Parameterized Simulation and Optimization of High-Speed Systems and Interconnects.

by

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Abstract

Increased levels of integration and increase in the operating frequencies have highlighted interconnect effects previously neglected during circuit simulation. Accurate prediction of these effects involve solution of large system of equations, the direct optimization and simulation of which is prohibitively expensive. A new algorithm to perform optimization of high speed interconnect networks efficiently using parameterized model reduction techniques is presented. The response and the sensitivity needed to perform optimization are efficiently computed using parameterized model reduction techniques. This method provides significant speed up to perform optimization when compared to the traditional methods available in literature. An algorithm to form closed-form expression for the response of interconnect networks is also presented in this thesis. The closed form expression of the response is formed as a function of the design parameters in the network. This is achieved by the use of parameterized model reduction techniques in conjunction with a popular rational approximation algorithm vector-fit. This method provides an efficient way to calculate the response of the network, compared to solving the system of equations representing the network, resulting in a significant reduction in CPU time.

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Contents

1 Introduction	1
1.1 Background and Motivation - - - - -	1
1.2 Contributions - - - - -	4
1.3 Organization of the Thesis - - - - -	6
 2 Literature Review	 7
2.1 High-Speed Interconnects - - - - -	9
2.2 Formulation of Network Equations - - - - -	10
2.3 Model-Reduction Based Interconnect Simulation Algorithms - - - - -	12
2.3.1 Direct Moment Matching. - - - - -	13
2.3.2 Indirect moment matching techniques - - - - -	14
2.3.3 The Arnoldi Algorithm - - - - -	14
2.3.4 Model -Reduction through Arnoldi - - - - -	16
2.4 Parameterized Model Reduction Techniques - - - - -	17
2.4.1 Computation of Moments - - - - -	18

2.4.2	Model Reduction through Congruent Transformation	18
2.5	Sensitivity Analysis	19
2.5.1	Adjoint Sensitivity Method	19
2.5.2	Sensitivity Analysis using Model-Reduction Techniques	21
3	Efficient Optimization of High-Speed Interconnects using Parameterized Model-Reduction Techniques	23
3.1	Introduction	23
3.2	Formulation of Network Equations	26
3.3	Optimization using Parameterized Model Reduction Techniques	27
3.3.1	Response Evaluation using Parameterized Model Reduction	28
3.3.2	Sensitivity Analysis using Parameterized Model Reduction	29
3.4	Numerical Examples	31
3.4.1	Example 1	31
3.4.2	Example 2	35
4	Closed-Form Expressions for Interconnect Networks using Parameterized Model-Reduction Techniques	38
4.1	Introduction	38
4.2	Formulation of Network Equations	40
4.3	Closed Form Expression for Response Evaluation	41
4.3.1	Parameterized Krylov Subspace	42
4.3.2	Closed Form Expressions for the Poles and Residues	43

4.3.3 Closed form expression for polynomial coefficients.-----	45
4.4 Numerical Examples -----	46
4.4.1 Example 1 -----	46
4.4.2 Example 2 -----	51
5 Summary and Future Work -----	55
5.1 Summary -----	55
5.2 Future Work -----	56
References	57

List of Tables

3.1 Comparison of CPU time for the 3-two coupled line example	35
3.2 Comparison of CPU time of the 9 coupled transmission line example	36
4.1 Comparison of CPU time for example 4.1	51
4.2 Comparison of CPU time for example 4.2	52

List of Figures

2.1 Computation of column q_2 through Arnoldi. - - - - -	16
3.1 Traditional optimization process - - - - -	24
3.2 Optimization using proposed method- - - - -	25
3.3 Original circuit for example 3.1 - - - - -	32
3.4 Results from original and reduced systems before optimization - - - - -	33
3.5 Percentage error before optimization - - - - -	33
3.6 Original system responses before and after optimization- - - - -	34
3.7 Reduced system responses before and after optimization - - - - -	34
3.8 Original circuit for example 3.2 - - - - -	36
3.9 Insertion loss before optimization - - - - -	37
3.10 Insertion loss after optimization - - - - -	37
4.1 Network under consideration for example 4.1 - - - - -	48
4.2 Comparison of variation in pole of the system response w.r.t width of a transmission line. - - - - -	49
4.3 Comparison of variation in a residue of the system response w.r.t width of a transmis-	

sion line. -----	49
4.4 Comparison of the frequency domain impulse response obtained from the closed-form expression and that obtained from the original network. -----	50
4.5 Comparison of the time-domain impulse response for two different values of line- widths. -----	50
4.6 Original System for example 4.2 -----	53
4.7 Comparison of response from the original system and using the proposed method	53
4.8 Percentage error of the response for example 4.2 -----	54

Abbreviations

CPU	-----	Central Processing Unit
VLSI	-----	Very Large Scale Integration
CAD	-----	Computer-Aided Design
EM	-----	Electromagnetic
MMT	-----	Moment Matching Techniques
PMR	-----	Parameterized Model Reduction
MNA	-----	Modified Nodal Analysis
PVL	-----	Padé via Lanczos
LU	-----	Lower-Upper
F/B	-----	Forward Backward
TEM	-----	Transverse Electromagnetic
RLC	-----	Resistor-Inductor-Capacitor
AWE	-----	Asymptotic Waveform Evaluation
CFH	-----	Complex Frequency Hopping
MR	-----	Model Reduction
MEMS	-----	Micro Electromechanical Systems

Chapter 1

Introduction

1.1 Background and Motivation

Rapid developments in VLSI technology, design and computer-aided design (CAD) techniques at both the chip and package level have pushed the central processor cycle times to the vicinity of 1ns and communication switches are being designed to transmit data that have bit rates faster than 1Gb/s. With higher signal speeds, the electrical length of interconnects becomes a significant portion of the wavelength. This gives rise to signal distorting effects that do not exist at lower frequencies. At high signal speeds the interconnect exhibits distributed capacitive and inductive effects which makes the receiver no longer a direct load of the driver [1]-[12]. This leads to various transmission line effects such as delay, distortion, reflections, overshoot, undershoot, crosstalk and modeling of these needs the blending of EM and circuit theory. Generally, high-speed interconnect

effects are neither handled efficiently nor accurately by conventional circuit simulators such as SPICE [13]. If not considered during the design stage, these interconnect effects can cause logic glitches which render a fabricated digital circuit inoperable, or they can distort an analog signal such that it fails to meet specifications. Since extra iterations in the design cycle are costly, accurate prediction of the effects is a necessity in high-speed designs. Hence it becomes extremely important for designers to simulate the entire design along with interconnect subcircuits as efficiently as possible while retaining the accuracy of simulation.

One of the main problems encountered in trying to predict interconnect effects is that in general, interconnect networks result in large systems of equations, direct simulation and optimization of which is prohibitively CPU expensive. Model reduction techniques were presented in literature [14]-[30] to overcome this issue. Model-reduction is a paradigm where a large system of equations is reduced to a small set of equations such that the response obtained from the smaller system resembles that of the original system.

There have been numerous publications using moment-matching techniques (MMT) and Padé approximation for reducing the order of the linear system [14]-[19]. However, Padé approximation has an ill-conditioning problem associated with it. In order to overcome this problem, several techniques based on Krylov-subspace methods and congruent transformations were developed [20]-[30]. Approaches like the Padé via Lanczos (PVL) and Arnoldi algorithm are based on Krylov-subspace methods and congruent transformations.

All the above-mentioned techniques perform model-reduction with respect to a single parameter (frequency). However, there is also necessity in many applications to include variations due to environmental effects and changes in interconnect parameters such as width and height. In such cases, the reduced model has to be repeatedly generated a large number of times corresponding to each variation in the design parameter. Parameterized model reduction (PMR) techniques were introduced in the literature [31]-[33] to overcome this computational inefficiency. PMR techniques perform analysis of large circuits simultaneously with respect to frequency and other design parameters of the circuit.

Generally, optimization typically requires repeated evaluation of response and gradients of the system. Optimization in circuit design literature still uses traditional methods to find responses and gradients of the network at every iteration. This can be very expensive, especially for interconnect networks owing to large sizes of the modified nodal analysis (MNA) matrices. Attempts to improve optimization techniques through traditional model reduction methods are not generally efficient because the evaluation of responses and gradients would involve regeneration of the reduced models for every change in the optimization parameters.

