Modeling of Differential Stripline in the Presence of Oblique Crossing Traces

by

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Abstract

Due to continually increasing operating frequencies previously neglected effects of interconnects such as delay, crosstalk and attenuation are becoming major bottlenecks in high-speed designs. Effect of crossing trace angle on signal integrity of differential edge-coupled stripline is studied in this thesis. The closed-form solutions exist for simple TEM geometries such as symmetric or asymmetric differential striplines. In the literature analytical modeling for the case of 90° trace crossings is available. However, for more complicated structures, such as differential stripline with crossing trace at oblique angles, computationally expensive 3D numerical methods are often employed. In this thesis, an efficient modeling and analysis method is presented for high-speed interconnects represented by differential asymmetrical transmission lines with oblique crossings. The method is based on dividing the structure into several sections, then using an appropriate technique for estimating the capacitance matrix of each section and subsequently evaluating the impedance matrix using the chain parameters. The thesis also investigates the implication of different angles for crossing traces on signal integrity metrics.

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Chapter 1

Introduction

1.1 Background and Motivation

The rapidly increasing demand for high-speed applications coupled with significant advances in VLSI and PCB design present enormous challenges for design of next generation CAD tools for the chip, package and board levels. These challenges are diverse, such as higher transistor density, higher frequencies, sharper signal edges and lower supply voltage levels, which results in smaller timing and noise margins.

At higher frequencies and with sharpening edge rates, the interconnects can no longer be treated as simple wires, they become transmission lines propagating electromagnetic waves. In today's designs the interconnects become the major source of signal degradation due to such signal integrity effects as attenuation, delay, skew, jitter, reflections, ringing, crosstalk, skin, edge and proximity effects as well as ground bounce (see Figure 1.1). Therefore, it is increasingly important to provide accurate models of interconnects in CAD

Figure 1.1: Signal Integrity Effects of Interconnects

tools and aid the designers to efficiently tackle the signal integrity and power integrity issues.

In the past, the interconnect design process heavily relied on using rules of thumb. For example, to determine if the signal will have signal integrity issues, the following rule of thumb was used: the interconnect is considered electrically long if its length is greater than the wavelength of the highest desired frequency component of the signal divided by 10. For electrically long interconnects signal integrity effects are often considered while the electrically short interconnects are treated as shorts (without considering signal integrity effects).

For simple transmission lines such as single-ended microstrip and stripline, approximate formulas were often used to calculate their parameters, giving

them more accuracy than simple rules of thumb [4]. Per-unit length parameters (R , L , C , G), characteristic impedance, and propagation delay formulas are available for simple transmission lines from many sources. For example, the formulas for single-ended microstrip and stripline are available in [2].

For more complicated transmission lines with multiple conductors and non-uniform sections, use of formulas becomes cumbersome. Therefore, various modeling and analysis methods were developed and employed in the CAD simulators to accurately predict the performance of the transmission line. The numerical methods used in CAD simulators provide the most accuracy, but computational time becomes a trade-off [4].

Until recently, CAD signal integrity simulators took into account only major features of the interconnect, such as trace, via and pad. However, with the increasing edge rates, other design trends and features such as differential signaling, trace bends and crossing junctions start to have an effect on signal integrity and need to be accurately and efficiently modeled as well.

Differential signalling is widely used in modern designs to improve the noise related figure of merit. Striplines are often used for implementing differential signaling. Stripline is a widely-used type of transmission line structure in carrying high-speed signals on PCBs. In the stripline structure, transmission line traces are routed between reference planes (power or ground) surrounded by a dielectric material, see Figure 1.2. Modern PCBs tend to have high routing density due to large number of ICs and each IC in turn with large number of pins. Therefore, new routing layers are added to the stackup; however it increases the cost of PCBs. To reduce the stackup height it is common to include two routing layers between reference planes in a dual

Figure 1.2: Cross-section of a Differential Stripline Structure

stripline configuration, see Figure 1.3. The work in this thesis focuses on differential signaling with striplines. Differential stripline is typically routed on one of the layers of dual stripline and the crossing trace at any angle is routed on the other layer of dual stripline. For the purpose of illustration, 3D representations of the differential stripline in the presence of crossing trace at 90° and 45° angles are shown in Figure 1.3 and 1.4, respectively.

The effect of crossing trace junctions at 90° angle on differential stripline was further modeled and studied in [6] and [7]. A structure with a dielectric layer sandwiched between two solid planes electrically behaves like a microwave cavity resonator. To minimize the coupling, tracks are often routed at 90° to each other in the adjacent signal layers. This is because, mutual inductance is considerably minimized due to the 90° angle between the two current directions and associated magnetic field cancellation. Also, from an electrical point of view, the perpendicular crossing gives the smallest overlap area, leading to lesser electrical field coupling and hence lower junction ca-

capitance. Despite the relatively low coupling between two crossing signals (that are perpendicularly routed with respect to each other), discontinuities at the crossing junctions can compromise the signal integrity of the driven lines.

In [7], the issue of crossing lines and the related discontinuities was handled in entirety, with signal integrity analysis in focus. In the proposed method, coupled asymmetrical strip lines were considered as the fundamental blocks constituting the signal path (sections with and without the impact of crossing junctions).

Figure 1.3: Differential Stripline with Crossing Trace at 90° Angle

The work in this thesis is to further advance the research on the modeling of crossing junctions and to develop an efficient model for differential stripline in the presence of crossing traces at oblique angles ($0^\circ \leq \theta \leq 180^\circ$) and to study the effect of crossing traces at oblique angles on the signal integrity associated with the differential striplines. The method developed in this thesis can be easily extended to model any number of crossing traces or

Figure 1.4: Differential Stripline with Crossing Trace at 45° Angle

other conductors in the vicinity of the differential stripline.

1.2 Contributions

In this thesis, modeling of differential stripline with crossing trace junctions has been further advanced. Particularly,

- (a) an efficient model for differential asymmetrical striplines with oblique crossing junctions has been developed.
- (b) effect of the crossing trace angle on the signal integrity metrics has been investigated in detail.

1.3 Organization of the Thesis

This thesis has been organized as follows. Chapter 1 provides an introduction to the topic of signal integrity as it applies to the differential stripline transmission line. Chapter 2 presents a detailed review of theory on Multi-conductor Transmission Line Analysis. Chapter 3 provides a review of the previous research on 90° crossings. Chapter 4 presents the proposed solution for oblique crossings. Chapter 5 presents the validating numerical results of the transmission line analysis and Chapter 6 presents the conclusions.

Chapter 2

Literature Survey:

Multiconductor Transmission

Line Analysis

Signal integrity effects in transmission lines are becoming a critical aspect of high-speed digital performance. The transmission line theory is well-established in the field of electrical engineering. The transmission line equations can be directly derived from Maxwell equations or from an extracted RLGC model [8]. Introductory material on signal integrity and transmission lines are available by many authors, such as [2], [3] and [4]. Widely used transmission line structures include: twisted-pair, single-ended microstrip and stripline. The relations for calculating per-unit length (PUL) parameters, characteristic impedance and propagation delay for twisted-pair, single-ended microstrip and stripline are available in [2].

The analysis of both two-conductor [5] and multiconductor transmission

lines is covered in great detail in [8]. The study of two-conductor differential stripline with crossing trace junctions at 90° angle in [7] uses novel approach by treating the crossing trace at 90° angle as a reference plane. Consequently, the analysis in [7] uses formulations for two-conductor transmission lines to model the differential stripline. The work presented in this thesis treats the crossing trace as a third conductor in addition to the two conductors of the differential stripline and exploits the formulations for multiconductor transmission lines to arrive at an efficient model.

This section presents the underlying theoretical background for the solution of the multiconductor transmission line. The fundamental assumption in the transmission line formulations is that the electric field intensity vector and the magnetic field intensity vector lie in a plane transverse or perpendicular to the direction of the line (the Z-axis) resulting in TEM mode of propagation.

2.1 The Transmission-Line Equations for Multiconductor Lines

The transmission line equations for n -conductor line in matrix form are given below, where n is the number of conductors excluding the reference plane [8]

$$\frac{\partial}{\partial z} \mathbf{v}(z, t) = -\mathbf{R}\mathbf{i}(z, t) - \mathbf{L} \frac{\partial}{\partial t} \mathbf{i}(z, t) \quad (2.1)$$

$$\frac{\partial}{\partial z} \mathbf{i}(z, t) = -\mathbf{G}\mathbf{v}(z, t) - \mathbf{C} \frac{\partial}{\partial t} \mathbf{v}(z, t) \quad (2.2)$$

where $\mathbf{v}(z, t)$ and $\mathbf{i}(z, t)$ are $n \times 1$ column vectors with voltages and currents of each conductor as a function of line position z and time t

$$\mathbf{v}(z, t) = \begin{bmatrix} v_1(z, t) \\ v_2(z, t) \\ \dots \\ v_n(z, t) \end{bmatrix} \quad (2.3)$$

$$\mathbf{i}(z, t) = \begin{bmatrix} i_1(z, t) \\ i_2(z, t) \\ \dots \\ i_n(z, t) \end{bmatrix} \quad (2.4)$$

The $\mathbf{R}$, $\mathbf{L}$, $\mathbf{G}$, $\mathbf{C}$ are per-unit-length resistance, inductance, conductance, and capacitance matrices, respectively. The resistance matrix is described as [8]

$$\mathbf{R} = \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & r_n \end{bmatrix} + \begin{bmatrix} r_0 & r_0 & \dots & r_0 \\ r_0 & r_0 & \dots & r_0 \\ \dots & \dots & \dots & \dots \\ r_0 & r_0 & \dots & r_0 \end{bmatrix} \quad (2.5)$$

where r_i are self-resistances of the conductors and r_0 are due to the resistance of the ground conductor. The per-unit-length resistance matrix in (2.5) adequately represents the case of reference conductors that are finite-sized such as wires. For ground conductor in the form of large ground plane, the return currents for the conductors will spread out on the ground plane beneath them as shown in Figure 2.1, possibly overlapping and having different cross-sectional areas. This will result in different ground resistances, causing

Figure 2.1: Return Current Spreading in Ground Plane

the resistance matrix to be of the form

$$\mathbf{R} = \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & r_n \end{bmatrix} + \begin{bmatrix} \hat{r}_{11} & \hat{r}_{12} & \dots & \hat{r}_{1n} \\ \hat{r}_{12} & \hat{r}_{22} & \dots & \hat{r}_{2n} \\ \dots & \dots & \dots & \dots \\ \hat{r}_{1n} & \hat{r}_{2n} & \dots & \hat{r}_{nn} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{12} & r_{22} & \dots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{1n} & r_{2n} & \dots & r_{nn} \end{bmatrix} \quad (2.6)$$

where r_i are self-resistances of the conductors and $\hat{r}_{ij}$ are due to the resistance of the ground plane.

The inductance matrix is of the form:

$$\mathbf{L} = \begin{bmatrix} l_{11} & l_{12} & \dots & l_{1n} \\ l_{12} & l_{22} & \dots & l_{2n} \\ \dots & \dots & \dots & \dots \\ l_{1n} & l_{2n} & \dots & l_{nn} \end{bmatrix} \quad (2.7)$$

where l_{ii} are self-inductances and l_{ij} , $i \neq j$ are mutual inductances.

The conductance matrix is of the form:

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & \dots & G_{1n} \\ G_{12} & G_{22} & \dots & G_{2n} \\ \dots & \dots & \dots & \dots \\ G_{1n} & G_{2n} & \dots & G_{nn} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^n g_{1k} & -g_{12} & \dots & -g_{1n} \\ -g_{12} & \sum_{k=1}^n g_{2k} & \dots & -g_{2n} \\ \dots & \dots & \dots & \dots \\ -g_{1n} & -g_{2n} & \dots & \sum_{k=1}^n g_{nk} \end{bmatrix} \quad (2.8)$$

where g_{ii} are self-conductances, g_{ij} , $i \neq j$ are mutual conductances, $g_{ij} = g_{ji}$, $i \neq j$ and the sums are expanded as follows

$$G_{11} = \sum_{k=1}^n g_{1k} = g_{11} + g_{12} + \dots + g_{1n} \quad (2.9)$$

$$G_{22} = \sum_{k=1}^n g_{2k} = g_{21} + g_{22} + \dots + g_{2n} \quad (2.10)$$

$$G_{nn} = \sum_{k=1}^n g_{nk} = g_{n1} + g_{n2} + \dots + g_{nn} \quad (2.11)$$

The capacitance matrix is of the form:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{12} & C_{22} & \dots & C_{2n} \\ \dots & \dots & \dots & \dots \\ C_{1n} & C_{2n} & \dots & C_{nn} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^n c_{1k} & -c_{12} & \dots & -c_{1n} \\ -c_{12} & \sum_{k=1}^n c_{2k} & \dots & -c_{2n} \\ \dots & \dots & \dots & \dots \\ -c_{1n} & -c_{2n} & \dots & \sum_{k=1}^n c_{nk} \end{bmatrix} \quad (2.12)$$

where c_{ii} are self-capacitances and c_{ij} , $i \neq j$ are mutual capacitances, $c_{ij} = c_{ji}$, $i \neq j$ and the sums are expanded as follows

$$C_{11} = \sum_{k=1}^n c_{1k} = c_{11} + c_{12} + \dots + c_{1n} \quad (2.13)$$

$$C_{22} = \sum_{k=1}^n c_{2k} = c_{21} + c_{22} + \dots + c_{2n} \quad (2.14)$$

$$C_{nn} = \sum_{k=1}^n c_{nk} = c_{n1} + c_{n2} + \dots + c_{nn} \quad (2.15)$$

The total charge on the i th conductor per unit length (q_i) is given by

$$\mathbf{Q} = \mathbf{C}\mathbf{V} \quad (2.16)$$

where $\mathbf{Q}$ is the $n \times 1$ column vector of the total per-unit-length charges

$$\mathbf{Q} = \begin{bmatrix} q_1 \\ q_2 \\ \dots \\ q_n \end{bmatrix} \quad (2.17)$$

The equation in (2.16) can be expanded as follows:

$$q_1 = C_{11}V_1 + C_{12}V_2 + \dots + C_{1n}V_n \quad (2.18)$$

$$q_2 = C_{21}V_1 + C_{22}V_2 + \dots + C_{2n}V_n \quad (2.19)$$

$$q_n = C_{n1}V_1 + C_{n2}V_2 + \dots + C_{nn}V_n \quad (2.20)$$

where C_{ij} are elements of the capacitance matrix $\mathbf{C}$. The above equations can be written in terms of self-capacitances and mutual capacitances as follows:

$$q_1 = c_{11}V_1 + c_{12}(V_1 - V_2) + \dots + c_{1n}(V_1 - V_n) \quad (2.21)$$

$$q_2 = c_{21}(V_2 - V_1) + c_{22}V_2 + \dots + c_{2n}(V_2 - V_n) \quad (2.22)$$

$$q_n = c_{n1}(V_n - V_1) + c_{n2}(V_n - V_2) + \dots + c_{nn}V_n \quad (2.23)$$

From these equations self-capacitance of a conductor can be defined as the coefficient of voltage on this conductor contributing to the total charge on the conductor. Mutual capacitance between two conductors can be defined as the

coefficient of the difference of voltages between these conductors contributing to the total charge on the conductor.

