
91    READ(LCON,5002,END=2000,ERR=2005)

C
C 95  READ IN ALL POINTS ON THIS CONTOUR.
C
95    ISP=95
      I=1
96    READ(LCON,*,END=2000,ERR=2005) ILO(I),IPO(I)
      IF(ILO(I).NE -1) GOTO 97
      IF (ILO(I-1).EQ ILO(1)) THEN
      ILO(I)=ILO(2)
      IPO(I)=IPO(2)
      ILO(I+1)=ILO(3)
      IPO(I+1)=IPO(3)
      IPT=I-2
      GOTO 100
      END IF
      IPT=I-3
      GOTO 100
97    I=I+1
      GOTO 96
```



```

C
C 100. CLOSE CONTOUR FILE.
C
100    ISP=100
        CLOSE(UNIT=LCOE,ERR=2005)

C
C    FOR EACH POINT ON THIS CONTOUR DO
C
        DO 111 IPW=2,IPT+1

C
C 110    READ IN THREE POINTS
C        LEFT ONE, RIGHT ONE AND THE POINT ITSELF
C
110    ISP=110
        ILL=ILO(IPW-1)
        ILP=IPO(IPW-1)
        IML=ILO(IPW)
        IMP=IPO(IPW)
        IRL=ILO(IPW+1)
        IRP=IPO(IPW+1)

C
C 115    CALCULATE AZIMUTH.
C
115    ISP=115
        CALL AMFOAZ(IML,IMP,ILL,ILP,IML,IMP,IRL,IRP,'M',
1          XXXXX,IER,RV1,RV2,RAL,RAR)
        IF (IER.NE.1) GOTO 2000

C
C 120    CALCULATE RUN1 AND RUN2
C
120    ISP=120
        CALL AMFORM(RV1,RAL,XXXXX,IER,LIN1,PIX1,RRV1,BTH1)
        IF (IER.EQ.(-1)) GOTO 111
        IF (IER.NE.1) GOTO 2000
121    CALL AMFORM(RV2,RAR,XXXXX,IER,LIN2,PIX2,RRV2,BTH2)
        IF (IER.EQ.(-1)) GOTO 111
        IF (IER.NE.1) GOTO 2000

D        CALL IODETA(0)
D        WRITE(LTER,4010) IPW,IML,IMP,BTH1,BTH2,RAL,RAR
D4010    FORMAT(1X,'8,LN,PI,TM1,TM2,A1,A2',1X,
D        1          I3,2I5,2(I4),2(F8.2)) -
D        CALL IOATTA(0)

C
C    IF NONE OF THE TWO NEIGHBOURS FRINGE # IS KNOWN, SKIP THIS POINT
C    IF BOTH OF THE TWO NEIGHBOURS FRINGE # ARE KNOWN, SKIP THIS POINT
C
        IF (RFR(BTH1).EQ.0 AND RFR(BTH2).EQ.0) GOTO 111
        IF (RFR(BTH1).NE.0 AND RFR(BTH2).NE.0) GOTO 111

C
C 130    CALCULATE (N,L) AT POINT
C        N=(1/2(LIN1+IML),1/2(PIX1+IMP))
C
130    ISP=130
        IF (RFR(BTH1).NE.0) GOTO 131

```

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

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```

      IF (RFR(BTH2).NE.0) GOTO 132
131  LIN=LIN1
      PIX=PIX1
      LLL=LLL1
      PPP=PPP1
      RAZ=RA1
      BTM=BTM1
      BTH=BTH2
      GOTO 133

132  LIN=LIN2
      PIX=PIX2
      LLL=LLL1
      PPP=PPP1
      RAZ=RA2
      BTM=BTM2
      BTH=BTH1
      GOTO 133

133  IF (RAZ.LT.180) RAZ=RAZ-180
      IF (RAZ.GT.180) RAZ=MOD(RAZ,180 0)

D      CALL XODETA(0)
D      WRITE (LTER,4040) BTM,RAZ,RFR(BTM),RFR(M)
D4040  FORMAT(1X,'BTM,RAZ,RFR(BTM),RFR(M)',2X,I4,2X,F12 4,2(F8 2))
D      CALL XOATTA(0)

      IF (RFR(BTM).GT.RFR(M)) RIA=RAZ+180
      IF (RFR(BTM).LE.RFR(M)) RIA=RAZ
      CALL AMFONL(LIN,PIX,IML,IMP,RIA,RFR(BTM),RFR(M),XXXXXX,IER,RNL)

C
C 135  ESTIMATE REFLECTANCE VALUE AT POINT
C      M=(1/2(LIN+IML),1/2(PIX+IMP))
C
135  ISP=135
      CALL AMFORV(LIN,PIX,IML,IMP,XXXXXX,IER,RV,RUM)

C
C 140  CALCULATE MODEL PARAMETER AT POINT
C      M=(1/2(LIN+IML),1/2(PIX+IMP))
C
140  ISP=140
      RO=RV/RNL

D      CALL XODETA(0)
D      L=0.5*(LIN+IML)
D      P=0.5*(PIX+IMP)
D      WRITE(LTER,4014) L,P,RV,RIA,RNL,RO
D4014  FORMAT(1X,'LM,PX,RL,AZ,RL,RO',
D      1      2X,2I4,F8 2,F8 2,F12 8,2X,F14 4)
D      CALL XOATTA(0)

C
C 145  ESTIMATE NEXT FRINGE NUMBER
C
145  ISP=145
      CALL AMFORV(IML,IMP,LLL,PPP,XXXXXX,IER,RN,RH)

      RA1=RAZ
      CALL AMFONL(IML,IMP,LLL,PPP,RA1,RFR(M),
D      1      RFR(M)+1,XXXXXX,IER,RNL1)
      RE1=(RN-RO+RNL1)*(RN-RO+RNL1)

