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**Cost Function Regularity and Economies of Scale, Scope,
and Total Factor Productivity:
An Application to Class I Canadian Railways, 1956-81**

by

Ashish Lall, B.A.(Hons.), M.A., M.A.

**A thesis submitted to
The Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy**

Department of Economics

Carleton University

Ottawa, Ontario

April 21, 1992

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and Total Factor Productivity: An Application to
Class 1 Canadian Railways, 1956-81"**

submitted by

Ashish Lall, B.A., M.A., M.A.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy.

Chairman, Department of Economics

Thesis Supervisor

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ABSTRACT

More than a decade ago, in the most widely quoted study on productivity in Canadian railways, Caves and Christensen (1980b) found that, contrary to the property rights literature, the publicly owned firm Canadian National outperformed the private firm Canadian Pacific. This result was attributed to the competitive environment ushered in by the passage of the *National Transportation Act* in 1967.

Caves and Christensen used output cost elasticities from a cost function for U.S. railways to derive their measure of productivity. This thesis derives a parametric measure of technical change from a theoretically consistent flexible cost function. The estimation is based on Canadian data for the period 1956-81. The data set is manipulated in a careful manner with particular attention being paid to the formulation of the user cost of capital.

The cost function is assumed to take the Translog parametric form. This specification is found to violate the theoretical consistency requirement of concavity in factor prices. Classical and Bayesian methods are used to constrain the cost function to be globally and locally concave, respectively. Estimates of economies of scope are derived from a globally concave Symmetric Generalized McFadden cost function.

Models that constrain the production technology of both firms to be identical, indicate that Canadian railways exhibit increasing returns to scale (1.15). The degree of economies of scope is found to be 0.8, indicating significant economies from the joint provision of freight and passenger transportation service.

Inferences on technical change are drawn from models that allow both the level of total cost and the rate of cost diminution to differ across firms. The impact of input aggregation is also examined. The preferred model indicates that at identical factor prices and output levels total costs of Canadian National are 45% higher than those of Canadian Pacific. No statistically significant difference exists in the rate of cost diminution across firms. The average annual rate of technical change is approximately 1%. Over the period 1956-81, technical change accounts for 33% of the rate of growth of total factor productivity. The results obtained here are consistent with the property rights hypothesis; they do not support the conclusions of Caves and Christensen.

DEDICATION

To my parents Dr. Krishan Lall and Mrs. Indu Prabha Lall

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CHAPTER 1

INTRODUCTION

1.1 Motivation and Intent

The motivation behind this study is to shed some light on the characteristics of the production technology and efficiency of Class I Canadian railways. The conventional Divisia index of Total Factor Productivity (TFP) is often used as a measure of productive efficiency. The literature on productivity measurement demonstrates that this measure is derived from a well-behaved production or cost function, and provides a measure of technical change under the assumptions of constant returns to scale and marginal cost pricing. This thesis derives a parametric measure of technical change from a theoretically consistent flexible cost function. The cost function is estimated using panel data on Canadian National and Canadian Pacific railways for the period 1956-81¹. The cost function yields local estimates of output cost elasticities and returns to scale. This information is used to analyze the sources of TFP growth in Canadian railways.

Existing studies on TFP in Canadian railways either utilize revenue shares to form a measure of aggregate output or rely on estimates of output cost elasticities derived using parameters of cost functions estimated for U.S. railroads. Freeman et al.

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¹ Data problems prevent the use of a larger sample.

(1985b and 1987) use the former approach while Caves and Christensen (1980b) use the latter. They reach opposite conclusions both with regard to relative TFP and the rate of growth of TFP. Freeman et al. (1985b and 1987) find that Canadian Pacific (CP) has a higher rate of growth of TFP than Canadian National (CN). The relative TFP comparison shows that CN has lower TFP than CP. Caves and Christensen (1980b) reach the opposite conclusion; the TFP of CN grows at a faster rate than that of CP. Their comparison of TFP levels shows that CP is more productive than CN only for the period prior to the *National Transportation Act* of 1967. Both studies have shortcomings. Freeman et al. (1985b and 1987) use revenue shares to weight their measure of aggregate output; this imposes *a priori* restrictions² on the production technology. Caves and Christensen (1980b) use output cost elasticities inferred from a cost function for U.S. railways; it is preferable to use output cost elasticities based on Canadian data.

Some attempts have been made at estimating cost functions for Canadian railways, but they have encountered violation of theoretical consistency requirements. Studies which explicitly discuss such violations find them to be quite severe, others do not discuss this issue at all. It makes little sense to derive estimates of TFP using cost elasticities from an irregular cost function because, theoretically, measures of TFP are

---

² The use of revenue shares assumes constant returns to scale and marginal cost pricing.

derived from well-behaved cost functions. The duality between cost and production functions also requires the assumption of cost minimization³.

A conscious effort is made to manipulate the raw data in a careful manner⁴. Compared to previous research, the user cost of capital is refined considerably. ARIMA methods are utilized to derive a forward looking expectation of the price of new capital goods, the fact that Canadian Pacific relies on debt and equity finance is recognized, and the return on equity is derived from a CAPM model. A more representative price index is used for materials input. The derivation of the price of fuel does not assume perfect substitutability between various types of fuel, and data on the labour input are re-classified in an attempt to remove inconsistencies arising from accounting changes. The sensitivity of the results to the data is also examined. Four data alternatives are maintained throughout the thesis.

Initial estimates based on the transcendental logarithmic (translog) cost function indicate violation of the theoretical consistency requirement of concavity in input prices. Two methods are utilized to resolve this problem. The first is based on the sampling theoretic or classical approach and is used by Diewert and Wales (1987). The use of this method destroys the flexibility of the translog approximation. The second approach utilizes Bayesian methods, treating the inequality restrictions of

---

³ As demonstrated by Shephard (1953 and 1970), Uzawa (1964), and McFadden (1978b).

⁴ One data error that has gone unnoticed from Caves and Christensen to Freeman et al. is also corrected.

concavity in factor prices as prior information. Application of Bayesian methods to flexible cost functions and systems of demand equations is of recent origin and existing studies apply these techniques to situations where Maximum Likelihood estimates exhibit inconsequential violations of concavity and monotonicity constraints. This study provides some evidence on the effectiveness of Bayesian methods in situations where the violation of regularity is significant.

Finally, the sensitivity of results to changes in the specification of the cost function is examined. A multi-output Symmetric Generalized McFadden cost function is estimated. This is a globally concave flexible cost function. Its use provides the first estimates of economies of scope in Canadian railways based on a theoretically consistent cost function.

The following section provides a primer on cost concepts for multi-product firms. Sections 1.3 and 1.4 review the literature on railway cost functions and productivity. Section 1.5 provides a discussion of the contribution of Caves and Christensen (1980b) to the issue of the relative efficiency of privately owned versus government-owned firms. The final section provides an outline of the thesis.

1.2 Primer on Cost Concepts

This section provides a brief review of concepts typically used in studies on cost characteristics of multi-product transportation firms. The following implicit transformation function provides a general representation of any production technology:

$$f(Y, X, T) = 0 \quad (1.1)$$

where Y is a vector of outputs, X is a vector of inputs and the vector T is a vector of technology variables. McFadden (1978b) has demonstrated that if the transformation function in Equation (1.1) has a strictly convex input structure, then the following dual cost function provides a complete and equivalent characterization of the production structure:

$$C = g(Y, W, T) \quad (1.2)$$

where C is total cost, and W is a vector of prices at which inputs are purchased.

For a railway firm, the general cost function in Equation (1.2) may take the following form:

$$C = h(Y_{TM}, Y_{PM}, Y_H, Y_T, N, W, t) \quad (1.3)$$

where C is total cost, Y_{TM} is freight ton-miles, Y_{PM} is passenger-miles, Y_H is average length of haul (freight), Y_T is average trip-length, N is a measure of network size, W is a vector of factor prices, and t is a measure of technical change.

An increase in freight ton-miles can be brought about either by an increase in the tonnage carried or by shipping the existing tonnage over a longer distance. Of course, ton-miles can also increase due to an increase in both distance and weight. The inclusion of average length of haul as an output serves to isolate the impact of weight, and distance on total costs⁵. The inclusion of a measure of network size permits the calculation of both returns to density and returns to scale; these are defined below⁶.

Returns to density are defined as the proportional increase in all outputs made possible by a proportional increase in all inputs, with network, input prices and technology held fixed. Two alternative measures of returns to density may be derived from Equation (1.3); they are⁷:

---

⁵ A similar argument applies to passenger-miles and average trip-length.

⁶ The discussion of these two concepts is drawn from Caves and Christensen (1988), Caves, Christensen and Tretheway (1984), and Caves, Christensen, Tretheway and Windle (1985).

⁷ Even though returns to density/scale are formally defined here as relationships between inputs and outputs, the concepts are easily interpreted as relationships between outputs and costs. Suppose there is only one output; there is no distinction between returns to scale and returns to density. In this case, returns to scale are defined as the inverse of the elasticity of cost with respect to output or, the elasticity of output with respect to cost. The proportional increase in output due to a proportional increase in all inputs is identical to the proportional increase in output due to a proportional increase in total costs. This is because input prices are held fixed. For example, a two-fold increase in the use of all inputs will result in a two-fold increase in total costs if input prices are constant.

$$RTD_1 = (\epsilon_{TM} + \epsilon_{PM})^{-1} \quad RTD_2 = (\epsilon_{TM} + \epsilon_{PM} + \epsilon_H + \epsilon_T)^{-1} \quad (1.4)$$

where ϵ_{TM} is the elasticity of total cost with respect to ton-miles, and similarly, ϵ_{PM} , ϵ_H , and ϵ_T represent the elasticity of total cost with respect to passenger-miles, average length of haul, and average trip-length, respectively. RTD_1 is the more popular measure of measure of returns to density; it holds average length of haul and average trip-length fixed. Any increase in ton-miles (passenger-miles) is assumed to come from an increase in tons (passengers). RTD_2 allows average length of haul and average trip-length to increase in the same proportion as ton-miles and passenger-miles⁸. In this case, tons and passengers are held constant, with the increase in ton-miles and passenger-miles coming solely from increased distance⁹. Returns to density are said to be increasing, constant, or decreasing when RTD_1 (RTD_2) is greater than unity, equal to unity, or less than unity, respectively.

Returns to scale are defined as the proportional increase in the overall scale of operation (that is, outputs and network size) made possible by a proportional increase in all inputs, holding input prices and technology fixed. Two measures of returns to scale can be derived from the cost function in Equation (1.3); they are:

---

⁸ Studies of U.S. railways generally find ϵ_H and ϵ_T to be negative. As a consequence, RTD_2 exceeds RTD_1 .

⁹ Many studies do not include average length of haul and/or average trip-length in the output vector, therefore it is not always possible to make a distinction between RTD_1 and RTD_2 .

$$RTS_1 = (\epsilon_{TM} + \epsilon_{PM} + \epsilon_N)^{-1} \quad RTS_2 = (\epsilon_{TM} + \epsilon_{PM} + \epsilon_H + \epsilon_T + \epsilon_N)^{-1} \quad (1.5)$$

where ϵ_N is the elasticity of total cost with respect to network size. RTS_1 holds distance constant while RTS_2 holds weight and passengers constant. Returns to scale are said to be increasing, constant, or decreasing when RTS_1 (RTS_2) is greater than unity, equal to unity, or less than unity, respectively.

Finally, the degree of economies of scope (SC) may also be derived from the cost function. SC measures the relative cost increase that would result from fragmentation of the railway firm into two specialized firms. Such a division increases, decreases or leaves total costs unchanged as SC is greater than, less than or equal to zero, respectively. A positive value of SC indicates the presence of economies of scope, or cost savings from the joint provision of both freight and passenger service. SC is defined as follows:

$$SC = \frac{[C(Y_{TM}, 0) + C(0, Y_{PM}) - C(Y_{TM}, Y_{PM})]}{C(Y_{TM}, Y_{PM})} \quad (1.6)$$

where $C(Y_{TM}, 0)$ is the stand alone cost of providing freight service, $C(0, Y_{PM})$ is the stand alone cost of providing passenger transportation service and $C(Y_{TM}, Y_{PM})$ is the joint cost of providing both services¹⁰.

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¹⁰ Other variables in the cost function are suppressed for convenience.

1.3 Literature on Cost Functions for Canadian Railways

One of the earliest attempts at estimating returns to scale¹¹ for Canadian railways¹² using flexible cost functions is due to Ridgeway (1980). Ridgeway estimates a translog cost function for the railway industry as a whole, using time series data for the period 1952-76. The translog cost function is defined over two inputs; labour and capital, and four outputs; ton-miles of statutory grain, agricultural products, products of forests and mines, and manufactured goods¹³. Average hourly compensation is used as the price of labour and the wholesale price index is used as the price of capital. Though the costs of provision of passenger service are included, passenger output is ignored. Ridgeway suggests that the bias from the exclusion of passenger services is minimal since railway operations are dominated by freight movements. The two input specification rules out any complementarity between inputs.

For the period 1952-76, mean returns to scale are estimated to be 1.24. Returns to scale are declining in magnitude over the sample, the average over the period 1952-68 is 1.31, while that for the later period 1969-76 is 1.11. For the earlier

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¹¹ No distinction between scale and density can be made in models that do not include a measure of network size. Thus, returns to scale are defined as the inverse of the sum of output cost elasticities. Further, if average length of haul and average trip-length are excluded, a distinction cannot be made between RTS_1 and RTS_2 .

¹² Ridgeway utilizes publicly available data.

¹³ Disaggregated ton-mile data as utilized by Ridgeway are only available for the industry as a whole and not on a firm specific basis.

period, the 95% confidence interval admits values from 1.03 to 2.14, while over the period 1969-76, the interval admits both increasing and decreasing returns to scale; the range being 0.84 to 1.44.

