KNOWLEDGE OF NUMBER INTEGRATION

By

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ABSTRACT

What are the roles of cardinal and ordinal processing in the development of arithmetic? In the present dissertation, cardinal knowledge is defined as the ability to determine the quantity of number symbols whereas ordinal knowledge is defined as the ability to determine the relative relationship among number symbols. This dissertation includes two studies examining the development of the relations among cardinal, ordinal and arithmetic skills, both concurrently and predictively, for children in the early grades of elementary school. In both studies, children completed a number comparison task (e.g., which number is bigger, 4 or 5?) as an index of their cardinal knowledge. They also completed two novel order tasks: (a) missing number (e.g., which number is missing, 1 _ 3 4?), and (b) number ordering (i.e., order the three digits from the smallest to the largest, e.g., 4 5 3 or 2 7 9). Furthermore, children completed two measures of inhibitory control. Last, children's arithmetic skill (e.g., solving problems such as 4 + 5 or 7 + 6) was measured.

In Study 1, I evaluated the internal consistency and validity of the novel order measures for children entering grades 1 to 3 (n = 70). In Study 2, multi-group path analysis showed that for children in grade 1 (n = 66), number ordering was strongly predicted by number comparison, but not by the missing number task or inhibitory control. Moreover, performance on the number comparison and missing number task independently predicted addition. Further, performance on the number comparison task uniquely predicted the growth of addition. In contrast, for children in grade 2 (n = 80), variance in the number ordering task was shared among the number comparison, missing number, and inhibitory control tasks. Number ordering uniquely predicted addition

concurrently and it also predicted the growth of addition.

I interpret the different patterns of results from grades 1 to 2 as reflecting different ongoing processes of integration of symbolic numerical associations. These findings suggest that development of number competence involves the integration of cardinal, ordinal, and arithmetic associations in an extensive network of relations among numbers.

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CHAPTER 1: INTRODUCTION

Children develop considerable fundamental mathematical competence early, and this competence supports the development of their later mathematical knowledge (Resnick, 1989). One central aspect of this initial mathematical competence involves the acquisition of basic symbolic representations, such as comparing and ordering number words and digits (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013; Holloway & Ansari, 2009; Lyons & Beilock, 2009); basic representations support advanced ways of representing number, such as numerical operations (Cirino, Tolar, Fuchs, & Huston-Warren, 2016). Several large longitudinal studies have shown that mathematical performance as early as kindergarten and grade 1 continues to predict children's performance years later (Duncan et al., 2007; Geary et al., 2018; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Thus, an understanding of how children integrate the different forms of basic symbolic representations into a coherent associative network is critical for designing appropriate instruction that will optimally support mathematical development.

Children and adults' knowledge of number symbols involves a variety of associations. Consider the numbers 1, 2 and 3. These numbers have cardinal associations (i.e., how many items are in a set; e.g., $3 > 1$; $3 > 2$), ordinal associations (i.e., the relative position of an item in a sequence; e.g., 2 comes after 1 and before 3), and arithmetic associations (i.e., addition, subtraction, multiplication and division operations with symbols; e.g., $1 + 2 = 3$; $3 – 1 = 2$; $1 \times 2 = 2$; $3 \div 1 = 3$). With increasing numerical experience, children add more advanced symbolic associations (e.g., fractions, ½; decimals, 2.3; and algebra, $2x + 1 = 3$) to their symbolic associative networks. All of these associations are hierarchical; that is, the more basic associations are fundamental to the acquisition of more advanced associations (Cirino et al., 2016; Entwisle & Alexander, 1990; Hiebert, 1988; Núñez, 2017; Siegler & Lortie-Forgues, 2014; Resnick, 1989). Thus, understanding how children integrate the basic associations among number symbols that scaffold the acquisition of higher-level mathematical competencies is an important goal of research.

Importance of Cardinal and Ordinal Processing in Arithmetic Development

Cardinal and ordinal processing of numbers are fundamental components of the development of symbolic number knowledge (Sury & Rubinsten, 2012). Cardinal processing of numbers is defined as the ability to access the quantitative information or magnitude of number symbols; in contrast, there is no widely accepted definition of ordinal processing of numbers in the literature on numerical cognition (Sury & Rubinsten, 2012). On the one hand, ordinal knowledge refers to the unique position of an item in a sequence (e.g., the third person in the line; Jacob & Nieder, 2007; Nieder, 2005). On the other hand, ordinal knowledge refers to the relative relations between symbolic representation of numbers (e.g., what comes before or after the number 3? Brannon & Van de Walle, 2001; Franklin & Jonides, 2008; Kaufmann, Vogel, Starke, Kremser, & Schocke, 2009; Turconi & Seron, 2002; Turconi, Campbell, & Seron, 2006). Other researchers have defined ordinal knowledge as the ability to understand individual members of a set in relation to other members of that set (Lyons, Vogel, & Ansari, 2016). Thus, a clear specification of what ordinal processing of numbers means is needed to frame the present research.

It is also important to distinguish between ordinal or cardinal *knowledge* and ordinal or cardinal *tasks*. As discussed further below, certain tasks have been used as indices of ordinal or cardinal knowledge. However, all of these suffer to some extent from the problem of task impurity; that is, any specific measure of a construct will necessarily involve other cognitive processes (e.g., Clark et al., 2016; Friedman, 2016; Miyake et al., 2000).

In the present dissertation, ordinal processing of numbers is assumed to rely on the knowledge of sequential and other relational associations that allow children to determine the relative position of symbolic numbers in ordinal tasks. Sequential associations are defined as the relations among adjacent numbers in the counting sequence. That is, for every natural number n in the count list, the next number on the list represents the cardinality "n + 1", also known as the "successor function" (Carey, 2004; Sarnecka & Carey, 2009). On this view, the acquisition of the sequential knowledge of number involves not only memorizing the counting string by rote, but also involves some understanding of the meaning of the order of the number words in the count list and the ability to indicate which number comes before or after a given number (Carey, 2004; Sarnecka & Carey, 2009). However, for number sequences that are not successive, children cannot rely on their sequential knowledge of numbers or even stored relational patterns to decide whether they are in order or not. Rather, they may need to also invoke cardinal processing to determine the relative ordinal relationships among different numbers, for example, 9 is bigger than 7, thus 9 comes after 7 (Lyons et al., 2016). Taken together, ordinal processing of numbers is defined as the application of knowledge of sequential and cardinal associations to determine relative position of symbolic numbers.

Cardinal processing is often measured by having participants indicate which of two presented digits is numerically larger (symbolic number comparisons; e.g., which is larger, 4 or 5? e.g., Castronovo & Gobel, 2012; Holloway & Ansari, 2009; Moyer & Landauer, 1967). Performance on the symbolic number comparison task is related to children's arithmetic skills as early as kindergarten (see Schneider et al., 2017 for a meta-analysis). In contrast, ordinal processing is often measured by having participants judge the relative order of a set of digits (symbolic order judgments; e.g., 1 2 3 is in order whereas 1 3 2 is not; Lyons & Beilock, 2009, 2011; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Vos, Sasanguie, Gevers, & Reynvoet, 2018; Turconi et al., 2006). Symbolic order judgment becomes increasingly more predictive of other mathematical tasks as children develop more advanced number system knowledge (Lyons & Ansari, 2015; Lyons et al., 2014; Vogel, Remark, & Ansari, 2015). Despite strong correlations between these tasks, they are assumed to measure distinct aspects of number knowledge (Lyons et al., 2016).

The pattern of development of cardinal and ordinal knowledge supports the view that these aspects of symbolic number processing are distinct (Lyons et al., 2014; Lyons & Ansari, 2015; Sansaguie & Vos, 2018). Symbolic order judgment seems to replace symbolic number comparison as the key index of symbolic numerical processing by the mid-elementary grades (Lyons et al., 2014). In particular, for children in grade 1, symbolic number comparisons were the strongest predictor of arithmetic performance; however, starting from grade 2, symbolic order judgments became a stronger predictor of arithmetic performance (Lyons et al., 2014; Sasanguie & Vos, 2018). Thus, the time between grades 1 and 2 is presumably crucial for the development of symbolic associations, where children appear to move from relying on cardinal processing to ordinal processing. However, it is still unclear how the processing of number symbols changes between grades 1 and 2 to support the development of mental arithmetic. Thus, my dissertation focused on how the integration of sequential and cardinal associations into a complex network of symbolic associations supports the development of mental arithmetic for children in grades 1 and 2.

The present research is built upon the theoretical account that development emerges as a result of a continuous process of differentiation and integration (the orthogenetic principle; Werner, 1957). Considering the context of numerical development, differentiation can be viewed as a process of distinguishing various kinds of numerical associations (Siegler & Chen, 2008). For example, 3 and 4 are differentially associated with 5, 7, and 12 through counting, addition and multiplication associations in an interrelated mental network. In contrast, integration can be viewed as a process of combining the acquired associations together to construct a more advanced associative network (Siegler & Chen, 2008). On this view, when children learn a new concept of numerical association, they initially go through a process of differentiation and then integration as it becomes connected with the previously learned associations in the mental network. Thus, the acquisition of the more advanced associations does not simply replace the influence of the more basic associations; rather, all of the acquired associations work interactively, allowing coordination among the associations within a single, unified mental network (Werner & Kaplan, 1956). Prior knowledge of the earlier acquired associations is the building block of mastering the higher-level associations (Hiebert, 1988). Thus, if children have failed to master any of the basic numerical associations early on, they may have difficulty with advanced mathematical concepts that are built upon the integration of previously acquired skills (Hiebert, 1988).

Hierarchical Symbol Integration Model

In the present dissertation, I used the *Hierarchical Symbol Integration* (HSI) model proposed by Xu, Feng, Newman, and LeFevre (under review) as the theoretical framework for understanding how individual differences in the acquisition of cardinal and ordinal associations are related to arithmetic development. The HSI model is built upon the assumption that the acquisition of associations among numbers forms a cohesive hierarchical mental network: the higher-level associations are learned on the basis of lower-level associations, an integrated network of symbol number knowledge (Hiebert, 1988). This assumption is common to many views of number symbol acquisition. Support for the underlying assumptions of the HSI model comes from the theoretical models for arithmetic associations (e.g., Associative Network model, Ashcraft, 1982; Distribution of Association model, Siegler, 1988). More specifically, the arithmetic problems and answers are linked in an interrelated associative network (e.g., Ashcraft, 1995; Campbell, 1995; Siegler, 1988; De Visscher & Noël, 2014). The relative strength of associations between the problems and the answers in the mental network depends on the frequency of access to each problem (for alternative views see Verguts & Fias, 2005; De Brauwer & Fias, 2009). For example, the smaller problems are often overlearned and thus have stronger associations with the correct answer rather than the wrong answers, compared to the large ones (Ashcraft, 1995; Siegler, 1988). On this view, the accessibility of arithmetic associative network is essential for the development of arithmetic for children.

The novel assumption of the HSI model is that development of number symbol knowledge reflects the integration of cardinal and ordinal knowledge, specifically, that cardinal and sequential associations serve as the base of the mental network and ordinal associations are built upon these relations (Xu et al., under review). Through extensive experience activating cardinal and ordinal associations in various numerical activities, individuals *integrate* these associations into a unified mental network; that is, access to cardinal associations becomes nested within ordinal associations, which then serve as a building block of the development of more advanced arithmetic associations (Xu et al., under review; see Figure 1).

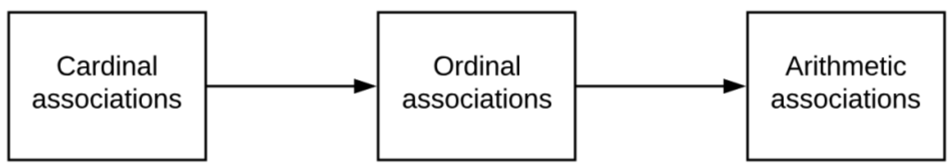

Figure 1.1. Central framework of the *Hierarchical Symbol Integration* model

Xu et al. tested some predictions of the integration assumption by comparing patterns of associations among more- and less-skilled adults. First, as shown in Figure 1.2, performance on the order judgment task mediated the relations between number comparisons and arithmetic fluency for both skill groups, suggesting an integrated cardinal and ordinal associative network. Furthermore, we also found that for the more-skilled Chinese adults, arithmetic fluency fully mediated the relations between symbolic order judgments and more advanced symbolic number knowledge (e.g., fraction and algebra arithmetic, number line estimation, and word problem solving), suggesting that the accessibility of arithmetic associations superseded the influence of cardinal and ordinal associations (see Figure 1.2). In contrast, for the less-skilled Canadian adults, the mediation was partial: both order judgments and arithmetic fluency uniquely predicted advanced symbolic number knowledge, suggesting a less integrated associative network (see Figure 1.2). These results suggest that that cardinal, ordinal and arithmetic associations become increasingly integrated with the increases in relative numerical skill.

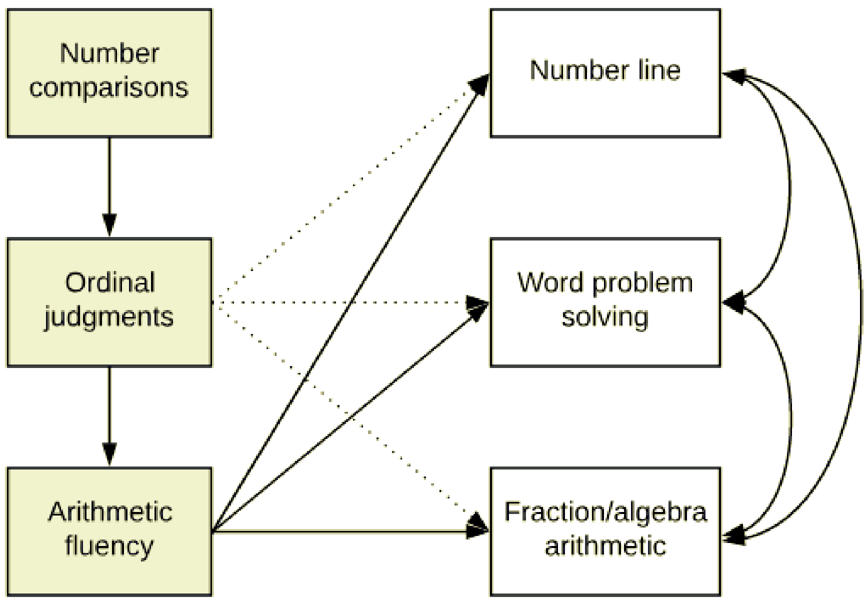

Figure 1.2. Findings in Xu et al. (under review). The mediating effect of order judgments on the relations between number comparisons and arithmetic fluency was found for both groups (left side of figure). The direct relations between order judgments and number line, fraction/algebra arithmetic, and word problem solving are shown with dotted lines for the Canadians (right side of figure) but these links were mediated by arithmetic fluency for the Chinese (solid lines).

According to the HSI model, performance on symbolic number comparisons and symbolic order judgments reflects individual differences in the fluency of access to cardinal and ordinal associations among symbolic numbers (Xu et al., under review). On this view, arithmetic fluency (speed and accuracy of solutions) relies directly on the accessibility of cardinal and ordinal associations. That is, for children who have developed a set of integrated associations, variations in the accessibility of ordinal associations should supersede variability in cardinal associations to become the main predictor of arithmetic performance. In contrast, for children who have not yet integrated ordinal associations into their associative network, variations in the accessibility of the more basic cardinal associations should predict arithmetic performance. On this view, ordinal processing reflects the ongoing processes of integration of previously learned cardinal associations, which allows children to fluently retrieve the easily accessible associations when they solve arithmetic problems.

In the present dissertation, I conducted a short-term longitudinal study with the goal of expanding the HSI model to explore the process of development with regards to the integration of various associations of number symbols for children. According to the HSI model, the operational definition of integration is that ordinal associations mediate the relation between cardinal associations and arithmetic fluency. Ordinal associations, for the young children in the present research, represent the highest level of network integration. In particular, I compared children in grades 1 and 2, who are known to differ in the degree of their integration of symbolic associative networks (Sasanguie & Vos, 2018), by assessing the relations among cardinal, sequential, ordinal, and arithmetic skills both concurrently and predictively.

In the next chapter, I provide an overview of the literature on the development of cardinal, sequential, ordinal, and arithmetic associations. In Chapter 3, I present Study 1, which was mainly focused on examining the reliability and validity of the measures used to measure children's fluency to access numerical associations for children in early grades of elementary school. In Chapter 4, I present the findings of Study 2, where the goal was to test an expanded the HSI model by examining changes in the development of integration among the various aspects of symbolic number knowledge for children in grades 1 and 2 over a four-month time frame. Finally, in Chapter 5, I discuss the implications of these data for the expanded model of hierarchical symbol integration.

CHAPTER 2: LITERATURE REVIEW

Knowledge of verbal counting sequence serves as the building block for young children as they develop initial understanding of cardinality (Jiménez Lira, Carver, Douglas, & LeFevre, 2017) and ordinality (Brannon & Van De Walle, 2001). The development of number symbol associations occurs slowly, starting as early as two years of age. More specifically, around the age of two or three, children learn to recite a sequence of number words by rote without any awareness of how those number words are related to their corresponding quantities (Wynn, 1992). They also learn to label some Arabic digits with number words (Mix, 2009). These mapping skills may develop independently and may not initially be closely linked to an understanding of numbers as representative of quantities (Bialystok, 1992; Case et at., 1996). As children develop an understanding of cardinality, they rely on counting as the means to connect the meanings of the number words with the objects they count (i.e., the last number counted determines the quantity of a set of items; e.g., Bialystok, 1992; Bialystok & Codd, 1996, 2000). Thus, during the early phase of children's learning about number symbols, children's knowledge of counting sequence is the precursor skill that allows children to link the quantitative information or magnitudes of number to the symbolic representations of number.

Through practice of counting in everyday experiences, by around the age of six, children acquire the understanding of successor function, demonstrated by the ability to reason about the successor function for all possible numbers: the successor of any number can be created by adding 1 (Cheung, Ruberson, & Barner, 2017). The understanding of successor function implies that children acquire the simplest aspect of symbolic ordinal associations, that is, knowledge of the *sequential relations* among successive numbers (e.g., 3 comes before 4, and after 2; Carey, 2004). However, the acquisition of sequential associations does not imply that children have fully developed ordinal associations. For number sequences that go beyond successive numbers, children may need to rely on cardinal processing to find out which number comes before and after another number through inferential reasoning (e.g., 9 is bigger than 7, 6 is smaller than 7, and therefore 6 → 7 → 9; Lyons et al., 2016). Thus, children also need to have some knowledge of cardinality (i.e., comparing the magnitudes of number symbols) to develop the more advanced understanding of ordinality (i.e., judging the relative order of sequences that are not successive). Taken together, the integration of sequential knowledge and cardinal knowledge of numbers may be the precursor skills that allow children to develop an understanding of how number symbols are related to other number symbols, an understanding of ordinal knowledge.

Despite the overlap between the development of cardinality and ordinality for symbolic numbers, cardinal and ordinal processing are conceptually different; knowing that 9 is greater in magnitude than 7 is fundamentally different from judging whether 7 comes before 9 (e.g., Delazer & Butterworth, 1997; Lyons & Beilock, 2013; Turconi & Seron, 2002; Turconi, Jemel, Rossion, & Seron, 2004). In this chapter, I first provide an overview of the existing neurological and behavioral research with respect to cardinal and ordinal processing of number. Next, I present the research with respect to the relations among cardinal, ordinal and early arithmetic development. Lastly, I present the goals and hypotheses of my dissertation, with an attempt to examine changes in the development of integration between cardinal and ordinal associations for children in grades 1 and 2.

Neurological Studies

Research has shown that brain-damaged patients can have selective impairments of either the cardinal or ordinal aspect of numbers (Delazer & Butterworth, 1997; Turconi & Seron, 2002). In particular, a patient with a quantity deficit was unable to decide which of two presented digits was numerically larger, but showed no difficulties in processing ordinal aspects of numbers (e.g., which number comes next; Delazer & Butterworth, 1997). In contrast, a patient with an ordinal deficit was unable to judge the ordinal relations among numbers, but showed no difficulties in processing the cardinal aspect of numbers (Turconi & Seron, 2002). The results from these two studies provide evidence that quantity processing and ordinal processing are neurologically distinct.

Similar results were obtained for healthy adults using functional magnetic resonance imaging (fMRI; Lyons & Beilock, 2013) and event-related potentials (ERPs; Turconi et al., 2004). In particular, when adults performed symbolic number comparison tasks (i.e., which of the two presented digits is numerally larger) and symbolic order judgment tasks (i.e., are two numbers in an ascending or descending order), no overlapping regions were found in any regions of the brain; rather, differences in the activation patterns in parietal and prefrontal cortices were observed (Lyons & Beilock, 2013; Turconi et al., 2004). These results suggest that distinct operational mechanisms are involved in cardinal and ordinal processing of symbolic representations of number.

Taken together, the results of neuroimaging studies of healthy adults and those with brain-damaged patients show distinctions in brain circuits when processing cardinality and ordinality for symbolic numbers. These results support the view that cardinal and ordinal processing for symbolic representations of number can be differentiated for adults when they are performing the symbolic number comparison and order judgment tasks (Lyons et al., 2016). However, for children, there is no neurological study examining whether cardinal and ordinal processing of numbers are also distinct. It is possible that at the initial phase of development of symbolic number knowledge, cardinal and ordinal processing of numbers can be intertwined to some extent, because knowledge of the counting sequence is the foundation for developing the understanding of cardinality and ordinality (Brannon & Van De Walle, 2001; Jiménez Lira et al., 2017). Thus, for a complete understanding of the similarities and differences in cardinal and ordinal processing of numbers, next, I provide an overview of behavioural research regarding how children develop the knowledge of cardinality and ordinality.

Behavioural Studies

The development of cardinal knowledge. Cardinality is defined as the ability to determine numerical quantities or magnitudes of a set of items, that is, answers to the question "how many?" Children develop symbolic representations of number in a sequence of stages (Bialystok, 1992; Bialystok & Codd, 1996, 2000; Case et al., 1996). At first, children learn to recite a sequence of number words by rote without any awareness of how those number words are related to their corresponding quantities. Next, children develop knowledge of the written forms of symbolic digits (1, 2, 3…) that correspond to each number-word in the sequence without understanding its relation to the quantity (Bialystok, 1992). At this stage, children perceive the symbolic digits as physical objects, not as symbols that hold meaning (Bialystok, 1992). Concurrently, they also learn to count sets of objects correctly, initially without understanding that the last number counted represents the quantity of the set (Wynn, 1992). In particular, they can assign each number word to a unique concrete referent (i.e., *one-to-one correspondence* principle), and they can also consistently use the number words in a fixed order (i.e., *stable-order* principle; Gelman & Gallistel, 1978).

Through practice of counting objects from everyday experiences, around the age of four, children start to understand that the last number word counted indicates the quantity of the whole set of objects, showing that they are able to map each number word of the counting sequence to the meaning of those numbers, that is, understand *cardinality* (Gelman, 1978; Mix, 2002; Mix, Sandhofer, & Baroody, 2005). The components of counting, that is, one-to-one correspondence, stable-order, and cardinality, allow children to operationally map between number words and concrete quantities. Thus, being able to compare sets of 3 and 5 objects and choose the set with the larger number indicates that children understand the cardinal principle in the sense that they use counting successfully and understand the relation between the last number counted and the size of the set. More generally, however, cardinal associations are assumed to operate on symbolic referents, either words or digits, so that children can select the larger set based on the words (e.g., which is larger, three or five) or the written symbols (e.g., which is larger 3 or 5) without having actual sets of objects to count; that is, symbolic comparisons are assumed to also rely on children understanding the cardinal principle. For Canadian children, number comparison using words develops earlier than number comparison using written digits (Jiménez Lira et al., 2017).

Around the age of five or six, children gradually integrate their knowledge of number words and quantities into a cohesive mental representation that allows them to connect this structure to the knowledge of Arabic symbols (Case et al., 1996). At this point, children start to focus on what a symbolic digit represents (i.e., the cardinality of a given set), rather than just on the number's physical features as a concrete object (Bialystok, 1992) and thus they can successfully perform number comparisons on digits (Jiménez Lira et al., 2017). In the present dissertation, children are assumed to have acquired cardinal associations when they are able to manipulate the abstract symbols independently of the referent system of concrete objects.

Symbolic number comparisons. The symbolic number comparison task is used to index individual differences in numerical processing skills, particularly the activation of cardinal representation of number symbols (i.e., the quantitative information of the symbolic digits; e.g., Castronovo & Gobel, 2012; De Smedt, Verschaffel, & Ghesquiere, 2009; Holloway & Ansari, 2008; Maloney, Ansari, & Fugelsang, 2011; Sasanguie & Vos, 2018; Schneider et al., 2016). For this task, participants need to select the numerically larger (or smaller) of a pair of numbers, typically single digits such as 3 and 4, or they may be asked to determine whether a target number is greater or larger than a referent. Performance on the number comparison task, such as mean accuracy, mean reaction time, or a combined score of both, are often used to index individual differences in cardinal processing (De Smedt et al., 2009; Holloway & Ansari, 2008; Schneider et al., 2017).

Performance on the number comparison task is strongly correlated with a range of other numerical and mathematical measures for typically-developing children (e.g., Bugden & Ansari, 2011; Holloway & Ansari, 2009; De Smedt, Verschaffel, & Ghesquière, 2009; Lyons et al., 2014; Sasanguie, De Smedt, Defevrer, & Reynvoet, 2012; Vogel, Remark, Ansari, 2015; Schneider et al., 2017), for children with mathematical learning difficulties (Brankaer, Ghesquière, & De Smedt, 2014; Rousselle, & Noël, 2007; Vanbinst, Ansari, Ghesquière, & De Smedt, 2014), and for adults (e.g., Castronovo & Gobel, 2012; Lyons & Beilock, 2011; Xu et al., under review). Thus, an individual's ability to understand and compare numerical magnitudes of symbolic digits, assumed to index cardinal processing of symbolic numbers, is persistently related to higher-level mathematical competencies (Schneider et al., 2017).

Distance or ratio effect. In addition to overall effects of performance as measured by speed and accuracy, digit comparison can also be indexed with two task-specific effects: a) a *distance effect* such that people are faster and more accurate to compare numbers that are farther apart (1 vs. 8) than numbers that are closer together (1 vs. 2), and b) a *ratio effect* such that high ratio pairs (i.e., numbers that are closer to each other, e.g., 7 vs. 8) are judged more slowly and less accurately than low ratio pairs (numbers that are farther apart, e.g., 1 vs. 2). The presence of the distance effect or the ratio effect is sometimes assumed to reflect children's ability to map between the symbols and an underlying representation of quantity or magnitude, a representation that has been characterized as a mental number line (Dehaene, 1997). That is, people convert the written digits into analog magnitudes, which is similar to comparing two physical lines varying in length (Moyer & Landauer, 1967). The internal representations of numbers that are closer to each other (e.g., 1 and 2) overlap more than those numbers that are farther apart (1 and 8), and therefore are judged more slowly and less accurately (Gallistel & Gelman, 1992).

Other researchers challenged the assumption that children's internal representations of number can be directly mapped onto the preexisting non-symbolic ones (e.g., Santens & Gevers, 2008; White, Szucs, & Soltesz, 2012). Instead, they argue that symbolic and non-symbolic representations of number become independent with experience (e.g., Lyons, Ansari, & Beilock, 2012; Noël & Rouselle, 2011). The extreme version of this view is that there is symbolic estrangement – the idea that an independent network of symbolic associations develops that is distinct from the representation or processes involved in non-symbolic relations (Lyons et al., 2012). Thus, symbolic number comparisons may involve the activation of both symbolic and non-symbolic representations of number, rather than reflecting a direct mapping between the two systems (Goffin & Ansari, 2016). However, the degree of overlap between these two types of number representations in individuals remains an open question (Schneider et al., 2017).

Automaticity of cardinal associations. When children are able to connect quantities to digits automatically, that is, in the absence of a conscious attempt to retrieve the meaning, they should have cognitive resources available to be allocated to more complicated mathematical procedures such as doing mental arithmetic (Chan, 2014). To evaluate the automatic activation of cardinal knowledge, participants are often asked to process non-numerical information about numbers (e.g., parity judgment; physical size judgment), but the task-irrelevant numerical magnitude is automatically activated (Berch, Foley, Hill, & Ryan, 1999; Girelli, Lucangeli, Butterworth, 2000; Noël, Rousselle, & Mussolin, 2005; Rubinsten, Henik, Berger, & Shahar-Shalev, 2002). For example, when participants are asked to choose the larger number between two digits based on the physical size, the irrelevant numerical dimension may facilitate or interfere with the relative judgment of the physical size depending on the congruity of numbers (i.e., size congruity effect). That is, when the physical size matches with numerical size (e.g., 4 vs. 7), participants respond faster compared to numerically neutral trials (e.g., 4 vs. 4). In contrast, when the physical size does not match with numerical size (e.g., 4 vs. 7), participants respond more slowly compared to physically neutral trials (e.g., 4 vs. 7). The presence of a size congruity effect indicates that the meaning of the symbolic digit is automatically accessed, although it is not part of the task requirements (Girelli et al., 2000; Henik & Tzelgov, 1982).

By the end of grade 1, children should be able to connect quantities to written digits fluently, although automaticity in accessing number magnitude may not occur in children from North America (Berch et al., 1999) or other Western cultures (Girelli et al., 2000; Mussolin & Noël, 2007; White et al., 2012) until later grades. In contrast, children in China achieve automaticity of the connections between written digits and quantities as early as kindergarten (Zhou et al., 2007). The factors that may influence children's onset of automaticity include the amount of exposure to numbers, differences in educational practices across different cultures, general processing efficiency, and the ability to suppress interference from task-irrelevant information (e.g., physical properties of the symbolic digits; Chan, 2014; Girelli et al., 2000).

