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Force Control of Flexible Manipulators

by

ARIA ALASTY. M.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

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Carleton University

Ottawa, Ontario

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acceptance of the thesis.

Force Control of Flexible Manipulators

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Aria Alasty, M.Sc.

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for the degree of Doctor of Philosophy

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May 1997

Abstract

The problem of force control of a flexible link manipulator is addressed in this research. The main objective is to design, simulate, and experimentally demonstrate a control scheme which provides good servo performance of this nonminimum phase system and simultaneously suppresses the effects of severe friction in the joint. A simplified pole/zero preserving methodology is suggested to obtain reliable models for the system in unconstrained and constrained (in contact with environment) motions. The impact phenomenon is addressed by modelling the transition process between the two states. The unknown parameters of the system are identified by off-line experiments. A “PI-tip force plus hub velocity feedback” (PIf+v) control scheme is designed based on optimal state feedback gains, and the stability of the system with discontinuous contact is investigated. To improve the system performance a Unified Predictive force Control (UPC) is applied to it. Although the simulation results suggest a significant system performance improvement for UPC vs the PIf+v control, experimental results show otherwise, attributable to a weakness of UPC in friction suppression. To obviate this drawback a two-stage (inner/outer loop) control scheme is suggested. A PID hub angle feedback in the inner loop is responsible for suppressing the friction effect, and simultaneously UPC force controller in the outer loop is tuned to achieve maximum performance. Simulation and experimental results are in very good agreement, and confirm that the research objective of simultaneous friction suppression and force control performance improvement has been achieved quite successfully. During the final UPC-PID tuning some modifications of the design rules of UPC are suggested. Also some practical notes for experimental implementation are presented.

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“... O my Lord! grant me that I may be grateful for Thy favor which Thou hast bestowed upon me ...” *Holy Quran, 46(15)*

Dedication

*To Imam Mahdy,
the promised saviour of mankind
(peace be upon him and his forefathers)*

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Nomenclature

Signs

$\dot{}$	$\frac{\partial}{\partial t}$
$\prime$	$\frac{\partial}{\partial x}$
sgn	Sign

Abbreviations

B.C.	Boundary Condition
DOF	Degrees OF Freedom
LQR	Linear Quadratic Regulator
MBPC	Model-Based Predictive Control
MP	Minimum Phase
NMP	Non Minimum Phase
PIF+V	Proportional Integral Force Plus Velocity feedback control

RHP	Right Half Plane
TF	Transfer Function
UPC	Unified Predictive Control

English Symbols

a	Hub-to-Link moment of inertia ratio ($\frac{J_h}{6J_b}$), (Eqn. 3.26).
b_i	Distribution coefficient of input for each flexural mode,(Eqn. 3.42).
c_h	Equivalent damping coefficient at joint. (Eqn. 3.49).
$k_{b/c}$	Beam-to-Contact stiffness ratio, (Eqn. 3.63).
k_{dc}	DC gain of system transfer function. (Eqn. 3.102).
k_t	Equivalent motor torque constant, (Eqn. 3.49).
$k_{v/s}$	Velocity signal calibration factor, (Eqn.4.13).
m	Mass per unit length of the link.
m_p	Pay load mass (force sensor + load).
n	Number of assumed modes in constrained motion.
n_u	Number of assumed modes in unconstrained motion.
p	Number of points where kinematic compatibility are satisfied.
$p_{j\pm}$	j^{th} pair of system transfer function's poles, (Eqn. 3.100).
q_i	i^{th} generalized coordinate of flexible system, (Eqn. 3.11).
q^{-1}	Backward shift operator.

u	Command voltage.
v	Absolute velocity of a point on the link, (Eqn. 3.5).
x	States vector, (Eqn. 3.48).
x	Link length variable in rigid body coordinate x-y.
y	Output vector in state space representation.
$y(x, t)$	Deflection of a point on the link (body coordinate x-y).
$y_L(t)$	Deflection of link tip.
$z_{j\pm}$	j^{th} pair of system transfer function's zeros. (Eqn. 3.94).
A	System matrix. (Eqn. 3.48).
A_b	System matrix including the joint damping effect, (Eqn. 3.52).
B	Control matrix, (Eqn. 3.48).
C	Observation matrix for force and velocity.
C_F	Observation matrix for contact force, (Eqn. 3.72).
$C_{\dot{j}}$	Observation matrix for motor velocity, (Eqn. 3.51).
D	Direct transmission matrix.
$\mathcal{D}$	Dissipation function, (Eqn. 3.37).
E_{kin}	Kinetic energy, (Eqn. 3.6).
E_{pot}	Potential energy, (Eqn. 3.6).
EI	Link stiffness.
$\mathcal{E}$	Event detector function. (Eqn. 3.81).
$F(t)$	The reaction force of the environment on the beam tip.

$G_f(s)$	Tip force transfer function for constrained motion.
$G_v(s)$	Motor velocity transfer function.
G_w	Tip velocity transfer function for unconstrained motion.
H_c	Control horizon in UPC.
H_m	Minimum-cost horizon in UPC.
H_p	Prediction horizon in UPC.
J_b	Link moment of inertia around joint. ($\frac{mL^3}{3}$).
J_h	Hub (motor) moment of inertia.
K_j	j^{th} generalized stiffness of the system. (Eqn. 3.32).
K_f	Force feedback gain in PIf+v control system.
K_i	Integral Force feedback gain in PIf+v control system.
K_v	Velocity feedback gain in PIf+v control system.
K_r	Feedforward gain in PIf+v control system.
L	Length of the link.
$\mathcal{L}$	Lagrangian function.
$M(x, t)$	Bending moment in link.
M_j	j^{th} generalized inertia of the system, (Eqn.s 3.32 and 3.33).
$\mathcal{P}$	Phase indicator function, (Eqn. 3.81).
$\mathcal{Q}$	External work function, (Eqn. 3.6).
$\mathcal{R}(q^{-1})$	Control polynomial in UPC. (Eqn. 6.13).
$\mathcal{S}(q^{-1})$	Output polynomial in UPC, (Eqn. 6.13).

$\mathcal{T}(q^{-1})$	Reference polynomial in UPC, (Eqn. 6.13).
T_s	Sampling time.
$V(x, t)$	Shear force in link.
V_{VELMON}	Motor velocity monitoring signal (voltage).
$W(x, t)$	Composite displacement of a point on the link (inertial coordinate X-W).
$W_L(t)$	Composite displacement of link tip.
X	Link length variable in inertial coordinate X-W.

Greek Symbols

α	The Stribeck effect in Tustin friction model, (Eqn. E.1).
α_0	Coulomb friction level in Tustin friction model, (Eqn. E.1).
α_1	$\alpha_1 + \alpha_0 =$ Static friction level, (Eqn. E.1).
α_2	The viscous friction effect in Tustin model, (Eqn. E.1).
β_i	Non-dimensionalized eigenvalues of system, (Eqn. 3.26).
δ	Logarithmic decrement of oscillations.
ϵ_g	Small neighborhood of zero velocity, (Eqn. E.2).
γ	The beam structural damping coefficient, (Eqn. 3.36).
ζ_j	j^{th} mode damping ratio, (Eqn. 3.43).
$\phi_i(x)$	i^{th} mode shape function of the flexible link, (Eqn. 3.11).
$\theta(t)$	The motor overall angle (rigid body + flexural motion), (Eqn. 3.3).

λ_i	i^{th} eigenvalue of the flexural motion, (Eqn. 3.12).
$\tau(t)$	The available torque on hub-beam inertia, (Eqn. 3.50).
$\tau_e(t)$	The torque produced by electrical part of motor, (Eqn. 3.49).
τ_{fr}	Friction torque, (Eqn. E.1).
τ_{res}	Resultant torque applied to the system, except friction, (Eqn. E.2).
ω_c	Cut off frequency of actuator, (Eqn. 3.45).
ω_i	i^{th} natural frequency of flexible link. (Eqn. 3.15).
$\Gamma_{u2c}, \Gamma_{c2u}$	Conversion matrices from unconstrained to constrained motion and vice versa.
$\Theta(t)$	The motor rigid body angular motion.
Λ	Lagrangian density function, (Eqn. 3.7).

Chapter 1

Introduction

During the past decade, the design of large, light weight robotic systems has received much attention, particularly for space applications. These systems usually possess flexible links, which confer some advantages over rigid link manipulators, viz., low weight, reduced energy consumption, high maneuverability, easier transportability, and smaller, lighter actuators required. Furthermore, designers of robot systems for ground-based activity are increasingly confronted with the challenges of increased operational speeds, greater payloads, and more stringent accuracy requirements of manipulators to meet the demands of productivity and flexible manufacturing. The manipulation accuracy of robots operating at high speeds or interacting with large contact forces is significantly affected by their link and joint flexibility.

Flexural behavior in an already highly nonlinear robotic system considerably complicates analysis and design, and demands elaborate control schemes with attendant requirements for sophisticated hardware/software. This has resulted in

increased research activity into methods of actively influencing flexible manipulator behaviour. Among these, end point position control has been a very active topic in recent years. Control of contact force of flexible manipulators, also a significant challenge, has received much less attention. The problem of designing a force control algorithm arises when contact between a robot and its environment occurs, e.g. in grasping parts, assembling in production lines or space applications, grinding, deburring, etc. .

High accuracy end point positioning and/or force control of flexible manipulators generally requires direct measurement of end effector position or force, with actuators at link bases usually for revolute robots. This creates noncollocated actuator/sensor configurations, which when accompanied by dispersive behavior of a link's lateral vibration produces nonminimum phase (NMP) transfer functions (see Appendix-F). NMP systems possess zeros in the right-half-plane (RHP), and suffer from inherent limitations on achievable control performance, especially when conventional controllers are used.

1.1 Research Objective

The main objective of this thesis is to develop a means of imposing a desired force trajectory onto a surface through a flexible manipulator link. This is to be accomplished accurately while rejecting disturbances, some of which, such as joint friction, would otherwise dominate system behaviour, and in spite of discontinuous surface contact. The manipulator considered in this thesis consists of a single flexible link: although this may appear at first glance to be too simple and well understood to

merit significant effort, closer examination reveals that this sort of system possesses some uniquely challenging problems. The following chapters describe the efforts toward achievement of the goals of this research.

1.2 Thesis Outline

A literature review of issues related to this research is presented in chapter 2. Chapter 3 is devoted to modelling and dynamic analysis required in the remainder of the thesis. In this chapter the partial differential equations (PDE) of motion are derived via Lagrange's method, then spatially discretized using the modal summation approach. A previously unreported issue of dependence of solution on number of modes and its relation to the selection of boundary conditions is discussed. State space representations of the system are developed using pinned-inertia/pinned B.C.'s for constrained motion and pinned-inertia/free-mass B.C.'s for unconstrained motion. A simple method is suggested to include the link structural damping, joint viscous damping, and motor back emf effects. The main output equation in the constrained motion is obtained from contact force model. By presenting a quantitative criterion it is shown that the spring-like soft contact model, which is very common, is not always an appropriate model, and therefore a new stiff contact model is introduced using the Lagrange Multiplier approach. A novel model for the process of transition between constrained and unconstrained motion is introduced based on kinematic compatibility of the manipulator. Defining phase indicator and event detector functions, a switching algorithm is also designed to be used in simulation. After conducting the first set of experimental results, significant differences between

simulation and experiment were detected, and it was soon revealed that the main cause of this difference is the presence of severe friction in the manipulator joint. Therefore the effect of joint friction on the open loop response of a single link flexible arm is also studied.

It is well known that the modal summation approach is a pole preserving but not a zero preserving method. Another method is the Infinite Product Expansion (IPE) by use of which an exact formulation for the infinite roots of the numerator and denominator can be found. The important weakness of this method is that solutions for only certain standard boundary conditions are available, and adding complication such as motor dynamics or structural and hub damping to the system renders the solution very difficult. In this research an analytical solution is obtained for numerator roots (zeros), and numerical algorithms are used to obtain the denominator roots (poles). The model which is obtained from the IPE method is called "exact" because it is closer to the real system than that obtained from the modal expansion (summation) method. Finally, a simplified procedure which is a combination of IPE and modal summation is introduced to obtain the exact model.

By means of experiments, some unknown system parameters for the motor, link, and friction models are obtained in chapter 4. Also the validity of system natural frequencies and transfer functions obtained theoretically is examined by experiment.

The first attempt to control the contact force is in chapter 5. Hub-velocity (collocated with actuator) feedback is added to proportional integral tip-force feedback (PIf+v) to improve the system performance. The gains of the controller are designed based on optimal state feedback gains. The effects of parameter uncertainties, joint

friction, and approach velocity are studied by means of simulation and experiments. Contact stability and existence of limit cycles are investigated for the controlled system with discontinuous contact.

To obtain better servo performance, Unified Predictive Control (UPC) is applied to the system in chapter 6. A brief review of the theory of UPC is presented and followed by the control design procedure. Simulation results show that UPC which uses only force feedback (noncolocated) improves the servo performance compared with the PIf+v control which uses not only tip force but also hub velocity feedback (colocated). The minimum number of assumed modes is determined such that spillover instability does not occur. UPC also shows satisfactory robustness in the presence of parameter uncertainties. It is shown that when the UPC is designed based on the modal summation model (which is not a zero preserving model) the experimental closed loop system becomes unstable. It is therefore suggested that the exact model be used in UPC design. Simulation and experimental results presented in this chapter also demonstrate that a UPC designed based on rules of thumb in the literature is not capable of overcoming the very severe friction in the joint, so that a modification in the controller filter of the UPC is suggested. This modification improves the friction suppression considerably but is incapable of cancelling the friction completely due to the latter's severity. An important issue on the effect of round-off errors on the stability of the controlled system is also discussed.

As the UPC is not completely effective in overcoming joint friction, a two-stage control algorithm is introduced in chapter 7 which is based on an inner/outer loop configuration. The inner loop, which is a colocated system, is selected to be a

PID hub-angle controller. The controller of this loop is designed to possess good disturbance rejection properties and is capable of suppressing the friction effects. The outer loop is the UPC force control system whose plant has been modified to incorporate the overall inner loop system. The final tuning of the UPC is done by modifying the parameters of tracking error filter. Both simulation and experimental results shows a significant improvement in servo performance of the system in the presence of severe friction.

Concluding remarks, a summary of thesis contributions, and future work suggestions are presented in chapter 8.

The experimental set up is explained in appendix A. and the governing equations of the motor are written briefly in appendix B. In appendix C it is shown that consideration of the axial force in dynamic modelling of the system is not necessary. In appendix D it is explained why the decomposition method, which is a well-known systematic approach, was not used in the modelling of the constrained system. A brief review of a number of available friction models is presented in appendix E. and a short note on noncolocated and nonminimum phase systems is presented in appendix F. For this thesis most simulations were performed using SIMULINK, and the details of these simulation programs are presented in appendix G. A sample of various computer programs used in the experiments is given in appendix H. A list of publications which resulted from this research up to date of preparation of this manuscript is in appendix I.

Chapter 2

Literature Survey

This chapter contains a review of the literature deemed most pertinent to the subject of this thesis. The first section deals with the dynamics of flexible manipulators, and the second with control of flexible manipulators, including both position and force control, and concluding with a discussion of flexible link control in the presence of joint friction, of particular significance to this thesis. Finally, the third section includes a review of predictive control, one of the methods used to solve the problem posed in the introduction.

2.1 Dynamics of Flexible Manipulator

The dynamics of a flexible manipulator system can be described by a set of Partial Integro-Differential Equations which represent a Distributed Parameter System. This is effectively an infinite dimensional system, generally arrived at via the extended Hamilton principle or Lagrangian dynamics, e.g. Low and Vidyasagar (1988), Ding, *et al.* (1989), Meirovitch (1991).

To convert a PDE so obtained to a set of Ordinary Differential Equations, which represents a spatially discretized system, many well-known methods exist, such as:

- The Finite Element Method, as used by Usoro. *et al.* (1986), Bayo and Moulin (1989), Chang and Hamilton (1991-a).
- Nicosia. *et al.* (1986, 1990) used the Ritz-Kantrovitch method and derived a discrete time model. Nicosia, *et al.* (1996) applied this approach to a two link flexible manipulator and validated the model by comparing its frequency response with experimental data.
- Another common approach is the Assumed Mode method (Modal Expansion), which is the method used in most papers on flexible manipulators, e.g. Gordon. *et al.* (1986), Banati and Morro (1988), Chait, *et al.* (1990).
- An interesting comparison between the assumed modes and finite element methods has been presented by Theodore and Ghosal (1995) in which they showed that fewer mathematical operations are required for inertia matrix computation in the finite element model than with the assumed modes formulation; however, the numerical simulation time may be greater for finite element models. They finally recommend the assumed modes model formulation for discretization of manipulator links with uniform cross-sectional geometries and for single-link flexible manipulators, and the finite element model formulation for manipulator links with complex cross-sectional geometries and for flexible multilink manipulator systems.

Apart from control schemes specifically designed for distributed parameter sys-

tems, most control approaches require reduction of a model to a finite number of degrees of freedom, and many schemes exist for determining an appropriate model order. Some researchers have used “actuator frequency bandwidth” limitations to determine the number of degrees of freedom of the model. Su, *et al.* (1991) used a Krylov- based reduced order model. Krishnan and Vidyasagar (1988) used a Hankel norm minimization method. and Tsujisawa, *et al.* (1989) applied Model Cost analysis techniques.

Some other methods of obtaining dynamical models for flexible manipulators are:

- The Extended Kane method as used by Everett (1989) and Du and Ling (1995).
- Recursive Extended Lumped Mass/Spring Approximation Model (ELMSAM) based on the Newton-Euler formulation and the Pseudo Joint concept. as used by Huang and Lee (1988).
- Petroka and Chang (1989) model the system by means of an “Equivalent Rigid Link System” (ERLS). Chang and Hamilton (1991-b) solve this model equation using a sequential integration method, and they derive the kinematic equation of flexible manipulators using the same method in (1991-c).
- Yen and Masada (1994) use Extended Bond Graphs (EBG) to simulate rigid and flexible multibody systems. This approach has the ability to solve the forward and inverse dynamic problems using the same bond graph.
- Xi and Fenton (1994) introduced a new kinematic notation called Algebra

of Rotations, and based on this derived a complete model of the elasto-kinematics. They showed that the combination of elasto-kinematics with elasto-dynamics can account for nonlinear coupling between joint variables and link deflections, and thus improve the inverse elasto-dynamics.

The dynamical characteristics of flexible manipulators and difficulties of their control have been studied from several points of view, e.g.:

- Forrest-Barlach (1986) noted that endpoint position control of flexible link robots is a nonhomogeneous, time-dependent boundary condition problem and suggested an approach to transform this problem into a modified problem with homogeneous B.C.s and distributed input force.
- Barbieri and Özgüner (1988) compared constrained and unconstrained modal expansions and investigated the effect of rigid body motion on the modal expansion of flexible manipulators. They showed that when the beam-to-hub inertia ratio is small enough, a finite expansion based on the constrained modes (such as clamped-free modes) will approximate the true solution (unconstrained expansion) with good accuracy: but as the hub inertia becomes less significant compared to the beam, unconstrained dynamics (such as hub-free B.C.s) must be considered to model the beam.
- Wang and Vidyasagar (1989) showed that when the number of modes is increased for more accurate modeling, the relative degree of the transfer function of a slewing beam becomes ill-defined.
- Spector and Flashner (1989) investigated the validity of different PDE models

for flexible beams and verified that the Euler-Bernoulli model is adequate for long thin beams, and for low frequency modes. If a thick, short beam or high frequency beam dynamics are investigated, the Timoshenko model is preferred.

- Morris and Vidyasagar (1990) investigated the suitability of various damping models for flexible beams from the stand point of designing finite- dimensional controllers to damp vibrations. In order to have any hope of designing finite-dimensional stabilizing controllers, they suggested the usage of the Rayleigh damping model.
- Spector and Flashner (1990) demonstrated the high sensitivity of transfer function zeros to errors in model and sensors location, which they attributed to inadequacies of the modal summation method in preserving both the qualitative and quantitative characteristics of transfer function zeros. They proposed using the Infinite Product Expansion Method (IPE) as an alternative. Parks and Pak (1991) modelled a flexible link using the IPE method and investigated the effect of a payload on the system characteristics. Lindner, *et al.* (1993) also gave a very detailed description of the zero behavior of modal models, and showed that in distributed parameter systems with noncollocated sensor/actuator configurations the zeros can vary widely depending on the number of assumed modes in the finite dimensional model.
- In some applications, robotic systems with flexible links are equipped with rotating tools, such as drills or grinding wheels. In an interesting study, Yamanaka, *et al.* (1995) showed that these gyro-elastic systems can display both static instabilities (divergence) and dynamic instabilities (flutter), and that

the configurations of stability regions depend on stiffness distributions.

On the subject of transfer function structure, Miu (1991) presented a very detailed explanation of the physical meanings of poles, conjugate zeros and real zeros of flexible systems. It was described that a noncolocated sensor/actuator configuration in conjunction with dispersive behavior of beam lateral vibration introduces some Right Half Plane (RHP) zeros in the system, resulting in a nonminimum phase (NMP) transfer function with its well-known difficulties of control, some of which are inherent limitations on the achievable performance of NMP systems, as has been demonstrated by Kwakernaak (1972). To overcome this limitation, some researchers have suggested modifications to the actuator/sensor configuration:

- Shamsa and Flashner (1990) suggested using co-located sensors and actuators to solve the non-minimum phase problem.
- Khorrami, *et al.* (1994) mounted a torque wheel at the end-effector of the flexible link manipulator to be used as an inertial actuator. The bending moment provided by this actuator is related to the velocity of the torque wheel and is used for vibration suppression and pointing purposes. Since the actuator and the sensor (an accelerometer at the tip) are colocated the system is minimum phase with no RHP zeros.
- Park and Asada (1994) designed and implemented a transmission mechanism to transform the actuator torque (at the hub) into a combination of force and torque which are then transmitted to an appropriate point on the link. By relocating the actuator near the endpoint, the system can be modified to approximate a minimum phase system.

Cetinkunt and Ittoop (1992) developed the dynamic equations of a flexible manipulator in a non recursive symbolic form. A more general symbolic formulation for manipulators with arbitrary rigid and flexible links was provided by Lin and Lewis (1994). The inverse dynamics of flexible manipulators in symbolic form was obtained by Korayem. *et al.* (1994) using a recursive Lagrangian assumed mode method.

A review of some early issues in modeling, design, and control of flexible manipulators was provided by Book (1990).

2.2 Control of Flexible Manipulators

2.2.1 Position Control

In the flexible manipulator field, position control has been a very active research topic in recent years. The manipulator end effector is desired to follow a reference trajectory or simply to maintain a fixed position. Various control schemes from very different view points have been tried for this purpose. Since position control is not the core subject of this thesis, only a brief review of the related literature will be presented in this section.

In many applications considering flexibility and motion to be linear, and using **linear control** systems, is quite reasonable. A work on this was presented by Book. *et al.* (1975) which compared three different linear controllers. Book and Majette (1983) used pole placement methods to design a linear controller. One of the first experimental results on this topic was published by Cannon and Schmitz (1984) in

which they introduced an experimental methodology to identify the coefficients of a system's nonminimum phase transfer function based on frequency response analysis and designed and implemented an LQG controller. The idea of link tip acceleration feedback was proposed by Kotnik, *et al.* (1988). Nicosia, *et al.* (1989) applied a pseudo-linearization method with a pole placement approach. In an interesting work Cetnikunt and Yu (1991) used tip position PD control and showed that the mode shapes of the closed loop system are neither the mode shapes of pinned-free nor clamped-free B.C.s. but rather a function of the feedback control. They also showed that if one were to use one or the other, the clamped free BC's provide a more accurate model for closed loop system than the pinned-free. For a linear system, Castelazo and Lee (1990) suggested using a nonlinear feedback controller to provide light damping initially and high damping toward the end of a trajectory. Lafontaine and Stieber (1986) investigated controllability and observability conditions, and their relations to the locations of sensors and actuators.

Nonlinear feedback control based on **inversion algorithms** has been used in the robotics field extensively; however, these techniques are not directly applicable to flexible link manipulators with their well-known nonminimum phase nature, without considering any remedy for stabilizing their zero dynamics. Singh and Schy (1986) suggested a force actuator at the tip to handle the unstable zero dynamics of a control system which has been designed based on an input-output inversion algorithm. This method requires a very complicated actuator system and is thus not easily implemented. De Luca and Siciliano (1989) also designed a joint-based inversion control scheme. Since only joint angle feedback was used in this method,

it showed good tracking results for certain joint trajectories, but it could not be extended to end-point trajectory following control. Solving the governing equations of motion to determine the required control signal which leads the end point to follow a desired trajectory is called the inverse dynamics problem, which is much more difficult in robots with flexible links than in those with rigid links. Kokkinis and Sahraian (1993) solved the inverse dynamics problem for flexible links using an optimal control approach. They showed that by choosing a proper performance index and weighting parameters it is possible to obtain bounded anticausal torques which cause the tip of the arm to track its desired trajectory. Kwon and Book (1994) suggested a time-domain inverse dynamic tracking control scheme. By dividing the inverse system equation into its causal and anticausal part, they successfully calculated a feedforward torque which is applied to the joint to produce a desired end point trajectory. Ata. *et al.* (1996) suggested use of Liapunov's second method to obtain causal inverse dynamics, and they implemented a successful PD controller accompanied by a computed torque method (using the above mentioned inverse dynamics) for tracking control purposes.

Khorami and Özgüner (1988) divided the flexible manipulator system into a fast (flexible) and slow (rigid body) subsystem using the **singular perturbation approach**. The slow subsystem generally has the usual nonlinear form of robotic systems while the fast part has the linear form of flexible link systems. Siciliano and Book (1988) proposed a composite control algorithm for these two subsystems, and reported some improvements especially in the trajectory control problem. Siciliano, *et al.* (1992) used the computed torque technique for control of the slow (rigid body)

subsystem and a linear PD controller for the fast (flexible) subsystem. Yesildirek, *et al.* (1994) proposed the use of neural networks for feedback linearization in the slow subsystem where no off-line training is needed.

Some effort has been devoted to the design of robust controllers for flexible manipulators. Franke (1986) presented a robust non-sliding mode (soft) Variable Structure Controller in the presence of plant uncertainties for a distributed parameter system. Baruh and Tadikonda (1989) considered the elastic terms of flexible manipulator equation to be deterministic disturbances, and reported on the robustness of point-wise-optimal control law in the presence of parameter uncertainties. Usually the controller of a flexible structure is designed based on its reduced-order model (ROM). In some applications the residual modes of system produce the well-known “spillover instability” problem. To avoid this problem Lin (1993) introduced a robust controller based on addition of a parallel or cascade connected residual mode filter (RMF) with an observer. Mohl (1993) verified the success of the ROM-RMF control approach through experiment. The excitation of residual modes by the control signal is called “control spillover”, which can cause unwanted, noise-like vibration in the system. One approach to reduce these residual vibrations is “shaped input” control. Using the system mode frequencies and damping ratios. Singh and Heppler (1993) shaped an impulse sequence which limited the vibrations of a multiple mode system. Magee and Book (1993) demonstrated that the impulse shaping scheme induces vibration in time-varying systems and then created a modified command filtering technique to eliminate the undesirable motion of flexible manipulators. Experimental results, presented by Drapeau and Wang (1993),

showed that a shaped-input controller can be effective and robust enough to reduce vibrations, even for nonlinear systems. Following their work, Zuo, *et al.* (1995) published experimental results for closed-loop shaped-input control and claimed the robustness of this approach to large magnitude disturbances.

Rovner and Cannon (1987) proposed an **adaptive control** algorithm based on the self-tuning regulator (STR) concept, for a very flexible link with an identification algorithm based on the recursive least-square method. Rovner and Franklin (1988) reported the experimental results of a load adaptive control algorithm based on a STR and Linear Quadratic Gaussian synthesis procedure. Hanagud, *et al.* (1990) developed explicit self-tuning adaptive control of flexible structures, using artificial intelligence techniques. Lucibello and Bellezza (1990) applied an STR scheme, least-square identification for the payload together with a nonlinear controller. Siciliano, *et al.* (1986) designed a robust controller based on Model Reference Adaptive Control (MRAC), for a trajectory following of flexible manipulators. Chen and Piontek (1992) proposed a decentralized adaptive scheme for on-line estimation of the “flexible arm uncertainty bound”. They then applied a robust control based on this estimation.

Following the successful application of **fuzzy logic** concepts in some engineering areas, they have been adopted in the control of robotic systems in general and flexible manipulators in particular in the past few years. A two part control law was designed by Lin and Lee (1993) in which the first part is responsible for system linearization by feedback, and the second uses a Fuzzy Logic Controller (FLC) to control the end-point position. Kubica and Wang (1993) suggested use of two parallel fuzzy

controllers, where one is designed to govern the rigid motion of the flexible arm and the second FLC is designed to attenuate the flexural vibrations. They used an existing model for a single flexible link robot for initial development of the FLCs. Lee, *et al.* (1994) proposed a general FLC design procedure to control the tip position of a very flexible link, without using a system mathematical model. Another pure model-free knowledge-based FLC which has a two level hierarchical structure was introduced by Akbarzadeh-T., *et al.* (1994). The higher level monitors link behavior and extracts features such as straight, oscillatory, gently curved. The lower level uses this informations to control the system. Moudgal, *et al.* (1994) presented an interesting fuzzy model reference learning controller, which can automatically synthesize a rule-base for a FLC and then tune it, so it can adapt to variations in the payload.

Another model-free approach which can be used in system identification and control is the **Artificial Neural Network (ANN)**, e.g., Newton and Xu (1993) introduced a neural network approach to on-line learning control of a flexible space manipulator. They used the concept of feedback-error-learning as a method of training a neural network. They claimed an 85% reduction in tracking error compared with conventional PID control. Takahashi and Yamada (1994) introduced an adaptive-type direct neural controller, and through experimental results they showed that the proposed ANN can control a flexible manipulator solely through its learning ability and without exact a priori knowledge of the system even under existing nonlinear disturbances such as dry friction. Donne and Özgüner (1994) addressed the control of flexible manipulators with partially known dynamics. They used the rigid body

dynamics to form a model of the known plant and controlled it by a model-based predictive control scheme. They utilized a neural net in a supervisory training mode to identify the unknown flexible dynamics, and then used an unsupervised neural net control scheme to provide corrective control signals. A similar identification and control scheme was also applied to a two link flexible manipulator by Lin and Yih (1996). In a significant paper Register, *et al.* (1996) claimed that most previously designed neural networks fail to converge for lightly damped flexible manipulators with NMP behavior. To overcome this limitation, they proposed a new cost function that includes a term related to the hub velocity which allows the amount of closed loop damping to be adjusted.

