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Performance of the Multicarrier Direct Sequence Spread Spectrum Uplink in the Presence of Frequency and Timing Synchronization Errors

by

László Háy

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
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Carleton University

May 2002

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**“Performance of the Multicarrier Direct Sequence Spread
Spectrum Uplink in the Presence of Frequency and Timing
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submitted by

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in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

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Abstract

New developments in wireless communications are occurring at an ever increasing rate. Demand for better and novel wireless services is also accelerating due to the fact that the typical user of the 21st century is becoming more and more mobile. As a result, there is a considerable worldwide effort being undertaken in order to close the gap between the wired and wireless services in terms of performance and availability. In this regard, physical layer solutions involving efficient mitigation of channel impairments, multiuser support, and low interference generation are receiving significant attention. In particular, multicarrier and spread spectrum technologies have already been adopted or are being considered for future wireless air interfaces. Recent evidence also shows increased interest in hybrid multicarrier spread spectrum systems.

In this research work one of these multicarrier spread spectrum systems is analyzed with the wireless uplink in mind. In particular, we are interested in analytically quantifying the sensitivity of the system to carrier frequency and timing errors for both the single user and multiple user cases. Our goal is to provide the analytical tools necessary to the undertaking of possible future design and research work.

Our research has revealed the interplay between the multicarrier and spread spectrum components of the signaling with respect to the sensitivity to synchronization errors, has established mathematical relations describing performance degradation, and quantified the effects for some particular cases which then can be used for benchmarking possible future work.

Acknowledgments

I would like to thank professor Mohammed El-Tanany for his support and guidance throughout this work. His insights and experience were of invaluable help. Further, I would also like to acknowledge the financial support provided by the National Sciences and Engineering Research Council of Canada (NSERC) and by the Ministry of Training, Colleges and Universities through the Ontario Graduate Scholarship Program.

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List of Acronyms

A/D	analog to digital
ADSL	asynchronous digital subscriber line
AWGN	additive white Gaussian noise
CDMA	code division multiple access
CE	controlled equalization
DFE	decision feedback equalization
DFT	discrete fourier transform
DSP	digital signal processing
DS-SS	direct sequence spread spectrum
EGC	equal gain combining
EHF	extra high frequency
FDD	frequency division duplex
FDMA	frequency division multiple access
GII	global information infrastructure
IDFT	inverse discrete fourier transform
IF	intermediate frequency
ISI	inter symbol interference
LAN	local area network
LOS	line of sight
LMCS	local multipoint communication system (same as LMDS)
LMDS	local multipoint distribution system (same as LMCS)
MC	multicarrier
MC-CDMA	multicarrier code division multiple access
MC-DS-CDMA	multicarrier spread spectrum code division multiple access

MC-SS	multicarrier spread spectrum
MMSE	minimum mean square error
MRC	maximum ratio combining
MT-CDMA	multitone code division multiple access
MUD	multiuser detection
MUI	multiuser interference
NII	national information infrastructure
OFDM	orthogonal frequency division multiplex
OFDMA	orthogonal frequency division multiple access
ORC	orthogonality restoring combining
P/S	parallel to serial
QAM	quadrature amplitude modulation
RF	radio frequency
SNR	signal to noise ratio
S/P	serial to parallel
SS	spread spectrum
TDD	time division duplex

List of Symbols

$a(N_s), b(N_s)$	degradation coefficients (frame sync error)
$c_k(t), c_k(n), c_k$	spreading signal and sequence of the k^{th} user
$d_k(t), d_k(n)$	information (data) signal and symbol of the k^{th} user
$D_k(i)$	decision variable for the k^{th} user on the i^{th} subcarrier
$e_{\pm 1}(\gamma), e_0(\gamma)$	error signals
$E_{\pm 1}(i), E_0(i)$	the DFTs of $e_{\pm 1}(\gamma)$ and $e_0(\gamma)$
$h(n)$	stationary channel impulse response
$h_{\beta}^{(\alpha)}(\gamma), h(\alpha; \gamma)$	time variant channel impulse response
$H(\alpha; i)$	channel frequency response on the i^{th} subcarrier
L	length of cyclic prefix
$MUI(i)$	multiuser interference on the i^{th} subcarrier
N	number of subcarriers
N_c	length of spreading sequence
N_f	normalized block synchronization error
N_s	normalized frame synchronization error
$N_{k,l}$	normalized time alignment error between the k^{th} and l^{th} user
N_u	number of (active) users
PG	processing gain
p	chip index

$p(t)$	rectangular pulse of unit amplitude and duration of T
$P_h(\gamma_\tau)$	channel power delay profile
P_{ICI}	power of the inter-carrier interference
$P_H(i)$	power of the channel coefficient on the i^{th} subcarrier
P_u	power of the useful component of the desired user
q	sample index within the OFDM frame
$r(t), r(n)$	low-pass equivalent received signal
$R_c(n, \delta f)$	time-frequency correlation function
$R_c(n)$	auto-correlation function
$R_{c_{k,l}}(n)$	cross-correlation function
$R(n)$	common notation for $R_{c_{k,l}}(n)$ and $R_{c_{l,k}}(n)$
$s_k(t), s_k(n)$	low-pass equivalent transmitted signal of the k^{th} user
$s_{\text{ofdm}}(n)$	discrete time OFDM signal, without cyclic prefix
$s_{\text{ofdm_cp}}^{(\alpha)}(\gamma)$	discrete time OFDM signal, including the cyclic prefix
SNR_0	signal-to-noise ration without degradation
T	duration on information symbol
T_c	chip duration
T_d	desired data term
T_f	OFDM frame duration, same as the chip duration T_c
T_{Nyq}	inverse of the Nyquist rate
T_{oi}	other-interference term
T_s	sample duration
T_{si}	self-interference term

W	bandwidth
$\delta f, \delta f'$	normalized carrier frequency synchronization error
δf_i	integer part of δf
δf_f	fractional part of δf
$\delta f_{k,l}$	relative normalized carrier frequency error between the k^{th} and l^{th} user
Δf	carrier frequency synchronization error
Δf_{coh}	coherence bandwidth of the channel
$\Delta f_{k,l}$	relative carrier frequency error between the k^{th} and l^{th} user
Δf_{sc}	subcarrier spacing
Δt	timing synchronization error
$\Delta t_{k,l}$	relative timing error between the k^{th} and l^{th} user
α	block index for the OFDM-DS-SS signal
β	frame index within the OFDM-DS-SS block
γ	sample index within the OFDM-DS-SS frame
$\Gamma, \Gamma_k, \Gamma_{\text{acorr}}$	SNR degradation
$\zeta(m; \delta f_f)$	inter-carrier interference coefficient
$\xi_k(i)$	equalization coefficient on the i^{th} subcarrier for the k^{th} user
$\Lambda(i; \delta f_f)$	inter-carrier interference seen on the i^{th} subcarrier
ε	relative OFDM overhead
σ_d^2	power (variance) of information symbols

σ_n^2	noise power (variance)
σ_{oi}^2	power of other-interference
σ_{si}^2	power of self-interference
τ_m	maximum excess delay of the channel
$\star$	linear convolution
$\circledast$	periodic convolution
$\mathcal{E}\{\cdot\}$	statistical expectation

Terminology

block	A collection of samples representing one MC-DS-CDMA signaling interval. This is associated with N data symbols, where N is the number of subcarriers and is a succession of N_c OFDM frames, with N_c being the length of the spreading sequence.
data symbol	A binary or non-binary information symbol at the input of the MC-SS transmitter.
frame	A substructure embedded into the block (defined above) and which is made up of samples representing the OFDM signal associated with N data symbols. Also referred to as an OFDM frame.
OFDM frame	Same as “frame”.
sample	A sample of the transmitted or received signal.

Introduction

In recent years research work directed towards the definition of future broadband wireless communication services has received much attention worldwide. The need for supporting ever increasing data rates while maintaining high quality of service is the result of the penetration of digital technology into formerly analog services (e.g. radio and television broadcasting), of the demand for mobility in case of typically fixed services (e.g. LAN and Internet access), of the long outstanding promise for certain wideband services to the home (e.g. video on demand and video conferencing) and of the evolution of currently existing cellular communication systems. The generally accepted system architecture of these new systems involves a wired backbone with wireless access to the users' terminals which, depending on the application, can be hand-held, portable or fixed. It is this "last mile" that has been attracting focus lately.

To illustrate, there is an ongoing research effort for defining and/or developing such systems as LMCS/LMDS, HyperLAN, wireless LAN, the NII/GII¹, third and fourth generation mobile communications, ad-hoc personal area communications, and communications in the unlicensed frequency bands. Although each of them has particular requirements, some general features such as high data rate, wide bandwidth, very high carrier frequency, hostile wireless channel, receiver imperfections specific to the considered frequency band, and the requirement for multiple access are common. In addition, physical layer solutions involving computationally efficient digital signal processing algorithms are becoming more and more important due to the gradual paradigm shift towards the soft-

¹. National Information Infrastructure and Global Information Infrastructure

ware radio concept.

1.1 Research topic and objectives

For the wireless communication systems of the future a transmission technique suitable for high speed (wideband) communications which can cope with the typical impairments of a wireless channel (frequency and possibly time selective fading, multi-access interference and phase noise) and provides multiple access support is needed. These same requirements would hold for a large class of planned broadband wireless services including mobile communication systems and wireless LANs. Therefore, the search for an adequate transmission system and the development of the corresponding signal processing algorithms meant to deal with the wireless channel, with the multiple access and with receiver imperfections at carrier frequencies of tens of GHz is seen as an important undertaking.

Considering the known simplicity and anti-fading properties of OFDM, and the well established performance and multiple access support of direct sequence spread spectrum (DS-SS), multicarrier spread spectrum (MC-SS) techniques are considered promising in achieving the stated objective. As a result, interest in these systems has recently been increasing. As discussed later on, there are three fundamental multicarrier spread spectrum techniques among which MC-CDMA is more suitable for the downlink while MC-DS-CDMA and MT-CDMA for the uplink.

MC-CDMA can be viewed as a basic OFDM or alternatively as a basic DS-CDMA transmission [25], and since both have already been well studied individually and recent work has already focused on the MC-CDMA system, further research seems to be less justified. Also, the performance of MC-CDMA in the presence of multiaccess interference was the subject of several studies [30], [28], [36] and [29].

On the other hand, MT-CDMA seems to be too complex compared to MC-DS-CDMA and its performance is much degraded in the presence of narrowband interference due to the largely overlapping nature of the subcarrier spectra. A system based on employing both MC-CDMA and MC-DS-CDMA principles has already been proposed for LMCS in [15] and performance aspects related to power control were studied assuming perfect recovery of the signal.

In the technical literature, MC-DS-CDMA is considered to be the preferred uplink transmission method. Although the performance with and without multiaccess and other in-band interference has already been investigated by other researchers [23], [21] (and shown to present improvement with respect to DS-CDMA), implementation issues, and more specifically synchronization, have not yet received significant attention.

Therefore, the study the MC-DS-CDMA transmission scheme for the wireless uplink is important and we set out to analytically quantify the sensitivity of the system to carrier frequency and timing synchronization errors for both the single user and multiple user cases. Our goal is to provide the analytical tools necessary to the undertaking of possible future design and research work.

To the best of our knowledge, there are only two research results published so far in the open literature which deal with synchronization issues related to MC-DS-CDMA:

- In [12] the authors undertake a comparative analysis of the sensitivity of MC-DS-CDMA to frequency errors for the downlink and the uplink. Their work has focused exclusively on Walsh codes and therefore, the analytical results are limited in scope. Further, the MC-DS-CDMA signal is also multiplied with a complex valued random scrambling sequence the presence of which affects the final results.

- In [16] the effect of frequency errors and small timing errors is analyzed for the case where the MC-DS-CDMA subcarriers have non-overlapping spectra, similar to the Kondo&Milstein type of MC-DS-CDMA (which is discussed briefly in chapter 2, section 2.1.2). Therefore, the results presented in this work are not directly applicable to our system. However, the authors do point out the important role played by the two dimensional time-frequency auto-correlation and cross-correlation functions and conclude that at large frequency errors the spreading codes behave like random sequences. A statistical characterization of the two dimensional time-frequency cross-correlation function is also presented.

1.2 Summary of contributions

In this work we consider the MC-DS-CDMA uplink. Our analysis of the signaling scheme and of the effect of synchronization errors lead to the following contributions:

- Based on the mathematical expression of the MC-DS-CDMA signal it is established that the signal can be represented in terms of a series of OFDM frames grouped into a larger structure we call an MC-DS-CDMA block. It is also established that there can be two distinct transmitter and receiver architectures and we introduce the DFT-DS and DS-DFT receivers. Further, it is shown that the main difference between the two architectures is the speed at which DFT/IDFT processing has to occur.
- We analytically evaluate some design considerations (approximate capacity and overhead) for the MC-DS-CDMA system and compare them with those of the related DS-CDMA and OFDMA multiple access methods. Based on the results, it is concluded that MC-DS-CDMA successfully combines DS-SS and multicarrier principles in order to provide a viable modulation and multiple access scheme for the wireless uplink.

- The sensitivity of MC-DS-CDMA to carrier frequency errors for both the single and multiple user cases is analytically derived and it is found that using an SNR based performance degradation measure is appropriate. We establish that for the single user case performance degradation is attributable to the OFDM-specific intercarrier interference and to the DS-CDMA-specific attenuation of the auto-correlation function. In the multiuser case the degradation is due to the non-coherent cross-correlation functions characterizing the spreading code family. We conclude that continuous tracking of a downlink carrier pilot might be necessary in order to accommodate the sensitivity to carrier frequency errors.
- For code phase alignment synchronization it is analytically established that MC-DS-CDMA exhibits a sensitivity similar to the DS-CDMA system (for both single and multiple user cases). We show that an SNR based degradation measure for the single user case is not appropriate and instead, a measure based on distance reduction in the received signal constellation is suggested. We also advance the idea of an inter-block guard interval or a decision feed-back algorithm as possible techniques to deal with the acquisition and possibly tracking of the code phase synchronization error.
- Regarding frame synchronization errors, it is found that in the single user case the interference affecting the received signal can be grouped into two components: a channel estimation error like self-interference and a Gaussian other-interference. For small errors we consider the aggregate effect and characterize it through an SNR degradation measure. To this end a degradation coefficient which characterizes the sensitivity of the OFDM component is introduced, while the dependence on the DS-SS component is shown to be through the partial auto-correlation function of the spreading code. For the multiuser case we find that the SNR degradation exhibits an oscillatory behavior with respect to the subcarrier index. The location of the peaks and troughs depends in a complicated way on a

number of system parameters and therefore, separating the effect of individual users on the SNR degradation is not always possible and upper-bounding is necessary.

- Finally, the effects of carrier frequency, code phase and frame synchronization errors is quantified for the particular case when cyclic-shift m-sequences and when Walsh codes are employed for spreading in both the single user and multiuser cases.

1.3 Organization of thesis

Throughout this thesis the reader is assumed to be reasonably familiar with the principles of direct sequence spread spectrum and with those of orthogonal frequency division multiplex. Based on these principles we then introduce multicarrier spread spectrum in chapter 2 and present the three currently known system configurations along with a list of their advantages and disadvantages, described mainly through comparison with direct sequence spread spectrum and orthogonal frequency division multiplex.

In chapter 3 we deal with the discrete time representation of the multicarrier direct sequence spread spectrum system by deriving the discrete time equivalent signal as well as the basic transmitter and receiver structures for ideal and multipath channels. We also briefly compare the theoretical capacity limit of this system with that of DS-SS and OFDMA.

General considerations on the synchronization of digital communication systems as well as more specific details for the MC-DS-CDMA system in question are given in chapter 4. Here, we also establish a mathematical model in order to describe carrier frequency synchronization and various timing errors. We use this model in the following chapters, where the individual effect are discussed in detail.

We investigate the effect carrier frequency synchronization errors have on the MC-DS-CDMA signal and on the corresponding multiple access scheme in chapter 5. We also quantify the analytical results for some particular cases and then discuss our findings.

Block level (code phase) synchronization errors are dealt with in chapter 6 and we show that the effects are similar to the ones encountered in DS-CDMA systems.

Chapter 7 is where we consider the effect of frame synchronization errors. We derive analytical results for the single user and multiple user cases, quantify these results for the same particular situations as with carrier frequency errors and then discuss our findings.

Finally, in chapter 8 we present the main conclusions drawn from this work and then suggest some topics for further study.

Multicarrier spread spectrum

Multicarrier spread spectrum (MC-SS) systems were introduced in an attempt to combine classical DS-SS principles with OFDM while preserving the main advantages of both. This transmission method encompasses modulation and multiple access and has been proposed for third generation mobile communication systems. However, it can also be employed for modulation only with multiple access provided through time or frequency division.

Typical DS-SS systems resolve multipath with a resolution higher than the symbol duration and obtain significant performance improvement over non-spread single carrier systems by employing a RAKE receiver which properly combines the signal received through each path¹. In addition, spread spectrum offers narrowband interference mitigation advantages over non-spread techniques. However, DS-SS systems suffer from the following drawbacks:

- Cannot handle large Doppler shifts.
- Code acquisition and tracking is complex and expensive to implement.
- The RAKE receiver requires continuous monitoring of each multipath component and updating of the receiver coefficients.

¹. It is important to understand that a RAKE filter is the optimal receiver (in fact, it is a matched filter) if the channel is flat with respect to the information symbol. The DS-SS signal expands the transmission bandwidth and hence, the spread signal will see a channel which is frequency selective. Mathematically, the channel excess delay should satisfy $\frac{1}{W} < \tau_m \ll T$ where T is the information symbol duration and W the DS-SS signal bandwidth.

- When used to implement CDMA, performance deteriorates as the number of users increases due to the imperfect cross-correlation of the spreading codes and as a result, the maximum allowable number of users is much less than the theoretical maximum (given by the number of orthogonal codes available for a given code length) although some of this loss can be recovered using coding and diversity.

Multicarrier systems, and in particular OFDM, have a set of highly desirable features compared with single carrier transmissions. ISI can be easily and efficiently mitigated due to the increased symbol duration in combination with the use of a guard interval (see appendix A for details), thus making OFDM an attractive choice for high data rate applications in a multipath environment by allowing the use of low complexity (1-tap) equalizers. These systems also offer good immunity to impulse noise given the spreading effect of the DFT processing in the receiver. Multicarrier techniques allow for superior bandwidth utilization through partial overlap of the subcarriers' spectra and by assigning a different signal constellation and bit rate to each subcarrier. Alternatively, it is possible to employ non-contiguous frequency bands for the transmission. Finally, using adaptive power and data rate allocation algorithms at the subcarrier level information throughputs close to capacity can be achieved. The disadvantages of these systems are:

- Orthogonality among subcarriers is lost due to uncompensated frequency offsets and high Doppler (rapid time variations of the channel) which results in severe interference among subcarriers.
- The envelope of the OFDM signal has a very large dynamic range and therefore it is very sensitive to typical nonlinear power amplification and requires a significant power back-off of the amplifier or the use of some kind of amplitude compression technique.

The goal of combining these two transmission techniques is to obtain a bandwidth

efficient multiple access system with good performance under multipath conditions which allows for a relatively simple receiver implementation.

2.1 MC-SS configurations

Generally speaking, one can obtain a MC-SS scheme by spreading the data of a multicarrier system in either the frequency or time domain. In the first case, one would spread the original data stream and then apply the multicarrier modulation, while for time domain spreading the multicarrier modulation would precede the spreading. Depending on exactly how the spreading process and multicarrier modulation are combined [24], three basic system configurations are arrived at [19]. In these systems code division multiple access is implemented over multicarrier transmission and they all allow for a reduced complexity discrete time implementation using IDFT/DFT processing.

2.1.1 MC-CDMA

In MC-CDMA systems [22], [33] spreading is performed before modulation (frequency domain spreading), as shown in figure 2.1-a, and the length of the spreading code coincides with the number of subcarriers.

A typical block diagram of the transmitter is presented in figure 2.2 along with the main parameters: subcarrier spacing Δf_{sc} , chip duration T_c (defined based on the structure of figure 2.1-a), aggregate bandwidth W , and processing gain PG . The data symbol $d_k(n)$ (binary or not, depending on the mapping) of the k^{th} user is of duration T and is transmitted on all subcarriers while different users are assigned orthogonal spreading codes (real or complex valued) $\mathbf{c}_k = [c_k(1) \dots c_k(N)]$ where N is the number of subcarriers. For the purposes of the foregoing presentation it is useful to view the spreading codes as periodic sequences. The subcarrier spacing is chosen to insure orthogonality among

these subcarriers, thus allowing for partial overlap of their individual spectra. The quantity $s_k(t)$ represents the complex envelope of the MC-CDMA signal and all pulse shaping is assumed rectangular. The transmitter is equivalent to the cascade of DS-SS and MC where the number of subcarriers is the same as the number of chips of the spreading sequence. The receiver corresponding to the basic transmitter of figure 2.2 is shown in figure 2.3.

For highly frequency selective channels where subcarriers would be subject to fre-

quency selective fading the transmitter can be modified to include M parallel branches, each of them similar to the basic system in figure 2.2. All M branches would use the same spreading code but will be fed by different data streams (obtained after S/P conversion of the user's data) and would use a different set of subcarrier frequencies. This way the subcarrier spacing will be reduced since the symbol duration of each branch has increased while the overall bandwidth is preserved. As a result of the reduced spacing subcarriers can undergo flat fading. To maximize frequency diversity, the carrier frequencies associated to each transmission branch should be chosen with maximum spacing, that is frequency domain interleaving should be employed for the M sets of subcarriers. This way the same frequency diversity is achievable as for the original configuration with flat fading subcarriers. Such a system, which also employs OFDMA, is presented in [29].

The most important properties of MC-CDMA are as follows:

- An alternative way [25] of looking at this signaling format is to consider it a DS-SS with orthogonal spreading codes derived by applying IDFT to the original spreading sequence $\mathbf{c}_k$. This configuration is shown in figure 2.1-b. Given this interpretation, all time domain reception techniques known for DS-SS (simple despreading, matched filtering, zero forcing equalization, MMSE equalization)

are directly applicable using the modified code, and it is shown that these techniques are all equivalent to well known frequency domain techniques.

- Compared to DS-SS, both systems have a similar performance for a low number of users however, MC-CDMA significantly outperforms DS-SS when the number of users increases assuring a more graceful degradation of performance [29].
- MC-CDMA achieves frequency diversity since the same data symbol $d_k(n)$ is sent on all subcarriers.
- Multiple frequency shifted versions of the signal can be recovered using a frequency domain RAKE receiver which combines all the signal energy scattered in the frequency domain. This property allows for detection under severe Doppler spread, exhibiting much improvement over DS-SS. In fact, this is a result of the duality theorem [31] since the Doppler spread channel is the dual of the delay spread channel and the MC-CDMA signal is a frequency domain DS-SS.
- At the receiving end, in principle, all combining/equalization methods can be employed [28] in the frequency domain: maximum ratio combining (MRC), equal gain combining (EGC), multiuser detection (MUD), ML detection, iterative detection [36], controlled equalization (CE) [32], orthogonality restoring combining (ORC) also called zero forcing, MMSE combining.
- Given a frequency selective channel, orthogonality among users is lost due to the unequal scaling of different subcarriers. For this reason, MC-CDMA seems more feasible for the downlink, in which case at a given receiver the signals for all users are received through the same equivalent channel. Alternatively, for an uplink scenario either MUD should be used in order to remove inter-user interference or the subcarriers of each signal should be predistorted in order to preserve orthogonality between users at the receiver.

- Does not need separate code acquisition nor tracking, sample and MC-frame synchronization being the only required timing to recover. As such, the synchronization problem is the same as for any basic MC system.
- Considering that there is only one data symbol transmitted during one MC-frame, the effective throughput in the presence of a long channel impulse response is severely limited by the need for a cyclic prefix.

2.1.2 MC-DS-CDMA

By spreading each subcarrier of a multicarrier modulation system and choosing a frequency spacing that keeps the individual subcarriers orthogonal² after spreading, one arrives to a MC-DS-CDMA scheme [19]. Conceptually, it is equivalent to a MC-SS system with time domain spreading (modulation before spreading) with some appropriately chosen subcarrier spacing.

The transmitter is represented in figure 2.4 along with the same parameters and notations as for MC-CDMA. The spreading signal $c_k(t)$ is assumed to be shaped with rectangular pulses, while the pulse $p(t)$ is also considered rectangular. As opposed to the previous system, here the number of chips N_c does not have to agree with the number of subcarriers N . If we assume $N_c = N$ then the listed parameters will have the same values as for MC-CDMA, although with MC-DS-CDMA different data symbols are transmitted on each subcarrier. The receiver is shown in figure 2.5.

In order to provide a certain amount of frequency diversity, a modified transmission scheme [23] is possible where each transmission branch is split into M parallel branches carrying the same data symbol $d_k(n)$. The subcarrier frequencies corresponding

² As it will become evident later, this means having a subcarrier spacing equal to the chip rate.

to the same-bit branches would then be chosen with maximum spacing (frequency interleaving). However, it should be noted that this is not a mitigation technique against frequency selectivity (as it was for MC-CDMA) since the subcarrier spacing remains unchanged while the overall bandwidth is increased.

A variation of the system is presented by Kondo and Milstein in [21] and [34] where the same data symbol is transmitted on each subcarrier and the spacing of these

subcarriers is such that there is no spectral overlap. Actually, this variant is a narrowband DS-SS system with frequency diversity implemented via multicarrier techniques.

In the following we list the more important properties of this transmission method:

- There is no explicit frequency diversity as for MC-CDMA however, coding and interleaving can produce an equivalent gain.
- Resistance to Doppler spread is similar to the capabilities of DS-SS.
- Each subcarrier must undergo nonselective fading in order to benefit from the advantage of simple equalization of MC systems. As a result, only short spreading sequences can be used.
- Although there is a DS-SS signal on each subcarrier, RAKE receivers are not needed due to the above mentioned flat fading requirement which also means that one cannot always recover all the signal energy scattered across a subcarrier. Nevertheless, it is possible to allow frequency selective fading of subcarriers and use Rake receivers to recover the information symbols. However, this leads to a greatly increased complexity since the independent fading of subcarriers requires independently updated Rake filters.
- Frequency selective channels won't destroy the orthogonality among users since each subcarrier is constrained to undergo flat fading. Therefore, MC-DS-CDMA is well suited for the uplink, as opposed to MC-CDMA which is considered more suitable for the downlink.
- Both MC-frame and chip synchronization is necessary. However, due to the longer chip duration (given by $T_c = N \cdot \frac{T}{N_c}$) code synchronization is easier compared to DS-SS.

2.1.3 MT-CDMA

Multitone CDMA, as described in [22] and [26], is also a time spread MC-SS system. It differs from MC-DS-CDMA by the subcarrier spacing which doesn't insure orthogonality in this case.

The MT-CDMA transmitter can be visualized as shown in figure 2.6 but a transmitter as in figure 2.4 is also possible. In this second case each subcarrier carries a wide-band DS-SS signal the individual spectra of which largely overlap. This implementation is shown in figure 2.7 and a typical receiver which uses RAKE filters on each subcarrier's processing branch is shown in figure 2.8.

MT-CDMA has the following characteristics:

- Resistance to Doppler spread is similar to MC-DS-CDMA.

- Resistance to narrowband interference is reduced compared to basic MC and to the other MC-SS systems since such an interferer will affect most subcarriers as opposed to only a small subset.
- Each subcarrier is a wideband DS-SS signal therefore fading cannot be flat. Further, because of the significant overlap of the subcarrier's spectra they don't fade independently.
- As already mentioned, RAKE receivers are needed on each subcarrier, and due to the lack of orthogonality there is interference between subcarriers. The cost involved by the presence of these RAKE receivers is generally considered to be one of the biggest disadvantages of this transmission system.
- Given the longer spreading codes (compared to MC-DS-CDMA) and hence higher chip rate, the processing gain and therefore the number of possible users is increased.
- Frequency selective fading destroys the orthogonality among users and therefore multiuser detection (MUD) is unavoidable. As a result, given the involved processing complexity, this transmission technique is more feasible for the uplink.

- Synchronization is difficult, and both code and multicarrier frame sync is required. Although the receiver can use all subcarriers to establish code synchronization, the lack of orthogonality and of independent fading of subcarriers makes the task more difficult.

