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ESSAYS ON CONSUMPTION WITH HABIT FORMATION

by

ZENNUBE AYLIN SECKIN, M.A.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

Department of Economics

Carleton University

Ottawa, Ontario

CANADA

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The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis,

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submitted by

Aylin Seckin, B.A., M.A.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Economics

Thesis Supervisor

External Examiner

Carleton University
April 1999

Dedication

To my father Nevzat Seçkin
and to my mother Ayla Seçkin,
without whom I would not find the courage and possibility
to achieve this objective,
without whom I would not have the chance
to fulfill my dreams.
Their love and compassion
for me cannot be translated in words.

Abstract

The aim of this thesis is to examine the implications of habit formation in a consumption-saving model with income uncertainty. By introducing habit formation, we have shown that habit formation itself leads to prudent behavior, in addition to the usual precautionary saving motive against income uncertainty.

We have shown that with habit formation, even in the quadratic case where the precautionary saving premium is zero, there is habits-induced saving. In the exponential case, we have obtained a closed form solution for consumption, which will provide a better framework for empirically testing the behavior of consumption over the business cycle. By allowing habit-forming preferences in consumption, we did not need to assume higher levels of income uncertainty to explain the excess sensitivity, the excess smoothness and the excess growth puzzles of consumption. We have also shown that the greater is the strength of habit formation the lower is the effect of income uncertainty on consumption.

By introducing liquidity constraints into the model with habit formation, we have shown that the effects of present and future shadow prices of liquidity constraint will be present and rise with the strength of habit formation.

In a consumption-leisure choice model with a Cobb-Douglas utility function and habit formation in consumption, the intertemporal marginal rate of substitution between consumption and leisure and the intertemporal marginal rate of substitution of leisure are higher the higher is the strength of habit formation. This result implies that the individual is less willing to intertemporally substitute current leisure with future leisure and with

current consumption. With the liquidity constraints depending on wage earnings, the intratemporal marginal rate of substitution between consumption and leisure decreases whenever the constraint binds.

Finally, in a two-period model, we have shown that habit formation leads to a larger prudence premium and greater precautionary saving, provided that the individual has decreasing absolute prudence. However, we have found that it is not possible to conclude, even with the utility function exhibiting a simple form of habit formation, that habits increase or decrease the precautionary premium “in the sense of Kimball” when there exists multiple periods.

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Chapter 1

INTRODUCTION

The literature on the consumption function aims to provide a theory that explains some of the time-series and cross-section correlations between consumption and income as well as correlations among consumption, wealth and interest rates. Being the largest component of aggregate demand in the economy, understanding the factors that motivate consumption behavior of households may bring about a better understanding of changes in aggregate demand over the business cycle.

One of the most striking facts about aggregate consumption behavior is that aggregate consumption is smooth relative to aggregate income. Shifts in aggregate income cause relatively small shifts in aggregate consumption, and variations in consumption about trend are smaller than variations in income about trend. The explanation of these facts is that consumption is determined by permanent income not by current income, and permanent income is smooth relative to current income. Innovations to income generate relatively small innovations to permanent income, and thus, to consumption. However, if current income is positively autocorrelated, the innovation variance of permanent income will be greater than that of current income. Deaton (1986) argues that permanent income is noisier than current income in such a case so that the Permanent Income Theory fails to explain the excess smoothness puzzle of consumption.

Several studies have shown that changes in consumption are highly correlated with anticipated changes to income. That is, changes in consumption can be explained by averages of past innovations to income. For the U.S., using macroeconomic data, Flavin (1981), Hall and Mishkin (1982), and many subsequent authors have found evidence that changes in consumption are positively related to predictable changes in income. This is the excess sensitivity puzzle of consumption.

There is also the excess expected growth puzzle of consumption. Hall and Mishkin (1982), Deaton (1986), among others, have pointed out that there have been long periods of time in which average U.S. aggregate consumption growth has been positive despite real interest rates that were very low (close to zero), and rates of time preference that were assumed to be zero.

The aim of this thesis is to examine the implications of habit formation in a consumption-saving model with uncertainty. Basically, we will drop an assumption of the basic Life-cycle Income Hypothesis/Permanent Income Hypothesis (LIH-PIH) model, i.e., the time-separability of preferences. Instead, we will adopt an assumption implying that individual preferences are intertemporally non-additive in a way that has been described as exhibiting habit persistence (or equivalently habit formation) in order to bring theoretical explanations to the behavior of consumption¹.

Campbell and Deaton (1989) and Deaton (1992) suggest habit formation as one of several potential explanations for the excess smoothness puzzle, without however actually exploring its implications in a theoretical analysis. We argue that habit formation itself leads to prudent behavior in addition to the usual precautionary saving motive against income uncertainty. However, we will show that with habit formation, the effect of income uncertainty on consumption is lower. The higher the habit formation parameter, i.e., the stronger the habits, the lower the effect of income uncertainty on consumption.

¹In the literature, these preferences have also been referred as time-non-separable. See Dunn and Singleton (1986), Eichenbaum, Hansen and Singleton (1988), Muellbauer (1988), Ferson and Constantinides (1991) and Dynan (1993a). We are following here the terminology of Altug and Labadie (1994), p.66.

More specifically, in chapter 3, with habit formation assumption in a representative agent model of intertemporal consumption-saving, we will first show that even in the quadratic case, where the precautionary saving premium is zero, there is habit formation induced saving. We will show that consumption depends not only on permanent income but also on past consumption, and that the excess sensitivity, the excess smoothness, and the excess growth puzzles of consumption can be solved with the habit formation assumption.

Next, in an exponential utility function model, by allowing habit-forming preferences in consumption, we will be able to explain consumption puzzles with only independently and identically distributed (i.i.d.) innovations to income². We will show that the marginal propensity to consume out of unanticipated income, (in the manner defined by Zeldes (1989a) and others), is lower, but the marginal propensity to consume out of anticipated income is positive, hence, consumption growth is higher with habit persistence. We will also show that habit formation induces a different kind of precautionary saving, and decreases precautionary saving against income uncertainty.

In chapter 4, we will introduce liquidity constraints to the intertemporal consumption-saving model with habit formation. The most recent work on liquidity constraints with non-separable preferences are the empirical studies of Meghir and Weber (1996) and Alessie, Devereux and Weber (1997). Meghir and Weber (1996) propose a method to test for liquidity constraints in a model with a general non-separable preference structure so that one can differentiate empirically between the effects of non-separability and liquidity constraints. Alessie, Devereux and Weber (1997) empirically test the Euler equation for consumption of durable and non-durable goods in an intertemporal framework, which allows the borrowing limit to depend on the stock of durables owned. They find evidence for non-separable preferences in consumption and liquidity constraints for young households. While these articles focus on empirical issues and do not consider

²Whereas Caballero (1990), in his model without habit formation, needed to assume uncertainty about higher moments of the distribution of income to explain these puzzles.

habit formation explicitly, we will focus on theoretical implications.

The motivation behind our analysis in this section is to explore the effects of habit formation on consumption choices of an individual facing income uncertainty. Based on the previous section's result we know that habit formation smooths consumption more than the LIH-PIH predicts. Then, it is interesting to argue for a precautionary motive played by habit-forming preferences in consumption together with binding liquidity constraints³.

First, we will theoretically analyze the response of lifetime maximizing consumers with habit-forming preferences facing liquidity constraints. Following Zeldes (1989b), we will first present a general infinite horizon model with liquidity constraints, where the consumer faces a liquidity constraint each period. Then, we will allow habit-forming preferences in consumption to examine the combined effect of habit formation and liquidity constraint. We will show that the presence of liquidity constraints increases the current marginal utility of consumption, implying a lower level of consumption than the case with time-separable preferences. We will, in particular, show that if the liquidity constraint is binding, not only the current period but all the future shadow prices of the liquidity constraint will affect current marginal utility of consumption. Due to the fact that the individual has habit-forming preferences in consumption, the future shadow prices of liquidity constraint will be present, and their effects will be higher the higher the strength of habit formation.

Next, we will relax a little the liquidity constraint assumption by allowing limited borrowing to the individual. We will reconstruct the model by adding an exogenously given debt limit, up to which, the individual can borrow each year. We will show that the current marginal utility of consumption will be higher when the current borrowing constraint is binding, implying much lower consumption than in the case with no borrowing

³By liquidity constraints, we mean that when the shadow price of the liquidity constraint is positive, then the individual is bound to consume current income without the option to borrow to smooth out consumption. Rather, the individual will save to loosen future binding liquidity constraints.

constraint and no habit formation. Since the individual is allowed to borrow up to a limit, this enables him or her to better smooth out consumption in low realizations of income. If the individual has accumulated a high amount of debt, he or she decreases current consumption to relax future borrowing constraints. In the next section, we will introduce borrowing constraints, another form of capital market imperfection, and determine the effects of habit formation on consumption.

In chapter 5, we will introduce leisure choice into the utility function with habit-forming preferences in consumption. The motivation behind this setting is to examine the effects of habit-forming preferences in consumption on current and future consumption and leisure choices. Our paper differs from several papers in this literature. While Hotz, Kydland and Sedlacek (1988) assume durability in leisure choices, Kennan (1988) assumes habit formation in leisure, Eichenbaum, Hansen and Singleton (1988) introduce durability in consumption and habit formation in leisure, we set up a consumption-leisure choice model with a habit-forming consumer to examine the behavior of the intertemporal marginal rate of substitution of leisure and that of the intratemporal marginal rate of substitution of leisure and consumption. We want to investigate, in particular, whether only the assumption of habit formation in consumption is capable of generating intertemporal substitution effects.

To this end, first, we will present the general model with no habit formation in consumption choices, in order to use it as a benchmark model to compare next, with the habit formation model. Then, we will introduce habit formation into consumption. We will show that, with a Cobb-Douglas utility function incorporating habit formation only in consumption, the marginal rate of substitution between consumption and leisure as well as the intertemporal marginal rate of substitution between current and future leisure are higher the higher the strength of habit formation. This result implies that the individual is less willing to intertemporally substitute current leisure for future leisure and current consumption.

Then, we will introduce liquidity constraints into the benchmark model and to the

model with habit formation in consumption, and endogenize the liquidity constraints by making them dependent on wage earnings. We will show that when the liquidity constraints depend on wage earnings, the intratemporal marginal rate of substitution between consumption and leisure decreases whenever the shadow price of the liquidity constraint is positive. More specifically, we will show that when considering taking one extra unit of leisure, the individual must take into account that the borrowing limit will decrease. Hence, the net marginal benefit of one unit of increase in leisure is less than the marginal utility of leisure. Current income falls with an increase in leisure, and leads to greater borrowing (including the interest cost) this period.

Finally, in chapter 6 we will examine the relationship between habit formation and Kimball's concept of prudence. More specifically, we will search for an answer whether habit formation leads to prudent behavior, and thus, induces precautionary savings in the sense of Kimball. Using Kimball's two-period model, we will show that habit formation leads to a larger prudence premium and greater precautionary saving, provided that the individual has decreasing absolute prudence. Finally, we will investigate the relationship between prudence and habit formation in a multi-period framework.

Chapter 2

LITERATURE REVIEW

In this chapter, we present a survey of the literature on consumption first by identifying some consumption puzzles, and reviewing the relevant empirical and theoretical work to solve those puzzles. Next, we will discuss the importance of the habit formation assumption as a type of non-separability of the utility function, and offer a brief account of studies on this issue. Then we will cover the liquidity constraints and borrowing constraints assumptions as other means to solve those consumption puzzles, and we will review the most important studies and their results. Finally, we will examine the literature on the intertemporal consumption and leisure choices. We will summarize the main findings of the most recent studies and the current state of this topic.

2.1 CONSUMPTION PUZZLES

Initial consumption theories were inspired by the Keynesian consumption function of the form $C_t = c(Y_t, r_t)$ which is a structural relationship between current consumption, current disposable income and the current interest rate. The empirical failure of the Keynesian consumption function consists of two types of conflicting evidence that made their appearance in the late 1940's. The first type of evidence came from estimates of the standard consumption function. They imply that the average propensity to consume

falls as the level of income rises.

The second piece of evidence was the finding by Kuznets (1946), using averages of data over long periods-10 and 30 years- that there was near proportionality between consumption and income. Kuznets's work suggests that, using long-term averages, there is little variation in the ratio of consumption to income and, in particular, that there is no tendency for the average propensity to consume to decline as disposable income rises. Kuznets's findings suggest that the average propensity to consume (APC) is constant over long periods, whereas the estimates of the consumption function based on the short-run data suggest that the APC falls as income rises. This also implies that the short-run marginal propensity to consume (MPC) is less than the MPC in the long-run.

Moreover, in aggregate time series, consumption was much smoother over the business cycle than income, which was not the case with the Keynesian consumption function. To better model consumption statistically, consumption was presented as a function of distributed lag of income. making consumption smoother than income, as the data required. This led to new theories on consumption, inspired by Irving Fisher's view that the consumption-saving behavior is not motivated by current disposable income alone, but by a household's lifetime income. Hence, in Fisher's theory, consumption-saving is an intertemporal problem rather than an atemporal problem one as in Keynesian formulation. The Modigliani-Ando-Brumberg life-cycle income hypothesis (LIH) and Friedman's permanent income hypothesis (PIH), developed in the late fifties in reaction to the empirical failure of the Keynesian consumption theory, draw their inspiration from Irving Fisher's (1930) dynamic theory of consumption. Both theories, based on the notion of a broader measure of income than just current income, relate current consumption to the expected present value of the lifetime income stream of individuals, including their accumulated assets to date, i.e., their "permanent income". Their prediction is that current consumption should not be very sensitive to current income, the latter being important only to the extent that it yields information about future income, thereby leading to a revision of expectations about the entire lifetime income, and hence to a

change in today's consumption through this channel. Instead, taking advantage of their ability to borrow and lend freely at the market interest rate, consumers aim to consume an approximately constant amount in relation to their expectations of their permanent income. That is, they tend to borrow whenever current income is lower than the lifetime average, and accumulate savings whenever current income is higher than the lifetime average in order to maintain a smooth consumption stream.

The life-cycle hypothesis views savings as resulting mainly from individuals' desires to provide consumption in old age. Consumption plans are made so as to achieve a smooth or even level of consumption by saving during high periods of income and dissaving during periods of low income. This is an important departure from the treatment of consumption as being based on current income, because the whole future profile of income enters into the calculation of lifetime consumption. The life-cycle approach explains both the long-run stability of the consumption-income ratio and its short-run fluctuations, by pointing out that people want to maintain a smooth profile of consumption even if their lifetime income profile is uneven, and thus, emphasizes the role of wealth in the consumption function.

Friedman's permanent income theory argues that people make their consumption decisions on the basis of their permanent or long-term income opportunities, not on their current income alone. Permanent income is the average and steady consumption rate a person could maintain for the entire lifetime, given the present value of wealth and income earned now and in the future. The permanent income hypothesis implies that there is a difference between the short-run and long-run marginal propensity to consume.

The reason is that when current income rises, the individual is not sure that the increase in income will be maintained over the longer period on which consumption is based. Accordingly, the full adjustment of consumption to a higher level of income occurs only if the increase in income turns out to be permanent and not transitory. Consumers increase their consumption only if the changes are permanent. They react only minimally to transitory changes in income. Therefore, unexpected changes in income

affect consumption only to the extent that they affect permanent income and their effects are expected to be permanent. Hence, consumption is less volatile than income.

Consumption functions based on the life-cycle income hypothesis and permanent income hypothesis (LIH-PIH) that were empirically estimated in the sixties, and even simulated, took a rigid view of expectations¹. The models and their empirical methods were vigorously criticized by Lucas (1976). His critique was that there was no such thing as a consumption function. There is a structural relation between permanent income and consumption, but the consumption function imposes a structural relation between observed income and permanent income although there is no reason to expect that relation to remain invariant. Changes elsewhere in the economy- for example, in monetary or fiscal policy- could alter the optimal way for a consumer to form expectations about permanent income based on observed income. According to Lucas, a model for policy evaluation based on this consumption function was misleading and wrong. Lucas's view of the consumption model is based on the intertemporal choice of consumers, seeking to maximize their expected lifetime utility, given all the information about current and future income and prices. The individual uses all available knowledge about the behavior of other factors in the economy, including the government, in the expectation forming process.

In a seminal paper (1978), Hall answers the Lucas's critique by proposing an alternative way of testing PIH, that is, by using directly the Euler equation instead of estimating the final consumption function. Hall neither tried to improve the traditional consumption function nor tried to estimate the major parameters of the utility function. Rather, he formulated a simple empirical test of the idea that consumers maximize the expected value of lifetime utility subject to a budget constraint, and face a constant real interest rate. The implication is that lagged real disposable income will have little predictive power for consumption and since innovation to income is unpredictable at time t , the

¹These early versions of LIH-PIH models assume that the agents have adaptive expectations, and capital markets are perfect in the sense that agents can lend or borrow against expected future income at the same interest rate.

best estimate of future consumption is current consumption. All other past values of consumption should have no predictive power.

Following Hall's work, Flavin (1981), Hall and Mishkin (1982) and others found that consumers do not smooth out consumption as much as predicted by the LIH-PIH. In fact, current consumption seems to be excessively sensitive to current and lagged income². Flavin (1981), using a testing method similar to the one proposed by Hall, examined the empirical relationship between income and consumption in the rational expectations framework and found a positive correlation, (a correlation that would be zero if the PIH model were true), between the change in consumption and lagged changes in income. She found the parameter describing the excess response of consumption to the anticipated changes in income to be 0.36. This has come to be known as the excess sensitivity puzzle of consumption³.

Besides the excess sensitivity puzzle of consumption, there are also the excess smoothness and excess growth puzzles of consumption, both of which were noticed by Deaton (1986). The excess smoothness of consumption is the slowness of the response of consumption to unanticipated changes in income. Deaton observed that consumption exhibited this type of behavior when labor income is an integrated process, that is, when labor income has a unit root. He concluded that the representative agent version of the permanent income hypothesis can be rejected because it fails to predict the fact that consumption is smooth, the very reason that it was proposed for in the first place. This anomaly in the joint behavior of consumption and income is known as "Deaton's paradox"⁴. Campbell and Deaton (1989) show that consumption is slow to adjust to

²Hayashi (1985b) conducts a panel study of Japanese households using a model with durability in consumption where he finds that lagged income has a significant impact on the change in consumption, thus, finding further evidence for excess sensitivity.

³Mankiw and Shapiro (1985) criticized Flavin's work on the grounds that her detrending procedure was responsible for the finding of excess sensitivity of current consumption to anticipated changes in income. The use of detrended data biases the test toward rejection of the LIH-PIH if income is a random walk. Since consumption innovation is proportional to income innovation, the consumption and income series will be highly correlated random walks.

⁴The excess smoothness puzzle has also been challenged in different papers. Christiano (1987) studies a simple general equilibrium real business cycle model that allows the interest rate to change over

innovations in income in the sense that changes in consumption are related to averages of previous innovations. This exposes both the smoothness and the sensitivity puzzles. Finally, regarding the excess growth puzzle, Hall and Mishkin (1982) found a growth rate of consumption 2 percent in excess the rate predicted by the PIH.

These are then the puzzles that have triggered a growing literature. One strand of that literature follows new empirical methodologies and econometric methods to test and explain these puzzles. Another strand of the literature tries to explain some of these puzzles by relaxing some of the assumptions (like perfect capital markets, constant interest rate, etc.) underlying the LIH-PIH.

2.2 HABIT FORMATION

Habit formation is a form of non-separability in preferences. It has been used in several contexts in economics. The implications of habit formation were first discussed in Duesenberry's work (1949). His proposition was that families are willing to sacrifice saving in order to protect their living standards. In the event of a fall in income, consumption will not fall proportionately, producing a ratchet effect. This was reflected by including the individual's previous high income in the utility function. Thus, the ratchet effect creates an inertia in consumers' responses to current income changes in terms of consumers' memories of past peaks of real income. Duesenberry assumes that the decline of the effect of past habits is discontinuous over time rather than continuous as in the case of more recent models of habit formation.

Whereas time-separable preferences imply that current utility depends only on current consumption, intertemporally non-separable preferences with habit formation imply that

time. He finds that small influences on consumption through intertemporal substitution together with variations in real returns could explain an apparent excess smoothness of consumption. Christiano argues that, if the intertemporal elasticity of substitution is high, then the consumption response to an unexpected permanent change may be very low. That may be the reason of excess smoothness.

past real consumption patterns and levels form consumer habits persist long enough to reduce the effects of current income changes on current consumption. For a given level of current expenditures, past purchases contribute to a habit stock. Hence, it is an increase of current consumption over and above the habit stock which raises current utility. An individual with preferences exhibiting habit formation who consumes a lot in period $t - 1$ will get used to that high standard of consumption, and would like to consume more. The stronger is the habit persistence, the more averse is the consumer to a fall in consumption, and will require larger consumption in order to have a positive utility.

An increase in current consumption in response to an increase in wealth or permanent income has two effects: it increases current utility, holding habit stock fixed; but (everything else equal) decreases utility at time $t + 1$. Since increasing consumption today generates a future externality, the rational consumer will respond to an increase in wealth or permanent income with a more moderate increase in consumption. In the presence of habit formation, an increase in current consumption increases the marginal utility of future consumption. There is, thus, an adjacent complementarity in consumption.

Ryder and Heal (1973) were the first to formally model habit formation in a continuous time model⁵. Then, Becker and Murphy (1988) examined rational addiction in a model with habit formation using the same specification. Constantinides (1990) also used the continuous time form and introduced habit formation in a rational expectations model to resolve the equity premium puzzle. By doing that, he was able to generate the necessary high variability in marginal rate of substitution in consumption with relatively low variability in the consumption growth⁶. Carroll, Overland, and Weil (1995) study the implications of habit formation in an endogenous growth model with no uncertainty. They find that an economy that starts off in the lucky state of having a low habit stock (in our model a low initial consumption) relative to its capital stock will save more and grow

⁵Ryder and Heal (1973) studied the implications of habit formation in a two-factor growth model.

⁶In his model, the presence of habit formation drives a wedge between the relative risk aversion of the representative agent and the intertemporal elasticity of substitution in consumption.

quickly, thus, lengthening the time during which the level of consumption will be high relative to habits. Habit formation in consumption allows them to explain the positive correlation between saving and growth rates observed in the data. They show that habit formation can lead to a positive short-run response of saving to a favorable productivity shock, even when the long-run relation between growth and saving is negative.

Another specification of habit formation is the one with discrete time. This form has been used by Hayashi (1985b), Muellbauer (1988), Deaton (1992), Dynan (1993a). In this specification, current utility depends on current net consumption, i.e., current consumption minus the effects of current habit stock.

Several empirical papers in the consumption literature have argued that habits may play an important role in determining consumption. Ferson and Constantinides (1991), Deaton and Paxson (1992), Dynan (1993a), Carroll and Weil (1994), Carroll, Overland and Weil (1995), Heaton (1995), Fuhrer and Klein (1998) are among these. Ferson and Constantinides test the combined effects of durability and habit formation in consumption choices. They find that habit formation dominates durability in monthly, quarterly and annual data. Dynan finds that habit formation has a moderate influence on consumption, and argues that habit formation may explain excess smoothness of aggregate consumption. Heaton examines the implications of various forms of intertemporal non-separability of total non-durable consumption on the behavior of asset prices. He finds evidence for the local intertemporal substitution of consumption with habit formation occurring over longer periods of time. In a recent work, Fuhrer and Klein confirm the evidence of habit-forming preferences for G-7 countries⁷.

Habit formation induces behavior akin to prudence. It has been suggested by Deaton (1992) and by Carroll, Overland and Weil (1995) that habit formation motivates pre-

⁷On the other hand, Dunn and Singleton (1986) and Eichenbaum, Hansen and Singleton (1988), Muellbauer (1988), study the U.S. aggregate monthly consumption data, and find no evidence for habit formation.

cautionary saving behavior, because consumers save more in order to achieve a higher consumption growth, whereas consumers without habit persistence want consumption across periods to be similar, and resist growth in their consumption profiles.

Most of the empirical research on the LIH-PIH uses the certainty equivalence assumption, since that enables one to find a closed form solution for consumption. With a quadratic utility function, however, it is not possible to capture all of the uncertainty involved in future income, since consumers only take into account the mean of future income, and are not affected by the higher moments of the distribution of income. Yet, uncertainty about the variability of future income may substantially alter consumption behavior by causing greater saving from a risk averse consumer. In addition, a quadratic utility function has the undesirable property of exhibiting increasing absolute risk aversion, implying that consumers' risk aversion increases in wealth.

Leland (1968), in his analysis of the theory of precautionary saving, has shown that whenever we depart from the certainty equivalence (CE) framework, by allowing the utility function to be time-separable and have a positive third derivative, an increase in labor income uncertainty will reduce current consumption and change the slope of the consumption function when insurance markets are not complete. In a two-period model with a deterministic interest rate. Sandmo (1970) shows that if current consumption is a normal good and temporal risk aversion is decreasing, saving is higher under uncertainty⁸. Kimball (1990) introduced the concept of prudence to characterize "the sensitivity of a decision variable to risk". He argues that income risk leads to precautionary saving when the individual has non-increasing absolute risk aversion. However, if the interest rate is stochastic but income is deterministic, Sandmo shows that the response of saving to risk is ambiguous. In a non-expected utility model with both income and interest rate risk, Demers and Demers (1992) show that the impact on saving of facing uncertainty is

⁸Let $U(c_1, c_2)$ denote the two-period utility function where c_t is consumption at time $t=1,2$.

Sandmo assumes that $\frac{-\partial^2 U / \partial c_2^2}{\partial U / \partial c_2}$ is decreasing in c_2 and increasing in c_1 . He defines this condition as decreasing temporal risk aversion.

equivalent to facing a lower income and interest rate with certainty.

Skinner (1988) suggests that precautionary savings can account for as a large fraction as 56 percent of aggregate savings⁹. Prudent behavior under uncertainty may also be one of the reasons why very young households do have more savings despite higher expected incomes¹⁰. Zeldes (1989a) explored the implications of a constant relative risk aversion (CRRA) utility function, and thus, departed from the certainty equivalence framework used by Hall and others. Zeldes also attributes Flavin's (1981) excess sensitivity of consumption result to the precautionary effect of uncertain labor income. While it was not possible to find a closed form solution for consumption with CRRA preferences, he was able, using numerical methods, to explain both the excess sensitivity and the excess growth rate puzzles.

Caballero (1990) used a constant absolute risk aversion (CARA) utility function and was able to obtain a closed form solution for consumption and precautionary behavior on the part of the consumer¹¹. He studied the interactions between precautionary savings and different stochastic processes for labor income.

Hubbard, Skinner and Zeldes (1994) examine predictions of a LIH-PIH model in which individuals face uncertainty with respect to their lifetime earnings, unexpected medical expenditures, and imperfect insurance and lending markets. Their model explains well the characteristics of aggregate wealth, cross-sectional differences in wealth-age and consumption-age profiles by education group and short-run time-series, and comovements of consumption and income.

⁹Aiyagari (1992), however, argues that the effects of uncertainty are smaller in a general equilibrium model. Guiso, Jappelli and Terlizzese (1992) find a smaller role for precautionary saving.

¹⁰Skinner assumes that agents display constant relative risk aversion. One appealing feature of the approximation is that consumption depends negatively on the ratio of uncertain to certain household resources.

¹¹Caballero who assumes constant absolute risk aversion derives an approximation to optimal consumption which is equal to the certainty equivalence plus a precautionary element which depends on the variability of income. This additive term is due to the assumption of CARA preferences while it is far easier to work with than the more complex form of CRRA.

2.3 LIQUIDITY CONSTRAINTS

One explanation for the rejection of the LIH-PIH is that some households are liquidity constrained¹². It was Tobin who first used the term “liquidity constraint”¹³. For consumers who cannot borrow or can only do so at higher interest rates, current consumption is closely tied to current income.

The widely accepted definition of the liquidity constraint term is that consumers are liquidity constrained if they face quantity constraints on the amount of borrowing (credit rationing) or if the loan rates available to them are higher than the rates at which they could lend (differential interest rates)¹⁴.

Liquidity constraints impose some price or quantity restrictions on holding assets. If an individual is constrained from transferring additional resources from tomorrow to today, due to liquidity constraints, the marginal utility of consumption must be higher today relative to tomorrow than would be predicted in an unconstrained model. In that case, the multiplier representing liquidity constraints will be positive, and the Euler equation between period t and period $t + 1$ will be violated. If the current liquidity constraint is not binding but there is a positive probability that the future constraints will bind sometime in the future, this will lower the current consumption of any risk averse individual. In this case, although the current Euler equation is satisfied with equality the consumer is “globally constrained”¹⁵.

Following Hall’s work, Flavin (1981), Hall and Mishkin (1982) and others found that consumers do not smooth out consumption as much as predicted by the LIH-PIH. In fact, current consumption seems to be excessively sensitive to current and lagged income¹⁶.

¹²See Blundell (1988) for a survey on this literature.

¹³See Tobin (1980, p.57-58), for a discussion on liquidity constraints and their macroeconomic implications.

¹⁴See Hayashi (1985a).

¹⁵See Zeldes (1989b).

¹⁶Pissarides (1978) examines the implications of liquidity for the optimal consumption plan and in particular for the hump saving function obtained by the life-cycle hypothesis and for the relation between

Hall and Mishkin (1982), by directly examining the question of excess sensitivity of consumption to innovations of income in a certainty equivalence framework, find that 20 per cent of consumption responds much more strongly to transitory rather than to permanent movements of income. They conclude that consumption does not adjust in the same mechanical way to every change in income. In addition, they find a negative relation between lagged changes in income and current changes in consumption, as would be generated by the existence of liquidity constraints.

Hubbard and Judd (1986) simulate a model with a constraint on net worth which shows that 19 percent of U.S. consumers are liquidity constrained. Mariger (1986) derives a life-cycle model with endogenous liquidity constraints, and using a cross-section data, estimates that 19.4 percent of U.S. families are liquidity constrained.

