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**Effect of Void Damage
and Shear Band Development on the Bendability of
AA6111 Automotive Aluminum Alloy Sheet**

by

W. Brent Liewers

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Master of Engineering

Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

May 9, 2001

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**Effect of Void Damage and Shear Band Development on the Bendability of
AA6111 Automotive Aluminum Alloy Sheet**

submitted by W. Brent Lievers
in partial fulfillment of the requirements for
the degree of Master of Engineering
in Mechanical Engineering

A.K. Pilkey, Thesis Supervisor

Chair, Department of Mechanical and Aerospace Engineering

Carleton University

May 9, 2001

"Sometimes a scream is better than a thesis."

Ralph Waldo Emerson (1803 - 1882)
Journals, 1836

*To Jen(n),
for her love, her heart, her smile*

*to my parents,
for their endless support and encouragement*

*and to Him
who blessed me with them.*

Abstract

Iron (Fe) has been shown to greatly influence the formability of aluminum alloys. Increased Fe results in a greater number of second-phase particles, which act as initiation sites for void damage. Four AA6111 Fe-variants have been studied, using a cantilever bend test apparatus, to determine the effect of void damage on bendability. To simulate the development of void damage and shear band localization, the kinematic hardening Gurson-Tvergaard-Needleman (GTN) equations have been reformulated for use in LS-DYNA, a commercial finite element method (FEM) code.

Bendability was found to be a function of reduced grain size and increased void damage, with increasing Fe content. Numerical predictions show that the greater curvature of the kinematic hardening yield surface allows shear bands to initiate more easily than under isotropic hardening conditions. Furthermore, the predictions were also found to be highly dependent upon the level of triaxial stress, the element size and aspect ratio, as well as the void nucleation parameters of the GTN equations.

Acknowledgements

I would like to thank and acknowledge:

- my thesis supervisor, Prof. A.K. Pilkey, for his support, guidance, and assistance during the past three years;
- Prof. M.J. Worswick, my academic grandfather, for kindly allowing me to build upon his Gurson-Tvergaard-Needleman LS-DYNA user material subroutine;
- my colleagues Cheryl Thomson and Jacobus Franciscus Adrianus Maria van Hoof for their LS-DYNA tutorials, their L^AT_EX advocacy and their patience;
- my colleague Shiva Bissoon for his invaluable assistance during the bend testing and for mounting, polishing and capturing images of the specimens;
- the technical assistance and endless help of Dan Evans of Alcan KRDC;
- Drs. D.J. Lloyd and M. Jain, both of Alcan KRDC, for their interest and involvement in this work;
- the financial support provided by the Kingston Research and Development Centre of Alcan International Limited, the Natural Sciences and Engineering Research Council of Canada and the Department of Mechanical and Aerospace Engineering at Carleton University.

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Nomenclature

α_{ij}	centre of the kinematic yield surface
δ_o	initial magnitude of the thickness inhomogeneity in the Tvergaard bend model
ε	true strain
ε_o	initial strain
ε_n	mean strain for void nucleation
$\bar{\varepsilon}^p$	effective plastic strain
$\dot{\bar{\varepsilon}}^p$	rate of effective plastic strain
η	distance to neutral axis from the outer surface, as a function of sheet thickness t
θ	angular displacement of the roller
θ_0	initial angular position of the roller
θ_{min}	angle at which the cantilever bend test roller begins to contact the bend specimen
$\dot{\mu}$	kinematic hardening yield surface centre evolution parameter
ν	Poisson's Ratio
σ	stress
σ_e	von Mises equivalent stress
σ_f	radius of the kinematic hardening yield surface
σ_{ij}	Cauchy stress

σ_m	matrix material flow stress
σ_n	mean stress for void nucleation
σ_R	residual circumferential stress
Φ	Gurson yield surface equation
A	void nucleation parameter
A_r	reduction in area
B	void nucleation parameter
b	parameter to select degree of kinematic and isotropic behaviour, in the range [0,1]
c	adiabatic speed of sound in the material
d_{FEM}	linear displacement extracted from the FEM model
$d_{crosshead}$	linear displacement of the crosshead
E	Young's modulus
F	index related to anisotropy and stress-strain states
f	porosity in the GTN equations
f^*	accelerated porosity in the GTN equations
f_c	critical porosity to cause coalescence
f_f	porosity at final fracture
F_{FEM}	force extracted from the FEM cantilever bend model
f_n	volume fraction of void nucleating particles
f_u^*	ultimate porosity at which all material strength is lost, $1/q_1$
F_x	contact force in the x-direction
F_y	contact force in the y-direction
$\dot{f}$	rate of change of porosity

$\dot{f}_{growth}$	rate of change of porosity by void growth
$\dot{f}_{nucleation}$	rate of change of porosity by void nucleation
G	shear modulus
h	instantaneous hardening modulus
h_o	initial height/thickness of the Tvergaard bend model
$h_{clearance}$	additional roller clearance for cantilever bend test
h_{gap}	gap height for the cantilever bend test
K	power law constant
l_o	initial width of the Tvergaard bend model
l_e	characteristic element length
N	Aravas power law exponent
n	power law hardening exponent
q_1, q_2, q_3	Tvergaard coefficients for GTN equation
R	minimum bend radius
$\bar{R}$	normal (transverse) anisotropy
l_o	initial radius of curvature of the Tvergaard bend model
R_o	die radius plus half-thickness of sheet
R_f	radius of curvature of sheet mid-section after springback
R_{drum}	radius of the drum
$R_{mandrel}$	radius of the mandrel
R_{roller}	radius of the roller
s_{ij}	deviatoric stress
s_n	standard deviation of strain for normal distribution of nucleating particles
t	sheet thickness

Δt_e	minimum time step for an element
w_{model}	width of the specimen in the FEM model
$w_{specimen}$	width of the cantilever bend test specimen
y	distance from the centre of the sheet

Chapter 1

Introduction

For any given research to be properly understood, it must be viewed within the proper framework which may be comprised of historical, economic, political and other factors. It is only within this context that the relevance of a given piece of work can be understood and evaluated. Accordingly, the following sections will outline a number of issues that have made the bendability of aluminum sheet a topical area of study, including:

- the motives for and methods of achieving automotive weight reduction;
- the role that aluminum alloys can and do play as a means to achieve these reductions;
- the way in which an improved understanding of the bending behaviour of aluminum alloy sheet can facilitate its use by the automotive industry.

1.1 Automotive Weight Reduction

The pursuit of automotive weight reduction is primarily a by-product of the need for improved fuel efficiency and reduced emissions. Increased efficiency can be achieved by a number of means, such as the reduction of drag through more aerodynamic design or the re-engineering of the engine. Further improvements can be achieved by reducing the overall weight of the vehicle.

1.1.1 Motivations for Automotive Weight Reduction

There are a number of forces motivating the improvement of automotive fuel efficiency and, by extension, weight reduction. For example, there are societal pressures. The preservation of the environment is a priority for a number of individuals and interest groups. Accordingly, they seek to persuade auto-makers to produce lighter, more fuel efficient, automobiles with reduced emissions.

The auto-makers also perceive a level of economic benefit in weight reduction. There are primary benefits in that lighter vehicles will, in a broad sense, require less material and therefore cost less to produce. Secondary weight reduction can then be achieved through the down-sizing of the brakes, engine, suspension, and other components made possible by a structurally lighter vehicle [1].

There are also legislative pressures, both national and international, behind the push for automotive weight reduction. In 1975, for example, during the peak of the oil crisis, the U.S. government enacted the Corporate Average Fuel Economy (CAFE) standards as part of the Energy Policy and Conservation Act [2]. The intent of CAFE was to reduce U.S. reliance on foreign oil through prescribed increases in the fuel efficiency of vehicles.

Compliance is measured by calculating the fuel efficiency of an auto-maker's product line by a sales-weighted mean formula, with domestically produced and imported vehicles measured separately. Financial penalties are then imposed for failure to comply.

Furthermore, there are international agreements affecting vehicle emissions that must also be considered, such as the Kyoto Protocol [3]. In 1997, as part of the United Nations Framework Convention on Climate Control (UNFCCC), 38 developed nations agreed to reduce their greenhouse gas emissions by a combined amount of 5% from 1990 levels by the years 2008 to 2012. One of the six particularly targeted gases is carbon dioxide, which places further emphasis on the need for increased automotive fuel efficiency and greater weight reduction. The extent to which this will affect the automotive industry remains unclear, however, as a number of countries have yet to ratify the agreement. In fact, the future of the entire Protocol is in question following an announcement by President Bush, on March 28, 2001, that the U.S. will not sign the treaty [4].

An even more recent example would be WP29 [5]. Since 1958, Europe has had a harmonized set of regulations known as the United Nations/European Economic Commission (UN/EEC) Working Party on the Construction of Vehicles (WP29). However, on August 24, 2000, a 1998 agreement, signed by the governments of the United States, Canada, Japan, Germany, France, the United Kingdom, Russia and the E.U., transformed WP29 to a global entity known as the World Forum for Harmonization of Vehicle Regulations. An international set of automotive standards, that includes restrictions on emissions, may force all auto-makers to consider additional methods of weight reduction.

1.1.2 Methods of Automotive Weight Reduction

There are two fundamental ways in which automotive weight reduction can be accomplished. One method is known as down-gauging, a process by which a design is re-engineered to remove unnecessary material. This method, however, has a maximum lower limit that is determined by safety, strength and performance. A second possibility is the use of alternative materials as a substitute for steel. Materials such as composites, polymers and aluminum alloys are currently being investigated to determine where and how they may be used.

1.2 Automotive Aluminum

In order to achieve the needed weight reduction, auto-makers have been considering a number of materials as alternatives to steel. Aluminum has received a great deal of attention due to its lower density, higher strength-to-weight ratio, excellent corrosion resistance and recyclability.

1.2.1 Current Automotive use of Aluminum

A typical family car has about 7% aluminum by weight, according to the 1998 Ward's Automotive Yearbook [6]. A specific breakdown is given by material in Figure 1.1, with similar data being presented by Muraoka and Miyaoka [7].

The bulk of this aluminum is used in the form of castings, as illustrated by Figure 1.2, for parts such as wheels, engine blocks, and transmission housings. A number of manufacturers, however, have realized large overall weight reductions through incorporation of higher levels of aluminum. This has spawned an increase in the use of aluminum

beyond castings to skin panels, the space-frame and other body structures. Some example automobiles include the Acura NSX, the Audi A2 and A8 (see Figure 1.3) and the Lincoln LS.

1.2.2 Potential of Aluminum Sheet

Aluminum sheet is under-used in a typical automobile. Instead, as shown in Figure 1.2, the majority of the aluminum is used in the form of castings. Automotive sheet is a market with great growth potential for the aluminum industry, although it must compete directly with steel.

The reason that aluminum sheet is not currently being more widely used is that it has a few disadvantages relative to steel. Apart from its higher cost, the use of aluminum is hampered primarily by its relatively poorer forming properties. Aluminum exhibits reduced bendability, greater springback due its lower elastic modulus and has a lower forming limit diagram (FLD) than steel. Furthermore, aluminum has highly anisotropic behaviour and an oxide layer which, although invaluable for corrosion resistance, is very hard and can abrade tools during forming [8].

Aluminum is at a further disadvantage because of the great deal of knowledge and experience already available in industry for designing dies and operations for steel sheet. It is imperative, therefore, that the behaviour of aluminum under various forming processes, such as stamping, stretch-flanging and bending, be better understood if it is to be used extensively in automobiles.

1.3 Role of the Present Work

One forming process of particular interest is hemming, a mechanical joining process involving the bending of one sheet of material, in a very tight radius, around a second. Two example hems are shown in Figure 1.4. This procedure is often used for joining automotive closure panels, such as doors, liftgates and decklids, when an outer panel is hemmed around an inner panel. The tight bend radii are an extreme test of the bendability of the material, as it can lead to failure by tearing or cracking of the outer surface, as shown in Figure 1.5.

Iron (Fe) levels have been shown to greatly influence the forming performance of an aluminum alloy [9]. For example, second phase particles, which result from Fe combining with aluminum, can act as initiation sites for void damage during deformation. The nucleation, growth and coalescence of voids can severely reduce the formability of a given alloy.

Since greater reductions of Fe content in aluminum alloys becomes increasingly expensive, particularly during recycling, an improved understanding of the failure mechanisms involved will allow for the balance between cost and performance to be optimized. Accordingly, this thesis will study the effect of void damage on the bendability of AA6111 aluminum alloy sheet by studying a series of four AA6111 Fe-variants. The bending behaviour will then be modelled numerically to facilitate an improved understanding of the role of void damage in automotive aluminum alloy sheet bendability.

Chapter 2

Literature Review

The bending of sheet metal is an enormous research area and one with a great deal of importance to industry. This chapter presents a review of the major research areas pertaining to the plastic bending of sheet, and will cover:

- an introduction to commonly used industrial bending processes;
- the issue of bendability;
- experimental studies focused on measurement of strain and residual stresses;
- theoretical bending models;
- finite element method (FEM) bending models.

2.1 Background

The bending of sheet metal is a widely used and important industrial forming process. Therefore, a brief introduction to various industrial processes is of interest here. While

numerous different bending methods exist, most can generally be classified as either pure bending, press brake bending, or wrap bending. The difference between each of these methods, which are illustrated in Figure 2.1, is explained in the following sections.

2.1.1 Pure Bending

Pure bending, as shown in Figure 2.1a, involves the bending of material due exclusively to the application of a bending moment. This represents an idealized case, one that has no parallel in industry. It will be shown, however, that it is an extremely popular assumption for theoretical analyses. In fact, a great deal of effort has gone into the creation of an experimental apparatus that can create a pure bending moment in order to verify these theoretical predictions.

2.1.2 Press Brake Bending

Press brake bending is used extensively in industry for forming sheet metal parts. An actuated press is used, along with a punch of prescribed geometry, to bend the sheet into a lower die. One such process is illustrated in Figure 2.1b. In fact, this process is so common that it is worthwhile to further distinguish between different press brake operations, usually done through consideration of the punch shape and the role of the lower die.

Punch Shape

References are often made in the literature to either V-die or U-die bending, both of which are press brake operations. The distinguishing feature is the shape of the punch and die set. Examples of V- and U-dies are shown in Figure 2.2.

It should be noted that these are simplifications, as many industrial processes build on these principles to create more complicated or hybrid methods. For example, UO-bending is a two-step process used to create tubing, as illustrated in Figure 2.3b. The sheet is initially U-bent and then a second stage is used to form the U into an O shape. The stretch bending process (see Figure 2.3a) can be thought of as an extension of press brake bending principles.

Role of Lower Die

Press brake bending also requires a female or lower die into which the sheet is bent. Distinction is often made as to whether or not the sheet is forced to come into full contact with the lower die, as illustrated in Figure 2.4. Both air bending and closed-die bending can be thought of as two consecutive parts of the bending process [10].

Air bending involves bending into a die, such that the sheet does not contact the die bottom. This is the first phase of the bending process, and allows for overbending of the sheet beyond the desired shape to account for springback. It is also advantageous since it decreases the number of dies required, thereby increasing the flexibility of a machine and die set. Unfortunately, there is less control of the final shape of the bend as the sheet is not forced to conform closely to the shape of the punch.

If the bending procedure is continued, the sheet will begin to contact the bottom of the die, a process known as closed-die bending but more commonly referred to as bottoming or coining. In this case, the final shape of the product conforms more closely to that of the die, however the die set must be designed to account for springback after unloading.

2.1.3 Wrap Bending

As the name suggests, wrap bending is a process whereby material is wrapped around a mandrel to create a bend. An example is shown in Figure 2.1c. Distinction is sometimes made for the method in which the wrapping force is exerted, hence the terms wrapping and wiping, as shown in Figure 2.5. Another process that could be classified as wrap bending is hemming; however, it is of such importance it will be addressed separately.

Hemming

By definition, hemming produces a bend angle greater than or equal to 180° , where bend angle refers to the angle through which the sheet's free end is swept. It is of particular interest to the automotive industry since it is the means by which outer and inner closure panels are joined. Finelli and Kelly [11] have pointed out that 80 spot welds can be eliminated from a typical hood by using this mechanical joining technique. Some examples of hemming were shown in Figure 1.4.

Hemming is also of great interest because it represents an extreme measure of material bendability. Failure during hemming is usually associated with cracking along the outer surface of the bend line. Not only do hems have structural importance with regard to vehicle integrity, but Klimi and Hattori [12] also noted that the overall appearance of the car will depend on the quality of the hemmed edges.

2.2 Bendability

Bendability is a measure of the extent to which a material can be bent without failure. This section examines various factors that affect the bendability of materials, such as

width effects, shear band and void damage development, as well as the influence of metallurgy and the resulting microstructure. The traditional measure of bendability – the minimum bend radius – is also examined, along with other methods that have been proposed and employed by various researchers to quantify bendability.

2.2.1 Factors Affecting Bendability

It has been shown that bendability can be affected by a number of factors, including specimen width, shear band and void damage development, metallurgy and microstructure.

Specimen Width

It is important that a bendability test specimen has an appropriate width to thickness ratio (w/t) in order to ensure that an appropriate stress state is being imposed.

One of the first studies on the effect of specimen width on minimum specimen radius was conducted by Gerard [13], who noted that bendability decreased with increasing width. This trend was attributed to the increasing stress biaxiality developing at the mid-width of the sheet. Other researchers [14, 15] have since expanded on this explanation, noting that as the width increases, the ratio of transverse strains to circumferential strains on the bend surface approaches zero. The highest region of biaxiality occurs at the centre of the sheet width, which is where failure occurs in wide specimen experiments. On the other hand, in narrow specimens, anticlastic curvature [15] (ie. flaring of the cross-section) produces larger circumferential strains at the edges, thereby causing cracks to initiate there. This means that for narrow width specimens, the stress state at the outer sheet surface is essentially uniaxial tension. As the specimen width increases, the increasing

level of stress biaxiality shifts the stress state towards plane strain, where the lowest forming limits are typically exhibited.

Therefore, when conducting bendability experiments, it is necessary to have an adequately wide specimen so that width effects can be ignored. The minimum width to thickness (w/t) ratio was initially specified, by Sachs and Espey [14], as roughly 20. However, the critical w/t ratio has since been reduced to 8 [16, 17], a value that is currently reported in the literature [18].

Shear Band and Void Damage Development

The bendability of a material is further influenced by the development of shear bands and void damage during the bending process. Since there is a coupling of these two phenomena, they are discussed together.

Akeret [19] remarked that failure of a bend specimen is very different from that of a tensile test. In a tensile test, high stress triaxiality develops in the centre of the neck, causing pores to initiate and eventually coalesce, ultimately creating a crack which runs perpendicular to the tensile direction. Conversely, in a bend specimen, failure is initiated at the outer surface and propagates inward. Figure 2.6 shows the typical progression of failure in a bend specimen. Initially, the surface exhibits roughness (1). With continued bending, the areas of roughness form valleys on the outer surface that run parallel to the bend line (2). Due to a type of selective growth, a few of these valleys continue to grow (3), until shear fracture initiates at one of the deeper valleys (4).

Dillamore *et al.* [20] examined the development of shear bands in rolled sheet from a macroscopic standpoint. It was determined that shear bands form because of load instability caused by geometric softening. Furthermore, the strain at which this occurs

is determined by the requirement of developing a sufficiently strong preferred orientation for crystallographic slip.

