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PROOF-LOADING EFFECTS ON LUMBER

by

Richard I. Cunliffe, B. Eng.

**A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfillment of the requirements**

for the degree of

MASTER OF ENGINEERING

Faculty of Engineering

Carleton University

Ottawa, Ontario

December, 1988

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The undersigned recommend to the Faculty of Graduate Studies
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submitted by Richard I. Cunliffe, B. Eng., in the partial fulfillment of
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MASTER OF ENGINEERING

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Abstract

Two test methods which can be used to eliminate or exclude weak pieces from a sample of lumber are considered. The effects that various proof-loads and modulus of elasticity exclusion limits have on the strength distribution of lumber are examined.

The Canadian Damage model is used to evaluate the damage caused during proof-loading and also to account for the viscoelastic behaviour of lumber. The theory presented accounts for different proof-load and strength test histories and can be used for tension and compression proof-loading as well as for bending.

The modulus of elasticity exclusion method, which is mostly non-destructive, is compared with the proof-load method. The MOE exclusion method depends on the correlation between lumber strength and stiffness to sort lumber in a similar way as Machine Stress Rating does. The strength distributions produced by the proof-load and MOE exclusion methods are compared.

Criteria are established to evaluate the profitability of the test methods. Optimum proof-load and MOE exclusion limits are determined as well as profitable ranges.

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To my father, Leonard H. Cunliffe. (1923-1986)

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List of Symbols

a	Canadian damage model parameter. Barrett and Foschi (1978a) model II parameter.
A	Gerhards damage model parameter.
A_n	Incremental area under proof-loaded lumber strength pdf.
A_s	Incremental area under initial strength pdf.
b	Canadian damage model parameter. Barrett and Foschi (1978a) model II parameter.
B	Gerhards damage model parameter.
c	Stress wave propagation speed. Canadian damage model parameter.
d	Bending member depth.
E_d	Dynamic modulus of elasticity.
F	Fraction of lumber excluded or eliminated.
f_{all}	Allowable strength.
f_b	Allowable bending strength (WSD).
F_b	Specified bending strength (LSD).
F_{HI}	Largest fraction excluded that is profitable.
F_{Low}	Smallest fraction excluded that is profitable.
f_Q	Load-effect probability density function.
f_r	Strength pdf.
F_R	Strength cumulative distribution function.
f_y	Yield strength of steel.
f'_c	Concrete compressive strength at 28 days.
f_{r_n}	After proof-load strength pdf.
F_{r_n}	After proof-load strength cdf.
f_{r_s}	Initial strength pdf.
F_{r_s}	Initial strength cdf.

G	Performance function.
k_b	Size effect adjustment factor for bending.
K_d	Code load duration factor.
k_{dFS}	Factor for duration of load and factor of safety.
K_G	Code specific grade factor for sawn lumber.
k_{grade}	Grade adjustment factor.
K_H	Code load sharing factor.
$k_{Harrison}$	75% lower confidence limit adjustment factor.
K_L	Code lateral stability factor for bending.
k_p	Proof-load test rate of loading.
k_r	Ramp load rate of loading.
k_s	Rate of loading used for short-term strength test.
K_{sb}	Code service condition factor for bending.
$k_{size\ 8 \rightarrow 12}$	Size effect adjustment from 8 in. to 12 in. deep lumber.
K_T	Code treatment factor.
k_{wd}	Moisture content adjustment factor.
K_{zb}	Code size factor for bending.
L	Span
m_Q	Mean load-effect.
m_R	Mean resistance.
MOE	Modulus of elasticity.
MOR	Modulus of Rupture.
n	Canadian damage model parameter. Number of pieces meeting test requirements.
N	Total number of tests.
n_e	Number of pieces excluded or eliminated.

P	Proof-load stress. Lognormally distributed random number. Non-dimensional profitability.
P_E	Probability that a piece of lumber will be eliminated by a proof-load test.
P_f	Probability of failure.
P_n	Profit from distribution altering method yielding n pieces meeting test requirements.
Q	Demand or load-effect.
R	Capacity or strength.
R_N	Standard normal variate.
S	Internal stress caused by applied load.
S_{LSD}	Section modulus determined using limit states design.
S_p	Maximum proof-load stress.
S_{WSD}	Section modulus determined using working stress design.
$S_{.05}$	Fifth percentile strength.
$S_{.05 \text{ MC}=15\%}$	Fifth percentile strength of lumber at 15% m.c.
$S_{75\%LCL}$	Strength based on 75% lower confidence limit.
$S'_{.05}$	New fifth percentile strength.
SL	Stress level.
t	Time.
T	Cost of testing.
t_f	Time to failure.
$t_{1 \text{ stage}}$	Computing time taken for a one stage sort.
$t_{2 \text{ stage}}$	Computing time taken for a two stage sort.
V	Bechtel's value of lumber. Coefficient of variation. Initial value or worth of a piece of lumber.
V_N	Standard normal variate.

V_D	Coefficient of variation of the load-effect.
V_r	Residual value or worth of a piece of lumber not meeting test requirements.
V_R	Coefficient of variation of the resistance.
V'	New value or worth of lumber meeting test requirements.
u_1, u_2	Uniformly distributed random number.
α	Damage.
α_D	Dead load factor.
α_L	Live load factor.
α_P	Damage that occurs during proof-loading.
β	Reliability index.
β_b	Reliability index for bending failure mode.
β_c	Reliability index for compression failure mode.
β_d	Reliability index for deflection serviceability requirement.
β_T	Target reliability index.
β_V	Reliability index for shear failure mode.
β'	New reliability index.
ϕ	Resistance or performance factor.
Φ	Standard normal cumulative distribution function.
γ	Residual fractional lifetime.
γ_P	Residual fractional lifetime after proof-loading.
λ	Lognormal distribution parameter. Canadian damage model parameter. Barrett and Foschi (1978a) model II parameter.
μ	Mean value.
μ_G	Mean value of the performance function.
ρ	Mass density.

σ	Standard deviation. Stress ratio.
σ_G	Standard deviation of the performance function.
σ_0	Threshold stress ratio.
τ	Internal stress caused by applied load.
τ_c	Internal stress at failure caused by constant applied load.
τ_n	Strength after proof-load test.
τ_s	Short-term strength.
$\tau_{S_{min}}$	Minimum initial strength of lumber to survive the proof-load test.
τ_0	Threshold stress.
ω_D	Uniformly distributed dead load.
ω_L	Uniformly distributed live load.
	Barrett and Foschi (1978b) variability parameters.
ζ	Lognormal distribution parameter.

Chapter 1

Introduction

Forest lands cover more than one-third of Canada's land area and the many structural wood products manufactured here play a vital role in Canada's economy. Wood products exports bring more than fourteen billion dollars (1987) into the Canadian economy annually and one in ten Canadian jobs depend on our forest industry. Along with the economic importance there is also an environmental benefit. Timber is a renewable resource which requires very little energy in order to be converted into lumber for structural use. In addition, there is little waste from the tree. Products such as flakeboard and particle board can be manufactured to utilize parts that cannot be sawn into structural sizes.

The increased demand for forest products has caused the stock of large dimension timber to decrease, particularly in eastern Canada. New engineered products have become economically feasible because of the increased cost of large dimension lumber. Composite joists with small dimension lumber flanges and waferboard, plywood, or pressed metal webs have recently come into use and are competing with trusses manufactured with small dimension lumber and pressed metal plates, glued-laminated beams and sawn lumber. The wood industry is not only very competitive within, but faces strong competition from other traditional building materials such as structural steel and concrete.

Lumber is a nonhomogeneous, anisotropic material and, unlike other structural materials such as steel and concrete, man has little control over its composition. The lumber producer has some control over which timbers are selected and how the log is converted to lumber but its basic composition cannot be altered. Knots, sloping grain, splits, wane, checks and other growth characteristics, and their location, affect the structural behaviour of lumber. Lumber graders use these visual factors to sort the lumber into grades for which CSA CAN3-086.1-M84, Engineering Design in Wood, specifies strength properties. Due to the inherent difficulty with an accurate prediction of strength, based on visual inspection of the lumber exterior and the short amount of time the grader is able to spend on each piece of lumber, the variability of strength associated with each lumber grade is large. This large variability results in code-specified strengths, based on a fifth percentile strength value, which are very low when compared with the strengths of most of the population. A typical strength probability density function for No.2 grade and better Western Hemlock lumber is shown in Figure 1-1. Having a coefficient of variation of 40.8%, the variability in lumber strength is large when compared with other structural materials. The fifth percentile short-term strength, equal to 3366 psi (23.2 MPa), is less than one-half of the average strength value of 6937 psi (47.8 MPa). Almost one-half of the lumber population is twice as strong as the fifth percentile strength. Elimination of the exceptionally weak pieces from the population would increase the allowable stresses and permit the lumber to be used in more demanding structural applications. This could lead to a more efficient use of lumber, either to be used alone or in engineered products such as glued-laminated beams, trusses or composite joists.

Fig. 1-1 Strength Probability Density Function for
2" x 6" Western Hemlock Lumber, Graded No. 2 and Better
Lognormal distribution parameters based on Madsen (1976)

Machine stress rating (MSR) is a "non-destructive testing" method which provides a better estimate of the strength than visual grading does. The method is based on the correlation between lumber strength and stiffness, and the accuracy of the prediction depends on the degree of correlation. MSR lumber is loaded on the flat in both directions and the deflections are measured to determine the modulus of elasticity. MSR lumber is assigned a stress grade based on one of its physical properties, the modulus of elasticity, not on its appearance alone. As a result, the code-specified strengths for MSR lumber in bending, tension, and compression parallel to grain, and the modulus of elasticity can be up to double those of visually graded lumber. This method is advantageous in that there is little material that is destroyed and

hence lost for sale; however, it is not efficient at eliminating those extraordinarily weak pieces which make up the lowest percentiles of the strength distribution which could have considerable stiffness.

Proof-loading is a more direct method that can be used to eliminate the exceptionally weak pieces from a population of lumber. In general, proof-loading implies the application of a load to a structural member to determine whether it can meet a given demand. When lumber is proof-loaded, those pieces that cannot meet the demand, fracture and are removed from the outgoing population. Proof-loading can result in the censorship of the lower tail of the strength distribution. This would allow the surviving lumber to be used in more demanding structural tasks. The benefits of an improved strength distribution brought about by proof-loading must be weighed against the loss of the lumber that fails the test.

Proof-loading is believed to be more efficient at removing the exceptionally weak pieces from a lumber population than non-destructive testing methods. Lumber failing a proof-load is fractured while those pieces not meeting the non-destructive test requirements are less likely to be damaged.

1.1 Objectives

The objectives of this study are:

- 1) To evaluate the strength distribution of lumber that has survived a proof-load test, taking into consideration:
 - i) the dependence of lumber strength on load history, and
 - ii) the possibility that damage to the lumber has occurred as a result of the test.
- 2) To evaluate the changes in the strength distribution brought about by varying the magnitude of the proof-load.
- 3) To evaluate the strength distribution which may result from using a non-destructive testing method (which excludes lumber that does not meet a stiffness level) and compare it with that from proof-loading.
- 4) To set appropriate criteria for profitability and determine optimum proof-load and stiffness exclusion limits and profitable ranges.

1.2 Proof-loading Literature Review

Proof-loading was first used to determine if a product could withstand an imposed demand comparable to that expected in service. It was in this context that the first papers dealing with proof-loading of wood products were written. An early paper by Freas (1949) considered the safety of ladders. The work was discontinued because of a lack of

proper testing methods. More recently, Strickler and Pellerin (1971) used proof-loading to test finger joints of lumber to be used in glued laminated beams.

The use of proof-loading to improve the strength distribution of lumber and expand its use in highly engineered systems is a more recent idea. The effect that proof-loading has on the strength distribution of lumber is still a contentious issue for researchers. There is uncertainty as to the degree of damage that is done to the lumber passing the proof-load test. Researchers generally agree that proof-loading will not produce a strength distribution which is less desirable than the original distribution.

Madsen (1976) investigated the "Degree of Damage Due to Proof Loading Lumber" as part of a larger In-grade testing program in 1976. Madsen questioned the method used to derive the code-specified strengths of lumber from adjustments made to the strength of small clear specimens. He proposed In-grade testing of full-sized lumber that had been visually graded as a better alternative for determining lumber strength. Madsen chose a bending proof-load that would cause approximately only 10% of the sample to fail. This allowed Madsen to use a large sample size as he could return the unbroken pieces for credit and reduce costs. The evaluation of the fifth percentile strength was his prime objective, not the description of the entire strength distribution. In order that he could return the unfractured lumber, Madsen had to ensure that the lumber returned would not adversely affect the overall strength distribution of the mill's lumber.

Madsen wrote in the summary of report no. 17 :

"Testing of some 600 pc of 2"x 6" Hem Fir of '2 and better' grade was undertaken to assess degree of damage which might occur by subjecting the material to a proof-load of 4000 psi (27.6 MPa).

Four different configurations were investigated and it was found that in two cases some damage did take place. However, the degree of damage was compensated for by removing from the sample those boards which broke during the proof-load.

It was thus found that returning the boards which did not break during proof-loading to the mill production would not reduce the strength distribution of the end product."

The four loading configurations mentioned above relate to the placement of the "weak edge" (edge with most knots and sloping grain) in tension or compression during the bending proof-load test and when the survivors were tested, to determine their strength after proof-loading. The four configurations listed in Table 1-1 were divided into two groups.

Table 1-1 MADSEN'S FOUR BENDING TEST CONFIGURATIONS
(placement of weakest edge)

	PROOF-LOAD	STRENGTH TEST
GROUP 1	T	T
	C	C
GROUP 2	T	C
	C	T

The first group had the survivors of the proof-load tested to destruction in the same direction as the proof-load was applied. Approximately 8% of the proof load survivors fractured at loads less than the maximum proof-load level of 4000 psi (27.6 MPa). This would indicate

that some damage was introduced by proof-loading. While some pieces were reduced in strength, the elimination of the weakest pieces in the proof-loading process resulted in an improved strength distribution. The fifth percentile strength value increased from 2100 psi (14.5 MPa) to 3700 psi (25.5 MPa) for the category of weak side in tension and from 2500 psi (17.2 MPa) to 3600 psi (24.8 MPa) for the category of weak side in compression.

The second group had the survivors of the proof-load tested to destruction in the opposite direction to that of the proof-load. This would be the worst case scenario for the returned lumber. No significant change in the strength distribution was apparent. The strength of lumber in the two loading orientations is not perfectly correlated because lumber strength is dependent on the size and location of knots and other growth characteristics. The elimination of the weakest pieces from one loading orientation will not guaranty the elimination of the weakest pieces from the other loading orientation.

Woeste et al (1987) tested both single and reverse proof-loading of lumber in bending to determine their effect upon bending strength and tensile strength parallel to grain. The single-proof load caused bending in one direction only with point loads located at one-fifth of the span from the simple end supports. Reverse proof-loading was carried out in one direction and then the other so that both edges were tested in tension. Reverse proof-loading eliminated 38% more pieces than did the single proof-load at the same load level.

The bending reverse proof-load caused a significant shift in the lower tail of the bending strength distribution. The fifth percentile

bending strength was increased by 24%. The results were less impressive for the single proof-load case, as the fifth percentile bending strength increased only by 5%. However, the lumber was tested for new strength in the opposite direction to which the single proof-load was applied. This was similar to the worst case scenario considered by Madsen (1976) when returning proof-loaded lumber to the mill. Proof-loading in the opposite direction to the intended use will not necessarily eliminate the weak elements in the end use orientation and is likely to cause damage to the lumber.

Results from tensile strength tests performed on those pieces of lumber that passed the bending proof-load tests showed a modest improvement in the tensile strength distribution. The single bending proof-load increased the fifth percentile strength by 10% and the reverse proof-load increased the fifth percentile strength by 15%. The point loads causing bending were located at a distance equal to one-fifth of the span from each simple support. This loading arrangement results in maximum bending stresses over three-fifths of the length of the lumber. Reverse proof-loading allows twice as much of the lumber cross section to be tested in tension as a single proof-load and so a better tensile strength distribution is expected. Proof-loading lumber on the flat may be as productive at improving the tensile strength distribution as proof-loading the lumber on edge. Testing on the flat is easier, requiring smaller loads and no lateral support while maintaining the same cross sectional area stressed in tension.

Proof-loading the lumber in tension is a direct method of improving the tensile strength distribution and is likely to be more efficient. However, bending tests are easier and more economical to conduct than

are tension tests.

McLain and Woeste (1986) studied the effect that the rate of loading has on the tensile strength of lumber. Tension proof-loading would be particularly useful to the wood truss industry where designs are often limited by the tensile strength of the lumber used. Proof-load histories of twenty seconds or less were proposed, which were approximately twenty times faster than the ASTM Standard. McLain and Woeste found that the tensile strength was "moderately" higher when tested at the faster, more economical rate of loading. The stronger lumber in the upper percentiles of the strength distribution was found to be most affected by the faster rate of loading.

Later, McLain and Woeste (1988) attempted to evaluate the damage caused by proof-loading 2" x 6" southern pine lumber in tension. A sample of 150 pieces was used with equal numbers of Select Structural, No. 1 and No. 2 grades. Sixty-six of the 150 pieces failed under the proof-load. The 88 survivors were then re-tested with the same load history used for the proof-load. Only one piece failed the re-test and it failed at a stress equal to 99.69% of the maximum proof-load stress. McLain and Woeste were uncertain if the apparent strength reduction was due to damage or test machine inaccuracy. In conclusion they wrote, "the 0.31% apparent strength reduction is negligible and apparently this proof load did not reduce strength in any significant way". McLain and Woeste made an implicit assumption in the analysis of the results which they did not explain. They assumed that the strength of the piece of lumber that failed the re-test was equal to the maximum proof-load stress. They really could not know what the initial strength was and,

therefore, could not know what the strength reduction was. The 0.31% figure can only be considered to be a lower bound to the reduction in strength caused by damage.

Gerhards (1979) used a linear cumulative damage model to predict the effect of proof-loading on the strength distribution of lumber in bending. The damage model allowed Gerhards to account for the time-dependent strength of lumber. Lumber could theoretically be proof-loaded to a stress higher than its standard static strength and not fail if the proof-load duration was shorter than that of the static strength test.

A parameter called "residual fractional lifetime" was used to account for the damage that occurs during time under load. The residual fractional lifetime is equal to 1.0 at the beginning of service life and 0.0 at failure. The residual fractional lifetime is expressed in Equation 1.1 as a function of the load history and two constant parameters A and B, used to calibrate the model to existing clear wood data. The stress term, SL, is non-dimensional and represents the applied stress relative to the standard strength of a particular piece of lumber.

$$\gamma = 1 - \int_0^t 10^{\frac{A-SL}{B}} dt \quad (1.1)$$

The residual fractional lifetime after proof-loading, γ_p , could be determined from Equation 1.1. Lumber that passes a proof-load test will have residual fractional lifetime greater than zero. The standard short-term strength of the lumber that passes the proof-load test can be determined by setting Equation 1.1 equal to γ_p and solving for SL.

Gerhards, using this damage accumulation model, estimated the effect of a ramp proof-load on the strength distribution of a presumed population of lumber. A ramp load to 110% of the fifth percentile strength applied in 10 seconds would theoretically destroy 4.7% of the initial lumber population. Approximately one-half percent of the survivors had residual strengths lower than the proof-load level. However, the fifth percentile strength of the surviving population increased by 28%.

Although Gerhards' work showed an important application of damage models, the use of clear wood data for lumber strength estimation and the inflexible linear cumulative damage model have since been criticized.

While considerable research focused on the effects that proof-loading had on lumber strength, Bechtel (1983) was the first to attempt an answer to the question "What should the value of the proof-load be?" When the severity of the proof-load is increased, the distribution of the strength of the surviving pieces tends to improve at the expense of lost lumber.

Bechtel used Equation 1.2 to express the price of the proof-loaded lumber as a function of the proof-load magnitude, P , and parameters, k and b , which reflect market conditions. He found that the price was proportional to the square root of the proof-load magnitude based on the asking prices of three MSR grades.

$$V(P) = k P^b \quad (1.2)$$

A specified strength, directly applicable in design, may be a better base for the estimate of the worth of the proof-loaded lumber than the magnitude of the proof-load itself.

Bechtel's equation for the expected economic value of the proof-loaded lumber is given in Equation 1.3 as the product of the economic value determined from Equation 1.2 and the probability that the lumber survives the proof-load.

$$E[V(P)] = V(P) [1-F(P)] \quad (1.3)$$

A proof-load magnitude which would optimize the expected economic value could be determined by setting the derivative of the expected value to zero and solving for P.

Bechtel used a simplified treatment of the new strength distribution assuming that the lower tail would be truncated at the proof-load level. The truncation of the strength distribution implies that no damage is done to the lumber passing the proof-load and that the proof-load and strength test histories are the same. Proof-loading expenses, such as the fixed cost of the testing machine, maintenance and labour, were not considered. Optimum proof-loads were determined but not profitable ranges.

Bechtel found that the "choice of proof load-level is not critical over a fairly broad range, so that one need get only close to the optimum load value to achieve nearly maximum economic return from the material." Bechtel found that the optimum proof-loads depend on the strength variability of the lumber population. An optimum proof-load for lumber with a high strength variability may eliminate as much as 10% of the lumber tested. Lower proof-loads, which cause less destruction, are recommended for lumber with lower variability in strength.

Suddarth (1985) measured the results of proof-loading by probabilistic means. He believed that the fifth percentile strength was not

necessarily a good indicator of the strength distribution. Two strength distributions could have the same fifth percentile strength (and hence the same specified code strength) but have very different lower tails of their probability density functions. Suddarth used the theoretical probability of failure to reflect the changes that occur in the lower tail of the strength distribution as a result of proof-loading.