In this thesis, a new efficient algorithm to perform optimization is presented in Chapter 3, using parameterized model reduction (PMR) techniques. As mentioned before, PMR techniques generate reduced models of the interconnect network as a function of circuit parameter and frequency. The responses of the system required during optimization are obtained by evaluating the reduced system generated using PMR techniques. The gradient information essential to the optimization process is evaluated by performing sensi-

tivity analysis using parameterized model reduction techniques. This is performed by using the concept of PMR technique on the adjoint system of equations which involves projecting the adjoint network equations on the reduced subspace with respect to frequency and optimization variables of the circuit. As a result of using this algorithm for optimization, a significant reduction of CPU time is obtained.

While the model-reduction techniques discussed above give reduced system of the large linear systems with respect to design parameters, they still require the process of solving the given reduced system to find the response of the system. Chapter 4 presents an algorithm to form closed-form expressions for the response of interconnect circuits as a function of a design parameter in the network. The response of the network is found as an explicit closed-form function of the design parameter. This eliminates the need to find the solution using LU decomposition and forward/backward substitution resulting in significant speed-up. The algorithm couples the merits of parameterized reduction with that of vector-fit [34], a popular algorithm to form rational approximations from given data points.

1.2 Contributions

The main subject of the thesis is simulation and optimization of high-speed interconnect networks. The main contributions of this thesis are as follows:

1. Sensitivity analysis is performed using the concept of parameterized model reduction techniques on the adjoint system. This eliminates the need for reducing the adjoint system at every iteration of the optimization algorithm, which

would be necessary for sensitivity analysis using any other model reduction technique. A significant speed up is obtained when compared to the traditional adjoint sensitivity method.

2. An efficient optimization algorithm using parameterized model reduction techniques is presented. Optimization in the circuit design literature still uses traditional methods to find responses and gradients of the network at every iteration. This can be very expensive, especially for interconnect networks owing to large sizes of the modified nodal analysis (MNA) matrices. The proposed algorithm generates reduced models with respect to frequency and design parameters to evaluate the response and gradients of the system. A significant speedup in the optimization process compared to traditional optimization is obtained.
3. Algorithm to automatically generate closed form expressions are developed for high speed interconnect examples by vector-fitting the coefficients of the numerator and denominator polynomials of complex conjugate pole-residue pairs. The efficiency of forming closed form expressions is significantly improved and the sensitivity of the polynomial coefficients to the circuit response is much less than that of the poles and residues.

1.3 Organization of the Thesis

This thesis is organized as follows: Chapter 2 presents a brief overview of the existing model reduction techniques. This chapter also gives a detailed review of the Arnoldi algorithm and parameterized model reduction techniques. Sensitivity analysis through traditional adjoint technique and sensitivity analysis using model reduction techniques are also explained in this chapter. In Chapter 3, a new efficient optimization algorithm using parameterized model reduction techniques is presented. Chapter 4 presents a technique to obtain a closed form expression of the response of the network with respect to any design parameter of the network. Finally the conclusions and future work are presented in Chapter 5.

Chapter 2

Literature Review

As the rise times of digital signals drop into sub-nanosecond range, the effects of off-chip circuitry become increasingly important. Advances in fabrication methods have made interconnect analysis imperative for diverse technologies such as packages, multi-chip modules, printed circuit boards and backplanes. Shrinking device sizes coupled with increasing operating speeds are highlighting the effects, such as ringing, signal delay, distortion, and reflections on single interconnect lines as well as crosstalk between adjacent lines [1]-[12]. In general we find that high-speed effects are not always handled appropriately by conventional circuit simulators such as SPICE [13]. Interconnect effects if not considered at early design stages can cause logic glitches that render a fabricated digital circuit inoperable or distort an analog signal enough to fail the desired specifications. Since extra iterations in the design cycle are costly, accurate prediction of these effects is a necessity in high-speed designs. Hence, it becomes extremely important for designers to

simulate their design along with interconnect effects as accurately as possible without a loss in computational efficiency. One of the main problems encountered in trying to predict interconnect effects is that in general interconnect networks result in large systems of equations, direct optimization and simulation of which is prohibitively CPU expensive. Model order reduction techniques have been proposed in the circuit simulation literature as a way to deal with the large system of equations that result from packaging and interconnect structures [14]-[30]. Interconnect networks generally tend to have a large number of poles, spread over a wide-frequency range. Even though the majority of these poles would normally have very little effect on simulation results, they make the simulation CPU expensive by forcing the simulator to take smaller step sizes. Model reduction takes advantage of this fact by approximating the original large system with a reduced order model that captures the essential characteristics, or the dominant poles of the original system.

Presented in this chapter is a brief overview of the existing model-reduction techniques. This chapter also provides some background on traditional sensitivity analysis and sensitivity analysis using model reduction techniques for large interconnect networks. The organization of this chapter is as follows: Section 2.1 defines high-speed interconnects. Section 2.2 illustrates the set-up of the modified nodal analysis (MNA) matrices of the system. A brief overview of the existing model reduction techniques are given in Section 2.3. Section 2.4 explains the parameterized model reduction technique. Traditional sensitivity analysis and sensitivity analysis using model reduction techniques are discussed in Section 2.5.

2.1 High-Speed Interconnects

In a broad sense a “high-speed interconnect” is an interconnect where the time taken by the propagating signal to travel between its end points cannot be neglected. An important but obvious factor that influences this definition is the physical extent of the interconnect; the longer the interconnect, the more time the signal takes to travel between its end points. The smoothness of the signal is also affected as it propagates through the line. The interconnect exhibits distributed capacitive and inductive effects which makes the receiver no longer a direct load of the driver [1]-[12]. This leads to various transmission line effects such as reflections, overshoot, undershoot, and crosstalk, and modeling of these needs the blending of EM and circuit theory. The term “high-speed” can also be understood from a different view point; in terms of the frequency content of the signal. At low frequencies, an interconnect behaves effectively as a short circuit. However, this is not the case at higher frequencies. The same interconnect exhibits inductive/capacitive effects at higher frequencies. Faster clock speeds and sharper slew rates tend to add a considerable amount of high-frequency content.

An important criterion used for classifying interconnects is the electrical length of an interconnect. An interconnect is “electrically short” if, at the highest operating frequency of interest, the interconnect length is physically shorter than approximately one-tenth of the wavelength. Otherwise, the interconnect is referred to as “electrically long” [1], [6]. Electrically short interconnects can be represented by lumped models whereas electrically long interconnects need quasi-TEM distributed or full-wave models. In most applications, the desired highest operating frequency of interest is governed by the rise/fall

time of the propagating signal. A practically useful relationship between the maximum frequency of interest (f_{max}) and the rise/fall time of the signal (t_r) can be expressed as [1], [35]-[37]

$$f_{max} \approx 0.35/t_r \quad (2.1)$$

In view of the various problems encountered through interconnect effects it becomes imperative to form interconnect models that capture high-frequency responses efficiently and accurately.

2.2 Formulation of Network Equations

A general linear network with interconnects consisting of lumped RLC and distributed components can be described through its MNA equations as

$$\mathbf{C}_\phi \frac{d\mathbf{x}_\phi}{dt} + \mathbf{G}_\phi \mathbf{x}_\phi + \sum_{k=1}^{N_t} \mathbf{D}_k \mathbf{i}_k(t) = \mathbf{B}_\phi \quad (2.2)$$

$$V = \mathbf{d}_\phi^T \mathbf{x}_\phi \quad (2.3)$$

- $\mathbf{x}_\phi(t, \lambda_1 \dots \lambda_n) \in \mathbb{R}^{N_\phi}$ is the vector of node voltage waveforms and currents in the system;
- $\mathbf{C}_\phi \in \mathbb{R}^{N_\phi \times N_\phi}$, $\mathbf{G}_\phi \in \mathbb{R}^{N_\phi \times N_\phi}$ are matrices describing the lumped memory and memoryless elements of the network;
- $\mathbf{B}_\phi \in \mathbb{R}^{N_\phi}$ is the vector with entries determined by the independent voltage and cur-

rent sources;

- N_ϕ is the total number of variables in the Modified Nodal Analysis (MNA) formulation;
- $\mathbf{D}_k$ is a selector matrix that maps $\mathbf{i}_k(t) \in \mathbb{R}^{2n_k}$, the vector of currents entering the interconnect subnetwork k , into the node space of the network,
- N_t is the total number of distributed coupled interconnect sets and n_k is the number of coupled conductors in the linear subnetwork k .
- V is the output response variable of interest;
- $\mathbf{d}_\phi$ is a selector vector with nonzero entries corresponding to the output variable; and
- n is the number of optimization variables in the network.