The success of **predictive controllers** in handling difficult systems such as those with time delays, open-loop instability, and nonminimum phase transfer functions, suggests their use in control of flexible manipulators. Cetinkunt and Wu (1992) developed a Predictive Adaptive Control (PAC) for tip position based on the non-dimensionalized dynamical model of a flexible robot arm. An adaptive Generalized Predictive Controller (GPC) was designed by Dugard, *et al.* (1992) for a two link flexible arm with significant variations in payload, and they presented some promising experimental results. This was followed by M'Saad, *et al.* (1993) who claimed that the linear quadratic control criterion is not suitable for tracking control when the plant model is unknown, and stated an objective based on pole placement concepts. For trajectory control of a Macro-micro manipulator which consists of a small rigid robot (micro) mounted on the tip of a long range flexible (Macro) arm., Yim (1993) suggested a combination of Variable Structure Control (VSC) for

the rigid part and predictive controller for the flexible part of the system. Yim and Singh (1996) developed a nonlinear predictive controller for a multilink flexible robot. They reported that this controller does not suffer from the instability of zero dynamics encountered in the VSC and feedback linearization end point controllers.

2.2.2 Force Control

An important problem in robotic control is that of controlling force when the robot makes contact with its environment, which can occur in many space and ground based applications such as grasping, grinding, assembly, etc.. Whitney (1987) presented a brief review of different types of force control strategies for rigid link robotic systems, and also discussed applications, stability and several unsolved problems in this area. Another relatively complete review of robotic force control was presented by Patarinski and Botev (1993).

Despite its importance, to date, much less attention has been paid to force control of flexible manipulators than to position control. Some early work of note in this area is that of Eppinger and Seering (1988) who developed models for tip force control based on linear second order concepts, and studied stability and control issues from this view point. In another work Eppinger and Seering (1992) explained how sensor/actuator noncollocation will introduce unstable modes in flexible force controlled systems. They modelled all types of flexibility by means of simple mass-spring-damper systems. Although these elements do create noncollocated sensor/actuator configurations, it is described in Appendix-F of this thesis that they do not possess nonminimum phase transfer functions. With much more accurate mod-

elling Li (1990) demonstrated an inherent limitation on the achievable performance of force controllers for flexible manipulators due to the structural flexibility of links, which are noncollocated sensor/actuator systems and also have nonminimum phase transfer functions.

Contact modelling and force control designs of flexible manipulators are based primarily on two approaches. The more common of these, is through the use of spring-like *soft contact models*. In this method the flexibility of the force sensor and/or environment is assumed to be significant. Choi and Krishnamorthy (1991) modelled a link using pinned-free B.C.s and then used a spring-like contact. They then applied a Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) scheme to control the force and position of a two-link flexible arm. In an effort toward contact control, Book and Kwon (1992) used inverse dynamics to plan the tip motion so that the free motion can halt very near the contact surface without collision. The B.C.s they used were pinned-inertia/pinned-mass, which provides more accurate models than does previous work, and they used a spring-damper contact model. Lin and Lee (1992) used a Timoshenko beam model with spring contact and assumed only the first flexural mode of the system. Latornell and Cherchas (1992) designed a force controller for a single flexible link with a significant approach velocity, and modeled the contact phenomenon as a spring-damper-mass system. They used Generalized Predictive Control (GPC), and the Recursive Least Square (RLS) method for system identification and reported difficulties in the use of adaptive control techniques when making and breaking contact.

It will be shown in this thesis that soft contact modelling is effective only when

the motor to link inertia ratio is not too high, and contact flexibility is at least as large as link flexibility. Chiou and Shahinpoor (1990) showed that the stability of a force controlled flexible manipulator strongly depends on contact stiffness when the controller has been designed using a soft contact model, and as stiffness increases the region of stability decreases, so that for infinitely stiff contact the system becomes unstable. This suggests that when the contact is stiff, rather than soft, a *stiff contact model* is more appropriate for control design purposes.

To obtain a stiff contact model, one approach is the Lagrange Multiplier method, which produces a set of differential and algebraic equations, with the latter representing the system constraints. Some researchers who used this method are: Yim and Singh (1993), Matsuno and Yamamoto (1994), Krishnan (1995), Pfeiffer, *et al.* (1996). Among the methods used to address this type of system is the decomposition method, which was first introduced by McClamroch and Wang (1988) for control of constrained rigid robots and then used by Hu and Ulsoy (1994-a,b) for force modelling and control of flexible link manipulators. Although this approach is very systematic, it is shown in Appendix-D of this thesis that the method is not always applicable for contact force modelling. To eliminate the dependent variables and reduce the order of differential equations in a constrained flexible manipulator system, Lew and Book (1993) used the Singular Value Decomposition (SVD) approach, which is limited to constraints with linear geometry. Another approach was introduced by Rocco and Book (1996) based on a coordinate partitioning procedure. In this method the dynamic equations of the constrained system are written in terms of a subset of the original generalized coordinates. One of advantages of this method

is that it can be applied to constraints with nonlinear geometries.

Some researchers have concentrated more on simultaneous position/force control of elastic manipulators than on control of contact only. Pfeiffer (1990) suggested a combination of feedforward and feedback for path and force control of elastic manipulators, with the feedforward evaluated off-line and only minor effects evaluated on-line and inserted into the feedback loop. He used only joint angle and strain gauge outputs in the feedback loop, and did not use any direct force sensor. Pfeiffer, *et al.* (1996) used a similar controlled flexible arm system for surface finishing purposes. Tzes and Yurkovich (1990) used a hybrid impedance control scheme for force control of a two-link flexible manipulator. They suggested an inner/outer loop with the inner loop decomposed into slow and fast components by means of singular perturbation theory. The slow part is then linearized using nonlinear feedback and input shaping is used for the fast part. The outer loop is responsible for robustness, disturbance rejection, etc. Borowiec, *et al.* (1993) used a similar idea of feedforward/feedback control with the feedforward component negating the effects of the rigid dynamics, and an adaptive PID feedback control which keeps the poles in specified locations. With almost the same structure, Borowiec and Tzes (1994) designed an implicit force controller (with no force sensing) in which they used an LQR in the feedback loop. In their next work, Borowiec and Tzes (1995) changed the feedback controller to a frequency-shaped implicit controller to eliminate the effects associated with control and observation spillover.

Matsuno and Yamamoto (1994) suggested a hybrid controller using a B-spline approximation and singular perturbation method which decompose the system into

a slow and a fast component. Since they used a state feedback controller for the fast component, they determined the minimum number of necessary strain gauges as $\frac{n+2}{3}$ to reduce the risk of unobservability, where n is the number of assumed modes. Later Matsuno and Tanaka (1995) compared the performances of an LQ controller with a low-pass characteristic, and an H_∞ controller for force control of a single link flexible manipulator. Both controllers were observer based without the use of a force sensor. They reported that the performance of the H_∞ controller was better than that of the LQ controller with low-pass characteristic, and also showed that the standard LQ controller without this low pass characteristic destabilizes the system due to control and observation spillover.

Yim and Singh (1993) designed a force-position control based on nonlinear inversion of an input-output map, and linear feedback stabilization. To avoid the instability of the zero dynamics (due to inversion of a NMP system), they suggested control of a parameterized output vector corresponding to a point close to the end point. They claimed that the zero dynamics would be stable or almost stable as long as the elements of this vector do not exceed some critical values. In a later work, Yim and Singh (1994) added a sliding mode controller to their previous controller to increase the robustness of the controlled system in the presence of flexible robot parameter and environmental constraint geometry uncertainties.

Instead of using a distributed parameter model, some researchers have designed the hybrid position/force controller based on the system's lumped-parameter model, e.g., Kim, *et al.* (1996), and Bossert, *et al.* (1996). The latter implemented an LQG controller. Modelling the link flexibility and inertia as lumped spring-mass system,

Yoshikawa. *et al.* (1996) designed a hybrid controller for a flexible-macro/rigid-micro manipulator. In this control algorithm, the macro part is controlled roughly to realize the desired trajectory, and suppress vibration. The micro part is controlled to compensate for the position and force errors due to elasticity in the macro part.

An interesting PhD thesis was presented by Latornell (1992) in the same field as the present work: however, fundamental differences exist between them. Some characteristics of Latornell's work are as follows. In dynamic modelling he assumed clamped-free B.C.s, and developed a spring-like soft contact model. for a two-link flexible manipulator. He used a GPC to control the contact force. To address the problem of an unknown contact stiffness he used an adaptive GPC algorithm. He presented extensive simulation results in his thesis. For comparison, some characteristics of the present work are listed. In dynamic modeling a pinned-inertia/free(pinned)-mass B.C.s is assumed, and a stiff contact model is developed for a single link flexible manipulator. UPC is used to control the contact force. The problem of presence of severe friction at the joint is addressed and a UPC-PID controller is introduced to solve this problem. Simulation and experimental results are presented in this work.

2.2.3 Control of Flexible Manipulators with Friction in the Joints

Although friction is unavoidable and sometimes severe, very little effort has been devoted to control of flexible link manipulators when static and dynamic friction are present in the joints. Since in this thesis all the proposed control systems are

verified experimentally, the effect of severe friction in the joint can not be neglected. and it is appropriate to briefly review the literature on this subject.

The study, modelling, and compensation of friction in general have attracted much attention throughout the history of science and engineering. A very good survey on this subject was presented by Armstrong-Hélouvry, *et al.* (1994). Haessig, Jr. and Friedland (1991) presented two friction models, one very accurate but found to be inefficient numerically, the other with a simpler structure and almost the same performance as many previous models suggested in the literature.

Many physical phenomena have been identified as friction effects, and combining all these effects into a single model is quite difficult. One of most complete models with good numerical efficiency has been suggested by Canudas de Wit, *et al.* (1993), who also recommend the use of friction observers for compensation. A model, linear in parameters, was introduced by Canudas de Wit, *et al.* (1991) which allowed on-line friction identification used in an adaptive compensation system. Popović, *et al.* (1993) proposed a step-by-step procedure for off line parameter identification of friction in a direct drive robotic arm. They used a feedforward model-based compensation technique.

Yoshida and Tanaka (1993) used a dither signal to linearize the friction effect and then controlled the position of a flexible arm. They assumed only the rigid body mode and the first vibration mode, so that the important effect of dither, excitation of the higher modes, was ignored. Another friction compensation method is the use of a high gain position feedback loop, which has provided good performances in some cases. Feliu, *et al.* (1993) modified this technique to control a nonminimum

phase flexible single arm system. They later applied the same approach to a two-link flexible arm. see Feliu, *et al.* (1994). In both works they modelled the flexible arm as a rigid link with one lumped flexibility (spring-like) effect. In the friction model they assumed only the Coulomb effect. Ignoring higher modes of the flexible link and also ignoring the important negative damping effect of friction at low velocity (sudden decrease of friction from static level to Coulomb level at threshold of motion), reduces the validity of the results.

2.3 Predictive Control

The class of Long Range Predictive Controllers (LRPC), which are able to predict system response multi-step time ahead and determine the required control sequence, has demonstrated some success in handling difficult systems, such as those with time delays, and even systems with open-loop instability, e.g., see Qi and Fisher (1993). They also have been quite successful at overcoming many of the well-known difficulties inherent in nonminimum phase (NMP) systems such as flexible structures. Morari and Lee (1991) discuss the benefits and drawbacks of predictive controllers. Some good surveys in this field were prepared by Garcia, *et al.* (1989), and Ricker (1991). Also recent developments were quoted in Clarke (1994).

A predictive controller, in general, determines future sequences of control signal such that the predicted values of a process output are close to a future reference trajectory. Any predictive controller must have four basic elements: 1. A process model which is used to predict the process output over a *prediction horizon*. 2. A reference trajectory. 3. A criterion function which is minimized to obtain the

controller output sequences. 4. A minimization procedure. To date, many predictive control algorithms have been proposed based on different choices of these elements. In this section a very brief overview is presented of this variety of controllers, without excessive detail.

“Model Predictive Control” (MPC) is a generic term which covers a large number of control algorithms. Classes of MPCs differ in detail but are usually formulated using a non-parametric, impulse or step response model to describe the process. Some proposed controllers in this class are: Model Predictive Heuristic Control (MPHC) proposed by Richalet, *et al.* (1978) and applied to several industrial processes; this controller with minor modifications was called Model Algorithm Control (MAC) by Bruijn and Verbruggen (1984), and its adaptive version Predictive Adaptive Control (PAC). Cutler and Ramaker (1980) introduced Dynamic Matrix Control (DMC). Internal Model Control (IMC) was suggested by Garcia and Morari (1982).

Another class of LRPCs is adaptive predictive control schemes. Even though the philosophy behind this class and MPCs is similar, there are significant differences between the models used. Adaptive schemes use *parametric* models such as ARMA, ARIMA, CARIMA, and transfer functions. Some well-known schemes in this class are: Extended Horizon Adaptive Control (EHAC) suggested by Ydstie (1984), Extended Prediction Self-Adaptive Control (EPSAC) proposed by De Keyser and Van Cauwenberghe (1985), MULTISTEP Multivariable Adaptive Regulator (MUSMAR) by Greco, *et al.* (1984), Predictive Bounding Control (PBC) by Veres and Norton (1993), Minimax PC by Tse, *et al.* (1993), and the most popular, Generalized Predictive Control (GPC) by Clarke, *et al.* (1987).

A comparative study between MAC, DMC, EHAC, and EPSAC was presented by De Keyser *et al.* (1988). Bruijn. *et al.* (1986) compared PAC and DMC with two other iterative predictive controllers called IDCOM and IPC. Also the relationship between GPC and proportional integral plus control (PIP) was discussed in Taylor. *et al.* (1994).

This wealth of choices in predictive control algorithms can complicate the problem of controller selection. Soeterboek. *et al.* (1990) therefore proposed *Unified Predictive Control* (UPC) which unifies a number of the well-known predictive controllers mentioned above. Using UPC the designer has access to most of the desirable properties of these controllers in a single framework simply by selecting the UPC parameters. Some design hints and tuning rules for UPC were presented by Soeterboek. *et al.* (1991-a). Soeterboek. *et al.* (1991-b), also suggested some modifications for use of UPC in nonlinear systems. Later, De Vries and Verbruggen (1994) extended the UPC for multivariable systems (MUPC). The control design procedure in the presence of process input/output constraints has been also discussed in Soeterboek (1991) and (1992). Soeterboek *et al.* (1991-c) implemented a unified predictive controller in a transonic wind tunnel to control the Mach number. Jeronymo and Muto (1995) used UPC for position and tracking control of an electro-hydraulic servo system. UPC is the main controller used in this thesis.

Although most predictive control design procedures use linear, discrete-time, and input/output models, it is possible to extend them to nonlinear, continuous and state space representations. For example, Lee *et al.* (1994) presented a state-space based predictive controller. Demircioglu (1994), Lu (1995), and Soroush and

Kravaris (1996) derived predictive controllers for systems described by continuous-time models. Some of the many researchers who suggested different PC design procedures for nonlinear systems are: Feng, *et al.* (1991), Biegler (1991), Abu El Ata-Doss, *et al.* (1992), and Pröll and Karim (1994).

In recent years many predictive controllers have been introduced which do not use the conventional models. This is usually due to a lack of exact knowledge of the system dynamics and/or due to the presence of severe nonlinearities in the system. For example Hangos *et al.* (1994) introduced a predictive controller scheme based on qualitative models (QMBPC). Use of fuzzy models (predictors) has been suggested by some researchers, e.g., Skrjanc and Matko (1994), Linkens and Kandiah (1996). Other group of control system designers such as Tan and De Keyser (1994), and Schenker and Agarwal (1995), has used neural network models as predictors. Two comparative studies on the efficiency of different neural network models in predictive controllers were prepared by Declercq and De Keyser (1996), and Botto and Costa (1996).

Richalet, *et al.* (1987) proposed predictive functional control (PFC) where the future control signal is assumed to be a linear combination of a priori known functions. The selection of these basis functions depends on the reference trajectory and on the process. A new design technique termed Evolutionary Predictive Control (EPC) was introduced by Goggos and King (1996). This technique uses evolutionary algorithms to compute the optimum values of the predictive controller parameters.

The presence of constraints on different parts of system such as actuator saturation or change rate (speed) limitation, usually degrades the performance dramati-

cally and can even cause instability in the controlled system. Solving such a problem requires an on-line optimization that produces a nonlinear controller. This problem has been addressed by many researchers in the field of predictive control, e.g.. MacArthur (1993) considered process and control output, and end-time constraints. Zafriou (1991) examined the constraint effects on the DMC robustness. De Nicolao and Scattolini (1994) studied the stability of predictive controllers in the presence of output constraints. Rossiter and Kouvaritakis (1994) proposed a constrained GPC algorithm.

During the last decade, the class of predictive controllers has been successfully implemented in many industrial applications. Some of these were mentioned earlier in this section, and some of their applications in position and force control of flexible robots were described in a previous section. To illustrate the wide industrial application of these controllers the following brief list is presented. Predictive controllers have been applied in the NASA deep space network antennas by Gawronski (1994), in a crude oil distillation tower by Richalet (1993), in impact-contact motion of robots by De Carufel and Neculescu (1995), in underwater vehicles by Kodogianis, *et al.* (1996), in multilink robot manipulator by Kaymaz and Mitra (1995), in control of motor drives by Dumur *et al.* (1996), in the flight control system by Singh (1995), and in combined sewer system by Gelormino and Ricker (1994).

Chapter 3

Dynamic Model

3.1 System Description

Three major sources of flexibility in robot manipulators performing contact tasks are joint flexibility, link flexibility, and environment flexibility. To study the effects of these sources, it is a common simplification to consider their influences separately, and since link flexibility is the concern of the present study, the joint and environment are considered to be rigid.

As mentioned earlier, inclusion of link flexibility greatly complicates the equations of motion, particularly with many links, and as the objective of this study is an understanding of control issues associated with forces imposed by flexible links, it is deemed sufficient to begin the analysis and design with only a single flexible link. Here a single flexible link with a Direct Drive dc torque motor and an incremental encoder at the base and with a force sensor at the tip, is the system of study, see Figure-3.1

Figure 3.1: Schematic of system

In the figure $\Theta(t)$ denotes the planar rotational motion of an imaginary rigid system, with an $(x - y)$ coordinate system attached to it. The overall hub displacement is denoted by $\theta(t)$, which includes both the rigid body and flexural effects. $y(x, t)$ denotes the deflection of the beam from its imaginary rigid situation (x -axis), while $W(x, t)$ is the composite displacement (deflection + rigid body motion). J_h is hub (motor) moment of inertia, m_p is payload mass (load cell+load). $F(t)$ is the reaction force of the environment on the beam tip. γ is the beam structural damping coefficient and c_h is the joint damping coefficient. Sensor compliance is ignored here as its order of magnitude is much less than that of the link.

The overall motion of the system is divided into two major phases. The first is with no contact between the beam and its environment. This is called *Unconstrained*

motion, and tip position is to be controlled here. The second phase is when link and environment are in contact for *Constrained motion*. A force controller is to be designed for this phase. In the case of switching between these two phases, a collision will occur if the link has an approach velocity. This causes a bouncing or making/breaking contact phenomenon.

The studies in this research are based on an experimental set up which is described on Appendix-A.1. The physical parameters of this set up were obtained either from manufacturer's manuals or by means of experimental identification in chapter-4.

The position of a point on the beam can be written in inertial coordinates (attached to the ground) as:

$$\begin{Bmatrix} X \\ W \end{Bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \quad (3.1)$$

Since control of force is the main objective of this research, the system is mostly studied around the contact position ($\Theta = 0$), and it is quite reasonable to linearize equation (3.1) in this neighborhood:

$$\begin{Bmatrix} X \\ W \end{Bmatrix} = \begin{bmatrix} 1 & -\Theta \\ \Theta & 1 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \quad (3.2)$$

For linear deflections and small rigid body motions around the contact position where $(y\Theta)$ is small, the relation between the angles can be written as:

$$\theta(t) \cong \left. \frac{\partial W}{\partial X} \right|_{x=0} = \left. \frac{\partial W}{\partial x} \frac{\partial x}{\partial X} \right|_{x=0} \cong \Theta(t) + y'(0, t) \quad (3.3)$$

where prime (') is the partial derivative with respect to x , $(\frac{\partial}{\partial x})$.

Using equation (3.2), it also can be concluded that

$$\frac{\partial^i W}{\partial x^i} = \frac{\partial^i y}{\partial x^i} \quad \text{for : } i \geq 2 \quad (3.4)$$

After differentiation with respect to time and recognizing that $\dot{x} = 0$, the absolute velocity of the point can be written as:

$$v(x, t) = \dot{W}(x, t) = \dot{\Theta}(t)x + \dot{y}(x, t) \quad (3.5)$$

where dot (·) is the partial derivative with respect to time ($\frac{\partial}{\partial t}$).

3.2 Distributed Parameter Model

The kinetic and potential energies, and external work for this system are:

$$\begin{aligned} E_{kin} &= \frac{m}{2} \int_0^L v(x, t)^2 dx + \frac{J_h}{2} \dot{\theta}(t)^2 \\ E_{pot} &= \frac{EI}{2} \int_0^L y''(x, t)^2 dx = \frac{EI}{2} \int_0^L W''(x, t)^2 dx \\ \mathcal{Q} &= \tau(t)\theta(t) - F(t)W_L(t) \end{aligned} \quad (3.6)$$

where W_L is the link tip displacement. The second term in the work equation represents the virtual work. Some of the most important applications of flexible manipulators are in space, where robots work under the zero gravity condition. To simulate this situation the motion of the flexible link has been assumed to be in the horizontal plane. For this reason gravity does not appear in the potential energy equation (3.6).

The Lagrangian is defined as: $\mathcal{L} = E_{kin} - E_{pot}$. Because the parameters have been distributed along the link, using the *Lagrangian density* is convenient to derive

the flexural motion equation (Nicosia, *et al.* 1986, 1990). Using equations (3.4),(3.5) and (3.6), the Lagrangian density can be obtained as

$$\Lambda = \frac{d\mathcal{L}}{dx} = \frac{m}{2}\dot{W}^2 - \frac{EI}{2}W''^2 \quad (3.7)$$

The density version of Lagrange's equations for a distributed parameter system is:

$$\frac{d}{dt}\left(\frac{\partial\Lambda}{\partial\dot{W}}\right) - \left[\frac{\partial\Lambda}{\partial W} + \frac{d^2}{dx^2}\left(\frac{\partial\Lambda}{\partial W''}\right)\right] = \frac{\partial Q}{\partial W} \quad (3.8)$$

The partial differential equation of motion can be obtained by applying equation (3.7) to (3.8) as:

$$EIW''''(x, t) + m\ddot{W}(x, t) = 0 \quad (3.9)$$

Equation (3.9) is called the *Euler-Bernoulli* beam model. Two well-known models for beams are the Timoshenko beam model, and the Euler-Bernoulli model. In the first the effects of rotary inertia and shear deformation are considered, while they are not considered in the Euler-Bernoulli model. Spector and Flashner (1989) have shown that the Euler-Bernoulli model is valid for long and thin beams, where $\frac{\omega_c}{\omega_n} \gg 20$ where ω_c is the Timoshenko shear mode cutoff frequency, and ω_n is the n^{th} natural frequency of the Euler- Bernoulli model. The above ratio for the beam used in this work is about 7000, (Appendix-A.2). Therefore the effects of rotary inertia and shear deformation are not included in equations (3.6) and (3.9).

Using equations (3.4) and (3.5), the Euler-Bernoulli model can be written in terms of rigid body coordinates as:

$$EIy''''(x, t) + m\ddot{y}(x, t) + mx\ddot{\Theta}(t) = 0 \quad (3.10)$$

During constrained motion where $\Theta = \dot{\Theta} = \ddot{\Theta} = \dots = 0$, the inertial and rigid body coordinate systems coincide.

3.3 Spatially Discretized Model

Using the separation principle partial differential equation (3.9) can be separated into space and time dependent functions. Then by means of the *Modal Expansion* method the general solution (free response) of the equation can be written as a linear combination of its basic solutions:

$$W(x, t) = \sum_{i=0}^{\infty} \phi_i(x) q_i(t) \quad (3.11)$$

Where $\phi_0(x) = x$, and $q_0(t) = \Theta(t)$ are used only for unconstrained motion. Inserting the above equation into (3.9) results in

$$\frac{\phi_i''''(x)}{\phi_i(x)} = -\frac{m}{EI} \frac{\ddot{q}_i(t)}{q_i(t)} = \lambda_i^4 \quad i = 0, \dots, \infty \quad (3.12)$$

Where λ_i s are the eigenvalues of the system. Because $\phi_0'''' = 0$, it is concluded that for free oscillation $\ddot{\Theta} = 0$ and the eigenvalue related to rigid body motion of the system is zero: $\lambda_0 = 0$. The above equation leads to an infinity of ordinary differential equations:

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = 0 \quad i = 1, \dots, \infty \quad (3.13)$$

$$\phi_i''''(x) - \lambda_i^4 \phi_i(x) = 0 \quad i = 1, \dots, \infty \quad (3.14)$$

The first set represents the free undamped dynamics of the system, where the ω_i s are the system natural frequencies, defined as:

$$\omega_i = \sqrt{\frac{EI}{m}} \lambda_i^2 \quad (3.15)$$

A general solution for the second equation is:

$$\phi_i(x) = A_1 \cos(\lambda_i x) + A_2 \sin(\lambda_i x) + A_3 \cosh(\lambda_i x) + A_4 \sinh(\lambda_i x) \quad (3.16)$$

Where A_1 to A_4 are determined based on the system boundary conditions.

3.3.1 Constrained Motion

It should be noted again that for constrained motion $\Theta = 0$ so that $W(x, t) = y(x, t)$.

The Problem of Symmetrical Boundary Conditions:

The easiest method of finding the overall dynamical model of the system is by finding the model of each main component: motor, beam, sensor; and then combining them. For constrained motion with contact between link and environment, this method leads to the simply supported beam B.C.s, as:

$$W(0, t) = 0, \quad W(L, t) = 0, \quad M(0, t) = 0, \quad M(L, t) = 0 \quad (3.17)$$

Where $M(x, t) = EIW''(x, t)$ is the internal moment in the link. These are *Symmetrical Boundary Conditions* and result in a set of symmetrical eigenfunctions. The eigenvalues and eigenfunctions for these B.C.s are:

$$\begin{aligned} \lambda_i &= \frac{i\pi}{L} \\ \phi_i(x) &= \sin(\lambda_i x) \end{aligned} \quad (3.18)$$

It will be seen that to model the force, some integration along the length of the beam is necessary and because the eigenfunctions are symmetrical some terms will cancel each other out. This introduces a problem of dependence on the number of modes. To quickly outline this problem the shear force relation is used:

$$F(t) = -V(L, t) = EIW'''(L, t) = EI \sum_{i=1}^{\infty} \phi_i'''(L) q_i(t) \quad (3.19)$$

Using equation (3.18) in (3.19) produces a model for contact force as:

Figure 3.2: Step response of a simply supported beam model. The severe number of mode dependency of response is observable.

$$\begin{aligned}
 F(t) &= EI \sum_{i=1}^{\infty} \left(\frac{i\pi}{L} \right)^3 \cos(i\pi) q_i(t) \\
 &= EI \left(\frac{\pi}{L} \right)^3 \sum_{i=1}^{\infty} i^3 (-1)^i q_i(t)
 \end{aligned} \tag{3.20}$$

It is seen that the sign of the steady state value of F depends on the number of assumed modes through the term $(-1)^i$. Figure (3.2) shows the response of system for a 3rd and 4th order model, which significantly differ.

Figure 3.3: The first four mode shapes of system in constrained motion.

Normal Modes:

To address this problem, the effect of motor inertia is included, introducing a set of *unsymmetrical boundary conditions*:

$$W(0, t) = 0, \quad W(L, t) = 0, \quad M(0, t) = J_h \ddot{\theta}(t), \quad M(L, t) = 0 \quad (3.21)$$

The first, second, and fourth B.C.s can be rewritten in the following form:

$$\phi_i(0) = 0, \quad \phi_i(L) = 0, \quad \phi_i''(L) = 0 \quad (3.22)$$

Substituting into equation (3.16), and solving for the coefficients produces the *Mode Shape Equation*:

$$\phi_i(x) = \frac{\sin[\lambda_i(L - x)]}{\sin(\lambda_i L)} - \frac{\sinh[\lambda_i(L - x)]}{\sinh(\lambda_i L)} \quad (3.23)$$

As can be seen, the mode shape functions do not have a symmetrical form, (Figure-3.3). thus they will not cause the number of mode dependency problem in the following.

Characteristic Equation:

Using equations (3.13) and (3.15), the third B.C. can be written as:

$$\begin{aligned}
 EI W''(0, t) &= J_h \bar{W}''(0, t) \\
 EI \sum_{i=1}^{\infty} \phi_i''(0) q_i(t) &= J_h \sum_{i=1}^{\infty} \phi_i'(0) \tilde{q}_i(t) \\
 EI \phi_i''(0) &= -J_h \phi_i'(0) \omega_i^2 \\
 \phi_i''(0) &= -\frac{J_h \lambda_i^4}{m} \phi_i'(0)
 \end{aligned} \tag{3.24}$$

Substituting equation (3.23) into (3.24) results in the following *characteristic equation*:

$$\frac{2mL^3}{J_h} = (\lambda_i L)^3 \left(\frac{1}{\tanh(\lambda_i L)} - \frac{1}{\tan(\lambda_i L)} \right) \tag{3.25}$$

This equation has an infinite number of solutions, and the system therefore has an infinite number of eigenvalues and degrees of freedom (actually an idealization of reality). Equation (3.25) was numerically solved noting that a stable numerical solution of this equation required that it be rewritten in a new form:

$$\begin{aligned}
 \tan(\beta_i) &= \frac{a(\beta_i)^3 \tanh(\beta_i)}{a(\beta_i)^3 - \tanh(\beta_i)} \\
 \beta_i &= \lambda_i L \\
 a &= \frac{J_h}{6J_b}
 \end{aligned} \tag{3.26}$$

Where the β_i are the *non-dimensionalized eigenvalues* of the system, and $J_b = \frac{mL^3}{3}$

Figure 3.4: Roots of characteristic equation of constrained motion. Dotted and solid lines represents the left and right side of equation (3.26), respectively.

is the link moment of inertia about the joint. The results for the first five eigenvalues of the system are presented in Figure (3.4) and Table (3.1). The values of J_h , m , and L are based on the experimental set up, Table (A.1). These values show that the effect of motor inertia has shifted the natural frequencies of all modes to the right. It is also observed that

$$\lambda_{i+1}L = \lambda_iL + \pi \quad \text{for } i \geq 4 \quad (3.27)$$

Table 3.1: Natural frequencies of constrained system.

i	$\lambda_i L$	λ_i	ω_i rad/s	f_i Hz
1	2.5	1.91	16.15	2.57
2	4.25	3.24	46.45	7.39
3	7.12	5.44	131	20.84
4	10.22	7.80	270	42.95
5	13.36	10.19	460	73.28

Orthogonality Property Of Mode Shapes:

The mode shapes of (3.23) possess an orthogonality property as follows:

$$\begin{aligned}
 EI \int_0^L \phi_i(x) \phi_j''''(x) dx &= EI \phi_i \phi_j''' \Big|_0^L - EI \int_0^L \phi_i' \phi_j''' dx \\
 &= EI \phi_i \phi_j''' \Big|_0^L - EI \phi_i' \phi_j'' \Big|_0^L + EI \int_0^L \phi_i'' \phi_j'' dx \quad (3.28)
 \end{aligned}$$

Using the third line of equation (3.24) and B.C.s (3.22), the above equation can be rewritten as:

$$EI \int_0^L \phi_i \phi_j'''' dx + J_h \omega_j^2 \phi_i'(0) \phi_j'(0) = EI \int_0^L \phi_i'' \phi_j'' dx \quad (3.29)$$

Interchanging indices i and j in equation (3.29), and subtracting the resulting equation from (3.29) while noting that $EI \phi_i'''' = m \omega_i^2 \phi_i$ (concluded from 3.9), results in

$$(\omega_j^2 - \omega_i^2) \left[J_h \phi_i'(0) \phi_j'(0) + m \int_0^L \phi_i(x) \phi_j(x) dx \right] = 0 \quad (3.30)$$

For $i \neq j$ the term within the brackets is zero, and for $i = j$ it is a positive value $\triangleq M_i$. From equation (3.29) it can be concluded that:

$$EI \int_0^L \phi_i'' \phi_j'' dx = \left[J_h \phi_i'(0) \phi_j'(0) + m \int_0^L \phi_i(x) \phi_j(x) dx \right] \omega_j^2 \quad (3.31)$$

Therefore the orthogonality property of the mode shapes can be written as:

$$\begin{aligned} M_j \delta_{ij} &= J_h \phi_i'(0) \phi_j'(0) + m \int_0^L \phi_i(x) \phi_j(x) dx \\ K_j \delta_{ij} &= M_j \omega_j^2 \delta_{ij} = EI \int_0^L \phi_i''(x) \phi_j''(x) dx \\ \delta_{ij} &= \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} \end{aligned} \quad (3.32)$$

Where M_j and K_j are called the j^{th} *generalized inertia and stiffness* of the system.