The tensile strengths of a sample of ten thousand pieces of lumber were simulated by computer to investigate the effect of proof-loading. Suddarth chose a proof-load of 2000 psi (13.8 MPa) which was approximately equal to the 6.6th percentile strength. The strength distribution of the lumber passing the proof-load was estimated assuming that some passing material would have strengths below the proof-load level. He chose a lower limit of 1900 psi (13.1 MPa) arbitrarily. A load distribution was chosen so that the theoretical probability of failure could be evaluated for both the proof-loaded lumber and the original sample. Suddarth numerically evaluated Equation 1.4 to determine the theoretical probability of failures.

$$p_f = \int_0^{\infty} f_L F_R dR \quad (1.4)$$

The convolution integral can only be used to calculate the probability of failure of a member if the load effect and strength are independent. This condition is not satisfied for lumber; however, the theoretical probability of failure can be used to compare strength distributions.

Suddarth found that the theoretical probability of failure of the lumber passing the proof-load was ten times lower than that of the original sample. To determine what improved use could be made from proof-loaded material, Suddarth shifted the load distribution up the

load scale relative to the proof-loaded lumber strength distribution so that the same probability of failure was obtained. With the proof-loaded lumber, the 99th percentile load could be increased 38% while maintaining the same probability of failure as the original case.

Suddarth suggested that the allowable strengths for proof-loaded lumber could be obtained in this manner by multiplying the non proof-loaded allowable strengths by the ratio of the 99th percentile loads.

1.3 Non-Destructive Testing Literature Review

Machine stress rating, discussed earlier, is the most widely used method of sorting lumber other than by visual grading. However, many other forms of non-destructive testing exist. Much attention has been given to procedures which will allow the location of knots and other defects to be detected in the log before the cutting operation begins. Tomography, using x-rays, gamma-rays, sound waves and microwaves can be used to get a picture of the cross-section of the log in a similar manner as x-rays or CAT scans are used to look inside our bodies. The potential use for tomographic techniques is large but at present the high cost of equipment and the slow speed of operation has limited their use to research studies.

Longitudinal stress wave behaviour is another non-destructive testing technique which can be applied to sawn lumber or wood products. Jayne (1959) determined that a material's energy dissipation characteristics are related to its static mechanical properties. The attenuation of particle velocity can be used as a measure of energy dissipation. Ross (1985) determined, experimentally, that particle velocity atten-

uated more rapidly with lumber containing defects than with clear wood specimens. Attenuation rate could, therefore, be used as an indicator of strength.

The dynamic modulus of elasticity of a homogeneous, isotropic, prismatic rod is given in Equation 1.5, and is the product of the square of the stress wave propagation speed and the mass density. Although lumber is a non-homogeneous and anisotropic material, Equation 1.5 has been found to be applicable provided that the stress wave length is large relative to the least transverse dimension of the lumber.

$$E_d = \rho c^2 \quad (1.5)$$

Both lumber strength and stiffness are correlated with the dynamic modulus of elasticity. Lumber could be sorted, based on the dynamic modulus of elasticity, in a similar manner as MSR lumber is sorted according to the static modulus of elasticity.

A non-destructive test method, which will be called "modulus of elasticity exclusion" (MOEE) in this study, is also based on the positive correlation between bending stiffness and strength. The modulus of elasticity is measured for each piece of lumber and those pieces falling below a specific MOEE limit are excluded from the passing lot. In this way an improved strength distribution can be obtained without destruction. The MOEE method is not as efficient as proof-loading in eliminating the extraordinarily weak elements. Snodgrass (1985) compared the use of proof-loading and the non-destructive MOEE method to provide tension laminae for Glulam beams. To qualify for tension laminae, the 2" x 6" Douglas fir lumber had to have a fifth percentile strength of 4000 psi (27.6 MPa) or better. Snodgrass used data from tests done on 387 pieces of "candidate" stock in which the

modulus of elasticity was determined in bending before its strength was determined in a tension test. Snodgrass used a scattergram of tensile strength vs bending MOEE to estimate that the fifth percentile strength could be increased from approximately 3000 psi (20.7 MPa) to 4000 psi (27.6 MPa) with a proof-load of 3800 psi (26.2 MPa) or by excluding pieces with a MOE less than 2.5×10^5 psi (17200 MPa). The proof-load was expected to eliminate 11% of the material tested and the MOEE method 56%. Although both methods provided the 4000 psi (27.6 MPa) fifth percentile strength, the lower tail of the proof-loaded lumber strength distribution was more severely censored and considerably better than with the MOE exclusion method.

1.4 Scope

Lumber's time-dependent strength and creep rupture behaviour complicate the assessment of the effect that proof-loading has on the strength of the lumber which survives the test. Gerhards (1979) demonstrated that a damage accumulation model can be an important tool for use in the prediction of the effects of proof-loading. The Canadian Damage Model, proposed by Foschi, has since emerged as the model which is best capable of representing the creep rupture behaviour of lumber.

Forintek Canada Corporation (1987) used interim results of a joint U.S. - Canada duration of load research program to calibrate the Canadian Damage Accumulation Model. Model parameters for small dimension Hemlock, Spruce and Douglas Fir in bending, and 2" x 4" Spruce in tension and compression are given in this interim report. This study will use the bending strength and stiffness properties of 2" x 6"

Hemlock lumber specifically. However, the theory developed is believed to be applicable to the tension, compression and bending strengths of all species.

The Canadian Damage Model will be used with calibration parameters from Forintek Canada Corporation to determine:

- (i) whether a piece of lumber will pass or fail a proof-load test.
- (ii) The degree of damage done to the pieces surviving the proof-load.
- (iii) The strength the surviving pieces of lumber would have if given a standard strength test.

The proof-load test history that will be used in this study will be a ramp load which starts from zero stress and increases linearly with time to cause a maximum internal stress in a period of 10 seconds. The proof-load should induce a stress pattern in the lumber which is similar to that which will occur in use. For example, in the joint Canada - U.S. duration of load study, joists were loaded with two point loads, placed at third point locations along the span, to simulate the bending stresses induced by a uniformly distributed load. Reverse proof-loading will not be considered. The problem of joists being used in a structure in the opposite direction to that in which they have been proof-loaded can be averted if a mark is placed on the joist indicating the preferred orientation. A random edge orientation will be assumed in this study based on observation of the well established practice of placing the crown of the joist up and not necessarily the edge containing the most knots.

The influence that proof-loading has on a population of lumber will be built up from many simulations of individual tests. The maximum magnitude of the proof-load will be varied to determine its influence on the new strength distribution. The prediction of the new strength distribution after proof-loading is the prime objective of this study.

A non-destructive test method, will also be studied. The modulus of elasticity exclusion method (MOEE) will be considered, primarily for comparison with the proof-load method. A probabilistic regression model will be developed to provide correlated modulus of rupture and modulus of elasticity values. A sample of correlated values can then be simulated which will be used to estimate the effect that various MOE exclusion limits have on the strength distribution. It will be assumed that no damage is caused during the evaluation of the bending stiffness and hence there will be no change in the strength of the lumber after the test. The improved strength distributions that can be produced with the MOEE and proof-load methods will be compared.

Appropriate criteria for comparing the profitability of the proof-loading and MOEE methods will be established. The increased worth of the lumber with improved strength distributions must be weighed against the cost of testing. The new worth of the lumber meeting the test requirements, the worth of the lumber rejected by the tests, and the testing costs will be evaluated relative to the original value of the lumber. Optimum and profitable ranges of proof-load and MOEE limits will be determined.

Chapter 2

Lumber Strength

Strength can be defined in many ways and it takes on special meanings when used in reference to different materials and the design philosophy employed. A decade ago, a structural engineer might have defined strength, in an allowable strength design context, as the maximum safe working stress. Today, many would define strength in terms of a limit state. The strength of concrete might be given as the maximum compressive stress determined from a standard cylinder test at 28 days; the strength of steel, as the yield stress determined from a tensile test performed on a steel coupon. The tests performed to determine lumber strengths have undergone many changes and many structural engineers are not familiar with the procedures followed to determine code-specified lumber strengths.

Steel and concrete are normally assumed to behave like isotropic materials, unlike wood, and only one set of strength properties is required for all directions. Wood is usually assumed to be orthotropic; its physical properties are considered to be different in orthogonal directions. The strength properties of lumber are normally defined in loading directions parallel and perpendicular to grain. Perpendicular to grain, the loading could be either tangential to the growth rings or radial. Often the radial and tangential properties are not differentiated and are grouped under the more general "perpendicular to grain" property heading.

The strengths of lumber are required in the orthogonal directions for the different loading conditions and must be determined for several different species groups and grades of lumber. While structural design with steel and reinforced concrete can be carried out with the basic information of one strength property parameter (f_y or f'_c), structural design with wood requires many tables of strength properties and modification factors.

Lumber strength is influenced by several conditions or phenomena, the most important being the load duration effect, size effect and moisture content. CAN3-086 Engineering Design in Wood specifies the strength of a 2" x 12" with 15% moisture content under a "normal" duration of loading of ten years as the base strength in bending for each species group and lumber grade. Adjustment factors are then applied to the base strengths to consider the particular size, use, location and loading conditions for the member.

2.1 Evolution of Strength Assessment by Code

Visually graded lumber used in structures that were designed in conformance with the code has performed well in the past, perhaps due to very conservative design practice. Codes for wood have evolved from those recommending good practice based on experience to current codes which are based on fundamental engineering principles. The change from a working stress to a limit states design philosophy put pressure on the wood engineering community to re-evaluate the factors affecting the

strength of lumber. As knowledge of the behaviour of lumber increases and uncertainties decrease, our designs can become less conservative and a more efficient use of our timber resource will result. Probability theory allows the variability in strength and load effects to be accounted for rationally without the loss of information associated with deterministic analysis. Probabilistic design procedures offer the greatest potential for the lumber industry to take advantage of an improved strength distribution of lumber brought about by proof-loading or non-destructive testing.

Many changes have been made to the CAN3-086 Engineering Design in Wood code in the last decade. Prior to the 1984 versions of the code, a working stress design format was used exclusively and the allowable strengths were based on tests performed on small clear wood specimens. Madsen (1975) questioned the practice of obtaining the strength of full size lumber (which includes growth characteristics such as knots, checks, crossgrain, etc.) by adjusting the strengths of small clear specimens free of defects. Madsen likened this practice to obtaining the strength of concrete from tests performed only on the cement paste. Madsen proposed the direct approach of testing full size visually graded lumber to determine the bending strength, modulus of elasticity, tension, compression parallel and perpendicular to grain, and shear capacity of lumber. This approach known as "In-Grade Testing" was used in a test program, which began in 1976, to provide suitable stresses for the 1984 working stress and limit states design codes.

Two versions of Engineering Design in Wood were published concurrently by CSA in 1984. CAN3-086.1-M84 was the first limit states design

version. CAN3-086-M84 (Working Stress Design) was published to help in the transition to Limit States Design. The procedure used to establish the allowable stresses and specified strengths for the 1984 code versions from the In-Grade Testing data is not widely known. Testing did not follow ASTM standards and no documentation is found in either code or commentary. The following description of the methods used to obtain the allowable and specified strengths in the last three versions of the code will highlight the complications encountered when determining the strength of lumber.

2.1.1 CAN3-086-M80 ALLOWABLE UNIT STRESSES

The allowable unit stresses for visually graded sawn lumber in M80 were based on procedures found in ASTM D245 "Establishing Structural Grades and Related Allowable Properties for Visually Graded Lumber". Testing was performed on clear wood specimens according to ASTM D143 "Standard Methods of Testing Small Clear Specimens of Timber". The test specimens, 51mm x 51mm in cross section, must be free of defects and have straight grain. The small specimens allow tests to be carried out with testing equipment commonly found in structural laboratories without the need for large set-ups. Bending tests are performed by loading the small clear specimens with a single midspan load on a span of 710 mm. The load is to be gradually increased so that failure occurs in approximately ten minutes. Deflection measured along with the applied loads allows the determination of a modulus of elasticity. The calculation for the allowable unit stress of full size lumber in the dry service

condition without fire treatment and used as a single member is given in Equation 2.1.

$$f_{all} = (f_r - 1.645\sigma_{f_r}) k_{grade} k_{dFS} k_{wd} k_b \quad (2.1)$$

The term in brackets would represent the fifth percentile strength determined from the tests, if the strength distribution of clear wood specimens were Gaussian. The Gaussian distribution is not well suited to represent strengths since it includes negative values.

The grade factor, k_{grade} , less than unity, decreases as the occurrence of defects increases causing a reduction in the allowable unit stress. This factor is proportional to the quality of the visual grade. Select structural lumber has the largest grade factor and No.3 has the lowest.

The second factor, k_{dFS} , serves two purposes: it accounts for the load duration effect and provides a factor of safety consistent with the working stress design philosophy. The base duration for the allowable unit stresses is called the "normal" duration of loading. It is that condition of loading such that the cumulative application of the full design load does not exceed ten years throughout the expected life of the member. Lyman Wood (1951) found that the strength of wood determined from a fast test to destruction in ten minutes is approximately 1.6 times greater than if loaded continuously for ten years. The factor of safety portion of k_{dFS} , depends on the loading condition and type of failure. Although Wood (1951) conducted his load duration experiments on wood in bending, the results have been applied to the other forms of loading also. Table 2-1 shows the k_{dFS} modification

factor for the different loading conditions broken down into the duration of load and factor of safety portions.

Table 2-1 THE BREAKDOWN OF k_{dFS}

LOADING CONDITION	k_{dFS}	k_d	k_{FS}	F.S.
Bending	1/2.1	1/1.6	1/1.3	1.3
Tension Parallel	1/2.1	1/1.6	1/1.3	1.3
Shear	1/4.1	1/1.6	1/2.6	2.6
Compression Parallel	1/1.9	1/1.6	1/1.2	1.2

The third factor, k_{wd} , is the only adjustment that is greater than unity. It is used to convert the strength of the green test specimens to the greater strength of "dry" lumber with a moisture content equal to 15%.

The fourth adjustment, k_b , applies only to bending strengths and accounts for the size effect. Lumber can be modelled with weakest link theory, the greater the depth of the member the greater the likelihood of encountering a defect which would initiate failure. Equation 2.2 expresses the bending adjustment, k_b , as a function of lumber depth, measured in millimetres.

$$k_b = \left(\frac{51}{d}\right)^{\frac{1}{9}} \quad (2.2)$$

The allowable strengths for eight groupings of species, four grades of lumber, and several sizes and uses of lumber are given in tables 50, 51 and 52 of CAN3-086-M80.

Adjustments are made to the allowable unit stresses by the designer in order to arrive at allowable working stresses which suit the particular design conditions. The duration of loading, service conditions (wet or dry), size of the member, and if the lumber is fire treated or not are important for the calculation of the allowable working stress.

2.1.2 CAN3-086-M84 WORKING STRESS DESIGN

Allowable unit stresses for lumber loaded in tension parallel to grain and bending were determined for the three most prevalent species groups in Canada from the In-Grade testing program described earlier. Douglas Fir-Larch, Hemlock-Fir and Spruce-Pine-Fir make up over 90% of all structural lumber produced in Canada. Full-size commercially graded lumber, from select structural to No.3, was tested at mill sites. The emphasis was on the determination of the low tail of the strength distribution. Test loads that would break between 10% and 15% of the lumber tested were used to get a description of the low tail of the strength distribution and estimate the fifth percentile strength. Estimates of the mean strength and coefficient of variation for the samples were obtained by loading between 25 and 40 pieces of the 200 piece samples to failure. The method used by the NLGA (1982) to determine the bending allowable unit stresses from the In-Grade tests is expressed by Equation 2.3. In this form it can be compared with the method used in the 1980 code for determining allowable unit stresses from clear specimen tests.

$$(f_{all})_{84} = (S_{.05,mc=15\%} k_{Harrison}) k_{Size8 \rightarrow 12} k_{dFS} \quad (2.3)$$

In-Grade testing eliminates a major adjustment that was previously necessary. The grade adjustment factor, k_{grade} , used to reduce the clear wood strength to account for the growth characteristics of the visually graded lumber, is no longer necessary since the graded lumber is tested directly. The lumber at the mills had varying moisture contents so the modulus of rupture values determined from the tests were adjusted to represent the strengths of lumber with a moisture content of 15%. This adjustment was much smaller than that used in Equation 2.1 since the clear specimens were converted from the green or wet state (30% M.C.) and the full size lumber had moisture contents much closer to 15%. Once the modulus of rupture values were converted to 15% moisture content, the fifth percentile strength of the samples was determined by ranking the strengths. The fifth percentile strength estimates, that were determined from each sample, varied. The best estimate of the population fifth percentile was obtained by combining the samples and selecting a ranked fifth percentile. To account for the uncertainty of the population fifth percentile modulus of rupture, the NLGA adopted the Harrison formula. The Harrison formula is based on a 75% lower confidence limit, which is that strength above which 75% of the sample fifth percentile estimates fall. The factor, k_{Harrison} , is used in this study to reflect the modification for sample variability. Equation 2.4 shows the k_{Harrison} modification factor as a function of the ratio of the 75% lower confidence limit divided by the combined sample 5th percentile estimate.

$$k_{\text{Harrison}} = \frac{S_{75\% \text{ LCL}}}{0.9 \times S_{.05}} \leq 1.0 \quad (2.4)$$

The allowable unit stresses in the 1984 version of the working stress design code represent those for 2" x 12" lumber. The stresses at failure of the 2" x 8" pieces tested were decreased to represent those of 2" x 12" lumber by the $k_{size8 \rightarrow 12}$ factor. The value of this factor is 0.857. It is considerably smaller than the k_b factor used in the 1980 allowable unit stress calculations discussed earlier.

Finally, the load duration difference between that of the rapid test and the "normal" load duration used in the code and the factor of safety must be accounted for. The code committee adopted the same k_{DFS} factor that was used in the previous code editions and which are shown in Table 2-1. It should be noted that the load duration portion of the factor was based on clear wood data and should be re-evaluated to reflect full size lumber duration of load effects. Table 2-2 shows the derivation of the allowable unit bending stresses for the Hemlock-Fir species group found in Table 50 of CAN3-086-M84.

Table 2-2 DERIVATION OF ALLOWABLE UNIT STRESSES FOR CAN3-086-M84

GRADE	$S_{.05}$ (psi)	$S_{75\%CL}$ (psi)	$k_{Harrison}$	$k_{size8 \rightarrow 12}$	k_{DFS}	f_b (psi)	f_b (MPa)
SS	3220	2950	1.0	0.857	1/2.1	1314	9.0
No. 1,2	2310	2130	1.0	0.857	1/2.1	943	6.5
No. 3	1590	1540	1.0	0.857	1/2.1	690	4.8

As a result of the In-Grade testing program, it was determined that there was little difference in the strength of No.1 and No.2 visual lumber grades. The same allowable unit stresses for these grades in bending are now used, unlike previous code editions in which the

allowable strength of No.1 was considerably higher than No.2 in bending. The testing of full size lumber also revealed that there was a smaller difference in strength between species groups than was previously found using small clear sample tests. The strength for the Douglas Fir-Larch group was reduced and the strengths of the Spruce-Pine-Fir and Hemlock-Fir groups increased in 1984 relative to the 1980 code.

2.1.3 CAN3-086.1-M84 LIMIT STATES DESIGN

CAN3-086.1-M84 Engineering Design in Wood employs the Limit States Design philosophy described in the National Building Code of Canada (1985). The load factors are the same as those used for structural steel and reinforced concrete design in Canada. This will allow the designer to use the same factored design loads for all structural materials. Separate load factors allow for the expression of the uncertainty with respect to the different load types which was not possible with working stress design. Since the load factors are set for all materials, the resistance factor must be used to calibrate the design equation to provide a suitable level of reliability. Elaborate probability studies may not be practical for the design office but should be used to calibrate the Limit States Design code.

The limit states design standard was not explicitly calibrated to a target reliability level. The 086.1-M84 Limit States Design code was calibrated to the 086-M84 Working Stress Design code. Excluding fastenings, the 1984 codes generally produce the same designs.

Reliability studies involving wood are complicated due to the dependance of the load-effect and resistance; but the next version of CSA CAN3-086.1 to be published, tentatively in 1989, will be calibrated to a given target reliability. Resistance factors used with LSD give the designer an indication of the uncertainty of the different failure modes. This was not possible with working stress design because the factors of safety (Table 2-1) were "hidden" in the allowable unit stresses.

The specified strengths used in CAN3-086.1-M84 were also based on the In-Grade test results. It is the author's belief that the specified strengths were obtained in the same manner as the allowable unit stresses of 086-M84, shown in Equation 2.3, with the exception that the adjustment for duration of load and factor of safety was not included. Equation 2.5 shows the calculations of the specified strengths.

$$F_{\text{spec}} = (S_{.05 \text{ MC}=15\%} k_{\text{Harrison}}) k_{\text{size8} \rightarrow 12} \quad (2.5)$$

$$F_{\text{spec}} = \frac{f_{\text{all 84}}}{k_{\text{dFS}}} \quad (2.6)$$

In examining the 1984 wood codes, it was found that Equation 2.6, with k_{dFS} equal to 1/2.1, held for all species groups in the bending and tension parallel to grain categories. This leads to an inconsistency because the specified strengths are meant to represent those for a normal duration of loading of ten years and not the very short duration of loading that was used for testing. There is no question that the factor of safety portion of k_{dFS} should not be included in the Limit States specified strength formulation but the duration of load modification should be included. Consider the calculations of the required section modulus for a simply supported beam using both the 1984 LSD and

WSD codes.

