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Design and Construction of a Variable Stiffness Prosthetic Arm Prototype

by

Megan C. McTavish, B.Eng.

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Applied Science

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

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Carleton University
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**DESIGN AND CONSTRUCTION OF A VARIABLE STIFFNESS
PROSTHETIC ARM PROTOTYPE**

submitted by

Megan C. McTavish, B.Eng.

in partial fulfillment of the requirements for
the degree of

Master of Applied Science

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Department of Mechanical and Aerospace Engineering

Professor D.L. Russell, Thesis Supervisor

Carleton University
May 4, 2005

Abstract

A variable stiffness elbow prosthesis should look and feel more natural and allow the amputee to perform constrained or contact movements with the arm at a low stiffness, and fast, precision movements at a high stiffness. Previous research suggests that it should simplify control issues, provide increased dexterity and greatly improved energy efficiency when interacting with the environment. Antagonistic nonlinear compliant actuators acting about the joint provides stiffness variation capabilities. A nonbackdrivable transmission allows external forces to be statically supported without using the motors.

In this work, a bench top prototype based on a combination of earlier research and numerous refinements was constructed. The configuration chosen for the prototype consists of two actuators each containing a dc motor and gearhead, a nonbackdrivable worm and worm gear set, pulleys and a nonlinear spring. A detailed design was completed and a prototype built. This prototype was tested and the results compared to simulations done in Matlab and to the design goals.

Results showed a good correspondence between simulation and prototype. The prototype was able to lift a 2 kg load through 135 degrees in 1.42 seconds and to vary its stiffness from 14 to 24 Nm/rad. Further refinements to the design are identified that will allow the construction of a wearable prototype.

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List of Symbols

Δ	deflection of arm from equilibrium, first used in Equation (2.1), p. 15..	[rad]
$\Phi(\delta)$	spring force, first used on p. 15	[N]
$\Psi(\delta)$	spring stiffness, first used in Equation (2.4), p. 16	[N/m]
Ψ_{max}	maximum spring stiffness, first used in Equation (3.66), p. 50.....	[N/m]
δ	spring deflection, first used on p. 15	[m]
δ_0	zero deflection of spring, first used on p. 40	[m]
δ_{lim}	limiting deflection of spring, first used on p. 40.....	[m]
δ_{max}	maximum spring deflection, first used on p. 42	[m]
δ_{min}	minimum spring deflection, first used on p. 42	[m]
λ	lead angle of worm gear tooth, first used in Equation (3.14), p. 29.....	[rad]
μ	coefficient of friction between gear teeth, first used on p. 29.	
ω_g	worm gear shaft angular velocity, first used on p. 77	[rpm]
ω_m	motor angular velocity, first used in Equation (3.5), p. 25.....	[rpm]
ω_w	worm shaft angular velocity, first used on p. 75	[rpm]
ω_{arm}	arm angular velocity, first used in Equation (3.5), p. 25	[rpm]

ω_{nl}	motor no load speed, first used in Equation (3.2), p. 24	[rpm]
ϕ	pressure angle of worm gear tooth, first used in Equation (3.14), p. 29 .	[rad]
$\rho(\delta)$	curvature of spring, first used in Equation (3.57), p. 44	[N/m ²]
τ_g	worm gear torque, first used in Equation (3.21), p. 29	[Nm]
τ_m	motor torque, first used in Equation (3.4), p. 25	[Nm]
τ_p	worm gear pulley torque, first used in Equation (3.22), p. 29	[Nm]
τ_s	power screw nut torque, first used in Equation (3.43), p. 37	[Nm]
τ_w	worm torque, first used in Equation (3.18), p. 29	[Nm]
τ_{arm}	arm torque, first used in Equation (2.3), p. 16	[Nm]
τ_{full}	torque causing springs to reach full limit, first used in Equation (3.52), p. 42	[Nm]
τ_{slack}	torque causing springs to go slack, first used in Equation (3.51), p. 42 .	[Nm]
τ_{stall}	motor stall torque, first used on p. 24	[Nm]
τ_{th}	motor thermal torque, first used in Equation (3.6), p. 25	[mNm]
θ_{arm}	arm range of motion, first used on p. 23	[rad]
b	linear damping coefficient, first used on p. 100	[Ns/m]
e	efficiency, first used in Equation (3.4), p. 25.	
e_i	bond graph effort variable, i is bond number, first used on p. 96.	
F_2	force applied to arm pulley, first used on p. 68	[N]
F_d	dynamic load, first used in Equation (3.32), p. 31	[lb]
F_f	worm/worm gear friction force, first used in Equation (3.14), p. 29	[N]
f_i	bond graph flow variable, i is bond number, first used on p. 96.	

F_s	strength capacity, first used in Equation (3.33), p. 31	[lb]
F_w	wear factor, first used in Equation (3.35), p. 33	[lb]
F_{ext}	external force applied to arm, first used on p. 42	[N]
F_{feq}	equilibrium friction force, first used on p. 112	[N]
F_{fk}	kinetic friction force, first used on p. 112	[N]
F_{fs}	static friction force, first used on p. 112	[N]
F_{ga}	worm gear axial force, first used in Equation (3.15), p. 29	[N]
F_{gr}	worm gear radial force, first used in Equation (3.16), p. 29	[N]
F_{gt}	worm gear tangential force, first used in Equation (3.14), p. 29	[N]
F_{load}	load applied to arm, first used on p. 68	[N]
F_{pt}	worm gear pulley tangential force, first used in Equation (3.23), p. 29 .	[N]
F_{sp}	cable/spring force, first used in Equation (3.42), p. 36	[N]
F_{wa}	worm axial force, first used in Equation (3.14), p. 29	[N]
F_{wr}	worm radial force, first used in Equation (3.16), p. 29	[N]
F_{wt}	worm tangential force, first used in Equation (3.15), p. 29	[N]
g	motor speed/torque gradient, first used in Equation (3.6), p. 25	[rpm/mNm]
G_T	total gear ratio, first used in Equation (3.4), p. 25.	
J_a	rotational inertia of arm, first used on p. 100	[kg m ²]
J_m	rotational inertia of motor, first used on p. 97	[kgm ²]
J_p	rotational inertia of worm gear pulley, first used on p. 99	[kg m ²]
J_w	rotational inertia of worm, first used on p. 99	[kg m ²]

J_{gh}	rotational inertia of gearhead, first used on p. 99.....	[kg m ²]
J_{wg}	rotational inertia of worm gear, first used on p. 99.....	[kg m ²]
K	effective joint stiffness, first used in Equation (2.5), p. 16.....	[Nm/rad]
K_v	motor speed constant, first used in Equation (3.2), p. 24.....	[rpm/V]
K_τ	motor torque constant, first used in Equation (3.3), p. 25.....	[mNm/A]
K_{max}	maximum arm stiffness, first used on p. 23.....	[Nm/rad]
K_{min}	minimum arm stiffness, first used on p. 23.....	[Nm/rad]
L	lead of power screw, first used in Equation (3.41), p. 36.....	[in]
L_a	motor inductance, first used on p. 97.....	[H]
L_{arm}	arm length, first used on p. 23.....	[m]
l_{spring}	spring length, first used in Equation (3.70), p. 51.....	[m]
m_{arm}	arm mass, first used on p. 23.....	[kg]
N	worm/worm gear normal force, first used in Equation (3.14), p. 29....	[N]
N_g	number of worm gear teeth, first used in Equation (3.13), p. 28.	
p_i	bond graph momentum variable, i is bond number, first used on p. 96.	
P_s	shear load, first used in Equation (3.78), p. 70.....	[N]
P_t	tensile load, first used in Equation (3.78), p. 70.....	[N]
P_m	motor power, first used in Equation (3.1), p. 24.....	[W]
q_i	bond graph displacement variable, i is bond number, first used on p. 96.	
R	effective elbow radius, first used in Equation (2.1), p. 15.....	[m]
R_a	motor resistance, first used on p. 97.....	[Ω]

τ_p	worm gear pulley radius, first used in Equation (3.13), p. 28	[m]
τ_s	spur gear ratio, first used in Equation (3.41), p. 36.	
τ_w	worm radius, first used in Equation (3.18), p. 29	[m]
τ_{gh}	gearhead ratio, first used in Equation (3.13), p. 28.	
τ_{wg}	worm gear radius, first used in Equation (3.21), p. 29	[m]
t_{flex}	arm flex time, first used on p. 23	[s]
t_{on}	time to thermal overload, first used in Equation (3.10), p. 26	[s]
V	motor voltage, first used in Equation (3.2), p. 24	[V]
V_g	worm gear tangential velocity, first used in Equation (3.25), p. 30	[m/s]
V_s	worm/worm gear sliding velocity, first used in Equation (3.26), p. 30	[m/s]
V_w	worm tangential velocity, first used in Equation (3.24), p. 30	[m/s]
x_k	motion of inputs corresponding to stiffness, first used in Equation (2.1), p. 15	[m]

Chapter 1

Introduction

The first recognizable prosthetic devices were made in Egypt, Rome and Greece. In 218 BC Marcus Serigus, a Roman General, lost his hand while fighting in the Punic War and had an iron hand designed to enable him to continue fighting [1]. Remains of prosthetic limbs have also been found in Egyptian mummies and prostheses are mentioned in some Greek mythology [1]. Through the Dark Ages prostheses were constructed mainly to help in battle and to cover up deformities. As science developed, so did the prosthetic devices. A lack of technology and demand, however, led to very little progress in the field.

The aftermath of the American Civil War and the industrial revolution allowed significant advancements in prosthetic design. Government funding was available to supply limbs for veterans. A relatively large number of British amputees in World War I shifted the advancements to European prosthetics. After World War II, demand in the U.S. had increased greatly. In response, the Americans set up the American Orthotics and Prosthetics Association [1].

The first prostheses after the war were made of wood with a rubber socket for the stump and were very uncomfortable. Most of the attention was focused on the lower extremity prostheses as two legs are necessary for mobility. In the late 1950s and early 1960s functional prostheses were introduced. All of the early models were body powered and, in the case of upper extremities, used shoulder movements to control the elbow and hand with a steel cable. Due to the unnatural movements involved, some found them difficult to use. The possibility of externally powered prostheses was researched with little success for many years. Finally, in the late 1960s significant progress in this area was made. Re-

searchers arrived at four main methods of controlling these prostheses, the most advanced being myoelectric control. Others are touch pads, switches and servo-control, which is a small cable operated device mounted to a harness [2]. The early models were quite bulky, heavy and noisy but this has been greatly improved on over the years [3].

Myoelectric prostheses are used for upper extremities more often than lower extremities because the improvements over conventional prostheses in this area are more evident. Myoelectric control operates hands, wrists and elbows driven by electric motors. Surface electrodes are used to measure the muscle action potentials and amplify them to turn on the motor. Multiple functions can be controlled with improved function as compared with conventional prostheses. The success rate of the myoelectric prosthetic arm/hand is still relatively low as 50% discontinue use after 2 years [4]. Much work is left in this field in order to improve this rather low rate of success. At present, the main objective in the field of myoelectrics is to improve the control aspect in order to make the prosthetics more user-friendly.

The current leaders in myoelectrically controlled prosthetic arms are, the Boston Elbow by Liberating Technologies [2] and the Utah Arm 3 by Motion Control [5]. The Boston Elbow is the stronger of the two arms and can lift 9 lbs with a 14 in forearm, giving a torque of over 10 ft-lbs. The Utah Arm 3 can only lift 1 kg, or 2.2 lbs. The no load speed of both arms is 1.2 s for flexion against gravity with a 1.2 lb hand. This speed slows to about 4 s with the full load applied. Both arms can support 50 lbs statically when locked at 90°. The weights are similar with the Boston Arm weighing in at 2 lbs 4 oz and the Utah Arm 3 weighing 2 lbs 2 oz. The range of motion is 135°.

The arms are operated by 12 V batteries, 1100 mAh for the Boston Elbow and 900 mAh for the Utah Arm. These batteries take between 1.5 and 2.5 hours to recharge. The 1100 mAh battery is limited at 13 A. When the Boston Elbow is lifting its full load of 10 ft-lbs, it only completes 4-5 cycles before it needs recharging. Regular use is at approximately 1.5 A, giving around 1 hour of active use before charging is necessary. For an active person, this is not sufficient for a full day.

Current prostheses are rigid in design and use a backdrivable transmission. A force applied to the arm will be absorbed in the motor(s) if the transmission is backdrivable. Therefore, to support a static external force a clutch or pin is used so that the motor cannot backdrive. An impact load in this case will either break the pin or clutch and the motor will backdrive, absorbing the load, or the stump will take the force. A backdriving motor causes a loud squealing noise if the applied force is large enough. Neither of these outcomes are desirable to the user.

Adding variable stiffness will allow this shock to be absorbed in the springs, making it more comfortable for the wearer. It has been shown [6] that there can be efficiency gains of up to 20 times by using a low stiffness during constrained movements. If the amputee, for example, opens a drawer, it is more efficient to apply the forces in the direction where motion is not allowed causing no work to be done by these forces. Also, a high stiffness is advantageous for quick or precision movements. Compliant actuators will be discussed in detail in the “Literature Review” section. The proposed prosthetic prototype will incorporate a variable stiffness in the design.

Nonbackdrivable transmissions, such as worm gear sets and power screws are very inefficient due to the large friction forces between the teeth. However, this inefficiency can be justified when the energy savings of nonbackdrivable versus backdrivable transmissions are studied [7]. A nonbackdrivable transmission is shown to be more efficient if the time spent statically supporting a load is at least twice the time spent lifting loads. The transmission will support static loads without having to engage a clutch mechanism. A nonbackdrivable transmission together with a nonlinear spring results in limb movement that appears natural. A nonbackdrivable transmission is used in this design.

1.1 Literature Review

Compliant actuators are systems that include flexible members which drive a mechanism. Compliance is the inverse of stiffness and together with damping and inertia makes up the mechanical impedance of a system. There are two main methods of implementing compliance, or impedance control. The first is the active method, using feedback control

and no mechanical elements [8, 9, 10]. The second is known as passive and uses mechanical parts for the stiffness, damping and inertia of the system. The literature review will focus on the second method.

Compliant actuators are found in the fields of robotics, prosthetics and orthotics. There are a number of scenarios in which the use of compliant actuators is advantageous. In the robot industry, robots are generally used in hot, hazardous or repetitive environments where heavy lifting is involved [11]. These robots are rigid and heavy which makes them unreasonable for new applications. Stiff robots are not safe for interaction with human beings due to the large, strong actuators used and the instabilities that can occur upon collision or failure of a component [12]. Compliance leads to less serious collisions because at low stiffness settings where energy can be absorbed in the springs, the mechanism will not be forced into a position against an external resistance [13]. It is also important to have low stiffness settings when handling delicate or easily damaged objects such as food. Using robots in the food industry will become feasible with these new soft grasps [14]. Grasp stability is improved with a high stiffness used for a better grip [15]. Also, when a robot is working in unstructured environments compliance allows it to have controlled contact and interaction with rigid objects [16, 17]. This is true for robots walking outdoors or up stairs when unexpected collisions can occur. "The speed, apparent lightness and ease of natural movements are so far unattained by conventional robotics" [13]. With elasticity, smooth rapid movements are possible. Without it robots are heavy and clumsy [13]. Finally, a low stiffness prevents undesirable transmission of shock and vibration to the support structures [15].

In the case of prostheses and orthoses, size, weight and energy supplies are limited. Compliant actuators provide increased energy efficiency [18, 6, 19, 20] as well as reduced weight and cost [21].

Despite the above reasoning for compliant robots, due to lack of actuators with good performance (like skeletal muscle) and a lack of adequate controller architectures and computer techniques needed to take advantage of such actuators, progress is slow [22].

The areas in which compliance is taken advantage of can be broken down into a few general subjects. The first is biorobotics; the interface between biology and robotics. These robots are either physical models of an animal used to study some aspect of biology, or are designed based on animals to perform a specified task (biomimetics) [23]. The second is the field of personal robots, which are slowly becoming a reality. In this area, it is notable that there are strict requirements for human-robot interaction. Safety is the key aspect in designing a robot which will most likely be an aid to the elderly and those with disabilities. Low levels of stiffness are an important feature for safe contact. In addition, accuracy and control must also be precise [10]. A third area of use is in walking robots. When walking in rough terrain or up stairs, they could have more dynamic mobility if animal like mechanisms (i.e. muscles) are included in the robot's design [24]. The final category in robotics is the humanoid. It has long been a dream to produce a robot that can think, move and act like a human. These robots should look, walk, and reach like humans. It is, therefore, imperative to include muscle-like compliant actuators in order for them to move as naturally as possible [25, 26]. It has been found that the human body is more stable for small oscillations if the body is more elastic [27]. This would also be true for the humanoid robot. In orthotics, compliance is used in an arm support for assisting arm movements. Mechanical impedance couples with impedance of the human to form a combined impedance. This is useful for people with neuro-muscular diseases that cause loss of strength (e.g. muscular dystrophy, multiple sclerosis), people with tremors (e.g. Parkinson's, MS, cerebral palsy) or people in rehabilitation following strokes [28]. Prosthetic design has generally not incorporated compliance extensively because of control challenges as well as space limitations. The few that do, however, have the goals of energy efficiency, low weight and more natural movements [29, 30].

Compliance can be incorporated into actuators in a number of ways. The two central methods are with springs and pneumatic actuators, both in series and in parallel. There are many designs using artificial muscles (called McKibben muscles, rubbertuators or Soft Arms) which are pneumatically driven [14]. Other methods used are hydraulic cylinders, fluid-filled bladders, pneumatic bladders, shape memory alloys [31] and ultrasonic motors [32].

1.1.1 Antagonist Compliant Actuators

Antagonistic actuators act about a joint where one actuator acts as antagonistic muscle and the other the agonistic muscle, much like in humans. This design is biologically inspired (e.g. biceps and triceps) and has an increased likeness to humans. In practice, this has meant better stiffness control.

Examples of these are found extensively in the previously mentioned areas of robotics in arms, legs and fingers as well as in a limited number of prosthetic designs [12, 14, 20, 13, 29, 30, 18, 33]. The main actuators used are springs and pneumatic artificial muscles, the most prominent being the pneumatic actuators. The design of the Variable Stiffness Upper Arm Prosthesis for this thesis uses antagonistically arranged, nonlinear springs as its compliant actuators. Four areas of research were found to be very relevant this thesis and they will be explored in detail.

1.1.2 Specific Cases

Antagonist actuators for a forearm prosthesis

Koganezawa et al have done extensive research on using antagonistic muscle type actuators for use in robotics. Koganezawa has also applied this phenomenon to prosthetic limbs [29]. His earlier design uses AMAs (antagonistic muscle-like actuators) as an approach to imitating a skeleto-muscular articulation system. It consists of 2 nonlinear conical springs, one DC motor which drives ball-screws and a pulley for the joint. Co-activation between the two nonlinear springs provides stiffness adjustment at the joint. For simplicity it is assumed that the springs are identical. Advantages of the antagonistic design include a wide-range of joint stiffness regulation as well as reduction of backlash, an increased range of motion, wear compensation and the option of positioning the actuators remotely.

The example application he chose is a forearm prosthesis with 6 d.o.f., 2 for the wrist and 4 for the fingers. It is driven by one DC motor and one stepping motor. The stiffness is adjusted before loading the arm with an external force, which gives fast response times. Two potentiometers are mounted at the universal joint of the wrist to sense joint position.

According to biological studies, muscles are nonlinear with the relationship between force and displacement being $F = kx^a$ [29]. Studies suggest that stiffness is approximately proportional to the square root of the force, therefore $a = 2$. The joint stiffness increases as the initial stretch length of the spring increases when $a > 1$. This is opposite when $a < 1$. If the spring is linear, $a = 1$, this property does not exist. Therefore, it is imperative that the springs are nonlinear.

Koganezawa concluded that simultaneous angular velocity feedback to both muscles, which loosely corresponds to the stretch reflex in humans, is a successful method of damping [29].

A new elastic element is proposed in a second paper by Koganezawa called a NonLinear Elastic Module (NLEM) which overcomes drawbacks of the previous model. These include the difficulty of designing specific nonlinear springs as well as the volume required to house the conical springs. NLEM is constructed with a torsion spring rolled around a guide rod which is aligned with the motor axis. A basic formula for controlling the stiffness and position of the end-point independently was derived and tested. It was found that the experimental results corresponded closely to theoretical results [30, 34].

A Bionic Robot Arm with Compliant Actuators

"A Bionic Robot Arm with Compliant Actuators" by Bernhard Moehl [13] describes a robot arm driven by electric motors. It has spiral springs and tendon-like wires antagonistically arranged around the shoulder and elbow joints for two arm links. The DC motor for each link stretches or releases a pair of spiral springs, producing forces to move the arm. The position is determined by the equilibrium of the springs as well as externally applied forces. There are two optomechanical pulse encoders to sense the motor and joint pulley positions. The arm is braced, ensuring the beams do not bend so the position of the arm tip can be determined very precisely.

The main problem encountered in this design is oscillation. This is compensated for actively using the motors. Movement control is performed by a signal processor. It computes

signals of the angular velocity of the arm and angular position of the motor based on information from the pulse encoders and calculates the appropriate values for motor positions. To prevent oscillations the final position, P_m , is temporarily replaced with an intermediate value, $\dot{P}_m$. $\dot{P}_m$ depends on the angular velocity of the arm and approaches P_m as the arm movement slows down:

$$\dot{P}_m = P_m - \text{const}V_a$$

where V_a is the angular velocity of the arm and *const* is a constant. Experimental results show that the oscillation damping is extremely effective, even when two links are attached together [13].

The above principle can only be applied if the load applied to the arm is constant so that the final position of the arm can be achieved with feedforward control. If a variety of loads with different weights require lifting, learning algorithms can be used in order to control each situation.

There are many advantages to this design. It is 'position tolerant', which means a certain arm position is not forced against an external resistance in the event of a collision resulting in less serious consequences. A collision can be sensed because the arm stops and motor is still going. Also, because the arm can be braced with wires to counteract bending forces, and the arm beams are subject to compressive stress along longitudinal axis only, it is lightweight yet strong. The bracing also ensures that there is no play in the joints and the arm position is very accurate.

CARL: A compliant articulated robot leg for dynamic locomotion

"CARL: A compliant articulate robot leg for dynamic locomotion" by G. Mennitto and M. Buehler is an autonomous robot quadruped designed for dynamic walking, trotting and bounding [18]. It includes an upper leg, lower leg and foot, all with revolute joints. Since it is intended for indoor use non-polluting battery based, electric actuation was chosen.

AnTagonistic LADD Actuation System (ATLAS) is used. ATLAS is an actuator system where a single electric motor drives an antagonistic pair of CLADDs, which are kinemati-

cally non-linear springs over a variable radius pulley. Essentially, it works like two muscles driving a joint. C/LADD transmissions are Linear-to-Angular Displacement Devices. They are specially designed springs with high strength, low weight and high efficiency.

Up to 50% of the energy used in a stiff legged robot can be saved by adding leg compliance [18]. Much of the movement required for walking is cyclic oscillatory, and this can be provided by the passive oscillation of the body's spring-mass system. The tendons, muscles and cartilage in animals store energy to provide this passive oscillation. Some robots have been designed with a single spring but the antagonistic structure suggested here would fully exploit this animal feature of dynamic walking.

Biped Walking Robot with Antagonistic Driven Joints Using Nonlinear Spring Mechanism

Research by Yamaguchi et al is being done with a goal of a biped humanoid robot with the most natural gait possible [33, 19]. Two reasons for this study are human science and the development of humanoid robots. The results from human science research can be used in engineering, science, medicine, rehabilitation engineering and sports science. Development of humanoids as personal robots gives the advantage of being able to adapt more easily to its working environment, a human's house or apartment.

The walking behaviour exhibited by humans is very efficient and can easily deal with uneven ground and obstacles. The stiffness of human joints can be varied by the antagonistic muscle groups, which have a nonlinear spring characteristic. The robot, WL-13, built by Yamaguchi et al takes this into account and a novel rotary type nonlinear spring mechanism made up of pulleys and linear springs is used to simulate muscles. Two AC servo motors with harmonic gears are fixed on the upper part of the joint drive of the mechanism and it is controlled by an Intel Pentium 133MHz. Kevlar tape is used for wires. The spring coefficient ranges from zero to infinity ($k_{wire} = \infty$) and sensors can be installed to measure the angle of the rotation in the rotation part, which determines the tension.

The software to antagonistically drive the joints consists of two subroutines applied to two motors controlling the joint. The first controls joint angle by feedback control and the

other controls the tension of the driving wire. Information from the sensors is used by both subroutines [33].

A second design, WL-14 [19], improves upon the first one by varying the stiffness of joints actively. This enables dynamic walking with less energy consumption. In order for a bipedal humanoid robot to be practical it must be self contained with no external connection, hence battery operated, which makes energy efficiency of utmost importance.

The energy consumption for the new model was improved by 25 percent during the swing phase of the walking gait. The subroutines were the same as WL-13 but feedforward control to add the joint angular velocity references to the control input was added. In dynamic walking, the deviation of the joint angle becomes too large when there is only feedback control, which makes continuous walking challenging [19]. Experiments proved the effectiveness of this control scheme. Research continues to improve on the walking control methods while using the current antagonistic structure.

As shown in the above papers, there are many obstacles when using compliance in both robotics and prosthetics. In nature, however, there is an extremely large range of "softness". Understanding the advantages and disadvantages of compliant mechanisms versus stiff ones, as well as learning how to successfully control these machines will lead to enormous advances in biomechanics and biorobotics [17].

