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Characterization of Classes of Parallel Programs and Their Use in Performance Evaluation of Scheduling Policies

by

Terrence Siu-Lun Au, B. C. S.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Computer Science

Ottawa-Carleton Institute for Computer Science
School of Computer Science
Carleton University
Ottawa, Ontario

October 8, 1992

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**"Characterization of Classes of Parallel Programs and Their Use in
Performance Evaluation of Scheduling Policies"**

submitted by

Terrence S. L. Au, B.C.S.

in partial fulfillment of the requirements for
the degree of Master of Computer Science

Thesis Supervisor

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October 8, 1992

ABSTRACT

Scheduling in multiprogrammed shared-memory multiprocessors has been studied for several years. Most of the previous studies modelled the workload by using a simple fork-and-join type of job structure. With this workload, the synchronization structure of parallel programs, which refers to the synchronization constraints among the tasks of the programs, is not considered. However, the performance of scheduling policies could be affected by the presence of synchronization.

An abstract model is introduced based on four different types of parallel programs: 1) fork-and-join, 2) divide-and-conquer, 3) Gaussian elimination and 4) state-space search with the possibility of premature task abortion. Each of these classes represents a different form of synchronization structure. Example programs from different application areas that belong to these classes are given to support the representativeness of the abstract model.

The performance of three scheduling policies, namely, coscheduling, round-robin job and dynamic partitioning are then studied with the new workload model to investigate how synchronization among tasks affects their performance.

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Finally, I would like to present this thesis to my beloved parents.

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CHAPTER I

INTRODUCTION

Advances in computing technology and the demands for faster computing systems have led to the development of different parallel processing systems. In these parallel systems, multiple processors work cooperatively on a parallel solution to a problem. The processors communicate through an interconnection network. The computation of a parallel job (or program) is divided into small chunks, each of which may potentially be executed in parallel. These chunks of a parallel computation are called tasks. Task granularity is an important issue in determining the performance of parallel systems. At one extreme, the entire job can be considered as a single task and at the other extreme we can consider each executable instruction as a task. Fine task granularity allows maximum exploitation of the parallelism that is inherently present in a program. The dataflow architecture is an example that supports task granularity at the instruction level. However, the overheads associated with the management of a large number of small tasks may cause implementation and performance problems. This thesis concentrates on systems that support medium grain size. These systems include the shared-memory parallel systems like the Sequent Symmetry [Lovett & Thakker 88], Deconler HEP [Simth 78] and the Stanford Dash [Lenoski et al. 92].

In a parallel system, perfect speed-up is not always achievable. A poor operating system design and synchronization among tasks are some of the major factors affecting the performance. In particular, the scheduling of tasks to run on multiple processors, an important mechanism of the operating system, directly affects the performance. As a

consequence, scheduling in multiprocessor systems has received considerable attention in the last decade.

Many approaches can be used to study the performance of scheduling algorithms. If the hardware and software are available at hand, we can implement the scheduling algorithms and use benchmarks to evaluate their performance. A benchmark is a complete program that is written in a high level language and is considered to be the representative of a given class of application programs [Kobayashi 81]. The major problem with this approach is that the results obtained are too specific and they can only be applied to that particular type of parallel machine. Another method is to use an abstract model to represent both the hardware and software. The abstract models can purely be mathematical where they can be formulated by using queuing theory. Such abstract model is called an analytical model. The values of the performance measures of an analytical model can be computed directly. However, the systems to be studied are often too complicated and their corresponding analytical models are not mathematically tractable [Kobayashi 81]. High-level simulation is the best alternative to the above techniques. There are two types of simulation: trace-driven and self-driven. Trace-driven simulation, as indicated by the name, refers to the use of traces as the inputs of the simulation program. A trace is a stream of events observed in an operational system. The problem of trace-driven simulation is that the traces of a computer system may not be available. In self-driven simulations, the inputs of the simulation programs are represented by a set of parameters and their values are generated from probability functions with appropriate statistical distributions. Identifying the set of parameters that represents the actual input sequences is important in performing the self-driven simulation.

1.1 Characterizing Parallel Programs

In this section, the methods to represent the parallel programs are discussed. Parallel programs can be characterized at a number of different levels of details [Sevcik 89]. At a higher level, they can be characterized by a single parameter. For example, the maximum parallelism or the maximum number of tasks that can be run in parallel can be used. At a lower level, the data dependency graphs can be used to characterize the parallel programs. High level characterization is not useful for performance evaluation, as a lot of information on parallel programs will be overlooked. For example, if the abstract model consists of a single parameter, i.e., the maximum number of tasks, it cannot be used to describe the synchronization among tasks. However, low level characterization may also be inappropriate because of the high cost of managing huge amounts of information. In the following sub-sections, a survey different approaches to characterize parallel programs is given.

1.1.1 Sevcik's Models

A number of parameters for characterizing parallel programs is defined and their use in decision making for processor allocation is discussed in [Sevcik 89]. The first parameter is the *average parallelism* (A), defined as the average number of busy processors during the execution of the application when an unlimited number of processors are available. It can also be defined as the ratio of the time to execute on a single processor to the time on an unlimited number of processors. The *minimum parallelism* (m) and *maximum parallelism* (M) of an application, are defined as the minimum and maximum number of busy processors during the execution of the application. The last parameter is the *variance of parallelism* (V) of an application, which represents the variance of the number of busy processors when the application is being executed, when there is no limitation on the

processing resources. It should be noted that Sevcik made use of these parameters to improve the performance of static scheduling in multiprocessor systems.

1.1.2 Leutenegger's Models

A relatively high level characterization of fork-and-join parallel programs¹ is presented in [Leutenegger 90]. In their studies, parallel programs are characterized by two parameters, the number of processes and service demand of parallel jobs. The service demand is the execution time of all the processes that a parallel program has. In addition, they assume that the service demand is distributed uniformly among all the processes of the job. Various statistical distributions are used to compute values for these two parameters.

1.1.3 Majumdar et al.'s Models

Another high level characterization of parallel programs is given in [Majumdar et al. 91]. Two parameters are introduced for the characterization. The first one is the *average parallelism* (A) of a job, defined as the average number of processors that are busy during its execution given an unbounded number of available processors. Another parameter is the execution time of jobs with A processors, $\omega(A)$, defined as the execution time attained by a job with A processors when the processors allocated to the job are shared equally by the currently active tasks in the job. The $\omega(A)$ is a measure of the variation in the job's parallelism. A high value of $\omega(A)$ represents a large variation in the parallelism. These two parameters together with the arrival rate form a characterization of parallel programs.

For a given value of A , four structures of parallel programs are used. Each of these structures is represented by a value of $\omega(A)$. The values of $\omega(A)$ of these structures and their descriptions are given in Table 1.1. Different workload models are derived by varying

¹ The meaning of fork-and-join parallel programs will be given in chapter.

the values of these parameters. The workload models are employed to study the performance of static and semi-static scheduling policies.

$\omega(A)$	Description
1	A single phase with parallelism A
$2 - \frac{1}{A}$	A sequential phase followed by a phase with unbounded parallelism
$1.5 - \frac{1}{2A}$	A sequential phase followed by a phase with parallelism $2A-1$
1.5	A phase with parallelism of $\frac{A}{2}$ and a phase of unbounded parallelism

Table 1.1 The values of $\omega(A)$ and their descriptions [Majumdar et al. 91]

1.1.4 Ghosal et al.'s Models

A single parameter for characterizing the behaviour of parallel programs is given in [Ghosal et al. 91]. This parameter is called the processor working set (*pws*).

A number of parameters are defined first, in order give a definition for *pws*. The first parameter is the aggregate execution time of a program $T(p)$ which is the sum of $T_{\text{comp}}(p)$ and $T_{\text{comm}}(p)$. $T_{\text{comp}}(p)$ represents the time for performing actual computation of the program and $T_{\text{comm}}(p)$ is the overhead in the program. Then the efficacy of a parallel program is given by $\eta(p) = \frac{S^2(p)}{p}$, where $S(p) = \frac{T(1)}{T(p)}$ is the speedup of the program given p processors.

Then, the parameter *pws* is defined as the minimum number of processors that achieves the absolute maximum of $\eta(p)$.

In [Ghosal et al. 91], the parameter *pws* is used to study the performance of four programs. Test runs of the programs on a transputer network are conducted to obtain

values for the parameters $T_{\text{comp}}(p)$ and $T_{\text{comm}}(p)$. The values for the efficacy is derived to study the performance of scheduling for parallel systems.

1.1.5 Zahorjan and McCann's Models

A method that makes use of the concept of benchmarking and synthetic programs are used in [Zahorjan & McCann 90]. Three classes of benchmark parallel programs are first selected. The task graph and execution time distribution are then derived for each class of parallel programs. Finally, a special parameter is derived for each class and a statistical distribution is used to generate values for the parameter so that different instances of the parallel programs can be generated. Zahorjan and McCann's approach is considered to be a low-level characterization of parallel programs.

An example of the first class is the *parallel Mean Value Analysis solution package (MVA)* for product form queuing networks containing two classes of N customers each. This is a parallel implementation of the dynamic programming [Horowitz & Sahni 78], commonly used to solve problems where their solutions may be viewed as the result of a sequence of decisions². This parallel program exhibits a wave-front computational structure, in a way that the number of tasks in the program changes according to the sequence 1, 2, 3, ..., N , $N+1$, N , ..., 3, 2, 1 during its execution. The computational structure of this program where N is equal to 5 is depicted in Figure 1.1. The parameter associated with this class of abstract models is *the number of customers N* .

The second class of parallel programs is called the *canonical fork-and-join*. These parallel programs cycle through between sequential phases and phases with K runnable tasks. The parameter associated with this class of abstract models is the number of runnable

² The problem of finding a shortest path from vertex i to vertex j in a directed graph G is another example of problems of this type. The problem is to decide the sequence of vertices that are to be traveled from i to j , where the path from i to j has minimum length.

tasks K . An example program of this type is a parallel implementation of the matrix multiplication given in [McCann et al. 90]. The matrices to be multiplied are divided into fixed-size blocks, and partial results are computed for the corresponding blocks in the result matrix. The parallel computation is done by creating tasks for each block in the result matrix. The structure of these programs is shown in Figure 1.2.

The third class of parallel programs is the *variable fork-and-join*. The task graph of this algorithm is similar to the canonical fork-and-join, that is, repeated cycles of sequential and parallel phases. The only difference is that the number of tasks is varied in each parallel phase. The number of tasks to be created at each parallel phase is decided based on the number of processors available. The structure of these programs is shown in Figure 1.3.

These three classes of parallel program models are employed to study the performance of static and dynamic scheduling policies.

Figure 1.1 The structure of parallel MVA programs [Zahorjan & McCann 90]

Figure 1.2 The structure of canonical fork-and-join programs [Zahorjan & McCann 90]

Figure 1.3 The structure of variable fork-and-join programs [Zahorjan & McCann 90]

1.2 Overview of The Thesis

The remainder of the thesis is organized as follows.

Chapter 2 gives a survey of scheduling policies for multiprogrammed shared memory multiprocessors. At the end of this chapter, we review the problems associated with the workload models in the previous studies to provide motivation for this thesis.

Chapter 3 gives a detailed description of the parallel implementation of four classes of algorithms. We then derive a parallel job model based on these parallel programs. This job model will be used to evaluate the performance of four scheduling policies.

Chapter 4 presents a detailed description of the selected scheduling policies. We also discuss the rationale for choosing these scheduling policies.

Chapter 5 presents the simulation results together with sensitivity analysis of the parameters of the parallel job model. We compare our results to the previous ones to see whether our new job model affects the performance of the scheduling policies.

Finally, Chapter 6 provides a summary of the thesis. The contributions of the thesis and directions for further research are also given.

CHAPTER II

A SURVEY OF SCHEDULING POLICIES

In this chapter, we review previous work in the area of processor scheduling for multiprogrammed shared-memory multiprocessors.

2.1 Previous Work on Performance Evaluation of Scheduling Policies

In this section, we give a brief introduction to the scheduling policies that have been studied in the literature. The scheduling policies are 1) the first-come-first-served policy *FCFS*, 2) the smallest number of tasks first policy *SNTF*, 3) the smallest cumulative demand first policy *SCDF*, 4) the task-based round robin policy *RRtask*, 5) the job-based round robin policy *RRjob*, 6) the dynamic partitioning policy *DP* and 7) the coscheduling policy *Cosched*. The scheduling policies can be divided into three categories, 1) Run-to-Completion, 2) Preemptive and 3) Dynamic.

Run-to-Completion Scheduling

Three *run-to-completion* scheduling policies have been considered extensively. By *run-to-completion*, we mean that when a processor is assigned to execute a task, it continues the execution uninterrupted until the termination. Each of the three policies has its own queuing discipline, the logical ordering of tasks in a queue and determines which task will be chosen for service when a server becomes free. One policy orders the tasks queue on a first come first served basis. The priority depends on some properties of the parallel jobs. These policies are described as follows.

- **First Come First Served (FCFS)** : When a parallel job arrives, its tasks are stored at the end of the queue. If there is a free processor available, a task at the head of the queue is scheduled.
- **Smallest Number of Tasks First (SNTF)** : The queue is ordered on a priority basis with the highest priority given to the tasks of the parallel job that has the smallest number of tasks. When two jobs have the same number of tasks, the one that arrives earlier gets a higher priority. When a processor becomes idle, the first task from the queue is scheduled.
- **Smallest Cumulative Demand First (SCDF)** : The queue is ordered on a priority basis with the highest priority given to the tasks of the parallel job which has the smallest cumulative demand. The cumulative demand is the sum of the service times of a job's tasks. When two jobs have the same cumulative demand, the one that arrives earlier gets a higher priority. When a processor becomes idle, the first task from the queue is scheduled.