The MC-DS-CDMA system

Here we derive the discrete time representation of the MC-DS-CDMA signal, the structure of the digital transmitter and that of the digital receiver, and finally, we discuss the multiuser capacity of MC-DS-CDMA.

3.1 The discrete time signal

Using the notations of figure 2.4 the lowpass equivalent MC-DS-CDMA signal for the m^{th} signaling interval, that is $t \in [mNT, (m+1)NT)$, $m \in \mathbb{Z}$, can be written as

$$s_k(t) = c_k(t) \cdot \sum_{i=0}^{N-1} d_k(i + mN) \cdot \exp[j2\pi i \Delta f_{sc} t] \quad (3.1)$$

Dropping the subscript k (denoting the k^{th} user), substituting $\Delta f_{sc} = 1/T_c$ and sampling at the rate of

$$R = 1/T_s = N/T_c \quad (3.2)$$

yields the discrete time representation

$$s(n) = c\left(\frac{nT_c}{N}\right) \cdot \sum_{i=0}^{N-1} d(i + mN) \cdot \exp\left[j2\pi \frac{in}{N}\right] = N \cdot c\left(\frac{nT_c}{N}\right) \cdot s_{\text{ofdm}}(n) \quad (3.3)$$

$$n = 0 \dots NN_c - 1$$

where $s_{\text{ofdm}}(n)$ is the lowpass representation of the OFDM signal associated with the data

$\{d(i)\}$ for the considered m^{th} signaling interval. Then, the discrete time MC-DS-CDMA signal is normalized by considering relation 3.3 without the constant factor N . Taking into account the periodicity by N of $s_{\text{ofdm}}(n)$, the rectangular pulse shaping of $c(t)$, and denoting by $c(p)$ the p^{th} chip we conclude that

$$\begin{aligned} s(pN + q) &= c(p) \cdot s_{\text{ofdm}}(q) \\ q &= 0 \dots N-1 \quad p = 0 \dots N_c - 1 \end{aligned} \quad (3.4)$$

where p is the chip index and q the sample index for the OFDM signal.

Based on equation 3.2, the MC-DS-CDMA signal has a structure as illustrated in figure 3.1 where samples are T_c/N spaced. The term **frame** refers to one OFDM symbol (containing N samples) and **block** refers to one MC-DS-CDMA symbol consisting of N_c OFDM frames and hence, of $N \cdot N_c$ samples. Furthermore, using equation 3.4 the MC-DS-CDMA transmitter can be regarded either as OFDM modulation applied to DS-SS subcarriers (the way figure 2.4 suggests) or as DS-SS applied to the OFDM signal and followed by block interleaving of $N \times N_c$. This interleaving is necessary to restore the proper order of the transmitted samples. In the first case, the digital transmitter is the discrete time version of the one in figure 2.4 and is shown in figure 3.2 where IDFT processing occurs at the chip rate. The second, alternative, implementation is shown in figure 3.3 and here

Figure 3.1: Structure of the MC-DS-CDMA signal.

IDFT processing is at an N_c times lower rate. In this latter case the spreading sequence $c(p)$ should have a chip duration of

$$T'_c = T_c/N = T/N_c \quad (3.5)$$

It is interesting to observe that equation 3.4 and figure 3.1 can be interpreted so that $c(p)$ plays the role of the data and $s_{\text{ofdm}}(q)$ plays the role of the spreading code. In this case, the transmitted data would stay the same over all transmitted blocks and the spreading code would be changed for every MC-DS-CDMA block. Nevertheless, the implementation of figure 3.3 allows for a more useful interpretation.

3.2 The basic receiver

In this section we consider the basic despreading receiver (optimal single user detector) and not the multiuser detector because the former is more suited to understanding the transceiver and to gaining insight into the effects of various synchronization errors (to be discussed in later chapters).

3.2.1 Ideal AWGN channel

For ideal, AWGN channels the optimal single user receiver is the matched filter receiver¹ which in case of multicarrier transmission means a matched filter on each subcarrier, as shown in figure 2.5. In this case, the decision variable at the i^{th} subcarrier is given by

$$D_k(i) = \int_0^{NT} r(t) c_k^*(t) \exp[-j2\pi i \Delta f_{sc} t] dt \quad (3.6)$$

with $r(t)$ denoting the complex envelope of the received signal. Given that for detection purposes Nyquist rate sampling results in sufficient statistics and considering perfect synchronization, the decision variable becomes (after dropping the subscript k)

$$\begin{aligned} D(i) &= \int_0^{NT} \left[\sum_n r(n) \delta\left(t - \frac{nT_c}{N}\right) \right] \cdot c^*(t) \cdot \exp[-j2\pi i \Delta f_{sc} t] dt \\ &= \sum_{n=0}^{NN_c-1} r(n) c^*\left(\frac{nT_c}{N}\right) \cdot \exp\left[-j2\pi \frac{in}{N}\right] = \sum_{p=0}^{N_c-1} c^*(p) \sum_{q=0}^{N-1} r_p(q) \exp\left[-j2\pi \frac{iq}{N}\right] \\ &= \sum_{p=0}^{N_c-1} c^*(p) \cdot R_p(i) = \sum_{q=0}^{N-1} \exp\left[-j2\pi \frac{iq}{N}\right] \sum_{p=0}^{N_c-1} c^*(p) \cdot r_p(q) \end{aligned} \quad (3.7)$$

¹. This is also optimal for the multiuser case with perfectly orthogonal spreading codes

where $r_p(q) = r(pN + q)$ and $R_p(i) = \text{FFT}\{r_p(q)\}$.

The above equations suggest two possible receiver implementations which are seen to form natural pairs with the corresponding transmitters. One possibility is demodulation followed by despreading on each subcarrier as in figure 3.4 where the DFT is performed at the chip rate. The other implementation is despreading before performing the multicarrier demodulation as in figure 3.5 with DFT processing at an N_c times lower rate.

In the latter case, the despreading described by $\sum_{p=0}^{N_c-1} c^*(p) \cdot r_p(q)$ requires a deinterleaving stage preceding the despreader which establishes the proper ordering of samples for the despreader. It should be noted however, that any of the two receiver structures can be used with any of the transmitters. The first implementation is essentially an OFDM receiver and we call it the DFT-DS implementation, while the second one closely resembles a DS-SS receiver and we name it the DS-DFT receiver.

3.2.2 Multipath channel

For a multipath (frequency selective) channel² the receiver has to be augmented

Figure 3.4: DFT-DS implementation of the MC-DS-CDMA receiver.

with a channel estimation block and an equalizer. Regarding the receiver of figure 3.4, which has an OFDM front-end, a cyclic prefix should be added to each transmitted OFDM frame and subsequently discarded at the receiver in order to remove interference between these frames and between the data symbols carried by one OFDM frame³. For an explanation of the cyclic prefix technique see appendix A. In terms of equalization, this translates into multiplication of $R_p(i)$ with an equalization coefficient $\xi(i)$ and, if the fading is slow enough, the equalization could take place at the level of the decision variable $D(i)$. The possibility of using such an one-tap equalization is due to the flat fading requirement at each subcarrier, which is expressed as

$$\Delta f_{sc} < \Delta f_{coh} \Rightarrow N_c < \frac{NT}{\tau_m} \Rightarrow \tau_m < T_c \quad (3.8)$$

and the equation clearly shows that there is a limitation on the length of the spreading sequence. The quantity Δf_{coh} denotes the channel coherence bandwidth, Δf_{sc} the subcarrier spacing, and τ_m the maximum excess delay of the channel.

². Unless we specifically refer to the physical channel, one should understand by “channel” the equivalent communications channel consisting of transmitter and receiver filters and the physical channel.

³. Note the difference between data symbols, which modulate subcarriers and hence are in the frequency domain, and signal samples, which are in the time domain.

The alternative implementation of figure 3.5 contains a DS-SS like front-end. The time resolution of the receiver is $1/W = T_c/N = T/N_c$. Based on the resemblance with a DS-SS front-end, one might argue that on a frequency selective channel with $\tau_m \ll N_c/W = T$ the despreader could be replaced with a RAKE. However, we show in appendix B that this is incorrect because the received signal contains the convolution between $s_{\text{ofdm}}(q)$ and the channel and not that between the spreading code and the channel. As a result, only the basic despreader is usable. In the appendix we also show that due to the multipath channel and the non-ideal autocorrelation function of the spreading sequence there is a self-interference term affecting the DS-DFT receiver and, that using a cyclic prefix attached to the OFDM frames, this interference can be mitigated.

We therefore conclude that, regardless of the chosen receiver implementation, employing a cyclic prefix is the most effective way of dealing with MC-DS-CDMA transmission over both $\tau_m < T$ and $\tau_m > T$ multipath channels. With respect to an OFDM transmission, the multipath resolution of MC-DS-CDMA is N_c -folds higher, which is analogous to the gain obtained with DS-SS in single carrier transmission. The number of resolved multipath components in MC-DS-CDMA is given by

$$M = \lfloor \tau_m W \rfloor + 1 = \left\lfloor \frac{\tau_m}{T} \cdot N_c \right\rfloor + 1 \quad (3.9)$$

3.3 Design considerations

In the general presentation of chapter 2 we found the bandwidth of our signal to be

$$W = \frac{N+1}{T_c} = \frac{N+1}{NT} \cdot N_c \approx \frac{N_c}{T} \quad (3.10)$$

and therefore, there is an N_c -fold increase in bandwidth with respect to an OFDM trans-

mission, increase which is due to the multiuser capability of MC-DS-CDMA. In the following discussion we point out some design considerations for MC-DS-CDMA and perform a comparison with the two related multiple access methods, DS-CDMA and OFDMA⁴, in terms of capacity and overhead.

As design parameters, we assume the same available bandwidth W , the same user data rate of $1/T$, the same channel (characterized by the maximum excess delay τ_m) for all three systems, and that N_u users can be supported⁵ with a spreading code length of N_c .

We define the amount of relative overhead due to cyclic prefix

$$\varepsilon = \frac{\text{time to transmit prefix}}{\text{time to transmit data and prefix}} \quad (3.11)$$

as a means to compare the efficiency of the OFDMA and MC-DS-CDMA signals.

3.3.1 Multipath fading channel

This case corresponds to the situation where the channel appears as multipath with respect to the data symbol and where the multipath is resolvable with a resolution better than the data symbol duration, which we can express as

$$\frac{1}{W} < T < \tau_m \quad (3.12)$$

MC-DS-CDMA signaling

In order to maintain flat fading of the subcarriers, their number should be chosen

⁴. In OFDMA users are allocated disjoint sets of subcarrier frequencies such that overall, the signals of various users add up to an OFDM signal. Basically, we have a frequency division multiple access system where the frequency bands are replaced with OFDM subcarriers.

⁵. For Walsh codes one could have $N_u = N_c$ while for other spreading sequence families N_u is some function of N_c .

such that the subcarrier separation Δf_{sc} is much less than the channel's coherence bandwidth Δf_{coh} :

$$\Delta f_{coh} > \Delta f_{sc} = \frac{W}{N} \Rightarrow N > W \cdot \tau_m \quad (3.13)$$

If we associate with the “greater than” condition a factor $\kappa > 1$, then 3.13 becomes

$$N = \kappa \cdot W \cdot \tau_m \quad (3.14)$$

In these conditions, with a cyclic prefix of length L and duration τ_m we can write

$$\frac{W}{N} = \frac{1}{\kappa \cdot \tau_m} = \Delta f_{sc} = \frac{1}{T_c} \cdot \left(1 + \frac{L}{N}\right) = \frac{1}{T_c - \tau_m} \quad (3.15)$$

and obtain a chip duration of $T_c = (1 + \kappa) \cdot \tau_m$. Since the various system parameters are connected through $NT = N_c T_c$, the length of the spreading sequences is given by

$$N_c = \frac{NT}{T_c} = \frac{\kappa}{\kappa + 1} \cdot W \cdot T < W \cdot T \quad (3.16)$$

Using equations 3.12, 3.14 and 3.16 find that

$$\frac{N_c}{N} = \frac{T}{\kappa \cdot \tau_m} < 1 \quad (3.17)$$

which is basically the same as relation 3.8. The duration of the MC-DS-CDMA block is $T_{ofdm-ds-ss} = N_c \cdot T_c = N_c \cdot (1 + \kappa) \cdot \tau_m$ and therefore, the relative overhead required in order to take advantage of the one tap equalization of the OFDM subcarriers is

$$\varepsilon = \frac{\tau_m N_c}{N_c T_c} = \frac{\tau_m}{T_c} = \frac{1}{1 + \kappa} \quad (3.18)$$

which can be made small if the number of subcarriers N is chosen such that $\Delta f_{sc} \ll \Delta f_{coh}$ which then implies $\kappa \gg 1$.

OFDMA signaling

The total number of subcarriers is $N = N' \cdot N_u$, with N' the number of subcarriers allocated to one user, and in order to maintain flat fading N should be chosen according to relation 3.13. As such, we obtain the same total number of subcarriers as for MC-DS-CDMA and equation 3.14 applies here as well. Then, with a cyclic prefix of length L and duration τ_m we can write

$$\frac{W}{N} = \frac{1}{\kappa \cdot \tau_m} = \Delta f_{sc} = \frac{1}{N' \cdot T} \cdot \left(1 + \frac{L}{N}\right) = \frac{1}{N' \cdot T - \tau_m} \quad (3.19)$$

and therefore the number of subcarriers allocated to one user N' and the number of users N_u are found to be given by

$$N' = (\kappa + 1) \frac{\tau_m}{T} \quad N_u = \frac{N}{N'} = \frac{\kappa}{\kappa + 1} \cdot W \cdot T < W \cdot T \quad (3.20)$$

where $N' > 1$ always in order to be able to use OFDM with cyclic prefix over the $T < \tau_m$ channel. Because the cyclic prefix is shared by all users and the duration of the OFDMA frame is $T_{ofdma} = N' \cdot T = (1 + \kappa) \cdot \tau_m$, the relative overhead is given by

$$\varepsilon = \frac{\tau_m}{N' \cdot T} = \frac{1}{1 + \kappa} \quad (3.21)$$

and it is seen to be the same as the relative overhead needed by MC-DS-CDMA.

DS-CDMA signaling

Given that we are considering the $T < \tau_m$ case, ISI would affect the symbols recovered by a RAKE receiver or a simple despreading front-end and therefore, an additional equalizer is required. As a result, multicarrier transmission schemes are preferable. Nevertheless, the spreading code length is given by

$$N_c = W \cdot T \quad (3.22)$$

which is comparable to that obtained for MC-DS-CDMA and hence, it results in a similar number of users being supported. However, MC-DS-CDMA offers a simpler receiver implementation where a RAKE is not necessary.

3.3.2 Flat fading channel

This case corresponds to the situation where the channel appears as flat fading with respect to the data symbol, but where the multipath is resolvable with a resolution higher than the maximum channel excess delay. In other words:

$$\frac{1}{W} < \tau_m < T \quad (3.23)$$

In general, the discussion and the results from the previous section apply here as well, with the following notes for the $\tau_m \ll T$ special situation:

- For MC-DS-CDMA the number of subcarriers N and the chip duration T_c can be reduced while the bandwidth W , the user data rate $1/T$ and the number of users are maintained. Therefore, we would effectively trade DFT complexity (the number of subcarriers) for chip rate.

- With OFDMA signaling, the number of subcarriers per user obtained in relation 3.20 can now be set to $N' = 1$.
- In DS-CDMA the RAKE receiver becomes the optimal receiver.

3.4 Conclusions

We conclude that MC-DS-CDMA makes the application of DS-SS principles possible to $\tau_m > T$ channels in a computationally efficient way by means of OFDM techniques. The following summarizes our conclusions:

- In terms of the maximum number of theoretical users, MC-DS-CDMA is similar to DS-CDMA however, complexity of the receiver is reduced by eliminating the RAKE. Further, OFDMA may support more users than MC-DS-CDMA depending on the particular relation linking the number of users N_u to the length N_c of the spreading sequence. In particular, equation 3.20 gives the maximum number of supported users in an OFDMA system while relation 3.16, similar to 3.20, gives the length of the spreading sequence for MC-DS-CDMA which in turn, is usually larger than the number of supported users.
- Both MC-DS-CDMA and OFDMA require the same amount of overhead, but MC-DS-CDMA transmits N_c times more information in a signaling period (an MC-DS-CDMA block) than OFDMA does (in an OFDMA frame). While the actual transmitted information rate is the same, this difference is due to the different “packaging of information” used in the two systems. Although this increase in the duration of the radio packet could result in equalization difficulties due to time varying channels, channel equalization in MC-DS-CDMA can

be performed at the frame level (that is at the same rate as in OFDMA) if so is needed.

- DS-CDMA either requires a RAKE receiver or an additional time domain equalizer, depending on the channel conditions. MC-DS-CDMA resorts to multicarrier techniques in order to deal with the channel delay spread in a computationally efficient manner.

Overall, MC-DS-CDMA successfully combines DS-SS and multicarrier principles in order to provide a viable modulation and multiple access scheme for the wireless uplink. Due to the particular mix of spread spectrum and OFDM employed by this technology, it is necessary to analyze and quantify its sensitivity to various synchronization errors. This task is undertaken in the remainder of the work.

Synchronization requirements

In this chapter we first briefly consider the general synchronization requirements of digital communications systems and then present specifics for MC-DS-CDMA. At the end, a mathematical model for dealing with the synchronization errors discussed in later chapters is established.

4.1 Synchronization in general

Regardless of the digital communications system considered, the received signal is first sampled at a rate larger than the Nyquist rate $1/T_s \geq 1/T_{Nyq}$. The sampling rate should be large enough to provide sufficient statistics for both data detection and synchronization purposes. In general, the timing for this sampler is derived from a free running oscillator and therefore T_s is usually incommensurate with T_{Nyq} , where T_{Nyq} is related to the sample duration of the transmitted data (serial equivalent data for parallel transmission formats). In conclusion, a timing synchronization is required in the digital processing chain for converting the T_s -spaced samples to T_{Nyq} -spaced ones. We will refer to this as sample synchronization and it involves both frequency and phase synchronization.

Another task is carrier¹ frequency and phase synchronization. This is required due to the mismatch between the transmitter and the receiver RF oscillators. The net effect of

¹. In general, the multicarrier signal is considered to have subcarriers rather than a common carrier as it would be the case for serial transmission methods. Nevertheless, the typical lowpass equivalent representation assumes a carrier frequency equal to the lowest subcarrier frequency while all the subcarriers are frequency locked to this lowest frequency subcarrier.

frequency offset is a translation of the received spectrum. If time variations of the channel are significant during one received symbol (sample in case of single carrier transmissions) the Doppler spread and the mean Doppler shift have to also be accounted for. The first will produce a broadening of the received spectrum while the second will contribute to the overall frequency offset. Accommodating this spectral shift and/or broadening is why the received signal has to be sampled at a rate larger than the Nyquist rate, and why frequency shift and spectral broadening have to be, in general, compensated for before down sampling to $1/T_{\text{Nyq}}$ can be performed. Typically, the carrier phase is a random variable (constant but unknown value) and it can be corrected later on, after down sampling, or totally ignored in case of differential modulation. Should this carrier phase be a stochastic process it is referred to as phase noise. In this case continuous estimation and compensation of the phase process is required which transforms carrier phase synchronization into a rather complex operation.

Channel estimation and equalization is usually considered as a separate issue but it may as well be considered within the general framework of synchronization since it refers to estimating and compensating for a stochastic process affecting the received signal. Furthermore, channel and timing estimation are not totally independent over multipath channels.

4.2 Synchronization of MC-DS-CDMA

In the following we describe the synchronization needs of the two MC-DS-CDMA receivers shown in figures 3.4 and 3.5 assuming no apriori knowledge of any synchronization parameters.

Establishing proper sample synchronization is the first requirement, and it has two aspects: sampling frequency and sampling phase. The latter is locked to a frame sync (in general, the OFDM frame sync) needed by the receiver and its effect is compensated for

through channel estimation and equalization and hence, it is usually of no further concern. On the other hand, obtaining an accurate sampling frequency is necessary in order to recover the information flow. This can be established in a number of ways, including:

- having a master clock synchronize the whole network or employing very stable clock generators at both the transmitter and receiver end,
- via side information transmitted by each user, or
- by locking it to a carrier frequency.

Tracking a small sampling frequency offset can be implemented via the phase information derived from a periodically acquired frame sync, assuming the maximum drift is less than one sampling clock cycle between two subsequent frame sync acquisitions.

As far as other timing synchronization issues are concerned, they are specific to the particular implementation of the receiver and therefore, they will be discussed in the context of the synchronization requirements of the DFT-DS and DS-DFT receivers.

Regarding carrier frequency synchronization, it is known that OFDM systems are sensitive to frequency errors. This will become apparent when we analyze the effect carrier frequency offsets have on the received OFDM-DS-SS signal. We do not consider a constant carrier phase as a separate issue since it can be included in the channel estimate.

Considering the acquisition of synchronization parameters, two approaches are possible: using separate training signals or embedding them into the OFDM frame. The second approach is known as the pilot tone technique in the OFDM literature. Given that the receiver has to have a DFT block, we may also regard the above two techniques as pre- and post-DFT acquisition. Finally, it is also mentioned that while tracking of synchronization parameters can employ these same methods, it is also possible to devise tracking algorithms if certain very particular effects can be identified at the level of the received signal

or at the decision variable, effects which can be clearly attributed to a given type of synchronization error.

4.2.1 DFT-DS receiver

In this implementation, the receiver has to acquire the timing of the OFDM frames for the desired user before performing the DFT. This means that OFDM frame sync should be derived from some sort of user specific side information originating from the transmitter. The receiver can be simplified if the transmission of individual users is synchronized, in which case a common OFDM frame sync has to be established. In particular, such a situation may arise if the CDMA component of the transmission is used to uplink different data streams from the same user on a code division basis, while user separation is achieved via frequency or time division.

After OFDM demodulation we obtain a DS-SS signal on each subcarrier which has to be despread. This operation requires code synchronization which is functionally equivalent to establishing OFDM-DS-SS block sync. Acquisition of this timing parameter can be done as in DS-SS by using a sliding correlator, serial search or similar techniques, although the presence of the N subcarriers which all share the same spreading code could lead to implementations specific to the MC-DS-CDMA signal.

Frequency synchronization and channel estimation errors would affect both the DFT and the despreading operation. While both pre- and post-DFT solutions are possible, the latter is clearly preferred for channel equalization out of complexity considerations. In general, once timing and carrier frequency have been acquired, a slowly varying channel will only affect the DFT and not the despreading, while a fast varying channel will affect both.

Figure 4.1 illustrates the synchronization requirements of the DFT-DS receiver

assuming pre-DFT carrier frequency synchronization and a fast varying channel (equalization before despreading).

4.2.2 DS-DFT receiver

The DS-DFT implementation of the receiver requires code phase synchronization (equivalent to block synchronization) of the desired user first in order to extract its signal from the composite multiple access signal through deinterleaving and despreading. While this bears some resemblance to despreading in DS-CDMA, in this case the sliding window² correlation technique is to be replaced by an analogous “sliding block” algorithm. The discussion on the OFDM frame sync from the DFT-DS receiver regarding user synchronicity also applies here, but this time to the code phase (equivalent to block) synchronization. At this point it should be noted that despreading in the DS-DFT receiver occurs at a much higher processing rate (effectively, at the N/T_c sample rate) than in the DFT-

² This is similar to the sliding correlator technique, but instead of running the locally generated spreading sequence at a slightly higher rate until sync is established, one attempts a real-time serial search. This is computationally feasible because MC-DS-CDMA systems can only use short spreading sequences.

DS receiver (at a rate of $1/T_c$).

Due to the despreading front-end, carrier frequency synchronization before despreading is required. In terms of channel estimation and equalization, the cyclic prefix technique is the solution of choice, as discussed in appendix B and in section 3.2.2 of chapter 3.

After despreading, an OFDM signal is obtained which may be processed further by employing typical OFDM techniques. However, acquisition of a separate OFDM frame sync may not be necessary since it can be derived directly from the block timing and the sampling clock. The phase of the spreading code is also related to the block timing and the sampling clock. Although it seems that this implementation requires one less synchronization function than the DFT-DS realization, this reduction can also be obtained for the DFT-DS structure if one somehow estimates block timing first. Figure 4.2 illustrates the synchronization requirements of the DS-DFT receiver.

4.3 Modeling synchronization errors

Considering the synchronization requirements detailed in section 4.2, in this work

we analyze the effect of carrier frequency, code phase and OFDM frame alignment errors. We shall not deal with sampling frequency errors because of the availability of low frequency oscillators with high stability. The problem of channel estimation is identical to the one for OFDM systems provided that the channel is slowly varying and can be regarded as constant for the duration of one or more OFDM-DS-SS blocks.

In this section the model used to analyze the effect of the above mentioned impairments is described, while in subsequent chapters the actual analysis is carried out. For each synchronization parameter we investigate, two cases are considered:

- the effect of tracking errors in a single user scenario, and
- the effect of relative synchronization errors between users (multiuser scenario).

For the purpose of our analysis we shall only consider the basic despreading receiver derived in section 3.2 and not the optimal multiuser detector because the former is more suited to gaining insight into the effects characterizing synchronization errors.

4.3.1 Carrier frequency errors

From the general transmission scheme of figure 4.3 the source of carrier frequency error (denoted by Δf) is identified to be the mismatch between the local oscillator of the transmitter and the receiver. The mean Doppler shift is another contributing factor. For convenience, the RF and IF blocks were replaced by one RF amplifier and one local oscillator. The filter with transfer function $F(\omega)$ models all the RF+IF filtering and is considered to be ideal in the signal's bandwidth. In fact, the effect of a more realistic filter on the information bearing signal is reversible by digital filtering as long as $F(\omega)$ satisfies certain conditions detailed in chapter 4 of [40].

In principle, the sampling rate $1/T_s$ at the receiver should be high enough to pre-

vent spectral aliasing in the presence of frequency offsets. However, we shall assume that it has the minimum value required in order to accommodate the cyclic prefix of length L and therefore, it is given by

$$1/T_s = (N + L)/T_c \quad (4.1)$$

This sampling rate is sufficient for detection purposes and for our analysis because we only pursue the estimation of the effects of frequency offsets at the decision device.

For purposes of mathematical modeling, we consider the received samples $r(n)$ to be given by

$$r(n) = s(n) \star h(n) \quad (4.2)$$

with $h(n)$ the equivalent end to end discrete time channel, $s(n)$ the transmitted samples (including the cyclic prefix), and where $\star$ denotes linear convolution. Then, as shown in figure 4.4, we account for the carrier frequency offset via

$$x(n) = r(n) \cdot e^{j2\pi n \Delta f T_s} \quad (4.3)$$

4.3.2 Timing errors

From the block diagram of figure 4.5 the source of the timing offset (denoted by Δt) is identified to be the misalignment and mismatch of the receiver and transmitter side timing references. The delay of the first path of a multipath channel is another contributing factor. For convenience, the RF and IF blocks were replaced by a single RF unit which is irrelevant for the purpose of discussing timing error.

With reference to figures 4.1 and 4.2, as well as to the discussion from section 4.2, the timing error can be split into sampling, OFDM frame synchronization and code phase (block) estimation errors. Since we only discuss the effects of timing misalignment and assume perfect sampling frequency synchronization, all timing errors can be modeled as sampling phase offsets and hence, taken into account at the front-end sampler

$$\Delta t = N_b T_b + N_f T_f + N_s T_s + \Delta T_s \quad (4.4)$$

The decomposition uses successive modulo- N_x operations and the following notations:

- N_b is the number of complete OFDM-DS-SS blocks contained in the timing offset, and T_b is the duration of such a block,

Figure 4.5: Timing error in communications.

- N_f is the number of OFDM frames contained in any incompletely received block within the timing error, and T_f is the duration of one frame,
- N_s is the number of samples within any incompletely received OFDM frame, and T_s is the sample duration, and
- ΔT_s is the fractional sampling time contained within the timing error.