The $\mathbf{R}$, $\mathbf{L}$, $\mathbf{G}$, $\mathbf{C}$ are generally frequency dependent, that is, they are function of frequency ω and can be written as $\mathbf{R}(\omega)$, $\mathbf{L}(\omega)$, $\mathbf{G}(\omega)$, $\mathbf{C}(\omega)$. To write the MTL equations in (2.1) and (2.2) in frequency domain instead of time-domain, $\frac{\partial}{\partial t}$ is substituted with $j\omega$, which gives:

$$\frac{\partial}{\partial z} \mathbf{V}(z, \omega) = -\mathbf{Z}(\omega) \mathbf{I}(z, \omega) \quad (2.24)$$

$$\frac{\partial}{\partial z} \mathbf{I}(z, \omega) = -\mathbf{Y}(\omega) \mathbf{V}(z, \omega) \quad (2.25)$$

where $\mathbf{Z}(\omega)$ and $\mathbf{Y}(\omega)$ are per-unit-length impedance and admittance matrices, respectively, and are given by

$$\mathbf{Z}(\omega) = \mathbf{R}(\omega) + j\omega \mathbf{L}(\omega) \quad (2.26)$$

$$\mathbf{Y}(\omega) = \mathbf{G}(\omega) + j\omega \mathbf{C}(\omega) \quad (2.27)$$

2.2 The PUL Parameters of Frequency-Dependent Multiconductor Lines

In the resistance matrix from (2.6) it is assumed that the resistances of the reference plane $\hat{r}_{ij}$ are equal to zero, therefore, the resistance matrix $\mathbf{R}$ becomes

$$\mathbf{R} = \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & r_n \end{bmatrix} \quad (2.28)$$

The conductor resistances in (2.28) can be calculated from [7] as DC and frequency-dependent components

$$R'_{dc} = \frac{1}{\sigma_{dc}wt} \quad (2.29)$$

$$R'(f) = \frac{1}{\eta(w+t)} \sqrt{\frac{\pi f \mu}{\sigma_{dc}}} \quad (2.30)$$

where w is the *width* of the conductor, t is the *thickness* of the conductor, σ_{dc} is *dc conductivity*, f is the *frequency*, and η is a *constant* such that $1 < \eta < 2$. The equation for the frequency-dependent component assumes that the width of the conductor w is much larger than the thickness of the conductor t .

The per-unit-length parameters for multiconductor line can be written as [8]

$$\mathbf{LC} = \mathbf{CL} = \mu \varepsilon \mathbf{1}_n \quad (2.31)$$

$$\mathbf{LG} = \mathbf{GL} = \mu \sigma \mathbf{1}_n \quad (2.32)$$

where ε is the electric permittivity obtained as the product of relative permittivity ε_r and permittivity of free space ε_0 : $\varepsilon = \varepsilon_r \varepsilon_0$; μ is the magnetic permeability obtained as the product of relative permeability μ_r and permeability of free space μ_0 : $\mu = \mu_r \mu_0$; σ is effective conductivity and $\mathbf{1}_n$ is $n \times n$ *identity matrix*

$$\mathbf{1}_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix} \quad (2.33)$$

If the $\mathbf{C}$ matrix is known, the $\mathbf{L}$ matrix can be found from (2.31) as follows

$$\mathbf{L} = \mu \varepsilon \mathbf{C}^{-1} \quad (2.34)$$

Substituting (2.34) into (2.32) gives the equation for $\mathbf{G}$ in terms of $\mathbf{C}$

$$\mathbf{G} = \frac{\sigma}{\varepsilon} \mathbf{C} \quad (2.35)$$

The effective conductivity, representing conductive and polarization losses is

$$\sigma = \omega \varepsilon \tan \delta \quad (2.36)$$

where $\tan \delta$ is the *loss tangent* of the dielectric material. Substituting (2.36) into (2.35) gives expanded expression for $\mathbf{G}$

$$\mathbf{G} = \omega \tan \delta \mathbf{C} \quad (2.37)$$

2.3 Frequency-Domain Analysis of Multiconductor Lines

For the purpose of conciseness, omitting writing the ω in the frequency domain equations (2.24) and (2.25) we have:

$$\frac{\partial}{\partial z} \mathbf{V}(z) = -\mathbf{Z}\mathbf{I}(z) \quad (2.38)$$

$$\frac{\partial}{\partial z} \mathbf{I}(z) = -\mathbf{Y}\mathbf{V}(z) \quad (2.39)$$

By differentiating these equations with respect to line position z and substituting (2.38) and (2.39), the following *second-order differential equations* are obtained

$$\frac{d^2}{dz^2} \mathbf{V}(z) = \mathbf{Z}\mathbf{Y}\mathbf{V}(z) \quad (2.40)$$

$$\frac{d^2}{dz^2} \mathbf{I}(z) = \mathbf{Y}\mathbf{Z}\mathbf{I}(z) \quad (2.41)$$

where impedance $\mathbf{Z}$ and admittance $\mathbf{Y}$ matrices are given by

$$\mathbf{Z} = \mathbf{R} + j\omega\mathbf{L} \quad (2.42)$$

$$\mathbf{Y} = \mathbf{G} + j\omega\mathbf{C} \quad (2.43)$$

The equations in (2.40) and (2.41) are *coupled* together because $\mathbf{ZY}$ and $\mathbf{YZ}$ are full matrices; that is, each set of voltages and currents $V_i(z)$ and $I_i(z)$ affects all the other sets of voltages and currents $V_j(z)$ and $I_j(z)$. The essential idea in the solution method is to *decouple* them with a *similarity transformation* [11]-[14]. To accomplish this, perform a change of variables in equations (2.40) and (2.41), apply the solution to the modified equations and then use change of variables to return to the original voltages and currents. The first step in the solution is to define the voltages and currents as transformations from *modal* voltages and currents ($\mathbf{V}_m(z)$ and $\mathbf{I}_m(z)$) to actual line voltages and currents ($\mathbf{V}(z)$ and $\mathbf{I}(z)$):

$$\mathbf{V}(z) = \mathbf{T}_V \mathbf{V}_m(z) \quad (2.44)$$

$$\mathbf{I}(z) = \mathbf{T}_I \mathbf{I}_m(z) \quad (2.45)$$

Substituting these in the equations (2.40) and (2.41) gives

$$\frac{d^2}{dz^2} \mathbf{V}_m(z) = \mathbf{T}_V^{-1} \mathbf{ZY} \mathbf{T}_V \mathbf{V}_m(z) = \gamma^2 \mathbf{V}_m(z) \quad (2.46)$$

$$\frac{d^2}{dz^2} \mathbf{I}_m(z) = \mathbf{T}_I^{-1} \mathbf{YZ} \mathbf{T}_I \mathbf{I}_m(z) = \gamma^2 \mathbf{I}_m(z) \quad (2.47)$$

The objective is to decouple these second-order equations by finding $\mathbf{T}_V$ and $\mathbf{T}_I$ that simultaneously diagonalize $\mathbf{ZY}$ and $\mathbf{YZ}$ matrix products via similarity transformations as follows

$$\mathbf{T}_V^{-1} \mathbf{ZY} \mathbf{T}_V = \gamma^2 \quad (2.48)$$

$$\mathbf{T}_I^{-1} \mathbf{Y} \mathbf{Z} \mathbf{T}_I = \gamma^2 \quad (2.49)$$

where γ^2 is an $n \times n$ diagonal matrix

$$\gamma^2 = \begin{bmatrix} \gamma_1^2 & 0 & \dots & 0 \\ 0 & \gamma_2^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \gamma_n^2 \end{bmatrix} \quad (2.50)$$

Thus, the equations with mode voltages and currents in (2.46) and (2.47) are *decoupled* and have the solution in the form of:

$$\mathbf{V}_m(z) = \mathbf{e}^{-\gamma z} \mathbf{V}_m^+ + \mathbf{e}^{\gamma z} \mathbf{V}_m^- \quad (2.51)$$

$$\mathbf{I}_m(z) = \mathbf{e}^{-\gamma z} \mathbf{I}_m^+ - \mathbf{e}^{\gamma z} \mathbf{I}_m^- \quad (2.52)$$

where $\mathbf{V}_m^\pm$ and $\mathbf{I}_m^\pm$ are $n \times 1$ vectors of unknown voltages and currents traveling in forward and backward directions along z . The *matrix exponentials* are defined as

$$\mathbf{e}^{\pm \gamma z} = \begin{bmatrix} \mathbf{e}^{\pm \gamma_1 z} & 0 & \dots & 0 \\ 0 & \mathbf{e}^{\pm \gamma_2 z} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mathbf{e}^{\pm \gamma_n z} \end{bmatrix} \quad (2.53)$$

Transforming back to actual line voltages and currents gives

$$\mathbf{V}(z) = \mathbf{T}_V (\mathbf{e}^{-\gamma z} \mathbf{V}_m^+ + \mathbf{e}^{\gamma z} \mathbf{V}_m^-) \quad (2.54)$$

$$\mathbf{I}(z) = \mathbf{T}_I (\mathbf{e}^{-\gamma z} \mathbf{I}_m^+ - \mathbf{e}^{\gamma z} \mathbf{I}_m^-) \quad (2.55)$$

These equations contain 4 undetermined vectors $\mathbf{V}_m^+$, $\mathbf{V}_m^-$, $\mathbf{I}_m^+$, $\mathbf{I}_m^-$. To reduce the number of undetermined vectors to 2, substitute (2.52) into (2.39) to give

$$\mathbf{V}(z) = -\mathbf{Y}^{-1} \frac{d}{dz} \mathbf{I}(z) = \mathbf{Y}^{-1} \mathbf{T}_I \gamma (\mathbf{e}^{-\gamma z} \mathbf{I}_m^+ + \mathbf{e}^{\gamma z} \mathbf{I}_m^-) \quad (2.56)$$

$$= \mathbf{Y}^{-1} \mathbf{T}_I \gamma \mathbf{T}_I^{-1} \mathbf{T}_I (\mathbf{e}^{-\gamma z} \mathbf{I}_m^+ + \mathbf{e}^{\gamma z} \mathbf{I}_m^-) \quad (2.57)$$

Define the characteristic impedance matrix as

$$\mathbf{Z}_C = \mathbf{Y}^{-1} \mathbf{T}_I \gamma \mathbf{T}_I^{-1} \quad (2.58)$$

Next, the line voltages and currents are expressed with 2 vectors $\mathbf{I}_m^+$ and $\mathbf{I}_m^-$

$$\mathbf{V}(z) = \mathbf{Z}_C \mathbf{T}_I (\mathbf{e}^{-\gamma z} \mathbf{I}_m^+ + \mathbf{e}^{\gamma z} \mathbf{I}_m^-) \quad (2.59)$$

$$\mathbf{I}(z) = \mathbf{T}_I (\mathbf{e}^{-\gamma z} \mathbf{I}_m^+ - \mathbf{e}^{\gamma z} \mathbf{I}_m^-) \quad (2.60)$$

If matrix multiplication $\mathbf{YZ}$ is performed by using (2.42), (2.43) and substitution is made by (2.31) and (2.32) the following relationship is obtained

$$\mathbf{YZ} = \mathbf{GR} + j\omega \mathbf{CR} + (j\omega\mu\sigma - \omega^2\mu\varepsilon) \mathbf{1}_n \quad (2.61)$$

Substituting (2.35) into (2.61) results in

$$\mathbf{YZ} = \left(\frac{\sigma}{\varepsilon} + j\omega \right) \mathbf{CR} + (j\omega\mu\sigma - \omega^2\mu\varepsilon) \mathbf{1}_n \quad (2.62)$$

From this equation it can be seen that only $\mathbf{CR}$ matrix product needs to be diagonalized as follows

$$\mathbf{T}_I^{-1} \mathbf{CR} \mathbf{T}_I = \Lambda^2 \quad (2.63)$$

where Λ^2 is an $n \times n$ diagonal matrix

$$\Lambda^2 = \begin{bmatrix} \Lambda_1^2 & 0 & \dots & 0 \\ 0 & \Lambda_2^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \Lambda_n^2 \end{bmatrix} \quad (2.64)$$

