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**SEISMIC ANALYSIS OF
GRAVITY DAM-RESERVOIR-FOUNDATION SYSTEM
USING AN EFFECTIVE HYBRID TECHNIQUE**

by

RAJAGOPALAN CHANDRASHAKER

B. Eng. - Bangalore University, India

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**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy**

May 1992

**Department of Civil Engineering
Faculty of Engineering
Carleton University
Ottawa, Canada**

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submitted by
Rajagopalan Chandrashaker
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Engineering

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ABSTRACT

The seismic response of a dam is strongly influenced by its interaction with the water reservoir and the foundation. The hydrodynamic forces in the reservoir are in turn affected by (1) radiation of waves towards infinity; (2) wave absorption at the reservoir bottom; (3) cross coupling between the foundation below the dam and the reservoir bottom. An efficient technique to compute the seismic response of a gravity dam, including the various interaction components resulting from the impounded reservoir and the underlying foundation is presented. The technique developed in this work is computationally efficient and imposes few restrictions on the geometry of the system analyzed. Analysis is carried out in the frequency domain and a substructure approach is used. A combination of finite element method (FFM) and boundary element method (BEM) is used in modeling the arbitrary shaped individual substructures. However, the BEM provides certain advantages in modeling domains of infinite extent, and in reducing the amount of data input.

The infinite reservoir substructure is divided into two parts: a near field of specified irregular geometry modeled by boundary elements and an infinite far field of uniform cross section modeled by 1D finite elements. The dam is discretized by finite elements. The foundation soil is idealized as an elastic/viscoelastic half plane and modeled by boundary elements. To enhance the computational efficiency of the boundary element procedure, a frequency independent fundamental solution is adopted for the reservoir analysis. The interpolation functions used for the approximate evaluation of the inertia integral terms in the frequency independent BEM model of the reservoir are examined. Also, the optimum number of interior nodes and their locations to be used for this purpose are studied. Both finite and infinite reservoir cases are considered. The equations of motion for the dam are obtained and then transformed to the first few modal coordinates of the dam-foundation system.

Preliminary analyses are carried out to demonstrate the effect on dam response of various interaction components: reservoir hydrodynamic forces, reservoir bottom absorption, foundation flexibility, and of material damping in the viscoelastic foundation halfplane.

The fluid-foundation interaction effect, i.e. the wave absorption along the reservoir bottom can be accounted for by using either an approximate 1D wave propagation model

or a rigorous analysis of interaction between the flexible soil along the reservoir base and the reservoir water. The rigorous approach requires enormous computational effort because of (a) cross coupling between the foundation of the dam and the soil below the reservoir, and (b) frequency dependence of the boundary condition along the fluid-foundation interface. The analysis can be simplified by neglecting the cross coupling and by using the approximate 1D wave propagation model. The effects of each of these two simplifications on the accuracy and computational efficiency of the procedure used for the response analysis of a dam subjected to random ground motion are closely investigated. The complex frequency response functions of an idealized triangular dam and for Pine Flat dam system are obtained. A response time history analysis is carried out for the Taft and ElCentro ground motions. The crest displacement of the dam and the maximum principal stress contours are studied.

A study is carried out to determine the extent of the infinite reservoir to be considered as irregular, while computing the response of a dam impounding infinite reservoir with sloped bottom. Also, the impact of varying reservoir geometry on the response of the dam is examined.

The results from the analytical models presented here are compared with classical solutions where available and with results obtained by other researchers.

To my teachers, with gratitude

and

To my parents, with affection

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TABLE OF CONTENTS

Abstract	iii
Acknowledgements	vi
Table of Contents	vii
List of Figures	x
List of Tables	xv
1 Introduction	1
1.1 General Remarks	1
1.2 State of the Art	2
1.2.1 <i>Water Compressibility and Dam-Water Interactions</i>	2
1.2.2 <i>Fluid-Foundation Interaction or Wave Absorption along Reservoir Bed</i>	4
1.2.3 <i>Flexibility of Foundation Soil and Foundation Modeling</i>	6
1.2.4 <i>Discretization of Dam and Reservoir Substructures</i>	8
1.2.5 <i>Method of Analysis</i>	10
1.3 Objectives of the Present Research	11
1.4 Layout of the Thesis	13
2. Boundary Element Formulation of Reservoir Model	17
2.1 Introduction	17
2.2 Description of the Model	18
2.3 Governing Equation of Motion of the Near Field	19
2.4 Boundary Conditions	19
2.4.1 <i>Along the Upstream Face of the Rigid Dam</i>	19
2.4.2 <i>Along the Reservoir Bottom: Fluid-Foundation Interaction</i>	20
2.4.3 <i>Along the Far Boundary</i>	21
2.5 Formulation of Boundary Integral Equations	22
2.5.1 <i>Frequency Independent Fundamental Solution</i>	22
2.5.2 <i>Evaluation of Inertia Integral</i>	23
2.5.3 <i>Discretization with Linear Boundary Elements</i>	25
2.5.4 <i>Hydrodynamic Mass Matrix</i>	29
2.5.5 <i>Inertia Interpolation Function</i>	30
2.5.6 <i>Choice of the Interpolation Points</i>	32
2.6 Radiation Condition at Far End of an Infinite Reservoir	32

2.6.1	<i>Concept of Far Field Rigid Reservoir Bottom</i>	33
2.6.2	<i>Boundary Conditions at the Interface of the Near and Far Field</i>	33
2.6.3	<i>Analytical Solution for the Far Field : Vertical Excitation</i>	34
2.7	<i>Final Equation for Reservoir Substructure</i>	35
2.8	<i>Corner Node Problem</i>	36
3.	Analysis of the Reservoir Model	41
3.1	<i>Introduction</i>	41
3.2	<i>Finite Reservoir study</i>	42
3.3	<i>Infinite Reservoir study</i>	43
3.3.1	<i>Reservoir with Non-Absorptive Bottom</i>	44
3.3.2	<i>Reservoir Bottom Absorption Effect</i>	45
4.	Dynamic Stiffness Matrix of Foundation Soil	71
4.1	<i>Introduction</i>	71
4.2	<i>BEM Modeling of Foundation Soil</i>	72
4.3	<i>Governing Differential Equations</i>	73
4.3.1	<i>Boundary Conditions</i>	73
4.4	<i>Boundary Integral Equation</i>	74
4.5	<i>Fundamental Solutions</i>	74
4.5.1	<i>Fundamental Solutions for Elastic Soil Half-plane</i>	75
4.5.2	<i>Fundamental Solutions for Visco-elastic Soil Half-plane</i>	76
4.6	<i>Boundary Element Discretization</i>	77
4.6.1	<i>Evaluation of Boundary Integrals</i>	79
4.6.2	<i>Size of Boundary Elements and Extent of the Mesh Discretization</i>	82
4.7	<i>Evaluation of Soil Impedance Matrix</i>	83
4.8	<i>Verification of BEM Model for Foundation Soil</i>	84
5.	Dam-Reservoir-Foundation Analyses by Simplified Approach	88
5.1	<i>Introduction</i>	88
5.2	<i>Analysis of the Dam Substructure</i>	89
5.3	<i>Equation of Motion of the Coupled System</i>	90
5.3.1	<i>Dam-Reservoir-Foundation System</i>	90
5.3.2	<i>Dam Alone</i>	96
5.3.3	<i>Dam-Reservoir System</i>	97
5.3.4	<i>Dam-Foundation System</i>	97

6. Dam-Reservoir-Foundation Analyses by Rigorous Approach	98
6.1 Introduction	98
6.2 Mathematical Formulations	99
6.3 Analysis of Foundation Substructure	100
6.4 Transformation of Equation Using Ritz Vector	101
6.5 Treatment of Fluid Domain	102
6.5.1 <i>Boundary Conditions</i>	104
7. Response Analysis of Dam-Reservoir-Foundation System	110
7.1 Introduction	110
7.2 Preliminary Study of Idealized Triangular Dam System	111
7.2.1 <i>Dam-Water Interaction and Reservoir Bottom Absorption Effect</i>	114
7.2.2 <i>Effect of Flexibility and Damping in Foundation Soil</i>	115
7.3 Comparison of Models based on Rigorous and Simplified Approach	116
7.3.1 <i>Study of Idealized Triangular Dam System</i>	116
7.3.2 <i>Analysis of Pine Flat Dam System</i>	118
7.4 Response of a Dam Impounding Reservoir with Sloped Bottom	121
7.4.1 <i>Location of Transmitting Boundary</i>	121
7.4.2 <i>Continuum Solution of an Absorptive Far Field: Vertical Excitation</i>	122
7.4.3 <i>Effect of Reservoir Bottom Slope on Response of the Dam</i>	123
8. Summary and Conclusions	158
8.1 Summary of the Research	158
8.2 Conclusions of the Present Study	160
8.3 Recommendations for the Future Work	163
References	165
Notation	174
Appendix A: Finite Element Formulation of Dam Substructure and Evaluation of Element Stress	181
Appendix B: 1D Model for Wave Absorption at Reservoir Bed and Infinite Wave Radiation along Reservoir Far Field	185

LIST OF FIGURES

- Fig. 1.1** Gravity Dam-Reservoir-Foundation System
- Fig. 2.1** Model of the Infinite Reservoir Substructure
- Fig. 2.2** Corner Nodes in an Infinite Rectangular Reservoir; Linear Boundary Element Formulations
- Fig. 3.1** BEM Model for the Reservoir Near Field
- Fig. 3.2** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = H$; $\Omega/\omega_{1res} = 2.5$
- Fig. 3.3** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 2H$; $\Omega/\omega_{1res} = 1.6$
- Fig. 3.4** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 2H$; $\Omega/\omega_{1res} = 2.0$
- Fig. 3.5** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 2H$; $\Omega/\omega_{1res} = 2.5$
- Fig. 3.6** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 3H$; $\Omega/\omega_{1res} = 1.2$
- Fig. 3.7** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 3H$; $\Omega/\omega_{1res} = 1.6$
- Fig. 3.8** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 3H$; $\Omega/\omega_{1res} = 1.85$
- Fig. 3.9** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 3H$; $\Omega/\omega_{1res} = 2.0$
- Fig. 3.10** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 3H$; $\Omega/\omega_{1res} = 2.5$
- Fig. 3.11** Hydrodynamic Pressure on a Rigid Dam; Finite Reservoir; Horizontal Excitation;
 $L = 3H$; $\Omega/\omega_{1res} = 3.8$
- Fig. 3.12a** Real Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 1.2$
- Fig. 3.12b** Imaginary Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 1.2$
- Fig. 3.13a** Real Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 1.6$
- Fig. 3.13b** Imaginary Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 1.6$
- Fig. 3.14a** Real Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 1.85$
- Fig. 3.14b** Imaginary Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 1.85$
- Fig. 3.15a** Real Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 2.0$

- Fig. 3.15b** Imaginary Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 2.0$
- Fig. 3.16a** Real Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 2.5$
- Fig. 3.16b** Imaginary Part of Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Horizontal Excitation; $\Omega/\omega_{1res} = 2.5$
- Fig. 3.17** Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Vertical Excitation; $\Omega/\omega_{1res} = 1.2$
- Fig. 3.18** Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Vertical Excitation; $\Omega/\omega_{1res} = 1.6$
- Fig. 3.19** Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Vertical Excitation; $\Omega/\omega_{1res} = 1.85$
- Fig. 3.20** Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Vertical Excitation; $\Omega/\omega_{1res} = 2.0$
- Fig. 3.21** Hydrodynamic Pressure on a Rigid Dam; Infinite Reservoir; Vertical Excitation; $\Omega/\omega_{1res} = 2.5$
- Fig. 3.22a** Real Part of Hydrodynamic Force on a Rigid Dam; Infinite Reservoir; - Horizontal Excitation; $\alpha_r = 1.0$
- Fig. 3.22b** Imaginary Part of Hydrodynamic Force on a Rigid Dam; Infinite Reservoir; - Horizontal Excitation; $\alpha_r = 1.5$
- Fig. 3.22c** Absolute Part of Hydrodynamic Force on a Rigid Dam; Infinite Reservoir; - Horizontal Excitation; $\alpha_r = 1.0$
- Fig. 3.23a** Real Part Hydrodynamic Force on a Rigid Dam; Infinite Reservoir; - Vertical Excitation; $\alpha_r = 1.0$
- Fig. 3.23b** Absolute Part of Hydrodynamic Force on a Rigid Dam; Infinite Reservoir; - Vertical Excitation; $\alpha_r = 1.0$
- Fig. 3.24a** Convergence Study of Bottom Absorption Effect; - Real Part of Hydrodynamic Force on a Rigid Dam; Horizontal Excitation; $\alpha_r = 0.85$
- Fig. 3.24b** Convergence Study of Bottom Absorption Effect; - Imaginary Part of Hydrodynamic Force on a Rigid Dam; Horizontal Excitation; $\alpha_r = 0.85$
- Fig. 3.24c** Convergence Study of Bottom Absorption Effect; - Absolute Part of Hydrodynamic Force on a Rigid Dam; Horizontal Excitation; $\alpha_r = 0.85$
- Fig. 3.25a** Far Field Treatment; Infinite Reservoir - Real Part of Hydrodynamic Force on a Rigid Dam; Vertical Excitation; $\alpha_r = 0.85$
- Fig. 3.25b** Far Field Treatment; Infinite Reservoir - Imaginary Part of Hydrodynamic Force on a Rigid Dam; Vertical Excitation; $\alpha_r = 0.85$
- Fig. 3.25c** Far Field Treatment ; Infinite Reservoir - Absolute Part of Hydrodynamic Force on a Rigid Dam; Vertical Excitation; $\alpha_r = 0.85$
- Fig. 3.26a** Reservoir Bottom Absorption Effect; Infinite Reservoir - Real Part of Hydrodynamic Force; Horizontal Excitation
- Fig. 3.26b** Reservoir Bottom Absorption Effect; Infinite Reservoir - Imaginary Part of Hydrodynamic Force; Horizontal Excitation
- Fig. 3.26c** Reservoir Bottom Absorption Effect; Infinite Reservoir - Absolute Part of Hydrodynamic Force; Horizontal Excitation

- Fig. 3.27a** Reservoir Bottom Absorption Effect; Infinite Reservoir - Real Part of Hydrodynamic Force; Vertical Excitation
- Fig. 3.27b** Reservoir Bottom Absorption Effect; Infinite Reservoir - Imaginary Part of Hydrodynamic Force; Vertical Excitation
- Fig. 3.27c** Reservoir Bottom Absorption Effect; Infinite Reservoir - Absolute Part of Hydrodynamic Force; Vertical Excitation
- Fig. 4.1** Variation of T_{ij}^* ; ($i \neq j$) at the Point of Singularity
- Fig. 4.2a** Comparison With Classical Solution Rigid Strip Footing; Horizontal Stiffness Coefficient
- Fig. 4.2b** Comparison With Classical Solution Rigid Strip Footing; Vertical Stiffness Coefficient
- Fig. 6.1** Dam-Reservoir-Foundation Model for Analysis by Rigorous Approach
- Fig. 6.2** Distribution of Hydrodynamic Forces Along the Fluid-Foundation Interface
- Fig. 7.1** Cross Sectional Property and Finite Element Model of Idealized Triangular Dam
- Fig. 7.2** Boundary Element Mesh for the Foundation Soil Analysis of Idealized Dam System
- Fig. 7.3** Boundary Element Mesh for Reservoir Substructure Analysis of Idealized Dam System
- Fig. 7.4** Dam-Reservoir Interaction and Reservoir Bottom Absorption; Horizontal Excitation
- Fig. 7.5** Dam-Reservoir Interaction and Reservoir Bottom Absorption; Vertical Excitation
- Fig. 7.6** Dam-Reservoir-Foundation Interaction and Reservoir Bottom Absorption; Horizontal Excitation
- Fig. 7.7** Dam-Reservoir-Foundation Interaction and Reservoir Bottom Absorption; Vertical Excitation
- Fig. 7.8** Dam-Foundation Interaction; Foundation Material Damping; Horizontal Excitation
- Fig. 7.9** Dam-Foundation Interaction; Foundation Material Damping; Vertical Excitation
- Fig. 7.10** Dam-Reservoir-Foundation Interaction; Foundation Material Damping and Reservoir Bottom Absorption; Horizontal Excitation
- Fig. 7.11** Dam-Reservoir-Foundation Interaction; Foundation Material Damping and Reservoir Bottom Absorption; Vertical Excitation
- Fig. 7.12** Frequency Response Function of Idealized Dam-Reservoir-Foundation System; Horizontal Excitation
- Fig. 7.13** Frequency Response Function of Idealized Dam-Reservoir-Foundation System; Vertical Excitation
- Fig. 7.14** Displacement Response of Idealized Dam System due to Taft N21E Component; S-Model and RC-Model
- Fig. 7.15** Displacement Response of Idealized Dam System due to Taft N21E Component; S-Model and RUC-Model
- Fig. 7.16** Displacement Response of Idealized Dam System due to Taft S69E Component; S-Model and RC-Model
- Fig. 7.17** Displacement Response of Idealized Dam System due to Taft S69E Component; S-Model and RUC-Model

- Fig. 7.18** *Displacement Response of Idealized Dam System due to Taft Vertical Component; S-Model and RC-Model*
- Fig. 7.19** *Displacement Response of Idealized Dam System due to Taft Vertical Component; S-Model and RUC-Model*
- Fig. 7.20** *Maximum Principal (tensile) Stress Envelope in Idealized Dam System due to Taft N21E Component*
- Fig. 7.21** *Maximum Principal (tensile) Stress Envelope in Idealized Dam System due to Taft S69E Component*
- Fig. 7.22** *Maximum Principal (tensile) Stress Envelope in Idealized Dam System due to Taft Vertical Component*
- Fig. 7.23** *Cross Sectional Geometry and Finite Element Mesh of Pine Flat Dam*
- Fig. 7.24** *Frequency Response Function of Pine Flat Dam-Reservoir-Foundation System; Horizontal Excitation*
- Fig. 7.25a** *Frequency Response Function of Absolute Crest Acceleration of Pine Flat Dam-Reservoir-Foundation System; Vertical Excitation*
- Fig. 7.25b** *Frequency Response Function of Real Part of Crest Displacement of Pine Flat Dam-Reservoir-Foundation System; Vertical Excitation*
- Fig. 7.25c** *Frequency Response Function of Imaginary Part of Crest Displacement of Pine Flat Dam-Reservoir-Foundation System; Vertical Excitation*
- Fig. 7.26** *Treatment of Reservoir Far Field; Pine Flat Dam-Reservoir-Foundation System; Simplified Model; Vertical Excitation*
- Fig. 7.27** *Displacement Response of Pine Flat Dam System due to Taft N21E Component; S-Model and RC-Model*
- Fig. 7.28** *Displacement Response of Pine Flat Dam System due to Taft N21E Component; S-Model and RUC-Model*
- Fig. 7.29** *Displacement Response of Pine Flat Dam System due to Taft S69E Component; S-Model and RC-Model*
- Fig. 7.30** *Displacement Response of Pine Flat Dam System due to Taft S69E Component; S-Model and RUC-Model*
- Fig. 7.31** *Displacement Response of Pine Flat Dam System due to Taft Vertical Component; S-Model and RC-Model*
- Fig. 7.32** *Displacement Response of Pine Flat Dam System due to Taft Vertical Component; S-Model and RUC-Model*
- Fig. 7.33** *Displacement Response of Pine Flat Dam System due to ElCentro S00E Component; S-Model and RC-Model*
- Fig. 7.34** *Displacement Response of Pine Flat Dam System due to ElCentro S00E Component; S-Model and RUC-Model*
- Fig. 7.35** *Displacement Response of Pine Flat Dam System due to ElCentro S90W Component; S-Model and RC-Model*
- Fig. 7.36** *Displacement Response of Pine Flat Dam System due to ElCentro S90W Component; S-Model and RUC-Model*
- Fig. 7.37** *Maximum Principal (tensile) Stress Envelope in Pine Flat Dam System due to Taft N21E Component*

- Fig. 7.38** Maximum Principal (tensile) Stress Envelope in Pine Flat Dam System due to Taft S69E Component
- Fig. 7.39** Maximum Principal (tensile) Stress Envelope in Pine Flat Dam System due to Taft Vertical Component
- Fig. 7.40** Maximum Principal (tensile) Stress Envelope in Pine Flat Dam System due to ElCentro S00E Component
- Fig. 7.41** Maximum Principal (tensile) Stress Envelope in Pine Flat Dam System due to ElCentro S90W Component
- Fig. 7.42** Dam-Reservoir Model for Reservoir with Sloped Bottom
- Fig. 7.43a** Treatment of Reservoir Far Field; Pine Flat Dam-Reservoir System; Vertical Excitation; $L_f = 1.0H$
- Fig. 7.43b** Treatment of Reservoir Far Field; Pine Flat Dam-Reservoir System; Vertical Excitation; $L_f = 2.0H$
- Fig. 7.43c** Treatment of Reservoir Far Field; Pine Flat Dam-Reservoir System; Vertical Excitation; $L_f = 2.5H$
- Fig. 7.44** Convergence Analysis of Pine Flat Dam-Reservoir System; 15 Degrees Bottom Slope; Horizontal Excitation
- Fig. 7.45** Convergence Analysis of Pine Flat Dam-Reservoir System; 15 Degrees Bottom Slope; Vertical Excitation
- Fig. 7.46** Convergence Analysis of Pine Flat Dam-Reservoir System; 5 Degrees Bottom Slope; Horizontal Excitation
- Fig. 7.47** Convergence Analysis of Pine Flat Dam-Reservoir System; 5 Degrees Bottom Slope; Vertical Excitation
- Fig. 7.48** Effect of Reservoir Bottom Slope; Pine Flat Dam-Reservoir System; Horizontal Excitation
- Fig. 7.49** Effect of Reservoir Bottom Slope; Pine Flat Dam-Reservoir System; Vertical Excitation

LIST OF TABLES

Table 7.1 *Conditions of Dam-Reservoir-Foundation System Analyzed*

Table 7.2 *Computational Time for the Analysis of Pine Flat Dam-Reservoir-Foundation System to Transverse and Vertical Ground Motion*

1.1 General Remarks

In a world experiencing rapid economic and industrial growth, the development of water resources has become a vital element of the infrastructure, because of the need to satisfy the ever increasing demand for power, irrigation and drinking water supply. This development requires both the design and construction of new facilities and the rehabilitation of existing older plants. Dams are usually the key element in such facilities and their economic construction and safety are of paramount importance.

It is not always feasible to locate the dams on sites that have no seismic activity. A number of major dams are, in fact, located in high seismic zones. Design of dams therefore demands proper consideration of the earthquake forces by a detailed dynamic analysis.

Failure of structures during earthquakes has been the cause of considerable loss of life and damage of property. The consequences of the failure of a dam can be particularly catastrophic, therefore the element of risk in designing a dam located in an active seismic zone must be minimized by using reliable techniques of analysis and design.

A rational earthquake analysis of a dam should account for all the interaction effects. These include the effect of interaction with the impounded reservoir and with the flexible foundation. During a seismic event, the water in the reservoir impounded by the dam induces dynamic forces on the dam, termed as 'hydrodynamic forces'. The dam motions are affected by these forces and these forces are in turn generated in part by the dam motions. The flexible foundation soil below the dam-reservoir system also influences the hydrodynamic forces and the dam response, making the interaction behavior more complicated.

It is not possible to utilize closed form mathematical solutions in the analysis of earthquake responses, except in a very few highly idealized cases; recourse must therefore be had to numerical methods. This is particularly so for the dam-reservoir-foundation system.

The Finite Element Method (FEM) and, more recently the Boundary Integral Equation Method, also known as Boundary Element Method (BEM), have become useful tools for such numerical modeling. In particular, the BEM has evolved as an effective means of treating infinite domains and 3D problems. The conventional BEM used in a dynamic analysis is based on a frequency dependent fundamental solution, implying that the boundary element matrices be formulated for each excitation frequency. This seriously affects the computational efficiency of the procedure. This problem can be circumvented by resorting to the concept of frequency independent fundamental solution.

The present study aims at developing an effective technique to evaluate the seismic response of gravity dams including the reservoir and the foundation interaction effects. A substructure method of analysis, in which individual substructures are modeled by FEM or BEM, is used. In particular, the reservoir is modeled by BEM using a frequency independent fundamental solution. Major emphasis is placed on the development of procedures for representing the fluid-foundation interaction effect.

1.2 State of the Art

1.2.1 Water Compressibility and Dam-Reservoir Interactions

In the traditional practice for the design of gravity dams (U.S. Bureau of Reclamation 1966), the effect of hydrodynamic forces was accounted for by taking the water pressure as the product of static pressure and a seismic coefficient, varying over the height of the dam. This is a highly simplified and crude method, as it ignores both the response characteristics of dam-reservoir-foundation system, and the characteristics of the earthquake ground motion, and essentially treats the dam as rigid and the water as incompressible. As a refinement of the technique for analyses of dams, U.S. Bureau of Reclamation (1976) recommended a dynamic, finite-element approach, in which the hydrodynamic effect was represented by an added mass concept and the foundation flexibility was accounted for approximately by considering a finite portion of the foundation. This frequency invariant added mass representation of the hydrodynamic force amounts to ignoring the water compressibility and treating the dam as rigid, and hence may lead to erroneous results while evaluating the response of a typical concrete gravity dam system (Chopra 1987).

Westergaard (1933) was the first to consider the water compressibility effect in an infinite rectangular reservoir while calculating the hydrodynamic pressure on a rigid dam with vertical upstream face subjected to horizontal harmonic cosine excitation and concluded that the hydrodynamic pressure could be represented as an added mass and an added load whose magnitudes depended on the frequency of excitation. Later, it was shown by Kotsu (1959) and Chopra (1967) that Westergaard's expression was valid only for excitation frequencies less than the fundamental frequency of the reservoir. For frequencies greater than the fundamental frequency, an added damping term exists in addition to the added mass and added load terms, all of which are frequency dependent. The importance of water compressibility was clearly highlighted in Chopra's (1967) work. The significant parameters controlling the importance of water compressibility in the response evaluation of a dam are the depth of the water in the reservoir, and the material and geometric properties of the dam cross section (Chopra, 1968).

Hall and Chopra (1980), Fenves and Chopra (1984) have established that when the water compressibility effect is ignored, the hydrodynamic pressure can be represented by frequency independent added mass and added load, and the response of the dam is essentially independent of the reservoir geometry. At the same time, the resonant response of the dam to horizontal ground motion is overestimated due to the fact that the incompressible water model does not allow for the radiation of energy along the upstream direction or along the reservoir bottom. It is therefore apparent that water compressibility must be accounted for in the dynamic analysis of the concrete gravity dams.

The significance of dam-water interaction in evaluating the hydrodynamic force on a flexible dam retaining compressible water reservoir was recognized by Chopra (1968) and Chakrabarti and Chopra (1973a,1973b,1974). These authors also developed an analysis procedure for the response of the dam to the vertical component of the ground motion. It was shown by them that the hydrodynamic forces due to vertical component of ground motion were comparable to those due to horizontal excitation. Also these authors found that the hydrodynamic forces resulting from the vertical ground motion were dependent on the modulus of elasticity of the dam material. They also established the importance of including the higher modes in the dam-water interaction analysis using mode superposition technique and concluded that, although the fundamental mode alone was capable of predicting the maximum displacements accurately, the stress responses were in considerable error, unless

sufficient number of modes were included. Bustamante (1959) and Chopra (1967) have demonstrated that the errors introduced by ignoring the surface wave effects in the reservoir analysis is negligible, thus facilitating a zero pressure boundary condition along the free surface of the reservoir.

1.2.2 Fluid-Foundation Interaction or Wave Absorption along the Flexible Reservoir Bed

Analytical studies have shown that when the reservoir bed is assumed to be rigid, the hydrodynamic force on the upstream face of the dam resulting from the vertical excitation can be enormously high, especially at excitation frequencies close to the natural frequencies of the reservoir. Rosenblueth (1968) relaxed this rigid bottom assumption, by accounting for the partial refraction and reflection of hydrodynamic waves at the bottom of the reservoir, and obtained more realistic results. Hall and Chopra (1980) proposed a simplified and approximate one dimensional (1D) model to account for the fluid-foundation interaction effects, both for horizontal as well as vertical excitation. Later, Fenves and Chopra (1984) argued that in actual practice, the sediment material does get deposited along the reservoir bed, which causes partial absorption of the incident waves, and therefore the wave reflection coefficient parameter of the 1D model must be related to the material properties of these sediments. From their studies, Fenves and Chopra have shown that reservoir bottom absorption plays a major role in reducing the response due to vertical ground motion, but has a relatively small effect on the response due to horizontal ground motion, especially if the dam-foundation interaction is included. They have also concluded that the cross coupling effect between the dam and the foundation below the reservoir has negligible influence on the overall response of the dam. In their formulation for displacements at the reservoir bottom, Fenves and Chopra used a static stiffness matrix of foundation to obtain the mode shapes corresponding to the reservoir bottom. It will be shown in this work that, in fact, frequency dependent impedance matrices must be used. It is therefore necessary that the aforesaid conclusion on the cross-coupling term be reconfirmed.

Lotfi et al. (1985, 1987) investigated the problem of fluid-foundation interaction effect by using a layered analysis and adopting the concept of fluid-solid hyperelement and auxiliary interface elements. They concluded that Chopra's approximate 1D model substantially underestimated the response. However, it should be mentioned here that, in their

work while calculating the value of so called *wave reflection coefficient* of the 1D model, Lotfi et al. erroneously used the formula for computing the shear wave velocity of a 2D model. Furthermore, the material properties of the sediment considered in their investigations were not consistent with the specifications of properties for a 1D model as outlined by Chopra and coworkers. Hence Lotfi's criticism of 1D model is not strictly valid (Chandrasekhar 1991b).

Medina et al. (1990) developed an analysis procedure for evaluating the complex frequency response function of gravity dams using BEM, considering both the layered and half-space idealization of the foundation. They treated the fluid-foundation interaction effect more rigorously by using BEM approach. They concluded that, although the 1D approximate model is capable of predicting correct responses to vertical excitation, it underestimates the peak value of crest acceleration caused by a harmonic horizontal excitation. Further, they found that the presence of fully saturated sediments along the reservoir bottom does not significantly influence the response of the dam to either component of the ground motions, when the foundation below the reservoir is treated as flexible. Cheng (1986) examined the effect of poroelastic sediment material on the hydrodynamic response of the rigid dam due to vertical excitations, and concluded that, for fully saturated sediment material, the damping along the reservoir bottom is mainly due to elastic foundation below the reservoir, but when the sediment is partially saturated, it significantly modifies the hydrodynamic response. Recognizing this fact, recently Bougacha et al. (1991) have studied the effect of fluid-filled poroelastic sediment material on the dam-reservoir-foundation system, following the lines of Lotfi's work, but by modeling the sediments as a two-phase medium, namely a fluid filled poroelastic solid. They have shown that in case of partially saturated sediments there is a substantial reduction in the peak acceleration of the dam under harmonic horizontal motion. However, there are a number of uncertainties associated with the nature and properties of the sediment materials.

Although BEM is capable of handling accurately the fluid-foundation interaction effect, the associated computational effort is enormous, especially when a larger portion of the reservoir-foundation interface is required to be modeled by BEM as in Medina et al. (1990). Therefore other simplified means of analysis capable of predicting reasonably accurate responses are strongly desired. Towards this end, the performance of 1D simplified model of Chopra and coworkers needs to be investigated more closely for its accuracy in

predicting the response for realistic earthquake ground motions.

1.2.3 Flexibility of Foundation Soil and Foundation Modeling

The fact that dam-foundation interaction alters the dynamic behavior of the system, thereby affecting the structural response considerably, was recognized by Vaish and Chopra (1974), Dasgupta and Chopra (1979), Dominguez (1978), Abdalla (1984), and Wolf (1986) and several other researchers. The possible influence of the soil-structure interaction are: modification of the natural frequencies of the system, and addition of damping into the system. These factors can cause attenuation or amplification of the structural response. Thus, the flexibility of the soil has a significant effect on the response of the dam and hence all practical dynamic response analyses should account for the flexibility of the underlying foundation soil.

The foundation soil below the dam needs to be analyzed so as to obtain the complex valued, frequency dependent impedance matrix corresponding to the degrees of freedom of the soil-structure interface nodes. The real part of the soil impedance matrix contributes to the stiffness, while the imaginary part provides damping in the associated structural system. As the properties of the structure are quite different from those of the soil on which it is founded, the soil-structure system is said to be a non-classically damped system.

The soil impedance matrix could be evaluated either by mathematical analysis of an idealized viscoelastic half plane as proposed by Dasgupta and Chopra (1975) or by treating a finite portion of soil as assemblage of finite elements (Hall 1986, Vaish and Chopra 1974, Gutierrez and Chopra 1978) or through a boundary element analysis of the soil half plane model (Dominguez 1978, Abdalla 1984, Wolf 1986, Manolis and Beskos 1988).

The major limitation of the mathematical analysis is that it is not suited for soil surface of arbitrary geometry or for structures with foundation embedment. Although the FEM can efficiently handle the irregular geometry, heterogeneity, and nonlinearity, the modeling of an infinite domain of soil requires a large FEM mesh, making the method computationally inefficient (Gutierrez and Chopra 1978). Further, FEM is beset with the problem of spurious wave reflections from the artificial boundary of FEM mesh, so that the infinite radiation or the geometric damping in the soil is not properly represented by a FEM model. For sites where layers of soft soil are underlain by rock at shallow depth, FEM idealization for foundation soil may be appropriate (Gutierrez and Chopra 1978). Dasgupta (1980, 1982)

studied the dynamic soil-structure interaction problem using FEM in conjunction with the concept of so called 'substructure deletion' and 'cloning' to account for the infinite wave radiation in soil.

The drawbacks of FEM and of mathematical treatment of half plane can be circumvented by resorting to the BEM, which is very efficient in modeling infinite media, and substantially reduces the input data preparation and discretization, as the dimensionality of the problem is reduced. For a soil-structure interaction problem, since only the quantities along the boundary are of interest, the interior nodal points of FEM are unnecessary. This drawback is overcome in BEM where only the surface is discretized. The drawback of BEM in comparison with FEM is that, the equation matrix in BEM formulation is unsymmetric and fully populated. Of course, the feasibility of using BEM depends on the availability of appropriate Green's function or the fundamental solution.

The formulations of BEM for soil are based on the theory of elastodynamics (Eringen and Suhubi 1975, Dominguez and Alarcon 1981). Cruse and Rizzo (1968) were first to obtain a solution in the frequency domain by using Green's function associated with a harmonic excitation in the unbounded linear elastodynamic media. This development has turned BEM into an effective tool for the analysis of the soil domain in the dynamic soil-structure interaction problems. Dominguez (1978) was first to successfully implement this frequency domain BEM in the dynamic soil-foundation interaction problem. The half space idealization would generally permit accurate modeling of the dam sites where, typically, similar rocks extend to great depths (Fenves and Chopra 1984).

The BEM can be applied for 2D as well as 3D analysis, both in time domain (Spyrakos and Patel 1986, Wolf 1986) and in frequency domain (Dominguez 1978, Abdalla 1984). The formulations for the solution of elastodynamic problem in frequency domain using BEM, both for 2D and 3D case is clearly outlined in Brebbia and Chuang (1982), who highlight various important aspects of the analysis. The BEM is capable of handling even a layered foundation medium, but in this case a discretization of a portion of the interface between two layers is necessary (Abdalla 1984, Medina et al. 1990). The infinite space fundamental solution for the soil domain requires discretization by boundary elements along the soil structure interface and over a finite extent of the traction free surface in the neighborhood. Although, the use of half space fundamental solution (Buchen 1978) requires discretization along the interface alone, the solution is highly complicated and hence not frequently

implemented.

It is possible to account for material damping introduced by a viscoelastic foundation soil, by using a fundamental solution which is a modification of its elastic counterpart. In a frequency domain analysis, this is achieved by the use of so called 'correspondence principle', wherein the frequency independent hysteretic damping in the foundation material is accounted approximately by replacing the elastic Lamé's constants by their complex equivalents (Veletsos and Verbic 1973, Chopra et al. 1976, Manolis and Beskos 1988, Beskos 1987). This principle implicitly assumes that the Poisson's ratio for viscoelastic materials is real valued, and is equal to that in the elastic medium. Experimental evidence suggests that material damping is almost independent of excitation frequency (Chopra et al. 1976, Israil and Banerjee 1990), thus conforming the validity of a hysteretic damping model.

1.2.4 Discretization of Dam and Reservoir Substructures

In the majority of the studies concerning the seismic response of dams, the dam substructure has been represented by a finite element discretization (Chopra 1968, 1970, Chakrabarti and Chopra 1973a, 1973b, 1974, Fenves and Chopra 1984). Medina et al. (1990), in their recent study have analyzed the complex frequency response function of the dam-reservoir-foundation system by employing BEM discretization of the dam substructure and have concluded that FEM representation of the dam will result in reduced responses at higher excitation frequencies, unless a very fine FEM mesh is adopted.

