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Nonlinear Analysis of Reinforced Concrete Space Frames under Combined Actions

Submitted by

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A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment
of the requirements for the degree of Master of Applied Science

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To my dear parents,

Eshrat and Iradj Rahmanian

&

To the pure soul of my dear friend,

Hootan Kasiri

Abstract

A method of analysis is developed, based on the stiffness method, to capture the complete materially nonlinear behaviour of reinforced concrete space frames under combined axial force, torsion, bending and shear. The axial/bending analysis is based on equilibrium, compatibility requirements and the constitutive laws of the materials, and it includes the coupling between axial and bending actions. For torsional analysis, the torque-twist relation of a section is modelled by a new multi-linear curve, the various segments of which correspond to the uncracked, cracked, yielded and post-yielded states of the section. To predict failure under combined actions, well-established failure criteria are used, but the values of the various actions used in these criteria are obtained by nonlinear analysis. Results of the proposed method of analysis are compared with available experimental data for a numbers of structures, and generally reasonable agreement is achieved between the numerical results and the experimental data.

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List of Important Notations

A_0	area enclosed by the centerline of the tube before cracking
A_{02}	effective area of cross-section resisting torsion immediately after cracking
A_{03}	effective area of cross-section resisting torsion at ultimate
A_{0h}	area of cross-section enclosed by the centerline of stirrup
A_c	area enclosed by outside perimeter of concrete cross-section including area of holes
A_g	total area of section excluding the hole area
A_L	total area of longitudinal steel
A_{Ln}	modified area of longitudinal steel
A_t	the area of one leg of a transverse steel bar
A_{tn}	modified area of transverse steel
A_{Lmin}	minimum required area of longitudinal steel
A_{tmin}	minimum required area of transverse steel
A_{si}	area of a single steel bar
A_w	effective concrete area resisting shear
$[A]$	transformation matrix
B	width of the rectangular grid used to define a cross-section
$[B]$	strain-displacement matrix
C_b	resultant compressive force due to bending
$[C]$	matrix defining geometry matrix of cross-section for bending purpose
$[E]$	constitutive matrix
E_{c0}	initial modulus of elasticity for concrete
E_c	tangent modulus of concrete
E_{s1}	modulus of elasticity of steel
E_{s2}	strain-hardening modulus of steel
$\{F\}_p^e$	nodal force vector due to body forces
$\{F\}_q^e$	nodal force vector due to surface tractions

$\{F\}_{\bar{e}}^e$	nodal force vector due to initial strains
$\{F\}_{\bar{\sigma}}^e$	nodal force vector due to initial stresses
$\{F_U\}$	unbalanced forces
G	shear modulus
GJ_1	initial torsional rigidity
GJ_2, GJ_3	torsional rigidity after cracking based on truss model
GJ_{3M}	modified torsional rigidity after cracking
H	height of the rectangular used to define the cross-section
K_s	support local stiffness matrix
$[K]$	structure stiffness matrix
$[K]^e = [K_{ee}]$	element stiffness matrix
$[K_{bb}]$	bending stiffness matrix
$[K_{aa}]$	axial stiffness matrix
$[K_{ab}] = [K_{ba}]$	coupled axial-bending stiffness matrix
$[K_{ne}] = [K_{ne}]$	matrix which couples the 13 th dof with the end dof
$[K_n]$	uncoupled stiffness matrix of the 13 th dof
$[K_T]$	torsional stiffness matrix
L	length of member
$M (M_U)$	bending moment (strength) of a section
$[N]$	shape function matrix
P_{0C}	ultimate strength of concrete member under pure compression
P_{0T}	ultimate strength of concrete member under pure tension
$P (P_U)$	axial force (strength) of a section
$\{R\}^e$	vector of external applied loads acting at the nodes
S	concrete effectiveness factor for diagonal strut
S'	concrete effectiveness factor for calculating shear
$T (T_U)$	torsional moment (strength) of a section
T_b	resultant tensile force due to bending
T_{cr}	torsional moment at cracking
T_y	torsional moment at yielding
$[T]$	matrix defining cross-section geometry for torsional analysis
V_t	total internal shear force due to shear and torsion
Y_{max}^+	maximum distance of filament center from the reference axis in y direction
Y_{min}^-	minimum distance of filament center from the reference axis in y direction
Z_{max}^+	maximum distance of filament center from the reference axis in z direction

$Z_{\min}^-$	minimum distance of filament center from the reference axis in z direction
a_0	equivalent tube wall thickness resisting constant shear stress
a	parameter dependent on type of cement
a_{ij}	tensor of direction cosines
b	parameter dependent on type of curing
b	the width of each filament in rectangular grid
e_{sy}	distance of steel bar center from the reference axis in y direction
e_{sz}	distance of steel bar center from the reference axis in z direction
f_d	compressive stress in the concrete struts
$f_{2\max}$	peak compressive strength of diagonally cracked concrete
f_{cp}	compressive stress in concrete due to prestressing
f'_c	concrete cylinder compressive strength
f''_c	compressive strength of concrete in a member
f_L	stress in the longitudinal reinforcing bar
f_t	stress in the transverse bar or stirrup
f'_t	modulus of rupture
f_y	yield stress of steel
f_{yt}	yield stress of transverse reinforcement
f_{ytn}	new yield stress of transverse reinforcement
f_{yL}	yield stress of longitudinal reinforcement
f_{yLn}	new yield stress of longitudinal reinforcement
f_u	ultimate steel stress
h	height of each filament in rectangular grid
k'_{rx}	rotational spring offering resistance around x' local axis
k'_{ry}	rotational spring offering resistance around y' local axis
k'_{rz}	rotational spring offering resistance around z' local axis
k'_x	translational spring offering resistance along x' local axis
k'_y	translational spring offering resistance along y' local axis
k'_z	translational spring offering resistance along z' local axis
m	number of rows in C matrix
n	number of columns in C matrix
P_c	outside perimeter of the concrete cross-section
P_0	perimeter of the area enclosed by centerline of tube
P_{02}	perimeter of the effective area enclosed by A_{02}
P_{03}	perimeter of the effective area enclosed by A_{03}
P_h	perimeter of the centerline of stirrup

q	shear flow
r	the ratio of yield force of the top to the bottom longitudinal steel in a section
r_t	coefficient relating concrete tensile strength to compressive strength
r'_t	coefficient relating concrete tensile strength to its density and compressive strength
r_c	parameter dependent on the shape and size of the member relative to the concrete test cylinder
s	spacing of transverse steel
s_i	vector connecting the support node to the auxiliary point S_i ($i=1, 3$)
s_L	spacing of longitudinal steel
t	time interval
t_e	the effective tube wall thickness.
u	axial displacement
v	transverse displacement in y direction
w	the density of concrete
w	transverse displacement in z direction
y	centroidal axis of cross-section
y_s	coordinate of the center of filament in y direction
z	centroidal axis of cross-section
z_t	coordinate of the center of filament in z direction
$\{\Delta\}$	displacement vector
$\{\Delta_i\}$	element nodal displacements vector
α_1	rectangular stress block uniform stress for bending and torsion
β_1	rectangular stress block depth parameter
ε_0	concrete strain at peak compressive stress
$\varepsilon_{2\max}$	maximum concrete strain on outside surface of torsional tube
$\varepsilon'_{2\max}$	modified maximum concrete strain on the outside surface of torsional tube
ε_c	concrete compressive strain
ε_{cr}	concrete cracking strain
ε_{cu}	concrete ultimate compressive strain
ε_p	tensile strain of concrete corresponding to the first unloading cycle
ε_{su}	the ultimate strain of steel
ε_t	concrete tensile strain
ε_{tu}	concrete ultimate tensile strain
ε_s	steel strain
ε_y	yield strain of steel

ϵ_{yM}	modified yield strain of steel
$\epsilon_{xa,b}$	total normal strain caused by bending and axial force
ϵ_{xc}	total amount of strain in concrete
ϵ_{xs}	total amount of strain in steel
$\{\epsilon\}$	strain vector
$\{\bar{\epsilon}\}$	vector of initial strains
$\bar{\eta}$	unit vector in local y direction
θ	angle of crack in the case of shear and torsion
θ_x	torsional rotation or twist
θ_y	rotation in y direction
θ_z	rotation in z direction
κ_y	curvature of cross-section with respect to the y axis.
κ_z	curvature of the cross-section with respect to the z axis
λ	factor to account for low density concrete
ν	torsional shear stress
ν_u	ultimate shear stress due to shear force
$\bar{\xi}$	unit vector in local x direction
$\bar{\rho}$	unit vector in local z direction
ρ_{bL}	balanced reinforcement ratio of longitudinal steel
ρ_{bt}	balanced reinforcement ratio of transverse steel
ρ_L	reinforcement ratio of longitudinal steel
ρ_t	reinforcement ratio of transverse steel
ρ_{tmin}	minimum reinforcement ratio of transverse steel
ρ_{tmax}	maximum reinforcement ratio of transverse steel
σ_c	compressive stress of concrete
σ_p	tensile stress of concrete corresponding to the first unloading cycle
σ_s	steel stress
σ_t	tensile stress of concrete
σ_y	yield stress of steel
$\{\bar{\sigma}\}$	vector of initial stresses
ϕ	curvature of the wall of tublar section
ϕ_c	resistance factor for concrete
φ	angle of twist per unit length(for different states of concrete member denoted as, φ_{cr1} , φ_{cr2} , φ_{cr3} , φ_y , φ_u and φ_{uF})
φ'	modified angle of twist per unit length(for different states of concrete member denoted as φ'_u and φ'_{uF})

Chapter 1

Introduction

1.1 General

Concrete structures are subjected to static, dynamic and environmental loads during their service life. Since safety and serviceability of structures are important, there is a great need for accurate analysis, which could predict the internal forces in a structure under prescribed loads. There are fundamentally two ways of finding these forces: experimental and analytical. Since for real structures experimental methods are too time-consuming and not often cost-effective, more attention is given to analytical methods. Frame analysis has been, and continues to be, a very popular method of analysis because a frame is a basic “cell” structure, which can be the constituent of very complicated structures, such as high-rise buildings and complex bridges.

Before the recent developments in computer hardware and software, structural analysis was generally limited to basic linear elastic analysis, but in reality, essentially all structures respond nonlinearly under high stresses and the linearization that is commonly used is in fact an approximation. Although such approximations are valid in many practical situations, there are also cases where a linearized treatment may be inadequate.

Moreover, nonlinear analysis may reveal unique phenomena, which the corresponding linearized analysis is incapable of showing. In addition, the growing need and expectation for more realistic and accurate modelling of structural responses have led to the development of techniques that heavily draw on nonlinear theories, e.g. the behaviour of concrete under high level of stresses. For this reason, more recently a great deal of attention has been given to the study of nonlinearity in ever-increasing complex structures. The study of nonlinearity involves the solution of nonlinear differential equations which can be systematically achieved by using the finite element method.

The finite element method is a numerical method used for the accurate solution of complex engineering problems, and is now fully accepted by the theoreticians and practitioners alike as an efficient and powerful technique. Discretization of structures by the finite element technique may involve beam, plane, plate bending, shell or combination of such elements. Beam elements are commonly employed to model frame structures and this represents a good compromise between simplicity and accuracy. Therefore, modelling of a frame with beam elements is practically more feasible as opposed to using plate/shell elements. Although modelling with the latter can be more accurate, they also render the analysis more complex and require more knowledge and experience to fully appreciate the complexities and nuances inherent in the theory and applications of plate and shell elements. Furthermore, in the case of frame structures, it can be argued that practically not much more is gained through the use of plate/shell elements vis-à-vis the use of three-dimensional beam elements.

Today the availability of fast and powerful computers and the advances in numerical techniques enable designers to model complex structures as full three-

dimensional frames. Although both the linear and nonlinear analysis of frames has been the subject of numerous investigations, there are still some issues related to the nonlinear analysis of reinforced concrete frames that require further investigation. In particular, the problem of nonlinear torsional deformation of reinforced concrete members and the interaction of torsion with other actions, such as axial, bending and shear forces, is an area that requires further enquiry.

1.2 Problem definition

With the reference to Fig.1.1, a space frame member in general may be subjected to the following internal actions: axial force, P , direct shear in the y and z directions, V_y and V_z , bending about the y and z axes, M_y and M_z , and torsion, T , where y and z are centroidal axes of the cross-section.

Fig.1.1: Reinforced concrete member subjected to all possible actions

The objective of structural analysis is to (1) determine the magnitude of each action and (2) determine whether the section is capable of resisting these actions without failure. The purpose of the analysis may also be the determination of the deformations and stresses corresponding caused by these actions.

In a statically determinate structure, whether behaving linearly or nonlinearly, it is sufficient to employ the equations of equilibrium in order to determine the internal actions. In statically indeterminate structures, we need additional equations, derived from the compatibility of deformations, in order to determine these actions. Determination of structural deformations requires knowledge of the constitutive laws of the materials of the structure, or alternatively knowledge of its force-deformation response. Stated differently, analysis of indeterminate structures requires knowledge of the stiffness of a member, which in the case of nonlinear behaviour is a function of the load and its variation with load.

In reinforced concrete analysis, both forms of the constitutive laws are used in practice. Methods, which calculate internal stresses and strains, use the stress-strain curves of concrete and steel to find the stiffness of members. Other methods apply the moment-curvature or torque-twist relation to relate forces and deformations. Whichever method is used, failure criteria based on forces (moment, axial force, torque and shear) or stresses (normal and shear stresses) are then used to check whether a member is able to sustain a certain level of combined forces or stresses. Although theoretically both methods are equivalent, practically it is easier to deal with one or the other, depending upon the analysis procedure and purpose. In the case of reinforced concrete frame structures both methods are prevalent. Refined analyses based on the finite element

method generally apply stress-strain type methods while methods based on conventional methods of structural analysis, the so-called stiffness or flexibility method; use the force-deformation approach.

Irrespective of the analysis procedure, to date the emphasis on the nonlinear analysis of concrete frame structures has been on their response to axial force and bending moments. Determination of the response of complex frame structures subjected to shear and torsion or to combinations of various actions has not received a great deal of attention. In part the reason is that bending problems lend themselves to rigorous theoretical analysis and therefore one can determine their-full response systematically without too many assumptions or empiricism. In contrast, determining the response of concrete structures to shear and torsion alone or in combination with other actions is not that straightforward. Hence, further research is needed to develop methods of analysis for reinforced concrete space frame structures subjected to different combinations of internal forces.

1.3 Objectives and Scope

The purpose of this investigation is to improve the existing nonlinear method of analysis of reinforced concrete space frames. An existing computer program, which is capable of capturing the full nonlinear response of concrete space frames subjected to axial force and biaxial bending, will be modified in order to endow it with the ability to capture the response of the same type of structures subjected to any combination of internal forces. More specifically:

1. The axial force and bending models will be improved by introducing “tension-stiffening”.
2. A torque-twist relation will be developed and implemented in the program to more realistically capture the torque-twist response of statically indeterminate space frames.
3. Failure criteria will be introduced in the program to predict the ultimate load capacity of space frames under combined actions.
4. The predictions of the improved models will be verified by comparing them with available experimental data.

The scope of this study is limited to concrete frame subjected to non-time-dependent loads. In other words, issues related to creep, shrinkage, and thermal effects are not the subject of this study. Also, the issue of bond and anchorage loss or failures due to local stresses in so-called disturbed regions is not dealt with. Finally, shear deformations are ignored, bending analysis is based on the Bernoulli-Euler hypothesis and twist is assumed to be governed by the Bredt's hypothesis for thin-walled structures.

The present study is based on the work of Mari (1984) and is extended here to enhance the capability of the computer program PCF3D with regard to prediction of the response of reinforced concrete space frames exposed to torsion and shear and also to combined actions. Additional modifications and improvements are introduced to further enhance the over-all accuracy of the program.

Chapter 2

Literature Review

2.1 General

This chapter gives an overview of some numerical methods and associated computer programs, developed for the analysis of concrete frame structures. These programs have employed different approaches with various degrees of accuracy to capture the full response of these types of structures. The programs vary in their scope and sophistication depending on the purpose for which they have been developed. It is important to point out that methods of analysis, or computer programs, which deal with plates and shells, are not within the scope of the current review.

2.2 Preview work on nonlinear analysis of concrete space frames

Since the beginning of the 20th century engineers have known that certain aspects of concrete behaviour could not be described or predicted based upon classical strength of materials techniques. Therefore, over the years extensive research has been carried out to model the actual behaviour of concrete, steel, and the bond between the concrete and steel reinforcement. The late 1950's and the early 1960's witnessed the earliest

application of the stiffness method to concrete structures, which was used in the analysis of large mass concrete dams being designed and built in the United States (Scordelis, 1972). Cracking, time-dependent, thermal and sequence of construction effects were traced in the analysis of concrete structures by Clough (1962), Ngo and Scordelis (1967) and Nilson (1967, 1968). However, the earliest published application of the method to reinforced concrete structures was by Ngo and Scordelis in March 1967. Beams with predefined “discrete cracks” were studied using a plane stress analysis in which the concrete and steel reinforcement were represented by two dimensional triangular finite element or axial bar elements, and special linkage elements were used to connect the steel to the concrete. Nilson (1967) modeled progressive discrete cracking using a strength-based criterion instead of fractured mechanical techniques. The resulting model accounted for (a) the influence of the reinforcement; (b) changing topology due to progressive cracking; (c) realistic bond stress transfer between concrete and reinforcement and; (d) nonlinear material properties. However, the redefinition of structural topology at the end of each increment made the analysis complicated; therefore, this approach fell out of favour. Instead another important approach for modelling of cracks in concrete structures, called the “smeared crack model” emerged (Rashid, 1968).

In 1970, the Berkeley nuclear laboratories developed the computer program BERSAFE (later updated in 1974, 1977 and 1980) for linear and nonlinear static and dynamic analysis of isotropic and anisotropic elasto-plastic materials and structures. The nonlinear materials version of the program included isotropic and kinematic hardening;

plasticity and creep, including concrete creep. It could also perform analysis under thermal transient loadings.

Spokowski et al. (1972) developed a program, called NFERCT, for nonlinear analysis of 2D reinforced concrete structures. It utilized triangular plane stress elements to discretize structures. Material nonlinearity was defined in terms of tensile cracking of concrete, yielding of main reinforcement or stirrups, yielding or crushing of concrete in compression and bond failure. Similar to other finite element programs using plane stress elements, it could not be used to analyze structures under biaxial bending and /or torsion.

A plane stress finite element program for the analysis of two dimensional reinforced concrete members was presented by Al-Mahaidi (1979). His program covered linear and nonlinear analysis of concrete structures under monotonically increasing loads. Material nonlinearities incorporated in this program were due to cracking, nonlinear compressive stress-strain response of concrete, yielding of reinforcement, bond-slip, and post cracking shear resistance due to aggregate interlock and dowel action. However, the program could only deal with frame members subjected to axial load, uniaxial bending and shear. Neither, biaxial bending nor torsion could be dealt with.

Kang and Scordelis (1980) developed a numerical procedure based on the finite element method for reinforced concrete structures. They included prestressing in the program PCFRAME. PCFRAME was written for the material and geometric nonlinear analysis of plane reinforced and prestressed concrete frames, including time-dependent effects due to load history, temperature history, creep, shrinkage, and aging of concrete and relaxation of prestressed steel. Structures were idealized as an assemblage of beam elements interconnected by joints. Three degrees of freedom, two translations (u , v) and

one rotation (θ) were assumed for each node. In order to calculate element stiffness, they employed a three point Gaussian quadrature along the length of each element, together with the direct summation of each concrete and steel layer contribution through the thickness. The cross-section was assumed to have an axis of symmetry, and the variations in material properties through the depth of each member was accounted for by dividing it into a number of discrete concrete and reinforcing steel layers. This program could only deal with plane frames under axial load and uniaxial bending.

Hsu et al. (1981) developed a computer program for the elastic-perfectly plastic analysis of reinforced concrete planar frames, based on an assumed rotational capacity of the “plastic hinges” and the formation of a “collapse mechanism” at ultimate limit state. This program was based on a general-purpose (matrix displacement formulation) computer program developed by Wang (1963) for steel plane frames. The program was modified by including the moment-curvature characteristics of a reinforced concrete section subjected to a constant axial load applied at the section centroid. Through this modification they could calculate the inelastic rotational capacity of the hinge based on Mattock (1964) and Corley (1966) equations. In a similar fashion, Cohn et al. (1983) developed a computer program STRUPL 1, which considered inelastic deformations by lumping the material nonlinearity at the elements nodes, where a piecewise linear moment-rotation law was assumed; the element themselves remained elastic. Material models included elastic-perfectly plastic or strain hardening/softening. Yielding criteria was defined as single or multi-stress (yield plane) yield condition. Later STRUPL 1 was improved (STRUPL-1C) by Cohn and Krzywiecki (1987) and also Riva and Cohn (1990) who added to the capability of the program by including analysis of prestressed and

partially prestressed concrete planar frames. They added a new feature in the form of a moment-curvature-rotation relation to automatically handle the conditions that determine the section response at all loading states automatically. The assumptions underlying the program are (a) Quasi-static monotonic loading; (b) negligible shear effect; (c) linear strain distribution; (d) known uniaxial material stress-strain relationships (analytical, experimental, etc.) valid for section analysis; and (e) only bonded tendons are considered. Once again no reference was made to failures other than those caused by axial load and uniaxial bending.

Mari (1984) developed a computer program, PCF3D, for the nonlinear, time-dependent analysis of 3D reinforced and prestressed concrete frames. This program is based on the finite element displacement formulation. The structural effects of the load and temperature histories, materials nonlinear behaviour, creep, shrinkage, aging of concrete and relaxation of prestressing steel and geometric nonlinearity were considered. He used a trilinear model to represent the torsional response of a reinforced and prestressed beam in terms of torque-twist relationship, but all the required parameters of the trilinear model was left as input data by the user. In order to incorporate the nonlinear and time-dependent behaviour of concrete, the time domain is divided into time intervals and a step forward integration is performed in which increments of displacements and strain are added successively to the previous total as one marches forward in the time domain. The frame consists of joints with six degrees of freedom at each node, interconnected by beam elements. Each element has six degrees of freedom at each end node, consisting of three displacements and three rotations. An additional axial degree of freedom is assigned to an internal node placed at mid length of the element to account for

the coupling of the axial and bending deformations in sections that are unsymmetric with respect to the centroidal axes of the cross-section. Each element is prismatic with an arbitrary cross-section, which is divided by a grid into cells. The program traces nonlinearity by calculating the normal stresses due to bending and axial load. In addition, from the equilibrium of the normal stresses within each element, the shear stresses are also calculated. However, it lacks any shear or combined shear and bending failure criterion. Similarly, no interaction among the various actions, i.e. shear, torsion and bending is considered.

Ghali and El-badry (1985), Abbas and Scordelis (1993), Murica and Herkenhoff (1994), Cruz et al. (1998) developed models and computer codes for segmental construction of plane frames. Once again all these programs essentially consider bending and axial loads. Mari (1999) released a new version of his program. His new model, which was implemented into a computer program called CONS, was designed to simulate most of the structural changes that take place during the construction process or during other event in the life of a structure, such as possible changes in structural geometry, boundary conditions and loading. The program in this case uses linear elasticity and the principle of superposition.

Vecchio (1985) introduced a computer program, TEMPEST, for nonlinear structural analysis of reinforced and prestressed concrete plane frames subjected to thermal and/or mechanical loads. Frame analysis was performed using an "effective stiffness" procedure, to account for the nonlinear behaviour of the concrete and the reinforcement. Each member was analyzed for its nonlinear response under axial force and moment, and then secant stiffness was determined. The procedure produced a new

matrix after each iteration and the process was continued until the change in stiffness in two consecutive iterations became sufficiently small. In this program, a cross-section can be an irregular section and it is divided into a number of concrete and steel layers. For members under thermal loads, the program determined nonlinear transient temperature profiles using standard one-dimensional heat flow principles.

Carol and Murica (1988) presented a new analytical model based on the mathematical solution of the equations of equilibrium in terms of variation of the cross-sectional forces (axial forces, shear forces and moment). They simply solved the equilibrium equations by hand, without considering either strain-displacement or material laws to define an interpolation function for these forces along the element. This would correspond to hybrid formulation of the FEM, but with the special property of being an exact interpolation, which produced two practical advantages (a) with the exact shape function the intrinsic error due to approximated interpolation vanished, (b) the limitations for beam size was removed. They used this formulation for linear and nonlinear material behaviour and first order equilibrium as well as the second order effects. They developed a program based on these theories for the general analysis of plane frames, including time-dependent, nonlinear behaviour of materials and second order effects, which had the capability of representing a result under complex interaction phenomena, such as creep-buckling. Based on the case of study, two different models were used for concrete. (a) the well-known Maxwell chain model (Carol and Murica, (1988)) with aging coefficient. This model is very time-consuming. (b) Sargin's (1971) stress-strain relationship for the pure instantaneous analysis. An elasto-plastic model was used for steel with strain

hardening and no time-dependent behaviour. The program could model symmetrical sections only.

A simple layered finite element model called “frame element” was presented by Zacharia and Krishnamoorthy (1990) to predict the behaviour of concrete frame structures. Their program, FELPRO, demonstrated the efficiency of a simplified finite element program in modelling the nonlinear behaviour of large structures such as buildings. In this method, the element stiffness matrix corresponding to the mid-depth of section was defined and a transformation matrix was used to account for the eccentricity of the neutral axis from the centroidal axis of the section. The structure stiffness was assembled only once using the element stiffness for the six degrees of freedom corresponding to the initial elastic moduli of concrete and steel. Subsequently, nonlinearity was considered by so-called pseudo-loads at each load step. Pseudo-load was computed from the stresses released due to cracking, nonlinear stress-strain law and yielding of concrete and steel in each iteration.

Ghosn et al. (1996) developed a nonlinear bridge analysis program, NONBAN, which can be routinely used to analyze the common type of bridge structures such as steel and concrete (reinforced concrete and prestressed concrete) slab on I-beam bridge as well as box girder bridges. Their method was based on modified stiffness matrix that accounts for nonlinearity due to bending by using moment versus plastic rotation curve. They assumed that beyond the elastic limit, the plastic zones caused by bending stress were concentrated at the ends of the beam element and rotational springs connected the elastic part of the beam to the plastic zone. The relationship between the moment and the rotation of plastic zone were represented by moment-rotation curves. In modelling the

shear behaviour of concrete members, a multi-linear shear deformation curve was considered. Since the nonlinear torsional behaviour of structural members was extremely difficult to model because of the interaction between the torsional capacity with shear and moment capacity, the program treated torsion only elastically by using an elastic torsional rigidity for each element.

Cruz et al. (1998) developed a step-by-step model for the material and geometric nonlinear and time-dependent analysis of reinforced concrete, prestressed concrete (post-tension and pretension) and composite steel-concrete planer frame structures. The model could determine the influence of the construction process in the course of short and long-term structural performance both at service life and ultimate load level of frame. It can evaluate the remaining capacity of damaged and repaired structure as well as the efficiency of a retrofitting of the system. Each of the frame elements could be added to or removed from the structure, or could undergo modification on its cross-section and also stress level at the stays and prestressing tendon can be modified at any instant of a structure's life. The analytical tool is based on displacement formulation of the finite element method. It incorporates two-noded conventional beam element and the Timoshenko three-noded isoparametric beam-element. Several kinds of connections such as hinge, rigid, and links with finite length are incorporated. A multi-stress-strain relationship with strain hardening was used for both reinforcing and prestressing steel. Two models were considered for concrete under instantaneous behaviour and rheologic behaviour. Cruz et al. (1998) developed a model for concrete both for damage and rheologic effects that were considered and treated by this model in an uncoupled way. The model was capable of representing numerous nonlinear and time-dependent aspects

of concrete behaviour, such as nonlinear creep and failure under sustained load. Note, however, that shear, torsion and biaxial bending could not be analyzed by this program.

Izzudin and Lloyd Smith (2000) presented a nonlinear method called “adaptive analysis concept” for analyzing three-dimensional reinforced concrete frames. In this method, analysis commences with only one elastic element per member, subsequently, upon development of inelasticity, automatic refinement (or subdivision) of elastic element into elasto-plastic elements was performed. Therefore, at the end of each step of nonlinear analysis, elastic elements were checked to see if they exceeded the elastic limits at selected points (Gauss points) within elements. Whenever an elastic element was subdivided, the solution corresponding to the current equilibrium step was re-established with the new mesh before continuing with the remainder of the incremental analysis. As there is a need for continuous checking of inelasticity at numerous locations within each element, it can take excessive computational effort to perform such analyses. Therefore, they adopted an efficient approach to perform the check more expeditiously. Before the start of the analysis, the interaction surface (biaxial bending and axial force) were determined for all the cross-sections to enable the program to check if the generalized stress state laid outside the interaction surface. To provide an accurate elastic formulation that could represent an entire member by only a single element, they proposed using quartic shape function for the two transfer displacements. Six Gaussian points were used for numerical integration of the stiffness matrix. To satisfy material and geometry nonlinearity due to large displacements and beam-column effects, a cubic shape function was assumed for an elasto-plastic member. They used for concrete the Karsan and Jirsa’s model (1969) and a bilinear strain hardening model for steel. In addition to these

computer programs and models, a number of other commercial and academic programs exist for the analysis of concrete space frames. These programs often utilize procedures similar to those applied in the programs already described.