The relationship between these various time durations is given by

$$T_b = N_c T_f \quad T_f = (L + N) T_s \quad T_f = T_c \quad (4.5)$$

where the meaning of N , N_c , T , T_c was introduced in chapter 2 and L is the length of the cyclic prefix. In light of equation 3.4 we note that

$$s(\alpha(N + L)N_c + \beta(N + L) + \gamma) = c(\beta) \cdot s_{\text{ofdm_cp}}^{(\alpha)}(\gamma) \quad (4.6)$$

and we have explicitly written out the dependence on the OFDM-DS-SS block index α . We have also denoted by β the OFDM frame index within the block and by γ the sample index within the $N + L$ sample frame³. The range of these indices is $\alpha \in \mathbb{N}$, $\beta \in [0, N_c - 1]$ and $\gamma \in [0, N + L - 1]$.

As it was already pointed out in section 4.2, fractional sampling time errors are automatically compensated for via channel estimation and equalization and as a result they are of no further concern. Hence, we can discard the ΔT_s term in relation 4.4. Fur-

³. Relation 3.4, which rewritten with our current notation is $s(\beta N + \gamma) = c(\beta) \cdot s_{\text{ofdm}}(\gamma)$ did not include the cyclic prefix and therefore, in that context we had N samples per frame and $\gamma \in [0, N - 1]$. In order to make a clear distinction, we shall use the $s_{\text{ofdm_cp}}^{(\alpha)}(\gamma)$ notation whenever we talk about the OFDM frame including the prefix.

ther, timing errors which are full multiples of the block duration, that is $\Delta t = N_b T_b$, result in a delay at the OFDM-DS-SS block level which then causes no degradation since the structure of the demodulated signal is not affected. For this reason, the $N_b T_b$ term is immaterial and can also be discarded from equation 4.4.

Figure 4.6: Timing error model.

For purposes of mathematical modeling, we consider the received samples $r(n)$ to be given by

$$r(n) = s(n) \star h(n) \quad (4.7)$$

and account for the timing offset via $x(n) = r(n - \Delta n)$ where the discrete timing error is

$$\Delta n = \frac{\Delta t}{T_s} = N_f \cdot (N + L) + N_s \quad (4.8)$$

and the samples $x(n)$ are then processed by the OFDM-DS-SS receiver. Figure 4.6 summarizes the model described above.

Effect of carrier frequency synchronization errors

For the purposes of the analysis in this chapter we consider that frame and code phase synchronization have already been established and we neglect at first the AWGN noise present in the received signal and account for it later by adding to the result a random variable with variance σ_n^2 .

5.1 Overview

We separately analyze the single user and the multiuser case by deriving the expression for the decision variable and then assessing the performance degradation. Then, in section 5.4 we justify our results and in 5.5 we draw the appropriate conclusions.

For the single user case we establish, based on the result of equation 5.7, that carrier frequency synchronization errors result in interference among the OFDM subcarriers and in attenuation due to the time-frequency auto-correlation function of the spreading sequence. We analyze the two effects individually using a Gaussian approximation to the subcarrier interference component. Finally, we combine the partial results of equations 5.19 and 5.23 to obtain the SNR degradation measure of relation 5.26. We also propose a technique which could be used to acquire an initial estimate based on the result of equation 5.18.

Regarding the multiuser case, we use the SNR degradation measure of equation

5.37 and study the degradation of the cross-correlation properties of the spreading sequence family due to relative carrier frequency errors. We use both worst-case maximum and average degradations and conclude that continuous tracking of a downlink pilot carrier might be necessary in order to maintain acceptable performance levels.

5.2 Single user case

With reference to the mathematical model established in section 4.3.1 of chapter 4 we now proceed with evaluating the effect of carrier frequency errors at the decision device. Because similar results for both receiver architectures are expected, a detailed analysis is carried out for the DFT-DS receiver, while for the DS-DFT receiver we show the important steps of the analysis for illustrative purposes only.

5.2.1 The decision variable

Considering the structure of the MC-DS-CDMA signal (figure 3.1 and relation 3.4) the γ^{th} sample of the β^{th} OFDM frame is given, after removal of the cyclic prefix, by

$$x_{\beta}(\gamma) = e^{j\varphi_{\beta}} c(\beta) \cdot [s_{\text{ofdm}}(\gamma) \circledast h_{\beta}(\gamma)] \cdot e^{j2\pi\gamma\Delta f T_s} \quad (5.1)$$

Here $\circledast$ denotes periodic convolution, φ_{β} the phase accumulation due to frequency offset at the beginning of the data component of the β^{th} OFDM frame, and $h_{\beta}(\gamma)$ is the possibly time varying channel. We normalize the frequency offset to the subcarrier spacing $\Delta f_{\text{sc}} = (1 + L/N)/T_c$ (see also equation A.1 in appendix A and relation 4.1) obtaining $\delta f = \Delta f T_s N$. Then, we split this normalized frequency offset into an integer part δf_i and a fractional part δf_f , take the DFT of relation 5.1 (with $X_{\beta}(i) = \text{DFT}\{x_{\beta}(\gamma)\}$) and obtain

$$X_{\beta}(i) = e^{j\varphi_{\beta}} c(\beta) \cdot \sum_{\gamma=0}^{N-1} \underbrace{[s_{\text{ofdm}}(\gamma) \odot h_{\beta}(\gamma)]}_{y(\gamma)} \cdot e^{j2\pi \frac{\gamma \cdot \delta f_i}{N}} \cdot e^{-j2\pi \frac{\gamma(i - \delta f_i)}{N}} \quad (5.2)$$

Then, since $Y_{\beta}(m) = \text{DFT}\{y(\gamma)\} = H_{\beta}(m) \cdot d(m)$ with $H_{\beta}(m) = \text{DFT}\{h_{\beta}(\gamma)\}$ and $d(m)$ representing the data, we can write

$$X_{\beta}(i + \delta f_i) = e^{j\varphi_{\beta}} c(\beta) \cdot \sum_{m=0}^{N-1} Y_{\beta}(m) \underbrace{\frac{1}{N} \cdot \sum_{\gamma=0}^{N-1} e^{j2\pi \frac{\gamma m}{N}} \cdot e^{j2\pi \frac{\gamma(\delta f_i - i)}{N}}}_{\zeta(i - m; \delta f_i)} \quad (5.3)$$

$$X_{\beta}(i + \delta f_i) = e^{j\varphi_{\beta}} c(\beta) \cdot \underbrace{\sum_{m=0}^{N-1} H_{\beta}(m) \cdot d(m) \cdot \zeta(i - m; \delta f_i)}_{\Lambda_{\beta}(i; \delta f_i)} \quad (5.4)$$

We note that the integer frequency offset δf_i produces a cyclic shift of the subcarriers (rotation of their index) while the fractional frequency offset δf_f results in inter-carrier interference as expressed by the $\sum_{m=0}^{N-1} (\cdot)$ term. The properties of the interference are discussed in appendix C.

The decision variable $D(i)$ on the i^{th} subcarrier is obtained after despreading which then yields

$$\begin{aligned} D(i + \delta f_i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} X_{\beta}(i + \delta f_i) \cdot c^*(\beta) = \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} |c(\beta)|^2 \cdot e^{j\varphi_{\beta}} \cdot \Lambda_{\beta}(i; \delta f_i) \\ &= e^{j\varphi_0} \cdot \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} |c(\beta)|^2 \cdot e^{j2\pi \beta \Delta f T_c} \cdot \Lambda_{\beta}(i; \delta f_i) \end{aligned} \quad (5.5)$$

where φ_0 is related to the phase at the beginning of the MC-DS-CDMA block and where we have exploited the

$$\varphi_\beta = \varphi_{\beta-1} + 2\pi\Delta f T_c \quad (5.6)$$

recurrence relation. Assuming the channel doesn't change for the duration NT of the MC-DS-CDMA block¹ ($\Lambda_\beta(i;\delta f_t)$ does not depend on β), relation 5.5 can be rewritten as

$$D(i + \delta f_t) = e^{j\varphi_0} \cdot \Lambda(i;\delta f_t) \cdot R_c(0;\delta f') \quad (5.7)$$

where

$$R_c(0;\delta f') = \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} |c(\beta)|^2 \cdot e^{j2\pi\beta\Delta f T_c} \quad (5.8)$$

and it represents the time-frequency auto-correlation function of the spreading sequence at time lag zero and $\delta f' = \Delta f T_c$.

5.2.1.1 Special case

In the multiuser case, which is the normal mode of operation, each subcarrier should be despread first in order to extract the desired user (i.e. only the decision variable $D(i)$ is available for further processing). However, if there is only one user (no code division multiple access), the receiver doesn't have to despread each subcarrier first in order to gain access to the user's signal. In practice, such a situation could arise during a base station controlled training phase during which only one, apriori known² user is allowed to transmit.

¹ This is the same as requiring a stationary channel for the duration of N_c OFDM frames.

² This can be enforced through an appropriate access protocol.

Considering the channel to be time invariant during two chip intervals

$$\Lambda_{\beta}(i; \delta f_f) = \Lambda_{\beta-1}(i; \delta f_f) \quad (5.9)$$

and using equation 5.4, the frequency offset can actually be estimated as

$$2\pi\hat{\Delta f}T_c = \arg\{X_{\beta}(i)c^*(\beta) \cdot [X_{\beta-1}(i)c^*(\beta-1)]^*\} \quad (5.10)$$

The estimate from relation 5.10 can be smoothed by appropriate filtering (averaging) over several consecutive values of β . However, values obtained when β and $\beta-1$ belong to different MC-DS-CDMA blocks should be discarded since in this case condition 5.9 doesn't hold (different $d(m)$ data).

5.2.2 The DS-DFT receiver

Given that the two receiver architectures are equivalent from an end-to-end signal processing point of view, we expect to obtain the same result as for the DFT-DS receiver. However, since the analysis is somewhat different, we summarize the steps involved in the calculation. Following the analysis found in appendix B but in the presence of a carrier frequency offset of Δf and considering a cyclic prefix of length L , relation B.3 becomes

$$\begin{aligned} r(\beta(N+L) + \gamma) = e^{j2\pi[\beta(N+L) + \gamma]\Delta f T_s} \cdot & \left[c(\beta) \cdot \sum_{\sigma=0}^{\gamma} s_{\text{ofdm}}(\gamma - \sigma) \cdot h(\sigma) + \right. \\ & \left. + c(\beta-1) \cdot \sum_{\sigma=\gamma+1}^L s_{\text{ofdm}}(N + \gamma - \sigma) \cdot h(\sigma) \right] \end{aligned} \quad (5.11)$$

Then, using equation 4.1 one can write relation B.5 as

$$\begin{aligned}
y(\gamma) &= \sum_{\beta=0}^{N_c-1} r(\beta(N+L)+\gamma) \cdot c^*(\beta) \\
&= e^{j2\pi\gamma\Delta f T_s} \cdot \left[\sum_{\beta=0}^{N_c-1} c^*(\beta)c(\beta) e^{j2\pi\beta\Delta f T_c} \cdot \sum_{\sigma=0}^{\gamma} s_{\text{ofdm}}(\gamma-\sigma) \cdot h(\sigma) + \right. \\
&\quad \left. + \sum_{\beta=0}^{N_c-1} c^*(\beta)c(\beta-1) e^{j2\pi\beta\Delta f T_c} \cdot \sum_{\sigma=\gamma+1}^L s_{\text{ofdm}}(N+\gamma-\sigma) \cdot h(\sigma) \right]
\end{aligned} \tag{5.12}$$

and by collecting the correlation terms associated with the spreading sequence we get

$$\begin{aligned}
y(\gamma) &= R_c(0; \delta f') \cdot e^{j2\pi\gamma\Delta f T_s} \sum_{\sigma=0}^{\gamma} s_{\text{ofdm}}(\gamma-\sigma) \cdot h(\sigma) + \\
&\quad + R_c(1; \delta f') \cdot e^{j2\pi\gamma\Delta f T_s} \sum_{\sigma=\gamma+1}^L s_{\text{ofdm}}(N+\gamma-\sigma) \cdot h(\sigma)
\end{aligned} \tag{5.13}$$

Finally, by removing the cyclic prefix relation B.8 becomes

$$z(\gamma) = R_c(0; \delta f') \cdot e^{j\varphi_0} \cdot e^{j2\pi\gamma\Delta f T_s} \cdot [s_{\text{ofdm}}(\gamma) \odot h(\gamma)] \tag{5.14}$$

At this point it is clear that we have obtained the despread version of equation 5.1, which we could have also arrived at by noticing that equation 5.1 applies to the DS-DFT receiver as well if the cyclic prefix is removed between the deinterleaving and despreading stages. Here the quantity φ_0 is the phase at the beginning of the MC-DS-CDMA block plus the phase accumulation over the duration of the removed prefix.

In conclusion, equations 5.7 and 5.8 describe the effect of frequency offsets regardless of which receiver is used.

5.2.3 Performance degradation

Relations 5.7 and 5.8 form the basis of our discussion on SNR degradation due to

carrier frequency offsets in MC-DS-CDMA systems. First, we note that $\Lambda(\cdot)$ and $R_c(\cdot)$ depend on different normalizations of the absolute frequency offset Δf , resulting in the quantities we have called δf (normalized to the subcarrier spacing) and $\delta f'$ (normalized to the chip rate) respectively. This is a consequence of the bandwidth expansion due to the cyclic prefix. The relation between the two normalized frequency offsets is given by

$$\delta f' = \left(1 + \frac{L}{N}\right) \cdot \delta f \quad (5.15)$$

and we recognize that equations 5.7 and 5.8 only depend on the fractional parts of δf and $\delta f'$, that is δf_f and $\delta f'_f$.

Bearing in mind the above and not considering the integer frequency offset induced cyclic shift of the subcarriers (that is, considering only small frequency errors), we simplify somewhat the notation in 5.7 by writing the explicit dependence on δf only. Also including the AWGN noise, the decision variable becomes:

$$D(i) = e^{j\varphi_0} \cdot \Lambda(i, \delta f) \cdot R_c(0; \delta f) + \text{noise} \quad (5.16)$$

where the noise power is σ_n^2 .

The phase φ_0 can be written as the sum of two components, the phase accumulation due to carrier frequency offset at the beginning of the α -th MC-DS-CDMA block, and a constant phase due to the duration LT_s of the cyclic prefix:

$$\varphi_0(\alpha) = 2\pi\Delta f(\alpha N_c T_c + LT_s) = 2\pi\alpha\left(1 + \frac{L}{N}\right)N_c\delta f + 2\pi\frac{L}{N}\delta f \quad (5.17)$$

The constant phase is automatically compensated for through channel estimation while the α -dependent component, which is a typical manifestation of the frequency offset, can be

compensated without knowing the actual frequency error through either monitoring of the received constellation on each subcarrier for a constant rotation across the subcarriers or, through block-by-block channel estimation. Therefore, in the following discussion we shall consider the effect of frequency offset on $R_c(0;\delta f)$, on $\Lambda(i;\delta f)$ and on the decision variable $D(i)$ disregarding the phase ϕ_0 . However, we point out that the α -dependence of ϕ_0 can be exploited for frequency offset estimation/tracking. In a fashion similar to the development in section 5.2.1.1, one can obtain

$$\hat{\Delta f} = \frac{\arg\{D(\alpha+1;j) \cdot [D(\alpha;i)]^*\}}{2\pi N_c T_c} \quad (5.18)$$

where we have explicitly written out the dependence of $D(i)$ on the block index α and assumed training with two identical MC-DS-CDMA data blocks. The estimate can be smoothed by some appropriate filtering of the individual estimates obtained at each subcarrier (note the dependence on the subcarrier index i in equation 5.18).

5.2.3.1 Attenuation of the autocorrelation

Recalling the definition of the time-frequency auto-correlation function from relation 5.8, the attenuation of the $R_c(0;0)$ autocorrelation for spreading sequences of constant magnitude³ can be written as

$$R_c(0;\delta f) = R_c(0;0) \cdot e^{j\pi(N_c-1)\delta f} \cdot \frac{\text{sinc}(\pi N_c \delta f)}{\text{sinc}(\pi \delta f)} \quad (5.19)$$

$$\Gamma_{\text{acorr}} = \left| \frac{R_c(0;0)}{R_c(0;\delta f)} \right| = \left| \frac{\text{sinc}(\pi \delta f)}{\text{sinc}(\pi N_c \delta f)} \right|$$

which is an even function of δf . The result is expressed graphically in figure 5.1 for vari-

³. This includes a large class of spreading sequences, such as m-sequences, Gold codes, Walsh codes, Lin-Chang sequences and Suehiro-Hatori polyphase sequences.

ous spreading sequence lengths and with the frequency offset normalized to the chip rate.

5.2.3.2 Inter-carrier interference in the OFDM component

As it was already observed in section 5.2.1, the frequency error produces inter-carrier interference (ICI) in the OFDM component of the signal. Recalling the definition of $\Lambda(i;\delta f_f)$ from relation 5.4 we write

$$\Lambda(i;\delta f_f) = H(i) \cdot d(i) \cdot \zeta(0;\delta f_f) + \underbrace{\sum_{m \neq i} H(m) \cdot d(m) \cdot \zeta(i-m;\delta f_f)}_{ICI} \quad (5.20)$$

From relation C.2 of appendix C it is clear that $\zeta(0;\delta f_f)$ accounts for a constant phase rotation and a constant attenuation, both of which are easily compensated for through channel

Figure 5.1: Attenuation of the autocorrelation with carrier frequency synchronization errors.

estimation. Therefore, the degradation is due to the additive ICI term which can be approximated as complex Gaussian due to the typically large number of subcarriers and hence, an SNR-based degradation measure is suitable. Assuming uncorrelated, zero mean data symbols with average power σ_d^2 and AWGN noise with variance σ_n^2 , the SNR on the i^{th} subcarrier without inter-carrier interference is

$$\text{SNR}_0(i) = P_H(i) \cdot \sigma_d^2 / \sigma_n^2 \quad (5.21)$$

with $P_H(i) = \mathcal{E} \{|H(i)|^2\}$ the average power of the channel coefficient ($\mathcal{E} \{\cdot\}$ denoting statistical expectation), and where we have neglected the dependence of the SNR on $R_c(0; \delta f)$ since in this section we are only interested in the effect of the ICI alone. Based on the result of C.14 obtained in appendix C, the power of the inter-carrier interference is

$$\begin{aligned} P_{\text{ICI}} &= \mathcal{E} \{| \text{ICI} |^2\} = \mathcal{E} \left\{ \left| \sum_{m \neq i} H(m) \cdot d(m) \cdot \zeta(i-m; \delta f_f) \right|^2 \right\} \\ &= \sigma_d^2 \cdot P_H \cdot (1 - |\zeta(0; \delta f_f)|^2) \end{aligned} \quad (5.22)$$

with P_H the average power of the channel coefficients $|H(m)|$ (which are assumed identically distributed across all subcarriers). Then, the ICI induced degradation is given by

$$\Gamma = \frac{\text{SNR}_0(i)}{\text{SNR}(i)} = \frac{1 + P_{\text{ICI}} / \sigma_n^2}{|\zeta(0; \delta f_f)|^2} = \frac{1 + \text{SNR}_0 \cdot [1 - |\zeta(0; \delta f_f)|^2]}{|\zeta(0; \delta f_f)|^2} \quad (5.23)$$

where SNR_0 is the average received SNR without inter-carrier interference. The SNR degradation is an even function of δf_f because, as it can be seen from relation C.2 of appendix C, $|\zeta(0; \delta f_f)|$ is even. The result of 5.23 is plotted in figure 5.2 with SNR_0 a parameter and the frequency offset normalized to the subcarrier spacing. We note that in light of equation

C.4 the SNR degradation is independent of the number of subcarriers.

5.2.3.3 Total SNR degradation

We now combine the results of section 5.2.3.1 and 5.2.3.2. It can be seen from equation 5.16 that the desired signal depends on the frequency offset via the $R_c(0;\delta f)$ factor. The same is true for the ICI and hence, we can write

$$\text{SNR}_0(i) = P_H(i) \cdot \sigma_d^2 / \sigma_n^2 \cdot |R_c(0;0)|^2 \quad (5.24)$$

and

$$\text{SNR}(i) = \frac{P_H(i) \cdot \sigma_d^2 \cdot |\zeta(0;\delta f_f)|^2 \cdot |R_c(0;\delta f)|^2}{\sigma_n^2 + P_{ICI} \cdot |R_c(0;\delta f)|^2} \quad (5.25)$$

where P_{ICI} is given in 5.22. Therefore, the SNR degradation due to frequency offset

Figure 5.2: SNR degradation due to ICI.

(including ICI and the attenuation of the autocorrelation) is given by

$$\Gamma = \frac{\text{SNR}_0(i)}{\text{SNR}(i)} = \Gamma_{\text{acorr}}^2 \cdot \frac{1 + \text{SNR}_0 \cdot [1 - |\zeta(0; \delta f_f)|^2] \cdot \Gamma_{\text{acorr}}^{-2}}{|\zeta(0; \delta f_f)|^2} \quad (5.26)$$

with Γ_{acorr} the attenuation of the autocorrelation function obtained in equation 5.19.

The relation above suggests that when the numerator is much larger than unity (large SNR_0 values and short spreading codes) the degradation is mainly due to ICI through the $|\zeta(0; \delta f_f)|^2$ terms. Figure 5.3 plots the result of relation 5.26 for various spreading sequence lengths with SNR_0 as a parameter. From equation 3.16 we see that different spreading sequence lengths really mean different user data rates or a different total bandwidth. In the second situation, the number of subcarriers should also change in order to maintain flat fading of the subcarriers (see relation 3.14). However, it is found that for the considered frequency offset values these degradation plots are insensitive to the number of subcarriers for all values of practical interest (64 or more subcarriers). Therefore, in the results of figure 5.3 the various spreading sequence lengths can be attributed to a change of either the user data rate or of the total bandwidth (and hence number of subcarriers). The insensitivity to the number of subcarriers can also be noted from the expression of $|\zeta(0; \delta f_f)|$ in equation C.2 which becomes independent of the number of subcarriers for large values of N .

Conclusions drawn upon the above results are presented at the end of the chapter, in section 5.4.

5.3 Multiuser case

Considering the frequency offset of the k^{th} user (the desired user) perfectly esti-

Figure 5.3: SNR degradation of MC-DS-CDMA with carrier frequency synchronization errors (single user case).

mated and tracked, we now investigate the effect of the multiuser interference. With $\Delta f_k = 0$ we rewrite the decision variable from equation 5.16 as

$$D_k(i) = H_k(i) \cdot d_k(i) \cdot R_{c_k}(0;0) + \text{noise} + \text{MUI}(i) \quad (5.27)$$

with $H_k(i)$ the channel and $d_k(i)$ the data of the k^{th} user on the i^{th} subcarrier, and $\text{MUI}(i)$ the multiuser interference on the i^{th} subcarrier.

5.3.1 Power of multiuser interference

Since synchronization errors are analyzed one at a time, we assume chip-synchronous users for the purpose of the following discussion. Then, using the simplified notation of equation 5.16 the multiuser interference is expressed as

$$\text{MUI}(i) = \sum_{\substack{l=1 \\ l \neq k}}^{N_u} \text{MUI}_l(i) = \sum_{\substack{l=1 \\ l \neq k}}^{N_u} e^{j\phi_{l,0}} \cdot \Lambda_l(i; \delta f_{k,l}) \cdot R_{c_{k,l}}(0; \delta f_{k,l}) \quad (5.28)$$

where $\delta f_{k,l} = \delta f_l - \delta f_k$ is the differential frequency offset of the l^{th} and k^{th} user (being a result of the frequency correction for the k^{th} user), and $\text{MUI}_l(i)$ is the interference due to the l^{th} user. The term $R_{c_{k,l}}(\bullet)$ is the time-frequency cross-correlation function given by

$$R_{c_{k,l}}(0; \delta f_{k,l}) = \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} c_l(\beta) \cdot c_k^*(\beta) \cdot e^{j2\pi\beta\delta f_{k,l}} \quad (5.29)$$

and the term $\Lambda_l(i; \delta f_{k,l})$ is the ICI affected OFDM signal of the l^{th} user on the i^{th} subcarrier and it is expressed as

$$\Lambda_l(i; \delta f_{k,l}) = \sum_{m=0}^{N-1} H_l(m) \cdot d_l(m) \cdot \zeta(i-m; \delta f_{k,l}) \quad (5.30)$$

Since users are independent of each other, the interferences add up in power

$$P_{\text{MUI}} = \sum_{l \neq k} P_{\text{MUI}}^{(l)} \quad (5.31)$$

$$P_{\text{MUI}}^{(l)} = \mathcal{E} \left\{ \left| \Lambda_l(i; \delta f_{k,l}) \cdot R_{c_{k,l}}(0; \delta f_{k,l}) \right|^2 \right\}$$

In this relation the expectation is over the data, the channel and the differential frequency offsets. Observing that the cross-correlation is random in the frequency offsets only, we first average over the other parameters. Following the line of reasoning in section C.2 of appendix C and using relation C.8 we can show that

$$\mathcal{E} \left\{ \left| \Lambda_l(i; \delta f_{k,l}) \right|^2 \right\} = E \{ |Y(i)|^2 \} \Big|_{\text{user } l} = P_{\text{av}} \Big|_{\text{user } l} \quad (5.32)$$

which is independent of the frequency offset. Then, assuming all users are received with the same power (by using power control as in DS-CDMA), the power of the multiuser interference is

$$P_{\text{MUI}} = P_H \cdot \sigma_d^2 \cdot \sum_{l \neq k} \mathcal{E} \left\{ \left| R_{c_{k,l}}(0; \delta f_{k,l}) \right|^2 \right\} \quad (5.33)$$

with the expectation over the frequency offsets.

5.3.2 Performance degradation

When there is a large enough number of simultaneous users the MUI may be approximated as Gaussian (law of large numbers). From relation 5.24 we find the average

power of the desired user to be $P_u = P_H \cdot \sigma_d^2 \cdot |R_{c_k}(0;0)|^2$. The power of the multiuser interference without frequency errors is given by

$$P_{MUI}^{(0)} = P_H \cdot \sigma_d^2 \cdot \sum_{l \neq k} |R_{c_{k,l}}(0;0)|^2 \quad (5.34)$$

Then, the average SNR degradation of user k is computed as

$$\Gamma_k = \frac{SNR_0}{SNR} \Big|_k = \frac{\frac{P_u}{\sigma_n^2 + P_{MUI}^{(0)}}}{\frac{P_u}{\sigma_n^2 + P_{MUI}}} \quad (5.35)$$

and we obtain

$$\Gamma_k = \frac{SNR_0}{SNR} \Big|_k = \frac{1 + \frac{P_H \cdot \sigma_d^2}{\sigma_n^2} \cdot \sum_{l \neq k} \mathcal{E} \left\{ |R_{c_{k,l}}(0; \delta f_{k,l})|^2 \right\}}{1 + \frac{P_H \cdot \sigma_d^2}{\sigma_n^2} \cdot \sum_{l \neq k} |R_{c_{k,l}}(0;0)|^2} \quad (5.36)$$

We express this in terms of the signal to noise ratio (including multiuser interference) when there is no frequency offset, $SNR_0|_k$ and get the result:

$$\Gamma_k = 1 + SNR_0|_k \cdot \frac{\sum_{l \neq k} \mathcal{E} \left\{ |R_{c_{k,l}}(0; \delta f_{k,l})|^2 \right\} - \sum_{l \neq k} |R_{c_{k,l}}(0;0)|^2}{|R_{c_k}(0;0)|^2} \quad (5.37)$$

with the expectation over the differential frequency offsets. In general, users' local oscillators are independent and the resulting individual frequency offsets seen at the receiver can be considered uniformly distributed over $[-\Delta f, +\Delta f]$, where Δf characterizes the stability of the individual oscillators. Therefore, $\delta f_{k,l}$ have a triangular distribution over the range

given by

$$\delta f_{k,l} \in [-2\Delta f T_c \dots 2\Delta f T_c] \quad (5.38)$$

It is now clear that the degradation is due only to the time-frequency cross-correlation properties of the employed spreading code family. In order to numerically quantify this result, we now generate degradation curves for the cases when cyclic-shift m-sequences and when Walsh codes are used as spreading codes.

First, we consider a worst case scenario when individual differential offsets are chosen within the range given by equation 5.38 such that Γ_k is maximal. We compute this maximum degradation for each desired user in part by conducting a full search and then maximizing over the choice of desired user.

