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Reversible Flight Control Identification

by

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fulfillment of the requirements for the degree of

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Abstract

Reversible Flight Control Identification is focused on the development of a high-fidelity simulator model for flight control systems. The model is developed for the elevator control system of the Cessna CJ1 aircraft, and is based on kinematic relations and state modelling. The model architecture is proposed based on a physical understanding of the control system mechanics and aerodynamics theory of hinge moments acting on the elevator surface. Flight data from a 75 hour flight test programme are used for system identification using multiple linear regression for parameter estimation. Verification is performed to demonstrate model compliance to the American Federal Aviation Administration Level D certification requirements.

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Nomenclature

Acronyms

AC	Advisory Circular
AIE	Aircraft Inertia Estimation
AoA	aircraft angle of attack
CAR	Canadian Aviation Regulation
DC	direct current
FAA	Federal Aviation Administration
FAR	Federal Aviation Regulation
FIR	Finite Impulse Response
FRL	Flight Research Laboratory
GCS	ground control sweep
IAR	Institute for Aerospace Research
IMU	Inertial Measurement Unit
IPSI	In-Flight Propulsion System Identification
IRS	inertial reference system
NASA	National Aeronautics and Space Administration
NRC	National Research Council Canada

PCU	power control unit
PID	proportional-integral-derivative
POM	Proof of Match
QTG	Qualification Test Guide
RAMI	Real-Time Aerodynamic Model Identification
ReFCI	Reversible Flight Control Identification
RMS	root mean square
SCADS	Simultaneous Calibration of Air Data Systems
TAS	true airspeed
TAT	total air temperature

Symbols

A	equation coefficient, acceleration
Alt	altitude
B	equation coefficient
$\bar{c}$	reference chord
C	equation coefficient
C_e	elevator aerodynamic hinge moment coefficient
CH	non-dimensional aerodynamic hinge moment
C_{SP}	slipstream coefficient
D	equation coefficient

F	force
g	acceleration of gravity at sea level
h	height above ground level
J	mass moment of inertia
Kts	nautical miles per hour
Lt	aircraft tail length
M	moment
Ma	Mach number
MH	aerodynamic hinge moment
mG	1/1000 of acceleration of gravity at sea level
n	polynomial order
N	number of data points
P	aircraft roll rate
P_D	dynamic pressure
P_S	static pressure
Q	aircraft pitch rate
r	degree of freedom
R	aircraft yaw rate
$RadAlt$	radio altitude
s	standard deviation of finite data, wing semi-span
S	reference surface area

std	standard deviation operator
t	time
$t_{r,95}$	Student's t-distribution factor for 95% confidence level
T_S	static air temperature
u	measurement uncertainty
v	dependent variable
V	true airspeed
α	pitch angle
β	yaw angle
δ	position
Δ	transformed variable
η	elevator position
η_{play}	elevator backlash
η_{TT}	elevator trim tab position
θ	aircraft pitch angle
ξ	aileron position
ξ_{TT}	aileron trim tab position
σ	standard deviation
σ_e	ground effect influence parameter
ϕ	aircraft roll angle

ψ aircraft yaw angle

Subscripts

breakout rate at which stiction begins to transition to coulomb friction

c column

cbl column-elevator linkage cable

clmb coulomb friction

cmd commanded position

colspr column stop spring

e elevator

end end time

err position or force model error

fric moment due to friction

i index of particular data point

inertia moment due to inertia

K flaps

max maximum

mcg minimum control speed ground

mdl modelled position or force

meas measured position or force

mu minimum unstick speed

s standard deviation

$start$	start time
$stic$	stiction friction
TD	trim dial
$trans$	rate above which all friction is due to coulomb effect
TT	trim tab
w	wheel
X	coordinate axis
ξ_{TT}	contribution of aileron trim position to aileron position
Y	coordinate axis
Z	coordinate axis

Aerodynamic Hinge Moment Subscripts

$C_{e,pri,sec}$	subscripts in <i>pri</i> position indicate the primary contribution to the hinge moment; subscripts in <i>sec</i> position indicate the secondary contribution to the hinge moment
0	naught term
α_e	horizontal stabiliser pitch angle of attack
$\dot{\alpha}_e$	horizontal stabiliser pitch angle of attack rate
β	sideslip angle of attack
η	elevator deflection
$\dot{\eta}$	elevator deflection rate
η_K	flaps position

η_{TT} elevator trim tab position

GE ground effect

Ma Mach number

Superscripts

$*$ modelled surface position including primary and trim contributions only

$**$ modelled surface position $*$ plus thermal and dynamic pressure effects

Chapter 1

Introduction

This thesis contains a description of the development of reversible flight control identification (ReFCI). The purpose of ReFCI is to create flight control system models for fixed-wing aircraft with reversible flight controls. Reversible flight controls are those where the aerodynamic forces acting on the flight control surfaces are translated back to the pilot through the control system linkages. This is the case for manually actuated flight control systems typically found on small aircraft. In contrast, irreversible flight control systems are those in which the aerodynamic hinge moments do not pass back through the controls to the pilot. The most common irreversible flight control systems use hydraulic actuators. The mechanism of hydraulic actuators prevent force feedback to the pilot.

The control system models that are generated with ReFCI form a component of what is known as the aircraft global model, which also includes models for rigid-body aerodynamics and propulsion systems. The aircraft global model provides the logic required to control high-fidelity flight simulators, such as those used for pilot training.

1.1 Flight Simulation for Flight Crew Training

Flight crew training is an expensive and time-consuming process. To become proficient in the skills required to safely control an aircraft requires extensive training and experience. A minimum of 200 hours of flight experience is required for a commercial pilot's license to be issued. For instrument flight rule certification, a pilot must have a minimum of 20 hours instrument flight time, of which up to 10 hours may be spent in flight simulators. (CAR Std 421)

Flight simulators are used to augment, and to an increasing amount, replace flying time in a real aircraft. There are two main reasons for using flight simulators during flight crew training. One is that flight simulators allow for training and familiarisation of the flight crew under simulated emergency conditions, such as how to handle various system failures. Through simulation, this training may be performed on the ground without risk to the crew, the public or property.

The other reason for the use of flight simulators is economics. It is significantly less expensive to accrue simulated flight hours than flight hours in an aircraft. For example, an hour of training in a business-jet type aircraft costs on the order of \$6000 per hour, while an hour in a flight simulator costs about \$500 per hour (Hui, 2005). When the number of hours required for flight crew training is considered, it is obvious that performing all training in aircraft would be economically prohibitive.

The more realistic the simulated flight experience is, the better that simulation will be for training flight crew for a specific aircraft. Increasing the fidelity of flight

simulation allows more extensive use of flight simulators in place of flight time during the flight training program, and this results in substantial cost savings. Companies in the business of flight training and flight simulation are keenly interested in offloading as many flight training hours as possible to flight simulators, and much work has gone into improving the fidelity of flight simulator technology.

To facilitate and regulate the development and use of high fidelity flight simulation technology, the Federal Aviation Administration (FAA) of the United States published the Advanced Simulation Plan in 1980 (Federal Aviation Administration, 1980). The FAA has also published Advisory Circular (AC) 120-40 Airplane Simulator and Visual System Evaluation that describes an acceptable means for receiving certification for four levels of simulation fidelity (Federal Aviation Administration, 1986). These levels are A through D, with D being the highest fidelity in the widest range of flight simulator functions, including motion system, visual and audio systems, and flight path simulation. The current standards are laid out in AC 120-40C, released in 1995 (Federal Aviation Administration, 1995). A table of the requirements governing certification of the flight control system models is given in Section 2.

1.2 Components of Aircraft Global Model

The logic that controls flight simulators is provided in the aircraft global model. The global model is a mathematical database containing all the relationships required to describe aircraft behaviour, such as the stability derivatives of aircraft rigid-body aerodynamics. The aircraft global model is composed of three parts; the aircraft

rigid-body motion model, the propulsion system model, and the flight control system model.

Within the global model, each component (or block) receives feedback information from the other components. For example, propulsive thrust is required to compute aircraft motion within the rigid-body aerodynamics block. At the same time, characteristics of the aircraft motion, such as angle of attack, affect the thrust calculated in the propulsion system block. A proper understanding of these feedback mechanisms is needed to create accurate models for flight simulation.

The Flight Research Laboratory (FRL) of the Institute for Aerospace Research (IAR) of the National Research Council of Canada (NRC) has developed processes for Real-Time Aerodynamic Model Identification (RAMI) (Hui et al, 2003), In-Flight Propulsion System Identification (IPSI) (Hui et al, 2005), and Aircraft Inertia Estimation (AIE) (Hui et al, 2003). To complete the capability of the FRL to produce complete high-fidelity aircraft global models requires the development of reversible flight control identification (ReFCI), which is the subject of this thesis. Figure 1.1 is a block diagram showing the aircraft global model development components.

This figure also shows the Proof of Match (POM) and industry collaboration activities involved in aircraft global model development. POM is used to validate the global model against measured aircraft behaviour to ensure that the design tolerances for flight simulation are met.

Figure 1.1: Level D flight simulator model development components

1.3 Evolution of Flight Control Modelling

Flight control system modelling for the purposes of flight simulation has been an engineering subject of study since the late 1950s (Hui, 2005). The initial models were simple, with control surface position given as a linear function of control input position (Hui, 2005). This method is straightforward, but does not take into account any dynamic characteristics of the control system, or any aerodynamic force feedback mechanism.

As flight control system modelling became more advanced, the effects of aerodynamic forces were recognised as the most difficult problem in control system parameter estimation. Three separate but complimentary techniques have been used for aerodynamic parameter estimation, comprising of analytical methods, wind-tunnel methods and flight test methods (Hamel et al, 1996). In modern work, the preferred parameter estimation techniques centre around flight test data because although it is the most expensive method, the data is not limited by wind tunnel scaling and environmental compromise nor analytical theoretical simplifications and assumptions (Hamel et al, 1996).

Much progress in flight control modelling was made during the 1980s. Modern control models were developed as simplified single degree of freedom systems. Kinematic models were created to relate control input forces and control surface hinge moments. These models included the effects of aerodynamics, cable stretch, friction, and inertia of the control system components (Hollander, 1982). In the first part of

the decade, parameter estimation for flight control models was performed using flight test data of small control inputs during straight and level flight (Mulder et al, 1981). Kalman filtering and Maximum Likelihood Estimation were used in the identification of vehicle aerodynamic characteristics, while multiple linear regression was the preferred method for flight control system parameter estimation (Hollander, 1982). The flight control system models developed during this time were limited in that it was exceedingly difficult to create a single model that would accurately predict flight control system behaviour throughout all the portions of the flight envelope (Hollander, 1982).

The latter 1980s saw a shift to short, large-amplitude dynamic manoeuvres (Mulder, 1989). These manoeuvres created a broader range of excitement to the aircraft and flight control systems, which allow for simultaneous identification of multiple parameters using the same flight data (Mulder, 1989). State models were made to represent the flight control systems, and Maximum Likelihood Estimation was applied for parameter identification (Mulder, 1989). The main drawback of this approach was the very high number of parameters required to model the flight control systems, which results in the requirement for a large number of flight manoeuvres to be performed (Mulder, 1989). The simultaneous identification of so many parameters was computationally intensive for the time, so the control system models were decomposed into problems of lower dimension for parameter estimation (Mulder, 1989).

In 1991, the Federal Aviation Administration (FAA) released Advisory Circular AC 120-40B, which details the requirements for Level D high-fidelity flight simulators (Federal Aviation Administration, 1991). This was then followed by AC 120-40C in 1995 (Federal Aviation Administration, 1995). This document set the standard for meeting the FAA certification requirements for advanced flight simulators. Simulators meeting this standard allow more effective ground training of air crew, benefiting in a reduction in the number of flight hours in a training program.

The stringent standards set for Level D flight simulators required an improvement in control system parameter estimation. As wind tunnel test data proved to be inadequate, new flight test programs were specifically designed for the purpose of aircraft model development. These included short, large-amplitude manoeuvres, ground effect, stall hysteresis, and other manoeuvres designed to cover the entire flight envelope of the aircraft.

In the 1990s, work on the Dornier 328 aircraft included a flight test program consisting of 1270 manoeuvres, and implemented single degree of freedom models. An Output Error Method was used for parameter identification, with some difficulty. Although modelling of the control system friction required “hand optimisation”, the mathematical databases created received Level D certification in 1998 (Jategaonkar et al, 1998 and Weiss et al, 1998).

Modern work in parameter identification includes real-time or near-real-time on-board identification systems utilising neural networks or Kalman filtering. In 2003,

NASA fit a F-15 with neural network equipment capable of determining the aerodynamic rigid body parameters in flight, and linked this database generation to adaptable flight control systems for flight path optimisation (Hageman et al, 2003).

Work for the future includes developing more automated schemes for aircraft, engine, and control system modelling for high fidelity flight simulation. Real-time, onboard systems promise enhancement in aircraft and mission health monitoring. Autopilot systems programmed with knowledge of the aircraft systems' dynamic behaviours can more accurately calculate the required system inputs to take the aircraft along a desired flight path. This, coupled with programmable control logic, will allow aircraft to adapt to system failures or flight surface contamination, increasing safety and survivability.

1.4 System Identification Process

System identification begins with the proposal of models that are designed to replicate the behaviour of the observed system. For aircraft dynamics, the global model consists of the rigid-body aerodynamics model, propulsion model, and flight controls model. The flight controls model is divided into components representing the primary and secondary flight control systems, including elevator, ailerons, rudder, flaps and spoilers.

Once a model has been proposed for a given component, the required parameters are estimated from flight data. Modelled parameters, such as aerodynamic hinge moment coefficients, moments of inertia, and coefficients of friction are estimated

using a variety of flight data covering the flight envelope of the aircraft. The methods used to determine the mentioned parameters are covered in the following sections. More information about the flight manoeuvres and data calibrations are found in Chapter 3.

When all the parameters required to complete a model have been identified, the model's performance is tested against measured aircraft behaviour for standard flight manoeuvres. The set of these manoeuvres are called the Qualification Test Guide (QTG), and the model must match the measured aircraft behaviour within given tolerances for all QTG data to receive Level D certification. If the proposed model does not meet the requirements, then the model parameters and/or the model architecture must be adjusted until all validation tests are satisfied. Once the model has passed the validation tests, it is ready for integration in a flight simulator. The above process is summarised in the flowchart given in Figure 1.2.

1.4.1 Parameter Estimation Techniques

There are a variety of techniques that can be used to find estimates of system parameters, including, in order of increasing complexity, trial and error, regression analysis, output error method, Kalman filtering, and artificial neural network algorithms. Trial and error, combined with root-finding algorithms such as bisection or Newton's method, can be an invaluable tool for fine-tuning of model parameter values to achieve the desired model behaviour. Output error methods and Kalman filtering are advanced statistical methods. Artificial neural network techniques involve learning

Figure 1.2: Flowchart of reversible flight control identification and simulation process

algorithms that can be programmed to converge to optimised estimates of system parameters. Both Kalman filtering and artificial neural networks can be designed to perform real-time system identification.

For the current study of ReFCI, the high quality of the available flight data allows the use of multiple regression analysis, as this technique concentrates on minimising the final equation error. In cases where measurement noise is greater, it may become necessary to use maximum likelihood estimation or Kalman filtering, as these techniques are more proficient at handling noisy data. A combination of regression analysis and trial and error is used in this thesis.

1.4.2 Multiple Regression Analysis

Regression analysis involves using the least-squares technique to find the relation between one dependent variable with one or multiple independent variables. This technique is suitable for producing graphs of the proposed relationships, and allows for an intuitive study of the causes of a given phenomenon. Consider dependent variable, v , as a function of measured independent variables x , y , and z , as represented in Equation (1.1). There are N data points of the measured variables, and N is greater than the number of independent variables.

$$v = A + Bx + Cy + Dz \quad (1.1)$$

For a given data point i , the square of the error of the above equation is represented by the following.

$$\begin{aligned}
 [A + Bx_i + Cy_i + Dz_i - v_i]^2 &= A^2 + 2ABx_i + 2ACy_i + 2ADz_i - 2Av_i \\
 &\quad + B^2x_i^2 + 2BCx_iy_i + 2BDx_iz_i - 2Bx_iv_i \\
 &\quad + C^2y_i^2 + 2CDy_iz_i - 2Cy_iv_i \\
 &\quad + D^2z_i^2 - 2Dz_iv_i \\
 &\quad + v_i^2
 \end{aligned} \tag{1.2}$$

Because the error squared term of each data point has the same form, we may sum them to produce the sum-squares. To find the minimum of the sum-squares, we take the partial derivative with respect to each of the coefficients, A , B , C , and D , and set each of these expressions equal to zero. Letting x^* represent the sum of all x_i , and equivalent expressions for the other variables, this set of equations is given as:

$$\begin{aligned}
 2AN + 2Bx^* + 2Cy^* + 2Dz^* &= 2v^* \\
 2Ax^* + 2B[x^2]^* + 2C[xy]^* + 2D[xz]^* &= 2[xv]^* \\
 2Ay^* + 2B[xy]^* + 2C[y^2]^* + 2D[yz]^* &= 2[yv]^* \\
 2Az^* + 2B[xz]^* + 2C[yz]^* + 2D[z^2]^* &= 2[zv]^*
 \end{aligned} \tag{1.3}$$

Dividing by two, and writing in matrix form, we have Equation (1.4).

$$\begin{bmatrix} N & x^* & y^* & z^* \\ x^* & [x^2]^* & [xy]^* & [xz]^* \\ y^* & [xy]^* & [y^2]^* & [yz]^* \\ z^* & [xz]^* & [yz]^* & [z^2]^* \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} v^* \\ [xv]^* \\ [yv]^* \\ [zv]^* \end{bmatrix} \tag{1.4}$$

Solving this matrix equation for the coefficient vector is performed by multiplying the right-hand-side vector by the inverse of the 4 x 4 matrix on the left. Thus the least-squares solution for the coefficients A , B , C and D are found. This process may be extended to any number of independent variables. However, there are limitations that must be considered when using multiple regression analysis.

The number of independent variables, called regressors, that may be included in the investigation of the behaviour of the dependent variable must always be less than

the number of data points available for study. If the number of regressors equals the number of points minus one, then the regression will result in a perfect correlation. An example of this case is a line fit to two points. The correlation may be exact, but there is little confidence that a repeatable trend has been identified. Therefore, the number of data points should always greatly exceed the number of regressors, preferably by a factor of 10 to 20.

Multicollinearity is a condition that occurs in multiple regression when one regressor is a linear function of another regressor or set of regressors. This results in matrix ill-conditioning, and the regression analysis will give unreliable results. An example of multicollinearity is trying to estimate how much fuel will be burned during a trip based on distance in kilometres and distance in miles. Because kilometers and miles are linear functions of each other, the distance is twice represented in the regression, and the resulting matrices will be ill-conditioned.

The limitation of regression analysis that is most difficult to correct is the simple fact that correlation does not mean causal connection. Regression may be used to provide evidence supporting expected trends and behaviours, but it can also produce misleading results. It is important that a trend should be reasonably expected before it is revealed by investigation of the data.

1.4.3 Sign Convention

The thesis work follows the standard sign convention for airplanes. The aircraft coordinate axes x , y and z are defined as positive towards the forward, starboard

and downward directions, respectively. The angular positions about the x , y and z directions follow the right-hand rule, and are called θ , ϕ and ψ , respectively. These coordinate axes and angular references are displayed in Figure 1.3.

Other important sign conventions pertain to the forces and moments acting on the flight control systems. The general scheme for the flight control system sign convention is that positive control deflections cause negative moments with respect to the aircraft coordinate system. Therefore, positive elevator position corresponds to deflection downwards, and positive aileron position is deflection downwards on the starboard aileron and deflection upwards on the port aileron. Correspondingly, positive control column deflection is forwards, and positive control wheel deflection is counter-clockwise from the pilots point of view. With respect to the flight control system, all forces and moments acting on the system are positive if they are in line with the positive displacement direction of the subject body. The sign convention of the elevator and aileron flight control systems is presented in Figure 1.4, parts (a) and (b), respectively.

Figure 1.3: Aircraft coordinate axes

Figure 1.4: Flight control system sign convention for the elevator (a) and aileron (b) systems. Arrows point in the positive direction.

Chapter 2

Requirements for Level D Flight Control System Modelling

The goal of the study of reversible flight control identification (ReFCI) is to develop a procedure that may be used to create mathematical relationships that accurately describe the physical behaviour of flight control systems on an aircraft. These mathematical relationships are to be programmed into a high-fidelity full-motion flight simulator. Once integrated, it is required that the simulator receive Level D certification as defined by the American Federal Aviation Administration (FAA).

In 1995, the FAA released Advisory Circular AC 120-40C (Federal Aviation Administration, 1995), which outlines acceptable criteria that a simulator must meet to receive Level D certification. This document covers requirements for all aspects of the pilots' experience in the simulator. The requirements specific to flight control systems are provided in condensed form in Table 2.1. The numbering scheme used in this table matches that of AC 120-40C, so there are missing numbers corresponding to requirements not relevant to flight control systems. It is important to note that the

flight control model must not only meet the requirements in this table, but must also be accurate enough to avoid causing the integrated aircraft model to stray beyond the tolerances specified for aircraft behaviour.

It is convenient to have a fidelity target not just for the integrated aircraft model, but also for stand-alone testing of the sub-models so that verification of model behaviour can be performed. As a general rule, if the stand-alone flight control model can match tolerance levels half that of the requirements for the global model, then the stand-alone model is acceptable (Hui, 2006). The more strict requirements on the stand-alone model allow for error propagation to and from the other global model components.

For validation, the flight control system models receive as inputs the measured column, wheel, and pedal positions, and simulated flight path and aircraft configuration characteristics. The outputs of the flight control system models are the positions of the primary aerodynamic control surfaces, namely the elevator, ailerons, and rudder, and the corresponding control force feedback to the column, wheel, and pedals.

Table 2.1: Excerpt of Level D Qualification Requirements from AC 120-40C

TESTS	TOLERANCE	CONDITIONS	COMMENTS
1. Performance			
b. Takeoff			
(2) Minimum Control Speed Ground (V_{mcg}) Aerodynamic Controls Only per Applicable Airworthiness Standard	$\pm 25\%$ of Maximum Airplane Lateral Deviation or ± 5 Feet	Ground/ Takeoff	Airplanes with reversible flight controls must also plot Rudder Pedal Force ($\pm 10\%$ or ± 5 lb).
(3) Minimum Unstick Speed (V_{mu}) or equivalent as provided by airplane manufacturer	± 3 Knots Airspeed ± 1.5 deg Pitch	Ground/ Takeoff	
(4) Normal Takeoff	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack ± 20 Feet Altitude	Ground/ Takeoff and First Segment Climb	Airplanes with reversible flight control systems must also plot Stick/Column Force ($\pm 10\%$ or ± 5 lb).
(5) Critical Engine Failure on Takeoff	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack ± 20 Feet Altitude ± 2 deg Bank and Sideslip Angle	Ground/ Takeoff and First Segment Climb	Airplanes with reversible flight control systems must also plot Stick/Column Force ($\pm 10\%$ or ± 5 lb), Wheel Force ($\pm 10\%$ or ± 3 lb), Rudder Pedal Force ($\pm 10\%$ or ± 5 lb).