Morris *et al.* [21] examined shear bands in bend specimens and noted that the bands intersected the surface at an angle of 45° , the angle of maximum shear stress. Grooves were seen to form running parallel to the bending axis, resulting from the intersection of the shear bands with the sheet surface. Cracks were observed to initiate at the base of these grooves and propagate down one shear band and, in some cases, even change direction to follow a conjugate band with a higher shear stress. It was also found that there was a higher density of voids near developing shear bands and that crack formation involved the linking up of voids nucleated within the shear band.

Much of the above observations have been confirmed by Steninger and Melander [22], who described the process of failure in bending. Initially the material work hardens strongly, void growth has only a small influence, and the deformation is stable everywhere. As the strain increases, however, the volume fraction of voids reaches a level where the work hardening ability of the material is counteracted by the softening due to void growth. This causes the work hardening ability of the material as a whole to reduce to zero. Deformation then concentrates locally because it requires a similar, and eventually a lower, expenditure of energy than that of continued homogeneous deformation. This leads to rapid void growth within a local band of material, and eventually, to final fracture.

While determining strain distributions in bent specimens, Geist and Parker [23] remarked that the regions of largest plastic strain are some distance deeper into the sheet than the surface grooves. Their findings suggest that these areas of high strain are due

to strain localization associated with shear banding rather than due to the geometry of the grooves themselves.

Metallurgy and Microstructure

Several metallurgical considerations, such as alloy temper, microstructure, and second phase particles have an effect on a material's bendability. Some of these results are summarized in the following paragraphs. Findings for materials other than aluminum have been included where appropriate.

During the development of the AA6111 aluminum alloy, Fortin *et al.* [9] found that its bendability was very much dependent upon the content level of certain elements. For example, increased Fe levels were found to adversely affect bendability due to the increased number and size of constituent second phase particles. Excess Mn was also found to increase the size of Fe-containing particles, thereby reducing bendability. Furthermore, lower levels of Mg and Si were found to improve the minimum bending radius, a measure of bendability discussed in the next section. Embury [24] describes how different quench rates can affect the bendability of aluminum by changing the degree of intergranular fracture.

The effect of various metallurgical variables on the bendability of steels has also been examined. Okamoto *et al.* [25] and Takahashi *et al.* [26] have shown that raising the annealing temperature above A_{c3} greatly improves the bendability in both transverse and longitudinal sheet directions. This improvement was attributed to the disappearance of the microstructural banding observable at lower annealing temperatures and to the lower carbon content in the second phase, specifically martensite. Hosoya *et al.* [27] studied the effect of chemical composition and tempering temperature on the bendability of ultra-

high-strength steel sheet. The balance between chemical composition and tensile strength is plotted for bendability and results in a nose shaped graph, shown in Figure 2.7.

The correlation between sharpness of texture and bendability in Zn is discussed by Diot *et al.* [28]. Specifically, they demonstrate that a sharper intensity of the 20° tilted c axes, measured from the normal direction in the rolling plane, results in better bendability behaviour at low temperatures. A texture gradient is shown to develop through the thickness of the material, resulting from shear stress and friction at the surface during rolling, with friction identified as having the greater influence.

2.2.2 Methods for Evaluating Bendability

In order to objectively compare the bending properties of different materials, a quantitative measure of bendability is required. This section will examine the minimum bend radius, the most common of bendability measures, and other alternative methods that have been proposed.

Minimum R/t Ratios

The minimum bend ratio, generally reported as a ratio of bend radius to sheet thickness (R/t), has long been used to measure material bendability. In this test, the sheet is bent around mandrels of successively smaller radii. After being wrapped around the mandrel, the outer surface is examined for cracks. The smallest radius at which no failure is observed determines the minimum R/t ratio. Hence, a lower value of minimum R/t ratio indicates superior bendability. This critical ratio has been found to be independent of sheet thickness for most metals [17]; however, aluminum alloys produce some exceptions [15, 19]. Despite the development of alternative criteria, this remains the most common

measure of bendability. Curiously, given their long history of use, there are few standards governing these tests [18], and more detailed descriptions must be found elsewhere [29].

Despite its widespread use, minimum bend radius testing has a number of shortcomings that have long been recognized [13, 30]. The most fundamental problem is the subjectivity inherent in detecting the occurrence of fracture, making repeatability amongst multiple operators, and even a single operator, an important issue. Furthermore, the resolution of the measurement is limited by the increments of mandrel radii that are available.

Alternate Methods to Evaluate Bendability

In light of the limitations of minimum bend radius testing, a number of authors have attempted to develop various experimental procedures to give improved measures of material bendability.

A number of investigators have attempted to develop machines that apply a pure moment, in order to measure the bendability of a material and to confirm various pure bending theories. While it must be recognized that a pure moment is impossible throughout the entire specimen due to clamping effects, attempts have been made to obtain pure moments in the centre of the sheet [31–34]. For example, Tozawa *et al.* [35] created an apparatus which assumes that, given a long enough bending arm, the bending moment in a short specimen is nearly constant, as illustrated in Figure 2.8. Another technique, proposed by Perduijn and Hoogenboom [33] and shown in Figure 2.9, consists of four identical rollers connected by hardened steel tape. The test piece is clamped between rolls B and C in Figure 2.9. The design is such that when tension is applied to the setup a pure bending moment is applied to the specimen. As a result, the bending moment

versus curvature relationship can be determined for the material. Finally, a pure bending machine has also been proposed by Duncan *et al.* [36, 37]. This apparatus, depicted in Figure 2.10, is mounted in a tensile frame, where load and displacement measurements can be recorded and then used to obtain a moment-curvature diagram. Duncan *et al.* claim that this data provides important information about material behaviour that is not available from a tensile test. A similar device was also discussed by Marciniak and Duncan [38], as is shown in Figure 2.11.

Takenaka [39] proposed the use of the rupture strain as a bendability criterion. A rectangular specimen, with two holes drilled along the centre line, is bent using a stretch-bending apparatus. The bend angle at fracture, under a stretching stress to ultimate strength ratio of 0.5, is reported as the critical bend angle θ_c . The rupture strain can be calculated using the final distance between the two holes.

Since the bending of material often results in structural damage prior to the point at which it becomes visible, a few authors have proposed methods to quantify this damage and evaluate bendability. For example, Yoshida *et al.* [40] measured the fracture strength under tensile unbending. The sample is first bent with a 45° V-die and then a tensile test is performed on the bent specimen, as illustrated in Figure 2.12. They propose that the minimum bend radius be defined as the bend radius at which the tensile strength of the bent sample is the same as that of the virgin material. Rasmussen [41] applied the same principles in the development of a Z-shaped shear test. The jig, as depicted in Figure 2.13, is capable of measuring bends of 0° to 95°. Here, the work required to shear the bending zone would be the quantification of bending damage, after the maximum shearing load is normalized with respect to specimen width and thickness.

Another test of note is the cantilever bend test developed by Selçuk and Rawlings [30]. A modified version has been created at the Kingston Research and Development Centre of Alcan International Limited (Alcan KRDC) and is shown in Figure 2.14. In this test, the sample is clamped to a mandrel of prescribed radius and a free-spinning roller sweeps the material around the radius. The entire apparatus is attached to a standard tensile frame with a belt wrapped around the drum and fixed to the displacing crosshead. Force and displacement data is recorded at the crosshead and subsequently converted to force versus bend angle. The authors also proposed a thickness correction factor to aid the comparison of different gauge materials.

2.3 Experimental Analysis

The strain and residual stresses resulting from bending play important roles in determining the failure and shape change of the material. It is extremely difficult, however, to obtain experimental measures of these values.

2.3.1 Strain Measurement

One of the greatest difficulties in evaluating and working with bends is the challenge of measuring and quantifying the strains caused during bending. Below are some examples of techniques that have been used.

Strains can be measured through the use of a photogrid technique. By applying a grid of known dimensions to the surface of a sample, the shapes of the grid during or after deformation can be used to calculate the strain undergone by the sample. While

this method has been used by a number of authors [13, 15, 21, 22, 42, 43], it is limited to measurements at the surface of a specimen.

Etching has been used to give a qualitative measure of strain. Morris *et al.* [21], for example, used low temperature annealing to precipitate particles at the dislocations in AA5182 and AA2036 aluminum alloy. An etching technique, similar to that of Sowerby and Johnson [44] and others, is then used which causes these particles to appear dark. Thus, strain intensity is shown by gray to black shading. However, according to Geist and Parker [23], the technique is limited to certain alloys and heat treatments.

An alternative method that has been used involves taking microhardness measurements. Morris *et al.* [21] compared deformed and undeformed hardness measurements to obtain an indication of relative strain through the sheet thickness. Geist and Parker [23] went slightly further and created a calibration curve to convert microhardness measurements into a value of plastic strain. Measurements were then taken in a grid pattern (with a spacing of $\approx 160 \mu\text{m}$) through the thickness of a 1 mm sheet of 5082 aluminum. These measurements were then converted to strain and used to form contour plots.

2.3.2 Residual Stress Measurement

The development of residual stresses during bending is important because of their effect on springback. Theoretical predictions of residual stress distributions can be made, but it is quite difficult to obtain experimental data to confirm these models. Two options are the layer removal method and a hole drilling method.

Treuting and Read [45] observed the change in shape following the removal or sectioning of a part and were able to convert that to residual stresses. This was originally performed on flat specimens to determine the residual stresses resulting from forming,

but later adopted by Queener and De Angelis [46] for bent sheet. Tan *et al.* [47] expanded the theory for relating curvature to residual stresses to include the material's properties, by combining x-ray diffraction with successive removal of material layers. This has also been used by Forcellese *et al.* [48].

Schilling [49] used a hole drilling method to analyze residual stresses. A ring-shaped hole was eroded from the sheet using abrasive grain blasting, while a strain gauge rosette recorded the change in shape of the hole. This information provided the basis for residual stress calculations. In order to determine the residual stresses across the entire thickness of the sheet, two specimens were analyzed; one beginning at the inner surface, and the other starting from the outer surface.

2.4 Theoretical Models

A number of theoretical models have been proposed to predict the plastic bending behaviour of different metals. The major distinctions between these models are the plasticity model employed and consideration of the Bauschinger effect. A summary of the theoretical predictions of minimum bend ratios, pure bending, press brake bending, bend allowance, springback and residual stresses, and void and shear development, is given below.

2.4.1 Bauschinger Effect

One particular material characteristic that is important in bending is the Bauschinger effect. This behaviour in a metal refers to the different strengths observed when the

loading path is reversed. While the principles of the Bauschinger effect are described in detail elsewhere [50, 51], they will be briefly summarized here.

The importance of this phenomenon in bending can be seen in Figure 2.15. Initially, the neutral axis (r_n) is at the mid-plane of the material (r_m). As the material is bent, however, the neutral axis begins to shift towards the centre of curvature. At the neutral axis, the material is in neither tension nor compression. Thus, the shifting of the neutral axis means that some material, which was previously in compression, will then be placed in tension. It follows then that the material can be placed in one of four zones. The first zone, from r_o to r_c , is material in tension that has been in tension throughout. The fibers from r_c to r_u have been compressed and then stretched more than they were originally compressed, whereas the fibers from r_u to r_n have been compressed then stretched less than their initial degree of compression. Therefore, r_u is the location at which the fibers have been compressed and then stretched equal amounts. Finally, the fourth zone, from r_n to r_i , represents the fibers that have only been compressed. At any given point during bending, the material will be in one of these four zones. It should be noted, however, that some authors refer to only three zones by combining the second and third zones and making no distinction regarding the ratio to which fibers have been compressed and then stretched.

Since the Bauschinger effect is the change in behaviour upon reversal of loading, it applies to the central two zones of the sheet where the material has gone from compression to tension. The manner in which this phenomenon is treated in theoretical models varies among researchers.

2.4.2 R/t Ratio

As discussed in Section 2.2.2, bendability has traditionally been evaluated based on an R/t ratio. This corresponds to the minimum radius (R) over which a given thickness of sheet (t) can be bent without fracture occurring at the outer surface.

A number of authors have attempted to link bendability, as measured by minimum bend radii, to various material parameters. This following relation was proposed by Datsko and Yang [52] based on reduction in area A_r :

$$\frac{R}{t} = \frac{(100 - A_r)^2}{200 - A_r^2}. \quad (2.1)$$

More recently, Wang *et al.* [53, 54] developed this equation which also accounted for material anisotropy and the shift of the neutral axis:

$$\frac{R}{t} = \frac{(1 - A_r)^{2F}}{1 - (1 - A_r)^2}. \quad (2.2)$$

Kals and Veenstra [55] developed a model based on the strain instability of a power law hardening material. The equation was:

$$\frac{R}{t} = \frac{1}{2 \{ \exp(n - \epsilon_o \sqrt{3}) - 1 \}}. \quad (2.3)$$

which can be approximated as:

$$\frac{R}{t} = \frac{1}{2n}. \quad (2.4)$$

A similar value was found by Akeret [19]:

$$\frac{R}{t} = \frac{1-n}{2n}. \quad (2.5)$$

However, Akeret demonstrated that the above equation gave very conservative estimates of bendability, and therefore, the exponent of strain hardening was an unsuitable reference quantity. Furthermore, Kals and Veenstra's equation is even more conservative at larger strain hardening exponents. Hence, its applicability is also questionable. Another model, proposed by Leu [56], employs a power-law hardening material in pure bending and also accounts for sheet anisotropy:

$$\frac{R}{t} = \frac{1}{2} \left(\frac{1}{\exp\left(\frac{n(1+R)}{2\sqrt{1+2R}}\right)} - 1 \right) \quad (2.6)$$

Rolf and Patrick [42] suggested the following:

$$\frac{R}{t} = \frac{\eta}{\varepsilon} - 1 + \eta \quad (2.7)$$

The major shortcoming of Equation 2.7 is that it requires prior knowledge of the amount of neutral axis shift, η , during bending.

2.4.3 Pure Bending

The simplest form of bending is pure bending, due exclusively to the application of a bending moment. Many theoretical models have been based on this principle. Most theoretical pure bending models are based on the same general assumptions. As men-

tioned earlier, the main distinctions amongst models are in the complexity of the material model, consideration of the shift of the neutral axis, sheet thinning, and the Bauschinger effect. The majority of the assumptions, however, remain the same and include:

- plane sections remain plane during bending;
- sheet is sufficiently wide so that plane strain conditions exist;
- stress-strain behaviour is the same in tension and compression;
- bending is a result of bending moments only and no other external forces.

These models cannot be summarized as succinctly as those for minimum R/t ratios. As a result, no formulae will be presented. Instead, a brief description of the various models will be given below and an attempt has been made to summarize the main distinctions of each in Table 2.1. It will be left for the reader to further investigate any model that appears to be of interest.

A very useful overview of items that must be considered is given by Miyauchi [57]. These items include the Bauschinger effect, latent hardening effect, strain aging, texture and microstructural transformation and evolution.

Although both earlier and contemporary examples of research dealing with plastic bending exist [58, 59], the most influential work is certainly that of Hill [60], who provides a solution for pure bending of a perfectly plastic sheet of constant thickness. Thinning of a similar material is accounted for by Lubhann and Sachs [59]; however, only the stress distribution could be solved algebraically and the rest required graphical solutions. Models involving power law plasticity were created by Denton [61], who limited the solution to moderate bending so that the shifting of the neutral axis could be ignored. Perduijn and Hoogenboom [33] also developed three models based on a power-law model.

The three models correspond to an elastic core with an elastic-plastic outer layer, an elastic core with a plastic outer layer and finally a fully plastic sheet.

Verguts and Sowerby [31] incorporated the Bauschinger effect into a power law hardening material. While the primary focus was on bending of composites, two models were developed for a homogeneous metal, both with and without the Bauschinger effect, to study the effect of its consideration on material thinning. It was found that the Bauschinger effect causes the sheet to thin more quickly, lowers the bending moment and results in an instability being reached earlier in the bending process. A similar study by Tan *et al.* [32] confirmed these results. Dadras and Majlessi [62] investigated the Bauschinger effect using both linear stress-strain behaviour and a power law model. He found that the difference between these two approaches was negligible due to the relatively small amount of material being subjected to reverse loading.

The models discussed thus far assume isotropic material behaviour, yet some recent efforts have attempted to incorporate anisotropy. Wang *et al.* [53, 54] and Leu [56] have both developed theories for pure bending which incorporated true strain, power law hardening and Hill's (1979) non-quadratic yield function for materials with normal and planar anisotropy (but ignoring the Bauschinger effect). Tan *et al.* [32] proposed a pair of anisotropic models, including and omitting the Bauschinger effect. In this case, Bauschinger behaviour was introduced through a kinematic hardening theory.

2.4.4 Press Brake Bending

A slightly more complicated situation to treat theoretically is that of press brake bending. It is of great interest, however, due to the prevalence of press brake bending in industrial

forming operations. Here, the goal is to predict major characteristics of the process such as shape, springback and forming load.

Proper modelling of the sheet geometry during the bending process is important for obtaining an accurate solution. Often the sheet blank is simply modelled as being circular under the punch and rigid-straight over the die shoulders [63]. Stelson [64] developed a model for a press brake operation that would predict the moment-curvature relationship and springback of the sheet (based on the geometry) for a power law hardening material. This model was incorporated into a control algorithm and provided improved sheet modelling. Pourboghrat and Stelson [65, 66] proposed a model to consider nonuniformities in the punch-sheet interface, assuming rigid-plastic material properties. This model was then applied to successfully predict the development of gaps in contact, similar to those which are observed experimentally. De Vin [67, 68] developed a three-section model for press brake bending consisting of a circular section around the punch, a section of varying radii and an elastically deformed section in the neighbourhood of the die shoulder.

Some specific press brake models have also been created for V-die air-bending [63], and stretch-bending [69, 70]. Wang's anisotropic pure bending theory is applied specifically to V-die air-bending, but the authors claim that it could easily be used for U-die and wipe bending as well, provided that no coining occurs.

2.4.5 Bend Allowance

Bend allowance is the length of the unstretched fiber in the bent portion of a sheet. This value is used to predict the final shape and dimensions after bending to allow for the insertion of important features such as holes. Accurate predictions are important in high precision applications.

Initially, this problem was dealt with in an empirical fashion. In his book on metal forming, Sachs [17] gives two empirical relations for determining the bend allowance. The first assumes the neutral axis remains at the centre of the sheet. He explains, however, that better results are obtained if one assumes the neutral axis to be a distance of $0.45t$ from the inner surface of the bend. This work was later analyzed by Stelson [64] where it was reported that use of Sachs' rule is limited because it cannot successfully account for differences in bend geometries.

One of the earliest attempts at addressing this issue analytically was that of Dickason [71, 72]. He demonstrated the importance of making allowances for the altered location of the neutral axis in the bent sheet to properly calculate the final shape of a bent part. He highlighted the need for accurate data on the location of the neutral axis for his methods to be successful, and noted that no such data existed at that time.

Prasad and Somasundaram [73] attempted a much more comprehensive evaluation. Their model accounted for such factors as thinning effects, strain hardening, bend radius and bend angle. They developed non-linear equations for the bend allowance factor, k , and the ratios of sheet thickness, t/t_0 , based on rigid-plastic strain-hardening material behaviour.