1.2 Background

This thesis is a continuation of work done by Russell, English and Burke [6, 31, 35, 7]. The key factors in the resulting design proposal are variable stiffness actuators and a nonbackdrivable transmission. The concept of variable stiffness was first introduced by Russell [6] in order to improve energy efficiency for constrained tasks. This was expanded on by English [35] who studied spring characteristics in detail and proved the need for nonlinear springs in the system. His conclusion was to use a quadratic spring. Burke's work focused on the dynamic behaviour of nonbackdrivable transmissions and he created a computer simulation of the system including detailed dynamics of the gears.

English proposed four configurations for possible prototypes. From these, Burke narrowed it down to Configuration 1 and Configuration 3.

Configuration 1 consists of two actuators arranged antagonistically around the elbow joint of the prosthesis. Each actuator consists of a motor, nonbackdrivable transmission and a nonlinear spring. The motors are run in opposite directions to change arm position and the same direction to vary the joint stiffness.

Configuration 3 also has two springs arranged antagonistically around the elbow joint. The difference is that one motor, located at the end of the springs changes the joint stiffness and another motor controls the arm position. Figure 1.1 shows the setup of both configurations.

This thesis will expand on the previous research to design, construct and test a bench top prototype.

Figure 1.1: Schematics of configurations 1 and 3

1.3 Thesis Outline

1.3.1 Chapter 2: Theory

In this section, a detailed analysis of nonlinear springs is conducted. The difference between hardening, softening and linear springs is explained and the resulting joint types examined.

1.3.2 Chapter 3: Detailed Design

A discussion on the method of choosing all parts and the resulting decisions are given in this chapter. The parts chosen include: motor, nonbackdrivable transmission, springs, miscellaneous components and electronics.

1.3.3 Chapter 4: Simulation

First, the dynamics of the system are examined and state equations developed for cases with and without friction. From these equations, a Matlab simulation is written. Three tasks are performed and results are given. These are compared with the tests performed on the actual prototype.

1.3.4 Chapter 5: Prototype Testing

The same three tasks are performed with the prototype. Results are presented. A discussion on the simulation results versus the test results for each task is presented. Other difficulties encountered are also explained.

1.3.5 Chapter 6: Conclusions and Recommendations

This chapter contains concluding remarks and future recommendations.

Chapter 2

Theory

2.1 Spring Theory

In general, springs can behave locally in three different ways. A hardening spring is a spring in which the stiffness, Ψ , (derivative of spring force, Φ , with respect to displacement, δ) increases with displacement, a softening spring has a stiffness which decreases with displacement and a linear spring has a stiffness that remains constant with displacement [36, 37, 35]. This means that the "hardness" of a spring is the second derivative of the spring force where a hardening spring has a positive value, softening a negative value and a linear spring has a second derivative equal to zero. A simple mass-spring system is stable if the spring stiffness is positive and unstable if the spring stiffness is negative. For multi-degree of freedom systems, more information is necessary to classify the stability of the system. These properties are illustrated in Figures 2.1 and 2.2.

2.1.1 Spring and Joint Stiffness

In the variable stiffness prosthetic elbow joint [36, 38], an antagonistic spring pair acts about a joint to simulate the bicep and tricep muscles. By moving the two springs in opposite directions the position of the joint will be altered and by moving the springs in the same direction the stiffness will change. To study the behaviour of the springs themselves, this analysis is limited to simply joint deflection as a result of applied external forces to the hand. In this case, movement will occur which will cause one spring to stretch and the other to compress. The spring deflections are given as:

$$\delta_1 = x_k - \Delta R \quad (2.1)$$

$$\delta_2 = x_k + \Delta R \quad (2.2)$$

where δ_1 and δ_2 are the deflections of the flexor and extensor springs, x_k is the spring stretch due to the stiffness setting, Δ is the deflection of the elbow joint due to the applied load and R is the radius of the joint. The individual actuator spring force is denoted by $\Phi(\delta)$. Using this, the joint torque is found as:

$$\tau_{arm} = R(\Phi_2(\delta_2) - \Phi_1(\delta_1)) \quad (2.3)$$

Figure 2.1: Hardening spring properties

Figure 2.2: Softening spring properties

The spring stiffness, $\Psi(\delta)$, is defined as the derivative of the spring force:

$$\Psi(\delta) = \frac{d\Phi(\delta)}{d\delta} \quad (2.4)$$

Also, the joint stiffness, K , is the derivative of the joint torque, τ_{arm} . Taking the derivative with respect to joint deflection, Δ :

$$K = \frac{d\tau_{arm}}{d\Delta} \quad (2.5)$$

Using the chain rule and Equations (2.1), (2.2) and (2.3) the resulting effective joint stiffness is defined as:

$$K = R^2(\Psi_1(\delta_1) + \Psi_2(\delta_2)) \quad (2.6)$$

When the load is applied, the joint could react in three different ways depending on the actuator springs chosen. It was found that to decouple the joint stiffness from deflection, creating a linear joint stiffness, a spring with a quadratic force function must be used [38]. However, it is also of interest to look at the possibilities of hardening joints (joint stiffness increases with displacement) and softening joints (joint stiffness decreases with displacement) [35, 36]. In order to achieve these various joint characteristics, the properties of the actuator springs must be studied. We will look at quadratic springs with linear spring stiffness curves, which we know from above to give us a joint stiffness which does not change with deflection, and non-quadratic springs with stiffness curves which are both concave up and concave down. This is done for both hardening and softening actuator springs.

A joint displacement, Δ , from the original position has been shown in Figure 2.3. The actuator spring deflections are found from equations 2.1 and 2.2 and are shown on the graph with x_k denoting the initial spring deflection. The initial joint stiffnesses are denoted as K_{Ao} , K_{Bo} , and K_{Co} . Initially, the two actuator springs (assumed identical) have the same stiffness and deflection, therefore, from Equation (2.6), each initial joint stiffness can be expressed as:

$$K_{Ao} = R^2 2\Psi_{Ao} \quad (2.7)$$

$$K_{Bo} = R^2 2\Psi_{Bo} \quad (2.8)$$

$$K_{Co} = R^2 2\Psi_{Co} \quad (2.9)$$

where Ψ_{Ao} , Ψ_{Bo} , and Ψ_{Co} are the initial actuator stiffnesses. Using Figure 2.3, each curve is analyzed according to resulting joint stiffness.

Figure 2.3: Force and stiffness curves for various hardening springs. Case A: linear stiffness curve, Case B: stiffness curve concave up, Case C: stiffness curve concave down

Case A: Linear stiffness curve

From Equation (2.6)

$$K_A = R^2(\Psi_{1A}(\delta_{1A}) + \Psi_{2A}(\delta_{2A})) \quad (2.10)$$

From Figure 2.3, $\Psi_{1A}(\delta_{1A})$ and $\Psi_{2A}(\delta_{2A})$ can be substituted as:

$$K_A = R^2((\Psi_{A0} - A^-) + (\Psi_{A0} + A^+)) \quad (2.11)$$

since $A^+ = A^-$;

$$K_A = R^2 2\Psi_{Ao} = K_{Ao} \quad (2.12)$$

This shows that the joint displacement caused no change in joint stiffness. This is due to the linear relationship of the spring stiffness (quadratic force-displacement relation) and agrees with previous research [38].

Case B: Stiffness curve concave up

$$K_B = R^2((\Psi_{Bo} - B^-) + (\Psi_{Bo} + B^+)) \quad (2.13)$$

where $B^+ > B^-$;

$$K_B = R^2(2\Psi_{Bo} + (B^+ - B^-)) > K_{Bo} \quad (2.14)$$

Therefore the joint stiffness increases with displacement. From the above definition, this is a hardening joint.

Case C: Stiffness curve concave down

$$K_C = R^2((\Psi_{Co} - C^-) + (\Psi_{Co} + C^+)) \quad (2.15)$$

where $C^+ < C^-$;

$$K_C = R^2(2\Psi_{Co} + (C^+ - C^-)) < K_{Co} \quad (2.16)$$

Here, the joint stiffness decreases with displacement and consequently the joint is softening [35].

The same reasoning can be used for the softening springs in Figure 2.4 and the same equations apply. It can be concluded that a hardening joint requires actuator stiffness curves that are concave up, a softening joint requires actuator stiffness curves that are concave down and a linear joint requires linear stiffness curves. Whether the actuator springs are hardening or softening is irrelevant to this result.

Figure 2.4: Force and stiffness curves for various softening springs. Case A: linear stiffness curve, Case B: stiffness curve concave up, Case C: stiffness curve concave down

2.1.2 Stability with respect to Spring and Joint Stiffness

Much research has been done on the effect of joint stiffness on the stability of the human arm [39, 40]. In the experiments done by McIntyre, results showed that "human subjects

must increase the stiffness at the joints in order to maintain limb stability in the presence of applied external forces at the hand" [39]. In other words, the joint must act as a hardening joint. Their goal was to find out ways that the central nervous system controls this stiffness and stability. He reports that "a local strategy may be used to achieve limb stability, in which the muscle stiffness increases with muscle force" [39]. This criteria fits the description of a stable hardening spring. From the above proof, however, we know that a hardening actuator spring can produce either a hardening or softening joint and a hardening joint can be made up of hardening or softening springs. This contradicts the above statement by McIntyre. It is the specific shape of the force and stiffness functions of the springs that will control the stability of the joint, i.e. create a hardening joint, not whether the springs themselves are hardening or softening. This is useful information when designing compliant actuators such as artificial joints as well as studying natural human joints.

Chapter 3

Detailed Design

3.1 Configuration

Figure 1.1 shows the two possible configurations for this prototype. Configuration 1 is similar to the natural elbow joint where one actuator acts as the bicep and the other as the tricep whereas Configuration 3 controls the stiffness and position with separate motors.

The advantage of Configuration 1 is that its natural setup would make a prototype that is very useful for future work on controls as well as testing different types of actuators. The disadvantage is that the motor has to support the spring force for the set stiffness while also lifting the applied load. Both motors need to be run to change stiffness as well as to change position. In the second case, one motor is doing all the lifting while the other is just releasing cable; this results in wasted energy.

Configuration 3 has one motor that controls stiffness and the other position. This means that the stiffness can be set and the stiffness motor turned off. The position motor can then lift external load without supporting spring stiffness at the same time. This saves energy as only one motor needs to run at a time and the stiffness is supported by a nonbackdrivable transmission. A larger weight could be lifted with same motor as in Configuration 1. The disadvantage is that it is not set up like a natural arm and is instead a very specific setup that would not work for different actuators such as artificial muscles. Therefore, controls cannot be investigated as thoroughly.

For the purposes of this prototype, it was determined that the first configuration would provide a more useful test device due to the more natural setup.

3.2 Basic Design

The major components in the variable stiffness prosthesis are the motors, nonbackdrivable transmission, cable, nonlinear springs, elbow pulley, power supply, sensors and circuitry. The basic design criteria for a wearable prosthesis are listed in the following table. The goal of this thesis is to make a bench top prototype for evaluation purposes. Therefore, all of the design goals may not be achieved at this point.

Item	Symbol	Units	Value
Maximum Torque	τ_{arm}	Nm	13.56
Flex Time:with 4kg load	t_{flex}	s	1.2
Maximum Stiffness	K_{max}	Nm/rad	80
Minimum Stiffness	K_{min}	Nm/rad	1
Mass	m_{arm}	kg	0.9
Actual range of joint motion	θ	°	135 (2.36 rad)
Length of forearm	L_{arm}	m	0.33

Table 3.1: Design criteria

These criteria is largely based on the current design of the two leading upper arm prostheses, namely, the Boston Elbow [2] and the Utah Arm 3 [5]. The Boston Elbow has the same maximum torque of 10 ft-lbs or 13.56 Nm but the flexion speed of 1.2 s is only achievable for a load of 1.2 lbs (the addition of a hand). With an arm weight of approximately 2 lbs or 0.9 kg, the lifting ability is about 9 lbs (4 kg). The forearm length is based on an average male's forearm.

3.3 Motors

The main criteria for the motors is to handle the required arm torque and speed. These values were determined by using specifications of existing prosthetic arms [5, 2]. The arm is required to exert torque in both directions. It is assumed that these torques are equal, creating a symmetric system. Therefore, the motors should be identical. Therefore only one side must be investigated in the design procedure. First, the steady state power required to move $\tau_{arm} = 13.56$ Nm through 135° in 1.2 s is calculated:

$$P_m = 13.56 \frac{(135\pi)}{180(1.2)} = 26.63 \text{ W} \quad (3.1)$$

In order for a safe design, the maximum power rating for the motor should be 1.5 to 2 times larger than the required power output [7] so the motor must have a power rating of at least 55 W.

The motor should have a nominal voltage of 12 V to be consistent with the current batteries used in prosthetic arms. Since the goal for a wearable prosthetic limb is 0.9 kg, mass is very important. Motors over 350 g will not be considered. Originally brushless motors were chosen based on their long life, quiet operation and low vibration characteristics [7]. It was found, however, that regular DC motors deliver more torque and draw less current.

Motor data were collected for each motor that fit the size and power constraints above from Maxon Motors [41] and MicroMo [42]. Due to some inconsistencies in the published motor data, the values for stall torque, τ_{stall} , and the torque constant, K_t , were used from the specifications and the no load speed, ω_{nl} and speed constants, K_v , were calculated.

$$\omega_{nl} = V K_v \quad (3.2)$$

where

$$K_v = \frac{30000}{\pi K_\tau} \quad (3.3)$$

with K_v (rpm/V) and K_τ (mNm/A). V is the voltage rating of the motor.

The total gear ratio, G_T , must be calculated in order to determine if the motor will meet the torque and speed requirements of the arm (τ_{arm} and ω_{arm}).

$$\tau_{arm} = G_T \tau_m e \quad (3.4)$$

where e is the efficiency of the transmission, and:

$$\omega_{arm} = \frac{\omega_m}{G_T} \quad (3.5)$$

where τ_m and ω_m are the motor torque and speed.

The desired speed that the arm should lift a 10 Nm through 136° is 1.2 s. To choose the motor we can estimate that it runs at the thermal torque, τ_{th} . Using the speed/torque gradient, g (rpm/mNm) we have:

$$\omega_m = \omega_{nl} - g \tau_{th} \quad (3.6)$$

The thermal torque is a safe operating torque for the motor. This is found by calculating the maximum power the motor can dissipate, P_{heat} . This is based on the operating and ambient temperatures, T_{op} and $T_{ambient}$, as well as the total thermal resistance from the rotor to the ambient surroundings, R_{R-A} :

$$P_{heat} = \frac{(T_{op} - T_{ambient})}{R_{R-A}} \quad (3.7)$$

The maximum current, i_{load} , is determined using P_{heat} , the no load current, i_{nl} , and the armature resistance, R_a :

$$i_{load} = \sqrt{\frac{P_{heat}}{R_a}} - i_{nl} \quad (3.8)$$

Finally, the thermal torque is calculated using the maximum current (or thermal current) and the given torque constant of the motor.

$$\tau_{th} = K_{\tau} i_{load} \quad (3.9)$$

Although this is the calculated safe operating torque of the motor, it can be run over this thermal torque for a period of time [41]. The following equation calculates the time to thermal overload, t_{on} , of a cold motor at 25° C running above the maximum continuous current, $i_{max,cont}$. A motor can run at $i_{max,cont}$ indefinitely without overheating.

$$t_{on} = t_{th} \ln \frac{K^2}{K^2 - 1} \quad (3.10)$$

where;

$$K = \frac{i_m}{i_{max,cont}} \sqrt{\frac{R_{th1}}{(R_{th1} + R_{th2})}} \quad (3.11)$$

$$= \frac{\tau_m}{\tau_{max,cont}} \sqrt{\frac{R_{th1}}{(R_{th1} + R_{th2})}} \quad (3.12)$$

where i_m is the operating current of the motor and τ_m is the motor torque. $\tau_{max,cont}$ is the maximum continuous torque rating of the motor. Note that in applying this equation $K > 1$ since $i_m > i_{max,cont}$. R_{th1} is the thermal resistance from the rotor to the housing and R_{th2} is from the housing to ambient. t_{th} is the thermal time constant of the winding.

Selection Process

The first step in the selection process was to calculate the gear ratio between the motor and the arm to give the desired flex time of 1.2 s for 135° using Equation (3.5). From this gear ratio, the arm torque at 100%, 50% and 25% efficiency is calculated using Equation (3.4) based on the assumption that the motor will run at the thermal torque (3.9). This is an important consideration since nonbackdrivable transmissions are known to have efficiency values of around 50% or less. The chosen motor should be capable of creating the desired arm torque at 50% efficiency.

	Maxon RE35(15V)	Maxon RE30	MicroMo 3557	MicroMo 3257
Maximum Power (W)	90	60	70	80
τ_{th} (Nm)	96.77	91.53	95.37	90.80
G_T	350.18	389.89	234.72	271.55
mass (g)	340	238	290	242
diameter (mm)	35	30	35	32
$\tau_{arm}(100\%e)$	33.89	35.69	22.39	24.66
(50%e)	16.94	17.84	11.19	12.33
(25%e)	8.47	8.92	5.60	6.16

Table 3.2: Motor selection

The motors shown in Table 3.2 are 12 V unless otherwise indicated. A maximum diameter of 35 mm was used as a constraint so that the chosen motor would be feasible in a real prosthetic arm. It is also important that the motor exceed the torque lifting criteria of 13.56 Nm (10 ft-lbs) to allow for the spring force that will be present in the real system.

From this table, the obvious choice is the Maxon RE30 motor as it gives the highest torque for the 1.2 s flexion speed as well as being the smallest and lightest. However, since this is a new model, the shipping time was extremely long so the RE35 is chosen for the prototype. Note that the RE30 would be recommended over the RE35 in the future.

3.4 Nonbackdrivable Transmission

Two types of nonbackdrivable transmissions were considered; power screws and worm/worm gear sets. The selection is based on which transmission is the lightest, most compact and has the highest efficiency while meeting the performance criteria.

3.4.1 Worm/Worm Gear Sets

This transmission setup consists of a worm and worm gear, a pulley and possibly a gearhead. The total gear ratio from motor to arm is:

Figure 3.1: Worm and worm gear free body diagrams

$$G_T = \frac{RN_g r_{gh}}{r_p} \quad (3.13)$$

where R is the effective radius of the elbow joint, r_p is the worm gear pulley radius, N_g is the number of teeth on the worm gear and r_{gh} is the gearhead ratio. This section will deal with choosing an appropriate worm and worm gear set with or without a gearhead. $\frac{R}{r_p}$ will be used when determining the appropriate spring.

Three companies were considered for worm and worm gear sets. Boston Gear [43], Berg Gear [44] and QTC [45]. In the end, Boston Gear was chosen due to the materials used in their gears which are hardened steel and bronze. It was found that the determining factor in the force analysis was the surface wear so the hardened steel was preferred. The other companies did not use hardened steel. First, the geometry of the worm and worm gear pair was examined in Figure 3.1. In this figure, ϕ is the pressure angle of the gear tooth and λ is the lead angle.

The forces between the worm and worm gear teeth are:

$$F_{gt} = N \cos \phi \cos \lambda - F_f \sin \lambda = F_{wa} \quad (3.14)$$

$$F_{wt} = N \cos \phi \sin \lambda + F_f \cos \lambda = F_{ga} \quad (3.15)$$

$$F_{gr} = N \sin \phi = F_{wr} \quad (3.16)$$

where

$$F_f = \mu N \quad (3.17)$$

with μ as the coefficient of friction between the gear teeth.

Also

$$F_{wt} = \frac{\tau_w}{r_w} \quad (3.18)$$

where

$$\tau_w = \tau_m r_{gh} \quad (3.19)$$

and

$$F_{gt} = F_{wt} \frac{\cos \phi \cos \lambda - \mu \sin \lambda}{\cos \phi \sin \lambda + \mu \cos \lambda} \quad (3.20)$$

The gear and pulley torques can also be calculated:

$$\tau_g = F_{gt} r_{wg} \quad (3.21)$$

$$\tau_p = \tau_g \quad (3.22)$$

$$F_{pt} = \frac{\tau_p}{r_p} \quad (3.23)$$

where τ_p is the pulley torque and F_{pt} is the tangential force on the pulley.

The worm tangential velocity, gear tangential velocity and the sliding velocity are defined as:

$$V_w = \frac{\omega_m r_w}{r_{gh}} \quad (3.24)$$

$$V_g = \tan \lambda V_w \quad (3.25)$$

$$V_s = \frac{V_w}{\cos \lambda} \quad (3.26)$$

where V_w and V_g are shown in Figure 3.1 and V_s is the sliding velocity between the worm and worm gear teeth.

The efficiency of the worm and worm gear with the worm as the input is defined as:

$$e = \frac{F_{gt} V_g}{F_{wt} V_w} \quad (3.27)$$

Using the above relations, this can also be expressed as:

$$e = \frac{\cos \phi \cos \lambda - \mu \sin \lambda}{\cos \phi \sin \lambda + \mu \cos \lambda} \tan \lambda \quad (3.28)$$

Nonbackdrivable Criteria:

In order for the motors not to be run as generators the worm and worm gear set must be nonbackdrivable (i.e. the worm gear cannot drive the worm). If a weight is placed on the arm, the motor should stay locked while the arm moves to a new equilibrium position created by the springs stretching from the applied load. It was found that in order for power to flow from the worm to the worm gear [7],

$$\mu < \frac{1}{\tan \lambda \cos \phi} \quad (3.29)$$

This is required for the motor to turn the worm gear. To get power flow from the worm gear to the worm [7],

$$\mu < \tan \lambda \cos \phi \quad (3.30)$$

This power flow is not wanted for a nonbackdrivable gear set. Therefore, in order to have a gearset that is nonbackdrivable [7]:

$$\tan \lambda \cos \phi < \mu < \frac{1}{\tan \lambda \cos \phi} \quad (3.31)$$

The locking is necessary in order to stop an external load from driving the motor. Locking would only occur when the gears are at zero velocity. Even if the motor is running, every time the motor changes direction it must pass through zero velocity. From Figure 3.2 [46], the coefficient of friction for worm gears at around zero velocity is approximately 0.15. Therefore if the calculated μ for nonbackdrivability is greater than this value the gear set will not be nonbackdrivable. In general, the gear set will satisfy this criteria if the lead angle, λ , is less than 5 degrees.

Load Capacity:

There are a number of ways that are used to determine the load capacity of worm gears. For most gear types, bending and surface fatigue strengths are found by comparing estimated bending and surface stresses with the estimated material fatigue strengths. For the worm gear this is done in a slightly different manner and the total gear-tooth tangential load is compared to tooth loads based on bending and surface fatigue strengths [46]. First the dynamic load or total tooth load is calculated;

$$F_d = F_{gt} \frac{1200 + V_g}{1200} \quad (3.32)$$

where V_g must be in ft/min. F_{gt} is found using Equations (3.18), (3.19) and (3.20), then V_g is calculated using (3.24) and (3.25). Using Figure 3.2 [46] and V_s from Equation (3.26), μ is estimated.

The bending fatigue limiting load or strength capacity, F_s is found:

$$F_s = S_n w P_c y \quad (3.33)$$

Figure 16.23 Worm gear coefficient of friction. (From ANSI/AGMA Standard 6034-A87.)

Figure 3.2: Worm gear friction coefficient values [46]

where the gear circular pitch is:

$$P_c = \frac{\pi}{D_p} \quad (3.34)$$

and D_p is the gear diametral pitch. S_n is the bending fatigue strength of the gear material (24 ksi for bronze), w is the gear face width and y is the Lewis form factor (a function of the normal pressure angle, Table 16.2[46]).

Finally, the surface fatigue limiting load or wear factor, F_w , is:

$$F_w = d_g w K_w \quad (3.35)$$

where K_w is a material and geometry factor (Table 16.3 [46]) and d_g is the worm gear pitch diameter.

For a safe design:

$$F_d \leq F_s \quad (3.36)$$

and

$$F_d \leq F_w \quad (3.37)$$

The forces F_s and F_w are found for a selection of worm and worm gears from Boston Gear [43]. Only single thread worms were considered because they are the only worms to meet the criteria of a lead angle less than 5° . Worm gears with a pitch diameter of over 2.5 in were also rejected due to size constraints. The mass is also an important factor in the design. The density used for the bronze worm gear is 8700 kg/m^3 and 7700 kg/m^3 for the steel worm. This data is tabulated in Tables 3.3 and 3.4.

The maximum recommended angular velocity of the worm is approximately 1800 rpm, at which the worm is pushing one tooth of the gear 30 times per second. Since a worm gear drive functions as a friction device, the gearing does need an opportunity to rid itself of the heat developed [44]. Therefore, a gearhead must be used in addition to the worm gears. Each motor is paired with a gearhead to reduce the worm speed to this maximum. The gearhead should be one stage and must be compatible with the chosen motor. This is given in the motor specifications [41, 42].

Each of the four motors discussed in the previous section were revisited and the best worm/worm gear combination was found for each. The coefficient of friction, μ , in Equation (3.20) was found using Figure 3.2 for the calculated sliding velocities of each particular transmission/motor combination. The dynamic load, F_d , was then found using Equation (3.32). This load was compared to the strength capacity and wear capacity for that gear.