Substituting (3.23) into (3.32) the values for M_j can be determined as:

$$M_j = \frac{mL}{2} \left[\frac{3}{a\beta_j^4} + \frac{1}{\sin^2 \beta_j} - \frac{1}{\sinh^2 \beta_j} \right] \quad (3.33)$$

Where a and β_j were defined in (3.26).

Ordinary Differential Equations of Motion:

Characteristics of the unforced system such as natural frequencies and mode shapes have now been determined, but in order to deal with forced oscillations equation (3.13) must account for the effect of external forces and torques. Since the external torque and force act point wise on the system, an integral version of equation (3.9) is used.

Substituting equation (3.11) into (3.6) and using equations (3.3) and (3.5), the Lagrangian and external work derivatives with respect to generalized coordinates q_j

become

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \dot{q}_j} &= \underbrace{\left[J_h \phi_j'(0)^2 + m \int_0^L \phi_j(x)^2 dx \right]}_{M_j} \dot{q}_j + \underbrace{\sum_{i=1, i \neq j}^{\infty} \left[J_h \phi_i'(0) \phi_j'(0) + m \int_0^L \phi_i(x) \phi_j(x) dx \right]}_0 \dot{q}_i \\
\frac{\partial \mathcal{L}}{\partial q_j} &= - \underbrace{\left(EI \int_0^L \phi_j''(x)^2 dx \right)}_{K_j} q_j - \sum_{i=1, i \neq j}^{\infty} \underbrace{\left(EI \int_0^L \phi_i''(x) \phi_j''(x) dx \right)}_0 q_i \quad (3.34) \\
\frac{\partial \mathcal{Q}}{\partial q_j} &= \phi_j'(0) \tau(t) - \underbrace{\phi_j(L)}_0 F(t) = \phi_j'(0) \tau(t)
\end{aligned}$$

The partial differential equation of motion can be spatially discretized into an infinite number of ordinary differential equations as:

$$M_j \ddot{q}_j + K_j q_j = \phi_j'(0) \tau \quad \text{for: } j = 1, \dots, \infty \quad (3.35)$$

Where: $\phi_j'(0) = \frac{1}{2L3_j^2}$.

Rayleigh's Distributed Damping Model:

Morris and Vidyasagar (1990) show that design of a finite dimensional stabilizing controller is facilitated by incorporating Rayleigh's damping model into the Euler-Bernoulli beam. In this model, distributed external damping is assumed to be proportional to link deflection rate $\dot{W}$. Usually this is applicable to systems in viscous fluid environments such as water or oil. Since flexible manipulators are usually not used in such environments, no distributed external damping is considered in this study. The distributed internal damping, which is taken to be structural damping, is assumed to be proportional to strain rate $\dot{W}''$, (Humar 1990). To satisfy the orthogonality properties, we can take

$$\text{Structural Damping} = \gamma EI \dot{W}'' \quad (3.36)$$

Figure 3.5: An element of the link.

One may add this effect to the bending moment as shown in Figure- 3.5. The dissipation function $\mathcal{D}$, can be written as:

$$\mathcal{D} = \frac{\gamma EI}{2} \int_0^L \dot{y}''(x, t)^2 dx = \frac{\gamma EI}{2} \int_0^L \dot{W}'''(x, t)^2 dx \quad (3.37)$$

The dissipation term which is to be added to the Lagrange Density Equation (3.8) is:

$$\left[\frac{\partial \mathcal{D}'}{\partial \dot{W}} + \frac{d^2}{dx^2} \left(\frac{\partial \mathcal{D}'}{\partial \dot{W}''} \right) \right] \quad (3.38)$$

Where $\mathcal{D}' = \frac{\gamma EI}{2} \dot{W}'''(x, t)^2$ is the derivative of (3.37) with respect to x . This term modifies the Euler-Bernoulli equation (3.9) to:

$$EI W''''(x, t) + \gamma EI \dot{W}''''(x, t) + m \ddot{W}(x, t) = 0 \quad (3.39)$$

In addition, the dissipation term for the Lagrange equation with respect to $\dot{q}_j$ is:

$$\frac{\partial \mathcal{D}}{\partial \dot{q}_j} = \underbrace{\gamma \left(EI \int_0^L \phi_j''(x)^2 dx \right)}_{K_j} \dot{q}_j + \gamma \sum_{i=1, i \neq j}^{\infty} \underbrace{\left(EI \int_0^L \phi_i''(x) \phi_j''(x) dx \right)}_0 \dot{q}_i \quad (3.40)$$

Adding this term to the Lagrange equation modifies the set of ordinary differential equations (3.35) to:

$$M_j \ddot{q}_j + \gamma K_j \dot{q}_j + K_j q_j = \phi'_j(0) \tau \quad \text{for: } j = 1, \dots, \infty \quad (3.41)$$

Using equation (3.32), the above set of equations can be rewritten as:

$$\ddot{q}_j + 2\zeta_j \omega_j \dot{q}_j + \omega_j^2 q_j = b_j \tau \quad \text{for: } j = 1, \dots, \infty \quad (3.42)$$

Where $b_j = \frac{\phi'_j(0)}{M_j}$, and ζ_j is the j^{th} modal damping ratio of flexible system, and is defined as:

$$\zeta_j = \frac{\gamma \omega_j}{2} \quad (3.43)$$

It is seen in this damping model that the damping ratio of each mode is proportional to the natural frequency of the mode.

Reduced Order Model

In order to facilitate the design of a finite-dimensional controller, the number of equations in (3.42) is truncated from infinity to a finite number n :

$$\ddot{q}_j + 2\zeta_j \omega_j \dot{q}_j + \omega_j^2 q_j = b_j \tau \quad \text{for: } j = 1, \dots, n \quad (3.44)$$

Here, the actuator (motor) frequency bandwidth is used to determine n . (Krishnan and Vidyasagar 1988). If the frequency bandwidth of the actuator is $[0, \omega_c]$, where ω_c is the cut off frequency, n is taken such that

$$\omega_n < \omega_c < \omega_{n+1} \quad (3.45)$$

where ω_n is the n^{th} natural frequency of system (link + motor). The reason for omitting oscillatory modes beyond the n^{th} is that these natural frequencies lie beyond

the motor bandwidth , so the actuator is unable to excite them. This is appropriate when disturbances are ignored, and when n is small.

The system transfer function from actuator torque input to overall displacement of a point at x can now be written as:

$$\frac{W(x, s)}{\tau(s)} = \sum_{j=1}^n \frac{\phi_j(x)b_j}{s^2 + 2\zeta_j\omega_j s + \omega_j^2}. \quad (3.46)$$

and equations (3.44) can be presented in state space form as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\tau \quad (3.47)$$

where:

$$\mathbf{x} = \begin{Bmatrix} q_1 \\ \dot{q}_1 \\ \vdots \\ q_n \\ \dot{q}_n \end{Bmatrix}, \mathbf{A} = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ -\omega_1^2 & -2\zeta_1\omega_1 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix}, \mathbf{B} = \begin{Bmatrix} 0 \\ b_1 \\ \vdots \\ 0 \\ b_n \end{Bmatrix} \quad (3.48)$$

Joint Viscous Damping Effect:

For a complete system model the effect of motor dynamics must be considered. From Appendix-B the dynamics of the motor and the link is:

$$\tau_e = k_t u = J_h \ddot{\theta} + c_h \dot{\theta} - M_0 \quad (3.49)$$

where: $\tau_e = k_t u$ = motor electrical torque; u = command voltage; M_0 = beam bending moment at the base ($x=0$). k_t and c_h (the combined effect of joint damping and back emf) are described in Appendix-B. The effect of motor inertia, $J_h \ddot{\theta}$, has

already been included in the hub-beam dynamics. equation (3.21). Therefore

$$\tau = J_h \ddot{\theta} - M_0 = \tau_e - c_h \dot{\theta} \quad (3.50)$$

The output equation for motor angular velocity can be written as

$$\dot{\theta} = C_{\dot{\theta}} x \quad (3.51)$$

$$C_{\dot{\theta}} = [0 \quad \phi'_1(0) \quad \dots \quad 0 \quad \phi'_n(0)]$$

Substituting equation (3.50) into (3.47) and using the above equation, the state space representation of the entire system becomes:

$$\dot{x} = A_b x + B \tau_e \quad (3.52)$$

where: $A_b = A - c_h B C_{\dot{\theta}}$.

3.3.2 Unconstrained Motion

Barbieri and Özgüner (1988) showed when the beam-to-hub inertia ratio is not sufficiently small (when $\frac{J_b}{J_h} \not\ll 0.1$), the clamped/free mode shapes and natural frequencies do not approximate the true solution with good accuracy. Since this ratio is $\frac{J_b}{J_h} \approx 6$ here, pinned-inertia B.C. is appropriate for the link base. The effect of payload mass at the free end of the link is also considered here. The pinned-inertia/free-mass B.C.s are therefore:

$$W(0, t) = 0, \quad M(0, t) = J_h \ddot{\theta}(t), \quad V(L, t) = -m_p \ddot{W}(L, t), \quad M(L, t) = 0 \quad (3.53)$$

Using equation (3.11) these B.C.s can be rewritten in the following form:

$$\phi_i(0) = 0, \quad \phi_i''(0) = -\frac{J_h \omega_i^2}{EI} \phi_i'(0), \quad \phi_i''(L) = 0, \quad \phi_i'''(L) = -\frac{m_p \omega_i^2}{EI} \phi_i(L) \quad (3.54)$$

Table 3.2: Special cases for unconstrained motion characteristic equation of (3.56), where $\beta_i = \lambda_i L$.

CASE	J_h	m_p	ϱ_i	ϑ_i	Characteristic Equation
Pinned-Free	0	0	∞	0	$\tan(\beta_i) = \tanh(\beta_i)$
Clamped-Free	∞	0	0	0	$1 + \cos(\beta_i) \cosh(\beta_i) = 0$
Simply Supported	0	∞	∞	∞	$\sin(\beta_i) = 0$
Barbieri and Özgüner (1988)	$\checkmark$	0	$\checkmark$	0	$[1 + \cos(\beta_i) \cosh(\beta_i)] +$ $+\varrho_i[\sin(\beta_i) \cosh(\beta_i) - \sinh(\beta_i) \cos(\beta_i)] = 0$
Constrained Motion	$\checkmark$	∞	$\checkmark$	∞	$\frac{1}{\tanh(\beta_i)} - \frac{1}{\tan(\beta_i)} = 2\varrho_i$ equation (3.25)

The first, second, and third B.C.s result in the following mode shapes:

$$\begin{aligned} \phi_i(x) &= \cos(\lambda_i x) - \cosh(\lambda_i x) + (\varrho_i - \varepsilon_i) \sin(\lambda_i x) + (\varrho_i + \varepsilon_i) \sinh(\lambda_i x) \\ \text{where: } \varrho_i &= \frac{3}{(\lambda_i L)^3} \frac{J_b}{J_h} \\ \text{and: } \varepsilon_i &= \frac{\cos(\lambda_i L) + \cosh(\lambda_i L) + \varrho_i[\sin(\lambda_i L) - \sinh(\lambda_i L)]}{\sin(\lambda_i L) + \sinh(\lambda_i L)} \end{aligned} \quad (3.55)$$

From the fourth B.C. the characteristic equation is obtained as

$$\begin{aligned} &[1 + \cos(\lambda_i L) \cosh(\lambda_i L)] + 2\varrho_i \vartheta_i \sin(\lambda_i L) \sinh(\lambda_i L) \\ &+ (\varrho_i - \vartheta_i)[\sin(\lambda_i L) \cosh(\lambda_i L) - \sinh(\lambda_i L) \cos(\lambda_i L)] = 0 \end{aligned} \quad (3.56)$$

where: $\vartheta_i = \frac{m_p}{mL} \lambda_i L$. Some special cases for this equation are shown in Table-3.2.

A numerical solution of equation (3.56) was performed, and the results for the first

Table 3.3: Natural frequencies of unconstrained system.

i	$m_p = 0.7 K_g$		$m_p = 0 K_g$		Difference%
	$\lambda_i L$	$\omega_i \frac{rad}{sec}$	$\lambda_i L$	$\omega_i \frac{rad}{sec}$	
0	0	0	0	0	N/A
1	2.67	18.42	2.93	22.18	9.8%
2	4.39	49.79	4.88	61.52	11.2%
3	7.25	135.9	7.89	161.0	8.9%
4	10.32	275.1	11.01	313.2	6.6%
5	13.44	466.7	14.14	516.8	5.27%

five eigenvalues are presented in Table- 3.3. The eigenvalue for $i = 0$ represents the rigid body motion of system. The payload affects primarily the first four modes, with the greatest influence on the second.

Since the kinetic energy of the unconstrained motion with the payload is:

$$E_{kin} = \frac{m}{2} \int_0^L v(x,t)^2 dx + \frac{J_h}{2} \dot{\theta}(t)^2 + \frac{m_p}{2} \dot{W}_L^2 \quad (3.57)$$

the orthogonality property becomes:

$$M_j \delta_{ij} = m_p \phi_i(L) \phi_j(L) + J_h \phi'_i(0) \phi'_j(0) + m \int_0^L \phi_i(x) \phi_j(x) dx \quad (3.58)$$

The other equation of orthogonality is similar to the second row in equations (3.32). Determining ω_i s and M_i s and using the eigenfunctions of (3.55), the differential equations of motion can be written as in (3.44), and the state space representation of the unconstrained system is the same as in (3.47), where the state vector, system.

and control matrices are:

$$\mathbf{x} = \begin{Bmatrix} \Theta \\ \dot{\Theta} \\ q_1 \\ \dot{q}_1 \\ \vdots \\ q_n \\ \dot{q}_n \end{Bmatrix}, \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & -\omega_1^2 & -2\zeta_1\omega_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & \dots & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix}, \mathbf{B} = \begin{Bmatrix} 0 \\ b_0 \\ 0 \\ b_1 \\ \vdots \\ 0 \\ b_n \end{Bmatrix} \quad (3.59)$$

where: $b_0 = \frac{1}{J_h + J_b + m_p L^2}$. The output equation for unconstrained motion is:

$$W_L = C_{w_L} \mathbf{x} \quad (3.60)$$

$$C_{w_L} = [L \ 0 \ \phi_1(L) \ 0 \ \dots \ \phi_n(L) \ 0]$$

3.4 Contact Model

3.4.1 Contact Classification Criteria

As mentioned earlier, there are many applications in which contact must be considered stiff. The opposite situation also occurs, so that it is important to have a quantitative parameter whose value determines whether a contact is to be considered soft or stiff.

Assuming that the composite contact stiffness (end-effector and environment) is k , see Figure-3.6, the first, third, and fourth B.C.s of (3.21) are valid here and the

Figure 3.6: Soft contact schematic.

second condition changes to:

$$V(L, t) = -EIW'''(L, t) = -kW(L, t) \quad (3.61)$$

which can be simplified to:

$$\phi_i(L) = \frac{EI}{k} \phi_i'''(L) \quad (3.62)$$

Based on these B.C.s the characteristic equation becomes:

$$(k_{b/c} - a) \left[\frac{1}{\tanh(\beta_i)} - \frac{1}{\tan(\beta_i)} \right] = \frac{1}{\beta_i^3} - 2ak_{b/c}\beta_i^3 \left[\frac{1 + \cos(\beta_i) \cosh(\beta_i)}{\sin(\beta_i) \sinh(\beta_i)} \right] \quad (3.63)$$

where: $k_{b/c} = \frac{EI}{2kL^3}$. Some special cases for this equation are shown in Table-3.4.

Equation (3.63) was numerically solved, and in order to have a stable numerical algorithm it was rewritten in the form of

$$\tan(\beta_i) = \frac{(k_{b/c} + a) \tanh(\beta_i)}{(k_{b/c} + a) - \tanh(\beta_i) f_{RHS}} \quad (3.64)$$

where f_{RHS} is the right hand side of equation (3.63). In Figure-3.7 the dashed and solid lines represent the left and right hand sides of equation (3.64) respectively. It is seen that for $k \geq 10^5 \frac{N}{m}$ (or $k_{b/c} \leq 0.5 \times 10^{-4}$) the eigenvalues of the system are

Table 3.4: Special cases for characteristic equation of (3.63), where $\beta_i = \lambda_i L$.

CASE	$k_{b/c}$	a	Characteristic Equation
Pinned-Free	∞	0	$\tan(\beta_i) = \tanh(\beta_i)$
Clamped-Free	∞	∞	$1 + \cos(\beta_i) \cosh(\beta_i) = 0$
Simply Supported	0	0	$\sin(\beta_i) = 0$
Barbieri and Özgüner (1988)	∞	$\checkmark$	$[1 + \cos(\beta_i) \cosh(\beta_i)] +$ $+ \varrho_i [\sin(\beta_i) \cosh(\beta_i) - \sinh(\beta_i) \cos(\beta_i)] = 0$
Constrained Motion	0	$\checkmark$	$\frac{1}{\tanh(\beta_i)} - \frac{1}{\tan(\beta_i)} = 2\varrho_i$ equation (3.25)

very close to those of a stiff contact system. The value of k is within this range in many applications including the experimental apparatus of this work where $k = 10^7 \frac{N}{m}$ or $k_{b/c} = 0.5 \times 10^{-6}$, see Appendix-A.3.

Soft contact modelling has been widely used by researchers, such as Tzes and Yurkovich (1990), Chiou and Shahinpoor (1990), Choi and Krishnamurthy (1991), Book and Kwon (1992), Latornell and Cherchas (1992). In this approach a free-end boundary condition is used, and after obtaining the tip position a spring-like model is used to calculate the contact force. As is seen in Figure-3.7 when $k \leq 10^3 \frac{N}{m}$ (or $k_{b/c} \leq 0.5 \times 10^{-2}$), the eigenvalues approach those of the non-contact system (free end B.C), so that using the soft contact modelling approach is valid only in this range of k .

Figure 3.7: Roots of characteristic equation of constrained motion, for various contact stiffness. Dash and solid lines represents the left and right side of equation (3.64), respectively. $\otimes$: $k = 10^3$, $\ominus$: $k = 10^4$, Δ : $k = 10^5$, and ∇ : $k = 10^6$ to $10^8 \frac{N}{m}$.

3.4.2 Stiff Contact Model

In constrained motion the most important element of the output vector of the model developed here is the reaction force F . However, in constrained motion W_L (tip position) is zero, so that it can be considered a virtual displacement, allowing F to be interpreted as a *Lagrange Multiplier* in the external work equation (3.6):

$$\mathcal{Q} = \tau\theta - FW_L, \quad (3.65)$$

and the *Constraint Equation* is:

$$W_L = W(L, t) = y(L, t) + L\Theta(t) = 0 \quad (3.66)$$

Since equation (3.66) is in the integrated form of its *Pfaffian*, the system is *Holonomic*. Substituting (3.66) into (3.65) and using equation (3.3) results in

$$\mathcal{Q} = \tau\theta - F[y_L + L(\theta - y'_e)] \quad (3.67)$$

In Appendix-D it is explained how the well known decomposition method is not applicable for this specific application if the Lagrange equation is written with respect to the system states (q_i). Thus another Lagrange-Multiplier-based method will be introduced in this section.

Lagrange's equation can be written in terms of a linear combination of the independent generalized coordinates such as θ . It is recalled here that θ is a dependent variable, see equation (3.3). Therefore the moment dynamic equilibrium at the joint becomes:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} &= \frac{\partial \mathcal{Q}}{\partial \theta} \\ J_h \ddot{\theta} + m \int_0^L x \ddot{W}(x, t) dx &= \tau - FL \\ \sum_{i=1}^{\infty} \tilde{q}_i \left[J_h \phi'_i(0) + m \int_0^L x \phi_i(x) dx \right] &= \tau - FL \end{aligned} \quad (3.68)$$

The integral term can be simplified as follows

$$\begin{aligned} m \int_0^L x \underbrace{\phi_i(x)}_{Eqn(3.14)} dx &= \frac{m}{\lambda_i^4} \int_0^L x \phi_i'''' dx = \frac{m}{\lambda_i^4} x \phi_i''' \Big|_0^L - \frac{m}{\lambda_i^4} \int_0^L \phi_i''' dx \\ &= \frac{m}{\lambda_i^4} [L \phi_i'''(L) - \underbrace{\phi_i''(L)}_{Eqn(3.22)} + \underbrace{\phi_i''(0)}_{Eqn(3.24)}] \\ &= \frac{mL}{\lambda_i^4} \phi_i'''(L) - J_h \phi'_i(0) \end{aligned} \quad (3.69)$$

Substituting (3.69) into (3.68), and obtaining $\tilde{q}_i$ from equation (3.42) results in

$$F = \left[\sum_{i=1}^n EI \phi_i'''(L) (q_i + \gamma \dot{q}_i) \right] + \underbrace{\left[\frac{1}{L} - \sum_{i=1}^n \frac{m \phi_i'(0) \phi_i'''(L)}{\lambda_i^4 M_i} \right]}_D \tau \quad (3.70)$$

Figure 3.8: In the contact force equation of (3.70), the direct forward term tends to zero rapidly as the number of assumed modes increases.

Figure-3.8 shows numerically that when the number of assumed modes increases, the value of the term $[D]$ tends to zero rapidly. It is conjectured that in an accurate model when n tends to infinity, $[D]$ vanishes:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{L} - \sum_{i=1}^n \frac{m \phi_i'(0) \phi_i'''(L)}{\lambda_i^4 M_i} \right) \rightarrow 0 \quad (3.71)$$

In this work for $n \geq 3$, the maximum error in equation (3.70) would be less than 2.7% caused by neglecting this direct feedforward term. Therefore for the remainder of the thesis it is assumed that $D = 0$. With this assumption, the output equation for constrained motion can be written in the standard form of:

$$F = C_F x \quad (3.72)$$

$$C_F = EI [\phi_1'''(L) \quad \gamma \phi_1'''(L) \quad \dots \quad \phi_n'''(L) \quad \gamma \phi_n'''(L)]$$

Figure 3.9: Open loop response of the constrained system (3.52) and (3.72), for $n \geq 4$. The system is initially in contact.

Figure-3.9 shows an open loop response of the constrained system when it is initially in contact. It is seen that the accuracy is quiet satisfactory for $n \geq 4$. Also the problem of dependence on number of modes which was described in section 3.3.1. does not exist in this model.

The output equation (3.72) satisfies the shear force condition at the tip of the link:

$$V(L, t) = -F = -EI(W_L'''' + \gamma \dot{W}_L''') \quad (3.73)$$

This is a modified shear force equation which considers the effect of structural damping by means of equation (3.36).

3.5 Discontinuous Contact

3.5.1 Transition Process

The objective of this section is to present a means of transition between the state vectors $\mathbf{x}^c$ of constrained motion and $\mathbf{x}^u$ of unconstrained motion and vice versa, during the making or breaking of contact. To avoid confusion, superscripts u and c are used to refer to unconstrained and constrained motions, respectively. When the system has an approach velocity toward the contact surface, at the threshold of contact the unconstrained state variables of equation (3.59) are known and are to transition smoothly to the immediate post contact state variables of equation (3.48). It is clear that the geometry and kinematics of the manipulator are the same for both models for an infinitesimal time pre and post contact. The only discontinuity occurs for tip velocity, which falls to zero suddenly. A portion of the system kinetic energy converts to heat during an impact through this discontinuity. As the manipulator is originally modeled as a distributed parameter system, a p dimensional subset of its modes are considered here. This necessitates the satisfaction of kinematic conditions at at least p points:

$$x_i = i \frac{L}{p+1} \quad \text{for: } i = 1, \dots, p \quad (3.74)$$

The position and velocity compatibility conditions for these points can be written

Figure 3.10: Points which geometrical and kinematic compatibility must be satisfied.

as:

$$\begin{Bmatrix} \theta^c \\ \dot{\theta}^c \\ W^c(x_1) \\ \dot{W}^c(x_1) \\ \vdots \\ W^c(x_p) \\ \dot{W}^c(x_p) \end{Bmatrix} = \begin{Bmatrix} \theta^u \\ \dot{\theta}^u \\ W^u(x_1) \\ \dot{W}^u(x_1) \\ \vdots \\ W^u(x_p) \\ \dot{W}^u(x_p) \end{Bmatrix} \quad (3.75)$$

It can be seen that for a transition to contact it should be $p \geq n - 1$ and for a transition from contact it should be $p \geq n$. In order to write these conditions in terms of state variables, the following output matrices are obtained from the previous sections:

$$\begin{aligned} C_{\theta}^c &= \begin{bmatrix} \phi_1^{c'}(0) & 0 & \dots & \phi_p^{c'}(0) & 0 \end{bmatrix} \\ C_{\theta}^u &= \begin{bmatrix} 1 & 0 & \phi_1^{u'}(0) & 0 & \dots & \phi_p^{u'}(0) & 0 \end{bmatrix} \\ C_{\dot{\theta}}^c &= \begin{bmatrix} 0 & \phi_1^{c'}(0) & \dots & 0 & \phi_p^{c'}(0) \end{bmatrix} \\ C_{\dot{\theta}}^u &= \begin{bmatrix} 0 & 1 & 0 & \phi_1^{u'}(0) & \dots & 0 & \phi_p^{u'}(0) \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
C_{\dot{W}_{z_i}}^c &= \begin{bmatrix} \phi_1^c(x_i) & 0 & \dots & \phi_p^c(x_i) & 0 \end{bmatrix} \\
C_{\dot{W}_{z_i}}^u &= \begin{bmatrix} x_i & 0 & \phi_1^u(x_i) & 0 & \dots & \phi_p^u(x_i) & 0 \end{bmatrix} \\
C_{\dot{W}_{x_i}}^c &= \begin{bmatrix} 0 & \phi_1^c(x_i) & \dots & 0 & \phi_p^c(x_i) \end{bmatrix} \\
C_{\dot{W}_{x_i}}^u &= \begin{bmatrix} 0 & x_i & 0 & \phi_1^u(x_i) & \dots & 0 & \phi_p^u(x_i) \end{bmatrix}
\end{aligned} \tag{3.76}$$

where $\phi_j^c(x)$ and $\phi_j^u(x)$ are obtained from equations (3.23) and (3.55) respectively.

Therefore equation (3.75) can be written in term of state variables as:

$$\Gamma^c \mathbf{x}^c = \Gamma^u \mathbf{x}^u \tag{3.77}$$

where:

$$\begin{aligned}
\Gamma^c &= \begin{bmatrix} [C_\theta^c]^T & [C_{\dot{\theta}}^c]^T & [C_{\dot{W}_{z_1}}^c]^T & [C_{\dot{W}_{z_1}}^c]^T & \dots & [C_{\dot{W}_{z_p}}^c]^T & [C_{\dot{W}_{z_p}}^c]^T \end{bmatrix}^T \\
\Gamma^u &= \begin{bmatrix} [C_\theta^u]^T & [C_{\dot{\theta}}^u]^T & [C_{\dot{W}_{z_1}}^u]^T & [C_{\dot{W}_{z_1}}^u]^T & \dots & [C_{\dot{W}_{z_p}}^u]^T & [C_{\dot{W}_{z_p}}^u]^T \end{bmatrix}^T
\end{aligned} \tag{3.78}$$

Superscript T denotes matrix transpose. Using a pseudo inverse the transition rules for making and breaking contact are obtained as:

$$\mathbf{x}^c = \Gamma_{u2c} \mathbf{x}^u \tag{3.79}$$

$$\mathbf{x}^u = \Gamma_{c2u} \mathbf{x}^c$$

where Γ_{u2c} is a $[2n \times 2(n+1)]$ matrix and Γ_{c2u} is a $[2(n+1) \times 2n]$ matrix:

$$\begin{aligned}
\Gamma_{u2c} &= (\Gamma^{cT} \Gamma^c)^{-1} \Gamma^{cT} \Gamma^u \\
\Gamma_{c2u} &= (\Gamma^{uT} \Gamma^u)^{-1} \Gamma^{uT} \Gamma^c
\end{aligned} \tag{3.80}$$

3.5.2 Switching Algorithm

When the system is in the constrained condition F has a non zero positive value, and $\dot{W}_L$ is zero. As soon as F becomes zero, the contact breaks and the constrained

to unconstrained conversion should be done, which is the second equation of (3.79). For the opposite case, when the system is not in contact, W_L has a non zero value and F is zero. As soon as W_L becomes zero the contact occurs and the unconstrained to constrained conversion, and the first equation of (3.79), should be applied. In order to implement this switching algorithm the following functions are introduced:

$$\begin{aligned} \text{Phase Indicator: } \mathcal{P}(t_k) &= \begin{cases} 1 & \text{Constrained Motion} \\ -1 & \text{Unconstrained Motion} \end{cases} \\ \text{Event Detector : } \mathcal{E}(t_k) &= \begin{cases} 1 & \text{u2c Making Contact.} \\ 0 & \text{-- No Switching.} \\ -1 & \text{c2u Breaking Contact.} \end{cases} \end{aligned}$$

Based on these definitions the following switching algorithm proves appropriate:

$$\begin{aligned} \mathcal{P}(t_k) &= \left(\frac{\mathcal{P}(t_{k-1}) - 1}{-2} \right) (1 - 2\text{sgn}^2(W_L(t_k))) + \left(\frac{\mathcal{P}(t_{k-1}) + 1}{2} \right) (2\text{sgn}^2(F(t_k)) - 1) \\ \mathcal{E}(t_k) &= \frac{\mathcal{P}(t_k) - \mathcal{P}(t_{k-1})}{2} \end{aligned} \quad (3.81)$$

A discontinuous model of the system based on two constrained and unconstrained models, and the switching algorithm just described were simulated in SIMULINK (see Appendix-G).

The simulation result of contact force response when the system is initially in contact is shown in Figure-3.11. It is seen that the link tip moves away from the contact surface with a short breaking and then re-establishing of contact. This small impact results in a 20% increase in the value of the overshoot, (compare Figures-3.9 and 3.11). This phenomenon can be explained by conservation of momentum physically, and the nature of nonminimum phase transfer functions mathematically.