Pourboghrat and Stelson [74] noted that Prasad and Somasundaram still neglect to take into account the deformation of the entire shape and its unloaded shape due to springback. Therefore, a press brake model was developed that accounts for strain hardening, friction, tooling and the possible formation of a gap between the sheet and the punch. This model and Sachs' models were compared with experimental data.

2.4.6 Springback and Residual Stresses

Springback is an important consideration during a bending operation to obtain the desired final shape. Therefore, any overview of the subject of sheet metal bending would be remiss not to include some of the many works devoted to this topic. A brief description of the mechanism of springback is first given, followed by a brief overview of some of the empirical and theoretical models used to predict springback and residual stresses. An attempt has been made to tabulate a summary of the various analytical theories in Table 2.2.

When a material is plastically deformed, there is always some elastic recovery following the removal of the applied load. This phenomenon is particularly important in bending where a stress gradient exists through the sheet, from tension at the outer surface to compression at the inner surface. The removal of a bending load causes an elastic recovery that results in a stress redistribution throughout the cross-section. This results in a change in the final shape of the product, known as springback or springforward, depending on the direction of elastic recovery [38].

Originally, when dealing with springback, no analytical solutions were available. Instead, designers were forced to rely on tables or empirical relations, such as those by Chapman *et al.* [75], for predictions of springback. Even more recently, Levy [76] has developed an empirical relation for steel as a function of bending radius, yield strength and thickness ratios. Tan *et al.* [77] have also developed empirical relations that can be calibrated using three trial bends for a given sheet metal and tool combination. This model was incorporated into a process control system to control the amount of overbending required to obtain the desired shape after springback.

Much of the pioneering work in this area was conducted by Schroeder [78] and Gardiner [79]. One of the benchmark theories, however, is that of Queener and De Angelis [46]. Here, analytical expressions for both springback and residual stress distribution are given as a function of sheet thickness, die radius and material stress-strain behaviour for a power law hardening material. The simplified expressions for springback and residual stress distribution, respectively, are as follows:

$$\frac{R_o}{R_f} \approx 1 - \frac{3K(1-\nu^2)}{E(2+n)(3/4)^{(1+n)/2}} \left(\frac{2R_o}{t} \right)^{1-n} \quad (2.8)$$

and

$$\sigma_R \approx \frac{K}{(3/4)^{(1+n)/2} R_o^n} \left[\pm |y|^n \frac{3y}{(2+n)(t/2)^{1-n}} \right] \quad (2.9)$$

assuming a power law of:

$$\sigma = K\epsilon^n. \quad (2.10)$$

Equations 2.8 and 2.9 were verified using experimental springback data and the material layer removal technique of Treuting and Read [45]. It should be noted that this theory is limited to only large bend radii.

In a manner similar to that of pure bending theories, most springback theories differ because of their material models. However, some do model different bending processes. While most theories have been verified experimentally, using some sort of press brake bending operation, many still use pure bending as a basic assumption. Thus, models incorporating power law hardening have been proposed by Queener and De Angelis [46]

and Rolf and Patrick [42]. Sidebottom and Gebhardt [80] developed an analytical solution based on a piece-wise linear stress-strain curve, which compared favorably to a similar solution of Queener and De Angelis [46]. A cyclic model was developed by Zhang and Hu [81] to predict residual stresses due to repeated bending and unbending of a power law hardening material.

Anisotropy was eventually incorporated into springback models, generally using Hill's non-quadratic yield function, for a power law hardening material in pure bending [56]. Similar models also addressed neutral axis shift [53] and sheet thinning [82].

Consideration of the geometry of the bending operation must also be taken into account. Raghupathi *et al.* [83] developed a prediction of springback based on an elastic linear-plastic material model and die geometry for press brake bending. Likewise, Zhang and Lin [84] have developed an analytical solution for the springback of the U-die bending of an elastic/perfectly plastic material using a rigid punch and an elastic die.

Some springback models were developed specifically for stretch bending which account for the imposed tensile stress. El-Megharbel *et al.* [85] created a model based on a power law hardening material to study the effect of several variables on predicted springback behaviour. Another model was developed by Yuen [86] which reduces to Gardiner's [79] equation when the tension term is zero. In Yuen's study, particular attention was given to the correlation of fluting, due to the upper and lower material yield points, and springback. A more recent model is that of Xue *et al.* [87], which applies the principle of stretch bending material with power law hardening behaviour to a double-curvature forming operation – stamping and stretch bending.

The effect of crystallographic texture on springback is accounted for in the anisotropic model of Zeng [88]. Verification involved the V-die bending of AA8090 aluminum alloy

and the application of neutron diffraction to obtain the average texture values used in the theoretical prediction.

In an overview of existing springback literature, Sanchez *et al.* [34] point out that there is little experimental data for springback with a low bending ratio ($R/t < 10$). Experiments were conducted and the results were compared with the predictions of the Gardiner [79] theory and Timoshenko-Goodier theory of elastic springback. Gardiner's equations were found to greatly underestimate springback. The Timoshenko-Goodier expression also tended to underestimate springback, although it was successful in correctly predicting the trends in behaviour.

2.4.7 Shear and Void damage

It has been shown that shear and void damage play an important role in the bendability of sheet. While no theoretical bending models incorporating these phenomena have been found in the literature, some general theoretical models for these failure mechanisms are given. A detailed summary of the Gurson-Tvergaard Needleman (GTN) constitutive softening model is also presented because it can facilitate a combination of shear band and void damage development in a deforming material. The implementation of these models, particularly the GTN model, within an FEM formulation is discussed further in Section 2.5.3.

Void Damage

The evolution of void damage in a deforming material is usually broken down into three stages: void nucleation, growth and coalescence. Typically, voids nucleate at second phase particles or inclusions in the material through particle cracking or through de-

cohesion from the matrix material. Void growth has been shown to be exponentially dependent upon hydrostatic stress [89, 90]. The exact conditions for void coalescence has yet to be determined, but many authors refer to a critical void size [90] or a critical void volume fraction [91].

Ductile fracture criteria based on void growth have been proposed by McClintock [89] and Steninger and Melander [22]. Gurson [91, 92] developed a widely used model for porous solids. Thomason [93] has presented a plastic limit-load criterion. A comparison of the ductile fracture predictions provided by the works of Rice and Tracy, Gurson and Thomason has been conducted by Zhang and Niemi [94], but only void growth was considered. Considerable differences were found in the predictions, but no conclusions were drawn as to whether one was superior to the others. Tang *et al.* [95] presents an anisotropic theory of damage for the prediction of fracture limit curves in aluminum, along with an extensive list of other damage research.

Shear Band Localization

A review article by Cowie and Tuler [96] gives a thorough introduction to various approaches employed for the prediction of shear band initiation.

For example, textural or geometric softening can be achieved through the development of a vertex on the yield surface at the point of loading [97–99]. Similar results can be achieved through the use of kinematic hardening [100, 101]. Since kinematic hardening results in a translation of the yield surface in stress space, rather than a dilation, it retains a greater degree of curvature, as shown in Figure 2.16. Thus, for the same stress increment, the kinematic scheme is more sensitive to a change in loading direction.

Void-nucleation softening is another frequently used method for creating a shear localization event, as a sudden burst of nucleation can have a destabilizing effect on plastic flow [96]. Different approaches to modelling void damage have been discussed in the preceding section.

Bifurcation analysis provides a means of predicting the strain level at which localization will occur [102–106]. An initial geometric or material inhomogeneity is assumed inside a thin slice of material, as illustrated in two dimensions in Figure 2.17. The stress state both inside and outside the slice are assumed to remain homogeneous throughout the deformation history. The predicted angle of shear band development, ψ , corresponds to the one that occurs at the lowest strain. Frequently, the inhomogeneity is a difference in initial porosity levels assumed inside and outside the band, f_b and f_o respectively. The results of Yamamoto [102] suggest that, at least for small volume fractions, shear localization is more dependent upon the difference in these values, Δf , rather than their initial values. This observation has been confirmed by other authors [106]. Saje *et al.* [103] report that shear localization in solids can be either void growth dominated or void nucleation dominated, depending upon the stress state.

Gurson-Tvergaard-Needleman (GTN) Equations

One of the most widely used approach for modelling void nucleation, growth and coalescence was developed by Gurson [91, 92]. Subsequent improvements have been proposed by many authors, most notably by Tvergaard and Needleman, such that it is more properly referred to as the Gurson-Tvergaard-Needleman (GTN) constitutive softening model.

Under isotropic hardening conditions, the yield function for the macroscopic stresses is specified as:

$$\Phi = \left(\frac{\sigma_e}{\sigma_m} \right)^2 + 2f^* q_1 \cosh \left(\frac{q_2}{2} \frac{\sigma_{kk}}{\sigma_m} \right) - 1 - q_3 f^{*2} = 0 \quad (2.11)$$

where $\sigma_e = (3/2 s_{ij} s_{ij})^{1/2}$ is the von Mises equivalent stress, $s_{ij} = \sigma_{ij} - \sigma_{kk}/3$ is the deviatoric component of σ_{ij} , and σ_m is the flow stress of the matrix material. The coefficients q_1 , q_2 and q_3 were introduced by Tvergaard [107] to better match material behaviour at low porosity levels. When $f^* = 0$, the yield surface reduces to that of von Mises. The effect of porosity on the shape of the yield surface is shown in two-dimensions in Figure 2.18 and in three-dimensions in Figure 2.19.

Tvergaard and Needleman [108] also introduced the following function, $f^*(f)$, to model the accelerated loss of material strength once the coalescence of neighbouring voids begins:

$$f^*(f) = \begin{cases} f & \text{for } f \leq f_c \\ f_c + \frac{f_u^* - f_c}{f_f - f_c} (f - f_c) & \text{for } f \geq f_c \end{cases} \quad (2.12)$$

where f_c is the porosity level at which void coalescence commences, f_f is the porosity level at final fracture, and $f_u^* = 1/q_1$.

The evolution of porosity in the model is driven by the growth of existing voids and the nucleation of new voids, as described by:

$$\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation} \quad (2.13)$$

where the growth of voids is a function of the plastic strain rate, such that:

$$\dot{f}_{growth} = (1 - f) \dot{\bar{\epsilon}}^p \quad (2.14)$$

and voids are assumed to nucleate according to:

$$\dot{f}_{nucleation} = \mathcal{A} \dot{\sigma}_m + \mathcal{B} \dot{\sigma}_{kk}/3. \quad (2.15)$$

As suggested by Chu and Needleman [109], nucleation is assumed to follow a normal distribution. If strain-controlled nucleation is used then the terms $\mathcal{A}$ and $\mathcal{B}$ are set at:

$$\mathcal{A} = \left(\frac{1}{h} \right) \frac{f_n}{s_n \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\bar{\epsilon}^p - \epsilon_n}{s_n} \right)^2 \right], \quad \mathcal{B} = 0. \quad (2.16)$$

However, if stress-controlled nucleation is used then:

$$\mathcal{A} = \mathcal{B} = \frac{f_n}{s_n \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{(\sigma_m - \sigma_{kk}/3) - \sigma_n}{s_n} \right)^2 \right] \quad (2.17)$$

where ϵ_n and σ_n are the average void nucleation strain or stress, respectively, f_n is the volume fraction of void nucleating particles and s_n is the standard deviation of void nucleating strains or stresses.

The original Gurson equations were limited to an isotropic hardening material. Subsequently, work by Mear and Hutchinson [110] proposed modifications to model a kinematic hardening material with an initial void volume fraction. Their method was further expanded by Tvergaard [111, 112] to account for void nucleation.

A family of isotropic/kinematic hardening yield surfaces was created using a parameter b , such that the radius of the yield surface, σ_f , is specified by:

$$\sigma_f = (1 - b) \sigma_y + b \sigma_m \quad (2.18)$$

where σ_y and σ_m are the initial yield stress and the matrix flow stress, respectively. The parameter b is a constant from 0 to 1 which specifies the relative amount of kinematic and isotropic hardening behaviour. However, it should be noted that evidence in the literature suggests that a b parameter that evolves as a function of the stress or strain history is more appropriate, particularly for modelling cyclic behaviour [113].

The current centre of the yield surface is represented by the tensor, α_{ij} , as shown in Figure 2.20, then:

$$\tilde{\sigma}_{ij} = \sigma_{ij} - \alpha_{ij}. \quad (2.19)$$

Equation 2.11 can then be generalized and written as:

$$\Phi = \left(\frac{\tilde{\sigma}_e}{\sigma_f} \right)^2 + 2f^* q_1 \cosh \left(\frac{q_2}{2} \frac{\tilde{\sigma}_{kk}}{\sigma_f} \right) - 1 - q_3 f^{*2} = 0. \quad (2.20)$$

According to Tvergaard [112], the macroscopic plastic strain increment, $\dot{\varepsilon}_{ij}^p$, and the effective plastic strain increment, $\dot{\varepsilon}^p$, can be related to one another by:

$$\tilde{\sigma}_{ij} \dot{\varepsilon}_{ij}^p = (1 - f) \sigma_f \dot{\varepsilon}^p. \quad (2.21)$$

Substituting the uniaxial stress-strain curve for the matrix material, $\dot{\epsilon}^p = (1/h)\dot{\sigma}_m$ into Equation 2.21 gives:

$$\dot{\sigma}_m = h \frac{\tilde{\sigma}_{ij} \dot{\epsilon}_{ij}^p}{(1-f)\sigma_f}. \quad (2.22)$$

The macroscopic plastic strain rate is:

$$\dot{\epsilon}_{ij}^p = \frac{1}{H} m_{ij}^G m_{ij}^F \dot{\sigma}_{kl} \quad (2.23)$$

where

$$H = \frac{\sigma_m}{2} \left[3\alpha(1-f) \frac{\partial \Phi}{\partial f} - \left\{ \frac{\partial \Phi}{\partial f} \mathcal{A} + \frac{\sigma_m}{\sigma_f} \frac{\partial \Phi}{\partial \sigma_f} \right\} \frac{1}{h} \right] \frac{1}{1-f} \left(\frac{\tilde{\sigma}_e^2}{\sigma_f} + \alpha \frac{\tilde{\sigma}_{kk}}{\sigma_f} \right), \quad (2.24)$$

$$m_{ij}^G = \frac{3\tilde{s}_{ij}}{2\sigma_f} + \alpha \delta_{ij}, \quad (2.25)$$

$$m_{ij}^F = \frac{3\tilde{s}_{ij}}{2\sigma_f} + \beta \delta_{ij}, \quad (2.26)$$

$$\alpha = \frac{1}{2} f^* q_1 q_2 \sinh \left(\frac{q_2 \tilde{\sigma}_{kk}}{2 \sigma_f} \right), \quad (2.27)$$

$$\beta = \alpha + \frac{B\sigma_m}{6} \frac{\partial \Phi}{\partial f}, \quad (2.28)$$

and continued plastic loading requires that

$$\Phi = 0 \quad (2.29)$$

and

$$\frac{1}{H} m_{ij}^F \dot{\sigma}_{kl} \geq 0. \quad (2.30)$$

Based on Ziegler's [114] modification of Prager's [115] hardening rule, the equation for the evolution of the yield surface centre during a plastic increment is given by Tvergaard [111, 112] as:

$$\dot{\alpha}_{ij} = \dot{\mu} \bar{\sigma}_{ij}. \quad (2.31)$$

By enforcing the consistency equation, $\dot{\Phi}=0$, and rearranging to solve for $\dot{\mu}$ we obtain

$$\begin{aligned} \dot{\mu} = & (1-b) \frac{1}{2} \left[\frac{\bar{\sigma}_e^2}{\sigma_f} + \alpha \frac{\bar{\sigma}_{kk}}{\sigma_f} + \frac{\partial \Phi}{\partial f} \frac{\mathcal{B}}{6} \bar{\sigma}_{kk} \right]^{-1} \left[\frac{2}{\sigma_f} m_{kl}^G \dot{\sigma}_{kl} + \frac{\sigma_y}{\sigma_f} \frac{\partial \Phi}{\partial f} \frac{\mathcal{B}}{3} \dot{\sigma}_{kl} \delta_{kl} \right. \\ & \left. + \frac{\sigma_y}{\sigma_f} \frac{1}{H} \frac{\partial \Phi}{\partial f} \left\{ 3\alpha(1-f) + \mathcal{A} \frac{1}{h} \frac{1}{1-f} \left(\frac{\bar{\sigma}_e^2}{\sigma_f} + \alpha \frac{\bar{\sigma}_{kk}}{\sigma_f} \right) \right\} m_{kl}^F \dot{\sigma}_{kl} \right] \end{aligned} \quad (2.32)$$

When no nucleation is used, $\mathcal{A}$ and $\mathcal{B}$ are both zero and the above equation simplifies to that of Mear and Hutchinson [110].

2.5 Finite Element Analysis

The finite element method (FEM) is an important analytical tool and is particularly useful in the study of bending. Numerical predictions for a number of press brake bending set-ups have been performed, although the major emphasis in the literature is towards the prediction of springback. Hemming has been studied as well using FEM and those studies are summarized along with attempts to model both shear and void damage development. Finally, various element formulations that have been developed specifically for problems possessing a large bending component have also been included for interest.

2.5.1 Press Brake Bending

There have been a number of attempts to numerically model various bending processes, including various press brake bending set-ups and hemming. A summary of some basic information regarding each of these models is presented in Table 2.3.

Oh and Kobayashi [116] were among the earliest researchers to apply the finite element method to press brake bending operations. A 2D analysis of U-die bending was performed using rigid-plastic and elasto-plastic formulations. Assuming the unloading conditions can be well defined, the final loading state can be used to calculate springback and residual stresses. This can be done for most sheet bending operations. However, since the calculations are more straightforward with elasto-plastic formulations than with rigid-plastic, the former approach has been used almost exclusively. It has been applied to U-die bending [49, 117–120], V-die bending [48, 49, 119–124] and even UO-bending [125], which is explained in Section 2.1.2 and is shown in Figure 2.3b. Most of these examples consist of air-bending, although coining is considered by Forcellese *et al.* [120]. Unlike the other researchers, Forcellese *et al.* [48] used an explicit loading, implicit unloading approach to examine V- and U-die bending of AA5083 aluminum.

The majority of these authors employ their own FEM codes, thus enabling them to focus on a particular area, with contact being of particular interest. As well, Makinouchi [117] incorporated an evolving Young's modulus (E) into his formulation. Experimental springback values for a U-bend operation were compared with numerical predictions based on an evolving modulus (E) and on a constant modulus (E_0). Accounting for the change in modulus was found to have little effect on the predicted stress distribution, but it did provide a considerable increase in the predicted springback accuracy. Von Finckenstein [122] applied FEM modelling to study the bending of vibration damping steels, which

consists of two layers of steel with a thin layer of polymer sandwiched between them. Failure was incorporated into the model to allow for the sheet splitting that had been observed experimentally.

Some researchers have used commercial FEM codes to examine press brake processes. Schilling [49] examined V- and U-die bending of steel sheet and compared residual stress predictions with experimental values. The Bauschinger effect was also incorporated into Schilling's model. Nilsson *et al.* [123] used NIKE2D to examine the V-die bending of eight different materials, including the aluminum alloys AA6016-T4 and AA5182. Experimental data was compared with simulations conducted using actual stress-strain data.

All of these models have been limited to 2D analyses because of the simplifications possible when dealing with a large sheet in a press brake situation. FEM has also been applied to more complex formed parts such as that of Chou and Hung [126] who used the commercial ABAQUS code to study a more complex 3D sheet bending configuration.