WSD: Allowable bending moment > Maximum bending moment

$$f_b S_{WSD} (k_D k_{Sb} k_T k_{Zb} k_G) k_H k_L > (\omega_D + \omega_L) L^2/8$$

$$S_{WSD} > \frac{(\omega_D + \omega_L) L^2/8}{f_b (k_D k_{Sb} k_T k_{Zb} k_G) k_H k_L} \quad (2.7)$$

LSD: Factored bending moment resistance > Factored bending moment

$$\phi F_b S_{LSD} (k_D k_{Sb} k_T k_{Zb} k_G) k_H k_L > (\alpha_D \omega_D + \alpha_L \omega_L) L^2/8$$

$$S_{LSD} > \frac{(\alpha_D \omega_D + \alpha_L \omega_L) L^2/8}{\phi F_b (k_D k_{Sb} k_T k_{Zb} k_G) k_H k_L} \quad (2.8)$$

Since the limit state code was calibrated to the working stress code, the WSD section modulus in Equation 2.7 is set equal to the LSD section modulus in Equation 2.8:

$$S_{WSD} = S_{LSD}$$

$$\frac{\omega_D + \omega_L}{f_b} = \frac{\alpha_D \omega_D + \alpha_L \omega_L}{\phi F_b}$$

Rearranging,

$$\frac{F_b}{f_b} = \frac{\alpha_D \omega_D + \alpha_L \omega_L}{\phi (\omega_D + \omega_L)}$$

Rearranging and substituting Equation 2.6 for the left term:

$$\frac{1}{k_{dFS}} = \frac{\alpha_D \omega_D + \alpha_L \omega_L}{\phi (\omega_D + \omega_L)}$$

From the relationship presented in Table 2-1:

$$k_{dFS} = k_d k_{FS} = \frac{1}{k_{dol} F.S.}$$

$$k_{dol} F.S. = \frac{\alpha_D \omega_D + \alpha_L \omega_L}{\phi (\omega_D + \omega_L)} \quad (2.9)$$

Equation 2.9 shows that the limit states design load and reduction factors (α, ϕ) are involved in the conversion of the short-term strength to the strength for a "normal" duration of loading. The WSD

factor of safety, alone, should be a function of the the load and resistance factors and the relative magnitudes of the dead and live loads. Rearranging Equation 2.9 and dividing the numerator and denominator by the dead load gives:

$$F.S. = \frac{1}{k_{dol}} \frac{\alpha_D + \alpha_L(\omega_L/\omega_D)}{\phi (1 + \omega_L/\omega_D)}$$

$$F.S. = \frac{1}{1.6} \frac{1.25 + 1.50(\omega_L/\omega_D)}{0.7 (1 + \omega_L/\omega_D)} \quad (2.10)$$

Equation 2.10 expresses the factor of safety (in a WSD context) of CAN3-086.1-M84 LSD as a function of the relative magnitudes of the dead and live loads. Figure 2-1 is a plot of Equation 2.10 and it shows that when the ratio of the live load to dead load is approximately equal to seven that the WSD and LSD codes will produce equivalent designs. Live to dead load ratios of approximately seven are common in wood design.

Fig. 2-1 Comparison of the Factors of Safety Associated with CAN3-086-M84 WSD and 086.1-M84 LSD

The omission of the DOL adjustment in the LSD specified strength calculation, shown by Equation 2.5, ignores accepted theory and should be included. If the DOL adjustment, k_{dol} , equal to 1.6 were used with the fifth percentile strength, a reduction factor greater than unity would be required to calibrate the LSD to the WSD code. This, too, is not acceptable. A tenth percentile strength may have to be used as the base strength or the duration of load adjustment may not be as high for lumber as was found for clear wood. This would allow for the DOL adjustment and a reduction factor less than unity to be calibrated to reliability studies.

2.2 Reliability Evaluation

Many uncertainties exist in structural design. Codified design procedures are necessarily deterministic; however, reliability studies can be used to calibrate limit states design procedures to achieve a targeted level of safety. The employment of probability theory is a rational way of accounting for the uncertainties which exist in structural engineering. The uncertainty regarding the maximum imposed load-effect and the strength or resistance of the member can be represented with their respective probability density functions. Figure 2-2 shows the relationship between the strength and load-effect probability density functions, f_R and f_Q . The probability of failure is the probability that the load-effect will exceed the strength of the member. Equation 2.11 expresses the probability of failure in terms of a convolution integral.

$$p_f = \int_0^{\infty} F_Q(s) f_R(s) ds \quad (2.11)$$

INTERACTION OF LOAD-EFFECT AND STRENGTH

Fig. 2-2 Initial Relationship Between Load-effect and Strength

INTERACTION OF LOAD-EFFECT AND STRENGTH

Fig. 2-3 New Relationship Between Load-effect and Strength

The area created by the overlapping of the right tail of f_Q and the left tail of f_R , shown shaded in Figure 2-2, gives an indication of the probability of failure. Generally, the larger the overlapping area, the greater the probability of failure. Proof-loading can alter the important left tail of the strength probability density function shifting it to the right. This can significantly decrease the probability of failure of a design, as shown graphically by the reduction in the overlapping shaded area in Figure 2-3.

The probability of failure calculated using the convolution integral of Equation 2.11 is accurate only if the load-effect and the strength variables are statistically independent. Since the strength of lumber is dependent on the load history used to test the member, the probability calculated using Equation 2.11 is not strictly correct. The probability of failure calculated in this way is still useful for the comparison of strength distributions with respect to a reference load-effect distribution. The load-effect distribution represents the maximum loads on a member that occur over its expected life. The load-effects at the high end of the distribution are not only rare but are likely only to occur over a very short duration. If the member considered was a floor joist, these extraordinarily high load-effects may be caused by a large gathering that might occur for a short duration, possibly only once or twice a year. A lumber member may conceptually fail due to a very high load near the beginning of its service life or it may fail at a lower load much later in its life. This phenomenon, alluded to earlier, is the load-duration effect and has been the topic of considerable research in wood engineering during the

past decade. Load duration effect, also known as creep-rupture behaviour or static fatigue, is not peculiar to lumber alone but it is much more evident in wood than in most of the conventional structural materials. The load-duration effect results in a decrease of lumber strength with time in service. The left tail of the strength distribution tends to shift down the strength scale with time, increasing the probability of failure. In this way, time in service has the opposite effect on the strength distribution and probability of failure that proof-loading can have. Cunliffe and Salinas (1987) have discussed the concept of reliability degradation with time.

2.2.1 RELIABILITY INDEX

The reliability index, β , is a good indicator of the level of safety of a system. If the demand and capacity of a system can be formulated using an analytical expression, the state of such a system can be characterized by a "performance function". The reliability index is defined as the ratio of the mean to the standard deviation of the performance function. The simplest performance function expression is given by Equation 2.12.

$$G = \text{Capacity} - \text{Demand}$$

$$G = R - Q \quad (2.12)$$

The following table shows how the performance function defines the state of the structure.

$R > Q$	$G > 0$	Safe State
$R = Q$	$G = 0$	Limit State
$R < Q$	$G < 0$	Failure State

If the load and resistance can be modelled with Gaussian or log-normal distributions, the probability density function (pdf) for G may be obtained analytically. Figure 2-4 shows the pdf of the performance function G and some of its properties. The reliability index, β , is expressed in terms of the mean and standard deviation of the performance function in Equation 2.13.

$$\beta = \frac{\mu_G}{\sigma_G} \quad (2.13)$$

Fig. 2-4 Performance Function Probability Density Function

CSA Special Publication S408-1981 recommends Equation 2.14 for the calculation of the reliability index of structural members. In this equation, m refers to the mean value and V refers to the coefficient of variation.

$$\beta = \frac{\ln(m_R/m_L)}{\sqrt{V_R^2 + V_Q^2}} \quad (2.14)$$

Equation 2.14 is a good approximation of the exact equation for log-normal variables given by 2.15, provided that the coefficients of variation are less than 0.3.

$$\beta = \frac{\lambda_R - \lambda_Q}{\sqrt{\zeta_R^2 + \zeta_Q^2}} \quad (2.15)$$

The probability of failure is related to the reliability index for the case of log-normal or normal variables in Equation 2.16. The probability of failure is equal to the value of the standard normal distribution function evaluated at $-\beta$.

$$p_f = \Phi(-\beta) \quad (2.16)$$

Table 2-3 shows the recommendation of Special Publication S408-1981 for the target reliability indices of steel and reinforced concrete buildings based on a 30 year life. These target reliability indices should be constant for all structural materials to provide consistent levels of safety for all buildings and to provide equal opportunity for the various construction materials involved. The probability of failure associated with the reliability calculated, using Equation 2.16, is shown in Table 2-3. The small numbers associated with the probability of failure, the negative connotation of failure and the difficulty in evaluating the convolution integral have made the reliability index a more convenient indicator of safety than the probability of failure itself.

Table 2-3 S408-1981 TARGET RELIABILITY INDICES
AND ASSOCIATED PROBABILITIES OF FAILURE

SAFETY CLASS *	TYPE OF FAILURE			
	GRADUAL		SUDDEN	
	β_T	P_f	β_T	P_f
Not Serious	2.5	6.21×10^{-3}	3.0	1.35×10^{-3}
Serious	3.5	2.33×10^{-4}	4.0	3.17×10^{-5}
Very Serious	4.0	3.17×10^{-5}	4.5	3.40×10^{-6}

*	Not Serious	Risk to life negligible and economic consequences are small Normal buildings
	Serious	
	Very Serious	Risk to life is great or economic consequences are considerable

The use of a First Order Second Moment (FOSM) formulation for non-linear performance functions is only approximate when the load and resistance cannot both be modelled by normal or log-normal probability distributions. The change in the reliability of the system brought about by censoring the left tail of the strength distribution cannot be adequately represented by the FOSM formulation because small changes to the left tail of the strength distribution will not be adequately reflected in the standard deviation of the strength. In addition to the lack of sensitivity to small changes in the distribution, the FOSM formulation requires the independence of the load and resistance and, therefore, will not be used in this study.

In this study the benefit that proof-loading can bring to lumber strength distributions will be evaluated by numerical integration of the convolution integral given by Equation 2-11. The strength distributions

of the proof-loaded lumber will be compared against a reference load-effect distribution to determine optimum proof-loading histories.

Probabilities of failure determined in this way are not correct because of the load and strength dependence, but they give a good indication of how changes in the strength distribution can improve the reliability of lumber. Evaluating the convolution integral ensures that no information about the strength distribution will be lost due to approximations using the first and second moments (mean and standard deviation) of the distribution. The FOSM methods approximate the distributions with the first and second moments but these cannot adequately represent the strength distribution of proof-loaded lumber. Reliability indices can be determined from the probability of failure using the inverse of Equation 2.16.

Determination of an accurate probability of failure which incorporates the dependence of strength on load history can be accomplished with the aid of a damage accumulation model. A Monte Carlo simulation of a load history can be evaluated with a Damage Model to determine whether failure will occur in the member's lifetime. Repeating the simulation will allow the calculation of the fraction of members that fail, giving indication of the probability of failure. The accurate description of the load duration effect with a damage accumulation model is fundamental to meaningful reliability calculations in lumber.

Chapter 3

Load Duration Effects

The strength of wood and lumber tends to decrease as the duration of the applied load is increased. The dependence of strength on time is known as the load duration effect, creep rupture behaviour or static fatigue. In addition to strength, serviceability properties of lumber, creep, and relaxation, are also time dependent.

In the past four decades many attempts have been made to account for the time-dependent strength of lumber. The work of Wood (1951) was an early attempt to investigate this phenomenon. He fit an empirical curve to the median times to failure of small, clear wood specimens that were loaded in bending at different constant stress levels.

Interest in reliability studies, just over a decade ago, highlighted the need for a model that could evaluate the load duration effect for random loading conditions. The majority of the more recent load duration models have taken the form of damage accumulation models.

Damage accumulation is an irreversible process throughout the life of a member that ultimately leads to failure. This process is due to creep rupture behaviour. The propagation of cracks or flaws can lead to fracture under service loads at stress levels that may be lower than those required to cause fracture in a short term test. Most damage accumulation models are based on a phenomenological approach to creep rupture since this behaviour is not understood well enough to base the models on fundamental physical laws.

3.1 Wood's Curve

Lyman Wood (1951) presented the first mathematical expression relating the bending strength of wood to duration of load. He intended the expression to be used by structural engineers in order that they may take advantage of the greater strength of wood under short term loading. L. Wood conducted long-term bending tests on small clear Douglas-Fir specimens. Constant loads with magnitudes which ranged from 60% to 95% of the loads that caused failure in a standard static test (failure in approximately five minutes) were used. The times to failure ranged from just over five minutes for the more heavily loaded specimens to more than five years for the specimens loaded at 60% of the standard short-term strength.

L. Wood compared his long-time constant load test results with earlier rapid loading test results from a study by Liska (1950) and impact test results from Elmendorf (1916). A single curve was fitted to these points that represented the trends of the long-time and rapid loading data and which passed close to the impact load data. The curve is represented by Equation 3.1 where the percentage of the applied stress relative to the static short-term strength, y , is expressed as a function of the duration of load, x , measured in seconds.

$$y = 108.4 x^{-.04635} + 18.3 \quad (3.1)$$

or,

$$\frac{S}{S_s} \times 100\% = 108.4 t^{-.04635} + 18.3 \quad (3.2)$$

converting from percentages to fractions

$$\frac{S}{S_s} = 1.084 t^{-.04635} + 0.183 \quad (3.3)$$

The term $\frac{S}{S_s}$ is known as the "stress ratio" and is denoted σ ,

$$\sigma = 1.084 t^{-.04635} + 0.183 \quad (3.4)$$

WOOD'S CURVE

Fig. 3-1 The Load Duration Behaviour of Wood
As Determined by L. Wood (1951)

Fig. 3-2a Real Time Load Duration Behaviour of Wood
With Time Units Measured in Years

Fig. 3-2b Real Time Load Duration Behaviour of Wood
With Time Units Measured in Minutes

Equation 3.4 was used to plot Wood's curve, shown in Figure 3-1, with the exception that the times have been converted from seconds to the logarithm of time, measured in hours. Plots where time is expressed as the logarithm of time are useful for portraying the load duration effect; however, the distorted representation of time may not give a true impression of the load duration effect. Figures 3-2a and b show Wood's curve where the time axes are plotted in real time units.

The Engineering Design in Wood codes use the load duration factor, k_D , to adjust the specified strengths for the "normal" duration of loading to the strengths relevant for a particular project's load duration. The load duration factor is one of the largest adjustments made to the specified strength and is based on Wood's work. Evaluating the stress ratio at the "normal" duration of loading equal to ten years gives:

$$\sigma_{\text{normal}} = 0.6206$$

An equation for the load duration factor can be obtained by dividing the stress ratio, given by Equation 3.4, by the stress ratio for the normal duration of loading.

$$k_D = \frac{\sigma(t)}{\sigma_{\text{normal}}} = \frac{1.084 t^{-.04635} + .183}{0.6206} \quad (3.5)$$

$$k_D = 1.747 t^{-.04635} + 0.2949 \quad (3.6)$$

Table 3-1 compares the derived load duration factor, Equation 3.6, with those found in Table 1 of the Engineering Design in Wood codes. While the load duration factor was based on bending tests performed on small, clear wood, Douglas Fir specimens in 1951, it is currently applied in the code to full size lumber of all grades, glue laminated members, plywood, and fastenings in all failure modes.

Table 3-1 LOAD DURATION FACTORS, k_D

LOADING CONDITION	DURATION	$k_D(3.6)$	$k_D(\text{code})$
Continuous	100 years	0.929	0.90
Normal	10 years	1.000	1.00
Snow	2 months	1.148	1.15
Temporary, Formwork	7 days	1.237	1.25
Wind or Earthquake	1 day	1.326	1.33
Impact	10 minutes	1.594	1.60

3.2 Load Duration Effect in Lumber

Madsen and Barrett (1976) studied the load duration effect on full size Douglas-fir lumber and found the behaviour of lumber to be considerably different from that exhibited by the clear wood specimens studied by Wood (1951). Foschi and Barrett (1982) studied the load duration effect of full size 2" x 6" Western Hemlock lumber and used the data to calibrate the damage accumulation model that they had developed. The median times to failure under constant loading obtained by Foschi and Barrett (1982) for Western Hemlock fell within the range reported by Madsen and Barrett (1976) for Douglas Fir lumber. Foschi and Barrett (1982) reported that this might imply that "the duration of load effect in lumber may be relatively species-independent." A comparison of Wood's curve and the mean duration of load behaviour of full sized lumber is shown in Figure 3-3. Wood's curve tends to overestimate the times to failure of lumber loaded with very high stress ratios. However, not all of this difference is due to the different behaviour of clear wood and lumber. Wood tried to fit, with single curve, not only

Fig. 3-3 A Comparison of the Load Duration Behaviour of Wood and Lumber as Described by Wood (1951) and Barrett and Foschi (1982)

the constant load data but also the rapid loading data of Liska and the impact loading of Elmendorf. As will be seen later, the different load histories play an important role in the load duration effect and should not be grouped together.

In the working range of stresses, Wood's curve overestimates the mean load duration behaviour of lumber by underestimating the times to failure. The variability of the load duration effect is not accounted for in either of the plots in Figure 3-3 but will be considered later.

Another major difference in load duration behaviour of wood and lumber involves the "threshold stress ratio". The "threshold stress ratio" is the fraction of the standard static strength that may be applied indefinitely without causing fracture of the member. Tests on lumber indicate that the threshold stress ratio is approximately 0.5 while Wood's tests on clear wood indicate that it is approximately 0.18.

The duration of load behaviour of lumber in bending was better understood after Madsen, Barrett and Foschi's tests; however, the extrapolation of bending results to other failure modes and species, as well as to other structural wood products, still needed to be investigated. In 1983 Canada and the USA began an extensive Duration of Load (DOL) research program. Douglas-Fir, Southern Pine, and Canadian White Spruce, small dimension sawn lumber with varying qualities, were tested under ramp and constant loads in bending, tension, and compression. This testing program is still in progress; however, Forintek Canada Corporation (1987) has provided an up-to-date report on the tests. The interim report concluded that Wood's (Madison) curve overestimated the

load duration effect for constant load durations between one day and two months but underestimated the load duration effect for a constant load of ten year duration. The quality (grade) of the lumber was found to have an effect on the load duration behaviour. The low quality lumber was less affected by the load duration than the high quality lumber. The species of lumber was not observed to affect the load duration behaviour of the lumber, as was predicted by Foschi and Barrett (1982). In addition to species, the load duration effects were relatively "insensitive" to size differences. A very important finding was that lumber loaded in tension and compression was found to exhibit a more pronounced load duration effect than when loaded in bending. The results of the joint Canada/USA DOL experiment will reduce much of the uncertainty concerning the load duration effect in lumber. There is still a need for the study of the load duration effect in other structural wood products.

3.3 Load Duration Models

In the mid 1970's, interest in reliability-based design created a need for the evaluation of the load duration effect in lumber subjected to actual load histories. Wood's curve was not satisfactory for the study of the safety of wood structures because it could not incorporate time-varying loads and was based on tests performed on small, clear wood specimens. The first load duration models for wood, proposed by Gerhards (1979) and Barrett and Foschi (1978a,1978b), could incorporate varying loads. The models measured the load duration effect in

different terms; Gerhards linear cumulative damage theory measured residual fractional lifetime, while Barrett and Foschi measured the damage that was accumulated. Both models have undergone changes over the past decade. The first modelling attempts were calibrated to Wood's clear wood data because experimental programs with lumber were not complete. Gerhards (1987) now measures damage instead of residual fractional lifetime and the models are now calibrated to lumber test results. Nielson (1980,1985) took a different approach; he based his load duration model on fracture mechanics and crack propagation theory. Nielson incorporated the theories of viscoelasticity and fracture mechanics in developing his model. The model includes an extensive mathematical formulation which makes it difficult to use.

Foschi and Yao (1986) examined the three duration of load models mentioned above and found that both the damage accumulation model by Foschi (an evolved form of Barrett and Foschi's earlier model) and Nielson's model "were suitable to represent accurately the experimental trends in Western Hemlock lumber in bending". Gerhards damage accumulation model was found to be too rigid to follow the observed experimental trends. The Canadian Damage Accumulation model by Foschi and Yao will be used in this study to evaluate the effect that proof-loading has on the strength of lumber.

3.3.1 EVOLUTION OF THE CANADIAN DAMAGE ACCUMULATION MODEL

The Canadian damage accumulation model by Foschi and Yao (1986) has evolved from Barrett and Foschi (1978a) model II. The effect of the load duration on lumber is measured with the damage accumulation parame-

ter, α . The measure of damage, α , varies between 0 and 1.0, where 0 corresponds to no damage and 1.0 corresponds to total damage or failure. Damage accumulation models are generally presented, in equation form, as a rate of damage accumulation which is a function of the current stress level and the previous damage. This formulation, expressed by Equation 3.7, was first presented by Bolotin (1969).

$$\frac{d\alpha}{dt} = f(\sigma, \alpha) \quad (3.7)$$

The existence of long standing wood structures, such as 800 year old stave churches in Norway, suggests that a stress threshold must be exceeded for creep rupture to occur. A threshold stress ratio, σ_0 , was first used by Wood (1951) and is incorporated in all damage accumulation models proposed by Foschi. Damage model II of Barrett and Foschi (1978a) is expressed by Equation 3.8.

$$\begin{aligned} \frac{d\alpha}{dt} &= a(\sigma - \sigma_0)^b + \lambda\alpha & \sigma > \sigma_0 \\ \frac{d\alpha}{dt} &= 0 & \sigma \leq \sigma_0 \end{aligned} \quad (3.8)$$

The applied stresses are non-dimensionalized by dividing by the short-term strength, shown by Equation 3.9.

$$\sigma = \frac{\tau(t)}{\tau_s} \quad (3.9)$$

The parameters a , b and λ are used to calibrate the model to experimental data. Once calibrated, the damage model can be integrated over the load history to estimate the amount of damage at any point in time. The time to failure can also be estimated by setting the damage equal to unity and solving for t . Closed form integration for damage is only possible for the simplest of load histories, such as constant or ramp loads, and numerical integration is often necessary.