Next, $\mathbf{C}$ can be diagonalized

$$\mathbf{U}^t \mathbf{C} \mathbf{U} = \theta^2 \quad (2.65)$$

where θ^2 is an $n \times n$ diagonal matrix

$$\theta^2 = \begin{bmatrix} \theta_1^2 & 0 & \dots & 0 \\ 0 & \theta_2^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \theta_n^2 \end{bmatrix} \quad (2.66)$$

In equation (2.65) n columns of $\mathbf{U}$ are eigenvectors of $\mathbf{C}$ and n values of θ^2 are eigenvalues of $\mathbf{C}$. Next, $\mathbf{C}\mathbf{R}$ matrix product is diagonalized as follows:

$$\mathbf{S}^t [\theta \mathbf{U}^t \mathbf{R} \mathbf{U} \theta] \mathbf{S} = \Lambda^2 \quad (2.67)$$

where n columns of $\mathbf{S}$ are eigenvectors of $\theta \mathbf{U}^t \mathbf{R} \mathbf{U} \theta$ and n values of Λ^2 are eigenvalues of $\theta \mathbf{U}^t \mathbf{R} \mathbf{U} \theta$. The result shows that the desired transformations are:

$$\mathbf{T}_I = \mathbf{U} \theta \mathbf{S} \quad (2.68)$$

$$\mathbf{T}_V = \mathbf{U} \theta^{-1} \mathbf{S} \quad (2.69)$$

$$\mathbf{T}_I^{-1} = \mathbf{S}^t \theta^{-1} \mathbf{U}^t = \mathbf{T}_V^t \quad (2.70)$$

$$\mathbf{T}_V^{-1} = \mathbf{S}^t \theta \mathbf{U}^t = \mathbf{T}_I^t \quad (2.71)$$

Therefore, the matrix product $\mathbf{Y}\mathbf{Z}$ is diagonalized as follows from (2.49), (2.62) and (2.63)

$$\mathbf{T}_I^{-1} \mathbf{Y} \mathbf{Z} \mathbf{T}_I = \left(\frac{\sigma}{\varepsilon} + j\omega \right) \Lambda^2 + (j\omega\mu\sigma - \omega^2\mu\varepsilon) \mathbf{1}_n = \gamma^2 \quad (2.72)$$

From this equation the propagation constant diagonal matrix γ is calculated as

$$\gamma = \sqrt{\left(\frac{\sigma}{\varepsilon} + j\omega\right) \Lambda^2 + (j\omega\mu\sigma - \omega^2\mu\varepsilon)\mathbf{1}_n} \quad (2.73)$$

If the resistance matrix is already diagonal as indicated in (2.28), there is no need to diagonalize it as in (2.67), and the equation for the propagation constant diagonal matrix becomes

$$\gamma = \sqrt{\left(\frac{\sigma}{\varepsilon} + j\omega\right) \mathbf{R}\theta^2 + (j\omega\mu\sigma - \omega^2\mu\varepsilon)\mathbf{1}_n} \quad (2.74)$$

The voltages and currents at the two ends of the transmission line are related by chain parameters as follows:

$$\begin{bmatrix} \mathbf{V}(L) \\ \mathbf{I}(L) \end{bmatrix} = \mathbf{\Phi}(L) \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{I}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{11}(L) & \mathbf{\Phi}_{12}(L) \\ \mathbf{\Phi}_{21}(L) & \mathbf{\Phi}_{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{I}(0) \end{bmatrix} \quad (2.75)$$

where L is length of the uniform transmission line. The chain parameters are calculated from (2.59) and (2.60) at $z = 0$ and $z = L$ to result in:

$$\mathbf{\Phi}_{11}(L) = \frac{1}{2} \mathbf{Y}^{-1} \mathbf{T}_I (\mathbf{e}^{\gamma L} + \mathbf{e}^{-\gamma L}) \mathbf{T}_I^{-1} \mathbf{Y} \quad (2.76)$$

$$\mathbf{\Phi}_{12}(L) = -\frac{1}{2} \mathbf{Y}^{-1} \mathbf{T}_I \gamma (\mathbf{e}^{\gamma L} - \mathbf{e}^{-\gamma L}) \mathbf{T}_I^{-1} \quad (2.77)$$

$$\mathbf{\Phi}_{21}(L) = -\frac{1}{2} \mathbf{T}_I (\mathbf{e}^{\gamma L} - \mathbf{e}^{-\gamma L}) \gamma^{-1} \mathbf{T}_I^{-1} \mathbf{Y} \quad (2.78)$$

$$\mathbf{\Phi}_{22}(L) = \frac{1}{2} \mathbf{T}_I (\mathbf{e}^{\gamma L} + \mathbf{e}^{-\gamma L}) \mathbf{T}_I^{-1} \quad (2.79)$$

The above calculations are performed for all frequencies in the range of interest. The chain parameters for individual sections of the transmission line are multiplied in appropriate order to obtain chain parameters of the whole transmission line using the following chain parameter property:

$$\mathbf{\Phi}(L_1 + L_2) = \mathbf{\Phi}(L_2) \mathbf{\Phi}(L_1) \quad (2.80)$$

The multiplication for multiple sections is performed as shown below:

$$\Phi(L) = \Phi_N(L_N) \times \Phi_{N-1}(L_{N-1}) \times \dots \times \Phi_1(L_1) \quad (2.81)$$

where the order of the sections of the transmission line is from 1 to N . The chain parameters of the whole transmission line are then transformed to impedance parameters or Z parameters as follows:

$$Z_{11} = -\Phi_{21}^{-1}\Phi_{22} = -\Phi_{11}\Phi_{21}^{-1} \quad (2.82)$$

$$Z_{12} = -\Phi_{21}^{-1} \quad (2.83)$$

$$Z_{21} = \Phi_{12} - \Phi_{11}\Phi_{21}^{-1}\Phi_{22} = -\Phi_{21}^{-1} \quad (2.84)$$

$$Z_{22} = -\Phi_{11}\Phi_{21}^{-1} = -\Phi_{21}^{-1}\Phi_{22} \quad (2.85)$$

where the Z parameters are defined in terms of voltages and currents at the near-end and far-end of the line, as follows:

$$\begin{bmatrix} \mathbf{V}(0) \\ \mathbf{V}(L) \end{bmatrix} = \mathbf{Z}(L) \begin{bmatrix} \mathbf{I}(0) \\ -\mathbf{I}(L) \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}(0) \\ -\mathbf{I}(L) \end{bmatrix} \quad (2.86)$$

2.4 Differential and Common-Mode Impedances for Lossless Case

Due to the presence of crossing trace, for evaluating self-capacitances of differential stripline traces, the structure is treated as an asymmetric stripline or as a nonsymmetrical directional coupler [16]. Let the odd and even mode single-ended impedances of the two conductors of the differential stripline (see Figure 2.2) be defined as follows:

Z_{0o}^a - odd-mode impedance of conductor A

Z_{0e}^a - even-mode impedance of conductor A

Z_{0o}^b - odd-mode impedance of conductor B

Z_{0e}^b - even-mode impedance of conductor B

Figure 2.2: Definition of Even and Odd Modes of Asymmetrical Differential Stripline

The odd and even mode single-ended impedances can be calculated from per-unit length capacitances according to [15] for the special case of homogeneous medium using the following equations:

$$Z_1 = \sqrt{\frac{z_1}{y_1}} \quad Z_2 = \sqrt{\frac{z_2}{y_2}} \quad (2.87)$$

$$Z_c = \sqrt{Z_1 Z_2} \sqrt{\frac{1 + y_m / \sqrt{y_1 y_2}}{1 - y_m / \sqrt{y_1 y_2}}} \quad (2.88)$$

$$Z_\pi = \sqrt{Z_1 Z_2} \sqrt{\frac{1 - y_m / \sqrt{y_1 y_2}}{1 + y_m / \sqrt{y_1 y_2}}} \quad (2.89)$$

$$Z_{0e}^a + Z_{0o}^a = \sqrt{\frac{Z_1}{Z_2}} (Z_c + Z_\pi) \quad (2.90)$$

$$Z_{0e}^a - Z_{0o}^a = Z_{0e}^b - Z_{0o}^b = Z_c - Z_\pi \quad (2.91)$$

$$Z_{0e}^b + Z_{0o}^b = \sqrt{\frac{Z_2}{Z_1}} (Z_c + Z_\pi) \quad (2.92)$$

where y_1 , y_2 are the PUL self-admittances, and y_m is the PUL mutual admittance of the two conductors of differential stripline in the presence of the crossing trace and are equal to the capacitance matrix elements C_{11} , C_{22} and $-C_{12}$.

$$y_1 = C_{11} \quad (2.93)$$

$$y_2 = C_{22} \quad (2.94)$$

$$y_m = -C_{12} \quad (2.95)$$

and z_1, z_2 are the PUL self-impedances of the two conductors of differential stripline in the presence of the crossing trace and are equal to the inductances L_{11}, L_{22} .

$$z_1 = L_{11} \quad (2.96)$$

$$z_2 = L_{22} \quad (2.97)$$

where inductances are calculated from capacitances as follows:

$$L_{11} = \mu\epsilon C_{11}^{-1} \quad (2.98)$$

$$L_{22} = \mu\epsilon C_{22}^{-1} \quad (2.99)$$

The differential Z_d and common-mode Z_c impedances can be obtained from the single-ended impedances as follows:

$$Z_d = Z_{0o}^a + Z_{0o}^b \quad (2.100)$$

$$Z_c = Z_{0e}^a \parallel Z_{0e}^b \quad (2.101)$$

An alternative way to calculate differential and common-mode impedances is by first calculating the differential and common-mode capacitances and then calculating the impedances. First, differential and common-mode capacitances are calculated by using series and parallel capacitances rules as follows:

$$C_d = \frac{C_{11} \times C_{22}}{C_{11} + C_{22}} - \frac{C_{12}}{2} \quad (2.102)$$

$$C_c = C_{11} + C_{22} + 2 \times C_{12} \quad (2.103)$$

Next, differential and common-mode inductances are calculated as:

$$L_d = \mu\epsilon C_d^{-1} \quad (2.104)$$

$$L_c = \mu\varepsilon C_c^{-1} \quad (2.105)$$

Subsequently, the differential and common-mode impedances are calculated as

$$Z_d = \sqrt{\frac{L_d}{C_d}} \quad (2.106)$$

$$Z_c = \sqrt{\frac{L_c}{C_c}} \quad (2.107)$$

Chapter 3

Literature Survey: Crossing Junctions with 90° Intersection

3.1 Introduction

Significant research has been previously done on the problem of striplines in the presence of crossing trace [17]-[27]. The previous work can be categorized into the following two groups.

In the first group, the problem is solved as a transmission line problem in the presence of crossing discontinuities [17]-[21]. For example, transient analysis of transmission lines (single-ended and differential) is used to solve the problem in [17] and [18]. In this work, the discontinuities are modeled as infinitely thin conducting strips. Frequency domain analysis is used to solve the problem in [19] and [20] using similar approach as in [17].

In the second group, the problem is formulated from EM equations and then solved using EM numerical methods [22], [23]. The discontinuity is

modeled by using transverse resonance technique in [22]. The discontinuity is characterized by method of lines in [23]. All of the research work in [17]–[23] are concerned with 90° angle crossings between two striplines. Coupling at oblique angles in free space of ideal, infinitely thin transmission lines is studied in [27].

3.2 Modeling of Asymmetric Differential Striplines Including Crossing Junction Discontinuities

The problem of differential stripline in the presence of crossing trace for signal integrity applications was addressed in [7]. In this approach, the entire transmission line consisting of multiple sections with and without the crossing junctions is modeled as follows:

1. An analytical method was developed to enhance the accuracy of per unit length (PUL) parameters [6] of asymmetric stripline (ASL) configurations, including the finite thickness of the conductors. DC- and frequency dependent losses are also appropriately incorporated.
2. An analytical method was proposed to evaluate the PUL parameters of the resulting ASL sections including fringing capacitances for the regions with crossing junctions.
3. A systematic and modular method was developed to combine the sections of ASL blocks constituting the signal path, in the form of cascaded

$ABCD$ matrices.

The main approach to solving the 90° crossing junction problem taken in [7] can be summarized as follows: A formula for even-mode and odd-mode PUL capacitances of ASL represented in Figure 3.1 is derived by decomposing the ASL into two symmetrical striplines shown in Figure 3.2 and combining the existing formulas for symmetrical stripline from [5].

Figure 3.1: Asymmetric Coupled Stripline (ASL) Topology

Next step is to solve the crossing junction topology shown in Figure 3.3 by using the derived equation for ASL and treating the crossing trace as reference plane. With solutions now available for ASL sections with and without crossing junctions, the entire transmission line can be modeled. The solution is enhanced by taking into account the trace thickness, frequency-dependent losses and fringing capacitances. The $ABCD$ or chain parameters are calculated for the sections with and without the crossing traces using two-conductor analysis method described in [8] and then combined into $ABCD$ parameters of the whole structure by multiplication using the property (2.80) of the chain parameters. $ABCD$ parameters of the whole structure are then

Figure 3.2: Modeling of ASL as Two Symmetrical Striplines: (a) Modeling of Top Part of the Traces and (b) Modeling of Bottom Part of the Traces.

Figure 3.3: Cross-section of the ASL in the Presence of Crossing Trace at 90° Angle.

transformed to Z parameters to obtain a complete model of the structure to be used in simulations.

