A Cross-Cultural Examination of the *Pathways to Mathematics Model* in Adults

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Abstract

The *Pathways to Mathematics* model (LeFevre et al., 2010) demonstrated the relations among cognitive precursors and mathematical outcomes in children. In this study, I extended the model to adults and compared the model across cultures. Participants included 71 native English speakers and 71 native Chinese speakers. Mathematical outcomes included calculation, word problems, and a number line task. These were predicted by four pathways: linguistic skill, quantitative knowledge, working memory, and spatial ability. Results showed similarities and differences in the model in relation to children. A major difference in the results for adults compared to children was that linguistic skill did not predict adults' performance on calculation, suggesting that linguistic ability is no longer related to symbolic number system knowledge in adults due to the developed ability of automatized number naming. Culture had a moderating effect on contribution of quantitative knowledge towards number line task performance. Better quantitative knowledge was related to better number line estimation for English speakers, but not for Chinese speakers, suggesting different strategy choices across culture. These findings indicate that the relative contributions of linguistic skill, quantitative knowledge, working memory, and spatial ability vary depending on the demands of specific task, and these contributions are generally universal cross-culturally. Overall, the model showed that math development is componential in both children and adults, and across cultures.

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Table of Contents

List of Tables

List of Figures

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Mathematics is a fundamental subject in education, and is essential to many fields of science. A number of cognitive abilities have been found to relate to and predict mathematical outcomes, and have been beneficial to the understanding of individual differences in math performance in children and adults. These cognitive abilities can be broadly categorized into linguistic skills, quantitative knowledge, working memory, and spatial ability.

Cognitive Precursors and Their Roles in Mathematical Performance

Linguistic skills. Linguistic skills are associated with math abilities: For example, vocabulary and phonological awareness predict children's performance on a number of math tasks, such as calculation, geometry, counting, magnitude comparison, and number line estimation (Alloway et al., 2005; Kleemans, Segers, & Verhoeven, 2011; LeFevre et al., 2010; Simmons, Singleton, & Horne, 2007; Sowinski et al., 2015). Researchers reasoned that remembering number words shares the same cognitive processes as learning everyday vocabulary, and therefore, receptive vocabulary is linked to children's ability to learn number words (LeFevre et al., 2010; Negen & Sarnecka, 2012). Furthermore, phonological awareness is a beneficial underlying ability that helps a person learn to match sounds with written symbols, which is similar to a child's ability to link number names to number symbols (LeFevre et al., 2010). Because linguistic skills are important for the acquisition of words in general, and learning the number system involves learning words and relations among words, it may be the reason for the association between linguistic skills and math abilities in children.

Quantitative knowledge. Quantitative knowledge allows individuals to identify and discriminate quantities rapidly and relatively effortlessly (LeFevre et al., 2010). Quantitative knowledge can be measured using a subitizing task, which assesses a person's ability to rapidly

and automatically enumerate a small set of items (≤ 4 items), and this ability has been shown to correlate with mathematical skills (Yun et al., 2011; Nguyen et al., 2016). Counting is another indicator of quantitative knowledge. It is correlated with mathematical achievements in children (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Johansson, 2005; Stock, Desoete, & Roeyers, 2009). Finally, quantitative knowledge is also involved in magnitude comparison tasks, which require individuals to first enumerate (for non-symbolic stimuli such as ** vs. ****) or evaluate (for symbolic stimuli such as 2 vs. 4) quantities, and then compare the quantities in terms of magnitude. The symbolic comparison task is assumed to be an extension of subitizing and counting in that it requires participants to represent quantities. Accordingly, it is related to mathematical achievement in children (De Smedt, Verschaffel, & Ghesquière, 2009; Fazio, Bailey, Thompson, & Siegler, 2014; Vanbinst, Ansari, Ghesquière, & De Smedt, 2016) and in adults (Sowinski, 2016).

Working memory. Working memory is a person's ability to temporarily hold information while performing complex tasks (Baddeley, 2001). Baddeley's (2001) working memory model includes three components: the phonological loop, the visual-spatial sketchpad, and the central executive. Solvers' performance on many math tasks is correlated with working memory ability (De Smedt et al., 2009; Raghubar, Barnes, & Hecht, 2010). The phonological loop is the part of working memory that processes verbal information. It is involved in solving math problems (DeStefano & LeFevre, 2004), both symbolic calculation problems such as 34 + 9 (Trbovich & LeFevre, 2003) and math word problems (Zheng, Swanson, & Marcoulides, 2011). The visual-spatial sketchpad (VSSP) is the component of working memory that is engaged in processing visual information. The VSSP is related to individuals' performance on mathematical tasks, such as mental arithmetic and word problems (Bull, Espy, & Wiebe, 2008; Krajewski &

Schneider, 2009; Raghubar et al., 2010; Reuhkala, 2001; Simmons, Singleton, & Horne, 2007), especially in preschoolers (Rasmussen & Bisanz, 2005). The role of the central executive is to control and regulate cognitive processing. Individual differences in central executive correlates with performance on many math tasks, including number line task (Geary, Hoard, Nugent, & Byrd-Craven, 2008), arithmetic (Andersson, 2008), and problem-solving accuracy (Zheng, Swanson, & Marcoulides, 2011). In summary, researchers assume that almost all math tasks require some contribution of working memory, or an interaction of all components of the working memory system (DeStefano & LeFevre, 2004).

Spatial ability. Spatial ability is a person's capacity to visualize and use their mental representations to reason about the spatial relations among objects (Hegarty & Kozhevnikov, 1999). Research has shown positive relations between spatial ability and achievements in science, technology, engineering, and mathematics (known as STEM; Lubinski, 2010; Newcombe, 2010; Shea, Lubinski, & Benbow, 2001; Wai, Lubinski, & Benbow, 2009; Webb, Lubinski, & Benbow, 2007; reviewed by Bishop, 2008). For example, Hegarty and Kozhevnikov (1999) found that participants who visualized the spatial relations described in a problem (which requries spatial ability) were more successful in mathematical problem solving than those who visualized the appearance of objects described in a problem, such as shape, color, or brightness (which requires visual imagery). Performance on spatial tasks (e.g., mental rotation tasks) correlates with math achievement in children and teenagers, and is predictive of scores on the mathematical portion of the SAT (Cheng & Mix, 2014; Lachance & Mazzocco, 2006; Mazzocco & Myers, 2003; Mix et al., 2016; Markey, 2010; Johnson, 1998; Rohde & Thompson, 2007).

Given the importance of these cognitive predictors to numerous mathematical achievements, researchers have proposed models to organize cognitive precursors to

mathematical learning (Cirino, 2011; Durand, Hulme, Larkin, & Snowling, 2005; Kleemans, Segers, & Verhoeven, 2011; Krajewski & Schneider, 2009; LeFevre et al., 2010; Sowinski et al., 2015; Zhang et al., 2014; Zhang & Lin, 2015). Different frameworks focus on varied domains of interest, but all models examined the relations of cognitive abilities and mathematical outcomes in children. For example, Zhang et al. (2014) examined how early linguistic and spatial skills predicted arithmetic performance in children. Kleemans et al. (2011) compared children with specific language impairment (SLI) with normally developing children, and investigated the roles of working memory and linguistic ability on a list of early numeracy tasks, including counting, quantities linking, numeral estimations, and so on. Cirino (2011) evaluated quantitative, linguistic, and spatial attention as precursors of single-digit addition problems that summed to 10 or less. Krajewski and Schneider (2009) used a longitudinal design and examined the relations between kindergarteners' linguistic skills, working memory, and quantitative knowledge on their mathematical school achievement in third graders. Durand et al. (2005) assessed linguistic skill and quantitative knowledge and how these two cognitive abilities would predict children's arithmetic and reading. Zhang and Lin (2015) tested the predictive roles of spatial ability and linguistic skill towards children's calculation ability and performance on word problems. Finally, LeFeve et al. (2010)'s *Pathways to Mathematics* model examined linguistic skills, quantitative knowledge and spatial ability, and using a longitudinal design, this model investigated the relations between these three cognitive precursors and children's early numeracy knowledge as well as their mathematical achievements two years later. Mathematical achievements were operationalized as children's performance on a list of math tasks, including geometry, measurement, calculation, number line, magnitude comparison, and so on. See Table 1 for a summary and comparison of characteristics and features of these models.

Table 1
Summary and Comparison of Models

	Linguistic Skill	Quantitative Knowledge	Working Memory	Spatial Ability	Multiple Outcome Measures	Number of ✔
Zhang et al. (2014)	✔			✔		2
Kleeman et al. (2011)	✔		✔		✔	3
Cirino (2011)	✔	✔	✔			3
Krajewski & Schneider (2009)	✔	✔	✔			3
Durand et al. (2005)	✔	✔			✔	3
Zhang & Lin (2015)	✔			✔	✔	3
LeFevre et al. (2010)	✔	✔	✔		✔	4

Note. Checkmarks indicate the presence of a specific feature.

This comparison revealed that the *Pathways to Mathematics* model developed by LeFevre et al. (2010) examined the most cognitive predictors and their relations with multiple math outcomes, which provides us with the most insight on the organization of cognitive abilities and math performance, thus making it the most comprehensive model so far. More specifically, the *Pathways to Mathematics* model investigated the role of different cognitive abilities in children's mathematical development and provided guidance for future research. The Pathways model includes three types of cognitive predictors, linguistic ability, quantitative skills, and

spatial attention, each contributing separately to different aspects of children's mathematical performance. LeFevre et al. used a 2-year longitudinal design with children aged 4- through 8-years and demonstrated the independent contribution of each of the three pathways to children's early numeracy knowledge (i.e., symbolic number system knowledge and numerical magnitude processes). In addition, they also showed that depending on the demands of mathematical tasks, linguistic, quantitative, and spatial attention pathways contributed differently towards children's performance in these tasks two years later. For instance, linguistic and spatial attention pathways contributed to children's performance in a geometry test, whereas the quantitative pathway did not. One goal of the present study is to extend the model to adults, and to determine whether similar pathways exist that can help us to understand adults' mathematical performance. The *Pathways to Mathematics* model was selected to organize the present research because it covers more aspects of cognitive abilities (i.e., linguistic, quantitative, and spatial) and links these cognitive abilities with more math outcomes than other existing models.

Sowinski et al. (2015) adopted the *Pathways to Mathematics* model and refined it with data from children in Grade 2 and Grade 3 (ages 8 through 10). More specifically, Sowinski et al. defined the quantitative pathway differently than in the LeFevre et al. (2010) version. LeFevre et al. used subitizing latency (i.e., the time to rapidly and accurately identify the exact quantity of a small set of items ($n \leq 4$)) as an indicator of quantitative skills for preschoolers, whereas Sowinski et al. (2015) expanded the quantitative pathway for older children to include measures of counting and symbolic magnitude comparison as well as subitizing as domain-specific components of the quantitative pathway.

In addition to the modifications to the quantitative pathway, Sowinski et al. (2015) also refined the spatial attention pathway. In LeFevre et al. (2010), the spatial attention pathway only

included a visual-spatial span task to investigate children's visual-spatial working memory, whereas Sowinski et al. (2015) included three working memory measures to examine more components of Baddeley's (2001) working memory model (i.e., visual-spatial sketchpad, phonological loop, and central executive), and renamed the spatial attention pathway the working memory pathway. Because different components of working memory are associated with different math outcomes, including more components of working memory in the attentional pathway made the model even more comprehensive.