In summary, the development of cardinal associations is assumed to reflect an integration of children's knowledge about non-symbolic quantities and their knowledge of the number symbols (e.g., Case et al., 1996; Bialystok, 1992; Dehaene, 1997). During the first few years of elementary school, children in Western countries develop an automaticity in accessing numerical magnitude, an indicator of activation of cardinal associations when they see numerical symbols (e.g., Berch et al., 1999; Girelli et al., 2000; White et al., 2012). To measure individual differences in children's fluency to access cardinal associations among numbers, the symbolic number comparison task in which children have to identify the numerically larger number of two Arabic digits is typically used. Performance on the number comparison task has been shown to be a robust predictor of children's mathematical competencies (Schneider et al., 2017).

The development of ordinal knowledge. The development of the ordinal associations between symbolic digits is also a fundamental skill for numerical cognition (Sury & Rubinsten, 2012). Ordinality is usually defined as the ability to determine the relations among numbers, that is, it answers the question what comes next (e.g., 2 comes before 3 and after 1; Brannon & Van De Walle, 2001)? This ability to distinguish relative order information among symbolic representations of numbers is essential to mathematical learning (Rubinsten & Sury, 2011).

Research suggests that there is a lag between the development of cardinality and ordinality for symbolic numerals such that the development of cardinality precedes the development of ordinal knowledge (Colomé, & Noël, 2012; Knudsen, Fischer, Henning, & Aschersleben, 2015; Xu & LeFevre, 2016). Xu and LeFevre (2016) found that 3- to 5-year-old children showed poor performance on the tasks that require ordinal understanding of numbers from 1 to 10 (i.e., ordering three single-digit numbers from smallest to largest, such as 2 7 1; and estimating the position of a target number on a visual line, e.g., where does 3 go on a 0-10 number line?) unless they had some training on sequential associations. Moreover, Colomé and Noël (2012) found that three- to five-year-old children performed better at the same numerical tasks when phrased in cardinal context (i.e., "How many?") than in ordinal context (i.e., "What position?). In particular, three- and four-year-old children could produce cardinal number words (e.g., three cars), however, they were not able to say the number word corresponding to an item at a given position either using the ordinal number words (e.g., the *third* car) or using cardinal number words to indicate the ordinal position of the item (e.g., car *three*) until the age of five (Colomé & Noël, 2012). Further, Knudsen et al. (2015) found that four- to seven-year-old children showed cardinal understanding (i.e., being able to map between number words and symbolic digits) prior to ordinal understanding (i.e., correctly ordering a set of symbolic digits from small to large). Together, these studies suggest that cardinal processing of symbolic representations of number develop first, followed by ordinal processing of numbers.

Symbolic order judgments. Speeded order judgments have been used to index ordinal processing for adults (e.g., Douglas & LeFevre, 2017; Lyons & Beilock, 2009, 2011; Sasanguie, Lyons, De Smedt, & Reynvoet, 2017) and for children (e.g., Lyons et al., 2014; Vogel et al., 2016; Sasanguie & Vos, 2018). For this task, participants judge whether a set of digits is in order by making a two-alternative forced choice (e.g., is 2 7 3 in order or not?). Note that this order judgment task requires that children both recognize ordered sequences (e.g., 1 2 3; increasing) but reject sequences that are not correctly ordered (e.g., 2 1 3; not increasing) even though these sequences have the same numbers in them. Thus, the relationships that exist among the numbers have to be evaluated in relation to each other, presumably making the task about more than just recognizing ordinal associations among numbers.

Performance on the order judgment task (i.e., accuracy, reaction time, or a combined score) is used to measure children's ability to access ordinal associations among symbolic numbers (e.g., Lyons & Beilock, 2012; Lyons et al., 2014; Sasanguie et al., 2017; Sasanguie & Vos, 2018). For adults, Lyons and Beilock (2011) were the first to show that the relation between performance on number comparison tasks and multi-digit arithmetic was mediated by symbolic order judgment performance. Sasanguie et al. (2017) extended this finding by showing that performance on symbolic order judgments fully mediated the relationship between symbolic digit comparisons and arithmetic in adults, above and beyond the influence of general non-numerical ordinal processing (measured by a serial order working memory task). Furthermore, Xu et al. (under review) found that symbolic order judgments mediated the relations between symbolic number comparisons and arithmetic fluency for both more-skilled adults who were educated in China and less-skilled adults who were educated in Canada. Thus, for adults, performance on the order judgment task mediates the relations between performance on the number comparison task and arithmetic fluency.

For children, the relations among number comparison, order judgments, and arithmetic performance vary with age. For example, performance on an order judgment task was a stronger predictor of arithmetic performance than number comparisons for children in grades 2 through 6; in grade 1, however, performance on number comparisons was the best predictor of children's arithmetic performance (Lyons et al., 2014; Sasanguie & Vos, 2018). By grade 2, however, performance on symbolic order judgments mediated the relationship between symbolic number comparisons and arithmetic (Sasanguie & Vos, 2018), similar to the pattern shown with adults.

In summary, these studies provide converging evidence to show that the ability to judge the relative order of numerals (i.e., ordinal processing) was a stronger correlate of arithmetic performance than the ability to compare two numbers and choose the larger (i.e., cardinal processing) for both adults (Lyons & Beilock, 2011; Sasanguie et al., 2017; Xu et al., under review) and for children in grades 2 through 6 (Sasanguie & Vos, 2018). These results suggest that, starting from grade 2, the fluent access and activation of ordinal associations may supersede that of cardinal associations as the single best predictor of subsequent arithmetic processing.

Reversed distance effect. A "reversed" distance effect has been sometimes found for order judgments, that is, numbers that are farther apart (e.g., 1 4 7 or 4 7) are judged more slowly and less accurately than numbers that are closer together (e.g., 1 2 3 or 2 3; e.g., Goffin & Ansari, 2016; Franklin & Jonides, 2009; Lyons & Beilock, 2013; Lyons & Ansari, 2016). Goffin and Ansari (2016) found that the distance effect, from the symbolic digit comparison task, and the reversed distance effect, from the order judgment task, were unrelated to each other, and that each captured unique variance in adult's arithmetic performance, which is further support for the view that cardinal and ordinal processing of symbolic numerals are dissociable.

There are at least two distinct theoretical accounts of the reversed distance effect in ordinal processing. One possibility is that judgments of numerical order involve a *serial scanning* process such that participants mentally search the counting sequence one by one to decide whether the numbers are in order or not (Turconi et al., 2004, 2006). As a result, it takes longer for participants to make order judgments for sequences with larger distances (e.g., 1 4 7) than sequences with smaller distances (e.g., 1 2 3) because there are more digits intervening in the serial search process for the former (e.g., 1-2-3-4-5-6-7) than for the latter (e.g., 1-2-3). The other possibility is that participants may process number sequences through the activation of *associative chaining* between the digits retrieved from long-term memory (e.g., Caplan, 2015; Serra & Nairne, 2000). For example, counting sequences are easier to judge because of the stronger associations between them (e.g., 1 2 3), compared to sequences with larger distance (e.g., 1 4 7; weaker associations). Both the serial scanning and associative-chaining accounts suggest that fluency of access to counting sequences (either through scanning or activation of associations between the digits) is relevant for the presence of reversed distance effect.

Although the presence of the reversed distance effect in symbolic order judgments is often used to index ordinal processing, it is not as robust and consistent as the distance effect found for cardinal judgments (symbolic number comparison task; e.g., Sasanguie et al., 2017; Sasanguie & Vos, 2018; Vogel et al., 2015). Two main factors may account for individual differences in the reversed distance effect for order judgments. First, in different versions of order judgment tasks, the digits that need to be judged as ascending order may be pairs (e.g., 4 5) or triplets (e.g., 1 2 3). A reversed distance effect is more likely to occur for order judgment tasks with number triplets (e.g., Goffin & Ansari, 2016; Lyons & Ansari, 2015), whereas a standard distance effect (as observed in the symbolic number comparison task) is often present for order judgment tasks with number pairs (e.g., Sasanguie et al., 2017; Vogel et al., 2015). These results suggest that judging the order of pairs of digits may promote reliance on cardinal strategies (i.e., successive number comparisons; e.g., 5 is larger than 4, and thus 4 5 is in ascending order). Therefore, in an attempt to distinguish processing mechanisms for cardinal and ordinal processing, using triplets of digits (e.g., 1 2 3 or 3 5 9) for the order judgment task may be more appropriate.

The types of sequence used in the order judgment task also vary across studies, which may be related to the individual differences in the reversed distance effect. For example, in the standard binary choice version of the order judgment task, by necessity participants see both the ordered (e.g., 1 2 3, 1 3 5, 1 4 7) and un-ordered versions of number sequences (e.g., 3 1 2, 5 1 3, 4 1 7). Research shows that the reversed distance effect is only present for trials that are correctly ordered, not for stimuli that are not ordered (e.g., Franklin & Jonides, 2009; Lyons & Ansari, 2015; Vos et al., 2017). Furthermore, for the correctly ordered sequences, the numerical distance between digits can be one (i.e., counting sequences; e.g., 1 2 3 or 7 8 9) or more than one (non-counting sequences; e.g., 1 2 5 or 2 5 9). Specifically, the reversed distance effects are largely driven by the trials with counting sequences, that is, participants are fastest and most accurate at judging the correctly ordered counting sequences than any other types of sequences (e.g., Vos et al., 2017; Lyons & Ansari, 2015). Similar results were found using pairs of digits: the reverse distance effect was only present with sequential number pairs (e.g., 4 5) not with non-sequential pairs (e.g., 3 7; Turconi et al., 2006). Taken together, these results suggest that the fluency of access to counting sequences is relevant for the presence of reversed distance effect in the symbolic order judgment task.

Sequential associations. One possibility for why the reversed distance effect exists, specifically, the finding that counting sequences are easier to judge than other types of sequences in symbolic order judgment tasks, is that counting sequences are more familiar to solvers than other types of sequences, resulting in stronger associations between these adjacent numbers and thus faster recognition (Bourassa, 2014; LeFevre & Bisanz, 1986; Turconi et al., 2006). For example, Bourassa (2014) found that adults show the fastest activation of related numerical information on ordered counting sequences (e.g., 4 5 6), whereas they are slow and least accurate on sequences that closely resemble these familiar sequences but are incorrectly ordered (5 6 4). This finding suggests that the ability to fluently recognize familiar number associations may be important for supporting the development of ordinal processing. Because correctly ordered counting sequences are overlearned, solvers are able to directly retrieve the sequential associations stored in long-term memory (LeFevre & Bisanz, 1986), explaining why the reversed distance effect is driven by correctly ordered counting sequences (Lyons et al., 2016). Therefore, order judgments for the counting sequences may involve the activation of sequential associations among number symbols.

In contrast, if the sequence of numbers to be judged is not in order (e.g., 2 1 3 or 9 7 8), or if the distance between digits is more than 1 (i.e., non-counting sequences; e.g., 1 2 8 or 7 3 4), then it is unlikely that solvers, particularly young children, would rely on direct retrieval of sequential associations to make order judgments. If so, then how do children make decisions about order for those unfamiliar number sequences?

Cardinal associations. One possibility is that magnitude-based mechanisms of assessing relative order may take over the retrieval-based mechanisms when people make order judgments for unfamiliar number sequences (Lyons et al., 2016). For example, on sequences that are not part of the sequential counting string (e.g., 4 2 7 or 2 7 4), adults were faster when the order violation occurred in the first two digits when the task was to judge if the sequence of numbers was in increasing order. For example, in 4 2 7 the violation of increasing order occurred in the first two digits; these sequences were rejected more quickly than ones such as 2 7 4 where the order violation occurred in the second and third digits (Bourassa, 2014). These results indicate that participants were using sequential magnitude comparisons to decide whether sequences were ordered. Therefore, the order judgment task may involve both recognizing sequential associations among number symbols and cardinal processing (i.e., successive number comparisons), depending on the types of sequences and the instructions used in the order judgment task.

Furthermore, the order judgment task requires participants to accept counting sequences such as 3 4 5 as correctly ordered but reject incorrectly ordered counting sequences such as 4 5 3. Research with adults shows that the latter decision is difficult (Lyons & Beilock, 2009), presumably because solvers may need to suppress responses that may activate the elements of a counting sequence even when those elements are not ordered (e.g., 4 5 3; Bourassa, 2014), and thus the order judgment task may also involve a strong requirement for *inhibitory* processing.

Inhibitory processing. The development of inhibitory control has received a great amount of attention in the literature because it has shown to be related to children's mathematical development (Bull & Scerif, 2001; Espy et al., 2004; Friso-van den Bos, van der Ven, Kroesbergen, van Luit, 2013; Laski & Dulaney, 2015). During the early years, children's inhibitory skills develop rapidly along with other executive functions[1] such as working memory/updating, and cognitive flexibility/shifting/task switching (Carlson, 2005). The concept of inhibitory control has been defined and measured in several ways (e.g., Fridman & Miyake, 2004; James, Choi, Woene, & Espy, 2016; Logan & Cowan, 1984; Nigg, 2000). Notably, the construct validity of inhibitory processing is low because the inhibitory control tasks involve a range of different cognitive processes,

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[1] Executive functions refer to the self-regulatory, general cognitive processes that are used to monitor and for control of thought and action.

which makes it difficult to isolate the specific construct from other executive functioning skills. More generally, this limitation is associated with the problem of task impurity in studies of executive functions, that is, there is no agreement on a set of tasks that tap onto a single underlying construct because any executive task requires the coordination of multiple processes (e.g., Clark et al., 2016; Friedman, 2016; Miyake et al., 2000; Welsh, 2002).

In the present dissertation, I focused on one type of inhibition, *response inhibition*, which is defined as the ability to suppress an automatic (prepotent) behaviour depending on task demands (Friedman & Miyake, 2004; Simmonds, Pekar, & Mostofsky, 2008). A great number of tasks can be used to measure “response inhibition”, such as stop-signal task and go/no-go task (i.e., participants are asked to either respond or withhold a response depending on instructions; Logan & Cowan, 1984; Simpson & Riggs, 2006), Stroop tasks (i.e., participants are asked to name the actual colour of the word, such as saying “blue” when they see RED printed in blue colour ink, while ignoring the meaning of the words; Stroop, 1935), and Eriksen flanker tasks (i.e., participants are asked to press the button that corresponds to the direction that a target stimulus is facing when other stimuli are facing the same or opposite directions; Eriksen & Eriksen, 1974). Response inhibition may be involved in number ordering because children may need to withhold an automatic response to accept an unordered counting sequence (e.g., 2 3 1).

More broadly, how is inhibitory processing involved in the development of symbolic number knowledge? Numbers have various associations that need to be differentially activated depending on the context. For example, consider the numbers 3 and 4: they are differentially associated with 5 (through counting associations), with 7 (through addition associations), and with 12 (through multiplication associations). Thus, children may need to suppress the interference from earlier acquired knowledge of the counting sequence (Siegler, 1988; Siegler & Shrager, 1984) to associate 7 with 3 and 4. Further, when children initially learn multiplication, they need to suppress the interference from addition associations (Miller & Paredes, 1990) to associate 12 with 3 and 4. The ability to inhibit earlier developed knowledge may be related to children's mathematical development. In one study, LeFevre and colleges had children in grade 3 through 5 perform a number-matching task where they saw a pair of numbers followed by a target and decided if the target matched one of the numbers in the pair (e.g., they were given "7 + 2" and were asked to verify the presence of a target digit, e.g., 7; LeFevre, Kulak, & Bisanz, 1991). LeFevre et al. (1991) found that children showed an interference effect (i.e., made slower and less accurate responses) on target digits that were closer to the initial pair (e.g., 6), suggesting an automatic activation of counting-based strategy for addition problems. In contrast, older and more-skilled children also showed interference from addition associations (e.g., 4 5 followed by 9; Lemaire, Barrett, Fayol, & Abdi, 1994) and adults also showed interference from multiplication associations (Galfano et al., 2011; Thibodeau, LeFevre, & Bisanz, 1996). Similarly, adults who were more skilled at arithmetic showed more interference effect for numbers with arithmetic associations in the number-matching task than less-skilled adults (LeFevre & Kulak, 1994). These findings support the view that the use of more advanced knowledge when children perform tasks may require the inhibition of earlier acquired knowledge.

In the context of the Hierarchical Symbol Integration (HSI) model, a hierarchy of activation of relevant associations may exist in which access to more basic associations is inhibited when access to higher-level associations is appropriate (Xu et al., under review). As suggested by my review of the literature, sequential and cardinal associations are important building blocks of ordinal processing. For children who have fewer numerical experiences, inhibitory processing may not be relevant in the decision-making process because the symbolic representations of sequential associations (e.g., 2 3 4 or 4 5 6) are not automatically activated and thus children are more likely to rely on cardinal mechanisms via successive number comparisons to make order judgments. Conversely, with mastery of the sequential associations, children are more likely to show interference effects for the incorrectly ordered counting sequences (e.g., 2 3 1 or 4 5 3) because they may need to inhibit responses that activate the elements of a counting sequence (e.g., 2 3 followed by 4; 4 5 followed by 6), indicating that they have developed strong sequential associations of numbers. Thus, inhibitory processing may be heavily involved in the symbolic order judgment task (judging the unordered counting sequences), particularly for children who have fluent access to the symbolic representations of sequential knowledge of numbers. To some extent, therefore, using the order judgment task that combines ordered and unordered sequences may partially confound measurement of ordinal knowledge with measurement of inhibitory processes and thus inflate the relations between the order judgment task and other mathematical tasks that also require inhibitory skills.

In summary, ordinal associations among numbers are assumed to integrate with sequential and cardinal associations as children advance their symbolic number knowledge through activating these associations in numerical activities. Inhibitory control may be also involved when children have to suppress the activation of sequential associations for the incorrectly ordered counting sequences. One implication is that the order judgment task that is typically used with adults may involve a range of cognitive abilities, including cardinal associations (successive magnitude comparisons), sequential associations (activated automatically for pairs of numbers) and ordinal knowledge (combinations of numbers that can be described as 'ascending' or 'descending' sequences). In the next section, I provided an overview of how the acquisition of cardinal, sequential, and ordinal associations are related to the initial development of arithmetic skills (i.e., mental addition).

Early Arithmetic Development

During the early development of mental addition, children gradually progress from using less efficient calculation strategies (e.g., counting with fingers) to directly retrieving the answer from long-term memory (Ashcraft, 1982; Carpenter & Moser, 1984; Siegler & Shrager, 1984). Prior to formal schooling, children use their fingers (or other objects) to represent the quantity of both sets when they solve addition problems and use a *counting-all strategy* (or *sum strategy*) by counting all of their fingers (Siegler & Jenkins, 1989). For example, to solve an addition problem such as 3 + 5, children who use a *sum strategy* hold up three fingers on one hand representing the first addend, then on the other hand, hold up five fingers representing the second addend, and lastly count all of these fingers (e.g., 1-2-3-4-5-6-7-8). These strategies require that children can represent both quantities separately with an external referent and then use their counting skills to determine the total quantity.

Once children understand that it is not necessary to represent both of the addends externally and count them all to solve addition problems, they begin to count forward from the first addend by the second addend using a *counting-on-first strategy*. For example, to solve 3 + 5, they would count on from the first addend (3) by five numbers (i.e., 4-5-6-7-8). Use of this strategy implies that children have a mental representation of one quantity, even if they need an external referent for the second quantity.

Later, children learn to use a counting-on-larger strategy (or *min strategy)*, in which they determine the larger number first and then count forward by the smaller number (Groen & Parkman, 1972). For example, to solve 3 + 5, children who use a *min strategy* would determine the larger addend first (5), and then directly count on three numbers from 5 (6-7-8), giving the final number word counted as the answer. Use of the min strategy implies that children are able to compare the relative quantities of the two symbols, decide which is larger, and then count-on by the smaller. As described above, the use of the min strategy for addition problems presumably involves number comparisons (i.e., activation of cardinal associations) and reciting the number sequence as children count on from the base number (i.e., activation of sequential associations). Thus, the *min strategy* involves both cardinal knowledge (i.e., to choose the starting point), and sequential knowledge (i.e., which number comes next). On this view, children who have better access to cardinal and sequential associations should be able to use the min strategy more efficiently than other children whose associations are weaker.

Through extensive practice, children start to memorize the number facts for familiar number combinations such as tie problems (e.g., 2 + 2 = 4; 3 + 3 = 6), allowing them to directly retrieve the answer without a counting strategy (Groen & Parkman, 1972). This implies that children have stored some of the addition associations in their mental network, which serves as the basis for them to learn and apply more sophisticated arithmetic strategies.

Later, when children learn addition problems with sums larger than 10, children learn a *decomposition strategy* in which they transform the unfamiliar problem into a familiar one, for example, by decomposing a larger number into two smaller sets (e.g., 9 + 4 = (9 + 1 = 10) + 3; Siegler, 1987). Decomposition strategies require children to manipulate number sets based on their understanding of relations among symbolic numbers, and to use their stored addition associations to simplify the computation, for example, their knowledge of the pairs of numbers which sum to 10 (Geary, Hoard, Byrd-Cravern, & DeSoto, 2004). Thus, the mental manipulation of number sets that involve the execution of the decomposition strategy presumably requires a strong ordinal knowledge. Taken together, the use and the development of these arithmetic strategies may activate children's cardinal associations, sequential associations, ordinal associations, and eventually arithmetic associations to produce fast and accurate solutions to the range of problems from 1 + 1 through 9 + 9. Thus, the development and integration of children's understanding of the various associations among symbolic numbers may be important for their improvement in strategy choice in arithmetic problem solving.

According to Siegler's adaptive strategy choice model, children's strategy choice at any given age on mental arithmetic is variable (Siegler, 2000, 2007; Siegler & Shrager, 1984). During the first two years of elementary school, children use three main types of addition strategies: counting, decomposition, and direct retrieval to solve single-digit addition problems (Geary et al., 2004; Lindberg, Linkersdorfer, Lehmann, Hasselhorn, &

Lonnemann, 2013; Shrager & Siegler, 1998). The development of addition fluency is reflected in the more-frequent use of more efficient strategies (i.e., min strategy, decomposition and direct retrieval), and the less-frequent use of the less efficient strategies (i.e., sum strategy; Siegler, 1987). Because arithmetic associations that allow direct access to addition facts develop slowly, the fluency of executing the most efficient counting strategy (i.e., *min strategy)* and decomposition strategy is assumed to be an important building block of early addition.

In the present dissertation, the acquisition of ordinal associations (i.e., ordered associations that go beyond sequential number pairs) is assumed to involve direct access to cardinal (i.e., activation of the associations between the symbolic numerals and the quantitative information they represent) and sequential knowledge (i.e., activation of the associations between the adjacent numbers from the count list), the same kind of knowledge required for successful execution of the min strategy and decomposition strategies. Therefore, the integration of cardinal and sequential associations into a coherent associative network may be critical for learners as preparation for understanding and development of mental arithmetic skills.

Present Research

The main goal of the present dissertation was to explore the process of how children in early grades of elementary school integrate cardinal and sequential associations to form ordinal associations of number, on the view that this process forms the foundation for the development of higher-level numerical processing (e.g., mental arithmetic). Children in grades 1 and 2 were compared because previous research has shown that the relative importance of symbolic number processing for predicting individual differences in arithmetic changes from cardinal to ordinal tasks between grades 1 and 2 (Sasanguie & Vos, 2018). In addition, children's mathematical skills were assumed to vary dramatically between grades 1 and 2 given that solving simple arithmetic problems is one foci of mathematics education in the early grades of elementary school in Ontario, Canada[2].

I used the Hierarchical Symbolic Integration (HSI) model proposed by Xu et al. (under review) as the central framework for modeling the integration process of cardinal and ordinal associations. That is, cardinal and sequential associations serve as the base of the mental network, followed by ordinal associations, and arithmetic associations join later (Figure 1; Xu et al., under review). The main goal of the present dissertation was to expand the HSI model by exploring the integration process of various number associations for children in grades 1 and 2. In particular, what are the building blocks of the acquisition of ordinal associations? Based on the earlier literature review, the order judgment task is assumed to reflect the extent to which individuals have strong associative relations among number symbols, that is, cardinal and sequential associations among numbers. Inhibitory processes may be also involved in certain tasks, allowing participants to reject a specific type of non-ordered sequence (i.e., incorrectly ordered counting sequences such as 2 1 3 or 4 5 3). Thus, the acquisition of ordinal associations for children in grades 1 and 2 is expected to involve three components: 1) sequential associations among numbers, 2) cardinal associations among numbers, and 3) inhibitory processing. To evaluate these questions, I needed to develop several new measures of

[2] According to the Ontario Mathematics Curriculum, children are expected to learn to "solve problems involving the addition and subtraction of single-digit [grade 1], or single- and two-digit [grade 2] whole numbers, using a variety of strategies".

sequential and ordinal processing. Cardinal processing was measured with a standard number comparison task (i.e., choose the larger of two digits).

Scoring task performance. One of the most commonly used scoring methods for the speeded number comparison and number order tasks is the ‘inverse efficiency’ score which combines response time (RT) and accuracy to account for potential speed-accuracy trade-offs. For example, Sasanguie and Vos (2018) used “RT/Accuracy” whereas Lyons and Ansari (2015) use “RT + RT x [2 x Percentage Error]”. These measures are applied to mean scores in a given condition. Both measures are equal to RT when accuracy is perfect. At 50% accuracy, however, a response time of 2000 ms would be scored as 4000 ms (2000/50%) for the first measure and as 4000 ms (2000 + 2000 x 2 x 50%) for the second measure. Thus, both of the adjusted RT scores increase proportionally to the number of errors. There is no consensus regarding which type of combined score is the best, nor have strong arguments been proposed that explain why a particular way of correcting for errors is better or whether, across tasks, these adjusted scores have similar properties. In the present research, I first examined both accuracy and response time for each type of numerical measures. Given that the tasks are designed to be easy enough for children in grades 1 and 2 to be successful, I expected that most of the children should reach ceiling based on the accuracy scores. Thus, for the analyses regarding the expanded HSI model, I used the response time measure on correct trials as the index of the fluency to access individual differences in the accessibility of various associations.

Cardinal associations. The *symbolic number comparison* task was used as the index of cardinal processing in the present research. As shown in Figure 2.1, children were asked to indicate the numerically larger of two single-digit numbers (e.g., 3 6) as fast as possible by touching the larger on the screen of an iPad. The speed that children accurately select the *larger number* was used to measure children's fluency of accessing cardinal associations of numbers.

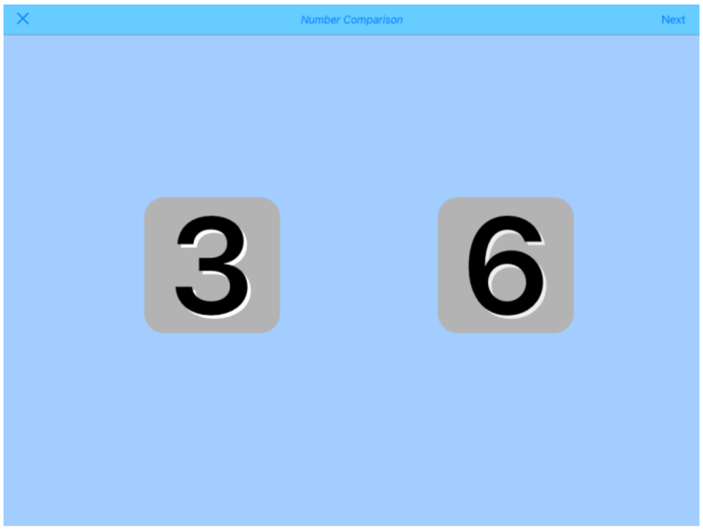

Figure 2.1. An example of an item from the symbolic number comparison task.

Sequential associations. The *missing number* task was used to assess children's acquisition of the sequential associations of numbers. As shown in Figure 2.2, children were shown consecutive numbers with one missing and asked to verbally indicate which number was missing (e.g., 1 __ 3 4; see a similar missing number task in Lee & Lembke, 2016). The speed with which children accurately stated the missing number was used to measure fluency of recognizing sequential patterns of numerical symbols. The assumption was that the role of cardinal processing (i.e., number comparisons) and other cognitive mechanisms (e.g., inhibitory control) was minimized in the missing number task and thus that is it a purer and easier measure of children's sequential number knowledge than typical order judgment tasks.

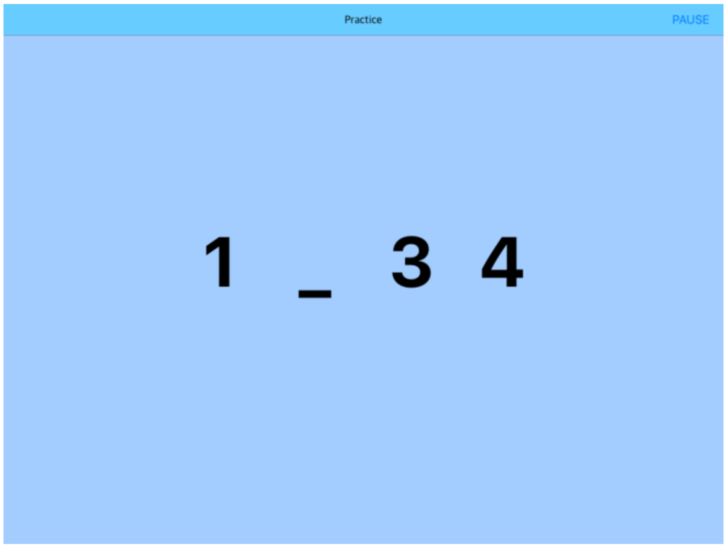

Figure 2.2. An example of an item from the missing number task.