```

```

      RA2=RAZ
      CALL AMFOWL(IML,IMP,LLL,PPP,RA2,RFR(M),RFR(M),XXXXXX,IER,RNLO)
      REO=(RM-RO+RNLO)*(RM-RO+RNLO)

      RA3=RAZ+180
      CALL AMFOWL(IML,IMP,LLL,PPP,RA3,RFR(M),RFR(M)-1,XXXXXX,IER,RNL2)
      RE2=(RM-RO+RNL2)*(RM-RO+RNL2)

      RT1(BTH)=RT1(BTH)+RE1
      RTO(BTH)=RTO(BTH)+REO
      RT2(BTH)=RT2(BTH)+RE2
      NTH(BTH)=NTH(BTH)+1

D      CALL IODETA(O)
D      L=O 5*(IML+LLL)
D      P=O 5*(IMP+PPP)
D      WRITE(LTER,4022) RA1,RA2,RA3,RN1,RNLO,RNL2
D4022  FORMAT(1X,'A1,A2,A3,NL1,NLO,NL2',2I,3(F8 2),1I,3(F12 4))
D      WRITE(LTER,4018) L,P,RM,RE1,REO,RE2
D4018  FORMAT(1X,'LN,PX,RL,RE1,REO,RE2',2I4,F8 2,3(F12 4))
D      CALL IOATTA(O)

111    CONTINUE

C
C      ONE CONTOUR MAY HIT SEVERAL DIFFERENT CONTOURS
C      CHOSE THE CONTOUR WHICH HAS BEEN HITTED MOST
C      OF THE TIMES IF A CONTOUR HAS BEEN HITTED ONLY
C      2 OR 3 TIMES, OMIT IT
C

D      CALL IODETA(O)

      MAX=O
      DO 112 I=1,NCF
D      WRITE(LTER,3010) I,NTH(I)
      IF ((NTH(I) LT MAX) OR (NTH(I) GT 500)) GOTO 112
      MAX=NTH(I)
      IDI=I
112    CONTINUE

D      CALL IOATTA(O)

D3010  FORMAT(1X,'NTH(',I2,')',2I,I6)

      IF (MAX EQ O OR NTH(IDI) LT 5) GOTO 80

      RTE1(IDI)=RT1(IDI)
      RTEO(IDI)=RTO(IDI)
      RTE2(IDI)=RT2(IDI)
      REM=MIN(MIN(RT1(IDI),RTO(IDI)),RT2(IDI))
      IF (RT1(IDI) EQ REM) RFR(IDI)=RFR(M)+1
      IF (RTO(IDI) EQ REM) RFR(IDI)=RFR(M)
      IF (RT2(IDI) EQ REM) RFR(IDI)=RFR(M)-1

      CALL IODETA(O)
D      WRITE(LTER,3000) M,IDI
D3000  FORMAT(1X,'M=',I6,2X,'IDI=',I6)
      WRITE(LTER,4044) RTE1(IDI),RTEO(IDI),RTE2(IDI),REM
4044  FORMAT(1X,'RE1,REO,RE2,REM',4(F14 2,1X))

      DO 113 I=1,NCF
      WRITE(LTER,4016) I,RFR(I)

```

```

4016  FORMAT(1X,'RFR(',I2,')',2X,F6.2)
113   CONTINUE
      CALL IOATTA(0)

80    CONTINUE

C
C    EXAMINE THE FRINGE & BUFFER
C

      DO 160 I=1,NCF
      IF (RFR(I) EQ 0) GOTO 78
160   CONTINUE

C
C    DISPLAY FINAL RESULTS
C

      CALL XODETA(0)
      DO 990 I=1,NCF
      WRITE(LTER,4020) BTH(I),RFR(I),RTE1(I),RTE0(I),RTE2(I)
4020  FORMAT(1X,'CONTOUR',I2,1X,'FRINGE',F6.2,1X,'ERROR+',0,-',
1      3(F14.2,1X))
990   CONTINUE
      CALL IOATTA(0)

C
C 999  RETURN
C
999   IERROR=IER
      RETURN

C
C 2005  ERROR REPORTING
C
2005  CALL ERRSES(IER)
      GOTO 2000

C
C 2000  ERROR REPORTING
C
2000  MSG=20
      CALL IOERRM('AMFOPR',ISP,IER,MSG)
      GOTO 999

C
C-----
C
C    FORMAT STATEMENTS
C
5002  FORMAT(1X)
5004  FORMAT(1X,I3,10X,13A1,6X,1A1,5X,I3)
C
C-----
      END

```

D.3.8 AMFORN.FTN

```

C+
C    ADDRESS      DR2 [171,110]40
C    NAME         AMFORN.FTN
C    PURPOSE      CALCULATE THE DISTANCE FROM A POINT ON ONE
C                  CONTOUR TO THE NEAREST CONTOUR ALONG THE
C                  NORMAL VECTOR DIRECTION
C

```

```

C      LANGUAGE      FORTRAN
C      AUTHOR        DING XIBONG
C      DATE          3-DEC 85
C      FIRST USED ON  ARIES-II IMAGE ANALYSIS SYSTEM
C
C-
C-----

```

```

C      DESCRIPTION
C

```

```

C      THIS IS A SUBROUTINE OF TASK 'JFO'. IT CALCULATES THE DISTANCE FROM
C      A POINT ON ONE CONTOUR TO THE NEAREST CONTOUR ALONG THE NORMAL VECTOR
C      DIRECTION. IT FIRST ESTIMATES THE FARTHEST POINT THE NORMAL VECTOR CAN
C      GO. THEN IT CALLS ANOTHER SUBROUTINE TO EXAMINE THE THEME VALUES ALONG
C      THE VECTOR DIRECTION. IF IT FINDS A PIXEL WITH THEME VALUE NOT EQUALS
C      0, THEN THE DISTANCE BETWEEN THE START POINT AND THE NO 0 POINT IS
C      THE REQUIRED 'RUV' VALUE
C