Continued on next page

TESTS	TOLERANCE	CONDITIONS	COMMENTS
(6) Crosswind Takeoff	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack ± 20 Feet Altitude ± 2 deg Bank and Sideslip Angle	Ground/ Takeoff and First Segment Climb	Airplanes with reversible flight control systems must also plot Stick/Column Force ($\pm 10\%$ or ± 5 lb), Wheel Force ($\pm 10\%$ or ± 3 lb) and Rudder Pedal Force ($\pm 10\%$ or ± 5 lb).
(7) Rejected Takeoff	$\pm 5\%$ Time or ± 1.5 sec $\pm 7.5\%$ Distance or ± 250 ft	Ground/ Takeoff	
(8) Dynamic Engine Failure After Takeoff	$\pm 20\%$ Body Rates	First Segment Climb	
2. Handling Qualities			
a. Static Control Checks			
(1) Column Position vs. Force and Surface Position Calibration	± 2 lb Breakout ± 5 lb or $\pm 10\%$ Force ± 2 deg Elevator	Ground	Uninterrupted control sweep, stop to stop.
(2) Wheel Position vs. Force and Surface Position Calibration	± 2 lb Breakout ± 3 lb or $\pm 10\%$ Force ± 1 deg Aileron ± 3 deg Spoiler Angle	Ground	Uninterrupted control sweep, stop to stop.

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TESTS	TOLERANCE	CONDITIONS	COMMENTS
(3) Rudder Pedal Position vs. Force and Surface Position Calibration	± 5 lb Breakout ± 5 lb or $\pm 10\%$ Force ± 2 deg Rudder Angle	Ground	Uninterrupted control sweep, stop to stop.
(4) Nosewheel Steering Force and Position	± 2 lb Breakout ± 3 lb or $\pm 10\%$ Force ± 2 deg Nosewheel Angle	Ground	Uninterrupted control sweep, stop to stop.
c. Longitudinal			
(6) Longitudinal Trim	± 1 deg Pitch Control (Stabiliser and Elevator) ± 1 deg Pitch Angle $\pm 5\%$ Net Thrust or Equivalent	Cruise Approach Landing	May be snapshot tests.
(7) Longitudinal Manoeuvring Stability	± 5 lb or $\pm 10\%$ Column Force or Equivalent Surface Position	Cruise Approach Landing	Test at approximately 20 deg and 30 deg of bank for approach and landing configurations. Test at approximately 20 deg, 30 deg and 45 deg of bank for the cruise configuration. May be a series of snapshot tests.
(8) Longitudinal Static Stability	± 5 lb or $\pm 10\%$ Column Force or Equivalent Surface Position	Approach	Data for at least 2 speeds above and 2 speeds below trim speed. May be a series of snapshot tests.

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TESTS	TOLERANCE	CONDITIONS	COMMENTS
(9) Stick Shaker, Airframe Buffet, Stall Speeds	± 3 Knots Airspeed ± 2 deg Bank for speeds higher than stick shaker or initial buffet	Second Segment Climb and Approach or Landing	Airplanes exhibiting a sudden pitch attitude change or "g break" must demonstrate this characteristic. Airplanes with reversible flight control systems must also plot Stick/Column force ($\pm 10\%$ or ± 5 lb).
d. Lateral Directional			
(2) Roll Response (Rate)	$\pm 10\%$ or ± 2 deg/sec Roll Rate	Cruise and Approach or Landing	Test with normal wheel deflection (about 30%). Airplanes with reversible flight control systems must also plot Wheel Force ($\pm 10\%$ or ± 3 lb).
(5) Engine Inoperative Trim	± 1 deg Rudder Angle or ± 1 deg Tab Angle or Equivalent Pedal ± 2 deg Sideslip Angle	Second Segment Climb and Approach or Landing	May be snapshot tests.

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TESTS	TOLERANCE	CONDITIONS	COMMENTS
(8) Steady State Sideslip	For a given rudder position ± 2 deg Bank ± 1 deg Sideslip $\pm 10\%$ or ± 2 deg Aileron $\pm 10\%$ or ± 5 deg Spoiler or Equivalent Wheel Position or Force	Approach or Landing	May be a series of snapshot tests using at least two ruder positions. Airplanes with reversible flight control systems must also show Wheel Force ($\pm 10\%$ or ± 3 lb) and Rudder Pedal Force ($\pm 10\%$ or ± 5 lb).
e. Landings			
(1) Normal Landing	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack $\pm 10\%$ Altitude or ± 10 Feet	Landing	Airplanes with reversible flight control systems must also plot Stick/Column Force ($\pm 10\%$ or ± 5 lb).
(2) Minimum/No Flap Landing	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack $\pm 10\%$ Altitude or ± 10 Feet	Minimum Certified Landing Flap Configuration	Airplanes with reversible flight control systems must also plot Stick/Column Force ($\pm 10\%$ or ± 5 lb).
(3) Crosswind Landing	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack $\pm 10\%$ Altitude or ± 10 Feet ± 2 deg Bank Angle ± 2 deg Sideslip Angle	Landing	Airplanes with reversible flight control systems must also plot Wheel Force ($\pm 10\%$ or ± 3 lb) and Rudder Pedal Force ($\pm 10\%$ or ± 5 lb).

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TESTS	TOLERANCE	CONDITIONS	COMMENTS
(4) One Engine Inoperative Landing	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack $\pm 10\%$ Altitude or ± 10 Feet ± 2 deg Bank Angle ± 2 deg Sideslip Angle	Landing	
(6) Go Around	± 3 Knots Airspeed ± 1.5 deg Pitch ± 1.5 deg Angle of Attack	Go Around	
f. Ground Effect			
(1) A Test to Demonstrate Longitudinal Ground Effect	± 1 deg Elevator or Stabiliser Angle $\pm 5\%$ Net Thrust or Equivalent ± 1 deg Angle of Attack $\pm 10\%$ Height/Altitude or ± 5 Feet ± 3 Knots Airspeed ± 1 deg Pitch Attitude	Landing	A rationale must be provided with justification of results.

Chapter 3

Flight Test Data

Flight control system identification is performed using flight test data. This data is recorded on board the aircraft during the flight test program, using precision instrumentation. All instrumentation, data collection and data reduction has been performed at the National Research Council of Canada (NRC) prior to the work documented in this thesis. The following sections describe the data acquisition and calibrations of the measurement of the Citation X and CJ1 aircraft characteristics. Following these are sections describing the manoeuvres that are used for model development.

3.1 Flight Data Acquisition

The Flight Research Laboratory (FRL) of the National Research Council Canada (NRC) installs and calibrates dash potentiometers (dash pots) to perform the control system measurements. Analogue signals from the potentiometer are sampled at 1024 Hertz (Hz) and filtered using a 48 tap Finite Impulse Response (FIR) low-pass digital filter with a cutoff frequency of 10 Hz (Ricciardi et al, 2003). The filtered data is

recorded at 64 Hz for the Citation X (Ricciardi et al, 2003), and 100 Hz for the CJ1 aircraft (Auriti et al, 2006). The dash pot signal is in the form of a Direct Current (DC) voltage that varies with the monitored position. The DC voltage signal is measured by a computer that converts the analogue signal into a 12 bit digital signal (Ricciardi et al, 2003).

At the beginning of the flight test programme, the potentiometers are calibrated using control measurements on the ground. For example, the rudder position measurement is calibrated according to measurement of the rudder position using a digital protractor. Measurements of the aileron and elevator positions are calibrated according to accelerometer readings, so that zero is when the surfaces are horizontal compared to the local gravity field (Ricciardi et al, 2003). This method is known as the accelerometer-gravity method of calibration.

An important factor of the calibration of the potentiometers is the quality of the fit of physically measured data to the digital signal. The quality of this fit is quantified by the root mean square (RMS) of the error of the calibration. The smaller the RMS of the error, the better the fit of the calibration curve. Errors in the calibration are a function of measurement noise and errors due to the calibration method. For example, the accelerometer-gravity method of calibration assumes that the acceleration of the position being measured is negligible compared to the local gravity field. If the measured object is accelerating at a more significant rate, then

the reading of the accelerometer will indicate a skewed sense of vertical, and the calibration angle measurement will be erroneous.

Measurement noise reflects a combination of the ultimate accuracy of the potentiometer reading and calibration measurement technique. This component of the error represents the level of confidence in the recorded flight test data. Where calibrations have been performed at the NRC, the order of the polynomial fit, n , the RMS error, and the number of points used in calibration, N , may be used to determine an estimate of measurement uncertainty. The degree of freedom, r , is determined from N and n . Next, RMS is converted to standard deviation of the sample, s , and then this standard deviation is multiplied by Student's t-distribution factor for one-sided, 95% confidence level, $t_{r,95}$. The result is the measurement uncertainty, u . The measurement uncertainty results are included in Tables 3.1 and 3.2.

$$r = N - n - 1 \quad (3.1)$$

$$s = RMS \sqrt{\frac{N}{r}} \quad (3.2)$$

$$u = t_{r,95} \cdot s \quad (3.3)$$

Flight data post-processing is performed to generate derived information from primary measurements, as described in the following sections. Other post-processing may include correction for signal delay, as described in Appendix A.

3.1.1 HG1700 Inertial Measurement Unit

For both the Citation X and CJ1 aircraft, the inertial measurements are performed using a Honeywell HG1700 Inertial Measurement Unit (IMU). This IMU uses laser-ring gyrometers to accurately determine angular accelerations and calculate angular rates and directionality. The uncertainty in this measurement is quoted at $3.0^\circ/hr$ (1σ) (Hui et al, 2003). A Student's t-distribution value of 1.6449 is used since σ indicates a true standard deviation, and the 95% once-sided confidence for the uncertainty in the angular rates is $\pm 4.93^\circ/hr$. This translates to $\pm 1.37E-3^\circ/sec$ (95%).

The other components of the Honeywell HG1700 IMU measurement are the linear accelerations. The uncertainty for these measurements are quoted as 2.0 mG (1σ) (Hui et al, 2003). This translates to $\pm 0.106 ft/sec^2$ (95%).

3.1.2 Aerodynamic Data

The primary aerodynamic measurements are dynamic pressure, P_D , static pressure, P_S , and total air temperature, TAT . The measurements for P_D and P_S are performed using Honeywell sensors, model number PPT015-1A-X-2V. The pressure measurements suffer from position error based on aerodynamic interference of the aircraft body. The NRC developed Simultaneous Calibration of Air Data Systems (SCADS) to correct for position error (Hui et al, 1996). The SCADS report indicates that the final uncertainty in P_D and P_S measurements is $\pm 0.0025 psi$. The same document reports that the derived pitch (alpha) and yaw (beta) angles of attack each

have an uncertainty of ± 0.25 deg, and the uncertainty of the TAT measurement is $\pm 2^\circ C$ (Hui et al, 1996).

Many aerodynamic measurements are derived from P_D , P_S , and TAT . These include pressure altitude (Alt), Mach number (M), static temperature (T_S) and true airspeed (TAS). The equations for the derivation of these parameters are listed in Equations (3.4) through (3.7), respectively (Ricciardi et al, 2003).

$$Alt(ft) = 145447.2023 \left(1 - \left(\frac{P_S(psi)}{14.695944} \right)^{0.190255132} \right) \quad (3.4)$$

$$M = \sqrt{5 \left(\left(1 + \frac{P_D}{P_S} \right)^{0.2857142857} - 1 \right)} \quad (3.5)$$

$$T_S(K) = \frac{TAT(^{\circ}C) + 273.15}{1 + 0.2M^2} \quad (3.6)$$

$$TAS(kts) = 38.968M\sqrt{T_S(K)} \quad (3.7)$$

The uncertainty for any derived value is calculated by the chain rule, as described in Equation (3.8).

$$u_{v(x,y,...)} = \frac{\partial v(x,y)}{\partial x} u_x + \frac{\partial v(x,y)}{\partial y} u_y + \dots \quad (3.8)$$

The derived measurements, then, have variable uncertainties to be calculated as per Equations (3.4) through (3.8).

3.1.3 JR3 Six Degree of Freedom Force Sensor

The six components of force, F_X , F_Y and F_Z , and moments, M_X , M_Y and M_Z , are measured at the control column/wheel using a sensor supplied by JR3 Inc. Multi

- Axis Load Cell Technologies. The manufacturer-supplied calibration matrix is given in Equation (3.9) (Ricciardi et al, 2003).

$$\begin{bmatrix} 47.9597 & -0.0454 & 0.0028 & -0.1483 & -0.2979 & -0.3715 \\ -0.0780 & 48.267 & 0.0048 & 0.2703 & -0.3607 & 0.7446 \\ 0.793 & -0.3513 & 48.6464 & -0.6214 & -1.4965 & -1.1557 \\ 1.1636 & 2.6616 & -0.0394 & 109.8856 & -0.0721 & -1.3914 \\ -3.0353 & 1.0915 & -1.6935 & 0.1081 & 110.5787 & 0.8132 \\ 0.127 & 1.4298 & 0.3026 & -0.2964 & 0.1908 & 112.2336 \end{bmatrix} = A \quad (3.9)$$

$$\begin{bmatrix} F_X(lbs) \\ F_Y(lbs) \\ F_Z(lbs) \\ M_X(lbs \cdot in) \\ M_Y(lbs \cdot in) \\ M_Z(lbs \cdot in) \end{bmatrix} = A \cdot \begin{bmatrix} F_X(volts) \\ F_Y(volts) \\ F_Z(volts) \\ M_X(volts) \\ M_Y(volts) \\ M_Z(volts) \end{bmatrix}$$

For the CJ1, the column and wheel force measurements are calculated according to Equations (3.10) and (3.11), respectively. The contribution to column force from M_X and to wheel force from M_Z are divided by moment arms measured on the aircraft during instrumentation at the NRC (Auriti et al, 2006).

$$F_c = -F_Z - \frac{M_X}{30.5in} \quad (3.10)$$

$$F_w = \frac{M_Z}{2 \cdot 11.75in} \quad (3.11)$$

The measurement uncertainty for the JR3 sensor is developed at length in Appendix B. For data containing hands-free events, there are two components of uncertainty for the column and wheel forces: precision and bias. The precision and bias uncertainties for F_c are $\pm 0.044 \text{ lb}$ (95%) and $\pm 0.266 \text{ lb/min}$ (95%), respectively, and the precision and bias uncertainties for F_w are $\pm 0.0053 \text{ lb}$ (95%) and $\pm 0.0216 \text{ lb/min}$ (95%), respectively. The bias uncertainty is given as a rate, as the bias can be calibrated for each hands-free event, and the uncertainty is a function of a bias drift

before and after such a calibration. Because the force bias is calibrated once for each data file, the bias uncertainty for that file is the bias rate multiplied by the time duration recorded in the file.

3.1.4 Calibration Tables

Table 3.1 lists the calibrations and uncertainties for the Citation X measurements (Ricciardi et al, 2003). Table 3.2 lists the calibrations and uncertainties for the CJ1 measurements (Auriti et al, 2006).

3.2 Flight Manoeuvres

Flight test data are collected for a large number of flight and ground manoeuvres. Data from some of these manoeuvres are used to develop the flight control system models, and other data are used for model validation. The following sections describe the types of manoeuvres used for model development.

3.2.1 Ground Control Sweep Manoeuvre

The ground control sweep (GCS) manoeuvre is frequently performed before takeoff for a test flight, as a checkout procedure. This manoeuvre is performed twice for each primary control system; once for the primary control input, and once for the trim control. For each manoeuvre, the system input is slowly moved through its full range of motion and then restored to neutral, and so the respective control surface may be observed to be operating nominally before the aircraft enters flight. As an example,

Table 3.1: Citation X Flight Data Calibration Table

$$\text{CalibratedParameter} = C0 + C1 \cdot \text{Bits} + C2 \cdot \text{Bits}^2 + C3 \cdot \text{Bits}^3$$

Ch. No.	Parameter	Units	C0	C1	C2	C3	RMS Error	N	u 95%
4	Column Position	deg	30.7986	-0.0099501	0	0	0.13625	>200	0.225
5	Wheel Position	deg	-99.4006	0.046436	0	0	0.38853	>200	0.703
10	Total Temperature	°C	-51.36235	0.02442	0	0	0.076161	14	2*
11	Pedal Position	deg	-31.8123	0.019621	0	0	0.15717	>200	0.327
12	Left Pedal Force	lb	0.0234	0.0456067	0	0	0.3641	8	0.757
13	Right Pedal Force	lb	-3.8235	0.0495462	0	0	0.51905	9	1.079
29	Ax IRS	ft/sec ²	0	1.22E-04	0	0	-	-	0.106
30	Ay IRS	ft/sec ²	0	1.22E-04	0	0	-	-	0.106
31	Az IRS	ft/sec ²	0	1.22E-04	0	0	-	-	0.106
32	Q IRS	deg/sec	0	3.94E-03	0	0	-	-	1.37E-3
33	P IRS	deg/sec	0	3.91E-03	0	0	-	-	1.37E-3
34	R IRS	deg/sec	0	3.91E-03	0	0	-	-	1.37E-3
44	Left Aileron Angle	deg	-18.2202	0.0090975	0	0	0.20671	>200	0.430
45	Right Aileron Angle	deg	-16.9862	0.0089083	0	0	0.10497	>200	0.218
46	Left Elevator Angle	deg	78.9095	0.035843	5.3505E-6	1.8033E-9	0.09775	>200	0.245
47	Right Elevator Angle	deg	113.096	0.072031	-1.7063E-5	1.8033E-9	0.13345	>200	0.335
48	Lower Rudder Angle**	deg	-	-	-	-	0.21581	28	0.727
50	Aileron Trim	deg	-2.0163	8.86E-4	0	0	0.01915	14	0.040
51	Rudder Trim	deg	4.336	-1.68E-3	0	0	0.0465	10	0.097
60	Static Pressure	psi	0	0.001	0	0	-	-	0.0025*
61	Dynamic Pressure	psi	0	0.0001	0	0	-	-	0.0025*
64	Horizontal Stabiliser Angle	deg	-14.1374	0.0041141	0	0	0.02723	>200	0.566
65	Upper RudderAngle	deg	-36.9	0.0234	0	0	0.13982	27	0.291
-	Alpha AoA	deg	-	-	-	-	-	-	0.25*

Continued on next page

Ch. No.	Parameter	Units	C0	C1	C2	C3	RMS Error	N	u 95%
-	Beta AoA	deg	-	-	-	-	-	-	0.25*

* Uncertainty value as per SCADS report (Hui et al, 1996)

**The lower rudder calibration does not follow the general format, as there are two calibration regions.

Bit Range	C0	C1	C2	C3
< 2500	39.481	0.015275	0	0
≥ 2500	181.69	0.16223	-5.1025E-5	6.0043E-9

Table 3.2: CJ1 Flight Data Calibration Table

$$\text{CalibratedParameter} = C0 + C1 \cdot \text{Bits} + C2 \cdot \text{Bits}^2 + C3 \cdot \text{Bits}^3$$

Ch. No.	Parameter	Units	C0	C1	C2	C3	RMS Error	N	u 95%
15	Q IRS	deg/sec	0	3.94E-03	0	0	-	-	1.37E-3
16	P IRS	deg/sec	0	3.91E-03	0	0	-	-	1.37E-3
17	R IRS	deg/sec	0	3.91E-03	0	0	-	-	1.37E-3
18	Ax IRS	ft/sec ²	0	1.22E-04	0	0	-	-	0.106
19	Ay IRS	ft/sec ²	0	1.22E-04	0	0	-	-	0.106
20	Az IRS	ft/sec ²	0	1.22E-04	0	0	-	-	0.106
49	Wheel Position	deg	-163.4	0.008036	6.48E-08	-2.60E-12	0.4074	>200	0.680
50	Column Position	deg	20.3479	0.001094	0	0	0.1479	>200	0.246
51	Left Pedal Position	deg	30.5464	-0.00579	2.59E-7	-1.30E-11	0.2385	>200	0.398
52	Right Pedal Position	deg	30.5464	0.005788	-2.59E-7	1.30E-11	0.2385	>200	0.398
57	Roll Trim Wheel	deg	2003.6	-0.1203	0	0	4.1475	7	9.889
58	Pitch Trim Wheel	deg	1366.13	-0.08044	0	0	1.9176	12	3.807
59	Yaw Trim Wheel	deg	1116.73	0.070094	0	0	4.7939	7	11.43

Continued on next page

Ch. No.	Parameter	Units	C0	C1	C2	C3	RMS Error	N	u 95%
61	Left Aileron Angle	<i>deg</i>	22.9251	-0.00163	-9.68E-9	0	0.1212	>200	0.202
62	Right Aileron Angle	<i>deg</i>	38.1408	0.002113	-6.66E-9	0	0.1078	>200	0.180
63	Elevator Angle	<i>deg</i>	29.9713	0.001716	0	0	0.0814	>200	0.135
64	Rudder Angle	<i>deg</i>	42.231	-0.00244	0	0	0.1853	13	0.362
73	Left Pedal Force	<i>lb</i>	-6.4863	0.008044	0	0	1.4198	13	2.772
74	Right Pedal Force	<i>lb</i>	-31.887	0.012499	0	0	1.0678	11	2.164
87	Right Flap Angle	<i>deg</i>	56.4733	-0.00204	0	0	0.2744	>200	0.456
88	Left Flap Angle	<i>deg</i>	-7.734	0.002007	0	0	0.2857	>200	0.475
89	Aileron Trim Angle	<i>deg</i>	46.9556	-0.00359	7.66E-8	-1.83E-12	0.0556	>200	0.093
90	Elevator Trim Angle	<i>deg</i>	14.1519	-0.0017	1.10E-8	-2.32E-13	0.0748	>200	0.125
91	Rudder Trim Angle**	<i>deg</i>	-	-	-	-	0.3238	5	0.984
125	Rad Alt	<i>ft</i>	0	1	0	0	-	-	1
164	TAT	<i>°C</i>	0	0	0	0	-	-	2*
169	Static Pressure	<i>psi</i>	0	0.001	0	0	-	-	0.0025*
170	Dynamic Pressure	<i>psi</i>	0	0.0001	0	0	-	-	0.0025*
-	Alpha AoA	<i>deg</i>	-0.2989	1.46541	0	0	0.01	12	0.25*
-	Beta AoA	<i>deg</i>	-	-	-	-	-	-	0.25*

* Uncertainty value as per SCADS report (Hui et al, 1996)

**The rudder trim follows a calibration as follows:

$$ruddertrim = C0 + ruddertrim.bits * C1$$

$$C0 = 5.3728E - 03 * rudder^2 + 2.2618E - 01 * rudder - 3.885E01$$

$$C1 = -2.3904E - 07 * rudder^2 + 3.1623 * rudder + 1.7812E - 3$$

Figure 3.1 displays the time history of one control column GCS manoeuvre for the Citation X, with measured control column position plotted against elapsed time.

The GCS control system analysis is done to determine the gearing between the surface position and its control position and trim inputs. GCS data is used in this analysis because of the full-range motion performed in the control sweeps, and the slow speed at which these sweeps are executed. The full-range motion allows the entire envelope of control position and trim inputs to be investigated. Also, the slow speed at which the control sweeps are done minimises dynamic effects such as inertia and friction. Therefore, the analysis performed for the GCS manoeuvre is isolated from these dynamic effects that would otherwise complicate the regression.

3.2.2 Frequency Sweep Hangar Test

Many tests are performed on the flight control systems while the aircraft is in the NRC hangar facility. The frequency sweep manoeuvre is one such test, and the data collected from this manoeuvre is used to supplement the flight test data. During the frequency sweep, the aircraft is supported on jacks, so that the landing gear are unloaded. This is specifically important for the rudder flight control system, as the nosewheel is also actuated by the rudder pedals.

The frequency sweep manoeuvre consists of large amplitude excitation of the subject flight control system, starting with slow sweeps and gradually increasing the frequency until the pilot cannot move the controls any faster. A time history plot of one control column frequency sweep manoeuvre is presented in Figure 3.2. For

Figure 3.1: Control column position time history for one ground control sweep manoeuvre on the Citation X

Figure 3.2: Control column position time history for one frequency sweep hangar test

the purposes of ReFCI, the frequency sweep manoeuvre is useful for determining the mechanical forces of friction and inertia in isolation of large aerodynamic forces.