3.3 Analytical Relations for the PUL Parameters of Asymmetrical Stripline Structures

By decomposing the ASL structure into two symmetrical differential striplines as shown in the previous section, the odd and even mode (C_{oe}) per-unit-length capacitances of the original structure can be expressed as follows [6]

$$C_{oe} = 2\varepsilon \left[\frac{K(\phi_{Ioe})}{K(\phi'_{Ioe})} + \frac{K(\phi_{IIoe})}{K(\phi'_{IIoe})} \right] \quad (3.1)$$

where

o - refers to odd mode,

e - refers to even mode,

K - complete elliptic integral of the first kind [5]

The modulus of the elliptic integral ϕ and ϕ' for even and odd modes of propagation can be expressed as follows:

$$\phi_{io} = \tanh \left(\frac{\pi w}{2b_i} \right) \times \coth \left(\frac{\pi(w+s)}{2b_i} \right) \quad (3.2)$$

$$\phi_{ie} = \tanh \left(\frac{\pi w}{2b_i} \right) \times \tanh \left(\frac{\pi(w+s)}{2b_i} \right) \quad (3.3)$$

$$\phi'_{i,oe} = \sqrt{1 - \phi_{i,oe}^2} \quad (3.4)$$

where $i = I, II$ and

w - width of the conductor,

s - spacing between the conductors,

b_i - thickness of the symmetrical striplines as defined in Figure 3.2.

The above expressions for the even and odd mode capacitances assumes infinitely thin conductors. To account for the finite thickness of the conductors, the correction factor is introduced as follows from [7]

$$C'_e = C_e + 2C_q(t) - 2C_q(0) \quad (3.5)$$

$$C'_o = C_o + 2\Delta C_1 + 2C_q(t) - 2C_q(0) \quad (3.6)$$

where

$$\Delta C_1 = \varepsilon \left[\frac{t}{s} + \frac{2}{\pi} \left(1 + \ln \frac{\pi}{8} \right) \right] \quad (3.7)$$

$$C_q(t) = \frac{\varepsilon}{2\pi} \left[\frac{2}{1 - \frac{t}{b}} \ln \left(1 + \frac{1}{1 - \frac{t}{b}} \right) - \left(\frac{1}{1 - \frac{t}{b}} - 1 \right) \ln \left(\frac{1}{(1 - \frac{t}{b})^2} - 1 \right) \right] \quad (3.8)$$

where t is trace thickness and b is thickness of the dielectric $b = h_1 + h_2 + t$.

The even and odd mode inductances are calculated from the corresponding capacitances similar to (2.34) for two-conductor case as

$$L'_{oe} = \mu \varepsilon C'^{-1}_{oe} \quad (3.9)$$

The resistance is calculated as DC and frequency dependent losses according to (2.29) and (2.30). The conductance is calculated similar to (2.37) for two-conductor case as

$$G'_{oe} = \omega \tan \delta C'_{oe} \quad (3.10)$$

Both resistance and conductance are frequency-dependent, thus the accuracy of the model at high frequencies is improved.

3.4 Modeling 90° Crossing Discontinuity

The diagram of the ASL with 90° crossing trace as viewed from the side is shown in Figure 3.4.

Figure 3.4: View of the 90° Crossing Trace From the Side

The per-unit-length capacitance of the crossing trace at the junction C_{pp} can be calculated as parallel combination of the parallel-plate capacitance between the crossing trace and top reference plane C_{pr} and the capacitance between the crossing trace and the signal trace C_{ps} .

$$C_{pp} = C_{pr} + C_{ps} = \varepsilon \left(\frac{w}{b - t - d} + \frac{w}{d} \right) \quad (3.11)$$

The C_{pp} capacitance does not include the fringing capacitance. The total per-unit-length capacitance including the parallel-plate and fringing capacitance can be calculated from [5] as

$$C_t = 2\varepsilon \left(\frac{K(\phi_I)}{K(\phi_I')} + \frac{K(\phi_{II})}{K(\phi_{II}')} \right) \quad (3.12)$$

where

$$\phi_I = \tanh \left(\frac{\pi w}{4(b - d - t/2)} \right) \quad (3.13)$$

$$\phi_{II} = \tanh \left(\frac{\pi w}{4(d + t/2)} \right) \quad (3.14)$$

and $\phi'_i = \sqrt{1 - \phi_i^2}$; with $i = I, II$. The finite thickness of the metal strip can be accounted for using the relations in the section 3.3. Next, the fringing capacitance can be calculated from the total capacitance and parallel-plate capacitance as

$$C_q = C_t - C_{pp} \quad (3.15)$$

Since C_q only accounts for the static capacitance due to fringing fields, the correction factor for the higher order modes can be approximated by adding the modal term as

$$B_q = 2\pi f C_q + A \quad (3.16)$$

where A is function of the wave number and the eigenvalues of the system given in [7].

3.5 Transmission Line Representation as $ABCD$ and Z parameters

The shunt admittance of the crossing trace section can be expressed as

$$Y_{x,oe} = sw_x C_{x,oe} + j2wB_q \quad (3.17)$$

The series impedance of the crossing trace section is

$$Z_{x,oe} = sw_x L_{x,oe} \quad (3.18)$$

where $s = j2\pi f$, f is the frequency, w is the width of the stripline conductor, w_x is the width of the crossing trace and $C_{x,oe}$ and $L_{x,oe}$ are per-unit-length parameters calculated using the equations for the ASL described in section 3.3. The $ABCD$ parameters of the crossing trace junction can be expressed from the admittance and impedance as follows

$$\Psi_{\mathbf{x},oe} = \begin{bmatrix} 1 + Z_{x,oe}Y_{x,oe} & Z_{x,oe} \\ Y_{x,oe} & 1 \end{bmatrix} \quad (3.19)$$

The $ABCD$ parameters of the n th transmission line section without the crossing trace can be obtained from [8] as

$$\Psi_{\mathbf{n},oe} = \begin{bmatrix} \cosh(\gamma_{oe}L_n) & -\sinh(\gamma_{oe}L_n)Z_{c,oe} \\ -\sinh(\gamma_{oe}L_n)Z_{c,oe}^{-1} & \cosh(\gamma_{oe}L_n) \end{bmatrix} \quad (3.20)$$

where

L_n is the length of the n th section,

γ_{oe} is propagation constant for odd and even modes,

$Z_{c,oe}$ is characteristic impedance for odd and even modes

The propagation constant and characteristic impedance can be obtained as follows

$$\gamma_{oe} = \sqrt{(R_{oe} + j\omega L_{oe})(G_{oe} + j\omega C_{oe})} \quad (3.21)$$

$$Z_{c,oe} = \sqrt{\frac{R_{oe} + j\omega L_{oe}}{G_{oe} + j\omega C_{oe}}} \quad (3.22)$$

By multiplying matrices in (3.19) and (3.20), the $ABCD$ parameters of the total transmission line can be computed as follows

$$\Psi_{oe} = \begin{bmatrix} A_{oe} & B_{oe} \\ C_{oe} & D_{oe} \end{bmatrix} = \Psi_{1,oe} \Psi_{\mathbf{x}1,oe} \Psi_{2,oe} \Psi_{\mathbf{x}2,oe} \cdots \Psi_{\mathbf{n},oe} \Psi_{\mathbf{x}n,oe} \quad (3.23)$$

The even and odd $ABCD$ matrices can be transformed into a Z-parameter matrix using equations in [7]. The Z-parameter matrix can be used in signal and power integrity circuit simulators to analyze the performance of the transmission line.

Method in [7] also investigated the effect of discontinuities of crossing junctions at 90° on signal integrity in PCIe Gen III signaling standard using the differential pair topology with and without crossing junctions.

Chapter 4

Proposed Method for Analysis of Differential Transmission Lines in the Presence of Oblique Crossings

In this chapter, an efficient method is presented for calculation of impedance matrix ($\mathbf{Z}$) of the differential trace in the presence of oblique crossings. The method involves combining the analytical and EM methods, where appropriate, to arrive at the final solution.

4.1 Outline of the Proposed Method

For the purpose of illustration, an asymmetrical differential stripline in the presence of an oblique crossing trace (at 45° angle) is shown in Figure 4.1. It

is a 3D structure that is non-uniform in the direction of signal propagation along the Z-axis. The approach taken in this thesis to model this 3D structure is outlined as follows:

1. *Partition the Structure into N_s Non-uniform Sections.* Divide the 3D non-uniform structure into N_s number of non-uniform sections by slicing the 3D structure with uniform spacing in perpendicular direction to the signal propagation as shown in Figure 4.2 (which shows an illustrative case of segments with $N_s = 50$).
2. *Modeling of Each Non-uniform Section.* Approximate each of the resulting non-uniform sections by uniform transmission line sections with three conductors (two conductors of the differential stripline and one conductor for the crossing trace) and two reference planes. An example cross-section of such uniform section is shown in Figure 4.3 (which corresponds to the circled area in Figure 4.2).
3. *Computation of Capacitance Matrix.* Calculate the Capacitance Matrix for each uniform section by using Method of Moments (MoM) numerical method.
4. *Computation of Chain Parameters.* Calculate the Chain Parameters Matrix from Capacitance Matrix for each uniform section.
5. *Concatenation of Chain Parameters of Each Section.* Calculate the Chain Parameters Matrix for the whole transmission line (original structure) by multiplication of individual section Chain Parameter Matrices.

6. *Computation of Impedance Parameters.* Calculate the Impedance Parameters Matrix for the whole transmission line by transforming the Chain Parameters Matrix from previous step into Impedance Parameters.

The above steps are given as a flow chart in Figure 4.4.

Figure 4.1: An Illustrative Differential Stripline with Crossing Trace at 45° Angle

Figure 4.2: Slicing 3D Structure into N_s Sections in XZ-plane and Approximating them with Uniform Sections. Circled (3) represents a sample section of the crossing trace and the corresponding nearest influenced region on the differential stripline is marked by entries (2) and (1).

Figure 4.3: Cross-sectional View of One Sample Section in XY-plane: (1) and (2) Correspond to the Differential Stripline & (3) Represents the Crossing Trace (see Figure 4.2 for additional information).

Figure 4.4: Proposed Solution for Oblique Crossing Problem

4.2 Computation of Capacitance Matrix

Each section of Figure 4.3 consisting of three conductors and two reference planes can be considered as an uniform TEM structure and therefore can be solved using MoM solution to the integral equation for EM fields in that structure. The details of MoM solution for this structure [9] are described in this section.

4.2.1 Solution of Integral Equation

MoM is a method for solving the integral equation of the form:

$$\int_{Z_L}^{Z_U} f(z')G(z, z')dz' = g(z) \quad (4.1)$$

where

$f(z)$ is unknown function

$g(z)$ is known function

$G(z, z')$ is Green's function, and

$Z_L \leq z \leq Z_U$

The expression (4.1) is an integral equation for the unknown function $f(z)$.

In order to find an approximate solution for $f(z)$ we first expand $f(z)$ in terms of some N selected known *expansion* functions $f_n(z)$ as

$$f(z) \approx \sum_{n=1}^N a_n f_n(z) \quad (4.2)$$

The coefficients $a_n, n = 1, 2, \dots, N$ are unknown and MoM is used to find the coefficients in order to find the unknown function. If (4.2) is substituted into

(4.1) the following expression is obtained

$$\int_{Z_L}^{Z_U} \left(\sum_{n=1}^N a_n f_n(z') \right) G(z, z') dz' \approx g(z) \quad (4.3)$$

Next, the summation of the coefficients can be moved out of the integral as

$$\sum_{n=1}^N a_n \left(\int_{Z_L}^{Z_U} f_n(z') G(z, z') dz' \right) \approx g(z) \quad (4.4)$$

which can be written as

$$\sum_{n=1}^N a_n D_n(z) = g(z) \quad (4.5)$$

where

$$D_n(z) = \int_{Z_L}^{Z_U} f_n(z') G(z, z') dz' \quad (4.6)$$

The function $D_n(z)$ can be calculated for any value of z since $f_n(z')$ and $G(z, z')$ are known. Next, we select a set of N *weighting* functions $g_m(z)$, $m = 1, 2, \dots, N$, multiply both sides of (4.5) by each of these weighting functions and take the integral with respect to z as

$$\sum_{n=1}^N a_n \left(\int_{Z_L}^{Z_U} D_n(z) g_m(z) dz \right) = \int_{Z_L}^{Z_U} g(z) g_m(z) dz \quad (4.7)$$

The equation (4.7) can be written in a matrix form as

$$\begin{bmatrix} \mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{V} \end{bmatrix} \quad (4.8)$$

where

$$Z_{mn} = \int_{Z_L}^{Z_U} D_n(z) g_m(z) dz \quad (4.9)$$

$$b_m = \int_{Z_L}^{Z_U} g(z) g_m(z) dz \quad (4.10)$$

The known impedance matrix is

$$[\mathbf{Z}] = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \dots & \dots & \dots & \dots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix} \quad (4.11)$$

The known excitation column vector is

$$[\mathbf{V}] = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix} \quad (4.12)$$

The column vector of unknown coefficients is

$$[\mathbf{I}] = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_N \end{bmatrix} \quad (4.13)$$

The goal of MoM is to find the a_n coefficients given the $\mathbf{Z}$ and $\mathbf{V}$. Therefore, the equation (4.8) can be rewritten as:

$$[\mathbf{I}] = [\mathbf{Z}]^{-1} [\mathbf{V}] \quad (4.14)$$

4.2.2 Segmentation of the Geometry

As a first step in MoM based solution, the surfaces of each conductor and reference plane are segmented into a number of segments. The number of segments can be different for each conductor and reference plane. Larger

number of segments give more accurate results at the expense of the computation time. For each segment, the x, y coordinates for the start x_n^1, y_n^1 and end of segment x_n^2, y_n^2 are stored, where n is the segment number. An example of segmented geometry for one section of the differential stripline in the presence of crossing trace is shown in Figure 4.5. It should be noted that in this example the conductors are assumed to be infinitely thin, segmenting the lines representing their surfaces. However, in case of a conductor with rectangular cross-section, it is possible to segment the contour of the rectangle to improve accuracy. Also, additional conductors can be modelled using this approach.