Although LeFevre et al. (2010) and Sowinski et al. (2015) established a well-developed *Pathways to Mathematics* model, both of these studies were conducted with children, and the model has not yet been examined in adults. It is interesting and important to test whether basic cognitive skills predict math performances in adults because it shows how cognitive abilities can translate into academic achievements in adults. Furthermore, whether the contributions of cognitive abilities towards academic outcomes are similar in children and in adults is important because it may tell us how differences in experience and changes in cognitive capacities over time are related to mathematical performance. Hence, in this study we tested whether the model could be used to understand individual differences in adults' math performance.

I proposed four cognitive predictor pathways for the adult version of the Pathways model: linguistic skills, quantitative knowledge, spatial ability, and working memory. Note that neither LeFevre et al. (2010) nor Sowinski et al. (2015) directly measured participants' spatial ability. Because spatial ability and working memory are two separate cognitive skills and require different cognitive processes, and because spatial ability is proven to be related to numerous math achievements, the current study includes spatial as well as attentional measures (i.e., working memory) as separate pathways in the model. Each predictor pathway was assessed with

several measures. Moreover, this study also intended to investigate the influence of cultural difference on the pathways model for English-compared to Chinese-speaking participants, and to thus establish a more comprehensive model that takes into account populations from not only different age groups, but also different cultural groups.

How Cognitive Differences in Children and Adults and May Affect the Model

Because I proposed pathways that were similar to those in LeFevre et al. (2010) and Sowinski et al. (2015) but with adults, the differences between children and adults in terms of the three cognitive precursors (i.e., linguistic skills, quantitative knowledge, and working memory/attentional span) is an important factor in how differences might affect the existing *Pathways to Mathematics* model.

Linguistic skills. Linguistic skills, such as phonological awareness and receptive vocabulary have been shown to contribute to word reading and reading comprehension, respectively, in both children and adults (Garlock, Walley, & Metsala, 2001; Pratt & Brady, 1988; Kim, Otaiba, Puranik, Folsom, & Gruelich, 2014). Although children may have a relatively more limited use of vocabulary and comprehension of words (Boyden & Ennew, 1997; Punch, 2002), the underlying cognitive mechanisms of linguistic abilities of children and adults share many of similarities. For instance, fMRI studies have demonstrated that semantic verbal fluency processing activates similar brain regions in children and adults, predominantly in left inferior frontal cortex (Broca's area) and left middle frontal gyrus (dorsolateral prefrontal cortex) (Gaillard et al., 2003; Gaillard et al., 2000). In addition, in a study that evaluated predictive processing of languages (i.e., rapidly processing information from speech to anticipate and predict upcoming words) in children and adults, the speed of anticipatory fixations did not vary with age (Borovsky, Elman, & Fernald, 2012). Thus, similar to adults, children are able to make

use of presented information in ongoing speech and anticipate upcoming language (Borovsky et al., 2012).

Although children and adults share similar cognitive mechanism in terms of linguistic skills, experience and practice also plays a crucial role in development and manifestation of language ability (Lyon, 1998). Studies have shown that alphabet knowledge fluency and spelling skills are related to letter writing automaticity in children (Kim, Otaiba, Puranik, Folsom, & Gruelich, 2014), and alphabet knowledge fluency and spelling ability are skills that can be acquired through practice. In addition, studies with adults showed that repetitive practice on word recognition tasks and external learning experience could lead to automaticity of word reading, and this was true for second language learners as well (Segalowitz & Segalowitz, 1993; Segalowitz, Segalowitz, & Wood, 1998).

Given that practice and experience can lead to automaticity of reading and writing, and because the symbolic number system contains new vocabulary and symbols for children to learn, children's lack of automaticity in symbol processing could be the reason for the heavy contribution of linguistic pathway to various mathematical outcomes in LeFevre et al. (2010) and Sowinski et al. (2015). Previous studies have also indicated that although 5- to 7-year-old children are familiar with the number symbols, they do not have direct and automatic access to their meanings (Gebuis, Kadosh, Haan, & Henik, 2009; Berch, Foley, Hill, & Ryan, 1999; Girelli, Lucangeli, & Butterworth, 2000; Rubinsten, Henik, Berger, & Shahar-Shaiev, 2002). Automatized number recognition comes with repeated exposure to number symbols (Butterworth, Zorzi, Girelli, & Jonckheere, 2001). In the current study with adults, it is hypothesized that because symbol recognition, and in particular digit recognition, should be an automatized process for adults, the linguistic pathway should not be strongly related to participants' symbolic

number system knowledge, and therefore, should not contribute to mathematical tasks that do not contain an explicit language component. Participants' performance on mathematical tasks that do have language components, however, such as word problems, should be predicted by the linguistic pathway. In addition, linguistic skills should also predict adult participants' performance on reading outcome measures, such as reading speed, reading comprehension, and word reading.

Quantitative knowledge. Quantitative knowledge, which can be measured using magnitude comparison or an enumeration tasks, is an indication of a person's ability to evaluate and differentiate quantities (LeFevre et al., 2010). Similar to linguistic skills, quantitative knowledge can be gained and automatized through experience.

Previous studies have demonstrated that children and adults do not differ much in terms of the underlying mechanisms of quantity processing (Chi & Klahr, 1975; Gebuis, Kadosh, Haan, & Henik, 2009; Temple & Posner, 1998). For example, both children and adults subitize errorlessly, and count with similar accuracy (Chi & Klahr, 1975), and some children who performed well in mathematics could already automatically process numerosities at the age of 5 (Gebuis, Kadosh, Haan, & Henik, 2009). In addition, brain imaging studies have also indicated that children and adults activate similar brain areas during symbolic and non-symbolic numerical processing (Cantlon, Brannon, Carter, & Pelphrey, 2006; Temple & Posner, 1998).

However, when performing quantitative tasks, there are substantial differences between children and adults in response time. For example, children were much slower (3 to 4 times slower) than adults in enumeration and magnitude comparison tasks (Chi & Klahr, 1975; Temple & Posner, 1998; Gebuis et al., 2009). This difference in response times presumably reflects

differences in the maturation and automaticity of underlying quantitative representations as well as differences in familiarity with number symbols (Ansari, Garcia, Lucas, & Dhital, 2005).

Given that the core difference between children and adults in enumeration and magnitude comparison tasks is response time rather than the underlying mechanisms of accessing quantity, the contribution of the quantitative pathway towards mathematical outcomes measures in this study with adults should be similar to previous studies with children (LeFevre et al., 2010; Sowinski et al., 2015). Thus, the quantitative pathway should predict all tasks that require quantitative knowledge, including calculation tasks and word problems. In LeFevre et al. and Sowinski et al., speeded responding was used as the measure of the quantitative pathway and a similar approach was taken in the current study.

Working memory. Children's working memory capacity increases with development (Thomason et al, 2009; Fry & Hale, 2000; Fry & Hale, 1996; Gathercole, Pickering, Ambridge, & Wearing, 2004; Zheng, Swanson, & Marcoulides, 2011). Thus, compared to children, adults can maintain more information when working memory load increased. However, Fry and Hale (1996, 2000) demonstrated that the increase in working memory capacity in adults is mostly due to developmental changes in processing speed, and the increase in processing speed improves speed on working memory (also see Kail, 1992; Kail & Park, 1994). Thus, the basic underlying mechanisms of working memory in children and adults may not differ much. Similarly, other studies have illustrated that children at the age of 6 have similar basic modular structure of working memory as adults (Gathercole, Pickering, Ambridge, & Wearing, 2004). They exhibit capacity in each component of the Baddeley's (2001) working memory model. Furthermore, brain imaging studies have demonstrated similar effects. Both children and adults activate the lateral areas of the prefrontal cortex (LPFC) when they utilize working memory (Tsujimoto,

Yamamoto, Kawaguchi, Koizumi, & Sawaguchi, 2004). This pattern holds true whether they are performing a visual-spatial working memory task (Thomas et al., 1999), a phonological working memory task (Casey et al., 1995), or an inhibition task that requires executive control (Casey et al., 1997).

Because working memory is associated with the performance of almost all tasks, and because children and adults do not vary much in terms of the underlying cognitive mechanisms of working memory, I hypothesized that the working memory pathway in this study should be related to the performance of all complex mathematical tasks that are not automatized, such as calculations, similar to the contributions of the working memory pathways in LeFevre et al. (2010) and Sowinski et al. (2015). However, the specific amount of contribution of working memory towards outcome measures depends on the cognitive requirements of the specific tasks.

In summary, based on literature, I hypothesized that there should be similarities and differences in the contributions of cognitive abilities towards math outcomes in children and in adults. Specifically, the contributions of linguistic ability should differ in these two groups, and the differences should demonstrate the development in number system knowledge due to practice and experience. In contrast, I hypothesized that the relation between the outcome measures and the cognitive predictors of working memory and quantitative knowledge should be similar for children and adults.

Cognitive Differences Across Cultures and How They Might Affect the Model

Due to linguistic, cultural, and education differences, Chinese and English speakers differ in their mathematical competency and choice of strategy when solving math problems. These differences are reflected in neural activations (Cantlon & Brannon, 2006). Because I included English-speaking and Chinese-speaking participants the present study and compared the fit of the

pathways model in these two cultural groups, it was crucial to have an overview of the differences between English-speaking and Chinese-speaking individuals in terms of the four cognitive precursors that will be examined in this study, linguistic skills, quantitative knowledge, spatial ability, and working memory. Based on the differences of these cognitive precursors across cultures and the relations of these cognitive abilities to mathematical performance, the predictions regarding the *Pathways to Mathematics* model were modified accordingly.

Linguistic. Chinese and English are fundamentally different languages and they have different number language systems. The Chinese spoken number language is rule-based and it corresponds closely to the base-10 Arabic digit system. For instance, in Chinese, eleven is expressed as shi-yi (ten-one), twenty is expressed as er-shi (two-ten), and twenty-one is expressed as er-shi-yi (two-ten-one). Research has shown that the 10-based numbering system in Chinese provides an advantage in early mathematics, especially in counting tasks (Chan, 2014; Geary, Bow-Thomas, Liu, & Siegler, 1996; Ho & Fuson, 1998; Mark & Dowker, 2015; Miller, Kelly, & Zhou, 2005; Miller & Stigler, 1987; Wang & Lin, 2005). Furthermore, children in China have greater exposure to numbers in everyday life because the Chinese language uses numbers to name days of the week and months of the year (Kelly, Miller, Fang, & Feng, 1999). For example, Chinese has *xingqi-yi* (weekday-one) for *Monday*, *xingqi-er* (weekday-two) for *Tuesday*, *yi-yue* (one-month) for *January*, and *er-yue* (two-month) for *February*. Furthermore, ordinal numbers in Chinese are also expressed using cardinal number words, and there is an auxiliary word for all ordinal numbers (i.e., "*di*"). For instance, *first* is *di-yi* (aux-one), *second* is *di-er* (aux-two), *third* is *di-san* (aux-three), and so on. The greater exposure to numbers in everyday life may also provide advantages to children learning math.

