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**MODELING AND EVALUATION OF NONBACKDRIVABLE
TRANSMISSIONS FOR VARIABLE STIFFNESS PROSTHESES**

by

TIMOTHY BURKE, B. Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research Department
in partial fulfillment of
the requirement for the degree of
Master's of Mechanical Engineering

**Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering**

Carleton University
Ottawa, Ontario
April 13, 2000
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**"MODELING AND EVALUATION OF NONBACKDRIVABLE TRANSMISSIONS
FOR VARIABLE STIFFNESS PROSTHESES"**

submitted by Timothy Burke, B. Eng.
in partial fulfillment of the requirements for
the degree of Master's of Mechanical Engineering

A handwritten signature in cursive script, appearing to read "David Russell", is written over a horizontal line.

Thesis Supervisor

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Abstract

The nonlinear dynamic behavior of nonbackdrivable transmissions, typically associated with low lead angle gear components, has not been established in literature. To study the dynamic behavior of two proposed variable stiffness upper arm prostheses [English, 1996] which require nonbackdrivable transmissions, a simple dynamic model of non-intersecting, non-parallel axes gears was developed. The model includes the nonlinear effects of backlash, Hertzian damping and friction and can be used to study both backdrivable and nonbackdrivable gear sets. The development of this model involved investigations of DC dynamo operation and nonbackdrivable characteristics of several gear types. Dynamic simulations of the two proposed variable stiffness upper arm prostheses were conducted involving nonbackdrivable components and motors that were selected in this thesis. The results from these simulations using the gear model appeared reasonable and accurate.

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Nomenclature

a_{eff}	Acceleration of effective worm and motor shaft mass
$a_{eff.norm}$	Normal component of a_{eff}
a_G	Acceleration of the effective gear mass
a_p	Acceleration of the effective pinion mass
a_w	Acceleration of effective worm mass
A	System Matrix for state space model
b	Damping coefficient
b_t	Approximate length of gear tooth base
B	Rotational damping coefficient
B	Input coefficient matrix
C	Compliance
C_{tooth}	Compliance between mating gear teeth
$d_{backlash}$	Backlash between teeth
d_d	Height of tooth dedendum
d_G	Pitch diameter of gear
d_{iw}	Inside diameter of worm
d_s	Pitch diameter of lead screw
d_w	Pitch diameter of worm
D	Nonlinear Hertzian damping coefficient
DV	Half of effective "zero" velocity range
e	Transmission power efficiency
e_G	Efficiency of gear driving the pinion
e_p	Efficiency of pinion driving the gear
e_R	Coefficient of restitution
e_w	Efficiency of worm driving worm gear
$e_w _{static}$	Ratio of applied gear torques when static friction is maximum
E	Modulus of elasticity
E_g	Back emf of a dynamo
E_k	Energy dissipated by motor while moving at steady state
E_s	Energy dissipated by motor holding a load in static equilibrium
F	General applied force
F_d	Dynamic load of gear teeth
F_f	Force of friction
$F_{f, equil}$	Required static friction force to maintain equilibrium
F_G	Force applied to gear tooth
$F_{G,ml}$	Massless force applied to the gear tooth
F_{Gt}	Maximum tangential force on worm gear teeth
FH	Maximum static friction force
F_{Live}	Maximum live lifting force
F_{max}	Maximum force produced by spring
F_p	Force applied to pinion tooth
$F_{p,ml}$	Massless force applied to pinion tooth
F_s	Strength capacity of worm
F_w	Wear capacity of worm gear

gh	Gear ratio of motor gearhead
G	Ratio of radii
GR	Gear ratio
GR_{C1}	Gear ratio of Configuration 1 transmission.
$GR_{C3, pos}$	Gear ratio of Configuration 3 position motor transmission
$GR_{C3, stiff}$	Gear ratio of Configuration 3 stiffness motor transmission
h	Height of gear tooth
i	Current in a dynamo
i_{load}	Motor current resulting from an external torque
i_{NL}	Motor no load current
I	Moment of inertia
J_{arm}	Rotational inertia of the prosthetic arm
J_G	Rotational inertia of gear
J_{G+arm}	Combined rotational inertia of gear and prosthetic arm
J_{motor}	Rotational inertia of motor (perpendicular to motor shaft)
J_s	Rotational inertia of motor shaft
J_w	Rotational inertia of worm
k	Spring stiffness
K^*	Torsional joint stiffness
K^*_{max}	Maximum torsional joint stiffness
K^*_{min}	Minimum torsional joint stiffness
K_{bend}	Bending stiffness of tooth
K_{eq}	Equivalent stiffness between mating teeth
K_G	Stiffness of gear tooth
$K_{material}$	Hertzian stiffness of tooth
K_p	Stiffness of pinion tooth
K_t	Torque constant of a dynamo
K_w	Material geometry factor
l_{lim}	Limiting length of spring operation
L	Lead of lead screw
L_a	Armature inductance of a DC dynamo
L_{arm}	Length of prosthetic arm
L_c	Thickness of cylinders
L_w	Length of worm
m_{arm}	Mass of prosthetic arm
m_G	Effective mass of gear
m_p	Effective mass of pinion
M	Bending moment
N	Normal force
N_G	Number of teeth on a gear
N_{worm}	Normal force between mating worm and worm gear teeth
p	Momentum
p_d	Diametral pitch
P	Pressure between lead screw and nut
$P_{arm, max}$	Maximum power output of the prosthetic arm
P_c	Circular pitch

$P_{Cl,max}$	Maximum power output of one of the Configuration 1 transmissions
P_{heat}	Maximum steady state heat dissipation rate of motor
P_k	Power loss to inefficiencies
P_{max}	Maximum output power of motor
P_{motor}	Power flow at dynamo voltage terminals
P_{Ra}	Power dissipated across motor armature resistance
r_{bG}, r_{bp}	Base radius, gear and pinion respectively
r_G, r_p	Pitch radius, gear and pinion respectively
r_{oG}, r_{op}	Outside radius, gear and pinion respectively
r_w	Pitch radius of a worm
R	Effective elbow radius
R_a	Armature resistance of a DC dynamo
R_G	Radius from worm gear center to spring/damper
R_{R-A}	Thermal resistance from motor rotor to ambient
S_n	Fatigue strength
SF	Safety factor
$t_{ext,NL}, t_{ext,1.2lb}$	Extension time with gravity, no load and 1.2 lb load respectively
$t_{flex,NL}, t_{flex,1.2lb}$	Flexion time against gravity, no load and 1.2 lb load respectively
t_{impact}	Time of impact
$T_{ambient}$	Ambient temperature
$T_{operating}$	Manufacturer's specified maximum internal motor operating temperature
U	Input vector
v	surface or general velocity
v_G	Pitch line velocity of gear tooth
v_t	Relative velocity at impact
v_p	Pitch line velocity of pinion tooth
v_{max}	Maximum velocity of spring ends
v_{slide}	Tangential relative velocity component
v_w	Velocity of effective worm tooth mass
V	Input voltage to a dynamo
Vel	Input velocity to end of spring/damper
w	Gear face width
W	Weight of prosthetic arm
x	Radial position on gear tooth
x_{ext}	Deflection of spring
x_{rel}	Relative displacement between mating gear teeth
$\mathbf{X}$	State vector
y	Lewis form factor
α	Mathematical simplification variable for (5-1), no physical meaning
α_G	Angular acceleration of worm gear
α_m	Angular acceleration of motor shaft
α_w	Angular acceleration of worm
β_{aG}, β_{ap}	Approach arc, gear and pinion respectively
$\beta_{aGmax}, \beta_{apmax}$	Maximum approach arc, gear and pinion respectively
β_{rG}, β_{rp}	Recess arc, gear and pinion respectively
$\beta_{rGmax}, \beta_{rpmax}$	Maximum recess arc, gear and pinion respectively

δ	Surface penetration of mating gear teeth
Δ	Angular deflection of joint from equilibrium
Δ_t	Deflection of gear tooth tip
Δt_k	Time of kinetic operation
Δt_s	Time of static operation
ϕ, ϕ_n	Pressure angle, Normal pressure angle
Φ	Nonlinear spring stiffness function
η	Ratio of maximum negative to positive displacement of springs
η_a	Ratio of approach arc to maximum approach arc
η_m	Mass ratio
η_r	Ratio of recess arc to maximum recess arc
φ	Friction angle
φ_{worm}	Friction angle for a worm and worm gear set
λ	Flux linkage across an inductor, Lead angle
μ	Coefficient of friction
μ_a	Average coefficient of friction during approach
μ_k	Kinetic coefficient of friction
μ_r	Average coefficient of friction during recess
μ_s	Static coefficient of friction
ν	Poisson's ratio
Π	Nonlinear backlash and tooth stiffness function
θ_G	Angular displacement of worm gear
θ_w	Angular displacement of worm
Θ_{lim}	Maximum range of motion of joint
ρ_w	Density of worm
τ	Torque output of a dynamo
τ_{aG}, τ_{ap}	Sum of applied torques during approach, gear and pinion respectively
τ_{arm}	Applied torque to joint
$\tau_{arm. max}$	Maximum applied torque to joint
τ_G	Torque applied to gear
$\tau_{G.ml}$	Massless torque applied to gear
τ_{max}	Maximum torque output of a dynamo
τ_{rG}, τ_{rp}	Sum of applied torques during recess, gear and pinion respectively
τ_{stall}	Stall torque of motor
τ_{th}	Maximum thermal torque of a motor
τ_w	Torque applied to worm
$\tau_{w.ml}$	Massless torque applied to worm
ω	Angular velocity of a dynamo shaft
ω_{arm}	Angular velocity of prosthetic arm
ω_G	Angular velocity of a gear
ω_{NL}	No load speed of motor
ω_p	Angular velocity of a pinion
Ξ	Nonlinear Hertzian damping function
ψ	Helix angle

Chapter 1

Introduction

The following thesis involves continuing research into a variable stiffness prosthetic arm, first presented by English [1996]. In his work, English established the fundamental designs of several variable stiffness prosthesis configurations, two of which are used in this thesis. One of the fundamental design concepts of both of these designs was the use of a nonbackdrivable transmission.

Most engineers are familiar with the concept of "nonbackdrivable" or "self-locking" in the typical context of static analysis of worm gear drives and lead screws. However, the dynamic characteristics of such nonbackdrivable transmission components have not been well established in literature. Though gear dynamics has been a common area of research since the '40's (see Ozguhen and Houser, 1988 for a literature survey prior to 1988), an extensive literature review revealed no existing dynamic models for nonbackdrivable gears. This was the fundamental motivation behind the research conducted in this thesis which focused on the development of a simple and accurate dynamic model for non-parallel, non-intersecting axes gears such as a worm and worm gears. The intention was to be able to use this nonlinear model to further investigate the dynamic behavior of the two proposed variable stiffness prosthesis designs.

In most common applications, including the prosthesis designs in this thesis, motors are coupled directly to geared transmissions. For this reason, the research in this thesis began with a review of the basic operation of a DC dynamo (ie. motor/generator). The fundamental concepts of nonbackdrivable gears were then examined, including the introduction of a simple model for non-parallel, non-intersecting axes gears. This background work concludes by examining the behavior of dynamos coupled to backdrivable and nonbackdrivable transmissions.

Preliminary design of the nonbackdrivable systems, including selection of motors and gearing components for the two proposed variable stiffness prosthesis designs was conducted followed by an extensive review of the development of the non-intersecting, non-parallel axes gear model. These components selected for the prosthesis designs, along with the nonbackdrivable model developed in this thesis, were then incorporated into the dynamic models of the prosthesis designs. Simulations were then performed to observe and study the dynamic behavior of the nonbackdrivable transmissions, motors, and prosthesis designs as a whole.

1.1 Modeling Technique: Bond Graphs

Bond graphs are a dynamic modeling technique that are used extensively in this thesis. The reader is referred to Karnopp et. al. [1990] as one reference to this topic. A complete discussion of bond graphs is outside the scope of this thesis, but a brief description of their fundamental concepts are now presented.

Bond graphs are a dynamic modeling technique which are based on the fundamentals of energy conservation and power flow or exchange between system components. One advantage to bond graphs is that they are not domain specific; in other words, models of mechanical (both linear and rotational), electrical, thermal, and fluid dynamics can all be modeled using the same techniques and incorporated into a single bond graph if a system consists of multiple domains (ie. an electrical circuit driving a motor in a thermo/fluid system). Another major advantage to bond graphs are that they can be used to model nonlinear systems with no functional change in the structure or interpretation of the model.

There are five main concepts in bond graphs: 1) power variables, 2) system elements and sources, 3) power bonds, 4) junction structures, 5) causality. Since in this thesis only the electrical and mechanical domains are used in the models, the examples and discussion will be restricted to these domains. The reader is referred to Karnopp et. al. [1990] for further discussions on the other possible domains. To assist in the explanation, a simple mass/spring/damper system with its corresponding bond graph is shown in Figure 1.1 with each of the five concepts highlighted on the bond graph.

The power variables consist of effort (ie. force, torque, voltage) and flow (ie. velocity, angular velocity, current) variables for each domain whose product is power. The fundamental system elements are inductance (ie. mass, rotational inertia, inductance), compliance (ie. linear springs, rotational springs, capacitors) and resistance (ie, linear damper, rotational damper, resistors). Source elements represent perfect flow and effort

(a)

(b)

Figure 1.1 (a) A simple mass/spring/damper system, (b) Bond graph of the mechanical system in (a).

"sources" such as an input current or external force. The power flow between these elements is modeled using power bonds, with the half arrow indicating the direction of positive power flow. Active bonds, not shown in Figure 1.1, represent information flow (no power) between bonds and are represented by a bond with a full arrow head.

Junction structures consist of four main types: 1) "0" junction, 2) "1" junction, 3) transformer, "TF", junction, 4) gyrator, "GY", junction. The "0" junction represents a common effort between all connecting power bonds, where the flow variables sum. The "1" junction represents a common flow between all connecting power bonds, where the effort variables sum. Multiple bonds can converge at either a "0" or "1" junction. The transformer, "TF", junction relates the effort and flow variables of one power bond to the effort and flow variables of the other power bond respectively: an example is a gear ratio. The gyrator, "GY", junction relates the effort and flow variables of one power bond to the flow and effort of the other power bond respectively. Both the transformer and gyrator junction can only have two power bonds attached and can be modulated by a changing (possibly nonlinear) variable. Causality, in brief, indicates the type of relationship between the flow and effort variables on a system element, being either derivative or integral. There are several rules and interpretations associated with causality which are beyond the scope of this simple introduction.

Once a model is developed in bond graphs, there are well defined methods used to develop the system of state equations associated with the energy storage elements in the system. In the case of the example in Figures 1.1, two state equations could be derived

for the momentum of the mass and the deflection of the spring, which can be used to completely define the state of the system.

1.2 The Nonbackdrivable Concept

One of the main focuses of this thesis is the development, modeling, and dynamic behavior of the nonbackdrivable transmissions required in the proposed prosthesis designs. In most literature, "nonbackdrivable", "non-overhauling", and "self-locking" are used synonymously and typically refer to a gear set in which the gear cannot "drive" the pinion. Note that in this thesis, the terms "pinion" and "gear" are used to describe the gears that are connected to the motor-side and load-side of a given gear set, respectively. These terms are typically referenced to a static situation, in which the gear set is "locked", and are rarely discussed in terms of a dynamic (moving) scenario. Since no formal definition for "nonbackdrivable", "non-overhauling" or "self-locking" was found in the literature the author proposes the following, general, definition:

Nonbackdrivable: a property of an interface between two contacting surfaces across which power can flow in only one direction.* (Def. 1-1)

It should be noted that this definition is not specific to gear sets. However, based on (Def. 1.1), a formal definition of nonbackdrivable gears and nonbackdrivable transmissions can be made:

Nonbackdrivable gears: a pair of gears whose contact between conjugate teeth is considered nonbackdrivable. (Def. 1-2)

* Impulsive forces may behave differently due to the need for detailed surface models.

Nonbackdrivable transmission: a power transmission that contains at least one set of nonbackdrivable gears. (Def. 1-3)

From this point on, the terms "nonbackdrivable", "nonbackdrivable gears" and "nonbackdrivable transmission" will refer to these three definitions. Since both definitions (Def. 1-2) and (Def. 1-3) are based on (Def. 1-1), it can be inferred that power flow through nonbackdrivable gears and nonbackdrivable transmissions can only be in one direction. Conversely, power flow in backdrivable gears and transmissions can be in both directions.

Using a nonbackdrivable worm and worm gear set as an example, the worm can "drive" the worm gear but the reverse is not true. Power is restricted to flow in only one direction, in this case from the worm to the worm gear. There is no such power restrictive bond associated with bond graphs making this concept difficult to model. The issue of nonbackdrivable gear trains and development of a nonbackdrivable gear model are both addressed in this thesis.

1.3 Proposed Prosthesis Designs

In his thesis, English [1996] proposed four different prosthesis designs, of which two showed particular potential and are further investigated in this thesis. These two designs, consistent with the terminology used by English, are called Configuration 1 and 3 and are presented schematically in Figures 1.2 and 1.3, respectively. Before describing the individual characteristics of these designs, several general concepts must be addressed.

These two designs were developed in anticipation that they would be controlled using an open loop scheme. There is no intention of using a closed loop between the prosthetic arm and the motors, and thus the issues of low bandwidth associated with the use of the springs need not be addressed. This thesis does not attempt to develop such a controller.

It was determined by English [1996] that in order to maintain constant stiffness when the prosthesis designs are in contact with the environment, the springs must have a quadratic stiffness function. The design of these springs is not addressed in this thesis. The design and analysis of the transmission of these two configurations is independent of the spring function characteristics and therefore the results can be used regardless of the final spring design. In both configurations, the torque developed at the prosthetic elbow is given by,

$$\tau_{arm} = K^* \cdot \Delta \quad (1-1)$$

where τ_{arm} is the torque, K^* is the effective torsional stiffness of the joint, and Δ is the angular deflection from the joint equilibrium. Notice that K^* is a function of the individual spring stiffness in each branch, but is independent of the specific spring stiffness function.

The fundamental design concepts and equations for each configuration are discussed separately. The reader is encouraged to review the following publications for additional details of these designs [English, 1996; English and Russell, 1999 (a), (b)].

1.3.1 Configuration 1

A schematic of the proposed design for Configuration 1 is presented in Figure 1.2 and consist of two springs arranged antagonistically around the prosthetic elbow joint (dampers, which are in parallel with these springs are not shown in Figure 1.2). To maintain consistent terminology to English [1996], each of the motor and spring systems is referred to as a branch. This spring arrangement in this configuration is very similar to that of a natural joint, where muscles are arranged antagonistically such that stiffness and displacement can be controlled. One of the main features of this configuration is the required nonbackdrivable transmission located between the motor and springs of each branch. The nonbackdrivable transmissions permit power to flow in only one direction: from the motor to the spring and not vice versa. This prohibits the motor from absorbing power from the springs and behaving as a generator, which is discussed in detail in Chapter 2. This nonbackdrivable transmission is an essential part of this design because it utilizes the energy storage ability of the springs in each branch. Once the motor-end of the spring is set, the power to the motors can be turned off and any external load that may be applied to the prosthesis results in the springs stretching and storing that energy. This energy can then be returned when the external load is removed, which is anticipated to result in dramatic energy savings for the prosthesis. If a backdrivable transmission was connected to the motor, the force in the springs would cause the motor to backdrive (operate as a generator) and the energy within the springs would then be dissipated as heat in the motor.

Figure 1.2 Schematic of Configuration 1 prosthetic arm.

In this design, displacement and stiffness of the elbow joint are related to the displacements of the motor-end of the springs of each branch. To change only the stiffness, both motors must move the ends of the springs in the same direction by the appropriate amounts. For pure displacement with no change in joint stiffness, the motors must move the springs in opposite directions by the appropriate amounts. In general, movement of the motor-end of the springs, other than these two special cases, results in changes in displacement and stiffness simultaneously.

The specific design equations for this configuration, which are critical to the transmission design, will be highlighted as they are needed.

1.3.2 Configuration 3

A schematic of the proposed design for Configuration 3 is presented in Figure 1.3. This configuration, though functionally similar to Configuration 1, is different in terms of motor and transmission arrangement. As in Configuration 1, there are two springs antagonistically arranged about the effective elbow joint (dampers which are in parallel with these springs are not shown in Figure 1.3). This configuration also requires two motors, but each motor is functionally different unlike the symmetric arrangement of Configuration 1. In this configuration, the motor located at the effective elbow radius is used to change position of the joint only: this motor will be referred to as the position motor. The motor located at the end of the springs is used to change stiffness only: this motor will be referred to as the stiffness motor. In this sense, the stiffness and displacement of the joint are independently controlled, unlike the coupling found in

Figure 1.3 Schematic of Configuration 3 prosthetic arm.

Configuration 1. The position control motor requires a nonbackdrivable transmission between it and the prosthetic arm for the same reasons mentioned in Configuration 1. A unique problem with this portion of the transmission is that the housing, containing the motor and nonbackdrivable worm and worm gear, must be able to rotate relative to the spring ends. This allows energy from external loads to be stored in springs, when the nonbackdrivable transmission is "locked". To help understand this section of the transmission, imagine an external weight is placed on the arm with the stiffness of the elbow joint remaining constant (the position of the stiffness motor is held constant). Since the nonbackdrivable transmission of the position motor is "locked", the entire motor and gearing within the housing would rotate as a rigid body which results in the top spring stretching and bottom spring compressing until the torque about the elbow joint (resulting from the forces in the springs) was equal and opposite to the torque caused by the external load. To lift this load, the position motor would need to be activated and the output shaft of the worm gear would rotate raising the arm while the housing and springs ends would remain static (since the equivalent torque to the external load must be maintained). This situation is examined in more detail in Chapter 4 where the motors and gear ratios for Configuration 3 are selected.

The additional spring fastened between the ends of the antagonist springs and ground is used as a "counter" force and is sized so that when the arm is in equilibrium with no external force, the force of the "counter" spring is equal and opposite to the combined force of the antagonist springs. In this way, theoretically, it would require no force by the stiffness motor to move the ends of the springs, ie. the spring arrangement is always in

equilibrium. When an external load is applied a force imbalance between the two antagonist springs would arise, and a torque that would presumably be taken up by the bearings located at the ends of the antagonistic springs, would result. The design of the "counter" springs as well as the antagonistic springs is beyond the scope of this thesis. The selection of the stiffness motor and gearing, though, are examined. For the same reasons discussed for Configuration 1, this configuration requires a nonbackdrivable transmission for the stiffness motor.

1.4 Chapter Outline

The following is a brief outline of the remaining chapters in this thesis. The next three chapters examine fundamental dynamo and nonbackdrivable issues, while the last four involve the applications of these fundamentals to the proposed prosthesis designs.

Chapter 2

This chapter reviews the basic operation of a dynamo (motor and generator) and presents a simplified graphical approach to assist in the understanding of its operation.

Chapter 3

The issues of nonbackdrivability of gears are examined in detail for several popular gear types. A simple model for nonintersecting, nonparallel axes gears is also presented.

Chapter 4

This chapter examines the operation of backdrivable and nonbackdrivable transmissions coupled to dynamos. The graphical dynamo representation used in Chapter 2 is applied to examples in this chapter to highlight the differences in dynamo operation when backdrivable and nonbackdrivable transmissions are used.

Chapter 5

A review of commercially available upper arm prostheses is presented and design goals for the Configuration 1 and 3 are established. The selection procedures for the motor and gearing of both configurations are explained along with convenient general graphical tools which can assist in determining the total required gear ratio of a variable speed/load transmission.

Chapter 6

A literature review of gear models along with the development of a novel bond graph model of a nonbackdrivable gear pair is presented. Simple simulations are used to demonstrate the nonbackdrivable model.

Chapter 7

Simulations of both configurations including the nonbackdrivable gear model are performed. Results from the two simulated tasks were compared against the design goals.

Chapter 8

Chapter 8 includes conclusions and recommendations for this thesis.

Chapter 2

Dynamo Operation

A basic electromagnetic-mechanical device which can convert either electrical power to mechanical power, or mechanical power to electrical power is called a dynamo [Thorpe, 1989]. When a dynamo converts electrical input power into mechanical output power, it is operating as a motor and when it converts mechanical input power into electrical output power, it operates as a generator. A simple bond graph [Karnopp, et. al., 1990] and schematic model of a series dynamo are shown in Figure 2.1. Both the general electromechanical actuator bond graph model developed by Karnopp [1985, 1991] and the general actuator bond graph model developed by Seth and Flowers [1990] reduce to this simple dc dynamo model when the transformer properties of the actuator are ignored, and the dynamic properties of the external load, including shaft inertia, are omitted.

The arrows on the bond graph indicate the direction of positive power flow. In this given configuration the dynamo is operating as a motor when positive input power flows from the electrical source and is gyrated into mechanical output power. This is also portrayed in the motor schematic where the terminal voltage and current (input power) and the output speed and torque of the mechanical port are in the same directions, indicating that positive power is flowing from the terminals to the mechanical domain (output power).

The main components of this model consist of an electrical source (V), inductance of the

(a)

(b)

Figure 2.1 Series model of a DC dynamo, (b) Bond graph of the series DC dynamo model in (a).

armature (L_a), resistance of the armature (R_a), and the back emf (E_g). The gyrator modulus of the bond graph, K_t , relates voltage to shaft speed and current to torque by:

$$E_g = K_t \omega \quad (2-1)$$

$$\tau = K_t i \quad (2-2)$$

where E_g represents the back emf. In this sense, K_t represents the back emf and torque constant of the dynamo.

The state equation for this dynamo model, following the sign convention chosen in the model, is:

$$\dot{\lambda} = V - iR_a - E_g \quad (2-3)$$

where $\dot{\lambda}$ is the rate of change of flux linkage of the inductor. L_a , otherwise written as,

$$\dot{\lambda} = \frac{1}{L_a} \frac{di}{dt} \quad (2-4)$$

Rearranging (2-3) gives:

$$V = \dot{\lambda} + iR_a + E_g \quad (2-5)$$

which indicates that the terminal voltage, V , equals the sum of voltage drops across all the other elements in the circuit. Because of the typically fast time constants of the electrical circuit, the system reaches steady state quickly, at which time:

$$V = iR_a + E_g \quad (2-6)$$

It is important to recognize the significance of the relative voltage drop across the resistor, iR_a , and the back emf, E_g , since these quantities are directly related to the mechanical output of the motor. For example, if the voltage drop across the resistor and

the back emf are known, the torque and speed of the motor shaft can be determined from (2-1) and (2-2) by:

$$\tau = K_t \left(\frac{iR_a}{R_a} \right) = K_t i \quad (2-7)$$

$$\omega = \frac{E_g}{K_t} \quad (2-8)$$

Most motor performance data includes a motor's no load speed (ω_{NL}) and its stall torque (τ_{stall}) at a nominal voltage. From (2-7) and (2-8), the theoretical no load speed of a motor occurs when the speed of the motor is maximized, which occurs when $V = E_g$. The theoretical stall torque occurs when the torque of the motor is maximized, which occurs when $V = iR_a$. Thus, no load speed and stall torque are the two extreme conditions of (2-6).

2.1 Graphical Representation of Steady State Dynamo Operation

The steady state operation of a dc dynamo can be portrayed graphically, as in Figures 2.2 and 2.3. Figure 2.2 uses the electrical variables, terminal voltage (V) and voltage drop across the resistor ($iR_a = \tau R_a / K_t$), as its axes variables, whereas Figure 2.3 uses the mechanical variables, τ and ω .

In Figure 2.2, there are four distinct regions of operation: two motor and two generator regions. In the motor regions, the torque and speed of the dynamo are in the same directions and power is being transferred out of the dynamo; however, in the generator regions [Welch Jr., R. H., 1997, Welch Jr. R. H., et. al., 1997] , they are in opposite

Figure 2.2 Graphical representation of steady state dynamo operation (electrical variables).

Figure 2.3 Graphical interpretation of steady state dynamo operation (mechanical variables)

directions and power is being absorbed by the dynamo. This corresponds to the sign convention which was chosen earlier for the dynamo model. Seth and Flowers [1990] refer to the motor regions as active quadrants and the generator regions as regenerative or passive operation regions in which the dynamo absorbs external mechanical power. If this absorbed power can be returned to the system as mechanical power then the process is called regeneration and the dynamo is referred to as a regenerative actuator [Seth and Flowers, 1990].

Other features of Figure 2.2 include a thin horizontal section which indicates the maximum current range of the motor. This range is typically dictated by the maximum thermal capacity of the dynamo representing a “safe” range of continuous operation without thermally damaging any internal components of the dynamo. This maximum current range is often small, sometimes 10-30% of the stall torque current for a dynamo operating at its nominal voltage. Within this thesis, this limit on the current and corresponding torque for safe, continuous dynamo operation will be referred to as the *maximum thermal current* and *maximum thermal torque*, respectively.

The motor and generator regions are separated by the theoretical stall torque line (ie. where $V = iR_a$) and the theoretical no load line (ie. where $V = E_g = K_t\omega$). The value of E_g can be determined anywhere on the graph by taking the difference between the terminal voltage and iR_a :

$$E_g = V - iR_a \quad (2-9)$$

The constant speed (ω) lines run parallel to the theoretical stall torque line ($\omega = 0$), with ω increasing in the indicated directions. Constant torque lines on Figure 2.2 correspond to horizontal lines which run parallel to the theoretical no load line.

Within the generator regions, there are two sub-regions labeled *Overhaul type I* and *Overhaul type II*. The Overhaul type I is also referred to as regenerative braking [Thorpe, 1989] or plugging [Heinzman, Seth and Turi, 1992]. The two sub-regions distinguish the two possible means in which a dynamo can operate as a generator. To highlight the difference between these two regions, first consider the dynamo operating somewhere in the motor regions. If the external opposing torque is increased (heavier load) and the terminal voltage is held constant, the operating point begins to move vertically. This means that the current in the dynamo has increased, causing the voltage drop across the resistor to increase and the speed of the dynamo shaft to decrease. Eventually, the operating point reaches the theoretical stall torque line where terminal voltage, V , equals the voltage drop across the resistor, iR_a , and the shaft of the dynamo has stopped. If the external load is increased even further, still holding terminal voltage constant, the operating point passes into the Overhaul type I region. In this case the speed, ω , of the dynamo shaft has changed direction, while the torque of the motor remains in the same direction. Since the torque and speed of the dynamo are now in opposite directions, mechanical power is now being transformed into electrical power and the dynamo is operating as a generator. In terms of the model in Figure 2.1, Overhaul type I corresponds to reversing the direction of ω and therefore the polarity on the back emf. Because the direction of the current remains the same, the terminal voltage and

back emf are equivalent to two voltage sources hooked in series. As a result, both the electrical source (voltage terminal) and the mechanical port are supplying positive power to the dynamo, all of which is dissipated by the resistor through heat. In terms of regeneration, this mode of operation is undesirable since none of the input power is being converted into reusable power, and as Seth and Flowers [1990] noted, a brake could be used to achieve the same operating point while expending less power from the dynamo terminals.

Again, consider the dynamo operating somewhere in the motor regions, but this time the external opposing torque is decreased, while the terminal voltage is held constant. In this case, the operating point will begin to move toward the no load line, since decreasing external opposing torque results in lower current in the dynamo and thus lower drops across the resistor. The speed of the motor shaft will continue to increase until the operating point reaches the theoretical no load line occurring when the external torque is zero. Now, if the external torque is applied in the same direction in which the shaft is moving, the dynamo torque will reverse direction to oppose the external torque and the shaft speed will increase beyond the theoretical no load speed for the given terminal voltage. In this case, the operating point moved into the Overhaul type II region. Since the dynamo torque and speed are again in opposite directions, the dynamo is absorbing external power and therefore is operating as a generator. So, in the case of Overhaul type II, the motor torque changes direction (not speed, as is the case of Overhaul type I), while the speed remains in the same direction. In terms of the model in Figure 2.1, Overhaul type II corresponds to reversing the direction of the motor torque and thus, the direction

of the current in the dynamo. In this circumstance, some of the power is being absorbed by the terminals and the remainder is being dissipated by the resistance of the dynamo, thereby highlighting the difference between Overhaul type I and II. In both cases, the external load is supplying power to the dynamo and therefore the dynamo is operating as a generator, but only in the Overhaul type II region is power being absorbed at the dynamo voltage terminals.

Figure 2.3 involves a transformation of variables from Figure 2.2. The axes in Figure 2.3 are the conventional mechanical variables, τ and ω . In this figure, quadrants I and III represent the regions in which the dynamo operates as a motor and quadrants II and IV are those in which the dynamo operates as a generator. As in Figure 2.2, the motor and generator regions are separated by the theoretical no load and stall torque lines which in this case, are the abscissa and ordinate of the graph. The constant terminal voltage lines of this figure are parallel to the line connecting the stall torque and no load speed of the dynamo. Each of these lines has a slope of $-\omega_{NL}/\tau_{stall}$ and assumes that, for any given terminal voltage, the speed of the dynamo decreases linearly with increasing torque. Constant current lines, though not shown, are horizontal lines corresponding to constant torque. The thin maximum torque range corresponds to the maximum current range from Figure 2.2, by (2-2). The Overhaul type I and II regions from Figure 2.2 are also indicated in Figure 2.3. As in Figure 2.2, Overhaul type I corresponds to the region between the theoretical stall torque line and the zero terminal voltage line, while the Overhaul type II region corresponds to the region between the no load line and the zero terminal voltage line.

It should be noted that (2-3) and (2-6), initially derived based on the motor model configuration in Figure 2.1, apply when the dynamo is operating as a motor or a generator. As long as the initial sign convention chosen for the model is obeyed, (2-3) and (2-6) represent general equations for the operation of a dynamo. Therefore, in terms of (2-3) and (2-6), Overhaul type I causes the sign on the back emf term, E_g , to reverse while Overhaul type II results in a sign reversal on the iR_a term (since the current direction reverses).

It should be emphasized that both figures represent the motor performance only at steady state. When considering the effects of moving the operating point on either curve, the transient voltage loss across the inductor, L_a , must be considered. Though Figures 2.2 and 2.3 can be used to portray dynamo operation during transients, the dynamo operating point may momentarily cross boundaries in the graphs due to transient effects. As mentioned earlier, the time constants for small DC motors are typically very fast, and therefore the dynamo tends to reach steady state very quickly.

In Chapter 4, the graphical presentations of dynamos in Figures 2.2 and 2.3 will be extended to dynamos coupled to backdrivable and nonbackdrivable transmissions.

Chapter 3

Nonbackdrivability and Efficiency of Gears

An important feature of the proposed variable stiffness upper arm prostheses (both Configuration 1 and 3) are their non-conventional transmissions. Nonbackdrivable gearing is required at the motors, but any gearing between the prosthetic arm and the quadratic springs must be backdrivable. Before any gear selection was made for the prosthesis designs, a study on backdrivable versus nonbackdrivable gears was necessary. The reader is encouraged to review the author's definitions of nonbackdrivable (Def. 1-1), nonbackdrivable gears (Def. 1-2), and nonbackdrivable transmission (Def. 1-3) before proceeding. In this chapter and later chapters, the terms worm and worm gear are used frequently. Within this thesis, *worm* will refer to the "screw"-like gear and the *worm gear* will refer to the helical-type gear of the worm gear set.

Typically, non-intersecting, non-parallel axes gear arrangements, such as a worm and worm gear, are associated with nonbackdrivable transmissions. It is common knowledge, though, that even simple parallel gear arrangements (eg. spur gears) with a single stage, high gear ratio can become nonbackdrivable. This topic of nonbackdrivability (single stage, high gear ratio gears) was not found in the reviewed gear literature, while that of

non-intersecting, non-parallel axes gears is commonly discussed [Buckingham, 1949; Davis, 1970; Dudley, 1984, 1991; Juvinall, Marshek, 1991; Merrit, 1971].

A primary concern with nonbackdrivable gear trains is their low efficiencies, often less than 50%. The design of nonbackdrivable gear sets usually results in high sliding velocities and coefficients of friction and thus, high power losses. To understand the relationships between friction coefficients, gear ratio, nonbackdrivability, and efficiency, a study of several common gear types was conducted. This study examined the nonbackdrivable conditions of parallel axes as well as intersecting and non-intersecting, non-parallel axes gear trains.

A working knowledge of gears and gear terminology has been assumed in the following analysis. A brief review of basic gear terminology and fundamentals can be found in the in any of these gear references [Buckingham, 1949; Davis, 1970; Dudley, 1984, 1991; Juvinall, Marshek, 1991; Merrit, 1971].

3.1 Parallel Axes Gears

This review begins with the study of parallel axes gears such as spur and helical gears. The issues of nonbackdrivability are first addressed, followed by the efficiencies of these gear sets.

3.1.1 Nonbackdrivability

The two common types of parallel axes gears are spur and helical (which includes crossed helical or herringbone). The most simple arrangement of these gear types is a meshing pinion and gear. Figure 3.1 exhibits such an arrangement with the base, pitch, and outside radius of both pinion and gear labeled. The path of contact is a locus of all points of contact of mating teeth. Only with the involute profile does this coincide with the tangent to the pinion and gear base circles, passing through the pitch point. The nominal pressure angle, ϕ , of the gear train is the angle enclosed by path of contact and common tangent to the pinion and gear pitch circles. The maximum path of contact which provides no interference or undercutting is the line connecting the base circle tangency points A and B. Note that the angle between the center line (connecting OP and OG) and the base circle radius drawn to the tangency points A and B is theoretically equal to the pressure angle, ϕ . The path of contact is typically divided into two regions, approach and recess. The approach is the line segment where the conjugate teeth first come in contact to the pitch point, and the recess is the line segment from the pitch point to where the teeth disengage. It is convenient to divide the path of contact in this fashion, since the direction of relative sliding velocity (and thus the frictional force) changes direction on either side of the pitch point.

Since we are concerned with the nonbackdrivable nature of spur gears, the following analysis will assume that the gear is driving the pinion, as indicated in Figure 3.1 by having the torque and angular velocity on the gear in the same direction. Under these conditions, the segment AP represents the maximum approach path and PB is the

Figure 3.1 Schematic of meshed spur gears

maximum recess path (assuming no interference between gears). Based on dimensions of the gears in Figure 3.1, the conjugate gear teeth first engage at C and disengage at D.

Based on these two points, both Buckingham [1949] and Dudley [1991] define an arc of approach and recess for the driver (the gear in this case) and the follower (the pinion in this case) as:

Driver (Gear):

$$\beta_{aG} = \frac{CP}{r_{bG}} = \frac{\sqrt{r_{op}^2 - r_{bp}^2} - r_p \sin(\phi)}{r_{bG}} \quad (3-1)$$

$$\beta_{rG} = \frac{PD}{r_{bG}} = \frac{\sqrt{r_{oG}^2 - r_{bG}^2} - r_G \sin(\phi)}{r_{bG}}$$

Follower (pinion):

$$\beta_{ap} = \frac{CP}{r_{bp}} = \frac{\sqrt{r_{op}^2 - r_{bp}^2} - r_p \sin(\phi)}{r_{bp}} \quad (3-2)$$

$$\beta_{rp} = \frac{PD}{r_{bp}} = \frac{\sqrt{r_{oG}^2 - r_{bG}^2} - r_G \sin(\phi)}{r_{bp}}$$

where r_{op} , r_{bp} , and r_p are the outside, base and pitch radii of the pinion and r_{oG} , r_{bG} , and r_G are the outside, base, and pitch radii of the gear. Based on the definitions in (3-1) and (3-2)... it is obvious that:

$$\frac{\beta_{ap}}{\beta_{aG}} = \frac{\beta_{rp}}{\beta_{rG}} = \frac{r_{bG}}{r_{bp}} = GR \quad (3-3)$$

where GR is the gear ratio of the mating gears. From the geometry in Figure 3.1, the following definitions for the maximum approach angle and maximum recess angle can be made:

Approach:

$$\beta_{aG_{max}} = \frac{1}{GR} \beta_{ap_{max}} = \frac{AP}{r_{bG}} = \tan(\phi) \quad (3-4)$$

Recess:

$$GR \beta_{rG_{max}} = \beta_{rp_{max}} = \frac{PB}{r_{bp}} = \tan(\phi) \quad (3-5)$$

The author defines the ratio of the actual approach (CP) and recess (PD) path to the maximum approach (AP) and maximum recess (PB) path as:

Approach:
$$\eta_a = \frac{CP}{AP} = \frac{\beta_{aG}}{\beta_{aG_{max}}} = \frac{\beta_{ap}}{\beta_{ap_{max}}} \quad 0 \leq \eta_a \leq 1 \quad (3-6)$$

Recess:
$$\eta_r = \frac{PD}{PB} = \frac{\beta_{rG}}{\beta_{rG_{max}}} = \frac{\beta_{rp}}{\beta_{rp_{max}}} \quad 0 \leq \eta_r \leq 1 \quad (3-7)$$

where η_a and η_r equal unity when the approach and recess angle are equivalent to the maximum values listed in (3-4) and (3-5). With the definitions given in (3-1) through (3-7), a static analysis was conducted to determine the influence of gear ratio, friction coefficient and arc lengths on the nonbackdrivability of spur gears. To begin, a sum of the external torques acting on the pinion and gear at any arbitrary point on the approach was determined:

$$\tau_{aG} = Nr_{bG} [1 - \mu(\tan(\phi) - \eta_a \beta_{aG_{max}})] \quad (3-8)$$

$$\tau_{ap} = Nr_{bp} [1 - \mu(\tan(\phi) + \eta_a \beta_{ap_{max}})] \quad (3-9)$$

where τ_{aG} and τ_{ap} are the applied torque on the gear and pinion during the approach, respectively, N and μ are the normal force and coefficient of friction respectively, and the arcs of approach are defined using the convention in (3-6). The force of friction is perpendicular to the path of contact and is in the same direction as the applied torque on the pinion and gear during approach. Next, the sum of the torques applied to the pinion and gear at any arbitrary point on the recess path was determined:

$$\tau_{rG} = Nr_{bG} [1 + \mu(\tan(\phi) + \eta_r \beta_{rG_{max}})] \quad (3-10)$$

$$\tau_{rp} = Nr_{bp} [1 + \mu(\tan(\phi) - \eta_r \beta_{rp_{max}})] \quad (3-11)$$

where τ_{rG} and τ_{rp} are the applied torque of the gear and pinion during the recess path respectively, and the arcs of recess are defined using the convention in (3-7). The frictional force changes direction after the mating teeth pass through the pitch point, and it now opposes the applied torques during recess. The ratio of torques acting on the pinion and gear during approach and recess was then evaluated:

$$\frac{\tau_{ap}}{\tau_{aG}} = \frac{1}{GR} \frac{[1 - \mu(\tan(\phi) + \eta_a \beta_{ap_{max}})]}{[1 - \mu(\tan(\phi) - \eta_a \beta_{aG_{max}})]} \quad (3-12)$$

$$\frac{\tau_{rp}}{\tau_{rG}} = \frac{1}{GR} \frac{[1 + \mu(\tan(\phi) - \eta_r \beta_{rp_{max}})]}{[1 + \mu(\tan(\phi) + \eta_r \beta_{rG_{max}})]} \quad (3-13)$$

The gears are nonbackdrivable when either of these ratios becomes zero or negative (indicating that torque on the pinion had reversed direction). This is not possible in (3-13) when the gear teeth are in recess since this would require:

$$\tan(\phi) - \eta_r \beta_{rp_{max}} < 0 \quad (3-14)$$

Substituting the definition from (3-5) into (3-14) and simplifying gives:

$$\tan(\phi)(1 - \eta_r) < 0 \quad (3-15)$$

Since $\tan(\phi)$ is always positive, (3-15) would require η_r to be greater than unity in order to be true. This contradicts the definition of η_r in (3-7), and therefore the torque ratio of the gears in recess in (3-13) can never be zero or negative. This indicates that contact between mating gear teeth of parallel axes gears during recess can never cause the gear set to become nonbackdrivable. However, contact between conjugate gear teeth during approach can result in the gear set becoming nonbackdrivable. This situation is now considered.