It has been assumed in much of previous research that for the purpose of hydrodynamic force evaluation the upstream face of the dam can be considered as vertical and the reservoir bottom as perfectly horizontal along its infinite length. With these assumptions, the reservoir water can be modeled as a continuum, for which a classical solution exists. The assumption of the vertical dam face is based on Zanger's (1953) conclusion that the hydrodynamic forces acting on the inclined face of a dam are almost the same as those on a vertical face, provided the face is inclined only over a portion less than half the dam height and vertical over the remaining. However, Zanger's studies were based on an incompressible water model and a rigid dam.

When the water compressibility effect is included, the dam response becomes sensitive to the reservoir geometry (Hall and Chopra 1980). In case of arbitrary reservoir geometry and compressible water, the reservoir has to be represented by a suitable numerical model

either by FEM model as in Saini et al. (1978), Sharan (1978), Humar and Roufaiel (1983), Hall and Chopra (1980) or by using BEM discretization as demonstrated by Hanna and Humar (1982), Humar and Jablonski (1988), Medina et al. (1990) and Tsai (1987). An alternative approach is the use of hyperelement and layered medium as in Lotfi (1985), and Bougacha et al. (1991). For a reservoir of infinite extent, the finite element or boundary element discretization is limited to a finite part of the reservoir adjacent to the dam termed as 'the near field', which usually contains all the arbitrary reservoir geometry, if any. The remaining part 'the far field' of the reservoir is assumed to possess a constant cross section extending to infinity. The wave radiation at the far end of an infinite reservoir is accounted for either by using an appropriate boundary condition as proposed by Sharan (1988), Hanna and Humar (1982), Humar and Roufaiel (1983), or by using the concept of infinite elements as in Saini et al. (1978) or through a transmitting boundary with continuum solution, as given in Hall and Chopra (1980), Humar and Jablonski (1988) and Tsai (1987). Medina et al. (1990) did not account for the wave radiation separately, but modeled a substantial length of the reservoir by BEM, so as to ensure the absence of any wave reflection from the reservoir far end.

In modeling the irregular reservoir geometry, it is preferable to adopt BEM rather than FEM, as the former method requires discretization only along the boundary of the domain, resulting in less data preparation. The number of equations to be solved is also smaller in comparison to those in the FEM. However, the matrices involved in the BEM equations are unsymmetric and fully populated. The efficiency of BEM can be further increased by adopting an half-space fundamental solution, which accounts for the free surface boundary condition along the reservoir, thereby avoiding the need for a discretization of the free surface. The implementation of this half space fundamental solution poses no additional complication for a reservoir problem.

The conventional BEM proposed by Hanna and Humar (1982), Humar and Jablonski (1988) and Medina et al. (1990) is based on a frequency dependent fundamental solution, which implies that the BEM matrices must be evaluated for each excitation frequency, greatly increasing the amount of computation. This problem can be overcome by using the newly developed dual reciprocity approach (Nardini and Brebbia 1982,1984,1985, Loeffler and Mansur 1987, Banerjee et al. 1988) which uses the concept of 'frequency independent fundamental solution' or 'static fundamental solution', wherein the domain integral repre-

senting the inertia term is transformed into its equivalent boundary integrals by means of some suitable approximating function. Tang and Brebbia (1989) have presented ways to transform the domain integral to the boundary, through two different sets of approximating functions viz. the conical type functions and the Fourier series method. Tsai (1987) has successfully implemented this technique in analyzing the dam-reservoir interaction problems, with a rectangular infinite reservoir and ignoring the foundation interaction. However, in his work, Tsai modeled only a very small region of the reservoir near field by BEM, representing the far field by a continuum solution. Also, his studies were limited to the case of horizontal harmonic excitation.

The effect of the dam's upstream slope on the response has been investigated by Hall and Chopra (1980), Tsai (1987) and Jablonski (1989), Hall and Chopra (1980), Antes and Estroff (1987) and Jablonski (1989). The last author also studied the influence of the reservoir bottom slope on the dam's response. Jablonski (1990) initiated a study relating to the effect of location of the transmitting boundaries in infinite reservoirs on the hydrodynamic pressures on rigid dams subjected to harmonic horizontal excitations. However, the influence of truncating the reservoir near field in an infinite reservoir with sloped bottom on the response of gravity dam system has not yet been fully analyzed.

From the foregoing review, it is quite clear that, a reliable technique for the seismic response analysis of concrete gravity dams should consider: water compressibility effect, dam-reservoir, reservoir-foundation, and dam-foundation interaction, the arbitrary nature of the dam, reservoir and foundation geometry, and the radiation damping in the foundation and the reservoir.

1.2.5 Method of Analysis

The concept of 'substructure approach' is very popular and has been extensively used in complex problems of structural analysis in the last few decades. It is found to be much superior to the so called 'direct approach' or 'the one pass method'. The major advantages of substructure approach are that: (a) it provides greater flexibility in analyzing individual substructures by specific numerical methods appropriate to them, (b) it permits the direct use of the free field ground motion as input along the soil-structure interface, thus avoiding the complicated deconvolution procedure and its related assumptions (Vaish and Chopra 1974, Gutierrez and Chopra 1978). (c) it allows the specification of different damping

properties for different substructures, i.e. in case of a non-classically damped systems.

The analysis of the dam-reservoir-foundation system is usually carried out in the frequency domain, since the substructure method of analysis is best formulated in that domain (Gutierrez and Chopra 1978). Analysis in frequency domain is very effective in the case of infinite domains, where the physical characteristics of the systems, for example, the impedance matrix of the foundation soil; the added mass, the added damping and the added load representing the reservoir hydrodynamic pressure, as well as the infinite radiation conditions for both the reservoir and soil substructure, are all dependent on the excitation frequency. Also, analysis in frequency domain makes the problem equivalent to that of a static case, by transforming a non-elliptic governing differential equation into an elliptic one, which is amenable to numerical treatment by BEM. Further, the availability of speedy computers and efficient Fast Fourier Transform (FFT) techniques, makes the frequency domain approach quite practicable. However, frequency domain analysis is only applicable to time-invariant, linear systems for which the principle of superposition is valid (Humar 1990). The time history of response to a realistic earthquake ground motions can be evaluated by first performing the analysis in frequency domain, and then transforming the frequency domain results to time domain by using a discrete Fourier transform approach along with an FFT algorithm.

For the case of a typical gravity dam system, the solution of the equations of motion in their original form would mean substantial computational effort. Fortunately, for a linear structural system, the Ritz concept is very effective in reducing the size of the original equations set. This is accomplished by expressing the displacements as the superposition of a few mode shapes of the associated dam-foundation system, each weighted by a factor known as generalized coordinate (Clough and Penzien, 1985). The number of modes to be considered depends upon the geometric and material properties of the structural system, the frequency content of the earthquake ground motion, and the response quantity that is of interest.

1.3 Objectives of the Present Research

The present study is aimed at developing an effective technique capable of predicting the response of concrete gravity dams subjected to earthquake ground motion, including

the simultaneous effects of dam-water interaction, dam-foundation interaction and wave radiation through the flexible reservoir bed. The technique so developed is computationally efficient and imposes fewer restrictions over the geometry of the system analyzed. Figure 1.1 shows the section of a typical dam-reservoir-foundation system, studied in this work.

The present research also examines the following important aspects of gravity dam analysis:

1) The role of interpolation function and the interior points in the approximate evaluation of the inertia integral, while computing the hydrodynamic forces in a reservoir, using BEM with a frequency independent fundamental solution.

2) The validity of the approximate 1D simplified model (Hall and Chopra 1980) in representing the wave absorption along the reservoir bed. This can be verified by comparing the results of a response analysis of the dam for a realistic earthquake ground motion with those obtained by using an exact approach in which the fluid-foundation interaction is represented by a BEM model. The relative difficulty in formulation and the computational effort associated with each model is also discussed.

3) The influence of the cross-coupling terms between the foundation below the dam and the reservoir bottom, on the response behavior of dam. This will be investigated through a new formulation, in which the mode shapes of the fluid-foundation interface nodes are obtained by the use of a frequency dependent soil impedance matrix, instead of a static stiffness matrix as used by other researchers.

4) The effect of location of transmitting boundary, that is, truncating the far end of the near field, for an infinite reservoir with continuous sloped bottom. The objective is to find out the length up to which the reservoir geometry should be considered irregular in the case of an infinite reservoir with sloped bottom. The study also helps in determining the impact of the varying reservoir geometry on the response of a gravity dam.

In order to achieve the above mentioned goals, a two dimensional analysis is carried out in the frequency domain, using a substructure approach. The response of the dam is evaluated by resorting to an efficient hybrid technique, in which a combination of FEM, BEM and continuum solution is utilized so as to exploit the efficiency of various numerical and analytical methods in modeling particular types of domains. The time history of response to a random ground motion is obtained by FFT techniques.

The dam material is assumed to be isotropic, linearly elastic and possessing hysteretic damping. The foundation soil below the dam is treated as a homogenous, isotropic semi-infinite (elastic/viscoelastic) half plane with any arbitrarily shaped surface. The reservoir water which might have any arbitrary geometry, is assumed to be non-viscous, but compressible; surface wave effects in water are ignored and small vibration theory is considered to be applicable. Earthquake waves are assumed to arrive simultaneously all along the base of the dam.

In the case of an infinite reservoir of arbitrary geometry, the reservoir is assumed to be bifurcated into two regions viz. "near field" and the "far field". The near field, which is of finite length and close to the upstream face of the dam is considered to contain all the geometric irregularities of the reservoir (Fig. 1.1). The far field is of infinite length with constant cross sectional geometry.

The system to be analyzed comprises three main substructures — the dam, the reservoir and the foundation. The dam substructure is represented by an assembly of finite elements. The soil substructure is analyzed using the boundary element method which is capable of modeling soil of any arbitrary geometry, requiring discretization only along a finite length of the soil surface. The infinite radiation condition in the foundation soil is automatically accounted for through the fundamental solution (Green's function). The reservoir near field of arbitrary geometry is modeled by BEM with frequency independent fundamental solution. The far field of the reservoir is idealized as a continuum, whose solution is obtained by modeling the interface between the near and the far field (transmitting boundary) by 1D finite elements. Two different approaches are adopted to account for the fluid-foundation interaction (1) a simplified analysis using an approximate 1D wave absorption model proposed by Hall and Chopra and (2) a rigorous analysis, wherein the base soil, idealized as a viscoelastic half plane is discretized by boundary elements located along the fluid-foundation interface, and the cross-coupling between the dam foundation and the reservoir bottom is included.

1.4 Layout of the Thesis

A general introduction, review of relevant literature, the objective of the current research, a brief description of the problem and method of analysis are contained in the

present Chapter. Chapter 2 presents the formulations relating to the reservoir substructure. The equation of motion governing the reservoir and the associated boundary conditions, boundary element formulations with frequency independent fundamental solution, boundary element discretization, treatment of the far field and other related aspects of the reservoir analysis are included.

The accuracy of the reservoir model so developed is verified, for both finite and infinite reservoirs, in Chapter 3, by studying the hydrodynamic pressure distribution along the upstream face of the dam at different excitation frequencies and comparing these results with those obtained by classical solutions and with the results of other researchers. The significance of the inertia interpolation functions and interior points are investigated. The concept of reservoir near field absorptive reservoir bottom for an infinite reservoir is demonstrated through a convergence analysis. Also the efficiency of the frequency independent reservoir model is illustrated by comparing its performance against a conventional frequency dependent BEM model.

The analysis of the foundation soil is dealt with in Chapter 4, which describes BEM treatment of the foundation substructure and the evaluation of the foundation impedance matrix. The results obtained by BEM approach are compared with the classical solutions and with the results of other researchers. Formulations for the analysis of coupled dam-reservoir-foundation system are presented in Chapters 5 and 6. Chapter 5 describes the formulation of the analytical model for a coupled system based on a simplified approach, while the procedures involved in the analyses of model using a more involved rigorous approach are presented in Chapter 6.

The performance of the model based on the simplified approach in comparison to the more rigorous models, is investigated in Chapter 7. The complex frequency response function, displacement time history, and principal stress distribution of an idealized dam and Pine Flat dam, subjected to various components of Taft and ElCentro earthquake ground motions are considered for this part of the study. Also the influence of cross coupling between the foundation below the dam and reservoir bed as in the case of a rigorous analysis, is examined. The role of each interaction component, and the effect of material damping in the foundation soil are highlighted. The results from the analytical formulations presented here are compared with those obtained from classical solutions where available and with results of other researchers. A study is carried out on the effect of the location of the

transmitting boundary, i.e. truncation of reservoir near field in case of a dam impounding an infinite reservoir with a sloped bottom. A summary and conclusions of the current research, and suggestions for future work are presented in Chapter 8.

Fig. 1.1 Gravity Dam-reservoir-foundation System

2 BOUNDARY ELEMENT FORMULATION OF RESERVOIR MODEL

2.1 Introduction

As explained in Chapter 1, it is well established that the response of the dam is influenced by the impounded reservoir, and that the reservoir analysis should include the water compressibility, flexibility of the dam, dam-water interaction, infinite wave radiation, and the bottom absorption due to fluid-foundation interaction. The method of analyzing the reservoir substructure is summarized in this chapter, considering the various important features of the reservoir induced hydrodynamic effects during an earthquake ground motion.

The procedure adopted in the formulation of the two-dimensional(2D) model of the reservoir is discussed in the following sections. Both finite and infinite reservoir cases are presented. A reservoir with length greater than thrice the depth, could be regarded as an infinite reservoir for the purpose of analysis (Bustamante 1963). A finite reservoir consists only of near field and is modeled completely by boundary elements, while an infinite reservoir, as discussed in Chapter 1, is assumed to be bifurcated into two regions: the *near field* and the *far field*. The near field which maps all the geometric irregularities of the reservoir, and is located in the neighborhood of the upstream face of the dam, is analyzed by BEM, while the far field which possess a uniform cross section is idealized by continuum solution. The 2D analysis can evaluate responses only due to excitations along upstream-downstream and vertical directions, but not along the cross-stream direction. However a 2D analysis can effectively predict the response of a gravity dam-reservoir-foundation system, as demonstrated by the behavior of gravity dams during the past earthquakes (Chakrabarti and Chopra, 1972).

The assumptions made in analyzing the hydrodynamic effect are:

- a) The reservoir water is non-viscous, but compressible.

b) Surface wave effects in reservoir are negligible; the associated errors were found to be negligible by other researchers (Bustamante 1959, Chopra 1967).

c) The motion of water particles is limited to small amplitudes.

d) In the case of an infinite reservoir, the far field is assumed to possess constant geometric properties and only for the analysis due to horizontal excitation, the fluid-foundation interaction in the far field is neglected.

The reason for the latter part of the last assumption and its justification are presented in the subsequent sections of this chapter.

2.2 Description of the Model

Figure 2.1 shows the model for the analysis of an infinite reservoir. The reservoir near field contains the geometric irregularities arising from a sloping upstream face of the dam and an irregular reservoir bottom, if any. These surfaces together with the fictitious interface (always vertical) between the near field and the far field of the reservoir, are modeled by boundary elements with frequency independent fundamental solutions. The far field is analyzed by discretizing the near field-far field interface (also termed as the transmitting boundary, as the infinite wave radiation phenomenon in the reservoir occurs along this boundary) by 1D finite elements. Alternatively the far field could be also analyzed by using analytical expressions of the continuum solution for a regular shaped water body.

In the analysis including the dam-water interaction, the hydrodynamic pressures are dependent on the motion of the dam's face. This motion is represented as the combination of a rigid body motion, equal to the free field motion of the dam and a motion relative to the free field, expressed by superposition of mode shapes of the dam-foundation system. As the formulation in this chapter deals with reservoir alone, a rigid dam-foundation case is considered. The flexible dam-foundation study will be undertaken in the subsequent chapters.

The hydrodynamic forces are also affected by the flexible foundation below the reservoir, which causes partial absorption of waves, thereby modifying the boundary conditions along the reservoir bottom. This phenomenon is known as the fluid-foundation interaction effect.

2.3 Governing Equation of Motion of the Near Field

The two-dimensional wave equation governing the fluid flow is given by:

$$\nabla^2 \bar{p} = \frac{1}{c_w^2} \frac{\partial^2 \bar{p}}{\partial t^2} \quad (2.1)$$

wherein $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; x and y are the cartesian coordinates; t is the time variable; $\bar{p}$ is the hydrodynamic pressure in excess of the hydrostatic pressure; and c_w is the compression wave velocity in water.

For an unit harmonic excitation, the hydrodynamic pressure can be expressed as $p^l(x, y, t) = p^l(x, y, \Omega) e^{i\Omega t}$, where Ω is the frequency of excitation, and $p^l(x, y, \Omega)$ is complex frequency response function for the hydrodynamic pressure due to the l th component of ground motion; $l = x, y$.

Equation 2.1 now transforms into a Helmholtz equation type,

$$\nabla^2 p + k^2 p = 0 \quad (2.2)$$

in which $k = \Omega/c_w$ is the wave number, and $p = p(x, y, \Omega)$ denotes the hydrodynamic pressure as a function of excitation frequency.

Equation 2.2 represents a transformed governing differential equation in frequency domain i.e. the Fourier transformed domain. The equation is amenable to a numerical solution.

2.4 Boundary Conditions

At the free surface of the reservoir, if the surface wave effect is neglected a zero pressure condition results. This is automatically satisfied in the near field of the reservoir through the half-space (mirror image concept) fundamental solution of BEM model. The conditions along the other boundaries are:

2.4.1 Along the upstream face of the rigid dam

The expression for the pressure gradient is

$$q(s, \Omega) \Rightarrow \frac{\partial p}{\partial n}(s, \Omega) = -\rho_w a_n^l(s) \quad (2.3)$$

where n is unit outward normal; $a_n^x = -\cos \theta$, $a_n^y = -\sin \theta$; ρ_w the mass density of water; θ the angle of inclination of the dam's upstream face with the vertical (Fig. 2.1); a_n^l is the acceleration along the outward normal of the dam's face due to l th component of excitation; s is the coordinate along the upstream face of the dam; and l is the excitation component with $l = x, y$.

2.4.2 Along the reservoir bottom: Fluid-Foundation interaction

Using a simplified 1D model for reservoir bottom (Hall and Chopra, 1980), the expression for the pressure gradient along the reservoir bottom is given by (Appendix B)

$$q(s', \Omega) \Rightarrow \frac{\partial p}{\partial n}(s', \Omega) = -\rho_w b_n^l(s') - i\Omega\gamma p(s', \Omega) \quad (2.4)$$

with $\gamma = \frac{\rho_w}{\rho_r c_r}$, $c_r = \sqrt{\frac{E_r}{\rho_r}}$; $b_n^x = \sin \beta$, $b_n^y = -\cos \beta$; where β is angle of inclination of the reservoir bottom with the horizontal (Fig. 2.1); b_n^l is the acceleration along the outward normal to the reservoir bottom due to l th component of excitation; γ is the wave absorption parameter at the reservoir bottom; ρ_r and E_r are respectively the mass density and elastic modulus of the material of the reservoir bed; s' is the coordinate along the reservoir bottom; and $i = \sqrt{-1}$, the imaginary number.

The first part of the right hand side expression of Eq. 2.4 denotes the free field ground acceleration along the normal to the surface s' and the second part represents the wave absorption along the reservoir bed, which is a consequence of the fluid-foundation interaction. This interaction effect can be either approximately represented by a 1D simplified model, as in the case here, or rigorously evaluated by boundary element modeling of the foundation below the reservoir. Both approaches are considered in this research. However, for the purpose of the reservoir formulation of this chapter, only the 1D simplified model is considered. The rigorous approach is investigated in later chapters.

A wave reflection coefficient for the reservoir bottom α_r can be defined as (Hall and Chopra 1980, Fenves and Chopra 1984)

$$\alpha_r = \frac{1 - \gamma c_w}{1 + \gamma c_w} \quad (2.5)$$

This coefficient serves as the measure of wave absorption by the foundation. A value of $\alpha_r = 1.0$ signifies a fully rigid reservoir bottom. This value may also be used when the fluid-foundation interaction is to be ignored. A value of $\alpha_r < 1.0$ represents a flexible reservoir

bed. The 1D wave absorption model yields a simplified frequency independent expression, which is easily incorporated in the formulation and is also computationally more efficient.

The 1D wave absorption model for representing the 2D fluid-foundation interaction, is based on the assumption that the pressure wave energy dissipates by a partial absorption of the incident compression waves traveling in the direction normal to the surface of the reservoir bed. The 1D model could be visualized by considering the foundation beneath the reservoir to be sliced into columns of infinite length and infinitesimal thickness, extending along the perpendicular direction to the fluid-foundation interface. The columns vibrate independent of each other and continuity is maintained with respect to the normal displacement and stress across the fluid-soil interface. The detailed derivation of the expression for this approximate 1D model has been presented by Hall and Chopra (1980) is included in Appendix B.

Considering the uncertainties involved in characterizing the properties of the sediments, approximate 1D model should be usually satisfactory for most of the investigations. The adequacy of the simplified 1D model will be examined in subsequent chapters by comparing its performance with a more accurate and rigorous approach for bottom absorption.

2.4.3 Along the far boundary

A finite reservoir consists of only the near field, modeled completely by BEM. The far end boundary condition for this case is dependent on whether that boundary is fixed or free to move.

For an infinite reservoir, the far boundary is located at the interface of the near and far field of the reservoir. Unlike foundation soil analysis, the nature of the fundamental solution of the reservoir formulation, does not ensure automatic infinite radiation damping along the reservoir far field, because the boundary integral terms of the pressure and pressure gradient do not decay rapidly with increasing distance of the transmitting boundary from the dam. Unless an excessively large region of the near field is considered as in Medina et al. (1990), the radiation boundary condition at the far field of the infinite reservoir has to be separately accounted for. The formulation for energy loss due to radiation damping in a infinite region is presented in a later section of this chapter, dealing with the treatment of reservoir far field.

2.5 Formulation of the Boundary Integral Equations

The analysis of arbitrary shaped near field of the reservoir has to be carried out through a suitable numerical technique. In the present study, a modified BEM is adopted and its formulation is as follows:

Using the weighted residual technique, the governing Eq. 2.2 is satisfied in an average sense over the 2D fluid domain, yielding

$$\int_A \int (\nabla^2 p + k^2 p) p^* dx dy = 0 \quad (2.6a)$$

or

$$\int_A \int \nabla^2 p \cdot p^* dx dy + k^2 \int_A \int p \cdot p^* dx dy = 0 \quad (2.6b)$$

in which p^* is the weighting function or the Green's function, also termed as the fundamental solution governing the reservoir medium, and A is the near field fluid domain. The hydrodynamic pressure satisfying the first part of the Eq. 2.6b alone is known as the complementary function, while the hydrodynamic pressure satisfying the latter part is termed as the particular integral.

2.5.1 Frequency Independent Fundamental Solution

The weighting function p^* is now selected so as to satisfy the following equation

$$\nabla^2 p_i^* + \Delta^i = 0 \quad (2.7)$$

The semi-infinite space or the half-space fundamental solution p^* satisfying Eq. 2.7 corresponds to the solution of a Laplace equation for a two dimensional case and is given by

$$p_i^* = -\frac{1}{2\pi} \ln \left(\frac{r_{ij}}{r'_{ij}} \right) \quad (2.8)$$

In Eqs. 2.7 and 2.8, Δ^i is a Dirac Delta function centered at point i (the source point or the collocation point) having coordinates (x_i, y_i) , r and r' are the distance to the field point j respectively from the source point i and the mirror image of the source point i' (Fig. 2.1).

Since the fundamental solution of the present study is independent of excitation frequency, it is referred to as the "static or frequency independent fundamental solution" ¹. The technique that employs such a solution is also referred to as BEM using particular integrals (Ahmad and Banerjee 1986, Tsai 1987), since the wave equations are solved by the concept of the complementary function and the particular integral. The main advantage of using such a solution, is that the boundary element matrices need to be constructed only once for the entire range of frequencies to be analyzed, thereby greatly enhancing the computational efficiency of the procedure. Also, since these fundamental solutions are frequency independent, they can be effectively applied even in a time domain analysis. Further, since a semi-infinite space Green's function is employed, the free surface boundary condition is automatically satisfied, requiring no discretization of the reservoir free surface. This further reduces the input data requirement and increases the computational efficiency.

Integration by parts of the first part of Eq. 2.6b results in

$$\begin{aligned} \int_A \int \nabla^2 p_i p_i^* dx dy &= \int_A \int \nabla^2 p_i^* p_i dx dy + \int_S p_i^* q ds - \int_S p q_i^* ds \\ &= -c_i p_i + \int_S p_i^* q ds - \int_S p q_i^* ds \end{aligned} \quad (2.9)$$

where $q^* = \frac{\partial p^*}{\partial n}$; $q = \frac{\partial p}{\partial n}$; S is the boundary of the reservoir near field; and n is the outward normal to the boundary. Parameter $c_i = 1 - \frac{\beta_i}{2\pi}$ when i is a corner node on the boundary, and β_i is the external angle in radians at the i th corner node (Fig. 2.1); $c_i = 1$ for point i within the domain (interior points); $c_i = 1/2$ for a point i on a smooth boundary; $c_i = 0$ for a point outside the domain; and p_i is the hydrodynamic pressure in excess of hydrostatic pressure, at point i .

2.5.2 Evaluation of Inertia Integral

The second part of Eq. 2.6b, viz. $k^2 \int_A p_i p_i^* dx dy$ represents the inertia term containing the integral over domain A , (also termed as the particular integral part of the hydrodynamic pressure). This domain integral can also be transformed into its equivalent boundary integrals by using suitable approximation (Nardini et al. 1985, Ahmad et al. 1986, Loeffler et

¹ The p_i^* in the conventional BEM satisfies the equation $\nabla^2 p_i^* + k^2 p_i^* + \Delta^i = 0$, resulting in a fundamental solution dependent on the excitation frequency

al.1987) for the hydrodynamic pressure term p contained in this integral. The expressions used for p is

$$p(x, y, \Omega) = \sum_{m=1}^M \alpha_m(\Omega) f_m(x, y) \quad (2.10)$$

in which $\alpha_m(\Omega)$ is an unknown coefficient, a function of the exciting frequency; $f_m(x, y)$ is a suitably chosen harmonic function, which is dependent on the geometry and is also referred to as the 'inertia interpolation function'; and M is the number of such harmonic functions. It is useful to think of function $f_m(x, y)$ as representing the pressure generated at point (x, y) by a source located at point m . For efficiency in computation, these points are taken to be the same as the boundary element nodes. Additional interior nodes may have to be introduced to improve the accuracy. The choice of such functions and locations of source points will be discussed in the following sections.

Since $f_m(x, y)$ is harmonic, it can be written as

$$f_m(x, y) = \nabla^2 \psi^m(x, y) \quad (2.11)$$

where ψ^m is another function of x and y . Substitution of Eq. 2.11 in Eq. 2.10 gives

$$p(x, y, \Omega) = \sum_{m=1}^M \nabla^2 \psi^m(x, y) \alpha_m(\Omega) \quad (2.12)$$

Using Eq 2.12, the second part of Eq. 2.6b can be expressed as

$$\begin{aligned} \int_A \int p_i^* dx dy &= \sum_{m=1}^M \alpha_m \int_A \int \nabla^2 \psi^m \cdot p_i^* dx dy \\ &= \sum_{m=1}^M \alpha_m \left(-c_i \psi_i^m + \int_S p_i^* \eta^m ds - \int_S \psi^m q_i^* ds \right) \end{aligned} \quad (2.13)$$

where $\eta^m = \frac{\partial \psi^m}{\partial n}$.

Substituting the results of Eqs. 2.9 and 2.13 in Eq. 2.6b, we obtain

$$-c_i p_i + \int_S p_i^* q ds - \int_S p q_i^* ds = -k^2 \sum_{m=1}^M \alpha_m \left(-c_i \psi_i^m + \int_S p_i^* \eta^m ds - \int_S \psi^m q_i^* ds \right) \quad (2.14)$$

An alternative approach to evaluate the inertia integral term on the left hand side of Eq. 2.13 is by discretizing the domain using appropriate interior elements (similar to finite

elements) as proposed by Providakis and Beskos (1987). However, this requires additional data preparation and computational effort, making the method relatively less efficient.

2.5.3 Discretization with Linear Boundary Elements

Equation 2.14 is the boundary integral equation governing the near field of the fluid domain, which serves as the tool to evaluate the unknown hydrodynamic pressure and pressure gradients along the boundary. Analytical methods are not always feasible for evaluating the integrals involved in these equations, hence suitable numerical technique must be adopted.

In the technique used here the boundary of the reservoir near field (except the free surface of the reservoir) is divided into a series of segments, termed boundary elements, with the assumption that the pressure and the pressure gradient vary in a specified fashion over each element. Three different types of variations viz. constant, linear and quadratic, with their respective BEM formulations are considered in the present study. Only the formulation of linear boundary element discretization is presented.

For a suitably chosen interpolation function f , ψ and η are known quantities, and hence require no approximation within an element. However, the exact integration of the integrals involving these functions will require substantial computational effort. Further, the integrals on the right hand side of Eq. 2.14 are very similar to those on the left, therefore, if identical variations are assumed for all p , q , ψ and η over an element, there will be no need for evaluating any additional integrals (Nardini and Brebbia 1982,1984,1985).

The numerical model using linear boundary element discretization will be discussed below.

Linear boundary elements consist of straight line segments, bounded by two nodes at the ends of each segment, in which the quantities p , q , ψ and η are assumed to vary in a linear fashion between the two nodes, along a given boundary element. For example, parameters p and q over the j th linear element can be written as

$$\begin{aligned} p &= [N_1 \quad N_2] \begin{Bmatrix} p_j \\ p_{j+1} \end{Bmatrix} \\ q &= [N_1 \quad N_2] \begin{Bmatrix} q_j \\ q_{j+1} \end{Bmatrix} \end{aligned} \quad (2.15)$$

where N_1 and N_2 are the shape functions corresponding to the nodes j and $(j + 1)$ of the

j th boundary element, and are given by:

$$N_n = \frac{1}{2} (1 + \xi \xi_n) \quad (2.16)$$

in which ξ is the natural coordinate along the length of the boundary element, and ξ_n is defined by $\xi_1 = -1, \xi_2 = +1$, corresponding to the two nodes ($n=1, n=2$) of the linear boundary element.

Substitution of Eq. 2.15 into Eq. 2.14 yields the i th equation in a linear element formulation:

$$\begin{aligned} -c_i p_i + \sum_{j=1}^{nelr} \int_{s_j} [N_1 \quad N_2] p_i^* ds_j \left\{ \begin{matrix} q_j \\ q_{j+1} \end{matrix} \right\} - \sum_{j=1}^{nelr} \int_{s_j} [N_1 \quad N_2] q_i^* ds_j \left\{ \begin{matrix} p_j \\ p_{j+1} \end{matrix} \right\} \\ + k^2 \sum_{m=1}^M \alpha_m \left(-c_i \psi_i^m - \sum_{j=1}^{nelr} \int_{s_j} [N_1 \quad N_2] q_i^* ds_j \left\{ \begin{matrix} \psi_j^m \\ \psi_{j+1}^m \end{matrix} \right\} \right. \\ \left. + \sum_{j=1}^{nelr} \int_{s_j} [N_1 \quad N_2] p_i^* ds_j \left\{ \begin{matrix} \eta_j^m \\ \eta_{j+1}^m \end{matrix} \right\} \right) = 0 \end{aligned} \quad (2.17)$$

$i = 1, 2, \dots, nequr$

where $nelr$ is the number of boundary elements in the reservoir near field and $nequr$ is the total number of boundary element equations in the reservoir analysis.

Defining

$$h_{ij}^n = \int_{s_j} N_n q_i^* ds_j \quad (2.18a)$$

$$g_{ij}^n = \int_{s_j} N_n p_i^* ds_j \quad (2.18b)$$

Eq. 2.17 can be represented in the matrix form as

$$[\mathbf{H}]\{\mathbf{p}\} - [\mathbf{G}]\{\mathbf{q}\} = k^2 (-[\mathbf{H}][\psi] + [\mathbf{G}][\eta]) \{\alpha\} \quad (2.19)$$

The elements of $[\mathbf{H}]$ and $[\mathbf{G}]$ are given by

$$H_{ij} = \left(1 - \frac{\beta_1}{2\pi}\right) \delta_{ij} + h_{ij}^1 + h_{i(j-1)}^2 \quad (2.20a)$$

$$G_{ij} = g_{ij}^1 + g_{i(j-1)}^2 \quad (2.20b)$$

Evaluation of Boundary Integrals

The fundamental solution p_i^* is given by Eq. 2.8, while the derivative of the fundamental solution q_i^* is given by

$$q_i^* = \frac{1}{2\pi} \left(\frac{1}{r'_{ij}} \frac{dr'_{ij}}{dn_j} - \frac{1}{r_{ij}} \frac{dr_{ij}}{dn_j} \right) \quad (2.21)$$

The boundary integral terms therefore reduce to

$$\begin{aligned} h_{ij}^n &= \int_{s_j} N_n q_i^* ds_j \\ &= \frac{l_j}{4\pi} \int_{-1}^1 (1 + \xi \xi_n) \left(\frac{1}{r'_{ij}} \frac{dr'_{ij}}{dn_j} - \frac{1}{r_{ij}} \frac{dr_{ij}}{dn_j} \right) d\xi \end{aligned} \quad (2.22a)$$

$$\begin{aligned} g_{ij}^n &= \int_{s_j} N_n p_i^* ds_j \\ &= \frac{l_j}{4\pi} \int_{-1}^1 (1 + \xi \xi_n) (\ln r'_{ij} - \ln r_{ij}) d\xi \end{aligned} \quad (2.22b)$$

For non-singular terms, the above integrals could be obtained using numerical integration by Gauss quadrature scheme. However, singularity problem arises in the integral expressions when $j = i$, and $j = (i - 1)$, since both i th and $(i - 1)$ th linear boundary element contain the common node i . The portion of an integral containing the singular term could be best evaluated by analytical method for the limiting case.

It should be noted that while an analytical treatment of singularity in the integral terms is possible for constant and linear boundary element discretization, such may not be the case for quadratic element formulation, particularly when the boundary is curved. A special numerical approach has to be adopted (Tan, 1987, Banerjee and Butterfield, 1981) in the integration of singularity terms of quadratic elements.

Gauss quadrature for non-singular terms for $j \neq i$, and $j \neq (i - 1)$

Adopting a four point Gauss integration scheme, the integrals can be obtained numerically as follows:

$$h_{ij}^n = \frac{l_j}{8\pi} \sum_{m=1}^4 (1 + \xi_m \xi_n) \left(\frac{1}{r'_{ij}} \frac{dr'_{ij}}{dn_j} - \frac{1}{r_{ij}} \frac{dr_{ij}}{dn_j} \right)_m w_m \quad (2.23a)$$

$$g_{ij}^n = \frac{l_j}{8\pi} \sum_{m=1}^4 (1 + \xi_m \xi_n) (\ln r'_{ij} - \ln r_{ij})_m w_m \quad (2.23b)$$

with w_m representing the Gauss weight corresponding to the m th Gauss point.

Singularity case for $j = i$, and $j = (i - 1)$

Since r_{ij} is measured from the collocation point, namely, node i , r_{ii} and $r_{i(i-1)}$ are measured along the length of i th and $(i - 1)$ th element respectively, therefore $\frac{dr_{ii}}{dn_i} = 0$ and $\frac{dr_{i(i-1)}}{dn_{(i-1)}} = 0$ hence, the second part of the h_{ii}^n and $h_{i(i-1)}^n$ vanishes, not requiring any analytical expression to be used, and leaving only the non-singular terms containing r' which could be evaluated as

$$h_{ii}^n = \frac{l_i}{8\pi} \sum_{m=1}^4 (1 + \xi_m \xi_n) \left(\frac{1}{r'_{ii}} \frac{dr'_{ii}}{dn_i} \right)_m w_m \quad (2.24a)$$

$$h_{i(i-1)}^n = \frac{l_{(i-1)}}{8\pi} \sum_{m=1}^4 (1 + \xi_m \xi_n) \left(\frac{1}{r'_{i(i-1)}} \frac{dr'_{i(i-1)}}{dn_{(i-1)}} \right)_m w_m \quad (2.24b)$$

The near diagonal elements of $[G]$, namely $G_{i(i+1)}$, G_{ii} , and $G_{i(i-1)}$ will contain the g_{ij}^n type singularity to be discussed next, while the elements of $[H]$, namely $H_{i(i+1)}$, H_{ii} , and $H_{i(i-1)}$ need modified expressions for h_{ij}^n given by Eqs. 2.24. Note that the second suffix j in H_{ij} and G_{ij} represents the j th boundary element node, while, the j in h_{ij}^n and g_{ij}^n represents the j th boundary element, over which the numerical integration is performed.

The analytical expression for the singularity terms g_{ii}^n and $g_{i(i-1)}^n$ is discussed below. Singularity occurs only in the second part of the expression, and hence, the first non-singular part containing the image term r' can be evaluated by Gauss quadrature method.