Figure 3.11: Open loop response of system with discontinuous contact, when system is initially in contact. Solid line denotes contact force and dotted line represents tip position.

Fig-3.12 shows the contact force and tip position responses when the system is not initially in contact. The unconstrained motion toward the constraint surface takes about 0.23 seconds. Because the system has an approach velocity at the time of first contact, an impact occurs. This causes the manipulator to bounce three times between 0.3 to 0.65 seconds. Including the contact discontinuity makes the simulation results much more realistic.

Figure 3.12: Open loop response of system with discontinuous contact. when system is not initially in contact. The initial angle is: $\theta_{t=0} = 15^\circ$. Solid line denotes contact force and dotted line represents tip position.

3.6 Joint Friction Effect

The experimental set up described in Appendix-A was used to verify the accuracy of the dynamic model. However, following experiments on the open loop system, considerable differences were observed between the experimental and simulation results. For example the frequency of oscillations in the simulated response. Figure-3.11, is 50% less than that of the experimental results, as is seen in Figure-3.13.

Joint friction is one of the major causes of this difference. In this experimental set up the effect of the stick-slip regime is much more prominent than other friction effects. To simulate the stick-slip regime two approaches were implemented, the first

Figure 3.13: Experimental open loop response of system, when system is initially in contact.

being the Tustin model for dynamic friction in combination with Karnopp's model for static friction, and a direct stick-slip model as an alternative approach. Each model exhibits some advantages and drawbacks over the other. The first method is easy to apply and understand, and also has fewer parameters, allowing their easy identification by experiment. The second approach employs a more complete model of friction and it therefore has more parameters, rendering their experimental identification more difficult. The most important disadvantage of the second method is its dependence on integration time step, which sometimes results in very slow simulations.

These models are described briefly in Appendix-E, and the friction coefficients

Figure 3.14: Open loop response of system when system is initially in contact. Solid and dash lines represent the responses with and without friction.

were determined experimentally, (see section 4.1.4). These models were also simulated by means of SIMULINK, (see Appendix-G). As joint viscous damping has already been considered, α_2 in equations (E.1), (E.7), and (E.8) must be ignored. It should also be mentioned that in the Karnopp model of equation (E.2) for the static situation, the value of τ_{res} can be obtained from equation (3.50) such as:

$$\tau_{res} = \tau + M_0 \quad (3.82)$$

The results of simulation including these frictional effects are shown in Figure-3.14. Comparing Figures-3.13 and 3.14, it is seen that the experimental results are very similar to the simulation results when friction is included. This suggests that friction in the joint increases the frequency of tip force response, and this should be

accounted for in control design.

3.7 Exact Modeling

Modal Expansion is a common method of converting partial differential equations governing a system into a set of ordinary differential equations. It is well known that this approach is a pole preserving but not a zero preserving method. (Spector and Flashner 1990). With this method the values of transmission zeros depend on the number of assumed modes. This inaccuracy in zeros of the system generally does not cause problems for most conventional controllers. In chapter 5, however, it will be shown that because of right hand zeros in the system transfer function, the performance of a closed loop system using a conventional controller is limited. In chapter 6 a model based predictive controller will be suggested to overcome this difficulty. It will be shown that the closed loop poles of the suggested predictive control system are dependent on plant zeros, and large inaccuracies in the zeros of the model (which may result from the modal expansion method) increases the risk of instability in this controlled system.

In this thesis the *Infinite Product Expansion* (IPE) is used for its zero preserving property, thus avoiding this source of instability. In this method the partial differential equation of motion is converted into a transfer function in the Laplace domain. Using IPE exact formulations for the infinite number of roots of the numerator and denominator can be found. The main weakness of this method is that solutions for only a limited group of standard boundary conditions are available, e.g. in Goodson (1970), and adding any complication such as motor dynamics or structural and

hub damping to the system makes solution very difficult. In such cases numerical algorithms are used to obtain the roots of the numerator and/or denominator.

In this thesis the model which is obtained from the IPE method is termed “exact” because it does not depend on the number of assumed modes, and it is closer to the real system than that obtained via the modal expansion method.

Characteristic Equation:

Considering the damped Euler-Bernoulli equation of (3.39) and using the separation method modifies equation (3.12) as:

$$\frac{\phi''''(x)}{\phi(x)} = -\frac{m}{EI} \frac{\ddot{q}(t)}{q(t) + \gamma \dot{q}(t)} = \lambda^4 \quad (3.83)$$

where without loss of generality the subscript j has been dropped in $\phi(x)$, $q(t)$, and λ . Therefore the time dependent differential equation of (3.13) is accordingly modified, and is written in the Laplace domain as:

$$(s^2 + \gamma\omega^2 s + \omega^2)q(s) = 0 \quad (3.84)$$

where s denotes the Laplace operator. The relation between s and the nondimensionalized eigenvalues of the system, β , is

$$s = \left[-\frac{\gamma}{2}\delta\beta^2 \pm i\sqrt{1 - \left(\frac{\gamma}{2}\delta\beta^2\right)^2} \right] \delta\beta^2 \quad (3.85)$$

where:

$$\begin{aligned} \delta &= \sqrt{\frac{EI}{mL^4}} \\ \omega &= \delta\beta^2 \end{aligned} \quad (3.86)$$

and $i = \sqrt{-1}$. If $\gamma = 0$ then $s = \pm i\delta\beta^2 = \pm i\omega$, which is used in most of the literature.

Applying the first, second, and fourth B.C.s of (3.21) results in the same mode shapes as equation (3.23). The third boundary condition can be written in a more complete form as:

$$\begin{aligned}
 \tau(t) &= J_h \ddot{\theta} + c_h \dot{\theta} - M(0, t) \\
 &= J_h \ddot{W}'(0, t) + c_h \dot{W}'(0, t) - EI[W'''(0, t) + \gamma \dot{W}'''(0, t)] \\
 &= [J_h \ddot{q}(t) + c_h \dot{q}(t)]\phi'(0) - EI\phi''(0)[q(t) + \gamma \dot{q}(t)] \\
 \tau(s) &= [(J_h s^2 + c_h s)\phi'(0) - EI(1 + \gamma s)\phi''(0)] q(s)
 \end{aligned} \tag{3.87}$$

The term within the brackets is the characteristic equation when the external torque is zero. Extracting $s^2 = -(\gamma\omega^2 s + \omega^2)$ from equation (3.84) and substituting in equation (3.87), the characteristic equation can be written as:

$$[\delta^2 \beta^4 J_h \phi'(0) + EI\phi''(0)] + s[(\delta^2 \beta^4 J_h - c_h)\phi'(0) + EI\gamma\phi''(0)] = 0 \tag{3.88}$$

The distributed and joint damping effects are gathered in the second bracket. For an undamped system when $\gamma = c_h = 0$ the first bracket determines the system characteristic equation, which is equal to equation (3.24). Substituting s from equation (3.85) into the above equation results in the characteristic equation in terms of only one variable β , as:

$$f_u(\beta) + s f_d(\beta) = 0 \tag{3.89}$$

where:

$$f_u(\beta) = 2\beta^2 - \bar{J}_h \beta^5 f_1(\beta)$$

$$\begin{aligned}
f_d(\beta) &= 2\gamma\beta^2 + (\bar{c}_h - \gamma\bar{J}_h\beta^4)\beta f_1(\beta) \\
f_1(\beta) &= \frac{1}{\tan(\beta)} - \frac{1}{\tanh(\beta)}
\end{aligned}$$

and $\bar{J}_h = \frac{J_h}{mL^3}$, $\bar{c}_h = \frac{c_h L}{EI}$. The first of these represents undamped behavior, while the f_d term includes the effect of damping on the characteristic equation of the system. Solving equation (3.89) for β and using equation (3.86), the natural frequencies of the system, ω_i , can be obtained.

Transfer Function:

Considering a stiff contact, the Laplace transform of the reaction force at the tip can be written using equation (3.73) as:

$$\begin{aligned}
F(s) &= -V(L, s) = M'(L, s) \\
&= EI[W''''(L, s) + \gamma s W''''(L, s)] \\
&= EI(1 + \gamma s)\phi'''(L)q(s)
\end{aligned} \tag{3.90}$$

Dividing equation (3.90) by (3.87), the exact transfer function for the force can be obtained as:

$$G(s) = \frac{F(s)}{\tau(s)} = \frac{\beta^3[\sin(\beta) + \sinh(\beta)](1 + \gamma s)}{L\sin(\beta)\sinh(\beta)[f_u(\beta) + sf_d(\beta)]} \tag{3.91}$$

This is the general form of the transfer function, which will be now expressed in terms of rational functions.

Exact Zeros:

The exact zeros of this transfer function can be determined by an infinite product expansion, (Goodson 1970), as follows:

$$\sin(\beta) + \sinh(\beta) = 2\beta \prod_{j=1}^{\infty} \left(1 + \frac{\beta^4}{4d_j^4}\right) \quad (3.92)$$

where d_j s are the positive real roots of:

$$\tan(d_j) + \tanh(d_j) = 0 \quad (3.93)$$

Using equations (3.84) and (3.86), the zeros can be obtained as

$$z_{j\pm} = 2d_j^2 \delta \left[d_j^2 \delta \gamma \pm \sqrt{d_j^4 \delta^2 \gamma^2 + 1} \right] \quad (3.94)$$

For $1 \leq j \leq n-1$, where n denotes the number of assumed modes and z represents the zeros of the transfer function. It is seen that all zeros are on the real axis in pairs of opposite sign, which makes the system nonminimum phase. From equation (3.91) another zero is at:

$$z = -\frac{1}{\gamma} \quad (3.95)$$

Note that the values of the zeros are analytically exact and do not depend on the number of assumed modes.

Poles and Eigenvalues:

It is quite difficult to solve the equation (3.89) by means of an infinite product expansion, so that a numerical approach is used. Using equation (3.85) in (3.89), the characteristic equation can be written in the following form:

$$f_2(\beta) + i f_3(\beta) = 0 \quad (3.96)$$

$$\begin{aligned}
f_2(\beta) &= f_u(\beta) - \frac{\delta^2 \gamma}{2} \beta^4 f_d(\beta) \\
f_3(\beta) &= \delta \beta^2 \sqrt{1 - \frac{\delta^2 \gamma^2}{4} \beta^4} f_d(\beta)
\end{aligned}$$

Because β is a complex number $f_2(\beta)$ and $f_3(\beta)$ are complex functions. Using the cartesian form of

$$\beta = \beta_{re} + i\beta_{im} \quad (3.97)$$

in equation (3.96) produces

$$f_{re}(\beta_{re}, \beta_{im}) + i f_{im}(\beta_{re}, \beta_{im}) = 0 \quad (3.98)$$

The problem is now reduced to the solution of a set of two real nonlinear equations in two real unknowns:

$$\begin{aligned}
f_{re}(\beta_{re}, \beta_{im}) &= 0 \\
f_{im}(\beta_{re}, \beta_{im}) &= 0
\end{aligned} \quad (3.99)$$

The Newton-Raphson numerical algorithm was used to solve this set of equations. Determining the system eigenvalues and using equation (3.85), the poles of the system are:

$$p_{j\pm} = -\frac{\delta^2 \gamma}{2} \beta_j^4 \pm i \delta \beta_j^2 \sqrt{1 - \frac{\delta^2 \gamma^2}{4} \beta_j^4} \quad (3.100)$$

for $1 \leq j \leq n$.

dc Gain:

To complete the rational transfer function the dc gain of the system is required.

Applying l'Hôpital's rule to equation (3.91) it is found that:

$$G(0) = \lim_{\beta \rightarrow 0} \frac{\frac{\partial^4 NUM}{\partial \beta^4}}{\frac{\partial^4 DEN}{\partial \beta^4}} = \frac{1}{L} \quad (3.101)$$

where NUM and DEN are the numerator and denominator of equation (3.91). The dc gain is then

$$k_{dc} = \frac{\gamma \prod_{j=1}^n p_{j-} p_{j+}}{L \prod_{j=1}^{n-1} z_{j-} z_{j+}} \quad (3.102)$$

so that the final truncated transfer function can be written as:

$$G(s) = k_{dc} \frac{(s + \frac{1}{\gamma}) \prod_{j=1}^{n-1} (s - z_{j-})(s - z_{j+})}{\prod_{j=1}^n (s - p_{j-})(s - p_{j+})} \quad (3.103)$$

In equation (3.103) the values of the zeros and poles were obtained separately, and do not depend on the number of assumed modes, n .

Model Comparison

An exact model and one based on modal expansion were developed based on the physical parameters of Table-A.1. These values were obtained from an experimental set up for force control of flexible manipulators in the labs of the Canadian Space Agency.

Table-3.5 shows the zeros and poles of the two models when 4 modes of the system were assumed. The zeros of the exact model do not depend on the number of assumed modes, while the values of zeros in the modal expansion model do change with n . For example if 3 modes are assumed, the zeros of the modal expansion model become (-811.62, -32.41, 33.75, -73.56, 80.89), where except the first zero all others are completely different from the zeros of model with 4 assumed modes. It must be noted that the zero at -811.62 is related to the system structural damping and does not depend on system modes. It is also seen from the table that all the exact model zeros are on the real axis while the zeros of the other model include some complex elements.

Table 3.5: Poles and zeros of exact and modal expansion models, for $n=4$.

n	Zeros $z_{j\pm}$	Poles $p_{j\pm}$	Modal Natural Frequencies (ω_j)	Modal Damping Ratios (ζ_j)
MODAL EXPANSION				
1	-811.62	-2.24±16.11i	16.3	0.14
2	-27.17 , 28.11	-2.21±46.31i	46.4	0.05
3	-113.79±71.88i	-10.68±130.64i	131	0.08
4	122.44±96.22i	-45.02±266.49i	270	0.17
EXACT MODEL				
1	-811.59	-2.34±16.07i	16.2	0.14
2	-28.39 , 29.42	-2.56±46.18i	46.2	0.06
3	-141.9 , 171.9	-11±130.53i	131	0.08
4	-304.8 , 488.0	-45.1±266.5i	270	0.17

The frequency responses of both models are shown in Figure-3.15. The models are in good agreement in phase and also on gain up to the fourth mode. For higher frequencies a 20 dB shift is seen in the gain plot.

The time response of both models are close and difficult to distinguish from one another, so that Figure-3.9 serves for both. This is possibly the main reason for most researchers using the modal expansion method without hesitation which, as pointed out, maybe inadequate for some controllers, and creates problems in the design procedure of Predictive Controllers.

Figure 3.15: Bode plot for open loop frequency responses of the exact model and modal expansion model. $n=4$.

3.7.1 Suggested Procedure

As indicated above, the numerator of the transcendental transfer function often has a simple structure which can be solved by an infinite product expansion. On the other hand the denominator of this transfer function is very complicated, particularly if sensor and actuator dynamics are included, and solving the characteristic equation by means of IPE or even numerical algorithms is difficult. Since the modal expansion method preserves the poles, obtaining the poles by this approach, while the zeros are determined using the IPE method, preserves both zeros and poles and also is

easier to apply. The combination of the two approaches preserves the accuracy of the resulting model, and simplifies the implementation process dramatically. As an example, this procedure can be applied for constrained motion as:

- System poles = eigenvalues of matrix $\mathbf{A}_b$ in equation (3.52), obtained from modal expansion method.
- System zeros: use equations (3.94) and (3.95) which were obtained from IPE method.
- DC gain: use equation (3.102).
- System transfer function: use equation (3.103).

Chapter 4

Experimental Off Line Identification

4.1 System Parameters

For good fidelity of a system model, an appropriate mathematical structure should be selected, which was extensively discussed in the previous chapter. In addition, the model parameters should be determined as accurately as possible. This chapter presents the efforts made to identify the unknown parameters of the system and also to verify the accuracy of some system characteristics.

4.1.1 Motor Characteristics

The core of the experimental setup is a Direct Drive motor used as an actuator. The manufacturer's manual provides only the motor inertia, see Table-A.1. For other important parameters, there is either no information or what there is, is of poor

quality.

Torque Constant

The direct drive motor operates in 3 modes, one of which is torque mode. In this mode the produced torque is proportional to the input voltage (step 5 in Appendix-B). In the manufacturer's manual it is stated that in the static condition, and for a maximum input of 8 volts, the produced torque would be 150 N.m. for a torque factor of $k_t = 18.75 \frac{\text{N.m}}{\text{volts}}$. It was observed, however, during initial experiments, that this factor is not correct in the working range of [0-30] N.m.

To obtain more reliable data for this parameter, a virtually rigid link was mounted on the motor, with a force sensor connected to the link tip. Several harmonic input voltages were applied to the system in contact, such as:

$$u = 2.2 \sin(0.314t + 0.091) \quad \text{volts} \quad (4.1)$$

The tip force was measured and plotted versus input voltage in Figure- 4.1. The hysteresis form of the plot is caused by friction. The torque relation can be written as:

$$\tau_e = k_t u = \tau_{fr} + LF \quad \Rightarrow \quad F = \frac{k_t}{L} u - \frac{\tau_{fr}}{L}. \quad (4.2)$$

so that the motor torque constant can be obtained from the slope of the plot. Since the input voltage of (4.1) moves the system slowly, it is quite reasonable to ignore the viscous friction term and consider only Coulomb friction in Figure-E.1. Thus for increasing and decreasing voltages the friction would be $+\alpha_c$ and $-\alpha_c$ respectively, and the difference between the two branches ($\Delta F_{\pm}$), is used to calculate the Coulomb

Figure 4.1: Experimental identification of motor torque constant.

friction level as:

$$\alpha_0 = \frac{L}{2}(\Delta F_{\pm}) \quad (4.3)$$

The average of the results obtained from seven experiments. is:

$$\alpha_0 = [5.5 \sim 6.5] \text{ N.m} \quad , \quad \boxed{k_t \cong 29.5 \frac{\text{N.m}}{\text{volts}}} \quad (4.4)$$

This value of k_t is valid and verified for torque in the range of [0-30] N.m and differs substantially from the value stated in the manual.

Damping and Back emf

c_h is defined in Appendix-B as the equivalent damping coefficient of the motor. This is a combination of viscous damping and back emf effects in the motor. To identify

Figure 4.2: Motor damping identification using a simple feedback controller.

this important parameter, a proportional feedback controller was applied to motor position without the link. The signal of a built-in encoder was used to measure the motor angular position.

Since this experiment was done on the motor without the link, M_0 in Appendix-B would be zero. Thus the motor transfer function can be written as shown in Figure-4.2 with closed loop transfer function (denoted G).

$$\frac{\theta(s)}{\theta_d(s)} = \frac{k_t k_p}{J_h s^2 + c_h s + k_t k_p} = \frac{\omega_o^2}{s^2 + 2\omega_o \zeta_o s + \omega_o^2} \quad (4.5)$$

where $\omega_o = \sqrt{\frac{k_p k_t}{J_h}}$ and $\zeta_o = \frac{c_h}{2\sqrt{J_h k_t k_p}}$. The unknowns here are k_t and c_h , while J_h is given in the manual, see Table-A.1. By trial and error k_p was tuned to provide an underdamped closed-loop system. Figure-4.3 shows the oscillatory response of the system for $k_p = 0.1 \frac{\text{volts}}{\text{deg}}$ or $5.73 \frac{\text{volts}}{\text{rad}}$. This response can be used to determine the unknowns.

Using the logarithmic decrement of motor angle, the damping ratio can be determined as from

$$\delta = \ln \left(\frac{e_1}{e_2} \right) = \ln \left(\frac{\theta_1 - \theta_d}{\theta_2 - \theta_d} \right) = \frac{2\pi \zeta_o}{\sqrt{1 - \zeta_o^2}} \Rightarrow \zeta_o = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (4.6)$$

From Figure-4.3 two successive overshoots can be obtained as $\theta_1 = 40.95^\circ$ and $\theta_2 = 38.56^\circ$, where $\theta_d = 30^\circ$. Therefore $\delta = 0.5788$ and then $\zeta_o = 0.0917$. From

Figure 4.3: Experimental identification of motor damping coefficient. Time response of closed loop system for $k_p = 0.1 \frac{\text{volts}}{\text{deg}}$.

measurement of the time between two successive overshoot as 0.1876 second. the damped frequency of system is determined to be $\omega_d = \frac{2\pi}{0.1876} = 33.49 \frac{\text{rad}}{\text{sec}}$. and the natural frequency of the closed loop system becomes:

$$\omega_0 = \frac{\omega_d}{\sqrt{1 - \zeta_0^2}} = 33.63 \frac{\text{rad}}{\text{sec}} \quad (4.7)$$

Using the definitions following (4.5), the two unknowns are calculated as:

$$\begin{aligned} k_t &= \frac{J_h \omega_0^2}{k_p} = \frac{0.142 \times 33.63^2}{5.73} = 28.03 \frac{\text{N.m}}{\text{volts}} \\ c_h &= 2\zeta_0 \sqrt{J_h k_p k_t} = 2 \times 0.0917 \sqrt{0.142 \times 5.73 \times 28.03} = 0.876 \frac{\text{N.m.sec}}{\text{rad}} \end{aligned} \quad (4.8)$$

The values of k_t in (4.8) and (4.4), which were obtained from two different experiments are in good agreement. It is also seen that the torque constant obtained for the stall condition is 5% greater than that for the moving motor.

Bandwidth

From the motor velocity transfer function, it can be concluded that

$$G_v(s) \triangleq \frac{s\theta(s)}{u(s)} = \frac{k_t}{J_h s + c_h} \Rightarrow |G_v(j\omega)| = \frac{k_t}{\sqrt{J_h^2 \omega^2 + c_h^2}} \Rightarrow |G_v(0)| = \frac{k_t}{c_h} \quad (4.9)$$

A magnitude function can be defined as $M(\omega) = \frac{|G_v(j\omega)|}{|G_v(0)|}$. The cutoff frequency is defined as the frequency at which $M(\omega_c) = 0.708$ or

$$20 \times \log(M_b) = 20 \times \log \left[\left(\frac{J_h}{c_h} \right)^2 \omega_c^2 + 1 \right]^{-\frac{1}{2}} = -3 \quad dB \quad (4.10)$$

Therefore the motor cutoff frequency is obtained as

$$\omega_c = \frac{c_h}{J_h} = \frac{[0.876 \text{ or } 1.021]}{0.142} = [6.169 \text{ or } 7.188] \quad \frac{rad}{sec} \quad (4.11)$$

The two values of c_h are obtained from equation (4.8) and from section-4.1.4.

The motor cutoff frequency can also be obtained directly from experiment, as follows:

- The experiment is done on the motor without a link or any feedback loop.
- Choose a voltage which produces a torque greater than the stiction level, and less than the saturation limit. $u_0 = 0.35$ volts satisfies these conditions.
- Apply a step command of u_0 volts and measure the velocity. This will give the output amplitude for $\omega = 0$. Here the result is $\dot{\theta}_0 = 230 \frac{deg}{sec}$ or $4.01 \frac{rad}{sec}$.
- Apply harmonic inputs of $u = u_0 \sin(\omega t)$ for a number of different ω 's and measure the amplitude of velocity oscillations. The following table shows the results for this step.

Figure 4.4: Experimental identification of motor frequency bandwidth. Motor velocity response for $u_0=0.35$ volts and $\omega=1.12$ Hertz.

ω [Hz]	0	0.4	0.6	0.8	1	1.1	1.12
ω $\left[\frac{\text{rad}}{\text{sec}}\right]$	0	2.51	3.77	5.03	6.28	6.91	7.04
$\dot{\theta}_{max}$ $\left[\frac{\text{rad}}{\text{sec}}\right]$	4.01	3.85	3.71	3.44	3.17	2.99	2.81
$M(\omega) = \frac{\dot{\theta}_{max}}{\dot{\theta}_0}$	1	0.96	0.92	0.86	0.79	0.75	0.70

- Find the cutoff frequency where the magnitude function is $M(\omega)=0.7$. From the table of results it is obtained that $\omega_c=7.04 \left[\frac{\text{rad}}{\text{sec}}\right]$. The time response for this experiment is shown in Figure-4.4.

This result verifies the larger of the two values of (4.11). It also can be concluded that the value of c_h obtained in section-4.1.4 is more reliable. Therefore the motor

frequency bandwidth may be written as:

$$\text{Motor Frequency Bandwidth} = 7.1 \frac{\text{rad}}{\text{sec}} \quad (4.12)$$

4.1.2 Beam Characteristics

Structural Damping

It is quite difficult to find any damping information on industrial metals, especially steel, and the damping coefficients of materials are usually just guesses in the literature. Since in this study the model was used to design a controller, it was necessary to build as realistic a model as possible. A procedure for obtaining link damping is therefore suggested in this section.

Although modal damping ratios ζ_j are proportional to modal natural frequencies ω_j , the structural damping coefficient γ is independent of modes and is a property of link material. (see equations 3.36 and 3.43). Therefore the magnitude of γ does not depend on the system boundary conditions and can be obtained via experiments on the same link with simpler B.C.s. In fact, γ can be easily obtained using the beam oscillations with a clamped-free boundary conditions. The experimental procedure is as follows:

- Fix one end of the beam, and allow the other end to move freely. The force sensor is not needed in this experiment.
- Attach a strain gauge to the beam very close to the clamped end to have the maximum sensitivity. Figure-4.5 shows a schematic of the experimental setup for this purpose.

Figure 4.5: Schematic of experimental setup for structural damping identification.

- Introduce a tip displacement as an initial condition. Release the tip and during the beam oscillation read the strain gauge signal. Particular care is needed to assure that the oscillation is primarily in the first mode of motion. A time response of the system for such an initial condition is seen in Figure-4.6.
- From the time response, find the period of oscillations and then the natural frequency of motion. The result is: $\omega_1 = 5.1 \frac{rad}{sec}$.
- Calculate the logarithmic decrement of the time response.

$$\delta = \frac{1}{7} \ln \left(\frac{0.4414}{0.3464} \right) = 0.0346$$

- Using equation (4.6), calculate the first mode damping ratio. Result: $\zeta_1 = 0.0055$
- Using equation (3.43), calculate the structural damping:

$$\gamma = \frac{2\zeta_1}{\omega_1} = 0.0022$$

4.1.3 Velocity Signal Calibration

An accurate built-in encoder is used to measure the angular position of the motor. To monitor the velocity, an analog filter of the encoder signal derivative (V_{VELMON})

Figure 4.6: Free oscillation of the beam with clamped-free boundary conditions. A tip displacement initial condition was applied.

has been provided by the manufacturer. This voltage is proportional to the velocity:

$$\dot{\theta} = k_{v/\theta} \times V_{VELMON} \quad (4.13)$$

Where $k_{v/\theta}$ is the calibration factor. From the motor user's guide, this signal should be 5 volts when the velocity is 1 rps. which results in a calibration factor of $72 \frac{\text{degrees}}{\text{sec.volts}}$. In early experiments it was observed that this factor is not correct and a series of experiments was done to calibrate it.

For this purpose, the motor was run by an arbitrary varying input voltage. Meanwhile the angular position (encoder) and the velocity signal were measured. To obtain the velocity, a numerical derivative of encoder signal was calculated as

$$\dot{\theta}_{num} = \frac{\theta_k - \theta_{k-1}}{\Delta t}$$

Figure 4.7: Velocity signal is measured and compared with the velocity obtained from numerical derivative of position signal. The calibration factor is averaged as $k_{v/\dot{\theta}} = 67 \frac{\text{degrees}}{\text{volts} \cdot \text{sec}}$.

This numerically obtained velocity is plotted versus the velocity signal in Figure-4.7. An average value can be obtained for the calibration factor as

$$\boxed{k_{v/\dot{\theta}} = 67 \frac{\text{degrees}}{\text{volts} \cdot \text{sec}}} \quad (4.14)$$

Figure-4.8 shows a comparison between the numerically obtained velocity and the monitored velocity with calibration factor of 67. Accuracy appears satisfactory. Numerical differentiation of the noisy encoder signal introduces some wild points, which of course are not seen in the monitored velocity.

Figure 4.8: Using the calibration ratio of $k_{v/\theta} = 67 \frac{\text{degrees}}{\text{volts} \cdot \text{sec}}$, the velocity signal (solid line) shows a much smoother behavior than the numerical obtained velocity (dot line).

4.1.4 Friction Parameters

To obtain the motor friction characteristics a model and experimental data are required. The Tustin model was chosen for its ease of parameter identification and simulation. This series of experiments was without a link or any other loads. Therefore at the threshold of motion and in steady state motion, the torque produced by the motor is equal to the friction torque: $\tau_{fr} = k_t u$. After division by k_t , equation (E.1) gives:

$$u = \left[\bar{\alpha}_0 + \bar{\alpha}_1 e^{\frac{-|\dot{\theta}|}{\alpha}} + \bar{\alpha}_2 |\dot{\theta}| \right] \text{sgn}(\dot{\theta}) \quad (4.15)$$

where $\bar{\alpha}_i = \frac{\alpha_i}{k_t}$ for $i = 0, 1, 2$. To identify these parameters, the following procedure from Popović, *et al.* (1993) is suggested:

- **High Velocity:** Apply various input voltages and measure the steady state velocity of the motor for each input. These inputs must be greater than the breakaway point and less than the saturation level. Figure-4.9 shows some responses of the motor to different voltages, where inputs 0.25 to 0.34 volts are in the desired speed range. In this range the friction equation can be simplified to

$$u = \bar{\alpha}_0 + \bar{\alpha}_2 \dot{\theta} \quad (4.16)$$

Measuring the motor velocity for two different inputs, the above parameters can be calculated as

$$\bar{\alpha}_2 = \frac{u_2 - u_1}{\dot{\theta}_2 - \dot{\theta}_1} \quad \bar{\alpha}_0 = \frac{\dot{\theta}_1 u_2 - \dot{\theta}_2 u_1}{\dot{\theta}_1 - \dot{\theta}_2} \quad (4.17)$$

An average of three experimental results gives

$$\bar{\alpha}_0 = 0.19 \quad \bar{\alpha}_2 = 0.035 \quad (4.18)$$

- **Breakaway Point:** In several experiments, it was observed that the size of the breakaway point depends on the manner of application of the voltage to the motor. Figure-4.10 illustrates this. When the desired voltage is applied as a ramp with mild slope, the breakaway occurs at $u_s=0.57$ volts, and when the voltage is applied suddenly as a step, the breakaway is at $u_c=0.23$ volts. The slope of applied voltage is approximated inversely proportional to the motor sticking time, called the dwell time, i.e. the shallower the slope the larger

Figure 4.9: Motor velocity step response for different input voltages.

the dwell time. Fig. 16 in Armstrong-Hélouvry, *et al.* (1994) shows that the breakaway point for a very slow ramp input is equal to the static friction level, and the breakaway point for a step input is equal to the Coulomb Friction Level. The latter can also be obtained by decreasing the input voltage very slowly while the system is rotating and detecting the voltage when the motor becomes stuck. This point is also equal to $u_c=0.23$ volts here. The static friction is equal to:

$$u_s = \bar{\alpha}_0 + \bar{\alpha}_1 \quad (4.19)$$

giving $\bar{\alpha}_1=0.38$.