Figures 5.4-a and 5.4-b plot the maximum degradation of the cross-correlation properties of the spreading sequences for a fully loaded system, that is the term

$$\max_k \left\{ \frac{\sum_{l \neq k} |R_{c_{k,l}}(0; \delta f_{k,l})|^2 - \sum_{l \neq k} |R_{c_{k,l}}(0; 0)|^2}{|R_{c_k}(0; 0)|^2} \right\}$$

of equation 5.37 for $N_u = N_c$ users, versus the normalized frequency stability of the local oscillators $\Delta f T_c$. When this cross-correlation degradation reaches a value of 1 we say that the spreading codes have lost their multiuser rejection capability since we have $\Gamma_k \approx \text{SNR}_0|_k$ which then implies a remaining effective signal-to-noise ratio of $\text{SNR}|_k = \text{SNR}_0|_k / \Gamma_k \approx 0$ dB. We see that this already happens at very low frequency errors in the order 2% or less. However, we are dealing with a very unlikely scenario: a fully loaded system and a worst-case combination of individual frequency errors coupled with the choice of the most affected user as the desired user.

(a) maximum degradation, Walsh codes

(b) maximum degradation, cyclic-shift m-sequences

(c) average degradation, Walsh codes

(d) average degradation, cyclic-shift m-sequences

Figure 5.4: Degradation of the cross-correlation properties of spreading codes due to independent carrier frequency errors.

We can estimate the probability of this worst-case fully-loaded scenario assuming independent users, each with an uniformly distributed frequency error δf and find

$$P_{\text{worst-case}} \approx \left(\frac{1}{N_0} \right)^{N_u - 1} \quad (5.39)$$

The parameter N_0 is related to the precision a particular combination of frequency errors has to be achieved in order to obtain the worst-case degradation with a certain accuracy. To be more precise, N_0 is the number of frequency bins the uniformly distributed frequency offset of each user has to be divided into in order to achieve the worst-case degradation with the considered accuracy. Using a ± 0.5 dB accuracy for the worst-case degradation and considering individual local oscillators with $\Delta f T_c = 0.01$ precision, we find via computer search that $N_0 = 8$ for spreading sequences of length $N_c \leq 32$. Using $N_u = N_c = 15$ users we find $P_{\text{worst-case}} \approx 2 \times 10^{-13}$, that is the fully loaded worst-case scenario is a very rare event.

Considering a more likely scenario and hence, a more useful measure, we plot in figures 5.4-c and 5.4-d the average degradation of the cross-correlation properties of spreading sequences for a fully loaded system. Here we allow $\delta f_{k,l}$ to be random and perform the averaging in equation 5.37 over the uniform distribution the range of which is given in relation 5.38 and over the choice of the desired user. Based on the results we note a significant improvement since the cross-correlation degradation stays below the critical value of one. Then, in figure 5.5 we plot the maximum and average SNR degradation in the fully loaded system with SNR_0 a parameter.

For a partially loaded system we plot the maximum and average SNR degradation in figure 5.6 using different spreading sequence lengths while keeping the ratio N_c/N_u

Figure 5.5: Performance degradation of fully loaded MC-DS-CDMA due to carrier frequency synchronization errors.

constant. With respect to the partially loaded worst-case degradation we find that $N_0 = 8$ is still sufficient for $\Delta f T_c = 0.01$ and for spreading sequences of length $N_c \leq 32$. Then, assuming $N_u = 5$ users a probability $P_{\text{worst-case}} \approx 2 \times 10^{-4}$ is obtained.

5.4 Verification and justification of results

In order to support the analytical relations derived in this chapter, we provide justification for and verification of the consistency of the results. Equation 5.26 describes the SNR degradation in MC-DS-CDMA due to carrier frequency errors for the single user case. Substituting $N_c = 1$ implies $\Gamma_{\text{acorr}} = 1$ and one reobtains equation 5.23 characterizing a pure OFDM system, while substituting $N = 1$ we have $|\zeta(0; \delta f_f)| = 1$ and obtain $\Gamma = \Gamma_{\text{acorr}}^2$, consistent with a pure DS-CDMA system. Further, the expression for the inter-channel interference $\zeta(0; \delta f_f)$ in equation 5.3 is consistent with the results published in [12], equations 8 through 11.

In regards of the multiuser case, equation 5.37 suggests that performance degradation is a purely DS-CDMA effect. Given that the considered carrier frequency errors between users are much smaller than the transmission bandwidth, there is no power loss in the received interference (no spectral clipping due to RF filtering). Then, the power of the multiuser interference, as seen at the decision device, should be as in a perfectly time synchronized non-coherent DS-CDMA system and equation 5.33 properly captures this expectation.

5.5 Conclusions

In this chapter we have discussed the effects of uplink carrier frequency errors in

Figure 5.6: Performance degradation of partially loaded MC-DS-CDMA due to carrier frequency synchronization errors.

an MC-DS-CDMA system. The cases of single and multiple users have been analyzed. The results from the single user case are relevant to the required performance of the algorithm employed for tracking the frequency offset of individual users while the results from the multiuser case are relevant to the network synchronization, that is carrier frequency alignment of the active users.

5.5.1 Single user case

We have found that degradation due to carrier frequency errors can be split into two components: intercarrier interference and attenuation of the autocorrelation function of the employed spreading sequence. We conclude that:

- Except for very small frequency offsets, in the order of a small percent of the intercarrier spacing, estimation and tracking is necessary. Upper bounds on the precision required for the tracking algorithm can be established starting from the degradation curves in figure 5.3.
- Frequency offset induced degradation is more severe for longer⁴ spreading sequences and for higher SNR_0 values. Moreover, the attenuation of the autocorrelation results in SNR degradation poles at multiples of $\delta f = 1/N_c$. Finally, we also note that $R_{c_k}(0;0.5) = 0$ for all constant modulus spreading sequences of even length and therefore, at this frequency offset value the user cannot be detected.
- As explained in section 5.2.3.3, the SNR degradation is, for all practical purposes, independent of the number of subcarriers.

⁴ In accordance to the discussion following relation 5.26, longer spreading sequences can mean lower data rates or a larger total bandwidth.

- We have also suggested a method for obtaining an initial carrier frequency error estimate via relation 5.18.

5.5.2 Multiuser case

From the analysis of the multiuser case we conclude that the degradation of the cross-correlation function with differential frequency offsets and hence, the DS-SS component of the signaling, is the limiting factor. For small differential frequency offsets the degradation is monotonically increasing while for large offsets the SNR degradation tends towards an asymptotic value. Based on our results we note that:

- Regardless of the used spreading code family, or whether worst-case or average degradation is considered, and regardless of the fully or partially loaded state of the system, the SNR degradation increases with the length of the spreading sequence.
- Considering the worst-case scenario, the fully loaded system is very sensitive to carrier frequency errors since both spreading code families lose their multiuser rejection capability at frequency errors in the order of $\delta f \geq 2.5\%$. For the still usable frequency stabilities of approximately $\delta f \leq 0.5\%$ performance degradation with Walsh codes is very close to the one obtained with m-sequences. For partially loaded systems and for short spreading sequences with $N_c \leq 32$ the results show that Walsh codes significantly outperform cyclic-shift m-sequences for frequency stabilities better than approximately 0.5-1%. However, this worst-case situation has a very low probability of occurrence.
- Regarding the average degradation measure, the two code sets yield a virtually indistinguishable performance for the fully loaded case. However, at frequency stabilities of $\delta f \geq 5\%$ both codes lose most of their multiuser interference rejection.

tion capabilities and the asymptotic SNR (SNR_0/Γ) tends towards 0 dB. For the partially loaded system with $N_c/N_u \approx 3$ the asymptotic SNR tends towards 5 dB and hence, some multiuser rejection capability is preserved. Also, in the partially loaded system Walsh codes are seen to outperform cyclic-shift m-sequences for frequency stabilities better than approximately 2.5%.

- If, for example, we consider the partially loaded MC-DS-CDMA system using Walsh codes with $\text{SNR}_0 = 20$ dB, $N_c = 16$ and a maximum tolerable SNR degradation of 5 dB, we find that the normalized frequency stability of the local oscillators $\delta f_{\max} = \Delta f T_c|_{\max}$ has to be better than 1%. Then, for typical commercial oscillator stabilities of $\Delta f \approx \pm 5$ ppm, for carrier frequencies f_c in the GHz range (1-30 GHz, depending on the envisioned application), it results that the chip duration T_c should be very small (e.g. 2 μ s at 1 GHz). As a consequence, with free running oscillators we lose the advantage of longer chips (and therefore easier time synchronization of users) offered by the multicarrier signal. On the other hand, the value of T_c is lower bounded by the requirement that $1/T_c$, which is approximately equal to the subcarrier spacing, should be less than the coherence bandwidth of the channel. Therefore, the chip duration T_c is constrained by the following:

$$\frac{1}{\Delta f_{\text{coh}}} < T_c < \frac{\delta f_{\max}}{\Delta f \cdot f_c} \quad (5.40)$$

- Given the pronounced sensitivity of the MC-DS-CDMA system to differential carrier frequency errors, we conclude that continuous tracking of a downlink carrier pilot by all users might be necessary.

Effect of code phase synchronization errors

In this chapter the timing error is due to uncertainty with respect to the phase of the spreading code, which is equivalent to not knowing at which OFDM frame does the MC-DS-CDMA block start. Therefore, one may also regard code phase errors as block alignment errors. With respect to the mathematical model of chapter 4, we have $\Delta t = \pm N_f T_f$ and $\Delta n = \pm N_f \cdot (N + L)$, and we recall that $T_f = T_c$. Considering that there is a DS-SS data stream on each OFDM subcarrier, code phase errors in the MC-DS-CDMA system result in timing errors at the subcarrier level which are multiples of the chip duration. Therefore, our results should only contain effects specific to DS-SS systems.

6.1 Overview

We separately analyze the single user and the multiuser case by deriving the expression of the decision variable and then assessing the performance degradation. Then, in section 6.4 we justify our results and in 6.5 we draw the appropriate conclusions.

For the single user case we establish, based on the result of equation 6.8, that code phase synchronization errors result in inter-block interference that manifest in a peculiar constellation spreading effect. This type of degradation is severe and we suggest a measure based on distance reduction in the received signal constellation. We also propose two techniques to deal with the acquisition and possibly tracking of the code phase synchronization error. We conclude that synchronization should be established within one chip.

Regarding the multiuser case, we use the SNR degradation measure of equation 6.21 and study its dependence on the cross-correlation properties of the spreading sequence family. We use both worst-case maximum and average degradations and conclude that the requirements on code phase synchronization among users strongly depend on the employed spreading code family, and that errors of only a few chips may be tolerable without incurring a significant loss of performance.

6.2 Single user case

We first consider the single user case and derive the expression of the decision variable as a function of the subcarrier index and the timing error. The calculations are carried out for both positive and negative values of the timing offset.

6.2.1 The decision variable

Positive timing errors (delay)

Considering the positive values of Δt and Δn above, upon decomposing the index n in a manner similar to the decomposition in relation 4.6 we obtain

$$x(n) = x_{\beta}^{(\alpha)}(\gamma) = r_{\beta - N_f}^{(\alpha)}(\gamma) = [c(\beta - N_f) \cdot s_{\text{ofdm_cp}}^{(\alpha)}(\gamma)] \star h_{\beta - N_f}^{(\alpha)}(\gamma) \quad (6.1)$$

If we assume the channel static for the duration of one MC-DS-CDMA block, then $h_{\beta}^{(\alpha)}(\gamma) = h^{(\alpha)}(\gamma) \equiv h(\alpha; \gamma)$ where we define the first argument to represent the time varying nature of the channel while the second argument is the delay variable. Since the convolution is in the γ variable, the term $c(\beta - N_f)$ can be factored out and we obtain the convolution between the OFDM frame $s_{\text{ofdm_cp}}^{(\alpha)}(\gamma)$ and the channel. Removing the cyclic prefix this transforms into periodic convolution

$$x_{\beta}^{(\alpha)}(\gamma) = c(\beta - N_f) \cdot [s_{\text{ofdm}}^{(\alpha)}(\gamma) \odot h(\alpha; \gamma)] \quad (6.2)$$

where $\gamma \in [0, N-1]$. Taking the DFT in the γ variable one gets

$$X_{\beta}^{(\alpha)}(i) = \text{DFT}\left\{x_{\beta}^{(\alpha)}(\gamma)\right\} = c(\beta - N_f) \cdot d(\alpha; i) \cdot H(\alpha; i) \quad (6.3)$$

with $d(\alpha; i)$ the data and $H(\alpha; i)$ the channel's frequency response on the i^{th} subcarrier during the α^{th} MC-DS-CDMA block. The decision variable on the i^{th} subcarrier is obtained after despreading and yields

$$\begin{aligned} D(\alpha; i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} X_{\beta}^{(\alpha)}(i) \cdot c^*(\beta) = d(\alpha; i) \cdot H(\alpha; i) \cdot \sum_{\beta=N_f}^{N_c-1} c^*(\beta) c(\beta - N_f) \\ &\quad + d(\alpha - 1; i) \cdot H(\alpha - 1; i) \cdot \sum_{\beta=0}^{N_f-1} c^*(\beta) c(\beta + N_c - N_f) \\ &= d(\alpha; i) \cdot H(\alpha; i) \cdot R_c(N_f) + d(\alpha - 1; i) \cdot H(\alpha - 1; i) \cdot R_c^*(N_c - N_f) \end{aligned} \quad (6.4)$$

Here we have introduced the partial auto-correlation function $R_c(\cdot)$ of the spreading sequence $c(\beta)$ while the formal definition and the properties are given in appendix D.

Negative timing errors (advance)

Considering the negative values of Δt and Δn we obtain step-by-step

$$x_{\beta}^{(\alpha)}(\gamma) = c(\beta + N_f) \cdot [s_{\text{ofdm}}^{(\alpha)}(\gamma) \odot h(\alpha; \gamma)] \quad (6.5)$$

$$X_{\beta}^{(\alpha)}(i) = \text{DFT}\left\{x_{\beta}^{(\alpha)}(\gamma)\right\} = c(\beta + N_f) \cdot d(\alpha; i) \cdot H(\alpha; i) \quad (6.6)$$

$$\begin{aligned}
D(\alpha; i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} x_{\beta}^{(\alpha)}(i) \cdot c^*(\beta) = d(\alpha; i) \cdot H(\alpha; i) \cdot \sum_{\beta=0}^{N_c-N_f-1} c^*(\beta) c(\beta + N_f) \\
&\quad + d(\alpha + 1; i) \cdot H(\alpha + 1; i) \cdot \sum_{\beta=N_c-N_f}^{N_c-1} c^*(\beta) c(\beta + N_f - N_c) \\
&= d(\alpha; i) \cdot H(\alpha; i) \cdot R_c^*(N_f) + d(\alpha + 1; i) \cdot H(\alpha + 1; i) \cdot R_c(N_c - N_f)
\end{aligned} \tag{6.7}$$

This result has a similar structure to the one obtained for positive timing errors.

6.2.2 Performance degradation

Assuming a slowly varying channel, we can approximate $H(\alpha - 1; i) \approx H(\alpha; i)$ in equation 6.4 and $H(\alpha + 1; i) \approx H(\alpha; i)$ in relation 6.7. Then, we can write our results as

$$\begin{aligned}
D(\alpha; i) &\approx \left[d(\alpha; i) \cdot \frac{R_c(N_f)}{R_c(0)} + d(\alpha - 1; i) \cdot \frac{R_c^*(N_c - N_f)}{R_c(0)} \right] \cdot H(\alpha; i) \cdot R_c(0) \quad \Delta t > 0 \\
D(\alpha; i) &\approx \left[d(\alpha; i) \cdot \frac{R_c^*(N_f)}{R_c(0)} + d(\alpha + 1; i) \cdot \frac{R_c(N_c - N_f)}{R_c(0)} \right] \cdot H(\alpha; i) \cdot R_c(0) \quad \Delta t < 0
\end{aligned} \tag{6.8}$$

In both cases we observe that the effect of code phase synchronization errors is a common scaling of the signal constellation by $|R_c(N_f)/R_c(0)|$, a rotation by $\pm \arg\{R_c(N_f)\}$, depending on the sign of the timing error, and the appearance of inter-block interference. This interference manifests itself as the superposition of an identical signal constellation scaled by $|R_c(N_c - N_f)/R_c(0)|$ and rotated by $\mp \arg\{R_c(N_c - N_f)\}$. What this means, is that in place of any given constellation point we actually obtain a scaled replica of the whole signal constellation.

Figure 6.1 illustrates the situation for the case $N_f = 1$, and hence $\Delta t = T_c$, using a 15-chip m-sequence and a 4-QAM constellation, and where the signal constellation has

been compensated for the $R_c(N_f)$ complex scaling. The received 16-QAM like signal constellation is a severe distortion and we conclude that the MC-DS-CDMA system is very sensitive to this type of timing errors and therefore, precise tracking of the code phase is required.

The type of distortion shown in figure 6.1 clearly indicates that an SNR based degradation measure is not appropriate. Rather, one might consider a worst-case distance reduction between adjacent constellation points, reduction which can be expressed as

$$\frac{d}{d_0} \approx 1 - \lambda \cdot \left| \frac{R_c(N_c - N_f)}{R_c(N_f)} \right| \quad (6.9)$$

with λ a measure of the “extent” of the signal constellation with respect to the minimum nominal distance d_0 . For the 4-QAM example above $\lambda = 1$. Equation 6.9 also places a limit on the maximum supportable code phase error without incurring an irreducible error floor, which occurs when:

Figure 6.1: Received signal constellation in the presence of code phase synchronization errors (single user case).

$$\left| \frac{R_c(N_c - N_f)}{R_c(N_f)} \right| \geq \frac{1}{\lambda} \quad (6.10)$$

Given the severity of the distortion, as well as its dependence on the signal constellation, we shall not pursue this quantitative study any further.

We conclude this section by pointing out that use of a guard interval between blocks would eliminate the inter-block interference. This solution could then be employed in the acquisition stage of code phase synchronization. While it is possible to use this approach for tracking purposes as well, the price paid would be a reduction in the effective data rate. Yet another sync acquisition or tracking technique, applicable when the minimum distance reduction is not severe, is the use of a decision feed-back algorithm that attempts to estimate the code phase synchronization error from the structure of the signal constellation spread. This technique would identify a synchronization error that gives partial auto-correlation values which result in the observed constellation spread, effectively implementing a reverse lookup of the partial auto-correlation values.

6.3 Multiuser case

For the purpose of this discussion we consider a particular form of synchronous MC-DS-CDMA. We assume the block sync of the desired user (the k^{th} user) perfectly acquired and we evaluate the effect of the multiuser interference when interfering users have their OFDM frames synchronized but suffer from relative code phase alignment errors. Therefore, these relative timing errors are modeled as

$$\Delta t_{k,l} = t_l - t_k = \pm N_{k,l} \cdot T_f \quad (6.11)$$

6.3.1 Power of multiuser interference

When the relative code phase error is positive we start with relation 6.3 written for

the l^{th} user and despread the signal with the desired user's code in order to obtain the multiuser interference due to this user:

$$\begin{aligned}
 \text{MUI}_l(i) &= \sum_{\beta=0}^{N_c-1} x_{\beta,l}^{(\alpha)}(i) \cdot c_k^*(\beta) = d_l(\alpha;i) \cdot H_l(\alpha;i) \cdot \sum_{\beta=N_{k,l}}^{N_c-1} c_k^*(\beta) c_l(\beta - N_{k,l}) \\
 &\quad + d_l(\alpha-1;i) \cdot H_l(\alpha-1;i) \cdot \sum_{\beta=0}^{N_{k,l}-1} c_k^*(\beta) c_l(\beta + N_c - N_{k,l}) \\
 &= d_l(\alpha;i) \cdot H_l(\alpha;i) \cdot R_{c_{k,l}}(N_{k,l}) + d_l(\alpha-1;i) \cdot H_l(\alpha-1;i) \cdot R_{c_{k,l}}(-N_c + N_{k,l})
 \end{aligned} \tag{6.12}$$

Here we have introduced the partial cross-correlation function $R_{c_{k,l}}(\cdot)$ of the two spreading sequences, and the relative code phase error $N_{k,l}$ of user l with respect to the desired user. The formal definition and properties of the partial cross-correlation are presented in appendix D.

If the relative code phase error is negative then, starting with equation 6.6, we obtain

$$\text{MUI}_l(i) = d_l(\alpha;i) \cdot H_l(\alpha;i) \cdot R_{c_{k,l}}^*(N_{k,l}) + d_l(\alpha+1;i) \cdot H_l(\alpha+1;i) \cdot R_{c_{k,l}}^*(-N_c + N_{k,l}) \tag{6.13}$$

The total multiuser interference is the sum of the individual interferences and can typically be approximated as Gaussian and hence, characterized by its power. Since users are independent of each other, the interferences add up in power, so that

$$P_{\text{MUI}} = \sum_{\substack{l=1 \\ l \neq k}}^{N_u} P_{\text{MUI}}^{(l)} \quad P_{\text{MUI}}^{(l)} = \mathcal{E} \left\{ |\text{MUI}_l(i)|^2 \right\} \tag{6.14}$$

where the expectation is over the data, the channel and the relative timing errors. We first compute the expectation over the data and the channel by assuming uncorrelated, zero mean data symbols with average power σ_d^2 and that the channel is slowly varying

$$P_{H_l(i)} = \mathcal{E} \left\{ |H_l(\alpha, i)|^2 \right\} = \mathcal{E} \left\{ |H_l(\alpha \pm 1, i)|^2 \right\} \quad (6.15)$$

Further, invoking the uncorrelated scattering channel model as in C.2 of appendix C, we may drop the dependence on the subcarrier index i . Also, assuming that all users are received with the same power (by using power control as in DS-CDMA) we can also drop the dependence of $P_{H_l(i)}$ on the user index l . Finally, we normalize the channel so that

$$P_{H_l(i)} = \sum_{n=0}^L \mathcal{E} \left\{ |h_l(n)|^2 \right\} = 1 \quad (6.16)$$

and adopt the following simplified notation

$$R(n) = \begin{cases} R_{c_{k,l}}(n) & \Delta t_{k,l} > 0 \\ R_{c_{l,k}}(n) & \Delta t_{k,l} < 0 \end{cases} \quad (6.17)$$

in order to be able to write our results for both positive and negative timing errors in a common form. We obtain

$$P_{MUI} = \sigma_d^2 \cdot \sum_{l \neq k} \left[\mathcal{E} \left\{ |R(N_{k,l})|^2 \right\} + \mathcal{E} \left\{ |R(-N_c + N_{k,l})|^2 \right\} \right] \quad (6.18)$$

with the expectation over the relative timing errors.

6.3.2 Performance degradation

Using the Gaussian approximation for the multiuser interference we calculate an SNR degradation due to relative code phase synchronization errors. Given the channel normalization in equation 6.16, the power of the desired user is $P_u = \sigma_d^2 \cdot |R_{c_k}(0)|^2$, and the power of the multiuser interference without timing errors is given by

$$P_{\text{MUI}}^{(0)} = \sigma_d^2 \cdot \sum_{l \neq k} |R_{c_{k,l}}(0)|^2 \quad (6.19)$$

Then, also considering the additive white Gaussian noise, the SNR degradation is computed as

$$\Gamma_k = \frac{\text{SNR}_0}{\text{SNR}} \bigg|_k = \frac{\frac{P_u}{\sigma_n^2 + P_{\text{MUI}}^{(0)}}}{\frac{P_u}{\sigma_n^2 + P_{\text{MUI}}}} \quad (6.20)$$

and the result is

$$\Gamma_k = 1 + \text{SNR}_0 \big|_k \cdot \frac{\sum_{l \neq k} \mathcal{E} \left\{ |R(N_{k,l})|^2 + |R(-N_c + N_{k,l})|^2 \right\} - \sum_{l \neq k} |R(0)|^2}{|R_{c_k}(0)|^2} \quad (6.21)$$

where the statistical expectation is over the relative timing errors, and where $R_{c_k}(0) = 1$ for unit modulus spreading sequences. We assume that in the MC-DS-CDMA system under consideration users are synchronized such that

$$\Delta t_{k,l} \in [-N_0 T_f, N_0 T_f] \quad (6.22)$$

with $\Delta t_{k,l}$ is a uniformly distributed random variable and $T_f = T_c$.

It is now clear that the degradation is due only to the partial cross-correlation properties of the employed spreading code family and the result is similar to the one obtained for a DS-CDMA system. In order to numerically quantify these results, we now generate degradation curves for the cases when cyclic-shift m-sequences and when Walsh codes are used as spreading codes.

First, we consider a worst-case scenario when individual users are offset in time

within the range given by equation 6.22 such that Γ_k is maximal. We compute this maximum degradation for each desired user in part by conducting a full search and then maximizing over the choice of desired user.

Figures 6.2-a and 6.2-b plot the maximum degradation of the cross-correlation properties of spreading sequences for a fully loaded system, that is the term

$$\max_k \left\{ \frac{\sum_{l \neq k} |R(N_{k,l})|^2 + |R(-N_c + N_{k,l})|^2 - \sum_{l \neq k} |R(0)|^2}{|R_{c_k}(0)|^2} \right\}$$

of equation 6.21 for $N_u = N_c$ users, versus the code phase synchronization accuracy. When this cross-correlation degradation reaches a value of 1 we say that the spreading codes have lost their multiuser rejection capability since we have $\Gamma_k \approx \text{SNR}_{0|k}$ which implies an effective remaining signal-to-noise ratio of $\text{SNR}_{|k} = \text{SNR}_{0|k} / \Gamma_k = 0 \text{ dB}$. We see that this already happens for a synchronization error of one chip. While this suggests extreme sensitivity, we are in fact dealing with a very unlikely scenario: a fully loaded system and a worst-case combination of individual timing errors coupled with the choice of the most affected user as the desired user.

We can estimate the probability of this worst-case fully-loaded scenario assuming independent users, each with an uniformly distributed relative code phase error $\Delta t_{k,l}$ within the range given in equation 6.22 and find

$$P_{\text{worst-case}} \approx \left(\frac{1}{2N_0 + 1} \right)^{N_u - 1} \quad (6.23)$$

Considering $N_0 = 1$ and $N_u = N_c = 16$ users we find $P_{\text{worst-case}} \approx 10^{-7}$, that is the fully loaded worst-case scenario is a very rare event.

(a) maximum degradation, Walsh codes

(b) maximum degradation, cyclic-shift m-sequences

(c) average degradation, Walsh codes

(d) average degradation, cyclic-shift m-sequences

Figure 6.2: Degradation of the cross-correlation properties of spreading codes due to code phase synchronization errors.

In figures 6.2-c and 6.2-d we plot a more likely scenario and hence, a more useful measure: the average degradation of the cross-correlation properties of spreading sequences for a fully loaded system. Here we allow $\Delta t_{k,l}$ to be random and perform the averaging in equation 6.21 over the uniform distribution the range of which is given in relation 6.22 and over the choice of the desired user. Based on the results we note the significant improvement in tolerance to code phase errors however, from the plot of the corresponding average SNR degradation of figure 6.3 it is seen that the fully loaded system suffers from excessive performance loss.

For a partially loaded system we plot the maximum SNR degradation (worst-case scenario) in figures 6.4-a and 6.4-b using different spreading sequence lengths while keeping the ratio N_c/N_u constant. Then, in figures 6.4-c and 6.4-d we plot the corresponding average SNR degradations, where the averaging process involves the same steps as for the average cross-correlation degradation above. With respect to the partially loaded worst-

Figure 6.3: Performance degradation of fully loaded MC-DS-CDMA due to code phase synchronization errors.

case degradation we find for $N_0 = 1$ and $N_u = 5$ users a probability of $P_{\text{worst-case}} \approx 0.01$.

6.4 Verification and justification of results

In order to support the analytical relations derived in this chapter, we provide justification for and verification of the consistency of the results. Considering that code phase synchronization errors of multiples of the chip duration should only yield performance degradation as in imperfectly synchronized DS-CDMA, equations 6.8 (single user) and 6.21 (multiuser) properly reflect this expectation. Further, the graphical result shown in figure 6.1 and obtained via computer simulation, is consistent with equations 6.8 and 6.9. For a 4-QAM system we have $\lambda = 1$ and $d_0 = 2$, while the m-sequence of length 15 used in the simulation was characterized by $|R_c(1)| = 2/15$. Then, as predicted by relation 6.9, the minimum distance in the received signal constellation is reduced to $d = 1$ from the original $d_0 = 2$.