In general, liquidity constraints are treated as exogenous¹⁷. Hayashi (1985a) and Zeldes (1989b) investigate the correlation among consumption, wealth, and income for various population groups, and infer that liquidity constraints are important for younger families with low levels of wealth and savings¹⁸. However, in their data set, constrained consumers are not observable, and have to be identified through indirect evidence (sample selection bias).

Hayashi defines the liquidity constraint as some exogenously given upper bound for consumption so that consumption cannot exceed this limit. If the upper bound is sufficiently large, it reduces to the permanent income hypothesis. Through sample selection bias, he divides households between high saving and low saving households, and assumes that high saving households are not bound by the liquidity constraint. He finds that the coefficients in the reduced form equation for consumption for high saving households ap-

the mpc out of transitory income and the mpc out of permanent income. He concludes that if the individual can purchase and sell assets with different liquidities, then the chosen consumption-wealth ratio is not independent of the timing of income, even with lifetime utility maximization and homothetic preferences. Thus, the correlation between consumption and current income should be greater than the one suggested by wealth theories.

¹⁷See also Deaton (1991), Hubbard, Skinner and Zeldes (1993).

¹⁸Hayashi (1987) argues that even though the consumer's planning horizon is infinite, the likelihood of future liquidity constraints effectively shortens the horizon.

pear to be quite different from those of the rest of the sample. He interprets this finding as evidence of liquidity constraints.

As Hayashi, Zeldes also uses a sample splitting technique to identify liquidity constrained households. Both presume that the Euler equations implied by the PIH are likely to hold for high wealth households under the assumption that this group is not constrained. Hayashi splits the sample according to the average propensity to save, whereas Zeldes uses the ratios of wealth and financial assets to disposable income.

Jappelli (1990) adopts a different approach which makes use of data where constrained consumers are observable, and explicitly links their existence to credit market imperfections and personal characteristics. He finds that 19 percent of families in the U.S. are liquidity constrained. They are on the average younger than the rest of the population, and they account for 12.7 percent of total income and 7 percent of wealth. He shows that this fraction depends on consumers' and lenders' behavior and that it is likely to vary in response to changes in current and future income, demographic variables and financial intermediaries.

One of the explanations of the excess sensitivity could be myopia in consumption, i.e., the consumer could behave according to the Keynesian consumption rule. Hall (1978), Hayashi (1987), Muellbauer (1988), Campbell and Mankiw (1989) have suggested that the excess sensitivity of consumption to predictable changes in income might be attributable to a fraction of the population following a Keynesian non-optimizing rule of thumb. Such behavior occurs when the liquidity constraint is binding at time t and $t - 1$. If the person starts and ends the period with no wealth, then consumption will be equal to income during that period: $c_t = y_t$. This is the case where the consumer is constrained to borrow each successive period.

However, liquidity constraints do not imply in general Keynesian behavior. The consumer can always smooth out consumption by saving more during high income re-

alizations, and dissaving with low income realizations¹⁹. If the consumer anticipates a higher income, thus, a loosening of liquidity constraints, he chooses a higher consumption in the current period by running down his assets.

Flavin (1985) attempts to determine the cause of excess sensitivity of consumption to current income: whether it is capital market imperfections which prevent some individuals from realizing their desired consumption path or the myopic behavior of the representative agent. She examines, as a specific alternative to PIH, a Keynesian consumption function in which the mpc out of transitory income is non-zero. By using the aggregate unemployment rate as a proxy for liquidity constraints, she finds that the estimated mpc out of transitory income is explained almost entirely by liquidity constraints. She concludes that the simple Keynesian consumption function with non-zero mpc out of transitory income is an incomplete model, and argues that the observed excess sensitivity of consumption to current income can be explained by liquidity constraints.

Campbell and Mankiw (1989) find striking evidence against the PIH. Using instrumental variables method, they have estimated the fraction of the population that consumes its current income about 50 percent.

Antzoulatos (1994) shows how to distinguish between the model with rule of thumb consumers and the model with liquidity constraints. He finds that under liquidity constraints, consumption growth will depend not only on contemporaneous expected income, as the model with the rule of thumb consumers predict, but also on future expected income.

More recently, Garcia, Lusardi and Ng (1997) model the probability of being liquidity constrained, and split the sample according to income. They find evidence of excess sensitivity among the liquidity constrained and among the unconstrained. They reject

¹⁹... "Correspondingly, given the standard earnings profiles, net asset holdings for liquidity constrained consumers will always be at least as high as those of unconstrained consumers whether or not the constraints are actually binding at that moment. However, perhaps the most important effect of borrowing constraints is to increase the marginal propensity to consume for the economy as a whole".(Deaton and Muellbauer (1980), p.318).

the rule of thumb behavior, but suggest the role of non-separable preferences for the excess sensitivity of the unconstrained people.

While so many studies find support for the existence of liquidity constraints, Bernanke (1985), Runkle (1991) and Flavin (1994) detect no such evidence. Bernanke studies a simultaneous determination of non-durables and durables consumption in a framework with a non-separable utility function and with adjustment costs. He argues that excess sensitivity of consumption can be attributable to durable goods rather than to liquidity constraints. Similarly, using panel data, Runkle examines the validity of PIH by explicitly modeling individual household's consumption decision and tests for liquidity constraints by using a linearized Euler equation. He finds strong evidence for the validity of the PIH, and no evidence for any persistent household specific effect in the data. His results suggest that measurement error in consumption is a serious problem in panel data.

More recently, models of precautionary saving combined with liquidity constraints have been used to examine the time-series features of consumption as well as the distribution of wealth in the U.S. economy²⁰. Deaton (1991) and Carroll (1992, 1997) and Carroll and Samwick (1997) argue that existing precautionary savings models have failed to predict the large fraction of the population with extremely low wealth accumulation. In their models of precautionary saving with liquidity constraints, they assume that the rate of time preference is high relative to the real interest rate. In response to earnings uncertainty together with liquidity constraints, individuals maintain a "buffer stock" or contingency fund against income downturns. Impatience keeps these buffer stocks small, thus, providing a possible explanation of why so many households save little throughout their life.

Similarly, Xu (1995) proves that there exists a unique level of wealth below which precautionary savings caused by liquidity constraints have no effects at all. He emphasizes the importance of precautionary savings caused by liquidity constraints over the usual

²⁰Hubbard, Skinner and Zeldes (1993) assume uncertain lifetime. They show that the elderly has a high precautionary saving to cushion uncertain health expenditures.

precautionary savings due to income uncertainty. An upper and lower bound to income are necessary for the separability of precautionary saving due to income uncertainty from the precautionary saving caused by liquidity constraints. With the assumption of impatient consumers, when borrowing is prohibited, the agents save more than they would in the pure life-cycle model as long as wealth is below its critical level. This is not only because the constrained agents are forced to save more, but they might want to save more even if liquidity constraints are not currently binding. What makes them more prudent is the possibility that they may not be able to choose consumption as desired sometime in the future because of future binding liquidity constraints²¹. They accumulate enough liquid wealth so that they will not be affected by future liquidity constraints, and can implement their own optimal consumption plan even in the low realizations of income²².

In an empirical intertemporal consumption-saving model with durable goods and liquidity constraints, Chah, Ramey and Starr (1995) find evidence for liquidity constraints²³. In their model, consumers anticipate a future increase in their income so that they increase only their durable goods holdings since they cannot prefinance non-durables consumption. Thus, the anticipatory movement of durables reveals more information about the future change in the marginal utility of non-durables than does the predicted change in income.

Following their work, Alessie, Devereux and Weber (1997) model consumption of durables and non-durable goods in an intertemporal framework with varying financial conditions. The borrowing limit is assumed to depend on the stock of durables owned. They find evidence for non-separability in preferences.

²¹This is also a point raised by Zeldes (1989a).

²²Bewley (1977) shows that, in an infinite horizon economy with uncertainty about income or tastes and with the equality of rate of interest and rate of time preference, individuals will tend to accumulate enough wealth over time so that they eventually act as if there is no uncertainty or borrowing constraint. Given many consumers with finite initial wealth, however, there will be periods in which some individuals hit the borrowing constraint while others do not.

²³Their model has a time-separable utility function in durable and non-durable goods.

Guiso, Jappelli and Terliezzese (1996) find support for the economic theory suggesting that uninsurable income risk and the expectation of future borrowing constraints can reduce the share of risky assets in a household's portfolio. If the utility function exhibits decreasing absolute risk aversion and decreasing prudence, the individuals will reduce their exposure to rate of return risk when confronted with other independent risks. If there are transaction costs, the expectation of future borrowing constraints should induce individuals to keep a lower proportion of their wealth in the form of illiquid and risky assets. Income risk affects the relation between borrowing constraints and the composition of the household portfolio. Many of the determinants of the probability of being liquidity constrained are time-invariant, or evolve slowly in time so that liquidity constraints persist. Moreover, the theory suggests that credit history plays an important role in the allocation of credit, which is another source of persistence.

Finally, in a general equilibrium setting, as illustrated by Stiglitz and Weiss (1981), liquidity constraints may arise from imperfect information. Furthermore, Scheinkman and Weiss (1986) examine a general equilibrium model with individual specific uncertainty and imperfect markets, where lenders may have difficulties in assessing the ability of potential clients to repay their loans. They show that imposing an exogenous restriction that individuals cannot borrow against their future labor income can induce price and output fluctuations that are similar to actual business cycles. More recently, in a general equilibrium framework, Diaz-Gimenez (1997) finds that in an economy with idiosyncratic risk and liquidity constraints, fluctuations in output are 20 percent larger than an economy with no liquidity constraints²⁴.

²⁴Diaz-Gimenez also finds that the liquidity constrained economy has a higher volatility of hours of work and productivity than the economy without liquidity constraint.

2.4 BORROWING CONSTRAINTS

Borrowing constraint is a more relaxed form of liquidity constraints which allows the consumer to borrow a limited amount from the capital markets. One of the uses of external finance is sheltering consumption from fluctuations in income, whether they are due to internal or external factors. In this section, we will especially discuss the implications of borrowing constraints in the context of international borrowing.

In the international context, first, there is the additional complication of the existence of sovereign risk, that is, the possibility of the non-payment of the due debt by the borrower country, while maintaining control of its associated assets. The sovereign risk arises because of the lack of enforcement in international borrowing. International debt has no clear-cut rules and regulations as domestic debt which is a legal obligation and regulated by the domestic law of the country.

Second, repayment of international debt however depends on the willingness of the borrowers to pay their debt. Therefore, the penalty imposed on a country is indirect since there is no direct mechanism by which the creditor is fully guaranteed.

The debt crisis of the 1980's opened a vast literature analyzing sovereign lending²⁵. The question of credit rationing in international markets is considered first in Jaffee and Modigliani (1969), Eaton and Gersovitz (1981a, 1981b)²⁶.

Jaffee and Modigliani (1969) argue that if the borrower's resources available to repay a loan are limited then, there is clearly an upper bound on the amount that a lender will be willing to lend. They show how credit rationing can arise when the borrower's insolvency threatens a loan. Even though the lender seizes all the borrower's assets if default occurs, the borrower can affect the probability of being able to repay.

²⁵See also McDonald (1982), Folkerts-Landau (1982), Sachs (1984), Edwards (1984), Cooper and Sachs (1985), Dornbusch and Fischer (1985), Kaletsky (1985), Krugman (1985), Eaton, Gersovitz and Stiglitz (1986), Cohen and Sachs (1986), Eichengreen and Portes (1986), Cohen (1988a, 1988b), Cohen (1991), Cohen (1992).

²⁶There are many other debt issues such as debt overhang, debt relief, secondary market for debt, debt rescheduling that are not addressed in this work.

Eaton and Gersovitz (1981a) provided a seminal paper on the theory of sovereign country borrowing in which the country has alternating bad and good years. They take an approach that allows for the possibility of rationing. Lenders set the credit ceiling such that the amount is the maximum loan that had been paid in the past by the borrower. Actual borrowing in a given year is the minimum of the credit ceiling and the amount that maximizes expected utility of the country. The country borrows in a bad year, and repays with interest in the following good year. The model is based on consumption smoothing under a utility function with declining marginal utility such that the present discounted value of consumption is maximized by being able to supplement domestic output with borrowed consumption in the bad years, and forgoing some extra consumption in the good years. In their model, the risk of default leads to the denial of future access to credit.

The amount the country desires to borrow rises with the growth rate of income, because future marginal utility at the time of repayment is smaller. The credit ceiling also rises with the growth rate of income, unless there is an unusually large rate of decline in marginal utility. When the perceived probability of default exceeds a given level, a credit ceiling will be reached and that particular country is completely excluded from the market.

The credit ceiling is determined such that the total benefits of default (which increase with debt) just equal the total cost of default, and rises with the variability of income. When the country's income is stochastic, an increase in its variability has an ambiguous effect on credit ceiling, raising it by increasing the penalty of exclusion, but lowering it by increasing the probability of low incomes. A higher income growth rate raises the desired level of debt. However, a higher growth rate may or may not raise the credit ceiling. Furthermore, a borrower expecting a higher future income may have less to fear from the future effects of a cut-off lending, thus, lowering the debt ceiling. They estimate the demand and supply schedules for debt, and analyze the probability of credit rationing, and obtain that 80 percent of their sample is constrained by a probability of more than

0.5.

Edwards (1984) explains the determinants of the interest rate differential for developing countries. Cooper and Sachs (1985) examine optimal borrowing of the debtor countries. Their solvency constraint on borrowing implies that debt cannot exceed the present discounted value of the country's potential future trade surpluses. Insolvency arises when assets are less than liabilities; this is the bankruptcy situation for a country, the violation of the intertemporal budget constraint. Liquidity problems, which are cash flow problems, arise even if the country is solvent, when available cash is not sufficient to meet current obligations. They assume that creditors can impose sanctions against default: debt penalty is a constant fraction of the present discounted value of its future income. Default of debt incurs a loss to the creditors and a gain to debtors. Therefore, creditors set a credit ceiling which depends on the debt penalty. Lenders will not lend beyond that point at which it is in the interests of borrowers to repudiate. There will be a point at which increasing the interest rate will not compensate for the increased probability of default, and further loans will not be extended. They show if there is only solvency constraint, the country's optimal borrowing smooths out consumption over time, i.e., at all times the marginal utility of consumption, discounted by the net of the time preference rate, and the international borrowing rate is constant and equal to the marginal utility of wealth. Transitory shocks have little impact on the long-term marginal utility of wealth, whereas permanent shocks reduce wealth, and raise its marginal utility. Hence, the optimal borrowing implies financing a temporary shock, and adjusting to a permanent shock by reducing consumption. Cooper and Sachs argue that both liquidity risk and default risk are likely to limit the borrowing potential of the debtor far below the solvency levels.

Assuming that lenders are risk neutral and borrowers risk averse, Cohen and Sachs (1986) propose several determinants of the debt ceiling. The debt ceiling is higher, the greater the borrower's elasticity of marginal utility, the higher the variability of its income, the lower the cost of smoothing by foreign finance, and the more limited are domestic

resources for consumption smoothing.

More recent research on borrowing constraints of developing countries are the ones of Lewis (1997) and Adda and Eaton (1998). Lewis examines the differences in consumption sensitivity to output growth across various measures of international restrictions²⁷. She finds that consumption is more sensitive to income changes for restricted countries than unrestricted countries. Adda and Eaton (1998) develop a methodology for estimating the optimal borrowing behavior by an intertemporal utility maximizing agent subject to a borrowing constraint. They estimate the preference parameters as well as the parameters of the distribution of the credit ceilings. The previous literature on borrowing focused on tests of liquidity constraints rather than the estimation of their level and their determinants. They find that the credit ceiling is about 5 to 50 percent of mean income over 29 countries in their sample. They also find that one percentage increase in a country's mean income raises its credit ceiling by about 1.6 percent.

2.5 CONSUMPTION-LEISURE CHOICE

Intertemporal substitution of goods and leisure is an important topic in modern macroeconomic theory²⁸. The basic result of general labor supply models is that labor supply will respond to temporary movements in the real wage even though it may not respond to permanent real wage changes²⁹.

²⁷Bilateral trade agreements, foreign borrowing contracts, etc. are the type of restrictions that Lewis assumes.

²⁸See Heckman and MaCurdy (1980), MaCurdy (1981, 1983), Altonji (1982, 1986) and Ghez and Becker (1975). Ghez and Becker assume a household production function with time and goods as inputs. The individual produces consumption goods with leisure and goods and derives utility from the production of consumption goods.

²⁹The behavior of the labor supply can be divided into three types of effects: *efficiency effect*; making individuals work more in periods when the wage is higher, *interest rate effect*; making individuals work much at first and less later on and *time preference effect*; making individuals work little at first and more later on. If the rate of interest is equal to the rate of time preference, then work and wage rate always move in the same direction over the life-cycle. If the rate of interest is greater than the rate of time preference, then work and wage rate may move in opposite directions for at least part of the life-cycle. (See Killingsworth (1983)).

The intertemporal substitution of leisure hypothesis of Lucas and Rapping (1969) is that differences in the relative price of consumption and leisure between the present and future periods induce workers to alter their supply of labor in the current period. The behavioral assumption is that leisure in the current period is highly substitutable with leisure and goods in other periods. Consequently, movements in the current real wage rates relative to the expected discounted future real wage rates provoke a large labor supply response³⁰. The deviation of labor supply from its trend level is interpreted as a measure of individual unemployment. Cyclical movements in labor supply and unemployment are, therefore, attributed to changes in real interest rates and real wage rates.

Empirically, using an adaptive expectations framework, Lucas and Rapping are reasonably successful in explaining labor supply and unemployment between 1930-1965 as functions of the current real wage rate, the expected future real wage rate, and the expected interest rates. They find that the elasticity of labor supply with respect to temporary wage changes is around 4.6. Hence, relatively small transitory movements in the perceived real wage rate can have large effects on the path of labor supply as individuals time their participation to coincide with periods of high transitory wages. On the other hand, permanent changes, because they do not affect the timing decision, are expected to have a much smaller effect on participation. Thus, unless leisure is very substitutable across periods, large observed cyclical variations in employment could not possibly be caused by the response of labor supply to the relatively small fluctuations which are found in real wages and real interest rates.

However, MaCurdy (1981) presents a structural intertemporal model of labor supply and estimates the intertemporal elasticity of substitution of labor to be between 0.1 and 0.5. Ashenfelter and Card (1982) find that the intertemporal substitution hypothesis by

³⁰In response to transitory wage fluctuations, individuals intertemporally substitute their work effort, and this can generate large fluctuations in labor supply. Intertemporal substitution generates a negatively serially correlated response of labor supply and employment to wage shocks. Increases in labor supply today reflect expected decreases in the labor supply later.

itself is not capable of describing the aggregate time-series data on wages, prices, interest rates, and unemployment. Furthermore, Altonji (1982) also does not find any evidence of intertemporal substitution of labor. In a subsequent paper, Altonji (1986) examines the response of labor supply to variations in wage rates over time. His results indicate that the intertemporal elasticity of substitution is between 0 and 0.35. Pencavel (1986), in his survey, states that the focus of the life-cycle research has been directed towards the labor supply responses to wage movements, and the evidence of several studies indicates that these labor supply responses are very inelastic with respect to life-cycle changes in wages. In other words, the large part of the variations in male labor supply across workers and over time remains unexplained.

Several authors, then, have argued that temporal non-separabilities in preferences may be introduced to explain these findings. Kydland and Prescott (1982) incorporate an intertemporally non-separable utility function in their general equilibrium model. They argue that allowing for intertemporally non-separable preferences in leisure can generate intertemporal elasticities of substitution of labor supply consistent with observed aggregate employment fluctuations.

Clark and Summers (1982) find that work may be habit-forming: if experience in employment persists so that the decision to work is a relatively long-term commitment, it is not surprising that transitory wage changes have no discernible effect on labor supply. If the elasticity of current labor supply with respect to past labor supply is positive, this is the sign of habit formation in work, that is, a higher past work increases current work. Clark and Summers find (except for those over 65) no evidence for the notion that transitory changes in opportunities play a significant role in labor supply decisions.

Barro and King (1984), compare the theoretical implications of time-separable and time-non-separable utility functions of consumption and leisure to technology shocks. In a certainty model, they assume once and for all shifts in the production technology that leaves unchanged the marginal productivity of labor. When preferences are time-separable and consumption and leisure are normal goods, they show that consumption

and leisure move in the same direction. On the other hand, with time-non-separable preferences they argue that consumption and leisure may move in opposite directions consistent with the observed procyclicality of aggregate hours worked.

Browning, Deaton, and Irish (1985) present a general theoretical framework for the analysis of the life-cycle model of consumption and leisure under uncertainty. Using duality theory, (profit functions) to represent additive preferences and constant marginal utility of wealth, the model can explain the life-cycle behavior of consumption and leisure separately but not simultaneously. The life-cycle interpretation of consumption expenditures requires that consumption and leisure be substitutes, while their estimates imply that leisure and consumption are complements. They find a considerable amount of evidence to refute the life-cycle hypothesis.

Altug and Miller (1990), in a general equilibrium framework with a complete set of markets with stochastic interest rate and wage rate, find evidence for aggregate shocks on household consumption and leisure choices. They argue the findings of Browning et al. cannot be consistent estimates of the expectation of future marginal utilities since the error term depends on both idiosyncratic and aggregate shocks which cannot be separately identified. They also detect the presence of non-separabilities in consumption-leisure choices.

Mankiw, Rotemberg and Summers (1985) study an additively time-separable utility function with stochastic future real interest rates. They argue that over the business cycle, consumption and leisure move in opposite directions. However, their empirical results imply highly implausible parameters for the utility function. They suggest that the failure of their model can be attributed to the absence of non-separability in preferences.

Following Mankiw, Rotemberg and Summers, the paper of Eichenbaum, Hansen and Singleton (1988) explores the empirical properties of representative agent models with time-non-separable preferences in consumption and leisure with technology shocks. The non-separability of preferences accommodates intertemporal substitution or complementarity of leisure. They find substantial evidence for time-non-separable preferences, both

with respect to consumption and leisure, i.e., durability in consumption and habit formation in leisure.

Kennan (1988), allowing preferences of the consumer to be time-non-separable, analyzes a simple equilibrium model of the labor market to obtain testable restrictions on the reduced form time-series for real wages and employment. He introduces uncertainty in the form of random taste and technology shocks. He finds that non-separability in preferences is an important assumption in explaining the observed time-series behavior of hours worked and the wage rate. He also considers models with habit formation in leisure.

Hotz, Kydland and Sedlacek (1988) estimate a model of life-cycle labor supply and consumption under uncertainty in which the structure of preferences is allowed to be intertemporally non-additive in leisure. Their model implies that lagged leisure and thus, lagged hours of work will be correlated with changes in consumption. Income being equal to the product of wages and hours of work, the model is capable of explaining the observed correlation between changes in consumption and changes in lagged income. They find evidence that the standard assumption of intertemporally separable preferences for leisure is not consistent with the data.

Chapter 3

HABIT FORMATION

In this chapter we will be examining several models of habit formation.

3.1 THE GENERAL MODEL

In this section, we will describe the general model of habit formation. The specification of continuous time habit formation was first introduced by Ryder and Heal (1973), where the utility function depends also on a subsistence level of consumption, or habit stock variable $x(t)$. Then, in a model with uncertain income, the lifetime expected utility of the consumer is defined as:

$$E_0 \int_0^{\infty} e^{-\rho t} U(c(t), x(t)) dt$$

where

$$x(t) \equiv e^{-at} x_0 + b \int_0^t e^{a(s-t)} c(s) ds$$

with U increasing in $x(t)$, the subsistence level of consumption which is an exponentially weighted sum of past consumption; ρ is the rate of time preference. The special case $x_0 \equiv b = 0$ corresponds to time-separable utility. This formulation of habit formation has been used then by Becker and Murphy (1988), Constantinides (1990), Carroll, Overland

and Weil (1995).

Another formulation of a utility function exhibiting habit formation is the one of the discrete form with intertemporally non-additive preferences in consumption. In this case, the individual's expected lifetime utility can be given as:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha x_t)$$

where

$$\begin{aligned} x_t &\equiv (1 - \zeta) [c_{t-1} + \zeta c_{t-2} + \zeta^2 c_{t-3} + \dots] \\ &= (1 - \zeta) \sum_{j=0}^{\infty} \zeta^j c_{t-1-j} \\ &= \frac{(1 - \zeta)}{(1 - \zeta L)} c_{t-1}. \end{aligned}$$

x_t is referred as habit stock, a weighted average of past consumption where the weights add to one. $(1 - \zeta)$ is the depreciation rate of habits, $0 \leq \zeta \leq 1$, and α , $0 < \alpha < 1$, is the habit formation parameter. It gives the proportion of this average that is compared to the current level of consumption, c_t . When the depreciation rate of habit stock is equal to one, so that $\zeta = 0$, i.e., the case where past values of consumption before $t - 1$ do not affect the habit stock, we have the simple model which reflects one-period habit formation. This is the case where there is total depreciation of past consumption levels. Then, we can reformulate the individual's lifetime expected utility as:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha x_t)$$

where $x_t \equiv c_{t-1}$. In this setting, the consumer derives utility from current consumption levels which are higher than a fraction of past consumption. The larger the habit stock, the less pleasure from a given amount of consumption, and the larger must be purchases to generate the same benefit. Any level of actual consumption represents the cumulation

of all past experience. For our purposes, it will be sufficient to assume a single lag of consumption. In this context, there is an indirect but no direct impact of past lags of consumption on current consumption. This form has been used by Hayashi (1985b), Muellbauer (1988), Eichenbaum, Hansen and Singleton (1988), Ferson and Constantinides (1991), Deaton (1992), Dynan (1993a). For simplicity, we will use this formulation of habit formation.

The utility function is assumed to have the following properties:

$$U(0) = 0, \quad U'(0) = \infty$$

The consumer chooses current consumption c_t , and future level of assets, A_{t+1} , to maximize expected lifetime utility:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}) \\ & \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \tag{3.1}$$

subject to :

$$A_{t+1} = (1 + r)[A_t + y_t - c_t] \tag{3.2}$$

$$\text{with } \lim_{t \rightarrow \infty} A_t(1 + r)^{-t} = 0$$

A_0, c_0 given together with an exogenous expected time pattern of income y_t , β is the discount factor, $\beta = \frac{1}{1+\rho}$, $0 < \beta < 1$, where ρ is the rate of time preference and α is the habit formation parameter, $0 < \alpha < 1$.

The individual makes decisions about current period consumption, c_t , and the next period asset holdings, A_{t+1} , subject to the budget constraint. Given the individual's expectations of the future (assuming that all present and past variables are observable),

we can solve the maximization problem using dynamic programming.