In conclusion it can be stated that although all the programs listed above have achieved specific objectives, they essentially focus on the flexural behaviour and capacity of concrete frames and they tend to either ignore the shear and torsional capacity or treat them minimally in a linear manner. Since space frames can be subjected to any combination of the various actions, it is essential that we investigate modes of failure other than flexural in their analysis. Shear and torsional failures can be rather dangerous and if the interaction of the different actions is ignored, the results of the analysis may be on the unsafe side. Therefore, there is a clear need for modelling shear and torsion and for considering the interaction of the different forces and moments acting on any member in a reinforced concrete space frame.

Chapter 3

Constitutive Laws and Finite Element Formulation

3.1 General

This chapter describes modelling of the constitutive laws of concrete and steel reinforcement, the structural idealization method and the nonlinear analysis procedures. In the first section, modelling of stress-strain relationship of concrete as a nonlinear material, and steel as an elasto-plastic material is presented. Modelling of the geometry of the structure is described in Section 3.2. In this section the required information for modelling the cross-section, the beam-element and the boundary conditions will be described. Section four explains the adopted finite element procedure for beam-element and specifically focuses on the stiffness matrix formulation, considering axial force, biaxial bending and torsion. The procedure for determining the nonlinear response of structures is given in the latter section, and the relations needed for calculating the strains, stresses, internal resisting loads and unbalance forces are presented. In Section five, solution techniques for nonlinear problem will be discussed and the adopted method in this study will be explained in detail. The finite element procedures and constitutive laws of concrete and steel that are used for axial force and bending analysis in the present

study are the same as in program PCF3D. It may be recalled that this program was written by Mari (1984) to perform nonlinear analysis of three-dimensional concrete frames. The program is modified in the current study and is called 3DRCF-CL (3D Reinforced Concrete Frames Analysis under Combined Loading).

3.2 Material properties

3.2.1 General

This section studies the constitutive relationships of concrete and steel under monotonic and cyclic states of stress. The behaviour of reinforced concrete is complex due to time-dependent and environmental effects, the nonlinear material behaviour of steel and concrete, the imperfect bond between steel and concrete and the cracking of concrete; therefore, some assumptions are made in the present study as follows:

- Concrete is defined as a homogeneous and isotropic material
- Uniaxial stress condition is assumed for concrete, recognizing that this is an approximation and that failures initiated by multi-axial states of stress cannot be dealt with by such models
- Perfect bond is assumed between concrete and steel. Clearly, failures initiated by bond failure cannot be dealt with due to this assumption.

It is important to mention that program PCF3D also can analyze prestressed concrete frames. However, since this subject is not within the scope of the present study, the properties of prestressed steel are not explained here. For further information the reader can refer to the report by Mari (1984).

3.2.2 Concrete in compression

The concrete model described here is the same as the one in program PCF3D. The ascending portion of the stress-strain relationship of concrete in compression is assumed to follow Hognestad's (Park and Paulay, 1975) stress-strain curve represented by a second-degree parabola, while the descending part is linear, as shown in Fig.3.1. The parameters necessary to define Hognestad's parabola are taken from ACI Committee 209 (1970) at time t as follows:

Fig.3.1: Idealized stress-strain curve for concrete in uniaxial compression based on Hognestad's model.

$$f'_c = \frac{t}{a + bt} (f'_c)_{28} \quad (3.1)$$

$$f''_c = r_c \cdot f'_c \quad (3.2)$$

$$E_{c0} = 0.043 w^{1.5} \sqrt{f'_c} \quad (3.3)$$

where

f'_c = the compressive cylinder strength (MPa)

w = the density of concrete (kg/m^3)

E_{c0} = initial modulus of elasticity for concrete (MPa)

and a , b are constants that are function of both the type of cement used and the type of curing method applied, according to ACI Committee 209. Since in this study time-dependent effects are not considered, a and b will not be discussed. The coefficient r_c depends on the shape and size of the member relative to the concrete test cylinder (Neville, 1995).

Softening normally occurs when the compressive stress reaches or exceeds the maximum compressive strength, f''_c , and concrete is assumed to have crushed when its compressive strain reaches the ultimate compressive strain, ϵ_{cu} . The effect of dynamic cyclic loading, such as seismic load or wind load, are not considered in this study, but loading and reloading due to live load is assumed via a simple model as shown in Fig.3.1. Linear unloading parallel to the initial elastic loading path is assumed. It should be remarked that improvement of the constitutive laws of concrete under compression is not an objective of this study.

3.2.3 Concrete in tension

A major characteristic of concrete is its weakness in tension. Cracking occurs at low stresses when the principal tensile stress exceeds the uniaxial tensile strength of concrete. The tensile strength of concrete may be obtained indirectly from the so-called split cylinder test or indirectly from a bending test conducted on plain concrete prisms. The tensile strength in flexure is known as the modulus of rupture, which is commonly used to estimate the tensile strength. The modulus of rupture can be obtained approximately from the relationship

$$f_t' = r_t' \sqrt{w f_c'} \quad (3.4)$$

where

f_t' = modulus of rupture

and r_t' can be considered as $r_t' = 0.12-0.2$ for the unit weight of concrete in SI units ($r_t' = 0.6-1.0$ for the unit weight of concrete in US standard units).

Since the modulus of rupture gives results that are substantially higher than direct tension test (Mindess and Young, 1981), in studying torsion and shear, the tensile strength of concrete can be considered as 50% to 75% of the modulus of rupture (Park and Paulay, 1975). Furthermore, the diagonal concrete struts which carry the stress caused by torsion and shear are subjected to both compression and shear (this concept will be discussed in Chapter four in detail) and this effect reduces both the compressive and tensile strength of concrete. Therefore, in this study two different moduli of rupture must be defined, one for bending and another for shear and torsion.

Cracking reduces the stiffness of a concrete member substantially and causes highly nonlinear behaviour in reinforced concrete members. After cracking, the reinforcing steel takes over any stress that is released by the cracked concrete while the concrete between cracks still resists some tensile stress. The contribution of concrete tensile resistance between cracks is known as “tension-stiffening” and significantly reduces the post-cracking deformations of reinforced concrete structures. Fig.3.2 illustrates the distribution of stresses in a reinforced concrete tensile member, after cracking. Between the initiation of the first crack and a fully developed cracked state, stresses between two cracks can be carried by the combined action of concrete and steel as show in Fig.3.2 (b), while at the cracked section only the steel carries the entire stress.

Fig.3.2: Tension-stiffening feature, a) A concrete member under tension; b) Stress in concrete and steel between the cracks

In nonlinear finite element analysis, it is necessary to include, post-cracking resistance of concrete for accurate predictions of deflections, crack width, bond transfer and shear transfer phenomena. Scanlon (1971) first accounted for this effect and modelled softening of concrete in tension in terms of steel stress-strain relation, Fig.3.3(a). Lin and Scordelis (1976) assumed a cubic polynomial relationship between stress and strain, Fig.3.3(b) while Kabir (1976) presented a linear model for cracked concrete, which is a special case of Lin's model, Fig.3.3(c). Mazar (1981) proposed a continuous damage-based formulation to represent the tensile softening response, Fig.1.3 (d), where damage D , an internal state variable, was considered as zero prior to peak stress and a function of strain thereafter. Gylltoft (1983) assumed a bilinear stress-strain relationship in the post-peak region, Fig.3.3(e). Based on fracture mechanics, Bazant and Oh (1983) proposed a linear-softening behaviour with a negative softening modulus E , which is related to the fracture energy, peak stress, initial modulus, and the characteristic length of the fractured zone, Fig.3.3(f). Gopalaratnam and Shah (1985) presented an analytical model which assumes a unique stress-strain relationship in the ascending part only and another relationship between stress and crack width in the descending part, Fig.3.3 (g).

Although each of these models have certain advantages and disadvantages and none has been demonstrated to capture the exact behaviour of tension-stiffening in every situation, for the purpose of this analysis Kabir's model is adopted. It is recognized that unlike Bazant and Oh's model, this model shows mesh-size dependency.

Fig. 3.3: Comparison of some of the existing analytical models used to represent tension-stiffening and the softening behaviour of concrete in uniaxial tension.

However, while Bazant and Oh's model is theoretically independent of mesh size, in practice, the characteristic length of the fracture size cannot be determined in a unique fashion, consequently some degree of mesh-dependency will still exist. It should be noted that, although the unloading curve in Kabir's model is a straight line, Fig.3.4, the actual curve followed is a stepped function. This is used in order to avoid numerical difficulties, which may arise when a negative stiffness is used to model the unloading curve.

Fig.3.4: Stress-strain relationship of concrete adopted based on Kabir's model (1976)

Therefore, within a load step or iteration, the elastic modulus is assumed to be zero when it is in the tension-stiffening region of the stress-strain curve. The value of the maximum tensile strain, ϵ_{tu} , depends on the user and is defined in terms of the cracking strain, ϵ_{cr} , (Fig.3.4). As a default value for ϵ_{tu} in this program, $10\epsilon_{cr}$ is considered. A simple

model base on the experimental studies of Gopalaratnam and Shah (1985) is assumed for load reversal, as represented by line PO in Fig.3.4

3.2.4 Details of Constitutive relationships

In this study, the concrete material behaviour in compression and in tension is characterized by 12 different possible points as shown in Fig.3.5. Broadly speaking these states represent the tensile or compressive loading and unloading of either cracked or uncracked concrete. Specifically, they can be described as follows:

1. In primary tension (path OA or AO), i.e. before cracking
2. In compression before yielding or peak stress (path OC)
3. In compression and yielded (path CE) or post-peak response
4. After cracking and loading (path AJ)
5. Crushed (beyond point E), zero stress and zero stiffness
6. Load reversal from state 2 (path BG or GB)
7. Load reversal from state 3 (path DI or ID)
8. Under compression, not yielded and initially cracked (path OC or BC)
9. Under compression, yielded and initially cracked (path CE or DE)
10. Stress reversal from state 8 and initially cracked (path BF or FB)
11. Stress reversal from state 9 and initially cracked (path DH or HD)
12. Stress reversal from state 4 (path KO or OK)

Table 3.1: Stress-strain relationships of concrete corresponding to different states in compression and in tension

Compression	State <u>2</u> and <u>8</u>	$\sigma_c = f_c'' \frac{\varepsilon_c}{\varepsilon_0} (2 - \frac{\varepsilon_c}{\varepsilon_0})$	$E_c = E_{c0} (1 - \frac{\varepsilon_c}{\varepsilon_0})$
	State <u>3</u> and <u>9</u>	$\sigma_c = -0.15 f_c'' (\frac{\varepsilon_c - \varepsilon_0}{\varepsilon_{cu} - \varepsilon_0}) + f_c''$	$E_c = 0$
	State <u>6,7,10</u> and <u>11</u>	$\sigma_c = E_c (\varepsilon_c - \varepsilon_r)$	$E_c = E_{c0}$
Tension	State <u>1</u>	$\sigma_t = E_c (\varepsilon_t - \varepsilon_r)$	$E_c = E_{c0}$
	State <u>4</u>	$\sigma_t = -\frac{f_t'}{\varepsilon_{tu} - \varepsilon_{cr}} (\varepsilon_t - \varepsilon_{cr})$	$E_c = 0$
	State <u>12</u>	$\sigma_t = \frac{\sigma_p}{\varepsilon_p} \varepsilon_t$	$E_c = \frac{\sigma_p}{\varepsilon_p}$

3.2.5 Reinforcing steel

In order to model the stress-strain relationship of reinforcing steel, a bilinear model with strain hardening is used. In general the stress-strain curves for steel in tension and compression are assumed to be identical. Fig.3.6 shows the details of the adopted model for steel. This model is defined by four parameters: the yield strength, f_y ; the

elastic modulus of elasticity, E_{s1} ; the strain-hardening modulus, E_{s2} ; and the ultimate strain, ϵ_{su} . A simple model is considered for load reversal. The unloading path follows the initial elastic slope. The dotted line in Fig.3.6 shows the behaviour of steel under cyclic loading. The stress-strain behaviour of reinforcing steel can be divided into the following states:

1. Loading or unloading in tension or compression without yielding
2. Yielded, strain-hardening
3. Load reversal or unloading after yielding
4. Failure or rupture

Fig.3.6: Uniaxial stress-strain relationship for reinforcing steel used in the present study

The latter four states are mathematically described by the equations given in Table 3.2. In this table, σ_s , ε_s , E_{s1} and E_{s2} are the steel stress, strain, elastic modulus and strain-hardening modulus, while σ_y and ε_y denote the yield stress and strain, respectively.

Table 3.2: Stress-strain equations for steel in different states

Tension and compression	State <u>1</u>	$\sigma_s = E_{s1} \varepsilon_s$	$E_s = E_{s1}$
	State <u>2</u>	$\sigma_s = E_{s2} \varepsilon_s \pm (f_y - E_{s2} \varepsilon_y)$	$E_s = E_{s2}$
	State <u>3</u>	$\sigma_s = E_{s1} (\varepsilon_s - \varepsilon_r)$	$E_s = E_{s1}$
	State <u>4</u>	$\sigma_s = 0$	$E_s = 0$

3.3 Geometry of the structure

3.3.1 General

In this section, the geometry of a structure and the way that the user needs to define the required information as input to the program will be discussed. Although the majority of this information is as described by Mari (1984), due to the changes made in the current study with respect to shear and torsion and for sake of completeness, it is recapitulated here.

3.3.2 Definition of cross-section geometry

The member cross-section in program 3DRCF-CL can be defined in two different manners, depending on the type of internal action, i.e. for bending and axial load on the one hand, and torsion and shear on the other. In both cases, the cross-section can be defined as solid or hollow section with any arbitrary shape.

For bending and axial load, the cross-section is plotted on a rectangular grid with dimensions H and B , divided into m rows and n columns. The H and B are the largest dimensions of the cross-section along its height and breadth, respectively (Fig.3.7). The cross-section can be expressed as an integer number of concrete areas (filaments). The area of each filament has dimensions of $b=B/n$ and $h=H/m$. The reference axes of the cross-section are y and z and are assumed to be parallel to the sides of the rectangle and their origin can be located at any location within the rectangle and not necessarily at its center. As indicated in Fig.3.7, the distance (eccentricity) of the reference axes from the edge of the rectangle are specified by the quantities $Z_{\max}^+, Z_{\max}^-, Y_{\max}^+, Y_{\max}^-$. It may be noticed in Fig.3.7 that accurate representation of curved cross-sections requires a small grid size.

The position of each filament is described by the coordinates (y_s, z_t) of its center where s and t show the s^{th} column and t^{th} row of the rectangular grid.

$$y_s = -\frac{B}{n}(s - 0.5) + y_{\max}^+ \quad (3.5)$$

$$z_t = -\frac{H}{m}(t - 0.5) + z_{\max}^+ \quad (3.6)$$

Fig.3.7: Actual and idealized cross-section and corresponding C matrix

To define the divided cross-section, a topology matrix is defined by the user. The terms of this matrix, called the **C** matrix, are associated only with the filaments contained within the cross-section. The **C** matrix has four columns and its rows are dependent on the geometry of the cross-section. Each row of the **C** matrix defines the property and geometry of one row of grid cells. Each row of the **C** matrix provides the following information:

- First column: row number
- Second column: it gives the column number in the rectangular grid which defines the left boundary of the cross-section
- Third column: gives the column of rectangular grid that defines the right boundary of the section. For hollow sections, this number specifies the cell number where the left wall of the hollow section terminates.
- Fourth column: gives the material property code of filaments which are bounded by the second and the third column of each row.

It should be noted here that the number of rows of **C** matrix is not necessarily equal to the number of physical layers of the rectangular grid. In the cases of the presence of a hole or discontinuity in the shape of the cross-section, a layer of rectangular grid can be defined more than once in the matrix. For example, the **C** matrix of the cross-section shown in Fig.3.7 is given by assuming a constant property for the entire section. Once the **C** matrix is known, moments of inertia, the element stiffness, internal resisting load vector, initial strain load vector and internal forces can be obtained by numerical integration over the filaments of the cross-section.

As stated in the beginning of this thesis, improved torsional analysis is one of the primary objectives of this study. Hence, it is necessary to describe how sections under torsion are defined in this investigation. There are two reasons, why a different method is needed to define a cross-section under torsion or shear. Fig.3.8 shows two cross-sections where only the hatched portions are deemed to be effective in resisting torsion (CPCA, 1998). The effective area resisting shear and torsion are defined by the user in accordance with prevailing codes or methods of design. As will be described later, calculating the torsional capacity of a section is based on the concept of an equivalent tubular section, which does not have the same dimensions as the actual cross-section.

Fig. 3.8: Effective area for shear flow

Based on thin-walled theory, the shear flow around the perimeter of the section is constant, so there is no need for considering a layered section for shear and torsion. For torsion the cross-section is defined by another topology matrix. This matrix, designated as $\mathbf{T}$, has 3 columns and a number of rows equal to the number of walls of the hollow

tube. Each wall is defined by the latitude and departure of its exterior face, and by its thickness. Walls are numbered sequentially moving in a counter clockwise direction. For illustration, consider the cross-section in Fig.3.9, which has four walls, with thickness h_1 to h_4 , respectively. The matrix T for this section is shown in the same figure. Each row of the matrix pertains to one wall where column one in that row gives the departure, column two the latitude and column three the thickness of the particular wall. Note that the wall numbering is arbitrary, but it must proceed counter clockwise.

Fig.3.9: A hollow section considered for torsion and relative T matrix

In the case of curved arbitrary sections, similar to the bending model, the cross-section is replaced by a polygon to closely approximate the shape of the real cross-section. In the case of circular section, only the exterior radius and thickness of the wall are required.

There are two different types of reinforcement considered in this study, longitudinal and transverse (stirrups). The position of each longitudinal reinforcing bar is defined by its eccentricities e_{sy} and e_{sz} in the element local coordinates and by its area A_{si} and its property code. The property code contains the steel stress-strain characteristics and strength. The transverse steel is defined by the area of one leg of a stirrup, its average concrete cover, its spacing and by its property code. It should be noted that the spacing of stirrups in an element is constant and if the spacing of transverse steel varies over the length of a member, the member should be divided into several elements, each with constant stirrup spacing.

3.3.3 Definition of beam-element geometry

In this study, the structure is modeled by one-dimensional beam-elements connected by joints. Each element has a local coordinate system x, y, z , which identifies the cross-section orientation. Unit vectors $\bar{\xi}, \bar{\eta}, \bar{\rho}$ are used to transform the element properties from the local to the global system. An element is defined by the coordinates of its first node, i , and its last node, j , in the global system. In addition to the nodal coordinates, the orientation of the local axes must be known. Hence by using an auxiliary node k in conjunction with joints i and j , the member orientation is uniquely defined. With reference to Fig.3.10, the unit vectors are defined as follows:

$$\bar{\xi} = \frac{IJ}{|IJ|} \quad (\text{unit vector in local x direction})$$

$$\bar{\eta} = \frac{IJ \times IK}{|IJ \times IK|} \quad (\text{unit vector in local y direction})$$

$$\bar{\rho} = \bar{\xi} \times \bar{\eta} \quad (\text{unit vector in local z direction})$$

Vector IJ connects node i to j and vector IK joins node i to auxiliary node k. Note that node k must not lie along the x axis. Fig.3.10 shows the position of an element in the global and local systems.

Fig.3.10: Three-dimensional beam-element geometry and displacement components

3.3.4 Boundary conditions

The support conditions are specified using support springs. At a support node, three translational springs, (k'_x, k'_y, k'_z) , and three rotational springs, $(k'_{rx}, k'_{ry}, k'_{rz})$, may be specified. The springs are oriented in the directions of the specified restraints. The latter directions are defined by three vectors in the direction of the displacement springs. These vectors are oriented along lines iS_1 , iS_2 and iS_3 , where i is the support node and S_1, S_2 , and S_3 are the points whose global coordinates must be specified by the user.

The extent of restraint in the direction of a spring is enforced by specifying the stiffness of that spring. Extremely large stiffness values correspond to fixity while extremely small values correspond to a free support condition.

Fig.3.11: Local spring system for supported joint

Support stiffness matrix, K_s , is added to the total global stiffness matrix of structure. In order to transform the support stiffness to the global system, the following procedure is used:

$$[K_s]^G = [A]^T [K_s] [A] \quad (3.7)$$

where

$$[K_s] = \begin{bmatrix} k'_x & & & & & \\ & k'_y & & & & \\ & & k'_z & & & \\ & & & k'_{rx} & & \\ \text{Symmetric} & & & & k'_{ry} & \\ & & & & & k'_{rz} \end{bmatrix}$$

where

$$[A] = \begin{bmatrix} a_{ij} & 0 \\ 0 & a_{ij} \end{bmatrix}$$

and a_{ij} is the matrix of the direction cosines of each support vector with respect to the three global axes, i.e.,

$$a_{ij} = \cos(s_i, x_j) \quad i, j=1, 3$$

Note that k'_x, k'_y , etc denote the spring stiffnesses and s_i refers to the vector connecting the support node to the point s_i ($i=1, 3$).

3.4 Layered finite element idealization

3.4.1 General

The computer program PCF3D uses the displacement finite element method for the geometric and material nonlinear analysis of three-dimensional concrete frames. For this reason the principle of virtual work is employed. It not only has the advantage of being simple but it is very general in that it can handle easily complex structural behaviour and material properties. This study focuses on material nonlinearity only, but the computer program PCF3D is also capable of calculating the deformations of the structure based on geometric nonlinearity (Mari, 1984).

A brief derivation of the displacement formulation is given and the numerical integration procedure, which is used to compute elements of the stiffness matrix, is presented. Following Mari (1984), each frame element is divided into a number of laminas or filaments. Each filament is assumed to be in a state of uniaxial stress and perfect bond between adjacent filaments is assumed. At any cross-section, the material properties of each filament can vary to accommodate material nonlinearities. Two Gauss integration points are used and material nonlinearity within each element is monitored at these sampling points.

3.4.2 Finite element displacement formulation

Concrete deformations are caused by different effects. The total deformation of a member consists of two different parts: mechanical and non-mechanical deformations.

Immediate application of load is assumed to cause mechanical deformations while non-mechanical deformations are produced by long-term changes such as creep, shrinkage, and thermal strains. In this study we are concerned with mechanical deformations only.

The key to the finite element formulation is the “displacement shape functions” which defines the displacement $\{\Delta\}$ at any point within the element in terms of the displacements at the nodes.

$$\{\Delta\} = [N]\{\Delta_i\} \quad (3.8)$$

where

$\{\Delta\}$ = displacement at any point

$[N]$ = shape functions

$\{\Delta_i\}$ = element nodal displacement

The strain can then be determined by differentiation of the displacements in Eq. (3.9),

$$\{\epsilon\} = [B]\{\Delta_i\} \quad (3.9)$$

where

$\{\epsilon\}$ = strain at any point within the element

$[B]$ = strain-displacement matrix

The constitutive relation is then applied to find the stresses:

$$\{\sigma\} = E(\{\epsilon\} - \{\bar{\epsilon}\}) + \{\bar{\sigma}\} \quad (3.10)$$

where

E = constitutive matrix

$[\bar{\epsilon}]$ = vector of initial strains

$\{\bar{\sigma}\}$ = vector of initial stresses

The total virtual work performed during the virtual deformation of the structure is comprised of an internal component and an external component, the latter being due to body forces, P , surface tractions, q , and external concentrated forces, R , respectively.

This may be expressed as follows:

$$W = W_i + W_e = - \int_v \{\varepsilon\}^T \{\sigma\} dv + \int_v \{\Delta\}^T \{P\} dv + \int_s \{\Delta\}^T \{q\} ds + \{\Delta_i\}^T \{R\} \quad (3.11)$$

If we substitute for $\{\Delta\}$, $\{\varepsilon\}$ and $\{\sigma\}$ on the right-hand sides of equations (3.9), (3.10) and (3.11), respectively, we obtain:

$$\{\Delta_i\} \int_v [B]^T E [B] dv = \quad (3.12)$$

$$\int_v [N]^T \{P\} dv + \int_s [N]^T \{q\} ds + \{R\} + \int_v [B]^T E \{\bar{\varepsilon}\} dv - \int_v [B]^T \{\bar{\sigma}\} dv$$

Eq. (3.12) can be written more succinctly as:

$$[K]^e \{\Delta_i\}^e = \{F\}_p^e + \{F\}_q^e + \{R\}^e + \{F\}_\varepsilon^e + \{F\}_\sigma^e \quad (3.13a)$$

where the superscript e refers to the element, or member, rather than structure. In Eq. (3.13a)

$[K]^e$ = element stiffness matrix

$$= \int_v [B]^T E [B] dv \quad (3.13b)$$

$\{F\}_p^e$ = nodal force vector due to body forces

$$= \int_v [N]^T \{P\} dv \quad (3.13c)$$

$\{F\}_q^e$ = nodal force vector due to surface tractions

$$= \int_v [N]^T \{q\} dv \quad (3.13d)$$

$\{R\}^e$ = vector of external applied loads acting at the nodes

$\{F\}_\varepsilon^e$ = nodal force vector due to initial strains

$$= \int_v [B]^T E \{\bar{\varepsilon}\} dv \quad (3.13e)$$

$\{F\}_{\bar{\sigma}}^e$ = nodal force vector due to initial stresses

$$= - \int_v [B]^T \{\bar{\sigma}\} dv \quad (3.13f)$$

Eq. (3.13) is the equations of equilibrium at the element level. The corresponding equations for the entire structure are obtained by superposition of the element equations and can be written as:

$$\sum_e [K]^e \{\Delta_i\}^e = \sum_e \{F\}_p^e + \sum_e \{F\}_q^e + \sum_e \{R\}^e + \sum_e \{F\}_\varepsilon^e + \sum_e \{F\}_{\bar{\sigma}}^e \quad (3.14)$$

The stiffness matrix of a member comprises bending, axial and torsional stiffness. It is important to recall that in this study direct shear deformations are ignored. The bending stiffness, which is dealt with adequately by Mari in program PCF3D, is calculated based on the concept of reinforced concrete filament. Furthermore, Mari, does not consider torsional calculation in any detail and he ignores the interaction between torsion and bending and/or axial load. In this study not only a more refined method for torsional stiffness is introduced, but also a procedure is introduced to consider the interaction among the various internal forces.

3.4.3 Beam-element displacement formulation

There are 13 degrees of freedom for the beam element illustrated in Fig.3.10; Six degrees of freedom are considered at each end, namely $u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \theta_{zi}$, and an internal mid-length axial degree of freedom u_n , which is later eliminated by static condensation at the element level. As Chan (1982) demonstrated, the incompatible axial displacement degree of freedom u_n is necessary for the correct modelling of the shift in the neutral axis due to cracking and other material nonlinearities in a reinforced concrete beam. Eq. (3.15) represents the equations of equilibrium for the beam-element.

$$[K] \cdot \{\Delta_i\} = \{F\} \quad (3.15a)$$

where

$[K]$ = 13×13 stiffness matrix

$\{\Delta_i\}$ = 13×1 nodal displacement matrix

$$= [u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \theta_{zi}, u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \theta_{zi}, u_n] \quad (3.15b)$$

$\{F\}$ = 13×1 nodal loads matrix.

For simplicity the superscript e has been dropped from the element based matrices. The shape functions representing bending are cubic Hermitian polynomials while those for axial displacements are parabolic. Since solid and hollow members with bulky cross-sections are most commonly used in reinforced concrete structures, torsional warping is negligible. Therefore, in this study the torsional degrees of freedom are completely independent of the other degrees of freedom and the torsional or twist shape

functions are linear. Based on the above degrees of freedom, the element displacement functions are as follows:

$$u = \lambda_1 u_1 + \lambda_2 u_2 + \lambda_3 u_3 = [\lambda] \{u\}$$

(axial displacement)

$$\theta_x = \alpha_1 \theta_{x1} + \alpha_2 \theta_{x2} = [\alpha] \{\theta\}$$

(torsional rotation or twist)

(3.16)

$$v = \phi_1 v_1 + \phi_2 \theta_{z2} + \phi_3 v_3 + \phi_4 \theta_{z4} = [\phi] \{v\}$$

(transverse displacement in y direction)

$$w = \phi_1 w_1 - \phi_2 \theta_{y2} + \phi_3 w_3 - \phi_4 \theta_{y4} = [\phi] \{w\}$$

(transverse displacement in z direction)

where x, y and z are local coordinates as illustrated in Fig.3.10. u, v, w, θ_x , θ_y and θ_z are nodal displacements and rotations in the x,y and z directions and subscripts 1,2, etc. refer to nodal numbers. The shape functions are defined as follows:

$$\begin{aligned} \lambda_1 &= 1 - \frac{x}{L} & \lambda_2 &= \frac{x}{L} & \lambda_3 &= 4 \frac{x}{L} \left(1 - \frac{x}{L}\right) \\ \alpha_1 &= 1 - \frac{x}{L} & \alpha_2 &= \frac{x}{L} \\ \phi_1 &= 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3 & \phi_2 &= x - 2\left(\frac{x^2}{L}\right) + \frac{x^3}{L^2} \\ \phi_3 &= 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3 & \phi_4 &= -\frac{x^2}{L} + \frac{x^3}{L^2} \end{aligned} \quad (3.17)$$

in which L is the element length. Notice that λ_3 is a so-called bubble function.