Preemptive Scheduling

The run-to-completion scheduling policies have the problem that when a task is assigned to a processor, it must be executed until its termination. Consider the *SCDF* policy. At time 0, there is one job on the queue with 4 units of service demand. We assume that all processors are used by this job. Later on, say at time 1, a new job comes with 1 unit of service demand. The new job will only be served after the first job terminates, even though the new job has smaller service demand than the old job.

To solve this problem, the system can deschedule some tasks of the running jobs so that other jobs can be served. This is called the preemption of tasks from processors. There are two types of preemptive scheduling policies: *variable* and *fixed*.

For variable preemptive scheduling, the processor reallocation only occurs when a new job arrives or an old job terminates. We consider the variable preemptive scheduling policies with *SNTF* and *SCDF* queuing discipline.

- **Preemptive Smallest Number of Tasks First (*PSNTF*)** : This is a variation of the *SNTF*. A priority is assigned to each job on the queue. The priority value depends on the number of tasks in the parallel jobs. The decision of preempting tasks from the processors is done by comparing the priority (number of tasks) of the new job to the jobs that are currently being executed.
- **Preemptive Smallest Cumulative Demand First (*PSCDF*)** : This is a variation of the *SCDF*. The task queue is organized on a priority basis. The priority value of the jobs depends on the remaining cumulative demand associated with them. A newly arrived job can preempt processors from the tasks that have lower priority (smaller cumulative service demand).

Round-Robin Scheduling

In a round-robin policy, services occur in increments of some parameter α , say $\alpha = 2$ time units. When a task gets to the head of the queue, the task receives $\alpha = 2$ time units of service. After being served for $\alpha = 2$ time units, the task is descheduled from the processor and placed back in the queue. The parameter α is called a quantum. Three variations of the round-robin scheduling have been studied intensively. They are described as follows.

- **Task-based Round robin (*RRtask*)** : When a job arrives, it is divided into a set of tasks that are placed at the end of the shared task queue. The round robin scheduling is applied to the task queue.

- **Job-based Round robin (*RRjob*)** : This policy is similar to the Task-based Round robin policy but instead of sharing a task queue, there is a shared job queue. Each entry in the job queue has a queue maintaining its own tasks. Scheduling is done round robin on the parallel jobs. The job at the front of a queue receives P quanta of size q , where P equals to the number of processors in the system.
- **Coscheduling (*Cosched*)** : The queuing discipline is like the one for the task-based round robin policy. Scheduling is done by moving a window of length equal to the number of processors in the system along the task queue. Each task in the window gets one quantum of service on a processor. At the end of the quantum, the window is moved down the queue until the first slot of the window is over the first task of a job that was not scheduled in the previous quantum. The quantum completes for each processor at the same time.

Dynamic Scheduling

Under the dynamic scheduling policies, processors are reallocated among jobs in response to changes in the jobs' parallelism, arrival or termination of parallel jobs. The goal of this scheduling policy is to allocate processors to parallel jobs as evenly as possible. Processors may be preempted from one job to another, if it is required. The policy is called dynamic partitioning as it partitions the processors among the jobs in the system.

- **Dynamic Partitioning (*DP*)** : When a job arrives, the following actions are taken: 1) if there are idle processors allocate them to the job. If the job needs fewer processors than the free processors, just allocate as many as are needed. 2) If there is no idle processor and there is a job that holds more than one processors, a processor is preempted from this job and is allocated to the new job, 3) If no processor is idle and no job has two or more processors, the new job must wait until a scheduled job departs the system.

When a task finishes, the following actions are taken: 1) if the job has other tasks still not scheduled, schedule one of them on the processor; 2) if the job has no unscheduled tasks allocate the processor to the first job that has unallocated tasks, based on their arrival time.

Figure 2.1 Performance of the scheduling policies [Leutenegger and Vernon 90]

Figure 2.1 shows the result of an experiment conducted by Leutenegger and Vernon that illustrates the performance of the policies. In Leutenegger and Vernon's paper, processes are assumed to be the smallest schedulable unit and a process may execute several tasks serially. In our case, we assume that tasks are the smallest schedulable units and there is a single queue of runnable tasks in the system. We will discuss their results based on some assumptions: 1) a parallel job is a collection of independent tasks and there

is no communication among tasks, 2) parallel jobs are modeled by two parameters, number of tasks and service demand and 3) there is a positive correlation between these parameters. Therefore, jobs with large service demand always have large number of tasks.

Results of the run-to-completion scheduling

- First Come First Served (*FCFS*) : *FCFS* is the worst policy as shown in Figure 2.1. The major problem of *FCFS* is that it allows jobs with large service demand to monopolize the processing power, making the smaller jobs wait unnecessarily. A longer waiting time results in a longer response time.
- Smallest Number of Tasks First (*SNTF*) : *SNTF* performs poorly and it is the second worst policy. *SNTF* suffers from the same problem as the *FCFS*, by permitting large jobs to dominate over the smaller ones.

Results of variable preemptive scheduling

- Preemptive Smallest Number of Tasks First (*PSNTF*) : *PSNTF* performs poorly, even though preemption is used. This is because the number of tasks in a parallel job does not give accurate information on the size of a parallel job. That is a large job can have small number of tasks with large service demand for each task of the job. Hence, this policy performs similar to the *FCFS* and *SNTF* policies.
- Preemptive Smallest Cumulative Service Demand First (*PSCDF*) : it is the best scheduling policy. Since the cumulative service demand is used for determining which tasks are to be scheduled, the processing power of the system can be distributed equally among the parallel jobs.

Results of the round robin policies

In general, the round robin policies perform better than the static and the preemptive static policies, except for the *PSCDF* policy. In a round robin policy, if both the task-based and job-based, is applied, the effect of large jobs dominating over the smaller ones can be reduced. The result of each policies is discussed below.

- Tasks-based Round Robin (*RRtask*) and Coscheduling (*Cosched*) : Since *RRtask* and *Cosched* perform similarly, we will discuss the result of these policies together. Note that *Cosched* performs slightly worse than *RRtask* which implies that it is not worth to have coscheduling without process communication. Due to the ability to time-slice the processing power among the jobs in the system, the effect of the large jobs can be reduced. As a result, *RRtask* performs better than the non-preemptive policies, like *FCFS* and *SNTF*. However, the allocation of processing power is still unfair, as large jobs can still dominate if the number of tasks is correlated to the size of the jobs. Hence, the jobs with large number of tasks may monopolize the system leading to a performance degradation. This problem is overcome by the job-based Round Robin policy.
- Job-based Round Robin (*RRjob*) : Since the policy varies the quantum size according to the number of tasks in a job, the allocation of processing power does not depend on the job size. The improvement makes *RRjob* the best in the group round robin policies.

Results of Dynamic Scheduling

- Dynamic Partitioning (*DP*) : Since *DP* can vary the number of processors allocated to parallel jobs according to the number of executable tasks in the job, the dynamic partitioning performs well. Its performance is comparable to that of the *RRjob* policy. However, this policy performs poorly when the system load is at around 90%, because

the number of parallel jobs exceeds the number of processors in the system which results in a large number of jobs waiting in the queue.

The performance results of the above algorithms is summarized in Table 2.1. We assign a rank to each policy and order the table according to their rank.

Scheduling Policies	Categories	Rank
<i>PSCDF</i>	Preemptive (fixed)	1
<i>RRjob</i>	Preemptive (variable)	2
<i>DP</i>	Dynamic	3
<i>RRtask</i>	Preemptive (variable)	4
<i>Cosched</i>	Preemptive (variable)	5
<i>PSNTF</i>	Preemptive (fixed)	6
<i>SNTF</i>	Run-to-completion	7
<i>FCFS</i>	Run-to-completion	8

Table 2.1 Performance of Scheduling Policies

2.2 Motivation for The Thesis

It can be seen from the review of the models that many of them cannot be used to specify the synchronization structures of parallel jobs. Some exceptions are [Zahorjan & McCann 90] and [Majumdar et al. 91], where some forms of synchronization structures are considered. Synchronization is a fundamental issue in parallel processing and it is not clear whether changing from one synchronization structure to another would affect the performance of scheduling policies. In the thesis, four classes of parallel programs are considered, each of which represents a synchronization structure. The classes of parallel programs include, the fork-and-join, divide-and-conquer, gaussian elimination and state-space search. These classes will be discussed in Chapter 3. The objective of the thesis is to investigate the effects of changing the synchronization structures on the performance of multiprocessor scheduling.

CHAPTER III

AN ABSTRACT MODEL OF PARALLEL PROGRAMS

In this chapter, a survey of some parallel implementations of algorithms for shared-memory multiprocessors is presented. The structure of these parallel programs is discussed. Then an abstract model of parallel programs that is based on the structure and other characteristics of the parallel programs surveyed is proposed. The abstract model is used to drive the simulation experiments to compare performance of scheduling algorithms. The remainder of this chapter is organized as follows. The next four sections, present various parallel implementations of algorithms. The abstract model is then presented in Section 3.6.

3.1 Parallel Fork-and-Join Programs

In this section, the algorithms which can be decomposed into independent sub-computations are considered. A simple example is the algorithms that consist of a single loop and each instance of the loop can be executed independently. The parallel implementation of these types of algorithms can be done by creating tasks to work on the sub-computations.

Example 1: Root-Search

Given a continuous function f having opposite signs at the end-points of an interval of length L , locate a zero of the function f within a unit interval. The parallel implementation of this algorithm is proposed in [Akl 89] and [Kung 76]. The interval to be evaluated is divided into N sub-intervals and are assigned to the tasks. Each task attempts to find a root within their assigned sub-interval by using the Newton method. The tasks are

synchronized when all of them finish and a root is found. The parallel program is shown in Figure 3.1. Since $f(x)$, $f'(x)$, and $f''(x)$ are continuous and bounded on each interval (a, b) , at least one interval will embrace a root of the function f .

```

PROCEDURE Root_Search (  $f, f', a, b, c, r, \text{ROOT}$  )

     $s := (b - a) / (N + 1)$ 
    FOR  $k := 1$  TO  $N$  DO
        create task  $k$ 
    ENDFOR

    TASK  $k$ 
         $X_{old} := a + k s$ 
        iteration := 0
        WHILE (iteration <  $r$  ) AND ( $\text{ROOT} = \infty$  ) DO
            iteration := iteration + 1
             $X_{new} := X_{old} - f( X_{old} ) / f'( X_{old} )$ 
            IF abs(  $X_{new} - X_{old}$  ) <  $c$  THEN  $\text{ROOT} := X_{new}$ 
             $X_{old} := X_{new}$ 
        END WHILE
    END TASK  $k$ 

END PROCEDURE

```

Figure 3.1 A parallel root-search (or zero-search) program

Example 2: The Parallel n-body Algorithm

Another example algorithm with the fork-and-join structure is the parallel n-body algorithm given in [Almasi & Gottlieb 89]. The basic problem can be stated as follows: given a collection interaction energy between bodies, predict the change in the interaction energy of the bodies over a period of time. These bodies are considered to be spherical with center-of-mass coordinate vectors $r_1, r_2, \dots, r_n$.

We define the interaction energy between two bodies to be

$$E_{ij}(r) = B_i(r^*/r)^9 + z_i z_j B_j/r$$

where $z_i = \pm 1$ is the charge on ion i , the parameters B_1 , r^* , and B_2 are special constants and $r = r_i - r_j$. And the binding energy of one of the bodies is defined as

$$B_i = \sum_{j \neq i} E_{ij}$$

It is assumed that the initial configuration and values of the initial energy are available. The bodies are moved randomly to generate a Markov chain of configurations. In this algorithm, the bodies are moved one at a time and the energy of the system after a move is given in terms of the energy prior to the move by

$$E_i^{\text{new}} = E_i^{\text{old}} + B_i^{\text{new}} - B_i^{\text{old}} \quad 1$$

The bottleneck of the algorithm involves the computations required to calculate the binding energy of the moved body, following and preceding its actual move. This computation of the binding energy involves evaluating the interaction of the moved body with each of the other bodies. However, the interactions can be evaluated in parallel with no communication between the computation. The algorithm is shown in Figure 3.2. In Figure 3.2, the function `barriersync()` specifies the points where parallel tasks perform their synchronization. The structure of this program is shown in Figure 3.3.

PROCEDURE n-body ()

WHILE (1)

 select a body to be moved

FOR $j = 1$ **TO** n **DO**

1 i is the moved body.

E_i^{old} (E_i^{new}) is a vector of interaction energy between the moved body i and all the other bodies before

(after) the move.

B_i^{new} (B_i^{old}) is the binding energy of i before (after) the move.

```

        create task j
    ENDFOR

    TASK j
        /* compute the binding energy between body j and the moved body i.
           (before and after the move)
           that is  $B_i^{new}$  and  $B_i^{old}$  */
    ENDTASK

    barriersync ();
    compute  $E_i^{new} = E_i^{old} + B_i^{new} - B_i^{old}$ 

ENDWHILE
ENDPROCEDURE

```

Figure 3.2 A parallel n-body algorithm

Example 3: A Parallel Event Driven Simulator

Another example of the fork-and-join structure is a parallel, distributed time, event driven simulator for verifying the behaviour of a logic circuit. This simulator is based on a distributed-time algorithm given in [Chandy & Misra 81].

In this simulator, a logic circuit is modeled by a collection of elements, represented by its inputs, outputs and internal state. These elements are interconnected by wires. An element can be any functional block in the circuit. It can be as simple as a two input gates such as AND-gate or OR-gate or it can be as complicated as a CPU. The goal of performing logic circuit simulation is to test the circuit by feeding different inputs and see whether the circuit has the expected output behaviour. An event in the simulation corresponds to a computation of the new output behaviors of a gate by feeding in the given inputs.