Taking the state variables at time t to be current assets, A_t , past consumption, c_{t-1} , and current income, y_t , the value function for the current period is defined as:

$$V(A_t, c_{t-1}, y_t) = \underset{c_t, A_{t+1}}{\text{Max}} U(c_t - \alpha c_{t-1}) + \beta E_t V(A_{t+1}, c_t, y_{t+1}) \quad (3.3)$$

subject to :

$$A_{t+1} = (1 + r)[A_t + y_t - c_t] \quad (3.4)$$

Replacing for the budget constraint, (3.3) becomes:

$$V(A_t, c_{t-1}, y_t) = \underset{c_t}{\text{Max}} U(c_t - \alpha c_{t-1}) + \beta E_t V((1 + r)[A_t + y_t - c_t], c_t, y_{t+1}) \quad (3.5)$$

The first order condition with respect to c_t is:

$$U'(c_t - \alpha c_{t-1}) + \beta E_t \left[\frac{\partial V}{\partial c_t} + \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] = 0 \quad (3.6)$$

Using the envelope theorem, we can find:

$$E_t \frac{\partial V}{\partial c_t} \text{ and } E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right]$$

First, differentiating the value function with respect to c_{t-1} :

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha U'(c_t - \alpha c_{t-1}) \quad (3.7)$$

By forwarding one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t U'(c_{t+1} - \alpha c_t) \quad (3.8)$$

Substituting (3.8) into the first order condition (3.6), we obtain:

$$U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U'(c_{t+1} - \alpha c_t) - \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} (1 + r) \right] = 0 \quad (3.9)$$

Differentiating the value function with respect to A_t gives:

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} \quad (3.10)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1 + r)$$

From (3.9), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$ gives:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1 + r)} [U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U'(c_{t+1} - \alpha c_t)] \quad (3.11)$$

Substituting (3.11) into (3.10), we obtain:

$$\frac{\partial V}{\partial A_t} = U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U'(c_{t+1} - \alpha c_t) \quad (3.12)$$

Forwarding (3.12) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t [U'(c_{t+1} - \alpha c_t) - \alpha \beta U'(c_{t+2} - \alpha c_{t+1})] \quad (3.13)$$

Substituting (3.13) into (3.9), we obtain the Euler equation for consumption:

$$\begin{aligned} & U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U'(c_{t+1} - \alpha c_t) \\ &= \beta(1 + r) E_t [U'(c_{t+1} - \alpha c_t) - \alpha \beta U'(c_{t+2} - \alpha c_{t+1})] \end{aligned} \quad (3.14)$$

Now, let $\widehat{c}_t \equiv c_t - \alpha c_{t-1}$. For future use, it will be convenient to express the budget constraint in terms of $\widehat{c}_t$. Then,

$$\widehat{c}_t = c_t - \alpha c_{t-1} \Rightarrow c_t = \widehat{c}_t + \alpha c_{t-1}$$

Similarly,

$$\begin{aligned} \widehat{c}_{t+1} &= c_{t+1} - \alpha c_t \Rightarrow c_{t+1} = \widehat{c}_{t+1} + \alpha [\widehat{c}_t + \alpha c_{t-1}] \\ \widehat{c}_{t+2} &= c_{t+2} - \alpha c_{t+1} \Rightarrow c_{t+2} = \widehat{c}_{t+2} + \alpha [\widehat{c}_{t+1} + \alpha \widehat{c}_t + \alpha^2 c_{t-1}] \end{aligned}$$

Then,

$$c_{t+i} = \widehat{c}_{t+i} + \alpha \widehat{c}_{t+i-1} + \dots + \alpha^i \widehat{c}_t + \alpha^{i+1} c_{t-1} \quad (3.15)$$

or

$$c_{t+i} = \left(\sum_{j=0}^i \alpha^j \widehat{c}_{t+i-j} \right) + \alpha^{i+1} c_{t-1}$$

Therefore, we can write

$$\sum_{i=0}^{\infty} \beta^i c_{t+i} = \sum_{i=0}^{\infty} \beta^i \left[\left(\sum_{j=0}^i \alpha^j \widehat{c}_{t+i-j} \right) + \alpha^{i+1} c_{t-1} \right] \quad (3.16)$$

or,

$$\begin{aligned} \sum_{i=0}^{\infty} \beta^i c_{t+i} &= \widehat{c}_t + \alpha c_{t-1} + \beta [\widehat{c}_{t+1} + \alpha \widehat{c}_t + \alpha^2 c_{t-1}] \\ &\quad + \beta^2 [\widehat{c}_{t+2} + \alpha \widehat{c}_{t+1} + \alpha^2 \widehat{c}_t + \alpha^3 c_{t-1}] + \\ &\quad + \beta^3 [\widehat{c}_{t+3} + \alpha \widehat{c}_{t+2} + \alpha^2 \widehat{c}_{t+1} + \alpha^3 \widehat{c}_t + \alpha^4 c_{t-1}] + \dots + \\ &\quad + \beta^i [\widehat{c}_{t+i} + \alpha \widehat{c}_{t+i-1} + \dots + \alpha^{i+1} c_{t-1}] \\ &= [1 + \beta\alpha + \beta^2\alpha^2 + \dots] \widehat{c}_t + \beta [\dots] \widehat{c}_{t+1} + \beta^2 [\dots] \widehat{c}_{t+2} \\ &\quad + \dots + \alpha c_{t-1} + \beta\alpha^2 c_{t-1} + \beta^2\alpha^3 c_{t-1} + \dots \end{aligned}$$

$$\sum_{i=0}^{\infty} \beta^i c_{t+i} = \left(\frac{1}{1 - \alpha\beta} \right) \sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i} + \frac{\alpha}{1 - \alpha\beta} c_{t-1} \quad (3.17)$$

Given the transversality condition, the lifetime budget constraint in this problem,

$$\sum_{i=0}^{\infty} \beta^i c_{t+i} = A_t + \sum_{i=0}^{\infty} \beta^i y_{t+i} \quad (3.18)$$

Substituting $\sum_{i=0}^{\infty} \beta^i c_{t+i}$ from (3.17) in (3.18), and taking expectations as of time t on both sides,

$$\left(\frac{1}{1 - \alpha\beta} \right) \sum_{i=0}^{\infty} \beta^i E_t \widehat{c}_{t+i} = \left\{ A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i} \right\} - \left(\frac{\alpha}{1 - \alpha\beta} \right) c_{t-1} \quad (3.19)$$

or,

$$\sum_{i=0}^{\infty} \beta^i E_t \widehat{c}_{t+i} = (1 - \alpha\beta) \left\{ A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i} \right\} - \alpha c_{t-1} \quad (3.20)$$

Equation (3.20) expresses the lifetime budget constraint of the individual as a function of the expected income stream. Note that, the first order condition together with the lifetime budget constraint are sufficient to solve for the time path for consumption. More specific statements about the form of this path will require a more precise description of the form of the utility function and of the stochastic income process.

3.2 THE CERTAINTY EQUIVALENCE MODEL

Starting with Hall's seminal paper (1978), the Certainty Equivalence (CE) model with quadratic utility has been widely used in the consumption literature for its convenience in finding a closed form solution. Before introducing habit formation into the CE model, we briefly review Hall's model.

3.2.1 HALL'S MODEL

Hall's main purpose was to look at the Euler equation describing the optimal behavior of a consumer, where the Euler equation characterized the equality of the intertemporal marginal rate of substitution between consumption this and next period to the relative price of the two. That relative price is simply the present discounted cost of one unit of future consumption.

More generally, the consumer seeks to maximize:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \\ & \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \\ \text{subject to} \quad & : \\ & A_{t+1} = (1+r)[A_t + y_t - c_t] \end{aligned}$$

where y_t is real stochastic earnings; A_t represents real current assets; c_t is current consumption and U is a strictly concave utility function.

Assuming that the present is observable and the future is not, the ratio of adjacent first order conditions yields:

$$E_t U'(c_{t+1}) = \frac{1}{\beta(1+r)} U'(c_t)$$

Then, letting $\beta = \frac{1}{1+r}$, and assuming rational expectations, current marginal utility of consumption equals the expectation of next period's marginal utility of consumption. That is,

$$E_t U'(c_{t+1}) = U'(c_t)$$

where E_t denotes expectations given the information set at time t . Since

$$U'(c_{t+1}) = E_t U'(c_{t+1}) + \varepsilon_{t+1}$$

where ε_{t+1} is a surprise or a non-forecastable innovation, i.e. is independently and identically distributed (*i.i.d.*) with $E_t \varepsilon_{t+1} = 0$,

$$U'(c_{t+1}) = U'(c_t) + \varepsilon_{t+1}$$

In addition, by assuming a quadratic utility function, Hall showed that consumption would also follow a random walk. In this case, it could be written as:

$$c_{t+1} = c_t + \varepsilon_{t+1}$$

The implication of this form is that lagged real disposable income will have little predictive power for consumption, and since ε_{t+1} is unpredictable at time t , the best estimate of the future consumption is the current consumption. All other past values of consumption should have no predictive power.

3.2.2 THE CERTAINTY EQUIVALENCE MODEL WITH HABIT FORMATION

Whereas the CE model led to Hall's random walk hypothesis, due to habit formation, c_t is no longer a random walk in our model.

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}) \\ & \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (3.21)$$

subject to :

$$A_{t+1} = (1+r)[A_t + y_t - c_t]$$

and with $\lim_{t \rightarrow \infty} A_t(1+r)^{-t} = 0$. Using the definition of the net consumption,

$$\widehat{c}_t = c_t - \alpha c_{t-1}$$

the quadratic utility function is given as:

$$U(\widehat{c}_t) = \nu_0 + \nu_1 \widehat{c}_t - \frac{1}{2} \nu_2 \widehat{c}_t^2$$

where ν_0, ν_1 , and $\nu_2 > 0$.

Using the dynamic programming formulation given earlier, the Euler equation for this problem is:

$$\begin{aligned} & \nu_1 - \nu_2 \widehat{c}_t - \alpha \beta [\nu_1 - \nu_2 E_t \widehat{c}_{t+1}] \\ = & \beta(1+r) E_t \{ \nu_1 - \nu_2 \widehat{c}_{t+1} - \alpha \beta \nu_1 + \alpha \beta \nu_2 \widehat{c}_{t+2} \} \end{aligned} \quad (3.22)$$

Combining terms,

$$\begin{aligned}
& -[\nu_2 \widehat{c}_t - \alpha\beta\nu_2 E_t \widehat{c}_{t+1}] + (\nu_1 - \alpha\beta\nu_1) \\
& = -\beta(1+r)E_t [\nu_2 \widehat{c}_{t+1} - \alpha\beta\nu_2 \widehat{c}_{t+2}] + \beta(1+r)(\nu_1 - \alpha\beta\nu_1)
\end{aligned} \tag{3.23}$$

Factoring out ν_2 :

$$\begin{aligned}
-\nu_2 [\widehat{c}_t - \alpha\beta E_t \widehat{c}_{t+1}] & = -\nu_2 \beta(1+r)E_t [\widehat{c}_{t+1} - \alpha\beta \widehat{c}_{t+2}] \\
& + \nu_1 [\beta(1+r) - 1] [1 - \alpha\beta]
\end{aligned} \tag{3.24}$$

Dividing both sides by ν_2 ,

$$\begin{aligned}
[\widehat{c}_t - \alpha\beta E_t \widehat{c}_{t+1}] & = \beta(1+r)E_t [\widehat{c}_{t+1} - \alpha\beta \widehat{c}_{t+2}] \\
& - \frac{\nu_1}{\nu_2} [\beta(1+r) - 1] [1 - \alpha\beta]
\end{aligned} \tag{3.25}$$

Let us define κ as:

$$\kappa = \frac{\nu_1}{\nu_2} [\beta(1+r) - 1] [1 - \alpha\beta] \tag{3.26}$$

which is a constant term. When the rate of interest is equal to the rate of time preference, i.e., $r = \rho$, we have $\beta(1+r) = 1$, the constant term κ vanishes. Then, the Euler equation becomes:

$$\widehat{c}_t - \alpha\beta E_t \widehat{c}_{t+1} = E_t \{\widehat{c}_{t+1} - \alpha\beta \widehat{c}_{t+2}\} \tag{3.27}$$

There are two solutions to equation (3.27). Rewriting (3.27) as:

$$\widehat{c}_t - (1 + \alpha\beta)E_t \widehat{c}_{t+1} + \alpha\beta E_t \widehat{c}_{t+2} = 0 \tag{3.28}$$

Equation (3.28) can be factored as;

$$E_t \widehat{c}_{t+1} [1 - \alpha\beta L^{-1}] [1 - L] = 0 \quad (3.29)$$

where L represents the lag operator.

The solution to equation (3.29) is:

$$E_t \widehat{c}_{t+1} [1 - L] = 0 \quad (3.30)$$

or,

$$E_t \widehat{c}_{t+1} [1 - \alpha\beta L^{-1}] = 0 \quad (3.31)$$

Under reasonable assumptions that, $-1 < \alpha < 1$ and $0 < \beta < 1$, and constant interest rates, the second solution, (3.31), can be ruled out since (3.31), (in combination with the restrictions on α and β) implies that marginal utility increases over time¹. Therefore, the Euler equation for the problem with habit formation in consumption is:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} \quad (3.32)$$

which indicates that $\widehat{c}_t$ (but not c_t) follows a random walk.

Using equation (3.32) together with the lifetime budget constraint (3.20), we can solve for $\widehat{c}_t$ as:

$$\widehat{c}_t = (1 - \alpha\beta) \left[(1 - \beta)(A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i}) \right] - \alpha(1 - \beta)c_{t-1} \quad (3.33)$$

Now, let us solve for c_t and rewrite (3.33) as:

$$c_t = (1 - \alpha\beta)y_t^p + \alpha\beta c_{t-1}$$

¹See Hayashi (1985b), and Dynan (1993a).

where y_t^p denotes permanent income, i.e.

$$y_t^p \equiv (1 - \beta)(A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i}) \quad (3.34)$$

Two effects are important here: first, the appearance of c_{t-1} in the last term due to the habit formation (the last term would be zero if $\alpha = 0$), second, the impact of changes in permanent income is smaller since:

$$(1 - \alpha\beta) < 1$$

Updating (3.33),

$$\widehat{c}_{t+1} = (1 - \alpha\beta) \left[(1 - \beta)(A_{t+1} + \sum_{i=0}^{\infty} \beta^i E_t y_{t+1+i}) \right] - \alpha(1 - \beta)c_t \quad (3.35)$$

We can also write the period budget constraint as:

$$A_{t+1} - A_t - y_t = rA_t + ry_t - (1 + r)c_t \quad (3.36)$$

Subtracting (3.33) from (3.35), using (3.36) and regrouping terms, we obtain:

$$\begin{aligned} \widehat{c}_{t+1} - \widehat{c}_t &= (1 - \beta)(1 - \alpha\beta)[rA_t + ry_t - (1 + r)c_t - \frac{\alpha}{1 - \alpha\beta}c_t] \\ &\quad + \frac{\alpha}{1 - \alpha\beta}c_{t-1} + (1 - \beta) \sum_{i=0}^{\infty} \beta^i E_t y_{t+i+1} \\ &\quad + \sum_{i=0}^{\infty} \beta^i [E_{t+1} y_{t+i+1} - E_t y_{t+i+1}] \end{aligned} \quad (3.37)$$

Note that, the two terms involving c_t on the right-hand side of equation (3.37) can

be simplified as:

$$-(1+r)c_t - \frac{\alpha}{1 - \frac{\alpha}{1+r}}c_t = -\frac{(1+r)^2}{1+r-\alpha}c_t = -\frac{(1+r)}{1-\alpha\beta}c_t \quad (3.38)$$

and

$$\frac{(1+r)}{1-\alpha\beta}c_t = [rA_t + ry_t + \frac{r}{(1+r)}E_t y_{t+1} + \dots] + \frac{\alpha}{1-\alpha\beta}c_{t-1}$$

Substituting for $-\frac{1+r}{1-\alpha\beta}c_t$ into the expression for $\hat{c}_{t+1} - \hat{c}_t$, we obtain:

$$\hat{c}_{t+1} - \hat{c}_t = (1-\beta)(1-\alpha\beta) \sum_{i=0}^{\infty} \beta^i [E_{t+1} y_{t+i+1} - E_t y_{t+i+1}] \quad (3.39)$$

or,

$$\begin{aligned} c_{t+1} - c_t &= (1-\alpha\beta)(1-\beta) \sum_{i=0}^{\infty} \beta^i [E_{t+1} y_{t+i+1} - E_t y_{t+i+1}] \\ &\quad + \alpha (c_t - c_{t-1}) \end{aligned} \quad (3.40)$$

Assuming that y_t follows a first order autoregressive process, i.e.,

$$y_{t+1} = \mu y_t + \omega_{t+1}, \quad 0 < \mu < 1 \quad (3.41)$$

with $\omega_{t+1} \sim i.i.d.$ with mean zero and variance σ_{ω}^2 , and the term

$$E_{t+1} y_{t+i+1} - E_t y_{t+i+1} = \mu^i \omega_{t+1} \quad (3.42)$$

represents the innovation in income.

Then, substituting for (3.42) into (3.40),

$$\begin{aligned} c_{t+1} - c_t &= (1-\alpha\beta)(1-\beta) \sum_{i=0}^{\infty} \beta^i \mu^i \omega_{t+1} + \alpha (c_t - c_{t-1}) \\ &= \frac{(1-\alpha\beta)(1-\beta)}{1-\beta\mu} \omega_{t+1} + \alpha (c_t - c_{t-1}), \end{aligned} \quad (3.43)$$

or ,

$$[1 - \alpha L] (c_{t+1} - c_t) = \frac{(1 - \alpha\beta)(1 - \beta)}{1 - \beta\mu} \omega_{t+1}. \quad (3.44)$$

Solving for $c_{t+1} - c_t$, we obtain:

$$c_{t+1} - c_t = \frac{(1 - \alpha\beta)(1 - \beta)}{1 - \beta\mu} \sum_{i=0}^{\infty} \alpha^i \omega_{t+1-i}. \quad (3.45)$$

or,

$$c_{t+1} - c_t = \frac{(1 - \alpha\beta)(1 - \beta)}{1 - \beta\mu} \omega_{t+1} + \frac{\alpha(1 - \alpha\beta)(1 - \beta)}{1 - \beta\mu} \sum_{i=0}^{\infty} \alpha^i \omega_{t-i}. \quad (3.46)$$

We observe that, because $(1 - \alpha\beta) < 1$, the marginal propensity to consume out of a current innovation in income is less than in the model without habit formation. Furthermore, in addition to the unanticipated changes in income, ω_{t+1} , changes in consumption now respond to past changes in income, thus helping to explain the excess sensitivity of consumption to lagged income observed in the data.

Note that when $\alpha = 0$, we obtain

$$c_{t+1} - c_t = \frac{(1 - \beta)}{1 - \beta\mu} \omega_{t+1}$$

or

$$c_{t+1} - c_t = \frac{r}{1 + r - \mu} \omega_{t+1}$$

in the absence of habit formation so that the change in consumption at time $t + 1$ is only affected by the innovation to income at $t + 1$.

3.3 EXPONENTIAL UTILITY MODEL

Here, we introduce a utility function with CARA (constant absolute risk aversion) preferences into the analysis of habit formation. The use of CARA preferences was dictated by the fact that this is the only specification (other than the Certainty Equivalence) which permits a closed form solution.

The problem has our familiar set up, that is:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}) . \\ & \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (3.47)$$

subject to :

$$A_{t+1} = (1 + r)[A_t + y_t - c_t]$$

$$\text{with } \lim_{t \rightarrow \infty} A_t(1 + r)^{-t} = 0$$

A_0 , and c_0 , and $\{y_t, y_{t+1}, \dots, y_{t+i}, \dots\}$ given. Now, the utility function is assumed to be exponential:

$$U(c_t - \alpha c_{t-1}) = -\frac{1}{\Theta} e^{-\Theta[c_t - \alpha c_{t-1}]} \quad (3.48)$$

As is well known this utility function is CARA, with:

$$-\frac{U''}{U'} = \Theta$$

where $\Theta > 0$, is the coefficient of absolute risk aversion. We will again assume that the rate of interest is certain and equal to the rate of time preference.

$$\frac{1}{1 + r} \equiv \frac{1}{1 + \rho} = \beta$$

We will assume that y_t follows an autoregressive process with a normal distribution². That is, $y_t = \phi y_{t-1} + \omega_t$, $0 < \phi < 1$ where $\omega_t \sim i.i.d.$ with mean zero and variance σ_ω^2 .

The first order condition for the dynamic programming problem can be stated as:

$$e^{-\Theta \hat{c}_t} = E_t [(1 + \alpha\beta)e^{-\Theta \hat{c}_{t+1}} - \alpha\beta e^{-\Theta \hat{c}_{t+2}}] \quad (3.49)$$

where

$$\hat{c}_{t+i} = c_{t+i} - \alpha c_{t+i-1} \quad i = 0, 1, 2, \dots \quad (3.50)$$

Taking inspiration from Caballero (1990), we conjecture a solution for $\hat{c}_{t+i}$ as follows:

$$\hat{c}_{t+i} = \Gamma_{t+i-1} + \Phi_{t+i-1} \hat{c}_{t+i-1} + v_{t+i} \quad i = 0, 1, 2, \dots \quad (3.51)$$

where Γ , Φ and v are terms to be determined. v_{t+i} is the innovation to consumption.

We use our conjecture for $\hat{c}_{t+1}$ and $\hat{c}_{t+2}$,

$$\hat{c}_{t+1} = \Gamma_t + \Phi_t \hat{c}_t + v_{t+1} \text{ and } \hat{c}_{t+2} = \Gamma_{t+1} + \Phi_{t+1}(\Gamma_t + \Phi_t \hat{c}_t + v_{t+1}) + v_{t+2} \quad (3.52)$$

and substitute into the first order condition (3.49) to find:

$$\begin{aligned} e^{-\Theta \hat{c}_t} = E_t \{ & (1 + \alpha\beta)e^{-\Theta[\Gamma_t + \Phi_t \hat{c}_t + v_{t+1}]} \\ & - \alpha\beta e^{-\Theta[\Gamma_{t+1} + \Phi_{t+1}(\Gamma_t + \Phi_t \hat{c}_t + v_{t+1}) + v_{t+2}]} \}. \end{aligned} \quad (3.53)$$

Factoring out $e^{-\Theta[\Gamma_t + \Phi_t \hat{c}_t + v_{t+1}]}$, the first order condition becomes:

²One could write this process with a constant level of income: $y_t = \bar{y} + \phi y_{t-1} + \omega_t$. However, that would not change the result. The constant can always be incorporated into the asset level at time zero.

$$e^{-\Theta \hat{c}_t} = E_t[e^{-\Theta[\Gamma_t + \Phi_t \hat{c}_t + v_{t+1}]} \times \{(1 + \alpha\beta) - \alpha\beta e^{-\Theta[\Gamma_{t+1} + (\Phi_{t+1} - 1)\Gamma_t + (\Phi_{t+1} - 1)\Phi_t \hat{c}_t + (\Phi_{t+1} - 1)v_{t+1} + v_{t+2}]}\} \quad (3.54)$$

Now, we divide both sides by $e^{-\Theta[\Gamma_t + \Phi_t \hat{c}_t]}$:

$$e^{-\Theta \hat{c}_t(1 - \Phi_t) + \Theta \Gamma_t} = E_t[e^{-\Theta v_{t+1}} \times \{(1 + \alpha\beta) - \alpha\beta e^{-\Theta[\Gamma_{t+1} + (\Phi_{t+1} - 1)\Gamma_t + (\Phi_{t+1} - 1)\Phi_t \hat{c}_t + (\Phi_{t+1} - 1)v_{t+1} + v_{t+2}]}\} \quad (3.55)$$

Then, matching coefficients of $\hat{c}_t$ on both sides yields:

$$\begin{aligned} -\Theta(1 - \Phi_t) &= -\Theta(\Phi_{t+1} - 1)\Phi_t \\ 1 - \Phi_t + \Phi_t - \Phi_{t+1}\Phi_t &= 0 \\ 1 &= \Phi_{t+1}\Phi_t \\ \Phi_{t+1} &= \frac{1}{\Phi_t} \end{aligned} \quad (3.56)$$

This gives us a relationship between Φ_{t+1} and Φ_t .

Therefore, we substitute for $\Phi_{t+1} = \frac{1}{\Phi_t}$ into (3.55) and divide through by $e^{-\Theta \hat{c}_t(1 - \Phi_t)}$ to get:

$$e^{\Theta \Gamma_t} = E_t[e^{-\Theta v_{t+1}} \times \{(1 + \alpha\beta)e^{\Theta \hat{c}_t(1 - \Phi_t)} - \alpha\beta e^{-\Theta[\Gamma_{t+1} + (\Phi_t^{-1} - 1)\Gamma_t + (\Phi_t^{-1} - 1)v_{t+1} + v_{t+2}]}\} \quad (3.57)$$

Matching coefficients of $\hat{c}_t$ on both sides:

$$(1 - \Phi_t) = 0 \Rightarrow \Phi_t = 1 \quad (3.58)$$

Since we have assumed that the innovations to income are normally distributed, the

innovations to consumption will also be normally distributed. Hence, we can write:

$$E_t e^{-\Theta v_{t+1}} = e^{-\Theta E_t v_{t+1} + \frac{\Theta^2}{2} \sigma_v^2}$$

Having found $\Phi_t = \Phi_{t+1} = 1$, we can rewrite (3.57) as:

$$\begin{aligned} e^{\Theta \Gamma_t} &= E_t \{ e^{-\Theta v_{t+1}} [(1 + \alpha\beta) - \alpha\beta e^{-\Theta \Gamma_{t+1} - \Theta v_{t+2}}] \} \\ &= E_t \{ (1 + \alpha\beta) e^{-\Theta v_{t+1}} - \alpha\beta e^{-\Theta \Gamma_{t+1} - \Theta v_{t+1} - \Theta v_{t+2}} \} \\ &= (1 + \alpha\beta) e^{-\Theta E_t v_{t+1} + \frac{\Theta^2}{2} \sigma_v^2} - \alpha\beta e^{-\Theta \Gamma_{t+1} - \Theta E_t v_{t+1} - \Theta E_t v_{t+2} + \Theta^2 \sigma_v^2} \end{aligned} \quad (3.59)$$

where we have assumed that Γ_t is time-varying, but known. Then,

$$e^{\Theta \Gamma_t} = e^{-\Theta E_t v_{t+1} + \frac{\Theta^2}{2} \sigma_v^2} \left[(1 + \alpha\beta) - \alpha\beta e^{-\Theta \Gamma_{t+1} - \Theta E_t v_{t+2} + \frac{\Theta^2}{2} \sigma_v^2} \right] \quad (3.60)$$

Divide both sides by $e^{-\Theta E_t v_{t+1} + \frac{\Theta^2}{2} \sigma_v^2}$ to get:

$$e^{\Theta [\Gamma_t + E_t v_{t+1} - \frac{\Theta}{2} \sigma_v^2]} = \left[(1 + \alpha\beta) - \alpha\beta e^{-\Theta [\Gamma_{t+1} + E_t v_{t+2} - \frac{\Theta}{2} \sigma_v^2]} \right] \quad (3.61)$$

The above equality will hold when:

$$\Gamma_t = -E_t v_{t+1} + \frac{\Theta}{2} \sigma_v^2 \text{ and } \Gamma_{t+1} = -E_t v_{t+2} + \frac{\Theta}{2} \sigma_v^2. \quad (3.62)$$

We will now proceed to substitute our conjectured solution for $\hat{c}_t$ into the lifetime budget constraint. First, note that,

$$\hat{c}_{t+i} = \Gamma_{t+i-1} + \hat{c}_{t+i-1} + v_{t+i} \quad (3.63)$$

can be written in terms of $\hat{c}_t$ as follows:

$$\hat{c}_{t+i} = \sum_{j=1}^i \Gamma_{t+j-1} + \sum_{j=1}^i v_{t+j} + \hat{c}_t \quad (3.64)$$

Therefore,

$$\sum_{i=0}^{\infty} \beta^i \hat{c}_{t+i} = \hat{c}_t \sum_{i=0}^{\infty} \beta^i + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \Gamma_{t+j-1} + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i v_{t+j} \quad (3.65)$$

Substituting (3.65) into the lifetime budget constraint (3.18) yields:

$$\begin{aligned} & \frac{1}{1-\beta} \hat{c}_t + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \Gamma_{t+j-1} + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i v_{t+j} + \alpha c_{t-1} \\ & - [1 - \alpha\beta] \left\{ A_t + E_t \sum_{i=0}^{\infty} \beta^i \{y_{t+i} - E_t y_{t+i}\} + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i} \right\} \\ & = 0 \end{aligned} \quad (3.66)$$

where we have added and subtracted the terms $E_t y_{t+i}$.

$$\begin{aligned} \Rightarrow & \frac{1}{1-\beta} \hat{c}_t + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \Gamma_{t+j-1} + \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i v_{t+j} + \alpha c_{t-1} \\ & - (1 - \alpha\beta) A_t - (1 - \alpha\beta) \sum_{i=0}^{\infty} \beta^i E_t y_{t+i} \\ & - (1 - \alpha\beta) \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \phi^{i-j} \omega_{t+j} = 0 \end{aligned} \quad (3.67)$$

where we have used the fact that

$$y_{t+i} - E_t y_{t+i} = \sum_{j=1}^i \phi^{i-j} \omega_{t+j} \quad (3.68)$$

since income was assumed to follow a first order autoregressive process³.

Taking expectations on both sides of the budget constraint to obtain:

$$\begin{aligned}\hat{c}_t = & (1 - \beta) [1 - \alpha\beta] \left\{ A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i} \right\} \\ & - (1 - \beta) \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \Gamma_{t+j-1} - (1 - \beta) \alpha c_{t-1}\end{aligned}\tag{3.69}$$

where we have used the fact that since ω_{t+i} is an *i.i.d.* random variable with zero mean, the innovation to consumption v_{t+i} will also have a mean of zero.

Now, we substitute for $\hat{c}_t$ from (3.69) back into equation (3.67). Since the resulting expression must be equal to zero, we have:

$$\sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i [v_{t+j} - (1 - \alpha\beta) \phi^{i-j} \omega_{t+j}] = 0\tag{3.70}$$

This term can be opened as:

$$\begin{aligned}& \beta[v_{t+1} - (1 - \alpha\beta)\omega_{t+1}] + \beta^2[v_{t+1} - (1 - \alpha\beta)\phi\omega_{t+1} + v_{t+2} \\ & - (1 - \alpha\beta)\omega_{t+2}] + \dots \\ = & \sum_{i=1}^{\infty} \beta^i [v_{t+1} - (1 - \alpha\beta)\phi^{i-1}\omega_{t+1}] \\ & + \sum_{i=2}^{\infty} \beta^i [v_{t+2} - (1 - \alpha\beta)\phi^{i-2}\omega_{t+2}] + \dots \\ = & 0\end{aligned}\tag{3.71}$$

³Note that,

$$\sum_{j=1}^i \phi^{i-j} \omega_{t+j} = \omega_{t+i} + \phi \omega_{t+i-1} + \dots + \phi^{i-1} \omega_{t+1}.$$

For this sum to be equal to zero, each of the terms must be zero.

Therefore,

$$\frac{\beta}{1-\beta}v_{t+1} = \sum_{i=1}^{\infty} \beta^i (1-\alpha\beta)\phi^{i-1}\omega_{t+1} \quad \forall t \quad (3.72)$$

or

$$v_{t+1} = \left[(1-\alpha\beta)(1-\beta) \sum_{i=0}^{\infty} \beta^i \phi^i \right] \omega_{t+1} \quad (3.73)$$

or

$$v_{t+1} = \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \omega_{t+1} \quad (3.74)$$

We have solved for the explicit relationship between v_{t+1} and ω_{t+1} .

By using (3.34) and (3.69), c_t can be written as:

$$c_t = (1-\alpha\beta)y_t^p + \alpha\beta c_{t-1} - (1-\beta) \sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \Gamma_{t+j-1} \quad (3.75)$$

where

$$\sum_{i=1}^{\infty} \beta^i \sum_{j=1}^i \Gamma_{t+j-1} = \frac{\beta}{(1-\beta)^2} \Gamma \quad (3.76)$$

Given our assumption that ω_{t+i} 's are *i.i.d.* with zero mean, so that the v_{t+i} 's are *i.i.d.* with zero mean⁴. That is, $\Gamma_t = \Gamma = \frac{\Theta}{2}\sigma_v^2 \forall t$. Hence, from (3.74) we obtain,

$$\Gamma = \frac{\Theta}{2} \left[\frac{(1-\alpha\beta)(1-\beta)}{(1-\beta\phi)} \right]^2 \sigma_{\omega}^2 \quad (3.77)$$

Then (3.75) becomes:

$$c_t = (1-\alpha\beta)y_t^p + \alpha\beta c_{t-1} - \frac{\beta}{(1-\beta)} \Gamma \quad (3.78)$$

or solving the difference equation for c_t in (3.78) ,

$$c_t = (1-\alpha\beta) \sum_{i=0}^{\infty} (\alpha\beta)^i y_{t-i}^p - \frac{\beta}{(1-\alpha\beta)(1-\beta)} \Gamma \quad (3.79)$$

⁴On the other hand, if v_t is serially correlated, then Γ_t will be time-varying.

or

$$c_t = (1 - \alpha\beta)y_t^p + (1 - \alpha\beta)\alpha\beta \sum_{i=0}^{\infty} (\alpha\beta)^i y_{t-i-1}^p - \frac{\beta}{(1 - \alpha\beta)(1 - \beta)}\Gamma \quad (3.80)$$

This gives us the solution for period t consumption. Before analyzing it, we first verify our original conjecture about $\widehat{c}_t$, and then we derive the change in consumption $c_{t+1} - c_t$.