Inhibitory processing. In the present research, I included two types of inhibitory control tasks, namely the *black/white Stroop* task, and the *go/no-go* task, to index one aspect of children's inhibitory control skills, that is, response inhibition (Simmonds et al., 2008). Although both of tasks are assumed to measure response inhibition, different measures of inhibitory control do not always tap a single construct (Brocki & Bohlin, 2004; James et al., 2016). In particular, the correlations between the go/no-go task and Stroop-like tasks vary across studies for preschool-aged children (James et al., 2016). Similarly, for school-aged children, the go/no-go task and the Stroop-like task have been shown to be loaded on different latent factors, suggesting that they may tap into different constructs (Brocki & Bohlin, 2004). One possibility for the lack of relationship between the go/no-go and Stroop-like tasks is that they may require different levels of cognitive demand (Pennington, 1997). That is, the go/no-go task requires children to withhold a response while doing nothing (i.e., when they see a cat, they are instructed to do nothing) while other types of cognitive demands are minimized (Simmonds et al., 2008). In contrast, the Stroop-like task requires children to withhold a response and then to switch to a new response (i.e., for the congruent trials, they are instructed to touch the button with the same color; for the incongruent trials, they are instructed to touch the button with the different color). Thus, the black/white Stroop task may also tap into other aspects of executive functions that are required to guide response inhibition, such as working memory and shifting attention/cognitive flexibility (Brocki & Bohlin, 2004; Conway & Engle, 1994; Simmonds et al., 2008). Moreover, the black/white Stroop requires children to suppress their previously overlearned knowledge (e.g., the automatic activation of the meaning of the colour word) whereas the go/no-go task does not involve any suppression of overlearned knowledge. In summary, these two types of inhibitory control tasks may tap into different constructs, although both tasks are assumed to measure "response inhibition". There is no research in the literature on which type(s) of inhibitory processing is involved in ordinal tasks, thus, in the present research, I included both types of inhibitory tasks with the goal of choosing the better measure as the index of children's inhibitory processing based on reliability and face validity for this age group.

Black/white Stroop. Stroop tasks, in which a well-learned response interferes with a target task (e.g., reading interferes with naming the colour of a word; MacLeod, 1991) are often used to index individual differences in inhibitory control (see Laird et al., 2005 for a review). A *black/white Stroop* task has been suggested as a good measure of children's inhibitory control skills because it does not pose extraneous memory demands as some other similar Stroop tasks such as day-night Stroop and colour-word Stroop tasks (Vendetti, Kamawar, Podjarny, & Astle, 2015). In the black/white *Stroop* task, children hear a colour name (black or white) and they are asked to press the same (congruent) or different (incongruent) colored button as quickly as they can on the screen of an iPad (see an example in Figure 2.3). An interference score was used to index children's inhibitory control (see Chapter 3 for detailed explanations of scoring).

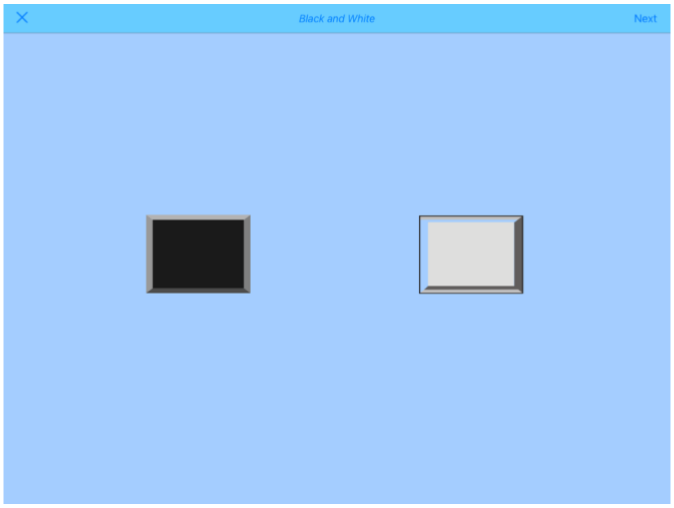

Figure 2.3. An example of an item from the black/white Stroop task.

Go/no-go. So-called, go-no go tasks in which participants respond selectively to certain stimuli are also often used to index inhibitory control (Simmonds et al., 2008). An adapted version of the *go/no-go* task developed by Simpson and Riggs (2006) was used for children in the present research as the second measure of inhibition skills (Gray & Reeve, 2014). In this task, children develop an automatic (prepotent) motor response to more frequently appearing target stimuli (e.g., by touching the stimuli or pressing a specific button), and then they need to inhibit this response when a less frequently appearing stimuli (no-go trials) appears. More specifically, an image of a mouse (go trials) or a cat (no-go trials) appears on the iPad screen, and children were asked to touch the stimulus on go trials while withholding responses on no-go trials. A *d* prime score is used to index children's inhibitory control (see Chapter 3 for detailed explanations of scoring).

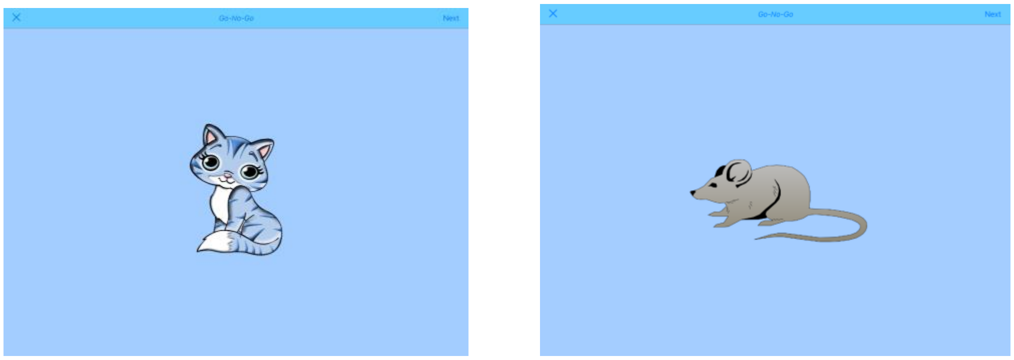

Figure 2.4. An example of an item from the go/no-go task. Children are instructed to touch the mouse (to catch it) but to not respond to the cat.

Ordinal associations. Children's acquisition of ordinal associations of numbers was measured by a modified (simpler) version of the order judgment task, a *number order* task. As shown in Figure 2.5, children were asked to touch the three digits in an ascending order one by one on the screen of an iPad rather than making a binary decision about whether three digits are in order or not as in the traditional symbolic order judgment task (e.g., Lyons & Beilock, 2011; Lyons et al., 2014; Sasanguie & Vos, 2018). In a pilot study for the present research, young children found it challenging to understand the concept of "in order" when it was explained as touching the numbers that "comes first, next, and last in a sequence". As a result, in the present research, the concept of "in order" was explicitly instructed to children as "from smallest to the largest" in a sequence. Thus, it seems that children needed to be told that the type of ordering was by size to comprehend the instructions.

This number order task is similar to the order judgment task used by other researchers (e.g., Lyons et al., 2014) in that it includes a sequence of numbers but it differs from those measures in that it does not require children to reject some sequences and accept others; instead, they are consistently required to determine how to order the numbers. Moreover, this number order task might activate inhibitory processing for children who have fluent access to the sequential associations of numbers. However, the inhibitory demand of the number order task is assumed to be smaller than that in the order judgment task, because the former has fewer cognitive demands (i.e., less need to inhibit unordered sequences). Together, the assumption is that the number order task involves three elements of cognitive processing: a) the activation of sequential associations among number symbols, b) the activation of cardinal associations between number symbols through successive number comparisons, and c) some requirement for inhibitory control.

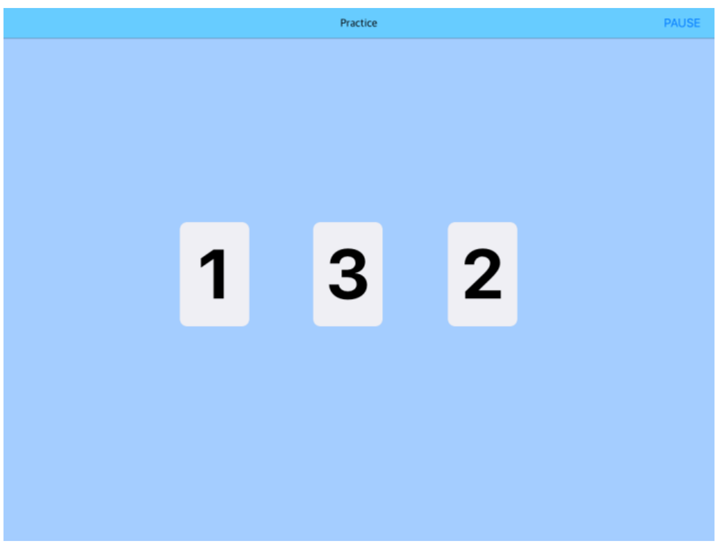

Figure 2.5. An example of an item from the number order task.

Design. Two studies were carried out to examine the integration process for children in early grades of elementary school. Study 1 was conducted to refine the measures that were used in Study 2. In particular, children who were entering grades 1 to 3 were tested on their sequential, cardinal, and ordinal knowledge of numbers, as well as inhibitory control skills. Arithmetic skills were measured by two speeded addition tasks (see Chapter 3 for details). Based on the preliminary findings of Study 1, in Study 2, the same measures of sequential cardinal, ordinal associations of numbers, and a modified addition task were used for children in grades 1 and 2. Specifically, children were tested at two time points: near the beginning (T1) and near the end of the school year (T2), with four months between testing times, to examine the developmental course of number integration in relations to the development of mental addition. The methodology and results for Study 2 are presented in Chapter 4.

Notably, the present dissertation only focuses on one aspect of children's developmental trajectory from fundamental associations (i.e., cardinal, sequential, and ordinal associations) to arithmetic development. However, there are other types of mathematical skills that are plausibly linked with these fundamental associations through access to symbolic number skills. For example, children's ability to place numbers on a line with endpoints marked (i.e.. number line estimation) is consistently correlated with a wide range of mathematical competence that involves knowledge of the interrelations among symbolic numbers (Schneider et al., 2018). Solving word problems also involves fundamental symbolic number knowledge as well as other kinds of cognitive skills, particularly language skills (e.g., Fuchs et al., 2008). In the present research, children also completed a number line estimation task and a problem solving task, as well as four other cognitive tasks in Study 2. However, these measures were collected to test other hypotheses that are not related to the specific predictions of the present dissertation, and thus they were not included in the main text of the present dissertation (see detailed information about these additional measures in Appendix B).

Hypotheses. My hypotheses for the main study (Study 2) are presented in Figure 2.6 and Figure 2.7 and explained as follows. First, I examined integration status for ordinal associations for children in grades 1 and 2. I hypothesized that children in grade 1 are unlikely to have integrated cardinal and sequential associations into ordinal associations and thus only cardinal knowledge would be relevant for the number order task, because children were expected to rely on successive number comparisons to order numbers (*Hypothesis 1a; Figure 2.6*). In contrast, for children in grade 2 who are more likely to have integrated cardinal and sequential associations, sequential, cardinal and inhibitory processing would predict ordinal processing (*Hypothesis 1b; Figure 2.7*).

Second, I examined how various forms of associations (sequential, cardinal and ordinal) were concurrently related to arithmetic associations for children in grades 1 and 2. For children in grade 1, whose numerical associations are not integrated yet, cardinal and sequential associations, rather than ordinal associations, should uniquely predict arithmetic associations (*Hypothesis 2a; Figure 2.6*). In contrast, for children in grade 2, who were expected to have integrated cardinal and sequential associations, ordinal associations should supersede the more fundamental forms of associations (cardinal and sequential) as the best predictor of arithmetic (*Hypothesis 2b; Figure 2.7*).

Lastly, I examined which specific form of associations would predict the growth of arithmetic associations. I hypothesized that, for children in grade 1, cardinal and sequential associations, the more basic forms of associations of the hierarchical mental network, would predict the growth of arithmetic, after controlling for initial arithmetic performance (*Hypothesis 3a; Figure 2.6*). This hypothesis is based on the assumption that children are using counting strategies to solve most problems, especially the min strategy, which involves both number comparisons and counting. In contrast, for children in grade 2, the higher-level of numerical associations of the hierarchical mental network, ordinal associations, would uniquely predict the growth of arithmetic, after controlling for their Time 1 arithmetic performance (*Hypothesis 3b; Figure 2.7*). This hypothesis is based on the assumption that the ordinal task involves manipulation of numerical symbols through the activation of sequential and cardinal associations, the same kind of knowledge required for successful execution of the min strategy and decomposition strategy that children commonly use to solve mental addition tasks.

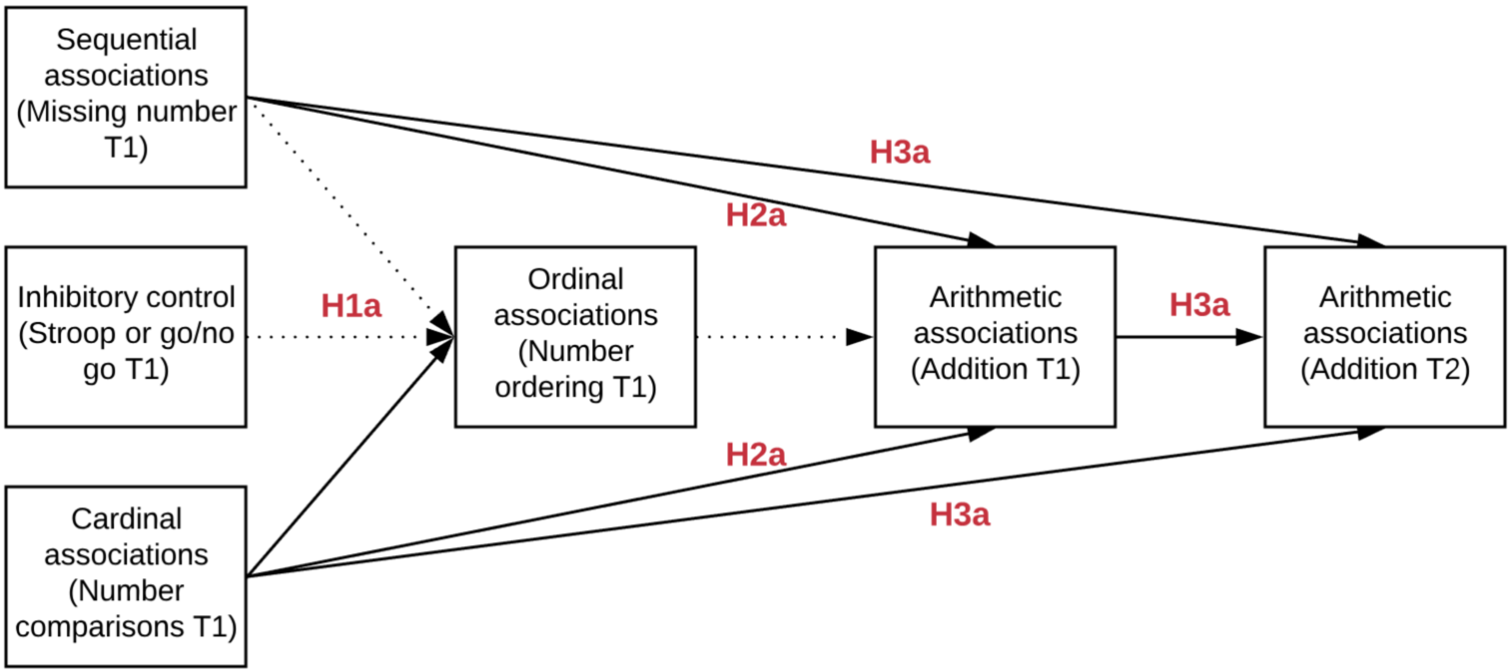

Figure 2.6. Hypotheses for the Expanded *HSI* model for children in grade 1

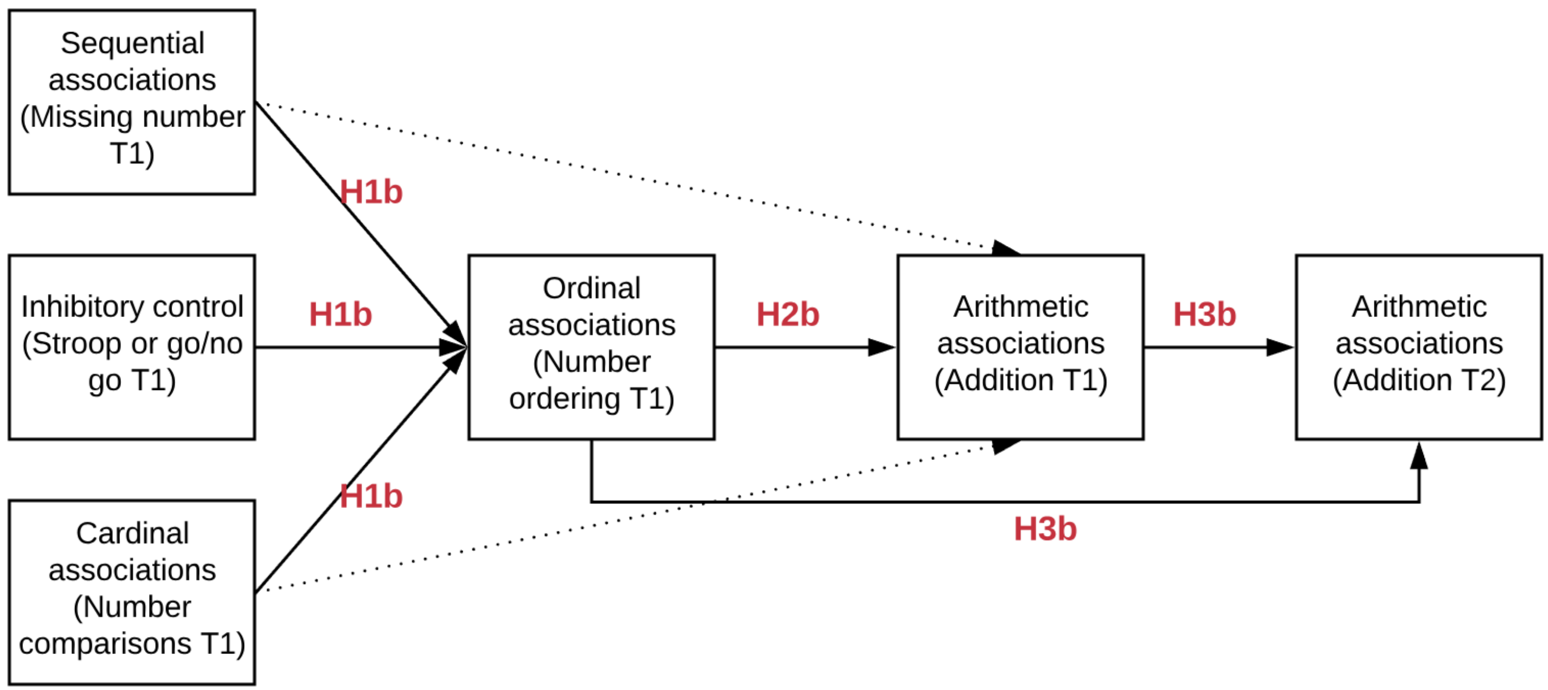

Figure 2.7. Hypotheses for the Expanded *HSI* model for children in grade 2

CHAPTER 3: STUDY 1

The primary goal of Study 1 was to examine the internal consistency and validity of the new measures that were assumed to tap into children's fluency to access numerical associations for children in early grades of elementary school. In particular, I used two simpler measures of ordinal processing than the order judgment task used by other researchers (Lyons et al., 2014; Sasanguie & Vos, 2018) in an attempt to isolate sequential knowledge (i.e., missing number task) from ordinal knowledge (i.e., number order task) and minimize the role of inhibitory processing in the ordinal task.

Method

Participants. Data were collected during the summer months for children who were transitioning into grades 1, 2, or 3. A sample of 70 children was recruited from a local community centre, libraries, Saturday language schools, or through direct contact with parents in Ottawa, Canada[3]. Thirty children were entering grade 1 (*M* = 77 months, *SD* = 6 months, range from 5:10 to 8:1 year:months; 12 girls); 24 children were entering grade 2 (*M* = 85 months, *SD* = 3 months, range from 6:9 to 7:9 year:months; 11 girls); and 16 children were entering grade 3 (*M* = 99 months, *SD* = 3 months, range from 7:8 to 9:4 year:months; 7 girls).

Parents were asked to indicate, "How often does your child speak English/Chinese/French at home?" based on a five-point scale ("always", "often", "about half of the time", "sometimes" or "occasionally"). The majority of children (96%) spoke English fluently: 59% of the children spoke English at home "always" or

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[3] Note that because of the recruiting strategy, 48 of the participating children had parents who had been born and educated in China. Thus, they were probably relatively high skilled in comparison to more diverse samples of Canadian children (Xu & LeFevre, 2018). For the present purposes, this characteristic of the present sample was not a problem because the main goal of Study 1 was to pilot test the new measures of ordinal processing for a wide age range of children.

"often". The majority of the children also spoke another language at home in addition to English: 57% of the children spoke Chinese at home at least half of the time (i.e., "always" or "often" or "half of the time"), and 48% of the children spoke French at home "occasionally" or "sometimes".

Information about mother's highest education level was collected from the consent form for 67 of 70 participants. Of those who responded to the questionnaire, 36% had received a postgraduate degree, 37% had received an undergraduate degree, 16% had received a community college degree, 3% had received a high school diploma, and 3% had received less than a high school diploma (*Mdn* = undergraduate degree). This relatively high level of education is consistent with other Canadian samples (e.g., Xu & LeFevre, 2016, 2018; Jiménez Lira et al., 2017; Skwarchuk, Sowinski, & LeFevre, 2014).

Procedure. All of the children met individually with one of six female experimenters for about 25 minutes at a quiet space at a public library near the child's home, in a laboratory at Carleton University, or at the child's home. Children received stickers, a participation award certificate and a $10 gift card as compensation for their participation.

Measures. The data were collected as part of a different project that had a different purpose, and thus children were also assessed on other cognitive measures, and their parents completed a questionnaire regarding home numeracy environment. Only the measures that were relevant to the research goals in the present dissertation and the relevant analyses are presented here.

Number comparison task. Children completed a symbolic number comparison task on an iPad as the index of their knowledge of cardinal associations of number. For each trial, children were presented with a pair of single-digit numbers (ranging from 1 to 9) on the screen, and they were asked to choose the numerically larger number as quickly as possible. Children who failed to respond after 3 seconds were presented with the next trial. There were 26 trials, and the distance between the two numbers was manipulated such that half of the trials had a small distance between the digits (i.e., the difference ranged from 1 to 3), whereas the other half had a large distance (i.e., the difference ranged from 4 to 7). The ratio between the digits was also manipulated such that each ratio was repeated once in random order, and each number was counterbalanced for the side of presentation (see the full stimuli set in Appendices Table A.1). The scoring was based on accuracy and mean RT on correct trials. Internal reliability of the number comparison task was excellent (split-half, $r = .97$, based on the odd/even items model; Cronbach's $\alpha = .94$).

Missing number task. Children completed a missing number task on an iPad as the index of their knowledge of sequential associations of numbers. Children were asked to find out the missing number from a set of four numbers in a counting sequence. In particular, for each trial, four single-digit numbers were presented in ascending order with one of them missing (e.g., 1 __3 4). The position of the blank in the number sequence was the second position for half the sequences and the third position for the other half of the sequences (see the full stimuli set in Appendices Table A.2). Children were asked to verbally report the missing number as quickly as possible, and the experimenter entered their response using a separate keyboard that was connected to an iPad. Children who failed to respond after 7 seconds were presented with the next trial. Children were given two practice trials to familiarize them with the procedure. There were a total of 10 experimental trials. Scoring was based on accuracy and mean RT on correct trials. Internal reliability of the missing number task was excellent (split-half, $r = .90$, based on the odd/even items model;

Cronbach's α = .89).

Number order task. Children completed a number order task on an iPad as the index of their knowledge of ordinal associations of number. For each trial, children were shown three numbers on the screen (e.g., 5 7 2), and they were asked to touch the number that comes first, next and last in sequence as fast as possible. For example, the experimenter said: "Look at these numbers. I want you to touch them in order from the smallest to the largest as fast as you can". Children were given three practice trials to ensure that they understood the instructions. Feedback was given on the practice trials. If the child hesitated or was not able to order the numbers 5 7 2 correctly, guidance was provided: "Which number is the smallest? Which number comes after 2? Which number comes last in this sequence?"

After the practice trials, 24 sequences (with digits ranging from 1 to 9) were presented to children (see the full stimuli set in Appendices Table A.3). Children who failed to respond after 7 seconds were presented with the next trial. Twelve of these sequences had adjacent numbers (counting sequences; e.g., 1 2 3 or 2 3 1) and 12 sequences had non-adjacent numbers (non-counting sequences; e.g., 4 5 7 or 5 4 7). Half of the sequences were presented in order (e.g., 1 2 3 or 4 5 7) and the other half of the sequences were not presented in order (e.g., 2 3 1 or 5 4 7). Reaction time was recorded from the time that the child touched the first number to the last number. Scoring was based on accuracy and mean RT on correct trials. Internal reliability of the number order task was excellent (split-half, r = .99, based on the odd/even items model; Cronbach's α = .95).

Black/white Stroop task. The black/white Stroop task adapted from Vendetti et al. (2015) was used to measure children's inhibitory control skills using an iPad. In this task, children saw a fixation cross in the center of the screen for 500 ms, then a blank screen for 500 ms, followed by a stimulus (black square or white square) on the screen. Children were asked to touch the square that was the same or different from the color name they heard as quickly as possible. Children who failed to respond after 3 seconds were presented with the next trial. Half of the trials were congruent trials (e.g., if they hear "white", they should press the "white" square), whereas the other half of the trials were incongruent trials (e.g., if they hear "white", they should press the "black" square). Feedback was given on the practice trials to ensure that the child understood the instructions. There were two blocks of congruent trials and two blocks of incongruent trials (8 trials each). Thus, each child completed a total of 32 trials for the black/white Stroop task.

The accuracy for the four blocks of trials on the Stroop task was high: 98% correct on the first (congruent) block, 90% correct on the second (incongruent) block, 96% correct on the third (congruent) block, and 88% correct on the last (incongruent) block. The score of this task was calculated based on the RT difference (on correct trials) between the average RT of congruent and incongruent trials as a measure of interference cost (mean RT of incongruent trials – mean RT of the congruent trials). Higher score on this Stroop task indicates higher interference cost, indicating poorer inhibitory control skills. The reliability of the interference score was calculated based on all possible pairs of difference score[4] (Cronbach's $\alpha = .63$).

Go/no-go task. The go/no-go task was also used to assess children's inhibitory control skill. This task required children to attend to stimuli presented individually on an iPad, and children were asked to respond as quickly as possible by touching the screen on go trials (mouse), while withholding responses to no-go trials (cat). In

[4] There are four possible interference (difference) scores between incongruent and congruent trials: (1) Block 2 – Block 1, (2) Block 4 – Block 3, (3) Block 2 – Block 3, and (4) Block 4 – Block 1.

particular, children saw a fixation cross in the center of the screen for 500 ms, then a blank screen for 500 ms, followed by a stimulus (a cat or a mouse) presented for 1 second. Children were given four practice trials to ensure that they understood the instructions. After the practice trials, children completed 40 experimental trials (30 go trials and 10 no-go trials).

The scoring of the go/no-go task was based on a *d*' score from signal detection theory (Schmidt & Vorberg, 2006), which assesses one's sensibility to discriminate the target stimulus (i.e., signal; go responses) from the alternative stimulus (i.e., noise; no go responses). More specifically, the d' score is the standardized difference between the hit rate (probability of correct go responses; touching the mouse) and the false alarm rate right tail *p*-values (probability of incorrect go responses; touching the cat) using the formula: *d*' = [z(Hit rate) – z(False alarm rate)][5] (James et al., 2016). The higher d' score, the easier it is to discriminate signal from noise, thus an indicator of better inhibitory control (Schmidt & Vorberg, 2006). The d' score is used in the present research because it is considered as an effect size measure which allows for comparisons across studies (Schmidt & Vorberg, 2006). Internal reliability of the go/no-go task was calculated based on accuracy of the test items (split-half, r = .50, based on the odd/even items model; Cronbach's α = .51).

Simple addition. The fluency of single-digit addition with sums less than or equal to nine was measured using an iPad application (MadMinute; see a screenshot in Appendix A.4). In this task, children were asked to complete two practice problems prior to completing the 1-minute timed task. They were asked to solve as many addition problems as possible by touching the answer on the screen. Scoring was based on the number of correct sums completed per minute. Internal reliability for the

[5] The d' scores were calculated using:
www.aston.ac.uk/EasySiteWeb/GatewayLink.aspx?alId=98347

simple addition task is not available because there was no RT data on individual items.

Complex addition. Children who were able to complete the simple arithmetic task were given a paper-and-pencil addition task where complex addition skill was tested. Problems consisted of 60 single-digit additions with sums less than or equal to 17, arranged in three columns (20 problems each). Children were given one minute to solve as many problems as possible in order, from top to bottom, starting from the left column to the right column (see the stimuli in Appendix A.5). Scoring was based on the number of correctly written sums completed per minute. This task was complex relative to simple addition because it included problems with sums of 10 or greater and because children had to print the answer. Internal reliability for the complex addition task is not available because there was no RT data on individual items.