3.2.3 2311 Manoeuvre

The 2311 (pronounced “two-three-one-one”) manoeuvre is a large-amplitude manoeuvre performed during flight. 2311 manoeuvres are performed separately for pitch, roll, and yaw, using the elevator, aileron, and rudder systems, respectively. The 2311 manoeuvre consists of a sudden control input in one direction, held for two seconds, followed by a sudden control input in the opposite direction held for three seconds, followed by two more reversals of one second duration each. Prior to and following the manoeuvre are a minimum of ten seconds of level, trimmed flight, so that the disturbances to the aircraft motion can be isolated to the 2311 control inputs. 2311 maneuvers are used because of their ability to excite all of the aircraft handling qualities, without excessive deviations from the trim condition, and avoiding data contamination from cross-coupled inputs.

Figure 3.3 displays the time history of one pitch 3211 manoeuvre for the Citation X, with measured control column on the y-axis and elapsed time on the x-axis. The 3211 manoeuvre is a slight variation on the 2311 manoeuvre, with the only difference being interchanging durations of the first two displacements. Figure 3.3 clearly shows the square wave pattern of the 2311 manoeuvre. The varying square wavelength pattern that is used results in a broad-spectrum low frequency excitation of the aircraft control systems. The broad-spectrum excitation is useful for the identification of dy-

namic effects on the control system, and allows for accurate spectral analysis of the data. The input frequency content of an idealised 3211 manoeuvre is displayed in Figure 3.4 (deLeeuw et al, 1987).

For reversible flight control systems, aerodynamic hinge moments on the control surface play an important role in system behaviour. It is necessary to analyse the performance of each control system over a range of flight conditions that adequately represent the flight envelope of the host aircraft. During the flight test program, the 2311 manoeuvres are repeated over a wide range of flight conditions, including airspeed, Mach number, pressure altitude, and temperature. Therefore, the data from the 2311 manoeuvres accommodate the analysis of aerodynamic effects on the control systems.

3.2.4 Flight in Ground Effect Manoeuvre

Flight in ground effect manoeuvres are performed to enable the study of aerodynamic ground effects on the aircraft rigid body dynamics and flight control systems. While in ground effect, the circulation pattern and vortex shedding of the wing are mirrored by equivalent vortices below the ground surface. This happens in order to satisfy the boundary condition that there is no air flow through the ground. The virtual vortices induce an upwash that partially counters the wing-generated downwash. The result of ground effect on control surfaces is an increased lifting force and associated hinge moment.

Figure 3.3: Control column position time history for 3211 manoeuvre

Figure 3.4: Input power spectrum density for ideal 3211 manoeuvre

Data for flight in ground effect is collected for very low-level flyby manoeuvres with altitudes ranging from less than one wingspan to several wingspans above ground level. These manoeuvres are performed at various speeds, landing gear settings and flap positions.

3.2.5 Flap Change Dynamics Manoeuvre

The flap change dynamics manoeuvre is simply the changing of flap positions during flight. Flaps have a powerful effect on the aerodynamic circulation patterns about the wing and therefore alter the flow field. Flap positions influence the effective angle of attack at the tail location, so the elevator surface aerodynamic hinge moment is sensitive to flap setting.

3.2.6 Trim Sweep Manoeuvre

The pitch trim sweep manoeuvre consists of sweeping through the trim positions of the subject flight control system during flight. Trim tabs exist to produce a desired aerodynamic hinge moment on the flight control surface, so the trim sweep manoeuvre is useful for determining the characteristics of this effect.

3.2.7 Steady State Sideslip Manoeuvre

The steady state sideslip manoeuvre involves maintaining a sideslip angle of attack, β . β and the pitch angle of attack, α , are defined in Figure 3.5. Data collected during the steady state sideslip manoeuvre will highlight the aerodynamic effects of sideslip

on the aircraft and flight control system dynamics. A time history of β during one such manoeuvre is presented in Figure 3.6.

Figure 3.5: Definition of α and β angles of attack

Figure 3.6: Time history plot of β during steady state sideslip manoeuvre

Chapter 4

Cessna Citation X: Irreversible Flight Control

The Flight Research Laboratory (FRL), of the Institute for Aerospace Research (IAR), National Research Council of Canada (NRC), was contracted by CAE Inc. to conduct the instrumentation and flight testing for the Cessna model 750, Citation X aircraft. These activities were carried out in the period between 11 February and 10 June of 2003, during which 126.6 hours of flight time were accumulated over 37 test flights (Ricciardi et al, 2003). Figure 4.1 is a picture of the Citation X on its visit to the NRC.

The Cessna model 750, Citation X is a high performance business jet aircraft. It has a wingspan of 63 ft 7 in, length of 74 ft 4 in, and overall height of 19 ft 2 in (Jane's, 2004). The maximum takeoff weight is 35,700 lb, and the standard-equipment empty weight is 21,600 lb (Jane's, 2004). The Citation X has a maximum operating Mach number of 0.92, a service ceiling of 51,000 ft, and a range of 3,216 nautical miles (Jane's, 2004).

Figure 4.1: Cessna model 750, Citation X on the tarmac by the NRC hangar

Figure 4.5: Measured elevator versus column positions for the ground control sweep manoeuvre, 9924 data points

Figure 4.6: Elevator versus column regression for region below deadband

ues and 95% confidence level uncertainties for the three regions of the η versus δ_c regression are listed in Equation (4.8).

$$\begin{aligned}
 A_{0,1} &= \#.5 \quad \pm \#.6 \quad [deg] \quad (95\%) \\
 A_{1,1} &= \#.7 \quad \pm \#.8 \quad (95\%) \\
 A_{0,2} &= \#.17 \quad \pm \#.18 \quad [deg] \quad (95\%) \\
 A_{1,2} &= \#.19 \quad \pm \#.20 \quad (95\%) \\
 A_{0,3} &= \#.9 \quad \pm \#.10 \quad [deg] \quad (95\%) \\
 A_{1,3} &= \#.11 \quad \pm \#.12 \quad (95\%)
 \end{aligned} \tag{4.8}$$

Finally, the saturation limits of motion are identified by bisection method, with the optimisation being to minimise the standard deviation of $\eta - \eta_{\delta_c}$. The result of this process is that η_{min} and η_{max} are defined as #.21 and #.22 degrees, respectively.

The match of η_{δ_c} to the measured η versus δ_c is presented in Figure 4.8.

4.1.3 Stabiliser - Elevator Gearing

The effect of the horizontal stabiliser position on the measured elevator position is identified using stabiliser ground control sweep manoeuvres, listed in Table D.2. The measured time history of a stabiliser sweep is shown in Figure 4.9. It is important to note that during the stabiliser GCS manoeuvre, the control column is not moved, so that the motion of the elevator surface is a function of the stabiliser motion only.

To avoid counting the elevator - column relationship bias into the following analysis, the η_{δ_c} is calculated and subtracted from measured η according to Equation (4.9). $\eta_{\delta_{hstab}}$ versus δ_{hstab} is presented in Figure 4.10.

$$\eta_{\delta_{hstab}} \simeq \eta - \eta_{\delta_c} \tag{4.9}$$

Figure 4.7: Elevator versus column regression for region above deadband

Figure 4.8: Elevator versus column match for column ground control sweep

Figure 4.9: Time history of stabiliser ground control sweep manoeuvre

Figure 4.10: η versus δ_{hstab} for stabiliser ground control sweep manoeuvre

In the current analysis, $\eta_{\delta_{hstab}}$ will be regressed against δ_{hstab} . Based on the explanation that the elevator position is sensitive to stabiliser position, and that the stabiliser is a rotating body, it is reasonable that the relationship would be a trigonometric function. This function has been proposed in Equations (4.2) and (4.3), repeated here as Equations (4.10) and (4.11). In order to identify the parameters δ_{offset} , B_0 , B_1 , and B_2 , the following approach is taken.

$$\eta_{\delta_{hstab}} = B_0 + B_1\Delta_{hstab} + B_2\Delta_{hstab}^2 \quad (4.10)$$

$$\Delta_{hstab} = \cos\left(\frac{\pi}{180}(\delta_{hstab} + \delta_{offset})\right) - 1 \quad (4.11)$$

First, δ_{offset} is estimated. From the data in Figure 4.10, it appears that δ_{offset} is approximately eight degrees, as the peak of the curve occurs near $\delta_{hstab} = -8$ [deg]. Using this estimation, Δ_{hstab} is calculated, and a quadratic fit is made for the regression of $\eta_{\delta_{hstab}}$ versus Δ_{hstab} . Then the value of δ_{offset} is optimised using bisection method to adjust δ_{offset} to find the greatest R-squared value for the $\eta_{\delta_{hstab}}$ versus Δ_{hstab} regression. This results in the optimal δ_{offset} value of 8.1 degrees.

The regression of $\eta_{\delta_{hstab}}$ versus Δ_{hstab} is presented in Figure 4.11. The parameters identified in this regression are listed in Equation (4.12). Finally, the match of $\eta_{\delta_{hstab}}$ versus δ_{hstab} is presented in Figure 4.12.

$$\begin{aligned} B_0 &= 0.16 & \pm 0.103 & [deg] & (95\%) \\ B_1 &= 32.7 & \pm 9.03 & & (95\%) \\ B_2 &= -5500 & \pm 682 & [deg^{-1}] & (95\%) \end{aligned} \quad (4.12)$$

Figure 4.11: $\eta_{\delta_{hstab}}$ versus Δ_{hstab} regression, stabiliser ground control sweep manoeuvre

Figure 4.12: $\eta_{\delta_{hstab}}$ versus δ_{hstab} match, stabiliser ground control sweep manoeuvre

4.1.4 Thermal Effects

It is observed that the measured elevator position signal is sensitive to the control system temperature. This observation is made through comparison of the behaviour of the elevator system during the ground control sweep and 2311 manoeuvres. The 2311 manoeuvre data files used here are listed in Table D.3.

There are two distinct possible causes of the thermal sensitivity of the measurements: either the control system behaviour is affected by temperature, or the potentiometer measurement system is affected by temperature. Potentiometer thermal sensitivity is ruled out in the investigation described in Appendix C.

The thermal effects on the elevator control system are identified by plotting measured η versus the modelled η^* calculated by Equation (4.4). For an ideal model match, there would be a “one-to-one” regression of these two sets, with a unity slope and zero offset. Deviation from unity slope indicates that temperature affects the gearing ratios between the control column and elevator positions, and deviation from zero offset indicates that temperature creates a bias in the elevator position.

The slope and bias adjustment parameters have been defined as C and D , respectively. The migration of these parameters with temperature may be investigated. Figure 4.13 shows the regression of the measured η versus the modelled η^* for one 2311 manoeuvre. For the migration of C and D with temperature, a suitable temperature measurement must be selected. There is no direct system temperature measurement available, and so air temperature is used. Total air temperature (TAT) more closely

Figure 4.13: Match of measured η versus modelled η^*

represents the temperature condition of the environment close to the aircraft, as it takes into account heating effects due to compression, whereas static air temperature does not.

Figures 4.14 and 4.15 show the migration of offset and slope, respectively, with total air temperature. In these figures it is observed that the gearing ratio between column and elevator positions is insensitive to temperature, while there is a significant elevator position bias created by temperature change. Equations (4.5) through (4.7) are repeated here as Equations (4.13) through (4.15), respectively, and the identified parameter values for these equations are listed in Equation (4.16).

$$\eta^{**} = C + D\eta^* \quad (4.13)$$

$$C = C_0 + C_1TAT \quad (4.14)$$

$$D = D_0 + D_1TAT \quad (4.15)$$

$$\begin{aligned} C_0 &= -0.058 \pm 0.314 \quad (95\%) \\ C_1 &= -0.01 \pm 0.01 \quad (95\%) \\ D_0 &= 0.955 \pm 0.109 \quad (95\%) \\ D_1 &= 0.002 \pm 0.0034 \quad (95\%) \end{aligned} \quad (4.16)$$

An explanation for these trends is variability of the stiffness of the centering spring, as seen in the elevator control diagram, Figure 4.2, with temperature. A stiffening of this spring would cause the power control unit (PCU) input to be pulled outboard, and this offset would be magnified through the PCU to cause a noticeable bias in the elevator position. The gearing ratio of the control system would be largely unaffected by the variable spring constant of the centering spring, which is in agreement with the observed trend.

Figure 4.14: Identified bias adjustment C versus total air temperature

Figure 4.15: Identified slope adjustment D versus total air temperature

4.1.5 Elevator System Backlash

The elevator system backlash model component is defined by η_{play} , which is the magnitude of the backlash observed in the elevator control system in degrees of elevator deflection. It is difficult to directly identify the best estimate for η_{play} from the data using regression analysis, so a root-finding method is employed instead. In this procedure, the elevator control system is modelled all the way up to η_{BL} , which is the output from the backlash model component. This model is run for the control column ground control sweep manoeuvre data. The error between measured η versus η_{BL} is determined, as per Equation (4.17), for this data set. Many model runs are performed using different values for η_{play} , and bisection method is used to optimise this value to achieve a minimum Error.

$$Error = std(\eta - \eta_{BL}) \quad (4.17)$$

The optimal backlash occurs for a setting of η_{play} of 0.48 degrees, as can be seen in Figure 4.16. In this figure, *Error* is plotted against η_{play} . Including backlash effects improves the standard deviation of the modelled η fit for the column GCS manoeuvre. The standard deviation of $(\eta - \eta_{\delta_c})$ is 0.336, whereas the standard deviation calculated in Equation (4.17) is reduced to 0.249 with the optimal η_{play} .

4.1.6 Citation X Elevator Model Summary

The final Citation X elevator control system model is summarised here. The model summary is presented according to the block diagram as repeated in Figure 4.17.

Figure 4.16: Improvement in η match error with η_{play} optimisation

Figure 4.17: Citation X Elevator Model Architecture

The Gearing model component has input δ_c and output η_{δ_c} , as defined in Equation (4.18).

$$\eta_{\delta_c} = \begin{cases} A_{0,1} + A_{1,1}\delta_c, & \delta_c < \delta_{c,DBmin}; \\ A_{0,2} + A_{1,2}\delta_c, & \delta_{c,DBmin} \leq \delta_c \leq \delta_{c,DBmax}; \\ A_{0,3} + A_{1,3}\delta_c, & \delta_{c,DBmax} < \delta_c \end{cases} \quad (4.18)$$

The Horizontal Stabiliser Effects Adjustment component has inputs η_{δ_c} and δ_{hstab} , and output η^* . The governing equations for this block are Equations (4.19) through (4.21).

$$\eta_{\delta_{hstab}} = B_0 + B_1\Delta_{hstab} + B_2\Delta_{hstab}^2 \quad (4.19)$$

$$\Delta_{hstab} = \cos\left(\frac{\pi}{180}(\delta_{hstab} + \delta_{offset})\right) - 1 \quad (4.20)$$

$$\eta^* = \eta_{\delta_c} + \eta_{\delta_{hstab}} \quad (4.21)$$

The Thermal Effects Adjustment component has inputs η^* and TAT , and output η^{**} . The governing equations for this block are Equations (4.22) through (4.24).

$$\eta^{**} = C + D\eta^* \quad (4.22)$$

$$C = C_0 + C_1TAT \quad (4.23)$$

$$D = D_0 + D_1TAT \quad (4.24)$$

The Backlash model component has input η^{**} and output η_{BL} . This block tracks the state of the input motion with respect to a range of input play, η_{play} , and updates the output η_{BL} only when the input has moved through all of the available free play. This block cannot be represented by a simple equation.

The Saturation component has input η_{BL} and output η_{mdl} . This block limits the range of the output to within the boundaries η_{min} and η_{max} .

The parameters required for the elevator control system model have values identified from the flight data, and these are listed in Table 4.1.

4.1.7 Elevator Model Verification

The performance of the Citation X elevator model is verified through the comparison of the model output, η_{mdl} versus measured elevator position, η for 20 2311 manoeuvres. The match of the first case is presented in Figure 4.18 as a time history. The standard deviation of the model error is calculated for each 2311 manoeuvre, and this error is 0.157 ± 0.0683 degrees (95%). This means that for the 2311 manoeuvre, verification indicates that the model error will be less than 0.225 degrees 95% of the time. This match is well within tolerance specification by AC 120-40C for flight manoeuvres.

Although the 2311 manoeuvre is not an official part of the Qualification Test Guide manoeuvre set, the purpose here is to observe the the elevator model architecture and parameter identification process results in an accurate model. The purpose is not to obtain Level D certification. This is because the modelling performed on the Citation X flight control systems is a stepping stone toward development of reversible flight control system identification on other aircraft.

It is interesting to consider the effect of turning off one of the components of the elevator model on the model match. For example, what happens if the thermal effects adjustment block is omitted? The resulting increase in model error due to the exclusion of model blocks is listed in Table 4.2. It appears that the Thermal Effects

Table 4.1: Citation X elevator model parameter values

Parameter	Value	Uncertainty (95%)
$A_{0,1}$	#.5	$\pm \#.6$
$A_{1,1}$	#.7	$\pm \#.8$
$A_{0,2}$	#.17	$\pm \#.18$
$A_{1,2}$	#.19	$\pm \#.20$
$A_{0,3}$	#.9	$\pm \#.10$
$A_{1,3}$	#.11	$\pm \#.12$
$\delta_{c,DBmin}$	#.2	-
$\delta_{c,DBmax}$	#.3	-
B_0	0.16	± 0.103
B_1	32.7	± 9.03
B_2	-5500	± 682
δ_{offset}	8.1	-
C_0	-0.058	± 0.314
C_1	-0.01	± 0.01
D_0	0.955	± 0.109
D_1	0.002	± 0.0034
η_{play}	0.48	-
η_{min}	#.21	-
η_{max}	#.22	-

Figure 4.18: Elevator model match time history for one 2311 manoeuvre. The tolerance are indicates the 0.5 degree maximum error limitation.

block does not improve the overall model match, so it may be recommended that this block not be included in the final control system model.

It is important to note that the above analysis is a simple verification only. The elevator control system model has not been integrated into a global aircraft model for validation tests using qualification test guide data to the satisfaction of Federal Aviation Administration requirements. This is the case because the rigid body aerodynamics model for the Citation X is not available for integration. Because of this limitation, the model error tolerance must be made more stringent. A maximum elevator position error of 0.5 degrees in flight must be satisfied.

4.2 Lateral Control: Ailerons and Aileron Trim

The ailerons and aileron trim are the primary flight controls responsible for lateral roll control of the Citation X. There are two ailerons: one near the tip of each wing. Aileron trim is performed by adjusting a bias to the aileron position. Control of the aileron position, ξ , is performed through inputs to the control wheel position, δ_w , and aileron trim buttons. The control input position is applied to hydraulic power control units located in the lower fuselage near the root of the wing. The PCU output is translated to the ailerons through cabling running along the leading edge of each wing.

4.2.1 Aileron Control System Model Architecture

The aileron control system model architecture is very similar to the elevator control system architecture, as both systems are dominated by the power of hydraulic actuation. The main differences between the aileron and elevator models is that the aileron model thermal effects block has been replaced with the dynamic pressure effects block, and the addition of the rate limitation block. Because the aileron position is separated from the hydraulic output position by a length of cable, this cable will have an effective spring rate. Hinge moments at the aileron will cause measurable deflection. Rate limitation is a reality for any hydraulically actuated position, and this effect is more apparent for the aileron control system than for the elevator control system on the Citation X. The block diagram for the Citation X aileron control system model is presented in Figure 4.19.

The following paragraphs outline the functional form of each of the blocks in Figure 4.19, and list the parameters required to define them.

Aileron Hydraulics: The aileron hydraulics block differs slightly from the elevator hydraulics block in that there is no noticeable deadband region. The function describing wheel - aileron gearing is given in Equation (4.25).

$$\xi_{\delta_w} = A_0 + A_1 \cdot \delta_w \quad (4.25)$$

Table 4.2: Elevator model error with modules deactivated

Deactivated Module	Model Error [deg] (95%)
None	0.225
Thermal Effects	0.224
Backlash	0.300
Thermal Effects & Backlash	0.327

Figure 4.19: Citation X Aileron Model Architecture

Aileron Trim Effects Adjustment: The contribution to aileron position by the aileron trim position, ξ_{TT} , is described in Equation (4.26).

$$\xi_{\xi_{TT}} = B_0 + B_1 \cdot \xi_{TT} \quad (4.26)$$

Dynamic Pressure Effects Adjustment: The modelled aileron position is adjusted for dynamic pressure effects in similar fashion to the elevator model thermal effects adjustment block. The adjustment is described in Equations (4.27) through (4.29).

$$\xi^* = \xi_{\delta_w} + \xi_{\xi_{TT}} \quad (4.27)$$

$$\xi^{**} = C + D\xi^* \quad (4.28)$$

$$D = D_0 + D_1 P_D \quad (4.29)$$

Backlash: For the aileron control system model, backlash is defined by the amount of play in the aileron position, ξ_{play} .

Rate Limitation: All hydraulically actuated control surfaces have a maximum rate of travel, defined by the volume flow rate capacity of the hydraulic pumps and the area of the hydraulic ram head. The effect of rate limitation is found to be significant for the Citation X aileron control system, so the addition of the rate limitation block is necessary for accurate model performance. The rate limitation is defined by a single parameter, $\dot{\xi}_{max}$.

Saturation: The saturation limitations for the aileron control system model are defined by the minimum and maximum aileron travel limits, ξ_{min} and ξ_{max} .

The identification of the parameters listed in the above paragraphs is presented in Appendix E. The flight data files used for the system parameter estimations are listed in Tables D.4 to D.6.

4.2.2 Aileron Model Verification

Verification of the Citation X aileron control model is performed in the same manner as for the elevator control model. Figure 4.20 displays the time history of one 2311 manoeuvre, with the modelled ξ_{mdl} plotted along with measured ξ . The model error statistics for all 2311 manoeuvres are listed in Table 4.3, including the effect of disabling the dynamic pressure and backlash model blocks. The maximum aileron model error target for stand-alone verification is 0.5 degrees. Thus, the aileron control system model including all model components and the model excluding backlash are candidates for Level D class model fidelity.

Figure 4.20: Aileron model match time history for one 2311 manoeuvre. The tolerance are indicates the 0.5 degree maximum error limitation.

Table 4.3: Aileron model error with modules deactivated

Deactivated Module	Model Error [deg] (95%)
None	0.342
Dynamic Pressure Effects	0.648
Backlash	0.337
Dynamic Pressure Effects & Backlash	0.638
Rate Limitation	1.07

Chapter 5

Cessna CJ1: Reversible Flight Control

The Flight Research Laboratory (FRL), of the Institute for Aerospace Research (IAR), National Research Council of Canada (NRC), was contracted by Mechtronix Systems Incorporated (MSI) to conduct the instrumentation and flight testing for the Cessna CitationJet CJ1 aircraft. These activities were carried out in the period between 24 May and 27 August of 2005, during which 75 hours of flight time were accumulated over 32 test flights. (Auriti et al, 2006) Figure 5.1 is a picture of the CJ1 on its visit to the NRC.

The Cessna CitationJet is an entry-level business jet aircraft. It has a wingspan of 46 ft 9 ¹/₂ in, length of 42 ft 7 in, and overall height of 13 ft 9 ¹/₄ in. The maximum takeoff weight is 10,600 lb, and the standard equipment empty weight is 6,670 lb. The CJ1 has a maximum operating Mach number of 0.71, a service ceiling of 41,000 ft, and a range of 1,475 nautical miles. (Jane's, 2005)

Figure 5.1: Cessna CitationJet CJ1 on the tarmac by the NRC hangar

The identification process for reversible flight control systems is developed for the CJ1 elevator control system. The elevator surfaces are the primary longitudinal flight controls for the Cessna CJ1 aircraft. The two elevators are located at the trailing edge of the left and right halves of the horizontal stabiliser. The elevators incorporate an aerodynamic horn balance and are each equipped with a tab for longitudinal trimming.

As the CJ1 has reversible flight controls, the elevators are motivated by human force, applied by the pilot through the control column. The control column is connected to the elevator through a system of cables and pushrods. The trim tab is controlled via the pitch trim dial.