Denoting the part of terms g_{ii}^n and $g_{i(i-1)}^n$ containing the singularity by $\bar{g}_{ii}^n$ and $\bar{g}_{i(i-1)}^n$, we could now write the analytical expressions as follows:

$$\bar{g}_{ii}^n = \frac{1}{4\pi} \int_{s_i} (1 + \xi \xi_n) \ln(r_{ii}) ds_i \quad (2.25)$$

Since r_{ii} is measured along the element length l_i , replacing both r and s by a common term x , and using the boundary values, ξ can be expressed in terms of x as

$$\xi = \left(\frac{2x}{l_i} - 1 \right) \quad (2.26)$$

where l_i is the length of the i th element. Using this relation in the previous equation and integrating analytically, the final expression reduces to

$$g_{ii}^n = \frac{l_i}{4\pi} (\ln l_i + 0.5\xi_n - 1) \quad (2.27)$$

Similarly,

$$g_{i(i-1)}^n = \frac{l_{(i-1)}}{4\pi} (\ln l_{(i-1)} - 0.5\xi_n - 1) \quad (2.28)$$

Now, the overall expressions for g_{ii}^n and $g_{i(i-1)}^n$ can be written as:

$$g_{ii}^n = \frac{l_i}{8\pi} \left(\sum_{m=1}^4 (1 + \xi_m \xi_n) (\ln r'_{ii})_m w_m - 2(\ln l_i + 0.5\xi_n - 1) \right) \quad (2.29)$$

and

$$g_{i(i-1)}^n = \frac{l_{(i-1)}}{8\pi} \left(\sum_{m=1}^4 (1 + \xi_m \xi_n) (\ln r'_{i(i-1)})_m w_m - 2(\ln l_{(i-1)} - 0.5\xi_n - 1) \right) \quad (2.30)$$

2.5.4 Hydrodynamic Mass Matrix

Since the inertia interpolation functions f may be related to both boundary and interior points, integral equations additional to those with boundary collocation points (equations with collocation points in the interior of the domain) must also be developed. Hence, the total number of boundary element equation *neqnr* is the sum of the boundary nodes and the interior points.

The value of the coefficient α_m , as yet an unknown, could be expressed in terms of the hydrodynamic pressure using Eq. 2.10, which in its matrix form is

$$\mathbf{p} = \mathbf{F}\alpha \quad (2.31a)$$

or

$$\alpha = \mathbf{F}^{-1}\mathbf{p} \quad (2.31b)$$

Note that for $\mathbf{F}$ to possess an inverse, it must be a square matrix, hence the number of equations in the Eq. 2.31 must be equal to the total number of points selected to evaluate the inertia interpolation functions $neqnr = M$, which is also the number of boundary element equations.

Substituting for $\{\alpha_m\}$ from Eq. 2.31b in Eq. 2.19, we obtain

$$(\mathbf{H} - k^2 \tilde{\mathbf{M}}) \mathbf{p} = \mathbf{G} \mathbf{q} \quad (2.32)$$

where the hydrodynamic mass matrix $\tilde{\mathbf{M}}$ is given by,

$$\tilde{\mathbf{M}} = (\mathbf{G} \boldsymbol{\eta} - \mathbf{H} \boldsymbol{\psi}) \mathbf{F}^{-1} \quad (2.33)$$

It is worth mentioning here that matrices $\mathbf{H}$, $\mathbf{G}$, and $\tilde{\mathbf{M}}$ are all independent of the frequency of excitation and hence need to be evaluated only once for the given reservoir geometry unlike in conventional BEM, where $\mathbf{H}$ and $\mathbf{G}$ must be computed for each excitation frequency.

Equation 2.32 can be partitioned into two sets of equations, one corresponding to the boundary nodes and the other to the interior points, as below:

$$\left(\begin{pmatrix} \mathbf{H}_{bb} & \mathbf{0} \\ \mathbf{H}_{db} & \mathbf{I} \end{pmatrix} - k^2 \begin{pmatrix} \tilde{\mathbf{M}}_{bb} & \tilde{\mathbf{M}}_{bd} \\ \tilde{\mathbf{M}}_{db} & \tilde{\mathbf{M}}_{dd} \end{pmatrix} \right) \begin{pmatrix} \mathbf{p}_b \\ \mathbf{p}_d \end{pmatrix} = \begin{pmatrix} \mathbf{G}_{bb} \\ \mathbf{G}_{db} \end{pmatrix} \mathbf{q}_b \quad (2.34)$$

In Eq. 2.34, suffixes b and d indicate respectively the boundary and interior points, and $\mathbf{I}$ is the unit matrix with a size equal to the number of interior points, and matrices $\mathbf{H}_{bd}$, $\mathbf{H}_{dd}$ and vector $\mathbf{q}_d$ are defined as follows:

$\mathbf{H}_{bd} = \mathbf{0}$: this submatrix corresponds to the collocation points along the boundary and the field points at the interior points. In this case q^* , the derivative of p^* along the outward normal to the boundary at the field point, is zero, because the field points correspond to the interior points. Further, as the matrix contains the non-diagonal term of $\mathbf{H}$, c_i term is also equal to 0. In other words, as evident from the right hand side of Eq. 2.17, the interpolation is only along the boundary, hence only the fixed points lying on the boundary contribute anything to the equation.

$\mathbf{H}_{dd} = \mathbf{I}$: this submatrix corresponds to collocation points and the field points that are both in the interior of the domain. In this case, contributions from the boundary integrals vanish for the reasons stated above; however, the diagonal elements will contribute $c_i = 1.0$ (for the collocation point in the interior domain). This will result in an unit diagonal matrix of size n_{ip} , equal to the number of interpolation points inside the domain.

$\mathbf{q}_d$ the vector of pressure gradients along the normal direction of the boundary is not defined with respect to the interior points, nor does it appear in the boundary integrals.

2.5.5 Inertia Interpolation Function

The fact that the inertia integral does not contain any derivatives provides a greater flexibility in choosing the interpolation function f required to approximate the inertia integral term (Nardini and Brebbia 1984). This very fundamental property of the inertia term is also utilized in evaluating the mass matrix of the FEM formulation, where the shape functions used for evaluating the mass matrix can be of a lower order than those used in evaluating the stiffness matrix. Refer, for example, to the concept of diagonal mass matrix.

Simple interpolation functions are quite effective in predicting the responses in at least the lower few modes. More sophisticated functions are recommended when higher mode responses are to be included. Several choices of these functions have been proposed by Nardini and Brebbia (1982,1984,1985). Tsai (1987), in his recent investigation for hydrodynamic pressures in an infinite reservoir, has adopted simple functions and has obtained good results.

The interpolation function in Tsai's model is:

$$\nabla^2 \psi_j^m \Rightarrow F_{mj} \Rightarrow \frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) \Rightarrow f_m(x_j, y_j) = r'_{mj} - r_{mj} \quad (2.35)$$

where (x_j, y_j) denotes the coordinate of the field point j ; m is the point chosen to represent the interpolation function; r_{mj} represents the radius vector from m to j and r'_{mj} that from the mirror image of m to j .

Integration of the harmonic function f twice results in

$$\psi_{jm} \Rightarrow \psi^m(x_j, y_j) = \frac{1}{9} (r'_{mj}{}^3 - r_{mj}^3) \quad (2.36)$$

$$\eta_{jm} \Rightarrow \eta^m(x_j, y_j) \Rightarrow \frac{\partial \psi_j^m}{\partial n} = \frac{1}{3} (r'_{mj}{}^2 \frac{\partial r'}{\partial n} - r_{mj}^2 \frac{\partial r}{\partial n}) \quad (2.37)$$

Tsai in his analysis assumed a perfectly horizontal reservoir bottom, and therefore modeled only a very small portion (near field length=5% of reservoir depth) of the infinite reservoir close to upstream face of the dam by BEM, and the remaining portion by continuum solution. Also, Tsai has reported the results corresponding to horizontal excitations alone. Further, the number, location and significance of the interior points representing the interpolation functions are not clearly outlined in Tsai's work. The accuracy of Tsai's model at higher frequency of excitations, particularly for considerable amount of near field

region is investigated in the present work, through the analysis of finite reservoir cases with stationary far end, and also for an infinite reservoir with fairly larger size of the near field. The results obtained are compared with those of a classical solution. As an improvement to Tsai's model a new function to evaluate the inertia integral term is proposed and its performance is examined.

The inertia interpolation function containing the first three terms of the natural logarithmic expansion series as given in equations 2.38 to 2.40, is tried as an alternative to Tsai's model. Tsai's function could be viewed as a model that considers only the first term of this series.

$$\nabla^2 \psi_j^m \Rightarrow F_{mj} \Rightarrow f_j^m = (2r'^3 - 9r'^2 + 18r') - (2r^3 - 9r^2 + 18r) \quad (2.38)$$

$$\psi_j^m = \left(\frac{2}{25}r'^5 - \frac{9}{16}r'^4 + 2r'^3 \right) - \left(\frac{2}{25}r^5 - \frac{9}{16}r^4 + 2r^3 \right) \quad (2.39)$$

$$\eta_j^m \Rightarrow \frac{\partial \psi_j^m}{\partial n} = \left(\frac{2}{5}r'^3 - \frac{9}{4}r'^2 + 6r' \right) r' \frac{\partial r'}{\partial n} - \left(\frac{2}{5}r^3 - \frac{9}{4}r^2 + 6r \right) r \frac{\partial r}{\partial n} \quad (2.40)$$

2.5.6 Choice of the Interpolation Points

From the preceding discussion, it is evident that the interpolation function f is dependent on r , the distance from the field point to a suitably chosen point X representing the interpolation function. As already stated, the obvious choice for these points will be that corresponding to the nodes of the boundary elements, since this avoids additional data input and storage associated with any other new set of points and results in a simplified form for the final equations. Because the boundary points serve the purpose of evaluating the inertia integral term over only the boundary of the domain, additional interior points located inside the reservoir domain are required for better representation of the inertia integral, increasing the number of dynamic degrees of freedom. These interior points were shown to be significant in the response calculations by Nardini and Brebbia (1982,1984,1985). However, an excessive number of such interior points tends to excite the higher modes, thereby causing the results to fluctuate (Loeffler et al., 1987). It should be noted here that for collocation corresponding to the interior points, the c_i value will be 1.0 in the boundary integral equation Eq. 2.14.

2.6 Radiation Condition at the Far End

For an infinite reservoir, both the pressure and pressure gradients are unknowns at

the interface of the near and the far field. This interface is also termed as transmitting boundary, as the energy dissipation due to infinite wave radiation in the reservoir occurs through this boundary. The frequency domain analysis of infinite layered media can be satisfactorily dealt with through a continuum representation in the infinite direction (Hall and Chopra 1982a) and a finite element discretization along the depth of the reservoir. This technique is generally more efficient and also more accurate than those employing infinite elements (Saini et al. 1978). In case of horizontal excitation, an analytical solution for the infinite rectangular part can be used to set up a relationship between the pressure and pressure gradients along the transmitting boundary, but a 1D FEM modeling of this interface makes the transfer matrices diagonal, and leads to a computationally efficient formulation.

2.6.1 Concept of Far Field Rigid Reservoir Bed

Using the 1D finite element discretization, the pressure and pressure gradients along the transmitting boundary are expressed in terms of a set of generalized coordinates. Then by a separation of variables, the governing equation of the far field together with the associated boundary conditions, can be transformed into an eigenvalue problem (Hall and Chopra 1980) as illustrated in Appendix B. This eigenvalue problem is complex valued and frequency dependent for an absorptive far field reservoir bottom, but real valued and frequency independent otherwise.

A near field consisting of an absorptive bottom and a rigid bottom far field will make the problem computationally more efficient, owing to the fact that the associated eigenvalue problem is real valued and needs to be evaluated only once for the entire range of frequencies analyzed. In fact, provided a suitable length is selected for the near field, the hydrodynamic pressures obtained with the absorptive bottom restricted to the near field are not significantly different from those obtained with an absorptive bottom of infinite extent. This is demonstrated by comparing the results of the present model with those of the classical solutions for an absorptive infinite reservoir bottom condition.

2.6.2 Boundary Condition at the Interface of Near and Far Field

The derivation of the special boundary condition has been presented (Appendix B) by Hall and Chopra (1980), and only the final results corresponding to the case of far field

with non-absorptive reservoir bottom are given below.

The total far field pressure and pressure gradients at the finite element nodes, with linear finite element formulations of the interface boundary, due to horizontal or vertical component of excitation, can be expressed in terms of the eigenvalues λ_n and eigenvector $\tilde{\phi}_n$ as:

$$\hat{\mathbf{p}} = \tilde{\Phi} \mathbf{v} + \rho_w \tilde{\Phi} \tilde{\mathbf{K}}^{-2} \tilde{\Phi}^T \mathbf{d} \quad (2.41a)$$

$$\hat{\mathbf{q}} = -\tilde{\Phi} \tilde{\mathbf{K}} \mathbf{v} \quad (2.41b)$$

in which $\mathbf{v}$ is a vector of modal coordinates, $\tilde{\Phi}$ represents the matrix containing the eigenvectors, $\tilde{\mathbf{K}}$ is a diagonal matrix with elements $\kappa_1, \kappa_2, \dots, \kappa_{n3}$, $n3$ is the number of finite element nodes along the interface boundary, $\kappa_n = \sqrt{\lambda_n - k^2}$; k is the wave number, a frequency dependent parameter as explained in Section 2.3; and $\mathbf{d}$ is a vector with only one non-zero entry equal to an input acceleration a_y corresponding to the base node. For the unit harmonic excitation case, since the free field acceleration along the vertical direction, $a_y=0$, the second term of Eq. 2.41a will vanish.

When κ is real valued, it is taken with a positive sign to ensure that the pressure tends to zero as $x \rightarrow \infty$. When κ is complex, both real and the imaginary parts should be positive; a positive imaginary part indicates that the waves radiate towards infinity without any reflection (Humar and Jablonski 1988).

2.6.3 Analytical Solution for the Far Field: Vertical Excitation

The hydrodynamic terms for the vertical excitation case obtained with a non-absorptive reservoir far field will still be infinite at the natural frequencies (corresponding to the depth of the truncation boundary) of the reservoir. These peaks are sharp and narrow compared to those obtained with non-absorptive reservoir bottom for both near and far field. One way to overcome these infinite resonant responses is to adopt a FEM treatment of far field corresponding to an absorptive far field, but as already mentioned, this will result in a complex valued and frequency dependent eigenvalue problem, thus drastically reducing the efficiency of the procedure. The other remedial approach is to use an analytical solution for the case where the bottom absorption of the far field is accounted for by simplified 1D wave absorption model. The analytical expression for the hydrodynamic pressure terms along this interface, due to the far field continuum for vertical excitation case is readily available

(Fenves and Chopra 1984), and can be easily implemented in the numerical solution of the reservoir domain. These expressions for hydrodynamic pressure and pressure gradient at the i th boundary element node along the transmitting boundary are respectively given by (Fenves and Chopra 1984):

$$\tilde{p}_3 = \frac{\rho\omega}{k} \frac{\sin k(H - y_i)}{(\cos kH + i\gamma c_\omega \sin kH)} \quad (2.42a)$$

$$\tilde{q}_3 = 0 \quad (2.42b)$$

where H is the reservoir depth at the transmitting boundary, and y_i the ordinate of the i th boundary element node from the reservoir bottom, along the transmitting boundary. Thus, the vertical excitation treatment of reservoir analysis requires neither the concept of absorptive near field and rigid far field, nor the treatment of far field by an eigenvalue problem.

It should be noted that, a necessary condition for the continuum solution of the far field to be valid, is that the pressure gradient along the transmitting boundary should vanish. This may not be satisfied in case of a reservoir near field with a non-rectangular geometry. But even under such circumstances the use of Eqs. 2.42 may give reasonable results, if the transmitting boundary is placed at a sufficiently large distance from the upstream face of the dam. This will be illustrated in Chapter 7, under the investigation of reservoirs with sloped bottom.

2.7 Final Equation for Reservoir Substructure

Equation 2.34 could be further partitioned and written in the following form:

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & -k^2 \tilde{M}_{14} \\ B_{21} & B_{22} & B_{23} & -k^2 \tilde{M}_{24} \\ B_{31} & B_{32} & B_{33} & -k^2 \tilde{M}_{34} \\ B_{41} & B_{42} & B_{43} & (I - k^2 \tilde{M}_{44}) \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \\ G_{41} & G_{42} & G_{43} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad (2.43)$$

wherein $B_{ij} = H_{ij} - k^2 \tilde{M}_{ij}$ and suffixes 1, 2, 3 and 4 represent respectively the upstream face of the dam, the reservoir bottom, the transmitting boundary, and the interior interpolation points.

Substitution for the boundary conditions along the upstream face and the reservoir bottom, together with the expression for pressure and pressure gradient at the far boundary, obtained from equations 2.3, 2.4, 2.41 or 2.42 respectively, in Eq. 2.13 gives the final BE equations as follows

For horizontal or vertical excitation

When the far field with rigid reservoir bottom is modeled by 1D FEM, the final boundary element equation reduces to:

$$\begin{pmatrix} \mathbf{B}_{11} & (\mathbf{B}_{12} + i\Omega\gamma\mathbf{G}_{12}) & (\mathbf{B}_{13}\tilde{\Phi} + \mathbf{G}_{13}\tilde{\Phi}\tilde{\mathbf{K}}) & -k^2\mathbf{M}_{14} \\ \mathbf{B}_{21} & (\mathbf{B}_{22} + i\Omega\gamma\mathbf{G}_{22}) & (\mathbf{B}_{23}\tilde{\Phi} + \mathbf{G}_{23}\tilde{\Phi}\tilde{\mathbf{K}}) & -k^2\mathbf{M}_{24} \\ \mathbf{B}_{31} & (\mathbf{B}_{32} + i\Omega\gamma\mathbf{G}_{32}) & (\mathbf{B}_{33}\tilde{\Phi} + \mathbf{G}_{33}\tilde{\Phi}\tilde{\mathbf{K}}) & -k^2\mathbf{M}_{34} \\ \mathbf{B}_{41} & (\mathbf{B}_{42} + i\Omega\gamma\mathbf{G}_{42}) & (\mathbf{B}_{43}\tilde{\Phi} + \mathbf{G}_{43}\tilde{\Phi}\tilde{\mathbf{K}}) & (\mathbf{I} - k^2\mathbf{M}_{44}) \end{pmatrix} \times$$

$$\begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{v} \\ \mathbf{p}_4 \end{pmatrix} = -\rho_w \begin{pmatrix} \mathbf{G}_{11}\mathbf{a}_n^l + \mathbf{G}_{12}\mathbf{b}_n^l + \mathbf{B}_{13}\tilde{\Phi}\tilde{\mathbf{K}}^{-2}\tilde{\Phi}^T\mathbf{d} \\ \mathbf{G}_{21}\mathbf{a}_n^l + \mathbf{G}_{22}\mathbf{b}_n^l + \mathbf{B}_{23}\tilde{\Phi}\tilde{\mathbf{K}}^{-2}\tilde{\Phi}^T\mathbf{d} \\ \mathbf{G}_{31}\mathbf{a}_n^l + \mathbf{G}_{32}\mathbf{b}_n^l + \mathbf{B}_{33}\tilde{\Phi}\tilde{\mathbf{K}}^{-2}\tilde{\Phi}^T\mathbf{d} \\ \mathbf{G}_{41}\mathbf{a}_n^l + \mathbf{G}_{42}\mathbf{b}_n^l + \mathbf{B}_{43}\tilde{\Phi}\tilde{\mathbf{K}}^{-2}\tilde{\Phi}^T\mathbf{d} \end{pmatrix} \quad (2.44)$$

For vertical excitation - analytical solution for the far field

When the analytical solution of absorptive far field is adopted, the boundary element equation corresponding to the vertical excitation case becomes

$$\begin{pmatrix} \mathbf{B}_{11} & (\mathbf{B}_{12} + i\Omega\gamma\mathbf{G}_{12}) & \mathbf{B}_{13} & -k^2\mathbf{M}_{14} \\ \mathbf{B}_{21} & (\mathbf{B}_{22} + i\Omega\gamma\mathbf{G}_{22}) & \mathbf{B}_{23} & -k^2\mathbf{M}_{24} \\ \mathbf{B}_{31} & (\mathbf{B}_{32} + i\Omega\gamma\mathbf{G}_{32}) & \mathbf{B}_{33} & -k^2\mathbf{M}_{34} \\ \mathbf{B}_{41} & (\mathbf{B}_{42} + i\Omega\gamma\mathbf{G}_{42}) & \mathbf{B}_{43} & (\mathbf{I} - k^2\mathbf{M}_{44}) \end{pmatrix} \times$$

$$\begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \\ \mathbf{p}_4 \end{pmatrix} = -\rho_w \begin{pmatrix} \mathbf{G}_{11}\mathbf{a}_n^y + \mathbf{G}_{12}\mathbf{b}_n^y \\ \mathbf{G}_{21}\mathbf{a}_n^y + \mathbf{G}_{22}\mathbf{b}_n^y \\ \mathbf{G}_{31}\mathbf{a}_n^y + \mathbf{G}_{32}\mathbf{b}_n^y \\ \mathbf{G}_{41}\mathbf{a}_n^y + \mathbf{G}_{42}\mathbf{b}_n^y \end{pmatrix} \quad (2.45)$$

Since an expression for $\mathbf{p}_3$ in Eq. 2.45 is known (Eq. 2.42a) the entire equation set corresponding to $\mathbf{p}_3$ can be dropped from Eq. 2.45.

Equation 2.44 or 2.45 can now be solved for the unknowns on the left hand side. It should be noted that $\mathbf{p}_1$, the vector of hydrodynamic pressures along the upstream face of the dam, is the unknown of primary interest in our study.

2.8 Corner Node Problem

In the BEM formulations of the reservoir substructure, each node is associated with two parameters, viz. a pressure and pressure gradient. One of these is known, while other needs to be determined by solving the boundary element equations.

In the case of a linear element formulation, when two successive boundary elements do not form one straight line, the junction between two such elements will give rise to a corner node. At such a node, although the hydrodynamic pressure values are unique, the pressure gradients are non-unique, due to the different normal directions of the two elements shared by the corner node. This poses some numerical problems, requiring special treatment.

Referring to Fig. 2.2, let c and d represent the two corner nodes of the rectangular reservoir. For simplicity, the interior points can be ignored for this treatment, as they pose no corner node problem. The i th equation of reservoir model (from equation system Eq. 2.44 or 2.45) can be written as:

$$\begin{pmatrix} B_{i1} & B_{i2} & B_{i3} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} G_{i1} & g_{ic}^L & g_{ic}^R & G_{i2} & g_{id}^L & g_{id}^R & G_{i3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_c^L \\ q_c^R \\ q_2 \\ q_d^L \\ q_d^R \\ q_3 \end{pmatrix} \quad (2.46)$$

Suffixes 1, 2 and 3 denote the quantities corresponding to the upstream face of the dam, reservoir bottom, and the transmitting boundary, respectively.

B_{i1} and p_1 include the respective terms corresponding to corner node c , B_{i3} and p_3 include the terms corresponding to corner node d , while G_{i1} and q_1 exclude the terms corresponding to corner node c , and G_{i3} and q_3 exclude the terms corresponding to corner node d .

B_{i2} , G_{i2} , p_2 and q_2 exclude both corner nodes c and d . q_c^L and q_c^R are respectively the pressure gradient along the normal direction of the boundary element situated to the left and right of the corner node c , and q_d^L and q_d^R are the corresponding quantities with respect to the corner node d . Similarly, g_{ic}^L , g_{ic}^R , g_{id}^L , g_{id}^R are the i th row elements of G , corresponding to the left and right boundary elements of the two corner nodes c and d .

For FEM formulation of rigid far field

The pressure gradients at the upstream face of the rigid dam can be written, using Eq. 2.3 as

$$\begin{pmatrix} q_1^L \\ q_c^L \end{pmatrix} = -\rho_w \begin{pmatrix} a_n^L \\ a_c^L \end{pmatrix} \quad (2.47)$$

The pressure gradients along the horizontal absorptive reservoir bottom can be expressed in terms of the respective hydrodynamic pressure, using Eq. 2.4,

$$\begin{pmatrix} q_c^{Hl} \\ q_2^{Ll} \\ q_d^{Ll} \end{pmatrix} = -\rho_w \begin{pmatrix} b_n^{lc} \\ b_n^l \\ b_n^{ld} \end{pmatrix} - (\iota\Omega\gamma) \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} \quad (2.48)$$

Note that p_c is the last entry of the vector $\mathbf{p}_1$, while p_d is the first entry of the vector $\mathbf{p}_3$.

The expression for pressure gradients and pressure along the transmitting boundary of the far field with non-absorptive bottom can be re-written as

$$\begin{pmatrix} q_d^R \\ q_3 \end{pmatrix} = -\tilde{\Phi} \tilde{K} v + \rho_w \tilde{\Phi} \tilde{K}^{-2} \tilde{\Phi}^T d \quad (2.49a)$$

and

$$\mathbf{p}_3 = \tilde{\Phi} v \quad (2.49b)$$

Substituting Eqs. 2.47, 2.48 and 2.49 in Eq. 2.46, and transferring all the unknowns to one side, Eq. 2.46 reduces to the following form, which is now devoid of any problems posed by the corner nodes.

$$\begin{aligned} & \left(\{ \mathbf{B}_{i1} + \iota\Omega\gamma g_{ic}^R \} \quad \{ \mathbf{B}_{i2} + \iota\Omega\gamma \mathbf{G}_{i2} \} \quad \{ (\mathbf{B}_{i3} + \iota\Omega\gamma g_{id}^L) \tilde{\Phi} + (g_{id}^R \quad \mathbf{G}_{i3}) \tilde{\Phi} \tilde{K} \} \right) \begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ v \end{pmatrix} \\ &= -\rho_w \left(\mathbf{G}_{i1} \mathbf{a}_n^l + g_{ic}^L a_n^c + g_{ic}^R b_n^c + \mathbf{G}_{i2} \mathbf{b}_n^l + g_{id}^L b_n^{ld} + (\mathbf{B}_{i3} + \iota\Omega\gamma g_{id}^L) \tilde{\Phi} \tilde{K}^{-2} \tilde{\Phi}^T d \right) \end{aligned} \quad (2.50)$$

For vertical excitation far field continuum solution

A similar treatment for the corner node is also valid for the BEM equations of vertical excitation modeled with continuum solution of absorptive far field, and the final equation set (ignoring the quantities corresponding to the interior points, for simplicity) is given by

$$\begin{aligned} & \left(\{ \mathbf{B}_{i1} + \iota\Omega\gamma g_{ic}^R \} \quad \{ \mathbf{B}_{i2} + \iota\Omega\gamma \mathbf{G}_{i2} \} \quad \{ \mathbf{B}_{i3} + \iota\Omega\gamma g_{id}^L \} \right) \begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{pmatrix} \\ &= -\rho_w \left(\mathbf{G}_{i1} \mathbf{a}_n^y(s) + g_{ic}^L a_n^y(s) + g_{ic}^R b_n^y(s') + \mathbf{G}_{i2} \mathbf{b}_n^y(s') + g_{id}^L b_n^y(s') \right) \end{aligned} \quad (2.51a)$$

and for the special case of rectangular reservoir near field, the above equation reduces to

$$\begin{aligned} & \left(\{ \mathbf{B}_{i1} + \iota\Omega\gamma g_{ic}^R \} \quad \{ \mathbf{B}_{i2} + \iota\Omega\gamma \mathbf{G}_{i2} \} \quad \{ \mathbf{B}_{i3} + \iota\Omega\gamma g_{id}^L \} \right) \begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{pmatrix} \\ &= \rho_w \left(g_{ic}^R + \mathbf{G}_{i2} + g_{id}^L \right) \end{aligned} \quad (2.51b)$$

The formulations developed in the present chapter will be investigated through numerical examples, by comparing with the results of classical solutions in Chapter 3, and will be subsequently used in the analyses of the coupled dam system, including the reservoir interaction effects.

Fig. 2.1 Model of the Infinite-Reservoir Substructure

Fig. 2.2 Corner Nodes in an Infinite Rectangular Reservoir Linear Boundary Element Formulations

3 ANALYSIS OF THE RESERVOIR MODEL

3.1 Introduction

The boundary element formulations for the evaluation of hydrodynamic pressures in a reservoir were discussed in detail under Chapter 2. Computer programs with algorithms that implement the formulations have been developed. The results of numerical studies carried out for both finite and infinite reservoirs, impounded by a rigid dam, are presented in this chapter. For the case of a reservoir with an absorptive bottom, the fluid-foundation interaction effect is accounted for by means of 1D approximate model of Hall and Chopra (1980).

The effectiveness of different interpolation functions, and layouts of interior points, namely, their number and location, in representing the inertia term for the reservoir model, obtained from a BEM using the static fundamental solution, is studied for both the finite and infinite reservoir cases. The use of a far field rigid reservoir bottom, adopted in the infinite reservoir analysis, is closely examined through numerical studies. Finally, the role of wave reflection coefficient in the 1D reservoir bottom absorption model is demonstrated for both horizontal and vertical excitations. The results are compared with the classical solution where available, and with results obtained by other researchers.

Number of boundary elements and their size

The boundary element mesh refinement required for the reservoir model is strongly dependent on the upper limit of the excitation frequency to be analyzed. For adequate representation of the pressure variation, the size of the element should be a function of the smallest wave length, which corresponds to the largest excitation frequency. The element size should be such that a minimum of 8 to 10 elements are contained within this wave length (Humar and Jablonski, 1988). This criterion is also applicable in choosing the size

of the finite elements located along the transmitting boundary of the far field. An optimum mesh can also be arrived at through a convergence study.

3.2 Finite Reservoir Study

As described in Chapter 2, two types of inertia interpolation functions were selected, one with only the first term in the natural logarithmic expansion series (Tsai, 1987), and the other containing the first three terms of the same series. The purpose of this study is primarily to ascertain the accuracy of these models, especially for larger length of the finite reservoir, modeled by BEM. Also, the number of additional inertia interpolation points required inside the finite reservoir domain, and their location is investigated. The numerical results corresponding to Tsai's interpolation function were obtained by the author from an implementation of Tsai's algorithms (Tsai, 1987).

For this study, finite rectangular reservoirs with three different lengths, $L = H$, $L = 2H$ and $L = 3H$, as shown in Fig. 3.1, are considered, where H is the depth of the reservoir, taken here as equal to 100 m (330 ft). The velocity of sound in water c_w is 1440 m/sec (4720 ft/sec), which gives a reservoir fundamental frequency of $\omega_{1res} = \frac{\pi c_w}{2H} = 22.62$ rad/sec (3.6 Hertz). The density of water is taken as 1000 kg/m³ (62.4 lb/ft³). A stationary far end boundary condition is adopted. A uniform boundary element mesh, with the length of each boundary element $= 0.1H$ is used in the discretization. This mesh contains 30, 40 and 50 boundary elements respectively for the three cases analyzed, and is capable of analyzing the results accurately for excitation frequencies upto $4 \times \omega_{1res} = 90.48$ rad/sec (14.5 Hertz). The reservoir bottom is considered to be rigid for this analysis, which amounts to absence of wave absorption along the reservoir bed. This assumption implies that the fluid foundation interaction effect is ignored.

Based on a trial study, the number and location of the interior points were arrived at. Figure 3.1 shows the distribution of interior points in the finite reservoir. The variations of normalized hydrodynamic pressure $\frac{p}{\rho_w a H}$ along the upstream face of a rigid dam, resulting from a unit horizontal harmonic ground motion (parameter a denotes the amplitude of the harmonic ground acceleration which in the present case is unity), at selected excitation frequencies are shown in Figs. 3.2 to 3.11. Constant element BEM model is adopted for this preliminary study. The results of the corresponding classical solutions (Hanna, 1980) are

also included to compare the performance of the two models under consideration. While doing so the values of hydrodynamic pressures are evaluated at discrete points corresponding to the boundary element nodes of the upstream face of the dam.

From this study, it can be concluded that, as the length of the finite reservoir increases, the error associated with the Tsai's model which uses only the first term of the logarithmic series increases, for most of the excitation frequencies analyzed. On the other hand, the proposed model using the first three terms seem to yield results that are closer to the classical solutions for the frequencies, and reservoir near field lengths analyzed.

A trial and error analysis showed that the location of interior nodes obtained by placing such nodes at a distance of one quarter the reservoir depth from the reservoir boundary, as well as from each other was found to be convenient and effective in the analysis of the reservoir near field region.

3.3 Infinite Reservoir Study

Initial studies showed that for the reservoir analysis, all the three BEM formulations, namely constant, linear and quadratic boundary elements gave results of similar quality. Only the linear boundary element formulation is considered for the subsequent investigations as it offers the advantage of a direct coupling of the BEM model of the reservoir near field with the linear finite elements along the transmitting boundary of the reservoir far field, as well as with the linear finite elements of the dam substructure along the upstream face of dam, when the dam-reservoir interaction effects are included.

The complex frequency response functions of the hydrodynamic force acting on the upstream face of a rigid dam, are compared with classical solutions (Chopra, 1967) for a rectangular infinite reservoir with rigid reservoir bottom. The hydrodynamic pressure distributions along the dam face are also evaluated using the proposed BEM model and compared with the corresponding classical solutions for accuracy, and with the frequency dependent BEM model to estimate the computational efficiency of the procedure. Both horizontal and vertical harmonic excitations are considered for this study. Also, the effect of reservoir bottom absorption on the hydrodynamic response is investigated and the results are compared with those obtained by Hall and Chopra (1980).

A study similar to that of the finite reservoir case was carried out for infinite reservoir,

with different lengths of the near field modeled by BEM. The purpose of this study was to ascertain the accuracy of the different interpolation function type, and to establish an appropriate choice for the interior points. This study revealed that the interpolation function containing only the first term of natural logarithmic expansion series, i.e. the one corresponding to Tsai's model, gave fairly accurate results for most of the frequencies and near field lengths of the infinite reservoir analyzed. Owing to the simple nature of the function used, and because it gives good results for the infinite reservoir case, Tsai's model will be used in all future analyses of infinite reservoir. Also, it was found that, the same configuration of interior points as used in a finite reservoir model is appropriate for the BEM analysis of the near field of an infinite reservoir, for both components of excitation.

3.3.1 Reservoir With Non-Absorptive Bottom

For an infinite reservoir with horizontal bed, and with no bottom absorption ($\alpha_r = 1.0$), a continuum solution can be used in this case, since the reservoir is rectangular. However, in order to test the effectiveness of the boundary element solution, the reservoir is divided into a near field and an infinite far field. A boundary element solution is used for the near field while a finite element treatment is adopted for the far field. To demonstrate the accuracy of BEM model of reservoir, the length of the near field region is chosen to be equal to $L_f = 2.0H$. The numerical model for this case is same as shown in Fig. 3.1. The reservoir water properties are identical to those considered in the finite reservoir study. The variation of normalized hydrodynamic pressure along the upstream face of a rigid dam, resulting respectively due to a unit horizontal harmonic and vertical excitations, at selected excitation frequencies are shown in Figs. 3.12 to 3.21. The corresponding classical solution results are also included for comparison. From these plots it is clear that the reservoir BEM model developed in the present study provides estimates of hydrodynamic pressure distribution which are in reasonable comparison with the results of classical solution. The quantitative differences are somewhat large in isolated cases. However, it will be seen from results presented later that the total hydrodynamic pressures predicted by the BEM are in much better agreement with the classical solution.

The total hydrodynamic force normalized with respect to the total hydrostatic force along the upstream face of a rigid dam is plotted against the excitation frequency normalized with respect to the fundamental frequency of the reservoir, for both a horizontal and a

vertical excitation. Again, a near field region of $L_f = 2.0H$ is considered for this analysis. Figures 3.22 show the real, imaginary, and the absolute values of the frequency response function of normalized hydrodynamic force due to unit horizontal harmonic excitation, while Figs. 3.23 show the response functions of the real and absolute values of hydrodynamic force for a vertical harmonic excitation. The results obtained from a BEM model are in good agreement with the corresponding classical solutions.

3.3.2 Reservoir Bottom Absorption Effect

The significance of the concept of far field rigid bottom, that is, confining the absorption effect to the near field, in the case of infinite reservoir possessing bottom absorption was explained in Section 2.6.1. Based on a convergence study, the minimum length of the absorptive near field to be considered, which will be capable of predicting results close to those of the infinite absorptive bottom reservoir case is arrived at. A classical solution for an infinite reservoir with an absorptive bottom based on the work of Hall and Chopra (1980) is utilized to provide bench mark results for purpose of comparison.