A well known and general method (Popov, 1998) is adopted in program PCF3D for the axial and biaxial bending of members with arbitrary cross-section. One of the advantages of this method is that the analysis can be performed assuming any reference axis, not necessarily the principal and /or centroidal axes, which is advantageous from the computer programming point of view. According to Bernoulli-Euler's theory, the longitudinal strain ε_x at any point with coordinates (x,y,z) on the beam-element can be written as:

$$\varepsilon_x = \left[\frac{du}{dx} \right] - \left[z \frac{d^2w}{dx^2} + y \frac{d^2v}{dx^2} \right] + \frac{1}{2} \left[\left(\frac{dv}{dx} \right)^2 + \left(\frac{dw}{dx} \right)^2 \right] \quad (3.18)$$

or

$$\varepsilon_x = \varepsilon_0 - z\kappa_y - y\kappa_z + \varepsilon_{NL} \quad (3.19)$$

where

$$\varepsilon_0 = \frac{du}{dx} = \text{strain of reference axis due to axial force.}$$

$$\kappa_y = \frac{d^2w}{dx^2} = \text{curvature of cross-section with respect to the y axis.}$$

$$\kappa_z = \frac{d^2v}{dx^2} = \text{curvature of the cross-section with respect to the z axis.}$$

The last term on the right-hand side of equations (3.18) and (3.19) represents second order strains due to large displacements or geometric nonlinearity. As mentioned earlier program PCF3D deals with geometric nonlinearity but this topic is not the focus of the current study. We will therefore drop ε_{NL} from further consideration.

Now by substituting for u , v , and w and θ from Eq. (3.16) in to equations (3.18) and (3.19), we can write:

$$\varepsilon_0 = \frac{d}{dx}[\lambda]\{u\} = [B_x]\{u\} \quad (3.20)$$

$$\varepsilon_{by} = -z \frac{d^2}{dx^2}[\phi]\{w\} = -z[B_y]\{w\} \quad (3.21)$$

$$\varepsilon_{bz} = -y \frac{d^2}{dx^2}[\phi]\{v\} = -y[B_z]\{v\} \quad (3.22)$$

$$\varphi = \frac{d}{dx}[\alpha]\{\theta\} = [B^*]\{\theta\} \quad (3.23)$$

where $[\lambda]$, $[\phi]$ and $[\alpha]$ are the shape function matrices whose elements are given by Eq. (3.17). Accordingly,

$$\varepsilon_x = \{[B_x] - z[B_y] - y[B_z]\} \begin{Bmatrix} \{u\} \\ \{w\} \\ \{v\} \end{Bmatrix} \quad (3.24)$$

The matrices $\{u\}$, $\{w\}$ and $\{v\}$ are nodal degrees of freedom pertaining to axial displacements and to bending about the y and z axes, respectively. Note also that $[B^*]$ pertains to torsion and it depends on x only and the twist φ is independent of the normal strain ε_x . In the program the nodal degrees of freedom are numbered from 1 to 13 in the following order.

$$\left\{ \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}, u_3 \end{matrix} \right\}$$

Corresponding to the above order, elements of the $[B]$ matrix are given as:

$$\begin{aligned}
B_1 &= B_4 = -B_7 = -B_{10} = -\frac{1}{L} & B_6 &= -\frac{2y}{L} \left(\frac{3x}{L} - 2 \right) \\
B_2 &= -B_8 = \frac{6y}{L^2} \left(1 - \frac{2x}{L} \right) & B_{11} &= \frac{2z}{L} \left(\frac{3x}{L} - 1 \right) \\
B_3 &= -B_9 = \frac{6z}{L^2} \left(1 - \frac{2x}{L} \right) & B_{12} &= -\frac{2y}{L} \left(\frac{3x}{L} - 1 \right) \\
B_5 &= \frac{2z}{L} \left(\frac{3x}{L} - 2 \right) & B_{13} &= 4 \left(\frac{1}{L} - \frac{2x}{L^2} \right)
\end{aligned}
\tag{3.25}$$

3.4.4 Beam-element stiffness matrix

The stiffness of a reinforced concrete beam element, excluding geometric nonlinearity can be expressed as:

$$[K]_e = \int_v [B]^T E [B] dv + \int_0^L [B^*]^T GJ [B^*] dx \tag{3.26}$$

where E is the modulus of elasticity and GJ is the torsional rigidity of a member, both of which are nonlinear functions of the load. Note that we have dropped the notation for E because in the case of a beam it has a single element. The axial and flexural terms of the stiffness matrix can be expressed by Eq. (3.13b), rewriting the Eq. (3.13b):

$$[K] = \int_v [B]^T E [B] dv \tag{3.27}$$

where

$$[B] = [[B_x] - z[B_y] - y[B_z]] \tag{3.28}$$

and

$$\begin{aligned}
 [K] &= \int \begin{Bmatrix} [B_x] \\ -z[B_y] \\ -y[B_z] \end{Bmatrix} E \{ [B_x] \quad -z[B_y] \quad -y[B_z] \} dv \\
 &= \int \begin{bmatrix} E[B_x]^T[B_x] & -Ez[B_x]^T[B_y] & -Ey[B_x]^T[B_z] \\ -Ez[B_y]^T[B_x] & Ez^2[B_y]^T[B_y] & Eyz[B_y]^T[B_z] \\ -Ey[B_z]^T[B_x] & Eyz[B_z]^T[B_y] & Ey^2[B_z]^T[B_z] \end{bmatrix} dv \quad (3.29)
 \end{aligned}$$

Terms of the above integral may be integrated in two separate steps because the $[B]$ matrices are only a function of x while the Ez , Ey , etc. are essentially a function of y and z . Variation of E along x is accounted for by assuming it to be constant along the tributary length of each quadrature point. The integration over the area is replaced by a summation over all the layers or filaments in a cross-section. For instance,

$$\int_v Ez^2 [B_y]^T [B_y] dv = \int_A Ez^2 dA \int_0^L [B_y]^T [B_y] dx \quad (3.30)$$

The area integral is written as:

$$\int_A Ez^2 dA = \sum_{i=1}^{n_c} E_{ci} A_{ci} z_{ci}^2 + \sum_{j=1}^{n_s} E_{sj} A_{sj} z_{sj}^2 + \sum_{i=1}^{n_c} \frac{1}{12} E_{ci} \Delta y_{ci} \Delta z_{ci}^3 = EI_y \quad (3.31)$$

where n_c is the total number of filaments and n_s is the total number of steel rebars. The last integral is the conventional flexural rigidity of the section about the y -axis, and the symbol EI_y in the case of a homogeneous and isotropic material represents the EI of the section bending about the y axis. The quantities E , A and z in the above equation represent the modulus, area and z coordinate of each layer. The subscripts c and s denote concrete and steel layers and n is the number of layers. The last term, in Eq. (3.31) represents the moment of inertia of each concrete layer about its own middle plane.

Using the same approach, the remaining terms of the $[K]$ matrix are integrated as follows:

$$EA = \sum_{i=1}^{n_c} E_{ci} A_{ci} + \sum_{j=1}^{n_s} E_{sj} A_{sj} \quad (3.32)$$

$$EY = -\sum_{i=1}^{n_c} E_{ci} A_{ci} z_{ci} - \sum_{j=1}^{n_s} E_{sj} A_{sj} z_{sj} \quad (3.33)$$

$$EZ = -\sum_{i=1}^{n_c} E_{ci} A_{ci} y_{ci} - \sum_{j=1}^{n_s} E_{sj} A_{sj} y_{sj} \quad (3.34)$$

$$EI_z = \int_A E y^2 dA = \sum_{i=1}^{n_c} E_{ci} A_{ci} y_{ci}^2 + \sum_{j=1}^{n_s} E_{sj} A_{sj} y_{sj}^2 + \sum_{i=1}^{n_c} \frac{1}{12} E_{ci} \Delta z_{ci} \Delta y_{ci}^3 \quad (3.35)$$

$$EI_{yz} = \int_A E y z dA = \sum_{i=1}^{n_c} E_{ci} A_{ci} y_{ci} z_{ci} + \sum_{j=1}^{n_s} E_{sj} A_{sj} y_{sj} z_{sj} \quad (3.36)$$

The above parameters are basically equivalent to $EA, EQ_y, EQ_z, EI_z, EI_y$ and EI_{yz} for a homogenous section. The quantities Q_y and Q_z are the first moment of area about the y and z axes, while I_y and I_z are the second moments of area about the same axes and I_{yz} is the product of inertia.

Next the integrations over the length are performed to obtain elements of $[K]$. For example,

$$K_{11} = EA \int_0^L \left(-\frac{1}{L}\right) \left(\frac{1}{L}\right) dx = \frac{EA}{L}$$

$$K_{13,13} = EA \int_0^L 4\left(\frac{1}{L} - \frac{2x}{L^2}\right) \left(4\left(\frac{1}{L} - \frac{2x}{L^2}\right)\right) dx = 16EA \int_0^L \left(\frac{1}{L} - \frac{2x}{L^2}\right)^2 dx = \frac{16EA}{3L}$$

The other terms are similarly integrated and are summarized in appendix B. The resultant stiffness matrix can be written as:

$$K = \begin{bmatrix} [K_{ee}] & [K_{en}] \\ [K_{ne}] & [K_{nn}] \end{bmatrix}_{13 \times 13} \quad (3.37)$$

where

$[K_{ee}] = 12 \times 12$, element stiffness matrix considering the two end nodes

$[K_{en}] = [K_{ne}]^T = 1 \times 12$, matrix which couples the 13th degree of freedom (dof) with the end nodes dof

$[K_{nn}] = 1 \times 1$, uncoupled stiffness matrix of the 13th degree of freedom.

Therefore, the element equilibrium equations can be expressed as

$$\begin{bmatrix} [K_{ee}] & [K_{en}] \\ [K_{ne}] & [K_{nn}] \end{bmatrix} \begin{bmatrix} \{\Delta_e\} \\ \{\Delta_n\} \end{bmatrix} = \begin{bmatrix} \{F_e\} \\ \{F_n\} \end{bmatrix} \quad (3.38)$$

Since the 13th degree of freedom is related to an internal element node, it is eliminated by static condensation in each element before transforming the local stiffness matrix to the global stiffness matrix. Since $\{F_n\} = \{0\}$, $\Delta_n = -[K_{nn}]^{-1}[K_{ne}]\{\Delta_e\}$, which when substitute in Eq. (3.38) renders the stiffness matrix $[K_{ee}]$ as follows:

$$[K_{ee}] = \begin{bmatrix} [K_{aa}] & [K_{ab}] & 0 \\ [K_{ab}] & [K_{bb}] & 0 \\ 0 & 0 & [K_T] \end{bmatrix} \quad (3.39)$$

where

$[K_{ab}]_{2 \times 2}$ = axial stiffness matrix

$[K_{ab}]_{2 \times 8}$ = coupled axial-bending stiffness matrix

$[K_{ba}]_{8 \times 2} = [K_{ab}]^T$

$[K_{bb}]_{8 \times 8}$ = biaxial bending stiffness matrix

$[K_T]_{2 \times 2}$ = torsional stiffness matrix.

For clarity, the above matrices are shown in somewhat different order than they appear in the actual computer program. The torsional stiffness matrix is given by,

$$[K_T] = \begin{bmatrix} \frac{GJ_{11}}{L} & 0 \\ 0 & \frac{GJ_{22}}{L} \end{bmatrix} \quad (3.40)$$

where GJ_{ii} is the torsional rigidity of the section at node i of the element. The calculation of the torsional rigidity, which is the main focus of this study, is discussed in detail in the next chapter.

3.4.5 Calculation of strains, stresses and unbalanced loads

After solving Eq. (3.14) using the nonlinear analysis procedure, which will be given in the next section, at the end of each iteration, incremental displacements corresponding to the six degrees of freedom at each node are obtained. In order to calculate stresses and strains, following procedure is applied.

- The incremental displacements are transformed from the global to the local coordinate system.

- The strain increments in an element can be calculated using Eq. (3.24)

$$\delta\epsilon_{x0} = \delta\left(\frac{du_0}{dx}\right) = B_1\delta u_1 + B_7\delta u_7 + B_{13}\delta u_{13} \quad (3.41)$$

$$\delta\kappa_y = \delta\left(\frac{d^2w}{dx^2}\right) = B_3\delta w_1 + B_5\delta\theta_{y1} + B_9\delta w_2 + B_{11}\delta\theta_{y2} \quad (3.42)$$

$$\delta\kappa_z = \delta\left(\frac{d^2v}{dx^2}\right) = B_2\delta v_1 + B_6\delta\theta_{z1} + B_8\delta v_2 + B_{12}\delta\theta_{z2} \quad (3.43)$$

where the B matrix is evaluated at the two Gauss points along each element. Notice that ϵ_{x0} is the axial strain along the reference axis and κ_y and κ_z are the curvatures about the y and z axes and δ denotes increment. By substituting the calculated amount of axial strain and curvatures in Eq. (3.24), the total strain increment for the current iteration can be obtained from:

$$\delta\epsilon_{xc} = \delta\epsilon_{x0} - \delta\kappa_y z - \delta\kappa_z y \quad (\text{concrete}) \quad (3.44)$$

where the z and y are the centroidal coordinates of each concrete filament. The steel strain is calculated as:

$$\delta\epsilon_{xs} = \delta\epsilon_{x0} - \delta\kappa_y e_{sz} - \delta\kappa_z e_{sy} \quad (\text{steel}) \quad (3.45)$$

where e_{sz} and e_{sy} are eccentricities of each steel rebar from the reference axes. Therefore, the total amount of strain can be obtained by adding the current strain increment to the previous total strain.

$$\epsilon_{xci} = \delta\epsilon_{xc} + \epsilon_{xc(i-1)} \quad (3.46)$$

$$\epsilon_{xsi} = \delta\epsilon_{xs} + \epsilon_{xs(i-1)} \quad (3.47)$$

It should be noted here that a modification was made in this study in order to improve the accuracy of the results. In program PCF3D the effect of the 13th-degree of freedom had been neglected and it caused an unbalanced axial strain at the reference axis

levels, which resulted in the wrong estimation of the position of the neutral axis. The extra axial strain in the element affected the calculation of axial force and moments in that element. Comparing the results of a simple problem with the results of program PCF3D, the existence of this mistake was detected. The mistake was corrected by calculating the displacements at the middle node and including them in the strain calculation. The increment of axial displacement at the middle node is calculate at each iteration using Eq. (3.38)

$$[K_{ne}]\{\delta\Delta_e\} + [K_{nn}]\{\delta\Delta_n\} = F_n = 0 \quad (3.48)$$

therefore:

$$\delta\Delta_n = -[K_{nn}]^{-1}[K_{ne}]\{\delta\Delta_e\} \quad (3.49)$$

The definition of the terms of the equation above was given in Section 3.4.4.

- After calculating the strains, the stress and the new modulus of elasticity of each filament and each steel rebar can be calculated using its stress-strain relationship, (Tables 3.1 and 3.2).
- Knowing the stress distribution in a section, the stress resultants comparing the axial force and two moments and the two shear forces are obtained by numerical integration of the stresses over the cross-section.
- With respect to torsion, the procedure of finding the torque and twist at each iteration is totally different from bending and axial actions. As mentioned before, the way the program handles the torsion problem is not based on stresses and strains. The adopted model for torsion is based on torque-twist curve. After solving Eq. (3.14), the rotations of

each node are known; therefore, by using the B matrix the relative twist of each node for the entire section can be defined, as:

$$\delta\phi = B_4\theta_{x1} + B_7\theta_{x2} \quad (3.50)$$

where $\delta\phi$ is the relative twist between the two ends of a member and θ_{x1} and θ_{x2} are the angles of twist at the ends 1 and 2, respectively. Similar to bending, the total twist is obtained by adding the twist increment to the total twist of the previous iteration. Knowing the twist, the torque and torsional stiffness can be found from the torque-twist curve.

-As long as the displacements do not satisfy the convergence criteria, the program continues the iteration process. At each iteration, the internal resisting forces due to the stresses in the concrete, and in the steel, can be calculated using Eq. (3.13f).

$$\{F_R\} = \int_{V^e} [B]^T \{\sigma\} dv = \int_0^L [B^*]^T \int_A \{\sigma\} \phi(y, z) dA dx = \int_0^L [B^*]^T \cdot F_i dx \quad (3.51)$$

where the B^* is the matrix defined based on function x only and $\phi(y, z)$ is a function of y and z only. The component B_{13} is not included in B matrix because it is assumed that no external force acts at this node. For the remaining 12 degrees of freedom the F_i can be written explicitly as:

$$F_1 = F_7 = \sum_{i=1}^{n_s} A_{ci} \sigma_{ci} + \sum_{j=1}^{n_s} A_{sj} \sigma_{sj} \quad (3.52)$$

$$F_4 = F_{10} = T_x \quad (3.53)$$

$$F_2 = F_6 = F_8 = F_{12} = \sum_{i=1}^{n_c} A_{ci} \sigma_{ci} y_{ci} + \sum_{j=1}^{n_s} A_{sj} \sigma_{sj} y_{sj} \quad (3.54)$$

$$F_3 = F_5 = F_9 = F_{11} = \sum_{i=1}^{n_c} A_{ci} \sigma_{ci} Z_{ci} + \sum_{j=1}^{n_s} A_{sj} \sigma_{sj} Z_{sj} \quad (3.55)$$

-The above internal resisting forces with reverse sign are added to the external nodal loads and the results constitute the unbalanced forces at the nodes. Then unbalanced forces contributed by all the members joining at node are added to obtain the net unbalanced nodal forces respect to each node. If all the unbalanced forces satisfy the convergence criteria, the program starts the next load step; otherwise, another iteration is commenced to reduce the unbalanced forces.

3.5 Nonlinear analysis method

3.5.1 General

In this section, a brief review of the nonlinear stress analysis techniques is given. In this study we deal with material nonlinearity of reinforced concrete structures. The procedure of nonlinear analysis applied and the convergence criteria for this method are presented as follows.

3.5.2 Techniques of nonlinear analysis

The equations of the equilibrium of the finite element method are given by:

$$[K]\{\Delta\} = \{F\} \quad (3.56)$$

In linear problems the stiffness matrix $[K]$ and forces $\{F\}$ are constant and independent of displacements $\{\Delta\}$, because the properties of materials are not a function of displacement or stress. In contrast, in nonlinear problems $[K]$ and $\{F\}$ are functions of $\{\Delta\}$. Therefore, the solution of nonlinear equations requires an iterative process.

In general, there are three basic methods in use for solving nonlinear equations: (a) Incremental method, (b) Newton-Raphson or Iterative method, and (c) Incremental-Iterative method. A brief review of these methods is presented here and the third method is used in the present study.

a. Incremental method

In this method, the total applied load F is sub-divided into a number of increments, δF_i such as:

$$\{F\} = \sum_{i=1}^k \{\delta F_i\} \quad (3.57)$$

where k is the number of increments. As indicated in Fig.3.12, for the first increment of load, an initial stiffness matrix $[K_0]$, which is calculated based on the initial constitutive relation at zero level of load, is used. For the i^{th} increment the stiffness matrix, is evaluated based on the stress-strain relation prevailing at the end of $(i-1)^{\text{th}}$ load increment, i.e.:

$$[K_{i-1}]\{\delta \Delta_i\} = \{\delta F_i\} \quad (3.58)$$

where $[K_{i-1}]$ is a function of $\{\Delta_{i-1}\}$. The total displacement vector at the end of the i^{th} increment is then given by:

Fig.3.12: a) Purely incremental method, b) Modified incremental method

$$\{\Delta_i\} = \sum_{j=1}^i \{\delta\Delta_j\} \quad (3.59)$$

One of the disadvantages of this method is that the approximate solution drifts further from the exact solution within each increment (Fig.3.12). In order to eliminate this deficiency, a modification is applied by adding a corrective term to the load term. The corrective term is based on the calculated unbalanced load in each increment. The unbalance load, $\{F_U\}$, is the difference between the applied increment load, $\{F_i\}$, and the force $\{F_R\}$ calculated as the product of the stiffness matrix and the current displacement. For example, for increment i :

$$\{\Delta_{i+1}\} = \{\Delta_i\} + [K_i]^{-1} [\{\delta F_{i+1}\} + (\{F_i\} - \{F_R\})] \quad (3.60)$$

By applying the unbalanced force to the system, in each increment the applied load is corrected and major drift from the actual response is reduced.

b. Newton-Raphson (N-R) or Iterative method

In this method the total load is applied and iterations are performed until the specified convergence criterion is satisfied. Similar to the previous method, the first iteration starts with the initial stiffness matrix and for subsequent iterations the stiffness matrix is adjusted based on the results of the preceding iteration. The calculated stiffness is called, the “tangent stiffness”. After each iteration, the unbalanced forces are calculated and applied in the next iteration as load. For the i^{th} iteration, the unbalanced force vector is given by:

$$\{F_U\} = \{F\} - \{F_R\} \quad (3.61)$$

Fig.3.13: a) Tangent stiffness procedure, b) Initial stiffness procedure

where $\{F\}$ is the total applied load and $\{F_R\}$ is the force calculated on the previous stiffness matrix multiplied by the current displacement. Knowing the unbalanced force, the displacement increment $\{\delta\Delta_i\}$, corresponding to the $\{F_U\}$, is then computed using:

$$[K_{i-1}]\{\delta\Delta_i\} = \{F_U\} \quad (3.62)$$

and the total displacement after the i^{th} iteration, similar to the incremental method, is given by:

$$\{\Delta_i\} = \sum_{j=1}^i \{\delta\Delta_j\} \quad (3.63)$$

There is another method that is called “modified Newton-Raphson.” In this method the tangent stiffness which is basically the initial stiffness for the entire process is either not updated or is updated infrequently. Thus, in complex structures, it avoids the expensive repetition of forming the stiffness matrix in each iteration. As one can observe in Fig.3.13 this method requires more iterative cycles in order to reach convergence.

c. Incremental-Iterative method

This method combines the incremental and the iterative methods. In implementing this procedure, the load is divided into a number of increments. Iterations are performed after application of each load increment. After the application of a load increment i , for the first iteration, the tangent stiffness evaluated at the end of the previous load increment, $(i - 1)$, is used to calculate the incremental displacement for the current iteration.

Using the displacement increment, the incremental strains, $\delta\epsilon_i$, are calculated based on Eq. (3.9), and the total strain is defined by adding the incremental strain to the previous total amount of strain, $\{\epsilon_i\}$. Corresponding to the total strain, the stress and internal nodal forces are found, and then equilibrium between the internal and external forces is checked. If there are any residual forces $\{\delta F_U\}$, another iteration is required and the unbalanced forces with reversed sign are applied as new nodal forces. The procedure of iterating is repeated until equilibrium between the internal and external forces is satisfied within the prescribed limits. More details of nonlinear analysis are given by Zienkiewicz (1983) and Bathe (1981).

For this study the Incremental-Iterative method is used and also an option is provided to use either the tangent stiffness or constant stiffness during the iteration. At the end of each iteration, a certain displacement ratio ρ is compared with a tolerance t_c for changing stiffness. If $\rho > t_c$, a new stiffness is formed for the next iteration, otherwise it is not changed.

Numerous studies have been carried out (Mari, 1984) to avoid the difficulties arising from the non-positive terms on the diagonal of the stiffness matrix. A non-positive term appears on the diagonal when a structure experiences strain softening. In this study following Mari (1984), the method proposed by Simmons (1982) and proven reliable in a comparative study by Chan (1982), is adopted. However, tracing the softening descending portion of the structural response requires the specification of incremental nodal displacements instead of nodal loads. Such an analysis is beyond the scope of the current study.

Fig.3.14: Incremental-Iterative method

3.5.3 Convergence criteria and termination of the analysis

Two convergence criteria, which were originally proposed by Kang (1977), are considered in the present study.

- Displacement criterion
- Unbalanced force criterion

In order to check the first criterion, the program compares a calculated displacement ratio ρ with a specified displacement ratio or tolerance defined by the user. If the displacement ratio ρ for all the degrees of freedom after the iteration is less than the specified tolerances, the program proceeds to the next load increment, otherwise it

continues the iteration process. A similar procedure is used in the case of the unbalanced force convergence criterion. If all the unbalanced forces are less than the specified tolerance, convergence is assumed and no more iteration is performed with the current load increment. If the number of iterations exceeds the maximum allowable number, defined by user, the program proceeds to the next load increment. Therefore, it is wise to specify a reasonable maximum number of iterations because otherwise the solution of the next load step may be affected by the unbalanced forces remaining at the end of the unconverged load step.

In order to terminate the analysis, two different criteria are used, depending on different requirements, i.e. serviceability and safety. To find the ultimate capacity of the structure, the solution stops once the stiffness matrix is found to contain zero or negative diagonal elements. In the case of checking serviceability, the program stops when the displacement exceeds the allowable value for displacements as specified by the user.

Chapter 4

Modelling Torsion and Combined Loading

4.1 General

This chapter focuses on the development of a semi-empirical torque-twist model and on the behaviour of the structure under combined bending, shear, torsion and axial force. The chapter is divided into three sections as follow:

- In Section one, the procedure for developing the torque-twist curve from the uncracked stage up to the ultimate stage will be presented.
- In Section two, the procedures for calculating ultimate strength of a reinforced concrete section subjected to single actions, i.e. axial load, bending, shear or torsion will be discussed.
- In Section three, the adopted method for predicting the response of the concrete frame to combined actions will be presented.

4.2 Torsion

4.2.1 General

In 1958, ACI Committee 438 studied the torsion problem for three major reasons and recommended research in this area.

- The ultimate strength design (USD) method was accepted as a replacement for the working stress design method, which resulted in more slender members in which neglecting torsional effects were no longer acceptable.
- New demands by architects to build modern buildings that had curved beams, spandrel beams, skew structures and many kinds of irregular shapes that may be subjected to substantial torsion.
- The remarkable advances made in electronic computer applications for structural analysis allowed engineers to consider many more design factors such as torsion.

Since that time the problem has been extensively studied and a great deal of knowledge has been gained with respect to this problem. However, computer-based analyses still do not treat the problem adequately when torsion is applied in combination with other actions.

4.2.2 A brief history of torsion

Before the motivation for the current study is presented, a brief history of torsion analysis of concrete members is presented. At the present time there are a number of methods available for analysis and design of reinforced concrete elements subjected to torsion. These methods are based on two theories: the truss analogy and the skew-bending theory. Below the basis and the development of each theory is briefly described.

Lessig (1958) proposed the skew-bending concept as the basis for torsion of concrete members, and the method was incorporated into the Russian code in 1962. In this method, a failure mechanism is assumed, and the equilibrium conditions on the assumed failure surface are considered using the equality of external and internal energies. Lessig identified two modes of failure; mode 1 considers the compression zone near the top surface of the beam, while mode 2 focuses on the compression zone along a side face. She presented a set of the three basic equations for the analysis of each mode of failure. These equations can be solved by a trial-and-error procedure. Contributions were made to the further development of this approach by other researchers such as Yudin (1962) who considered the contribution of stirrups and further refined the equilibrium equations. Collins et al (1965, 1968) were able to combine the three sets of equations in to one equation for each failure mode and also discovered a third mode of failure, which involves a compression zone near the bottom face of the beam. Collins et al's theory served as a basis of the new Australian Code torsion provisions in 1974. Goode and Helmy (1968), McMullen and Warwaruk (1970) and Elfgren et al (1972, 1974) continued working on this method. They dealt primarily with the interaction of torsion and bending

or the interaction of torsion, bending and shear. The details of this method will be presented later in this chapter.

An early design procedure for reinforced concrete members under shear was based on the truss analogy, developed by Ritter (1899) and Morsch (1909). This theory assumes that concrete is not capable of resisting tension, that a cracked reinforced concrete beam acts as a truss with compression chords of concrete inclined at 45-degree and tension chords of steel reinforcement which tie the member together. The truss analogy predicts that a beam needs both stirrups and longitudinal steel in order to resist shear. Experience with 45-degree truss model has shown that the results of this model are generally rather conservative, especially for beams with low web reinforcement ratio.

The truss analogy expressions for torsion were first developed by Rausch (1929). As in shear, it was assumed that after cracking the concrete is incapable of carrying any tension and that the beam acts as a truss with longitudinal chords and with walls composed of diagonal concrete struts and transverse steel ties. Rausch also assumed the angle of the diagonal concrete struts to be 45-degree and he assumed that the shear flow path coincides with the centerline of the closed stirrups.

This approach, which was later further developed, based on the work of Hsu (1968) and Mattock (1968), consists of adding an empirical "concrete contribution" related to the diagonal cracking load and considering an empirical coefficient which is a function of the shape of the beam. The ACI Code 318-77 (1977) adopted this modified form of 45-degree truss method. In 1969, the 45-degree model was generalized by Lampert and Thurliman (1969) for members subjected to torsion and combined torsion and bending. They assumed that the angle of inclination of the concrete struts may

deviate from 45-degree, and that the theory of plasticity is applicable to reinforced concrete members. Accordingly, they were able to demonstrate that both longitudinal and transverse steel may yield under pure torsion even if their volume ratios are not equal. Since the angle of the concrete struts is not 45-degree, they called their theory the variable-angle truss model. The European Code (CEB, 1970) and Canadian Code CSA A23.3 (1972) adopted the truss analogy as the basis for their torsion design provisions.