5.1 Elevator Trim Tab Model

The elevator trim tab control mechanism is adequately modelled by a linear algebraic relationship. This relationship is constant throughout the operational regime of the CJ1, as it is a matter of a gearing ratio between the trim tab, η_{TT} , and the pitch trim dial, δ_{TD} . This relationship is proposed in Equation (5.1).

$$\eta_{TT} = B_0 + B_1\delta_{TD} \quad (5.1)$$

The coefficients of Equation (5.1) are defined from the longitudinal manoeuvre flight test data listed in Table D.9 of Appendix D. The resulting regression of η_{TT} versus δ_{TD} is displayed in Figure 5.2, and the coefficients are identified in Equation (5.2).

$$\begin{aligned} B_0 &= 0.229 \quad \pm 0.157 \quad [deg] \quad (95\%) \\ B_1 &= -0.0159 \quad \pm 1.12E-4 \quad (95\%) \end{aligned} \quad (5.2)$$

Figure 5.2: Regression of η_{TT} versus δ_{TD} , in flight

5.2 Elevator Control System Model Architecture

The elevator control system model is required to reproduce the elevator position and column force behaviour measured from the CJ1 aircraft during various manoeuvres, as described in Chapter 2.

The Cessna CJ1 elevator control system is modelled as a kinematic model that determines elevator motion as resulting from the various forces and moments acting on the elevator body. The development of the elevator model is therefore a study of the forces that act on the elevator. The column force model is therefore an integral part of the elevator model.

A simplified diagram of the elevator control system is shown in Figure 5.3. This diagram displays the sign conventions of the positions and forces of the elevator system. Column position and force are positive forwards, and elevator position, trim tab position, and hinge moments are positive clockwise as viewed from port side. From this figure, it is seen that hinge moments acting on the elevator are relayed to the control column through the elevator cable. This cable couples the two main bodies of the elevator control system; the control column and the elevator body.

The block model representation of the CJ1 elevator control model is displayed in Figure 5.4. This model shows how position is calculated by double-integrating acceleration, which itself is calculated from the forces or hinge moments acting on the elevator body. This modelling introduces internal feedback loops for friction and spring force modelling, and so it is a departure from the classical “straight-

Figure 5.3: CJ1 Elevator control system model

Figure 5.4: CJ1 elevator control model block diagram

through” modelling approach taken with the Citation X irreversible flight control system models.

The elevator model includes a control column model in order to capture the friction and inertia forces of the column body. This model is constructed so that control column motion is calculated from the forces acting on the column body. This part of the model, therefore, responds to column force, and outputs a modelled column position. However, the simulator model requirement is to use column position as an input and column force as an output. This inversion of concepts is accounted for by using the force feedback block, where an error between the input column position and the internally modelled column position is used to determine what column force is required to drive the column according to the desired position.

A second diagram, Figure 5.5, shows an alternative elevator control model in which control column friction and inertia are included. Added to this model are the column friction and inertia feedback loops, and a force feedback block. The complications of using feedback loops to model friction and inertia effects are accepted because the alternative is to account for friction and inertia as functions of the first and second time derivatives of body position. This involves multiplying a derivative of a noisy signal by a gain, and this tends to create more noise in the output than useful force modelling. Other advantages of using the feedback loop modelling technique are discussed in the friction model paragraph below.

Figure 5.5: CJ1 elevator control with column model block diagram

5.2.1 Column - Elevator Gearing

The column elevator gearing is the relationship between elevator and column position. For the CJ1 aircraft, this relationship is slightly non-linear, due the combination of rotational and translational bodies within the elevator control system. The longitudinal displacement of the elevator cable is not precisely linear with the rotational displacement of the control column. Similarly, the rotational displacement of the elevator surface is not precisely linear with the longitudinal displacement of the elevator cable. The net result is a small degree of nonlinearity, which can be accommodated by a second-order polynomial fit as given in Equation (5.3). The parameters of this equation are identified according to the methods described in Section 5.3.

$$\eta_{cmd} = A_0 + A_1\delta_c + A_2\delta_c^2 \quad (5.3)$$

5.2.2 Elevator Position Saturation

The elevator position range of motion is physically limited. These limits are defined at η_{min} and η_{max} . The elevator limits of motion are defined in Section 5.3.

5.2.3 Elevator Cable

The hinge moment transmitted by the elevator cable between the control column and elevator is in the form of spring force. Therefore, this hinge moment is a function of the displacement of the elevator cable, which is defined in Equation (5.4). The relationship between cable displacement and hinge moment observed at the control column is slightly non-linear, so a second order polynomial is fit to the data. The

proposed relation for column hinge moment, $M_{c,cbl}$, is given in Equation (5.5). The elevator cable spring rate parameters are defined in Section 5.4.

$$\delta_{cbl} = \eta - \eta_{cmd} \quad (5.4)$$

$$M_{c,cbl} = C_1 \delta_{cbl} + C_2 \delta_{cbl}^2 \quad (5.5)$$

5.2.4 Column Spring

The control column has limits of motion in similar fashion as the elevator position saturation. There is an effective spring rate that applies once the control column is forced against one of these limits. For the CJ1, only one limit of motion appears to be engaged, $\delta_{c,min}$. The stretch in the column stop is then defined in Equation (5.6), and the contribution to column hinge moment is defined in Equation (5.7). The column spring rate and motion limit are identified in Section 5.4.

$$\delta_{colspr} = \delta_c - \delta_{c,min} \quad (5.6)$$

$$M_{c,colspr} = D_1 \delta_{colspr}, \quad (\delta_{colspr} < 0) \quad (5.7)$$

5.2.5 Column Friction, Inertia and Force Feedback

The control column is a physical body and so is subject to friction and inertia effects. Inertia is accounted for by the mass moment of inertia of the control column, J_c , which governs the column angular acceleration response to hinge moments, M_c , according to Equation (5.8).

$$\ddot{\delta}_c = \frac{M_c}{J_c} \quad (5.8)$$

The column friction hinge moment is a function of column angular rate, $\dot{\delta}_c$. The friction is a combination of stiction and coulomb friction. For very small angular rates, stiction dominates, and there is a relatively large resistance to motion. As column deflection rate increases, the friction transitions to coulomb, or sliding, friction, which is roughly a constant opposing moment. This friction moment profile is displayed in Figure 5.6. (Weiss et al, 1998)

The friction profile from Figure 5.6 is defined by four parameters. The magnitude of the maximum amount of resistance that stiction provides is $M_{c,stic}$, and the magnitude of the sliding friction is $M_{c,clmb}$. The deflection rate at which breakout occurs is $\dot{\delta}_{c,breakout}$, and the deflection rate at which transition is complete and sliding friction takes over is $\dot{\delta}_{c,trans}$.

Replicating column friction and inertia effects in the model is difficult because column position, not force, is the input to the model. Although it is possible to take the time derivatives of the input column position and use these to calculate friction and inertia forces, these derivatives will be noisy. Considering especially that the friction profile presented in Figure 5.6 is very much like a step function, a noisy $\dot{\delta}_c$ signal will result in a large band of noise in the output column force, and this is unacceptable. Low-pass filtering to reduce noise in the time derivatives can be used to reduce this noise, but this will result in phase lag in the modelled friction and inertia force outputs.

Figure 5.6: Friction model profile (Weiss et al, 1998)

Driving the force feedback block with a modelled column position error signal to create column force allows the column motion to be modelled as a response to all the moments acting on the column body. This avoids attempting to capture column friction and inertia forces by amplifying $\dot{\delta}_c$ and $\ddot{\delta}_c$ signals. However, this approach does not alleviate all the difficulties associated with using column position as an input signal. The force feedback block is modelled a proportional-integral-derivative (PID) controller, which includes amplifying the derivative of the modelled column position error signal. This method is still preferable, however, because $\dot{\delta}_{c,err}$ is not being passed to a step function, and there is no amplification of the second time derivative of a signal. The model output column force is passed through a low-pass filter to reduce noise.

A final disadvantage to having column position as the input to the flight control system model is that this situation contradicts the natural direction of the flow of information in the physical system being modelled. The column position is not directly commanded by the pilot. Rather, the pilot desires a certain column position, and then he applies the necessary force to achieve this goal. There is a feedback loop on the human side of the equation, which is beyond the scope of this report. The force feedback loop included in the elevator control model does not represent this human feedback loop, as the desired column position in the pilots mind is not recorded.

The main problem with the flow of information problem can be explained using two examples. The first example is for the case of stiction and breakout behaviour.

Consider that the pilot wishes to move the control column, which is initially at rest. In order to start the column moving, the pilot must first overcome the stiction force, $M_{c,stic}$. If the pilot should apply any force less than this amount, the column will remain stuck. In this case, the column position signal contains no information about column motion, but there are friction forces at work. The pilot could just as easily been applying force in the opposite direction, so the best guess from a modelling perspective is zero frictional forces. This situation becomes a limitation to maximum fidelity of the computer simulation part of the flight simulator. In reality, a Level D simulator will have a physical control column that may be designed to replicate the aircraft control column friction behaviour.

The second example is for when the model is excited at high frequency relative to the natural frequency of the system. For second-order systems, the system output will naturally lag the input as a function of the input frequency. Thus, at high frequencies, the column position will lag the column force. However, with column position as the input, the model does not have the information to reconstruct the leading column force signal, so there is necessarily an error. This problem is not significant, however, for input frequencies well below the natural frequency of the elevator control system.

5.2.6 Elevator Friction and Inertia

The elevator friction and inertia are managed in the same fashion as for the control column, though no force feedback block is required. Elevator mass moment of inertia

is given by J_e , and the elevator friction parameters are $M_{e,static}$, $M_{e,climb}$, $\dot{\eta}_{breakout}$ and $\dot{\eta}_{trans}$.

5.2.7 Aerodynamic Hinge Moments

The aerodynamic hinge moments acting on the elevator form the aerodynamic force feedback that differentiates reversible from irreversible flight control systems from the perspective of pilot force feel. The elevator hinge moments are given in the form of Equation (5.9) (Weiss et al, 1998). In this equation, MH_e is the total aerodynamic hinge moment on the elevator, CH_e is the non-dimensional elevator hinge moment coefficient, P_D is the free-stream dynamic pressure, and S_e and $\bar{c}_e$ are the area and reference chord of the elevator. In this case, the full chord of the elevator is used, as opposed to the chord of the portion of the elevator behind the hinge line.

$$MH_e = \frac{1}{2}CH_e P_D S_e \bar{c}_e \quad (5.9)$$

The elevator area and reference chord are $24.7[ft^2]$ and $1.42[ft]$, respectively, as measured from a scaled 3-view drawing in Jane's All the World's Aircraft 2005-2006 edition. CH_e may then be calculated by rearranging Equation (5.9), once Mh_e is known. The non-dimensional elevator hinge moment coefficient is itself a combination of several parts, each of which is a function of a few parameters. The possible components of CH_e are listed in Equation (5.10) (Weiss et al, 1998).

$$CH_e = \frac{2MH_e}{P_D S_e \bar{c}_e} \quad (5.10)$$

$$\begin{aligned}
CH_e = & CH_{e,0}(\alpha_e, Ma, C_{SP}, \eta_K) \\
& + CH_{e,\eta}(\alpha_e, Ma, C_{SP}, \eta, \eta_K) \\
& + CH_{e,\eta_{TT}}(\alpha_e, Ma, \eta_{TT}, \eta_K) \\
& + CH_{e,\dot{\alpha}}(\eta_K) \dot{\alpha} \bar{c} / TAS \\
& + CH_{e,\dot{\eta}}(\eta_K) \dot{\eta} \bar{c} / TAS \\
& + CH_{e,\beta}(\beta_e, \eta_K) \\
& + CH_{e,GE}(\eta, \eta_K) \sigma_e
\end{aligned} \tag{5.11}$$

The parameters that contribute to CH_e are α_e (angle of attack at the elevator), Ma (Mach number), C_{SP} (slipstream coefficient), η (elevator position), η_K (flaps position), η_{TT} (elevator trim tab position), $\dot{\alpha}_e$ (rate of change of α_e , $\dot{\eta}$ (elevator deflection rate), β (yaw angle of attack), and σ_e (ground effect influence parameter). Of these, Ma , η , η_K , η_{TT} , $\dot{\eta}$ and β are straightforward. The remainders require some further calculations.

α_e is adjusted to include the effect of aircraft pitching. As the aircraft is pitching up, the tail is moving downward relative to the incoming air stream, so the angle of attack is effectively increased. Although α_e is an improved approximation over using α for the horizontal stabiliser angle of attack, effects such as downwash from the wing have not been included. Later in this section, it will be seen that the effect of wing-induced downwash at the tail is accounted for through an additional aerodynamic hinge moment coefficient, $C_{e,0,0}$. The local angle of attack at the elevator is calculated according to Equations (5.12) through (5.14). In Equation (5.13), Q is the pitch rate of the aircraft in rad/sec , and the tail length, L_t , is $19.6[ft]$, as measured from the wing aerodynamic centre to the aerodynamic centre of the tail from a 3-view diagram in Jane's All the World's Aircraft 2005 -2006 edition. In the following equations, the

variables have been converted to radians, feet, and seconds units. $\dot{\alpha}_e$ is calculated as the time derivative of α_e .

$$V_{x,e} = T A S \cos(\beta) \cos(\alpha) \quad (5.12)$$

$$V_{z,e} = T A S \cos(\beta) \sin(\alpha) + Q L_t \quad (5.13)$$

$$\alpha_e = \text{atan} \left(\frac{V_{z,e}}{V_{x,e}} \right) \quad (5.14)$$

C_{SP} is the slipstream coefficient, and its presence in Equation (5.11) is meant to capture the effects of engine jet or propwash contacting the elevator body. This effect is especially important for aircraft with large turboprop engines upstream of the elevator, such as the DeHavilland DCH-8. However, the CJ1 has small jet engines located well away from the streamlines that interact with the elevator, so all C_{SP} terms can be dropped from Equation (5.11).

σ_e is the ground effect influence factor. When the CJ1 is flying in ground effect, such as during takeoff and landing manoeuvres, ground effect will cause a significant change in the circulation of airflow about the wings. Downwash at the tail should be greatly reduced, so that the effective angle of attack at the elevator body is increased. The ground effect influence factor is calculated according to Equation (5.15) (Weiss et al, 1998). In this equation, h_e is the height of the elevator above the ground, and s is the wing semi-span, 23.5[ft] (Jane's, 2006).

$$\sigma_e = 1 - \tanh(4h_e/2s) \quad (5.15)$$

Before any parameters are identified, it is important to have an understanding of the underlying physics. It is important to predict the expected contribution of each term of Equation (5.11) in direction and relative magnitude. The following paragraphs provide some discussion of the aerodynamics about the elevator control surface. A cross-section diagram of the horizontal stabiliser is given in Figure 5.7. Let us step through each of the parts of Equation (5.11) and discuss the hinge moment contribution of the parameters of each. The first is $CH_{e,0}$, which for the CJ1 has parts α_e , Ma , η_K , and a naught term, as represented in Equation (5.16).

$$CH_{e,0} = C_{e,0,0} + C_{e,0,\alpha_e}\alpha_e + C_{e,0,Ma}Ma + C_{e,0,\eta_K}\eta_K \quad (5.16)$$

Increasing α_e will reasonably cause a negative hinge moment, as this will tend to push the elevator upwards. The α_e effect is expected to be relatively strong, and $C_{e,0,\alpha_e}$ should be negative. Increasing Mach number will move the centre of pressure backwards on the horizontal stabiliser, which will increase the magnitude of aerodynamic hinge moments already present on the elevator. The Mach number effect is expected to be relatively weak, and it is difficult to predict the sign of $C_{e,0,Ma}$. Increasing flap position will increase downwash at the horizontal stabiliser, which is equivalent to decreasing α_e . Since the α_e effect is negative, the η_K effect should be opposite, and therefore increasing η_K will increase MH_e . Flap effect is expected to be relatively weak, and the sign of $C_{e,0,\eta_K}$ should be positive. Finally, the naught term is effectively the effect of increasing dynamic pressure on MH_e . Increasing dynamic pressure will result in a smaller required lift coefficient at the wing to maintain level flight. The

Figure 5.7: Elevator aerodynamic hinge moment definition

small wing lift coefficient results in less wing-induced downwash at the tail, which effectively increases the angle of attack at the horizontal stabiliser. By this reasoning, the $C_{e,0,0}$ parameter actually captures an α_e effect. The wind downwash effect modelled in this way because it is simpler to identify the effect independently rather than to try to predict the actual downwash produced by the wing through additional equations.

The second part of Equation (5.11) is the contribution from the elevator position. $CH_{e,\eta}$ is broken down into components in Equation (5.17).

$$CH_{e,\eta} = C_{e,\eta,\eta}\eta + C_{e,\eta,\alpha_e}(\eta)(\alpha_e) + C_{e,\eta,Ma}(\eta)(Ma) + C_{e,\eta,\eta_K}(\eta)(\eta_K) \quad (5.17)$$

Firstly, increasing elevator position will cause an increasingly negative hinge moment, which provides a strong negative aerodynamic feedback to elevator motion. This effect will be very strong, and the sign of $C_{e,\eta,\eta}$ will be negative. It is also possible that other effects, such as flow separation, will require a non-linear fit be used, so that the $C_{e,\eta,\eta}$ term will become more complicated. The hinge moment caused by elevator position may be effected by α_e and Ma , and η_K , as each of these change the static pressure distribution around the horizontal stabiliser. Increasing α_e and η together may trigger separation earlier than increasing either separately, so such nonlinearities must be investigated. Increasing Ma will amplify the hinge moments caused by η . This effect is expected to be relatively weak, and the sign of $C_{e,\eta,Ma}$ is expected to be negative, in line with $C_{e,\eta,\eta}$. Increasing flap position will reduce the effective α_e , and this effect is expected to be small and of uncertain sign.

Next we have the $CH_{e\eta_{TT}}$ contribution, expanded in Equation (5.18).

$$CH_{e,\eta_{TT}} = \begin{aligned} &C_{e,\eta_{TT},\eta_{TT}}\eta_{TT} + C_{e,\eta_{TT},\alpha_e}(\eta_{TT})(\alpha_e) \\ &+ C_{e,\eta_{TT},Ma}(\eta_{TT})(Ma) + C_{e,\eta_{TT},\eta_K}(\eta_{TT})(\eta_K) \end{aligned} \quad (5.18)$$

Increasing η_{TT} position will cause a negative hinge moment on the elevator, and this effect will be relatively strong. The effects of α_e , Ma and η_K on the $CH_{e,\eta_{TT}}$ sensitivity coefficient will be analogous to their effect on $CH_{e,\eta}$.

The $CH_{e,\dot{\alpha}_e}$ and $CH_{e,\dot{\eta}}$ are aerodynamic effects. These represent the hinge moments caused by the amount of displacement α_e and η go through in the time it takes for the average fluid particle to travel over the elevator surface, $\bar{c}_e/TAS$. Both of these effects are sensitive to the circulation pattern about the elevator, so that the effect of η_K is included in each. $CH_{e,\dot{\alpha}_e}$ and $CH_{e,\dot{\eta}}$ are expanded in Equations (5.19) and (5.20), respectively. The primary terms are relatively weak, and negative in sign. The terms including η_K are to be even weaker, and uncertain in sign.

$$CH_{e,\dot{\alpha}_e} = C_{e,\dot{\alpha}_e,\dot{\alpha}_e}\dot{\alpha}_e + C_{e,\dot{\alpha}_e,\eta_K}(\dot{\alpha}_e)(\eta_K) \quad (5.19)$$

$$CH_{e,\dot{\eta}} = C_{e,\dot{\eta},\dot{\eta}}\dot{\eta} + C_{e,\dot{\eta},\eta_K}(\dot{\eta})(\eta_K) \quad (5.20)$$

Sideslip effects are taken into account by the $CH_{e,\beta}$ term, as expanded in Equation (5.21).

$$CH_{e,\beta} = C_{e,\beta,\beta}\beta + C_{e,\beta,\eta_K}(\beta)(\eta_K) \quad (5.21)$$

Sideslip may have an effect because it will cause an increase in the lift generated by the vertical stabiliser, and therefore there will be an associated circulation pattern.

Since the horizontal stabiliser is placed at the top of the vertical stabiliser in a “T-tail” configuration, it acts like an end-wall to the vertical stabiliser. The high-pressure side of the flow about the vertical stabiliser will cause the neighboring portion of the elevator to lift, while the low-pressure side of the flow will pull down the neighboring portion of the elevator. If one of these two competing effects is dominant, then the elevator hinge moment will be sensitive to sideslip. The main trend of $CH_{e,\beta}$ with β is likely to be nonlinear, as it should be an even function about $\beta = 0$. As usual, the effect of flaps is included in the analysis.

Finally, $CH_{e,GE}$ incorporates the aerodynamic effects of flight in ground effect. In the ground effect condition, the wing trailing vortices are mirrored below ground level by virtual vortices to satisfy the boundary condition of no air flow through the ground plane. The virtual vortices counteract the wing trailing vortices, and reduce the amount of downwash at the horizontal stabiliser. Therefore, flight in ground effect will cause an angle of attack that effectively increases at the horizontal stabiliser. If the elevator hinge moment is sensitive to α_e , then there should also be a trend with σ_e . The ground effect on the aerodynamic hinge moment is potentially sensitive to η and η_K , so these terms are included in the $CH_{e,GE}$ definition in Equation (5.22).

$$CH_{e,GE} = C_{e,GE,0}\sigma_e + C_{e,GE,\eta}(\sigma_e)(\eta) + C_{e,GE,\eta_K}(\sigma_e)(\eta_K) \quad (5.22)$$

5.2.8 Stick Pusher

The stick pusher is a device attached to the elevator control system that shakes the control column when the aircraft is at a high angle of attack. The force produced

by this device is not the result of an aerodynamic hinge moment or a natural force mechanism of the elevator control system. The stick pusher is a programmable black box. The stick pusher model used here has been developed by Josh Brinkerhoff, a student worker at the NRC.

The stick pusher device model has a sinusoidal force output with a magnitude of 1.88 [lb] and a frequency of 33 [Hz]. The stick pusher is activated for any of the conditions listed in Table 6.2.

5.3 Column - Elevator Gearing and Saturation

For the purposes of propriety, the numerical values from the control column to elevator gearing relationship have been omitted from this thesis. The numbers have been replaced with the symbol #.

Data from the ground control sweep (GCS) manoeuvres provides measurements of the elevator surface and control column positions under negligible aerodynamic load and minimal effects from system friction and inertia. Therefore, 7704 data points from four ground control sweep manoeuvres listed in Table D.7 is combined for the η versus δ_c regression. The data used for this regression is limited to the range where the elevator position is not saturated. This filtering is done on the basis of column position, passing the data with column positions between $\delta_c = \#.29$ and $\delta_c = \#.30$ degrees. These limits have been defined by bisection method of maximisation of the R-squared correlation of the polynomial fit to the η versus δ_c trend. The regression of this trend is presented in Figure 5.8. The coefficients of the relationship proposed

in Equation (5.3), repeated here as Equation (5.23), are defined in Equation (5.24).

$$\eta_{cmd} = A_0 + A_1\delta_c + A_2\delta_c^2 \quad (5.23)$$

$$\begin{aligned} A_0 &= \#.31 \pm \#.32 \text{ [deg]} & (95\%) \\ A_1 &= \#.33 \pm \#.34 & (95\%) \\ A_2 &= \#.35 \pm \#.36 \text{ [deg}^{-1}] & (95\%) \end{aligned} \quad (5.24)$$

The elevator position saturation limits are determined by calculating η_{cmd} according to Equations (5.23) and (5.24) for the entire range of δ_c positions measured during the GCS manoeuvres. Saturation limits are applied to this trend, and bisection method is used to determine the optimal values for the limits, minimising the standard deviation of the error. The saturation limits η_{min} and η_{max} are thus defined as #.37 and #.38 degrees, respectively.