```

```

C      ERROR MESSAGES
C

```

```

C      IERROR = -1      NOT HIT ANY CONTOUR IN GIVEN DIRECTION
C      IERROR = -2      NO THEME BIT IN THIS IMAGE
C      IERROR = -3      VECTOR HAS 0 LENGTH
C
C-----

```

```

C      VARIABLE DEFINITION SECTION
C

```

```

C      INPUT  VARIABLES
C

```

```

C      RVECTR  ARRAY CONTAINING START AND POINTS
C              OF NORMAL VECTOR
C              RVECTR(1,1) FIRST PIXEL
C              (1,2) LAST PIXEL
C              (2,1) FI
C              (2,2) LAST LINE
C      RAZNTH  AZIMUTH FOR THE NORMAL VECTOR
C

```

```

C      OUTPUT VARIABLES
C

```

```

C      IERROR  ERROR FLAG
C      LINECO  LINE COORDINATE
C      PIXLCO  PIXEL COORDINATE
C      RRUVVL  RUV VALUE
C      BTHNUM  THEME NUMBER
C

```

```

C      COMMON VARIABLES
C

```

```

C      BXLD    IXEL DEPTH - 1
C      I STL   START LINE OF DISPLAY IMAGE
C      I STP   START PIXEL OF DISPLAY IMAGE
C      NDIL    NUMBER OF DISPLAY LINES
C      NDIP    NUMBER OF DISPLAY PIXELS
C      BTH     NUMBER OF THEMES, ASSUMING PACKED FORMAT
C      BTH     THEME TYPE
C      ICBS    CURSOR STATUS
C

```

```

C      LOCAL VARIABLES
C

```

```

C      BNV     (8)MASK VALUE
C      BTH     THEME BITS
C      BTH     (128)THEME NUMBER
C      BUF     (128)THEME BITS
C      EDI     DISTANCE
C

```

```

C      EPM      MASK TO SELECT WANTED BITS
C      EPV      PIXEL VALUE
C      IDP      IXEL DEPTH
C      IXO      OLD IXV VALUE
C      IYO      OLD IYV VALUE
C      IXV      X VALUE INTERPOLATED
C      IYV      Y VALUE INTERPOLATED
C      IXX      X VALUE INTERPOLATED
C      IYY      Y VALUE INTERPOLATED
C      R1       tan(.)
C      R2       ctan(.)
C      RXY      (2,2)VECTOR
C      SDL      BEST LINE LENGTH ESTIMATE
C      SX       BEST LINE LENGTH ESTIMATE X DIRECTION
C      SY       BEST LINE LENGTH ESTIMATE Y DIRECTION
C      I        INTERPOLANT X
C      INC      INCREMENT IN X
C      Y        INTERPOLANT Y
C      YNC      INCREMENT IN Y
C
C-----
C      DECLARATIONS
C
C      SUBROUTINE ANFORM(RVECTR,RAZMTH,IXXXXI,IERROR,
1          LINECO,PIXLCO,RNUVVL,BTHNUM)
C
C      IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C      IMPLICIT LOGICAL*1(A),INTEGER*2(F-Q),REAL(R-Z)
C
C      DIMENSION RVECTR(2,2),RXY(2,2),BTH(128),BMV(8),BUF(128)
C
C      INCLUDE 'ANFOLU INC'
C
C      COMMON /XZ1IPR/ BIME, BILD, BARE(4), RDEC,
1          ISTL, ISTD, NDIL, NDIP,
2          ESAD, BNFE, BMTB, BFSP(8,10),
3          IDEC, JJJJ, NWPL, ICRS, RASP,
4          NBYT, BYBT, BMTF,
5          BRED, BGRN, BBLU,
6          BBB1, BTCD(128), BBB2, BCSP, IYAR, IXAR, ITAT,
7          ESTL, ESTP, EBYT, BUWI, BLOC(11)
C
C      DATA BMV/1,3,7,15,31,63,127,255/
C
C-----
C      PROCESS
C
C      10  SET INITIAL VALUES                                12
C
C      IERROR=1
C      BFL='F'
C      BKK='T'
C      ICRS='C'
C
C      12  SET IXEL DEPTH                                    15
C
C      IDP=BILD+1
C
C      15  CALCULATE THE BITS IF THERE                        999/20
C      BITS IS 0, SET ERROR FLAG AND RETURN
C
C      15  BTB=BFSP(7,BNFE+1)

```

```

      IF (BTB.NE 0) GOTO 20
      IERROR=-2
      GOTO 999

C 20  CALCULATE IXL MASK                                25
C
20    EPM=BMV(BTB)
      IF (ICRS EQ 'N') GOTO 25
      EPM=EPM+2

C
C 25  SET START POINT OF SEARCHING.                      30
C
25    RXY(1,1)=RVECTR(1,1)
      RXY(2,1)=RVECTR(2,1)

C
C 30  CALCULATE R1=tan(RAZMTH)and R2=ctan(RAZMTH)        35
C
      R2=0
      IF (RAZMTH EQ 90 OR RAZMTH EQ 270) GOTO 35
30    R1=ABS((RVECTR(2,2)-RVECTR(2,1))/(RVECTR(1,2)-RVECTR(1,1)))
      R2=0
      IF (R1.EQ 0) GOTO 35
      R2=1/R1

C 35  ESTIMATE THE FARTHEST POINT THE NORMAL             40/45/50/55/60/65/70/75
C        VECTOR CAN GO ACCORDING TO DIFFERENT AZIMUTH
C
35    IF (RAZMTH.GE.0.AND RAZMTH.LT.45) GOTO 40
      IF (RAZMTH.GE.45.AND RAZMTH.LT.90) GOTO 45
      IF (RAZMTH.GE.90.AND RAZMTH.LT.135) GOTO 50
      IF (RAZMTH.GE.135.AND RAZMTH.LT.180) GOTO 55
      IF (RAZMTH.GE.180.AND RAZMTH.LT.225) GOTO 60
      IF (RAZMTH.GE.225.AND RAZMTH.LT.270) GOTO 65
      IF (RAZMTH.GE.270.AND RAZMTH.LT.315) GOTO 70
      IF (RAZMTH.GE.315.AND RAZMTH.LT.360) GOTO 75

C 40  0 <= RAZMTH < 45                                80
C
40    RXY(1,2)=ESTP+WDIP-1
      RXY(2,2)=RXY(2,1)-ABS((RXY(1,2)-RXY(1,1))*R1)
      GOTO 80

C
C 45  45<= RAZMTH < 90                                80
C
45    RXY(2,2)=ESTL
      RXY(1,2)=RVECTR(1,1)+(RVECTR(2,1)-RXY(2,2))*R2
      GOTO 80

C
C 50  90<= RAZMTH < 135                                80
C
50    RXY(2,2)=ESTL
      RXY(1,2)=RVECTR(1,1)-(RVECTR(2,1)-RXY(2,2))*R2
      GOTO 80

C
C 55  135 <= RAZMTH < 180                              80
C
55    RXY(1,2)=ESTP
      RXY(2,2)=RVECTR(2,1)-(RVECTR(1,1)-RXY(1,2))*R1
      GOTO 80

C 60  180 <= RAZMTH < 225                              80
C