6.5 Conclusions

In this chapter we have discussed the effects of uplink code phase (block) synchronization errors in an MC-DS-CDMA system. The cases of single and multiple users have been analyzed and, considering our results, we conclude that synchronization should be established within one chip duration.

6.5.1 Single user case

In view of the results obtained in section 6.2.2 it is concluded that the degradation due to code synchronization is severe and results in inter-block interference which manifests itself in a “constellation replication” effect. Therefore, an SNR based degradation

Figure 6.4: Performance degradation of partially loaded MC-DS-CDMA due to code phase synchronization errors.

measure for the single user case is not appropriate and instead, we suggested a measure based on distance reduction in the received signal constellation. We also advanced the idea of an inter-block guard interval or a decision feed-back algorithm as possible techniques to deal with the acquisition and possibly tracking of the code phase synchronization error.

6.5.2 Multiuser case

Using a Gaussian approximation to the multiuser interference, the following conclusions can be drawn based on the results of section 6.3.2:

- In the worst-case scenario the fully loaded system has no tolerance to code phase synchronization errors because both code families lose their multiuser interference rejection capability even at a synchronization error of one chip. This is due to the very large maximum partial cross-correlation value $|R_{c_k,1}(\pm 1)|$ which is unity (or close to unity) for the studied spreading codes. However, this worst-case situation has a very low probability of occurrence. On an average degradation basis, performance loss is still excessive and hence, the fully loaded system cannot reliably operate if there are code phase errors.
- For partially loaded systems cyclic-shift m-sequences yield a significantly better performance up to a hard limit, which in the case represented in figure 6.4 is a misalignment of $N_0 = 2$ chips. This is due to the fact that $N_c/N_u \approx 3$, that is the m-sequences assigned to different users are at least three cyclic shifts away. Once the separation of the spreading sequences of individual users is violated via time shifts arising from code phase errors, the cross-correlation properties quickly degrade. There is also a sizeable difference between the worst-case maximum and the average degradation, difference that in some cases may be in excess of 5 dB.

- In the partially loaded system, for synchronization errors below the above mentioned hard limit, the SNR degradation increases with the length of the spreading sequence for Walsh codes, but it decreases for cyclic-shift m-sequences. This difference in behavior is consistent with the dependence on the spreading sequence length of the $|R_{c_k, l}(\pm 1)|$ partial cross-correlation values.

Effect of frame synchronization errors

We investigate here the effect of timing errors that are shorter than the duration of one OFDM frame. In DS-CDMA terminology this would correspond to code synchronization within a chip period while in OFDM terminology it is called frame synchronization. With regards to our mathematical model from chapter 4, we are considering the situation when $\Delta t = \pm N_s T_s$ and hence $\Delta n = \pm N_s$.

7.1 Overview

We separately analyze the single user and the multiuser case by deriving the expression of the decision variable and then assessing the performance degradation. Then, in section 7.4 we justify our results while in 7.5 we draw the appropriate conclusions.

For the single user case we establish that frame synchronization errors result in a self-interference term and a term we call other-interference. The first is shown to be equivalent to a channel estimation error while the second is due to interference from the other subcarriers and from the adjacent MC-DS-CDMA block. For errors not larger than the length of the cyclic prefix we introduce the SNR degradation measure of relation 7.59 which has two components: an OFDM related degradation coefficient, and the effect of the partial auto-correlation of the spreading code.

Regarding the multiuser case, we use the SNR degradation measure of equation 7.88 and find that it exhibits an oscillatory behavior with respect to the subcarrier index. However, the location of the peaks and troughs depends in a complicated way on a number

of system parameters and therefore, separating the effect of individual users on the SNR degradation is not always possible and upper-bounding is necessary. We then proceed with evaluating the worst-case maximum degradation and the average degradation.

7.2 Single user case

As with the code phase synchronization error calculation, we first consider the single user case and derive the expression of the decision variable as a function of the subcarrier index and the timing error. The calculations are carried out for both positive and negative values of the timing offset, but this time we expect different results as a consequence of the asymmetry introduced by the cyclic prefix.

We differentiate between two situations: when the cyclic prefix is long enough to accommodate both the channel delay spread and a frame synchronization error, and when it can only accommodate the channel. In the first situation the prefix has to be of length

$$L \geq 2 \cdot N_s^{\max} + \frac{\tau_m}{T_s} \quad (7.1)$$

with τ_m the maximum excess delay of the channel and $\pm N_s^{\max}$ the maximum synchronization error to be accommodated. By designing the nominal frame synchronization position accordingly, after removal of the prefix we obtain

$$x_{\beta}^{(\alpha)}(\gamma) = c(\beta) \cdot [s_{\text{ofdm}}^{(\alpha)}(\gamma) \odot h_{\beta}^{(\alpha)}(\gamma)] \Big|_{(\gamma \pm N_s)_N} \quad (7.2)$$

where $(\gamma \pm N_s)_N$ stands for modulo- N addition and subtraction. Clearly, after DFT this translates into a constant phase rotation in the amount $\pm 2\pi i(N_s/N)$ on the i^{th} subcarrier. However, since this phase can be automatically compensated for through channel estimation and equalization, it does not represent a degradation.

On the other hand, if the cyclic prefix is only designed for combatting multipath spread, samples from adjacent frames will corrupt the detection process. In the following we quantify this type of degradation.

7.2.1 The decision variable

Positive timing errors (delay)

Figure 7.1 illustrates the situation. Samples from the tail of the previous OFDM frame enter the detection window and therefore the prefix seen by the receiver is no longer cyclic. Our approach is to add and subtract the missing samples that would make the demodulated data frame become a periodic delay (rotation of samples) of the transmitted one. Then, the subtracted samples and the foreign ones constitute an interference signal which we shall characterize. A special situation arises at the beginning of the MC-DS-CDMA block since in that case the foreign samples belong to a different block.

Upon the α - β - γ decomposition of the index n as in relation 4.6 we can write

$$x(n) = x_{\beta}^{(\alpha)}(\gamma) = r_{\beta}^{(\alpha)}(\gamma - N_s) = [c(\beta) \cdot s_{\text{ofdm_cp}}^{(\alpha)}(\gamma)] \star h_{\beta}^{(\alpha)}(\gamma)|_{\gamma - N_s} \quad (7.3)$$

Assuming the channel static for the duration of one MC-DS-CDMA block and using the $h^{(\alpha)}(\gamma) \equiv h(\alpha; \gamma)$ notation where we define the first argument to represent the time varying nature of the channel while the second argument is the delay variable, we have

$$\begin{aligned}
 x_{\beta}^{(\alpha)}(\gamma) &= c(\beta) \cdot \sum_{\gamma_{\tau}=0}^L s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} - N_s) \cdot h(\alpha; \gamma_{\tau}) \\
 &= c(\beta - 1) \cdot \sum_{\gamma_{\tau} > \gamma - N_s} s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} - N_s + N + L) \cdot h(\alpha; \gamma_{\tau}) \\
 &\quad + c(\beta) \cdot \sum_{\gamma_{\tau} \leq \gamma - N_s} s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} - N_s) \cdot h(\alpha; \gamma_{\tau}) = c(\beta - 1)T_{-1} + c(\beta)T_0
 \end{aligned} \tag{7.4}$$

The useful term is $c(\beta)T_0$ and it only exists for $\gamma \geq N_s$ while the interference $c(\beta - 1)T_{-1}$ is non-zero for $\gamma < L + N_s$.

We now add and subtract additional terms such that, in terms of $s(n)$ and with reference to figure 7.1, we extend the cyclic prefix beyond its left margin, into the preceding OFDM frame. Then, the received OFDM frame $x_{\beta}^{(\alpha)}(\gamma)$ contains for $\gamma \in [0, N + L - 1]$ the convolution between the channel and the periodically delayed (by N_s) transmitted samples. Because at the receiver we will eventually remove the first L samples of $x_{\beta}^{(\alpha)}(\gamma)$, we only need to concern ourselves with the interference due to those samples which enter the detection window at $\gamma \in [0, N_s - 1]$ (see figure 7.1). Therefore, we have to extend the prefix in $s(n)$ with the N_s samples $s_{\text{ofdm_cp}}^{(\alpha)}(N - N_s + k)$, $k \in [0, N_s - 1]$. In terms of $x_{\beta}^{(\alpha)}(\gamma)$, this results in the $c(\beta)T_0$ term where

$$\begin{aligned}
T_0(\gamma) &= \sum_{k=0}^{N_s-1} [s_{\text{ofdm_cp}}^{(\alpha)}(N - N_s + k) \cdot \delta(\gamma - k)] \star h(\alpha; \gamma) \\
&= \sum_{k=0}^{N_s-1} s_{\text{ofdm_cp}}^{(\alpha)}(N - N_s + k) \cdot h(\alpha; \gamma - k) \\
&= \sum_{\gamma_\tau = \gamma - N_s + 1}^{\gamma} s_{\text{ofdm_cp}}^{(\alpha)}(N - N_s + \gamma - \gamma_\tau) \cdot h(\alpha; \gamma_\tau)
\end{aligned} \tag{7.5}$$

and we obtain

$$x_\beta^{(\alpha)}(\gamma) = c(\beta)T_0 + c(\beta)T_0 - c(\beta)T_0 + c(\beta - 1)T_{-1} \tag{7.6}$$

The sum of the first two terms contains the desired convolution with the periodically delayed transmitted samples. We now remove the first L samples, which mathematically means the $\gamma \rightarrow \gamma + L$ substitution and a new range of $\gamma \in [0, N - 1]$, and obtain

$$\begin{aligned}
x_\beta^{(\alpha)}(\gamma) &= c(\beta) \cdot [s_{\text{ofdm}}^{(\alpha)}(\gamma) \odot h(\alpha; \gamma)] \Big|_{(\gamma - N_s)_N} + c(\beta - 1)e_{-1}(\gamma) - c(\beta)e_0(\gamma) \\
e_{-1}(\gamma) &= T_{-1}(\gamma + L) = \sum_{\gamma_\tau > \gamma + L - N_s} s_{\text{ofdm_cp}}^{(\alpha)}(N + 2L - N_s + \gamma - \gamma_\tau) \cdot h(\alpha; \gamma_\tau) \\
e_0(\gamma) &= T_0(\gamma + L) = \sum_{\gamma_\tau > \gamma + L - N_s} s_{\text{ofdm_cp}}^{(\alpha)}(N + L - N_s + \gamma - \gamma_\tau) \cdot h(\alpha; \gamma_\tau)
\end{aligned} \tag{7.7}$$

The term $e_{-1}(\gamma)$ is the previous-frame interference and $e_0(\gamma)$ is the same-frame interference. Both are non-zero for $\gamma \in [0, N_s - 1]$ only. Then, we take the DFT and despread the result in order to arrive at the expression of the decision variable:

$$\begin{aligned}
D(\alpha; i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} x_\beta^{(\alpha)}(i) \cdot c^*(\beta) = d(\alpha; i) \cdot H(\alpha; i) \cdot R_c(0) \cdot e^{-j2\pi \frac{iN_s}{N}} \\
&\quad - E_0(\alpha; i) \cdot R_c(0) + E_{-1}(\alpha; i) \cdot R_c(1) + E_{-1}(\alpha - 1; i) \cdot R_c^*(N_c - 1)
\end{aligned} \tag{7.8}$$

where $E_0(\cdot)$ and $E_{-1}(\cdot)$ are the DFTs of the corresponding interference terms from equa-

tion 7.7. We have introduced in these equations the partial auto-correlation function $R_c(\cdot)$ of the spreading code which is discussed in appendix D. At this point we note that the calculations above are valid for $N_s \leq N$.

Negative timing errors (advance)

The derivation of the decision variable follows a similar line of reasoning as for positive timing errors. As shown in figure 7.2, foreign samples enter the detection window from the following OFDM frame. Therefore, we can write

$$\begin{aligned}
 x_{\beta}^{(\alpha)}(\gamma) &= c(\beta) \cdot \sum_{\gamma_{\tau}=0}^L s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} + N_s) h(\alpha; \gamma_{\tau}) \\
 &= c(\beta - 1) \cdot \sum_{\substack{\gamma_{\tau} > \gamma + N_s \\ \gamma + N_s}} s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} + N_s + N + L) h(\alpha; \gamma_{\tau}) \\
 &\quad + c(\beta) \cdot \sum_{\gamma_{\tau} = \gamma + N_s - N - L + 1} s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} + N_s) h(\alpha; \gamma_{\tau}) \\
 &\quad + c(\beta + 1) \cdot \sum_{\gamma_{\tau} \leq \gamma + N_s - N - L} s_{\text{ofdm_cp}}^{(\alpha)}(\gamma - \gamma_{\tau} + N_s - N - L) h(\alpha; \gamma_{\tau}) \\
 &= c(\beta - 1)T_{-1} + c(\beta)T_0 + c(\beta + 1)T_{+1}
 \end{aligned} \tag{7.9}$$

The desired $c(\beta)T_0$ term exists for $\gamma < N + 2L - N_s$, the next-frame interference $c(\beta + 1)T_{+1}$ for $\gamma \geq N + L - N_s$, and the previous-frame interference $c(\beta - 1)T_{-1}$ for $\gamma < L - N_s$. Because at the receiver we will eventually remove the first L samples of $x_{\beta}^{(\alpha)}(\gamma)$, we need not concern ourselves with the previous frame interference.

We now add and subtract a term $c(\beta)T_0$ for $\gamma \geq N + L - N_s$ in order to properly complete the tail of the OFDM frame. That is, in terms of $s(n)$ and with reference to fig-

Figure 7.2: Frame synchronization error (advance).

Figure 7.2, we extend the transmitted data frame to the right with a cyclic postfix of length N_s . Then, the received OFDM frame $x_\beta^{(\alpha)}(\gamma)$ contains for $\gamma \in [0, N + L - 1]$ the convolution between the channel and the periodically time advanced (by N_s) transmitted samples. We write:

$$\begin{aligned}
 T_0(\gamma) &= \sum_{k=0}^{N_s-1} [s_{\text{ofdm_cp}}^{(\alpha)}(L+k) \cdot \delta(\gamma-k-N-L+N_s)] \star h(\alpha; \gamma) \\
 &= \sum_{k=0}^{N_s-1} s_{\text{ofdm_cp}}^{(\alpha)}(L+k) \cdot h(\alpha; \gamma-k-N-L+N_s) \\
 &= \sum_{\gamma_t = \gamma-N-L+1}^{\gamma} s_{\text{ofdm_cp}}^{(\alpha)}(-N+N_s+\gamma-\gamma_t) \cdot h(\alpha; \gamma_t)
 \end{aligned} \tag{7.10}$$

and obtain

$$x_\beta^{(\alpha)}(\gamma) = c(\beta)T_0 + c(\beta)T_0 - c(\beta)T_0 + c(\beta-1)T_{-1} + c(\beta+1)T_{+1} \tag{7.11}$$

Then, we remove the first L samples, that is $\gamma \rightarrow \gamma + L$ with a new range of $\gamma \in [0, N - 1]$, and get

$$\begin{aligned}
x_{\beta}^{(\alpha)}(\gamma) &= c(\beta) \cdot [s_{\text{ofdm}}^{(\alpha)}(\gamma) \odot h(\alpha; \gamma)] \Big|_{(\gamma + N_s)_N} + c(\beta + 1)e_{+1}(\gamma) - c(\beta)e_0(\gamma) \\
e_{+1}(\gamma) &= T_{+1}(\gamma + L) = \sum_{\gamma_{\tau} \leq \gamma - (N - N_s)} s_{\text{ofdm_cp}}^{(\alpha)}(-N + N_s + \gamma - \gamma_{\tau}) \cdot h(\alpha; \gamma_{\tau}) \\
e_0(\gamma) &= T_0(\gamma + L) = \sum_{\gamma_{\tau} \leq \gamma - (N - N_s)} s_{\text{ofdm_cp}}^{(\alpha)}(L - N + N_s + \gamma - \gamma_{\tau}) \cdot h(\alpha; \gamma_{\tau})
\end{aligned} \tag{7.12}$$

The term $e_{+1}(\gamma)$ is the next-frame interference and $e_0(\gamma)$ is the same-frame interference. Both are non-zero for $\gamma \in [N - N_s, N - 1]$ only. Then, we take the DFT and despread the result in order to arrive at the expression of the decision variable:

$$\begin{aligned}
D(\alpha; i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} X_{\beta}^{(\alpha)}(i) \cdot c^*(\beta) = d(\alpha; i) \cdot H(\alpha; i) \cdot R_c(0) \cdot e^{j2\pi \frac{iN_s}{N}} \\
&\quad - E_0(\alpha; i) \cdot R_c(0) + E_{+1}(\alpha; i) \cdot R_c^*(1) + E_{+1}(\alpha + 1; i) \cdot R_c(N_c - 1)
\end{aligned} \tag{7.13}$$

where $E_0(\cdot)$ and $E_{+1}(\cdot)$ are the DFTs of the corresponding interference terms from equation 7.12. Again, we note the presence of the partial auto-correlation function $R_c(\cdot)$ and that the calculations above are valid for $N_s \leq N$.

7.2.2 Structure of the interference terms

By inspecting the results of relation 7.8 and 7.13 we see that the effect of frame synchronization errors is a subcarrier index dependent phase rotation and the appearance of a number of interference terms. While the phase rotation can be automatically compensated for through channel estimation, the interference terms will cause spreading of the signal constellation.

In order to be able to quantify the performance degradation we first have to evaluate the three interference terms. We shall write out explicitly the $E_0(\cdot)$ and $E_{\pm 1}(\cdot)$ terms

and then split the interference into two parts: one that depends on the data being transmitted on the considered i^{th} subcarrier, $d(\alpha; i)$, and call it *self-interference*, and one that depends on the data of the other subcarriers as well as on the data from the previous/next block, and call it *other-interference*.

Given that $e_0(\gamma)$ and $e_{\pm 1}(\gamma)$ have different expressions depending on the sign of the timing error, the analysis has to be carried out separately for the two cases. During the calculations that follow it is useful to recall the relation

$$s_{\text{ofdm_cp}}^{(\alpha)}(\gamma) = \frac{1}{N} \sum_{n=0}^{N-1} d(\alpha; n) e^{j2\pi \frac{n(\gamma-L)}{N}} \quad \gamma \in [0, N+L-1] \quad (7.14)$$

Positive timing errors

Based on equation 7.7 we may write:

$$E_0(\alpha; i) = \text{DFT}\{e_0(\alpha; \gamma)\} \quad (7.15)$$

$$\begin{aligned} &= \sum_{\gamma=0}^{N_s-1} e^{-j2\pi \frac{i\gamma}{N}} \sum_{\gamma_\tau > \gamma + L - N_s} s_{\text{ofdm_cp}}^{(\alpha)}(N+L-N_s+\gamma-\gamma_\tau) \cdot h(\alpha; \gamma_\tau) \\ &= \sum_{\gamma=0}^{N_s-1} e^{-j2\pi \frac{i\gamma}{N}} \sum_{\gamma_\tau > \gamma + L - N_s} h(\alpha; \gamma_\tau) \cdot \frac{1}{N} \sum_{n=0}^{N-1} d(\alpha; n) e^{j2\pi \frac{n(N-N_s+\gamma-\gamma_\tau)}{N}} \end{aligned}$$

We then change the order of summation and perform the $\kappa = N_s - \gamma$ change of variable:

$$\begin{aligned} E_0(\alpha; i) &= e^{-j2\pi \frac{iN_s}{N}} \cdot \frac{1}{N} \sum_{n=0}^{N-1} d(\alpha; n) \sum_{\kappa=1}^{N_s} e^{j2\pi \frac{\kappa(i-n)}{N}} \sum_{\gamma_\tau > L-\kappa} h(\alpha; \gamma_\tau) e^{-j2\pi \frac{n\gamma_\tau}{N}} \\ &= e^{-j2\pi \frac{iN_s}{N}} \cdot \sum_{n=0}^{N-1} d(\alpha; n) \cdot \Lambda_\alpha(N_s; n) \end{aligned} \quad (7.16)$$

where we have defined

$$\Lambda_{\alpha}(N_s; n) = \frac{1}{N} \sum_{\kappa=1}^{N_s} e^{j2\pi \frac{\kappa(i-n)}{N}} \sum_{\gamma_{\tau} > L-\kappa} h(\alpha; \gamma_{\tau}) e^{-j2\pi \frac{n\gamma_{\tau}}{N}} \quad (7.17)$$

In exactly the same manner, we also obtain

$$E_{-1}(\alpha; i) = e^{-j2\pi \frac{iN_s}{N}} \cdot \sum_{n=0}^{N-1} d(\alpha; n) e^{j2\pi \frac{nL}{N}} \cdot \Lambda_{\alpha}(N_s; n) \quad (7.18)$$

Let us denote by T_{si} the interference terms in 7.8 that depend on $d(\alpha; i)$ (the self-interference), and by T_{oi} those which depend on any other data symbols (the other-interference). This separation yields:

$$T_{si} = d(\alpha; i) \cdot e^{-j2\pi \frac{iN_s}{N}} \left[R_c(1) e^{j2\pi \frac{iL}{N}} - R_c(0) \right] \Lambda_{\alpha}(N_s; i) \quad (7.19)$$

$$T_{oi} = e^{-j2\pi \frac{iN_s}{N}} \cdot \left\{ \sum_{\substack{n=0 \\ n \neq i}}^{N-1} \left[R_c(1) e^{j2\pi \frac{nL}{N}} - R_c(0) \right] \Lambda_{\alpha}(N_s; n) d(\alpha; n) \right. \\ \left. + R_c^*(N_c - 1) \sum_{n=0}^{N-1} e^{j2\pi \frac{nL}{N}} \Lambda_{\alpha-1}(N_s; n) d(\alpha - 1; n) \right\} \quad (7.20)$$

while the desired data term is

$$T_d = d(\alpha; i) \cdot H(\alpha; i) \cdot R_c(0) \cdot e^{-j2\pi \frac{iN_s}{N}} \quad (7.21)$$

Negative timing errors

Starting from equations 7.12 and 7.13 and following through with derivations as

for positive timing errors, we find:

$$E_0(\alpha; i) = \text{DFT}\{e_0(\alpha; \gamma)\} \quad (7.22)$$

$$\begin{aligned} &= \sum_{\gamma = N - N_s}^{N_s - 1} e^{-j2\pi \frac{i\gamma}{N}} \sum_{\gamma_\tau \leq \gamma - (N - N_s)} s_{\text{ofdm_cp}}^{(\alpha)}(L - N + N_s + \gamma - \gamma_\tau) \cdot h(\alpha; \gamma_\tau) \\ &= e^{j2\pi \frac{iN_s}{N}} \cdot \sum_{n=0}^{N-1} d(\alpha; n) \cdot \Lambda_\alpha(N_s; n) \end{aligned}$$

and

$$E_{+1}(\alpha; i) = e^{j2\pi \frac{iN_s}{N}} \cdot \sum_{n=0}^{N-1} d(\alpha; n) e^{-j2\pi \frac{nL}{N}} \cdot \Lambda_\alpha(N_s; n) \quad (7.23)$$

with

$$\Lambda_\alpha(N_s; n) = \frac{1}{N} \sum_{\kappa=1}^{N_s} e^{-j2\pi \frac{(\kappa-1)(i-n)}{N}} \sum_{\gamma_\tau < \kappa} h(\alpha; \gamma_\tau) e^{-j2\pi \frac{n\gamma_\tau}{N}} \quad (7.24)$$

after a $\kappa = \gamma - (N - N_s) + 1$ change of variable. Then, the data and interference terms are written as

$$T_d = d(\alpha; i) \cdot H(\alpha; i) \cdot R_c(0) \cdot e^{j2\pi \frac{iN_s}{N}} \quad (7.25)$$

$$T_{si} = d(\alpha; i) \cdot e^{j2\pi \frac{iN_s}{N}} \left[R_c^*(1) e^{-j2\pi \frac{iL}{N}} - R_c(0) \right] \Lambda_\alpha(N_s; i) \quad (7.26)$$

$$T_{oi} = e^{j2\pi \frac{iN_s}{N}} \cdot \left\{ \sum_{\substack{n=0 \\ n \neq i}}^{N-1} \left[R_c^*(1) e^{-j2\pi \frac{nL}{N}} - R_c(0) \right] \Lambda_\alpha(N_s; n) d(\alpha; n) \right. \\ \left. + R_c(N_c - 1) \sum_{n=0}^{N-1} e^{-j2\pi \frac{nL}{N}} \Lambda_{\alpha+1}(N_s; n) d(\alpha+1; n) \right\} \quad (7.27)$$

7.2.3 Performance degradation

Given that the self-interference is correlated with the desired data while the other-interference is not, we need to separately quantify the performance degradation due to these two interference terms.

7.2.3.1 Self-interference

Combining the self-interference T_{si} with the desired data term T_d yields

$$T_d + T_{si} = d(\alpha; i) R_c(0) e^{\mp j2\pi \frac{iN_s}{N}} \cdot \left\{ H(\alpha; i) + \left[\frac{R_c(1)}{R_c(0)} e^{\pm j2\pi \frac{iL}{N} \pm j\phi_c^{(1)}} - 1 \right] \Lambda_\alpha(N_s; i) \right\} \quad (7.28)$$

where the $\pm$ sign depends on the sign of the timing error and we have defined

$$\phi_c^{(1)} \equiv \arg\{R_c(1)\} \quad (7.29)$$

We can then see that the effect of the self-interference is equivalent to a channel estimation error which is given by

$$\Delta H(\alpha; i) = \left[\frac{R_c(1)}{R_c(0)} e^{\pm j 2\pi \frac{iL}{N} \pm j \phi_c^{(1)}} - 1 \right] \Lambda_\alpha(N_s; i) \quad (7.30)$$

The common phase rotation by $\mp 2\pi(iN_s)/N$ in equation 7.28 is not an impairment because it is automatically compensated by the channel estimate which is assumed to be

$$\hat{H}(\alpha; i) = H(\alpha; i) \cdot e^{\mp j 2\pi \frac{iN_s}{N}} \quad (7.31)$$

If pilot tone based channel estimation is performed then the estimate is

$$\hat{H}(\alpha; i) = [H(\alpha; i) + \Delta H(\alpha; i)] \cdot e^{\mp j 2\pi \frac{iN_s}{N}} \quad (7.32)$$

and there would be no self-interference during detection. In this case, the following discussion does not apply. We briefly note that the estimate of equation 7.31 is suboptimal with respect to the one in 7.32 because it does not exploit the extra SNR gain which can be derived from the self-interference term.

In the following we consider that the channel estimate is given by relation 7.31 and we characterize the resulting channel estimation error conditioned on the value of $H(\alpha; i)$ for the considered i^{th} subcarrier. We are interested in evaluating the penalty due to imperfect frame synchronization errors and therefore, we shall only consider small values¹ for the timing error, i.e. $N_s \leq L$. From equation 7.30 results that the variance of the channel estimation error is written, regardless of the sign of the timing error, as

$$\sigma_{\Delta H}^2 = \left| 1 - \frac{R_c(1)}{R_c(0)} \cdot e^{j 2\pi \frac{iL}{N}} \right|^2 \cdot \sigma_\Lambda^2 \quad (7.33)$$

¹. Otherwise, for $N_s > L$, we find that $\Lambda_\alpha(N_s; i)$ depends on $H(\alpha; i)$ which would then imply a non-zero bias for the channel estimation error. The foregoing discussions would then apply with respect to the total power of the channel estimation error.