The torque ratio of the gears in approach given in (3-12), can equal zero or become negative when:

$$\mu \geq \frac{1}{\tan(\phi) + \eta_a \beta_{ap_{\max}}} = \frac{1}{\tan(\phi) + GR \eta_a \beta_{aG_{\max}}} = \frac{1}{\tan(\phi)(1 + \eta_a GR)} \quad (3-16)$$

Figure 3.2 graphically depicts (3-16) in which friction coefficient is plotted versus gear ratio for a 20° pressure angle and varying values of η_a . If η_a is known for a given gear set, then any friction coefficient which lies above the corresponding η_a line will cause the gear set to become nonbackdrivable. Figure 3.2 can also be used as a design tool if GR and the friction coefficient are known then the dimensions of the mating gears can be designed to provide a value of η_a that will either insure backdrivability or nonbackdrivability. In a general sense, gear pairs with lower values of η_a are less likely to become nonbackdrivable. However, this requires a reduction in contact ratio between the gear teeth which is typically undesirable. Figure 3.2 also demonstrates why single stage, high gear ratio, parallel gear sets tend to become nonbackdrivable even with low coefficients of friction. Figure 3.3 depicts (3-16) for a constant value of $\eta_a = 0.6$ with varying pressure angles. Notice that for a given gear ratio, the required friction coefficient to cause the gear pair to become nonbackdrivable decreases with increasing pressure angle.

The fact that parallel axis gears are prone to being nonbackdrivable during approach is of particular importance since the friction coefficient is typically higher in approach than in recess [Buckingham, 1949]. Caution should therefore be used when designing a backdrivable parallel axes gear set based on an average coefficient of friction (eg. from

Figure 3.2 Spur gear driving pinion--Nonbackdrivable conditions ($\phi=20^\circ$).

Figure 3.3 Spur gear driving pinion--Nonbackdrivable conditions (variable ϕ)

experimental results) since this may result in a nonbackdrivable gear set due to the coefficient of friction in approach being larger than the average value

Performing the same static analysis using the pinion as the driving member and the gear as the follower for comparison reveals that the gear pair can lock during approach only with extreme values of friction coefficients given by:

$$\mu \geq \frac{1}{\tan(\phi) + \eta_a \beta_{Ga_{max}}} = \frac{1}{\tan(\phi) + \frac{\eta_a}{GR} \beta_{pa_{max}}} = \frac{1}{\tan(\phi) \left(1 + \frac{\eta_a}{GR} \right)} \quad (3-17)$$

Figure 3.4 depicts the results in (3-18) for a 20° pressure angle and varying values of η_a .

Again, for a given gear ratio and value of η_a , coefficients of friction that lie above this line indicate that the gear set is nonbackdrivable. It is apparent from Figure 3.4 that only under extreme coefficients of friction would the gear set ever lock in this direction.

However, if the friction coefficient is high enough to cause the gear to "lock" according to (3-17), then the *entire* gear set can become "locked" with neither the gear nor pinion being able to turn. This is due to the fact that extreme coefficients of friction which satisfy (3-17) will also satisfy (3-16) for most values of η_a . Under this special circumstance, the gear set is not considered nonbackdrivable according to (Def. 1.1) since power cannot flow in either direction.

Since arcs of approach and recess are calculated in an identical fashion for spur and helical gears, and since the 'locking' conditions in (3-16) and (3-17) are independent of helix angle, ψ , the nonbackdrivable conditions in (3-16) and (3-17) apply to both spur and helical gears.

Figure 3.4 Pinion driving gear--Nonbackdrivable conditions ($\phi=20^\circ$)

3.1.2 Multiple Gear Stages

Up to this point, the discussion of nonbackdrivability for parallel axes gears has been restricted to a single gear set. The following is a brief discussion related to the nonbackdrivability of transmissions consisting of multiple gear states.

As stated in Section 1.2, if a transmission consists of multiple gear sets of which one is nonbackdrivable, then the entire transmission is nonbackdrivable. Equation (3-16) and Figure 3.2 indicate that, for a given value of η_a and friction coefficient, a gear set is more likely to become nonbackdrivable as its gear ratio increases. Therefore, to minimize the possibility of a transmission becoming nonbackdrivable, the number of gear stages within the transmission should be maximized and the total transmission gear ratio should be distributed evenly amongst the gear stages. Both of these measures minimize the gear ratio of all the gear sets within the transmission, and thus reduce the risk of any of these gear sets being nonbackdrivable. On the other hand, if a nonbackdrivable transmission is required then the number of gear stages should be minimized and the total transmission gear ratio should be placed on one gear set within the transmission. These "rules of thumb" assume that each gear set has similar properties (ie. η_a and coefficients of friction).

For example, if a transmission requires a 1000:1 gear ratio and the maximum number of gear stages that can be used is three, then the gear ratios of each of the three stages should be 10:1 to minimize the possibility of the transmission being nonbackdrivable.

Conversely, to maximize the possibility of the transmission being nonbackdrivable, a single gear stage with a gear ratio of 1000:1 should be used.

In summary, based on this analysis, to minimize the possibility of a transmission consisting of parallel gear sets being nonbackdrivable, the following steps should be taken:

- minimize the coefficient of friction between conjugate teeth.
- use multiple gear stages when high gear ratios are required.
- minimize the pressure angle.
- minimize approach arc ratio (η_a), though this results in a lower contact ratio.

3.1.3 Spur Gear Efficiency

Buckingham [1949] and Dudley [1991] both derive the efficiencies of parallel axes gears by integrating (3-8) - (3-11) over the entire path of contact. If the coefficient of friction can be assumed constant over the entire approach and recess then both Buckingham and Dudley show that the efficiency of the pinion driving the gear is given by:

$$e_p = 1 - \left[\frac{1 + \frac{1}{GR}}{\beta_{ap} + \beta_{rp}} \right] \frac{\mu}{2} (\beta_{ap}^2 + \beta_{rp}^2) \quad (3-18)$$

where e_p represents the efficiency of the pinion driving the gear. This equation of efficiency can also be used when the gear is driving the pinion, as long as the approach and recess arcs of the gear are used and the GR is inverted (eg. for a 10:1, $GR = 0.1$ when the gear is driving the pinion). The assumption of constant coefficient of friction is

inaccurate since, as mentioned earlier, the coefficient is typically lower during recess than approach in which case the efficiency is defined using average coefficients over the approach (μ_a) and recess (μ_r):

$$e_p = 1 - \left[\frac{1 + \frac{1}{GR}}{\beta_{ap} + \beta_{rp}} \right] \left(\frac{\mu_a}{2} \beta_{ap}^2 + \frac{\mu_r}{2} \beta_{rp}^2 \right) \quad (3-19)$$

This expression also applies to the gear driving the pinion as long as the changes mentioned earlier are made.

3.1.4 Helical Gear Efficiency

In a similar fashion, Buckingham [1949] and Dudley [1991] derived the efficiencies for helical gears, which differ from (3-18) and (3-19) by a single term which compensates for the helix angle, ψ , and normal pressure angle, ϕ_n . Again, if the coefficient of friction is considered constant:

$$e_p = 1 - \left[\frac{\cos(\phi)}{\cos(\phi_n) \cos(\psi)} \right] \left[\frac{1 + \frac{1}{GR}}{\beta_{ap} + \beta_{rp}} \right] \frac{\mu}{2} (\beta_{ap}^2 + \beta_{rp}^2) \quad (3-20)$$

and if the average value of the coefficient of friction on the approach and recess is used then the efficiency is defined by:

$$e_p = 1 - \left[\frac{\cos(\phi)}{\cos(\phi_n) \cos(\psi)} \right] \left[\frac{1 - \frac{1}{GR}}{\beta_{ap} + \beta_{rp}} \right] \left(\frac{\mu_a}{2} \beta_{ap}^2 + \frac{\mu_r}{2} \beta_{rp}^2 \right) \quad (3-21)$$

As in the case of spur gears, (3-20) and (3-21) can be used to determine the efficiency of the gear driving the pinion as long as the gear's arc of approach and recess is used and GR is inverted.

3.2 Non-intersecting, Non-parallel Axes Gear Model

The remainder of this study involves the investigation of the nonbackdrivability and efficiency of non-intersecting, non-parallel axes gears. First, a simple model is introduced in this section which is used to develop the non-intersecting, non-parallel axes gear dynamic model later in this thesis. The results from this study of the simple model are then applied specifically to worm and worm gears.

3.2.1 Nonbackdrivability

The issues of nonbackdrivability and efficiency of non-intersecting, non-parallel axes gears can be investigated by using a simple model, similar to that presented by Mansfield [1988], as seen in Figure 3.5. In his thesis, Mansfield used a similar model to determine the efficiencies and backdrivable conditions for several non-intersecting, non-parallel axes gear types but his presentation did not include any of the force diagrams nor the derivation of the nonbackdrivable conditions presented in this thesis.

For simplicity, the normal pressure angle in the model in Figure 3.5 is assumed to be zero and two consecutive teeth of the pinion surround a single tooth of the gear. The lead angle of the pinion is λ and the positive directions of pinion and gear forces and velocities are shown. The pinion and gear teeth roll on frictionless bearings, though

Figure 3.5 Non-intersecting, non-parallel axes gear model.

friction exists on the faces of the meshed gear teeth. Depending on the direction of the external force, the tooth of the gear will be in contact with one of the pinion teeth, but never both at the same time due to backlash. Based on this restriction and with the given symmetry of the model, the fundamental equations for nonbackdrivability and efficiency can be generated using only one of the pinion and gear teeth interactions.

Based on (Def. 1-2), nonbackdrivable gears are defined as a gear set whose interface between mating gear teeth is considered nonbackdrivable (based on (Def. 1-1)). Thus, to develop the nonbackdrivable criteria for this gear model, the dynamics at the interface between the mating gear teeth must first be examined. To do so, a "reduced" model of Figure 3.5 is presented in Figure 3.6 (a). This model is conceptually the same as that in Figure 3.5 except the mass of the pinion (m_p) and gear (m_G) are now represented as "lumped" masses connected to massless "wedges" by infinitely rigid links. The contacting massless wedges, in this model, represent the interface between the mating gear teeth. Figure 3.6 (b) depicts a free body diagram of only the mating wedges. The forces applied to the wedges in Figure 3.6 (b) are referred to in this thesis as the *massless* forces and are defined as:

$$F_{p,ml} = F_p - m_p a_p \quad (3-22)$$

$$F_{G,ml} = F_G - m_G a_G \quad (3-23)$$

where F_p and F_G are the external forces applied to the pinion and gear, respectively, and a_p and a_G are the accelerations of the pinion and gear, respectively. It is important to note that when the system is operating in steady state (e.g. $a_p = a_G = 0$), the massless force and external force of the pinion and gear are equal. This fact is used later in this chapter and

(a)

(b)

Figure 3.6 (a) Reduced non-intersecting, nonparallel axes gear model, (b) free body diagram of massless gear teeth interface.

in subsequent chapters in this thesis. Having established a model for the "massless" interface between mating gear teeth (Figure 3.6 (b)), the next step was to use it to establish the criteria for nonbackdrivable gears.

Figure 3.7 (a) and (b) presents four free body diagrams of the massless pinion wedge in equilibrium under four unique loading conditions. In this discussion, the massless pinion wedge will simply be referred to as the pinion. In each of the four scenarios, the input massless force applied to the pinion, $F_{p,ml}$, and the lead angle, λ , are constant. The other forces represented in the vector diagrams are the normal force, N , the frictional force, F_f , and the reaction force, $F_{G,ml}$, which is equal to the massless force applied to the gear but in the opposite direction. The angle enclosed by the normal and frictional force vectors, referred to here as the friction angle, is defined as:

$$\phi = \tan^{-1} \left(\frac{F_f}{N} \right) \quad (3-24)$$

If the system is in static equilibrium, the maximum friction angle is given by:

$$\phi = \tan^{-1} \left(\frac{\mu_s \cdot N}{N} \right) = \tan^{-1}(\mu_s) \quad (3-25)$$

where μ_s is the static coefficient of friction. Similarly, if the system is in motion, the friction angle is given by:

$$\phi = \tan^{-1} \left(\frac{\mu_k \cdot N}{N} \right) = \tan^{-1}(\mu_k) \quad (3-26)$$

where μ_k is the kinetic coefficient of friction.

Scenario 1 **Scenario 2**

Figure 3.7 (a) Free body diagrams of the pinion tooth in equilibrium (scenarios 1 and 2).

Figure 3.7 (b) Free body diagrams of the pinion tooth in equilibrium (scenarios 3 and 4).

Each of the four scenarios shows a velocity vector that represents the actual (kinetic) or pending (static) direction of motion. The direction of the frictional force is drawn such that it opposes motion. Scenarios 1 and 2 indicate the actual or pending motion of the pinion is to the right (positive direction) while scenarios 3 and 4 have the actual or pending motion of the pinion directed to the left (negative direction). The following discussion uses the power flow across the interface between the pinion and gear wedges, and the definition of nonbackdrivable given in (Def. 1-1), to establish the nonbackdrivable criteria for non-intersecting, non-parallel axes gear sets. Though this discussion is based on the assumption that the system is in motion, the nonbackdrivable criteria can be extended to the static case where motion is pending. As mentioned in Section 1.2, power flow across the interface of mating gear teeth of a nonbackdrivable gear set is only possible in one direction, however, power flow across the interface between mating gear teeth of a backdrivable gear set is possible in both directions. In the following development, it will be assumed that power can always flow across the interface from the pinion to the gear for both backdrivable and nonbackdrivable gear sets. However, power can only flow across the interface from the gear to the pinion in the case of a backdrivable gear set.

In scenario 1, the velocity vector and the massless force on the pinion are in the same direction indicating that power is flowing from the pinion to the gear (since the force and velocity vectors on the gear are in opposite directions). This scenario is only possible when:

$$\begin{aligned}
\lambda + \varphi &< 90^\circ \\
\lambda &< 90^\circ - \varphi \\
\tan(\lambda) &< \frac{1}{\mu} \\
\frac{1}{\tan(\lambda)} &> \mu
\end{aligned} \tag{3-27}$$

Based on the above assumption that power can always flow across the interface from the pinion to the gear for both a backdrivable and nonbackdrivable gear set, the restriction in (3-27) is satisfied by both types of gear sets at all times.

In scenario 2, the direction of all the forces on the pinion are the same as scenario 1, except for the reaction force, $F_{G.mt}$. In this scenario, the coefficient of kinetic friction is high (resulting in a large friction angle, φ) and as a result the massless force on the gear must be in the same direction of motion in order for the system to be in equilibrium. In this case, no power is being transferred across the interface in either direction. Instead possitive input power from both the pinion and gear is being dissipated by friction between the mating gear teeth. This scenario only occurs when:

$$\begin{aligned}
\lambda + \varphi &> 90^\circ \\
\lambda &> 90^\circ - \varphi \\
\tan(\lambda) &> \frac{1}{\mu} \\
\frac{1}{\tan(\lambda)} &< \mu
\end{aligned} \tag{3-28}$$

This contradicts the restriction from (3-27), so as long as (3-27) is satisfied scenario 2 is never possible.

Scenario 3 is similar to scenario 1, except the frictional force has changed direction due to the change in direction of the pinion velocity. In this scenario, power is being transferred across the interface from the gear to the pinion. As a result, the massless force on the pinion and its velocity are in opposite directions. Based on earlier assumptions, this scenario is only possible for a backdrivable gear set. From the force polygon, scenario 3 is only possible when:

$$\begin{aligned}\lambda &> \varphi \\ \tan(\lambda) &> \mu\end{aligned}\tag{3-29}$$

Therefore, for the gear set to be nonbackdrivable:

$$\begin{aligned}\lambda &< \varphi \\ \tan(\lambda) &< \mu\end{aligned}\tag{3-30}$$

In scenario 4, as in scenario 2, both the pinion and gear are supplying positive power. As a result, no power is being transferred across the interface and all the input power from both the pinion and gear is being dissipated by friction between the mating teeth. From the force polygon, this scenario is possible when:

$$\begin{aligned}\lambda &< \varphi \\ \tan(\lambda) &< \mu\end{aligned}\tag{3-31}$$

This is the same restriction as (3-30), and therefore, scenario 4 occurs only in a nonbackdrivable system. This scenario highlights the fact that in a nonbackdrivable gear set, power cannot flow across the interface from the gear to the pinion and thus the power flow is restricted to only one direction. This also indicates that in order for nonbackdrivable gears to move in any direction, the pinion must *always* input positive power.

As mentioned earlier, the criteria developed in (3-27) to (3-31) are valid for both kinetic and static equilibrium conditions, even though they were developed assuming a kinetic state. In this sense, the gear set model (Figure 3.5) is considered nonbackdrivable when both:

$$\tan(\lambda) < \mu_s < \frac{1}{\tan(\lambda)} \text{ and } \tan(\lambda) < \mu_k < \frac{1}{\tan(\lambda)} \quad (3-32)$$

and similarly, it is a backdrivable gear set when:

$$\tan(\lambda) > \mu_s < \frac{1}{\tan(\lambda)} \text{ and } \tan(\lambda) > \mu_k < \frac{1}{\tan(\lambda)} \quad (3-33)$$

It is important to note that based on the nonbackdrivable definition in (Def. 1-1), both the static and kinetic coefficients of friction between the mating gear teeth must satisfy (3-32) in order for the gear set to be considered nonbackdrivable under both static and kinetic conditions. In most cases, though, the static and kinetic (kinetic is typically lower than static) coefficients of friction are not equal in which case it is possible to have the static coefficient satisfy (3-32) while the kinetic coefficient satisfies (3-33) such that:

$$\tan(\lambda) < \mu_s < \frac{1}{\tan(\lambda)} \text{ but } \tan(\lambda) > \mu_k < \frac{1}{\tan(\lambda)} \quad (3-34)$$

This situation is worth examining in some detail. Under these conditions, the interface between the mating gear teeth is considered nonbackdrivable based on its static coefficient of friction (eg. power can flow in only one direction) however, it is considered backdrivable (eg. power can flow in both directions) based on its kinetic coefficient of friction. In this sense, while the gear set is in motion power can flow in both directions across the gear teeth mesh but from rest, the gear cannot transfer power through the teeth mesh to start the system in motion. A gear set whose coefficients of static and kinetic

coefficients satisfy (3-34) will be referred to in this thesis as *partially* nonbackdrivable gears. Table 3.1 summarizes the conditions from (3-32) to (3-34).

Table 3.1 Backdrivable and nonbackdrivable conditions for the non-intersecting, non-parallel axes gear model.

Gear Set Characterization	Coefficients of friction (μ) and Lead Angle (λ) Requirements	
Backdrivable	$\tan(\lambda) > \mu_s < \frac{1}{\tan(\lambda)}$	$\tan(\lambda) > \mu_k < \frac{1}{\tan(\lambda)}$
Nonbackdrivable	$\tan(\lambda) < \mu_s < \frac{1}{\tan(\lambda)}$	$\tan(\lambda) < \mu_k < \frac{1}{\tan(\lambda)}$
<i>Partially</i> Nonbackdrivable	$\tan(\lambda) < \mu_s < \frac{1}{\tan(\lambda)}$	$\tan(\lambda) > \mu_k < \frac{1}{\tan(\lambda)}$

As mentioned earlier, in order for a nonbackdrivable set of gears to move in any direction positive power must be supplied by the pinion. In other words, the massless force applied to the pinion and the velocity of the pinion must *always* be in the same direction in the case of nonbackdrivable gears. This, however, is not always the case for the external force applied to the pinion and the pinion velocity. To help clarify this issue, a brief discussion on the unsteady state operation of a nonbackdrivable gear set is now presented.

3.2.2 Unsteady State Operation of Nonbackdrivable Gears

Under steady state conditions, the external force and the massless force applied to the pinion are equal. This, however, is not true during unsteady state conditions when the pinion is accelerating or decelerating. When the pinion deceleration rate becomes large, the massless and external force can be in opposing directions. To illustrate this effects of unsteady state operation, Figure 3.8 presents three free body diagrams and force polygons

Figure 3.8 Free body diagrams of the pinion tooth in unsteady state conditions.

of the pinion (including its mass) during accelerating and decelerating conditions. Let's assume that this particular gear set is nonbackdrivable according to the criteria specified in Table 3.1. In all three cases shown in Figure 3.8 the direction of the normal force, frictional force, and massless gear reaction force ($F_{G,ml}$) are held constant as well as the pinion velocity vector. In Figure 3.8 (a), the pinion is accelerating and its external force and velocity vector are in the same direction indicating that some of the positive input power is increasing the momentum of the pinion and some is being transferred across the interface to the gear. In Figure 3.8 (b), the pinion is decelerating (losing energy) but the external force on the pinion and its velocity are still in the same direction. This indicates that both the positive input power to the pinion and the power loss from its decreasing momentum is flowing across the interface to the gear. Notice that in both (a) and (b) the external and massless force of the pinion are in the same direction. The case shown in Figure 3.8 (c), however, depicts a decelerating pinion and in this case the external force on the pinion its velocity are now in opposing directions. At first glance, this may appear to indicate that power is flowing across the interface from the gear to the pinion and thus violating the definition of nonbackdrivable gears in (Def. 1-2). This, of course, is not the case. In Figure 3.8 (c), as the pinion decelerates rapidly, its stored energy is being released quickly. As a result, some of this released power is flowing across the interface to the gear (indicated by the fact that the massless force and pinion velocity are in the same direction) and the remainder is being absorbed by the external pinion load (indicated by the opposing external force and pinion velocity vectors). This indicates that the power flowing out of the pinion can only come from energy stored in the pinion inertia alone, and not from power flowing across the interface from the gear. This

situation occurred frequently in the simulations in Chapters 6 and 7 and is discussed in further detail at that time.

3.2.3 Non-Intersecting, Non-parallel Axes Gear Model Efficiency

The efficiencies of the three scenarios above can be determined by dividing the output power by the input power. First, the relationship between the pinion velocity, v_p , and the gear velocity, v_G , is given by:

$$\frac{v_p}{v_G} = \frac{1}{\tan(\lambda)} \quad (3-35)$$

This relationship indicates that $1/\tan(\lambda)$ is the effective gear ratio between the pinion and gear. Hence the efficiency of scenario 1, is the ratio of the output gear power to input pinion power given by:

$$e_p = \frac{|F_{G.mt} v_G|}{|F_{p.mt} v_p|} = \frac{\tan(\lambda)}{\tan(\lambda + \phi)} = \frac{1 - \mu \tan(\lambda)}{1 + \mu \cot(\lambda)} \quad (3-36)$$

This equation can be represented graphically as seen in Figure 3.9, where efficiency is plotted against the lead angle, λ . The coefficient of friction can vary widely in such transmissions, but typical values of μ (static and kinetic) range between 0.01 and 0.15 [Juvinall, Marshek, 1991]. For this reason, the efficiency curves of μ values between 0 and 0.3 are shown in Figure 3.9. Also depicted in this figure is the borderline $\mu = \tan(\lambda)$ that separates the nonbackdrivable and backdrivable gear regions. The region under this line represent nonbackdrivable conditions (where $\mu > \tan(\lambda)$) and above this line is the backdrivable region (where $\mu < \tan(\lambda)$). It is important to note that the nonbackdrivable region has a maximum efficiency of 0.5 or 50% while a backdrivable transmission can

Figure 3.9 Efficiency vs. lead angle for the non-intersecting, non-parallel gear model.

theoretically reach 100% efficiency. This low efficiency is an undesirable property of nonbackdrivable gears and is considered in more detail in Chapter 6. Transmissions that must remain either backdrivable or nonbackdrivable should operate at a safe distance from the borderline area since crossing into the opposing region could have detrimental effects (eg.. a nonbackdrivable gear set unexpectedly becoming backdrivable).

The efficiency for scenario 3 is given by:

$$e_G = \left| \frac{F_{G.ml} v_G}{F_{P.ml} v_P} \right| = \frac{\tan(\lambda - \phi)}{\tan(\lambda)} = \frac{1 - \mu \cot(\lambda)}{1 + \mu \tan(\lambda)} \quad (3-37)$$

where e_G represents the efficiency of the gear driving the pinion. An efficiency for scenario 4 cannot be defined since both the pinion and gear are inputting power.

As expected, the efficiency for both scenario 1 and 3 decreases with increasing friction coefficient. These gear efficiencies should be taken into consideration when selecting the type of gearing required for an application (backdrivable versus nonbackdrivable).

3.2.4 Force Transmission Ratios

Referring back to the kinetic steady state conditions shown in Figure 3.7, it is interesting to note that the backdrivable and nonbackdrivable gear sets differ in what the author refers to as the *force transmission ratios* (FTR): the relative magnitude of $F_{p.ml}$ to $F_{G.ml}$ required to maintain equilibrium. The value of the FTR in scenario 1, common to both backdrivable and nonbackdrivable gear sets, is:

$$\frac{F_{p.ml}}{F_{G.ml}} = -\tan(\lambda + \phi) = \frac{-1}{G \operatorname{Re}_p} \quad (3-38)$$

where $GR = 1/\tan(\lambda)$ and e_p is defined in (3-36). The negative sign implies that the force applied to the gear is in the negative direction based on the sign convention shown in Figure 3.5. Under steady state conditions, the external forces are equal to the massless forces and therefore the FTR in (3-38) also applies:

$$\left. \frac{F_p}{F_G} \right|_{ss} = -\tan(\lambda + \varphi) = \frac{-1}{G Re_p} \quad (3-39)$$

In scenario 3, the FTR for the massless forces, as well as the external forces under steady state conditions, is given by:

$$\frac{F_{p,ml}}{F_{G,ml}} = \left. \frac{F_p}{F_G} \right|_{ss} = -\tan(\lambda - \varphi) = \frac{-1}{G Re_G} \quad (3-40)$$

where $(\lambda - \varphi) > 0$, from (3-30), and e_G is defined by (3-37). The FTR of the massless forces, as well as the external forces under steady state conditions, for the nonbackdrivable gear set in scenario 4 is:

$$\frac{F_{p,ml}}{F_{G,ml}} = \left. \frac{F_p}{F_G} \right|_{ss} = \tan(\lambda - \varphi) = \frac{1}{G Re_G} \quad (3-41)$$

where $(\lambda - \varphi) < 0$, from (3-31). Notice that this ratio, unlike the two previous, is positive indicating that both the pinion and gear forces are applied in the same direction.

Equations (3-40) and (3-41) indicate that as the absolute difference between λ and φ approaches 90° , for a given $F_{G,ml} \neq 0$, the required $F_{p,ml}$ for equilibrium approaches infinity.

The static coefficient of friction is typically one third higher than kinetic coefficients [Juvinall, Marshek, 1991] so the static friction angle is greater than the kinetic friction

angle. As a result the force required to overcome static friction and start the transmission moving is greater than those required to maintain dynamic equilibrium described in (3-38)-(3-41).

3.3 Non-intersecting, Non-parallel Axes Gears - Worm and Worm Gear

3.3.1 Nonbackdrivability

The conditions of nonbackdrivability of the model can easily be extended to cover worm and worm gears. In the above model, the pressure angle was assumed to be zero, which is not the case in real gears. In order to apply the conditions of nonbackdrivability from the model to an actual worm and worm gear, it must first be recognized that:

$$N_{\text{worm}} = N \cos(\phi_n) \quad (3-42)$$

where N is the normal force from the model, N_{worm} is the actual normal component of force on the worm and worm gear teeth, and ϕ_n is the normal pressure angle measured perpendicular to the gear teeth. Substituting (3-42) into the equations derived for the model results in a new friction angle:

$$\phi_{\text{worm}} = \tan^{-1} \left(\frac{\mu N_{\text{worm}}}{N_{\text{worm}} \cos(\phi_n)} \right) = \tan^{-1} \left(\frac{\mu}{\cos(\phi_n)} \right) \quad (3-43)$$

where μ represents both static and kinetic coefficients of friction. Substituting this friction angle into the backdrivable, (3-33), and nonbackdrivable, (3-32), conditions of the model results in a backdrivable condition of:

$$\tan(\lambda) \cos(\phi_n) > \mu < \frac{1}{\tan(\lambda) \cos(\phi_n)} \quad (3-44)$$

and nonbackdrivable condition of:

$$\tan(\lambda)\cos(\phi_n) < \mu < \frac{1}{\tan(\lambda)\cos(\phi_n)} \quad (3-45)$$

3.3.2 Worm and Worm Gear Efficiency

The efficiency of a worm and worm gear is addressed in many gear texts [Buckingham, 1949; Davis, 1970; Dudley, 1984; Dudley, 1991; Juvinall, Marshek, 1991; Merrit, 1971] and can easily be derived by substituting the friction angle from (3-43) into the efficiency equations from the model. The efficiency of the worm driving the worm gear (analogous to scenario 1 of the model) is then:

$$e_w = \frac{\cos(\phi_n) - \mu \tan(\lambda)}{\cos(\phi_n) + \mu \cot(\lambda)} \quad (3-46)$$

And the efficiency of the worm gear driving the worm is thus:

$$e_G = \frac{\cos(\phi_n) - \mu \cot(\lambda)}{\cos(\phi_n) + \mu \tan(\lambda)} \quad (3-47)$$

The efficiency of the worm driving the gear, with a pressure angle of 14.5° , is shown in Figure 3.10. Comparing Figure 3.9 to 3.10, note that as the pressure angle increases (0° for the model in Figure 3.9 to 14.5° in Figure 3.10) the efficiency of the worm gear decreases for fixed lead angle and coefficient of friction. Also, from (3-45) it is apparent that the coefficient of friction required to make the worm and worm gear nonbackdrivable decreases with increased pressure angle, for a fixed lead angle. In other words, for a constant coefficient of friction, worm gear trains with low pressure angles will become nonbackdrivable at lower lead angles than those with high pressure angles.

Figure 3.10 Efficiency vs. lead angle for a worm and worm gear with $\phi_n = 14.5^\circ$.

3.3.3 Torque Transmission Ratios

Prior to developing the torque transmission ratios (TTR) for the worm and worm gear set, the analogous "massless" torques of the worm and worm gear are defined as:

$$\tau_{w.ml} = \tau_w - J_w \alpha_w \quad (3-48)$$

$$\tau_{G.ml} = \tau_G - J_G \alpha_G \quad (3-49)$$

where τ_w , J_w and α_w are the externally applied torque, rotational inertia and angular acceleration of the worm and τ_G , J_G , and α_G are the externally applied torque, rotational inertia, and angular acceleration of the worm gear, respectively. Analogous to the force transmission ratios defined for the gear model, torque transmission ratios can be defined specifically for a worm and worm gear set. For the case when the worm is driving the worm gear, the TTR of the massless torques, as well as the externally applied torques under steady state conditions, is given by:

$$\frac{\tau_{w.ml}}{\tau_{G.ml}} = \frac{\tau_w}{\tau_G} \bigg|_{ss} = \frac{-1}{GR e_w} \quad (3-50)$$

where e_w is the efficiency of the worm driving the gear given in (3-46) and GR in this case is found to be:

$$GR = \frac{r_w}{r_G \tan(\lambda)} \quad (3-51)$$

where r_w and r_G represent the pitch radii of the worm and worm gear, respectively. This gear ratio is also equivalent to the ratio of the number of teeth on the worm gear to the number of starts on the worm. Notice the negative sign in (3-50) indicates that the torques are opposing each other.

The TTR for the massless torques, as well as the externally applied torques under steady state conditions, for the case of the worm gear driving the worm in a backdrivable gear set is:

$$\frac{\tau_{w.ml}}{\tau_{G.ml}} = \frac{\tau_w}{\tau_G} \bigg|_{ss} = \frac{-1}{G Re_G} \quad (3-52)$$

where e_G represents the efficiency of the gear driving the worm in a backdrivable gear set given in (3-47).

Chapter 4

Dynamo and Transmission Operation

Having examined the basic operations of a dynamo and backdrivable / nonbackdrivable gear sets separately, the next step is to examine their combined operation as an actuator. The following discussions in this chapter, including the examples, deal only with steady state operation of the dynamo and transmissions. Under this condition, as mentioned in Chapter 3, the external and massless torques applied to the transmission are equal. Therefore, to simplify the discussions and examples, only the externally applied torques will be used. Unsteady state operation of dynamos and transmissions will be examined in Chapters 6 and 7.

Before introducing several qualitative examples, a few important points need to be made. First, when speaking of a dynamo connected to a transmission the states of "backdriving" the transmission and "overhauling" the motor become coincident under steady state conditions. When the transmission backdrives, power flows from the external load through the transmission to the dynamo shaft. As a result, the dynamo must absorb this power and thus be in a state of overhauling. In this sense, overhauling the dynamo and backdriving the transmission in steady state cannot be performed independently -- they can only occur simultaneously. In a similar sense, a nonbackdrivable transmission connected to a dynamo prohibits a dynamo from ever being overhauled by an external

load since the transmission can not be backdriven. This enables us to use Figures 2.2 and 2.3, which were initially presented to describe dynamo operation, to describe the operation of an entire dynamo and transmission system.

The following is a graphical qualitative description of the modes of steady state operation for a dynamo with either a backdrivable or nonbackdrivable gear set. The actuator in these examples is a dynamo coupled to a worm and worm gear transmission, loaded by an external spring (Figure 4.1). In the following discussion, the worm gear will simply be referred to as the "gear". The static and kinetic coefficients of friction between the mating gear teeth are assumed equal in the following discussion. For the nonbackdrivable gear set in these examples, the static and kinetic coefficients of friction are assumed to satisfy the nonbackdrivable criteria in (3-50). The operating points in all the following examples are based on steady state operation of the system in Figure 4.1.

Figures 4.2 - 4.5 used in this discussion, are similar to the dynamo Figures 2.2 and 2.3, except in this discussion both the operating points of the worm (labeled 'x') and gear (labeled 'o') are displayed simultaneously. Note that the ordinate axis labels in Figures 4.2-4.5 include both the torque applied to the worm (τ_w) and the negative of the torque applied to the gear ($-\tau_G$). In this way, under typical operation, both the worm and gear operating points exist in the same quadrant while maintaining the initial sign convention chosen for the gear model in Figure 3.5.

Figure 4.1 Model of a dynamo and worm gear transmission.

Since the worm is rigidly connected to the shaft of the dynamo, the operating point of the worm also represents operating point of the dynamo. In this sense, the region in which the worm operating point is located indicates the mode of operation of the dynamo (ie, motor vs. generator). When the system is in motion, τ_w and τ_G are dependent and their ratio is defined by the absolute value of their steady state torque transmission ratios defined in (3-50) and (3-52). When the transmission is stationary or "locked", τ_w and τ_G are independent. The axes labels in Figures 4.2-4.5 are similar to those in Figures 2.2 and 2.3 except they are defined with respect to the torque and angular velocities of the worm and gear.

In the (a) plots of Figures 4.2 - 4.5 are based on Figure 2.2 where the axes are defined by electrical variables of the dynamo. The variable on the abscissa is the terminal voltage of the dynamo, and that on the ordinate is the voltage drop across the resistor. The vertical position of the worm and gear operating points is determined by calculating the current in the dynamo corresponding to their respective applied torques using (2-2) and then multiplied by the armature resistance.

$$iR_a = \frac{\tau_w R_a}{K_t}, \frac{-\tau_G R_a}{K_t} \quad (4-1)$$

The constant angular velocity lines are parallel to the theoretical stall torque line and are labeled $\omega_w = \omega_G GR$, where is the gear ratio defined in (3-51). In steady state, therefore, both the worm and gear operating points lie on the same constant velocity line at all times.

The (b) plots of Figures 4.2 - 4.5 are correspond to Figure 2.3, where the axes are labeled using the mechanical variables, torque and angular speed. In these examples, the abscissa is labeled $\omega_w = \omega_G GR$ and corresponds to the no load line. The ordinate axis is labeled with τ_w and $-\tau_G$ and corresponds to the theoretical stall torque line. As before, the worm and gear operating points lie on the same constant speed lines which, in these figures, are vertical lines. As in the (a) figures, the ratio of the vertical coordinates of the worm and gear operating points represent the torque transmission ratios from (3-50) and (3-52). As in the (a) figures, the operating point of the worm also corresponds to the dynamo's operation and the electrical variables shown in these figures correspond to the dynamo electrical variables. For example, if the worm's operating point remains on a constant voltage line it indicates that the dynamo is operating a constant voltage. In this way, as mentioned earlier, the region in which the worm operating point lies (ie. motor, generator, overhaul type I and II) implies the mode of operation for the dynamo.

4.1 Dynamo with a Backdrivable Transmission

The following two examples demonstrate the steady state operation of a dynamo with a backdrivable under various loading conditions.

Example 1. Increasing External Load

In Figure 4.2 (a) and (b), the worm and gear operating points are initially in the motor region at ①. If the free end of the spring is fixed, the dynamo is stretching the spring and therefore the external load is increasing. Assuming the terminal voltage of the dynamo is held constant, the two operating points begin to rise and move toward the theoretical stall

line. The worm operating point remains on the constant voltage line and the ratio of the applied torque on the worm and gear remains constant, governed by (3-50), as long as points remain in the motor region. If the force in the spring becomes large enough to stall the dynamo at the fixed voltage, the two operating points reach ②. Once stalled, there are four possible options: i) increase the external load, ii) decrease external load, iii) increase terminal voltage, iv) decrease terminal voltage. Each of these situations is now examined:

- i) While stalled, the forces of the worm and gear are no longer dependant. Initially, the external torque can be increased without effecting the worm operating point at ②. Eventually, the τ_G will become large enough such that the ratio of τ_w to τ_G will satisfy (3-52), the system will begin to move into the generator overhaul type I region (i), and the direction of motion will reverse. Assuming the terminal voltage is still held constant, the worm operating point will continue to follow the constant voltage line and the two operating points will remain on the same constant speed line; however, their torque transmission ratio is now governed by (3-52).
- ii) If the external load is decreased when the operating points are at ②, the two points will move back into the motor region, following the same path from which they came (assuming terminal voltage is held constant). This return path is not shown in Figure 4.2.

Figure 4.2 (a) Example 1: Dynamo and backdrivable transmission operation with an increasing external load (Electrical variables).

Figure 4.2 (b) Example 1: Dynamo and backdrivable transmission operation with an increasing external load (Mechanical variables).

- iii) If the external load is held constant and the terminal voltage is increased, from ②, the two operating points will move horizontally back into the motor region, (iii), and the motor shaft will gradually increase in speed.
- iv) Decreasing the terminal voltage of the dynamo from ② has much the same effect as increasing the load in (i). However, in this case, the worm point will descend along the stall line without initially influencing the gear operating point. Eventually the ratio of the worm and gear forces will satisfy (3-52) and any further drop in terminal voltage will move the two operating points into the generator overhaul type I region. If the external load is held constant, the two points will move horizontally to (iv), as illustrated in Figure 4.2.

Example 2. Decreasing External Load

In Figure 4.3 (a) and (b), the two operating points begin in the motor region at ①. If the terminal voltage is held constant and the external load is decreased (ie. spring force is decreased), the two operating points begin to descend. As before, the worm operating point remains on the constant voltage line and the ratio of the worm to gear torques satisfies (3-50) while their operating points are in the motor region. If the load is decreased sufficiently, the operating points will reach the theoretical no load line where the two points meet. At any point along the no load line, $\tau_w = \tau_G = 0 \text{ Nm}$ indicating that the mating gear teeth are theoretically no longer in contact as shown in Figure 3.5. From ②, four possible changes can take place: i) increase external load, ii) decrease external load, iii) increase terminal voltage, iv) decrease terminal voltage.

- i) If the external load is increased and the terminal voltage is held constant, the two points move back into the motor region, following the same path from which they came (not shown).

- ii) While holding the terminal voltage constant, if the external load is decreased so that the direction of τ_G changes direction, the gear becomes in contact with the opposing worm tooth. At this instant, the two points move into the generator overhaul type II region where the ratio of their forces is now governed by (3-52). In this case the load is forcing the dynamo to rotate faster than its no load speed.

- iii) If the terminal voltage is increased, the speed of the worm and gear will both increase while the two points move together along the no load speed line. This may first appear impossible but it is based on the original assumption of continuous steady state operation (no transient states are included in these examples). As a result, the inertia of the worm and gear have been ignored and thus the torques on the worm and gear are not required to change in order to change the speed of the system.

- iv) Decreasing the voltage results in the speed of the worm and gear to decrease, and the two points again move together horizontally along the no load speed line. As mentioned in (iii), no external force is required to change the speed of the system.

Figure 4.3 (a) Example 2: Dynamo and backdrivable transmission operation with a decreasing external load (Electrical variables).

Figure 4.3 (b) Example 2: Dynamo and backdrivable transmission operation with a decreasing external load (Mechanical variables).

4.2 Dynamo with a Nonbackdrivable Transmission

The following two examples use the same loading conditions as Examples 1 and 2 except the dynamo is now coupled to a nonbackdrivable transmission.

Example 3. Increasing External Load

Figure 4.4 (a) and (b) shows the worm and gear operating points initially in the motor region at ①. The free end of the spring is initially fixed. The spring is initially being stretched by the dynamo causing the external load to increase and as a result the two operating points begin to rise and move towards the theoretical stall torque line. The terminal voltage of the dynamo is held constant so the worm operating point moves along the constant voltage line. The two operating points remain on the same constant velocity line at all times and the torque transmission ratio remains constant, governed by (3-50), as long as both points remain in the motor region. When the force in the spring is large enough to stall the dynamo, for the given terminal voltage, the operating points arrive at ②. Once the system stalls, the ratio between the torques of the two operating points becomes independent. From ②, there are four possible options: i) increase external load, ii) decrease external load, iii) increase terminal voltage, iv) decrease terminal voltage.

Figure 4.4 (a) Example 3: Dynamo and nonbackdrivable transmission operation with an increasing external load (Electrical variables).

Figure 4.4 (b) Example 3: Dynamo and nonbackdrivable transmission operation with an increasing external load (Mechanical variables).

- i) Since the transmission is nonbackdrivable, the external load can be increased indefinitely and the two operating points will remain on the stall line, with the worm operating point unchanged. For example, the gear operating point can move from its position at ② to a higher position (i) without effecting the position of the worm operating point at ②, since the two points are independent when the system is in static equilibrium.

- ii) If the external load is decreased at all, the worm and gear operating points begin to move back into the motor region, following the same path from which they arrived (assuming constant terminal voltage at the terminals). This return path is not illustrated in Figure 4.4.

- iii) If the terminal voltage of the dynamo is increased while maintaining a constant external load, the worm and gear operating points move horizontally back into the motor region to points (iii). If the external load was first increased, and then the terminal voltage was increased, the worm operating point would move up the stall line until the ratio of τ_w between τ_G satisfied (3-50) at which time the operating points would again move horizontally back into the motor region (not shown).

- iv) If the terminal voltage is decreased, the worm operating point moves down the stall torque line, while the gear operating point remains unchanged at ②. If the terminal voltage reverses, the worm operating point continues to descend on the stall line. In this region, at some fixed point on the stall line, the worm and gear torque (at ②) will

satisfy (3-52). If the terminal voltage continues to decrease and the external load is kept constant, the worm operating point (iv) will move horizontally into the negative motor region, while the gear operating point (iv) will move horizontally into the generator overhaul type I region. The system is again in motion and the worm and gear operating points remain on the same constant velocity line. Caution should be exercised in interpreting this result. First, this situation demonstrates the means in which the dynamo and nonbackdrivable transmission would “unwind” the stretched spring, by forcing the direction of velocity of the worm to reverse. The fact that the gear operating point has moved into the overhaul type I region has no significance on the dynamo’s operation, since the dynamo’s operation is only related to the location of the worm’s operating point and not the gear’s operating point. The location of the gear’s operating point (iv) does indicate that it is no longer absorbing energy since the sign on the gear force and velocity are in the same direction. This corresponds to Configuration 4 from Figure 3.6 in which both that worm and the gear are inputting positive power in order to maintain equilibrium.