Figure 4.10: Observation of dwell time effect on breakaway torque. schematic. The breakaway torque rises when the input torque increases smoothly and slowly.

- Saddle Point: This is the point where the negative damping (the Stribeck effect in Figure-E.1) becomes positive viscous damping, and is approximately equal to the Coulomb friction level. The main property of this point is $\frac{\partial \tau_{fr}}{\partial \dot{\theta}} = 0$. Applying equation (4.15) results in $\dot{\theta}_c = \alpha \ln \left(\frac{\bar{\alpha}_1}{\bar{\alpha}_2 \alpha} \right)$. Furthermore,

$$u_c = \bar{\alpha}_0 + \bar{\alpha}_2 \alpha \left[1 + \ln \left(\frac{\bar{\alpha}_1}{\bar{\alpha}_2 \alpha} \right) \right] \quad (4.20)$$

$$0.23 = 0.19 + 0.035\alpha \left[1 + \ln \left(\frac{0.38}{0.035\alpha} \right) \right]$$

Solving this for α gives: $\alpha = 0.24$

- Using the $\bar{\alpha}_i$'s here, and the $k_t = 29.5$ previously described, the Tustin model can be written as:

$$\tau_{fr} = \left[5.6 + 11.2e^{\frac{-|\dot{\theta}|}{0.24}} + 1.02|\dot{\theta}| \right] \text{sgn}(\dot{\theta}) \quad (4.21)$$

This model is illustrated in Figure-4.11 and is compared with experimental data. Beyond the specified speed range, saturations effects dominate the friction, and this model no longer represents the system characteristics.

Figure 4.11: Identification of Tustin friction model. (*) denotes the experimental data and solid line shows the Tustin friction model. The vertical axis represents the voltage proportional to the friction torque in the motor.

The parameters $(\alpha, \alpha_0, \alpha_1, \alpha_2)$ in the stick-slip model of Appendix-E.3 are equal to those of the Tustin model. Other parameters used in the simulation are adopted from Canudas de Wit, *et al.* (1993), as:

$$\varsigma_\theta = 0.01, \quad \varsigma_v = 0.001, \quad k_\theta = 10000, \quad d_\theta = 1100 \quad (4.22)$$

4.2 Model Determination

To verify the modeling results of chapter 3 an off line experimental identification was performed. A frequency based method was selected to identify the system natural frequencies, poles and zeros. The hub-link system was set to be initially in contact

with the environment for each experiment. A series of harmonic voltages was then applied to the system of the form

$$u = u_s + u_a \sin(\omega t) \quad (4.23)$$

To avoid the discontinuity in contact a constant bias voltage of $u_s=0.4$ volts was applied to the motor. The amplitude of the harmonic voltage must be greater than the stiction level but less than u_s , and here $u_a=0.3$ volts was chosen. To avoid possible damage to the system, the exciting frequency ω did not exceed the second resonance, and was thus in the range of 0.3 to 8 Hertz. For each experiment the hub velocity and tip force were measured. The system response was in the form of

$$F = F_s + F_a \sin(\omega t + \phi_f)$$

$$\dot{\theta} = \dot{\theta}_s + \dot{\theta}_a \sin(\omega t + \phi_v)$$

A selected experimental result is shown in Figure-4.12. Using a series of figures (not shown here) similar to Figure-4.12 the amplitudes F_a and $\dot{\theta}_a$, and also the phase lags ϕ_f and ϕ_v were obtained for their respective frequencies.

The bode plots of the velocity and force responses are shown in Figures-4.13 and 4.15. respectively. These experimental frequency responses can be compared with the frequency responses of the model obtained in chapter 3, which are shown in Figures-4.14 and 4.16. It should be noted that in the experimental results the magnitude ratios are $\frac{F_a}{u_a}$ and $\frac{\dot{\theta}_a}{u_a}$, while they are dimensionless, $\frac{F_a}{\tau_a} \times L$ and $\frac{\dot{\theta}_a}{\tau_a} \times \frac{3EI}{L}$ in the simulation results. The overall shapes are in good agreement with simulation.

Two peaks at 16 and 46 $\frac{rad}{sec}$ are seen in both the force and velocity responses. These values are approximately equal to the system natural frequencies, and are

Figure 4.12: System response to a harmonic voltage with $u_s=0.4$. $u_a=0.3$ volts and $\omega=2\pi \frac{\text{rad}}{\text{sec}} = 1$ Hertz.

very close to the simulation results, (compare with Table-3.1). System poles can be roughly estimated using these frequency values. From equation (3.43) and $\gamma = 0.0022$, identified in section 4.1.2, we have:

$$2\zeta_1\omega_1 = \gamma\omega_1^2 = 0.0022 \times 16^2 = 0.56 \quad , \quad 2\zeta_2\omega_2 = \gamma\omega_2^2 = 0.0022 \times 46^2 = 4.66$$

Therefore the denominators of both the force and velocity transfer functions are approximately

$$T.F_{den} = (s^2 + 0.56s + 16^2)(s^2 + 4.66s + 46^2) \quad (4.24)$$

Figure 4.13: Experimental frequency response of motor velocity transfer function $\left(\frac{s\theta(s)}{u(s)}\right)$. (o) denotes the experimental data.

In the frequency analysis method (e.g. see Ogata 1982), the exciting frequency is varied, and when it approximately coincides with a lightly damped system zero, it creates a minimum value (valley) in the magnitude plot and also a positive jump occurs in the phase plot. A real valued zero distant from the imaginary axis creates neither a valley in the magnitude plot nor a positive jump in the phase plot, because it does not coincide with the exciting frequency ($j\omega$). Thus, pure imaginary zeros can be approximately determined by studying frequency responses, and little information can be obtained about the value of real zeros directly from frequency plots.

A valley is seen in the bode plot of the velocity response at $40 \frac{\text{rad}}{\text{sec}}$, see Figure 4.13.

Figure 4.14: Frequency response of motor velocity transfer function $\left(\frac{s\theta(s)}{\tau(s)} \times \frac{3EI}{L}\right)$. simulation result.

The phase has a $+90$ degree jump at this point which shows the existence of an imaginary zero. So this zero can easily be obtained from the valley frequency. The velocity transfer function estimated from experimental data can be written in pole-zero and partial fraction forms as:

$$\begin{aligned}
 G_v(s) &= g_v \frac{s(s^2 + \xi_v s + 40^2)}{(s^2 + 0.56s + 16^2)(s^2 + 4.66s + 46^2)} \\
 &= g_v \left[\frac{g_{v,1}s}{s^2 + 0.56s + 16^2} + \frac{g_{v,2}s}{s^2 + 4.66s + 46^2} \right] \quad (4.25)
 \end{aligned}$$

where

$$1 = g_{v,1} + g_{v,2}$$

$$\begin{aligned}\xi_v &= g_{v,1} \times 4.66 + g_{v,2} \times 0.56 \\ 40^2 &= g_{v,1} \times 46^2 + g_{v,2} \times 16^2\end{aligned}$$

Solving this set of equations gives:

$$g_{v,1} = 0.72 \quad , \quad g_{v,2} = 0.28 \quad , \quad \xi_v = 3.51 \quad (4.26)$$

To non-dimensionalize the transfer function, the dc gain must be $g_v = \frac{16^2 \times 46^2}{40^2} = 338.56$. Therefore the experimental velocity transfer function is:

$$G_v(s) = 338.56 \times \frac{s(s^2 + 3.51s + 40^2)}{(s^2 + 0.56s + 16^2)(s^2 + 4.66s + 46^2)} \quad (4.27)$$

A valley is detected at $30 \frac{\text{rad}}{\text{sec}}$, between the two peaks of force response, see Figure-4.15. Since the phase plot of force transfer function remains constant around the valley frequency of $30 \frac{\text{rad}}{\text{sec}}$, it is concluded that no imaginary zeros exist at this valley. This indicates that the zeros of the force transfer function must be $\pm$ real valued (agrees with equation 3.94), which produces a non minimum phase system, and are not therefore identifiable via force frequency plots directly. To find zeros of the force transfer function a *Ratiometry Technique* was used. In this technique the zeros of a NMP system are calculated from the zeros of a minimum phase system, (Cannon and Schmitz 1984, Maples 1985).

The force transfer function can be written in pole-zero and partial fraction formats as:

$$\begin{aligned}G_f(s) &= g_f \frac{s^2 + \xi_f s - \omega_f^2}{(s^2 + 0.56s + 16^2)(s^2 + 4.66s + 46^2)} \\ &= g_f \left[\frac{-g_{f,1}}{s^2 + 0.56s + 16^2} + \frac{g_{f,2}}{s^2 + 4.66s + 46^2} \right] \quad (4.28)\end{aligned}$$

Figure 4.15: Experimental frequency response of tip force transfer function $\left(\frac{F(s)}{u(s)}\right)$. (o) denotes the experimental data.

Since the peaks of this lightly damped system are well separated it can be concluded that for $\omega = \omega_i$

$$|G_v(j\omega_i)| = \left| \frac{\dot{\theta}}{\theta_d} \right|_{\omega_i} \cong g_v \frac{g_{v,i}}{2\zeta_i \omega_i} \quad , \quad |G_f(j\omega_i)| = \left| \frac{F}{F_d} \right|_{\omega_i} \cong g_f \frac{g_{f,i}}{2\zeta_i \omega_i^2} \quad (4.29)$$

Dividing $|G_v(j\omega_i)|$ by $|G_f(j\omega_i)|$ results in

$$(g_f g_{f,i}) = (g_v g_{v,i}) \omega_i k_{v/f} \left| \frac{F}{\theta} \right|_{\omega=\omega_i} \quad (4.30)$$

where $k_{v/f} = \left| \frac{\dot{\theta}_d}{F_d} \right| = \frac{L^2}{3EI} = 0.025$ is a non-dimensionalization coefficient. Equation

Figure 4.16: Frequency response of tip force transfer function $\left(\frac{F(s)}{\tau(s)} \times L\right)$, simulation result.

(4.30) is the main relationship used in the ratiometry technique with

$$\left|\frac{F}{\theta}\right|_{\omega=\omega_i} = \left[\frac{\text{Force Sensor Output}}{\text{Velocity Sensor Output}}\right]_{at:\omega=\omega_i} \quad (4.31)$$

At $\omega = \omega_1 = 16$ and $\omega = \omega_2 = 46 \frac{\text{rad}}{\text{s}}$ the experimental values for the force/velocity ratio are

$$\left|\frac{F}{\theta}\right|_{\omega_1} = \left|\frac{9.53}{1.88}\right| = 5.07 \quad , \quad \left|\frac{F}{\theta}\right|_{\omega_2} = \left|\frac{7}{0.7}\right| = 10 \quad (4.32)$$

Substituting into equation (4.30) gives

$$\begin{aligned} (g_f g_{f,1}) &= (338.56 \times 0.72) \times 16 \times 0.025 \times 5.07 = 488 \\ (g_f g_{f,2}) &= (338.56 \times 0.28) \times 46 \times 0.025 \times 10 = 1053 \end{aligned} \quad (4.33)$$

Table 4.1: Natural frequencies and zeros of simulation and experimental TF.

	ω_1	ω_2	poles	z_v	z_f
Experiment	16	46	$-0.28 \pm 16j$ $-2.33 \pm 46j$	$-1.76 \pm 40j$	-35.2, 38.2
Simulation	16.3	46.4	$-2.34 \pm 16.07j$ $-2.56 \pm 46.18i$	$-1.76 \pm 39.8j$	-28.4, 29.8

From equation (4.28) it is concluded that

$$g_f = -(g_f g_{f,1}) + (g_f g_{f,2}) = -488 + 1053 = 565 \quad (4.34)$$

and thus $g_{f,1} = \frac{488}{565} = 0.86$ and $g_{f,2} = \frac{1053}{565} = 1.86$. Also from equation (4.28) the following relations are obtained:

$$\xi_f = -4.66 \times g_{f,1} + 0.56 \times g_{f,2} = -2.97 \quad (4.35)$$

$$\omega_f^2 = g_{f,1} \times 46^2 - g_{f,2} \times 16^2 = 1343.6 \Rightarrow \omega_f = 36.7$$

Calculating all necessary coefficients the experimental force transfer function is therefore estimated as:

$$G_f(s) = 565 \times \frac{(s^2 - 2.97s - 36.7^2)}{(s^2 + 0.56s + 16^2)(s^2 + 4.66s + 46^2)} \quad (4.36)$$

The results of this section are illustrated briefly in Table-4.1, where z_v and z_f denote zeros of velocity and force transfer functions, respectively. A difference between first model damping of the experiment and simulation appears, because the effect of joint viscous damping and motor back emf were not considered during

the experimental model determination. These factors affect only damping of the first mode. Since the zeros of the force transfer function were not determined directly from experimental data, and the ratiometry technique was used to calculate them, approximately a 20% difference between simulation and experiment is seen. In other parameters such as natural frequencies, poles, velocity zeros, and second mode damping, good agreement between experimental and simulation results is observed. This verifies the reliability of the system model obtained in chapter 3.

Chapter 5

Composite Colocated and Noncolocated Control

In previous chapters the system dynamic model was obtained and its parameters were identified experimentally. Based on this information the poles and zeros of the system were calculated, and presented in Table-3.5. This table displayed some Right Half Plane (RHP) zeros in the system transfer function. For a set of transfer functions with the same magnitude function $|G(j\omega)|$, the phase angles for those with RHP zeros are greater than that with only LHP zeros, giving the name “NonMinimum Phase systems (NMP)” to the former, (Ogata 1982).

The presence of link flexibility causes the actuator (at the base) to be noncolocated with the sensor (at the tip). In much current literature, this noncolocation has been cited as the cause of RHP zeros for this type of system. However, via a simple example it is shown in Appendix-F that noncolocation does not necessarily imply the existence of RHP zeros. A system with a noncolocated actuator/sensor

configuration, possesses RHP zeros (is NMP) when it also exhibits *dispersive* behavior, such as in the bending of a flexible link. The axial and torsional deformation of a flexible beam are *non-dispersive* by contrast, and are thus not NMP. (Miu 1991).

Nonminimum phase systems are sluggish in response due to their initially moving in a direction opposite to that desired. They possess limitations in the achievable performance of a feedback loop independent of the design methodology used (Freudenberg and Looze 1985). Kwakernnak and Sivan (1972) discuss inherent performance limitations for linear NMP control systems, and Slotine and Li (1991) describe how RHP zeros introduce unstable internal dynamics and cause problems in tracking control. The RHP zeros are not modifiable and they remain unchanged in a conventional feedback loop. This is one of the major limitations on achievable bandwidth.

In this chapter we attempt to improve the control system performance by means of adding colocated feedback.

5.1 (PI force + velocity) Feedback Control

Since contact force control of a flexible beam by means of an actuator at its base is a noncolocated control problem which introduces the problems of nonminimum phase systems, a possible remedy might be an augmentation of sensing capability so as to produce a combination of colocated and noncolocated control. Motor velocity (colocated) plus tip force (noncolocated) feedback control is suggested here. Figure-5.1 shows the block diagram of the control system.

Figure 5.1: Block diagram of PI force + Velocity feedback Control system.

5.1.1 Suboptimal Design

As a preliminary step, the Linear Quadratic Regulator (LQR) method, e.i. Kirk (1970), was used to design a set of optimal controller gains. At this stage no feedforward (K_r) and integral action (K_i) gains were considered. Via well-known methods the feedback gain matrix $\bar{K}$ in the control law:

$$u = -\bar{K}x \quad (5.1)$$

which minimizes the objective function:

$$J = \int_0^t (x^T \bar{Q}x + u^T Ru)dt = \int_0^t (y^T Qy + u^T Ru)dt \quad (5.2)$$

is determined. $\bar{Q}$ and R are weighting and penalty matrices respectively. u is the control signal and x is the state vector defined in equation (3.48). The output vector is defined as:

$$y = [F \quad \dot{\theta}]^T = Cx = [C_F^T \quad C_{\dot{\theta}}^T]^T x \quad (5.3)$$

Allowing the weighting function to be written as: $\bar{Q} = C^T Q C$.

From the output vector definition it can be seen that the full state vector is unavailable for use in equation (5.1). However it was observed in this study that a *static output feedback controller*, based on this optimal state controller can never-

theless produce good results. This static gain is determined as:

$$K \equiv [K_f \ K_v] = \bar{K} C^T (C C^T)^{-1} \quad (5.4)$$

which is used in the control law

$$u = -K y \quad (5.5)$$

This control scheme produces a transfer function matrix for the overall closed loop in the following form:

$$\begin{Bmatrix} F(s) \\ \dot{\theta}(s) \end{Bmatrix} = \begin{bmatrix} G_{F/F} & G_{F/\theta} \\ G_{\theta/F} & G_{\theta/\theta} \end{bmatrix} \begin{Bmatrix} F_d(s) \\ 0 \end{Bmatrix}$$

Where $F_d(s)$ is the desired contact force, and the desired motor velocity is zero.

Therefore:

$$F(s) = G_{F/F}(s) F_d(s) \quad (5.6)$$

where $G_{F/F}(s)$ can be determined by block diagram manipulation as:

$$G_{F/F}(s) = \frac{k_t K_f G_f(s)}{1 + k_t K_v G_v(s) + k_t K_f G_f(s)} \quad (5.7)$$

G_f and G_v are the plant transfer functions from applied torque to tip force and hub velocity respectively. $G_f(s=0)$ can be physically interpreted as the steady state relation between joint torque and tip force. Equation (3.101) shows that $G_f(0) = \frac{1}{L}$.

Thus the steady state response of the system to a unit step is seen to be

$$F_{ss} = G_{F/F}(0) = \frac{K_f k_t}{L + K_f k_t} \quad (5.8)$$

This is a steady state error in the unit step response of the closed loop system. To address this error one could use a feed forward gain such as

$$K_r = \frac{1}{G_{F/F}(0)} = \frac{L + K_f k_t}{K_f k_t} \quad (5.9)$$

5.1.2 Simulation Results

Although the LQR method maintains system stability, the modification using the pseudo inverse in equation (5.4) no longer allows this assurance. Nevertheless, the results of applying this procedure on the system described on chapter 3, is a set of stable gains (verified by examining the closed loop poles).

$$K_f = 0.0013 \frac{volt}{V} \quad K_v = 0.415 \frac{volt.s}{rad} \quad K_r = 34.5 \quad (5.10)$$

where the weighting and penalty matrices were taken to be $Q = \begin{bmatrix} 100 & 0 \\ 0 & 10 \end{bmatrix}$. $R = 10$. Since the main concern in this research is controlling the contact force, weight of minimization for the force error is selected to be 10 times greater than those for the velocity error and energy consumption.

We now examine the effect of these gains on the controlled system. The SIMULINK diagram on which a simulation was based is shown in Figure-G.12. To study the effect of various controller components as well as their performance in the presence of friction and uncertainty, several cases were simulated, with the legend of results shown in Table-5.1. These are now discussed.

It is desired to impose a force of 10 Newton on surface when the system is initially in contact. Figure-5.2, case A, shows the closed loop response of the system when only K_f is used and no friction and uncertainty are assumed. It illustrates how the p-controller suffers from a significant steady state error. This problem can be solved by using a feedforward gain as in case B, which has a large overshoot in its response and has thus replaced one problem with another. To reduce this overshoot and otherwise improve the performance, velocity feedback is added in case C which

Table 5.1: Legend of different Plf+v step responses.

Label	Figure	K_r	$K_f \frac{\text{volt}}{N}$	$K_i \frac{\text{volt}}{N.s}$	$K_v \frac{\text{volt.s}}{\text{rad}}$	Friction	Uncertainty
A	5.2	1	0.0013	0	0	NO	NO
B	5.2	34.6	0.0013	0	0	NO	NO
C	5.2	34.6	0.0013	0	0.4	NO	NO
D	5.2	34.6	0.0013	0	0.4	NO	YES
E	5.2 & 5.3	1	0.0013	0.1	0.4	NO	NO
F	5.2	1	0.0013	0.1	0.4	NO	YES
G	5.3	1	0.0013	0.1	0	NO	NO
H	5.3	1	0.0013	0.1	0	YES	NO
I	5.3	1	0.0013	0.1	0.4	YES	NO
J	5.4	1	0.007	0.1	0.4	NO	NO
K	5.4	1	0.007	0.1	0.4	YES	NO

shows much improved behavior.

The controller with feedforward gain K_r is insufficiently robust when uncertainty and/or friction is considered in the system. As an example, case D shows the system response when a 37% uncertainty exists in the motor torque constant k_t . Another (standard) approach for correcting the steady state error is by using integral action, $\frac{K_i}{s}$, case E. Although the controller with integral action is slower, it does show good robustness in the presence of parameter uncertainty, case F. Here K_i is used as a fine tuning parameter.

Figure 5.2: Effects of forward gain, velocity feedback, integral action, and parameter uncertainty on the time response of PI+ v feedback system.

In Figure-5.3, cases E and G show again how adding the velocity feedback to a PI-force feedback improves the performance and behavior of the system. Cases H and I show the system response in the face of severe joint friction. It is seen that stiction produces unwanted oscillations around the desired point. Comparison between these cases shows how adding velocity feedback reduces the amplitude and increases the frequency of these unwanted oscillations.

As a final gain tuning example, K_f was increased to 0.007. Case J in Figure-5.4 shows that this change results in a faster response with lower overshoot.

The response of the final PI+ v controller (case J), when the system is not

Figure 5.3: Joint friction effect on the PIf+v feedback system.

initially in contact is also shown in the figure. The initial angle is $\theta_{t=0} = 15^\circ$, which results in an approach velocity and an impact at the instant of contact. Case K shows the response when $\theta_{t=0} = 15^\circ$ and friction exists in the joint. Contact breaking is seen in both cases. It is observed that friction not only postpones the first contact time, but also increases the impact effect.

5.2 Contact Stability

Analysis of contact stability is of importance due to the inherent discontinuity of the system. It should be noted that the controller is desired to handle both the contact

Figure 5.4: Effect of approach velocity on PIf+v feedback system.

and non-contact cases. Its stability must therefore be studied in both cases.

Figure-5.5 shows the PIf+v block diagrams for the system *in* contact and *not in* contact. From these diagrams the transfer functions of the two systems are:

$$G^u(s) = \frac{sW_L(s)}{F_d(s)} = \frac{(K_f s + K_i)k_t G_w}{s(1 + k_t K_v G_v^u)} \quad (5.11)$$

$$G^c(s) = \frac{F(s)}{F_d(s)} = \frac{(K_f s + K_i)k_t G_f}{(K_f s + K_i)k_t G_f + s(1 + k_t K_v G_v^c)} \quad (5.12)$$

where G_w and G_v^u are the open loop transfer functions of the unconstrained system for tip and hub velocity respectively. G_f and G_v^c are the open loop transfer functions of the constrained system for contact force and hub velocity. The following can be concluded from these transfer functions:

Figure 5.5: Closed loop block diagram for PI+ v control system. a) PI+ v for unconstrained system, b) PI+ v for constrained system.

1. Since G_w and G_v^u are both of type 0, G^u is at least of type 1. This implies the existence of pure integrators in the unconstrained closed loop system. Therefore this system is *not* BIBO (bounded input-bounded output).
2. In the non-contact situation the force feedback signal is zero and only hub velocity feedback exists (see Figure-5.5.a), therefore the stability of the closed loop system depends only on K_v (see the denominator of equation 5.11). The root locus of G^u shown in Figure-5.6 reveals that the system is stable.
3. G^c is conditionally stable and its stability depends on the values of controller gains K_f , K_i , and K_v .
4. When the system is initially in contact, if G^c is stable a stable contact is established, e.g. see Figure-5.4.J.
5. If G^c is unstable, the amplitude of contact force oscillations increases, and because motion is not constrained to the surface, after some oscillations contact will be broken. Because the unconstrained closed loop is stable, the system will

Figure 5.6: Root locus of unconstrained system with velocity feedback.

return toward the wall and contact will be made again. Empirical observations obtained from several simulations show that the making-breaking contact cycle is repeated several times until the hybrid system (constrained-unconstrained) reaches a limit cycle. This limit cycle is observed as a chattering in the real system. It is concluded that, even if G^c is unstable, oscillations will be bounded by a stable limit cycle.

Figure-5.7.top. shows an example of an unstable G^c . In this simulation the gains are $K_f = 0.068 \frac{\text{volt}}{\text{N}}$, $K_i = 0.1 \frac{\text{volt}}{\text{N.s}}$, and $K_v = 0.4 \frac{\text{volt.s}}{\text{rad}}$, and G^c has a pair of unstable poles at: $+5.6 \pm 38.9i$. A limit cycle with a period of 0.214 second, frequency of 4.67 Hertz, force amplitude of 35 Newton, and tip position amplitude of 45 millimeter, is evident. Through several simulations it was observed that the frequency and magnitude of these limit cycles depend on the values of desired force and controller

Figure 5.7: Limit cycle (chattering) observation in PI+D controlled system. Top: without friction, bottom: with friction effect. $K_f = 0.068 \frac{\text{volt}}{\text{N}}$, $K_i = 0.1 \frac{\text{volt}}{\text{N.s}}$, and $K_v = 0.4 \frac{\text{volt.s}}{\text{rad}}$. System initially in contact.

gains. Figure-5.7.bottom. shows that friction increases the frequency and introduces irregularities in the magnitude of the limit cycle.

Using a numerical algorithm the stability boundaries were found. This is shown in Figure-5.8. and can be used to determine the contact stability condition of the system.

Figure 5.8: Region of stability for PIf+v feedback system.

5.3 Experimental Results

Before discussing the experimental results of the controlled system, an important observation on the open loop response of the system should be noted. Figure-5.9 shows a step response of the open loop system when it is not initially in contact. The initial angle between the link and the wall is $\theta_0 = 15^\circ$. It can be seen that during the non-contact period ($t < 0.3$ sec.), the force sensor produces some non-zero signals which are caused by sensor inertia. This is one difference between experiment and simulation, where sensor inertia was not considered. This phenomenon invalidates the phase indicator and event detector functions of (3.81) which were used in simu-

Figure 5.9: Force sensor inertia effect. Experimental result. Open loop response for system with approach velocity. $\theta_0 = 15^\circ$.

lation. However, in the proposed Plf+v control, when the system switches between contact and non-contact situations, the controller remains unchanged and there is no need of such functions in the experiments.

With use of just proportional force control (when $K_v = K_i = 0$), it should be noted that the control voltage at the threshold of motion (u_0), must exceed the joint static friction ($u_s = 0.57$ volts). Using the following control voltage transfer function

$$\frac{u(s)}{F_d(s)} = \frac{K_r K_f}{1 + K_f k_i G_f(s)} \quad (5.13)$$

Figure 5.10: P-action force control. Experimental result. $K_r = 2.11$. $K_f = 0.04 \frac{\text{volt}}{\text{N}}$. System initially in contact.

and also substituting in K_r from equation (5.9), u_0 can be obtained as

$$u_0 = u_{t=0} = K_r K_f F_d = \left(\frac{L}{k_t} + K_f \right) F_d \quad (5.14)$$

Thus a condition for threshold of motion is determined as

$$\text{for } F_d = 10 \text{ [N]} : \quad K_f > \left(\frac{u_s}{F_d} - \frac{L}{k_t} \right) = 0.012 \frac{\text{volt}}{\text{N}} \quad (5.15)$$

This criterion explains why the actual system did not move when the simulation gains ($K_f = 0.007$ and $K_r = 34.5$) were used in experiments.

An experiment result with $K_f = 0.04 \frac{\text{volt}}{\text{N}}$ and $K_r = 2.11$ is seen in Figure-5.10.

Figure 5.11: Velocity feedback effect. Experimental result. $K_r = 2.11$. $K_v = 0.4 \frac{\text{volt.s}}{\text{rad}}$. Top: $K_f = 0.07 \frac{\text{volt}}{\text{N}}$, Bottom: $K_f = 0.04 \frac{\text{volt}}{\text{N}}$. System initially in contact.

and shows a 200% overshoot. A steady state error caused by the system itself and also by joint friction is seen. Following the transient response ($t > 0.8$ sec), an unwanted oscillation, mostly excited by stiction, is observed.

Figure-5.11.bottom, shows how adding a velocity feedback with $K_v = 0.4 \frac{\text{volt.s}}{\text{rad}}$ improves the system behavior, with lower overshoot, faster response and lower magnitude of stiction excited oscillations. A steady state error still remains in the

Figure 5.12: Effect of approach velocity on Pf+v feedback system. Experimental result. $K_r = 2.11$. $K_v = 0.4 \frac{\text{volt}\cdot\text{s}}{\text{rad}}$, dash: $K_f = 0.07 \frac{\text{volt}}{\text{N}}$, solid: $K_f = 0.04 \frac{\text{volt}}{\text{N}}$. $\theta_0 = 15^\circ$.

system, but can be decreased by increasing K_f . Figure-5.11.top, shows that this approach would, however, dramatically worsen other aspects of system response. Figure-5.12 shows the system response with an initial approach velocity.

The step responses of a PI+f+v control with $K_i = 0.1 \frac{\text{volt}}{\text{N}\cdot\text{s}}$ are shown in Figure-5.13. Since integral action has been used, K_r is set to one. Comparing the upper and lower figures shows that increasing K_f from 0.04 (top) to 0.07 (bottom) improves the system performance in this case. It is seen that between the times of 0.5 second and

Figure 5.13: Force integral effect on PIf+v feedback system. Experimental result. $K_r = 1$, $K_v = 0.4 \frac{\text{volt}\cdot\text{s}}{\text{rad}}$, $K_i = 0.1 \frac{\text{volt}}{\text{N}\cdot\text{s}}$, Top: $K_f = 0.04 \frac{\text{volt}}{\text{N}}$, Bottom: $K_f = 0.07 \frac{\text{volt}}{\text{N}}$. System initially in contact.

1 second, the system was stuck because of friction, and the integral action caused the system to jump up above the desired point. This stick-slip mode of motion introduces a small, low frequency limit cycle around the desired point. The noise in the response is from sensor bit resolution, which is about ± 0.3 Newton.

Figure 5.14: Effect of approach velocity on PI+D feedback system. Experimental result. $K_r = 1$, $K_v = 0.4 \frac{\text{volt}\cdot\text{s}}{\text{rad}}$, $K_f = 0.07 \frac{\text{volt}}{\text{N}}$, $K_i = 0.1 \frac{\text{volt}}{\text{N}\cdot\text{s}}$, $\theta_0 = 15^\circ$.

Figure-5.14 shows how this controller acts when the system is not initially in contact. The response of this system shows a significant decrease in overshoot and also in steady state error in comparison with the previous experiments (see Figures- 5.9 and 5.12).

Chapter 6

Unified Predictive Force Control

As mentioned in previous chapters the existence of RHP zeros limits the achievable performance of control systems. Although adding colocated feedback improved control significantly, we wish to improve performance still further.

One means of addressing this problem is via predictive controllers, which have demonstrated some success in handling difficult systems, such as those with time delays, open-loop instability, and NMP transfer functions, (Soeterboek 1992).