Convergence Study

For the horizontal excitation case, this is examined by considering different absorptive near field lengths, $L_f = H, 2H, 2.5H, \& 3H$, with the coefficient of wave reflection $\alpha_r = 0.85$. The layout of interior points is taken to be similar to that used in the finite reservoir case (Fig. 3.1). In all the cases analyzed, the reservoir far field is assumed to possess a rigid bottom, implying that the eigenvalue problem associated with the infinite radiation of the far field is real and independent of the excitation frequencies. The response function of hydrodynamic force on the upstream face of a rigid dam, with an absorptive reservoir bottom, due to horizontal harmonic excitation is shown in Figs. 3.24, where only the results corresponding to $L_f = 2H, 2.5H, \& 3H$ have been plotted. This figure show that the response converges to the classical solution at a certain length of the near field. It is seen that for an horizontal excitation case, the results corresponding to both $L_f = 2.5H$ and $L_f = 3.0H$ are close to the classical solution for an infinite absorptive bottom (Hall and Chopra, 1980). Therefore, for a horizontal excitation, it is sufficient to consider a near field length of $L_f = 2.5H$, while modeling an infinite reservoir with absorptive bottom.

Using a near field length of $L_f = 2.5H$ arrived on the basis of convergence analysis for horizontal excitation, the concept of absorptive near field is verified for the vertical

harmonic excitation. The reservoir far field is assumed to be non-absorptive and is modeled by 1D FEM. The response results are compared against the hydrodynamic force obtained by continuum solution for an infinite rectangular reservoir with absorptive bottom. From Figs. 3.25, it is clear that the hydrodynamic force distribution obtained from the two methods compare well, except for the sharp and narrow peak values corresponding to the natural frequency of the reservoir, obtained for the FEM modeling of the far field with non-absorptive bottom. This implies that in the analysis of a reservoir subject to either components of ground motion, it is enough to consider a near field length of $L_f = 2.5H$ as absorptive and to treat the far field as non-absorptive.

If the reservoir is rectangular, an analytical solution exists for the hydrodynamic pressures and there is no need to use a boundary element discretization. In the analytical studies referred to above, the reservoir was assumed to be rectangular so that BEM results could be compared with a classical solution. Specifically, for the case of vertical excitation, the rectangular reservoir may be modeled by a near field discretized by boundary elements and the far field by continuum solution for an absorptive bottom. Such a continuum solution is simple and straight forward, unlike in the case of horizontal excitation. The response results obtained for a vertical excitation using a near field length $L_f = 2.5H$ and a far field continuum solution for absorptive reservoir bed are plotted in Fig. 3.25 and almost match the classical solution.

As stated above for vertical excitation case, the FEM treatment of the reservoir far field having non-absorptive bottom will give sharp and narrow peaks in the complex frequency response functions corresponding to the natural frequencies of the reservoir. These peaks will not appear if the far field was considered to have an absorptive bottom. However in that case the finite element treatment leads to a complex eigenvalue problem whose solution will certainly take more computational effort than the solution of the corresponding real eigenvalue problem. If a continuum solution for absorptive bottom could be used, this problem will be overcome. However, the validity of continuum solutions of an absorptive far field is subject to the condition that the pressure gradient vanish along the transmitting boundary. This may not be the case for a reservoir near field with non-rectangular geometry, but even under such circumstances a continuum solution may be applicable for response evaluation under vertical excitation, provided the transmitting boundary is located sufficiently far from the upstream face of the dam. This fact is demonstrated for a

dam-reservoir system with sloped reservoir bottom in Chapter 7.

The role of reservoir bottom absorption effect on the reservoir response, is illustrated by a plot of frequency response function for horizontal and vertical excitation cases, for three different values of α_r namely, 1.0 (non-absorptive), 0.85 and 0.5 (absorptive). The plots are shown in Figs. 3.26 and 3.27. It is clear from these sketches that a lower value of α_r leads to a higher reduction in response at excitation frequencies close to the resonant frequencies of the reservoir and that in the absence of such bottom absorption, the resonant responses are unbounded.

Fig. 3.1 Boundary Element Model for the Reservoir Near Field

Fig. 3.2 Hydrodynamic Pressure on a Rigid Dam; $L = H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{1res} = 2.5$

Fig. 3.3 Hydrodynamic Pressure on a Rigid Dam; $L = 2H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{1res} = 1.6$

Fig. 3.4 Hydrodynamic Pressure on a Rigid Dam; $L = 2H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{res} = 2.0$

Fig. 3.5 Hydrodynamic Pressure on a Rigid Dam; $L = 2H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{res} = 2.5$

Fig. 3.6 Hydrodynamic Pressure on a Rigid Dam; $L = 3H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{lres} = 1.2$

Fig. 3.7 Hydrodynamic Pressure on a Rigid Dam; $L = 3H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{lres} = 1.6$

Fig. 3.8 Hydrodynamic Pressure on a Rigid Dam; $L = 3H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{lres} = 1.85$

Fig. 3.9 Hydrodynamic Pressure on a Rigid Dam; $L = 3H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{lres} = 2.0$

Fig. 3.10 Hydrodynamic Pressure on a Rigid Dam; $L = 3H\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{lres} = 2.5$

Fig. 3.11 Hydrodynamic Pressure on a Rigid Dam; $L = 3H$; $\alpha_r = 1.0$
Finite Reservoir; Horizontal Excitation; $\Omega / \omega_{lres} = 3.8$

Fig. 3.12a Real Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$, $\Omega / \omega_{res} = 1.2$

Fig. 3.12b Imaginary Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$, $\Omega / \omega_{res} = 1.2$

Fig. 3.13a Real Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 1.6$

Fig. 3.13b Imaginary Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 1.6$

Fig. 3.14a Real Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$, $\Omega / \omega_{1res} = 1.85$

Fig. 3.14b Imaginary Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$, $\Omega / \omega_{1res} = 1.85$

Fig. 3.15a Real Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 2.0$

Fig. 3.15b Imaginary Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 2.0$

Fig. 3.16a Real Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$, $\Omega / \omega_{res} = 2.5$

Fig. 3.16b Imaginary Part of Hydrodynamic Pressure on a Rigid Dam;
Infinite Reservoir; Horizontal Excitation $\alpha_r = 1.0$, $\Omega / \omega_{res} = 2.5$

Fig. 3.17 Hydrodynamic Pressure on a Rigid Dam; $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 1.2$
Infinite Reservoir; Vertical Excitation

Fig. 3.18 Hydrodynamic Pressure on a Rigid Dam; $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 1.6$
Infinite Reservoir; Vertical Excitation

Fig. 3.19 Hydrodynamic Pressure on a Rigid Dam; $\alpha_r = 1.0$, $\Omega / \omega_{res} = 1.85$
Infinite Reservoir; Vertical Excitation

Fig. 3.20 Hydrodynamic Pressure on a Rigid Dam; $\alpha_r = 1.0$, $\Omega / \omega_{res} = 2.0$
Infinite Reservoir; Vertical Excitation

Fig. 3.21 Hydrodynamic Pressure on a Rigid Dam; $\alpha_r = 1.0$; $\Omega / \omega_{1res} = 2.5$
Infinite Reservoir; Vertical Excitation

Fig. 3.22a Real Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 1.0$
Infinite Reservoir; Harmonic Horizontal Excitation

Fig. 3.22b Imaginary Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 1.0$
Infinite Reservoir; Harmonic Horizontal Excitation

Fig. 3.22c Absolute Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 1.0$
Infinite Reservoir; Harmonic Horizontal Excitation

Fig. 3.23a Real Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 1.0$
Infinite Reservoir; Harmonic Vertical Excitation

Fig. 3.23b Absolute Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 1.0$
Infinite Reservoir; Harmonic Vertical Excitation

Fig. 3.24a Real Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 0.85$
Convergence Study of Bottom Absorption Effect; Horizontal Excitation

Fig. 3.24b Imaginary Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 0.85$
Convergence Study of Bottom Absorption Effect; Horizontal Excitation

Fig. 3.24c Absolute Part of Hydrodynamic Force on a Rigid Dam; $\alpha_r = 0.85$
Convergence Study of Bottom Absorption Effect; Horizontal Excitation

Fig. 3.25a Real Part of Hydrodynamic Force on a Rigid Dam;
Treatment of Reservoir Far Field; Vertical Excitation $\alpha_r = 0.85$

Fig. 3.25b Imaginary Part of Hydrodynamic Force on a Rigid Dam;
Treatment of Reservoir Far Field; Vertical Excitation $\alpha_r = 0.85$

Fig. 3.25c Absolute Part of Hydrodynamic Force on a Rigid Dam;
Treatment of Reservoir Far Field; Vertical Excitation $\alpha_r = 0.85$

Fig. 3.26a Real Part of Hydrodynamic Force on a Rigid Dam;
Reservoir Bottom Absorption Effect; Horizontal Excitation

Fig. 3.26b Imaginary Part of Hydrodynamic Force on a Rigid Dam;
Reservoir Bottom Absorption Effect; Horizontal Excitation

Fig. 3.26c Absolute Part of Hydrodynamic Force on a Rigid Dam;
Reservoir Bottom Absorption Effect; Horizontal Excitation

Fig. 3.27a Real Part of Hydrodynamic Force on a Rigid Dam;
Reservoir Bottom Absorption Effect; Vertical Excitation

Fig. 3.27b Imaginary Part of Hydrodynamic Force on a Rigid Dam;
Reservoir Bottom Absorption Effect; Vertical Excitation

Fig. 3.27c Absolute Part of Hydrodynamic Force on a Rigid Dam;
Reservoir Bottom Absorption Effect; Vertical Excitation

4

DYNAMIC STIFFNESS MATRIX OF FOUNDATION SOIL

4.1 Introduction

The seismic response of a dam is expected to be affected by the flexibility of the foundation soil and the resulting interaction between the dam and the soil. The soil-structure interaction effect becomes more predominant as either the flexibility of the foundation soil or the stiffness of the superstructure increases (Wolf, 1986). The modulus of elasticity of the foundation material at a dam site is often of the same magnitude as that of the dam material, emphasising the error involved in the rigid foundation assumption, and the importance of dynamic soil-structure interaction effects in the response evaluation of dams.

This chapter describes the procedures used in the numerical treatment of the governing differential equations of elastodynamics related to the soil domain, and of the corresponding boundary conditions. Also presented, is the approach used in obtaining the dynamic stiffness matrix of the soil substructure, required for the analysis of dam resting on a flexible foundation. A frequency domain procedure using the substructure approach is used in evaluating the soil impedance matrix by BEM. The accuracy of the procedures described is verified by evaluating the dynamic stiffness coefficients of a rigid strip footing and comparing the results with the classical solutions of Karasudhi et al. (1968). The model developed here will be utilized to analyze the foundation soil below the dam structure, in the coupled analysis of dam system in the subsequent chapters and also in the BEM treatment of fluid-foundation interaction by a rigorous approach as discussed in Chapter 6.

4.2 BEM Modeling of Foundation Soil

The foundation soil substructure is analyzed by an appropriate technique to evaluate the complex valued frequency dependent impedance matrix, also referred to as the dynamic soil-stiffness matrix, corresponding to the degrees of freedom of the soil-structure interface nodes. The real part of the soil impedance matrix contributes to the stiffness, while the imaginary part provides damping into the associated structural system. Damping in the foundation arises from infinite radiation phenomenon in soil half plane, also known as the geometric damping. In case of viscoelastic soil, additional damping, termed as material damping, exists whose magnitude depends on the material properties of the soil.

The BEM is an attractive tool for modeling the unbounded soil domains for the following reasons:

- (a) BEM can efficiently handle arbitrary geometry of the soil surface, including foundation embedments,
- (b) the infinite radiation condition is automatically satisfied by the fundamental solution of the BEM, circumventing the wave reflection problems posed by the artificial boundary in FEM,
- (c) because in a soil-structure interaction problem only the quantities along the boundary are of interest, the interior nodal points of FEM are redundant. On the other hand, with the BEM, only the surface needs to be discretized. The BEM thus reduces the dimensionality of the problem by one, thereby increasing the computational efficiency and decreasing the data input,
- (d) In comparison to FEM, the BEM involves fewer approximations in representing the solution parameters, and hence possess better resolution.

The direct BEM formulation is preferred as it deals with real physical variables and is more versatile. The formulation is made possible through the application of weighted residual techniques, or alternatively, reciprocal identities viz. Betti, Maxwell, together with the concept of fundamental solution for the governing differential equation (Tan, 1987).

In using the BEM to model the soil domain, the following assumptions have been made

1. The soil-substructure below the dam is idealized by a two-dimensional, semi infinite half

plane model, in plane stress or plane strain. However, for compatibility, both the dam and foundation soil should follow the same idealization for stress or strain condition. Since the present research deals with the study of 2D gravity dam systems, which is best represented by plane stress idealization (Chopra and Chakrabarthi, 1972), hence the same plane stress idealization is also extended to the foundation soil analysis as well.

2. Soil material is assumed to be homogeneous, isotropic, and linear. Both elastic and viscoelastic half plane treatment of the soil substructure are considered.
3. In the case of viscoelastic soil, material damping is assumed to be constant hysteretic type, independent of the excitation frequency.

4.3 Governing Differential Equations

The elastodynamic governing equation in the transformed frequency domain for a homogeneous, isotropic, and linear elastic region is given in an indicial form by (Dominguez 1978, Wolf 1986)

$$(c_1^2 - c_2^2) u_{j,ij} + c_2^2 u_{i,jj} + \Omega^2 u_i + b_i = 0. \quad (4.1)$$

where c_1 and c_2 are respectively the dilatational and shear wave (also known as P-wave and S-wave) velocities, obtained from

$$c_1^2 = \frac{(\lambda + 2\mu)}{\rho_f} ; c_2^2 = \frac{\mu}{\rho_f} \quad (4.2)$$

ρ_f is the mass density of the foundation soil, λ and μ the Lamé's constant for the soil material, u the displacement vector, Ω the excitation frequency, and b_i the body force per unit volume divided by the mass density of the soil. For a 2D case, the values of i and j vary between 1 and 2, corresponding to the two Cartesian co-ordinate directions.

4.3.1 Boundary Conditions

The boundary conditions along the surface of the soil domain are such that, tractions are specified along a part of the boundary, where displacements are unknown, while displacements are specified along the rest of the boundary, where the tractions are unknown. The far end boundary condition is automatically accounted for by the fundamental solution adopted for the foundation substructure.

In the present analysis, for the evaluation of impedance matrix by flexibility method, the traction is fully specified all along the finite length of the free surface of the soil boundary, and it has either a prescribed value or is zero (traction free boundary).

4.4 Boundary Integral Equation

Assuming zero body forces, and applying the weighted residual technique, Eq. 4.1 transforms to

$$C_{ij}u_j(P) + \int_S T_{ji}^*(P, Q)u_i(Q) ds - \int_S t_i(Q) U_{ji}^*(P, Q)ds = 0 \quad (4.3)$$

where P is called a source point and Q is a field point located on the boundary. Notations u_i and t_i denote the displacement and traction vectors along the soil surface, U_{ji}^* and T_{ji}^* are the respective Green's functions or the fundamental solutions, and S represents the boundary of the soil domain. Constants C_{ij} depends on the location of P , when P is located on a smooth surface, $C_{ij} = 0.5\delta_{ij}$.

Equation 4.3 is referred to as the boundary integral equation governing the soil domain. This equation is the dynamic equivalent of Somigliana's identity, analogous to the Green's third identity in potential theory. The weighted residual technique is a more general method of deriving the boundary element equation and is equally applicable to FEM as well as to hybrid methods which combines BEM and FEM.

4.5 Fundamental Solutions

The fundamental solution to 2D elastodynamic problem represents the dynamic displacement response due to a unit dynamic concentrated load applied at a point in the domain, and acting along each of the two Cartesian co-ordinate directions.

The weighting function for displacement u_i^* , is chosen, so as to satisfy the governing differential equation

$$(c_1^2 - c_2^2) u_{j,ij}^* + c_2^2 u_{i,jj}^* + \Omega^2 u_i^* + \Delta P \epsilon_i = 0. \quad (4.4)$$

where ϵ_i is the unit vector along the i th Cartesian direction, and ΔP is a Dirac delta function centred at source point P .

The fundamental solution for displacement and associated traction are expressed in tensor form, as follows

$$u_i^* = U_{ji}^* e_j ; t_i^* = T_{ji}^* e_j \quad (4.5)$$

4.5.1 Fundamental Solutions for Elastic Soil Half plane

The infinite space fundamental solution tensors, for displacements and tractions in a 2D plane strain field, for the case of elastic soil half plane, are given by (Cruse and Rizzo 1968, Dominguez 1978, Abdalla 1984)

$$U_{ji}^* = \frac{t}{4\mu} [\Theta \delta_{ij} - \Upsilon r_{,i} r_{,j}] \quad (4.6)$$

$$T_{ji}^* = \frac{t}{4} \left[\left(\frac{d\Theta}{dr} - \frac{\Upsilon}{r} \right) \left(\delta_{ij} \frac{\partial r}{\partial n} + r_{,i} n_{,j} \right) - \frac{2\Upsilon}{r} \left(n_{,i} r_{,j} - 2r_{,i} r_{,j} \frac{\partial r}{\partial n} \right) - 2 \frac{d\Upsilon}{dr} r_{,i} r_{,j} \frac{\partial r}{\partial n} + \frac{\lambda}{\mu} \left(\frac{d\Theta}{dr} - \frac{d\Upsilon}{dr} - \frac{\Upsilon}{r} \right) n_{,i} r_{,j} \right] \quad (4.7)$$

where

$$\Theta = -H_0^{(2)}(k_2 r) + \frac{1}{k_2 r} \left[H_1^{(2)}(k_2 r) - \Gamma H_1^{(2)}(k_1 r) \right] \quad (4.8)$$

$$\Upsilon = H_2^{(2)}(k_2 r) - \Gamma^2 H_2^{(2)}(k_1 r) \quad (4.9)$$

In Eqs. 4.6 through 4.9, $k_n = \frac{\Omega}{c_n}$ represents the wave number, $\Gamma = \frac{c_2}{c_1}$, $H_m^{(2)}$ is the Hankel function of m th order and second kind, r is the radius vector measured from the source point P to the field point Q located on a soil surface, $r_{,i} = \frac{\partial r}{\partial x_i}$, and n indicates the outward normal to the surface.

Differentiating Θ and Υ with respect to r and substituting in Eq. 4.7, the expression for fundamental solution of traction can be written in the following form

$$T_{ji}^* = \frac{t}{4} \left[D_1 \left(\delta_{ij} \frac{\partial r}{\partial n} + r_{,i} n_{,j} \right) + D_2 r_{,i} r_{,j} \frac{\partial r}{\partial n} + D_3 n_{,i} r_{,j} \right] \quad (4.7a)$$

where

$$D_1 = k_2 H_1^{(2)}(k_2 r) - 2 \frac{\Upsilon}{r} \quad (4.10a)$$

$$D_2 = 2 \left[4 \frac{\Upsilon}{r} + k_2 \left(\Gamma^3 H_1^{(2)}(k_1 r) - H_1^{(2)}(k_2 r) \right) \right] \quad (4.10b)$$

$$D_3 = \left(1 - 2\Gamma^2 \right) k_1 H_1^{(2)}(k_1 r) - 2 \frac{\Upsilon}{r} \quad (4.10c)$$

It is worth mentioning that the above fundamental solutions are expressed in terms of the Hankel function of second kind, as these functions in association with unit harmonic excitation $e^{i\Omega t}$, correctly represents the outgoing waves (Hanna and Humar, 1982), which accounts for proper infinite wave radiation in the foundation and prevents any spurious wave reflection from infinity, a problem usually encountered in FEM formulation of the soil. Since the fundamental solution satisfies the radiation condition at infinity, only the soil-structure interface along with a small finite region of the soil surface needs to be discretized, substantially increasing the efficiency of the procedure.

The fundamental solutions for a 2D plane stress idealization could be derived from the above expressions by using a modified Poisson's ratio $\bar{\nu}$ and Lamé's constant $\bar{\lambda}$ given by $\bar{\nu} = \frac{\nu}{(1+\nu)}$ and $\bar{\lambda} = \frac{2\mu\nu}{1-2\nu}$ in place of ν and λ . Note that the value of μ remains unchanged in both plane stress and plane strain idealization, while the Young's modulus of elasticity for plane stress case changes to $\bar{E} = E(1 - \nu^2)$.

Fundamental Solutions for Horizontal Soil Surface

For the special case of foundation soil with a horizontal surface, the expression of fundamental solutions for displacement and traction reduces to a very simplified form, and can be written as a 2×2 matrix

$$\mathbf{U}^* = \frac{t}{4\mu} \begin{pmatrix} (\Theta - \Upsilon) & 0 \\ 0 & \Theta \end{pmatrix} \quad (4.11)$$

$$\mathbf{T}^* = \frac{t}{4} \frac{x_1}{|x_1|} \begin{pmatrix} 0 & D_3 \\ D_1 & 0 \end{pmatrix} \quad (4.12)$$

where $\frac{x_1}{|x_1|}$ represents the sign of x_1 .

4.5.2 Fundamental Solutions for Visco-elastic Soil Half plane

It is convenient to represent the material damping in the soil through a simple visco-elastic model. Although it may not be the most accurate way, this approximation leads to fairly correct representation (Beskos, 1987). The effect of material damping increases with a decrease in the foundation radius, and is most pronounced for a shallow foundation or a layered soil medium. In the case of semi-infinite soil half plane, the role of infinite radiation damping is more prominent as compared to material damping (Novak and Hifnaway 1983). This will be also demonstrated in the future chapters. Experimental evidence suggests that

the material damping is almost independent of excitation frequency (Chopra et al. 1976, Israil and Banerjee 1990), thus establishing the validity of a hysteretic damping model.

The material damping in a viscoelastic foundation soil may be accounted for by using a fundamental solution which is a modified form of its elastic counterpart. In frequency domain, this can be obtained by the use of so-called 'correspondence principle', wherein the frequency independent hysteretic damping in the foundation material is accounted approximately by replacing the elastic Lamé's constants λ and μ , by their complex equivalents λ^* and μ^* (Veletsos and Verbic 1973, Chopra et al. 1976, Manolis and Beskos 1988, Beskos 1987). This principle implicitly assumes that the Poisson's ratio for viscoelastic materials is real valued, and is equal to the value of the corresponding elastic problem. This assumption is quite valid for $\nu=0.5$, and the error associated with other values of ν are negligible for most engineering purposes (Veletsos and Verbic, 1973).

For a viscoelastic soil, adopting a constant hysteresis damping model with a frequency independent damping constant η_s , the complex valued, modified Lamé's constants are defined by

$$\lambda^* = \lambda(1 + i\eta_s) ; \mu^* = \mu(1 + i\eta_s) \quad (4.13)$$

Damping constant η_s is also referred to as coefficient of energy loss or structural damping coefficient.

Note that the modified Lamé's constants enter in the fundamental solutions directly, as well as indirectly through the expression for the wave velocities c_1^* and c_2^*

$$c_1^{*2} = \frac{(\lambda^* + 2\mu^*)}{\rho_f} ; c_2^{*2} = \frac{\mu^*}{\rho_f} \quad (4.14a)$$

Also, it can be shown that

$$c_1^* = c_1(1 + i\eta_s)^{1/2} ; c_2^* = c_2(1 + i\eta_s)^{1/2} \quad (4.14b)$$

4.6 Boundary Element Discretization

As discussed under reservoir modeling, the integrals of the boundary integral equation, Eq. 4.3 have to be evaluated by using appropriate numerical methods. For this purpose, the boundary of the soil domain is discretized into a series of boundary elements and an appropriate variation over each element is assumed for the displacements and tractions. For

the elastodynamic analysis of soil substructure, since the fundamental solution implicitly accounts for the infinite radiation damping, only the soil-structure interface along with a finite region of soil surface close to the structure, needs to be discretized.

Using constant element formulation, the boundary integral equation, Eq. 4.3, can be written in a discretized form as

$$0.5u_j(P) + \sum_{l=1}^{nels} u_l(Q) \int_{S_l} T_{ji}^*(P, Q) ds - \sum_{l=1}^{nels} t_l(Q) \int_{S_l} U_{ji}^*(P, Q) ds = 0 \quad (4.15)$$

where $nels$ is the total number of boundary elements in soil discretization, and S_l under the integration sign implies integration over the l th boundary element. By locating the source point P successively over the mid nodes of each constant element, $2 * nels$ number of boundary integral equations can be generated, corresponding to the two degrees of freedom (DOF) per boundary element node.

In matrix form, the above referred equations can be represented as

$$\mathbf{Q}\mathbf{u} = \mathbf{T}\mathbf{t} \quad (4.16)$$

where $\mathbf{u}$ and $\mathbf{t}$ respectively denote the displacement and traction vectors. The elements of the matrices $\mathbf{Q}$ and $\mathbf{T}$ are defined by

$$Q_{mn} = 0.5\delta_{mn} + \int_{S_l} T_{m'n'}^*(P, Q) ds \quad (4.17)$$

$$T_{mn} = \int_{S_l} U_{m'n'}^*(P, Q) ds \quad (4.18)$$

$m' = 1$ for even values of m and $m' = 2$ for odd m , the value of n' depends on n in a similar manner; P and Q denote the nodes containing the m th and n th DOF respectively, the values of m and n ranges between 1 and $2 * nels$. The integrations in Eqs. 4.17 and 18 are evaluated over the l th element containing the field point Q .

Since in the present study, for the evaluation of impedance matrix of foundation soil, $\mathbf{t}$ is fully known and hence the right hand side of the matrix equation reduces to a vector containing known quantities. Matrix $\mathbf{Q}$ is fully populated and unsymmetric, but the system of resulting equations is well conditioned.

Since the fundamental solution is dependent on the excitation frequency, matrices $\mathbf{Q}$ and $\mathbf{T}$ must be assembled for each frequency analyzed, and hence the resulting impedance

matrix are also frequency dependent. At this stage, it is important to note two important differences in the BEM formulation of soil half plane in comparison to that of the reservoir model, viz. (a) the concept of frequency independent fundamental solution is not effective in case of elastodynamic formulation of soil domain, due to the nature of the geometry of the semi-infinite soil half plane (b) in the treatment of infinite wave radiation in soil, separate consideration of far field is not essential, since the associated fundamental solutions are such that, the quantities under the boundary integrals decay rapidly with increasing distance from the soil-structure interface, automatically accounting for the infinite radiation. More precisely, in case of foundation soil analysis "the kernels of the boundary integrals satisfy the regularity conditions, meaning that the product of the fundamental solutions for traction and the actual displacement T^*u , and vice-versa U^*t decay at the rate of $\frac{1}{r^3}$, as a result the integrals over the far boundary vanish" (Dominguez, 1978). This is not the case in reservoir analysis.

Equation 4.16 represents the final boundary element equation governing the foundation soil substructure. Its solution, satisfying the appropriate boundary condition, yields the unknown displacements and tractions along the surface of the foundation soil. Equation 4.16 contains complex unknowns and can be expressed as twice as many equations in real valued unknowns. Using the theory of complex algebra, the expanded matrices in Eq. 4.16 can be written as

$$\mathbf{T}_{mn} = \text{real} \int_{S_i} U_{m'n'}^*(P, Q) ds \quad (4.19a)$$

$$\mathbf{T}_{(m+2 \bullet nels),n} = \text{imaginary} \int_{S_i} U_{m'n'}^*(P, Q) ds \quad (4.19b)$$

$$\mathbf{T}_{(m+2 \bullet nels),(n+2 \bullet nels)} = \mathbf{T}_{mn} \quad (4.19c)$$

$$\mathbf{T}_{m,(n+2 \bullet nels)} = -\mathbf{T}_{(m+2 \bullet nels),n} \quad (4.19d)$$

Matrix $\mathbf{Q}$ is expanded in a similar manner.

4.6.1 Evaluation of Boundary Integrals

The boundary integrals contained in the integral equations can be conveniently evaluated by Gauss quadrature integration. However, when the source point P and the field point Q coincide, i.e. when the integration has to be performed over the boundary element that also contains the source point, singularity problem arises, and the integration requires

a special treatment as explained below. In the present formulation, constant boundary elements are used so that displacements and tractions are assumed to remain constant over a given boundary element.

The procedures used in the integration of both non singular and singular boundary integrals, are illustrated in the following two sections.

Gauss quadrature for non-singular terms

When the source point and field point are located in different boundary elements, the integration can be performed using a four point Gauss quadrature as below:

$$\int_{S_l} T_{mn}^*(P, Q) ds = |J| \sum_{k=1}^4 T_{mn}^*(P, Q_k) w_k \quad (4.20a)$$

$$\int_{S_l} U_{mn}^*(P, Q) ds = |J| \sum_{k=1}^4 U_{mn}^*(P, Q_k) w_k \quad (4.20b)$$

where $|J|$ is the determinant value of the Jacobian matrix, equal to half the length of the boundary element; w_k is the Gauss weight corresponding to the k th Gauss point, and Q_k is the field point located at the k th Gauss sampling point of the l th boundary element.

Singularity treatment

Singularity problem will arise, when the load point and the field point lie on the same boundary element. For the constant boundary elements, this integration can be performed through analytical technique combined with the Gauss numerical integration, so as to increase the accuracy of the integration procedure (Huang and Jablonski, 1988).

To demonstrate the treatment of singularity in the present work as involved in the evaluation of diagonal elements of matrix **Q** and **T**, consider the special case of elastic soil half plane with a horizontal surface, whose fundamental solution are defined by Eqs. 4.11 and 4.12. The non-zero expressions of U_{ij}^* and T_{ij}^* entering the singular terms are

$$U_{11}^* = \frac{1}{4\mu} (\Theta - \Upsilon) \quad (4.21a)$$

$$U_{22}^* = \frac{1}{4\mu} \Theta \quad (4.21b)$$

$$T_{12}^* = \frac{1}{4} \frac{x_1}{|x_1|} D_3 \quad (4.22a)$$

$$T_{21}^* = \frac{1}{4} \frac{x_1}{|x_1|} D_1 \quad (4.22b)$$

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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Evaluation of T_{mm}

The behavior of Θ and Υ under the limiting conditions of the radius vector $r \rightarrow 0$ can be observed by using the property of Bessel's function in the limiting case. Thus

$$\lim_{r \rightarrow 0} \Theta = -1 + i \frac{2}{\pi} \ln(k_2 r) \quad (4.23)$$

$$\lim_{r \rightarrow 0} \Upsilon = 0 \quad (4.24)$$

Substituting Eqs. 4.23 and 4.24 in Eq. 4.21, the expression for U^* becomes

$$U_{11}^* = U_{22}^* = \frac{i}{4\mu} \left(-1 + i \frac{2}{\pi} \ln(k_2 r) \right) \quad (4.25)$$

Hence the singularities associated with U_{ii}^* are of the order $O(\ln r)$, and the diagonal term of matrix $\mathbf{T}$ can be written as

$$\mathbf{T}_{mm} = \frac{i}{4\mu} \int_{S_i} \left(-1 + i \frac{2}{\pi} \ln(k_2 r) \right) dr \quad (4.26)$$

Eq. 4.26 will yield fairly accurate results when the length of the boundary element S_i is small (as discussed in the section dealing with the element size and mesh refinement). However, the accuracy of $\mathbf{T}_{mm}$ can be improved by numerically integrating by Gauss quadrature the residual part viz. $\frac{i}{4\mu} \int_{S_i} (\Theta - \Upsilon) - \left(-1 + i \frac{2}{\pi} \ln(k_2 r) \right) dr$ when m is odd, and $\frac{i}{4\mu} \int_{S_i} \Theta - \left(-1 + i \frac{2}{\pi} \ln(k_2 r) \right) dr$ when m is even, and combining with the analytical integration expression of Eq. 4.26.

Evaluation of Q_{mm} , $Q_{m,(m+1)}$, $Q_{(m+1),m}$

Since $T_{11}^* = T_{22}^* = 0$, from Eq. 4.17 it follows that all the diagonal elements Q_{mm} will have a value of $\frac{1}{2}$. Under the limiting case, the expression for D_1 and D_3 simplifies to

$$\lim_{r \rightarrow 0} D_1 = \frac{2i}{\pi r} \quad (4.27a)$$

$$\lim_{r \rightarrow 0} D_3 = (1 - 2\Gamma^2) \frac{2i}{\pi r} \quad (4.27b)$$

Substituting for D_1 and D_3 from Eq. 4.27 in Eq. 4.22, and using the resulting expressions for T_{12}^* and T_{21}^* , Eq. 4.17 reduces to

$$\begin{aligned} Q_{m,(m+1)} &= \int_{S_i} \lim_{r \rightarrow 0} T_{12}^* dr \\ &= - \frac{x_1}{|x_1|} \frac{(1 - 2\Gamma^2)}{2\pi} \int_{S_i} \frac{1}{r} dr \end{aligned} \quad (4.28a)$$

and

$$\begin{aligned}
 Q_{(m+1),m} &= \int_{S_i} \lim_{r \rightarrow 0} T_{21}^* dr \\
 &= -\frac{x_1}{|x_1|} \frac{1}{2\pi} \int_{S_i} \frac{1}{r} dr
 \end{aligned} \tag{4.28b}$$

Hence the singularities associated with T_{ij}^* are of the order $O(\frac{1}{r})$. Referring to Fig.4.1, the variation of the term T_{ij}^* is antisymmetric with respect to the singular point. On excluding the point of singularity, it is observed that the integral value or area to the left of the singular point is equal to that on the right side, but opposite in sign. The resultant integral therefore vanishes, and both $Q_{m,(m+1)}$ and $Q_{(m+1),m}$ are zero.

4.6.2 Size of Boundary Elements and Extent of the Mesh Discretization

The following criteria need to be established while choosing a boundary element model for the foundation soil analysis (Abdalla, 1984):

- (1) the extent of boundary element discretization, in other words, the appropriate size of the model to accurately represent the semi-infinite extent of the foundation region,
- (2) the optimum boundary element mesh for the discretization, i.e. the coarsest mesh capable of yielding reasonably accurate response,
- (3) boundary element discretization in the vicinity of the point of application of the load.

In essence, the BEM model for the soil analysis is dependent on the geometric and material properties of the foundation. Also, the number and size of elements in the boundary element discretization is dependent on the maximum value of excitation frequency analyzed, and at least six nodes are required to represent accurately a wavelength (Wolf 1983), and preferably 10 elements are to be used to span one full length of the shear wave (Vaish and Chopra, 1974).

The optimum extent of the mesh discretization, number, size and distribution of the boundary elements can be decided on the basis of a trial and error procedure, by comparing the results of the BEM solution obtained from a chosen model with a classical solution results if available, or with the results of the other well established methods.

Another method to determine the extent up to which the foundation surface needs discretization is by observing the off-diagonal terms of the flexibility matrix of the foundation

corresponding to the maximum excitation frequency. One may not require any boundary elements beyond a particular point, provided the off diagonal term of the flexibility matrix corresponding to that point with respect the nearest soil-structure interface node is of negligible value.

4.7 Evaluation of Soil Impedance Matrix

The complex valued, frequency dependent impedance matrix is best obtained by first evaluating the analogous flexibility matrix, corresponding to the DOF of the dam-foundation soil interface nodes along the structure (also along the fluid-foundation interface nodes for the analysis by rigorous approach), and then by obtaining its inverse. This method of evaluating the impedance matrix using flexibility approach is fairly straight forward. Furthermore, it ensures both compatibility of displacements and equilibrium of stresses at the soil-structure interface (Dasgupta and Chopra 1979). The size of the impedance matrix will correspond to the number of DOF of the interface nodes between the soil and the associated structure (or fluid).

For the foundation soil possessing a horizontal surface, and with equally spaced interface nodes (nodes common to both dam and the soil), the entire flexibility matrix can be constructed from just two sets of analyses, one for a traction equivalent to a unit horizontal excitation $e^{i\Omega t}$ and other for a traction equivalent to a unit vertical harmonic excitation $e^{i\Omega t}$, applied at the location of say, the first interface node of a massless foundation half-space and solving for the displacements at the boundary element nodes mapping the finite element nodes at the base of the dam. A traction free boundary condition is adopted for all the boundary elements over the soil surface, except for the one along which the prescribed traction is applied. This corresponds to a welded contact condition at the soil-structure interface. However, Abdalla (1984) has shown that both welded and relaxed contact condition yield almost similar results. The relaxed condition corresponds to the case wherein, the vertical components of displacement due to horizontal load are negligible, and vice versa. The boundary conditions of the problem are such that only tractions, but not a load, can be prescribed along the boundary of the soil, hence the unit point load can be simulated through the application of a uniform traction over a relatively small element length, the magnitude of the traction being equal to the reciprocal of the length of the

boundary element over which the traction is applied.

The displacements obtained from the two sets of analysis, form the first two columns of the complex valued flexibility matrix corresponding to the horizontal and vertical DOF of the first dam-soil interface node, at which the unit traction is applied. The rest of the flexibility matrix is constructed by making use of the fact that the deflection pattern of the soil surface is independent of the point at which the traction is applied, and shifting this load point will result in a mere shift of the displacement pattern. The elements of the other columns of the flexibility matrix can therefore be determined from those of the first two columns by using the following simple relations

$$E_{i,j} = E_{(i-j+1),1}, \text{ for all odd values of } j ; i \geq j > 2$$

$$E_{i,j} = E_{(i-j+2),2}, \text{ for all even values of } j ; i \geq j > 2$$

where $E_{i,j}$ denotes complex valued element of the flexibility matrix.