Results and Discussion

Preliminary analyses indicated that there were no effects of gender for any of the measures, *p*s > .05. Further, the normality assumption for RT (correct) and accuracy measures were met for most of the tasks. There were three exceptions. First, the RT scores of the number order task for children entering grade 1 were skewed ($z = 5.20$), but further examination of the distribution revealed no extreme outliers and therefore it suggests that there were generally more children who were relatively fast at ordering numbers than the children who were very slow. Second, performance on the complex addition task for children entering grade 1 was skewed ($z = 5.56$) and leptokurtic ($z = 9.61$), due to one outlier (with an extreme high score). When the outlier was removed, no difference was found in the overall analyses (correlations and regressions), and thus this child's data was kept in the dataset. Lastly, the accuracy scores for the missing number task for children entering grades 2 and 3 were skewed

($z = -8.63$) and leptokurtic ($z = 11.88$). The majority of the children were highly accurate on the missing number task, suggesting a ceiling effect for accuracy. This was not a concern, and in fact supports the decision to use RT scores as the index of accessibility of sequential associations in further analyses.

Performance by grade. The descriptive statistics for each grade are shown in Table 3.1. Accuracy scores for the basic number tasks were approaching ceiling for all children, and thus they were not sensitive measures of children's performance. In contrast, RT on correct trials had great variability for children in each grade, and thus it was used as the index of children's performance on cardinal, sequential and ordinal associations of number.

Performance on the simple addition task shows good variability for children at each grade. However, performance on the complex addition task for children entering grade 1 was positively skewed ($z = 5.56$) and leptokurtic ($z = 9.61$). Further examination of the distribution revealed that children entering grade 1 generally performed poorly on this task, except for one child. Two factors might be related to the poor performance. First, the complex addition task involved problems with sums equal to a value between 10 and 17, which might be too challenging for these young children. Second, children completed the complex addition task using paper-and-pencil. Based on observations from the experimenters, many of the younger children experienced difficulty writing the answers down correctly given the time pressure, possibly due to limited motor coordination or limited practice printing numbers. Thus, further adaptation is needed for the complex addition task in Study 2.

Children's schooling (which grade the child was entering in the fall: 1, 2, or 3) and age (in months) was highly correlated with all of the measures, ps $< .001$. Thus, correlations among the various measures controlling for children's schooling or age are shown in Table 3.2. There was no difference in the patterns of correlations among measures when controlling for schooling vs. age. All of the basic numerical measures were correlated with each other. In particular, the two novel order measures were highly correlated with each other, indicating great concurrent validity. Each of the inhibitory control tasks (black/white Stroop and go/no-go tasks) was marginally correlated with all of the numerical measures; however, they were not related to each other. Note that the two inhibitory tasks also had lower internal reliability than the numerical tasks. Such problems of measuring inhibitory processing are consistent with the literature (e.g., Brocki & Bohlin, 2010; James et al., 2016; Miyake et al., 2000).

Table 3.1

Descriptive Statistics for the Measures used in Study 1 (n = 70)

	Grade 1 ($n = 30$)			Grade 2 ($n = 24$)			Grade 3 ($n = 16$)		
Measures	*M*	*SD*	*Range*	*M*	*SD*	*Range*	*M*	*SD*	*Range*
Number comparison RT[a]	1.20	0.23	0.72 – 1.74	1.14	0.21	0.83 – 1.69	0.91	0.13	0.68 – 1.14
Number comparison Correct[b]	0.94	0.06	0.73 – 1.00	0.95	0.04	0.85 – 1.00	0.91	0.13	0.88 – 1.00
Missing number RT[a]	2.85	0.73	1.51 – 4.59	2.50	0.68	1.19 – 4.15	1.87	0.33	1.16 – 2.40
Missing number Correct[b]	0.97	0.05	0.80 – 1.00	0.97	0.06	0.78 – 1.00	0.99	0.03	0.90 – 1.00
Number order RT[a]	3.17	0.56	1.83 - 5.09	3.01	0.80	1.96 – 4.97	2.25	0.38	1.51 – 2.71
Number order Correct[b]	0.82	0.12	0.46 – 1.00	0.88	0.08	0.71 – 1.00	0.87	0.08	0.71 – 1.00
Black/white Stroop[c]	0.39	0.14	0.15 – 0.72	0.34	0.14	0.05 – 0.65	0.26	0.11	0.01 – 0.44
Go/no-go task *d'* score	2.89	0.63	1.54 – 3.77	3.10	0.64	1.36 – 3.77	3.38	0.35	2.65 – 3.77
Simple Addition[d]	9.73	5.41	3 - 30	14.33	7.74	4 - 30	18.19	5.93	7 - 30
Complex Addition[d]	6.00	6.30	0 - 31	12.25	8.53	1 - 34	17.38	7.71	4 - 32

Note. [a] response time on correct trials, [b] proportion correct, [c] response time difference between congruent and incongruent trials, [d] total correct

Table 3.2

Correlations on the Various Measures for children controlling for children's schooling (grade) below the diagonal and controlling for children's age (n = 70) above the diagonal

	1	2	3	4	5	6	7
1. Black/white Stroop[b]	-	-.119	.208†	.298*	.230†	-.238*	-.224†
2. Go/no-go *d'* score	-.124	-	-.284*	-.252*	-.209†	.256*	.247*
3. Number comparison [a]	.229†	-.319**	-	.559***	.720***	-.562***	-.478***
4. Missing number [a]	.322**	-.290*	.628***	-	.657***	-.538***	-.507***
5. Number order [a]	.252*	-.266*	.757***	.712***	-	-.655***	-.580***
6. Simple Addition[c]	-.256*	.278*	-.606***	-.608***	-.688***	-	.853***
7. Complex Addition[c]	-.239*	.277*	-.537***	-.593***	-.635***	.868***	-

Note. [a] response time on correct trials, [b] response time difference between congruent and incongruent trials, [c] total correct;

$p < .10$†, $p \leq .05$*, $p \leq .01$**, $p \leq .001$***

Predictors of number ordering. First, hierarchical multiple regressions were conducted to examine whether children's performance on number comparisons (cardinal associations), missing number (sequential associations), inhibitory tasks (inhibitory processing) would uniquely predict number ordering, after controlling for schooling. As shown in Table 3.3, both number comparisons and missing number tasks were uniquely related to number ordering, suggesting that the number order task was related to both successive number comparisons and sequential associations of number knowledge. Moreover, neither of the inhibitory tasks was uniquely related to number ordering, after controlling for schooling. The lack of unique relationship between inhibitory control tasks and number ordering may be related to the wide age range of children involved in the this study who are expected to have different integration status.

Table 3.3

Hierarchical Regression Analyses Predicting Performance Number Order Task

	Predictors	**ΔR^2**	**B**	**SE**	**ß**	***t***	***p***
1	Grade	.22***	-.03	.07	-.03	-.38	.706
2	Black/white Stroop	.04*	.04	.36	.01	.10	.918
3	Go/no-go	.04*	.01	.09	.01	.16	.872
4	Missing number	.30***	.384	.096	.397	3.99	**< .001**
5	Number comparisons	.13***	1.61	.292	.521	5.51	**< .001**
	Total R^2	.74***					

*$p \leq .05$, **$p \leq .01$, ***$p \leq .001$; bolded values indicate significance at $p \leq .05$*

Predictors of arithmetic performance. Hierarchical multiple regressions were then conducted to examine the relations between the fluency of basic associations (i.e., cardinal, sequential, and ordinal associations) and arithmetic performance (i.e., simple addition and complex addition), after controlling for schooling. In particular, for each of the arithmetic outcomes (i.e., simple and complex addition measures), two different models were tested using different ordinal measures. Specifically, after controlling for schooling, in Model 1, I tested whether performance on the number comparison and missing number tasks were uniquely related to arithmetic performance. Conversely, in Model 2, I tested whether performance on the number comparison and number order tasks were uniquely related to arithmetic performance.

The pattern of results was similar for the simple (see Table 3.4) and complex arithmetic tasks (see Table 3.5). That is, when the missing number task was used as the measure of ordinal processing (Model 1), performance on both of the number comparison and missing number tasks uniquely predicted addition fluency. In contrast, when number order was used as the measure of ordinal processing (Model 2), only performance on the number order task uniquely predicted addition fluency, whereas performance on the number comparison task did not contribute unique variance, instead variance was shared between the two measures. These results support the view that the number order task involves both successive number comparisons and sequential associations of number knowledge. In contrast, the missing number task seems to be a more pure measure of ordinal (i.e., sequential) knowledge.

Table 3.4

Hierarchical Regression Analyses Predicting Performance on Simple Addition Fluency for the Model 1 and Model 2

	Predictors	**ΔR^2**	Final Model Statistics				
Model 1			**B**	**SE**	**ß**	***t***	***p***
1	Grade	.22***	.91	.86	.10	1.06	.293
2	Number comparison	.28***	-11.17	3.59	-.36	-3.11**	**.003**
3	Missing number	.07***	-3.81	1.15	-.40	-3.32**	**.001**
	Total R^2	.58***					
Model 2							
1	Grade	.22***	1.16	.81	.13	1.43	.158
2	Number comparison	.28***	-6.20	4.17	-.20	-1.49	.142
3	Number order	.09***	-5.30	1.35	-.53	-3.93**	**< .001**
	Total R^2	.60***					

$p \leq .05$*, $p \leq .01$**, $p \leq .001$***

Table 3.5

Hierarchical Regression Analyses Predicting Performance on Complex Addition Fluency for Model 1 and Model 2

	Predictors	Final Model Statistics					
Model 1		ΔR^2	**B**	**SE**	**ß**	***t***	***p***
1	Grade	.28***	2.15	1.05	.20	2.05*	**.044**
2	Number comparisons	.21***	-9.69	4.41	-.26	-2.20*	**.032**
3	Missing number	.08**	-4.87	1.43	-.42	-3.42**	**.001**
	Total R^2	.56***					
Model 2							
1	Grade	.28***	2.47	1.01	.23	2.43*	.018
2	Number comparisons	.21***	-4.70	5.19	-.13	-.91	.369
3	Number order	.09***	-6.18	1.68	-.52	-3.68**	**< .001**
	Total R^2	.58***					

* $p \leq .05$, **$p \leq .01$, ***$p \leq .05$

In summary, the preliminary results of Study 1 show that, as early as grade 1, both cardinal and ordinal processing were uniquely associated with individual differences in arithmetic skill when the missing number task was used. In contrast, only ordinal processing was uniquely associated with individual differences in arithmetic skill when the number order task was used. These results suggest that sequential associations of number can be isolated from ordinal associations of number using a simpler missing number task than the traditional order judgment task. Moreover, performance on the number order task is a unique predictor of arithmetic fluency, presumably because it reflects the integration of the lower level cardinal and sequential associations.

Moreover, the results of Study 1 show that the two types of inhibitory control tasks were not correlated with each other, although they are assumed to measure similar types of inhibitory processing. It is still unclear about which type of inhibitory control measures is more relevant to number ordering based on the present results. Therefore, in Study 2, both of the inhibitory control measures were used.

The results of this study also show that the new measures of ordinal knowledge are appropriate in that they have great reliability. They also show correlations with the other early numeracy tasks, suggesting adequate concurrent validity. High levels of accuracy indicate that these tasks are manageable for children at this age level and assess primarily fluency of access, rather than accuracy.

In contrast, the results suggest that the complex addition task using paper-and-pencil responses may be inappropriate for younger children given the complexity of the calculations required and the extra demand of motor coordination skills. Furthermore, reliability was not available for either of the addition tasks given the design of these tasks. Thus, in Study 2, the addition tasks were presented via an iPad application to ensure that internal reliability could be calculated. To be consistent with the children's knowledge level, children in grade 1 completed a simple addition task, and children in grade 2 completed both a simple addition and a complex addition task (see details in Chapter 4).

CHAPTER 4: STUDY 2

Study 2 had three main goals. The first goal was to examine the initial integration status for ordinal associations of children in grades 1 and 2 at the beginning of the school year. If children have developed integrated ordinal associations, then both performance on the number comparison and missing number tasks should be related to number ordering concurrently. In contrast, if children have not yet fully acquired ordinal associations, then they are expected to rely on quantitative information to order numbers through successive number comparisons, and thus number comparisons should be related to number ordering concurrently. Inhibitory control should be involved in number ordering only for children who have integrated ordinal associations. This prediction is based on the assumption that the symbolic representations of sequential associations are automatically activated if they are integrated with cardinal knowledge. I hypothesized that children in grade 1 would not yet have integrated ordinal associations of numbers and thus inhibitory control would not be a unique predictor of their performance on the number order task (*Hypothesis 1a*), whereas children in grade 2 would have integrated ordinal associations and thus inhibitory control would uniquely predict their performance on the number order task (*Hypothesis 1b*).

The second goal of Study 2 was to examine how cardinal, sequential and ordinal associations were concurrently related to arithmetic associations for children in grades 1 and 2. For children in grade 1, I hypothesized that the fluency with which they access both cardinal and sequential associations would be related separately to the fluency with which they access arithmetic associations, based on the prediction that children in grade 1 have not integrated cardinal and sequential associations to form ordinal associations (*Hypothesis 2a*). In contrast, for children in grade 2, I hypothesized that the fluency of access to higher-level ordinal associations would become the only unique predictor of arithmetic associations for children in grade 2 because they would have integrated ordinal associations into their associative networks (*Hypothesis 2b*).

The last goal of Study 2 was to examine which specific form of associations would predict the growth of arithmetic associations for children in grades 1 and 2. I tested different models based on the hypothesized integration status for children in grades 1 and 2. For children in grade 1, I predicted that the fluency with which they access both cardinal and sequential associations would independently predict the growth of arithmetic fluency (*Hypothesis 3a*). This hypothesis was based on the assumption that continued development of arithmetic associations for children in grade 1 was linked to the efficient use of counting strategies, especially min counting, which involves number comparisons and reciting the number sequence. Thus, their access to cardinal knowledge and sequential knowledge are expected to support their execution of the better counting strategies. In contrast, for children in grade 2, I predicted that the fluency with which they access ordinal associations would predict the growth of arithmetic fluency (*Hypothesis 3b*). This hypothesis is based on the assumption that continued development of arithmetic associations for children in grade 2 was linked to the increasing use of more efficient strategies such as decomposition, which requires mental manipulation of numerical symbols (e.g., Geary et al., 2004; Siegler, 2007). Thus, children's access to the higher-level ordinal knowledge that captures representations of cardinal and sequential knowledge is expected to support their selection and execution of the more advanced arithmetic strategies.

Data were collected during grades 1 and 2 twice (Time 1 and Time 2), approximately four months apart. This short-term longitudinal design is important because it allows me to examine the changes in the development of integration among the various aspects of symbolic number knowledge. I expected that rapid change in children's ordinal knowledge in grades 1 and 2 would provide important information about the development of their associative networks.

The data were collected as part of a larger project that had several different purposes. More specifically, children completed six additional measures at both of the time points: 1) quantity-digit mapping, 2) rapid automatic naming, 3) spatial span, 4) 2-D mental rotation, 5) number line estimation, and 6) problem solving (i.e., the mathematics reasoning subset of the WIAT [Wechsler Individual Achievement Test-III]). These measures were not included in the main text of this dissertation because they were not directly related to the specific predictions with respect to the developmental trajectory from cardinal → ordinal → arithmetic associations of the expanded HSI model. Nevertheless, detailed descriptions of these measures and preliminary analyses were included in Appendix B. In this chapter, only the measures that are relevant to the research goals and following analyses are presented and analyzed.

Method

Participants. One hundred and forty-six children from five public schools were recruited in Ottawa, Canada. Sixty-six children were in grade 1 (M = 78 months, SD = 3 months, range from 6:0 to 7:0 year:months; 29 girls) and 80 children were in grade 2 (M = 90 months, SD = 3 months, range from 7:0 to 8:0 year:months; 34 girls). The majority of children (90%) spoke English as their primary language at home; 7% spoke Arabic and 3% spoke Chinese as their primary language at home. Further, 56% of the children spoke another language at home in addition to English. The most frequent secondary language was French (31%), followed by Chinese (3%), Arabic (2%), Tamil (2%), Punjabi (1%), and a variety of other languages (one child for each type).[6]

Information about parents' highest education level was collected from the consent form for 144 mothers and 140 fathers. Of those who responded to the questionnaire, 20% of the mothers (18% of the fathers) had received a postgraduate degree, 41% of the mothers (36% of the fathers) had received an undergraduate degree, 29% of the mothers (25% of the fathers) had received a community college degree, 8% of the mothers (14% of the fathers) had received a high school diploma, and 2% of the mothers (3% of the fathers) had received less than a high school diploma (*Mdn*= undergraduate degree for both mothers and fathers). As in Study 1, the parents in this study were generally from middle-class families, with similar levels of education as other Canadian samples.

Procedure. To assess the developmental trajectories of number knowledge integration, children were tested twice during the school year: Time 1 (November to January) and Time 2 (March to May), with an approximately four-month gap between testing for each child. Three children changed schools during the year, which resulted a sample of 143 children in Time 2. All children were tested individually at school by one of four trained experimenters. Children were given stickers as a token of appreciation for their effort.

Measures. Children completed a number of numerical and cognitive tasks twice (at both Time 1 and Time 2). Most of the measures were identical to the ones used in Study 1 (i.e., number comparisons, missing number, number order, and black/white Stroop) and thus are only briefly described here. The revised measures

[6] The other languages were Spanish, Portuguese, Albanian, Serbian, Russian, Korean, Vietnamese, Romanian, Hindi, and Polish.

(i.e., addition tasks) are described fully.

Number comparison task. Children completed the same symbolic number comparison task on an iPad as in Study 1. The scoring was based on accuracy and mean RT on correct trials, but only the mean RT correct score was used as the index of children's fluency of access to cardinal associations of number.

Missing number task. Children completed the same missing number task on an iPad as in Study 1. The scoring was based on mean accuracy and mean RT on correct trials, but only the mean RT correct score was used as the index of children's fluency of access to sequential associations of number.

Number order task. Children completed the same number order task on an iPad as in Study 1. The scoring was based on accuracy and mean RT on correct trials, but only the mean RT correct score was used as the index of their knowledge of ordinal associations of number.

Black/white Stroop task. Children completed the same black/white Stroop task on an iPad as in Study 1 to index their inhibitory control skills. The accuracy for the four blocks of trials on the Stroop task was high for both children in grades 1 and 2. In particular, for children in grade 1, the accuracy from first to the last blocks of trials (i.e., congruent, incongruent, congruent, and incongruent) was 97%, 90%, 97%, 87% at Time 1, respectively, and 97%, 94%, 96%, and 90% at Time 2, respectively. For children in grade 2, the accuracy from first to the last blocks of trials was 98%, 91%, 97%, 92% at Time 1, and 97%, 93%, 97%, and 91% at Time 2. Accuracy was lower on the incongruent blocks (2 and 4) than on the congruent blocks (1 and 3), as expected.

The score of this task was calculated based on the RT difference (on correct trials) between the average RT of congruent and incongruent trials as a measure of interference cost (mean RT of congruent trials – mean RT of the incongruent trials). Higher score indicates higher interference cost, indicating poorer inhibitory control skills.

Go/no-go task. Children completed the same go/no-go task on an iPad as in Study 1 to index their inhibitory control skills. This scoring of the go/no-go task was based on a *d*' score that was calculated in the same way as in Study 1. Higher *d*' score indicates better inhibitory control (i.e., better discrimination between hit and false alarm).

Addition tasks. Fluency of single-digit addition was measured using an iPad application (see an example in Appendix A.6). In this task, children were asked to say out loud the answer for each addition question as quickly as possible, and the experimenter recorded the answer using a keyboard connected to the iPad. Children in Grade 1 completed the simple version of the addition task (i.e., with sums less than or equal to nine), whereas children in Grade 2 completed both the simple version (i.e., with sums less than or equal to nine) and the complex version of the addition task (i.e., with sums ranged from 10 to 16 inclusive).

Children completed two practice trials prior to completing the task. There were 16 experimental trials for both the simple and complex versions of the task (see Appendix A.7 for the full stimuli set). Each trial had a 10-second time limit such that the child proceeded to the next trial if he or she did not answer the question within the 10-second limit. Questions were presented in a different random order for each child. The test was terminated if the child made five errors in a row.

Missing data was present for children who failed to answer the first five addition problems correctly or did not cooperate on the task and thus did not have any correct trials. All of the other children solved at least five trials correctly and were included in further analyses: 143 children for the simple addition task and 77 children for the complex addition task at Time 1; 138 children for the simple addition task and 73 children for the complex addition task at Time 2.

For children in grade 1, the RT (correct trials) was used as the index of children's fluency to access simple addition associations. For children in grade 2, an average RT (correct trials) score of the simple and addition task was used as the index of their fluency to access addition associations.

Reliability. As shown in Table 4.1, all of the tasks had reasonable internal reliability. Note that the two inhibitory control tasks had lower test-retest reliability than the numerical tasks. There are at least two kinds of explanations for this difference. First, it is possible that these tasks are simply not very reliable in the sense that children may develop different strategies during the course of the test or between testing times. Second, it is possible that variability in these tests does not capture consistent individual differences in the underlying cognitive processes of interest, but rather reflects moment-to-moment influences of other factors, such as fatigue or inattention. These difficulties with measuring inhibitory control are not specific to this thesis (Friedman, 2016; Müller, Kerns, & Konkin, 2012).

Table 4.1

Reliability for measures at Time 1 and Time 2 in Study 2

Measures	Time 1		Time 2		Test-retest
	Split-half [c]	Cronbach's alpha	Split-half [c]	Cronbach's alpha	
1. Number comparison [a]	.94	.94	.85	.94	.81
2. Missing number [a]	.83	.82	.85	.82	.77
3. Number order [a]	.92	.89	.96	.90	.81
4. Black/White Stroop [b]	-	.81	-	.77	.49
5. Go/no-go *d'* score [d]	.77	.68	.86	.73	.58
5. Simple Addition [a]	.84	.75	.73	.79	.84
6. Complex Addition [a]	.64	.78	.63	.75	.66

Note. [a] response time on correct trials, [b] average response time of congruent and incongruent trials, [c] split half reliability was calculated based on odd/even split for all tasks, except for the Stroop task (based on the blocks of congruent or incongruent trials) as in Study 1

Results and Discussion

There were no significant differences between boys and girls for any of the measures for children in grade 2 or for most of the measures for children in grade 1 (Bonferroni correction for multiple comparisons was used at $p < .007$). In particular, the exception for children in grade 1 was that boys were faster at comparing digits ($M = 1.25$ s, $SD = .22$) than girls ($M = 1.49$ s, $SD = .33$) at Time 1, $t(47) = -3.53$, $p = .001$, Cohen's $d = .86$. Given that this minor gender difference is unlikely to be influential to the interpretation of the results, and that the gender effect was not the main focus of the present research, gender effects were not further considered.

Most of the measures were normally distributed. There were two exceptions. First, the simple addition performance for children in grade 2 at Time 2 was skewed (z = 3.93): the majority of the children were quite fast at solving simple single-digit problems and thus there was a potential ceiling effect. This was not considered as an issue because an average RT of both simple and complex addition (normally distributed) was used for the main analyses for these children. Second, performance on the black/white Stroop task for children in grade 2 at Time 2 was leptokurtic (z = 5.60). Two outliers were observed (one at either end of the distribution), however, this was not a concern because the Time 2 data for the black/white Stroop task was not used for the main analyses.

In contrast, violations of the assumption of normality for the accuracy measures on most of the tasks were found, except for the two harder tasks (i.e., number order and complex addition tasks). For children in both grades, the accuracy scores on all other tasks (i.e., number comparison, missing number, and simple addition tasks) were negatively skewed (zs > 3.29), suggesting the accuracy scores on most of the tasks have a ceiling effect (see Table 4.2 for means and SDs). This is not a concern given that the accuracy scores were not used in the main analyses. The tasks were deliberately kept simple to allow children to succeed.

Performance by grade and time. To examine the performance and improvements on the various measures for children in grade 1 and grade 2, performance on each task as a function of grade and time was analyzed in separate 2 (Grade: 1 vs. 2) x 2 (Time: 1 and 2) mixed analyses of variance (ANOVA). Post hoc tests used Bonferroni adjustments for multiple comparisons. The means are shown in Table 4.2 and the ANOVA results are shown in Table 4.3.

Children in grade 2 showed better performance on all of the measures than children in grade 1, except for the accuracy score on the number comparison task and the go/no-go task (see Table 4.2). Note that children in both grades reached ceiling given the highly accurate performance on the number comparison task at both time points (>.90 proportion correct). The lack of difference between children in grades 1 and 2 on the performance of go/no-go task was probably due to a ceiling effect. Moreover, children improved over time on all of the measures except for inhibitory control measures (see Table 4.3). These patterns suggest that four months might not be sufficient for children to show significant improvements on the general cognitive skills such as inhibitory control whereas school-based number tasks are improving due to children's experiences.

In contrast, performance did not improve differentially from Time 1 to Time 2 with a few exceptions. First, children in grade 1 showed improvements over time on the accuracy of the missing number task (*M* difference = .05), $p < .001$, whereas children in grade 2 did not (*M* difference = .004), $p = .708$. Children in grade 2 showed a ceiling effect at both Time 1 and Time 2 (see Figure 4.1). Second, for the simple addition task, children in grade 1 showed more improvements over time on both of the RT (*M* difference = .49), $p < .001$, and accuracy (*M* difference = .08), $p < .001$, whereas children in grade 2 showed less improvements on RT (*M* difference = .08), $p = .012$, and no improvement on accuracy (*M* difference = .01), $p = .438$ (see Figure 4.1 and 4.2). These results suggest a ceiling effect for the simple addition task for children in grade 2, and thus the simple addition task alone might not be an appropriate measure of the arithmetic fluency for them. In the following analyses, the average RT correct based on both of the simple and complex addition tasks was used as the index of fluency to access arithmetic associations for children in grade 2.

Table 4.2

Mean Performance on Various Measures from Time 1 to Time 2 for Grade 1 and Grade 2 Children (SDs in Parentheses)

	Grade 1		Grade 2	
Measure	Time 1	Time 2	Time 1	Time 2
Predictors				
Number comparison RT[a]	1.35 (.30)	1.29 (.23)	1.09 (.18)	1.06 (.16)
Number comparison	0.92 (.10)	0.94 (.08)	0.94 (.09)	0.96 (.05)
Missing number RT[a]	2.99 (.72)	2.72 (.64)	2.63 (.63)	2.40 (.54)
Missing number Correct[b]	0.94 (.11)	0.98 (.04)	0.98 (.05)	0.98 (.08)
Number order RT[a]	3.47 (.52)	3.45 (.54)	3.01 (.54)	2.85 (.44)
Number order Correct[b]	0.78 (.15)	0.83 (.12)	0.86 (.09)	0.89 (.09)
Black/White Stroop[c]	0.38 (.21)	0.38 (.17)	0.32 (.17)	0.30 (.18)
Go/no-go *d'* score	2.74 (.72)	2.88 (.59)	2.93 (.69)	3.02 (.79)
Addition performance				
Average Addition RT[a]	4.16 (1.21)	3.67(1.04)	3.94 (.85)	3.66 (.85)
Average Addition Correct[b]	0.76 (.27)	0.84 (.25)	0.80 (.18)	0.83 (.18)
Simple Addition RT[a]	4.16 (1.21)	3.67 (1.04)	3.12 (.83)	2.90 (.85)
Simple Addition Correct[b]	0.76 (.27)	0.84 (.25)	0.92 (.14)	0.94 (.14)
Complex Addition RT[a]	-	-	4.71 (1.14)	4.46
Complex Addition Correct[b]	-	-	.69 (.26)	.74 (.25)

[a] response time on correct trials, [b] proportion correct total correct, [c] response time difference between congruent and incongruent trials

Table 4.3

ANOVA Results of Grade x Time for All Measures in Study 2

		Independent variable					
		Grade (1 vs. 2)		Time (1 vs. 2)		Time x Grade	
Dependent variable	*df*	*F*	η_p^2	*F*	η_p^2	*F*	η_p^2
Number comparison RT	1,141	56.74***	.29	10.41**	.07	2.06	.01
Number comparison Correct	1,141	1.66	.01	7.73**	.05	.17	.001
Missing number RT	1,141	13.77***	.09	30.74***	.18	.34	.001
Missing number Correct	1.141	3.78*	.03	5.83*	.04	8.51**	.06
Number order RT	1,138	47.39***	.26	7.01**	.05	2.70	.02
Number order Correct	1,138	17.72***	.11	14.36***	.09	.42	.003
Black/White Stroop	1,141	7.68**	.05	.17	.001	.22	.002
Go/no-go *d’* score	1,141	2.77	.02	2.55	.02	.17	.001
Average Addition RT [a]	1,134	-	-	28.37***	.18	-	-
Average Addition Correct [a]	1,134	-	-	14.26***	.09	-	-

Simple Addition RT	1,134	54.93***	.21	28.10***	.17	3.93*	.03
Simple Addition Correct	1,140	18.12***	.12	12.61***	.08	6.32*	.04
Complex Addition RT	1,70	-	-	3.53†	.05	-	-
Complex Addition Correct	1,75	-	-	3.98*	.05	-	-

$p < .10$†, $p < .05$*, $p < .01$**, $p < .001$***

[a] The score for grade 1 children was solely based on performance on the simple version of the task, whereas the score for grade 2 children was based on the average performance of simple and complex versions of the task, and therefore they are not comparable.