6.1 Theory Review

In this section $u(k)$, $y(k)$, $\hat{y}(k)$, and $r(k)$ denote the control signal, measured process output, predicted value of process output, and reference trajectory signals respectively, at sample time k in the discrete time domain, see Figure–6.1. A predictive controller, in general, determines future sequences of $u(k)$ such that the predicted values of $\hat{y}(k)$ are close to the future reference trajectory $r(k)$. Only the first element of this controller output sequence ($u(k)$) is applied to the system at any time. At

Figure 6.1: Illustration of some of the UPC's parameters. H_p : prediction horizon. H_m : minimum-cost horizon, H_c : control horizon.

the next sample the whole procedure is repeated using the latest measured information ($y(k)$). This is called the *Receding Horizon* principle. This method allows the compensation of disturbances and/or modeling errors.

Any predictive controller has four basic elements: 1. A process model which is used to predict the process output over the *prediction horizon* H_p . 2. A reference trajectory. 3. A criterion function which is minimized to obtain the controller output sequences. 4. A minimization procedure. Based on how these elements are selected, many predictive controllers have been introduced to date, such as Generalized Predictive Control (GPC), Dynamic Matrix Control (DMC), Extended Prediction Self-Adaptive Control (EPSAC), etc.. This wealth of choices can complicate the

problem of controller selection. Therefore Soeterboek, *et al.* (1990) proposed *Unified Predictive Control* which unifies a number of well-known predictive controllers such that the designer has access to most of the desirable properties of these controllers in a single framework.

6.1.1 Unified Prediction Model

Predictive controllers are model based controllers. Although some predictive controllers have been designed for nonlinear and continuous systems, most well known MBPC have been designed based on linear, discrete time system models. A linear process can be represented as a discrete time transfer function, and the *certainty equivalence principle* allows us to replace the true values of the plant parameters by their estimated values, (see Åström and Wittenmark 1989). A process model, which we can use for prediction, can be written as:

$$\hat{y}(k) = \frac{q^{-\hat{d}}\hat{B}(q^{-1})}{\hat{A}(q^{-1})}u(k-1) + \frac{T(q^{-1})}{D(q^{-1})}e(k) \quad (6.1)$$

where the hat denotes estimate and q^{-1} is the backward shift operator. $\hat{A}$ and $\hat{B}$ are the estimated polynomials in q^{-1} with $\hat{A}$ monic. $\hat{d}$ is the estimated delay. $e(k)$ is a disturbance. T and D are disturbance model polynomials which are very difficult to identify, so are usually used as tuning parameters.

Using the following *Diophantine equations*:

$$\begin{aligned} \frac{\hat{B}}{\hat{A}} &= G_i + q^{-i+\hat{d}}\frac{H_i}{\hat{A}} & i \geq \hat{d} + 1 \\ \frac{T}{D} &= E_i + q^{-i}\frac{F_i}{D} \end{aligned} \quad (6.2)$$

the *unified i-step-ahead predictor* can be obtained as:

$$\hat{y}(k+i) = \underbrace{G_i u(k+i-\hat{d}-1)}_{future} + \underbrace{\frac{H_i}{\hat{A}} u(k-1) + \frac{F_i}{T} [y(k) - \hat{y}(k)]}_{past} \quad (6.3)$$

The elements of $\hat{y}(k+i)$ for $i = 1$ to H_p , are then written in matrix form as:

$$\hat{\mathbf{y}} = \underbrace{\mathbf{G}\mathbf{u}}_{future} + \underbrace{\mathbf{H}\check{\mathbf{u}} + \mathbf{F}\mathbf{c}}_{past} \quad (6.4)$$

where $\hat{\mathbf{y}}$, $\mathbf{u}$, $\check{\mathbf{u}}$, $\mathbf{c}$ are vectors with respective elements: predicted sequences of $y(k)$ up to H_p , future sequences of $u(k)$ up to $(H_p - \hat{d} - 1)$, past sequences of $\check{u} = u(k)/\hat{A}$, and past sequences of $c(k) = [y(k) - \hat{y}(k)]/T$. $\mathbf{G}$, $\mathbf{H}$, and $\mathbf{F}$ are matrices constructed of the coefficients of the polynomials G_i , H_i , and F_i respectively.

6.1.2 Unified Criterion Function

A criterion function J is used to monitor how close the predicted process output is to the reference trajectory. If this criterion is quadratic and the model is linear, an analytical solution to the minimization problem exists. Otherwise if any constraint or nonlinearity is present in the model, or a non-quadratic criterion is used, an iterative minimization procedure should be employed.

The quadratic criterion function which is used in UPC can be written in matrix notation as:

$$\begin{aligned} J &= \sum_{i=H_m}^{H_p} \left[P(q^{-1})\hat{y}(k+i) - P(1)r(k+i) \right]^2 + \rho \sum_{i=1}^{H_p-\hat{d}} \left[\frac{Q_n(q^{-1})}{Q_d(q^{-1})} u(k+i-1) \right]^2 \\ &= (\hat{\mathbf{y}}^* - \mathbf{r}^*)^T (\hat{\mathbf{y}}^* - \mathbf{r}^*) + \rho \mathbf{u}^{*T} \mathbf{u}^* \end{aligned} \quad (6.5)$$

where:

$$\mathbf{r}^* = P(1)[r(k+H_m), \dots, r(k+H_p)]^T$$

$$\begin{aligned}\hat{\mathbf{y}}^* &= P(q^{-1})[\hat{y}(k + H_m), \dots, \hat{y}(k + H_p)] \\ \mathbf{u}^* &= \frac{Q_n(q^{-1})}{Q_d(q^{-1})} \mathbf{u}\end{aligned}\quad (6.6)$$

This criterion function is minimized subject to the following constraint:

$$\phi P u(k + i - 1) = 0 \quad 1 \leq H_c < i \leq H_p - \hat{d} \quad (6.7)$$

Where H_p , H_m , and H_c are respectively prediction, minimum-cost, and control horizons, and all are integers. ϕ is a polynomial in q^{-1} and is designed to eliminate steady state error. $P(q^{-1})$ is a filter for tracking error, and its usage is essential for tuning the servo behavior of the control system. $Q_n(q^{-1})$ and $Q_d(q^{-1})$ are filters for controller output. ρ is a weighting factor and is introduced to allow a trade-off between minimization of tracking error and minimization of controller output.

The inclusion of minimum-cost and control horizons in the criterion function, ensures that the process output is close to the reference trajectory within H_m samples and by H_c distinct controller outputs. This consideration reduces the calculation time significantly.

6.1.3 Unified Predictive Control Law

The elements of the controller output sequence which must be calculated are included in vector $\bar{\mathbf{u}}$ as:

$$\bar{\mathbf{u}} = [u(k), \dots, u(k + H_c - 1)]^T \quad (6.8)$$

The criterion function J must be minimized with respect to $\bar{\mathbf{u}}$ as

$$\frac{\partial J}{\partial \bar{\mathbf{u}}} = 0 \quad (6.9)$$

On solving equation (6.9). $\bar{\mathbf{u}}$ can be determined as:

$$\bar{\mathbf{u}} = 2\mathbf{J}^{-1}\mathbf{M}^T \left[\mathbf{G}^T(\mathbf{r}^* - \mathbf{H}\bar{\mathbf{y}} - \mathbf{F}\mathbf{c} - \mathbf{G}\mathbf{N}\bar{\mathbf{u}}) - \rho\Phi^T(\Omega\bar{\mathbf{u}} + \Phi\mathbf{N}\bar{\mathbf{u}}) \right] \quad (6.10)$$

where the *Hessian* $\mathbf{J}$ is defined as:

$$\mathbf{J} = 2\mathbf{M}^T(\mathbf{G}^T\mathbf{G} + \rho\Phi^T\Phi)\mathbf{M} \quad (6.11)$$

Elements of matrices Φ and Ω are the coefficients of polynomials Φ_i and Ω_i respectively. These polynomials are obtained from the following diophantine equation:

$$\frac{Q_n}{Q_d} = \Phi_i + q^{-i} \frac{\Omega_i}{Q_d} \quad (6.12)$$

Vectors $\bar{\mathbf{u}}$ and $\bar{\mathbf{u}}$ are defined as

$$\begin{aligned} \bar{\mathbf{u}} &= \left[\frac{u(k-1)}{Q_d}, \dots, \frac{u(k-n_\Omega)}{Q_d} \right]^T \\ \bar{\mathbf{u}} &= [u(k-1), \dots, u(k+H_c - n_o - n_P)]^T \end{aligned}$$

Matrices $\mathbf{M}$ and $\mathbf{N}$ contain the coefficients of M_i and N_i which are obtained from solving the diophantine equation for $1/oP$. as:

$$\begin{aligned} \frac{1}{oP} &= U_{i-H_c} + q^{-i+H_c} \frac{V_{i-H_c}}{oP} \\ V_{i-H_c} &= M_{i-H_c} + q^{-H_c} N_{i-H_c} \end{aligned}$$

The unified predictive control law of equation (6.10) can be rewritten in polynomial form as:

$$\mathcal{R}(q^{-1})u(k) = -\mathcal{S}(q^{-1})y(k) + \mathcal{T}(q^{-1})r(k+H_p) \quad (6.13)$$

Where $\mathcal{R}$, $\mathcal{S}$, and $\mathcal{T}$ are polynomials in q^{-1} . Figure-6.2 shows the UPC block diagram using the final form of equation (6.13). Details of this section can be found in (Soeterboek 1992).

Figure 6.2: UPC block diagram.

6.2 Control Design Procedure

The system model of equations (3.52) and (3.72) can be easily written in the form of equation (6.1). As mentioned earlier, estimation of polynomials T and D is difficult in practice, and they are therefore often used as design parameters.

Prediction Horizon: Figure-6.3 shows the pole-zero map of the discrete process transfer function with the parameter values of Table-A.1. Sampling time is $T_s = 0.01$ second. It is seen that the system is NMP and poorly-damped. For poorly-damped processes the prediction horizon can be chosen as:

$$H_p = \text{integer}\left(\frac{2f_s}{f_b}\right) \quad (6.14)$$

where f_s is the sampling frequency and f_b is the frequency bandwidth of the open-loop process. From the bode plot of the process, Figure-4.16, it can be seen that $f_b = 10H_z$. The prediction horizon of (6.14) is set to $H_p = 20$, as about this value the controller is not sensitive to changes in H_p .

Minimum-Cost Horizon: Changing H_m may cause unpredictable system behavior, especially for NMP systems, so that H_m is set to $\hat{d} + 1$ and is not used as a

Figure 6.3: Pole zero map of open loop system in discrete time domain.

design parameter. The estimated delay is zero so the minimum-cost horizon is set to be $H_m = 1$ here.

Control Horizon: Many researchers have reported that choosing $H_c = n_A$ (where n_A is the degree of polynomial $\hat{A}(q^{-1})$), yields a compromise between robustness and performance of the closed-loop system. The relation between the number of assumed modes in flexible beam model n , and n_A is $n_A = 2n$. Assuming 3 modes, the control horizon is set to be $H_c = 6$. However after doing several simulations it was concluded that this value is the best possible value for H_c regardless of how many modes are considered in the prediction model (n_A).

Tracking Error Filter: $P(q^{-1})$ can be used to define the desired servo behavior of the closed-loop system. Often it is desirable for the closed-loop system to have a second-order response to a step change in the set point: $P(q^{-1}) = 1 + p_1 q^{-1} + p_2 q^{-2}$.

Choosing the damping as $\xi = 0.7$, and selecting a desired peak time t_p , the natural frequency can be found as $\omega_n = \pi / (t_p \sqrt{1 - \xi^2})$. The coefficients of P can then be obtained as $p_1 = -2e^{-\xi\omega_n T_s} \cos(\omega_n T_s \sqrt{1 - \xi^2})$ and $p_2 = e^{-2\xi\omega_n T_s}$. Choosing $t_p = 0.5$ second, the tracking error filter becomes:

$$P(q^{-1}) = 1 - 1.877q^{-1} + 0.884q^{-2} \quad (6.15)$$

Control Output Filter: For processes without integrators, to eliminate steady-state error it has been suggested that Q_n be selected as $1 - q^{-1}$, which can, however, result in a poorly damped closed-loop system. This problem is avoided by choosing Q_d as $Q_d = 1 - 0.95q^{-1}$. (Soeterboek, *et al.* 1991).

Disturbance Model: It can be shown that if $\hat{A}$ is a factor of D , such as $D = \hat{A}D_1$, then $\hat{A}$ would be no longer a factor of the closed loop characteristic equation. Therefore the UPC controller would be stable even if the open loop plant $\hat{A}$ were unstable. On the other hand, $\phi(q^{-1})$ must be a factor of D_1 to prevent steady state error, and P must also be a factor of D_1 to increase stability robustness margins. Choosing $\phi = 1 - q^{-1}$ results in a polynomial for D_1 as $D_1 = P(1 - q^{-1})$. Since the open loop plant is poorly damped, the value suggested by Robinson and Clarke (1991): $T = (1 - 0.8q^{-1})^{n_A}$, is used in this study.

Weighting Factor: Usually increasing ρ , the weighting factor of equation (6.5), slows the system, and for a stable process, when ρ is increased, an increase in robustness and a corresponding decrease in performance can be expected. In this work ρ is used as a fine tuning parameter.

Assuming 3 modes, this set of design parameters results in the following set of

Figure 6.4: Step response comparison between UPC and PIF+V control.

controller polynomials

$$\begin{aligned} \mathcal{R} = & 1 - 5.6q^{-1} + 12.9q^{-2} - 15.3q^{-3} + 9.1q^{-4} - 1.5q^{-5} - 1.2q^{-6} + 0.6q^{-7} \\ & - 0.05q^{-8} - 0.01q^{-9} \end{aligned} \quad (6.16)$$

$$\begin{aligned} \mathcal{S} = & 0.09 - 0.7q^{-1} + 2.3q^{-2} - 4.4q^{-3} + 5.2q^{-4} - 3.9q^{-5} + 1.8q^{-6} - 0.4q^{-7} \\ & - 0.006q^{-8} + 0.01q^{-9} \end{aligned}$$

$$\mathcal{T} = 0.01 \times (0.2 - 0.9q^{-1} + 2q^{-2} - 3q^{-3} + 2.5q^{-4} - 1q^{-5} + 0.3q^{-6} - 0.04q^{-7})$$

Figure 6.5: Tracking behavior comparison between UPC and PIF+V control.

6.3 Simulation Results

The discontinuous system model with a unified predictive controller was simulated using SIMULINK, see Figure-G.13. A UPC design program was prepared and executed prior to the simulation by double clicking on the “Redesign” block. In this section the UPC results are compared with the PIF+v feedback control of chapter 5. It is to be noted that the UPC uses only noncolocated feedback while the PIF+v benefits from colocated feedback.

Figure-6.4 shows the step response of the UP Controlled system. The system is initially in contact and no uncertainty and friction have been considered. The initial delay is due to the NMP behavior of the system. The response is faster and smoother than that of PIF+V.

Figure 6.6: Step response of UPC and PIF+V systems with approach velocity.

6.3.1 Performance & Robustness Study

Figure-6.5 shows the tracking and command following behavior of UPC and PIF+V controllers. The UPC shows superior tracking performance of the system, and this without any additional colocated (velocity) feedback.

The simulation was also run for a situation with the system initially not in contact, with an approach velocity and a consequent impact. The assumption of very stiff link-environment contact makes the control problem very difficult. Figure-6.6 shows the response of the UP Controlled system for this situation. In comparison with the PIF+V control scheme, the UPC demonstrates faster sustained contact, and it damps oscillations due to link flexibility more quickly.

Figure 6.7: Performance comparison between UPC and PIF+V control.

Figure-6.7 shows the magnitude of the complementary sensitivity function as a measure of the performance and stability robustness of the UPC and PIF+V controlled systems. These functions for the UPC and PIF+V are:

$$(C.S.F.)_{UPC} = \frac{q^{-\hat{d}-1} \hat{B}S}{\hat{A}\mathcal{R} + q^{-\hat{d}-1} \hat{B}S}$$

$$(C.S.F.)_{PIF+V} = \frac{G_f(sK_f + K_i)}{sG_f + sK_vG_{\dot{\theta}} + G_f(sK_f + K_i)}$$

where K_f, K_i, K_v are the control gains for force PI-action and hub velocity feedback. G_f and $G_{\dot{\theta}}$ are the system transfer functions from control command to contact force and hub velocity respectively. A large peak value of magnitude of this function indicates better performance but reduced stability robustness of the system. The results show a noticeable performance improvement for UPC using only force feedback(noncolocated) over the PIF+V scheme in which additional hub velocity

Figure 6.8: Model truncation effect on UPC.

feedback (colocated) was used to improve the overall performance.

Figure-6.8(bottom left), shows the pole-zero map of the system when it was assumed that the orders of model and plant were equal. In practice, the order of a model of a real plant can be very large, and is generally truncated to obtain a manageable model. To study the effect of the higher modes not included in the model, known as spillover effects, the order of plant was assumed to be 10, while only 3 modes were used to design the UPC. Figure-6.8(bottom right) illustrates the pole-zero map of this case, and shows that the closed loop system is stable. It

Figure 6.9: UPC stability robustness in presence of sampling time uncertainty.

was empirically determined that to avoid spillover problems the number of assumed modes should be at least 2.

A UPC is very sensitive to the sampling time (T_s). In most conventional controllers, a smaller T_s results in a better control system, but in UPC's this rule is in general not valid. For example the stable UPC, designed based on $T_s = 0.01$ second in the last section, would become unstable if a sampling time of $T_s = 0.001$ second were chosen. The UPC does, however, show a robustness up to 35% uncertainty in this important parameter. Figure-6.9 shows two different cases of T_s . The system with 30% (< 35%) variation is still stable while that with 40% (> 35%) shows unstable behavior.

Figure-6.10 compares the pole-zero maps of the UPC when there is no parameter

Figure 6.10: UPC stability robustness in presence of parameter uncertainties. (Top): no uncertainty, (Bottom): with uncertainties of 8% in m , 100% in γ , 3% in L , and 10% in J_h .

uncertainty (left), and when the uncertainties are 8% in m , 100% in γ , 3% in L , and 10% in J_h (right). The Figures indicate good stability robustness for the worst case combination within these ranges.

It can be concluded that, although the PIf+v shows greater stability robustness, the UPC still has a satisfactory robustness in the presence of physical and geometrical parameter uncertainties.

6.3.2 Zero Preserving Property & UPC Stability

The overall closed loop transfer function of the UPC can be written as:

$$\frac{y(q^{-1})}{r(q^{-1})} = \frac{q^{-d}BT}{A\mathcal{R} + q^{-d}BS} \quad (6.17)$$

The set (d, B, A) represents the plant model. A corresponding set, $(\hat{d}, \hat{B}, \hat{A})$ represents the prediction model which is used in the design of the set $(\mathcal{T}, \mathcal{S}, \mathcal{R})$. The method by which the prediction and plant models are obtained, can be selected separately in simulation.

In chapter 3, two different models were prepared for the contact force of a single link flexible manipulator, using the modal expansion method and an exact modeling method. It was shown that while both models have the same poles, they have different sets of zeros, with the zeros of the exact modeling method much more representative of the real system. These models can be used as prediction and plant models in the design procedure and simulation. Therefore four different combinations are possible:

1. Prediction: modal expansion model, Plant: modal expansion model;

2. Prediction: exact model, Plant: modal expansion model;
3. Prediction: modal expansion model, Plant: exact model;
4. Prediction: exact model, Plant: exact model.

As can be seen in Figure-6.10(left), the simulation results show a stable closed loop system for the first combination. The problem with this combination is that the model obtained from modal expansion preserves neither the qualitative nor quantitative characteristics of real zeros of the system. Since the UPC was designed based on a prediction model with large errors in its zeros, and the same errors were repeated in the plant model, the overall closed loop system was stable, which might not be the case if the controller were applied to the real system.

Since the model from modal expansion has a large error in its zeros, it can not describe the real plant precisely and combinations 1 and 2 are not useful in practice. Combination 3 with the UPC designed based on a modal expansion prediction model and then applied to an experimental system (represented by an exact model) also has problems. It can be seen from equation (6.17) that the unstable zeros of the plant affect the poles of the closed loop system, and a large mismatch between the model used in control design and the plant, can make the system unstable. Figure-6.11 shows the pole-zero map of the system with this combination. Three unstable poles exist. In this situation the flexible arm will break contact and never again touch the wall. Thus if the modal expansion model is used in UPC design and then this controller is implemented experimentally, the simulation predicts an unstable system.

We are therefore left with the conclusion that to design a UPC which can be

Figure 6.11: The pole-zero map of Unified Predictive Controlled system, with plant and prediction models obtained from exact and modal expansion methods respectively. $n=4$

implementable on a real system, the controller should be designed based on the model obtained from the 'exact modeling method', i.e. combination 4. The pole-zero map of the closed loop system with this combination can be seen in Figure-6.12. The step response of the closed loop system is the same as that shown in Figure-6.4, but the controller is completely different from the one in equation (6.16). The new controller is:

$$\begin{aligned} \mathcal{R} = & 1 - 5q^{-1} + 9.7q^{-2} - 8.2q^{-3} + 1.3q^{-4} + 2.7q^{-5} - 1.6q^{-6} \\ & + 0.03q^{-7} + 0.18q^{-8} - 0.03q^{-9} \end{aligned} \quad (6.18)$$

$$\mathcal{S} = 0.07 - 0.48q^{-1} + 1.2q^{-2} - 1.4q^{-3} - 0.09q^{-4} + 2.3q^{-5} - 3q^{-6}$$

Figure 6.12: The pole-zero map of Unified Predictive Controlled system, with both plant and prediction models obtained from the exact modeling methods. $n=4$

$$-2q^{-7} - 0.68q^{-8} + 0.1q^{-9}$$

$$\mathcal{T} = 0.01 \times (0.3 - 2q^{-1} + 4q^{-2} - 5q^{-3} + 4q^{-4} - 2q^{-5} + 0.6q^{-6} - 0.07q^{-7})$$

Note that these results do not necessarily imply the UPC possesses poor robustness properties, as the zeros obtained via modal expansion greatly differ from those obtained from the exact model. In the latter, stability of the closed loop system in the face of substantial excursions of the zeros from their nominal positions indicates that the UPC is, in fact, robust.

Simulation results show that using the modal expansion model in the controller design procedure can destabilize the closed loop system. It is therefore recommended that the exact model in UPC or other model based controller design procedures be used, especially when the controller is to be implemented experimentally.

Figure 6.13: Effect of severe friction on UPC step response. (dotted): $\alpha_0 = 5.6$ & $\alpha_1 = 11.2$; (solid): $\alpha_0 = 2$ & $\alpha_1 = 4$.

6.3.3 Friction Effect

A dynamic friction model introduced in Appendix-E. was included in the simulation to study the effect of stiction on the designed UPC. For a more realistic simulation, the effects of force sensor quantization and motor saturation were also included. Figure-6.13 shows the step response of UPC with two sets of friction parameters. The friction parameters of the system with the dotted line response were obtained from experimental identification in chapter 4. The solid line represents the response of the system with 36% less friction. It can be seen that UPC is unable to damp the oscillations caused by stiction.

It must be noted that the Coulomb friction of 5.6 (N.m) and static level of 16.8

(N.m) are very large compared with the desired torque of 13.1 (N.m) at the motor (10 N at tip). The presence of such a large nonlinear disturbance in a highly sensitive (improved servo performance) closed loop system is the main reason for these oscillations. This result predicts a failure in implementing the UPC experimentally, if no other remedy is considered for the stiction problem.

6.4 Experimental Results

Implementation of $\mathcal{R}$: As mentioned earlier $\mathcal{R}(q^{-1})$ is monic, and thus can be written as

$$\begin{aligned}\mathcal{R} &= 1 + r_1 q^{-1} + \dots + r_{n_r} q^{-n_r} \\ &= 1 + q^{-1} (r_1 + r_2 q^{-1} + \dots + r_{n_r} q^{-n_r+1}) \\ &= 1 + q^{-1} \mathcal{R}_1\end{aligned}\tag{6.19}$$

Substituting into equation (6.13), the present control signal is obtained as:

$$u(k) = \mathcal{T}(q^{-1})r(k + H_p) - \mathcal{S}(q^{-1})y(k) - \mathcal{R}_1(q^{-1})u(k - 1)\tag{6.20}$$

This equation can be easily implemented in experiment. It must be noted that for a step input such as: $r(k) = r_d$, the first term would become: $\mathcal{T}(1)r_d$.

Effect of sensor inertia: The designed UPC of equations (6.18) were implemented experimentally, and very strange behavior was observed. Figure-6.14 illustrates the step response of the experimental set up and shows the oscillatory behavior predicted by simulation in Figure-6.13. Surprisingly it was observed that *all these oscillations occurred after the link had escaped from the wall and while it*

Figure 6.14: Experimental results. Force sensor inertial effect on UPC response.

was not in contact with the environment. Due to high speed oscillations, the effect of force sensor inertia was significant, and it was measured and fed back to the controller erroneously as the contact force signal. Consequently the system began oscillating around a virtual desired force of 10 Newton, although it was not in contact. The main cause for this problem is weakness of the designed UPC in overcoming the severe stiction which imposes undesired high frequency oscillations into the system.

Redesigning the UPC: Some effort was expended in tuning the UPC to deal with stiction. The final results are as follows: in the previous design the controller filter $\frac{Q_n}{Q_d}$ was selected as $\frac{1-q^{-1}}{1-0.95q^{-1}}$, which is a high pass filter. Using equation (6.5), this filter minimizes the high frequency elements of the controller. Choosing this filter as $Q_n = Q_d = 1$ allows a higher controller bandwidth and to therefore act

faster than before. The new controller, assuming 2 flexural modes ($n = 2$) is then

$$\begin{aligned}
 \mathcal{R} &= 1 - 4.1289q^{-1} + 6.4163q^{-2} - 4.2512q^{-3} + 0.5544q^{-4} \\
 &\quad + 0.6244q^{-5} - 0.2150q^{-6} \\
 \mathcal{S} &= 0.0168 - 0.2439q^{-1} + 1.0862q^{-2} - 2.3675q^{-3} + 2.8958q^{-4} \\
 &\quad - 2.0392q^{-5} + 0.7759q^{-6} - 0.1240q^{-7} \\
 \mathcal{T} &= 0.0052 - 0.0251q^{-1} + 0.0502q^{-2} - 0.0535q^{-3} + 0.0321q^{-4} \\
 &\quad - 0.0103q^{-5} + 0.0014q^{-6}
 \end{aligned} \tag{6.21}$$

The experimental result for this controller is shown in Figure-6.15 (solid line), which is very similar to the simulation result (dotted line). Although the response shows great improvement, however the overall behavior is still not satisfactory.

Effect of round-off errors: An important issue related to experimental implementation is that of round off errors which are produced in the controller polynomials. To illustrate this one can write the control signal in the following form:

$$u(k) = \frac{\mathcal{T}(q^{-1})r(k + H_p) - \mathcal{S}(q^{-1})y(k)}{\mathcal{R}(q^{-1})} \tag{6.22}$$

To have a control signal within the saturation bounds, this equation implies that $\mathcal{R}(q^{-1})$ must be stable, which means that all the roots of $\mathcal{R}$ must be inside the unit circle. The roots of $\mathcal{R}$ in equation (6.21) are:

$$roots(\mathcal{R}) = [-0.37 \quad 0.76 \quad 0.87 \quad 0.99 \quad 0.94 \pm 0.06i] \tag{6.23}$$

which shows that $\mathcal{R}$ is stable. If for the sake of implementation simplicity the coefficients of $\mathcal{R}$ are rounded off with 3 decimal digits of accuracy, the polynomial becomes:

$$\bar{\mathcal{R}} = 1 - 4.129q^{-1} + 6.416q^{-2} - 4.251q^{-3} + 0.554q^{-4} - 0.624q^{-5} - 0.215q^{-6} \tag{6.24}$$

Figure 6.15: Step response of tuned UPC. Comparing experimental result (solid line), and simulation result (dotted line).

This polynomial possesses the following roots:

$$\text{roots}(\bar{\mathcal{R}}) = [-0.37 \quad 0.71 \pm 0.11i \quad 0.96 \pm 0.22i \quad 1.16] \quad (6.25)$$

One unstable root at 1.16 is detected. Figure-6.16(left) shows the pole-zero map of the closed loop system with accurate $\mathcal{R}$ and the figure at (right) shows the map for the system with rounded off $\bar{\mathcal{R}}$. It is seen that the designed UPC is stable while the implemented one is unstable. It is concluded that here the coefficients of UPC polynomials must be implemented with an accuracy greater than 3 decimal digits to preserve system stability.

Figure 6.16: Effect of round off error on stability of implemented UPC. (Top): with exact $\mathcal{R}$. (Bottom): with $\mathcal{R}$ rounded off to 3 decimal digits.

Chapter 7

Hybrid (Two Stage) Control

The final simulation and experimental results of chapter 6 indicated that although the UPC improved system performance considerably, this was insufficient to overcome the severe joint friction. This chapter presents attempts to remedy this problem.

7.1 Control Configuration

It is desired to design a controller which provides good servo performance of this NMP system in spite of joint friction. These two actions are physically separate: friction affects the system at the joint, which is colocated with the actuator, while the overall objective is to control the contact force, which is at the tip and thus noncolocated with the motor. In other words, the friction occurs at the joint and the main controlled signal is measured at the tip. This suggests the separation of these two problems into distinct stages.

Regardless of the type of controllers used, Figure-7.1 describes the general idea

Figure 7.1: Block diagram of two-stage control configuration.

of an appropriate two stage control scheme. In this approach the control signal u controls the joint angle θ , while θ is considered as the input to the flexible arm. Therefore reducing the effect of friction on θ automatically reduces its effect on the contact force. The desired angle θ_d is determined from an outer loop which controls the force.

A PID controller was selected for the inner loop for the following reasons: **1-** Since the inner loop process ($\triangleq G_{\theta/u}$) is minimum phase it is possible to use conventional controllers without having to deal with performance limitation due to RHP zeros, so there is no need for more complicated controllers. **2-** The main objective in the inner loop is to design a controller with a good disturbance rejection which is achievable through good tuning of a PID. **3-** Well established procedures exist for PID tuning.

The overall inner loop is still NMP and acts as the process for the outer loop. For reasons explained in the previous chapter, such as improving the NMP system performance, UPC was selected as the outer loop controller.

7.2 Inner Loop Design & Simulation

Since both joint angle and angular velocity are available by measurement, the controller is selected as PI action on angular error plus velocity feedback, so that this controller is a PID controller. The effects of the various controller gains on the system characteristics can be roughly summerized as follows: K_p is the position error feedback gain and affects both the disturbance rejection and command following characteristics of the system. K_d is the velocity feedback gain, and the ratio of $\frac{K_d}{K_p}$ affects the servo performance (command following) of the system. K_i is the position error integral gain and $\frac{K_i}{K_p}$ influences the disturbance rejection of the system. Finally K_r is an input gain to the inner loop to adjust the system DC gain. (R. M. De Santis 1994).