For the case of non-horizontal foundation soil surface or in case of soil-structure interface with unequally spaced common interface nodes, the impedance matrix obtained may not be symmetric, due to the approximations involved in the BEM. However, the error due to this effect could be minimized by averaging the unsymmetric off-diagonal terms (Gaitanaros and Karabalis 1988).

The dynamic soil stiffness matrix needs to be computed only once for each excitation frequency analyzed. The resulting matrix could be stored for future use in the response analysis for any earthquake ground motion. This matrix is independent of the geometry of the superstructure, the nature of the reservoir and the excitation component, provided the positions of the finite element nodes corresponding to the structure along the soil-structure interface (and also the nodes along the fluid-foundation interface in case of the rigorous model analysis) remain unaltered and the soil properties are unchanged.

4.8 Verification of BEM Model for Foundation Soil

In order to compare the results of the procedure described in the foregoing sections with available classical solutions for the foundation soil stiffness, the case of a rigid strip foundation resting on an elastic soil half plane is analyzed. The rigid strip has two degrees of freedom; a vertical and a horizontal displacement corresponding to a relaxed contact condition. The complex valued, frequency dependent dynamic stiffness coefficient of the

rigid strip footing is defined as the sum of the forces developed at interface nodes along the i th DOF of the footing, due to rigid body displacements of the same interface nodes caused by a unit displacement of the footing along the j th DOF, with no displacements along the other degrees of freedom. The values of i and j are either 1 or 2, representing the horizontal and vertical DOF respectively.

The dynamic stiffness coefficients of the foundation soil can be expressed in a non-dimensional form, as

$$K_{ij}^s(\Omega) = \frac{\sum_{k=1}^{nib} F_{ki}(\Omega)}{\pi \mu u_j} \quad (4.29)$$

where nib is the number of nodes along the soil-structure interface, F_{ki} is the force per unit width of the footing, developed at the k th interface node along the i th DOF due to the application of a rigid body harmonic displacement of amplitude u and excitation frequency Ω along the j th DOF of all nodes.

The accuracy of the present formulation can be verified by plotting the variation of this non-dimensional dynamic stiffness coefficients K_{ij}^s versus the dimensionless frequency a_o , and comparing it with the closed form solutions for the rigid strip footing resting on a elastic semi-infinite half plane, corresponding to a relaxed contact condition, given by Karasudhi et al. (1968). The dimensionless frequency a_o is defined as $\frac{\Omega b}{c_2}$; and b is half width of the rigid strip footing.

For the present example, the properties of the soil medium are taken as Young's Modulus = 96.5Mpa (14 ksi), Poisson's ratio=0.25, and the density = 2163 kg/m³ (135 lb/ft³), with a 2D plane strain idealization. A BEM modeling of the foundation soil is adopted, wherein the soil-structure interface along with a finite portion of soil surface, in the neighborhood of the structure is discretized. The boundary element mesh used in the analysis, i.e. the extent of discretization and the length of the boundary elements was arrived at on the basis of a convergence study.

Figure 4.2 shows the variation of non-dimensional dynamic horizontal and vertical stiffness coefficients K_{11}^s and K_{22}^s with the dimensionless frequency. The results obtained in the present work are in good agreement with the closed form solutions of Karasudhi et al. (1968).

Fig. 4.1 Variation of T_{ij}^* ($i \neq j$) at the Point of Singularity

Fig. 4.2a Horizontal Stiffness Coefficients of a Rigid Strip Footing; Comparison with Classical Solution

- 1 — -ve of Imag. Part; Karasudhi
- 2 — -ve of Imag. Part; BEM Soln.
- 3 — Real Part; Karasudhi
- 4 — Real Part; BEM Solution

Fig. 4.2b Vertical Stiffness Coefficients of a Rigid Strip Footing; Comparison with Classical Solution

5 DAM-RESERVOIR-FOUNDATION ANALYSES BY SIMPLIFIED APPROACH

5.1 Introduction

A comprehensive response analysis of a gravity dam should include the interactions resulting from the impounded reservoir, as well as the flexible foundations below the dam and the reservoir. This chapter describes the procedure involved in the response analysis of gravity dams by a simplified approach, using the techniques developed in the previous chapters for the evaluation of reservoir induced hydrodynamic forces and impedance matrix of the foundation soil. Only an infinite reservoir is considered in the analysis.

The analysis by simplified approach includes the dam - reservoir interaction, dam - foundation interaction, and fluid - foundation interaction, but the cross coupling effect between the foundation below the dam and the reservoir bottom is ignored. The fluid-foundation interaction effect, also referred to as the wave absorption along the reservoir bed, is accounted for by modeling the foundation below the reservoir using the 1D approximate model for wave propagation proposed by Hall and Chopra (1980). A more rigorous analysis procedure in which the cross coupling effect is included and the flexible foundation below the reservoir is represented by BEM is presented in the next chapter. The performance of these two models are compared in Chapter 7.

For both simplified and rigorous analysis, in modeling the fluid-foundation interaction, it is assumed that only the flexible foundation below the reservoir is responsible for the wave absorption, and the effect of the sediment that could have possibly deposited along the reservoir bed is ignored. This assumption is justified by the study carried out by Medina et al.(1990), who concluded that the presence of fully saturated sediments along the reservoir bottom does not significantly influence the response of the dam to either component

of the ground motions, when the foundation below the reservoir is treated as flexible. Furthermore, Bougacha et al.(1991) using a poro-elastic model of the sediments, have shown that the response of the dam with the reservoir bottom conditions corresponding to the fully saturated sediments are larger than those obtained with partially saturated sediments. In addition, owing to the many uncertainties associated with the nature and properties of the sediment materials, a fully saturated sediment assumption is quite reasonable.

5.2 Analysis of the Dam Substructure

The dam substructure is modeled by four noded isoparametric, 2D quadrilateral finite elements. In such an element two DOF are defined for each node, and it is assumed that the parameters vary in a linear fashion between two successive nodes. The details regarding the formulation of the mass and stiffness matrices for a structure by finite element approach are available in standard texts (Bathe et al. 1987, Zienkiewicz 1981), and hence will not be repeated here. The dam could also be analyzed by BEM or other suitable techniques. The FEM procedure leads to a banded and symmetric structural stiffness matrix, which makes the computation more efficient. Further, the use of FEM makes it easier to extend the formulation to include the material non-linearity in dam due to concrete cracking or water cavitation, although such a study is not within the scope of the present work.

For the present research, the material of the dam is assumed to be isotropic and linear elastic with hysteretic damping (Humar, 1990). A 2D plane stress idealization is adopted for the gravity dam analysis. During a severe earthquake sliding takes place along the construction joints between the monoliths of the concrete gravity dam. These monoliths thus tend to vibrate independently with no force in the direction of the dam axis being transferred across the joints. The plane stress idealization is thus appropriate for representing the state of stress in the dam. This has been borne out by the experience of the behavior of Koyna dam in India during 1967 (Chakrabarti and Chopra, 1972). The analyses of both the dam and the foundation soil should follow the same 2D idealization, viz. the plane stress, so as to satisfy the compatibility requirements. Hence a plane stress idealization is adopted for the foundation analysis as well, although a plane strain would be more appropriate for the foundation-soil substructure.

5.3 Equations of Motion of the Coupled System

5.3.1 Dam-Reservoir-Foundation System

The equations of motion of the dam-reservoir-foundation system in frequency domain, in its matrix form, can be written as

$$(-\Omega^2 \mathbf{M} + i\Omega \mathbf{C} + \mathbf{K}) \mathbf{U}^l(\Omega) = -\mathbf{E}^l + \mathbf{R}^l(\Omega) \quad (5.1)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ represent respectively the mass, damping and the stiffness matrices of the dam substructure including the dam-foundation interface. The mass matrix is developed using the lumped mass model of the FEM (Bathe et al., 1974). The resulting mass matrix which is diagonal in form, offers computational advantage and is sufficiently accurate at the same time. Also, in Eq. 5.1 $\mathbf{U}^l(\Omega)$ is the vector of nodal displacements relative to the free field, $\mathbf{R}^l(\Omega)$ is the nodal load vector, having non-zero quantities corresponding to the upstream face (hydrodynamic force terms) and the base degrees of freedom of the dam, Ω is the excitation frequency, and $l = x, y$ is the direction of excitation. Vector $\mathbf{E}^l$ contains the inertia loads arising from a unit harmonic free field ground motion along l th coordinate direction, and is given by

$$\{\mathbf{E}^x\}^T = \{m_1 \ 0 \ m_2 \ 0 \ \dots \ m_N \ 0\}; \ \{\mathbf{E}^y\}^T = \{0 \ m_1 \ 0 \ m_2 \ \dots \ 0 \ m_N\},$$

where m_i is the lumped mass associated with the i th finite element node of the dam and N is the total number of finite element nodes in the dam structure, including those along the dam-foundation soil interface; superscript T indicates the transpose of a vector or a matrix. In obtaining Eq. 5.1, it is assumed that the free field ground motion is identical at all the base nodes of the dam.

Assuming constant hysteretic damping instead of viscous damping in the dam, Eqn. 5.1 can be rewritten in partitioned form as,

$$\left(-\Omega^2 \begin{bmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}_b \end{bmatrix} + (1 + i\eta_d) \begin{bmatrix} \mathbf{k} & \mathbf{k}_b \\ \mathbf{k}_b^T & \mathbf{k}_{bb} \end{bmatrix} \right) \begin{bmatrix} \mathbf{u}^l(\Omega) \\ \mathbf{u}_b^l(\Omega) \end{bmatrix} = -\mathbf{E}^l + \begin{bmatrix} \mathbf{R}_h^l(\Omega) \\ \mathbf{R}_r^l(\Omega) + \mathbf{R}_b^l(\Omega) \end{bmatrix} \quad (5.2)$$

where η_d is the frequency independent hysteretic damping factor for the dam material; suffix b indicates quantities associated with the dam-foundation interface nodes, $\mathbf{R}_h^l$ is a load vector having non-zero terms corresponding to the statically equivalent hydrodynamic forces along the DOF of the nodes on the dam face in contact with the reservoir water

excluding the node common with the foundation; $\mathbf{R}_b^l$ is the vector of forces along the dam-soil interface nodes; and $\mathbf{R}_r^l$ is the vector of hydrodynamic forces on the node c common to the upstream face of the dam and the foundation. Note that the size of $\mathbf{R}_r^l$ is expanded to be the size of $\mathbf{R}_b^l$, and that $\mathbf{R}_r^l$ has only two non-zero terms R_r^{cx} and R_r^{cy} . The size of $\mathbf{R}_h^l$ is equal to the number of DOF of the dam nodes, excluding the dam-soil interface nodes.

The earthquake induced displacements experienced by the foundation soil surface, in the absence of the structure, are termed free field displacements, and under this condition no external forces exist on the soil surface. The displacements of the foundation relative to the free field are therefore due to the forces exerted by the superstructure on the foundation soil. Utilizing the equilibrium of forces and compatibility of displacements along the dam-soil interface, an expression for the soil-structure interface load vector $\mathbf{R}_b^l$ could be obtained in the following form

$$\mathbf{R}_b^l(\Omega) = -\mathbf{S}_{bb}(\Omega) \mathbf{u}_b^l(\Omega) \quad (5.3)$$

where $\mathbf{S}_{bb}(\Omega)$ is frequency dependent, complex valued soil impedance matrix, corresponding to the base DOF of the dam, obtained by BEM analysis of the foundation soil substructure as discussed under Chapter 4. The effect of hydrodynamic forces along the reservoir bottom are reported to be negligible (Fenves and Chopra, 1984), and hence ignored in the present analysis, allowing the use of a traction free boundary condition along the reservoir bottom for the analysis of the foundation soil.

Substitution of Eq. 5.3 in Eq. 5.2, and transferring of the terms containing $\mathbf{u}_b^l$ to the left side of the equation, yields

$$\left(-\Omega^2 \begin{bmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}_b \end{bmatrix} + (1 + i\eta_d) \begin{bmatrix} \mathbf{k} & \mathbf{k}_b \\ \mathbf{k}_b^T & \mathbf{k}_{bb} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{bb}(\Omega) \end{bmatrix} \right) \begin{bmatrix} \mathbf{u}^l(\Omega) \\ \mathbf{u}_b^l(\Omega) \end{bmatrix} = -\mathbf{F}^l + \begin{bmatrix} \mathbf{R}_h^l(\Omega) \\ \mathbf{R}_r^l(\Omega) \end{bmatrix} \quad (5.4)$$

Equation 5.4 represents a set of large number of equations, the solution of which can be quite time consuming. By using a transformation based on appropriately selected Ritz vectors, the size of Eq. 5.4 can be substantially reduced. If mode shapes are used as Ritz vectors this concept is referred to as 'mode superposition technique'. In the present study the first n_{mod} mode shapes of the undamped free vibration of the associated dam-foundation system (Clough and Penzien, 1985) are used as Ritz vectors, so that the nodal displacements can be expressed as linear combination of first few modes, and can be represented as

$$U^l(\Omega) = \sum_{j=1}^{nmodes} Z_j^l(\Omega) \phi_j \quad (5.5)$$

where $Z_j^l(\Omega)$ is the j th modal coordinate corresponding to a motion in direction l . The number of modes to be included in the analysis depends upon the structural properties, boundary conditions (rigid or flexible foundation) and the frequency content of the ground motion. For example, in the case of a dam on flexible foundation a larger number of modes need to be included, as compared to those for a dam on rigid foundation. The natural frequencies ω_j and the orthonormalized Ritz vectors ϕ_j are obtained from the following eigenvalue problem of the associated dam-foundation system

$$\begin{pmatrix} k & k_b \\ k_b^T & k_{bb} + S_{bb}^r(\Omega_0) \end{pmatrix} \{\phi_j\} = \omega_j^2 \begin{pmatrix} m & 0 \\ 0 & m_b \end{pmatrix} \{\phi_j\} \quad (5.6)$$

where $S_{bb}^r(\Omega_0)$ is the real part of the soil impedance matrix, evaluated at a small frequency value Ω_0 , say 1 rad./sec.

Substitution of Eq. 5.5 in Eq. 5.4 and premultiplication by Φ^T gives

$$(-\Omega^2 \mathbf{I} + (1 + i\eta_d)\mathbf{A} + \Phi^T \mathbf{S}_{0f}(\Omega) \Phi) \mathbf{Z}^l(\Omega) = -\Phi^T \mathbf{E}^l + \Phi_h^T \mathbf{R}_h^l(\Omega) + \Phi_b^T \mathbf{R}_r^l(\Omega) \quad (5.7)$$

where

$$\mathbf{S}_{0f}(\Omega) = \begin{pmatrix} 0 & 0 \\ 0 & (S_{bb}^r(\Omega) - S_{bb}^r(\Omega_0)) + i(S_{bb}^i(\Omega) - \eta_d S_{bb}^r(\Omega_0)) \end{pmatrix}$$

$$\Phi^T = \{\Phi_h^T \quad \Phi_b^T\}$$

$S_{bb}^r(\Omega)$ and $S_{bb}^i(\Omega)$ are the real and imaginary parts of the foundation impedance matrix respectively, $\mathbf{A}$ is the diagonal matrix of the square of the natural frequencies ω_j^2 ; Φ_h is the matrix containing the mode shapes corresponding to the DOF of dam, excluding the base DOF, and Φ_b represents the mode shapes corresponding to the DOF along the dam-soil interface nodes.

Evaluation of Hydrodynamic Force: $\mathbf{R}_h^l$, $\mathbf{R}_r^l$

The hydrodynamic nodal force along the upstream face of the dam are the static equivalents of the corresponding hydrodynamic pressure, computed directly by using the principle of virtual work, with an assumed linear variation of parameter between the nodal points.

In the formulations for hydrodynamic pressure terms of Chapter 2, the dam reservoir interaction was not considered, and hence a rigid dam boundary condition was adopted. However, since the formulation presented here includes the dam-reservoir interaction, the corresponding hydrodynamic terms must be evaluated for modified boundary conditions. The governing differential equation is still Eq. 2.2; however the boundary conditions are as given below:

along the upstream face of the dam

$$\begin{pmatrix} \mathbf{q}_1^l \\ q_c^{L,l} \end{pmatrix} = -\rho_w \left[\mathbf{a}_n^l - \Omega^2 \sum_{j=1}^{nmod} \bar{\mathbf{A}}_n \psi_j Z_j^l(\Omega) \right] \quad (5.8a)$$

where

$$\bar{\mathbf{A}}_n = - \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cos\theta_2 & \sin\theta_2 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cos\theta_3 & \sin\theta_3 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cos\theta_i & \sin\theta_i \end{pmatrix}$$

$\psi_j^T = \{\psi_{jx}^1, \psi_{jy}^1, \psi_{jx}^2, \psi_{jy}^2, \psi_{jx}^3, \psi_{jy}^3, \dots, \psi_{jx}^c, \psi_{jy}^c\}$, and $Z_j^l(\Omega)$ is the j th modal displacement of the dam due to l th excitation component with $l = x, y$; $nmod$ is the number of modes included in the analysis; ψ_{jx}^i and ψ_{jy}^i refer to the mode shapes along the horizontal and vertical DOF of i th upstream dam face (dam-reservoir interface) node of the dam-foundation system vibrating in the j th mode; θ_i is the angle of inclination of the upstream face of the dam with the vertical at the location of this i th node; and $\mathbf{a}_n^l$ is the vector of $\mathbf{a}_n^l$ corresponding to the boundary elements along the dam reservoir interface.

Note that, if the upstream face comprises of multiple slopes, the node corresponding to the junction of two boundary elements with different inclination will not possess a unique outward normal. In such cases, a more involved treatment similar to the formulation at corner nodes c and d as explained in Chapter 2 may be adopted, or if the difference in inclination angle between the two adjacent boundary element is small, an approximate procedure wherein the unit normal is obtained with respect to the average of the two angles of inclination at the node under consideration may be used in the computation of $\mathbf{a}_n^l$ and $\bar{\mathbf{A}}_n$ of Eq. 5.8a.

along the reservoir bottom

$$\begin{pmatrix} q_c^{Rl} \\ q_2^l \\ q_d^{Ll} \end{pmatrix} = -\rho_w \mathbf{b}_n^l - i\Omega\gamma \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} \quad (5.8b)$$

where the superscripts L and R denote respectively the boundary elements to the left and right side of the corner node under consideration, and subscripts 1 and 2 correspond to the upstream face of the dam, and the reservoir bottom respectively as in Section 2.8. c is the node common to the dam, foundation and the reservoir, d is the node on the far field boundary common to the reservoir near and far fields, and the foundation; $\mathbf{b}_n^l$ is the vector of b_n^l corresponding to the boundary elements along the reservoir-foundation interface of the near field, and other notations are the same as used in Chapter 2.

Since the governing equation and the boundary conditions are linear, the principle of superposition holds and the complex frequency response functions for the hydrodynamic pressure vector $\mathbf{p}^l(\Omega)$ can be expressed as

$$\mathbf{p}^l(\Omega) = \mathbf{p}_o^l(\Omega) + \sum_{j=1}^{nmod} Z_j^l(\Omega) \mathbf{p}_j^f(\Omega) \quad (5.9)$$

where $Z_j^l(\Omega)$ is the j th modal displacement of the dam due to l th excitation component and $\mathbf{p}_o^l(\Omega)$ is the pressure vector obtained by solving governing equation of reservoir, in discretized form as in Eq. 2.32, with boundary conditions corresponding to unit harmonic ground acceleration of a rigid dam along the l th direction, i.e.

along the upstream face of the dam

$$\begin{pmatrix} q_1^l \\ q_c^{Ll} \end{pmatrix} = -\rho_w \mathbf{a}_n^l \quad (5.10a)$$

along the reservoir bottom

$$\begin{pmatrix} q_c^{Rl} \\ q_2^l \\ q_d^{Ll} \end{pmatrix} = -\rho_w \mathbf{b}_n^l - i\Omega\gamma \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} \quad (5.10b)$$

Also, in Eq. 5.9 $\mathbf{p}_j^f(\Omega)$ is the solution of Eq. 2.32, with boundary conditions corresponding to that of horizontal acceleration of dam-foundation system vibrating in the j th mode, with no motion of the reservoir bed, i.e.

along the upstream face of the dam

$$\begin{pmatrix} q_1 \\ q_c^L \end{pmatrix} = \rho_w \Omega^2 \mathbf{A}_n \psi_j \quad (5.11a)$$

along the reservoir bottom

$$\begin{pmatrix} q_c^R \\ q_2 \\ q_d^L \end{pmatrix} = -i\Omega\gamma \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} \quad (5.11b)$$

In the evaluation of both $\mathbf{p}_o^l(\Omega)$ and $\mathbf{p}_j^f(\Omega)$, the boundary conditions along the free surface is automatically satisfied in the formulation, while the boundary conditions along the interface between the near and far field of the infinite reservoir are identical to those of Eqs. 2.41 and 2.42, respectively for horizontal and vertical excitations. Further, the vector $\mathbf{p}_j^f(\Omega)$ is independent of the excitation direction, and hence need to be evaluated only once, for the analysis of response to both components of ground motion. Using these boundary conditions appropriately in Eqs. 2.44 and 2.55, $\mathbf{p}_o^r(\Omega)$, $\mathbf{p}_o^y(\Omega)$ and $\mathbf{p}_j^f(\Omega)$ can be obtained. Referring to Eq. 5.9 it may be noted that, even under a vertical excitation, the expression for hydrodynamic pressure $\mathbf{p}^l(\Omega)$ will possess damping terms due to infinite radiation phenomenon contained in $\mathbf{p}_j^f(\Omega)$ for excitation frequencies greater than the fundamental frequency of the transmitting boundary.

In Eq. 5.9 the cross coupling term between the foundation below the dam and the reservoir bottom $\mathbf{p}_j^b(\Omega)$ is ignored, and the fluid-foundation interaction in Eqs. 5.10b and 5.11b is accounted for by means of an 1D approximate wave absorption model proposed by Hall and Chopra (1980). This part of the formulation will differ substantially in the rigorous approach to be developed in the next chapter.

Following Eq. 5.9, the hydrodynamic force vector $\mathbf{R}_h^l$ and $\mathbf{R}_r^l$ can now be expressed as

$$\mathbf{R}_h^l(\Omega) = \mathbf{R}_{ho}^l(\Omega) + \sum_{j=1}^{nmod} Z_j^l(\Omega) \mathbf{R}_{hj}^f(\Omega) \quad (5.12a)$$

$$\mathbf{R}_r^l(\Omega) = \mathbf{R}_{ro}^l(\Omega) + \sum_{j=1}^{nmod} Z_j^l(\Omega) \mathbf{R}_{rj}^f(\Omega) \quad (5.12b)$$

where the non-zero terms of $\mathbf{R}_{ho}^l(\Omega)$, $\mathbf{R}_{ro}^l(\Omega)$, $\mathbf{R}_{hj}^f(\Omega)$ and $\mathbf{R}_{rj}^f(\Omega)$ denote the nodal forces along dam's upstream face. These forces are statically equivalent to the hydrodynamic pressures $\mathbf{p}_o^l(\Omega)$ and $\mathbf{p}_j^f(\Omega)$ respectively.

Substitution for $\mathbf{R}_h^l(\Omega)$ and $\mathbf{R}_r^l(\Omega)$ from Eq. 5.12 in Eq. 5.7 and grouping of all terms containing $\mathbf{Z}^l(\Omega)$ to the left, yields

$$\mathbf{K}^*(\Omega)\mathbf{Z}^l(\Omega) = \mathbf{F}^{*l}(\Omega) \quad (5.13)$$

The elements of the effective stiffness matrix of the coupled system are given by

$$K_{ij}^*(\Omega) = \left(-\Omega^2 + (1 + i\eta_d)\omega_i^2\right) \delta_{ij} + \phi_i^T \mathbf{S}_{0f}(\Omega) \phi_j - \left(\phi_{hi}^T \mathbf{R}_{hj}^l(\Omega) + \phi_{bi}^T \mathbf{R}_{rj}^l(\Omega)\right) \quad (5.14a)$$

and those of the effective force by

$$F_i^{*l}(\Omega) = -\phi_i^T \mathbf{E}^l + \phi_{hi}^T \mathbf{R}_{ho}^l(\Omega) + \phi_{bi}^T \mathbf{R}_{ro}^l(\Omega) \quad (5.14b)$$

where ω_i and ϕ_i , are the solutions of eigenvalue problem of the associated undamped dam-foundation system with empty reservoir. It should be noted that the hydrodynamic force vectors $\mathbf{R}_{hj}^l$ and $\mathbf{R}_{ho}^l$ have non-zero terms corresponding to the degrees of freedom of upstream face nodes of the dam only excluding the node common to the base and the dam face. Vectors $\mathbf{R}_{rj}^l$ and $\mathbf{R}_{ro}^l$ have only two non-zero terms corresponding to the common node c . Unlike in a classical modal analysis, the matrix $\mathbf{K}^*(\Omega)$ in Eq. 5.13 is non-diagonal, because the displacement vectors i.e. the mode shapes ϕ_j are not the normal modes of the coupled dam-reservoir-foundation system, but they simply are the mode shapes of the associated dam-foundation system with an empty reservoir.

By examining Eqs. 5.13 and 5.14, the reservoir induced hydrodynamic effects can be interpreted as added mass term represented by the real part of vectors $\mathbf{R}_{hj}^l$ and $\mathbf{R}_{rj}^l$, and the added damping terms represented by the imaginary part of these vectors. The complex valued added load vector is denoted by $\mathbf{R}_{ho}^l$ and $\mathbf{R}_{ro}^l$. Thus only the added load term is dependent on the excitation direction, while the added mass and added damping terms are independent of the excitation direction. The hydrodynamic terms are of course dependent on the excitation frequency. Equation 5.13 represents the *nmod* complex valued frequency dependent simultaneous equations for a general case of the coupled dam-reservoir-foundation system. Special cases, corresponding to the dam response for various conditions of reservoir water and foundation soil can be deduced from this general equation.

5.3.2 Dam Alone

For the dam on rigid foundation soil, with empty reservoir, the elements of the effective stiffness matrix and the effective load vectors can be deduced from Eq. 5.14 as

$$K_{ij}^*(\Omega) = \left(-\Omega^2 + (1 + i\eta_d)\omega_i^2\right) \delta_{ij} \quad (5.15a)$$

$$F^{*l}_i(\Omega) = -\phi_i^T \mathbf{E}^l \quad (5.15b)$$

where ω_i and ϕ_i , are the solutions of eigenvalue problem of the undamped free vibration of dam on rigid foundation with empty reservoir and the matrix $\mathbf{K}^*(\Omega)$ is a diagonal matrix.

5.3.3 Dam-Reservoir System

For the dam-reservoir system with a rigid foundation below the dam, the governing equations are given by

$$K_{ij}^*(\Omega) = \left(-\Omega^2 + (1 + i\eta_d)\omega_i^2\right) \delta_{ij} - \phi_{hi}^T \mathbf{R}_{hj}^f(\Omega) \quad (5.16a)$$

$$F^{*l}_i(\Omega) = -\phi_i^T \mathbf{E}^l + \phi_{hi}^T \mathbf{R}_{ho}^l(\Omega) \quad (5.16b)$$

where ω_i and ϕ_i , are the solutions of eigenvalue problem of the undamped free vibration of dam on rigid foundation with empty reservoir and the matrix $\mathbf{K}^*(\Omega)$ is non-diagonal.

5.3.4 Dam-Foundation System

The governing equations for the dam-foundation system with empty reservoir are

$$K_{ij}^*(\Omega) = \left(-\Omega^2 + (1 + i\eta_d)\omega_i^2\right) \delta_{ij} + \phi_i^T \mathbf{S}_{of}(\Omega) \phi_j \quad (5.17a)$$

$$F^{*l}_i(\Omega) = -\phi_i^T \mathbf{E}^l \quad (5.17b)$$

where ω_i and ϕ_i , are the solutions of eigenvalue problem of undamped free vibration of the associated dam-foundation system with empty reservoir. The matrix $\mathbf{K}^*(\Omega)$ is not diagonal, except for $\Omega = \Omega_0$.

6

DAM-RESERVOIR-FOUNDATION ANALYSES BY RIGOROUS APPROACH

6.1 Introduction

A formulation for the analysis of coupled dam-reservoir-foundation system was developed in Chapter 5 using a simplified approach. Two different approximations were involved in that approach. First, the interaction between the fluid and reservoir bottom was represented by a simplified 1D model suggested by Hall and Chopra (1980). Second, the effect of coupling between the dam foundation and the reservoir bottom was ignored. Medina et al.(1990) analyzed the complex frequency response function of absolute value of horizontal crest acceleration of a dam with triangular cross section and concluded that the representation of fluid-foundation interaction by approximate 1D model, although capable of predicting responses under vertical excitation accurately, underestimates the peak response due to harmonic horizontal excitation. However, the real test of the effectiveness of a method is in knowing how well it can predict the response of a dam to random ground motion. Significant differences in complex frequency response need not necessarily cause large difference in the response to real earthquakes. Therefore, to examine the accuracy of the results predicted by the simplified approach for the response of the dam to realistic earthquake ground motion, particularly its transverse component, a procedure for the analysis of the coupled system by using a more rigorous approach is developed in this chapter, and a comparative analysis of the two models is carried out in Chapter 7. The model based on rigorous approach eliminates some of the key assumptions made in the simplified approach. The major difference between the two models lies in the inclusion of the cross coupling between the foundation below the dam and the reservoir bottom, and in the treatment of the fluid-foundation interaction.

As stated earlier, the simplified approach totally ignores the cross coupling term between the foundation below the dam and the reservoir, while accounting for the wave

absorption along the flexible reservoir bed through an approximate 1D wave propagation model. On the other hand, the model based on a more rigorous approach considers the cross coupling effect. In addition, it accounts for the fluid-foundation interaction effect by modeling the foundation soil below the reservoir as a half plane using a boundary element discretization of the fluid-foundation interface. It should be noted that, in a rigorous approach, the same soil half plane extends under the reservoir and the dam as shown in Fig. 6.1. However, the sediment material that could possibly get deposited along the reservoir bed is not accounted for separately, for the reasons already discussed in Chapter 5.

Due to the more elaborate treatment involved in a rigorous approach, the storage requirements and computational effort associated with that approach is much larger than those in a numerical model based on a simplified approach (refer Table 7.2). If the results of the simplified model are reasonably accurate in predicting the response to a realistic ground motion, it would be sufficient and more advantageous to adopt the simplified model for the analysis because of its less complex formulation and greater computational efficiency.

6.2 Mathematical Formulation

A rigorous analysis of the dam-reservoir-foundation system is best carried out in the frequency domain by a substructure approach in which the dam, reservoir and foundation are treated as separate substructures. The BEM treatments of the reservoir and foundation substructures were dealt with in Chapters 2 and 4, and the FEM modeling of the dam was described in Chapter 5.

The equations of motion of the coupled dam system are given by Eq. 5.2, rewritten here for ease of reference

$$\left(-\Omega^2 \begin{bmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}_b \end{bmatrix} + (1 + i\eta_d) \begin{bmatrix} \mathbf{k} & \mathbf{k}_b \\ \mathbf{k}_b^T & \mathbf{k}_{bb} \end{bmatrix} \right) \begin{bmatrix} \mathbf{u}^t(\Omega) \\ \mathbf{u}_b^t(\Omega) \end{bmatrix} = -\mathbf{E}^t + \begin{bmatrix} \mathbf{R}_h^t(\Omega) \\ \mathbf{R}_r^t(\Omega) + \mathbf{R}_b^t(\Omega) \end{bmatrix} \quad (6.1)$$

The notations used in Eq. 6.1 are as explained in Chapter 5. The hydrodynamic forces along the upstream face of the dam, $\mathbf{R}_h^t$ and $\mathbf{R}_r^t$ are obtained by performing an appropriate analysis of reservoir domain, while the soil structure interaction forces along the dam base $\mathbf{R}_b^t$ are computed from an analysis of the foundation soil. Note that vector $\mathbf{R}_r^t$ represents the hydrodynamic forces along the common node c and has only two non-zero terms R_r^x and

R_b^y . The evaluation of these two sets of forces and other related quantities are discussed in the subsequent sections of this chapter.

6.3 Analysis of Foundation Substructure

In an analysis by the rigorous approach, the dam-soil interface, the near field of the reservoir-soil interface, and a small finite portion of the foundation soil surface are all discretized by boundary elements, as shown in Fig. 6.1. For the special case of reservoir with a horizontal bed, it is sufficient to define only the vertical DOF for the soil below the reservoir.

The force displacement relation for the soil substructure can be expressed using the complex valued, frequency dependent soil impedance matrix $\mathbf{S}$ as:

$$\begin{pmatrix} \mathbf{S}_{bb}(\Omega) & \mathbf{S}_{bf}(\Omega) \\ \mathbf{S}_{bf}^T(\Omega) & \mathbf{S}_{ff}(\Omega) \end{pmatrix} \begin{pmatrix} \mathbf{r}_b^l(\Omega) \\ \mathbf{r}_f^l(\Omega) \end{pmatrix} = \begin{pmatrix} \mathbf{R}_b^l(\Omega) \\ \mathbf{Q}_h^l(\Omega) \end{pmatrix} \quad (6.2)$$

where suffix b denotes DOF along the foundation below the dam; and suffix f indicates those corresponding to the reservoir bottom. Vector $\mathbf{r}$ represents the displacement of the soil, $\mathbf{R}_b^f$ the interaction forces in the soil along the dam-foundation interface nodes and $\mathbf{Q}_h$ the interaction forces along the fluid-foundation interface.

The second part of Eq. 6.2, yields (after dropping the excitation direction term l , for simplicity)

$$\mathbf{r}_f = \mathbf{S}_{ff}^{-1} \mathbf{Q}_h - \mathbf{S}_{ff}^{-1} \mathbf{S}_{bf}^T \mathbf{r}_b \quad (6.3)$$

It should be noted that all quantities in Eq. 6.3 are frequency dependent.

Using the equilibrium and compatibility conditions along the dam-foundation interface we get

$$\mathbf{r}_b = \mathbf{u}_b \quad (6.4a)$$

$$\mathbf{R}_b + \mathbf{R}_b^f = 0 \quad (6.4b)$$

Substituting Eq. 6.3 in the first part of Eq. 6.2, and using Eq. 6.4 we get

$$\mathbf{S}_{bb} \mathbf{u}_b + \mathbf{S}_{bf} \mathbf{S}_{ff}^{-1} \mathbf{Q}_h - \mathbf{S}_{bf} \mathbf{S}_{ff}^{-1} \mathbf{S}_{bf}^T \mathbf{u}_b = -\mathbf{R}_b \quad (6.5a)$$

or

$$\mathbf{R}_b = -\mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{Q}_h - (\mathbf{S}_{bb} - \mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{S}_{bf}^T)\mathbf{u}_b \quad (6.5b)$$

or

$$\mathbf{R}_b = -\mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{Q}_h - \tilde{\mathbf{S}}_{bb}\mathbf{u}_b \quad (6.5c)$$

in which $\tilde{\mathbf{S}}_{bb} = (\mathbf{S}_{bb} - \mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{S}_{bf}^T)$

On including the excitation direction, Eq. 6.5c can be written as

$$\mathbf{R}_b^l = -\mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{Q}_h^l - \tilde{\mathbf{S}}_{bb}\mathbf{u}_b^l \quad (6.5d)$$

Substitution of Eq. 6.5d in Eq. 6.1 and transferring of the terms containing $\mathbf{u}_b$ to the left yields

$$\begin{aligned} \left(-\Omega^2 \begin{pmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}_b \end{pmatrix} + (1 + i\eta_d) \begin{pmatrix} \mathbf{k} & \mathbf{k}_b \\ \mathbf{k}_b^T & \mathbf{k}_{bb} \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{S}}_{bb}(\Omega) \end{pmatrix} \right) \begin{pmatrix} \mathbf{u}^l \\ \mathbf{u}_b^l \end{pmatrix} \\ = -\mathbf{E}^l + \begin{pmatrix} \mathbf{R}_h^l \\ \mathbf{R}_r^l - \mathbf{S}_{bf}\mathbf{S}_{ff}^{-1}\mathbf{Q}_h^l \end{pmatrix} \end{aligned} \quad (6.6)$$

In Eq. 6.6, matrix $\mathbf{S}$, and vectors $\mathbf{u}$, $\mathbf{R}$, $\mathbf{Q}$ are all dependent on the excitation frequency, Ω .

6.4 Transformation of the Equations Using Ritz Vector

The undamped free vibration version of the Eq. 6.6 represents an eigenvalue problem, whose solution gives eigenvalues ω_j^2 and eigenvectors ϕ_j . Thus

$$\left(-\omega_j^2 \begin{pmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}_b \end{pmatrix} + \begin{pmatrix} \mathbf{k} & \mathbf{k}_b \\ \mathbf{k}_b^T & \mathbf{k}_{bb} + \tilde{\mathbf{S}}_{bb}^r(\Omega_o) \end{pmatrix} \right) \{\phi_j\} = \mathbf{0} \quad (6.7)$$

where $\tilde{\mathbf{S}}_{bb}^r(\Omega_o)$ is the real part of the soil impedance matrix, corresponding to the dam base nodes, at a small value of frequency Ω_o .