In modeling the channel we consider uncorrelated scattering and zero mean Gaussian channel taps. We also normalize the power delay profile $P_h(\gamma_\tau)$ of the channel so that

$$P_H(i) = \mathcal{E} \{ |H(\alpha; i)|^2 \} = \sum_{\gamma_\tau=0}^L \mathcal{E} \{ |h(\alpha; \gamma_\tau)|^2 \} = \sum_{\gamma_\tau=0}^L P_h(\gamma_\tau) = P_h^{\text{tot}} = 1 \quad (7.34)$$

Positive timing errors

The expression for $\Lambda_\alpha(N_s; i)$ in equation 7.17 can be rewritten by counting the number of occurrences of $h(\alpha; \gamma_\tau)$ for a given value of γ_τ when the index κ is varied, resulting in

$$\Lambda_\alpha(N_s; i) = \frac{1}{N} \sum_{\kappa=1}^{N_s} \sum_{\gamma_\tau > L-\kappa} h(\alpha; \gamma_\tau) e^{-j2\pi \frac{\kappa \gamma_\tau}{N}} = \frac{1}{N} \sum_{\gamma_\tau \in \Psi(N_s)} h(\alpha; \gamma_\tau) e^{-j2\pi \frac{\gamma_\tau}{N} (N_s - L + \gamma_\tau)} \quad (7.35)$$

where we have defined the interval $\Psi(N_s) = [L+1-N_s, L]$. Then, the mean value of $\Lambda_\alpha(N_s; i)$ is zero² and its variance is

$$\sigma_\Lambda^2 = \mathcal{E} \left\{ |\Lambda_\alpha(N_s; i)|^2 \right\} = \frac{1}{N^2} \sum_{\gamma_\tau \in \Psi(N_s)} P_h(\gamma_\tau) \cdot (N_s - L + \gamma_\tau)^2 \quad (7.36)$$

We now search for the power delay profile $P_h(\gamma_\tau)$ that maximizes this variance, subject to the constraint of relation 7.34. In appendix E we find that this happens when $P_h(\gamma_\tau) = \delta(\gamma_\tau - L)$, that is a flat fading channel with delay of L , and

$$\max \left\{ \sigma_\Lambda^2 \right\} = \left(\frac{N_s}{N} \right)^2 \quad (7.37)$$

² If $N_s > L$ then we would have obtained $\mathcal{E} \{ \Lambda_\alpha(N_s; i) | H(\alpha; i) \} = H(\alpha; i) \cdot (N_s - L)/N$.

Negative timing errors

Starting from relation 7.24 and proceeding as for positive timing errors we get

$$\Lambda_{\alpha}(N_s; i) = \frac{1}{N} \sum_{\kappa=1}^{N_s} \sum_{\gamma_{\tau} < \kappa} h(\alpha; \gamma_{\tau}) e^{-j2\pi \frac{n\gamma_{\tau}}{N}} = \frac{1}{N} \sum_{\gamma_{\tau} \in \Theta(N_s)} h(\alpha; \gamma_{\tau}) e^{-j2\pi \frac{i\gamma_{\tau}}{N}} (N_s - \gamma_{\tau}) \quad (7.38)$$

where we defined the interval $\Theta(N_s) = [0, N_s - 1]$. We again find the mean value of $\Lambda_{\alpha}(N_s; i)$ to be zero while its variance is

$$\sigma_{\Lambda}^2 = \mathcal{E} \left\{ |\Lambda_{\alpha}(N_s; i)|^2 \right\} = \frac{1}{N^2} \sum_{\gamma_{\tau} \in \Theta(N_s)} P_h(\gamma_{\tau}) \cdot (N_s - \gamma_{\tau})^2 \quad (7.39)$$

Maximization of the variance (see appendix E) yields $P_h(\gamma_{\tau}) = \delta(\gamma_{\tau})$ and

$$\max \left\{ \sigma_{\Lambda}^2 \right\} = \left(\frac{N_s}{N} \right)^2 \quad (7.40)$$

which is identical to relation 7.37 obtained for positive timing errors.

Summary

In conclusion, the channel estimation error is a Gaussian random variable (due to the Gaussian channel taps) with zero mean and a maximum variance (power) of

$$\sigma_{\Delta H}^2 \leq \left| 1 - \frac{R_c(1)}{R_c(0)} \cdot e^{j2\pi \frac{iL}{N}} \right|^2 \cdot \left(\frac{N_s}{N} \right)^2 \quad (7.41)$$

We may upper bound this relation by considering the phase of $R_c(1)$ to be $\pi - 2\pi(iL)/N$ for a particular subcarrier index i . Then, the upper bound is

$$\sigma_{\Delta H}^2 \leq \left[1 + \left| \frac{R_c(1)}{R_c(0)} \right| \right]^2 \cdot \left(\frac{N_s}{N} \right)^2 \approx \left(\frac{N_s}{N} \right)^2 \quad (7.42)$$

where the approximation in the last step assumes a “good” spreading sequence, one for which $|R_c(1)| \ll R_c(0)$. Comparing this upper bound on the variance of the channel estimation error with the average power of the channel tap, which is $\mathcal{E} \{ |H(\alpha; i)|^2 \} = 1$, we conclude that for the above class of spreading sequences and for timing errors significantly smaller than the duration of the OFDM frame, $N_s \ll N$, the self-interference introduced degradation is negligible.

7.2.3.2 Other-interference

This interference term, as given by equations 7.20 and 7.27, is uncorrelated with the desired data and the self-interference since it comprises data symbols from all other subcarriers and from the previous/next MC-DS-CDMA block. We assume³ no coding across nor along the subcarriers. Also assuming that data on various subcarriers have the same statistical distribution and considering the relatively large number of subcarriers in a typical multicarrier system, it results that the other-interference is Gaussian with zero mean (because the data is zero mean) and a variance that we shall determine.

Positive timing errors

We split relation 7.20 into three terms based on the partial autocorrelation factors and then rewrite it as

$$T_{oi} = -R_c(0) \cdot A_0 + R_c(1) \cdot A_1 + R_c^*(N_c - 1) \cdot A_{N_c - 1} \quad (7.43)$$

³ The use of coding with interleaving would not affect our results as long as the interleaver removes the correlation between two successive blocks and among the subcarriers.

where we have defined

$$\begin{aligned}
 A_0 &= e^{-j2\pi \frac{iN_s}{N}} \cdot \sum_{\substack{n=0 \\ n \neq i}}^{N-1} \Lambda_\alpha(N_s; n) d(\alpha; n) \\
 A_1 &= e^{-j2\pi \frac{iN_s}{N}} \cdot \sum_{\substack{n=0 \\ n \neq i}}^{N-1} e^{j2\pi \frac{nL}{N}} \Lambda_\alpha(N_s; n) d(\alpha; n) \\
 A_{N_c-1} &= e^{-j2\pi \frac{iN_s}{N}} \cdot \sum_{n=0}^{N-1} e^{j2\pi \frac{nL}{N}} \Lambda_{\alpha-1}(N_s; n) d(\alpha-1; n)
 \end{aligned} \tag{7.44}$$

Then, the variance of the other-interference is computed as

$$\begin{aligned}
 \sigma_{oi}^2 &= |R_c(0)|^2 \cdot \mathcal{E} \left\{ |A_0|^2 \right\} + |R_c(1)|^2 \cdot \mathcal{E} \left\{ |A_1|^2 \right\} \\
 &\quad + |R_c(N_c-1)|^2 \cdot \mathcal{E} \left\{ |A_{N_c-1}|^2 \right\} - 2 \cdot \text{real} \left\{ R_c(0) \cdot R_c^*(1) \cdot \mathcal{E} \left\{ A_0 A_1^* \right\} \right\}
 \end{aligned} \tag{7.45}$$

Based on the assumption of uncorrelated data across the subcarriers we can write

$$\begin{aligned}
 \mathcal{E} \left\{ |A_0|^2 \right\} &= \mathcal{E} \left\{ |A_1|^2 \right\} = \sigma_d^2 \sum_{\substack{n=0 \\ n \neq i}}^{N-1} \mathcal{E} \left\{ |\Lambda_\alpha(N_s; n)|^2 \right\} \\
 \mathcal{E} \left\{ |A_{N_c-1}|^2 \right\} &= \sigma_d^2 \sum_{n=0}^{N-1} \mathcal{E} \left\{ |\Lambda_{\alpha-1}(N_s; n)|^2 \right\} \\
 \mathcal{E} \left\{ A_0 A_1^* \right\} &= \sigma_d^2 \sum_{\substack{n=0 \\ n \neq i}}^{N-1} e^{-j2\pi \frac{nL}{N}} \mathcal{E} \left\{ |\Lambda_\alpha(N_s; n)|^2 \right\}
 \end{aligned} \tag{7.46}$$

Assuming a slowly varying channel statistics we also have

$$\mathcal{E} \left\{ |\Lambda_\alpha(N_s; n)|^2 \right\} = \mathcal{E} \left\{ |\Lambda_{\alpha-1}(N_s; n)|^2 \right\} \quad (7.47)$$

and then

$$\mathcal{E} \left\{ |A_{N_c-1}|^2 \right\} = \mathcal{E} \left\{ |A_0|^2 \right\} + \sigma_d^2 \cdot \mathcal{E} \left\{ |\Lambda_\alpha(N_s; i)|^2 \right\} \quad (7.48)$$

We define the degradation coefficient $c(N_s) \equiv \mathcal{E} \left\{ |\Lambda_\alpha(N_s; i)|^2 \right\}$, and note that this was already computed in section 7.2.3.1 as σ_Λ^2 . We also define and compute using the uncorrelated scattering assumption the degradation coefficient $a(N_s)$:

$$\begin{aligned} a(N_s) &\equiv \sum_{n=0}^{N-1} \mathcal{E} \left\{ |\Lambda_\alpha(N_s; n)|^2 \right\} = \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{\kappa=1}^{N_s} \sum_{\kappa'=1}^{N_s} \sum_{\gamma_\tau \in \Psi(\kappa, \kappa')} P_h(\gamma_\tau) e^{j2\pi \frac{(\kappa - \kappa')(i-n)}{N}} \\ &= \frac{1}{N} \sum_{\kappa=1}^{N_s} \sum_{\gamma_\tau \in \Psi(\kappa)} P_h(\gamma_\tau) = \frac{1}{N} \sum_{\gamma_\tau \in \Psi(N_s)} P_h(\gamma_\tau) \cdot (N_s - L + \gamma_\tau) \end{aligned} \quad (7.49)$$

with $\Psi(\kappa, \kappa') = [L + 1 - \min(\kappa, \kappa'), L]$, and the coefficients $b_1(N_s)$ and $b_2(N_s)$:

$$\begin{aligned} \sum_{n=0}^{N-1} e^{-j2\pi \frac{nL}{N}} \mathcal{E} \left\{ |\Lambda_\alpha(N_s; n)|^2 \right\} &= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{\kappa=1}^{N_s} \sum_{\kappa'=1}^{N_s} \sum_{\gamma_\tau \in \Psi(\kappa, \kappa')} P_h(\gamma_\tau) e^{j2\pi \frac{(\kappa - \kappa')(i-n) - nL}{N}} \\ &= \frac{1}{N} \sum_{\kappa=1}^{N_s} \sum_{\kappa'=1}^{N_s} \sum_{\gamma_\tau \in \Psi(\kappa, \kappa')} P_h(\gamma_\tau) e^{-j2\pi \frac{iL}{N}} \cdot \left[\underbrace{\delta(\kappa - \kappa' + L)}_{\rightarrow b_1(N_s)} + \underbrace{\delta(\kappa - \kappa' + L + \lambda N)}_{\rightarrow b_2(N_s)} \right] \\ &\equiv [b_1(N_s) + b_2(N_s)] \cdot \exp \left(-j2\pi \frac{iL}{N} \right) \end{aligned} \quad (7.50)$$

The second Dirac delta is due to the periodicity by N of the complex exponential and λ is any non-zero integer. Given the range of κ and κ' , the expression $\kappa - \kappa' + L$ takes on val-

ues from $-(N_s - 1) + L$ to $(N_s - 1) + L$. Then, considering that our derivations in section 7.2.1 are valid for $N_s \leq N$, we find that only $\lambda = -1$ and hence $\delta(\kappa - \kappa' + L - N)$ is possible in the second delta function. From these considerations we also deduce that $b_1(N_s) = 0$ for $N_s \leq L$ and that $b_2(N_s) = 0$ for $N_s \leq N - L$. Since we are concerned with small errors only the condition $N_s \leq L$ stands and $b_1(N_s) = b_2(N_s) = 0$. Then, the variance of the other-interference is written

$$\begin{aligned} \sigma_{oi}^2 = & \sigma_d^2 \cdot |R_c(0)|^2 \cdot \left\{ \left[1 + \left| \frac{R_c(1)}{R_c(0)} \right|^2 \right] [a(N_s) - c(N_s)] + \left| \frac{R_c(N_c - 1)}{R_c(0)} \right|^2 a(N_s) \right. \\ & \left. + 2 \left| \frac{R_c(1)}{R_c(0)} \right| c(N_s) \cdot \cos \left(2\pi \frac{L}{N} + \phi_c^{(1)} \right) \right\} \end{aligned} \quad (7.51)$$

with $\phi_c^{(1)}$ defined in equation 7.29.

Negative timing errors

Following the same procedure as for positive timing errors, relation 7.27 is rewritten as

$$T_{oi} = -R_c(0) \cdot A_0 + R_c^*(1) \cdot A_1 + R_c(N_c - 1) \cdot A_{N_c - 1} \quad (7.52)$$

with A_0 , A_1 and $A_{N_c - 1}$ having the same definition except for the sign in the complex exponential and for the $(\alpha - 1) \rightarrow (\alpha + 1)$ substitution. We then obtain

$$\begin{aligned} a(N_s) & \equiv \sum_{n=0}^{N-1} \mathcal{E} \left\{ |\Lambda_\alpha(N_s; n)|^2 \right\} = \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{\kappa=1}^{N_s} \sum_{\kappa'=1}^{N_s} \sum_{\gamma_\tau \in \Theta(\kappa, \kappa')} P_h(\gamma_\tau) e^{-j2\pi \frac{(\kappa - \kappa')(i - n)}{N}} \\ & = \frac{1}{N} \sum_{\kappa=1}^{N_s} \sum_{\gamma_\tau \in \Theta(\kappa)} P_h(\gamma_\tau) = \frac{1}{N} \sum_{\gamma_\tau \in \Theta(N_s)} P_h(\gamma_\tau) \cdot (N_s - \gamma_\tau) \end{aligned} \quad (7.53)$$

where $\Theta(\kappa, \kappa') = [0, \min(\kappa, \kappa') - 1]$, and

$$\begin{aligned}
 \sum_{n=0}^{N-1} e^{j2\pi \frac{nL}{N}} \mathcal{E} \left\{ \left| \Lambda_{\alpha}(N_s; n) \right|^2 \right\} &= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{\kappa=1}^{N_s} \sum_{\kappa'=1}^{N_s} \sum_{\gamma_{\tau} \in \Theta(\kappa, \kappa')} P_h(\gamma_{\tau}) e^{-j2\pi \frac{(\kappa - \kappa')(i-n) - nL}{N}} \quad (7.54) \\
 &= \frac{1}{N} \sum_{\kappa=1}^{N_s} \sum_{\kappa'=1}^{N_s} \sum_{\gamma_{\tau} \in \Theta(\kappa, \kappa')} P_h(\gamma_{\tau}) e^{j2\pi \frac{iL}{N}} \cdot \left[\underbrace{\delta(\kappa - \kappa' + L)}_{\rightarrow b_1(N_s)} + \underbrace{\delta(\kappa - \kappa' + L + \lambda N)}_{\rightarrow b_2(N_s)} \right] \\
 &\equiv [b_1(N_s) + b_2(N_s)] \cdot \exp\left(j2\pi \frac{iL}{N}\right)
 \end{aligned}$$

Regarding the non-zero range for $b_1(N_s)$ and $b_2(N_s)$, the same result applies as for positive timing errors since we have the same delta functions. Hence, for the considered small errors $b_1(N_s) = b_2(N_s) = 0$ and the variance of the other-interference is also given by

relation 7.51, with $a(N_s)$ defined in equation 7.53 and $c(N_s) \equiv \mathcal{E} \left\{ \left| \Lambda_{\alpha}(N_s; i) \right|^2 \right\} = \sigma_{\Lambda}^2$ according to equation 7.39.

7.2.3.3 Total interference

As it was established in section 7.2.3.1, the self-interference produces a channel estimation error which is a zero mean Gaussian random variable for $N_s \leq L$ timing errors, while section 7.2.3.2 shows that the other-interference is also Gaussian. Then, we may calculate a total interference by incorporating both the self- and other-interference. Based on equations 7.28, 7.30 and 7.33 we write the variance of the self-interference as

$$\begin{aligned}
 \sigma_{si}^2 &= \sigma_d^2 \cdot |R_c(0)|^2 \cdot \sigma_{\Delta H}^2 \quad (7.55) \\
 &= \sigma_d^2 \cdot |R_c(0)|^2 \cdot c(N_s) \cdot \left[1 + \left| \frac{R_c(1)}{R_c(0)} \right|^2 - 2 \left| \frac{R_c(1)}{R_c(0)} \right| \cdot \cos\left(2\pi \frac{iL}{N} + \phi_c^{(1)}\right) \right]
 \end{aligned}$$

The total interference is also a zero mean Gaussian random variable with mean power

$$\sigma_{\text{tot}}^2 = \sigma_{\text{si}}^2 + \sigma_{\text{oi}}^2 = \sigma_d^2 \cdot |R_c(0)|^2 \cdot a(N_s) \cdot \left[1 + \left| \frac{R_c(1)}{R_c(0)} \right|^2 + \left| \frac{R_c(N_c - 1)}{R_c(0)} \right|^2 \right] \quad (7.56)$$

where the degradation coefficient $a(N_s)$ is computed as

$$a(N_s) = \begin{cases} \frac{1}{N} \sum_{\gamma_\tau \in \Psi(N_s)} P_h(\gamma_\tau) \cdot (N_s - L + \gamma_\tau) & \Delta t > 0 \\ \frac{1}{N} \sum_{\gamma_\tau \in \Theta(N_s)} P_h(\gamma_\tau) \cdot (N_s - \gamma_\tau) & \Delta t < 0 \end{cases} \quad (7.57)$$

and where $\Psi(N_s) = [L + 1 - N_s, L]$ and $\Theta(N_s) = [0, N_s - 1]$. The SNR degradation due to OFDM frame synchronization errors is then written as

$$\Gamma = \frac{\text{SNR}_0(i)}{\text{SNR}(i)} = \frac{P_u / \sigma_n^2}{P_u / (\sigma_n^2 + \sigma_{\text{tot}}^2)} = 1 + \text{SNR}_0 \cdot \frac{\sigma_{\text{tot}}^2}{\sigma_d^2 \cdot |R_c(0)|^2} \quad (7.58)$$

where we used the $P_u = \sigma_d^2 \cdot P_H(i) \cdot |R_c(0)|^2$ notation, and where, according to the normalization stated in relation 7.34, $P_H(i) = \mathcal{E} \{ |H(i)|^2 \} = 1$. Therefore, the SNR degradation works out to be

$$\Gamma = 1 + \text{SNR}_0 \cdot \left[1 + \left| \frac{R_c(1)}{R_c(0)} \right|^2 + \left| \frac{R_c(N_c - 1)}{R_c(0)} \right|^2 \right] \cdot a(N_s) \quad (7.59)$$

It is then easily seen that the degradation has two components: the OFDM related degradation coefficient $a(N_s)$, and the effect of the partial cross-correlations characterizing the spreading code family.

In order to obtain an upper bound on the degradation, we carry out a constrained maximization procedure with respect to the channel power delay profile in a manner simi-

lar to the one in appendix E and obtain

$$a(N_s) \leq \frac{N_s}{N} \leq 1 \quad (7.60)$$

Hence, the worst-case SNR degradation is

$$\Gamma_{\max} = 1 + \text{SNR}_0 \cdot \left[1 + \left| \frac{R_c(1)}{R_c(0)} \right|^2 + \left| \frac{R_c(N_c - 1)}{R_c(0)} \right|^2 \right] \cdot \frac{N_s}{N} \quad (7.61)$$

which is linear in N_s .

For unit modulus spreading sequences the SNR degradations further simplify to

$$\begin{aligned} \Gamma &= 1 + \text{SNR}_0 \cdot \left[1 + |R_c(1)|^2 + \frac{1}{N_c^2} \right] \cdot a(N_s) \\ \Gamma_{\max} &= 1 + \text{SNR}_0 \cdot \left[1 + |R_c(1)|^2 + \frac{1}{N_c^2} \right] \cdot \frac{N_s}{N} \end{aligned} \quad (7.62)$$

We shall plot the worst-case degradation for cyclic-shift m-sequences and for Walsh codes. In appendix D the values the partial autocorrelation $R_c(1)$ can take on are determined. In light of those results we find for cyclic-shift m-sequences that

$$1 + \text{SNR}_0 \cdot \left(1 + \frac{1}{N_c^2} \right) \cdot \frac{N_s}{N} < \Gamma_{\max} < 1 + \text{SNR}_0 \cdot \left(1 + \frac{5}{N_c^2} \right) \cdot \frac{N_s}{N} \quad (7.63)$$

and we approximate the SNR degradation as

$$\Gamma_{\max} \approx 1 + \text{SNR}_0 \cdot \frac{N_s}{N} \quad (7.64)$$

regardless of the actual spreading code within the family. For Walsh codes we obtain

$$1 + \text{SNR}_0 \cdot \left(1 + \frac{2}{N_c^2}\right) \cdot \frac{N_s}{N} < \Gamma_{\max} < 1 + 2 \cdot \text{SNR}_0 \cdot \left(1 - \frac{N_c - 1}{N_c^2}\right) \cdot \frac{N_s}{N} \quad (7.65)$$

where the upper and lower bounds can be further simplified so as to write

$$1 + \text{SNR}_0 \cdot \frac{N_s}{N} < \Gamma_{\max} < 1 + 2 \cdot \text{SNR}_0 \cdot \frac{N_s}{N} \quad (7.66)$$

For both spreading code families the approximations are tight if the $N_c \geq 15$ condition on the spreading sequence length is satisfied. Considering the result of relation 7.66, we see that when SNR_0 is large there is a 3 dB spread in the worst-case degradation due to the spread in the $|R_c(1)|$ partial autocorrelation values of Walsh codes. We also note that the lower bound coincides with the approximate value obtained for cyclic-shift m-sequences.

In figure 7.3 we plot the worst-case SNR degradation given in equation 7.62, which is obtained for the $P_h(\gamma_\tau) = \delta(\gamma_\tau)$ and $P_h(\gamma_\tau) = \delta(\gamma_\tau - L)$ channel power delay profiles. On the graph we have normalized the frame sync accuracy N_s to the number of subcarriers N . The plot for Walsh codes represents the upper bound derived in relation 7.66 while the graph for cyclic-shift m-sequences can be used, according to our observation above, as the lower bound for the Walsh code case. We also recall that our result is for small errors, where the frame synchronization error is less than the duration of the cyclic prefix, that is $N_s \leq L$. This means that the plots are to be used only for normalized frame sync errors of less than L/N which represents the OFDM overhead due to the cyclic prefix.

The above worst-case degradation is seen to occur for the rather unlikely⁴ flat fading channel. As such, it is instructive to also plot the SNR degradation Γ in relation 7.62

⁴ We say that the flat fading channel is unlikely because OFDM type transmissions are typically used over multipath channels.

(a) cyclic-shift m-sequences

(b) Walsh codes

Figure 7.3: Maximum SNR degradation of MC-DS-CDMA with frame synchronization errors (single user case).

for other channel power delay profiles, while keeping in mind our previous considerations regarding the worst and best-case partial cross-correlations. Namely, we shall consider the equal rays (uniform) channel profile $P_h(\gamma_\tau) = 1/(L+1)$, and the exponential profile $P_h(\gamma_\tau) = k \cdot \exp[-\lambda\gamma_\tau]$, with λ the rate of decay and k a normalization coefficient. The results are plotted in figure 7.4 for an SNR_0 of 20 dB using two different values for the L/N OFDM overhead, 12.5% and 25%. As before, the graph for cyclic-shift m -sequences can be used as the lower bound for Walsh codes. Considering that both the flat fading and the uniform profiles are particular extreme cases of the exponential profile, the obtained result is consistent with the dependence, given in equation 7.57, of the $a(N_s)$ degradation coefficient on the channel power delay profile.

Conclusions we draw based on the above result are presented at the end of the chapter, in section 7.4.

7.3 Multiuser case

We now turn our attention to the effect multiuser interference has on the decision variable. In particular, we consider a form of quasi-synchronous MC-DS-CDMA where interfering users have their code phase aligned within one chip period. Therefore, if the desired user is the one with index k , an interfering user with index l is offset in time by an amount

$$\Delta t_{k,l} = t_l - t_k = \pm N_{k,l} \cdot T_s \quad (7.67)$$

and we shall limit the timing error to $|\Delta t_{k,l}| \leq N_{k,l} T_s < T_c$. The frame sync of the desired user is assumed perfectly acquired.

Figure 7.4: SNR degradation limits of MC-DS-CDMA with frame synchronization errors for various channel power delay profiles (single user case).

7.3.1 Power of multiuser interference

Although we do not have to, it is simpler to start with the results of section 7.2.1 as applied to the interfering user's signal. In particular, we use equations 7.7 and 7.12 written for the l^{th} user. Therefore, after despreading with the k^{th} user's code, we obtain

$$\begin{aligned} \text{MUI}_l(i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} X_{\beta,l}^{(\alpha)}(i) \cdot c_k^*(\beta) = d_l(\alpha;i) \cdot H_l(\alpha;i) \cdot R_{c_{k,l}}(0) \cdot e^{-j2\pi \frac{iN_{k,l}}{N}} \\ &\quad - E_{0,l}(\alpha;i) \cdot R_{c_{k,l}}(0) + E_{-1,l}(\alpha;i) \cdot R_{c_{k,l}}(1) + E_{-1,l}(\alpha-1;i) \cdot R_{c_{k,l}}(-N_c+1) \end{aligned} \quad (7.68)$$

when $\Delta t_{k,l} > 0$ and

$$\begin{aligned} \text{MUI}_l(i) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-1} X_{\beta,l}^{(\alpha)}(i) \cdot c_k^*(\beta) = d_l(\alpha;i) \cdot H_l(\alpha;i) \cdot R_{c_{k,l}}(0) \cdot e^{j2\pi \frac{iN_{k,l}}{N}} \\ &\quad - E_{0,l}(\alpha;i) \cdot R_{c_{k,l}}(0) + E_{+1,l}(\alpha;i) \cdot R_{c_{k,l}}^*(1) + E_{+1,l}(\alpha+1;i) \cdot R_{c_{k,l}}^*(-N_c+1) \end{aligned} \quad (7.69)$$

for $\Delta t_{k,l} < 0$. Bearing in mind that $R_{c_{k,l}}(0) = R_{c_{l,k}}^*(0)$, in the following derivations we shall adopt the simplified notation introduced in chapter 6

$$R(n) = \begin{cases} R_{c_{k,l}}(n) & \Delta t_{k,l} > 0 \\ R_{c_{l,k}}(n) & \Delta t_{k,l} < 0 \end{cases} \quad (7.70)$$

which will allow us to write the results for both positive and negative timing errors in a common form.