A second and more recent attempt at estimating returns to scale is that of Roy and Cofsky (1985). This study uses panel data on CN and CP for the years 1956-81. The cost function is defined over five inputs and two outputs. The five inputs are labour, fuel, materials, equipment, and way and structures. The two outputs are passenger-miles and ton-miles of freight¹⁴. The measure of capital used in this study has been subject to much criticism. Total kilometres of first main track is used as a measure of way and structures capital. The stock of equipment is a weighted index of the nominal capacity of freight equipment and the number of passenger cars in service. The weights are the respective shares of freight and passenger service in total revenue. A generalized translog cost function is used in the estimation; this replaces the natural logarithms of the two outputs with a Box-Cox transformation. Technical change is assumed to be neutral. The major advantage of the generalized translog is that it does not collapse if there are zero outputs and therefore permits the estimation of economies of scope.

Roy and Cofsky do not obtain very encouraging results. They find that the theoretical consistency requirement of concavity in input prices is not satisfied at any

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¹⁴ Given the two outputs, returns to density are not calculated and there is only one measure of returns to scale.

point in the sample. The cost function is estimated over two time periods; 1956-78, and 1956-81. The results differ substantially, passenger specific economies of scale are 0.26 and -61.5, respectively. Freight specific economies of scale are 0.17 and - , 6.08, overall returns to scale are 1.17 and 3.35, and economies of scope are 0.86 and 4.36. Clearly, use of the longer time period results in negative marginal costs and therefore, a further deterioration in the results.

The third study is due to Freeman et al. (1985a). This is a study based on panel data¹⁵ for CN and CP covering the same time period as Roy and Cofsky. Like the latter, Freeman et al. (1985a) also assume that the railways produce two outputs by employing five inputs. In an attempt to estimate both economies of scale and economies of density, route-miles is used as a measure of network size¹⁶. Based on these variables, the authors experiment with various specifications of the cost function, starting with a Cobb-Douglas and expanding to a translog with non-neutral technical change. They also experiment with input aggregation, estimating both three and four input models.

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¹⁵ The raw data used in this thesis and those used by Freeman et al. (1985a) are drawn from the same sources.

¹⁶ Since average length of haul and average trip-length are not included, RTS_1 and RTS_2 are identical; so are RTD_1 and RTD_2 .

Only point estimates¹⁷ of returns to scale and density are reported, therefore the local behaviour of these measures cannot be examined. Further, there is no discussion of theoretical consistency requirements such as monotonicity and concavity in input prices, though in some models the coefficient of freight has a statistically significant negative sign, hence indicating negative marginal costs.

For the Cobb-Douglas specification in models without a measure of network, there are increasing returns to scale ranging from 1.06 to 1.96, depending on the input aggregation¹⁸. In models with a network variable, returns to scale¹⁹ vary from 1.22 to 1.41 and returns to density range from 2.37 to 4.12. In the three and five input versions, the hypothesis of constant returns to scale cannot be rejected at the 5% level of significance. An "arbitrarily chosen"²⁰ four output model is investigated beyond the Cobb-Douglas specification. In models without network effects, two models violate the requirement of positive output cost elasticities. For those that do not, returns to scale vary from 0.92 to 1.95. In models that include route-miles as a measure of network, again, two models produce negative marginal costs for one of the two outputs. For the remaining, returns to scale vary from 1.07 to 1.52 and returns to

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¹⁷ These results are calculated at the point of approximation of the cost function.

¹⁸ Without a network variable, returns to scale are defined to be the inverse of the sum of passenger and freight cost elasticities.

¹⁹ In this case, returns to scale are defined as the inverse of the sum of cost elasticities with respect to freight service, passenger service and network size. Returns to density are defined as the inverse of the sum of cost elasticities with respect to freight and passenger service.

²⁰ See Freeman et al. (1985a) page 26.

density vary from 1.96 to 4.46. Due to the offending sign on freight output, a general specification of technical change is rejected and Freeman et al. conclude that the "correct"²¹ specification for the cost function is translog with neutral technical change. This model suggests decreasing returns to scale (0.93) if the network measure is excluded. The inclusion of the network variable suggests increasing returns to both scale (1.19) and density (2.91). The lack of robustness of the results is not unexpected; this is primarily due to the limitations of the data²². Unlike studies on U.S. railways that are based on large rich panels, Canadian data offer fewer observations²³ and very few cross-sections. The paucity of Canadian studies is primarily due to the proprietary nature of some of the information²⁴.

The production technology of U.S. railways has been investigated quite thoroughly. No attempt is made to provide a survey, but some issues deserve mention. An overwhelming majority of the studies are based on the translog specification of the cost function. Though railways appear to be, and probably are prime candidates for a short run specification of the cost function, almost all studies use a long run cost function hence assuming that firms are able to instantaneously adjust all factors of production to their optimal levels in response to price changes.

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²¹ See Freeman et al. (1985a), page 28.

²² In the earliest study based on this data, Caves and Christensen (1978) observe that the data "are not of uniformly high quality". See page xii.

²³ Changes in accounting methods prevent the construction of a long consistent time series.

²⁴ The Canadian industry is comprised primarily of two large firms which. Both CN and CP, and the National Transportation Agency are inordinately protective of the data.

As mentioned earlier, the large data sets on U.S. railways permit much richer specifications of the cost function. Since many studies include a measure of network they are able to provide estimates of both economies of density and scale. Most studies also include variables such as average length of haul²⁵ thus enabling them to isolate the effects of distance and weight on total cost. Table 1.1²⁶ summarizes the results from some influential studies.

Table 1.1			
Returns to Scale and Density in U.S. Railroads			
Study	Returns to Density (RTD₁)	Returns to Scale	
		Fixed Haul & Trip (RTS₁)	Increased Haul & Trip (RTS₂)
Friedlaender and Spady (1981)	1.16	0.88-1.08	1.07-1.37
Caves, Christensen & Swanson (1981)	-	1.01	1.13
Harmatuck (1979)	1.92	0.93	1.02
Harris (1977)	1.72	1.03	1.17
Keeler (1974)	1.79	1.01	-
Caves et al. (1985)	1.76	0.98	1.09

In the above table, estimates of returns to density hold average length of haul and average trip-length constant, therefore increases in ton-miles (passenger-miles) are

²⁵ This is defined as ton-miles divided by tons.

²⁶ This table is reproduced from Caves, Christensen, Tretheway and Windle (1985), page 112. Detailed references are available in that paper.

assumed to come solely from increased tonnage (passengers). Caves et al. (1985) also estimate RTD_2 for U.S. railways; it is found to be 3.0²⁷. The estimate of RTD_1 in the last row of the Table 1.1 indicates that a 1% increase in both ton-miles and passenger-miles (caused by increased tonnage and passengers) increases total costs by 0.57%²⁸. The estimate of RTD_2 indicates that total costs increase by 0.33%²⁹ if the increase in ton-miles and passenger-miles is caused by increased distance.

So far as returns to scale are concerned, the last row of Table 1.1 shows that a 1% increase in the scale of operations (network, ton-miles, and passenger-miles; with distance held fixed) causes total costs to increase by 1.02%³⁰. The estimate of RTS_2 shows that a 1% increase in the scale of operations (network, passenger-miles, ton-miles, average length of haul, and average trip-length; tonnage and passengers held fixed) increases total costs by 0.92%³¹. Since studies on U.S. railways generally find the cost elasticities with respect to average length of haul, and average trip-length to be negative, estimates of RTD_2 (RTS_2) exceed those of RTD_1 (RTS_1).

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²⁷ See footnote 9 on page 109 of Caves, Christensen, Tretheway and Windle (1985).

²⁸ Inverse of 1.76.

²⁹ Inverse of 3.0.

³⁰ Inverse of 0.98.

³¹ Inverse of 1.09.

1.4 Literature on Productivity in Canadian Railways

The most influential study on productivity in Canadian railways is that of Caves and Christensen (1980b)³². They use Total Factor Productivity (TFP) to examine the relative efficiency of the government-owned firm: Canadian National and the privately owned firm: Canadian Pacific, over the period 1956-75³³.

The primary productivity estimates of Caves and Christensen are based on a two output, five input specification of railway technology³⁴. The average annual rate of growth of TFP was found to be 3.1% for CN and 2.5% for CP over the period 1956-75. Relative TFP³⁵ (CN/CP) was less than one prior to 1967, thereafter it exceeded or was equal to unity. The measure of aggregate output constructed by Caves and Christensen uses output cost elasticities as weights. Output cost elasticities are derived from the parameters of a translog cost function estimated by Caves, Christensen and Swanson (1980) for U.S. railways. The cost function is estimated using cross-section data for the years 1955, 1963, and 1974. Parameters from the U.S. cost function and data from CN and CP are used to obtain output cost elasticities for

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³² This is the revised and published version of their 1978 study done for the Canadian Transport Commission.

³³ This issue is examined in more detail in the next section.

³⁴ The two outputs are freight ton-miles and passenger-miles. The five inputs are labour, fuel, way and structures capital, equipment capital, and materials.

³⁵ Based on the bilateral translog productivity index due to Jorgenson and Nishimizu (1978), using 1963 as the base year.

Canadian firms for the three years. The cost elasticities are then interpolated between years to yield annual weights. Their U.S. cost function results also suggest that in the region of output levels produced by CN and CP, the hypothesis of constant returns to scale cannot be rejected³⁶.

In order to examine the robustness of their productivity results, Caves and Christensen provide two additional measures of TFP. The first alternative productivity estimates include two additional outputs: average length of haul and average trip length. The second alternative productivity estimates utilize the two output specification of the primary productivity estimates, but the two outputs are constructed from very disaggregated data which classify various types of freight by many mileage bands³⁷. The two alternative measures indicate that the rate of growth of CN's productivity was 2.9% and 3.6% respectively, while that of CP's was 2.4% and 2.6%. A comparison of the level of TFP provides similar results to those obtained in the primary productivity estimates. The authors conclude that contrary to the property rights argument, the government-owned firm outperforms the private firm primarily due to the existence of competition.

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³⁶ See footnote 12 on page 965 in Caves and Christensen (1980b).

³⁷ The disaggregated data are from the Waybill Analysis and are not very reliable. Revenue shares are used to aggregate up to the two output level, then, output cost elasticities are used to aggregate the two outputs freight ton-miles and passenger-miles.

In a later study, Caves, Christensen, Swanson and Tretheway (1982) extend the first alternative productivity estimates³⁸ of Caves and Christensen for an additional four years. As expected, their results merely reaffirm the earlier conclusions.

The most recent study of productivity in Canadian railways is by Freeman et al. (1987)³⁹. It covers the period 1956-81 and uses the two output, five input specification employed in the primary productivity estimates of Caves and Christensen. Freeman et al. find that CP is more productive than CN for every year in the sample. Relative TFP (CN/CP) ranges from 0.8 to 0.95, the average annual rate of growth of TFP is 3.4% and 3.5% for CN and CP respectively. The authors do not make any attempt to reconcile their results with those obtained earlier, they state that the results are not comparable. Freeman et al. use a more restrictive measure of TFP. Their assumptions of marginal cost pricing and constant returns to scale permit the use of revenue shares rather than output cost elasticities as weights in the construction of an index of aggregate output. They use the translog multilateral index to make comparisons over time and across firms. They also correct some data errors committed by previous researchers.

Despite the poor performance of the cost function, Roy and Cofsky also provide productivity estimates for CN and CP. They derive two measures of TFP, one

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³⁸ This is the four output, five input specification.

³⁹ The main results of this book are also available in Freeman et al. (1985b).

based on revenue shares and the other on output cost elasticities. The use of revenue share yields an average annual rate of growth of TFP of 3.25% for CN and 3.09% for CP over the period 1956-81. Relative TFP is not calculated, but these results seem to be in conformity with those of Caves and Christensen and at odds with those of Freeman et al. The use of output cost elasticities produces discouraging results. The average annual rate of growth of TFP for CN is a phenomenal 9.8% whereas CP lags behind at 3.8%.

1.5 The Relative Efficiency of Government-Owned and Privately Owned Firms

Caves and Christensen (1980) propose that there are two separate themes in the economics literature on productive efficiency. Drawing on the property rights literature, they argue that privately owned firms are likely to be more efficient than government-owned firms. Their reading of the industrial organization literature suggests that the existence of effective competition between firms promotes efficiency. Canadian railways exist in an environment which reflects a mixture of the two effects, CN is a government-owned firm whereas CP is privately owned. Further, Caves and Christensen identify two sources of "effective competition". Since World War II both firms have faced increasing competition from other modes, particularly trucking. In 1967, the *National Transportation Act* fostered competition between the two railways by relaxing regulations and allowing more freedom in pricing. Though the *National Transportation Act* did not permit widespread track abandonment, it did provide for

considerable freedom in rate setting. Market and modal competition determined rates, with the floor being set by the variable costs of the particular movement.

Caves and Christensen found that average annual TFP growth was higher for CN than for CP over the entire sample. A comparison of TFP levels revealed that the performance of CN was comparable to, or superior to that of CP after 1967. Based on these results, they concluded⁴⁰:

"Contrary to what is predicted in the property rights literature, we find no evidence of inferior efficiency performance by the government-owned railroad. In fact, our evidence indicates that the CN has achieved larger gains in productivity than the CP since 1956...We conclude that the beneficial effects of competition have been sufficient to overcome any tendency toward inefficiency resulting from public ownership."