The linguistic advantage of the Chinese language has been shown to be beneficial for children first learning about numbers, because the numbering system simplifies number words for children to learn and remember (e.g., Miller et al., 1995). However, because the present study involved adults, who have automatized the number words already, it is hypothesized that Chinese-speaking and English-speaking adults do not differ in the contributions of linguistic pathway with respect to digit names towards mathematical outcomes. Because both Chinese-speaking and English-speaking participants have direct and automatic access to the number words in their mental lexicon, the linguistic pathway should not strongly relate to participants' symbolic number system knowledge, and therefore, should not contribute to mathematical tasks unless they involve a non-numerical language component. Accordingly, for participants from both language groups, their performance on mathematical tasks that require language processing that is not automatized, such as word problems where the mathematics has to be extracted from the verbal problem, should be predicted by the linguistic pathway.

Quantitative knowledge. Numerous studies have demonstrated that the Chinese students outperform their counterparts from North America in a wide variety of mathematical tasks, including counting and place values (Miller & Stigler, 1987), computation (Campbell & Xue, 2001; Huntsinger, Jose, Liaw, & Ching, 1997; LeFevre & Liu, 1997), measurement and scaling (Stevenson et al., 1990), number line estimation (Siegler & Mu, 2008), and word problems (Stevenson et al., 1990). Although there might be different reasons for the better mathematical skills in Chinese children and adults, such as a linguistic advantage (Miller & Stigler, 1987; Ho & Fuson, 1998; Wang & Lin, 2005; Geary, Bow-Thomas, Liu, & Siegler, 1996; Miller, Kelly, & Zhou, 2005; Mark & Dowker, 2015), sociocultural influences (Huntsinger, Jose, Liaw, & Ching, 1997; Starkey & Klein, 2008; Hess, Chang, & McDevitt, 1987), and educational approach (Cai

& Nie, 2007; Jiang & Eggleton, 1995; Paine & Ma, 1993), the end result is the relatively better quantitative knowledge in Chinese-speaking individuls. Zhou et al. (2007) found that Chinese kindergartners (mean age of 5.8 years) can already automatically process numbers, compared to the emergence of automatic number processing in children at the age of 7 to 8 in North America (Girelli, Lucangeli, & Butterworth, 2000; Rubinsten, Henik, Berger, & Shahar-Shaiev, 2002). Such findings support the view that automatization of access to numbers and quantities develops sooner in Chinese students. Automaticity in calculation tasks may also be stressed more in mathematics education in China, where calculators are typically not available until later than in Canadian education. Thus, Chinese students may develop automaticity of access to quantitative knowledge and continue to show increasing automaticity over their schooling.

I hypothesized that individuals who are more skilled and fluent in math and calculation will use quantitative-related strategies to solve problems, and those who are not fluent or confident about math may avoid quantitative-related strategies. For instance, Newman (2017) found that when adults were asked to locate 4829 on a 0-7000 number line, individuals with better math skills might divide 4829 by 7000 to get a fraction, and then utilize spatial ability to locate that specific proportion on a number line. In contrast, individuals with poorer math ability may have used an anchoring approximation either above or below the midpoint, then moved upwards to estimate final location, suggesting less use of proportional reasoning and mental calculation than the more skilled solver. Therefore, it is hypothesized that both the quantitative pathway and the spatial pathway will contribute to Chinese-speaking participants' performance on the number line task, whereas the quantitative pathway will contribute less for the English-speaking participants.

Working memory. There are working-memory-related explanations for Chinese students' better performance on math tasks. Because Chinese digits have shorter pronunciation duration than the digits in English (a mean duration of 320 ms for Chinese digits vs. 420 ms for English digits), a greater number of digits can be stored in verbal working memory (Stigler, Lee, & Stevenson, 1986). Consistent with this difference, Chinese participants have longer digit-spans than English-speaking participants because digit span tasks require phonological codes to contain information in working memory (Chen & Stevenson, 1988). Similarly, because it is easier for Chinese participants to contain more digits in working memory, Chinese students learn multiplication facts by memorizing a short rhyme that contains the entire multiplication table. Research has shown that Chinese adults can perform faster mental multiplication because they store and access multiplication facts automatically using phonological codes (LeFevre, Lei, Smith-Chant, & Mullins, 2001; LeFevre & Liu, 1997; Campbell & Xue, 2001).

However, in terms of the involvement of specific components of working memory in math problem solving, research has not shown differences between Chinese-speaking and English-speaking participants. Imbo and LeFevre (2010) found that participants from both language groups use both phonological and visual-spatial working memory when solving complex subtraction and multiplication problems. Similarly, Lan et al. (2011) showed that Chinese children and American children had comparable working memory performance, and the relationship between executive functions and mathematical achievement was similar for the English-speaking and Chinese-speaking children.

Therefore, I hypothesized that due to shorter pronunciation durations of Chinese number names, Chinese participants should demonstrate better performance on working memory tasks that involve digit names (e.g., digit span task) than English-speaking participants, but not on

those working memory tasks that do not involve number names (e.g., visual-spatial span task). Moreover, working memory should be involved in all mathematical outcome measures, and the involvement of working memory should not differ for participants from the two cultural groups. That is to say, even though Chinese speakers might exhibit better performance on some working memory tasks, the relations between working memory and outcome measures should not differ across these two cultural groups.

Spatial ability. A few studies have examined the difference in spatial ability for Chinese-speaking and English-speaking individuals. Li, Nuttall, and Zhou (1999) showed that learning to write Chinese characters is beneficial for geometry learning and is helpful to the development of the perception of space. In addition, Li et al. (2001) found that undergraduate students who could write in Chinese were able to achieve better performance on mental rotation tasks and obtain higher SAT-mathematics scores than individuals who could not write in Chinese. However, there has been no study that showed different types of contributions of spatial ability towards mathematical skills in English-speaking and Chinese-speaking participants.

Spatial ability is hypothesized to predict participants' performance on the number line task for both English-speaking and Chinese-speaking participants. A number line is a spatial representation of numbers, presented in a form of a numerical straight line divided into linear unit intervals in an ascending or a descending order in correspondence to the magnitude of numbers. Performance on the number line task may reflect an individual's number-space association (Rugani & Hevia, 2016). Studies have shown a positive relationship between spatial ability and accuracy on linear number lines (Cheng & Mix, 2014; Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011; Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013; Cheng & Mix, 2014; Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011). Thus, I hypothesized

that the spatial pathway will predict participants' performance on other numerical tasks, such as calculation and word problems (Mix et al, 2016; Cheng & Mix, 2014). Taken together, it is hypothesized that in the current study, the spatial pathway will contribute to the outcome of number line tasks, calculation tasks, as well as word problems.

To summarize, based on past research, it is hypothesized that there will be differences between Chinese and English-speaking participants in terms of adults' performance on math tasks. These differences may relate to language (e.g., resulting in a difference in digit span) or to experiences (e.g., resulting in differences in arithmetic performance, as well as other math measures). However, the contributions of cognitive abilities towards these math outcome measures should generally be similar across the two cultures.

Expected Results

To conclude the introduction above, Figure 1 and Figure 2 are illustrative figures developed to capture the hypotheses. Figure 1 represents hypotheses for English-speaking participants, and Figure 2 represents hypotheses for Chinese-speaking participants. Arrows indicate hypothesized contributions of cognitive precursors towards mathematical outcomes. These mathematical outcomes (i.e., word problems, calculation, and number line) were chosen because they are relatively simple math tasks that most adults are familiar with but nevertheless show individual differences in performance. As is illustrated in the figures, it is hypothesized that the linguistic pathway will predict participants' performance on reading outcome measures, including reading comprehension, word reading, and reading speed, as well as mathematical measures that have a language component (i.e., word problems). Reading outcome measures were included as control measures to examine the possibility that all basic cognitive abilities are involved in all complex tasks, and not just the mathematical ones. The quantitative pathway is

hypothesized to predict performance on computation and calculation problems for both English-speaking and Chinese-speaking participants. Furthermore, the quantitative pathway is hypothesized to predict Chinese-speaking participants' performance on mental number line tasks. The spatial pathway should predict participants' performance on number line tasks, calculation tasks, and word problems for both English-speaking and Chinese-speaking participants. Finally, the working memory pathway should predict participants' performance on all mathematical outcome measures for both English-speaking and Chinese-speaking participants.

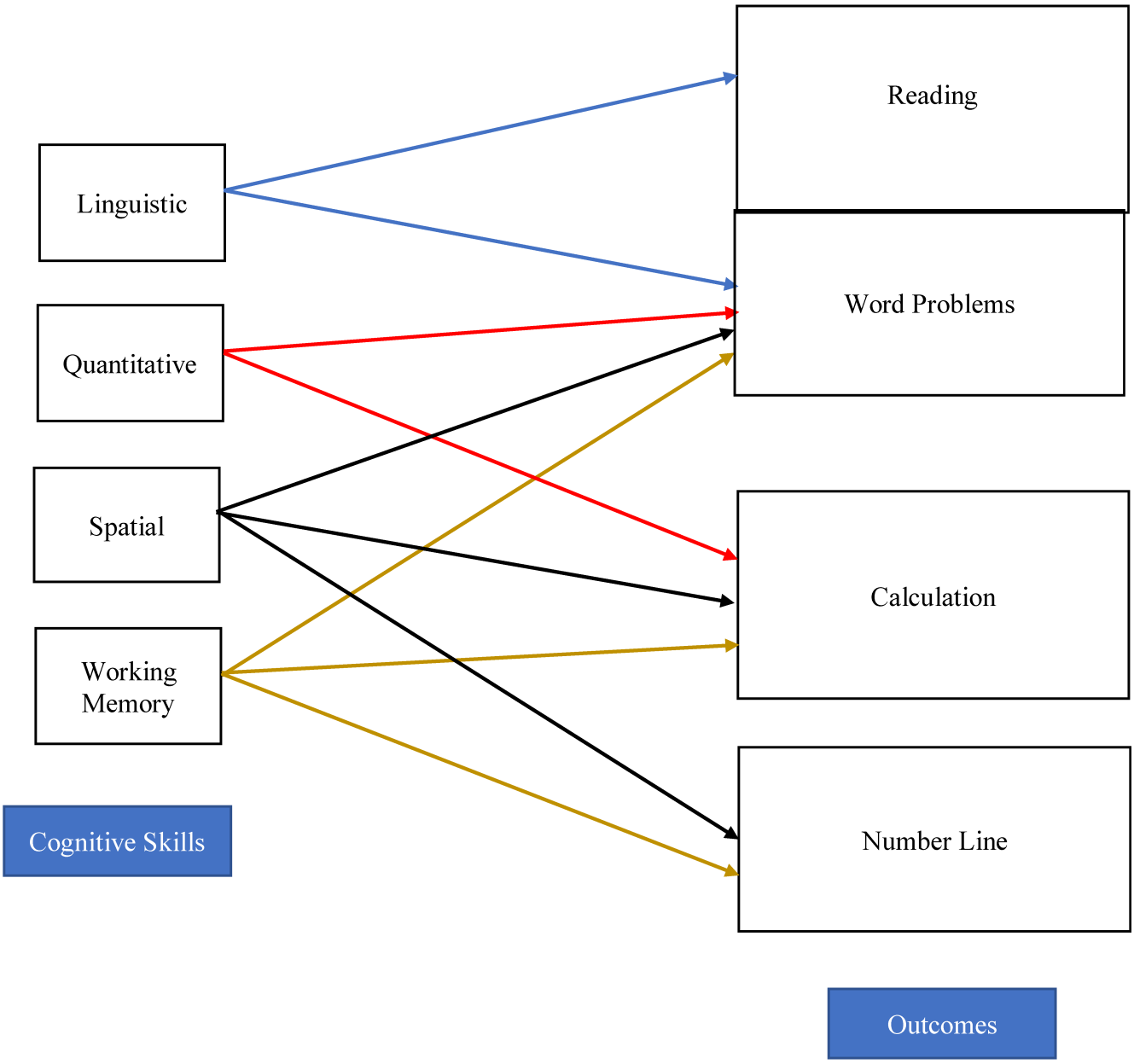

Figure 1. Hypotheses for English-speaking participants. Arrows indicate hypothesized contributions of cognitive precursors towards mathematical outcomes.