The frequency-domain equations of interconnect subnetwork k containing distributed interconnects can be written in the form

$$\mathbf{I}_k(s) = \mathbf{Y}_k(s)\mathbf{V}_k(s) = \mathbf{Y}_k(s)\mathbf{D}_k^T\mathbf{X}_\phi(s) \quad (2.4)$$

where $\mathbf{V}_k(s)$, $\mathbf{I}_k(s)$ and $\mathbf{Y}_k(s)$ represent the Laplace-domain terminal voltages, currents and the admittance parameters for the subnetwork k respectively. $\mathbf{X}_\phi(s)$ is the Laplace transform of $\mathbf{x}_\phi(t)$.

The distributed interconnect subnetwork described in (2.2) can be discretized using an appropriate discretization method [38], [39]. The vector of unknowns $\mathbf{x}_\phi(t)$ is

expanded to $\mathbf{x}(t)$ to accommodate the new variables introduced due to discretization.

Similarly, the matrices and vectors, $\mathbf{C}_\phi$, $\mathbf{G}_\phi$, $\mathbf{d}_\phi$, $\mathbf{V}_\phi$ and $\mathbf{B}_\phi$ are expanded to the form $\mathbf{C}$, $\mathbf{G}$, $\mathbf{d}$, $\mathbf{V}$ and $\mathbf{B}$. The MNA equations of the network can now be written as

$$\mathbf{C} \frac{d\mathbf{x}}{dt} + \mathbf{G}\mathbf{x} = \mathbf{B} \quad (2.5)$$

$$\mathbf{V} = \mathbf{d}^T \mathbf{x} \quad (2.6)$$

where, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{C} \in \mathbb{R}^{N \times N}$, $\mathbf{G} \in \mathbb{R}^{N \times N}$, $\mathbf{B} \in \mathbb{R}^N$ and $N > N_\phi$.

The network equations described in (2.5), (2.6) can be expressed in frequency domain as

$$\mathbf{A}\mathbf{x} = \mathbf{B} \quad (2.7a)$$

$$\mathbf{V} = \mathbf{d}^T \mathbf{x} \quad (2.7b)$$

where $\mathbf{A} = \mathbf{G} + s\mathbf{C}$ and s represents complex frequency.

2.3 Model-Reduction Based Interconnect Simulation Algorithms

Model-reduction techniques are based heavily on the fact that the behaviour of a large linear system is governed mostly by a small set of so-called “dominant poles” of the system. “Dominant poles” are poles that are close to the imaginary axis and significantly influence the time as well as the frequency characteristics of the system. The reduction

methods in the circuit simulation literature can be classified under two main classes: 1) Direct moment matching based on Padé approximation, and 2) Indirect approximations based on finding the leading eigenvalues (those with the largest magnitude).

2.3.1 Direct Moment Matching

Direct moment-matching techniques (MMT) [14]-[19] are based on directly finding a rational approximation of the impulse response in the frequency domain, typically using Padé approximation. Obtaining a lower order approximation of the network transfer function using a single Padé expansion is commonly referred to as asymptotic waveform evaluation (AWE) in the literature [15]. However, due to inherent limitations of Padé approximants, MMTs based on single expansion often gave inaccurate results. This leads to the development of multipoint expansion techniques such as complex frequency hopping (CFH). CFH extends the process of moment matching to multiple expansion points in the complex plane near or on the imaginary axis using a binary search algorithm [18]. However, Padé approximation has an ill-conditioning problem associated with it. In addition, there is no guarantee that the reduced model obtained is passive. Passivity implies that a network cannot generate more energy than it absorbs and no passive termination of the network will make the system unstable. Passivity is important because stable but not passive macromodels can lead to unstable system when connected to other passive systems. To address these issues of ill-conditioning and passivity, indirect moment-matching techniques were developed.

2.3.2 Indirect Moment Matching Techniques

The indirect moment-matching algorithms are based on what is known as Krylov-subspace formulation and Congruent transformation [20]-[30]. One of the main features of these algorithms is that they construct the reduced-model based on the extraction of leading eigenvalues of the given system compared to extracting the dominant poles of the given system.

Padé via Lanczos (PVL) was suggested to overcome the difficulties due to ill-conditioning [20]. However, the passivity of the reduced-order model generated by PVL is ensured for only networks involving two elements and not for general RLC networks. A popular approach called Arnoldi's algorithm [22]-[30] became available in literature which addressed both the ill conditioning and the passivity issues. Arnoldi based reductions could also achieve higher order approximations, hence better accuracy could be achieved. In the next section, a detailed review of the Arnoldi algorithm is given.

2.3.3 The Arnoldi Algorithm

The Arnoldi algorithm [27] starts by computing a Krylov-subspace which spans the same space as the moments of the system of equations (2.7). This subspace is denoted by $\kappa(\mathbf{A}, \mathbf{R}, n)$. Any vector that is a linear combination of the moments of the system, can be expressed as a linear combination of the columns of $\kappa(\mathbf{A}, \mathbf{R}, n)$. This can be mathematically expressed as

$$\kappa(\mathbf{A}, \mathbf{R}, n) = \text{colsp}(\mathbf{K}) \quad (2.8)$$

where,

$$\mathbf{K} = (\mathbf{R} \mathbf{A} \mathbf{R} \dots \mathbf{A}^{n-1} \mathbf{R}) \quad (2.9)$$

$$\mathbf{A} = -\mathbf{G}^{-1} \mathbf{C} \quad \text{and} \quad \mathbf{R} = \mathbf{G}^{-1} \mathbf{B}$$

The matrix $\mathbf{K}$ typically suffers from ill-conditioning since the columns of $\mathbf{K}$ are formed based on the sequence $\mathbf{A}^i \mathbf{R}$, which quickly converges to the eigenvector of the largest eigenvalue [27]. To overcome the ill-conditioning problem, the Arnoldi algorithm computes an orthogonal matrix $\mathbf{Q}$ such that

$$\text{colsp}(\mathbf{Q}) = \text{colsp}(\mathbf{K}) \quad (2.10)$$

In contrast to the matrix $\mathbf{K}$, matrix $\mathbf{Q}$ is well conditioned. This matrix can be computed using techniques such as the *modified Gram-Schmidt algorithm* [27]. A simple implementation of the Arnoldi algorithm which uses modified Gram-Schmidt is given in the next page.

The columns q_j computed by the Arnoldi algorithm are called Arnoldi vectors. The loop over i updating z in the pseudo-code, corresponds to the modified Gram Schmidt algorithm, which subtracts the components of $\mathbf{A}q_j$ in the directions q_1 to q_j away from z , leaving them orthogonal to z . This is demonstrated in Fig. 2.1. Computing a total of “ k ” Arnoldi vectors costs k matrix vector multiplications involving $\mathbf{A}$, plus $O(k^2 n)$ related cost.

function *Arnoldi*
 $q_1 = \mathbf{R} / \|\mathbf{R}\|_2$ // Get the direction of q_1
for $j = 1$ to k // k is the desired number of columns of the $\mathbf{Q}$ matrix

 $z = \mathbf{A}q_j$
for $i = 1$ to j
 $h_{ij} = q_i^T z$
 $z = z - h_{ij}q_i$
end for
 $h_{j+1,j} = \|z\|$; **if** $h_{j+1,j} = 0$, **quit**
 $q_{j+1} = z / h_{j+1,j}$;

end for
end function

Fig. 2.1 Computation of column q_2 through Arnoldi.

2.3.4 Model-Reduction through Arnoldi

In this section, we use matrix $\mathbf{Q}$ found using the Arnoldi algorithm to perform model-reduction on the original circuit equations [27]-[30]. The reduced-order circuit equations can be formed through a change of variables by mapping the vector x of dimen-

sion N into a smaller vector $\hat{x}$ of dimension q ($q \ll N$) using the orthogonal matrix $\mathbf{Q}$. This mapping is performed as

$$\mathbf{x} = \mathbf{Q}\hat{\mathbf{x}} \quad (2.11)$$

and is known as the congruent transformation.