Discrete event simulation is inherently sequential because of the presence of time ordering. However, it can be parallelized by having the elements in the circuit maintain their own time independently of other elements. Different elements might have different notions

of the current simulated time. If something occurs at each of the inputs of an element, it will receive a time-stamped event on its outputs. In actual implementation, an element is activated for evaluation when it receives a new event. An activation of an element will generate a task that computes the outputs behaviour of the element. In [Singh et al. 91], the algorithm is implemented in a way that allows the possibility of deadlocks signaled by having no task on the queue. The computation of tasks can be done independently. If a deadlock occurs, the algorithm will resolve it and continue the execution as usual. The parallelism in the algorithm reduces to 1, when a deadlock occurs. The structure of this parallel program is the same as the one for the parallel n-body program shown in Figure 3.3.

Figure 3.3 The task graph of the parallel n-body

3.2 Parallel Divide-and-Conquer Programs

In this section, a parallel implementation of the divide-and-conquer algorithms is considered. Large numbers of parallel programs have been developed that are based on this strategy, especially from the area of symbolic computation, where the computation works on a discrete data structure. In general, the divide-and-conquer strategy does not restrict the number of sub-problems into which a given problem is partitioned. However, we will restrict ourselves into the binary case, where a given problem will only be divided into two

subproblems. Thus, the parallelism in these programs steps through the sequence $2^0, 2^1, \dots, 2^{n-1}, 2^n, 2^{n-1}, \dots, 2^1, 2^0$ during their execution. Efficient parallel implementations of divide-and-conquer algorithms have been developed in many areas of application, such as, database management [Bitton et al. 83], [Deminet 82], [Evans & Yousif 85], numerical integration [Grit & McGraw 85], computational geometry [Knecht 90] and VLSI circuit design [Hachtel & Moceyunas 88] and combinatorial problems [Knecht 89], [Shekhawat & Chakravarty 90]. We will consider the implementation of a subset of these algorithms.

3.2.1 Implementing Divide-and-Conquer Algorithms in Parallel

An implementation of the divide-and-conquer algorithms on parallel systems has received significant attention in the last decade. One way to parallelize the divide-and-conquer algorithm is to use the COBEGIN/COEND construct. This implementation is given in [Horowitz & Zorat 83], where a schema of the parallel implementation of divide-and-conquer algorithms is presented. A reproduction of the schema is shown in Figure 3.4.

```

PROCEDURE DaC ( I : array; O : array )
  local S1, S2, ..., Sk : array;
  local T1, T2, ..., Tk : array;
  IF SMALL ( I, O )
    return ( ANSWER( I : O ) );
  ELSE
    SPLITINPUT( I; S1, S2, ..., Sk );
    COBEGIN
      DaC( S1; T1 ); DaC( S2; T2 ); ...; DaC( Sk; Tk );
    COEND
    COMBINE( T1, T2, ..., Tk, I; O )
  ENDIF
ENDPROCEDURE

```

Figure 3.4 A schema for implementing divide-and-conquer in parallel
[Horowitz & Zorat 83]

The function SMALL determines whether further partition of the array I is needed. The function SPLITINPUT generates k input arrays for the next recursive step based on the original input I . Within the COBEGIN/COEND, k independent tasks are created and

executed simultaneously. The function COMBINE groups together the outputs of the recursive calls and produces the output.

When the above program is executed, k independent inputs $S_1, S_2, \dots, S_k$ are generated from the original input I . Thus, there is no data dependence among tasks that work on the input $S_1, S_2, \dots, S_k$. The data for $S_1, S_2, \dots, S_k$ can either consist of one (or more) unmodified copy (copies) of the data in I , partitioned among k structures, or it may consist of data that is actually computed from the data in I . The above program represents a general divide-and-conquer where k recursive calls will be made at each level. However, we are only interested in the binary case where there are at most only two recursive calls at each level.

3.2.2 Examples of the Parallel Implementation of Divide-and-Conquer Algorithms

In this section, we present some example parallel implementations of divide-and-conquer algorithms on multiprocessors.

Example 1: Parallel QuickSort

The first one to be discussed is the quick sort given in [Deminet 82], [Evans & Yousif 85]. It first creates tasks to partition the input array using a median element, then generates more tasks to further divide or sort the subarrays sequentially, if the size of the subarrays is less than or equal to a predefined limit. Note that the program does not require to merge any of the subarrays. The parallel implementation of the quicksort algorithm has received considerable attention. The earliest one is the implementation on the Cm* multiprocessor. It uses a stack to store the descriptors for a repeated subset of the array that has not yet been sorted. A descriptor is a pointer to the first element of the sub-array to be sorted. In fact, a descriptor is same as a task which holds the information about the

indivisible computation. A similar implementation of the quicksort algorithm is in [Evans & Yousif 85]. In Evans and Yousif's work, a task queue is used to store the tasks generated at each level of the recursive call. The merge tasks of the parallel quicksort program are trivial and they are not present in the program. A parallel implementation of the Quick Sort algorithm is shown in Figure 3.5.

```

PROCEDURE Quick_Sort ( A, i, j )
  IF the size of A[ i, . . . , j ]  $\geq$  bound
    THEN

```

Let v be the larger of the 1st two distinct keys found PERMUTE A[i], . . . , A[j] so that $A[i, \dots, k-1] < v$ and $A[k+1, \dots, j] \geq v$
--

← The input array is divided into two subarrays. Tasks will be created to sort these subarrays.

```

  COBEGIN

```

```

    Quick_Sort ( A, i, k )

```

```

    Quick_Sort ( A, k, j )

```

← The subarrays will be sorted concurrently.

```

  COEND

```

← The combine operation in this procedure is trivial.

```

  ELSE

```

```

    sort A[i, . . . ,j] sequentially on a single CPU

```

```

  ENDIF

```

```

ENDPROCEDURE

```

Figure 3.5 A parallel implementation the Quicksort algorithm

Example 2 : Parallel Bubble-Merge Sort

Merge sort is a common sorting algorithm. In [Hagan et al. 85], a sorting algorithm called bubble-merge sort is implemented on the Denelcor HEP multiprocessor. Its parallel implementation is depicted in Figure 3.6. To accomplish the sorting, the original n key population is partitioned into d sets. For each of these sets, the bubble sort program is used to sort these subarrays. Then, the parallel merge sort is used to merge the subarrays together until a single array is formed.

Figure 3.6 A parallel implementation of the mergesort algorithm

Example 3: Parallel Adaptive Quadrature

In this section, we present a parallel implementation of an algorithm for solving the quadrature problem discussed in [Grit & McGraw 85]. Given a continuous, nonnegative function $f(x)$, the quadrature problem is to compute the area bounded by $f(x)$, the x axis, and vertical lines through l and r . An illustration of the problem is shown in Figure 3.7. The algorithm begins with one interval from l to r and computes the midpoint m between l and r . It then calculates the areas of three trapezoids: (1) the large one bounded by l , r , $f(l)$ and $f(r)$; (2) the small one bounded by l , m , $f(l)$, and $f(m)$; and (3) the small one bounded by m , r , $f(m)$, and $f(r)$. Next, the area of the larger trapezoid is compared with the sum of the areas of the two smaller ones. If these are sufficiently close, the area of the larger trapezoid is an acceptable approximation of the area under f . Otherwise, the process is repeated by solving the two subproblems of computing the area from l to m and from m to r . The recursion continues until the approximation becomes acceptable. Figure 3.7 illustrates how the problem is subdivided.

Figure 3.7 An illustration quadrature problem

This is called adaptive quadrature since the solution adapts itself to the shape of the curve. In particular, in places where the curve is flat, a wide trapezoid will closely approximate the area. Hence new subproblems will not be generated. In places where the curve is changing shape, smaller and smaller subproblems will be generated as needed. An implementation of the parallel adaptive quadrature algorithm is given in Figure 3.8.

The procedure `Adaptive_Quadrature` has five input parameters. A left point a , a right point b , the values of $f(a)$ and $f(b)$, and the area of the trapezoid defined by these four points. (The area is included to avoid repeating the computation). It keeps on dividing the inputs until an acceptable answer is found. It is similar to the parallel Quicksort as they do not have an explicit combine operation.

PROCEDURE Adaptive_Quadrature (a, b, fa, fb, area : real)

```
tarea := (f (l) + f (r) )*(l + r) / 2
m := (a+b)/2;
fm := f(m)
compute larea and rarea using trapezoids
diff := tarea - (larea + rarea)
```

← This divide operation computes the three different areas for the current iteration. And decides whether further division is needed

```
IF (diff <= epsilon)
  return tarea
ELSE
```

COBEGIN

```
Adaptive_Quadrature(a, m, fa, fm, larea)
Adaptive_Quadrature(m, b, fm, fb, rarea)
```

← The sub-problems are solved concurrently

COEND

← The combine operation in this procedure is trivial and the synchronization point is implicit

```
ENDIF
END PROCEDURE
```

Figure 3.8 A parallel implementation of the adaptive quadrature algorithm

Example 4 : Parallel Convex Hull

A parallel implementation of the convex hull algorithm is given in [Knecht 90]. The problem is stated as follows: given a point set S consisting of n points in two dimensions, the convex hull is the boundary of the smallest convex domain in $(E^2)^2$ containing the point set S [Knecht 90]. To compute the convex hull, the point set S is subdivided with respect to their x coordinates into two subproblems which can then be solved independently. The point set S is recursively divided into sub-sets until they contain at most three points. At each level of the recursive call, the convex hull of each subsets are combined together to form a convex hull. The parallel program is shown in Figure 3.9.

² E^2 is the 2-D Euclidean space, that is the plane. Each point in E^2 consists of two co-ordinates that is the x and y coordinates.

PROCEDURE Parallel_CH (Input: a point set S)

Subdivide the point S into $S_1 = \{a_1, \dots, a_{\frac{1}{2}n}\}$ and

$S_2 = \{a_{\frac{1}{2}n+1}, \dots, a_n\}$

COBEGIN

Parallel_CH(S_1), Parallel_CH(S_2)

COEND

Combine (S_1 , S_2)

ENDPROCEDURE

Figure 3.9 A parallel implementation of the convex hull algorithm

Figure 3.10 Illustration of the planar merge procedure

The combine operation proposed in [Preparata & Hong 77] is used. It is assumed that the point set S is sorted by their x-coordinates. Given two sub-convex hulls, the combine operation finds the ordered convex hull of a set of two-dimensional points, i.e.

not only the set of vertices of the convex polygon representing the hull but also the sequence of the vertices on the boundary of this polygon. The left and right tangents are found by computing the L_b and R_b points of the convex polygon B, as well as the L_a and R_a points of the convex polygon A. L_b is the leftmost point of the polygon B and R_b is the rightmost point of the polygon B. Then, the resulting convex hull is computed by deleting the points that are bounded by the two tangents. Figure 3.10 uses an example to illustrate how the combine operation works.

3.2.3 The Structure of Parallel Divide-and-Conquer

The parallel implementation of divide-and-conquer algorithms exhibits a partitioning structure [Madala & Sinclair 91]. The tasks created by this type of programs perform one of the following three operations: 1) splitting the inputs, 2) working on the inputs sequentially and 3) combining the outputs. Tasks that divide the input structures are called *DIVIDE* tasks. Tasks that work on the input structures sequentially are called *WORK* tasks. Tasks that merge the output are called *MERGE* tasks. The task system of parallel divide-and-conquer programs is depicted Figure 3.11. It gives a high-level view of the structure of the parallel implementation of divide-and-conquer algorithms.

Corresponding to the program scheme shown in Section 3.3.1, the *DIVIDE* tasks are those that performs the split-input operation. The *WORK* task executes the code of the *ANSWER* procedure. The *MERGE* tasks execute the *COMBINE* operation. Generally, the execution time of the tasks decreases level by level over the upper half of the structure, and increases level by level over the lower half of the structure. Synchronizations are done at the *MERGE* stage.

Figure 3.11 The task system of divide-and-conquer algorithm

In summary, we surveyed a number of parallel implementations of the divide-and-conquer algorithms for the shared-memory multiprocessors. The examples given in this section only represent a subset of algorithms that belong to this class.

3.3 Parallel Matrix Factorization

We are now going to discuss the implementation of a family of parallel algorithms for factorizing dense matrices. Factorization of dense linear matrices is an extremely important problem in the area of linear algebra. Once a matrix is factorized into some simpler form, its corresponding system of linear equations can be solved immediately. These algorithms use a method which computes the factorization directly from the matrix itself as opposed to finding an approximation to the factorization of the matrix.

3.3.1 Examples of Matrix Factorization Algorithm and Their Parallel Implementations

Example 1: The LU Decomposition Algorithm

To solve a matrix equation of the form $Ax = b$, one can decompose A into upper U and lower L triangular matrices and then solve it by solving $Ly = b$ and $Ux = y$. The most computation intensive part of the algorithm is to find the LU decomposition. A LU decomposition algorithm, proposed by [Lord et al. 83], for the shared-memory multiprocessors will be reviewed. The process of forming the upper and lower triangular matrices relates to the direct method for solving the above matrix equation. Consider the direct method of reducing a matrix by performing a series of elementary row operations. The only allowed operation is the addition of a multiple of one row to the rows below. We call this the *basic row operation*.

$$A_1 = \begin{bmatrix} 2 & 4 & -2 \\ 1 & -1 & 5 \\ 4 & 1 & -2 \end{bmatrix}$$

If $\frac{1}{2}$ times the first row is subtracted from the second and 2 times the first subtracted from the third row, the result will be the matrix

$$A_2 = \begin{bmatrix} 2 & 4 & -2 \\ 0 & -3 & 6 \\ 0 & -7 & 2 \end{bmatrix}$$

in which the first column of the second and third rows is reduced to zeros. The second column of the third row can be reduced to zero by subtracting $\frac{7}{3}$ times the second row. The result is as follows.

$$A_3 = \begin{bmatrix} 2 & 4 & -2 \\ 0 & -3 & 6 \\ 0 & 0 & -12 \end{bmatrix}$$

From the above row reduction, we obtain an upper triangular matrix A_3 . Each of the above reductions corresponds to a premultiplication of an elementary triangular matrix³, where

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \quad M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -\frac{7}{3} & -1 \end{bmatrix}$$

We can see that $A_3 = M_2 M_1 A_1$. As M_1 and M_2 are non-singular matrices and their inverse M_1^{-1} , M_2^{-1} exist. Thus, from the above equation, we have

$$A_1 = M_1^{-1} M_2^{-1} A_3$$

Therefore, a matrix can be reduced to an upper triangular matrix A_3 by premultiplying a series of elementary matrices. Since M_1 and M_2 are lower triangular matrices, the product of M_1 and M_2 must be a lower triangular matrix. If we let, $L = M_1^{-1} M_2^{-1} = M_1 M_2$ and $U = A_3$, we have $A_1 = LU$, which is the LU decomposition of the matrix A_1 .