Example 4. Decreasing External Load

In Figures 4.5 (a) and (b), the two operating points again start in the motor region at points ①. If the external load (force in the spring) is decreased, while maintaining a constant terminal voltage, the two operating points begin to descend with the worm operating point following the constant terminal voltage line. Eventually, if the spring reaches its neutral length, the external load diminishes to zero and the two operating points merge at ② on the theoretical no load line. At this location $\tau_w = \tau_G = 0 \text{ Nm}$ and the

Figure 4.5 (a) Example 4: Dynamo and nonbackdrivable transmission operation with a decreasing external load (Electrical variables).

Figure 4.5 (b) Example 4: Dynamo and nonbackdrivable transmission operation with a decreasing external load (Mechanical variables).

force polygon between mating gear teeth illustrated in scenario 1 of Figure 3.6, has been reduced to a point. The dynamo is now operating at its maximum motor speed and theoretically, the mating gear teeth are not in contact as shown in Figure 3.5. At ②, there are again four possible options: i) increase external load, ii) decrease the external load, iii) increase terminal voltage, iv) decrease terminal voltage.

- i) If the external load is suddenly increased, while maintaining constant terminal voltage, the two operating points move back into the motor region, following the same path from which they arrived. This return path is not shown on Figure 4.5.
- ii) If the external load is decreased, τ_G will change direction and the gear would now be in contact with the opposite worm tooth, pushing in the direction of motion. The transmission is now operating as scenario 4 from Figure 3.5 and both the worm and gear are inputting positive power to the system. As a result, the ratio of the worm force and gear force correspond to (3-52). In terms of the graphs, the gear operating point (ii) moves into the generator overhaul type II region while the worm operating point (ii) moves back into the motor region, following the constant voltage line (assuming terminal voltage was held constant). The two operating points remain on the same constant speed line, though the speed of the system has decreased.
- iii) Increasing the terminal voltage increases the speed of the system and the two operating points move together along the no load speed line. As in Example 2, the speed of the system can be changed without changing the applied torques since the

inertia of the gear and worm are not included in not considered in these steady state examples.

- iv) Decreasing the terminal voltage decreases the speed of the system and the two points move together along the no load speed line. As mentioned in (iii), no external force is required to change the speed of the system.

These four examples have highlighted several similarities and differences between backdrivable and nonbackdrivable transmissions. When both transmissions are stalled, a decrease in external load or an increase in terminal voltage forces both transmissions back into the motor regions. Similarly, if the transmissions are operating on the no load speed line, an increase in external load moves the transmissions back into the motor regions while changing terminal voltage moves the operating points along the no load speed line. Examples 1 and 2 highlight how a dynamo can be overhauled in both type I and type II under steady state conditions, while Examples 3 and 4 illustrate the fact that a dynamo with a nonbackdrivable transmission can never operate outside the motor regions in steady state. From this point of view, an actuator consisting of a dynamo and nonbackdrivable transmission is not a regenerative actuator [Seth and Flowers, 1990] since it can never absorb mechanical energy. It should be highlighted that if the static and kinetic coefficients of friction for a set of gears within the nonbackdrivable transmission satisfied the condition of *partial* nonbackdrivability as defined in Table 3.1, then the dynamo with the nonbackdrivable transmission in Example 4 could have theoretically operated in the overhaul type II under steady state conditions.

This graphical technique of presenting motor and transmission operation is used again in Chapters 6 and 7 to present simulation results.

Chapter 5

Motor and Gear Selection Techniques

The following describes the steps used in designing the transmissions of the two proposed prosthesis designs: Configuration 1 and 3. This chapter is arranged in the following fashion:

- Review of commercially available upper arm prostheses
- Definition of design goals for the two proposed prosthesis configurations
- Determination of required spring stiffness and effective elbow radii for both configurations, using the technique and design criteria from English [1996]
- Review general gearing arrangements for both configurations
- General gear and motor selection techniques
- Motor and gear selections for Configurations 1 and 3

Since the focus of the thesis involves the dynamics associated with the nonbackdrivable components of the transmission, most of the emphasis is placed on sizing and selecting the components of the nonbackdrivable gearing. The other backdrivable gearing components will be sized but final selection of the gears is not performed.

5.1 Commercially Available Upper Arm Prostheses

There are currently three major, commercially available powered upper arm prostheses, namely the *Boston Elbow*, *Utah Arm*, and the *Hosmer/NY Arm*. Within a publication by Liberty Technology [1997] is a performance and specification comparison table that can be found in Appendix A. These design comparisons provided the required data from which to develop the basic design criteria for Configurations 1 and 3.

The *Boston Elbow* [Liberty Technology, 1997, Williams, T. W. III, 1986] is a popular commercially available prosthetic arm. It weighs about 3 lb 5 oz, and boasts the strongest lifting torque of all three prostheses of more than 10 ft-lbs (13.56 Nm). The prosthesis uses a direct drive system and has a reverse locking clutch which automatically engages when the external load exceeds the stall torque of the motor. It disengages when the external load becomes less than the stall torque or can be intentionally unlocked under load enabling it to lower a heavy weight. The no load speed of the prosthesis is approximately 1.1 s, flexion against gravity, and 1.0 s for extension with gravity. It also has a mechanical free-swing mode that uses no power and assists in normal gait.

The *Utah Arm* [Jacobsen, S. C. et. al., 1982] is another popular powered prosthetic arm which weighs about 3 lb 3 oz. Its lifting torque is about a third of the *Boston Elbow* at 3 ft-lbs (4.07 Nm). This arm uses a pin lock mechanism to lock the arm in position which cannot be released under load. The no load speed of this arm is the same as the *Boston Elbow*, approximately 1.1 sec in flexion against gravity and approximately 1.0 sec in

extension with gravity. The drive train of this arm consists of a belt drive, and has a free-swing mode which requires power.

The *Hosmer/NY Arm* weighs slightly less than the other models at 2 lb 15 oz. Its lifting torque is slightly greater than that of the *Utah Arm* at 3.5 ft-lbs. (4.75 Nm). This model, like the *Utah Arm*, uses a pin lock mechanism to lock the arm in position which cannot be released under load. The no load speeds for this arm are slower than the other two models at 1.4 s flexion against gravity and 1.1 s extension with gravity. Like the *Utah Arm*, it uses a belt drive as its transmission.

5.2 Design Criteria

The basic design criteria for the two proposed prosthesis design configurations of this thesis are presented in Table 5.1 and were developed primarily from the specifications of the commercially available arm prostheses listed in Appendix A.

Table 5.1 Design criteria for the two proposed prosthesis configurations

Performance item	Variable Symbol	Design Goals
Maximum Torque	$\tau_{\text{arm, max}}$	10 ft-lbs ¹ (13.56 Nm)
Live Lift (with 14" forearm)	F_{Live}	9 lbs ¹ (40.0 N)
Speed: No Load		
Flexion (against gravity)	$t_{\text{flex, NL}}$	1.1 sec ²
Extension (with gravity)	$t_{\text{ext, NL}}$	1.0 sec ²
Speed: 1.2 lb load		
Flexion (against gravity)	$t_{\text{flex, 1.2 lb}}$	1.2 sec ¹
Extension (with gravity)	$t_{\text{ext, 1.2 lb}}$	0.9 sec ³
Maximum Stiffness	K_{max}	80 Nm/rad
Minimum Stiffness	K_{min}	1 Nm/rad
Weight of Elbow and forearm, no TD	W	≈2 lb (8.9 N)

¹ Boston Elbow performance² Boston Elbow and Utah Arm performance³ Utah Arm performance

The design specifications represent the best values of all three of the commercially available prostheses for each performance item. Since none of the commercially available prostheses have variable stiffness capabilities, the stiffness requirements for the proposed prosthesis designs were selected based on the normal ranges of stiffness of the natural arm of 1 to 80 Nm/rad [English, 1996]. Though not specified in Table 5.1, the maximum voltage for the power source of the prosthesis was assumed to be 12 Volts.

5.3 Effective Elbow Radii and Maximum Spring Forces

A detailed preliminary design for both Configuration 1 and 3 was conducted by English [1996] which involved the selection of key design parameters. A brief description of these parameters and their values is presented in Table 5.2.

Table 5.2 Design parameters for the two proposed prosthesis configurations

Parameter	Description	Value
η	Ratio of maximum negative to positive displacements of spring	1
$K_{\max}^*$	Maximum joint stiffness of prosthesis	80 Nm/rad
$K_{\min}^*$	Minimum joint stiffness of prosthesis	1 Nm/rad
$\Theta_{\lim}$	Maximum range of motion of joint	3.67 rad
$l_{\lim}$	Limiting length of spring operation	0.25 m
m_{arm}	Mass of prosthetic arm	0.28 kg
L_{arm}	Length of prosthetic arm	0.33 m

To remain competitive with the commercial prosthetic arms, a maximum lifting torque for the two proposed prosthesis configurations was assumed to be 13.56 Nm. Using the parameter values listed in Table 5.2, and the limiting torque of 13.56 Nm, the required maximum spring forces of both Configurations 1 and 3 (recognizing that $\Theta_{\lim} = 0$ rad for Configuration 3) can be derived by rearranging the following equations and solving for $F_{\max}$ [English, 1996]:

$$\tau_{\text{arm,max}} = \frac{K_{\max}^{*2} \alpha}{2(K_{\max}^* - K_{\min}^*)(1 + \eta)} \quad (5-1)$$

where α , which has no physical meaning, is used to simplify (5-1) and is defined as:

$$\alpha = \frac{8(1 + \eta)F_{\max}(K_{\max}^* - K_{\min}^*)l_{\lim}}{K_{\max}^* \sqrt{K_{\max}^{*2} \Theta_{\lim}^2 + 32(1 + \eta)F_{\max}l_{\lim}(K_{\max}^* - K_{\min}^*)} - K_{\max}^{*2} \Theta_{\lim}} - \frac{1}{2} \Theta_{\lim} \quad (5-2)$$

Knowing the maximum spring forces, the optimum effective elbow radius of both configurations can then be determined using [English, 1996]:

$$R = \frac{K_{\max}^{*2} \left(-\Theta_{\lim} + \sqrt{\Theta_{\lim}^2 + 32(1 + \eta)F_{\max}l_{\lim} \left(\frac{K_{\max}^* - K_{\min}^*}{K_{\max}^{*2}} \right)} \right)}{16(1 + \eta)F_{\max}(K_{\max}^* - K_{\min}^*)} \quad (5-3)$$

These maximum forces and effective elbow radius values are presented in Table 5.3.

Table 5.3 Maximum spring forces and effective elbow radii for Configurations 1 and 3

Configuration	Maximum Spring Force	Effective Elbow Radius
1	136 N	0.050 m
3	36 N	0.187 m

Due to the expected problems associated with developing high force quadratic springs, an early effort of this thesis involved determining whether low force springs could be used in Configurations 1 and 3 without loss of performance. Unfortunately, low force springs required large effective elbow radii to maintain high output torque performance. This results in dramatically limited deflections of the arm at low stiffness, which defeats the purpose of the variable stiffness prosthesis. As a result, the work in this thesis assumes that the maximum forces indicated in Table 5.3 are obtainable, and the corresponding effective elbow radii are used in the design of the geared prosthesis transmission.

5.4 Configuration Gearing

Figure 1.2 presents the conceptual design of Configuration 1, including specific locations of gearing. The gearing on both branches of Configuration 1 is symmetrical. A worm and worm gear were chosen to provide the nonbackdrivable component of these transmissions. The motor gearhead is coupled to the worm and worm gear, and the output shaft of the worm gear is coupled to a pinion which drives the rack and spring end. Though the details of the gearing at the prosthetic elbow are not shown, the gear ratio at this location must equal the required effective elbow radius for this configuration, given

in Table 5.3. Examining only one branch of this configuration, the total gear ratio between the motor and the prosthetic arm is defined by:

$$GR_{CI} = \frac{RN_G gh}{r_p} \quad (5-4)$$

where R is the effective elbow radius of the prosthesis, N_G is the number of teeth of the worm gear, gh is the gear ratio of the motor gearhead, and r_p is the radius of the pinion which drives the rack. Based on this definition of total gear ratio, the torque and angular speed of the motor is related to that of the prosthesis by:

$$\tau_{arm} = GR_{CI} \tau \quad (5-5)$$

$$\omega_{arm} = \frac{\omega}{GR_{CI}} \quad (5-6)$$

where τ_{arm} and τ are the torque of the prosthesis and the motor respectively and ω_{arm} and ω are the angular speeds of the prosthesis and motor, respectively.

Figure 1.3 displays the conceptual design of Configuration 3 and indicates the gearing locations. The nonbackdrivable gearing at the position motor, described briefly in Chapter 1, is unconventional. The position motor has a gearhead which is attached to a worm and worm gear. The output shaft of the worm gear is attached directly to the prosthetic arm. This motor and its nonbackdrivable gearing are housed in a casing which is mounted directly to the effective elbow radius. Between this housing and the springs is gearing that must have a gear ratio equal to the required effective elbow radius for this configuration, given in Table 5.3. As described in Chapter 1, this housing can rotate relative to the springs and the prosthetic arm. The total gear ratio between the position motor and the prosthetic arm in this configuration is simply:

$$GR_{C3-pos} = N_G gh \quad (5-7)$$

Notice that the effective radius of the elbow, R , does not appear in this equation since this gearing does not lie between the position motor and the prosthetic arm. The relationship between steady state motor torque and angular speed to the output prosthesis torque and angular speed is of the same form as (5-5) and (5-6).

The nonbackdrivable linear drive mechanism at the stiffness motor consists of a lead screw. The output shaft of the gearhead, attached to the stiffness motor, is coupled to a lead screw. Rotation of the lead screw results in linear travel of the screw nut which is attached to the springs. This lead screw scheme can provide remarkably high rotational-to-linear gear ratios. The total gear ratio between the stiffness motor and the lead screw nut is given by:

$$GR_{C3-stiff} = \frac{gh}{2\pi L} \quad (5-8)$$

where L is the lead of the lead screw. If the lead is in meters, the units of this gear ratio are in rad/m.

The ends of the springs are attached to the linear lead screw drive via linear slide rails, as depicted in Figure 1.3. Torque created by uneven forces in the two springs will be taken up by these sliders, and therefore will not be transmitted to the power screw plate (eliminating the risk of the lead screw plate getting "kinked"). The "counter balance" spring shown in Figure 1.3 was not considered when selecting the motor and required gear ratio for the stiffness motor transmission. In this sense the gear and motor selection for the stiffness transmission is a conservative design.

5.5 General Gear Ratio and Motor Selection Techniques

5.5.1 Gear Ratio Selection Techniques

The next step involves determining the overall gear ratio and selection of motors which will provide the desired performance for the prosthesis. The following describes the assumptions and methods use to select the overall gear ratio of the transmission and the motors for the individual configuration designs.

Selecting a motor and gear train for the transmission design, such as the proposed prostheses designs in this thesis, poses several difficulties. First, typical motor/gearhead selection methods assumes a stiff, rather than compliant transmission such as those in Configuration 1 and 3. Second, the performance data of motors and gearheads are usually based on their continuous operation, but for this application the transmission is to be used in an upper arm prosthesis which does not operate in this fashion. Third, the performance data of most motors is typically based on the motor operating at its nominal voltage, yet in the case of this prosthesis design it is desirable to operate at variable voltages levels between 0 V and the nominal voltage of the motor in order to obtain varying speeds for the prosthesis. Though a PWM drive could be used, we will assume that the "effective" voltage experienced by the motor is between 0 V and its nominal rated voltage. Several basic assumptions were used to rectify these design problems. It was assumed that the prosthesis operated at steady state which resolves the issue of a stiff versus compliant transmission since under steady state both function exactly the same. This assumption, however, is an oversimplification since the human arm rarely operates

at steady state. It will be assumed that, even though the prosthesis operates in an intermittent fashion, the continuous motor operation data can be used in the motor selection process for these designs. To resolve the issue of motor data being quoted at its nominal voltage, a linear representation was used to determine the operating conditions of the motors at voltage levels other than their nominal values.

In a general sense, the overall gear ratio, GR , relates the torque/speed of the motor shaft to the torque/speed of the prosthetic arm by (assuming 100% efficiency):

$$\begin{aligned}\tau_{arm} &= GR\tau \\ \omega_{arm} &= \frac{\omega}{GR}\end{aligned}\tag{5-9}$$

Therefore, as the gear ratio increases, the output torque increases while the output speed decreases. In the case of a 100% efficient transmission, the input power is equal to the output power. The following general graphical techniques illustrate the effects of gear ratio the on output torque and speed of the prosthesis given in (5-9). A Micro Mo Brushless 3564 motor (Micro Mo Electronics Inc., 1993), which has a nominal voltage rating of 12 V, is used in the development of these graphical techniques.

The first technique assumes the motor is operating at its nominal voltage (12 V in this case) and examines the effect of gear ratio on the output torque and speed of the prosthesis. For this example, three motor operating points are considered:

- i) $\tau = 20\%$ of Stall torque, $\omega = 80\%$ of No load speed
- ii) $\tau = 10\%$ of Stall torque, $\omega = 90\%$ of No load speed
- iii) $\tau = 0\%$ of Stall torque, $\omega = 100\%$ of No load speed

All three of these operating points would exist on the same constant voltage line from Figure 2.2 or 2.3. If the output torque and speed of the prosthesis is plotted for each of these three motor operating points and varying gear ratios, the result is a family of three curves as depicted in Figure 5.1. These curves are plotted on dimensionless axes, assuming a 100% transmission efficiency. Each of the three curves is specific to one of the above operating points of the motor (case (iii) is a line coincident with the abscissa), and represent the range of possible output speed and torque for the prosthetic using various gear ratios. For example, this figure shows three different constant gear ratio lines, namely 10:1, 15:1, and 20:1. The output torques and speeds of the prosthetic corresponding to the three motor operating points with these three gear ratios are indicated in Figure 5.1. The range of gear ratios used in this figure was $GR = 10:1$ to $100:1$, where $GR = 10:1$ corresponds to the points on the right end of the curves and $GR = 100:1$ to the points on the left end of the curves. The use of dimensionless axes provides an easy method of locating constant GR lines since they are straight lines connecting the value of GR on the ordinate to $1/GR$ on the abscissa. If $GR = 1:1$ was shown in this figure, its constant gear ratio line would have connected the motor's stall torque to its no load speed, similar to the constant voltage lines in Figure 2.3.

Figure 5.2 demonstrates this technique without using dimensionless axes labels. Note that this figure, like Figure 5.1, assumes a transmission power efficiency of 100% so to obtain the real output torque the values taken from this curve must be multiplied by the transmission efficiency. In this figure, the gear ratio was varied logarithmically from 100:1 to 1000:1, as indicated on the dotted constant gear ratio lines. The three operating

Torque vs. Speed – Varying Gear Ratios Micro Mo Brushless 3564 Motor

Figure 5.1 Torque vs. angular speed for varying motor loads and gear ratios [Micro Mo Electronics Inc., 1993].

Figure 5.2 Torque vs. angular speed for varying gear ratios and constant terminal voltage [Micro Mo Electronics Inc., 1993].

points used in this example represent the motor's maximum thermal torque, $\frac{1}{2}$ its maximum thermal torque, and its no load torque. Due to the increasing slope of the constant gear ratio lines, the range in speeds for the three operating torque diminishes. In other words, the output speed for a gear ratio of 1000:1 is approximately the same regardless if the motor is operating at its maximum, $\frac{1}{2}$ its maximum, or no load torque. Therefore, if the design criteria for the output (the prosthetic arm in this case) requires a large difference in loaded and unloaded speeds, a relatively low gear ratio is necessary. Figure 5.2 also demonstrates the trade off between high output torque at low speeds to high output speeds at low torque. The selection of a specific gear ratio typically requires compromising either the output torque or angular speed.

The second graphical technique used to illustrate the effects of gear ratio on the output torque and speed of the prosthesis assumes the motor is operating at a fixed output torque but with varying voltages applied to its terminals. For this example, the motor is assumed to be operating at its maximum thermal torque (60.3 mNm for this motor) under the following terminal voltages: 0 V, 1.25 V, 2.5 V, 4 V, 8 V, and 12 V. All of these operating points would lie on the same constant torque line in Figure 2.2 and 2.3. This selection is different from Figures 5.1 and 5.2 which selected several operating points with the motor operating at its nominal voltage. Figure 5.3 depicts the output torque and speed of the prosthesis for each of these six operating points with varying gear ratios (again assuming 100% transmission efficiency). The constant gear ratio lines in this figure are horizontal and are displayed in logarithmic increments between GR = 100:1 to 1000:1 as in Figure 5.2. Note that the curve corresponding to the 12 V terminal

Figure 5.3 Torque vs. angular speed for varying gear ratios and constant thermal torque [Micro Mo Electronics Inc., 1993].

voltage in this plot is the same as the 12 V maximum thermal torque curve in Figure 5.2 since both represent the same operating point. The 2.5 V terminal voltage curve is a straight line at 0 degrees/s. This operating point is the voltage at which the maximum thermal torque is the same as the stall torque of the motor (which is the “corner” of the safe operating region in Figure 2.2). The curves left of this vertical line have negative speeds because the operating points on the constant torque line are in the overhaul type I region at these low levels of voltage. For a motor with a nonbackdrivable transmission, the curves to the left would all line up with the 2.5 V line at speed = 0 degrees/s when the system is operating at steady state since the motor cannot be overhauled.

In summary, Figures 5.1 and 5.2 demonstrate the range of output speeds for a motor operating at a constant voltage at varying torque levels. Figure 5.3 portrays the possible range of output speeds for a motor operating at constant torque (ie. constant current) for varying voltage levels. These graphical interpretations of gear ratio versus output angular velocity and torque will be used in the selection of gearing for both Configurations.

5.5.2 Motor Selection Techniques

Several design requirements for the motors are common to both proposed prosthesis configurations and are presented before discussing the specific design requirements for each. One of the most important factors when selecting a motor is a thermal consideration. Under high torque levels, current in motor windings can reach unacceptably high levels which results in large i^2R_a losses (heat losses). This heat must be dissipated or else the motor may sustain permanent damage. For this reason, the

maximum continuous torque of a motor is not its rated stall torque rather a safe thermal operating level (usually 10-30% of stall torque). This thermal consideration was mentioned in Chapter 2 and is portrayed in Figures 2.2 and 2.3. In cases where the thermal resistance of the motor components is known, the safe operating torque of a motor can be determined by first calculating the maximum power the motor can dissipate:

$$P_{\text{heat}} = \frac{(T_{\text{operating}} - T_{\text{ambient}})}{R_{R-A}} \quad (5-10)$$

where P_{heat} is the maximum heat the motor can safely dissipate, $T_{\text{operating}}$ is the maximum operating temperature of the motor (specified by the manufacturer), T_{ambient} is the ambient temperature surrounding the motor, and R_{R-A} is the total thermal resistance from the rotor to the ambient surroundings. The maximum current is then determined using:

$$\begin{aligned} (i_{\text{load}} + i_{\text{NL}})^2 &= \frac{P_{\text{heat}}}{R_a} \\ i_{\text{load}} &= \sqrt{\frac{P_{\text{heat}}}{R_a}} - i_{\text{NL}} \end{aligned} \quad (5-11)$$

where i_{load} is the current resulting from an external load on the motor shaft, i_{NL} is the current the motor draws under no load, and R_a is the armature resistance of the motor. To maximize i_{load} and thus the torque capacity of the motor it is desirable to minimize the armature resistance, R_a . Knowing i_{load} , the maximum safe thermal torque of a motor can be determined using:

$$\tau_{\text{th}} = K_t i_{\text{load}} \quad (5-12)$$

where K_t is the torque constant for the motor. Since the value of maximum torque is an estimate, it is wise to apply a safety factor.

Having discussed the method for determining the safe operating torque for a motor, the next step was to begin selecting motors and gearing for both proposed prosthesis configurations. Based on the comparison table in Appendix A, the maximum voltage supply of the three commercially available prostheses is 12 V. For this reason, it was desirable to select a motor for the proposed prosthesis designs which was rated for 12 V or less. It was also desirable to select a brushless DC motor due to its smooth performance and long life. Though not specified in the Table 5.1, the selected motors should occupy as small a volume as possible so that they may fit within the volume of an average size prosthetic forearm. To assist in selecting the motors, an estimate of the maximum power required to operate the prosthesis is made by using the maximum torque and the average angular speed of the loaded prosthesis:

$$P_{\text{arm,max}} = (10 \text{ ft lb}) \left(\frac{1.356 \text{ Nm}}{1 \text{ ft lb}} \right) \left(\frac{135^\circ \pi}{180^\circ \cdot 1.2 \text{ s}} \right) = 26.63 \text{ W} \quad (5-13)$$

where 135° is the approximate angle of a prosthesis from extension to flexion. As a rule of thumb, the maximum power rating of a motor should be 1.5 to 2 times the maximum required power output (Maxon Motor Inc., 1998). With this in mind, the next step was to examine the specific motor requirements for each of the two proposed prosthesis configurations.

5.6 Motor and Gear Selection for Configuration 1

Configuration 1 consists of two actuators antagonistically arranged around the elbow joint. Due to the symmetry of this configuration, either motor need only support half the externally applied load. For this reason, we need only design one side of the transmission

and apply it to both actuating devices (consisting of a motor, gearing and spring). Note that in evaluating the motors and gearing for this configuration, only one "branch" of the actuation was considered.

To begin the motor selection process, the approximate maximum power required for one of the motors in this configuration, which is half that of the prosthesis (because of the above mentioned symmetry), is determined:

$$P_{Cl,max} \approx \frac{P_{arm,max}}{2} = 13.31 \text{ W} \quad (5-14)$$

As mentioned earlier, as a rule of thumb, motors should have a power rating at least 1.5 to 2.0 times greater than the maximum required power output which in this case is 19.97 W to 26.63 W. The next step was to obtain a list of motors which met the power requirement and which had nominal voltages less than or equal to 12 V. Based on the design weight of the prosthetic elbow and forearm reported in Table 5.1, the maximum weight of each motor in Configuration 1 can not exceed one half of the total weight (approximately 1 lb = 453.6 g) but should be as light as possible. A list of several DC brushless motors, manufactured by Micro Mo Motors and Maxon Motors, which met these requirements is given in Table 5.4. Though other motor companies were contacted, only these two manufacturers were considered based primarily on their reputation and their wide selection of compact, brushless DC motors. The choice of brushless motors for these prosthesis designs was based on their long life, quiet operation, and low vibration characteristics.

Table 5.4 Selected brushless DC motors by Micro Mo and Maxon Motor

Manufacturer	Motor Model	Nominal Voltage (V)	Maximum Output Power (W)	Stall Torque (mNm)	Maximum Thermal Torque (mNm)	Thermal Torque/ Stall Torque (%)	No Load Speed (rpm)	Maximum Efficiency (%)	Mass (g)	Diameter of Motor (mm)
MicroMo Motors	1628 012B	12	10	11	2.6	24.1	28650	68	31.1	16
	2036 012B	12	20	22	6.2	28.2	17600	70	49.9	20
	3056 012B	12	48	95	26.1	27.5	8790	73	189.9	30
	3564 012B	12	109	291	60.3	20.7	7850	81	309.9	35
Maxon Motor	118891	12	40	483	44.2	9.2	15160	78	263.0	32
	118894	12	100	543	94.7	17.4	6600	79	390.0	40
	118895	12	100	550	118.1	21.4	4500	80	390.0	40

It is interesting to note that the Micro Mo motors in Table 5.4 typically have thermal torque values which are larger percentages of their stall torque. This is due to the lower thermal resistances of the Micro Mo motors compared to the Maxon motors (ie. the Maxon motors heat up to higher temperatures than the Micro Mo motors for a given torque). The majority of the maximum output power levels listed in Table 5.4 are 3 to 7.5 times greater than the maximum output power, $P_{Cl,max}$, found in (5-14). These motors may appear to be excessively oversized for this design, but the efficiency of the transmission has not yet been taken into account. As was shown in Chapter 3, the maximum efficiency of a nonbackdrivable transmission, such as a simple rack and pinion gear design, is only 50%. Due to such low transmission efficiencies, the motors selected for these prosthesis designs will need to be several times more powerful than the estimate given in (5-14).

To assist in determining which motor is best suited for this design, the gear ratio between the motors and the prosthesis device was examined. The thermal torque levels given in Table 5.4 represent maximum allowable continuous operating torque levels for each motor, dictated by a maximum current level. A current limiter could be used to prevent the current in the motor from exceeding this maximum level but as further caution a safety factor of 1.25 was applied to the thermal torque of each of the motors in this design. Before determining the total gear ratio required for each motor, an estimate of the transmission efficiency was set at 25%. This efficiency may at first appear extremely low, but as was shown in Chapter 3, the use of a nonbackdrivable transmission typically results in extreme efficiency penalties and due to the multiple gearing stages a 25%

efficiency is a reasonable first estimate. This estimate will be revisited once the specific efficiencies of the individual gearing stages have been established. The required gear ratio of the transmission between one motor and the prosthetic device for Configuration 1 was then determined using:

$$GR_{Cl} = \frac{\left(\frac{\tau_{arm,max}}{2} \right)}{\frac{\tau_{th}}{SF} e} \quad (5-15)$$

where e is the efficiency of the transmission and SF is the safety factor of 1.25 applied to the thermal motor torque. Table 5.5 displays the values of the required transmission gear ratios, GR_{Cl} , for each of the motors listed in Table 5.4. Also in Table 5.5 are the approximate maximum loaded angular speeds (in °/s) associated with the given transmission gear ratios which were calculated using the same linear approximation between torque and speed as that shown in Figures 2.3 and 2.3:

$$\omega_{arm} = \frac{\omega_{NL} \cdot \left(1 - \frac{\tau_{th}}{\tau_{stall}} \right)}{GR_{Cl}} \quad (5-16)$$

where τ_{stall} is the stall torque of the motor. Table 5.5 also presents the approximate time for the prosthesis to move 135° (approximate range from full extension to full flexion) based on the loaded angular speed. These flexion times are underestimates since the periods of acceleration and deceleration are not considered.

Table 5.5 Required gear ratios and flexion speeds for Configuration 1

Manufacturer	Motor Model	Thermal Torque (mNm)	Gear Ratio (GR_{CI})	Motor No Load Speed ($^{\circ}/s$)	Loaded Speed ($^{\circ}/s$)	Loaded Flexion Time (s)
Micro Mo Motors	2036 012B	6.2	5467	105600	13.8	9.78
	3056 012B	26.1	1299	52740	29.4	4.59
	3564 012B	60.3	545	47100	68.5	1.97
Maxon Motors	118891	44.2	767	90960	107.7	1.25
	118894	94.7	358	39600	91.4	1.47
	118895	118.1	287	27000	73.9	1.83

Notice that the motor which provided the fastest loaded lifting time was the Maxon Motor model 118891. This is partly due to the low maximum thermal torque of the motor and its high no load speed.

When selecting gear ratios, a trade off between speed and torque is often required. The gear ratio for this prosthesis design was based on the load and torque requirements, since it was deemed more important to be able to lift large loads slowly rather than move quickly under low loads. This design choice, however, may possibly compromise the speed performance of the arm.

Only the last four motors listed in Table 5.5 provided acceptable flexion speeds. For this reason, the final motor selection is based on the last four motors in Table 5.5.

The prosthesis design is to be as light as possible and therefore we no longer consider the last two motors in Table 5.5, since they are the heaviest of four remaining motors. The motor selection was then between the MicroMo 3564 model and the Maxon Motor 118891 model.

Of the two remaining motors, the Maxon motor 118891 provides a faster flexion time than the Micro Mo 3564 model and it is also lighter with a smaller diameter. For these reasons, the Maxon Motor model 118891 with a total gear ratio of 767 was selected for Configuration 1. Table 5.6 presents some of the key details of this motor.

Table 5.6 Maxon Motor 118891 specifications

Parameter	Value
Nominal voltage	12 V
Maximum power output	40 W
Stall torque	0.483 Nm
No load speed	15160 rpm
Torque constant	0.0075 Nm/V
Maximum efficiency	78%
Terminal inductance (phase-to-phase)	0.03 mH
Terminal resistance (phase-to-phase)	0.19 Ω
Total mass	263 g
Length of motor	60 mm
Diameter of motor housing	32 mm

The diameter of this motor (32 mm) is the smallest of the motors in the final selection stage. Two of these motors, mounted side by side would occupy a minimum of a 64 mm (2.5 in) width. The purpose is to fit this entire design within the size of a reasonable forearm, which seems feasible with this selected motor.

Figures 5.4 (a) and (b) depict the relationship for torque and angular speed of the prosthetic arm using this motor and the graphical techniques developed earlier in section 5.5. The torque values in these figures include the assumed 25% efficiency of the

Figure 5.4 (a) Maxon Motor 118891 [Maxon Motor, Inc., 1998] performance for varying gear ratios and constant terminal voltage.

Figure 5.4 (b) Maxon Motor 118891 [Maxon Motor, Inc., 1998] performance for varying gear ratios and constant thermal torque.

transmission and only consider the torque output of one of the branches of this configuration. In Figure 5.4 (a), the maximum thermal and operating torque (maximum thermal torque/SF) are both indicated. The 767:1 gear ratio line in this figure depicts the operating angular velocity of the arm under varying torque loads, with the motor operating at its nominal voltage. Figure 5.4 (b) indicates the output torque and speed of the prosthetic arm with the motor operating at varying terminal voltage levels and constant output torque (maximum thermal torque). These figures are useful as a "quick check" to determine the changes in performance of the design if the final gear ratio wasn't exactly 767 or if the original efficiency estimate was shown to be incorrect.

The allocation of this total gear ratio is shown in Figure 1.2. The total gear ratio as well as the effective elbow radius are now known and thus the remaining gear ratio at the motor-end of the spring can be determined by rearranging (5-4) such that:

$$\frac{N_G gh}{r_p} = \frac{GR_{Cl}}{R} = \frac{767}{0.05 \text{ m}} = 15340 \text{ m}^{-1} \quad (5-17)$$

Effectively, there is an infinite combination of N_G , gh , and r_p which would satisfy (5-17) but to optimize the design for this application some assumptions must be made. This design should be as compact as possible, so we begin by assuming a radius of the pinion of 1/4" (6.35 mm). Substituting this value of r_p into (5-17) leaves:

$$N_G gh = 97.4 \quad (5-18)$$

This gear ratio dictates the output speed and torque to the pinion shaft. The selection of N_G and gh are closely related. The gearhead ratio attached to the motor dictates the speed and torque seen by the worm. This influences the size requirements of the worm and

worm gear. The gear ratio associated with the worm and worm gear, N_G , has several restrictions placed on it. First of all, in order that the worm and worm gear be nonbackdrivable a small lead angle is required ($<5^\circ$ [INCOM International Inc., 1985]). This requires the worm to be a single thread. In the case of a single thread worm, the gear ratio, N_G , simply represents the number of teeth on the worm gear.

Since the focus of this thesis is on the nonbackdrivable components of this transmission, a more thorough design of the worm and worm gear was conducted. Juvinall and Marshek [1991] present a simplified approach for sizing worm and worm gears, which is used in this design. Unlike most other gear types, load capacity of worm and worm gears is often limited by cooling capacity as opposed to strength considerations. In this gear selection procedure, it is assumed that the cooling requirements do not limit the design rather the strength of the gear mesh is limiting design factor. Typically, for good wear reasons, worms are made of hardened steel and worm gears from special gear bronze (SAE 65).

The sizing of the worm and worm gear for this design began with estimating the tangential dynamic load on the gear teeth, F_d , using:

$$F_d = F_{gt} \frac{1200 + v_G}{1200} \quad (5-19)$$

where F_{gt} is the maximum tangential force applied to the worm gear tooth and v_G is the pitch line velocity of the worm gear in feet per minute. Next, the limiting bending fatigue load of the worm gear, called the strength capacity, F_s , was determined using:

$$F_s = S_n w P_c y \quad (5-20)$$

where S_n was the zero-to-maximum bending fatigue strength of the worm gear bronze (assumed to be 24 ksi [Juvinal, Marshek, 1991]), w is the gear face width, P_c is the gear circular pitch and y is the Lewis form factor (assumed to be 0.100 for 14.5° pressure angle [Juvinal, Marshek, 1991]). Finally, the "wear" capacity, F_w , of the worm and worm gear mesh was calculated (assuming proper lubrication) using:

$$F_w = d_G w K_w \quad (5-21)$$

where d_G is the pitch diameter of the gear, b is its face width, and K_w is the material geometry factor (assumed to be 80 lb/in² for a hardened steel worm on a bronze gear with a lead angle less than 10° [Juvinal, Marshek, 1991]).

Though not all the failure-related factors have been addressed in this analysis, a safe worm and worm gear design requires that:

$$F_d \leq F_s \quad (5-22)$$

and

$$F_d \leq F_w \quad (5-23)$$

Having established the design criteria for the worm and worm gear, the next step was to look at possible gear sets and sizes that satisfied both (5-18) and (5-22)-(5-23). This selection procedure began by compiling all possible single thread worm and worm gear combinations from the Boston Gear catalog [INCOM International Inc., 1985], with worm gears no larger than 3.5" in diameter. This list is provided in Table 5.7.

Table 5.7 Boston Gear worm and worm gears (worm gears < 3.5")

Diametral pitch	Number of teeth - N_G	Pitch diameter (in.)	Face width (in.)	Strength capacity, F_s (N)	Wear capacity, F_w (N)
24	20	.833	.21875	306	65
	30	1.25	.21875	306	97
	40	1.667	.21875	306	130
	50	2.083	.21875	306	162
	60	2.5	.21875	306	195
	72	3	.21875	306	234
	80	3.333	.21875	306	259
16	20	1.25	.3125	655	139
	30	1.875	.3125	655	209
	40	2.5	.3125	655	278
	50	3.125	.3125	655	348
12	12	1.00	.5000	1397	178
	16	1.333	.5000	1397	237
	20	1.667	.5000	1397	297
	30	2.5	.5000	1397	445
	40	3.333	.5000	1397	593
10	16	1.6	.625	2096	356
	20	2.0	.625	2096	445
	30	3.0	.625	2096	667
8	8	1.000	.75	3144	267
	12	1.500	.75	3144	400
	16	2	.75	3144	534
	20	2.5	.75	3144	667
6	16	2.667	1.0	5589	949
	20	3.33	1.0	5589	1185

Next, the bending fatigue strength capacity, F_s , and wear capacity, F_w , were calculated for each gear pair and are shown in Table 5.7. The next step was to calculate the dynamic load capacity for the worm and worm gear for this configuration. The dynamic load capacity is a function of the gear ratio of the gearhead, since varying gh changes F_{Gt} and v_G . First, the pitch line velocity of the gear can be defined as,

$$v_G = \omega \left(\frac{d_w \tan(\lambda)}{2gh} \right) \quad (5-24)$$

where d_w is the pitch diameter of the worm and λ is the lead angle of the worm. If we assume the motor is putting its maximum power, $P_{\max}$, then the tangential force on the gear, F_{Gt} , can be approximated using.

$$F_{Gt} = \frac{P_{\max}}{v_G} \quad (5-25)$$

If we substitute (5-24) and (5-25) and (5-19), and account for the unit conversion, we arrive at an expression for the dynamic load capacity in terms of the gearhead ratio, gh ,

$$F_d = P_{\max} \left[\frac{2gh}{d_w \omega \tan(\lambda)} + 0.164 \right] \quad (5-26)$$

For the Maxon Motor 118891, its maximum power output, $P_{\max}$, is 40 W and its maximum angular velocity, ω , is 1587 rad/s. The dynamic load capacity for varying worm gear diametral pitch, worm pitch diameter, lead angles and gearhead ratios are presented in Table 5.8.

Table 5.8 Dynamic Load Capacities for varying worm pitch, lead, and gearhead sizes (Configuration 1)

Diametral pitch	Worm pitch diameter (in.)	Lead angle of single thread (deg)	Dynamic Load Capacity (N)				
			Available gearhead ratios (shown in bold)				
			3.71	4.80	5.75	13.80	17.83
48	0.333	3.583	360	463	554	1319	1703
32	0.438	4.083	242	311	371	882	1138
24	0.500	4.767	183	235	280	663	855
16	0.625	5.717	124	159	189	444	572
12	1.000	4.767	95	121	143	335	431
10	1.250	4.567	80	102	121	281	361
8	1.500	4.767	65	83	98	225	289

The final step of the selection process was to find worm and worm gear, along with a gearhead ratio which satisfied all three criterion presented in (5-18), (5-22) and (5-23).

Several combinations satisfied the criteria, but the final selection was a 12 DP, 20 teeth worm gear with a 1" pitch diameter worm and a corresponding gearhead ratio of 5.75 (model 114572). This combination provided a combined gear ratio of 115, close to the desired 97.4, the difference of which can be compensated for by using a pinion gear with a pitch diameter of 0.295" (7.5 mm). This final selection, though not necessarily the most compact gear combination, provided a safety factor on wear capacity of 2.08 which should compensate for addition load factors such as shock, intermittent operation, and general service factors. As well, the use of the 5.75 gearhead reduces the maximum speed of the worm gear to a reasonable level of 275 rad/s (43.9 rev/s). For the given worm and worm gear selection, the pitch line velocity, v_G , is approximately 57.4 ft/min which results in an approximate kinetic friction coefficient, μ_k , of 0.067 [Juvinal, Marshek, 1991]. Table 5.9 summarizes the gear selection results along with further details of the selected hardware.

Table 5.9 Configuration 1 gear selections

Gear Components	Parameters	Values
Required gear ratio		97.4
Actual gear ratio		115
Single thread worm	Manufacturer	Boston Gear
	Item code	12960
	Material	Hardened steel
	Diametral pitch	12
	Pitch diameter	1" (25.4 mm)
	Lead angle	4°46'
	Pressure angle	14 1/2°
Worm gear	Manufacturer	Boston Gear
	Item code	13626
	Material	Bronze
	Number of teeth	20
	Diametral pitch	12
	Pitch diameter	1.667" (42.3 mm)
	Pressure angle	14 1/2°
	Face width	1/2" (12.7 mm)
Motor gearhead	Manufacturer	Maxon Motor
	Model number	114572
	Type	Planetary
	Number of stages	1
	Absolute reduction	5.75
	Maximum efficiency	80%
	Diameter	32 mm
	Length	26.5 mm
Pinion	Pitch diameter	0.58" (14.7 mm)

Knowing how fast the stiffness can be changed in Configuration 1, while holding the prosthesis position constant, is of particular interest. For this motor and gearing arrangement, the fastest we can stretch the spring at steady state is approximately 103 mm/s. Since the springs being used are nonlinear, this value cannot be directly converted

into a rate of stiffening, but it will be helpful in sizing the stiffness-control motor in Configuration 3.

5.7 Motor and Gear Selection for Configuration 3

5.7.1 Position Motor and Gearing

Configuration 3's motor and gear requirements are quite different from that of Configuration 1, which has already been highlighted. The position motor will first be selected along with required gearing, then the stiffness motor will be finalized. In this configuration, the position motor must be capable of lifting the entire external load, therefore as a started the approximate maximum power required for this motor is the prosthesis maximum power given in (5-13) which is 26.63 W. To begin, the power ratings of this motor should be in the range of 39.94 W to 53.25 W (based on the aforementioned rule of thumb of 1.5 to 2 times the required power rating). Examining the brushless motors collected in Table 5.4 indicates that all the motors except the first Micro Mo motor (Model 2036) can be considered for the position motor. As in Configuration 1, we estimate the required gear ratio by assuming a safety factor of 1.25 and a transmission efficiency of approximately 40%. This efficiency is higher than that selected for Configuration 1 since the number of stages between the motor and the prosthesis in this case is lower. The required gear ratio for the applicable motors of Table 5.4 is then defined by:

$$GR_{C3-pos} = \frac{(\tau_{arm,max})}{\frac{\tau_{th}}{SF} e} \quad (5-27)$$

The resulting gear ratios and corresponding loaded speeds, which were determined using (5-9), are displayed in Table 5.10. The loaded flexion time in Table 5.10 represents the average time for the prosthesis move through 135° based on the loaded speed of the motor. Again, caution should be used when examining the flexion time since the period of acceleration and deceleration are not considered.