The circuit equations (2.7) can be rewritten using (2.11) as smaller system equation with respect to frequency as

$$\hat{\mathbf{A}}\hat{\mathbf{x}} = \hat{\mathbf{B}} \quad (2.12)$$

where

$$\hat{\mathbf{A}} = \mathbf{Q}^T \mathbf{A} \mathbf{Q} \quad \hat{\mathbf{B}} = \mathbf{Q}^T \mathbf{B} \quad (2.13)$$

The response obtained using the reduced system matches the response obtained from the original system depending on the number of moments matched during reduction. For a given frequency range, the number of moments can be evaluated to match the response accurately.

2.4 Parameterized Model Reduction Techniques

Model reduction techniques reduce systems of linear equations into smaller set of equations with respect to (w.r.t) a single parameter (frequency). However, in many applications, it is required to evaluate the response of the circuit as a function of other design parameters such as temperature and interconnect dimensions, in addition to frequency. In such cases, the reduced model has to be repeatedly generated a large number of times cor-

responding to each variation of the design parameter. To overcome this computational inefficiency, parameterized model reduction techniques were recently introduced in literature [31]–[33] to generate reduced models as a function of circuit parameters. These techniques avoid, the repeated generation of reduced model for changes in design parameter.

The first and most important step of the PMR technique is the computation of the multidimensional subspace ($\mathbf{Q}$) of the system in (2.7).

2.4.1 Computation of Moments

The moments of $\mathbf{x}$ w.r.t $\mathbf{s}$ (denoted by $\mathbf{Q}_s$) are computed using procedure described in Section 2.3.3 and [28]. Moments w.r.t λ (denoted by $\mathbf{Q}_\lambda$) are computed using the recursive relationship as given in [32]. In the special case where the variation of matrices $\mathbf{C}$ and $\mathbf{G}$ with respect to the parameter λ is linear, techniques such as Arnoldi (see Section 2.3.3) can be used to compute the moments with respect to λ to extract better efficiency.

2.4.2 Model Reduction through Congruent Transformation

Once all the required moments are evaluated, the congruent transformation matrix $\mathbf{Q}$ is constructed such that

$$\text{colsp}(\mathbf{Q}) = \text{colsp}[\mathbf{Q}_s \ \mathbf{Q}_\lambda] \quad (2.14)$$

where $\mathbf{Q} \in \mathbb{R}^{N \times q}$.

This is achieved by using a standard QR decomposition on $[\mathbf{Q}_s \ \mathbf{Q}_\lambda]$. It has to be noted that once computed, the multidimensional subspace $\mathbf{Q}$ is a constant real matrix independent of frequency and λ .

The reduced system is formed by using the multidimensional $\mathbf{Q}$ to perform congruent transformation on the original system (2.7) to produce a reduced order macromodel similar to the procedure described in section 2.3.4. The response of the reduced system is calculated by evaluating (2.12) and is transformed to the original system response through (2.11).

2.5 Sensitivity Analysis

Several techniques can be found in the literature to determine the sensitivity of the output with respect to any parameter λ in the network. Amongst these, the adjoint sensitivity method has been proven to be an efficient technique [40]. Section 2.5.1 briefly reviews the adjoint sensitivity method. Recently, sensitivity analysis using model reduction technique was presented in literature [41] which is detailed in Section 2.5.2.

2.5.1 Adjoint Sensitivity Method

With λ being a circuit parameter representing an element in the network, (2.7) can be written as

$$\mathbf{A}(s, \lambda)\mathbf{X}(s, \lambda) = \mathbf{B} \quad (2.15a)$$

$$\mathbf{V} = \mathbf{d}^T \mathbf{X} \quad (2.15b)$$

The sensitivity of V in (2.15b) with respect to λ_i , $1 \leq i \leq n$ is [40]

$$\frac{\partial V}{\partial \lambda_i} = \mathbf{X}_a^T \frac{\partial \mathbf{A}}{\partial \lambda_i} \mathbf{X} \quad (2.16)$$

where

- $\mathbf{X}$ is the solution of the network equation
- $\mathbf{X}_a$ is the solution of the adjoint equation

$$\mathbf{A}^T \mathbf{X}_a = -\mathbf{d} \quad (2.17)$$

$$\text{and } \frac{\partial \mathbf{A}}{\partial \lambda_i} = \frac{\partial \mathbf{G}}{\partial \lambda_i} + s \frac{\partial \mathbf{C}}{\partial \lambda_i} \quad 1 \leq i \leq n.$$

In traditional adjoint sensitivity method, the computational complexity of evaluating $\mathbf{X}_a$ is one forward-backward substitution (F/B) since the LU decomposition of $\mathbf{A}^T$ is already available at every frequency point from the solution of the network equations (2.15). However, in model reduction techniques the original system is reduced to yield computational efficiency. LU decomposition is performed on $\hat{\mathbf{A}}$ at every frequency point as opposed to $\mathbf{A}$. Thus, the process of evaluating the adjoint system requires evaluation of the original system repeatedly at every frequency to compute LU values of $\mathbf{A}$ which becomes CPU intensive for interconnect networks. To address this issue, sensitivity analysis using model reduction techniques were presented in literature [41]. The next section reviews the sensitivity analysis using model reduction technique.

2.5.2 Sensitivity Analysis using Model-Reduction Techniques

The sensitivity in terms of a design parameter λ using model order reduction is calculated using the same concept used by Arnoldi algorithm (section 2.3.3 and 2.3.4). Model reduction is directly applied to the adjoint system (2.17) through a congruent transformation matrix $\mathbf{Q}_a$ as opposed to the original system of equations. As in the case of model reduction of the original system, the moments of X_a are needed for the construction of the congruent transformation $\mathbf{Q}_a$ [41]. Reducing the adjoint circuit equation (2.17), we have

$$\hat{\mathbf{A}}_a^T(s) \hat{X}_a(s) = -\hat{d}_a \quad (2.18)$$

where

$$\hat{\mathbf{A}}_a = \mathbf{Q}_a^T \mathbf{A} \mathbf{Q}_a \quad \hat{d}_a = \mathbf{Q}_a^T d \quad (2.19)$$

The solution of the adjoint network can, therefore, be found by mapping $\hat{X}_a$ back to the original space through the congruent transformation

$$X_a(s) = \mathbf{Q}_a \hat{X}_a(s) \quad (2.20)$$

The sensitivity can then be found at any frequency by simply solving the reduced system in (2.12) and (2.18) and evaluating (2.16). It should be noted that system moments and the adjoint moments are computed using the same LU decomposition. However, there is no obvious relationship between $\mathbf{Q}$ and $\mathbf{Q}_a$. Therefore two separate QR decompositions

are required to find the two transformation matrices. Furthermore, $\mathbf{Q}$ and $\mathbf{Q}_a$ need not be the same order, although same order for both transformation is recommended.

Chapter 3

Efficient Optimization of High-Speed Interconnects using Parameterized Model Reduction Techniques

3.1 INTRODUCTION

As the feature sizes of integrated circuits (ICs) decrease and signal frequencies increase, many interconnect effects such as delay, coupling, attenuation and crosstalk, previously neglected during circuit simulation, have become prominent [1]-[12]. These interconnect effects, if not predicted at early design stages, can severely degrade system performance. Generally, interconnect networks result in large system of equations, direct simulation or optimization of which is prohibitively expensive. Forming a reduced linear system is the key to fast simulation of interconnect networks. Numerous model reduction techniques can be found in the literature that efficiently simulate circuits by reducing the

order of the linear system. These techniques reduce a large system of equations to a smaller set of equations and the reduced system can be simulated in a fraction of the time required to simulate the original system.

In addition to simulation, circuit designers are often interested in optimization of circuit responses with respect to optimization variables [3]. In general, optimization typically requires repeated evaluation of responses and gradients of the system (see fig 3.1). Traditional optimization process can be very expensive, especially for interconnect networks owing to large sizes of the modified nodal analysis (MNA) matrices. Attempts to improve optimization techniques through traditional model reduction methods are not generally efficient because the evaluation of responses and gradients would involve regeneration of the reduced models for every change in the optimization parameters. Recently, a new class of model reduction algorithms known as parameterized model reduction (PMR) techniques [31]-[33] has been presented to generate reduced models as a function of circuit parameters.