The nodal displacement vector can be expressed as a linear combination of first $nmod$ modes as

$$\begin{pmatrix} \mathbf{u}^l \\ \mathbf{u}_b^l \end{pmatrix} = \sum_{j=1}^{nmod} Z_j^l(\Omega) \begin{pmatrix} \phi_j^h \\ \phi_j^b \end{pmatrix} \quad (6.8)$$

In Eq. 6.8, the quantities $\mathbf{u}$ and ϕ_j^h correspond to the DOF of the dam excluding the dam-foundation nodes, while $\mathbf{u}_b$ and ϕ_j^b correspond to those along the dam-foundation interface

nodes. Collectively ϕ_j^h and ϕ_j^b represents the j th mode shape, ϕ_j of the coupled dam-foundation system. Noting that $\mathbf{r}_b = \mathbf{u}_b$, an expression for the former is obtained from Eq. 6.8. Substitution of this expression in Eq. 6.3 yields

$$\mathbf{r}_f = \mathbf{S}_{ff}^{-1}(\Omega) \mathbf{Q}_h(\Omega) - \sum_{j=1}^{nmod} \mathbf{Z}_j(\Omega) \chi_j(\Omega) \quad (6.9)$$

in which $\chi_j(\Omega) = \mathbf{S}_{ff}^{-1}(\Omega) \mathbf{S}_{bf}^T(\Omega) \phi_j^b$ represents the mode shape elements corresponding to the fluid-foundation interface, and ϕ_j^b is the j th mode shape corresponding to the DOF of nodes along the dam-foundation interface.

In their formulation for displacement at reservoir bottom, Chopra and coworkers adopted the static stiffness matrix of foundation to obtain χ_j , given by $\chi_j = \mathbf{S}_{ff}^{-1}(\Omega_o) \mathbf{S}_{bf}^T(\Omega_o) \phi_j^b$. However as shown above, χ_j is, in fact frequency dependent.

Using the transformation given by Eq. 6.8, Eq. 6.6 reduces to a system with only $nmod$ number of equations:

$$\begin{aligned} \left(-\Omega^2 \mathbf{I} + (1 + i\eta_d) \mathbf{A} + \Phi^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{S}}_{bb}(\Omega) - (1 + i\eta_d) \tilde{\mathbf{S}}_{bb}^r(\Omega_o) \end{bmatrix} \Phi \right) \begin{bmatrix} \mathbf{Z}' \\ \Phi \end{bmatrix} \\ = -\Phi^T \mathbf{E}^l + \Phi^T \begin{bmatrix} \mathbf{R}_r^l - \mathbf{S}_{bf} \mathbf{S}_{ff}^{-1} \mathbf{Q}_h^l \end{bmatrix} \end{aligned} \quad (6.10)$$

6.5 Treatment of Fluid Domain: Evaluation of $\mathbf{Q}_h^l$, $\mathbf{R}_h^l$ and $\mathbf{R}_r^l$

Referring to Fig. 6.2, and noting that linear boundary elements are used for discretization of reservoir, while constant boundary elements are used for foundation soil analysis, the reservoir hydrodynamic pressure (traction) along the reservoir bottom can be related to corresponding forces in soil along the fluid-foundation interface by the equilibrium conditions so that

$$\mathbf{Q}_h = -\mathbf{A} \begin{bmatrix} p_c \\ \mathbf{p}_2 \\ p_d \end{bmatrix} \quad (6.11)$$

where $\mathbf{Q}_h^T = \{Q^{1h} \ Q^{1v} \ Q^{2h} \ Q^{2v} \ Q^{3h} \ Q^{3v} \ \dots \ Q_d^h \ Q_d^v\}$,

$\mathbf{p}_2^T = \{p^1 \ p^2 \ p^3 \ \dots \ p^{(n-1)}\}$, n is the the number of nodes along the near field

reservoir bottom including the corner node d , but excluding the node c , and

$$A = \frac{l_{be}}{6} \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & -- & -- & 0 \\ a_{21} & a_{22} & a_{23} & 0 & -- & -- & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & -- & 0 \\ 0 & a_{42} & a_{43} & a_{44} & 0 & -- & 0 \\ 0 & 0 & a_{53} & a_{54} & a_{55} & -- & 0 \\ 0 & 0 & a_{63} & a_{64} & a_{65} & -- & 0 \\ -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- \\ 0 & 0 & 0 & -- & 0 & a_{(2n-1)n} & a_{(2n-1)(n+1)} \\ 0 & 0 & 0 & -- & 0 & a_{2n,n} & a_{2n(n+1)} \end{pmatrix}$$

in which

$$a_{11} = -\sin\beta_c; a_{12} = -2(\sin\beta_c + \sin\beta_1); a_{13} = -\sin\beta_1;$$

$$a_{21} = \cos\beta_c; a_{22} = 2(\cos\beta_c + \cos\beta_1); a_{23} = \cos\beta_1;$$

$$a_{32} = -\sin\beta_1; a_{33} = -2(\sin\beta_1 + \sin\beta_2); a_{34} = -\sin\beta_2;$$

$$a_{42} = \cos\beta_1; a_{43} = 2(\cos\beta_1 + \cos\beta_2); a_{44} = \cos\beta_2;$$

$$a_{53} = -\sin\beta_2; a_{54} = -2(\sin\beta_2 + \sin\beta_3); a_{55} = -\sin\beta_3;$$

$$a_{63} = \cos\beta_2; a_{64} = 2(\cos\beta_2 + \cos\beta_3); a_{65} = \cos\beta_3;$$

$$a_{(2n-1)n} = -\sin\beta_{n-1}; a_{(2n-1)(n+1)} = -2\sin\beta_{n-1};$$

$$a_{2n,n} = \cos\beta_{n-1}; a_{2n(n+1)} = 2\cos\beta_{n-1}$$

Q^{ih} and Q^{iv} are the horizontal and vertical components of the equivalent nodal forces at the i th boundary element node, p^i and p_d are the hydrodynamic pressures, and $\beta_{(i-1)}$ and β_i represent the angle of inclination of the elements located to the left and right side of the i th node respectively along the reservoir bottom with the horizontal direction as shown in Fig. 6.2.

In Eq. 6.11, A denotes the area matrix relating the forces and pressures at the fluid-foundation interface; p_2 is the vector of hydrodynamic pressures at the nodes of the linear boundary element along the reservoir bottom, excluding the corner node c at upstream face of the dam and node d along the transmitting boundary of the far field; and l_{be} is the length of boundary elements of the fluid-foundation interface, assumed to be equal all along the bottom of the reservoir near field.

While evaluating the reservoir induced hydrodynamic pressure by using rigorous ap-

proach, the governing differential equation, Eq. 2.2 or its discretized version Eq. 2.32, is solved in conjunction with the boundary conditions as explained below. The major difference between this model and the simplified approach is in the treatment of fluid-foundation interaction, and in the inclusion of cross coupling term between the foundation below the dam and the reservoir.

6.5.1 Boundary Conditions

along the upstream face of the dam

$$\begin{pmatrix} q_1^l \\ q_c^l \end{pmatrix} = -\rho_w \left[\mathbf{a}_n^l - \Omega^2 \sum_{j=1}^{nmod} \bar{\mathbf{A}}_n \psi_j Z_j^l(\Omega) \right] \quad (6.12a)$$

where

$$\mathbf{A}_n = - \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 & 0 & 0 & -- & -- & 0 \\ 0 & 0 & \cos\theta_2 & \sin\theta_2 & 0 & 0 & -- & -- & 0 \\ 0 & 0 & 0 & 0 & \cos\theta_3 & \sin\theta_3 & 0 & -- & 0 \\ -- & -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- & -- \\ 0 & 0 & 0 & 0 & 0 & -- & 0 & \cos\theta_c & \sin\theta_c \end{pmatrix}$$

$\psi_j^T = \{\psi_{jx}^1 \ \psi_{jy}^1 \ \psi_{jx}^2 \ \psi_{jy}^2 \ \psi_{jx}^3 \ \psi_{jy}^3 \ \dots \ \psi_{jx}^c \ \psi_{jy}^c\}$, and other notations are same as those defined in Chapters 2 and 5.

along the reservoir bottom

$$\begin{pmatrix} q_c^{Rl} \\ q_2^l \\ q_d^l \end{pmatrix} = -\rho_w \left[\mathbf{b}_n^l - \Omega^2 \begin{pmatrix} \bar{\mathbf{B}}_n^c \mathbf{r}_c \\ \bar{\mathbf{B}}_n \mathbf{r}_f \end{pmatrix} \right] \quad (6.12b)$$

where $\mathbf{B}_n^c = [\sin\beta_c \ \dots \ -\cos\beta_c]$,

$$\mathbf{B}_n = \begin{pmatrix} \sin\beta_1 & -\cos\beta_1 & 0 & 0 & 0 & 0 & -- & -- & 0 \\ 0 & 0 & \sin\beta_2 & -\cos\beta_2 & 0 & 0 & -- & -- & 0 \\ 0 & 0 & 0 & 0 & \sin\beta_3 & -\cos\beta_3 & 0 & -- & 0 \\ -- & -- & -- & -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & -- & -- & -- & -- \\ 0 & 0 & 0 & 0 & 0 & -- & 0 & \sin\beta_d & -\cos\beta_d \end{pmatrix}$$

$$\mathbf{r}_c^T = \{r_c^x \ r_c^y\}; \quad \mathbf{r}_f^T = \{r_1^x \ r_1^y \ r_2^x \ r_2^y \ \dots \ r_d^x \ r_d^y\}$$

superscripts R and L denote respectively the boundary elements located to the right and left side of the corner node under consideration, r_c is a displacement component of the

corner node c (at the dam-reservoir-foundation junction), angle β_c is as defined in Fig 6.2, (note that $\beta_d = \beta_{n-1}$) and $\mathbf{r}_f$ is the displacement vector of the foundation soil along the fluid-foundation interface as defined by Eq. 6.9

The first part of the b.c. of Eq. 6.12b corresponds to the free field acceleration of the reservoir bottom along the outward normal direction and the later part represents the acceleration of the flexible reservoir bottom along with the fluid-foundation interaction effect.

Using the principle of mode superposition, the displacements $\mathbf{r}_c$ and $\mathbf{r}_f$ can be expressed as linear combination of first few modes, giving

$$\begin{pmatrix} q_c^{Rl} \\ \mathbf{q}_2^l \\ q_d^{Ll} \end{pmatrix} = -\rho_w \left[\mathbf{b}_n^l - \Omega^2 \begin{pmatrix} \sum_{j=1}^{nmod} Z_j^l \mathbf{B}_n^c \psi_j^c \\ \mathbf{B}_n \mathbf{S}_{ff}^{-1} \mathbf{Q}_h^l - \sum_{j=1}^{nmod} Z_j^l \mathbf{B}_n \chi_j \end{pmatrix} \right] \quad (6.13)$$

in which $\chi_j(\Omega) = \mathbf{S}_{ff}^{-1}(\Omega) \mathbf{S}_{bf}^T(\Omega) \phi_j^b$, χ_j refers to the mode shapes corresponding to the horizontal and vertical DOF of reservoir bottom nodes vibrating in the j th mode, and ψ_j^c is the j th mode shape of the dam-foundation system corresponding to the DOF of node c ; $\bar{\mathbf{B}}_n \mathbf{S}_{ff}^{-1} \mathbf{Q}_h^l$ represents the fluid-foundation interaction component, and $\sum Z_j^l \mathbf{B}_n \chi_j$ denotes the cross coupling between the foundation below the dam and the reservoir bed. In a simplified model the fluid-foundation interaction is represented by approximate 1D bottom absorption model of Hall and Chopra (1980) as given in Eq. 5.11b, for example, while the cross coupling terms are ignored.

Using Eq. 6.11 to represent $\mathbf{Q}_h$, the force vector due to the fluid-foundation interaction, Eq. 6.13 can be written as

$$\begin{pmatrix} q_c^{Rl} \\ \mathbf{q}_2^l \\ q_d^{Ll} \end{pmatrix} = -\rho_w \left[\mathbf{b}_n^l - \Omega^2 \begin{pmatrix} \sum_{j=1}^{nmod} Z_j^l \mathbf{B}_n^c \psi_j^c \\ -\mathbf{B}_n \mathbf{S}_{ff}^{-1} \mathbf{A} \begin{pmatrix} p_c \\ \mathbf{p}_2 \\ p_d \end{pmatrix} - \sum_{j=1}^{nmod} Z_j^l \mathbf{B}_n \chi_j \end{pmatrix} \right] \quad (6.14)$$

The relationship between the pressure and the pressure gradient along the far end of the near field of the reservoir, viz. along the transmitting boundary, can be obtained through the continuum expressions of 1D FEM model which are given by Eqs. 2.41 and 2.42, for horizontal and vertical excitations respectively. As already stated, the zero pressure boundary condition along the free surface of the reservoir is accounted for through the fundamental solution of the BEM formulation of the reservoir fluid.

Taking advantage of the linear form of the governing equation and the associated boundary conditions, the hydrodynamic pressure vector $\mathbf{p}^l$ can be expressed as

$$\mathbf{p}^l(\Omega) = \mathbf{p}_o^l(\Omega) + \sum_{j=1}^{nmod} Z_j^l \{ \mathbf{p}_j^f(\Omega) + \mathbf{p}_j^b(\Omega) \} \quad (6.15)$$

In Eq. 6.15 $\mathbf{p}_o^l$ is the pressure vector obtained by solving Eq. 2.32, with boundary conditions corresponding to unit harmonic free field motion of the rigid dam along the l th component direction, i.e.

along the upstream face of the dam

$$\begin{pmatrix} q_1^l \\ q_c^l \end{pmatrix} = -\rho_w \mathbf{a}_n^l \quad (6.16a)$$

along the reservoir bottom

$$\begin{pmatrix} q_c^R \\ q_2^l \\ q_d^L \end{pmatrix} = -\rho_w \left(\mathbf{b}_n^l + \Omega^2 \begin{pmatrix} 0 \\ \bar{\mathbf{B}}_n \mathbf{S}_{ff}^{-1} \mathbf{A} \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} \end{pmatrix} \right) \quad (6.16b)$$

Also in Eq. 6.15, vector $\mathbf{p}_j^f(\Omega)$ is the solution of Eq. 2.32, with boundary conditions corresponding to that of horizontal acceleration of dam-foundation system vibrating in the j th mode, with no motion of the reservoir bed, i.e.

along the upstream face of the dam

$$\begin{pmatrix} q_1 \\ q_c^L \end{pmatrix} = \rho_w \Omega^2 \bar{\mathbf{A}}_n \psi_j \quad (6.17a)$$

along the reservoir bottom

$$\begin{pmatrix} q_c^R \\ q_2 \\ q_d^L \end{pmatrix} = -\rho_w \Omega^2 \begin{pmatrix} 0 \\ \bar{\mathbf{B}}_n \mathbf{S}_{ff}^{-1} \mathbf{A} \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} \end{pmatrix} \quad (6.17b)$$

where ψ_{jx} and ψ_{jy} refers to the mode shapes along the horizontal and vertical DOF of upstream dam face nodes of the dam-foundation system vibrating in the j th mode; and other notations are the same as used in Chapter 2.

Pressure vector $\mathbf{p}_j^b(\Omega)$ is the solution of Eq. 2.2, with boundary conditions corresponding to that of vibration of the fluid-foundation interface in the j th mode χ_j , with no motion of the dam, i.e.

along the upstream face of the dam

$$\begin{pmatrix} \mathbf{q}_1 \\ \mathbf{q}_c^L \end{pmatrix} = \mathbf{0} \quad (6.18a)$$

along the reservoir bottom

$$\begin{pmatrix} \mathbf{q}_c^{lt} \\ \mathbf{q}_2 \\ \mathbf{q}_d^L \end{pmatrix} = -\rho_w \Omega^2 \begin{pmatrix} -\mathbf{B}_n^c \zeta_j^c \\ \mathbf{B}_n \mathbf{S}_{jj}^{-1} \mathbf{A} \begin{pmatrix} p_c \\ p_2 \\ p_d \end{pmatrix} + \mathbf{B}_n \lambda_j \end{pmatrix} \quad (6.18b)$$

Using Eq. 6.15, the hydrodynamic force vector $\mathbf{R}_h^l$ and $\mathbf{R}_r^l$ can now be expressed as

$$\mathbf{R}_h^l(\Omega) = \mathbf{R}_{ho}^l(\Omega) + \sum_{j=1}^{nmod} Z_j^l(\Omega) (\mathbf{R}_{hj}^f(\Omega) + \mathbf{R}_{hj}^b(\Omega)) \quad (6.19a)$$

$$\mathbf{R}_r^l(\Omega) = \mathbf{R}_{ro}^l(\Omega) + \sum_{j=1}^{nmod} Z_j^l(\Omega) (\mathbf{R}_{rj}^f(\Omega) + \mathbf{R}_{rj}^b(\Omega)) \quad (6.19b)$$

where the non-zero terms of $\mathbf{R}_{ho}^l(\Omega)$, $\mathbf{R}_{ro}^l(\Omega)$, $\mathbf{R}_{hj}^f(\Omega)$, $\mathbf{R}_{rj}^f(\Omega)$, $\mathbf{R}_{hj}^b(\Omega)$ and $\mathbf{R}_{rj}^b(\Omega)$ denote the nodal forces along dam's upstream face. These forces are statically equivalent to the hydrodynamic pressures $\mathbf{p}_o^l(\Omega)$, $\mathbf{p}_j^f(\Omega)$ and $\mathbf{p}_j^b(\Omega)$ respectively.

Substituting for $\mathbf{R}_h^l$ and $\mathbf{R}_r^l$ in Eq. 6.10, the final equation reduces to the form

$$\mathbf{K}^*(\Omega) \mathbf{Z}^l(\Omega) = \mathbf{F}^{*l}(\Omega) \quad (6.20)$$

where the elements of the effective stiffness matrix of the coupled system are given by

$$K_{ij}^*(\Omega) = (-\Omega^2 + (1 + \eta_M)\omega_i^2) \delta_{ij} + \phi_i^T \mathbf{S}_{of}(\Omega) \phi_j - \phi_i^{ltT} (\mathbf{R}_{hj}^f(\Omega) + \mathbf{R}_{hj}^b(\Omega)) - \phi_i^{btT} (\mathbf{R}_{rj}^f(\Omega) + \mathbf{R}_{rj}^b(\Omega)) \quad (6.21a)$$

in which

$$\mathbf{S}_{of}(\Omega) = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{S}}_{bb}(\Omega) - (1 + \eta_M) \hat{\mathbf{S}}_{bb}^r(\Omega_0) \end{pmatrix}$$

and those of the effective force elements by

$$\mathbf{F}_i^{*l}(\Omega) = -\phi_i^T \mathbf{E}^l + \phi_i^{ltT} \mathbf{R}_{ho}^l(\Omega) + \phi_i^{btT} \mathbf{R}_{ro}^l(\Omega) \quad (6.21b)$$

Equations similar to those in Chapter 5 can be developed for the special cases of dam vibrating alone, dam-reservoir system on rigid foundation, and dam-foundation system with empty reservoir. Furthermore, analysis excluding the cross coupling between the foundation below the dam and the reservoir bottom can be performed by just dropping the $\mathbf{R}_j^b(\Omega)$ term in Eq. 6.21a.

Fig. 6.1 Dam-Reservoir-Foundation Model for Rigorous Analysis

Fig. 6.2 Distribution of Hydrodynamic Forces Along the Fluid-Foundation Interface

7

RESPONSE ANALYSIS OF DAM- RESERVOIR-FOUNDATION SYSTEM

7.1 Introduction

Formulations for the response evaluation of a dam-reservoir-foundation system were developed in Chapters 5 and 6. Results of numerical and parametric studies based on these formulations are presented in this Chapter.

Preliminary response analysis of a dam whose cross section is idealized as a triangular shape is carried out first in order to demonstrate the various interaction effects, namely, reservoir hydrodynamic forces, foundation flexibility, and reservoir bottom absorption effect, on the response of the dam. Complex frequency response functions are considered for this purpose. The role of material damping in the foundation soil on the response of the dam supporting it is also examined.

The performance of the model based on simplified approach (for ease of reference denoted as S-model) is studied by comparing the results obtained by using it with those obtained from a model that relies on a more rigorous approach in the response evaluation of a dam-reservoir-foundation system. The studies are carried out initially for the idealized dam cross section, and later extended to the Pine Flat dam system. The models based on rigorous approach are of two kinds, the first one includes the cross coupling between the foundation below the dam and the reservoir bed (denoted by RC-model), the other excludes this cross coupling effect (denoted by RUC-model). Comparison between the response results obtained from RC-model and RUC-model will illustrate the effect that cross coupling has on the response, while comparison between the results from RUC-model and S-model will throw light on the effectiveness of representing the fluid-foundation interaction by simplified 1D approximate model of Hall and Chopra(1980). The main objective of these

analytical studies is to examine how well the simplified model is capable of predicting the response of a dam-reservoir-foundation system.

In the studies presented, numerical results are obtained for the complex frequency response functions of the crest displacement and acceleration for a unit harmonic horizontal and vertical excitation. In addition, response time history of crest displacement, and the contours of principal tensile stress envelopes are obtained for the dam system subjected to transverse and vertical components of Taft and ElCentro ground motions. Response to transverse ground motion are given particular emphasis, since previous researchers (Medina et al. 1990) have claimed that the S-model underestimates the peak value of complex frequency response of the dam subjected to horizontal harmonic excitation. The computational effort and the accuracy associated with each model are also discussed. The time history of response to a random ground motion is obtained by FFT techniques.

In addition to the studies mentioned in the foregoing paragraph, the effect of location of the transmitting boundary in a reservoir with constant sloped bottom is examined by means of a convergence study on the response results for Pine Flat dam-reservoir system, where for simplicity the foundation below the dam is assumed to be rigid. This study is meant to provide some guidelines on the length of the irregular portion that must be included in the model of the infinite reservoir. Also, the effect of the reservoir bottom slope on the overall response of the dam is examined.

7.2 Preliminary Study of Idealized Dam System

Figure 7.1 shows the cross sectional dimension and the finite element mesh used in the study of an idealized gravity dam section of a triangular geometry. The material and geometric properties of the dam-reservoir-foundation system are chosen to be the same as those adopted by Fenves and Chopra (1984) in the analysis of a similar system. However, the finite element mesh used in the present research for the discretization of the dam is finer than that used by Fenves and Chopra. This is so partly because the number of finite elements along the upstream face of the dam are chosen to be equal to the number of boundary elements required along the dam-fluid interface for the reservoir analysis.

The complex valued frequency response functions of the absolute value of horizontal crest acceleration of the dam, subjected to a unit harmonic excitation, are presented in this section. In obtaining the response values, various interaction components of the impounded reservoir and underlying foundation are considered.

Table 7.1: Conditions of Dam-Reservoir-Foundation System Analyzed

Case	E_f/E_d	Reservoir	α_r	η_s	Excit. Direction
1	∞	empty	-	-	horizontal
2	∞	empty	-	-	vertical
3	∞	full	1.0	-	horizontal
4	∞	full	1.0	-	vertical
5	∞	full	0.75	-	horizontal
6	∞	full	0.75	-	vertical
7	1.0	empty	-	0.1	horizontal
8	1.0	empty	-	0.1	vertical
9	1.0	full	1.0	0.1	horizontal
10	1.0	full	1.0	0.1	vertical
11	1.0	full	0.75	0.1	horizontal
12	1.0	full	0.75	0.1	vertical
13	1.0	empty	-	0.0	horizontal
14	1.0	empty	-	0.0	vertical
15	1.0	full	1.0	0.0	horizontal
16	1.0	full	1.0	0.0	vertical
17	1.0	full	0.75	0.0	horizontal
18	1.0	full	0.75	0.0	vertical

Table 7.1 shows the various cases analyzed in the preliminary study. The properties of dam material are: Poisson's ratio = 0.2, unit weight = 2483 kg/m^3 (155 lb/ft^3), and the Young's modulus = 27580 Mpa ($4 \times 10^6 \text{ psi}$). The hysteretic damping factor is taken as η_d

$= 0.1$, which corresponds to a 5 % modal damping in the dam on rigid foundation and with an empty reservoir. A 2D plane stress idealization for the dam is adopted in the analysis for the reasons presented in the Chapter 5.

For the flexible foundation case, the elastic modulus of the foundation material is assumed to be same as that of the dam material. Although a plane strain idealization would be more appropriate in representing the foundation behavior, a plane stress idealization is adopted for the analysis for compatibility with the idealization adopted for the dam substructure. The Poisson's ratio of the foundation material is $1/3$, hysteretic damping factor for foundation soil is $\eta_s = 0.1$ when the soil is viscoelastic, and 0.0 when the soil is purely elastic. The unit weight of foundation soil is 26.43 kg/m^3 (165 lb/ft^3). The boundary element discretization of the foundation surface for the preliminary analysis is presented in Fig. 7.2.

The reservoir properties are the same as those described in Chapter 2. The full reservoir has a constant depth of $H=91 \text{ m}$ (300 ft), equal to the total depth of the dam cross section. For this part of the study, the fluid-foundation interaction effect is accounted for by 1D simplified model. Two different cases of reservoir bottom absorption conditions corresponding to the wave reflection coefficients $\alpha_r = 1.0$ and 0.75 are considered. The first value of α_r represents a non-absorptive reservoir bottom. Since the reservoir is rectangular, a continuum solution can be used in this case. However, in order to test the effectiveness of the boundary element solution, the reservoir is divided into a near field and an infinite far field. For the case of horizontal excitation, the near field bottom is assumed to be absorptive while the far field reservoir bottom is considered as being non-absorptive. A boundary element solution is used for the near field while a finite element treatment is adopted for the analysis of far field. Based on the convergence study carried out in Chapter 3, the length of the near field is chosen to be $L_f = 2.5H$. The same model is used for the case of vertical excitation. However, the far field reservoir bottom is considered to be absorptive and continuum solution is used for this purpose. This is a valid approach because the complete reservoir is rectangular and therefore the pressure gradients along the interface between the near and far field are zero. The details of the interior points are identical to those discussed under Section 3.2. Linear boundary element formulation is adopted for the reservoir analysis. The boundary element mesh for the reservoir analysis is as shown in Fig. 7.3.

Based on an initial study of the response for excitation frequencies upto 25Hz, first 5 modes were found to be adequate for the analysis of a dam on rigid foundation, while the first 10 modes had to be included when the flexibility of the foundation was taken into account. In all the cases analyzed, the absolute horizontal acceleration of the dam's crest, normalized with respect to the amplitude of the harmonic excitation, is plotted against the normalized excitation frequency normalized with respect to the fundamental frequency of the dam on rigid foundation with empty reservoir.

The frequency response functions of the crest accelerations for the different cases of Table 7.1, are shown in Figs. 7.4 to 7.11. The results clearly demonstrate the role of reservoir hydrodynamic pressure, reservoir bottom absorption, foundation flexibility, and material damping in the foundation soil, for both horizontal and vertical components of harmonic excitations. These effects are briefly discussed in the following sections.

7.2.1 Dam-Water Interaction and Reservoir Bottom Absorption Effect

Comparing the curves 1 and 2 of Figs. 7.4 to 7.7, it is clear that the hydrodynamic forces, i.e. the dam-reservoir interaction effect reduce the fundamental resonant frequency of the system and increase the dam's response for a reservoir with non-absorptive bottom, for both horizontal and vertical harmonic excitations. For the case of vertical excitation, the increased response approaches infinite values, for a range of frequencies in the vicinity of the resonant frequencies of the dam, whether or not the foundation is flexible. For horizontal excitation, finite, sharp and narrow peaks can be seen at the resonant frequencies. Also in this case the response at higher frequencies is smaller when there is water in the reservoir than when the reservoir is empty. This is because of the infinite wave radiation occurring in a water filled reservoir at excitation frequencies greater than the reservoir fundamental frequencies.

The effect of reservoir bottom absorption can be seen by comparing curves 2 and 3 of Figs. 7.4 to 7.7. In general, reservoir bottom absorption tends to decrease the response of the dam. In the case of horizontal excitation, the bottom absorption reduces the response at the fundamental resonant frequency of the dam. This reduction is significant for a dam on rigid foundation, but only marginal when the foundation below the dam is flexible. Also for a horizontal excitation, the bottom absorption effect is not pronounced at higher excitation frequencies, because of the fact that infinite radiation damping in the reservoir dominates

the energy dissipation at these frequencies.

For vertical excitation, reservoir bottom absorption eliminates the infinite peaks in the response and converts them to finite and smaller peak values, regardless of the flexibility of the foundation below the dam.

In summary, the reservoir bottom absorption effect is less significant in reducing the response of the dam under horizontal excitation, especially when the foundation below the dam is flexible. However, the absorption effect considerably reduces the response at resonant frequencies of the dam-reservoir system, when the system is subjected to vertical excitation. This makes the response to vertical excitation less important as compared to the response to horizontal component, particularly for a flexible foundation below the dam.

7.2.2 Effect of Flexibility and Material Damping in Foundation Soil

Figures 7.6 and 7.7 present the complex frequency response functions for dam-reservoir system with flexible foundation. Comparison of these plots with Figs. 7.4 and 7.5 shows that foundation flexibility causes an overall reduction in response and makes the response function vary smoothly, except at resonance in the case of a vertical excitation of a reservoir with non-absorptive bottom. The foundation flexibility also reduces the fundamental frequency of the dam-reservoir-foundation system. In the case of horizontal excitation, the dam-foundation interaction makes the effect of reservoir bottom absorption less pronounced particularly at the fundamental resonant frequency. In other words, geometric radiation phenomenon of the foundation half plane plays a more dominant role in energy dissipation process than either reservoir bottom absorption or infinite radiation in the reservoir at higher excitation frequencies.

Figs. 7.8 to 7.11 illustrates the effect of material damping in the foundation soil on the response of the dam. The response varies with frequency in almost identical manner for both a viscoelastic and an elastic half plane. However the response associated with viscoelastic foundation is smaller as compared to that with an elastic foundation, implying that the foundation material damping results in a reduction in the response. This effect is more significant at the fundamental resonant frequency of the dam than at other frequencies, for both components of excitation.

7.3 Comparison of Models Based on Simplified and Rigorous Approach

In a recent study Medina et al(1990) used a BEM modeling of the foundation to account for the fluid-foundation interaction. They obtained the complex frequency response function of absolute value of horizontal crest acceleration of a dam with triangular cross section and concluded that although the representation of fluid-foundation interaction by approximate 1D model provided a reasonably accurate estimate of responses to a vertical excitation, it led to an underestimation of the peak response to horizontal excitation. However, the real test of the effectiveness of a method is in knowing how well it can predict the response to a random ground motion. Significant differences in complex frequency response need not necessarily cause large difference in the response to real earthquakes. This section investigates the behavior of the 1D model for wave absorption, by comparing the results obtained from this simplified 1D model with those obtained from a more rigorous approach, for both the complex frequency response functions as well as the response to earthquake ground motion.

The response of the dam-reservoir-foundation system is analyzed using the three different models described under Section 7.1. Only viscoelastic foundation material is considered in this analysis. In order that the performance of the S-model may be compared with that of an equivalent model based on the rigorous approach, the wave reflection coefficient used for the S-model is calculated from the same material properties as used in the rigorous model. Two set of analyses, one with an idealized dam (coarser finite element mesh) and the other using a realistic dam (finer finite element mesh) are carried out to fully investigate the performance of each of the models, particularly under transverse component of excitation.

7.3.1 Study of Idealized Triangular Dam System

The cross section and material properties of the dam-reservoir system as well as the finite element and boundary element meshes used to represent the system are the same as those chosen for the preliminary analyses of the idealized dam section (similar to cases 11 and 12 of Table 7.1), except that for the S-model, a value of 0.7118 is used for α_r corresponding to the selected properties of the foundation soil. In addition, for the analysis

using a rigorous approach, the fluid-foundation interaction is accounted for by using the BEM to model the foundation below the reservoir as explained in Chapter 6.

Complex Frequency Response Functions

Figures 7.12 and 7.13 present the complex frequency response functions of the normalized absolute value of horizontal crest acceleration of the idealized dam system obtained by the three models, respectively for horizontal and vertical unit harmonic excitations. The response functions obtained by the three different approaches are in good agreement with each other, except for some difference in peak values corresponding to the fundamental resonant frequency. In case of horizontal harmonic excitation, the S-model underestimates this peak response by 7.7% in comparison with RC-model and 9.2% with respect to RUC-model, while for vertical excitation, the corresponding differences are 1.7% and 8.3%.

Time History Response and Principal Stress Envelope for Taft Ground Motion

The S69E, N21E and vertical components of ground motion of Taft Lincoln School Tunnel during the Kern County, California earthquake of 21 July 1952, are treated as the free-field input. S69E and N21E component acts along the horizontal direction, transverse to the axis of the dam. Response analysis is performed by considering the first ten mode shapes of the associated dam-foundation system. For the frequency domain analysis using Fast Fourier Transform (FFT) technique, 2048 time steps of 0.02 s are used. The first 20 s of the excitation is taken as equal to the ground acceleration values for the Taft motion, the remaining period is made up of a grace band of zeros to avoid the aliasing errors inherent in discrete Fourier transforms.

The effects of static loads in the dam due to self weight, hydrostatic loads, uplift forces or any other load are excluded from the present study for the computation of displacements and stresses, so that the interpretation of the effect of dam-reservoir interaction, reservoir bottom absorption, and foundation flexibility on the response of dam to random ground motion is not obscured (Fenves and Chopra, 1984); however, analysis including these static load effects are very straight forward.

The time histories of the horizontal crest displacement of the dam subjected to each of the ground motion components are evaluated by the three models and are plotted in Figs. 7.14 to 7.19. The response time histories match very well except for some minor discrepancies. For an input represented by the transverse component of Taft motion, the

maximum difference in the peak value of crest displacement between the S-model and RC-model is only 5.1%, while that between S-model and RUC-model is only 2.6%. For an excitation given by the vertical component of Taft motion corresponding differences are 4.8% and 7.0%.

The nodal displacements corresponding to the finite element nodes are evaluated at each time step in the computation of response history. These displacements are then utilized to compute the element principal stresses. The stresses at the geometric centroid of each finite element of the dam are computed by using the element stress-displacement relations in the FEM formulation. These relations are presented in Appendix A. The maximum principal stress (tensile) during the entire time history is obtained for each finite element. These maximum stresses are plotted to obtain the contours of maximum principal stress. Figures 7.20 to 7.22 show such contours for each of the three components of Taft motion obtained by three different approaches. Once again, as observed for the displacement time histories, the principal tensile stress envelopes corresponding to the three different models are in good agreement with each other.

7.3.2 Analysis of Pine Flat Dam System

The non-overflow monolith of Pine Flat Dam, is analyzed for its response to the Taft and ElCentro ground motions. The cross section dimensions and the finite element mesh for the analysis of this dam are given in Fig. 7.23. For the dam material, the Young's modulus of elasticity $E_d = 22500 \text{ Mpa}$ ($3.25 \times 10^6 \text{ psi}$), unit weight $= 2483 \text{ kg/m}^3$ (155 lb/ft^3), Poisson's ratio $= 0.2$. The hysteretic damping factor for the dam, η_d , and for the viscoelastic foundation soil, η_s , were both taken equal to 0.1. The properties of the foundation material are: $E_f = 22500 \text{ Mpa}$ ($3.25 \times 10^6 \text{ psi}$), so that the elastic moduli ratio $E_f/E_d = 1$; unit weight $= 2643 \text{ kg/m}^3$ (165 lb/ft^3); and Poisson's ratio $= 1/3$. The full reservoir has a constant depth of $H = 115.5 \text{ m}$ (381 ft). The pressure wave velocity in water is 1440 m/sec (4720 ft/sec), and the unit weight of water is 1000 kg/m^3 (62.4 lb/ft^3). The length of the reservoir near field is $L_f = 2.58 \times H$. For the selected properties of foundation soil, the value of wave reflection coefficient α_r in the 1D simplified wave absorption model works out to 0.6853. The boundary element meshes for the reservoir and foundation discretization were developed following the criteria similar to those in Figs. 7.2 and 7.3.

Complex Frequency Response Functions

The complex frequency response functions of the absolute value of horizontal acceleration of dam's crest, for both components of the excitation are evaluated using the three models and are plotted in Figs. 7.24 and 7.25a. For horizontal harmonic excitation, the response corresponding to the fundamental resonant frequency predicted by the RUC-model is largest, while that obtained by S-model is in between the responses predicted by the two other models. The peak response obtained from the S-model is 4.7% higher than that obtained by RC-model, and 5.1% lower than that obtained by RUC-model. The respective figures for the vertical harmonic excitation case are 9.1% and 6.4%.