Table 5.10 Gear ratios and loaded speeds for Configuration 3 position motor

Manufacturer	Motor Model	Thermal Torque (mNm)	Gear Ratio (GR_{C3-pos})	No Load Speed ($^\circ/s$)	Loaded Speed ($^\circ/s$)	Loaded Flexion Time (s)
Micro Mo Motors	3056 012B	26.1	1623	52740	29.2	4.6
	3564 012B	60.3	702	47100	60.3	2.2
Maxon Motors	118891	44.2	959	90960	86.1	1.6
	118894	94.7	447	39600	79.6	1.7
	118895	118.1	359	27000	67.7	2.0

Notice that in general, the speed performance of the prosthesis has suffered due to the increased gear ratio requirement for one motor. In Configuration 1, the lifting torque requirement was divided by two motors which lowered the required gear ratio for each motor and resulted in higher flexion speed times. As in Configuration 1, though, the Maxon Motor model 118891 provides the best flexion time even though its required gear ratio is the second highest of all the motors in Table 5.10. This, again, is due to the motors reasonably high thermal torque yet low thermal torque/stall torque ratio.

Since this motor is lighter and smaller than the other motors in Table 5.10 which have comparable flexion speed, it is selected along with a gear ratio of 959 for the position motor in this configuration. The key specifications for the Maxon Motor model 118891 can be found in Table 5.6.

Figures 5.5 (a) and (b) depict the relationship between torque and angular velocity using the graphical technique presented earlier in section 5.5. Figure 5.5 (a) includes both the maximum thermal torque and operating torque level along with various gear ratios. Figure 5.5 (b) indicates the performance of the prosthetic arm under constant output torque from the motor (maximum thermal torque) and for varying terminal voltages and gear ratios. The output torque values shown in both these figures include the assumed 40% transmission efficiency. These figures serve as a quick reference with respect to performance if the actual gear ratio is not 959 or if the assumed efficiency is determined to be incorrect.

The selection of the worm and worm gear for the position motor was conducted in the same fashion as that presented in the Configuration 1 case. The wear capacity and strength capacity values are unique to the gear pairs and therefore are unchanged for this selection. The only difference is the dynamic load capacity for the position motor since the total gear ratio is different from that of the Configuration 1 motors. Table 5.11 summarizes the dynamic load capacity for the position motor for varying worm/worm gear pairs and gearhead sizes. The gearheads listed in Table 5.11 are available from Maxon Motor and are compatible with the 118891 motor.

Figure 5.5 (a) Maxon Motor 118891 [Maxon Motor, Inc., 1998] performance for varying gear ratios and constant terminal voltage (position motor).

Figure 5.5 (b) Maxon Motor 118891 [Maxon Motor, Inc., 1998] performance for varying gear ratios and constant thermal torque (position motor).

Table 5.11 Dynamic load capacities for varying worm pitch, lead, and gearhead sizes (Config. 3 Position Motor)

Diametral pitch	Worm pitch diameter (in.)	Lead angle of single thread (deg)	Dynamic Load Capacity (N)				
			Available gearhead ratios at motor (shown in bold)				
			27.60	51.24	66.22	85.58	102.51
16	0.625	5.717	882.5	1632.8	2108.8	2723.4	3260.8
12	1.000	4.767	663.7	1226.6	1583.2	2044.2	2447.3
10	1.250	4.567	555.5	1025.8	1323.7	1708.8	2045.5
8	1.500	4.767	444.7	819.9	1057.7	1364.9	1633.8
6	2.000	4.767	335.2	616.56	794.9	1025.4	1226.9

As was the case in the gear selection for Configuration 1, several worm/worm gear combinations satisfy (5-22) and (5-23). The final selection was a 6 diametral pitch worm with a 20 tooth worm gear. This selected gear has a safety factor of 1.92 on wear capacity which should allow for reasonable shock loading without failure. The gearhead selected has an absolute gear ratio of 51.24 which combined with the 20:1 gear ratio of the worm/worm gear provides a total gear ratio of 1025, slightly higher than the required 959. This selection of gearhead provides a surface speed of approximately 12.9 ft/min which corresponds to a kinetic frictional coefficient between the worm and worm gear teeth of approximately 0.09 [Juvinall, Marshek, 1991]. Table 5.12 summarizes the details of the position motor gearing.

Table 5.12 Configuration 3 position motor gear selection

Gear Components	Parameters	Values
Required gear ratio		959
Actual gear ratio	Gear ratio	1025
Single thread worm	Manufacturer	Boston Gear
	Item code	12974
	Material	Hardened steel
	Diametral pitch	6
	Pitch diameter	2" (50.8 mm)
	Lead angle	4°46'
	Pressure angle	14 1/2°
Worm gear	Manufacturer	Boston Gear
	Item code	13670
	Material	Bronze
	Number of teeth	20
	Diametral pitch	6
	Pitch diameter	3.33" (84.5 mm)
	Pressure angle	14 1/2°
	Face width	1" (25.4 mm)
Motor gearhead	Manufacturer	Maxon Motor
	Model number	1114579
	Type	Planetary
	Number of stages	3
	Absolute reduction	51.24
	Maximum efficiency	70%
	Diameter	32 mm
	Length	26.5 mm

The worm gear has a pitch diameter of 3.33" which is much larger than desired, but the size requirements reflect the output torque requirements on this single gearing stage. In order to decrease the size requirement of this gearing stage, additional gearing stages or motors with a higher maximum thermal torques are required.

5.7.2 Stiffness Motor and Gearing

As mentioned previously, the selection of the motor and gearing for the stiffness motor in Configuration 3 assumed that the proposed "counter-balance" spring was not used. This is a conservative assumption in terms of the motor and gearing selection. The maximum power requirement of this motor occurs when the springs are stretched to their maximum force and the stiffness is changing at its maximum rate. As a first design approximation, the operating linear speed for the stiffness motor was selected to match that of Configuration 1 (103 mm/s). The maximum power requirement for the stiffness motor is thus defined as:

$$P_{\max} = 2F_{\max} v_{\max} = 7.4 \text{ W} \quad (5-28)$$

$F_{\max}$ is the maximum force in the springs (36 N) and $v_{\max}$ is the maximum linear velocity which the ends of the springs must move (103 mm/s). Due to the asymmetry of Configuration 3, the position motor power requirement was higher than the motors in Configuration 1 yet the stiffness motor power requirement is lower. Table 5.13 lists several motors that exceed the 11.3 W range (1.5 times $P_{\max}$).

The total required gear ratio between each of these motors and the spring ends is defined as:

$$GR_{C3\text{-stiff}} = \frac{2F_{\max}}{\frac{\tau_{th}}{SF} e} \quad (5-29)$$

where e is the transmission efficiency assumed to be 35% and SF is the safety factor (1.25). Notice that in this case the gear ratio has units of m^{-1} . The calculated gear ratios and maximum operating linear speeds for each motor are given in Table 5.13.

Table 5.13 Gear ratios and linear velocities for Configuration 3's stiffness motor

Manufacturer	Motor Model	Thermal Torque (mNm)	Gear Ratio, $GR_{C3-stiff}$ (m^{-1})	No Load Speed (rpm)	Maximum linear velocity (mm/s)
Micro Mo Motors	1628 012B	2.6	98901	28650	23
	2036 012B	6.2	41475	17600	32
	3056 012B	26.1	9852	8790	68
Maxon Motors	118891	44.2	5818	15160	248

Again, the Maxon Motor 118891 motor is selected, with a total gear ratio of $5818 m^{-1}$ and a loaded linear speed of 248 mm/s which is almost 2.5 times that of the design goal of 103 mm/s. Details of this motor can be found in Table 5.6.

Figures 5.6 (a) and (b) depict the output force and velocity for this selected motor and varying gear ratios neighboring the calculated value of $5818 m^{-1}$. The output force values in both of these figures incorporates the assumed 35% transmission efficiency. These figures, again, serves as a quick reference with respect to performance if the final gear ratio doesn't exactly meet the calculated value or if the efficiency estimate was wrong.

The next step was to determine the gearing for this motor. Equation (5-8) indicated that there were only two stages of gearing for the stiffness motor, namely: motor gearhead and the lead screw. We begin with the selection of the lead screw. Table 5.14 displays several lead screw assemblies by Nook Industries. The selected assemblies in this table are all single thread screws with small lead angles to achieve a nonbackdrivable design. Due to the low force levels required ($72 N = 16.2 lb$ maximum), all the listed lead screws employ a plastic nut design.

Figure 5.6 (a) Maxon Motor 118891 [Maxon Motor, Inc., 1998] performance for varying gear ratios and constant terminal voltage (stiffness motor).

Figure 5.6 (b) Maxon Motor 118891 [Maxon Motor, Inc., 1998] performance for varying gear ratios and constant thermal torque (stiffness motor).

Table 5.14 Nook Industries lead screw for Configuration 3's stiffness motor

Diameter (in)	Lead Angle (degrees)	Static load (lbs)	Efficiency (%)	Backlash (in)	Adjusted Pressure (psi)	Surface Velocity (ft/min)	PV Product (psi*ft/min)
1/4	3.64	156	49	0.0060	130	51	6,595
3/8	4.03	351	49	0.0060	58	46	2,659
1/2	3.64	625	46	0.0067	32	51	1,623
5/8	2.92	976	42	0.0090	21	64	1,332

Even the smallest of these plastic nuts and lead screws were oversized in terms of maximum operating static force. The key to the lead screw selection for these plastic nuts was the heat build up and wear. According to Nook Industries, Inc. [1998], the product of the pressure on the nut and the surface velocity (relative velocity between the lead screw OD and the nut) had to be less than 10,000 psi ft/min.

The operating loads shown in Table 5.14 were based on a nominal pressure of 1250 psi. Due to the low operating load in this design, this value of pressure was scaled down using:

$$P = \frac{\text{Actual operating load}}{\text{Chart operating load}} \times 1250 \text{ psi} \quad (5-29)$$

The surface velocity for each lead screw assembly was determined using:

$$v = 98d_s \frac{0.103 \text{ m/s}}{L} \quad (5-30)$$

where 0.103 m/s is the maximum desired linear speed of the nut, d_s is the diameter of the lead screw and 98 is the unit conversion factor.

Table 5.14 includes the calculated values of PV for each of the given lead screws. All the selected screw assemblies satisfy the heat and wear design criteria, therefore the smallest

lead screw assembly was selected for this application which was the 1/4", 0.050" lead assembly. Notice its efficiency is listed as 49% with a maximum backlash value of 0.0060". The friction coefficient for the lead screw and nut is listed by Nook Industries, Inc. [1998] as 0.075 lubricated and 0.16 unlubricated.

The final step was to select the gearhead for the stiffness motor. With a total gear ratio of 5818 m^{-1} required for this system, the gear ratio of the motor gearhead was determined by rearranging (5-8) such that:

$$gh = GR_{C3\text{-stiff}} \left(\frac{L}{2\pi} \right) = (5818 \text{ m}^{-1}) \left(\frac{0.050" \cdot 0.0254 \frac{\text{m}}{\text{inch}}}{2\pi} \right) = 1.18 \quad (5-31)$$

This indicates that the lead screw assembly almost provides all the required gear ratio for the stiffness motor. For this reason, no gearhead for the stiffness motor was selected.

The lead screw, in this sense, is connected directly to the motor shaft. Having chosen no gearhead at all results in a lower gear ratio than initially required (4947 m^{-1} compared to 5818 m^{-1}) and results in faster steady state stiffening speed (Figure 5.6) approximately three times that of Configuration 1 (321 mm/s compared to 103 mm/s). This arrangement, however, cannot output the required maximum force without exceeding the thermal torque of the motor for short periods of time. Its assumed that these short durations will not damage the motor.

A summary of the gearing components selected for Configuration 3's stiffness motor are given in Table 5.15.

Table 5.15 Configuration 3 stiffness motor gear selection

Gear Components	Parameters	Values
Required Gear Ratio		5818 m ⁻¹
Actual Gear Ratio		4947 m ⁻¹
Lead screw	Manufacturer	Nook Industries, Inc.
	Item code	SSRH 91020
	Material	304 Stainless
	Pitch diameter	1/4" (6.35 mm)
	Lead angle	3.46°
	Pressure angle	14 1/2°
	Length	24" (610 mm)
Lead screw nut	Manufacturer	Nook Industries, Inc.
	Item Code	RH 30024
	Material	plastic
	Pressure angle	14 1/2°
	Length	0.41" (10.4 mm)
	Backlash	0.006" (.152 mm)

Having selected the motors and gearing for both configurations, the next step was to develop an accurate non-intersecting, non-parallel axes gear model and simulate both configurations with the above selected hardware.

Chapter 6

Gear Models

6.1 Literature Review

A simple and accurate model of a nonbackdrivable gear train was required to study the dynamic characteristics of the proposed variable stiffness prosthesis. Literature on gear modeling dates back to the 1940's and continues to be an active area of research today, but none contained nonbackdrivable gear train models. An extensive survey of gear modeling literature prior to 1988 can be found in Ozguhen and Houser [1988].

In early studies, gear models were developed using lumped parameters such as masses, springs, and dampers to model gear masses, tooth compliances, and damping effects respectively. These models were primarily concerned with determining the dynamic factor of gear sets in order to find maximum stresses in the gear teeth. Tuplin [1953] was credited with the first gear model consisting of a mass-spring system which modeled gear errors with an oscillating wedge (Figure 6.1). Many of the studies that followed used a mass-spring-damper system which was called the "boxcar" [Goodman, 1963] or "impact pair" [Dubousky, Fruedenstein, 1971] and incorporated the effects of backlash and pioneered the early investigations of gear dynamics [Hebert, McWhannell, 1977; Hunt, Crossley, 1975], as demonstrated in Figure 6.2.

Figure 6.1 Tuplin [1953] gear error model.

Many of the more recent studies have moved away from the linear, lumped parameter models, to investigated the influence of nonlinearities in gear train dynamics such as backlash, clearances, gear tooth compliances, variable gear teeth mesh stiffness, damping and friction. It is interesting to note, as Ozguhen and Houser [1988] recognized, that the recent models do not show much difference from the early linear models apart from the inclusion of such nonlinearities.

Backlash in gears is inevitable and is required to avoid teeth interference and for proper lubrication distribution. It is highly prevalent in frequently reversing transmissions, such as the nonbackdrivable transmission under consideration in this thesis. Because of its importance, much of the reviewed gear modeling literature included backlash models [Chalhoub, Zhang, 1996; Hogan, 1988; Kahraman, Singh, 1990, 1991, 1991; Sarkar, et. al., 1997; Sepehri, et. al., 1996; Yang, Lin, 1987; Yang, Sun, 1985]. Typically, backlash was modeled as a function of relative displacement between meshing teeth as seen in Figure 6.2. With such a model, the mating gears dynamics' are dependent only when in contact. To get around this problem many of the authors wrote two sets of dynamic equations, namely contact and non-contact. The backlash function was then used as a "switch" and the equations were integrated using either piece-wise or numerical integration techniques. As a means to avoid the backlash "switching" problem, Hogan [1988] suggested that backlash could be modeled as an exponential stiffness function which would simplify numerical integration, but this would result in "artificial" forces between teeth when they were not in contact. A similar model to that of backlash was used to study the effects of clearances in joints by Dubousky and Freudenstein [1971].

Figure 6.2 "Boxcar" [Goodman, 1963] or "impact pair" [Dubousky, Fruedenstein, 1971] gear model, including backlash effects.

The stiffness of gear teeth is another contributor to the nonlinear characteristics of gear dynamics. Amabili and Rivola [1997] used the bending stiffness, while others estimated the Hertzian stiffness of the gear teeth [Dubowsky, Freudenstein, 1971; Hebert, McWhannell, 1977; Yang, Sun, 1985] to represent the gear teeth compliance in their studies. Yang and Lin [1987] chose to model both the bending and Hertzian stiffness in their model. They modeled these two stiffness in series and calculated an equivalent gear tooth stiffness.

In recent years several studies have incorporated both gear tooth stiffness and damping together in a varying gear teeth mesh stiffness models. The gear mesh was defined in several different ways: a displacement function with linear viscous damping [Kahraman, Sing, 1990, 1991 (a), 1991 (b)], stiffness and variable damping [Amabili, Rivola, 1997], harmonic function of time [Nadolski, Pielorz, 1998]. Models compensated for gear teeth compliance and variable load sharing between teeth (for contact ratios $\neq 1$). These studies attempted to model the compliant and damping effects of gear trains together and study their effects on the gear dynamics.

Several studies defined the damping in the model explicitly. Most authors modeled a damper in parallel with the tooth stiffness element. These dampers were modeled as material or Hertzian damping [Dubowsky, Freudenstein, 1971; Hebert, McWhannell, 1977; Hunt, Crossley, 1975; Sarkar, et. al., 1997; Yang, Lin, 1987; Yang, Sun, 1985], linear viscous dampers [Kahraman, Singh, 1990], or nonlinear viscous dampers [Kahraman, Singh, 1990, 1991, 1991]]. One study [Hunt, Crossley, 1975], examined the

relationship between the coefficient of restitution of the gear material and Hertzian damping and determined a function for this type of nonlinear damping. The use of linear viscous dampers in models has been shown to be undesirable because of their hysteresis loop [Hebert, McWhannell, 1977; Hunt, Crossley, 1975; Sarkar, et. al., 1997; Yang, Sun, 1985]. When two gear teeth suddenly collide, linear viscous dampers produce unreasonably high forces upon initial contact due to the non-zero impact velocity, as shown in Figure 6.3. This type of damping force is not realistic and thus nonlinear and Hertzian dampers are preferred in modeling gear damping.

Friction between gear teeth was ignored or considered negligible in much of the literature. Most studies compensate for friction by lumping the losses of friction into the damping effects. It is assumed that friction is ignored in most studies since many are models of spur gear trains in which friction losses are small. When friction was considered, as in Yang and Lin [1987], only kinetic friction was modeled and effects of static friction during start-up were ignored.

The majority of the literature on gear models focused on spur gear models [Ozguhen, Houser, 1988, Yang, Lin, 1987, Yang, Sun, 1985, Kahraman, Singh, 1990, 1991 (a), 1991 (b)], presumably because of their popularity. One article focused on the load distribution across worm gear teeth (Sudoh, et. al., 1996), but no literature was found that modeled the dynamic characteristics of worm gears or any type of nonbackdrivable gear train. Other than Hogan [1988], none of the aforementioned studies used the bond graph technique to model the gear trains, and none included the effects of static friction.

Figure 6.3 Linear and nonlinear gear teeth damping models (based on Yang and Sun, 1985).

The objectives of the non-intersecting, non-parallel axes gear model developed in this thesis was to be simple yet advanced enough to take backlash, material damping, and friction all into consideration. The following is the development of a novel non-intersecting, non-parallel axes gear model.

6.2 Non-intersecting, Non-parallel Axes Gear Model

In order that the gear model be as realistic as possible it had to contain such nonlinearities as backlash, gear tooth compliance, Hertzian damping and both static and kinetic frictional effects. The goal was to develop a bond graph model that was simple yet accurate, of low order, and which could be easily incorporated into any other bond graph with little or no modification.

The bond graph of our non-intersecting, non-parallel axes gear is shown in Figure 6.4 (a), accompanied by its corresponding word bond graph (Figure 6.4 (b)) and the gear model from Figure 3.5 (Figure 6.4 (c)). This model includes all the desired nonlinear effects previously mentioned and also met the general requirements of being a simple, accurate, low order and compatible model. A description of the general structure followed by a detailed discussion of the individual components of this model is now presented.

We begin by highlighting the major components of the model, the details of each component and state equations will be discussed later. The force applied to the pinion, F_p , and the force applied to the gear, F_G , are located on either side of the bond graph. The

Figure 6.4 (a) Bond graph of non-intersecting, non-parallel axes gear set.

(b)

(c)

Figure 6.4 (b) Word bond graph for gear model in (a). (c) Non-intersecting, non-parallel axes gear model (taken from Figure 3.5).

masses of the pinion and gear are designated as m_p and m_G and are located on either side of the inner hexagon structure. Two components, compliance and damping, in parallel constitute the normal force between mating teeth and are located on the top of the hexagon structure. The frictional force is designated as a resistance and is situated at the bottom of the hexagon structure. The active bonds, which indicate signal flow with no power, from the external forces and the "0" junction at the top of the hexagon structure to the resistive friction component on the bottom of the model are required for the frictional model and will be explained later. The last major features of the bond graph are the four transformers (TF) located on the corners of the hexagon. The discussion of the details of each component begins with the purpose of these transformers.

6.2.1 Transformer Structure

Before discussing the purpose of the four transformers, several of the underlying assumptions must be understood. First, it is assumed that the faces of the wedges are perfectly straight and that contact between mating faces is flush. Based on this assumption, the normal force acting between mating gear faces can be approximated by a force which acts perpendicular to the gear face (the normal direction). The frictional force thus acts in a direction which is parallel to the gear face (the tangential direction) and is perpendicular to the normal force. One of the unique problems in developing this model was that the normal and frictional forces were not collinear with the external forces. As a result, in order to apply Newton's first law to this problem, the normal and frictional forces had to be separated into components which were collinear with the external forces. This is the purpose of transformers located on four of the corners of the

inner hexagon structure. These transformers generate a coordinate transformation from the normal and tangential directions to directions which are collinear with the external forces. The modulus of these transformers are sine and cosine functions of lead angle (λ) and pressure angle (ϕ_n). Note that the pressure angle for the simple wedge model is 0° and thus the $\cos(\phi_n)$ terms reduce to unity in this special case.

6.2.2 Outer Structure

The outer structures of the bond graph are a simple application of Newton's first law. Figure 6.5 depicts this outer structure for the pinion and also indicates the free body diagram and force polygon for the pinion. The bond graph structure shown in Figure 6.5 (a) can easily be reduced to a single 'I' junction which satisfies Newton's first law in the x direction (ie. direction in which the external force is applied). The additional 'I' junction attached at the pinion and gear masses was used intentionally to separate the hexagon structure from the masses and external forces, but is not required. However, by doing so the forces associated with bond # 3 and #16 in Figure 6.4 (a) represent the massless pinion and gear force defined in (3-22) and (3-23), respectively. Note that the coefficients of the normal and frictional forces in Figure 6.5 (a) correspond to the transformer modulus in Figure 6.4 (a) . As mentioned previously, the active bonds from the external forces to the resistive frictional component at the bottom of the model are associated with the frictional model requirements and will be explained in more detail in the friction model description (section 6.2.5).

Figure 6.5 (a) Bond graph representing sum of forces on the pinion. (b) Free body diagram of the pinion. (c) Force polygon for free body diagram in (b).

6.2.3 Compliance Model

As mentioned previously, the normal force between the mating teeth is comprised of two components, compliance and damping, in parallel at the top of the hexagon structure. Both components are nonlinear and require some explanation. First, the compliance component of the normal force incorporated both the backlash and the stiffness effect of the gear teeth in a single nonlinear function depicted in Figure 6.6. The flat region of this function represents the backlash between the teeth and the slope of the outer regions is the stiffness of the mating gear teeth when in contact. Notice that the stiffness coefficient was assumed to be constant (linear force-displacement function) in this model. As mentioned previously in the literature review of this topic, most of the studies considered either the material compliance or the bending compliance of the gear teeth as the major stiffness component of their model. In this model, both were considered. The equivalent stiffness of the mating pinion and gear teeth was determined assuming that the material compliance and bending stiffness of the teeth were in series and thus:

$$K_p = \frac{(K_{\text{bend}} K_{\text{material}})_p}{(K_{\text{bend}} + K_{\text{material}})_p} \quad (6-1)$$

$$K_G = \frac{(K_{\text{bend}} K_{\text{material}})_G}{(K_{\text{bend}} + K_{\text{material}})_G} \quad (6-2)$$

where K_p and K_G are the equivalent stiffnesses of the pinion and gear teeth, respectively, and K_{bend} and K_{material} were the bending and material stiffnesses of the teeth, respectively. The total combined stiffness of mating gear teeth, assuming the stiffnesses to be in series, could now be defined as:

$$K_{\text{eq}} = \frac{(K_p K_G)}{(K_p + K_G)} \quad (6-3)$$

Figure 6.6 Nonlinear backlash and mating teeth stiffness model.

where K_{eq} is the total equivalent stiffness between two mating pinion and gear teeth. The bending stiffness of the teeth was determined by approximating the involute profile as a constant-strength parabolic gear tooth profile [Juvinal, Marshek, 1991] where the bending stresses throughout the tooth are equal to those at the fixed end. If the force on the gear tooth is assumed to be applied perpendicular to the tip of the gear tooth, Castigliano's Method can be used to determine the deflection at the tip by:

$$\Delta_t = \int_0^h \frac{M \left(\frac{\partial M}{\partial F} \right)}{EI} dx \quad (6-4)$$

where Δ_t is the deflection of the gear tooth tip, h is the approximate height of the gear tooth, M is the applied moment caused by the force F . E is the gear tooth material's modulus of elasticity, and I is the moment of inertia about the bending axis. The moment of inertia for this parabolic profile is given by,

$$I = \frac{wb_t^3}{12} \sqrt{\frac{x}{h}} \quad (6-5)$$

where w is the width of the gear face and b_t is the approximate length of the gear tooth base. Recognizing that M is equivalent to Fx and thus its partial derivative with respect to the force, F , is simply x , (6-4) could then be solved:

$$\Delta = \frac{8Fh^3}{Ewb_t^3} \quad (6-6)$$

Therefore, the resulting tooth stiffness can be expressed as:

$$K_{bend} = \frac{wE}{8} \left(\frac{b_t}{h} \right)^3 \quad (6-7)$$

This approximation requires some justification. The force between mating teeth is not typically applied to the tip of the gear tooth, which makes this approximation an

underestimate of the true gear tooth stiffness. Also, since a parabolic profile is "thinner" than a real involute profile, the bending stiffness defined in (6-7) is again an underestimate of a real gear tooth. Even though (6-7) underestimates a real gear tooth bending stiffness, it should provide a reasonable estimate.

The other component of stiffness was material or Hertzian stiffness. The expression used for Hertzian stiffness in this model was developed by Yang and Sun [1985] who approximated the mating gear teeth as two cylinders and applied the Hertzian law which states that the elastic compression of two isotropic elastic bodies can be approximated by two paraboloids in the area of the contact point. Their resulting expression was a linear function of force and displacement given by,

$$F = \frac{\pi E L_c}{4(1 - \nu^2)} \delta \quad (6-8)$$

where

$$K_{\text{material}} = \frac{\pi E L}{4(1 - \nu^2)} \quad (6-9)$$

where E was Young's modulus, L was the thickness of the cylinders, ν was Poisson's ratio for the cylinder, and δ is the depth of penetration. These expressions of K_{material} and K_{bend} were combined as in (6-3) to determine the equivalent tooth stiffness when mating teeth were in contact.

6.2.4 Hertzian Damping Model

Several studies have developed models for Hertzian damping [Dubousky, Freudenstein, 1971; Hebert, McWhannell, 1977; Hunt, Crossley, 1975; Sarkar, et. al., 1997; Yang, Lin,

1987; Yang, Sun, 1985]. For the gear model developed in this thesis, Yang and Sun's [1985] approach was adopted due to its simplicity and apparent accuracy. As mentioned previously, the use of linear viscous dampers has been criticized by several authors because of its unrealistic hysteresis loop (Figure 6.3). The approach of Yang and Sun [1985] was to develop a damping model which was both a function of velocity and displacement:

$$F_{\text{damping}} = D\delta\dot{\delta} \quad (6-10)$$

where D was the unknown damping factor and δ , $\dot{\delta}$ were the penetration displacement and velocity, respectively. From the dynamic equation of motion, they obtained an analytical approximation for D given by:

$$D = \frac{6(1-e)}{[(2e-1)^2 + 3]} \frac{K_{eq}}{v_i} \quad (6-11)$$

where v_i was the impact velocity, and e was the coefficient of restitution which itself was a function of impact velocity given by,

$$e = 1 - 0.022v_i^{0.36} \quad (6-12)$$

To simplify simulating such a nonlinear expression, D was considered constant.

The method used to select this nonlinear damping coefficient is presented later in the discussion of the gear model simulation.

6.2.5 Friction Model

One of the most difficult features to model in a real dynamic system is friction, and for this reason many studies have been devoted to developing a realistic friction model. A thorough survey of friction models prior to 1994 can be found in Armstrong-Helouvry,

et. al. [1994] and a brief survey of basic friction modeling techniques is presented in Berman, et. al. [1996]. Much of the work in this field has stemmed from controls research in which friction models were developed to study the effects of friction on control feedback systems [Harnoy, 1997, Dupont, 1994, Dupont, Dunlap, 1995]. Several friction models attempted to characterize the asperities between mating bodies in terms of stiffness and damping [Sherif, Bassioni, 1994, Zamm, et. al., 1994] or atomic behavior [Matsukawa, Fukuyama, 1994]. Starting with an analytical representation of friction, Dupont [1993] investigated its existence and uniqueness and Heck and Ferri [1996] applied perturbation techniques to study the effects of friction in their dynamic system. Mostaghel and Davis [1997] presented a method of approximating friction using multiple continuous functions.

There were problems with adopting any of the above mentioned friction models for our simulation purposes. They were either designed specifically for a particular system simulation, were highly complex, or did not model the switch from static to kinetic friction well. For these reasons none of these models were easily adaptable to our simulation. In fact, only one friction model, proposed by Karnopp [1985], was found in the literature which was derived using bond graphs and designed for computer simulations. In this paper, Karnopp presented a simple, but effective, model for Coulomb friction which was adopted for as our friction model. Figure 6.7 (a) shows a typical interpretation of Coulomb friction where the frictional force is plotted against the velocity of a mass. Static friction maintains the mass at zero velocity until the external force, F , exceeds the maximum static frictional force, $\pm F_H = \mu_s N$, at which time the mass

begins to accelerate. Due to the nonlinear nature of the frictional force-velocity relationship in Figure 6.7 (a), neither causal relationship of $F_f = F_f(v)$ nor $v = v(F_f)$ is unique in the vicinity of $v = 0$ m/s. As a result, numerically simulating Coulomb friction around $v = 0$ m/s becomes very difficult. To avoid this problem, Karnopp proposed taking an extremely small region on either side of $v = 0$ m/s, which he referred to as the 2DV region (shown in Figure 6.7 (b)), and accepting that any mass with a velocity within this region is not moving (ie. is static). This simplifies numerical simulations since instead of checking when $v=0$ m/s (which is very susceptible to numerical error), the check becomes whether the velocity of the mass falls within the 2DV region. To best understand how Karnopp's friction model works, a simple example is now presented. This example was also used in Karnopp's paper and is illustrated in Figure 6.7 (c). This simple example includes a horizontal external force, F , being applied to a mass, m , on a friction surface with the accompanying bond graph of the system. The state equation for this system is simply Newton's first law applied to the mass,

$$\dot{p} = F - F_f \quad (6-13)$$

where $\dot{p}$ is the rate of change of momentum of the mass, m . To determine the value of the frictional force, first it must be determined whether the velocity, v , of the mass falls within the 2DV range, $-DV < v < DV$. If it does not, then the mass is assumed to be moving and the frictional force is set equal to the kinetic frictional force, $F_f = \mu_k N$. If the velocity of the mass does fall within the 2DV range then it must be determined if the external force, F , is greater than or less than the maximum static frictional force, $\pm F_H$. If the external force, F , is less than the maximum static frictional force, $\pm F_H$, then $F_f = F$ and

Figure 6.7 (a) Conventional stick-slip friction model. (b) Karnopp's [1985] friction model. (c) Mass on a frictional surface, free body diagram and bond graph model.

the sum of the forces in (6-13) equals zero and the mass remains stationary. If the external force, F , exceeds the maximum static force, $\pm FH$, then the force of friction takes on the value of the maximum static force, $F_f = FH$, and the force unbalance causes the mass to begin to accelerate. To summarize, in order to determine the force of friction we require the following simplified logic statements,

```

If  $v < \text{abs}(DV)$ 
    If  $F < \text{abs}(FH)$ 
         $F_f = F$ 
    Else
         $F_f = \text{sign}(F)FH = \text{sign}(F)\mu_s N$  (6-14)
    End
Else
     $F_f = \text{sign}(V)\mu_k N$ 
End

```

From the above explanation of Karnopp's model, it is clear that both the values of the external force and normal force are required before assigning a value to the frictional force. This explains the reason for the active bonds from the external forces and "0" junction representing the normal force in the gear model from Figure 6.4 (a) are required. In the simple example from Figure 6.7 (c) the normal force is constant, so only one active bond from the external force to the resistive frictional component is required. In fact, in all the examples presented in Karnopp's paper the normal force was constant, which makes the application in this thesis an extension of his original model.

Though these active bonds are not "real" they permit the use of a single causality on the resistive friction bond, ie. $F_f = F_f(v, N, F)$. Without these active bonds, awkward causality switching is required since during "slip" the force of friction is a function of velocity only ($F_f = F_f(v)$) while during "stick" it is a function of the other parameters such

as normal force and external forces ($F_f = F_f(N, F)$). This justifies the reason for the active bonds, but not their relative placement in the model. In earlier developments in Chapter 3, the nonbackdrivable behavior of the gear tooth interface was established using the massless forces applied to the wedge model (Figure 3.7 (b)) and not the externally applied forces. This would seem to indicate that the active bonds in the gear model in Figure 6.4 (a) should originate from bonds #3 and #16 which represent these massless forces in the system. However, the causality on bonds #3 and #16 indicated that their input to the hexagon structure (representing the massless wedges from Figure 3.7 (b)) is a velocity and that the output on these bonds is a force. Based on this causality, the massless forces are defined by the frictional force (and normal force) and not vice versa. This is one reason why the massless forces cannot be used to define the frictional force. Also, to use of the massless forces as the input to the nonlinear friction function would require knowing the acceleration of the masses in the model (based on the definition of the massless forces in (3-22) and (3-23)). This is only possible if accelerations from the previous time step are used, which would result in a friction force that "lagged" the simulation by a single time step. For these reasons, the active bonds originate from the external forces on the system.

Figure 6.8 shows the graphical results of the numerical simulation of the above friction model, simulated in MATLAB, using a sinusoidal input for the external force, F . A mass of 1 kg was chosen for this simulation along with static and kinetic coefficients of friction of 0.45 and 0.3 respectively. Notice that when the external force exceeds that absolute maximum static friction force of 4.41 N, the friction force drops to its constant kinetic

Figure 6.8 Karnopp's [1985] friction model simulation results. (a) Forces on the mass, (b) velocity of mass, (c) displacement of mass.

value of 2.94 N. When the mass velocity falls back into the effective "zero" velocity range, the system "sticks" and the force of friction becomes static once more. Notice that in the "stick" regions, the resultant force on the mass is 0 N. These results highlight the stick-slip phenomena in Figure 6.8 (a) and (b), indicating the effectiveness of Karnopp's friction model.

Though easy to implement, Karnopp's model is sensitive to the accuracy and tolerances used in the numerical integration routine. For his model to work well in numerical simulations, the accuracy of the integration routine must be greater than the order of the 2DV region. otherwise the instances of "stick" or "slip" may be missed. If the 2DV region is made too large, noticeable velocities and displacements result making the simulation less accurate.

6.2.6 State Equation Development

The state equations for the model shown in Figure 6.4 were developed using standard bond graph techniques [Karnopp, et. al., 1990]. The subscripts in the state equations refer to particular bonds of Figure 6.4. There are three energy storage elements within the model, namely the pinion and gear masses and the mating gear teeth compliance. The rate of change of momentum of the pinion is defined by:

$$\dot{p}_2 = F_p - N \sin(\lambda) \cos(\phi_n) - F_f \cos(\lambda) \quad (6-15)$$

The normal force consists of the nonlinear compliance and damping models and the friction model is the nonlinear model discussed earlier. Substituting these nonlinear functions into (6-15) gives:

$$\dot{p}_2 = F_p - \left[\Pi\{q_7\} + \Xi\{\delta, \dot{\delta}\} \right] \sin(\lambda) \cos(\phi_n) - F_f \{N, F_p, F_G\} \cos(\lambda) \quad (6-16)$$

where Π and Ξ are the nonlinear functions of the normal force compliance and damping, respectively, and q_7 is the relative normal displacement between the mating gear teeth.

Similarly, the rate of change of momentum of the gear is given by:

$$\dot{p}_{17} = F_G + N \cos(\lambda) \cos(\phi_n) - F_f \sin(\lambda) \quad (6-17)$$

Making the same substitution for the nonlinear functions as before gives:

$$\dot{p}_{17} = F_G + \left[\Pi\{q_7\} + \Xi\{\delta, \dot{\delta}\} \right] \cos(\lambda) \cos(\phi_n) - F_f \{N, F_p, F_G\} \sin(\lambda) \quad (6-18)$$

The final state variable is the rate of change of normal relative displacement between the mating gear faces described by:

$$\dot{q}_7 = \frac{\sin(\lambda) \cos(\phi_n)}{m_p} p_2 - \frac{\cos(\lambda) \cos(\phi_n)}{m_G} p_{17} \quad (6-19)$$

where m_p and m_G are the mass of the pinion and gear respectively.

Though the system is completely defined with these equations, one last equation is required to determine the equilibrium force of friction required to maintain static equilibrium when the external forces are not large enough to overcome static friction.

This value of friction is determined using a technique presented by Karnopp [1985]

which first involves writing the equation for the relative sliding velocity between the mating gear teeth faces, which is the flow variable on bond 13 of the model bond graph, and differentiating it with respect to time:

$$\dot{v}_{slide} = \frac{\cos(\lambda)}{m_p} \dot{p}_2 + \frac{\sin(\lambda)}{m_G} \dot{p}_{17} \quad (6-20)$$

When the system is "locked" in static equilibrium, the rate of change of v_{slide} is zero.

Therefore, (6-16) and (6-18) can be substituted into (6-20) and by setting the left hand side equal to zero the force of friction which is required to maintain equilibrium can be solved explicitly in terms of the system variables such that:

$$F_{f,equl} = \frac{F_p \cos(\lambda) + \eta_m F_G \sin(\lambda) + N \sin(\lambda) \cos(\lambda) \cos(\phi_n)(\eta_m - 1)}{\eta_m \sin^2(\lambda) + \cos^2(\lambda)} \quad (6-21)$$

where η_m represents the mass ratio, m_p/m_G . Though this expression is not intuitive, it does satisfy (6-20) making the rate of change of v_{slide} zero. One must realize, though, that making the rate of change of v_{slide} equal to zero does not necessarily mean that the pinion and gear have zero velocities, it just means that the relative sliding velocity between the teeth face is not changing. More importantly, setting the force of friction based on (6-21) does not mean that the system is necessarily in static equilibrium. To show this, we rewrite (6-20) in terms of velocities:

$$\dot{v}_{slide} = \dot{v}_p \cos(\lambda) + \dot{v}_G \sin(\lambda) = 0 \quad (6-22)$$

Notice that this equation could easily have been obtained from a scalar analysis of the pinion and gear velocity diagrams as well. Solving (6-22) gives:

$$\dot{v}_p = -\dot{v}_G \tan \lambda \quad (6-23)$$

Therefore, (6-22) is satisfied if both the acceleration of the pinion and gear are zero, or when their accelerations are related by the ratio given in (6-23). In this way, both the pinion and gear can be accelerating even though the rate of change of v_{slide} is zero.

The advantage to having the expression in this form is that when the bond graph model is inserted into other models, the user need only calculate the equivalent F_p and F_G and substitute them into (6-21) to determine the equilibrium friction force (assuming N is known, which must be the case when implementing this model).

Equations (6-15)-(6-23) are the fundamental equations for this model, which satisfies the original goals of simplicity and low order.

6.3 Simulation Development

Having selected the models for tooth compliance, damping, and friction, the next step was to simulate the entire non-intersecting, non-parallel axes gear model. Figure 6.9 (a) depicts a simple arrangement including a motor and gearhead connected to a worm and worm gear. The motion of the worm gear is retarded by a spring and damper hooked to ground. The bond graph of this mechanical system is shown in Figure 6.9 (b). This simple system was used to evaluate the gear model. The gearhead on the motor is treated as a ideal (100% efficient) transformer in this model. This is intentional so that the focus of the results can be placed on the non-intersecting, non-parallel axes gear model.

In order to develop a realistic model of this mechanical system, the components selected for the Configuration I design were used including the motor, gearhead, worm and worm gear. For simplicity, the spring used in this simulation was assumed to be linear (unlike the quadratic springs designed for both proposed Configurations).

Figure 6.9 (a) Mechanical system including motor, gearing, and external spring/damper. (b) Bond graph for the system in (a).

The parameter values of each of the mechanical components needed to be determined including tooth compliance, tooth Hertzian damping, and worm and worm gear inertia. First, the equivalent inertia values for the nonbackdrivable model were determined. It should be noted that in the bond graph the masses are modeled as linear inertia and not rotational inertia. The reason for this is so that the basic nonbackdrivable model could be used and the equations developed previously for this model could be applied with as few changes as possible. This transfer from rotational inertia to linear inertia also demonstrates the power and simplicity of bond graph manipulation.

The mass on the motor-side of the bond graph represents the combined inertia associated with the worm and the motor shaft. The inertia of the motor shaft is included since inertial components closest to the motor have the most substantial effect on the overall effective inertia experienced by the motor. To determine the rotational inertia of the worm, its mass was first estimated by assuming the worm could be approximated as a solid tube and thus:

$$m_w \approx \frac{\pi}{4} (d_w^2 - d_{iw}^2) L_w \rho_w \quad (6-24)$$

where d_w was the pitch diameter of the worm, d_{iw} was the inside diameter of the worm, L_w was the length of the worm, and ρ_w was the density of the hardened steel. From (6-24), the mass of the worm was estimated at 69.3 g. Next, the rotational inertia of the worm was estimated using:

$$J_w \approx \frac{m_w}{8} (d_w^2 - d_{iw}^2) \quad (6-25)$$

which is the definition of the moment of inertia for a solid tube. From (6-25), the rotational inertia of the worm was estimated as $3.1\text{e-}6 \text{ kg.m}^2$. The effective inertia, as seen through the gearhead of the motor, of the motor shaft ($3.0\text{e-}6 \text{ kg.m}^2$ given in Maxon Motor[1998]) was then added to the worm inertia and this overall rotational inertia was then reduced to a single linear inertia by:

$$m_{w\text{eff}} = \frac{J_w + J_s gh^2}{r_w^2} \quad (6-26)$$

where r_w is the pitch radius of the worm (1/2"), J_w is the calculated inertia of the worm, J_s is the inertia of the motor shaft, and gh is the gear ratio of the motor gearhead. The result was an effective linear mass of 636 g.

In a similar fashion, the linear inertia representing the worm gear was determined. First, the mass of the worm gear was approximated using the same technique as (6-24) which equaled 133 g. The effective rotational inertia for the worm gear was then determined using (6-25). Finally, this rotational inertia was converted into a linear inertia using:

$$m_{g\text{eff}} = \frac{J_G}{r_G^2} \quad (6-27)$$

where J_G was the worm gear's rotational inertia, and r_G is the pitch radius of the worm gear. The effective worm gear inertia was determined to be 60.6 g.

The bending stiffness of the worm gear tooth was determined using (6-7). For Configuration 1, a 12 diametral pitch, 1.66" diameter worm gear was selected. The overall height of the worm gear tooth was approximated using [INCOM International Inc., 1985]:

$$h = \frac{2.157}{p_d} \quad (6-28)$$

where p_d is the diametral pitch of the worm gear (12 in this case). Next, the length of the base of the tooth was estimated using the calculated tooth thickness at the pitch circle and the normal pressure angle (14.5°) [INCOM International Inc., 1985]:

$$b_t = \frac{1.5708}{p_d} + 2d_d \sin(\lambda) \quad (6-29)$$

where d_d is the height of the dedendum defined as:

$$d_d = h - \frac{1}{p_d} \quad (6-30)$$

The width of the worm gear tooth was approximated as the face width of the gear which was 0.5". The Young's Modulus for the phosphor bronze worm gear is 110 GPa [Juvinall, Marshek, 1996]. Substituting these values into (6-7) determined a bending stiffness for the worm gear of $178.4e6$ N/m.