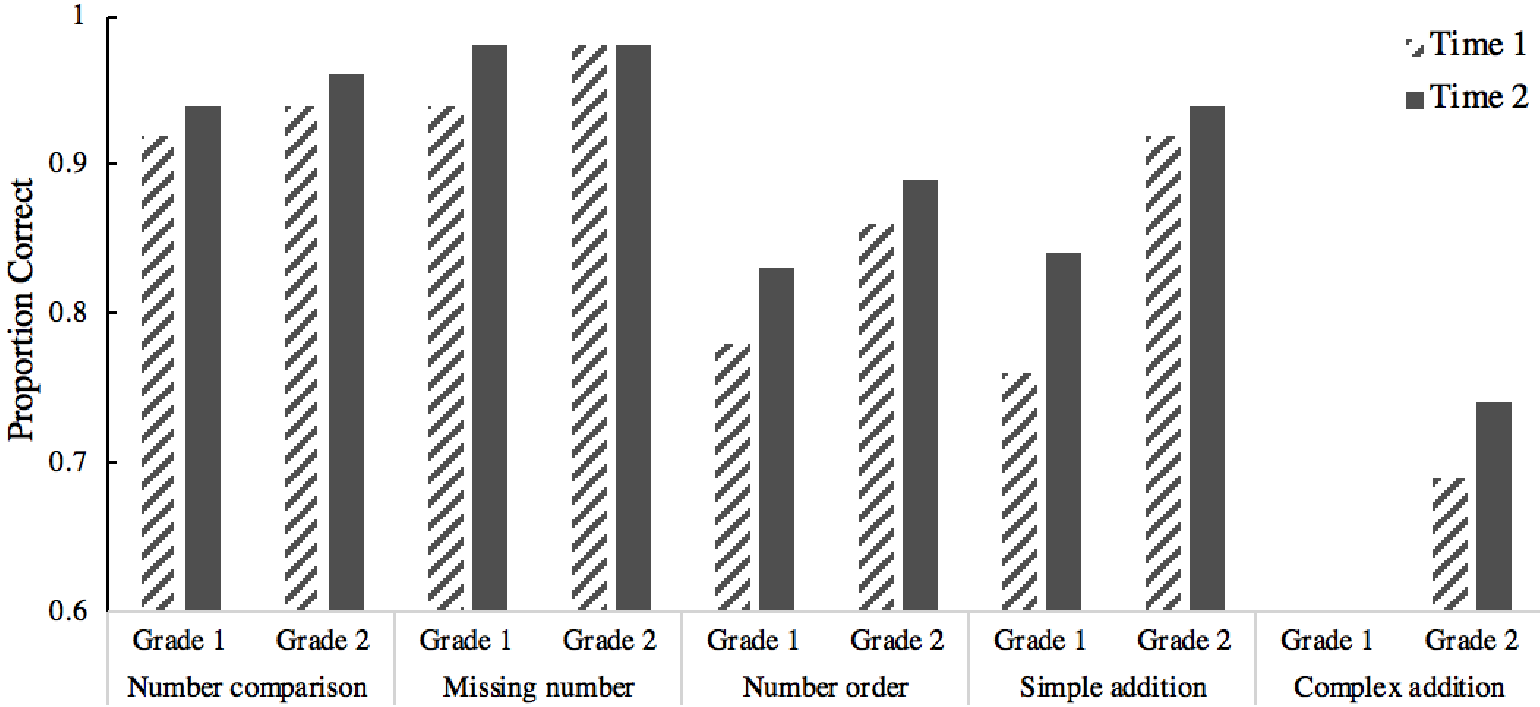

Figure 4.1. Proportion correct on measures from Time 1 to Time 2 for children in grades 1 and 2

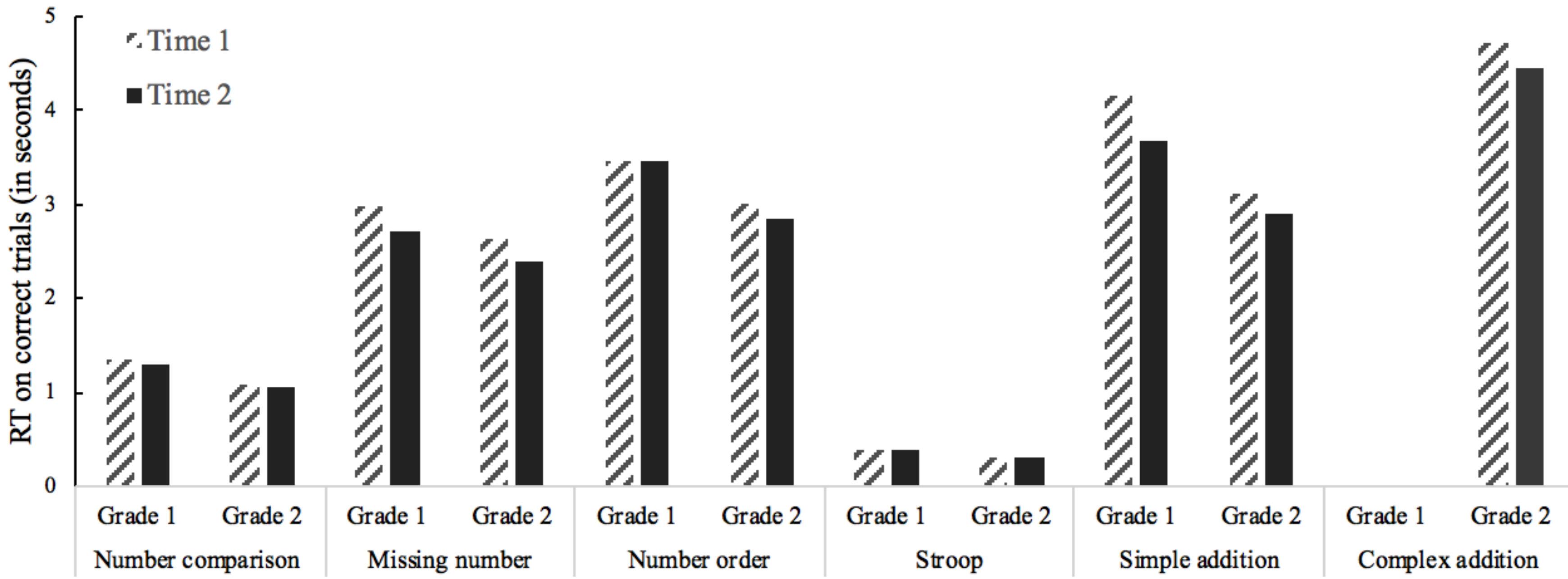

Figure 4.2. Reaction time on correct trials on measures from Time 1 to Time 2 for children in grades 1 and 2 except for the Stroop task (interference RT on correct trials)

Correlations. Correlations among the various measures (for RT measures only) for children in grade 1 and grade 2 are shown in Table 4.4 and Table 4.5. For all children, performance on the number comparison, missing number, and number order tasks were significantly correlated with each other at both time points.

As expected, performance on the two types of inhibitory control tasks was unrelated with each other at both time points for both grades of children, consistent with the results in Study 1. Of most interest, the relationship between inhibitory control measures and performance on the number order task was variable. In the present research, the testing of the expanded HSI model focuses on the basic numerical measures and inhibitory control measures at Time 1. As shown in Table 4.4, for children in both grades 1 and 2, performance on the black/white Stroop task was highly correlated with number ordering. In contrast, performance on the go/no-task was not related to number ordering for children in grade 1, and marginally correlated with number ordering for children in grade 2. Taken together, the two types of inhibitory control tasks used in the present research presumably tap into different aspects of cognitive processing.

Moreover, for children in grade 1, addition fluency was significantly correlated with performance on the number comparison, missing number, and number order tasks at Time 1 (see Table 4.4). At Time 2, addition fluency was correlated with the two ordinal measures and performance on the number comparison task (see Table 4.5). In contrast, for children in grade 2, addition fluency was correlated with performance on the ordinal measures and performance on the number comparison task at Time 1 (see Table 4.4). This pattern of correlation was similar to the one shown at Time 2 for children in grade 1 (see Table 4.5). Further, at Time 2, addition fluency was significantly correlated with all three measures of basic numerical associations for children in grade 2 (see Table 4.5). These results suggest a gradual shift in the relative strength of relationship between basic number processing and arithmetic from cardinal to ordinal processing over time.

Table 4.4

Correlations at Time 1 for children in Grade 1 (n = 66) below the diagonal and Grade 2 (n = 80) above the diagonal

Measure	1	2	3	4	5	6
1. Number comparison [a]	-	.34**	.60***	.23*	-.24*	.37***
2. Missing number [a]	.37**	-	.60***	.16	-.27*	.25*
3. Number order [a]	.68***	.34**	-	.38***	-.27*	.49***
4. Black/white Stroop [b]	.54***	.16	.43***	-	-.03	.12
5. Go/no-go *d*' score	-.22	-.17	-.19	-.13	-	-.08
6. Average addition [c]	.50***	.60***	.39**	.17	-.22	-

$p < .05$*, $p < .01$**, $p < .001$***

[a] Response time on correct trials, [b] response time difference between congruent and incongruent trials, [c] average RT for children in grade 1 reflected the mean RT for the simple addition task, whereas the average RT for children in grade 2 reflected the mean RT for both of the simple and complex addition tasks.

Table 4.5

Correlations on the Various Measures at Time 2 for children in Grade 1 (n = 64) below the diagonal and Grade 2 (n = 79) above the diagonal

Measure	1	2	3	4	5	6
1. Number comparison [a]	-	.37**	.56***	.12	-.35***	.46***
2. Missing number [a]	.52***	-	.47***	.08	-.32**	.40***
3. Number order [a]	.65***	.62***	-	.03	-.30**	.50***
4. Black/White Stroop [b]	.26*	.27*	.36**	-	-.12	.10
5. Go/no-go *d'* score	-.31*	-.41***	-.27*	.02	-	-.20
6. Average addition [c]	.35**	.57***	.49***	.37**	-.29*	-

p < .05*, *p* < .01**, *p* < .001***

[a] Response time on correct trials, [b] response time difference between congruent and incongruent trials, [c] average RT for children in grade 1 reflected the mean RT for the simple addition task, whereas the average RT for children in grade 2 reflected the mean RT for both of the simple and complex addition tasks.

Performance on the number order task across sequence types. Before evaluating the status of children's integration of ordinal associations using a number order task for children in grades 1 and 2, I examined the potential involvement of cognitive factors of the number order task by looking at the performance on different types of ordering sequences. As discussed in Chapter 2, the number order task is assumed to involve three different types of cognitive processing: cardinal, sequential and inhibitory processing, depending on the types of sequences that children ordered. In particular, four types of sequences were used: a) ordered counting sequence (e.g., 1 2 3), b) non-ordered counting sequence (e.g., 2 3 1), c) ordered non-counting sequence (e.g., 1 5 7), and d) non-ordered non-counting sequence (e.g., 1 7 5). In this section, I examined the accuracy and RT (correct) data on these four types of sequences of the number order task.

More specifically, I first examined the accuracy data to determine the involvement of inhibitory control in the number order task. Children are expected to be the least accurate at ordering counting sequences that are incorrectly ordered (e.g., 2 4 3) than other types of sequences, because children need to suppress the activation the counting strings even though the numbers are not in order. Next, I examined the RT (correct) data for different types of sequences. Children are expected to be fastest at ordering correctly ordered counting sequences (e.g., 1 2 3), if these sequences activate the sequential associations of number. Further, children are expected to be faster at ordering sequences that are in order (e.g., 1 2 3 or 1 5 7) than the sequences that are not in order (e.g., 2 3 1 or 1 7 5), because the latter decision requires more intermediate steps to make number comparisons. Lastly, some demand for inhibitory processing is expected to be involved for ordering the counting sequences that are not in order.

Accuracy. Accuracy data was first analyzed in a 4 (sequence: ordered count, non-ordered count, ordered non-count, non-ordered non-count) x 2 (grade: 1 vs. 2) mixed ANOVA for the number order task at Time 1. Overall, children in grade 2 (M = .86) were more accurate than children in grade 1 (M = .78). As expected, children were least accurate at the counting sequences that were not ordered (M = .70) compared to any other types of sequences, ps < .001, $F(3, 423) = 52.72$, $p < .001$, $\eta_p^2 = .27$. Children were less accurate on the non-ordered non-count sequences (M = .83) than both types of correctly ordered sequences, ps < .05, whereas no difference was found between the correctly ordered count (M = .88) and correctly ordered non-count sequences (M = .90), p = 1.00.

The two-way interaction between sequence and grade was significant, $F(3, 423) = 4.76$, $p = .003$, $\eta_p^2 = .03$. As shown in Figure 4.3, children in grade 1 were less accurate on the non-ordered count sequences than the other three types of sequences, ps < .001. Further, they were less accurate on the non-ordered non-count sequences than both types of the ordered sequences, ps < .001, whereas no difference was found between the two types of ordered sequences, ps >.05. In contrast, children in grade 2 also were least accurate on the non-ordered count sequences than the other three types of sequences, ps < .001, which did not differ, ps = 1.00 (see Figure 4.3).

The results of accuracy scores on the number order task show that children in both grades made the most errors on the non-ordered counting sequences, presumably because the elements of the counting strings were activated even though the numbers are not in order. These results support the view that inhibitory processing was involved in the number order task.

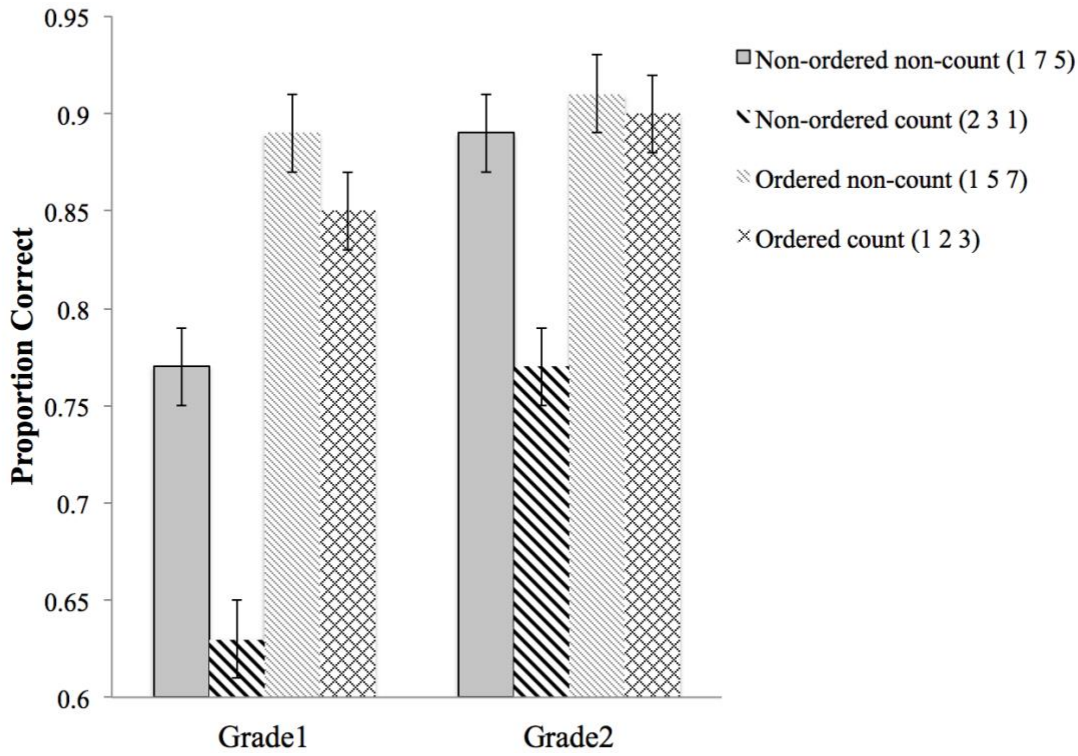

Figure 4.3. Mean proportion of accuracy on the number order task across sequence types for children in grades 1 and 2. Error bars represent 95% CIs calculated based on the MSEs from the 2-way interaction (Masson & Loftus, 2003).

Response time. Next, mean RT on correct trials was analyzed in a 4 (sequence: ordered count, non-ordered count, ordered non-count, non-ordered non-count) x 2 (grade: 1 vs. 2) mixed ANOVA for the number order task at Time 1. Post-hoc tests were conducted based on the Bonferroni adjustment for multiple comparisons. Overall, children in grade 2 (M = 3.01 s) were faster at ordering digits than children in grade 1 (M = 3.47 s). As expected, children responded fastest to ordered counting sequences (M = 2.98 s) compared to any other types of sequences, ps < .001, $F(3, 417) = 44.96$, $p < .001$, η_p^2 = .24. Children judged the correctly ordered non-counting sequences (M = 3.23 s) faster than both types of the incorrectly ordered sequences, ps < .001, whereas for non-ordered sequences, no difference in RT was found between counting (M = 3.44 s) and non-counting versions (M = 3.43 s), p =

1.00.

Of most interest, the two-way interaction between sequence and grade was significant, $F(3, 417) = 1.85$, $p < .001$, $\eta_p^2 = .08$. As shown in Figure 4.4, children in grade 1 judged ordered count sequences faster than any of the other types of sequences, *p*s < .001, and were faster on ordered non-counting sequences than on either type of non-ordered sequence (the latter did not differ, $p = 1.00$). In grade 2, children also judged ordered count sequences faster than any of the other types of sequences, *p*s < .003, but did not show differences among the other three types of sequences, *ps* = 1.00 (see Figure 4.3).

These results show that children in both grades were fastest at ordering numbers that come from the counting string in the correct sequence (e.g., 1 2 3) than any other type of sequences, consistent with the pattern of results as shown in adults (Bourassa, 2014; Vos et al., 2017). It is possible that children found it easier to process the counting sequences because each number in the sequence serves as a priming stimulus for the number that comes before and after it (e.g., 1 primes 2, and 2 primes 3; Serra & Nairne, 2000). In other words, children may have activated the sequential associations from their mental network on the number order task. Moreover, children in grade 1 were faster at ordering sequences that are in order (e.g., 1 2 3 or 1 5 7) than the sequences that are not in order (e.g., 2 3 1 or 1 7 5), suggesting that they might need to take a few extra steps to do *pairwise* number comparisons based on inferential reasoning for the later decision (e.g., 1 < 7, 7 > 5, 5 > 1, so 1 → 5 → 7) than the former one (e.g., 1 < 5, 5 < 7, so 1 → 5 → 7). In contrast, children in grade 2 were equally fast at ordering sequences regardless of whether they are in order or not, suggesting that they might activate the cardinal information of *all* of the numbers instead of pairwise comparisons (e.g., locating the smallest number → next number → largest number).

Taken together, the present results suggest that the number order task potentially involves three types of cognitive processing depending on the different types of sequences. In particular, sequential processing is involved for ordering the sequences that come from the counting strings in the correct sequence; cardinal processing is involved for ordering the sequences that do not come from the counting strings; and lastly, inhibitory processing is involved for ordering sequences that come from the counting sequences in the incorrect sequence. The relative activation of each of these processes depends on the integration status of children's number associations. For grade 1 children with less integrated network, the number order task might primarily involve cardinal processing through successive number comparisons. In contrast, for grade 2 children with more integrated network, the number order task might involve all of cardinal, sequential and inhibitory processing. In the next section, I tested the expanded HSI model using path analysis to evaluate the integration paths for children in grades 1 and 2.

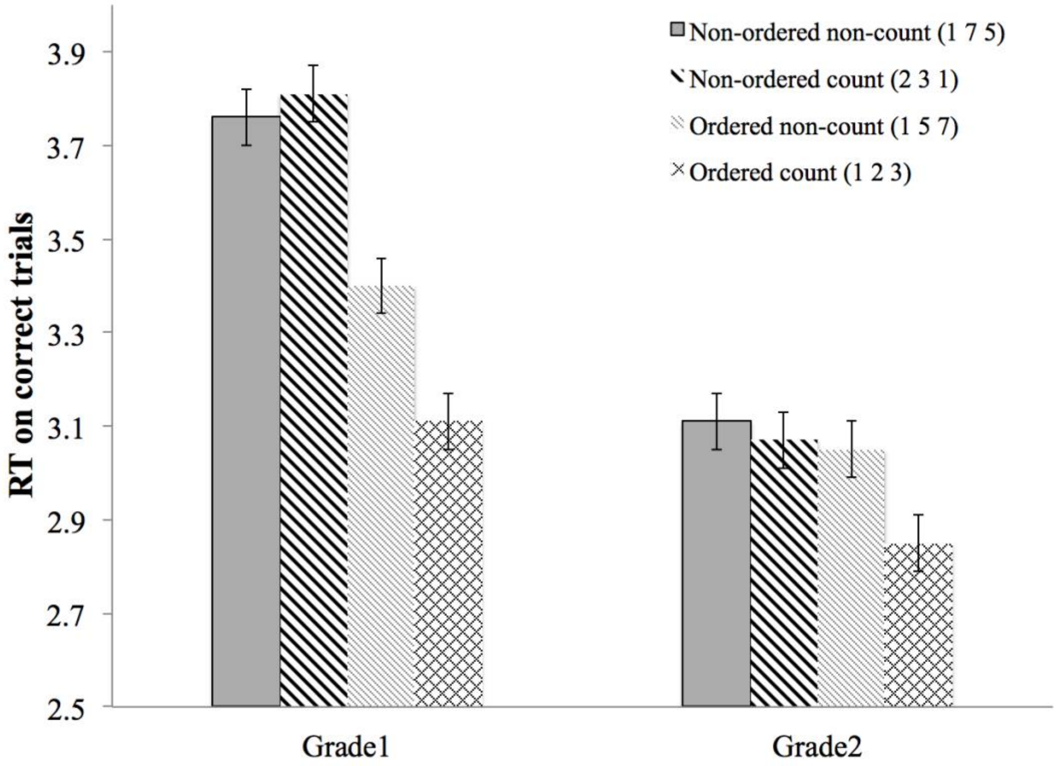

Figure 4.4. Mean RT on correct trials on the number order task across sequence types for children in grades 1 and 2. Error bars represent 95% CIs calculated based on the MSEs from the 2-way interaction (Masson & Loftus, 2003).

Testing the expanded Hierarchy of Symbol Integration model. I used path analysis to test the three main predictions. For ease of interpretation, I explain the path models for children in grades 1 and 2 in two steps. At the first step, I examined the integration status of ordinal associations at Time 1. In the present research, the number order task is assumed to reflect the extent to which individuals have activated associative relations among number symbols (i.e., cardinal associations and sequential associations). In particular, I tested a) whether the three candidate predictors (cardinal associations, sequential associations and inhibitory control) would be related to ordinal associations differentially for children in grades 1 and 2 (Hypothesis 1), and b) whether the patterns of concurrent relationship between basic associations (i.e., cardinal, sequential, and ordinal associations) and arithmetic fluency would be different for children in grades 1 and 2 (Hypothesis 2).

To examine the integration status for the acquisition of ordinal associations, I used multiple-group path analysis using Mplus Version 7 (Muthén & Muthén, 1998-2012) to test whether differences in the structural parameters were statistically significant between children in grades 1 and 2. In particular, testing for cross-group invariance involves comparing two nested models: 1) an unconstrained model in which all paths were specified in both of the groups, but the coefficients for each of the paths were estimated independently for each group, and 2) a constrained model where all paths and the coefficients were constrained to be equal across groups. The comparison of the two nested models allows me to statistically test whether the integration status of numerical associations are different for children in grades 1 and 2.

At the second step, I added children's arithmetic fluency at Time 2 into the model while controlling for the arithmetic fluency at Time 1 to examine which specific form of associations would predict the growth of arithmetic. Because the integration status for the acquisition of ordinal associations are expected to be different for children in grades 1 and 2, I tested two separate path models for each grade. Specifically, for children in grade 1, I tested whether cardinal and sequential associations would predict the growth of arithmetic. In contrast, for children in grade 2, I tested whether ordinal associations would predict the growth of arithmetic.

Step 1 (Hypotheses 1 and 2). Multi-group analysis was used to examine the integration status for children in grades 1 and 2. Although children in grades 1 and 2 are expected to have different integration status (i.e., differential paths from basic numerical processing to addition fluency), for the purpose of comparison across groups, all of the candidate predictors (i.e., number comparisons, missing number, inhibitory processing, and number ordering) were added into the model.

Inhibitory processing. Because the two types of inhibitory control tasks might tap into different aspects of inhibitory processing, I first tested a model where *both* of the inhibitory tasks were included to explore which specific type of inhibitory control task was relevant for number ordering (Model A). The unconstrained model had an excellent fit, $\chi^2(4) = 2.412$, $p = .660$, SRMR = .018, CFT = 1, RMSEA = 0 (90% CI = [0, .14]), whereas the constrained model fit poorly, $\chi^2(11) = 33.955$, $p < .001$, SRMR = .190, CFT = .851, RMSEA = .169 (90% CI = [.11, .24]). The comparison of the two nested models based on a likelihood ratio test showed that the constrained model had a statistically significantly poorer fit than the unconstrained model, $\Delta\chi^2(7) = 31.543$, $p < .001$. These results suggest that the integration status of numerical associations were different for children in grades 1 and 2. Thus, the fully unconstrained path models for each grade where the coefficients for each of the paths were estimated independently were retained. As shown in Figure 4.5 and Figure 4.6, performance on the black/white Stroop task was uniquely related to number ordering for grade 2 but not for grade 1, whereas performance on the go/no-go task was not uniquely related to number ordering for children in both grades. These results suggest that the type of inhibitory control skills involved in the black/white Stroop task was more relevant to number ordering. Next, I tested alternative models (Model B) where *only* the black/white Stroop task was used to index inhibitory processing.

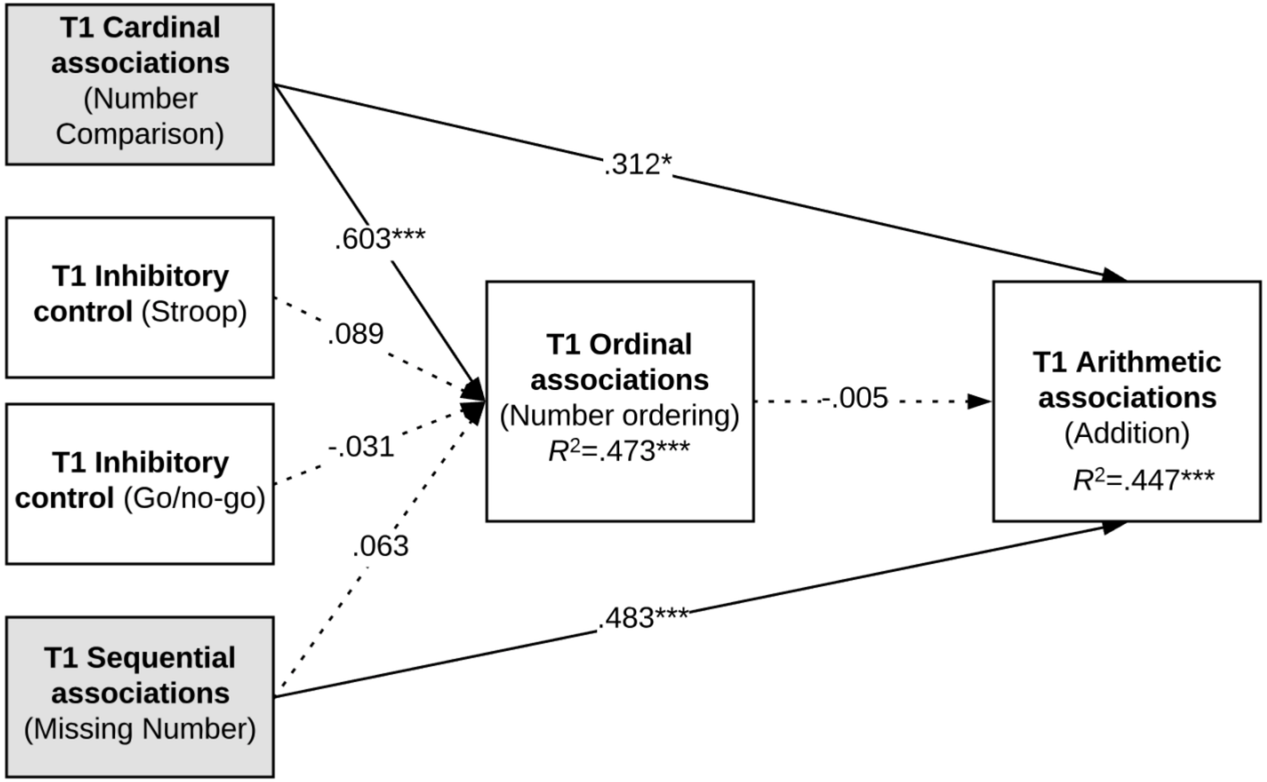

Figure 4.5. Path analyses show relations among variables for children in grade 1 when both of the inhibitory tasks were entered in the model (Model A). The numbers on the arrows are the standardized coefficients. Dotted lines indicate paths that are not significant.