```

```

60      RXY(1,2)=ESTP
      RXY(2,2)=RVECTR(2,1)+(RVECTR(1,1)-RXY(1,2))*R1
      GOTO 80

C 65.  225 <= RAZNTH < 270.                                80
C
65      RXY(2,2)=ESTL+NDIL-1
      RXY(1,2)=RVECTR(1,1)-(RXY(2,2)-RVECTR(2,1))*R2
      GOTO 80

C
C 70  270 <= RAZNTH < 315                                80
C
70      RXY(2,2)=ESTL+NDIL-1
      RXY(1,2)=RVECTR(1,1)+(RXY(2,2)-RVECTR(2,1))*R2
      GOTO 80

C
C 75  315 <= RAZNTH < 360                                80
C
75      RXY(1,2)=ESTP+NDIP-1
      RXY(2,2)=RVECTR(2,1)+(RXY(1,2)-RVECTR(1,1))*R1

C
C 80  ESTIMATE LINE LENGTH ALONG X AND Y DIRECTIONS        999/85
C      AND SELECT THE LONGER ONE
C
80      SX=ABS(RXY(1,2)-RXY(1,1))
      SY=ABS(RXY(2,2)-RXY(2,1))
      IF(SY.GE.SX) SDL=SY
      IF(SX.GE.SY) SDL=SX
      IF(SDL.EQ.0) GOTO 85
      IERROR=-3
      GOTO 999

C
C 85.  CALCULATE INCREMENTAL VALUES, ACCOUNTING FOR        90
C      LINK DIRECTION.
C
85      INC=(RXY(1,2)-RXY(1,1))/SDL
      YNC=(RXY(2,2)-RXY(2,1))/SDL

C 90.  INITIALIZE INTERPOLATION POINT.                      95
C
90      I=RXY(1,1)+0.5
      Y=RXY(2,1)+0.5

C
C 95.  SET INITIAL VALUES FOR EDI, IX0, IY0 AND           100
C      BEGING SEARCH ALONG THE GIVEN DIRECTION
C
95      EDI=0
      IX0=I+INC
      IY0=Y+YNC
      DO 96 I=1,SDL
          I=I+INC
          Y=Y+YNC
          IXV=I
          IYV=Y

C 100. IT IS POSSIBLE THAT A NORMAL VECTOR MAY NOT         105/125
C      HIT A CONTOUR EVEN IF THERE IS ONE. THIS CASE
C      CAN ONLY HAPPEN WHEN THE ITH IXV AND IYV VALUES
C      ARE BOTH DIFFERENT FROM THE (I-1)TH IXV AND IYV
C      VALUES. IN THESE CASES, WE SHOULD LOOK 2 MORE
C      POINTS AROUND THE (I-1)TH POINT BEFORE GOING
C      TO ITH POINT

```


APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

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```

C
100      IF (IXV.EQ.IXO.OR.IYV.EQ.IYO) GOTO 125

C
C 105. FIRST POSSIBLE POINT AROUND IXO AND IYO THAT          110
C      MAY HIT A CONTOUR
C
105      IXX=IXO+1
          IYY=IYO
          IF (IXV.GT.IXO.AND.IYV.LT.IYO) GOTO 110
          IXX=IXO
          IYY=IYO-1
          IF (IXV.LT.IXO.AND.IYV.LT.IYO) GOTO 110
          IXX=IXO-1
          IYY=IYO
          IF (IXV.LT.IXO.AND.IYV.GT.IYO) GOTO 110
          IXX=IXO
          IYY=IYO+1
          IF (IXV.GT.IXO.AND.IYV.GT.IYO) GOTO 110

C
C 110 EXAMINE THERE VALUE AT A SPECIFIED POINT                2000/115/140
C
110      ISP=110
          IF ((IXX.LE.ESTP) OR (IXX.GE.(ESTP+NDIP-1))) GOTO 135
          IF ((IYY.LE.ESTL) OR (IYY.GE.(ESTL+NDIL-1))) GOTO 135
          CALL AMFOEX(IXX,IYY,IDP,EPN,BFL,XXXXXX,IER,IPV)
          IF (IER.NE.1) GOTO 2000
          IF (IPV.EQ.0) GOTO 115
          RPIXL=REAL(IXX)-RVECTR(1,1)
          RLINE=REAL(IYY)-RVECTR(2,1)
          RRUNVL=SQRT(RLINE*RLINE+RPIXL*RPIXL)
          IF (RRUNVL.LE.3) GOTO 115
          LINECO=IYY
          PIXLCO=IXX
          GOTO 140