The remainder of this section takes issue with the work of Caves and Christensen. The literature provides weak support for the argument that product market competition promotes productive efficiency. It is debatable whether CN and CP operate in a competitive environment or whether tacit collusion exists in some aspects of their operations. Though the *National Transportation Act* brought about more freedom in rate setting, it did not unleash the forces of competition. Even if it did, the TFP results of Caves and Christensen cannot lead one to attribute the superior performance of CN to increased competition.

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⁴⁰ See pages 960-961 in Caves and Christensen (1980b).

The property rights literature asserts that efficiency differentials can be explained by ownership; private firms are expected to be more efficient than government-owned firms. Alchian and his associates are the main proponents of this view⁴¹. Private firms have lower net agency costs. The decisions made by private firms are capitalized in the markets for private ownership claims or the market for corporate control. Transferability of ownership permits the exercise of authority over how resources are employed. Better utilization of resources increases the value of the firm and therefore the value of the shares held by the controlling group⁴². This mechanism provides an incentive to acquire control. Transferability also serves as a dispute resolution mechanism whereby those who disagree have the option to "exit" or sell their shares. Internal rivalry between managers of private firms, performance incentives and the board of directors are other sources of lower agency costs.

Monitoring and bonding activity is much more complicated and expensive in government-owned firms. There is a hierarchy of principal-agent problems; made up of the general public, politicians, unelected civil servants and managers of government-owned firms. The market for corporate control is absent and prices are often set by "rules of thumb". Private firms on the other hand pursue pricing strategies that maximize the value of the firm. There is little incentive for the general public to monitor the activities of government-owned firms. The benefits of such

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⁴¹ This literature is summarized by various authors. A comprehensive treatment can be found in Borcharding (1983), and McFetridge (1985).

⁴² Alternatively, one can think of private firms as facing bankruptcy constraints.

activity are shared with society at large, while the costs are borne only by the group engaged in monitoring activity.

The theoretical underpinnings of the impact of market structure on productive efficiency are dubious. Caves and Christensen state⁴³:

"The industrial organization literature, predicts superior efficiency in markets characterized by effective competition among firms. The essential element of the theory is that in competitive markets productive efficiency is a prerequisite for survival-at least for privately owned firms."

Unfortunately, Caves and Christensen provide no citations from the "industrial organization literature". Their reference to the work of Spann, and De Alessi provides a mixed picture⁴⁴. Spann "expects" the product market to impose some discipline on discretionary behaviour on the part of bureaucrats. De Alessi asserts that access to subsidies may enable government firms to survive for long periods with grossly inefficient management.

On the topic of product market competition and agency costs, McFetridge writes⁴⁵:

"Competition in product markets is thought to induce managerial efficiency for a number of reasons, none of which are particularly persuasive. First it is thought that competition in the product market would be conducive to the evolution of superior techniques of managerial control. Second, it is argued that given competitive product markets, firms which fail to adopt techniques

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⁴³ Page 959 in Caves and Christensen (1980b).

⁴⁴ The references are available in Caves and Christensen (1980b).

⁴⁵ Page 198 in McFetridge (1985).

that minimize agency costs will disappear. In essence, the economics of natural selection is at work here."

Even if such Darwinian forces become operative in a competitive environment and cause both government-owned and private firms to become more efficient, private firms still have an edge on account of lower net agency costs. Caves and Christensen do not disagree with this interpretation, they state⁴⁶:

"There is little doubt that expert opinion would rank competitive private firms as most efficient..."

Strictly speaking, the two major Canadian railways do not fall under any of the four categories described by Caves and Christensen. These are: government-owned and competitive, government-owned and non-competitive, private and competitive, and private and non-competitive. Both CN and CP are regulated by the National Transportation Agency.

Surveys of empirical comparisons of the efficiency of private and government-owned firms disagree on the interpretation of the results obtained by Caves and Christensen. Borcharding (1983)⁴⁷ argues that the "impressive study by Caves and Christensen" is one of the:

"... few negative pieces of evidence for Alchian's hypothesis that public firms are more costly to operate than their private counterparts ..."

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⁴⁶ See page 959, Caves and Christensen (1980b).

⁴⁷ See pages 133-134 in Borcharding (1983).

According to Millward and Parker, the evidence from various industries suggests that regulation raises the costs of private firms and competition reduces the costs of government-owned firms. They do not find any general indication that private firms are more cost efficient than government-owned firms. With regard to Canadian railways, they state⁴⁸:

"There is at present, no general support for the proposition that public enterprises are less cost efficient than private firms. The Canadian railroads experience confirms that - it cannot, *per se*, be adduced in support of a role for competition."

Vickers and Yarrow (1989) interpret the study by Caves and Christensen to be "confirming evidence of the importance of competition ..." They are also critical of the general conclusions drawn by Millward and Parker. Vickers and Yarrow interpret the empirical literature as suggesting that private firms tend to be more efficient if there is effective competition in product markets. If market power is significant or if firms are subject to regulation, there is little empirical justification in favour of either type of ownership.

Boardman and Vining (1989) assert that the existing empirical evidence provides weak support for the property rights theory. Further, they argue that practically none of the existing research has tested the effect of ownership in a

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⁴⁸ See page 250 in Millward and Parker (1983).

competitive environment. In all the existing comparisons, at least one or more of the following limitations apply⁴⁹:

"(1) firms have a natural (spatial) monopoly (namely, electric and water utilities, nonrail transit, fire services, and refuse collection); (2) there is a regulated duopoly (for example, airlines railroads, nonrail transit, financial institutions); or (3) output is not or cannot be priced by competitive forces (this applies to all of the above plus health services)."

Given these limitations, they argue that the work of Caves and Christensen cannot be taken to imply that government-owned firms may be as efficient as private firms in a competitive environment.

Caves and Christensen point out that since World War II CN and CP have faced stiff competition from other modes of transportation, particularly the trucking industry. Thus both firms faced inter-modal competition. Over the years, relaxation of regulation has enhanced the ability of the two firms to compete with other modes and with each other. The *National Transportation Act* of 1967 did provide railways considerable freedom in pricing; it did not do away with collective rate making or the exchange of cost information on interline rates. The railways were not free to abandon track at will and were still obliged to carry grain at statutory rates. Thus it is important to emphasize that this was not "deregulation" in the true sense.

The results obtained by Caves and Christensen do not capture the impact of increased inter-modal competition in the post-war period since the entire sample

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⁴⁹ See page 1 in Boardman and Vining (1989), and page 7 for the discussion on Canadian railways.

relates to the post-war era. Further, prior to 1967, the average annual rate of growth of TFP is higher for CN than for CP. The faster growth helps CN to catch-up with CP by 1967. The relaxation of regulations in 1967 has no bearing on this result.

In a later article, Caves, Christensen, Swanson and Tretheway (1982), undertake a TFP comparison of Canadian, and their more regulated counterparts, the U.S. railways. Their results on productivity of Canadian railways are similar to those of Caves and Christensen (1980b). Their comparison of Canadian and U.S. firms indicates a superior performance by the less regulated Canadian firms both in terms of relative TFP and the rate of growth of TFP. Heaver and Waters provide the following comments on these results⁵⁰:

"... the years of highest annual growth of TFP are 1963 and 1964, before the passage of the NTA. This makes us a little uneasy in assuming the significance of a changed regulatory environment as the sole factor explaining the high performance of Canadian railways."

All that can be said on the basis of the results obtained by Caves and Christensen is that CN was less productive than CP in 1956. Over the period 1956-75, the average annual rate of growth of TFP was higher for CN than for CP. This allowed CN to catch-up to CP by 1967. The property rights argument would be clearly rejected had the TFP level of CN (in 1956) also exceeded that of CP. As they stand, the results of Caves and Christensen reject the property rights literature in so far

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⁵⁰ See page 154 in Heaver and Waters (1982).

as the growth in TFP of the government-owned firm was faster than that of the privately owned firm.

The results obtained by Freeman et al. (1987) support the property rights hypothesis since CN is always less productive than CP. CN also exhibits a marginally lower average annual rate of growth of TFP than CP. Unfortunately, these results are based on an inappropriate output index. The output index uses revenue shares as weights; it assumes marginal cost pricing and constant returns to scale in railway operations.

1.6 Outline of the Thesis

This thesis has eight chapters and three appendices. Chapter 2 provides the basic methodology relating to a long run translog cost function. It also provides some evidence of the superiority of the translog to other flexible cost functions such as the Generalized Leontief. Chapter 3 discusses the data. The discussion of some issues relating to the index of fuel use is relegated to Appendix 1. Appendix 2 provides data tables. Chapter 4 provides results from the basic specification and investigates other specifications such as three output models, models with a network variable, and a restricted cost function.

The parametric form of the cost function is changed in Chapter 5. A Symmetric Generalized McFadden (SGM) cost function is estimated in order to ensure theoretical consistency. This chapter also provides estimates of economies of scope. Since the SGM produces unreasonable estimates of returns to scale, the focus is shifted back to the translog in Chapter 6. Global concavity is imposed on the translog using the method outlined by Diewert and Wales. Further, Bayesian methods are used to impose minimum regularity on the cost function. The advantage of the Bayesian approach is that it allows the imposition of inequality constraints on the translog without sacrificing flexibility.

Chapter 7 allows both the level of total cost, and the rate of cost diminution to differ across firms. It provides a parametric measure of technical change in Canadian railways. Sensitivity of the results to the data, input aggregation, and the extent of regularity of the cost function is also examined. Information from the cost function is used to quantify the contribution of deviations from marginal cost pricing, variable returns to scale and pure technical change to the conventional index number measure of TFP. An attempt is also made to reconcile the results obtained here with those of previous research. Supplementary results from Chapters 4 through 7 are provided in Appendix 3. Chapter 8 summarizes the major conclusions.

CHAPTER 2

METHODOLOGY

The measurement of productivity is based on the neo-classical theory of production. Developments in duality theory in the mid 1970s have shown that there are numerous ways of examining the underlying production technology of firms and industries. Following these theoretical developments, the methods utilized in applied production analysis have also undergone significant change. This chapter begins with a description of the properties of cost functions in general¹ and then discusses the particular form of the cost function employed here.

2.1 Properties of Cost Functions

The production technology of a firm is generally characterized by a production function². This relationship between output and inputs is viewed as an engineering specification. If x is a N -dimensional vector of inputs utilized in any period, then the production function reveals the maximal output y that can be produced using this vector of inputs. The production function can be written as:

$$y = f(x) \tag{2.1}$$

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¹ This discussion is based on Diewert and Wales (1987).

² Alternatively, a transformation function can be used as the primitive.

Developments in duality theory have also enabled the representation of the production technology of a firm by means of a cost function. Given a vector of positive input prices, $p \gg 0_N$ the cost function is obtained as the solution to the following problem:

$$C(y, p) = \min_x [p'x : f(x) \geq y, x \geq 0_N] \quad (2.2)$$

The cost function $C(y, p)$ is assumed to be a positive real valued function in y and approaches infinity as y approaches infinity. Further, $C(\cdot)$ is twice continuously differentiable and non-decreasing in p , homogeneous of degree one and concave in p for all positive output levels. Given the above conditions and competitive factor markets, it has been shown³ that there exists a production function that is dual to the cost function $C(\cdot)$. Even though it may not be possible to express the dual production function in explicit parametric form, the nature of input substitution, returns to scale and other properties of the underlying technology can be determined from the cost function alone.

Since in a particular empirical application the functional form of the cost function is not known, it is approximated at a point. The algebraic functional form that is used to approximate the underlying technology should be flexible⁴, in that it

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³ See Shephard (1970) and Diewert (1971 and 1974).

⁴ For a discussion of flexibility see Lau (1974), pages 183-184 and Lau (1986), Section 2.3. A more rigorous treatment of flexibility and second order approximation can be found in Barnett (1983b).

should be capable of approximating any arbitrary but theoretically consistent cost function through an appropriate choice of parameters. Let $p^* \gg 0_N$ and $y^* > 0$. A cost function $C(y, p)$ provides a second-order differential approximation⁵ to the true cost function $C(\cdot)$ at the point (y^*, p^*) if and only if it contains enough free parameters to satisfy the following $1+(N+1)+(N+1)^2$ equations:

$$\begin{aligned} C(y^*, p^*) &= C^*(y^*, p^*) \\ \nabla C(y^*, p^*) &= \nabla C^*(y^*, p^*) \\ \nabla^2 C(y^*, p^*) &= \nabla^2 C^*(y^*, p^*) \end{aligned} \quad (2.3)$$

Imposing linear homogeneity in input prices on the approximating function $C(\cdot)$ implies the following $(N+2)$ restrictions:

$$\begin{aligned} p^{*'} \nabla_p C(y^*, p^*) &= C(y^*, p^*) \\ p^{*'} \nabla_{pp}^2 C(y^*, p^*) &= 0'_N \\ p^{*'} \nabla_{py} C(y^*, p^*) &= \nabla_y C(y^*, p^*) \end{aligned} \quad (2.4)$$

Further, symmetry of the second-order partial derivatives of $C(\cdot)$ implies the following $N(N+1)/2$ restrictions, where $\nabla^2 C(\cdot)$ is the matrix of second-order partial derivatives with respect to all the $(N+1)$ arguments of $C(\cdot)$:

$$\nabla^2 C(y^*, p^*) = [\nabla^2 C(y^*, p^*)]' \quad (2.5)$$

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⁵ This definition of flexibility is due to Diewert (1974), page 113. Lau (1974), page 184 refers to Diewert's concept as a "differential approximation". The term "second order" is used because the gradient and the hessian of the cost function with respect to input prices can assume arbitrary non-negative and negative semidefinite values respectively.