2.5.2 Hemming

Hemming represents an extreme case in bending behaviour. As such, it is worthwhile to examine how well finite element formulations can capture this event

Makinouchi [117] modelled a symmetric hemming process consisting of two stages: pre-hemming and hemming. The stress and strains along the outer surface were plotted. Hemming was more accurately modelled as an unsymmetrical process by Iwata *et al.* [127]. The material behaviour of the commercial finite element code JNIKE3D was modified to include a biquadratic yield function, friction and breakage evaluation. This study attempted to predict geometric inaccuracies such as hemming deflection and hemming

warp, and the numerical results were found to overestimate the experimental values in both cases.

Modifications to the constitutive behaviour of JNIKE3D were also undertaken by Muderrisoglu *et al.* [128] to study the effects of flange radius and flange height on the hemming of aluminum alloys. A three-stage operation, consisting of bending, pre-hemming and hemming, was modelled to determine how the variation of these parameters affected the hemming load. This work also examined springback after the first two stages, as well as the variation of the final hem from the desired hem angle of 180° .

Livatyali *et al.* [129] conducted an experimental investigation of the influence of pre-hemming path on the final hem quality in a two-stage process. Some of the experimental cases were simulated using the 2D finite element program DEFORM. The results are summarized in trend lines that indicate the effect of die radius, pre-hem path, pre-hem stroke and final hem stroke on the hemming defects of creep, recoil and warp.

2.5.3 Shear Band and Void Damage Development

The development of shear bands and void damage in a numerical code has been attempted by a number authors. The typical approaches reported in the literature are presented in the following section, followed by a discussion of various issues associated with modelling this type of material behaviour using FEM.

While the majority of shear band predictions have been limited to two dimensions [101, 130–133], some authors have considered the problem in three dimensions [134, 135]. The isotropic hardening GTN equations are frequently employed to model the material's constitutive behaviour. A few authors [101, 136–138] have applied the kinematic Gurson formulation of Mear and Hutchinson [110] and others [133, 139] have adopted the

formulation of Tvergaard [111, 112] which also accounts for void nucleation. Triantafylidis *et al.* [99], however, attempted to model the development of shear bands in a plate, having periodic sinusoidal thickness imperfections and subject to pure bending, using a corner theory material model. The wavelength component of the thickness imperfection was found to play the primary role in providing surface strain concentrations which eventually initiate shear bands.

The use of an initial geometrical or material inhomogeneity to initiate strain localization permeates the literature. One notable exception is a study by Becker [140]. Sheet bending was modelled using constitutive behaviour based on crystallographic slip in order to produce shear localization. In this case, plastic flow localization occurred without the need for a geometric imperfection.

As pointed out by Ortiz *et al.* [141], FEM localization models often employ quadrilaterals consisting of four triangle elements because triangular elements can resolve narrow bands of deformation in four directions: parallel with the quadrilateral sides and along the element diagonals. Conversely, quadrilateral elements can only resolve deformation in two directions. A bifurcation analysis is typically employed to first determine an aspect ratio for the elements which aligns the diagonals at the appropriate angle. Some authors, including Ortiz *et al.* [141], have attempted to create new elements capable of providing an improved response from four-noded elements.

The numerical predictions obtained from these FEM-based models have been found to be highly mesh dependent [111, 130, 131, 142, 143]. The width of a shear band, for example, is limited by the size of the elements. It has been noted by Tvergaard and Needleman [142] that the size of a shear band should be related to void radius. As a result, non-local formulations of the GTN equations [142] have been developed in order

to lessen mesh dependence and link the predictions to an appropriate material length scale.

A further difficulty in trying to implement the GTN equations within a general FEM code is the inherent instability of the formulation. For a von Mises material, an “elastic predictor-radial corrector” scheme can be employed; however, a forward integration scheme is unsuitable for use with the GTN model’s constricting yield surface. If the stress point is not returned to the yield surface after each plastic increment, numerical errors are introduced and accumulate, eventually leading to numerical instability. A solution proposed by Aravas [144] involves the use of a backwards Euler method which has been found to be unconditionally stable, even for large strains, and has been successfully incorporated into the commercial code ABAQUS. This scheme has been found to be numerically intense, and therefore, suitable only for an implicit FEM code. A less computationally expensive approach was developed by Worswick and Pelletier [145] for implementation in the explicit FEM code LS-DYNA. A “normal predictor - normal corrector” method was proposed which accounts for the strong dependence of the yield surface on hydrostatic stress and relies on an iterative Newton-Raphson scheme to ensure that the stress point remains on the yield surface.

It is worth noting that the phenomenon of voids nucleating and growing within shear bands is not limited to metallic materials, nor are numerical predictions limited to FEM. Iwashita and Oda [146] examined the generation of voids within shear bands in soils using a numerical approach called the distinct element method (DEM). This method models elements as two-dimensional circles which deform and interact according to prescribed behaviour. Using this approach, they were able to obtain simulations showing voids aligned along shear bands.

2.5.4 Finite Element Formulations

Speed, efficiency and accuracy are continuously being balanced in the use of FEM for modelling industrial processes. Membrane elements are often used in modelling tensile dominated sheet forming operations because of their speed. They are inaccurate, however, in forming processes dominated by bending. Shell elements perform this task much better, but with a corresponding increase in computation time. A number of authors have proposed hybrid membrane element methods to account for bending while obtaining superior speed.

Yoo *et al.* [147] developed a so-called Bending Energy Augmented Membrane (BEAM) element, claiming that it gives improved behaviour in cases involving considerable shape change and bending. The theory is applied to the stamping of a front fender. A similar article by Huh *et al.* [148] applies the same formulation to deep drawing of cylindrical and square cups. Choi and Huh [149] expands the formulation to model materials with planar anisotropy.

Pourboghraat and Chu [150] and Pourboghraat *et al.* [151] have developed a different hybrid membrane/shell element where pure bending is superimposed on stretches obtained from membrane calculations. This method, shown to be more cost effective than shell elements, is applied to the springback prediction of 2008-T4 aluminum. Finally, Esche and Kinzel [152] and Esche *et al.* [153] also created an axisymmetric hybrid membrane/shell element, which is compared with existing membrane elements using a round cup drawing simulation.

Chapter 3

Experimental Methods

Quantifying bendability is neither a trivial procedure nor is it as standardized as other mechanical tests, such as uniaxial tensile testing. This chapter will:

- examine the cantilever bend test as an alternative to the minimum R/t measurements traditionally used to evaluate bendability;
- explain the experimental procedure used in this thesis;
- discuss the materials used to evaluate the role of void damage in bending behaviour.

3.1 Cantilever Bend Test

There are a number of different ways in which the bendability of materials can be evaluated, many of which were outlined in Section 2.2.2. The traditional method of assessing bendability, the minimum R/t ratio, has been shown to be overly subjective. In the following sections, an introduction to the cantilever bend test apparatus, such as the

one commissioned by Alcan KRDC, will be presented and the information that can be obtained from this test will be explained.

3.1.1 Description of the Cantilever Bend Test

The cantilever bend test apparatus was originally developed by Selçuk and Rawlings [30], as an alternative to minimum R/t measurements which they found to be insensitive to changes in formability of Cu-Be-Co alloys. An apparatus, based on this design, was constructed at Alcan KRDC. A schematic of this device, which was used to obtain the experimental results presented in this thesis, is shown in Figure 3.1.

The entire apparatus is mounted on a standard tensile test frame, as seen in the pictures of Figures 3.2 and 3.3. A belt is wrapped around the drum and is connected to the load cell such that the belt causes the drum to rotate as the crosshead is raised. Attached to the drum is an adjustable arm at the end of which is a free spinning roller. As the drum rotates, the roller sweeps a specimen blank around a mandrel of prescribed radius, as seen in the photograph of Figure 3.4. Currently, Alcan KRDC only has a mandrel with a radius of 0.006 in., although there are plans for more to be machined.

3.1.2 Data from the Cantilever Bend Test

The data collected from these experiment consists of the force recorded at the load cell and the linear displacement of the crosshead. A sample of load versus displacement output is shown in Figure 3.5. The results exhibit characteristics similar to that of a tensile curve. There is an initially elastic region, followed by plastic hardening to a maximum load, and eventually a drop in force.

It is believed that there is a direct correlation between the rate of load drop and bendability, as the drop is accelerated by the occurrence of damage and cracking at the outer surface. Thus, a more rapid load drop would indicate poorer bendability.

3.2 Experimental Procedure

The test protocol used for the cantilever bend test experiments is given below. Each step is explained in detail, followed by an itemized summary of the procedure.

3.2.1 Setting the Gap Height

The position of the roller may be adjusted radially inwards or outwards from the centre of rotation. A prescribed clearance between the mandrel and the roller, known as the gap height (h_{gap}), is required. This gap height is equal to the thickness of the sheet plus some additional clearance ($h_{clearance}$), such that:

$$h_{gap} = t + h_{clearance} \quad (3.1)$$

where $h_{clearance}$ is usually expressed in term of a percentage of sheet thickness. A clearance of 5%, for example, would mean a gap height of $1.05t$.

To set the gap height, the drum of the apparatus must be rotated 90° in the clockwise direction from the upright position. It has been designed such that it may be locked in this orientation, as shown in Figure 3.6a. Feeler gauges, equal in thickness to the desired gap height, are placed between the mandrel and the roller. The radial position of the arm is then adjusted such that the roller is contacting the feeler gauges, yet not so tightly

that they cannot be easily removed. Having set the gap height, the drum is unlocked and returned to the upright position.

3.2.2 Zeroing Force and Displacement

A consistent and repeatable zero point for both force and displacement is critical to enable meaningful comparison amongst different sets of tests. The location of zero displacement is obtained by placing the drum such that the rotating arm is in the vertical position. Half of the circumference of the drum has been graduated from 0° to 180° , at 1° increments, with an indicator on the bend test frame to measure drum rotation. The vertical position of the arm corresponds to 180° on the indicator. With the drum in this position, the displacement reading is zeroed when the crosshead has been sufficiently raised to remove all of the slack from the belt, as shown in Figure 3.6b. The drum is then rotated slightly to slacken the belt, at which time the force reading is zeroed, as illustrated in Figure 3.6c. Once these two zero points have been set, they remain unchanged for ensuing series of the tests.

3.2.3 Test Specimens

The test specimens are simply rectangular blanks of the sheet material to be tested, cut at the desired orientation with respect to the rolling direction. In the current work, specimens were nominally 0.50 in. in width and 3.5 in. in length, although the exact length is not critical, with a sheet thickness of 0.040 in.

Prior to insertion in the apparatus, the specimen edges are filed to remove any burrs incurred during shearing. The specimens are also cleaned using a tissue moistened with

WD-40 to remove any debris. Once inserted, the specimen is clamped to the mandrel and positioned such that 6 mm of the material extends past the end of the mandrel.

3.2.4 Pre-loading the Specimens

One problem, stemming from the fact that the apparatus' drum rotation is belt-driven, is that the tests cannot begin with the roller arm in the vertical position. If this is attempted, the eccentricity of the arm would result in an accelerated rotation and impact with the specimen, as the drum begins to rotate. The belt would then be slack and the drum would cease rotating momentarily until this slack was removed. Hence, this accelerated rotation results in no correspondence between the displacement measured at the crosshead and the actual position of the drum. Additionally, the initial portion of the force-displacement curve would contain an unknown inertial effect. The magnitude of both of the effects would vary with clearance.

It was decided that the easiest solution involved purposefully pre-loading the specimen. By ensuring that all the belt slack is removed, better correspondence between the displacement and force readings is obtained. While this does result in the loss of the initial portion of the force curve, it is not seen as detrimental since the latter portion is of greatest interest. The pre-loading and the choice of a zero displacement point are the reasons that neither the force nor displacement data begin at zero, as can be seen in Figure 3.5. This adopted procedure did not seem to affect the repeatability of the test as demonstrated by the results depicted in Figure 3.7 for samples of production quality AA6111.

3.2.5 Converting from Displacement to Bend Angle

It is difficult to associate units of linear crosshead displacement with the deformation undergone by the specimen. A much more intuitive way to describe the bending of a strip is in terms of the bend angle, where 0° indicates a flat specimen and 180° indicates a full bend (see Figure 3.8). Accordingly, the linear displacement data obtained from the experiments was converted to bend angles, θ , using the equations given below and the apparatus dimensions shown in Figure 3.9.

Due to the gap height, the roller does not begin bending the sheet until it has rotated a given amount, θ_{min} :

$$\theta_{min} = \cos^{-1} \left(\frac{R_{roller} + R_{mandrel} + t_{sheet}}{R_{roller} + R_{mandrel} + h_{gap}} \right). \quad (3.2)$$

Subtracting this minimum angle from the total angle of drum rotation with respect to the vertical gives the angular displacement of the roller, which is equal to the bend angle of the specimen, such that:

$$\theta = \left(\frac{d_{crosshead}}{R_{drum}} - \theta_{min} \right) \quad (3.3)$$

where both θ and θ_{min} are in radians.

3.2.6 Summary of the Test Procedure

The overall test procedure used during the experimentation can be summarized in brief as follows:

- 1 – Set the gap size.

- 2 – Zero the force and displacement readings.
- 3 – Label and load the test specimen with the free end protruding 6 mm over the end of the mandrel.
- 4 – Pre-load the specimen.
- 5 – Conduct the test using a crosshead speed of 5 in./min.
- 6 – Save the force and displacement data following the completion or interruption of the test.
- 7 – Remove the specimen.
- 8 – Repeat items 3 to 7.
- 9 – Periodically clean the mandrel and roller, roughly every 10 specimens, with 600 grit emery paper.
- 10 – If desired, convert displacement measurements from linear displacement to bend angle using Equations 3.2 and 3.3.

3.3 Materials

One aluminum alloy that is of specific interest to the automotive industry is AA6111, which was designed specifically for use in outer closure panels [9]. The chemical composition of the alloy is presented in Table 3.1. Since it will frequently be hemmed to an inner panel, understanding the bendability of this material is particularly pertinent.

3.3.1 AA6111 Fe-Variants

AA6111 is a heat-treatable alloy. The attraction of heat-treatable alloys to the automotive industry is that they allow a lower strength during forming, which can then be raised during the paint bake cycle. To study the effect of void damage on the bendability of AA6111, a number of Fe-variants were produced by Alcan KRDC. The composition of these variants is summarized in Table 3.2.

During the development of the alloy, Fe content was found to greatly affect the formability [9]. Since Fe has a very low solubility in aluminum, the majority of Fe is present as second-phase particles, such as FeAl_3 . These particles act as nucleation sites for void damage, either through matrix-particle decohesion or particle cracking. The increasing levels of Fe will result in an increased volume fraction of second-phase particles in the AA6111 Fe-variant alloys shown in Table 3.2 and in turn, higher void damage rates. Therefore, the effect of void damage on sheet bendability can be examined.

A single tensile specimen of the 0.06, 0.26 and 0.68% Fe content alloys were tested by Alcan KRDC transverse to the rolling direction. Figure 3.10a contains the force and displacement data from these tests for standard uniaxial sheet specimens with nominal gauge dimensions of 2 in. x 0.5 in. x 0.04 in. The data is shown converted to true stress-true strain data in Figure 3.10b, although it should be noted that no Bridgman corrections were made.

3.3.2 Material Preparation

The specimens used in the bend tests were cut from a 0.040 in. sheet of the various AA6111 Fe-variant alloys, both parallel and transverse to the rolling direction. They were solutionized at 560°C for three minutes, followed by a forced air quench (FAQ) and

natural aging at room temperature for a period of weeks. The specimens were sheared to blanks with nominal dimensions of 0.5 in. x 3.5 in. for subsequent testing.

Chapter 4

Numerical Methods

The finite element method (FEM) has been used to obtain numerical predictions of bending behaviour. This chapter will provide:

- an introduction to LS-DYNA, the commercial FEM code used for this thesis;
- a discussion regarding the implementation and validation of an LS-DYNA user material subroutine (UMAT) which implemented the kinematic hardening Gurson-Tvergaard-Needleman (GTN) constitutive equations;
- an overview of the Tvergaard bend model;
- an overview of the cantilever bend test model.

4.1 LS-DYNA

Livermore Software Technology Corporation's (LSTC) commercial FEM code, LS-DYNA [154], has been utilized in this work to solve the FEM models. A brief overview of general

issues related to the use of LS-DYNA will be given, including model generation, the Courant condition, user material subroutines, contact and post-processing.

The creation of an FEM model requires the explicit definition of nodes and elements. In the present study, this has been accomplished in two ways. The first involved the use of Structural Dynamics Research Corporation's (SDRC) computer-aided design (CAD) software I-DEAS 7. Once a three-dimensional solid model had been created in I-DEAS, it was then meshed with the desired element type, either manually or using I-DEAS' automated mesh generation routines. The assembled models were then exported to an I-DEAS universal (.unv) file and converted into an LS-DYNA input deck using translation software developed by M.J. Worswick [155]. Alternatively, an ANSI C program was created to generate the LS-DYNA node and element information directly. It was found that this latter approach, while initially time consuming, enabled more rapid and straightforward modifications to dimensions and meshes for the relatively simple geometries involved.

LS-DYNA is a dynamic FEM code with both implicit and explicit capabilities, although only the latter has been used in this thesis. In an explicit formulation, the equations of motion are integrated at each time step whereas an implicit approach involves assembling and solving large stiffness matrices. The explicit technique results in a lower computational cost per time step relative to implicit codes; however, a greater number of total time steps is required due to the Courant condition, which states that the minimum time step must satisfy:

$$\Delta t_e \leq \frac{l_e}{c} \quad (4.1)$$

where l_e is the element characteristic length and c is the adiabatic speed of sound in the material. This is equivalent to saying that the minimum time step must be less than the time needed for a stress wave to traverse the smallest element [156]. The minimum time step is usually extremely small, which make explicit codes best suited to short-duration, high-velocity problems, such as those involving ballistics or metal forming. Solution times for low-velocity or quasi-static problems become unreasonable unless they are run at artificially high speeds because of the huge number of time steps that would otherwise be required. The use of elevated velocities has been shown to be acceptable, provided that inertial effects are kept small [157]. Regardless, great care must be taken to ensure that dynamic effects are not introducing artifacts into the results.

One of the advantages of LS-DYNA is that it allows for the incorporation of user-defined material subroutines (UMATs) which facilitates the use of material models not supported by the standard software release. For this work, an isotropic GTN subroutine developed by Worswick and Pelletier [145] has been modified to include a kinematic formulation.

To govern contact between tooling and parts, LS-DYNA makes use of a penalty function formulation which prevents penetration of one body into another. Normal interface springs are placed between nodes and the contact surface [158].

Visualization and post-processing of the results was accomplished using LS-POST [159]. In more complex cases, routines were written in ANSI C to read the LS-DYNA output files and extract predicted values of interest.

4.2 Kinematic Hardening Gurson-Tvergaard-Needleman (GTN) User Material Subroutine

Modifications were made to the isotropic GTN subroutine created by Worswick and Pelletier [145] to allow for the incorporation of kinematic hardening. The forward integration method used by Tvergaard [111, 112] and subsequent authors [133, 137–139], however, limited the range in which the solution remained valid. The equations were subsequently reformulated to make it compatible with the iterative method of the existing subroutine and to increase the stable range of applicability. The implementation and verification of the subroutine are presented below.