Next, the Hertzian stiffness of the worm gear was evaluated using (6-9). The effective length, L_c , of the "cylinders" in contact was assumed to be equivalent to the face width of the worm gear (0.5"). Using 0.35 for Poisson's ratio [Juvinall, Marshek, 1996] for the phosphor bronze revealed a Hertzian stiffness for the worm gear tooth of $1.206e9$ N/m. Combining these two effective stiffness in series revealed a total worm gear tooth stiffness of $155.4e6$ N/m.

As presented earlier, the tooth stiffness of the worm should also be considered in series with the tooth stiffness of the worm gear when the mating teeth are in contact. Since the

worm is made from hardened steel and has a shorter tooth, its overall stiffness was much greater than that of the worm gear. For this reason, (6-3) can be approximated as:

$$K_{eq} \approx K_G \text{ for } K_w \gg K_G \quad (6-31)$$

Therefore the stiffness of the mating worm and worm gear teeth was approximated as the stiffness of the worm gear tooth only. Notice also that this simulation assumes that only one worm gear tooth is in contact with the worm at any given time (ie. contact ratio = 1). This is a conservative assumption, and if the contact ratio was assumed to be greater than 1 (more than one set of mating teeth in contact at any time, on average), this would be equivalent the springs in parallel in which case the net stiffness of the mating gear teeth could be multiplied by the contact ratio to obtain a corrected stiffness.

The next parameter which was determined was the Hertzian damping of the gear teeth. To simplify simulating the nonlinear Hertzian damping, the value of the coefficient, D , was assumed to be constant and was determined in the following manner. From (6-11) and (6-12), it can be seen that both the coefficient of restitution, e , and the Hertzian damping coefficient, D , are functions of impact velocity. In simulating such nonlinear functions, it is almost impossible to obtain the instantaneous value of impact velocity. Therefore, for these simulations the impact velocity was assumed constant. This impact velocity was estimated based on the approximate speed with which the worm, initially at rest, would impact a stationary worm gear tooth if it was accelerated at its maximum rate. The following procedure was used to estimate this impact velocity, and thus, the value of the Hertzian damping coefficient, D .

Based on the calculated value of total combined inertia between the worm and motor shaft in (6-26), and knowing the maximum allowable thermal torque that the motor can deliver, the maximum motor shaft acceleration can be determined using:

$$\alpha_m = \frac{\tau_{\max}}{m_{w_{\text{eff}}} r_w^2} = 472 \text{ rad/s}^2 \quad (6-32)$$

where α_m is the unloaded maximum acceleration of the motor shaft, $\tau_{\max}$ is the maximum allowable thermal torque of the motor, and J is the combined total rotational inertia of the motor shaft and worm gear. This value of rotational acceleration can then be converted into an acceleration of the equivalent mass, $m_{w_{\text{eff}}}$, defined in (6-26) by:

$$a_{\text{eff}} = \alpha_m g r_w = 34.5 \text{ m/s}^2 \quad (6-33)$$

Recognizing that the acceleration normal to the worm tooth is equivalent to:

$$a_{\text{eff}_{\text{norm}}} = a_{\text{eff}} \sin(\lambda) = a_{\text{eff}} \sin(4^\circ 46'') = 2.86 \text{ m/s}^2 \quad (6-34)$$

Assuming the normal faces of the worm and worm gear teeth are separated initially by a distance equal to the maximum backlash (0.004") [INCOM International Inc., 1985], and that the worm gear tooth accelerates linearly from rest to this maximum acceleration rate then the time to impact is given by:

$$t_{\text{impact}} = \sqrt{\frac{2d_{\text{backlash}}}{a_{\text{eff}_{\text{norm}}}}} = 0.0084 \text{ s} \quad (6-35)$$

where t_{impact} is the time to impact and d_{backlash} is the maximum backlash between the teeth.

Finally, the impact velocity can be obtained using:

$$v_{\text{impact}} = a_{\text{eff}_{\text{norm}}} t_{\text{impact}} = 0.024 \text{ m/s} \quad (6-36)$$

Substituting this value and the tooth stiffness value into (6-11) and (6-12) gives a Hertzian damping coefficient of $56.1 \text{e}6 \text{ Ns/m}^2$. This assumed value will be validated in the simulations.

Having selected and calculated all the parameters, the next step was to run the simulation. The derivation of the state equations for the model in Figure 6.9 along with the Matlab code for this simulation can be found in Appendix B. Table 6.1 summarizes the values of each of the parameters in the Matlab code and provides a brief description of each variable. Notice that the friction coefficients do not have particular values since these will be changed in order to demonstrate the flexibility of the non-intersecting, non-parallel axes gear model.

Table 6.1 . Matlab simulation variables for the model in Figure 6.9

Variable Definition	Matlab Variables	Value
Motor terminal inductance	L_a	0.03e-6 H
Motor terminal resistance	R_a	0.19 Ω
Motor torque constant	K_t	7.5 mN/A
Gearhead ratio	gh	5.75
Pitch radius of the worm	r_w	0.0127 m
Pitch radius of the worm gear	r_G	0.0212 m
Effective mass of the worm	m_w	0.6360 kg
Effective mass of the worm gear	m_G	0.0607 kg
Effective compliance of mating worm and worm gear teeth	C_{teeth}	$(155.4e6)^{-1}$ m/N
Hertzian damping coefficient	D	56.1e6 Ns/m ²
Backlash between gear teeth	backlash	0.0001016 m
Static coefficient of friction	$\mu_s (\mu_s)$	Variable
Kinetic coefficient of friction	$\mu_k (\mu_k)$	Variable
Effective "Zero" velocity region for friction model	2DV	1e-5 m/s
Radius between worm gear center and external spring and damper	R_G	0.0074 m
Ratio of worm gear pitch radius to R_G	$G = r_G/R_G$	2.8741
External spring compliance	C	$(2000)^{-1}$ m/N
External damping coefficient	b	255 Ns/m
Lead angle of the worm	$\lambda (\lambda)$	4°46"
Pressure angle of the worm and worm gear	$\phi (\phi_n)$	14.5°
Voltage input to motor	V	Input
Velocity of external wall	Vel	Input

This simulation is fourth order, containing four energy storage elements namely the two masses, the compliance of the mating gear teeth, and the compliance of the external spring hooked to ground.

Before simulating this model, some interesting insight can be obtained by simplifying this nonlinear model. By making several simplifying assumptions, this simulation can be reduced to a linear model from which eigenvalues (ie. natural frequencies) can be

obtained. There are three main sources of nonlinearities in this model which are: backlash, friction, and Hertzian damping. If we assume the gear teeth are in contact, and remain in contact, then the nonlinearity associated with the backlash can be ignored. In this case, the masses represented the worm and worm gear are connected by a linear spring (tooth compliance) and the nonlinear Hertzian damping. As will be shown in simulation, the Hertzian damping is not very significant, and dissipates little energy. The reason being that typically the relative velocity and/or the relative penetration of the mating gear teeth is always small. As a result, the damping force expressed in (6-10), provided by the Hertzian damping effect is usually negligible. Therefore, for this simplifying case, the Hertzian damping nonlinear effect can be ignored. By ignoring this damping effect, the normal force between the gear teeth becomes a function of one of the states (ie. $N = q_{1,d}/C_{teeth}$). If the system is assumed to be in motion, the frictional force is kinetic and is thus a linear function of the normal force ($F_f = \mu_k N$). Therefore, the frictional force can also be expressed as a function of one of the states (ie. $F_f = \mu_k q_{1,d}/C_{teeth}$). Based on these simplifications, the entire system can be written in the following linear state space format:

$$\dot{\mathbf{X}} = \mathbf{A} \cdot \mathbf{X} + \mathbf{B} \cdot \mathbf{U} \quad (6-37)$$

$$\frac{d}{dt} \begin{Bmatrix} p_2 \\ p_8 \\ q_{14} \\ p_{23} \\ q_{27} \end{Bmatrix} = \begin{bmatrix} -\frac{Ra}{La} & -\frac{Kt \cdot gh}{rw \cdot mw} & 0 & 0 & 0 \\ \frac{Kt \cdot gh}{rw \cdot La} & 0 & -\left(\frac{\sin \lambda \cdot \cos \Phi + \cos \lambda \cdot \mu_k}{C_{teeth}}\right) & 0 & 0 \\ 0 & \frac{\sin \lambda \cdot \cos \Phi}{mw} & 0 & -\frac{\cos \lambda \cdot \cos \Phi}{mg} & 0 \\ 0 & 0 & \left(\frac{\cos \lambda \cdot \cos \Phi - \sin \lambda \cdot \mu_k}{C_{teeth}}\right) & -\frac{b}{mg \cdot G^2} & -\frac{1}{C \cdot G} \\ 0 & 0 & 0 & \frac{1}{G \cdot mg} & 0 \end{bmatrix} \cdot \begin{Bmatrix} p_2 \\ p_8 \\ q_{14} \\ p_{23} \\ q_{27} \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{b}{G} \\ 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} V \\ V_{el} \end{Bmatrix}$$

Notice that the states are numbered to correspond with the bond graph notation in Figure 6.9. Table 6.2 depicts what each of the five states represent in terms of the model:

Table 6.2 States of the linearized model

Symbol	State Description
p_2	Flux Linkage
p_8	Momentum of worm
q_{14}	Stretch of gear teeth spring
p_{23}	Momentum of worm gear
q_{27}	Stretch of external spring

As mentioned earlier, the motivation to simplify and linearize the nonlinear model was to conduct a linear analysis. For the following nonbackdrivable simulation, a kinetic friction coefficient value of 0.0825 was selected. Based on this friction coefficient and the parameter values given in Table 6.1, the eigenvalues, eigenvectors, natural frequencies and damping ratios of the $\mathbf{A}$ matrix from (6.38) was obtained, and are presented in Table 6.3.

Table 6.3 . Eigenvalues, Eigenvectors, natural frequencies, and damping ratios of linearized system matrix A

Natural Frequency (rad/s)	48682	48682	6237	97.4	0.01
Damping Ratio	0.005	0.005	1.000	1.000	1.000
Eigenvalue	-254 + 48681i	-254 - 48681i	6237	97.4	0.01
Eigenvector	0.0000 - 0.0000i	0.0000 - 0.0000i	0.0551	0.0009	-0.0007
	0.0187 + 0.1664i	0.0187 - 0.1664i	-0.9985	-1.0000	0.7760
	0.0003 - 0.0000i	0.0003 + 0.0000i	0.0000	0.0000	0.0000
	-0.0999 - 0.9808i	-0.0999 + 0.9808i	-0.0078	-0.0080	0.0062
	-0.0001 + 0.0000i	-0.0001 - 0.0000i	0.0000	0.0005	-0.6307

These values were obtained using the *eig* and *damp* commands in Matlab. Several observations regarding the natural vibratory modes of the system can be made based on the information in this table. First of all, the two largest eigenvalues are complex conjugates indicating a natural oscillatory mode that is dominated by the momentum of the worm and the worm gear. This mode is extremely underdamped and is associated with the large mating teeth stiffness between these two gear masses. The remaining three natural modes are all overdamped. The largest of these three remaining eigenvalues (6237) is predominantly associated with the worm momentum and the flux linkage in the motor. The second smallest eigenvalue (97.4) is dominated by the worm momentum, while the smallest eigenvalue represents a very low frequency mode associated with both the worm and worm gear momentum and the external spring. This linearized model provides some insight into the dynamic behavior of this nonlinear model. It is apparent that the system consists of very fast and relatively slow eigenvalues. For this reason, a stiff integration routine (ode23s) was selected in Matlab. This linearization also indicates that the momentum of the worm dominates several of the natural modes of the

system, and thus we would expect this parameter to influence the dynamics of the system the most.

6.3.1 Friction coefficient selection

To demonstrate the flexibility and accuracy of the gear model, two simulations were conducted using the same initial conditions and motor voltage input, but with varying frictional coefficients. In Chapters 2 and 4, the distinction between the terms overhauling and nonbackdrivable was made, as they pertain to a dynamo and geared transmission. Figures (2.2) and (2.3) depicting the regions of dynamo operation indicated a motor region and a generator region. The generator region was subdivided into overhaul type I and type II regions. In review, the overhaul type I region depicted a region in which an excessive external load would backdrive a dynamo (ie. external torque opposed the motor output torque). If a dynamo was coupled to a worm and worm gear, the dynamo could only operate in this region (ie. backdrive) under steady state conditions if the coefficient of *static* friction between the gear teeth satisfied:

$$\mu_s < \tan(\lambda) \cdot \cos(\phi_n) \quad (6-38)$$

This criteria emphasizes the fact that operation in the overhaul type I region is a function of the static coefficient of friction between the gear teeth only. The overhaul type II region depicted a dynamo operating condition in which the external load would drive the dynamo faster than it would operate without the external load (ie. external torque was in the same direction as the motor output torque). Operation of a dynamo in this region can only occur in *partially* nonbackdrivable (as defined in Table 3.1) and backdrivable

transmissions. If the transmission consists of a worm and worm gear, the dynamo can only operate in this region if the coefficient of *kinetic* friction satisfies:

$$\mu_k < \tan(\lambda) \cdot \cos(\phi_n) \quad (6-39)$$

To emphasize these friction criteria, two simulations were conducted. The first simulation involves a nonbackdrivable transmission and the second is a backdrivable transmission. For the gearing selecting for Configuration 1, which is used in these simulations, the "critical" friction value defined by $\tan(\lambda)\cos(\phi_n)$ was equal to 0.08073. Table 6.4 indicates the coefficients of friction used in the following two simulations.

Table 6.4 Coefficients of Friction used in the model illustrated in Figure 6-9

Coefficients of Friction	Case 1 Nonbackdrivable	Case 2 Backdrivable
Static	0.110	0.0367
Kinetic	0.0825	0.025
Permissible Overhauling under Steady State Conditions	Neither	Both Type I and Type II

Note that based on the "critical" friction value of 0.08073, the motor with the nonbackdrivable gearing in Case 1, while operating at steady state, can not be overhauled in either type I and II modes. The motor with the backdrivable gearing in Case 2 can be overhauled in both type I and type II modes. The coefficients for the backdrivable transmission are low in Case 2 for this type of gear arrangement, but these values were chosen intentionally to highlight the difference between the two types of transmissions.

6.3.2 Simulation Task

The task in both of the following simulations uses the motor and transmission to stretch the initially compressed external spring, until its force in tension is approximately equal

to its initial compressive force. An attempt is made to stay within the thermal torque limit of the motor ($44.2\text{e-}3$ Nm equivalent to a motor current of 5.9 A) during the entire simulation.

Both cases listed in Table 6.4 were run with the same initial conditions and motor voltage input, only their frictional coefficients were different. At $t = -0.25$ seconds, the external spring is initially in compression and as a result the gear teeth of the worm and worm gear start off in contact and loaded (ie. backlash is taken up). The initial normal and frictional force between the mating gear teeth for all three cases is the same. The velocity of the worm and worm gear masses is zero initially, and there is no current flowing in the motor nor any voltage applied to the motor terminals. At $t = 0$ seconds, the voltage across the motor terminals is ramped up from 0 V to 12 V in 0.15 seconds. This voltage is held constant for 1.6 seconds then ramped down from 12 V to 0 V in 0.15 seconds.

6.4 Simulation Results

6.4.1 Plotting Notes

Before presenting the results of the simulations, a quick note must be made regarding some of the output plots. Two pages of plots are output for each simulation along with one page of figures depicting the general forces on the worm (not to scale) at several stages during the simulation.

The first figure of plots depicts the mechanical output variables for geared transmission and is used to discuss the nonbackdrivable and backdrivable nature of each gear set. In this figure is a frictional force plot that is used to depict the frictional force between the mating teeth. Along with the solid line depicting the actual frictional force, there are several other lines which depict the maximum static friction (dashed) and kinetic friction (dash-dot) forces. A "mirror image" about the abscissa is used for both the maximum static and kinetic friction force to indicate the "boundary" of friction forces. These lines aid in demonstrating the transition from the static to kinetic and kinetic to static frictional values. Further explanations of this plot will be given in the discussion of the simulation results.

The second page of graphs includes plots that depict the electrical output variable for the motor along with two other plots that correspond to Figures 2.2 and 2.3 in Chapter 2 which were presented as graphical tools to observe dynamo and transmission operations (Chapter 4). One of these plots uses motor voltage and voltage drop across the motor resistor (as in Figure 2.2) to demonstrate the system operation in the motor and generator regions. The other plot uses the applied torque on the worm and its corresponding angular velocity (as in Figure 2.3) to again demonstrate the system operation in the motor and generator regions. Further discussion of these plots is given in the simulation results.

Case 1 Results -- Nonbackdrivable Transmission

Figure 6.10 presents the mechanical variable outputs for this simulation. Each of the plots in this figure will be referred to by its subplot lettering. Subplot (a) depicts the

torque profile on the worm and (b) shows the torque applied to the gear during the simulation. The numbers shown in (a) indicate the times corresponding to the free body diagrams and force polygons given in Figure 6.11. Before the voltage begins to be applied to the motor at $t = 0$ s, the system is in static equilibrium. During this time, the external spring is exerting a positive torque on the gear yet no torque is required on the worm to maintain equilibrium, indicating that the transmission is indeed nonbackdrivable. Referring to Figure 3.5, this initial configuration corresponds to the gear tooth in contact with the top worm tooth. At this time, the normal force between the gears is negative (j) and the friction force is positive (i) based on the selected sign convention in Figure 3.6. Figure 6.11 (a)-(1) shows these forces on the worm tooth at $t = -0.125$ s and the force polygon satisfying static equilibrium. At $t = 0$ s, the voltage to the motor (Figure 6.12 (a)) begins to increase rapidly from 0 V to 12 V over the 0.15 s. During this time, the torque on the worm begins to increase rapidly. At $t = 0.008$ s, torque on the worm is sufficient to overcome the maximum static friction force shown in (i) and the system begins in motion. The actual friction force in (i) is given by a solid line while the maximum static friction line is dashed and the kinetic coefficient line is given by the dash-dot line. Notice that once the actual friction force reached the maximum static friction force line in (i), the actual friction force "immediately" dropped onto the kinetic friction force line and the system started in motion. With the system now in motion, the sliding velocity (g) between the gear teeth, along with the worm (c) and gear (d) angular velocities become non-zero. Notice that the ratio of the angular velocities of the worm and gear throughout this simulation is equal to their gear ratio, 20. The torque on the worm (a) continues to climb initially as it accelerates the worm and

Figure 6.10 Case 1 results, mechanical output variables (nonbackdrivable transmission).

Figure 6.11 (a) Free body diagrams of the worm tooth corresponding to the noted times 1, 2, and 3 in Figure 6.10 (a).

Figure 6.11 (b) Free body diagrams of the worm tooth corresponding to the noted times 4, 5, and 6 in Figure 6.10 (a).

gear. It reaches a maximum torque of 0.24 Nm at $t = 0.115$ s. The forces on the worm tooth at this time are depicted in Figure 6.11 (a)-(2). As the system reaches steady state, around $t = 0.200$ s, the torque on the worm drops to a near zero value. This is due to the fact that initially the external compressed spring is being "unwound" and therefore the system is moving in a direction to lower the potential energy in this external spring. For this reason, the worm need only overcome the small force of friction (i) at this time. The forces on the worm at $t = 0.5$ s are shown in Figure 6.11 (a)-(3). As the system is moving the external torque on the gear (b), which is being applied by the external spring and damper, continues to decrease steadily. This is again due to the "unwinding" of the initially compressed external spring. At approximately $t = 0.872$ s, the external torque on the gear is insufficient to maintain contact between the worm and gear teeth, and the mating teeth separate. This is depicted in the relative normal displacement plot in (h), as the line transverses the backlash region between the worm and gear teeth. While the teeth are not in contact both the normal (j) and friction (i) forces go to zero. At $t = 0.878$ s, the worm and opposing gear teeth collide and transients ensue. These transients are most evident in (d), (i), and (j). The impact causes the normal force (j), and thus the frictional force (i), to increase dramatically. Figure 6.13 contains a "blown-up" image of the relative displacement between the two gear teeth during the impact transient. Notice the increasing frequency of consecutive "collisions" or impacts between the mating gear teeth until approximately $t = 0.895$ s, at which time the mating teeth are in continuous contact (represented by the gear tooth in contact with the bottom worm tooth in Figure 3.5). The normal force (j) has changed sign after the impact but the frictional force (i) remains positive. As the system continues to move, the external spring begins to get

stretched further from its neutral position, placing it in tension. This causes the torque on the gear to reverse direction. Now the system is storing potential energy in the spring and as a result the torque on the worm (a) increases. The forces on the worm tooth at $t = 1.5$ s are shown in Figure 6.11 (b)-(4). The increasing torque on the worm (a) results in a slightly decreased angular velocity for both the worm (c) and gear (d). At $t = 1.85$ s, the voltage applied to the motor begins to ramp down. As a result, the torque on the worm (a) drops off quickly. At $t = 1.878$ s, the torque on the worm (a) falls negative, and is now in the opposite direction to its angular speed. This is caused by the high deceleration rate of the system and is shown vectorily in Figure 6.11 (b)-(5). During this deceleration phase, the angular velocity of the worm (c) and gear (d) decrease rapidly. As the deceleration decreases, the torque on the worm (a) briefly becomes positive again (since the voltage input is still positive at this time) until $t = 1.994$ s when the sliding velocity (g) between the gear teeth falls back into the DV (effective "zero" relative sliding velocity) and the system "re-locks". This transition from kinetic to static is shown best in the friction plot (i) as the actual friction line drops from the positive kinetic value to the negative static value. At this point in time, the system is back in static equilibrium even though a nonzero torque is still being applied to the gear (b). This is presented vectorily in Figure 6.11 (b)-(6).

Figure 6.12 presents the power variable outputs for the motor in this simulation. Subplot (a) is the voltage input to the motor terminals and (b) is the current in the motor. The profile of the current plot is the same as that of the gear torque, as expected. This plot indicates that the thermal motor current of 5.9 A was not exceeded during simulation.

The current reaches its maximum during the initial acceleration of the system and rises after $t = 0.9$ s when the external spring is placed in tension. When the current slips negative at $t = 1.878$ s, so too does the power to the motor terminals shown in (c). At this time the dynamo is overhauling (type II) as energy flows from the decelerating worm and motor shaft inertia back into the motor. As indicated earlier, during this time the torque and angular velocity of the worm (and thus dynamo shaft) are in opposite directions. Prior to the system "re-locking", current in the motor briefly becomes positive as a result of the low deceleration rates of the system at this time.

The operation of the motor and transmission is further highlighted in (d), (e), and (f) which correspond to the graphical interpretations of dynamo and transmission presented in Figures 2.2 and 2.3. Subplot (d) plots the terminal voltage versus the voltage drop across the resistor in the dynamo. The dashed line in (d) represents the stall torque line from Figure 2.2. Subplot (e) plots the angular velocity of the worm versus its applied torque, while (f) plots the angular velocity of the worm versus its massless torque (defined in (3-22)). The numbers shown in these figures correspond to the numbering on the worm torque plot in Figure 6.10 (a).

In Figure 6.12 (d), the dynamo operating points starts at (1) at the origin and moves into the motor region towards (2) as both the voltage and current in the dynamo increase. As the torque requirement on the worm decreases, the dynamo operating point moves down to (3), still in the motor region. After the impact of the gear teeth has occurred, the worm torque increases and the dynamo operating point moves upward toward (4). As the

system decelerates, the operating point moves into the overhaul type II region towards (5). Finally, the voltage is decreased to zero and the dynamo operating point moves back to the origin at (6).

The graphical results in (e) and (f) are explained simultaneously to highlight the effects of inertia and unsteady state operation on the nonbackdrivable gear model. Recall from Chapter 4 that the location of the operating point of the worm in (e) corresponds to the region in which the dynamo is operating (ie. motor vs. generator regions). The operating point in (f) corresponds the behavior of nonbackdrivable gear interface, and not directly to the dynamo's operation. This is due to the fact that the torque "seen" by the dynamo is the applied torque not the massless torque (except when the system is operating at steady state at which time these two torques are identical). In both figures, the system starts at the origin at (1). As the applied torque on the worm increases, both operating points in (e) and (f) move into the motor region towards (2). It is interesting to note that the direction of travel of the operating point is clockwise in (e) and counterclockwise in (f). As the torque requirement on the worm decreases, the operating point in (e) moves downward to (3) while in (f) the operating point moves upward to (3). Both operating points remain in the motor region during this time. The impact, resulting in high decelerations, is evident in (f) from the collection of near-vertical lines near (3). When the applied torque on the worm begins to increase again (as the external spring is being stretched), the operating points in (e) and (f) move toward (4). When the terminal voltage begins to decrease, and the worm decelerates quickly and the direction of the applied torque changes. As a result, the operating point in (e) slips into the overhaul type

Figure 6.12 Case 1 results, electrical output variables (nonbackdrivable transmission).

Figure 6.13 Impact induced oscillations between mating gear teeth (taken from Figure 6.10 (h)).

II region towards (5) while the operating point in (f) remains in the motor region and also moves towards (5). The operating point in (e) slipping into the overhaul type II region, as mentioned earlier, is a result of the kinetic energy stored in the worm and motor shaft inertia being transferred to the dynamo. As the voltage of the motor is decreased to zero, the operating points in both (e) and (f) return to the origin. The operating point in (f) remaining in the motor region is further evidence that no power is being transferred across the interface from the gear to the worm during this simulation at any time.

Case 2 -- Backdrivable Transmission

Figure 6.14 presents the mechanical variable outputs for the backdrivable simulation. These results are similar to Case 1, but their differences will be highlighted in the following discussion.

The initial conditions in this simulation are identical to that of Case 1, except the friction coefficients have been changed. The numbers in Figure 6.14 (a) correspond to the worm free body diagrams and force polygons shown in Figure 6.15. Notice that as soon as the simulation starts ($t = -0.25$ s), a negative torque begins to develop on the worm (a) and the worm begins to be backdriven as indicated by the opposing directions of the worm torque (a) and its angular velocity (c). This is depicted in the force polygon shown in Figure 6.15 (a)-(1). During this time, some of the potential energy from the external spring is being transferred across the gear teeth interface, increasing the kinetic energy of the worm and the electrical energy in the dynamo. At this time, both the worm (c) and gear (d) are in motion and the sliding velocity between their mating teeth (g) is outside

the DV region. This causes the friction force (i) to become kinetic almost immediately. Before $t = 0$ s, the torque on the worm (a) has reached a near steady state value as well the worm (c) and gear (d) are moving at steady state. At $t = 0$ s, the voltage to the motor begins to ramp up from 0 V to 12 V over the next 0.15 s. As a result, the torque on the worm (a) moves back into the positive region and reaches a maximum of 0.20 Nm at $t = 0.120$ s. The forces on the worm at this time are shown in Figure 6.15 (a)-(2). The torque on the worm (a) begins to descend as the system reaches steady state around $t = 0.200$ s. At this time, the torque on the worm has again become negative as more potential energy is released from the external spring (b). As the external torque on the gear (b) decreases further, the torque on the worm moves toward zero. The forces on the worm tooth at this point are again shown in Figure 6.15 (a)-(3). At $t = 0.865$ s, the torque on the worm (a) becomes positive while the torque on the gear (b) becomes negative. At very short time after this, at $t = 0.892$ s, the worm and gear teeth lose contact (h), and the worm impacts the opposing gear tooth and transients ensue. After the transients die out, the system continues to stretch the external spring, storing more potential energy in it. As a result, the torque on the worm (a) continues to climb. The forces on the worm tooth at this stage are shown in Figure 6.15 (b)-(4). At $t = 1.85$ s, the voltage to the motor begins to descend. As in Case 1, the torque on the worm (a) falls negative during this time as the worm and gear decelerate quickly. The forces on the worm tooth are depicted in Figure 6.15 (b)-(5). Notice that unlike Case 1 in which the nonbackdrivable transmission "re-locked" once the voltage to the motor became 0 V, this backdrivable system remains in motion after $t = 2.00$ and the friction force (i) remains kinetic though it changes sign. At this time, the external torque on the gear (b) is negative resulting in a positive torque

Figure 6.14 Case 2 results, mechanical output variables (backdrivable transmission).

Figure 6.15 (a) Free body diagrams of the worm tooth corresponding to the noted times 1, 2, and 3 in Figure 6.14 (a).

Figure 6.15 (b) Free body diagrams of the worm tooth corresponding to the noted times 4, 5, and 6 in Figure 6.14 (a).

on the worm (a). The forces on the worm tooth at this time are shown in Figure 6.15 (b)-(6).

Figure 6.16 presents the power output variables for the motor in this simulation. Subplot (a) depicts the voltage input to the motor (same as Case 1) and (b) shows the current in the motor. As soon as the simulation begins, the current slips negative due to the transmission and motor being overhauled in type I mode. Once the voltage to the motor increased, the current became positive again but fell negative once more after the system had finished accelerating. As the potential energy in the spring was released, the current in the motor moved toward zero and became positive again at $t = 0.865$ s. When the terminal voltage descended, the current again fell negative until the system finished decelerating. At this time, the current became positive once more (even after the terminal voltage reached 0 V) as the external load overhauled the motor (type I) through the backdrivable gear set. The power flow at the motor terminals (c) is zero initially even though current is flowing prior to $t = 0$ s. This indicates that all the current in the motor prior to this point is being dissipated as heat through the resistor (which is part of the definition of overhaul type I). At $t = 0$ s, the power becomes positive but then follows the trend of the current and slips negative at $t = 0.160$ s. At $t = 2.000$ s, the power at the motor terminals becomes zero and the remaining current still flowing in the motor gets dissipated as heat by the armature resistance.

Subplots (d), (e), and (f) as explained in Case 1, portray the motor and worm operation in terms of power variables. In (d) the operating point starts at the origin but quickly moves

downward along the overhaul type I boarder (note that the boarder of the overhaul type I region *belongs* to the overhaul type I, ie. operating on its boarder indicates overhaul type I operation except at the origin) and operates at (1) until the motor voltage begins to increase. As the voltage increases, the operating point moves into the motor region towards (2). When the terminal voltage reaches 12 V, the operating point moves into the overhaul type II region at (3). As the external load increases (stretching the spring), the operating point moves back into the motor region momentarily at (4). The terminal voltage is then ramped down and the operating point moves back into the overhaul type II region towards (5). At the end of the simulation, the dynamo is operating in the overhaul type I region once again at (6).

Figure 6.16 (e) and (f) represent the applied and massless torque on the worm versus its angular speed, respectively. In both (e) and (f), after the simulation starts, the operating point quickly moves into the overhaul type I region. The fact that the operating point in (f) is located in an overhaul region at all indicates that the transmission is backdrivable. As the voltage is increased, the operating point in (e) moves into the motor region towards (2) while the operating point in (f) remains in the overhaul type II region also moving toward (2) (indicating that power is still being transferred from the gear to the worm across the gear teeth interface). Again, notice that the direction of the operating point in (e) is clockwise and in (f) is counterclockwise. The system reaches maximum speed at (3) with both operating points from (e) and (f) located in the overhaul type II region. As in Case 1, the decelerations associated with the gear teeth impact are evident in (f) from the vertical paths near (3). The increasing external torque caused the

Figure 6.16 Case 2 results, electrical output variables (backdrivable transmission).

operating points in (e) and (f) to move into the motor region to (4). As the voltage is ramped down, the operating point in (e) moves into the overhaul type II region towards (5) while the operating point in (f) remains in the motor region and also moves towards (5). The fact that the operating point in (f) remains in the motor region indicates that power is not being transferred from the gear to the worm across the gear teeth interface during this time. Therefore, the external load is not overhauling the motor at this time rather the motor absorbing energy from the decelerating worm and motor shaft inertia (as in Case 1). As the motor voltage ramps down to 0 V, both operating points in (e) and (f) move into the overhaul type I region towards (6).

6.5 Discussion

The previous simulations demonstrated the usefulness and flexibility of the non-intersecting, non-parallel axes gear model. Case 1 indicated that with the model can accurately model the nonlinear behavior of a nonbackdrivable transmission. Case 2 showed that with a simple change in friction coefficients, the model can be used to simulate backdrivable transmissions including frictional and other nonlinear effects. Again, it is worth noting that the gearhead on the motor was modeled as an ideal transformer and losses, as well as backlash, included in this gearing stage were ignored in order to maintain focus on the non-intersecting, non-parallel axes gear model.

At this point, we use the results from both of these simulations to check some of the assumptions made in the earlier theoretical developments and design work. We begin by

looking at the efficiencies of both the motor and each of the transmissions in Case 1 and 2. By dividing the power input to the motor by the power output of the worm (torque on the worm multiplied by its angular velocity), an estimated motor efficiency of approximately 98% was found (compared to the reported 78% [Maxon Motor, 1998]). This discrepancy is presumably caused by the linear motor model and the fact that no other losses (ie. friction in the bearings, eddy current losses, etc.) apart from that of the resistor were modeled. In a similar fashion, the power output of the gear was divided by that of the worm to estimate the efficiency of the non-intersecting, non-parallel axes gear set. In Case 1, for the nonbackdrivable transmission the efficiency was approximately 49% compared to the theoretical value of 49.1% using (3-38). Similarly, the efficiency for the backdrivable gear set in Case 2 was approximately 76% compared to theoretical value of 76.1% using (3-38).

In developing the non-intersecting, non-parallel axes gear model, an assumption was made regarding the value of the Hertzian damping coefficient. A constant value for D of $54.4e6 \text{ Ns/m}^2$ was based on an estimated impact velocity of 0.024 m/s in (6-36). Examining the data from Case 1 and 2, the impact velocities in these two simulations ranged between 0.058 and 0.060 m/s, approximately 2.3 times higher than initially estimated. Since this nonlinear Hertzian damping has very little effect the overall dynamics of the model, and since the Hertzian damping coefficient does not change dramatically with small changes in impact velocity (like those found in these results), the initially selected value was left unchanged in these simulations and those in the next chapter.

It is worth noting that though this model was initially developed to simulate non-intersecting, non-parallel axes gears, it can also be used to model other mechanical components such as lead and power screw assemblies. Under these circumstances, the lead angle of the wedges corresponds to the lead angle of the screw and the normal pressure angle in the model (shown on the top two transformers in the model) corresponds to the thread angle of the lead or power screw assembly. This highlights further usefulness for the model and emphasizes its simplicity and generality. In Chapter 7, the gear model developed in this chapter is used to simulate the lead screw assembly of the stiffness motor design in Configuration 3.

6.5.1 Energy Savings

Of particular interest to the two proposed prosthetic designs in this thesis, is the potential energy savings associated with using a nonbackdrivable transmission, as discussed in Chapter 1. The following discussion examines the trade-off between energy savings versus low efficiency associated with a nonbackdrivable transmission.

The results from Case 1 indicated that an external load on the gear caused the backdrivable transmission to overhaul. If overhauling is undesirable, as in the case of the two proposed prosthetic designs, then a voltage must be supplied to the motor terminal in order to hold the external load in static equilibrium. This is not the case when using a nonbackdrivable transmission, as indicated by the results in Case 1 earlier. Of particular interest is the amount of energy that must be used by the motor in the backdrivable case

in order to hold an external load in static equilibrium. To determine this energy requirement, we begin with (3-50) that describes the torque transmission ratio required to maintain equilibrium:

$$\frac{\tau_{w,ml}}{\tau_{G,ml}} = \frac{\tau_w}{\tau_G} \bigg|_{ss} = \frac{-1}{G Re_w} \quad (3-50)$$

This expression, including e_w , was originally developed based on the a moving (kinetic) system but it can also be used in a steady state, static scenario when the friction force is equal to its maximum static value, $\mu_s N$. When used in this way, e_w is defined using the coefficient of static friction so that (3-46) becomes:

$$e_w \big|_{static} = \frac{\cos(\phi_n) - \mu_s \tan(\lambda)}{\cos(\phi_n) + \mu_s \cot(\lambda)} \quad (6-40)$$

In this sense, (3-50) represents the ratio of applied torques required to hold the system in static equilibrium. Rearranging (3-50), using the steady state values of the applied torques, and solving for the applied torque on the worm gives:

$$\tau_w = -\frac{\tau_G}{G Re_w \big|_{static}} \quad (6-41)$$

This is the minimum torque which must be applied to the worm by the motor in order to maintain static equilibrium when using a non-intersecting, non-parallel axes gear transmission. Assuming the use of a gearhead, with a gear ratio of gh , between the motor and the worm then the minimum torque output required by the motor is:

$$\tau = \frac{\tau_G}{gh G Re_w \big|_{static}} \quad (6-42)$$

where τ represents the minimum output torque from the motor required to hold the transmission in static equilibrium. The current required in a DC motor to output this torque is given by:

$$i = \frac{\tau}{K_t} \quad (6-43)$$

where K_t represents the torque constant of the motor. Thus, the power loss associated with holding the transmission in static equilibrium is equal to the power dissipated across the resistance in the motor, give by:

$$P_{Ra} = i^2 R_a = \left[\frac{\tau_G}{K_t ghG Re_w|_{static}} \right]^2 R_a \quad (6-44)$$

where R_a is the resistance of the motor armature. For this simplified development, the external torque is assumed constant and therefore integrating (6-44) with respect to time gives the energy required to maintain the system in static equilibrium,

$$E_s = \left[\frac{\tau_G}{K_t ghG Re_w|_{static}} \right]^2 R_a \Delta t_s \quad (6-45)$$

where E_s is the energy dissipated by the motor's armature resistance while maintaining static equilibrium and Δt_s is the total time the system is maintained in static equilibrium (assuming the applied torque to the gear is constant during this time). This result is significant since it represents a source of energy savings when a nonbackdrivable transmission is used since the motor need not supply a torque to the worm in order to maintain static equilibrium.

There is, however, a trade-off associated with using a nonbackdrivable transmission: efficiency. Using a nonbackdrivable transmission means that the efficiency during operation of the transmission cannot exceed 50% (refer to Section 3.3.2). Recalling once again, the efficiency of a non-intersecting, non-parallel axes gear set was defined in (3-46) as:

$$e_w = \frac{\cos(\phi_n) - \mu_k \tan(\lambda)}{\cos(\phi_n) + \mu_k \cot(\lambda)} \quad (3-46)$$

where the coefficient of friction is kinetic assuming the system is moving under steady state conditions. The power loss associated with this inefficiency while the transmission is in motion is given by:

$$P_k = \tau_w \omega_w (1 - e_w) = \tau_G \omega_G \left(\frac{1}{e_w} - 1 \right) \quad (6-46)$$

Recognizing that the angular velocity of the gear, during steady state, is proportional to the angular velocity of the worm through their gear ratio, then (6-46) can be rewritten as,

$$P_k = \frac{\tau_G \omega_w}{GR} \left(\frac{1}{e_w} - 1 \right) \quad (6-47)$$

With the expression of power loss in this form, and assuming the applied torque on the gear and the worm angular velocity is constant then the energy loss can be determined by integrating (6-47) with respect to time,

$$E_k = \frac{\tau_G \omega_w}{GR} \left(\frac{1}{e_w} - 1 \right) \Delta t_k \quad (6-48)$$

where Δt_k is the time during which the transmission is moving.

With (6-45) and (6-48), several basic rules can be deduced. In general, a nonbackdrivable transmission should be used when the energy savings during static operation, (6-45), exceeds the energy dissipated due to inefficiencies during moving, (6-48), which is satisfied when:

$$E_s > E_k \quad (6-49)$$

Likewise, a backdrivable transmission should be used when the energy loss associated with holding the transmission in static equilibrium, (6-45), is less than the energy loss due to inefficiencies during moving, (6-48), which is satisfied when:

$$E_s < E_k \quad (6-50)$$

These criteria are obviously not easily established. They are functions not only of the transmission characteristics (ie. μ_s , μ_k , λ , ϕ_n , gh , r_w , r_G , K_t , R_a), but also on the external torque on the gear and the steady state operating speed of the transmission. The development so far has assumed steady state conditions only, constant applied torque on the gear, constant angular velocities of the worm, and maximum static frictional force. Though oversimplified, this type of analysis can be expanded so that these assumptions are no longer required. The above equations will, however, serve as a tool for this discussion.

The other major factor affecting this type of energy trade-off analysis involves the duty cycle of the transmission in question. In other words, what percentage of time is spent stationary and what percentage is spent moving.

To demonstrate the use of the criteria given in (6-45) and (6-48), several simple examples are used. First, examining the extreme conditions indicates that if the transmission is always in motion then obviously a backdrivable transmission should be used and if the transmission is always stationary a nonbackdrivable transmission should be used. To find the "break-even" point, in terms of energy savings, (6-45) and (6-48) are set equal and solving for the ratio of time gives:

$$\frac{\Delta t_s}{\Delta t_k} = \frac{\omega_w K_t^2 g h^2 G R e_w \left|_{\text{static}}^2}{\tau_G R_a} \left(\frac{1}{e_w} - 1 \right) \quad (6-51)$$

which is a unitless ratio. Applying the criteria in (6-49) shows that in order for this nonbackdrivable transmission to result in energy savings:

$$\frac{\Delta t_s}{\Delta t_k} > \frac{\omega_w K_t^2 g h^2 G R e_w \left|_{\text{static}}^2}{\tau_G R_a} \left(\frac{1}{e_w} - 1 \right) \quad (6-52)$$

And likewise, applying (6-50) indicates that a backdrivable transmission would result in energy savings if:

$$\frac{\Delta t_s}{\Delta t_k} < \frac{\omega_w K_t^2 g h^2 G R e_w \left|_{\text{static}}^2}{\tau_G R_a} \left(\frac{1}{e_w} - 1 \right) \quad (6-53)$$

As a practical example, the nonbackdrivable transmission selected for Configuration 1 (used in the previous simulations) is considered. Let's assume that the worm is operating at its maximum speed ($\omega_w \sim 275$ rad/s), and the prosthetic arm of Configuration 1 operating at its maximum lifting torque of 13.56 Nm. This torque corresponds to a torque of $\tau_G = 5.08$ Nm applied to the worm gear in the Configuration 1 design. Based on the above definitions, GR in this example is simply the ratio of the number of teeth of the worm gear to the worm (20), and gh is the gear ratio of the gear head (5.75). Using

the static and kinetic coefficients of friction from the previous Case 1 simulations, the static ($e_w|_{\text{static}}$) and kinetic efficiencies are $e_w|_{\text{static}} = 0.4193$ and $e_w = 0.4911$ respectively. Using the motor torque constant ($7.5\text{e-}3 \text{ Nm/A}$) and armature resistance (0.19 ohms), and substituting all these values into (6-51) gives a ratio of:

$$\frac{\Delta t_s}{\Delta t_k} = 1.93 \quad (6-54)$$

This indicates that, based on (6-52), in order for the selected nonbackdrivable transmission for Configuration 1 to result in energy savings the time spend in a static (stationary) mode must be 1.93 times greater than the time spent in a kinetic (moving) mode. Though this example and development was based on several simplified assumptions, it does provide some insight into the energy trade-offs associated with nonbackdrivable versus backdrivable transmissions.

In general, for a fixed duty cycle (ie. fixed ratio of times) a nonbackdrivable transmission becomes more energy efficient when the expression on the right of (6-52) is minimized. Likewise, for the same duty cycle, a backdrivable transmission becomes energy efficient when the expression on the right of (6-53) is maximized. As noted earlier, this is not a simple calculation but it brings insight into the issue of energy savings with respect to backdrivable versus nonbackdrivable transmissions.

The design criteria for the transmission given in (6-49) and (6-50) deal specifically with the energy saving potential of the nonbackdrivable transmission. These energy relationships only deal with the energy savings associated with the transmission and do

not consider the energy savings associated with the use of real springs in the prosthetic designs.