WORM GEAR				
Diametral Pitch	Number of Teeth	Strength Capacity	Wear Factor	Mass
D_p	N_g	F_s (N)	F_w (N)	m_g (kg)
24	20	349.63	74.11	0.031
	30	349.63	111.21	0.055
	40	349.63	148.30	0.087
	50	349.63	185.31	0.135
12	12	1397.45	177.93	0.086
	16	1397.45	237.18	0.149
	20	1397.45	296.61	0.215
	30	1397.45	444.82	0.378
10	16	2096.18	355.86	0.282
	20	2096.18	444.82	0.354

Table 3.3: Worm gear data

WORM			
Diametral Pitch	Lead Angle	Pressure Angle	Mass
D_p	$\lambda(^{\circ})$	$\phi(^{\circ})$	m_w (kg)
24	4.77	14.5	0.017
12	4.77	14.5	0.084
10	4.57	14.5	0.136

Table 3.4: Worm data

The resulting combinations shown in Table 3.5 are from a number of iterations to obtain the worm/worm gear and gearhead combination that satisfied Equations (3.36) and (3.37) while remaining as compact as possible. Also, the remaining R/r_p should be a reasonable value. R should fit in the elbow joint area and r_p should be as small as possible, but large enough so that a cable can easily wind around it. Assuming the ranges of R and r_p to be:

$$0.025 < R < 0.05 \quad (3.38)$$

$$0.008 < r_p < 0.02 \quad (3.39)$$

The required range of R/r_p is:

$$1.25 < R/r_p < 6.25 \quad (3.40)$$

Motor	RE30	RE35	3557	3257
Gearhead ratio, r_{gh}	4.8	4.8	3.71	3.71
Diametral Pitch, D_p	12	12	12	12
Sliding Velocity, V_s (ft/min)	400.1	359.4	311.6	360.5
Coefficient of Friction, μ	0.032	0.036	0.037	0.036
# Teeth, N_g	20	30	20	20
Dynamic Load, F_d (N)	276.4	309.5	212.6	204.7
Strength Capacity, F_s (N)	1397.5	1397.5	1397.5	1397.5
Wear Capacity, F_w (N)	296.6	444.8	296.6	296.6
R/r_p	4.06	2.43	3.16	3.66
Total mass (kg)	1.31	1.91	1.41	1.31

Table 3.5: Motor/transmission options

The determining factor in choosing a worm/worm gear set is the wear capacity, F_w . This is also the less important factor of the two, so no additional safety factor was added to Equation (3.37). The strength capacity of the chosen gears was at least four times greater than the dynamic load, giving a more than sufficient safety factor against tooth breakage.

The efficiency for the worm and worm gears depends on the coefficient of friction between them and the lead and pressure angles of the worm (see Equation (3.28)). From Figure 3.2, using the sliding velocity from Table 3.5, the running coefficient of friction is found. A starting and stopping friction coefficient of $\mu = 0.1$ is assumed and an average value can be computed. For starting and shut down times of 0.1 s and a running time of 1.0 s, $\mu = 0.0467$ for the RE35 motor and $\mu = 0.0433$ for the RE30 motor. The same worm, $12D_p$, was chosen for each motor. Using Equation (3.28) the efficiency is 63% for RE35

and 63% for RE30. Therefore both the Maxon RE30 and RE35 meet the torque criteria (see Table 3.2). In addition to lifting the load of 13.56 N, the RE35 can also handle 222.6 N of spring force.

Looking at the total mass, which is the mass of the motor, gearhead, worm and worm gear of two branches, the Maxon RE30 combination is the lightest. Combined with it producing the highest torque as discussed in the previous section, this makes it the ideal choice. Again, because of lengthy shipping time the RE35 will be used for the prototype. Note that the desired mass of the arm is 0.9 kg and the mass of the gears and motors alone exceeds this goal. This mass can be greatly reduced by drilling holes in the worm gear. Also, since the wear factor is the critical factor, a smaller gear could be used if special surface hardening was performed on the gear. These and other solutions to the problem of total mass will be addressed in the design of a future wearable device.

3.4.2 Power Screws

Another method considered for creating a nonbackdrivable transmission is to use a power screw. In this case, the total gear ratio is:

$$G_T = \frac{2\pi r_{gh} r_s R}{L} \quad (3.41)$$

where r_s is the spur gear ratio between a gear driven by the gearhead shaft and the nut of the power screw. $L = \left(\frac{\#threads}{in}\right)^{-1}$ is the lead of the power screw. For each rotation (2π rad) of the nut the screw will move L inches. Knowing the desired arm speed and the motor speed at τ_{th} , $\frac{r_s}{L}$ can be computed with Equations (3.5) and (3.41). Acme threads were chosen for analysis [46, 47]. The common screw sizes were listed and r_s was calculated for each one, based on a given L . The same gearheads as for the worm/worm gear setup are assumed.

The required torque for the power screw to achieve the required arm torque of 10 ft-lbs or 13.56 Nm is calculated. The force on the cable/spring is:

$$F_{sp} = \frac{\tau_{arm}}{R} \quad (3.42)$$

For an Acme thread screw, assuming a thrust bearing with negligible friction, the torque required to pull F_{sp} is [46, 47]:

$$\tau_s = \frac{F_{sp}d_m}{2} \left(\frac{f_s \pi d_m + L \cos \alpha_n}{\pi d_m \cos \alpha_n - f_s L} \right) \quad (3.43)$$

where f_s is the coefficient of friction between the screw and nut, d_m is the pitch diameter of the screw and α_n is the angle of the Acme thread. For oil-lubricated thread/nut combinations, $f_s = 0.15 \pm 0.05$ [47].

The desired nut torque τ_s is calculated for each screw using Equation (3.43). Using the calculated value for r_s and τ_{th} the actual torque τ_s is calculated:

$$\tau_s = \tau_{th} r_{gh} r_s e \quad (3.44)$$

where e is the power screw efficiency. The efficiency is calculated as:

$$e = \frac{L}{\pi d_m} \frac{\pi d_m \cos \alpha_n - f_s L}{\pi f_s d_m + L \cos \alpha_n} \quad (3.45)$$

which is simplified to:

$$e = \frac{\cos \alpha_n \cos \lambda - f_s \sin \lambda}{\cos \alpha_n \sin \lambda + f_s \cos \lambda} \tan \lambda \quad (3.46)$$

The calculated efficiency for the screws was low at approximately 30% for the smaller screws.

τ_s is then tabulated for each screw. This torque must be higher than the required torque in order to meet the goal of lifting 13.56 Nm.

It was found that for both RE30 and RE35 the arm flexion time had to be lengthened to 4 s and 3.8 s respectively in order to have the required τ_s (3.43) less than the available τ_s (3.44). This is mostly a result of the low efficiency. Therefore the speed performance of this type of transmission is much lower than the worm/worm gear set.

Finally, the strength of the screw must be tested against the applied torsional and axial stresses [46, 47]. Torsional shear stress is calculated as follows:

$$\tau = \frac{16\tau_s}{\pi d_r^3} \quad (3.47)$$

where d_r is the thread root diameter of the screw. This is assuming the collar friction is negligible and the full nut torque is transmitted.

The axial stress is simply:

$$\sigma = \frac{F_{sp}}{A_t} \quad (3.48)$$

where A_t is the tensile stress area:

$$A_t = \frac{\pi(d_m + d_r)^2}{16} \quad (3.49)$$

These two stresses are combined with the distortion energy theory:

$$S_y = \sigma' = \sqrt{(\sigma_x^2 + 3\tau_{xy}^2)} \quad (3.50)$$

where, $S_y = S_p$. S_p is the proof strength of the screw found in Table 10.5 from [46]. For a safe design, $3S_y \leq S_p$ using a safety factor of 3. A material of SAE 5.8 was chosen based on PIC lead screws being 303 Stainless steel. The smallest available screw, 1/4 in, has a total yield stress of $S_y = 29$ MPa. This screw easily meets the strength criteria and can be used.

It is clear from the above analysis that the worm and worm gear combination is a better choice. Although the power screw option is a more compact solution it does not meet the design criteria for speed and torque. The worm and worm gear set that will be used has a diametral pitch of 12. The worm is a hardened steel single thread worm, H1056RH, and the worm gear is bronze with 30 teeth, GB1051 [2].

3.5 Springs

3.5.1 Spring Design Criteria

Non-linear springs are difficult to design and produce and typical spring designs are linear. This causes spring selection for non-linear designs to be a challenge. In addition to the constraint of non-linear springs, they must also fit within design parameters of stiffness, force, deflection, size and weight. There are a variety of non-linear springs available and it is advantageous to have the flexibility to choose from both hardening and softening types regardless of the required joint properties.

A nonlinear spring is required in order to allow stiffness variation. The joint can be hardening, softening or linear. A linear joint should be the most simple in terms of controls, however either a softening or hardening joint can also be used without much error [38]. A natural human arm is a hardening joint, but this is not a criterion.

From Table 3.1 the desired spring stiffness range is 1 – 80 Nm/rad. This may not be possible with all springs. The key is to have a noticeable range of stiffness. The minimum stiffness should be less than 15 Nm/rad and the maximum as high as possible up to 80 Nm/rad. As described previously, this prototype configuration requires the motor to lift a weight at the same time as supporting a spring force. It was said that with the chosen gearset and motor, the arm can move 13.56 Nm and also support a 222.6 N spring force. If there is no load on the arm, the maximum spring force is less or equal to F_{pt} at τ_{th} . For a coefficient of friction of $\mu = 0.036$, using Equation (3.23), the maximum force allowed is 666.35 N.

One advantage of introducing variable stiffness into a prosthetic arm is that the arm will move when an external force is applied. There must be limits to this movement in order to have a usable limb. When the arm is at its low stiffness, or relaxed state, the deflection upon impact should be a minimum of 30 degrees. This should give the amputee time to react to the force. However, when the arm is at its higher stiffness settings, the arm should be able to lift approximately 4 kg (13.56 Nm including weight of arm) without any significant

spring deflection. For design purposes, the arm should be capable of lifting 4 kg with an arm deflection of less than 15 degrees.

The final, yet very important factor, in the design is spring weight and size. The spring should be as small and compact as possible.

Three springs that have been suggested previously are rolomites, belleville washers and different types and shapes of rubber. These are nonlinear and close to quadratic. The rolomites are difficult to manufacture and are therefore not desirable [31]. Both the belleville washers and rubber were studied in detail to choose an appropriate spring for the prototype. The belleville washers are a good choice since they give the flexibility of changing the spring characteristics by adding washers, trying series or parallel construction or varying sizes. On the other hand, the stacks are difficult to construct precisely and may require extra manufacturing time. Another idea was to use some sort of rubber such as bungee cords or rubber tubing. These are much simpler, lightweight and compact. However, the high end stiffness range becomes a problem. These are both studied in greater detail in the following sections.

3.5.2 Torque Limitations

An important quality of the chosen spring will be how much torque it can handle without going slack (spring is at its rest length) or becoming fully compressed or extended, depending on the spring type. These torque values can be calculated using Equation (2.3) but first it is important to clarify the notation used for spring deflection.

The springs used for this design can be either tension or compression springs as well as hardening or softening. Figure 3.3 shows the different positions of spring deflection for both types. The compression spring shown is a stack of belleville washers made to act like a tension spring by using a cylinder and piston setup. δ_{lim} and δ_0 are the two limiting positions of the spring. The maximum and minimum deflection positions must be within these limits to allow for movement of the joint when a force is applied. This will create a non-rigid joint, which is one of the goals of this thesis.

Figure 3.3: Deflection positions; (A) Tension spring, deflection to full (B) Compression spring, deflection to slack

In this thesis, δ_0 corresponds to the rest length of the spring and is therefore always zero. δ_{lim} is the maximum extension in the case of the tension spring and maximum compression in the case of the compression spring. δ_{min} and δ_{max} are shown in between. For a hardening spring, the maximum stiffness occurs at δ_{max} and for a softening spring, the maximum stiffness occurs at δ_{min} . It is important to remember this to avoid confusion.

In both tension and compression springs, the springs are slack at δ_0 and fully compressed or extended at δ_{lim} . Applying an external force, F_{ext} , the top spring could go slack if $x_k < \frac{1}{2}\delta_{lim}$ or the bottom spring reaches δ_{lim} if $x_k > \frac{1}{2}\delta_{lim}$. From Equation (2.3):

$$\tau_{slack} = R(\Phi_2(\delta_2) - \Phi_1(\delta_1)) = R(\Phi(2R\Delta) - \Phi(\delta_0)) \quad (3.51)$$

For the fully compressed/stretched torque:

$$\tau_{full} = R(\Phi_2(\delta_2) - \Phi_1(\delta_1)) = R(\Phi(\delta_{lim}) - \Phi(\delta_{lim} - 2R\Delta)) \quad (3.52)$$

These equations are valid for both tension and compression springs. The curves of these torques for a softening spring are shown in the Figure 3.4.

In the case of a hardening spring, the graph would look the same but the slack and full boundaries would be reversed. The point at which the curves cross is at $x_k = \frac{1}{2}\delta_{lim}$. At this point, the maximum deflection with an external applied torque can occur before the spring reaches its end conditions. The area between the curves can be called the stiffness controlled region and is the operating area of the springs. This area should be maximized to give large joint deflections, which is a requirement at minimum stiffness in order to have a realistic arm.

Figure 3.4: Joint torque vs. deflection: slack and full stretch boundaries for a softening spring

3.5.3 Belleville Washers

Belleville washers are stackable to create springs and are inherently non-linear. The basic parameters are shown in Figure 3.5.

Figure 3.5: Belleville washer [48]

From [48, 36], the force-displacement curve for a single washer (subscript w) is described by:

$$\Phi_w(\delta_w) = C \left[\frac{t}{2} \delta_w^3 - \frac{3ht}{2} \delta_w^2 + (h^2t + t^3) \delta_w \right] \quad (3.53)$$

where C is:

$$C = \frac{E\pi a^2 \ln(a)}{6(1 - \mu_p^2)r^2(a - 1)^2} \quad (3.54)$$

where $a = D/d$ and r is the contact radius. For washers with a small thickness, $t < 2$ mm, $r = D/2$. Therefore:

$$C \approx \frac{2E\pi \ln \frac{D}{d}}{3(1 - \mu_p^2)(D - d)^2} \quad (3.55)$$

where E is the modulus of elasticity, 200,000 MPa for steel, and μ_p is Poisson's ratio, 0.3 for steel.

Taking the derivative we get the washer stiffness:

$$\Psi_w(\delta_w) = \frac{d\Phi_w}{d\delta_w} = C \left[\frac{3t}{2} \delta_w^2 - (3ht) \delta_w + (h^2t + t^3) \right] \quad (3.56)$$

The second derivative is the curvature or "hardness":

$$\rho_w(\delta_w) = 3Ct(\delta_w - h) \quad (3.57)$$

Since $C > 0$, $\rho_w(\delta_w) < 0$ for $\delta_w < h$, the washers are softening springs. However, when the spring stiffness curve is plotted, it is found to be concave up (see Figure 3.6). From the proof in the Spring Theory section, this results in a hardening joint.

Belleville washers can be combined in either series, parallel or a combination of the two by stacking them. These stacking methods are shown in Figure 3.7.

For a series setup (subscript s), the washers meet in opposite directions giving an overall spring displacement as:

$$\delta_s = S\delta_w \quad (3.58)$$

where S is the number of series interfaces plus one. Therefore, the force-displacement curve can be found for the entire spring.

Figure 3.6: Belleville washer force and stiffness curves

$$\begin{aligned}
 \Phi_s(\delta_s) &= C \left[\frac{t}{2S^3} \delta_s^3 - \frac{3ht}{2S^2} \delta_s^2 + \frac{h^2t + t^3}{S} \delta_s \right] \\
 &= C \left[\frac{t}{2} \delta_w^3 - \frac{3ht}{2} \delta_w^2 + (h^2t + t^3) \delta_w \right]
 \end{aligned} \tag{3.59}$$

The stiffness curve is:

$$\Psi_s(\delta_s) = \frac{d\Phi_s}{d\delta_s}$$

Figure 3.7: Belleville washer stacking methods

Therefore:

$$\begin{aligned}\Psi_s(\delta_s) &= C \left[\frac{3t}{2S^3} \delta_s^2 - \frac{3ht}{S^2} \delta_s + \frac{h^2t + t^3}{S} \right] \\ &= \frac{C}{S} \left[\frac{3t}{2} \delta_w^2 - 3ht\delta_w + (h^2t + t^3) \right]\end{aligned}\quad (3.60)$$

So, the force is the same for a single washer or a series stack. The stiffness of a series stack is less than it is for a single washer. Figure 3.8 shows this relationship.

For a parallel setup (subscript p), the washers are stacked in the same direction. The overall displacement is the same:

$$\delta_p = \delta_w \quad (3.61)$$

but the forces add:

Figure 3.8: Belleville washer force and stiffness curves: series stack

$$\Phi_p(\delta_p) = P\Phi_w(\delta_w) \quad (3.62)$$

P is the number of parallel interfaces plus one.

$$\Phi_p(\delta_p) = CP \left[\frac{t}{2} \delta_w^3 - \frac{3ht}{2} \delta_w^2 + (h^2t + t^3) \delta_w \right] \quad (3.63)$$

$$\Psi_p(\delta_p) = CP \left[\frac{3t}{2} \delta_w^2 - 3ht \delta_w + (h^2t + t^3) \right] \quad (3.64)$$

The stiffness increases with the number of washers because the force increases and acts over the same displacement range.

Figure 3.9: Belleville washer force and stiffness curves: parallel stack

The washers can also be combined in a combination of series and parallel, as shown in Figure 3.10, but it is somewhat complicated. When a single series interface is combined with a single parallel interface, the stiffness is discontinuous. This discontinuity occurs when the series pair bottoms out. Since the parallel pair has a higher maximum force, the combination continues as a parallel set until the maximum stiffness is reached. This discontinuity would also appear if different sized washers were used in combination. If a third washer is added to make a second parallel pair, the curve becomes smooth. However this extra washer does not change the displacement range or maximum force.

Figure 3.10: Belleville washer force and stiffness curves: combination stack

The series setup is the most desirable for the prosthetic arm design due to the capability of large deflections. With parallel washers, the deflections would be so small that the differences between stiffness settings would be insignificant and therefore hard to control. The combination setup uses washers that add nothing to the overall performance and it is set up so that no springs bottom out. Therefore this design is not using the washers to their full potential. The series setup was investigated further.

3.5.4 Belleville Washer Selection

For large joint deflection, the area between the curves in Figure 3.4 must be maximized. The area is largest with a higher h/t value. It can be seen in Figure 3.11 [47] that with

Figure 3.11: Belleville washers: force curves [47]

$h/t = 1.414$ the curve is very close to quadratic. After this, the washers become unstable. Therefore we choose the Belleville Washers that are as close to quadratic as possible.

Since the maximum stiffness required is $K_{max} = 80 \text{ Nm/rad}$, this was used as a starting point in the design. From Equation (2.6), arm stiffness is defined as:

$$K = R^2(\Psi_1(\delta_1) + \Psi_2(\delta_2))$$

From Figure 3.6 it is noted that K_{max} will occur at δ_{min} , therefore;

$$K_{max} = R^2(\Psi_1(\delta_{min}) + \Psi_2(\delta_{min})) \quad (3.65)$$

Assuming that the springs are identical:

$$\Psi_1(\delta_{min}) = \Psi_2(\delta_{min}) = \Psi_{max} \quad (3.66)$$

So:

$$K_{max} = 2R^2\Psi_{max} \quad (3.67)$$

Using this equation and the design criteria of $K_{max} = 80 \text{ Nm/rad}$, the number of washers, S , can be calculated:

$$\Psi_{max} = \frac{K_{max}}{2R^2} = \frac{C}{S} \left[\left(\frac{3t}{2} \right) \delta_{min}^2 - (3ht) \delta_{min} + (h^2t + t^3) \right] \quad (3.68)$$

where δ_{min} is estimated as a fraction of h (example $0.1h$). This gives;

$$S = \frac{2R^2C}{K_{max}} \left[\left(\frac{3t}{2} \right) \delta_{min}^2 - (3ht) \delta_{min} + (h^2t + t^3) \right] \quad (3.69)$$

With S known, the minimum, maximum and limiting forces and stiffnesses can also be calculated using the above equations. Also, the resulting spring length, l_{spring} , is given as;

$$l_{spring} = Sh_{washer} \quad (3.70)$$

where h_{washer} is the washer height, shown as $O.H.$ in Figure 3.5.

With the spring size determined for the required stiffness range, the other design criterion are visited. Since this spring is a softening spring, the higher torque values are on the τ_{slack} border in Figure 3.4 and the maximum stiffness is at δ_{min} . To find out if the spring can support a 4 kg load with only 15° arm deflection, τ_{slack} is calculated with $\Delta = 15^\circ$ using Equation (3.51). Also, the joint deflection at K_{min} is calculated using:

$$\Delta = \frac{\delta}{R} \quad (3.71)$$

The variables in the final selection process are the amount of deflection left at each end of the spring to allow movement when a force is applied to the joint and the effective elbow radius. Space considerations are very important and the spring and elbow radius chosen will affect this greatly. The total arm length was calculated as follows:

$$L_{arm} = 0.14 + l_{spring} + hS + R\theta + 2R \quad (3.72)$$

where 0.14 m is the length from the RE 35 motor to the edge of the chosen worm gear plus 5 mm, hS is the length of cable required to pull the spring to full deflection, $R\theta$ is the length of cable required to move the arm through the full range of θ , and $2R$ is the diameter of the elbow pulley.

The effective elbow radius, R , also depends on the gear ratio for the chosen transmission components (3.13). With the current choice of the Maxon RE35 motor, $\frac{R}{r_p} = 2.43$ (Table 3.5). Using the criteria outlined in Equation (3.39):

$$0.025 < R < 0.0486$$

and

$$0.0103 < r_p < 0.02$$

the stiffness range, force at maximum and minimum stiffness, deflection at minimum stiffness, torque at 15° arm deflection and system length and size were optimized. This information is compiled in Table 3.6 for a value of $R = 35$ mm with $\delta_{min} = 0.1h$ and $\delta_{max} = 0.75h$ for various springs from Spaenaur [48].

None of the tabulated washer stacks met all of the design criteria. It was determined that the most important criterion was $\Phi(\delta_{max})$, the spring force at minimum arm stiffness, because if this force is too high, the motors will be unable to pull the spring to the minimum stiffness setting. The only two springs to meet the force limitation were M231207 and M321608.

To use the Belleville washers as a tension spring as shown in Figure 3.3, they will be housed in a cylinder and piston casing. This will be supported by the cable between the worm gear pulley and the arm pulley.

From an initial prototype, the actual force required to compress the washers is much larger than the calculated value, $\Phi_s(\delta_s)$. This is likely due to the large amount of friction present in the cylinder, both between the washers themselves and the washers and the casing. Therefore, from the above list of washers, the M231207 would be the best choice.

model#	M231207	M321608	M341210	M361809	M402010	M452213
D (mm)	25.0	31.5	34.0	35.5	40.0	45.0
h_{washer} (mm)	1.60	1.85	2.25	2.05	2.30	2.85
$\frac{h}{t}$	1.29	1.32	1.25	1.29	1.31	1.29
S	47	47	65	50	55	82
l_{spring} (mm)	75	87	146	102	127	237
K_{max} (Nm/rad)	79.53	78.79	79.74	79.91	80.08	79.55
K_{min} (Nm/rad)	11.59	10.22	13.44	11.91	10.86	11.74
$\tau_{slack15^\circ}$ (Nm)	16.51	17.35	20.36	18.62	19.83	22.10
$\Phi(\delta_{min})$ (N)	153.10	177.07	293.59	208.96	261.05	480.93
$\Phi(\delta_{max})$ (N)	595.93	678.38	1168.29	815.94	1005.95	1875.14
$\Delta K_{min} (^\circ)$	17.44	20.32	33.44	23.70	29.50	54.78
L_{arm} (m)	0.410	0.429	0.520	0.453	0.491	0.663

Table 3.6: Belleville washer data

3.5.5 Rubber Springs

Two types of rubber springs were tested; bungee cords and rubber tubing. A rubber spring would be significantly lighter, more compact and much more simple than the belleville washers. Some data were available for the rubber tubing from Thera-Band [49] as well as from Primeline Industries [50]. The data from Thera-Band showed an array of small diameter tubes with elongation up to 250%. This is shown in Figure 3.12.

The different colours correspond to varying wall thickness. Each tube has an inner diameter of 0.2 in and wall thickness increases from Yellow, 0.045 in to Silver, 0.125 in. Gold has the thickest wall but the actual thickness is not given. From this data, force versus deflection as well as stiffness vs. deflection curves were generated. These are shown in Figures 3.13, 3.14 and 3.15.

It can be seen that the curve is nonlinear and close to quadratic for the first 150%-175% elongation. Also, it behaves as a softening spring similar to the Belleville washers. From

Thera-Band® Force-Elongation

Page, et al. *JOSPT* 30(1):A47. 2000.

Elongation	Yellow	Red	Green	Blue	Black	Silver	Gold
25%	1.1	1.5	2	2.8	3.6	5	7.9
50%	1.8	2.6	3.2	4.6	6.3	8.5	13.9
75%	2.4	3.3	4.2	5.9	8.1	11.1	18.1
100%	2.9	3.9	5	7.1	9.7	13.2	21.6
125%	3.4	4.4	5.7	8.1	11	15.2	24.6
150%	3.9	4.9	6.5	9.1	12.3	17.1	27.5
175%	4.3	5.4	7.2	10.1	13.5	18.9	30.3
200%	4.8	5.9	7.9	11.1	14.8	21	33.4
225%	5.3	6.4	8.8	12.1	16.2	23	36.6
250%	5.8	7	9.6	13.3	17.6	25.3	40.1

Figure 3.12: Force elongation data for Thera-Band rubber tubing (lb) [49]

Figure 3.13: Force deflection curves for Thera-Band rubber tubing

Figure 3.14: Spring stiffness curves for black Thera-Band rubber tubing

Figure 3.15: Joint stiffness curves for black Thera-Band rubber tubing

the stiffness curve it can be seen that a pair of these tubes would produce a hardening joint. Only one curve is shown in Figures 3.14 and 3.15 because all of the tubes have very similar stiffness characteristics and the stiffness curves actually lie on top of each other.