The selection process of the PID gains is now outlined. For the first step, it was assumed that $K_i = 0$, and then based on a desired transient response, K_p and K_d is designed. In this case the inner loop transfer function is:

$$G_{in}(s) = \frac{\theta}{\tau_d} = \frac{K_p k_t G_{\theta/\tau}}{1 + k_t(K_p + sK_d)G_{\theta/\tau}} \quad (7.1)$$

where $G_{\theta/\tau}$ is the transfer function from torque to hub angle. Considering only the first flexural mode of the system, $G_{\theta/\tau}$ can be written as:

$$\tilde{G}_{\theta/\tau}(s) = \frac{\omega_1^2 g_1}{s^2 + 2\omega_1 \zeta_1 s + \omega_1^2} \quad (7.2)$$

where $g_1 = G_{\theta/\tau}(0) = \frac{L}{3EI}$. Substituting this into equation (7.1), the inner closed loop transfer function is approximately

$$\tilde{G}_{in}(s) = \frac{\omega_{ap}^2 g_{ap}}{s^2 + 2\omega_{ap} \zeta_{ap} s + \omega_{ap}^2} \quad (7.3)$$

Figure 7.2: Comparison between the step responses of the inner loop system with $K_p = 4$, $K_d = 0.7$, $K_i = 0$ (solid); and the approximated second order system with $\omega_{ap} = 28 \frac{rad}{sec}$ and $\zeta_{ap} = 1.7$ (dashed). The desired angle is $\theta_d = 1 rad$.

where:

$$\omega_{ap} \triangleq \omega_1 \sqrt{1 + k_t K_p g_1} \quad , \quad \zeta_{ap} \triangleq \frac{2\zeta_1 + k_t K_d \omega_1 g_1}{2\sqrt{1 + k_t K_p g_1}} \quad , \quad g_{ap} \triangleq \frac{K_p k_t g_1}{1 + K_p k_t g_1} \quad (7.4)$$

are the approximate natural frequency, damping ratio, and dc gain. Choosing $\omega_{ap} = 28 \frac{rad}{sec}$ and $\zeta_{ap} = 1.7$ by trial and error, results in a relatively fast over-damped system, with a rise time of $t_r = 0.27$ seconds. Solving equations (7.4) for K_p and K_d gives

$$K_p = 4 \quad , \quad K_d = 0.7 \quad (7.5)$$

Figure-7.2 shows the step responses of G_{in} and $\tilde{G}_{in}$ for $\theta_d = 1$ radians with these gains.

Figure 7.3: Friction effect on step responses of the PID inner loop system, with $K_p = 4$, $K_d = 0.7$, dash: $K_i = 9$, and solid: $K_i = 15$. $\theta_d = 0.5$ radian.

The next step is the tuning of the integral gain K_i for effective disturbance rejection and steady state error elimination. After simulating the controller and analyzing some experimental results, a range of $[9 - 15]$ was found to be reasonable for this gain. The hub angle and contact force transfer function of this inner closed loop now becomes

$$\begin{aligned} \frac{\theta}{\theta_d} &= \frac{k_t G_{\theta/\tau} \left(K_p + \frac{K_i}{s} \right)}{1 + k_t G_{\theta/\tau} \left(K_p + sK_d + \frac{K_i}{s} \right)} \\ \frac{F}{\theta_d} &= \frac{k_t G_{F/\tau} \left(K_p + \frac{K_i}{s} \right)}{1 + k_t G_{\theta/\tau} \left(K_p + sK_d + \frac{K_i}{s} \right)} \end{aligned} \quad (7.6)$$

The responses of this system for a step input of $\theta_d = 0.5$ radians are shown in Figure-7.3. The solid line shows the system response with $K_i = 15$ and the dashed line

Figure 7.4: Sensitivity to disturbance of the PID inner loop system, with $K_p = 4$, $K_d = 0.7$, and $K_i = 15$ (solid line), and for open loop system (dash). Top: $\frac{\ddot{\theta}}{d_i}$. Middle: $\frac{\dot{\theta}}{d_i}$. Bottom: $\frac{F}{d_i}$.

represents that with $K_i = 9$. The simulation programs used here are in Appendix G. Figures-G.14 and G.15. Note that joint friction is included in the simulation. The response of the system with $K_i = 9$ is more conservative in hub angle and the system with $K_i = 15$ is more successful in overcoming the stiction problem in contact force response. Therefore the integral gain of the inner loop system was taken to be $K_i = 15$. Figure-7.4 shows the magnitudes of $\frac{\ddot{\theta}}{d_i}(j\omega)$, $\frac{\dot{\theta}}{d_i}(j\omega)$, and $\frac{F}{d_i}(j\omega)$, which

represent the sensitivity to disturbance of hub velocity, hub angle, and contact force, respectively.

$$\begin{aligned}\frac{\theta}{d_i} &= \frac{G_{\theta/\tau}}{1 + k_t G_{\theta/\tau} \left(K_p + sK_d + \frac{K_i}{s} \right)} \\ \frac{F}{d_i} &= \frac{G_{F/\tau}}{1 + k_t G_{\theta/\tau} \left(K_p + sK_d + \frac{K_i}{s} \right)}\end{aligned}\quad (7.7)$$

In all cases sensitivity reduction is seen to be on the order of 100 $\frac{rad}{sec}$. This reduction is as high as 40 dB at low frequencies. Since the stiction is mostly excited by velocity changes, the velocity sensitivity to disturbances are of greater interest. For velocity, the peak value has been reduced about 30 dB, which means very good improvement in the disturbance rejection characteristics of the system.

7.3 Outer Loop Design & Simulation

For the next stage of controller design the outer loop, which is a force controller, is the subject. Since the system is still NMP, UPC is the suggested controller as discussed earlier. The design procedure of chapter 6 is valid here, the main difference being in the prediction models. In chapter 6 the plant (process) model was used as the prediction model, while in this Two Stage (Hybrid) Control scheme the inner loop transfer function, consisting of plant and hub angle controller, was used as the prediction model. This transfer function can be written as

$$G_{process}(s) = \frac{F}{u_{upc}} = \frac{K_r k_t G_{F/\tau} \left(K_p + \frac{K_i}{s} \right)}{1 + k_t G_{\theta/\tau} \left(K_p + sK_d + \frac{K_i}{s} \right)} \quad (7.8)$$

where K_r is a gain used for fine tuning of transient behavior. For the time being K_r is taken to be unity. This transfer function can be written in the discrete time domain

Figure 7.5: Nondimensionalized frequency responses of plant (dash-dot) and inner loop system (solid line).

in a form parallel to that of equation (6.1), and its bode plot is given in Figure-7.5. which shows a slight decrease in the frequency bandwidth of the inner loop system compared with that of the plant. Measuring $\omega_b = 9$ Hertz, and assuming a sampling frequency of $\omega_s = 100$ Hertz, the prediction horizon can be obtained from equation (6.14) as $H_p = 23$. The other UPC parameters used in section 6.4 are: $H_\pi = 1$, $H_c = 6$, $\phi = 1 - q^{-1}$, $P(\xi = 0.7, t_p = 0.5)$, $T = (1 - 0.8q^{-1})^6$, $D_1 = P\phi$, $\rho = 0.0001$, and $Q_n = Q_d = 1$. Using these parameters and applying the UPC design procedure to the prediction model of equation (7.8) produces the following controller:

$$\begin{aligned}
 \mathcal{R} &= 1 - 5.0340q^{-1} + 10.3140q^{-2} - 10.8127q^{-3} + 5.8164q^{-4} \\
 &\quad - 1.2013q^{-5} - 0.1615q^{-6} + 0.0792q^{-7} \\
 \mathcal{S} &= 0.0132 - 0.0928q^{-1} + 0.2838q^{-2} - 0.4851q^{-3} + 0.4965q^{-4} \\
 &\quad - 0.3026q^{-5} + 0.1012q^{-6} - 0.0143q^{-7}
 \end{aligned} \tag{7.9}$$

Figure 7.6: Comparison between the time responses of UPC system(dash-dot) and the proposed Hybrid UPC-PID system (solid line), in presence of a severe joint friction.

$$\begin{aligned} \mathcal{T} = & 0.0015 - 0.0071q^{-1} + 0.0142q^{-2} - 0.0151q^{-3} + 0.0091q^{-4} \\ & - 0.0029q^{-5} + 0.0004q^{-6} \end{aligned}$$

Figure-G.16 of Appendix-G displays the simulation program for this hybrid UPC-PID control system written in the SIMULINK environment of Matlab. The step response of this controlled system is shown in Figure-7.6, where the effect of severe joint friction was considered. Compared with the single loop UPC system, much better performance and disturbance rejection can be seen with this Two-Stage UPC-PID control system.

The high frequency noise-like oscillations in the response are due to the force

sensor bit resolution which is 0.3 Newton. In an attempt to eliminate this noise, two low pass filters were used in force and velocity feedback, see Figure-G.16. Use of these filters not only did not improve the response, but actually degraded the performance and in some cases even caused instability in the system. With these filters, the controller does not detect any fast signal and therefore acts sluggishly, losing its ability to suppress the effect of stiction in the system.

To reduce the observed overshoot in the system response one can change the parameters in the tracking error filter $P(q^{-1})$. In section 6.2 it was explained how P can be designed so as to have an under damped ($\xi < 1$) second order response with peak time t_p . To reduce the overshoot, P was changed to produce a critically damped response ($\xi = 1$). In this case there is no peak time in the response, so that the rise time t_r must be used instead. By definition rise time is the time required by the system to reach 90% of its steady state value. For a critically damped response $\xi = 1$, for any rise time t_r , the natural frequency of the response can be obtained from $\omega_n = \frac{3.89}{t_r}$. The coefficients of P then can be determined as $p_1 = -2e^{-\omega_n T_s}$ and $p_2 = e^{-2\omega_n T_s}$. For two values of $t_r = 0.5$ and 1 second the polynomial is as follows:

$$\xi = 1, \quad t_r = 0.5 : \quad P(q^{-1}) = 1 - 1.850q^{-1} + 0.856q^{-2} \quad (7.10)$$

$$\xi = 1, \quad t_r = 1.0 : \quad P(q^{-1}) = 1 - 1.924q^{-1} + 0.925q^{-2}$$

Figure-7.7-Top shows the step response of the system for three sets of P . As expected, the overshoot was reduced by increasing the damping and rise time in P . The middle figure shows the response of the system when it is not initially in contact. The system with $P(\xi = 1, t_r = 1)$ shows overshoot considerably reduced even with an approach velocity. This combination does, however, show sluggish command

Figure 7.7: Servo behavior tuning of Hybrid (UPC-PID) control system using polynomial $P(q^{-1})$. Solid: $\xi = 0.7, t_p = 0.5$ sec. Dash: $\xi = 1, t_r = 0.5$ sec. Dash-Dot: $\xi = 1, t_r = 1$ sec. Dotted: desired force. Top: Step response for $\theta_{t=0} = 0$. Middle: Step response for system with approach velocity, $\theta_{t=0} = 15^\circ$. Bottom: Command following behavior. All with severe friction at the joint.

Figure 7.8: Time response comparison between UPC and other MBPCs when acting as the outer loop controller. Solid: UPC. Dash: GPC. Dash-Dot: EPSAC.

following behavior, with a substantial delay, see Figure-7.7-Bottom.

As mentioned earlier, different settings of the UPC design parameters produces some well known model-based predictive controllers. On referring to Table-3.6 and 3.7 of Soeterboek (1992), one sees that by choosing $T = P = Q_d = 1$, and $D = \phi = Q_n = 1 - q^{-1}$ the controller becomes one of the following: Dynamic Matrix Control (DMC), Predictive Control Algorithm (PAC), or Model Algorithm Control (MAC). In the first, $H_c = 6$ as before, while in the second and third $H_c = H_p - \hat{d} = 23$. An Extended Horizon Adaptive Control (EHAC) is produced if the parameters are selected as $T = P = H_c = 1$, $D_1 = \phi = 1 - q^{-1}$, $H_m = H_p = 23$, and $\rho = 0$. For each case, other parameters not mentioned here would be the same as for UPC. These

controllers produce instability in this specific system.

The only stable controllers were Generalized Predictive Control (GPC) and Extended Prediction Self-Adaptive Control (EPSAC). In both of these $D_1 = \phi = 1 - q^{-1}$. In GPC: $P = Q_d = 1$, and $Q_n = 1 - q^{-1}$. While in EPSAC: $H_c = 1$ and $\rho = 0$. Figure-7.8 shows a comparison between the step responses of the Two Stage (Hybrid) control system when the outer loop is UPC (solid line), GPC (dash line), and EPSAC (dash-dot line). It is seen that UPC has the lowest overshoot and the fastest settling time. In addition UPC is capable of having its servo characteristics tuned simply through the P polynomial.

7.4 Experimental Results

The inner loop hub angle PID controller was implemented experimentally. The encoder signal and its derivative analog filter were used to measure the hub angle and velocity. The error integral was calculated numerically using the following trapezoidal formula:

$$I_k = I_{k-1} + \frac{e_{k-1} + e_k}{2} T_s \quad \text{for : } k > 1 \text{ and } I_0 = 0 \quad (7.11)$$

where $e_k = \theta_d(k) - \theta(k)$ is the error in the hub angle and I_k is the error integral at the k^{th} sampling time.

Figure-7.9-top shows the experimental step response of the inner loop system for the controller gain set of $(K_p = 4 \frac{\text{volt}}{\text{rad}}, K_d = 0.7 \frac{\text{volt.sec}}{\text{rad}}, K_i = 15 \frac{\text{volt}}{\text{rad.sec}})$ which was designed in the previous section. Since the angle is measured in degrees, the gain set is converted to $(K_p = 0.07 \frac{\text{volt}}{\text{deg}}, K_d = 0.012 \frac{\text{volt.sec}}{\text{deg}}, K_i = 0.262 \frac{\text{volt}}{\text{deg.sec}})$. The response

Figure 7.9: Step response of inner PID loop. Solid line: experimental result. dash-dot: simulation result. $\theta_d = 11^\circ$.

shows good agreement between simulation and experimental results. No overshoot or steady state error is seen in the response. The effect of disturbances and sensor noises are more visible in the bottom figure which shows the contact force response. Since no force feedback is used in the inner loop, the system between hub angle and contact force is open loop.

In the next step the outer loop was closed. This required force feedback which was provided by a 6 DOF universal force sensor at the tip of the link. The UPC polynomials of equation (7.9) and the above mentioned PID gains were used in the

experiment. Since the implemented PID gains are based on degrees the K_r value of unity was converted to $\frac{180}{\pi} = 57.3$. After implementation, this value of K_r was found to be high, and was therefore reduced to $K_r = 5.7$ which is equal to 0.1 when the other PID gains are in radians. Figure-7.10-Top shows the force step response for three K_r values of 0.1, 0.12, and 0.02 (PID gains in radians). The best response is that for $K_r = 0.1$ (in radians) equal to 5.7 (in degrees). The bottom figure compares the simulation and experimental results for the designed control system. These results are in good agreement.

Figure-7.11 shows the experimental and simulation results of the final UPC-PID controller when the system is not initially in contact. Although simulation predicts earlier contact, the settling time for both simulation and experiment are the same. The reason for this difference in initial contact time is that in experiment during non contact motion, the force sensor inertia is fed back. This reduces the force error and thus the control command; consequently the real system moves more slowly than does the simulated system.

Comparing Figures-5.13 and 6.15 with 7.10, and also Figure-5.14 with 7.11 reveals that the proposed UPC-PID control has improved the NMP system performance and also was quite successful in suppressing the friction effects. This controller shows a faster settling time with no overshoot, steady state error, or stick-slip motion (limit cycle). All these were observed by means of computer simulations and verified experimentally.

It was discussed earlier that using a low pass filter to reduce the sensor noises degraded the system behavior rather than improving it. This was verified experi-

Figure 7.10: Top: Experimental tuning of Two Stage Control system using K_r . Solid line: $K_r = 0.1$, dashed: $K_r = 0.02$, and dotted: $K_r = 0.12$. Bottom: Comparison between simulation (dash-dot) and experimental (solid) results for $K_r = 0.1$.

mentally by implementing some low pass filters for force and/or force and velocity signals. Depending on the filter cut-off frequency, some cases made no change in the system, some degraded the behavior, and some even destabilized the system. Figure-7.12 shows the results of experiment for a simple low pass filter: $\frac{1}{0.1s+1}$. This figure clearly illustrates that adding filters has degraded the system performance significantly.

In a previous section a range of $[9 - 15] \frac{\text{volt}}{\text{rad.sec}}$ was suggested for K_i , and by

Figure 7.11: Step response of Two stage control system with approach velocity. Solid line: experimental result. dashed line: simulation result. $K_r = 0.1$. $\theta_{t=0} = 15^\circ$.

means of simulation it was concluded that $K_i = 15$ was a better choice than 9 for force control. The experimental result of Figure-7.13-Top, verifies this conclusion.

To increase the stability robustness Soeterboek (1992) suggests choosing the controller output filters as

$$Q_n = \phi P \quad . \quad Q_d = (1 - 0.95q^{-1})(1 - r_{p1}q^{-1})(1 - r_{p2}q^{-1}) \quad (7.12)$$

where r_{p1} and r_{p2} are the roots of polynomial $P(q^{-1})$. The experimental result in Figure-7.13-Middle illustrates that not only does this selection of Q_n and Q_d not improve the system robustness but it actually makes this specific system unstable.

Figure 7.12: Filters effect on UPC-PID. Solid: no filter, dash: force signal filter, dash-dot: force and velocity filter.

In the last section, the effect of tracking error filter $P(q^{-1})$ on the system's servo behavior was discussed and investigated by means of simulation. This investigation was also implemented experimentally with one set of its results seen in Figure-7.13- Bottom for $P(\xi = 1, t_r = 1)$. (see equation 7.10). It can be concluded that the selection of $P(\xi = 0.7, t_r = 0.5)$ is probably close to the best achievable choice.

Figure 7.13: Top: Experimental tuning of Two Stage Control system using K_i . Solid line: $K_i = 15$, dash-dot: $K_i = 9$. Middle: Q_n and Q_d effect. Bottom: dash-dot: $P(\xi = 1, t_r = 1)$.

Chapter 8

Conclusion

The problem of force control of a flexible link manipulator was addressed in this research. The main objective of this thesis was to develop a means of imposing a desired force trajectory onto a surface through a flexible manipulator link. This was to be accomplished accurately while rejecting disturbances, some of which, such as joint friction, would otherwise dominate system behaviour, and in spite of discontinuous surface contact. These objectives have been very well met in this work, and a brief summary now follows.

To be able to simulate the system and also for design purposes two reliable dynamic models were developed for the system in constrained and unconstrained configurations. The contact was assumed to be stiff - a much more difficult control problem than that of soft contact. A stiff contact model was obtained via the Lagrange Multiplier approach. Impact phenomena and discontinuous contact were simulated by introducing a transition process model and a switching algorithm. A simplified methodology to obtain an exact model, a mixture of modal summation

and infinite product expansion methods was used. For a more realistic model, the effects of joint friction, motor saturation, and sensor quantization were included in the simulation.

Some experimental methods for identification of the unknown system parameters were presented. Experimental examination also verified the validity of the model.

To improve the performance of the force control system a combination of colocated (hub velocity) and noncolocated (tip force) feedback was implemented, and to increase system robustness, integral action was added to the controller (PIf+v). The results of simulation and experiment verified that the quality of the force control improved when velocity feedback was included in the control system. Contact stability and existence of limit cycles in the system with discontinuous contact were studied.

Simulation results for various inputs showed that the PIf+v controller for the noncolocated system does not possess satisfactory command following behaviour. This was expected, as the system is nonminimum phase and thus has inherent performance limitations. Therefore a Unified Predictive force controller (UPC) was designed and applied to the system. Simulation results showed considerable performance improvement. The UPC design demonstrated superior response and tracking behaviour than that of PIf+v feedback control, and this without using any additional colocated (e.g. hub velocity) sensor. It was also discussed that in practice the real plant is closer to the exact model than to the modal expansion model, and that using the modal expansion method in the controller design procedure can destabilize the closed loop system. It is therefore recommended that the exact model in

UPC or other model based controller design procedures be used, especially when the controller is to be implemented experimentally. Experimental results revealed the weakness of UPC in overcoming the problems caused by severe joint friction.

Finally, an inner/outer loop configuration was used, with a PID hub angle feedback inner loop responsible for suppressing the friction effect, and simultaneously a UPC force controller in the outer loop tuned to achieve the maximum performance. Simulation and experimental results were in very good agreement, and confirmed that the research objective of *simultaneous friction suppression and force control performance improvement* was achieved quite successfully. During final UPC-PID tuning some modifications in the design rules for UPC were proposed. Also some practical notes for experimental implementation were presented.

8.1 Summary of Contributions

A list of achievements and contributions resulting from this research is presented here, with the relative importance of each indicated by (*Major*) or (*minor*).

- Presentation of the problem of dependence on number of modes in contact force modelling and its relation to boundary conditions. (*minor*)
- Determining and solving the mode shapes and characteristic equations of the system with pinned-inertia/pinned (constrained) and pinned-inertia/pinned-mass (unconstrained) boundary conditions, and using them to build the state space representations. (*minor*)
- Studying the axial load effect on the system model. (*minor*)

- Suggesting a simple method to include the joint viscous damping effect in the system model. (*minor*)
- Obtaining and solving the characteristic equation of the system with pinned-inertia/spring boundary conditions, and then introducing a quantitative criterion for selecting the contact type. (*minor*)
- Introducing a methodology for stiff contact modelling. (*Major*)
- Introducing a methodology for transition process modelling in systems with discontinuous contact. (*Major*)
- Designing an algorithm for switching between constrained and unconstrained motions based on defining a function to determine the type of motion (phase indicator) and an event detector function. (*minor*)
- Deriving the exact transfer function of the constrained system and obtaining its exact poles and zeros. (*Major*)
- Suggesting a simple methodology to obtain the exact system model, based on a combination of modal summation and infinite product expansion methods. (*minor*)
- Setting up the experimental test bed and preparing the computer programs for data acquisition and control system. (*minor*)
- Performing experiments to identify all necessary system parameters. (*minor*)
- Suggesting PIf+v control scheme and designing the controller gains based on LQR state feedback gains. (*minor*)

- Stability analysis for the PIf+v system with discontinuous contact. (*minor*)
- Designing and implementing UPC as a force controller for the flexible system. (*Major*)
- Modification of UPC design rules for better suppression of joint friction. (*minor*)
- Analyzing the instability problem of the experimental UPC caused by use of the modal summation model, and providing a solution. (*Major*)
- Presenting some practical guidelines for experimental implementation of UPC. (*minor*)
- Developing the UPC-PID configuration for simultaneous performance improvement and friction suppression. (*Major*)
- Presenting the design and fine tuning procedure for the UPC-PID control scheme. (*minor*)
- Experimental implementation of PIf+v, UPC, and UPC-PID controllers successfully. (*Major*)

8.2 Suggestions For Future Work

The following subjects are suggested as possible continuations of this research:

- Experimental off-line identification of system model.

- Designing a contact detector mechanism (software or hardware) for use in experiment. It must be noted that the suggested switching algorithm in this thesis is for use only in simulation, and due to force sensor inertia it is not applicable directly to experiment.
- Using the above mechanism and design a control algorithm to reduce the impact effect.
- Study of the UPC-PID control scheme for a system with unknown contact: simulation and experiment. The contact might be stiff, very soft, medium soft, stiff but moving (such as in dual arm grasping).
- If previous experiment fails, modify the control algorithm to perform promptly for systems with unknown contact stiffness.
- Extending the experimental set up, models, control algorithms, and simulation programs to a two (or multi) link flexible manipulator, and studying the hybrid (position/force) control problem.
- Extending the experimental set up, models, control algorithms, and simulation programs to a dual arm flexible manipulator, and investigating the grasping control problem.

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Appendix A

Experimental Set Up

A.1 Schematic & System Parameters

An experimental test bed was set up in the Robotics Laboratories of the Canadian Space Agency. Directorate of Space Mechanics. Control Group. A schematic of this test bed is demonstrated in Figure-A.1.

This set up consists of a Dynaserv DM1150A *Direct Drive Motor*, with peak torque of 150 N.m, produced by Yukogawa Precision. An incremental encoder with maximum 1,024,000 step/rev was mounted in the motor by the manufacturer. The model of motor driver system is SD1150A. which also produces a filtered analog signal of motor velocity. The motor-driver system can be tuned in three different modes: position, velocity, and torque modes. The torque mode is used in these experiments. In this mode the torque produced in the motor is proportional to the command voltage to the driver, as: $\tau_e = k_t u$.

A JR3 six axis force/torque sensor is mounted at the link tip. with a range of

300 *lb* in force and 150 *lb.in* in moment. To decompose the six measured signals into distinct force and moment components digital signal decoupling is performed, requiring A/D and D/A conversions. The maximum achievable bit resolution is 140 *lb* in 12 bits which results an accuracy of ± 0.3 Newtons.

A 120Ω strain gauge with gauge factor of 2.130, was attached to the link, to be used in only off line parameter identification, see Chapter 4. A Datascan 7000 measurement and bridge unit is used to prepare the strain digital signal. This unit transfers the signal to the computer through a serial port using RS232 protocol.

A RTI 815 data acquisition card was used to read the velocity analog signal prepared by motor driver, the decoupled force/torque analog signals of JR3 processor, and also to send the control analog signal to the motor driver.

An encoder reader card from Technology 30 was used to read the encoder signal and determine the motor angular position.

Other system specifications are shown in Table-A.1.

Figure A.1: Schematic of experimental set up.

Table A.1: Physical parameters of system. Where superscripts § and ¶ represents data extracted from experiments (or measurement) and manuals, respectively.

PRIMARY			
Parameter		Value	unit
Beam material		Steel	
Density		7860 [¶]	K_g/m^3
Modulus of elasticity	E	206 [¶]	GPa
Length	L	1.31 [§]	m
Width		5 [§]	cm
Thickness		3 [§]	mm
Structural damping ratio	γ	0.0022 [§]	
Hub moment of inertia	J_h	0.142 [¶]	$K_g.m^2$
Joint damping	c_h	1.0207 [§]	N.m.s
Sensor plus load mass	m_p	0.7 [§]	K_g
Motor torque constant	k_t	29.5 [§] (18.75 [¶])	N.m/volts
SECONDARY			
Link's second moment of cross section area	I	112.5	mm^4
Link bending stiffness	EI	23.175	$N.m^2$
Link mass per unit length	m	1.179	K_g/m
Link moment of inertia around joint axis	J_b	0.8835	$K_g.m^2$

A.2 Calculation of $\frac{\omega_c}{\omega_n}$

The Timoshenko model is a well-known beam model, which considers the effect of rotary inertia and shear deformation:

$$EIW'''' + m\ddot{W} + \frac{mI}{A} \left[\frac{m}{\kappa GA} \frac{\partial^4 W}{\partial t^4} - \left(1 + \frac{E}{\kappa G} \right) \ddot{W}'' \right] = 0 \quad (\text{A.1})$$

Spector and Flashner (1989), have showed that the Euler-Bernoulli beam model is valid only if:

$$\frac{\omega_c}{\omega_n} \gg 20 \quad (\text{A.2})$$

where ω_c is the cutoff frequency of the Timoshenko beam model and ω_n is the largest assumed mode frequency of Euler-Bernoulli beam model. ω_c was obtained in the mentioned article as:

$$\omega_c = \sqrt{\frac{\kappa GA^2}{mI}} \quad (\text{A.3})$$

The shear coefficient κ is dimensionless and depends on Poisson's ratio and the shape of the cross section. For a steel beam with a rectangular section this parameter is $\kappa = \frac{5}{6}$. A is the cross section area, and G is the modulus of rigidity, which is equal to 73-79 GPa for steel. For this experimental set up the Timoshenko cutoff frequency is calculated as $\omega_c \approx 32 \times 10^5$. Assuming five modes in the Euler-Bernoulli model and from Table-3.1, it is obtained that $\boxed{\frac{\omega_c}{\omega_5} \approx 7000 \gg 20}$. Therefore using the Euler-Bernoulli model is quite reasonable in this study.

A.3 Contact Stiffness Calculation

Figure-A.2 shows a schematic of contact between the force sensor and its environment. Both sensor and wall are Aluminum. The stiffness for the sensor can be

Figure A.2: Schematic of contact.

roughly calculated as:

$$K_s = \frac{E_{Al} \pi r^2}{l_s} = \frac{69 \times 10^9 \times \pi \times (8 \times 10^{-3})^2}{3 \times 10^{-2}} = 462 \frac{MN}{m} \quad (A.4)$$

In Figure-A.2 the depth of the wall is about 30cm. therefore:

$$I_w = \frac{1}{12} (depth) t_w^3 = \frac{0.3 \times 0.015^3}{12} = 8.4 \times 10^{-8} \text{ m}^4 \quad (A.5)$$

The stiffness of the wall can be obtained from following equation:

$$K_w = \frac{F}{\Delta} = \frac{48 E_{Al} I_w}{l_w^3} = \frac{48 \times 69 \times 10^9 \times 8.4 \times 10^{-8}}{0.3^3} = 10.3 \frac{MN}{m} \quad (A.6)$$

Stiffnesses K_s and K_w are in series, thus the equivalent stiffness of contact is obtained from:

$$k = \frac{K_s K_w}{K_s + K_w} = 10.1 \frac{MN}{m} \quad (A.7)$$

Therefore the beam-to-contact stiffness ratio ($=k_{b/c}$) is:

$$k_{b/c} = \frac{EI}{2kL^3} = \frac{23.175}{10.0 \times 10^6 \times 1.31^3} = 5 \times 10^{-5} \quad (A.8)$$

From Table-A.1 the hub-to-beam inertia ratio is:

$$a = \frac{J_h}{6J_b} = 2.6 \times 10^{-2} \quad (\text{A.9})$$

As is seen, the order of magnitude for $k_{b/c}$ is 10^{-3} times smaller than that for a : the contact can therefore be confidently categorized as stiff.

Appendix B

Motor Dynamics

The equations governing the mechanical and electrical portions of the motor are:

1. dc motor principle: $\bar{\tau}_e = \bar{k}_t i_a$
where: $\bar{k}_t =$ motor torque constant
and: $i_a =$ armature current
2. back emf: $E_a = \bar{k}_t \dot{\theta}$
3. Kirchhoff's voltage law: $u = E_a + L_a \frac{di_a}{dt} + R_a i_a$
where: $L_a \approx 0$: small inductance.
and: $R_a =$ armature resistance.
4. Newton's second law at joint: $\bar{\tau}_e = J_h \ddot{\theta} + \hat{c}_h \dot{\theta} - M_0$
where: $\hat{c}_h =$ mechanical damping at joint.
and: $M_0 =$ beam bending moment at base.
5. Combining Eqn.s 1 to 4 results in $\tau_e = k_t u = J_h \ddot{\theta} + c_h \dot{\theta} - M_0$
where: $k_t = \frac{\bar{k}_t}{R_a} =$ equivalent torque constant.
and: $c_h = \hat{c}_h + \frac{\bar{k}_t^2}{R_a} =$ equivalent damping.