The first damage accumulation models were deterministic in nature. To account for the random nature of the load duration effect, Barrett and Foschi (1978b) introduced variability parameters, ω_1 and ω_2 , that were multiplied by standard normal variates, R_N and V_N , shown in Equation 3.10.

$$\begin{aligned} \frac{d\alpha}{dt} &= a e^{\omega_1 R_N} \left(\frac{\tau - \tau_0}{\tau_s} \right)^b + \lambda e^{\omega_2 V_N} \alpha & \tau > \tau_0 \\ \frac{d\alpha}{dt} &= 0 & \tau \leq \tau_0 \end{aligned} \quad (3.10)$$

The median of Equation 3.10 ($R_N = V_N = 0$) is Damage Model II expressed by Equation 3.8. The random model, Equation 3.10, was calibrated by Foschi and Barrett (1982) to results of tests performed on 2" x 6" Western Hemlock lumber.

Damage models are often compared by the times to failure they produce for constant load histories. Plots of constant stress ratio versus the logarithm of time to failure similar to Wood's curve are used. These comparisons overlook the most important function of the damage model, which is its ability to evaluate the load duration behaviour of time varying load histories. Damage model II was able to successfully represent the load duration behaviour of lumber for constant and ramp load histories but not for some random load histories. Damage model II tends to overestimate the damage rate when a significant amount of damage has occurred and the stresses are low, just above the threshold stress. The second term, $\lambda\alpha$, was modified by Foschi and Yao (1986) in the Canadian damage model to rectify this problem. The second term in the Canadian damage model, shown by Equation 3.11, is now a function of the stress level.

$$\begin{aligned} \frac{d\alpha}{dt} &= a[\tau(t) - \sigma_0 \tau_s]^b + c[\tau(t) - \sigma_0 \tau_s]^n \alpha(t) & \tau(t) > \sigma_0 \tau_s \\ \frac{d\alpha}{dt} &= 0 & \tau(t) \leq \sigma_0 \tau_s \end{aligned} \quad (3.11)$$

The Canadian damage model parameters a , b , c , n , and σ_0 , are considered to be constant for a particular piece of lumber but vary from piece to piece to account for the variability of the load duration effect. In the previous version of the model, given by Equation 3.10, only the parameters α and λ were considered to be variable while a and b were constant for all pieces. The number of model parameters appear to have increased from four (a , b , σ_0 and λ) to five (a , b , c , n and σ_0) in the latest model; however, the parameter "a" can be obtained approximately as a function of two of the other model parameters.

$$a \sim \frac{k_s(b+1)}{[\tau_s(1-\sigma_0)]^{(b+1)}} \quad (3.12)$$

The parameter "a" is obtained from the integration of Equation 3.11 over the short-term ramp load test that is used to determine the short term strength, τ_s . The short-term test proceeds at a constant rate of stress increase, k_s , which is expected to cause the mean time to failure to be approximately one minute. The short-term strength load history is represented by Figure 3-4.

Fig. 3-4 Ramp Load History Used to Determine Short-term Strength

The selection of the model form is only the first step in the development of a damage accumulation model. A duration of load testing program must be carried out to provide data for the calibration of the model. The joint Canada/USA duration of load research program, which started in 1982, will provide test results for lumber in bending, tension, and compression to which the Canadian damage model can be calibrated. As part of an up-to-date report, Forintek Canada Corporation (1987) calibrated the Canadian damage accumulation model to preliminary results from the co-operative DOL research program. The model parameters were assumed to be positive, independent, and lognormally distributed random variables. The calibration to the test data was made with a least squares minimization technique. Distribution parameters for each model parameter were selected and 1000 times to failure were then simulated. The minimization function, a measure of the difference between the predicted and measured times to failure, was then evaluated. The process of selecting model parameters for the lognormal distributions was then repeated until a best set of parameters was obtained. Table 3-2 shows the calibrated mean and standard deviation of the model parameters and the short-term strength of a control group, and the rate of loading used for the short-term strength test. These parameter values, evaluated by Forintek Canada Corporation (1987), require that time be measured in hours and stresses in psi.

Table 3-2 CANADIAN DAMAGE ACCUMULATION DATA

SPECIES Loading	b mean/s.d.	c mean/s.d.	n mean/s.d.	σ_o mean/s.d.	τ_s mean/s.d.	K_s psi/hr.
2x6 Hemlock Bending	37.161 10.430	0.2465×10^{-6} 0.8294×10^{-7}	1.290 0.097	0.533 0.159	6937 2833	388500
2x4 Spruce Bending	608.398 47.949	0.4368×10^{-5} 0.2419×10^{-6}	1.721 0.057	0.603 0.100	3964 1435	867000
2x4 Spruce Compression	65.392 4.966	0.1757×10^{-6} 0.4422×10^{-7}	1.415 0.179	0.247 0.053	3718 816	384000
2x4 Spruce Tension	17.058 5.386	0.7944×10^{-3} 0.9620×10^{-4}	0.060 0.044	0.519 0.037	2245 595	142800

3.3.2 TIME TO FAILURE UNDER CONSTANT LOADING

In this subsection the Canadian damage model, Equation 3.11, will be used to develop an equation for the damage that results from a constant applied load that exceeds the threshold stress. The time to failure under the application of a constant load can then be obtained by setting the damage equal to 1.0.

In mathematical notation: Let $\tau(t) = \tau_c > \sigma_o \tau_s$

$$\frac{d\alpha}{dt} = a(\tau_c - \sigma_o \tau_s)^b + c(\tau_c - \sigma_o \tau_s)^n \alpha(t) \quad (3.13)$$

The "variables separable" form, Equation 3.14, facilitates the solution.

$$\frac{d\alpha}{dt} = d + k\alpha \quad (3.14)$$

where

$$d = a(\tau_c - \sigma_o \tau_s)^b$$

$$k = c(\tau_c - \sigma_o \tau_s)^n$$

dividing by k,

$$\frac{d\alpha}{k dt} = \frac{d}{k} + \alpha$$

rearranging and integrating,

$$\int \frac{d\alpha}{d/k + \alpha} = \int k dt$$

$$\ln\left(\alpha + \frac{d}{k}\right) = kt + c_1$$

$$\alpha + \frac{d}{k} = e^{kt + c_1}$$

$$\alpha = c_2 e^{kt} - \frac{d}{k}$$

boundary condition, at $t=0 \rightarrow \alpha=0$

$$0 = c_2 - \frac{d}{k} \rightarrow c_2 = \frac{d}{k}$$

$$\alpha = \frac{d}{k} e^{kt} - \frac{d}{k}$$

substituting back for d and k ,

$$\alpha = \frac{a}{c} [\tau_c - \sigma_0 \tau_s]^{b-n} [e^{c(\tau_c - \sigma_0 \tau_s)^n t} - 1] \quad (3.15)$$

at failure, $t=t_f$ and $\alpha=1$

$$1 = \frac{a}{c} [\tau_c - \sigma_0 \tau_s]^{b-n} [e^{c(\tau_c - \sigma_0 \tau_s)^n t_f} - 1]$$

$$\frac{c}{a} [\tau_c - \sigma_0 \tau_s]^{n-b} + 1 = e^{c(\tau_c - \sigma_0 \tau_s)^n t_f}$$

Taking the natural logarithm of each side,

$$\ln\left\{\frac{c}{a} [\tau_c - \sigma_0 \tau_s]^{n-b} + 1\right\} = c [\tau_c - \sigma_0 \tau_s]^n t_f$$

$$t_f = \frac{\ln\left\{\frac{c}{a} [\tau_c - \sigma_0 \tau_s]^{n-b} + 1\right\}}{c [\tau_c - \sigma_0 \tau_s]^n} \quad (3.16)$$

Equation 3.16 gives the time to failure under constant loading; however, values for the parameters cannot be directly substituted because the stress term in the numerator is raised to a large power, n , and will lead to an arithmetic overflow. Equation 3.17 was used to prepare plots of constant applied stress versus time to failure, which will be discussed later.

$$t_f = \frac{\ln(e^Z + 1)}{c(\tau_c - \sigma_0 \tau_s)^n} \quad (3.17)$$

where $Z = \ln[c] + (b+1)\ln[\tau_s - \sigma_0 \tau_s] - \ln[k_s(b+1)] + (n-b)\ln[\tau_c - \sigma_0 \tau_s]$

3.3.3 PARAMETER STUDY

It would appear from Equation 3.11 that the damage rate and, therefore, the degree of load duration behaviour, was directly proportional to the parameters b , c and n and inversely proportional to the parameter σ_0 . The previous statement is true with the exception that parameter b , which is also used in the calculation of parameter a , is inversely proportional to the damage rate. Figures 3-5a to d show the influence of each model parameter on the load duration behaviour of lumber under constant loading. The middle curve in each of the figures represents the mean load duration behaviour of Western Hemlock lumber in bending. The curves on each side of the mean curve were obtained with the fifth and ninety-fifth percentiles of the parameter under investigation while the values of the other parameters were left equal to their mean values. Three facts concerning these plots may be observed:-

- (i) The threshold stress ratio, σ_0 , has the greatest influence on the load duration behaviour of the four model parameters.
- (ii) The parameters b and σ_0 are directly proportional to the time to failure, while c and n are inversely proportional to the time to failure.
- (iii) The parameters c and n do not have a noticeable influence for times to failure less than 100 hours.

An estimate of the range of the load duration behaviour under constant loading is shown in Figure 3-6. For very short durations of loading, the maximum constant applied stress that can be achieved approaches the static short-term strength (6937 psi). As the duration of load increases, the maximum constant stress approaches the threshold stress. The dispersion of times to failure tends to increase as the applied stress decreases.

Fig. 3-5a Influence of Parameter b on the Characterization of the Load Duration Effect

Fig. 3-5b Influence of Parameter c on the Characterization of the Load Duration Effect

Fig. 3-5c Influence of Parameter n on the Characterization of the Load Duration Effect

Fig. 3-5d Influence of Parameter σ_0 on the Characterization of the Load Duration Effect

Fig. 3-6 The Range of the Load Duration Behaviour;
Characterized by Extreme Values of the CDM Parameters

3.3.4 CDM FIRST TERM APPROXIMATION

The parameters c and n , which did not appear to influence the load duration behaviour for times to failure less than 100 hours, are both involved with the second term of the Canadian Damage model given by Equation 3.11. Perhaps a first term approximation to the CDM can be used to accurately represent the load duration behaviour of lumber for short periods of loading. The approximation, Equation 3.18, would simplify the evaluation of the load duration behaviour and allow closed form solutions to ramp load histories which would be used in proof-loading.

$$\begin{aligned} \frac{d\alpha}{dt} &= a[\tau(t) - \sigma_0 \tau_s]^b & \tau(t) > \sigma_0 \tau_s \\ \frac{d\alpha}{dt} &= 0 & \tau(t) \leq \sigma_0 \tau_s \end{aligned} \quad (3.18)$$

To evaluate the accuracy of the approximation, consider a comparison of the two models when used to evaluate the load duration behaviour of Western Hemlock under constant loading. Integration of Equation 3.18 over a constant load history yields the expression for the time to failure given by Equation 3.19.

$$t_f = \frac{[\tau_s - \sigma_0 \tau_s]^{b+1}}{k_s(b+1)[\tau_c - \sigma_0 \tau_s]^b} \quad (3.19)$$

Figure 3-7 shows the times to failure obtained using the CDM (Equation 3.16) and the approximation (Equation 3.19) for the load duration behaviour of Western Hemlock loaded in bending, for a wide range of parameter values. The approximation is very good for times to failure less than 10 hours. A damage model is needed to evaluate the damage that occurs as a result of short ramp loads used in proof-loading. Ramp loads are less severe than constant loads of the same maximum stress

COMPARISON OF C.D.M. WITH APPROXIMATION

WESTERN HEMLOCK IN BENDING

Fig. 3-7 A Comparison of the Characterization of the Load Duration Behaviour using the Canadian Damage Model and Approximation

level and, therefore, the approximation will be satisfactory for ramp loads used in proof-loading which are generally less than one minute.

3.3.5 CDM APPROXIMATION FOR RAMP LOAD HISTORIES

Consider the integration of the CDM approximation (Equation 3.18) to determine the damage, as a function of time, and the time to failure under ramp loading.

$$\text{Let } \tau(t) = k_r t$$

$$\frac{d\alpha}{dt} = a[k_r t - \sigma_o \tau_s]^b \quad k_r t > \sigma_o \tau_s$$

$$\frac{d\alpha}{dt} = 0 \quad k_r t \leq \sigma_o \tau_s$$

Rearranging and Integrating,

$$\alpha = \frac{a}{k_r(b+1)} [k_r t - \sigma_o \tau_s]^{b+1} + C_1$$

Boundary condition at $t = \frac{\sigma_o \tau_s}{k_r} \rightarrow \alpha = 0$ therefore $C_1 = 0$

$$\alpha = \frac{a}{k_r(b+1)} [k_r t - \sigma_o \tau_s]^{b+1} \quad t > \frac{\sigma_o \tau_s}{k_r} \quad (3.20)$$

At failure, $\alpha = 1$ and $t = t_f$

$$1 = \frac{a}{k_r(b+1)} [k_r t_f - \sigma_o \tau_s]^{b+1}$$

Rearranging,

$$t_f = \frac{1}{k_r} \left\{ \left[\frac{k_r(b+1)}{a} \right]^{\frac{1}{b+1}} + \sigma_o \tau_s \right\} \quad (3.21)$$

Now, consider a special ramp load, the ramp load history used to determine the short term strength.

$$\tau_s = k_s t_f \quad \text{or} \quad t_f = \frac{\tau_s}{k_s}$$

Substituting into Equation 3.21

$$\frac{\tau_s}{k_s} = \frac{1}{k_r} \left\{ \left[\frac{k_r(b+1)}{a} \right]^{\frac{1}{b+1}} + \sigma_o \tau_s \right\}$$

Solving for "a", Equation 3.12 can be derived.

$$a = \frac{k_s(b+1)}{[\tau_s - \sigma_o \tau_s]^{b+1}} \quad (3.12)$$

Substituting 3.12 into Equation 3.20 gives

$$\alpha = \frac{k_s}{k_r} [k_r t - \sigma_0 \tau_s]^{b+1} \quad t > \frac{\sigma_0 \tau_s}{k_r} \quad (3.22)$$

Substitute Equation 3.12 for "a" in Equation 3.21

$$t_f = \frac{1}{k_r} \left\{ \left[\frac{k_r(b+1)}{k_s(b+1)} [\tau_s - \sigma_0]^{b+1} \right]^{\frac{1}{b+1}} + \sigma_0 \tau_s \right\}$$

$$t_f = \frac{\tau_s}{k_r} \left\{ \left[\frac{k_r}{k_s} \right]^{\frac{1}{b+1}} [1 - \sigma_0] + \sigma_0 \right\} \quad (3.23)$$

The stress at failure, τ_r , is equal to $k_r t_f$.

$$\tau_r = \left\{ \left[\frac{k_r}{k_s} \right]^{\frac{1}{b+1}} [1 - \sigma_0] + \sigma_0 \right\} \tau_s \quad (3.24)$$

The stress at failure under ramp loading, given by Equation 3.24, is a function of the rate of loading relative to the short term test rate, the threshold stress ratio, the short term strength, and the model parameter b. When the rate of loading, k_r , exceeds the short term test rate, k_s , the stress at failure will exceed the short term strength.

Equation 3.23 was used to plot the load duration behaviour of Western Hemlock lumber ramp loaded to failure. Figure 3-8 shows the range of load duration behaviour obtained with the fifth and ninety-fifth percentile values for b and σ_0 . The curves all cross at the point which corresponds to the short-term test strength and time to failure. It would appear that those pieces of lumber which exhibit a strong load duration behaviour fail sooner than the norm at low stress levels but fail later than the norm when loaded at very high stress levels greater than the short term strength.

Fig. 3-8 The Range of Load Duration Behaviour of Western Hemlock Lumber Under Ramp Loading

3.3.6 LOAD HISTORY EQUIVALENCE

Load histories can be considered equivalent if they produce the same amount of damage in a member. The load histories that are represented by all points in the plot of stress versus duration of load can be considered equivalent since they all produce failure in a particular member. As was mentioned earlier, constant loads are more severe than ramp loads which have the same maximum stress level. The times to failure under constant loads are much shorter than those for ramp loads with the same maximum load. Equations 3.19 and 3.23 were used to plot Figure 3-9. Consider a particular piece of Western Hemlock with CDM model parameters equal to their mean values, represented by the middle curves of both the ramp and constant load series of plots. The load histories in Figure 3-10 have been taken from points along the middle curves in Figure 3-9 and can be considered equivalent because they all produce failure. The four load histories in Fig. 3-10 are very different and show the importance of defining strength with respect to a specific load history. The stress at failure for the particular piece of lumber, defined by mean values of the CDM parameters, could be as high as 7300 psi, if ramp loaded to failure in one second or as low as 6540 psi, if held at a constant load for 100 seconds. The short-term strength of the piece is 6937 psi.

Salinas and Gillard (1987) used the concept of equivalent load histories to recommend faster, more economical tests to determine the acceptance of new structural wood products.

BEHAVIOUR UNDER RAMP AND CONSTANT LOADS

WESTERN HEMLOCK IN BENDING

Fig. 3-9

A Comparison of the Load Duration Behaviour
Under Ramp and Constant Load Histories

Fig. 3-10 Equivalent Load Histories

Chapter 4

Proof-load Testing

Proof-load testing is a direct means of eliminating exceptionally weak structural elements from use. This can improve the strength distribution of lumber or other engineered wood products, thus increasing their utility.

An accurate analysis of the proof-loading process must address the viscoelastic and creep rupture behaviour of lumber. Economical proof-loading systems generally test at a faster rate of loading than standards prescribe for strength testing. Lumber appears stronger when tested at a fast rate; therefore, a proof-load test at a fast rate of loading is no guarantee that the surviving lumber has a "strength" (measured in a longer standard test) as great as the maximum proof-load stress. McLain and Woeste (1985) studied proof-load testing in tension at a rapid rate (test period of approximately 20 seconds) which was twenty times faster than the ASTM D198 standard rate. They proposed that an adjustment factor be used to reduce the strengths determined at the faster rate of loading to those strengths that would be determined at the slower ASTM rate.

While the effect of loading rate in proof-loading systems is generally accepted, whether lumber is damaged during proof-loading, and subsequently reduced in strength, is a controversial issue having significant commercial implications. After a literature review on this topic, Bechtel (1983) wrote: "At low loads, the evidence points toward negligible damage caused by either bending or tension proof testing".

In his 1983 work, which was primarily concerned with the determination of the proof-load that would yield maximum economic return, Bechtel assumed that proof-testing would truncate the lower tail of the strength distribution. He gave no consideration to the difference in the proof-load history and the load history used to define strength.

The approximation to the Canadian Damage Model will be used in this study to account for both the difference in the proof-load and strength test histories and any reduction in strength that may occur as a result of damage incurred during the test. The damage accumulation model will allow the assessment of these important issues in this study where physical experiments could not, due to financial restrictions. Since lumber cannot be tested to destruction twice (unlike with model analysis), physical experiments must rely on the change in the distributions of strength, before and after proof-loading, to evaluate the changes caused by proof-loading. Two groups have to be established, one to be proof-loaded and the other to serve as a control group. Hundreds of tests would be required to build up accurate strength distributions that could be used for comparison of the groups. Thousands of tests may be required to determine with certainty the amount of damage that has occurred at some proof-load levels. If an experimental program were attempted, it would need to be repeated for several proof-load histories with varying magnitudes in order to assess the effect of proof-load magnitude. Physical experimentation is very important for proof-load research; however, small scale testing programs may not clearly resolve the important issues of damage accumulation and rate of loading effects.

The proof-load analysis using the damage accumulation model which follows could be used to set up an economical proof-loading system. This analysis accounts for damage and different proof-load histories. It is more conservative, yet more realistic, than the truncation method presently used. The output of the proof-load regime should be sampled and a Bayesian approach could be used to update the model's predicted distribution.

4.1 Proof-load Test Model

It was shown in Chapter 3 that a first term approximation to the Canadian Damage Model provided results that were consistent with the parent model for short durations of loading. The damage model, expressed as a rate of damage accumulation, is a function of the load history, model parameters, and short term strength.

4.1.1 LUMBER STRENGTH AND CREEP RUPTURE

The time dependent behaviour of a piece of lumber can be evaluated with the approximation to the Canadian Damage Model provided that the short term strength, τ_s , and the model parameters, b , and the threshold stress ratio, σ_0 , are known.

The maximum internal stress at which a piece of lumber will fail will vary with the load history used for the test; however, the piece of lumber can only have one strength at a point in time. For the purpose of this study, "strength" will be defined as the maximum fibre stress which is calculated using fundamental strength of materials theory,

attained from tests at a rate that is expected to cause the mean time to failure of the population to be one minute. The same definition was used in the joint Canada/USA duration of load study, reported by Forintek Canada Corporation (1987). The short-term strength test load rates, k_s , are given in Table 3-2.