In general, given a matrix $A = M_1^{-1} M_2^{-1} \dots M_k^{-1} \dots M_n^{-1} A_n$ where the M_i 's are elementary lower triangular matrix and A_n is an upper triangular matrix, then $L = M_1^{-1} M_2^{-1} \dots M_k^{-1} \dots M_n^{-1}$ and $U = A_n$ is the LU decomposition of A .

The LU decomposition algorithm is shown in Figure 3.12. The main body of the algorithm is the k loop. In its k^{th} iteration, the k^{th} pivot column is first computed. The pivot column consists of the multipliers for performing the basic row operations⁴ in the k^{th} reduction of the matrix A . And, it is equal to $[a_{kk}, \frac{a_{(k+1)k}}{a_{kk}}, \dots, \frac{a_{nk}}{a_{kk}}]^T$. The multipliers are

³a matrix, obtained from an $n \times n$ identity matrix I_n by performing a basic row operation

⁴for the definition of basic row operation, see page 32.

then used to perform the *basic row operations* on the submatrix to its right. An illustration of the *reduction* is shown in Figure 3.13.

PROCEDURE LU_Decomp (A(n,n))

FOR k := 1 TO n - 1 DO

FOR l = k+1 TO n DO
 $A(l, k) = A(l, k) / A(k, k)$
ENDFOR

l loop
 Compute the kth
 pivot column

FOR j = k+1 TO n DO
FOR i = k+1 TO n DO
 $A(i, j) = A(i, j) - A(i, k) * A(k, j)$
ENDFOR
ENDFOR

j loop
 Use the pivot column
 to modify all the
 columns to its right

ENDFOR

ENDPROCEDURE

Figure 3.12 The LU Decomposition algorithm

Figure 3.13 An illustration of the reduction where $k=2$

This algorithm works fine as long as there is no small pivot⁵ during the reduction of the matrix A. Division of a small pivot results in a large rounding error that propagates through the execution. The problem will be very severe if the algorithm is implemented on

⁵A pivot is the first element in the pivot column.

computers that perform finite digit arithmetic. Consider the following example that illustrate the problem of having a small pivot. A small pivot is equal to 0.001, located in the upper-leftmost corner of the matrix A_1 .

$$A_1 = \begin{bmatrix} 0.001 & 2.000 & 3.000 \\ -1.000 & 3.712 & 4.632 \\ -2.000 & 1.072 & 5.643 \end{bmatrix}$$

Then a four-digit realization of the reduction of A to upper triangular form produces the following matrices.

$$M_1 = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 1000.0 & 1.000 & 0.000 \\ 2000.0 & 0.000 & 1.000 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.001 & 2.000 & 3.000 \\ 0.000 & 2004.0 & 3005.0 \\ 0.000 & 4001.0 & 6006.0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & -1.997 & 1.000 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.001 & 2.000 & 3.000 \\ 0.000 & 2004.0 & 2005.0 \\ 0.000 & 0.000 & 5.000 \end{bmatrix}$$

The (3,3)-element of the resulting matrix A_3 has very large round off error, that is the resulting value is 5.000 compared to the real value of 5.922. However, if the first and second row of the matrix are interchanged, the matrix can be reduced with very little numerical error. This new matrix is stated as follows. It is equivalent to A_1 and can be reduced without any difficulty.

$$A_3 = \begin{bmatrix} -1.000 & 3.712 & 4.623 \\ 0.001 & 2.000 & 3.000 \\ -2.000 & 1.072 & 5.643 \end{bmatrix}$$

From this observation, we see that the algorithm should be modified in such a way that it must search the *pivot column* for a maximum element and perform row interchange to make the maximum element to become the pivot. The algorithm for computing LU decomposition with partial pivoting is given in Figure 3.14.

Figure 3.14 The algorithm for LU decomposition with partial pivoting

A number of researchers have worked on parallelizing the above algorithm. The most common approach is to derive a task system for the above algorithm and perform static scheduling on this task system [Lord et al. 83].

An implementation of the LU decomposition on the DASH shared-memory multiprocessor is discussed in [Gupta et al. 91]. In this implementation, the matrix is decomposed on a columnwise basis. Columns are statically assigned to the processors in an interleaved fashion. Starting from left to right, columns are used to modify all columns to their right. Processors wait until the columns owned by them are ready to be modified.

Then they use the pivot column to modify all columns owned by them to the right of the pivot. When a processor finishes a column, it releases all the processors that are waiting for that column.

Another parallel implementation is to have processes for each column of the matrix. At each iteration, tasks that perform the operation on the columns are created and assigned to the processes. We assume that only one task is assigned to a process, in each iteration. There are two shared variables in this program, namely, the matrix A and an array of Pivot record. A pivot record contains two fields, flag and value. The flags in the pivot record are used to ensure that a pivot is computed before it is being used. We consider this implementation of the algorithm and it is shown in Figure 3.15.

The program begins its execution by creating processes for each column of the matrix. During the execution, tasks are created and assigned to the processes. When a processor becomes idle, a process (or a task) is scheduled to run on it. We assume that each processor has local memory. When it starts the computation for task *i*, it copies all the shared variables that will be accessed prior to performing the computation for task *i*. The concurrence control mechanism for this algorithm is locking. A task locks the column of the matrix before it performs any modification on it. The column will be released when the computation on that column is completed.

```

Pivot_Rec = {
    Flag : Boolean;
    Value : integer;
}

Shared Variables [A(1:n, 1:n) of real, Pivot(1:n) of Pivot_Rec]
Global initialize ( A, Pivot )

TASK Multiply_Column
    local Variables[ col(1:n), locPiv:Pivot_Rec]
    FOR k=1 TO n-1 DO
        IF (k==task_Id)
            lock(A(1:n, k))
            get(A(1:n,k), col(1:n))

```

```

        find l such that
        |col(l,k)| = max(|col(k,k)|, . . . ,|col(n,k)|)
        locPiv.Value = l
        Interchange(col(locPiv.Value, k), col(k,k))
        FOR i=k+1 TO n DO
            col(i,k) = col(i,k)/col(k,k)
        ENDFOR
        locPiv.Flag = TRUE
        put(v(i,k), A(1:n,k))
        unlock(A(1:n, k))
        lock(Pivot(k))
        put(locPiv, Pivot(k))
        unlock(Pivot(k))
    ELSE
        j = TASK_ID
        IF ( j ≥ k+1 )
            wait until Pivot(k).Flag == TRUE
            lock ( A(1:n, j) )
            get( A(1:n, j), col(1:n) )
            get( Pivot(k), locPiv )
            Interchange( col(locPiv.Value,j), col(k,j) )
            FOR i=k+1 TO n DO
                A(i,j) = A(i,j) - A(i,k) * A(k,j)
            ENDFOR
            put( col(1:n), A(1:n, j) )
            unlock ( A(1:n, j) )
        ENDIF
    ENDIF
ENDFOR
ENDTASK

```

Figure 3.15 A parallel implementation of the LU decomposition algorithm

Example 2: The Gauss-Jordan Algorithm

The Gauss-Jordan method is used to solve a linear system of equations. It is similar to the LU decomposition algorithm except that in the k^{th} *pivot column modification*, not only the row $i+1$ to n are modified but also the 1 to $i-1$ rows. At the end, it gives a diagonal matrix which is the result of solving a system of linear equations. The algorithm for computing the Gauss-Jordan method is shown in Figure 3.16.

PROCEDURE Gauss_Jordan (A(n,n))

FOR k := 1 **TO** n - 1 **DO**

Find l such that
 $|A(l,k)| = \max(|A(k,k)|, \dots, |A(n,k)|)$
 PIV(k) <- l
 EXCHANGE (A(PIV(k), k), A(k,k))

Find maximum
 element of the pivot
 column and perform
 row interchange

l loop **FOR** l = k+1 **TO** n **DO**
 $A(l, k) = A(l, k) / A(k, k)$
ENDFOR

Compute the pivot
 column

j loop **FOR** j = k+1 **TO** n **DO**
 EXCHANGE (A(PIV(k), k), A(k,j))
 $A(k, j) = A(k, j) / A(k, k)$
FOR i = 1 **TO** n **DO**
IF (i ≠ k)
 $A(i,j) = A(i,j) - A(i,k) * A(k,j)$
ENDFOR

Add multiple of the
 kth row to the 1,...,
 k-1, k+1, ..., n th
 rows.

ENDFOR

ENDPROCEDURE

Figure 3.16 The Gauss-Jordan algorithm

Since the Gauss-Jordan algorithm is more or less the same as the LU algorithm, its parallel version is very similar to the previous one. A parallel implementation of this algorithm is shown in Figure 3.17. The synchronization methods as well as the method for ensuring mutually exclusive access of a shared variable are the same as those LU decomposition.

```
Pivot_Rec = {
  Flag : Boolean;
  Value : integer;
}
```

Shared Variables [A(1:n, 1:n) of real, Pivot(1:n) of Pivot_Rec]
 Global initialize (A, Pivot)

```
TASK Multiply_Column
  local Variables[ col(1:n), locPiv:Pivot_Rec]
  FOR k=1 TO n-1 DO
    IF (k==task_Id)
      lock(A(1:n), k)
```

```

    get(A(1:n,k), col(1:n))
    find l such that
    |col(l,k)| = max(|col(k,k)|, ..., |col(n,k)|)
    locPiv.Value = l
    Interchange(col(locPiv.Value, k), col(k,k))
    FOR i=k+1 TO n DO
        col(i,k) = col(i,k)/col(k,k)
    ENDFOR
    locPiv.Flag = TRUE
    put(v(i,k), A(1:n,k))
    unlock(A(1:n), k))
    lock(Pivot(k))
    put(locPiv, Pivot(k))
    unlock(Pivot(k))
ELSE
    j = TASK_ID
    IF ( j ≥ k+1 )
        wait until Pivot(k).Flag == TRUE
        lock ( A(1:n, j) )
        get( A(1:n, j), col(1:n) )
        get( Pivot(k), locPiv )
        Interchange( col(locPiv.Value,j), col(k,j) )
        FOR i=1 TO n DO
            IF (i ≠ j)
                A(i,j) = A(i,j) - A(i,k) * A(k,j)
            ENDFOR
        put( col(1:n), A(1:n, j) )
        unlock ( A(1:n, j) )
    ENDIF
ENDIF
ENDFOR
ENDTASK

```

Figure 3.17 A parallel implementation of the Gauss-Jordan algorithm

Example 3: The Cholesky Decomposition Algorithm

A symmetric matrix A is positive definite, if $x \neq 0 \Rightarrow x^T A x > 0$. Let A be a positive definite matrix. The Cholesky decomposition of A is L and L^T , such that $A = LL^T$.

Consider the following positive definite matrix, $\begin{bmatrix} 6 & 11 \\ 11 & 34 \end{bmatrix}$.

Its Cholesky decomposition is given by

$$L = \begin{bmatrix} \sqrt{6} & 0 \\ \frac{11}{\sqrt{6}} & \sqrt{\frac{83}{6}} \end{bmatrix} \text{ and } L^T = \begin{bmatrix} \sqrt{6} & \frac{11}{\sqrt{6}} \\ 0 & \sqrt{\frac{83}{6}} \end{bmatrix}$$

The Cholesky factorization algorithm is derived based on the matrix equation shown below

$$A = \begin{bmatrix} a_{11} & \bar{w}^T \\ \bar{w} & B \end{bmatrix} = \begin{bmatrix} \sqrt{a_{11}} & 0 \\ \frac{\bar{w}}{\sqrt{a_{11}}} & I_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} \sqrt{a_{11}} & \frac{\bar{w}^T}{\sqrt{a_{11}}} \\ 0 & I_{n-1} \end{bmatrix}$$

$\bar{w}$ is a column vector which is the first column of A without a_{11} and by equating both sides of the equation we get $A_1 = B - \frac{\bar{w}\bar{w}^T}{a_{11}}$.

From the above equation, we see that if the Cholesky factorization of A_1 is $L_1 L_1^T$, where L is a lower triangular matrix then we can substitute A_1 by $L_1 L_1^T$. Hence, we have

$$A = \begin{bmatrix} \sqrt{a_{11}} & 0 \\ \frac{\bar{w}}{\sqrt{a_{11}}} & I_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & A_1 = L_1 L_1^T \end{bmatrix} \begin{bmatrix} \sqrt{a_{11}} & \frac{\bar{w}^T}{\sqrt{a_{11}}} \\ 0 & I_{n-1} \end{bmatrix}$$

and by some algebraic manipulations on the matrices, we have

$$A = \begin{bmatrix} \sqrt{a_{11}} & 0 \\ \frac{\bar{w}}{\sqrt{a_{11}}} & L_1 \end{bmatrix} \begin{bmatrix} \sqrt{a_{11}} & \frac{\bar{w}^T}{\sqrt{a_{11}}} \\ 0 & L_1^T \end{bmatrix}$$

Therefore, the Cholesky factorization of A is LL^T where

$$L = \begin{bmatrix} \sqrt{a_{11}} & 0 \\ \frac{\bar{w}}{\sqrt{a_{11}}} & L_1 \end{bmatrix} \quad \text{Equation 3.1}$$

Thus, the Cholesky factorization of A can be found by repeatedly performing the reduction over the submatrices with decreasing size given in the equation 3.1. The Cholesky factorization algorithm is shown in Figure 3.18. The algorithm is very similar to the LU decomposition, that is, they both make use of a pivot column to perform the basic row operations on the submatrix to its right. The only difference is the way the pivot column is defined. In this algorithm, the k^{th} pivot column is equal to $[\sqrt{a_{kk}}, \frac{a_{(k+1)k}}{\sqrt{a_{kk}}}, \dots, \frac{a_{nk}}{\sqrt{a_{kk}}}]^T$. The parallel implementation of this algorithm is shown in Figure 3.19.