5.4 Elevator Cable and Column Spring

The elevator cable and column spring parameters are identified using ground control sweep manoeuvre data. The greatest range of elevator cable stretch occurs while the elevator position is saturated, so that increases in control column input merely extend the elevator cable. The control column spring is engaged only when the column is pulled back well beyond the position where the elevator becomes saturated, so that if the column spring is engaged, the elevator cable is always in a state of high displacement. Because the column spring is never engaged independently of the elevator cable being significantly displaced, the parameters governing the spring behaviours must be identified together.

Table 5.1: Stick shaker engagement conditions

η_K [deg]	α [deg]	Stick Shaker Engaged
any	$> 11.749 - 0.034107\dot{\alpha}$	yes
< 13	> 11.25	yes
≥ 13 & < 31	> 10.93	yes
≥ 31	> 10.09	yes

Figure 5.8: Regression of η versus δ_c , ground control sweep manoeuvre

For the purpose of modelling, the control column force is converted into control column hinge moment, according to Equation (5.25) and Figure 5.9.

$$M_c = F_c \cdot \frac{30.5}{12} ft \quad (5.25)$$

During the GCS manoeuvre, friction and inertia forces of the column body are negligible, so that the total column moment is approximately equal to the sum of the contributions of the elevator cable moment and the column spring moment, as given in Equation (5.26).

$$M_c \simeq M_{c,cbl} + M_{c,colspr} \quad (5.26)$$

Figure 5.10 displays M_c versus δ_{cbl} . It is apparent from this graph that the uncertainty in the calculation of δ_{cbl} is much larger than the uncertainty of M_c . This is because δ_{cbl} contains uncertainty from both η and η_{cmd} , and this becomes large compared to the difference between these two signals.

The regression of M_c with δ_{cbl} is performed on the data excluding the region of δ_c where the column spring is engaged. To establish the limit of $\delta_{c,min}$ to be used, bisection method is used to maximise the R-squared correlation of the $M_{c,cbl}$ fit to the data. This results in the value of $\delta_{c,min}$ being identified as -4.72 degrees. The regression of M_c with δ_{cbl} for all $\delta_c > \delta_{c,min}$ is displayed in Figure 5.11. The resulting elevator cable spring parameters are given in Equation (5.28).

$$M_{c,cbl} = C_1 \delta_{cbl} + C_2 \delta_{cbl}^2 \quad (5.27)$$

$$\begin{aligned} C_1 &= -113 \pm 25.2 \text{ [ft} \cdot \text{lb} \cdot \text{deg}^{-1}] \text{ (95\%)} \\ C_2 &= 12.8 \pm 2.38 \text{ [ft} \cdot \text{lb} \cdot \text{deg}^{-2}] \text{ (95\%)} \end{aligned} \quad (5.28)$$

Figure 5.9: Forces acting on the control column (a), and their equivalent moments (b)

Figure 5.10: M_c versus δ_{cbl} , ground control sweep manoeuvre

To identify the column spring coefficient, the contribution of the elevator cable to the column moment, $M_{c,cb}$ is removed from measured M_c so that the remaining column moment is due to column spring moment, $M_{c,colspr}$, only. Regression is then performed for $M_{c,colspr}$ versus δ_{colspr} for all $\delta_c < \delta_{c,min}$. The results of this regression are displayed in Figure 5.12 and Equation (5.30).

$$M_{c,colspr} = D_1 \delta_{colspr} \quad (5.29)$$

$$D_1 = 400 \pm 83.0 \text{ [ft} \cdot \text{lb} \cdot \text{deg}^{-1}] \text{ (95\%)} \quad (5.30)$$

5.5 Column Friction and Inertia

Control column friction and inertia parameters are identified using data from frequency sweeps performed in the Flight Research Laboratory hangar, comprising 70964 points as listed in Table D.8. During this manoeuvre, contributions to the column hinge moment come from the elevator cable, column inertia, and friction. The column is never deflected enough to contact its limit of motion, so the column spring is never engaged.

Multiple regression is used to identify column inertia, J_c , and the coulomb friction magnitude, $M_{c,clmb}$. For the purpose of estimating $M_{c,clmb}$, a smoothly-transitioning step function is calculated from $\dot{\delta}_c$, according to Equation (5.31). This function produces a constant -1 for $\dot{\delta}_c < -5 \text{ } [\frac{deg}{sec}]$ and 1 for $\dot{\delta}_c > 5 \text{ } [\frac{deg}{sec}]$, with a smooth curve between these limits.

$$\Delta_c = \tanh\left(\frac{2\pi}{5} \dot{\delta}_c\right) \quad (5.31)$$

Figure 5.11: M_c versus δ_{cbl} regression Figure 5.12: $M_{c,colspr}$ versus δ_{colspr} regression

The control column moment regression definition is given in Equation (5.32). Elevator cable stretch is included as a regressor so that column inertia and friction moments can be isolated from elevator cable moment. The resulting regression plots for $M_{c,fric}$ versus $\dot{\delta}_c$ and $M_{c,inertia}$ versus $\ddot{\delta}_c$ are presented in Figures 5.13 and 5.14, respectively.

$$[\dot{\Delta}_c \quad \ddot{\delta}_c \quad \delta_{cbl}] [M_{c,clmb} \quad J_c \quad K]^T = M_c \quad (5.32)$$

To establish the column friction parameters $M_{c,stic}$, $\dot{\delta}_{c,breakout}$ and $\dot{\delta}_{c,trans}$, the data presented in Figure 5.13 is analysed more closely for the slower column deflection rates. Because the friction trend is one of odd symmetry, the data on the bottom half of the graph may be rotated to the top half, so that the close-up examination will result in an averaging of the positive and negative friction moment trends. Next, the density of data points is calculated for cells forming a grid with $\Delta\dot{\delta}_c = 0.5 \left[\frac{deg}{sec} \right]$ and $\Delta M_{c,fric} = 0.125 [ft \cdot lb]$. This data density distribution is represented by a contour plot for density thresholds of 20, 30 and 40 points per cell, out of a total 70946 points, overlying the data points themselves in Figure 5.15.

The shape of the contour for the density of 30 points/cell is used as a guide for laying out the maximum stiction force and the stiction to sliding friction transition. The proposed friction trend is presented as the thick solid line. The identified column friction and inertia parameters are listed in Equations (5.33) and (5.34), respectively.

$$\begin{aligned} M_{c,stic} &= 8 & [ft \cdot lb] \\ M_{c,clmb} &= 4.72 & [ft \cdot lb] \\ \dot{\delta}_{c,breakout} &= 0.6 & [deg \cdot sec^{-1}] \\ \dot{\delta}_{c,trans} &= 8 & [deg \cdot sec^{-1}] \end{aligned} \quad (5.33)$$

Figure 5.13: $M_{c,fric}$ versus $\dot{\delta}_c$ regression

Figure 5.14: $M_{c,inertia}$ versus $\ddot{\delta}_c$ regression

$$J_c = 0.0714 \pm 3.93E-3 \text{ [ft} \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{deg}^{-2}] \text{ (95\%)} \quad (5.34)$$

5.6 Elevator Friction and Inertia

Elevator friction and inertia parameters are identified using the same data from frequency sweeps as is used for the control column friction and inertia identification. First, we must identify what hinge moments are acting on the elevator body. The contributions to the total elevator hinge moment in the absence of aerodynamic forces are from the elevator cable, elevator inertia, and friction. Therefore, if the elevator cable moment contribution is known, it is possible to determine the friction and inertia hinge moment contributions.

The moment applied to the elevator body from the elevator cable is related to the moment of the elevator cable on the control column, through the column - elevator gearing. The torque transfer through the gearing is the inverse of the gearing ratio, as described in Equation (5.35). Equation (5.37) describes this inverse gear ratio as a function of the parameters of Equation (5.3), repeated here as Equation (5.36).

$$M_{e, cbl} = M_{c, cbl} \frac{\partial \delta_c}{\partial \eta} \quad (5.35)$$

$$\eta_{cmd} = A_0 + A_1 \delta_c + A_2 \delta_c^2 \quad (5.36)$$

$$\frac{\partial \delta_c}{\partial \eta} = \left(\frac{\partial \eta}{\partial \delta_c} \right)^{-1} = \frac{1}{A_1 + 2A_2 \delta_c} \quad (5.37)$$

The elevator cable moment acting on the control column during the frequency sweep manoeuvres is calculated according to Equation (5.5), repeated here as Equation (5.38).

$$M_{c,cbl} = C_1 \delta_{cbl} + C_2 \delta_{cbl}^2 \quad (5.38)$$

This moment is then translated into elevator hinge moment contribution by Equation (5.35). Finally, this elevator hinge moment is regressed against $\dot{\eta}$ and $\ddot{\eta}$ to determine the elevator mass moment of inertia and friction profile.

To determine the magnitude of coulomb friction, the elevator deflection rate is translated into a hyperbolic tangent according to Equation (5.39). The elevator hinge moment regression definition is then given in Equation (5.40). The resulting $M_{e,fric}$ versus $\dot{\eta}$ and $M_{e,inertia}$ versus $\ddot{\eta}$ graphs are presented in Figures 5.16 and 5.17, respectively.

$$\dot{\Delta}_e = \tanh\left(\frac{2\pi}{5}\dot{\eta}\right) \quad (5.39)$$

$$\begin{bmatrix} \dot{\Delta}_e & \ddot{\eta} \end{bmatrix} \begin{bmatrix} M_{e,clmb} & J_e \end{bmatrix}^T = M_e \quad (5.40)$$

Finally, the elevator stiction and transition parameters $M_{e,stic}$, $\dot{\eta}_{stic}$ and $\dot{\eta}_{trans}$ are established in the same fashion as for the equivalent control column parameters. The contour plot of the elevator friction profile and the proposed elevator friction model is presented in Figure 5.18. The identified values of the elevator friction and inertia parameters are listed in Equations (5.41) and (5.42), respectively.

$$\begin{aligned} M_{e,stic} &= 2.4 & [ft \cdot lb] \\ M_{e,clmb} &= 1.24 & [ft \cdot lb] \\ \dot{\eta}_{breakout} &= 0.6 & [deg \cdot sec^{-1}] \\ \dot{\eta}_{trans} &= 7 & [deg \cdot sec^{-1}] \end{aligned} \quad (5.41)$$

Figure 5.15: $M_{c,fric}$ versus $\dot{\delta}_c$ zoom view including data density contours

Figure 5.16: $M_{e,fric}$ versus $\dot{\eta}$ regression

Figure 5.17: $M_{e,inertia}$ versus $\ddot{\eta}$ regression

Figure 5.18: $M_{e,fric}$ versus $\dot{\eta}$ zoom view including data density contours

$$J_e = 7.58E-3 \pm 4.78E-4 \text{ [ft} \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{deg}^{-2}] \text{ (95\%)} \quad (5.42)$$

5.7 Aerodynamic Hinge Moments

Aerodynamic hinge moment sensitivities, as defined in Equation (5.11) and repeated here as Equation (5.43), are identified using data from a large number of flight manoeuvres listed in Table D.9. The data totals 368624 points.

$$\begin{aligned} CH_e &= CH_{e,0}(\alpha_e, Ma, C_{SP}, \eta_K) \\ &+ CH_{e,\eta}(\alpha_e, Ma, C_{SP}, \eta, \eta_K) \\ &+ CH_{e,\eta_{TT}}(\alpha_e, Ma, \eta_{TT}, \eta_K) \\ &+ CH_{e,\dot{\alpha}}(\eta_K) \dot{\alpha} \bar{c} / TAS \\ &+ CH_{e,\dot{\eta}}(\eta_K) \dot{\eta} \bar{c} / TAS \\ &+ CH_{e,\beta}(\beta_e, \eta_K) \\ &+ CH_{e,GE}(\eta, \eta_K) \sigma_e \end{aligned} \quad (5.43)$$

The main flight manoeuvre used is the 2311 manoeuvre, and 140 of these manoeuvres are used to provide an adequate variation in flight conditions, flap settings and trim settings for the regression analysis. As a large number of parameters is required to fully describe the aerodynamic hinge moment phenomena, a large quantity of data is needed to produce statistically meaningful parameter estimates. The 2311 manoeuvre data is combined with data from flap change dynamics, pitch trim sweep, steady state sideslip and flight in ground effect manoeuvres. A full listing of the data used for the aerodynamic hinge moment analysis is given in Appendix D.

The elevator hinge moment to be regressed is calculated according to the rule that the sum of all moments acting on the elevator must be zero, as represented in Equations (5.44) and (5.45).

$$\Sigma M_e = MH_e + M_{e,cbf} + M_{e,fric} + M_{e,inertia} = 0 \quad (5.44)$$

$$MH_e + M_{e,fric} + M_{e,inertia} = -M_{e,cbl} \quad (5.45)$$

In Equation (5.45), the friction and inertia terms are grouped together with MH_e , as the friction and inertia effects can be taken into account by including $\dot{\Delta}_e$ and $\ddot{\eta}$ as regressors. $M_{e,cbl}$ is calculated according to Equations (5.3), (5.5) and (5.35), repeated here as Equations (5.46), (5.47) and (5.48), respectively.

$$\eta_{cmd} = A_0 + A_1\delta_c + A_2\delta_c^2 \quad (5.46)$$

$$M_{c,cbl} = C_1\delta_{cbl} + C_2\delta_{cbl}^2 \quad (5.47)$$

$$M_{e,cbl} = M_{c,cbl} \frac{\partial \delta_c}{\partial \eta} \quad (5.48)$$

An initial regression is performed that includes all of the terms listed in Equations (5.16) through (5.22), repeated here as Equations (5.49) through (5.55), respectively, and $\dot{\Delta}_e$ and $\ddot{\eta}$ as regressors.

$$CH_{e,0} = C_{e,0,0} + C_{e,0,\alpha_e}\alpha_e + C_{e,0,Ma}Ma + C_{e,0,\eta_K}\eta_K \quad (5.49)$$

$$CH_{e,\eta} = C_{e,\eta,\eta}\eta + C_{e,\eta,\alpha_e}(\eta)(\alpha_e) + C_{e,\eta,Ma}(\eta)(Ma) + C_{e,\eta,\eta_K}(\eta)(\eta_K) \quad (5.50)$$

$$CH_{e,\eta_{TT}} = \begin{aligned} &C_{e,\eta_{TT},\eta_{TT}}\eta_{TT} + C_{e,\eta_{TT},\alpha_e}(\eta_{TT})(\alpha_e) \\ &+ C_{e,\eta_{TT},Ma}(\eta_{TT})(Ma) + C_{e,\eta_{TT},\eta_K}(\eta_{TT})(\eta_K) \end{aligned} \quad (5.51)$$

$$CH_{e,\dot{\alpha}_e} = C_{e,\dot{\alpha}_e,\dot{\alpha}_e}\dot{\alpha}_e + C_{e,\dot{\alpha}_e,\eta_K}(\dot{\alpha}_e)(\eta_K) \quad (5.52)$$

$$CH_{e,\ddot{\eta}} = C_{e,\ddot{\eta},\ddot{\eta}}\ddot{\eta} + C_{e,\ddot{\eta},\eta_K}(\ddot{\eta})(\eta_K) \quad (5.53)$$

$$CH_{e,\beta} = C_{e,\beta,\beta}\beta + C_{e,\beta,\eta_K}(\beta)(\eta_K) \quad (5.54)$$

$$CH_{e,GE} = C_{e,GE,0}\sigma_e + C_{e,GE,\eta}(\sigma_e)(\eta) + C_{e,GE,\eta_K}(\sigma_e)(\eta_K) \quad (5.55)$$

Any of the identified parameters that have an uncertainty equal to or greater than the value of the parameter itself are removed from the subsequent regression. The effects accounted for by these parameters are all weaker than the uncertainty due to the scatter of the data and the bias uncertainties of the associated flight data measurements. Thus, the aerodynamic hinge moment equation may be simplified to include fewer terms specific to the CJ1 elevator control system. The resulting aerodynamic hinge moment equation is presented in Equations (5.56) through (5.60).

$$\begin{aligned} CH_e &= CH_{e,0}(\alpha_e) \\ &+ CH_{e,\eta}(Ma, \eta, \eta_K) \\ &+ CH_{e,\eta_{TT}}(\alpha_e, Ma, \eta_{TT}) \\ &+ CH_{e,\dot{\eta}}\dot{c}/TAS \end{aligned} \quad (5.56)$$

$$CH_{e,0} = C_{e,0,0} + C_{e,0,\alpha_e}\alpha_e \quad (5.57)$$

$$CH_{e,\eta} = C_{e,\eta,\eta}\eta + C_{e,\eta,Ma}(\eta)(Ma) + C_{e,\eta,\eta_K}(\eta)(\eta_K) \quad (5.58)$$

$$CH_{e,\eta_{TT}} = C_{e,\eta_{TT},\eta_{TT}}\eta_{TT} + C_{e,\eta_{TT},\alpha_e}(\eta_{TT})(\alpha_e) + C_{e,\eta_{TT},Ma}(\eta_{TT})(Ma) \quad (5.59)$$

$$CH_{e,\dot{\eta}} = C_{e,\dot{\eta},\dot{\eta}}\dot{\eta} \quad (5.60)$$

It is notable that the aerodynamic damping due to $\dot{\alpha}_e$, the sideslip effects, the mixed $(\eta)(\alpha_e)$ effect, and the flight in ground effect parameters have been completely removed. The aerodynamic damping for the CJ1 elevator has been found to be a strong function of $\dot{\eta}$ only. For the CJ1, the competing effects of β on the elevator hinge moment appear to be roughly equal, so that no significant hinge moment contribution is found. The data supported no evidence of an effect on hinge moment by increasing both η and α_e together. Specifically, no evidence was found that flow separation occurs earlier when both η and α_e are at large values. Finally, the ground effect

influence was found to be too small in comparison to uncertainty in the data. This has much to do with the fact that no hands-free events occur during flight in ground effect manoeuvres because of the safety concerns for the very low altitude flight condition. No hands-free events means that the column force bias cannot be corrected, and the bias uncertainty becomes very large according to the analysis performed in Appendix B.

With the reduced aerodynamic hinge moment coefficient given in Equation (5.56), the regression definition equation may be built. The elevator hinge moment from the cable is normalised by the surface area and reference chord of the elevator according to Equation (5.61) so that the following investigation is in non-dimensional form. The elevator cable hinge moment is not divided by dynamic pressure because dynamic pressure may become very small for manoeuvres on the ground, and it is desirable to keep the option of including ground manoeuvres in the data. Dynamic pressure, then, is multiplied into each of the aerodynamic regressors. For plotting purposes, both the aerodynamic hinge moment and aerodynamic regressors are divided by dynamic pressure so that the graphs are truly non-dimensional. The regression definition is presented in Equations (5.62) through (5.68).

$$P_D C H_e = -\frac{2M_{e,cbl}}{S_e \bar{c}_e} \quad (5.61)$$

$$\begin{bmatrix} Reg_0 & Reg_\eta & Reg_{\eta_{TT}} & P_D \dot{\eta} \bar{c}/TAS & \dot{\Delta}_e & \ddot{\eta} & 1 \end{bmatrix} \begin{bmatrix} C_{e,0} \\ C_{e,\eta} \\ C_{e,\eta_{TT}} \\ C_{e,\dot{\eta}} \\ M_{e,clmb} \\ J_e \\ Bias \end{bmatrix} = P_D CH_e \quad (5.62)$$

$$Reg_0 = P_D \cdot \begin{bmatrix} 1 & \alpha_e \end{bmatrix} \quad (5.63)$$

$$Reg_\eta = P_D \cdot \begin{bmatrix} \eta & \eta_i & \eta_s & (\eta)(Ma) & (\eta)(\eta_K) \end{bmatrix} \quad (5.64)$$

$$Reg_{\eta_{TT}} = P_D \cdot \begin{bmatrix} \eta_{TT} & \eta_{TT,s} & (\eta_{TT})(\alpha_e) & (\eta_{TT})(Ma) \end{bmatrix} \quad (5.65)$$

$$C_{e,0} = \begin{bmatrix} C_{e,0,0} & C_{e,0,\alpha_e} \end{bmatrix}^T \quad (5.66)$$

$$C_{e,\eta} = \begin{bmatrix} C_{e,\eta,\eta} & C_{e,\eta,\eta_i} & C_{e,\eta,\eta_s} & C_{e,\eta,Ma} & C_{e,\eta,\eta_K} \end{bmatrix}^T \quad (5.67)$$

$$C_{e,\eta_{TT}} = \begin{bmatrix} C_{e,\eta_{TT},\eta_{TT}} & C_{e,\eta_{TT},\eta_{TT,s}} & C_{e,\eta_{TT},\alpha_e} & C_{e,\eta_{TT},Ma} \end{bmatrix}^T \quad (5.68)$$

The terms η_i , η_s and $\eta_{TT,s}$ exist to allow for nonlinearities in the elevator and elevator tab contributions to the aerodynamic hinge moment. These parameters are defined in Equations (5.69) through (5.71), respectively. The first term, η_i , is included to account for an increase in the sensitivity of MH_e to η for η values below η_1 . This increased sensitivity is likely due to the elevator producing a greater flow disturbance when its deflection becomes large with respect to the airflow boundary layer present on the horizontal stabiliser. While the elevator deflections are small, the elevator is to a certain degree blanketed by the boundary layer, but when the deflections become larger, the boundary layer and airflow patterns are more significantly disturbed, and so larger hinge moments are produced. The second term, η_s , accounts for flow separation at large negative η positions, below η_2 . Similarly, the $\eta_{TT,s}$ term

accounts for flow separation for large η_{TT} deflections. The values η_1 , η_2 , and $\eta_{TT,1}$ are all established using bisection method to minimise the standard deviation of the error $MH_e - MH_{e,mdl}$. This optimisation results in the values $\eta_1 = -4$ [deg], $\eta_2 = -10$ [deg] and $\eta_{TT,1} = 11$ [deg].

$$\eta_i = (\eta - \eta_1)^2, \quad \eta < \eta_1 \quad (5.69)$$

$$\eta_s = (\eta - \eta_2)^2, \quad \eta < \eta_2 \quad (5.70)$$

$$\eta_{TT,s} = (\eta_{TT} - \eta_{TT,1})^2, \quad \eta_{TT} > \eta_{TT,1} \quad (5.71)$$

The terms $M_{e,clmb}$ and J_e in Equation (5.62) do not represent the actual values of the equivalent parameters identified in Section 5.6. They will be off by a factor of $2S_e^{-1}\bar{c}_e^{-1}$. In any case, these parameters are included only to isolate the aerodynamic hinge moment parameter identification process from the elevator friction and inertia. The uncertainty of the flight data is larger than that for the ground frequency sweep data used in Section 5.6, and therefore the estimates for $M_{e,clmb}$ and J_e produced here are not as reliable as those produced using the frequency sweep data.

The *Bias* term in Equation (5.62) is included to account for a combination of aerodynamic hinge moment bias produced by gravity acting on any imbalance in the mass of the elevator body.

The graphs produced in the identification of the aerodynamic hinge moment parameters are presented in Figures 5.19 through 5.32. The identified parameter values are listed in Equation (5.72).