4.1.2 PROOF-LOAD HISTORY

The proof-load considered in this study will be a ramp load, defined by the maximum internal stress that is caused and the test duration. The duration must be considerably less than one minute for the proof-loading system to be economical. Any proof-load duration can be accommodated by the theory and the computer program developed; however, a test duration of 10 seconds will be used for future discussion of the effects of proof-loading on lumber strength. Figure 4-1 depicts the proof-load history and the symbols that will be used.

Fig. 4-1 Ramp Proof-load Test History

The proof-load should induce a stress pattern over the length of the lumber that is as close as possible to the pattern of stresses that

the lumber will see in service. For example, lumber to be used as joists could be loaded in bending with third point loads. This would create a stress pattern that is similar to that caused by uniformly distributed loads which are expected in service. While proof-loading in tension is more difficult to perform than in compression, both tests should stress as much of the length of the piece of lumber as possible. The organizers of the joint Canada/USA duration of load study, which was used to calibrate the Canadian Damage Model, observed these requirements.

4.1.3 EVALUATION OF DAMAGE CAUSED BY PROOF-LOADING

The damage model can be used to determine whether a piece of lumber, defined by specific values for the model parameters and short term strength, will pass or fail a proof-load test and to evaluate the amount of damage that occurred. If the maximum stress imposed by the proof-load is less than the threshold stress ($\sigma_o \tau_s$) of the piece of lumber, then no damage is expected to occur. However, if the threshold stress is exceeded, some damage will occur. Equation 3.22 was developed in Chapter 3 to determine the damage that occurs under ramp loading. Equation 4.1 will be used to determine the proof-load damage, α_p .

$$\begin{aligned} \alpha_p &= \frac{k_s}{k_p} \left[\frac{S_p / \tau_s - \sigma_o}{1 - \sigma_o} \right]^{b+1} & S_p > \sigma_o \tau_s \\ \alpha_p &= 0 & S_p \leq \sigma_o \tau_s \end{aligned} \quad (4.1)$$

If the proof-load damage calculated by Equation 4.1 is less than 1.0, then the piece of lumber will pass the test; otherwise it will be assumed to fail.

4.1.4 STRENGTH AFTER PROOF-LOADING

The strength of lumber that passes the proof-load test, but sustains some damage in the process, must be determined. The definition of strength, as the short term strength, must remain the same as that used to determine the initial strength. The lumber strength after proof-loading can be evaluated with the damage model by determining the stress that causes the damage to rise from α_p to 1.0 when loaded at the short-term strength test rate, k_s . The new strength after proof-loading, τ_n , will now be developed starting from the approximation to the Canadian Damage Model given by Equation 3.18.

$$\frac{d\alpha}{dt} = a [\tau(t) - \sigma_0 \tau_s]^b \quad \tau(t) > \sigma_0 \tau_s \quad (3.18)$$

$$\text{where } \tau(t) = k_s t$$

No additional damage will occur until $\tau(t)$ exceeds the threshold stress, $\sigma_0 \tau_s$, which will happen when $t = \sigma_0 \tau_s / k_s$. At failure $\alpha = 1.0$, $t = t_f$ and the new strength of the proof-loaded lumber, $\tau_n = \tau(t_f)$.

$$\int_{\alpha_p}^{1.0} d\alpha = a \int_{\frac{\sigma_0 \tau_s}{k_s}}^{t_f} [k_s t - \sigma_0 \tau_s]^{b+1} dt$$

$$1 - \alpha_p = \left[\frac{k_s t_f - \sigma_0 \tau_s}{\tau_s - \sigma_0 \tau_s} \right]^{b+1}$$

$$[1 - \alpha_p]^{\frac{1}{b+1}} = \frac{\tau_n - \sigma_0 \tau_s}{\tau_s (1 - \sigma_0)}$$

Solving for the new strength, τ_n :

$$\begin{aligned} \tau_n &= \left\{ \sigma_0 + (1 - \sigma_0)[1 - \alpha_p]^{\frac{1}{b+1}} \right\} \tau_s & \alpha_p < 1.0 \\ \tau_n &= 0 & \alpha_p = 1.0 \end{aligned} \quad (4.2)$$

The questions and calculations involved in the proof-loading analysis are put into flowchart format in Figure 4-2. Path No. 1

Fig. 4-2 Decisions and Calculations Involved in the Proof-load Test Analysis

Fig. 4-3 The Proof-load, Strength Test and Corresponding Damage Histories of a Piece of Lumber that is Subjected to Stress in Excess of its Threshold Strength but Survives the Proof-load.

involves those pieces of lumber that pass the proof-load and are not damaged. Path No. 3 involves those pieces of lumber that are fractured during the proof-load. Path No. 2 is the most interesting and is represented by Figure 4-3. It involves those pieces that pass the proof-load test but sustain some damage. Their new strength, τ_n , is lower than the initial strength, τ_s , and is calculated using Equation 4.2. In Figure 4-3 the initial strength test history and the corresponding damage are represented with a dashed line for comparison. The damage model permits the testing of the same piece of lumber to destruction twice to determine both the initial strength and its strength after proof-loading, where physical experiments could not.

4.2 Proof-loaded Lumber Strength Distribution Range

A major goal of this study is to determine the distribution of strength of the proof-loaded lumber. The preceding section described a model of the proof-load process that could be used in the development of a new strength distribution, provided the model parameters b and σ_0 are known. Like the short-term strength, the damage model parameters are not constant for the entire population of lumber. The parameters are random variables. They are assumed to be lognormally distributed and uncorrelated. Considering representative high and low values of the model parameters will give an indication of the range of the passing lumber strength distribution which could be expected. Figures 4-4a to c show the distribution of the short-term strength, and the model parameters b and σ_0 for 2" x 6" Western Hemlock lumber loaded in bending. There exists a problem with the description of the threshold

Fig. 4-4a Distribution of the Short-term Strength, τ_s

Fig. 4-4b Distribution of Model Parameter b

Fig. 4-4c Distribution of the Threshold Stress Ratio

stress ratio. A piece of lumber with a threshold stress ratio equal to 1.0 would not exhibit creep rupture behaviour and could theoretically sustain loads equal to its short-term strength forever. The lognormal probability density function extends beyond 1.0 and this would imply that the lumber strength could increase with time under load, which is not possible. The choice of a lognormal distribution to describe the distribution of the threshold stress ratio is, therefore, questionable. The threshold stress ratio pdf was modified for this study so that values greater than 0.9 were not accepted. The upper limit of the threshold stress ratio was arbitrarily chosen to be 0.9. Figure 4-4c shows the truncated threshold stress ratio pdf which will be used in this study.

4.2.1 DERIVED DISTRIBUTION OF NEW STRENGTH

Equation 4.2 represents the functional relationship which predicts the new strength after proof-loading, given values of the independent variables b , σ_0 and initial strength τ_s , and state variables representing the proof-load history k_s , k_p , and S_p . Distributions for the independent variables are known; the problem is to derive the probability distribution for the dependent variable, τ_n . The case of three independent variables is extremely difficult to solve; considering the damage model parameters b and σ_0 to be state variables, leaves only τ_s as the independent variable. Equation 4.3 denotes the one-to-one transformation between τ_s and τ_n .

$$\tau_n = fn(\tau_s) \quad (4.3)$$

If the transformation were monotonically increasing and an inverse function could be evaluated, then the cumulative distribution function for τ_n could be determined as follows:

$$\begin{aligned} F_{\tau_n}(s) &= P(\tau_n < s) \\ &= P(\tau_s < f_n^{-1}(s)) \\ &= F_{\tau_s}[f_n^{-1}(s)] \end{aligned}$$

A numerical evaluation of the new strength distribution will be used because the transformation function is monotonically increasing only over part of the initial strength range, and the calculation of the inverse function is extremely difficult to determine.

A new strength probability density function can be developed from the initial strength distribution and the transformation function, as shown in Figure 4-5. The probability that the initial strength is in the range $(\tau_{s_i}, \tau_{s_{i+1}})$ must equal the probability that the new strength is in the range $(\tau_{n_i}, \tau_{n_{i+1}})$.

In equation form,

$$P(\tau_{s_i} < \tau_s < \tau_{s_{i+1}}) = P(\tau_{n_i} < \tau_n < \tau_{n_{i+1}})$$

Shown in Fig 4-5 as $A_s = A_n$

If the distance between points i and $i+1$ is extremely small, then the function between these points can be taken to be linear and the following mathematical expression can be obtained:-

$$\frac{1}{2}[f_{\tau_s}(\tau_{s_i}) + f_{\tau_s}(\tau_{s_{i+1}})](\tau_{s_{i+1}} - \tau_{s_i}) = \frac{1}{2}[f_{\tau_n}(\tau_{n_i}) + f_{\tau_n}(\tau_{n_{i+1}})](\tau_{n_{i+1}} - \tau_{n_i})$$

proper testing methods. More recently, Strickler and Pellerin (1971) used proof-loading to test finger joints of lumber to be used in glued laminated beams.

The use of proof-loading to improve the strength distribution of lumber and expand its use in highly engineered systems is a more recent idea. The effect that proof-loading has on the strength distribution of lumber is still a contentious issue for researchers. There is uncertainty as to the degree of damage that is done to the lumber passing the proof-load test. Researchers generally agree that proof-loading will not produce a strength distribution which is less desirable than the original distribution.

Madsen (1976) investigated the "Degree of Damage Due to Proof Loading Lumber" as part of a larger In-grade testing program in 1976. Madsen questioned the method used to derive the code-specified strengths of lumber from adjustments made to the strength of small clear specimens. He proposed In-grade testing of full-sized lumber that had been visually graded as a better alternative for determining lumber strength. Madsen chose a bending proof-load that would cause approximately only 10% of the sample to fail. This allowed Madsen to use a large sample size as he could return the unbroken pieces for credit and reduce costs. The evaluation of the fifth percentile strength was his prime objective, not the description of the entire strength distribution. In order that he could return the unfractured lumber, Madsen had to ensure that the lumber returned would not adversely affect the overall strength distribution of the mill's lumber.

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Fig. 4-5 Derivation of the New Strength Distribution of the Lumber Surviving the Proof-load Test

Rearranging for $f_{\tau_n}(\tau_{n_{l+1}})$

$$f_{\tau_n}(\tau_{n_{l+1}}) = \left[f_{\tau_s}(\tau_{s_l}) + f_{\tau_s}(\tau_{s_{l+1}}) \right] \frac{\tau_{s_{l+1}} - \tau_{s_l}}{\tau_{n_{l+1}} - \tau_{n_l}} - f_{\tau_n}(\tau_{n_l}) \quad (4.4)$$

Using the above equation, the new strength probability density function could be built up. The equation is not correct for this particular case because the transformation function is not monotonically increasing over the range of initial strengths which will fail the proof-load. A lower bound on the minimum strength that a piece of lumber would require to pass the proof-load test can be found by evaluating the maximum initial strength that a failing piece of lumber could have. $\tau_{s_{min}}$ can be evaluated by setting the proof-load damage, α_p , equal to 1.0 in Equation 4.1.

$$\tau_{s_{min}} = \frac{S_p}{(1-\sigma_o) \left[\frac{k_p}{k_s} \right]^{\frac{1}{b+1}} + \sigma_o} \quad (4.5)$$

The fraction of the population that will fail the proof-load test is equal to the probability that the initial strength is less than or equal to $\tau_{s_{min}}$.

$$P_E = F_{\tau_s}(\tau_{s_{min}}) = \int_0^{\tau_{s_{min}}} f_{\tau_s}(\tau) d\tau \quad (4.6)$$

The area of the initial strength distribution between 0 and $\tau_{s_{min}}$, equal to P_E , will not be transferred to the new strength probability density function by the method described earlier. If Equation 4.4 were used, the area under the new strength pdf would equal $1 - P_E$, which is a violation of one of the criteria of probability density functions. To remedy the situation, Equation 4.4 is modified so that the incremental area shown shaded in the new pdf is set equal to $(1/1-P_E)$ times the area shown shaded in the initial strength pdf.

$$\frac{1}{1-F_{\tau_s}(\tau_{s_{\min}})} A_s = A_n$$

$$\frac{1}{1-F_{\tau_s}(\tau_{s_{\min}})} P(\tau_{s_i} < \tau_s < \tau_{s_{i+1}}) = P(\tau_{n_i} < \tau_n < \tau_{n_{i+1}})$$

$$f_{\tau_n}(\tau_{n_{i+1}}) = \frac{1}{1-F_{\tau_s}(\tau_{s_{\min}})} [f_{\tau_s}(\tau_{s_i}) + f_{\tau_s}(\tau_{s_{i+1}})] \frac{\tau_{s_{i+1}} - \tau_{s_i}}{\tau_{n_{i+1}} - \tau_{n_i}} - f_{\tau_n}(\tau_{n_i}) \quad (4.7)$$

Equation 4.7 can be used to build up a new strength pdf starting from $\tau_s = \tau_{s_{\min}}$ and stepping along the initial strength axis. At points where the initial strength is considerably greater than $\tau_{s_{\min}}$ (points with initial strength greater than or equal to the threshold stress), the slope of the transformation function is equal to 1.0 which indicates that there is no change in strength after proof-loading. A FORTRAN program called NEWSTRAP (New Strength After Proof-Loading) was written to perform the above transformation analysis. The increments of initial stress have been chosen to be small, close to $\tau_{s_{\min}}$, where the slope is the greatest. The program is deterministic in nature since constant values of b and σ_0 must be entered; however, its purpose is to evaluate the range of the new strength distribution functions.

Figures 4-6a and b depict a series of one-to-one transformation functions for 2" x 6" Hemlock in bending from initial strength to the strength after a proof-load to 4000 psi in 10 seconds. The three curves to the left of the truncation function represent the mean and extreme behaviour predicted with the damage model. The curves were evaluated using Equations 4.1 and 4.2 with the fifth, mean, and ninety-fifth percentile values of the model parameters b and σ_0 . The transformation function that will yield a truncation of the initial strength

Fig. 4-6a The Range of the Transformation Function

Fig. 4-6b The Range of the Transformation Function
At Points Near the Proof-load Stress

distribution at the maximum proof-load stress is shown for reference.

The minimum strength that is required for a piece of lumber to pass the proof-load test varies due to the variability in the viscoelastic behaviour of lumber, as expressed by the damage model parameters b and c_0 . There is a high probability that lumber with initial strengths lower than the maximum proof-load stress of 4000 psi will pass the proof-load test. These results contradict other arguments that the proof-load test will remove those pieces of lumber with strengths less than the maximum proof-load stress. The difference between the minimum initial strength which could pass the test and the maximum proof-load stress will tend to increase as the proof-load duration decreases.

The transformation functions deviate from the line of equality, $\tau_n = \tau_s$, indicating a reduction in strength after the proof-load test. This occurs as a result of damage that occurs during proof-load testing. Lumber with an initial strength less than the proof-load stress and those pieces with initial strengths slightly greater than the proof-load stress can be expected to lose some strength as a result of the test. As the initial strengths become larger, the functions tend to the line $\tau_n = \tau_s$, indicating that those pieces with strengths considerably greater than the proof-load will not be adversely affected by the proof-load test.

A series of probability density functions for the strength of Western Hemlock lumber passing the proof-load test are shown relative to the initial strength pdf in Figure 4-7. The curves were obtained from the transformation functions of Figure 4-6 using Equation 4.7. The

Fig. 4-7a The Range of Probability Density for Lumber
Surviving a Proof-load of 4000 psi

Fig. 4-7b The Range of the Left Tail of the New Strength Distribution
Brought About by a 4000 psi Proof-load

Fig. 4-8a The Range of the New Strength Cumulative Distribution Relative to the Initial Strength Cumulative Distribution

Fig. 4-8b The Range of the Lower Decile Strengths After Proof-loading

Fig. 4-7a The Range of Probability Density for Lumber Surviving a Proof-load of 4000 psi

Fig. 4-8b The Range of the Lower Decile Strengths After Proof-loading

curves show proof-loading's tremendous potential for eliminating the exceptionally weak pieces. The pdf obtained with the ninety-fifth percentile values of the damage parameters b and σ_0 is close to the truncation pdf and represents a best case scenario. The pdf obtained with the fifth percentile values of damage parameters represents a worst case scenario. There appears to be no danger that proof-load testing will yield a strength distribution less desirable than the initial, as once was feared.

The cumulative distribution functions are shown in Figures 4-8a and b relative to the initial cdf. These plots were determined by numerical integration of the probability density functions. The range of possible cumulative distribution functions for the strength after proof-loading appears quite narrow when the entire distribution is considered, as in Figure 4-8a. Figure 4-8b shows the improvement in the lowest percentile strengths which are very important. The fifth percentile strength of Western Hemlock proof-loaded to 4000 psi (27.6 MPa) in 10 seconds can be expected to increase from 3366 psi (23.2 MPa) to between 4208 and 4320 psi (29.0 and 29.8 MPa). The improvement in the strength distribution comes at the expense of the fractured lumber. Between 9.64% and 11.20% of the Western Hemlock lumber that is proof-loaded to 4000 psi in 10 seconds is expected to fracture and be eliminated.

The best estimate of the new distribution may be determined with mean values of the damage parameters. Using the mean parameters, the fifth percentile strength is expected to increase 915 psi (6.31 MPa) from 3366 psi (23.2 MPa) to 4281 psi (29.5 MPa) at the expense of 10.62% of the lumber tested.

4.3 Monte Carlo Simulation

A Monte Carlo simulation will now be used to evaluate the strength distribution of the proof-loaded lumber. The preceeding analysis using NEWSTRAP was deterministic in nature and could not account for the variability of the model parameters b and σ_o . A FORTRAN program called PROOF was written to perform the simulation of the effect that a specific proof-load would have on a sample of lumber. The following sections explain important details of the simulation process.

4.3.1 GENERATION OF LUMBER STRENGTH AND DAMAGE MODEL PARAMETER VALUES

The definition of the distribution of the initial strength and damage model parameters is the first step in the simulation. The random variables τ_s , b and σ_o are considered to be lognormally distributed. Subroutine DATAIN (DATA INput) allows the user to select distributions for the parameters that were reported by Forintek Canada Corporation (1987) for 2" x 6" Hemlock in bending, high quality 2" x 8" Spruce in bending, 2" x 4" Spruce in compression or 2" x 4" Spruce in tension or to select any mean and standard deviation for the parameters.

The simulation of a value of a lognormally distributed random variable requires the simulation of two independent values from a standard uniform distribution and the parameters of the lognormal distribution, λ and ζ . The mathematical expression for the calculation of a random variable, P , which is lognormally distributed, is shown by Equation 4.8 and is based on the inverse transform method as outlined by Ang and Tang (1984).

$$P = \text{EXP} \left[\lambda_p + \xi_p \sqrt{-2 \ln(u_1)} \cos(2 \pi u_2) \right] \quad (4.8)$$

Subroutine PARAGEN (PARAMeter GENERator) uses Equation 4.8 to calculate random values for τ_s , b and σ_o . The lognormal parameters of the random variables are calculated from the mean and standard deviations input in DATAIN; and the random numbers with standard uniform distribution are supplied by a function called RANDU. The standard uniform random numbers are obtained from the decimal portion of a parameter which undergoes recursive linear transformations. A seed number is needed to start the process and the date, which is input, serves this purpose.

The simulation of the time-dependent strength characteristics of a piece of lumber is accomplished with the random selection of values for τ_s , b and σ_o . Values for the damage parameters b and σ_o do not change during the lifetime of the piece of lumber. Figure 4-9 depicts the simulation of a sample of lumber of size n .

Fig. 4-9 Model Characterization of a Lumber Sample

4.3.2 PROOF-LOAD TEST ANALYSIS

The theory outlined in section 4.1 could be used to predict whether or not a piece of lumber with characteristics τ_{s1} , b_1 and σ_{o1} will pass a particular proof-load test, the damage caused by proof-loading, and the strength of the lumber after testing. The procedure used in PROOF is as shown by the flow chart diagram of Figure 4-2, with one exception. The new strength, τ_n , of those pieces which passed the proof-load and sustained damage less than 1×10^{-6} showed changes in strength which could not be detected with seven significant figures. The new strength of those pieces sustaining damage less than 1×10^{-6} is assumed to be equivalent to their initial strength.

From a sample of simulated lumber pieces, inferences can be made about the population. The strengths of the passing lumber are sorted and ranked by PROOF to estimate the cumulative distribution function. Sorting of the strengths of the passing lumber occurs in a two stage process. The first stage of the sorting process occurs after the simulation of the one-thousandth test. The strengths of the lumber that pass the first 1000 tests are sorted and ten strength categories of equal size are established. Simulation of the remaining tests continue and subroutine DIVIDE is used to place the new strengths into the appropriate strength categories. When the test simulation has finished, the strengths in the ten categories are then sorted using a "bubble sort" technique employed by subroutine SORT.

The two stage sorting process can be up to ten times faster than a one stage bubble sort. The time taken to sort a file can be considered to vary as the square of the number of elements in the file. The two stage process reduces the size of individual files by ten but there will be ten files instead of one. Consider the ratio of times taken to sort n numbers by the two methods.

$$\frac{t_{2 \text{ stage}}}{t_{1 \text{ stage}}} = \frac{10(n/10)^2}{n^2} = \frac{1}{10}$$

The sizes of the ten individual files are not likely to equal one-tenth of the total sample, due largely to the variability in strengths used to establish the ten categories. The unequal size of the individual files and the extra step required to set up the categories will reduce the efficiency of the two stage process relative to the idealized case; however, the two stage sorting process is still considerably faster than the one stage process.