Fig. 3.1 Traditional optimization process

Fig. 3.2 Optimization using proposed method

In this chapter, an efficient optimization algorithm using parameterized model reduction techniques is presented. As shown in fig 3.2, the responses of the system required during optimization are obtained by evaluating the reduced system generated using PMR techniques. The gradient information essential to the optimization process is evaluated by projecting the adjoint network equations on the reduced subspace with respect to frequency and optimization variables of the circuit. This chapter is organized as follows: Section 3.2 presents the formulation of the circuit equations. The proposed algorithm is presented in Section 3.3 with response and gradient calculation discussed in 3.3.1 and 3.3.2 respectively. Section 3.4 gives the numerical results to prove the accuracy and efficiency of the given algorithm.

3.2 Formulation of Network Equations

A general linear network with its elements as a function of parameters $\lambda_1 \dots \lambda_n$ can be written as

$$\mathbf{C}(\lambda_1 \dots \lambda_n) \frac{d\mathbf{x}}{dt} + \mathbf{G}(\lambda_1 \dots \lambda_n) \mathbf{x} = \mathbf{B} \quad (3.1a)$$

$$V = \mathbf{d}^T \mathbf{x} \quad (3.1b)$$

where,

- $\mathbf{x}(t, \lambda_1 \dots \lambda_n) \in \mathfrak{R}^N$ is the vector of node voltage waveforms and currents in the system;
- $\lambda_1 \dots \lambda_n$ are physical parameters in the circuit which need to be optimized such as temperature, circuit elements, and interconnect dimensions;
- $\mathbf{C}(\lambda_1 \dots \lambda_n) \in \mathfrak{R}^{N \times N}$, $\mathbf{G}(\lambda_1 \dots \lambda_n) \in \mathfrak{R}^{N \times N}$ are matrices describing the lumped memory and memoryless elements of the network dependent on $\lambda_1 \dots \lambda_n$;
- $\mathbf{B} \in \mathfrak{R}^N$ is the vector with entries determined by the independent voltage and current sources;
- N is the total number of variables in the Modified Nodal Analysis (MNA) formulation;
- V is the output response variable of interest;
- $\mathbf{d}$ is a selector vector with nonzero entries corresponding to the output variable; and

- n is the number of optimization variables in the network.
- s represents complex frequency;

The network equations described in (3.1) can be expressed in frequency domain as

$$\mathbf{A}(\lambda_1 \dots \lambda_n) \mathbf{x} = \mathbf{B} \quad (3.2)$$

$$\mathbf{V} = \mathbf{d}^T \mathbf{x} \quad (3.3)$$

where $\mathbf{A} = \mathbf{G} + s\mathbf{C}$.

3.3 Optimization using Parameterized Model Reduction Techniques

In general, optimization typically requires repeated evaluation of responses and gradients of the system. Owing to the large size of system equations for interconnect circuits, repeated evaluation of responses and gradients become prohibitively expensive. Model reduction (MR) techniques are generally used in literature (see Section 2.3), to speed up simulation of large networks. Traditional MR techniques reduce systems of linear equations into smaller set of equations w.r.t a single parameter (frequency). As smaller systems are evaluated instead of the original system, the CPU expense reduces. However, attempts to improve optimization techniques through traditional MR techniques are not generally efficient because the evaluation of response and gradient for every change in each optimization variable would involve regeneration of reduced model.

Recently, parameterized model reduction (PMR) techniques have been presented [31]-[33] to generate reduced models as a function of circuit parameters. These techniques

avoid the repeated generation of reduced models for changes in optimization variables. The proposed algorithm provides an efficient way to perform optimization using PMR techniques. The algorithm is presented in the following sections. Section 3.3.1 reviews the evaluation of circuit responses using parameterized model order reduction. Section 3.3.2 presents a new method to compute the sensitivity using the PMR techniques.

3.3.1 Response Evaluation using Parameterized Model Reduction

As mentioned before, Parameterized Model Reduction techniques form reduced order macromodels of large linear systems w.r.t frequency and other optimization variables of the network. The most important step of the parameterized model reduction techniques is the computation of the multidimensional space $\mathbf{Q}$ of the system (3.2). This multidimensional subspace is a constant real matrix independent of frequency and $\lambda_1 \dots \lambda_n$. The reduced system is formed by performing congruent transformation on the original system (3.2) to produce a reduced order macromodel. Now the response of the system can be found by solving the reduced system and transforming it to the original system through congruent transformation. As the evaluation of the reduced system takes much less CPU time when compared to direct simulation of the original system, repeated response evaluation required by optimization process is much more CPU efficient. The details to calculate the response using PMR technique is given in Section 2.4. In the next section, a new efficient method to compute the sensitivity analysis using the PMR technique is presented.

3.3.2 Sensitivity Analysis using Parameterized Model Reduction

The use of gradient information significantly improves the effectiveness of the optimization algorithm. Sensitivity analysis is performed to find the gradients of the system in terms of the optimization variables. Several techniques can be found in the literature to determine the sensitivity of the output w.r.t any parameter λ of the linear network. Amongst these, the adjoint sensitivity method has been proven to be an efficient technique [40]. The adjoint sensitivity method is detailed in Section 2.5.1. However, when using the reduced system for response evaluation, the process of evaluating the adjoint system requires repeated evaluation of the original system which becomes CPU intensive for interconnect networks. To address this issue, sensitivity analysis using model reduction was presented in the literature (see Section 2.5.2). However, model reduction techniques reduce the system in terms of a single parameter (frequency). This would result in regeneration of the reduced model for every variation in optimization variables. Hence we propose a new algorithm to perform sensitivity analysis using parameterized model reduction techniques.

To calculate the sensitivity in terms of the design parameters, the algorithm applies PMR directly on the adjoint system

$$\mathbf{A}^T \mathbf{x}_a = -\mathbf{d} \quad (3.4)$$

through a congruent transformation matrix $\mathbf{Q}_a$ evaluated such that

$$span(\mathbf{Q}_a) = span[\mathbf{M}_s^a \ \mathbf{M}_{\lambda_1}^a \ \dots \ \mathbf{M}_{\lambda_n}^a] \quad (3.5)$$

where, $\mathbf{M}_s^a$ represents the moments of x_a w.r.t frequency s and $\mathbf{M}_{\lambda_i}^a$ represents the moments of x_a w.r.t $\lambda_1 \dots \lambda_n$.

These moments are evaluated on the adjoint system (3.4) similar to the method described in Section 2.4 where the moments are found for the original system. Once the moments are obtained, $\mathbf{Q}_a$ is calculated using QR decomposition. The adjoint circuit (3.4) is reduced using the congruent transformation matrix $\mathbf{Q}_a$ to give,

$$\hat{\mathbf{A}}_a(s)\hat{\mathbf{x}}_a(s) = \hat{\mathbf{d}}_a \quad (3.6)$$

where

$$\hat{\mathbf{A}}_a = \mathbf{Q}_a^T \mathbf{A}^T \mathbf{Q}_a \quad \text{and} \quad \hat{\mathbf{d}}_a = \mathbf{Q}_a^T (-\mathbf{d}) \quad (3.7)$$

The solution of the adjoint network is evaluated by mapping $\hat{\mathbf{x}}_a$ back to the original space through the congruent transformation.

$$\mathbf{x}_a(s) = \mathbf{Q}_a \hat{\mathbf{x}}_a(s) \quad (3.8)$$

The gradient can then be found at any frequency point by solving

$$\frac{\partial V}{\partial \lambda_i} = \mathbf{x}_a^T \frac{\partial \mathbf{A}}{\partial \lambda_i} \mathbf{x}_a \quad (3.9)$$

for each optimization variable $\lambda_1 \dots \lambda_n$.

3.4 Numerical Examples

This section presents numerical examples to demonstrate the efficiency and accuracy of the algorithm presented in this chapter.

3.4.1 Example 1

An interconnect circuit, shown in fig. 3.3, consisting of 3 two-conductor transmission lines is considered for this example. The size of the MNA matrices of this original circuit is 1209x1209. Optimization is performed to reduce the signal delay of Vout1 and Vout2, and the crosstalk voltages Vcross1 and Vcross2, respectively [42] by imposing constraints through an error function given as,

$$\begin{bmatrix} err1 \\ \dots \\ err20 \\ err21 \\ \dots \\ err40 \\ err41 \\ \dots \\ err60 \\ err61 \\ \dots \\ err80 \end{bmatrix} = \begin{bmatrix} TGout1(\Phi, s_1) - 6 \\ \dots \\ TGout1(\Phi, s_{20}) - 6 \\ TGout2(\Phi, s_1) - 6 \\ \dots \\ TGout2(\Phi, s_{20}) - 6 \\ |Vcross1(\Phi, s_1)| - 0.07 \\ \dots \\ |Vcross1(\Phi, s_{20})| - 0.07 \\ |Vcross2(\Phi, s_1)| - 0.07 \\ \dots \\ |Vcross2(\Phi, s_{20})| - 0.07 \end{bmatrix}$$

where $s_k = j\omega_k = jk\omega_1$, $k=1,2,\dots, 20$ and $\omega_1 = 0.0837758 \times 10^9$ rad/sec. The optimization variables include capacitors C1, C2 and resistor R1.