C 115 SECOND POSSIBLE POINT AROUND IXO AND IYO THAT          120
C      MAY HIT A CONTOUR
C
115      IXX=IXO
          IYY=IYO-1
          IF (IXV.GT.IXO.AND.IYV.LT.IYO) GOTO 120
          IXX=IXO-1
          IYY=IYO
          IF (IXV.LT.IXO.AND.IYV.LT.IYO) GOTO 120
          IXX=IXO
          IYY=IYO+1
          IF (IXV.LT.IXO.AND.IYV.GT.IYO) GOTO 120
          IXX=IXO+1
          IYY=IYO
          IF (IXV.GT.IXO.AND.IYV.GT.IYO) GOTO 120

C
C 120. EXAMINE THERE VALUE AT A SPECIFIED POINT                2000/125/140
C
120      ISP=120
          CALL AMFOEX(IXX,IYY,IDP,EPN,BFL,XXXXXX,IER,IPV)
          IF (IER.NE.1) GOTO 2000
          IF (IPV.EQ.0) GOTO 125
          RPIXL=REAL(IXX)-RVECTR(1,1)
          RLINE=REAL(IYY)-RVECTR(2,1)
          RRUNVL=SQRT(RLINE*RLINE+RPIXL*RPIXL)

```

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

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```

      IF (BRUNVL.LE.3) GOTO 125
      LINECO=IYY
      PIXLCO=IIX
      GOTO 140

C
C 125. CONTINUE SEARCH ALONG GIVEN DIRECTION.                2000/130/140
C
125   ISP=125
      CALL AMFOEX(IIV,IYV,IDP,EPH,BFL,IXIIXI,IER,IPV)
      IF (IER.NE.1) GOTO 2000
      EDI=EDI+1
      IF (IPV.EQ.0) GOTO 130
      RPIXL=REAL(IIV)-RVECTR(1,1)
      RLINE=REAL(IYV)-RVECTR(2,1)
      BRUNVL=SQRT(RLINE*RLINE+RPIXL*RPIXL)
      IF (BRUNVL.LE.3) GOTO 130
      LINECO=IYV
      PIXLCO=IIV
      GOTO 140

C
C 130. BEFORE GOING TO NEXT POINT, SAVE THE OLD IIV AND IYV      96
C
130   IIO=IIV
      IYO=IYV
96    CONTINUE

C
C 135. NOT FIND ANY CONTOUR IN THIS DIRECTION                    999
C      SET ERROR FLAG AND RETURN
C
135   IERROR=-1
      GOTO 999

140   ISP=140

C
C 145. CALCULATE THE THEME NUMBER OF THAT CONTOUR.              150
C
145   IF (ICRS.EQ.'C') IPV=IPV/2
      BTHNUM=IPV

C
C 150. DISPLAY RESULTS ON TERMINAL.                                999
C
D150  CALL IODETA(0)
D      WRITE(LTER,5002) BRUNVL,BTHNUM
D      CALL IOATTA(0)

C 999  CLEAR THE CONTOUR
C
999   BKK='Q'
      IF (BKK.EQ.'Q') GOTO 991
      BFL='C'
      BKK='Q'
      GOTO 80

C
C RETURN.
C
991   BKK='T'
      RETURN

C 2000. ERROR REPORTING.
C

```

```

2000 MSG=20
      CALL XERRM('AMFORV',ISP,IER,MSG)
      GOTO 999

```

```

C

```

```

C      FORMAT STATEMENTS

```

```

C

```

```

5000 FORMAT(1X,'ESTL=',I4,/,
1       1X,'ESTP=',I4,/,
2       1X,'NDIL=',I4,/,
3       1X,'NDIP=',I4,/,
4       1X,'BETH=',I4)
5001 FORMAT(1X,'RVECTR(1,1)=' ,F6 2,/,
1       1X,'RVECTR(2,1)=' ,F6 2,/,
2       1X,'RVECTR(1,2)=' ,F6 2,/,
3       1X,'RVECTR(2,2)=' ,F6 2,/,
4       1X,'RAZMTH=' ,F8.2)
5002 FORMAT(1X,'ERRUNVL=' ,F6 2,/,
1       1X,'BTHNUM=' ,I3)

```

```

C

```

```

C-----

```

```

      END

```

D.3.9 AMFORV.FTN

```

C+
C      ADDRESS      DR2-[171,110]
C      NAME         AMFORV.FTN
C      PURPOSE      TO ESTIMATE REFLECTANCE VALUE AT
C                   GIVEN POINT
C      LANGUAGE     FORTRAN
C      AUTHOR       DING XINHONG
C      DATE         10-MARCH-86
C      FIRST USED ON ARIES-II SYSTEM FOR
C                   MOIRE FRINGE ANALYSIS
C
C
C-----

```

```

C
C      DESCRIPTION
C
C      THIS SUBROUTINE COMPUTES THE REFLECTANCE VALUE AT A GIVEN POINT
C      IT AVERAGES THE VMA IMAGE FEATURE VALUES INDISE A CIRCLE THAT HAS
C      THE GIVEN POINT AS THE CENTRE
C
C-----

```

```

C      VARIABLES
C
C      INPUT  VARIABLES
C
C      LINEST
C      PIXEST
C      LINEED
C      PIXEED
C
C      OUTPUT VARIABLES
C
C      IERROR  ERROR FLAG
C      REFVAL  REFLECTANCE VALUE
C      NUMPOT  NUMBER OF POINTS