Once the ten individual strength files have been sorted, they can be stacked together and ranked to obtain an estimate of the cumulative distribution function. Due to limitations in storage space and in order to facilitate plotting, the size of the ranked file to be output is reduced. Subroutine REDUCE combines data points, thus reducing the number of points to be output, and also serves to smooth the curve. The cumulative distribution function (cdf) is output to a file called PROOF.PRN, ready for plotting.

4.3.3 INDICATORS OF EFFECTS OF PROOF-LOADING

The cumulative distribution function provides the most important and complete description of the effects of the proof-load on lumber strength. The fifth percentile strength, which is often used as a basis for code specified strengths, is calculated by PROOF. The fifth percentile strength is not a good indicator of the benefit of a proof-load test since it does not always represent the true benefit in the elimination of the exceptionally weak elements. The probability of failure is the best single value indicator of the improvement in the strength distribution brought about by proof-loading since the entire distribution is used in the calculation, with much importance given to the left tail of the strength distribution.

The probability of failure is evaluated by numerical integration of the convolution integral expressed by equation 2.14. The cumulative distribution function for strength is determined by the simulation program and a reference load-effect probability density function must be provided. Program PROOF assumes that the load-effect distribution is lognormally distributed. The user may choose the mean and standard deviation of the load-effect distribution or select a default value.

The benefit of an improved strength distribution must be considered with respect to the loss of lumber that fails the test. The percentage of the lumber that fails the test is evaluated by PROOF. Statistics regarding the proof-load test and a sample of the initial strength, after proof-load strength, and corresponding damage, are output to file PROOF.OUT.

4.3.4 SIMULATION ACCURACY

Like physical experimentation, the accuracy of a computer simulation tends to increase when the sample size is increased. Unlike physical experimentation, the accuracy of the simulation result is only as good as the simulation model. As the number of tests in a physical experiment approaches infinity, the results approach an "exact" solution; however, there is not the same guarantee that the "exact" solution will be arrived at with the simulation process. Simulation accuracy is limited by the mathematical characterization of reality, the description of the probability distribution of the random variables and their correlation, the program's ability to generate pseudo-random numbers, and the number of simulations performed. The proof-load simulation model developed in this study is based on the Canadian Damage Model and Forintek Canada's calibration of the model to the Canada/USA duration of load experimental results. The accuracy of this study is, therefore, dependent on the accuracy of the CDM and its experimental calibration. With an infinite number of trials, the proof-load simulation will only be as accurate as the potentially imperfect mathematical model used. The effect of the number of simulations on sampling error will be considered in the next section.

4.3.5 SAMPLING ERROR AND NUMBER OF SIMULATIONS

The number of simulations required to accurately represent the proof-load process must be established within the capabilities of the computer and time constraints. A null proof-load, zero stress in a finite time (0,1), can be entered in program PROOF. The pieces

simulated will not be damaged; hence, their after proof-load strengths will equal their initial strengths. Results of this simulation can then be compared with the expected results to determine how the number of simulations affects accuracy.

Western Hemlock lumber with an initial bending strength distribution, represented by Figure 1-1, was used for the accuracy check. The lognormal distribution for strength has a mean of 6937 psi (47.8 MPa), standard deviation equal to 2833 psi (19.5 MPa), and a fifth percentile value equal to 3366 psi (23.2 MPa). An arbitrary load-effect distribution was chosen to calculate the probability of failure indicator. It was lognormally distributed and had a mean of 1400 psi (9.65 MPa) and standard deviation of 450 psi (3.10 MPa). The expected probability of failure is 0.756×10^{-3} . The reliability index was calculated to be 3.17 using Equation 2.15.

The effects that varying the number of simulations performed had on the estimates of the fifth percentile strength, probability of failure, reliability index, and the time taken for the simulation on an IBM XT work-alike with turbo switch on, are given in Table 4-1.

Table 4-1 RESULTS FROM VARYING NUMBER OF SIMULATIONS

TRIALS	$S_{.05}$	% ERROR	P_f	β	% ERROR	TIME
1000	3492	3.7%	$.488 \times 10^{-3}$	3.29	3.8%	10 min
2000	3430	1.9%	$.657 \times 10^{-3}$	3.21	1.3%	18 min
5000	3390	0.7%	$.759 \times 10^{-3}$	3.17	0%	42 min
10000	3375	0.3%	$.797 \times 10^{-3}$	3.16	0.3%	96 min
15000	3341	0.7%	$.843 \times 10^{-3}$	3.14	0.9%	170 min
20000	3338	0.8%	$.844 \times 10^{-3}$	3.14	0.9%	263 min
25000	3350	0.5%	$.853 \times 10^{-3}$	3.14	0.9%	367 min
30000	3351	0.4%	$.869 \times 10^{-3}$	3.13	1.3%	524 min
35000	3351	0.4%	$.838 \times 10^{-3}$	3.14	0.9%	635 min

In general, the fifth percentile strength error exhibited a greater reduction with the increase in the number of simulations performed than the reliability index error did. This may be so because the calculation of the reliability index is based on numerical integration which introduces error in addition to the sampling error. Error may also occur in the calculation of the reliability index due to the difficulty in defining the lowest values in the tail of the ranked strength distribution function.

It is interesting to note that even after 1000 simulations, which would constitute an enormous physical testing program, the sampling error may be as high as 3.7%. While single value indicators are convenient for comparison, the most important function of the simulation program is to produce an estimate of the cumulative distribution function. The cdf obtained from the simulation of the strength after a null proof-load with 1000 simulations is compared against the initial strength cdf determined from the input distribution in Figure 4-10.

The ranked cdf determined from 1000 simulations appears to accurately represent the initial strength cdf over the wide range of strengths shown in Figure 4-10 but meanders from the theoretical when viewed at the smaller scale shown in Figure 4-11. The ranked cdf's determined from simulations of size 5000 and 25000 are also shown in Figure 4-11. For this study, 25000 simulations will be considered acceptable to represent the strength distribution; and the running time, slightly in excess of six hours, will allow the program to be run overnight while leaving the terminal open during the daytime.

Fig. 4-10 The Simulated Initial Strength Cumulative Distribution

Fig. 4-11 Simulation of the Low Decile Portion of the Initial Strength Distribution

4.3.6 SIMULATION RESULTS AND MODEL BOUNDS

Five cumulative distribution functions for Western Hemlock lumber proof-loaded in bending to an internal stress of 4000 psi in ten seconds are shown in Figure 4-12a and b relative to the initial strength cdf. The proof-loaded lumber cumulative distribution functions range from a cdf determined using NEWSTRAP with fifth percentile values of the model parameters b and σ_0 to the cdf determined using the assumption that the distribution is truncated at the proof-load stress. The assumptions involved in the determination of the strength distribution of the proof-loaded lumber vary greatly but the distributions do not show a great variability in Figure 4-12a where the entire distribution is considered.

Figure 4-12b shows the lower decile strengths, which are of prime importance to the utility of the lumber. At this scale, the different cumulative distribution functions can be distinguished. The cdf determined with program PROOF using 25,000 simulations is within the bounds obtained with program NEWSTRAP using the fifth and ninety-fifth percentile values of the model parameters. The simulated cdf is closest to the cdf determined with the mean values of the model parameters. Program NEWSTRAP, using mean values of the model parameters, will be used to plot probability density functions of the proof-loaded lumber because probability density functions cannot be determined with program PROOF. Program PROOF, using a minimum of 25,000 simulations, will be used to estimate the strength distribution of the proof-loaded lumber and provide statistics with respect to the effects of proof-loading.

Fig. 4-12a The Range of New Strength Distribution After Proof-loading

Fig 4-12b A Close-up of the Range of New Strength Over the Lower Decile Strengths

4.4 Proof-loading Results

The cumulative distribution functions for Western Hemlock lumber, passing ten second ramp proof-loads to 2000, 3000, 4000 and 5000 psi (13.8, 20.7, 27.6, 34.5 MPa), are shown relative to the initial strength distribution prior to testing in Figures 4-13a and b. The 2000 psi proof-load had only a slight effect on the strength distribution and eliminated only 0.1% of the pieces tested, while the 5000 psi proof-load had a great effect on the strength distribution and eliminated 24.5% of the pieces tested. Proof-loading is effective in eliminating the exceptionally weak pieces; however, the strength distribution will not be truncated at the maximum proof-load stress. The simulations of the proof-load tests reveal that lumber with strengths lower than the proof-load stress could pass the proof-load test and lumber with strengths greater than the proof-load stress could be damaged by the test and their strengths reduced below the proof-load stress.

Figure 4-13b reveals that as the proof-load stress is increased, the fraction of the passing population with strengths below the proof-load stress can be expected to increase from 0.05% for the 2000 psi test to 2.53% for the 5000 psi test. Both damage and viscoelastic behaviour are responsible for the occurrence of lumber passing the proof-load tests with strengths less than the maximum imposed stress.

Viscoelastic behaviour is one reason for the increase in the percentage of the passing population with strengths below the proof-load stress. The greater the lumber strength, the longer the standard ramp strength test duration. Therefore, as the maximum proof-load stress

Fig. 4-13a The New Strength Distributions Brought About by Proof-loads of 2000, 3000, 4000, and 5000 psi

Fig. 4-13b The Distribution of the Lower Decile Strengths

is increased, the difference between the constant proof-load test duration (10 seconds) and the strength test duration increases. This has the effect of reducing the severity of the proof-load test with respect to the standard strength test. Another source for the increase in the fraction of the passing lumber with strengths below the proof-load stress with increasing maximum proof-load severity is the increase in the occurrence of damage. As the proof-load stress is increased, the strength probability density increases as well. This results in an increased probability of encountering pieces of lumber with a strength close to but greater than the maximum proof-load stress, which are the pieces that are most likely to be damaged.

The occurrence of damage and passing lumber with strengths less than the proof-load stress is very infrequent when low stress proof-load tests are used. The limitations on the sample size of physical proof-loading experiments and the use of low stress proof-load tests would make the detection of damage very difficult.

McLain and Woeste (1988) found that damage due to proof-loading southern pine lumber in tension was, "at worst, minimal or possibly non-existent". When a sample of 150, 2" x 6" pieces of lumber was proof-loaded in tension, 66 failed and 84 survived the tests. The surviving lumber was re-tested at the same rate that was used for the proof-load so that only damage would be detected and not viscoelastic behaviour. Only one piece failed the re-test at a stress equal to 99.7% of the maximum proof-load stress. McLain and Woeste considered this to be a strength reduction of only 0.3% which was "negligible". However, they assumed, without reason, that the initial strength of the piece of

lumber was equal to the maximum proof-load stress. The initial strength of the piece of lumber could have been greater than they assumed. Putting another perspective on their results, almost 1.2% of those pieces passing the proof-load test had strengths below the maximum proof-load stress. These results fall within the expected range predicted in this study.

Figure 4-13b would indicate that the error associated with the assumption that proof-loading will eliminate those pieces with strengths less than the proof-load, resulting in a truncated distribution, becomes greater as the proof-load stress is increased. The probability density functions, obtained with mean values of the damage parameters, are shown in Figure 4-14a. Figure 4-13a is included again for easy reference.

In Figures 4-15a and 4-15b, the strength probability density functions which are possible with proof-loading are shown relative to the load-effect pdf which was used to calculate the probability of failure indicator in program PROOF. The change in the overlapping region of the tails gives strong indication that the reliability can be greatly improved with proof-load testing. The benefit of the improved strength distributions comes at the expense of the lumber that was fractured during the proof-load test. Table 4-2 provides statistics regarding proof-load testing of 2" x 6" Western Hemlock lumber in bending. Three sets of results were simulated using three different random number seeds with program PROOF. The number of simulations was increased as the proof-load stress increased to provide a minimum of 25,000 simulations that would pass the test. The results of the three

Fig. 4-14a The New Strength Probability Density of Lumber Surviving Proof-loads of 2000, 3000, 4000, and 5000 psi.

Fig. 4-13a The New Strength Distributions Brought About by Proof-loads of 2000, 3000, 4000, and 5000 psi

Fig. 4-15a New Strengths After Proof-loading Relative to Load-effects

Fig. 4-15b New Relationships Between Strength and the Load-effect

sets are shown to provide an indication of the variability of the program results.

Table 4-2 PROOF-LOAD TEST RESULTS FROM PROGRAM PROOF

PROOF LOAD	TRIALS	No.	FAILURES	$S_{.05}$	p_F	β
2000	25000	1	0.10%	3363	0.692×10^{-3}	3.19
	25000	2	0.10%	3390	0.636×10^{-3}	3.22
	25000	3	0.10%	3386	0.655×10^{-3}	3.21
2500	30000	1	0.83%	3446	0.329×10^{-3}	3.41
	30000	2	0.75%	3456	0.325×10^{-3}	3.41
	30000	3	0.74%	3452	0.350×10^{-3}	3.39
3000	30000	1	2.53%	3611	0.138×10^{-3}	3.64
	30000	2	2.43%	3634	0.134×10^{-3}	3.64
	30000	3	2.55%	3633	0.134×10^{-3}	3.64
3500	30000	1	5.77%	3911	0.485×10^{-4}	3.90
	30000	2	5.61%	3912	0.478×10^{-4}	3.90
	30000	3	5.65%	3915	0.476×10^{-4}	3.90
4000	35000	1	10.77%	4296	0.149×10^{-4}	4.18
	35000	2	10.47%	4277	0.151×10^{-4}	4.17
	35000	3	10.49%	4275	0.153×10^{-4}	4.17
4500	35000	1	16.97%	4693	0.467×10^{-5}	4.43
	35000	2	16.68%	4685	0.480×10^{-5}	4.43
	35000	3	16.97%	4686	0.477×10^{-5}	4.43
5000	40000	1	24.51%	5122	0.144×10^{-5}	4.68
	40000	2	24.31%	5128	0.142×10^{-5}	4.68
	40000	3	24.53%	5116	0.145×10^{-5}	4.68
5500	40000	1	32.69%	5581	0.434×10^{-6}	4.92
	40000	2	32.41%	5577	0.434×10^{-5}	4.92
	40000	3	32.67%	5563	0.446×10^{-5}	4.92

The fifth percentile strengths, reliability indices, and the percentage losses of lumber due to failure statistics, are shown in graph form in Figures 4-16a, b and c. These figures show that proof-loading can significantly alter the distribution of lumber strength. Figure 4-16a shows that the fifth percentile strength tends to approach the maximum proof-load stress as the proof-load severity is increased. The

relationships between the fifth percentile strength and the maximum proof-load stress and the reliability index and the maximum proof-load stress (Figures 4-16a and b) appears to be approximately linear for the larger proof-loads while the percentage loss of lumber (Fig. 4-16c) tends to increase at a faster rate than the maximum proof-load stress. This would indicate that an upper limit to the usefulness of the proof-load process is likely to exist.

The improved strength distributions shown in Figure 4-13b must be considered with respect to the loss of lumber that results from proof-loading (Fig. 4-16c). The choice of an optimum proof-load is discussed in Chapter 6.

Fig. 4-16a The Expected Fifth Percentile Strengths as a Function of Proof-load Magnitude

Fig. 4-16b Expected Reliability Indices Possible with Proof-loading

Fig. 4-13b The Distribution of the Lower Decile Strengths

Fig. 4-16c Expected Loss of Lumber Due to Proof-loading

Chapter 5

Modulus of Elasticity Exclusion Method

The strength of lumber in bending, and tension and compression parallel to grain, are functionally related to the bending stiffness or modulus of elasticity. The positive correlation between the strength properties and the modulus of elasticity (MOE) can be used to estimate the strength of a piece of lumber from the MOE determined in a mostly non-destructive test. Machine stress rating (MSR) uses the correlation between the strength and the MOE of the lumber determined in the flatwise orientation. The Engineering Design in Wood codes recognize 10 f-E grades for trussed rafters and 4 f-E grades to be used as joists. MSR provides a better grading of lumber than standard visual grading does. The variability of strength and modulus of elasticity within a grade is reduced with machine stress rating and a better separation of strength categories can be achieved. Where only a few strength categories (Select Structural, No. 1 and 2) are available with visually graded lumber, up to ten different f-E grades are available with MSR lumber. MSR lumber is limited to a lesser extent by the weakest elements in the population than is visually graded lumber. MSR lumber provides the engineer with greater flexibility in design with increased allowable strengths.

The effectiveness of this method of strength estimation depends on the degree of correlation between the strength property and the lumber stiffness. The correlation between the bending strength or modulus of rupture (MOR) and the stiffness could be increased above that of MSR

lumber if the MOE were determined with the lumber oriented on edge, rather than on the flat, as would be the case in use. Tensile strength parallel to grain and tensile MOE have a higher correlation than tensile strength and bending MOE; however, the bending MOE is much easier to evaluate than the tensile MOE. Compression parallel to grain is likely to have higher correlation to the bending MOE determined on the flat rather than on the edge due to the failure mode of instability about the weak axis.

This chapter investigates the effect that excluding lumber with a stiffness less than a target bending MOE has on the bending strength distribution of a given population. The same Western Hemlock lumber bending strength distribution will be used in the evaluation of the modulus of elasticity exclusion (MOEE) method as was used in the proof load test analysis of the last chapter. The loads used to determine the MOE are considerably less than those used in the proof-loading process, in general, and are less likely to cause damage. In the MOEE analysis of Western Hemlock in bending, no reduction in strength or loss of lumber due to breakage under the loads used to determine the bending modulus of elasticity are assumed. The choice to examine the MOEE method using the bending correlation and neglecting damage caused by MOE determination represents the best possible conditions for the modulus of elasticity exclusion (MOEE) method.

5.1 Regression Analysis

An exact relationship between the MOR and bending MOE does not exist. The MOR and MOE can be considered to be random variables which are not perfectly correlated. When plotted, points representing the corresponding MOR and MOE of a sample of lumber are scattered but there is a general tendency for the MOR to increase with increasing values of MOE. The MOR and MOE are positively correlated. A probabilistic model is required to express one of the random variables in terms of the other. Generally, the MOE is the independent or measured attribute and a model is used to predict the MOR, the inferred attribute. This is known as the regression of MOR on MOE. For comparison of the proof-load and MOEE methods, it is desirable that the same bending strength distribution be used for the analysis of both methods. To achieve this, a probabilistic model must be developed for the regression of MOE on MOR.

Curry and Tory (1986) recommend a simple linear regression equation to represent the relation between the modulus of rupture and the bending modulus of elasticity. The probabilistic regression model will be linear and have deterministic and random or error components as shown by Equation 5.1.

$$\text{MOE} = a \text{ MOR} + b + \text{ERROR} \quad (5.1)$$

5.1.1 PROBABILISTIC REGRESSION MODEL

A scattergram of the strength versus modulus of elasticity of 2" x 6" Western Hemlock lumber loaded in bending presented by Madsen (1976) will

be the basis for the regression. The mean MOE reported by Madsen was 1.61 million psi (11100 MPa) which is equal to the published Engineering Design in Wood codes values for No.1 and No.2 grades. The code specified MOE value for Select Structural Western Hemlock is slightly higher and is equal to 1.70 million psi (11700 MPa). The mean MOE value used in this study is 1.64 million psi (11300 MPa) which assumes the lumber population contains approximately 30% Select Structural pieces. The strength distribution of the lumber Madsen tested was not the same as the lognormal distribution presented earlier in this study, particularly in the high percentile strength range. Madsen's highest population strengths were lower than those of the assumed distribution. This may be as a result of the different load test histories used to determine 'strength'. A line was fit through the scatter of Madsen's data points so that approximately an equal number of points fell above the line as below and the line passed through the point defined by the mean MOE and MOR values. The scattergram was oriented so that the MOE was the abscissa and the MOR the ordinate. The mean values of the MOE for several MOR intervals were also used as an aid. Equation 5.2 is the line that best fits the above criteria and which serves as the deterministic component of the probabilistic regression model.

$$\text{MOE} = 99 \text{ MOR} + 950000 \text{ psi} \quad (5.2)$$

For a given MOR range, the scatter of the MOE values about the regression line is assumed to be normally distributed. The random error component is added to the deterministic equation. From Madsen's data the standard deviation of MOE was determined for several MOR ranges and it was determined that the variance increased with the magnitude of the

MOR. The linear regression of the MOE standard deviation on the MOR was carried out. The equation of the MOE standard deviation as a function of the MOR was determined by using the method of least squares. Equation 5.3 expresses the MOE standard deviation as a function of the MOR.

$$\sigma_{\text{MOE/MOR}} = 8.4 \text{ MOR} + 17.6 \times 10^4 \text{ psi} \quad (5.3)$$

The generation of the MOE error component, which is assumed to be normally distributed, is carried out in a similar manner as lognormal random variables were generated in the proof load simulation program. A normally distributed random number, x , may be generated with Equation 5.4 provided that the mean and standard deviation are known and two uniformly distributed random numbers, u_1 and u_2 are provided.

$$x = \mu + \sigma \sqrt{-2\ln(u_1)} \cos(2\pi u_2) \quad (5.4)$$

The probabilistic regression model, given by Equation 5.5, is obtained from Equation 5.4 by substituting Equation 5.2 for the mean and Equation 5.3 for the standard deviation.

$$\text{moe}_i = 99\text{mor}_i + 950000 + (8.4\text{mor}_i + 176000) \sqrt{-2\ln(u_j)} \cos(2\pi u_{j+1}) \quad (5.5)$$

A scattergram produced from 1000 simulations of correlated MOE and MOR values is shown in Figure 5-1. A lognormally distributed random MOR value was first simulated; then the probabilistic regression model (Equation 5.5) was used to simulate a correlated MOE value. The deterministic regression line of MOE on MOR is shown in Figure 5-1 along with bounds which lie two MOE standard deviations either side of the regression line. The MOE dispersion increases with increasing MOR. The MOE is shown as the ordinate because of convention even though it is the MOR that is the independent random variable. A different regression line would result if the regression was MOR on MOE but both lines would

PROBABILISTIC LINEAR REGRESSION

WESTERN HEMLOCK IN BENDING

Fig. 5-1 Simulated Scattergram and Regression Characteristics

Fig. 5-2 Simulated Distribution of Bending Modulus of Elasticity

intersect at the point defined by the variable's mean values. The correlation coefficient of the plotted points is 0.77. The range of correlation coefficients published by Curry and Tory (1980) was 0.67 to 0.78 and tended to vary inversely with the depth of the piece of lumber.