If $C(\cdot)$ satisfies linear homogeneity in factor prices and symmetry, then the definition of flexibility requires it to have at least $[1+(N+1)+(N+1)^2]-(N+2)-[N(N+1)/2]$ or $(N+1)(N+2)/2$ parameters. In order that $C(\cdot)$ satisfies all the theoretical consistency requirements, it must be concave in input prices. This requires the N by N Hessian of the cost function with respect to input prices $\nabla_{pp}C(y^*, p^*)$ to be a negative semidefinite matrix.

The $(N+1)(N+2)/2$ parameters⁶ can be thought of as distinct economic effects that characterize the usual comparative static properties of a cost function. These are, the cost level, returns to scale, $(N-1)$ input shares, N own price elasticities of factor demand and $N(N-1)/2$ Allen-Uzawa partial elasticities of factor substitution⁷.

2.2 Choice of Functional Form

Two well-know flexible functional forms used in empirical applications are the Translog⁸ (TL) and the Generalized Leontief⁹ (GL). The former is the more popular of the two. As mentioned in Chapter 1, recent literature on the railways is based

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⁶ If the cost function were described over N inputs, M outputs and a measure of technical change, or $C=C(y, p, t)$, then flexibility would require at least $N(N+1)/2 + M(M+1)/2 + NM + N + M + 1$ free parameters.

⁷ Fuss, McFadden and Mundlak (1978), page 231 and Nadiri (1982), page 438 have identified these and other effects for a production function.

⁸ This is due to Christensen, Jorgenson and Lau (1971) and (1973).

⁹ Due to Diewert (1971).

almost entirely on the TL. This fact in itself may be sufficient justification for using the TL, however the literature on choice of functional form¹⁰ provides additional evidence on the superiority of the TL approximation.

Studies on choice of functional form can be classified on the basis of the tools they utilize. Existing studies use one of three approaches. The first can be termed theoretical or analytical, the second, empirical and the last utilizes Monte Carlo methods.

2.2.1 The Analytical Approach

Caves and Christensen (1980a) and Barnett, Lee and Wolfe (1985) use the analytical approach to examine various functional forms¹¹. Flexible cost functions have the capability of assuming arbitrary substitution elasticities at any particular data point. However, selection of a given set of parameter values, *a priori*, or by means of estimation, determines the pattern of these elasticities for all possible data points. Some of these values may not be plausible, for example, they may not satisfy the theoretical consistency requirements such as monotonicity and concavity and hence lie outside the region of regularity. It may also be the case that the regular region excludes the entire data set under consideration. Functional forms may therefore differ

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¹⁰ Lau (1986) provides a very readable discussion of ex-ante choice of functional form in the context of consumer and producer choice.

¹¹ Both papers examine consumer preferences rather than production technology.

with respect to the range over which they provide an acceptable characterization of the underlying technology or the underlying preferences. These differences may also be dependent on the nature of the underlying technology.

The theoretical approach therefore attempts to determine the range over which flexible functional forms provide an acceptable representation of producer or consumer behaviour. Caves and Christensen examine a two good TL and GL indirect utility function, with homothetic preferences¹². They find that the GL has a larger regular region when the elasticities of substitution are small and similar or dissimilar. The TL is found to have a larger regular region when the elasticities are similar and large. In the case of non-homothetic preferences neither dominates, whether substitution is easy or difficult.

Barnett, Lee and Wolfe are not surprised by these findings since the TL is known to satisfy global regularity conditions when preferences are Cobb-Douglas, and the GL does so in the Leontief special case, that is, when substitution possibilities do not exist and preferences are homothetic. They extend the approach taken by Caves and Christensen, and consider three goods with non-homothetic preferences. Further, they include in the comparison, the Minflex Laurent functional form¹³. They conclude that of the three, the Minflex Laurent is the only form capable of serving the

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¹² They also extend this to non-homothetic preferences and provide some examples of cases with more than two goods.

¹³ This functional form is due to Barnett (1985).

objectives that were originally contemplated for flexible functional forms. The TL and GL are found to have smaller and less stable regular regions. The most that can be expected of these functional forms is that they will be well-behaved in the neighbourhood of the point of approximation which is conventionally selected to be the median observation.

2.2.2 The Empirical Approach

Results based on the empirical performance of various functional forms using particular data sets are less informative. This is because results based on particular data sets cannot be generalized to all data sets. Further, this approach does not shed much light on the problem of ex-ante choice of functional form. Nonetheless, the main findings are summarized below.

Berndt, Darrough and Diewert (1977) examine three functional forms, the TL, GL, and a Generalized Cobb-Douglas in the context of consumer theory¹⁴. A "principal purpose" of their work is to "compare and discriminate" among the aforementioned forms¹⁵. They suggest that *a priori* discrimination either on theoretical or econometric grounds is not possible. All three forms have identical differential approximation properties, the estimation involves the same dependent

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¹⁴ These functions are taken as approximations to a non-homothetic reciprocal indirect utility function defined over three goods. Post-war Canadian household expenditure data is utilized.

¹⁵ See Berndt, Darrough and Diewert (1977), page 661-662.

variables and the same number of free parameters. Classical test procedures cannot be utilized since the models are non-nested. They choose among the functions *a posteriori*, using a Bayesian discrimination technique. This technique is equivalent to choosing the model which produces the smallest sample generalized residual variance, that is, the smallest value of the determinant of the error covariance matrix. Based on this test, the TL is the preferred specification.

An alternative approach is that employed by Applebaum (1979) and Berndt and Khaled (1979)¹⁶. Both papers use the Generalized Box-Cox functional form¹⁷, nested in which are the TL, GL and the Generalized Square-Root Quadratic (GSRQ) as special cases. Applebaum found the GL and GSRQ to provide the best representation of the direct and indirect production functions respectively. Berndt and Khaled cannot reject the GL form for the cost function. They reject the GSRQ and suggest that models close to the TL case would also be rejected. Clearly, the conclusions of this line of research are data dependent and cannot be generalized.

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¹⁶ This method of testing nested hypotheses also falls in the realm of ex-post rather than ex-ante choice of functional form.

¹⁷ This is due to Khaled (1978).

2.2.3 The Monte Carlo Approach

The Monte Carlo approach is perhaps the most telling of the three, since the true technology is known and the researcher is examining the ability of various functional forms to track the technology over a range of observations. This line of inquiry was initiated by Wales (1977) and has been extended by Guilkey and Lovell (1980) and Guilkey, Lovell and Sickles (1983).

Wales represents true preferences by a two good homothetic constant elasticity of substitution (CES) utility function. He then examines the approximation properties of the TL and GL reciprocal indirect utility functions. So far as curvature is concerned, the fundamental results are not different from those derived by Caves and Christensen. As the elasticity of substitution departs from unity in either direction, the performance of the TL worsens, while the GL does worse for large values of the elasticity of substitution. The curvature results also deteriorate when the variance of the independent variables increases. This finding is disturbing because obtaining efficient parameter estimates is more difficult if this variance is small. Another surprising finding is that the approximating function may fit the data very well as measured by R^2 even if curvature is violated. Both models provide good estimates of

the elasticity of substitution when the true value is less than or equal to one¹⁸. In conclusion, Wales notes¹⁹,

"Although the flexible functional forms are capable of a local second-order approximation to an arbitrary utility function, they do not necessarily provide a good approximation over a range of observations, as measured by their ability to satisfy the regularity conditions required for utility maximization"

The violation of theoretical consistency requirements need not imply the absence of an underlying utility maximization process, rather it is an indication of the inability of the function to approximate the true technology over the range of the data. With regard to choice of functional form he suggests that it may be wise to estimate both the GL and TL forms.

Guilkey and Lovell use Monte Carlo techniques to examine the ability of the TL functional form to track technologies of varying degrees of complexity. They also attempt to determine whether the TL is better suited to estimate scale economies rather than elasticities of substitution. Lastly, they compare the single equation and system of equations TL models.

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¹⁸ Of course, the TL yields exact results when the elasticity of substitution is unity and the GL does so when it is zero.

¹⁹ Wales (1977), page 191.

The performance of the TL is found to deteriorate as the technology becomes more complex, but by an "impressively small amount"²⁰. The TL model is not particularly superior in estimating scale economies rather than elasticities of substitution, so long as the latter lie between one third and three the TL provides good estimates of both scale economies and elasticities of substitution. The estimate of the mean returns to scale is biased upwards and there is a tendency for the estimates of scale economies to be clustered rather closely about the estimated mean. Guilkey and Lovell also find that the systems model does not outperform the single equation model.

This line of inquiry is broadened by Guilkey, Lovell and Sickles. They compare the ability of three approximating functions, TL, GL and the Extended Generalized Cobb-Douglas (EGCD) to track a known technology²¹. The TL systems model outperforms both the GL and EGCD and single equation models by a wide margin in its ability to provide good estimates of the returns to scale. This model not only estimates the true returns to scale with the smallest bias, but it also performs well in terms of tracking the path of the returns to scale. The dominance of the TL model is particularly obvious in cases where complementarity is present.

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²⁰ Guilkey and Lovell (1980) pages 143-144.

²¹ As in the case of the earlier paper, the focus is not on issues relating to regularity.

The TL has good approximating properties, provided the true technology is not too complex. If the elasticities of substitution diverge from each other, or if similar elasticities depart from unity, the performance of the TL deteriorates. If the partial elasticities are small and positive, the GL form is preferable. This is the only case in which the TL and EGCD provide poor results, in all other cases, the GL ranks last. The systems models are also found to perform better than single equation models particularly in case of complementarity and when the partial elasticities diverge from each other. The authors conclude that their "effort to turn up a flexible functional form more reliable than the TL form must be considered a failure."²² The Monte Carlo evidence clearly suggests that the TL is superior to the GL as an approximating function.

The above review also suggests that increased flexibility is not necessarily costless and may result in sacrifice of theoretical consistency. If theoretical consistency is of concern, Barnett, Lee and Wolfe suggest that the Minflex-Laurent is superior to both the TL and GL particularly if time series data is utilized. Not only is the Minflex-Laurent more complex algebraically, but estimation also requires the imposition of inequality restrictions on the parameters. This functional form has not seen popular use. The translog is therefore the obvious choice for the problem at hand. The translog cost function and the associated comparative static effects are discussed in the following section.

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²² See Section 6(3) on page 614.

2.3 The Translog Cost Function

The following is a translog cost function with general technical change, where y is an M -dimensional vector of outputs, p is an N -dimensional vector of factor prices and t is a measure of technical change.

$$\begin{aligned}
 \ln C(y, p, t) = & \alpha_0 + \sum_{i=1}^N \alpha_i \ln p_i + \sum_{k=1}^M \beta_k \ln y_k + \alpha_t t + (1/2) \alpha_{tt} t^2 \\
 & + (1/2) \sum_{i=1}^N \sum_{j=1}^N \gamma_{ij} \ln p_i \ln p_j \\
 & + (1/2) \sum_{k=1}^M \sum_{l=1}^M \beta_{kl} \ln y_k \ln y_l \\
 & + \sum_{i=1}^N \sum_{k=1}^M \delta_{ik} \ln p_i \ln y_k + \sum_{i=1}^N \alpha_{it} t \ln p_i \\
 & + \sum_{k=1}^M \beta_{tk} t \ln y_k
 \end{aligned} \tag{2.6}$$

The following restrictions are necessary and sufficient for the above cost function to be linearly homogeneous in input prices²³.

$$\begin{aligned}
 \sum_{i=1}^N \alpha_i &= 1 ; \quad \sum_{i=1}^N \alpha_{it} = 0 \\
 \sum_{i=1}^N \gamma_{ij} &= 0 \quad j = 1, \dots, N \\
 \sum_{i=1}^N \delta_{ik} &= 0 \quad k = 1, \dots, M
 \end{aligned} \tag{2.7}$$

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²³ Linear homogeneity in input prices can also be imposed by dividing total cost and $N-1$ input prices by one input price p_N . The parameters relating to this input can be derived using the restrictions in (2.7).

Symmetry restrictions on the cross-effects imply:

$$\gamma_{ij} = \gamma_{ji} \quad ; \quad \beta_{ki} = \beta_{ik} \quad (2.8)$$

Shephard's Lemma allows the derivation of the following N factor share equations obtained by partial differentiation of the cost function in (2.6) with respect to $\ln p_i$:

$$S_i = \alpha_i + \sum_{j=1}^N \gamma_{ij} \ln p_j + \sum_{k=1}^M \delta_{ik} \ln y_k + \alpha_{it} \quad (2.9)$$

Prior to estimation, an additive stochastic disturbance term is appended to the cost function and the factor share equations. One factor share equation is dropped in order to prevent the singularity of the error variance-covariance matrix. The cost function and N-1 factor share equations are estimated simultaneously using Zellner's iterative technique (IZEF) for seemingly unrelated regression equations²⁴.

The estimated cost function can be used to derive economic effects that are of interest; such as returns to scale, elasticities of substitution, and the nature of technical change. These and other effects are discussed below.

In general, returns to scale (RTS) can be derived as follows:

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²⁴ Kmenta and Gilbert (1968), in a series of Monte Carlo experiments, demonstrated that Maximum Likelihood and IZEF estimation led to identical estimates in all samples. The parameter estimates are invariant to the omitted factor share equation.

$$RTS = \left[\sum_{k=1}^M \left(\frac{\partial \ln C(.)}{\partial \ln y_k} \right) \right]^{-1} \quad (2.10)$$

which in the case of the translog cost function simplifies to:

$$RTS = \left[\sum_{k=1}^M \left(\beta_k + \sum_{l=1}^M \beta_{kl} \ln y_l + \sum_{i=1}^N \delta_{ik} \ln p_i + \beta_{tk} t \right) \right]^{-1} \quad (2.11)$$

Local overall economies (diseconomies) of scale are said to exist if $RTS > 1$

($RTS < 1$), whereas returns to scale are said to be constant if $RTS = 1$.