First, from (3.78),

$$\widehat{c}_{t+1} = c_{t+1} - \alpha c_t = (1 - \alpha\beta)y_{t+1}^p - \alpha(1 - \beta)c_t - \frac{\beta}{1 - \beta}\Gamma \quad (3.81)$$

If our conjecture was correct, $\widehat{c}_{t+1} - \widehat{c}_t$ as obtained from (3.81) should be equal to $\Gamma + v_{t+1}$ according to (3.51).

Hence,

$$\begin{aligned} \widehat{c}_{t+1} - \widehat{c}_t &= (1 - \alpha\beta)[y_{t+1}^p - y_t^p] - (1 - \beta)\alpha(c_t - c_{t-1}) \\ &\quad - \frac{\beta}{1 - \beta}[\Gamma - \Gamma] \\ &= (1 - \beta)\{(1 - \alpha\beta)[A_{t+1} + \sum_{i=0}^{\infty} \beta^i E_{t+1}y_{t+i+1}] - \alpha c_t \\ &\quad - (1 - \alpha\beta)[A_t + \sum_{i=0}^{\infty} \beta^i E_t y_{t+i}] - \alpha c_{t-1}\} \\ &= (1 - \beta)\{(1 - \alpha\beta)[A_{t+1} - A_t \\ &\quad + \sum_{i=0}^{\infty} \beta^i (E_{t+1}y_{t+i+1} - E_t y_{t+i})] - \alpha(c_t - c_{t-1})\} \\ &= (1 - \beta)(1 - \alpha\beta)\{A_{t+1} - A_t - y_t + [E_{t+1}y_{t+1} - \beta E_t y_{t+1}] \\ &\quad + \beta[E_{t+1}y_{t+2} - \beta E_t y_{t+2}] + \dots\} - (1 - \beta)\alpha(c_t - c_{t-1}) \end{aligned} \quad (3.82)$$

Adding and subtracting,

$$[E_{t+1}y_{t+1} - \beta E_t y_{t+1}] = [E_{t+1}y_{t+1} - E_t y_{t+1}] + (1 - \beta)E_t y_{t+1} \quad (3.83)$$

Therefore, we can rewrite (3.82) as:

$$\begin{aligned}\widehat{c}_{t+1} - \widehat{c}_t &= (1 - \beta)(1 - \alpha\beta)\{A_{t+1} - A_t - y_t \\ &\quad + \sum_{i=0}^{\infty} \beta^i [E_{t+1}y_{t+i+1} - E_t y_{t+i+1}] \\ &\quad + (1 - \beta) \sum_{i=0}^{\infty} \beta^i E_t y_{t+i+1}\} - (1 - \beta)\alpha(c_t - c_{t-1})\end{aligned}\quad (3.84)$$

Rewriting the period budget constraint as:

$$A_{t+1} - A_t - y_t = rA_t + ry_t - (1 + r)c_t \quad (3.85)$$

and since we can write c_t from (3.78) as:

$$c_t = (1 - \alpha\beta)\left(\frac{r}{1 + r}\right)[A_t + y_t + \frac{1}{1 + r}E_t y_{t+1} + \dots] - \frac{1}{r}\Gamma + \frac{\alpha}{1 + r}c_{t-1} \quad (3.86)$$

Then,

$$\begin{aligned}(1 + r)c_t &= (1 - \alpha\beta)\left[rA_t + ry_t + \frac{r}{1 + r}E_t y_{t+1} + \frac{r}{(1 + r)^2}E_t y_{t+2} \right. \\ &\quad \left. + \dots\right] - \frac{1 + r}{r}\Gamma + \alpha c_{t-1}\end{aligned}\quad (3.87)$$

Now, rewrite (3.84) by putting the $\alpha(1 - \beta)(c_t - c_{t-1})$ term inside the braces:

$$\begin{aligned}\widehat{c}_{t+1} - \widehat{c}_t &= (1 - \beta)(1 - \alpha\beta)[rA_t + ry_t - (1 + r)c_t - \frac{\alpha}{1 - \alpha\beta}c_t \\ &\quad + \frac{\alpha}{1 - \alpha\beta}c_{t-1} + (1 - \beta) \sum_{i=0}^{\infty} \beta^i E_t y_{t+i+1} \\ &\quad + \sum_{i=0}^{\infty} \beta^i [E_{t+1}y_{t+i+1} - E_t y_{t+i+1}]]\end{aligned}\quad (3.88)$$

Note that,

$$\begin{aligned}
-(1+r)c_t - \frac{\alpha}{1 - \frac{\alpha}{1+r}}c_t &= -(1+r)c_t - \frac{\alpha(1+r)}{1+r-\alpha}c_t \\
&= \frac{(1+r)}{1+r-\alpha}[-c_t(1+r-\alpha) - \alpha c_t] \\
&= \frac{(1+r)}{1+r-\alpha}[-c_t(1+r)]
\end{aligned} \tag{3.89}$$

Therefore,

$$-\frac{(1+r)^2}{1+r-\alpha}c_t = -\frac{(1+r)}{1-\alpha\beta}c_t \tag{3.90}$$

$$\begin{aligned}
\frac{(1+r)}{1-\alpha\beta}c_t &= [rA_t + ry_t + \frac{r}{(1+r)}E_t y_{t+1} + \dots] - \frac{(1+r)}{r(1-\alpha\beta)}\Gamma \\
&\quad + \frac{\alpha}{1-\alpha\beta}c_{t-1}
\end{aligned} \tag{3.91}$$

Substituting for $-\frac{1+r}{1-\alpha\beta}c_t$ into the expression (3.88), we obtain:

$$\begin{aligned}
\widehat{c}_{t+1} - \widehat{c}_t &= (1-\beta)(1-\alpha\beta) \left[\frac{(1+r)}{r(1-\alpha\beta)}\Gamma \right] \\
&\quad + (1-\beta)(1-\alpha\beta) \sum_{i=0}^{\infty} \beta^i [E_{t+1} y_{t+i+1} - E_t y_{t+i+1}] \\
&= \underbrace{\Gamma + (1-\beta)(1-\alpha\beta) \sum_{i=0}^{\infty} \beta^i \phi^i \omega_{t+1}}_{\equiv v_{t+1}}
\end{aligned} \tag{3.92}$$

That is,

$$\widehat{c}_{t+1} - \widehat{c}_t = \Gamma + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \omega_{t+1} \tag{3.93}$$

This verifies our conjecture.

Equation (3.93) can also be expressed in terms of $(c_{t+1} - c_t)$ as,

$$c_{t+1} - c_t = \Gamma + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \omega_{t+1} + \alpha(c_t - c_{t-1}) \quad (3.94)$$

$$\Rightarrow [1 - \alpha L] (c_{t+1} - c_t) = \Gamma + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \omega_{t+1} \quad (3.95)$$

Then, we can solve for $(c_{t+1} - c_t)$ as ,

$$c_{t+1} - c_t = \frac{1}{1-\alpha} \Gamma + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \sum_{i=0}^{\infty} \alpha^i \omega_{t+1-i} \quad (3.96)$$

or

$$\begin{aligned} c_{t+1} - c_t &= \frac{1}{1-\alpha} \Gamma + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \omega_{t+1} \\ &\quad + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \alpha \sum_{i=0}^{\infty} \alpha^i \omega_{t-i} \end{aligned} \quad (3.97)$$

where the first term represents the precautionary premium, the second term is the “unanticipated changes in income” and the third term is the “anticipated changes in income” or averages of previous income innovations. The difference of (3.97) from equation (3.46) in the CE case is that, now we have an additional term representing the precautionary savings. Unlike the quadratic utility case, the CARA utility leads to a precautionary behavior.

However, substituting for Γ ,

$$\begin{aligned} c_{t+1} - c_t &= \frac{\Theta}{2(1-\alpha)} \left[\frac{(1-\alpha\beta)(1-\beta)}{(1-\beta\phi)} \right]^2 \sigma_{\omega}^2 \\ &\quad + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \omega_{t+1} \\ &\quad + \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right] \alpha \sum_{i=0}^{\infty} \alpha^i \omega_{t-i} \end{aligned} \quad (3.98)$$

we see that the first term which is the precautionary premium against income uncertainty

is smaller with habit formation.

If there were no habit formation equation (3.98) would be written as:

$$c_{t+1} - c_t = \frac{\Theta}{2} \left[\frac{(1 - \beta)}{(1 - \beta\phi)} \right]^2 \sigma_\omega^2 + \left[\frac{(1 - \beta)}{(1 - \beta\phi)} \right] \omega_{t+1}$$

where the first term, the precautionary premium against income uncertainty, is larger than the one with habit formation, because the habit formation parameter affects the size of the precautionary term. The higher the parameter α , i.e., the stronger the habits, the lower the effect of income uncertainty on consumption. The reason is that the individual with habit-forming preferences in consumption saves more, i.e., consumes less out of permanent income in order to achieve higher consumption growth. More precisely, the consumer chooses to consume less, and takes into account the negative externality of higher current consumption on future utility level.

However, this does not imply that the consumer does not save against income uncertainty. On the contrary, the precautionary saving behavior against income uncertainty also does exist, together with the saving induced by habit formation, but the magnitude of such a saving is less than the one in the case with no habit formation. Therefore, the income uncertainty affects less the optimal consumption level of habit-forming individual because there is habits-induced precautionary saving to cushion him or her against income uncertainty. That is why the precautionary premium against income uncertainty does not need to be as high as the one of the individual with no habit formation.

We are now in a position to look at Zeldes's definition of the marginal propensity to consume to examine the excess sensitivity and the excess smoothness paradoxes. The marginal propensity to consume out of an unanticipated income change ($\frac{\Delta c}{\Delta y}$) with habit formation is defined as the change in current consumption from an innovation in income. Hence, from (3.97),

$$MPC_{HF} = \frac{(1 - \beta)(1 - \alpha\beta)}{(1 - \beta\phi)} < MPC_{NoHF} = \frac{(1 - \beta)}{(1 - \beta\phi)} \quad (3.99)$$

That is, the response to current changes in income is slower than would be expected without habit persistence, i.e., $\alpha = 0$. This implies that the mpc out of an unanticipated income change is smaller with habit-forming preferences in consumption.

Second, notice also that consumption responds to all past (anticipated) income innovations. With $\alpha = 0.8$ for example, current consumption responds relatively strongly to past anticipated innovations in income. This implies that, with a positive coefficient on ω_{t-i} , the mpc out of anticipated income changes is positive with habit formation. Hence, consumption will have a higher rate of growth. (With $\alpha = 0$, this term disappears). Both results are in conformity with the “excess sensitivity” puzzle and with the empirical findings of Campbell and Deaton (1989).

Now, let us look at variances rather than levels of change of consumption. Note that:

$$Var\Delta c_t^{HF} = \left[\frac{(1-\beta)(1-\alpha\beta)}{(1-\beta\phi)} \right]^2 \sigma_\omega^2 \quad (3.100)$$

In the case without habit formation we have,

$$Var\Delta c_t = \left[\frac{(1-\beta)}{(1-\beta\phi)} \right]^2 \sigma_\omega^2 \quad (3.101)$$

for a AR(1) process. As the income process approaches a random walk, $\phi \rightarrow 1$ and $Var\Delta c_t = Var\Delta y_t = \sigma_\omega^2$. Campbell and Deaton pointed out that this prediction of the theory was counterfactual in view of the evidence from the data which imply

$$Var\Delta c_t < Var\Delta y_t$$

This is referred to as the “excess smoothness” puzzle.

With habit formation, $Var\Delta c_t = (1-\alpha\beta)^2 \sigma_\omega^2$, i.e., even when $\phi = 1$, hence, we will have $Var\Delta c_t < Var\Delta y_t$ in conformity with the empirical evidence.

While these results are dependent on the specific form of the stochastic income process, this also holds true when y_t follows a AR(1) process in differences which, ac-

cording to Hansen and Singleton, Flavin among others, better characterizes the actual income process. Let us assume now that income is a first order autoregressive process in differences of the following form:

$$(y_{t+1} - y_t) = \delta(y_t - y_{t-1}) + \omega_{t+1} \quad \text{so that} \quad y_{t+1} = (1 + \delta)y_t - \delta y_{t-1} + \omega_{t+1}$$

Then, in the absence of habit formation,

$$Var\Delta c_t = \left[\frac{(1+r)}{(1+r-\delta)} \right]^2 Var\Delta y_t \quad (3.102)$$

where $\left[\frac{(1+r)}{(1+r-\delta)} \right]^2 > 1$ so that:

$$Var\Delta c_t > Var\Delta y_t$$

whereas with habit formation:

$$Var\Delta c_t = \left[\frac{(1+r-\alpha)}{(1+r-\delta)} \right]^2 Var\Delta y_t \quad (3.103)$$

so that:

$$Var\Delta c_t < Var\Delta y_t \quad \text{provided that } \alpha > \delta$$

This is not a big restriction if $\delta = 0.442$, as in Campbell and Deaton's empirical findings.

Looking at (3.80) directly, the excess sensitivity puzzle as well as the greater importance of the precautionary premium again become evident. Replacing for Γ equation (3.80) becomes:

$$\begin{aligned} c_t = & \underbrace{(1 - \alpha\beta)y_t^p}_1 + \underbrace{(1 - \alpha\beta)\alpha\beta \sum_{i=0}^{\infty} (\alpha\beta)^i y_{t-i-1}^p}_2 \\ & - \underbrace{\frac{\beta\Theta(1 - \alpha\beta)(1 - \beta)}{2(1 - \beta\phi)^2} \sigma_{\omega}^2}_3 \end{aligned} \quad (3.104)$$

Overall, with habit formation, consumption has a lower response to y_t^p than in the usual model, but also responds to a distributed lag of past permanent incomes, i.e., to past perceptions of the income stream, thus exhibiting a faster rate of growth.

The first term shows a smaller response to what is perceived to be permanent income. The second term stands for the response to lagged income through lagged permanent income. The last term in the expression is the precautionary saving against income uncertainty. Comparing this term with the usual case without habit formation, we observe that precautionary saving against income uncertainty is smaller than the case without habit-forming preferences in consumption. Thus, for a given value of the constant measure of risk aversion Θ , a consumer with habit formation will have smaller precautionary savings per unit of variance of change in consumption. Here, we note that optimizing behavior of the habit-forming consumer leads to a lower variance of change in consumption than that of the usual case, i.e.,

$$\sigma_v^2 |_{HF} = \left[\frac{(1 - \alpha\beta)(1 - \beta)}{(1 - \beta\phi)} \right]^2 \sigma_\omega^2 \quad (3.105)$$

whereas,

$$\sigma_v^2 |_{NoHF} = \left[\frac{(1 - \beta)}{(1 - \beta\phi)} \right]^2 \sigma_\omega^2 \quad (3.106)$$

so that $\sigma_v^2 |_{HF} < \sigma_v^2 |_{NoHF}$.

However, the precautionary saving term for the consumer with habit formation is:

$$\frac{\beta\Theta(1 - \alpha\beta)(1 - \beta)}{2(1 - \beta\phi)^2} \sigma_\omega^2$$

while, the consumer without habit formation has higher precautionary savings:

$$\frac{\beta\Theta(1 - \beta)}{2(1 - \beta\phi)^2} \sigma_\omega^2$$

Therefore, we can say that per unit of income risk faced by the consumer, σ_ω^2 , the precautionary saving term is smaller for the consumer with habit formation.

Conclusion:

In this section, we have introduced habit formation into the intertemporal consumption-saving model with income uncertainty and we have obtained a closed form solution for consumption. Similar to Caballero, our motivation was to assess the potential importance of precautionary savings. In contrast to Caballero, by allowing habit-forming preferences in consumption, we did not need to assume higher levels of labor income uncertainty to explain consumption puzzles.

When we introduce habit formation into the intertemporal consumption-saving model with income uncertainty, we obtain lower consumption than the case with no habit-forming preferences in consumption. This basically implies that preferences which exhibit habit formation in consumption can help explain the high level of precautionary savings observed in the data. However, a portion of these precautionary savings are due to the habits which make the individual more prudent toward the consumption risk. This is why we have obtained lower income uncertainty-induced precautionary savings per unit of variance of change in consumption. We have also shown that the extent of the effects of income uncertainty on income depend on the strength of habits, i.e., the larger is the habit formation parameter, the lower is the savings against income uncertainty.

Furthermore, the fact that we have obtained the closed form solution for consumption with habit formation will provide a better framework for empirically testing the behavior of consumption over the business cycle. This basically implies that intertemporally non-additive preferences which exhibit habit formation in consumption may help explain high precautionary savings observed in the data, as this model shows. This is the novelty of this section.

Chapter 4

CONSUMPTION WITH LIQUIDITY CONSTRAINTS

In this chapter, we will introduce liquidity constraints into the model of consumption with habit formation.

4.1 ZELDES'S MODEL

Following Zeldes (1989b), we present an infinite horizon model with period by period liquidity constraints¹. The individual faces a quantity constraint on net indebtedness each period, which prohibits borrowing today based on future available income.

While all individuals face the liquidity constraints, at any point in time the liquidity constraint will be binding for some individuals and not for others. The constraint will be binding for those who chose not to save to build up their wealth in earlier periods or to those who received exceptionally bad draws of income.

¹Zeldes (1989b) is an finite horizon model.

In this section, we will assume that the rate of interest r is equal or less than the rate of time preference ρ , $r \leq \rho$. This assumption implies that the individual is impatient and therefore is more likely to be liquidity constrained when a low realization of income occurs; and for consumers with $r > \rho$, the liquidity constraint is unlikely to bind. They are so patient that the liquidity constraint may become irrelevant². When consumers are impatient, and when the income is independently and identically distributed over time, assets act like a buffer stock, smoothing consumption against bad draws of income³.

The individual each period seeks to maximize expected lifetime utility by choosing current consumption and future level of assets. Net wealth, excluding future labor income is restricted to be nonnegative.

Then, the problem can be defined as:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \\ & \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \tag{4.1}$$

subject to :

$$A_{t+1} = (1+r)[A_t + y_t - c_t] \tag{4.2}$$

$$A_{t+1} \geq 0 \quad \forall t, t = 0, 1, 2, \dots \tag{4.3}$$

$$\lim_{t \rightarrow \infty} A_t(1+r)^{-t} = 0$$

That implies that, as time goes to infinity, the discounted value of assets will be zero.

The individual faces an uncertain income stream y_t , where (4.2) is the period-by-period budget constraint and (4.3) is the liquidity constraint for all periods.

We can formulate the problem in a dynamic programming framework as:

²Deaton (1991) assumes a strict inequality, which implies impatience of the individual to better characterize the consumption and saving behavior of a developing country.

³Deaton (1991) shows that when the income is positively autocorrelated assets are used less effectively to smooth out consumption, and in the limit when income follows a random walk, it is optimal for impatient liquidity constrained consumers simply to consume their incomes.

$$V(A_t, y_t) = \underset{c_t, A_{t+1}}{\text{Max}} U(c_t) + \beta E_t V(A_{t+1}, y_{t+1}) \quad (4.4)$$

subject to :

$$A_{t+1} = (1 + r)[A_t + y_t - c_t]$$

$$A_{t+1} \geq 0 \quad \forall t.$$

or alternatively,

$$\begin{aligned} V(A_t, y_t) = \underset{c_t}{\text{Max}} & U(c_t) + \beta E_t V((1 + r)[A_t + y_t - c_t], y_{t+1}) \\ & + \mu_t(1 + r)[A_t + y_t - c_t] \end{aligned} \quad (4.5)$$

where μ_t is the Lagrange multiplier associated with the liquidity constraint.

The Kuhn-Tucker conditions are:

$$U'(c_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] - \mu_t(1 + r) \leq 0 \quad (4.6)$$

$$(1 + r)[A_t + y_t - c_t] \geq 0 \text{ with equality if } \mu_t > 0 \quad (4.7)$$

$$c_t \geq 0; \mu_t \geq 0 \quad (4.8)$$

The first order condition (4.6) will always hold with equality due to non-satiation.

Using the envelope theorem, we can find:

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V(A_{t+1}, y_{t+1})}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} + \mu_t(1+r) \quad (4.9)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1+r) \quad (4.10)$$

In (4.6), solving for $\beta E_t \frac{\partial V(A_{t+1}, y_{t+1})}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r)} [U'(c_t) + \mu_t(1+r)] \quad (4.11)$$

Substituting (4.11) into (4.9), we can obtain:

$$\begin{aligned} \frac{\partial V}{\partial A_t} &= (1+r) \left[\frac{U'(c_t)}{(1+r)} - \mu_t \right] + \mu_t(1+r) \\ &= U'(c_t) \end{aligned} \quad (4.12)$$

Note that, at the optimum, the extra lifetime utility from being given an extra dollar in assets equals the extra utility from consuming it immediately, whether or not the constraint is binding. Forwarding (4.12) one period and plugging into (4.6) gives:

$$U'(c_t) = \beta(1+r)E_t U'(c_{t+1}) + (1+r)\mu_t \quad (4.13)$$

The marginal utility of current consumption will be higher, if the current liquidity constraint is binding. The constraint will be binding for those who chose not to save in earlier periods, or those who received extremely low levels of income for the current and past periods. μ_t is the increase in expected lifetime utility if the current constraint were relaxed by one extra unit of wealth. The individual can always save more, and consume less precisely because he or she cannot borrow. Thus, a binding liquidity constraint implies lower consumption.

This is the result of Mariger (1987) and Zeldes (1989b) that liquidity-constrained households need not have zero assets. As Zeldes (1989b) notes, the liquidity constrained

individual need not have zero assets at all times. That is, even though the liquidity constraint may be binding some periods, it need not bind every period. In this case, the individual saves precisely because he or she may be unable to borrow in the future if low realizations of income should occur. Thus, saving when the liquidity constraint is not binding permits the consumer to dissave, and to increase consumption when the constraint is binding. The liquidity constrained individual, therefore, may hold positive assets.

4.2 ZELDES'S MODEL WITH HABIT FORMATION

Now, we will allow for habit-forming preferences in consumption and reproduce infinite horizon Zeldes's model. We will again use the simple habit formation model with consumption depending on one lagged consumption. Then, the consumer seeks to maximize:

$$\begin{aligned} \text{Max } E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}) \\ \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (4.14)$$

subject to :

$$A_{t+1} = (1+r)[A_t + y_t - c_t] \quad (4.15)$$

$$A_{t+1} \geq 0 \quad t = 0, 1, 2, \dots \quad (4.16)$$

where $\lim_{t \rightarrow \infty} A_t(1+r)^{-t} = 0$ and with an uncertain income stream y_t , where (4.15) is the period-by-period budget constraint and (4.16) is the liquidity constraint for all periods. We assume that the rate of interest r is equal or less than the rate of time preference ρ , $r \leq \rho$.

We can formulate the problem in a dynamic programming framework as:

$$\begin{aligned} V(A_t, c_{t-1}, y_t) = \text{Max } U(c_t - \alpha c_{t-1}) + \beta E_t V(A_{t+1}, c_t, y_{t+1}) \\ c_t, A_{t+1} \end{aligned} \quad (4.17)$$

subject to :

$$A_{t+1} = (1+r)[A_t + y_t - c_t]$$

$$A_{t+1} \geq 0 \quad \forall t.$$

Substituting for the constraints:

$$\begin{aligned}
V(A_t, c_{t-1}, y_t) = & \text{Max}_{c_t} U(c_t - \alpha c_{t-1}) \\
& + \beta E_t V((1+r)[A_t + y_t - c_t], c_t, y_{t+1}) \\
& + \mu_t(1+r)[A_t + y_t - c_t]
\end{aligned} \tag{4.18}$$

where μ_t is the Lagrange multiplier associated with the liquidity constraint.

The Kuhn-Tucker conditions are:

$$U'(c_t - \alpha c_{t-1}) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} + \frac{\partial V}{\partial c_t} \right] - \mu_t(1+r) = 0 \tag{4.19}$$

$$(1+r)[A_t + y_t - c_t] \geq 0 \text{ with equality if } \mu_t > 0 \tag{4.20}$$

$$c_t \geq 0; \mu_t \geq 0 \tag{4.21}$$

Differentiating the value function with respect to c_{t-1} , and using the envelope theorem:

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha U(c_t - \alpha c_{t-1}) \tag{4.22}$$

Forwarding (4.22) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t U(c_{t+1} - \alpha c_t) \tag{4.23}$$

Substituting (4.23) into (4.19):

$$\begin{aligned}
& U'(c_t - \alpha c_{t-1}) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} - \alpha U(c_{t+1} - \alpha c_t) \right] \\
& - \mu_t(1 + r) \\
& = 0
\end{aligned} \tag{4.24}$$

Now, differentiating the value function with respect to A_t gives:

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} + \mu_t(1 + r) \tag{4.25}$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1 + r)$$

From (4.24), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$ gives:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{1 + r} [U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U(c_{t+1} - \alpha c_t)] - \mu_t \tag{4.26}$$

Substituting (4.26) into (4.25), we obtain:

$$\frac{\partial V}{\partial A_t} = [U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U(c_{t+1} - \alpha c_t)] \tag{4.27}$$

Forwarding (4.27) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t [U'(c_{t+1} - \alpha c_t) - \alpha \beta U(c_{t+2} - \alpha c_{t+1})] \tag{4.28}$$

Substituting (4.28) into (4.19), we obtain the Euler equation for consumption:

$$\begin{aligned}
& U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U'(c_{t+1} - \alpha c_t) \\
& = \beta(1 + r) E_t [U'(c_{t+1} - \alpha c_t) - \alpha \beta U'(c_{t+2} - \alpha c_{t+1})] \\
& \quad + (1 + r) \mu_t
\end{aligned} \tag{4.29}$$

Using the definition of $c_t - \alpha c_{t-1} = \widehat{c}_t$, let us assume that the utility function is quadratic:

$$U(\widehat{c}_t) = \nu_0 + \nu_1 \widehat{c}_t - \frac{1}{2} \nu_2 \widehat{c}_t^2$$

Then, the Euler equation will be:

$$\begin{aligned} [\widehat{c}_t - \alpha \beta E_t \widehat{c}_{t+1}] &= \beta(1+r) E_t [\widehat{c}_{t+1} - \alpha \beta \widehat{c}_{t+2}] \\ &\quad - \frac{\nu_1}{\nu_2} [\beta(1+r) - 1] [1 - \alpha \beta] \\ &\quad - \frac{(1+r)}{\nu_2} \mu_t \end{aligned} \quad (4.30)$$

Let us define κ as:

$$\kappa = \frac{\nu_1}{\nu_2} [\beta(1+r) - 1] [1 - \alpha \beta] \quad (4.31)$$

Rewrite equation (4.30) as:

$$\begin{aligned} [\widehat{c}_t - \beta(1+r) E_t \widehat{c}_{t+1}] &= \alpha \beta E_t [\widehat{c}_{t+1} - \beta(1+r) \widehat{c}_{t+2}] \\ &\quad - \kappa - \frac{(1+r)}{\nu_2} \mu_t \end{aligned} \quad (4.32)$$

where we can factor out $[\widehat{c}_t - \beta(1+r) E_t \widehat{c}_{t+1}]$ in (4.32) using the lag operator such that:

$$\begin{aligned} &E_t [\widehat{c}_t - \beta(1+r) \widehat{c}_{t+1}] (1 - \alpha \beta L^{-1}) \\ &= -\kappa - \frac{(1+r)}{\nu_2} \mu_t \end{aligned} \quad (4.33)$$

Dividing both sides by $(1 - \alpha \beta L^{-1})$:

$$E_t [\widehat{c}_t - \beta(1+r) E_t \widehat{c}_{t+1}] = -\frac{\kappa}{(1 - \alpha \beta)} - \frac{(1+r)}{\nu_2(1 - \alpha \beta L^{-1})} \mu_t \quad (4.34)$$

or,

$$E_t [\hat{c}_t - \beta(1+r)\hat{c}_{t+1}] = -\frac{\kappa}{(1-\alpha\beta)} - \frac{(1+r)}{\nu_2} E_t \sum_{\tau=0}^{\infty} (\alpha\beta)^\tau \mu_{t+\tau} \quad (4.35)$$

Rearranging, we obtain for $\hat{c}_t$:

$$\begin{aligned} \hat{c}_t = & \beta(1+r)E_t\hat{c}_{t+1} - \frac{(1+r)}{\nu_2}\mu_t - \frac{(1+r)}{\nu_2}E_t \sum_{\tau=1}^{\infty} (\alpha\beta)^\tau \mu_{t+\tau} \\ & - \frac{\kappa}{(1-\alpha\beta)} \end{aligned} \quad (4.36)$$

We can clearly see that optimal current consumption under binding liquidity constraints and with habit formation will be even lower than the level of optimal constrained consumption with time-separable preferences.

If there were only liquidity constraints but no habit formation in consumption, then in the context of a quadratic utility function the result would be:

$$c_t = \beta(1+r)E_t c_{t+1} - \frac{(1+r)}{\nu_2}\mu_t - \kappa \quad (4.37)$$

where κ is a constant term defined earlier.