The test of a probability regression model is its ability to reproduce the distribution of the inferred attribute. The regression line was chosen so that it passed close to the mean point, $((1.64 \times 10^6, 5937 \text{ psi}), (11300, 47.83 \text{ MPa}))$. The overall coefficient of variation, determined from the simulation of 1000 data points, was 0.23 which is close to the value of 0.20 published by Curry and Tory (1986). Figure 5-2 is a histogram of the MOE prepared to provide a visual check of the distribution. The general shape of the histogram agrees with those prepared by Curry and Tory (1986). The probabilistic regression model, expressed by Equation 5.5, will be used to evaluate the effect that sorting lumber with varied target stiffness requirements has on the strength distribution of the lumber meeting the required stiffness.

5.2 Modulus of Elasticity Exclusion Results

A sample of 7000 correlated Western Hemlock (MOE,MOR) data points was simulated, using Equation 5.5, for use in the estimation of the effect that MOE exclusion can have on the lumber strength distribution. A fraction of the sample is shown in Figure 5-3 along with the target MOE values of 1.3, 1.5, 1.7, 1.9 and 2.1 million psi (8960, 10340, 11720, 13100, and 14480 MPa). The lumber represented by data points to the left of the MOE limits would be excluded from the outgoing sample.

Fig. 5-3 Scattergram of Correlated Lumber Properties
and MOE Exclusion Limits

Fig. 5-4 New Strength Distributions Resulting From Exclusion Limits
of 1.3, 1.5 1.7, 1.9, and 2.1 Million psi

EFFECT OF EXCLUSION BASED ON STIFFNESS

Fig. 5-5a Comparison of Strength Distributions Before and After Exclusion of Lumber with a MOE Less Than 1.3×10^6 psi (8963 MPa)

EFFECT OF EXCLUSION BASED ON STIFFNESS

Fig. 5-5b Comparison of Strength Distributions Before and After Exclusion of Lumber with a MOE Less Than 1.5×10^6 psi (10342 MPa)

Fig. 5-5c Comparison of Strength Distributions Before and After
Exclusion of Lumber with a MOE Less Than 1.7×10^6 psi (11721 MPa)

Fig. 5-5d Comparison of Strength Distributions Before and After
Exclusion of Lumber with a MOE Less Than 1.9×10^6 psi (13100 MPa)

Fig. 5-5e Comparison of Strength Distributions Before and After
Exclusion of Lumber with a MOE Less Than 2.1×10^6 psi (14479 MPa)

A series of histograms in Figure 5-4 shows the change in the distribution of strength of the outgoing lumber relative to the initial strength distribution. Figures 5-5a to e provide a one-to-one comparison of strength distributions before and after the MOE exclusion is performed. The shift in the distributions can be quite significant, as shown by the plots, but the benefits must be weighed against the cost of testing and the devalued price of lumber that is rejected. The strength distribution produced with the 2.1 million psi MOE requirement (Fig. 5-5e) caused the exclusion of almost 9 out of 10 pieces tested. To produce a lot of 1000 pieces would require that nearly 10,000 pieces be tested.

The MOE exclusion method does not produce strength distributions with as sharp a censorship of the left tail as proof-loading does. The MOE Exclusion method also appears to require a greater amount of testing to produce a sample of lumber with a given improved strength distribution than proof-loading does.

To obtain statistics indicating the effect that MOE Exclusion has on the strength distribution of lumber, a FORTRAN program called "STIFF" was written. The results of this could later be used for comparison of the stiffness exclusion and proof-loading methods. "STIFF" performs the simulation of correlated Western Hemlock lumber bending strength and stiffness properties and it estimates the effect that excluding pieces with bending stiffnesses below a target MOE would have on the strength distribution. "STIFF" uses many of the subroutines that were created for program "PROOF" described in the previous chapter. The generation of uniformly distributed random numbers, dividing the strength values

into ten categories for faster sorting, and the reducing of the cumulative distribution function values were accomplished with subroutines RANDU, DIVIDE, SORT and REDUCE respectively.

Simulations were carried out for five different target modulus of elasticity exclusion limits. Simulation and new strength distribution statistics of 2" x 6" Western Hemlock lumber that is expected to meet various stiffness requirements are given in Table 5-1. Initial strength distribution statistics are included for comparison.

Table 5-1 "STIFF" RESULTS

MOE LIMIT psi	TRIALS	No.	TIME hrs.	EXCLUSION	$S_{.05}$ psi	P_f	β
0	-	-	-	-	3366	0.756×10^{-3}	3.17
1.3×10^6	25000	1	5.33	16.6%	3801	0.272×10^{-3}	3.46
	25000	2		17.1%	3812	0.303×10^{-3}	3.43
1.5×10^6	25000	1	5.67	38.1%	4420	0.708×10^{-4}	3.80
	25000	2		38.5%	4395	0.101×10^{-3}	3.72
1.7×10^6	25000	1	7.73	61.4%	5187	0.153×10^{-4}	4.17
	25000	2		61.6%	5218	0.176×10^{-4}	4.14
1.9×10^6	25000	1	9.43	79.2%	6261	0.171×10^{-5}	4.64
	25000	2		79.1%	6274	0.150×10^{-5}	4.67
2.1×10^6	20000	1	11.32	89.6%	7485	0.763×10^{-7}	5.26
	20000	2		89.6%	7445	0.699×10^{-7}	5.27

The strength cumulative distribution functions (CDF) for lumber meeting the 1.3, 1.5, 1.7, 1.9, and 2.1 million psi MOE requirements are shown relative to the initial strength distribution in Figure 5-6a. A close-up of the lower decile portion of the strength CDF curves is shown in Fig. 5-6b. Figures 5-6a and b can be compared with the estimate of

Fig. 5-6a Cumulative Strength Distributions of Lumber Meeting Various Stiffness Requirements Relative to the Initial Strength Distribution

Fig. 5-6b The Lower Decile Portion of the Strength Distribution

the cumulative distributions obtained by proof-loading, shown in Figures 4-13a and b. The slopes of the proof-loading CDF left tails are much steeper than those obtained from the MOE exclusion method. As an indicator of the strength of a population, the fifth percentile strength appears to be less conservative when used with the MOE exclusion method than when used with proof-loading. The differences in these series of plots makes a strong case for the use of reliability theory, which considers the shape of the important left tail of the strength distribution, for use in the evaluation of a strength distribution and not the fifth percentile strength alone.

Improvements to the strength distribution brought about by MOE exclusion are expressed by the fifth percentile strength and reliability index in Figures 5-7a and b. These improvements must be weighed against the cost of testing and the devalued price of the lumber that does not meet the MOE requirement. Figure 5-8 shows the expected percentage of the lumber that would be excluded as a function of the MOE requirement.

Figure 5-9 compares the number of tests that would need to be performed with proof-loading and MOE exclusion methods to obtain 100 pieces of lumber with a given reliability index. Greater economy of testing is provided by proof-loading, particularly as the reliability requirement increases; however, those pieces excluded from the sample with the MOEE method will not be broken and can be put to less demanding uses, whereas with proof-loading exclusion means destruction.

Both the MOE Exclusion and proof-loading methods have great potential for improving the strength distribution of lumber. The choice

of desirable MOE exclusion levels and proof-load histories will be considered in the next chapter.

Fig. 5-7a The Fifth Percentile Strengths of Lumber Meeting MOEE Limits

Fig. 5-7b The Reliability Indices of Lumber Meeting MOEE Limits

Fig. 5-8 The Fraction of Lumber Excluded As a Result of MOE Exclusion

Fig. 5-9 The Number of Pieces of Lumber Required to Produce a Lot of 100 Pieces With Given Reliability Index

Chapter 6

Test Method Profitability

It has been shown that the proof-load and modulus of elasticity exclusion methods can be effective in altering the strength distribution of lumber. The previous two chapters have identified the new strength distributions of 2" x 6" Western Hemlock meeting a variety of proof-load and MOE requirements.

The profitability of the proof-load and MOE exclusion methods will be evaluated in this chapter. The benefits of an improved strength distribution will be weighed against the cost of testing and the reduction in value of the lumber not meeting the test requirements. Criteria must be set that will allow a fair comparison of the proof-load and MOE exclusion methods. Optimum proof-load and MOE exclusion limits and profitable ranges of testing will be determined.

6.1 Profitability Criteria

Profit, defined as income minus expenses, will be used to determine which proof-loads and MOE exclusion limits will be profitable, which levels of load or MOE give optimum profit, and which test method gives the best results. Profit need not be in strictly economic terms alone but can also be evaluated in terms of increasing utility or reliability for the tested product. Income will be derived from the new value of the lumber passing test requirements and the residual value of the rejected lumber. Expenses include those incurred by the testing program and the lost sales of the lumber at its initial value.

Consider the profit, P_n , derived from the production of a sample of lumber of size n , whose pieces meet the test method requirements:

$$\text{PROFIT} = \text{INCOME} - \text{EXPENSES}$$

Let n = number of pieces meeting test requirements
 F = fraction excluded or eliminated
 N = total number of tests
 n_e = number of pieces excluded or eliminated
 V' = new value or worth of a piece of lumber meeting test requirements
 V = initial value or worth of a piece of lumber
 V_r = residual value or worth of a piece of lumber not meeting test requirements
 T = cost of testing

$$P_n = V' n + V_r n_e - TN - VN$$

Dividing both sides by the initial value allows the new and residual values and the cost of testing to be defined in relative, non-dimensional terms.

$$\frac{P_n}{V} = \frac{V'}{V} n + \frac{V_r}{V} n_e - \frac{T}{V} N - N$$

Dividing by the total number of tests, N

$$\frac{P_n}{VN} = \frac{V'}{V} \frac{n}{N} + \frac{V_r}{V} \frac{n_e}{N} - \frac{T}{V} - 1$$

Substituting $1-F$ for $\frac{n}{N}$ and F for $\frac{n_e}{N}$

$$\frac{P_n}{VN} = \frac{V'}{V}(1-F) + \frac{V_r}{V} F - \frac{T}{V} - 1$$

The non-dimensional profitability is re-written as Equation 6.1

$$P = \frac{V'}{V}(1-F) + \frac{V_r}{V} F - \frac{T}{V} - 1 \quad (6.1)$$

Calculation of the non-dimensional profit requires estimates of the new value, the residual value, and the cost of testing relative to the initial lumber value. The evaluation of these items cannot be carried

out in such an objective manner as the calculations of the new strength distributions were. Factors involved in the profit equation will be discussed in the following sections.

6.2 The New Value of the Proven Lumber

The economic value of lumber meeting proof-load or MOE exclusion test requirements will depend on market forces, the analysis of which are beyond the scope of this work. The value of the tested lumber can also be expressed in terms of its usefulness from a structural viewpoint. As an expression of the uncertainty in this area, three categories of the value of the lumber meeting test requirements, relative to the initial value, will be considered:- Market Value, Tested Property Merit and Utility. The relative fifth percentile strengths and reliability indices will be used as the basis for the calculation of the relative new value. Two equations, one involving the fifth percentile strengths and the other the reliability indices, will be given for each of the three new value categories. Figures 6-1a and b, included at the end of this section, are plots of the new value expressions that will be discussed in the following sub-sections.

6.2.1 MARKET VALUE

Bechtel (1983) attempted to determine the relationship between price and proof-load magnitude. He found that the price was approximately proportional to the square root of the proof-load magnitude based on February 1982 asking prices of three MSR grades of 2" x 6" Hem-Fir

lumber. The prices of four MSR grades of 2" x 6" Spruce lumber were obtained from the May 27, 1988 edition of Madison's Canadian Lumber Reporter. Table 6-1 gives the Canadian and U.S. dollar values of these MSR grades. The expected fifth percentile strengths of these grades were determined based on the current code allowable strengths. The fifth percentile strengths and reliability indices, estimated from the MOE exclusion analysis of 2" x 6" Western Hemlock performed in Chapter 5, are also shown.

Table 6-1 THE VALUE OF MSR LUMBER AND MOE EXCLUSION COMPARISON

MSR GRADE	VALUE \$/MBF*	f_b psi	$k_z k_{fs} f_b$ $1.3 f_b \times 2.1$	$S_{.05}$ psi	β
1650f-1.5E	360	1650	4505	4412	3.76
1800f-1.6E	372	1800	4914	4805	3.96
2100f-1.8E	433	2100	5733	5735	4.41
2400f-2.0E	498	2400	6552	6866	4.96

* Converted to Canadian wholesale prices, freight not included.

The fifth percentile strengths determined in Chapter 5 compare quite closely with those estimated from code allowable strengths. A relationship between the value and the fifth percentile strength or reliability index could now be established, but it is the new value relative to the initial value that is of interest. It will be assumed that the value of "No. 2 and better" 2" x 6" Western Hemlock is \$300 Cdn/MBF. The fifth percentile strength is 3366 psi (23.2 MPa) and the reliability index was calculated previously to be 3.17. Table 6-2

indicates the relative new values and the fifth percentile strengths and the reliability indices relative to their initial strengths.

Table 6-2 RELATIVE NEW VALUES, FIFTH PERCENTILE STRENGTHS AND RELIABILITY INDICES

$\frac{V'}{V}$	$\frac{S'_{.05}}{S_{.05}}$	$\frac{\beta'}{\beta}$
1.20	1.31	1.19
1.24	1.43	1.25
1.44	1.70	1.39
1.66	2.04	1.56

Equations 6.2 and 6.3 were determined to be the best-fit of the data points presented in Table 6-2. The equations represent straight lines and the square root relationship which Bechtel (1983) determined was not evident in this analysis.

$$\frac{V'}{V} = 0.625 \left(\frac{S'_{.05}}{S_{.05}} - 1 \right) + 1 \quad (6.2)$$

$$\frac{V'}{V} = 1.128 \left(\frac{\beta'}{\beta} \right) - 0.128 \quad (6.3)$$

6.2.2 TESTED PROPERTY MERIT

As was shown in Chapter 2, the allowable or specified strengths of lumber are based on a fifth percentile strength. A doubling of the fifth percentile strength as a result of proof-loading or MOE exclusion should then double the allowable strength, based on this scenario. It is assumed in this section that the lumber value is directly proportional to the tested strength property. Equation 6.4 expresses the relative new value as a linear function of the relative new fifth percentile strength.

$$\frac{V'}{V} = \frac{S'_{.05}}{S_{.05}} \left(= \frac{f'_{all}}{f_{all}} \right) \quad (6.4)$$

The reliability index is a better indicator of a strength distribution than the fifth percentile strength because it reflects the entire strength distribution and, most importantly, the weakest elements. A direct relationship between the relative new value and the relative reliability indices, given by Equation 6.5, will also be used.

$$\frac{V'}{V} = \frac{\beta'}{\beta} \quad (6.5)$$

A difference of one unit in the reliability index reflects a difference of approximately ten times in the probability of failure. A unit increase in the reliability index (a decrease in the probability of failure by a factor of ten) is more significant if the system is at a low reliability index initially. The benefit of testing would be greater if the reliability index increased from 3 to 4 rather than from 6 to 7.

6.2.3 UTILITY

Increased lumber strength may be translated into larger spacings, longer spans, smaller member sizes and possibly more efficient designs. The selection of a member for use requires that all code strength and serviceability requirements be satisfied. The study of the proof-load and MOE exclusion methods have focused only on the change in the bending strength to this point.

The modulus of rupture is the most restrictive consideration for joists; however, all failure modes and serviceability requirements must

be considered in design. The doubling of the bending strength of lumber is not likely to allow the doubling of the spacing of the new joists. Shear, lateral torsional buckling, bearing and deflection also limit the use of the lumber.

This study was primarily concerned with the identification and the evaluation of the effect of proof-loading on the bending strength of lumber. Although a bending proof-load test can be used as an overall indication of lumber quality, it may not completely assess the shear capacity, compression perpendicular to grain strength other mechanical properties. The proposed analysis, however, is applicable to all these effects and the physical test could be tailored to specifically control one of the failure or serviceability modes.

The uniformly distributed loads allowed by the code were determined for each failure mode for nine sizes of joists and typical spans. In all cases, the bending strength governed the design. Shear and deflection were the next most restrictive design considerations. After bending strength, shear and deflection governed the design approximately an equal number of times. Bearing strength (compression perpendicular to grain) was not considered restrictive.

The ratio of the allowable load of the second most restrictive design criteria to the allowable bending load was calculated for the cases considered. The average ratio by which the bending strength could be increased without the other criteria governing the design was 1.38 and the coefficient of variation was 12%.

Equation 6.4 which relates the relative new value directly to relative new fifth percentile bending strength may be adequate for fifth percentile strength ratios less than 1.38 but tends to overestimate the relative value with increasing fifth percentile strengths. As the bending strengths are increased, the likelihood of design criteria, other than the bending strength governing the design of the member, increases; therefore, the relative new value should depart from the one-to-one value-strength relationship as the fifth percentile strength increases.

The relationship between the relative new value and relative fifth percentile strength was established, assuming that:-

- (i) a doubling of the fifth percentile strength would cause a 50% increase in the value of the lumber.
- (ii) a 40% increase in the fifth percentile strength would result in a 30% increase in the value.

Equation 6.6 will be used to describe the relative new value of the tested lumber as a function of the relative bending strengths.

$$\frac{V'}{V} = -0.4 \left(\frac{S'_{.05}}{S_{.05}} \right)^2 + 1.7 \left(\frac{S'_{.05}}{S_{.05}} \right) - 0.3 \quad (6.6)$$

The probability of failure and reliability index calculated previously should be considered in terms of a bending failure mode and not the overall failure. A distinction between system and component reliability must be made. The failure modes of bending rupture, shear, and bearing and the deflection serviceability limit can be considered to be individual components of the lumber joist system. The failure modes can be represented by failure surfaces in a reduced variate space, as

THREE RELATIONS OF VALUE TO STRENGTH

Fig. 6-1a New Value of Lumber Based on Fifth Percentile Strengths

THREE RELATIONS OF VALUE TO RELIABILITY

Fig. 6-1b New Value of Lumber Based on the Reliability Index

Fig. 6-2a Initial Set of Failure Surfaces

Fig. 6-2b Revised Set of Failure Surfaces After Testing

defined by Ang and Tang (1984). Figure 6-2a illustrates this concept. Each surface defines a limit state equation which divides the safe state from the failure state. The minimum distance from the origin to the failure surface represents the reliability index for that failure mode. Proof-loading tends to push the bending failure surface away from the origin, increasing the reliability index. The shear failure surface and possibly others may move away from the origin but to a lesser extent. As the bending failure surface moves away from the origin, the influence of the other failure modes increases. Figure 6-2b depicts the possible reorganization of the failure surfaces after testing. The system reliability increases proportionately to the increase in the bending reliability initially but then tapers off as other failure modes become dominant. The next most restrictive mode, after bending, is likely to be shear for lumber tested with the MOE exclusion method and deflection when proof-loading is used. This is because MOE exclusion will weed out those pieces with low stiffness and proof-loading will eliminate those pieces weak in shear.

Equation 6.7 will be used to represent the new relative value of lumber meeting the test requirements as a function of the relative bending reliability index ratio.

$$\frac{V'}{V} = -0.45 \left(\frac{\beta'}{\beta} \right)^2 + 1.9 \left(\frac{\beta'}{\beta} \right) - 0.45 \quad (6.7)$$

6.3 Rejected Lumber Residual Value

The value of lumber that does not meet the test requirements varies greatly between testing methods. Lumber not meeting a MOE limit is assumed to have a higher value than lumber that fails a proof-load test and is fractured. Lumber that fails the proof-load test will be assigned an arbitrary residual value equal to 20% of its initial value. The pieces could be put to other uses such as:

- i) Middle laminations in Glulam Beams
- ii) Limited structural applications
- iii) Crates, Pallets, Non-Structural
- iv) Flakeboard

In general, lumber that is rejected by the MOE exclusion method will not be damaged. The value of these pieces will vary with the MOE exclusion limit or fraction rejected. Lumber that is rejected with a low MOE exclusion limit is very likely to have a poorer strength distribution than those rejected from a higher MOE exclusion limit. The value of those pieces rejected from MOE exclusion limits which result in all or none of the pieces being excluded, can be used for guidance. If the MOE limit is so high that all of the lumber is excluded, then the value of the rejected lumber should equal its initial value. As the MOE exclusion limit is lowered and the fraction of lumber rejected tends to zero, the value of the rejected lumber will tend to the value of the lumber rejected from the proof-load test.

It is expected that the residual value of the rejected lumber will rise sharply as the fraction of lumber that meets the MOE requirement

Increases from zero. It will be assumed that the residual value of the lumber not meeting the MOE requirement, which causes 10% of the lumber to be excluded, will be 80% of its initial value.

Equation 6.8 will be used to evaluate the residual value of the lumber relative to the initial value of the lumber as a function of the fraction of lumber that is excluded.

$$\frac{V_r}{V} = 0.2 + 0.8 F^{.125} \quad (6.8)$$

Figure 6-3 is a plot of the relative residual value that is expressed by Equation 6.8.