Appendix C

Axial Load Effect

It is desired to understand what the effect of axial force is on the dynamics of the system and when it must be considered. If the contact force has a component along the x axis, such as P in Figure-C.1, the moment and force equation for a beam element can be written as:

$$\begin{aligned} V(x,t) &= -M'(x,t) - P(x,t)W'(x,t) = -EIW'''' - PW' \\ -V' + m\ddot{W} &= 0 \end{aligned} \quad (C.1)$$

Eliminating $V(x,t)$ results in a modified Euler-Bernoulli equation:

$$EIW''''(x,t) + P(x,t)W''(x,t) + m\ddot{W}(x,t) = 0 \quad (C.2)$$

For the sake of simplicity the axial force $P(x,t)$ is considered to be constant along x and through the time. With this assumption and using the same B.C.s as in equation (3.21), the following mode shapes are obtained:

$$\phi_i(x) = \frac{\sin\left[\psi_i\left(1 - \frac{x}{L}\right)\right]}{\sin(\psi_i)} - \frac{\sinh\left[\varphi_i\left(1 - \frac{x}{L}\right)\right]}{\sinh(\varphi_i)} \quad (C.3)$$

Figure C.1: Axial force effect on an element of the link.

where:

$$\psi_i = \sqrt{\frac{\sqrt{\varpi^2 + 4\beta_i^4} + \varpi}{2}} \quad \varphi_i = \sqrt{\frac{\sqrt{\varpi^2 + 4\beta_i^4} - \varpi}{2}} \quad \varpi = \frac{L^2}{EI}P \quad (C.4)$$

where $\beta_i = \lambda_i L$ are the non-dimensionalized eigenvalues of the system. The characteristic equation of system is modified to:

$$\tan(\psi_i) = \frac{\psi_i \chi_i \tanh(\varphi_i)}{\varphi_i \chi_i - \tanh(\varphi_i)} \quad (C.5)$$

where

$$\chi_i = \frac{J_h \beta_i^4}{m L^3 \sqrt{\varpi^2 + 4\beta_i^4}} \quad (C.6)$$

and the orthogonality property of the mode shapes is

$$K_j \delta_{ij} = M_j \omega_j^2 \delta_{ij} = EI \int_0^L \phi_i''(x) \phi_j''(x) dx + P \int_0^L \phi_i'(x) \phi_j'(x) dx \quad (C.7)$$

where the second equation of orthogonality is similar to that of equations (3.32). It is seen that the axial force affects only the stiffness properties of the system.

It can be checked that if $P = 0$ then $\psi_i = \varphi_i = \beta_i$ and $\chi_i = a\beta_i^2$, therefore equations (C.3) and (C.5) would produce the same mode shapes and characteristic function as in equations (3.23) and (3.26), respectively.

To study the effect of axial force on the system dynamics, the characteristic equation of (C.5) was solved for β_i s numerically. Figure-C.2 show the six primary

Figure C.2: Effect of axial force (P) on modal eigenvalues.

eigenvalues of the system for an axial force variation of between 0 to 100 Newton. A major change in the first eigenvalue is seen while the changes in higher modes are not large.

Figure-C.3 shows the deviation of each eigenvalue in percentage ($\Delta\beta_i$) and for two different axial forces of 20 and 100 Newtons. where:

$$\Delta\beta_i = 100 \times \frac{\beta_i^{(P=0)} - \beta_i^{(P=\bar{P})}}{\beta_i^{(P=0)}} \quad (\text{C.8})$$

In this figure (*) and (o) represent the results when $\bar{P} = 100$ and 20 Newtons.

Figure C.3: Eigenvalue deviation for each mode caused by 100 N axial force (*) and 20 N (o).

respectively. In the first eigenvalue and for $\bar{P} = 100$ Newtons, a deviation of 27% is seen while it is reduced to 4% for $\bar{P} = 20$.

In general it can be concluded that the axial force has a major effect only on the first mode of the system. It can be also added that this effect strongly depends on the magnitude of the axial force itself. For this study where the magnitude of contact force does not exceed 30 Newtons and the angle between the X -axis and the contact surface is small, it is quite reasonable not to consider the axial force effects on the system dynamics, as in section 3.3.1. It should be noted that this assumption only make the modeling simpler and has no effect on the results and validity of control systems.

Appendix D

Decomposition Method for Constrained Systems

Assume an n^{th} order constrained system with state vector $\{\mathbf{x}\}_{n \times 1}$. For the sake of simplicity only one input and one constraint are assumed. The constraint equation can be written as:

$$g(\mathbf{x}) = 0 \quad (D.1)$$

To find the constraint reaction effect on the system, the Lagrange Multiplier concept can be used. This effect may be added to the external work as:

$$Q = h(\mathbf{x})u + g(\mathbf{x})\lambda \quad (D.2)$$

where λ is the Lagrange Multiplier and represents the constraint reaction effect.

The following set of Lagrange equations:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i} \right) - \frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial Q}{\partial x_i} \quad , \quad i = 1, \dots, n \quad (D.3)$$

results a set of n differential equations of motion as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u, \lambda) \quad (\text{D.4})$$

with $n + 1$ unknowns.

The *decomposition method*, used in e.g. McClamroch and Wang (1988) and Hu and Ulsoy (1994), is a systematic approach which utilizes the algebraic constraint equation of (D.1) to eliminate the dependent variable and to decrease the number of differential equations, to a form of:

$$\{\dot{\bar{\mathbf{x}}}\}_{(n-1) \times 1} = \bar{\mathbf{f}}(\bar{\mathbf{x}}, u) \quad (\text{D.5})$$

and also determines an algebraic equation for the desired unknown as:

$$\lambda = \bar{g}(\bar{\mathbf{x}}) \quad (\text{D.6})$$

In the present study, where a single flexible link is in contact with the environment, the contact force is assumed as the Lagrange Multiplier $F = \lambda$. The results of equation (D.1) for this system have been used in equations (3.34) and (3.35). As is seen the coefficients of F are $\phi_i(L)$, which are always equal to zero due to the second B.C. of (3.22). Therefore the Lagrange Multiplier (F) does not appear in the system's differential equation of motion, unlike in equation (D.4). It is concluded that the so-called decomposition method is not applicable for the contact force modeling purpose of this study. In section 3.4.2 another approach has been suggested to determine the contact force by means of the Lagrange Multiplier method.

Appendix E

Review of Friction Models

In order to consider the effects of friction in the design and analysis of moving systems, several models have been introduced by researchers in this field, see Armstrong-Hélouvry, *et al.* (1994). None of these models include all effects of the friction.

E.1 Tustin Dynamic Model

Figure-E.1 shows some important friction parameters. A well known friction model was introduced by Tustin as:

$$\tau_{fr} = \left[\alpha_0 + \alpha_1 e^{\frac{-|\dot{\theta}|}{\alpha}} + \alpha_2 |\dot{\theta}| \right] \text{sgn}(\dot{\theta}) \quad (\text{E.1})$$

where α_0 denotes the Coulomb friction level. When $\dot{\theta} = 0$, the level of $\alpha_0 + \alpha_1$ shows the maximum static friction or so called break away point. α_2 represents the viscous damping effect. The Stribeck effect is included using another coefficient α . This effect is a result of transition between dry and viscous frictions. This effect introduces a severe problem in applications with low velocity and/or with velocity

Figure E.1: Schematic of major parameters in friction models.

reversals, for further details see Popović, *et al.* (1993).

E.2 Karnopp Static Model

The Tustin model can not predict the value of friction for static situations. It only determines the break away point of system. It is known that in static situations the friction is in equilibrium with all other internal and external forces (torques) applied to the system. As soon as the resultant of applied forces exceeds the break away point the system switches to dynamic situation and the amount of friction falls down dramatically. This is shown in Figure-E.2. One of models for static friction was introduced by Karnopp (1985) as:

Figure E.2: Description of static friction (stiction).

$$\tau_{fr} = \begin{cases} \tau_{fr}^{(dynamic)} & \text{if: } |\dot{\theta}| > \epsilon_{\dot{\theta}} \\ \tau_{fr}^{(static)} = -sgn(\tau_{res})min(|\tau_{res}|, (\alpha_0 + \alpha_1)) & \text{if: } |\dot{\theta}| \leq \epsilon_{\dot{\theta}} \end{cases} \quad (E.2)$$

A small neighborhood of zero velocity is defined by $\epsilon_{\dot{\theta}}$. Outside this neighborhood, friction is a function of velocity and can be obtained using the Tustin model. Inside this neighborhood, velocity is considered to be zero and friction is torque dependent. As long as the resultant torque τ_{res} is less than the maximum stiction torque $(\alpha_0 + \alpha_1)$, the small velocity remains constant or is set to zero. Using this small neighborhood $\epsilon_{\dot{\theta}}$ enables the simulation to employ a longer step size without creeping or jumping through zero velocity, and thus simulating the stick-slip phenomena correctly.

E.3 Stick-Slip Model

Canudas de Wit, *et al.* (1993), have suggested a model to show the stick-slip regime in one single framework. This model includes most of the relevant properties that friction has been observed to display, e.g. Stribeck effect, hysteresis, spring-like characteristics in stiction, and stick-slip regime. Here, stiction ($\tau_{fr}^{(static)}$) and dynamic friction ($\tau_{fr}^{(dynamic)}$) are modeled separately. The total friction torque (τ_{fr}) is assumed to be a combination of these two components:

$$\tau_{fr} = (1 - \vartheta)\tau_{fr}^{(static)} + \vartheta\tau_{fr}^{(dynamic)} \quad (E.3)$$

where ϑ is used to determine in which frictional regime (stiction or slipping), the system is operating. $\vartheta=1$ implies sliding and $\vartheta=0$ implies sticking. The variable ϑ is defined by:

$$\varsigma_{\vartheta}\dot{\vartheta} = -\vartheta + \left(1 - e^{-(\frac{\dot{\theta}}{\alpha})^2}\right) \quad (E.4)$$

ς_{ϑ} is time constant to account for frictional lag, and α describes the rate of the Stribeck effect. The stiction effect is modeled as saturation function:

$$\tau_{fr}^{(static)} = \begin{cases} \text{sgn}(k_{\vartheta}\nu + d_{\vartheta}\dot{\theta}) \cdot (\alpha_0 + \alpha_1) & \text{if } |k_{\vartheta}\nu + d_{\vartheta}\dot{\theta}| \geq (\alpha_0 + \alpha_1) \\ k_{\vartheta}\nu + d_{\vartheta}\dot{\theta} & \text{if } |k_{\vartheta}\nu + d_{\vartheta}\dot{\theta}| \leq (\alpha_0 + \alpha_1) \end{cases} \quad (E.5)$$

k_{ϑ} and d_{ϑ} describe the spring-like characteristic of stiction. ν is an internal variable representing the relative displacement from the position where stiction occurred. It is modeled by:

$$\dot{\nu} = (1 - \vartheta)\dot{\theta} - \vartheta\frac{\nu}{\varsigma_{\nu}} \quad (E.6)$$

where ς_v is a time constant. The dynamic friction is described with classical Coulomb and viscous friction as:

$$\tau_{fr}^{(dynamic)} = \alpha_0 \cdot sgn(\dot{\theta}) + \alpha_2 \dot{\theta} \quad (E.7)$$

E.4 Linear Parameter Model

Canudas de Wit. *et al.* (1991) has proposed a simple dynamic model that captures the exponential friction characterization at low velocities and is linear in parameters. as:

$$\tau_{fr} = [\alpha_0 + \alpha_1 \sqrt{|\dot{\theta}|} + \alpha_2 |\dot{\theta}|] \cdot sgn(\dot{\theta}) \quad (E.8)$$

Although this model is nonlinear with respect to velocity, it is linear in parameters. so that it is suitable for the identification procedure.

Appendix F

Noncollocated vs NMP Systems

In this appendix it is shown that a noncollocated system is not necessarily nonminimum phase.

Figure-F.1 shows two 2-DOF systems. Force (F) and torque (τ) are the inputs for the left and right side systems respectively. The reaction force (R) is the output for both systems. The transfer functions for these systems are:

$$\begin{aligned} G_1(s) &= \frac{R(s)}{F(s)} = \frac{k_1 k_2}{(m_1 s^2 + k_1)(m_2 s^2 + k_1 + k_2) - k_1^2} \\ G_2(s) &= \frac{R(s)}{\tau(s)} = \frac{\frac{k_2}{L-d_2} \left[\left(c - \frac{b\beta}{\alpha} \right) - \frac{m_2}{\alpha} s^2 \right]}{\frac{1}{b\alpha} (m_1 d_1 s^2 + \gamma)(m_2 s^2 + b\beta) + (m_2 d_1 s^2 + \delta)} \end{aligned} \quad (F.1)$$

where:

$$\begin{aligned} a &= \frac{d_2}{d_1(d_2 - d_1)} \quad , \quad b = \frac{1}{d_2 - d_1} \quad , \quad c = \frac{L - d_1}{(L - d_2)(d_2 - d_1)} \quad , \quad \alpha = k_1 a + k_2 c \\ \beta &= \frac{k_1 b^2 + k_2 c^2}{b} \quad , \quad \gamma = k_1 a - d_1(c - b)k_2 b \quad , \quad \delta = d_1(c - b)k_2 c - k_1 b \end{aligned}$$

There is no zero for the first transfer function while a pair of zeros can be obtained

Figure F.1: Illustrating example for distinction between noncollocated and NMP systems.

for the second transfer function, as:

$$z_{1,2} = \pm \sqrt{\frac{k_1 L}{d_1(L - d_2)(d_2 - d_1)m_2}} \quad (\text{F.2})$$

which includes a RHP zero. Therefore the second transfer function is nonminimum phase. While both systems are noncollocated, only the right side system in Figure-F.1 is NMP. This example shows that noncollocation does not always make a system nonminimum phase.

Miu (1991) explained that if a continuous elastic system is noncollocated and *dispersive* then it will also be nonminimum phase. In non-dispersive systems wave speeds are constant and are governed by a second order partial differential equation, the well known wave equation. Problems of this type include transverse deformation of strings and membranes, longitudinal deformation of rods and beams, and torsional deformation of beams. In these systems even if the actuator/sensor configuration is noncollocated, the system is not nonminimum phase. In dispersive systems the wave speed is a function of wave frequency. In these systems each frequency component

contained in the initial waveform will propagate at a different speed. Dispersive systems such as bending deformation of beams are governed by fourth order partial differential equations. If such a system is noncolocated it will also be nonminimum phase.

Appendix G

Simulation Algorithms

Figure G.1: Simulink program for open loop simulation.

Figure G.2: Details of “Plant-Friction Model” block, from Figure-G.1.

1
F

ip
Initial contact

Initial time

I.C. detection

Phase detection

Next Phase determination

Transition detection

phase signal

Unit Delay3

1
transition signal

2

1/z

Gain

Unit Delay2

1/z

Gain1

2

W(L)

Ts : sampling time must be known.

Figure G.4: Details of “P-T detection” block, from Figure-G.3.

Figure G.5: Details of “Modified Tustin” block, from Figure-G.1.

Figure G.6: Details of “Tustin Friction” block, from Figure-G.5.

Figure G.7: Details of “Stick-Slip Friction” block, equation (E.3).

Figure G.8: Details of “xi-equ” block, from Figure-G.7, and equation (E.6).

Figure G.9: Details of “s-equ” block, from Figure-G.7. and equation (E.4).

Figure G.10: Details of “Dynamic Friction” block, from Figure-G.7. and equation (E.7).

Figure G.11: Details of “Static Friction” block, from Figure-G.7. and equation (E.5).

Figure G.12: Simulation program of PI+ v control system.

Figure G.13: Simulation program of Unified Predictive Control system.

Figure G.14: Simulation program of PID Control inner loop system.

Figure G.15: Details of “PID-Position Control” block, from Figure-G.14.

UPC-PID Force Control Of a Single Link Flexible Manipulator with Discontinuous Contact

Aria Alastv. April 1996

All polynomials in ascending power of $1/z$.

Figure G.16: Simulation program of Hybrid UPC-PID control system.

Appendix H

Computer Programs

The following is one of many computer programs used in the experimental sections of this thesis. Programs were written for open loop analysis, closed loop control of the various types described, system identification, etc...

```
/*      UPC_PID.C      */
/*      -----      */
/* Closed Loop Control      */
/* Inner Loop: PID Hub Angle Controller      */
/* Outer Loop: UPC Tip Force Controller      */
/*      Response To the Step Input      */
/*      */
/*      Aria Alasty      */
/*      March 1996      */
/*-----*/

#include <dos.h>
#include <stdio.h>
#include <conio.h>
#include <bios.h>
#include <stdlib.h>
#include <math.h>
#include <process.h>
#include <sys/timeb.h>
#include <time.h>
```



```

#include <sys/types.h>
#include "c:\inc\jr3.h" /*Include file for JR3 sensor*/
#include "c:\encoder\c\te5312.h"

#define N 1200
#define N1 1200
#define V_LIM 2.0 /* controller output voltage limits */
#define F_LIM 20.0 /* Maximum Desired force limit */
#define AXIS_1 0
#define wBoardAddr 0x300 // The base address the 5312 is strapped at
                        // in hexadecimal

#define GLOBAL -1
#define VEL_GAIN 1.169 /* converts input voltage to velocity in rad/sec */
#define VEL_CONV 57.296 /* converts velocity from rad/s to degree/sec */
#define CALB_FAC 2.786 /* calibrate the force sensor signal */
#define TIMESTEP 0.01 /* time step using in integration. */
#define LOOP 17000 /* NL=no. of loops for introducing delay */
/* NL=2*LOOP */

extern int aot815();
extern int aot();
extern int ain();

int erstat;
int board, gain, DA_lchan[1], AD_lchan[15];
int ch, choice;
float dif_time, step_time, F, v_output;
float Fz_force;
float ang_jr3(), sigmoidal(), control_output();
double f_time, s_time;

FILE *stream;
FILE *stream1;
FILE *stream2;
char file[12], file1[12], file2[12];

struct timeb timebuffer;

/* ***** MAIN ***** */

main()

```

```

{
    int i;

    main_init();
    /* Zero the analog outputs on both D/A physical channels */
    for (ch=0 ; ch<=1 ; ch++)
        aot (DA_lchan[ch], 0, &erstat);

    for(i=0;i<=3;i++)
        jr3_write(COM1, "R0 \r", 4);
        /* R0 is a command for sensor to set*/

    te5312InitSw();          // initialize the software
    te5312InitBoard(wBoardAddr, 4);    // initialize the board

    for(;;) {
        choice= main_menu();
        switch(choice)
        {
            case 1 :
                test();
                break;
            case 2 :
                for (ch=0 ; ch<=1 ; ch++)
                    aot (DA_lchan[ch], 0, &erstat);

                exit(0);
                break;
        }
    }

}

/* ##### TEST ##### */
test()
{
    long    preset_cnt,cnt;
    int     i,j,k,m,n,ns,nr,ntaw,nx,is,ir,it,ix;
    float   velocity[N], hub_ang[N], F_act[N];
    float   v_cmd[N],volt_out,v_new;
    float   K_r, K_p, K_v , K_i;
    double  S[50],R[50],Taw[50],Taw_1,TW,SY,RU;

```

```

float   ang_des[N], ang_err_p, err_integ, ang_err,F_des;
float   v_offset, F_offset;

printf("\n  FILENAME FOR READING THE UPC GAINS :");
scanf("%s",file2);
if ( (stream2=fopen(file2,"r")) == NULL)
{
fprintf(stderr, "Error opening UPC gains file.");
exit(1);
}

printf("\n 3) INFO FILENAME :");
scanf("%s",file);
stream=fopen(file,"w");
printf("\n 4) DATA STORAGE FILENAME FOR MATLAB: (*.m)");
scanf("%s",file1);
stream1=fopen(file1,"w");

Taw_1=0;
fscanf(stream2, "%d %d %d", &nr , &ns , &ntaw);
for (ir=0 ; ir<=nr ; ir++)
{ fscanf(stream2, "%lf" , &R[ir]); }
for (is=0 ; is<=ns ; is++)
{ fscanf(stream2, "%lf" , &S[is]); }
for (it=0 ; it<=ntaw ; it++)
{ fscanf(stream2, "%lf" , &Taw[it]);
Taw_1=Taw_1+Taw[it];      /* When desired force is constant */
}

printf("\nR= \n--- \n");
for (i=0 ; i<=nr ; i++)
{ printf("\n    %23.20lf",R[i]); }
printf("\n press any key to continue ...");
while(!kbhit()) {}
if (!getch())
(void)getch();
printf("\n");

printf("\nS= \n--- \n");
for (i=0 ; i<=ns ; i++)
{ printf("\n    %23.20lf",S[i]); }
printf("\n press any key to continue ...");
while(!kbhit()) {}

```

```

        if (!getch())
(void)getch();
        printf("\n");

        printf("\nTaw= \n--- \n");
        for (i=0 ; i<=ntaw ; i++)
{ printf("\n      %23.20lf",Taw[i]); }
        printf("\n press any key to continue ...");
        while(!kbhit()) {}
        if (!getch())
(void)getch();
        printf("\n");

ef: printf("\n      ENTER DESIRED CONTACT FORCE :");
        printf("\n !! MAXIMUM CONTACT FORCE  = 20 [N] ");
        scanf("%f",&F_des);
        if(F_des<0) /*only +Forces accepted */
            F_des=-F_des;
        if(fabs(F_des) > F_LIM )
            { printf("\n***** Error entering contact force ! *****");
goto ef;
            }

        printf("\n      7) ENTER K_r [180/pi=57.3] -> :");
        scanf("%f",&K_r);
        printf("\n      7) ENTER K_p [0.07] -> :");
        scanf("%f",&K_p);
        printf("\n      8) ENTER K_v [0.012] -> :");
        scanf("%f",&K_v);
        printf("\n      9) ENTER K_i [0.262] -> :");
        scanf("%f",&K_i);

        preset_cnt= 1024000; /* preset encoder count value to be loaded */
        te5312LoadCntr(GLOBAL, preset_cnt); // load counter value

        start();
        printf("\n ");

        if(ns>=nr) nx=ns;
        else nx=nr;
        for(ix=0 ; ix<nx ; ix++) {
F_act[ix]=0;
v_cmd[ix]=0;

```

```

hub_ang[ix]=0;
velocity[ix]=0;
ang_des[ix]=0;
}
k=nx;
TW =(double) F_des * Taw_1 ;

F_offset = ang_jr3();
v_offset = ain (AD_lchan[7], &erstat)*10.0 *VEL_GAIN*VEL_CONV/ 2048.0 ;
ftime(&timebuffer);
s_time=(double)timebuffer.time+timebuffer.millitm/1000.0;

while(!kbhit() && k<N ) {
for (j=0 ; j<=LOOP ; j++) {}
for (j=0 ; j<=LOOP ; j++) {}

F_act[k] = ang_jr3() - F_offset;
SY=0;
RU=0;
for (is=0 ; is<=ns ; is++)
    { SY =(double)SY + S[is] * F_act[k-is]; }
for (ir=1 ; ir<=nr ; ir++)
    { RU =(double)RU + R[ir] * ang_des[k-ir]; }
ang_des[k] =TW - SY - RU ;

velocity[k] = (ain (AD_lchan[7], &erstat)*10.0
               *VEL_GAIN*VEL_CONV/ 2048.0)- v_offset ;
cnt = te5312ReadCntr(AXIS_1);
hub_ang[k] =(float) (cnt - preset_cnt) * 360.0/1024000.0;
ang_err=ang_des[k]*K_r - hub_ang[k];
if(k==nx) err_integ=0;
else err_integ=err_integ+(ang_err + ang_err_p)*Timestep/2;
volt_out = ang_err*K_p - velocity[k]*K_v + err_integ * K_i;
        /*control voltage output*/

v_cmd[k]=control_output(DA_lchan[0],volt_out);
ang_err_p=ang_err;

k=k+1;
}

k=k-1;
n=k-nx;

```

```

ftime(&timebuffer);
f_time=(double)timebuffer.time+timebuffer.millitm/1000.0;
dif_time=f_time-s_time;
step_time=dif_time/n;

printf(" \n Zero output voltage command ..... ");
for (ch=0 ; ch<=1 ; ch++)
aot (DA_lchan[ch], 0, &erstat);

fprintf(stream," Test for the Force command = %.2f\n",F_des);
fprintf(stream," Test time = %.4f [s] for %d loops ", dif_time,n);
fprintf(stream,"\nTime of 1 loop : %.4f [s]", step_time);

for(m=nx; m<=k; m++) {
fprintf(stream1,"\n %.5f  %6.2f  %6.2f %6.2f %6.2f",
(m-nx)*TIMESTEP,F_act[m],hub_ang[m],velocity[m],v_cmd[m] );
}

fclose(stream);
fclose(stream1);
fclose(stream2);
}

/* ##### ang_jr3 ##### */

float ang_jr3()

{

float Fz_conv;
float const_1, const_2;
float Fz_volt;
float conv;
int AD_ch[3],value_in[3];

conv = 4.448 ; /*converts lbs to N */
Fz_conv = 300/10 ; /* 300 lbs per 10 input volt */
const_2 = 10.0 / 2048.0 ;

```

```

AD_ch[2] = 6 ;

value_in[2] = ain (AD_lchan[AD_ch[2]], &erstat);

if( value_in[2] >=0.0)
    Fz_volt = 0;
else
    Fz_volt = value_in[2] * const_2;

Fz_force= Fz_volt * Fz_conv *conv/CALB_FAC;

F=fabs(Fz_force);

    return(F);
}

/* ##### MAIN_INIT ##### */

main_init()

{
    jr3_init(COM1,A_CODE);
    rti_init();
}

/* ##### RTI_INIT ##### */

rti_init()

{
    /*    Parameter initialization    */

board=1;                /* RTI-815 Board reference number */
DA_lchan[0] = 0;
DA_lchan[1] = 1;
AD_lchan[0] = 2;
AD_lchan[1] = 3;
AD_lchan[2] = 4;
AD_lchan[3] = 5;
AD_lchan[4] = 6;
AD_lchan[5] = 7;
AD_lchan[6] = 8;
AD_lchan[7] = 9;

```

```

gain = 1;

/*      D/A & A/D board initialization      */

    initialize(&erstat);
    if(erstat) {

printf("*****\n");
printf("**  initialization error: %2d  **\n", erstat);
printf("*****\n\n");
exit(0);
}

        for (ch=0;ch<=1;ch++)
        {
aot815(DA_lchan[ch], board, ch, &erstat);

if(erstat)
    {
        printf("*****\n");
        printf("**  D/A setup error: %2d  **\n", erstat);
        printf("*****\n\n");
        exit(0);
    }
        for (ch=0;ch<=7;ch++)
        {
ain815(AD_lchan[ch], board, ch, gain, &erstat);

if(erstat)
    {
        printf("*****\n");
        printf("**  A/D setup error: %2d  **\n", erstat);
        printf("*****\n\n");
        exit(0);
    }
        }
    }

/* ##### JR3_INIT ##### */

jr3_init(port,code)

```



```

    int port;
    unsigned char code;

{
    union REGS r;

    r.x.dx = port;
    r.h.ah = 0;
    r.h.al = code;

    int86(0x14, &r, &r);
}

/* ##### MAIN_MENU ##### */

int main_menu()

{ int mode;
  printf("\n          ***** ");
  printf("\n          *      PROGRAM  UPC_PID.C      * ");
  printf("\n          ***** ");
  printf("\n\n 1 DEGREE OF FREEDOM MANIPULATOR FORCE CONTROL TEST");
  printf("\n  2DOF SCHEME: INNER LOOP PID POSITION CONTROL - ");
  printf("\n          OUTER LOOP UPC FORCE CONTROL. \n");
  retry: printf("\n***** \n");
  printf(" SELECT TEST MODE \n\n");
  printf("    1.  -->  RUN THE TEST \n");
  printf("    2.  -->  EXIT \n");
  printf(" \nENTER # :  -->  ");
  scanf("%d", &mode);
  if ( !((mode==1)|| (mode==2)))
    goto retry;
  printf("***** \n\n");
  return(mode);

}

/* ##### START ##### */

start()

{

```

```

char ans;
again: printf("\n\t PRESS 's' TO START JOINT MOTION");
      printf("\n\t PRESS   'e' TO EXIT -->  ");
      scanf("%s",&ans);
      if (ans == 's' || ans == 'e')
      {
if (ans == 'e')
          exit(0);
      }
      else
      goto again;
}

/* ##### PORT_PUTCH ##### */

int port_putch(port,c)

int port;
char c;
{
    union REGS r;

    r.x.dx = port;
    r.h.al = c;
    r.h.ah = 1;

    int86(0x14, &r , &r);

    if(r.h.ah & 128)    /* check bit 7 */
return(-1);
    else
return(0);
}

/* ##### JR3_WRITE ##### */

int jr3_write(port, buf, cnt)

int port;
char *buf;
int cnt;
{

```

```

    char *byte_ptr;
    int loop;

    byte_ptr = buf;

    for(loop=0;loop<cnt;loop++)
    {
if(*byte_ptr == '\n')
    *byte_ptr = '\r';
if(port_putch(port, (*byte_ptr)))
    return(loop);
byte_ptr++;
    }
    return(cnt);
}

/* ##### CONTROL_OUTPUT ##### */

/* Function to limit and generate the Controller Output
   voltage to the motor drive amplifier */

float control_output (logchan, v_out)

int    logchan;
float  v_out;

{
int int_v_out;
    float  v_out_scaled;

if (v_out > V_LIM)
    v_out = V_LIM;
if (v_out < -V_LIM)
    v_out = -V_LIM;

v_output=v_out;
/* The voltage cmd is changed to an integer value
   in range of +/- 0 to 2048. This corresponds to
   a 12 bit range of possible analog outputs. */

v_out_scaled = 2048.0/10.0 * v_out;

```

```
int_v_out = (int) v_out_scaled;

    /* Account for chop-off error */
if (v_out_scaled > 0.0)
{
    if ((v_out_scaled - int_v_out) > 0.5)
        int_v_out += 1;
}
    else
{
    if ((v_out_scaled - int_v_out) < -0.5)
        int_v_out -= 1;
}

    /* Perform the D to A conversion */

aot(logchan, int_v_out, &erstat);
return(v_output);
}
```

Appendix I

Publications

- Alasty, A., Vukovich, G., "An Exact-Model Based Unified Predictive Force Control of Flexible Manipulators," *Proceedings of AIAA Conference on Guidance, Navigation, and Control*, July 29-31, 1996, San Diego, CA, USA.
Submitted to AIAA Journal of Guidance, Navigation, and Control.
- Alasty, A., Vukovich, G., "Unified Predictive Control of Contact Force in Flexible Link Manipulators," *Proceedings of 5th IEEE International Conference on Control Applications*, September 15-18, 1996, Dearborn, Michigan, USA.
- Alasty, A., Vukovich, G., "Preliminary Experiments On Force Control of Flexible Manipulators," *Proceedings of Japan-USA Symposium on Flexible Automation (ASME & ISCE)*, July 7-10, 1996, Boston, Massachusetts, USA.
- Alasty, A., Vukovich, G., "Experiments on Force Control of Flexible Manipulators with Stiff Contact," *Presented in CASI 43rd Annual Conference, 9th Symposium on Aerospace Structures & Materials*, April 29-May 1, 1996, Ottawa, Ontario, Canada.

- Alasty, A., Vukovich, G., "Force Control of Flexible Manipulators with Discontinuous Contact and Joint Friction," *Proceedings of 3rd IFAC Symposium on Nonlinear Control Systems Design*, June 26-28, 1995, Tahoe City, CA, USA.
- Alasty, A., Sasiadek, J.Z., Vukovich, G., "Dynamics of One Link Flexible Manipulators for Contact Force Control," *Proceedings of 10th CISM-IFTOMM Symposium on Theory and Practice of Robots and Manipulators (RoManSy'10)*, Sept. 12-15, 1994, Gdansk, Poland.