From the second data source, Primeline Industries [50], the data are for higher elongation values, starting at 100% and going to 900%. A spreadsheet was supplied that generated force elongation curves for different values of inner diameter and wall thickness. A sample of this data is given in Figure 3.16 for tubes with an inner diameter of 1/4 in and varying wall thicknesses. While another nonlinear curve is produced, this time it is a hardening spring.

Figure 3.16: Force elongation curves for Primeline rubber tubing

Further testing was done in order to come up with a complete curve. A Thera-Band black tube (inner diameter 0.2 in, wall thickness 0.098 in) was tested as well as a bungee cord and a rectangular rubber cord. The resulting curves are shown in Figures 3.17 and 3.18. These tests were completed by hanging the bungee or rubber tube from a metal frame and using a hook, adding weights at small increments to stretch the rubber. At each increment,

Figure 3.17: Force elongation test data for bungee

Figure 3.18: Force elongation test data for Thera-Band rubber tubing

a measurement using callipers was taken and both the mass and elongation were recorded. It is noted that the trend discussed above holds true for these practical tests. The curves start out showing softening behaviour, followed by a linear section and finally a hardening section. The top curve shows the behaviour of the spring upon release. It follows a similar curve but remains slightly more stretched at the same load. This indicates that the spring has a memory. If left for a few minutes with no load it did return to its original length.

Spring Selection:

Due to the available data for the rubber tubing versus the bungee cords, an attempt was made to find an appropriate tube to fit the specifications of the required spring. This was done using a combination of data from lab tests, Thera-Band and Primeline [49, 50]. The result was detailed information up to 25% elongation, readings every 25% up to 250% followed by reading every 100% after that. This was done in order to get the most information possible for the two nonlinear sections of the curve, especially at the relaxed endpoint (0 – 25%). Figure 3.19 shows the spring force curve and Figure 3.20 shows the resulting joint stiffness for a pair of rubber tubes.

From Equations (2.6) and (2.4) and Figure 3.3 we get:

$$K_{max} = 2R^2 \frac{d\Phi(\delta_{min})}{d\delta_{min}} \quad (3.73)$$

Similarly:

$$K_{min} = 2R^2 \frac{d\Phi(\delta_{max})}{d\delta_{max}} \quad (3.74)$$

The required length of rubber at rest can be found for either a set value of K_{max} or K_{min} at a designated elongation percentage using:

$$l_{spring} = \frac{2R^2}{K} \frac{d\Phi}{d\epsilon} \quad (3.75)$$

where ϵ is the % elongation of the tube. Once l_{spring} is determined, K can be calculated at every data point using equation (2.6).

It is assumed, as with the belleville washers, that the minimum joint stiffness should be a maximum of 15 Nm/rad. Using this criteria, the maximum stiffness achievable for the

Figure 3.19: Combined force elongation curve for rubber tubing

Figure 3.20: Combined joint stiffness curve for rubber tubing

softening section of the spring is 61 Nm/rad giving a range of 46 Nm/rad. This is using a range of 6.35% to 175% elongation. For the hardening section, 450% to 700% elongation, a very large range can be achieved (15 Nm/rad to 260 Nm/rad), however, the spring forces are extremely large and unacceptable for this design.

There is a large variety of tubing to choose from with varying inner diameters and wall thickness values. A complete list from Primeline can be found on their website [50]. A minimum rest length of 5 cm was chosen for ease of handling and to make it possible to attach to cables.

The final criterion that the spring must meet is the arm torque. One difference with this type of spring is that there is no real τ_{full} since the spring can stretch to 900%. The value of τ_{slack} is calculated for each tube for a 15° arm deflection. The desired value is 13.56 N to comply with the current Boston Arm's lifting ability.

Variables in the spring design are; number of tubes, tube size and elbow radius, R . As outlined in the Belleville Washer selection, the allowable range of R and r_p , given the chosen motor and gearhead is (from Equation (3.39):

$$0.025 < R < 0.0486$$

and

$$0.0103 < r_p < 0.02$$

Through trial and error, the smallest tube that could give a reasonable stiffness range while having a minimum length of 5 cm had an inner diameter of 3/8 in and a wall thickness of 3/8 in. This is very short and thick. Another, more practical alternative is to use multiple tubes of a smaller size. The blue and black Thera-Band tubes can be used in varying lengths to create reasonable stiffness ranges.

Tables 3.7 and 3.8 show three spring choices for two different values of R . The first table is for an elbow radius of $R = 0.035$ m which gives the worm gear pulley size of $r_p = 0.0144$ m. This is the more compact solution. The stiffness range for each spring is

Tube Size (in)	0.2IDx0.085wall (blue)	0.2IDx0.098wall (black)	3/8IDx3/8wall
number of tubes	10	10	1
l_{spring} (mm)	50	50	50
K_{max} (Nm/rad)	32.65	39.36	37.91
K_{min} (Nm/rad)	7.96	9.59	9.24
$\tau_{slack15^\circ}$ (Nm)	6.25	7.54	7.26
$\Phi(\delta_{max})$ (N)	477.6	599.9	577.7

Table 3.7: Rubber tubing choices: $R = 0.035$ m

Tube Size (in)	0.2IDx0.085wall (blue)	0.2IDx0.098wall (black)	3/8IDx3/8wall
number of tubes	10	10	1
l_{spring} (mm)	50	60	58
K_{max} (Nm/rad)	61.41	61.70	61.46
K_{min} (Nm/rad)	14.97	15.04	14.98
$\tau_{slack15^\circ}$ (Nm)	10.72	11.74	11.309
$\Phi(\delta_{max})$ (N)	477.6	599.9	577.7

Table 3.8: Rubber tubing choices: $R = 0.048$ m

about 25 – 29 Nm/rad. This is smaller than the maximum possible range of 46 Nm/rad. To get the larger range, the spring length would have to be less than 3 cm. The spring force for each choice is below the maximum allowable force of 666.35 N. Finally, the torque that each spring can support with a 15° deflection without going slack is lower than the required 13.56 N. With these springs, the arm's lifting performance would be decreased to about 2 kg rather than the desired 4 kg.

The second table (Table 3.8) shows the same three spring choices with a larger elbow radius, $R = 0.48$ m. In this case, the worm gear pulley radius is $r_p = 0.02$ m. Except in the blue Thera-Band case, the tubing length is longer, closer to 6 cm rather than the minimum value of 5 cm. Due to this and the larger pulley radii, the resulting design is larger overall. The stiffness range, however, is also larger at 46 Nm/rad for each combination. Also, the

$\tau_{slack15^\circ}$ values are higher. Although they do not meet the criteria of 13.56 Nm they are much closer. The maximum spring forces $\Phi(\delta_{max})$ are the same in both cases.

Comparing the belleville washers and the Thera-Band tubing, the rubber appears to be a better choice due to simplicity and size. Although the stiffness range and lifting torque capacity are not as large, the spring forces are smaller and the additional friction much less. There are many different ways to combine short sections of tube to create a spring to meet most of the design criteria.

For this prototype, the blue Thera-Band is used due to the smaller spring forces. The performance of this spring is only slightly lower than the black tubes which has a significantly larger spring force at K_{min} . An elbow radius of $R = 0.035$ m is used in order to test both the Thera-Band spring and the belleville washers. This is something that can be altered in the future to test different spring combinations. The tubes will be combined in parallel using two aluminum plates as support and 1/4 in machine screws to hold the tubes. The screw will be coated with super glue and inserted into the tube. Extra support is added by using zip ties around the tubing.

Figure 3.21: Spring force for blue Thera-Band

Figure 3.22: Spring stiffness for blue Thera-Band

Figure 3.23: Arm stiffness for 10, 5cm blue Thera-Band tubes

Figures 3.21, 3.22 and 3.23 show the spring characteristics for the chosen spring.

3.6 Miscellaneous Components

In addition to the main components, there are many other important parts to consider. These include shafts, screws, pins and keys, bearings, mounts and cables. In order to properly choose them a force analysis must be conducted. A safety factor of three was used throughout this selection process.

3.6.1 Transmission Force Analysis

In order to make the appropriate selections of the hardware, the maximum forces are determined based on kinematic and dynamic analysis.

The forces are calculated at the thermal torque of the motor, which is the maximum desired operating torque. Using the data for the Maxon RE35 motor;

$$\tau_m = \tau_{th} = 0.09677 \text{ Nm}$$

Therefore, from Equation (3.6):

$$\omega_m = 6565.79 \text{ rpm}$$

The velocities of the teeth are found using Equations (3.24), (3.25) and (3.26) in ft/min. V_s is then used to find the coefficient of friction between the worm and worm gear using Figure (3.2) [46]. This gives:

$$V_w = 358.11 \text{ ft/min}$$

$$V_g = 27.93 \text{ ft/min}$$

$$V_s = 359.20 \text{ ft/min}$$

$$\mu = 0.036$$

The forces on the worm and worm gear are determined. The worm torque is found with Equation (3.19).

$$\tau_w = 0.4645 \text{ Nm}$$

From (3.18):

$$F_{wt} = 36.57 \text{ N}$$

Using Equations (3.14), (3.15) and (3.16) and rearranging we get:

$$F_{wa} = F_{wt} \frac{(\cos \phi \cos \lambda - \mu \sin \lambda)}{(\cos \phi \sin \lambda + \mu \cos \lambda)} \quad (3.76)$$

$$F_{wr} = F_{wt} \frac{\sin \phi}{(\cos \phi \sin \lambda + \mu \cos \lambda)} \quad (3.77)$$

where $\phi = 14.5^\circ$ and $\lambda = 4.77^\circ$ [43].

$$F_{wa} = 302.22 \text{ N}$$

$$F_{wr} = 78.67 \text{ N}$$

For the worm gear, the gear forces are again determined using Equations (3.15), (3.14) and (3.16).

$$F_{gt} = F_{wa} = 302.22 \text{ N}$$

$$F_{gr} = F_{wr} = 78.67 \text{ N}$$

$$F_{ga} = F_{wt} = 36.57 \text{ N}$$

The gear torque is found with Equation (3.21).

$$\tau_g = 9.60 \text{ Nm}$$

Also, the pulley torque and tangential force are found with (3.22) and (3.23).

$$\tau_p = \tau_g = 9.60 \text{ Nm}$$

$$F_{pt} = 666.35 \text{ N}$$

Using these forces and Newton's Law, the resulting reaction forces on the shaft bearings are found for the shafts rotating in both the clockwise and counter-clockwise directions. The free body diagrams for the worm and worm gear shafts are shown in Figures 3.24 and 3.25. The maximum forces at each bearing are shown in Table 3.9.

Figure 3.24: Free body diagram: worm and worm gear, clockwise direction

Figure 3.25: Free body diagram: worm and worm gear, counter-clockwise direction

Bearing	Force (N)	Direction
Worm Shaft		
R_{w1x}	18.28	clockwise
R_{w1y}	96.62	clockwise
R_{w1z}	302.22	clockwise
R_{w2x}	-18.28	counter-clockwise
R_{w2y}	96.62	counter-clockwise
R_{w2z}	302.22	counter-clockwise
Worm Gear Shaft		
R_{g1x}	36.57	clockwise
R_{g1y}	51.03	clockwise
R_{g1z}	-390.09	clockwise
R_{g2x}	36.57	counter-clockwise
R_{g2y}	61.29	counter-clockwise
R_{g2z}	390.09	counter-clockwise

Table 3.9: Maximum forces on bearings

For the arm shaft, the free body diagram is shown in Figure 3.26. The maximum allowable forces for the arm shaft will occur when the force on the arm pulley is equal to F_{pt} applied by the worm gear pulley and the load force F_{load} acts in the same direction.

$$F_2 = F_{pt} = 666.35 \text{ N}$$

With an arm pulley radius, $R = 0.035 \text{ m}$ the resulting arm torque is:

$$\tau_{arm} = 666.35(0.035) = 23.3 \text{ Nm}$$

From Table 3.1 the arm length is given as 0.33 m. This gives a maximum arm force of:

$$F_{load} = 70.7 \text{ N}$$

Using Newton's Law, the following forces are calculated. Note: There are no axial forces that act on this shaft. The resulting forces are given in Table 3.10.

Figure 3.26: Free body diagram: arm shaft

Bearing	Force (N)
R_{a1y}	0
R_{a2y}	0
R_{a1z}	225.06
R_{a2z}	515.29

Table 3.10: Maximum forces on arm shaft bearings

3.6.2 Screws

All of the mounts require machine screws to hold them together and/or secure them to a base. It is important that these are sized properly to prevent damage to the prototype. The design is done according to Juvinall's "Fundamentals of Machine Component Design" [46].

The bolts on all four bearing mounts will carry both a tensile and a shear force. The tensile stress and shear stress are used to select an appropriate screw. The effective area is the tensile area, A_t which is given by Juvinall in Table 10.1 [46]. The equivalent tensile stress is calculated using the distortion energy equation:

$$\sigma_e = \sqrt{(\sigma^2 + 3\tau^2)} = \frac{\sqrt{(P_t^2 + 3P_s^2)}}{A_t} \quad (3.78)$$

where P_t is the tensile load and P_s is the shear load. These loads are the total loads divided by the number of screws. σ_e is equal to the proof strength of the screw material, S_p [46]. A screw of SAE Class 5.8 is assumed, giving a proof strength of $S_p = 380$ MPa.

The length of thread engagement into a tapped hole in order to prevent shear stripping for steel screws in aluminum is a minimum of two times the screw diameter [47].

Worm Shaft:

There are two mounts for the worm shaft and they are both attached to the worm gear shaft mount with four screws. The highest forces are the same for both mounts, therefore the calculations are only done for mount 1.

$$P_t = 3 \frac{R_{w1z}}{4} = 226.67 \text{ N}$$

$$P_s = 3 \frac{\sqrt{R_{w1x}^2 + R_{w1y}^2}}{4} = 73.75 \text{ N}$$

Rearranging Equation (3.78) to solve for A_t :

$$A_t = 0.685 \text{ mm}^2 = 0.00106 \text{ in}^2$$

Therefore, from Juvinall's Table 10.1 [46] a UNC 1 machine screw is required.

Worm Gear Shaft:

The worm gear shaft mount supports both the worm gear shaft as well as half of each of the worm shafts attached to it. The forces are highest on mount 2 in the counter-clockwise

direction. Using these forces and the directions shown on the free body diagram, the tensile and shear forces are:

$$P_t = 3 \frac{R_{g2y} - 0.5(R_{w1y} + R_{w2y})}{2} = 52.99 \text{ N}$$

$$P_s = 3 \frac{(R_{g2x} + 0.5(R_{w1x} + R_{w2x}))^2 + (R_{g2z} + 0.5R_{w2z})^2}{2} = 813.65 \text{ N}$$

which gives:

$$A_t = 3.71 \text{ mm}^2 = 0.0058 \text{ in}^2$$

Therefore, a UNC 4 machine screw is required.

Arm Shaft:

The forces on the arm shaft are higher at mount 2, therefore the screws are designed for that side.

$$P_s = 3 \frac{R_{a2z}}{2} = 772.93 \text{ N}$$

$$A_t = 3.52 \text{ mm}^2 = 0.0055 \text{ in}^2$$

Therefore, a UNC 4 machine screw is required.

The sizes of screws required are much too small for easy machining and assembly. For all bearing mounts, UNC 8 machine screws will be used. For the motor mounts, UNC 6 will be used.

3.6.3 Pins and Keys

Pins and keys are essential for attaching pulleys and gears to the shafts. They must withstand the forces applied to the system and are sized accordingly.

Worm Key: From Norton's "Machine Design: An Integrated Approach" [47] for a safe design, the key length L_k should be $L_k = 1.8d$, where d is the shaft diameter. For the worm shaft, $d = 0.5$ in, the appropriate key length is 0.9 in. The standard keyway size for the worm according to Boston Gear [43] is $W_k = 1/8$ in and $d_k = 1/16$ in, where W_k is the

key width and d_k is the depth. The keyway in the shaft will be endmilled.

Pins: The pins that will be used are rolled pins, or "Standard Slotted Type Spring Pins"[51]. Table 10, p1452 in the Machinery's Handbook [51] shows the dimensions and load capabilities for the pins in double shear. The shear force is calculated for each pin and the appropriate pin size is chosen from the table.

Worm Gear Pin: The shear force, with a safety factor of 3 is:

$$P_s = 3\sqrt{F_{ga}^2 + F_{gt}^2} = 913.27 \text{ N} = 205.31 \text{ lb}$$

Therefore the 1/16 in diameter spring pin can be used.

Pulley Pin:

$$P_s = 3F_{pt} = 1999.05 \text{ N} = 449.40 \text{ lb}$$

Therefore the 5/64 in diameter spring pin can be used.

Arm Pulley Pin:

$$P_s = 3F_2 = 1999.05 \text{ N} = 449.40 \text{ lb}$$

Therefore the 5/64 in diameter spring pin can be used.

Arm Attachment Pin:

$$P_s = 3F_{load} = 212.1 \text{ N} = 47.9 \text{ lb}$$

Therefore the 1/16 in diameter spring pin can be used.

For simplicity of assembly, pins with 3/32 in diameter will be used for all locations.

3.6.4 Bearings

The chosen bearings on the worm and worm gear shafts must take both radial and thrust loads. Also, they must be as small as possible. As SKF [52] is a well known bearing company, the bearings were chosen from their online catalogue. Figure 3.27 [52] shows the different types of bearings available and the types of forces they are designed for.

The matrix can only provide a rough guide to that in each individual case it is necessary to make a more qualified selection referring to the information given in the catalogue

Symbols
 +++ excellent
 ++ good
 + fair
 -- poor
 - unsuitable
 ← single direction
 ↔ double direction

	Design					Characteristics																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Suitability of bearings for					Compensation for errors of alignment (initial)										
1 Tapered bore		a																																	
2 Shields or seals																																			
3 Self-aligning																																			
4 Non-separable																																			
5 Separable																																			
6 Purely radial load																																			
7 Purely axial load																																			
8 Combined load																																			
9 Moment load																																			
10 High speed																																			
11 High running accuracy																																			
12 High stiffness																																			
13 Quiet running																																			
14 Low friction																																			
15 Compensation for misalignment in operation																																			
16 Compensation for errors of alignment (initial)																																			
17 Locating bearing arrangements																																			
18 Non-locating bearing arrangements																																			
19 Axial displacement possible in bearing																																			
Deep groove ball bearings																																			
Angular contact ball bearings																																			
Self-aligning ball bearings																																			
Cylindrical roller bearings																																			
Needle roller bearings																																			
Taper roller bearings																																			
Spherical roller bearings																																			
GARF bearings																																			
Thrust ball bearings																																			
Needle roller thrust bearings																																			
Cylindrical roller thrust bearings																																			
Spherical roller thrust bearings																																			

Figure 3.27: SKF bearing table [52]

From the table it would appear that the taper roller bearings are the best for combined loads, followed by angular contact and deep groove ball bearings. Of these, the deep groove come in the smallest sizes and were, therefore, considered for this design.

The bearings are sized based on dynamic loading. First, the load on the bearing, P , must be calculated :

$$P = XF_r + YF_a \quad (3.79)$$

where F_r is the radial force on the bearing and F_a is the axial force. The process is an iterative one. First, using Table 3.11 [52], values of X and Y are chosen based on an estimated value of e . If $F_a/F_r > e$, then X and Y are given in the table. Otherwise, $X = 1$ and $Y = 0$. These values are for normal clearance and intermediate values are found through interpolation.

$f_0 \frac{F_a}{C_0}$	e	X	Y
0.172	0.19	0.56	2.30
0.345	0.22	0.56	1.99
0.689	0.26	0.56	1.71
1.03	0.28	0.56	1.55
1.38	0.30	0.56	1.45
2.07	0.34	0.56	1.31
3.45	0.38	0.56	1.15
5.17	0.42	0.56	1.04
6.89	0.44	0.56	1.00

Table 3.11: Calculation factors for single row deep groove ball bearings with normal clearance [52]

The required bearing life, L_{10} , is calculated using the dynamic load rating and the load on the bearing.

$$L_{10} = \frac{C^p}{P} \quad (3.80)$$

where $p = 3$ for ball bearings and $p = \frac{10}{3}$ for roller bearings. C is the bearing's dynamic load rating.

$$L_{10} = L_{10h} \frac{60\omega}{10^6} \quad (3.81)$$

where L_{10} is the basic rating life at 90% reliability in millions of revolutions, ω (rpm) is the rotational velocity of shaft and L_{10h} is the basic rating life at 90% reliability in hours.

A bearing is chosen from the SKF product tables [52] to satisfy C and the shaft diameter it is intended for. Following the selection of a bearing, the assumptions of e , X and Y are verified based on the value of $f_0 \frac{F_a}{C_0}$ for the chosen bearing. f_0 and C_0 can be found in the bearing specifications. If the value for e is different from the assumed, a second iteration is performed and a new bearing is chosen.

Worm Shaft:

Since this shaft is symmetric, both bearings will be the same. At no load speed, $\omega_{nl} = 7383.48$ rpm:

$$\omega_w = \frac{\omega_{nl}}{r_{gh}} = 1538.25 \text{ rpm}$$

For a life $L_{10h} = 500$ hrs, Equation (3.81) gives:

$$L_{10} = 46.15$$

Choosing $e = 0.38$, we get $X = 0.56$ and $Y = 1.15$. Using calculated forces and a safety factor of three:

$$F_r = 3\sqrt{(R_{w2x}^2 + R_{w2y}^2)} = 308.51 \text{ N}$$

$$F_a = 3R_{w2z} = 949.83 \text{ N}$$

Therefore from Equation (3.79):

$$P = 1207.86 \text{ N}$$

Solving for the required dynamic load rating by rearranging Equation (3.80):

$$C = 4.33 \text{ kN}$$

The smallest available bearing with this C rating is 629 shown in Figure 3.28. The dynamic load rating is $C = 4.75 \text{ kN}$.

Figure 3.28: SKF bearing 629 [52]

Checking the value of e for the chosen bearing:

$$C_0 = 1.96 \text{ kN}$$

which gives:

$$f_0 \frac{F_a}{C_0} = 5.55$$

Interpolating from Table 3.11, $e = 0.424$, $X = 0.56$ and $Y = 1.031$. This gives a new value of P :

$$P = 1099.97 \text{ N}$$

so:

$$C = 3.95 \text{ kN}$$

Therefore the bearing shown in Figure 3.28 can still be used.

Worm Gear Shaft:

This shaft is not symmetrical so the bearings must be sized for bearing 2, which is the side with a larger load. Ignoring friction, the angular velocity of the worm gear shaft is calculated:

$$\omega_g = \frac{\omega_{nl}}{r_{gh} N_g} = 51.27 \text{ rpm}$$

For a life $L_{10h} = 500$ hrs, Equation (3.81) gives:

$$L_{10} = 1.538$$

Choosing $e = 0.38$, we get $X = 0.56$ and $Y = 1.15$. Using calculated forces and a safety factor of three;

$$F_r = 3\sqrt{(R_{g2z}^2 + R_{g2y}^2)} = 1184.62 \text{ N}$$

$$F_a = 3R_{g2x} = 109.71 \text{ N}$$

Therefore from Equation (3.79):

$$P = 789.55 \text{ N}$$

and

$$C = 0.911 \text{ kN}$$

The bearing chosen is 619/6 shown in Figure 3.29. The dynamic load rating is $C = 1.24$ kN.

Figure 3.29: SKF bearing 619/6 [52]

Checking the value of e for the chosen bearing:

$$C_0 = 0.475 \text{ kN}$$

which gives:

$$f_0 \frac{F_a}{C_0} = 2.29$$

Interpolating from Table 3.11 $e = 0.348$, $X = 0.56$ and $Y = 1.27$. This gives a new value of P :

$$P = 802.72 \text{ N}$$

so:

$$C = 0.927 \text{ kN}$$

Therefore, the bearing shown in Figure 3.29 can still be used. The chosen bearing, however, is not readily available. As a result, a slightly larger bearing, SKF 626, shown in Figure 3.30 is used.

Figure 3.30: SKF bearing 626 [52]

Checking the value of e , with $C_0 = 0.95$ for this bearing:

$$f_0 \frac{F_a}{C_0} = 1.49$$

which gives $e = 0.306$, $X = 0.56$ and $Y = 1.427$. A new value of $P = 819.94$ and $C = 0.946$ shows that this bearing is appropriate for the design.

Arm Shaft:

Although there is no axial load, the deep groove ball bearings will be used. Since bearing 2 has the highest loads, the design is based on its loading. Therefore:

$$F_r = 3R_{a2z} = 1545.87 \text{ N}$$

$$P = F_r$$

since there is no axial load. Again, neglecting friction;

$$\omega_{arm} = \frac{\omega_{nl} r_p}{r_{gh} N_g R} = 21 \text{ rpm}$$

For a life $L_{10h} = 500$ hrs, Equation (3.81) gives:

$$L_{10} = 0.63$$

$$C = 1.325 \text{ kN}$$

The same bearing, SKF 626, as for the worm gear shaft will be used.