TP is a measure of disembodied technical change:

$$TP = \frac{\partial \ln C(.)}{\partial t} \quad (2.12)$$

A negative sign on TP indicates that the cost function is shifting downward over time.

For the translog, TP takes the following specific form:

$$TP = \beta_t + \beta_{tt} t + \sum_{i=1}^N \alpha_{ti} \ln p_i + \sum_{k=1}^M \beta_{tk} \ln y_k \quad (2.13)$$

The own (ϵ_{ii}) and cross (ϵ_{ij}) price elasticities of factor demand are defined as follows:

$$e_{ii} = \left[\left(\frac{\partial x_i}{\partial p_i} \right) \left(\frac{p_i}{x_i} \right) \right] \quad e_{ij} = \left[\left(\frac{\partial x_i}{\partial p_j} \right) \left(\frac{p_j}{x_i} \right) \right] \quad (2.14)$$

where x_i is the demand for input i . Use of Shephard's Lemma and some simple manipulation yields the following alternative expressions:

$$\begin{aligned} e_{ii} &= S_i \sigma_{ii} & i &= 1, \dots, N \\ e_{ij} &= S_j \sigma_{ij} & i, j &= 1, \dots, N ; i \neq j \end{aligned} \quad (2.15)$$

where S_i is the share of input i in total cost and σ_{ij} is the Allen-Uzawa partial elasticity of substitution defined below. The cross price elasticities of factor demand are asymmetric, so that $e_{ij} \neq e_{ji}$ but they have the same signs due to symmetry of the cross partial derivatives of the cost function. Further, the sign of e_{ij} (and e_{ji}) is identical to the sign of σ_{ij} .

In the two factor case, the notion of the elasticity of substitution is due to Hicks (1932). It provides a measure of the curvature of the isoquant. It also provides information about changes in relative factor shares due to changes in either the input ratio or the input price ratio. In the two factor case this measure is always positive. In the case of more than two inputs, there is some ambiguity about the appropriate generalization²⁵.

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²⁵ Chambers (1988) provides a good discussion of the various generalizations.

The most popular multi-factor measure is the Allen-Uzawa partial elasticity of substitution. It is a generalization of Hicks' measure in the sense that it coincides with it when there are two variables. This measure has traditionally been used to examine substitutability and complementarity between inputs even though such information is easily and perhaps more intuitively deduced from the factor demand elasticities. The Allen-Uzawa elasticity of substitution is derived as follows:

$$\sigma_U = \frac{\left[C(.) \left(\frac{\partial^2 C(.)}{\partial p_i \partial p_j} \right) \right]}{\left[\left(\frac{\partial C(.)}{\partial p_i} \right) \left(\frac{\partial C(.)}{\partial p_j} \right) \right]} \quad (2.16)$$

In the case of the TL cost function, the own and cross elasticities of substitution take the following form:

$$\begin{aligned} \sigma_{ii} &= (\gamma_{ii} + S_i^2 - S_i) / S_i^2 & i = 1, \dots, N \\ \sigma_{ij} &= (\gamma_{ij} / S_i S_j) + 1 & i, j = 1, \dots, N ; i \neq j \end{aligned} \quad (2.17)$$

Blackorby and Russell (1981 and 1989) suggest that the Allen-Uzawa elasticity does not preserve any of the salient features of the original two variable measure. They correctly point out that a classification of pairs of inputs as complements and substitutes can be accomplished without knowledge of the Allen-Uzawa elasticity. The comparative static information about absolute factor shares can be derived from the shares themselves along with the factor demand elasticities. Further, they suggest

that symmetry may not be a natural property of an n-dimensional elasticity of substitution since in the case of more than two inputs, there are an innumerable number of directions in which curvature of the isoquant can be measured. Thus, Blackorby and Russell introduced a new elasticity of substitution: the Morishima elasticity of substitution. Unlike the Allen-Uzawa elasticity, this measure is not symmetric. It can be derived as follows:

$$\begin{aligned}\omega_{ji} &= \epsilon_{ji} - \epsilon_{ii} \\ \omega_{ij} &= \epsilon_{ij} - \epsilon_{jj}\end{aligned}\tag{2.18}$$

The asymmetry results from the recognition of the fact that a change in the price ratio p_i / p_j can be brought about by either a change in p_i or p_j . Thus, ω_{ij} reflects the effect on the input quantity ratio if the source of the price change is p_j , whereas ω_{ji} is relevant when the change in the factor price ratio is caused by p_i . As with the Allen-Uzawa measure, no one input can be a Morishima complement with all other inputs²⁶. If two inputs are Allen-Uzawa substitutes then this implies that they are Morishima substitutes, but the converse is not true. If σ_{ij} is positive then ϵ_{ij} (and ϵ_{ji}) is positive, concavity of the cost function implies that ϵ_{jj} (and ϵ_{ii}) is negative, clearly, ω_{ij} (and ω_{ji}) is positive. If on the other hand, ω_{ij} is positive, this implies that $(\epsilon_{ij} - \epsilon_{jj})$ is positive. Since $-\epsilon_{jj}$ is positive, if it is greater in value than $-\epsilon_{ij}$, then inputs i and j will be Allen-Uzawa complements despite being Morishima substitutes.

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²⁶ This is implied by linear homogeneity in input prices and concavity of the cost function.

The Morishima elasticity has yet to see popular use despite its introduction almost a decade ago²⁷. In their more recent paper, Blackorby and Russell provide a simple yet extremely convincing example of how misleading the Allen-Uzawa measure can be.

Prior to deriving the comparative static effects or testing hypotheses, it is important to ensure that the estimated cost function behaves in a manner required by economic theory. Some structure is imposed prior to estimation, but it is important to check for other theoretical consistency requirements. For example, the estimated cost function should be increasing in y , or, marginal costs should be positive.

Monotonicity is satisfied if the predicted factor shares in total cost are positive. The second-order conditions of cost minimization require the cost function to be concave in factor prices. The cost function has the appropriate local curvature if the matrix $\nabla_{pp}C(y^*, p^*, t)$ is negative semidefinite (NSD). The translog cost function does not satisfy global concavity; local concavity can be ascertained at each observation. Given the Monte Carlo evidence discussed in Section 2.2.3 above, violations at some points in the sample should not be surprising. Most empirical pieces mention such violations in footnotes or endnotes. Others do not discuss them at all. The problem remains pervasive and even using "good" data may not ensure appropriate curvature at each

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²⁷ Ball and Chambers (1982) calculate Morishima elasticities. They also calculate a share weighted average of the two Morishima elasticities and call this the shadow elasticity of substitution.

observation²⁸. Such violations may prove problematic since they can result in upward sloping input demand functions. Attempts have been made to impose concavity on the translog, but the results have either not been satisfactory on theoretical grounds²⁹ or the complexity of the computational methods has prevented popular use³⁰.

2.4 Standard Errors and Confidence Intervals

Most studies in applied production analysis measure various summary characteristics of the cost function. These include returns to scale, elasticities of factor demand and elasticities of substitution. Some of these are non-linear functions of the parameters of the model, others are derived using the parameters and the data. Some studies do not report any associated statistical measures of precision for these economic effects. This situation may change due to the introduction of some new computer intensive methods of deriving standard errors.

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²⁸ Diewert and Wales (1987) use the Berndt and Wood KLEM data to examine the curvature of various flexible functional forms. The Translog violates concavity at 6 out of 26 observations and the Generalized Leontief is not concave at any data point.

²⁹ See the discussion of Jorgenson and Fraumeni (1981) in Diewert and Wales (1987).

³⁰ See Hazilla and Kopp (1986a and 1986b).

There are primarily three methods of deriving standard errors for elasticities.

The first method³¹ relies on a linearization of the non-linear function of interest.

This is accomplished by means of a first order Taylor series approximation. Suppose $\phi(\theta)$ is an elasticity expressed as a function of the parameter vector θ . If Δ is the gradient of $\phi(\theta)$, θ^* is the estimate of the parameter vector and Σ is the associated covariance matrix, then the elasticity is approximated as follows:

$$\phi(\theta) \approx \phi(\theta_M) + (\theta^* - \theta_M)' \Delta \quad (2.19)$$

where $\theta_M = E(\theta)$. The variance of the elasticity is:

$$\text{var}(\phi(\theta^*)) \approx \Delta' \Sigma \Delta \quad (2.20)$$

The second method is implemented in the context of a translog cost function by Krinsky and Robb (1986, 1990 and 1991). They refer to it as a simulation method, but it is also known as the parametric bootstrap³². This method assumes that the disturbance terms follow a multivariate normal distribution and the parameter estimates follow their multivariate normal limiting distribution. Using a normal random number generator, a large number of sets of parameter vectors are drawn. The vectors are

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³¹ The first reference is to Klein (1953). Toevs (1980 and 1982) derives approximate variance formulas for a three input translog production and cost function. See also Anderson and Thursby (1986) who find the distribution of the Allen-Uzawa elasticities to be non-normal.

³² See Section 5.2 in Efron (1982). Gallant and Golub (1984) use a slightly different method, they draw a new set of errors from a normal distribution.

assumed to have mean θ^* and covariance matrix³³ Σ . Using each draw of the parameter vector, the elasticities of interest can be calculated, thus yielding an empirical distribution of the elasticities. Standard errors and/or confidence intervals are easily derived from the empirical distribution. Krinsky and Robb (1990) compare the results of the linear approximation and the parametric bootstrap. They find that the two methods yield similar results.

The third method is the bootstrap, this is a nonparametric method of deriving confidence intervals due to Efron³⁴. Freedman and Peters (1984a and 1984b) implement this procedure for simultaneous equation models and find that asymptotic standard errors are too optimistic by almost a factor of three³⁵. Implementation of the bootstrap is the most computer intensive of the three methods. The first step involves estimation of the cost function and (N-1) factor share equations using IZEF, this yields a T by K matrix of errors, where T is the number of observations and K is the number of equations. T draws with replacement are then made from the empirical distribution of the errors³⁶. A K-dimensional row vector is drawn each time in order

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³³ This is the parameter vector derived from the IZEF estimation of the cost function and the associated covariance matrix.

³⁴ See Efron (1979, 1981 and 1982), Efron and Gong (1983), Efron and Tibshirani (1986) and Wu (1986).

³⁵ Green, Hahn and Rocke (1987) compare the linear approximation and the bootstrap for a Linear Expenditure System and find that for income elasticities of demand, the former may be too optimistic by a factor of two. This could be due to the fact that they used only 50 bootstrap replications.

³⁶ Freedman and Peters suggest inflating the errors prior to drawing in the case of an OLS model. No variance inflation is used here since this issue is not resolved for seemingly unrelated regression models where the number of independent variables differs across equations.

to preserve the contemporaneous correlation of the errors. In this manner, a new T by K matrix of errors is constructed, this is added to the T by K matrix of dependent variables, thus yielding a matrix of pseudo-dependent variables. These dependent variables are used to re-estimate the model using IZEF which yields a new set of parameter estimates and elasticities of interest. Using the base IZEF error matrix, this procedure is repeated a large number of times. Clearly this is very time consuming since the model has to be re-estimated each time.

Krinsky and Robb (1991) compare the results from all the three procedures described above in the context of a translog cost function and find that there are no major differences among the three. The bootstrap is preferred primarily because it does not rely on the asymptotic normality of the model, instead it relies on the empirical distribution of the errors.

This thesis uses the bootstrap to construct 95% confidence intervals for local estimates of economies of scale, and all the input elasticities³⁷. Efron and Tibshirani suggest the use of 1000 bootstrap replications for the implementation of this method. The model is estimated 1000 times and the parameters from these replications are sorted in ascending order. Observations 26 and 975 provide the endpoints of the 95% confidence interval. Standard errors may also be readily obtained since they are just the standard deviations calculated using the bootstrap replications.

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³⁷ This is identical to the percentile method used by Eakin, McMillen and Buono (1990) for a multi-product hybrid translog cost function.

CHAPTER 3

DATA

The Canadian railway industry is dominated by two firms, Canadian National Railways (CN) and Canadian Pacific Railways (CP). In 1981, these firms accounted for 91.6% of the freight revenues and 83.6% of the total revenues of all railways in Canada. In 1988, total revenues from railway operations were in excess of 3.5 billion dollars for CN and in excess of 2.5 billion for CP. These revenues are comparable to those earned by large U.S. railways such as Southern Pacific and Union Pacific.

CN and CP, like all railway companies in Canada, are under a statutory obligation to submit annual reports to the National Transportation Agency¹ (NTA). Copies of these reports are also submitted to the Chief Statistician of Statistics Canada. These reports are the primary data source for firm specific information on the railways in Canada². Statistics Canada publications relating to railway transport are based on these annual reports but some information is either withheld or reported after aggregation across firms in order to meet confidentiality requirements. In

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¹ Previously, the Canadian Transport Commission.

² Caves and Christensen (1978 and 1980b), Caves, Christensen, Swanson and Tretheway (1982) and Freeman et al. (1985a, 1985b and 1987) and this thesis use data from this source.

particular, data on investment are not reported³ and data on fuel use are reported after aggregation across all Class I and Class II railways⁴.

The information in the annual reports conforms to the uniform classification of accounts. This accounting system was changed in 1955, and more recently in 1982. Cross-reference tables are available from the NTA⁵. In principle, these tables provide a key to re-classify the data from the old classification (pre-1982) to the new classification (post-1981) and vice versa. Given the nature of the changes, any attempt to undertake either exercise would not only be very time consuming, but may also fail to yield a consistent time series. Due to these limitations and the proprietary nature of crucial data, the sample size is restricted to the twenty-six year period 1956-81.