Note: $p \leq .001$***; $p \leq .05$*

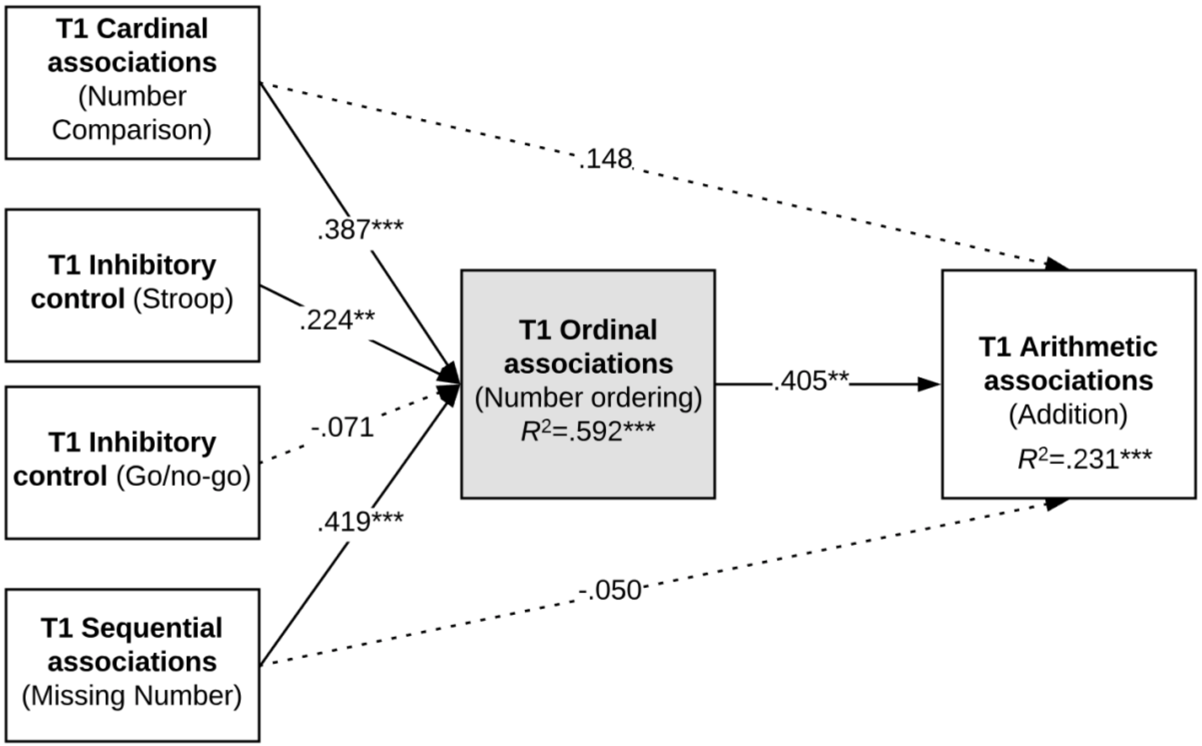

Figure 4.6. Path analyses show relations among variables for children in grade 2 when both of the inhibitory tasks were entered in the model (Model A). The numbers on the arrows are the standardized coefficients. Dotted lines indicate paths that are not significant. Note: $p \leq .001$***; $p \leq .05$*

When only the black/white Stroop task was used to index inhibitory processing (Model B), the unconstrained model had an excellent fit, $\chi^2(2)= 1.302$, $p = .522$, SRMR = .015, CFT = 1, RMSEA = 0 (90% CI = [0, .21]), whereas the constrained model fit poorly, $\chi^2(8)= 33.104$, $p < .001$, SRMR = .221, CFT = .973, RMSEA = .207 (90% CI = [.14, .28]). The comparison of the two nested models showed that the constrained model had a statistically significantly poorer fit than the unconstrained model, $\Delta\chi^2(6)= 31.802$, $p < .001$, suggesting different integration status of numerical associations for children in grades 1 and 2. Thus, the fully unconstrained path models for each grade where the coefficients for each of the paths were estimated independently were retained.

Furthermore, I compared two unconstrained models based on a likelihood ratio test where both of the inhibitory control tasks were entered (Model A) and only the black/white Stroop task was entered (Model B). The results show that there was no statistical difference between these two models, $\Delta\chi^2(2)= 1.12$, $p = .571$. Thus, the more parsimonious model where the black/white Stroop task was used as the index of inhibitory processing (Model B) was retained for interpretation.

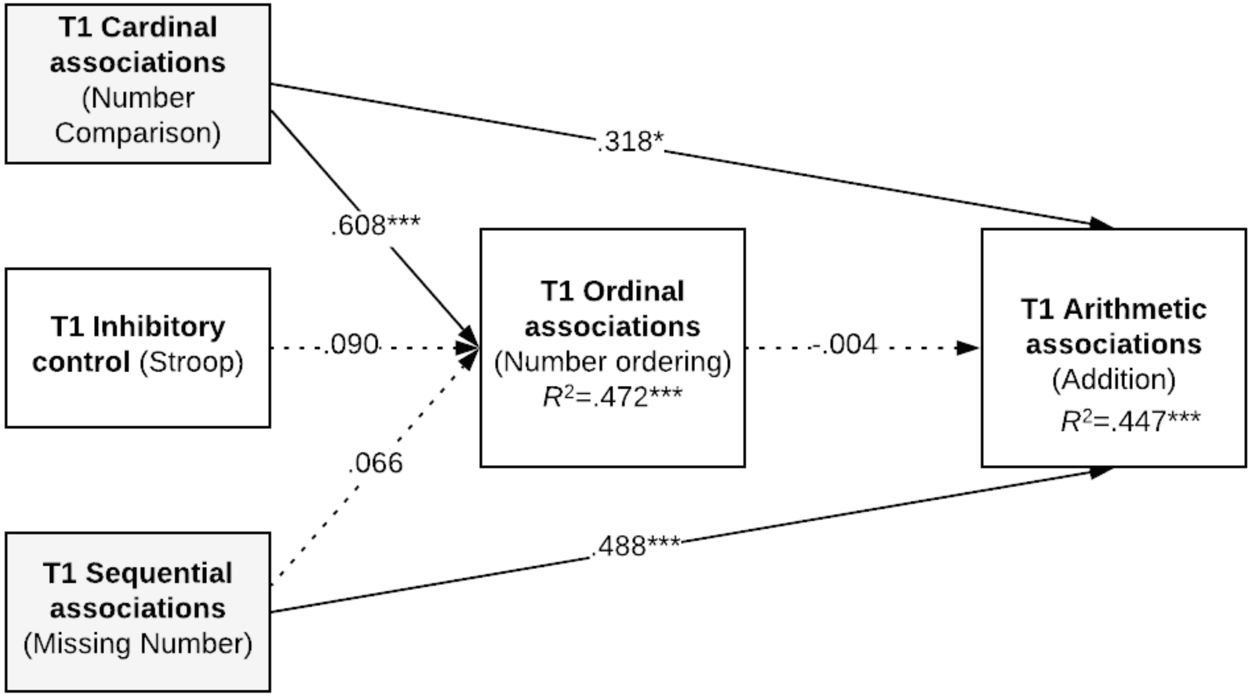

Figure 4.7. Path analyses show relations among variables for children in grade 1 when the black/white Stroop task was used to index inhibitory processing (Model B). The numbers on the arrows are the standardized coefficients. Dotted lines indicate paths that are not significant. Note: $p \leq .001$***; $p \leq .05$*

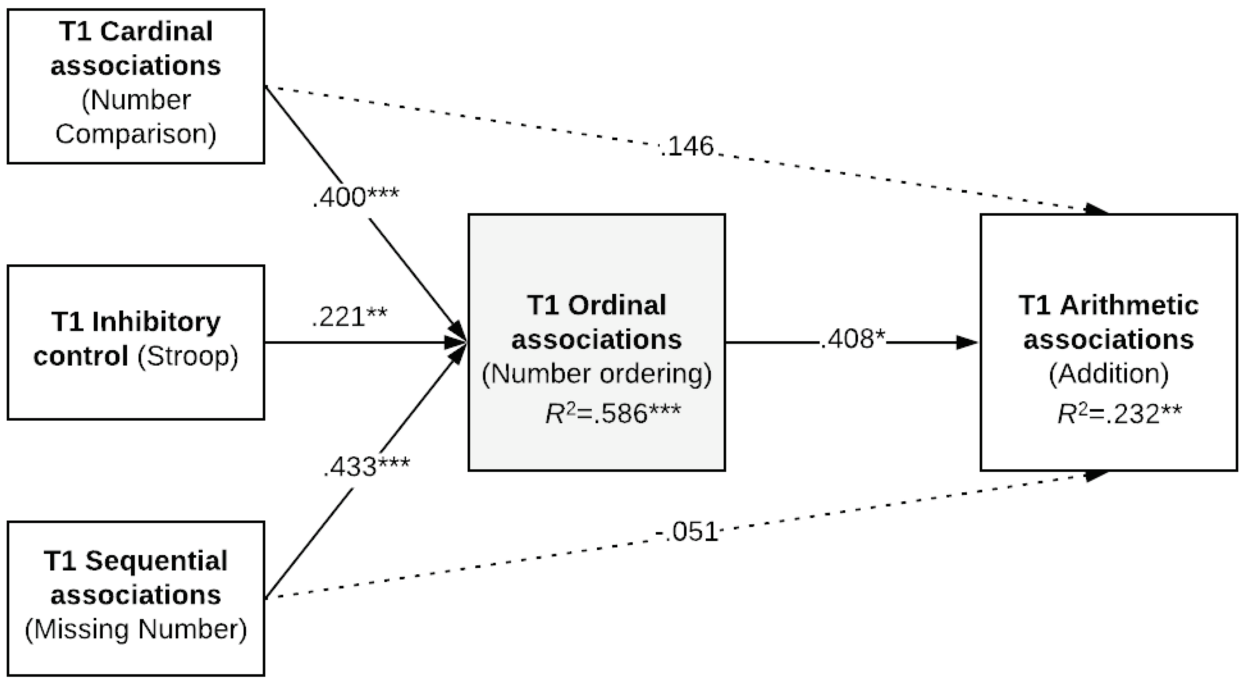

Figure 4.8. Path analyses show relations among variables when the black/white Stroop task was used to index inhibitory processing for children in grade 2 (Model B). The numbers on the arrows are the standardized coefficients. Dotted lines indicate paths that are not significant. Note: $p \leq .001$***; $p \leq .01$**; $p \leq .05$*

Hypothesis 1a. Because performance on the number order task is assumed to reflect individual differences in the accessibility of cardinal and ordinal associations, the patterns of relations among performance on the number comparison task, missing number task, inhibitory control, and number order task are expected to be different for children in grade 1 and grade 2. As shown in Figure 4.7, for children in grade 1, number ordering was strongly (and only) predicted by number comparison (β = .608), whereas performance on the missing number task (β = .066) and inhibitory control task (β = .090) was unrelated to the performance on the number order task. This pattern suggests that children were mostly likely to rely on successive number comparisons when they did the number order task.

Hypothesis 1b. In contrast, as shown in Figure 4.8, for children in grade 2, variance on the number order task was shared among three measures: the number comparison task (β = .400), missing number task (β = .433), and inhibitory control (β = .221). These results support the view that the number order task involves not only successive number comparisons (cardinal processing), as it did for grade 1 children, but also recognizing ordinal associations among numerical symbols (ordinal processing) for individuals with more integrated ordinal associations. Inhibitory control was also a unique predictor of number ordering for children in grade 2, possibly because children needed to suppress responses when the elements of counting sequences are not in order (e.g., 2 3 1). That is, their performance was influenced by associations activated automatically by the relations among numbers in counting sequences.

Summary. Consistent with the first hypothesis, children in grade 1 have not yet integrated cardinal and sequential associations, evidenced by the strong relation between number comparisons and number ordering. In contrast, children in grade 2 have integrated cardinal and sequential associations, evident by the shared variance of number ordering among the performance on number comparisons, missing number, and inhibitory control tasks.

Hypothesis 2a. As predicted, for children in grade 1, performance on the number comparison task and missing number task independently predicted addition fluency at Time 1, whereas number ordering was not related to addition fluency at Time 1 (see Figure 4.6). Previous research has shown that number comparison was the best predictor of arithmetic for children in grade 1 (Lyons et al., 2014; Sasanguie & Vos, 2018). The results of the present research showed that number comparison was a unique predictor of addition fluency ($\beta = .318$), however, performance on the missing number task was also a unique predictor of addition fluency for children in grade 1 ($\beta = .488$). The measure of ordinal associations (number ordering), which is similar to the order judgment task used in Lyons et al. (2014), was not a significant predictor of arithmetic fluency in the current study. These results support the view that the acquisition of cardinal and sequential associations of number is the precursor knowledge that supports the development of children's ordinal associations. For children in grade 1, both of the cardinal and sequential associations (number comparisons and missing number) uniquely predicted addition fluency, suggesting that they have not yet integrated to form the more advanced ordinal associations.

Hypothesis 2b. In contrast, as shown in Figure 4.8 for children in grade 2, performance on the number comparison task and missing number task were not uniquely predictive of addition fluency at Time 1 ($\beta = .146$ and $\beta = -.051$, respectively), whereas number ordering uniquely predicted addition fluency at Time 1 ($\beta = .408$). These results support the view proposed in the HSI model that speeded processing in both the number order task and arithmetic reflect individual differences in the accessibility of previously acquired associations among numerical symbols for individuals with integrated numerical knowledge (Xu et al., under review). The results of the present research show that children in grade 2 have integrated cardinal and sequential associations into a unified associative network, allowing them to select the specific and relevant associations efficiently to solve arithmetic problems.

Summary. Consistent with the second hypothesis, children in grade 1 have not yet integrated cardinal and sequential associations into an integrated network, and thus each of them separately predicted the arithmetic fluency in the concurrent analysis. In contrast, children in grade 2 have integrated cardinal and sequential associations into a unified associative network, and thus ordinal associations superseded the "cardinal" influence to be a better predictor of arithmetic than number comparison, accounting for the finding that ordinal processing was a better predictor than cardinal processing of arithmetic in this study, as in previous work (Lyons et al., 2014; Sasanguie & Vos, 2018).

Step 2 (Hypothesis 3). Given the different integration status for children in grades 1 and 2 (as shown in Step 1), two separate path models were tested for each grade to examine which specific form of numerical associations would predict the *growth* of addition fluency. This approach is based on the assumption that children in these grades accomplish arithmetic processing differently because considerable change in their overall skill and in their selection of strategies on addition problems occurs in this educational time frame.

Hypothesis 3a. For children in grade 1, I tested whether performance on the number comparison and missing number tasks would predict addition fluency at Time 2, controlling for children's addition fluency at Time 1. This hypothesis was partially supported (see Figure 4.5). The model fit was excellent, $\chi^2(3)= .724$, $p = .868$, SRMR

= .014, CFT = 1, RMSEA = 0 (90% CI = [0, .11]). As predicted, performance on the number comparison task marginally predicted the growth of addition fluency for children in grade 1 (β = .202), suggesting the importance of the accessibility of cardinal associations in the development of addition fluency in the second half of grade 1 (see Figure 4.9). However, performance on the missing number task did not directly predict the growth of arithmetic fluency for children in grade 1 (β = .158). Note that the indirect effects from number comparisons and missing number performance to addition fluency at Time 2 were significant through addition fluency at Time 1 (β = .138, p = .048, and β = .211, p = .004, respectively), suggesting that children's initial addition fluency mediated the relations between cardinal or sequential knowledge and addition fluency at a later time point. These results suggest that the growth in arithmetic fluency in the latter half of grade 1 for this group of children was probably reflected in faster and more accurate execution of counting strategies, particularly min counting, since number comparisons were the only direct predictors of growth.

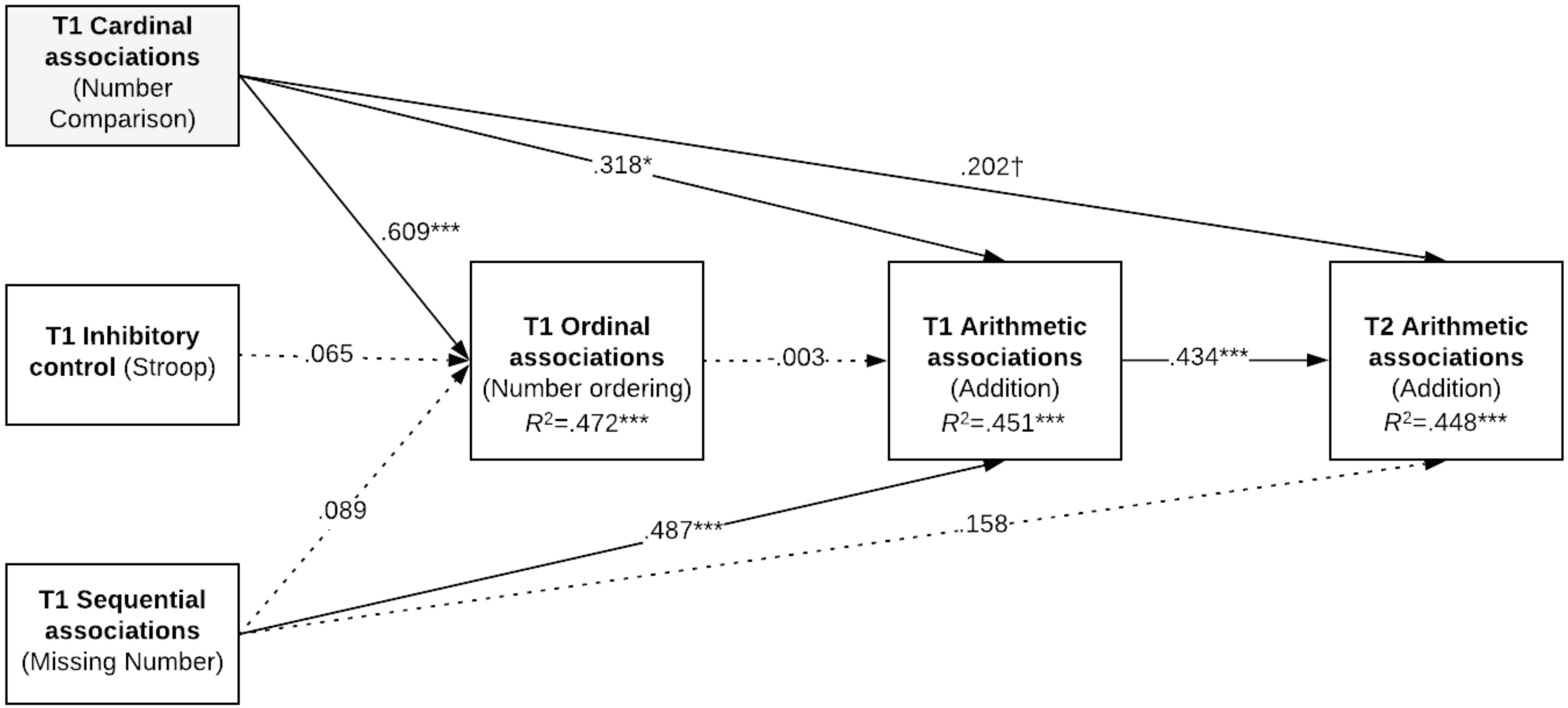

Figure 4.9. Final path model shows relations among variables for children in grade 1. The numbers on the arrows are the standardized coefficients. Note: $p \leq .001$***; $p \leq .05$*; $p = .077$†

Hypothesis 3b. In contrast, for children in grade 2, I hypothesized that performance on the number order task would predict addition fluency at Time 2 controlling for children's addition fluency at Time 1. Hypothesis 3b was supported with an excellent model fit, $\chi^2(4)= 1.316$, $p = .859$, SRMR = .018, CFT = 1, RMSEA = 0 (90% CI = [0, .09]). As shown in Figure 4.8, performance on the number order task predicted the growth of addition fluency for children in grade 2 ($\beta = .251$), suggesting that the fluency of access to integrated associations predicted the development of addition fluency (Figure 4.10). Further, the indirect effect from number ordering to addition fluency at Time 2 was also significant through addition fluency at Time 1 ($\beta = .216$, $p = .009$), suggesting that children's addition fluency at Time 1 partially mediated the relations between ordinal knowledge and addition fluency at Time 2. These results suggest that for children in grade 2, their accessibility of ordinal associations predicted the growth of arithmetic fluency, suggesting a more integrated network among cardinal, sequential, and ordinal associations.

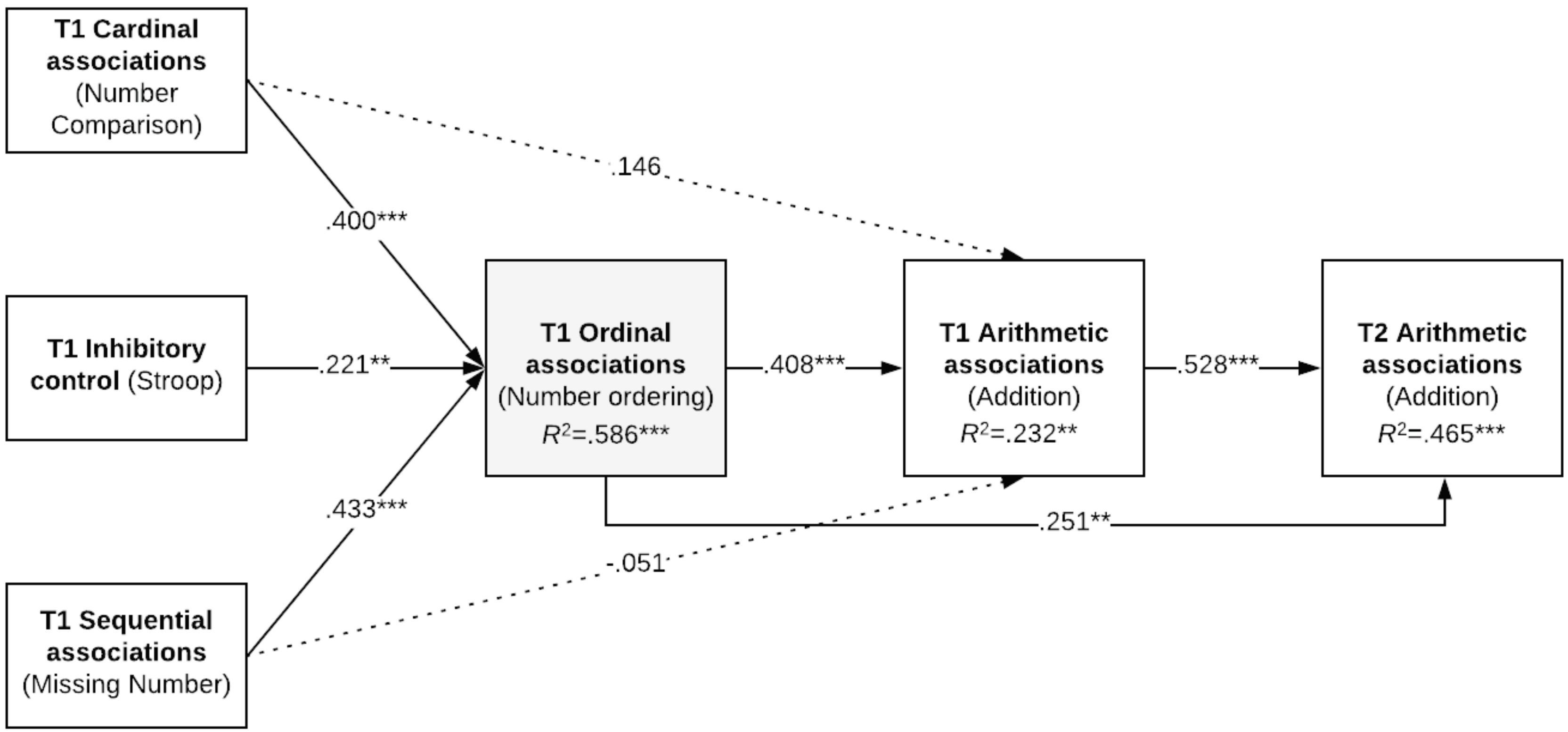

Figure 4.10. Final path model shows relations among variables for children in grade 2. The numbers on the arrows are the standardized coefficients. Note: $p \leq .001$***; $p \leq .01$*

Note that children in grade 2 completed both the simple and complex versions of the addition task; as a result, the addition fluency reflected a mixture of both types of addition skills. Thus, I present two additional analyses of the expanded HSI model for children in grade 2 when the average addition fluency was broken down into simple and complex addition. Children in grade 2 are expected to use different strategies to solve different types of addition problems (Siegler, 1987). In the present research, for addition problems that do not involve carrying (i.e., sums less than or equal to nine), children in grade 2 probably mainly relied on direct retrieval (Siegler, 1987); this conclusion is supported by the lack of change in performance on the simple addition task over time (Table 4.3). In contrast, for problems that involve carrying (i.e., sums ranged from 10 and 17), children in grade 2 are more likely to use the min strategy or decomposition strategies (Siegler, 1987). The accessibility of ordinal associations (number ordering) is presumably related to the efficiency to select and implement calculation strategies (i.e., min strategy and decomposition) for the complex addition task but not for the simple addition task. Therefore, I hypothesized that the unique contribution of number ordering in the growth of addition fluency would be different for the model using simple addition as outcomes and the model using complex addition as the outcomes: number ordering should predict the growth of complex addition fluency but not of simple addition fluency for children in grade 2.

As shown in Figure 4.11, performance on the number order task uniquely predicts simple addition concurrently (β = .470), however, it did not predict the growth of simple addition fluency (β = -.005). The model fit was excellent, $\chi^2(4)$= 2.844, p = .584, SRMR = .020, CFT = 1, RMSEA = 0 (90% CI = [0, .15]). The significant relationship between number ordering and simple addition fluency at Time

1 is consistent with the results when the average addition performance was used, supporting the view that children in grade 2 have integrated ordinal associations of number. Further, in contrast to the finding that number ordering predicted the growth of addition fluency when the average performance was used, in the present analysis, number ordering was not directly related to simple addition fluency at Time 2 after controlling for children's initial simple addition fluency at Time 1. Nevertheless, the indirect effects from number ordering to simple addition fluency at Time 2 was significant through simple addition fluency at Time 1 ($\beta = .354$, $p < .001$). One possibility is that children in grade 2 reached ceiling on the simple addition task at the beginning of the year, and thus there was little room for improvement over time. Thus, the lack of predictive power of number ordering in the growth of simple addition fluency is presumably constrained by the limited improvement on the simple addition performance from Time 1 to Time 2.

In contrast, as shown in Figure 4.12, performance on the number order task was related to complex addition both concurrently ($\beta = .299$), and predictively ($\beta = .359$), consistent with the patterns of results when the average addition performance was used. The model fit was excellent, $\chi^2(4) = 1.499$, $p = .829$, SRMR = .019, CFT = 1, RMSEA = 0 (90% CI = [0, .10]). The indirect effects from number ordering to complex addition fluency at Time 2 was marginally significant through complex addition fluency at Time 1 ($\beta = .122$, $p = .082$). These results suggest that children's initial complex addition fluency at Time 1 explained some portion of the total effect from number ordering to complex addition fluency at Time 2. However, the remaining effect of the number ordering on complex addition at Time 2 suggests that the fluency to access ordinal associations contributed to the improvement of complex addition fluency over time for children in grade 2.

Taken together, the results of the additional analyses suggest that the finding that number ordering predicted the growth of addition fluency when the averaged addition performance was used as the outcome was primarily driven by complex addition performance rather than simple addition performance. Note that children are expected to use calculation strategies to solve complex addition problems whereas they probably rely on direct retrieving the answers for the simple addition problems (e.g., Siegler, 1987). Thus, the results of the present analyses suggest that children's accessibility of ordinal associations may allow them to more efficiently (faster and more accurately) execute the min strategy and decomposition strategy to solve the complex addition problems.