If there were no habit formation and no liquidity constraints, then we would obtain the usual result of Hall's random walk hypothesis:

$$c_t = \beta(1+r)E_t c_{t+1} - \kappa \quad (4.38)$$

We can also write an expression for the general case:

$$U'(\hat{c}_t) = \beta(1+r)E_t U'(\hat{c}_{t+1}) + (1+r)\mu_t + (1+r)E_t \sum_{\tau=1}^{\infty} (\alpha\beta)^\tau \mu_{t+\tau} \quad (4.39)$$

Thus, we expressed equation (4.29) in terms of current and future shadow prices of liquidity constraints. This is an interesting result, because we can better analyze the

effect of liquidity constraints and habit formation on current consumption.

The presence of liquidity constraints increases the current marginal utility of consumption, implying a lower level of consumption than the case with a non-binding liquidity constraint case with habit-forming preferences in consumption. In the current period, the liquidity constraint has an independent effect on current marginal utility. However, since there is habit formation in consumption, all future expected shadow prices of the liquidity constraint will also affect current marginal utility. As a result, current effective consumption is lower when the liquidity constraint is binding. If we did not have habit formation but only liquidity constraint assumption, the joint effect of future expected shadow prices would not appear in the Euler equation.

Facing an uncertain income and trying to keep a certain living standard, the individual decreases consumption and increases savings. The introduction of habit formation rationalizes a precautionary saving, additional to the one arising from uncertain income and to the one against liquidity constraints. Comparing to the previous literature, this model does not have to rely only on the effects of income uncertainty and binding liquidity constraints to explain precautionary savings, which account for a large proportion of total savings, but also on intertemporally non-additive preferences exhibiting habit formation in consumption. Since habit-forming consumers prefer a higher growth rate of consumption to a stable one, they save even more in the presence of income uncertainty and constraints to borrowing.

4.3 BORROWING CONSTRAINTS

Borrowing constraints are a more relaxed form of liquidity constraints which allow the consumer to borrow a limited amount from the capital markets. One of the uses of external finance is sheltering consumption from fluctuations in income, whether they are due to internal or external factors. In this section, we will especially discuss the implications of borrowing constraints in the context of international borrowing.

Let us assume the basic intertemporal consumption-saving problem of an economy (or equivalently a representative agent) with borrowing constraints, income uncertainty, and habit-forming preferences in consumption. The economy will be assumed to behave like an individual consumer, maximizing the discounted flow of utility from a consumption stream over the infinite horizon subject to a budget constraint and borrowing constraint. Each period the economy chooses a consumption level and next period's asset (debt) level. The essence of the problem is the income uncertainty.

The representative agent seeks to maximize:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}) \\ & \{c_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (4.40)$$

subject to :

$$A_{t+1} = (1+r)[A_t + y_t - c_t] \quad (4.41)$$

$$A_{t+1} \geq -\bar{A} \quad t = 0, 1, 2, \dots \quad (4.42)$$

$$\lim_{t \rightarrow \infty} A_t(1+r)^{-t} = 0, \quad -\bar{A} > -\infty \text{ or } \bar{A} < \infty$$

with an uncertain income stream y_t , current period of interest rate r , current period asset level A_t , next period asset level A_{t+1} , current consumption c_t , past consumption c_{t-1} , habit formation parameter α , $0 < \alpha < 1$, the discount factor $\beta = \frac{1}{1+\rho}$, the rate of time preference ρ . We assume that the rate of interest r is equal or less than the rate of

time preference ρ , $r \leq \rho$.

The individual chooses current consumption c_t , and future asset level A_{t+1} subject to the period-by-period budget constraint (4.41), and (4.42) is the borrowing constraint for each period. The debt (negative assets) cannot exceed the exogenously given limit $\bar{A}$. Each period the individual can only borrow up to a certain exogenously given debt ceiling $\bar{A}$. This is decided by the lender. The consumer is constrained from borrowing if the desired borrowing is greater than the debt ceiling $\bar{A}$. In this case, the individual will only consume total currently available resources: current income, and currently available debt which is equal to its ceiling level.

We can formulate the problem in a stochastic dynamic programming framework with the value function V and the state variables A_t , and c_{t-1} , and y_t such that:

$$V(A_t, c_{t-1}, y_t) = \underset{c_t, A_{t+1}}{\text{Max}} U(c_t - \alpha c_{t-1}) + \beta E_t V(A_{t+1}, c_t, y_{t+1}) \quad (4.43)$$

subject to :

$$\begin{aligned} A_{t+1} &= (1+r)[A_t + y_t - c_t] \\ A_{t+1} &\geq -\bar{A} \quad \forall t \quad t = 0, 1, 2, \dots \end{aligned}$$

Substituting for the constraints:

$$\begin{aligned} V(A_t, c_{t-1}, y_t) &= \underset{c_t}{\text{Max}} U(c_t - \alpha c_{t-1}) \\ &\quad + \beta E_t V((1+r)[A_t + y_t - c_t], c_t, y_{t+1}) \\ &\quad + \mu_t [(1+r)[A_t + y_t - c_t] + \bar{A}] \end{aligned} \quad (4.44)$$

where μ_t is the Lagrange multiplier associated with the borrowing constraint.

The Kuhn-Tucker conditions are:

$$U'(c_t - \alpha c_{t-1}) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} + \frac{\partial V}{\partial c_t} \right] - (1+r)\mu_t = 0 \quad (4.45)$$

$$(1+r)[A_t + y_t - c_t] + \bar{A} \geq 0 \text{ with equality if } \mu_t > 0 \quad (4.46)$$

$$c_t \geq 0; \mu_t \geq 0 \quad (4.47)$$

Let us assume that c_t solves (4.45) and (4.46). Then, differentiating the value function with respect to c_{t-1} we obtain:

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha U'(c_t - \alpha c_{t-1}) \quad (4.48)$$

Forwarding (4.48) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t U'(c_{t+1} - \alpha c_t) \quad (4.49)$$

Substituting (4.49) into (4.45):

$$\begin{aligned} & U'(c_t - \alpha c_{t-1}) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] - \alpha \beta E_t U'(c_{t+1} - \alpha c_t) \\ & - (1+r)\mu_t \\ & = 0 \end{aligned} \quad (4.50)$$

From (4.50), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r)} [U'(c_t - \alpha c_{t-1}) - \alpha \beta U'(c_{t+1} - \alpha c_t) - \mu_t] \quad (4.51)$$

Now, differentiating (4.44) with respect to A_t ,

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} + (1+r)\mu_t \quad (4.52)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1+r)$$

Substituting (4.51) in (4.52):

$$\begin{aligned} \frac{\partial V}{\partial A_t} &= \frac{1}{(1+r)} [U'(c_t - \alpha c_{t-1}) - \alpha \beta U'(c_{t+1} - \alpha c_t) - \mu_t] (1+r) \\ &\quad + (1+r)\mu_t \\ &= [U'(c_t - \alpha c_{t-1}) - \alpha \beta U'(c_{t+1} - \alpha c_t)] \end{aligned} \quad (4.53)$$

This is the envelope condition. Forwarding now (4.53) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = [U'(c_{t+1} - \alpha c_t) - \alpha \beta U'(c_{t+2} - \alpha c_{t+1})] \quad (4.54)$$

Substituting (4.54) into the first order condition (4.50) one can obtain the Euler equation for consumption:

$$\begin{aligned} &U'(c_t - \alpha c_{t-1}) - \alpha \beta E_t U'(c_{t+1} - \alpha c_t) \\ &= \beta(1+r)E_t [U'(c_{t+1} - \alpha c_t) - \alpha \beta U'(c_{t+2} - \alpha c_{t+1})] + (1+r)\mu_t \end{aligned} \quad (4.55)$$

There are three factors leading to a decrease in current consumption: habit formation, borrowing constraint (when it is binding), and the precautionary savings premium associated with income uncertainty. As we have found in the previous section, (4.39), marginal utility of current consumption can be expressed as:

$$U'(\widehat{c}_t) = \beta(1+r)E_t U'(\widehat{c}_{t+1}) + (1+r)\mu_t + (1+r)E_t \sum_{\tau=1}^{\infty} (\alpha\beta)^\tau \mu_{t+\tau}$$

The only difference between this model and the model where no borrowing is allowed is the level of utilities. In this model, the consumer is allowed to borrow up to a limited exogenously given amount $\bar{A}$. The borrowing constraint binds whenever the consumer's desired level of borrowing exceeds $\bar{A}$. This enables the individual to better smooth out consumption in low realizations of income and to reach a higher level of lifetime utility.

The current marginal utility of consumption will be higher when the current borrowing constraint is binding, implying lower consumption than in the case with no borrowing constraint. The individual with habit-forming preferences in consumption, and with an uncertain future income stream decreases consumption and borrows less if the borrowing constraint is not binding this period. If the realization of current income is high enough, the individual may even save to reduce the accumulated debt in order to relax future borrowing constraint.

Conclusion:

There are three factors leading to a decrease in current consumption: habit formation, the borrowing constraint (when it is binding), and expected future borrowing constraint. Each of them will make the consumer more cautious about consuming too much today.

Hence, one can explain the fact that high precautionary savings observed by several economists may not only be the result of pure income uncertainty but also of habit-forming preferences in consumption together with binding borrowing constraints.

Chapter 5

CONSUMPTION-LEISURE CHOICE

In this chapter, we will examine the impact of habit formation on consumption and leisure choices of the individual. We will also introduce liquidity constraints in this framework.

5.1 THE GENERAL MODEL

We first present a simple consumption-leisure model with wage uncertainty where the consumer seeks to maximize:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \\ & \{c_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \tag{5.1}$$

subject to :

$$A_{t+1} = (1+r)[A_t + w_t(1-l_t) - c_t] \tag{5.2}$$

with $\lim_{t \rightarrow \infty} A_t(1+r)^{-t} = 0$

In each period t , total time available for leisure l_t and work h_t is assumed to be limited to one unit, that is a day. So, for all t , $h_t = 1 - l_t$, and w_t is current known real wage rate and A_{t+1} future asset holdings, A_t , current asset holdings. Each period, the consumer chooses current consumption, current leisure (hours of work) and future asset holdings.

We can formulate the problem in a dynamic programming framework such as:

$$\begin{aligned} V(A_t, w_t) = \underset{c_t, l_t, A_{t+1}}{\text{Max}} & U(c_t, l_t) + \beta E_t V(A_{t+1}, w_{t+1}) \end{aligned} \quad (5.3)$$

subject to :

$$A_{t+1} = (1 + r)[A_t + w_t(1 - l_t) - c_t]$$

Substituting for the constraint:

$$\begin{aligned} V(A_t, w_t) = \underset{c_t, l_t}{\text{Max}} & U(c_t, l_t) \\ & + \beta E_t V((1 + r)[A_t + w_t(1 - l_t) - c_t], w_{t+1}) \end{aligned} \quad (5.4)$$

The two first order conditions are:

$$U_1(c_t, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] = 0 \quad (5.5)$$

$$U_2(c_t, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial l_t} \right] = 0 \quad (5.6)$$

Differentiating the value function with respect to A_t :

$$\frac{\partial V}{\partial A_t} = \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} \right] \quad (5.7)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1 + r)$$

From (5.5) solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1 + r)} U_1(c_t, l_t) \quad (5.8)$$

Substituting (5.8) into (5.7):

$$\frac{\partial V}{\partial A_t} = U_1(c_t, l_t) \quad (5.9)$$

This is the envelope condition. Forwarding (5.9) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t U_1(c_{t+1}, l_{t+1}) \quad (5.10)$$

Plugging (5.10) into the first order condition (5.5), we obtain the Euler equation for consumption:

$$U_1(c_t, l_t) = \beta(1 + r) E_t [U_1(c_{t+1}, l_{t+1})] \quad (5.11)$$

Similarly, from (5.6) solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1 + r)w_t} U_2(c_t, l_t) \quad (5.12)$$

Substituting (5.12) into (5.7):

$$\frac{\partial V}{\partial A_t} = \frac{1}{w_t} U_2(c_t, l_t) \quad (5.13)$$

This is the envelope condition. Forwarding by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_t} = E_t \frac{1}{w_{t+1}} U_2(c_{t+1}, l_{t+1}) \quad (5.14)$$

Plugging (5.14) into the first order condition (5.6), we will obtain the Euler equation for leisure:

$$U_2(c_t, l_t) = \beta(1+r)E_t \left\{ \left[\frac{w_t}{w_{t+1}} \right] U_2(c_{t+1}, l_{t+1}) \right\} \quad (5.15)$$

The marginal rate of substitution between current leisure and future leisure will depend on the ratio of the current and future wage rate. Since the wage rate is not constant but time-varying and stochastic, the ratio of wages will affect the marginal rate of substitution between current and future leisure. If the expectations of the future wage rate are higher, then the marginal rate of substitution between current and future leisure decreases. implying a higher leisure for today.

By plugging (5.10) into (5.6), we can also express $U_2(c_t, l_t)$ as:

$$U_2(c_t, l_t) = \beta(1+r)w_t E_t [U_1(c_{t+1}, l_{t+1})] \quad (5.16)$$

Combining (5.11) and (5.16), we can show that $U_2(c_t, l_t)$ is equal to:

$$U_2(c_t, l_t) = w_t U_1(c_t, l_t) \quad (5.17)$$

Thus, we can write the intratemporal marginal rate of substitution between consumption and leisure as:

$$\frac{U_2(c_t, l_t)}{U_1(c_t, l_t)} = w_t$$

where the wage rate is the relative price of consumption and leisure.

5.2 THE GENERAL MODEL WITH HABIT FORMATION

Now, we modify the general model to incorporate habit formation into consumption. As usual, we will assume that current utility depends on current and one lagged consumption. Then, the consumer seeks to maximize:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}, l_t) \\ & \{c_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (5.18)$$

subject to :

$$A_{t+1} = (1 + r_t)[A_t + w_t(1 - l_t) - c_t] \quad (5.19)$$

with h_t being current labor, l_t is current leisure, w_t is current known real wage rate and A_{t+1} future asset holdings, A_t , current asset holdings. Each period, the consumer chooses current consumption, current leisure (hours of work) and future asset holdings. In this section, we assume time-varying interest rates, r_t , to give the model a more realistic setup.

We can formulate the problem in a dynamic programming framework as:

$$\begin{aligned} V(A_t, c_{t-1}, w_t) = \quad & \text{Max}_{c_t, l_t, A_{t+1}} U(c_t - \alpha c_{t-1}, l_t) + \beta E_t V(A_{t+1}, c_t, w_{t+1}) \end{aligned} \quad (5.20)$$

subject to :

$$A_{t+1} = (1 + r_t)[A_t + w_t(1 - l_t) - c_t]$$

Substituting for the constraint:

$$\begin{aligned}
V(A_t, c_{t-1}, w_t) &= \text{Max}_{c_t, l_t} U(c_t - \alpha c_{t-1}, l_t) \\
&\quad + \beta E_t V((1 + r_t)[A_t + w_t(1 - l_t) - c_t], c_t, w_{t+1})
\end{aligned} \tag{5.21}$$

The two first order conditions are:

$$U_1(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} + \frac{\partial V}{\partial c_t} \right] = 0 \tag{5.22}$$

$$U_2(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial l_t} \right] = 0 \tag{5.23}$$

Differentiating the value function with respect to c_{t-1} , we will obtain:

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha U_1(c_t - \alpha c_{t-1}, l_t) \tag{5.24}$$

Forwarding (5.24) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \tag{5.25}$$

Plugging (5.25) into (5.22), the first order condition for consumption becomes:

$$\begin{aligned}
&U_1(c_t - \alpha c_{t-1}, l_t) + \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \\
&\quad - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \\
&= 0
\end{aligned} \tag{5.26}$$

From (5.26) solving for $\beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \right]$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1 + r_t)} [U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta U_1(c_{t+1} - \alpha c_t, l_{t+1})] \tag{5.27}$$

where

$$\frac{\partial A_{t+1}}{\partial c_t} = -(1 + r_t)$$

Then, differentiating the value function with respect to A_t :

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} \quad (5.28)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1 + r_t)$$

Plugging (5.27) into (5.28):

$$\begin{aligned} \frac{\partial V}{\partial A_t} &= \frac{1}{(1 + r_t)} [U_1(c_t - \alpha c_{t-1}, l_t) \\ &\quad - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1})] \frac{\partial A_{t+1}}{\partial A_t} \\ &= [U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1})] \end{aligned} \quad (5.29)$$

This is the envelope condition. Forwarding (5.29) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t [U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha \beta U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \quad (5.30)$$

Replacing (5.30) into the first order condition for consumption. (5.26), we can obtain the Euler equation for consumption:

$$\begin{aligned} &U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \\ &= \beta(1 + r_t) E_t [U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha \beta U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \end{aligned} \quad (5.31)$$

(5.31) is the Euler equation for consumption. The left-hand side is the net marginal utility cost of forgoing one unit of consumption expenditure in period t . Since there is habit formation in consumption, the future leisure and consumption choices will be also

taken into account in the current net marginal utility of consumption. The right-hand side is the discounted marginal benefit of decreasing consumption by one unit in period t , and increasing it by one unit in period $t + 1$. Utility in period $t + 1$ increases, whereas utility in period $t + 2$ decreases because the habit stock is higher.

In that case, it is clear that current consumption decision will not only affect consumption and leisure choices in period t and period $t + 1$ but also, expectations of future consumption and leisure. That is, current consumption decisions are affected by expectations of consumption and leisure choices up to and including period $t + 2$. Clearly, if the form in which habit formation was introduced, had included further lags, the Euler equation would have had even richer dynamics.

To find the Euler equation for leisure, in (5.23) we solve for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1 + r_t)w_t} U_2(c_t - \alpha c_{t-1}, l_t) \quad (5.32)$$

Now, replacing (5.32) into (5.28):

$$\begin{aligned} \frac{\partial V}{\partial A_t} &= \frac{1}{(1 + r_t)w_t} U_2(c_t - \alpha c_{t-1}, l_t)(1 + r_t) \\ &= \frac{1}{w_t} U_2(c_t - \alpha c_{t-1}, l_t) \end{aligned} \quad (5.33)$$

This is the envelope condition. Forwarding (5.33) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t \left\{ \frac{1}{w_{t+1}} U_2(c_{t+1} - \alpha c_t, l_{t+1}) \right\} \quad (5.34)$$

Replacing (5.34) into the first order condition for leisure, (5.23), we obtain the Euler equation for leisure:

$$U_2(c_t - \alpha c_{t-1}, l_t) = \beta(1 + r_t) E_t \left\{ \left[\frac{w_t}{w_{t+1}} \right] U_2(c_{t+1} - \alpha c_t, l_{t+1}) \right\} \quad (5.35)$$

The left-hand side is the marginal cost of forgoing one unit of leisure in period t .

The right-hand side is the discounted expected marginal benefit of decreasing leisure by one unit in period t . The higher w_{t+1} is expected to be relative to w_t , the smaller the expected marginal benefit of substituting leisure intertemporally, and foregoing leisure today and increasing leisure in period $t + 1$.

By replacing (5.30) into (5.23), we can also express $U_2(c_t - \alpha c_{t-1}, l_t)$ as:

$$\begin{aligned} U_2(c_t - \alpha c_{t-1}, l_t) = & \beta(1 + r_t)w_t E_t[U_1(c_{t+1} - \alpha c_t, l_{t+1}) \\ & - \alpha\beta E_t U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \end{aligned} \quad (5.36)$$

or

$$\begin{aligned} U_2(c_t - \alpha c_{t-1}, l_t) = & w_t[U_1(c_t - \alpha c_{t-1}, l_t) \\ & - \alpha\beta E_t U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \end{aligned} \quad (5.37)$$

Alternatively,

$$\frac{U_2(c_t - \alpha c_{t-1}, l_t)}{U_1(c_t - \alpha c_{t-1}, l_t) - \alpha\beta U_1 E_t(c_{t+1} - \alpha c_t, l_{t+1})} = w_t \quad (5.38)$$

The left-hand side of (5.38) is the marginal rate of substitution between current leisure and current consumption, which is equal to w_t , the relative price of leisure.

Since there is a positive habit formation parameter, $\alpha > 0$, (5.38) indicates that the optimal choice of leisure will be affected by habit-forming consumption expenditures. Because of the intertemporal non-additivity of preferences, even the intratemporal marginal rate of substitution between consumption and leisure is affected by past and future consumption choices. This is not the case where preferences are even weakly separable as has been noted by Meghir and Weber.

We would like to explore further the impact of habit formation on the intertemporal marginal rate of substitution of leisure and the intratemporal marginal rate of substitution between consumption and leisure. However, it is not possible to obtain determinate

results in the general case. Therefore, we consider a special form of the preference functional in the next section.

5.3 MULTIPLICATIVE FORM

In this section, we undertake to show, with an example of multiplicative utility function, that the intratemporal marginal rate of substitution (MRS) between consumption and leisure, as well as the intertemporal MRS of leisure are higher in the case of habit formation.

The individual makes decision about next period assets A_{t+1} , current leisure l_t and current consumption c_t subject to the budget constraint and to the time constraint. In each period t , total time available for leisure l_t and work h_t is assumed to be limited to one unit, that is a day. So, for all t , $h_t = 1 - l_t$. In this section as well, we assume time-varying interest rates, r_t . The consumer's maximization problem is stated as follows:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{\theta} [c_t^\theta c_{t-1}^{-\theta\alpha}]^{\frac{1}{\eta}} l_t^\eta \\ \text{subject to} \quad & \{c_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty}, \end{aligned} \quad (5.39)$$

subject to :

$$A_{t+1} = (1 + r_t) [A_t + w_t(1 - l_t) - c_t] \quad (5.40)$$

The utility function is a Cobb-Douglas utility function where $0 < \theta < 1$, and $0 < \eta < 1$, w_t is the real wage rate and with the consumption good being numeraire. We assume that real wages facing the individual are exogenous, i.e., they do not depend on past labor supply choices. As the beginning of period t , the realizations of the real wage rate w_t are known to the individual but future realizations are unknown and random.

Based on the above structure, we can characterize the individual's maximization problem as a dynamic programming problem.

The value function at time t for this problem is:

$$V(A_t, c_{t-1}, w_t) = \underset{c_t, l_t, A_{t+1}}{\text{Max}} \frac{1}{\theta} [c_t^\theta c_{t-1}^{-\theta\alpha}] \frac{1}{\eta} l_t^\eta + \beta E_t V(A_{t+1}, c_t, w_{t+1}) \quad (5.41)$$

subject to :

$$A_{t+1} = (1 + r_t) [A_t + w_t(1 - l_t) - c_t]$$

with the state variables A_t , and c_{t-1} and w_t .

Substituting for the constraint:

$$\begin{aligned} V(A_t, c_{t-1}, w_t) = \underset{c_t, l_t}{\text{Max}} & \frac{1}{\theta} [c_t^\theta c_{t-1}^{-\theta\alpha}] \frac{1}{\eta} l_t^\eta \\ & + \beta E_t V((1 + r_t) [A_t + w_t(1 - l_t) - c_t], c_t, w_{t+1}) \end{aligned} \quad (5.42)$$

The first order conditions with respect to c_t , and l_t are :

$$c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} + \frac{\partial V}{\partial c_t} \right] = 0 \quad (5.43)$$

$$c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\theta} l_t^{\eta-1} + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial l_t} \right] = 0 \quad (5.44)$$

Taking the derivative of the value function with respect to c_{t-1} :

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha c_t^\theta c_{t-1}^{-\theta\alpha-1} \frac{1}{\eta} l_t^\eta \quad (5.45)$$

Forwarding (5.45) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right\} \quad (5.46)$$

Substituting (5.46) for $E_t \frac{\partial V}{\partial c_t}$ into (5.43), the first order condition for consumption becomes:

$$\begin{aligned} c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] \\ - \alpha \beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right\} = 0 \end{aligned} \quad (5.47)$$

From (5.47), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r_t)} \left[c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta - \alpha \beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right\} \right] \quad (5.48)$$

By taking the derivative the value function with respect to A_t :

$$\frac{\partial V}{\partial A_t} = \beta(1+r_t) E_t \frac{\partial V}{\partial A_{t+1}} \quad (5.49)$$

Substituting (5.48) into (5.49):

$$\frac{\partial V}{\partial A_t} = \left[c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta - \alpha \beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right\} \right] \quad (5.50)$$

This is the envelope condition. Forwarding (5.50) one period, and taking expectations we obtain:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t \left[c_{t+1}^{\theta-1} c_t^{-\theta\alpha} \frac{1}{\eta} l_{t+1}^\eta - \alpha \beta c_{t+2}^\theta c_{t+1}^{-\theta\alpha-1} \frac{1}{\eta} l_{t+2}^\eta \right] \quad (5.51)$$

Substituting equation (5.51) into equation (5.47), the F.O.C. for consumption, we obtain the Euler equation for consumption:

$$\begin{aligned} c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta - \alpha \beta^2 E_t \left[c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right] \\ = \beta(1+r_t) E_t \left[c_{t+1}^{\theta-1} c_t^{-\theta\alpha} \frac{1}{\eta} l_{t+1}^\eta - \alpha \beta c_{t+2}^\theta c_{t+1}^{-\theta\alpha-1} \frac{1}{\eta} l_{t+2}^\eta \right] \end{aligned} \quad (5.52)$$

Now, let us find the Euler equation for leisure. Solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$ in (5.44):

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r_t)w_t} c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\theta} l_t^{\eta-1} \quad (5.53)$$

Substituting (5.53) into (5.49):

$$\frac{\partial V}{\partial A_t} = \frac{1}{w_t} c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\theta} l_t^{\eta-1} \quad (5.54)$$

This is the envelope condition. Forwarding (5.54) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t \left[\frac{1}{w_{t+1}} c_{t+1}^\theta c_t^{-\theta\alpha} \frac{1}{\theta} l_{t+1}^{\eta-1} \right] \quad (5.55)$$

Plugging (5.55) into the first order condition for leisure (5.44), we will obtain the Euler equation for leisure:

$$c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\theta} l_t^{\eta-1} = \beta(1+r_t) E_t \left[\frac{w_t}{w_{t+1}} c_{t+1}^\theta c_t^{-\theta\alpha} \frac{1}{\theta} l_{t+1}^{\eta-1} \right] \quad (5.56)$$

Alternatively, by substituting (5.51) into (5.44), we can write:

$$c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\theta} l_t^{\eta-1} = (1+r_t) \beta w_t E_t \left[c_{t+1}^{\theta-1} c_t^{-\theta\alpha} \frac{1}{\eta} l_{t+1}^\eta - \alpha \beta c_{t+2}^\theta c_{t+1}^{-\theta\alpha-1} \frac{1}{\eta} l_{t+2}^\eta \right] \quad (5.57)$$

Now, let us define:

$$\frac{1}{\theta} c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta \equiv U(\widehat{c}_t, l_t) \quad (5.58)$$

so that the marginal utility of leisure, $U_l(\widehat{c}_t, l_t)$, can be expressed as:

$$U_l(\widehat{c}_t, l_t) = \frac{1}{\theta} c_t^\theta c_{t-1}^{-\theta\alpha} l_t^{\eta-1} \quad (5.59)$$

and $MU_c(\widehat{c}_t, l_t)$, the net marginal utility of consumption, can be written as:

$$MU_c(\widehat{c}_t, l_t) = \left[c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta - \alpha \beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right\} \right] \quad (5.60)$$

or

$$MU_c(\widehat{c}_t, l_t) = \theta c_t^{-1} [U(\widehat{c}_t, l_t) - \alpha \beta E_t U(\widehat{c}_{t+1}, l_{t+1})]$$

If there were no habit formation in consumption, the intratemporal marginal rate of substitution between consumption and leisure would be:

$$\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = \frac{\eta c_t}{\theta l_t}$$

Then, we can write the intratemporal marginal rate of substitution between consumption and leisure, $MRS_{|\alpha>0}^{intra}$, as:

$$\begin{aligned} MRS_{|\alpha>0}^{intra} &= \frac{U_l(\widehat{c}_t, l_t)}{MU_c(\widehat{c}_t, l_t)} \\ &= \frac{\frac{1}{\theta} c_t^\theta c_{t-1}^{-\theta\alpha} l_t^{\eta-1}}{\left[c_t^{\theta-1} c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta - \alpha \beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1} \frac{1}{\eta} l_{t+1}^\eta \right\} \right]} \end{aligned} \quad (5.61)$$

After factoring out l_t^{-1} in the numerator and multiplying and dividing the numerator by η , and factoring out c_t^{-1} in the denominator and multiplying and dividing the denominator by θ , we obtain:

$$MRS_{|\alpha>0}^{intra} = \frac{l_t^{-1} \eta \left[\frac{1}{\theta} c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta \right]}{c_t^{-1} \theta \left[\frac{1}{\theta} c_t^\theta c_{t-1}^{-\theta\alpha} \frac{1}{\eta} l_t^\eta - \alpha \beta E_t \left\{ \frac{1}{\theta} c_{t+1}^\theta c_t^{-\theta\alpha} \frac{1}{\eta} l_{t+1}^\eta \right\} \right]} \quad (5.62)$$

or

$$\begin{aligned} MRS_{|\alpha>0}^{intra} &= \frac{U_l(\widehat{c}_t, l_t)}{MU_c(\widehat{c}_t, l_t)} = \frac{\eta c_t}{\theta l_t} \frac{U(\widehat{c}_t, l_t)}{[U(\widehat{c}_t, l_t) - \alpha \beta E_t U(\widehat{c}_{t+1}, l_{t+1})]} \\ &= MRS_{|\alpha=0}^{intra} \frac{U(\widehat{c}_t, l_t)}{[U(\widehat{c}_t, l_t) - \alpha \beta E_t U(\widehat{c}_{t+1}, l_{t+1})]} \end{aligned} \quad (5.63)$$

The denominator of the term on the right-hand side is positive since MU_c is positive. Hence, we can rewrite (5.63) as:

$$\begin{aligned}\frac{MU_c(\widehat{c}_t, l_t)}{U_l(\widehat{c}_t, l_t)} &= \left[\frac{\theta}{\eta} \frac{l_t}{c_t} \right] \left\{ 1 - \frac{\alpha\beta E_t U(\widehat{c}_{t+1}, l_{t+1})}{U(\widehat{c}_t, l_t)} \right\} \\ &< \frac{U_c(c_t, l_t)}{U_l(c_t, l_t)}\end{aligned}\quad (5.64)$$

where $\frac{U_c(c_t, l_t)}{U_l(c_t, l_t)}$ is the inverse of the intratemporal marginal rate of substitution between consumption and leisure when there is no habit formation in consumption, i.e., when $\alpha = 0$.