Elfgren (1972) applied the variable-angle truss model to members subjected to torsion, bending, and shear. He observed that this model was very similar to Wagner's tension field theory (1929). Wagner (1929) dealt with the post-buckling shear resistance of thin-walled metal girders. He postulated that after buckling, the thin webs would not resist compression and that the shear would be carried by a field of diagonal tension. To determine the angle of inclination of the diagonal tension, he assumed that the angles of inclination of the diagonal tensile stress and the principal tensile strain coincide. This approach has been termed the tension field theory.

Before the equilibrium equations of the truss analogy can be used to analyze a member for shear or torsion, the inclination of the diagonal compression chord must be known. To determine this angle, Collins (1973) approached the problem differently and further developed the variable-angle truss model. Instead of using the theory of plasticity, he focused on strain compatibility in the truss model. He derived, based on an assumed compatibility condition, an equation to determine the angle of the compression stress field. The assumed compatibility condition made it possible to use the simple Mohr's circle in order to determine the strain field within the member. Collins called this theory the "diagonal compression field theory." This method serves as a basis for the so-called

accurate method in the CEB-FIP Code (1978) and the “general method” of shear and torsion design in the Canadian standard CSA A23.3 (1994).

In this theory in addition to the compatibility and equilibrium equations in the variable angle truss model, the compressive stress-strain curve of the concrete struts must be assumed because the prediction of torsional strength based on the standard concrete compression cylinder was found to be very unconservative. Hsu and Mo (1984) used a “softened” stress-strain curve which resulted from the diagonal shear cracking and were able to predict the torsional strength as well as the deformations and strains throughout the loading history. Gradually this method was further refined by considering important parameters such as, spalling of concrete cover and the thickness of the concrete diagonal struts for the better prediction of the real torsional behaviour of concrete member. However, these models are often applied in isolation, i.e. to members subjected to pure torque. When a member is subjected to combined actions, the results are not as satisfactory. Furthermore, the change in member stiffness under combined actions has not been the prime focus of these models. Since the “truss model” is the basis for the development of a practical torque-twist relationship, this model will be discussed in more detail in the next section.

4.2.3 Torque-twist relationship

Fig.4.1 illustrates typical experimental torque-twist curves for a number of rectangular sections with identical dimensions and concrete strengths but with different transverse reinforcement ratios, ρ_t (Hsu, 1968).

Fig.4.1: Typical torque-twist relationship for reinforced concrete section by Hsu (1968)

From these curves and other similar test results it can be demonstrated that a reinforced concrete member under pure torsion experiences three distinct states before failure: uncracked, cracked not yielded and finally cracked yielded. Before cracking, the member closely follows the elastic behaviour, the concrete exhibits almost linear behaviour and the steel is virtually unstressed. After cracking, the twist suddenly increases and the section torsional rigidity decreases. The extent of the increase in twist and the reduction in stiffness depends upon the amount of both longitudinal and transverse reinforcement. We notice in Fig.4.1 that for small transverse reinforcement ratios, i.e. for $0.189\% \leq \rho_t \leq 0.392\%$, immediately after cracking both the jump in the angle of twist and the reduction in torsional rigidity is quite dramatic and noticeable. On the other hand, for higher ratios, i.e. $\rho_t \geq 0.540\%$, the jump in twist is much smaller, but the reduction in rigidity is still appreciable.

Between torsional cracking and yielding of reinforcement, the torque-twist curve is practically linear, while after yielding it turns more nonlinear up to the peak torque. For the smaller transverse reinforcement ratios, softening occurs after the peak torque while for the larger reinforcement ratios we do not observe such a response. From a practical point of view, the torsional softening is not that significant (less than 10% difference between the peak torque and the torque at failure); therefore, it is sufficiently accurate to assume a constant torque after the peak or ultimate torque.

Fig.4.1 also shows the lines representing the theoretical torsional stiffness of the tested sections after cracking (cracked stiffness). We observe that these dashed lines severely over-estimate the torsional stiffness after cracking, particularly at higher loads.

The dashed lines in Fig.4.1 seem to be representing the tangent stiffness immediately after torsional cracking.

In view of these observations, in this study a new semi-empirical torque-twist relation will be introduced in order to better capture the nonlinear torsional response of reinforced concrete frame structures. It must be emphasized that the proposed model is based on both empirical observations and some theoretical analysis of the torsional resistance mechanism of concrete frame members. Further refinements and calibration may be needed to improve its accuracy.

For a better understanding of the proposed torque-twist curve, consider Fig.4.2. The solid line OADCE in the figure schematically illustrates the applied torque, T , versus the angle of twist ϕ relation for a section. Below each part of this curve is described:

1. Line OA, represents the behaviour of the concrete section before cracking and its slope is obtained using elastic theory. Point A corresponds to the cracking torsion whose calculation will be described later.
2. Line AB shows the jump that occurs suddenly after cracking. To determine point B, the well-known “space truss analogy” will be used.
3. Line BC shows a simplified model of actual behaviour of a concrete member after cracking subjected to torsion. As mentioned before, the experimental curves follow mostly a linear path. Therefore, in this study a linear path is considered and to find point C, a simplified form of the “modified compression field theory” will be used. This theory is based on St. Venant’s (1958) torsion theory and does not consider warping effects.

Hsu (1984) showed that reinforced concrete hollow and solid beams have similar ultimate torsional strength. Consider a small length of a thin tube with variable wall thickness as shown in Fig.4.3. By reference to any standard mechanics of materials book (Popov, 1998), we can write the following relations:

Fig.4.3: Thin-walled tube subjected to torsion

$$T = 2qA_0 \quad (4.1)$$

$$v = \frac{T}{2A_0 t_e} \quad (4.2)$$

$$\varphi = \frac{TP_0}{2A_0^2 G} \oint \frac{ds}{t_e} \quad (4.3)$$

or

$$\phi = \frac{T}{GJ_1} \quad (4.4)$$

where

$$GJ_1 = G \int \frac{4A_0^2}{t_e} ds \quad (4.5)$$

For a tube with a constant wall thickness t_e ,

$$GJ_1 = G \frac{4A_0^2 t_e}{p_0} \quad (4.6)$$

where

v = torsional shear stress

T = torsional moment

A_0 = the area enclosed by the centerline of the tube

q = shear flow

ϕ = angle of twist per unit length

G = shear modulus

p_0 = the perimeter of the area enclosed by centerline of tube

t_e = the effective tube wall thickness.

For a prismatic tube of length L subjected to a constant torque T , ϕ would be constant and the total angle of twist is $\theta = \phi L$. Hence, the angle of twist is related to the applied torque as:

$$T = \frac{GJ_1}{L} \theta = K_T \theta \quad (4.7)$$

where

K_T = torsional stiffness of the tube.

For a member subjected to pure torque, the shear stress calculated according to Eq. (4.2) can be used to monitor the advent of cracking. Since under a state of pure shear, the principal tensile stress is equal to the shear stress, cracking may be assumed when $v \geq f_t$, where the f_t is the tensile stress of concrete. Alternatively, we can use the equation for the cracking torsion, provided in CSA Standard 23.3 (1994) to predict torsional cracking. Hence as soon as the resisting torque at any section in a member reaches T_{cr} , the member is assumed to have cracked. The advent of cracking leads to a noticeable reduction in torsional stiffness and a sudden increase in the angle of twist. According to the CSA A23.3

$$T_{cr} = \left(\frac{A_c^2}{p_c} \right) 0.4 \lambda \phi_c \sqrt{f'_c} \sqrt{1 + \frac{\phi_c f_{cp}}{0.4 \lambda \phi_c \sqrt{f'_c}}} \quad (4.8)$$

where

A_c = area enclosed by outside perimeter of concrete cross-section including area of holes,

if any

p_c = outside perimeter of the concrete cross-section

λ = factor to account for low density concrete

ϕ_c = resistance factor for concrete

f'_c = compressive strength of concrete

f_{cp} = compressive stress in concrete due to prestressing.

This equation includes design factors and also the prestressing effect. Since this study only focuses on analysis of reinforced concrete, Eq. (4.8) can be simplified as:

$$T_{cr} = \left(\frac{A_c^2}{p_c}\right) r_t \sqrt{f'_c} \quad (4.9)$$

The $r_t \sqrt{f'_c}$ expresses the tensile strength of concrete. As mentioned before, the tensile strength of concrete for shear and torsion is not defined based on modulus of rupture because the modulus of rupture shows higher value compared to the actual tensile strength of concrete. Hsu and Mo (1984) suggested $0.41\sqrt{f'_c}$ MPa ($5\sqrt{f'_c}$ psi) as a reasonable value for cracking shear stress of concrete. While ACI Committee 438 adopted the value $0.5\sqrt{f'_c}$ MPa ($6\sqrt{f'_c}$ psi) for the estimation of diagonal cracking load. We notice that, a unique value for tensile strength of concrete cannot be given and the subject is still controversial. Consequently, in the present study, the choice of the actual value is left to the user.

To generalize this method to all kinds of sections besides the thin-walled tube sections, Collins and Mitchell (1980) presented an approximate procedure based on the concept that most of the torsion is resisted by the high shear stresses near the outer perimeter of the section. The thickness of the tube depends on the amount of longitudinal and transverse reinforcement, the concrete strength and the geometry of the section. In this approach an equivalent thin-walled tube section, with the wall thickness t_e , having the same external dimensions as the actual cross-section, is used to represent the actual cross-section. This method was adopted by the CSA Standard 23.3 (1984). The wall thickness is given by:

$$t_e = \frac{3 A_c}{4 p_c} \quad (4.10)$$

where

A_c = the area enclosed by the outside perimeter of the cross-section

p_c = the outside perimeter of concrete cross-section

Later Rahal and Collins (1996) found that the average value of the thickness of the concrete that is effective in resisting the torsional moment is:

$$t_e = 0.5 \frac{A_c}{p_c} \quad (4.11)$$

and it is this equation that has been adopted as the equivalent tube thickness equation in the current study.

b. After cracking

After cracking, there is a remarkable loss in torsional stiffness, which loss is much more severe than the loss in flexural stiffness due to flexural cracking. After torsional cracking, it is mainly the longitudinal and transverse steel which controls the stiffness. Based on space truss analogy, after cracking the torsion is resisted by diagonal concrete compressive stresses that spiral around the beam at an angle θ and by longitudinal and transverse reinforcement which hold the beam together.

The analysis of a torsionally cracked member requires the determination of four quantities:

- The stress in the longitudinal reinforcing bars, f_L
- The stress in the transverse bars or stirrups, f_t

- The stress in the concrete struts, f_2
- The inclination θ of the struts.

Since there are only three equations of equilibrium available at any section, we require an additional relation among the above unknowns in order to obtain a unique solution. The additional equation is often generated by imposing an equation of constraint, e.g. that the directions of principal compressive stress and principal compressive strain at a given location and at any load level coincide. To satisfy this condition, the constitutive laws of concrete and steel are used in order to calculate the steel and concrete stresses and strains. Since determination of those stresses involves the angle θ , and since θ depends on the level of stresses, an iterative process is needed to solve the problem of torsion in cracked reinforced concrete members. Alternatively, the angle θ may be assumed without checking the compatibility of strains, which is the basis for the 45° space truss model in the ACI 318-02 Code (ACI 2002). Theoretically, it is incorrect to assume a single angle of inclination for the concrete struts, but in practice such an assumption is justified by empirical data from many tests and some adjustments to the shear resisting mechanisms.

The preceding iterative process, using stresses and strains, could produce the full torque-twist response of a section, including its torsional rigidity. The process, however, suffers from two shortcomings. First, the assumption that the directions of principal stress and principal strain in cracked reinforced concrete coincide, albeit reasonable, is an approximation and cannot be proven theoretically. Second, the notion that shear stresses induced by torsion in any section are constant is not theoretically exact. Both of these approximations, in addition to the fact that in the current study we are interested in the

torsional moment and not torsional shear stresses, are motivations for the construction of a simpler torque-twist response curve, but one that is more accurate than those currently used in concrete frame analysis programs. Furthermore, it is important to note that research to date on the torque-twist response involving iterative techniques have been limited to sections subjected to pure torque. When a section is subjected to combined moment, shear, torsion and axial load, the response of the section may be quite different from those due to each action applied individually. In the light of the preceding comments, we will introduce an approximate semi-empirical torque-twist relation.

As indicated in Fig.4.2, two points identify the cracking level of a member under torsion. Point A represents first cracking and point B the full cracking. Physical tests show a sudden increase in twist at the onset of diagonal cracking, after which elastic theory is no longer valid. Applying the truss analogy enables one to calculate the torsional stiffness and angle of twist of a member after cracking, which are mainly affected by the reinforcement properties. A linear behaviour is assumed for the cracked reinforced concrete up to the ultimate torsional capacity of the member. The slope of this linear curve represents the post-cracking torsional rigidity, which will be calculated later in this section. For calculating the ultimate torsional strength and the corresponding deformation, a nonlinear method based on Rahal and Collins (1996) work is adopted in the present study.

Since the truss analogy constitutes the physical basis for modelling the behaviour of a torsionally cracked member, a brief description of the truss model and its underlying assumptions are presented in the following section.

Equilibrium equations of truss model

Fig.4.4 shows a twisted beam under an applied torque T larger than the cracking torque. The applied torque is resisted by a field of compressive stresses that spiral around the section at an angle θ to the axis of the beam (Fig.4.4). The tangential component of these stresses is equilibrated by shear stresses (or shear flow q) in the cross-section.

Several points need to be clarified about the truss model representation of beams under torsion. The actual beam is replaced by an equivalent hollow tube of wall thickness t_e and the stresses across the wall thickness are assumed to be uniformly distributed. The determination of the wall thickness is part of the modelling procedure. In addition, it is assumed that aggregate interlock along the crack plane is negligible and that the torsion is equilibrated by a field of compressive stresses in the concrete struts (see shaded diagonal elements, Fig.4.4) and by tensile stresses in the longitudinal and transverse reinforcement.

For the purpose of analysis, there are four unknowns in this model; namely, the compression in the concrete struts, the tension in the longitudinal and transverse reinforcements and the angle of inclination of the struts. Since in a plane problem only three equations of equilibrium are available, this problem is internally statically indeterminate and a fourth equation must be found to obtain a unique solution.

Fig.4.4: Cracked concrete member subjected to torsion

In normal stress analysis, one would appeal to the compatibility requirements to generate the additional equation, but the determination of strains/ deformations requires knowledge of rigidity/ stiffness, which depends on the stiffness of the materials involved. Since reinforced concrete is a nonlinear material, determination of this quantity for the truss model is not a simple task. Nevertheless, the truss model is good for assessing the effect of the various parameters on stresses and deformations of diagonally cracked concrete.

In order to visualize the equilibrium conditions in the space truss in Fig.4.4, let us consider a typical element from the walls of the hollow beam. The free-body diagram of the shaded square element shown in Fig.4.4 is shown in Fig.4.5. The element has thickness t_e and is assumed to be of unit side length. Due to torsion, the element will be

subjected to shear flow q (Fig.4.5 (a)) according to analysis using elementary thin-walled beam theory.

Fig.4.5: a) Direction of shear flow on shear element, b) Truss model of shear element.

Upon cracking and the formation of the diagonal struts, the element resists the applied shear by the mechanisms shown in Fig.4.5 (b). For the purpose of analysis the actual number of the diagonal cracks is not germane. Therefore, if we imagine that the element in Fig.4.5 (a) is intercepted by one diagonal crack, forming an angle θ with the axis of the beam, as indicated in this figure, then we can consider the equilibrium of the lower triangular portion by drawing its free body diagram in Fig.4.6 (a).

Fig.4.6: Free-body diagram of forces in a strut

From the equilibrium of horizontal forces in Fig.4.6 (a),

$$\frac{A_L f_L}{s_L} = q \times 1$$

or

$$f_L = \frac{q s_L}{A_L} \quad (4.12)$$

and from the equilibrium of vertical forces in Fig.4.6 (b), we have:

$$\frac{A_t f_t}{s} = q \tan \theta$$

or

$$f_t = \frac{q s}{A_t} \tan \theta \quad (4.13)$$

Similarly from vertical equilibrium of the wedge in Fig.4.6 (b):

$$f_d = \frac{q}{t_e} \frac{\tan \theta}{\sin^2 \theta}$$

or

$$f_d = \frac{q}{t_e} \left(\frac{1}{\sin \theta \cos \theta} \right) \quad (4.14)$$

We can see from Equations (4.12) to (4.14) that if θ were known, then the steel and concrete stresses in the diagonally cracked concrete can be uniquely determined. By substituting for q (shear flow) from Eq. (4.1) into equations (4.12) to (4.14) and rearranging the terms, we obtain:

$$T = 2A_0 \sqrt{\frac{A_L f_L}{p_0} \cdot \frac{A_t f_t}{s}} \quad (4.15)$$

$$\tan \theta = \sqrt{\frac{A_t f_t}{s} \cdot \frac{p_0}{A_L f_L}} \quad (4.16)$$

where

A_L = the area of one longitudinal steel bar

f_L = the stress in longitudinal steel

s_L = spacing of longitudinal steel.

A_t = the area of one leg of transverse steel bar

f_t = the stress in transverse steel

s = spacing of transverse steel.

and A_0 and p_0 are the effective area and perimeter of the cross-section. Since in each stage after cracking, the definition of the effective area is different, these variables will be introduced properly when describing the appropriate stages.

The two equations (4.15) and (4.16) serve as a basis for both the softened truss model and the compression field theory. They will be used to calculate the resisting torque in the post-crack stages, including the ultimate torque. What distinguishes one stage from the other is the stress in the steel reinforcement and the effective concrete area. It is, however, evident that the above equations are helpful in design, but are not particularly useful for obtaining the torsional stiffness of a diagonally cracked beam due to torsion.

There is another model available to treat the problem of torsion in reinforced concrete. In studying the deformation of a member subjected to torsion, Lampert and Thurliman (1969) observed that the plane walls of the beam before twisting become hyperbolic surfaces after twisting, indicating that the principal compressive strain varies within the depth t_e , the stress being the highest at the exterior surface of each wall. The concrete below the depth t_e is virtually unstressed and does not affect the strength of the section. Hsu (1984) and Mitchell and Collins (1974) have shown that the torsional strength of the core of a solid section is negligible and a solid section can be considered as a hollow section with the same outer dimensions and reinforcement. This concept is used in the present study in order to find the points B and C on the torque-twist diagram in Fig.4.2. In the next sections the derivation of point B, by using truss model, and of point C, by using the compression field theory, is presented.

Fig. 4.7: Equivalent hollow tube and centerline of shear flow

Derivation of the coordinates of point B in Fig. 4.2

Collins (1971) compared Eq. (4.15), defining the torque by the truss analogy, and Eq. (4.2) giving the shear flow in a tube, and the similarity in form as well as in behaviour was found remarkable. This implies that to assess the torsional stiffness reliably the space truss could be replaced by a tube of similar dimensions, having a wall thickness of

$$t_e = \sqrt{\left(\frac{A_t}{s} \cdot \frac{A_L}{p_{02}}\right)} \quad (4.17)$$

By making the approximation that $G=1/2E_s$ and assuming the $A_{02} \approx 0.85A_{0h}$ and $p_{02} \approx 0.90p_h$ (CSA A23.3 1998) and substituting t_e from the Eq. (4.17) into the elastic torsional rigidity expressed by Eq. (4.6), the torsional rigidity of a diagonally cracked reinforced concrete member becomes:

$$GJ_2 = 2E_s \frac{A_{02}^2}{p_{02}} \sqrt{\left(\frac{A_t}{s} \cdot \frac{A_L}{p_{02}}\right)} \quad (4.18)$$

where A_{0h} and P_h are the area and perimeter of the centerline of stirrup, respectively. It must be noted here that the terms A_{02} , area of shear flow, and p_{02} , perimeter of shear flow path, in the CSA A23.3 Code are the same as A_0 and p_0 . To avoid confusion with the code symbols, a new set of symbols is adopted for the effective section perimeter and area in torsion.

The angle of twist of a crack member can be obtained using:

$$\phi_{cr2} = \frac{T_{cr}}{GJ_2} \quad (4.19)$$

where T_{cr} is calculated according to Eq.(4.9). Quantities T_{cr} and ϕ_{cr} define the coordinates of point B on the torque-twist curve in Fig.4.2.

Derivation of the coordinates of point C in Fig.4.2

Since the truss analogy consistently overestimates the torsional strength of a concrete member, the compression field theory will be adopted. The compression field theory, in addition to applying the equilibrium equations of the truss model, considers the continuity of deformations (compatibility equations) of a member and also the constitutive laws for concrete, which is actually subjected to biaxial compression- tension stresses. In the adopted model some simplifying assumptions are made to avoid the usual iterative process required by this theory. In the model the following effects are considered:

- Effective thickness of concrete diagonal stress field
- Spalling of concrete cover
- Concrete softening due to diagonal cracking

The first two phenomena play a significant role in the definition of shear flow path. In other words, they affect the calculation of A_0 and p_0 . The effect of these phenomena and the assumptions, which are made in the present study, will be discussed during the model derivation. It should be noted here that in describing “the compression field theory” only those concepts which are germane to the development of the simplified model in the present study will be discussed. For other aspects of this theory, reference can be made to Collins and Mitchell (1991).

1. Effective thickness of diagonal field

At ultimate state, both the concrete stresses and their distribution vary from those prevailing immediately after cracking. To be able to determine the effective wall thickness at ultimate state, we will first determine the portion of the wall thickness subjected to a uniform stress of $\alpha_1 f_{2\max}$. This equivalent thickness is denoted as a_0 , following the concept of a rectangular stress block in bending. Thus a_0 is related to t_e in the same way as the depth of equivalent rectangular block is related to c , the depth of the neutral axis in bending. Once a_0 is known, we can easily determine the effective area at ultimate.

To determine a_0 consider Fig.4.8, where the principal compressive strain is maximum at the surface and decreases with depth. Mitchell and Collins (1974) observed that the change in the compressive strains is almost linear. Knowing the strain

distribution and stress-strain relationship of concrete, the magnitude and position of the resultant force can be defined. Since the actual stresses would be distributed non-uniformly, they can be replaced by an equivalent rectangular stress block of depth a_0 (Fig.4.8).

Fig. 4.8: Equivalent rectangular stress distribution.

Fig.4.8 shows the assumed strain and stress distribution within depth t_e . The rectangular stresses block factors, α_1 and β_1 , can be calculated from the stress-strain curve of the concrete. The stress resultant can be calculated as:

$$f_d t_e = \alpha_1 f_{2max} \cdot a_0 \quad (4.20)$$

where

f_{2max} = peak compressive strength of diagonally cracked concrete.

Re-writing the truss model equations, the following expression can be obtained:

$$\frac{A_L f_L}{p_0} + \frac{A_t f_t}{s} = f_d t_e = \alpha_1 f_{2max} \cdot a_0 \quad (4.21)$$

To relate the term a_0 (effective thickness of diagonal field) to the steel stresses f_L and f_t , an iterative procedure is needed. For calculating the thickness of concrete subjected to the diagonal stress field at ultimate state, Rahal and Collins (1996) used Eq. (4.21) and made some simplifying assumptions to arrive at the following equation for finding a_0

$$a_0 = \beta_1 t_e = \frac{\left[\frac{\rho_{bL}}{\rho_L} \right] A_L f_{yL}}{0.5\alpha_1 f'_c p_c} + \frac{\left[\frac{\rho_{bL}}{\rho_L} \right] A_t f_{yt}}{0.55\alpha_1 f'_c s} \quad (4.22)$$

They stated that $f_{2\max}$ and p_0 can be approximated by $0.55f'_c$ and $0.9p_c$, respectively. Since the force in the reinforcement is limited to the yield force, the ratios in the brackets are limited to 1. In Eq. (4.22), ρ represents the reinforcement (ratio), and subscripts L, t, and b refer to longitudinal, transverse, and balanced, respectively.

$$\rho_t = 100 \frac{A_t p_h}{A_c s} \quad (4.23)$$

$$\rho_L = 100 \frac{A_L}{A_c} \quad (4.24)$$

$$\rho_{tb} = \frac{5340f'_c}{f_{yt}(f_{yt} + 193)} \text{ (in MPa)} \quad (\rho_{tb} = \frac{780,000f'_c}{f_{yt}(f_{yt} + 28,000)} \text{ in psi}) \quad (4.25)$$

$$\rho_{Lb} = \frac{5100f'_c}{f_{yL}(f_{yL} + 193)} \text{ (in MPa)} \quad (\rho_{Lb} = \frac{740,000f'_c}{f_{yL}(f_{yL} + 28,000)} \text{ in psi}) \quad (4.26)$$

where

f'_c = compressive strength of concrete

f_{yt} = yield stress of transverse reinforcement

f_{yL} = yield stress of longitudinal reinforcement

p_h = perimeter of closed stirrups

A_c = gross area of concrete.

The depth $a_0/2$, acts as the centerline of the resulting tube subjected to uniform stress and defines the shear flow path. Therefore, the effective area of the cross-section can be calculated, using $a_0/2$.

2. *Spalling of concrete cover*

At the corner elements of a beam under torsion, the compression in the concrete tends to push-off the corner while the tension in the stirrups holds it on (Fig.4.9). As the stirrups are unable to embrace the concrete outside the stirrups, at high level of loading these tensile stresses exceed the tensile strength of concrete and the concrete splits. Splitting usually takes place along the weak plane and the effect of bond deterioration is significant (Mitchell and Collins, 1974). This phenomenon causes reduction in the outer dimensions of the section and reduces them to the centerline of the stirrups. This reduction in A_0 results in lower torsional capacity of the section.

Fig. 4.9: Spalling of concrete cover

Mitchell and Collins (1974) and Nagataki et al (1988) conducted a series of tests and they attained very similar results showing that increase in the thickness of concrete cover causes spalling and reduction in torsional strength. Because the larger the cover and the higher the level of load, the larger compressive force and the higher the potential for spalling, Rahal and Collins (1996) suggested that spalling takes place if the thickness of the concrete cover exceeds 30 percent of A_c / p_c .

$$\text{cover} \geq 0.3 \frac{A_c}{p_c} \quad (4.27)$$

If Eq. (4.27) is satisfied, then A_c and p_c should be replaced by A_{oh} and p_h , respectively. Since there is inadequate experimental data available with regard to spalling stresses and other spall-induced conditions, we adopt here Rahal and Collins (1996) suggestion but recognize that further research is needed to fully assess the spalling problem due to torsion. In fact, this aspect is optional in the program and entirely depends on user's choice.

Constitutive laws of concrete

The compressive stress-strain relationship of concrete is usually defined based on the response of a standard cylinder. This relationship can be approximated by a second or higher order parabola. In formulating a stress-strain relationship for the cracked concrete in the web of a beam subjected to torsion, it must be noted that the diagonal compressive struts which carry the compression force are also subjected to large tensile strain orthogonal to the direction of compression. This condition does not exist in a cylinder test after the formation of cracks. Due to orthogonal tensile strain, concrete is weaker and

softer than the concrete in the cylinder. This phenomenon is referred to as “tension softening” of concrete compressive strength.

Vecchio and Collins (1982) proposed a relationship for the principal compressive stress in concrete, which is not only a function of compressive strain, but also of the coexisting principal tensile strain. They suggested:

$$f_2 = f_{2 \max} \left[2 \left(\frac{\varepsilon_2}{\varepsilon'_c} \right) - \left(\frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right] \quad (4.28)$$

where

$$f_{2 \max} = S f'_c \quad (4.29)$$

$$S = \frac{1}{0.8 + 170 \varepsilon_1} \leq 1.0 \quad (4.30)$$

where S depends on the principal tensile strain ε_1 , which is a function of the level of load. Since in this thesis the strain due to torsion is not directly calculated, Eq. (4.30) cannot be used to find S . To overcome this problem, the expression suggested by Rahal and Collins (1996) will be used. According to their investigation, S is related to the total amount of reinforcement in a section, as follows:

$$S = 0.7 - 0.1 R_r \geq 0.40 \quad (4.31)$$

The term R_r is an indicator of the amount of reinforcement in the section and is given by:

$$R_r = \frac{\rho_{tb}}{\rho_t} + \frac{\rho_{Lb}}{\rho_L} \quad (4.32)$$

The stress-strain relationship of concrete is shown in Fig.4.10. It is assumed that both concrete maximum strength and the corresponding strain are reduced by factor S . Hence,

$$\epsilon_{2\max} = 1.5(S\epsilon'_c) \quad (4.33)$$

Rahal and Collins (1996) have suggested that the concrete strain at failure is approximately 1.5 the strain at its peak concrete stress.

Fig.4.10: Stress-strain relationship for concrete in diagonal direction

Compatibility equations

As mentioned earlier, the thin-walled tube walls undergo bending due to torsion. This fact can relate the angle of inclination of the struts, θ , to the angle of the twist, ϕ , and the curvature of the diagonal chords, ϕ . Mitchell and Collins (1974) have shown that

$$\phi = \phi \sin 2\theta \quad (4.34)$$

As shown in Fig.4.8, the curvature ϕ can also be related to the surface strain, $\epsilon_{2\max}$, by following equation:

$$\phi = \frac{\epsilon_{2\max}}{t_e} \quad (4.35)$$

Therefore:

$$\phi = \frac{\epsilon_{2\max}}{\sin 2\theta \cdot t_e} \quad (4.36)$$

Eq. (4.36) gives the twist at ultimate torque and since the ultimate torque can be easily calculated using standard equations of the Code (CSA A23.3, 1994), the coordinates of point C on the torque-twist curve are now easily determined.