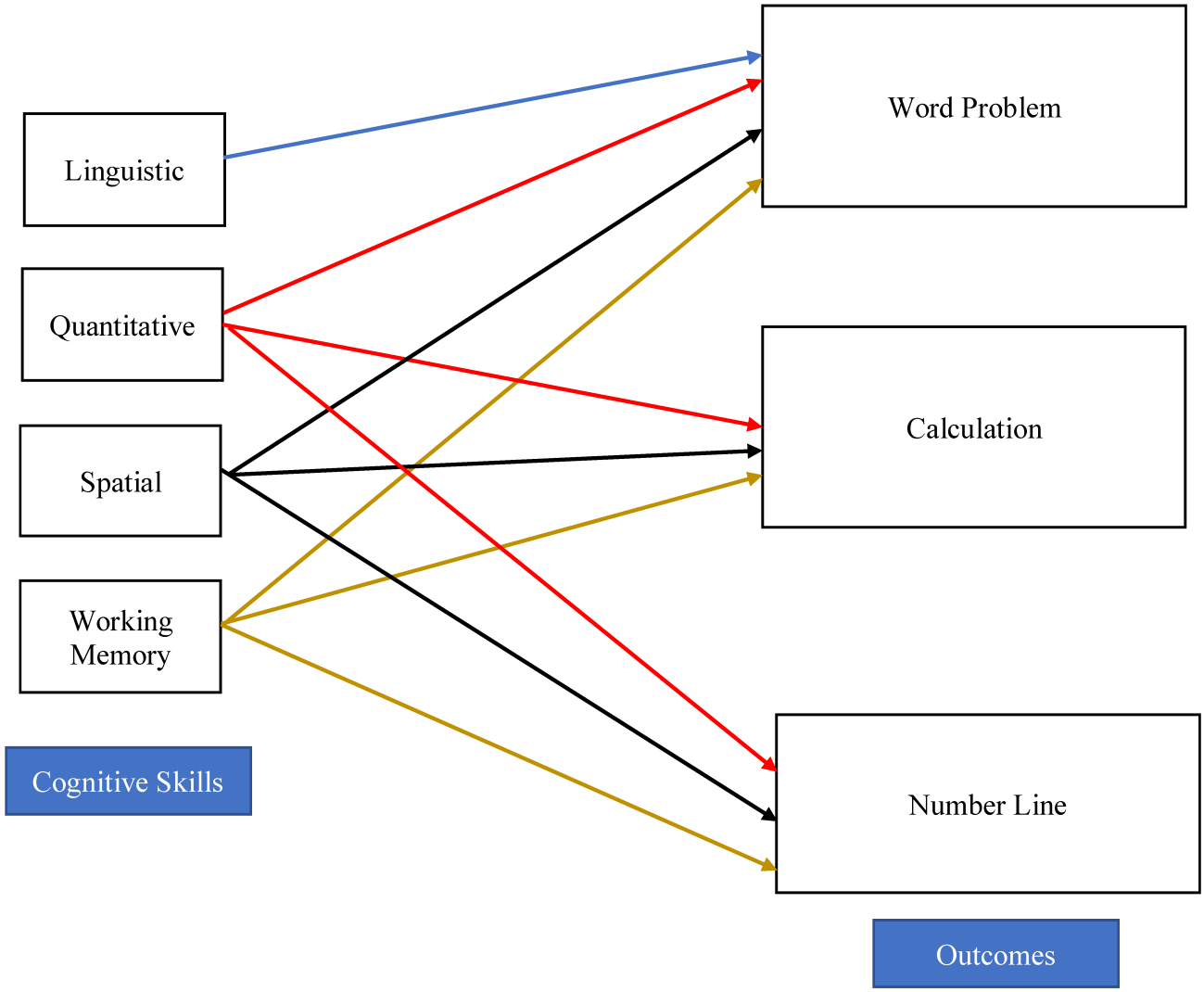

Figure 2. Hypotheses for Chinese-speaking participants. Arrows indicate hypothesized contributions of cognitive precursors towards mathematical outcomes.

Method

Participants

In the Winter of 2016, 142 participants were recruited from introductory classes in Psychology and Cognitive Science or were paid to participate. The participants included 129 undergraduate students, six Master's students, and seven PhD students. 71 of the participants learned Chinese (Mandarin) as their first language and attended elementary schools instructed in Mandarin. The other half of the participants learned English as their first language and received elementary education in English. All participants were proficient in English and English was used for the informed consent, instructions and debriefing forms. Participants took part in this study in exchange of 2% course credit or $20 monetary reward.

Materials and procedures

All participants were asked to complete tasks in two sequential sessions. The first session took about 50-60 minutes to complete, and the second session took about 40-50 minutes to complete. Within each session, participants were given short breaks periodically, and between the two sessions, participants were given a 10-minute break with snacks and drinks provided. The entire experiment took about 2 hours to complete. Participants received oral instructions in the participants' first language. The Chinese-speaking students were tested by one of two bilingual Chinese-English experimenters; the English-speakers were tested by one of these two experimenters or by a monolingual English speaker.

Paper-and-pencil tasks that required participants' written input were completed in the first session, including quantity speeded processing, computational fluency test, brief math assessments, Nelson-Denny reading test (only for English-speaking participants), and the spatial orientation test.

Tasks that contained audio presentations and required participants' verbal input or inputs on iPads were completed in the second session. These tasks included digit span forward task, digit span backward task, elision, KeyMath applied problems, word reading (for English-speaking participants only), vocabulary, number line task, simple arithmetic task, and the spatial span task. See Table 2 for a summary of measures used in this study.

Table 2
Summary of Tasks Used in This Study

Cognitive Skills				Outcome Measures	
Linguistic Skill	Quantitative Knowledge	Working Memory	Spatial Ability	Mathematical Outcomes	Reading outcomes
PPVT**	Quantity speeded processing (subitize)	Digit span forward	Spatial orientation test	Simple Arithmetic Test	Reading speed*
NIH Picture Vocabulary*	Symbolic magnitude comparison	Digit span backward		Computational Fluency Test	Reading comprehensi-on*
Elision	Non-symbolic magnitude comparison	Spatial span		Brief Math Assessment	Word reading fluency*
				word problems	
				Number line	

Note. * For English-speaking participants only. **For Chinese-speaking participants only.

Predictor Measures

There are hypothesized four pathways in this study: linguistic skills, quantitative knowledge, spatial ability, and attentional span. Within each pathway, a number of tasks were included as predictor measures.

Linguistic skills. Consistent with LeFevre et al. (2010), receptive vocabulary and phonological awareness tests were selected as measures of linguistic skills in adults.

Receptive vocabulary of English-speaking participants was measured using the Picture Vocabulary Test from the NIH tool box (Weintraub et al., 2013). Participants were presented with an audio recording of an English word and four photographic images on an iPad. Participants were instructed to touch the picture that most closely matched the meaning of the word. The specific words and the number of words presented to each participant depended on the participant's age and performance on this task. This task took about 5 minutes and contained about 25 items for each participant. This task was administered by the iPad in an untimed fashion until the task is completed. Age-corrected standardized scores provided by the NIH tool box app were used for data analyses. Test-Retest reliability of the NIH Toolbox Picture Vocabulary Test reported by Gershon et al. (2014) was high, intra-class correlation (ICC) = .80.

The Peabody Picture Vocabulary Test – Third Edition (PPVT-III) (Dunn & Dunn, 1997) was translated into Chinese and adopted as a measure of receptive vocabulary for Chinese-speaking participants. The Chinese translations of the English words in the PPVT test matched the difficulty levels of the words and their frequencies of use in everyday life and in academic situations. The participants were presented with a written Chinese word and four pictures on a page, and were instructed to point to the picture that best depicted the meaning of the word. This task started from Set 13 of the PPVT test, which is the most appropriate level for adults, and

there were 8 sets in total, with 12 items per set. The test was terminated after 8 errors in a single set. This task was administered by an experimenter in an untimed fashion, and it took about 10 minutes to complete. Total number of correct answers obtained on this test was used for data analyses. Because this test produced eight subscores for the eight sets, and because these sets were increasing in difficulty level, it was not appropriate to conduct an internal reliability test on this measure.

Elision task was adopted to measure phonological awareness of participants. The Elision subtest of the Comprehensive Test of Phonological Processing (CTOPP; Wagner, Torgensen, & Rashotte, 1999) was adopted to examine English-speaking participants' ability to remove phonological segments from spoken words. This task was presented using audio recordings that instructed participants to first say a word, and then remove a phoneme (i.e., a phonetic sound) from the word and say it again. One example of the instruction can be, "Say flame. Now say flame without saying /f/." In this case, the expected response from a participant would be to first correctly pronounce "flame", and then correctly articulate "lame". This task contained 20 individual items, but the test would be terminated after 3 consecutive errors, as supervised by the experimenter. The content presented in the recording was in English. This task was untimed, and it took up to 3 minutes to complete. Scores on this task for data analyses were total number of correct responses from participants across all trials. The internal reliability of the CTOPP elision subtest reported by the developers of this test exceeded .80 (Wagner, Torgensen, & Rashotte, 1999).

For Chinese-speaking participants, phonological awareness was measured using a phonological elision task developed by Newman, Tardif, Huang and Shu (2011). Similar to the English elision test, this task presented participants with recordings of instructions asking

participants to remove a phoneme from a word and pronounce the remaining segments. The entire task contained 32 items, but the test was terminated after 3 consecutive errors. The content of the recordings was in mandarin Chinese. This task was untimed, and it took up to 10 minutes to complete. Scores on this task for data analyses were total number of correct responses from participants. The internal reliability of this Chinese phonological awareness test was high, Cronbach's $\alpha = .95$, as was reported by Newman et al. (2011).

Quantitative knowledge. Expanding from LeFevre et al. (2010), Sowinski et al. (2015)'s quantitative pathway included enumeration and magnitude comparison as domain-specific numerical components to examine children's ability to evaluate and differentiate quantities with minimal additional cognitive demand. Deriving from LeFevre et al. and Sowinski et al.'s quantitative pathways, four tasks were used in the present study to investigate adults' numerical abilities. Consistent with LeFevre et al. and Sowinski et al., a measure of subitizing was adopted to examine participants' ability to quickly and accurately distinguish the quantities of small sets of items (≤ 4) without counting. In addition, similar to Sowinski et al., measures of magnitude comparison were also included in this study. As an extension from Sowinski et al., besides the symbolic magnitude comparison, the present study also used a non-symbolic magnitude comparison to test participants' ability to quickly evaluate and distinguish numerosities.