3.6.5 Mounts

The bearing mounts are designed to take the thrust load of the outer race of the bearing. This means that each bearing will take thrust in one direction. The mounts are designed so that the outer race is supported with a lip on the bearing hole. The thickness of this lip must be checked for shear strength.

For the worm shaft, the shear force, with a safety factor of 3, is:

$$P_s = 3(302.22) = 906.66 \text{ N}$$

Assuming a bearing mount thickness of $t_m = 3/8$ in, a bearing width of $b = 8$ mm, and a bearing diameter of $d_b = 26$ mm the shear area can be calculated:

$$A_{shear} = \pi d_b (t - b) = 1.245(10^{-4})$$

The yield strength is calculated by:

$$0.58S_y = \frac{P_s}{A_{shear}}$$

Therefore for the mount to be safe in shear, the yield strength must be:

$$S_y = 12.55 \text{ MPa}$$

Aluminum property charts indicate that all aluminum alloys have yields strengths above this value, making this a safe design.

The shear force from the worm gear shaft is:

$$P_s = 3(36.57) = 109.71 \text{ N}$$

Assuming a bearing mount thickness of $t_b = 3/8$ in, a bearing width of $b = 6$ mm and a bearing diameter of $d_b = 19$ mm the shear area can be calculated:

$$A_{shear} = \pi d_b(t - b) = 2.10(10^{-4})$$

Therefore for the mount to be safe in shear, the yield strength must be:

$$S_y = 0.90 \text{ MPa}$$

Aluminum property charts indicate that all aluminum alloys have yields strengths above this value. This too, therefore, is a safe design.

The arm shaft has no axial forces and therefore no shear load on the bearing mounts.

3.6.6 Shafts

The shafts' sizes must be verified to ensure they are safe in bending, torsion and axial strength [46, 47]. Assuming the shafts will be made from 1020 cold rolled steel, the material properties are:

$$S_{ut} = 469 \text{ MPa}$$

$$S_y = 393 \text{ MPa}$$

$$S_{ys} = 0.58S_y$$

where S_{ut} is the ultimate tensile strength, S_y is the yield strength and S_{ys} is the shear yield strength. To find the endurance strength, S_e , the fatigue strength, S'_e must be found.

$$S'_e = 0.5S_{ut} \quad (3.82)$$

This value is multiplied by various correction factors to give:

$$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S'_e \quad (3.83)$$

where:

$$\begin{aligned} C_{load} &= 1 && \text{for bending} \\ &= 0.70 && \text{for axial load} \\ C_{size} &= 1 && \text{for } d \leq 8 \text{ mm} \\ &= 0.869d^{-0.097} \\ C_{surf} &= 0.88 && \text{for cold drawn or machined surface} \\ C_{temp} &= 1 && \text{for } T = 450^\circ\text{C} \\ C_{reliab} &= 0.814 && \text{for 90\% reliability} \end{aligned}$$

The required shaft diameter is calculated at each critical point, considering torsional, axial and bending loads for all three shafts. The bending stress is calculated as:

$$\sigma_b = \frac{32K_f M_a}{\pi d^3} \quad (3.84)$$

with K_f being the axial stress concentration, M_a the axial moment and d_s the resulting shaft size.

The torsional stress is:

$$\tau = \frac{16K_{fsm} T}{\pi d^3} \quad (3.85)$$

where K_{fsm} is the torsional stress concentration and T is the applied torque.

Finally, the axial stress is:

$$\sigma = \frac{4P_t}{\pi d^2} \quad (3.86)$$

and P_t is the tensile force.

Combining the stresses:

$$\left(\frac{\sigma}{S_y}\right)^2 + \left(\frac{\sigma_b}{S_e}\right)^2 + \left(\frac{\tau}{S_{ys}}\right)^2 = 1 \quad (3.87)$$

Worm Shaft:

The worm shaft is 12.7 mm (0.5 in), governed by the size of the worm. Checking this shaft size:

The axial load on this shaft is:

$$P_t = F_{wa} = 302.22 \text{ N}$$

The torque on the worm shaft is:

$$\tau_w = 0.4645 \text{ Nm}$$

Stress Concentrations:

The main stress concentrations are at the keyway and the pins. It is noted that the steps are very close to the edge where bending moments are small and there is no torsion. Therefore, the shaft size at this point is determined by the chosen bearing size.

Keyway:

for $r = 0.01 \text{ in}$

$$a^{\frac{1}{2}} = 0.093$$

for $S_{ut} = 68 \text{ ksi}$ (Table 6-6) [47]:

$$q = \frac{1}{(1 + (\frac{r}{a})^{1/2})} = 0.48$$

From Figure 9-16 [47]:

$$\begin{aligned}
\frac{r}{d} &= 0.02 \\
K_t &= 2.2 \\
K_{ts} &= 2.6 \\
K_f &= 1 + q(K_t - 1) = 1.576 \\
K_{fsm} &= 1 + q(K_{ts} - 1) = 1.768
\end{aligned}$$

Pins:

From Figure E-7 and E-8 [47]:

$$\begin{aligned}
\frac{d}{D} &= 0.1875 \\
K_t &= 2.05 \\
K_{ts} &= 2.725 \\
K_f &= 1 + q(K_t - 1) = 1.504 \\
K_{fsm} &= 1 + q(K_{ts} - 1) = 1.828
\end{aligned}$$

where $d = 3/32$ in is the pin diameter and $D = 1/2$ in is the shaft diameter at the pin.

Bending Moments:

The bending moments are determined in both the clockwise and counter-clockwise directions.

Clockwise:

$$\begin{aligned}
M_y &= -R_{w1x} \langle z \rangle^1 + F_{wt} \langle z - 33.5 \rangle^1 \\
M_x &= -R_{w1y} \langle z \rangle^1 + F_{wr} \langle z - 33.5 \rangle^1 + F_{wa}(r_w) \langle z - 33.5 \rangle^0 \\
M_{xy} &= \sqrt{M_x^2 + M_y^2}
\end{aligned}$$

Counter-clockwise:

$$\begin{aligned}
M_y &= -R_{w1x} \langle z \rangle^1 - F_{wt} \langle z - 33.5 \rangle^1 \\
M_x &= -R_{w1y} \langle z \rangle^1 + F_{wr} \langle z - 33.5 \rangle^1 - F_{wa}(r_w) \langle z - 33.5 \rangle^0 \\
M_{xy} &= \sqrt{M_x^2 + M_y^2}
\end{aligned}$$

Figure 3.31: Moment diagrams: combined moment for worm shaft

Since the shaft is symmetrical, the critical points are the same in both directions. They occur at the key and the pin closest to the bearing that takes the axial load. For the clockwise direction:

Key: 33.5 mm

$$M_{xy} = 3.29 \text{ Nm}$$

Pin: 14.5 mm

$$M_{xy} = 1.43 \text{ Nm}$$

In the counter-clockwise direction the moments are the same but pin 2 will have the higher moment rather than pin 1.

The endurance strength (from Equation (3.83)) is:

$$S_e = 109.28 \text{ MPa}$$

To check the shaft size, Equation (3.87) with $d = 12.7 \text{ mm}$ is calculated and if the value is less than or equal to 1, the proposed shaft diameter is safe. A safety factor, S_f , of 3 is added to each term.

Key:

$$\frac{\sigma^2}{S_y^2} + \frac{\sigma_b^2}{S_e^2} + \frac{\tau^2}{S_{ys}^2} = 0.502$$

Therefore the shaft diameter of 0.5 in is safe at the key.

Pin:

$$\frac{\sigma^2}{S_y} + \frac{\sigma_b^2}{S_e} + \frac{\tau^2}{S_{ys}} = 0.087$$

Therefore the shaft diameter of 0.5 in is safe at the pin. The proposed 0.5 in shaft diameter will be used.

Worm Gear Shaft:

The worm gear shaft is 12.7 mm (0.5 in), governed by the size of the gear. Checking this shaft size:

The axial load on the shaft is:

$$P_t = F_{ga} = 36.57 \text{ N}$$

The torque on the worm gear shaft between the worm gear and pulley is:

$$\tau_g = 9.60 \text{ Nm}$$

Stress Concentrations:

The stress concentrations on this shaft occur at the two pins.

Pins: From Figure E-7 and E-8 [47]:

$$\frac{d}{D} = 0.1875$$

$$K_t = 2.05$$

$$K_{ts} = 2.725$$

$$K_f = 1 + q(K_t - 1) = 1.504$$

$$K_{fsm} = 1 + q(K_{ts} - 1) = 1.828$$

where $d = 3/32$ in is the pin diameter and $D = 1/2$ in is the shaft diameter at the pin.

Bending Moments:

The bending moments are determined in both the clockwise and counter-clockwise directions.

Clockwise:

$$\begin{aligned} M_y &= R_{g1z} \langle x \rangle^1 + F_{pt} \langle x - 15 \rangle^1 - F_{gt} \langle x - 39 \rangle^1 \\ M_z &= R_{g1y} \langle x \rangle^1 - F_{gr} \langle x - 39 \rangle^1 - F_{ga}(r_{wg}) \langle x - 39 \rangle^0 \\ M_{yz} &= \sqrt{M_z^2 + M_y^2} \end{aligned}$$

Counter-clockwise:

$$\begin{aligned} M_y &= R_{g1z} \langle x \rangle^1 - F_{pt} \langle x - 15 \rangle^1 + F_{gt} \langle x - 39 \rangle^1 \\ M_z &= R_{g1y} \langle x \rangle^1 - F_{gr} \langle x - 39 \rangle^1 + F_{ga}(r_{wg}) \langle x - 39 \rangle^0 \\ M_{yz} &= \sqrt{M_y^2 + M_z^2} \end{aligned}$$

Figure 3.32: Moment diagrams: combined moment for worm gear shaft

From Figure 3.32, the critical points occur at the gear pin in the counter-clockwise direction and the pulley pin in the clockwise direction.

Gear Pin: 53.2 mm

$$M_{yz} = 1.05 \text{ Nm}$$

Pulley Pin: 28.2 mm

$$M_{yz} = 2.63 \text{ Nm}$$

The pulley pin is the most critical point. With the same endurance strength as above, the shaft size is verified at this point.

$$\frac{\sigma^2}{S_y} + \frac{\sigma_b^2}{S_e} + \frac{\tau^2}{S_{ys}} = 0.621$$

Therefore the shaft diameter of 0.5 in is safe and will be used.

Arm Shaft:

There is no axial load on this shaft. Only torsion and bending must be considered. A shaft diameter of 5/8 in is assumed. This size will be verified for strength.

From above, the torque on the arm shaft between the pulley and the arm is:

$$\tau_{arm} = 23.3 \text{ Nm}$$

Stress Concentrations:

Again, the stress concentration will occur at one of the pins.

Pins:

From Figure E-7 and E-8 [47]:

$$\begin{aligned} \frac{d}{D} &= 0.15 \\ K_t &= 2.15 \\ K_{ts} &= 2.79 \\ K_f &= 1 + q(K_t - 1) = 1.552 \\ K_{fsm} &= 1 + q(K_{ts} - 1) = 1.859 \end{aligned}$$

where $d = 3/32$ in is the pin diameter and $D = 5/8$ in is the shaft diameter at the pin.

Bending Moments:

$$M_y = -R_{a1z} \langle x \rangle^1 + F_{load} \langle x - 17.75 \rangle^1 + F_2 \langle x - 51 \rangle^1$$

Figure 3.33: Moment diagrams: combined moment for arm shaft

The critical points are the arm pin and the arm pulley pin.

Arm Pin: 17.75 mm

$$M_y = 3.99 \text{ Nm}$$

Arm Pulley Pin: 32.2 mm

$$M_y = 6.18 \text{ Nm}$$

Therefore the arm pulley pin is the most critical point. With the same endurance strength as above; Check $d = 15.875 \text{ mm}$ (5/8 in) using a safety factor of 3:

$$\frac{\sigma^2}{S_y} + \frac{\sigma_b^2}{S_e} + \frac{\tau^2}{S_{ys}} = 0.976$$

The shaft diameter of 5/8 in is safe and will be used.

3.6.7 Cable

The maximum force of the cable when the motor is at the thermal torque is 666.35 N and the stiffness of the Thera-Band tubing is 13.3 kN/m. For a starting point, 3 times the maximum cable force is approximately 2000 N and this will be used as a guideline for cable strength. The types of cable investigated were flat band cable and aircraft cable. In order for the cable to not affect the stiffness of the system, it must be significantly stiffer than

the spring. From previous tests [53], the 7x7, 3/32 in steel aircraft cable has a stiffness of 233 kN/m and failed at 4800 N. This is sufficiently stiff to cause no major variations in the chosen stiffness.

The main advantage of using the aircraft cable over the steel strapping is for safety. The steel strapping, has a very sharp edge and when in motion acts like a knife. This is not desirable for an open concept prototype. Information on steel strapping can be found from Signode Packaging Systems [54].

The cable that is attached to the spring and pulleys must withstand the tangential force from the worm gear pulley when the motor is run at the thermal torque. This gives:

$$F_{cable} = F_{pt} = 666.35\text{N} = 149.80\text{lb}$$

From Suncor Stainless [55], the 7x19 wire rope is chosen for its strength as well as flexibility. The recommended *WLL* (working load) is 20% of the *BL* (breaking load). The 3/32 in 316 steel has a working load of 160 lb and the 3/32 in 304 stainless steel has a working load of 180 lb. The 304 steel is chosen to give a working load of 16.6% of the breaking load (900 lb or 4000 N).

The length of cable that must be wound around the pulley is:

$$l_{cable} = R\theta_{arm} + 0.1 = 0.134 \text{ m}$$

where 0.1 m is a 200% elongation of the 5 cm long Thera-Band tube. For the worm gear pulley, $r_p = 0.0144 \text{ m}$, this means the cable must wind around it 1.48 times. The aircraft cable has a diameter of 3/32 in which, if the cable winds on top of itself, would greatly affect the desired gear ratio. Therefore, the pulley is designed so that the cable will lie next to itself, not on top. Also, the actual pulley radius will be the required radius minus half the cable diameter for the correct overall gear ratio to be achieved.

3.7 Electronics

The electronics for this prototype were kept as simple as possible. The goal was to create a setup that would allow tests to be run that would mimic simulations done in Matlab and to gain data from those tests to compare to the simulations. In order to do this, two basic components are required. First, a system to run the two motors is necessary. These motors should be controlled separately and at variable speeds. Secondly, sensors must collect data to monitor the motion of the arm shaft as well as the worm gear shaft in order to obtain information on the springs. Power supplies must also be chosen to run these systems.

The two motors will be run simultaneously. They each have a voltage of 15 V and run at a maximum continuous current of 4 A. A power supply to drive them must have a voltage of at least 15 V. A current of 10 A should be sufficient. The start up current can be accommodated for in the electronics, i.e. the H-bridge and the use of pulse width modulation. An Instek power supply, GPR-1810HD with 18 V and 10 A can be used.

Alternatively, a more portable solution is to use a rechargeable battery. Nickle Metal Hydride (NiMH) batteries have a low internal resistance which makes them ideal for applications with high current. Also, they can maintain close to the same voltage as they are discharged, dropping sharply only at the end of the charge. They are lightweight with a high energy density compared with other types such as Ni-Cad and lead acid [56].

3.7.1 Motor Driver

The MAXI Dual H-Bridge 2 Motor Driver Kit from the Robot Store [57] will be used to drive the motors. This kit will control two 10-40 V motors, each with up to 10 A continuous current. It uses pulse width modulation for variable speed and allows for both forward and reverse motion.

The kit came with soldering required. The kit was assembled according to the instructions given. Because the maximum continuous current per motor is 4 A, a heatsink was added according to the specifications [57].

Figure 3.34: MAXI-Dual 2 connection diagrams [57]

The motors are run via a PC parallel port. A 5 V power supply is required to run the motor driver. Figure 3.34 shows the set up for the connections.

As shown, pins A1 and A2 control Motor A while Motor B is controlled by B1 and B2. It should be noted that the actual board used is labelled backwards and Motor A is actually controlled by B1 and B2 and vice versa for Motor B. The signal given is a digital one of 5 V. This signal is then amplified through the H-bridges. Table 3.12 outlines the required signal for each motor to run in the clockwise and counter-clockwise directions. Signals

MOTOR A		
Pin	A1	A2
CW	0	1
CCW	1	0
Off	0	0
MOTOR B		
Pin	B1	B2
CW	0	1
CCW	1	0
Off	0	0

Table 3.12: Pin assignments for motor driver [57]

are sent to these pins using pulse width modulation. For example, if the desired motor voltage is 5 V, the signal will be sent so that it is on (15 V) for 33.3% of the time and off for 66.6%. The speed of this signal is variable. This particular motor driver can handle signals of approximately 2 – 16 kHz.

In order to minimize current spikes upon motor start up, the motor voltage will be ramped up to the desired running voltage. Using pulse width modulation, the length of the signal is gradually increased until it reaches the appropriate percentage. This can be shown in Figure 3.35.

A program written in Labview sends signals to the driver through a National Instruments 6024E DAQ board. The program allows the user to select the amount of time to wait before starting for each motor, the start up time, running time and shut down time. The pulse width in ms and the duty cycle (%) are also selected by the user. The motor direction

Figure 3.35: Pulse width modulated signal

is also variable by selecting the appropriate channel. This allows simple setup for various tests and experimentation to be carried out.

3.7.2 Sensors

Precision potentiometers are used as position sensors. A potentiometer converts rotary motion into a measure of resistance. They can be used to control electrical devices, such as radios, or as sensors. A potentiometer basically acts as a voltage divider where:

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in} \quad (3.88)$$

$R_1 + R_2$ is the total resistance of the potentiometer and R_2 is the resistance at a position along the resistor where the measurement is being taken. As the potentiometer turns, V_{out} will decrease until it is equal to zero. This occurs when the potentiometer reaches the end of the resistor.

Potentiometers come with various types of resistors, wirewound, nonwirewound, conductive plastic or hybrid. Wirewound potentiometers are the most common type. The

Figure 3.36: Voltage divider

resistive element is a tight coil of wire and measurements occur in steps. They are excellent in terms of accommodating high temperature and power. The nonwirewound type is capable of unlimited resolution. Conductive plastic potentiometers have a smooth film, unlimited resolution, low friction, low noise and long operational life. They are temperature sensitive, however, and have low power dissipation abilities. A hybrid potentiometer has a wirewound core covered with conductive plastic and redeems some of the better qualities of the two types.

The 6024E DAQ board from National Instruments will read the information from the potentiometers. This board has a 2.44 mV resolution for an input voltage of 5 V. Ideally the resolution should correspond to less than 1° of shaft rotation to maximize the quality of the information. The worm gear shaft will rotate a maximum of 1.5 times in each direction. To be safe, a 5-turn potentiometer is suggested. The most common potentiometer resistance value is $10000\ \Omega$. For a 5-turn pot, this gives $2000\ \Omega/\text{turn}$ or $5.56\ \Omega/^\circ$. With $V_{in} = 5\ \text{V}$ and $V_{out} = 2.44\ \text{mV}$ (one step on A/D board) the resistance for this step can be determined using Equation (3.88).

$$2.44(10^{-3}) = \frac{R_2}{10000} 5$$

This gives:

$$R_2 = 4.88 \, \Omega$$

At 0° , the resistance reading is $0 \, \Omega$. Therefore, for a potentiometer resolution of $5.56 \, \Omega/^\circ$, the position of the potentiometer shaft at R_2 is:

$$\frac{4.88 \, \Omega}{5.56 \, \Omega/^\circ} = 0.88^\circ$$

Therefore it is possible to measure rotations as small as 0.88° , which is satisfactory.

A single turn potentiometer will be used for the arm shaft. Using the same logic, the angular resolution is 0.175° .

Multi-turn potentiometers were difficult to obtain. The desired shaft diameter of $1/8$ in was not available. A potentiometer with a $1/4$ in shaft will be used instead and a coupling was constructed to fasten it to the worm gear shaft. The chosen potentiometer is Spectrol 535, $10 \, \text{K}\Omega$ with 5 turns. It has a wirewound resistance. The single turn potentiometer chosen was a Spectrol 149 Series with a resistance of $10 \, \text{K}\Omega$ [58]. It was chosen for its $1/8$ in shaft which will fit into a hole at the end of the arm shaft and is held in using set screws. The resistive element is cermet which is a hybrid type.

Chapter 4

Simulation

A computer simulation was performed in Matlab and the results of this simulation will be compared with actual data from the prototype. The first step was to derive the system's dynamic equations.

4.1 Dynamics

The system includes electrical and mechanical elements with both translational and rotational motion. This makes it difficult to obtain equations of motion using traditional Newton-Euler techniques. For this thesis, a bond graph modelling technique is used. For detailed information refer to Karnopp et al [59]. The advantage of using bond graphs is that many domains can be combined into a single equation (i.e. electric, hydraulic and mechanical). Also, it is easy to model nonlinear systems. It is based on energy conservation and power flow between system components. The generalized variables are effort, e , flow, f , momentum, p and displacement, q . Table 4.1 shows the values of these variables for different domains.

Variable	Mechanical Translation	Mechanical Rotation	Electric
e	force (N)	torque (Nm)	voltage (V)
f	velocity (m/s)	angular velocity (rad/s)	current (A)
p	momentum (Ns)	angular momentum (Nms)	flux linkage (Vs)
q	displacement (m)	angular displacement (rad)	charge (C)

Table 4.1: Bond graph variables for mechanical translation, rotation and electrical ports

Bond graphs consist of basic 1-port elements which include the resistor, R , capacitor, C and inertia, I . There are two 2-port elements, the transformer TF and the gyrator, GY which allow conversions due to gear ratios and motor constants, etc. In the following bond graphs, the constitutive relations for the gyrator are such that $f_1 = Ce_2$ and $e_1 = CF_2$ where C is a constant. For a transformer, $f_1 = Cf_2$ and $e_2 = Ce_1$. Finally, there are 3-port junction elements, 1 and 0. For a 1 junction, the efforts are equal and the flows add. The opposite is true for a 0 junction. Using these features, bond graphs can create state equations for any system.

The general system setup from Figure 1.1 is shown in the block diagram in Figure 4.1. The detailed dynamics of each of the seven blocks in Figure 4.1 are given in the following sections.

Figure 4.1: Bond graph block diagram

4.1.1 Motors

The chosen motor, the Maxon RE35, is a DC electric motor. Figure 4.2 shows the characteristics of a DC motor. L_a is the motor's inductance, R_a is the resistance and V is the input voltage. J_m is the rotational inertia of the motor, K_v is the motor's speed constant and K_τ is the torque constant.

Using this schematic the bond graph for a DC motor is given in Figure 4.3 where $K_{v,\tau}$ is the motor's torque and speed constants.

Figure 4.2: Motor schematic [60]

Figure 4.3: Bond graph; motor

4.1.2 Transmission

The chosen transmission consists of a gearhead, worm, worm gear and pulley. r_{gh} is the gearhead ratio, r_w is the pitch radius of the worm, r_{wg} is the pitch radius of the worm gear and r_p is the radius of the pulley. Also, the rotational inertias of the gearhead, worm, worm gear and pulley are J_{gh} , J_w , J_{wg} and J_p respectively. Two simulations will be performed, one with friction and one without. For the simulation without friction the bond graph is shown in Figure 4.4. Here N_g is the number of worm gear teeth. Taking friction into account we get Figure 4.5. In this case, the gear ratio between the worm and worm gear is calculated based on the lead angle, λ , and pressure angle, ϕ , of the gears. The bond graph

is based on Figure 3.1.

Figure 4.4: Bond graph; transmission with no friction

Figure 4.5: Bond graph; transmission with friction

4.1.3 Springs

Nonlinear springs will be used which means that the spring force is dependant on spring deflection. The spring force is $\Phi(q)$ where q is deflection. A damper may be placed in

parallel where the damping coefficient is b . The bond graph for this section is shown in Figure 4.6.

Figure 4.6: Bond graph; spring

4.1.4 Arm

The arm consists of a pulley of radius R and a shaft of length L_{arm} . The arm rotational inertia is given as J_a and includes both the pulley and the shaft. The applied force at the end of the arm (at the hand) is F_{ext} .

Figure 4.7: Bond graph; arm

4.1.5 State Equations

Two sets of state equations are developed. The first is a simple model which does not include friction between the gear teeth. The second includes friction and the nonbackdrivable condition.

No Friction

For the case with no friction, Figures 4.8 to 4.11 show the detailed bond graph. From these figures, the state equations are derived.

In order to simplify the equations of motion, the transmission section of the bond graph is simplified, combining the inertias of the gear train into one.

Figure 4.9 gives the constitutive relationships.

Constitutive Relationships:

$p_9 = (J_{gh} + J_w)f_9$; where p_9 is the angular momentum of the gearhead and worm
and f_9 is the angular velocity

$p_{12} = (J_{wg} + J_p)f_{12}$; where p_{12} is the angular momentum of the worm gear and pulley
and f_{12} is the angular velocity

Solving for e_8 (worm torque) and f_8 (worm angular velocity) the combined inertia is found to be:

$$J_{gt} = \frac{N_g^2(J_{gh} + J_w) + J_{wg} + J_p}{N_g^2} \quad (4.1)$$

Using this combined inertia, the new simplified bond graph is shown in Figure 4.9.

To find the equations of motion, the states must first be identified and defined.