It is assumed that Class I railways provide two services, passenger transportation and freight transportation. The definition of output for a service industry is a much debated issue. The conduct of empirical economic analysis requires quantification or the ability to measure. The output of any firm or industry, whether it be a tangible product or service, can be thought of as a bundle of characteristics. This is a particularly attractive approach for the transportation sector. The "ideal" measure of output for the transportation sector should not only account for

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³ This has changed since 1982, investment data can be found in the railway property accounts published by Statistics Canada.

⁴ CN, CP, and VIA are classified as Class I and all other railways are classified as Class II.

⁵ *Uniform Classification of Accounts and Related Railway Records*, November 24, 1980.

the two basic characteristics of weight and distance but should also incorporate other "attributes" like some measure of speed of service. The quality of service is particularly important in the case of passenger transportation. Together, the various characteristics form the "output" of the transportation sector. Two other distinguishing features of the transportation sector should be mentioned. Firstly, the output of the transportation sector cannot be stored. Unutilized capacity is lost forever. Secondly, the provision of service from point A to B creates the capacity to provide service from B to A.

In practice it may not be possible to accurately measure some of these characteristics. Studies on U.S. railways utilize a variety of output measures. A majority of them are able to isolate the effect on total cost of both weight and distance. One study has treated service between each origin and destination pair as a distinct output⁶. Limitations of sample size and data availability prevent the inclusion of a wide variety of service attributes in this thesis.

The provision of the passenger and freight transportation service is assumed to require the employment of five factors of production. These are labour, way and structure capital, equipment and machinery capital, fuel and a residual category called

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⁶ See Jara-Diaz and Winston (1981), this is a study of two Class III firms in the U.S.

materials. The procedure used in the construction of the prices and quantities of outputs and inputs is described below⁷.

3.1 Labour

Labour force data are classified in five broad categories. These are:

- (a) General
- (b) Road maintenance
- (c) Equipment maintenance
- (d) Transportation
- (e) Other

The last category contains statistics relating to highway and express operations and other activities that are outside the scope of this thesis. This category is therefore ignored.

For the period 1956-63 two measures of labour input are available. These are:

- Average number of employees
- Time on duty (in hours)

For the remainder of the sample, the following measures are reported:

- Average number of employees
- Service hours actually worked
- Service hours paid for

The last measure includes paid leave and vacations. Time on duty (1956-63) and service hours actually worked (1964-81) are utilized here as a measure of labour input.

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⁷ The general method used here is similar to that used by Caves and Christensen (1978 and 1980b) and Freeman et al. (1985a, 1985b and 1987), but the particular prices and quantities of all inputs are derived differently.

In 1964, the approximately eighty sub-categories of labour were re-classified between the five broad categories due to a change in the accounting system⁸. As is evident from the first two columns of the following tables, this re-classification causes a discontinuity in the data over the period 1963-64. Table 3.1 presents the share of each category of labour in total hours on duty (1963) and total hours actually worked (1964) for Canadian Pacific Railway. Canadian National data also conform to this pattern.

TABLE 3.1				
Labour Category Share in Total Hours on Duty/Hours Worked				
	Re-allocated			
CP Rail	1963	1964	1963	1964
General	0.22	0.13	0.24	0.24
Road	0.20	0.19	0.19	0.18
Equipment	0.24	0.26	0.22	0.23
Transportation	0.34	0.42	0.34	0.35

Table 3.2 shows the share of each of the four categories of labour in total compensation. Like the previous table, the information relates to Canadian Pacific for the two years 1963, and 1964. In order to rectify this problem, the data are re-allocated within the four broad categories in the manner discussed below. The re-allocated data are presented in the last two columns of the above two tables, it is clear that the discontinuity has been removed.

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⁸ Freeman et al. (1987) use a Multilateral index of labour input and suggest that the reclassification does not affect their aggregate.

TABLE 3.2				
Labour Category Share in Total Compensation				
			Re-allocated	
CP Rail	1963	1964	1963	1964
General	0.23	0.13	0.26	0.26
Road	0.16	0.17	0.16	0.15
Equipment	0.23	0.25	0.20	0.21
Transportation	0.38	0.45	0.38	0.38

The re-allocation is accomplished by employing a third classification, one which conforms to neither the pre-1964, nor the post-1963 classification. The use of this system provides consistency over the entire sample. This classification is reported by Statistics Canada as a key⁹. It requires the incorporation under the general category, certain supervisory and clerical workers normally included under the remaining three categories. For the period 1964-81, certain labour categories reported under transportation labour are excluded all together since they were classified under other operations prior to 1964.

The re-allocation, though tedious, is fairly straightforward. The period 1956-63 poses no problem since time on duty is reported for all the eighty sub-categories of labour¹⁰. For the remaining period, the available measure of labour input at the required level of disaggregation is service hours paid for and not service hours actually

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⁹ See Table 7 in Statistics Canada Catalogue No. 52-212, 1964.

¹⁰ Except for Canadian National for the year 1956.

worked. It is assumed that for any particular year, the ratio of hours actually worked to hours paid for, is invariant within each of the four broad categories of labour. For example, in the year 1960, this ratio takes a value of 0.8 for the general labour category. The assumption implies that the ratio 0.8 holds for each sub-category of labour belonging to the general labour category. The following formula is used to convert hours paid for into hours actually worked:

$$H^w_{it} = (H^w_{kt} / H^p_{kt}) * H^p_{it} \quad k = 1...4, i \in k \quad (3.1)$$

In the above formula, H^w_{it} represents hours actually worked by labour in sub-category i belonging to category k at time t . Similarly, H^p_{it} represents hours paid for. Further, k is the category of origin. So if a particular sub-category i of labour were being re-allocated from transportation to general, then k would represent transportation and H^w_{it} would be subtracted from transportation and added to the general category.

Disaggregated data are not available for Canadian National for the first year in the sample. Estimates for 1956 are derived as follows: Consider, for example, the general labour category. The ratio of reported to re-allocated general labour can be calculated for the period 1957-63. It is assumed that the ratio prevailing in 1957 holds for the previous year for both labour quantity and compensation for the four main categories of labour. Applying this ratio to the reported values for 1956 yields the requisite measures. Both the quantity and compensation are then summed across the four categories; these deviated from the reported totals by -0.19% and -0.18%,

respectively. To compensate for these marginal errors, the data were scaled to conform to the reported totals.

The re-allocated labour input and compensation data are aggregated to form an index of labour input. The discrete approximation to the continuous Divisia index is used to form this aggregate. This index is also known as the translog index or the Tornqvist index. It takes the following form:

$$\ln\left(\frac{L_t}{L_{t-1}}\right) = \sum_{j=1}^4 \left(\frac{(S_{jt} + S_{jt-1})}{2} \right) \ln\left(\frac{L_{jt}}{L_{jt-1}}\right) \quad (3.2)$$

where S_{jt} is the share in total compensation of labour category j at time t and L_{jt} is the quantity of labour input of type j at time t .

Caves and Christensen (1978) use a less tedious procedure to adjust for the change in the classification of labour data. They compute an aggregate index of labour input for the periods 1956-63, and 1964-75. The two series are linked using the growth in total labour hours over the period 1963-64. They make a further adjustment for the years 1962-75 to account for the fact certain labour sub-categories in other operations were being reported as rail labour¹¹.

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¹¹ Note that the method employed here excludes these sub-categories explicitly. See pages 24-25 in Caves and Christensen (1978) for further details of their method.

An attempt was made to derive the aggregate labour input index using the Caves and Christensen method and, interestingly enough, the shorter route is as effective as the more tedious method employed here. The average annual growth rate of the price of labour for Canadian Pacific and Canadian National over the period 1956-81 is 7.69% and 8.09%, respectively. Using the method due to Caves and Christensen results in respective growth rates of 7.72% and 8.13%.

In order to derive total labour compensation, labour taxes are added to the total wage bill. These include unemployment and pension contributions on behalf of the employer in Canada and the U.S. One further adjustment is made to both total compensation and aggregate labour input. They are multiplied by $(1-V_t)$, where V_t is the ratio of compensation charged to capital account to total compensation in year t . This is to account for the fact that labour is used both for operating purposes and for the installation of capital goods. Given the value and quantity of the labour input, the price is readily obtainable.

3.2 Fuel

The railways use different types of fuel for motive power; such as coal, gasoline, electricity, and diesel. The annual reports contain data on quantity purchased and total cost for each type of fuel. Coal is no longer utilized since all operations were fully converted to diesel by the mid 1960s. The observations for coal use are

positive before the mid 1960s and zero thereafter. This is the case for most fuel types with the possible exception of diesel, which is used throughout the sample period. A Divisia index cannot be utilized to aggregate across fuel types due to the presence of zero observations in part of the sample.

Various second-best solutions may be used to form an index of fuel use. The first, and most popular in previous research, is to convert all fuel types into their respective British Thermal Unit (BTU) equivalents. Having solved the problem of units, the quantity of fuel input can be derived by aggregating across all fuel types. The average cost per BTU is used as the price of fuel. This approach does not distinguish between a BTU from coal and a BTU from diesel. It implies that all fuel types are perfect substitutes.

A second alternative is to treat the price of diesel as the representative price of fuel for the railways¹². The quantity of 'fuel' can then be derived using the total expenditure on all fuel types. This may not be a poor approximation; after 1962, expenditure on diesel represents 93-100% of the total expenditure on fuel.

This study utilizes a Fisher ideal index to aggregate the BTUs derived from various types of fuel¹³. Like the Tornqvist index, the Fisher ideal index is a

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¹² This approach is followed by the National Energy Board.

¹³ The BTU equivalents are reported in Appendix I.

superlative index number¹⁴. Its use also leads to approximate consistency in aggregation. As shown by Diewert (1980), the presence of zero observations leads to biases in the Fisher aggregate. Diewert discusses biases in the Fisher ideal price index in the context of the new goods problem. By means of a simple example, his results are extended here to obsolete goods and what might be termed "fashion goods" that are "in" one year and "out" in another. For details, the interested reader is directed to the discussion in Appendix 1. The Fisher ideal quantity index can be constructed using Equation (3.3) below where P_{jt} is the price of fuel type j at time t and X_{jt} is the quantity purchased of fuel type j at time t :

$$\ln\left(\frac{F_t}{F_{t-1}}\right) = 1/2 \ln\left(\frac{\sum_{j=1}^n P_{jt} X_{jt}}{\sum_{j=1}^n P_{jt} X_{j,t-1}}\right) + 1/2 \ln\left(\frac{\sum_{j=1}^n P_{j,t-1} X_{jt}}{\sum_{j=1}^n P_{j,t-1} X_{j,t-1}}\right) \quad (3.3)$$

The price of fuel is derived implicitly from total cost and the aggregate quantity of fuel.

Table 3.3 provides information on the differences between the three alternatives. For the period 1961-81, the average annual rate of growth of all three series is practically identical. For CN, the Fisher price index and the price of diesel are in close conformity with each other for the entire sample. The price per BTU

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¹⁴ Diewert (1976 and 1978) defines a superlative index number as one that is exact for a flexible aggregator function.

registers significant growth in the first five years of the sample because total BTUs of fuel are declining at an average annual rate of 35% over the same period. This is due to decreased use of fuels such as coal and fuel oil. By 1960, over 95% of the BTU's were accounted for by diesel.

Table 3.3						
Average Annual Rate of Growth of Fuel Prices (%)						
Time Period	Canadian National			Canadian Pacific		
	BTU	Diesel	Fisher	BTU	Diesel	Fisher
1956-60	16.59	-0.60	-0.56	16.34	-0.93	0.62
1961-81	9.72	9.73	9.73	10.50	10.37	10.27
1956-81	10.82	8.08	8.09	11.43	8.56	8.73

3.3 Capital

This study distinguishes between two types of capital, these being ways and structures capital and equipment capital. The total capital stock for the two capital inputs is obtained by an addition of the owned and rented capital. Owned capital stock is derived from the history of gross real investment, using the perpetual inventory method. The quantity of rented capital is obtained by dividing net rental expenditures by the user cost of capital. Details are provided below.

3.3.1 Rented Capital

The value of rented equipment is equivalent to the net rents paid for equipment; this is divided by the user cost of capital for equipment and machinery in order to derive the quantity of rented equipment and machinery. For way and structures capital; other rent income, and income from lease of buildings and other properties are deducted from the sum of, net rental payments for joint facilities and the lease of road and equipment¹⁵. The quantity of rented way and structures capital is derived by deflating the value with the respective user cost of capital.

As an alternative, rental capital is excluded entirely. There are various reasons for doing so. Firstly, the above method for calculating net rental expenditures is a second-best method since it is not based on actual rental/lease agreements. Secondly, there is a fair amount of volatility in the net rental expenditures. In the case of equipment capital, net rental payments are negative over the period 1956-68 for both firms. Not only is there a sign reversal in the middle of the sample, but there are considerable year to year fluctuations in the series. Previous research has adopted a somewhat arbitrary solution to this problem. All previous studies on Canadian railways use the following approach: in calculating the stock of rented capital, net rental expenditures are divided by the user cost of capital for the period 1956-64, and by a five year moving average of the user cost of capital for the remainder of the

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¹⁵ This follows Freeman et al. (1987), for details, see page 250.

sample. Having calculated the stock, the value of rented capital is recalculated by multiplying the stock by the user cost of capital for the entire period 1956-81.

Freeman et al. (1987) determine from "experimentation" that 1965 is an appropriate year to begin smoothing¹⁶. They favour smoothing because most rental agreements are made for periods exceeding one year at locked-in rates. If this is appropriate, then the smoothing should be done for the entire sample rather than just for the period 1965-81. Given these problems encountered in previous research, it was considered best to maintain two data alternatives, one that includes rented capital and one that excludes it. Note that no smoothing is undertaken here in the derivation of the stock of rented capital.