Let each conductor surface be a contour, denoted by $C_n : C_1, C_2, \dots, C_M$, where M is the number of conductors. In the example uniform structure (of Figure 4.5) there are 5 conductor contours:

- C_1 - top reference conductor with N_1 segments
- C_2 - bottom reference conductor with N_2 segments
- C_3 - left conductor of stripline with N_3 segments
- C_4 - right conductor of stripline with N_4 segments
- C_5 - crossing conductor with N_5 segments

The total number of segments in this case:

$$N = N_1 + N_2 + N_3 + N_4 + N_5$$

Next, define

- ΔC_n to be the portion of the contour on any conductor that is occupied by $n - th$ segment
- ρ to be the vector from the coordinate system origin to a point on the contour of any conductor.

- h_n to be the length of the $n - th$ segment.

Figure 4.5: Segmentation of One Section of the Differential Stripline in the Presence of Crossing Trace

The reference diagram for the segmentation step with two segments is shown in Figure 4.6.

Figure 4.6: Reference Segmentation Diagram with Focus on Two Segments

4.2.3 Selection of the Expansion Functions

In the proposed algorithm, "pulse" expansion functions are selected as:

$$P_n(\boldsymbol{\rho}) = \begin{cases} 1 & \boldsymbol{\rho} \in \Delta C_n \\ 0 & \boldsymbol{\rho} \notin \Delta C_n \end{cases} \quad (4.15)$$

The unknown function is surface charge density and can be expressed using (4.2) as

$$\sigma(\boldsymbol{\rho}) = \sum_{n=1}^N a_n P_n(\boldsymbol{\rho}) \quad (4.16)$$

4.2.4 Selection of the Weighting Functions

For this purpose, delta functions are selected as weighting functions:

$$g_m(\boldsymbol{\rho}) = \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_m) \quad (4.17)$$

4.2.5 Evaluation of the Excitation Matrix Terms

The excitation matrix elements are expressed by (4.10). With the selected weighting functions, this expression evaluates to

$$V_m = \int_{C_{eq}} V(\boldsymbol{\rho}) g_m(\boldsymbol{\rho}) dC = \int_{C_{eq}} V(\boldsymbol{\rho}) \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_m) dC = V(\boldsymbol{\rho}_m) = \begin{cases} V_1 & \boldsymbol{\rho}_m \in C_1 \\ V_2 & \boldsymbol{\rho}_m \in C_2 \\ \dots & \dots \\ V_M & \boldsymbol{\rho}_m \in C_M \end{cases} \quad (4.18)$$

where M is the number of conductors.

4.2.6 Evaluation of the Impedance Matrix Terms

The impedance matrix $\mathbf{Z}$ is evaluated using the expression (4.9) as [9]:

$$Z_{mn} = -\frac{h_n}{4\pi\varepsilon} \int_0^1 \ln \left(\left\{ x_m - x_n^{(1)} - \xi(x_n^{(2)} - x_n^{(1)}) \right\}^2 + \left\{ y_m - y_n^{(1)} - \xi(y_n^{(2)} - y_n^{(1)}) \right\}^2 \right) d\xi \quad (4.19)$$

where x_m and y_m are the coordinates of the middle of the segment m :

$$x_m = (x_m^{(1)} + x_m^{(2)}) / 2 \quad (4.20)$$

$$y_m = (y_m^{(1)} + y_m^{(2)}) / 2 \quad (4.21)$$

The diagonal elements of the $\mathbf{Z}$ matrix are calculated as follows:

$$Z_{nn} = -\frac{h_n}{2\pi\varepsilon} \left(\ln \frac{h_n}{2} - 1 \right) \quad (4.22)$$

Since all the segments in the structure of interest are horizontal (see Figure 4.5), the following relations can be simplified as

$$x_n^{(2)} - x_n^{(1)} = h_n \quad (4.23)$$

$$y_n^{(2)} - y_n^{(1)} = 0 \quad (4.24)$$

$$y_m = y_m^{(1)} = y_m^{(2)} \quad (4.25)$$

With these simplifications, the equation (4.19) can be written as

$$Z_{mn} = -\frac{h_n}{4\pi\varepsilon} \int_0^1 \ln \left(\left\{ x_m - x_n^{(1)} - \xi h_n \right\}^2 + \left\{ y_m^{(1)} - y_n^{(1)} \right\}^2 \right) d\xi \quad (4.26)$$

4.2.7 Evaluation of the PUL Charge

Once the a_n coefficients are known from the equation (4.14), the charge per unit length for each conductor can be calculated with substitution from (4.16) as follows:

$$q_i = \int_{C_n} \sigma(\boldsymbol{\rho}) dC = \int_{C_n} \left(\sum_{n=1}^{N_i} a_n P_n(\boldsymbol{\rho}) \right) dC = \sum_{n=1}^{N_i} a_n h_n = a_1 h_1 + a_2 h_2 + \dots + a_{N_i} h_{N_i} \quad (4.27)$$

where N_i is the number of segments on conductor i .

4.2.8 Evaluation of the PUL Capacitance

With the charge per unit length known using (4.27), elements of the capacitance matrix can be calculated using the equation (2.16), as follows:

$$q_1 = C_{11}V_1 + C_{12}V_2 + \dots + C_{1N}V_N \quad (4.28)$$

$$q_2 = C_{21}V_1 + C_{22}V_2 + \dots + C_{2N}V_N \quad (4.29)$$

$$q_n = C_{N1}V_1 + C_{N2}V_2 + \dots + C_{NN}V_N \quad (4.30)$$

where C_{ij} are elements of the capacitance matrix $\mathbf{C}$. To calculate the capacitance matrix elements, we set the voltage on conductor j to $V_j = 1V$ and on the other conductors and reference planes to $0V$ to define the excitation matrix $\mathbf{V}$. Subsequently, $\mathbf{Z}$ matrix is calculated and the unknown a_n coefficients are found to calculate the charge per unit length q_i as described in the previous sections. Next, the elements of the capacitance matrix can be calculated as follows:

$$C_{ij} = \frac{q_i}{V_j} = q_i; V_j = 1; V_{k \neq j} = 0 \quad (4.31)$$

where i, j, k belong to symbol 1, 2, 3.

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \quad (4.32)$$

Chapter 5

Numerical Results

This chapter presents the validating numerical results for the proposed method for efficient analysis of asymmetrical differential striplines in the presence of oblique crossing. The results are organized to provide the validation for each step of the algorithm. First, a differential stripline with practical dimensions in the presence of crossing trace at different angles is described and is divided into sections along the Z-axis. The diagrams of the sectionalized transmission lines with crossing traces are provided in Section 5.1. The capacitance matrix elements are calculated using the MoM and are plotted along the Z-axis while comparing with Ansys Maxwell 2D tool in Section 5.1.1. Section 5.1.2 provides the differential and common-mode impedances for lossless case along the Z-axis for crossing traces at different angles. Section 5.1.3 provides the comparison of Z parameters for four different experiments using the proposed method and Ansys SI Wave tool.

Section 5.2 presents an investigation of impact of crossing junctions on signal integrity performance. Results are provided in terms of eye diagrams

in the presence of crossings at 45° , 90° and no crossings.

5.1 Example 1

For the experiments in Section 5.1, differential stripline in the presence of crossing trace is considered with the cross-sectional setup in XY-plane as shown in Figure 5.1. The stripline dimensions were chosen to be the same as in [6] and [7] which represent the typical dimensions of traces on PCBs. The length of the crossing trace segment for this experiment is 200 mils, the width is 8 mils and the thickness is 0.7 mils. The selected number of uniform sections is $N_s = 50$ to give reasonable accuracy and computational time. The crossing trace angles of 0° , 15° , 30° , 45° , 60° , 75° and 90° were considered in the investigation. The plots of the sectionalized transmission line with crossing trace at these angles are shown in Figures 5.2 - 5.3.

Figure 5.1: Cross-sectional View of One Sample Section in XY-plane: (1) and (2) Correspond to the Differential Stripline & (3) Represents the Crossing Trace

Figure 5.2: Illustration of Differential Stripline with Crossing Trace at Different Angles (0° , 15° , 30° , 45°) and its Segmentation into $N_s = 50$ Sections

Figure 5.3: Illustration of Differential Stripline with Crossing Trace at Different Angles (60° , 75° , 90°) and its Segmentation into $N_s = 50$ Sections

5.1.1 Capacitance Matrix Elements at Different Angles

The capacitance matrix along the Z-axis is computed using the proposed method. The plots of different capacitance matrix elements along the Z-axis for the differential ASL in the presence of crossing trace at different angles are shown in Figures 5.4 - 5.9 (when using $N_s = 50$ number of sections). Also, an error comparison of the capacitance matrix elements using the proposed method with the Ansys Maxwell 2D is made and the results are given in Tables 5.1 - 5.6.

In these tables, results are organized with respect to varying sample angles as given in Column 1. For each angle, capacitances are measured with respect to different (sample) section position numbers (which are given in Column 2), and the corresponding capacitance values using the proposed method are reported in Column 3. For example, for crossing angle 15° (please refer to Figure 5.2(b)), the sections are numbered from left-bottom (Section #1) to top-right (Section #50). Similar approach is taken while considering other angles and also other tables. In column 4 capacitance values obtained using Ansys Maxwell 2D are reported. In column 5 the percent error between the two methods is given. The results show reasonably good matching between the proposed approach and the Ansys Maxwell 2D.

Figure 5.4: Variation (of C_{11} Capacitance Matrix Element) for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.1: Comparison of Capacitance Matrix Element C_{11}

Trace Angle	Section Position Number	C_{11} (pF)		% Error
		Proposed	Maxwell 2D	
0 °	1	123.27	124.72	1.17
15 °	1	117.20	118.54	1.13
15 °	20	129.86	131.46	1.21
15 °	32	117.69	119.05	1.14
15 °	50	117.08	118.42	1.13
45 °	1	117.08	118.57	1.26
45 °	20	122.43	124.10	1.34
45 °	32	117.09	118.58	1.26
45 °	50	117.08	118.57	1.26
75 °	1	117.08	118.76	1.41
75 °	20	128.19	130.26	1.59
75 °	32	117.10	118.78	1.42
75 °	50	117.08	118.76	1.41
90 °	1	131.49	133.56	1.55

Figure 5.5: Variation (of C_{22} Capacitance Matrix Element) for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.2: Comparison of Capacitance Matrix Element C_{22}

Trace Angle	Section Position Number	C_{22} (pF)		% Error
		Proposed	Maxwell 2D	
0 °	1	123.27	124.72	1.17
15 °	1	117.08	118.42	1.13
15 °	20	118.05	119.41	1.14
15 °	32	130.01	131.61	1.21
15 °	50	117.20	118.54	1.13
45 °	1	117.08	118.57	1.26
45 °	20	117.10	118.58	1.25
45 °	32	119.47	121.04	1.30
45 °	50	117.08	118.57	1.26
75 °	1	117.08	118.71	1.37
75 °	20	117.21	118.84	1.38
75 °	32	122.71	124.62	1.53
75 °	50	117.08	118.71	1.37
90 °	1	131.49	133.48	1.49

Figure 5.6: Variation (of C_{33} Capacitance Matrix Element) for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.3: Comparison of Capacitance Matrix Element C_{33}

Trace Angle	Section Position Number	C_{33} (pF)		% Error
		Proposed	Maxwell 2D	
0 °	1	163.56	164.98	0.86
15 °	1	152.17	153.30	0.74
15 °	20	169.80	171.26	0.85
15 °	32	169.62	171.08	0.85
15 °	50	152.17	153.30	0.74
45 °	1	183.65	185.29	0.89
45 °	20	190.11	191.79	0.87
45 °	32	186.43	188.05	0.86
45 °	50	183.65	185.33	0.91
75 °	1	281.28	283.78	0.88
75 °	20	401.27	404.77	0.87
75 °	32	392.24	395.56	0.84
75 °	50	281.28	283.86	0.91
90 °	1	2180.91	2195.60	0.67

Figure 5.7: Variation (of C_{13} Capacitance Matrix Element) for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.4: Comparison of Capacitance Matrix Element C_{13}

Trace Angle	Section Position Number	C_{13} (pF)		% Error
		Proposed	Maxwell 2D	
0 °	1	-2.79E+01	-2.84E+01	1.60
15 °	1	-3.54E+00	-3.63E+00	2.37
15 °	20	-4.59E+01	-4.66E+01	1.41
15 °	32	-7.82E+00	-7.96E+00	1.83
15 °	50	-1.03E-01	-1.05E-01	1.82
45 °	1	-1.07E-03	-1.11E-03	4.19
45 °	20	-2.64E+01	-2.70E+01	2.09
45 °	32	-6.14E-01	-6.27E-01	2.00
45 °	50	-2.90E-05	-3.03E-05	4.21
75 °	1	-5.49E-05	-5.73E-05	4.12
75 °	20	-4.40E+01	-4.50E+01	2.25
75 °	32	-1.28E+00	-1.32E+00	3.24
75 °	50	-1.50E-06	-1.55E-06	3.35
90 °	1	-6.14E+01	-6.24E+01	1.54

Figure 5.8: Variation (of C_{23} Capacitance Matrix Element) for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.5: Comparison of Capacitance Matrix Element C_{23}