```

```

C
C      LOCAL VARIABLES 4
C
C-----

C      ,DECLARATIONS
C
C      SUBROUTINE ANFORV(LINEST,PIKEST,LINEED,PIKEED,
1          XXXXX,IEROR,REFVAL,NUMPOT)
C
C      IMPLICIT BYTE(B),COMPLEX(C),DOUBLE PRECISION(D),INTEGER*4(E)
C      IMPLICIT LOGICAL*1(A),INTEGER(F-Q),REAL(R-Z)
C
C      DIMENSION BVBF(8128),IAD(2),BRY(2),BIL(2)
C
C      INCLUDE 'AMFOLU INC'
C
C      COMMON /DPICOM/ IVBF(4064),IIII(30),IAOR(2)
C      COMMON /XZ1IPR/ BINB, BILD, BARE(4), RDEC,
2          ISTL, ISTD, EDIL, EDIP,
3          ESAD, BNFE, BETH, BFSP(8,10),
4          IDEC, JJJJ, NVPL, ICRS, RASP,
5          EFLC, EFPC, ELLC, ELPC, NBYT, BETH, BETH,
6          BBED, BGRN, BBLU,
7          BBB1, BTCD(128), BBB2, BCSP, IYAR, IYAR, ITAT,
          ESTL, ESTP, EBYT, BUWI, BLOC(11)
C
C      EQUIVALENCE (EAD,IAD(1))
C      EQUIVALENCE (IVBF(1),BVBF(1))
C      EQUIVALENCE (IIL,BIL(          EQUIVALENCE (GRY,BRY(1))
C
C-----

C      PROCESS
C
C      10. SET ERROR FLAG
C
C      IER=1
C      ICRS='C'
C
C      25. CALCULATE MIDDLE POINT COORDINATE
C
C      PIXH=0.5*(PIKEST+PIKEED)
C      LNH=0.5*(LINEST+LINEED)
C
C      30. CALCULATE R
C
C      RR=(PIKEST-PIKEED)*(PIKEST-PIKEED)+(LINEST-LINEED)*(LINEST-LINEED)
C      QQ=INT(RR/4)
C      R=SQRT(RR)
C      Q=INT(R/2)
C
C      35. INTEGRATE ALL THE REFLECTANCE VALUES INSIDE THE CIRCLE
C
C      IL1=LNH-Q
C      IL2=LNH+Q
C      IP1=PIXH-Q
C      IP2=PIXH+Q
C      RS=0
C      NUMPOT=0
C      DO 36 ILN=IL1,IL2
C      DO 37 IPX=IP1,IP2

```

```

      QLP=(IPX-PIH)*(IPX-PIH)+(ILH-LWH)*(ILH-LWH)
      IF (QLP.GT.QQ) GOTO 37

      PI=IPI-ESTP+1
      ELI=ILH
      ENL=ELI-ESTL
      EAD=ESAD+(ENL*WVPL*2)
      IAOB(1)=IAD(1)
      IAOB(2)=IAD(2)
      IXL=IVBF(PI)
      BRY(1)=BIL(2)
      RV=REAL(GRY)
      RS=RS+RV
      NUMPOT=NUMPOT+1
      IVBF(PI)=IOB(IVBF(PI),1)

C
37      CONTINUE
36      CONTINUE
      REFVAL=RS/NUMPOT

D      CALL IODETA(0)
D      WRITE(LTER,4006) REFVAL,NUMPOT
D4006  FORMAT(1X,'REFVAL,NUM',5X,F10.2,2X,18)
      CALL IOATTA(0)

C
C      RETURN
C
999      RETURN
C
C 2000  ERROR REPORTING
C
2000      MSG=20
      CALL IOERRM('AMFORV',ISP,IER,MSG)
      GOTO 999

C-----
C
C      FORMAT STATEMENTS
C
C-----
      END

```

D.4 Fortran Include-Type Files for Task JFO

D.4.1 AMFOLU.INC

```

C+
C
C      NAME          AMFOLU INC
C      PURPOSE       DEFINE ALL LOGICAL UNITS USED IN THE TASK
C      LANGUAGE      FORTRAN
C      AUTHOR        DING XINHONG
C      DATE          10-MARCH-86
C      FIRST USED ON  ARIES II IMAGE ANALYSIS SYSTEM FOR THE
C                    AUTOMATED NOISE FRINGE ORDERING TASK
C
C-----
C
C      DESCRIPTION

```

```

C
C   THIS COMMON BLOCK IS CALLED /AMFOLU/ WHICH CONTAINS THE
C   LOGICAL UNIT NAMES REQUIRED BY THIS TASK.
C

```

```

C           LCFG      CONFIGURATION FILE
C           LVMA      VMA DISK
C           LIDI      INDEX FILE
C           LCON      CONTOUR POINT FILE
C           LCEF      TRANSFORMATION COEFFICIENT FILE
C           LMOR      MOIRE INSTRUMENT PARAMETER FILE
C           LTER      LOGICAL UNIT FOR TERMINAL
C           LPRT      LOGICAL UNIT FOR LINE PRINTER
C

```

```

C-----
C
C   DECLARATIONS
C
C   COMMON /AMFOLU/ LCFG, LVMA, LIDI, LCON, LCEF, LMOR, LTER, LPRT
C
C-----
C

```

D.4.2 AMFOMR.INC

```

C+
C
C   NAME           AMFOMR INC
C   PURPOSE        DEFINE MOIRE INSTRUMENT PARAMETERS USED IN THE TASK
C   LANGUAGE       FORTRAN
C   AUTHOR         DING YISHONG
C   DATE           10-MARCH-86
C   FIRST USED ON  ARIES II IMAGE ANALYSIS SYSTEM FOR THE
C                   AUTOMATED MOIRE FRINGE ORDERING TASK
C
C-

```

```

C-----
C
C   DESCRIPTION
C
C   THIS COMMON BLOCK IS CALLED /AMFOMR/ WHICH CONTAINS THE
C   MOIRE INSTRUMENT PARAMETERS
C
C   Model: OTAL Moire Contourgraph SS-20-40
C
C   All values are in millimeters
C
C   RIMP      RYMP      RZMP
C   <x>        <y>        <z>
C   0          0          0      (origin @ intersection of fiducial strings)
C   0          2          0      (direction and magnitude of grid pitch)
C   0          0          1700   (camera position)
C   0          710        1700   (lamp position)
C   -237.5     350         0      (fiducial target: upper left)
C   237.5      350         0      (fiducial target: upper right)
C   -237.5     -350        0      (fiducial target: lower left)
C   237.5      -350        0      (fiducial target: lower right)
C   ?          ?          ?      (step tablet, left side ID 85-1)
C   ?          ?          ?      (step tablet, right side ID 85-2)
C
C-----