So far, all of the relationships needed to predict the response of a member at ultimate level (point C) subjected to pure torsion have been discussed. In summary, to calculate the full torque-twist curve, we follow the following steps:

1. Calculate a_0 by using Eq. (4.20)
2. Calculate A_{03} , the area enclosed by the centerline of effective wall thickness, a_0 , and p_{03} , the perimeter of the area, A_{03} .

3. Calculate the torque T and inclination θ using equations (4.15) and (4.16), respectively, using A_{03} and p_{03} and assuming that both longitudinal and transverse steel have yielded before ultimate strength is reached.
4. Calculate $\varepsilon_{2\max}$ using Eq. (4.33)
5. Calculate the ultimate twist using Eq. (4.36)
6. Having calculated the torque and twist at points B and C, the torsional stiffness GJ_3 of the system after cracking will be obtained as:

$$GJ_3 = \frac{T_C - T_B}{\phi_C - \phi_B} \quad (4.37)$$

Fig.4.11: Existing model for torque-twist relationship without applied modifications

By comparing the results of the proposed model and of other available models with experimental results, their limitations may be realized. Although they can predict the response of a member at some loading levels precisely, such as cracking and ultimate load, they fail to predict the complete response over the entire loading range. It can be observed that in order to predict the response of a member more accurately, some modifications are needed in the proposed model. Fig.4.11 shows the region A-B on the torque-twist curve which needs further deliberation. In the following, suggestions are made for how to ameliorate the situation.

Fig.4.12: Torque-twist curve of a reinforced concrete T-beam tested by Mitchell and Collins (1974)

Before torsional cracking, the load is mainly carried by plain concrete and the effect of reinforcement is negligible, while after cracking, it is generally assumed that only the longitudinal and transverse steel control the stiffness (Collins and Mitchell, 1991). If the concrete contribution to the torsional stiffness is neglected after cracking, a drastic loss in torsional stiffness occurs as illustrated in Fig.4.11. However, comparisons of test results with the existing models predictions do not support the assumption that concrete contribution is negligible.

In order to address this shortcoming of the proposed model, in the present study the concept of “tension-stiffening” is applied to the torque-twist response. As mentioned earlier in Chapter three, the concrete between cracks in reinforced concrete members does not completely lose its stiffness upon cracking because some stress is transferred to it by bond action. Therefore, from the practical point of view, a member does not reach full cracking upon the occurrence of the first crack. The reinforcement ratio, bond slip, crack width, load level and aggregate interlock, among other factors, affect tension-stiffening and post-crack stiffness.

Essentially, there are two ways to model tension-stiffening: changing the concrete properties or modifying the steel stress-strain behaviour. Since twist is largely governed by elongation of the reinforcing bars in the section, the method adopted in the present study is based on the transverse steel reinforcement ratio in the section. As mentioned before, the applied torque must be carried by compression struts of concrete and by longitudinal and transverse steel ties. The two components of the diagonal compression force acting on each wall of a hollow section are balanced by tensile forces in the longitudinal and the transverse steel. Therefore, the forces in the longitudinal and the

transverse steel are not completely independent. The latter is recognized by the equations offered by the CSA A23.3 Code (1998) and by Hsu (1997) when calculating the amount of transverse and longitudinal steel for torsion. Based on these expressions, the amount of longitudinal steel is directly dependent on the amount of transverse steel and is also limited by it, therefore, the ratio of the transverse steel will be adopted as an indicator of a section torque-twist response after cracking.

The amount of transverse reinforcement in a member is limited by maximum and minimum permissible values in the CSA A23.3 Code. It is here assumed that the larger the ratio of transverse steel, the smaller the increase in twist will be immediately after cracking. For convenience, we assume that a member with minimum or less than minimum transverse reinforcement will experience the maximum jump in twist, while one with the maximum transverse steel will experience no jump, albeit its stiffness will still change. For members with reinforcement ratios between the preceding limits, we will calculate the extent of the jump (twist at point D in Fig. 4.13) by linear interpolation as follows:

$$\varphi_{cr3} = (\varphi_{cr1} - \varphi_{cr2}) \left(\frac{\rho_t - \rho_{tmin}}{\rho_{tmax} - \rho_{tmin}} \right) + \varphi_{cr2} \quad (4.38)$$

where $\rho_t, \rho_{tmin}, \rho_{tmax}$ represent the ratio of transverse steel and subscripts t, min and max refer to the actual, minimum, and maximum, respectively. Generally, the minimum reinforcement in a concrete section is provided to avoid a brittle torsional failure. Therefore, by equating the post-cracking strength to the cracking strength of a section, the minimum amount of longitudinal and transverse steel is calculable. By using this theory and the space truss model, Hsu (1997) derived the following simplified equations for solid and hollow sections:

$$A_{tmin} = \frac{2.5\sqrt{f'_c}A_g s}{f_{yt}p_h} \quad (4.39)$$

$$A_{Lmin} = \frac{5\sqrt{f'_c}A_g}{f_{yL}} - \left(\frac{A_{tmin}}{s}\right)p_h\left(\frac{f_{yt}}{f_{yL}}\right) \quad (4.40)$$

where A_g is the entire cross-sectional area and in the case of hollow section, does not include the area of the hole, and A_{tmin} and A_{Lmin} are the minimum area of transverse and longitudinal steel, respectively. To satisfy the value of A_{Lmin} , the transverse steel area per unit length, A_{tmin}/s , needs to be less than $25b_w/f_{yt}$.

Based on the ultimate torsional strength of section, the maximum amount of transverse steel can be calculated using equations (4.13) and (4.14):

$$A_{tmax} = \frac{f_{2max}t_e s \sin^2 \theta}{f_{yt}} \quad (4.41)$$

in which the angle θ and the maximum compression in the strut f_{2max} can be obtained using equations (4.16) and (4.29), respectively. The ratio of minimum and maximum transverse steel can be calculated using Eq. (4.23).

Once the extent of increase in twist at cracking has been established, the torsional stiffness (the slope of line DC in Fig.4.13) for a cracked member can be calculated as follows:

$$GJ_3 = \frac{T_u - T_{cr}}{\phi_u - \phi_{cr3}} \quad (4.42)$$

Fig.4.13: Modified torque-twist relationship.

Comparison of theoretical and experimental rigidities of torsionally cracked member in Fig 4.12 reveals that the slope of the actual torque-twist curves after cracking is much smaller than its theoretical prediction. This observation points to the need for further modification of the torque-twist relationship. As shown in Fig. 4.13 the proposed rigidity, GJ_3 , is smaller than the rigidity, GJ_2 , which is based on existing expressions for torsionally cracked members.

The next point to be considered relates to the actual angle of twist at failure (twist at point E in Fig. 4.13). From the experimentally measured torque-twist curve of a standard section, (see Fig. 4.1), it can be seen that the section does not fail right after reaching its ultimate torque but instead it follows a descending path up to failure.

Between the points of peak torque and failure, the resisting torque decreases while the angle of twist concurrently increases. This is similar to the phenomenon observed in a moment-curvature relationship of a section under bending. This phenomenon is related to the ductility of a section, or in other words to the amount of reinforcement in a section. In the case of torsion, the ratios of longitudinal and transverse steel have a significant role on this behaviour.

Since the ductility of concrete sections subjected to torsion, has not been extensively studied in the past, insufficient information is available for precisely determining it. Using different test results, a simple method is proposed in the present study. Based on empirical observations, the ultimate twist value is almost two or three times greater than the twist corresponded to the peak torque (see Fig.4.1). Therefore, the $\phi_{ur} = 3\phi_u$ is adopted as a default value for ultimate twist in this study, but this value can be changed if a better value is known. In fact, the length of line CE (Fig.4.13) is not very important, because it was assumed that both longitudinal and transverse steel has yielded at this stage and the concrete member is not able to resist any more torque, therefore, it experiences increased twist at constant torque and fails due to large displacement. To avoid any problem in the numerical procedure in the computer program, because of zero stiffness of line CE, a small percentage of post-cracking torsional stiffness is used as the slope of line CE.

As illustrated in Fig.4.13, right after cracking a vertical drop in the resisting torque occurs. This drop presents the actual behaviour of a concrete member right after the first cracking. To observe this phenomenon in tests the structure must be loaded in displacement control and the displacement increments must be small so the data

acquisition system can capture it. This phenomenon also occurs for a concrete member subjected to other types of loads, i.e. bending, shear and torsion. In this study, the post-cracking stiffness, GJ_3 , and the cracking angle of twist, ϕ_{cr3} , define the extent of the drop (torsional moment at point F in Fig. 4.13).

In the present torque-twist model, a simple load reversal path is defined because study of the behaviour of the concrete member under cyclic torque is out of the scope of this thesis. To enable indeterminate structures to redistribute the moments at the joints, the simple load reversal model in Fig. 4.14 is adopted.

Fig.4.14: Adopted load reversal path for torque-twist model

4.3 Ultimate strength of reinforced concrete sections

4.3.1 General

To predict the ultimate capacity of a reinforced concrete section under combined actions, the usual procedure is to determine the capacity of the same section under each action applied singly.

As mentioned before, program PCF3D like many other frame analysis programs, focuses on bending and axial effects, thus the program cannot predict shear or torsion failure. Since the interactions of different actions in a member and the prediction of the ultimate capacity of a member subjected to these actions is one purpose of this study, the procedure used for the ultimate strength calculation will be explained in this section. The method used to calculate the ultimate torsional capacity of a member was presented in Section 4.2 of this chapter.

4.3.2 Ultimate axial capacity of a section

The most common structural member which is subjected to axial force is a column. In real concrete structures columns subjected to pure axial load rarely exists. However, it can be assumed that axially loaded members are those with a relatively small axial load eccentricity, e , of about $0.1h$ or less, where h is the total depth of the column and e is the eccentric distance from the centroid of the section.

The ultimate load of a column under compression does not vary significantly with the history of loading. For under-reinforced sections, when the load is increased, the steel

will normally reach the yield strength before the concrete reaches its full strength. After that, concrete can carry further load because the steel sustains the yield stress while the deformations and the load increase until concrete reaches its full strength. In a over-reinforced section, however if the concrete reaches its strength before the yielding of steel, the increase deformation of the concrete, when near its peak stress, allows the steel to reach yielding. It can be concluded from the above that the ultimate capacity of a member under compression is governed by both concrete and steel. The ultimate strength of concrete in a column is affected by several factors such as the specimen shape and size, and the variations in the construction of an actual column compared with a test column. Accordingly, both the CSA A23.3 (1998) and the ACI Code (2002) recommended the ultimate concrete strength in a column to be $\alpha_1 f'_c$, where f'_c is the standard cylinder strength of the same column. Accordingly, the ultimate strength, P_{oc} , of a concentrically load reinforced concrete column is given by:

$$P_{oc} = \alpha_1 f'_c (A_g - A_s) + f_y A_s \quad (4.43)$$

where

A_g = the gross area of the cross-section

A_s = the total area of longitudinal steel

f_y = the yield strength of steel

α_1 = equivalent rectangular stress block parameter (This will be elaborated on in the following section)

To predict the ultimate strength of a member under tension, it can be assumed that the entire tension force is resisted by steel. Because the tensile strength of concrete is

about one-tenth of its compressive strength, even considering the tension stiffening, concrete will still reach its tensile strength very quickly and cracks will develop across the entire section. Therefore, we can obtain the ultimate strength under tension, P_{0T} , as follows:

$$P_{0T} = A_s f_y \quad (4.44)$$

Another important phenomenon which affects the strength of columns under axial load is the amount of confining, or transverse steel. It is well-known that transverse steel in the form of spiral significantly increase both the ductility and strength of circular columns (Park and Paulay, 1973). The increase due to rectilinear transverse reinforcement is still appreciable, but not as significant. In this study, the effect of confinement of concrete is not included, but it is recommended that this effect be included in the future modifications of the current frame analysis program.

4.3.3 Ultimate bending strength

Use of the equivalent stress block is a well-known procedure for calculating the ultimate strength of a member subjected to uniaxial bending. This method is adopted by CSA A23.3 (1998) and ACI Code (2002), which was first proposed by Whitney (1937) and checked by Mattock et al (1961). This method is very well-established and it can be found in any standard concrete design book, it will be only briefly described here.

A concrete stress $\alpha_1 f'_c$ is assumed to be uniformly distributed over an equivalent compression zone bounded by the edges of the cross-section and the line parallel to the

neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain, where c is the distance between the top of the compressive section and the neutral axis.

Fig.4.15: Equivalent distribution at failure.

The coefficient α_1 and β_1 are given by the CSA 23.3 as follows:

$$\alpha_1 = 0.85 - 0.0015f'_c \geq 0.67$$

$$\beta_1 = 0.97 - 0.0025f'_c \geq 0.67$$

where f'_c is the specified compressive strength of concrete in MPa. Other standards such as the ACI Code (2002) assumes $\alpha_1 = 0.85$ and $\beta_1 = 0.85$ for concrete strengths ≤ 4000 psi, but is then reduced linearly at a rate of 0.05 for each 1000 psi of stress greater than 4000 psi, with a minimum value of 0.65. The ultimate bending moment is equal to

$$M_u = C_b \left(d - \frac{a}{2}\right) = T_b \left(d - \frac{a}{2}\right)$$

The calculation for finding the neutral axis is performed by the program 3DRCF-CL using an iterative procedure. The program starts with a depth of the neutral axis and

iterates until the equilibrium condition $C_b = T_b$ is satisfied. The program is able to consider any arbitrary cross-section, and for each local direction, the negative and positive moment capacities are calculated.

4.3.4 Ultimate shear capacity of a section

Braestrup (1974) developed a method to predict the shear capacity of a section in terms of the concrete strength and the amount of reinforcing steel in a section. This method is based on the mathematical theory of plasticity, assuming that cracks in concrete may transmit shear stresses. This method is adopted in the present study and according to this method the ultimate shear stress v_u is given in four distinct categories based on the level of reinforcement in the section:

$$v_u = f'_c \sqrt{\omega_L \omega_t} \quad \text{for } \omega_L \omega_t \leq S' \quad (4.45a)$$

$$v_u = f'_c \sqrt{\omega_t (S' - \omega_t)} \quad \text{for } \omega_L > S' - \omega_t \text{ and } \omega_t \leq \frac{S'}{2} \quad (4.44b)$$

$$v_u = f'_c \sqrt{\omega_L (S' - \omega_L)} \quad \text{for } \omega_t > S' - \omega_L \text{ and } \omega_L \leq \frac{S'}{2} \quad (4.44c)$$

$$v_u = f'_c \frac{S'}{2} \quad \text{for } \omega_t > \frac{S'}{2} \text{ and } \omega_L \leq \frac{S'}{2} \quad (4.44d)$$

where ω_L, ω_t are two non-dimensional indices defined as:

$$\omega_L = \frac{\rho_L f_{yL}}{f'_c} \quad (4.46)$$

$$\omega_t = \frac{\rho_t f_{yt}}{f'_c} \quad (4.47)$$

and S' is the concrete effectiveness factor, ρ and f_y are the reinforcement ratio and yield

stress, respectively and the subscript L and t refer to the longitudinal and transverse reinforcement. To calculate the parameter S' , Braestrup used Nielsen et al (1978) simple empirical formula

$$S' = 0.8 - f_{ck} / 200 > 0.4 \quad (4.48)$$

where f_{ck} is the characteristic concrete cylinder strength in MPa which is akin to f'_c in North American practice. The Danish code 1999 (Braestrup, 2000) suggested a lower limit of 0.4 for S' and Nielsen (1994) confirmed the value 0.4 as a reasonable estimate for high-strength concrete.

These equations predict the four different failure modes. Eq. (4.44a) is valid for under-reinforced members where failure occurs by yielding of both reinforcements. Equations (4.44b) and (4.44c) correspond to the member which is over-reinforced in one direction and failure occurs because of yielding and crushing in the concrete. Eq. (4.44d) is valid for the member over-reinforced in both directions which would lead to the crushing of concrete before any yielding.

Braestrup (2000) compared his method with a new method that was developed by Rahal (2000). Rahal's method predicts ultimate shear strength of a member based on a simplified and non-iterative "modified compression field theory." The comparison of the results of the two methods revealed good agreement. Braestrup's method is more suitable for computer programming because Rahal presented his method in the form of figures or graphs which is simple for manual calculation but not for programming.

A member is assumed to have reached its pure shear capacity if at any load step

$$V \geq v_u A_w \quad (4.49)$$

where A_w is the effective concrete area (normally effective depth $\times$ width) resisting shear.

4.4 Combined action of bending, shear, torsion and axial force

4.4.1 General

The behaviour of reinforced concrete members under different types of isolated actions was studied. In this section the behaviour of reinforced concrete members under combined actions will be analyzed. To illustrate the accuracy of the adopted method, the next section will be dedicated to describing the nonlinear behaviour of concrete members under combined actions and to the way that the program deals with this issue.

4.4.2 Nonlinear behaviour of a member under combined action

Study of the interaction of different structural actions has been of some interest to researchers and design code developers because it has an important impact on the safety of structures. However, the extent of these studies have been limited, insofar as the nonlinear behaviour of concrete structures is concerned. As mentioned before, in case of combined actions, there are two different approaches to check the response of a structure. The first approach is mainly based on plate and shell finite element analyses and it deals with stresses and strains and the constitutive laws of concrete and reinforcement. Although this approach yields accurate results, it is often complicated and extremely time-consuming. For design purposes, this approach is rarely applied. The second approach deals with stress-resultants, such as axial force, moment, etc., and the deformations corresponding to these actions. This approach is less time-consuming and can be applied to large structures. The latter approach utilizes the force-deformation

response of a section to obtain the various rigidities or stiffnesses of a member. At the same time, the failure criteria are expressed in terms of interaction equations or diagram.

Hence the accuracy of the results of such an analysis depends largely on the accuracy of the adopted force-deformation curves (e.g. moment-curvature, torque-twist, etc.) and of the interaction-equations. Eq. (4.50) represents a typical interaction equation involving axial force, P , shear force, V and moment, M . Similar equations exist for other combinations of internal forces (Hsu,1993). The numerators in this equation represent the actual values of each action acting on a section while each denominator denotes the ultimate capacity of the same section under that action alone. The ultimate capacity of a section is entirely dependent on the cross-section geometry, reinforcement and material properties.

$$\left(\frac{P}{P_U}\right)^\alpha + \left(\frac{V}{V_U}\right)^\beta + \left(\frac{M}{M_U}\right)^\gamma = A \quad (4.50)$$

The key to the development of an accurate interaction equation is the determination of the appropriate values of the exponents α, β and γ and the parameter A . Various empirical interaction formulas have been proposed in the literature (see Section 4.2.4 of this chapter); however, their limitations have not been theoretically established.

The program PCF3D is capable of predicting the failure of a member subjected to pure axial force/bending or their combined action by checking the allowable strains and stresses at each step of loading. But the program suffers from lack of shear and torsion failure criteria as well as their combination with axial loads. For instance, if a

concentrated load is applied laterally to a short beam, the predicted failure is flexural not shear even though in reality such a beam is more likely to fail in shear.

In current design practice, the interaction between shear and torsion is considered as well as the interaction between biaxial moments and axial load, but interaction among all these effects acting concurrently is neglected. The calculation of the ultimate capacity of a section under isolated actions is straightforward and well-established. Similarly, if linear-elastic behaviour is assumed, the value of the shear, moment, axial load, etc. acting on a section can be easily determined by established methods of structural analysis. Once these qualities are known, the interaction equation can be applied to check the safety of a section at ultimate load.

In this method, there are two fundamental points that deserve some discussion. First, determination of the exponents of the interaction equation is not theoretically simple and hence existing empirical values need further scrutiny. Secondly, the assumption that sectional forces can be obtained from elastic analysis is incorrect in the case of statically indeterminate structures experiencing nonlinear behaviour and stress redistribution.

Since due to material nonlinearity the stiffness of members and joints changes, the change leads to redistribution of internal forces and the redistributed force values differ from their corresponding values based on elastic analysis. This difference brings into question the utility of the interaction equations.

In this study we examine the behaviour and strength of reinforced concrete frame structures subjected to combined actions. The objective is not to introduce new

interaction equations, but rather to replace the internal forces based an elastic analysis by those obtained from inelastic analysis using the computer program 3DRCF-CL.

4.4.3 Combined stresses

Fig.4.16 depicts a reinforced concrete member subjected to axial load, P , bending, M , shear, V , and torsion, T . The interaction of these forces contributes to the yielding of the longitudinal and the transverse steel. In this section, the effect of the interaction of forces on the yielding of the longitudinal and transverse steel will be discussed.

Fig.4.16: Combined action on reinforced concrete element

Longitudinal steel

In general the longitudinal bars in a section are partially resisting all the actions. The axial force and bending cause direct tension/compression in the longitudinal bar

(Fig.4.17). According to the space truss model, torsion and shear cause compression force in the diagonal concrete struts and the horizontal component of the compression force in the struts is balanced by the tension force in the longitudinal bars. Fig.4.17 illustrates the effect of several actions on stresses in longitudinal bars.

Fig.4.17: Equilibrium for axial force-moment-shear-torsion interaction.

In Fig.4.17, V_t represents the total shear in one wall caused by shear and torsion. The shear flows due to shear and torsion are additive on one side and subtractive on the other side of the web because the torsional shear flow circulates around the perimeter of the section. Therefore, it is wise to consider the combined shear stress for checking failure. According to Fig.4.17, the total longitudinal strain at the level of the flexural tension reinforcement due to all actions can be approximately by:

$$\epsilon_x = \frac{\frac{M}{d_v} + \frac{P}{2} + \frac{V_t}{2} \cot \theta}{E_s A_s} \quad (4.51)$$

Program PCF3D calculates the first two terms in the numerator of Eq. (4.51), which relate to bending and axial force only. The additional strain in the longitudinal bars caused by torsion and shear can be expressed by the following equation based on the space truss model:

$$\epsilon_{x_{s,v}} = \frac{\cot \theta (V + \frac{Tp_0}{2A_0})}{E_s A_s} \quad (4.52)$$

For design purpose, the CEB-FIP Code (1978) requires that the transverse and longitudinal steel due to torsion and shear should be simply added together. However, this provision is very conservative because the shear flow caused by shear and torsion, as mentioned earlier, is additive on one side of the web and subtractive on the other. Collins and Mitchell (1980) observed that the Eq. (4.52) is accurate for hollow sections but is somewhat conservative for solid sections, where significant redistribution of the stresses is possible. Therefore, they proposed Eq. (4.53) as a more appropriate expression valid for any kind of section. The tensile strain due to shear and torsion was written as:

$$\epsilon_{x_{s,v}} = \frac{\cot \theta \sqrt{V^2 + (\frac{Tp_0}{2A_0})^2}}{E_s A_s} \quad (4.53)$$

Therefore, a general expression for strain in the longitudinal steel may be written as follows:

$$\varepsilon_{xt} = \varepsilon_{xa,b} + \frac{\cot \theta \sqrt{V^2 + \left(\frac{Tp_{03}}{2A_{03}}\right)^2}}{E_{sl}A_L} \quad (4.54)$$

The strain $\varepsilon_{xa,b}$ is due to axial force and biaxial bending which is calculated using:

$$\varepsilon_{xa,b} = \varepsilon_a - z\kappa_y - y\kappa_z \quad (4.55)$$

where ε_a is the strain due to axial force, κ_y and κ_z are the section curvatures about the y and z axes and y and z are the coordinates of the centroid of the reinforcing bar of interest in the cross-section. The shear and torsion values in Fig.4.17 and Eq. (4.54) can be obtained from the nonlinear analysis results for any load level. The quantities A_L and E_{sl} express the total amount of longitudinal steel and the modulus of elasticity of steel, respectively. The plasticity truss model is based on the assumption that both the longitudinal and the transverse steel must yield before failure. According to this theory Hsu (1993) suggested a method in order to estimate the angle of inclination θ . He divided the elements into two types, under-reinforced and over-reinforced and the state of stresses which differentiated the under-reinforced mode of failure from the over-reinforced was called the balance condition. At the balance condition, both steels yield, whereas the concrete crushes at an effective stress of Sf'_c . The criteria for these conditions are given in Table 4.1.

Table 4.1: Various states of an element based on plasticity truss model

Section Type	Condition	Inclined Strut Angle
Underreinforced element	$\omega_L + \omega_t < 1$	$\tan \theta = \sqrt{\frac{\omega_t}{\omega_L}}$
Balance Condition $\omega_L + \omega_t = 1$	$\omega_L = \omega_t$	$\theta = 45^\circ$
	$\omega_L < 0.5$	$\tan \theta = \sqrt{\frac{(1 - \omega_L)}{\omega_L}} \quad \theta > 45^\circ$
	$\omega_t < 0.5$	$\tan \theta = \sqrt{\frac{\omega_t}{(1 - \omega_t)}} \quad \theta < 45^\circ$
Overreinforced element	$\omega_L + \omega_t > 1$	$\theta = 45^\circ^*$

* If the steel does not yield, then it is reasonable to assume that the diagonal cracks will not rotate from their initial direction.

where

$$\omega_L = \frac{\rho_L f_{yL}}{S f'_c} = \text{longitudinal reinforcement index}$$

$$\omega_t = \frac{\rho_t f_{yt}}{S f'_c} = \text{transverse reinforcement index}$$

The calculated angle of inclination is limited to $3/5 < \tan \theta < 5/3$ based on the CEB-FIP Code (1978). The effectiveness factor S was given in Section 4.2.3.

Transverse steel

According to the truss analogy the stirrups play a significant role in maintaining the structural integrity of the truss in transverse direction while it is subjected to the shear and torsion. Under bending and axial force the stirrups do not contribute to transferring the loads and are mostly needed to maintain the position of the longitudinal bars (note

that confinement effect of stirrups is not the subject of this investigation). The vertical component of the compressive force in the concrete struts needs to be balanced by the tension force in the stirrups as indicated in Fig.4.18.

Fig.4.18: Shear element subjected to shear-torsion interaction

In the above figure V_t is the total amount of shear caused by torsion and shear in a stirrup. Eq. (4.56) gives the total amount of force in a stirrup resisting torsion.

$$\frac{A_t f_t}{s} = q \tan \theta = \frac{T}{2A_{03}} \tan \theta \quad (4.56)$$

By considering the force polygon in Fig.4.18, the total force in one leg of a stirrup is $V_t \tan \theta$ and the corresponding strain can be expressed as

$$\epsilon_t = \frac{\tan \theta \sqrt{\left(\frac{1}{n} \frac{V}{d_v} s\right)^2 + \left(\frac{T s}{2A_{03}}\right)^2}}{E_{st} A_t} \quad (4.57)$$

where n is the number of legs of stirrups in the cross-section. In fact there is not ample data available regarding the total measured strain due to shear and torsion in members

subjected to combined action. Therefore, to evaluate the equivalent strain in stirrups, here the same assumptions as those made by Collins and Mitchell (1980) for longitudinal steel are made, and the total strain, ϵ_t , in the stirrup is expressed per Eq. (4.57).

In the present nonlinear analysis the total strain in each longitudinal bar and each stirrup is evaluated by using equations (4.54) and (4.57) respectively, and if either longitudinal strain, ϵ_x , or transverse strain, ϵ_t , exceeds the yield strain of steel, the following modifications are introduced in the nonlinear analysis procedure:

1- Changing stress-strain curve of steel

In the case of yielding of longitudinal steel, the stress-strain relationship of steel is modified as indicated in Fig.4.19. The modified curve has the same basic shape as the original stress-strain curve of the steel, but it has a lower yield stress and strain. The new yield strain, ϵ_{yM} in Fig.4.19, corresponds to the longitudinal strain caused by the current axial load and bending only. Once ϵ_{yM} is known, the new yield stress is given by $f_{yM} = E_{sl}\epsilon_{yM}$. It must be mentioned that the strain in each longitudinal bar within a section is separately evaluated and that each may yield at different load level. Based on this modification, the yield stress is adjusted, which affects the magnitude of displacements, axial force, bending and shear forces.

In the case of yielding of stirrups, the same procedure is applied and the stress-strain curve of transverse steel and its yield strain are altered.