Figure 3.18 The Cholesky factorization algorithm

```

Pivot_Rec = {
  Flag : Boolean;
  Value : integer;
}
  
```

```

Shared Variables { A(1:n, 1:n) of real, Pivot(1:n) of Pivot_Rec }
Global initialize ( A, Pivot )
  
```

```

CKfactor_Rec = {
  Flag : Boolean;
}
  
```

```

TASK Multiply_Column
  local Variables [ col(1:n), locCKfac : CKfactor_Rec ]
  FOR k = 1 TO n-i DO
    IF (k==Task_ID) THEN
      lock(A(1:n, k))
      get(A(1:n,k) col(1:n))
      col(k,k) =  $\sqrt{\text{col}(k,k)}$ 
      FOR i = k+1 TO n DO
        col(i,k) = col(i,k)/col(k,k)
      ENDFOR
      locCKfac.flag = TRUE
      put(v(i,k), A(1:n,k))
      unlock(A(1:n, k))
    ELSE
      j = Task_Id
      IF (j≥k+1) THEN
        wait until CKfactor(k).Flag == TRUE
        lock ( A(1:n, j) )
        get( A(1:n, j), col(1:n) )
        FOR i=j TO n DO
          A(i,j) = A(i,j) - A(i,k) * A(k,j)
        ENDFOR
        put( col(1:n), A(1:n, j) )
        unlock ( A(1:n, j) )
      ENDIF
    ENDIF
  ENDFOR
ENDTASK

```

Figure 3.19 A parallel implementation of the Cholesky factorization algorithm

3.3.2 The Structure of Parallel Matrix Factorization

The structure of these parallel programs is represented by a task system shown in Figure 3.20. T_i^j represents the task that works on column i in the j^{th} iteration of the outermost loop.

Figure 3.20 The task graph of matrix factorization

However, the actual function of the tasks are different for each of the above mentioned algorithms. In the LU decomposition and Gauss Jordon algorithms, T_k^k finds the pivot element. For the Cholesky factorization algorithm, T_k^k computes the square root of the diagonal element and then divides the lower part of column k by that square root. In all the three algorithms, T_k^j uses the elements of column k to modify those in column j .

As mentioned in [Lord et al. 83], T_k^k should have additional importance because they have a large number of outgoing edges. Much research has been done to find an optimal static scheduling for this type of parallel computation. However, our major concern is the performance of scheduling policies for general-purpose parallel systems. Since matrix factorization is a way of performing Gaussian elimination on a matrix, from now on we call this class of programs as Gaussian elimination.

3.4 Parallel Search

Searching is a common method for solving problems in artificial intelligence. In general, a search system consists of three main components. The first of these is a *database*, which describes both the current state and the goal. Consider the problem of proving a theorem. In this case, the current state consists of the assertions representing the axioms, lemma, and semantic networks. The goal is an assertion representing the theorem to be proved. The second component of the search system is a set of operators that are used to manipulate the database. In the case of a theorem-proving system, operations are the rules of inference such as modus ponens or resolution. The third component is the control strategy for deciding the next action, what operator to apply and where to apply it. The choice of a control strategy affects the contents and organization of the database. The process of problem solving is to bring the problem state forward from its initial configuration to one satisfying a goal condition by repeatedly applying an operator.

3.4.1 Breadth-first on State-Space Graphs

The search system mentioned above advances the search by applying an operator to produce a new state in the database. A *state-space representation* of a problem employs two kinds of entities: *states*, which are data structures that give "snapshots" of the problem at each stage of the problem solving, and *operators* applied to transform the problem from one state to another. We use the 8-puzzle game to illustrate the basic concept of the state-space representation. An 8-puzzle is a square tray containing eight square tiles of equal size, numbered 1 to 8. The space for the ninth tile is vacant.

2	1	6
4		8
7	5	3

Figure 3.21 An example of the 8-puzzle game

The tiles can be moved by sliding them vertically or horizontally into the empty square. The problem is to transform some initial tile configuration to a special final configuration. The state-space representation of the problem is given as follows. A *state* is a particular configuration or arrangement of tiles; each state might be represented by a 3 X 3 matrix. The *operators* are the possible moves that are made by the empty square. Thus, four operators are to be used.

UP	Move the blank up one square,
DOWN	Move the blank down one square,
LEFT	Move the blank left one square,
RIGHT	Move the blank right one square.

The four operators defined for the 8-puzzle form a set of partial functions on the state space. A state space problem can be interpreted as a triple (S, O, G), where O represents the set of operators, S represents the set of initial states and G the set of goal states. A *solution* to the problem is a finite sequence of applications of operators that changes an initial state into a goal state.

A state space can be treated as a directed graph whose nodes are states and whose arcs are operators transforming from one state to another. Consider the 8-puzzle example. Its state-space graph representation has the following property. For each node, if it does

not represent a goal state, it will in turn create four nodes corresponding to the four operators as mentioned above. In Figure 3.22, node 1 represents the initial state of the game, and four different nodes will be created depending on the four different operators. Nodes 2, 3, 4 and 5 are called the successors of node 1. Note that the node 2, 3, 4 and 5 will in turn generate their successors and this process continues until the goal state is reached.

Figure 3.22 A state-space graph

The search for a solution is conducted by making explicit just enough of the state-space graph to contain a solution path. If the order in which potential solution paths are considered is arbitrary, using no domain-specific information to judge where the solution is likely to lie, the search is called *blind search*. There are mainly two *blind search* methods: the *breadth-first* and the *depth-first* search. They differ from one another mainly in the order in which nodes are examined. In each case, it is assumed that there is a procedure for finding all the successors of a given node; that is, all the states that can be reached from the current state by a single operator application. Such a procedure is said to *expand* the given node. However, we will only consider the *breadth-first* search of a state-space graph. The algorithm makes the following two assumptions.

- 1) The state-space graph is a tree. The implication is that there is only one start state (the root) and that the path from the start node to any other node is unique.

- 2) Whenever a node is expanded, creating a node for each of its successors, the successor nodes contain pointers back to the parent node. When a goal node is finally generated, this feature makes it possible to trace the solution path.

The *breadth-first* search method expands nodes in order of their proximity to the start node, measured by the depth of the node. The depth of a node is the number of arcs between the start node and the given node. In other words, it considers every possible operator sequence of length n , before any sequence of length $n+1$. Thus, although the search may be an extremely long one, it is guaranteed eventually to find the right solution sequence if any solution exists.

The breadth-first search is described by the following algorithm:

1. Place the root node on a list, called OPEN, of unexpanded nodes. If the start node is a goal node, a solution has been found.
2. If OPEN is empty, no solution exists.
3. Remove the first node, n , from OPEN and place it in a list, called CLOSED, of expanded nodes.
4. Expand node n . If it has no successors, go to (2).
5. Place all the successors of node n at the end of the OPEN list.
6. If any of the successors of node n is a goal node, a solution has been found. Otherwise, it goes to (2).

We consider a simple problem that involves changing the order of three blocks. The initial state of the world is that blocks 2 and 3 are on the table, and block 1 is on top of block 2. We want to reach a goal state in which the three blocks are stacked with block 1 on

top, block 2 in the middle, and block 3 on the bottom. The problem is depicted in Figure 3.23.

Figure 3.23 The configuration of the problem of changing three blocks

The only operator is **MOVE X to Y**, which moves object **X** on top of another object, **Y**. As preconditions to applying the operator, it is required (a) that **X**, the object to be moved, be a block with nothing on top of it and (b) that if **Y** is a block, there must be nothing on **Y**. Finally, the operator is not to be used to generate the same state more than once. Figure 3.24 shows the search tree generated by the breadth-first algorithm.

Figure 3.24 The state-space graph of the block rearrangement problem

The nodes are labelled S_0 through S_{10} . Each node is obtained by the ordering of the three blocks. For example, node S_1 corresponds to the successor state of S_0 reached by "MOVE block 1 to the table". The nodes are generated and expanded in the order given by their labels. When the algorithm terminates, finding S_{10} to be the goal, the list of expanded nodes contains S_0 through S_5 , and the OPEN list still contains S_6 through S_{10} .

3.4.2 Implementing The Breadth-first Search in Parallel

To accomplish the implementation of the breadth-first search in parallel, note that at each level of the search, tasks are created that correspond to every node of the state-space graph. We assume that these tasks can be executed independently. Consider the above example. An initial task is created in correspondence to the node S_0 . Then tasks that correspond to S_1 , S_2 and S_3 are created and placed on the task queue. The search for the state-space graph continues until the final state is reached. Once the final state is reached, the algorithm enters the wait & abort stage where the tasks that do not yield the results will be aborted. The implementation of the breadth-first search is similar to the divide-and-conquer algorithms, except that the merge tasks in this case also perform task abortion when needed. A task system which gives a high-level view of the structure of the parallel breadth-first search is shown in Figure 3.25. Since this class of programs are essentially performing the state-space search on a graph, from now they are referred to as state-space search.

Figure 3.25 A task graph of the parallel breath-first search

3.5 Parameters of The Abstract Model

This section presented an abstract model for parallel programs. The abstract model is a set of input parameters for the simulation model. They are listed in Table 3.1.

Parameters	Description
mp	Maximum Parallelism
CV_{mp}	Coefficient Variation of Maximum Parallelism
num_phases	number of parallel phases
d	Mean Service Demand
CV_d	Coefficient Variation of Service Demand
t	Probability of Task abortion

Table 3.1 Input parameters for the simulation model (an abstract model)

Maximum Parallelism (mp)

The maximum parallelism of a parallel program refers to its maximum number of tasks that can be run in parallel, if an unlimited number of processors is provided. Since we assume that the tasks in a fork-and-join job can be executed independently, its maximum parallelism is always equal to the number of tasks the job has. For the divide-and-conquer job model, the maximum parallelism is achieved when the job is in its work stage. For the matrix factorization, the maximum parallelism is at the second stage of the parallel program, where independent tasks are created to work on each column of the matrix. However, as the execution progress is, the number of parallel tasks decreases as the number of columns to be solved decreases. For the parallel state-space search jobs, maximum parallelism is reached when the search enters the final state.

Number of parallel phases (num_phases)

This parameter specifies the number of parallel phases in the input jobs. For the fork-and-join job model, this parameter represents the number of parallel phases which are separated by a synchronization point in the program. For the divide-and-conquer job model, this parameter represents the sum of the number of divide phases, work phases and merge phases. For the gaussian elimination job model, this parameter represents the number of iterations in the program. Finally, for the state-space search job model, this parameters corresponds is the sum of the number of expand phases, final state phases and the synchronization and abortion phases.

Mean Service Demand (d)

There are two approaches to model the service demand of a parallel program. One is to compute the overall service demand of a parallel program and distribute the overall service demand among the tasks of the program. We can

compute the service demand of each task of a parallel program and sum them up to get the overall service demand. The workload created from the first approach is called *uncorrelated* because the overall service demand of a parallel program is independent of the maximum parallelism. On the other hand, the model based on the second approach is called *correlated* because the overall service demand of a job is directly proportional to the maximum parallelism.

For the uncorrelated model, the parameter d represents the mean service demand of a parallel program. For example, if it is set to 20.0, then, on average, the parallel program will impose 20.0 units of service demand onto the system.

For the correlated model, d is the mean service demand of the tasks in the parallel programs. For example, if d is equal to 1.0, then on average, every job has a mean task service time equal to 1.0. The mean task service time is the sum of the service time of the tasks divided by the number of tasks in the job.

Coefficient of Variation Service Demand (CV_d)

CV_d is the coefficient of variation of service demand of parallel programs, if the first approach is used. It is the coefficient of variation of service demand of the tasks, if the second approach is used.

Probability of Task Abortion

The probability of task abortion is only used by the state-space search job model. It is used to decide whether a task should be aborted by the wait & abort tasks.

These parameters along with the job structure defines the workload. The four types of job structures considered in the thesis are fork-and-join (Figure 3.3), divide-and-

conquer (Figure 3.11), Gaussian Elimination (Figure 3.20), state-space search (Figure 3.25).

Summary

This chapter presented a survey of the parallel implementations of four classes of algorithms. Examples of the implementation were described. An abstract model, a set of input parameters for our simulation, was presented at the end of this chapter. The model was capable of specifying parallel programs studied in this chapter.

CHAPTER IV

SCHEDULING POLICIES

This chapter discusses the scheduling policies used in the simulation experiments: These are the round robin job policy (*RRjob*) [Leutenegger & Vernon 90], the dynamic partition policy (*DP*) [Zahorjan & McCann 90] and the coscheduling policy (*CoSched*) [Leutenegger & Vernon 90], [Ousterhout 82]. Each of these policies has its advantages and disadvantages. In addition, we will also evaluate the performance of the preemptive smallest cumulative demand first policy (*PSCDF*). *PSCDF* provides good performance and can serve as a benchmark for comparison. *RRjob* aims at providing equal shares of the processing power to the jobs in the system, regardless of their number of tasks. This is done by varying the quantum size for each job in the system. Since *RRjob* provides equal allocation per job, it results in good performance in multiprocessor systems [Leutenegger & Vernon 90]. *DP* has the same objective as *RRjob* in that it tries to make the jobs share the processing power equally [Leutenegger & Vernon 90]. In addition, *DP* also aims at minimizing the number of context switches and reacting dynamically to the changes in the job's parallelism. If the tasks of the same job require coordination among them, it would be better if these tasks can be scheduled to run at the same time. The *CoSched* policy is designed in light of this requirement [Leutenegger & Vernon 90], [Ousterhout 82]. Even though the Cosched policy is shown to perform worse than the DP and RRjob policies, the Cosched policies is considered in this thesis because of the presence of synchronization in the workload.