MOTOR 1:

MOTOR 2:

Figure 4.8: Bond graph; motors - no friction

TRANSMISSION 1:

SIMPLIFY

TRANSMISSION 2:

Figure 4.9: Bond graph; transmission - no friction

SPRING 1:

SPRING 2:

Figure 4.10: Bond graph; spring - no friction

Figure 4.11: Bond graph; arm - no friction

States:

$p_2 = L_a f_2$; where p_2 is the flux linkage of motor 1 and f_2 is the current

$p_6 = J_m f_6$; where p_6 is the angular momentum of motor 1 and f_6 is the angular velocity

$e_{14} = \Phi(q_{14})$; where e_{14} is the spring force and q_{14} is the spring deflection for spring 1

$p_{17} = J_a f_{17}$; where p_{17} is the arm angular momentum and f_{17} is the angular velocity

$e_{24} = \Phi(q_{24})$; where e_{24} is the spring force and q_{24} is the spring deflection for spring 2

$p_{30} = J_m f_{30}$; where p_{30} is the angular momentum of motor 2 and f_{30} is the angular velocity

$p_{33} = L_a f_{33}$; where p_{33} is the flux linkage of motor 2 and f_{33} is the current

Next, the constitutive relationships and inputs are defined.

Constitutive Relationships:

$e_3 = R_a f_3$; where e_3 is the voltage drop across the resistor of motor 1 and f_3 is the current

$p_9 = J_{gt} f_9$; where p_9 is the angular momentum of transmission 1 and f_9 is the angular velocity

$e_{13} = b f_{13}$; where e_{13} is the force on damper 1 and f_{13} is the velocity

$e_{23} = b f_{23}$; where e_{23} is the force on damper 2 and f_{23} is the velocity

$p_{27} = J_{gt} f_{27}$; where p_{27} is the angular momentum of transmission 2 and f_{27} is the angular velocity

$e_{34} = R_a f_{34}$; where e_{34} is the voltage drop across the resistor of motor 2 and f_{34} is the current

Inputs:

$$e_1 = V_1$$

$$e_{35} = V_2$$

$$e_{19} = F_{ext}$$

Derivation of the state equations gives seven coupled first order differential equations:

$$\dot{p}_2 = V_1 - \frac{R_a}{L_a} p_2 - \frac{1}{K_v J_m} p_6 \quad (4.2)$$

$$\begin{aligned} \dot{p}_6 = & \frac{K_\tau r_{gh}^2 J_m}{L_a (r_{gh}^2 J_m + J_{gt})} p_2 - \frac{r_p^2 b}{N_g^2 (r_{gh}^2 J_m + J_{gt})} p_6 + \\ & + \frac{r_{gh} J_m R r_p b}{N_g J_a (r_{gh}^2 J_m + J_{gt})} p_{17} - \frac{r_p r_{gh} J_m}{N_g (r_{gh}^2 J_m + J_{gt})} \Phi(q_{14}) \end{aligned} \quad (4.3)$$

$$\dot{q}_{14} = \frac{r_p}{N_g r_{gh} J_m} p_6 - \frac{R}{J_a} p_{17} \quad (4.4)$$

$$\begin{aligned} \dot{p}_{17} = & \frac{R b r_p}{N_g r_{gh} J_m} p_6 - \frac{2 R^2 b}{J_a} p_{17} + R \Phi(q_{14}) - \frac{R b r_p}{N_g r_{gh} J_m} p_{30} - \\ & - R \Phi(q_{24}) + L_{arm} F_{ext} \end{aligned} \quad (4.5)$$

$$\dot{q}_{24} = \frac{R}{J_a} p_{17} + \frac{r_p}{N_g r_{gh} J_m} p_{30} \quad (4.6)$$

$$\begin{aligned} \dot{p}_{30} = & \frac{K_\tau r_{gh}^2 J_m}{L_a (r_{gh}^2 J_m + J_{gt})} p_{33} - \frac{r_p^2 b}{N_g^2 (r_{gh}^2 J_m + J_{gt})} p_{30} - \\ & - \frac{r_{gh} J_m R r_p b}{N_g J_a (r_{gh}^2 J_m + J_{gt})} p_{17} - \frac{r_p r_{gh} J_m}{N_g (r_{gh}^2 J_m + J_{gt})} \Phi(q_{24}) \end{aligned} \quad (4.7)$$

$$\dot{p}_{33} = V_2 - \frac{R_a}{L_a} p_{33} - \frac{1}{K_v J_m} p_{30} \quad (4.8)$$

Friction

The second simulation's bond graphs are shown in Figures 4.12 to 4.15. The friction between the worm and worm gear teeth is included in this case. Note that there are more bonds and consequently the numbering of the bonds has changed.

MOTOR 1:

MOTOR 2:

Figure 4.12: Bond graph; motors - friction

Figure 4.13: Bond graph; transmission - friction

SPRING 1:

SPRING 2:

Figure 4.14: Bond graph; spring - friction

Figure 4.15: Bond graph; arm - friction

Again, the system's states are defined.

States:

$p_2 = L_a f_2$; where p_2 is the flux linkage of motor 1 and f_2 is the current

$p_6 = J_m f_6$; where p_6 is the angular momentum of motor 1 and f_6 is the angular velocity

$e_{28} = \Phi(q_{28})$; where e_{28} is the spring force and q_{28} is the spring deflection for spring 1

$p_{31} = J_a f_{31}$; where p_{31} is the arm angular momentum and f_{31} is the angular velocity

$e_{38} = \Phi(q_{38})$; where e_{38} is the spring force and q_{38} is the spring deflection for spring 2

$p_{58} = J_m f_{58}$; where p_{58} is the angular momentum of motor 2 and f_{58} is the angular velocity

$p_{62} = L_a f_{62}$; where p_{62} is the flux linkage of motor 2 and f_{62} is the current

The constitutive relationships and inputs are given.

Constitutive Relationships:

$e_3 = R_a f_3$; where e_3 is the voltage drop across the resistor of motor 1 and f_3 is the current

$p_9 = (J_w + J_{gh}) f_9$; where p_9 is the angular momentum of the gearhead and worm of transmission 1 and f_9 is the angular velocity

$e_{18} = \mu e_{13}$; where e_{18} is the friction force, F_{f1} , between the worm and worm gear teeth of transmission 1, e_{13} is the normal force and μ is the coefficient of friction

$p_{23} = (J_{wg} + J_p) f_{23}$; where p_{23} is the angular momentum of the worm gear and pulley of transmission 1 and f_{23} is the angular velocity

$e_{29} = b f_{29}$; where e_{29} is the force on damper 1 and f_{29} is the velocity

$e_{37} = b f_{37}$; where e_{37} is the force on damper 2 and f_{37} is the velocity

$p_{41} = (J_{wg} + J_p)f_{41}$; where p_{41} is the angular momentum of the worm gear and pulley of transmission 2 and f_{41} is the angular velocity

$e_{50} = \mu e_{46}$; where e_{50} is the friction force, F_{f2} , between the worm and worm gear teeth of transmission 2, e_{50} is the normal force and μ is the coefficient of friction

$p_{55} = (J_w + J_{gh})f_{55}$; where p_{55} is the angular momentum of the gearhead and worm of transmission 2 and f_{55} is the angular velocity

$e_{61} = R_a f_{61}$; where e_{61} is the voltage drop across the resistor of motor 2 and f_{61} is the current

Inputs:

$$e_1 = V_1$$

$$e_{63} = V_2$$

$$e_{33} = F_{ext}$$

Derivation of the state equations for Branch 1 gives:

$$\dot{p}_2 = V_1 - \frac{R_a}{L_a} p_2 - \frac{1}{K_v J_m} p_6 \quad (4.9)$$

$$\begin{aligned} \dot{p}_6 = & \frac{K_r r_{gh}^2 J_m}{L_a J} p_2 - \left(\frac{b r_p r_w \tan \lambda}{J r_{wg}} \right)^2 p_6 + \frac{J_m R b r_p r_{gh} r_w \tan \lambda}{J J_a r_{wg}} p_{31} - \\ & - \frac{J_m r_{gh} r_w r_p \tan \lambda}{J r_{wg}} \Phi(q_{28}) - \frac{J_m r_w r_{gh}}{J \cos \lambda} F_{f1} \end{aligned} \quad (4.10)$$

where J is an equivalent inertia to simplify the equation:

$$J = J_m r_{gh}^2 + J_{wgh} + J_{gp} \left(\frac{r_w \tan \lambda}{r_{wg}} \right)^2 \quad (4.11)$$

where

$$J_{wgh} = (J_w + J_{gh})$$

and

$$J_{gp} = (J_{wg} + J_p)$$

$$\dot{q}_{28} = \frac{r_p r_w \tan \lambda}{r_{wg} r_{gh} J_m} p_6 - \frac{R}{J_a} p_{31} \quad (4.12)$$

$$\begin{aligned}
\dot{p}_{31} = & \frac{Rbr_p r_w \tan \lambda}{r_{wg} r_{gh} J_m} p_6 + R\Phi(q_{28}) - \frac{2R^2 b}{J_a} p_{31} - \\
& - R\Phi(q_{38}) - \frac{Rbr_p r_w \tan \lambda}{r_{wg} r_{gh} J_m} p_{58} + L_{arm} F_{ext}
\end{aligned} \quad (4.13)$$

The friction force between the worm and worm gear teeth must be calculated to complete Equation (4.10). Using Dean Karnopp's theory of stick-slip friction [61], three different values of friction can be found. A very small velocity region, $-DV < V < DV$, is chosen and a zero velocity is assumed inside this region. This is necessary when using digital computation because an exact zero value will not be computed. The friction force in this region, F_{feq} is determined using the forces present in the system and keeping the sliding velocity equal to zero. This will remain the friction force until a maximum value, F_{fs} , is reached. This will cause the system to slip into motion. The kinetic friction, F_{fk} , is the friction between the worm and worm gear teeth while the system is in motion. This is a function of velocity.

The kinetic friction, F_{fk1} , is calculated by using the constitutive relationship $e_{18} = \mu e_{13}$ with $\mu = \mu_k$. Moving to the right on the bond graph we get:

$$F_{fk1} = \frac{\mu_k}{C_k} F_{gt} \quad (4.14)$$

where F_{gt} , the tangential force on the worm gear tooth, is e_{21} on the bond graph and:

$$C_k = \cos \phi \cos \lambda - \mu_k \sin \lambda \quad (4.15)$$

From previous calculations, F_{gt} is found in terms of $\dot{p}_6$, p_6 , p_{31} and q_{28} . Substituting $\dot{p}_6$ for equation (4.10) and solving (4.14) for F_{fk1} we get:

$$\begin{aligned}
F_{fk1} = f_k \left(\frac{r_w r_{gh} J_{gp} K_\tau \tan \lambda}{r_{wg}^2 J L_a} p_2 + \frac{r_p^2 r_w b J_t \tan \lambda}{r_{wg}^4 r_{gh} J J_m} p_6 - \right. \\
\left. - \frac{r_p b R J_t}{r_{wg}^3 J J_a} p_{31} + \frac{r_p J_t}{r_{wg}^3 J} \Phi(q_{28}) \right)
\end{aligned} \quad (4.16)$$

where f_k is:

$$f_k = \frac{\mu_k r_{wg}^2 J \cos \lambda}{r_{wg}^2 J C_k \cos \lambda + \mu_k r_w^2 J_{gp} \tan \lambda} \quad (4.17)$$

and J_t :

$$J_t = J r_{wg}^2 - J_{gp} r_w^2 \tan^2 \lambda \quad (4.18)$$

The maximum static friction is found by taking (4.16), substituting μ_s for μ_k and setting $p_6 = 0$.

$$F_{fs1} = f_s \left(\frac{r_w r_{gh} J_{gp} K_\tau \tan \lambda}{r_{wg}^2 J L_a} p_2 - \frac{r_p b R J_t}{r_{wg}^3 J J_a} p_{31} + \frac{r_p J_t}{r_{wg}^3 J} \Phi(q_{28}) \right) \quad (4.19)$$

where f_s is:

$$f_s = \frac{\mu_s r_{wg}^2 J \cos \lambda}{r_{wg}^2 J C_s \cos \lambda + \mu_s r_w^2 J_{gp} \tan \lambda} \quad (4.20)$$

and C_s is:

$$C_s = \cos \phi \cos \lambda - \mu_s \sin \lambda \quad (4.21)$$

The equilibrium friction, F_{feq1} , is the friction force required to keep the system in equilibrium. It is found by calculating p_6 from the bond graph when:

$$p_6 = p_9 = p_{12} = 0$$

Setting $p_6 = 0$ and solving for F_{feq1} :

$$F_{feq1} = \frac{r_{gh} K_\tau \cos \lambda}{r_w L_a} p_2 + \frac{r_p b R \sin \lambda}{r_{wg} J_a} p_{31} - \frac{r_p \sin \lambda}{r_{wg}} \phi(q_{28}) \quad (4.22)$$

Due to symmetry, the equations for the second branch are the same as the first with the exception of the sign of R . Therefore:

$$\dot{q}_{38} = \frac{r_p r_w \tan \lambda}{r_{wg} r_{gh} J_m} p_{58} + \frac{R}{J_a} p_{31} \quad (4.23)$$

$$\begin{aligned} \dot{p}_{58} = & \frac{K_r r_{gh}^2 J_m}{L_a J} p_{62} - \left(\frac{b r_p r_w \tan \lambda}{J r_{wg}} \right)^2 p_{58} - \frac{J_m R b r_p r_{gh} r_w \tan \lambda}{J J_a r_{wg}} p_{31} - \\ & - \frac{J_m r_{gh} r_w r_p \tan \lambda}{J r_{wg}} \phi(q_{38}) - \frac{J_m r_w r_{gh}}{J \cos \lambda} F_{f2} \end{aligned} \quad (4.24)$$

$$\dot{p}_{62} = V_2 - \frac{R_a}{L_a} p_{62} - \frac{1}{K_v J_m} p_{58} \quad (4.25)$$

The friction values for this branch are:

$$\begin{aligned} F_{fk2} = f_k \left(\frac{r_w r_{gh} J_{gp} K_t \tan \lambda}{r_{wg}^2 J L_a} p_{62} + \frac{r_p^2 r_w b J_t \tan \lambda}{r_{wg}^4 r_{gh} J J_m} p_{58} + \right. \\ \left. + \frac{r_p b R J_t}{r_{wg}^3 J J_a} p_{31} + \frac{r_p J_t}{r_{wg}^3 J} \Phi(q_{38}) \right) \end{aligned} \quad (4.26)$$

$$F_{fs2} = f_s \left(\frac{r_w r_{gh} J_{gp} K_t \tan \lambda}{r_{wg}^2 J L_a} p_{62} + \frac{r_p b R J_t}{r_{wg}^3 J J_a} p_{31} + \frac{r_p J_t}{r_{wg}^3 J} \Phi(q_{38}) \right) \quad (4.27)$$

$$F_{feq2} = \frac{r_{gh} K_t \cos \lambda}{r_w L_a} p_{62} - \frac{r_p b R \sin \lambda}{r_{wg} J_a} p_{31} - \frac{r_p \sin \lambda}{r_{wg}} \Phi(q_{38}) \quad (4.28)$$

Spring Force, Stiffness and Damping

The actual equation of the Thera-Band is not known, therefore a quadratic force curve of a softening spring is used instead. It is assumed that the stiffness at 200% is zero, giving the required low spring stiffness at 175% deflection. With a spring length of 0.05 m, the force curve should be flat at 0.1 m. This gives the following curve:

$$\Phi(\delta) = -\delta^2 + 0.2\delta$$

To achieve approximately the same force values, this is multiplied by a constant of 50000. The resulting equation is:

$$\Phi(\delta) = 50000(-\delta^2 + 0.2\delta) \quad (4.29)$$

Figure 4.16 compares this spring force equation with the actual force curve of the ten blue Thera-Band tubes.

Figure 4.16: Force curves - theoretical spring and actual spring

The spring stiffness is simply the derivative of the force with respect to deflection which gives:

$$\Psi(\delta) = 50000(-2\delta + 0.2) \quad (4.30)$$

The range of joint stiffness possible using this theoretical spring is $K_{min} = 1$ Nm/rad and $K_{max} = 24$ Nm/rad.

Although no outside damping exists in the real prototype, there is internal damping in the springs themselves. A damping coefficient is used in this simulation to prevent oscillations of the spring during the tasks. For a well damped system at K_{max} , from [7]:

$$B = 1.414\sqrt{J_a K_{max}} \quad (4.31)$$

$$b = \frac{B}{2R^2} \quad (4.32)$$

where B is the rotational damping coefficient and b is the linear damping coefficient.

External Force

The external force, F_{ext} , applied to the arm is comprised of the applied load, F_{load} , as well as the impact force of the arm hitting the end points of the possible range of deflection. In this simulation, the deflection limits are -25° and 200° . When these limits are hit, a large force of 10000 N is applied in the opposite direction of the arm's motion. This force is applied gradually for 0.1° . The equation is given below:

$$\begin{aligned}
 F_{ext} &= F_{load} \\
 &\quad \text{for } -25 < \theta_{arm} < 200 \\
 F_{ext} &= F_{load} + \frac{-10000}{0.1}(\theta_{arm} - 200) \\
 &\quad \text{for } 200 < \theta_{arm} < 200.1 \\
 F_{ext} &= F_{load} - 10000 \\
 &\quad \text{for } \theta_{arm} > 200.1 \\
 F_{ext} &= F_{load} - \frac{-10000}{0.1}(\theta_{arm} - (-25)) \\
 &\quad \text{for } -25.1 < \theta_{arm} < -25 \\
 F_{ext} &= F_{load} + 10000 \\
 &\quad \text{for } \theta_{arm} < -25.1
 \end{aligned} \tag{4.33}$$

4.2 Matlab Simulation

The state equations derived above are used in Matlab to simulate the movement and forces of the arm. The code can be found in Appendix A.

4.2.1 Task Description

Three tasks were performed for both the simulation and the prototype tests. The first task was to change the arm position while keeping the stiffness constant. The second was

to change the stiffness, keeping the arm position constant, and the third was to lift a 2 kg weight. Approximate times, initial stiffness settings and initial position were kept constant between the simulations and testing. Due to the setup of the prototype, the range of motion is from 22.5° to 157.5° .

4.2.2 Position Change: Simulation

This task is run for 1.0 s ramping the voltage up to 15 V for 0.1 s, running at 15 V for 0.8 s and ramping back down to 0 V for 0.1 s. The stiffness is initially set at 16 Nm/rad. To match the prototype tests, the initial position is set at 0.7199 rad or 41.2° . Figures 4.17 to 4.22 show the results for the friction model.

Figure 4.17 shows the motor power is well below the maximum of 90 W and the motor current is continuous at 2 A, peaking at 5 A. In Figure 4.18 it can be seen that the arm runs with the continuous motor torque less than half the thermal torque limit ($\tau_m < \tau_{th}$), peaking at τ_{th} during motor start up. The thermal torque is shown with a horizontal line on the graph. Also in this figure, the no load speed of the motor is shown with a line on the motor speed graph. Motor 1 runs slightly below this speed while motor 2 is slightly above. This velocity difference can also be seen in the worm/worm gear sliding velocity in Figure 4.20.

The arm position changes from 41° to approximately 148° as shown in Figure 4.19. The arm stiffness is also shown in this figure. It increases very slightly throughout the arm movement.

Looking at the spring position in Figure 4.21, spring 1 starts at 17.28 cm and ends at 11.14 cm moving a total distance of 6.14 cm. Comparatively, spring 2 starts at 12.86 cm and moves to 19.57 cm which is a difference of 6.71 cm. Spring 2 covers slightly more distance than spring 1 in the same amount of time. This is consistent with the difference in motor speeds. Through other position change simulations done at different arm stiffness values, it is noted that this difference increases as the stiffness decreases. Therefore, the main reason for this speed difference is the spring force. A higher spring force (which

Figure 4.17: Simulation: position change - motor voltage, current and power

Figure 4.18: Simulation: position change - motor speed and torque

Figure 4.19: Simulation: position change - arm displacement, velocity and stiffness

Figure 4.20: Simulation: position change - gear sliding velocity and friction force

Figure 4.21: Simulation: position change - spring position and deflection

Figure 4.22: Simulation: position change - spring force and stiffness

Figure 4.23: Simulation: position change - motor speed and torque, no friction

Figure 4.24: Simulation: position change - arm displacement, velocity and stiffness, no friction

occurs at a lower stiffness) will put more torque on the motor. In motor 1's case, this will cause it to slow down. However, for motor 2, the force is pulling the same direction as the motor is turning, which decreases the work required by the motor, decreases friction in the transmission and allows the motor to turn faster.

This inconsistency in motor speed is also evident in the spring deflection. As the motors start, spring 1 deflects and spring 2 relaxes. This is due to the inertia of the arm. Once the motors reach their running speed, both springs gradually relax due to the small difference in motor speed. When the motors stop, spring 1 becomes slightly more relaxed while spring 2 deflects a small amount toward the initial position. Upon reaching a new equilibrium, the arm stiffness is slightly higher because, overall, both springs have a decreased deflection value.

The final figure, Figure 4.22 shows the spring force and deflection which are consistent with the spring deflection graphs. As the deflection increases, so does the force, while the stiffness decreases.

Comparing these results with the model without friction, it is found that the motor torque, τ_m is higher and the arm moves slower. Without friction, the final arm position for the same running time is approximately 152° . This is due to the higher forces in the gear teeth with friction. Therefore, friction is a large factor in the performance of this system. Figures 4.23 and 4.24 illustrate this.

4.2.3 Stiffness Change: Simulation

This task runs for the amount of time required to pull the spring through the full range of stiffness. In the friction case, it runs for 1.5 s and for the case with no friction, 1.4 s. Again, the voltage is ramped up and down for 0.1 s and is run at 15 V. The initial stiffness is 24 Nm/rad which is K_{max} for this theoretical spring. The initial position is 1.01 rad or 58° to match prototype tests. Figures 4.25 to 4.30 show the results for the friction model.

Figure 4.30 shows that as the spring is stretched to the low stiffness setting, the spring force increases. This causes an increase in motor current, power and torque as well as the

Figure 4.25: Simulation: stiffness change - motor voltage, current and power

Figure 4.26: Simulation: stiffness change - motor speed and torque

Figure 4.27: Simulation: stiffness change - arm displacement, velocity and stiffness

Figure 4.28: Simulation: stiffness change - gear sliding velocity and friction force

Figure 4.29: Simulation: stiffness change - spring position and deflection

Figure 4.30: Simulation: stiffness change - spring force and stiffness

Figure 4.31: Simulation: stiffness change - motor speed and torque, no friction

Figure 4.32: Simulation: stiffness change - arm displacement, velocity and stiffness, no friction

friction force between the worm and worm gear teeth (see Figures 4.25, 4.26 and 4.29). These values do, however, remain below the limits of the motor.

As the arm stiffness decreases, Figure 4.27 shows that the arm position gradually decreases. This is due to the torque caused by the arm's mass. As the arm stiffness decreases, the same torque will cause a larger arm deflection from the no load equilibrium position. Therefore a change of equilibrium position occurs as the springs are stretched. A simulation done with a massless arm showed no change in arm position. Also, when the motors start, there is an initial arm movement with gravity. After the motors turn off, the arm settles at its new equilibrium position and the arm velocity returns to zero.

Figure 4.29 gives the deflections and positions of the two springs. The position change of spring 1 is 9.29 cm while spring 2 moves 9.64 cm. Both springs start at the same deflection, but spring 2 deflects less throughout the simulation. This is due to the arm mass which stretches spring 1 and relaxes spring 2. This causes spring 2 to have a lower spring force and allows motor 2 to turn slightly faster. This also results in a slight decrease of arm position.

Comparing these results with the model without friction, again the motor torque, τ_m , is higher and the spring is stretched more slowly. This is shown in Figures 4.31 and 4.32.

4.2.4 Lifting 2 kg Weight: Simulation

This final task shows the ability of the arm to lift a 2 kg weight. Although the original requirement was 4 kg, the chosen spring can only handle 2 kg with an arm deflection less than 15°.

This task runs for the amount of time required to lift the weight through the arm's full range of motion (135°). In the friction case it runs for 1.3 s and 1.2 s without friction. This is very close to the design criteria of 1.2 s. The running voltage is 15 V and is ramped up and down as before. The initial stiffness is 22 Nm/rad which is calculated to be the arm stiffness that allows only 15° deflection before one of the springs becomes slack. The initial

position is 0.393 rad or 22.5°, the bottom limit of the arm's range of motion. Figures 4.33 to 4.38 show the results for the friction model.

Figure 4.37 shows the movement of the springs during this task. Motor 1 is doing the work lifting the weight whereas motor 2 simply releases the cable at the same speed. The arm stiffness is high which means that the spring force is low. Figure 4.38 shows an initial spring force of 100 N for each spring. Figure 4.37 also shows the deflection characteristics of the springs. As motor 1 pulls one end of spring 1 and the applied load on the arm adds a force to the other, spring 1 stretches. Spring 2 compresses and becomes slack. Once the arm is moving, the spring deflection remains relatively constant until the θ_{arm} gets close to 90°. At this point, the arm torque is at its maximum and spring 1 continues to stretch while spring 2 remains slack. When θ_{arm} passes 90°, spring 1 begins to relax while spring 2 stretches again and they both return to close to the original position when the motors stop. The arm continues to move after the motors are turned off because of this. Figure 4.35 shows that as the arm goes through 90°, it slows down due to the higher torque on the arm.

The motor current and power are well within the limits for both motors (Figure 4.33). Motor 2 has extremely low values because the spring force is zero when the spring becomes slack. As this motor is simply letting out the cable for motor 1 to pull in, there is virtually no load on the motor for this task. This is also shown in Figure 4.34 in the motor torque values. The motor speed is at the no load speed for motor 2 and slightly below for motor 1.