Trace Angle	Section Position Number	C_{23} (pF)		% Error
		Proposed	Maxwell 2D	
0 °	1	-2.79E+01	-2.84E+01	1.58
15 °	1	-1.03E-01	-1.06E-01	3.46
15 °	20	-9.92E+00	-1.01E+01	1.80
15 °	32	-4.64E+01	-4.70E+01	1.42
15 °	50	-3.54E+00	-3.63E+00	2.37
45 °	1	-2.90E-05	-3.03E-05	4.20
45 °	20	-1.21E+00	-1.23E+00	2.05
45 °	32	-1.68E+01	-1.72E+01	2.29
45 °	50	-1.07E-03	-1.11E-03	4.19
75 °	1	-1.50E-06	-1.55E-06	3.19
75 °	20	-3.48E+00	-3.60E+00	3.27
75 °	32	-2.77E+01	-2.85E+01	2.85
75 °	50	-5.49E-05	-5.73E-05	4.11
90 °	1	-6.14E+01	-6.23E+01	1.48

Figure 5.9: Variation (of C_{12} Capacitance Matrix Element) for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.6: Comparison of Capacitance Matrix Element C_{12}

Trace Angle	Section Position Number	C_{12} (pF)		% Error
		Proposed	Maxwell 2D	
0 °	1	-4.986	-5.150	3.18
15 °	1	-8.357	-8.599	2.81
15 °	20	-5.948	-6.133	3.01
15 °	32	-6.285	-6.478	2.97
15 °	50	-8.357	-8.599	2.81
45 °	1	-8.360	-8.633	3.16
45 °	20	-8.085	-8.348	3.15
45 °	32	-8.266	-8.536	3.17
45 °	50	-8.360	-8.633	3.16
75 °	1	-8.360	-8.658	3.44
75 °	20	-7.332	-7.586	3.34
75 °	32	-8.064	-8.348	3.40
75 °	50	-8.360	-8.658	3.44
90 °	1	-4.618	-4.828	4.34

5.1.2 Differential and Common-Mode Impedances for Lossless Case at Different Angles

Using the capacitance matrices computed in the previous section, differential and common-mode impedances are computed. The plots for the differential and common-mode impedances along the Z-axis for the transmission line with the crossing trace at different angles are shown in Figure 5.10 and 5.11, respectively. Also, an error comparison for the differential and common-mode impedances between the proposed method and the Ansys Maxwell 2D are shown in Tables 5.7 and 5.8, respectively. Results in these tables are organized in a similar way as that in Table 5.1 (as described in Section 5.1.1). As seen from the tables, a reasonably good matching between both methods is observed.

Figure 5.10: Comparative Study of Variation of Differential Impedance Z_d for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.7: Comparison of Differential Impedance Z_d

Trace Angle	Section Position Number	Z_d (Ohm)		% Error
		Proposed	Maxwell 2D	
0 °	1	109.04	107.68	1.26
15 °	1	111.43	110.04	1.26
15 °	20	107.88	106.52	1.28
15 °	32	107.71	106.35	1.28
15 °	50	111.43	110.04	1.26
45 °	1	111.48	109.94	1.40
45 °	20	109.43	107.88	1.44
45 °	32	110.52	108.97	1.42
45 °	50	111.48	109.94	1.40
75 °	1	111.48	109.77	1.56
75 °	20	107.75	106.04	1.61
75 °	32	109.33	107.59	1.62
75 °	50	111.48	109.77	1.56
90 °	1	102.74	101.08	1.64

Figure 5.11: Comparative Study of Variation of Common-Mode Impedance Z_c for Different Crossing Trace Angles (with $N_s = 50$)

Table 5.8: Comparison of Common-Mode Impedance Z_c

Trace Angle	Section Position Number	Z_c (Ohm)		% Error
		Proposed	Maxwell 2D	
0 °	1	29.56	29.24	1.09
15 °	1	32.14	31.82	1.01
15 °	20	29.63	29.30	1.10
15 °	32	29.74	29.41	1.09
15 °	50	32.14	31.82	1.01
45 °	1	32.16	31.80	1.12
45 °	20	31.30	30.94	1.17
45 °	32	31.78	31.42	1.15
45 °	50	32.16	31.80	1.12
75 °	1	32.16	31.76	1.25
75 °	20	30.30	29.89	1.39
75 °	32	31.26	30.84	1.35
75 °	50	32.16	31.76	1.25
90 °	1	27.56	27.17	1.43

5.1.3 Computation of Z Parameters

In this section, the ASL cross-section of Figure 5.1 is used in four different experimental setups. In these experiments, using the capacitance matrices computed in section 5.1.1, Z parameters are computed as described in Section 2.3 and compared with the Ansys SI Wave tool.

Experiment 1: ASL with One Crossing Junction at 45° .

In this experiment, the proposed model for ASL is investigated in the presence of one crossing junction at 45° segmented with $N_s = 50$. The setup for this experiment is shown in Figure 5.12. Z parameters for the differential stripline in the presence of one 45° crossing are computed and compared with the Ansys SI Wave. The plots of computed Z_{11} and Z_{12} parameters are compared with the Ansys SI Wave in Figures 5.13 and 5.14, respectively. As can be seen, the results from both methods match reasonably well. Similar results with comparable accuracy were observed for other Z -parameter matrix elements such as Z_{13} , Z_{14} , and Z_{21} , etc.

Figure 5.12: Experimental Setup with ASL with 1 Trace Crossing at 45° Angle (Experiment 1)

Figure 5.13: Comparison of Z_{11} Parameter of the ASL with 1 Trace Crossing at 45° Angle (Experiment 1)

Figure 5.14: Comparison of Z_{12} Parameter of the ASL with 1 Trace Crossing at 45° Angle (Experiment 1)

Experiment 2: ASL with 51 Crossing Junctions at 45°.

In this experiment, the proposed model for ASL is investigated in the presence of 51 equally-spaced crossing junctions at 45°. The setup for this experiment is shown in Figure 5.15. The Z parameters for the transmission line with 51 crossings were obtained by multiplying the chain parameters for the transmission line with one crossing junction from the previous experiment 50 times and transforming the resulting chain parameters to Z parameters. The Z_{11} and Z_{12} parameters using the proposed method are compared with the Ansys SI Wave in Figures 5.16 and 5.17, respectively. As can be seen from the comparison plots, the computed and simulated plots are in good agreement, further validating the proposed model.

Figure 5.15: Experimental Setup with ASL with 51 Trace Crossings at 45° Angle (Experiment 2)

Figure 5.16: Comparison of Z_{11} Parameter of the ASL with 51 Trace Crossings at 45° Angle (Experiment 2)

Figure 5.17: Comparison of Z_{12} Parameter of the ASL with 51 Trace Crossings at 45° Angle (Experiment 2)

Experiment 3: ASL with 51 Crossing Junctions at 90°.

For this experiment, the differential stripline transmission line with total length of 7.5" with 51 equally-spaced crossing traces at 90° angle was considered. The setup for this experiment is shown in Figure 5.18. The Z parameters for the differential stripline in the presence of 51 crossings at 90° are computed and compared with the Ansys SI Wave. The Z_{11} and Z_{12} parameters using the proposed method are compared with the Ansys SI Wave in Figures 5.19 and 5.20, respectively. As can be seen from the comparison plots, the results from both methods match reasonably well.

Figure 5.18: Experimental Setup with ASL with 51 Trace Crossings at 90° Angle (Experiment 3)

Figure 5.19: Comparison of Z_{11} Parameter of the ASL with 51 Trace Crossings at 90° Angle (Experiment 3)

Figure 5.20: Comparison of Z_{12} Parameter of the ASL with 51 Trace Crossings at 90° Angle (Experiment 3)

Experiment 4. ASL with 45°, 90°, and no crossings.

In this experiment, a relatively large and mixed configuration is considered. The hybrid structure consists of ASLs with no crossing traces, with 45° crossing traces and also with 90° crossing traces. The setup for this experiment is shown in Figure 5.21. Figures 5.22 - 5.23 show the comparison of Z parameters for 1 crossing trace. The figures show that the difference between the Z parameters is negligible, so the effect of 1 crossing trace on Z parameters is very small. Figures 5.24 - 5.25 show the comparison of Z parameters for transmission line with length of 7.5" with 51 equally-spaced crossing traces. Figures 5.26 - 5.27 show the comparison of Z parameters for the transmission line with length of 7.5" with 102 equally-spaced crossing traces. The plots indicate that at higher frequencies, Z parameters for the case with large number of crossings differ significantly from those with no crossings or less number of crossings.

Figure 5.21: Experimental Setup for Comparison of ASLs with No Crossings, with 45° Crossings and 90° Crossings (Experiment 4)

Figure 5.22: Comparison of Z_{11} Parameter of the ASL with 1 Trace Crossing (Experiment 4)

Figure 5.23: Comparison of Z_{12} Parameter of the ASL with 1 Trace Crossing (Experiment 4)

Figure 5.24: Comparison of Z_{11} Parameter of the ASL with 51 Trace Crossings (Experiment 4)

Figure 5.25: Comparison of Z_{12} Parameter of the ASL with 51 Trace Crossings (Experiment 4)

Figure 5.26: Comparison of Z_{11} Parameter of the ASL with 102 Trace Crossings (Experiment 4)

Figure 5.27: Comparison of Z_{12} Parameter of the ASL with 102 Trace Crossings (Experiment 4)

5.2 An Investigation of Impact of Crossing Junctions at Various Angles on Signal Integrity Performance

In the following experiments, the effect of discontinuities of crossing junctions on signal integrity is demonstrated for the PCIe serial link at 16 Gbps for differential stripline with 102 equally-spaced crossing traces at 45° , 90° and no crossings with an ASL of length 7.5" (as described in Figure 5.21). Four experiments were performed based on different ASL cross-section dimensions defined in Figure 5.28 and given in Table 5.9 as will become evident in the results.

Figure 5.28: ASL Cross-section Dimension Definitions

Table 5.9: Cross-section Dimensions (in mils) for the Experiments 5-8

Parameter	h1	h2	h3	w	s	t
Experiment 5	5	5	5	5	8	0.7
Experiment 6	10	5	10	5	8	0.7
Experiment 7	3	3	3	5	8	0.7
Experiment 8	3	3	3	5	4	0.7

Experiment 5: $h1=h2=h3=5\text{mils}$, $s=8\text{mils}$.

The resulting eye diagrams for this experiment are shown in Figures 5.29 – 5.31. The eye diagram measurements are shown in Table 5.10. For this example, results for the case of 45° provide degradation in eye height and width compared to the case of no crossings or 90° crossings. Also, the jitter in the case of 45° crossings is higher compared to the case of no crossings or 90° crossings.

Table 5.10: Eye Diagram Measurements

Parameter	No Crossings	90° Crossings	45° Crossings
Eye Height, mV	245.87	244.18	230.14
Eye Width, ps	44.88	44.13	43.5
Jitter, p2p, ps	18.25	19.13	19.75

Figure 5.29: Eye Diagram for 7.5'' ASL with No Crossings (Experiment 5)

Figure 5.30: Eye Diagram for 7.5'' ASL with 102 Crossings at 90° angle (Experiment 5)

Figure 5.31: Eye Diagram for 7.5'' ASL with 102 Crossings at 45° angle (Experiment 5)

Experiment 6: $h1=h3=10\text{mils}$, $h2=5\text{mils}$, $s=8\text{mils}$ (increased distance between the reference planes and the signal conductors).

The eye diagram measurements for this experiment are shown in Table 5.11. As can be seen from the table, in this experiment too, the case of 45° crossing performed inferiorly compared to both other cases.

Table 5.11: Eye Diagram Measurements

Parameter	No Crossings	90° Crossings	45° Crossings
Eye Height, mV	266.79	241.17	219.22
Eye Width, ps	47.13	45.0	43.75
Jitter, p2p, ps	16.13	18.25	19.63

Experiment 7: $h1=h2=h3=3\text{mils}$, $s=8\text{mils}$ (decreased distance between the reference planes and the signal conductors as well as decreased distance between the crossing conductor and the differential stripline).

The eye diagram measurements for this experiment are shown in Table 5.12. In this example too, results show a significant degradation in the eye diagram for the cases with crossings compared to the case without crossings. In this experiment too, the case with 45° oblique crossing performed poorly compared to both other cases. Such an information needs to be taken into account in the timing budget of high-speed designs while designing the layouts.

Table 5.12: Eye Diagram Measurements

Parameter	No Crossings	90° Crossings	45° Crossings
Eye Height, mV	195.6	190.74	166.92
Eye Width, ps	36.13	37.13	34
Jitter, p2p, ps	27.25	26.38	29.5

Experiment 8: $h1=h2=h3=3\text{mils}$, $s=4\text{mils}$ (decreased distance between reference planes and signal conductors, decreased distance between the crossing conductor and the differential stripline as well as decreased distance between the conductors of differential stripline).

The resulting eye diagrams for this experiment are shown in Figures 5.32 – 5.34. The eye diagram measurements are shown in Table 5.13. In this example too, results show a significant degradation in the eye diagram for the cases with crossings compared to the case without crossings. In this experiment, the case with 90° oblique crossing performed poorly compared to both other cases. Such an information needs to be taken into account in the timing budget of high-speed designs while designing the layouts.

Table 5.13: Eye Diagram Measurements

Parameter	No Crossings	90° Crossings	45° Crossings
Eye Height, mV	178.65	100.74	162.66
Eye Width, ps	33.63	29.5	31.7
Jitter, p2p, ps	30.0	34.13	31.75

Figure 5.32: Eye Diagram for 7.5'' ASL with No Crossings (Experiment 8)

Figure 5.33: Eye Diagram for 7.5'' ASL with 102 Crossings at 90° angle (Experiment 8)

Figure 5.34: Eye Diagram for 7.5'' ASL with 102 Crossings at 45° angle (Experiment 8)

Chapter 6

Conclusion

In this thesis, modeling of differential stripline with crossing trace junctions has been advanced. Particularly,

- (a) an efficient model for differential asymmetrical striplines with oblique crossing junctions has been developed.
- (b) effect of the crossing trace angle on the signal integrity metrics has been investigated.