In the subsequent sections, the four scheduling policies are described in detail and show the additional requirements of these policies in order to schedule the parallel jobs that are considered in this thesis.

4.1 The CoScheduling Policy

It is assumed that a linked list is used to implement the task queue. A processor window of size equal to the number of processors in the system is moved along the linked list. When a job arrives into the system, its tasks are appended to the end of the linked list.

At each quantum, tasks located in the processor window are scheduled to run. Some tasks may have service demand less than a quantum. When these small tasks are completed, their processors remain idle until the end of this quantum. All processors do the context switch at the same time. Example runs of a system with four processors are given as follows.¹

Example 1:

1st quantum: A1 A2 A3

2nd quantum: A1 A2 A3 B1 B2 B3 B4

3rd quantum: A1 A2 A3 B1 B2 B3 B4 C1 C2

4th quantum: A1 A2 A3 B1 B2 B3 B4 C1 C2

5th quantum: A1 A2 A3 B1 B2 B3 B4 C1 C2

•
•

In example 1, the system begins with only job A which consists of tasks A₁, A₂ and A₃. In the first quantum, the scheduling window is placed on the tasks of job A.

¹In all the examples presented below, a box is used to represent the processor window. We assume that there are 4 processors in the system. Capital letters are used to represent a job (or program). The tasks *i* of job A is represented by A_i.

Hence, the tasks A_1 , A_2 and A_3 are scheduled to run on these processors. Since all the tasks of job A have been scheduled at the first quantum, in the second quantum the processor window is moved to job B. In the third quantum, tasks of job C are scheduled. Since job C has only two tasks, the remaining processors in the window will be used to execute the tasks of job A. In the 4th quantum, tasks of job A is scheduled because not all the tasks of job A were scheduled in the previous quantum; and this shows the effect of a job that is not being completely coscheduled. A job is completely coscheduled, if all its tasks have been scheduled to run in the previous quantum. At the end of each quantum, the processor window is moved along the linked list until the head of the window is at the first task of the job that is not completely coscheduled in the previous quantum.

However, this definition cannot be applied to the fork-and-join parallel jobs when the number of tasks P is greater than the number of processors N in the system. For the fork-and-join parallel jobs consisting of $P > N$ tasks, it is impossible to schedule all the tasks of the job in one quantum. In this case, the job is completely coscheduled through the schedules in the previous consecutive quanta. An example scheduling sequence of fork-and-join parallel jobs of this type is given below.

Example 2:

1st quantum: A1 A2 A3 A4 A5 A6 A7

2nd quantum: A1 A2 A3 A4 A5 A6 A7 B1 B2 B3

3rd quantum: A1 A2 A3 A4 A5 A6 A7 B1 B2 B3

4th quantum: A1 A2 A3 A4 A5 A6 A7 B1 B2 B3

•
•

From the above example, we see that job A has more tasks than the number of processors in the system. Thus, job A is completely coscheduled if and only if all its tasks

are scheduled in two or more consecutive quanta, as in the case of quantum 1 and 2, and quantum 3 and 4.

The basic definition of complete coscheduling may not be applicable to the job models where new tasks can be created dynamically, as the tasks of the same job may not be grouped together in the queue. Consider the scheduling sequence of the parallel divide-and-conquer jobs that is given in example 3.

Example 3:

1st quantum: A1 B1 C1 D1
 2nd quantum: B1 C1 D1 A2 A3
 3rd quantum: C1 D1 A2 A3 B2 B3
 4th quantum: C1 D1 B3 A4 A5 B4 B5 A6 A7
 •
 •

At the 4th quantum, the tasks of job A and B are interleaved and these jobs cannot be completely coscheduled, even though their number of tasks is less than the number of processors. Under these circumstances, a loose definition of complete coscheduling is adapted. A job is considered to be completely coscheduled, if a group of its tasks that have the same ancestor is completely coscheduled. Thus, in the above example, jobs A and B are both completely coscheduled by the fourth quantum. This definition does not violate the spirit of coscheduling, because only the processes that are created by the same task are required to synchronize with each other.

4.2 The Preemptive Smallest Cumulative Demand First Policy

A priority queue is used to implement this policy. When a new job arrives, the system will preempt processors that are currently running the tasks of jobs having larger cumulative demand than the new job. Thus, jobs with small service demand have higher

priority than those with large service demand. Since the service demand of the parallel divide-and-conquer jobs changes during their execution, due to task creation and termination, we use their total service demand as their job size. By total service demand, we mean the sum of the service demand of all the tasks of the given job. Their total service demand decreases when their tasks are completed. In reality, this policy cannot be implemented, as the total service demand of a job is not known in advance.

4.3 The Round Robin Job Policy

The RRjob policy was originally defined by [Leutenegger & Vernon 90]. The goal of the RRjob policy is to provide equal allocation of processors to jobs temporally. This is done by giving jobs with a small number of tasks a longer time slice. In addition, RRjob is a dynamic scheduling policy in that the processor allocation changes dynamically.

A shared queue of parallel jobs is used to implement this policy. Each entry in the queue consists of a list of tasks of the job. Scheduling is done round robin on this queue. Tasks of the first job in the queue is scheduled to run, when there is at least one processor available. If there are M processors available, where $M \leq P$ (the number of processors in the system) and the job at the front of the queue (call it job f) has N tasks waiting for service, where $N \leq M$, these N tasks are scheduled to run in the next quantum. If $N < M$, the remaining $M - N$ processors will be given to the next job on the queue. On the other hand, if $N \geq M$, only M tasks will be scheduled and the remaining tasks will have to wait for the next quantum of job f .

The quantum size of this policy varies from job to job and it depends on the number of tasks the job has. The quantum size is equal to $(P * q) / N$, with q being the unit size for a quantum, if the number of tasks in the job is less than P ; otherwise, the quantum size is equal to q .

Since RRjob is a dynamic policy, scheduling is done when a new job arrives or a task finishes its execution. A formal description of this policy is shown below.

P = number of processors in the system
 M = number of idle processors
 N_i = number of tasks in job i
 W_i = number of waiting tasks in job i

Suppose that job i is the job to be scheduled next and there are M idle processors available.

RRjob

WHILE ((processorsCount > 0) && (there exist a waiting task))

Job i is the job at the head of the queue

IF ($W_i \geq P$)

Schedule the first M tasks of job i with quantum size = q

ELSE IF (($N_i \leq P$) && ($W_i \leq M$))

Schedule all the waiting tasks of Job i

with quantum size = $(P * q) / N_i$

IF ($W_i \leq M$)

Use RRjob to schedule the next job to run on the remaining

$M - W_i$ processors

ENDIF

ENDIF

move the queue-head pointer to the one following job i

END WHILE

End RRjob

An example run of this policy is stated as follows. Suppose that the system has 4 jobs A, B, C and D. Program A has 4 tasks A_1 , A_2 , A_3 and A_4 . Program B has two tasks B_1 and B_2 . Program C has three tasks C_1 , C_2 and C_3 . Assume that the service demand of these tasks are large and $q = 1.0$ time unit. There are 3 processors in the system, namely, X, Y and Z.

The allocation of processors between time 0.0 to time 6.5 is shown in Figure 4.1. As job A has more tasks than the number of processors in the system, its first three tasks will get 1.0 time unit of the processing power. Once the quantum for job 1 is expired, processors will be switched to the next job in the queue, namely, job B. Since job B only has two tasks, its quantum size is equal to $3 * 1.0 / 2 = 1.5$. The remaining processor, namely, processor Z is allocated to job 3. As job 3 has 3 tasks, its quantum size is equal to 1.0. Programs in the system will be scheduled in this way as long as there is an idle processor and a task waiting for service. By varying the quantum size of the jobs, this policy tends to provide an equal share of processing power among all jobs.

If the parallel job is not fork and join, the number of tasks per job changes dynamically. In these cases, the current number of tasks in a job is used for computing the quantum size.

Processor Time	X	Y	Z
0.0 - 0.5	a1	a2	a3
0.5 - 1.0			
1.0 - 1.5	b1	b2	c1
1.5 - 2.0			
2.0 - 2.5			a1
2.5 - 3.0	c3	a4	
3.0 - 3.5			a1
3.5 - 4.0	a2	b1	b2
4.0 - 4.5			
4.5 - 5.0	c1		
5.0 - 5.5		c3	
5.5 - 6.0			
6.0 - 6.5			

Figure 4.1: Processor allocation of the RRjob policy

4.4 The Dynamic Policy

The dynamic policy is a two level scheduling policy [Leutenegger 90] and [Zahorjan & McCann 90]. At the upper level, processors are partitioned among the jobs in the system. At the lower level, the task dispatcher assigns processors to the parallel jobs. The task dispatcher may release processors when the parallel jobs do not have enough parallelism to utilize them. The actions of the processor allocator is described as follows.

When a job requests one or more processors, the following actions is taken

1. If there are idle processors, use them to fulfill the requests.
2. Otherwise, if the job making the requests is a new job, allocate it a single processor by preempting a processor from a job that currently owns more than two processors.

3. If any part of the requests for processors cannot be satisfied, it remains outstanding until either a processor becomes available for it or the task dispatcher releases processors from other parallel jobs.

When a parallel job releases its processors, the processor allocator will search for jobs that have not yet been assigned a processor (i.e. the unsatisfied new arrivals) and assign a processor to each of these jobs. If there are more free processors after this allocations, scan the queue again, and allocate the rest of the processors on a *FCFS* basis.

When a processor finishes a task, the dispatcher will be invoked to either assign another task of the same job to the idle processor or hand it over to the processor allocator. The function of the dispatcher is described as follows.

4. If there is a newly arrived job that has not allocated a processor, give the processor to this job.
5. Otherwise, let q_j be the number of job j 's tasks waiting for processors. If $q_j = 1$, the dispatcher simply dequeues the task and starts its execution.
6. If $q_j > 1$ the task dispatcher will begin executing the first task in the queue
7. Finally, if $q_j = 0$, the dispatcher will release its processors to the allocator.

CHAPTER V

SIMULATION EXPERIMENTS

This chapter presents the results of the simulation experiments to evaluate the performance of scheduling policies introduced in the previous chapter. The job models given in chapter 3 are used. The output parameters are shown in Table 5.1.

Parameters	Description
<i>MRT</i>	mean job response time
ρ	system utilization
n	mean number of tasks per job
CV_n	CV of number of tasks per job
d	mean job service demand
CV_d	CV of job service demand
<i>syn_delay</i>	synchronization delay per job
<i>ctxt</i>	number of context switch per job

Table 5.1 The output parameters and their descriptions

All the simulations are parameterized for a system with $p=20$ processors. The quantum length (Q) for the coscheduling and rrjob policies is equal to 0.1. The overhead for the context switch (*ctxtOverhead*) is assumed to be 1.5% of the quantum, that is 0.0015. The overhead also applies to the pscdf and dynamic policies, when a processor is preempted from one task to another.

5.1 The Impact of Job Structure on the Performance of Scheduling Policies

The purpose of the first set of experiments is to compare the performance of the scheduling policies for different job models and utilization levels. In [Leutenegger 90], an experiment using the fork-and-join model was conducted. In this section, the experiment is

repeated over different job models to see whether the performance is affected by the change.

To provide a fair comparison, the output values of the average number of tasks and the average total service demand of the jobs must have the same values for all the job models. We call these the *control parameters* of the simulation. This is done by setting appropriate values for the input parameters. The utilization level is varied by changing the arrival rate of the parallel jobs. If U is the target system utilization, D_T is the average total service demand of the jobs in the workload, and there are P processors in the system, the arrival rate $\lambda = \frac{UP}{D_T}$.

Since there are eight workload models in our experiments, a vector with two components is used to represent the models. The first one shows whether the job model is correlated or uncorrelated by using the abbreviations *Cor* or *UnCor*. The second component shows the type of the job models: Fork-and-Join model (FJ), Divide-and-conquer model (DC), Gaussian elimination model (GE)¹ and Breadth first search model (SSS). For example, correlated fork-and-join workload is represented by (Cor, FJ).

Two types of workload, defined in the previous chapter, are considered (correlated and uncorrelated). The default values of the uncorrelated and correlated workload are shown in Table 5.2 and Table 5.3.

¹Since matrix factorization is in basic solving the Gaussian Elimination problem, we use GE to represent the matrix factorization algorithms.

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	20.0	12.444	5.667	12.444
CV_{mp}	1.0	1.0	1.0	1.0
d	20.0	20.0	20.0	20.0
CV_d	5.0	5.0	5.0	5.0
P_T	0.0	0.0	0.0	0.5

Table 5.2 Default values of the input parameters
(uncorrelated workload)
(Impact of job models on the performance of scheduling policies)

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	20.0	12.444	5.667	12.444
CV_{mp}	1.0	1.0	1.0	1.0
d	1.0	1.0	1.0	1.0
CV_d	5.0	5.0	5.0	5.0
P_T	0.0	0.0	0.0	0.5

Table 5.3 Default values of the input parameters
(correlated workload)
(Impact of job models on the performance of scheduling policies)

The simulation experiments were run with system utilizations of 0%, 40%, 60%, 70%, 80% and 90% for the uncorrelated and correlated workload for different job models. To reduce the effect of transients, the information is only collected after the measured throughput had reached 99.5% of the arrival rate. The batch method is then used to compute the confidence interval (in our case, 30 batch runs were used for the results reported here and each batch contains more than 50000 jobs). This method gives 95% confidence intervals that were less than 1% of the mean response times when the system utilization is low to moderate and less than 20% for high utilization level (for example 90%). This technique is also applied for all the experiments described in this chapter.

5.1.1 Results of the Fork-and-Join Job Model

Figures 5.1 and 5.2 present the results of the fork-and-join model for the correlated and uncorrelated workload. The results shown here are consistent with the ones reported in [Leutenegger 90].