3.3.2 Owned Capital

Data on investment are available in Schedule 8 of the annual reports of the railways. The various types of capital are grouped under two broad headings:

- Way and Structures
- Equipment

Data on nominal additions to equipment are used as reported, but investment in land is omitted from the way and structures category because reliable information is not available for the period preceding 1956.

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¹⁶ See page 251 in Freeman et al. (1987). This method is also used by Caves and Christensen (1980b).

The annual reports provide information for the period 1956-81. So far as the preceding years are concerned, a series created for internal use by CN provides data for investment in way and structures (excluding land) for the period 1890-1955¹⁷. CN's nominal gross additions to machinery and equipment capital and CP's additions to both categories of capital are available from Haritos (1973) for the period prior to 1956.

It may be useful to mention that the components of way and structures capital and equipment capital differ from previous research. This thesis defines the two types of capital as they are defined in the Annual Reports of the railways. The data are used as reported. Previous studies exclude investment in commercial communications from way and structures. They also classify certain components of equipment¹⁸ as structures, in particular, investment in shop machinery and power plant machinery is classified as structures. The reason for this re-classification is that some of the previous work was also dealing with U.S. railways and Canadian data were re-classified to conform to U.S. standards¹⁹.

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¹⁷ Previous research inadvertently attributes all pre-1956 data to Haritos (1973). In fact, all previous studies utilize this internal CN series which was provided to Christensen in a letter from George Hariton of the CTC.

¹⁸ Freeman et al. (1987) include government provided hopper cars in their stock of equipment capital. This approach is not adopted here since the attempt is at measuring private productivity based on private rather than social outputs and inputs.

¹⁹ Freeman et al. (1987) suggest there were some data errors in Caves and Christensen (1980b). Unfortunately, one crucial error was overlooked. All previous research on Canadian railways maintains that land is excluded from the definition of way and structures capital. This is true so far as CP is concerned; for CN, all previous studies include land for the period 1956-75 and exclude it thereafter.

Investment price deflators for railways are published by Statistics Canada²⁰.

The deflator used for way and structures capital is available from 1871 onwards. The index for machinery and equipment begins in 1896, for the remaining five years 1890-95, the growth rate of the wholesale price index²¹ is applied to the asset price deflator for machinery and equipment. Both deflators are set equal to one in 1967 and the following version of the perpetual inventory method is used to construct owned capital stocks:

$$K_{it} = I_{it} + (1 - \delta_i) * K_{it-1} \quad (3.4)$$

where K_{it} is the capital stock of type i in year t , I_{it} is the real investment in capital of type i in year t and δ_i is the economic decay rate applicable to capital of type i .

Following previous work on the railways in Canada, a depreciation rate of 3% is used for way and structures capital and 6% for machinery and equipment.

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²⁰ See Catalogue 13-568.

²¹ This is from a supplement to The Labour Gazette.

3.3.3 Price of Capital

The formula for the user cost of capital employed here can be found in Boadway, Bruce and Mintz (1987)²². This is based on the original formulation of Jorgenson (1967). The incorporation of tax effects is discussed in Hall and Jorgenson (1967) and Christensen and Jorgenson (1969).

$$\left[\left(\frac{(1-\phi)(1-uz)}{(1-u)} \right) \left(r_f + \delta - \left[\frac{q_t^* - q_{t-1}}{q_{t-1}} \right] \right) \right] + t_p q_t \quad (3.5)$$

In the above equation, u is the federal corporate income tax rate²³; historical rates are reported by Boadway and Kitchen (1984)²⁴. z is the present value of capital consumption allowance (CCA) deductions on a dollar's worth of investment. If CCA write-offs occur at a rate α on a declining balance basis then:

$$z = \int_0^{\infty} \alpha e^{-(\alpha+r_f)t} dt = \frac{\alpha}{\alpha + r_f} \quad (3.6)$$

This formulation also implies that capital purchases made in the beginning of the year do not receive CCA benefits till the end of the year. The value of α for way and

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²² See Jung (1989) for an application of this approach.

²³ Note that over the entire sample CN has paid no income tax due to carrying forward accumulated losses, therefore the tax terms are not applicable to it.

²⁴ See Table 3.3.

structures capital is 6% for the entire sample²⁵. The rate for equipment is 6% for the period 1956-76. For the period 1977-81, the rate is a share (share in total equipment investment) weighted average of the rate on locomotives, cars, and other equipment (10%, 7% and 6%).

As discussed earlier, δ is the depreciation rate, it assumes a value of 3% for way and structure capital and 6% for machinery and equipment. Railways became eligible for investment tax credits on November 16, 1978, the amount ϕ is 7% of the capital cost of investment. Nearly all components of equipment capital, and some items from the way and structures category are eligible for the tax credit. It is assumed that all railway capital is eligible for the credit since data on investment in way and structures are not available at the required level of disaggregation.

The property tax rate is represented by the term t_p . All non-income tax payments made by the railways to provincial and municipal governments are assumed to represent payments for property taxes. Property tax payments are derived by adding provincial and municipal tax payments and deducting from the total, Ontario and Quebec income tax payments and payments to the Quebec Pension Plan. The tax base is the nominal stock of way and structure capital, re-defined to include shop and power plant machinery²⁶. The latter two items are usually reported under the

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²⁵ These rates can be found in the *Income Tax Act*.

²⁶ This re-definition is only used here to calculate the property tax rate.

equipment category. The real way and structures stock is multiplied by the asset price deflator for way and structures to derive the nominal stock. The tax payment divided by the nominal stock yields the rate of property taxes.

In the expression for the user cost of capital, q_t is the price of new capital goods. The investment price deflators for way and structures and machinery and equipment are taken to represent the price of new capital goods. The term q_t^* is the expectation of q_t formed at time $t-1$. Expected nominal capital gains are therefore represented by $(q_t^* - q_{t-1})$. The empirical treatment of this term is another source of controversy in the literature. It is often assumed that expectations are myopic and expected capital gains are zero, or that $q_t^* = q_t$. Jorgenson (1963 and 1965) and Hall and Jorgenson (1967) assume that all capital gains are regarded as transitory, the expected gains term is therefore set equal to zero. Jorgenson and Siebert (1968a and 1968b) empirically compare myopic expectations and perfect anticipation, and suggest that the latter may be somewhat superior. Denison (1969) argues that it may be appropriate to use long term averages of realized capital gains since there are large year to year fluctuations. Diewert²⁷ provides the following comments on this issue:

"...from our rather narrow viewpoint, which concentrates on the measurement of capital in the context of production function estimation and the measurement of total factor productivity, it seems clear that the capital gains term belongs in the rental price formula - what is not clear is the Jorgenson-Griliches perfect anticipation assumption..."

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²⁷ See Diewert (1980) page 476.

The use of perfect anticipation causes two problems. Firstly, the capital gains term becomes volatile. The second is aptly posed by Harper, Berndt and Wood (1986)²⁸:

"...the recent 1974-81 time period has been one of substantial inflation and movements in the relative growth rates of capital asset-specific investment deflators, resulting in a non-trivial number of instances where the measured capital rental price becomes negative."

Previous literature on Canadian railways alleviates both problems by using long term moving averages of realized capital gains. This approach has some disturbing aspects; it is not clear what the order of the moving average should be. If the aim is to drive the user cost positive, and this can be accomplished by a three year moving average, should one stop there or use a five year moving average which may also result in a positive user cost? Further, it is not clear when the volatility in the capital gains term becomes "unacceptable".

Following Epstein (1977), and Boadway, Bruce and Mintz (1982), ARIMA methods are employed to derive a forward looking expectation of the price of new capital goods. For the way and structures price deflator, the forecasts are based on the first differences of the natural logarithm of the original series. An AR(1) model is found to be appropriate for all time periods. In the case of the equipment deflator, the forecasts for the period 1956-74 are based on the first differences of the series and for

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²⁸ See page 2. There is one of the few papers that examines these problems at some length. Note that the above quote has been excluded from the (1989) version of their paper.

the remainder of the sample they are based on the first differences of the natural logarithm of the original series²⁹. In this case an MA(1) model is found to be appropriate³⁰. Note that a new forecast is formed for each period, so in order to forecast the 1956 value of the price deflator, data is utilized for the period 1928-55. For the subsequent year, 1957, a new model is estimated using data for the period 1928-56, and so on. In cases where alternative specifications seem reasonable, model selection is based on the minimization of the Akaike Information Criterion³¹. The resultant user cost is found to be positive in all periods but somewhat volatile after the mid 1970s.

The term r_t in the user cost formula is the marginal opportunity cost of capital. Earlier studies, particularly those relating to the railways, assume that this is most clearly reflected in the cost of debt. They utilize the yield to maturity on debt as a measure of r_t . The series is obtained from Moody's Transportation Manual. The bond rating for CN is Aa for the entire sample, the rate for CP is Baa for the period 1956-64, and A for the remainder. Diewert³² provides the following comments on the use of an exogenous bond rate to represent the cost of finance r_t :

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²⁹ The use of the logarithms is helpful in series with non-stationary variance. It also permits a more parsimonious specification.

³⁰ Though for the first few years, the model could be random.

³¹ See Judge et al. (1985), Section 7.5 for details.

³² See Diewert (1980) pages 476-77.

"Which r should be used? If the firm is a net borrower, then r should be the nominal cost of borrowing an additional dollar for one period, while if the firm is a net lender, then r should be the one period interest it receives on its last loan. In practice r is taken to be either (a) an exogenous bond rate that may or may not apply to the firm under consideration, or (b) an internal rate of return. I tend to use the first alternative, while...Jorgenson and his co-workers use the second. As usual, neither alternative appears to be correct from a theoretical a priori point of view; so again, reasonable analysts could differ on which r to use in order to construct a capital aggregate."

The method used here is that of Boadway, Bruce and Mintz (1987). They define r_f as follows:

$$r_f = \lambda r_b(1-u) + (1-\lambda)r_e \quad (3.7)$$

where λ is the fraction of an extra dollar raised through debt. r_b is the market rate of interest per dollar of borrowing; it is taken to be the Moody's bond rate discussed above. As before, u is the tax rate on corporate income, and r_e is the nominal yield per dollar of equity. The latter is assumed to be the same for retained earnings and new equity finance.

For the period 1956-72, λ is easily derived from the annual financial reports of The Canadian Pacific Railway Co³³. The perpetual consolidated debenture stock and bank loans are included in debt as are the collateral trust bonds and equipment trust certificates. On July 3, 1971, the company changed its name to Canadian Pacific Ltd.

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³³ Note that this only applies to CP, in the case of CN it is assumed that all financing is through debt and the relevant Moody's bond rate is used.

and in 1973, the accounts of CP Rail and other companies such as Canadian Pacific Investments were consolidated. Despite this accounting change, the details in the annual financial reports permit a separation of the debt instruments of CP Rail. The only item that needs to be separated is the long term debt maturing within one year. For the year 1972, information is reported following two alternative accounting systems; the old, and the new. It is assumed that the 1972 share of CP Rail in long term debt maturing within one year holds for the remainder of the sample. So far as equity is concerned, the only item that needs to be split between CP Rail and the remainder of CP Ltd. is the retained income. Here again, the 1972 share of CP Rail in total retained income is assumed to hold for the remainder of the sample. The calculations yield an average share of equity of 72.9% over the period 1956-72, and 73.1% over the entire sample 1956-81.

The return on equity r_e is derived by using the following version of the Capital Asset Pricing Model:

$$r_e = r_{rf} + \beta (r_m - r_{rf}) \quad (3.8)$$

where r_{rf} is the risk free rate of return, β is the beta or firm specific risk for CP, and the term in brackets is the excess return on the market portfolio. The value of β is

taken to be 1.161 for the entire sample³⁴. This value was derived in the CTC Railway Costing Study³⁵ and is estimated using monthly data for the period 1974-79³⁶. The risk free rate of return, and the excess return on the TSE-300 are taken from Gordon and Gould (1984).

As an alternative, the user cost of capital used in previous studies on Canadian railways is also constructed. This formulation is based on Christensen and Jorgenson (1969); it takes the following form:

$$\left[\left(\frac{(1-\phi)(1-uz)}{(1-u)} \right) (r_b q_{t-1} + \delta q_t - (q_t - q_{t-1})) \right] + t_p q_t \quad (3.9)$$

As before, u is the federal income tax rate, r_b is the market rate of interest per dollar of borrowing (Moody's bond rate), q_t is the price of new capital goods (investment price deflator), δ is the depreciation rate of capital, t_p is the property tax rate, ϕ is the investment tax credit rate, and z is the present value of CCA deductions on a dollar's worth of investment. In Equation (3.9), z is defined as follows:

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³⁴ The National Transportation Agency undertakes cost of capital calculations to determine compensation under the Western Grain Transportation Act. The calculation for 1988-89 uses 1.15 as the value of β .

³⁵ See Appendix 5.A in Volume 2 of the study.

³⁶ The CTC study also attempts to separate the systematic risk between CP Investments Ltd. and the synthetic remainder of CP Ltd. It finds that the null hypothesis of equal levels of systematic risk for CP Investments Ltd. and CP remainder cannot be rejected.

$$z = \frac{\alpha}{\alpha + r_b} \quad (3.10)$$

where CCA write-offs occur at a rate of α on a declining balance basis.

In the conventional formulation of user cost of capital in Equation (3.9), the capital gains term ($q_t - q_{t-1}$) represents realized rather than anticipated capital gains. As expected, this formulation results in negative values of the user cost of capital in the mid 1970s. Following previous work, realized capital gains are replaced by their five year moving average in order to derive a positive price of capital.