The results of the analysis were compared to the commercial EM and signal integrity tools and showed good matching, validating the proposed solutions. A study of the effect of crossing traces on the performance of the differential stripline transmission line was conducted. It was shown for some ASL cross-sections the effect of crossing traces on the eye diagram is significant and should be taken into account when designing high-speed differential transmission lines. This is particularly the case when the distance between the crossing traces and the differential stripline is reduced or when the distance

between the signal traces in differential stripline is reduced.

Future Work: Future research in this area can be done based on the work in this thesis to develop:

- (a) An alternative efficient approach based on Schwarz-Christoffel transformation for modeling of ASL with oblique crossings.
- (b) Sensitivity analysis of high-speed systems with ASLs having oblique crossings.
- (c) Use of the proposed model for optimization of signal integrity metrics.

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Appendix A

Matlab Code for Calculation of Per-Unit-Length Capacitance Matrices

```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 16, 2020
% Description: Oblique Crossing of Dual Stripline
% Author: Peter Bliznyuk-Kvitko

clear all;
close all;
clc;

% Define the structure parameters
w = 5; % width of dual stripline conductor in mils
s = 8; % Spacing between dual stripline conductors in mils
wc = 8; % Width of crossing conductor in mils
L = 200; % Length of crossing conductor in mils
N = 50; % Number of Z direction segments

% figure number
fnum = 1;

% Output file for the C matrix elements
fileID = fopen('output_Ccoeff.txt','w');

% Generate the C matrix elements and plots for angles 0, 15, 30, 45, 60,
% 75 and 90 degrees between crossing conductor and differential stripline
for ang=0:15:90

    fprintf("Solving MoM for %d deg angle ...\n", ang);

    a = degtorad(ang); % Convert the angle from degrees to radians

    % Calculate start coordinates for crossing conductor
    xc1_start = - 0.5*(L*sin(a) + wc/cos(a));
    yc1_start = - 0.5*(L*cos(a) + wc*sin(a));

    xc2_start = - 0.5*(L*sin(a) - wc/cos(a));
    yc2_start = - 0.5*(L*cos(a) + wc*sin(a));

    % Calculate end coordinates for crossing conductor
    xc1_end = 0.5*(L*sin(a) - wc/cos(a));
    yc1_end = 0.5*(L*cos(a) + wc*sin(a));

    xc2_end = 0.5*(L*sin(a) + wc/cos(a));
    yc2_end = 0.5*(L*cos(a) + wc*sin(a));

    w2d = xc2_start - xc1_start;

    if (xc2_end > L/2)
        m = tan(pi/2 - a);
        b = 0.5*wc/sin(a);

        xc1_new = -L/2;

```

```

    yc1_new = -m*L/2 + b;
    xc1_mid = -L/2;
    yc1_mid = yc1_start;

    xc2_new = L/2;
    yc2_new = m*L/2 - b;
    xc2_mid = L/2;
    yc2_mid = yc1_end;
end

% Plot dual stripline and crossing conductors
figure(fnum);

plot([-s/2 -s/2], [yc2_start yc1_end]);
hold on;
plot([-s/2-w -s/2-w], [yc2_start yc1_end]);
hold on;
plot([-s/2 -s/2-w], [yc2_start yc2_start]);
hold on;
plot([-s/2 -s/2-w], [yc1_end yc1_end]);

plot([s/2 s/2], [yc2_start yc1_end]);
hold on;
plot([s/2+w s/2+w], [yc2_start yc1_end]);
hold on;
plot([s/2 s/2+w], [yc2_start yc2_start]);
hold on;
plot([s/2 s/2+w], [yc1_end yc1_end]);

if (xc2_end < L/2)
    plot([xc1_start xc2_start], [yc1_start yc1_start]);
    hold on;
    plot([xc1_start xc1_end], [yc1_start yc1_end]);
    hold on;
    plot([xc1_end xc2_end], [yc1_end yc1_end]);
    hold on;
    plot([xc2_start xc2_end], [yc2_start yc2_end]);
    hold on;
end

% Segmentation

% Calculate segment length in Y and X directions
seg_y = 2*yc1_end / N;
seg_x = (abs(xc1_start) + xc1_end) / N;

% Plot segments of crossing conductor
for k=1:N
    if ((xc1_start+seg_x/2+(k-1)*seg_x) > -L/2)
        left = xc1_start+seg_x/2+(k-1)*seg_x;
    else
        left = -L/2;

```

```

end

if ((xc2_start+seg_x/2+(k-1)*seg_x) < L/2)
    right = xc2_start+seg_x/2+(k-1)*seg_x;
else
    right = L/2;
end

x(1) = left;
x(2) = left;
x(3) = right;
x(4) = right;
x(5) = left;

y(1) = yc1_start+(k-1)*seg_y;
y(2) = yc1_start+k*seg_y;
y(3) = yc1_start+k*seg_y;
y(4) = yc1_start+(k-1)*seg_y;
y(5) = yc1_start+(k-1)*seg_y;

plot(x, y);
end

str = sprintf('Crossing Segmentation, %d deg', ang);
title(str);
xlabel('x, mils');
ylabel('z, mils');
xlim([-L/2 L/2]);
ylim([-L/2 L/2]);

% Plot 2D geometry for each segment of crossing conductor
for k=1:N
    if ((xc1_start+seg_x/2+(k-1)*seg_x) > -L/2)
        left = xc1_start+seg_x/2+(k-1)*seg_x;
    else
        left = -L/2;
    end

    if ((xc2_start+seg_x/2+(k-1)*seg_x) < L/2)
        right = xc2_start+seg_x/2+(k-1)*seg_x;
    else
        right = L/2;
    end

    c2le = left;

    % Calculate the C matrix for this 2D geometry using MOM
    Cm = cross2D(w*2.54e-5, s*2.54e-5, (right-left)*2.54e-5, ...
        c2le*2.54e-5, 2*L*2.54e-5, k);

    c12(k) = Cm(1,2);
    c13(k) = Cm(1,3);
    c23(k) = Cm(2,3);

```



```

    c11(k) = Cm(1,1);
    c22(k) = Cm(2,2);

    % Output C matrix data to output file
    fprintf(fileID, '%d %d %f %f %f\n', ang, k, (right-left), ...
        1500 + left, seg_y);
    fprintf(fileID, '%f %f %f\n', Cm(1,1), Cm(1,2), Cm(1,3));
    fprintf(fileID, '%f %f %f\n', Cm(2,1), Cm(2,2), Cm(2,3));
    fprintf(fileID, '%f %f %f\n', Cm(3,1), Cm(3,2), Cm(3,3));
end

m=1:N;
m = (m-1)*seg_y+seg_y/2 + (L/2 - seg_y*25);

% Plot C matrix elements versus the position along Z axis
figure(100);
plot(m, c12);
hold on;

figure(101);
plot(m, c13);
hold on;

figure(102);
plot(m, c23);
hold on;

figure(103);
plot(m, c11);
hold on;

figure(104);
plot(m, c22);
hold on;

fnum = fnum + 1;
end

% close output file
fclose(fileID);

% Add title, labels and legend to the C matrix elements plots
figure(100);
title('C12 from Capacitor Matrix');
xlabel('Z, mils');
ylabel('C12, pF/m');
legend('0', '15', '30', '45', '60', '75', '90');

figure(101);
title('C13 from Capacitor Matrix');
xlabel('Z, mils');
ylabel('C13, pF/m');
legend('0', '15', '30', '45', '60', '75', '90');

```

```

figure(102);
title('C23 from Capacitor Matrix');
xlabel('Z, mils');
ylabel('C23, pF/m');
legend('0', '15', '30', '45', '60', '75', '90');

figure(103);
title('C11 from Capacitor Matrix');
xlabel('Z, mils');
ylabel('C11, pF/m');
legend('0', '15', '30', '45', '60', '75', '90');

figure(104);
title('C22 from Capacitor Matrix');
xlabel('Z, mils');
ylabel('C22, pF/m');
legend('0', '15', '30', '45', '60', '75', '90');

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 16, 2020
% Description: Method of Moments
% Author: Peter Bliznyuk-Kvitko

% Function cross2D
% Input parameters
%   w - width of dual stripline conductor in m
%   s - Spacing between dual stripline conductors in m
%   wc - Width of the crossing conductor in XY plane in m
%   c2le - Center to left edge of the crossing conductor on X axis
%          in m
%   L - Length of the reference conductor in m
%   k - Slice number
% Output parameter:
%   Cmatrix - Capacitance Matrix

function Cmatrix=cross2D(w, s, wc, c2le, L, k);

t = 0.7 * 2.54e-5; % thickness of dual stripline conductor
h1 = 5 * 2.54e-5; % height between bottom to dual stripline conductors
h2 = 5 * 2.54e-5; % height between dual stripline and crossing conductors
h3 = 5 * 2.54e-5; % height between crossing and top conductors
tc = 0.7 * 2.54e-5; % thickness of crossing conductor

N1 = 100; % number of segments for top conductor
N2 = 100; % number of segments for bottom conductor
N3 = 30; % number of segments for left conductor of dual stripline
N4 = 30; % number of segments for right conductor of dual stripline
N5 = 100; % number of segments for crossing conductor

% Top conductor segment coordinates
for q=1:N1
    xt1(q) = (q-1) * L/N1;
    zt1(q) = h1 + t + h2 + tc + h3;

    xt2(q) = q * L/N1;
    zt2(q) = h1 + t + h2 + tc + h3;
end

% Bottom conductor segment coordinates
for q=1:N2
    xb1(q) = (q-1) * L/N2;
    zb1(q) = 0;

    xb2(q) = q * L/N2;
    zb2(q) = 0;
end

% Left conductor of dual stripline segment coordinates
for q=1:N3

```

```

    xp1(q) = L/2 - s/2 - w + (q-1)*w/N3;
    zp1(q) = h1 + t/2;

    xp2(q) = L/2 - s/2 - w + q*w/N3;
    zp2(q) = h1 + t/2;
end

% Right conductor of dual stripline segment coordinates
for q=1:N4
    xn1(q) = L/2 + s/2 + (q-1)*w/N4;
    zn1(q) = h1 + t/2;

    xn2(q) = L/2 + s/2 + q*w/N4;
    zn2(q) = h1 + t/2;
end

% Crossing conductor segment coordinates
for q=1:N5
    xc1(q) = L/2 + c2le + (q-1)*wc/N5;
    zc1(q) = h1 + t + h2 + tc/2;

    xc2(q) = L/2 + c2le + q*wc/N5;
    zc2(q) = h1 + t + h2 + tc/2;
end

% Concatenate segment coordinates for all conductors and reference planes
xq1 = cat(2, xt1, xb1, xp1, xn1, xc1);
xq2 = cat(2, xt2, xb2, xp2, xn2, xc2);

yq1 = cat(2, zt1, zb1, zp1, zn1, zc1);
yq2 = cat(2, zt2, zb2, zp2, zn2, zc2);

% Store number of segments for each conductor and reference plane
N(1) = N1;
N(2) = N2;
N(3) = N3;
N(4) = N4;
N(5) = N5;

% Store the segment length for each conductor and reference plane
SL(1) = L/N1;
SL(2) = L/N2;
SL(3) = w/N3;
SL(4) = w/N4;
SL(5) = wc/N5;

% Clear C matrix
c = zeros(3,3);

% Calculate [Z] matrix elements
Z = populate_Zmatrix(xq1, xq2, yq1, yq2, N);

% Get the total number of segments

```

```

Na = sum(N);

% For each of the three conductors (2 of stripline and 1 of crossing trace)
for j=1:3
    Vj = zeros(5); % clear all voltages
    Vj(j+2) = 1; % set the voltage on one conductor to 1V
    V = populate_Vmatrix(N, Na, Vj); % populate [V] matrix
    I = zeros(Na, 1); % clear [I] matrix
    I = Z^-1 * V; % calculate [I] matrix
    Qpu = solve_qpu(I, N, Na, SL, j); % calculate charge per unit length
    for i=j:3
        c(i,j) = Qpu(i+2); % assign C matrix elements to charge pul
    end
end

% assign remaining C matrix elements
c(1,2) = c(2,1);
c(1,3) = c(3,1);
c(2,3) = c(3,2);

% Convert C matrix to pF
Cmatrix = c .* 1e12;

end % end function

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 16, 2020
% Description: [Z] Matrix Calculation
% Author: Peter Bliznyuk-Kvitko

% Function populate_Zmatrix
% Input parameters
%   x1 - start x coordinates for all segments
%   x2 - end x coordinates for all segments
%   y1 - start y coordinates for all segments
%   y2 - end y coordinates for all segments
%   Na - Total number of segments
% Output parameter:
%   Z - [Z] (Na x Na) Matrix

function Z=populate_Zmatrix(x1, x2, y1, y2, Na);

er = 4.2;
e0 = 1e-9/36/pi;
e = e0 * er;

N = sum(Na);
Z = zeros(N,N);

for m=1:N
    for n=1:N
        hn = x2(n) - x1(n); % segment length
        xm = (x2(m) + x1(m)) / 2; % x of the middle of segment m
        ym = y1(m); % y of the middle of segment m

        if (m == n) % diagonal elements
            Z(m, n) = - (hn/2/pi/e) * (log(hn/2) - 1);
        else % non-diagonal elements
            f = @(x) log((xm - x1(n) - x*hn).^2 + (ym - y1(n)).^2);
            Z(m,n) = - (hn/4/pi/e) * integral(f, 0, 1);
        end
    end
end
end

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 16, 2020
% Description: [V] Matrix Calculation
% Author: Peter Bliznyuk-Kvitko

% Function populate_Vmatrix
% Input parameters
%   N - Array with number of segments for each conductor
%   Na - Total number of segments
%   Va - Array with voltages on each conductor
% Output parameter:
%   V - [V] (Na x 1) Matrix with voltages on each segment

function V=populate_Vmatrix(N, Na, Va)

V = zeros(Na, 1);

for m = 1:Na
    if (m <= N(1))
        V(m, 1) = Va(1);
    elseif (m <= (N(1) + N(2)))
        V(m, 1) = Va(2);
    elseif (m <= (N(1) + N(2) + N(3)))
        V(m, 1) = Va(3);
    elseif (m <= (N(1) + N(2) + N(3) + N(4)))
        V(m, 1) = Va(4);
    elseif (m <= (N(1) + N(2) + N(3) + N(4) + N(5)))
        V(m, 1) = Va(5);
    end
end

end % end function

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 16, 2020
% Description: Charge per unit length Calculation
% Author: Peter Bliznyuk-Kvitko

% Function solve_qpu
% Input parameters
%   Io - [I] matrix with an coefficients
%   N - Array with number of segments for each conductor
%   Na - Total number of segments
%   SL - Array with segment lengths for each conductor
%   j - column of the capacitance matrix
% Output parameter:
%   Q - Array with Charge per unit length for each conductor

function Q=solve_qpu(Io, N, Na, SL, j);

Q = zeros(5, 1);

if (j == 1)
    Q(3,1) = sum(Io(N(1)+N(2)+1:Na-N(5)-N(4))*SL(3));
    Q(4,1) = sum(Io(N(1)+N(2)+N(3)+1:Na-N(5))*SL(4));
    Q(5,1) = sum(Io(N(1)+N(2)+N(3)+N(4)+1:Na)*SL(5));
elseif (j == 2)
    Q(4,1) = sum(Io(N(1)+N(2)+N(3)+1:Na-N(5))*SL(4));
    Q(5,1) = sum(Io(N(1)+N(2)+N(3)+N(4)+1:Na)*SL(5));
else
    Q(5,1) = sum(Io(N(1)+N(2)+N(3)+N(4)+1:Na)*SL(5));
end