Fig. 6-3 The Residual Value of Lumber Not Meeting
MOE Exclusion Requirements

6.4 Expenses

Two expense categories were identified in the derivation of the profitability Equation 6.1. Expenses associated with the proof-load or MOE exclusion testing programs include those for the machinery, manpower, overhead and maintenance. The cost of one test, T , is assumed to be equivalent for both methods. The testing expense will vary over time and will likely fall when the test method operation becomes established. It is difficult to predict the cost of testing. The cost of one test may range from less than 1% to as much as 15% of the initial value of the piece of lumber. The cost of testing will be assumed to be 5% of the initial lumber value in this study.

Standard sawn lumber is used in the test processes and its cost is an expense incurred as part of the testing process. Lumber could be purchased specifically for upgrading using the test methods presented or testing could be set up as part of the mill production. In either case the associated expense is equal to the initial value of the lumber, V . In Equation 6.1 this expense was divided by the initial lumber value and is shown as unity.

Both the testing and lumber input expenses are independent of the fraction of lumber that is rejected by the test in the profitability equation. These expenses will not affect the choice of the optimum proof-load or MOE exclusion limit. They will be involved in the determination of which ranges of proof-loads and MOE exclusion limits will be profitable.

6.5 Profitability Results

The profitability of the proof-load and MOE exclusion methods will be discussed in this section. Equation 6.1 is the basis for the profitability calculations with the values of lumber meeting and failing the test and the expense involved, as outlined previously. A great deal of uncertainty exists in the value functions, especially in the value of lumber that meets the test requirements. This uncertainty led to the use of six new relative value functions based half on the fifth percentile strength (an indicator of allowable strength) and half on the reliability index. Effort was made to develop criteria that would allow a fair comparison of the two methods. The non-dimensional profitability, expressed by Equation 6.1, is modified to account for the expenses and is re-written as Equation 6.9.

$$P = \frac{V'}{V}(1-F) + \frac{V_r}{V} F - 0.05 - 1 \quad (6.9)$$

Due to the different residual values of the lumber failing the proof-load and MOE exclusion tests, Equation 6.9 becomes Equation 6.10 for the proof-load method and Equation 6.11 for the MOE exclusion method.

$$P = \frac{V'}{V}(1-F) + 0.2 F - 1.05 \quad (6.10)$$

$$P = \frac{V'}{V}(1-F) + (0.2 + 0.8 F^{.125}) F - 1.05 \quad (6.11)$$

The fifth percentile strength and the reliability indices presented in Chapters 4 and 5 are summarized in Table 6-3a and b in the profitability Equation.

Table 6-3a SUMMARY OF PROOF-LOAD RESULTS

PROOF-LOAD	F	$S'_{.05}$	β'	$S'_{.05}/S_{.05}$	β'/β
0	0	3366	3.17	1.0	1.0
2000	.0010	3380	3.21	1.004	1.012
2500	.0077	3451	3.40	1.025	1.074
3000	.0250	3626	3.64	1.077	1.148
3500	.0568	3913	3.90	1.162	1.230
4000	.1058	4283	4.17	1.272	1.318
4500	.1687	4688	4.43	1.393	1.397
5000	.2445	5122	4.68	1.522	1.476
5500	.3259	5577	4.92	1.656	1.552

Table 6-3b SUMMARY OF MOE EXCLUSION RESULTS

MOE LIMIT	F	$S'_{.05}$	β'	$S'_{.05}/S_{.05}$	β'/β
0	0	3366	3.17	1.0	1.0
1.3×10^6	0.1685	3807	3.45	1.131	1.087
1.5×10^6	0.3830	4408	3.76	1.309	1.193
1.7×10^6	0.6150	5203	4.16	1.546	1.311
1.9×10^6	0.7915	6268	4.66	1.862	1.468
2.1×10^6	0.8960	7465	5.27	2.218	1.661

Figures 6-4a to c are plots of the profitability as a function of the fraction of lumber excluded. The new relative values V'/V are from market value, tested property merit, and utility categories respectively. Curves for the proof-load method are to the left of the MOE exclusion method, in general due to the lower fractions excluded.

Figure 6-4a shows the proof-load method to be either considerably more or less profitable than the MOE exclusion method depending on

Fig. 6-4a The Profitability Of Proof-loading and MOE Exclusion
Based on the Market Value of MSR Lumber

Fig. 6-4b The Profitability Of Proof-loading and MOE Exclusion
Based Directly on Bending Strength Indicators

Fig. 6-4c The Profitability Of Proof-loading and MOE Exclusion
Based on the Utility of the Tested Lumber

whether Equation 6.2 or 6.3 is used to estimate the new relative value. The two curves representing the profitability of the MOE exclusion method are quite consistent because the market values of MSR lumber were calibrated to the MOEE results. Although the ranges of profitable proof-loads are quite different, depending on whether Equation 6.2 or 6.3 is used, it is important to note that the optimum fractions excluded are quite close.

Figure 6-4b shows the profitability of the proof-load and MOE exclusion methods when the relative new value of the tested lumber is based directly on the relative values of the reliability indices and fifth percentile strengths. The maximum profitability of the proof-load method is approximately equivalent when using Equations 6.4 and 6.5, although the optimum fraction excluded varies. The MOE exclusion profitability curves, based on relative fifth percentile strengths and reliability indices, are considerably different. Concern was expressed previously that the fifth percentile strength was not a good indicator of the strength distribution. Figure 6-5 is included to show how the indicators, $S'_{.05}/S_{.05}$ and β'/β , rated the improvements in the strength distribution. The fifth percentile strength appears to provide a closer characterization of the proof-load strength distributions than the MOE exclusion method. As the proof-load and MOE exclusion limits are increased, the tails of the strength distributions of the lumber passing the tests tend to extend farther to the left of the fifth percentile strength. The reliability index which uses the entire tail can account for this whereas the fifth percentile strength cannot. The fifth percentile strength indicator tends to overestimate the effect of the MOE

Fig. 6-5 The Assessment of the New Strength Distributions Based on the Ratio of the Fifth Percentile Strengths and Reliability Indices

exclusion method.

Figure 6-4c shows the profitability of the methods based on the relative new utility value of lumber meeting the test requirements. The profitability of the MOE exclusion method may be overestimated and the profitability of the proof-load method may be underestimated due to its basis on the fifth percentile strengths in Equation 6.6. However, the ranges of profitable proof-load and MOE exclusion levels are quite consistent for the two Equations considered.

Figures 6-6a and b show the entire set of profitability curves for the proof-load and MOE exclusion methods as functions of the severity of the tests. The general shapes of the profitability curves are similar although some are more skewed to the left or to the right. Those unprofitable proof-load and MOE exclusion limits on the left are unprofitable because the tests did not have a significant effect on the strength distribution to compensate for the expense of testing. Those unprofitable levels on the right of the figures exist because the loss of lumber rejected by testing outweighed the benefits of the improved strength distribution.

Figures 6-7a and 6-7b show the profitability of the two methods as functions of the lumber that is eliminated or excluded during the test. The profitable ranges shown by Figures 6-7a and b, as well as by 6-6a and b, are quite wide. The choice of an optimum proof-load and MOE limit does not appear to be critical because the slopes of the curves at the crest are shallow. Tables 6-4a and b provide statistics on the optimum proof-load and MOE exclusion limits.

Fig. 6-6a The Profitability of Proof-load Levels as Evaluated With Six Expressions For the New Value of Tested Lumber

Fig. 6-6b The Profitability of MOE Exclusion Limits as Evaluated With Six Expressions For the New Value of Tested Lumber

Fig. 6-7a Profitable Fractions of Lumber Eliminated by Proof-loading

Fig. 6-7b Profitable Fractions of Lumber Excluded by MOE Exclusion

Table 6-4a PROOF-LOAD PROFITABILITY

VALUE CATEGORY	V'/V EQ.	OPTIMUM LOAD	OPTIMUM F	PROFITABLE F _{LOW}	RANGE F _{HI}	MAX. PROFIT
MARKET VALUE	6.2	4300	0.138	0.060	0.25	0.025
	6.3	4300	0.140	0.004	0.43	0.191
TESTED PROPERTY	6.4	4950	0.225	0.020	0.50	0.150
	6.5	4200	0.135	0.003	0.40	0.151
UTILITY VALUE	6.6	4300	0.137	0.026	0.29	0.062
	6.7	3950	0.100	0.007	0.29	0.110

The optimum proof-load magnitude for 2" x 6" Western Hemlock in bending (when a 10 second ramp load is used) varies from 3950 psi to 4950 psi and the optimum percentage eliminated varies from 10.0% to 22.5%. The mean optimum proof-load is 4333 psi which would cause the elimination of approximately 14.6% of the pieces tested.

As is shown by the F_{LOW} and F_{HI} columns in Table 6-4a, a wide range of proof-loads is profitable. The tightest range of profitable proof-loads is that which would eliminate between 6.0% and 25.0% of the pieces tested.

Table 6-4b MODULUS OF ELASTICITY EXCLUSION PROFITABILITY

VALUE CATEGORY	V'/V EQ.	OPTIMUM MOE LIMIT	OPTIMUM F	PROFITABLE F _{LOW}	RANGE F _{HI}	MAX. PROFIT
MARKET VALUE	6.2	1.75X10 ⁶	0.65	0.21	0.97	0.053
	6.3	1.65X10 ⁶	0.55	0.15	0.94	0.059
TESTED PROPERTY	6.4	1.71X10 ⁶	0.62	0.10	0.94	0.131
	6.5	1.68X10 ⁶	0.58	0.18	0.94	0.041
UTILITY VALUE	6.6	1.60X10 ⁶	0.51	0.12	0.88	0.072
	6.7	1.61X10 ⁶	0.50	0.19	0.84	0.025

The optimum MOE exclusion limit for 2" x 6" Western Hemlock varies from 1.60×10^6 to 1.75×10^6 psi and the optimum percentage excluded varies from 50% to 65%. The mean optimum proof-load is 1.67×10^6 psi which would cause approximately 57% of the pieces tested to be excluded.

As with the proof-load method, a wide range of MOE exclusion limits is profitable. The tightest range of profitable MOE limits is that which would eliminate between 21% and 84% of the pieces tested.

The average maximum profits were calculated to be 0.115 for proof-loading and 0.064 for the MOE exclusion methods. Table 6-5 provides data for the comparison of the profitability of the two methods with respect to the equation chosen for the relative new lumber value. The proof-load method was the most profitable method in four of the six cases that were examined. For the two occasions where the MOE exclusion method was calculated to be the most profitable, the equations for the new relative value were based on the fifth percentile strength.

Table 6-5 COMPARISON OF PROOF-LOAD AND MOE EXCLUSION PROFITABILITY

V'/V EQ.	EQ. BASIS	PROOF-LOAD MAX. PROFIT	MOE EXCLUSION MAX. PROFIT	MOST PROFITABLE
6.2	$S_{.05}$	0.025	0.053	MOEE
6.3	β	0.191	0.059	PL
6.4	$S_{.05}$	0.150	0.131	PL
6.5	β	0.151	0.041	PL
6.6	$S_{.05}$	0.062	0.072	MOEE
6.7	β	0.110	0.025	PL

Chapter 7

Summary and Conclusions

7.1 Summary

This study has considered two test methods which can be used in an attempt to eliminate or exclude weak pieces from a sample of lumber. The lumber meeting the test requirements will have a strength distribution which is improved relative to the initial strength distribution. This lumber could be put to more demanding structural uses and it is hoped that a more efficient use of our natural resource can be obtained.

The assessment of "strength" is made difficult due to the load duration, viscoelastic or creep rupture behaviour of lumber. The strengths of lumber presented were given with respect to a specific load history that would be used for the strength assessment.

The Canadian damage model has emerged as the model which is best able to account for the load duration behaviour of lumber while having a mathematical formulation that is tractable. Numerical integration of the Canadian damage model is most often necessary except for the simplest of load histories, a constant load. It was discovered that a first term approximation to the Canadian damage model provided results that were consistent with the parent model for the short durations of loading that were considered in this study.

A methodology was established for the evaluation of the proof-load process using the approximation to the Canadian damage model. The theory presented accounts for different proof-load and strength test

histories. It can also be used for tension and compression proof-loading as well as bending. A very important feature of the analysis is that it accounts for damage that may occur during proof-loading and the corresponding reduction in lumber strength.

Program PROOF was written to perform the simulation of the proof-load process. Model parameters for 2" x 6" Western Hemlock lumber loaded in bending were used to provide data by which the proof-load method could be assessed. Lumber strength distributions before and after proof-loading were plotted and statistics regarding the distributions were presented.

The modulus of elasticity exclusion method, which is mostly non-destructive, was also considered. The MOE exclusion method uses the positive correlation between strength and stiffness in an attempt to exclude the weakest elements from the sample. The same experimental strength distribution was used for the analysis of the MOE exclusion method as was used with the proof-load method so that the two methods could be compared. A probabilistic model was developed for the regression of the modulus at rupture on the modulus of elasticity. Program STIFF was written to perform the simulation of the MOE exclusion method and to provide new strength distribution statistics in an analogous way as program PROOF was used to simulate the proof-loading method.

Lastly, criteria were set to evaluate the profitability of the proof-load and MOE exclusion methods. The increased value of the tested lumber with an improved strength distribution was weighed against the cost of testing and the reduced value of the lumber not meeting test

requirements. Optimum proof-load levels and MOE exclusion limits were determined as well as profitable ranges.

7.2 Conclusions

Important conclusions which have been arrived at throughout this report are summarized below:-

1. Both the proof-load and MOE exclusion methods can be effective in improving the strength distribution of lumber.
2. Damage can be caused by proof-loading and should not be neglected.
3. Damage is most likely to be caused to those pieces of lumber which have strengths very close to the demand of the proof-load test.
4. The occurrence of damage to the surviving pieces of lumber increases as the proof-load magnitude is increased.
5. Fewer tests are required to produce a sample of lumber with a given improved strength distribution using the proof-load method than are required using MOE exclusion.
6. A wide range of proof-load magnitudes and MOE exclusion limits are profitable.
7. An optimum ten second ramp proof-load for 2" x 6" Western Hemlock lumber, loaded in bending, is expected to be in the range of 3950 to 4950 psi (27.2 to 34.1 MPa) which would cause the elimination of 10.0% to 22.5% of the pieces tested.
8. An optimum MOE exclusion limit is expected to be in the range of 1.60×10^6 to 1.75×10^6 psi (11030 to 12070 MPa) which would exclude 50% to 65% of the pieces tested.
9. Proof-loading appears to be more profitable than MOE exclusion.

7.3 Recommendations

A method of testing lumber which utilizes the desirable aspects of both the proof-loading and MOE exclusion methods may be possible. The advantage of proof-loading is that it results in a more sharply censored left tail of the strength distribution than can be attained with MOE exclusion. MOE exclusion has the advantage over proof-loading in that the pieces rejected by the test are not usually fractured.

It is possible that MOE exclusion could be used to select lumber to be proof-loaded. This may result in less breakage and a desirable new strength distribution. Perhaps there exists a combination of MOE exclusion and proof-loading that is very profitable. The theory and computer programs developed in this study could be used to investigate the combined test method.

References

Ang, A.H-S., and Tang, W.H., 1975, Probability Concepts in Engineering Planning and Design, Volume I-Basic Principles, John Wiley & Sons.

Ang, A.H-S., and Tang, W.H., 1984, Probability Concepts in Engineering Planning and Design, Volume II-Decision, Risk, and Reliability, John Wiley & Sons.

ASTM, American Society for Testing Materials, D245, Establishing Structural Grades and Related Allowable Properties for Visually Graded Lumber, ASTM, Philadelphia, P.A.

ASTM, American Society for Testing Materials, D143, Standard Methods of Testing Small Clear Specimens of Timber, ASTM, Philadelphia, P.A.

Barrett, J.D., and Foschi, R.O. 1978a. Duration of load and probability of failure in wood. Part I. Modelling creep rupture. Canadian Journal of Civil Engineering, Vol. 5, No. 4, pp. 505-514.

Barrett, J.D., and Foschi, R.O. 1978b. Duration of load and probability of failure in wood. Part II. Constant, ramp, and cyclic loadings. Canadian Journal of Civil Engineering, Vol. 5, No. 4, pp. 515-532.

Bechtel, F.K. 1983. Proof Test Load Value Determination for Maximum Economic Return. Forest Products Journal 33(10) pp.30-38.

Bolotin, V.V. 1969. Statistical methods in structural mechanics. Holden-Day Inc., San Francisco, CA. pp. 171-175.

Canadian Standards Association, 1980. Engineering Design in Wood. CAN3-086-M80. C.S.A. 178 Rexdale Blvd., Rexdale, Ontario, Canada.

Canadian Standards Association, 1984. Engineering Design in Wood. (Working Stress Design). CAN3-086-M84. C.S.A. 178 Rexdale Blvd., Rexdale, Ontario, Canada.

Canadian Standards Association, 1984. Engineering Design in Wood. (Limit States Design). CAN3-086.1-M84. C.S.A. 178 Rexdale Blvd., Rexdale, Ontario, Canada.

Canadian Standards Association, 1981. Guidelines for the Development of Limit States Design, Special Publication. S408-1981, C.S.A. 178 Rexdale Blvd., Rexdale, Ontario, Canada.

Cunliffe, R.I and Salinas, J.J., 1987, "Lifetime Performance and Load Tests of Wood Structures". Proceedings of the Fifth International Conference on Applications of Statistics and Probability In Soil and Structural Engineering, pp. 246-252.

Curry, W.T., and Tory, J.R. 1976. The Relation Between the Modulus of Rupture (Ultimate Bending Stress) and Modulus of Elasticity of Timber. Building Research Establishment, Princes Riseborough Laboratory CP30/76.

Forintek Canada Corporation 1987. Duration of Load Effects in Lumber in Bending, Compression and Tension: An Analysis Report on the Up-to-Date Experimental Data.

Foschi, R.O. and Yao, Z.C., 1986, Another Look at Three Duration of Load Models, Paper presented at nineteenth meeting of International Council for Building Research Studies and Documentation. Working commission W18 - Timber Structures, Florence, Italy, September 1986.

Foschi, R.O., and Barrett, J.D., Load Duration Effects in Western Hemlock Lumber., Journal of the Structural Division, ASCE, Vol. 108, No. ST7, July, 1982.

Freas, A.D., 1949, Don't Test That Ladder, USDA Forest Products Laboratory, Madison, Wisconsin.

Gerhards, C.C., 1979. Time-related Effects of Loading on Wood Strength: A Linear Cumulative Damage Theory, Wood Science 11(3) pp.139-144.

Gerhards, C.C., and Link, C.L. 1988. A Cumulative Damage Model to Predict Load Duration Characteristics of Lumber., Wood and Fibre Science, 19(2), 1987, pp.147-164.

Keenan, F.J., 1986, Limit States Design of Wood Structures, First Edition, Copyright 1986 by Morrison Hershfield Limited.

Jayne, B.A., 1959. Vibrational Properties of Wood as Indices of Quality. Forest Products Journal 9(11).

Madsen, B., 1976, In-Grade Testing, Degree of Damage Due to Proof-Loading of Lumber in Bending, University of British Columbia, Dept. of Civil Engineering, Report No.17, I.S.S.N. 0318-3378.

Madsen, B., and Barrett, J.D. 1976. Time-strength relationship for Lumber. University of British Columbia, Vancouver B.C., Structural Research Series Report No. 13.

Madsen, B., and Sexsmith, R., 1982, Recent Developments in Timber Engineering, 1982-83 National Lecture Tour sponsored by CSCE.

McLain, T.E., and Woeste, F.E., 1986, Rate of Loading Adjustment for Proof Testing of Lumber in Tension, Forest Products Journal 36 (9) 51-54.

McLain, T.E., and Woeste, F.E., 1988, Proof Test Damage Evaluation with Southern Pine Lumber., (In press) Forest Products Journal.

National Building Code of Canada, 1985, National Research Council of Canada.

National Lumber Grades Authority, 1981, NLGA Submission to CSA Technical Committee of Engineering Design in Wood Re Bending Strength Properties of Dimension Lumber, June 1981.

Pellerin, R.F., and Strickler, M.D., 1971, Tension Proof Loading of Lam Stock for Laminated Beams. Forest Products Journal 21(5) pp.50-55.

Salinas, J.J., and Gillard, R.G. "Equivalence Criteria in Acceptance Testing". Proceedings of the Fifth International Conference on Applications of Statistics and Probability In Soil and Structural Engineering, pp. 325-332.

Snodgrass, De Los V. 1985. Proof Loading of Lumber for Use in Reliability Based Design. Reliability Based Wood Design Conference., May 1985.

Strickler, M.D., and Pellerin, R.F., 1971, Tension Proof Loading of Finger Joints for Laminated Beams. Forest Products Journal 21(6) pp. 19-24.

Suddarth, S.K. 1985 Measuring the Results of Proof Loading of Lumber. Reliability Based Wood Design Conference, Purdue University, May 1985.

Ross, R.J. 1985. Stress Wave Propagation in Wood Products. Proceedings of the Fifth Nondestructive Testing of Wood Symposium, Washington State University, Sept. 9-11, 1985.

Woeste F.E., Green, D.W., Tarbell, K.A., and Marin, L.A. 1987. Proof Loading to Assure Lumber STrength. Wood and Fiber Science 19(3) pp. 283-297.

Woeste, F.E., Suddarth, S.K., and Galligan, W.L. 1979. Simulation of Correlated Lumber Properties Data - A Regression Approach. Wood Science Vol.12, No. 2.

Wood, L. 1951. Relation of Strength of Wood to Duration of Load. United States Forest Products Laboratory, Madison WI., No. R1916.

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