Comparing these results with the model without friction, the motor torque, τ_m is higher and the weight is lifted more slowly. Without friction, the arm speed meets the required 1.2 s. With friction it is slightly over this time. This is shown in Figures 4.39 and 4.40.

Figure 4.33: Simulation: lifting 2 kg weight - motor voltage, current and power

Figure 4.34: Simulation: lifting 2 kg weight - motor speed and torque

Figure 4.35: Simulation: lifting 2 kg weight - arm displacement, velocity and stiffness

Figure 4.36: Simulation: lifting 2 kg weight - gear sliding velocity and friction force

Figure 4.37: Simulation: lifting 2 kg weight - spring position and deflection

Figure 4.38: Simulation: lifting 2 kg weight - spring force and stiffness

Figure 4.39: Simulation: lifting 2 kg weight - motor speed and torque, no friction

Figure 4.40: Simulation: lifting 2 kg weight - arm displacement, velocity and stiffness, no friction

Chapter 5

Prototype Testing

5.1 Prototype

A prototype was built according to the drawings in Appendix B. This prototype was then used to perform the same three tasks as the simulation.

Figure 5.1: Bench top prototype

Figure 5.2: Bench top prototype - motor

Figure 5.3: Bench top prototype - transmission

Figure 5.4: Bench top prototype - spring

Figure 5.5: Bench top prototype - elbow joint

5.2 Test Procedure

The first step in the test procedure was to calibrate the potentiometers so that the final graphs were given in actual position rather than a voltage reading. This was done for both the arm position and the spring position at the motor end.

First, with the arm in the bottom position (22.5°) and the springs at their rest length, the distance between the motor mount and the spring was measured for both springs. This number was used as a starting point for spring deflection equations. For spring 1, $x_1 = 26.7$ cm and for spring 2, $x_2 = 19.6$ cm.

To calibrate the arm angle, a reading was taken at the two end points of the arm deflection (22.5° and 157.5°). A constant was calculated:

$$C_{arm} = \frac{\theta_{157.5} - \theta_{22.5}}{2.36}$$

where 2.36 rad is the range of motion. For each arm measurement recorded, V_{arm} , the actual angle in radians is found by:

$$\theta_{arm} = \frac{V_{arm} - \theta_{22.5}}{C_{arm}} + 0.393$$

with 0.393 rad as the bottom limit (22.5°).

The spring position calibration was done in a similar manner. Two readings were taken from the potentiometers at the worm gear with each spring in a different position. Measurements of the distances between the motor mounts and the springs were recorded as $x_{m1_1} = 21.1$ cm, $x_{m1_2} = 17.8$ cm, $x_{m2_1} = 22.2$ cm and $x_{m2_2} = 18.9$ cm. The difference between the two positions was 3.3 cm. A spring position constant was calculated for each motor:

$$C_{x_1} = \frac{x_{m1_1} - x_{m1_2}}{3.3}$$

$$C_{x_2} = \frac{x_{m2_1} - x_{m2_2}}{3.3}$$

For each recorded spring measurement, V_{spring_1} and V_{spring_2} , the actual position can be found:

$$x_{spring_1} = \frac{V_{spring_1} - x_{m1_1}}{C_{x_1}} + x_{m1_2}$$

$$x_{spring_2} = \frac{V_{spring_2} - x_{m2_1}}{C_{x_2}} + x_{m2_2}$$

Each task follows the same procedure for testing. First, the initial stiffness must be measured. To do this, one reading of arm position is taken when it is set at the desired initial position and stiffness for the particular task. A 0.4 kg load is applied to the end of the arm, and another reading is taken. From these two readings, an approximate value of the arm stiffness can be obtained. The arm angle, θ_{arm} , is recorded for each position, the difference between the two being Δ . The torque at the final position can be calculated using the angle:

$$\tau_{arm} = F_{load} \sin(\theta_{arm}) L_{arm}$$

where L_{arm} is the arm length, 0.33 m. The arm stiffness can then be approximated:

$$K = \frac{\tau_{arm}}{\Delta}$$

Once the stiffness has been determined, the task can be performed and the potentiometers will record the movements. When the task is complete, the stiffness measurement must be repeated to find the final arm stiffness value.

5.3 Test Results

5.3.1 Position Change: Test

For this task, the arm position starts at 41.25° . The arm is raised for approximately 1 s and the movement recorded using the potentiometers. The initial and final stiffness values are measured as described above.

K_{init} (Nm/rad)	15.75
K_{final} (Nm/rad)	16.78
$\theta_{init}(^\circ)$	41.25
$\theta_{final}(^\circ)$	133.47
Test time (s)	0.98

Table 5.1: Position change data

Figure 5.6: Test: position change - arm position

Figure 5.7: Test: position change - spring position and spring deflection

5.3.2 Stiffness Change: Test

For this task, the springs are initially stretched to approximately 0.5 cm from the rest length. The arm position is 58.06° . The springs are stretched to 175% deflection. The arm and spring movement is recorded using the potentiometers. The initial and final stiffness values are measured as described above.

K_{init} (Nm/rad)	24.15
K_{final} (Nm/rad)	14.01
$\theta_{init}(^\circ)$	58.06
$\theta_{final}(^\circ)$	52.91
Test time (s)	2.27

Table 5.2: Stiffness change data

Figure 5.8: Test: stiffness change - arm position

Figure 5.9: Test: stiffness change - spring position and spring deflection

5.3.3 Lifting 2 kg Weight: Test

In this task, the initial stretch of the springs is approximately 1 cm so that the arm deflection will be 15° when the 2 kg weight is added to the endpoint of the arm. The weight is hung from a string approximately 10 cm in length. The initial arm position is the bottom of the range of motion, 22.5° . The arm then lifts the weight to the top position of 157.5° and the time is noted. The movement is recorded using potentiometers. A stiffness measurement is taken at an intermediate arm angle to approximate arm stiffness for this task.

K (Nm/rad)	23.65
$\theta_{init}(^\circ)$	22.5
$\theta_{final}(^\circ)$	157.5
Test time (s)	1.42

Table 5.3: Lifting 2 kg weight data

Figure 5.10: Test: lifting 2 kg weight - arm position

Figure 5.11: Test: lifting 2 kg weight - spring position and spring deflection

Figure 5.12 shows the arm deflection when a 2 kg load is applied. It is shown to be approximately 16.5° , slightly over the desired 15° .

Figure 5.12: Test: lifting 2 kg weight - arm deflection

5.4 Discussion

The electrical setup described in the detailed design section proved to be insufficient for this prototype. The motor driver worked sporadically and was not dependable. When a load was applied to the arm, or the springs were pulled past about 100% deflection, the motors would not run at all. Therefore, either the current present in the system is higher than the

allowable 10 A per motor, or the motor driver cannot actually handle its rated current. The Instek power supply also had difficulty handling the current. If the current spiked over 10 A total (5 A per motor) it would switch to current mode and would no longer supply the required 15 V. It can be concluded from this that there are current spikes higher than 5 A per motor.

The final setup for the tests was simply to run the motors from the NiMH battery, which can handle current spikes, and to physically switch the wires around to change motor direction. Although this was fine for the preliminary tests performed for this thesis, a better motor driver is necessary in the future.

5.4.1 Position Change

This task shows the system changing arm position while keeping the stiffness constant. This means that the motors run at the same time, but in opposite directions. Results of the simulation are given in Figures 4.17 to 4.22 and Figures 5.6 and 5.7 for the tests.

The simulation shows less friction than there is in the real system since the arm moves about 15° more than the test in the same amount of time. A large part of this is likely the starting and stopping friction of the gears. Both the simulation and the test show that the stiffness increases slightly when changing arm position.

Figure 5.7 shows the spring positions and deflections for the test. The same characteristics occur here as in the simulation (Figure 4.21). During the time of the test, spring 1 moves a total of 5.77 cm while spring 2 moves 6.38 cm. This is a difference of 0.61 cm which is very close to the difference noted in the simulation of the same task. This difference can be explained in the same manner as the simulation; motor 2 moves faster because the spring force is applied in the same direction as the motor motion. This causes the motor speed to increase. Motor 1 has the opposite effect and is therefore slower. Other reasons that may appear in the test and not in the simulation that may account for some of this motor speed difference are inconsistencies in spring length, motor parameters, electrical connections and drive friction. The deflection of spring 1 increases slightly at first

due to the arm inertia, then it remains relatively constant. Spring 2 relaxes when spring 1 stretches, and continues to relax as the task is carried out due to the difference in motor speed and the inertia of the arm.

5.4.2 Stiffness Change

The stiffness is varied in this task while the arm position is kept constant. This is done by running both motors at the same time in the same direction. Figures 4.25 to 4.30 show the results of the simulation while Figures 5.8 and 5.9 show the test results. This task stretches both springs from their highest to lowest stiffness. The simulation takes much less time than the real test. Figure 5.9 shows that the motors start to slow down as the spring stretches due to the increasing spring force, which in turn increases the frictional force between the gears. The calculated spring force at the lowest stiffness is much lower than the allowable force for a motor torque less than the thermal limit. The difficulty the motors had in stretching the springs can be explained by friction in the transmission as well as possibly a higher spring force than expected.

As the springs reach the lowest stiffness (175% deflection), both the simulation and test show the arm angle decreasing slightly. This is mostly due to the arm mass, which causes greater arm deflections from the no load equilibrium position as the arm stiffness decreases. Figure 5.9 shows that spring 2 deflects less than spring 1. Also, the motor end of spring 2 moves slightly further (9.66 cm compared with 9.47 cm). This is also consistent with the simulation results. Again, spring 2 is deflected slightly due to the arm inertia, which causes a small decrease in spring force. This causes motor 2 to run slightly faster and therefore the arm position decreases. The decrease in arm position is less in the test than the simulation due to a lighter arm mass.

In the test, actual arm stiffness at the low stiffness end is shown to be 14 Nm/rad. According to the design, it should theoretically be closer to 8 Nm/rad. This is most likely due to inaccuracies in the method of measurement as well as differences in individual tube length within each set of ten. The spring data given in Table 3.7 assumes identical tube sections. Each spring set should be tested as a unit for accurate force and stiffness curves.

Also it is noted that it takes a bit of time for the spring to fully relax, as can be seen in Figure 3.18 where the relaxation curve is slightly higher than the extension curve. If tests are done quickly one after the other, the maximum stiffness will decrease from test to test.

5.4.3 Lifting 2kg Weight

For the final test, a 2 kg load was lifted through the entire range of motion of the arm. Figures 4.33 to 4.38 show the results of the simulation while Figures 5.10 and 5.11 show the test results. The test took approximately 0.2 s longer to reach the top than the simulation did, again showing that the simulation did not account for as much friction as is actually present in the system. Both simulation and test show spring 2 going slack (by 0.5 cm). This is due to the fact that the applied load of 2 kg is slightly higher than the calculated torque that the spring can handle (1.9 kg) especially when the arm mass is included. This occurred when the arm went through 90° where the torque is the highest.

Figure 5.11 shows spring positions and deflections for the test. Spring 2 moves faster than spring 1 in this task, it covers 10.75 cm versus the 9.75 cm of spring 1. This difference can be attributed to the spring force in the first part of the test and the fact that there is no load on the motor for the part of the test that the spring is slack. The spring deflections are also shown in this figure and they are similar to the simulation deflections. Spring 1 is initially stretched and continues to stretch as the arm passes through 90°. At this point, there is a dip in the curve which can be accounted for due to swinging of the weight at the end of the arm. As the arm position reaches the top, spring 1 begins to relax and spring 2 is stretched once again. In the test, the motors actually run for a short time after the arm hits the endpoint. This is why the spring deflections make a sudden turn at 2.25 s. If the arm stop wasn't there, the springs would simply return to their original position as they did in the simulation.

Figure 5.10 also shows evidence of the swinging weight as the arm speed increases just after 1.5 s. It then slows just after 90° when the arm torque is at its highest.

The final figure, Figure 5.12 shows the arm deflection when a 2 kg weight is added to the endpoint at an initial position of approximately 90° . The resulting deflection is shown to be 16° which is close to the estimated 15° at this stiffness setting.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

A variable stiffness prosthetic arm bench top prototype was designed, constructed, mathematically modelled and tested. The chosen configuration consisted of two actuators arranged antagonistically around the elbow joint. Each actuator was made up of a dc motor, gear-head, nonbackdrivable transmission, nonlinear spring and two pulleys.

Using design criteria based on currently available upper arm prostheses, the components were chosen. A Maxon RE35 dc brush motor with a 4.8:1 gearhead was chosen for its torque and speed capabilities as well as the quick delivery time. A worm and worm gear set from Boston Gear made up the transmission. A single start hardened steel worm and 30 tooth bronze worm gear with a diametral pitch of 12 met the strength and wear criteria for the chosen motor running at its thermal torque limit.

It was shown that both a hardening spring or a softening spring could be used in this design. The two main springs studied were the belleville washer and rubber tubing. Both of these act as softening springs that produce a hardening joint. It was determined that although the belleville washers have ideal spring force and stiffness characteristics, there is too much friction between the washers, creating a much higher spring force than originally calculated. This causes problems with the motor's ability to deflect the spring. The rubber tubing is chosen and rather than one very thick tube, ten 5 cm lengths of blue Thera-Band tubing were used to create an arm stiffness within the required range.

The radii of the two remaining pulleys were determined to create the desired total gear ratio required to move the arm through 135° in 1.2 s. The elbow pulley radius was 0.035 m while the worm gear pulley radius was 0.0144 m.

This system was mathematically modelled using bond graphs and Matlab. Two simulations were written, one that included the transmission friction and one that did not. The models did not include gear tooth backlash or chattering in order to get a clear picture of the overall system dynamics, taking the focus away from the gear teeth themselves. Three tasks were performed using these simulations. The first involved changing the arm position, the second changing the stiffness and the third lifting a 2 kg weight through the entire range of motion. The results were compared with the same tasks performed using the prototype.

The intended method of control for these tests was to run the motors using a Maxi Dual H-bridge motor driver. A program in Labview was written to send a pulse width modulated signal to the motor driver for a predetermined amount of time. The arm was to be powered by either a 10 A, 18 V power supply from Instek or a 14.4 V, 4500 mAh NiMH rechargeable battery. Current spikes proved to be too high for the driver as well as the Instek power supply. Instead, the battery was simply plugged directly into the wires connected to the motors and the wires were physically switched in order to change motor direction. This method was not as accurate as the voltage could not be gradually ramped up to 15 V and the test could not be run for a very specific amount of time. However, it was sufficient for the requirements of this thesis. The NiMH battery performed well and can be used for future testing. The three tests were done and the arm and motor position was recorded. Also, the initial and final stiffness values were measured for each test.

Testing has shown that the prototype works as expected and basically agrees with the Matlab simulations. Notable differences are higher current spikes that caused the motor driver and power supply to be unable to perform as expected and higher friction which caused the simulation to be optimistic with arm speeds.

The arm's performance when lifting the 2 kg load was 1.42 s for the full range of flexion. This is very close to the design goal of 1.2 s. The speed was not as good for the stiffness

change. The motors could not handle the load of the springs at the fully stretched length and gradually slowed to a stop. A spring with a lower overall force would solve this problem.

Both the simulation and the tests show a relatively constant stiffness can be maintained during a position change. The stiffness increases a small amount during the task due to the arm's inertia. This increase is higher for a lower initial arm stiffness, which gives a high spring force. When the stiffness is decreased from K_{max} to K_{min} , the arm position lowers slightly. This is also due to the inertia of the arm creating a changing equilibrium position.

6.2 Recommendations

The first issue that must be dealt with is designing a more sophisticated electrical setup that can handle the current values present in the motors. A motor driver that can handle current spikes over 10 A per motor is required. This motor driver should be controlled through pulse width modulation to decrease the current spike amplitude.

The configuration for this prototype was chosen due to its natural setup (i.e. motors acting like the biceps and triceps of a natural arm). This prototype can now be used as a test device to determine methods of control. As discussed in the Literature Review, control is the biggest challenge in designing devices with compliant actuators. Control systems need to be designed to allow an amputee to easily control the arm movement and stiffness using the most natural muscle contractions possible. The prototype should be tested using myoelectric control.

This prototype can also provide a bench model for testing different types of actuators and springs. The maximum spring force of the current spring has been shown to be too high for the chosen motor. Since this motor is at the maximum allowable motor size, more research is required to find other spring possibilities. A spring with a lower force and higher torque capabilities would be ideal. Also, damping may be required and should be considered. An alternative that should also be studied is combining the actuators and the stiffness, using different types of artificial muscles and active polymers. This would replace the current actuation system and would greatly reduce the size and weight of the entire system.

To create a wearable prototype using this design, the main requirement would be to reduce the size and weight of the system. The worm gear is the largest component and the large size is due to the wear factor. A smaller worm gear could be used if the material geometry factor, K_w , was higher. This would require a harder material for both the worm and worm gear. Also, the Maxon RE30 motor should replace the RE35. Lightweight materials for pulleys, mounts and casing are necessary to replace the aluminum parts found in the bench top model.

The main problem with the current design is that there are high forces when the arm is at a low stiffness. This takes away the energy savings of having a low stiffness. If a hardening spring is used instead of the current softening spring, there would be very low forces at the low stiffness, but at the high stiffness the motor would have to support both the weight the arm is lifting and the full spring force at the same time. With the size of motor currently being used, that would not be possible. The energy requirements of the softening spring setup versus the hardening one can be studied and hardening springs should be tested. Ideally, if the setup of motors, nonbackdrivable transmission and nonlinear springs are used in a wearable arm, they should be set up like Configuration 3, where one motor controls the stiffness and the other the position.

References

- [1] *Northwestern University Prosthetics-Orthotics Center. Prosthetics History.* <<http://www.nupoc.northwestern.edu>>. [Accessed February 21, 2005].
- [2] *Liberating Technologies.* <<http://www.liberatingtechnologies.com>>. [Accessed Feb 21, 2005].
- [3] J. Dailami. The myoelectric arm: It's electrifying. <ilumin.usc.edu>, 2002.
- [4] D.H. SilcoxIII, M.D. Rooks, R.R Vogel, and L.L. Fleming. Myoelectric prostheses. *The Journal of Bone and Joint Surgery*, 75(12), 1993.
- [5] *Utah Arm.* <<http://www.utaharm.com>>. [Accessed February 21, 2005].
- [6] D.L.R. Russell. Energy expenditure in upper limb prostheses performing constrained movements. *UNB's International Symposium on Myoelectric Control*, 1993.
- [7] T. Burke. Modeling and evaluation of nonbackdrivable transmissions for variable stiffness prostheses. Master's thesis, Carleton University, April 2000.
- [8] J.H. Park. Impedance control for biped robot locomotion. *IEEE Transactions on Robotics and Automotion*, 7(6), 2001.
- [9] N. Hogan. Impedence control: An approach to manipulation: Part 1-Theory, Part 2-Implementation, Part 3-Applications. *Journal of Dynamic Systems, Measurement and Control*, 107:1–24, 1985.

- [10] L. Zollo, B. Siciliano, C. Laschi, G. Teeti, and P. Dario. Compliant control for a cable-actuated anthropomorphic robot arm: An experimental validation of different solutions. *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*, 2002.
- [11] N. Boubekri and P. Chakraborty. Robotic grasping: gripper designs, control methods and grasp configurations-a review of research. *Integrated Manufacturing Systems*, 13(7):520–531, 2002.
- [12] G.J.M. Tuijthof and J.L. Herder. Design, actuation and control of an anthropomorphic robot arm. *Mechanism and Machine Theory*, 35(945-962), 2000.
- [13] B. Moehl. A bionic robot arm with compliant actuators. *Proceedings of SPIE*, 4196:82–85, 2000.
- [14] D.G. Caldwell and N. Tsagarakis. "soft" grasping using a dextrous hand. *Industrial Robot: An International Journal*, 27(3):194–199, 2000.
- [15] N. Hogan. Adaptive control of mechanical impedance by coactivation of antagonist muscles. *IEEE Transactions on Automatic Control*, 29(8), 1984.
- [16] G.H. Pfreundschuh, V. Kumar, and T. Sugar. Design and control of a 3 d.o.f. in-parallel actuated manipulator. *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, 1991.
- [17] M. Cutkosky. Biomimetic robots for robust operation in unstructured environments. <<http://www-cdr.stanford.edu/biomimetics/pdf/biomimetics.pdf>>, October 1997.
- [18] G. Mennitto and M. Buehler. Carl: A compliant articulated robot leg for dynamic locomotion. *Robotics and Autonomous Systems*, 18:337–344, 1996.
- [19] J. Yamaguchi, D. Nishino, and A. Takanishi. Realization of dynamic biped walking varying joint stiffness using antagonistic driven joints. *Proceedings of the 1998 International Conference on Robotics and Automation*, 1998.

- [20] R.Q. van der Linde. Design, analysis, and control of a low power joint for walking robots by phasic activation of mckibben muscles. *IEEE Transactions on Robotics and Automation*, 15(4), 1999.
- [21] K.Y. Yi. Walking of a biped robot with compliant ankle joints: Implementaion with KUBCA. *Proceedings of the 39th Conference on Decision and Control*, 2000.
- [22] Y. Tamura and M. Saito. Adaptive compliant motion control of pseudo-muscular visco-elastic actuator: Simulation and experiments. *11th Conference of the ESB*, 1998.
- [23] R.D. Beer, H.J. Chiel, R.D. Quinn, and R.E. Ritzmann. Biorobotic approaches to the study of motor systems. *Current Opinion in Neurobiology*, 8(6):777–782, 1998.
- [24] R.D. Quinn, G.M. Nelson, R.J. Bachmann, D.A. Kingsley, J. Offi, and R.E. Ritzmann. Insect designs for improved robot mobility. *Proceedings of the 4th International Conference on Climbing and Walking Robots, Berns and Dillmann eds., Prof. Eng. Pub.*, pages 69–76, 2001.
- [25] P. Dario, E. Guglielmelli, and C. Laschi. Humanoids and personal robots: Design and experiments. *Journal of Robotic Systems*, 18(12):673–690, 2001.
- [26] G. Cheng, A. Nagakubo, and Y. Kuniyoshi. Continous humanoid interaction: An integrated perspective - gaining adaptivity, redundancy, flexibility - in one. *Robotics and Autonomous Systems*, 37:161–183, 2001.
- [27] T. Yamamoto and Y. Kuniyoshi. Harnessing the robot’s body dynamics: a global dynamics approach. *Proceedings of the 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2001.
- [28] W.S. Harwin. Design and control of a compliant mobile arm support for assisting arm movements. *Advancement of Assistive Technologies*, 3:320–325, 1997.
- [29] K. Koganezawa, Y. Watanabe, and N. Shimizu. Antagonist muscle-like actuator and its application to multi-d.o.f. forearm prosthesis. *Advanced Robotics*, 12(7):771–789, 1999.

- [30] K. Koganezawa and M. Yamazaki. Mechanical stiffness control of tendon-driven joints. *Proceedings of the 1999 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1999.
- [31] C. English. Mechanics and design of a variable stiffness actuator for use in impedance control. Master's thesis, Carleton University, 1996.
- [32] M. Pecson, K. Ito, Z. Luo, A. Kato, T. Aoyama, and M. Ito. Compliance control of an ultrasonic motor powered prosthetic forearm. *IEEE International Workshop on Robot and Human Communication*, 1993.
- [33] J. Yamaguchi and A. Takanishi. Development of a biped walking robot having antagonistic driven joints using nonlinear spring mechanism. *Proceedings of the 1997 IEEE International Conference on Robotics and Automation*, 1997.
- [34] K. Koganezawa and S. Ban. Stiffness control of antagonistically driven redundant d.o.f. manipulator. *Proceedings of the 2002 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2002.
- [35] C. English. *Stiffness Behaviour in Two Degree of Freedom Mechanisms*. PhD thesis, Carleton University, May 2000.
- [36] C English. Spring characteristics. Correspondance, August 1999.
- [37] C. Felippa. Nonlinear finite element methods. <<http://caswww.colorado.edu>>, 2001.
- [38] C. English and D. Russell. Mechanics and stiffness limitations of a variable stiffness actuator for the use in prosthetic limbs. *Mechanism and Machine Theory*, 34:7–25, 1999.
- [39] J. McIntyre, F.A. Mussa-Ivaldi, and E. Bizzi. The control of stable postures in the multijoint arm. *Experimental Brain Research*, 110:248–264, 1996.
- [40] T.E. Milner. Contribution of geometry and joint stiffness to mechanical stability of the human arm. *Experimental Brain Research*, 143:515–519, 2002.