Using a decomposition of $\text{MUI}_l(i)$ into data and self- and other-interference as we did in section 7.2.2, we may write

$$MUI_l(i) = T_d + T_{si} + T_{oi} \quad (7.71)$$

where T_{oi} is uncorrelated with the other two terms. Then, in order to compute the power

$$P_{MUI}^{(l)} = \mathcal{E} \left\{ |MUI_l(i)|^2 \right\} \text{ we need the variance of } T_d, T_{si} \text{ and } T_{oi}, \text{ as well as the}$$

$$\mathcal{E} \left\{ T_d T_{si}^* \right\} \text{ cross-correlation.}$$

With the simplified notational convention in place, the variance of the data component is obviously given by

$$\mathcal{E} \left\{ |T_d|^2 \right\} = \sigma_d^2 \cdot |R(0)|^2 \quad (7.72)$$

while the variance of the self-interference component is found from the multiuser version of equation 7.55 as

$$\begin{aligned} \sigma_{si}^2 &= \mathcal{E} \left\{ |T_{si}|^2 \right\} \\ &= \sigma_d^2 \cdot |R(0)|^2 \cdot c(N_{k,l}) \cdot \left\{ 1 + \left| \frac{R(1)}{R(0)} \right|^2 - 2 \left| \frac{R(1)}{R(0)} \right| \cdot \cos \left(2\pi \frac{lL}{N} + \phi^{(1)} \right) \right\} \end{aligned} \quad (7.73)$$

where

$$\phi^{(1)} = \arg\{R(1)R^*(0)\} = \begin{cases} \phi_{k,l}^{(1)} \equiv \arg\{R_{c_{k,l}}(1)\} - \arg\{R_{c_{k,l}}(0)\} & \Delta t_{k,l} > 0 \\ \phi_{l,k}^{(1)} \equiv \arg\{R_{c_{l,k}}(1)\} - \arg\{R_{c_{l,k}}(0)\} & \Delta t_{k,l} < 0 \end{cases} \quad (7.74)$$

Equations 7.19 and 7.26 rewritten for the multiuser case are used in finding the cross-correlation

$$\mathcal{E} \left\{ T_d T_{si}^* \right\} = \sigma_d^2 \cdot |R(0)|^2 \cdot \left[\frac{|R(1)|}{|R(0)|} e^{\mp j 2\pi \frac{1L}{N} \mp j \phi^{(1)}} - 1 \right] \cdot \mathcal{E} \left\{ H_l(\alpha; i) \cdot \Lambda_\alpha^*(N_{k,l}; i) \right\} \quad (7.75)$$

Employing equations 7.35 and 7.38 we derive that

$$\mathcal{E} \left\{ H_l(\alpha; i) \cdot \Lambda_\alpha^*(N_{k,l}; i) \right\} = a(N_{k,l}) \quad (7.76)$$

and therefore

$$\mathcal{E} \left\{ T_d T_{si}^* \right\} = \sigma_d^2 \cdot |R(0)|^2 \cdot \left[\frac{|R(1)|}{|R(0)|} e^{\mp j 2\pi \frac{1L}{N} \mp j \phi^{(1)}} - 1 \right] \cdot a(N_{k,l}) \quad (7.77)$$

Finding the variance of T_{oi} requires that we reconsider the derivation from section 7.2.3.2 for the case when the timing error is not constrained to $N_{k,l} \leq L$. In particular, we need to find from relations 7.50 and 7.54 a general expression for $b(N_{k,l}) = b_1(N_{k,l}) + b_2(N_{k,l})$. Extracting from these equations the expression for $b_1(N_{k,l})$ and $b_2(N_{k,l})$ results in

$$b_1(N_{k,l}) = \begin{cases} \frac{1}{N} \sum_{\kappa=1}^{N_{k,l}-L} \sum_{\gamma_\tau \in \Psi(\kappa)} P_{h_l}(\gamma_\tau) & \Delta t_{k,l} > 0 \\ \frac{1}{N} \sum_{\kappa=1}^{N_{k,l}-L} \sum_{\gamma_\tau \in \Theta(\kappa)} P_{h_l}(\gamma_\tau) & \Delta t_{k,l} < 0 \end{cases} \quad (7.78)$$

$$b_2(N_{k,l}) = \begin{cases} \frac{1}{N} \sum_{\kappa'=1}^{N_{k,l}-(N-L)} \sum_{\gamma_\tau \in \Psi(\kappa')} P_{h_l}(\gamma_\tau) & \Delta t_{k,l} > 0 \\ \frac{1}{N} \sum_{\kappa'=1}^{N_{k,l}-(N-L)} \sum_{\gamma_\tau \in \Theta(\kappa')} P_{h_l}(\gamma_\tau) & \Delta t_{k,l} < 0 \end{cases} \quad (7.79)$$

In the discussion following relation 7.50 we have established the non-zero range of $b_1(\cdot)$

and $b_2(\cdot)$. By comparing the results of equations 7.78 and 7.79 with the definitions we had for $a(N_s)$ in equations 7.49 and 7.53 we find:

$$b(N_{k,l}) = \begin{cases} 0 & , N_{k,l} \leq L \\ a(N_{k,l} - L) & , N_{k,l} \in (L, N - L] \\ a(N_{k,l} - L) + a(N_{k,l} - (N - L)) & , N_{k,l} \in (N - L, N] \end{cases} \quad (7.80)$$

where the $N - L \geq L$ inequality (true for all practical multicarrier systems) was assumed. Then, making use of the results of section 7.2.3.2, the variance of the other-interference is

$$\begin{aligned} \sigma_{oi}^2 = \mathcal{E} \left\{ |T_{oi}|^2 \right\} &= \sigma_d^2 \cdot |R(0)|^2 \cdot \left\{ \left[1 + \left| \frac{R(1)}{R(0)} \right|^2 \right] [a(N_{k,l}) - c(N_{k,l})] \right. \\ &\quad \left. + \left| \frac{R(-N_c + 1)}{R(0)} \right|^2 a(N_{k,l}) - 2 \left| \frac{R(1)}{R(0)} \right| [b(N_{k,l}) - c(N_{k,l})] \cdot \cos \left(2\pi \frac{iL}{N} + \phi^{(1)} \right) \right\} \end{aligned} \quad (7.81)$$

Combining the results from relations 7.73, 7.72, 7.77 and 7.81, $P_{MUI}^{(l)}$ is found to be

$$P_{MUI}^{(l)} = \mathcal{E} \left\{ |T_d|^2 \right\} + \sigma_{si}^2 + \sigma_{oi}^2 + 2 \cdot \text{real} \left\{ T_d T_{si}^* \right\} = \sigma_d^2 \cdot \vartheta(N_{k,l}) \quad (7.82)$$

where we have defined

$$\begin{aligned} \vartheta(N_{k,l}) &= |R(0)|^2 + |R(0)|^2 \cdot \left\{ \left[\left| \frac{R(1)}{R(0)} \right|^2 + \left| \frac{R(-N_c + 1)}{R(0)} \right|^2 - 1 \right] \cdot a(N_{k,l}) \right. \\ &\quad \left. + 2 \cdot \left| \frac{R(1)}{R(0)} \right| \cdot [a(N_{k,l}) - b(N_{k,l})] \cdot \cos \left(2\pi \frac{iL}{N} + \phi^{(1)} \right) \right\} \end{aligned} \quad (7.83)$$

Therefore, the multiuser interference is seen to depend on the index i of the subcarrier under consideration. Finally, also averaging over the relative timing errors the total multiuser interference power is given by

$$P_{\text{MUI}} = \sigma_d^2 \cdot \sum_{l \neq k} \mathcal{E} \{ \vartheta(N_{k,l}) \} \quad (7.84)$$

with the statistical expectation over the relative timing errors. We should keep in mind that $R(n)$ and $\phi^{(1)}$ in equation 7.83 also depend on the user indices k and l via relations 7.70 and 7.74.

7.3.2 Performance degradation

Using a Gaussian approximation for the multiuser interference, we can calculate the SNR degradation due to relative frame synchronization errors. The power of the desired user is given by $P_u = \sigma_d^2 \cdot |R_{c_k}(0)|^2$ while the power of the multiuser interference without timing error is

$$P_{\text{MUI}}^{(0)} = \sigma_d^2 \cdot \sum_{l \neq k} |R_{c_{k,l}}(0)|^2 \quad (7.85)$$

Also considering the additive white Gaussian noise, the sought degradation is computed as

$$\Gamma_k(i) = \frac{\text{SNR}_0}{\text{SNR}(i)} \Big|_k = \frac{P_u / (\sigma_n^2 + P_{\text{MUI}}^{(0)})}{P_u / (\sigma_n^2 + P_{\text{MUI}})} \quad (7.86)$$

and found to be

$$\Gamma_k(i) = 1 + \text{SNR}_0 \Big|_k \cdot \frac{\sum_{l \neq k} \mathcal{E} \{ \vartheta(N_{k,l}) \} - \sum_{l \neq k} |R_{c_{k,l}}(0)|^2}{|R_{c_k}(0)|^2} \quad (7.87)$$

For unit modulus spreading sequences $R_{c_k}(0) = 1$ and $|R(-N_c + 1)| = \frac{1}{N_c}$, and the SNR degradation is expressed as

$$\begin{aligned} \Gamma_k(i) = 1 + \text{SNR}_{0|k} \cdot \mathcal{E} \left\{ \sum_{l \neq k} \left\{ \left[|R(1)|^2 + \frac{1}{N_c^2} - |R(0)|^2 \right] \cdot a(N_{k,l}) \right. \right. \\ \left. \left. + 2 \cdot |R(1)| \cdot |R(0)| \cdot [a(N_{k,l}) - b(N_{k,l})] \cdot \cos\left(2\pi \frac{iL}{N} + \phi^{(1)}\right) \right\} \right\} \end{aligned} \quad (7.88)$$

where the statistical expectation is over the relative timing errors. We assume that in the quasi-synchronous MC-DS-CDMA system under consideration users are synchronized such that

$$\Delta t_{k,l} \in [-N_0 T_s, N_0 T_s] \quad (7.89)$$

and $\Delta t_{k,l}$ is a uniformly distributed random variable.

Each element in the summation of relation 7.88 can be attributed to the interference contribution from a particular user, and this is seen to have an oscillatory behavior with respect to the subcarrier index i . The property carries over to the SNR degradation $\Gamma_k(i)$ and therefore, there are subcarriers on which the multiuser interference is at a minimum and subcarriers where it is at a maximum. In general however, the location of these subcarriers varies with the set of active users, the spreading code family and the channel power delay profiles of the individual users and as a result, we shall only consider the most affected subcarrier. For this purpose, we rewrite equation 7.88 as

$$\begin{aligned} \Gamma_k(i) &= 1 + \text{SNR}_{0|k} \cdot \mathcal{E} \left\{ \sum_{l \neq k} \left[\text{dc}(N_{k,l}) + \text{ampl}(N_{k,l}) \cdot \cos\left(2\pi \frac{iL}{N} + \phi^{(1)}\right) \right] \right\} \\ &= 1 + \text{SNR}_{0|k} \cdot \mathcal{E} \left\{ \text{dc} + \text{ampl} \cdot \cos\left(2\pi \frac{iL}{N} + \chi\right) \right\} \end{aligned} \quad (7.90)$$

where the $\text{dc}(N_{k,l})$ and $\text{ampl}(N_{k,l})$ terms are easily identifiable from relation 7.88, and where

$$\begin{aligned}
dc &= \sum_{l \neq k} dc(N_{k,l}) \\
\text{ampl} &= \left| \sum_{l \neq k} \text{ampl}(N_{k,l}) \cdot \exp(j\phi^{(1)}) \right| \\
\chi &= \arg \left\{ \sum_{l \neq k} \text{ampl}(N_{k,l}) \cdot \exp(j\phi^{(1)}) \right\}
\end{aligned} \tag{7.91}$$

Therefore, we find that the SNR degradation is

$$\Gamma_k \leq 1 + \text{SNR}_0|_k \cdot \mathcal{E} \{dc + \text{ampl}\} \tag{7.92}$$

with equality for the most affected subcarrier.

In order to obtain numerical results, we now generate SNR degradation curves for the quasi-synchronous MC-DS-CDMA system. We consider the cases when cyclic-shift m-sequences and when Walsh codes are used as spreading codes.

First, we consider a worst-case scenario when individual users are offset in time within the range given by equation 7.89 such that Γ_k is maximal. We compute this maximum degradation for each desired user in part by conducting a full search and then maximizing over the choice of desired user.

This maximum SNR degradation is plotted versus the timing accuracy N_0 normalized to the number of subcarriers with SNR_0 a parameter. For the fully loaded system where $N_u = N_c$ the result is illustrated in figures 7.5-a and 7.5-b, and for the partially loaded system in figures 7.6-a and 7.6-b. Maximization is also carried out with respect to the channel power delay profiles⁵ of the individual users. The results are found to be independent of the number of subcarriers N as long as the ratio L/N (the OFDM overhead) is maintained constant.

We can estimate the probability of this worst-case maximum degradation assuming inde-

pendent users, each with an uniformly distributed relative frame sync error $\Delta t_{k,1}$ within the range given in equation 7.89 and find

$$P_{\text{worst-case}} \approx \left(\frac{1}{2N_0 + 1} \right)^{N_u - 1} \quad (7.93)$$

Considering a very loose control of the frame sync, that is $N_0 = N = 64$, and $N_u = 5$ users we find $P_{\text{worst-case}} \approx 4 \times 10^{-9}$, that is the worst-case scenario is a very rare event. Assuming a very strict control of the frame synchronization error in the order of 1 sample, that is $N_0 = 1$, we find $P_{\text{worst-case}} \approx 0.01$ when $N_u = 5$. However, these estimates do not account for the probability of each user independently encountering the worst-case channel power delay profile.

Next, we allow $\Delta t_{k,1}$ to be random and we perform the averaging in equation 7.88 over the uniform distribution the range of which is given in relation 7.89. This would describe a more likely scenario and hence, give a more useful degradation measure. Averaging over the choice of the desired user is also performed. The results are plotted in figures 7.5-c and 7.5-d for the fully loaded system, and in figures 7.6-c and 7.6-d for the partially loaded system. These average degradations were found for the same channel profiles used in plotting the maximum degradations⁶ above.

5. This was possible because of the particular partial cross-correlation properties of the spreading code families concerned. Namely, Walsh codes satisfy the $R(0) = 0$ property while for cyclic-shift m-sequences the phases $\phi^{(1)}$ of all non-zero $R(1) \cdot R^*(0)$ products are null. As a result, we can separate the effect of different users:

$$dc + \text{ampl} = \sum_{l \neq k} [dc(N_{k,l}) + \text{ampl}(N_{k,l})]$$

and maximize with respect to the channel on a per-user basis. However, in general such a separation is not possible due to the $\phi^{(1)}$ phases. In such cases the upper bounding

$$\text{ampl} = \left| \sum_{l \neq k} \text{ampl}(N_{k,l}) \cdot \exp(j\phi^{(1)}) \right| \leq \sum_{l \neq k} |\text{ampl}(N_{k,l})|$$

could be used in order to force the separation.

In figure 7.7 we compare the average SNR degradation for an SNR_0 of 20 dB for the power delay profiles we used in the single user case. Namely, we consider the flat fading, the exponential and the uniform channels. The graphs are obtained under the assumption that all users are characterized by the same channel power delay profile and we consider two different values for the L/N OFDM overhead, 12.5% and 25%.

Conclusions are presented at the end of the chapter, in section 7.5.

7.4 Verification and justification of results

In order to support the analytical relations derived in this chapter, we provide justification for and verification of the consistency of the results. We resort to computer simulation of the single user case. Considering a MC-DS-CDMA system with 4-QAM modulation over $N = 128$ subcarriers, cyclic prefix of $L = 6$, a channel power delay profile $P_h(\gamma_\tau) = \delta(\gamma_\tau)$, m-sequence based spreading where $|R_c(1)| = 2/15$ and a frame synchronization error of $\Delta t = -2T_s$ we obtain the received signal constellation shown in figure 7.8. The constellation shows that characterizing the interference by its power (spherical distribution) is a good approximation (in this case, we used a Gaussian approximation). The power of the interference is monitored on each subcarrier and is also plotted in the figure, where the interference power is expressed in σ_d^2/N units (normalization). Spectral decomposition of the interference power as a function of the subcarrier index shows that for all practical purposes it can be considered a constant. The analytically predicted values and the experimentally obtained ones are compared in table 7.1 and we note the very good agreement of theory with simulation.

⁶. For the particular case of Walsh codes and cyclic-shift m-sequences these channels are the $P_h(\gamma_\tau) = \delta(\gamma_\tau)$ and $P_h(\gamma_\tau) = \delta(\gamma_\tau - L)$ flat fading channels.

(a) maximum degradation, Walsh codes

(b) maximum degradation, cyclic-shift m-sequences

(c) average degradation, Walsh codes
(same channels as for maximum degradation)(d) average degradation, cyclic-shift m-sequences
(same channels as for maximum degradation)

Figure 7.5: Performance degradation of fully loaded MC-DS-CDMA due to frame synchronization errors.

Figure 7.6: Performance degradation of partially loaded MC-DS-CDMA due to frame synchronization errors.

Figure 7.7: Average SNR degradation of MC-DS-CDMA with frame synchronization errors for various channel power delay profiles.

Figure 7.8: Received signal constellation and interference due to small frame synchronization errors.

Table 7.1: Analytical and simulation results, $N_s \leq L$.

Channel power delay profile and frame synchronization error	Normalized power of interference: $N \cdot \sigma_{\text{tot}}^2 / \sigma_d^2$	
	theoretical	simulation
$P_h(\gamma_\tau) = \delta(\gamma_\tau), \Delta t = -2T_s$	2.04	2.00
$P_h(\gamma_\tau) = \delta(\gamma_\tau), \Delta t = -4T_s$	4.088	4.13
$P_h(\gamma_\tau) = \delta(\gamma_\tau - 2), \Delta t = -2T_s$	0	5×10^{-29}

In the multiuser case the analytical relations predicted an oscillatory behavior of the interference power with the subcarrier index. The same phenomenon is obtained in the single user case if the $N_s \leq L$ restriction is removed. Therefore, we carried out a computer simulation of the single user case for $N_s > L$. In order to extrapolate the result in equation 7.56 to the $N_s > L$ case, we note the appearance of the following additional term:

$$\text{additional term} = 2 \cdot |R_c(1)| \cdot \sigma_d^2 \cdot b(N_s) \cdot \cos\left(2\pi \frac{iL}{N} + \phi_c^{(1)}\right) \quad (7.94)$$

This term arises from the evaluation of

$$\mathcal{E} \{A_0 A_1^*\}$$

the fact that $b(N_s)$ is no longer zero.

Therefore, it can be concluded that in the $N_s > L$ case we may use the result of equation 7.56 as a dc term (mean value with respect to the subcarrier index) and the new component in equation 7.94 as an oscillatory term. Simulation results presented in figure 7.9 confirm the oscillatory behavior. Spectral decomposition of

Figure 7.9: Power of interference due to large frame synchronization errors (extrapolated single user case).

the interference power as a function of the subcarrier index allows us to determine the dc component and the amplitude and frequency of the oscillatory component of the interference power. The analytical predictions and the experimentally determined values are given in table 7.2 for a representative case. We note the good agreement between theory and simulation. From the graph the normalized frequency of the oscillatory component is found to be equal to the length of the cyclic prefix, $L = 6$, as predicted by the additional term in equation 7.94.

Finally, we also mention that it is possible to obtain frame synchronization results for the case of pure OFDM if we set $N_c = 1$ and hence, $R_c(1) = 0$ in the result of equation 7.62. It is then apparent that SNR degradation curves for pure OFDM will closely follow those obtained for MC-DS-CDMA with worst auto-correlation Walsh codes. In both cases we get $\Gamma \approx 1 + 2 \cdot \text{SNR}_0 \cdot a(N_s)$.

Table 7.2: Analytical and simulation results, $N_s > L$.

Channel power delay profile and frame synchronization error	Normalized power of interference: $N \cdot \sigma_{\text{tot}}^2 / \sigma_d^2$	
	theoretical	simulation
$P_h(\gamma_\tau) = \delta(\gamma_\tau), \Delta t = -8T_s$	dc=8.17 ampl=0.53	dc=7.9 ampl=0.57

7.5 Conclusions

In this chapter we have discussed the effects of frame synchronization errors on an MC-DS-CDMA uplink. The cases of single and multiple users have been analyzed. The results from the single user case are relevant to the accuracy required in establishing the OFDM frame timing reference, while the results from the multiuser case bear on the network synchronization, that is time alignment of active users.

7.5.1 Single user case

Regarding the single user results of section 7.2.3, we draw the following conclusions:

- The effect of frame synchronization errors is a channel estimation error-like self-interference and a Gaussian other interference. For small errors (less than the length of the cyclic prefix) these two terms can be combined together and used to quantify the effect through an SNR degradation based measure. This SNR degradation has two components: an OFDM related degradation coefficient $a(N_s)$, and the effect of the partial auto-correlation of the spreading code.

- Walsh codes give an approximately 3 dB additional performance degradation over cyclic-shift m-sequences due to the large range of values the partial auto-correlation $|R_c(1)|$ may take on.
- The results for the studied spreading code families are practically independent of the length of the spreading sequence as long as $N_c \geq 15$.
- The degradation, as plotted versus the normalized frame sync accuracy, is found not to depend on the number of subcarriers. However, the spread between the degradations obtained for the different channel types is sensitive to the OFDM overhead.
- Due to the steepness at the origin of the SNR degradation graphs for the worst-case channel and worst-case auto-correlation, we conclude that MC-DS-CDMA system can be very sensitive to frame synchronization errors. This sensitivity is due to the OFDM component and, at large number of subcarriers, an error of only a few samples might be tolerable. For example, for the flat fading channel and cyclic-shift m-sequences a normalized error of 0.01 results in about 3 dB degradation at an SNR_0 of 20 dB and $16/128=12.5\%$ cyclic prefix overhead.

Considering $N = 128$ subcarriers, we obtain $N_s^{\max} = 1$, that is a tolerable error of one sample. However, for the uniform channel we can handle $N_s^{\max} = 6$ for the same 3 dB degradation. The results can be justified if we consider that in the worst-case, for a timing error of N_s there are that many “foreign samples” entering the detection window and hence, the ratio of interference power to signal power should be proportional to the ratio of samples used to compute these powers, that is N_s and N respectively.

7.5.2 Multiuser case

Regarding the multi-user case, the following conclusions are drawn based on the results of section 7.3.2:

- Using a Gaussian approximation to the multiuser interference, we employ an SNR based degradation measure, and find that it exhibits an oscillatory behavior with respect to the subcarrier index. However, the location of the peaks and troughs depends in a complicated way on the number of active users, on the employed spreading sequences, on the individual channel power delay profiles and on the relative frame synchronization errors.
- In general, separating the effect of individual users on the SNR degradation is not possible and upper-bounding is necessary. However, due to particular properties of the studied spreading code families, separation could be achieved.
- Considering the partially loaded system, cyclic-shift m-sequences are more robust than Walsh codes due to their much better partial cross-correlation properties. For example, using a worst-case maximum degradation of 3 dB at SNR_0 of 20 dB with spreading sequences of length 31 and cyclic prefix overhead of $16/128=12.5\%$, we obtain tolerable normalized frame synchronization errors of 0.2 and 0.15 respectively. For a 128-carrier system, these translate into relative frame synchronization accuracies of 25 and 19 samples respectively. However, on a 3 dB average rather than maximum degradation basis we obtain acceptable performance for up to a normalized frame synchronization accuracy of 1 for cyclic-shift m-sequences and of 0.4 for Walsh codes. Using 128 subcarriers, this translates into 128 and 51 samples respectively.
- We further note that while the fully loaded case does not exhibit sensitivity to the spreading sequence length, the partially loaded system does. Namely, we see that

shorter Walsh codes and longer cyclic-shift m-sequences perform better. This difference in behavior is consistent with the dependence on the spreading sequence length of the $|R_{c_{k,l}}(\pm 1)|$ partial cross-correlation values.

- Regarding the dependence of the average degradation on the channel power delay profiles, we notice that at large frame synchronization errors the various channel profiles become indistinguishable. At lower sync errors the spread between the degradations for the different channel types shows some, although very little, sensitivity to the OFDM overhead. In the cases illustrated in figure 7.7, the spread is at most 0.5 dB at an SNR degradation of about 1 dB based on an SNR_0 of 20 dB. Therefore, the channel power delay profiles and the OFDM overhead play only a small role in determining the average performance degradation.

Conclusions

In this thesis we studied the sensitivity of the MC-DS-CDMA uplink to various synchronization parameters. In this regard, the effect of carrier frequency errors, code phase alignment errors and frame synchronization errors was considered. We have investigated both single user and multiuser cases. The results from the single user case are relevant to the synchronization accuracy required for individual users, while the results from the multiuser case are relevant to network synchronization.

8.1 Conclusions

In view of the discussion in chapter 3 and of the results established in chapters 5, 6 and 7 we draw the following conclusions:

- Based on the mathematical expression of the MC-DS-CDMA signal it is established that the signal can be represented in terms of a series of OFDM frames grouped into a larger structure we named an MC-DS-CDMA block. It is also established that there can be two distinct transmitter and receiver architectures, the DFT-DS and DS-DFT receivers. Further, we have shown that the main difference between the two architectures is the speed at which DFT/IDFT processing has to occur. Throughout this thesis we have mainly focused on the first architecture, although occasional references to the DS-DFT receiver are also made.
- The effect of carrier frequency and frame timing synchronization errors can be split into OFDM and spread spectrum specific effects.

- It is shown that MC-DS-CDMA systems are characterized by pronounced sensitivity to carrier frequency errors due to the fact that both the multicarrier and spread spectrum components exhibit significant sensitivity to this type of errors. For the single user case, performance degradation is attributable to the OFDM-specific intercarrier interference and to the DS-CDMA-specific attenuation of the autocorrelation function. In the multiuser case our results show that performance degradation is due to the non-coherent cross-correlation functions characterizing the spreading code family and we note that spreading codes tend to behave like random codes in the presence of frequency errors. Based on the numerical results, we conclude that carrier tracking and accurate network wide carrier frequency synchronization is required and the use of a downlink carrier pilot might be necessary.
- Code phase (block) synchronization effects are dependent on the cross-correlation properties of the spreading codes and hence, are purely DS-CDMA related. It is shown that an SNR based degradation measure for the single user case is not appropriate and instead, a measure based on distance reduction in the received signal constellation is suggested. We have also advanced the idea of an inter-block guard interval or a decision feed-back algorithm as possible techniques to deal with the acquisition and possibly tracking of the code phase synchronization error. Both single user and multiuser cases suggest that in order to prevent significant performance degradation synchronization might have to be established within one chip duration.
- Regarding frame synchronization errors, it is found that in the single user case the interference affecting the received signal can be grouped into two components: a channel estimation error-like self-interference and a Gaussian other-interference. For small errors, the aggregate effect was considered and characterized through an SNR degradation measure. To this end we have introduced a

degradation coefficient which characterizes the sensitivity of the OFDM component, while the dependence on the DS-SS component was through the partial auto-correlation function of the spreading code. For the multiuser case, it is found that the SNR degradation exhibits an oscillatory behavior with respect to the subcarrier index. The location of the peaks and troughs depends in a complicated way on a number of system parameters and therefore, separating the effect of individual users on the SNR degradation is not always possible and upper-bounding is necessary. Based on the numerical results we conclude that the OFDM component shows some tolerance (although quite limited for the single user case) while the spread spectrum component has the capability of maintaining good performance provided well designed spreading codes are used which have low partial cross-correlations at time shifts of ± 1 .

8.2 Future research

In the present work we derived analytical relations describing the effect carrier frequency and timing synchronization errors have on the MC-DS-CDMA uplink. Along our presentation a number of remarks and suggestions were made. We now summarize these and propose them for further study.

- In the discussion on frequency errors we have suggested a technique for estimating a given user's frequency offset via equation 5.18 in conjunction with the use of two successive blocks carrying training data. Given that the so obtained estimate depends on the subcarrier index, it can be smoothed by some appropriate filtering of the individual estimates obtained at each subcarrier. We consider further development of this idea to have merit since it could lead to a simple and practical frequency error estimation algorithm.

- Given the sensitivity of the MC-DS-CDMA uplink to carrier frequency errors among users we suggested that using a downlink carrier pilot might be necessary. However, it is also possible to use a closed loop control system over the radio channel, where individual frequency errors are estimated on the uplink and the control loop is closed by means of individual error signals computed at the base station and relayed to users on a downlink control channel. To this end, we advance the idea of pilot subcarriers on the uplink which then can be used to generate an error signal for frequency tracking purposes and suggest the study of appropriate synchronization algorithms.
- While discussing the effects of code phase alignment errors in chapter 6 for the single user case, we suggested a decision feed-back algorithm that attempts to estimate the code phase synchronization error from the structure of the signal constellation spread. This technique would identify a synchronization error that gives partial auto-correlation values which result in the observed constellation spread, effectively implementing a reverse lookup of the partial auto-correlation values. We also noted that this technique would only be applicable when the minimum distance reduction in the signal constellation is not severe. A feasibility study emphasizing on the reverse lookup requirement of the partial auto-correlation values could provide insight into whether such a technique is feasible.
- We have seen that performance degradation in the presence of frame timing errors depends on the auto and cross-correlation properties of the spreading codes for time arguments ± 1 . Therefore, being able to choose specific spreading sequences out of the employed family such that these properties are optimized is important in reducing the sensitivity of the system to frame synchronization errors.

- If we are to consider frequency and time synchronization errors simultaneously, we would find that good time-frequency auto and cross-correlation properties are needed in order to minimize performance degradation. As such, searching for such spreading code families would be a useful undertaking.
- We also suggest investigating if and how could a receiver using multiuser detection and iterative interference cancellation be modified in order to improve upon the performance degradation due to small residual synchronization errors between active users. In this context, we suggest exploring the possibility of simultaneously dealing with both relative carrier frequency and timing errors.