The proposed algorithm using PMR techniques was used to reduce the network as a function of frequency and the optimization parameters of the network mentioned above. The size of the reduced system formed was 22×22 . A comparison of results of the network obtained from the original system as well as the reduced system before optimization are shown in fig. 3.4. The percentage error between the original system and the reduced system is plotted on fig. 3.5

Fig. 3.6 shows the results of optimization performed on the original circuit. The time taken for optimization of the design parameters using the original system was 498 sec. Optimization performed on the reduced system takes 2.39 sec with a speed up of 19.5 and the results are shown in fig. 3.7. Table 3.1 gives the CPU time comparison for optimization of the original and reduced system.

Fig. 3.3 Original Circuit

Fig. 3.4 Results from original and reduced systems before optimization

Fig. 3.5 Percentage error between original and reduced system before optimization

Fig. 3.6 Original system responses before and after optimization

Fig. 3.7 Reduced system responses before and after optimization

Table. 3.1 Comparison of CPU Time for the 3-two coupled line example

	Size	Setup time	Optim. Time	Speed-up
Org. System	1209×1209	-	498 s	-
Red. System	22×22	23.11s	2.39 s	19.5

3.4.2 Example 2

A distributed transmission line network consisting of 9 coupled transmission lines and RLC components [43]-[44] was chosen to demonstrate the efficiency of the proposed algorithm (see fig 3.8). The distributed network was discretized using lumped segmentation. The size of the MNA (Modified Nodal Analysis) equations representing the network was 1813×1813 . The proposed algorithm using parameterized model reduction techniques was used to reduce the network as a function of frequency and optimization variables of the network. For this example, 3 parameters $[L_1 \ L_2 \ C]$ were chosen as the optimization variables. Moments w.r.t frequency as well as each design parameter were used to form the reduced order networks for both response calculation and sensitivity analysis. The size of the equations representing the reduced systems was 32×32 . The insertion loss of the circuit of the original circuit before optimization is shown in fig. 3.9. The following design specifications were imposed on the insertion loss (IL):

$$IL(\omega) \geq 25dB, \text{ if } \omega \geq 2.5 \text{ rad/sec}$$

$$IL(\omega) \leq 1.5dB, \text{ if } \omega \leq 1 \text{ rad/sec.}$$

Fig. 3.10 shows a comparison of the insertion loss of the original system and the reduced system after optimization. The optimization results obtained from the proposed algorithm match the results obtained from the original system accurately. The percentage error between the optimized variables $[L_1 \ L_2 \ C]$ obtained from optimization of the original system and the reduced system is $\{0.45\%, 0.46\%, 0.0092\%\}$. A comparison of the CPU time taken to optimize the design parameters to obtain the insertion loss of the circuit is shown in Table 3.2. A speed up of 357.22 was achieved using the proposed algorithm.

Table. 3.2 CPU time comparison of the 9 coupled transmission line example

	Size	Setup. Time	Optim. Time	Speed up
Org. System	1813×1813	-	164540 s (approx. 45 hrs)	-
Red. System	32×32	81.467	379.14 s (approx. 6 min.)	357.22

Fig. 3.8 Circuit under consideration. Unless indicated, resistance and capacitance values are $100\text{M}\Omega$ and 1pF

Fig. 3.9 Insertion loss before optimization **Fig. 3.10** Insertion loss after optimization

Chapter 4

Closed-Form Expressions for Interconnect Networks using Parameterized Model-Reduction Techniques

4.1 Introduction

As mentioned in chapters 1 to 3, numerous algorithms can be found in the literature to efficiently simulate circuits by reducing the order of the linear system [14]-[30]. Traditional model order reduction (MOR) techniques perform reduction w.r.t (with respect to) a single parameter (frequency). The reduction process of the MOR techniques captures the dominant nature of the original system using moment-matching techniques. Once the reduction is performed, the response of the network is obtained in the form of dominant poles and their corresponding residues. These poles and residues can then be used to com-

pute the time-domain response of the network. In addition to reducing the CPU expense of regular simulations, it is also important to predict the response of the network due to other parameters in the network. Parameterized model reduction (PMR) techniques [31]-[33] were introduced to perform reduction in terms of frequency and other design parameters. Though PMR techniques are efficient, they still involve computation of the reduced equations to find the response of the system.

In this chapter, a technique to obtain closed-form expressions for circuit responses of the network w.r.t frequency and any design parameter of the network is presented. Using this algorithm, closed form expressions for the system are formed by tracking the behaviour of poles and residues w.r.t the design parameter of interest. This method eliminates the need to solve sets of equations at each time/frequency point to find the response of the system. The proposed algorithm couples the merits of parameterized reduction with that of vector-fit [34], a popular algorithm to form rational approximations from given data points.

However, since the circuit response can be sensitive w.r.t variation in poles and residues, undue computational effort is spent in trying to ensure extreme accuracy in tracking the poles and residues. Another algorithm is also proposed in this chapter that improves the efficiency of the above proposed method. This algorithm tracks the coefficients of the numerator and denominator polynomials of complex conjugate pole-residue pairs. This chapter is organized as follows: Section 4.2 presents the formulation of the network equations. Section 4.3 gives the proposed algorithms with the review of parameterized model reduction and pole-residue formulation in Section 4.3.1, the algorithm to form

closed form expressions for poles and residues is given in Section 4.3.2. Section 4.3.3 proposed an algorithm to further improve the efficiency of the method in Section 4.3.2. Section 4.4 gives the numerical results to prove the accuracy and efficiency of the given algorithm.

4.2 Formulation of Network Equations

A general linear network with its elements as a function of parameters λ can be written as

$$\mathbf{C}(\lambda)\frac{d\mathbf{x}}{dt} + \mathbf{G}(\lambda)\mathbf{x} = \mathbf{B} \quad (4.1a)$$

$$V = \mathbf{d}^T \mathbf{x} \quad (4.1b)$$

where,

- $\mathbf{x}(t, \lambda) \in \mathfrak{R}^N$ is the vector of node voltage waveforms and currents in the system;
- λ is the physical/electrical parameter of interest;
- $\mathbf{C}(\lambda) \in \mathfrak{R}^{N \times N}$, $\mathbf{G}(\lambda) \in \mathfrak{R}^{N \times N}$ are matrices describing the lumped memory and memoryless elements of the network dependent on λ ;
- $\mathbf{B} \in \mathfrak{R}^N$ is the vector with entries determined by the independent voltage and current sources;
- N is the total number of variables in the Modified Nodal Analysis (MNA) formulation;
- V is the output response variable of interest;

- $\mathbf{d}$ is a selector vector with nonzero entries corresponding to the output variable; and
- s represents complex frequency;

The network equations described in (4.1) can be expressed in frequency domain as

$$\mathbf{A}(\lambda)\mathbf{x} = \mathbf{B} \quad (4.2a)$$

$$\mathbf{V} = \mathbf{d}^T \mathbf{x} \quad (4.2b)$$

where

$$\mathbf{A} = \mathbf{G} + s\mathbf{C}$$

4.3 Closed Form Expression for Response Evaluation

In this section, an algorithm to form closed form expressions for the response of the system (4.2) as a function of any design parameter is presented. Section 4.3.1 reviews the Parameterized model reduction technique and computation of the pole- residue form. Section 4.3.2 gives the algorithm to compute rational approximations for the poles and residues in terms of the design parameters such that for any change in design parameter λ , response can be found using the explicit closed form expression. In Section 4.3.3, an algorithm to further improve the efficiency of the closed form method (described in Section 4.3.2) is proposed.