Chapter 7

Proposed Prosthetic Arm Simulations

In the previous chapter, the non-intersecting, non-parallel axes gear model was demonstrated using a simplified model. In this chapter this model is used and is incorporated into the bond graph dynamic models of Configuration 1 and 3 which were initially studied by English [1996] and several simulations were conducted. The reader is encouraged to review the work by English [1996], specifically Chapter 5 which pertains to the dynamic simulations of Configuration 1 and 3, since much of the following work is an extension of this material. This chapter begins with a description of each of the models and their parameters, followed by a description of the simulation task, and ends with a discussion of the simulation results.

7.1 Prosthetic Arm Parameters

This section highlights several common prosthetic arm parameters that are found in both the Configuration 1 and 3 dynamic models. In an attempt to remain consistent with English's [1996] work, several values of parameters that he had selected are used in these models. These parameters and values are listed in Table 7.1.

Table 7.1 Common parameters to Configuration 1 and 3

Parameter	Variable	Value/Definition
Mass of the prosthetic arm	m_{arm}	0.280 kg
Rotational inertia of the arm	J_{arm}	$\frac{1}{3} m_{\text{arm}} L_{\text{arm}}^2 = 0.0102 \text{ kg} \cdot \text{m}^2$
Length of the prosthetic arm	L_{arm}	0.33 m
Maximum Stiffness	K_{max}	80 Nm/rad
Minimum Stiffness	K_{min}	1 Nm/rad
Effective Rotational Damping Coefficient	B	$1.414 \sqrt{J_{\text{arm}} K_{\text{max}}} = 1.275 \frac{\text{Nm} \cdot \text{s}}{\text{rad}}$

Included in this list is the overall mass (0.280 kg) of the prosthetic arm. Based on the selected hardware in this thesis, this value would appear to be optimistic. It is used in these simulations, though, since it is a design goal and since a more accurate estimate is only possible once more of the prosthetic arm components have been designed. As shown in Table 7.1, the rotational inertia of the arm, J_{arm} , is approximated as that of a uniform cylinder rotating about its end. Since the majority of the prosthetic arm mass would be close to the elbow, this estimate of rotational inertia would tend to be high.

The absolute value of damping is not given in this table, rather an equation which defines the rotational damping coefficient with respect the maximum arm stiffness and arm inertia. Based on the linear definition of damping ratio given by:

$$\xi = \frac{B}{2\sqrt{J_{\text{arm}} K^*}} \quad (7.1)$$

where B is the rotational damping coefficient, J_{arm} is the prosthetic arm inertia and K^* is the rotational arm stiffness, the definition of damping coefficient given in Table 7.1 provides a well damped system at maximum arm stiffness ($\xi = 0.707$ at $K = 80 \text{ Nm/rad}$)

and an overdamped system at minimum stiffness ($\xi = 6.32$ at $K = 1 \text{ Nm/rad}$). It should be noted that this damping value is based on the inertia of the arm alone; if a load is added to the arm, the effective inertia will increase thereby decreasing these damping ratios. Defining the damping coefficient as in Table 7.1 provides desirable non-oscillatory behavior throughout the stiffness range of the prosthetic arm (unloaded) but reduces its response time and dissipates considerable power. Note that this value of damping is implemented in the simulations as two linear dampers arranged antagonistically around the effective elbow radius. In this sense, each linear damper has a damping coefficient given by,

$$b = \frac{B}{2R^2} \quad (7.2)$$

For Configuration 1, $b = 255 \text{ Ns/m}$ and for Configuration 3, $b = 19 \text{ Ns/m}$. Even though these two linear damping values vary between configurations, they result in the same effective rotational damping and thus the same amount of energy is dissipated by each set of dampers for a given prosthetic arm velocity.

7.2 Configuration 1 Model

The complete bond graph for Configuration 1 is presented in Figures 7.1. The complete bond graph, including bond numbers, and the derivation of the state equations for this system is given in Appendix C. The total model for Configuration 1, including the nonbackdrivable models, has a total of twelve states. In Figure 7.1 there are two distinct

Figure 7.1 (a) Word Bond graph model of Configuration 1 including nonbackdrivable transmissions.

Figure 7.1 (b) Bond graph model of spring/damper, (c) bond graph model of prosthetic arm, (d) bond graph model of external impedance.

"branches" to the bond graph which model the two branches of Configuration 1. The motor model used is the same as the dynamo model illustrated in Figure 2.1. The motor model is hooked directly to the nonbackdrivable gear model which in turn is connected to the nonlinear spring and damper in parallel. These elements make up one of the branches to the bond graph model. Each branch is then connected to the prosthetic arm inertia through an effective radius of the elbow.

For the simulations conducted in this chapter, the coefficients of static and kinetic friction were selected such that the transmissions were nonbackdrivable (as defined in Table 3.1). All the gearing properties including lead angle, pressure angle, backlash, and effective masses were based on the gear selection for this configuration which are summarized in Table 5.13 and 5.16.

7.3 Configuration 3 Model

The bond graph for Configuration 3 is shown in Figure 7.2. The complete bond graph, including bond numbers, and derivations of the state equations for this model are presented in the Appendix D. This complete model of Configuration 3, including the nonbackdrivable transmission model, has a total of thirteen states. Note that the "counter" spring that was originally proposed by English [1996] for this design, was not included in this model. The motor and gearing for the stiffness actuator were designed and selected without consideration of this spring (a conservative design in that sense) and

therefore in order to compare simulated results to those developed earlier, this "counter" spring was left out of this model.

The bond graph model for this design is slightly different than that for Configuration 1. The top branch of this bond graph represents the position motor with its nonbackdrivable worm/worm gear set. The inertia of the worm gear of the position motor transmission and the inertia of the prosthetic arm are summed together in a single inertia (J_{G+arm}) since the two masses are rigidly connected in this design. The bottom branch of the bond graph models the stiffness motor with its nonbackdrivable lead screw design along with the quadratic springs and dampers of the variable stiffness actuator. Notice that in this model, the nonbackdrivable model for the position motor does not convert the worm and worm gear inertia into equivalent linear masses, instead they are left as rotational inertia. The derivation of the forces of static friction required to maintain equilibrium was conducted in the same fashion as (6-20) and (6-21) and is presented in Appendix D. The output torque from the nonbackdrivable transmission of the top branch is hooked to a "0" junction and not directly to the worm gear inertia as it usually is. This "0" junction indicates that the output torque from the position actuator is common to the other two bonds attached to the "0" and that their angular velocities sum to equal zero. This models the unique characteristic of the Configuration 3 in which the position motor is hooked between the effective elbow radius and the prosthetic arm and is free to rotate relative to either of these components. Since the position motor and its gearhead and worm are free to rotate relative to an inertial reference frame, their rotational inertia about the center of

Figure 7.2 (a) Word bond graph model of Configuration 3 including nonbackdrivable transmissions.

Figure 7.2 (b) Bond graph model of position motor and elbow inertia.

rotation (J_{motor}) had to be included in this model, which increased the number of states. For simplicity, the coefficients of friction for both the position motor transmission and the stiffness motor transmission were assumed to be the same and were chosen such that both transmissions were nonbackdrivable. The friction coefficients are slightly higher than those used in Configuration 1 since the lead angles in this configuration were slightly higher.

7.4 Dynamic Simulations

7.4.1 Task Definitions

To demonstrate the these two models, two tasks were designed. The two tasks were chosen to demonstrate the performance of the proposed prostheses and to study the operation of the motor/gearing selections and the nonbackdrivable gear model. The results of these simulations will be compared against design assumptions made in Chapter 5 and the nonbackdrivable characteristics of the transmission will be related to the earlier theoretic development from Chapters 3 and 4. All simulations were conducted using Matlab, and the its built-in Runge-Kutta integration routine, ODE.

Task 1

The first task involves changing the prosthetic arm stiffness from an initial value of 1 Nm/rad to 80 Nm/rad as quickly as possible without exceeding the maximum thermal

current limit of the motors. In this task, the arm acts in a horizontal plane (no gravitational effects).

Task 2

This task involves lifting 9 lb in a gravitational field as quickly as possible without exceeding the maximum thermal current limit in the motors. The initial arm stiffness is 30 Nm/rad, and the arm moves from -80° (approximately fully extended) to 55° (approximately fully flexed). The total flexion time required differs between configurations due to the difference in motor configurations.

7.4.2 Output Format

The results from the following simulations are presented in graphical form. For the most part, the graphs are similar to those presented in the previous chapter for the simple simulation. For each simulation there is typically a total of five pages of graphs. Table 7.2 summarizes the subplots of each of these figures and indicates what each subplot represents.

Table 7.2 Graphical results output index

Figure Type	Subplots	Subplot Representation
Nonbackdrivable gear set mechanical outputs (1 figure/motor)	(a)	External torque applied to the worm (τ_w)
	(b)	External torque applied to the worm gear (τ_G) -- this graph includes the force exerted by the quadratic springs and dampers
	(c)	Angular velocity of the worm (ω_w)
	(d)	Angular velocity of the worm gear (ω_G)
	(e)	Angular displacement of the worm (θ_w)
	(f)	Angular displacement of the worm gear (θ_G)
	(g)	Relative sliding (tangential) velocity between the mating gear teeth
	(h)	Relative normal displacement between the mating gear teeth
	(i)	Frictional forces between mating gear teeth (F_f)
	(j)	Normal force between the mating gear teeth (N)
Electrical Variable Output from the Motors (1 figure/motor)	(a)	Voltage input to the motor terminals (V)
	(b)	Current in the motor (i)
	(c)	Power flow at the motor terminals (P_{motor})
	(d)	iR_a vs. V plot (similar to Figure 2.2)
	(e)	τ_w vs. ω_w plot (similar to Figure 2.3)
	(f)	$\tau_{w,mf}$ vs. ω_w plot (similar to Figure 2.3)
Prosthetic Arm Mechanical Outputs (1 figure)	(a)	Prosthetic arm angular velocity
	(b)	Prosthetic arm angular displacement
	(c)	Prosthetic arm rotational stiffness
	(d)	Prosthetic elbow angular velocity (Config. 3 only)
	(e)	Prosthetic elbow angular displacement (Config. 3 only)

Two figures are devoted to the mechanical outputs from each of the nonbackdrivable transmissions. Two other figures depict the power and motor variable outputs for each motor, including the graphs that are similar to those shown in Figures 2.2 and 2.3. The final figure of the set presents the mechanical output variables for the prosthetic arm including arm velocity, position and stiffness. For Configuration 3, two additional plots are added to this final figure which include the angular velocity and displacement of the prosthetic elbow. The large number of plots is required to fully explain the operation of the nonbackdrivable drives along with their effect on the prosthetic arm operation. In the discussion of the graphical results, detailed times are reported that are beyond the

resolution of the graphs. This is done since many of the occurrences in the simulations happen over very short time scales.

Figures 3.5 and 3.7 of previous chapters are referred to frequently in the discussion of the task results. The reader is encourage to re-familiarize themselves with these figures and to mark them for later reference.

The results for simulations of Task 1 and 2 for Configuration 1 are presented first followed by the results for Task 1 and 2 for Configuration 3.

7.4.3 Motor Voltage Command Signals

The reader should recall that the variable stiffness prosthetic designs of Configuration 1 and 3 are designed to be controlled in an open loop fashion. The user of this prosthetic arm will "close" the loop by adjusting the command signals to the motors, ideally through the use of myoelectric control. Therefore, the voltage commands to the motors in all of the simulations presented in this chapter were obtained by trial-and-error and are not unique. A smooth "S" voltage profile was used in the simulations in order to obtain relatively smooth motor shaft accelerations. The S-curves were designed using parabolic profiles for the initial and final 20% of the overall rise time, with the linear ramp region consisting of 60% of the overall rise time. Note that the derivative of this function (rate of change of voltage) is a trapezoidal profile. A "maximum" rate of change of voltage is not defined, instead it is directly related to the time span specified between the initial and

final voltage values. The specific voltage input to each of the motors is presented graphically in each of the simulation results.

7.5 Simulation Results

7.5.1 Configuration 1 -- Task 1 Results

This involved increasing the arm stiffness from 1 Nm/rad to 80 Nm/rad in the horizontal plane (no gravitational effects). The results for this simulation are presented in Figures 7.3 through 7.5. For this simulation, since the results were symmetric for both branches of the design, only the results for the top branch presented.

Figure 7.4 (a) indicates that the voltage input to each of the motors was trapezoidal. Figure 7.3 (b) indicates that the quadratic springs, that are initially in compression, are exerting a positive torque on the worm gear that results in a negative normal force (j). Referring to Figure 3.5, this situation represents contact between the worm and mating worm gear teeth on the top mating surface, which is shown as a negative relative normal displacement in subplot (h). The voltage input command begins at $t = 0$ s though from subplot (i) it is apparent that motion of the worm is delayed slightly until the maximum static friction can be overcome at $t = 0.008$ s and the sliding velocity (g) exceeds the effective zero velocity region, DV. To initially overcome the static friction and then to accelerate the worm, the worm torque in (a) climbs steadily until it reaches a maximum value of approximately 0.21 Nm. Notice that the torque on the worm shown in (a) and the motor current shown in Figure 7.4 (b) are directly proportion, as expected. At $t =$

0.169 s the voltage input to the motor, shown in Figure 7.4 (a), reaches a steady value of 12 V and the worm torque (a) begin to decrease as the worm's angular velocity approaches a steady value. Since the quadratic springs are initially in compression, the positive displacement of the worm and worm gear are effectively "unwinding" the springs and releasing potential energy initially stored in the springs. This explains the near zero torque region between $t = 0.200$ to $t = 0.310$ s in (a) since in this region the system is operating near steady state and the worm need only overcome a small frictional force (i).

Before $t = 0.305$ s, the worm and worm gear are in continuous contact and the worm gear angular velocity and displacement are proportional to that of that of the worm. As the worm and worm gear move in the positive direction, the torque exerted by the quadratic springs on the worm gear decreases as indicated in (b) and the normal force between the mating teeth approaches 0 N (j). At $t = 0.305$ s, the combined force of the quadratic spring and damper in parallel is not sufficient to maintain contact between the worm and worm gear teeth and the worm and mating worm gear lose contact as indicated by the relative normal displacement in (h) as well by the small "flat" 0 N regions in (i) and (j). Notice that when the worm and worm gear separate, the worm gear's angular velocity (d) decreases slightly due to the small negative external torque still being supplied. As expected, the friction and normal force between the gear teeth goes to zero until the worm impacts the next adjacent worm gear tooth at $t = 0.308$ s. At this time, a high frequency, nonlinear, transient occurs (similar to those shown earlier in Figure 6.13) as the mating gear teeth impact one another. The sliding velocity is nonzero upon each impact of the

Figure 7.3 Task 1, Configuration 1 mechanical output variables.

transient and therefore the frictional force in (i) remains kinetic while the teeth are in contact. Referring again to Figure 3.5, the contact between worm and worm gear is now on the bottom contact surface. Now with the worm gear tooth in contact with the opposing worm gear tooth, the worm torque begins to increase again (a) as it stores potential energy in the springs while the external torque on the worm gear increases (b). At $t = 0.331$ s, the voltage applied to the motors begins to descend (Figure 7.4 (a)), and the torque on the worm decreases resulting in deceleration of the worm and worm gear. Initially, when the deceleration is low the forces on the worm correspond to those depicted in Figure 3.8 (a). At $t = 0.342$ s, the torque on the worm drops below 0 Nm corresponding to a sign change in the motor current (Figure 7.4 (b)). This situation is depicted by Figure 3.8 (b) in which the deceleration of the worm has increased to the point where the torque on the worm must reverse direction in order to maintain equilibrium. Even though the motor voltage doesn't reach 0 V until $t = 0.500$ s, the nonbackdrivable transmission "re-locks" at $t = 0.495$ s due to reduced sliding velocity between the mating gear teeth. This "re-locking" is indicated by the dramatic change in the frictional force in (i) which suddenly goes from a kinetic to a static frictional value. Again, the nonbackdrivable behavior of this model is indicated by the fact that no motion occurs even after the terminal voltage has decreased to 0 V.

Figure 7.4, which has already been referenced several times, includes the power output variables for each of the motors. Of particular interest is the motor current (b) and the terminal power shown in (c). The motor current is positive for the majority of the simulation but falls negative when the worm is decelerating. Note that the current in the

Figure 7.4 Task 1, Configuration 1 electrical output variables.

motor never exceeds the thermal current limit of 5.89 A. Subplot (c) indicates that from $t = 0.342$ s to $t = 0.495$ s, power is being supplied to the motor terminals. Subplots (d) and (e) both support this fact, showing that the motor slips into the overhaul type II region during this time. Subplot (f), which plots the massless torque of the worm against its angular velocity, indicates that the transmission is nonbackdrivable since its operating point remains in the motor region throughout the simulation. This shows that at no time during the simulation did power transfer across the mating gear interface from the gear to the worm, as expected with a nonbackdrivable transmission.

Figure 7.5 depicts the mechanical output of the prosthetic arm. The prosthetic arm stiffness successfully ramps from 1 Nm/rad to 80 Nm/rad as shown in (c). Subplots (a) and (b) indicate that the prosthetic arm did move a very small amount, even though the voltage commands to the symmetric branches of the arm were identical. These arm motions are a result of marginally different actuating velocities from both branches in the configuration which are assumed to be the result of computation error. It should be noted that these arm motions are of the same order as the specified tolerance for the simulations.

7.5.2 Configuration 1 -- Task 2 Results

This task involved lifting a 9 lb load from extension to flexion in a gravitational field as quickly as possible. The results from this simulation are presented in Figures 7.6 through 7.10. The results of this simulation are significantly more complex than that of Task 1 due primarily to the fact that the results are non-symmetric. Therefore, the results for

Figure 7.5 Task 1, Configuration 1 prosthetic arm mechanical output variables.

each branch must be presented separately. The voltage input to both motors was identical except one is the inverse of the other. The total simulation time was from -0.1 s to 1.75 s in this case. Note that the choice of damping coefficient, displayed in Table 7.1, to obtain a well damped system at high stiffness was based on the unloaded arm. As will be shown in these results, with a 9 lb load on the arm the effective damping ratio at maximum stiffness is only 0.16. This results in severe oscillations of the prosthetic after the movement command from the motors has ceased.

Figures 7.6 and 7.7 depict the results for the motor and transmission of the top branch. We begin by examining the mechanical outputs of the nonbackdrivable gear set given in Figure 7.6. Subplot (b) indicates that under load and at a stiffness of 30 Nm/rad, the quadratic springs are initially in compression and exert a positive torque on the worm gear which in turn resulted in an initial negative normal force (j). This initial condition corresponds to gear teeth in contact on the bottom mating surface in Figure 3.5. At $t = 0$ s, the motor voltage begins to increase (Figure 7.7 (a)), which resulted in an increase in motor current (Figure 7.7 (b)) and thus an increase in the torque applied to the worm (a). As in task 1, there is a small delay caused by static friction before the system starts in motion. The initial trapezoidal-like profile in the worm torque, between $t = 0$ s to $t = 0.240$ s, corresponds to the acceleration profile of the worm. As the worm and worm gear begin to move, the external torque on worm gear begins to decrease (b) as the potential energy in the compressed springs is released. The damping force in this branch is in the opposite direction to that of the springs. Since, the worm is "unwinding" the initially compressed springs, its required torque, after bringing the system to a near

steady state operation, is very low (a) and represents the torque to overcome friction alone. At $t = 0.222$ s, the external torque on the worm gear (b) approaches 0 Nm and is insufficient to maintain contact with the worm and the gear teeth lose contact (h). At this time the normal (j) and frictional (i) forces both go to zero. When the worm gear tooth loses contact with the worm tooth, it slows down slightly due to the small negative external torque still being applied by the quadratic spring and damper. At $t = 0.230$ s, the worm impacts the adjacent worm gear tooth and transients ensue, lasting for approximately 0.020 s. These transient are most evident in the friction (i) and normal (j) force plots but are also evident in (b), (d), and (h). Referring to Figure 3.5, the worm and worm gear teeth are now in contact on the right side. After the transients have died off, the worm (c) and worm gear (d) reach near steady state angular velocities. At $t = 0.310$ s, the external torque on the worm gear (b) reaches a local minimum and then begins to increase again moving towards (but not reaching) zero at $t = 0.472$ s. This is a result of the prosthetic arm angular velocity (Figure 7.10 (a)) getting high enough that the elbow-end of the springs beginning to move faster than the motor end of the springs which caused the spring length to decrease slightly and the damping force to change sign (b). Notice that the gear teeth remain in contact during this low external torque situation. After $t = 0.310$ s, the worm gear velocity becomes large enough to cause the springs to begin to stretch again and as a result the external torque on the worm gear and the torque on the worm both begin to increase steadily. The external torque on the worm gear (b) and the torque on the worm (a) both reach their absolute maximum at $t = 0.975$ s which corresponds closely to when the prosthetic arm passes through the 0 degree angle. At this time the load on the arm is exerting its maximum torque and the spring is stretched to its

Figure 7.6 Task 2, Configuration 1 mechanical output variables (top branch).

maximum. Notice that the angular velocity of the worm gear (c) falls slightly under these high torque conditions. As the prosthetic arm begins to move away from the 0 degree level and closer to full flexion, the external torque requirement on the worm gear (b) and thus the worm (a) decreases steadily. At $t = 1.280$ s, the voltage applied to the motor begins to decrease and the worm and worm gear begin to decelerate. This results in a deceleration of the prosthetic arm (Figure 7.10 (a)). At $t = 1.313$ s, the torque on the worm gear falls negative due to the high deceleration command from the motor. This corresponds to the schematic shown in Figure 3.8 (c). At $t = 1.444$ s, the combined torque of the spring and damper on the worm gear are again insufficient to maintain contact with the worm gear tooth and the teeth separate (h). Notice that in this case, while separated, the worm gear accelerates due to the slightly positive torque still being supplied by the spring and damper. At $t = 1.449$ s, the worm and worm gear teeth collide on the bottom surface, referring to Figure 3.5, and again transients occur. Though it is not obvious in the friction force plot (i), the system "re-locks" at $t = 1.542$ s as the sliding velocity (g) falls within the "effective" zero range. After this time, due to the low damping ratio associated with the loaded prosthetic arm, the arm continues to oscillate as shown in Figure 7.10 (a) and (b). This results in a fluctuating external torque on the worm gear shown in (b). By the time the simulation has ended, the prosthetic arm oscillations have not ceased. It is worth noting, though, that at no time during these oscillations does the transmission "unlock" or begin to backdrive.

Figure 7.7 presents the power variable output data for the motor of the top branch. Subplot (a), as mentioned earlier, depicts the voltage input at the motor terminals.

Subplot (b) portrays the current in the motor during the simulation. Notice, again, the current does not exceed the thermal torque limit of the motor in this simulation. As the motor initially brings the transmission up to speed, the current in the motor peaks around 3.75 A then drops off to a near zero value once the worm and worm gear reached a near steady state velocity. During this time ($t = 0.300$ to $t = 0.450$ s), the motor torque is only required to overcome the frictional and normal force between the gear teeth. Between $t = 0.450$ to $t = 1.000$ s, the current increases due to the increased torque on the worm gear. As the external torque on the gear reduces, the current in the motor also reduces. At $t = 1.280$ s, the motor voltage begins ramping down and shortly after the current slips negative. During this negative current time, the power (c) also slips negative as the motor absorbs power from the decelerating worm and motor shaft. This stage is also evident in (d) and (e) as the curves move into the overhaul type II region. It is also worth noting that even though the motor voltage and power go to zero at $t = 1.5$ s, the current in the motor does not decrease to 0 A until the worm and transmission have "re-locked" at $t = 1.542$ s (due to back emf). Since the motor terminal is not absorbing power beyond $t = 1.500$ s, the excess current must be dissipated across the resistor (overhaul type I operation). At this time, the curves in (d) and (e) have moved from the overhaul type II region to the boarder of the overhaul type I region indicating that the motor terminals are no longer absorbing power and that all the power between $t = 1.500$ and 1.580 s is being dissipated across the resistor. Subplot (f) portrays the massless worm torque versus its angular velocity. Again, the fact that the operating point remains in the motor region at all times indicates that power is only flowing across the mating gear interface from the

(a)

(d)

(b)

(e)

(c)

(f)

Figure 7.7 Task 2, Configuration 1 electrical output variables (top branch).

worm to the worm gear. This unidirectional power flow, as described in (Def. 1-1), indicates the use of a nonbackdrivable gear set.

Next we examine the mechanical operation of the motor and transmission in the bottom branch of the prosthetic arm as depicted in Figures 7.8 and 7.9. These results are not as complex as those for the top branch since the springs remain in compression throughout the simulation and thus the external torque on the worm gear remains positive. At $t = 0$ s, the voltage to the motor terminals begins to ramp down towards -12 V as depicted in Figure 7.9 (a). The current and thus the worm torque increases until the frictional force exceeds the negative maximum static frictional force shown in (i). Notice that in this case, since the frictional force was initially positive and goes negative, the delay in motion (approximately 0.024 s) in this branch is longer than the top branch. Once in motion, the worm torque (a) increases as it accelerates the system. Once the worm and worm gear have reached a near steady state velocity at $t = 0.240$ s, the current (Figure 7.9 (b)) and thus worm torque reduce. Notice that unlike the top branch where the current almost diminished to zero after accelerating the system, the current and worm torque (a) in this branch maintain a nonzero value since the worm in this case is compressing the spring even further than its initial state and thus storing more potential energy. Since the worm gear remains "loaded" against the worm tooth throughout the simulation, the mating teeth never lose contact (h). As the worm and worm gear begin to decelerate at $t = 1.280$ s, the damping force changes sign (since the relative velocity between the motor end of the spring and the elbow becomes negative) and the external torque on the worm gear decreases (b). This translates into slightly decreased normal (j) and frictional forces

Figure 7.8 Task 2, Configuration 1 mechanical output variables (bottom branch).

(i) between the mating teeth. At $t = 1.495$ s, before the voltage to the motor has reached 0 V, the system "re-locks" as indicated by the near zero sliding velocity in (g) and the jump in the frictional force from a negative kinetic value to a positive static value. Due to the oscillations of the prosthetic arm (Figure 7.10 (a) and (b)) after the transmission has "re-locked", the external torque on the worm gear (b), along with the frictional (i) and normal (j) force, have not reached equilibrium.

Figure 7.9 depicts the power variable outputs for the lower branch. Subplot (a) shows the voltage input to the motor which is the same as the top branch only inverted. Subplot (b) presents the motor current during the simulation. The current peaks at -6.2 A, slightly over the thermal torque current limit of 5.89 A, during the initial acceleration of the system. It then settles out to a value near -2.5 A before the decelerating stage of the simulation. At $t = 1.32$ s, the current goes positive while the motor operates in the overhaul type II region, reducing the worm's inertia. This operating stage is indicated by the curves in (d) and (e) moving into the overhaul type II region. Unlike the top branch, this current in the motor of the bottom branch does not exist beyond $t = 1.500$ s and therefore the motor never slips into the overhaul type I region as portrayed in (d) and (e). Subplot (f) exists entirely in the third quadrant which, based on Figure 2.3, is also a motor region. This shows the nonbackdrivable nature of the mating gear interface. The "tightness" of the path in this figure indicates that the massless worm torque remains relatively constant throughout the simulation. The peak positive power (c) delivered by the motor in this branch reaches a level approximately 1.75 times that of the top branch

Figure 7.9 Task 2, Configuration 1 electrical output variables (bottom branch).

(due to the high external torque on the worm gear), though its maximum negative power is less than that of the top branch.

Figure 7.10 presents the mechanical output variables for the prosthetic arm during this simulation. The initial task involved moving a 9 lb weight from extension to flexion in a gravitational field without a change in stiffness. Subplot (a) presents the velocity profile for the prosthetic arm. The arm initially accelerates to a velocity of 131 °/s but then "dips" to a local minimum velocity of 65 °/s at 0.725. This reduced angular velocity is a result of the arm moving through its 0° position, which corresponds to the maximum torque on the arm in the gravity field. The oscillations, evident in (a) die out slightly as the arm approaches full flexion. The arm doesn't begin to decelerate until shortly after the voltage to the motors has started ramping down. It is obvious that even after the simulation has ended, the arm continues to oscillate around the 52° position. Notice that in (b), the actual starting (-83°) and ending position (52°) of the arm is slightly lower than the initial task definition. This is a result of the finite arm stiffness (30 Nm/rad) which, by design, deflects under the application of an external load. It is apparent, in (c) that the stiffness of the arm changed slightly over the 1.5 seconds of motion which is due to the fact that the transmissions of the top and bottom branch were unable to move the ends of the spring/dampers at exactly the same absolute velocity (remember, the voltage inputs to the motor were chosen as open loop inputs). This mismatch in velocity of the spring/damper ends, which is evident when comparing the worm gear velocities from Figures 7.6 (c) and 7.8 (c), resulted in this slight change in stiffness and the apparent oscillatory behavior. It is worth noting that by the time both nonbackdrivable

Figure 7.10 Task 2, Configuration 1 prosthetic arm mechanical output variables.

transmissions had "re-locked" (around $t = 1.5$ s), the stiffness of the arm becomes constant as it should. Further oscillations of the arm do not effect the stiffness setting of the arm once the transmissions have "locked".

7.5.3 Configuration 3 -- Task 1 Results

The results for Task 1, Configuration 3 are presented in Figures 7.11 - 7.13. In this case, only three figures are presented, since stiffening the prosthetic arm using Configuration 3 design only requires actuating the stiffness motor. Therefore, the position motor and transmission are idle during this task and thus their two figures are not presented.

Figure 7.11 presents the mechanical outputs from the nonbackdrivable transmission (lead screw in this case) of the stiffening motor. The voltage input to the stiffening motor is presented in Figure 7.12 (a). Examining the details of Figure 7.11, (a) depicts the torque applied to the lead screw of this transmission. The torque on this lead screw begins to increase at $t = 0$ s though the system does not beginning moving until the maximum static friction component has been exceeded (at $t = 0.014$ s). Subplot (b) indicates that the force of the two quadratic springs on the nut of the lead screw is initially positive (springs initially in compression) resulting in a negative normal force (j) and a positive initial static frictional force (i). The force on the nut of the lead screw, (b), begins to decrease as the nut starts in motion and initially the external force and velocity of the nut are both positive. Notice that lead screw at the beginning of this simulation is "releasing" the initially compressed springs and, therefore, the torque on the lead screw in (a) is mainly a result of accelerating the system and is not caused by an increasing potential energy being

stored in the springs. The torque on the lead screw, (a), reaches a maximum around $t = 0.080$ s once the system has stopped accelerating. When the lead screw and nut begin to decelerate (around $t = 0.130$ s) the lead screw torque begins to diminish. Once the system deceleration is complete, and the lead screw and nut are moving at a near steady state velocity, the torque on the lead screw reduces even further due to decreasing frictional (i) and normal forces (j) (a result of a decreasing external load on the nut). Notice that the force applied to the nut in (b) is dominated by the quadratic spring force, and is only slightly affected by the damping force (remember the damping requirement in this configuration was less than Configuration I due to a larger required effective elbow radius). At $t = 0.342$ s, the external force on the nut (b) approaches zero and the normal force between nut and lead screw teeth (j) becomes zero, which results in the teeth losing contact (h). Before impacting the opposing teeth, the normal and frictional forces on both the lead screw and the nut go to 0 N and yet the nut decelerates due to the small external force still being applied by the quadratic springs and damper (b). At this same time, the torque on the lead screw (a) reaches a local, non-zero, minimum due to the decreased external load. At 0.349 s, the lead screw tooth impacts the opposing nut tooth and transients ensue. These transients are most evident in (d), (h), (i) and (j) but can also be seen in the damping and overall force exerted on the nut in (b). These transients are more severe than the those seen in Configuration I due to the lower damping coefficients for the dampers used in parallel with the quadratic springs in this configuration. By approximately $t = 0.384$ s, the transient behavior has completed, and the lead screw and nut teeth are firmly in contact (h). At $t = 0.365$ s, the force of the quadratic springs changes sign (the springs are now in tension). As a result, further motion of the nut

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causes an increase in the potential energy in the springs. From this point on, the lead screw is "stretching" the springs and the external force and velocity of the nut are in opposing directions. This increasing external load results in an increasing torque on the lead screw (a), until the motor command begins decelerating the system at $t = 0.400$ s. As depicted in Figure 3.7 (a) and (c), initially when the lead screw begins to decelerate the torque on the lead screw remains positive (similar to Figure 3.8 (a)) but soon the deceleration gets large ($t = 0.425$ s) enough to cause the torque on the lead screw to change direction (similar to Figure 3.8 (a)). At $t = 0.535$ s, the deceleration of the lead screw has reduced to the point where its torque (a) momentarily becomes positive again due to the low deceleration rates of the system and the fact that the a positive voltage is still being applied across the motor terminals. The worm torque reaches a local peak just as the system "re-locks" at $t = 0.547$ s. Notice that the frictional force (i) at this time goes from a kinetic value to a static value within the bounds of the maximum static frictional force. After the system has "re-locked", the lead screw torque (a) drops to zero as the motor voltage decreases to 0 V at $t = 0.560$ s. After this point, the entire system is in static equilibrium.

Figure 7.12 depicts the power variables output from the stiffness motor for Task 1.

Subplot (a), as mentioned earlier, shows the voltage profile applied to the motor. Subplot (b) depicts the current in the motor, which at no time during the simulation exceeds the thermal current limit of the motor. It is worth noting that Configuration 3 required more time (approximately 0.06 s) then Configuration 1 to achieve the task of increasing stiffness from 1 to 80 Nm/rad since accelerations above those used in Configuration 3

Figure 7.12 Task 1, Configuration 3 electrical output variables (stiffness motor).

would have resulted in higher, undesirable, current levels. As expected, the current profile in (b) is the same as the lead screw torque profile from the previous figure. The increasing current near the end of the simulation, which peaks at $t = 0.547$ s, is a result of decreasing deceleration rates in the system while the terminal voltage remained positive. This current "spike" corresponds to the peak in the worm torque seen in the previous figure. The power plot shown in (c) indicates the highest power demand occurred just as the motor was decelerating the lead screw in the initial ramp up phase ($t = 0.132$ s). The power went negative during the same time that the current is negative, corresponding to the final deceleration of the lead screw when the lead screw torque became negative in the previous figure. Subplots (d), (e) and (f) show the power flow of the stiffening motor during the execution of task 1. Subplot (d) shows the electric variables within the motor, while (e) and (f) portray the mechanical variables of the worm. In subplots (d) and (e), both operating curves slip into the Overhaul type II region momentarily when the power to the motor went negative. However, the operating points in subplot (f), which plots the massless torque versus angular velocity of the worm, remain in the motor region throughout the simulation indicating that the transmission was not backdriven at any time. This indicates the successful modeling of a nonbackdrivable lead screw using the non-intersecting, non-parallel axes gear model.

Figure 7.13 (c) depicts the mechanical variable outputs from the prosthetic arm. Subplot (c) indicates that the arm stiffness did increase from 1 to 80 Nm/rad during the simulation. The remaining subplots, which deal primarily with the angular velocity and

Figure 7.13 Task 1, Configuration 3 prosthetic arm mechanical output variables.

displacement of the arm, are zero throughout the simulation which indicates the prosthetic arm didn't move during this stiffening task.

7.5.4 Configuration 3 -- Task 2 Results

Task 2 involved lifting a 9 lb weight, from extension ($\sim 80^\circ$) to full flexion ($\sim 55^\circ$), arm in a gravity field as quickly as possible. The results for this simulation are presented in Figures 7.14 to 7.18. Note the significantly larger time scale (0.1 s to 3.5 s) required in this simulation. As was discussed in the Task 2 results for Configuration 1, the low effective damping ratio (0.18) of the prosthetic arm in this case results in severe oscillations of the prosthetic after the movement command from the position motor has ceased. As a result, the prosthetic arm is not in equilibrium upon completion of the simulation, though each of the nonbackdrivable transmissions is in equilibrium. The results for the position motor and transmission are presented first.

Before describing the results, the reader should be aware that the angular velocity of the worm gear in subplot (d) represents a relative angular velocity between the prosthetic arm and the effective arm elbow. This stems directly from the bond graph model of this configuration in Figure 7.2. This relative angular velocity of the worm gear, and not its absolute angular velocity, is what is of interest in terms of the nonbackdrivable transmission model.

Figure 7.14 presents the mechanical variable outputs for the position motor transmission. Subplot (b) indicates that the external load exerts an initial negative torque on the worm gear which results in a positive normal force (j) and negative initial friction (i) force between the gear teeth. Referring to Figure 3.5, this initial condition represents the gear teeth in contact on the bottom face. Subplot (h) indicates that these same gear teeth remain in contact throughout the simulation. At $t = 0$ s, the voltage to the motor begins to ramp up (Figure 7.15 (a)). The commencement of motion is delayed approximately 0.008 s until the torque on the worm (a) is large enough to overcome the maximum static friction force (i). Due to the large time scale, this initial friction force change is hard to detect in (i). Once the system starts in motion, the torque on the worm (a) continues to ramp up while accelerating the system from rest. At $t = 0.129$ s, this torque begins to descend as the system approaches near steady state operation. The external torque applied to the worm gear increases steadily in the negative direction due to the increasing moment caused by the external load on the arm (as the arm approaches the 0° point). In this case, the worm is "driving" the worm gear in the opposite direction to the force on the worm gear (the worm is thus absorbing power). This absorbed power by the worm gear is translated into increasing potential energy being stored in the rising external load. Since the magnitude of the external torque on the worm gear (b) continues to increase prior to $t = 1.320$ s, the torque exerted on the worm (a) also continues to climb over this time. As the external torque increases (b), so too does the normal (j) and friction (i) forces between the gear teeth. At $t = 1.320$ s, the prosthetic arm is crossing the 0° point (Figure 7.18 (b)) at which time the external load is creating its largest moment on the worm gear. The normal (j) and friction (i) forces both are maximum at this time. As the

Figure 7.14 Task 2, Configuration 3 mechanical output variables (position motor).

arm moves further into the flexion region, this external torque begins to decrease and as a result so too does the torque applied to the worm (a). At $t = 1.710$ s, the voltage to the motor (Figure 7.15 (a)) begins to ramp down to 0 V. Due to the high deceleration rate of the system, the torque on the worm falls negative at $t = 1.710$ s, which is similar to the situation depicted in the force polygon of Figure 3.7 (c). At $t = 1.876$ s, the relative sliding velocity (g) between the gear teeth falls within the effective zero velocity range and the system "re-locks". This is best indicated by the dramatic change in frictional force (i) as the friction between the teeth goes from a kinetic to static value. Notice that after the system has "re-locked", the oscillations of the prosthetic arm (Figure 7.18 (b)) cause the external torque on the worm gear (b), the normal force (j), and friction force (i) to oscillate. The important point to recognize is that even with these external oscillations occurring, the worm and worm gear never "un-lock" and the torque on the worm remains at 0 Nm indicating the success of the nonbackdrivable drive model.

Figure 7.15 presents the power output variables for the position motor. Subplot (a) represents the voltage command to the motor. The profile of the current in the motor (b) is the same as that of the torque on the worm from the previous figure, as expected. The current in the motor never exceeds the thermal current limit during any point in the simulation. For the majority of the simulation, the current is positive and so too is the power shown in (c). When the current slips negative during the deceleration phase of the simulation, so too does the power (c). This, again, is due to the dynamo absorbing energy from the decelerating motor shaft and worm inertia. In this simulation, both the motor voltage and current reach zero at the same time ($t = 1.876$ s). Subplots (d), (e), and (f)

Figure 7.15 Task 2, Configuration 3 electrical output variables (position motor).

depict the operation of the motor and drive during the simulation. In subplots (d) and (e), the operating point remains in the motor region for the majority of the simulation, but falls into the overhaul type II region briefly when the current goes negative at $t = 1.740$ s. Again the operating point in (f), depicting the massless torque versus angular speed of the worm, remains in the motor region during the entire simulation indicating no power was transferred across the gear teeth interface from the worm gear to the worm during the simulation.

Figure 7.16 depicts the mechanical output variables to the stiffness motor. The changing torque requirements of the prosthetic arm during this simulation resulted in changes in the spring forces. This caused some dynamic results for the stiffness motor transmission, even though there was no voltage command to this motor. There are two incidents ($t = 0.891$ and $t = 1.741$ s) when the load from the springs on the nut of the lead screw crosses the 0 N line. Around the same time of these two zero crossings, the mating teeth on the lead screw and nut lose contact and impact opposing teeth. Since the lead screw and nut were not being driven, the transmission quickly "re-locks" after each impact, though there is a noticeable displacement of the nut during these occurrences. As expected, the nut travels a distance equivalent to the backlash between the mating teeth. Though this motion is small in itself, it results in small effective "step" changes in the arm stiffness as indicated in the Figure 7.18. More importantly, however, is the small resulting motion of the lead screw depicted in (c) and (e). Motion of the lead screw would indicate that this nonbackdrivable transmission has backdriven. This phenomena is easier to explain in the context of the power output variables in Figure 7.17.

Figure 7.16 Task 2, Configuration 3 mechanical variables (stiffness motor).

Figure 7.17 illustrates the electrical output variables for the stiffness motor. No voltage is applied to this motor and as a result, the current "spikes" are an indication of short periods of the overhaul type I. This is also indicated in subplots (d), (e) and (f) as the operating points in these figures slip into the overhaul type I region several times. It would appear from these results that the nonbackdrivable definition in (Def. 1-1) has been violated. It is believed that these results indicate a limitation of the non-intersecting, non-parallel axes gear model and not a direct violation of the definition in (Def. 1-1). As noted in this original definition, impulsive forces similar to those seen in the results of this simulation, may behave differently due to the need for a detailed surface model of the gear teeth. Details of this sort are not included in the non-intersecting, non-parallel axes gear model developed in this thesis. Part of this limitation is based on the simplifications of the selected friction model. Based on the fundamentals of this model, impact of two gear teeth will result (in most cases) in a kinetic coefficient of friction between the mating teeth since their relative velocity will be outside the effective zero velocity region (2DV region). This, however, is not always the case during oblique impacts in which static friction can exist between the impacting surfaces [Johnson, 1985]. A detailed theory of oblique impacts, including the effects of friction between mating surfaces, is presented by Johnson [1985]. This theory involves nonlinear, microscopic effects such as regions of "micro-slipping" which are beyond the details of the gear model presented in this thesis. For these reasons, we acknowledge this limitation of the non-intersecting, non-parallel axes gear model and must be cautious interpreting simulation results when impact is involved.

Figure 7.17 Task 2, Configuration 3 electrical output variables (stiffness motor).

Figure 7.18 presents the mechanical output variables of the prosthetic arm. In addition to arm angular velocity (a), displacement (b), and stiffness (c) this figure also shows the effective arm elbow angular velocity (d) and displacement (e). These additional plots are significant since difference between the angular velocities and displacements of the arm and elbow are the angular velocity and displacement of the position motor worm gear, as portrayed in the bond graph in Figure 7.2. The system initially starts from rest, and at $t = 0$ s the angular velocity of the elbow (d) begins to fall negative. Shortly after this, the angular velocity of the arm (a) begins to increase. Prior to $t = 1.5$ s, the oscillatory behavior in the prosthetic arm is already evident in (a). Note that while the worm gear of the position motor is operating at a near steady state velocity (Figure 7.14 (d)) the profiles of the arm angular velocity (a) and the elbow angular velocity (d) become similar (since their difference is almost constant during this time). The position of the prosthetic arm (b) climbs steadily from its initial position of -83° . The additional 3° deflection of the arm beyond the -80° set point is due to the finite, moderately low, stiffness. The displacement of the prosthetic elbow (e) reaches a minimum when the arm passes through the 0° point (b) and the external load is delivering its maximum torque to the elbow. It is interesting to note that the elbow joint deflection never returns to zero during the simulation due to the continuously applied external torque. Once the voltage command to the position motor has ceased ($t = 1.870$ s), the prosthetic arm and elbow continue to oscillate due to the low effective damping ratio of 0.18. Subplot (c) of this figure shows the stiffness of the arm during the simulation. The two sudden small "step" change in the stiffness are due to the loss of contact between mating teeth of the lead screw and nut of the stiffness motor transmission discussed earlier.