Cyclic prefix in OFDM

In this appendix we discuss the need for a cyclic prefix in DFT based multicarrier systems which include OFDM and various multicarrier spread spectrum configurations where data frames are generated through IDFT at the transmitter. As a result of the IDFT processing at the transmitter, DFT processing has to be performed at the receiver in order to recover the transmitted data.

Given a multipath channel, ISI and ICI¹ will generally affect the received samples of a MC signal and an equalizer would be required. By inserting between the transmitted MC symbols a guard interval of duration $T_g = \tau_m$, where τ_m is the maximum channel excess delay, during which the complex exponentials associated with the individual subcarriers are extended in time, it can be shown that the ISI and ICI will only affect the first T_g seconds of the received signal [39]. By first discarding the affected part and then performing matched filtering on the remaining part of the signal, one can recover the transmitted data without ISI/ICI. The recovered data is scaled with a complex channel coefficient corresponding to samples of the channel's frequency response (see [39]) and therefore, an one tap equalizer at each subcarrier will suffice. However, the data rate has to be reduced in order to accommodate the cyclic prefix. In conclusion, equalization has been greatly simplified by trading processing complexity against useful data rate.

Considering the discrete time implementation of the MC system, sampling the transmitted MC signal at a rate equal to the data rate one obtains a frame structure com-

¹. Inter-symbol and inter-carrier interference.

posed of the IDFT of the data symbols and of a cyclic prefix, as shown in figure A.1, and where the OFDM frame will contain $N_{\text{total}} = L + N$ samples. At the receiver, removing the guard interval becomes equivalent to removing the cyclic prefix, while the effect of the channel transforms into the periodic convolution of the discrete time channel with the IDFT of the data symbols. Performing a DFT on the received samples after the cyclic prefix is discarded, the periodic convolution is transformed into multiplication, as it was the case for the analog MC receiver. Therefore, a discrete time implementation of the MC transmitter will use a cyclic prefix to emulate the guard interval from the analog transmission.

It should be noted that the cyclic prefix technique can also be employed for single carrier transmissions as well, resulting in the use of an one tap equalizer in the frequency domain, and hence also requiring an IDFT stage at the receiver in order to revert to time domain after equalization [38]. The price paid is the need for both a DFT and IDFT at the receiver and a reduction of the useful data rate. Since the gain in complexity with respect to time domain equalization is arguable while throughput is reduced, this technique is not commonly used in conjunction with single carrier systems. However, it is interesting to note that if the mentioned DFT-IDFT pair is distributed between the transmitter and the receiver, then a single carrier system equivalent to the OFDM system is obtained.

At this point we should note that for a data symbol duration of T before the multi-carrier modulation (and hence the S/P conversion), the subcarrier spacing has to be chosen

as

$$\Delta f_{sc} = \left(1 + \frac{L}{N}\right) \cdot \frac{1}{NT} \quad (\text{A.1})$$

that is, in order to preserve the data rate we have to expand the subcarrier spacing and hence the transmission bandwidth by a factor of $(1 + L/N)$. In order for the cyclic prefix technique to be applicable the amount of overhead introduced should be small, that is

$$\tau_m = T_g < NT \quad (\text{A.2})$$

where NT is the duration of the prefixed OFDM frame. For a given data rate and a given channel this will set a minimum number of required subcarriers. Considering that the channel coherence bandwidth can be approximated by $\Delta f_{coh} \approx 1/\tau_m$ and neglecting the $(1 + L/N)$ factor in relation A.1, we obtain

$$\tau_m < NT \Leftrightarrow \Delta f_{sc} < \Delta f_{coh} \quad (\text{A.3})$$

Since the bandwidth associated with a given subcarrier (main lobe of $\text{sinc}(\cdot)$ function) is $W_{sc} = 2 \cdot \Delta f_{sc}$, it is now clear that the applicability of the cyclic prefix is related to flat fading conditions at the subcarriers. Should this flat fading condition not be satisfied, severe ISI will affect the symbols received on each subcarrier or equivalently, the span of the channel's impulse response would be larger than the duration of one OFDM frame.

Despreading in the DS-DFT receiver

The DS-DFT receiver has a DS-SS like front-end following the deinterleaver. However, the $\tau_m \ll T$ condition mentioned in section 3.2.2, where $T = N_c/W = T$, usually does not hold for high data rate transmissions. Then, the channel impulse response is longer than, or comparable to, the symbol duration T and hence, the despreading receiver employed in DS-SS is no longer optimal since there would be ISI after despreading. In this appendix we derive the details of this ISI, and we also show that arguments based on the resemblance with the DS-SS front-end are misleading because even for $\tau_m \ll T$ a RAKE front-end is not usable due to the particular structure of the MC-DS-CDMA signal.

The presence of the deinterleaver in figure 3.5 requires some attention. In a general case, passing the signal through an interleaver at the transmitter and a deinterleaver at the receiver will cause the apparent channel after deinterleaving to be time variant and non-causal which in turn can cause difficulties in equalization. However, we show that due to the particular frame structure of the MC-DS-CDMA signal and to limitations imposed on the maximum excess channel delay no such problems arise.

Let us first consider the transmission of an isolated MC-DS-CDMA block where no cyclic prefix is appended to the constituent OFDM frames. Using the notations from chapters 2 and 3, the T_c/N spaced samples of the received signal are given by

$$\begin{aligned}
 r(n) &= \sum_{l=0}^L s(n-l)h(l) & n &= pN + q \\
 q &= 0 \dots N-1 & p &= 0 \dots N_c - 1
 \end{aligned} \tag{B.1}$$

with $h(l)$ the channel impulse response and L the maximum excess channel delay expressed in T_c/N units (there are $L+1$ channel taps). Assuming the channel time invariant for the duration of an MC-DS-CDMA block and writing the index l as $l = \rho N + \sigma$, we get

$$r(\rho N + q) = \sum_{\rho} \sum_{\sigma} s((\rho - \rho)N + q - \sigma) \cdot h(\rho N + q) \quad (\text{B.2})$$

Assuming the channel impulse response shorter¹ than the OFDM frame, we have the restriction $L < N$ which means $\rho = 0$ always. Taking also into account relation 3.4, equation B.2 becomes

$$\begin{aligned} r(\rho N + q) = & c(\rho) \cdot \sum_{\sigma=0}^q s_{\text{ofdm}}(q - \sigma) \cdot h(\sigma) + \\ & + c(\rho - 1) \cdot \sum_{\sigma=q+1}^L s_{\text{ofdm}}(N + q - \sigma) \cdot h(\sigma) \end{aligned} \quad (\text{B.3})$$

which is then rewritten as

$$r(\rho N + q) = \sum_{\rho=0}^1 c(\rho - \rho) \cdot h_{\text{eq}}(\rho; q) \quad (\text{B.4})$$

At this point it is clear that the received signal is in the form of a convolution between the spreading sequence and an equivalent channel $h_{\text{eq}}(\rho; q)$. However, this new channel has a non trivial dependence on the OFDM symbol $s_{\text{ofdm}}(q)$ and hence, a RAKE receiver using $h_{\text{eq}}(\rho; q)$ as channel taps is not feasible.

¹. This assumption will allow us later to apply the cyclic prefix technique. Actually, as explained in appendix A, the maximum channel excess delay should be much less than the OFDM frame duration out of overhead considerations.

By inspecting equation B.3 we see that $r(pN + q)$ really contains the convolution between the channel $h(q)$ and the transmitted OFDM frames and hence, interference is as in OFDM: inter-sample and inter-frame interference (ISI and IFI). As a result, despreading using a RAKE receiver is not possible and only a basic despreader followed by a time domain equalizer can be employed.

B.1 Basic despreader front-end

After deinterleaving and despreading the received signal in the DS-DFT receiver we obtain

$$\begin{aligned}
 y(q) &= \sum_{p=0}^{N_c-1} r(pN + q) \cdot c^*(p) = \sum_{p=0}^{N_c-1} c^*(p)c(p) \cdot \sum_{\sigma=0}^q s_{\text{ofdm}}(q - \sigma) \cdot h(\sigma) + \\
 &+ \sum_{p=0}^{N_c-1} c^*(p)c(p-1) \cdot \sum_{\sigma=q+1}^L s_{\text{ofdm}}(N + q - \sigma) \cdot h(\sigma) \\
 &= R_c(0) \cdot \sum_{\sigma=0}^q s_{\text{ofdm}}(q - \sigma) \cdot h(\sigma) + R_c(1) \cdot \sum_{\sigma=q+1}^L s_{\text{ofdm}}(N + q - \sigma) \cdot h(\sigma)
 \end{aligned} \tag{B.5}$$

where $R_c(m)$ is the autocorrelation function of the spreading sequence. Therefore, in general, there will be a self interference term due to $R_c(1) \neq 0$ and it will affect the first L samples of $y(q)$. Physically, this interference is due to the preceding frame belonging to the same block. For an ideal spreading code the autocorrelation function is given by $R_c(m) = R_c(0) \cdot \delta(m)$ and the second term in relation B.5 becomes zero. Thus, the equation simplifies to

$$y(q) = R_c(0) \cdot \sum_{\sigma=0}^q s_{\text{ofdm}}(q-\sigma) \cdot h(\sigma) = R_c(0) \cdot [s_{\text{ofdm}}(q) \star h(q)] \quad (\text{B.6})$$

and it is apparent that we obtain the convolution of the OFDM frame with the channel.

If we now consider the transmission of a series of MC-DS-CDMA blocks, equations B.3 through B.6 should be modified to include the interference from the previous block which would affect samples $r(n)$, $n < L$ of each block and therefore, after despreading, samples $y(q)$, $q < L$. Eliminating this undesired effect would require an all zero guard interval of duration L samples preceding each MC-DS-CDMA block or the use of a time domain equalizer.

As suggested in section 3.2.2, a cyclic prefix of L samples could be added to each OFDM frame $s_{\text{ofdm}}(q)$ at the transmitter of figure 3.3. In this case, the first L samples of $y(q)$ are related to this prefix and they entirely contain the self interference term due to $R_c(1)$ as well as the interference from the previous block. For analysis purposes, it is convenient to view $s_{\text{ofdm}}(q)$ as having $N+L$ samples including the prefix and indexed $-L \dots 0 \dots N-1$. By rewriting the equations accordingly and then discarding the first L samples of $y(q)$ we get instead of equation B.6

$$y(q) = R_c(0) \cdot \sum_{\sigma=0}^q s_{\text{ofdm}}(q-L-\sigma) \cdot h(\sigma) \quad q = L \dots N+L-1 \quad (\text{B.7})$$

Taking into account that the channel spans $L+1$ samples, that is $\sigma = 0 \dots L$, the summation limits are 0 and L regardless of q , and then relation B.7 is written as

$$z(q) = y(q+L) = R_c(0) \cdot [s_{\text{ofdm}}(q) \star h(q)] \quad q = 0 \dots N-1 \quad (\text{B.8})$$

thus obtaining the desired periodic convolution. Therefore, further processing of the signal can proceed as a standard OFDM demodulation. In order to accommodate the cyclic pre-

fix, the interleaver/deinterleaver size should be increased to $(N + L) \times N_c$. Finally, by interchanging the order of operations and removing the first $L \times N_c$ samples right after the deinterleaver the same result is obtained but with fewer operations in the despreaders.

Properties of the ICI

C.1 General properties of $\zeta(m; \delta f_f)$

In chapter 5 (relations 5.3 and 5.4) it was shown that the inter-carrier interference due to frequency offset can be expressed as

$$\Lambda(i; \delta f_f) = \sum_{m=0}^{N-1} Y(m) \cdot \zeta(i-m; \delta f_f) \quad (C.1)$$

where $Y(m) = H(m) \cdot d(m)$ and

$$\zeta(m; \delta f_f) = \frac{1}{N} \cdot \sum_{\gamma=0}^{N-1} e^{j2\pi \frac{\gamma(\delta f_f - m)}{N}} = \frac{1}{N} \cdot \frac{\sin(\pi \delta f_f)}{\sin\left(\frac{\pi(\delta f_f - m)}{N}\right)} \cdot e^{j\pi \frac{(N-1)\delta f_f + m}{N}} \quad (C.2)$$

In particular, when the frequency offset tends towards zero the ICI vanishes because

$$\lim_{\delta f_f \rightarrow 0} \zeta(m; \delta f_f) = \delta(m) \quad (C.3)$$

We can also readily establish the property

$$|\zeta(0; \delta f_f)| = \frac{1}{N} \cdot \left| \frac{\sin(\pi \delta f_f)}{\sin\left(\frac{\pi \delta f_f}{N}\right)} \right| \approx \left| \frac{\sin(\pi \delta f_f)}{\pi \delta f_f} \right| \quad (C.4)$$

showing that $|\zeta(0; \delta f_f)|$ is, for all practical purposes, independent of the number of subcarriers N .

Since $\zeta(m; \delta f_f)$ is periodic in m with period N , we can rewrite equation C.1 as a periodic convolution:

$$\Lambda(i; \delta f_f) = Y(m) \odot \zeta(m; \delta f_f) \Big|_{m=i} \quad (C.5)$$

By denoting $\lambda(n; \delta f_f) = \text{IDFT}\{\Lambda(i; \delta f_f)\}$ and $\xi(n; \delta f_f) = \text{IDFT}\{\zeta(m; \delta f_f)\}$ we find

$$\lambda(n; \delta f_f) = N \cdot y(n) \cdot \xi(n; \delta f_f) \quad (C.6)$$

with $\xi(n; \delta f_f)$ being given by

$$\xi(n; \delta f_f) = \frac{1}{N} \cdot \sum_{m=0}^{N-1} \left[\frac{1}{N} \cdot \sum_{\gamma=0}^{N-1} e^{j2\pi \frac{\gamma(\delta f_f - m)}{N}} \right] \cdot e^{j2\pi \frac{mn}{N}} = \frac{1}{N} \cdot e^{j2\pi \frac{n \cdot \delta f_f}{N}} \quad (C.7)$$

Given the DFT relationship between $\xi(n; \delta f_f)$ and $\zeta(m; \delta f_f)$ we also find

$$\sum_{m=0}^{N-1} |\zeta(m; \delta f_f)|^2 = N \cdot \sum_{n=0}^{N-1} |\xi(n; \delta f_f)|^2 = 1 \quad (C.8)$$

$$\sum_{m=0}^{N-1} \zeta(m; \delta f_f) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \frac{1}{N} e^{j2\pi \frac{n \cdot \delta f_f}{N}} \cdot e^{-j2\pi \frac{mn}{N}} = 1 \quad (C.9)$$

which are used in the computation of the frequency offset induced degradation of system performance.

The result of C.7 is to be expected in an OFDM system since the demodulator DFT and the above IDFT are pairs and hence, we got back the original time domain. However, in MC-DS-CDMA one may take advantage of the above result and implement an all feed-forward frequency offset correction scheme after the subcarriers are despread.

C.2 The power of the ICI

From relation C.1 the ICI on the i^{th} subcarrier is

$$\text{ICI}(i) = \sum_{m \neq i} Y(m) \cdot \zeta(i-m; \delta f_f) \quad (\text{C.10})$$

and therefore, its power is

$$P_{\text{ICI}} = \mathcal{E} \{ |\text{ICI}|^2 \} = \mathcal{E} \left\{ \sum_{m \neq i} \sum_{n \neq i} Y(m) Y^*(n) \zeta(i-m; \delta f_f) \zeta^*(i-n; \delta f_f) \right\} \quad (\text{C.11})$$

Considering uncorrelated, zero mean data symbols and the fact that the channel coefficients and the data are independent, we can take the expectation with respect to the data first and get

$$\mathcal{E}_d \{ Y(m) Y^*(n) \} = \sigma_d^2 \cdot |H(m)|^2 \cdot \delta_{m,n} \quad (\text{C.12})$$

Next, we take the expectation value with respect to the channel and obtain

$$P_{\text{ICI}} = \sigma_d^2 \cdot \sum_{m \neq i} P_H(m) \cdot |\zeta(i-m; \delta f_f)|^2 \quad (\text{C.13})$$

with $P_H(m)$ the average power of the m^{th} channel coefficient. We note that the same result is obtained if we relax the requirement on uncorrelated data symbols (e.g. coding across subcarriers) and exploit the fact that $H(m)$ are decorrelated due to the subcarrier spacing. Assuming that $P_H(m)$ are all equal, we finally get the result

$$P_{\text{ICI}} = \sigma_d^2 \cdot P_H \cdot (1 - |\zeta(0; \delta f_f)|^2) \quad (\text{C.14})$$

which is independent of the subcarrier index. The assumption that on the average all channel coefficients have equal power is a consequence of uncorrelated scattering. For other

types of channels one can employ adaptive power allocation on the subcarriers in order to equalize (on the average) the SNR over all subcarriers. Therefore, the assumption that $P_H(m)$ is independent of the subcarrier index m is reasonable.

Partial correlation functions

In this appendix we define the partial auto-correlation and cross-correlation functions for an arbitrary set of spreading sequences. We consider the set $\{c_k\}$ where each spreading sequence has a length of N_c chips.

D.1 Definitions and properties

The definition of the partial cross-correlation of two spreading sequences, c_k and c_l , is illustrated in figure D.1 and given by

$$\begin{aligned} R_{c_k, l}(q) &= \frac{1}{N_c} \cdot \sum_{\beta=q}^{N_c-1} c_k^*(\beta) c_l(\beta-q) \\ R_{c_k, l}(-q) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-q-1} c_k^*(\beta) c_l(\beta+q) \end{aligned} \quad (D.1)$$

with the property that

$$R_{c_{k,l}}(-q) = R_{c_{l,k}}^*(q) \quad (D.2)$$

We note that in general, $R_{c_{k,l}}(q)$ is unrelated to $R_{c_{k,l}}(-q)$ and therefore, $R_{c_{k,l}}(q)$ is unrelated to $R_{c_{l,k}}(q)$. Put in different words, the left and right cross-correlations are different, and it matters whether we correlate c_k to c_l or c_l to c_k . A particular case of equation D.2 is obtained for the zero argument

$$R_{c_{k,l}}(0) = R_{c_{l,k}}^*(0) \quad (D.3)$$

which then implies $|R_{c_{k,l}}(0)| = |R_{c_{l,k}}(0)|$.

In order to arrive at the partial auto-correlation we set $k = l$ and obtain

$$\begin{aligned} R_c(q) &= \frac{1}{N_c} \cdot \sum_{\beta=q}^{N_c-1} c^*(\beta) c(\beta-q) \\ R_c(-q) &= \frac{1}{N_c} \cdot \sum_{\beta=0}^{N_c-q-1} c^*(\beta) c(\beta+q) \end{aligned} \quad (D.4)$$

with the easily established property that

$$R_c(-q) = R_c^*(q) \quad (D.5)$$

Clearly, the left and right auto-correlations are not independent. It is also worth mentioning that $R_c(0)$ is always real.

D.2 Particular cases

It is shown in chapter 7 that the performance degradation due to frame synchronization errors can be written as a function of the absolute value of the auto- and cross-correlations of the spreading sequence family computed for arguments in the $\{0, \pm 1, \pm(N_c - 1)\}$ set. Therefore, we find here the expression for these correlation values for the spreading sequence families considered in chapter 7.

Based on the definition of equation D.4 it is easy to see that for all unit modulus spreading sequences $R_c(0) = 1$, $|R_c(\pm(N_c - 1))| = 1/N_c$ and $|R_{c_{k,l}}(\pm(N_c - 1))| = 1/N_c$.

Cyclic-shift m-sequences

For this family of spreading codes we exploit the property that the periodic auto-correlation function of m-sequences is $R_c^{(p)}(k) = -1/N_c$ for all $k \neq 0$. Since the spreading code family is made up of periodic shifts of a given m-sequence, the periodic cross-correlation is also $R_{c_{k,l}}^{(p)}(k) = -1/N_c$ for all k except for one particular value k_0 , where $R_{c_{k,l}}^{(p)}(k_0) = 1$. The later situation arises due to the fact that all the spreading codes in the family are derived via cyclic shift from one given m-sequence. If we design¹ the code set such that $k_0 \neq \pm 1$, then it is straightforward that

$$R_c(1) = R_c^*(-1) = R_c^{(p)}(1) - \frac{c(0)c(N_c - 1)}{N_c} = \frac{-1 \pm 1}{N_c} \in \left\{0, -\frac{2}{N_c}\right\} \quad (D.6)$$

and that

¹. In practical terms, this means that we select the spreading codes to be at least a cyclic-shift of 2 apart, and if we are to also consider the acquisition of code synchronization, then the minimum distance should be 3.

$$\begin{aligned}
R_{c_{k,l}}(1) &= R_{c_{k,l}}^{(p)}(1) - \frac{c_k(0)c_l(N_c - 1)}{N_c} = \frac{-1 \pm 1}{N_c} \in \left\{0, -\frac{2}{N_c}\right\} \\
R_{c_{k,l}}^*(-1) &= R_{c_{l,k}}(1) \in \left\{0, -\frac{2}{N_c}\right\} \\
R_{c_{k,l}}(0) &= R_{c_{l,k}}(0) = -\frac{1}{N_c}
\end{aligned} \tag{D.7}$$

Walsh codes

Walsh codes are obtained as rows in a Haddamard matrix whose elements are +1 and -1. The fundamental property that the rows of the matrix are orthogonal translates into

$$R_{c_{k,l}}(0) = 0 \tag{D.8}$$

In order to derive the other auto- and cross-correlation values of interest, we start with the expression of the Haddamard matrix:

$$H_{2n} = \begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix} \tag{D.9}$$

$$\begin{aligned}
H_2 &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & H_4 &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} & H_8 &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}
\end{aligned} \tag{D.10}$$

Computing the partial auto-correlation in ± 1 for the three particular cases yields

$$R_c(\pm 1) = \frac{1}{N_c} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad R_c(\pm 1) = \frac{1}{N_c} \cdot \begin{bmatrix} 3 \\ -3 \\ 1 \\ -1 \end{bmatrix} \quad R_c(\pm 1) = \frac{1}{N_c} \cdot \begin{bmatrix} 7 \\ -7 \\ 1 \\ -1 \\ 5 \\ -5 \\ 3 \\ -3 \end{bmatrix} \quad (D.11)$$

Given the recurrence structure of the Haddamard matrices in relation D.9 we derive that $R_c^{(n)}(\cdot)$, the partial auto-correlation for a length n Walsh code, satisfies the recurrence

$$R_c^{(2n)}(\pm 1) = 2 \cdot R_c^{(n)}(\pm 1) \pm 1 \quad (D.12)$$

Then, using the particular cases we developed for $n \in \{2, 4, 8\}$ and the principle of mathematical induction, it is straightforward to show that

$$R_c(\pm 1) \in \left\{ \pm \frac{1}{N_c}, \pm \frac{3}{N_c}, \dots, \pm \frac{N_c - 1}{N_c} \right\} \quad (D.13)$$

Derivation of the partial cross-correlations in ± 1 is based on the following observations:

- If we compute $R_{c_{k,l}}(\pm 1)$ for the case when the two spreading sequences belong to different halves of the Haddamard matrix (say, one to the top half and the other to the bottom half), the result is always $1/N_c$ or $-1/N_c$.
- If we compute $R_{c_{k,l}}(\pm 1)$ when the two spreading sequences belong to the same half of the Haddamard matrix, then the recurrence relation below can be established

$$R_{c_{k,l}}^{(2n)}(\pm 1) = 2 \cdot R_{c_{k,l}}^{(n)}(\pm 1) \pm 1 \quad (D.14)$$

- For the particular cases of $n \in \{2, 4\}$ we obtain

$$R_{c_{k,l}}^{(2)}(\pm 1) \in \left\{ \pm \frac{1}{N_c} \right\} \quad R_{c_{k,l}}^{(4)}(\pm 1) \in \left\{ \pm \frac{1}{N_c}, \pm \frac{3}{N_c} \right\} \quad (\text{D.15})$$

Therefore, using mathematical induction it is readily found that

$$R_{c_{k,l}}(\pm 1) \in \left\{ \pm \frac{1}{N_c}, \pm \frac{3}{N_c}, \dots, \pm \frac{N_c - 1}{N_c} \right\} \quad (\text{D.16})$$

Maximization of the self-interference power

It was found in chapter 7 that the variance of the self-interference depends on the actual power delay profile of the channel. In this appendix we find the channel that maximizes this variance. We had:

$$\begin{aligned} \mathcal{E} \left\{ |\Lambda_{\alpha}(N_s; i)|^2 \right\} &= \frac{1}{N^2} \sum_{\gamma_{\tau} \in \Psi(N_s)} P_h(\gamma_{\tau}) (N_s - L + \gamma_{\tau})^2 & \Delta t > 0 \\ \mathcal{E} \left\{ |\Lambda_{\alpha}(N_s; i)|^2 \right\} &= \frac{1}{N^2} \sum_{\gamma_{\tau} \in \Theta(N_s)} P_h(\gamma_{\tau}) (N_s - \gamma_{\tau})^2 & \Delta t < 0 \end{aligned} \quad (E.1)$$

with $\Psi(N_s) = [L + 1 - N_s, L]$ and $\Theta(N_s) = [0, N_s - 1]$, and subject to the restriction

$$\sum_{\gamma_{\tau}=0}^L P_h(\gamma_{\tau}) = 1 \quad P_h(\gamma_{\tau}) \geq 0 \quad (E.2)$$

In each case, the maximization procedure is carried out for two situations: $N_s \leq L$ and $N_s > L$.

a) The $N_s > L$ case

For positive (negative) timing errors we express $P_h(0)$ (respectively $P_h(L)$) from equation E.2 and substitute it into relation E.1, obtaining

$$\begin{aligned}
\mathcal{E} \left\{ \left| \Lambda_{\alpha}(N_s; i) \right|^2 \right\} &= \frac{1}{N^2} \cdot \left\{ 1 + \sum_{\gamma_{\tau}=1}^L P_h(\gamma_{\tau}) [(N_s - L + \gamma_{\tau})^2 - (N_s - L)^2] \right\} & \Delta t > 0 \\
\mathcal{E} \left\{ \left| \Lambda_{\alpha}(N_s; i) \right|^2 \right\} &= \frac{1}{N^2} \cdot \left\{ 1 + \sum_{\gamma_{\tau}=0}^{L-1} P_h(\gamma_{\tau}) [(N_s - \gamma_{\tau})^2 - (N_s - L)^2] \right\} & \Delta t < 0
\end{aligned} \tag{E.3}$$

where each difference of squares term is positive. These new functions are to be maximized with the constraint

$$\begin{aligned}
\sum_{\gamma_{\tau}=1}^L P_h(\gamma_{\tau}) &= 1 - P_h(0) & \Delta t > 0 \\
\sum_{\gamma_{\tau}=0}^{L-1} P_h(\gamma_{\tau}) &= 1 - P_h(L) & \Delta t < 0
\end{aligned} \tag{E.4}$$

Because of the positivity of the coefficients multiplying $P_h(\gamma_{\tau})$ in equation E.3, we can maximize our new functions by setting $P_h(0)$ (respectively $P_h(L)$) to zero. Then, going back to our original maximization problem in relations E.1 and E.2, we have reduced their order from L to $L - 1$.

We repeat the steps outlined above in an iterative manner by always substituting in for the variable having the smaller coefficient in equation E.1. In the end, we find that maximization is achieved for

$$\begin{aligned}
P_h(\gamma_{\tau}) &= \delta(\gamma_{\tau} - L) & \Delta t > 0 \\
P_h(\gamma_{\tau}) &= \delta(\gamma_{\tau}) & \Delta t < 0
\end{aligned} \tag{E.5}$$

b) The $N_s \leq L$ case

In this situation $\Psi(N_s)$ and $\Theta(N_s)$ do not span the entire range of $[0, L]$ and hence, we may set those $P_h(\gamma_{\tau})$ which do not appear in equation E.1 to zero. For positive

timing errors we then eliminate $P_h(\gamma_\tau)$ for $\gamma_\tau \in [0, L - N_s]$, while for negative timing errors those for $\gamma_\tau \in [N_s, L]$. Then, we apply the same procedure as for $N_s > L$ to the remaining variables $P_h(\gamma_\tau)$ and obtain the result of equation E.5.

In conclusion, relation E.5 gives the channel power delay profile resulting in maximum self-interference power (variance), and this maximum value is given through

$$\max \left\{ \sigma_\Lambda^2 \right\} = \left(\frac{N_s}{N} \right)^2 \quad (\text{E.6})$$

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