4.3.1 Parameterized Krylov Subspace

As mentioned in Section 2.4, Parameterized model reduction techniques can generate reduced models as a function of circuit parameters. The information pertaining to the dominant eigen-values of the system in (4.2) is extracted with respect to frequency and the parameter λ by forming a Krylov subspace in multiple dimensions. Moments of x w.r.t frequency and w.r.t λ are computed (see Section 2.4) to form the parameterized subspace $\mathbf{Q}$. Congruent transformation is performed using

$$\mathbf{x} = \mathbf{Q}\hat{\mathbf{x}} \quad (4.3)$$

where $\hat{\mathbf{x}} \in \mathbb{R}^q$ and q is the reduced vector of unknowns. It has to be noted that $q \ll N$. Using (4.3), a congruent transformation is performed on the original system (4.2) to give the reduced system

$$\hat{\mathbf{A}}\hat{\mathbf{x}} = \hat{\mathbf{B}} \quad (4.4)$$

$$\mathbf{V}_\phi = \hat{\mathbf{d}}^T \hat{\mathbf{x}} \quad (4.5)$$

where,

$$\hat{\mathbf{A}} = \mathbf{Q}^T \mathbf{A} \mathbf{Q}; \quad \hat{\mathbf{B}} = \mathbf{Q}^T \mathbf{B}; \quad \hat{\mathbf{d}} = \mathbf{Q}^T \mathbf{d}$$

The response of the system for a certain $\lambda = \lambda_i$ can be computed by evaluating its poles-residue form in the frequency domain as

$$\mathbf{V}_\phi(s, \lambda_i) = \sum_{k=1}^q \frac{r_k(\lambda_i)}{s - p_k(\lambda_i)} \quad (4.6)$$

where, $r_k(\lambda_i) = (\mu_k \gamma_k) / \Lambda_k$ and $p_k(\lambda_i) = (-1) / \Lambda_k$; μ_k is the k -th element in the row vector $\hat{\mathbf{d}}^T \mathbf{S}$; γ_k is the k -th element in the column vector $\mathbf{S}^{-1} \mathbf{G}^{-1}(\lambda_i) \hat{\mathbf{B}}$; Λ_k and $\mathbf{S}$ are obtained through an eigenvalue decomposition of $\mathbf{G}^{-1}(\lambda_i) \hat{\mathbf{C}}(\lambda_i)$ such that

$$\mathbf{G}^{-1}(\lambda_i) \hat{\mathbf{C}}(\lambda_i) = \mathbf{S} \mathbf{D} \mathbf{S}^{-1} \quad (4.7)$$

where, $\mathbf{D} = \text{diag}(\Lambda_1, \Lambda_2, \dots, \Lambda_q)$. The response in the time domain can be expressed in terms of $r_k(\lambda_i)$ and $p_k(\lambda_i)$ in the time-domain by performing an inverse Laplace transform on (4.6). From (4.6) it is clear that the computation of the poles and residues has to be repeated to evaluate the response of the system for another $\lambda = \lambda_j$ where $\lambda_j \neq \lambda_i$. In the next section, a technique to obtain a closed form expression for the response as a function of the design parameter λ is presented.

4.3.2 Closed Form Expressions for the Poles and Residues

As seen from the previous subsection, the poles and residues corresponding to the response V_ϕ are computed at a certain $\lambda = \lambda_i$ and for every change in λ , the procedure of finding poles and residues has to be repeated which can be time consuming. In order to address this issue, a rational approximation is used to capture the variation of each pole and residue of the response at V_ϕ . It should be noted that the variation of poles/residues can be captured by using any curvefitting algorithm. However in this thesis, vector-fit algorithm is used to form rational approximations of poles and residues. The rational approximation is constructed with the help of data points collected by sampling the values

of poles and residues for different values of λ as described in the previous subsection and applying the vector-fit algorithm [10]. As a result of the rational approximation each pole and residue of V_ϕ can be expressed as

$$r_k(\lambda) = \frac{P_k^x(\lambda)}{Q_k^x(\lambda)}; p_k(\lambda) = \frac{P_k^y(\lambda)}{Q_k^y(\lambda)} \quad (4.8)$$

where, $P_k^x(\lambda)$, $Q_k^x(\lambda)$, $P_k^y(\lambda)$ and $Q_k^y(\lambda)$ are polynomials in λ .

Using (4.8), a closed-form function for V_ϕ can be explicitly written in terms of frequency and λ as (assuming only real poles for now)

$$V_\phi(s, \lambda) = \sum_{k=1}^q \frac{P_k^x(\lambda)/Q_k^x(\lambda)}{s - P_k^p(\lambda)/Q_k^p(\lambda)} \quad (4.9)$$

Applying inverse Laplace transform on (4.9), a closed-form expression for the time-domain response $v_\phi(t, \lambda)$ can be obtained explicitly in terms of the parameter λ of the network as

$$v_\phi(t, \lambda) = \sum_{k=1}^q \frac{P_k^x(\lambda)}{Q_k^x(\lambda)} e^{-\frac{P_k^p(\lambda)}{Q_k^p(\lambda)} t} \quad (4.10)$$

In the case where the system has complex poles and residues, the real and imaginary part of the poles and residues have to be fit by different rational approximations separately. The algorithm constructs rational approximations to capture the variation of each pole and residue of the response at V_ϕ . The real and imaginary parts of poles and residues

have to be fit by different rational approximations separately. However, the circuit response can be sensitive w.r.t variation in poles and residues. Undue computational effort is spent in trying to ensure extreme accuracy in tracking the poles and residues. This is overcome in the next section, where an algorithm is presented to further improve the efficiency of the algorithm to form closed form expressions of the response.

4.3.3 Closed form expression for polynomial coefficients.

Poles and residues mostly occur as complex conjugate pairs. Taking advantage of this fact, complex conjugate pairs of poles and residues of the system are formulated as rational functions. The response $V_\phi(s, \lambda_i)$ can be written as

$$V_\phi(s, \lambda_i) = \sum_{k=1}^m \frac{r_k(\lambda_i)}{s - p_k(\lambda_i)} + \sum_{k=1}^n \frac{r_k^o(\lambda_i) + r_k^1(\lambda_i)s}{1 + p_k^1(\lambda_i)s + p_k^2(\lambda_i)s^2} \quad (4.11)$$

where r_k and p_k are pole-residue pairs corresponding to poles with no conjugate pair; r_k^o , r_k^1 , p_k^1 and p_k^2 are the coefficients of rational functions of the pole-residue pairs of complex-conjugate poles. The coefficients of the polynomial are fitted using the vector-fit algorithm [34] to track their variation w.r.t λ . As a result of the rational approximation each coefficient can be expressed as

$$r_k^a(\lambda) = \frac{P_k^a(\lambda)}{Q_k^a(\lambda)}; p_k^b(\lambda) = \frac{P_k^b(\lambda)}{Q_k^b(\lambda)} \quad (4.12)$$

where, $P_k^a(\lambda)$, $Q_k^a(\lambda)$, $P_k^b(\lambda)$ and $Q_k^b(\lambda)$ are polynomials in λ . Once the rational approximation is evaluated, the coefficients of the polynomial can be explicitly found for

every change in the design parameter. Similarly, we can also find rational approximations for the poles and residues that do not have complex conjugate pairs in the system. The sensitivity of the polynomial coefficients to the circuit response is much less than that of the poles and residues. In the next section, numerical results are presented that demonstrate the efficiency and accuracy of the proposed techniques.

4.4 Numerical Examples

This section presents numerical examples to demonstrate the efficiency and accuracy of the algorithm presented in this chapter. Example 1 illustrates the accuracy and efficiency of the algorithm in finding the response using rational approximations of the poles and residues. Example 2 demonstrates the increase in the efficiency of finding the response using the rational approximation of the coefficients of the polynomials of the complex conjugate pole-residue pair.