4.2.1 Implementation of the Subroutine

According to Tvergaard [111, 112], as described in Section 2.4.7, a plastic increment produces the following change to the yield surface centre:

$$\dot{\alpha}_{ij} = \dot{\mu} \tilde{\sigma}_{ij}. \quad (2.31)$$

Furthermore, the value of the parameter $\dot{\mu}$ must satisfy the consistency condition $\dot{\Phi} = 0$. Tvergaard solved for $\dot{\mu}$ by expanding $\dot{\Phi}$ in accordance with the chain rule to obtain:

$$\dot{\Phi}(\sigma_{ij}, \alpha_{ij}, \sigma_f, f) = \frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial \Phi}{\partial \alpha_{ij}} \dot{\alpha}_{ij} + \frac{\partial \Phi}{\partial \sigma_f} \dot{\sigma}_f + \frac{\partial \Phi}{\partial f} \dot{f} = 0. \quad (4.2)$$

However, for a fully kinematic case, where $\sigma_f = \sigma_y$ and $\dot{\sigma}_f = 0$, Equation 4.2 reduces to:

$$\dot{\Phi} = \frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial \Phi}{\partial \alpha_{ij}} \dot{\alpha}_{ij} + \frac{\partial \Phi}{\partial f} \dot{f} = 0. \quad (4.3)$$

By recognizing that:

$$\begin{aligned} \frac{\partial \Phi}{\partial \alpha_{ij}} &= \frac{\partial \Phi}{\partial \tilde{\sigma}_{ij}} \frac{\partial \tilde{\sigma}_{ij}}{\partial \alpha_{ij}} \\ &= \left[3 \frac{\tilde{s}_{ij}}{\sigma_f^2} + \frac{f^* q_1 q_2}{\sigma_f} \sinh \left(\frac{q_2}{2} \frac{\tilde{\sigma}_{kk}}{\sigma_f} \right) \delta_{ij} \right] [-1] \\ &= -2 \left(\frac{3 \tilde{s}_{ij}}{2 \sigma_f^2} + \frac{\alpha}{\sigma_f} \delta_{ij} \right) \end{aligned} \quad (4.4)$$

and substituting Equations 2.31 and 4.4 into Equation 4.3, it is found that:

$$\begin{aligned} 0 &= \frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} - 2 \left[\frac{3 \tilde{s}_{ij}}{2 \sigma_f^2} + \frac{\alpha}{\sigma_f} \delta_{ij} \right] [\dot{\mu} \tilde{\sigma}_{ij}] + \frac{\partial \Phi}{\partial f} \dot{f} \\ 2 \left(\frac{\tilde{\sigma}_e^2}{\sigma_f^2} + \frac{\tilde{\sigma}_{kk}}{\sigma_f} \alpha \right) \dot{\mu} &= \left(\frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial \Phi}{\partial f} \dot{f} \right) \\ \dot{\mu} &= \frac{1}{2} \left(\frac{\tilde{\sigma}_e^2}{\sigma_f^2} + \frac{\tilde{\sigma}_{kk}}{\sigma_f} \alpha \right)^{-1} \left(\frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial \Phi}{\partial f} \dot{f} \right). \end{aligned} \quad (4.5)$$

To express Equation 4.5 in terms of a family of isotropic/kinematic hardening yield surfaces the terms $(1 - b)$ and σ_y/σ_f can be introduced to obtain:

$$\dot{\mu} = (1 - b) \frac{1}{2} \left(\frac{\tilde{\sigma}_e^2}{\sigma_f^2} + \frac{\tilde{\sigma}_{kk}}{\sigma_f} \alpha \right)^{-1} \left(\frac{\partial \Phi}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\sigma_y}{\sigma_f} \frac{\partial \Phi}{\partial f} \dot{f} \right). \quad (4.6)$$

Use of the “elastic predictor – normal corrector” approach of Worswick and Pelletier [145] ensures that the stress point is always returned to the current yield surface. From

this return, the increment of effective plastic strain, $\dot{\bar{\epsilon}}^p$, can then be calculated. In turn, $\dot{\bar{\epsilon}}^p$ can be used to determine the change in flow stress, $\dot{\sigma}_f$, and porosity, $\dot{f}$.

The equivalent plastic work expression was given in Section 2.4.7 as:

$$\tilde{\sigma}_{ij} \dot{\bar{\epsilon}}_{ij}^p = (1 - f) \sigma_f \dot{\bar{\epsilon}}_m^p. \quad (2.21)$$

Likewise, the change in plastic strain was given as:

$$\dot{\bar{\epsilon}}_{ij}^p = \frac{1}{H} m_{ij}^G m_{kl}^F \dot{\sigma}_{kl}. \quad (2.23)$$

Multiplying both sides of Equation 2.23 by $\tilde{\sigma}_{ij}$ gives:

$$\tilde{\sigma}_{ij} \dot{\bar{\epsilon}}_{ij}^p = \frac{1}{H} \tilde{\sigma}_{ij} m_{ij}^G m_{kl}^F \dot{\sigma}_{kl} \quad (4.7)$$

which can be then be equated with Equation 2.21 and simplified to obtain

$$m_{kl}^F \dot{\sigma}_{kl} = \frac{H (1 - f) \sigma_f}{\tilde{\sigma}_{ij} m_{ij}^G} \dot{\bar{\epsilon}}_m^p. \quad (4.8)$$

Recalling, from Equations 2.25 and 2.26, that $m_{ij}^F = m_{ij}^G$ for strain controlled nucleation, Equation 4.8 then becomes:

$$\begin{aligned} \frac{\partial \Phi}{\partial \sigma_{kl}} \dot{\sigma}_{kl} &= \left(\frac{2}{\sigma_f} m_{ij}^G \right) \dot{\sigma}_{kl} \\ &= \frac{2}{\sigma_f} \frac{H (1 - f) \sigma_f}{\tilde{\sigma}_{ij} m_{ij}^G} \dot{\bar{\epsilon}}_m^p \\ &= \frac{2H (1 - f)}{\tilde{\sigma}_{ij} m_{ij}^G} \dot{\bar{\epsilon}}_m^p. \end{aligned} \quad (4.9)$$

While this limits the solution to only being valid for strain controlled nucleation, it enables Equation 4.6 to be reduced to:

$$\dot{\mu} = (1 - b) \frac{1}{2} \left(\frac{\tilde{\sigma}_e^2}{\sigma_f^2} + \frac{\tilde{\sigma}_{kk}}{\sigma_f} \alpha \right)^{-1} \left(\frac{2H(1-f)}{\tilde{\sigma}_{ij} m_{ij}^G} \dot{\epsilon}_m^p + \frac{\sigma_y}{\sigma_f} \frac{\partial \Phi}{\partial f} \dot{f} \right). \quad (4.10)$$

4.2.2 Validation of the Subroutine

A number of single element test cases were conducted in order to confirm that the subroutine had been implemented properly. Unfortunately, no published data could be found that gives results of test cases involving a kinematic hardening GTN material as implemented by Tvergaard [111, 112]. Tsuta *et al.* [139] have recently published results for a variant of the constitutive model; however, their modifications make direct comparison impossible and the absence of nucleation in their results make it undesirable. Instead, the subroutine was validated by comparing it with a voidless kinematic hardening von Mises material (ie. equivalent to a voidless GTN model) as well as with published isotropic GTN results.

Comparison with Kinematic von Mises

The kinematic von Mises behaviour was verified by comparison with the linear hardening kinematic material model available in LS-DYNA. A reversed uniaxial tension single-element test case was used to compare the behaviour of the subroutine with the linear hardening material model available in the standard release of LS-DYNA. A four-noded plane-strain element was stretched to 50% of its original height and then compressed an equal amount. The boundary conditions are as shown in Figure 4.1a and the material variables are given in Table 4.1.

The resulting axial stress versus axial strain predictions are plotted in Figure 4.2 for fully isotropic ($b = 1$), combined isotropic/kinematic ($b = 0.5$) and fully kinematic ($b = 0$) conditions. The results are in good agreement and indicate that the subroutine is valid in the absence of porosity ($f=0$).

Comparison with Isotropic Hardening Gurson

As explained by Tvergaard [111, 112], the results for isotropic and kinematic hardening GTN models should be identical under proportional, non-reversed loading. Noting that the subroutine developed by Worswick and Pelletier [145] corresponds to fully isotropic conditions ($b = 1$), their subroutine was employed as a basis for verifying the correct implementation of kinematic hardening.

Three three-dimensional test cases were studied by Worswick and Pelletier [145], as illustrated schematically in two dimensions in Figure 4.1a-c. The first two test cases, uniaxial tension and volumetric expansion, were compared with published results by Aravas [144]. The third test, constrained uniaxial tension, was introduced by the authors. The material's hardening behaviour follows a power law function specified by Aravas [144] as:

$$\frac{\sigma}{\sigma_y} = \left(\frac{\sigma}{\sigma_y} + \frac{3G}{\sigma_y} \bar{\epsilon}^p \right)^N \quad (4.11)$$

and using the parameter settings given in Table 4.2.

These three test cases were applied to the fully kinematic formulation developed in the present study. The kinematic predictions are compared to their isotropic counterparts in Figures 4.3 to 4.5. Only the fully isotropic and fully kinematic cases are shown as they represent the maximum disparity in hardening behaviour.

A graph of axial stress, normalized with respect to initial yield stress, versus axial strain is plotted in Figure 4.3 for the uniaxial tension case. Porosity versus axial strain is also shown. The agreement for both parameters is excellent. Figure 4.4 presents the findings for volumetric expansion. Graphs of normalized hydrostatic stress and porosity are shown relative to volumetric strain. Under these conditions there is less agreement, although both the isotropic and kinematic hardening results exhibit the same trends. Finally, normalized axial stress data and porosity are plotted against axial strain in Figure 4.5 for the constrained uniaxial tension case with a good level of agreement.

The evolving shape of the Gurson yield surface make visualization of the stress path under a given set of loading conditions difficult. To aid in understanding, the stress paths for the three cases, under fully isotropic conditions, have been plotted in terms of equivalent and hydrostatic stresses normalized with respect to the current flow stress. These curves are depicted in Figure 4.6. Agreement between the isotropic and kinematic cases appears to decrease with increasing hydrostatic stress (or stress triaxiality). The stress paths in Figure 4.6 illustrate that the imposed level of hydrostatic stress increases from uniaxial tension to constrained uniaxial tension to volumetric expansion.

Since porosity is exponentially dependent upon triaxiality, slight differences in hydrostatic stress will greatly influence the void growth rate. Furthermore, larger porosity values produce larger values of effective plastic strain, as can be seen by rearranging Equation 2.21 to obtain:

$$\dot{\bar{\epsilon}}^p = \frac{\bar{\sigma}_{ij} \dot{\epsilon}_{ij}^p}{(1 - f)\sigma_f}. \quad (4.12)$$

Evidence that the kinematic hardening case is predicting greater amounts of plastic strain is demonstrated in Figure 4.5b. Voids are nucleating earlier, suggesting that the effective

plastic strain is slightly higher since strain controlled nucleation is being employed. This trend is not present in Figure 4.4b because the high initial porosity ($f_o = 0.04$) of the volumetric expansion case renders the effect imperceptible.

As the majority of the work in this thesis will involve two-dimensional plane strain conditions, these single element test cases were repeated in this way and the outcome is shown in Figures 4.7 to 4.9. Uniaxial tension and biaxial tension cases are presented in Figures 4.7 and 4.8. The agreement between the isotropic and kinematic cases is excellent. Results for constrained uniaxial tension in plane strain two dimensions are comparable to those in three dimensions (Figures 4.9 versus 4.5), which is expected given that the boundary conditions are identical.

It is evident, based on Figure 4.6, that there remains a form of behaviour that has yet to be fully tested. Volumetric expansion provides one extreme, in that it measures a fully hydrostatic case. However, none of the test cases address a fully deviatoric condition. Therefore, a fourth test was conducted that involved pure shear, as shown graphically in Figure 4.1d. The results of this simulation are presented in Figure 4.10, noting that the agreement for isotropic and kinematic conditions is excellent. The stress paths for all four two-dimensional cases, under fully isotropic conditions, are provided in Figure 4.11.

The agreement under plane strain conditions is much improved relative to that of the three-dimensional cases. Again, the two-dimensional predictions follow the general trends found in the three-dimensional models, in which better agreement is achieved at lower levels of triaxiality. While the source of this disagreement has not been completely resolved, it is not expected to greatly influence the predictions presented in this thesis as plane strain conditions will be employed throughout.

4.3 Tvergaard Model

The kinematic hardening GTN equations were applied by Tvergaard [111] to the study of ductile shear fracture at the surface of a bent specimen. This model has been repeated because of its relevance to the present research. Furthermore, it serves as a qualitative point of comparison for the application of the subroutine to a multi-element problem. The original model, as documented in the literature, will be described in the following sections, along with the modifications needed to simulate this problem in LS-DYNA.

4.3.1 Original Model

Tvergaard [111] utilized a rectangular portion of material, as shown in Figure 4.12, in which the upper surface is given an initial thickness inhomogeneity defined by:

$$w = -\frac{1}{2}\delta_o \cos \frac{\pi x^1}{l_o} \quad (4.13)$$

The boundary conditions for the four surfaces, expressed in terms of reference coordinates (x^i), displacement coordinates (u^i), and traction components (T^i), were each specified as functions of a prescribed bend angle, θ :

$$\left. \begin{aligned} T^i &= 0 && \text{for } x^2 = w(x^1) \\ u^1 &= 0, T^2 = 0 && \text{for } x^1 = 0 \\ (u^1 + x^1) \cos \theta - (u^2 + x^2 + R + h_o) \sin \theta &= 0, \\ T^1 \sin \theta + T^2 \cos \theta &= 0 \end{aligned} \right\} \text{for } x^1 = l_o \quad (4.14)$$

$$\left. \begin{aligned} (x^1 + u^1)^2 + (u^2 + R)^2 - R^2 &= 0, \\ T^1 \frac{R+u^2}{R} + T^2 \frac{x^1+u^1}{R} &= 0 \end{aligned} \right\} \text{for } x^2 = -h_o$$

where the radius, R , at the surface $x^2 = -h_o$ is given as:

$$R = R_o + l_o/\theta. \quad (4.15)$$

Tvergaard's simulation, which has been repeated here, had an initial aspect ratio of $h_o/l_o = 10.2$ and an initial imperfection of $\delta_o/l_o = 0.005$. It was discretized using an 48×8 mesh of quadrilaterals, each consisting of four linear displacement triangular elements. The initial radius of curvature, R_o , was chosen to be zero. Constitutive material behaviour was modeled using the GTN equations with the parameters given by Tvergaard [111] in Table 4.3. A piecewise power law of the following form was employed:

$$\varepsilon = \begin{cases} \frac{\sigma}{E}, & \text{for } \sigma \leq \sigma_y \\ \frac{\sigma_y}{E} \left(\frac{\sigma}{\sigma_y} \right)^n, & \text{for } \sigma > \sigma_y \end{cases} \quad (4.16)$$

where $\sigma_y/E = 0.0033$, $n = 10$ and Poisson's ratio was set at $\nu = 0.3$. The value of σ_y was chosen to be 127 MPa based on the yield strength of the AA6111 alloy.

4.3.2 Modifications for LS-DYNA

Since LS-DYNA does not allow for the prescription of boundary conditions as described in Equations 4.14 and 4.15, alterations are needed to obtain the desired displacements. A pin-and-slider-type mechanism was created, as shown in Figure 4.13. The slider consisted of two offset surfaces, on either side of the free end (at $x^1 = l_o$). Frictionless contact was then defined between these two surfaces and the free end nodes. The thicknesses of the slider surfaces were chosen such that a small gap was left between them, thereby imposing a rotation on the nodes while allowing them the freedom to translate towards

the centre of rotation. A third rigid body, with a fixed position, was needed to allow for the definition of a rotating joint.

The model was created using two-dimensional plane strain three-noded triangular elements. The Gurson parameters were set to the values used by Tvergaard [111] and summarized in Table 4.3. The slider and fixed part were both constructed from four-noded shell elements with rigid body properties.

4.4 Alcan Cantilever Bend Test

The Alcan KRDC cantilever bend test, described earlier in Section 3.1, has been modelled in two dimensions. The simplified schematic in Figure 4.14a serves to illustrate the important features. The details of the geometry will be discussed in the proceeding sections, followed by an explanation of the means by which force-displacement data was extracted for comparison with experimental data.

4.4.1 Model

The test apparatus was represented using five parts: the specimen, roller, mandrel, clamp and rotating link, configured as shown in Figure 4.14a. Initially, the geometry was represented in three dimensions; however, this limited the resolution that could be used through the thickness of the sheet without incurring unreasonable computation times. As the number of elements increases, their size decreases, and the total computation time rises dramatically due to the reduction in the minimum time step governed by Equation 4.1. In the end, a two-dimensional plane strain approach was adopted to obtain a reasonable compromise between computation time and mesh resolution. Thus,

the model corresponds to a strip of plane strain material down the centreline of the bend specimen.

The specimen was modelled using three- and four-noded two-dimensional plane strain elements, employing material properties described by the GTN constitutive equations, as implemented in the LS-DYNA user-material subroutine. All of the remaining parts were meshed with four-node shell elements, with material properties corresponding to steel (see Table 4.4).

The model was created in the x-y plane. The mandrel position was completely fixed, whereas the roller and rotating link were allowed to translate in the x-y plane. Two rotational joints were created between the link and the mandrel, as well as the link and the roller. Hence, rotational displacement of the link about the z-axis caused the roller to be swept around the mandrel in the desired manner. Both joints were assumed to be frictionless. Automatic two-dimensional contact was defined between the roller and the upper surface of the specimen.

Initially, the clamp was given a small downward displacement prior to the commencement of roller rotation in order to secure the specimen to the mandrel. It was found, however, that an appropriate amount of clamping was difficult to determine. If too large a displacement was specified, the free end of the specimen was caused to flare upwards off of the mandrel. Conversely, if an insufficient amount of displacement was used, then the specimen would slide across the mandrel as it was being bent. While more realistic behaviour could have been achieved through the modelling of the entire clamped length of the specimen, this would have added unnecessarily to the computational cost. Instead, the position of the clamp was fixed and contact was defined between the clamp and the specimen. The presence of the clamp was necessary to account for the clamp-specimen

interaction during the bending process. The nodes at the clamped end of the specimen were then given the boundary conditions shown in Figure 4.14b. The arrows in that diagram indicate restraint imposed by the contact of the specimen with the fixed mandrel and clamp.

4.4.2 Force versus Displacement Data

As discussed in Section 3.1.2, the data obtained from the cantilever bend test is in the form of force versus displacement data. This information must be extracted from the FEM simulations to allow for quantitative comparison with the experimental data.

Rigid body displacement data and the contact forces for designated parts can be stored by LS-DYNA at prescribed time intervals. An ANSI C utility was written to extract the rotational displacement of the rotating link and the contact force components for the roller-specimen interface. Knowing the dimensions of the model and those of the actual bend test apparatus, as shown in Figure 4.15, the force and displacement can be calculated as follows:

$$F_{FEM} = \{F_x \cos(\theta + \theta_0) - F_y \sin(\theta + \theta_0)\} \frac{R_{roller}}{R_{drum}} \frac{w_{specimen}}{w_{model}} \quad (4.17)$$

$$d_{FEM} = R_{drum} (\theta + \theta_0) \quad (4.18)$$

Results were then converted from metric to imperial units for comparison with the experimental data.