We see that:

$$\frac{MU_c(\widehat{c}_t, l_t)}{U_l(\widehat{c}_t, l_t)} < \frac{U_c(c_t, l_t)}{U_l(c_t, l_t)}$$

Therefore,

$$MRS_{|\alpha>0}^{intra} = \frac{U_l(\widehat{c}_t, l_t)}{MU_c(\widehat{c}_t, l_t)} > \frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} = MRS_{|\alpha=0}^{intra} \quad (5.65)$$

$$\frac{MU_c(\widehat{c}_t, l_t)}{U_l(\widehat{c}_t, l_t)} = \frac{\theta}{\eta} c_t^{-1} l_t - \theta\alpha\beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta\alpha-1-\theta} \frac{1}{\eta} l_{t+1}^\eta l_t^{1-\eta} c_{t-1}^{\alpha\theta} \right\} \quad (5.66)$$

Factoring out $\frac{\theta}{\eta} c_t^{-1} l_t$:

$$\begin{aligned}\frac{MU_c(\widehat{c}_t, l_t)}{U_l(\widehat{c}_t, l_t)} &= \frac{\theta}{\eta} c_t^{-1} l_t \left[1 - \alpha\beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta(\alpha+1)} c_{t-1}^{\alpha\theta} l_{t+1}^\eta l_t^{-\eta} \right\} \right] \\ &= \frac{\theta}{\eta} c_t^{-1} l_t \left[1 - \alpha\beta E_t \left\{ c_{t+1}^\theta c_t^{-\theta} l_{t+1}^\eta l_t^{-\eta} \right\} \left(\frac{c_t}{c_{t-1}} \right)^{-\alpha\theta} \right]\end{aligned}\quad (5.67)$$

To find out the effect of higher habit formation on the inverse of the intratemporal marginal rate of substitution between consumption and leisure, we take the derivative of

the right hand side of (5.67) with respect to α , habit formation parameter.

$$\begin{aligned}
\frac{d \frac{MU_c(\widehat{c}_t, l_t)}{U_l(\widehat{c}_t, l_t)}}{d\alpha} &= -\frac{\theta}{\eta} \beta E_t \{ c_{t+1}^\theta c_t^{-\theta-1} l_{t+1}^\eta l_t^{1-\eta} \} \left(\frac{c_t}{c_{t-1}} \right)^{-\alpha\theta} \\
&\quad - \alpha \beta \frac{\theta}{\eta} E_t \{ c_{t+1}^\theta c_t^{-\theta-1} l_{t+1}^\eta l_t^{1-\eta} \} \left(\frac{c_t}{c_{t-1}} \right)^{-\alpha\theta} \ln \left(\frac{c_t}{c_{t-1}} \right) (-\theta) \\
&= -\frac{\theta}{\eta} \beta E_t \{ c_{t+1}^\theta c_t^{-\theta-1} l_{t+1}^\eta l_t^{1-\eta} \} \left(\frac{c_t}{c_{t-1}} \right)^{-\alpha\theta} \\
&\quad \times \left[1 - \alpha \theta \ln \left(\frac{c_t}{c_{t-1}} \right) \right]
\end{aligned} \tag{5.68}$$

Thus:

$$\frac{d(MRS_{|\alpha>0}^{intra})^{-1}}{d\alpha} < 0 \tag{5.69}$$

Then,

$$\frac{d(MRS_{|\alpha>0}^{intra})}{d\alpha} > 0$$

This result implies that the intratemporal marginal rate of substitution between consumption and leisure is higher when there is habit formation. The individual takes more leisure today, i.e., he or she is less willing to intratemporally substitute between consumption and leisure. From this analysis, we can find the effect of greater habit formation on the intratemporal marginal rate of substitution between consumption and leisure. That is, the stronger is habit formation, the lower is the intratemporal elasticity of substitution between current consumption and current leisure. Thus, the higher the effect of past consumption on current utility, the lower is the cost of current leisure. In this case, the individual will prefer to consume less of consumption goods and more of leisure.

We can also calculate the intertemporal marginal rate of substitution between current and future leisure as:

$$\frac{U_l(\widehat{c}_t, l_t)}{E_t U_l(\widehat{c}_{t+1}, l_{t+1})} = \left[\frac{l_t^{\eta-1} \frac{1}{\theta} c_t^\theta c_{t-1}^{-\alpha\theta}}{E_t \{ l_{t+1}^{\eta-1} \frac{1}{\theta} c_{t+1}^\theta \} c_t^{-\alpha\theta}} \right] \tag{5.70}$$

Then, (5.70) can be divided into two terms:

$$\begin{aligned}
MRS_{|\alpha>0}^{inter} &= \underbrace{\left[\frac{l_t^{\eta-1} \frac{1}{\theta} c_t^\theta}{E_t \{l_{t+1}^{\eta-1} \frac{1}{\theta} c_{t+1}^\theta\}} \right]}_{MRS_{|\alpha=0}^{inter}} \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta} \\
&= MRS_{|\alpha=0}^{inter} \times \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta}
\end{aligned} \tag{5.71}$$

where the first term is the intertemporal marginal rate of substitution between current leisure and future leisure when there is no habit formation, and the second term is a function of the growth rate of consumption.

To find the effect of an increase in the strength of habit formation α on the intertemporal marginal rate of substitution between leisure today and leisure tomorrow, we take the derivative of (5.71) with respect to α .

$$\frac{dMRS_{|\alpha>0}^{inter}}{d\alpha} = MRS_{|\alpha=0}^{inter} \times \frac{d \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta}}{d\alpha} \tag{5.72}$$

Now, let:

$$\left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta} \equiv a^x = e^{x \ln a} \tag{5.73}$$

where $x \equiv \alpha\theta$ and $\alpha \equiv \frac{c_t}{c_{t-1}}$. Then.

$$a^x = \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta} \tag{5.74}$$

$$\begin{aligned}
\frac{da^x}{dx} &= e^{x \ln a} \ln a \, dx \\
&= \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta} \ln \left(\frac{c_t}{c_{t-1}} \right)^\theta > 0.
\end{aligned} \tag{5.75}$$

That is,

$$\frac{d \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta}}{d\alpha} = \left(\frac{c_t}{c_{t-1}} \right)^{\alpha\theta} \ln \left(\frac{c_t}{c_{t-1}} \right)^{\theta} > 0 \quad (5.76)$$

Therefore, $MRS_{\alpha>0}^{inter}$ is greater than $MRS_{\alpha=0}^{inter}$. Thus, habit formation increases the intertemporal marginal rate of substitution between current and future leisure.

$$\frac{U_l(\widehat{c}_t, l_t)}{E_t U_l(\widehat{c}_{t+1}, l_{t+1})} > \frac{U_l(c_t, l_t)}{E_t U_l(c_{t+1}, l_{t+1})} \quad (5.77)$$

This implies that the individual would consume less leisure in the future, and take more leisure today in the model with habit formation. Thus, such an individual is less willing to intertemporally substitute between current and future leisure.

We have obtained that, habit formation increases both the intratemporal marginal rate of substitution between consumption and leisure and the intertemporal marginal rate of substitution between current and future leisure, when we consider a preference functional of a multiplicative form.

5.4 CONSUMPTION-LEISURE CHOICE WITH LIQUIDITY CONSTRAINTS

In this section, we will introduce liquidity constraints into the consumption-leisure model. But first, let us assume that intertemporally non-additive preferences are not allowed in consumption choices, i.e., there is no habit formation in consumption, to better understand the pure effect of liquidity constraints on the consumption and leisure choices of the individual.

This will be our benchmark model for this section, and provide a comparison tool for the results of the next model with habit formation in consumption and with liquidity constraints. The individual makes decisions about next period assets A_{t+1} , current leisure l_t and current consumption c_t subject to the budget constraint and to the time constraint. In each period t , total time available for leisure l_t and work h_t is assumed to be limited to one unit. So, for all t , $h_t = 1 - l_t$. The individual has an uncertain wage profile, with current wage rate known as of the beginning of the period. Then, the consumer's maximization problem is stated as follows:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \\ & \{c_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (5.78)$$

subject to :

$$A_{t+1} = (1 + r)[A_t + w_t(1 - l_t) - c_t] \quad (5.79)$$

$$A_{t+1} \geq 0 \quad (5.80)$$

with $\lim_{t \rightarrow \infty} A_t(1 + r)^{-t} = 0$, and with h_t being current labor(work), l_t is current leisure, w_t is current known real wage rate and A_{t+1} future asset holdings, A_t , current

asset holdings. We assume that the rate of interest r is equal or less than the rate of time preference ρ , $r \leq \rho$.

We can formulate the problem in a dynamic programming framework as:

$$\begin{aligned} V(A_t, w_t) = \underset{c_t, l_t, A_{t+1}}{\text{Max}} & U(c_t, l_t) + \beta E_t V(A_{t+1}, w_{t+1}) \end{aligned} \quad (5.81)$$

subject to :

$$A_{t+1} = (1 + r)[A_t + w_t(1 - l_t) - c_t]$$

$$A_{t+1} \geq 0$$

Substituting for the constraints:

$$\begin{aligned} V(A_t, w_t) = \underset{c_t, l_t}{\text{Max}} & U(c_t, l_t) \\ & + \beta E_t V((1 + r)[A_t + w_t(1 - l_t) - c_t], w_{t+1}) \\ & + \mu_t E_t [(1 + r)(A_t + w_t(1 - l_t) - c_t)] \end{aligned} \quad (5.82)$$

where μ_t is the Lagrange multiplier associated with the liquidity constraint.

The Kuhn-Tucker conditions are:

$$U_1(c_t, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] - \mu_t(1 + r) = 0 \quad (5.83)$$

$$U_2(c_t, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial l_t} \right] - \mu_t(1 + r)w_t = 0 \quad (5.84)$$

$$(1 + r)[A_t + w_t(1 - l_t) - c_t] \geq 0 \text{ with equality if } \mu_t > 0 \quad (5.85)$$

$$l_t \geq 0; \quad c_t \geq 0; \quad \mu_t \geq 0 \quad (5.86)$$

Differentiating the value with respect to A_t , we find:

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} + (1+r)\mu_t \quad (5.87)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1+r)$$

From (5.83), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r)} [U_1(c_t, l_t) - \mu_t(1+r)] \quad (5.88)$$

Plugging (5.88) into (5.87):

$$\frac{\partial V}{\partial A_t} = U_1(c_t, l_t) \quad (5.89)$$

Forwarding (5.89) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t U_1(c_{t+1}, l_{t+1}) \quad (5.90)$$

Substituting (5.90) into (5.83), we can obtain the Euler equation for consumption:

$$U_1(c_t, l_t) = \beta(1+r)E_t [U_1(c_{t+1}, l_{t+1})] + \mu_t(1+r) \quad (5.91)$$

Similarly, from (5.84), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r)w_t} [U_2(c_t, l_t) - \mu_t(1+r)w_t] \quad (5.92)$$

Substituting (5.92) into (5.87):

$$\frac{\partial V}{\partial A_t} = \frac{1}{w_t} U_2(c_t, l_t) \quad (5.93)$$

Forwarding (5.93) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t \left[\frac{1}{w_{t+1}} U_2(c_{t+1}, l_{t+1}) \right] \quad (5.94)$$

Plugging (5.94) into (5.84), we obtain the Euler equation for leisure:

$$U_2(c_t, l_t) = \beta(1+r)E_t \left\{ \left[\frac{w_t}{w_{t+1}} \right] U_2(c_{t+1}, l_{t+1}) \right\} + (1+r)w_t\mu_t \quad (5.95)$$

By substituting (5.90) into (5.84), and solving for $U_2(c_t, l_t)$:

$$U_2(c_t, l_t) = \beta(1+r)w_tE_t [U_1(c_{t+1}, l_{t+1})] + (1+r)w_t\mu_t \quad (5.96)$$

Combining (5.91) and (5.96), we can express the constrained intratemporal marginal rate of substitution between consumption and leisure as:

$$\frac{U_2(c_t, l_t)}{U_1(c_t, l_t)} = w_t \quad (5.97)$$

which is the same whether or not the liquidity constraint is binding. That means, the liquidity constraint has no effect on the relative price of leisure and consumption for the same period. However, intertemporal marginal rate of substitution of consumption today and consumption tomorrow will be affected. From (5.91), we can say that the current marginal utility of consumption will be higher if the liquidity constraint is binding, and hence current consumption will be lower. Similarly, from (5.96), the marginal utility of leisure today will be higher, that is leisure today will be lower since it is more costly to consume more leisure and consumption today if the liquidity constraint is binding.

5.5 CONSUMPTION-LEISURE CHOICE WITH LIQUIDITY CONSTRAINTS AND HABIT FORMATION

Now, repeating the same exercise with the additional assumption of habit formation in consumption, the problem becomes the following:

The consumer seeks to maximize:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}, l_t) \\ & \{c_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (5.98)$$

subject to :

$$A_{t+1} = (1 + r_t)[A_t + w_t(1 - l_t) - c_t] \quad (5.99)$$

$$A_{t+1} \geq 0 \quad t = 0, 1, 2, \dots \quad (5.100)$$

with h_t being current labor(work), l_t is current leisure, w_t is currently known exogenous stochastic real wage rate, and α is the habit formation parameter. In this section as well, we assume time-varying interest rates, r_t .

We can reformulate the problem in a dynamic programming framework as:

$$\begin{aligned} V(A_t, c_{t-1}, w_t) = \text{Max} \quad & U(c_t - \alpha c_{t-1}, l_t) + \beta E_t V(A_{t+1}, c_t, w_{t+1}) \\ & c_t, l_t, A_{t+1} \end{aligned} \quad (5.101)$$

subject to :

$$A_{t+1} = (1 + r_t)[A_t + w_t(1 - l_t) - c_t]$$

$$A_{t+1} \geq 0$$

Substituting for the constraints:

$$\begin{aligned}
V(A_t, c_{t-1}, w_t) &= \text{Max}_{c_t, l_t} U(c_t - \alpha c_{t-1}, l_t) \\
&\quad + \beta E_t V((1 + r_t)[A_t + w_t(1 - l_t) - c_t], c_t, w_{t+1}) \\
&\quad + \mu_t [(1 + r_t)(A_t + w_t(1 - l_t) - c_t)]
\end{aligned} \tag{5.102}$$

where μ_t is the Lagrange multiplier associated with the liquidity constraint.

The Kuhn-Tucker conditions are:

$$U_1(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} + \frac{\partial V}{\partial c_t} \right] - \mu_t(1 + r_t) = 0 \tag{5.103}$$

$$U_2(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial l_t} \right] - \mu_t(1 + r_t)w_t = 0 \tag{5.104}$$

$$(1 + r_t)[A_t + w_t(1 - l_t) - c_t] \geq 0 \text{ with equality if } \mu_t > 0. \tag{5.105}$$

$$l_t \geq 0; \quad c_t \geq 0; \quad \mu_t \geq 0 \tag{5.106}$$

Differentiating the value function with respect to c_{t-1} :

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha U_1(c_t - \alpha c_{t-1}, l_t) \tag{5.107}$$

Forwarding (5.107) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \tag{5.108}$$

Plugging (5.108) into (5.103), the first order condition for consumption becomes:

$$\begin{aligned} & U_1(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] \\ & - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \mu_t(1 + r_t) \\ & = 0 \end{aligned} \tag{5.109}$$

From (5.109), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\begin{aligned} \beta E_t \frac{\partial V}{\partial A_{t+1}} &= \frac{1}{(1 + r_t)} [U_1(c_t - \alpha c_{t-1}, l_t) \\ & - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \mu_t(1 + r_t)] \end{aligned} \tag{5.110}$$

Now, differentiating the value function with respect to A_t :

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} + \mu_t(1 + r_t) \tag{5.111}$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1 + r_t)$$

Then, plugging (5.110) into (5.111):

$$\frac{\partial V}{\partial A_t} = U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \tag{5.112}$$

At the optimum, the extra lifetime utility from being given an extra dollar in assets equals the extra utility from consuming it immediately, whether or not the current liquidity constraint is binding.

Forwarding (5.112) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t [U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha \beta E_t U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \tag{5.113}$$

Plugging (5.113) into (5.109), we will obtain the Euler equation for consumption:

$$\begin{aligned}
& U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \\
= & \beta(1 + r_t) E_t [U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha \beta U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \\
& + \mu_t(1 + r_t)
\end{aligned} \tag{5.114}$$

Using the definition of $c_t - \alpha c_{t-1} = \widehat{c}_t$, we can rewrite (5.114) as:

$$\begin{aligned}
& U_1(\widehat{c}_t, l_t) - \beta(1 + r_t) E_t U_1(\widehat{c}_{t+1}, l_{t+1}) \\
= & \alpha \beta E_t [U_1(\widehat{c}_{t+1}, l_{t+1}) - \beta(1 + r_t) U_1(\widehat{c}_{t+2}, l_{t+2})] + \mu_t(1 + r_t)
\end{aligned} \tag{5.115}$$

We can factor out $U_1(\widehat{c}_t, l_t) - \beta(1 + r_t) E_t U_1(\widehat{c}_{t+1}, l_{t+1})$ such that:

$$[U_1(\widehat{c}_t, l_t) - \beta(1 + r_t) E_t U_1(\widehat{c}_{t+1}, l_{t+1})] [1 - \alpha \beta L^{-1}] = \mu_t(1 + r_t) \tag{5.116}$$

Dividing both sides by $[1 - \alpha \beta L^{-1}]$:

$$\begin{aligned}
[U_1(\widehat{c}_t, l_t) - \beta(1 + r_t) E_t U_1(\widehat{c}_{t+1}, l_{t+1})] &= \frac{1}{[1 - \alpha \beta L^{-1}]} \mu_t(1 + r_t) \\
&= (1 + r_t) E_t \sum_{\tau=0}^{\infty} (\alpha \beta)^\tau \mu_{t+\tau}
\end{aligned} \tag{5.117}$$

Rearranging,

$$\begin{aligned}
U_1(\widehat{c}_t, l_t) &= \beta(1 + r_t) E_t U_1(\widehat{c}_{t+1}, l_{t+1}) + (1 + r_t) \mu_t \\
&\quad + (1 + r_t) E_t \sum_{\tau=1}^{\infty} (\alpha \beta)^\tau \mu_{t+\tau}
\end{aligned} \tag{5.118}$$

Hence, we have expressed the Euler equation for current consumption in terms of current

and future shadow prices of liquidity constraints. The last term in (5.118) stands for the effects of habit formation when the liquidity constraint is binding.

Now, let us find the Euler equation for leisure, from (5.104) we solve for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r_t)w_t} [U_2(c_t - \alpha c_{t-1}, l_t) - \mu_t(1+r_t)w_t] \quad (5.119)$$

Plugging (5.119) into (5.111):

$$\frac{\partial V}{\partial A_t} = \frac{1}{w_t} U_2(c_t - \alpha c_{t-1}, l_t) \quad (5.120)$$

Forwarding (5.120) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t \frac{1}{w_{t+1}} U_2(c_{t+1} - \alpha c_t, l_{t+1}) \quad (5.121)$$

Substituting (5.121) into (5.104), we will find the Euler equation for leisure:

$$\begin{aligned} U_2(c_t - \alpha c_{t-1}, l_t) &= \beta(1+r_t)E_t \left\{ \left[\frac{w_t}{w_{t+1}} \right] U_2(c_{t+1} - \alpha c_t, l_{t+1}) \right\} \\ &\quad + \mu_t(1+r_t)w_t \end{aligned} \quad (5.122)$$

Using the definition of $c_t - \alpha c_{t-1} = \widehat{c}_t$, we can rewrite (5.122) as:

$$\begin{aligned} U_2(\widehat{c}_t, l_t) &= \beta(1+r_t)E_t \left\{ \left[\frac{w_t}{w_{t+1}} \right] U_2(\widehat{c}_t, l_{t+1}) \right\} \\ &\quad + \mu_t(1+r_t)w_t \end{aligned} \quad (5.123)$$

By plugging (5.110) into (5.104), we can also express $U_2(c_t - \alpha c_{t-1}, l_t)$, the marginal utility of leisure, as:

$$U_2(c_t - \alpha c_{t-1}, l_t) = \beta(1 + r_t)w_t E_t[U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha\beta U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \quad (5.124)$$

Then, the constrained intratemporal marginal rate of substitution between consumption and leisure as:

$$\frac{U_2(c_t - \alpha c_{t-1}, l_t)}{U_1(c_t - \alpha c_{t-1}, l_t) - \alpha\beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1})} = w_t \quad (5.125)$$

which is the same whether or not the liquidity constraint is binding. That means, the liquidity constraint has no effect on the relative price of leisure and consumption for the same period, even when there is habit formation in consumption.

Moreover, the intertemporal marginal rate of substitution of consumption and the intertemporal marginal rate of substitution of leisure will also be affected. From (5.114), we see that the current marginal utility of consumption will be even higher with habit formation together with a binding liquidity constraint, which implies that current consumption will be much lower than the case without habit formation and no liquidity constraints. Thus, a binding liquidity constraint induces a further decrease in consumption with habit-forming preferences.

From (5.122), we observe that if the current liquidity constraint is binding, the current marginal utility of leisure will be higher, thus implying a lower current leisure.

Conclusion:

An individual with habit formation in consumption and with uncertain labor income, decreases current consumption when facing a binding liquidity constraints. Habit formation, liquidity constraints together with income uncertainty make the individual more cautious about consuming too much of consumption and of leisure. When the liquidity constraint is binding, leisure as well as consumption will become more expensive today.

This is a situation in which the individual feels as if he or she has less wealth to spend on consumption and leisure so that the optimal decision is to decrease both consumption and leisure.

5.6 EARNINGS-RELATED CONSTRAINTS

LIQUIDITY

In this section, we will introduce endogenous borrowing constraints. Borrowing is allowed but up to a certain positive limit. This limit depends on the current wage earnings of the individual. Following Alessie, Melenberg and Weber (1988), the borrowing constraint can be expressed as:

$$A_{t+1} \geq \psi_0 + \psi_1 w_t (1 - l_t)$$

where ψ_0 and ψ_1 are parameters, $\psi_0 > 0$ and $\psi_1 < 0$, implying that the borrowing limit is inversely related to current earnings. For simplicity, let $\psi_0 = 0$.

The individual makes decisions about next period assets A_{t+1} , current leisure l_t and current consumption c_t subject to the budget constraint and to the time constraint. In each period t , total time available for leisure l_t and work h_t is assumed to be limited to 1, that is a day. So, for all t , $h_t = 1 - l_t$. The individual has an uncertain wage profile, with current wage rate, w_t , known as of the beginning of the period. Then, the consumer's maximization problem can be stated as follows:

$$\begin{aligned} \text{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \alpha c_{t-1}, l_t) \\ & \{c_t\}_{t=0}^{\infty}, \{l_t\}_{t=0}^{\infty}, \{A_{t+1}\}_{t=0}^{\infty} \end{aligned} \quad (5.126)$$

subject to :

$$A_{t+1} = (1 + r_t)[A_t + w_t(1 - l_t) - c_t] \quad (5.127)$$

$$A_{t+1} \geq \psi_1 w_t (1 - l_t) \quad (5.128)$$

with α being the habit formation parameter. We assume that the rate of interest, r_t , is time-varying and is equal or less than the rate of time preference ρ , $r_t \leq \rho$.

In this setting, we can reformulate the problem in a dynamic programming framework as:

$$V(A_t, c_{t-1}, w_t) = \underset{c_t, l_t, A_{t+1}}{Max} U(c_t - \alpha c_{t-1}, l_t) + \beta E_t V(A_{t+1}, c_t, w_{t+1}) \quad (5.129)$$

subject to :

$$\begin{aligned} A_{t+1} &= (1 + r_t)[A_t + w_t(1 - l_t) - c_t] \\ l_t + h_t &= 1 \\ A_{t+1} &\geq \psi_1 w_t(1 - l_t) \end{aligned}$$

Substituting for the constraints:

$$\begin{aligned} V(A_t, c_{t-1}, w_t) &= \underset{c_t, l_t}{Max} U(c_t - \alpha c_{t-1}, l_t) \\ &\quad + \beta E_t V((1 + r_t)[A_t + w_t(1 - l_t) - c_t], c_t, w_{t+1}) \\ &\quad + \mu_t [(1 + r_t)[A_t + w_t(1 - l_t) - c_t] \\ &\quad - \psi_1 w_t(1 - l_t)] \end{aligned} \quad (5.130)$$

where μ_t is the Lagrange multiplier associated with the borrowing constraint.

The Kuhn-Tucker conditions are:

$$U_1(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} + \frac{\partial V}{\partial c_t} \right] - (1 + r_t)\mu_t = 0 \quad (5.131)$$

$$U_2(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial l_t} \right] - \mu_t w_t(-\psi_1 + 1 + r_t) = 0 \quad (5.132)$$

$$(1 + r_t)[A_t + w_t(1 - l_t) - c_t] - \psi_1 w_t(1 - l_t) \geq 0, \text{ with equality if } \mu_t > 0 \quad (5.133)$$

$$l_t \geq 0; c_t \geq 0; \mu_t \geq 0 \quad (5.134)$$

Differentiating the value function with respect to c_{t-1} :

$$\frac{\partial V}{\partial c_{t-1}} = -\alpha U_1(c_t - \alpha c_{t-1}, l_t) \quad (5.135)$$

Forwarding (5.135) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial c_t} = -\alpha E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \quad (5.136)$$

Plugging (5.136) into the first order condition (5.131):

$$\begin{aligned} & U_1(c_t - \alpha c_{t-1}, l_t) + \beta E_t \left[\frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial c_t} \right] \\ & - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) - (1 + r_t) \mu_t \\ & = 0 \end{aligned} \quad (5.137)$$

From (5.137), solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\begin{aligned} \beta E_t \frac{\partial V}{\partial A_{t+1}} &= \frac{1}{(1 + r_t)} [U_1(c_t - \alpha c_{t-1}, l_t) \\ & - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \mu_t(1 + r_t)] \end{aligned} \quad (5.138)$$

Now, differentiating the value function with respect to A_t :

$$\frac{\partial V}{\partial A_t} = \beta E_t \frac{\partial V}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial A_t} + \mu_t(1 + r_t) \quad (5.139)$$

where

$$\frac{\partial A_{t+1}}{\partial A_t} = (1 + r_t)$$

Then, plugging (5.138) into (5.139) to replace $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\frac{\partial V}{\partial A_t} = U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \quad (5.140)$$

At the optimum, the extra lifetime utility from being given an extra dollar in assets equals the extra utility from consuming it immediately, whether or not the current liquidity constraint is binding.

Forwarding (5.140) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t [U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha \beta U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \quad (5.141)$$

Plugging (5.141) into (5.137), we will obtain the Euler equation for consumption:

$$\begin{aligned} & U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) \\ &= \beta(1 + r_t) E_t [U_1(c_{t+1} - \alpha c_t, l_{t+1}) - \alpha \beta U_1(c_{t+2} - \alpha c_{t+1}, l_{t+2})] \\ & \quad + \mu_t(1 + r_t) \end{aligned} \quad (5.142)$$

which is exactly the same one that we have obtained for the previous case.

Hence, (5.142) is equivalent to:

$$\begin{aligned} U_1(c_t - \alpha c_{t-1}, l_t) &= \beta(1 + r_t) E_t U_1(c_{t+1} - \alpha c_t, l_{t+1}) + (1 + r_t) \mu_t \\ & \quad + (1 + r_t) E_t \sum_{\tau=1}^{\infty} (\alpha \beta)^\tau \mu_{t+\tau} \end{aligned}$$

as we have found in the previous section.

However, earnings-related liquidity constraint specification will affect the solution of

the Euler equation for leisure. From (5.132) solving for $\beta E_t \frac{\partial V}{\partial A_{t+1}}$:

$$\beta E_t \frac{\partial V}{\partial A_{t+1}} = \frac{1}{(1+r_t)w_t} [U_2(c_t - \alpha c_{t-1}, l_t) - \mu_t w_t (-\psi_1 + 1 + r_t)] \quad (5.143)$$

Plugging (5.143) into (5.139):

$$\begin{aligned} \frac{\partial V}{\partial A_t} &= \frac{1}{w_t} U_2(c_t - \alpha c_{t-1}, l_t) - \mu_t (-\psi_1 + 1 + r_t) + \mu_t (1 + r_t) \\ &= \frac{1}{w_t} U_2(c_t - \alpha c_{t-1}, l_t) + \mu_t \psi_1 \end{aligned} \quad (5.144)$$

Forwarding (5.144) by one period, and taking expectations:

$$E_t \frac{\partial V}{\partial A_{t+1}} = E_t \left[\frac{1}{w_{t+1}} U_2(c_{t+1} - \alpha c_t, l_{t+1}) + \mu_{t+1} \psi_1 \right] \quad (5.145)$$

Substituting (5.145) into (5.132), we will obtain the Euler equation for leisure:

$$\begin{aligned} U_2(c_t - \alpha c_{t-1}, l_t) &= \beta(1+r_t) E_t \left\{ \left[\frac{w_t}{w_{t+1}} \right] U_2(c_{t+1} - \alpha c_t, l_{t+1}) \right\} \\ &\quad + (-\psi_1 + 1 + r_t) \mu_t w_t \\ &\quad + \beta(1+r_t) \psi_1 w_t E_t \mu_{t+1} \end{aligned} \quad (5.146)$$

With endogenous borrowing constraints, the intratemporal marginal rate of substitution between consumption and leisure is expressed differently, and obtained by solving (5.131) and (5.137) simultaneously:

$$\begin{aligned} &\frac{U_2(c_t - \alpha c_{t-1}, l_t)}{U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1})} \\ &= w_t \left[1 - \frac{\psi_1 \mu_t}{U_1(c_t - \alpha c_{t-1}, l_t) - \alpha \beta E_t U_1(c_{t+1} - \alpha c_t, l_{t+1})} \right] \end{aligned} \quad (5.147)$$

This is a very interesting result, because with endogenous liquidity constraints the

Euler equation for leisure contains the current as well as next period's shadow price of liquidity constraint. We can express (5.146) as:

$$\left\{ \frac{1}{w_t} U_2(c_t - \alpha c_{t-1}, l_t) + \psi_1 \mu_t \right\} = \beta(1 + r_t) E_t \left\{ \frac{1}{w_{t+1}} U_2(c_{t+1} - \alpha c_t, l_{t+1}) + \psi_1 E_t \mu_{t+1} \right\} + (1 + r_t) \mu_t$$

This equation represents the trade-off in intertemporally substituting leisure. The left-hand side of this equation represents the net marginal benefit of increasing leisure this period. The first term is the marginal utility of leisure weighted by the current price of one unit of leisure $\frac{1}{w_t}$. The second term is negative (since $\psi_1 < 0$), and represents the impact on the borrowing limit of taking one extra unit of leisure today. Thus, when considering taking one extra unit of leisure, the individual must take into account that the borrowing limit will decrease. Hence, the net marginal benefit of one unit of increase in leisure is less than the marginal utility of leisure.