The ability to subitize was measured using the quantity version of the speeded processing task. In this task, participants were presented with small quantities within the subitizable range represented by dots (i.e., 1 to 3 dots). All dots were of the same size. Participants were instructed to name the quantities out aloud as quickly and as accurately as possible. Participants completed a practice sheet prior to this task. During the task, for each page, the experimenter measured and recorded the rapid naming speed in seconds using a stop watch (i.e., how long it took the

participant to name the quantities) as well as accuracy (i.e., the number of errors committed by the participant). Each page had 24 sets of dots, and there were two pages to ensure reliability. Scores on this task were the mean number of correct items per second for the two pages. The entire task took up to 2 minutes. All participants used their first language when completing this task. Internal reliability on this task was calculated using participants' scores on the two pages, Cronbach's $\alpha = .93$.

The symbolic magnitude comparison task was a paper-and-pencil task that presented participants with pairs of single digits (i.e., 1 to 9), and required participants to cross out the greater digit in each pair. The task began with instructions and a demonstration of solutions to four sample items from the experimenter. Participants then proceeded to eight practice items, after which they continued to the actual test. For each page of the test, the experimenter measured the time of completion in seconds (i.e., speed) using a stop watch as well as the number of errors (i.e., accuracy). Each page contained 30 items, and this task had two pages to ensure reliability. Scores on this task were the mean number of correct items per second for the two pages. This task took up to 2 minutes to complete. Internal reliability calculated based on performance on the two pages was high, Cronbach's $\alpha = .95$.

The procedure of administration for the non-symbolic magnitude comparison task was same as its symbolic counterpart. As to the stimuli, the non-symbolic version required participants to compare pairs of dots (1 to 9 dots), and cross out the one with greater quantity. This task also took up to 2 minutes to complete. Score calculation procedures were also the same. Internal reliability based on performance on the two pages of the non-symbolic magnitude comparison task was also high, Cronbach's $\alpha = .89$.

Working Memory. Working Memory was examined in LeFevere et al. (2010) using a visual-spatial span task to measure spatial attention. In Sowinski et al. (2015)'s expanded pathway, they tapped all three components from Baddeley and Hitch (1974). This study included three tasks to examine the three components of working memory. The digit span forward task reflected phonological loop, the digit span backward task examined phonological loop and central executive, and the spatial span task studied visual-spatial sketchpad and central executive.

The digit span forward task presented participants with audio recordings of sequences of single-digits, and asked participants to orally repeat what they had heard in the recordings. This test started with a sequence of four digits (e.g., 2-9-5-7), and participants were required to repeat the four digits in order, starting from the first digit and ending with the last digit. There were three trials for each span (i.e., the number of digits in each sequence). If the participant could answer any of the three trials correctly, the span size increased by one digit. The test would finish if the participant was unable to recall any of the three trials for a given span size, or if the participant was able to correctly answer at least one trial for all span sizes. The greatest digit span size for this task was 12 digits. The total number of correct trials was kept as this participant's score on this task. Depending on participants' performance on this task, it took up to 2 minutes to finish this task. All audio recordings were in participants' first language, and participants used their first language to respond on this task. Internal reliability was calculated based on the subscores of first, second, and third trials at each span (up to 10 digits), Cronbach's $\alpha = .59$.

Similar to the digit span forward task, the digit span backward task also presented participants with audio recordings of sequences of single-digits. In this task, participants were instructed to orally repeat the digits in reverse order, starting from the last digit and ending with

the first digit. For example, if the sequence of digits was “5-9-2” in the recording, then the correct response from the participant would be “2-9-5”. This test started with a sequence of two digits (e.g., 2-5), and there were three trials for each span size (i.e., the number of digits in each sequence). If the participant answered any of the three trials correctly, the span size increased by one digit. The greatest digit span size for this task was six digits. The test would terminate if the participant was unable to recall any of the three trials for a given span size, or if the participant was able to correctly answer at least one trial for all span sizes. The total umber of correct trials was kept as this participant’s score on this task. Depending on participants’ performance on this task, it took up to 2 minutes to finish this task. All audio recordings were in participants’ first language, and participants used their first language to respond on this task. Internal reliability was calculated based on the subscores of first, second, and third trials at each span (up to 6 digits), Cronbach’s $\alpha = .80$.

Visual-spatial working memory was measured using a spatial span task. This task was administered on an iPad with the application *PathSpan*. Participants were shown a random array of nine green circles on the screen. These green circles would light up in sequence, and participants were instructed to watch and remember the sequence, and then repeat it by touching the circles in the same order in which they lit up (i.e., starting with the first and ending with the last). This test started with a sequence of two circles lighting up, and there were three trials for each span size (i.e., the number of lit-up circles in each sequence). If the participant answered any of the three trials correctly, the span size increased by one circle. The greatest span size for this task was 9. The test would terminate if the participant was unable to repeat the sequence for any of the three trials for a given span size, or if the participant was able to correctly answer at least one trial for all span sizes. The total number of correct trials was kept as this participant’s

score on this task. Depending on participants' performance on this task, it took up to 5 minutes to finish this task. Internal reliability was calculated based on the subscores of first, second, and third trials at each span size (up to the span size of seven), Cronbach's $\alpha = .70$.

Spatial ability. A spatial orientation test developed by Hegarty, Kozhevniko, and Waller (also see Hegarty & Waller, 2004) was used to examine participants' ability to estimate locations of objects from different perspectives and angles, and to mentally orient and navigate towards different directions. It consisted of 12 problems. Each problem was presented on a sheet of paper with a picture of an array of objects and an "arrow circle". See example in Figure 3. For each problem, the participants were instructed to imagine standing at the position of a specific object in the array (the name of the object was labeled in the centre of the circle), and facing another object in the array (the name of the facing object is shown at the top of the circle), and then draw an arrow from the centre of the circle indicating the direction to a third object from the facing orientation. An example of a question in this test would be, "Imagine you are standing at the **flower** and facing the **tree**. Point to the **cat**." The flower, tree and cat are three of the objects presented in the array. Participants were not allowed to physically rotate the testing sheets or make any marks on the diagram showing the configuration of objects while completing the test. Each problem was scored as the absolute value of the difference between the participants' drawn angle and the correct angle of the direction of the target (i.e., absolute directional error; Hegarty & Waller, 2004). In this timed task, participants were given 5 minutes to complete this task, and were told to solve as many problems as possible. The instructions of this task were in simple English and were understood by all participants. The scoring of this task used the absolute values of the differences between participants' indicated degrees and correct degrees. Internal reliability was calculated based on participants' performance across the 12 problems, Cronbach's $\alpha = .70$.

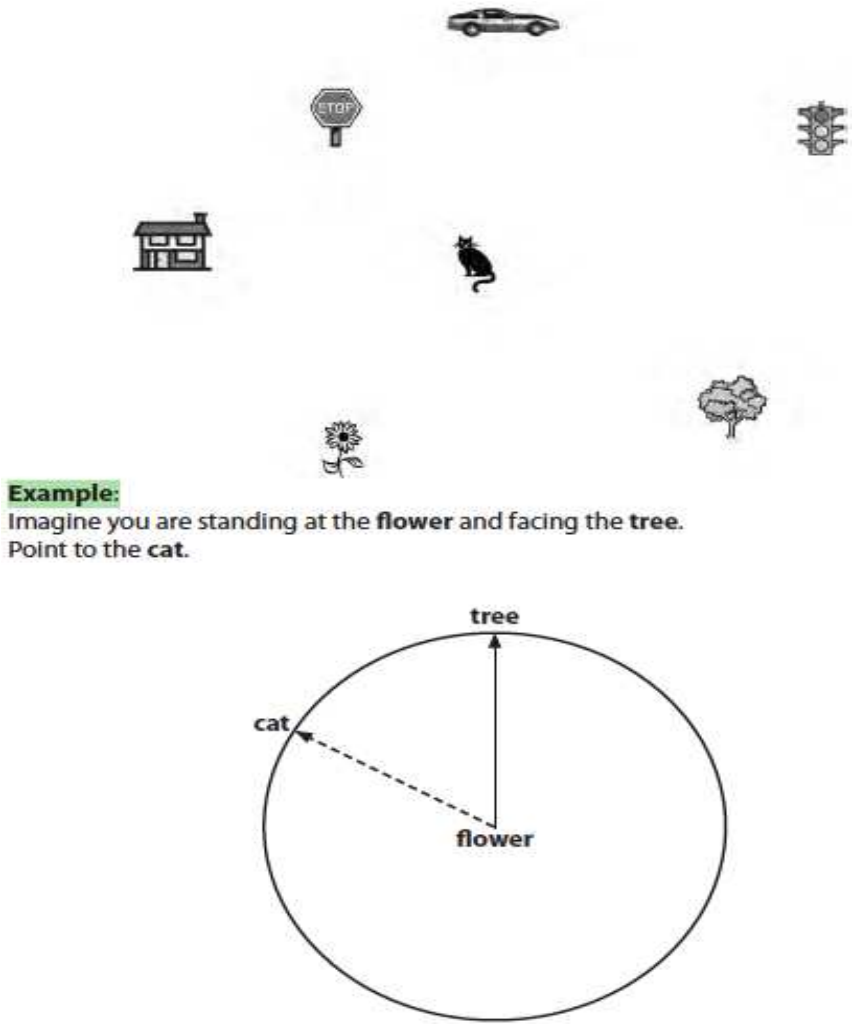

Figure 3. An example of the spatial orientation task.

Mathematical Outcomes

Five mathematical outcome measures were included in this study, a Computational Fluency Test (CFT), a simple arithmetic test, a Brief Math Assessment (BMA), a KeyMath applied problem solving test, and a number line task. The four pathways are hypothesized to have different contributions towards these five tasks, depending on the specific demands of these tasks.

Computational Fluency Test (CFT) was a paper-and-pencil test that consisted of three sheets of math problems with 60 problems on each sheet. The first sheet contained 60 double-digit addition problems (e.g., 19+24), the second sheet had 60 double-digit subtraction problems (e.g., 79-32), and the third sheet was a sheet of multiplication problems (e.g., 16×8). This task began with two practice questions for each operation and during practice, the participants may ask the experimenter for help. Participants were then given one minute per sheet to complete as many questions as they could. Scoring of this task was the sum of the numbers of correct answers a participant could achieve for each operation. Internal reliability based on performance on the three subsets was high, Cronbach's $\alpha = .92$.

Simple arithmetic test was an examination of participants' ability to calculate single digit addition with sums less than or equal to nine. This task was completed using an iPad application. Participants first completed two practice questions, and then continued to the one-minute timed task. The number of correct answers attained by the participants within one minute was kept as their score on this task. Because this test kept the total correct answers as the scores, it was not appropriate to conduct a reliability test on this single-item variable.

Brief Math Assessment (BMA) was a task developed by Steiner and Ashcraft (2012) based on the Wide Range Achievement Test: Third Edition (WRAT3). In this paper-and-pencil task, participants were provided with ten progressively more difficult math problems ranging from whole number addition and subtraction (much like the ones in CFT), to fractions, decimals, and algebra. Participants were allowed 6 minutes to complete this task, and the number of correct answers were kept as their scores on this task. Internal reliability was calculated based on the 10 questions, Cronbach's $\alpha = .62$.

KeyMath applied problem solving test was a subset of items (i.e., *Applied Problem Solving*) from the KeyMath Numeration test developed by Connolly (2000). This task in this experiment started from Item 3 of the Applied Problem Solving subset because this was the appropriate level for undergraduate students. Participants were presented with audio recordings of 16 questions in sequence, from the easiest to the hardest. Paired with the audio recording of each question, participants were shown a picture describing the scenario of the applied problem and some written key information mentioned in the problem. Participants were not provided with any papers or pencils to take notes or to write on for this task, and participants were supposed to mentally form an answer and orally respond to each question. Participants were allowed to have 7 minutes in total for this task, and were instructed to answer the questions as quickly and accurately as possible. Scoring of this task based on the total number of correct answers. For the Chinese participants in this study, questions were translated into Mandarin Chinese and were recorded by a native Mandarin speaker. Internal reliability was calculated based on the 16 questions, Cronbach's $\alpha = .69$.