Figure 5.1 Performance of scheduling policies for the (Cor,FJ) model

Figure 5.2 Performance of scheduling policies for the (UnCor,FJ) model

Since rrjob provides the jobs with an equal share of the processing power, large jobs (in terms of total job service demand) do not dominate small jobs. This results in a shorter average response time for small jobs and therefore provides better overall job response time. On the other hand, the coscheduling policy performs poorly because it gives large jobs a larger share of processing power.

Dynamic policy allocates processors to jobs on a first-come-first-served basis. It does not adapt well to balance the processor allocation. Hence, fair allocation of processors to parallel jobs cannot be made at low utilization level. The problem becomes serious if the workload is correlated. Assume that the system has six processors. Consider the following sequence of jobs in the queue, where each task is assumed to require 2 time units of service.

At time 0: $A_1 A_2$

Time	<u>Processors</u>					
	P_1	P_2	P_3	P_4	P_5	P_6
0-1	A_1	A_2				

At time 1: $A_1 A_2 B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8 B_9 B_{10}$

Time	<u>Processors</u>					
	P_1	P_2	P_3	P_4	P_5	P_6
0-1	A_1	A_2				
1-2	A_1	A_2	B_1	B_2	B_3	B_4

At time 2: $A_1 A_2 B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8 B_9 B_{10} C_1 C_2$

Time	<u>Processors</u>					
	P_1	P_2	P_3	P_4	P_5	P_6
0-1	A_1	A_2				
1-2	A_1	A_2	B_1	B_2	B_3	B_4
2-3	C_1	B_5	B_1	B_2	B_3	B_4

At time 3: $B_5 B_6 B_7 B_8 B_9 B_{10} C_1 C_2$

Time	<u>Processors</u>					
	P_1	P_2	P_3	P_4	P_5	P_6
0-1	A_1	A_2				
1-2	A_1	A_2	B_1	B_2	B_3	B_4
2-3	C_1	B_5	B_1	B_2	B_3	B_4
3-4	C_1	B_5	B_6	B_7	B_8	B_9

At time 4: B₅ B₆ B₇ B₈ B₉ B₁₀ C₁ C₂ D₁ D₂

Time	<u>Processors</u>					
	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆
0-1	A ₁	A ₂				
1-2	A ₁	A ₂	B ₁	B ₂	B ₃	B ₄
2-3	C ₁	B ₅	B ₁	B ₂	B ₃	B ₄
3-4	C ₁	B ₅	B ₆	B ₇	B ₈	B ₉
4-5	C ₂	D ₁	B ₆	B ₇	B ₈	B ₉

It can be seen from the above example that more processors are allocated to job B, as it has more tasks than the other jobs. At low utilization level, where the multiprogramming level is low, the jobs with large number of tasks tend to get more processors allocated than those with smaller number of tasks. In the above example, job B tends to get more processors than the other jobs. It can be seen from Figure 5.2 that at low utilization levels, say between 0% to 70%, the dynamic policy performs worse than coscheduling. However, as the utilization level gets higher than 70% the dynamic policy starts performing better than coscheduling. An increase in utilization means an increase in the number of jobs in the system. As a result of this, more jobs are run in parallel. This leads to a more equal distribution of processing power and a better performance of the dynamic policy.

5.1.2 Results of the Divide-and-Conquer Job Model

Results of the divide-and-conquer job model are presented in Figures 5.3 and 5.4.

Figure 5.3 Performance of scheduling policies for the (Cor, DC) model

Figure 5.4 Performance of scheduling policies for the (UnCor, DC) model

Due to the dynamic nature of the job model, processors will be released by parallel jobs during their execution. Thus, the processor allocation is balanced by the dynamic

change of job parallelism and the performance of dynamic resembles that of the rrjob. Coscheduling performs poorly compared to the other two policies mainly because jobs with a larger number of tasks get a larger fraction of processing power than the jobs with a small number of task, thus increasing the waiting times of the small jobs.

5.1.3 Results of the Gaussian Elimination Job Model

Figures 5.5 and 5.6 prese it the results of the Gaussian Elimination job model.

Figure 5.5 Performance of scheduling policies for the (Cor, GE) model

Figure 5.6 Performance of scheduling policies for the (UnCor,GE) model

Due to the presence of the pivot tasks² at each parallel phase of the GE job model, it is not always possible for a parallel phase to finish in a single quantum of the rrjob. As a result, multiple quanta are required. At high utilization levels, multiple quanta incurs higher queueing delay per job which results in longer job execution times and significantly degrades the performance. Consequently, the rrjob performs very similar to the dynamic policy up to 80% utilization level as shown in figures 5.5 and 5.6.

5.1.4 Results of the Breadth First Search Job Model

Figure 5.7 and 5.8 present the results for the breadth-first search job model.

²A pivot task is the task for computing the pivot column which is used to modify the other columns of the matrix. Its execution is larger than the other tasks in the same parallel phase.

Figure 5.7 Performance of scheduling policies for the (Cor,SSS) model

Figure 5.8 Performance of scheduling policies for the (UnCor,SSS) model

The difference between the fork-and-join and divide-and-conquer models is the additional synchronization delay introduced by the divide and merge phases of the parallel

jobs. For the SSS job model, the synchronization delay induced by the merge tasks are reduced by having task abortion. This is due to the termination of the aborted tasks. The divide-and-conquer job model can be treated as a SSS model without task abortion (i.e. probability of task abortion $p=0.0$). Note that, at the other extreme, when $p=1.0$, there is no synchronization in the SSS job model because the other task is always aborted.

Figures 5.7 and 5.8 show the results of the task abortion on the performance of scheduling policies. The value of p in this experiment is set equal to 0.5, so that the synchronization time induced by task abortion is in between 0.0 to 1.0. Then, on the average, there will be a decrease in mean job response time for all the scheduling policies by increasing the probability of task abortion. This change of the response time can be seen by comparing Figures 5.3 and 5.4 (within $p=1.0$) with Figures 5.7 and 5.8 (within $p=0.5$).

5.1.5 Summary of The Results

Based on the results of this set of experiments, the following observations can be made.

1. Changing the job structure only affects the performance of the dynamic policy
2. Coscheduling (a fixed-preemptive) policy performs poorly for all the job models
3. For the DC, GE and SSS job models, dynamic performs as good as the rrjob, up to 70% of the utilization
4. Performance of policies is not affected by the presence of task abortion

5.2 Sensitivity of the Performance to CV_d

In this section, we study the effect of CV_d on the performance of the scheduling policies. The input parameters for different job models are set such that the output parameters are $n=20.0$, $CV_n=1.0$, $d=20.0$ and $CV_d=1.0, 2.0, 3.0, 4.0$ and 5.0 . The utilization level is fixed at around 70% by setting $\lambda=0.7$. Table 5.4 and 5.5 show the default values of the input parameters for the correlated and uncorrelated workload.

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	20.0	12.444	5.667	12.444
CV_{mp}	1.0	1.0	1.0	1.0
d	1.0	1.0	1.0	1.0
CV_d	vary	vary	vary	vary
P_T	0.0	0.0	0.0	0.5

Table 5.4 Default values of the input parameters
(correlated workload)
(Sensitivity of the Performance to CV_d)

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	20.0	12.444	5.667	12.444
CV_{mp}	1.0	1.0	1.0	1.0
d	20.0	20.0	20.0	20.0
CV_d	vary	vary	vary	vary
P_T	0.0	0.0	0.0	0.5

Table 5.5 Default values of the input parameters
(uncorrelated workload)
(Sensitivity of the Performance to CV_d)

Figure 5.9 Sensitivity of performance to CV_d (Cor, FJ)

Figure 5.10 Sensitivity of performance to CV_d (UnCor, FJ)

Figure 5.9 and 5.10 presents the results of the fork-and-join model. As CV_d increases, the ratio between the service demand of large jobs to that of small jobs increases. Dynamic policy is sensitive to the change of CV_d . An increase in CV_d results in an increase

in the variation of service demand among the jobs. Since the dynamic policy does not provide time-sharing of the processors to the jobs in the system, small jobs can be held up by some large jobs that had arrived earlier. This problem gets more serious as CV_d increases. The other policies are insensitive to CV_d . rrjob allocates the processing power based on job parallelism and time-sharing of the processors. An increase in CV_d only increases the variation of job service demand and does not affect the processor allocation decision. For a similar reason, the coscheduling policy is also insensitive to CV_d . Since the pscdf policy always gives priority to the jobs that have smallest cumulative demand, varying the coefficient of variation of the service demand does not affect the performance of this policy.

Figure 5.11 Sensitivity of performance to CV_d (Cor, DC)

Figure 5.12 Performance of scheduling policies to CV_d (UnCor, DC)

Figures 5.11 and 5.12 present the results for the divide-and-conquer job model. All four policies including the dynamic policy are all insensitive to CV_d . Due to the dynamic nature of this job model, processor reallocation can be used to meet the change in parallelism of a job. Thus, varying CV_d does not degrade the performance on the dynamic policies. As rrjob and Coscheduling allocate the processing power on a time sharing basis, varying the CV_d does not have effect on their performance.

Figure 5.13 through Figure 5.16 presents the results of the GE and SSS models. The results are similar to the ones for the divide-and-conquer model.

Figure 5.13 Sensitivity of Performance to CV_d (Cor, GE)

Figure 5.14 Sensitivity of performance to CV_d (UnCor, GE)

Figure 5.15 Sensitivity of performance to CV_d (Cor, SSS)

Figure 5.16 Sensitivity of Performance to CV_d (UnCor, SSS)

5.3 Sensitivity of Performance to n

In this section, we present the sensitivity analysis of n . We fix the system utilization level at 70%. Other parameters are set as follows: $CV_n \approx 1.0$, $d \approx 20.0$ and $CV_d \approx 5.0$ for all the job models. We perform the experiments were conducted for values of $n \approx 10.0, 15.0, 20.0, 30.0$ and 40.0 for both the correlated and uncorrelated workload.

5.3.1 Results for the Uncorrelated Workload

The simulation results for the fork-and-join job model is shown in Figure 5.17. The default values of the input parameters for this experiment are given in Table 5.6.

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	vary	vary	vary	vary
CV_{mp}	1.0	1.0	1.0	1.0
d	20.0	20.0	20.0	20.0
CV_d	5.0	5.0	5.0	5.0
P_T	0.0	0.0	0.0	0.5

Table 5.6 Default values of the input parameters
(uncorrelated workload)
(Sensitivity of performance to n)

Figure 5.17 Sensitivity of performance to n (UnCor, FJ)

Similar experiments have also been conducted by [Leutenegger 90]. We extend the experiment to consider the cases where $n \geq 20$ (the number of processors in the system). The results of the uncorrelated workload is first presented in this subsection. As n increases the mean job response time decreases up to $n \approx 20.0$ and the mean job response time remains steady after that. An increase in n introduces additional job parallelism and hence a decrease in the response time.

For the coscheduling policy, an increase in job parallelism results in a shorter execution time per job. However, when $n \approx 20.0$ the advantage of additional parallelism is limited by the existing resources and, hence, no further performance improvement can be obtained. The analysis for the rrjob is similar to the coscheduling. For the dynamic policy, an increase in job parallelism improve the ability of each job to utilize the system's

resources. However, the advantage of additional parallelism is limited by the number of processors that can be allocated to each job.

Figure 5.18 plots n versus the average job response time for the divide-and-conquer job model.

n	10.3627	16.0435	20.038	31.052	41.464
mp	7.5	8.0	12.444	16.0	22.5

Figure 5.18 Sensitivity of performance to n (UnCor, DC)

For the divide-and-conquer job model, within the range of n values considered were, the average maximum parallelism is approximately 16.0. When n is 40.0, the parallelism per job is still less than the number of processors. Thus, the advantage of additional parallelism is not affected by the system's resources. For the coscheduling policy, the decrease in mean job response time is only marginal. This shows that coscheduling is not suitable for divide-and-conquer jobs, when jobs have higher degree of parallelism.

The results of the GE and SSS job models are shown in Figure 5.19 and 5.20. The results are similar to one for the divide-and-conquer job model.

<i>n</i>	10.1695	16.0435	19.647	29.573	41.5694
<i>mp</i>	3.5	5.0	5.7	7.1	8.2

Figure 5.19 Sensitivity of performance to n (UnCor, GE)

PM-1 3" x 4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

<i>n</i>	10.3627	16.0435	20.038	31.052	41.464
<i>mp</i>	7.5	8.0	12.444	16.0	22.5

Figure 5.20 Sensitivity of performance to n (UnCor, SSS)

5.3.2 Results of Correlated Workload

The sensitivity analysis of n is also performed for correlated workload. The results are presented in Figure 5.21 through 5.24. Note that, in this workload, there is a positive correlation between the service demand and the number of tasks of a job. This is reasonable, since in real situations the service demand of individual tasks should stay unaffected by the number of tasks. Consider the case of a program for Gaussian elimination. Tasks are responsible for the computation of the dot-product of their associated rows and columns. Thus, an increase in the number of tasks corresponds to an increase in the size of the input matrices, which implies an increase in job service demand. A similar argument can also be made for the divide-and-conquer job model. In the experiment, n is set equal to 10.0, 15.0, 20.0, 30.0 and 40.0. We vary the arrival rate so that the utilization

level of the system is approximately equal to 70%, for all values of n . The default values of the input parameters are given in Table 5.7.