The first term on the right-hand side is the discounted present value of the marginal utility of next period's leisure normalized by the wage rate at $t + 1$ net of the impact of a one-unit reduction in leisure on next period's borrowing limit (which would increase), and represents the net marginal cost of reducing leisure next period. The second term on the right-hand side is the additional cost of increasing leisure today, namely the negative impact on current income. Current income falls with an increase in leisure, and leads to greater borrowing (including the interest cost) this period.

Chapter 6

THE RELATIONSHIP BETWEEN HABIT FORMATION AND PRUDENCE IN THE SENSE OF KIMBALL

It has been argued that habit formation makes consumers more prudent. The validity of this notion depends on the meaning of prudence. In an article published in 1990, Kimball introduced the concept of prudence to characterize the “sensitivity of a decision variable to risk”. He argues that risk leads, in the presence of non-increasing absolute risk aversion, to a precautionary premium. Kimball defines prudence as “the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to risk aversion which is how much one dislikes uncertainty and would turn away from uncertainty if possible”.

Then, Kimball and Weil (1991) extended Kimball’s analysis to Kreps-Porteus preferences. Subsequently, Eeckhoudt and Schlesinger (1994) further explored the implications of the link between risk aversion and prudence while Demers and Demers (1997) developed a non-expected utility analysis which allows a separation between risk aversion and

prudence.

In this chapter, we will first present Kimball's prudence premium in the context of a two-period optimal consumption-saving problem, which he used to investigate prudence. We will then introduce habit formation in the same two-period problem. Finally, we will investigate the relationship between prudence and habit formation in a multi-period framework.

6.1 KIMBALL'S MODEL

In Kimball's model, the consumer makes an optimal consumption-saving decision subject to a risky second period income.

Define c_t as time t consumption, y_t as time t income, s_t as time t saving, $R \equiv 1 + r$ where r is the real interest rate. Kimball considers a separable two-period utility function $v(c_1) + \beta v(c_2)$ where v is increasing and concave and β is the discount factor. The optimization problem is:

$$\begin{aligned} & \underset{c_1}{\text{Max}} \quad v(c_1) + \beta E v(\tilde{c}_2) \\ & \text{subject to} \quad : \\ & \quad c_1 = y_1 - s_1 \\ & \quad \tilde{c}_2 = s_1 R + \tilde{y}_2 \end{aligned} \tag{6.1}$$

where a tilde indicates a random variable.

We can restate the optimization problem as:

$$\begin{aligned} & \underset{s_1}{\text{Max}} \quad v(y_1 - s_1) + \beta E v(s_1 R + \tilde{y}_2) \end{aligned} \tag{6.2}$$

The first order condition becomes :

$$-v'(y_1 - s_1^*) + R\beta E v'(s_1^* R + \tilde{y}_2) = 0 \tag{6.3}$$

Define χ as the precautionary premium, i.e., the amount of income such that if the individual has $E\tilde{y}_2 - \chi$ with certainty he or she chooses the same s_1^* as the one when

facing the risk:

$$-v'(y_1 - s_1^*) + R\beta Ev'(s_1^*R + \tilde{y}_2) = -v'(y_1 - s_1^*) + R\beta v'(s_1^*R + E\tilde{y}_2 - \chi) \quad (6.4)$$

The first terms and $R\beta$'s cancel each other.

Now, following Pratt (1964), we take a second order Taylor approximation of the left-hand side (LHS) around $E\tilde{y}_2$:

$$\begin{aligned} Ev'(s_1^*R + \tilde{y}_2) &\cong v'(s_1^*R + E\tilde{y}_2) + E[v''(s_1^*R + E\tilde{y}_2)(\tilde{y}_2 - E\tilde{y}_2)] \\ &\quad + \frac{1}{2}E[v'''(s_1^*R + E\tilde{y}_2)(\tilde{y}_2 - E\tilde{y}_2)^2] \end{aligned} \quad (6.5)$$

Taking a first order Taylor approximation of the right-hand side (RHS) around $E\tilde{y}_2$:

$$v'(s_1^*R + E\tilde{y}_2 - \chi) \cong v'(s_1^*R + E\tilde{y}_2) + v''(s_1^*R + E\tilde{y}_2)(E\tilde{y}_2 - \chi - E\tilde{y}_2) \quad (6.6)$$

Let LHS=RHS:

$$\frac{1}{2}Ev'''(s_1^*R + E\tilde{y}_2)(\tilde{y}_2 - E\tilde{y}_2)^2 \cong v''(s_1^*R + E\tilde{y}_2)(-\chi) \quad (6.7)$$

Then, solving for χ :

$$\chi \cong -\frac{1}{2}\sigma_y^2 \frac{v'''(s_1^*R + E\tilde{y}_2)}{v''(s_1^*R + E\tilde{y}_2)} \quad (6.8)$$

where χ is the prudence premium.

We can also express the premium as :

$$\chi \cong \frac{1}{2} \sigma_y^2 \eta (s_1^* R + E \tilde{y}_2) \quad (6.9)$$

where η is the measure of prudence¹:

$$\eta \cong - \frac{v'''(s_1^* R + E \tilde{y}_2)}{v''(s_1^* R + E \tilde{y}_2)} \quad (6.10)$$

Since, in this two-period model c_1 is a normal good, period 1 saving, s_1 , is a decreasing function of second period income. Therefore, since $\chi > 0$, the saving level of a prudent consumer is larger. Prudence has the same impact on saving as having a lower second period income.

¹Recall that the absolute risk aversion measure is:

$$\Theta = - \frac{v''}{v'}$$

whereas the absolute prudence measure is:

$$\eta = - \frac{v'''}{v''}$$

6.2 KIMBALL'S MODEL WITH HABIT FORMATION

We now introduce habit formation in Kimball's two-period problem. The optimization problem becomes:

$$\begin{aligned}
 & \underset{c_1}{Max} \quad v(c_1 - \alpha c_0) + \beta E v(\tilde{c}_2 - \alpha c_1) \\
 & \text{where } 0 < \alpha < 1 \\
 & c_1 = y_1 - s_1 \\
 & \tilde{c}_2 = s_1 R + \tilde{y}_2
 \end{aligned} \tag{6.11}$$

After substituting the constraints the problem becomes:

$$\begin{aligned}
 & \underset{s_1}{Max} \quad v(y_1 - s_1 - \alpha c_0) + \beta E v(s_1 R + \tilde{y}_2 - \alpha y_1 + \alpha s_1) \\
 & s_1
 \end{aligned} \tag{6.12}$$

The first order condition is:

$$-v'(y_1 - s_1^* - \alpha c_0) + \beta (R + \alpha) E v'(s_1^* R + \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) = 0 \tag{6.13}$$

Define χ as the precautionary premium, i.e., the amount of income such that if the individual has $E\tilde{y}_2 - \chi$ with certainty he or she chooses the same s_1^* as the one when facing the risk:

$$\begin{aligned}
& -v'(y_1 - s_1^* - \alpha c_0) + (R + \alpha) \beta E v'(s_1^* R + \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) \\
= & -v'(y_1 - s_1^* - \alpha c_0) + (R + \alpha) \beta v'(s_1^* R + E \tilde{y}_2 - \chi - \alpha y_1 + \alpha s_1^*)
\end{aligned} \tag{6.14}$$

Taking a second order approximation of the LHS around $E \tilde{y}_2$ we get:

$$\begin{aligned}
E v'(s_1^* R + \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) & \cong v'(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) + \\
& E v''(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) \\
& \times (\tilde{y}_2 - E \tilde{y}_2) \\
& + \frac{1}{2} E v'''(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) \\
& \times (\tilde{y}_2 - E \tilde{y}_2)^2
\end{aligned} \tag{6.15}$$

Taking a first order approximation of the RHS, we obtain:

$$\begin{aligned}
v'(s_1^* R + E \tilde{y}_2 - \chi - \alpha y_1 + \alpha s_1^*) & \cong v'(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) + \\
& v''(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*) \times \\
& (E \tilde{y}_2 - \chi - E \tilde{y}_2)
\end{aligned} \tag{6.16}$$

When we equate LHS and RHS, we find the prudence premium χ as:

$$\chi \cong -\frac{1}{2} \sigma_y^2 \frac{v'''(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*)}{v''(s_1^* R + E \tilde{y}_2 - \alpha y_1 + \alpha s_1^*)} \tag{6.17}$$

and the prudence measure η as:

$$\eta \cong -\frac{v'''(s_1^* R + E \tilde{y}_2 - \alpha(y_1 - s_1^*))}{v''(s_1^* R + E \tilde{y}_2 - \alpha(y_1 - s_1^*))} \tag{6.18}$$

The effect of an increase in the habit formation parameter α on the prudence measure is obtained as follows:

$$\begin{aligned}
\frac{d\eta}{d\alpha} &= \frac{(y_1 - s_1^*)[v''v'''' - (v''')^2]}{[v'']^2} \\
&= c_1^* \frac{[v''v'''' - (v''')^2]}{[v'']^2} \\
&= -c_1^* \frac{d\eta}{d\widehat{c}_2}
\end{aligned} \tag{6.19}$$

where $\widehat{c}_2 = \widetilde{c}_2 - \alpha c_1$,

Therefore,

$$\frac{d\eta}{d\widehat{c}_2} \leq (\geq) 0 \Rightarrow \frac{d\eta}{d\alpha} \geq (\leq) 0 \tag{6.20}$$

The impact of habit formation on prudence is determined according to whether the individual has decreasing, constant or increasing absolute prudence. If absolute prudence is decreasing with an increase in the second period income, then labor income uncertainty will raise the marginal propensity to consume at a given level of consumption. An important example of DAP is the exponential utility function. Note that with a quadratic utility function $v''' = 0$ so that the prudence premium is zero. Under the hypothesis of decreasing absolute prudence (DAP), habit formation makes the individual more prudent: since the prudence premium is higher, there is precautionary saving induced by habit formation.

Therefore, we conclude that in a two-period model with uncertainty in the second period income, under the hypothesis of decreasing absolute prudence, habit formation increases precautionary premium in the sense of Kimball.

6.3 PRUDENCE IN AN INFINITE HORIZON PROBLEM

In this section, we extend the analysis to an infinite horizon problem. The individual faces a stochastic income stream $\{y_t\}$ which is *i.i.d.* The sequence of real interest rate is known with certainty. We assume that current utility depends on lagged consumption in a general form, as captured by the non-separable utility function $U(c_t, c_{t-1})$. We will thereafter consider a specific functional form, namely $U(c_t - \alpha c_{t-1})$.

$$V(W_t, c_{t-1}) = \underset{W_{t+1}, c_t}{\text{Max}} \{U(c_t, c_{t-1}) + \beta E_t V(W_{t+1}, c_t)\} \quad (6.21)$$

subject to :

$$W_{t+1} = R_{t+1}(W_t - c_t) + \tilde{y}_{t+1} \quad (6.22)$$

Substituting for the constraint:

$$V(W_t, c_{t-1}) = \underset{c_t}{\text{Max}} \{U(c_t, c_{t-1}) + \beta E_t V(R_{t+1}(W_t - c_t) + \tilde{y}_{t+1}, c_t)\} \quad (6.23)$$

The first order condition is:

$$\begin{aligned} \frac{\partial U(c_t^*, c_{t-1})}{\partial c_t} + \beta E_t V_c(R_{t+1}(W_t - c_t^*) + \tilde{y}_{t+1}, c_t^*) \\ - \beta R_{t+1} E_t V_w(R_{t+1}(W_t - c_t^*) + \tilde{y}_{t+1}, c_t^*) = 0 \end{aligned} \quad (6.24)$$

where for simplicity we let :

$$\begin{aligned} V_c &= \frac{\partial V(W_{t+1}, c_t)}{\partial c_t} \\ V_w &= \frac{\partial V(W_{t+1}, c_t)}{\partial W_{t+1}} \end{aligned}$$

Similarly,

$$\begin{aligned} V_{cw} &= \frac{\partial^2 V(W_{t+1}, c_t)}{\partial c_t \partial W_{t+1}} \\ V_{ww} &= \frac{\partial^2 V(W_{t+1}, c_t)}{\partial W_{t+1}^2} \end{aligned}$$

and

$$\begin{aligned} V_{cww} &= \frac{\partial^3 V(W_{t+1}, c_t)}{\partial c_t \partial W_{t+1}^2} \\ V_{www} &= \frac{\partial^3 V(W_{t+1}, c_t)}{\partial W_{t+1}^3} \end{aligned}$$

Define χ by :

$$\begin{aligned} & \frac{\partial U(c_t^*, c_{t-1})}{\partial c_t} + \beta E_t V_c(R_{t+1}(W_t - c_t^*) + \tilde{y}_{t+1}, c_t^*) - \\ & \beta R_{t+1} E_t V_w(R_{t+1}(W_t - c_t^*) + \tilde{y}_{t+1}, c_t^*) \\ \cong & \frac{\partial U(c_t^*, c_{t-1})}{\partial c_t} + \beta V_c(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1} - \chi, c_t^*) - \\ & \beta R_{t+1} V_w(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1} - \chi, c_t^*) \end{aligned} \tag{6.25}$$

Note that $\frac{\partial U(c_t^*, c_{t-1})}{\partial c_t}$ and β' 's cancel.

Expanding the RHS of (6.25) around $E_t \tilde{y}_{t+1}$ with a first order Taylor expansion, we get:

$$\begin{aligned}
& V_c(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1} - \chi, c_t^*) \\
& - R_{t+1} V_w(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1} - \chi, c_t^*) \\
\cong & V_c(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1}, c_t^*) \\
& - R_{t+1} V_w(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1}, c_t^*) \\
& - \chi [V_{cw}(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1}, c_t^*)] \\
& + \chi R_{t+1} [V_{ww}(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1}, c_t^*)]
\end{aligned} \tag{6.26}$$

Expanding the LHS of (6.25) around $E_t \tilde{y}_{t+1}$ with a second order Taylor expansion, we obtain:

$$\begin{aligned}
& E_t V_c(R_{t+1}(W_t - c_t^*) + \tilde{y}_{t+1}, c_t^*) \\
& - E_t R_{t+1} V_w(R_{t+1}(W_t - c_t^*) + \tilde{y}_{t+1}, c_t^*) \\
\cong & V_c(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1}, c_t^*) \\
& - R_{t+1} V_w(R_{t+1}(W_t - c_t^*) + E_t \tilde{y}_{t+1}, c_t^*) \\
& + [V_{cw}(\cdot, \cdot) - R_{t+1} V_{ww}(\cdot, \cdot)] E_t (\tilde{y}_{t+1} - \tilde{y}_{t+1}) \\
& + \frac{1}{2} [V_{cww}(\cdot, \cdot) - R_{t+1} V_{www}(\cdot, \cdot)] E_t (\tilde{y}_{t+1} - E_t \tilde{y}_{t+1})^2
\end{aligned} \tag{6.27}$$

Letting LHS=RHS, we obtain:

$$\begin{aligned}
\chi & \cong \frac{1}{2} \sigma_y^2 \frac{(V_{cww} - V_{www} R_{t+1})}{(V_{ww} R_{t+1} - V_{cw})} = \frac{1}{2} \sigma_y^2 \\
& \times \underbrace{\left[\left(-\frac{V_{www} R_{t+1}}{V_{ww} R_{t+1} - V_{cw}} \right) + \left(\frac{V_{cww}}{V_{ww} R_{t+1} - V_{cw}} \right) \right]}_{\eta,}
\end{aligned} \tag{6.28}$$

where the measure of prudence is η :

$$\eta = \left(-\frac{V_{www}R_{t+1}}{V_{ww}R_{t+1} - V_{cw}} \right) + \left(\frac{V_{cww}}{V_{ww}R_{t+1} - V_{cw}} \right)$$

η now consists of two terms. Without habit formation η reduces to:

$$\eta' = -\frac{V_{www}}{V_{ww}}$$

Hence, habit formation introduces a second term in the precautionary premium. It also modifies the denominator of the first term. In what follows, we will attempt to determine the sign of each term.

The value function of the infinite horizon problem is the limit function of a sequence of value functions $\{V^n(W, c)\}$. In order to determine the signs of the derivatives of the value function V_{ww} , V_{cw} , V_{www} , and V_{cww} we proceed by induction. We first examine the last two periods of a finite horizon T period problem. We then make the induction assumption and finally consider the initial period.

6.3.1 THE LAST TWO PERIODS

Define $V_T(W_T, c_{T-1})$ as the value with zero periods to go (i.e. for the last period). The subscript now denotes the time period reached.

$$V_T(W_T, c_{T-1}) \equiv U(c_T^*, c_{T-1}) \quad (6.29)$$

where

$$\begin{aligned} c_T^* &= W_T \\ &= R_{T-1}(W_{T-1} - c_{T-1}) + y_T \end{aligned} \quad (6.30)$$

$$\begin{aligned} V_{T-1}(W_{T-1}, c_{T-2}) &= \text{Max } U(c_{T-1}, c_{T-2}) + \beta E_{T-1} V(W_T, c_{T-1}) \\ &\quad \{W_T, c_{T-1}\} \end{aligned} \quad (6.31)$$

$$\text{subject to } W_T = R_{T-1}(W_{T-1} - c_{T-1}) + y_T \quad (6.32)$$

Define

$$c_{T-1}^* \equiv g(W_{T-1}, c_{T-2}) \quad (6.33)$$

as the optimal decision rule.

We take the derivative of the value function for the period $T - 1$ with respect to its state variables W_{T-1} , and c_{T-2} . After making use of the envelope theorem, we find:

$$\begin{aligned} \frac{\partial V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}} &= \beta E_{T-1} \frac{\partial V_T(W_T, c_{T-1})}{\partial W_T} \frac{\partial W_T}{\partial W_{T-1}} \\ &= \beta E_{T-1} \frac{\partial V_T(W_T, c_{T-1})}{\partial W_T} R_{T-1} \end{aligned} \quad (6.34)$$

Using (6.29) and (6.30), we can rewrite (6.34) as:

$$\frac{\partial V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}} = \beta E_{T-1} R_{T-1} \frac{\partial U(W_T, c_{T-1})}{\partial c_T}$$

where

$$\frac{\partial U}{\partial c_T} \equiv \frac{\partial V(W_T, c_{T-1})}{\partial W_T}$$

Now, recalling that

$$W_T = (R_{T-1}(W_{T-1} - g(W_{T-1}, c_{T-2}) + y_T)$$

and

$$c_{T-1} = g(W_{T-1}, c_{T-2})$$

we take the derivative of the left-hand side of (6.34) with respect to its state variables c_{T-2} and W_{T-1} respectively. We obtain:

$$\begin{aligned} & \frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1} \partial c_{T-2}} \\ = & \beta E_{T-1} R_{T-1} \frac{\partial g}{\partial c_{T-2}} \left[-\frac{\partial^2 U}{\partial c_T^2} R_{T-1} + \frac{\partial^2 U}{\partial c_T \partial c_{T-1}} \right] \end{aligned} \quad (6.35)$$

and

$$\begin{aligned} & \frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2} \\ = & \beta E_{T-1} [(R_{T-1})^2 \frac{\partial^2 U}{\partial c_T^2} (1 - \frac{\partial g}{\partial W_{T-1}}) \\ & + R_{T-1} \frac{\partial^2 U}{\partial c_T \partial c_{T-1}} (\frac{\partial g}{\partial W_{T-1}})] \end{aligned} \quad (6.36)$$

Repeating the same procedure, now we take the derivative of the left-hand side of (6.36) with respect to its state variables W_{T-1} and c_{T-2} respectively.

Taking the derivative with respect to W_{T-1} , we have:

$$\begin{aligned}
& \frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^3} \\
= & \beta E_{T-1} \left\{ [(R_{T-1})^3 \frac{\partial^3 U}{\partial c_T^3} (1 - \frac{\partial g}{\partial W_{T-1}})^2] \right. \\
& - [(R_{T-1})^2 \frac{\partial^2 U}{\partial c_T^2} (\frac{\partial^2 g}{\partial W_{T-1}^2})] + [R_{T-1} \frac{\partial^2 U}{\partial c_T \partial c_{T-1}} (\frac{\partial^2 g}{\partial W_{T-1}^2})] \\
& + [(R_{T-1})^2 \frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (1 - \frac{\partial g}{\partial W_{T-1}}) (\frac{\partial g}{\partial W_{T-1}})] \\
& \left. + [R_{T-1} \frac{\partial^3 U}{\partial c_T \partial c_{T-1}^2} (\frac{\partial g}{\partial W_{T-1}})^2] \right\} \tag{6.37}
\end{aligned}$$

Next, we take the derivative of (6.36) with respect to c_{T-2} :

$$\begin{aligned}
& \frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2 \partial c_{T-2}} = \beta E_{T-1} (R_{T-1}) \\
& \times \left\{ [\frac{\partial^3 U}{\partial c_T^3} (R_{T-1})^2 (-\frac{\partial g}{\partial c_{T-2}}) (1 - \frac{\partial g}{\partial W_{T-1}})] \right. \\
& + [\frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (R_{T-1}) (\frac{\partial g}{\partial c_{T-2}}) (1 - \frac{\partial g}{\partial W_{T-1}})] \\
& - [\frac{\partial^2 U}{\partial c_T^2} (R_{T-1}) (\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}})] \\
& + [\frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (R_{T-1}) (-\frac{\partial g}{\partial c_{T-2}}) (\frac{\partial g}{\partial W_{T-1}})] \\
& + \frac{\partial^3 U}{\partial c_T \partial c_{T-1}^2} (\frac{\partial g}{\partial c_{T-2}}) (\frac{\partial g}{\partial W_{T-1}}) \\
& \left. + [\frac{\partial^2 U}{\partial c_T \partial c_{T-1}} (\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}})] \right\} \tag{6.38}
\end{aligned}$$

As can be seen, the expressions involved in determining the sign of the precautionary premium are highly complex. In order to obtain a specific results, we will consider a utility function of the form $U(c_t - \alpha c_{t-1})$.

However, we proceed further we examine the premium when there is no habit forma-

tion so that c_{T-2} disappears as a state variable in the value function for period $T - 1$. As a result, η reduces to:

$$\eta' = - \frac{\frac{\partial^3 V_{T-1}(W_{T-1})}{\partial W_{T-1}^3}}{\frac{\partial^2 V_{T-1}(W_{T-1})}{\partial W_{T-1}^2}} \quad (6.39)$$

where

$$\frac{\partial^2 V_{T-1}(W_{T-1})}{\partial W_{T-1}^2} = \beta E_{T-1} \left\{ (R_{T-1})^2 \frac{\partial^2 U(c_T^*)}{\partial c_T^2} \left(1 - \frac{\partial g}{\partial W_{T-1}}\right) \right\} < 0 \quad (6.40)$$

by the concavity of U , and since the marginal propensity to consume, $\frac{\partial g}{\partial W_{T-1}}$ is less than one.

$$\begin{aligned} \frac{\partial^3 V_{T-1}(W_{T-1})}{\partial W_{T-1}^3} &= \beta E_{T-1} \left\{ (R_{T-1})^3 \frac{\partial^3 U(c_T^*)}{\partial c_T^3} \left(1 - \frac{\partial g}{\partial W_{T-1}}\right)^2 \right. \\ &\quad \left. - (R_{T-1})^2 \frac{\partial^2 U(c_T^*)}{\partial c_T^2} \left(\frac{\partial^2 g}{\partial W_{T-1}^2}\right) \right\} \end{aligned} \quad (6.41)$$

The prudence measure without habit formation, η' , is equal to:

$$\eta' = - \left[\frac{[\beta (R_{T-1})^2 E_{T-1} U'' (1 - \frac{\partial g}{\partial W_{T-1}})]}{[\beta (R_{T-1})^3 E_{T-1} U''' (1 - \frac{\partial g}{\partial W_{T-1}})^2 - (R_{T-1})^2 U'' (\frac{\partial^2 g}{\partial W_{T-1}^2})]} \right]^{-1} \quad (6.42)$$

or

$$\eta' = - \frac{E_{T-1} [R_{T-1} U''' (1 - \frac{\partial g}{\partial W_{T-1}})^2 - U'' (\frac{\partial^2 g}{\partial W_{T-1}^2})]}{E_{T-1} U'' (1 - \frac{\partial g}{\partial W_{T-1}})}$$

η' will be greater than zero if:

$$R_{T-1} U''' (1 - \frac{\partial g}{\partial W_{T-1}})^2 - U'' (\frac{\partial^2 g}{\partial W_{T-1}^2}) > 0 \quad (6.43)$$

i.e. if:

$$-U'''[R_{T-1}(-\frac{U'''}{U''})(1 - \frac{\partial g}{\partial W_{T-1}})^2 + (\frac{\partial^2 g}{\partial W_{T-1}^2})] > 0$$

i.e. if:

$$R_{T-1}(-\frac{U'''}{U''})(1 - \frac{\partial g}{\partial W_{T-1}})^2 + (\frac{\partial^2 g}{\partial W_{T-1}^2}) > 0$$

i.e. if:

$$R_{T-1}(-\frac{U'''}{U''}) > \frac{-(\frac{\partial^2 g}{\partial W_{T-1}^2})}{(1 - \frac{\partial g}{\partial W_{T-1}})^2} \quad (6.44)$$

Since $U''' > 0$ under decreasing or constant absolute risk aversion, and since $U'' < 0$, the sign of $\frac{\partial^3 V_{T-1}(W_{T-1})}{\partial W_{T-1}^3}$ and of the prudence premium is only unambiguously positive if $\frac{\partial^2 g}{\partial W_{T-1}^2} = 0$. That is, it is only unambiguously positive if the marginal propensity to consume out of wealth is constant.

However, according to Carroll and Kimball (1996), the consumption function is generally a concave function of wealth. This implies that the derivative of the marginal propensity to consume with respect to wealth, $\frac{\partial^2 g}{\partial W_{T-1}^2}$, is negative in general. Two exceptions include the exponential utility function when the interest rate is deterministic but income is random, and the constant relative risk aversion utility function when income is deterministic while the interest rate is random.

Therefore, from (6.44) we see that even without habit formation the prudence premium is not unambiguously positive. Rewriting (6.44) a sufficient condition for

$$\frac{\partial^3 V_{T-1}(W_{T-1})}{\partial W_{T-1}^3} > 0$$

can be obtained as:

$$-\frac{\frac{\partial^3 U}{\partial c_T^3}}{\frac{\partial^2 U}{\partial c_T^2}} > \frac{-\left(\frac{\partial^2 g}{\partial W_{T-1}^2}\right)}{\left(1 - \frac{\partial g}{\partial W_{T-1}}\right)^2} \left(\frac{1}{R_{T-1}}\right) \quad (6.45)$$

That is, the individual must be sufficiently prudent with respect to period T consumption risk for him or her to be prudent with respect to wealth risk.

Now, in order to obtain specific results we assume a utility function which we have considered in section 6.2, $U(c_t - \alpha c_{t-1})$.