Chapter 5

Results

Following the experimental and numerical procedures described in Chapters 3 and 4, this chapter will present:

- the experimental results obtained from the bend specimens of the AA6111 Fe-variants;
- the results of two parametric studies involving the Tvergaard bend model;
- preliminary results from numerical simulations of the cantilever bend test.

5.1 Experimental Results for the 6111 Fe-variants

Specimens of the AA6111 Fe-variants were tested using the cantilever bend test apparatus to produce a series of force versus bend angle graphs. Following these tests, the nature of cracking at the outer specimen surfaces was examined to draw correlations with the experimental data. Selected specimens were also mounted and examined microscopically

to obtain second-phase particle measures, to assess the degree of thinning, and to evaluate the development of surface roughness.

5.1.1 Bend Test Data

Multiple samples of the AA6111 Fe-variants were tested, using the cantilever bend test, in both the rolling and transverse sheet directions. Interrupted specimens were also obtained to evaluate intermediate states of deformation. The force versus bend angle graphs for each combination of material and orientation are shown in Figures 5.1 to 5.4, in order of increasing Fe content.

As expected, the bendability for all the alloys is superior in the longitudinal direction as compared to the transverse direction. The 0.06% Fe alloy exhibits behaviour that is slightly different from the remaining alloys. For example, the maximum load achieved in both directions for the 0.06% Fe alloy is lower than those obtained by the other variants. Furthermore, the maximum load achieved by the 0.06% Fe alloy in the transverse direction is significantly higher than in the longitudinal direction. The reasons for these differences is not known.

One explanation for the inferior bendability of the 0.06% Fe alloy is illustrated by the micrographs in Figure 5.5. An increase in Fe content leads to smaller grains, as verified by the average grain size measures recorded in Table 5.1. The same trend has also recently been reported by Sarkar *et al.* [160] for AA5754 Fe-variants. As a result, the AA6111 Fe-variants contain two competing effects. Higher levels of Fe causes finer grains which improve the overall performance of the material. However, this benefit is counteracted by an increase in void damage rates resulting from more and larger second-phase particles.

Another observation is that the load curves of the Fe-variants exhibit more variability than the production level AA6111 material shown in Figure 3.7. The production quality material displays minimal variability in either the maximum load achieved by each repeated trial or in the bend angle at which the final load drop occurs. It is believed, therefore, that the variability in the load curves for each of the Fe-variants can be attributed to material inhomogeneity rather than the repeatability of the test apparatus itself. The production material was created under closely controlled and monitored conditions, whereas the Fe-variants were created in small laboratory-scale batches conducted specifically for research purposes. A greater degree of inhomogeneity is likely under these conditions, in terms of sheet gauge, alloy composition and microstructural features (eg. second-phase particles).

Bendability does not appear to be a simple linear function of Fe level. Instead, the results indicate that the materials reach maximum bendability at an intermediate level of Fe, around 0.35%. This is in agreement with minimum bend radius measurements conducted by Alcan KRDC as a function of aging time, as summarized for three Fe levels in Table 5.2. This result is important because it indicates that some Fe content is tolerable in terms of bending performance, an observation that has serious implications with respect to recycling.

5.1.2 Bend Test Surface

To investigate the cause of the final drop in the experimental load curves, images of the surfaces of each bend specimen were taken, at a resolution of 2048 x 1536 pixels, using a Nikon E990 digital camera mounted on a stereographic microscope. These digital images

were then compared with their respective load curves to obtain correlations between the surface characteristics and distinguishing load curve features.

The maximum load appears to coincide with a substantial roughening of the outer bend surface, as demonstrated by the surfaces of the interrupted 0.26% Fe specimens shown in Figure 5.6. This roughening phenomenon, often referred to as orange peel, is caused by the rotation of grains at the free surface of the material, as discussed with respect to AA6111 by Ramage *et al.* [161].

More interesting is the correlation between the nature of the load drop and the shape of the cracks that develop at the bend surface. In Figure 5.7, three specimens are shown whose tests were all interrupted at approximately the same final load. However, the bend angles at which these loads were achieved is quite different. Although specimen 3 was bent to the largest angle and surface roughening is evident, no predominant cracks are visible at the outer surface. On the other hand, specimens 2 and 4 show major cracks of approximately the same severity, yet they were achieved at different bend angles. These differences are attributed to the ways in which the cracks propagated along the specimen surface. Specimen 2 has a very discontinuous crack shape relative to that of specimen 4. This suggests that the development of a continuous crack results in a more rapid load drop than when a discontinuous crack develops on several planes, meaning that the severity of the load drop provides an indication of the nature of final failure.

5.1.3 Microscopic Examination

Samples of the undeformed material were cut and mounted so that the planar, longitudinal and transverse microstructures could be observed. A 10 x 10 matrix of digital images was taken and assembled to create a single large-scale, high-resolution image using a

custom suite of metallographic image analysis software [162]. These composite images were analyzed to extract microstructural information such as volume fractions of particles and clustering measures. The benefit of this procedure is that the results are based on a statistically significant sample of the material.

Unfortunately, the amount of particles in the 0.06% Fe alloy was so dilute that reliable metallographic image analysis measures could not be obtained. Accordingly, the data from the low and high Fe-variants are summarized in Table 5.3. As noted in Table 5.3, the effective first-order cluster area fraction includes both the area of the particles themselves and the area of the matrix material within the first-order cluster. This parameter has been shown to correlate well with the nucleating potential of the material [163].

Table 5.3 demonstrates that an increase in Fe level results in an increase in both the number and size of the second-phase particles. Furthermore, the number of first-order clusters increases. This is reinforced by the inter-particle dilational spacing (IPDS) frequencies that are plotted in Figure 5.8 for the 0.26% and 0.68% Fe alloys in the longitudinal sheet plane. The magnitude of the first peak in these graphs ($\text{IPDS} \approx 2\mu\text{m}$) indicates that an increase in Fe level produces a higher percentage of particles contained in first-order clusters.

Interrupted samples were also examined microscopically. It was hoped that examination of bent specimens could be used to provide an experimental basis for the void nucleation parameters, ϵ_n and s_n , used in the GTN equations. Half of the bend was removed and mounted so that the behaviour at the centre of the sheet could be examined, as shown in Figure 5.9. Due to the geometry of the bend, only 3×10 matrices of images could be acquired, as illustrated in Figure 5.10.

Unfortunately, there are a number of problems associated with the analysis of deformed bend specimens. The main obstacle is the non-linear gradient of plastic strain through the thickness of the sheet. Determining the amount of plastic strain at any given point in the acquired 3×10 grid of images is very difficult. Furthermore, spring-back would have caused a redistribution of stress and strain following the removal of the specimen from the cantilever bend rig. These factors make it extremely difficult to extract meaningful void measures and associate them with their actual strain state. As a result, the analysis of deformed specimens was limited to a purely qualitative assessment of bent specimens. Namely, the lower Fe content alloys exhibited very little damage near the outer bend surface, whereas the 0.68% Fe-variant contained many voids associate with both clusters of particles and larger particles. Representative metallographic images for the 0.26% and 0.68% Fe alloys are shown in Figures 5.11 to 5.14.

5.1.4 Low Magnification Examination

Low magnification images were also taken of the mounted specimens to make qualitative observations of the degree of thinning in the material and the frequency of undulations on the outer surface (see Figure 5.15).

The observed thinning best explains the load curve drop observed in the experimental load versus bend angle data. During the test the specimen geometry is thinning while the material is hardening. At some point the hardening behaviour of the material is saturated, at which point thinning becomes dominant and results in a drop in the total load. This explains why a load drop is still evident in materials with excellent bendability. As well, the images clearly illustrate an impingement of the mandrel into the underside of the sheet which further accelerates the thinning process.

Closer examination of the outer surface shows that the frequency of the surface undulations decreases as the bend progresses, which is in keeping with the observations of Akeret [19]. Akeret also reported that the amplitude or magnitude of these imperfections should also increase with bend angle; however, in the samples of Figure 5.15 this is not immediately visible. No effort was made to quantify the amplitude of the evolving undulations.

In order to obtain some insight into the frequency of undulations at the outer surface of the material, some very simple measurements were conducted based on the images shown in Figure 5.15. It is recognized that there is an undeniable level of subjectivity in these measurements. However, their purpose was only to obtain a general sense of the frequency for later use in the finite elements models. The frequencies are reported in Figure 5.15 as a length scale l_0 (see Figure 4.12), normalized with respect to the original sheet thickness, h_o , of 1 mm.

5.2 Parametric Studies using the Tvergaard Bend Model

Two parametric studies were undertaken using the Tvergaard bend model, described in Section 4.3, to evaluate the sensitivity of the FEM prediction to the various model parameters. The first study utilized stress-strain behaviour in accordance with the piece-wise power law of function of Equation 4.16. The second analysis employed piece-wise stress-strain data obtained experimentally from commercial grade AA6111 sheet.

Force versus bend angle curves have been extracted from the model, as illustrated in Figure 5.16, to provide a basis for comparing the FEM predictions. Element deletion has

been employed such that the elements are removed from the mesh when a final porosity level of $0.9f_f$ is reached, following the example of Tvergaard [111]. The deletion of elements produces a noticeable drop in load that will be used to indicate the initiation of fracture from the outer surface. It should be noted, however, that the deletion of elements and the associated drop in load does not necessarily signify the formation of a shear band. As shown in Figure 5.17, the crack shape can either be vertical, producing a “zipper-like” fracture, or diagonal, as would be expected from a shear fracture. Furthermore, various combinations of these two crack types can occur. For example, the crack may first descend vertically some distance and then propagate diagonally. It is the initial behaviour that is of greatest interest, particularly since the imposed boundary conditions can eventually induce artifacts such as the crack reflections seen in Figure 5.17b. A superscript * has been added to the legend entries for load curves associated with an initially vertical fracture.

5.2.1 Piece-wise Power Law

The parameters employed by Tvergaard [111], and the piece-wise power law defined in Equation 4.16, were used as a baseline to study the influence of these parameters on the FEM predictions. A yield stress of 127 MPa was chosen based on the nominal yield strength of AA6111 aluminum. The ratio of $\sigma_y/E = 0.0033$ was maintained throughout. The parameters studied include the:

- isotropic/kinematic hardening parameter, b ;
- volume fraction of nucleation particles, f_n ;
- strain-controlled nucleation parameters, ϵ_n and s_n ;

- hardening exponent, N ;
- coalescence and failure porosities, f_c and f_f ;
- mesh size and aspect ratio;
- magnitude of the initial imperfection, δ_o/l_o .

Hardening Parameter, b

It should be recalled that a family of hardening yield surfaces was developed such that the degree of isotropic and kinematic hardening behaviour can be varied linearly as a function of the parameter b . Thus, $b=1$ corresponds to fully isotropic behaviour and $b=0$ to fully kinematic.

The Tvergaard bend model was examined under fully isotropic ($b=1$), half isotropic / half kinematic ($b=0.5$), and fully kinematic hardening conditions ($b=0$). Following Tvergaard [111], the geometric and GTN parameters were chosen to be $h_o/l_o=10.2$, $\delta_o/l_o=0.005$, $f_n=0.04$, $\epsilon_n=0.4$, $s_n=0.1$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

The slider force versus bend angle predictions are shown in Figure 5.18. As expected, the load drops indicate that cracking initiates much earlier under fully kinematic conditions. Fringe plots of porosity for these three cases, depicted in Figure 5.19 at the same bend angle of $\theta=6^\circ$, illustrate the increased propensity for developing shear bands for a yield surface with higher curvature, as is the case for kinematic hardening.

Volume Fraction of Nucleating Particles, f_n

Voids can nucleate from inclusions or second-phase particles in a given material through decohesion or particle cracking. Since the overall purpose of this study is to determine the

effect of Fe content on formability, particularly bendability, this parameter is extremely important because Fe forms Fe-rich constituent particles in AA6111. It is also one of the few parameters that can be established reliably from experiments.

The values of $f_n=0.02$ and $f_n=0.08$ were chosen such that they bracketed the Tvergaard value of $f_n=0.04$. With the other parameters held constant, FEM simulations were conducted under fully isotropic and fully kinematic conditions to study the behaviour at these extremes. The force versus bend angle data is presented in Figure 5.20.

As expected, shear bands initiate more rapidly when a higher volume fraction of particles are void nucleation sites. Since the study of the effect of b showed that shear bands occur more easily with increased curvature of the yield surface, this is not surprising. An increased value of f_n results in more porosity, which leads to greater constriction of the yield surface, and therefore, a greater propensity to initiate shear bands.

A somewhat surprising result is that for $f_n=0.08$ there is very little difference in the behaviour of the isotropic and kinematic results. Again, this is likely due to the increased curvature of a constricting yield surface in the presence of voids. Given that the power law material exhibits very little strain hardening, it is possible that the softening effect of voids, when $f_n=0.08$, overcomes hardening under these conditions, resulting in the similarities between the isotropic and kinematic predictions.

Nucleation Parameters, ϵ_n and s_n

As with the investigation of f_n , values were chosen to bracket Tvergaard's values for ϵ_n and s_n . The results for fully isotropic and kinematic hardening conditions are shown in Figure 5.21 for $\epsilon_n=0.6$ & $s_n=0.2$, $\epsilon_n=0.3$ & $s_n=0.1$, and $\epsilon_n=0.15$ & $s_n=0.05$.

It can be seen that the nucleation of voids at lower strain levels results in more rapid softening of the material for both isotropic and kinematic hardening conditions, as evidenced by the lower forces that are obtained. In general, nucleation appears to have a large effect on shear band development. It is difficult, however, to distinguish the effect of mean nucleating strain, ϵ_n , from the shape of the distribution as defined by the standard deviation, s_n . As a result, these parameters were investigated separately. Results in Figure 5.22 illustrate the effects of various mean strain values with a constant standard deviation of $s_n=0.05$. When the standard deviation is varied around a mean strain of $\epsilon_n=0.04$, a different set of curves are generated as shown in Figure 5.23.

The kinematic predictions appear much more sensitive to ϵ_n than those with isotropic hardening. Particularly interesting is the fact that failure occurs earliest at the intermediate value of $\epsilon_n=0.3$. This suggests that the timing of the nucleation relative to the hardening of the material is an important factor. By looking at the power law stress-strain curve used in these calculations (see Figure 5.24), the hardening rates at effective plastic strains of 0.15, 0.3 and 0.45 can be compared. If most of the voids are nucleated during the early period of great hardening, the softening effect of voids is minimal compared to the case where most of the void nucleation occurs after strain hardening has been exhausted. This suggests that localization is promoted by rapid periods of void nucleation, which intensify the porosity gradients between neighbouring elements, thereby allowing localization to develop more quickly.

Fringe plots of nucleated particles for the case of $\epsilon_n=0.15$ and $\epsilon_n=0.3$ in Figure 5.24 are shown in Figure 5.25 for a constant value of $s_n=0.05$. It is clear that the majority of particles have nucleated by this stage of deformation when $\epsilon_n=0.15$. Conversely, a very clear band of intense nucleation is seen for $\epsilon_n=0.30$. Since it is this second case in

which fracture occurs first, Figure 5.25 suggests that the onset of nucleation creates the porosity gradient that encourages shear band development.

This theory is reinforced by the findings in Figure 5.23 for constant values of ϵ_n with varying levels of s_n , in which localization occurs earlier for smaller standard deviations. With a more dispersed normal distribution function, voids are introduced at an earlier stage of deformation, but the rate of nucleation is slower, and hence, localization is delayed. This is particularly evident under isotropic conditions, where no localization is observed except in the case where void nucleation proceeds within a very tight distribution (ie. $s_n = 0.033$).

Hardening Exponent, N

Three different strain hardening exponents, N , were investigated, based on the power law hardening behaviour of Equation 4.16. The stress-strain behaviour is plotted in Figure 5.26, where it can be seen that $N=8$ has the greatest amount of strain hardening and $N=13$ the least. The force versus bend angle predictions obtained with these three parameter settings are presented in Figure 5.27.

Based on the previously presented observations concerning void nucleation, it is anticipated that localization will occur earlier with lower hardening. Indeed, this behaviour is observed, although it is only evident in the kinematic hardening case. However, it is possible that void growth is confounding these results. Void growth rates are exponentially dependent upon hydrostatic stress, meaning greater levels of strain hardening induce higher levels of hydrostatic stress and more rapid void growth. Consequently, this difference in void growth rate may be masking the true sensitivity of the modelling predictions to the exponent N . It is unclear, however, how these effects can be decoupled.

Coalescence and Failures Porosities, f_c and f_f

The porosity values at coalescence and failure, f_c and f_f , were varied while maintaining the nominal values for the other model parameters. The findings for the fully isotropic and fully kinematic cases are depicted in Figure 5.28. It is evident that a decrease in f_c and f_f invokes earlier fracture, as indicated by an earlier load drop.

The effect of these critical porosity parameters on the path of crack propagation, however, is of more interest. For example, the crack initially grows vertically, as indicated by the superscript *, when $f_c=0.10$ and $f_f=0.20$ in a fully isotropic formulation. The lower values of $f_c=0.05$ and $f_f=0.10$ result in an initially diagonal crack, although it does become vertical at its first reflection. This is the opposite behaviour seen under fully kinematic conditions, where the lowest values of f_c and f_f result in a vertical crack, with the higher values being diagonal.

Variation of the two porosity parameters, f_c and f_f , was then isolated for separate study. Figure 5.29 plots the outcome for a constant value of $f_c=0.05$ and different failure porosities, while different coalescence values with a constant value of $f_f=0.20$ are given in Figure 5.30. It can be seen that the value of f_c is predominantly responsible for determining the onset of final failure. This is evident for both hardening schemes, but it is particularly evident under isotropic conditions.

Mesh Size and Aspect Ratio

As discussed by other authors [111, 142], FEM simulations involving either the GTN equations or shear band localization are extremely mesh dependent. Accordingly, the effects of mesh size and element aspect ratio have been studied in the current work.

The meshing scheme originally used by Tvergaard [111] involved 48×8 quadrilaterals made up of four triangular elements. Therefore, an additional mesh of 96×16 elements was constructed to study the effect of element size. The outcome is shown in Figure 5.31. It should be noted that the refined mesh caused localization to initiate from the upper right hand corner during the kinematic case. This behaviour was viewed as an artifact of the way in which the boundary conditions were applied in the model, so the four triangles of the quadrilateral in the upper right hand corner were given isotropic hardening properties to alleviate this problem.

One observation from Figure 5.31 is that larger loads are attained in the model with more elements. This is primarily a result of the increase in elements through the thickness of the specimen. The increase from 48 to 96 elements improves the discretization of the geometry and allows the strain and porosity gradients to develop more fully, while the remaining bulk of the material softens more slowly.

The second trend in Figure 5.31 is that a more refined mesh promotes earlier localization and crack initiation. This is most clearly illustrated in the fully isotropic case.

Another important meshing parameter is the selection of an appropriate aspect ratio for the elements. As discussed earlier, the aspect ratio of the elements was chosen by Tvergaard [111] such that the element diagonals corresponded to the predicted angle of shear band development as obtained by a bifurcation analysis. Accordingly, the aspect ratio of elements was varied to establish the parameter's influence.