Note that in Fig. 7.25a, at higher excitation frequencies the complex frequency response function for crest acceleration due to vertical harmonic excitation corresponding to the rigorous approach are quite larger as compared with those of the simplified approach. Figures 7.25b and 7.25c shows the frequency response functions of the real and imaginary part of the crest displacement corresponding to the three models, and only small difference in displacement is seen at higher frequencies, between the results of the three approaches. These differences are magnified severely in case of the acceleration response plot, due to the fact the pseudo acceleration are the square of the excitation frequencies times the corresponding displacement, and the difference being occurring at larger excitation frequency range. However, it is only the real and imaginary part of the crest displacement, and not the acceleration, that enters in the FFT analysis for the computation of displacement time history and the stress contours, due to a given random motion. Furthermore, the predominant frequency content of most of the earthquake are smaller than the frequency level at which these difference occur. It will be shown in the following sections that the responses due to vertical component of earthquake ground motion computed by three models are almost similar, inspite of the discrepancy in complex frequency response values of the crest acceleration at higher excitation frequencies.

In the evaluation of frequency response to vertical excitation, since the inclined upstream face of the Pine Flat dam makes the reservoir near field a non-rectangular one, theoretically the continuum solution for the treatment of infinite far field region is not applicable. However, when the interface between the reservoir near and far field is located at sufficiently larger distance from the dam, even under the situation of irregular reservoir near field, the pressure gradient along this interface turns out to be negligibly small, making it

possible to adopt the continuum solution corresponding to an absorptive reservoir bottom in the far field analysis. This is illustrated by Fig. 7.26, the response to vertical excitation using simplified approach using both the FEM treatment of the far field with non-absorptive bottom and the continuum solution of the far field with an absorptive bottom are almost identical, except for some sharp peaks at resonant frequency of the system exhibited by the FEM solution of the rigid far field. This argument is further confirmed in the next section dealing with the study of reservoirs with sloping bed, wherein if the transmitting boundary is located from the dam at a distance of at least $2.5 \times$ the depth of reservoir at the dam's face, then for vertical excitation case the response of the dam predicted by the FEM treatment of the far field and those predicted using the continuum solution are not different. Therefore for the future analysis of Pine Flat dam in this section (since the interface is located at $2.58 \times H$), continuum solution of absorptive far field is adopted in the analysis of vertical excitation.

Time History Response and Principal Stress Envelopes for Taft and ElCentro Ground Motions

The Pine Flat dam described in the previous section is analyzed for its response to the S69E, N21E and vertical components of Taft earthquake ground motion of 1952, and S00E and S90W components of ElCentro ground motion of 1942.

The time histories of the horizontal crest displacement of the dam subjected to each of the ground motion components evaluated by the three models are plotted in Figs. 7.27 to 7.36. In all the cases analyzed, the S-model seems to slightly overestimate some of the peak responses compared to the responses predicted by RC and RUC models. Comparison of S-model and RC-model gives a maximum peak crest displacement difference of 13.4% (Fig. 7.29) for S69E component of Taft, while the corresponding difference is 12.8% (Fig. 7.31) for the vertical component of Taft. This difference is due to the approximations involved in ignoring the cross coupling effect, as well as in representing the fluid-foundation interaction by 1D simplified model.

Comparison of the S-model and the RUC-model indicates that difference in maximum peak crest displacement is only 2.6% (Fig. 7.34) for transverse components of ElCentro and 8.4% (Fig. 7.32) for the Taft vertical component. This difference is completely due to the approximation in representing the fluid-foundation interaction by 1D simplified model

instead of a more rigorous approach.

Figures 7.37 to 7.41 present the contours of maximum principal tensile stress envelopes of Pine Flat dam subjected to Taft and ElCentro ground motions, obtained by three different approaches. Here again the principal stress envelopes corresponding to all the three different models are in fairly good agreement.

Table 7.2: Computational Time for the Analysis of Pine Flat Dam-Reservoir-Foundation System to Transverse and Vertical Ground Motion

Description of the Model	C.P.U time in Minutes*
Simplified approach	467
Rigorous uncoupled approach	691
Rigorous coupled approach	777

* Sun Spark Station II Computer

The computer time required in obtaining the complex frequency response functions for the Pine Flat dam-reservoir-foundation system to transverse and vertical ground motions is presented in Table 7.2 for each of the three different approaches of analysis discussed in Chapters 5 and 6.

It is evident from the figures presented, that the simplified approach is computationally more efficient than the other two rigorous approaches of analysis.

7.4 Response of a Dam Impounding Reservoir with Sloped Bottom

7.4.1 Location of Transmitting Boundary

In modeling an infinite reservoir of irregular geometry, the reservoir has been represented as consisting of two regions, a near field of irregular geometry and a far field of rectangular section. It is believed that if the interface between these two regions is located at sufficient distance from the dam, the model will provide a reasonable representation of the original reservoir. In order to examine the validity of this assumption, a study is carried out of the effect of location of the interface on the response of the dam system. For

simplicity the reservoir is assumed to have a bottom of constant slope and the foundation below the dam is treated as rigid. Results presented earlier in this work show that for the present analyses a 1D simplified model is adequate for representing the fluid-foundation interaction effect for both components of excitations.

The objective of this study is to find the optimum position for the location of the transmitting boundary, in other words the extent of the irregular region of the reservoir, for the analysis of dam-reservoir system where the infinite reservoir possesses a sloped bottom throughout its length. This is achieved by means of a convergence analysis of the complex frequency response functions for the absolute value of crest acceleration of the dam, subjected to unit harmonic horizontal and vertical excitations separately.

Response analyses are carried out for Pine Flat dam. Two different reservoir bottom slopes, 5 degrees and 15 degrees, are considered. The wave reflection coefficient α_r is selected as 0.75. For each bottom slope, response analysis is carried out for both components of excitation with the transmitting boundary placed at 5 different locations: $1.0H$, $1.5H$, $2.0H$, $2.5H$, and $3.0H$ (where H represents the reservoir depth at the dam-fluid interface) respectively from the upstream face of the dam as shown in Fig. 7.42. First 5 modes are included in the evaluation of frequency response function.

7.4.2 Use of Continuum Solution of an Absorptive Far Field in Response Analysis for Vertical Excitation

In an analysis for vertical excitation, the FEM treatment of the reservoir far field having non-absorptive bottom will result in sharp and narrow peaks in the complex frequency response functions corresponding to the natural frequencies of the reservoir calculated for the depth of the far field. Continuum solutions of an absorptive far field are quite straightforward in such cases, however their validity is subject to the condition that the pressure gradient vanish along the transmitting boundary. This may not be the case for a reservoir near field with non-rectangular geometry, but even under such circumstances a continuum solution may be applicable for response evaluation under vertical excitation, provided the transmitting boundary is located at a distance of about $2.5H$ or more from the upstream face of the dam. The validity of the last statement can be verified by referring to Figs. 7.43.

Figures 7.44 and 7.45 illustrate the complex frequency response functions for five different locations of the transmitting boundary for a reservoir with a bottom slope of 15 degrees

subjected to horizontal and vertical excitation respectively. Figures 7.46 and 7.47 show the corresponding response functions for the reservoir bottom slope of 5 degrees. These figures exhibit promising convergence as the distance at which the transmitting boundary is located increases, and it is evident that convergence in response function occurs when the transmitting boundary is located at about $2.5H$ from the upstream face of the dam, for the two bottom slopes and excitation components analyzed.

7.4.3 Effect of Reservoir Bottom Slope on Response of the Dam

Finally, the effect of the reservoir bottom slope on the overall response of the dam is examined, by considering 3 different reservoir bottom slopes of 0, 5 and 15 degrees. In all the cases the transmitting boundary is placed at a distance of $2.5H$ from the upstream face of the dam. Figures 7.48 and 7.49 show the complex frequency response functions for the Pine Flat dam-reservoir system subjected to horizontal and vertical harmonic excitations. On the basis of these plots it can be stated that the response reduces as the bottom slope increases, except in the response at the second resonant frequency, where the trend seems to be just the opposite.

Fig. 7.1 Cross Sectional Properties and Finite Element Model of the Idealized Triangular Dam

Fig. 7.2 Boundary Element Mesh for the Foundation Soil Analysis of the Idealized Triangular Dam System.

Fig. 7.3 Boundary Element Mesh for Reservoir Analysis of the Idealized Dam System

Fig. 7.4 Dam-Reservoir Interaction; Horizontal Excitation
Reservoir Bottom Absorption Effect;

Fig. 7.5 Dam-Reservoir Interaction; Vertical Excitation
Reservoir Bottom Absorption Effect;

Fig. 7.6 Dam-Reservoir -Foundation Interaction;and Reservoir Bottom Absorption Effect; Horizontal Excitation; $\eta_s = 0.1$

Fig. 7.7 Dam-Reservoir -Foundation Interaction;and Reservoir Bottom Absorption Effect; Vertical Excitation; $\eta_s = 0.1$

Fig. 7.8 Dam-Foundation Interaction; Horizontal Excitation
Effect of Foundation Material Damping

Fig. 7.9 Dam-Foundation Interaction; Vertical Excitation
Effect of Foundation Material Damping

Fig. 7.10 Foundation Damping and Reservoir Bottom Absorption Effect; Dam-Reservoir-Foundation System; Horizontal Excitation

Fig. 7.11 Foundation Damping and Reservoir Bottom Absorption Effect; Dam-Reservoir-Foundation System; Vertical Excitation

Fig. 7.12 Frequency Response Function of Idealized Triangular Dam-Reservoir-Foundation System; Horizontal Excitation

Fig. 7.13 Frequency Response Function of Idealized Triangular Dam-Reservoir-Foundation System; Vertical Excitation

Fig. 7.14 Displacement Response of Idealized Dam System to Taft N21E Component; Simplified and Rigorous Coupled

Fig. 7.15 Displacement Response of Idealized Dam System to Taft N21E Component; Simplified and Rigorous Uncoupled

Fig. 7.16 Displacement Response of Idealized Dam System to Taft S69E Component; Simplified and Rigorous Coupled

Fig. 7.17 Displacement Response of Idealized Dam System to Taft S69E Component; Simplified and Rigorous Uncoupled

Fig. 7.18 Displacement Response of Idealized Dam System to Taft Vertical Component; Simplified and Rigorous Coupled

Fig. 7.19 Displacement Response of Idealized Dam System to Taft Vertical Component; Simplified and Rigorous Uncoupled

Fig. 7.20 Maximum Principal Stress (in psi) Envelope of Idealized Dam System to Tilt N21E Component

Fig. 7.21 Maximum Principal Stress (in psi) Envelope of Idealized Triangular Dam System to Tilt S69E Component

Fig. 7.22 Maximum Principal Stress (in psi) Envelope of Idealized Triangular Dam: System to Tilt Vertical Component

Fig. 7.23 Cross Sectional Properties and Finite Element Mesh of Tallest, Non-overflow Monolith of Pine Flat Dam

Fig. 7.24 Frequency Response Function of Pine Flat Dam-Reservoir-Foundation System; Horizontal Harmonic Excitation

Fig. 7.25a Frequency Response Function of Pine Flat Dam-Reservoir-Foundation System; Vertical Harmonic Excitation

Fig. 7.25b Response Function of Real Part of Crest Displacement of Pine Flat Dam-Reservoir-Foundation System; Vertical Harmonic Excitation

Fig. 7.25c Response Function of Imaginary Part of Crest Displacement of Pine Flat Dam-Reservoir-Foundation System; Vertical Harmonic Excitation

Fig. 7.26 Frequency Response Function of Pine Flat Dam-Reservoir-Foundation System; Vertical Excitation; Treatment of the Reservoir Far Field

Fig. 7.27 Displacement Response of Pine Flat Dam System to Taft N21E Component; Simplified and Rigorous Coupled

Fig. 7.28 Displacement Response of Pine Flat Dam System to Taft N21E Component; Simplified and Rigorous Uncoupled

Fig. 7.29 Displacement Response of Pine Flat Dam System to Taft S69E Component; Simplified and Rigorous Coupled

Fig. 7.30 Displacement Response of Pine Flat Dam System to Taft S69E Component; Simplified and Rigorous Uncoupled

Fig. 7.31 Displacement Response of Pine Flat Dam System to Taft Vertical Component; Simplified and Rigorous Coupled

Fig. 7.32 Displacement Response of Pine Flat Dam System to Taft Vertical Component; Simplified and Rigorous Uncoupled

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Fig. 7.33 Displacement Response of Pine Flat Dam System to ElCentro S00E Component; Simplified and Rigorous Coupled

Fig. 7.34 Displacement Response of Pine Flat Dam System to ElCentro S00E Component; Simplified and Rigorous Uncoupled

Fig. 7.35 Displacement Response of Pine Flat Dam System to ElCentro S90W Component; Simplified and Rigorous Coupled

Fig. 7.36 Displacement Response of Pine Flat Dam System to ElCentro S90W Component; Simplified and Rigorous Uncoupled

Fig. 7.37 Maximum Principal Stress (in psi) Envelope of Pine Flat Dam to Tail N21E Component

Fig. 7.38 Maximum Principal Stress (in psi) Envelope of Pine Flat Dam System to Tail S69E Component

Fig. 7.39 Maximum Principal Stress (in psi) Envelope of the Pine Flat Dam system to Tail Vertical Component

Fig. 7.40 Maximum Principal Stress (in psi) Envelope of Pine Flat Dam System to ElCentro S00E Component

Fig. 7.41 Maximum Principal Stress (in psi) Envelope of the Pine Flat Dam System to Elcentro S90W Component

Fig. 7.42 Dam-Reservoir Model for the Study of Truncation of Near Field in Case of Reservoirs with Sloped Bottom

Fig. 7.43a Reservoir Far Field Treatment for Vertical Harmonic Excitation; Pine Flat Dam-Reservoir System; $L_f = 1.0 \cdot H$

Fig. 7.43b Reservoir Far Field Treatment for Vertical Harmonic Excitation; Pine Flat Dam-Reservoir System; $L_f = 2.0 \cdot H$

Fig. 7.45c Reservoir Far Field Treatment for Vertical Harmonic Excitation; Pine Flat Dam-Reservoir System; $L_f = 2.5 \cdot H$

Fig. 7.44 Location of Transmitting Boundary; 15 Degrees Bottom Slope Pine Flat Dam-Reservoir System; Horizontal Harmonic Excitation

Fig. 7.45 Location of Transmitting Boundary; 15 Degrees Bottom Slope Pine Flat Dam-Reservoir System; Vertical Harmonic Excitation

Fig. 7.46 Location of Transmitting Boundary; 5 Degrees Bottom Slope Pine Flat Dam-Reservoir System; Horizontal Harmonic Excitation

Fig. 7.47 Location of Transmitting Boundary; 5 Degrees Bottom Slope Pine Flat Dam-Reservoir System; Vertical Harmonic Excitation

Fig. 7.48 Effect of Reservoir Bottom Slope; Pine Flat Dam-Reservoir System; Horizontal Harmonic Excitation

Fig. 7.49 Effect of Reservoir Bottom Slope; Pine Flat Dam-Reservoir System; Vertical Harmonic Excitation; Absorptive Far Field

8.1 Summary of the Research

The primary objective of this research is to develop an effective analytical technique for predicting the seismic response of gravity dams, including the simultaneous effects of dam-water interaction, dam-foundation interaction and fluid-foundation interaction. The technique so developed should be computationally efficient and impose as few restrictions on the geometry of the system analyzed as possible.

The analysis of the dam-reservoir-foundation system is best carried out in frequency domain by a substructure approach in which the dam, reservoir and foundation are treated as separate substructures. Both finite element and boundary element method have been effectively applied in modeling the individual substructures. However, the BEM provides certain advantages in modeling domains of infinite extent and in reducing the amount of data input.

In the present study the dam substructure is discretized by finite elements. The foundation soil half-plane is analyzed using BEM, requiring discretization only along a finite length of the soil surface. The infinite radiation condition in the foundation soil is automatically accounted for through the elastodynamic fundamental solution.

An infinite reservoir bounded by the dam is divided into two regions: a finite region of arbitrary geometry, and an infinite region of rectangular cross section. A finite reservoir or the finite region (near field) of an infinite reservoir is modeled by BEM. To improve the computational efficiency, a frequency independent fundamental solution is used, so that the boundary element matrices do not vary with the excitation frequency. This, however, requires that a certain number of interior nodes be defined in addition to the boundary nodes. The role of the inertia interpolation function, and the number of interior nodes and their location, required in the approximate evaluation of inertia integral for the analysis of

both finite and infinite reservoirs are investigated. In case of an infinite reservoir, the far field of the reservoir is idealized as a continuum, whose solution is obtained by modeling the interface between the near and the far field (transmitting boundary) by 1D finite elements. The results of the frequency independent BEM model are compared with the classical solution where available, and with results obtained by other researchers.

Preliminary response analyses of the dam-reservoir-foundation system are carried out to demonstrate the effect of each of the interaction forces, namely the reservoir hydrodynamic forces, reservoir infinite wave radiation, wave absorption along the flexible reservoir bed, effect of foundation flexibility, and geometric and material damping phenomenon in foundation soil underlying the dam-reservoir system.

The wave absorption along the reservoir bottom can be accounted for by using either an approximate 1D wave propagation model proposed by Hall and Chopra (1980) or a rigorous analysis of interaction between the flexible soil along the base and the reservoir water, wherein the foundation soil below the reservoir is modeled by BEM and compatibility and equilibrium conditions are enforced along this fluid-foundation interface. The rigorous approach of analysis requires enormous computational effort because of (a) cross coupling between the foundation of the dam and the soil below the reservoir, and (b) frequency dependence of the boundary condition along the fluid-foundation interface. The analysis can be simplified by neglecting the cross coupling and by using the approximate 1D wave propagation model. The effects of each of these two simplifications on the accuracy and computational efficiency of the procedure used for the response analysis of a dam subjected to random ground motion are examined. These studies are carried out for both an idealized dam cross section, as well as for the Pine Flat dam.

Finally, a study of dam-reservoir system is carried out to determine the length upto which the reservoir geometry should be considered irregular in the case of an infinite reservoir with sloped bottom, and to establish the effect of reservoir geometry on the response of the dam.

The step by step procedure for obtaining the seismic response of a typical dam-reservoir-foundation system can be briefly summarized as below:

- a. The frequency dependent complex valued foundation impedance matrix is evaluated from the BEM analysis of the foundation media underlying the dam-reservoir

system

- b. The reservoir substructure is analyzed to compute the hydrodynamic forces, using appropriate boundary conditions along the upstream face of the dam, reservoir bottom and the transmitting boundary.
- c. The FEM model of the dam, in conjunction with dam-reservoir and dam-foundation interaction forces is analyzed to determine the complex frequency response function of the generalized displacement of the coupled dam system, for both horizontal and vertical component, corresponding to each excitation frequency. Only the first few mode shapes of the associated dam-foundation system are utilized in the analysis.
- d. The time history of displacement corresponding to each finite element node of the dam subjected to a given arbitrary ground motion is obtained by applying fast Fourier Transform techniques to the complex frequency response function of the dam. The stress response in the dam is evaluated by means of the stress-displacement relation of the FEM formulation, and from these results obtained the contours of maximum principal stress in the dam cross section due the earthquake loading are arrived at.

A series of computer programs were developed based on the detailed formulations for the analysis of the coupled dam system, so as to attain the above mentioned objectives.

8.2 Conclusions of the Present Study

The following important conclusions can be drawn from the studies undertaken in this research:

- The boundary element method using a frequency independent fundamental solution for the reservoir is an effective technique for the analysis of hydrodynamic forces in a reservoir, since it results in considerable saving in the computational effort as compared to the conventional boundary element method.
- When the impounded reservoir is finite and of a relatively larger size, the use of only the first term of the logarithmic series in the evaluation of the inertia integral may lead to significant errors, for most of the excitation frequencies analyzed. As proposed in this work, the use of the first three terms of the logarithmic series, seems to yield results

of better accuracy in the analysis of a finite reservoir. However, in the analysis of an infinite reservoir, a single term of the logarithmic series is adequate for the evaluation of inertia integral.

- In the analysis of the reservoir near field region the interior nodes arrangement obtained by placing them at a distance of one quarter the reservoir depth at the section under consideration from the reservoir boundary, as well as from each other is found to be convenient and effective. This setup of interior nodes should result in a fairly accurate evaluation of the hydrodynamic pressure in the reservoir, for most of the excitation frequencies of interest, and for both components of excitation.
- For the evaluation of hydrodynamic pressure in an infinite reservoir caused by horizontal excitation, a finite element representation of the infinite radiation at the reservoir far field is preferable to a continuum solution, since the former approach results in additional computational efficiency.
- The reservoir bottom absorption effect may be restricted to the near field in the evaluation of response to horizontal excitation. This simplifying assumption will not cause any significant error provided the near field portion is selected to be of a sufficient length - a minimum of 2.5 times the reservoir depth. On the other hand, the assumption of a rigid bottom for the far field will make the eigenvalue problem (associated with the far field) real and frequency independent leading to considerable saving in computations.
- For vertical excitation, the FEM treatment of far field possessing rigid reservoir bottom results in sharp and narrow peaks in the complex frequency response function at resonant frequencies of the reservoir (corresponding to the depth of the far field). Hence the continuum solution of an absorptive reservoir bottom, whose implementation is very simple and computationally efficient, is recommended instead. For the continuum solution to be valid, one of the necessary conditions is that the pressure gradients along the interface between the near and the far field must vanish. This will usually not be the case when the reservoir near field is non-rectangular. But, even under these circumstances, pressure gradients along the interface tends to be negligible, provided that this interface is located at sufficient distance from the dam - a minimum of 2.5 times the reservoir depth in the vicinity of the dam. This confirms the feasibility of the representation of the far field by continuum solution.

- Wave absorption at the reservoir bottom reduces the reservoir response, particularly at the resonant frequencies of the reservoir. For vertical excitation this reduction is considerable; and becomes more pronounced as the wave reflection coefficient used in the approximate 1D simplified model decreases. In the absence of such bottom absorption, the resonant responses are unbounded.
- The reservoir bottom absorption effect is less significant in reducing the response of the dam under horizontal excitation, especially when the foundation below the dam is flexible. On the other hand, as stated earlier, bottom absorption plays a major role in response reduction at resonant frequencies of the dam system subjected to vertical excitation. On account of such absorption the response to vertical excitation becomes less important when compared to the response due to horizontal excitation, particularly for a flexible foundation below the dam.
- The use of boundary element method to model the foundation soil is quite effective in the response evaluation of a dam on flexible foundation, both in terms of accuracy and computational efficiency. The material damping associated with the viscoelastic soil results in an overall reduction of the dam response, and this reduction is more predominant in the vicinity of the fundamental resonant frequency of the dam system.
- The results obtained by using a rigorous model to represent the fluid-foundation interaction effect are not much different from those obtained by the simplified approach, for both transverse as well as the vertical components of ground motions analyzed, for the cross sectional and material properties of the dams chosen. The small differences, if any, can be attributed to the fact that the cross coupling is neglected in the simplified approach. The 1D model is therefore satisfactory in the evaluation of response due to both components of ground motion. It should be noted that the rigorous model involves much greater computational effort.
- The effect of cross-coupling between the foundation of the dam and the foundation soil below the reservoir is not very significant in determining the response to either component of ground motion. Hence the cross coupling could be conveniently ignored thereby improving the computational efficiency of the analysis procedure. This implies that in the analysis of the seismic response of a dam located on a flexible foundation and impounding a reservoir, the simplified approach presented here and also used by earlier

researchers is fairly accurate. However, it is recommended that the wave reflection coefficient parameter for the approximate 1D model should be based on the material properties of the foundation underlying the reservoir, and not on the material properties of the sediment deposited along the reservoir bottom. Use of reflection coefficient based on the properties of the sediment could lead to erroneous response, especially, under horizontal excitation.

- In the analysis of a dam-reservoir system, where the reservoir has a sloped bottom it is sufficient to consider that the irregular part of the reservoir extends to a distance of 2.5 times the depth of the reservoir at the face of the dam, and to treat the rest of the infinite reservoir as being regular, that is, possessing a constant cross section. Thus it is sufficient to consider the geometric irregularity and the reservoir bottom absorption effect over a length of 2.5 times the reservoir depth, for the reservoir analysis to both components of excitation. For such a selection of near field length, a far field continuum solution corresponding to the absorptive far field can be used in the analysis of response due to vertical excitation, with only a small loss in accuracy.

8.3 Recommendations for Future Work

- A boundary element discretization of the dam substructure can be attempted using a frequency independent fundamental solution, and the accuracy and computational efficiency associated with such an approach compared with the one that uses a finite element model.
- The present formulation of the soil-substructure could be extended to include boundary element discretization with quadratic isoparametric shape functions.
- A numerical study based on rigorous analysis of dam-reservoir foundation for the case of the foundation with more irregular geometry could be carried out and the results compared with those obtained by using the simplified approach.
- A study of the effect of the location of transmitting boundary could be carried out for the case of an infinite reservoir possessing irregularly shaped bottom, considering the dam-foundation interaction, the dam-reservoir interaction and the fluid foundation interaction.
- There is also a need for carrying out an independent study on the effectiveness of the

simplified approach in representing the fluid-foundation interaction in the 3-dimensional problem of arch dam impounding a water reservoir.

REFERENCES

- Abdalla, M.H.M.(1984), "Dynamic Soil-Structure Analysis by Boundary Element Method", *Ph.D. Thesis, Carleton University, Ottawa, Canada.*
- Ahmad, S., Banerjee, P.K.(1986), " Free Vibration Analysis by BEM using Particular Integrals", *Journal of the Engineering Mechanics Division, ASCE, Vol. 112*, pp. 682-695.
- Antes, H., Estroff, O.V.(1987), " Analysis of Absorption Effects on the Dynamic Response of Dam-Reservoir System by BEM", *Earthquake Engineering and Structural Dynamics, Vol. 15*, pp. 1023-1036.
- Banerjee, P.K., Butterfield, R., eds.(1979), "Developments in Boundary Element Method - 1", *Applied Science Publishers Ltd., London.*
- Banerjee, P.K. Butterfield, R.(1981), "Boundary Element Methods in Engineering and Science", *McGraw-Hill Book Co. (U.K.) Ltd., Maidenhead, Berkshire, England.*
- Banerjee, P.K., Ahmad, S., Chen, K.(1988a), "Advanced Application of BEM to Wave Barriers in Multi-Layered 3D Soil Media", *Earthquake Engineering and Structural Dynamics, Vol. 16*, pp. 1041-1060.
- Banerjee, P.K., Ahmad, S., Wang, H.C.(1988b), "A New BEM formulation for the Acoustic Eigen Frequency Analysis Problem", *Inter. Jour. of Numerical Methods in Eng., Vol. 26*, pp. 1299-1309.
- Bathe, K.-J., Wilson, E.L., Peterson, F.E.(1974), "SAP 4 - A Structural Analysis Program for Static and Dynamic Response of Linear Systems", *Report No. UCB/EEERC-73/11, Earthquake Engineering Research Center, University of California, Berkeley, California.*
- Bathe, K.-J., Wilson, E. L.(1987), "Numerical Methods in Finite Element Analysis", *Prentice-Hall of India, New Delhi.*
- Beskos, D. E., ed.(1987), "Boundary Element Methods in Mechanics", *in Computational Methods in Mechanics, North Holland Publishers, New York.*

- Bougacha, S., Tassoulas, J.L.(1991), " Seismic Analysis of Gravity Dams: Modeling and Effects of Sediments I & II ", *Journal of the Engineering Mechanics Division, ASCE, Vol. 117, No. 8*, pp. 1826-1850.
- Brebbia, C.A.(1977), "Boundary Element Method for Engineers", *Pentech Press, Devon, England*.
- Brebbia, C.A., Chuang, P.(1982), "Application of Boundary Element Method for Solving Elastodynamic Problems", *Soil Dynamics and Earthquake Engineering Conference, Ed. Brebbia, C.A., Southampton, U.K.*, pp. 381-408
- Buchen, P.W.(1978), "Elastodynamic Green's Tensor for 2D Half-space", *Journal of Australian Mathematical Society, No. 20, Series B*, pp. 385-400.
- Bustamante, J.L., Rosenblueth, E., Herrera, I., and Flores, A. (1963), "Presion Hidrodinamica en Presas y Depositos", *Boletin Sociedad Mexicana de Ingenieria Sismica, Vol. 1, No. 2*.
- Bustamante, J.L., Flores, A.(1966), "Water Pressure on Dams Subjected to Earthquakes", *Journal of the Engineering Mechanics Division, ASCE, Vol. 92*, pp. 115-127.
- Chakrabarti, P., Chopra, A.K.(1972), "The Earthquake Experience at Koyna Dam and Stresses in Concrete Gravity Dams", *Earthquake Engineering and Structural Dynamics, Vol. 1*, pp. 151-164.
- Chakrabarti, P., Chopra, A.K.(1973a), "Hydrodynamic Pressure and Response of Gravity Dams to Vertical Earthquake Component", *Earthquake Engineering and Structural Dynamics, Vol. 1*, pp. 325-335.
- Chakrabarti, P., Chopra, A.K.(1973b), "Earthquake Analysis of Gravity Dams Including Hydrodynamic Interaction", *Earthquake Engineering and Structural Dynamics, Vol. 2*, pp. 143-160.
- Chakrabarti, P., Chopra, A.K.(1974), "Hydrodynamic Effects in Earthquake Response of Gravity Dams", *Journal of the Structural Mechanics Division, ASCE, Vol. 100*, pp. 1211-1221.
- Chandraseker, R.(1987), " Vibration Analysis of Dams ", *M. Eng. Thesis, University of Roorkes, Roorkes, India*.
- Chandraseker, R. and Humar, J.L.(1991a), " Boundary Element Analysis of the Seismic

- Response of Gravity Dams ", *Proc. Sixth Canadian Conference on Earthquake Eng., Toronto, Canada*, pp. 165-172.
- Chandraseker, R. (1991b), " Private Communications " with Prof. Gregory Fenves, University of California, Berkeley.
- Cheng, A.H.D.(1986), " Effect of Sediment on Earthquake-Induced Reservoir Hydrodynamic Response", *Journal of the Engineering Mechanics Division, ASCE, Vol. 112*, pp. 654-665.
- Chopra, A.K.(1967), "Hydrodynamic Pressures on Dams During Earthquakes", *Journal of the Engineering Mechanics Division, ASCE, Vol. 93*, pp. 205-223.
- Chopra, A.K.(1968), "Earthquake Response of Reservoir-Dam Systems", *Journal of the Engineering Mechanics Division, ASCE, Vol. 94*, pp. 1475-1499.
- Chopra, A.K.(1970), "Earthquake Response of Concrete Gravity Dams", *Report No. UCB / EERC-70/01, Earthquake Engineering Research Center, University of California, Berkeley, California*.
- Chopra, A.K., Chakrabarti, P., Dasgupta, G.(1976), "Dynamic Stiffness Matrices for Viscoelastic Half Plane Foundations", *Journal of the Engineering Mechanics Division, ASCE, Vol. 102, No. 3*, pp. 497-514.
- Chopra, A.K., Chakrabarti, P.(1981), "Earthquake Analysis of Concrete Gravity Dams Including Dam-Water-Foundation Rock Interaction", *Earthquake Engineering and Structural Dynamics, Vol. 9*, pp. 363-383.
- Chopra, A.K., Gupta, S.(1982), "Hydrodynamic and Foundation Interaction Effects in Frequency Response for Concrete Gravity Dams ", *Earthquake Engineering and Structural Dynamics, Vol. 10*, pp. 89-106.
- Chopra, A.K.(1987), "Earthquake Analysis Design and Safety Evaluation of Concrete Dams", *Proceedings, the 5th Canadian Conference on Earthquake Engineering, Ottawa*, pp. 39-62.
- Clough, R.W. and Penzien, J. (1985), "Dynamics of Structures", *McGraw-Hill Book Co., New Delhi*.
- Cruse, T.A., Rizzo, F.J.(1968), "A Direct Formulation and Numerical Solution of the General Transient Elastodynamic Problem". Part I and part II, *Journal of Math-*

- emtical Analysis and Applications, Vol. 22, pp. 249-259 and pp. 341-355.*
- Dasgupta, G., Chopra, A.K.(1979), "Dynamic Stiffness Matrices for Viscoelastic Half Planes", *Journal of the Eng. Mechanics Division, ASCE, Vol. 105*, pp. 729-745.
- Dasgupta, G.(1980), "Foundation Impedance Matrices by Substructure Deletion - Cloning", *Journal of the Engineering Mechanics Division, ASCE, Vol. 106*, pp. 517-523.
- Dasgupta, G.(1982), "Finite Element Formulation for Unbounded Homogeneous Continuum", *Journal of the Applied Mechanics Division, ASME, Vol. 49*, pp. 136-140.
- Dominguez, J.(1978), "Dynamic Stiffness of Rectangular Foundations", *Report, Massachusetts Institute of Technology, Cambridge, Massachusetts.*
- Dominguez, J., Alarcon, E.(1981), "Elastodynamics", *Chapter 7, Progress in Boundary Elements, Vol. 1, Ed. Brebbia, C.A., Pcntech Press*, pp. 213-257.
- Dominguez, J., Abascal, R.(1984), "On Fundamental Solutions for the Boundary Integral Equations Method in Static and Dynamic Elasticity", *Engineering Analysis with Boundary Elements, Computational Mechanics Publication, Vol. 1, No. 3*, pp. 128-134.
- Eringen, A.C., Suhubi, E.S.(1975), "Elastodynamics, Vol. 2 - Linear Theory", *Academic Press, New York.*
- Fenves, G., Chopra, A.K.(1984), "Earthquake Analysis and Response of Concrete Gravity Dams", *Report No. UCB/EERC-84/10, Earthquake Engineering Research Center, University of California, Berkeley, California.*
- Gaitanaros, A.P., Karabalis, D.L.(1988), "Dynamic Analysis of 3D Flexible Embedded Foundation by a Frequency Domain BEM-FEM", *Earthquake Engineering and Structural Dynamics, Vol. 16*, pp. 653-674.
- Gutierrez, J.A., Chopra, A.K.(1978), "A Substructure Method for Earthquake Analysis of Structure Including Structure-Soil Interaction", *Earthquake Engineering and Structural Dynamics, Vol. 6*, pp. 51-69.
- Hall, J.F., Chopra, A.K.(1980), "Dynamic Response of Embankment, Concrete Gravity and Arch Dams including Hydrodynamic Interaction", *Report No. UCB/EERC-80/39, Earthquake Engineering Research Center, University of California, Berkeley, California*, pp. 389-406.

- Hall, J.F.(1982), "An FFT Algorithm for Structural Dynamics", *Earthquake Engineering and Structural Dynamics*, Vol. 10, pp. 797-811.
- Hall, J.F.(1986), "Study of Earthquake Response of Pine Flat Dam", *Earthquake Engineering and Structural Dynamics*, Vol. 14, pp. 281-295.
- Hall, J.F.(1988), "The Dynamic and Earthquake behaviour of Concrete Dams: Review of Experimental behaviour and Observational Evidence", *Soil Dynamics and Earthquake Engineering*, Vol. 7.
- Hanna, Y.G. (1980), "Dynamic Soil-Structure Analysis by Boundary Element Method", *M.Eng. Thesis, Carleton University, Ottawa, Canada*.
- Hanna, Y.G., Humar, J.L.(1982), "Boundary Element Analysis of Fluid Domain", *Journal of Engineering Mechanics Division, ASCE*, Vol. 108, pp. 436-450.
- Humar, J.L., Roufaiel, M.(1983), "Finite Element Analysis of Reservoir Vibration", *Journal of Engineering Mechanics Division, ASCE*, Vol. 109, pp. 215-230.
- Humar, J.L., Jablonski, A.M.(1988), "Boundary Element Reservoir Model for Seismic Analysis of Gravity Dams", *Earthquake Engineering and Structural Dynamics*, Vol. 16, pp. 1129-1156
- Humar, J.L.(1990), "Dynamics of Structures", *Prentice-Hall, Englewood Cliffs, New Jersey*.
- Humar, J.L. and Chandrashaker, R.(1991), "Gravity Dam Response Including Interaction Effects", *ASCE Conference on Mechanics Computing in 1990 and Beyond, Columbus, Ohio, U.S.A*, Vol. 1, pp. 451-456.
- Humar, J.L. and Chandrashaker, R.(1992), "Treatment of Interaction Effects in the Seismic Response Analysis of Dams" submitted to *Tenth World Conference on Earthquake Eng., Madrid, Spain*
- Indian Standard 1893-1975. (1985), "Criteria for Earthquake Resistant Design of Structures", *ISI, New Delhi*.
- Indian Standard 6512-1972. (1981), "Criteria for Design of Solid Gravity Dams", *ISI, New Delhi*.