The value function with zero period to go becomes:

$$V_T(W_T, c_{T-1}) \equiv U(c_T^* - \alpha c_{T-1}) \quad (6.46)$$

where

$$\begin{aligned} c_T^* &= W_T \\ &= R_{T-1}(W_{T-1} - c_{T-1}) + y_T \end{aligned} \quad (6.47)$$

Defining the value function for one period before the last period:

$$\begin{aligned} V_{T-1}(W_{T-1}, c_{T-2}) &= \max_{W_T, c_{T-1}} U(c_{T-1} - \alpha c_{T-2}) \\ &\quad + \beta E_{T-1} V(W_T, c_{T-1}) \\ \text{subject to} \quad &: W_T = R_{T-1}(W_{T-1} - c_{T-1}) + y_T \end{aligned} \quad (6.48)$$

The first order condition for c_{T-1} can be expressed as:

$$U'(\widehat{c}_{T-1}) - \beta E_{T-1} U'(\widehat{c}_T) [R_{T-1} + \alpha] = 0 \quad (6.49)$$

where

$$\widehat{c}_{T-1} = c_{T-1} - \alpha c_{T-2} \quad (6.50)$$

and

$$\widehat{c}_T = R_{T-1}(W_{T-1} - c_{T-1}) + y_T - \alpha c_{T-1} \quad (6.51)$$

Differentiating the first order condition, we find:

$$\frac{\partial c_{T-1}}{\partial c_{T-2}} = \frac{\alpha U''(\widehat{c}_{T-1})}{U''(\widehat{c}_{T-1}) + \beta(R_{T-1} + \alpha)^2 E_{T-1} U''(\widehat{c}_T)} > 0 \quad (6.52)$$

$$\frac{\partial^2 c_{T-1}}{\partial c_{T-2}^2} = \frac{-\alpha\beta(R_{T-1} + \alpha)^2 E_{T-1} U''(\widehat{c}_T) U'''(\widehat{c}_{T-1})}{[U''(\widehat{c}_{T-1}) + \beta(R_{T-1} + \alpha)^2 E_{T-1} U''(\widehat{c}_T)]^2} > 0 \quad (6.53)$$

$$\frac{\partial c_{T-1}}{\partial W_{T-1}} = \frac{\beta(R_{T-1} + \alpha) R_{T-1} E_{T-1} U''(\widehat{c}_T)}{U''(\widehat{c}_{T-1}) + \beta(R_{T-1} + \alpha)^2 E_{T-1} U''(\widehat{c}_T)} > 0 \quad (6.54)$$

$$\frac{\partial^2 c_{T-1}}{\partial W_{T-1}^2} = \frac{U''(\widehat{c}_{T-1}) \beta E_{T-1} U'''(\widehat{c}_T) R_{T-1}^2 (R_{T-1} + \alpha)}{[U''(\widehat{c}_{T-1}) + \beta(R_{T-1} + \alpha)^2 E_{T-1} U''(\widehat{c}_T)]^2} < 0 \quad (6.55)$$

$$\frac{\partial^2 c_{T-1}}{\partial c_{T-2} \partial W_{T-1}} = \frac{\beta\alpha(R_{T-1} + \alpha) E_{T-1} U''(\widehat{c}_T) R_{T-1} U'''(\widehat{c}_{T-1})}{[U''(\widehat{c}_{T-1}) + \beta(R_{T-1} + \alpha)^2 E_{T-1} U''(\widehat{c}_T)]^2} < 0 \quad (6.56)$$

Now, using (6.33), and following the induction step, we take the derivative of the value function for the period $T - 1$ with respect to W_{T-1} :

$$\frac{\partial V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}} = \beta E_{T-1} R_{T-1} \frac{\partial U}{\partial c_T} \quad (6.57)$$

Taking the derivative of (6.57) with respect to c_{T-2} , we find:

$$\begin{aligned} \frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1} \partial c_{T-2}} &= \\ \beta E_{T-1} R_{T-1} \left(\frac{\partial g}{\partial c_{T-2}} \right) [-U'' R_{T-1} - \alpha U'''] & \\ > 0 \quad \text{since } \frac{\partial g}{\partial c_{T-2}} > 0 \end{aligned} \quad (6.58)$$

Then, taking the derivative of (6.57) with respect to W_{T-1} :

$$\begin{aligned} \frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2} &= \beta E_{T-1} \{ (R_{T-1})^2 U'' (1 - \frac{\partial g}{\partial W_{T-1}}) \\ &+ R_{T-1} (-\alpha U'') (\frac{\partial g}{\partial W_{T-1}}) \} \end{aligned} \quad (6.59)$$

i.e.,

$$\begin{aligned} \frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2} &= \beta E_{T-1} R_{T-1} U'' [R_{T-1} (1 - \frac{\partial g}{\partial W_{T-1}}) \\ &\quad - \alpha (\frac{\partial g}{\partial W_{T-1}})] \\ &= \beta E_{T-1} R_{T-1} U'' [R_{T-1} - (R_{T-1} + \alpha) (\frac{\partial g}{\partial W_{T-1}})] \end{aligned}$$

(6.59) is less than zero, and the value function is concave in wealth if:

$$\frac{\partial g}{\partial W_{T-1}} < \frac{R_{T-1}}{R_{T-1} + \alpha} \quad (6.60)$$

i.e. if the marginal propensity to consume is not too large. Equation (6.58) and (6.59) allow us to sign the denominator of the precautionary premium (χ). Since lagged consumption increases current consumption, the denominator of χ , is negative if the marginal propensity to consume is not too large as expressed in (6.60). Now, turning to the numerator of χ , we take the derivative of (6.59) with respect to W_{T-1} to obtain:

$$\begin{aligned} \frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^3} &= \beta E_{T-1} R_{T-1} \{ [(R_{T-1})^2 U''' (1 - \frac{\partial g}{\partial W_{T-1}})^2] \\ &\quad - [R_{T-1} U'' \frac{\partial^2 g}{\partial W_{T-1}^2}] + [R_{T-1} (-\alpha U''') (1 - \frac{\partial g}{\partial W_{T-1}}) (\frac{\partial g}{\partial W_{T-1}})] \\ &\quad + [\alpha^2 U''' (\frac{\partial g}{\partial W_{T-1}})^2] + [(-\alpha U'') (\frac{\partial^2 g}{\partial W_{T-1}^2})] \} \end{aligned} \quad (6.61)$$

Then,

$$\frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^3} > 0 \text{ if:}$$

$$\begin{aligned} & U'''(1 - \frac{\partial g}{\partial W_{T-1}})R_{T-1}[R_{T-1} - (R_{T-1} + \alpha)(\frac{\partial g}{\partial W_{T-1}})] \\ & - (\frac{\partial^2 g}{\partial W_{T-1}^2})U''[R_{T-1} + \alpha] + \alpha^2 U'''(\frac{\partial g}{\partial W_{T-1}})^2 \\ & > 0 \end{aligned}$$

and

$$\frac{\partial g}{\partial W_{T-1}} < \frac{R_{T-1}}{[R_{T-1} + \alpha]}$$

That is,

$$\begin{aligned} - \frac{U'''}{U''} < \\ - \frac{(R_{T-1} + \alpha)(\frac{\partial^2 g}{\partial W_{T-1}^2})}{(1 - \frac{\partial g}{\partial W_{T-1}})R_{T-1}[R_{T-1} - (R_{T-1} + \alpha)\frac{\partial g}{\partial W_{T-1}}] + \alpha^2(\frac{\partial g}{\partial W_{T-1}})^2} \end{aligned}$$

Since $\frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1} \partial c_{T-2}} > 0$ under these assumptions, the first term in the precautionary measure which is

$$\frac{\frac{\partial^3 V_{T-1}}{\partial W_{T-1}^3}}{\frac{\partial^2 V_{T-1}}{\partial W_{T-1}^2} R_{T-1} - \frac{\partial^2 V_{T-1}}{\partial W_{T-1} \partial c_{T-2}}}$$

is positive.

Then, we take the derivative of (6.59) with respect to c_{T-2} :

$$\begin{aligned} & \frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2 \partial c_{T-2}} = \\ & \{ (\frac{\partial g}{\partial c_{T-2}}) \beta E_{T-1} R_{T-1} [R_{T-1} (-\alpha U''') (1 - 2 \frac{\partial g}{\partial W_{T-1}}) \\ & - (R_{T-1})^2 U''' (1 - \frac{\partial g}{\partial W_{T-1}}) + \alpha^2 U''' (\frac{\partial g}{\partial W_{T-1}})] \} \end{aligned}$$

$$-\{\beta E_{T-1} \frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}} [U''(R_{T-1})^2 - R_{T-1}(-\alpha U'')]\} \quad (6.62)$$

Since lagged consumption increases current consumption, the first term in braces is negative if $U''' > 0$ and if:

$$\frac{\partial g}{\partial W_{T-1}} < \frac{R_{T-1}}{R_{T-1} + \alpha}$$

that is, if the marginal propensity to consume out of wealth is not too large. Since $\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}} < 0$, the second term in braces is positive.

Hence, under these conditions $\frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2 \partial c_{T-2}} < 0$. Therefore, the second term in η , which is

$$\frac{\frac{\partial^3 V_{T-1}}{\partial W_{T-1}^2 \partial c_{T-2}}}{\frac{\partial^2 V_{T-1}}{\partial W_{T-1}^2} R_{T-1} - \frac{\partial^2 V_{T-1}}{\partial W_{T-1} \partial c_{T-2}}}$$

is positive. The first step of the induction proof indicates that under the assumed utility function $U(c_t - \alpha c_{t-1})$ habit formation results in an additional precautionary premium.

Finally, for the sake of future reference we take the derivative of (6.58) with respect to c_{T-2} :

$$\begin{aligned} & \frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1} \partial c_{T-2}^2} = \\ & \beta E_{T-1} \left\{ (R_{T-1}) \left(\frac{\partial^2 g}{\partial c_{T-2}^2} \right) [-U'' R_{T-1} - \alpha U'''] \right\} \\ & + \beta E_{T-1} \left\{ R_{T-1} \left(\frac{\partial g}{\partial c_{T-2}} \right)^2 [U'''(R_{T-1})^2 \right. \right. \\ & \left. \left. + 2\alpha U''' R_{T-1} + \alpha^2 U'''] \right\} \\ & > 0 \end{aligned} \quad (6.63)$$

since $\frac{\partial^2 g}{\partial c_{T-2}^2} > 0$ and $U''' > 0$.

6.3.2 THE INITIAL PERIOD t

We make the induction assumption that:

$$\begin{aligned} \frac{\partial^2 V_j}{\partial W_j \partial c_{j-1}} &> 0; \quad \frac{\partial^2 V_j}{\partial W_j^2} < 0; \quad \frac{\partial^3 V_j}{\partial W_j^3} > 0; \quad \frac{\partial^3 V_j}{\partial W_j^2 \partial c_{j-1}} < 0; \\ \text{and } \frac{\partial^3 V_j}{\partial W_j \partial c_{j-1}^2} &> 0 \quad \text{for } j = t+1, \dots, T-1. \end{aligned} \quad (6.64)$$

We have to show that:

$$\begin{aligned} \frac{\partial^2 V_t}{\partial W_t \partial c_{t-1}} &> 0; \quad \frac{\partial^2 V_t}{\partial W_t^2} < 0; \quad \frac{\partial^3 V_t}{\partial W_t^3} > 0; \quad \frac{\partial^3 V_t}{\partial W_t^2 \partial c_{t-1}} < 0; \\ \text{and } \frac{\partial^3 V_t}{\partial W_t \partial c_{t-1}^2} &> 0 \end{aligned} \quad (6.65)$$

under the same assumptions needed to ensure (6.64).

We have then:

$$\begin{aligned} V_t(c_{t-1}, W_t) &= \text{Max}_{W_{t+1}, c_t} U(c_t - \alpha c_{t-1}) \\ &\quad + \beta E_t V_{t+1}(W_{t+1}, c_t) \end{aligned} \quad (6.66)$$

subject to:

$$W_{t+1} = R_t(W_t - c_t) + y_{t+1} \quad (6.67)$$

Substituting for the constraint:

$$\begin{aligned} V_t(c_{t-1}, W_t) &= \text{Max}_{c_t} U(c_t - \alpha c_{t-1}) \\ &\quad + \beta E_t V_{t+1}(R_t(W_t - c_t) + y_{t+1}, c_t) \end{aligned} \quad (6.68)$$

The first order condition is:

$$\frac{\partial U}{\partial c_t} + \beta E_t \frac{\partial V_{t+1}}{\partial W_{t+1}} (-R_t) + \beta E_t \frac{\partial V_{t+1}}{\partial c_t} = 0 \quad (6.69)$$

Differentiating with respect to c_t, c_{t-1} and W_t gives us:

$$\begin{aligned} & \frac{\partial^2 U}{\partial c_t^2} dc_t + \frac{\partial^2 U}{\partial c_t \partial c_{t-1}} dc_{t-1} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 dc_t \\ & - \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 dW_t + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} (-R_t) dc_t \\ & + \beta E_t \frac{\partial^2 V_{t+1}}{\partial c_t^2} dc_t + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t dW_t - \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t dc_t \\ & = 0 \end{aligned} \quad (6.70)$$

Regrouping terms,

$$\begin{aligned} & \left[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t \right] dc_t \\ & + \left[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t \right] dW_t \\ & + \frac{\partial^2 U}{\partial c_t \partial c_{t-1}} dc_{t-1} \\ & = 0 \end{aligned} \quad (6.71)$$

From (6.71), we can calculate:

$$\frac{\partial g}{\partial c_{t-1}} \equiv \frac{dc_t}{dc_{t-1}}$$

as

$$\begin{aligned} \frac{dc_t}{dc_{t-1}} &= \frac{\alpha U''}{[U'' + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]} \\ &> 0 \end{aligned} \quad (6.72)$$

by the induction assumption, and since $U'' < 0$.

Similarly, to calculate $\frac{\partial g}{\partial W_t}$:

$$\begin{aligned} \frac{\partial g}{\partial W_t} &\equiv \frac{dc_t}{dW_t} = - \frac{[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{[U'' + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]} \\ &> 0 \end{aligned} \quad (6.73)$$

by the induction assumption, and since $U'' < 0$.

Unfortunately, we cannot determine the signs of:

$$\frac{\partial^2 g}{\partial c_{t-1} \partial W_t}; \quad \frac{\partial^2 g}{\partial W_t^2}; \quad \text{and} \quad \frac{\partial^2 g}{\partial c_{t-1}^2}$$

on the basis of the induction assumption. (See appendix for the exact expressions).

$$\begin{aligned} \frac{\partial V_t(W_t, c_{t-1})}{\partial W_t} &= \beta E_t R_t \frac{\partial V_{t+1}(W_{t+1}, c_t)}{\partial W_{t+1}} \\ &= \beta E_t R_t \left[\frac{\partial V_{t+1}(R_t(W_t - c_t) + y_{t+1}, c_t)}{\partial W_{t+1}} \right] \end{aligned} \quad (6.74)$$

Then, taking the derivative of (6.74) with respect to c_{t-1} :

$$\frac{\partial^2 V_t(W_t, c_{t-1})}{\partial W_t \partial c_{t-1}} = \frac{\partial g}{\partial c_{t-1}} \beta E_t \left\{ -\frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 + \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t \right\} \quad (6.75)$$

(6.75) is greater than zero since:

$$\frac{\partial g}{\partial c_{t-1}} > 0$$

Now, taking the derivative of (6.74) with respect to W_t , we will obtain:

$$\begin{aligned}
\frac{\partial^2 V_t(W_t, c_{t-1})}{\partial W_t^2} &= \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (-R_t)^2 \frac{\partial g}{\partial W_t} \\
&\quad + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t \frac{\partial g}{\partial W_t} \\
&= \beta E_t R_t \left\{ \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} R_t \left(1 - \frac{\partial g}{\partial W_t}\right) + \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} \frac{\partial g}{\partial W_t} \right\} \quad (6.76)
\end{aligned}$$

Unfortunately, we cannot determine the sign of (6.76).

Taking the derivative of (6.76) with respect to c_{t-1} and W_t respectively, we find:

$$\begin{aligned}
\frac{\partial^3 V_t(W_t, c_{t-1})}{\partial W_t^2 \partial c_{t-1}} &= \beta \frac{\partial g}{\partial c_{t-1}} E_t R_t \left\{ R_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^2 \partial c_t} \left[1 - 2 \frac{\partial g}{\partial W_t}\right] \right. \\
&\quad \left. - R_t^2 \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} \left(1 - \frac{\partial g}{\partial W_t}\right) + \frac{\partial^3 V_{t+1}}{\partial W_{t+1} \partial c_t^2} \frac{\partial g}{\partial W_t} \right\} \\
&\quad - \beta \frac{\partial^2 g}{\partial c_{t-1} \partial W_t} E_t R_t \left\{ \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} R_t - \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} \right\} \quad (6.77)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^3 V_t}{\partial W_t^3} &= \beta \frac{\partial g}{\partial W_t} E_t R_t \left\{ \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^2 \partial c_t} R_t \left(1 - \frac{\partial g}{\partial W_t}\right) + \frac{\partial^3 V_{t+1}}{\partial W_{t+1} \partial c_t^2} \left(\frac{\partial g}{\partial W_t}\right) \right\} \\
&\quad - \beta \left(\frac{\partial^2 g}{\partial W_t^2}\right) E_t R_t \left\{ \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} R_t - \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} \right\} \\
&\quad + \beta E_t (R_t)^3 \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} \left(1 - \frac{\partial g}{\partial W_t}\right)^2 \quad (6.78)
\end{aligned}$$

We cannot find the signs of:

$$\frac{\partial^2 V_t}{\partial W_t^2}, \frac{\partial^3 V_t}{\partial W_t^2 \partial c_{t-1}}, \text{ and } \frac{\partial^3 V_t}{\partial W_t^3}$$

on the basis of the induction assumption. It is, therefore, not possible to conclude, even

by restricting the utility function to the form $U(c_t - \alpha c_{t-1})$, that habit formation increases the precautionary premium “in the sense of Kimball” when there exists multiple periods.

In order to find more definite results, we could attempt to do the above analysis with an even more specific form of a utility function. If we still cannot get definite results, then we could resort to numerical methods. This is left for future work.

Chapter 7

CONCLUSION

In this work, we have examined the implications of intertemporally non-additive preferences exhibiting habit formation in consumption in various models of consumption-saving with uncertainty.

By introducing habit formation into a representative agent model of intertemporal consumption-saving with income uncertainty, we have shown that habit formation itself leads to prudent behavior in addition to the usual precautionary saving motive against income uncertainty.

Basically, in chapter 3, first, we have shown that even in the quadratic case, where the precautionary saving premium is zero, there is habit formation induced saving. More specifically, we have shown that changes in consumption are less responsive to unanticipated changes in income, and more sensitive to anticipated changes in income, when we assume habit-forming preferences in consumption. This result, hence, explains the excess sensitivity, the excess smoothness, and the excess growth puzzles of consumption.

We wanted to show that habit formation in consumption can help explain the high level of precautionary savings observed in the data. Since habit-forming individual is more averse to consumption risk, this induces a precautionary behavior which further results in additional precautionary saving, apart from the one against income uncertainty.

However, the higher the habit formation parameter, i.e., the stronger the habits, the lower the effect of income uncertainty on consumption. The reason is that the individual with habit-forming preferences in consumption saves more, i.e., consumes less out of permanent income in order to achieve higher consumption growth. This result arises because of the fact that habits lead the consumer to take into account the negative externality of higher current consumption on future utility level. Therefore, for a given value of the constant coefficient of absolute risk aversion, a consumer with habit formation will have lower precautionary savings (against income uncertainty) per unit of variance of change in consumption. That is, the greater is the strength of habit formation the lower will be the effect of income uncertainty on consumption.

By allowing habit-forming preferences in consumption, we did not need to assume higher levels of labor income uncertainty to explain consumption puzzles. We have shown that the marginal propensity to consume out of an unanticipated income change is lower, while the marginal propensity to consume out of anticipated income changes is positive with habit-forming preferences. Hence, consumption growth is higher with habit formation. Moreover, we have found that the variance of change in consumption is less than the variance of change in income when there is habit formation, even in the case of a random walk income process. Overall, with habit formation, consumption has a lower response to permanent income than in the usual model, but also responds to a distributed lag of past permanent incomes, i.e., to past perceptions of the income stream, thus exhibiting a faster rate of growth.

Furthermore, the fact that we have obtained the closed form solution for consumption with habit formation will provide a better framework for empirically testing the behavior of consumption over the business cycle. This is the major contribution of this section of the thesis to the consumption literature.

In chapter 4, we have introduced liquidity constraints to the intertemporal consumption-saving model with habit formation in consumption, where we have theoretically analyzed the response of lifetime maximizing consumer. We have shown that the presence of liq-

liquidity constraints increases the current marginal utility of consumption, implying a lower level of consumption than the case with time-separable preferences. We have, in particular, demonstrated that, if the liquidity constraint is binding, not only the current period but all the future shadow prices of the liquidity constraint will affect current marginal utility of consumption. By the fact that the individual has habit-forming preferences in consumption, the future shadow prices of liquidity constraint will be present and their effects will be higher the higher the strength of habit formation. Next, we have relaxed the liquidity constraint assumption by allowing limited borrowing to the individual. If the accumulated amount of debt is high, the individual decreases current consumption to relax future borrowing constraints.

In chapter 5, we have introduced the choice of leisure into our model. We have allowed intertemporal non-additivity in preferences with habit formation in consumption, and with separability in leisure to examine the implications of habit formation on current and future consumption and leisure choices. To this end, first, we have presented the general model with no habit formation in consumption choices, in order to use it as a benchmark model to compare with the habit formation model.

Then, we have shown that, with a Cobb-Douglas utility function incorporating habit formation only in consumption, the intertemporal marginal rate of substitution between consumption and leisure as well as the intertemporal marginal rate of substitution between current and future leisure are higher the higher the strength of habit formation. This result implies that the individual is less willing to intertemporally substitute current leisure with future leisure and with current consumption.

Then, we have introduced liquidity constraints into the benchmark model and to the model with habit formation in consumption, and endogenized the liquidity constraints by making them dependent on wage earnings. We have shown that when the liquidity constraints depend on wage earnings, the intratemporal marginal rate of substitution between consumption and leisure decreases whenever the shadow price of the liquidity constraint is positive. Thus, when considering taking one extra unit of leisure, the

individual must take into account that the borrowing limit will decrease.

Finally, in chapter 6 we have examined the relationship between habit formation and Kimball's concept of prudence. Using Kimball's two-period model we have shown that habit formation leads to a larger prudence premium and greater precautionary saving, provided that the individual has decreasing absolute prudence. Finally, we have investigated the relationship between prudence and habit formation in a multi-period framework. However, we have found that it is not possible to conclude, even with the utility function exhibiting a simple form of habit formation, that habits increase or decrease the precautionary premium "in the sense of Kimball" when there exists multiple periods. In order to find more precise results, it would be interesting to apply numerical methods. This is left for future work.

APPENDIX

Taking the derivative of (6.73) with respect to c_{t-1} , we find:

$$\begin{aligned}
\frac{\partial^2 g}{\partial c_{t-1} \partial W_t} &= \frac{\frac{\partial[-\frac{\partial^2 U}{\partial c_t \partial c_{t-1}}]}{\partial W_t} [\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2} \\
&\quad + \frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{\partial W_t} \frac{\partial^2 U}{\partial c_t \partial c_{t-1}}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2} \\
&= \frac{-\frac{\partial^2 U}{\partial c_t^2 \partial c_{t-1}} \frac{\partial g}{\partial W_t} [\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2} \\
&\quad + \frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{\partial W_t} \frac{\partial^2 U}{\partial c_t \partial c_{t-1}}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2}
\end{aligned}$$

Taking the derivative of (6.73) with respect to W_t , we find:

$$\begin{aligned}
\frac{\partial^2 g}{\partial W_t^2} &= \frac{\frac{\partial[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{\partial W_t}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2} \\
&\quad \times [\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t] \\
&\quad - \frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{\partial W_t}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2} \\
&\quad \times [-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} R_t^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]
\end{aligned}$$

By taking the derivative of (6.72) with respect to c_{t-1} , we find:

$$\begin{aligned}
\frac{\partial^2 g}{\partial c_{t-1}^2} &= \frac{-\frac{\partial(\frac{\partial^2 U}{\partial c_t \partial c_{t-1}})}{\partial c_{t-1}} [\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2} \\
&\quad + \frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]}{\partial c_{t-1}}}{\frac{\partial^2 U}{\partial c_t \partial c_{t-1}}} \frac{\partial^2 U}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1} \partial c_t} R_t]^2}
\end{aligned}$$

When we take the derivative of (6.36) with respect to c_{T-2} , we find:

$$\begin{aligned}
&\frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2 \partial c_{T-2}} = \\
&\quad \beta E_{T-1} \{ (R_{T-1})^3 \frac{\partial^3 U}{\partial c_T^3} (-\frac{\partial g}{\partial c_{T-2}}) (1 - \frac{\partial g}{\partial W_{T-1}}) \\
&\quad + (R_{T-1})^2 \frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (\frac{\partial g}{\partial c_{T-2}}) (1 - \frac{\partial g}{\partial W_{T-1}}) \\
&\quad - (R_{T-1})^2 \frac{\partial^2 U}{\partial c_T^2} (\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}}) (1 - \frac{\partial g}{\partial W_{T-1}}) \\
&\quad + (R_{T-1})^2 \frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (-\frac{\partial^2 g}{\partial c_{T-1} \partial c_T}) (\frac{\partial^2 g}{\partial W_{T-1} \partial c_{T-2}}) \} \\
&= \beta E_{T-1} (\frac{\partial g}{\partial c_{T-2}}) (R_{T-1}) \\
&\quad \times \{ R_{T-1} [-\frac{\partial^3 U}{\partial c_T^3} (1 - \frac{\partial g}{\partial W_{T-1}}) + \frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (1 - \frac{\partial g}{\partial W_{T-1}}) \\
&\quad - \frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (\frac{\partial g}{\partial W_{T-1}})] + \frac{\partial^3 U}{\partial c_T \partial c_{T-1}^2} (\frac{\partial g}{\partial W_{T-1}}) \} \\
&\quad + \beta E_{T-1} (\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}}) [-(R_{T-1})^2 \frac{\partial^2 U}{\partial c_T^2} + R_{T-1} \frac{\partial^2 U}{\partial c_T \partial c_{T-1}}] \\
&= (\frac{\partial g}{\partial c_{T-2}}) \beta E_{T-1} \{ (R_{T-1})^2 \frac{\partial^3 U}{\partial c_T^2 \partial c_{T-1}} (1 - 2\frac{\partial g}{\partial W_{T-1}}) \\
&\quad - (R_{T-1})^3 \frac{\partial^3 U}{\partial c_T^3} (1 - \frac{\partial g}{\partial W_{T-1}}) + R_{T-1} \frac{\partial^3 U}{\partial c_T \partial c_{T-1}^2} \frac{\partial g}{\partial W_{T-1}} \} \\
&\quad - (\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}}) \beta E_{T-1} [(R_{T-1})^2 \frac{\partial^2 U}{\partial c_T^2} - R_{T-1} \frac{\partial^2 U}{\partial c_T \partial c_{T-1}}]
\end{aligned}$$

Equation (6.77) is:

$$\begin{aligned}
\frac{\partial^3 V_t(W_t, c_{t-1})}{\partial W_t^2 \partial c_{t-1}} &= \beta E_t \frac{\partial^3 V_{t+1}}{\partial c_t \partial W_{t+1}^2} (R_t)^2 \left(1 - \frac{\partial g}{\partial W_t}\right) \frac{\partial g}{\partial c_{t-1}} \\
&\quad - \beta E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} (R_t)^3 \left(1 - \frac{\partial g}{\partial W_t}\right) \frac{\partial g}{\partial c_{t-1}} \\
&\quad - \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 \frac{\partial^2 g}{\partial c_{t-1} \partial W_t} \\
&\quad + \beta E_t \frac{\partial^3 V_{t+1}}{\partial c_t \partial W_{t+1}^2} (R_t)^2 \frac{\partial g}{\partial W_t} \frac{\partial g}{\partial c_{t-1}} \\
&\quad - \beta E_t \frac{\partial^3 V_{t+1}}{\partial c_t^2 \partial W_{t+1}^2} (R_t) \frac{\partial g}{\partial W_t} \frac{\partial g}{\partial c_{t-1}} \\
&\quad - \beta E_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}} (R_t) \frac{\partial^2 g}{\partial W_t \partial c_{t-1}} \\
&= \frac{\partial g}{\partial c_{t-1}} \left\{ \beta \left[E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} - R_t E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} \right] R_t^2 \left(1 - \frac{\partial g}{\partial W_t}\right) \right. \\
&\quad \left. - \beta \left[E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^2 \partial c_t} R_t - E_t \frac{\partial^3 V_{t+1}}{\partial c_t^2 \partial W_{t+1}} R_t \right] \frac{\partial g}{\partial W_t} \right\} \\
&\quad - \beta \left[E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - E_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}} R_t \right] \frac{\partial^2 g}{\partial W_t \partial c_{t-1}} \\
&= \frac{\partial g}{\partial c_{t-1}} \left\{ R_t^2 \beta E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^2 \partial c_t} \left[1 - 2 \frac{\partial g}{\partial W_t}\right] - R_t^3 E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} \left(1 - \frac{\partial g}{\partial W_t}\right) \right. \\
&\quad \left. + R_t \beta E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1} \partial c_t^2} \frac{\partial g}{\partial W_t} \right\} \\
&\quad - \beta \left[E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} (R_t)^2 - E_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}} R_t \right] \frac{\partial^2 g}{\partial W_t \partial c_{t-1}}
\end{aligned}$$

Equation (6.78) is equal to:

$$\frac{\partial^3 V_t}{\partial W_t^3} = \beta E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} (R_t)^3 \left(1 - \frac{\partial g}{\partial W_t}\right)^2$$

$$\begin{aligned}
& +\beta E_t \frac{\partial^3 V_{t+1}}{\partial c_t \partial W_{t+1}^2} \left[R_t^2 \left(1 - \frac{\partial g}{\partial W_t} \right) \frac{\partial g}{\partial W_t} \right. \\
& \left. + \frac{\partial^3 V_{t+1}}{\partial c_t^2 \partial W_{t+1}} R_t \left(\frac{\partial g}{\partial W_t} \right)^2 \right] \\
& -\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} R_t^2 \left(\frac{\partial g}{\partial W_t} \right) \left(\frac{\partial g}{\partial W_t} \right)^2 \\
& +\beta E_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}} R_t \frac{\partial^2 g}{\partial W_t^2} \\
= & \frac{\partial g}{\partial W_t} \left\{ \beta E_t (R_t)^2 \frac{\partial^3 V_{t+1}}{\partial c_t \partial W_{t+1}^2} \left(1 - \frac{\partial g}{\partial W_t} \right) \right. \\
& \left. + R_t \beta E_t \frac{\partial^3 V_{t+1}}{\partial c_t^2 \partial W_{t+1}} \left(\frac{\partial g}{\partial W_t} \right) \right\} \\
& -\beta \left[E_t (R_t)^2 \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} - E_t R_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}} \right] \left(\frac{\partial^2 g}{\partial W_t^2} \right) \\
& +\beta E_t (R_t)^3 \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3} \left(1 - \frac{\partial g}{\partial W_t} \right)^2
\end{aligned}$$

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