- [41] *Maxon Motor USA*. <<http://www.maxonmotorusa.com>>. [Accessed February 21, 2005].
- [42] *Faulhaber - Micromo Electronics*. <<http://www.micromo.com>>. [Accessed February 21, 2005].
- [43] *Boston Gear*. <<http://www.bostongear.com>>. [Accessed February 21, 2005].
- [44] *W.M. Berg Inc.* <<http://www.wmberg.com>>. [Accessed February 21, 2005].
- [45] *Quality Transmission Components*. <<http://www.qtcgears.com>>. [Accessed February 21, 2005].
- [46] R. C Juvinall. *Fundamentals of Machine Component Design*. John Wiley and Sons, 2nd edition, 1991.
- [47] R.L. Norton. *Machine Design, An Integrated Approach*. Prentice Hall, 1998.
- [48] *Spaenaur*. <<http://www.spaenaur.com>>. [Accessed February 21, 2005].
- [49] *Thera-Band*. <<http://www.thera-band.com>>. [Accessed February 21, 2005].
- [50] *Primeline Industries*. <<http://www.primelineindustries.com>>. [Accessed February 21, 2005].
- [51] E. Oberg, F.D. Jones, and H.C. Horton. *Machine Design Handbook*. Industrial Press Inc., 23rd edition, 1989.
- [52] *SKF*. <<http://www.skf.com>>. [Accessed February 21, 2005].
- [53] S. Scott. Prosthetic limb prototype. Undergraduate Thesis, Carleton University, 2000.
- [54] *Signode Packaging Systems*. <<http://www.signode.com>>. [Accessed February 21, 2005].
- [55] *Suncor Stainless*. <<http://www.suncorstainless.com>>. [Accessed February 21, 2005].
- [56] *Lumicycle*. <<http://www.lumicycle.co.uk/nimh.html>>. [Accessed April 4, 2005].

-
- [57] *Robot Store*. <<http://www.robotstore.com>>. [Accessed February 21, 2005].
- [58] *Spectrol Electronics*. <http://www.nteinc.com/pot_web>. [Accessed February 21, 2005].
- [59] D. Karnopp, D. Margolis, and R. Rosenberg. *System Dynamics: A Unified Approach*. John Wiley & Sons, 2nd edition, 1990.
- [60] *Agilant Technologies - Educator's Corner. Introduction to the DC Motor*. <<http://www.educatorscorner.com/media/Exp78.doc>>. [Accessed February 21, 2005].
- [61] D. Karnopp. Computer simulation of stick-slip friction in mechanical dynamic systems. *Transactions of the ASME*, 107:100–103, March 1985.
- [62] K.Y. Yi. Locomotion of a biped robot with compliant ankle joints. *Proceedings of the 1997 IEEE International Conference on Robotics and Automation*, 1997.

Appendix A

Matlab Code

```

% arm.m

clear
global xki xmovei

%DEFINE VARIABLES%

initialize                                % calls initialize.m

%ARM EQUILIBRIUM POSITION
thetaeq=fzero('equilibrium',0)           % calls equilibrium.m and finds zero

%INITIAL SPRING POSITION
deltaeq = xki+xmovei-R*thetaeq           % (m) initial position of spring 1 - (2.1)
delta2eq = xki-xmovei+R*thetaeq          % (m) initial position of spring 2 - (2.2)

%INITIAL CONDITIONS
%Bond graph [p] initial conditions
p_init = [
    0           % p(1) (Vs) flux linkage of motor 1 inductor [p2=pm1]
    0           % p(2) (kg.m^2/s) angular momentum of motor 1 shaft [p6=pms1]
    deltaeq     % p(3) (m) deflection of branch 1 spring [q29=delta1]
    0           % p(4) (kg.m^2/s) angular momentum of prosthetic arm [p32=pa]
    delta2eq    % p(5) (m) deflection of branch 2 spring [q39=delta2]
    0           % p(6) (kg.m^2/s) angular momentum of motor 2 shaft [p60=pms2]
    0           % p(7) (Vs) flux linkage of motor 2 inductor [p63=pm2]
    0           % p(8) (rad) angular displacement of motor 1 shaft [thetam1]
    0           % p(9) (rad) angular displacement of motor 2 shaft [thetam2]
    thetaeq     % p(10)(rad) absolute angular displacement of prosthetic arm [theta]
];

%SIMULATION TIME
t_start = 0;
t_stop = 1.1;
t_span = [t_start t_stop];

% SET INTEGRATION TOLERANCES
options = odeset('reltol',1e-6,'abstol',1e-9,'OutputFcn','odeplot');

% CALL INTEGRATION ROUTINE
[tt,pp] = ode23s('arm_st',t_span,p_init,options);

% CALL POST PROCESSING ROUTINE
post_processing                                % calls post_processing.m

```

```

%arm_st.m

%This file integrates the state equations

function pdot = arm_st(t,p)

global La Kt Jm rp rw rwg rgh Jgp b R Ja ...
      A B A_non J lambda phi uk us DV Jt Ff_k1 Ff_k2 ...
      Ff_eq1 Ff_eq2 Ffeq3

F_spring1 = Phi(p(3)); % Force in branch 1 quadratic spring - calls Phi.m
F_spring2 = Phi(p(5)); % Force in branch 2 quadratic spring - calls Phi.m

Ck = cos(phi)*cos(lambda)-uk*sin(lambda); % (4.15)
Cs = cos(phi)*cos(lambda)-us*sin(lambda); % (4.21)
fk = uk*rwg^2*J*cos(lambda)/(Ck*rwg^2*J*cos(lambda)+uk*Jgp*rw^2*tan(lambda)); % (4.17)
fs = us*rwg^2*J*cos(lambda)/(Cs*rwg^2*J*cos(lambda)+us*Jgp*rw^2*tan(lambda)); % (4.20)

Ff_k1 = fk*(Jgp*rw*tan(lambda)*rgh*Kt*p(1)/(rwg^2*J*La)+...
      b*rp^2*tan(lambda)*rw*Jt*p(2)/(Jm*J*rwg^4*rgh)-...
      R*rp*b*Jt*p(4)/(J*Ja*rwg^3)+rp*Jt*F_spring1/(J*rwg^3)); % (4.16)

Ff_k2 = fk*(Jgp*rw*tan(lambda)*rgh*Kt*p(7)/(rwg^2*J*La)+...
      b*rp^2*tan(lambda)*rw*Jt*p(6)/(Jm*J*rwg^4*rgh)+...
      R*rp*b*Jt*p(4)/(J*Ja*rwg^3)+rp*Jt*F_spring2/(J*rwg^3)); % (4.26)

Ff_s1 = fs*(Jgp*rw*tan(lambda)*rgh*Kt*p(1)/(rwg^2*J*La)-...
      R*rp*b*Jt*p(4)/(J*Ja*rwg^3)+rp*Jt*F_spring1/(J*rwg^3)); % (4.19)

Ff_s2 = fs*(Jgp*rw*tan(lambda)*rgh*Kt*p(7)/(rwg^2*J*La)+...
      R*rp*b*Jt*p(4)/(J*Ja*rwg^3)+rp*Jt*F_spring2/(J*rwg^3)); % (4.27)

Ff_eq1 = rgh*cos(lambda)*Kt*p(1)/(La*rw)+...
      rp*b*R*sin(lambda)*p(4)/(rwg*Ja)-rp*sin(lambda)*F_spring1/rwg; % (4.22)

Ff_eq2 = rgh*cos(lambda)*Kt*p(7)/(La*rw)-...
      rp*b*R*sin(lambda)*p(4)/(rwg*Ja)-rp*sin(lambda)*F_spring2/rwg; % (4.28)

vs1 = (rw/(rgh*Jm*cos(lambda)))*p(2); % (3.26)
vs2 = (rw/(rgh*Jm*cos(lambda)))*p(6); % (3.26)

Ff_1 = (abs(vs1)>=DV).*(Ff_k1)+...
      (abs(vs1)<DV).*((abs(Ff_eq1)<abs(Ff_s1)).*(Ff_eq1)+...
      (abs(Ff_eq1)>=abs(Ff_s1)).*(Ff_s1));

Ff_2 = (abs(vs2)>=DV).*(Ff_k2)+...
      (abs(vs2)<DV).*((abs(Ff_eq2)<abs(Ff_s2)).*(Ff_eq2)+...
      (abs(Ff_eq2)>=abs(Ff_s2)).*(Ff_s2));

% Calculation of state derivatives
pdot = A*p + A_non*[F_spring1;F_spring2;Ff_1;Ff_2] + B*[V1(t);V2(t);Fext(p(10),t)];

% calls V1.m, V2.m and Fext.m

```



```

% initialize.m

global Ra La Kt Jm Jw Jwg Jp rp rw rwg rgh Jgh b R Ja L xki xmovei Kmax Kmin...
    deltapos K_init thetai lmax lrest theta_lim n Wload Warm A B A_non Kv Wnl...
    Tth lambda phi J uk us DV Jgp Jt

% Initial equilibrium position and stiffness of arm

thetai =0.7199; % (rad) initial position of arm
K_init =16; % (N.m/rad) initial joint stiffness

% Motor specs (Maxon RE30 DC Motor)

Ra = 0.334; % (Ohms) Armature resistance
La = 0.09e-3; % (H) Armature inductance
Kt = 19.4e-3; % (Nm/A) Torque constant of motor
Jm = 6.55e-6; % (kg.m^2) inertia of motor rotor
Kv = 1/Kt; % (1/Vs) speed constant of motor
Wnl = Kv*15; % (rad/s) no load speed at nominal voltage
Tth = 0.09677; % (Nm) calculated thermal torque for motor

%gearhead specs (maxon GP 32 C)

rgh = 4.8; % gearhead reduction ratio
Jgh = 1.5e-7; % (kg.m^2) mass inertia

% Gearing specs (Boston Gear 12DP, 30teeth)

Jw = 8.43e-6; % (kg.m^2) Inertia of worm
Jwg =1.84e-4; % (kg.m^2) Inertia of worm gear
Jp = 2.73e-6; % (kg.m^2) Inertia of pulley
Jgp = Jwg+Jp;
rp = 0.0144; % (m) Pitch radius of pulley
rw = 0.0127; % (m) pitch radius of worm
rwg = 0.03175; % (m) pitch radius of worm gear
lambda = 0.078; % (rad) lead angle of helix
phi = 0.253; % (rad) pressure angle of helix
uk = 0.036; % kinematic coefficient of friction
us = 0.15;
DV = 1e-5;

% arm

Kmax = 24; % (N.m/rad) Max elbow stiffness
Kmin = 1 ; % (N.m/rad) Min elbow stiffness
L = .33; % (m) forearm length
marm = .9; % (kg) mass of forearm
R = 0.035; % (m) effective radius of elbow joint
mload = 0; % (kg) load mass
Wload = mload*9.81; % (N) Weight of load
Warm = marm*9.81; % (N) Weight of arm
Jap = 1.32e-4; % (kg.m^2) mass inertia of arm pulley
Ja = Jap + marm*(L^2)/3 + mload*(L^2); % (kg.m^2) Mass moment of inertia
b = 0.707*sqrt(marm*(L^2)/3*Kmax)/(R^2) % (N.s/m) linear damping
% coefficient - (4.32)

J = Jm*rgh^2+Jw+Jgh+(Jwg+Jp)*(rw*tan(lambda)/rwg)^2; % combined inertia - (4.11)
Jt = J*rwg^2-Jgp*rw^2*tan(lambda)^2; % (4.18)

```

```

%Initial calculations
xki = (K_init-20000*R^2)/(-200000*R^2)           % (m) initial stretch of springs
xmovei = thetai*R;                               % (m) initial position of springs

% Dynamic State Equations - (4.9), (4.10), (4.12), (4.13), (4.25)
% Matrix [A]
A=zeros(10);
A(1,1) = -Ra/La;
A(1,2) = -1/(Jm*Kv);
A(2,1) = (Jm*Kt*rgh^2)/(J*La);
A(2,2) = (-b/J)*((rp*rw*tan(lambda))/rwg)^2;
A(2,4) = (Jm*rp*rw*rgh*b*R*tan(lambda))/(J*rwg*Ja);
A(3,2) = (rp*rw*tan(lambda))/(Jm*rwg*rgh);
A(3,4) = -R/Ja;
A(4,2) = (R*b*rp*rw*tan(lambda))/(Jm*rwg*rgh);
A(4,4) = (-2*(R^2)*b)/Ja;
A(4,6) = (-R*b*rp*rw*tan(lambda))/(Jm*rwg*rgh);
A(5,4) = R/Ja;
A(5,6) = (rp*rw*tan(lambda))/(Jm*rwg*rgh);
A(6,4) = (-Jm*rp*rw*rgh*b*R*tan(lambda))/(J*rwg*Ja);
A(6,6) = (-b/J)*((rp*rw*tan(lambda))/rwg)^2;
A(6,7) = (Jm*Kt*rgh^2)/(J*La);
A(7,6) = (-1/(Jm*Kv));
A(7,7) = -Ra/La;
A(8,2) = 1/Jm;
A(9,6) = 1/Jm;
A(10,4) = 1/Ja;

% Matrix [B]
B=zeros(10,3);
B(1,1) = 1;
B(4,3) = L;
B(7,2) = 1;

% Matrix [A_non]
A_non=zeros(10,4);
A_non(2,1) = -Jm*rp*rw*rgh*tan(lambda)/(J*rwg);
A_non(2,3) = -Jm*rw*rgh/(J*cos(lambda));
A_non(4,1) = R;
A_non(4,2) = -R;
A_non(6,2) = -Jm*rp*rw*rgh*tan(lambda)/(J*rwg);
A_non(6,4) = -Jm*rw*rgh/(J*cos(lambda));

```

```

% equilibrium.m

function eq = equilibrium(thetaact)

% This function calculates the equilibrium arm angle due to the weight of the arm and
% loading (for initial conditions).
% Finds theta that makes the difference between to two spring forces equal to zero.
% Spring force from equation (4.29)

global R xki xmovei

eq = 50000*(-((xki+xmovei-R*thetaact).^2)+0.2*(xki+xmovei-R*thetaact))-...
    50000*(-((xki-xmovei+R*thetaact).^2)+0.2*(xki-xmovei+R*thetaact));

```

```

% Fext.m

function F_ext = Fext(arm_pos,t);
global Wload Warm Fload

% External forces on arm including load and constraint forces

tload = 0;
theta_c1 = 200*pi/180;           % (rad) Angle of contact at top
theta_c2 = -25*pi/180;          % (rad) Angle of contact at bottom
theta_s = .1*pi/180;            % (rad) Angle of contact stiffening
F_c = -10000;                   % (N) Constant constraint force
theta_c_max = theta_c1 + theta_s; % (rad) angle of max stiff at top
theta_c_min = theta_c2 - theta_s; % (rad) angle of max stiff at bottom
Fload = -(Wload.*(t>tload)+Warm/2).*sin(arm_pos); % (N) load and arm

F_ext = Fload.*(arm_pos < theta_c1)+(Fload + (F_c/theta_s)).*(arm_pos-theta_c1)...
    .*(arm_pos > theta_c1 & arm_pos < theta_c_max)+(Fload + F_c).*(arm_pos >...
    theta_c_max)+(Fload - F_c/theta_s).*(arm_pos- theta_c2).*(arm_pos < theta_c2 &...
    arm_pos > theta_c_min)+(Fload - F_c).*(arm_pos < theta_c_min);

% (N) load force and constraint force - (4.33)

```

```

% Phi.m
function F = phi(delta);
F = 50000*(-(delta).^2+0.2*(delta)); %(4.29)

```

```

% psi.m
function k = psi(delta)
k = 50000*(-(2*delta)+0.2); %(4.30)

```

```

% V1.m
function Voltage1 = V1(t);
% This file calculates the voltage input to the motor in branch 1

t1 = 0;
t2 = 0.1;
t3 = 0.9;
t4 = 1.0;

V1 = 0;
V2 = 15;

Voltage1 = V1.*(t<t1)+(V2/(t2-t1)).*(t-t1).*(t > t1 & t < t2) +...
V2.*(t > t2 & t < t3)+(V2-(V2/(t4-t3)).*(t - t3)).*(t > t3 & t < t4)+...
V1.*(t > t4);

```

```

% V2.m
function Voltage2 = V2(t);
% This file calculates the voltage input to the motor in branch 2

t1 = 0;
t2 = 0.1;
t3 = 0.9;
t4 = 1.0;

V1 = 0;
V2 = -15;

Voltage2 = V1.*(t<t1)+(V2/(t2-t1)).*(t-t1).*(t > t1 & t < t2) +...
V2.*(t > t2 & t < t3)+(V2-(V2/(t4-t3)).*(t - t3)).*(t > t3 & t < t4)+...
V1.*(t > t4);

```

```

% Post_processing.m

global Ra La Kt Jm Jw Jwg Jgp rw rwg rgh rp b R Ja L xki xmovei Kmax Kmin...
deltapos K init thetai lmax theta_lim n Wload Warm A B A_non Wn1 Tth uk us...
lambda DV J phi

%calculate joint stiffness
K = (psi(pp(:,3))+psi(pp(:,5)))*(R^2);          % (N.m/rad) joint stiffness -(2.6)
F = Fext((pp(:,10)),tt);                        % (N) external force on arm
Pa = -L.*F.*pp(:,4)/Ja;                        % arm Power (W)
Fspring1 = Phi(pp(:,3));
Fspring2 = Phi(pp(:,5));
kspring1 = psi(pp(:,3));
kspring2 = psi(pp(:,5));

%motor voltages
Volt1 = V1(tt);
Volt2 = V2(tt);

%friction
Ck = cos(phi)*cos(lambda)-uk*sin(lambda);
Cs = cos(phi)*cos(lambda)-us*sin(lambda);
fk = uk*rwg^2*J*cos(lambda)/(Ck*rwg^2*J*cos(lambda)+ uk*Jgp*rw^2*tan(lambda));
fs = us*rwg^2*J*cos(lambda)/(Cs*rwg^2*J*cos(lambda)+ us*Jgp*rw^2*tan(lambda));

Ff_k1 = fk*(Jgp*rw*tan(lambda)*rgh*Kt*pp(:,1)/(rwg^2*J*La)+...
    b*rp^2*tan(lambda)*rw*Jt*pp(:,2)/(Jm*J*rwg^4*rgh)-...
    R*rp*b*Jt*pp(:,4)/(J*Ja*rwg^3)+rp*Jt*Fspring1/(J*rwg^3));

Ff_k2 = fk*(Jgp*rw*tan(lambda)*rgh*Kt*pp(:,7)/(rwg^2*J*La)+...
    b*rp^2*tan(lambda)*rw*Jt*pp(:,6)/(Jm*J*rwg^4*rgh)+...
    R*rp*b*Jt*pp(:,4)/(J*Ja*rwg^3)+rp*Jt*Fspring2/(J*rwg^3));

Ff_s1 = fs*(Jgp*rw*tan(lambda)*rgh*Kt*pp(:,1)/(rwg^2*J*La)-...
    R*rp*b*Jt*pp(:,4)/(J*Ja*rwg^3)+rp*Jt*Fspring1/(J*rwg^3));

Ff_s2 = fs*(Jgp*rw*tan(lambda)*rgh*Kt*pp(:,7)/(rwg^2*J*La)-...
    R*rp*b*Jt*pp(:,4)/(J*Ja*rwg^3)+rp*Jt*Fspring2/(J*rwg^3));

Ff_eq1 = rgh*cos(lambda)*Kt*pp(:,1)/(La*rw)+...
    rp*b*R*sin(lambda)*pp(:,4)/(rwg*Ja)-rp*sin(lambda)*Fspring1/rwg;

Ff_eq2 = rgh*cos(lambda)*Kt*pp(:,7)/(La*rw)-...
    rp*b*R*sin(lambda)*pp(:,4)/(rwg*Ja)-rp*sin(lambda)*Fspring2/rwg;

vs1 = (rw/(rgh*Jm*cos(lambda)))*pp(:,2);
vs2 = (rw/(rgh*Jm*cos(lambda)))*pp(:,6);

Ff_1 = (abs(vs1)>=DV).*(Ff_k1)+(abs(vs1)<DV).*((abs(Ff_eq1)<abs(Ff_s1)).*(Ff_eq1)+...
    (abs(Ff_eq1)>=abs(Ff_s1)).*(Ff_s1));

Ff_2 = (abs(vs2)>=DV).*(Ff_k2)+(abs(vs2)<DV).*((abs(Ff_eq2)<abs(Ff_s2)).*(Ff_eq2)+...
    (abs(Ff_eq2)>=abs(Ff_s2)).*(Ff_s2));

```

```

figure(1)
subplot(3,2,1)
axis([t_start t_stop 0 20])
hold on
plot(tt,Volt1)
ylabel('Motor 1 Voltage (V)')
xlabel('Time (s)')
grid

subplot(3,2,2)
axis([t_start t_stop -20 0])
hold on
plot(tt,Volt2)
ylabel('Motor 2 Voltage (V)')
xlabel('Time (s)')
grid

subplot(3,2,3)
axis([t_start t_stop -2 6])
hold on
plot(tt,pp(:,1)/La)
ylabel('Motor 1 Current (A)')
xlabel('Time (s)')
grid

subplot(3,2,4)
axis([t_start t_stop -2 6])
hold on
plot(tt,pp(:,7)/La)
ylabel('Motor 2 Current (A)')
xlabel('Time (s)')
grid

subplot(3,2,5)
axis([t_start t_stop -20 80])
hold on
plot(tt, (pp(:,1)/La).*Volt1)
ylabel('Motor 1 Power top (W)')
xlabel('Time (s)')
grid

subplot(3,2,6)
axis([t_start t_stop -80 20])
hold on
plot(tt, (pp(:,7)/La).*Volt2)
ylabel('Motor 2 Power (W)')
xlabel('Time (s)')
grid

figure(2)
subplot(2,2,1)
axis([t_start t_stop -50 1000])
hold on
plot(tt, pp(:,2)/(Jm))
hold on
plot(tt,Wn1, 'r--')
ylabel('Motor 1 Speed, Wm (rad/s)')
xlabel('Time (s)')
grid

```

```

subplot(2,2,2)
axis([t_start t_stop -1000 50])
hold on
plot(tt, pp(:,6)/(Jm))
hold on
plot(tt, -Wn1, 'r--')
ylabel('Motor 2 Speed, Wm (rad/s)')
xlabel('Time (s)')
grid

subplot(2,2,3)
axis([t_start t_stop -0.1 0.1])
hold on
plot(tt, pp(:,1)*Kt/La)
hold on
plot(tt, Tth, 'r--')
plot(tt, -Tth, 'r--')
ylabel('Motor 1 Torque, Tm (Nm)')
xlabel('Time (s)')
grid

subplot(2,2,4)
axis([t_start t_stop -0.1 0.1])
hold on
plot(tt, pp(:,7)*Kt/La)
hold on
plot(tt, Tth, 'r--')
plot(tt, -Tth, 'r--')
ylabel('Motor 2 Torque, Tm (Nm)')
xlabel('Time (s)')
grid

figure(3)
subplot(311)
axis([t_start t_stop 0 180])
hold on
plot(tt, pp(:,10)*180/pi)
ylabel('Arm Displacement (degrees)')
xlabel('Time (s)')
grid

subplot(312)
axis([t_start t_stop -50 150])
hold on
plot(tt, (pp(:,4)*180/pi)/Ja)
ylabel('Arm Velocity (degrees/s)')
xlabel('Time (s)')
grid

subplot(313)
axis([t_start t_stop 0 30])
hold on
plot(tt, K)
ylabel('Arm Stiffness (Nm/rad)')
xlabel('Time (s)')
grid

```

```

figure(4)
subplot(221)
axis([t_start t_stop -2 12])
hold on
plot(tt, vs1)
ylabel ('Worm/Worm Gear 1 Sliding Velocity (m/s) ')
xlabel('Time (s)')
grid

subplot(222)
axis([t_start t_stop -12 2])
hold on
plot(tt, vs2)
ylabel ('Worm/Worm Gear 2 Sliding Velocity (m/s) ')
xlabel('Time (s)')
grid

subplot(223)
axis([t_start t_stop -20 30])
hold on
plot(tt, Ff_1)
ylabel ('Worm/Worm Gear 1 Friction Force (N)')
xlabel ('Time (s)')
grid

subplot(224)
axis([t_start t_stop -20 30])
hold on
plot(tt, Ff_2)
ylabel ('Worm/Worm Gear 2 Friction Force (N)')
xlabel ('Time (s)')
grid

figure(5)
subplot(221)
axis([t_start t_stop 11 20])
hold on
plot(tt, (0.1375-pp(:,3)-R*(pp(:,10)-2.75))*100)
ylabel('Spring 1 Position (cm)')
xlabel('Time (s)')
grid

subplot(222)
axis([t_start t_stop 11 20])
hold on
plot(tt, (0.1375-pp(:,5)+R*(pp(:,10)))*100)
ylabel('Spring 2 Position (cm)')
xlabel('Time (s)')
grid

subplot(223)
axis([t_start t_stop 0 10])
hold on
plot(tt, pp(:,3)*100)
ylabel('Spring 1 Deflection (cm)')
xlabel('Time (s)')
grid

```



```
subplot(224)
axis([t_start t_stop 0 10])
hold on
plot(tt,pp(:,5)*100)
ylabel('Spring 2 Deflection (cm)')
xlabel('Time (s)')
grid

figure(6)
subplot(221)
axis([t_start t_stop 0 500])
hold on
plot(tt,Fspring1)
ylabel('Spring 1 Force (N)')
xlabel('Time (s)')
grid

subplot(222)
axis([t_start t_stop 0 500])
hold on
plot(tt,Fspring2)
ylabel('Spring 2 Force (N)')
xlabel('Time (s)')
grid

subplot(223)
axis([t_start t_stop 0 10000])
hold on
plot(tt,kspring1)
ylabel('Spring 1 Stiffness (N/m)')
xlabel('Time (s)')
grid

subplot(224)
axis([t_start t_stop 0 10000])
hold on
plot(tt,kspring2)
ylabel('Spring 2 Stiffness (N/m)')
xlabel('Time (s)')
grid
```

Appendix B

Drawings

VARIABLE STIFFNESS EMULATOR
 MEGAN MCTAVISH
 OCTOBER 5, 2004
 ARM SHAFT ⑨
 1020 STEEL
 ALL TOLERANCES : 0.003
 UNLESS OTHERWISE NOTED