Figure 7.18 Task 2, Configuration 3 prosthetic arm mechanical output variables.

7.6 Discussion

Configuration 1

In Chapter 5, it was estimated that the maximum rate of stiffening at steady state for Configuration 1 was approximately 103 mm/s. Based on the results from Task 1 in 7.5.1, the maximum angular velocity reached by the worm gear was approximately 13.7 rad/s. Having assumed a pinion radius (mated to the rack) of 7.4 mm, the actual maximum stiffening rate was 100.9 mm/s ($13.7 \text{ rad/s} \times 7.4 \text{ mm}$) which corresponds well with the initial estimate. An initial estimate for the value of the nonlinear damping coefficient for the gear tooth model was made in Chapter 6 based on a maximum estimated impact velocity of 17.6 mm/s. In the simulation results for Task 1 (Section 7.5.1), the impact velocity of the gear teeth at $t = 0.138 \text{ s}$ was approximately 25 mm/s (not shown in any of the figures). This value corresponds well with the estimate of 24 mm/s originally made in Chapter 6.

The simulation result from Task 2 in 7.5.2 indicated that the designed load of 9 lb could be lifted by the transmission selected for Configuration 1. The originally estimated extension-flexion time under load of 1.2 s, made in Chapter 5, was not met primarily due to current limitations of the motor and the maximum acceleration of the system inertia. As was evident from the results in the last simulation, the effective rotational damping of the arm should be increased in order for the system to be non-oscillatory under high loads. This, of course, results in more power being dissipated during any position or stiffness change of the arm.

Configuration 3

In Chapter 5, an estimate of 321 mm/s was made for the rate at which the stiffening motor could actuate the spring ends in Configuration 3. The results from Task 1 (Section 7.5.3) in Figure 7.10 (d) indicate a maximum linear velocity of the lead screw of 323 mm/s, which corresponds well with the original estimate. Figure 7.11 (b) indicates that the current in the motor did not exceed the maximum thermal torque for the motor even though the total gear ratio for this motor is slightly undersized (see Section 5.7.2). This is probably due to the fact that a safety factor was used when determining the required gear ratio.

The results from Task 2 with Configuration 3 indicated that the 9 lb load could be lifted but with a significantly greater extension-to-flexion time of approximately 2.75 s (including settling time) compared to 1.2 s for Configuration 1. The estimated lifting time for this configuration which was made in Chapter 5 was 1.6 s, which again did not take into consideration the slow acceleration rate. The under damped prosthetic arm in this case indicates that the damping in the system must be increased for better performance. Adding more damping will slow the rise time of the prosthetic arm but will decrease the final settling time. As mentioned before, increasing the damping results in more energy being dissipated during movement which isn't desirable but is required to maintain a stable and non-oscillatory system.

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

A nonlinear dynamic model of a non-intersecting, non-parallel axes gear set was successfully developed in this thesis. The model is simple, flexible, and of low order and can be easily incorporated into existing bond graph models. Simple changes in parameters allow for modeling the dynamic behavior of backdrivable and nonbackdrivable gear transmissions. Experimental verification of this model was not performed, though the simulation results of the model appear accurate. Specifically, this model was used to simulate the nonlinear dynamic behavior of nonbackdrivable components including worm and worm gears as well as lead screws. This non-intersecting, non-parallel gear model was developed based on static analysis of mating gear teeth, in which the nonbackdrivable criteria for both parallel and non-parallel, non-intersecting axes gears, operating at steady state, was established. A graphical interpretation of steady state operation of a dynamo was also developed and was extended to study the behavior of nonbackdrivable and backdrivable geared transmission coupled to a dynamo.

A simple analysis was conducted which examined the energy "trade-offs" associated with using a nonbackdrivable gear set. This analysis highlighted possible energy savings associated with the use of nonbackdrivable transmissions during static loaded conditions.

This energy "trade-off" analysis developed a set of basic criteria that can assist a designer in selecting the most energy efficient gearing (backdrivable vs. nonbackdrivable) for any specific application.

The original purpose for the development of the nonbackdrivable non-intersecting, non-parallel axes gear model was to use it to study the dynamic behavior within the proposed variable stiffness prosthetic designs first developed by English [1996]. To do so, components for the nonbackdrivable actuators, including motors, for the two proposed prosthetic designs were selected based on a preliminary design analysis. A majority of the components selected did not satisfy the initial space and weight requirements. These components along with the nonbackdrivable gear model were incorporated into two high order dynamic models of the proposed prosthetic designs. Simulation results from these models indicated that the system as a whole fell short of the velocity design goals which were developed based on the performance specifications of existing commercially available prosthetic arms. The dynamic simulations of the entire prosthetic system indicated that unsteady state conditions dominated the majority of the system's operation. For that reason, these transient states (instead of steady state) should have been considered in the preliminary design and selection of the motors and gearing for the nonbackdrivable actuator.

8.2 Recommendations

In future work, the non-intersecting, non-parallel axes gear model developed in this thesis should be verified experimentally. A simple test rig, similar to that illustrated in Figure 6.9, consisting of a DC motor, a non-intersecting, non-parallel axes geared transmission, and an external spring and damper could be instrumented and used to verify the model simulation results. It is anticipated that this model will be used to further study nonlinear dynamic effects in nonbackdrivable transmissions including topics such as: control issues involving "stick-slip" and nonbackdrivable transmissions, the effects of vibrations on nonbackdrivable components.

In terms of the proposed prosthetic designs, it is recommended that Configuration 1 be pursued in future designs and prototypes due to its simplified design and its symmetric distribution of power requirements on each motor, in comparison to Configuration 3. Alternative actuators, including linear motors, should be investigated in attempt to reduce the size requirements of the actuators.

To further establish the energy savings associated with using a nonbackdrivable transmission in the proposed prosthetic designs, research into an accurate estimate of prosthetic duty cycle would be warranted. The simple energy "trade-off" analysis should also be expanded to consider unsteady state conditions and varying loads and speeds.

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APPENDIX A

CONTENTS

Commercially Available Prosthetic Arm Comparison Table

Commercially Available Prosthetic Arm Comparison Table [Liberty Technology, 1997]

Powered Elbow Prostheses Performance/Specifications

Factor/measure	Boston Elbow	Utah Arm	Hosmer/NYU Arm
Torque	10- ft-lbs	3- ft-lbs	3.5 ft-lbs
Weight Lifting Ability (@14")	9 lbs	1.5 lbs	3 lbs (in 5 sec)
Clutch	reverse locking	pin lock	pin lock
Clutch release under load	yes	no	no
Speed: no load			
flexion (against gravity)	1.1 sec	1.1 sec	1.4 sec
extension (with gravity)	1.0 sec	1.0 sec	1.1 sec
Speed: with Greifer (1.2 lbs)			
flexion (against gravity)	1.2 sec	1.4 sec	2.0 sec
extension (with gravity)	1.0 sec	0.9 sec	1.0 sec
Weight of Elbow & forearm, no TD	2 lb 4 oz	2 lb 2 oz	1 lb 14 oz
Weight of Elbow & forearm with OB TD	3 lb 5 oz	3 lb 3 oz	2 lb 16 oz
Weight with light weight TD	2 lb 15 oz	Not Available	2 lb 8 oz
Center of Mass (from rear of housing)	3.50 inches	3.38 inches	3.92 inches
Control Options:			
myo-mve	yes	yes	Not Available
servo	1 or 2 separate servos	servo-servo	Not Available
servo-servo	no	yes	Not Available
switches	yes	Not Available	yes
Touch Pads (FSR) - up to 5	yes	Not Available	Not Available
Mode Selection	various switches	on-contraction	Not Available
Battery Capacity	850 mAh, 12 V	600 mAh, 12 V	900 mAh, 6 V
Removable battery	yes	yes	no
On-board Charging	yes	Not Available	yes
Slow charger	yes	yes	yes
Fast Charger	yes (1.25 hrs)	Not Available	Not Available
Patent adjustable humeral rotation friction	yes	wrench req'd	no
Forearms:			
wrist sizes	4 standard (plus custom)	1	2
number of colors	11	2	3
range of lengths	8.5" to 14.5"	9.5" to 12.75"	7.75" to
maximum circumference	9.25"	9.25"	9.33"
maximum coupling collar length	2.80"	4.75"	2.8"
Terminal Device Compatibility:			
Oso Back electric hand	yes	yes	yes
Carrin electric hand	yes	no	yes
Steeber electric hand	yes	no	yes
Oso Back Greifer	yes	yes	yes
Steeber Powered Greifer	yes	no	yes
Oso Back electric wrist rotator	yes	yes	yes
Hosmer hands & split-hooks	yes	yes	yes
Modular construction (for easy servicing)	yes	yes	no
Surface Mount Technology (SMT) circuits	yes	yes	no
Drive Train	direct drive (noise generation)	belt drive	belt drive
Free-wheeling mode	yes (mechanical)	yes (powered)	yes (mechanical)
Myoelectrodes/amplifier size	2" x 1 1/16" x 3 1/16" thick	2" x 5/8" x 1 1/2" thick	NA
Warranty	2 year	2 year	6 months
HCFA L-code (effective January 1, 1997)	L7180	L7180	L7173
Medicare Reimbursement (1996 median)	\$26,000.84	\$26,000.34	\$4,000.34

NA=Not Applicable

APPENDIX B

CONTENTS

Mechanical System and Complete Bond Graph for System in Figure 6.9

State Equation Development for the Sytem in Figure 6.9

Matlab Code Used to Simulate the Model in Figure 6.9

Mechanical System and Complete Bond Graph for System in Figure 6.9

State Equation Development for the System in Figure 6.9

States: p_2 -- Flux linkage of motor inductance (AH)
 p_8 -- Linear momentum of worm (kgm/s)
 q_{13} -- Relative deflection of the mating worm and worm gear teeth (m)
 p_{23} -- Linear momentum of worm gear (kgm/s)
 q_{28} -- Deflection of external spring (m)

Constitutive Laws: $f_2 = \frac{1}{L_a} p_2$ $f_{23} = \frac{1}{m_G} p_{23}$
 $f_8 = \frac{1}{m_G} p_8$ $e_{28} = \frac{1}{C_{spring}} q_{28}$
 $e_{13} = \frac{1}{C_{tooth}} q_{13}$

Nonlinear Component Definitions:

$$N\{\dot{q}_{13}\} = \Pi\{q_{13}\} + D(\dot{\delta})$$

where

$$\Pi\{q_{13}\} = \begin{cases} K_{tooth} \left(q_{13} - \left(\frac{\text{backlash}}{2} \right) \text{sign}(q_{13}) \right) & \text{when } \left(q_{13} \geq \left| \frac{\text{backlash}}{2} \right| \right) \\ 0 & \text{when } \left(q_{13} < \left| \frac{\text{backlash}}{2} \right| \right) \end{cases}$$

$$\delta = \begin{cases} \left(q_{13} - \left(\frac{\text{backlash}}{2} \right) \text{sign}(q_{13}) \right) & \text{when } \left(q_{13} \geq \left| \frac{\text{backlash}}{2} \right| \right) \\ 0 & \text{when } \left(q_{13} < \left| \frac{\text{backlash}}{2} \right| \right) \end{cases}$$

$$\dot{\delta} = \begin{cases} \dot{q}_{13} & \text{when } \left(q_{13} \geq \left| \frac{\text{backlash}}{2} \right| \right) \\ 0 & \text{when } \left(q_{13} < \left| \frac{\text{backlash}}{2} \right| \right) \end{cases}$$

Ff is defined by (6-21) where: $F_w = \frac{K_t g h}{r_w L_a} p_2$

$$F_G = -g r \left(\frac{1}{C_{spring}} q_{28} + \frac{b g r}{m_G} p_{23} + b \text{Vel}(t) \right)$$

Derivation of State Equations:

$$\begin{aligned}
 \dot{p}_2 &= V - e_3 - e_4 \\
 &= V - \left(\frac{R_a}{L_a} \right) p_2 - (K_t f_5) \\
 &= V - \left(\frac{R_a}{L_a} \right) p_2 - \left(\frac{K_t gh}{r_w} f_6 \right) \\
 &= V - \left(\frac{R_a}{L_a} \right) p_2 - \left(\frac{K_t gh}{r_w \cdot m_w} \right) p_8
 \end{aligned}$$

$$\begin{aligned}
 \dot{p}_8 &= e_7 - e_9 \\
 &= e_6 - (e_{10} + e_{17}) \\
 &= \left(\frac{gh}{r_w} e_5 \right) - (\sin(\lambda) \cos(\phi) e_{11} + \cos(\lambda) e_{17}) \\
 &= \left(\frac{gh K_t}{r_w} f_6 \right) - (\sin(\lambda) \cos(\phi) e_{12} + \cos(\lambda) e_{19}) \\
 &= \left(\frac{gh K_t}{r_w L_a} \right) p_2 - \left(\sin(\lambda) \cos(\phi) N\{q_{13}, \dot{q}_{13}\} + \cos(\lambda) F_f\{F_w, F_g, N\} \right)
 \end{aligned}$$

$$\begin{aligned}
 \dot{q}_{13} &= f_{12} \\
 &= f_{11} - f_{15} \\
 &= (\sin(\lambda) \cos(\phi) f_{10}) - (\cos(\lambda) \cos(\phi) f_{16}) \\
 &= (\sin(\lambda) \cos(\phi) f_9) - (\cos(\lambda) \cos(\phi) f_{22}) \\
 &= \left(\frac{\sin(\lambda) \cos(\phi)}{m_w} \right) p_8 - \left(\frac{\cos(\lambda) \cos(\phi) f_{16}}{m_g} \right) p_{23}
 \end{aligned}$$

$$\begin{aligned}
\dot{\mathbf{p}}_{23} &= \mathbf{e}_{22} - \mathbf{e}_{24} \\
&= (\mathbf{e}_{16} - \mathbf{e}_{21}) - \mathbf{e}_{25} \\
&= (\cos(\lambda)\cos(\phi)\mathbf{e}_{15} - \sin(\lambda)\mathbf{e}_{20}) - \left(\frac{r_p}{r_g} \mathbf{e}_{26} \right) \\
&= (\cos(\lambda)\cos(\phi)\mathbf{e}_{12} - \sin(\lambda)\mathbf{e}_{19}) - \left(\frac{r_p}{r_g} (\mathbf{e}_{27} + \mathbf{e}_{28}) \right) \\
&= \left(\cos(\lambda)\cos(\phi)\mathbf{N}\{\mathbf{q}_{13}, \dot{\mathbf{q}}_{13}\} - \sin(\lambda)\mathbf{F}_f\{\mathbf{F}_w, \mathbf{F}_g, \mathbf{N}\} \right) - \left(\frac{r_p}{r_g} \left(\mathbf{b}\mathbf{f}_{27} + \frac{1}{C}\mathbf{q}_{28} \right) \right) \\
&= \left(\cos(\lambda)\cos(\phi)\mathbf{N}\{\mathbf{q}_{13}, \dot{\mathbf{q}}_{13}\} - \sin(\lambda)\mathbf{F}_f\{\mathbf{F}_w, \mathbf{F}_g, \mathbf{N}\} \right) - \left(\frac{r_p}{r_g} \left(\mathbf{b}\mathbf{f}_{26} + \frac{1}{C}\mathbf{q}_{28} \right) \right) \\
&= \left(\cos(\lambda)\cos(\phi)\mathbf{N}\{\mathbf{q}_{13}, \dot{\mathbf{q}}_{13}\} - \sin(\lambda)\mathbf{F}_f\{\mathbf{F}_w, \mathbf{F}_g, \mathbf{N}\} \right) - \left(\frac{r_p}{r_g} \left(\frac{\mathbf{b}r_p}{r_g} \mathbf{f}_{24} + \frac{1}{C}\mathbf{q}_{28} \right) \right) \\
&= \left(\cos(\lambda)\cos(\phi)\mathbf{N}\{\mathbf{q}_{13}, \dot{\mathbf{q}}_{13}\} - \sin(\lambda)\mathbf{F}_f\{\mathbf{F}_w, \mathbf{F}_g, \mathbf{N}\} \right) - \left(\frac{\mathbf{b}r_p^2}{r_g^2 m_g} \right) \mathbf{p}_{23} - \left(\frac{r_p}{r_g \cdot C} \right) \mathbf{q}_{28}
\end{aligned}$$

$$\begin{aligned}
\dot{\mathbf{q}}_{28} &= \mathbf{f}_{26} \\
&= \frac{r_p}{r_g} \mathbf{f}_{25} \\
&= \frac{r_p}{r_g} \mathbf{f}_{24} \\
&= \left(\frac{r_p}{r_g m_g} \right) \mathbf{p}_{23}
\end{aligned}$$

Matlab Code used to Simulate the Model in Figure 6.9

GEARS. M

```
%
%           SIMPLE GEAR MODEL
%
% This simulation was developed to model the mechanical system
% shown in Figure 6.9. The explicit integration was conducted
% using Matlab's ODE function.
%
%
clear

global La Ra Kt r_w G gh mw mg gr eta lambda phi D tooth_stiffness backlash DV ...
      mu_s mu_k b C A A_non B v_slide_coeff v_normal_coeff

% ***** PARAMETER DEFINITIONS *****

% ----- Motor Variables -----

Ra = 0.19; % (Ohms) Armature resistance (phase-to-phase)
La = 0.03e-3; % (H) Armature inductance (phase-to-phase)
Kt = 7.5e-3; % (Nm/A) Torque constant of motor
gh = 5.75; % motor gearhead ratio

% ----- Rotational Inertias -----
J_rotor = 3.0e-6; % (kg.m^2) Inertia of motor rotor
J_worm = 3.4e-6; % (kg.m^2) Approx. inertia of worm
J_gear = 27.2e-6; % (kg.m^2) Approx. inertia of gear

J_total_1 = J_rotor*gh^2 + J_worm; % (kg.m^2) Effective inertia

% ----- Worm and Gear Parameters -----
r_w = (1*0.0254)/2; % (m) pitch radius of worm
r_g = (1.667*0.0254)/2; % (m) pitch radius of worm gear
mw = (J_total_1)/(r_w^2); % (kg) equivalent linear mass of worm and motor rotor
mg = (J_gear)/(r_g^2); % (kg) equivalent linear mass of worm gear
eta = mw/mg; % ratio of equiv. worm mass to and gear mass
lambda = (4+46/60)*pi/180; % (rad) Lead angle of worm and rack
phi = 14.5*pi/180; % Pressure angle of worm and rack (rad)

% ----- Gear Teeth Paramaters -----
D = 26.99e6; % (N.s/m^2) Nonlinear damping coefficient of Hertzian (tooth)
           % damping
tooth_stiffness = 155.4e6; % (N/m) Equivalent stiffness of mating gear teeth
```

backlash = 1.016e-4; % (m) Total backlash between the pinion and gear teeth
 DV = 1e-5; % (m/s) Velocity which is considered "zero" velocity

% ----- Friction Coefficients -----

mu_s = .11; % static friction coefficient
 mu_k = 0.0825; % kinetic friction coefficient
 b = 255; % (Ns/m) damper to ground attached to gear
 K = 2000; % (N/m) spring to ground attached to gear
 C = 1/K; % (m/N) compliance of spring to ground

% ----- Gear Shaft Parameters -----

R = 0.58*0.0254/2; % (m) radius of gear shaft where spring/damper are attached
 G = r_g/R; % Ratio of gear radius to shaft radius

% ***** MATRIX DEFINITIONS *****

% ----- A matrix -----

A = zeros(7);
 A(1,1) = -Ra/La;
 A(1,2) = -Kt*gh/(r_w*mw);
 A(2,1) = Kt*gh/(r_w*La);
 A(3,2) = sin(lambda)*cos(phi)/mw;
 A(3,4) = -cos(lambda)*cos(phi)/mg;
 A(4,4) = -b/(G^2*mg);
 A(4,5) = -1/(G*C);
 A(5,4) = 1/(G*mg);
 A(6,2) = 1/mw;
 A(7,4) = 1/mg;

% ----- B matrix -----

B = zeros(7,2);
 B(1,1) = 1;
 B(4,2) = -b/G; % -b*gr;
 B(5,2) = 1;

% ----- A_non matrix (coeff. of nonlinear N, Ff terms) -----

A_non = zeros(7,2);
 A_non(2,1) = -sin(lambda)*cos(phi);
 A_non(2,2) = -cos(lambda);
 A_non(4,1) = cos(lambda)*cos(phi);
 A_non(4,2) = -sin(lambda);

% ***** PRE-PROCESSING *****

% ----- Velocity coefficients (used to determine the normal and sliding velocity
 % ----- between the pinion and gear teeth)-----

```

v_slide_coeff = [cos(lambda)/mw, sin(lambda)/mg];
v_normal_coeff = [sin(lambda)*cos(phi)/mw, -cos(lambda)*cos(phi)/mg];

% ----- Variable Initialization -----
x_spring_init = -0.093; % (m) Initial stretch in the spring
F_spring_init = (-1/C*x_spring_init); % (N) Initial force exerted by the spring

x_n_init = -F_spring_init*cos(lambda)/(G*cos(phi)*tooth_stiffness)-
sign(F_spring_init)*(backlash/2);
% (m) Determines initial gear teeth relative deflection under load

% ----- Initial conditions of states -----
p_init = [0;0;x_n_init;0;x_spring_init;0;0];

% ----- Time Span -----
t_start = -0.25;
t_stop = 2.25;
t_span = [t_start t_stop];

options = odeset('reltol',1e-7,'abstol',1e-7);

% ----- Call Integration Routine -----

[tt,pp] = ode23s('gears_st',t_span,p_init,options);

% ----- Call Post Processing Routine -----
post

```

GEARS_ST.M

% File - gears_st.m

% This routine computes the derivatives of the state equations for Figure 6.9

```
function pdot = gears_st(t,p)
```

```
global La Ra Kt r_w G gh mw mg gr eta lambda phi D tooth_stiffness backlash DV ...
mu_s mu_k b C A A_non B v_slide_coeff v_normal_coeff
```

```
gr = 1/G;
```

```
% Define velocity in the normal and sliding direction
```

```
v_n = v_normal_coeff*[p(2);p(4)]; % Normal vel. between gear teeth
```

```
v_t = v_slide_coeff*[p(2);p(4)]; % Sliding vel. between gear teeth
```

```
delta = Di(p(3)); % One of the nonlinear damping terms
```

```
delta_dot = Di_dot(p(3),v_n); % Second of the nonlinear damping terms
```

```
tooth_damp = D*delta*delta_dot; % Damping force of nonlinear damper between gear
teeth (Hertzian)
tooth_stiff = N_spring(p(3)); % Spring force of nonlinear spring between gear teeth
```

```
N = tooth_damp + tooth_stiff; % Normal force between gear teeth
Fg = -gr*(1/C*p(5) + b*gr/mg*p(4) + b*Vel(t)); % Force applied to gear by external
spring and damper
Fw = Kt*gh/(r_w*La)*p(1); % Input force on the pinion

Ff_max = abs(N*mu_s); % Maximum static frictional force
Ff_k = abs(N*mu_k); % Kinetic friction
Ff_eq = (Fw - N*sin(lambda)*cos(phi)*(1-eta) + Fg*eta*tan(lambda))/...
(cos(lambda)*(1+eta*tan(lambda)^2)); % Force of friction required to maintain static
equilibrium
```

```
% Calculation of actual friction force
Ff = (N~=0).*...
((abs(v_t)>=DV).*(Ff_k.*sign(v_t))+...
(abs(v_t)<DV).*((abs(Ff_eq)<abs(Ff_max)).*(Ff_eq)+...
(abs(Ff_eq)>=abs(Ff_max)).*(Ff_max.*sign(Ff_eq))));
```

```
% Calculate Rates of Change for the States
pdot = A*p + B*[V(t);Vel(t)] + A_non*[N;Ff];
```

N_SPRING.M

```
% File - N_spring.m
% Calculating the nonlinear spring force between the gear teeth
```

```
function Force_k = tooth_spring(x);
```

```
global backlash tooth_stiffness
```

```
Force_k = sign(x).*(tooth_stiffness*(abs(x)-
backlash/2*ones(size(x)))).*(abs(x)>backlash/2);
```

DI.M

```
% File - di.m
% Calculating the penetration displacement
```

```
function d = di(x);
```

```
global backlash
```

```
d = (abs(x)>=backlash/2*ones(size(x))).*(abs(x)-backlash/2*ones(size(x)));
```

DI_DOT.M

% File - di_dot.m

% Calculating the interpenetration velocity

function d_dot = di_dot(x,x_dot);

global backlash

d_dot = (x_dot).*(abs(x)>=backlash/2);

V.M

% File - V.m

function Voltage = V(t);

t1 = 0;

t2 = .15;

t3 = 1.85;

t4 = 2;

Vi = 0;

Vf = 12;

dV = Vf-Vi;

dt = t2-t1;

V_avg = (Vi+Vf)/2;

a = (Vi-V_avg)/(0.2*(0.2*dt^2 + dt*t1 - dt*t2));

Vo = (V_avg) - a*(0.2*dt)*(t2+t1);

Voo = (V_avg) + a*(0.2*dt)*(t2+t1);

V_line1 = (.4*a*dt)*t + Vo*ones(size(t));

V_line2 = -(0.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));

V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));

V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));

V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));

V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));

Voltage = (t>=t1).*(t<(0.2*dt+t1)).*V_line1 +
 (t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...
 +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +
 (t>t3).*(t<(0.2*dt+t3)).*V_para3...
 + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;

APPENDIX C

CONTENTS

Schematic of Configuration 1 (taken from Figure 1.2)

Complete Bond Graph Model of Configuration 1

State Equation Development for Configuration 1 Model (Figure 7.1)

Matlab Code Used to Simulate Configuration 1 Model

Complete Bond Graph Model of Configuration 1 (cont. on next page)

Complete Bond Graph Model of Configuration 1 (cont. from previous page)

State Equation Development for Configuration 1 Model (Figure 7.1)

States:

$$\begin{aligned}
 p_2 &= L_a \dot{i}_1 & p_{33} &= L_a \dot{i}_2 \\
 p_8 &= m_w v_{w1} & p_{39} &= m_w v_{w2} & p_{63} &= J_{\text{arm}} \omega_{\text{arm}} \\
 q_{13} &= x_{\text{normal}1} & q_{44} &= x_{\text{normal}2} & q_{68} &= x_{\text{ext.spring}} \\
 p_{23} &= m_G v_{G1} & p_{54} &= m_G v_{G2} \\
 q_{29} &= x_{\text{spring}1} & q_{60} &= x_{\text{spring}2}
 \end{aligned}$$

Nonlinear Normal and Friction Definitions:

$$\begin{aligned}
 N_1 &= \Pi\{q_{13}\} + \Xi\{\delta_1, \dot{\delta}_1\} & N_2 &= \Pi\{q_{44}\} + \Xi\{\delta_2, \dot{\delta}_2\} \\
 F_{f1} &= F_f\{N_1, F_{w1}, F_{G1}\} & F_{f2} &= F_f\{N_2, F_{w2}, F_{G2}\}
 \end{aligned}$$

State Equations:

$$\begin{aligned}
 \dot{p}_2 &= V_1 - \frac{R_a}{L_a} p_2 - \frac{K_t g h}{r_w m_w} p_8 \\
 \dot{p}_8 &= \frac{K_t g h}{r_w L_a} p_2 - [N_1 \sin(\lambda) \cos(\phi_n) + F_{f1} \cos(\lambda)] \\
 \dot{q}_{13} &= \frac{\sin(\lambda) \cos(\phi_n)}{m_w} p_8 - \frac{\cos(\lambda) \cos(\phi_n)}{m_G} p_{23} \\
 \dot{p}_{23} &= [N_1 \cos(\lambda) \cos(\phi_n) - F_{f1} \sin(\lambda)] - \frac{b}{G^2 m_G} p_{23} + \frac{Rb}{GJ_{\text{arm}}} p_{63} - \frac{\Phi\{q_{29}\}}{G} \\
 \dot{q}_{29} &= \frac{1}{Gm_G} p_{23} - \frac{R}{J_{\text{arm}}} p_{63} \\
 \dot{p}_{33} &= V_2 - \frac{R_a}{L_a} p_{33} - \frac{K_t g h}{r_w m_w} p_{39} \\
 \dot{p}_{39} &= \frac{K_t g h}{r_w L_a} p_{33} - [N_2 \sin(\lambda) \cos(\phi_n) + F_{f2} \cos(\lambda)] \\
 \dot{q}_{44} &= \frac{\sin(\lambda) \cos(\phi_n)}{m_w} p_{39} - \frac{\cos(\lambda) \cos(\phi_n)}{m_G} p_{54} \\
 \dot{p}_{54} &= [N_2 \cos(\lambda) \cos(\phi_n) - F_{f2} \sin(\lambda)] - \frac{b}{G^2 m_G} p_{54} - \frac{Rb}{GJ_{\text{arm}}} p_{63} - \frac{\Phi\{q_{60}\}}{G} \\
 \dot{q}_{60} &= \frac{1}{Gm_G} p_{54} + \frac{R}{J_{\text{arm}}} p_{63}
 \end{aligned}$$

$$\begin{aligned}\dot{p}_{63} &= \left[\frac{-2R^2b}{J_{\text{arm}}} p_{63} + R(\Phi\{q_{29}\} - \Phi\{q_{60}\}) + \frac{Rb}{Gm_G}(p_{23} - p_{54}) \right] - L(F_{\text{in}} + K_c\{q_{68}\}) \\ \dot{q}_{68} &= \text{Vel} + \frac{L}{J_{\text{arm}}} p_{63}\end{aligned}$$

Matlab Code Used to Simulate Configuration 1 Model

The following code was used to simulate Configuration 1. The code presented below simulates the 1 to 80 Nm/rad stiffness change. In order to simulate the -80 to 55 degree raising load case, the following must be changed:

- in Config1.m, the time span must be increased to 1.75 s
- in Initialize.m, K_init must be set to 30 and theta_i set to -80
- in Equilibrium.m, the external torque due to the load and arm must be un-commented
- in Fin.m, the external force must be set equal to the arm and load

CONFIG1.M

```
% File - config1.m
% This is a dynamic simulation of the Configuration 1 model in Figure 7.1.
% The state equations for this model were developed in Appendix A. %
%
clear

global Ra La Kt gh mw mg rw G eta lambda phi mu_s mu_k D tooth_stiffness ...
    backlash DV v_slide_coeff v_normal_coeff theta_contact ...
    theta_contact_stiffen constraint_stiffness V x_max theta_max ...
    Fmax n b Kmax Kmin L marm R mload Wload Warm Ja pmax theta_contact_max ...
    x_contact A A_non B xki xmovei K_init

% VARIABLE DEFINITIONS
%*****

Initialize
%
%*****

% DETERMINE ARM EQUILIBRIUM POSITION
%*****

thetaeq = fzero('equilibrium',0);

% DETERMINE INITIAL QUADRATIC SPRING STRETCHES AND GEAR TEETH
DEFLECTIONS
delta1eq = xki + xmovei -R*thetaeq;
delta2eq = xki - xmovei +R*thetaeq;
tooth_init1 =
Phi(delta1eq)*cos(lambda)/(G*cos(phi)*tooth_stiffness)+sign(Phi(delta1eq))*(backlash/
2);
tooth_init2 =
Phi(delta2eq)*cos(lambda)/(G*cos(phi)*tooth_stiffness)+sign(Phi(delta2eq))*(backlash/
2);
```

% INITIAL CONDITIONS

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Bond graph state notation shown in square brackets []
```

```
p_init = [
    0          % p(1) Flux linkage of Motor 1 inductor [p2]
    0          % p(2) Effective linear momentum of motor 1 shaft and worm [p8]
    tooth_init1 % p(3) Relative deflection of branch 1 worm/gear teeth [q13]
    0          % p(4) Effective linear momentum of gear 1 tooth [p23]
    delta1eq   % p(5) Deflection of Branch 1 quadratic spring [q29]
    0          % p(6) Flux linkage of Motor 2 inductor [p33]
    0          % p(7) Effective momentum of motor 2 shaft and worm [p39]
    tooth_init2 % p(8) Relative deflection of Branch 2 worm/gear teeth [q44]
    0          % p(9) Effective linear momentum of gear 2 tooth [p54]
    delta2eq   % p(10) Deflection of Branch 2 quadratic spring [q60]
    0          % p(11) Angular momentum of prosthetic arm [p63]
    0          % p(12) Deflection of environmental spring [q68]
    0          % p(13) Linear displacement of Motor 1 shaft and worm [--]
    0          % p(14) Linear displacement of gear 1 tooth [--]
    0          % p(15) Linear displacement of Motor 2 shaft and worm [--]
    0          % p(16) Linear displacement of gear 2 [--]
    thetaeq    % p(17) Abs. angular displacement of prosthetic arm [--]
];
```

% SIMULATION TIME

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
t_start = -.1;
```

```
t_stop = .5;
```

```
t_span = [t_start t_stop];
```

```
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

% SET INTEGRATION TOLERANCES

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
options = odeiset('reitol',1e-6,'abstol',1e-6);
```

```
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

% CALL INTEGRATION ROUTINE

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
[tt,pp] = ode23s('config1_st',t_span,p_init,options);
```

```
%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

% CALL POST PROCESSING ROUTINE

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
post_processing
```

```

%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

CONFIG_ST.M
% File - config1_st.m
% This routine integrates the state equations for Configuration 1 model

function pdot = config1_st(t,p)

global Ra La Kt gh mw mg rw G eta lambda phi mu_s mu_k D tooth_stiffness ...
backlash DV v_slide_coeff v_normal_coeff theta_contact ...
theta_contact_stiffen constraint_stiffness V x_max theta_max ...
Fmax n b Kmax Kmin L marm xkmax R mload Wload Warm Ja pmax
theta_contact_max ...
x_contact A A_non B

% BRANCH 1 CALCULATIONS OF NORMAL AND FRICTIONAL FORCES IN
THE WORM/GEAR PAIR

% Define velocity in the normal and sliding direction
v_n1 = v_normal_coeff*[p(2);p(4)]; % Normal vel. between gear teeth of branch 1
v_t1 = v_slide_coeff*[p(2);p(4)] ; % Sliding vel. between gear teeth of branch 1
F_spring1 = Phi(p(5)); % Force in branch 1 quadratic spring

delta1 = Di(p(3)); % One of the nonlinear damping terms
delta_dot1 = Di_dot(p(3),v_n1); % Second of the nonlinear damping terms
tooth_damp1 = D*delta1*delta_dot1; % Damping force of nonlinear damper between
gear teeth (Hertzian)
tooth_stiff1 = N_spring(p(3)); % Spring force of nonlinear spring between gear teeth

N1 = tooth_damp1 + tooth_stiff1; % Normal force between gear teeth
F_11 = -(F_spring1/G + b*(1/(G^2*mg)*p(4) - R/(G*Ja)*p(11))) ; % Force applied to gear

% external spring and damper
Fw1 = Kt*gh/(La*rw)*p(1); % Input force on the pinion

Ff_max1 = abs(N1*mu_s); % Maximum static frictional force
Ff_k1 = abs(N1*mu_k); % Kinetic friction force
Ff_eq1 = (Fw1 - N1*sin(lambda)*cos(phi)*(1-eta) + Fg1*eta*tan(lambda))/...
(cos(lambda)*(1+eta*tan(lambda)^2)); % Force of friction required to maintain static
equilibrium

% Calculation of actual friction force
Ff1 = (N1~=0).*...
((abs(v_t1)>=DV).*(Ff_k1.*sign(v_t1))+...
(abs(v_t1)<DV).*((abs(Ff_eq1)<abs(Ff_max1)).*(Ff_eq1)+...

```

```

(abs(Ff_eq1)>=abs(Ff_max1)).*(Ff_max1.*sign(Ff_eq1))));

% BRANCH 1 CALCULATIONS OF NORMAL AND FRICTIONAL FORCES IN
THE WORM/GEAR PAIR

% Define velocity in the normal and sliding direction
v_n2 = v_normal_coeff*[p(7);p(9)]; % Normal vel. between gear teeth of branch 2
v_t2 = v_slide_coeff*[p(7);p(9)]; % Sliding vel. between gear teeth of branch 2
F_spring2 = Phi(p(10)); % Force in branch 2 quadratic spring

delta2 = Di(p(8)); % One of the nonlinear damping terms
delta_dot2 = Di_dot(p(8),v_n2); % Second of the nonlinear damping terms
tooth_damp2 = D*delta2*delta_dot2; % Damping force of nonlinear damper between
gear teeth (Hertzian)
tooth_stiff2 = N_spring(p(8)); % Spring force of nonlinear spring between gear teeth

N2 = tooth_damp2 + tooth_stiff2; % Normal force between gear teeth
Fg2 = -(F_spring2/G + b*(1/(G^2*mg)*p(9) + R/(G*Ja)*p(11))); % Force applied to gear
by
                                % external spring and damper
Fw2 = Kt*gh/(La*rw)*p(6); % Input force on the pinion

Ff_max2 = abs(N2*mu_s); % Maximum static frictional force
Ff_k2 = abs(N2*mu_k); % Kinetic friction
Ff_eq2 = (Fw2 - N2*sin(lambda)*cos(phi)*(1-eta) + Fg2*eta*tan(lambda))/...
(cos(lambda)*(1+eta*tan(lambda)^2)); % Force of friction required to maintain static
equilibrium

% Calculation of actual friction force
Ff2 = (N2~=0).*...
((abs(v_t2)>=DV).*(Ff_k2.*sign(v_t2))+...
(abs(v_t2)<DV).*((abs(Ff_eq2)<abs(Ff_max2)).*(Ff_eq2)+...
(abs(Ff_eq2)>=abs(Ff_max2)).*(Ff_max2.*sign(Ff_eq2))));

F_ke = constraint(p(17)); % Force of External contact constraint

% Calculation of state derivatives
pdot = A*p + A_non*[N1;Ff1;F_spring1;N2;Ff2;F_spring2;F_ke] +
B*[V1(t);V2(t);Fin(p(17));Vel(t)];

CONSTRAINT.M
% File Constraint.m
% This function defines the applied force of the external constraint

function cons = constraint(position)

```

```
global theta_contact theta_contact_max constraint_stiffness
```

```
cons = 0*(position<theta_contact*pi/180)+...
    constraint_stiffness/(theta_contact_max-theta_contact)*...
    (180/pi*position-theta_contact).*...
    (position>theta_contact*pi/180 & position<theta_contact_max*pi/180)+...
    constraint_stiffness*(position>theta_contact_max*pi/180);
```

DI.M

```
% File - Di.m
```

```
% Calculating the interpenetration displacement of mating gear teeth
```

```
function d = di(x);
```

```
global backlash
```

```
d = (abs(x)>=(backlash/2)).*(abs(x)-backlash/2*ones(size(x)));
```

DI_DOT.M

```
% File - di_dot.m
```

```
% Calculating the interpenetration velocity of mating gear teeth
```

```
function d_dot = di_dot(x,x_dot);
```

```
global backlash
```

```
d_dot = (x_dot).*(abs(x)>=(backlash/2));
```

EQUILIBRIUM.M

```
function eq = equilibrium(thetaact)
```

```
% This function calculates the equilibrium arm angle due to the weight of
% the arm and loading (for initial conditions).
```

```
global Kmax Kmin xkmax Wload Warm R xki xmovei L
```

```
eq = (Kmax-Kmin)/(4*R^2*xkmax)*((xki+xmovei-R*thetaact).^2)...
    +Kmin/(2*R^2)*(xki+xmovei-R*thetaact)-...
    xkmax*(Kmax^2-2*Kmin^2)/(8*R^2*(Kmax-Kmin))...
    -(Kmax-Kmin)/(4*R^2*xkmax)*((xki-xmovei+R*thetaact).^2)...
    -Kmin/(2*R^2)*(xki-xmovei+R*thetaact)...
    +xkmax*(Kmax^2-2*Kmin^2)/(8*R^2*(Kmax-Kmin)); % ...
% -(Wload*L+Warm*L/2)/R*cos(thetaact);
```

FIN.M

```
% File - Fin.m
% This function calculates the external force applied to the arm
```

```
function F_in = Fin(arm_pos);
```

```
global Wload Warm
```

```
if 1
    F_in = 0;
end
```

```
if 0
    F_in = (Wload + Warm/2)*cos(arm_pos);
end
```

INITIALIZE.M

```
% File: Initialize.m
% This file defines the variables and parameters and initializes constants
% for Configuration 1's simulation
```

```
global Ra La Kt gh mw mg rw G eta lambda phi mu_s mu_k D tooth_stiffness ...
    backlash DV v_slide_coeff v_normal_coeff theta_contact ...
    theta_contact_stiffen constraint_stiffness V x_max theta_max ...
    Fmax n b Kmax Kmin L marm R xkmax mload Wload Warm Ja pmax
theta_contact_max ...
    x_contact A A_non B xki xmovei K_init
```

```
% INITIAL EQUILIBRIUM POSITION OF ARM AND STIFFNESS OF ARM
```

```
*****
```

```
thetai = 0;
K_init = 1;
```

```
%
```

```
*****
```

```
% MOTOR SPECIFICATIONS (MicroMo 118891 Brushless
motor)*****
```

```
Ra = 0.19; % (Ohms) Armature resistance (phase-to-phase)
```

```
La = 0.03e-3; % (H) Armature inductance (phase-to-phase)
```

```
Kt = 7.5e-3; % (Nm/A) Torque constant of motor
```

```
gh = 5.75; % gear ratio of motor gear head
```

```
%
```

```
*****
```


% WORM AND GEAR SPECIFICATIONS

J_rotor = 3.0e-6; % (kg.m^2) Inertia of motor shaft

J_worm = 3.4e-6; % (kg.m^2) Inertia of worm

J_gear = 27.2e-6; % (kg.m^2) Inertia of worm gear

J_total = J_rotor*gh^2 + J_worm; % (kg.m^2) Effective inertia of worm and rotor as
seen

% by the motor

rw = (1*0.0254)/2; % (m) Pitch radius of worm

rg = (1.667*0.0254)/2; % (m) Pitch radius of worm gear

mw = (J_total)/(rw^2); % (kg) Total equivalent linear mass of worm and motor assembly

mg = (J_gear)/(rg^2); % (kg) Total equivalent linear mass of worm gear

eta = mw/mg; % () Mass ratio of equiv. worm inertia to and rack inertia

lambda = (4+46/60)*pi/180; % (rad) Lead angle of worm and rack

phi = 14.5*pi/180; % (rad) Normal Pressure angle of worm and rack

Rp = 0.58*0.0254/2; % (m) Pitch radius of pinion gear

G = rg/Rp; % () Ratio of gear pitch radius to pinion gear pitch radius

%

% FRICTION COEFFICIENTS BETWEEN GEARS

mu_s = 0.11; % () static friction coefficient between worm and rack

mu_k = 0.0825; % () kinetic friction coefficient between worm and rack

%

% NONLINEAR TEETH DAMPING, FRICTION AND STIFFNESS PARAMETERS

D = 26.99e6; % (N.s/m^2) Nonlinear damping coefficient of Hertzian (tooth) damping

tooth_stiffness = 155.4e6; % (N/m) Equivalent stiffness of mating gear teeth

backlash = 1.02e-4; % (m) Total backlash between the pinion and gear teeth

DV = 1e-5; % (m/s) Effective "zero" velocity (m/s) -- used in Friction model

%

% DEFINITION OF NORMAL AND TANGENTIAL VELOCITY COEFF BETWEEN WORM AND GEAR

v_slide_coeff = [cos(lambda)/mw, sin(lambda)/mg];

v_normal_coeff = [sin(lambda)*cos(phi)/mw, -cos(lambda)*cos(phi)/mg];

%

% CONSTRAINT DEFINITION

```
*****
theta_contact = 200;           % (degrees) Angle of contact
theta_contact_stiffen = .1;    % (degrees) Angle of contact stiffening
constraint_stiffness = 10000;  % (N/m) Constant constraint stiffness
%
*****
```

% QUADRATIC SPRING PARAMETERS

```
*****
xmax = .25;                    % (m) spring length at full stretch
theta_max = 210;               % (degrees) maximum angle between extremes
Fmax = 136;                    % (N) Max Force possible
n = 1;                         % n*xkmax = maximum compression
%
*****
```