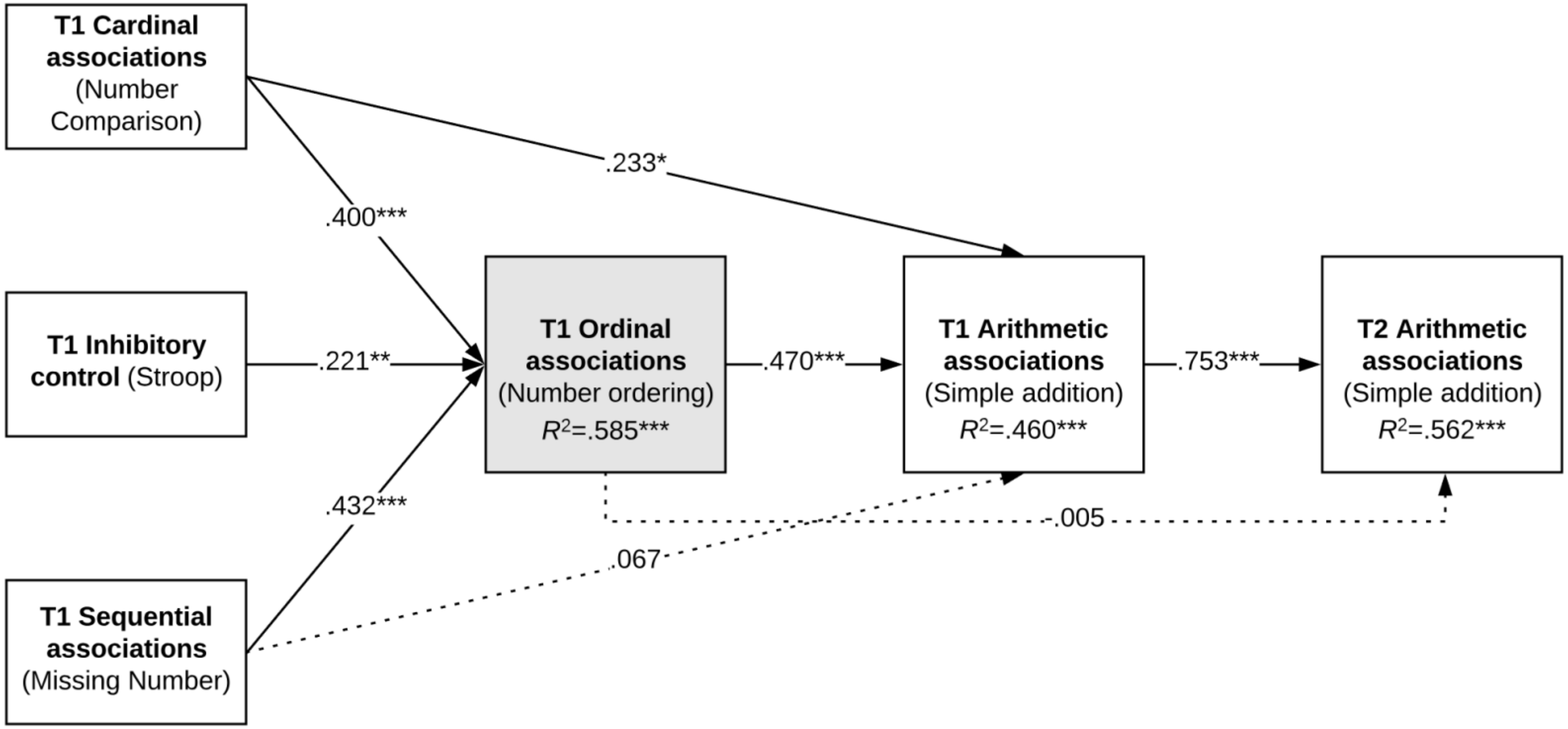

Figure 4.11. Additional path model using simple addition as the outcome for children in grade 2. The numbers on the arrows are the standardized coefficients. Note: $p \leq .001$***; $p \leq .01$**; $p < .05$*

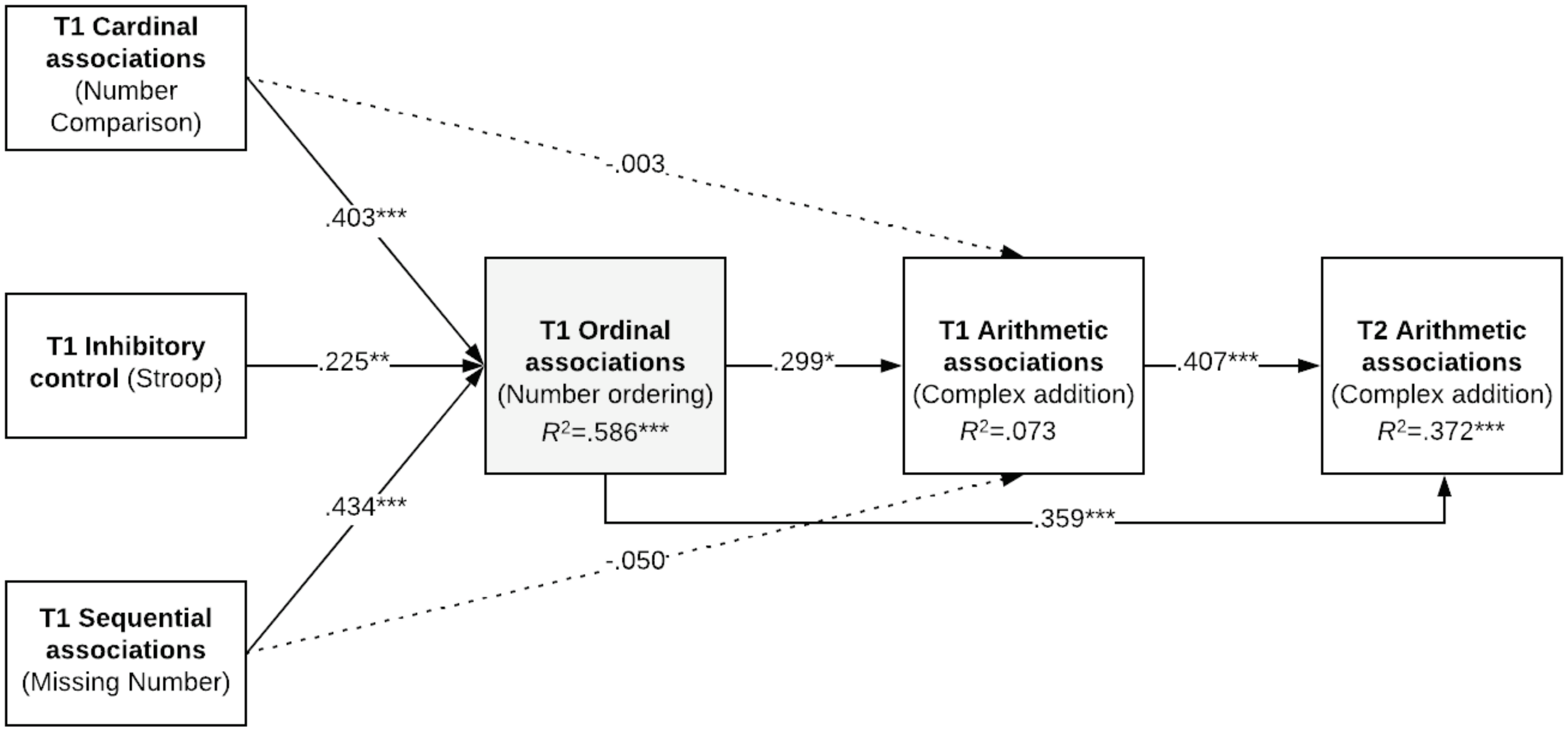

Figure 4.12. Additional path model using complex addition as the outcome for children in grade 2. The numbers on the arrows are the standardized coefficients. Note: $p \leq .001$***; $p \leq .01$**; $p = .058$*

Summary. These results suggest that for grade 2 children who have a more integrated associative network, individual differences in the accessibility of ordinal associations determine growth of arithmetic fluency. The accessibility of ordinal associations is predicted by the combination of cardinal and sequential processing, also consistent with the notion that ordinal processing reflects the integration of these associations. In contrast, for grade 1 children, who have a less integrated network, individual differences in the accessibility of cardinal associations is the only direct predictor the growth of arithmetic fluency. Extrapolating across grades, the results of the present research suggest that children progress from a reliance on cardinal processing to ordinal processing when developing arithmetic fluency.

CHAPTER 5: GENERAL DISCUSSION

How are individual differences in the acquisition of fundamental numerical competencies related to children's arithmetic development? Cardinal knowledge and ordinal knowledge have been suggested as two fundamental components in the development of symbolic number knowledge (Sury & Rubinsten, 2012). In the present dissertation, cardinal knowledge is defined as the fluency of access to magnitudes of numerical symbols, whereas ordinal knowledge is defined as the fluency of access to the ordered relations among numerical symbols. Previous research suggested that the development of cardinal knowledge precedes the development of ordinal knowledge (Colomé, & Noël, 2012; Lyons et al., 2014; Knudsen et al., 2015; Sasanguie & Vos, 2018). In the present research, I postulated that the acquisition of ordinal knowledge requires the integration of sequential knowledge (i.e., the associations among successive numbers, e.g., 3 comes before 4, and after 2) and cardinal knowledge of numbers as the precursor skills. Moreover, the acquisition of ordinal knowledge as measured in order judgment or decision tasks also involves inhibitory processing based on the assumption that children may need to suppress the activation of the symbolic representations of sequential associations while making order judgments. Taken together, ordinal processing of numbers may involve a range of cognitive abilities, including cardinal and sequential associations, and inhibitory processing.

The development of numerical associations is hierarchical such that the earlier acquired associations are the building blocks of mastering the higher-level associations (Cirino et al., 2016; Entwisle & Alexander, 1990; Hiebert, 1988; Núñez, 2017; Siegler & Lortie-Forgues, 2014; Resnick, 1989). The present research is built up upon the

Hierarchical Symbol Integration (HSI) model: cardinal knowledge develops first and thus serves as the base of the mental network, followed by ordinal associations, and then arithmetic associations (Xu et al., under review). The HSI model implies that the earlier developed associations are not replaced but are integrated with as more associations are acquired in a unified mental network that can be accessed according to the demands of the specific task. Thus, it is the increasing complexity of symbolic associations that determines the hierarchical relations among them. On this view, ordinal processing can be viewed as the higher-level processing in the associative network than cardinal processing, because it involves a wider range of cognitive skills (cardinal, sequential and inhibitory processing). Further, arithmetic associations are integrated into the hierarchical network as arithmetic solutions require children to access their sequential and cardinal associations and thus arithmetic fluency is built upon fluent access to ordinal associations (Geary et al., 2004; Lindberg et al., 2013; Shrager & Siegler, 1998). Thus, the integration of cardinal and sequential associations into ordinal associations is critical for the development of mental arithmetic skills for children.

The main goal of the present research was to examine the precise developmental trajectories of cardinal, ordinal, and arithmetic knowledge. Hence, I examined changes in the relations among symbolic number processes for children in grades 1 and 2 using a short-term longitudinal design. The present research comprises two studies. In both studies, I measured children's completion of a) a number comparison task as an index of their cardinal knowledge, b) a missing number task as an index of the simplest form of ordinal processing, that is, access to the sequential associations among numbers, c) a number ordering task as the index of acquisition of fully integrated ordinal associations, and d) a mental addition task as the index of arithmetic skills. The first study served as a pilot study for the second study to test the reliabilities and validities of the measures, and thus it was not further discussed in this chapter.

Hypothesis 1: Integration Status of Ordinal Associations

First, I examined the integration status for the acquisition of ordinal associations for children in grades 1 and 2. As predicted, for children in grade 1, only cardinal knowledge was highly associated with the number ordering task, suggesting that children relied on successive number comparisons to order sequences. Moreover, inhibitory control processing was also not related to ordinal processing, suggesting that the activation of the symbolic representations of counting sequences was not obligatory for most of the children in grade 1. In contrast, for children in grade 2, all of the sequential, cardinal, and inhibitory processing tasks were uniquely associated with ordinal processing, suggesting that these children have integrated cardinal and ordinal associations into a cohesive network. Together, these results suggest that for children in grade 1, the number order task reflects children's accessibility of cardinal associations because they relied on successive number comparisons as the main strategy to order the number sequences. Conversely, for children in grade 2, the number order task captures individual differences in a range of cognitive abilities beyond cardinal processing.

Inhibitory processing. In the present research, inhibitory processing was measured by two tasks (i.e., the black/white Stroop task and the go/no-go task) that are assumed to measure the same construct "response inhibition", that is, the ability to withhold an automatic (prepotent) behaviour (Friedman & Miyake, 2004; Simmonds et al., 2008). However, the results of the present research show that the two inhibitory tasks are not correlated with each other in either Study 1 or Study 2. This finding is consistent with the literature on inhibitory processing such that the different measures of inhibitory control do not always tap a single construct (Brocki & Bohlin, 2004; James et al., 2016).

Moreover, the results of Study 2 show that performance on the black/white Stroop task was a unique predictor of number ordering for children in grade 2 but not for children in grade 1, whereas performance on the go/no-go task was not uniquely related to number ordering for children in either grade. These results suggest that the type of inhibitory control involved in the black/white Stroop task may be relevant to number ordering for children in grade 2 who have more integrated number associations. The black/white Stroop task requires children to suppress the automatic activation of previously overlearned knowledge in the incongruent conditions (i.e., the activation of the meaning of the black or white colour). This type of inhibitory processing is similar to the inhibitory processing involved in the number ordering in that children need to suppress the automatic activation of previously acquired sequential associations for the unordered counting sequences (e.g., 2 3 1). Similarly, the performance on the traditional order judgment task also requires that participants suppress the automatic activation of sequential associations when they need to reject the unordered counting sequences. In contrast, no automatic activation of previously learned knowledge is needed for the go/no-go task. Therefore, the black/white Stroop task may be a more appropriate measure of the type of inhibitory processing involved in the ordinal tasks.

Cognitive processes involved in the number order task. Children's performance on the different types of sequences on the number order task was examined, to test the assumption that different cognitive processes may be used depending on the characteristics of the sequence (Lyons et al., 2016; Vos et al., 2017). The results of the present research suggested that there are three possible cognitive pathways when children performed the number order task, as illustrated in Figure 5.1. In particular, if the sequence to be judged is a correctly ordered counting sequence (e.g., 4 5 6), then children can activate the symbolic representations of sequential associations directly. However, if the sequence to be judged is an unordered counting sequence that shares numbers with ordered counting sequences (e.g., 5 6 4), then the activation of sequential associations may need to be suppressed first, followed by the activation of cardinal associations based on inferential reasoning (3 is smaller than both 4 and 5, so 3 should come first, followed by 4 5). On this view, children's performance reflects the extra inhibitory demand needed to process the unordered sequence. Lastly, if the sequence to be judged is not a counting sequence, then children activate the symbolic representations of cardinal associations directly, regardless of whether the numbers are correctly ordered or not (e.g., 4 1 8 or 3 5 8). Together, three types of cognitive processing (cardinal, sequential, and inhibitory processing) are assumed to be involved in the number order task, activated differentially depending on the characteristics of the sequence.

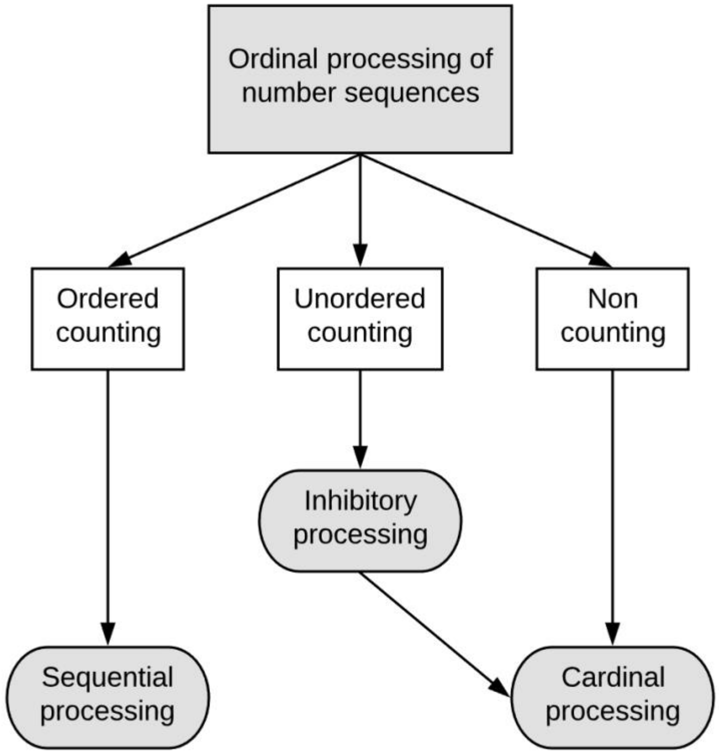

Figure 5.1. Theoretical pathways for number ordering for children in grades 1 and 2

The results of the present research show that children in both grades 1 and 2 were faster at ordering the sequences that are part of the counting string (see Figure 4.2) than any other types of sequences. Recall that this pattern of results is usually interpreted as the "reversed distance effect": numbers that are farther apart are judged more slowly and less accurately than numbers that are closer together (Franklin & Jonides, 2009; Goffin & Ansari, 2016; Lyons & Ansari, 2016; Lyons & Beilock, 2013). On this view, the size of the numerical distance between numbers should be the main factor affecting all order judgments, because participants activate associations between the adjacent digits from their long-term memory more easily than the associations between the non-adjacent digits, resulting in the reversed distance effect. However, this interpretation seems unlikely based on the results of the present research for the following reasons.

First, if the size of the numerical distance between numbers affects order judgments, then children should be equally fast and accurate at ordering sequences with the same intervals between numbers regardless of whether the digits are correctly ordered or not. However, the results of the present research show that all children found it substantially harder to process counting sequences that were incorrectly ordered (e.g., 4 5 3 or 3 5 4) than the sequences that were correctly ordered (e.g., 4 5 6 or 3 4 5; Figure 4.3 and 4.4). This pattern of results is consistent with the research with adults using the order judgment task where participants are asked to accept sequences such as 3 4 5 but reject sequences such as 4 5 3, in that adults also find the latter decision difficult (Bourassa, 2014; Lyons & Beilock, 2009). These results suggest that solvers need to suppress the interference effects resulting from activating the elements of a counting sequence when those elements are not ordered. Notably, such interference effects were larger for children in grade 1 than children in grade 2, evidenced by the error patterns as shown in Figure 4.3, suggesting that children in grade 2 were more successful at inhibiting the interference effects resulting from obligatory activation of sequential associations when the elements of counting sequences are not in order. Thus, effective inhibition of task-irrelevant information is also related to performance on the number order task. Furthermore, these results show that the size of the numerical distance between numbers themselves might not be the primary reason why children showed an advantage at ordering sequences that are part of the counting string. Rather, it is possible that counting sequences are more familiar to children than other types of sequences, resulting in stronger associations between these adjacent numbers and thus faster decisions when the sequences are in ascending order.

Second, if the size of the numerical distance between numbers affects order judgments, then children should perform similarly on non-counting sequences regardless of whether they are correctly ordered or not. This pattern was true for children in grade 2 but not for children in grade 1. In particular, children in grade 2 performed similarly on both types of non-counting sequences (Figure 4.3 and 4.4). However, children in grade 1 were faster and more accurate on non-counting sequences that were correctly ordered (e.g., 3 5 8) than on non-counting sequences that were incorrectly ordered (e.g., 4 1 8). Previous research suggests that exposure to Arabic digits during the first year of formal education supports automaticity of cardinal activation (Rubinsten et al., 2002; White et al., 2012). Therefore, in the present research, it is possible that children in grade 1 had not achieved automaticity in activating cardinal associations, and thus they were slower and less accurate on the non-counting sequences that were incorrectly ordered (e.g., 4 3 8) because additional mental processing might be needed to obtain the correct response on those sequences (e.g., $4 > 1$; $1 < 8$; $4 < 8$, therefore $3 \rightarrow 4 \rightarrow 8$), than on ones that are correctly ordered (e.g., 3 4 8; $3 < 4$; $4 < 8$, therefore $3 \rightarrow 4 \rightarrow 8$). In contrast, children in grade 2 might have achieved automaticity of cardinal processing (White et al., 2012), and thus they were equally fast at ordering the non-counting sequences regardless the order of the numbers, showing adult-like performance on these sequences (Bourassa, 2014).

In summary, the results of the present research do not support the interpretation of reversed distance effects in the literature. Instead, the results of the current research suggest that children can rely on number comparisons (cardinal associations) to order all types of sequences; however, when the associations between the digits are familiar (i.e., part of the counting sequence), some children activated the symbolic representations of sequential associations directly. In particular, the results show that the number comparison strategy is more dominant for children in grade 1, evidenced by the strong relation between number comparisons and number ordering. In contrast, different mechanisms might be used depending on the characteristics of the sequences in the number order task for children in grade 2, evidenced by the shared variance of number ordering among the performance on number comparisons, missing number, and inhibitory control tasks.

Hypothesis 2: Concurrent Relations Among Basic Associations and Arithmetic Fluency

Second, I examined how various forms of associations (sequential, cardinal and ordinal associations) were concurrently related to arithmetic associations for children in grades 1 and 2. As predicted for children in grade 1 whose numerical associations are not integrated yet, cardinal and sequential associations, rather than ordinal associations, uniquely predicted arithmetic associations. In contrast, for children in grade 2, ordinal associations superseded the more fundamental forms of associations (cardinal and sequential) as the best predictor of arithmetic. These results suggest that ordinal associations among numbers are integrated with cardinal and sequential associations for children in grade 2.

Based on the assumption of HSI model, children who are more fluent in accessing the symbolic associations have more integrated symbolic networks than children who are less fluent in accessing these associations (Xu et al., under review). Performance on the number comparison, missing number, and number order tasks all reflect individual differences in the fluency of access to associations among symbolic numbers, namely, cardinal associations, sequential associations and ordinal associations, respectively. According to the HSI model, the pattern of relations among these tasks and arithmetic indicate the integration status for children in the hierarchical associative network. In particular, if children have not yet fully integrated the more basic forms of associations (cardinal and sequential associations) together, then each of these associations should uniquely predict arithmetic fluency. In other words, these children are more likely to show individual differences in the lower-level numerical associations in the HSI model (rather than the higher-level ordinal associations) as predictors of the more advanced arithmetic task. The results of present research show that both cardinal and sequential associations uniquely predicted arithmetic for children in grade 1, further supporting the view that these children have not fully integrated the basic numerical associations together into a cohesive network.

In contrast, if children have integrated the more basic forms of associations, then ordinal associations should mediate the relations between the more basic forms of associations and arithmetic. On this view, ordinal associations represent the higher level of the symbolic network, as is suggested by the findings for older children and adults in previous research (e.g., Lyons & Beilock, 2009; Lyons et al., 2014; Sasanguie & Vos, 2018), and thus, these children are more likely to show individual differences in the higher-level ordinal processing in the HSI model as the unique predictor of arithmetic. The results of the present research show that by grade 2, the relations among cardinal, sequential and arithmetic associations were completely mediated through ordinal associations, supporting the view that increasing levels of numerical experiences and skills are associated with the development of a more integrated network.

Taken together, the results of the present research support the theoretical account that the development of cardinal associations precedes the development of ordinal associations (Colomé & Noël, 2012; Lyons et al., 2014; Knudsen et al., 2015; Sasanguie & Vos, 2018; Xu et al., under review). During the early phase of development, processing of cardinal and ordinal aspects of numbers is intertwined (Lyons et al., 2016), presumably because children need to rely on cardinal processing of numbers to extract ordinal information of numbers for most of the sequences. However, when the symbolic representations of the associations between the numbers in a sequence become sufficiently strong, ordinal processing of numbers activates both cardinal and sequential associations. The activation of both cardinal and sequential aspects of number when children are ordering numbers is assumed to reflect an integrated associative network, which serves as the building block for their understanding of development of arithmetic skill.

Hypothesis 3: Predictive Relations Among Basic Associations and Arithmetic Fluency

Lastly, I examined which specific form of associations would predict the *growth* of arithmetic associations over a four-month period. For children in grade 1, after controlling for their initial arithmetic performance, cardinal associations uniquely predicted the growth of arithmetic, whereas sequential associations did not. In contrast, for children in grade 2, ordinal associations uniquely predicted the growth of arithmetic.

When children initially learn addition, they count using a sum strategy (i.e., counting both addends starting from one) or a min strategy (i.e., determining the larger of two addends to serve as the base, with the smaller operand used as the increment Groen

& Parkman, 1972; Lindberg et al., 2013). Children do not consistently rely on more efficient strategies such as direct retrieval and decomposition to solve addition problems until grade 3 in North American samples (e.g., Geary et al., 2004). Relatedly, according to Siegler's adaptive strategy choice model (Siegler, 2000; Siegler & Shrager, 1984), strategy development in various contexts is not stage-like, but rather can be viewed as a series of overlapping waves. That is, children at any grade level do not necessarily solve arithmetic problems uniformly; instead they choose a strategy on a given problem from a range of available options. The particular strategy chosen depends on various factors, such as the recency with which they solved that problem. Nevertheless, the overall speed and accuracy with which children execute each strategy and the sophistication of strategy choice generally improves with age and educational experiences (Siegler, 1987). For children in grades 1 and 2, number fact retrieval is not readily available for majority of the children, and the min strategy and decomposition are usually the most dominant addition strategies (Geary et al., 2004; Lindberg et al., 2013; Shrager & Siegler, 1998; Siegler & Araya, 2005).

The improvement in strategy choice in arithmetic problem solving may be related to the development of children's understanding of basic numerical associations. More specifically, children's use of the min strategy (i.e., counting from the larger addend) involves knowledge of cardinal associations to choose which number is the larger and involves sequential associations as they count on from the larger number. Further, children's use of the decomposition strategy (e.g., solving 6 + 5 as 6 + 4 = 10 + 1) involves manipulation of number sets based on their conceptual understanding of how number symbols are related to others (i.e., ordinal association), and retrieval of partial sums based on their stored addition associations (Geary et al., 2004). On this view, the development of the min strategy and decomposition strategies depends on children's fluency of access to cardinal, sequential, and ordinal associations among symbolic numbers.

The results of the present research show that for children in grade 1, individual differences in the fluency with which numbers can be compared (measured by a number comparison task) predicted improvements in addition fluency in the second half of grade 1. However, the fluency with which children were able to insert a number into a sequence (measured by a missing number task) did not predict improvements in addition fluency. The indirect effects from number comparisons and missing number performance to addition fluency at Time 2 were significant through addition fluency at Time 1. These results suggest that the growth in arithmetic fluency for children in grade 1 may reflect better execution of the min strategy. In contrast, for children in grade 2, the fluency with which they were able to determine the relative order of symbolic digits (measured by a number order task) predicted improvements in addition fluency. These results suggest that ordinal associations capture the representations of cardinal and sequential associations as an integrated building block of arithmetic development. Thus, the growth in arithmetic fluency for children in grade 2 may be reflected in better execution of both min and decomposition strategies. Notably, in the present research, I did not collect children's strategy data for the addition tasks, thus, the assumption that the improvement of the arithmetic fluency for children in grades 1 and 2 resulted from the increasing use and execution of more efficient counting and decomposition strategies was not directly tested because there is already a wealth of evidence showing how children's strategies develop as they become skilled at simple addition (e.g., Geary et al., 2004; Lindberg et al., 2013; Siegler, 1987; Shrager & Siegler, 1998; Siegler & Araya, 2005).

Implications of the Present Research

Hierarchical Symbol Integration Model. Previous research on adults provides converging evidence that cardinal and ordinal processing of numbers can be dissociated, evidenced by the differential activation patterns in brain circuits based on neurological studies (Delazer & Butterworth, 1997; Lyons & Beilock, 2013; Turconi et al., 2004; Turconi & Seron, 2002), as well as the presence of two distinct effects (distance effect vs. reversed distance effect) when people are performing cardinal and order judgment tasks in behavioural studies (e.g., Goffin & Ansari, 2016; Lyons & Ansari, 2015). For children, the results of the present research suggest that cardinal and ordinal processing of numbers are not dissociable for children in grades 1 and 2, at least for the types of order tasks used, because the ordinal task (e.g., order judgment task and number order task) activated a range of cognitive skills that included cardinal processing of numbers, as well as sequential processing of numbers and inhibitory processing.

The findings of the present research support the view that the acquisition of fundamental associations among numbers forms a hierarchical mental network that allows one to fluently and selectively activate the specific associations depending on the task demands (Xu et al., under review). This view is built on the theoretical models for arithmetic associations (e.g., Ashcraft, 1995; Campbell, 1995; Siegler, 1988; De Visscher & Noël, 2014). In these models, arithmetic facts are stored in a complex associative network. The associations between problems and the answers become stronger through extensive experience of activating these associations (Ashcraft, 1995; Siegler & Shrager, 1984). Similarly, the fundamental associations among numbers (cardinal, sequential and ordinal associations) are also stored in the same associative network, which start to develop before symbolic arithmetic associations. More specifically, true automaticity of the connections between symbolic digits and quantities, that is, the automatic processing of numerical magnitude without intention, reflects strong cardinal associations among numbers (e.g., Girelli et al., 2000; Noël et al., 2005). Obligatory activation of symbolic representations of the counting sequence even when the sequences are not in order (e.g., 2 3 1) reflects strong sequential associations among numbers (also see Bourassa, 2014; LeFevre & Bisanz, 1986; Lyons & Beilock, 2009). In the present research, I postulate that the acquisition of ordinal associations and their dominance in predicting arithmetic processes captures an ongoing process of integration of the previously acquired cardinal and sequential associations into a unified network, allowing children to selectively activate the appropriate associations for the ordinal task versus the cardinal and arithmetic tasks.

Further, according to the HSI model, the unified associative network that includes a collection of fundamental associations is the building block of the development of arithmetic skills (Xu et al., under review). On this view, people who have a larger associative network have a greater flexibility to choose more efficient strategies to solve arithmetic problems (see also Siegler, 2007), presumably because they have a greater number of associations available in their mental network. For example, there are many different ways to solve the problem 6 + 7, such as a) sum strategy (counting from 1 until 13), b) count-on-first strategy (counting up from 6 by 7 numbers), c) min strategy (counting up from 7 by 6 numbers), d) decomposition (6 + 6 + 1 = 13, 7 + 7 - 1 = 13, or 6

+ 4 + 3 = 13, or 6 x 2 + 1…), and e) direct retrieval. The use of the decomposition or direct retrieval requires a sufficiently large mental associative network that includes not only the most basic cardinal, sequential and ordinal associations, but also the higher-level arithmetic associations (e.g., 6 + 6 = 12, 6 + 4 = 10, 6 x 2 = 12, 6 + 7 = 13). In contrast, people who have a limited associative network are constrained to the less efficient counting strategies when they are performing mental calculations. More broadly, the findings of the present research support the view that integrating the different forms of symbolic associations into a large, cohesive mental associative network is crucial for the development of the acquisition of higher-level mathematical competencies (Cirino et al., 2016; Entwisle & Alexander, 1990; Lyons et al., 2016; Hiebert, 1988; Núñez, 2017; Siegler & Lortie-Forgues, 2014; Resnick, 1989).

Future Research

The present research is an important step in understanding the development of symbolic associations that support numerical processing. However, there are many related questions that were not directly addressed in this work. First, although the expanded HSI model provides a framework for understanding children's developmental progression of how various associations integrated over time in relation to the development of arithmetic, averaging performance by grade results in a summary score that may not accurately represent the changes that occur for an individual child. Children at the same grade level may have different integration status given their different numerical experiences outside of school. Children's early academic achievement is related to variables such as parents' education, family income, socioeconomic status, and attitudes and beliefs about mathematics (e.g., Baharudin & Luster, 1998; Benavides-

Varela et al., 2016; Niklas & Schneider, 2017; Zippert & Ramani, 2017). Thus it is also possible that the integration may happen earlier or later than the first year of schooling depending on other variables that are not tested in the present research or for different groups of children. Nevertheless, given similar educational circumstances as those experienced by the children in this research, it seems reasonable to assume that the integration of ordinal associations generally occurs at some time between grades 1 and 2. This conclusion is consistent with White et al. (2012) who suggested that one year of formal schooling was sufficient for children to show obligatory activation of cardinal knowledge in the number comparison task. Future studies should model quantitative individual differences using a latent variable approach to examine whether different latent classes (i.e., integration status) emerge within each grade, and what factors would contribute to the transition from cardinal to ordinal integration.

Another important issue that requires further research concerns the construct validity of the various cognitive processes that are implicated in numerical development. In the present research, I used a single task to index each of the numerical processes included in the expanded HSI model, which potentially restricts the generalizability from each of the measures to the underlying construct that these measures reflect. For example, the number order task was used to index young children's ordinal processing of numbers, a task which was adapted from the order judgment task used in other studies (e.g., Lyons & Beilock, 2012; Lyons et al., 2014; Sasanguie & Vos, 2018). These tasks may measure somewhat different aspects of ordinal processing.