Fig.4.19: Modified stress-strain curve for yielded steel under combined action

2- Changing torque-twist curve

The second modification is applied to the torsional stiffness of the member. With the new yield stress, either due to the transverse or longitudinal steel, a new GJ is calculated using the truss model. The effective amount of longitudinal or transverse steel corresponding to the new yield stress can be calculated using the truss model, i.e.

$$q = \frac{A_t f_{yt}}{s} \cot \theta = \frac{A_L f_{yL}}{p_{03}} \tan \theta \quad (4.58)$$

therefore:

$$A_{tn} = \frac{A_L f_{yLn} s \tan^2 \theta}{p_{03} f_{yt}} \quad (\text{if yielding occurs in longitudinal steel}) \quad (4.59)$$

or

$$A_{Ln} = \frac{A_t f_{yt} p_{03} \cot^2 \theta}{s f_{yL}} \quad (\text{if yielding occurs in transverse steel}) \quad (4.60)$$

where f_{yLn} and f_{ytn} are the new yield stress of longitudinal and transverse steel, respectively and A_{Ln} and A_{tn} are the modified steel areas. The modified areas will be less than the original area, hence when substituted in the torsional stiffness equation; they would cause reduction in the stiffness. The new or reduced torsional stiffness can be calculated using

$$GJ_4 = 2E_s \frac{A_{03}^2}{p_{03}} \sqrt{\left(\frac{A_t}{s} \frac{A_{Ln}}{p_{03}}\right)} \quad \text{or} \quad GJ_4 = 2E_s \frac{A_{03}^2}{p_{03}} \sqrt{\left(\frac{A_{tn}}{s} \frac{A_L}{p_{02}}\right)} \quad (4.61)$$

In order to define the point C' on the torque-twist curve (Fig.4.20), the value of the ultimate twist of the section will be re-calculated using the latest yield stress

$$\phi'_u = \frac{\epsilon'_{2max}}{\sin 2\theta.t_e} \quad (4.62)$$

where ϵ'_{2max} is calculable by using the procedure mentioned in the previous section, and by using the new yield stress for either the longitudinal or the transverse steel and the same θ as that used to check the occurrence of yielding in the steel. The strain corresponded to the yield stress will be defined as yielding twist, ϕ_y , thus the twist at point E' in Fig.4.20, i.e., ϕ'_{uF} , similar to the point E , is calculate as

$$\phi'_{uF} = 3.0\phi_y \quad (4.63)$$

Note that the coefficient 3.0 in the above equation is based on empirical observation and can be changed if a better value can be justified.

The program calculates the total strain in both the longitudinal and the transverse steel at each step of the iterative process. It should be pointed out that the change in the

skew bending theory and space truss analogy. Based on skew bending theory, and using the results of numerous experimental work conducted by researchers such as, Liao and Ferguson (1969), McMullen and Warwaruk (1970), Kirk and Lash (1971), Elfgren (1972), Lampert and Collins (1972), a number of empirical interaction equations considering different combination of bending-shear-torsion were derived. Also extensive work has been performed on this issue by applying the space truss model and the compression field theory, which focus on stress and strain interaction due to these actions. Collins and Mitchell (1980), Rahal and Collins (1995), Rahal (2000) and Recupero et al (2003) have carried out further studies in this field.

In this section, some of the existing interaction equations or failure criteria will be given. The equations can be classified into the three separate groups. These equations are listed below and for further information the reader can see the references cited above.

a. First group

A simple elliptical relation between torsion and bending moment based on the work of Kirk and Lash (1971), and between torsion and shear, based on the work of Liao and Ferguson (1970) is given below:

$$\left(\frac{T}{T_u}\right)^2 + \left(\frac{M_{y,z}}{M_{yu,zu}}\right)^2 \geq 1 \quad (4.64)$$

$$\left(\frac{T}{T_u}\right)^2 + \left(\frac{V_{y,z}}{V_{yu,zu}}\right)^2 \geq 1 \quad (4.65)$$

where

T = torsional strength of member under combined action

V_y, V_z = shear strength of member under combined action in y and z direction
respectively

M_y, M_z = flexural strength of member around y axis and z axis under combined
action, respectively

T_u = ultimate torsional strength of member subjected to pure torsion

V_{yu}, V_{zu} = ultimate shear strength of member subjected to pure shear in the y and z
direction respectively

M_{yu}, M_{zu} = ultimate flexural strength of the member under pure bending
about y axis and z axis, respectively

Since no effort has been previously made to study combined biaxial bending and torsion, or biaxial bending and shear, here in the case of biaxial bending and biaxial shear the moments and shears will be treated separately.

b. Second group

Lampert and Collins (1972) suggested two modes of failure for combined torsion and bending. The first failure mode is caused by the yielding of the bottom stringer (bottom steel) and of the transverse steel, which can be expressed as:

$$r \left[\frac{T}{T_u} \right]^2 + \left[\frac{M_{y,z}}{M_{yu, zu}} \right] \geq 1 \quad (4.66)$$

and the second failure mode occurs due to yielding in the top stringer (top steel) and in the transverse steel and the pertinent equation is given by:

$$\left[\frac{T}{T_u} \right]^2 - \frac{1}{r} \left[\frac{M_{y,z}}{M_{yu,zu}} \right] \geq 1 \quad (4.67)$$

where r is the ratio of the yield force of the top ($A'_s f_y$) to the bottom ($A_s f_y$) longitudinal steel. The parameters in the equations (4.66) and (4.67) are the same as those in equations (4.64) and (4.65).

c. Third group

The equations in this group was proposed by Hsu (1984), which are basically an extension of the equations in group two. Hsu studied the interaction of M - V - T and P - M - V based on the space truss analogy and proposed the following.

M-V-T interaction

The first failure mode is caused by yielding in the bottom stringer and in the transverse reinforcement on the side where shear flow due to shear and torsion are additive. The equation is expressed as:

$$\left[\frac{M_{y,z}}{M_{yu,zu}} \right] + \left[\frac{V_{y,z}}{V_{yu,zu}} \right]^2 r + \left[\frac{T}{T_u} \right]^2 r \geq 1 \quad (4.68)$$

The second failure mode is caused by yielding in the top stringer and in the transverse reinforcement on the side where shear flows due to shear and torsion are additive.

$$-\left[\frac{M_{y,z}}{M_{yu,zu}} \right] \frac{1}{r} + \left[\frac{V_{y,z}}{V_{yu,zu}} \right]^2 + \left[\frac{T}{T_u} \right]^2 \geq 1 \quad (4.69)$$

and the third failure mode is caused by the yielding in the stirrups at top, bottom and on the side where shear flows due to shear and torsion are additive. This mode of failure is independent of the bending M .

$$\left[\frac{V_{y,z}}{V_{yu,zu}} \right]^2 + \left[\frac{T}{T_u} \right]^2 + \left[\frac{V_{y,z} T}{V_{yu,zu} T_u} \right] 2 \sqrt{\frac{2d_v}{P_0}} \geq \frac{(1+r)}{2r} \quad (4.70)$$

P-M-V interaction

The effect of axial tension P on the yield strength of a member can be included easily. The axial tension is resisted only by the longitudinal steel bars. Its affect is only included as an addition stress in both the top and the bottom stringers. Therefore, only the first two modes of failure corresponding to the yielding of longitudinal steel is applicable to the interaction of axial tension, bending and shear. The first mode of failure can be expressed as:

$$\left[\frac{P}{P_u} \right] + \left[\frac{M_{y,z}}{M_{yu,zu}} \right] + \left[\frac{V_{y,z}}{V_{yu,zu}} \right]^2 r \geq 1 \quad (4.71)$$

and the second mode of failure can be defined as:

$$\frac{1}{r} \left[\frac{P}{P_u} \right] - \frac{1}{r} \left[\frac{M_{y,z}}{M_{yu,zu}} \right] + \left[\frac{V_{y,z}}{V_{yu,zu}} \right]^2 \geq 1 \quad (4.72)$$

During nonlinear analysis, when anyone of these interaction equations is satisfied, the concrete frame is assumed to have failed. In order to stop the nonlinear process, the

stiffness of the structure needs to be reduced by dividing it on a large number, such as 400, and the nonlinear process will be terminated based on large deflection criterion. As stated earlier, it is not the objective of the current study to develop new failure criteria for combined action, but rather to use existing criteria within the framework of nonlinear frame analysis.

In summary, we follow the following steps to check the possibility of failure in each load step

1. Calculation of strain in the longitudinal steel due to bending and axial load
2. Calculation of strain in the longitudinal steel and stirrups due to shear and torsion
3. Calculation of total normal strain
4. Checking if the longitudinal or transverse steel exceeded the yield strain
5. In the case of yielding of either the longitudinal or transverse steel, the two modifications described earlier are applied. First, changing the stress-strain curve of the longitudinal or the transverse steel and secondly, changing the torque-twist curve.
6. Checking the interaction equations at the beginning of each load step. If the interaction equations satisfy the failure criteria, the stiffness of the member will be drastically reduced, which then terminates the analysis.

Chapter 5

Numerical Examples

5.1 General

In this chapter the applicability and accuracy of the proposed numerical method is assessed via the solution of several numerical examples and their results are compared with the corresponding experimental data. The examples serve as a means to demonstrate the accuracy and the capability of the computer program developed to predict the nonlinear behaviour of concrete frames under different type of actions, applied singly or in combination. Briefly, the numerical examples are as follows:

- In Section one, the application of the proposed tension stiffening model is tested. One portal frame and one simply supported beam subjected to two concentrated loads are modeled. The model results are compared with the corresponding experimental data, or with results of a more refined analysis.
- In Section two, the accuracy of the proposed method to generate torque-twist relationship for a concrete member will be demonstrated. For this

purpose four different concrete beams with various section sizes, reinforcing and material properties, subjected to pure torsion, are modeled.

- In Section three the behaviour and ultimate capacity of various reinforced concrete beams subjected to combined bending, shear and torsion are studied. The results in this section are intended to demonstrate the validity of the proposed approach for nonlinear analysis under combined actions.

In all these analyses the applied load was increased gradually and in small increments in order to observe the expected changes in the stiffness and deformation of the members. The increment ratio, defined as the ratio of the load increment to the failure load, was selected between 0.001 and 0.0035 during the analysis. A maximum number of 40 iterations was allowed in each load step and in most examples the procedure converges within four iterations.

In the cases that the initial or elastic modulus of concrete could not be obtained from the reported experimental data, it was calculated based on the equation suggested by the ACI Code (2002), i.e. $E_{c0} = 4780\sqrt{f'_c}$ (MPa) or $E_{c0} = 57000\sqrt{f'_c}$ (psi) and the shear modulus, G , was assumed to be $(0.4 - 0.5)E_{c0}$ as suggested by Collins and Mitchell (1991) and Lampert (1986). In general, those values which are highlighted in tables have been calculated based on existing theories due to lack of experimental data.

The computer used to compile the program 3DRCF-CL is a Compaq Presario 2500 carrying an Intel Pentium4, 2.4 GHz processor and has 448 MB memory space. The total time to run the different used examples in this chapter varied between 60 to 90 seconds.

5.2 Tension stiffening examples

It was mentioned earlier in this thesis that program PCF3D, upon which program 3DRCF-CL is based, can analyse frames under axial load and bending rather well. However, the one feature that the program lacked was modelling “tension-stiffening”. Therefore, a tension-stiffening model was introduced here and the following two examples are presented to demonstrate the improvement in response prediction due to the introduction of the proposed “tension-stiffening” model.

5.2.1 Portal frame

A single bay portal frame, used by Van Mier (1987) to demonstrate the accuracy of a well-known three-dimensional finite element program for concrete structures, is analyzed to illustrate the improved accuracy of bending analysis in the current program. Fig.5.1 illustrates the frame dimensions and support conditions. Due to different stirrup spacings, particularly in the joints regions, and different concrete properties, four different cross-sections are modeled in this study. The dimensions of the cross-sections, as well as the concrete and reinforcing details, are given in Table 5.1. Since the members are subjected to uniaxial bending, each cross-section is defined by a grid of one column and 10 rows. It should be pointed out that due to the tension-stiffening effect, the accuracy of the results is affected by the number of layers used to model a section. Since each layer is assumed to be under uniform stress, cracking of an entire layer is assumed to occur at once.

Table 5.1: Cross-section details and material properties of portal frame tested by Van Mier (1987)

Cross section type	Cross-section dimension(mm)		Cover (mm)	Longitudinal steel (mm)		Stirrups (mm)	Material properties (N/mm ²)			
	Width b	Height h		Top	Bottom		f'_c	f_t	f_{yL}	f_{yT}
C ₁	70	120	5	2 ϕ 6	2 ϕ 6	ϕ 2.8@20	37.5	3.5	450	360
C ₂	70	120	5	2 ϕ 6	2 ϕ 6	ϕ 2.8@30	25	2.2	450	360
C ₃	70	120	5	2 ϕ 6	2 ϕ 6	ϕ 2.8@50	25	2.2	450	360
C ₄	70	120	5	2 ϕ 6	2 ϕ 6	ϕ 2.8@104	25	2.2	450	360

Fig.5.1: Portal frame dimensions and elements arrangement

To prevent premature crack propagation, the thinner the layers, the more accurate the results will be. The frame is divided into 20 elements as shown in Fig. 5.1 and the type of cross-section for each element is indicated. Note that although the original experiment was carried out to study the behaviour of structural joints, enough information is not given about the reinforcement arrangement, the tensile strength of the concrete and the yield strength of the stirrups in the joint regions. Therefore, in this study, some reasonable values are assumed as highlighted in Table 5.1. The frame is loaded by two concentrated loads causing negative moment at the beam-column joints. In the analysis the loads were applied in 40 increments.

Fig.5.2 illustrates the load-midspan deflection curve of the beam based on results of three different computer programs, including the program developed in this study. The other results are from the original program PCF3D and from the three-dimensional shell finite analyses using the computer program DIANA (Van Mier, 1987). The results in this figure highlight the effect of “tension-stiffening”, especially at first cracking. By accounting for this behaviour, instead of a sudden large increase in deformation after cracking, a gradual change in stiffness is introduced without any jump in deformation. As the load increases; the effect of tension-stiffening diminishes. With the development of cracks, the concrete tensile contribution decreases and only the reinforcement resists the internal tensile forces.

It was observed in the physical test (Strobrand and Kolpa, 1983) that the first cracks appeared at a load of $P=4.0$ (KN) in the midspan region of the beam, and very shortly afterwards in the column near the joint. A similar series of events took place in the existing analysis, while the program DIANA predicted cracking of the column near

the joints at $P=8.0$ (KN). For this reason, after the first set of cracks, the present predicted deflection is larger than that predicted by program DIANA. At load $P=21$ (KN) stirrups at both corners yield and shortly after at 22 (KN) the longitudinal steel at midspan started to yield. After three more load steps ($P=23.8$ KN), following the yielding of the majority of the tensile longitudinal bars in the beam, equilibrium was no longer possible and the frame collapsed.

The writer was not able to access the actual test results and that is why the test results could not be plotted. However, Van Mier (1987) reported good comparison between the results of DIANA and the test results. In the present analysis, a lower failure load than DIANA's value is predicted. However, since the actual experimental failure load is not known, we cannot be certain about the difference between DIANA's prediction and the actual failure load.

Fig. 5.2: Load-midspan deflection curve of portal frame (total load against deflection at midspan)

5.2.2 Simple beam

A simply supported beam (Fig.5.3), subjected to two concentrated loads, was tested by Alami and Ferguson (1963). It is modeled here to study the effect of “tension-stiffening” on its numerically calculated response. The two different cross-sections details, including the reinforcing layout, are given in Table 5.2. It should be noted that since no information is available about the stirrup properties in the actual test specimen, some reasonable values are assumed in this analysis.

Table 5.2: Cross-section details and material properties of Alami and Ferguson (1963) beam

Cross-section Designation	Cross-section dimension(in.)		Cover (in.)	Longitudinal steel		Stirrups	Material properties (ksi)			
	Width b	Height h		Top	Bottom		f'_c	f_t	f_{yL}	f_{yt}
D_1	7.5	9.0	2	0	2#9	#3@4.0in.	3.10	.31	48.5	48.5
D_2	7.5	9.0	2	0	2#9	#2@5.0in.	3.1	.31	48.5	48.5

In the analysis the beam was divided into 22 elements of 6" length each. Each element was divided into 10 equal layers through its cross-section. Since the beam is subjected to bending about one axis only, only one layer system would suffice. The load was applied in 40 increments and 30 iterations were allowed in each load step.

Fig.5.3: Dimensions and loading of tested beam by Alami and Ferguson (1963)

Fig.5.4 shows the load-midspan deflection curve of the beam. In the figure the predicted load-deflection curves according to the present model and the program PCF3D are compared with the corresponding experimental curve. In this case, good agreement can be observed between the present predicted response and the experimentally measured response. Similar to the previous example, no sudden drop occurs in the stiffness of the beam during the nonlinear analysis, and the predicted result follows the same behaviour as the test result. The beam failed in flexure, due to the yielding of the longitudinal bars at midspan. Considering the results of these two examples, it can be stated that “tension-stiffening” affects the behaviour of a reinforced concrete member after cracking and leads to a gradual diminution of stiffness, but it has little effect on the ultimate capacity of the member.

Fig.5.4: Load-midspan deflection curve of beam tested by Alami and Ferguson (1963)

5.3 Pure torsion

In this part four different concrete beams under pure torsion are modeled. The Beam B_1 was tested by Klus (1968), Beams B_2 and B_3 were tested by McMullen and Vijaya (1978), and Beam B_4 was tested by Hsu (1980). All the beams were prismatic and had rectangular cross-sections. The concrete strength and reinforcement is the same at all sections in each beam. Table 5.3 shows the dimensions of cross-sections, reinforcing arrangement and material properties which are used in the computer program.

Table 5.3: Beam details and material properties for Beams B_1 , B_2 , B_3 and B_4

Beam	Cross-section dimension (in)		Length L (in)	Cover (in)	Longitudinal steel		Stirrups	Material properties (ksi)			
	Width b	Height h			Top	Bottom		f'_c	f_t	f_{yL}	f_{yt}
B_1	8.0	12.0	62.0	.50	2 ϕ 11/16+ 1 ϕ 7/8	2 ϕ 11/16+ 1 ϕ 7/8	ϕ 5/16@4 .in.	3.12	.261	62.3	38.4
B_2	8.75	8.75	144	.50	2#3	2#3	#2@3.125 in.	5.74	.611	52.2	41.4
B_3	8.63	8.63	144	.50	2#6	2#6	#3@2.25 in.	5.68	.588	50.9	52.2
B_4	8.5	13.5	100	.75	2#6	2#6	#3@4.125 in.	4.43	.27*	47.7	51.8

* Assumed due to the lack of experimental data

Fig.5.5: Typical support condition for tested Beams B_1 , B_2 , B_3 and B_4

Since the tensile strength of the concrete is not known for Beam B_4 , a value of $4\sqrt{f'_c}$ is assumed in the present analysis. All the beams are subjected to a constant torsional moment over the length. The support condition for the last three beams is shown in Fig.5.5. The beams are simply supported and the left support is restrained against twist or rotation around the local x axis. In order to create constant torsion along the beam, a torsional moment, T , is applied to the free end. Beam B_1 is a semi-continuous beam and only the part of the beam between the two supports is subjected to a constant torque. Therefore, in the analysis, modelling of the two overhangs is neglected and this beam is modeled similar to the other three beams. Each beam is divided into 5 equal elements. The applied torque was applied in 40 increments and 30 iterations were permitted in each increment.

Fig.5.6: Torque-twist curve for Beam B_1 tested by Klus (1968)

Figures (5.6) to (5.8) illustrate the predicted and experimentally observed torque-twist curves of the four beams. Each the lines represent the predicted response while the symbols represent the experimental values. It is important to point out that the solid lines in these figures were drawn by joining a large number of points obtained in the analysis. The torque-twist curves demonstrate the different stages of the response of a concrete member, including the uncracked, the cracked, the yielded (steel) and finally the ultimate state. Comparison of the predicted curves and the experimental results demonstrate the relative accuracy of the proposed method for modelling more realistically the torsional stiffness of reinforced concrete members. As mentioned before, the amount of steel in a section determines the amount of jump in the twist immediately after cracking. The response of Beam B_3 and Beam B_4 demonstrate this aspect of the behaviour.

The Beams B_2 and B_3 were designed as under-reinforced and over reinforced beams, respectively. As it can be observed in Fig.5.7, there is no significant jump after cracking for Beam B_3 due to the large amount of its steel while for Beam B_2 there is a large jump, leading to failure. Fig.5.7 shows two different behaviour, the brittle failure of Beam B_2 immediately after cracking and the ductile failure of Beam B_3 . The present numerical analysis showed yielding of both the longitudinal and the transverse steel in Beam B_2 right after cracking, which caused failure. In the case of Beam B_3 , yielding of stirrups coincided with the failure load of the beam while in the experiment, yielding of steel happened after the peak torque. Since the current program is not capable of predicting the descending part of the torque-twist curve, it models the behaviour of the beam after ultimate state as plastic flow.

Fig. 5.7: Torque-twist curve for Beam B_2 and B_3 tested by McMullen and Vijaya (1978)

Fig.5.8: Torque-twist curve for Beam B_4 tested by Hsu (1980)

In Fig.5.8, results of Beam B_4 show good agreement with experimental data. The yielding of the stirrups in the actual experiment and in the analysis happened at a torque of approximately 318 (kip-in) and it is manifested by a visible reduction in the slope of the torque-twist curve.

There are some additional features that were investigated during the analysis of these beams. Spalling of the concrete cover and the ultimate torsional capacity are two properties that need to be discussed. Among the four beams, the behaviour of Beam B_4 is strongly affected by the spalling phenomenon. The predicted response of Beam B_4 , as shown in Fig.5.8, was obtained without considering the spalling of the concrete cover, while allowing for spalling to occur, the beam will fail at a torque $T = 308$ (kip-in.), which compared to experimental ultimate torque, is much lower. The other three beams are not affected by spalling because, Beams B_1 and B_2 do not satisfy the spalling criteria, and Beam B_3 is only marginally affected by spalling because of the section geometry. Since there is no information given about spalling by the researchers who tested Beam B_4 , no conclusion can be drawn regarding the applicability of the current criteria for initiation of cover spalling under torsion. It would be appear that more study is needed to further refine the existing criterion.

As mentioned in Chapter four, in order to calculate the ultimate torsional capacity of a member, Rahal and Collins's method (1996) has been adopted in this study. In this method they assumed that yielding of longitudinal and transverse steel occur at the same time as the crushing of concrete. But in Section 4.3, some modifications were made to the torque-twist model base on the yielding of either longitudinal or transverse steel before failure. This modified model enables the program to estimate the ultimate torsional

strength of the member more accurately. For instance, based on Rahal and Collins's method, the ultimate torsional strengths of Beams B_1 , B_2 , B_3 and B_4 are 240, 100, 318 and 377 kip-in., respectively, while the presented results in the figures show the ultimate torque as 155, 100, 318 and 369 kip-in. From the behaviour of the tested beams, it can be concluded that the amount of change is strongly influenced by the yielding of steel. If the yielding occurs in early load steps, such as Beam B_1 , a significant drop is observed in the ultimate strength of the beam. Since no yielding was observed in Beam B_3 , Rahal and Collin's method shows good agreement with the experimental results. Although the predicted strength of Beam B_1 is higher than its actual ultimate strength, nevertheless overall the results show that the program predicts reasonably well the ultimate strength of reinforced concrete structures subjected to pure torsion.

5.4 Combined loading

5.4.1 Spandrel beam under combined bending, shear and torsion

Fig.5.9 schematically illustrates a portion of an indeterminate three-dimensional structural frame, comprising a spandrel beam and a floor beam. This frame was experimentally tested by Hsu and Burton (1974) to study the effect of cracking in the floor beam on the torsional moment in the spandrel beam. If a load P is applied to the floor beam on the torsional moment in the spandrel beam. If a load P is applied to the floor beam, it will produce a rotation at its ends which in turn induces a torsional moment in the spandrel beam because one end of the beam is monolithic with the spandrel. After cracking, the torsional stiffness of a reinforced concrete member drops significantly and

is only a small fraction of the uncracked stiffness, this leads to redistribution of internal forces. The calculation of the torsional moment in the spandrel requires satisfaction of both equilibrium and compatibility conditions.

Fig.5.9: Dimensions, support condition, load details and elements arrangement for frame tested by Hsu and Burton (1974)

This example is intended to demonstrate the capability of the proposed model with respect to predicting redistribution of moments after cracking. For this purpose two of Hsu and Burton's frames will be analysed. These frames are geometrically similar frames but with different reinforcement and concrete properties. The frames are designed as FR1 and FR2.

Table 5.4 shows the properties of three different cross-sections which were used in this frame; namely, the cross-sections of the spandrel Beam, E_1 , the floor beam joints, E_2 , and the floor beam midspan region, E_3 . Each cross-section is divided into a grid of 1×10 or 10 layers.

Table 5.4: Cross-section details and material properties for frames FR1 and FR2

Beam	Type	Cross-section dimension(in.)		Cover (in.)	Longitudinal steel		Stirrups	Material properties (ksi)			
		Width b	Height h		Top	Bottom		f'_c	f_t	f_{yL}	f_{yt}
FR1	Midspan floor beam E_3	6.0	9.0	.5	2#2	2#5	#2@4.0in.	4.38	.26	57.	50.
	Joint floor beam E_2	6.0	9.0	.5	2#4	2#5	#2@4.0 in	4.38	.26	72.	50.
	Spandrel beam E_1	6.0	12.0	.5	2#3	2#6	#3@4.6in.	4.38	.26	65.	70.
FR2	Midspan floor beam E_3	6.0	9.0	.5	2#2	2#5+1#3	#2@4.0in.	4.12	.25	40.	50.
	Joint floor beam E_2	6.0	9.0	.5	2#3	2#5	#2@4.0 in	4.12	.25	40.	50.
	Spandrel beam E_1	6.0	12.0	.5	2#2	2#5	#2@5.0in.	4.12	.25	56.	50.

The locations of the different cross-sections are shown in Fig. 5.9. The spandrel beam is modeled by 18 equal elements with cross-section E_1 , and the floor beam is discretized into 18 elements of different length, with shorter elements used in the vicinity of loading points and joints. Since there is no information provided for the length of the

joint elements, a length of 18" was assumed for the joint region and this region was assumed to have cross-section type E_2 . Yield strength of longitudinal steel in each cross-section is an average of different yield strengths corresponding to different steel bars in each section. Since there is no information about the tensile strength of concrete, $4\sqrt{f'_c}$ is used as a reasonable approximation. The support conditions are illustrated in Fig.5.9 and in order to prevent the rotation of the spandrel beam, its ends are torsionally fixed. The floor beam was uniformly loaded which is simulated by four concentrated loads. In the analysis, the point loads were applied in small increments to allow the program to redistribute the moments according to the changes in the structural stiffness.

Fig.5.10 shows the variation of the torque with the applied load in the spandrel beam of frame FR1. The figure shows both the experimentally measured and the presently calculated torque variation. It can be seen from the figure that, the predicted behaviour of the frame is in a good agreement with its actual behaviour. Torsional cracking of the spandrel beam occurred at a torque of 49 (kip-in.) while the experimental data show a cracking torque of 28 (kip-in.). As mentioned before, the tensile strength of concrete cannot be predicted precisely; therefore, if the tensile strength of concrete based on the experimental results is used in the analysis, the predicted results approach the actual test results. The torque-twist relationship of the spandrel beam for frame FR1 is given in Fig.5.11. Since the spandrel beam was designed as an over-reinforced beam, no significant jump in twist occurred after cracking.

Fig.5.10: Midspan torque-load relationship of spandrel beam for Frame FR1

Fig.5.11: Midspan torque-twist relationship of spandrel beam in frame FR1

At a total load of 21 (kip), the stirrups of the floor beam near the support yield because the floor beam support takes most of the shear force of the system. At 23 (kip), the midspan longitudinal steel in the floor beam yields, causing reduction in the stiffness of the floor beam. The latter reduction causes more of the load to be transferred to the spandrel beam and the spandrel beam will be subjected to a larger twist (Fig.5.11). Shortly after this large twist and its corresponding torsional moment, both the longitudinal and the transverse steel in the spandrel beam yielded. The angle of twist in the spandrel beam is large enough to cause flexural failure at midspan of the floor beam. The three sets of interaction equations for each element show that due to the specific design of the various cross-sections the spandrel beam is still able to carry more load, while the floor beam has reached its ultimate capacity.

Fig.5.12 shows the variation of torque with the applied load at the midspan of the spandrel beam in frame FR2. Once again the tensile strength of concrete has an important effect on the shape of the predicted response. The predicted crack is at a torque of approximately 35(kip-in.) while the test result shows cracking at a torque of 22 (kip-in.). By looking at the torque-twist curve of the spandrel beam in this frame (Fig.5.13), it can be observed that there is a drastic drop in torsional stiffness after cracking. After the cracking of the spandrel beam, loads are redistributed according to the new torsional stiffness, therefore, the spandrel beam releases some load, which is reflected by the sudden drop.

Fig.5.12: Midspan torque-load relationship of spandrel beam in frame FR2

Fig.5.13: Midspan torque-twist relationship of spandrel beam in frame FR2

The numerical results exhibits a response similar to that observed in the actual test. In particular, we can observe in Fig.5.12 that the torque initially increases with the applied load, but then suddenly decreases, followed by a gradual increase thereafter. The calculated response shows a similar set of events, but overall it underestimates the actual strength of the structure. At load level 16 (kip), the total stress due to torsion and bending moments causes yielding in the longitudinal steel at mid span of the spandrel beam and therefore the spandrel beam is not able to receive more moment as indicated by line CD in Fig.5.12. Thereafter, it gradually loses more stiffness up to the yielding of longitudinal steel in the floor beam, which is denoted by point E in the figure. At a load of 19 (kip) the yielding of longitudinal steel in floor beam occurs, which transfers more load to the spandrel beam and it causes the yielding of all the longitudinal and transverse steel in the spandrel beam. Consequently, the combined bending and shear failure criterion is satisfied in the floor beam for combined bending and shear and the frame fails.