The number line task was completed using an iPad app. This app was developed based on the number line task proposed by Siegler and Opfer (2003). In this task, participants were provided with a number line with 0 on the left end and 7,000 on the right end. For each trial, participants were provided with a number (e.g., 4783) and were asked to locate the number on the number line by touching the appropriate position on the line. 30 target numbers were presented in a random order, and participants were asked to respond to each trial as quickly and as accurately as possible. After participants finished all 30 trials, they were required to report strategies used when completing this number line task. It took about five minutes for participants to complete this task. Each target number was scored using Percent Absolute Error (PAE)

suggested by Siegler and Booth (2004) with this equation: | (Estimate – Presented Number) / Scale of the Estimate | x 100. For example, if a participant marked the location of 500 on the 0-7000 number line at the position that corresponded to 1000, the PAE for this response would be | (1000 – 500) / 7000 | x 100 = 7%. Internal reliability was calculated based on the 30 trials, Cronbach's α = .92.

Reading outcomes

Three reading outcome measures were included in this study to measure participants' reading speed, reading comprehension ability, and word reading. Because there were no suitable and comparable standardized language tests in Chinese, these reading outcome measures were only administered in English for English-speaking participants.

Reading speed was examined using a subtest of the *Nelson-Denny Reading Test* (Brown, Fishco, & Hanna, 1993). Participants were given a passage to read, and next to each line of text, a number was printed to indicate the number of words read. the experimenter would start timing using a stopwatch as soon as the participant had begun reading. The experimenter would call "stop" when one minute had elapsed and ask participants to circle the number printed next to the line that they were reading. This number was participants' reading speed (i.e., number of words read per minute). Because this test produced a single-item variable, it was not appropriate to conduct a reliability test on this measure.

Reading comprehension was measured using the second subtest of the *Nelson-Denny Reading Test* (Brown, Fishco, & Hanna, 1993). Participants were assigned three passages to read and were told to respond to questions following each passage before proceeding to the next one. There were 18 multiple choice questions in total. About half of the questions tested participants on factual content, while the other half of the questions were inferential in nature. Participants

were allowed 12 minutes to complete this task. Score was based on the number of correct responses, and there was no penalty for incorrect answers. Internal reliability of the *Nelson-Denny* reading comprehension measure was calculated using participants' subscores on first, second, and third passage, Cronbach's $\alpha = .55$.

Word reading fluency was assessed using the *Oral Reading Recognition Test* provided in the NIH toolbox app on iPad (Weintraub et al., 2013). Participants were instructed to pronounce words appeared on the screen as accurately as possible. Participants were not required to know the word or its meaning. This test used age and education to determine which items would present initially, and the computer adjusted difficulty level of items depending on the participants' performance. Therefore, every participant was presented with a different set of items. Age-corrected standardized scores provided by the NIH tool box app were used for data analyses. Items were presented one by one, and in an order of increasing difficulty in an untimed fashion until the test was completed. Testing would automatically discontinue if a participant made seven consecutive errors. For most participants, this test took approximately three minutes and contained about 25 items. The test–retest intra-class correlation (ICC) for the NIH Toolbox Oral Reading Recognition Test was 0.90, as was reported by Gershon et al. (2014).

Results

Data Cleaning

The first step of data analysis was to examine outliers for each variable. For each of the variables, scores with a z-value greater or equal to 3 or -3 were considered outliers. 9 variables had 1 to 3 outliers, and there were 16 outliers in total in the data set. These outliers were replaced with the nearest values that were not extreme. Descriptive statistics of all measures after removing outliers are shown in Table 3 and Table 4.

Descriptive Statistics and Comparisons Between Groups

The means and standard deviations for each group are shown in Tables 3 and 4. In general, the variables were normally distributed although the number line task, the problem solving task, and the Brief Math Assessment were positively skewed; some participants had very high or perfect scores on these tasks. For the latter two measures, the positive skew was greater for the Chinese than English participants, suggesting that in future studies measures with greater range should be included.

The Chinese- and English-speaking participants were not significantly different on spatial working memory or on the spatial ability task. They also did not differ on the non-symbolic number comparison task or on the number line task. However, on all of the other symbolic numerical measures and mathematical skill tasks, the Chinese group performed better than the English group. This advantage for the Chinese participants was expected; the effect sizes varied across measures, with the largest differences on the tasks that involved arithmetic.

Data Standardization

Because participants in the two language groups were given different measures for vocabulary and phonological awareness, z-scores for each measure within each language group were first calculated and then combined to create a new variable. Therefore, vocabulary and phonological awareness were created using z-scores from the respective measure in each language group.

Furthermore, because Chinese digits have shorter pronunciation duration than the digits in English (Stigler, Lee, & Stevenson, 1986), scores on tasks that can be affected by this factor were standardized within groups. For example, Chinese participants had better performance on the quantity speeded processing task (as is shown in Table 3) because they were able to articulate

digits faster, and not necessarily because they had better quantitative knowledge. Moreover, because pronunciation durations are shorter for each digit, Chinese participants were able to hold more digits in working memory in digit span forward task and digit span backward tasks, yielding better performance on these two tasks (see Table 3). Thus, for each of the language groups, z-scores were calculated for speeded processing task, digit span forward task, and digit span backward task. New variables for these three tasks were created using these z-scores.

Finally, because scores on the spatial orientation task and number line task are absolute values of deviation scores, greater scores indicate poorer performance on these two tasks. To make the results more intuitive and consistent, the values of these deviation scores were multiplied by -1. These newly obtained scores were used for data analyses so that greater values index better performance.

Table 3
Descriptive Statistics for Predictor Measures

	English speakers		Chinese speakers		Overall		*t* between groups
Measures	*M*	*SD*	*M*	*SD*	*M*	*SD*	
Quantitative Measures							
Quantity speeded processing	2.42	.43	2.80	.46	2.61	.48	-5.07***
Symbolic magnitude comparison	1.47	.31	1.59	.34	1.53	.33	-2.21*
Non-symbolic magnitude comparison	1.00	.21	1.07	.34	1.03	.29	-1.55
Working Memory Measures							
Digit span forward	10.19	3.41	17.28	4.03	13.79	5.15	-11.23***
Digit span backward	11.07	2.61	12.92	2.25	12.01	2.60	-4.48***
Spatial span	15.79	3.18	15.48	3.19	15.63	3.18	.57
Spatial Measure							
Spatial orientation task	-44.59	41.46	-39.26	25.39	-41.92	34.36	-.92

Note. Quantitative measures are the mean number of correct items per second. Working memory measures are total number of correct sequences. Spatial measure is the differences between participants' indicated degrees and correct degrees.
*$p < .05$. **$p < .01$. ***$p < .001$.

Table 4
Descriptive Statistics for Outcome Measures

Measures	English speakers		Chinese speakers		Overall		*t* between groups
	M	*SD*	*M*	*SD*	*M*	*SD*	
Math Outcomes							
Simple arithmetic task (number correct)	43.31	7.29	46.38	6.07	44.86	6.85	-2.72**
Calculation fluency test (number correct)	33.72	15.89	58.10	14.52	45.91	19.48	-9.54***
Brief math assessment (number correct)	6.41	1.80	8.25	1.49	7.33	1.89	-6.65***
Number line task (percent error)	-6.21	3.41	-5.25	2.66	-5.73	3.08	-1.86
Word Problems (number correct)	11.01	2.61	12.82	1.88	11.93	2.44	-4.70***
Reading outcomes							
Reading speed (words per min)	240.34	87.79					
Reading comprehension (number correct)	15.25	2.49					
Word reading (age-corrected standardized score)	118.26	16.69					

Note. *$p < .05$. **$p < .01$. ***$p < .001$.

Composite Measures

Three Principal Component Analyses were then conducted create component scores that would reflect the shared variance among variables within each of the three pathways that contained more than one measure (i.e., linguistic, quantitative, working memory). Because spatial ability pathway included only one measure, it was not possible to conduct a Principal Component Analysis on this pathway. The PCA produces a factor score which is a weighted average across the measures in the analysis. These three component scores for linguistic, quantitative and working memory pathways were used in subsequent multiple regression analyses.

The two factors in the linguistic pathway were receptive vocabulary and phonological awareness. The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy was .5, which barely met the commonly recommended value, indicating that 50% of variance in these two variables might be caused by an underlying factor. Bartlett's test of sphericity was also significant, χ^2 (1) = 12.28, $p < .001$, indicating that these two measures have covariance, and therefore suitable for structure detection. In addition, the communalities were .65 for these two factors, which were above the .3 cut-off value, further confirming that the two items shared common variance. These tests indicated that it was appropriate to conduct a Principal Component Analysis on the two variables in the linguistic pathway. These two items accounted for 64.66% of variance in the factor, and the component loadings for these two items were .80 each.

The three factors in the quantitative pathway were speeded processing (subitizing), symbolic magnitude comparison and non-symbolic magnitude comparison. The working memory pathway included digit span forward, digit span backward, and a spatial span measure. Furthermore, Principal Component Analyses were also conducted to investigate the shared

variances among variables in outcome measures. Specifically, two principal component analyses examined calculation outcomes and reading outcomes. Calculation outcome component included the simple arithmetic task, computational fluency test, and brief math assessment. Reading outcome component included the reading speed measure, reading comprehension test, and word reading task. Note that these measures were only conducted with the English-speaking participants. See Table 5 for a summary of the assumption tests of these pathways and outcome measures. As is shown in Table 5, all assumptions are satisfied, and therefore it was appropriate to conduct Principal Component Analyses on these pathways and outcomes. Component scores for the pathways, calculation outcomes and reading outcomes were adopted in subsequent multiple regression analyses. See Table 6 for a summary of bivariate correlations within each component, as well as component loadings for variables in each pathway and in each outcome variable.

Pearson bivariate correlations were then calculated to evaluate relations among cognitive predictors (i.e., the four predictor pathways) and outcome variables. See Table 7 for the relations among these variables. As shown in Table 7, other than the moderate correlation between the spatial pathway and working memory pathway, the four pathways are not strongly correlated with each other, whereas the math outcome measures are moderately to highly correlated. The correlation table also indicates that the linguistic pathway has the strongest relationship with the reading outcome, the quantitative pathway has the strongest relation with the calculation outcome, and working memory and spatial pathways correlate with several of the outcome measures.

Table 5
Assumption Tests for Principal Component Analyses

Components and Measures	KMO	Bartlett's test of sphericity (χ^2)	communalities
Linguistic (*N* = 139)	.50	12.28 ***	
1. Vocabulary			.65
2. Phonological awareness			.65
Quantitative (*N* = 142)	.64	164.69***	
1. Subitizing			.56
2. Symbolic magnitude Comparison			.83
3. Non-symbolic magnitude comparison			.77
Working memory (*N* = 138)	.50	30.59***	
1. Digit span forward			.43
2. Digit span backward			.71
3. Spatial span			.34
Calculation outcomes (*N* = 141)	.57	93.60***	
1. Simple arithmetic test			.35
2. Calculation fluency test			.79
3. Brief math assessment			.72
Reading outcomes (*N* = 68)	.60	22.62***	
1. Reading speed			.55
2. Reading comprehension			.45
3. Word reading			.68

Note. Recommended value for KMO is .50. Recommended cut-off value for communalities is .30. ***$p < .001$.