Figure 5.19: Regression of $P_D CH_{e,0,0}$ versus P_D for $C_{e,0,0}$ identification

Figure 5.20: Regression of $CH_{e,0,\alpha_e}$ versus α_e for $C_{e,0,\alpha_e}$ identification

Figure 5.21: Regression of $CH_{e,\eta,\eta}$ versus η for $C_{e,\eta,\eta}$ identification

Figure 5.22: Regression of CH_{e,η,η_i} versus η_i for C_{e,η,η_i} identification

Figure 5.23: Regression of CH_{e,η_s} versus η_s for C_{e,η_s} identification

Figure 5.24: $CH_{e,\eta}$ versus η for total η aerodynamic hinge moment trend

Figure 5.25: Regression of $CH_{e,\eta Ma}$ versus ηMa for $C_{e,\eta Ma}$ identification

Figure 5.26: Regression of $CH_{e,\eta \eta_K}$ versus $\eta \eta_K$ for $C_{e,\eta \eta_K}$ identification

Figure 5.27: Regression of $CH_{e,\eta_{TT},\eta_{TT}}$ versus η_{TT} for $C_{e,\eta_{TT},\eta_{TT}}$ identification

Figure 5.28: Regression of $CH_{e,\eta_{TT},\eta_{TT,s}}$ versus $\eta_{TT,s}$ for $C_{e,\eta_{TT},\eta_{TT,s}}$ identification

Figure 5.29: $CH_{e,\eta_{TT}}$ versus η_{TT} for total η_{TT} aerodynamic hinge moment trend

Figure 5.30: Regression of $CH_{e,\eta_{TT},\alpha_e}$ versus $\eta_{TT}\alpha_e$ for C_{e,η_{TT},α_e} identification

Figure 5.31: Regression of $CH_{e,\eta_{TT},Ma}$ versus $\eta_{TT}Ma$ for $C_{e,\eta_{TT},Ma}$ identification

Figure 5.32: Regression of $CH_{e,\dot{\eta}}$ versus $\dot{\eta} \bar{c}_e / TAS$ for $C_{e,\dot{\eta}}$ identification

$$\begin{aligned}
C_{e,0,0} &= 5.44E-5 \quad \pm 1.73E-5 \quad (95\%) \\
C_{e,0,\alpha_e} &= -5.26E-4 \quad \pm 2.27E-4 \quad (95\%) \\
C_{e,\eta,\eta} &= -1.68E-3 \quad \pm 1.42E-4 \quad (95\%) \\
C_{e,\eta,\eta_i} &= 6.37E-3 \quad \pm 1.05E-3 \quad (95\%) \\
C_{e,\eta,\eta_s} &= -1.79E-2 \quad \pm 3.34E-3 \quad (95\%) \\
C_{e,\eta,Ma} &= -1.86E-3 \quad \pm 5.41E-4 \quad (95\%) \\
C_{e,\eta,\eta_K} &= -5.80E-4 \quad \pm 2.70E-4 \quad (95\%) \\
C_{e,\eta_{TT},\eta_{TT}} &= -1.85E-3 \quad \pm 1.09E-4 \quad (95\%) \\
C_{e,\eta_{TT},\eta_{TT,s}} &= 4.24E-3 \quad \pm 2.41E-3 \quad (95\%) \\
C_{e,\eta_{TT},\alpha_e} &= 1.18E-3 \quad \pm 7.15E-4 \quad (95\%) \\
C_{e,\eta_{TT},Ma} &= -2.11E-3 \quad \pm 5.02E-4 \quad (95\%) \\
C_{e,\dot{\eta},\dot{\eta}} &= -1.08E-2 \quad \pm 2.50E-3 \quad (95\%) \\
Bias &= 0.191 \quad \pm 0.260 \quad (95\%)[lb \cdot ft^2]
\end{aligned} \tag{5.72}$$

As expected, the dominant contributions to MH_e are from η and η_{TT} . As seen in Figures 5.24 and 5.29, these trends contribute about $7E-4$ and $9E-4$, respectively, to the non-dimensional hinge moment coefficient, CH_e . The remaining trends contribute on the order of $2E-4$ to $3E-4$ each.

5.8 CJ1 Elevator Model Summary

The CJ1 elevator model is summarised here, presented in reference to the block model diagram, Figure 5.4, repeated here as Figure 5.33.

The Elevator Gearing component has input δ_c and output η_{cmd} , governed by Equation (5.73).

$$\eta_{cmd} = A_0 + A_1\delta_c + A_2\delta_c^2 \tag{5.73}$$

Elevator cable stretch is calculated by according to Equation (5.74), where η is a system state, taken from the output η_{mdl} of the previous timestep.

$$\delta_{cbl} = \eta - \eta_{cmd} \tag{5.74}$$

Figure 5.33: CJ1 elevator control model block diagram

The Cable Spring model component receives δ_{cbl} and has outputs $M_{c,cbl}$ and $M_{e,cbl}$. Elevator cable stretch is used to calculate the cable spring moment on the control column by Equation (5.75).

$$M_{c,cbl} = C_1 \delta_{cbl} + C_2 \delta_{cbl}^2 \quad (5.75)$$

The cable stretch moment is translated through to the elevator body via Equations (5.76) and (5.77).

$$M_{e,cbl} = M_{c,cbl} \frac{\partial \delta_c}{\partial \eta} \quad (5.76)$$

$$\frac{\partial \delta_c}{\partial \eta} = \left(\frac{\partial \eta}{\partial \delta_c} \right)^{-1} = \frac{1}{A_1 + 2A_2 \eta} \quad (5.77)$$

The Column Spring has input δ_c and output $M_{c,colspr}$, as governed by Equations (5.78) and (5.79).

$$\delta_{colspr} = \delta_c - \delta_{c,min} \quad (5.78)$$

$$M_{c,colspr} = D_1 \delta_{colspr}, \quad (\delta_{colspr} < 0) \quad (5.79)$$

The moments acting on the column, $M_{c,cbl}$ and $M_{c,colspr}$, are summed and filtered. The filter works by taking the average of the current column moment prediction and the prediction from previous timestep. Next, the control column moment is converted into control column force according to Equation (5.80)

$$F_c = M_c \cdot \frac{12}{30.5} ft \quad (5.80)$$

The final stage for the column force model is the Stick Pusher model component. The stick pusher creates a sine wave force output with a magnitude of 1.88 [lb] and

a frequency of 33 [Hz] when any of the conditions listed in Table 5.2 are satisfied. The stick pusher force component is added to the column force and the output is the modelled F_c .

Returning to the elevator position portion of the model, all the moments acting on the elevator body must be calculated and summed before the elevator deflection acceleration is calculated. These moments include the cable spring moment, $M_{e,cb}$, the elevator friction moment, $M_{e,fric}$, and the aerodynamic hinge moment, MH_e .

The Elevator Friction Model component calculates the friction moment using input $\dot{\eta}_{mdl}$, which is a system state and must be read from the previous timestep. The friction moment is applied according to Figure 5.34. When the friction model is in the stiction state, $M_{e,fric}$ is calculated as the greater of the sum of all external moments acting on the elevator or the maximum stiction moment, $M_{e,stic}$. In this way, the friction model prevents the elevator from moving until the stiction moment is overcome by external moments.

The Aerodynamic Hinge Moment model component is calculated according to Equations (5.81) through (5.86).

$$MH_e = \frac{1}{2} CH_e P_D S_e \bar{c}_e \quad (5.81)$$

$$\begin{aligned} CH_e &= CH_{e,0}(\alpha_e) \\ &+ CH_{e,\eta}(Ma, \eta, \eta_K) \\ &+ CH_{e,\eta_{TT}}(\alpha_e, Ma, \eta_{TT}) \\ &+ CH_{e,\dot{\eta}} \dot{\eta} \bar{c} / TAS \end{aligned} \quad (5.82)$$

$$CH_{e,0} = C_{e,0,0} + C_{e,0,\alpha_e} \alpha_e \quad (5.83)$$

$$CH_{e,\eta} = C_{e,\eta,\eta} \eta + C_{e,\eta,\eta_i} \eta_i + C_{e,\eta,\eta_s} \eta_s + C_{e,\eta,Ma} \eta Ma + C_{e,\eta,\eta_K} \eta \eta_K \quad (5.84)$$

Table 5.2: Stick shaker engagement conditions

η_K [deg]	α [deg]	Stick Shaker Engaged
any	$> 11.749 - 0.034107\alpha$	yes
< 13	> 11.25	yes
≥ 13 & < 31	> 10.93	yes
≥ 31	> 10.09	yes

Figure 5.34: Friction model profile (Weiss et al, 1998)

$$CH_{e,\eta_{TT}} = C_{e,\eta_{TT},\eta_{TT}}\eta_{TT} + C_{e,\eta_{TT},\eta_{TT},s}\eta_{TT,s} + C_{e,\eta_{TT},\alpha_e}\eta_{TT}\alpha_e + C_{e,\eta_{TT},Ma}\eta_{TT}Ma \quad (5.85)$$

$$CH_{e,\dot{\eta}} = C_{e,\dot{\eta},\dot{\eta}}\dot{\eta} \quad (5.86)$$

The Elevator Body Acceleration component receives all the moments acting on the elevator body and produces $\ddot{\eta}_{mdl}$ according to Equation (5.87).

$$\ddot{\eta}_{mdl} = \frac{M_{e,cbl} + M_{e,fric} + MH_e}{J_e} \quad (5.87)$$

Elevator deflection rate, $\dot{\eta}_{mdl}$, is calculated through integration of $\ddot{\eta}_{mdl}$ over the period of one timestep and adding this value to $\dot{\eta}_{mdl}$ from the previous timestep. Integration is used in similar fashion to calculate η_{mdl} using $\dot{\eta}_{mdl}$. Finally, η_{mdl} is subject to the limits of position saturation.

The CJ1 elevator control system model parameters are listed in Table 5.3.

Table 5.3: Citation X elevator model parameter values

Parameter	Value	Uncertainty (95%)
Aircraft Geometry		
S_e	24.7[ft ²]	-
$\bar{c}_e$	1.42[ft]	-
L_t	19.6[ft]	-
s	23.5[ft]	-
Gearing and Spring Rates		
A_0	#.31	$\pm$ #.32
A_1	#.33	$\pm$ #.34
A_2	#.35	$\pm$ #.36
η_{min}	#.37	-
η_{max}	#.38	-
C_1	-113	$\pm$ 25.2
C_2	12.8	$\pm$ 2.38
$\delta_{c,min}$	-4.72	-
D_1	400	$\pm$ 83.0
Friction and Inertia		
$\dot{\eta}_{breakout}$	0.6	-
$\dot{\eta}_{trans}$	7	-
$M_{e,stic}$	2.4	-
$M_{e,clmb}$	1.24	-
J_e	7.58E-3	$\pm$ 4.78E-4
Aerodynamic Sensitivities		
$C_{e,0,0}$	5.44E-5	$\pm$ 1.73E-5
$C_{e,0,\alpha_e}$	-5.26E-4	$\pm$ 2.27E-4
$C_{e,\eta,\eta}$	-1.68E-3	$\pm$ 1.42E-4
C_{e,η,η_i}	6.37E-3	$\pm$ 1.05E-3
C_{e,η,η_s}	-1.79E-2	$\pm$ 3.34E-3
$C_{e,\eta,Ma}$	-1.86E-3	$\pm$ 5.41E-4
C_{e,η,η_K}	-5.80E-4	$\pm$ 2.70E-4
$C_{e,\eta_{TT}\eta_{TT}}$	-1.85E-3	$\pm$ 1.09E-4
$C_{e,\eta_{TT},\eta_{TT,s}}$	4.24E-3	$\pm$ 2.41E-3
C_{e,η_{TT},α_e}	1.18E-3	$\pm$ 7.15E-4
$C_{e,\eta_{TT},Ma}$	-2.11E-3	$\pm$ 5.02E-4
$C_{e,\dot{\eta},\dot{\eta}}$	-1.08E-2	$\pm$ 2.50E-3

Chapter 6

Model Verification

Verification tests are performed on the stand-alone CJ1 elevator model for model match analysis using qualification test guide (QTG) manoeuvres. These include take-offs, landings, stalls, ground effect, manoeuvring and static stability, and ground control sweep manoeuvres. For the verification tests, measured column position, aircraft motion parameters and aerodynamic data are used to simulate elevator position and column force. In a fully integrated simulator only measured column position would be used, and simulated aircraft motion and aerodynamic parameters would form the inputs to the elevator control system model. These limitations must be respected when viewing the outcome of the verification study, as the fully integrated model will incur errors due to the simulated inputs.

Verification is done using a specific set of data, and this is listed in Table 6.1. In general, this data is a unique set that has not been used for the parameter estimation as described in Chapter 5. The exception is that the ground effect manoeuvre data is used both for parameter estimation and model verification. This exception is

Table 6.1: CJ1 elevator model QTG data set

Test	Filename	t_{start}	t_{end}
	(* .cmb)	[sec]	[sec]
Minimum Control Speed Ground	cj04080203	30.5	45
Minimum Unstick Speed	cj13080303	21	41
Normal Takeoff	cj08080107	55	86
Critical Engine Failure on Takeoff	cj08080105	35	75
Crosswind Takeoff	cj19080116	20	50
Rejected Takeoff	cj06080103	25	68
Dynamic Engine Failure After Takeoff	cj17080104	0.5	31
Column Position vs. Force and Surface Position Calibration	colswp0201	8	80
Longitudinal Maneuvering Stability (Cruise)	cj13080421	20	26
Longitudinal Maneuvering Stability (Approach)	cj13080421	50	55
Longitudinal Maneuvering Stability (Landing)	cj13080421	65	70
Longitudinal Static Stability (Approach)	cj07080109	470	480
Stick Shaker, Airframe Buffet, Stall Speeds (Climb)	cj07080107	50	97
Stick Shaker, Airframe Buffet, Stall Speeds (Approach)	cj06080133	34	64
Normal Landing (Light)	cj28070137	115	153
Normal Landing (Medium)	cj11080220	54	84
Normal Landing (Heavy)	cj08080104	56	76
Minimum/No Flap Landing	cj08080106	1	52
Crosswind Landing	cj19080115	17	57
One Engine Inoperative Landing	cj12080116	26	55.5
Go Around (Engine Out - A/P Off)	cj12080104	4	41
Go Around (Both Engines - A/P On)	cj12080107	65	91
Longitudinal Ground Effect, flap 0, 5 ft	cj04080208	53	56
Longitudinal Ground Effect, flap 0, 10 ft	cj04080207	60.5	62.5

Continued on next page

Test	Filename	t_{start}	t_{end}
	(* .cmb)	[sec]	[sec]
Longitudinal Ground Effect, flap 0, 14 ft	cj03080126	30.75	35.75
Longitudinal Ground Effect, flap 0, 42 ft	cj04080211	47	52
Longitudinal Ground Effect, flap 0, 52 ft	cj04080212	57	60
Longitudinal Ground Effect, flap 0, 74 ft	cj04080213	35	36.6
Longitudinal Ground Effect, flap 15, 7 ft	cj03080109	56	59
Longitudinal Ground Effect, flap 15, 10 ft	cj03080108	55.5	60.5
Longitudinal Ground Effect, flap 15, 15 ft	cj03080107	52	55
Longitudinal Ground Effect, flap 15, 34 ft	cj03080106	48	51
Longitudinal Ground Effect, flap 15, 44 ft	cj03080111	59	62
Longitudinal Ground Effect, flap 15, 62 ft	cj03080114	52.2	56
Longitudinal Ground Effect, flap 35, 5 ft	cj03080120	41	45
Longitudinal Ground Effect, flap 35, 10 ft	cj03080118	45	48.5
Longitudinal Ground Effect, flap 35, 14 ft	cj03080116	65.5	68.5
Longitudinal Ground Effect, flap 35, 30 ft	cj03080122	44	47
Longitudinal Ground Effect, flap 35, 50 ft	cj03080124	83	86

necessary because the ground effect manoeuvre is the only manoeuvre in which flight in ground effect occurs while the aircraft is not in landing or takeoff configurations.

This verification study involves some model optimisation before the final verification results are produced. First, the model is run over all the QTG data, and the aerodynamic hinge moment parameters, listed in Equation (5.72) and repeated here as Equation (6.1), are varied randomly within the 95% uncertainty bounds as the simulation is performed multiple times over the QTG data.

$$\begin{aligned}
 C_{e,0,0} &= 5.44E-5 \quad \pm 1.73E-5 \quad (95\%) \\
 C_{e,0,\alpha_e} &= -5.26E-4 \quad \pm 2.27E-4 \quad (95\%) \\
 C_{e,\eta,\eta} &= -1.68E-3 \quad \pm 1.42E-4 \quad (95\%) \\
 C_{e,\eta,\eta_i} &= 6.37E-3 \quad \pm 1.05E-3 \quad (95\%) \\
 C_{e,\eta,\eta_s} &= -1.79E-2 \quad \pm 3.34E-3 \quad (95\%) \\
 C_{e,\eta,Ma} &= -1.86E-3 \quad \pm 5.41E-4 \quad (95\%) \\
 C_{e,\eta,\eta_K} &= -5.80E-4 \quad \pm 2.70E-4 \quad (95\%) \\
 C_{e,\eta_{TT},\eta_{TT}} &= -1.85E-3 \quad \pm 1.09E-4 \quad (95\%) \\
 C_{e,\eta_{TT},\eta_{TT,s}} &= 4.24E-3 \quad \pm 2.41E-3 \quad (95\%) \\
 C_{e,\eta_{TT},\alpha_e} &= 1.18E-3 \quad \pm 7.15E-4 \quad (95\%) \\
 C_{e,\eta_{TT},Ma} &= -2.11E-3 \quad \pm 5.02E-4 \quad (95\%) \\
 C_{e,\dot{\eta}} &= -1.08E-2 \quad \pm 2.50E-3 \quad (95\%) \\
 Bias &= 0.191 \quad \pm 0.260 \quad (95\%)[lb \cdot ft^2]
 \end{aligned} \tag{6.1}$$

The combination of parameter values that produces the best match is selected for the final elevator model. 200 randomisation tests are run, and the resulting aerodynamic hinge moment coefficients are listed in Table 6.2.

The coefficient C_{e,η,η_s} represents the decrease in hinge moment as the airflow separates from the elevator surface at large elevator deflection angles. The value given by the parameter estimation in the previous chapter proved to be much too large, so that the aerodynamic hinge moment trailed off too quickly during the stall manoeuvre tests. A halving of the initial estimate produced satisfactory results.

Table 6.2: Final CJ1 elevator hinge moment coefficient values

Coefficient	Estimation	Uncertainty (95%)	Final Value
$C_{e,0,0}$	5.44E-5	$\pm 1.73E - 5$	$7.10E - 5$
$C_{e,0,\alpha_e}$	$-5.26E - 4$	$\pm 2.27E - 4$	$-5.27E - 4$
$C_{e,\eta,\eta}$	$-1.68E - 3$	$\pm 1.42E - 4$	$-1.59E - 3$
C_{e,η,η_i}	$6.37E - 3$	$\pm 1.05E - 3$	$6.41E - 3$
C_{e,η,η_s}	$-1.79E - 2$	$\pm 3.34E - 3$	$-8.95E - 3^*$
$C_{e,\eta,Ma}$	$-1.86E - 3$	$\pm 5.41E - 4$	$-1.64E - 4$
C_{e,η,η_K}	$-5.80E - 4$	$\pm 2.70E - 4$	$-4.42E - 4$
$C_{e,\eta_{TT},\eta_{TT}}$	$-1.85E - 3$	$\pm 1.09E - 4$	$-1.76E - 3$
$C_{e,\eta_{TT},\eta_{TT,s}}$	$4.24E - 3$	$\pm 2.41E - 3$	$4.45E - 3$
C_{e,η_{TT},α_e}	$1.18E - 3$	$\pm 7.15E - 4$	$1.00E - 3$
$C_{e,\eta_{TT},Ma}$	$-2.11E - 3$	$\pm 5.02E - 4$	$-2.56E - 3$
$C_{e,\dot{\eta}}$	$-1.08E - 2$	$\pm 2.50E - 3$	$-8.78E - 3$
<i>Bias</i>	0.191	± 0.260	0

* This value was set manually through investigation of the stall manoeuvres.

The performance of the elevator model for the verification tests is summarised in Table 6.3. For each test, the model error tolerance specified by AC 120-40C is listed along with the tested model error. Where no specific tolerance exists, it is generally required to maintain an error no greater than 2 degrees elevator position and 10 lb or 20% column force. Graphs of the model match for representative tests are presented in Figures 6.1 through 6.7. In these figures, the modelled parameter, either column force, $F_{c,mdl}$, or elevator position, η_{mdl} , is plotted with the measured parameter, again either column force, $F_{c,meas}$, or elevator position, η_{meas} . The measured parameter is bracketed by a tolerance region within which the modelled parameter must stay to meet the certification requirements. Also present on each graph are two statistics, the standard deviation of the error between the modelled and measured parameters, either $F_{c,err,s}$ or $\eta_{err,s}$, and the maximum error between the modelled and measured parameters, either $F_{c,err,max}$ or $\eta_{err,max}$.

Of the verification matches, the worst appears to be case 1. b. (3), minimum unstick speed. However, all instances of column force model error exceeding 10 lb occur when the column has hit the hard-stop or column spring as described in Section 5.4. The model is not required to accurately match the measured column force for this condition, so the model passes for this test case.

Table 6.3: CJ1 elevator control model verification summary

	Test	Error Tolerance		Model Error	
		[deg]	[lb] (%)	[deg]	[lb]
1. Performance					
b. Takeoff					
(2)	Minimum Control Speed Ground	-	-	0.201	1.60
(3)	Minimum Unstick Speed	-	-	0.550	16.72
(4)	Normal Takeoff	-	5 (10)	0.361	7.34
(5)	Critical Engine Failure on Takeoff	-	5 (10)	0.381	7.85
(6)	Crosswind Takeoff	-	5 (10)	0.317	6.86
(7)	Rejected Takeoff	-	-	0.295	8.03
(8)	Dynamic Engine Failure After Takeoff	-	-	0.277	5.06
2. Handling					
a. Static Control Checks					
(1)	Column Position vs. Force and Surface Position Calibration	2	5 (10)	0.242	7.24
c. Longitudinal					
(7)	Longitudinal Maneuvering Stability (Cruise)	-	5 (10)	0.181	4.02
(7)	Longitudinal Maneuvering Stability (Approach)	-	5 (10)	0.160	3.03
(7)	Longitudinal Maneuvering Stability (Landing)	-	5 (10)	0.232	2.51
(8)	Longitudinal Static Stability (Approach)	1	5 (10)	0.138	2.12
(9)	Stick Shaker, Airframe Buffet, Stall Speeds (Climb)	-	5 (10)	0.570	9.81
(9)	Stick Shaker, Airframe Buffet, Stall Speeds (Approach)	-	5 (10)	0.466	9.88
e. Landings					
(1)	Normal Landing (Light)	-	5 (10)	0.341	7.62
(1)	Normal Landing (Medium)	-	5 (10)	0.240	7.33
(1)	Normal Landing (Heavy)	-	5 (10)	0.216	4.02
(2)	Minimum/No Flap Landing	-	5 (10)	0.201	4.71

Continued on next page

	Test	Error Tolerance		Model Error	
		[deg]	[lb] (%)	[deg]	[lb]
(3)	Crosswind Landing	-	5 (10)	0.518	11.28
(4)	One Engine Inoperative Landing	-	-	0.206	4.71
(6)	Go Around (Engine Out - A/P Off)	-	-	0.249	4.82
(6)	Go Around (Both Engines - A/P On)	-	-	0.271	5.14
f. Ground Effect					
(1)	Longitudinal Ground Effect, flap 0, 5 ft	1	-	0.189	4.06
(1)	Longitudinal Ground Effect, flap 0, 10 ft	1	-	0.185	3.59
(1)	Longitudinal Ground Effect, flap 0, 14 ft	1	-	0.157	3.68
(1)	Longitudinal Ground Effect, flap 0, 42 ft	1	-	0.206	4.32
(1)	Longitudinal Ground Effect, flap 0, 52 ft	1	-	0.170	4.36
(1)	Longitudinal Ground Effect, flap 0, 74 ft	1	-	0.219	2.91
(1)	Longitudinal Ground Effect, flap 15, 7 ft	1	-	0.193	4.16
(1)	Longitudinal Ground Effect, flap 15, 10 ft	1	-	0.220	3.98
(1)	Longitudinal Ground Effect, flap 15, 15 ft	1	-	0.181	3.70
(1)	Longitudinal Ground Effect, flap 15, 34 ft	1	-	0.236	3.15
(1)	Longitudinal Ground Effect, flap 15, 44 ft	1	-	0.177	3.89
(1)	Longitudinal Ground Effect, flap 15, 62 ft	1	-	0.182	3.42
(1)	Longitudinal Ground Effect, flap 35, 5 ft	1	-	0.196	3.48
(1)	Longitudinal Ground Effect, flap 35, 10 ft	1	-	0.219	3.71
(1)	Longitudinal Ground Effect, flap 35, 14 ft	1	-	0.196	2.93
(1)	Longitudinal Ground Effect, flap 35, 30 ft	1	-	0.214	3.24
(1)	Longitudinal Ground Effect, flap 35, 50 ft	1	-	0.207	2.89

Figure 6.1: Column force model match, normal takeoff manoeuvre

Figure 6.2: Column force model match, crosswind takeoff manoeuvre

Figure 6.3: Column force model match, ground control sweep manoeuvre

Figure 6.4: Column force model match, stall on approach manoeuvre

Figure 6.5: Column force model match, normal landing - medium weight manoeuvre

Figure 6.6: Column force model match, crosswind landing manoeuvre

Figure 6.7: Elevator position model match, ground effect manoeuvre with flaps 0 and at 5 feet above ground level

It can be seen from Table 6.3 that the Reversible Flight Control Identification (ReFCI) CJ1 elevator model does not meet every requirement specified by Advisory Circular (AC) 120-40C, and especially does not meet the more stringent requirement for stand-alone verification that all tolerances must be halved. However, the approval of the CJ1 elevator control system model is done on the basis of comparison to an independently developed model that has been accepted by the customer, Mechtronix Systems Incorporated (MSI). The previously accepted model has been developed using a more classical algebraic form, where aerodynamic, flight path and pilot control inputs are translated directly into column force and elevator position through sets of equations. Further discussion of the comparison of the classical model and the state model used for Reversible Flight Control Identification (ReFCI) is provided in Chapter 7.