Although the results of frame FR2 are not in good agreement with the test results, the predicted behaviour of this frame follows the same basic path as the test shows. If we compare the reinforcement arrangement in the cross-sections of frames FR1 and FR2, the predicted results for frame FR2 do not seem to be unexpected, because as a weaker frame, it should not carry the same load as frame FR1. However, this example emphasizes the need for further improvement of the torque-twist model, especially at cracking. As mentioned before, right after cracking, the torque-twist curve indicates a sharp drop in stiffness, depending on the amount of reinforcement (Fig.5.13). The analysis of the indeterminate frames, FR1 and FR2, using this type of torque-twist curve, seems to indicate that after cracking the torsional stiffness of the member subjected to

torsion suddenly drops and it needs further loading to eventually adjust to the new stiffness distribution. The experimental data do not exhibit this phenomenon (vertical drop) but this may be either due to the inability of the data acquisition system or due to the fact that some experiments are load controlled. In either case, one may miss this sudden drop. This discrepancy between the experimental and numerically calculated responses requires further investigation.

5.4.2 Beam under combined bending, shear and torsion

In order to study the interaction of bending, shear and torsion, two beams tested by Behra and Ferguson (1970) were modelled. They investigated the effect of combined actions on spandrel beams using three different cross-sections shape: as, L-shape, T-shape and rectangular. In this study, one L-shape and one rectangle beam are analyzed. Since the test setup is not clearly given in the preceding reference, a loading system was assumed for the model. The assumed system completely satisfies the description of the test setup in the mentioned reference. The beams under combined loading were subjected to positive moment at the midspan diaphragm and to negative moment at the supports, the two moments being equal, and a constant torque and shear is applied to the each half span. Fig.5.15 illustrates the support and loading conditions for the tested beams. The details of the properties of the two beams are given in Table 5.5 while the full dimensions of the L-shape cross-section are shown in Fig.5.14. Based on the described loading condition, the ratio T/V in both beams is 5.0 in.

Table 5.5: Cross-section details and material properties of Behra and Ferguson (1970) beams

Cross-section type	Cross-section dimension(in.)		Cover (in.)	Longitudinal steel		Stirrups (wire)	Material properties (ksi)			
	Width b	Height h		Top	Bottom		f'_c	f_t	f_{yL}	f_{yt}
L – shape	3	6.13	7/16	2#3	2#3	1#8@2.5in.	4.44	.23	49.2	44.7
Rectangle	3	6.13	7/16	2#3	2#3	1#8@2.5in.	4.06	.23	49.2	44.7

Fig.5.14: L-shape cross-section size details for

Fig.5.15: Support and load conditions for Behra and Ferguson (1970) beams

The ultimate value of moment, shear and torsion based on the interaction of bending-shear-torsion are given in Table 5.6. The tabulated values are those obtained in the tests and the corresponding values, found in the present analysis. The last three columns are the values obtained using the three interaction equations, namely (4.64), (4.66) and (4.68) given in Section 4.4. Notice that a value of 1.0 or greater indicates failure under the specified actions, otherwise the structure is assumed to be able to safely carry the load producing those actions. According to the results in Table 5.6, the first set of interaction equations is unconservative, while the second set is unconservative for the rectangular beam only. The third set indicates that under the specified combination of loads, both sections will fail. The results indicate good agreement between the test results and the predicted results based on the current computer program. Fig.5.16 and 5.17 illustrate the torque-twist relationship for both beam models. In the case of the rectangular beam, the experimental torque-twist curve was not reported by Behra and Ferguson (1970) but they did report the failure torque.

Table 5.6: Comparison of predicted result with tests values of beams tested by Behra and Ferguson (1970)

Beam	Test value			Predicted value by present analysis			Predicted based on interaction equations*		
	M_u (kip-in)	V_u (kip)	T_u (kip-in)	M_u (kip-in)	V_u (kip)	T_u (kip-in)	Set -1	Set-2	Set-3
L – shape	44.0	2.63	13.10	42.43	2.72	13.77	.78	1.02	1.08
Rectangle	42.2	2.53	12.60	40.5	2.16	12.42	.63	.87	.98

*Equations (4.64), (4.66) and (4.68) of Section 4.4.

** $M_U=73.4$, $T_U=20.83$, $V_U=14.46$ for L-shaped beam, $M_U=70.3$, $T_U=20.83$,

$V_U=11.26$ for rectangular beam

Once again the predicted and experimentally observed torque-twist curves in Fig.5.16 agree quite well. These examples demonstrate the capability of the program to handle combined actions. For instance, the ultimate capacity of the rectangular beam under pure moment, shear and torsion is 72.4 (kip-in.), 14.4 (kip) and 19.62 (kip-in.) respectively. Without considering the interaction equations, although the yielding of both longitudinal and transverse steel is observed during the nonlinear iterative procedure, the program continues the analysis until reaching either the ultimate moment or ultimate torque. This would be clearly contrary to the observed behaviour and unsafe.

Fig.5.16: Torque-twist relationship for L-shaped cross-section tested by Behra and Ferguson (1970)

Fig.5.17: Torque-twist relationship for rectangular beam tested by Behra and Ferguson (1970)

5.4.3 Beam under combined torsion & shear and moment& shear

A continuous beam tested by Klus (1968), is considered to study the interaction of combined torsion and shear in some sections, and combined moment and shear in the other sections. Similar to the other examples, all the relevant information, including cross-sectional dimensions, reinforcement layout, material properties and support conditions are given in Table 5.7 and Fig.5.18.

Table 5.7: Cross-section details and material properties of Klus (1968) beam

Beam	Cross-section dimension(in.)		Cover (in.)	Longitudinal steel (in.)		Stirrups (in.)	Material properties (ksi)			
	Width b	Height h		Top	Bottom		f'_c	f_t	f_{yL}	f_{yt}
SC3	8.0	12.0	.5	2 ϕ 11/16 + 1 ϕ 7/8	2 ϕ 11/16 + 1 ϕ 7/8	ϕ 5/16 @4.0in	3.12	.26	62.3	38.4

As it can be seen in Fig.5.18, the beam model is similar to the previous example and it is subjected to three concentrated loads and a concentrated torque at midspan. This condition provides a positive moment at midspan, negative moment at the supports, and a constant torque and shear in each half span. Considering the ultimate moment capacity of the section and the length of the beam, one can surmise that the interaction of bending and torsion and its effect on the failure load are important. Hence failure is expected to be initiated by combined torsion and shear. The maximum applied P (vertical load at midspan) and T (concentrated torque at midspan) on the beam are 12.5% and 87.5% of the ultimate shear and ultimate torsional capacity of the beam, respectively.

The ultimate torsional capacity of the beam was calculated, as in Section 6.3, to be $T_u = 166$ (kip-in.). In order to find the maximum flexural capacity of the beam, the concentrated torque is removed from the system and the beam is studied under combined shear and bending. The nonlinear procedure stops at a load of $P=84.5$ (kip), where the

third set of interaction equation (Eq.4.68) gives a non-dimensional value of 1.11 due to the high amount of shear in the beam. The ultimate capacity under pure shear can be calculated as $V_u = 42.5$ (kip) while the test result shows a maximum shear capacity of $V_u = 35.70$ (kip). Although the predicted value for ultimate shear and torque show higher value than the test results, the ratio of T/V for the predicted results ($T/V=3.90$) is almost the same as in the test result ($T/V=3.82$).

Fig.5.18: Support condition and loading of continuous beam SC3 tested by Klus (1968)

To study the interaction of shear and torsion, 12.5% and 87.5% of the calculated ultimate shear and torsion are applied to the beam, respectively.

Fig.5.19: Torque-twist relationship for Beam SC3 tested by Klus (1968)

The nonlinear iterative procedure stops at $T = 153$ (kip-in.), $V = 8.98$ (kip) and $M = 130$ (kip), because the third set of interaction equations gives a non-dimensional value of 1.03. From Fig.5.19, it can be observed that the numerical procedure shows higher capacity for the tested beam than the corresponding experimental values. This discrepancy between the experimental and theoretical results may be partly attributed to the difference between the actual and the specified material properties for this beam. Because in the case of ultimate capacity of the beam under pure torque and pure shear the predicted values are exactly 20% higher than the test values. Also Fig.5.19 indicates good comparison between the measured and calculated torque-twist curves, therefore one can only surmise that the concrete may have had a much higher strength than reported. Of course, further investigation is needed to examine more carefully the accuracy of the interaction equations. This subject is beyond the scope of the current study, but it is an

issue that deserves further study. In this study well-known equations (see Section 4.4) have been adopted, but it is not the aim of the present study to compare the accuracy of the various interaction equations. We have shown how these interaction equations could be incorporated in a nonlinear analysis. It would appear that it is too risky to generalize these equations for all possible loading conditions. It is concluded that the response of structures to combined loading is complex and further study is needed to determine how one can predict the failure of three-dimensional reinforced concrete frames under complex loading conditions. The problem may be even more complicated if one were to consider non-proportional loading or load reversals.

Chapter 6

Summary, Conclusion and Recommendations

6.1 Summary

The current study is primarily focused on the torsional analysis of reinforced concrete members and on the prediction of the response of concrete frames subjected to combined loading, i.e., axial, bending, shear and torsion. For this purpose, a nonlinear space frame analysis program, called 3DRCF-CL, has been developed. This program is composed of two major parts. The first part, which covers axial and bending analysis, is based on a program called PCF3D, developed by Mari (1984). The second part, which was developed by the writer, deals with torsion, shear and combined loading analysis of concrete frames.

A non-iterative procedure is proposed to construct the torque-twist curve for any arbitrary cross-section. The torque-twist curve takes into account the different states of concrete; namely, uncracked, cracked and crushed, and of elastic and yielded steel. The uncracked section torque-twist curve is developed using “Bredth’s theory” assuming elastic behaviour, while the “space truss analogy” is utilized as the basis for modelling the torque-twist response after torsional cracking. To determine the ultimate torsional

capacity of the section, a simplification of the “compression field theory” has been adopted and the effects of some important phenomena such as, spalling of the concrete cover, effective thickness of concrete subjected to diagonal stress field and concrete softening due to diagonal cracking have been considered. In order to obtain a more accurate torque-twist relation, partly based on empirical observations, some other aspects of the torsional behaviour of reinforced concrete members are also incorporated in the model. The first aspect is consideration of the effect of tensile stresses carried by cracked concrete, i.e. the “tension-stiffening” effect. Introduction of tension-stiffening in the model prevents a sudden drop in torsional stiffness immediately after cracking. The second aspect pertains to the calculation of the ultimate angle of twist. Finally, although the study of cyclic torque is not within the scope of the present study, a simple load reversal model has been included in the torque-twist model.

Program 3DRCF-CL uses two main steps to predict the failure of the concrete structure under single or combined loading. The first step involves calculation of the correct stiffness of member. In this step, the total strain in the longitudinal and the transverse steel is calculated to check their yielding. In the case of yielding, the stress-strain relationship of steel for bending analysis and the torsional stiffness of the section are modified. In this fashion, in each iteration of a load increment, the stiffness of the structure is determined under combined actions. In the second step, using well-known interaction equations, failure of the structure is checked. In order to utilize these equations, the ultimate capacity of each member under individual actions is calculated. These actions include axial force, bending moment about both cross-sectional axes, shear parallel to both cross-sectional axes and torsion. Therefore, in each load step, if any of the

interaction equations satisfies the failure criterion, the member at that particular location is assumed to have failed. Failure of all members at a joint causes a zero diagonal stiffness and the program uses this criterion to terminate the analysis.

In order to improve the accuracy of the results under bending and axial load, two other modifications were introduced to the procedures of program PCF3D. First, the tensile stress-strain model of concrete was modified to account for “tension-stiffening”. Similar to the torque-twist model, tension-stiffening prevents a drastic drop in the flexural stiffness after initial cracking of a reinforced concrete member, and allows for a gradual change in the stiffness of the cracked member. The second modification was applied to the finite element procedure of the program PCF3D. In PCF3D, a straight beam element with an arbitrary cross-section is used to model the members of a frame. Each element has six degrees of freedom at each end and one internal axial degree of freedom at its mid-length. This 13th degree of freedom properly controls the shift in the position of the neutral axis due to cracking and other nonlinearities. In the program PCF3D, the effect of this degree of freedom on the total axial strain is completely ignored, which leads to incorrect location of the neutral axis. The consequence is the wrong magnitude of stresses and stress resultants, i.e. axial force and moments. To rectify this error, the effect of the 13th degree of freedom on axial strain in the member was introduced.

Finally, a series of examples are analyzed by the program 3DRCF-CL based on the above principles and the results are compared with corresponding experimental data to demonstrate the capability and validity of the adopted methods and their assumptions.

6.2 Conclusions

1. The application of the tension-stiffening model introduced in this study improved the accuracy of the predicted response of reinforced concrete frames subjected to bending and axial load.
2. The inclusion of a new torque-twist relation in the program 3DRCF-CL enabled it to predict the full torque-twist response of statically determinate and indeterminate frames reasonably well.
3. Practically all existing nonlinear frame analysis programs lack the ability to predict the full response and ultimate capacity of concrete frame structures under combined loads.
4. The system of interaction equations used in this study is one possible method for overcoming the deficiency stated in Conclusion 3. However, further study is required to refine and improve the current interaction equations which are based on limited empirical results.
5. When checking the stress in the longitudinal steel reinforcement under combined actions, one must include the effect of shear and torsion on the normal stress in the longitudinal steel. Failing to take this effect into account may overestimate the actual stiffness and strength of a structure.
6. The method that is adopted in this study for calculating the total strain in the longitudinal steel is adequate, but it needs further examination because it calculates the strain based on an empirical superposition method.
7. A cover spalling criterion was adopted in this study based on available information in the literature, but use of this criterion is not able to predict the

actual response of members under shear and torsion because upon spalling, it drastically reduces the section strength, which is contrary to the observed behaviour of such members.

6.3 Recommendations and Future Work

To further improvement the accuracy and capabilities of program 3DRCF-CL, the following recommendations are made for further work:

- i. The criterion for the initiation of concrete cover spalling and its effect on the strength and stiffness of a member require further study.
- ii. To develop a more accurate torque-twist curve; determining the cracked section stiffness requires further study, with emphasis on the effects of aggregate interlock and shear transfer across cracks.
- iii. In order to predict the failure of concrete frames subjected to any load condition, much more study is needed to establish better interaction equations or failure criteria for combined loading.
- iv. The present study can be extended to include the prestressing effect on the shear and torsional behaviour of concrete frame members.
- v. The effects of concrete confinement and multi-axial stresses on its strength and deformations require more study. Constitutive relations should be developed that include multi-axial stress states but the relations should be expressed in terms of stress resultants rather than stresses or strains.
- vi. Introduce a reinforcement bond model.
- vii. Include other types of load besides concentrate joint loads in the program.

- viii. Improve the nonlinear solution strategy of the program and define better convergence and divergence criteria.
- ix. Change the load application strategy, in order to enable the user to define different load factors for each load step.
- x. Investigate how the program can be extended to multicell cross-sections.
- xi. Investigate the possibility of developing the full stiffness of the entire member based on a refined space truss model. In this case one can follow the concepts of stringers and shear panels as is commonly applied to analyze thin skin structures.

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Appendix A

Computer Program

A.1 General

In this part the structure of program 3DRCF-CL, a nonlinear frame analysis program for analyzing three-dimensional concrete frames subjected to combined actions, will be discussed. This program is a substantially modified form of the program PCF3D, developed at the University of California at Berkeley (Mari, 1984). Program 3DRCF-CL differs from program PCF3D in at least two main respects; namely, program 3DRCF-CL uses a details method for calculating the torsional stiffness of three-dimensional concrete frames and it employs various failure criteria for determining the failure of a structure under combined actions. Program 3DRCF-CL also utilizes a more refined tension-stiffening model. Overall, this program can better trace the response of structure under complex loading and can predict their ultimate capacity more accurately.

- In the first section of this appendix the function of each subroutine is described in detail.

- In the second section, the entire nonlinear procedure of the program and the way that it calls the different subroutines, are summarized in a flowchart.
- In Section three, the User's Manual of the program 3DRCF-CL is presented.
- Finally an example input file is presented at the end of the section.

It should be pointed out that this appendix is mostly focused on those aspects of the program which are investigated in the present study. For more information about the other capabilities of program PCF3D, the reader can refer to Mari (1984).

A.2 Structure of the program

The program is composed of the main program, 11 subprograms and 18 subroutines. The main program manages the transfer of data between the different subprograms and in turn each subprogram performs a similar function among the subroutines. Fig. A.1 illustrates the architecture of the main program with its subprograms and subroutines. In Fig. A.1, different colors and patterns are used to highlight important aspects of this diagram. The dark grey indicates the parts of the program which have been developed in the present study, the light grey indicate those aspects of the program PCF3D which have been improved in this study, the hatched pattern shows those subroutines of the program PCF3D which are related to the present study but which have not been modified here, and the white color presents those parts of

the program PCF3D that are related to prestressed and to time-dependent phenomena which are outside the scope of present study and are shown here only for the sake of completeness.

The functions of the subprograms and their subroutines are as follows:

MAIN subroutine	Initializes vectors and matrices; manages data between the subroutines; performs the nonlinear strategies and checks the convergence and failure criteria.
Subroutine INPUT	Reads the general data about the geometry of the structure and the material properties.
Subroutine TORQ	Develops the multi-linear torque-twist model for each cross-section. Calls subroutines PERIM-AREA and OFFSET to obtain the perimeter and area of the cross-section for the different states of concrete section from the uncracked stage up to failure. Calls subroutine MINS to obtain for each cross-section the minimum required amount of longitudinal and transverse steel for pure torsion.
Subroutine PERIM-AREA	Calculates the perimeter and area of any arbitrary shape using the coordinates of the corners of the shape.

Fig. A.1: Structure of program 3DRCF-CL

Subroutine OFFSET	Finds the required coordinates of the effective portion of cross-section for any level of concrete stresses.
Subroutine MINS	Obtains the minimum required longitudinal and transverse steel considering the geometry and material properties of a cross-section subjected to pure torsion.
Subroutine SHEAR	Obtains the cracking and ultimate shear capacity of each cross-section.
Subroutine FLEXURE	Obtains the ultimate moment capacity of the cross-section with respect to the y and z local axes for both positive and negative moments. In fact this subroutine prepares the required data for each direction and transfers the data to subroutine BALANCE to obtain the ultimate moment capacity in each direction.
Subroutine BALANCE	Obtains the plastic center, maximum axial capacity of the section and also performs a trial and error loop to find the neutral axis of the cross-section about both the y and the z local axes.

Subroutine BLOCKSTRS	Obtains the resultant concrete and steel forces and their moments corresponding to any reference axis.
Subroutine LOAD	Reads the joint loads, load factors and the nonlinear analysis option (i.e. refer to variable ITAN in the User's Manual).
Subroutine INTERACT	Checks the possibility of failure of each element subjected to single or combined actions, using the interaction equations at the beginning of each load step. If the existing internal forces satisfy the failure criteria, this subroutine calls subroutines STIFF to notify it of failure of the element.
Subroutine UPDATE	Accumulates external joint loads and load factors for each load step.
Subroutine STIFF	Receives an input from subroutine INTERACT. When informed about failure, it reduced the stiffness of failed element to basically zero; transforms the local stiffness matrix to the global coordinates, assembles the structure stiffness matrix and checks for structural failure by

monitoring the presence of zero along the diagonal of the global stiffness matrix.

- Subroutine SYMSOL   This subroutine is the solver within the program. It obtains the increment of displacement in global coordinates. It triangularizes the stiffness matrix, and then through back substitution calculates the increment of joint displacements. SYMSOL (1) uses the newly formed stiffness matrix, while SYMSOL (2) uses the last triangularized stiffness matrix.
- Subroutine STRESS   Transforms the global increments of displacements to local coordinates of each element and obtains the total strains and stresses for each element under single or combined action. In the case of yielding of either longitudinal or transverse steel changes the torque-twist model, calculates support reactions, and unbalanced load vector.
- Subroutine CONCRT   Using the stress-strain relationship of concrete, it obtains stresses in each concrete filament and determines the concrete state.
- Subroutine STEELF   Obtains the stress in each longitudinal bar from the steel stress-strain relationship and determine the state of each

steel bar. This stress is only affected by combined axial force and biaxial bending.

Subroutine BMTORQ	Obtains torque in each element by using the torque-twist model and checks the state of the element under torsion.
Subroutine SHEARSTRS	Calculates the strain in the both longitudinal and transverse steel, induces by shear and torsion.
Subroutine STEELTL	Finds the total longitudinal strain due to axial force, biaxial bending, shear and torsion, which were separately calculated in subroutines STRESS, STEELF and SHEARSTRS. It determines the state of shear in the bar based on total strain.
Subroutine STEELTT	Obtains stress in transverse steel from stress-strain relationship and determines the stirrup state.
Subroutine TORQ1	In the case of yielding of either longitudinal or transverse steel, calculates new torsional stiffness, ultimate torsional capacity and ultimate angle of twist for the member. It should be noted that only one yielding condition

(longitudinal or transverse steel) is considered for each element.

Subroutine RESIST	Obtains internal resisting force vectors for each element and assembles the total resisting force vector for the entire structure.
Subroutine OUTPUT	Prints result, which includes joint displacement, joint forces, and...

A.3 Flow chart

The entire nonlinear procedure of the computer program 3DRCF-CL, which is controlled by the main program (main subroutine) is shown in the Fig.A.2. In order to illustrate the relation among the subroutines in subprogram STRESS, another flowchart is given in Fig.A.3. The variables which are used in the flowchart can be defined as follows:

ITIME	Time step counter
NTIME	Number of time steps
LST	Load step counter
NSLT	Number of load steps for the current time step
ITER	Iteration counter
NITER	Maximum number of iterations allowed
KCNT	Code of nonlinear strategy adopted in the current time step. If KCNT =1, displacement control. If KCNT=0 , load control.

NM	Number of elements
NNM	Element counter
NY	Yielding condition counter. This parameter is initialized as zero in the main subroutine
TCR1	Cracking torque
TORR	Existing torque for current load step
VCR	Cracking shear
VS	Existing shear for current load step.

A.4 3DRCF-CL

This section shows the User's Manual of the program and the proper format of the required input data. Depending on the nature of the required data, the input value should be entered as a real value or an integer. In each section of the manual, enough information has been given to help the user input required value. The symbols used for data format are as follows:

Iw	Integer data
Fw.d	Real data in decimal notation
Ew.d	Real data in scientific notation
A	Character data

where w is positive integer specifying the field width and d is non-negative integer specifying the minimum number of digits to be displayed.

Fig.A.2: Flowchart of main
subroutine of program 3DRCF-CL

Fig.A.3: Flowchart of subprogram STRESS

Note that all the input data is in free format. Some sections of the input file are defined only in one line others depending on the number of variables, such as number of concrete types or number of cross-section types, must be defined in several sets. Therefore, in the description of each section, the information about the number of required lines has been provided. The input file is format free. For the sake of completeness, complete User's Manual is presented, despite the fact that some parts are outside the scope of this study.

SECTION-1   **Title (one line)**

TITLE (80A)   Title of the problem

SECTION-2   **Control Information (one line)**

NJ (I5)   Number of joints

NSJ (I5)   Number of supported joints

NM (I5)   Number of elements

NSEC (I5)   Number of different types of cross sections

IPB (I5)   Types of bending problem:

IPB=0 Biaxial bending

IPB=1 Uniaxial bending

NCNC (I5)   Number of different concretes

NSNS (I5)   Number of different reinforcing steel materials

KPRT (I5)   Prestressing code:

KPRT=0 There is no prestressing

KPRT=1 There is prestressing

KCNC (I5)   Input code for material properties:

KCNC=1 Input ACI parameters

KCNC=2 Input experimental results

AGE (F10.0)   Age of concrete in days at time of initial loading

TZERO (F10.0)   Reference temperature (in degrees centigrade)

ALPHA (F10.0)   Coefficient of thermal expansion (constant and equal for steel and concrete)

SPAL (I5)   Input code for spalling of concrete cover in torsion

                                 SPAL=0   No spalling

                                 SPAL= 1   Spalling considered

SECTION-3   Nodal Coordinates (one line per node)

I (I5)   Joints number

X(I) (F10.0)   X-coordinate in global

Y(I) (F10.0)   Y-coordinate in global

Z(I) (F10.0)   Z-coordinate in global

*Node numbers should be in order.

SECTION-4   Support Orientation Matrix (one line per each support)

I (I8)   Support joint number

XS(1) (F8.0)   X-coordinate for auxiliary node in spring 1

YS(1) (F8.0)   Y-coordinate for auxiliary node in spring 1

ZS(1) (F8.0)   Z-coordinate for auxiliary node in spring 1

XS(2) (F8.0)   X-coordinate for auxiliary node in spring 2

YS(2) (F8.0)   Y-coordinate for auxiliary node in spring 2

ZS(2) (F8.0)   Z-coordinate for auxiliary node in spring 2

XS(3) (F8.0)   X-coordinate for auxiliary node in spring 3

YS(3) (F8.0)   Y-coordinate for auxiliary node in spring 3

ZS(3) (F8.0)   Z-coordinate for auxiliary node in spring 3

*The auxiliary nodes should be defined in global coordinates. The arrangement of the springs is left to the user preference. For more information please see Section 3.3.2.

SECTION-5   Spring Constants (one line per each supported node)

NS(I) (I5)   Supported joint number

SP(I,J) (E10.0)   Value of the spring constant at node I, J=1,6

*Each support has six springs, the first three are translational springs and the last three are rotational springs. The order of springs is determined by the spring arrangement in Section four of manual.

SECTION-6   Cross Section Data (one set of lines per each section type)

PART-6.1   General data (one line)

ISEC (I5)   Section type number

NRT(I) (I5)   Number of sides

NRT(I)=0 Circle

NRT(I)= n Polygon (n= number of sides)

MSHP(I) (I5)   MSHP(I)=0 Hollow section

MSHP(I)= 1 Solid section

NCLY(I) (I5)   Number of concrete layers normal to y-axis

NCLZ (I) (I5)   Number of concrete layers normal to z-axis

NFB(I) (I5)   Number of rows of **C** matrix used to define the cross-section shape (**C** matrix is called **IB** in the program)

YMP(I) (F10.0)	Maximum positive distance from center of reference to the circumscribed rectangle, in y direction
ZMP(I) (F10.0)	Maximum positive distance from center of reference to the circumscribed rectangle, in z direction
YMN(I) (F10.0)	Maximum negative distance from center of reference to the circumscribed rectangle, in y direction
ZMN(I) (F10.0)	Maximum negative distance from center of reference to the circumscribed rectangle, in z direction
NRC(I) (I5)	Concrete condition code NRC(I)= 0 Plane concrete NRC(I)= 1 Reinforced concrete

PART-6.2   C matrix definition(one line for each row of the matrix)

IB(I,J,1) (I5)	Number of physical layer
IB(I,J,2) (I5)	Column in which concrete starts
IB(I,J,3) (I5)	Column in which concrete ends
IB(I,J,4) (I5)	Concrete material code for the current layer

If NRT(I)=0 skip part 6.3a

PART-6.3a   T matrix definition(one line for each wall of section)

DELTAY(I,J) (F10.0)	Departure of middle line wall J in section type I

DELTAZ(I,J) (F10.0) Latitude of middle line wall J in section type I

ACOV(I,J) (F10.0) Reinforcement cover in wall J in section type I

FTH(I,J) (F10.0) Thickness of tube

FTH=0 For solid section

FTH=t For hollow section (t=thickness of tube corresponding to the wall)

*The numbering of the walls must be counterclockwise.

If $NRT(I) \neq 0$ skip part 6.3b

PART-6.3b Special case for circular section (one line)

DIAM(I) (F10.0) Diameter of the section

DCOV(I) (F10.0) Reinforcement cover

DTH(I) (F10.0) Thickness of tube

DTH=0 For solid section

DTH=t For hollow section (t= thickness of tube)

If $NRC(I)=0$ skip part 6.4 and 6.5

PART-6.4 General steel properties (one line)

NFS(I) (I5) Number of reinforcing steel filament

NSF(I) (I5)	Material property code for longitudinal steel
AL(I) (F10.0)	Percentage of total longitudinal steel specified for torsion design
NST(I) (I5)	Material property code for transverse steel
AV(I) (F10.0)	Area of one stirrup
NLEGY(I) (I5)	Number of stirrup leg perpendicular to y axis
NLEGZ(I) (I5)	Number of stirrup leg perpendicular to z axis
ATEP(I) (F10.0)	Stirrup spacing

PART-6.5   **Properties of each longitudinal bar (one line per each bar)**

AS(I,J) (F10.0)	Area of one or a bundle of steel bars
ESY(I,J) (F10.0)	Y-eccentricity of the centroid of the bar bundle
ESZ(I,J) (F10.0)	Z-eccentricity of the centroid of the bar bundle

SECTION-7   **Member Data (one line per element)**

L (I5)	Element number
NODI(L) (I5)	Node at I end
NODJ(L) (I5)	Node at J end
MSEC(L) (I5)	Cross section type of the current element
XA(L) (F10.0)	Global X coordinate of orientation auxiliary node
YA(L) (F10.0)	Global Y coordinate of orientation auxiliary node
ZA(L) (F10.0)	Global Z coordinate of orientation auxiliary node

*The number of elements must be in order.