% ARM STIFFNESS DESIGN PARAMETERS

```
*****
Kmax = 80;                     % (N.m/rad) Max elbow stiffness
Kmin = 1;                      % (N.m/rad) Min elbow stiffness
%
*****
```

% ARM AND LOAD DEFINITION

```
*****
L = .33;                       % (m) forearm length
marm = .280;                   % (kg) mass of forearm
R = 0.05;                      % (m) effective radius of elbow joint
mload = (9-marm/2)/2.2;        % (kg) load mass
%
*****
```

% WEIGHT LOAD CALCULATIONS

```
*****
Wload = mload*9.81;            % (N) Weight of load
Warm = marm*9.81;              % (N) Weight of arm
Ja = marm*(L^2)/3 + mload*(L^2); % (kg*m^2) Mass moment of inertia
pmax = theta_max*pi/180;       % convert max angle to radians
xkmax = (xmax-R*pmax)/(1+n);    % (m) Maximum stretch of springs
b = 0.707*sqrt(marm*(L^2)/3*Kmax)/(R^2); % (N.s/m) Damping in series with
quadratic spring
%
*****
```

% CONTACT CALCULATIONS

theta_contact_max = theta_contact + theta_contact_stiffen; % angle of max stiff

x_contact = theta_contact*pi/180*R; % rest position of constraint spring

xki = (K_init-Kmin)/(Kmax-Kmin)*xkmax;

xmovei = thetai*pi/180*R;

%

% [A] MATRIX DEFINITION

A= zeros(17);

A(1,1) = -Ra/La;

A(1,2) = -Kt*gh/(rw*mw);

A(2,1) = Kt*gh/(rw*La);

A(3,2) = sin(lambda)*cos(phi)/mw;

A(3,4) = -cos(lambda)*cos(phi)/mg;

A(4,4) = -b/(G^2*mg);

A(4,11) = R*b/(G*Ja);

A(5,4) = 1/(G*mg);

A(5,11) = -R/Ja;

A(6,6) = -Ra/La;

A(6,7) = -Kt*gh/(rw*mw);

A(7,6) = Kt*gh/(rw*La);

A(8,7) = sin(lambda)*cos(phi)/mw;

A(8,9) = -cos(lambda)*cos(phi)/mg;

A(9,9) = -b/(G^2*mg);

A(9,11) = -R*b/(G*Ja);

A(10,9) = 1/(G*mg);

A(10,11) = R/Ja;

A(11,4) = R*b/(G*mg);

A(11,9) = -R*b/(G*mg);

A(11,11) = -2*R^2*b/Ja;

A(12,11) = L/Ja;

A(13,2) = 1/mw;

A(14,4) = 1/mg;

A(15,7) = 1/mw;

A(16,9) = 1/mg;

A(17,11) = 1/Ja;

%

% [A_NON] MATRIX DEFINITION (COEFF. OF NONLINEAR VARIABLES)

A_non = zeros(17,7);

```

A_non(2,1) = -sin(lambda)*cos(phi);
A_non(2,2) = -cos(lambda);
A_non(4,1) = cos(lambda)*cos(phi);
A_non(4,2) = -sin(lambda);
A_non(4,3) = -1/G;
A_non(7,4) = -sin(lambda)*cos(phi);
A_non(7,5) = -cos(lambda);
A_non(9,4) = cos(lambda)*cos(phi);
A_non(9,5) = -sin(lambda);
A_non(9,6) = -1/G;
A_non(11,3) = R;
A_non(11,6) = -R;
A_non(11,7) = -L;
%

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

% [B] MATRIX DEFINITION

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

B = zeros(17,4);
B(1,:) = [ 1 0 0 0];
B(6,:) = [ 0 1 0 0];
B(11,:) = [ 0 0 -L 0];
B(12,:) = [ 0 0 0 1];
%

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

N_SPRING.M

```

% File - N_spring.m

```

```

% Calculating the nonlinear spring force between the gear teeth

```

```

function Force_k = tooth_spring(x);

```

```

global backlash tooth_stiffness

```

```

Force_k = sign(x).*(tooth_stiffness*(abs(x)-
backlash/2*ones(size(x)))).*(abs(x)>backlash/2);

```

PHI.M

```

% File phi.m

```

```

% This function calculates force in a spring based on stretch (delta).

```

```

function F = phi(delta);

```

```

global Kmax Kmin xkmax R

```

```

% QUADRATIC SPRING

```

```
F = (Kmax-Kmin)/(4*R^2*xkmax)*(delta.^2)+Kmin/(2*R^2)*delta-...
    xkmax*(Kmax^2-2*Kmin^2)/(8*R^2*(Kmax-Kmin));
```

PSI.M

```
% File psi.m
```

```
% This function calculates the stiffness of a spring based on stretch (delta).
```

```
function k = psi(delta)
```

```
global Kmax Kmin R xkmax Kset
```

```
% QUADRATIC SPRING
```

```
k = (Kmax-Kmin)/(2*R^2*xkmax)*delta+Kmin/(2*R^2);
```

V1.M

```
% File - V1.m
```

```
% This function calculates the voltage input to the branch 1 motor
```

```
function Voltage1 = V1(t);
```

```
if 0
```

```
    Voltage1 = 0;
```

```
end
```

```
% Routine for 1-80 Nm/rad stiffness change
```

```
if 1
```

```
    t1 = 0;
```

```
    t2 = .1688;
```

```
    t3 = .3312;
```

```
    t4 = .5;
```

```
    Vi = 0;
```

```
    Vf = 12;
```

```
    dV = Vf-Vi;
```

```
    dt = t2-t1;
```

```
    V_avg = (Vi+Vf)/2;
```

```
    a = (Vi-V_avg)/(.2*(.2*dt^2 + dt*t1 - dt*t2));
```

```
    Vo = (V_avg) - a*(.2*dt)*(t2+t1);
```

```
    Voo = (V_avg) + a*(.2*dt)*(t2+t1);
```

```
    V_line1 = (.4*a*dt)*t + Vo*ones(size(t));
```

```
    V_line2 = -(.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));
```

```

V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));
V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));
V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));
V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));

Voltage1 = (t>=t1).*(t<(0.2*dt+t1)).*V_para1 +
(t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...
    +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +
(t>t3).*(t<(0.2*dt+t3)).*V_para3...
    + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;
end

% Routine for -80 to 55 degree movement routine (loaded arm with gravity)

if 0
    t1 = 0;
    t2 = .22;
    t3 = 1.28;
    t4 = 1.5;

    Vi = 0;
    Vf = 12;

    dV = Vf-Vi;
    dt = t2-t1;

    V_avg = (Vi+Vf)/2;

    a = (Vi-V_avg)/(.2*(.2*dt^2 + dt*t1 - dt*t2));
    Vo = (V_avg) - a*(.2*dt)*(t2+t1);
    Voo = (V_avg) + a*(.2*dt)*(t2+t1);

    V_line1 = (.4*a*dt)*t + Vo*ones(size(t));
    V_line2 = -(.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));
    V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));
    V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));
    V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));
    V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));

    Voltage1 = (t>=t1).*(t<(0.2*dt+t1)).*V_para1 +
(t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...
    +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +
(t>t3).*(t<(0.2*dt+t3)).*V_para3...
    + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;
end

```

V2.M

```
% File - V2.m
```

```
% This file calculates the voltage input to the branch 2 motor
```

```
function Voltage2 = V2(t);
```

```
if 0
```

```
    Voltage2 = 0;
```

```
end
```

```
% Routine for 1-80 Nm/rad stiffness change
```

```
if 1
```

```
    t1 = 0;
```

```
    t2 = .1688;
```

```
    t3 = .3312;
```

```
    t4 = .5;
```

```
    Vi = 0;
```

```
    Vf = 12;
```

```
    dV = Vf-Vi;
```

```
    dt = t2-t1;
```

```
    V_avg = (Vi+Vf)/2;
```

```
    a = (Vi-V_avg)/(.2*(.2*dt^2 + dt*t1 - dt*t2));
```

```
    Vo = (V_avg) - a*(.2*dt)*(t2+t1);
```

```
    Voo = (V_avg) + a*(.2*dt)*(t2+t1);
```

```
    V_line1 = (.4*a*dt)*t + Vo*ones(size(t));
```

```
    V_line2 = -(.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));
```

```
    V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));
```

```
    V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));
```

```
    V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));
```

```
    V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));
```

```
    Voltage2 = (t>=t1).*(t<(0.2*dt+t1)).*V_para1 +  
    (t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...  
    +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +  
    (t>t3).*(t<(0.2*dt+t3)).*V_para3...  
    + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;  
end
```

```
% Routine for -80 to 55 degree movement routine (loaded arm with gravity)
```

```

if 0
    t1 = 0;
    t2 = .22;
    t3 = 1.28;
    t4 = 1.5;

    Vi = 0;
    Vf = -12;

    dV = Vf-Vi;
    dt = t2-t1;

    V_avg = (Vi+Vf)/2;

    a = (Vi-V_avg)/(.2*(.2*dt^2 + dt*t1 - dt*t2));
    Vo = (V_avg) - a*(.2*dt)*(t2+t1);
    Voo = (V_avg) + a*(.2*dt)*(t2+t1);

    V_line1 = (.4*a*dt)*t + Vo*ones(size(t));
    V_line2 = -(.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));
    V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));
    V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));
    V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));
    V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));

    Voltage2 = (t>=t1).*(t<(0.2*dt+t1)).*V_para1 +
(t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...
    +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +
(t>t3).*(t<(0.2*dt+t3)).*V_para3...
    + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;
end

```

VEL.M

% File - Vel.m

% This function calculates the velocity of the external constraint

```
function V_constraint = Vel(t);
```

```
V_constraint = 0;
```


APPENDIX D

CONTENTS

Schematic of Configuration 3 (taken from Figure 1.3)

Complete Bond Graph Model of Configuration 3

State Equation Development for Configuration 3 Model (Figure 7.2)

**Derivation of equilibrium friction force for the rotational inertia and torque of the
position motor transmission**

Matlab Code Used to Simulate Configuration 3 Model

Schematic of Configuration 3 (taken from Figure 1.3)

Complete Bond Graph Model of Configuration 3 (cont. on next page)

Complete Bond Graph Model of Configuration 3 (cont. from previous page)

State Equation Development for Configuration 1 model in Figure 7.2

States:

$$\begin{aligned}
 p_2 &= L_{ap} \dot{i}_p & q_{36} &= x_{normal_s} \\
 p_8 &= J_{wp} \omega_{wp} & p_{46} &= m_{Gs} v_{Gs} \\
 q_{13} &= x_{normal_p} & q_{52} &= x_{spring1} & p_{65} &= J_{arm+Gear} \omega_{arm+Gear} \\
 p_{25} &= L_{as} \dot{i}_s & q_{58} &= x_{spring2} & q_{70} &= x_{ext.spring} \\
 p_{31} &= m_{ws} v_{ws} & p_{62} &= J_{motor} \omega_{motor}
 \end{aligned}$$

Nonlinear Normal and Friction Definitions:

$$\begin{aligned}
 N_p &= \Pi\{q_{13}\} + \Xi\{\delta_p, \dot{\delta}_p\} & N_s &= \Pi\{q_{36}\} + \Xi\{\delta_s, \dot{\delta}_s\} \\
 F_{fp} &= F_f\{N_p, \tau_{wp}, \tau_{Gp}\} & F_{fs} &= F_f\{N_s, F_{ws}, F_{Gs}\}
 \end{aligned}$$

Note: The expression for the equilibrium value of F_{fp} is derived later in this appendix.

State Equations:

$$\begin{aligned}
 \dot{p}_2 &= V_p - \frac{R_{ap}}{L_{ap}} p_2 - \frac{K_{tp} g h_p}{J_{wp}} p_8 \\
 \dot{p}_8 &= \frac{K_{tp} g h_p}{L_{ap}} p_2 - r_{wp} (N_p \sin(\lambda_p) \cos(\phi_n) + F_{fp} \cos(\lambda_p)) \\
 \dot{q}_{13} &= \frac{\sin(\lambda_p) \cos(\phi_n) r_{wp}}{J_{wp}} p_8 - \cos(\lambda_p) \cos(\phi_n) r_{Gp} \left(\frac{1}{J_{arm+Gear}} p_{65} - \frac{1}{J_{motor}} p_{62} \right) \\
 \dot{p}_{25} &= V_s - \frac{R_{as}}{L_{as}} p_{25} - \frac{K_{ts} g h_s}{m_{ws} r_{ws}} p_{31} \\
 \dot{p}_{31} &= \frac{K_{ts} g h_s}{L_{as} r_{ws}} p_{25} - (N_s \sin(\lambda_s) \cos(\phi_n) + F_{fs} \cos(\lambda_s)) \\
 \dot{q}_{36} &= \frac{\sin(\lambda_s) \cos(\phi_n)}{m_{ws}} p_{31} - \frac{\cos(\lambda_s) \cos(\phi_n)}{m_{Gs}} p_{46} \\
 \dot{p}_{46} &= (N_s \cos(\lambda_s) \cos(\phi_n) - F_{fs} \sin(\lambda_s)) - \frac{1}{G_s} \left(\Phi\{q_{52}\} + \Phi\{q_{58}\} + \frac{2b}{G_s m_{Gs}} p_{46} \right) \\
 \dot{q}_{52} &= \frac{1}{G_s m_{Gs}} p_{46} - \frac{R}{J_{motor}} p_{62} \\
 \dot{q}_{58} &= \frac{1}{G_s m_{Gs}} p_{46} + \frac{R}{J_{motor}} p_{62}
 \end{aligned}$$

$$\dot{p}_{62} = R\Phi\{q_{52}\} - R\Phi\{q_{58}\} - \frac{2R^2b}{J_{\text{motor}}} p_{62} - r_{Gp} (N_p \cos(\lambda_p) \cos(\phi_n) - F_{fp} \sin(\lambda_p))$$

$$\dot{p}_{65} = r_{Gp} (N_p \cos(\lambda_p) \cos(\phi_n) - F_{fp} \sin(\lambda_p)) - L(F_{in} + K_{env}\{q_{70}\})$$

$$\dot{q}_{70} = Vel - \frac{L}{J_{\text{arm+Gear}}} p_{65}$$

Derivation of equilibrium friction force for the rotational inertia and torque of the position motor transmission:

First, to simplify the following expressions let:

$$\begin{aligned}\tau_{wp} &= \frac{K_{tp} \cdot gh_p}{L_{ap}} \cdot p_2 \\ \tau_{Gp} &= L \cdot (F_{in} \cdot K_{env} \{q_{70}\}) \\ \tau_m &= R \cdot (\Phi\{q_{52}\} - \Phi\{q_{58}\}) - \frac{2 \cdot R^2 \cdot b}{J_{motor}} \cdot p_{62}\end{aligned}$$

Similar to (6-20):

$$\begin{aligned}\dot{v}_{slide} = 0 &= \frac{\cos(\dot{\lambda}_p) \cdot r_{wp}}{J_{wp}} \cdot \dot{p}_8 + \sin(\dot{\lambda}_p) \cdot r_{Gp} \cdot \left(\frac{1}{J_{arm+Gear}} \cdot \dot{p}_{65} - \frac{1}{J_{motor}} \cdot \dot{p}_{62} \right) \\ &= \frac{\cos(\dot{\lambda}_p) \cdot r_{wp}}{J_{wp}} \cdot (\tau_w - r_{wp} \cdot (N_p \cdot \sin(\lambda_p) \cdot \cos(\phi_n) + F_{fp} \cdot \cos(\lambda_p))) + \\ &\quad \frac{\sin(\dot{\lambda}_p) \cdot r_{Gp}}{J_{arm+Gear}} \cdot (r_{Gp} \cdot (N_p \cdot \cos(\lambda_p) \cdot \cos(\phi_n) - F_{fp} \cdot \sin(\lambda_p)) - \tau_G) - \\ &\quad \frac{\sin(\dot{\lambda}_p) \cdot r_{Gp}}{J_{motor}} \cdot (\tau_m - r_{Gp} \cdot (N_p \cdot \cos(\lambda_p) \cdot \cos(\phi_n) - F_{fp} \cdot \sin(\lambda_p)))\end{aligned}$$

Simplifying and solving for F_{fp} gives,

$$F_{fp} = \frac{\left[\frac{\tau_{wp}}{J_{wp}} \cdot r_{wp} \cdot \cos(\dot{\lambda}_p) - \left(\frac{\tau_G}{J_{arm+Gear}} + \frac{\tau_m}{J_{motor}} \right) \cdot r_{Gp} \cdot \sin(\dot{\lambda}_p) \right] - \left[\frac{r_{wp}^2}{J_{wp}} - r_{Gp}^2 \cdot \left(\frac{1}{J_{arm+Gear}} + \frac{1}{J_{motor}} \right) \right] \cdot N_p \cdot \sin(\dot{\lambda}_p) \cdot \cos(\lambda_p) \cdot \cos(\phi_n)}{\left[\sin^2(\dot{\lambda}_p) \cdot r_{Gp}^2 \cdot \left(\frac{1}{J_{arm+Gear}} + \frac{1}{J_{motor}} \right) + \cos^2(\dot{\lambda}_p) \cdot r_{wp}^2 \cdot \frac{1}{J_{wp}} \right]}$$

Matlab Code Used to Simulate Configuration 3 Model

The following code was used to simulate Configuration 1. The code presented below simulates the 1 to 80 Nm/rad stiffness change. In order to simulate the -80 to 55 degree raising load case, the following must be changed:

- in Config3.m, the time span must be increased to 2.00 s
- in Initialize.m, K_init must be set to 30 and theta_i set to -80
- in Equilibrium.m, the external torque due to the load and arm must be uncommented
- in Fin.m, the external force must be set equal to the arm and load

CONFIG3.M

```
% File - config3.m
%
% This is a dynamic simulation of the Configuration 3 model in Figure 7.2.
% The state equations for this model were developed in this Appendix.
%
clear

global Ra_p La_p Kt_p gh_p rw_p Jw_p rg_p Jg_p Jm Gp eta_p lambda_p phi_p ...
v_slide_coeff_p v_normal_coeff_p ...
Ra_s La_s Kt_s gh_s rw_s mw_s rg_s mg_s Gs eta_s lambda_s phi_s ...
v_slide_coeff_s v_normal_coeff_s ...
mu_s mu_k D tooth_stiffness_p tooth_stiffness_s backlash_p backlash_s DV
theta_contact ...
theta_contact_stiffen constraint_stiffness x_max theta_max ...
Fmax n b Kmax Kmin L marm R xkmax mload Wload Warm Ja pmax
theta_contact_max ...
x_contact A A_non B xki xmovei K_init thetai

% VARIABLE DEFINITIONS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Initialize
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% DETERMINE ARM EQUILIBRIUM POSITION
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
thetaeq = fzero('equilibrium',0);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% DETERMINE INITIAL QUADRATIC SPRING STRETCHES AND GEAR TEETH
DEFLECTIONS
delta1eq = xki + xmovei - R*thetaeq;
```



```

delta2eq = xki - xmovei + R*thetaeq;
tooth_init_p
=(1/2*Warm*L+Wload*L)*cos(thetaeq)/(rg_p*tooth_stiffness_p)+backlash_p/2;
tooth_init_s =
(Phi(delta1eq)+Phi(delta2eq))*cos(lambda_s)/(cos(phi_s)*tooth_stiffness_s)+sign(Phi(delta1eq)+Phi(delta2eq))*(backlash_s/2);
%
*****

```

% INITIAL CONDITIONS

```

*****
% Bond graph numbers are in []
p_init = [
0 % p(1) Flux linkage of Position Motor inductor [p2]
0 % p(2) Rotational Momentum of Position Motor shaft and worm [p8]
tooth_init_p % p(3) Relative deflection of Position worm & worm gear teeth [q13]
0 % p(4) Flux linkage of Stiffness motor inductor [p25]
0 % p(5) Momentum of Stiffness motor lead screw [q31]
tooth_init_s % p(6) Relative deflection of lead screw and nut mating teeth [p36]
0 % p(7) Momentum of lead screw nut [p46]
delta1eq % p(8) Deflection of branch 1 quadratic spring [q53]
delta2eq % p(9) Deflection of branch 2 quadratic spring [q59]
0 % p(10) Rotational momentum of motor and elbow joint [q62]
0 % p(11) Rotational inertia of prosthetic arm [p65]
0 % p(12) Deflection of environmental spring [q70]
0 % p(13) Abs. ang. displacement of position worm tooth [--]
thetaeq % p(14) Abs. ang. displacement of position worm gear tooth [--]
0 % p(15) Abs. displacement of stiffness lead screw tooth [--]
0 % p(16) Abs. displacement of stiffness lead screw nut tooth [--]
0 % p(17) Abs. ang. displacement of motor and elbow joint [--]
];

```

% SIMULATION TIME

```

*****
t_start = -.1;
t_stop = 2.5;
t_span = [t_start t_stop];
%
*****

```

% SET INTEGRATION TOLERANCES

```

*****
options = odeset('reltol',1e-6,'abstol',1e-6);
%
*****

```

```
% CALL INTEGRATION ROUTINE
```

```
*****
```

```
[tt,pp] = ode23s('config3_st',t_span,p_init,options);
```

```
%
```

```
*****
```

```
% CALL POST PROCESSING ROUTINE
```

```
*****
```

```
post_processing
```

```
%
```

```
*****
```

CONFIG3_ST.M

```
% File - config3_st2.m
```

```
% This routine integrates the state equations for Configuration 3 model
```

```
function pdot = config3_st(t,p)
```

```
global Ra_p La_p Kt_p gh_p rw_p Jw_p rg_p Jg_p Jm Gp eta_p lambda_p phi_p ...
```

```
    v_slide_coeff_p v_normal_coeff_p ...
```

```
    Ra_s La_s Kt_s gh_s rw_s mw_s rg_s mg_s Gs eta_s lambda_s phi_s ...
```

```
    v_slide_coeff_s v_normal_coeff_s ...
```

```
    mu_s mu_k D tooth_stiffness_p tooth_stiffness_s backlash_p backlash_s DV
```

```
theta_contact ...
```

```
    theta_contact_stiffen constraint_stiffness x_max theta_max ...
```

```
    Fmax n b Kmax Kmin L marm R xkmax mload Wload Warm Ja pmax
```

```
theta_contact_max ...
```

```
    x_contact A A_non B xki xmovei K_init thetai
```

```
% Define quadratic spring forces, external forces, and constraint forces
```

```
F_spring1 = Phi(p(8));
```

```
F_spring2 = Phi(p(9));
```

```
F_ke = constraint(p(12));
```

```
F_in = Fin(p(14));
```

```
% DEFINE NORMAL AND FRICTIONAL FORCES FOR THE POSITION MOTOR  
(WORM AND WORM GEAR)
```

```
% Define velocity in the normal and sliding direction
```

```
v_np = v_normal_coeff_p*[p(2);p(10);p(11)]; % Normal vel. between gear teeth of  
position motor
```

```
v_tp = v_slide_coeff_p*[p(2);p(10);p(11)]; % Sliding vel. between gear teeth of position  
motor
```

```
delta_p = Di_p(p(3)) ;%% One of the nonlinear damping terms
```

```
delta_dot_p = Di_dot_p(p(3),v_np); % Second of the nonlinear damping terms
```

```
tooth_damp_p = D*delta_p*delta_dot_p; % Damping force of nonlinear damper between
gear teeth (Hertzian)
tooth_stiff_p = N_spring_p(p(3)); % Spring force of nonlinear spring between gear teeth
```

```
N_p = tooth_damp_p + tooth_stiff_p; % Normal force between gear teeth
Tg_p = L*(F_in + F_ke); % External torque on the gear
Tw_p = Kt_p*gh_p/(La_p)*p(1); % Torque applied to the worm
Tm = R*(F_spring1 - F_spring2) - 2*b*R^2/Jm*p(10); % Torque applied to elbow joint
```

```
Ff_max_p = abs(N_p*mu_s); % Maximum static frictional force
Ff_k_p = abs(N_p*mu_k); % Kinetic friction
% Required frictional force to maintain equilibrium
Ff_eq_p = ((Tw_p/Jw_p*rw_p*cos(lambda_p) - (Tg_p/Jg_p +
Tm/Jm)*rg_p*sin(lambda_p)) - ...
(rw_p^2/Jw_p - rg_p^2*(1/Jg_p +
1/Jm))*N_p*sin(lambda_p)*cos(phi_p))/...
(sin(lambda_p)^2*rg_p^2*(1/Jg_p + 1/Jm) + cos(lambda_p)^2*(rw_p^2)/Jw_p);
```

```
% Calculation of actual friction force
Ff_p = (N_p~=0).*(...
((abs(v_tp)>=DV).*(Ff_k_p.*sign(v_tp))+...
(abs(v_tp)<DV).*((abs(Ff_eq_p)<abs(Ff_max_p)).*(Ff_eq_p)+...
(abs(Ff_eq_p)>=abs(Ff_max_p)).*(Ff_max_p.*sign(Ff_eq_p)))));
```

```
% DEFINE NORMAL AND FRICTIONAL FORCES FOR THE STIFFNESS MOTOR
(LEAD SCREW AND NUT)
```

```
% Define velocity in the normal and sliding direction
v_ns = v_normal_coeff_s*[p(5);p(7)]; % Normal vel. between gear teeth of stiffness
motor
v_ts = v_slide_coeff_s*[p(5);p(7)]; % Sliding vel. between gear teeth of stiffness motor
```

```
delta_s = Di_s(p(6)); % One of the nonlinear damping terms
delta_dot_s = Di_dot_s(p(6),v_ns); % Second of the nonlinear damping terms
tooth_damp_s = D*delta_s*delta_dot_s; % Damping force of nonlinear damper between
gear teeth (Hertzian)
tooth_stiff_s = N_spring_s(p(6)); % Spring force of nonlinear spring between gear teeth
```

```
N_s = tooth_damp_s + tooth_stiff_s; % Normal force between gear teeth
Fg_s = (-2*b/(Gs^2*mg_s)*p(7) - 1/Gs*F_spring1 - 1/Gs*F_spring2); % Force applied to
nut quadratic springs and dampers
Fw_s = Kt_s*gh_s/(La_s*rw_s)*p(4); % Input force on the lead screw tooth
```

```
Ff_max_s = abs(N_s*mu_s); % Maximum static frictional force
Ff_k_s = abs(N_s*mu_k); % Kinetic friction
```

```
Ff_eq_s = (Fw_s - N_s*sin(lambda_s)*cos(phi_s)*(1-eta_s) +
Fg_s*eta_s*tan(lambda_s))/...
(cos(lambda_s)*(1+eta_s*tan(lambda_s)^2)); % Force of friction required to maintain
static equilibrium
```

```
% Calculation of actual friction force
```

```
Ff_s = (N_s~=0).*...
((abs(v_ts)>=DV).*(Ff_k_s.*sign(v_ts))+...
(abs(v_ts)<DV).*((abs(Ff_eq_s)<abs(Ff_max_s)).*(Ff_eq_s)+...
(abs(Ff_eq_s)>=abs(Ff_max_s)).*(Ff_max_s.*sign(Ff_eq_s))));
```

```
% CALCULATE STATE VARIABLE DERIVATIVES
```

```
pdot = A*p + A_non*[N_p;Ff_p;N_s;Ff_s;F_spring1;F_spring2;F_ke] +
B*[Vp(t);Vs(t);F_in;Vel(t)];
```

CONSTRAINT.M

Same as Config1 simulation (Appendix C)

DI_P.M

```
% File - Di_p.m
```

```
% This function calculates the interpenetration displacement of mating gear teeth
% for the position motor
```

```
function d = di_p(x);
```

```
global backlash_p
```

```
d = (abs(x)>=(backlash_p/2)).*(abs(x)-backlash_p/2*ones(size(x)));
```

DI_S.M

```
% File - Di.m
```

```
% This function calculates the interpenetration displacement of mating gear teeth
% for the position motor
```

```
function d = di_s(x);
```

```
global backlash_s
```

```
d = (abs(x)>=(backlash_s/2)).*(abs(x)-backlash_s/2*ones(size(x)));
```

DI_DOT_P.M

```
% File - di_dot_p.m
```

```
% This function calculates the interpenetration velocity of the mating teeth
% of the position motor
```

```
function d_dot = di_dot_p(x,x_dot);
```

```
global backlash_p
```

```
d_dot = (x_dot).*(abs(x)>=(backlash_p/2));
```

DI_DOT_S.M

```
% File - di_dot_s.m
```

```
% This function calculates the interpenetration velocity of the mating teeth
```

```
% of the stiffness motor
```

```
function d_dot = di_dot_s(x,x_dot);
```

```
global backlash_s
```

```
d_dot = (x_dot).*(abs(x)>=(backlash_s/2));
```

EQUILIBRIUM.M

```
Same as Config1 simulation in Appendix C
```

FIN.M

```
Same as Config1 simulation in Appendix C
```

INITIALIZE.M

```
% File: Initialize.m
```

```
% This file defines the variables and parameters and initializes constants
```

```
% for Config3.m simulation
```

```
global Ra_p La_p Kt_p gh_p rw_p Jw_p rg_p Jg_p Jm Gp eta_p lambda_p phi_p ...
```

```
    v_slide_coeff_p v_normal_coeff_p ...
```

```
    Ra_s La_s Kt_s gh_s rw_s mw_s rg_s mg_s Gs eta_s lambda_s phi_s ...
```

```
    v_slide_coeff_s v_normal_coeff_s ...
```

```
    mu_s mu_k D tooth_stiffness_p tooth_stiffness_s backlash_p backlash_s DV
```

```
theta_contact ...
```

```
    theta_contact_stiffen constraint_stiffness x_max theta_max ...
```

```
    Fmax n b Kmax Kmin L marm R xkmax mload Wload Warm Ja pmax
```

```
theta_contact_max ...
```

```
    x_contact A A_non B xki xmovei K_init thetai
```

```
% //////////////////////////////////// PARAMETERS ////////////////////////////////////
```

```
thetai = -80;
```

```
K_init = 30;
```

```
% POSITION MOTOR SPECIFICATIONS -- "p" subscript (MicroMo 118891 Brushless  
motor)*****
```

```

Ra_p = 0.19; % Armature resistance (phase-to-phase) (Ohms)
La_p = 0.03e-3; % Armature inductance (phase-to-phase) (H)
Kt_p = 7.5e-3; % Torque constant of motor (Nm/A)
gh_p = 51.24; % 66+(54/245); % gear ratio of motor gear head
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% ARM AND LOAD DEFINITION
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

L = .33; % forearm length (m)
marm = .280; % mass of forearm (kg)
R = 0.187; % effective radius of elbow joint (m)
mload = (9-marm/2)/2.2; % load mass (kg)

%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% ////////////////////////////////// CALCULATIONS //////////////////////////////////

% WEIGHT LOAD CALCULATIONS
Wload = mload*9.81; % weight of load (N)
Warm = marm*9.81; % weight of arm (N)

% DESIGN PARAMETERS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Kmax = 80; % max elbow stiffness (N.m/rad)
Kmin = 1; % min elbow stiffness (N.m/rad)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% ARM CALCULATIONS
Ja = marm*(L^2)/3 + mload*(L^2); % mass moment of inertia (kg*m^2)
b = 0.707*sqrt(marm*(L^2)/3*Kmax)/(R^2); % damping in series with quadratic
spring (N.s/m)

% POSITION WORM AND WORM GEAR SPECIFICATIONS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

J_rotor_p = 3.0e-6;
J_worm_p = 209e-6;
J_gear_p = 919e-6;
Jm = .263*(3.33*0.0254/2)^2; % 336.6e-6;

Jw_p = J_rotor_p*gh_p^2 + J_worm_p; % J_rotor_p + J_worm_p/(gh_p^2);
Jg_p = J_gear_p + Ja;

rw_p = 1*0.0254; % radius of worm (m)

```

```
% mw_p = J_total_pw/(rw_p^2); % worm, 1 in. long, made of bronze (8700 kg/m^3)
```

```
rg_p = 3.33*0.0254/2;
```

```
% mg_p = J_total_pg/rg_p^2; % mass of mating worm rack (kg)
```

```
% mg_p = mg_p*4;
```

```
% eta_p = mw_p/mg_p; % ratio of equiv. worm inertia to and rack inertia ()
```

```
Gp = 1;
```

```
lambda_p = (4+46/60)*pi/180; % Lead angle of worm and rack (rad)
```

```
phi_p = 14.5*pi/180; % Pressure angle of worm and rack (rad)
```

```
%
```

```
*****
```

```
% b = 0.707*sqrt(J_total_pg*Kmax)/(R^2); % damping in series with quadratic  
spring (N.s/m)
```

```
% DEFINITION OF NORMAL AND TANGENTIAL VELOCITY COEFF BETWEEN  
WORM AND WORM GEAR
```

```
v_slide_coeff_p = [cos(lambda_p)*rw_p/Jw_p,-  
sin(lambda_p)*rg_p/Jm,sin(lambda_p)*rg_p/Jg_p];
```

```
v_normal_coeff_p =
```

```
[sin(lambda_p)*cos(phi_p)*rw_p/Jw_p,cos(lambda_p)*cos(phi_p)*rg_p/Jm,-  
cos(lambda_p)*cos(phi_p)*rg_p/Jg_p];
```

```
%
```

```
*****
```

```
% STIFFNESS MOTOR SPECIFICATIONS -- "s" subscript (MicroMo 118891 Brushless  
motor)*****
```

```
Ra_s = 0.19; % Armature resistance (phase-to-phase) (Ohms)
```

```
La_s = 0.03e-3; % Armature inductance (phase-to-phase) (H)
```

```
Kt_s = 7.5e-3; % Torque constant of motor (Nm/A)
```

```
gh_s = 1; % 3+5/7; % gear ratio of motor gear head
```

```
%
```

```
*****
```

```
% STIFFNESS WORM AND RACK SPECIFICATIONS
```

```
*****
```

```
J_rotor_s = 3.0e-6;
```

```
m_screw = pi*(0.20*0.0254/2)^2*0.125*7700;
```

```
J_screw = 1/2*m_screw*(0.20*0.0254/2)^2;
```

```
% J_worm_s = 3.4e-6;
```

```
J_total_s = J_rotor_s*gh_s^2 + J_screw;% J_worm_s;
```

```
rw_s = 0.25/2*0.0254; % radius of worm (m)
```

```
mw_s = J_total_s/rw_s^2; % worm, 1 in. long, made of bronze (8700 kg/m^3)
```

```
mg_s = .104; % mass of mating worm rack (kg)
```

```
eta_s = mw_s/mg_s; % ratio of equiv. worm inertia to and rack inertia ()
```

```

Gs = 1;
lambda_s = 3.64*pi/180; % Lead angle of worm and rack (rad)
phi_s = 14.5*pi/180; % Pressure angle of worm and rack (rad)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% DEFINITION OF NORMAL AND TANGENTIAL VELOCITY COEFF BETWEEN
WORM AND RACK
v_slide_coeff_s = [cos(lambda_s)/mw_s,sin(lambda_s)/mg_s];
v_normal_coeff_s = [sin(lambda_s)*cos(phi_s)/mw_s,-cos(lambda_s)*cos(phi_s)/mg_s];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% FRICTION COEFFICIENTS BETWEEN GEARS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mu_s = 0.11; % static friction coefficient between worm and rack ()
mu_k = 0.09;%0.2: % kinetic friction coefficient between worm and rack ()
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% NORMAL FORCE PARAMETERS BETWEEN WORM AND RACK
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
D = 26.99e6; % Nonlinear damping coefficient of Hertzian (tooth) damping (Ns/m^2)
tooth_stiffness_p = 2.685e8;% 155.4e6;% 1.4044e9;% 155.4e6; % Equivalent stiffness of
mating gear teeth (N/m)
tooth_stiffness_s = 1.984e8;
backlash_p = 1.016e-4;% 1.5240e-4;% 1e-4; % Total backlash between the pinion and
gear teeth (m)
backlash_s = 1.524e-4;
DV = 1e-5; % Velocity bandwidth which is considered "zero" velocity (m/s) -- used in
Friction model
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CONSTRAINT DEFINITION
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
theta_contact = 200; % angle of contact (degrees)
theta_contact_stiffen = .1; % angle of contact stiffening (degrees)
constraint_stiffness = 10000; % constant constraint stiffness (N/m)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% QUADRATIC SPRING PARAMETERS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
xmax = .25; % spring length at full stretch (m)
theta_max = 0; % maximum angle between extremes (deg.)

```



```

Fmax = 36.3;           % max Force possible (N)
n = 1;                % n*xkmax = maximum compression
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

pmax = theta_max*pi/180;      % convert max angle to radians
xkmax = (xmax-R*pmax)/(1+n);  % maximum stretch (m)

% CONTACT CALCULATIONS
theta_contact_max = theta_contact + theta_contact_stiffen; % angle of max stiff
x_contact = theta_contact*pi/180*R; % rest position of constraint spring

xki = (K_init-Kmin)/(Kmax-Kmin)*xkmax;
xmovei = thetai*pi/180*R;

% [A] MATRIX *****
A= zeros(17);
A(1,1) = -Ra_p/La_p;
A(1,2) = - Kt_p*gh_p/(Jw_p);
A(2,1) = Kt_p*gh_p/(La_p);
A(3,2) = sin(lambda_p)*cos(phi_p)*rw_p/Jw_p;
A(3,10) = cos(lambda_p)*cos(phi_p)*rg_p/Jm;
A(3,11) = -cos(lambda_p)*cos(phi_p)*rg_p/Jg_p;
A(4,4) = -Ra_s/La_s;
A(4,5) = -Kt_s*gh_s/(mw_s*rw_s);
% A(4,11) = -2*R^2*b/(Gp*Ja);
A(5,4) = Kt_s*gh_s/(rw_s*La_s);
A(6,5) = sin(lambda_s)*cos(phi_s)/mw_s;
A(6,7) = -cos(lambda_s)*cos(phi_s)/mg_s;
A(7,7) = -2*b/(Gs^2*mg_s);
A(8,7) = 1/(Gs*mg_s);
A(8,10) = -R/(Jm);
A(9,7) = 1/(Gs*mg_s);
A(9,10) = R/(Jm);
A(10,10) = -2*R^2*b/Jm;
A(12,4) = L/(Jg_p);
A(13,2) = 1/Jw_p;
A(14,11) = 1/Jg_p;
A(15,5) = 1/mw_s;
A(16,7) = 1/mg_s;
A(17,10) = 1/Jm;

```

```
% [A_NON] MATRIX (Coeff. of nonlinear terms of state equations)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
A_non = zeros(17,7);
A_non(2,1) = -rw_p*sin(lambda_p)*cos(phi_p);
A_non(2,2) = -rw_p*cos(lambda_p);
A_non(10,1) = -rg_p*cos(lambda_p)*cos(phi_p);
A_non(10,2) = rg_p*sin(lambda_p);
A_non(10,5) = R;
A_non(10,6) = -R;
A_non(11,1) = rg_p*cos(lambda_p)*cos(phi_p);
A_non(11,2) = -rg_p*sin(lambda_p);
A_non(5,3) = -sin(lambda_s)*cos(phi_s);
A_non(5,4) = -cos(lambda_s);
A_non(7,3) = cos(lambda_s)*cos(phi_s);
A_non(7,4) = -sin(lambda_s);
A_non(7,5) = -1/Gs;
A_non(7,6) = -1/Gs;
A_non(12,7) = -L;
```

```
% [B] MATRIX (Coeff. of Input variables of State equations)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
B = zeros(17,4);
B(1,:) = [ 1 0 0 0];
B(4,:) = [0 1 0 0];
B(11,:) = [ 0 0 -L 0];
B(12,:) = [ 0 0 0 1];
```

N_SPRING_P.M

```
% File - N_spring_p.m
```

```
% This function calculates the nonlinear spring force between mating gear teeth  
% for the position motor gears
```

```
function Force_k = tooth_spring(x):
```

```
global backlash_p tooth_stiffness_p
```

```
Force_k = sign(x).*(tooth_stiffness_p*(abs(x)-  
backlash_p/2*ones(size(x)))).*(abs(x)>backlash_p/2);%(100000*x).^9;
```

N_SPRING_S.M

```
% File - N_spring_p.m
```

```
% This function calculates the nonlinear spring force between mating gear teeth  
% for the stiffness motor lead screw
```

```
function Force_k = tooth_spring(x):
```

```
global backlash_s tooth_stiffness_s
```

```
Force_k = sign(x).*(tooth_stiffness_s*(abs(x)-  
backlash_s/2*ones(size(x)))).*(abs(x)>backlash_s/2); % (100000*x).^9;
```

PHI.M

Same as Config1 simulation in Appendix C

PSI.M

Same as Config1 simulation in Appendix C

VEL.M

Same as Config1 simulation in Appendix C

VP.M

```
% File - Vp.m
```

```
% This function calculates the input voltage to the position motor
```

```
function Voltage1 = Vp(t);
```

```
if 1
```

```
    Voltage1 = 0;
```

```
end
```

```
% Input for the -80 to 55 degree position change (arm loaded in gravity field)
```

```
if 0
```

```
    t1 = 0;
```

```
    t2 = .16;
```

```
    t3 = 2.25*.76;
```

```
    t4 = t2+t3;
```

```
    Vi = 0;
```

```
    Vf = 12;
```

```
    dV = Vf-Vi;
```

```
    dt = t2-t1;
```

```
    V_avg = (Vi+Vf)/2;
```

```
    a = (Vi-V_avg)/(.2*(.2*dt^2 + dt*t1 - dt*t2));
```

```
    Vo = (V_avg) - a*(.2*dt)*(t2+t1);
```

```
    Voo = (V_avg) + a*(.2*dt)*(t2+t1);
```

```
    V_line1 = (.4*a*dt)*t + Vo*ones(size(t));
```

```
    V_line2 = -(.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));
```

```
    V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));
```

```

V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));
V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));
V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));

Voltage1 = (t>=t1).*(t<(0.2*dt+t1)).*V_para1 +
(t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...
    +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +
(t>t3).*(t<(0.2*dt+t3)).*V_para3...
    + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;
end

```

VS.M

% File - Vs.m

% This function calculates the voltage input to the stiffness motor

```
function Voltage2 = Vs(t);
```

```

if 0
    Voltage2 = 0;
end

```

% Routine for 1-80 Nm/rad stiffness change (no gravity)

```

if 1
    t1 = 0;
    t2 = .16;
    t3 = .40;
    t4 = t3+t2;

    Vi = 0;
    Vf = 12;

    dV = Vf-Vi;
    dt = t2-t1;

    V_avg = (Vi+Vf)/2;

    a = (Vi-V_avg)/(.2*(.2*dt^2 + dt*t1 - dt*t2));
    Vo = (V_avg) - a*(.2*dt)*(t2+t1);
    Voo = (V_avg) + a*(.2*dt)*(t2+t1);

    V_line1 = (.4*a*dt)*t + Vo*ones(size(t));
    V_line2 = -(.4*a*dt)*(t-t3*ones(size(t))) + Voo*ones(size(t));
    V_para1 = a*(t-t1*ones(size(t))).^2 + Vi*ones(size(t));
    V_para2 = -a*(t-t2*ones(size(t))).^2 + Vf*ones(size(t));
    V_para3 = -a*(t-t3*ones(size(t))).^2 + Vf*ones(size(t));

```

```

V_para4 = a*(t-t4*ones(size(t))).^2 + Vi*ones(size(t));

Voltage2 = (t>=t1).*(t<(0.2*dt+t1)).*V_para1 +
(t>=(0.2*dt+t1)).*(t<(0.8*dt+t1)).*V_line1...
    +(t>=(0.8*dt+t1)).*(t<=t2).*V_para2 + (t>t2).*(t<=t3).*Vf +
(t>t3).*(t<(0.2*dt+t3)).*V_para3...
    + (t>=(0.2*dt+t3)).*(t<(0.8*dt+t3)).*V_line2 + (t>=(0.8*dt+t3)).*(t<=t4).*V_para4;
end

```