These results are plotted in Figure 5.32 for elements with height to width ratios ($h_e:l_e$) of roughly 1.7:1, 1:1 and 0.6:1. Rather than decreasing the already limited number of elements in the width direction, the element dimensions were only varied in the vertical direction (ie. thickness). Thus, the aspect ratios of 1.7:1, 1:1, and 0.6:1 corresponds to

meshes of 48×8 , 82×8 and 136×8 , respectively. The graphs in Figure 5.32 indicate that the predictions are extremely sensitive to element aspect ratio. This is particularly emphasized by the fact that a greater number of elements should have resulted in earlier localization, similar to the results shown in Figure 5.31.

Magnitude of Initial Imperfection, δ_o/l_o

The influence of the magnitude of the imposed geometric imperfection on the model behaviour was also investigated. Figure 5.33 summarizes this study.

The largest imperfection, $\delta_o/l_o = 0.01$ results in the earliest crack formation, as expected. However, the differences are subtle. Since the geometric imperfection as defined by Equation 4.13 only alters the location of nodes along the upper surface, it only changes the geometry of the top row of elements. This means that the progression of deformation and damage is unchanged for the majority of elements.

5.2.2 AA6111 Stress-Strain Data

In order to apply the kinematic GTN equations to a full cantilever bend test simulation, it was desirable to first gain some insight into the behaviour of the AA6111 material by using the Tvergaard bend model. Thus, the piece-wise power law of Equation 4.16, was replaced by a stress-strain curve for AA6111 sheet. This curve was a composite of tensile and bulge test data that had been merged in order to obtain an accurate yield point and a large range of plastic strain.

Required Changes to the Tvergaard Bend Model

It was found that the FEM models became unstable under fully kinematic conditions at the commencement of element deletion. This is believed to be a function of the amount of hardening, as a similar instability occurs when the hardening exponent, N , in Equation 4.16 is changed from 10 to 5. One possible explanation is that the act of element deletion results in a dramatic change in the local stiffness of the model, which leads to the observed mesh instability. It is suggested [164] that gradually decreasing the strength of the element to zero, over a number of time steps, prior to deletion will alleviate this problem. This solution is currently being investigated.

Due to these difficulties, all simulations using AA6111 stress-strain data have been run with a mixed hardening scheme of $b=0.5$. This is not seen as a huge deficiency, as it is a more realistic value for b . Unfortunately, it prevents the exploration of the extreme behaviour that was possible with the lower hardening material.

It was also found that the coarser 48 x 8 mesh was relatively insensitive to the model parameters when using the AA6111 stress-strain curve. As a result, the 96 x 16 mesh was employed, with the triangles of the upper right quadrilateral being given isotropic hardening behaviour to prevent localization from that location (as explained in Section 5.2.1). Since no bifurcation analysis has been performed for this material, the optimum selection for the element aspect ratio is not known, so it remains unchanged from the analysis presented in Section 5.2.1.

Results Using f_n Measures from the AA6111 Fe-variants

As discussed in Section 5.1.3, clustering data obtained from microscopic examinations of the undeformed AA6111 Fe-variant sheets determined suitable values for f_n of 0.0069

and 0.0252 for the 0.26% Fe and 0.68% Fe alloys, respectively. These settings were then used to study the effects of five combinations of ϵ_n & s_n and three combination of f_c & f_f , as summarized in Figures 5.34 and 5.35.

Given that a volume fraction of void nucleating particles of 0.0069 is extremely small, it is not surprising that the graphs shown in Figure 5.34 show very little deviation in behaviour. With the exception of the most aggressive coalescence and failure schemes, in Figures 5.34a and b, there is no perceivable change.

The predicted load curves for various combinations of void nucleation, coalescence and failure values are shown in Figure 5.35 for $f_n = 0.0252$. It should be noted that the initial fracture path for all conditions in Figure 5.35 is vertical. Eventually, the cracks do venture diagonally, as illustrated by the example in Figure 5.36.

5.3 Preliminary Results from the Cantilever Bend Test Model

The kinematic hardening GTN subroutine has been applied to an FEM model of the Alcan KRDC cantilever bend test apparatus as described in Section 4.4.1. The bend predictions are explained and then contrasted with those obtained from the Tvergaard bend model.

5.3.1 Power Law Behaviour

In these simulations, the material stress-strain response for the specimen was defined by the power law behaviour of Equation 4.16. Equation 4.16 was chosen over experimental AA6111 stress-strain data for the preliminary investigations given the better

understanding of this implementation, based on the work of Tvergaard [111, 112] and the parametric study conducted in Section 5.2.1. Furthermore, the models remain stable under fully kinematic conditions which facilitates a more thorough study of the behaviour under the full range of hardening schemes. The specimen mesh was refined in the area of the mandrel to obtain 32 elements through the sheet thickness. The element aspect ratio in the refined area is identical to that used in Section 5.2.1 (ie. $h_e:l_e = 1.7:1$) to enable direct comparison with the trends outlined therein.

Figures 5.37 gives fringe plots of effective plastic strain, at different bend angles, for the model under von Mises conditions. The location of maximum strain can be seen to advance around the outer surface of the specimen. This is further illustrated by the graph of Figure 5.38 which plots the strain distribution along a section of the outer surface at various bend angles. It is worth noting that the predicted impingement of the mandrel into the underside of the sheet, as seen in Figure 5.37, is reflected in the experimental specimens (see Section 5.1.4).

Figure 5.39 presents force versus bend angle predictions for the von Mises case above. The noise evident in these curves is a result of dynamic effects in the FEM model. A sinusoidal velocity, as discussed in Section 4.1, is applied to the rotating link in order to move the roller. While the link velocity is increasing, the roller is kept in constant contact with the specimen. Once the link velocity starts to decrease, however, there is nothing to restrain the deformation of the specimen. Inertia causes the specimen to continue bending even though it is no longer in contact with the roller. As the specimen slows, it is impacted once the roller catches up. This pattern repeats itself during the deceleration of the roller and results in the spikes witnessed in the latter half of the force

values. Further noise is caused by the impingement of the specimen between the roller and mandrel.

The kinematic hardening GTN material parameters described in Table 4.3 were applied to the full cantilever bend test model for three values of b (ie. 0, 0.5, 1). Predictions were performed for both a model with an initial upper surface imperfection, defined by Equation 4.13 with $h_o/l_o = 10.2$ and $\delta_o/l_o = 0.005$, as well as a model without an initial imperfection. Fracture is predicted in all of these models at roughly the same time, irrespective of the value of the hardening parameter, b , or the presence of an initial imperfection. This similarity is evident in the images of Figure 5.40. It is anticipated that kinematic hardening would develop a shear band more quickly based on its increased sensitivity to shearing, an effect which would be further heightened by the presence of an imperfection. On the other hand, an isotropic hardening model with no imperfection is expected to initiate a shear band much later. However, the fact that this trend is not seen in Figure 5.40 suggests that the evolution of void damage is dominated by void growth in these particular results. In the Tvergaard bend model, described in Section 4.3, the block is allowed to thin dramatically during bending because there are no displacement restrictions in the through-thickness (ie. vertical) direction. In the cantilever bend model, however, thinning of the bent portion of the sheet is restrained by the adjacent undeformed bulk of material. When the sheet is not allowed to thin, the through-thickness stress level is raised, resulting in a higher level of stress triaxiality. Void growth is then greatly enhanced, due to its exponential dependence on stress triaxiality, such that growth dominates the damage behaviour regardless of the hardening regime or the presence of geometric irregularities.

To substantiate the above hypothesis, additional models were run so that void nucleation was dispersed over a larger strain range, by increasing ϵ_n from 0.3 to 0.6 and s_n from 0.1 to 0.2. While Figure 5.41 does exhibit a greater variation in the predicted behaviour, the differences are relatively modest when compared to those observed in Section 5.2.1.

Based on the work of Akeret [19], it is known that shear bands should develop from the valleys of surface undulations that develop during bending, similar to the sample shown in Figure 5.42. By exaggerating the shape of the imposed sinusoidal upper surface imperfection, the same behaviour is predicted by the FEM models. While a familiar periodic pattern of interlocking shear bands develops with accumulating strains (see Figure 5.43), a crack is ultimately initiated at one of these locations, as depicted in Figure 5.44.

5.3.2 Cantilever Bend Model versus the Tvergaard Bend Model

There are a few important differences between the Tvergaard bend model examined in Section 5.2.1 and the cantilever bend model discussed above. These differences are crucial to understanding why predictions of the cantilever bend test appear less sensitive to the selection of material parameters.

As discussed in the preceding subsection, one fundamental reason for the behavioural differences is the change from a stress state where void development is dominated by nucleation to one in which void development is dominated by growth. In Figure 5.20, the hardening scheme (ie. selection of b) was shown to be less important at high porosities. Therefore, in cases where void growth is rapid, it is not surprising that these same trends are revealed.

The movement of the region of maximum strain at the upper surface is an added complexity. In the Tvergaard bend model, the bend angles remained small, and a well-defined deformation gradient was imposed through the thickness of the sheet. The upper surface also had only one viable location from which localization could initiate. The cantilever bend model, however, has multiple imperfections and a region of maximum effective plastic strain that moves along the outer surface of the specimen as bending proceeds, such that favorable conditions for localization occur at different specimen locations over the course of the simulation.

Finally, the well-defined strain gradient of the Tvergaard bend test is no longer present, due to the more complex geometry of the cantilever bend test. The impingement of the mandrel into the specimen, for example, results in an area of very high compressive strain. The fringe plot of effective plastic strain in Figure 5.45 illustrates how the regions of intense strain from the upper and lower surfaces interact. The crack eventually follows that interaction path. Since the strain around the mandrel develops first, it may influence the development of the upper surface strains.

All of these factors indicate that the numerical modelling of the cantilever bend test represents a large increase in complexity over the Tvergaard bend model. More research is needed to identify and rationalize the influence of these additional factors on the FEM predictions.

Chapter 6

Discussion

Several issues have arisen from the results presented in this thesis. As such, additional discussion is warranted on the following topics:

- experimental results;
- performance of the cantilever bend test apparatus;
- numerical results;
- use of a continuum FEM approach to simulate shear band localization;
- performance of the kinematic GTN subroutine.

6.1 Experimental Results

One of the primary objectives of this research was to study the effect of Fe content on AA6111 sheet bendability. The experimental results for the AA6111 Fe-variants indicate that bendability is not a simple function of Fe level. Rather, at least two competing

factors appear to be at work. Grain size refinement due to increased Fe content acts to improve bending performance, but higher Fe levels also produce a greater number of larger second-phase particles which act as nucleation sites for void damage. The fact that a certain level of Fe is tolerable is of particular interest with respect to recycling. Fe pick-up from scrap alloys can lead to unacceptably high levels of Fe [160] that become increasingly expensive to reduce during the recycling process.

Sarkar *et al.* [160] have also studied the effect of Fe content on bendability in AA5754 Fe-variants. They report that minimum R/t performance in the low Fe alloy is superior to that of the higher Fe alloy. Unfortunately, only alloys with 0.08% and 0.30% Fe were studied, which makes it impossible to determine whether or not AA5754 Fe-variants display better bendability at intermediate levels of Fe, similar to the findings of the current work.

While iron content and grain size have been considered, there are a number of other factors that might influence bendability which have not been addressed. Other damage mechanisms, such as intergranular fracture, have been shown to greatly influence bending behaviour [24]. Furthermore, the effect of processing variables on bendability has not been addressed. Pilkey [162] has demonstrated that the nature of second-phase particle clustering is a very significant factor in the development of void damage. It may be possible, for example, to alter the casting, rolling, or heat treatment processes such that the second-phase particles are less clustered at higher Fe levels, in order to limit damage and improve manufacturing performance. For example, Embury [24] refers to the dramatic effect that changes in quench rate can have on bendability.

Furthermore, additional metallographic examination of deformed specimens could be used to obtain greater insight into the mechanisms that result in shear band development

and void growth. The effect of grain rotation at the upper surface on the evolution of surface roughness is of particular interest, including the influence of different grain sizes within the AA6111 Fe-variants. The role that such geometric changes play in shear band initiation could then be better understood. Lloyd [165] indicates that in heat treatable alloys, such as AA6111, dislocations can shear particles and cause preferred channels of deformation which ultimately lead to shear bands. These events could also be investigated more closely.

6.2 Performance of the Cantilever Bend Test Apparatus

The repeatability of results from the cantilever bend test has been shown to be highly dependent on material homogeneity. This is because of the localized nature of a bend test. In a tensile test, the entire gauge length is under the same nominal stress state and the results obtained prior to necking are an average of the entire cross-section of the specimen geometry. Conversely, in a bend test a very localized stress gradient is imposed so that only a specific portion of the material is being highly strained. Since failure begins at the outer tensile surface and develops inward, it is only the behaviour of a narrow region of material that determines the test result. All of these factors suggest that a greater degree of variability is to be expected in a bend test, as compared to a tensile test, and that this variability will be exacerbated by material inhomogeneity, as expected with the laboratory-scale produced AA6111 Fe-variants.

A few modifications to the test protocol are suggested for future work involving the cantilever bend test. Differences in blank width are inevitable, particularly if they have

been sheared to only nominal dimensions, as they have been in this work. These variations in geometry are not currently accounted for when presenting the data. It is suggested that future blanks be machined, in a manner similar to tensile specimens, to obtain more consistent dimensions amongst samples. Additionally, the thickness and width of the specimen should be recorded in the blank bend area, prior to insertion into the bend test apparatus. The force values obtained during the experiment should then be normalized with respect to the initial area of the blank, A_o , to account for any geometric differences.

6.3 Numerical results

Using the kinematic hardening GTN equations, it has been shown that strain localization is highly dependent upon yield surface curvature. At low porosity levels, the fully kinematic hardening yield surface is more prone to localization due to its greater curvature. At higher porosity levels, greater constriction of the yield surface results in less difference between the curvature of isotropic versus kinematic hardening surfaces, such that the deformation behaviour is more comparable.

Localization is also highly dependent on stress state. Under low triaxiality, void development is dominated by nucleation, whereas under high triaxiality it is dominated by void growth. Similar trends have been noted by Saje *et al.* [103] for three-dimensional bifurcation predictions.

As a final point, strain localization is facilitated by a large porosity gradient between elements outside and inside the shear band. Such gradients could develop, for example, from normal void nucleation distribution functions with very small standard deviations. This is in keeping with bifurcation predictions by a number of authors [102, 106, 166]

who have noted that localization is highly dependent upon the difference in void volume fractions, Δf , between the material inside and outside the shear band.

The boundary conditions under which the Tvergaard bend model simulations were performed did not impose the strong void growth component present in the cantilever bend test model, making it difficult to apply trends directly from the former model to the latter. Therefore, the parametric studies of the various material parameters should be repeated with boundary conditions chosen to prevent excessive material thinning in the Tvergaard bend model. These predictions could then form a basis for the selection of material parameters for use in the cantilever bend model.

The geometry of the cantilever bend test model has also been shown to produce more complex stress states. Specifically, the impingement of the mandrel into the underside of the sheet introduces an area of very intense plastic strain. It remains unclear precisely what effect this may be having, if any, on the evolution of strain at the upper surface. The interaction of these two high strain areas will also influence the path of shear crack development. Further investigation is needed to determine the significance of this impingement.

6.4 Use of a Continuum FEM Approach to Simulate Shear Band Localization

Admittedly, there are a number of limitations when trying to capture a highly localized event, such as the development of a shear band, using a continuum approach. In particular, important factors such as surface roughness evolution through grain rotation,

intergranular fracture, dislocation movement and other microstructural processes are not considered.

It is also recognized that the numerical predictions are extremely mesh dependent. As summarized by many authors [130, 131, 142, 143], the size of the shear band is limited to the size of the smallest element. Instead, Tvergaard and Needleman [142] have proposed that the characteristic material length-scale for shear band formation be related to void radius. Furthermore, the mesh must often be designed specifically to accommodate the shear band, meaning the behaviour must be known *a priori*. It is suggested that a bifurcation analysis be performed for the AA6111 material to improve mesh designs in future studies.

Given the presence of mesh dependencies, the use of a continuum approach for such a problem may be questioned. It could be argued that another predictive approach, such as crystal plasticity, might be more suitable for modelling strain localization. However, for practical reasons, crystal plasticity calculations are limited to relatively small problems due to their detailed nature. Hence, a crystal plasticity approach would not be suitable for numerical predictions involving a full scale model of an automotive hemming procedure. In order to help reduce mesh dependence, authors such as Tvergaard and Needleman [142] have developed non-local formulations which incorporate a specified material length scale directly into the constitutive equations. It has been found that a local approach provides a good approximation during the early stages of void growth, whereas a non-local formulation is needed to properly account for the localization which leads to void coalescence.

6.5 Performance of the subroutine

The most important numerical contribution in this work has been the reformulation of the kinematic GTN equations and their incorporation into the subroutine developed by Worswick and Pelletier [145] to allow predictions well beyond the plastic strain levels that can be attained using forward integration. The subroutine, as implemented in this thesis, displays excellent agreement with isotropic predictions under two-dimensional plane strain conditions. The source of the disagreement that becomes more prevalent in three-dimensional models has yet to be identified; however, the trends predicted under both schemes are comparable.

The fully kinematic hardening condition is unstable when used with high hardening materials such as AA6111. This not believed to be a result of the subroutine itself, but rather an instability stemming from the method of element deletion. Prior to element deletion the results are stable, but once the first element is deleted the mesh rapidly becomes unstable. It has been suggested that, at high hardening rates, the sudden loss of stiffness resulting from element deletion causes this instability [164]. A proposed solution involves the gradual reduction of element strength over a number of time steps prior to its deletion from the mesh. The validity of this proposal is currently being investigated.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

The following are the major conclusions that have resulted from this thesis:

- Bendability has been found to be a function of two competing effects. Increased Fe content results in smaller grain sizes which improves bendability. However, greater amounts of Fe also result in more and larger second-phase particles which induce more damage and ultimately reduce bendability.
- The kinematic hardening GTN equations exhibit increased sensitivity to the development of shear bands in comparison to an isotropic GTN formulation.
- The predicted formation of a shear band is highly dependent upon the void volume fraction of nucleating particles, f_n , the strain-controlled nucleation parameters, ϵ_N and s_n , the element size and aspect ratio, as well as the level of stress triaxiality.

7.2 Future Work

It is recommended that future research programs investigate the following areas:

- Experiments should be designed to assess the degree to which grain size and the spatial and size distribution of second-phase particles affect the bendability. A matrix of experiments, using different heat treatments at various Fe content levels, for example, would allow these parameters to be varied such that their relative effects may be better understood.
- A number of other factors that may be influencing bendability remain to be investigated, including:
 - intergranular fracture;
 - particle shearing;
 - the evolution of surface roughness at the outer surface.
- The kinematic GTN LS-DYNA user material subroutine should be reviewed to:
 - establish the source of the disagreement between isotropic and kinematic hardening predictions under non-reversed proportional loading;
 - improve its stability to enable high-hardening materials, such as AA6111, to be modelled stably under fully kinematic conditions;
 - expand the implementation to include stress-controlled void nucleation.
- The sensitivity of the FEM simulations to element aspect ratio should be considered by performing a bifurcation analysis that models material behaviour using the AA6111 stress-strain data. These predictions might also be compared with shear

bands observed experimentally to determine an appropriate element aspect ratio for incorporation into cantilever bend test FEM models.