4.4.1 Example 1

A network consisting 18 transmission lines (9 coupled transmission lines and 9 single transmission lines; see Fig. 4.1) was considered as an example to prove the concept proposed in the algorithm for finding the response using rational approximations of the poles and residues. When discretized, the transmission line network was of size 3551×3551 . The impulse response of this network is desired as the width of one of the transmission lines is changed from 2.6mm to 3.8mm. Using traditional methods, analysis of this network from 0-4GHz at 15 line-widths took 2,988.80s (approx. 50 mins). 150

moments w.r.t frequency and 5 moments w.r.t the width of the line along with 4 cross-moments were used to construct the parameterized reduced-order network. Next, poles and residues were computed at 15 line-widths and variation of each pole and residue w.r.t the width was captured as a rational approximation. A comparison of the variations captured from these rational approximations w.r.t the variation in the original system for a pole-residue pair is shown in Fig.4.2 and Fig.4.3. In total 159 rational approximations for the poles and another 159 rational approximations for the residues were performed to capture the behaviour of all poles and residues w.r.t the line-width. Using these rational approximations a closed-form expression of the response at the output voltage of interest was formed in terms of frequency and width of the transmission line. The computational cost involved in generating this closed-form expression was 320.31s (about 5 mins). Once the closed-form expression is calculated, the response can be calculated in a fraction of the time required to solve the original system. The frequency response of the network from 0-4GHz at 15 line-widths took 2.69s. The resulting speed-up in simulation is about 9.25. The impulse response in frequency domain obtained from the closed-form expression is compared with that obtained from the original system in Fig. 4.4. The results match accurately. The time domain impulse response obtained from the closed-form expression is compared with the time domain response calculated from the original system in Fig. 4.5. As it is seen the time-domain results also match accurately. The CPU comparison is demonstrated in Table 4.1.

Fig. 4.1 Network under consideration. The matrices L and C shown (in the next page) are the per-unit-length parameters of the 9 coupled transmission lines. The per-unit-length resistance matrix for the 9 coupled lines is a diagonal matrix with a value of $5\Omega/m$ on the diagonal. The single transmission lines have per-unit-length parameters of $R = 3.74\Omega/m$, $L = 284nH/m$, $C = 84.6pF/m$.

$$L = \begin{pmatrix} 5.04 & 0.182 & 0.315 & 0.057 & 0.315 & 0.182 & 0.315 & 0.057 & 0.315 \\ 0.182 & 5.04 & 0.315 & 0.182 & 0 & 0 & 0 & 0 & 0.315 \\ 0.315 & 0.315 & 5.04 & 0.315 & 0.182 & 0 & 0.057 & 0 & 0.182 \\ 0.057 & 0.182 & 0.315 & 5.04 & 0 & 0 & 0 & 0 & 0 \\ 0.315 & 0 & 0.182 & 0 & 5.04 & 0.315 & 0.182 & 0 & 0 \\ 0.182 & 0 & 0 & 0 & 0.315 & 5.04 & 0.315 & 0.182 & 0 \\ 0.315 & 0 & 0.057 & 0 & 0.182 & 0.315 & 5.04 & 0.315 & 0.182 \\ 0.057 & 0 & 0 & 0 & 0 & 0.182 & 0.315 & 5.04 & 0 \\ 0.315 & 0.315 & 0.182 & 0 & 0 & 0 & 0.182 & 0 & 5.04 \end{pmatrix} nH/cm$$

$$C = \begin{pmatrix} 187.6 & -0.385 & -2.23 & -0.0046 & -2.23 & -0.385 & -2.23 & -0.0046 & -2.23 \\ -0.385 & 187.6 & -2.23 & -0.385 & 0 & 0 & 0 & 0 & -2.23 \\ -2.23 & -2.23 & 187.6 & -2.23 & -0.385 & 0 & -0.0046 & 0 & -0.385 \\ -0.0046 & -0.385 & -2.23 & 187.6 & 0 & 0 & 0 & 0 & 0 \\ -2.23 & 0 & -0.385 & 0 & 187.6 & -2.23 & -0.385 & 0 & 0 \\ -0.385 & 0 & 0 & 0 & -2.23 & 187.6 & -2.23 & -0.385 & 0 \\ -2.23 & 0 & -0.0046 & 0 & -0.385 & -2.23 & 187.6 & -2.23 & -0.385 \\ -0.0046 & 0 & 0 & 0 & 0 & -0.385 & -2.23 & 187.6 & 0 \\ -2.23 & -2.23 & -0.385 & 0 & 0 & 0 & -0.385 & 0 & 187.6 \end{pmatrix} pF/m$$

Fig. 4.2 Comparison of variation in pole of the system response w.r.t width of a transmission line, captured from the rational approximation with that obtained from the original network.

Fig. 4.3 Comparison of variation in a residue of the system response w.r.t width of a transmission line, captured from the rational approximation with that obtained from the original network

Fig. 4.4 Comparison of the frequency domain impulse response obtained from the closed-form expression and that obtained from the original network.

Fig. 4.5 Comparison of the time-domain impulse response obtained from the closed-form expression and that obtained from the original network for two different values of line-widths.

	Setup Time	Sim.Time	Speed-up
Original System	-	2,988.80s	-
Closed-form exp.(poles/residues)	320.31s	2.69s	9.25

Table. 4.1 CPU comparison**4.4.2 Example 2**

A network containing 7 transmission lines shown in fig.4.6. was considered as an example to prove the concept proposed in this paper. The discretized transmission line network was of the size 713×713 . Using traditional methods, analysis of this network from $0-1.5GHz$ for 15 capacitance values took 213.75s. The original system was then reduced using parameterized model reduction techniques as a function of frequency and a capacitor in the network. 16 moments w.r.t frequency and 2 moments w.r.t the capacitor were used to construct the parameterized reduced order network. The poles and residues of the reduced system were computed. The complex conjugate pairs of poles and residues were combined to form rational polynomials. Variation of each coefficient of the polynomial w.r.t increasing capacitance values were captured as a rational approximation using vector-fit. The remaining poles and residues with no complex conjugate pairs were also fitted using vectorfit to form rational approximations that capture their variation w.r.t the parameter of interest. Using these rational approximations, a closed form expression of the response at the output voltage of interest was formed in terms of frequency and the design parameter. The computational cost involved in generating this closed- form expression

was 3.135s. Once the closed-form expression is calculated, the response can be calculated in a fraction of the time required to solve the original system. The frequency response of the network from $0-1.5\text{GHz}$ for 15 parameter values took 0.9s. The resulting speed-up in simulation is about 52.97. The CPU comparison is demonstrated in Table 4.2 and it can be noticed that the response evaluation using the rational approximation of the coefficients of the polynomial of the complex conjugate pole-residue pair gave a much better speed up than the response evaluation using the rational approximation of the poles and residues (31.26). The impulse response in frequency domain obtained from the closed-form expression compared with that obtained from the original system for 3 different values of capacitance (1pF, 2pF, and 4pF) is given in fig.4.7. The percentage error is given in fig 4.8.

	Setup Time	Sim.Time	Speed-up
Original System	-	213.75s	-
Closed-form exp.(poles/residues)	5.187s	1.65s	31.26
Closed-form exp.(polynomial coeff.)	3.135s	0.90s	52.97

Table. 4.2 CPU comparison of the original system, and the proposed algorithms

Fig. 4.6 Original System

Fig. 4.7 Response of the original system and response using the proposed method

Fig. 4.8 Percentage error between the response for $C=4\text{pF}$ from the original system and using the proposed method

Chapter 5

Summary and Future Work

5.1 Summary

This thesis presents solutions to problems related to simulation and optimization of high-speed interconnect networks. Several methods were presented in this work that enhance the accuracy and efficiency of simulation and optimization of large interconnect networks. First, an efficient algorithm was presented to perform optimization of high speed interconnect networks with respect to the design parameters using parameterized model reduction techniques. The response and gradients required for optimization were obtained using parameterized model reduction techniques. Significant speed up in optimization was observed using the proposed method when compared to the traditional optimization process. Next, a technique to obtain closed form expressions for circuit responses of the network by tracking the behaviour of the poles and residues w.r.t frequency and any design parameter of the network was presented. The response of the system is obtained by

evaluating the explicit closed form expressions formed by the proposed method. The algorithm couples the merits of parameterized reduction with that of vector-fit, a popular algorithm to form rational approximations from given data points. Finally, the efficiency of the closed form method proposed was further improved using an algorithm that tracks the coefficients of the numerator and denominator polynomials of complex conjugate pole-residue pairs.

5.2 Future Work

Though most of the work in this thesis was done on interconnect networks, the techniques presented here can be applied to any general circuit simulation and optimization. In this section, some suggestions for further research are listed for the proposed methods.

1. During the optimization process, response of the system can be computed in terms of the design parameter in the circuit using the closed form method which will result in significant speedup in CPU time.
2. The closed form method can also be applied to the adjoint system of equations to calculate the gradients (sensitivity) of the system with respect to the design parameter in the circuit.
3. The optimization algorithm and closed form methods can be applied to various areas, such as thermal analysis of electronic components and MEMS (micro electromechanical systems).

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