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	vary	vary	vary	vary
CV_{mp}	1.0	1.0	1.0	1.0
d	1.0	1.0	1.0	1.0
CV_d	5.0	5.0	5.0	5.0
P_T	0.0	0.0	0.0	0.5

Table 5.7 Default values of the input parameters
(correlated workload)
(Sensitivity of Performance to n)

Figure 5.21 Sensitivity of performance to n (Cor, FJ)

<i>n</i>	5.034	10.3627	16.0435	20.038	31.052	41.464
<i>mp</i>	3.54	7.5	8.0	12.444	16.0	22.5
<i>d(output)</i>	5.130	10.127	16.5638	20.34	30.993	40.939

Figure 5.22 Sensitivity performance to n (Cor, DC)

n	5.054	10.1695	16.0435	19.647	29.573	41.5694
mp	2.87	3.5	5.0	5.7	7.1	8.2
$d(output)$	5.204	10.1434	16.1133	19.754	29.773	41.3923

Figure 5.23 Sensitivity of performance to n (Cor, GE)

Figure 5.24 Sensitivity of performance to n (Cor, SSS)

Due to the correlation between the job service demand and number of tasks, the mean job response time increases as we increase the values of n . Furthermore, there is no change in the relative performance among the policies, as we increase the number of tasks.

5.4 Impact of CV_n on The Performance of Scheduling Policies

Experiments for different levels of utilization were performed with $CV_n=0.0$ and 5.0 . $CV_n=0.0$ represents the case where all jobs have the same number of tasks. $CV_n=5.0$ represents the situation where the workload consists of large number of jobs with small number of tasks and small number of jobs with a large number of tasks. The input parameters for this experiment are shown in Tables 5.8 and 5.9.

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	20.0	12.444	5.667	12.444
CV_{mp}	0.0 or 5.0	0.0 or 5.0	0.0 or 5.0	0.0 or 5.0
d	20.0	20.0	20.0	20.0
CV_d	5.0	5.0	5.0	5.0
P_T	0.0	0.0	0.0	0.5

Table 5.8 Default values of the input parameters
(uncorrelated workload)
(Impact of CV_n on The Performance of The Scheduling Policies)

Input Parameters	Default Values			
	FJ	DC	GE	SSS
mp	20.0	12.444	5.667	12.444
CV_{mp}	0.0 or 5.0	0.0 or 5.0	0.0 or 5.0	0.0 or 5.0
d	1.0	1.0	1.0	1.0
CV_d	5.0	5.0	5.0	5.0
P_T	0.0	0.0	0.0	0.5

Table 5.9 Default values of the input parameters
(uncorrelated workload)
(Impact of CV_n on The Performance of The Scheduling Policies)

5.4.1 Discussion of Results for $CV_n=0.0$

Figure 5.25 Performance of scheduling policies when $CV_n=0.0$ (Cor,FJ)

Figure 5.26 Performance of scheduling policies when $CV_n=0.0$ (UnCor,FJ)

Figures 5.25 and 5.26 present the results for the correlated and uncorrelated versions of the fork-and-join job models with $CV_n=0.0$. Since the correlation between the service demand and the number of tasks breaks down as a result of using a constant number of tasks per job (because $CV_n=0.0$), the results for the uncorrelated workload is similar to those for the correlated workload. This can be seen by comparing Figures 5.25 and 5.26.

In a general setting, coscheduling tends to give large jobs consecutive quanta and hence increases the waiting time for the small jobs. Such a problem does not exist when the number of tasks per job is fixed and the performance of the coscheduling gets closer to the rrjob. Under these circumstances, the rrjob maintains the same quantum size for each job. Thus, the scheduling sequence of this policy is similar to that in the coscheduling and, therefore, results in a similar performance.

Fixing the number of tasks per job does not yield performance benefits by using to the dynamic policy, as can be seen from the above results. At low to moderate system loads, the jobs with large service demand acquire more processors, as they stay in the system for a longer time. Small jobs that come after the large jobs will be allocated fewer processors than the large jobs. Such an unfair allocation of processors results in increased response time for the small jobs.

Figure 5.27 Performance of scheduling policies when $CV_n=0.0$ (Cor,DC)

Figure 5.28 Performance of scheduling policies when $CV_n=0.0$ (UnCor,DC)

Figures 5.27 and 5.28 present the results for the correlated and uncorrelated versions of the divide-and-conquer job model. Due to break down of the correlation between the

service demand and number of tasks of the parallel jobs, the results for the two versions of the workload are similar.

Under the divide-and-conquer workload, the coscheduling policy gives the worst performance among the four policies. It performs worse than the rrjob mainly due to a larger synchronization delay induced by it. Although coscheduling is designed to reduce the synchronization delay of parallel jobs, it fails to do so under the divide-and-conquer workload. The group of tasks created from different branches of a job may be scheduled in different quanta, as each group is placed at different places in the queue. As a result of this, the synchronization delay of the jobs increases. On the other hand, rrjob is implemented in such a way that the tasks of a job is placed together in the job queue, this results in minimizing synchronization delay of a job. In addition, the large context switch overhead associated with the coscheduling policy also affects its performance.

Coscheduling performs worse than the dynamic mainly because it incurs large amount of context switch overhead. On the other hand, dynamic performs preemption only when a new job arrives or a job changes its parallelism. Therefore, the amount of overhead will be much less than that in the coscheduling policy.

Dynamic has a smaller context switch overhead by minimizing the number of context switches per job. However, it yields a longer synchronization delay per job. On the other hand, although the rrjob gives a shorter synchronization delay per job, its context switch overhead is larger than that of the dynamic policy. These two policies are comparable up to the utilization level of 80%, then the performance of the dynamic degrades as the number of jobs exceeds the number of processors.

Figure 5.29 Performance of scheduling policies when $CV_n=0.0$ (Cor,GE)

Figure 5.30 Performance of scheduling policies when $CV_n=0.0$ (UnCor, GE)

Figures 5.29 and 5.30 present the results for the correlated and uncorrelated version of the workload for the gaussian elimination job model. The reason for the similar results for these two versions of the workload was discussed previously.

Figure 5.31 Performance of scheduling policies when $CV_n=0.0$ (Cor,SSS)

Figure 5.32 Performance of scheduling policies when $CV_n=0.0$ (UnCor,SSS)

The relative performance of the four policies under the state-space search job model is similar to that of the divide-and-conquer job model. As mentioned previously, the presence of task abortion does not change the relative performance. The reasons for the poor performance of coscheduling and the similar performance between rrjob and dynamic follow from the discussion of the divide-and-conquer job model.

5.4.2 Discussion of Results for $CV_n=5.0$

The results of the correlated and uncorrelated versions of the fork-and-join workload model with $CV_n=5.0$ are presented in Figure 5.33 and 5.34.

Figure 5.33 Performance of scheduling policies when $CV_n=5.0$ (Cor,FJ)

Figure 5.34 Performance of scheduling policies when $CV_n=5.0$ (UnCor,FJ)

The performance of the policies under the correlated workload is discussed first. Under this workload, there are 20.561% of sequential jobs, 66.159% of jobs with 1 to 10 tasks, 7.4066% of jobs with 10 to 20 tasks and 5.381% of jobs with more than 20 tasks.

When the workload is correlated, jobs with large number of tasks tend to have larger service demand. Thus, coscheduling performs poorly, as a result of providing disproportionate share of the system. As discussed in the previous section, the dynamic policy gives an unequal share of the processing power among the jobs. This causes the policy to perform worse than the rrjob.

On the other hand, under the uncorrelated workload, the performance of the coscheduling is closer to that of the rrjob policy. When $CV_n=5.0$, there are large number of jobs that are either sequential or have small amount of parallelism. Moreover, in this case, there is no correlation between the service demand and number of tasks, so the jobs with large number of tasks may not have large service demand. Hence, even though the jobs with large number of tasks may take up series of quanta, their effects will be very limited. The dynamic policy performs poorly at moderate to high utilization levels. Since dynamic only allows as many jobs into the system as there are processors, the remaining jobs are kept in the queue until a job finishes execution. At moderate to high loads, the number of jobs in the system often exceeds the number of processors. As a result, the dynamic policy behaves similar to the first-come-first-served discipline and provides performance that is worse than the coscheduling.

Figure 5.35 Performance of scheduling policies when $CV_n=5.0$ (Cor,DC)

Figure 5.36 Performance of scheduling policies when $CV_n=5.0$ (UnCor,DC)

The results of the correlated and uncorrelated versions of the divide-and-conquer workload model with $CV_n=5.0$ is shown in Figures 5.35 and 5.36.

When $CV_n=5.0$, there are 28.6718% of sequential jobs, 58.8366% of jobs with 1 to 10 tasks, 7.4066% of jobs with 11 to 20 tasks and 5.085% of jobs with more than 20 tasks.

Coscheduling performs poorly under the correlated workload. The disproportionate share of the system for the large jobs is the reason for its poor performance. Dynamic performs poorly at high utilization level because the number of jobs in the system often exceeds the number of processors.

When the workload is uncorrelated, the performance of the coscheduling is closer to that of the rrjob for the reasons discussed in the last section.

Figure 5.37 Performance of scheduling policies when $CV_n=5.0$ (Cor,GE)

Figure 5.38 Performance of scheduling policies when $CV_n=5.0$ (UnCor,GE)

Figures 5.37 and 5.38 plot the results of the correlated and uncorrelated versions of the gaussian elimination workload model. The relative performance of the policies under this workload model is similar to that in the fork-and-join model.

Figure 5.39 Performance of scheduling policies when $CV_n=5.0$ (Cor,SSS)

Figure 5.40 Performance of scheduling policies when $CV_n=5.0$ (UnCor,SSS)

Figures 5.39 and 5.40 present the results for the correlated and uncorrelated version of the state-space search job model. The relative performance of the policies under this

workload model is similar to that in the divide-and-conquer job model. Once again it shows that the presence of task abortion does not change the relative performance of the four policies.

The following summarizes that impact of CV_n on the performance of the four scheduling policies. For the fork-and-join model, changing the CV_n value effects the relative performance. Note that $CV_n=0.0$ is a special case where all jobs in the system have the same number of tasks and is not used for the comparison. However, the relative performance of the policies for $CV_n=1.0$ and $CV_n=5.0$ can be compared.

The coscheduling policy under the fork-and-join workload model is considered first. The performance of coscheduling is close to that of the dynamic policy when $CV_n=1.0$. However, it performs worse than the dynamic policy when $CV_n=5.0$. This shows that the coscheduling policy is sensitive to variance in job parallelism. Coscheduling gives consecutive quanta to large jobs (in terms of service demand and number of tasks). This results in a disproportionate share of the processing power and longer mean job response time. In general, the relative performance of the policies is not changed by varying the CV_n value. This can be seen by comparing the Figures 5.1 and 5.2 to Figures 5.33 and 5.34.

It can be seen by comparing Figures 5.3 and 5.4 to Figures 5.35 and 5.36 that the relative performance of the four policies is not significantly affected by CV_n . Except when $CV_n=5.0$ and at high utilization levels, the dynamic policy performs worse than the coscheduling. Since dynamic does not time-share the processing power among the jobs, at high utilization levels, parallel jobs need to wait on the queue for a longer time. This results in increased mean job response times.

For the gaussian elimination workload model, changing the CV_n values does not affect the relative performance of the policies. This can be seen by comparing Figures 5.5

and 5.6 to Figures 5.35 and 5.36. Finally, for the state-space search workload model, varying the CV_n values has no effect on the relative performance of the policies as well. This can be seen by comparing Figures 5.7 and 5.8 to Figures 5.37 and 5.38.

5.5 Conclusions

From the above experiments and discussions, we have reached some conclusions. They are listed as follows.

- Varying the job structure has significant impact on the performance of the dynamic policy
- Dynamic policy is sensitive to CV_d for the fork-and-join model but not for the other models
- Performance of the scheduling policies is not be affected by task-abortion
- Most of the experiments indicate that coscheduling performs worse than the other policies except when the job structure is fork-and-join and $CV_n=0.0$. In this case, the coscheduling policy performs better than the others.
- Varying the CV_n values does not have significant effect on the relative performance of the policies.
- rjob performs very well for all the job models
- For DC, GE and SSS job models, the performance of dynamic policy is similar to the rjob

CHAPTER VI

CONCLUSION

6.1 Summary

Characterizing parallel programs is an important issue for performance evaluation of multiprocessing systems. The goal of this thesis is to investigate the effects of job structures on the performance of scheduling policies for multiprogrammed multiprocessors.

Chapter 1, we discussed various methods of characterizing parallel programs. Chapter 2 is an introduction to scheduling policies and review of the performance of these policies for a simple fork and a simple fork-and-join type workload.

Chapter 3 presented a survey of four classes of algorithms and their parallel implementations. They are classified as 1) fork-and-join, 2) divide-and-conquer, 3) Gaussian Elimination and 4) state-space search. An abstract model is then constructed for evaluating the performance of four scheduling policies.

Chapter 4 described the scheduling policies that are selected for performance evaluation.

Simulation experiments are conducted to evaluate the performance of these policies with different job models and the results are reported in Chapter 5.

6.2 Contribution of the Thesis

The main contribution of the thesis is in Chapters 3 and 5. Chapter 3 presented an abstract model of parallel programs that incorporates the structure of four categories of parallel programs. Such an approach has never been attempted before. Chapter 5 presented

the simulation results that compared the performance of the four scheduling policies. We have found that varying the job models affects the scheduling policies only in some special cases. In a normal circumstance, the effect is very limited. In addition, the results also prove that task abortion does not have any significant effect of the performance of the scheduling policies.

In summary, the results presented show that varying the job structure does not significantly affect the relative performance of the scheduling policies considered. In addition, the dynamic policy is sensitive to CV_d for the fork-and-join model but not so for the other models. Moreover the relative performance of the scheduling policies is not affected by the premature task-abortion. For DC, GE and SSS job models, the performance of the dynamic policy is similar to that of the rrjob.

6.3 Further Research

In this thesis, we introduced an abstract model to characterize parallel programs with regular synchronization pattern. We believe that it can also be used to represent programs that belong to other classes. Further research is needed to investigate the representativeness of this model.

The presence of pivot tasks in the GE job model introduces the need for giving priority to these tasks at the application level. It is interesting to investigate its effect on processor scheduling.

Another research direction is to apply this abstract model to study issues other than scheduling such as task routing in parallel distributed systems as the issue of synchronization should have significant impact on the routing decision of such systems.

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