The model match performance for the QTG manoeuvres, as summarised in Table 6.3, is in all cases equal or better than the model match for the NRC classical model. Based on this, the ReFCI CJ1 elevator model may be recommended for Level D certification.

Chapter 7

Comparison of Classical and ReFCI Models

Two models have been independently developed to simulate the behaviour of the CJ1 elevator flight control system: the Reversible Flight Control Identification (ReFCI) model, as described in this thesis, and the “classical” model developed by the Flight Research Laboratory (FRL) at the National Research Council of Canada (NRC). The modelling technique used for the classical CJ1 elevator model follows the work documented by Pick (Pick, 1986). The classical model was developed for the customer Mechtronix Systems Incorporated (MSI), and has been accepted. MSI is in the process of having the model certified to Level D standards.

The classical type of modelling may be called data-based modelling, where the model is made to match the flight data without regard to the underlying physics of the flight control system. For this type of modelling, correlations between dependent and independent variables may be found and used without explanation. The equations resulting from the established correlations are combined to form a single equation

each for the elevator position model and control column force model. Often, separate equations are required to handle different flight regimes of the aircraft, although only two sets were needed for the CJ1 elevator model: one static ground model and one for flight.

By contrast, the ReFCI CJ1 elevator model is a state model, where the physical properties of position and velocity of the elevator body are the system states. This model is designed to best represent the physics of the elevator control system, with the goal of producing a high-fidelity model for simulation. The ReFCI CJ1 model is universal for all flight conditions, including ground. The disadvantages of physics-based modelling compared to data-based modelling are that it restricts the modeller from using trends that may be present in the data but do not have an underlying physical explanation and that if the control system is very complex, there is an extra challenge of proposing a sufficient model architecture.

The greatest advantage of the ReFCI modelling technique is that it should be applicable to reversible flight control systems on any aircraft. The general approach of proposing a model architecture based on kinematic relationships of the physical control system, then reducing the governing equations and estimating parameter values from flight data should produce similar results in other applications as long as there is sufficient understanding of the control path and aerodynamic mechanics, and there is a large quantity of high-quality flight data.

Chapter 8

Conclusions

In conclusion, the development of the CJ1 elevator model using Reversible Flight Control Identification (ReFCI) has been a success, as the final model passed Qualification Test Guide (QTG) verification tests with equivalent performance to an independent model that has been recommended for Level D certification. Additionally, there are advantages to the ReFCI modelling and parameter estimation techniques that make the technique recommendable for implementation in future reversible flight control modelling applications.

The most unique feature of the ReFCI CJ1 elevator model is that it incorporates the physical properties of the elevator control system in a state model as opposed to a set of algebraic models. This modelling allows for a more natural handling of the forces and motions of the elevator control system. A unified elevator model has been developed that covers the complete flight envelope of the CJ1 aircraft.

The use of multiple regression brings a potential drawback to the ReFCI methodology. Because of the large amount of flight data required to produce a unified control

system model and because multiple regression handles all of this data simultaneously, large computer resources are required to perform the parameter estimation calculations. Specifically, over 512 megabytes of memory was required for the CJ1 elevator control system identification. More advanced techniques, such as Kalman filtering, may be used to perform parameter estimation while handling the flight data incrementally, and thus reduce memory requirements. However, memory capacities of typical personal computers are now large enough to tackle the entire flight data set at once without resorting to more expensive computer solutions.

The next stage in the development of ReFCI is to apply the methodology to other aircraft modelling projects. The experience from these programmes will confirm the ReFCI process and aid in the development of automation, so that less human intervention is needed in the parameter estimation. It will also be possible to develop the parameter estimation procedures to allow for real-time in-flight system identification.

The modelling techniques used for ReFCI will be directly applicable to other reversible control systems, such as aileron and rudder flight controls, as well as reversible flight control systems on other aircraft. Examination of the control system may lead to adjustments to the basic model by incorporating new elements such as centering springs or more complicated control paths and trimming systems. However, so long as sufficient high-quality flight data is available, there is no reason to expect that models developed for other aircraft cannot achieve the same level of fidelity as the CJ1 elevator control model.

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Appendix A

Time Delay of Elevator Position Measurement Signal

Post-processing of the flight test data is performed to remove any identifiable errors introduced by the recording process. For the Citation X flight test data, the only data correction required is for measurement signal recording time delay. For example, the signals indicating the position of the flight control surfaces may be recorded a small time interval after they are measured. Srinivasan et al. discovered such a recording delay between the elevator position signal and the pitch rate signal during the analysis of the Dash 8 Series 300 aircraft flight test data. This delay was attributed to a filter in the signal path of the primary flight control system aboard the aircraft.

Figure A.1 shows the measured left elevator position, η , plotted against the measured control column position, δ_c , for one 2311 manoeuvre. Besides the strong correlation between the two measured positions, there are apparent looping patterns in the data that look similar to hysteresis. However, the effects of hysteresis and time delay may be easily separated because hysteresis is frequency dependant, while signal

time delay is a constant value for all frequencies. As stated in section 3.2.3, data from the 2311 manoeuvre contains broad-spectrum excitation of the control systems, and so this manoeuvre is useful for time delay identification.

The method employed for time delay identification is an optimisation study of the time advance applied to the elevator position signal as it is read from the recorded data. If a delay exists in the control surface position signal as it is recorded, then applying an equivalent advance to this signal will remove the large looping patterns noted in Figure A.1. With the looping patterns removed, the regression of the parameters will be better, and this is marked by a higher R-squared value. The R-squared value is plotted for varying time advances applied to the elevator position signal in Figure A.2, with R-squared on the y-axis and time advance, in counts¹, on the x-axis. It is thus determined that the time delay on the elevator position signal is three counts. A three count advance is applied to the elevator position signal for the remainder of the analyses.

Figure A.3 shows the improved regression between the delay corrected η and control δ_c . The remaining nonlinear patterns may be attributed to the dynamics of the control system behaviour.

¹The measurement signals are recorded at 64 Hz, and so one count is equivalent to 1/64 of a second.

Figure A.1: η with time delay verses δ_c , 2311 manoeuvre

Figure A.2: Optimisation of elevator position signal time delay correction

Figure A.3: Delay corrected η verses δ_c , 2311 manoeuvre

Appendix B

JR3 Sensor Drift

B.1 Introduction

The six components of force, F_X , F_Y and F_Z , and moments, M_X , M_Y and M_Z , are measured at the control column/wheel using a sensor supplied by JR3 Inc. Multi - Axis Load Cell Technologies. There is a notable variability in the bias of these measurements, especially F_Z . Through correspondence, JR3 Inc. has suggested that the sensor bias drift is due to temperature effects. The purpose of this report is to analyse the impact of the JR3 thermal drift on the uncertainties of recorded measurements.

This report will analyse the JR3 sensor bias drift during engine start-up records, between recording periods during flight, and for specific tests applied to the JR3 sensor at the FRL.

B.2 Hands-Free Events

The Flight Research Laboratory (FRL) has purchased a new JR3 sensor with integrated temperature correction. However, this sensor was not available at the time

of the CJ1 flight programme, so the recorded data is subject to significant thermal drift. Fortunately, the sensor drift may be quantified at certain times when the pilot takes his or her hands off the control column for a short period. During these "hands-free" events, the forces applied to the control column are known to be zero, so any steady offset signal output by the JR3 sensor is known to be bias.

Hands-free events are identified by searching for periods during which column and wheel force measurements contain the least amount of noise. These forces are calculated from F_Z , M_X and M_Z according to the following two equations.

$$F_c = -F_Z - \frac{M_X}{30.5in} \quad (B.1)$$

$$F_w = \frac{M_Z}{2 \cdot 11.75in} \quad (B.2)$$

The flight data for each of 869 files recorded during the test program is divided into two-second time "bins", and the noise within each bin is evaluated by calculating the standard deviation of the measurements of F_c and F_w . From inspection, it is found that the range of column forces is approximately six times greater than the range of wheel forces. Therefore, the noise of the wheel force measurement is weighted in the criterion formula for establishing hands-free events, given in Equation (B.3).

$$bin\ noise = std(F_{c,bin}) + 6 \cdot std(F_{w,bin}) \quad (B.3)$$

The algorithm that searches through each of the data files flags the two-second bin for which the least *bin noise* is found. If, for a given file, the minimum *bin noise* is greater than a maximum noise tolerance, then the file is marked as having no hands-

free event. For the data from the CJ1 test programme, the noise tolerance is set to 0.1 lb. This tolerance level eliminates false hands-free identifications as apparent from human inspection.

602 hands-free events are identified in this manner. The standard deviations of F_c and F_w during each of these two-second test intervals can be used to calculate the precision error of the respective measurements. The standard deviations of F_c are combined to find the pooled $\langle s_{Fc} \rangle$ for all M instances according to Equation (B.4).

$$\langle s_{Fc} \rangle = \sqrt{\frac{1}{M} \sum_{j=1}^M s_{Fc,j}^2} \quad (\text{B.4})$$

To find the total number of data points used in the calculation of $\langle s_{Fc} \rangle$, we need the number of points, N , in each two-second bin. Since the data is collected at a rate of 100 Hz, N is 200. The degrees of freedom of the pooled standard deviation is then given by Equation (B.5).

$$r = M \cdot N - 1 \quad (\text{B.5})$$

The resulting precision uncertainties, for a 95% confidence level with 120,399 degrees of freedom are 0.0440 lb for F_c and 0.0053 lb for F_w .

Some flight data files will contain no hands-free events, so that the measurement biases for these files must be interpolated from the biases established for the other files within the same flight. This process is described later in this report. For now, we turn to establishing some evidence in support of the theory that the sensor bias drift is a temperature phenomenon.

B.3 Engine Start-Up Files

At the beginning of each flight, the recording instruments are turned on to record the engine start-up. Typically, all the components of the aircraft and measurement systems will start at ambient temperatures, and the components will heat up over time. Therefore, if there is a thermal drift in the measured data, it should be exemplified within the engine start-up files.

The time history of the F_Z signal is presented for two engine start-up files in Figures B.1 and B.2. In the first figure, the F_Z measurement indicates a positive load of over 5.2 lb, which drops off to less than 4.6 lb over about 220 seconds, during which time there is no significant action in the control column or elevator. A similar drift is noted in Figure B.2, but in this case the sensor bias starts below -5.2 lb.

22 engine start-up files are examined in all, out of a total of 30 recorded flights. The missing 8 start-up files were either not recorded or represent flights that were not the first flight of the day. Only the files corresponding to cold starts are used, and for all of these the bias drift always tends towards negative over time. The engine start-up bias drift rate is -0.23 ± 0.151 lb/min, (95%).

The fact that the F_Z bias drift is always towards the negative for the engine start-up files is consistent with the proposition that the drift is a thermal phenomenon, as the measurement equipment will always be warming up during the start-up files. However, the trend in the bias drift should not be linear, as the start-up files seem to indicate, but rather should appear as a first-order response to a step input. If this is

Figure B.1: Time history of measurement F_Z during engine start-up cj12080101

Figure B.2: Time history of measurement F_Z during engine start-up cj29070101

the case, then the time constant of the response may be large, so that the 4-minute interval of recorded data for the engine start-up files is too short to capture first-order effects.

B.4 In-Flight Sensor Drift

The drift rate of the JR3 sensor is calculated by comparing the identified bias for hands-free events in different flight files. The F_c drift rate between files $n-1$ and n is then:

$$drift(F_c)_n = \frac{bias(F_c)_n - bias(F_c)_{n-1}}{t_n - t_{n-1}} \quad (B.6)$$

In-flight, the F_c bias drift does not appear to have a preferred trend, as the start-up files had. The F_c bias seems to trend upward as frequently as downward. For 599 calculated in-flight bias drift rates, the F_c bias drift rate is within ± 0.266 lb/min, (95%). The F_w bias drift rate is within ± 0.0216 lb/min (95%).

The in-flight drift rates, then, are in the same order of magnitude as the engine start-up drift rates, but with much greater scatter. The magnitude of the in-flight drift rate leads to a large degree of uncertainty in the sensor bias present in flight files in which there are no hands-free events. The following section describes the interpolation scheme used to estimate the sensor bias for these files, and provides an analysis of the uncertainty associated with this estimation process.

B.5 Laboratory Tests of JR3 Drift

Laboratory tests of the JR3 sensor have been performed in the attempt to identify the cause of the sensor drift. Two types of tests were done on the sensor: countertop self-warming and temperature-controlled tests.

B.5.1 Countertop Self-Warming Test

To establish a better idea of the start-up transient behaviour of the JR3 sensor, the sensor has been tested in a laboratory. The test was performed at room temperature, and the unloaded sensor was turned on and the output recorded for two hours. The time response of the F_Z output during this test is displayed in Figure B.3.

Besides a pair of brief disturbances during the first half-hour of testing, it appears that the F_Z signal follows a roughly first-order response curve. The disturbances were created when the lab technician manually compressed the JR3 sensor to provide an example of the magnitude of the sensor readout from the human grip. The data points corresponding to these disturbances have been removed from the set used for further analysis.

The time constant of the response curve in Figure B.3 is determined using the procedure described in Appendix F. The results of this determination are that τ is 52.4 minutes, and the final settling value of the response curve should be -2.94 lb. The initial value of F_Z is 7.93 lb, so the total range expected for the countertop self-warming test is 10.9 lb.

Using the definition of $z(t)$ given in Equation (B.7), the curve fit used for the identification of τ is shown in Figure B.4.

$$z(t) = \ln \left(\frac{F_Z(t) - F_{z,\infty}}{F_{Z,0} - F_{Z,\infty}} \right) \quad (\text{B.7})$$

Given that the JR3 sensor temperature at the beginning of the laboratory test was room temperature, and the operating environment stayed at room temperature throughout the test, it is reasonable to assume that any heating of the measurement system is due to the operation of the system itself. This heating trend will be very similar to the trends seen during operation of the system during the CJ1 flight test programme, as the operating environment within the fuselage is temperature-controlled near room temperature. Therefore, the time constant of the sensor should remain close to one hour during the flight tests.

One point of correlation between the laboratory experiment and the engine start-up files is that the F_Z bias drift rate within the first 200 seconds of the laboratory experiment is -0.29 lb/min, which fits within 0.7 of one standard deviation of the drift rates observed for the engine start-up files.

B.5.2 Temperature Controlled Test

The procedure followed for the temperature-controlled test was to place the unloaded JR3 sensor in the oven and initiate data recording. The oven was heated from an initial 25 °C to 35 °C, and held there for half an hour. Then, the oven was heated

Figure B.3: Time history of measurement F_Z during room-temperature test in laboratory

Figure B.4: Plot of $z(t)$ versus time for extraction of τ

to $45\text{ }^{\circ}\text{C}$ and held again for half an hour. Finally, the oven was cooled to $-15\text{ }^{\circ}\text{C}$. The output of the JR3 sensor was recorded continuously at 100 Hz sampling rate.

The heating of the oven takes very little time, so this input is essentially a step. The cooling of the oven happened very slowly, taking half an hour to adjust in temperature. This change in temperature can also be treated as a step response, with the recognition that the thermal capacity of the oven itself is significant in comparison to the rate of cooling provided by the machinery. The time constant of the JR3 first-order response is therefore expected to be much larger for the cooling phase, but the characteristic first-order response behaviour is maintained.

The time history of the F_Z recorded signal for the temperature-controlled test is presented in Figure B.5.

The half-hour of settling time is not necessarily enough to allow the F_Z value to reach a final value, so this value must be determined by extrapolation. The method used to determine this final value, and the time constants associated with each temperature change, are described in Appendix F. The final settling values and associated time constants for each temperature level are listed in Table B.1. As expected the time constant for the settling to $-15\text{ }^{\circ}\text{C}$ is much larger than the time constants for warming responses.

The static sensitivity of F_Z to temperature, κ , is calculated as the slope of the line fit by least-squares to the data in Table B.1. A graph of this fit is presented in Figure B.6, and κ is found to be $-0.32\text{ lb}/^{\circ}\text{C}$.

Figure B.5: Time history of F_Z response during temperature-controlled tests

Table B.1: Final values and time constants for temperature-controlled JR3 test

T [$^{\circ}C$]	F_Z [lb]	τ [min]
25	-2.79	-
35	-5.28	11.4
45	-6.36	6.6
-15	12.01	57.0

The range of bias seen in the F_Z measurement due to temperature variation is significant, but it is smaller than the drift observed throughout the flight test programme. Also, the temperature variation in the laboratory test is much greater than the variations seen in the CJ1 cabin environment. Therefore, environment temperature is known to have an effect on the JR3 sensor output, but this effect is not enough on its own to explain the F_Z bias observed in the flight data.

B.6 Interpolation of JR3 Bias

For flight files where no hands-free events occur, the JR3 bias is estimated through interpolation. The interpolation scheme that is used is a third-order polynomial fit to the four hands-free event points that bracket the point of interpolation. For example, to estimate the force bias at point $n = 5$, a third-order polynomial is fit to points $n = 3, 4, 6$ and 7 . The value of this curve at the time corresponding to point $n = 5$ is the predicted force bias.

The start and end points and their neighbours must be treated specially. The end points are extrapolated based on a linear fit to the two neighbouring points. The points next to the end points are interpolated based on a second-order polynomial fit to points $n = 1, 3$, and 4 or $n = n-3, n-2$, and n . The following uncertainty statistics are provided for the internal points, where a 3rd-order polynomial interpolation is performed. The uncertainties associated with the estimation of the bias in the pairs of files at the start and end of a flight are not of great interest, as these files either contain hands-free events or are of no value to ReFCI.

The uncertainty of the force bias correction predicted by the interpolation scheme must be quantified. For this purpose, the accuracy of interpolation scheme is measured by comparing the measured force bias of each hands-free event against the bias that interpolation predicts for the same GPS time with the data point of interest removed. For example, if there exists hands-free events in all files from $n = 1$ to $n = 10$, then the error in the bias interpolation is calculated for point $n = 3$ by comparing the bias identified from the hands-free event in file 3 with the bias estimated using interpolation based on points 1, 2, 4 and 5. The interpolation error is given in Equation (B.8).

$$E_{int}(F_Z)_n = \text{abs}(\text{bias}_{int}(F_Z)_n - \text{bias}(F_Z)_n) \quad (\text{B.8})$$

The interpolated force bias error is calculated for each hands-free event for 32 flights, so that statistical analysis may be performed. Because the temporal spacing of the data points is not constant, it is necessary to relate the interpolated force bias error to the position of the interpolated point relative to its neighbours.

It is expected that the magnitude of the interpolation error will be reduced when the interpolated point is in close proximity to established hands-free bias calibration events. Conversely, interpolation error will be greatest for points furthest from neighbouring hands-free events. It is proposed to correlate the calculated interpolated force bias error with an equivalent point remoteness factor, defined in Equation (B.9)

$$\text{remoteness}_n = \left[\sum_m \frac{1}{\text{abs}(\Delta t_n - \Delta t_m)} \right]^{-1}, \text{ where } m = n-2, n-1, n+1, n+2 \quad (\text{B.9})$$

In terms of interpolation error versus remoteness factor, additional data points may be generated by repeating the 3rd-order polynomial interpolation for a given point using different sets of 4 calibration points. For example, the interpolation for the 5th point in a series may be performed using points 3, 4, 6 and 7, but also by points 2, 3, 7 and 8. The second set of calibration points will yield a new interpolation estimate, with its own error and remoteness factor.

Figure B.7 shows the calculated force bias interpolation error versus remoteness factor, on log-log scales. A power curve is fit to the data, so that the error, interpolation uncertainty due to remoteness, is described as per Equation (B.10).

$$u_{F_z,0} = 0.000835 \cdot remoteness_n^{1.150} \quad (B.10)$$

The standard deviation of the data from the curve fit in Figure B.7 is 1.794 lb, with 2533 points. The Student t-distribution value for 2532 degrees of freedom at a 95 percent confidence level is 1.645, so that the uncertainty in the bias correction procedure due to the scatter in Figure B.7 is given as:

$$\mu_{F_z,1} = 2.974 \text{ [lb]} (95\%) \quad (B.11)$$

The total uncertainty in the F_Z bias correction for any file is then given by the root sum-squares (RSS) of the bias error calculated from the power fit with the point remoteness and the bias uncertainty from the data scatter.

As two examples, consider three flight files with no hands-free events, with remoteness factors of 100, 1000, and 10000 seconds, respectively. The F_Z force bias will

Figure B.6: Static sensitivity of F_Z to temperature

Figure B.7: Force bias prediction error versus remoteness factor for JR3 F_Z sensor drift

be approximated by the previously described interpolation scheme. The uncertainties associated with the bias interpolation are summed up in Table B.2, with a confidence level of 95 percent.

The interpolation process described in the preceding pages is applied to all the flight files where no hands-free event is identified. An example of the results of this process is presented in Figure B.8, where the hands-free calibration points are displayed as dots, and the interpolated bias corrections are triangles with error bars. The bias interpolation and uncertainty analysis described above is for the F_Z measurement. The same process is applied to the other five JR3 measurements. The uncertainties associated with the bias interpolation for each JR3 measurement is summarised in Table B.3.

$$\begin{aligned}\mu &= \sqrt{\mu_0^2 + \mu_1^2} \text{ (95\%)} \\ \mu_0 &= A \cdot (\text{remoteness})^B\end{aligned}\tag{B.12}$$

Table B.2: Total bias interpolation uncertainty for F_Z

Remoteness [sec]	$u_{F_z,0}$ [lb]	$u_{F_z,1}$ [lb]	u_{F_z} [lb]
100	0.167	2.607	2.613
1000	2.352	2.607	3.512
10000	33.23	2.607	33.33

Figure B.8: Force bias calibration points (dots) and interpolated points (triangles) for flight cj210802

Table B.3: Bias interpolation uncertainty for JR3

JR3 Channel	A	B	μ_1
F_X	0.00290	0.789	2.367
F_Y	0.00568	0.705	2.310
F_Z	0.000835	1.150	2.607
M_X	0.0101	0.706	8.414
M_Y	0.00643	0.699	6.963
M_Z	0.00466	0.720	13.43

Appendix C

Potentiometer Thermal Sensitivity Study

The precision potentiometers used for control system measurements operate by varying the position of a wiper on a resistor, and thus producing a variable resistance based on the position being measured. The voltage read from the potentiometer is a product of the ratio of resistance between one resistor equipped with the wiper and one reference resistor. According to this setup, there should be no thermal sensitivity of the potentiometer readings, because both resistors are proportionately affected by the sensor temperature, which does not affect the resistance ratio.