Another potential issue involved in measuring ordinal processing among young children is how to explain to the children what the task requires. Pilot testing of the number order task used in the present research showed that the experimenter needed to explicitly define the instruction “in ascending order” as “smallest to largest” during the practice phase to ensure that children understood the task. Similarly, the traditional order judgment task also includes magnitude-related phrases in the instructions such as “numbers are all increasing” to explain the concept “in ascending order” to young children (e.g., Lyons et al., 2014). These magnitude-related words “smallest”, “largest” and “increasing” used in the instructions might promote cardinal processing of numbers, at least for some children. As a result, the finding that children in grade 1 mainly relied on successive number comparisons to do the number order task might be confounded with the specific words used in the instructions. Therefore, future research should use multiple measures for each of the numerical processing measures to determine whether all of the measures tap the same underlying ordinal construct for children. To include multiple measures, it will be necessary for researchers to refine the constructs of cardinal and ordinal knowledge even more precisely than in the present research. For example, future research could manipulate the instructional words for the ordinal tasks (e.g., from smallest to largest, increasing, and in ascending order) to examine whether children show different performance depending on the instructions.

Finally, it is also possible that other relations among symbolic digits than the ones assessed in cardinal, sequential, and ordinal decision tasks may be relevant in understanding the complexity of the mental network that children acquire in the early grades of school. Sowinski et al. (2015) found that children’s knowledge of backwards counting (by 1s, 2s, or 5s) was related to basic quantitative and linguistic numerical skills and correlated at .51 or higher with complex numerical skills (i.e., arithmetic fluency, calculation, and number system knowledge). Thus, a broader perspective on the integrated mental network may inform future research.

Practical Implications

The present findings show that acquisition of symbolic knowledge is cumulative: Children need to first build strong cardinal and sequential associations that provide the basis for the development of integrated ordinal associations, which in turn support the development of arithmetic associations. Thus, it is possible that many of the children who show difficulties in learning and developing arithmetic skills may have impairments in mastering the lower-level numerical associations (Jordan & Dyson, 2016; Moore, vanMarie, & Geary, 2016; Rousselle & Noël, 2007; Vanbinst et al., 2016). Delineating this developmental trajectory is essential for designing appropriate teaching content for children (e.g., curriculum, assessments, and interventions). In particular, the results of the present research suggest that children need to have a thorough understanding of the basic associations (e.g., cardinal, sequential and ordinal associations) before they can use them to perform mental operations with number symbols (see also Lyons et al., 2016). Educational activities that guide children to understand and differentiate the various ways that number symbols are related to each other may be helpful, such as finding patterns in numbers (Clements & Sarama, 2014). Through extensive experience, children presumably integrate various associations into a unified structure, activating these associations selectively so that they can fluently retrieve the specific associations needed for particular numerical tasks.

Furthermore, the present findings suggest that the initial development of cardinal and ordinal processing can be differentiated because cardinal processing serves as a building block of the development of ordinal processing. They also suggest that the integration of cardinal knowledge and ordinal knowledge into a coherent associative network supports children's mathematical development. To facilitate the integration process, I postulate that teachers and educators should emphasize the interconnected relationship between the symbolic representations of cardinal and ordinal aspects of numbers, rather than instructing the concepts of cardinal and ordinal knowledge in isolation. For example, teachers could guide children to understand that 7 is bigger than 4 because 7 comes after 4 (e.g., through reciting the counting sequence 4, 5, 6, 7 or placing the numbers on a number line). Similarly, teachers could ask children to place a series of numbers in ascending order using the instructions from smallest to largest to guide them to apply their cardinal knowledge to make order judgments. I postulate that providing educational activities that emphasize the interrelated relationships among these associations can facilitate the integration of fundamental numerical association into a more cohesive network.

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APPENDICES

Appendix A. Stimuli set

Table A.1

Symbolic Number Comparison Stimuli Set

Number pair	Ratio	Distance	Distance size
1-8	0.13	**7**	Large
7-1	0.14	**6**	Large
1-6	0.17	**5**	Large
5-1	0.2	**4**	Large
2-9	0.22	**7**	Large
8-2	0.25	**6**	Large
2-7	0.29	**5**	Large
9-3	0.33	**6**	Large
3-8	0.38	**5**	Large
5-2	0.4	3	Small
3-7	0.43	**4**	Large
9-4	0.44	**5**	Large
4-8	0.5	**4**	Large
9-5	0.56	**4**	Large
4-7	0.57	3	Small
5-3	0.6	2	Small
5-8	0.63	3	Small
3-2	0.67	1	Small
5-7	0.71	2	Small
8-6	0.75	2	Small
7-9	0.78	2	Small
5-4	0.8	1	Small
5-6	0.83	1	Small
7-6	0.86	1	Small
7-8	0.88	1	Small
9-8	0.89	1	Small

Note: Small distance: difference ranged from 1 to 3 (Mean distance = 2); large distance (bolded): difference ranged from 4 to 7 (Mean difference = 5)

Practice trials: 1-2; 4-3; 6-9

Table A.2

Missing Number Stimuli Set

Second position missing	Third position missing
2 (3) 4 5	2 3 (4) 5
3 (4) 5 6	3 4 (5) 6
4 (5) 6 7	4 5 (6) 7
5 (6) 7 8	5 6 (7) 8
6 (7) 8 9	6 7 (8) 9

Practice trials: 1 (2) 3 4; 1 2 (3) 4

Table A.3

Number Order Stimuli Set

Counting (mean distance)		Non-counting (mean distance)	
Ordered (1.0)	Not Ordered (1.5)	Ordered (2.2)	Not Ordered (3.1)
1 2 3	2 3 1	1 4 6 (2.5)	1 6 4 (3.5)
2 3 4	4 2 3	2 4 7 (2.5)	7 2 4 (3.5)
3 4 5	5 3 4	3 6 8 (2.5)	6 3 8 (3.0)
4 5 6	4 6 5	4 7 9 (2.5)	4 9 7 (3.5)
5 6 7	5 7 6	5 6 8 (1.5)	5 8 6 (2.5)
6 7 8	8 6 7	6 8 9 (1.5)	9 6 8 (2.5)

Practice trials: 3-1-2; 1-3-7; 5-7-2

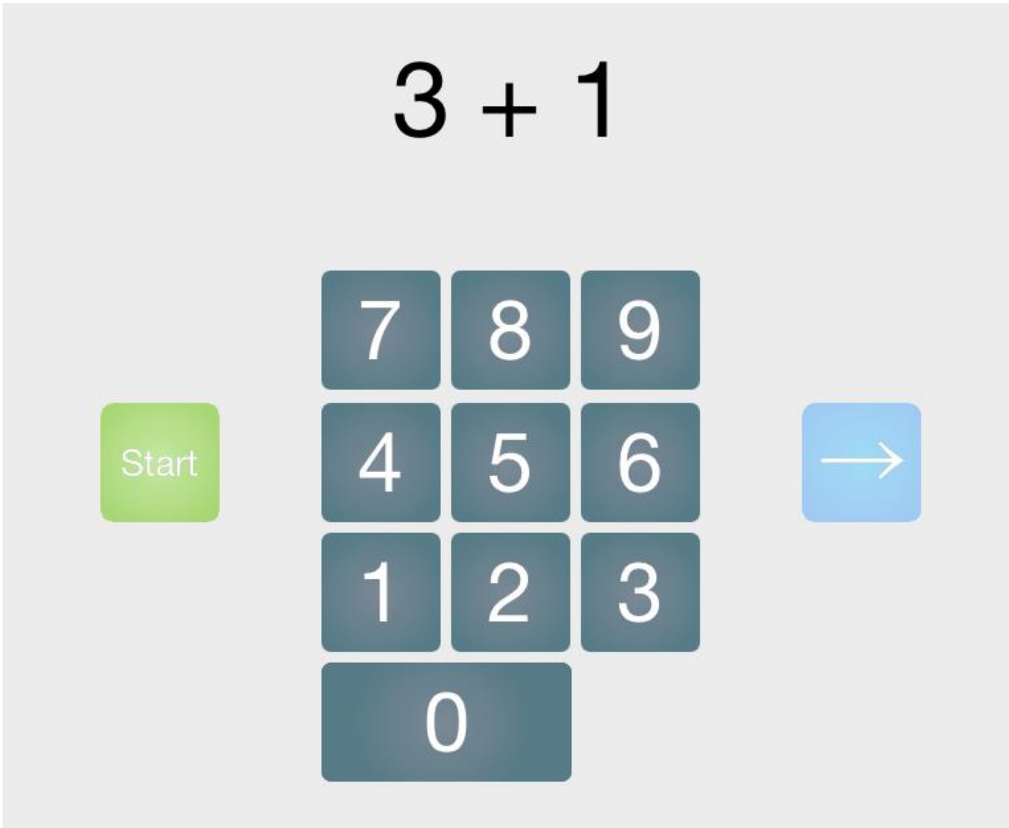

Figure A.4. An example of the simple addition task used in Study 1

2 + 5 = ________	5 + 5 = ________	8 + 6 = ________
5 + 7 = ________	3 + 4 = ________	1 + 9 = ________
6 + 3 = ________	5 + 3 = ________	7 + 3 = ________
2 + 2 = ________	8 + 7 = ________	6 + 8 = ________
3 + 8 = ________	9 + 2 = ________	9 + 5 = ________
7 + 2 = ________	1 + 5 = ________	7 + 4 = ________
4 + 8 = ________	9 + 4 = ________	4 + 1 = ________
5 + 4 = ________	3 + 5 = ________	6 + 2 = ________
3 + 3 = ________	7 + 7 = ________	5 + 8 = ________
7 + 5 = ________	8 + 5 = ________	6 + 5 = ________
1 + 2 = ________	5 + 2 = ________	5 + 9 = ________
9 + 6 = ________	9 + 7 = ________	4 + 5 = ________
1 + 7 = ________	6 + 6 = ________	7 + 6 = ________
8 + 8 = ________	2 + 3 = ________	3 + 9 = ________
4 + 7 = ________	4 + 4 = ________	1 + 6 = ________
1 + 3 = ________	9 + 3 = ________	8 + 3 = ________
9 + 8 = ________	6 + 7 = ________	2 + 9 = ________
8 + 1 = ________	4 + 2 = ________	4 + 6 = ________
2 + 7 = ________	6 + 9 = ________	7 + 9 = ________
3 + 6 = ________	2 + 8 = ________	8 + 4 = ________

Figure A.5. Complex addition stimuli set used in Study 1

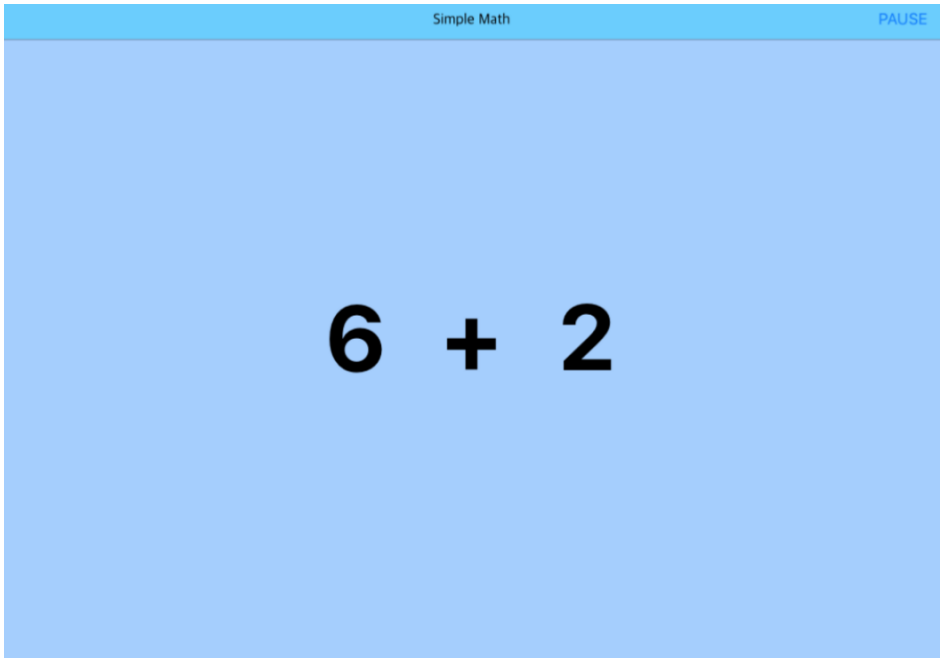

Figure A.6. An example of the simple addition task used in Study 2

Table A.7.

The full stimuli set for the simple and complex addition tasks used in Study 2

Simple addition problems	Answer	Problem type
1 + 1	2	Practice trials
2 + 1	3	Practice trials
1 + 4	5	First operand smaller
1 + 6	7	First operand smaller
2 + 3	5	First operand smaller
2 + 5	7	First operand smaller
2 + 7	9	First operand smaller
3 + 5	8	First operand smaller
3 + 1	4	First operand larger
5 + 1	6	First operand larger
4 + 2	6	First operand larger
6 + 2	8	First operand larger
4 + 3	7	First operand larger
6 + 3	9	First operand larger
5 + 4	9	First operand larger
2 + 2	4	Tie problems
3 + 3	6	Tie problems
4 + 4	8	Tie problems

Complex addition problems	Answer	Problem type
1 + 9	10	Practice trials
9 + 3	12	Practice trials
2 + 8	10	First operand smaller
3 + 7	10	First operand smaller
4 + 6	10	First operand smaller
4 + 8	12	First operand smaller
5 + 7	12	First operand smaller
6 + 8	14	First operand smaller
8 + 3	11	First operand larger
7 + 4	11	First operand larger
6 + 5	11	First operand larger
8 + 5	13	First operand larger
7 + 6	13	First operand larger
8 + 7	15	First operand larger
5 + 5	10	Tie problems
6 + 6	12	Tie problems
7 + 7	14	Tie problems
8 + 8	16	Tie problems

Appendix B. Additional measures

The additional measures were collected in Study 2 to test other hypotheses related to number line and problem solving outcomes, which were not related to the main focus of the present dissertation. However, for completeness, the descriptions of each of these measures and descriptive statistics are included here.

Descriptions of the Additional Measures

Quantity-digit mapping. A quantity-digit mapping task was used to measure children's ability to map between symbolic digits and quantities. In particular, children saw a quantity of shapes and a digit on the screen of an iPad. They were asked to decide whether the number of shapes matches the digit by touching the "yes" button (matching; e.g., ******* vs. 7) or the "no" button (non=matching; e.g., ** vs. 3). Because the task involves visual representation of quantities, children were allowed to count to determine the quantity. There was no time limit on the presentation of each trial, but children were encouraged to respond as quickly as possible. Three practice trials were presented at the beginning of the task. The experimental trials consisted of 27 trials with the numbers ranging from 2 to 9. A third of the trials were matching (i.e., the quantity of shapes matches the digit), whereas the two thirds of the trials were non-matching (i.e., the quantity of shapes does not match the digit). Half of the trials had small distance (+/-1), whereas the other half had medium distance (+/- 3 or 4). The scoring of this task was based on the response time on correct trials.

Rapid automatic naming. Children completed two versions of the rapid automatic naming (RAN) task that is modelled on naming tasks used in other research (cf. Denckla & Rudel, 1974). For RAN-Quantity task, children were asked to name sets of 1, 2, or 3 dots. For the RAN-digit task, they were asked to name the digits 1, 2, and 3. Each task consisted of two pages (8 ½ x 11 inch white paper) with rows of either digits or groups of dots. Each page consisted of 24 stimuli presented in four rows of six stimuli. The RAN-quantity pages consisted of 24 squares with 1, 2, or 3 dots (eight of each quantity) placed randomly within each square. The RAN-Digit pages each had 24 digits (1, 2 or 3; eight of each) arranged in a random order.

On the cover page for each task, children were shown a single row of stimuli (six digits or six sets of dots) and they were asked to name the digits or state the quantity of dots from left to right as quickly as possible. Feedback was provided if children named the stimuli incorrectly. After the practice items, the experimenter showed a full page of 24 stimuli. For each page, the child was timed and errors were noted. A mean score of correct item-per-second for each page was calculated using the formula: (number of correct items) / time in seconds. For example, if the child completed one page of this task in 20 seconds and made two errors (out of 24), his or her score for that page would be [(24-2) / 24 = .92] items-per-second. The mean correct item-per-second score was the average across the two pages.

Spatial span. In the spatial span task, nine green dots were presented on the screen of an iPad, and dots lit up one by one. For each trial, children were asked to reproduce the sequence of dots in the same order by touching them. At the beginning of the task, the experimenter demonstrated a practice trial, and children were asked to watch and remember a sequence of two locations. Then, children were given three more trials of sequences of two locations without any feedback. If they correctly reproduced at least one of those sequences, then the task proceeded to the next level (i.e., three trials with sequences of three locations). However, if children made errors on all three sequences for each length, the task was terminated. The score of the spatial span task was the total number of sequences that children completed correctly. Most children reproduced sequences up to five locations.

2-D mental rotation. In this task, each trial consisted of two separate pieces and a 2 x 2 array of target shapes formed by the two pieces presented together but in different orientations (http://spatiallearning.org/index.php/testsainstruments). Children were asked to find the correct picture in the choice array. Four types of items were included: 1) horizontal translation, 2) diagonal translation, 3) horizontal rotation, and 4) diagonal rotation. Sixteen trials were selected from the original stimuli set (four items per type of transformation). Two types of presentation order (adapted from original stimuli set) were used and counterbalanced at both time points of the study. The score was the total number of trials on which children correctly selected the shape that matches the two pieces.

Number line estimation. Children were asked to estimate the positions of 20 target numbers on a 0-100 line on an iPad application (EstimationLine; https://hume.ca/ix/estimationline.html), with "0" at the left end and "100" at the right end at Time 1 and Time 2. There were two parts of the number line task. The first part of the task included four target numbers (presented one at a time) in which children were asked to verbally report a strategy. The four numbers for strategy recording were 8, 48, 72 and 96, and these numbers were specifically chosen to promote various strategies on the number line task. For example, the experimenter said: "Look at this number (pointing to the number). Do you know what number is it? Where do you think number 8 would go on this number line? (No feedback regarding the accuracy of the estimation will be given). Why do you think number 8 goes there?" The experimenter recorded the strategy that children reported for each trial in the first part of the task. To help assess strategy use, two target numbers were located near the beginning and end of the number line (i.e., 8 and 96) whereas the other two target numbers were located near the middle of the number line (i.e., 48) and ¾ of the number line (i.e., 72). Next, in the second part of the task, children were presented 16 target numbers in which they were asked to estimate the presented number on the line without reporting a strategy (i.e., 3, 14, 16, 21, 29, 33, 39, 42, 53, 57, 61, 66, 79, 84, 88, and 91).

The percent of absolute error (PAE) was used as the index of how close (not considering direction) participants' placement of each number was to the actual location of that number. In particular, the percent of absolute error was calculated as: PAE = [|(Estimate – Presented Number) / Scale of the Estimate| x 100]. For example, if a child estimated the location of 48 at the position that corresponded to 50, the PAE would be 2% [|(48 – 50) / 100 | x 100)].

Problem solving. The mathematics reasoning subset of the WIAT (Wechsler Individual Achievement Test-III) was used to assess children's mathematical problem solving and reasoning ability (e.g., counting, identifying shapes, and solving verbally presented word problems). The first 20 questions from the subset were used. Testing was stopped if the child made three consecutive errors. The total number of questions that children correctly answer was the index of children's problem solving skill.

Descriptive Statistics

Comparisons for each task as a function of grade and time are shown in Table 1. Performance on each of the measures was analyzed in separate 2 (Grade: 1 vs. 2) x 2 (Time: 1 and 2) mixed analyses of variance (ANOVAs). The means are shown in Table 1 and the ANOVA results are shown in Table 2.

As shown in Table C.3 and C.4, correlational analyses show that all of the three outcome measures (i.e., addition, number line, and problem solving task) were highly correlated with each other. Moreover, all of the basic numerical measures were correlated with each other, suggesting a great degree of overlapping among these measures. More specifically, at Time 1, performance on the number line task was correlated with all measures except for the digit-quantity mapping task and Stroop task for children in grade 1, whereas it was correlated with a limited number of measures such as number comparisons, number ordering, mental rotation and RAN-Digit for children in grade 2. At Time 2, performance on the number line task was only correlated with missing number, number ordering and spatial span task for children in grade 1, whereas it was correlated with all of the measures for children in grade 2.

The differential patterns of correlations over time for children in grades 1 and 2 between number line task and other cognitive measures suggest that children used different strategies to locate the numbers on the line. In a different paper, I showed how ordinal knowledge contributed to strategy improvement for children in grades 1 and 2 on the number line task using a latent transition analytical approach (Xu, under review)[7]. In particular, two types of latent profiles were extracted based on children's estimation errors: the *variable* profile where children were more accurate at estimating the numbers

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[7] Because the paper was used as credit for the Concentration in Quantitative Methodology, it was not included as part of this thesis.

that are close to the endpoints and the middle of the line than other numbers, and the *uniform* profile where children were uniformly accurate across all target numbers. Children's verbal strategy reports provided support for the latent profile classification. Furthermore, transition between the variable and the uniform profiles was predicted by children's ordinal skills (as indexed by the number order task) after controlling for their grade level and spatial skills (as indexed by the mental rotation task). These results suggest that symbolic representations of ordinal associations may be a key skill for constructing and refining their solution strategies on the number line task, from relying less efficient counting-based strategies to the more efficient relational strategies.

Furthermore, as shown in Table C.3, and C.4, at Time 1, performance on the problem solving task was correlated with all measures except for digit-mapping, number ordering, and the two inhibitory control measures for children in grade 1, whereas it was correlated with all measures except for digit-quantity mapping, missing number and number ordering tasks for children in grade 2. At Time 2, performance on the problem solving task was correlated with the two rapid automatic naming tasks, missing number, number order, mental rotation and go/no-go tasks for children in grade 1, whereas it was correlated with most of the measures except for the number order and go/no-go tasks for children in grade 2. Note that the problem solving task involves a wide range of cognitive skills, particularly language skills such as vocabulary, listening comprehension, and oral expression (Fuchs et al., 2008). Thus, further research could examine whether the differential correlations among basic numerical skills and math problem solving performance for children in grades 1 and 2 at different time points were mediated by other variables (e.g., language skills).

Table B.1

Mean Performance on Various Measures from Time 1 to Time 2 for Grade 1 and Grade 2 Children (SDs in Parentheses)

	Grade 1		Grade 2	
Measure	Time 1	Time 2	Time 1	Time 2
RAN-Digit[a]	1.51 (.39)	1.64 (.44)	1.80 (.40)	1.93 (.42)
RAN-Quantity[a]	1.23 (.23)	1.31 (.24)	1.44 (.29)	1.51 (.30)
Digit-quantity RT[b]	3.67 (.74)	3.31 (.67)	3.09 (.74)	2.96 (.57)
Digit-quantity Correct[e]	.92 (.08)	.92 (.07)	.92 (.08)	.95 (.07)
Spatial span[c]	7.09 (2.57)	8.17 (2.98)	8.33 (2.74)	9.43 (3.1)
2D mental rotation[c]	12.09 (2.38)	13.23 (2.08)	12.91 (2.19)	13.80 (1.71)
Number line PAE[d]	16.18 (6.19)	13.65 (6.18)	10.42 (4.80)	9.08 (3.50)
Problem solving[c]	12.64 (3.50)	14.72 (3.39)	15.82 (3.02)	17.14 (2.64)

[a] Correct item per second, [b] response time on correct trials, [c] total correct, [d] percent absolute error, and [e] proportion correct

Table B.2

ANOVA Results for Various Measures

		Independent variable					
		Grade (1 vs. 2)		Time (1 vs. 2)		Time x Grade	
Dependent variable	*df*	*F*	η_p^2	*F*	η_p^2	*F*	η_p^2
RAN-Digit	1,141	19.46***	.12	42.78***	.23	.02	< .001
RAN-Quantity	1,141	23.11***	.14	25.20***	.15	.16	.001
Digit-quantity RT	1,141	23.03***	.14	17.062***	.11	3.60	.03
Digit-quantity Correct	1,141	2.39	.02	1.91	.01	2.86	.02
Spatial span	1,141	8.40**	.06	25.14***	.15	.003	< .001
2D mental rotation	1,141	5.80*	.04	24.92***	.15	.39	.003
Number line	1,140	42.87***	.23	26.04***	.16	2.44	.02
Problem solving	1,141	33.06***	.19	74.44***	.35	3.75	.03

$p < .05$*, $p < .01$**, $p < .001$***

Table B.3

Correlations of Measures at Time 1 for Children in Grade 1 (n = 66) below the Diagonal and Grade 2 (n = 80) above the Diagonal

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13
1. RAN-Digit[a]	-	**.78**	-.19	**-.33**	**-.54**	**-.59**	**.29**	.13	-.14	**.32**	**-.28**	**-.37**	**.33**
2. RAN-Quantity[a]	**.75**	-	**-.25**	**-.29**	**-.59**	**-.54**	**.32**	.20	-.11	**.39**	-.20	**-.32**	**.37**
3. Quantity-Digit[b]	**-.45**	**-.47**	-	**.58**	**.29**	**.46**	**-.22**	-.03	.12	**-.31**	.12	**.24**	-.14
4. Number compare[b]	**-.35**	**-.32**	**.49**	-	**.34**	**.60**	-.15	.03	**.23**	**-.24**	**.33**	**.37**	**-.24**
5. Missing number[b]	**-.55**	**-.64**	**.25**	**.37**	-	**.60**	**-.29**	-.06	.16	**-.27**	.17	**.25**	-.17
6. Number order[b]	**-.27**	-.20	**.32**	**.68**	**.34**	-	**-.28**	-.08	**.38**	**-.27**	**.28**	**.49**	-.16
7. Spatial span[c]	**.40**	**.36**	**-.27**	**-.25**	**-.38**	-.07	-	.06	-.05	.08	-.17	**-.33**	**.36**
8. Mental rotation[c]	**.38**	**.40**	-.12	-.14	**-.35**	-.13	**.35**	-	-.20	.13	**-.42**	-.11	**.45**
9. Stroop[b]	-.18	-.15	**.25**	**.54**	.16	**.43**	.04	-.18	-	-.03	.13	.12	**-.27**
10. Go/no-go d'	**.28**	**.34**	**-.41**	-.22	-.17	-.19	**.25**	.21	-.13	-	-.06	-.08	**.27**
11. Number line[d]	**-.32**	**-.25**	.23	**.36**	**.32**	**.26**	**-.38**	**-.34**	.11	**-.25**	-	**.32**	**.44**
12. Addition[b]	**-.37**	**-.35**	.18	**.50**	**.60**	**.39**	**-.41**	**-.38**	.17	-.22	**.46**	-	**-.34**
13. Problem solving[c]	**.51**	**.48**	-.20	**-.43**	**-.52**	-.21	**.43**	**.35**	-.08	.20	**-.48**	**-.59**	-

[a] Correct item per second, [b] response time on correct trials, [c] total correct, [d] percent absolute error. Bold values are significant at $p < .05$

Table B.4

Correlations of Measures at Time 2 for Children in Grade 1 (n = 64) below the Diagonal and Grade 2 (n = 79) above the Diagonal

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13
1. RAN-Digit[a]	-	.81	**-.42**	**-.41**	**-.54**	**-.43**	**.35**	.07	-.10	.21	**-.44**	**-.44**	**.43**
2. RAN-Quantity[a]	**.72**	-	**-.59**	**-.44**	**-.55**	**-.55**	**.39**	.19	-.14	**.32**	**-.44**	**-.50**	**.40**
3. Quantity-Digit[b]	-.19	-.18	-	**.41**	**.28**	**.38**	-.19	-.05	**.31**	**-.43**	**.37**	**.34**	**-.28**
4. Number compare[b]	**-.38**	**-.51**	.15	-	**.37**	**.56**	**-.30**	**-.28**	.12	**-.35**	**.43**	**.46**	**-.33**
5. Missing number[b]	**-.50**	**-.55**	.04	**.52**	-	**.47**	**-.36**	-.19	.08	**-.32**	**.40**	**.40**	**-.36**
6. Number order[b]	**-.48**	**-.54**	**.27**	**.65**	**.62**	-	**-.31**	-.12	.03	**-.30**	**.40**	**.50**	-.13
7. Spatial span[c]	**.29**	.21	-.14	**-.16**	**-.36**	**-.26**	-	**.32**	**-.31**	.14	**-.31**	**-.34**	**.38**
8. Mental rotation[c]	-.18	-.07	-.06	.15	-.02	.04	.19	-	-.02	.07	**-.27**	-.17	**.33**
9. Stroop[b]	-.21	-.23	-.11	**.26**	**.27**	**.36**	-.05	.14	-	-.12	**.25**	.10	**-.28**
10. Go/no-go d'	**.39**	**.45**	-.13	**-.31**	**-.41**	**-.27**	**.34**	-.03	.02	-	**-.24**	-.20	.21
11. Number line[d]	-.20	-.05	.20	.05	**.34**	**.27**	**-.43**	-.07	-.04	-.22	-	**.40**	**.51**
12. Addition[b]	**-.43**	**-.33**	.01	**.35**	**.57**	**.49**	**-.38**	.16	**.37**	**-.29**	**.51**	-	**-.34**
13. Problem solving[c]	**.37**	**.27**	-.21	-.22	**-.41**	**-.38**	**.49**	.08	-.18	**.30**	**-.60**	**-.70**	-

[a] Correct item per second, [b] response time on correct trials, [c] total correct, [d] percent absolute error. Bold values are significant at $p < .05$