If KPRT=0 skip section 8

SECTION-8   Prestressing Data(one set of information)

PART-8.1   General information (one line)

ITRANS (I5)	Time step when prestressing is transferred
NTEND (I5)	Number of tendons
NPS (I5)	Number of prestressing segments
NPT (I5)	Number of points used to define stress-strain curve of prestressing steel
FPSY (F10.0)	0.1% Offset yield stress of prestressing steel
ROZR (F10.0)	Wobble friction coefficient
ROZC (F10.0)	Curvature friction coefficient
CRPS (F10.0)	Coefficient for the relaxation loss
	Use CRPS=10 for ordinary prestressing steel

PART-8.2   Stress-strain curve for prestressing steel (one line)

PSF(I) (E8.0)	Stress of point I
PSE(I) (E8.0)	Strain of point I (I=1, NPT,NPT<=5)

PART-8.3   Tendon information (one line for each tendon)

I (I5)	Tendon number
NIN(I) (I5)	Number of element in which tendon starts

NFN(I) (I5)	Number of element in which tendon ends
INDP(I) (I5)	Jacking code: INDP(I)=1 Jacking from I end INDP(I)=2 Jacking from J end INDP(I)=3 Jacking from both ends
AREAT(I) (F10.0)	Steel area of tendon I
PAI(I) (F10.0)	End force when jacking from I end
PAJ(I) (F10.0)	End force when jacking from J end
DESL(I) (F10.0)	Anchorage slip

PART-8.4   Prestressing segment data (one line for each segment)

I (I5)	Segment number
MPS(I) (I5)	Element in which this segment is embedded
NTPS(I) (I5)	Prestressing tendon to which the segment belongs
EY1(I) (I5)	Y eccentricity of segment at I end
EZ1 (I) (I5)	Z eccentricity of segment at I end
EY2(I) (I5)	Y eccentricity of segment at J end
EZ2 (I) (I5)	Z eccentricity of segment at J end

*These eccentricities are in element local coordinate

SECTION-9   Material Properties(one set of information)

PART-9.1   Concrete properties (one line per each different concrete)

KCNC=1 ACI properties

- FPC28(K) (F10.0)   28 day strength in psi (enter with negative sign)
- WGT(K) (F10.0)   Weight per unit volume in lb / ft.³
- ACNC(K) (F10.0)   Coefficient “a” to compute f'_c
- BCNC(K) (F10.0)   Coefficient “b” to compute f'_c
- RCMP(K) (F10.0)   Ratio r_c in $f''_c = r_c f'_c$
- RTNS(K) (F10.0)   Ratio r'_t in $f'_t = r'_t \sqrt{w f'_c}$
- RCRP1(K) (F10.0)   Ratio $r_1 = f''_c / \sigma$ up to which $\sigma_e = \sigma$ in creep calculation
(usually=0.35)
- RCRP2(K) (F10.0)   Ratio $r_2 = \sigma_e / \sigma$ when $\sigma = f''_c$ in creep calculation (usually=1.865)
- ECU(K) (F10.0)   Ultimate compressive strain. Enter with negative sign
- G(K) (F10.0)   Shear modulus at the age of initial loading
- TCOEF(K) (F10.0)   Coefficient “c” value to compute ϵ_{ut} (tension-stiffening feature)

KCNC=2 Test results

- ECI(K) (F10.0)   Initial modulus at the age of initial loading
- G(K) (F10.0)   Shear modulus at the age of initial loading

FCDP(K) (F10.0)	Compressive strength of concrete at 28, f'_c . Enter with negative sign.
FTP(K) (F10.0)	Modulus of rupture
FTPT (F10.0)	Tensile strength
ECU(K) (F10.0)	Ultimate compressive strain (negative)
RCRP1(K) (F10.0)	Ratio $r_1 = f'_c / \sigma$ up to which $\sigma_e = \sigma$ in creep calculation (usually=0.35)
RCRP2(K) (F10.0)	Ratio $r_2 = \sigma_e / \sigma$ when $\sigma = f'_c$ in creep calculation (usually=1.865)
TCOEF(K) (F10.0)	Coefficient "c" value to compute ϵ_{ut} (tension-stiffening feature)
DUCT(K) (F10.0)	Ductility factor for torsion

*It should be noticed that FTP is used for analysis of bending, while FTPT is considered for analysis of torsion and shear where the combined compression and tension exist in a strut.

If NSNS=0 skip part 9.2

PART-9.2   Steel properties (one line per each different steel)

ES1(K) (F10.0)	First modulus
ES2(K) (F10.0)	Second modulus
FSY(K) (F10.0)	Yielding stress for longitudinal steel
FST(K) (F10.0)	Yielding stress for transverse steel
ESU(K) (F10.0)	Ultimate strain

SECTION-10   **Structural Analysis Control Data(one line)**

NTIME (I5)	Number of time steps
NITI (I5)	Number of iterations allowed for intermediate load steps
NITF (I5)	Number of iterations allowed for final load step
KOUT (I5)	Output code: KOUT=1 Output results after every iteration KOUT= 0 Output results only at load steps
ITAN (I5)	Type of analysis code: ITAN=0 Linear analysis ITAN=1 Nonlinear material, small displacements ITAN=2 Nonlinear geometry, elastic material ITAN=3 Nonlinear material and geometry
KSTR (I5)	Stress output code. Number of elements in which the stresses are to be output
JRESP (I5)	Output code for intermediate values (detailed information of nonlinear procedure in each iteration). Use JRESP=0
KTEMP (I5)	Code for temperature effects: KTEMP=0 No temperature effects are considered KTEMP=1 Constant variation of temperature in all the structure KTEMP=2 Planar variation of temperature over the cross-section, different for each element

KSHRNK (I5) Code for shrinkage effects:

KSHRNK=0 No shrinkage effects are considered

KSHRNK=1 Constant shrinkage in the structure

KSHRNK=2 Planar variation of shrinkage for each element

* If KPRT $\neq$ 0, it is convenient to use a specific time step for the introduction of prestressing force.

If KSTR=0 (no element) or KSTR=NM (all elements) skip section 11

SECTION-11 Chosen Element for required output (one line)

KELEM(I) (I5) Element numbers I=1,KSTR

SECTION-12 Convergence Ratio Tolerances (one line)

TOLI (F10.0) Displacement ratio tolerance for intermediate load steps

TOLF (F10.0) Displacement ratio tolerance for final load step

TOLC (F10.0) Displacement ratio tolerance for changing stiffness

TOLL (F10.0) Maximum unbalanced load allowed

TOLM (F10.0) Maximum unbalanced moment allowed

TOLD (F10.0) Maximum allowable displacement

TOLR (F10.0) Maximum allowable rotation

SECTION-13 Load Information (one set of line for each time step)

PART-13.1 Control load information (one line)

ITIME (I5)	Time step number
NLS (I5)	Number of load steps for the current time step
NLJ (I5)	Number of loaded joints
NDJ (I5)	Number of supported nodes with imposed displacements
KCNT (I5)	Nonlinear strategy code for the current time step: KCNT=0 Load control KCNT=1 Displacement control
NDC (I5)	Node with controlled displacement (if KCNT=0 put NDC=0)
LDEG (I5)	Controlled degree of freedom in node NDC. (if KCNT=0 put LDEG=0)
DDISP (I5)	Maximum value of controlled displacement. (if KCNT=0 put DDISP=0)

*LDEG=1 to 6 depending on the dof controlled

for axial displacement (u)	LDEG=1
for transverse displacement (v)	LDEG=2
for transverse displacement (w)	LDEG=3
for rotation about y axis (θ_y)	LDEG=5
for rotation about z axis (θ_z)	LDEG=6

If NLJ=0 skip part 13.2

PART-13.2 Nodal loads (one line per each node)

I (I5)	Loaded joint number

PLOAD(J) (F10.0) Loads in global coordinates (J=1,6) acting on the joint J

*Loads must be defined in global coordinates. This program only considers point loads applied at the joints

If NDJ=0 skip part 13.3

PART-13.3 Imposed displacement (one line per each node)

NDI (I5) Number of the node with imposed displacement

DES(NDI,J) (F10.0) Value of the imposed displacements for each of the six degree of freedom (J=1,6) of node number NDI

*Displacements must be defined in global coordinates

PART-13.4 Load factors (one line for each load or displacement step)

LS (I5) Load step number

FLOAD(LS) (F10.0) Factor for external loads

FINST(LS) (F10.0) Factor for initial strain loads

FDISP(LS) (F10.0) Factor for controlled displacement. This factor will scale the value of DDISP

*When controlled displacement is used, it is necessary to specify FLOAD=1. The program scales the external load in order to obtain the specified value of the controlled dof. Prestressing can be introduced gradually by load steps. Since prestressing is treated as an initial strain load vector, its value is controlled by factor FINST (LST).

If ITIME=1 skip parts 13.5, 13.6, 13.7

PART-13.5   Time increment(one line)

DTIME (F10.0)   Time increment, in days, for the current time intervals

PART-13.6   Creep coefficient (one line for each concrete type)

XA1(M) (E10.0)

XA2(M) (E10.0)   Creep coefficient a_1, a_2, a_3 , for the time step t_{n-1}

XA3(M) (E10.0)

PART-13.7   Current concrete properties (one line for each different concrete)

ECI(M) (E10.0)   Current initial modulus

FCDP(M) (E10.0)   Concrete compressive strength (with negative sign)

FTP(M) (E10.0)   Concrete tensile strength

If KSHRNK=0 skip part 13.8

PART-13.8   Shrinkage strain increment (one set of line)

KSHRNK=1 Constant value for all the structure (one line)

DEPSS (F10.0)   Shrinkage strain

KSHRNK=2 Planar variation (one line for each element)

I (I5)	Current element number
SY(1) (E8.0)	Local Y- coordinate, of first point
SZ(1) (E8.0)	Local Z- coordinate, of first point
SS(1) (E8.0)	Shrinkage strain increment at point 1
SY(2) (E8.0)	Local Y- coordinate, of second point
SZ(2) (E8.0)	Local Z- coordinate, of second point
SS(2) (E8.0)	Shrinkage strain increment at point 2
SY(3) (E8.0)	Local Y- coordinate, of third point
SZ(3) (E8.0)	Local Z- coordinate, of third point
SS(3) (E8.0)	Shrinkage strain increment at point 3

*Increments of shrinkage strain for the current time step are specified at three arbitrary points of the cross-section for each element. The shrinkage strain increment for each filament is automatically obtained by the program by fitting a plane surface to the specified of shrinkage strains in the section.

If KTEMP=0 skip part 13.9

PART-13.9   Temperature value

KTEMP=1 Constant value for all the structure (one line)

TEMP (F10.0)	Temperature in degrees centigrade

KTEMP=2 Planar variation (one line for each element)

I (I8)	Element number
YT(1) (F8.0)	Local Y-coordinate, of first point
ZT(1) (F8.0)	Local Z- coordinate, of first point
TT(1) (E8.0)	Temperature, in degrees centigrade of first point
YT(2) (F8.0)	Local Y-coordinate, of second point
ZT(2) (F8.0)	Local Z- coordinate, of second point
TT(2) (E8.0)	Temperature, in degrees centigrade of second point
YT(3) (F8.0)	Local Y-coordinate, of third point
ZT(3) (F8.0)	Local Z- coordinate, of third point
TT(3) (E8.0)	Temperature, in degrees centigrade of third point

*Temperature values for this option are specified at three arbitrary points of the cross-section for each element. The value of the temperature at each filament is automatically obtained by the program, by fitting a planar surface of temperature into the section.

A.4 Input Files

In this part the input file of frame FR1 discussed in Chapter five is given. It should be noticed that the dotted lines and the italicized phrases are used for clarification of input file format. They are not part of input file.

EXAMPLE HSU-BURTON FRAME MODEL FR1	<i>SECTION-1</i>
37,3,36,3,1,0,1,3,0,2,28,0.,0.00001,1	<i>SECTION-2</i>
1,0.,0.,0.	<i>SECTION-3</i>
2,6.,0.,0.	
3,12.,0.,0.	
4,18.,0.,0.	
5,24.,0.,0.	
6,30.,0.,0.	
7,36.,0.,0.	
8,42.,0.,0.	
9,48.,0.,0.	
10,54.,0.,0.	
11,60.,0.,0.	
12,66.,0.,0.	
13,72.,0.,0.	
14,78.,0.,0.	
15,84.,0.,0.	
16,90.,0.,0.	
17,96.,0.,0.	
18,102.,0.,0.	
19,108.,0.,0.	
20,54.,6.,0.	
21,54.,13.,0.	
22,54.,18.,0.	
23,54.,24.,0.	
24,54.,30.,0.	
25,54.,36.,0.	
26,54.,40.5,0.	
27,54.,48.,0.	
28,54.,54.,0.	
29,54.,60.,0.	
30,54.,67.5,0.	
31,54.,72.,0.	

32,54.,78.,0.
 33,54.,84.,0.
 34,54.,90.,0.
 35,54.,94.5,0.
 36,54.,102.,0.
 37,54.,108.,0.

1,1.,0.,0.,0.,1.,0.,0.,0.,1.
 19,110.,0.,0.,108.,1.,0.,108.,0.,1.
 37,54.,110.,0.,55.,108.,0.,54.,108.,1.

SECTION-4

1,1.E+40,2.E+40,3.E+40,4.E+40,5.E-40,6.E-40
 19,1.E+40,2.E+40,3.E+40,4.E+40,5.E-40,6.E-40
 37,1.E-40,2.E+40,3.E+40,4.E-40,5.E-40,6.E+40

SECTION-5

1,4,1,1,20,20,3.,4.5,-3.,-4.5,1

SECTION-6
PART-6.1

1,1,1,1
 2,1,1,1
 3,1,1,1
 4,1,1,1
 5,1,1,1
 6,1,1,1
 7,1,1,1
 8,1,1,1
 9,1,1,1
 10,1,1,1
 11,1,1,1
 12,1,1,1
 13,1,1,1
 14,1,1,1
 15,1,1,1
 16,1,1,1
 17,1,1,1
 18,1,1,1
 19,1,1,1
 20,1,1,1

PART-6.2

0.,-9.,.5,0.
 6.,0.,.5,0.
 0.,9.,.5,0.
 -6.,0.,.5,0.

PART-6.3a

4,1,100.,1.,048,2,2,0.,4.

PART-6.4

.31,-2.5,-4.	<i>PART-6.5</i>
.31,2.5,-4.	
.048,-2.5,4.	
.048,2.5,4.	
2,4,1,1,20,20,3.,4.5,-3.,-4.5,1	<i>PART-6.1</i>
1,1,1,1	<i>PART-6.2</i>
2,1,1,1	
3,1,1,1	
4,1,1,1	
5,1,1,1	
6,1,1,1	
7,1,1,1	
8,1,1,1	
9,1,1,1	
10,1,1,1	
11,1,1,1	
12,1,1,1	
13,1,1,1	
14,1,1,1	
15,1,1,1	
16,1,1,1	
17,1,1,1	
18,1,1,1	
19,1,1,1	
20,1,1,1	
0.,-9.,.5,0.	<i>PART-6.3a</i>
6.,0.,.5,0.	
0.,9.,.5,0.	
-6.,0.,.5,0.	
4,2,100.,2.,.048,2,2,0.,4.	<i>PART-6.4</i>
.31,-2.5,-4.	<i>PART-6.5</i>
.31,2.5,-4.	
.20,-2.5,4.	
.20,2.5,4.	
3,4,1,1,20,20,3.,6.,-3.,-6.,1	<i>PART-6.1</i>
1,1,1,1	<i>PART-6.2</i>
2,1,1,1	
3,1,1,1	
4,1,1,1	

5,1,1,1
 6,1,1,1
 7,1,1,1
 8,1,1,1
 9,1,1,1
 10,1,1,1
 11,1,1,1
 12,1,1,1
 13,1,1,1
 14,1,1,1
 15,1,1,1
 16,1,1,1
 17,1,1,1
 18,1,1,1
 19,1,1,1
 20,1,1,1

0.,-12.,.5,0.
 6.,0.,.5,0.
 0.,12.,.5,0.
 -6.,0.,.5,0.

PART-6.3a

4,3,100.,3.,11,2,2,0.,4.625

PART-6.4

.44,-2.5,-5.5
 .44,2.5,-5.5
 .11,-2.5,5.5
 .11,2.5,5.5

PART-6.5

1,1,2,3,1,5.,0.,10.
 2,2,3,3,1,5.,0.,10.
 3,3,4,3,1,5.,0.,10.
 4,4,5,3,1,5.,0.,10.
 5,5,6,3,1,5.,0.,10.
 6,6,7,3,1,5.,0.,10.
 7,7,8,3,1,5.,0.,10.
 8,8,9,3,1,5.,0.,10.
 9,9,10,3,1,5.,0.,10.
 10,10,11,3,1,5.,0.,10.
 11,11,12,3,1,5.,0.,10.
 12,12,13,3,1,5.,0.,10.
 13,13,14,3,1,5.,0.,10.
 14,14,15,3,1,5.,0.,10.
 15,15,16,3,1,5.,0.,10.
 16,16,17,3,1,5.,0.,10.
 17,17,18,3,1,5.,0.,10.

SECTION-7

18,18,19,3,1,5.,0.,10.
 19,10,20,2,1,54.,10.,10.
 20,20,21,2,1,54.,10.,10.
 21,21,22,2,1,54.,10.,10.
 22,22,23,1,1,54.,10.,10.
 23,23,24,1,1,54.,10.,10.
 24,24,25,1,1,54.,10.,10.
 25,25,26,1,1,54.,10.,10.
 26,26,27,1,1,54.,10.,10.
 27,27,28,1,1,54.,10.,10.
 28,28,29,1,1,54.,10.,10.
 29,29,30,1,1,54.,10.,10.
 30,30,31,1,1,54.,10.,10.
 31,31,32,1,1,54.,10.,10.
 32,32,33,1,1,54.,10.,10.
 33,33,34,1,1,54.,10.,10.
 34,34,35,2,1,54.,10.,10.
 35,35,36,2,1,54.,10.,10.
 36,36,37,2,1,54.,10.,10.

SECTION-9
 PART-9.1

3772.,1866.,-4.38,30,264,-0.0035,0.35,1.865,10.,3.

PART-9.2

29000.,120.,57.,50.,0.16
 29000.,120.,72.,50.,0.16
 29000.,120.,65.,70.,0.16

SECTION-10

1,30,30,1,1,36,0,0,0

SECTION-12

.5.,.5.,.5,5.,10.,.5,.5

SECTION-13
 PART-13.1

1,40,4,0,0,0,0,0.

PART-13.2

21,0.,0.,-15.,0.,0.,0.
 26,0.,0.,-15.,0.,0.,0.
 30,0.,0.,-15.,0.,0.,0.
 35,0.,0.,-15.,0.,0.,0.

PART-13.4

1,.01,.0,1
 2,.01,.0,1
 3,.01,.0,1
 4,.01,.0,1
 5,.01,.0,1
 6,.01,.0,1
 7,.01,.0,1

8,.01,.0,1
9,.01,.0,1
10,.01,.0,1
11,.01,.0,1
12,.01,.0,1
13,.01,.0,1
14,.01,.0,1
15,.01,.0,1
16,.01,.0,1
17,.02,.0,1
18,.02,.0,1
19,.02,.0,1
20,.01,.0,1
21,.01,.0,1
22,.01,.0,1
23,.01,.0,1
24,.01,.0,1
25,.01,.0,1
26,.01,.0,1
27,.01,.0,1
28,.01,.0,1
29,.01,.0,1
30,.015,.0,1
31,.015,.0,1
32,.015,.0,1
33,.015,.0,1
34,.015,.0,1
35,.015,.0,1
36,.015,.0,1
37,.015,.0,1
38,.02,.0,1
39,.02,.0,1
40,.02,.0,1

Appendix B

Derivation of Stiffness Matrix Terms

In this part, detailed derivations of the elements of the 13×13 stiffness matrix of a space frame element using the shape functions derivatives are presented.

$$K_{1,1} = EA \int_0^L \left(\frac{-1}{L}\right) \left(\frac{-1}{L}\right) dx = \frac{EA}{L}$$

$$K_{1,2} = -\frac{6EZ}{L^3} \int_0^L \left(-\frac{1}{L}\right) \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{1,3} = -\frac{6EY}{L^3} \int_0^L \left(-\frac{1}{L}\right) \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{1,4} = 0$$

$$K_{1,5} = -\frac{2EY}{L^2} \int_0^L \left(-\frac{1}{L}\right) \left(\frac{3x}{L} - 2\right) dx = -\frac{EY}{L}$$

$$K_{1,6} = \frac{2EZ}{L^2} \int_0^L \left(-\frac{1}{L}\right) \left(\frac{3x}{L} - 2\right) dx = \frac{EZ}{L}$$

$$K_{1,7} = EA \int_0^L \left(\frac{-1}{L}\right) \left(\frac{1}{L}\right) dx = -\frac{EA}{L}$$

$$K_{1,8} = \frac{6EZ}{L^3} \int_0^L \left(-\frac{1}{L}\right) \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{1,9} = \frac{6EY}{L^3} \int_0^L \left(-\frac{1}{L}\right) \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{1,10} = 0$$

$$K_{1,11} = -\frac{2EY}{L^2} \int_0^L \left(-\frac{1}{L}\right) \left(\frac{3x}{L} - 1\right) dx = \frac{EY}{L}$$

$$K_{1,12} = \frac{2EZ}{L^2} \int_0^L \left(-\frac{1}{L}\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{EZ}{L}$$

$$K_{1,13} = -\frac{4E}{L} \int_0^L \left(-\frac{1}{L}\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = 0$$

$$K_{2,2} = \frac{36EI_z}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{12EI_z}{L^3}$$

$$K_{2,3} = \frac{36EI_{yz}}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{12EI_{yz}}{L^3}$$

$$K_{2,4} = 0$$

$$K_{2,5} = \frac{12EI_{yz}}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 2\right) dx = -\frac{6EI_{yz}}{L^2}$$

$$K_{2,6} = -\frac{12EI_z}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 2\right) dx = \frac{6EI_z}{L^2}$$

$$K_{2,7} = \frac{6EZ}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{2,8} = -\frac{36EI_z}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right) dx^2 = -\frac{12EI_z}{L^3}$$

$$K_{2,9} = -\frac{36EI_{yz}}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = -\frac{12EI_{yz}}{L^3}$$

$$K_{2,10} = 0$$

$$K_{2,11} = \frac{12EI_{yz}}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{6EI_{yz}}{L^2}$$

$$K_{2,12} = -\frac{12EI_z}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = \frac{6EI_z}{L^2}$$

$$K_{2,13} = -\frac{24EZ}{L^2} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = -\frac{8EZ}{L^2}$$

$$K_{3,3} = \frac{36EI_y}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{12EI_y}{L^3}$$

$$K_{3,4} = 0$$

$$K_{3,5} = \frac{12EI_y}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 2\right) dx = -\frac{6EI_y}{L^2}$$

$$K_{3,6} = -\frac{12EI_{yz}}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 2\right) dx = \frac{6EI_{yz}}{L^2}$$

$$K_{3,7} = \frac{6EY}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{3,8} = -\frac{36EI_{yz}}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = -\frac{12EI_{yz}}{L^3}$$

$$K_{3,9} = -\frac{36EI_y}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = -\frac{12EI_y}{L^3}$$

$$K_{3,10} = 0$$

$$K_{3,11} = \frac{12EI_y}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{6EI_y}{L^2}$$

$$K_{3,12} = -\frac{12EI_{yz}}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = \frac{6EI_{yz}}{L^2}$$

$$K_{3,13} = \frac{24EY}{L^2} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = \frac{8EY}{L^2}$$

$$K_{4,4} = GJ \int_0^L \left(-\frac{1}{L}\right) \left(-\frac{1}{L}\right) dx = \frac{GJ}{L}$$

$$K_{4,5} = K_{4,6} = K_{4,7} = K_{4,8} = K_{4,9} = K_{4,11} = K_{4,12} = K_{4,13} = 0$$

$$K_{4,10} = GJ \int_0^L \left(-\frac{1}{L}\right) \left(\frac{1}{L}\right) dx = -\frac{GJ}{L}$$

$$K_{5,5} = \frac{4EI_y}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right)^2 dx = \frac{4EI_y}{L}$$

$$K_{5,6} = -\frac{4EI_{yz}}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right)^2 dx = -\frac{4EI_{yz}}{L}$$

$$K_{5,7} = \frac{2EY}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right) dx = -\frac{EY}{L}$$

$$K_{5,8} = -\frac{12EI_{yz}}{L^3} \int_0^L \left(\frac{3x}{L} - 2\right) \left(1 - \frac{2x}{L}\right) dx = \frac{6EI_{yz}}{L^2}$$

$$K_{5,9} = -\frac{12EI_y}{L^3} \int_0^L \left(\frac{3x}{L} - 2\right) \left(1 - \frac{2x}{L}\right) dx = \frac{6EI_y}{L^2}$$

$$K_{5,10} = 0$$

$$K_{5,11} = \frac{4EI_y}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right) \left(\frac{3x}{L} - 1\right) dx = \frac{2EI_y}{L}$$

$$K_{5,12} = -\frac{4EI_{yz}}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{2EI_{yz}}{L}$$

$$K_{5,13} = \frac{8EY}{L} \int_0^L \left(\frac{3x}{L} - 2\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = \frac{4EY}{L}$$

$$K_{6,6} = \frac{4EI_z}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right)^2 dx = \frac{4EI_z}{L}$$

$$K_{6,7} = -\frac{2EZ}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right) dx = -\frac{EZ}{L}$$

$$K_{6,8} = \frac{12EI_z}{L^3} \int_0^L \left(\frac{3x}{L} - 2\right) \left(1 - \frac{2x}{L}\right) dx = -\frac{6EI_z}{L^2}$$

$$K_{6,9} = \frac{12EI_{yz}}{L^3} \int_0^L \left(\frac{3x}{L} - 2\right) \left(1 - \frac{2x}{L}\right) dx = -\frac{6EI_{yz}}{L^2}$$

$$K_{6,10} = 0$$

$$K_{6,11} = -\frac{4EI_{yz}}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{2EI_{yz}}{L}$$

$$K_{6,13} = -\frac{8EZ}{L} \int_0^L \left(\frac{3x}{L} - 2\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = -\frac{4EZ}{L}$$

$$K_{6,12} = \frac{4EI_z}{L^2} \int_0^L \left(\frac{3x}{L} - 2\right) \left(\frac{3x}{L} - 1\right) dx = \frac{2EI_z}{L}$$

$$K_{7,7} = EA \int_0^L \left(\frac{1}{L}\right) \left(\frac{1}{L}\right) dx = \frac{EA}{L}$$

$$K_{7,8} = -\frac{6EZ}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{7,9} = -\frac{6EY}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) dx = 0$$

$$K_{7,10} = 0$$

$$K_{7,11} = \frac{2EY}{L^2} \int_0^L \left(\frac{3x}{L} - 1\right) dx = -\frac{EY}{L}$$

$$K_{7,12} = -\frac{2EZ}{L^2} \int_0^L \left(\frac{3x}{L} - 1\right) dx = \frac{EZ}{L}$$

$$K_{7,13} = \frac{4E}{L} \int_0^L \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = 0$$

$$K_{8,8} = \frac{36EI_z}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{12EI_z}{L^3}$$

$$K_{8,9} = \frac{36EI_{yz}}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{12EI_{yz}}{L^3}$$

$$K_{8,10} = 0$$

$$K_{8,11} = -\frac{12EI_{yz}}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = \frac{6EI_{yz}}{L^2}$$

$$K_{8,12} = \frac{12EI_z}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{6EI_z}{L^2}$$

$$K_{8,13} = -\frac{24EZ}{L^2} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = \frac{8EZ}{L^2}$$

$$K_{9,9} = \frac{36EI_y}{L^4} \int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{12EI_y}{L^3}$$

$$K_{9,10} = 0$$

$$K_{9,11} = -\frac{12EI_y}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = \frac{6EI_y}{L^2}$$

$$K_{9,12} = \frac{12EI_{yz}}{L^3} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{3x}{L} - 1\right) dx = -\frac{6EI_{yz}}{L^2}$$

$$K_{9,13} = -\frac{24EZ}{L^2} \int_0^L \left(1 - \frac{2x}{L}\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = -\frac{8EY}{L^2}$$

$$K_{10,10} = GJ \int_0^L \left(\frac{1}{L}\right) \left(\frac{1}{L}\right) dx = \frac{GJ}{L}$$

$$K_{10,11} = K_{10,12} = K_{10,13} = 0$$

$$K_{11,11} = \frac{4EI_y}{L^2} \int_0^L \left(\frac{3x}{L} - 1\right)^2 dx = \frac{4EI_y}{L}$$

$$K_{11,12} = -\frac{4EI_{yz}}{L^2} \int_0^L \left(\frac{3x}{L} - 1\right)^2 dx = -\frac{4EI_{yz}}{L}$$

$$K_{11,13} = \frac{8EY}{L} \int_0^L \left(\frac{3x}{L} - 1\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = \frac{8EY}{L}$$

$$K_{12,12} = \frac{4EI_z}{L^2} \int_0^L \left(\frac{3x}{L} - 1\right)^2 dx = \frac{4EI_z}{L}$$

$$K_{12,13} = -\frac{8EZ}{L} \int_0^L \left(\frac{3x}{L} - 1\right) \left(\frac{1}{L} - \frac{2x}{L^2}\right) dx = -\frac{8EZ}{L}$$

$$K_{13,13} = 16E \int_0^L \left(\frac{1}{L} - \frac{2x}{L^2}\right)^2 dx = \frac{16E}{3L}$$