To ensure that the measurement data is not affected by sensor thermal drift, the potentiometer used for the Citation X left elevator position measurement has been tested in a temperature controlled environment. The results of this test are recorded in Table C.1.

To determine if the potentiometer readings are affected by temperature, the correlation between output voltage and temperature is examined. The effect of the

variation in input voltage is addressed by dividing each output voltage by its input voltage and then multiplying by the input voltage for the measurement at 21.8°C , which will be the reference point. The voltage errors are then defined by the difference between the normalised output voltages and the reference point. The relative error is given in parts per million (ppm), and graphed against temperature in Figure C.1. The following equations demonstrate the data reduction process involved in calculating the error in parts per million.

$$V_{norm_i} = V_{out_i} * V_{in_{21.8}} / V_{in_i} \quad (\text{C.1})$$

$$V_{err_i} = V_{norm_i} - V_{out_{21.8}} \quad (\text{C.2})$$

$$V_{errppm_i} = V_{err_i} / V_{norm_i} * 1E6 \quad (\text{C.3})$$

It is found that the potentiometer output has a sensitivity of $4.6 \text{ ppm}/^{\circ}\text{C}$. The sensor is sufficiently insensitive to temperature, and so any observed thermal effect is attributed to real control system behaviour.

Table C.1: Thermal sensitivity test of measurement potentiometer

Temperature [$^{\circ}\text{C}$]	Vin	Vout
21.8	5.0126	2.4716
0	5.0125	2.4719
-20	5.0127	2.4723
-40	5.0125	2.4724
-60	5.0126	2.4725
-30	5.0128	2.4724

Figure C.1: δ_e with time delay verses δ_c , 2311 manoeuvre

Appendix D

Data Tables

This appendix lists the data files used in the parameter identification for the Citation X and CJ1 aircraft flight control models.

D.1 Citation X Flight Data Files

The flight data used in the flight control system identification analyses for the Citation X aircraft are listed in Tables D.1 through D.6. The start time, t_{start} , and end time, t_{end} columns of each table indicate the beginning and end of the sampled data signals in seconds relative to the beginning of the data file. The flight test programme and data collection are documented in NRC Report LTR-FR-200 (Ricciardi et al, 2003).

Table D.1: Citation X elevator ground control sweep manoeuvre files

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Elevator GCS	cx002002	55	105	cx003002	65	100
	cx017002	18	48	cx181002	18	58

Table D.2: Citation X stabiliser ground control sweep manoeuvre files

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Stabiliser GCS	cx002002	210	320	cx003002	100	180
	cx017002	120	210	cx181002	140	220

Table D.3: Citation X pitch 2311 manoeuvre files

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Pitch 2311	cx005021	14.5	23	cx005022	14	21
	cx007007	19.5	27	cx007030	30.5	37.5
	cx016012	11	19	cx016025	13	20
	cx016026	11	18	cx181007	12	21
	cx201039	12	21	cx221007	12	20
	cx001010	20	27	cx001011	16	22
	cx001012	8	15	cx002024	16	22
	cx002026	14	20	cx011006	13	20
	cx011013	13	20	cx003009	17	26
	cx003020	14	22	cx011012	12	20

Table D.4: Citation X aileron ground control sweep manoeuvre files

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Aileron GCS	cx002002	5	60	cx017002	45	85
	cx181002	10	45			

Table D.5: Citation X aileron trim ground control sweep manoeuvre files

Manoeuvre	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]
Aileron Trim GCS	cx002002	225	265	cx017002	135	185
	cx181002	145	190			

Table D.6: Citation X roll 2311 manoeuvre files

Manoeuvre	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]
Roll 2311	cx005021	130	138	cx005022	182	190
	cx016026	46	54	cx181007	60	69
	cx201039	75	85	cx221007	196	204
	cx007014	59	66	cx013022	40	48
	cx281034	112	121	cx001010	81	88
	cx001011	66	73	cx001012	56	62
	cx002009	64	71	cx002026	111	118
	cx002024	135	141	cx011006	83	90
	cx011016	94	101	cx011013	46	54
	cx003009	109	116	cx003020	127	134
	cx011012	81	89			

D.2 CJ1 Flight Data Files

The flight data used in the flight control system identification analyses for the CJ1 aircraft are listed in Tables D.7 through D.9. The start time, t_{start} , and end time, t_{end} columns of each table indicate the beginning and end of the sampled data signals in seconds relative to the beginning of the data file. The flight test programme and data collection are documented in NRC Report LTR-FR-260 (Auriti et al, 2006).

Table D.7: CJ1 elevator ground control sweep manoeuvre files, 7704 points

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Elevator GCS	cj02080102	65	95	cj03080201	35	55
	cj07080102	70	84	cj07080137	3	16

Table D.8: CJ1 elevator frequency sweep manoeuvre files, 70946 points

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Elevator Frequency Sweep	freqp00202	79	175	freqp00202	216.3	307.8
	freqp00202	361.5	462.4	colfrq0101	2.5	265
	colfrq0201	4.2	79.6	colfrq0301	5	88.1

Table D.9: CJ1 longitudinal flight manoeuvre files, 368624 points

Manoeuvre	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]	Filename (*.cmb)	t_{start} [sec]	t_{end} [sec]
Flap Change Dynamics	cj02080116	10	30	cj02080118	25	40
	cj02080126	18	28			
Ground Effect	cj03080105	30	60	cj03080106	20	50
	cj03080107	30	70	cj03080108	30	70
	cj03080109	40	90	cj03080111	40	90
	cj03080113	130	180	cj03080114	20	70
	cj03080115	70	120	cj03080116	35	75
	cj03080117	30	75	cj03080118	30	70
	cj03080119	20	65	cj03080120	20	60
	cj03080121	20	55	cj03080122	10	60
	cj03080123	40	80	cj03080124	55	90
	cj03080125	20	65	cj03080126	20	50
	cj03080104	55	75	cj04080207	30	70
	cj04080208	20	65	cj04080209	50	90
	cj04080210	10	60	cj04080211	25	65
	cj04080212	25	65	cj04080213	20	60
	cj04080214	20	60	cj04080215	55	75
	cj04080216	60	80	cj04080217	40	60
Pitch 2311	cj02080104	20	35	cj02080105	25	44
	cj02080106	30	47	cj02080108	5	35
	cj02080111	20	35	cj02080112	15	30

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Manoeuvre	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]
Pitch 2311	cj02080113	13	27	cj02080114	15	35
	cj02080115	13	28	cj02080119	20	35
	cj02080120	2	16	cj02080121	18	33
	cj02080122	33	48	cj03080110	10	25
	cj03080209	8	22	cj07080204	15	32
	cj07080205	14	28	cj07080206	8	24
	cj07080207	14	28	cj07080209	18	33
	cj07080210	8	22	cj07080211	8	22
	cj07080213	12	22	cj07080215	25	40
	cj07080216	10	25	cj07080217	17	32
	cj07080224	20	35	cj07080225	16	28
	cj07080226	6	21	cj07080228	12	26
	cj07080229	17	32	cj07080230	14	29
	cj07080231	21	34	cj07080232	14	26
	cj07080233	10	23	cj07080234	12	28
	cj10080104	17	32	cj10080105	8	22
	cj10080106	18	30	cj10080107	15	28
	cj10080108	15	30	cj10080109	25	37
	cj10080110	12	27	cj10080111	10	26
	cj10080112	17	30	cj10080113	17	32
	cj10080114	17	32	cj11080222	2	16
	cj13080104	4	18	cj13080105	4	18
	cj13080106	2	16	cj13080107	2	16
	cj13080108	2	16	cj13080109	2	16
	cj13080111	5	20	cj13080112	2	16
	cj13080113	2	18	cj13080202	3	17
	cj13080203	3	17	cj13080204	3	17
	cj13080206	3	17	cj13080207	3	17
	cj13080208	32	44	cj13080209	3	17
	cj13080210	3	17	cj13080212	3	17
	cj13080213	3	17	cj13080214	3	17
	cj13080215	3	17	cj13080216	5	22
	cj13080217	3	18	cj13080218	5	20
	cj28070105	5	20	cj28070113	5	20
	cj28070114	2	14	cj28070115	5	20
	cj28070116	2	17	cj28070116	42	57
	cj28070117	2	17	cj28070117	67	82
	cj28070119	2	17	cj28070120	5	20
	cj28070120	54	68	cj28070121	10	25

Continued on next page

Manoeuvre	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]	Filename (* .cmb)	t_{start} [sec]	t_{end} [sec]
Pitch 2311	cj28070122	25	40	cj28070123	5	20
	cj28070123	65	70	cj29070108	20	32
	cj29070109	8	22	cj29070110	14	28
	cj29070112	12	26	cj29070113	14	28
	cj29070114	18	32	cj29070116	20	35
	cj29070117	20	35	cj29070118	10	25
	cj29070131	14	28	cj29070132	10	25
	cj29070133	12	26	cj29070134	6	22
	cj29070135	32	48	cj29070136	12	26
	cj29070137	15	28	cj29070138	12	26
	cj29070306	4	16	cj29070307	20	35
	cj29070308	18	33	cj29070309	1	14
	cj29070313	3	18	cj29070317	2	17
	cj29070318	4	18	cj29070319	3	18
	cj29070327	3	18	cj30070109	15	30
	cj30070111	32	46	cj30070112	11	26
	cj30070113	20	35	cj30070117	8	23
	cj30070118	12	26	cj30070119	12	26
	cj30070120	10	25	cj30070121	12	27
	cj30070122	12	27	cj30070123	8	23
	cj30070124	10	25	cj30070125	12	27
	cj30070126	17	32	cj30070127	20	35
	cj30070128	37	52	cj30070204	27	42
	cj30070205	17	32	cj30070206	17	32
	cj30070207	13	28	cj30070208	16	31
	cj30070209	15	30	cj30070210	22	35
	cj30070211	10	25	cj30070212	16	31
Pitch Trim Sweep	cj07080110	100	350	cj07080111	150	400
Steady State Sideslip	cj06080120	35	320	cj06080121	24	365
	cj07080114	17	510			

Appendix E

Citation X Aileron Model Identification

This appendix includes the parameters and graphs produced in the identification of the parameters for the Citation X aileron control model.

E.1 Wheel - Aileron Gearing and Saturation

For the purposes of propriety, the numerical values from the control wheel to aileron gearing relationship have been omitted from this thesis. The numbers have been replaced with the symbol #.

To identify the wheel - aileron gearing, the data lying in the regions of aileron position saturation are excluded. These regions are defined by δ_w less than #.23 and greater than #.24 degrees. These limits are established to produce the greatest R-squared value for the regression of the remaining middle region of data. The regression of aileron position versus wheel position is presented in Figure E.1.

E.2 Aileron Trim - Aileron Gearing

The effect of aileron trim position on the aileron position is investigated using aileron trim ground control sweep manoeuvre data. The parameter identification is performed by calculating $\xi_{\xi_{TT}}$ according to Equation (E.1), and regressing this versus $\delta_{\xi_{trim}}$. The resulting regression is shown in Figure E.2.

$$\xi_{\xi_{TT}} \simeq \xi - \xi_{\xi_{TT}} \quad (E.1)$$

E.3 Dynamic Pressure Effects Adjustment

The sensitivity of the aileron position to the inputs from the wheel and aileron trim input positions is found to be sensitive to dynamic pressure. The match of ξ^* to ξ for one 2311 manoeuvre is presented in Figure E.3. The parameters C , D_0 and D_1 are identified as 0.079, 0.996 and -0.049, respectively. The trend of D with dynamic pressure is presented in Figure E.4.

E.4 Backlash

The optimal backlash is found by testing a range of ξ_{play} possibilities and selecting the best one. This analysis finds the optimal ξ_{play} to be 0.1 degrees. The change in the standard deviation of the error in the fit of ξ_{BL} to ξ measured for the wheel ground control sweep is presented in Figure E.5.

Figure E.1: ξ versus δ_w regression, ground control sweep manoeuvre

Figure E.2: $\xi_{\xi_{trm}}$ versus $\delta_{\xi_{trm}}$ regression, ground control sweep manoeuvre

Figure E.3: ξ versus ξ^* regression for one 2311 manoeuvre Figure E.4: Trend of ξ sensitivity adjustment with dynamic pressure

E.5 Rate Limitation

The maximum aileron deflection rate is identified by incorporating a rate limitation module into the aileron control model, and adjusting the maximum rate parameter, $\dot{\xi}_{max}$ using bisection method to minimise the modelled aileron position error for the 2311 manoeuvres. This process identifies $\dot{\xi}_{max}$ as 61 degrees per second.

E.6 Saturation

The limits of aileron motion are identified from the wheel ground control sweep data, using bisection technique. These limits ξ_{min} and ξ_{max} are defined as -15.23 and 15.95 degrees, respectively.

Figure E.5: Improvement in ξ match with ξ_{play} optimisation

Appendix F

Determination of First-Order Time Constant from Limited Data

The determination of the time constant of a first-order step response from recorded data is a fundamental problem. The process for this determination is complicated by the limited time during which data is collected, so that the system output may not have settled to a final value by the end of the recording period. This appendix describes the process that has been developed to determine system time constants under this constraint.

F.1 Governing Equation of First-Order Systems

The governing equation of first-order systems is given in Equation (F.1). Here, $x(t)$ represents the system output, G is the system gain, τ is the time constant of the system response, and $u(t)$ is the input to the system.

$$\frac{dx(t)}{dt} = \frac{Gu(t) - x(t)}{\tau} \quad (\text{F.1})$$

For simplicity, $Gu(t)$ is replaced with the final settled value of $x(t)$, or x_∞ . The solution of the system response to a unit step-input, $u(t)$ is then given in Equation (F.2).

$$x(t) = x_\infty - e^{-t/\tau}(x_\infty - x_0) \quad (\text{F.2})$$

Rearranging Equation (F.2), we have a simple relation, Equation (F.3), by which τ may be determined. Letting $z(t)$ represent the right hand side of this equation, it is seen that a graph of $z(t)$ versus τ is a straight line so long as the system is first-order. τ is the negative inverse of the slope of $z(t)$.

$$z(t) = -\frac{t}{\tau} = \ln \left(\frac{x_\infty - x(t)}{x_\infty - x_0} \right) \quad (\text{F.3})$$

F.2 Estimation of System Output Final Value

The final settling value is required to solve for the system time constant, τ . However, with n data points, the last recorded value of x is x_n , so an estimate of the final value must be made. This is an iterative process, where this final value is estimated as per Equation (F.4).

$$x_\infty = x_n + k(x_n - x_0) \quad (\text{F.4})$$

Here, k is the extrapolation factor, and it represents the amount x_∞ exceeds x_n as a fraction of the observed change in $x(t)$ from the data. k is determined from the fraction of the full response, y , defined in Equation (F.5). Because this fraction is not known, initially it is assumed to equal 0.6321, as this is the response fraction for the duration of one time constant. With a guess for the response fraction, k is calculated

by Equation (F.6).

$$y \equiv \frac{x_n}{x_\infty} \quad (\text{F.5})$$

$$k = \frac{1 - y}{y} \quad (\text{F.6})$$

New estimates of the response fraction and extrapolation factor are calculated for each iteration once a new estimate of τ is available. The response fraction is calculated from a modified form of Equation (F.2), where the system gain is set to unity and the initial value of the output is zero. The time value given to this equation is the time of the last data point. This is represented in Equation (F.7).

$$y = 1 - \exp\left(-\frac{t_n}{\tau}\right) \quad (\text{F.7})$$

The new estimate for y , therefore, is the expected response fraction that would be observed in the duration of the recorded data with the most recent estimate of τ . k is then recalculated from the new y as per Equation (F.6), and a new estimate for x_∞ is made for the next iterations evaluation of τ .

F.3 Evaluation of τ

As stated above, τ is the negative inverse of the slope of $z(t)$. However, if the value of x_∞ used to calculate $z(t)$ is wrong, the trend of $z(t)$ with time will not be linear. If the estimate of k is too small, there will be a negative curvature in this trend. Conversely, if the estimate of k is too large, the curvature of $z(t)$ will be positive. Examples of these nonlinearities are given in Figures F.1 and F.2, respectively.

Figure F.1: Negative curvature in $z(t)$ for k too small

Figure F.2: Positive curvature in $z(t)$ for k too large

The estimation of τ from curves such as those shown in Figures F.1 and F.2 involves a polynomial fit to the data using least-squares approximation. Because of the curvature in $z(t)$, a second-order polynomial is used to fit the data. If a first-order polynomial fit is used, the solution of τ will not converge. This is because the positive curvature shown in Figure F.2 will result in too shallow of a line of best fit, and therefore too large of an estimate in τ . This over-estimate will result in a lower expected response fraction, y , and this leads to an increase in the value of k , which increases the positive curvature in $z(t)$. This is a positive feedback loop and is clearly unstable.

Using a second-order polynomial fit avoids this problem, as much of the curvature observed in $z(t)$ is absorbed in the quadratic term. The value of τ is then calculated from the linear coefficient of the polynomial, and the iteration scheme becomes conditionally stable. Solution will diverge if the initial condition is poor. Here, choosing the initial value of y to be 0.6321 brings the iterative scheme within stability bounds. The effect of using a second-order polynomial fit to the data becomes negligible as the solution converges, because the magnitude of the quadratic term decreases as the appropriate value of k is approached and $z(t)$ becomes more linear.

F.4 Iteration Scheme Overview

The iteration scheme outlined above is summarised here for clarity. The steps are listed below.

- 1) Initialise $y = 0.6321$, therefore $k = 0.582$ by Equation (F.6).

- 2) Estimate x_∞ by Equation (F.4).
- 3) Calculate $z(t)$ by Equation (F.3).
- 4) Fit a second-order polynomial to $z(t)$.
- 5) The new estimate for τ is the negative inverse of the linear coefficient of the polynomial found in step 4).
- 6) Calculate the change in the estimate of τ . If this change is below the error threshold, convergence is complete.
- 7) The new estimate for k is calculated by Equation (F.6).
- 8) Repeat steps 2) through 7) until convergence criterion is met or set maximum number of iterations has been reached.

Convergence happens rapidly. For example, only eight steps are required to converge to a solution of τ for the first-order response of a sensor to temperature change. In this case, the duration of the data record is 2.35τ , and the noise in the data is about 0.2% of the full range of the observed response. The convergence criteria set for this case is change in τ less than $1e-4$ minutes. The iterative convergence of the error in τ for this case is shown in Figure F.3.

Figure F.3: Iterative convergence of τ for thermal response of sensor

Appendix G

Identification of Polynomial Fit Uncertainty

This appendix describes the process used to establish the 95% uncertainty bounds for the polynomial coefficients resulting from regression analysis. Consider a simple case with independent variable measurement, $[x]$, and dependent variable measurement $[y]$. A second-order polynomial fit is to be applied to the apparent trend of $y^* = f(x)$. The proposed relationship then has the form of Equation (G.1)

$$\begin{bmatrix} [x^2] & [x] & [1] \end{bmatrix} \cdot \begin{bmatrix} A_2 & A_1 & A_0 \end{bmatrix}^T = \begin{bmatrix} [y] \end{bmatrix} \quad (\text{G.1})$$

The data $([x], [y])$ will have a scatter about the curve $y^* = f(x)$, quantified by the standard deviation of $[y^* - y]$. This standard deviation and the degrees of freedom are used to calculate the data scatter uncertainty, u_{fit} , for a 95% confidence level using Student's t-distribution table. Also, the data will contain measurement uncertainties u_x and u_y . Because u_{fit} has been calculated in the vertical axis, it is convenient to consider a total vertical uncertainty u_Y , which is calculated by root sum-squares of u_{fit} and the average value of u_y .

Now, the curve $y^* = f(x)$ is identified using multiple regression analysis. We wish to find the uncertainty of each coefficient of the fit polynomial. This process is essentially done by finding the limits of the offset, slope, curvature, and any higher-order trends that permit y^* to lie within the total 95% confidence envelope of the data, including the scatter and measurement uncertainties.

For the offset parameter, A_0 , the uncertainty is simply u_Y . It is evident that the curve may be moved up or down by this amount and remain within the uncertainty bound of the data.

$$u_{A0} = u_Y \quad (\text{G.2})$$

For the slope parameter, A_1 , the uncertainty is calculated as follows. The contribution of the linear term to y^* is calculated according to Equation (G.3). Four points are plotted around both of the endpoints of the resulting line to represent the uncertainty boundaries of the line endpoints, according to Equations (G.4) through (G.7). Next, each of the new points surrounding the start of the line is matched to each of the points surrounding the end of the line and the slope is calculated. Of the sixteen combinations, the greatest and the least slopes are $A_{1,max}$ and $A_{1,min}$, respectively. The uncertainty in A_1 , u_{A1} , then, is the average of the differences between A_0 and $A_{1,max}$ and $A_{1,min}$.

$$[y_x^*] = A_0 * [x] \quad (\text{G.3})$$

$$(x_{1,j}, y_{1,j}) = (x_j - u_x, y_j - u_Y) \quad (\text{G.4})$$

$$(x_{2,j}, y_{2,j}) = (x_j + u_x, y_j - u_Y) \quad (\text{G.5})$$

$$(x_{3,j}, y_{3,j}) = (x_j - u_x, y_j + u_Y) \quad (\text{G.6})$$

$$(x_{4,j}, y_{4,j}) = (x_j + u_x, y_j + u_Y) \quad (\text{G.7})$$

where $j = \text{start}, \text{end}$

$$u_{A1} = \text{mean}(A_{1,\max} - A_1, A_1 - A_{1,\min}) \quad (\text{G.8})$$

The procedure for calculating the uncertainty of A_2 or higher terms follows in similar fashion as for u_{A1} , with a transformation of variables. Instead of finding the slopes of $y_{x^2}^*$ with respect to x , these slopes are found with respect to x^2 . Otherwise, the steps of the solution are the same.

$$u_{A2} = \text{mean}(A_{2,\max} - A_2, A_2 - A_{2,\min}) \quad (\text{G.9})$$

An example of the results of this process is for the elevator control gearing equation for the CJ1, for which a second-order polynomial fit is used. The independent variable is column position, δ_c , and the dependent variable is elevator position, η . Data, scatter, and parameter estimation uncertainties are described in Equations (G.10) through (G.16). A plot of the regression is given in Figure G.1, including sample data error bars and the uncertainty bounds of the fit curve.

$$\delta_c = A_0 + A_1 \cdot \eta + A_2 \cdot \eta^2 \quad (\text{G.10})$$

$$u_{\delta_c} = 0.246 \text{ [deg]} \text{ (95\%)} \quad (\text{G.11})$$

$$u_{\eta} = 0.135 \text{ [deg]} \text{ (95\%)} \quad (\text{G.12})$$

$$u_{fit} = 0.100 \text{ [deg]} \text{ (95\%)} \quad (\text{G.13})$$

$$u_{A0} = 0.168 \text{ [deg]} \text{ (95\%)} \quad (\text{G.14})$$

$$u_{A1} = 0.116 \text{ (95\%)} \quad (\text{G.15})$$

$$u_{A2} = 0.00397 \text{ [1/deg]} \text{ (95\%)} \quad (\text{G.16})$$

Figure G.1: Regression of η versus δ_c showing uncertainty bounds of the curve