- Israil, A.S.M., Ahmed, S.(1989), "Dynamic Vertical Compliance of Strip Foundations in Layered Soils", *Earthquake Engineering and Structural Dynamics*, Vol. 18, , pp. 933-950.
- Israil, A.S.M., Banerjee, P.K.(1990), "Effects of Geometrical and Material Properties on the Vertical Vibration of 3D Foundations by BEM ", *Inter. Jour. of Numerical Methods in Eng.*, Vol. 14, pp. 49-70.
- Jablonski, A.M.(1988), "Boundary Element Analysis of Earthquake Induced Hydrodynamic Pressures in a Water Reservoir", *Ph.D. Thesis, Carleton University, Ottawa, Canada*
- Jablonski, A.M.(1990), "Effect of Location of Transmitting Boundary on Seismic Hydrodynamic Pressures on Gravity Dams", *Proceeding of the Fourth U.S. Conference on Earthquake Engineering, Palm Spring, California*, Vol. 3, pp. 95-104.
- Karabalis, D.L.(1987), "Dynamic Soil Structure Interaction by BEM", *Proc., 8th International Conference on BEM*, , Ed. Brebbia, C.A., Springer-Verlag, New York, pp. 351-374.
- Karasudhi, P., Keer, and Lee, S.L.(1968), "Vibratory Motion of a Body on an Elastic Half Plane", *Jour. of Applied Mech.*, Vol. 35, pp. 697-705.
- Kobayashi, S., Kawakami, T.(1985), "Application of BE-FE Combined Method to the Analysis of Dynamic Interactions Between Structures and Viscoelastic Soils", *Proc., 7 th International Conference on BEM*, , Ed. Brebbia, C.A., Springer-Verlag, New York, pp. 6-3 to 6-12.
- Kotsubo, S.(1959), "Dynamic Water Pressures on Dams Due to Earthquakes", *Memoirs Faculty of Engineering, Kyushu University, Fukuoka, Japan*, Vol. 8, No. 4.
- Loeffler, C.F., Mansur, W.J.(1987), "Analysis of Time Integration Schemes for BE Applications to Transient Wave Propagation Problems", *B. E. Techniques, Application in Stress Analysis and Heat Transfer*, Ed. Brebbia, C.A., and Venturini, *Computational Mechanics Publication* , pp. 105-122.
- Lotfi, V.(1985), "Analysis of the Response of Dams to Earthquakes", *Ph.D. Thesis, University of Texas at Austin, Austin, Texas*.
- Lotfi, V., Roesset, J.M. and Tassoulas, J. L. (1987), "A Technique for the Analysis of

- the Response of Dams to Earthquakes", *Earthquake Engineering and Structural Dynamics*, Vol. 15, , pp. 463-490.
- Luco, J.E., Westmann, R.A.(1972), "Dynamic Response of a Rigid Footing Bonded to an Elastic Half Space ", *Jour. of Applied Mech.*, Vol. 39, pp. 527-531.
- MacLachlan, N.W.(1955), "Bessel Functions for Engineers", *Oxford at the Clarendon Press, Oxford, England*.
- Manolis, G.D., Beskos, D.E.(1988), " Boundary Element Methods in Elastodynamics", *Unwin Hyman Press, London, England*.
- Medina, F., Dominguez, J.(1989), "Boundary Elements for the Analysis of the Seismic Response of Dams Including Dam-Water-Foundation Interaction Effects: I & II ", *Engineering Analysis with Boundary Elements, Computational Mechanics Publication*, Vol. 6, No. 3, pp. 152-163.
- Medina, F., Dominguez, J. and Tassoulas, J. L.(1990), " Response of Dams to Earthquakes Including Effects of Sediments", *Journal. of the Structural Engineering Division, ASCE*, Vol. 116, No.11, pp. 3108-3121.
- Meek, J.W., Veletsos, A.S.(1972), " Dynamic Analysis by Extra Fast Fourier Transform", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 98, No. 2, pp. 367-384.
- Nardini, D., Brebbia, C.A.(1982), "A New Approach to Free Vibration Analysis by Boundary Elements", *Proc., 4th International Conference on BEM, Southampton, England*, Ed. Brebbia, C.A., Springer-Verlag, New York, pp. 313-326.
- Nardini, D., Brebbia, C.A.(1984), "Boundary Integral Formulation of Mass Matrices for Dynamic Analysis", *Topics in B.E. Research*, Vol. 16, Ed. Brebbia, C.A. Springer-Verlag, New York, pp. 191-208.
- Nardini, D., Brebbia, C.A.(1985), "The Solution of Parabolic and Hyperbolic Problems using an Alternative B.E. Formulation", *Proc., 7th International Conference on BEM, Italy*, Ed. Brebbia, C.A., Springer-Verlag, New York, pp. 387 to 398.
- Novak, M., Hifnaway, L.E.(1983), "Effect of Soil-Structure Interaction on Damping of Structures", *Earthquake Engineering and Structural Dynamics*, Vol. 11, pp. 595-621.

- Providakis, C.P., Beskos, D.E.(1989), "Free and Forced Vibrations of Plates by Boundary and Interior Elements", *Inter. Jour. of Numerical Methods in Eng.*, Vol. 28, pp. 1977-1994.
- Rosenblueth, (1968), "Presion Hidrodinimica en Presas Debida a la Acceleracion Vertical Con Refraccion en el Fondo", *Second National Cong. Seis. Eng.*, Veracruz, Mexico.
- Saini, S.S., Bettess, P., Zienkiewicz, O.C.(1978), "Coupled Hydrodynamic Response of Concrete Gravity Dams Using Finite and Infinite Elements", *Earthquake Engineering and Structural Dynamics*, Vol. 6, pp. 363-374.
- Selvadurai, A.P.S.(1980), "The Dynamic Response of a Rigid Circular Foundation Embedded in an Isotropic Medium of Infinite Extent", *International Symposium on Soils Under Cyclic and Transient Loading*, Ed. G.N. Pande and O.C. Zienkiewicz, Swansea, pp. 597-608.
- Sharan, S.K. (1978), "Earthquake Response of Dam-Reservoir-Foundation Systems", *Ph.D. Thesis, University of Waterloo, Ontario, Canada.*
- Sharan, S.K. (1992), "Efficient FEM Analysis of Hydrodynamic Pressures on Dams", *Jour. of Computers & Structures*, Vol. 42, No. 5, pp. 713-723.
- Singhal, A.C.(1991), "Comparison of Computer Codes for Seismic Analysis of Dams", *Jour. of Computers & Structures*, Vol. 38, No. 1, pp. 107-112.
- Spyrakos, C.C., Patel, P.N.(1986), "Dynamic Analysis of Flexible Embedded Foundations: Plane Strain Case", *Computation Methods in Experimental Measurements, Proc., 3rd Intern. Conf.*, pp. 615-624.
- Stroud, A., Secrest, D.(1966), "Gaussian Quadrature Formulas", *Prentice-Hall, Englewood Cliffs, New Jersey.*
- Tan, C.L.(1987), "The Boundary Element Method : A Short Course", *Dept. of Mech. and Aero. Eng., Carleton University, Ottawa, Canada.*
- Tang, W., Brebbia, C.A.(1989), "Critical Comparison between Two Transformation Methods for Taking BEM Domain Integrals to the Boundary", *Engineering Analysis with Boundary Elements, Computational Mechanics Publication*, Vol. 6, No. 4, pp. 185-191.

- Tsai, C.S.(1987)**, "An Improved Solution Procedure to the Fluid Structure Interaction Problem as Applied to the Dam-Reservoir System", *Ph.D. Thesis, State University of New York, Buffalo*.
- U.S. Bureau of Reclamation,(1966, 1974 and 1976)**, "Design Criteria for Concrete Arch and Concrete Gravity Dams", *Engineering Monograph No. 19, Washington, D.C., U.S. Government Printing Office*.
- Vaish, A.K., Chopra, A.K.(1974)**, " Earthquake Finite Element Analysis of Structure - Foundation System", *Journal of the Engineering Mechanics Division, ASCE, Vol. 100*, pp. 1101-1115.
- Veletsos, A.S., Verbic, B.(1973)**, "Vibration of Viscoelastic Foundations", *Earthquake Engineering and Structural Dynamics, Vol. 2*, pp. 87-102.
- Wepf, D.H., Wolf, J.P.(1988)**, " Hydrodynamic Stiffness Matrix based on BE for Time Domain Dam-Reservoir-Soil Analysis", *Earthquake Engineering and Structural Dynamics, Vol. 16*, pp. 417-432.
- Westergaard, H.M.(1933)**, "Water Pressures on Dams during Earthquake", *Transactions, ASCE, Vol. 98*, pp. 418-433.
- Wolf, J.P., Darbre, G.R.(1984)**, "Dynamic-Stiffness Matrix of Soil by the Boundary Element Method: Part 1-Conceptual Aspects, Part 2-Embedded foundation", *Earthquake Engineering and Structural Dynamics, Vol. 12*, pp. 385-416.
- Wolf, J.P.(1986)**, "Dynamic Soil-Structure Interaction", *Prentice-Hall, Englewood Cliffs, New Jersey*.
- Zagnar, C.N.(1953)**, "Hydrodynamic Pressures on Dams due to Horizontal Earthquakes", *Proceedings of the Society for Experimental Stress Analysis, Vol. 10*, pp. 93-102.
- Zienkiewicz, O.C.(1981)**, "The Finite Element Method ", *Tata McGraw-Hill, New Delhi, India*.

NOTATION

a_o	$= \frac{\Omega b}{c_2}$ dimensionless frequency
a_n^l	acceleration along the outward normal of the dam's face
$\mathbf{a}_n^l$	vector containing the values of a_n^l
A	near field fluid domain
$\mathbf{A}$	area matrix relating the hydrodynamic forces and pressures at the nodes located along the fluid-foundation interface
$\mathbf{A}_n$	boundary condition matrix at the upstream face of dam
b	half width of the rigid strip footing
b_i	body force per unit volume divided by the mass density of the soil
b_n^l	acceleration along the outward normal to the reservoir bottom
$\mathbf{b}_n^l$	vector of b_n^l
$\mathbf{B}_n$	boundary condition matrix at the reservoir bottom
$\mathbf{C}$	damping matrix of the dam substructure
c, d	corner nodes of the rectangular reservoir corresponding to the upstream face of the dam and the reservoir far field, respectively
c_1, c_2	dilatational and shear wave (also known as P-wave and S-wave) velocities in the soil media
c_1^*, c_2^*	complex wave velocities corresponding to a viscoelastic soil
c_t	$= 1 - \frac{\beta}{2\pi}$ constant parameter in the BEM formulation
c_r	$= \sqrt{\frac{E_r}{\rho_r}}$ compression wave velocity in the foundation material below the reservoir
c_w	compression wave velocity in water
$\mathbf{d}$	a vector with only one non-zero entry equal to an input acceleration a_y corresponding to the base node, for the FEM treatment of the reservoir far field
$e^{i\Omega t}$	unit harmonic excitation

- E_f Young's modulus of elasticity in foundation soil
- E_{ij} elements of the flexibility matrix of the foundation
- E^l inertia load Vector arising from a unit harmonic free field ground motion along l th coordinate direction
- $f_m(x, y)$ a suitably chosen harmonic function termed 'inertia interpolation function' in the analysis of reservoir near field
- F_{ki} force per unit width of the footing in the foundation analysis
- F matrix containing $f_m(x, y)$
- $F^*(\Omega)$ effective force vector of the dam system
- F_{dyn} total hydrodynamic force on upstream face of a rigid dam
- F_{st} total hydrostatic force on upstream face of a rigid dam
- g acceleration due to gravity
- g_{ic}^L, g_{ic}^R i th row elements of G , corresponding to the left and right boundary elements of the corner nodes c in the BEM formulation of the reservoir
- G, H boundary element matrices in the reservoir analysis
- H reservoir depth in the vicinity of the dam
- $H_m^{(2)}$ Hankel function of m th order and second kind
- I unit matrix
- $|J|$ determinant value of the Jacoby matrix
- $k_n = \frac{\Omega}{c_n}$ wave number corresponding to the P-wave and S-wave
- $k = \Omega/c_w$ wave number in the reservoir analysis
- $K_{ij}^s(\Omega)$ dynamic stiffness coefficients of the rigid footing
- K stiffness matrix of the dam substructure
- $\tilde{K}$ diagonal kappa matrix in the FEM treatment of the reservoir far field
- $K^*(\Omega)$ effective stiffness matrix of the coupled dam system
- l direction of the ground motion component = x, y
- l_{be} length of boundary elements of the fluid-foundation interface, in the analysis by rigorous approach

- L_f length of the reservoir near field
- m_i lumped mass associated with the i th finite element node of the dam
- $\tilde{\mathbf{M}}$ hydrodynamic mass matrix
- M number of such harmonic functions
- $\mathbf{M}$ mass matrix of the dam substructure
- n_{ctr} number of boundary elements in the reservoir near field
- n_{cls} total number of boundary elements in soil discretization
- n_{ib} number of nodes along the soil-structure interface
- n_{eqnr} total number of boundary element equations in the reservoir analysis
- n_{ip} number of interpolation points inside the reservoir domain
- n_{mod} number of mode shapes considered in the analysis
- $\mathbf{n}$ unit outward normal along a boundary
- N total number of finite element nodes in the dam structure
- N_1, N_2 shape functions of the linear boundary element in the reservoir
- p hydrodynamic pressure in excess of hydrostatic pressure
- p^* weighting function or the Green's function of the hydrodynamic pressure
- $\mathbf{p}_2$ vector of hydrodynamic pressures at the nodes of the linear boundary element along the reservoir bottom
- $p^l(x, y, \Omega)$ complex frequency response function for the hydrodynamic pressure due to the l th component of ground motion
- $\mathbf{p}_o^l(\Omega)$ pressure vector obtained with boundary conditions corresponding to unit harmonic ground acceleration of a rigid dam along the l th direction
- $\mathbf{p}_j^f(\Omega)$ pressure vector obtained with boundary conditions corresponding to that of horizontal acceleration of dam-foundation system vibrating in the j th mode, with no motion of the reservoir bed
- $\mathbf{p}_j^b(\Omega)$ Pressure vector obtained with boundary conditions corresponding to that of the fluid-foundation interface vibrating in the j th mode. This term represents the cross coupling term between the foundation below the dam and the reservoir

bottom

P, Q source point and the field point along the soil surface

Q^{ih}, Q^{iv} horizontal and vertical components of the equivalent nodal forces at the i th boundary element node

$\mathbf{Q}(\Omega), \mathbf{T}(\Omega)$ the boundary element matrices of foundation

$\mathbf{Q}_h(\Omega)$ interaction forces along the fluid-foundation interface

$q = \frac{\partial p}{\partial n}$: pressure gradient

$q^* = \frac{\partial p^*}{\partial n}$: weighting function corresponding to the pressure gradient

q_c^L, q_c^R the pressure gradient along the normal direction of the boundary element situated respectively to the left and right of the corner node c

r, r' the distance to the field point j respectively from the source point i and the mirror image of the source point i'

$\mathbf{r}$ vector representing the displacement of the foundation soil

$\mathbf{R}(\Omega)$ the nodal load vector, having non-zero quantities corresponding to the upstream face (hydrodynamic force terms) and the base degrees of freedom of the dam

$\mathbf{R}_h(\Omega)$ load vector having non-zero terms corresponding to the statically equivalent hydrodynamic forces along the DOF of the nodes on the dam face in contact with the reservoir water excluding the node common with the foundation

$\mathbf{R}_b(\Omega)$ vector of forces along the dam-soil interface nodes

$\mathbf{R}_r(\Omega)$ vector of hydrodynamic forces on the node c common to the upstream face of the dam and the foundation

$\mathbf{R}_j^b(\Omega)$ vector of nodal forces along dam's upstream face, statically equivalent to the hydrodynamic pressures $\mathbf{p}_j^b(\Omega)$

$\mathbf{R}_b^f(\Omega)$ interaction forces in the soil along the dam-foundation interface nodes

$\mathbf{R}_{ho}(\Omega), \mathbf{R}_{ro}(\Omega)$ nodal forces along dam's upstream face, statically equivalent to the hydrodynamic pressures $\mathbf{p}_o(\Omega)$

$\mathbf{R}_{hj}^f(\Omega), \mathbf{R}_{rj}^f(\Omega)$ nodal forces along dam's upstream face, statically equivalent to the hydrodynamic pressures $\mathbf{p}_j^f(\Omega)$

$\mathbf{R}_{hj}^b(\Omega), \mathbf{R}_{rj}^b(\Omega)$ nodal forces along dam's upstream face, statically equivalent to the hy

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- hydrodynamic pressures $p_j^b(\Omega)$
- s coordinate along the upstream face of the dam
- s' coordinate along the reservoir bottom
- S surface of the boundary element domain in the analysis of reservoir and foundation
- $S_{ff}(\Omega)$ impedance matrix corresponding to the fluid-foundation interface nodes, for the rigorous approach of analysis
- $S_{bb}(\Omega)$ frequency dependent, complex valued soil impedance matrix
- $S_{bb}^r(\Omega_0)$ real part of the soil impedance matrix, evaluated at a frequency Ω_0
- $S_{bb}^r(\Omega), S_{bb}^i(\Omega)$ real and imaginary parts of the foundation impedance matrix
- t is the time variable
- $u(\Omega), t(\Omega)$ respectively denote the displacement and traction vectors along the soil surface
- $U_{ji}^*(\Omega), T_{ji}^*(\Omega)$ Green's functions for displacement and traction in foundation
- $U^l(\Omega)$ vector of nodal displacements in dam, relative to the free field
- w_m gauss weight corresponding to the m th Gauss point
- $\frac{x}{|x|}$ sign of x
- x, y cartesian coordinates
- y_i ordinate of the i th boundary element node from the reservoir bottom, along the transmitting boundary of the reservoir far field
- $Z_j^l(\Omega)$ j th modal displacement corresponding to a motion in direction l
- $\alpha_m(\Omega)$ an unknown coefficient in the evaluation of hydrodynamic mass matrix
- α_r wave reflection coefficient for the reservoir bottom
- $\beta_{(i-1)}, \beta_i$ angle of inclination of the boundary elements located to the left and right side of the i th node respectively along the reservoir bottom with the horizontal direction
- $\bar{\beta}$ external angle in radians at the corner
- $\chi_j(\Omega)$ mode shape elements corresponding to the fluid-foundation interface
- Δ^i Dirac Delta function centered at the source point
- η_d hysteretic damping factor for the dam material

- η_s frequency independent constant hysteretic damping constant in foundation soil
 γ wave absorption parameter at the reservoir bottom for the 1D simplified model
 $\Gamma = \frac{c_f}{c_1}$ wave velocity ratio in the foundation analysis
 $i = \sqrt{-1}$, the imaginary number of a complex quantity
 $\kappa_n = \sqrt{\lambda_n - k^2}$ elements of the kappa matrix for FEM analysis of the reservoir far field
 λ_n eigen values of the reservoir far field
 λ, μ Lamé's constant for the soil material
 λ^*, μ^* complex Lamé's constants of viscoelastic foundation soil
 Λ diagonal matrix of the square of the natural frequencies ω_j^2
 ν Poisson's ratio of the material
 ν, λ, E elastic properties corresponding to plane stress foundation
 $\omega_{1res} = \frac{\pi c}{2H}$ first natural frequency of the reservoir
 ω_1 fundamental frequency of the dam on a rigid foundation with empty reservoir
 ω_j, ϕ_j natural frequencies and the orthonormalized Ritz vectors of the dam
 Ω frequency of excitation
 ϕ_j^b j th mode shape corresponding to the DOF of nodes along the dam-foundation interface
 $\tilde{\phi}_n$ eigen vectors of the reservoir far field
 ψ_{jx}^i, ψ_{jy}^i mode shapes along the horizontal and vertical DOF of i th upstream dam face node of the dam-foundation system vibrating in the j th mode
 Φ_h matrix containing the mode shapes corresponding to the nodal DOF of the dam, excluding the base nodes
 Φ_b mode shapes corresponding to the DOF along the dam-soil interface nodes
 $\tilde{\Phi}$ matrix containing the eigen vectors of the reservoir far field
 ρ_w mass density of water
 ρ_f mass density of the foundation soil
 ρ_r, E_r mass density and elastic modulus of the foundation material below the reservoir respectively

- θ_i angle of inclination of the i th node along the dam's upstream face with the vertical direction
- v a vector of modal coordinates
- ξ natural coordinate along the length of the boundary element

APPENDIX A

Finite Element Formulation of the Dam Substructure

The dam substructure is discretized using an assembly of two-dimensional, four noded linear isoparametric finite elements. The properties of the shape functions, stress-strain constitutive matrix, strain-displacement and stress-displacement relationship, the element stiffness and mass matrices, for such a finite element are presented below.

The displacement at any point (x, y) can be related to the nodal displacement at (x^e, y^e) , through the shape function as follows:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N] \{ \delta^e \} \quad (A - 1a)$$

Similarly the cartesian coordinates at any point can be expressed in terms of their nodal values through the same function by:

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = [N] \{ X^e \} \quad (A - 1b)$$

where

$$\{ \delta^e \}^T = \{ u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4 \}$$

$$\{ X^e \}^T = \{ x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4 \}$$

are the nodal displacements and nodal coordinates corresponding to the four finite element nodes, and

$$[N] = [\bar{N}_1, \bar{N}_2, \bar{N}_3, \bar{N}_4] \quad (A - 1c)$$

and

$$\bar{N}_i = \begin{Bmatrix} N_i & 0 \\ 0 & N_i \end{Bmatrix}$$

Shape Functions of a 4-Noded Isoparametric Elements

The shape function N_i corresponding to the i th finite element node, can be expressed in terms of the natural coordinates (ξ, η) as follows

$$N_i = \frac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i) \quad (A-1d)$$

in which (ξ_i, η_i) are the nodal values of the natural coordinate corresponding to the i th nodal point and are given by $\xi_i = -1, +1, +1, -1$, and $\eta_i = -1, -1, +1, +1$ respectively for the values of i varying from 1 to 4.

Strain-Displacement Relation

The strain components at a point in a finite element can be related to the corresponding displacement components, and hence the nodal displacements, as follows:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \quad (A-2a)$$

Using Eq. A-1a, Eq. A-2a reduces to

$$\{\epsilon\} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} [N] \{\delta^e\} \quad (A-2b)$$

or

$$\{\epsilon\} = [B_1, B_2, B_3, B_4] \{\delta^e\} \quad (A-2c)$$

where the strain-nodal displacement matrix is given by

$$B_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad (A-3)$$

The natural coordinates can be related to the Cartesian coordinates by a Jacobian matrix

$$\begin{Bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{Bmatrix} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} \quad (A-4a)$$

or

$$\begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} = \mathbf{J}^{-1} \begin{Bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{Bmatrix} \quad (A-4b)$$

where the Jacobian matrix is given by

$$\mathbf{J} = \begin{Bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{Bmatrix} = \begin{Bmatrix} \sum \frac{\partial N_i}{\partial \xi} x_i & \sum \frac{\partial N_i}{\partial \xi} y_i \\ \sum \frac{\partial N_i}{\partial \eta} x_i & \sum \frac{\partial N_i}{\partial \eta} y_i \end{Bmatrix} \quad (A-4c)$$

and the derivatives of the shape function can be written as

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = \mathbf{J}^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \quad (A-5)$$

Differentiating Eq. A-1d, the expression for the derivatives of the shape functions can be written as:

$$\frac{\partial N_i}{\partial \xi} = \frac{\xi_i}{4}(1 + \eta\eta_i); \quad \frac{\partial N_i}{\partial \eta} = \frac{\eta_i}{4}(1 + \xi\xi_i) \quad (A-6a)$$

At the centroid location of the finite element, where $(\xi, \eta) = (0, 0)$, the above expressions reduce to

$$\frac{\partial N_i}{\partial \xi} = \frac{\xi_i}{4}; \quad \frac{\partial N_i}{\partial \eta} = \frac{\eta_i}{4} \quad (A-6b)$$

Stress-Strain Relationship: Constitutive Matrix

The stress components are related to the corresponding strain components through an elasticity matrix or a constitutive matrix as:

$$\{\sigma(\xi, \eta)\} = \mathbf{D} \{\epsilon(\xi, \eta)\} = \mathbf{D}\mathbf{B} \{\delta^e\} \quad (A-7a)$$

where the constitutive matrix for a two-dimensional plane stress case considered in the present formulations, is defined by

$$\mathbf{D} = \begin{pmatrix} d_1 & d_2 & 0 \\ d_2 & d_1 & 0 \\ 0 & 0 & d_3 \end{pmatrix} \quad (A-7b)$$

with

$$d_1 = \frac{E}{(1-\nu^2)}; \quad d_2 = \frac{\nu E}{(1-\nu^2)}; \quad d_3 = \frac{E}{2(1+\nu)}$$

E and ν being respectively the Young's modulus and poisson's ratio of the material of the dam.

The expressions for the major and minor principal stresses are given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_{xy}^2} \quad (A-8)$$

Nodal Force-Displacement Relation: Stiffness Matrix

By using the principle of virtual work, the nodal load vector can be related to the nodal displacement components, and the elementary stiffness matrix can be defined as

$$\mathbf{K}^e = \int_{Vol.} \mathbf{B}^T \mathbf{D} \mathbf{B} d_v \quad (A-9)$$

The above integration can be performed using numerical integration with Gauss quadrature.

Diagonal Mass Matrix

A simple and yet fairly accurate representation of the mass characteristic is obtained by lumping the total mass of an element at its node. This results in a diagonal mass matrix, and reduces the computational effort significantly.

APPENDIX B

B-1: Representation of Fluid-Foundation Interaction by 1D Simplified Approximate Model

The one-dimensional simplified model to account for the wave absorption along the flexible reservoir bed was first proposed by Hall and Chopra (1980). Only brief discussions of the formulation and other associated aspects of this model are presented here. A more detailed treatment has been given by Hall and Chopra (1980), and Jablonski (1989).

The interaction problem is assumed to be one-dimensional, along the direction normal to the reservoir bottom, with no variation of any kind in the other direction. The excitation is initiated by an incident displacement wave travelling upward through the foundation.

Fluid Reservoir

Consider a rectangular reservoir with horizontal bottom. The hydrodynamic pressure in the reservoir resulting from a harmonic excitation is governed by a one-dimensional Helmholtz equation:

$$\frac{d^2 p}{dy^2} + k^2 p = 0 \quad (B - 1)$$

where $k = \frac{\Omega}{c_w}$ is the wave number corresponding to the reservoir.

The pressure must satisfy the following boundary condition along the fluid-foundation interface

$$\frac{dp}{dy} = -\rho_w \frac{\partial^2 u}{\partial t^2} \quad (B - 2)$$

where u is the displacement of the reservoir bed in the vertical direction. For an harmonic excitation, the boundary condition given by Eq. B-2 reduces to

$$\frac{dp}{dy} = \rho_w \Omega^2 u \quad (B - 3)$$

Foundation

Displacements within the foundation soil are also governed by a one-dimensional Helmholtz equation given by:

$$\frac{d^2 u}{dy^2} + k_r^2 u = 0 \quad (B-4)$$

where $k_r = \frac{\Omega}{c_r}$, and c_r is the compression wave velocity in the foundation soil below the reservoir bottom.

The general solution to Eq. B-4 is

$$u(y, \Omega) = A(\Omega) e^{i \frac{\Omega}{c_r} y} + B(\Omega) e^{-i \frac{\Omega}{c_r} y} \quad (B-5)$$

where $A(\Omega)$ is the unknown amplitude of the reflected displacement wave and $B(\Omega)$ is the specified amplitude of the incident displacement wave.

Amplitudes of the incident wave, $B(\Omega)$, and the free-field acceleration, a_y , can be related by the considering that at $y = 0$ the stress and hence the strain in the foundation must be zero.

$$\frac{du}{dy} = 0 \quad \text{at} \quad y = 0 \quad (B-6)$$

From Eq. B-5, it follows that:

$$A(\Omega) = B(\Omega) \quad (B-7)$$

Substituting Eq. B-7 into Eq. B-5 it follows that

$$u(y, \Omega, t) = \left(B(\Omega) e^{i \frac{\Omega}{c_r} y} + B(\Omega) e^{-i \frac{\Omega}{c_r} y} \right) e^{i \Omega t} \quad (B-8)$$

Differentiating Eq. B-8 twice with respect to time variable t , and taking the value of the differential at $y = 0$

$$\left. \frac{d^2 u}{dt^2} \right|_{y=0} = -2B(\Omega) \Omega^2 e^{i \Omega t} \quad (B-9)$$

Hence Eq. B-9 results in

$$a_y = -\Omega^2 2B(\Omega) \quad (B-10)$$

or

$$B(\Omega) = -\frac{a_y}{2\Omega^2} \quad (B-11)$$

Therefore, an upward propagating wave has an acceleration amplitude equal to $\frac{1}{2}a_y$, which doubles in its value upon reflection at a free boundary at $y = 0$.

In the presence of the reservoir water, the incident wave amplitude will still remain $B(\Omega)$, however the amplitude of the reflected wave will be altered.

$$u(y, \Omega) = D(\Omega)e^{i\frac{\Omega}{c_r}y} - \frac{a_y}{2\Omega^2}e^{-i\frac{\Omega}{c_r}y} \quad (B-12)$$

An expression for $D(\Omega)$ can be obtained by noting that the pressure at the fluid-foundation interface must equal the normal stress in the foundation soil.

$$p(0, \Omega) = -E_r \frac{du}{dy}(0, \Omega) \quad (B-13)$$

where $E_r = \rho_r c_r^2$ is the effective modulus of elasticity of the foundation material; ρ_r = the mass density of foundation material.

On combining Eqs. B-12 and B-13 one can find a solution for D , the amplitude of the reflected wave:

$$\left. \frac{du}{dy} \right|_{y=0} = +\frac{i\Omega}{c_r} D(\Omega) + \frac{i\Omega a_y}{2c_r \Omega^2} \quad (B-14)$$

$$p(0, \Omega) = -\rho_r c_r^2 \left(\frac{i\Omega}{c_r} D(\Omega) + \frac{i\Omega a_y}{2c_r \Omega^2} \right) \quad (B-15)$$

From Eq. B-15, the value for D is as follows:

$$D(\Omega) = -\left(\frac{a_y}{2\Omega^2} + \frac{1}{\rho_r c_r} \frac{1}{i\Omega} p(0, \Omega) \right) \quad (B-16)$$

Substituting Eqs. B-16 in B-12 gives

$$u(y, \Omega) = -\left(\frac{a_y}{2\Omega^2} + \frac{1}{\rho_r c_r} \frac{1}{i\Omega} p(0, \Omega) \right) e^{i\frac{\Omega}{c_r}y} - \frac{a_y}{2\Omega^2} e^{-i\frac{\Omega}{c_r}y} \quad (B-17)$$

Substituting Eq. B-17 for u , Eq. B-3 reduces to

$$\frac{dp}{dy}(0, \Omega) = -\rho_w a_y + i\gamma\Omega p(0, \Omega) \quad (B-18)$$

where $\gamma = \frac{\rho_w}{\rho_r c_r}$ is the wave absorption parameter corresponding to the 1D simplified model; ρ_w is the mass density of the water; ρ_r and c_r are respectively the mass density and compressive wave velocity of the foundation soil below the reservoir.

The wave reflection coefficient for the flexible reservoir bed is defined as (Hall and Chopra, 1980)

$$\alpha_r = \frac{1 - \gamma c_w}{1 + \gamma c_w} = \frac{1 - \frac{\rho_w c_w}{\rho_r c_r}}{1 + \frac{\rho_w c_w}{\rho_r c_r}} \quad (B-19)$$

This coefficient serves as a measure of wave absorption by the foundation. A value of $\alpha_r \approx 1.0$ (or $\gamma = 0$.) denotes a fully rigid reservoir bottom and amounts to ignoring the fluid-foundation interaction, while a value of $\alpha_r < 1.0$ (or $\gamma > 0.0$) represents a flexible reservoir bed.

The reservoir-foundation interface condition derived above does not consider two-dimensional type of excitation in both half-spaces. It considers only a one-dimensional form of excitation in y -direction due to incident displacement wave traveling in the direction normal to the interface. As an approximation it can be extended to reservoir bottoms of any arbitrary geometry by assuming that one directional wave propagation takes place along normals to the reservoir bed.

B-2: Finite Element Treatment of the Infinite Wave Radiation at Reservoir Far Field

This section briefly deals with the FEM formulation associated with the boundary condition at the interface between the near and far field of the infinite reservoir. As already discussed in Chapter 2, the far field reservoir bottom is assumed to be non-absorptive to improve the computational efficiency of the procedure.

The hydrodynamic pressure p in the uniform rectangular infinite region of the 2D reservoir can be expressed as $p = p_x p_y$ where the pressure component p_y satisfies the following equation

$$\frac{d^2 p_y}{dy^2} + \lambda^2 p_y = 0 \quad (B - 20)$$

with the boundary conditions

$$\begin{aligned} p_y &= 0 & \text{at } y &= H \\ \frac{dp_y}{dy} &= 0 & \text{at } y &= 0 \end{aligned} \quad (B - 21)$$

In Eq. B-20, κ = the separation constant and λ is given by

$$\lambda^2 = k^2 + \kappa^2 \quad (B - 22)$$

Equation B-20 is solved by a one-dimensional finite element discretization of the transmitting boundary, using a weighted residual technique

$$\int_0^H \left(\frac{d^2 p_y}{dy^2} + \lambda^2 p_y \right) \delta p_y dy = 0 \quad (B - 23)$$

The first term under integral in Eq. B-23 can be reduced by partial integration as follows

$$\int_0^H \frac{d^2 p_y}{dy^2} \delta p_y dy = \left. \frac{dp_y}{dy} \delta p_y \right|_0^H - \int_0^H \frac{dp_y}{dy} \frac{d}{dy} (\delta p_y) dy \quad (B - 24)$$

Substitution of the boundary conditions from Eq. B-21 gives

$$\int_0^H \frac{d^2 p_y}{dy^2} \delta p_y dy = - \int_0^H \frac{dp_y}{dy} \delta \left(\frac{dp_y}{dy} \right) dy \quad (B-25)$$

On substituting Eq. B-25 into Eq. B-23 the following relationship is obtained:

$$\delta \int_0^H \left[-\frac{1}{2} \left(\frac{dp_y}{dy} \right)^2 + \frac{1}{2} \lambda^2 p_y^2 \right] dy = 0 \quad (B-26)$$

or

$$\delta \chi = 0 \quad (B-27)$$

where χ is a functional given by

$$\chi = \int_0^H \left[\frac{1}{2} \left(\frac{dp_y}{dy} \right)^2 - \frac{1}{2} \lambda^2 p_y^2 \right] dy \quad (B-28)$$

The far boundary is now divided into a number of finite elements and the functional χ is expressed in a discrete form as

$$\chi = \sum_{n=1}^M \int_n \left[\frac{1}{2} \left(\frac{dp_y}{dy} \right)^2 - \frac{1}{2} \lambda^2 p_y^2 \right] dy \quad (B-29)$$

A linear variation of p_y is assumed in the present finite element formulation, and the far boundary is divided into a number of linear finite elements.

The hydrodynamic pressure variations along the far field can be expressed in terms of the nodal quantities as

$$p_y = \mathbf{N} \mathbf{p}_y \quad (B-30)$$

where $\mathbf{N}$ is the shape function matrix corresponding to linear finite elements similar to those discussed in the Appendix A, but for a one dimensional case.

Substitution of Eq. B-30 in Eq. B-29 follows

$$\chi = \sum_{n=1}^M \int_{-1}^{+1} \left(\frac{1}{2} \mathbf{p}_y^T \frac{d\mathbf{N}^T}{dy} \frac{d\mathbf{N}}{dy} \mathbf{p}_y - \frac{1}{2} \lambda^2 \mathbf{p}_y^T \mathbf{N}^T \mathbf{N} \mathbf{p}_y \right) \frac{dy}{d\xi} d\xi \quad (B-31)$$

The minimization of the functional χ required by Eq. B-27 can be expressed as

$$\frac{d\chi}{d\{\mathbf{p}_y\}} \delta\{\mathbf{p}_y\} = 0 \quad (B-32a)$$

or

$$\frac{d\chi}{d\{\mathbf{p}_y\}} = 0 \quad (B-32b)$$

Using Eq. B-31 for χ , Eq. B-32 gives

$$\sum_{n=1}^M \int_{-1}^{+1} \left(\mathbf{B}_n^T \mathbf{B}_n \mathbf{p}_y - \lambda^2 \mathbf{N}^T \mathbf{N} \mathbf{p}_y \right) \frac{dy}{d\xi} d\xi = 0 \quad (B-33)$$

where ξ is the natural coordinate and

$$\mathbf{B}_n = \frac{d\mathbf{N}}{dy} \quad (B-34)$$

Individual terms within Eq. B-33 can be obtained as follows

$$\int_{-1}^{+1} \mathbf{B}_n^T \mathbf{B}_n \frac{dy}{d\xi} d\xi = \frac{1}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (B-35a)$$

$$\int_{-1}^{+1} \mathbf{N}^T \mathbf{N} \frac{dy}{d\xi} d\xi = \frac{l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (B-35b)$$

where l is the length of the finite element under consideration.

Introducing the following notations

$$\mathbf{F}^e = l \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \quad (B-36a)$$

$$\mathbf{A}^e = \frac{1}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (B-36b)$$

END

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FIN