```

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Appendix B

Matlab Code for Calculation of Characteristic Impedance for Lossless Case

```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 23, 2020
% Description: Top file for calculating
% Differential Impedance Zd and Common-Mode Impedance Zc
% Author: Peter Bliznyuk-Kvitko

clear all;
close all;
clc;

% Define constants
N=50;           % number of sections
u0=4e-7*pi;     % permeability of the free space
e0=1e-9/36/pi;  % permittivity of the free space
Er=4.4;         % relative permittivity of dielectric

% read the file that contains per-unit-length capacitances
[segMap, lenMap] = read_Cdata('output_Ccoeff.txt');

% For angles: 0, 15, 30, 45, 60, 75, and 90 degrees
for a=1:7
    ang = (a-1) * 15;

    % for each section
    for n=1:N
        % get the C matrix and length
        C = segMap(strcat(num2str(ang), '_', num2str(n)));
        L = lenMap(strcat(num2str(ang), '_', num2str(n)));
        length(a) = L;

        % get C matrix elements
        C11 = C(1,1)*1e-12;
        C22 = C(2,2)*1e-12;
        C12 = -C(1,2)*1e-12;

        % get L matrix elements
        L11=u0*e0*Er/C11;
        L22=u0*e0*Er/C22;
        L12=u0*e0*Er/C12;

        % get impedances
        z1 = L11;
        z2 = L22;
        zm = L12;

        % get admittances
        y1 = C11;
        y2 = C22;
        ym = C12;

        % Calculate Odd and Even impedances of inductors A and B
    end
end

```

```

        Zoe = get_Zparameters_homog(z1, z2, y1, y2, ym);

        Zea(n) = Zoe(1);
        Zoa(n) = Zoe(2);
        Zeb(n) = Zoe(3);
        Zob(n) = Zoe(4);
    end

    m=1:N;
    m = ((m-1)*L+L/2 + (100 - L*25));

    % Calculate Differential Impedance Zd and Common-Mode Impedance Zc
    Zd = Zoa + Zob;
    Zc = Zea .* Zeb ./ (Zea + Zeb);

    figure(113);
    plot(m, abs(Zd));
    hold on;

    figure(114);
    plot(m, abs(Zc));
    hold on;

end

figure(113);
title('Zd Differential Impedance');
xlabel('Z, mils');
ylabel('Zd, Ohm');
legend('0', '15', '30', '45', '60', '75', '90', 'Location', 'southeast');

figure(114);
title('Zc Common-Mode Impedance');
xlabel('Z, mils');
ylabel('Zc, Ohm');
legend('0', '15', '30', '45', '60', '75', '90', 'Location', 'southeast');

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 23, 2020
% Description: Read the text file with per-unit-length capacitance data
% Author: Peter Bliznyuk-Kvitko

% Function read_Cdata
% Input parameters
%   file - Path/name of C data file
% Output parameter:
%   segMap - map that contains the section lengths
%   lenMap - map that contains the section C matrices

function [segMap, lenMap] = read_Cdata(file)

INfile = fopen(file, 'rt');

segMap = containers.Map;
lenMap = containers.Map;

line_num = 0;
while ~feof(INfile)
    line = fgetl(INfile);

    if (line_num == 0)
        [ang, rem] = strtok(line);
        [num, rem1] = strtok(rem);
        [width, rem2] = strtok(rem1);
        [loc, segy] = strtok(rem2);
        lenMap(strcat(ang, '_', num)) = str2double(segy);
        C = zeros(3, 3);
    else
        [cap, rem] = strtok(line);
        C(line_num, 1) = str2double(cap);
        [cap, rem] = strtok(rem);
        C(line_num, 2) = str2double(cap);
        C(line_num, 3) = str2double(rem);

        if (line_num == 3)
            segMap(strcat(ang, '_', num)) = C;
        end
    end

    if (line_num == 3)
        line_num = 0;
    else
        line_num = line_num + 1;
    end
end
end

```

```

% Version: 2.0
% Created: March 4, 2018
% Revised: Feb 23, 2020
% Description: Calculation of Odd and Even Impedances
% for asymmetrical conductors A and B in homogenous medium
% Author: Peter Bliznyuk-Kvitko

function Zoe = get_Zparameters_homog(z1, z2, y1, y2, ym)

Z1 = sqrt(z1/y1);
Z2 = sqrt(z2/y2);

Zc = sqrt(Z1*Z2) * sqrt( (1+ym/sqrt(y1*y2)) / (1-ym/sqrt(y1*y2)) );
Zp = sqrt(Z1*Z2) * sqrt( (1-ym/sqrt(y1*y2)) / (1+ym/sqrt(y1*y2)) );

Zoa = (sqrt(Z1/Z2)*(Zc + Zp) - Zc + Zp) / 2;
Zea = Zc - Zp + Zoa;

Zob = (sqrt(Z2/Z1)*(Zc + Zp) - Zc + Zp) / 2;
Zeb = Zc - Zp + Zob;

Zoe = [Zea Zoa Zeb Zob];

end

```

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Appendix C

Matlab Code for Calculation of Z parameters

```

% Version: 2.0
% Created: March 4, 2018
% Revised: March 3, 2020
% Description: Z parameter calculation for N crossings
%               at specified angle
% Author: Peter Bliznyuk-Kvitko

clear all;
close all;
clc;

u0=4e-7*pi;           % permeability of the free space
e0=1e-9/36/pi;        % permittivity of the free space
Er=4.4;               % relative permittivity of dielectric
Ur = 1;               % magnetic permeability
tand = 0.02;          % loss tangent
wd = 5 * 2.54e-5;     % width of stripline trace in XY plane, m
wdc = 11.313708 * 2.54e-5; % width of crossing trace in XY plane, m
th = 0.1 * 2.54e-5;   % thickness of the trace, m
Len = 2.941564 * 2.54e-5; % length of one section in z direction

nc = 3;               % number of conductors
N = 51;               % number of 45 degree trace crossings
Ns = 50;              % number of sections
Nf = 1501;             % number of frequency points
f = logspace(8.0,10.0,Nf); % frequency range from 100 MHz to 10 GHz

w = 2*pi*f;           % angular frequency
e = Er*e0;            % electrical permittivity of dielectric
u = Ur*u0;            % magnetic permeability of dielectric
sigma = w * e * tand; % effective conductivity

ang = 45;             % crossing trace angle

% Read the Capacitance Matrices for all the sections
[segMap, lenMap] = read_Cdata('output_Ccoeff.txt');

for i=1:Nf % for each frequency point
    for n=1:Ns % for each section

        % Get C Matrix for the 45 degree angle, section # 51-n
        C = segMap(strcat(num2str(ang), '_', num2str(51-n)));

        % Convert capacitance from pF to Farads
        C = C * 1e-12;

        % Calculate PUL Inductance Matrix
        L = u*e*C^-1;

        % find matrix U whose columns are eigenvectors of C
        % and theta_sq whose values are eigenvalues of C

```

```

[U, thetta_sq] = eig(C);

% calculate thetta
thetta = sqrt(thetta_sq);

% create identity matrix
n1 = diag(ones(nc,1));

% Calculate PUL Resistance Matrix
R = zeros(nc, nc);
R(1,1) = get_resistance(f(i), wd, th);
R(2,2) = get_resistance(f(i), wd, th);
R(3,3) = get_resistance(f(i), wdc, th);

% Calculate PUL Conductance Matrix
G = zeros(nc, nc);
G = sigma(i)/e * C;

% find matrix S whose columns are eigenvectors and
% gamma_sq whose values are eigenvalues of expression in brackets
[S, gamma_sq] = eig(thetta * U^-1 * R * U * thetta);

% Calculate Trasformation matrix Ti
Ti = U * thetta * S;

% Calculate Trasformation matrix Tv
Tv = U * thetta^-1 * S;

% Find gamma
gamma = sqrt(gamma_sq);

% Calculate Propagation Constant
y = sqrt( (sigma(i)/e + j*w(i)) * gamma_sq + ...
          (j*u*w(i)*sigma(i) - w(i)^2*u*e)*n1 );

% Calculate Admittance Matrix
Y = G + j*w(i)*C;

% Calculate Impedance Matrix
Z = R + j*w(i)*L;

% Calculate Chain Parameters
[F11 F12 F21 F22] = get_chain_matrix(Y, y, Ti, Len);

F = [F11 F12;
     F21 F22];

% Mulitply Chain Matrices of all sections
% and store them for each frequency point
if (n == 1)
    Fm(:, :, i) = F;
else
    Fm(:, :, i) = Fm(:, :, i) * F;

```



```

        end
    end
end

for i=1:Nf % for each frequency point
    for n=1:N % for each 45 degree crossing
        % Multiply Chain Matrices of all crossings
        if (n == 1)
            Ft = Fm(:, :, i);
        else
            Ft = Ft * Fm(:, :, i);
        end
    end

    % Get Chain Parameters from the Chain Matrix
    F11 = Ft(1:nc, 1:nc);
    F12 = Ft(1:nc, nc+1:2*nc);
    F21 = Ft(nc+1:2*nc, 1:nc);
    F22 = Ft(nc+1:2*nc, nc+1:2*nc);

    % Calculate Z-parameters for the whole transmission line
    Z11 = -F11\F21^-1;
    Z12 = -F21^-1;
    Z22 = -F21^-1 * F22;
    Z21 = -F21^-1;

    % Store Z parameters for each frequency point
    Z11t(i) = Z11(1,1);
    Z12t(i) = Z12(1,1);
    Z21t(i) = Z21(1,1);
    Z22t(i) = Z22(1,1);
end

% Plot Z parameters vs frequency
figure;
loglog(f, abs(Z11t), 'r-', f, abs(Z12t), 'b', 'Linewidth', 2);
title('Z-parameter Data');
set(gca, 'FontSize', 12);
ylabel('Impedance in Ohms');
xlabel('Frequency(Hz)');
legend('z11', 'z12');

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: March 3, 2020
% Description: Calculate DC- and frequency-dependent resistance
% Author: Peter Bliznyuk-Kvitko

% Function: get_resistance
% Inputs:
%     fq - frequency
%     w - conductor width
%     t - conductor thickness
% Output:
%     Zs - DC- and frequency-dependent resistance

function Zs = get_resistance(fq, w, t)
mu0=4e-7*pi;          % permeability of the free space
rho=1.736e-8;         % copper's resistivity
kc=1.01;              % 1<kc<2

skinDep=1./sqrt(pi*mu0*fq/rho); % skin depth
R0=rho/w/t; % dc PUL resistance
Rs=rho/kc./skinDep/(w+t); % frequency-dependent resistance
Zs=R0+Rs*(1+j);

end

```

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```

% Version: 2.0
% Created: March 4, 2018
% Revised: March 3, 2020
% Description: Calculate Chain Parameters
% for a uniform section of the line
% Author: Peter Bliznyuk-Kvitko

% Function: get_chain_matrix
% Inputs:
%     Y - admittance matrix
%     y - propagation constant matrix
%     Ti - transformation matrix
%     Len - length of the uniform section
% Outputs:
%     F11, F12, F21, F22 - chain parameters

function [F11 F12 F21 F22] = get_chain_matrix(Y, y, Ti, Len)
    nc = 3; % number of conductors
    n1 = diag(ones(nc,1)); % identity matrix

    % Calculate Chain Parameters
    F11 = 1/2*Y^-1*Ti*(exp(y*Len).*n1+exp(-y*Len).*n1)*Ti^-1*Y;
    F12 = -1/2*Y^-1*Ti*y*(exp(y*Len).*n1-exp(-y*Len).*n1)*Ti^-1;
    F21 = -1/2*Ti*(exp(y*Len).*n1-exp(-y*Len).*n1)*y^-1*Ti^-1*Y;
    F22 = 1/2*Ti*(exp(y*Len).*n1+exp(-y*Len).*n1)*Ti^-1;

end

```

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