Table 6
Bivariate Correlations Among Variables Within Each Component and Component Loadings for the predictor pathways and the outcomes

Components and Measures	Correlations within groups		Component loadings	Percentage of Variance Explained
	1	2		
Linguistic (*N* = 139)				64.66
1. Vocabulary	-		.80	
2. Phonological awareness	.30**	-	.80	
Quantitative (*N* = 142)				72.11
1. Subitizing	-		.75	
2. Symbolic magnitude Comparison	.53**	-	.91	
3. Non-symbolic magnitude comparison	.44**	.76**	.88	
Working memory (*N* = 138)				49.28
1. Digit span forward	-		.66	
2. Digit span backward	.35**	-	.84	
3. Spatial span	.04	.30**	.58	
Calculation outcomes (*N* = 141)				61.59
1. Simple arithmetic test	-		.59	
2. Calculation fluency test	.34**	-	.89	
3. Brief math assessment	.24**	.65**	.85	
Reading outcomes (*N* = 68)				55.99
1. Reading speed	-		.74	
2. Reading comprehension	.22	-	.67	
3. Word reading	.39**	.35**	.82	

Note. ** $p < .01$ level (two-tailed).

Table 7
Bivariate Correlations Among Predictors and Outcomes

	1	2	3	4	5	6	7
Predictors							
1. Linguistic	-						
2. Quantitative	.02	-					
3. Working Memory	.20*	.22*	-				
4. Spatial	.21*	.12	.47**	-			
Outcomes							
5. Reading	.53**	.13	.35**	.41**	-		
6. Calculation	.11	.36**	.40**	.40**	.43**	-	
7. Number Line	.33**	.21*	.30**	.35**	.30*	.43**	-
8. Word Problems	.37**	.29**	.40**	.42**	.29*	.61**	.47**

Note. * $p < .05$ level (two-tailed).
** $p < .01$ level (two-tailed).

Multiple Regression Analyses

To further examine the predictive values of these four pathways on various outcome measures, multiple linear regression analyses were conducted. As shown in Table 7, the four predictors are not highly correlated, so multicollinearity of the predictors was not a problem in this data set. Furthermore, scatter plots indicated that the relationship between the predictors and outcome variables were linear. Therefore, assumptions for multiple regression were satisfied and it was appropriate to conduct linear regression analyses. One regression test was conducted for each outcome measure. To assess the effects of predictors without accounting for the effects of culture, culture was first entered as a control variable in block 1. Four pathways (i.e., the independent variables) were entered in block 2. The third block consisted of interactions of each pathway with culture. Interactions were obtained using dummy coded variables for culture

(English = 0, Chinese = 1) multiplied by standardized scores of the pathways. Because linguistic, quantitative, and working memory pathways were composite scores obtained using Principal Component Analysis, these scores had means of 0 and standard deviations of 1. The z-scores of the spatial pathway measure were calculated and saved as standardized scores. Results of multiple regression are shown in Table 8, Table 9, and Table 10, for the dependent variables of calculation, number line, and word problems, respectively. Because the reading outcomes were only administered to English-speaking participants, a multiple regression analysis examining predictive effects of pathways on the reading outcome was conducted with English speakers only (n = 71), see Table 11.

As shown in the regression tables, quantitative ability, working memory, and spatial skill predicted participants' calculation performance. Participants' number line accuracy was predicted by linguistic skill, quantitative knowledge, and spatial ability. All four cognitive predictors predicted participants' performance on word problems. Finally, linguistic ability was the only significant predictor of reading outcomes.

The only significant moderating effect of culture occurred as an interaction between quantitative knowledge and culture on number line performance. A simple slope analysis was then conducted to examine this significant interaction and pinpoint the difference. As can be seen from the simple slope graph demonstrated in Figure 4, for English speakers, better quantitative ability was associated with better performance on the number line task, $\beta = .47$, $p < .01$, whereas for Chinese speakers, there was no significant relationship between quantitative ability and number line performance, $\beta = .038$, $p = .71$. A closer examination of strategies adopted by English speakers and Chinese speakers indicated four most popular strategies, including evenly dividing the 0-7000 number line into 7 units (Strategy 1), using midpoint to estimate location

(Strategy 2), dividing the number line into 4 quarters (Strategy 3), and combining two or more of the strategies above (Strategy 4). Among these four strategies, the first two strategies are the simplest and the most efficient strategies, because dividing a 0-7000 number line into 2 or 7 identical units is easier than diving it into 4 units, thus requiring fewer cognitive resources. Results showed that 71.9% of Chinese participants used Strategy 1 and Strategy 2 (n = 33 for Strategy 1, and n = 18 for Strategy 2), whereas 55.8% of English participants used these two simpler strategies (n = 23 for Strategy 1, and n = 16 for Strategy 2). The second most popular strategy among English speakers was Strategy 4. 31.4% of English speakers combined multiple strategies when completing the 0-7000 number line task (n = 22), whereas only 9.9% of Chinese speakers reported doing so (n = 7). The use of more complex strategies might be the reason for the relationship between quantitative skill and number line performance in English participants. To summarize the results of multiple regressions and this simple slope analysis, Figure 5 depicts the relations between the predictors and outcome variables. Finally, to better represent the relations between results and hypotheses, Table 12 presents a summary of hypotheses and whether these hypotheses were supported by the results of data analyses.

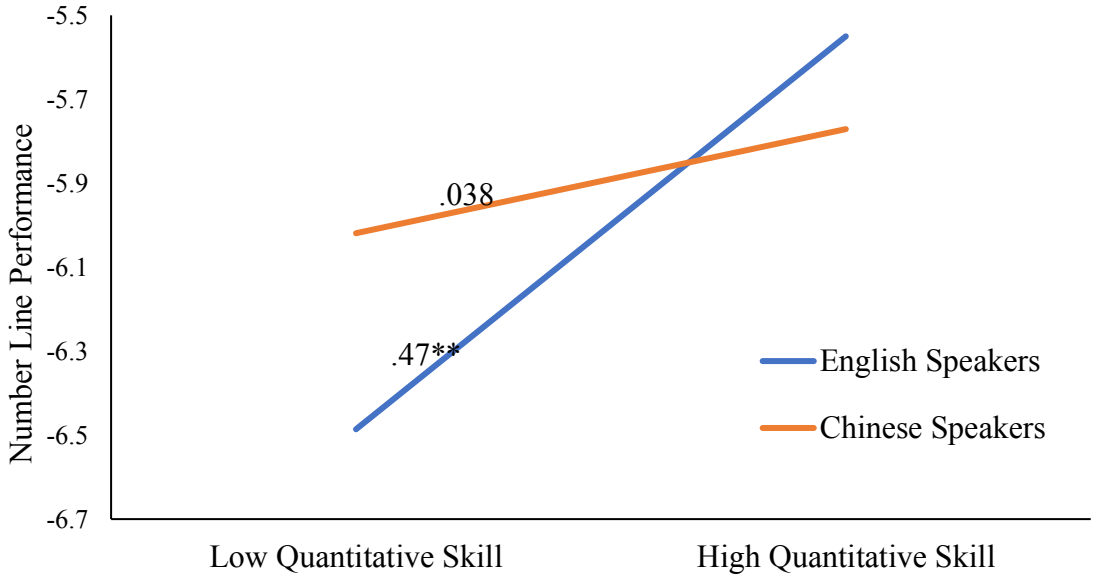

Figure 4. Relations between quantitative skill and number line performance for English speakers and Chinese speakers. Low quantitative skill indicates performance that was 1 *SD* below the mean, and high quantitative skill indicates 1 *SD* above the mean.

Table 8
Multiple Regression Analyses Predicting Calculation Performance

Block					
	β	*b*	*SE*	R^2	R^2 change
Block 1:					
Culture	.56***	1.12***	.11		
				.36	.36
Block 2:					
Linguistic	.05	.05	.08		
Quantitative	.32**	.32	.09		
Working Memory	.34***	.34	.09		
Spatial	.20**	.01	.002		
				.60	.24
Block 3: Interactions					
Linguistic × Culture	-.08	-.12	.12		
Quantitative × culture	-.13	-.16	.12		
Working memory × culture	-.09	-.13	.13		
Spatial × culture	-.06	-.11	.14		
				.62	.02
Total R^2				.62	

Note. *$p < .05$. **$p < .01$. ***$p < .001$.

Table 9
Multiple Regression Analyses Predicting Number Line Performance

Block	β	b	SE	R^2	R^2 change
Block 1: Controlled variable					
Culture	.13	.78	.48		
				.03	.03
Block 2: Independent variables					
Linguistic	.30**	.94**	.08		
Quantitative	.39**	1.21**	.09		
Working Memory	.12	.34	.36		
Spatial	.21*	.01*	.02		
				.24	.21
Block 3: Interactions					
Linguistic × Culture	-.09	-.40	.49		
Quantitative × culture	-.32*	-1.25*	.51		
Working memory × culture	.05	.21	.55		
Spatial × culture	-.05	-.31	.59		
				.28	.04
Total R^2				.28	

Note. *$p < .05$. **$p < .01$.

Table 10
Multiple Regression Analyses Predicting Word Problem Performance

Block	β	b	SE	R^2	R^2 change
Block 1: Controlled variable					
Culture	.33***	1.61***	.33		
				.14	.14
Block 2: Independent variables					
Linguistic	.32**	.77**	.23		
Quantitative	.21†	.52†	.27		
Working Memory	.27*	.66*	.26		
Spatial	.18†	.01†	.01		
				.45	.31
Block 3: Interactions					
Linguistic × Culture	-.06	-.21	.33		
Quantitative × culture	-.04	-.12	.35		
Working memory × culture	-.09	-.33	.38		
Spatial × culture	.04	.19	.41		
				.46	.01
Total R^2				.46	

Note. *$p < .05$. **$p < .01$. ***$p < .001$. †$p = .06$.

Table 11
Multiple Regression Analyses Predicting Reading outcome for English Speakers (n = 71)

	β	*b*	*SE*
Independent variables			
Linguistic	.45***	.43***	.10
Quantitative	.04	.05	.12
Working Memory	.19	.19	.12
Spatial	.17	.004	.003
R^2	.38		

Note. $^{**}p < .01$. $^{***}p < .001$.