```

```

C   DECLARATIONS

```

```

C
C      DIMENSION RXMP(10),RYMP(10),RZMP(10)
C
C      COMMON /AMFOTC/ RXMP,RYMP,RZMP,SCAL,AZMO
C-----
C

```

D.4.3 AMFOTC.INC

```

C+
C
C      NAME          AMFOTC.INC
C      PURPOSE       DEFINE TRANSFORMATION COEFFICIENTS USED IN THE TASK
C      LANGUAGE      FORTRAN
C      AUTHOR        DING XINHONG
C      DATE          10-MARCH-86
C      FIRST USED ON  ARIES II IMAGE ANALYSIS SYSTEM FOR THE
C                    AUTOMATED MOIRE FRINGE ORDERING TASK
C-----
C

```

```

C      DESCRIPTION
C
C      THIS COMMON BLOCK IS CALLED /AMFOTC/ WHICH CONTAINS THE
C      COEFFICIENTS REQUIRED FOR COORDINATE TRANSFORMATIONS BETWEEN
C      THE DIGITIZED IMAGE PLANE AND THE GRID PLANE OF MOIRE INSTRUMENT
C
C      THE FOLLOWING COORDINATE TRANSFORMATION IS USED
C
C              X = a0 + a1*x + a2*y
C              Y = b0 + b1*x + b2*y
C              x = c0 + c1*X + c2*Y
C              y = d0 + d1*X + d2*Y
C

```

```

C      WHERE  x IS THE HORIZONTAL IMAGE PLANE COORDINATE
C             y IS THE VERTICAL IMAGE PLANE COORDINATE
C             X IS THE HORIZONTAL GRID PLANE COORDINATE
C             Y IS THE VERTICAL GRID PLANE COORDINATE
C

```

```

C      This is a 1st order polynomial transformation, the coefficients of
C      which can be calculated by DIPIX Task "GC" It accounts for scale
C      change and rotation and maps the image plane into the grating plane of
C      the instrument
C

```

```

C      These coefficients for image ABOFFBBB taken from a listing of file
C      'ABOPTUTMO' as produced by task "GC" These parameters are useful
C      only for that image
C

```

```

C-----"FORWARD X TRANSFORM"-----
C      -0.9272037506E+01 = A0
C      0.1197631657E-01 = A1
C      -0.1080797083E-03 = A2
C-----"FORWARD Y TRANSFORM"-----
C      0.1991445557E+04 = B0
C      0.9134858556E-04 = B1
C      0.1203695778E-01 = B2
C-----"FORWARD X TRANSFORM"-----
C      -0.7194415894E+03 = C0
C      0.8349156189E+02 = C1

```

```

C          0.7500000000E+00 = C2
C-----"FORWARDY TRANSFORM"-----
C          -0.1654330937E+06 = D0
C          -0.6329114437E+00 = D1
C          0.8307142639E+07 = D2
C-----
C
C  DECLARATIONS
C
COMMON /ANFOTC/ TRA0,TRA1,TRA2,TRB0,TRB1,TRB2,
1             TRC0,TRC1,TRC2,TRD0,TRD1,TRD2
C-----
C

```

D.5 Example Data Files Required by Task JFO

D.5.1 Moiré Instrument Parameter File

```

1, 10 = # of records in header-----DR2 [200,200]MOIREINST IYZ
2
3   This file contains the IYZ-coordinates that define the spatial
4   relationship of the primary components of the moire contourgraph
5
6
7
8   All values are in millimeters
9
10  <x>      <y>      <z>
11
12      0      0      0      (origin @ intersection of fiducial strings)
13      0      2      0      (direction and magnitude of grid pitch)
14      0      0      1700    (camera position)
15      0      710    1700    (lamp position)
16      -237 5    350    0      (fiducial target upper left)
17      237 5    350    0      (fiducial target upper right)
18      -237 5   -350    0      (fiducial target lower left)
19      237 5   -350    0      (fiducial target lower right)
20      0      0      0      (step tablet, left side ID 85-1)
21      0      0      0      (step tablet, right side ID 85-2)
22
23-----EOF-----

```

D.5.2 Moiré Image Transformation Coefficients File

```

25 = # of header records-----[171,106]BSPFOR100 CEF
-1
0
1
2
3   This file contains the coefficients required coordinate transformations
4   between the digitized topogram and the contourgraph instrument
5
6   NOTE> The following coordinate transformation is used:
7
8   X = a0 + a1*x + a2*y      x is the horizontal image coordinate
9   Y = b0 + b1*x + b2*y      y is the vertical image coordinate
10  x = c0 + c1*X + c2*Y      X is the horizontal instrument coordinate
11  y = d0 + d1*X + d2*Y      Y is the vertical instrument coordinate
12

```



```

13 This is a 1st order polynomial transformation, the coefficients of
14 which can be calculated by DIPIX Task "GC" It accounts for scale
15 change and rotation and maps the image plane into the grating plane of
16 the instrument
17
18 These coefficients for image BX1FFR100 taken from a listing of file
19 'BX1PTUTMO' as produced by task "GC" These parameters are useful
20 only for that image
21

```

```

-----"FORWARD X TRANSFORM"-----
- 5140842285E+03 = a0
0 5969551802E+00 = a1
- 3573687281E-02 = a2
-----"FORWARD Y TRANSFORM"-----
0 4847613220E+03 = b0
- 2272936119E-02 = b1
- 5992960930E+00 = b2
-----"FORWARD Z TRANSFORM"-----
0 8660000000E+03 = c0
0 1675105453E+01 = c1
- 9999999776E-02 = c2
-----"FORWARD Y TRANSFORM"-----
0 8055999756E+03 = d0
- 6329114083E-02 = d1
- 1668571472E+01 = d2
-----eof-----