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Table 2.1: Summary of pure bending theories.

Author	Comparison with Experiment	Anisotropy	Neutral Axis Shift	Thickness Change	Bauschinger Effect	Year of Publication	Reference
Hill	x	x	✓	x	x	1950	[60]
Lubahn	✓	x	✓	✓	x	1950	[59]
Denton	x	x	x	x	x	1966	[61]
Verguts	x	x	✓	✓	x	1975	[31]
	x	x	✓	✓	✓		
Dadras	✓	x	✓	✓	✓	1982	[62]
	✓	x	✓	✓	✓		
Wang	✓	✓	✓	x	x	1993	[53]
Tan	✓	✓	✓	✓	x	1995	[32]
	✓	✓	✓	✓	✓		
Perduijn	✓	x	x	x	x	1995	[33]
	✓	x	x	x	x		
	✓	x	x	✓	x		
Leu	x	✓	✓	x	x	1997	[56]

Table 2.2: Summary of springback and residual stress theories.

Author	Comparison with Experiment	Anisotropy	Neutral Axis Shift	Thickness Change	Bauschinger Effect	Year of Publication	Reference
Schroeder	✓	×	×	×	×	1943	[78]
Gardiner	✓	×	×	×	×	1957	[79]
Queener	✓	×	×	×	×	1968	[46]
Rolf	✓	×	×	×	×	1978	[42]
Sidebottom	✓	×	×	×	×	1979	[80]
Raghupathi	✓	×	×	×	×	1983	[83]
El-Megharbel	×	×	×	×	×	1990	[85]
Yuen	✓	×	×	×	×	1990	[86]
Wang	✓	×	×	×	×	1993	[53, 54]
Zhang	✓	✓	×	✓	×	1995	[82]
Zeng	✓	✓	×	×	×	1995	[88]
Leu	×	✓	✓	×	×	1997	[56]
Zhang	×	×	×	×	×	1997	[84]
Zhang	×	×	×	×	×	1998	[81]
Xue	×	×	×	×	×	1999	[87]

Table 2.3: Summary of FEM press brake theories.

Author	Comparison with Experiment	Punch Load	Springback	Stress and Strain Distribution	Friction	Year of Publication	Reference
Oh	✓	✓	✓	✓	✗	1980	[116]
Makinouchi	✓	✗	✓	✓	✗	1985	[117]
Huang	✓	✓	✓	✓	✗	1992	[121]
Schilling	✓	✓	✓	✓	✓	1993	[49]
Huang	✗	✓	✓	✗	✓	1995	[125]
Huang	✓	✓	✗	✗	✓	1995	[119]
Huang	✗	✓	✓	✗	✓	1995	[118]
Forcellese	✓	✓	✓	✗	✓	1995	[120]
von Finckenstein	✓	✓	✗	✗	✗	1996	[122]
Forcellese	✓	✗	✓	✓	✗	1996	[48]
Nilsson	✓	✗	✓	✗	✗	1997	[123]
Huang	✓	✓	✓	✗	✓	1998	[124]

Table 3.1: Nominal composition of AA6111, as given by Fortin *et al.* [9], with the balance made up of Al.

Alloy	Si	Fe	Cu	Mn	Mg
6111	0.85	0.20	0.75	0.20	0.72

Table 3.2: Composition of the AA6111 Fe-variants with the balance made up of Al.

Alloy	Si	Cu	Mn	Mg	Ti	B
0.06% Fe	0.66	0.69	0.04	0.70	0.013	0.003
0.26% Fe	0.65	0.69	0.04	0.69	0.019	0.004
0.35% Fe	0.67	0.71	0.04	0.68	0.023	0.004
0.66% Fe	0.66	0.71	0.04	0.72	0.018	0.003

Table 4.1: Material properties for the von Mises kinematic hardening test case.

Parameter	Value
K (GPa)	46.7
G (GPa)	28.02
ρ (g/cm ³)	2.64
σ_y (MPa)	127.4
E_t (MPa)	1000

Table 4.2: Material properties used for the kinematic hardening GTN test cases.

Parameter	Value
K (GPa)	170.1
G (GPa)	78.5
ρ (g/cm ³)	7.89
σ_y (MPa)	680.3
N	0.1
q_1	1.5
q_2	1.0
q_3	$q_1^2 = 2.25$
f_N	0.10
ϵ_N	0.30
S_N	0.04
f_0	0.00/0.04
f_c	0.15
f_f	0.25

Table 4.3: Standard GTN material parameters used with the Tvergaard bend model.

Parameter	Value
σ_y (MPa)	127
N	10
q_1	1.5
q_2	1.0
q_3	$q_1^2 = 2.25$
f_n	0.04
ϵ_n	0.30
s_n	0.10
f_0	0.00
f_c	0.15
f_f	0.25

Table 4.4: Material properties, corresponding to steel, that were used for the rigid bodies in all of the FEM models.

Parameter	Value
E (GPa)	280.3
ν	0.3
ρ (g/cm ³)	7.7

Table 5.1: Average grain size measurements for the four AA6111 Fe-variants measured in the sheet rolling direction.

Alloy	Grain Size (μm)
0.06% Fe	113
0.26% Fe	47
0.35% Fe	42
0.68% Fe	26

Table 5.2: Results of minimum R/t measurements made using three AA6111 Fe-variants, that were solutionized at 560°C for 3 minutes, FAQ, and then naturally aged. Values are reported at various aging times.

Natural Ageing Period (h)	0.06% Fe	0.26% Fe	0.68% Fe
1	0.00	0.00	0.00
8	0.00	0.00	0.15
24	0.34	0.25	0.39
48	0.34	0.25	0.39
96	0.39	0.25	0.39
144	0.49	0.40	0.49

Table 5.3: Particle measures obtained from microstrutural evaluation of the 0.26% and 0.68% AA6111 Fe-variants.

Alloy	Direction	# of particles per mm ²	particle area fraction	# of first-order clusters per mm ²	effective* first-order cluster area fraction
0.26% Fe	long.	2361	0.005	235	0.007
	planar	1759	0.005	223	0.008
0.68% Fe	long.	4302	0.014	637	0.026
	planar	4155	0.013	761	0.035
	trans.	4290	0.010	648	0.023

* where effective first-order cluster area includes the area of the particles and the area of the matrix material within the cluster

Figure 1.1: Material makeup of a typical family vehicle in percentage of total weight and in total lbs. [6].

Figure 1.2: Aluminum content by form in 1999 U.S. passenger cars and light trucks [167].

(a) Audi A2 Space Frame [168]

(b) Audi A8 [169]

Figure 1.3: Examples of automobiles incorporating above average aluminum content.

Figure 1.4: Two schematic examples of a hem.

(a) successful bend

(b) failed bend

Figure 1.5: SEM micrographs of a successful and a failed bend [170].

Figure 2.1: Differences between pure bending, press brake bending and wrap bending [29].

(a) V-die

(b) U-die

Figure 2.2: Examples of U- and V-shaped press brake dies [120].

(a) stretch bending [69]

(b) UO-bending

Figure 2.3: Stretch bending and UO-bending.

(a) air bending

(b) closed-die bending

Figure 2.4: Air bending and closed-die bending [171].

(a) wrap bending

(b) wipe bending

Figure 2.5: Wrap and wipe bending [29].

Figure 2.6: Stages of bend failure as described by Akeret [19].

Figure 2.7: Effects of carbon equivalent and tensile strength on bendability of ultra-high-strength steel sheet [27].

(a) principle of the bending test

(b) bending apparatus

Figure 2.8: Bending moment apparatus as described by Tozawa *et al.* [35].

Figure 2.9: Pure bending apparatus as described by Perduijn and Hoogenboom [33].

Fig. 1. The bend test device.

Figure 2.10: Pure bending machine as described by Duncan *et al.* [36].

Figure 2.11: Pure bending machine as described by Marciniak and Duncan [38].

Figure 2.12: Tensile unbending test as described by Yoshida *et al.* [40].

Figure 2.14: Alcan KRDC cantilever bend test apparatus.

Figure 2.15: Example of Bauschinger effect in bending as given by Prasad and Somasundaram [73].

(a) two-dimensional isotropic and kinematic hardening yield surfaces

(b) differences in plastic increment direction. $d\epsilon_{ij}^p$, for a given stress increment. $d\sigma_{ij}$

Figure 2.16: Increased sensitivity of the kinematic hardening yield surface to changes in loading direction compared to an isotropic hardening yield surface.

Figure 2.17: Two-dimensional bifurcation model with an assumed initial material imperfection.

Figure 2.18: Effect of porosity on the GTN yield surface with $q_i=1$, illustrated in two dimensions.

Figure 2.19: Effect of porosity on the GTN yield surface with $q_i=1$, illustrated in three dimensions.

Figure 2.20: Illustration of the differences between isotropic and kinematic hardening.

Figure 3.1: Schematic of the cantilever bend test apparatus commissioned by Alcan KRDC.

Figure 3.2: Experimental set-up for the cantilever bend test apparatus.

Figure 3.3: Cantilever bend test apparatus shown mounted on a tensile test frame.

Figure 3.4: Cantilever bend test being performed.

Figure 3.5: Example of load versus displacement data obtained from the cantilever bend test. using AA6111-T4PB cut transverse to the rolling direction.

Figure 3.6: Setting of gap height and zeroing procedure used for the cantilever bend test.

Figure 3.7: Repeatability of the cantilever bend test results, using AA6111-T4PB cut transverse to the rolling direction.

Figure 3.8: Definition of bend angle used in this thesis.

Figure 3.9: Geometric parameters needed to determine the minimum bend angle, θ_{min} , and to convert linear crosshead displacement data to bend angles.

(a) force-displacement uniaxial tensile data

(b) true stress-true strain data with no Bridgman corrections

Figure 3.10: Uniaxial tensile test data for three AA6111 Fe-variants, tested transverse to the rolling direction.

Figure 4.1: Boundary conditions for the single element test cases.

Figure 4.2: Comparison of the kinematic hardening GTN subroutine predictions with those obtained using the linear kinematic plasticity material model in LS-DYNA.

(a) stress

(b) porosity

Figure 4.3: Comparison of isotropic and kinematic hardening predictions for a single 3D element subjected to uniaxial tension.

(a) stress

(b) porosity

Figure 4.4: Comparison of isotropic and kinematic hardening predictions for a single 3D element subjected to volumetric expansion.

(a) stress

(b) porosity

Figure 4.5: Comparison of isotropic and kinematic hardening predictions for a single 3D element subjected to constrained uniaxial tension.

Figure 4.6: Equivalent and hydrostatic components of the stress paths for the isotropic hardening 3D single element test cases.

(a) stress

(b) porosity

Figure 4.7: Comparison of isotropic and kinematic hardening predictions for a single 2D plane strain element subjected to uniaxial tension.

(a) stress

(b) porosity

Figure 4.8: Comparison of isotropic and kinematic hardening predictions for a single 2D plane strain element subjected to biaxial tension.

(a) stress

(b) porosity

Figure 4.9: Comparison of isotropic and kinematic hardening predictions for a single 2D plane strain element subjected to constrained uniaxial tension.

(a) stress

(b) porosity

Figure 4.10: Comparison of isotropic and kinematic hardening predictions for a single 2D plane strain element subjected to pure shear.

Figure 4.11: Equivalent and hydrostatic components of the stress paths for the isotropic hardening 2D single element test cases.

Figure 4.12: Geometry of the Tvergaard bend model.

Figure 4.13: Slider mechanism used to impose rotational boundary conditions in the LS-DYNA implementation of the Tvergaard bend model.

(a) geometry of the cantilever bend test model

(b) boundary conditions of the specimen

(c) location of the revolute joints

Figure 4.14: Simplified geometry of the Alcan KRDC cantilever bend test apparatus.

Figure 4.15: Extracting force versus displacement information from the FEM model of the cantilever bend test.

(a) L direction

(b) T direction

Figure 5.1: Force versus bend angle results for the 0.06% AA6111 Fe-variant.

(a) L direction

(b) T direction

Figure 5.2: Force versus bend angle results for the 0.26% AA6111 Fe-variant.

(a) L direction

(b) T direction

Figure 5.3: Force versus bend angle results for the 0.35% AA6111 Fe-variant.

(a) L direction

(b) T direction

Figure 5.4: Force versus bend angle results for the 0.68% AA6111 Fe-variant.

Figure 5.5: Comparison of grain sizes for the different AA6111 Fe-variants, where RD indicates the rolling direction and ST indicate the short transverse, or through-thickness, direction.

(a) specimen 5, $\theta = 54.3^\circ$

(b) specimen 8, $\theta = 73.2^\circ$

(c) specimen 7, $\theta = 84.7^\circ$

Figure 5.6: Evolution of surface roughness in the 0.26% L AA6111 Fe-variant.

(a) specimen 3, $\theta = 114.7^\circ$, $F = 7.32$ lb

(b) specimen 2, $\theta = 101.1^\circ$, $F = 7.30$ lb

(c) specimen 4, $\theta = 92.9^\circ$, $F = 6.42$ lb

Figure 5.7: Load drop differences resulting from crack formation in the 0.26% T AA6111 Fe-variant.

Figure 5.8: Frequencies of inter-particle dilational spacings obtained from two AA6111 Fe-variants measured in the longitudinal direction.

Figure 5.9: Material removed for specimen sectioning.

Figure 5.10: Area covered by the matrix of micrographic images taken using the deformed bend specimens.

Figure 5.11: Low magnification image of a 0.26% Fe specimen. bent to $\theta=84.7^\circ$. highlighting the region of interest.

Figure 5.12: High magnification image of a 0.26% Fe specimen, bent to $\theta=87.4^\circ$, showing minimal void damage.

Figure 5.13: Low magnification image of a 0.68% Fe specimen, bent to $\theta=95.4^\circ$, highlighting the region of interest.

Figure 5.14: High magnification image of a 0.68% Fe specimen, bent to $\theta=95.4^\circ$, showing the development of voids along a shear band prior to crack initiation.

(a) specimen 3, $\theta = 67.1^\circ$, $l_o/h_o \approx 0.10$

(b) specimen 2, $\theta = 79.5^\circ$, $l_o/h_o \approx 0.25$

(c) specimen 4, $\theta = 114.7^\circ$, $l_o/h_o \approx 0.40$

Figure 5.15: Low magnification images of 0.26% T Fe alloy. with estimates of the surface imperfection wavelength normalized to sheet thickness. l_o/h_o .

Figure 5.16: Extraction of force from the Tvergaard bend model.

(a) downward crack (b) diagonal crack

Figure 5.17: Different crack shapes resulting from element deletion.

Figure 5.18: Effect of b on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

Figure 5.19: Fringe plots of porosity, at a common bend angle of 6° , showing the effect of b on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.20: Effect of void volume fraction of nucleating particles, f_n , on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.21: Effect of void nucleation parameters ϵ_n and s_n on a Tvergaard model using parameters $N=10.0$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.22: Effect of void nucleation parameter ϵ_n , with a constant standard deviation of $s_n=0.05$, on a Tvergaard model using parameters $N=10.0$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.23: Effect of standard deviation, s_n , with a constant average strain of $\epsilon_n=0.3$, on a Tvergaard model using parameters $N=10.0$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

Figure 5.24: Different hardening rate at $\bar{\epsilon}^p = 0.15$, 0.3 and 0.45 for a power law with $\sigma_y/E = 0.0033$, $\sigma_y = 127$ MPa and $N = 10$.

Figure 5.25: Fringe plots of void volume fraction of nucleated particles for different values of ϵ_n , on a Tvergaard model using kinematic hardening and the parameters $N=10.0$, $s_n=0.05$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

Figure 5.26: Stress-strain behaviour for three N values, using a power law with $\sigma_y/E=0.0033$ and $\sigma_y=127$ MPa.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.27: Effect of hardening exponent on. N , on a Tvergaard model using parameters $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.28: Effect of coalescence and failure porosities, f_c and f_f , on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.29: Effect of failure porosity, f_f , with a constant value of coalescence, $f_c=0.05$, on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.30: Effect of coalescence porosity, f_c , with a constant value of failure, $f_f=0.20$, on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.31: Effect of element size on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$. The size has been varied by changing the number of elements in the height, N_h and width, N_l .

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.32: Effect of element aspect ratio, $h_e:l_e$, on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

(a) fully isotropic $b=1.0$ (b) fully kinematic $b=0.0$

Figure 5.33: Effect of the magnitude of the initial surface imperfection, δ_o/l_o , on a Tvergaard model using parameters $N=10.0$, $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.04$, $f_o=0.0$, $f_c=0.15$ and $f_f=0.25$.

Figure 5.34: Predictions from Tvergaard bend model simulations using AA6111 stress-strain data with $f_n=0.0069$ for combinations of ϵ_n & s_n and f_c & f_f .

(a) $\epsilon_n = 0.09$, $s_n = 0.03$ (b) $\epsilon_n = 0.15$, $s_n = 0.05$ (c) $\epsilon_n = 0.30$, $s_n = 0.10$ (d) $\epsilon_n = 0.45$, $s_n = 0.15$ (e) $\epsilon_n = 0.60$, $s_n = 0.20$

Figure 5.35: Predictions from Tvergaard bend model simulations using AA6111 stress-strain data with $f_n=0.0252$ for combinations of ϵ_n & s_n and f_c & f_f .

Figure 5.36: Example of the crack shape predicted from simulations using the AA6111 stress-strain data with parameters $\epsilon_n=0.3$, $s_n=0.1$, $f_n=0.0252$, $f_o=0.0$, $f_c=0.10$ and $f_f=0.20$.

Figure 5.37: Fringe plots of effective plastic strain at various bend angles for an isotropic von Mises material exhibiting power law behaviour with $N=10$.

(a) limits of the surface area to be examined

(b) effective plastic strain distribution along surface

Figure 5.38: Distribution of effective plastic plastic strain, for the selected location, at various bend angles. An isotropic von Mises material with power law hardening behaviour ($N=10$) was used.

Figure 5.39: Force versus bend angle information extracted from the isotropic hardening von Mises material with power law hardening behaviour ($N=10$).

(a) isotropic hardening ($b=1$). with no imperfection

(b) kinematic hardening ($b=0$). with an imperfection

Figure 5.40: Comparison of cantilever bend predictions at the same bend angle, with $\epsilon_n=0.3$ and $s_n=0.1$.

(a) isotropic hardening ($b=1$). with no imperfection

(b) kinematic hardening ($b=0$). with an imperfection

Figure 5.41: Comparison of cantilever bend predictions at the same bend angle. with $\epsilon_n=0.6$ and $s_n=0.2$.

Figure 5.42: Formation of shear bands from irregularities at the outer surface of a AA5754 cantilever bend test specimen [172].

Figure 5.43: Fringe plot of porosity for a cantilever bend test model, with an initial imperfection of $\delta_o/l_0=0.05$, at a bend angle of $\theta=37.1^\circ$.

Figure 5.44: Fringe plot of porosity for a cantilever bend test model, with an initial imperfection of $\delta_o/l_0 = 0.05$, at a bend angle of $\theta = 43.1^\circ$.

Figure 5.45: Interaction of the regions of high effective plastic strain at the upper and lower surfaces, at any angle of $\theta = 33.6^\circ$.