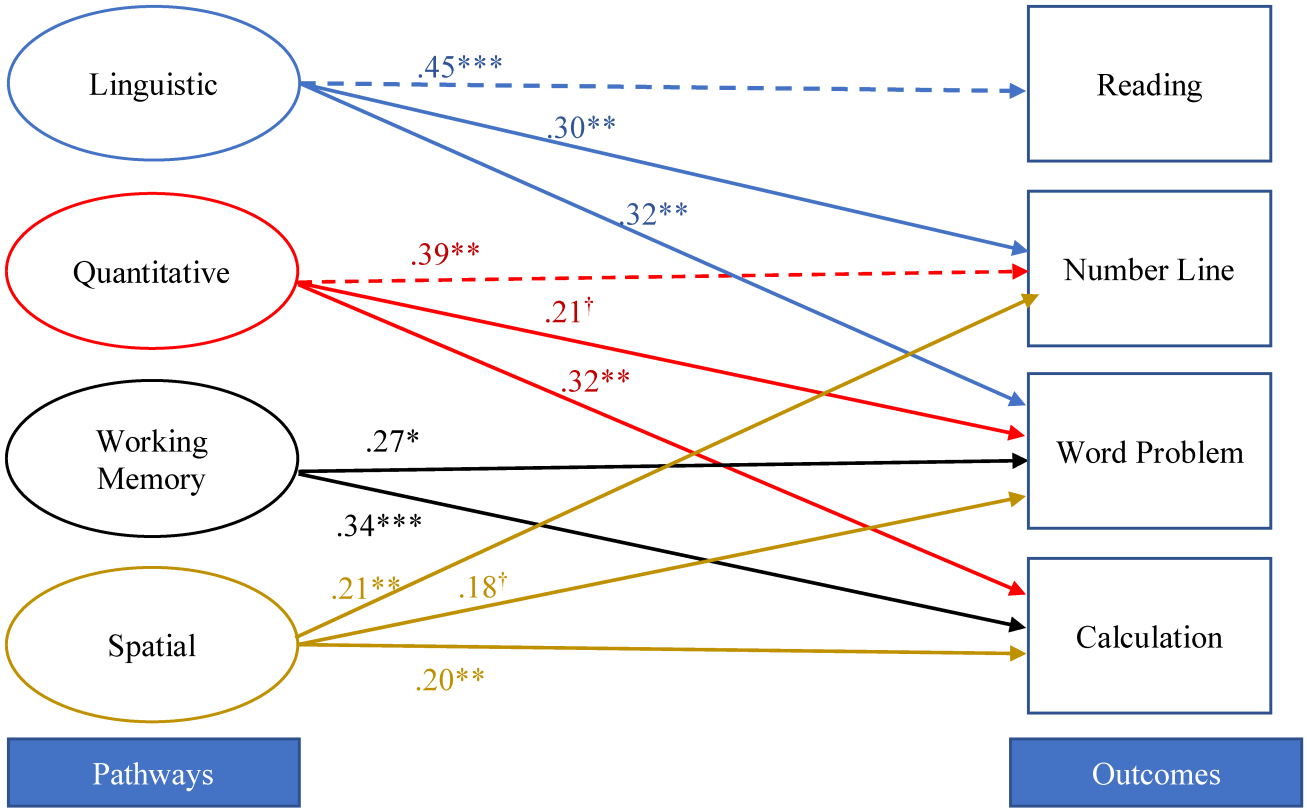

Figure 5. Relations between four cognitive predictors and various outcomes (N = 142). Arrows indicate contributions of cognitive precursors towards mathematical outcomes. Dotted arrows indicate relations for English speakers only. Labeled values are standardized coefficients (β). $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$. $^{\dagger}p = .06$.

Discussion

The *Pathways to Mathematics* model is a comprehensive model examining relations among cognitive abilities, early numeracy knowledge and mathematical outcomes in children. The current study built on and expanded the *Pathways to Mathematics* model among adults and across two cultures. More specifically, this study investigated four cognitive predictors, linguistic ability, quantitative knowledge, working memory, and spatial skill, and assessed the unique contributions of these predictors towards mathematical outcomes in English-speaking and Chinese-speaking adults.

Results indicated that similar to children, in adults from different cultures, linguistic, quantitative, working memory, and spatial pathways also vary in their contributions to mathematical performance, depending on the specific demands of tasks. However, outcome tasks have different cognitive demands for adults compared to children, and also vary from one culture to another. Accordingly, the patterns of relationships between predictors and outcomes were somewhat different for adults than has been previously observed for children. See Table 12 for a summary of hypotheses supported and not supported.

Table 12
Summary of Support of Hypotheses

	Hypotheses	Support of Hypotheses
Linguistic	predicts reading	√
	predicts word problems	√
	does not predict calculation	√
	does not predict number line	X
Quantitative	does not predict reading	√
	predicts word problems	√
	predicts calculation	√

	does not predict number line in English speakers, but predicts number line in Chinese speakers	X
Spatial	does not predict reading	√
	predicts word problems	√
	predicts calculation	√
	predicts number line	√
Working memory	does not predict reading	√
	predicts word problems	√
	predicts calculation	√
	predicts number line	X

Note. √ indicates that a hypothesis is supported. X indicates that a hypothesis is not supported.

Results for Children versus Adults

Consistent with the hypothesis that the contribution of linguistic pathway towards mathematical outcome measures differ in children and adults, in this study, linguistic pathway did not predict adult participants' performance on the calculation tasks. This finding supported the hypothesis that number recognition is an automatic process for adults. Linguistic ability was not related to adults' number system knowledge, and did not predict participants' performance on math outcome measures that did not have a non-numerical language component. In contrast, for outcome measures that do have a language component, such as word problems, linguistic ability was a significant predictor. Furthermore, perhaps not surprisingly, linguistic ability also significantly predicted English-speaking participants' reading outcomes.

However, contrary to my hypotheses, but consistent with previous findings for children in the original *Pathways to Mathematics* model (LeFevre et al., 2010), linguistic ability predicted adults' performance on the number line task. Because the number line task adopted in this study used number lines ranging from 0 to 7000, most of the numbers within this range have long

number names. Participants needed to mentally decompose the written numbers into verbal forms and hold the verbal information using phonological working memory while completing the task. Moreover, if a participant was using mental calculation to solve a number line problem, digit names would be transformed back and forth from symbols to sounds. This transformation between symbols and sounds is possibly why linguistic ability predicted participants' performance on this 0-7000 number line task.

For the quantitative pathway, consistent with my hypotheses, quantitative ability predicted adults' performance on word problems and calculation tasks, because both outcome measures require quantitative processes.

As hypothesized, working memory predicted adults' performance on calculation tasks and word problems. However, it did not predict participants' performance on the number line task. It is possible that adults do not require a lot of working memory on number line task because this task do not require mentally holding a large amount of information, as the number line task is not complex enough to require great devotion of working memory, as compared to calculation and word problems.

Finally, consistent with hypotheses, spatial skill predicted adults' performance on all math tasks. Past research has consistently found the link between spatial ability and math achievements, and previous studies proposed several reasons for this link from different aspects. Neurological evidence showed that spatial and mathematical abilities rely on partly overlapping neural networks (Hubbard, Piazza, Pinel, & Dehaene, 2005). In a twin study, Tosto et al. (2014) showed that genetic factors accounted for 60% of the relationship between spatial ability and mathematics, indicating genetic overlap between spatial and math abilities. Mix et al. (2016) found a great deal of overlapping variance in spatial and math abilities, and reasoned it was

possible that spatial ability and math performance are both associated with a higher-order, more general cognitive capacity, such as fluid intelligence.

Results for Culture

Overall, contributions of cognitive abilities towards math outcomes were very similar for Chinese and English speakers. For both groups, as hypothesized, linguistic ability predicted performance on word problems, due to the language component involved in this task. Moreover, quantitative skills predicted calculation and word problems for participants from both cultural groups, because both tasks require quantitative knowledge. The contribution of working memory was also the same for the two groups, and it predicted word problem and calculation outcomes. Finally, spatial predicted all math outcomes for both cultural groups.

However, opposite to what was hypothesized, quantitative ability did not predict Chinese participants' performance on the number line task, but predicted this outcome for English speakers. A closer examination of strategies reported by participants revealed that Chinese speakers generally adopted simpler and more efficient strategies (71.9%), while only 55.8% of English-speaking participants used these simpler strategies, and 31.4% of English speakers chose to combine multiple strategies together when solving number line problems, which led to the use of more quantitative skills. These individual differences in strategy choices are possibly why quantitative knowledge significantly predicted the number line performance of English but not Chinese participants. The simple slope analysis also demonstrated that English speakers with better quantitative knowledge performed better on the number line task than those with lower quantitative ability. This indicated that the requirement of cognitive skills of a specific task depended on the individual differences in choosing strategies. Strategy choices may be affected by an individual's familiarity and fluency with numbers. For example, on a 0-7000 number line

task, a person with more experience with numbers and calculations might immediately know what numbers 7000 are divisible by, and thus choosing a more efficient strategy, while a person with less experience with math might use less efficient strategies.

Summary

This study elaborated the *Pathways to Mathematics* model and examined the contributions of cognitive abilities to mathematical outcomes in adults across different cultures. There were both similarities and differences between children and adults, and between English-speaking and Chinese-speaking adults, in terms of amounts of cognitive requirements and levels of cognitive depletion when solving math problems. These differences between children and adults might reflect development of familiarity with number system, as well as development of other more general abilities, such as processing speed. Differences between participants from two cultural groups can be caused by strategy choices or different levels of familiarity with numbers and calculation fluency. Overall, the fact that the relations between cognitive predictors and outcome measures are similar across the two cultural groups indicate that even though there can be individual differences in the performance of cognitive and math tasks, such as magnitude comparison task and calculation task, the underlying relations among cognitive predictors and outcome tasks are generally universal. That is to say, the manifestation of better or worse performance on a math task is not due to differences in the amount or types of cognitive contributions. Furthermore, this study included an additional pathway, the spatial ability pathway, and showed the contributions of spatial ability to math outcome measures. Therefore, one implication of this study can be to inform literature and provide support for future research designs. Furthermore, examining developmental differences in the use of cognitive resources can

potentially provide directions for math education and future intervention studies, because knowing the destination is half the journey!

Limitations and Future Directions

This study showed that linguistic ability was not related to adults' calculation fluency. One possible reason is, as has been discussed above, that adults have reached automatized number naming, and linguistic ability is no longer related to adults' number system knowledge. Another reason can also be that adults have developed a much faster processing speed, so that transformation from number symbols to number names happens much faster. Future research can include a measure of processing speed to investigate the latter possibility.

Although this study demonstrated relationships between cognitive abilities and outcome measures, the results do not establish causality, nor was it the focus of this study. For example, it is not clear if better spatial ability causes better math performance, or if better math skills makes people improve spatial ability. Similarly, the relationships between cognitive abilities and outcome tasks showed by this study do not reveal more general, higher-order potential latent abilities that might affect these relationships, such as intelligence. Future research can use intervention designs to establish causal relationships between cognitive abilities and math outcome tasks, or to investigate latent abilities that affect both cognitive and math abilities, so that math education can be more effective and beneficial.

Finally, future studies can also examine this model in children and adults from other cultures, to investigate the contributions of cognitive precursors to mathematical outcomes. In addition, future research can also use this framework to assess relations between cognitive predictors and academic achievements in special populations of children and adults.

Conclusion

Building on *Pathways to Mathematics* Model (LeFevre et al., 2010), this study developed a framework that assessed the contributions of cognitive abilities to mathematical outcomes in adults across different cultures. This study examined four cognitive predictors, linguistic skill, quantitative knowledge, working memory, and spatial ability, and showed that these predictors differently predicted adults' performance on a number of math outcome tasks, depending on the specific cognitive requirements of a task. Combined with the *Pathways to Mathematics* model in children, the results of this study demonstrated differences in the amount and types of cognitive contributions towards math outcomes in children and in adults, which reflected developmental change in underlying cognitive processes when solving math problems. This study also showed that math development is componential in both children and adults, and across cultures, which further added weight of evidence to the *Pathways to Mathematics* model as a well-developed framework of organizing cognitive abilities and linking cognitive precursors to mathematical learning and development.

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