```

D.5.3 Moiré Image Map List

27 = # OF HEADER RECORDS ----- FILE NAME = BSPDCBBBB OUT

DESCRIPTION

THIS IS AN OUTPUT FILE FOR TASK JDC WHICH
 DRAWS CONTOURS USING QUADRATIC CURVE IN
 VMA IT RECORDS SOME PARAMETERS OF THE IMAGE ON
 WHICH THE CONTOURS ARE DRAWN IT ALSO RECORDS ALL
 THE CONTOUR NAMES, THEIR NUMBERS, TYPE AND THE
 THEME NUMBERS ASSIGNED TO THEME

THE FOLLOWING IS THE PARAMETERS OF THE IMAGE

IMAGE NUMBER = 2
 AREA NAME = BSP
 PIXEL DEPTH = 15
 NUMBER OF FEATURES = 1
 NUMBER OF THEME FILES = 1
 NUMBER OF THEMES = 0
 THEME TYPE = P
 CURSOR = C
 NUMBER OF DISPLAY LINES = 501
 NUMBER OF DISPLAY PIXELS = 348
 FIRST LINE = 560
 FIRST PIXEL = 693
 LAST LINE = 1060
 LAST PIXEL = 1040

```

-----END OF HEADER-----

```

CONTOUR #	FILE NAME	TYPE	THEME #	COLOUR
1	BSPSPBB10 SPF	C	1	
2	BSPSPBB01 SPF	C	2	
3	BSPSPBB02 SPF	C	3	
4	BSPSPBB03 SPF	C	4	

5	BSPSPBB04.SPF	C	5
6	BSPSPBB05.SPF	C	6
7	BSPSPBB07.SPF	C	7
8	BSPSPBB08.SPF	C	8
9	BSPSPBB09.SPF	C	9

D.5.4 Moiré Image Contour-Point Lists

File: BSPSPBB01.SPF

```

1, 7 = # of header records -----
2, file name = BSPSPBB01.SPF
3,      CONTOUR 1 (B)
4,
5,
6,
7, -----end of header-----

```

LINES	PIXEL #
805	928
836	923
862	897
869	863
855	833
823	814
785	821
757	855
762	901
805	928
-1	-1

File: BSPSPBB02.SPF

```

1, 7 = # of header records -----
2, file name = BSPSPBB02.SPF
3,      CONTOUR 2 (B)
4,
5,
6,
7, -----end of header-----

```

LINES	PIXEL #
805	952
844	946
873	921
891	888
890	850
869	816
833	793
793	792
756	812
736	847
733	869
758	930
805	952
-1	-1

File: BSPSPBB03.SPF

```

1, 7 = # of header records -----

```

APPENDIX D. PROGRAM LISTINGS FOR TASK JFO

485

```

2, file name = BSPSPBB03 SPF
3,          CONTOUR 3 (B)
4,
5,
6,
7, -----end of header-----

```

LINE#	PIXEL #
805	968
847	962
880	941
903	905
909	865
896	824
865	790
823	775
778	780
737	809
716	852
720	902
751	947
805	968
-1	-1

File: BSPSPBB04.SPF

```

1, 7 = 3 of header records -----
2, file name = BSPSPBB04 SPF
3,          CONTOUR 4 (B)
4,
5,
6,
7, -----end of header-----

```

LINE#	PIXEL #
805	981
847	977
881	958
907	927
920	884
918	844
901	805
870	777
831	762
791	761
752	777
718	813
702	852
704	895
722	936
745	959
775	974
805	981
-1	-1

File: BSPSPBB05.SPF

```

1, 7 = 3 of header records -----
2, file name = BSPSPBB05 SPF
3,          CONTOUR 5 (B)
4,
5,

```

6,
7, -----end of header-----

LINE#	PIXEL #
804	993
842	989
877	974
907	949
928	911
932	868
924	826
904	791
873	764
829	749
775	754
729	781
703	814
690	861
694	902
713	941
739	969
772	986
804	993
-1	-1

File: BSPSPBB07.SPF

1, 7 = # of header records -----
2, file name = BSPSPBB07.SPF
3,   CONTOUR 6 (B)
4,
5,
6,
7, -----end of header-----

LINE#	PIXEL #
909	959
920	946
936	918
939	905
943	881
942	854
939	836
929	809
909	781
887	763
856	747
820	739
779	743
747	756
715	782
693	816
682	850
682	887
692	923
714	958
742	982
776	997
821	1002
855	995
886	979
909	959
-1	-1

File: BSPSPBB08.SPF

```

1, 7 = # of header records. -----
2, file name = BSPSPBB08.SPF
3,          CONTOUR 7.(B)
4,
5,
6,
7, -----end of header-----

```

LINE#	PIXEL #
901	979
920	959
941	927
951	889
950	849
942	818
928	790
906	767
888	753
857	738
826	731
788	732
747	745
716	767
692	798
676	832
672	873
678	911
692	943
715	972
742	992
770	1004
814	1010
851	1006
886	989
894	984
901	979
-1	-1

File: BSPSPBB09.SPF

```

1, 7 = # of header records. -----
2, file name = BSPSPBB09.SPF
3,          CONTOUR 8.(B)
4,
5,
6,
7, -----end of header-----

```

LINE#	PIXEL #
910	982
922	969
941	944
951	921
958	892
960	863
966	840
949	816
939	795
925	776
907	758
890	746

866	733
837	725
812	723
779	726
748	737
722	754
701	773
687	791
675	816
667	845
665	873
667	898
673	923
682	940
695	959
714	981
740	999
772	1012
798	1017
835	1016
859	1011
887	998
910	982
-1	-1

File: BSPSPBB10.SPF

```

1, 7 = # of header records. -----
2, file name = BSPSPBB10.SPF
3,          CONTOUR 9.(B)
4,
5,
6,
7, -----end of header-----

```

LINE#	PIXEL #
822	881
827	876
830	868
828	861
817	857
808	857
800	865
799	873
805	882
814	884
822	881
-1	-1

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