The alternative definitions of the quantity of capital and the user cost of capital result in a total of four data options. These are termed model A, model AR, model B and model BR. The first two models, A and AR, utilize the user cost of capital as formulated by Boadway, Bruce and Mintz (Equation (3.5)). The last two models, B and BR, utilize the conventional user cost of capital presented in Equation (3.9) above. Models A and B exclude rented capital whereas models AR and BR include rented capital. Given the high variability in the value of rented capital and the failings of the conventional user cost of capital, Model A is the preferred data alternative. The data used in the estimation are provided in Appendix 2.

3.4 Materials

The price of materials is assumed to be identical for both firms³⁷. A Tornqvist price index of railway materials was obtained from Statistics Canada. This series is based on the Input-Output Tables and is available for the period 1961-81. For the remaining five years 1956-60, the growth rate of the implicit GNE deflator is applied to the Statistics Canada series. The quantity of materials is derived implicitly from the price and expenditures series.

Expenditures on materials are derived in a residual manner by subtracting the following items from total railway expenditures:

- road property depreciation
- rolling stocks and vessels depreciation
- other equipment and machinery depreciation
- pensions
- railway tax accruals
- net joint facility and equipment rents
- labour compensation charged to operation expenses
- the cost of fuel

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³⁷ Caves and Christensen use the wholesale price index for non-farm materials as the price of materials for both firms; Freeman et al. use the GNE deflator.

3.5 Outputs

There are two outputs, passenger-miles and freight ton-miles³⁸. Data on freight ton-miles are available from Statistics Canada publications, as are those on passenger-miles. The latter pose a problem over the period 1978-81. VIA Rail was formed in 1977, and assumed responsibility for the provision of rail passenger service in Canada. In the initial years, the two railways CN and CP continued to provide passenger services under contract to VIA. Though the dependence of VIA on CN and CP has declined over the years, VIA continues to make payments to CN and CP for various services such as, the use of track. Since 1978, non-commuter passenger data are not reported for CN and CP; this responsibility was assumed by VIA. Given VIA's almost total dependence on the other two railways in the early years of its formation, VIA's passenger-mile data are allocated between CN and CP for the period 1978-81. Any such allocation is arbitrary, but regrettably unavoidable. Due to the period they study, Caves and Christensen (1980b) do not face this problem. Freeman et al. resolve it by using data on passenger train fuel use. Their method cannot be used here since the relevant data are not available. VIA's passenger-miles are allocated between CN and CP based on a confidential *Thematic Overview of Rail Passenger Service* done by the *Nielsen Task Force* in 1985. This overview provides VIA's passenger-miles on a route by route basis for the year 1984. Since the track is

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³⁸ The railways provided information on ton-miles of statutory grain shipments, this permits the separation of freight into grain and non-grain freight. CN was unable to provide data relating to statutory grain for the year 1956. In order to derive the ton-miles of grain for 1956, it is assumed that the 1956 ratio of grain ton-miles to total freight ton-miles is identical to that prevailing in 1957.

owned by either CN or CP, the allocation is done on the basis of ownership of the track. On jointly owned track, passenger-miles are split equally between firms. The allocation results in CN assuming 76.5% of the passenger-miles; this proportion is assumed to hold for the period 1979-81. Another alternative is to divide VIA's output based on the 1977 shares of CN and CP in total passenger traffic. This is rejected because it leads to large fluctuations in CP's passenger-miles³⁹. Clearly the allocation used here is arbitrary, but the available information does not permit any sophistication.

In order to separate economies of density and scale, route-miles are used as a measure of network. This measure is available from Statistics Canada publications. For the period under consideration, there has been little change in route-miles of either railway. For the entire sample, CN's route-miles are approximately twice those of CP. Technical change is represented by a time trend from 1 to 26.

The estimation results from the two output, five input specification of the cost function are provided in the next chapter. As mentioned earlier, all four data alternatives, models A, AR, B and BR are utilized.

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³⁹ VIA passenger-miles are first divided between CN and CP, and then added to the data reported totals for CN and CP. The reported totals for CP are not zero in 1979; this results in volatility.

CHAPTER 4

RESULTS

This chapter presents results from the estimation of the translog cost function introduced in Chapter 2. All the specifications discussed in this chapter assume that railways provide two services, passenger transportation and freight transportation. The former is measured in units of passenger-miles and the latter in ton-miles. Five inputs are employed in the production process, these being labour, equipment capital, way and structures capital, fuel, and materials. For each specification of the production technology, four models are estimated. These alternatives are based on the data. Models A and AR utilize the user cost of capital constructed using the Boadway, Bruce and Mintz method. Models B and BR use the more conventional user cost of capital. Models AR and BR include rented capital while models A and B do not.

Various modifications are made to the general specification of the cost function in order to account for the particular characteristics of transportation firms. These include incorporating the potential quasi-fixed nature of way and structures capital and the inclusion of a measure of network size in the model. Three output models are also estimated. Since the results vary considerably across models, and specifications, summary tables of results are provided in Appendix 3.

4.1 General Translog Cost Function

The general translog model was presented in Equations (2.6) to (2.9) in Chapter 2. Summary results from the estimation are reported in Tables A3.1 to A3.4. None of the four models are concave anywhere. Model AR shows a minor monotonicity violation; the predicted share of structures input is negative in the year 1976 for CN. Both models A and AR show negative freight cost elasticities for six or fewer observations¹ and, in both cases, the problem is restricted to CP. Negative output cost elasticities are unacceptable since they imply negative marginal costs. They also result in extremely high local estimates of returns to scale. The maximum returns to scale are around 23 for model A and 5 for model AR. Though all local estimates of output cost elasticities for CN are positive, the returns to scale estimates seem unreasonable. They range from 0.7 to 2.3 in model A and from 0.7 to 1.6 in model AR and increase in a monotonic fashion from the mid 1960s to the end of the sample. For CP, if one ignores the observations for which freight cost elasticities are negative, the local estimates of returns to scale are still unacceptable. They range from about 0.8 to 8 in model A and 0.8 to 5 in model AR.

Use of the conventional user cost of capital helps alleviate this problem since all local estimates of output cost elasticities are positive for models B and BR. In model B, returns to scale range from 0.9 to 1.08 for CN, and from 0.9 to 3.0 for CP.

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¹ For model A, the last six observations have offending signs, whereas for model AR, the last three observations have negative freight cost elasticities.

In model BR, CN exhibits moderately decreasing returns to scale for the entire sample. CP, on the other hand, displays increasing returns to scale over the period 1960-81. From the mid to late 1970s however, the numbers are as high as 2.4. The mean returns to scale are lower for models that include rented capital than for those that exclude it.

The sample average rate of technical change is not sensitive to the exclusion of rented capital. All four models indicate technical progress in Canadian railways over the sample. Use of the conventional user cost of capital rather than that due to Boadway, Bruce and Mintz results in estimates that are about 50% higher.

None of the models satisfy the second-order conditions for cost minimization. This results in upward sloping demand curves and positive own price elasticities of factor demand². In model A, the demand schedules for labour and materials are upward sloping at every observation, the situation in model B is no better. In both cases inclusion of rented capital improves the results, the improvement being more significant if the conventional user cost of capital is used.

In an attempt to improve the performance of this specification, the set of four models is estimated assuming neutral technical change (that is, excluding the technology interaction terms). The prices of labour and materials are highly correlated

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² Note that the elasticities of substitution and the own and cross price elasticities of factor demand are calculated using the sample mean values of predicted factor shares.

with the time trend. This may be one of the factors contributing to the positive slope of the demand functions for these inputs. The results from the modified specification are discussed in the following section.

4.2 Translog Cost Function with Neutral Technical Change

Assuming neutral technical change modifies the specification presented in Equations (2.6) to (2.9). The modified specification excludes the squared time trend and all interaction variables with time and outputs, and time and input prices. The factor share equations no longer include the time trend as an explanatory variable. This modification provides more degrees of freedom, reducing the number of parameters in the model by seven. The two output, five input translog cost function with neutral technical change is shown in Equation (4.1) below. Linear homogeneity in input prices is imposed by dividing total cost and (N-1) input prices by the price of materials.

Summary results for models AR, B and BR are presented in Tables A3.5 through A3.7 in Appendix 3. Detailed results for model A are provided below in Tables 4.1 through 4.8.

$$\begin{aligned}
\ln\left(\frac{C}{p_M}\right) = & \alpha_0 + \alpha_L \ln\left(\frac{p_L}{p_M}\right) + \alpha_E \ln\left(\frac{p_E}{p_M}\right) + \alpha_S \ln\left(\frac{p_S}{p_M}\right) + \alpha_F \ln\left(\frac{p_F}{p_M}\right) \\
& + \beta_P \ln P + \beta_Q \ln Q + \alpha_t t + \gamma_{LL}/2 \left[\ln\left(\frac{p_L}{p_M}\right) \right]^2 \\
& + \gamma_{LE} \ln\left(\frac{p_L}{p_M}\right) \ln\left(\frac{p_E}{p_M}\right) + \gamma_{LS} \ln\left(\frac{p_L}{p_M}\right) \ln\left(\frac{p_S}{p_M}\right) \\
& + \gamma_{LF} \ln\left(\frac{p_L}{p_M}\right) \ln\left(\frac{p_F}{p_M}\right) + \gamma_{EE}/2 \left[\ln\left(\frac{p_E}{p_M}\right) \right]^2 \\
& + \gamma_{ES} \ln\left(\frac{p_E}{p_M}\right) \ln\left(\frac{p_S}{p_M}\right) + \gamma_{EF} \ln\left(\frac{p_E}{p_M}\right) \ln\left(\frac{p_F}{p_M}\right) \\
& + \gamma_{SS}/2 \left[\ln\left(\frac{p_S}{p_M}\right) \right]^2 + \gamma_{SF} \ln\left(\frac{p_S}{p_M}\right) \ln\left(\frac{p_F}{p_M}\right) \\
& + \gamma_{FF}/2 \left[\ln\left(\frac{p_F}{p_M}\right) \right]^2 + \beta_{PP}/2 (\ln P)^2 + \beta_{QQ}/2 (\ln Q)^2 \\
& + \beta_{PQ} (\ln P) (\ln Q) + \delta_{LQ} \ln\left(\frac{p_L}{p_M}\right) (\ln Q) + \delta_{EQ} \ln\left(\frac{p_E}{p_M}\right) (\ln Q) \\
& + \delta_{SQ} \ln\left(\frac{p_S}{p_M}\right) (\ln Q) + \delta_{FQ} \ln\left(\frac{p_F}{p_M}\right) (\ln Q) \\
& + \delta_{LP} \ln\left(\frac{p_L}{p_M}\right) (\ln P) + \delta_{EP} \ln\left(\frac{p_E}{p_M}\right) (\ln P) \\
& + \delta_{SP} \ln\left(\frac{p_S}{p_M}\right) (\ln P) + \delta_{FP} \ln\left(\frac{p_F}{p_M}\right) (\ln P)
\end{aligned} \tag{4.1}$$

where C represents total cost, Q represents freight ton-miles, P represents passenger-miles, t represents technical change and p_L , p_E , p_S , p_F , p_M , represent the prices of labour, equipment, way and structure, fuel, and materials, respectively.

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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The most striking difference between this, and the previous specification is that the demand schedules for labour and materials have the appropriate slope at every observation in the sample in all four models. Despite this significant improvement, the models remain non-concave everywhere. The problem of negative output cost elasticities has also been eliminated as has the minor monotonicity violation encountered in model AR.

In models AR, B and BR the trend in the estimates of returns to scale is reversed. This reversal is most noticeable in model AR. For CN, returns to scale are at their maximum value of 2.23 in 1960 and decline to 1.06 in 1981. A similar pattern is exhibited by CP. Local estimates of returns to scale exceed 2 over the period 1957-62, for CN and 1956-69, for CP. This model also produces the highest sample average returns to scale estimate.

Models that utilize the conventional cost of capital suggest that on average railways exhibit decreasing returns to scale. In the initial five to ten years of the sample both firms show moderately increasing returns. From the mid to late 1960s the opposite is true, with the local estimates declining in a somewhat monotonic fashion.

Irrespective of the method employed to construct the user cost of capital, the inclusion of rental capital results in higher sample average returns to scale estimates.

Models A and AR indicate that the estimated rate of technical change is not sensitive to the inclusion of rental capital while models B and BR show moderate sensitivity. As with the previous specification, all models indicate technical progress and models that use the conventional cost of capital indicate faster technical progress, in this case more so, almost by a factor of four.

The estimates of returns to scale are clearly too high in model AR, and it might be argued that the same is true for model A. Models B and BR are also unsatisfactory since they suggest decreasing returns to scale on average³. At this stage model A is the preferred model, however the other alternatives are not discarded just yet. All four models are retained for further investigation; the inferior performance of models B and BR will become more evident in Chapter 6.

Consider now, the detailed results provided for model A in Tables 4.1 through 4.8. The model provides good predictions of the factor shares in total cost, though this is also true for models AR, B and BR.

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³ One would expect increasing returns to scale in railways, particularly in cases where no distinction is made between returns to scale and returns to density. Research on U.S. railways clearly suggests that moderately decreasing or constant returns are encountered in models that include a measure of network. Such models also exhibit strong increasing returns to density.

Table 4.1		
Summary Results from Non-Concave Translog Cost Function		
R² Cost Function		0.935
R² Share of Labour		0.911
R² Share of Equipment		0.945
R² Share of Structures		0.913
R² Share of Fuel		0.818
Mean share of Labour: Observed (Predicted)		0.4716 (0.4721)
Mean share of Equipment: Observed (Predicted)		0.1338 (0.1335)
Mean share of Structures: Observed (Predicted)		0.1837 (0.1831)
Mean share of Fuel: Observed (Predicted)		0.0502 (0.0504)
Mean share of Materials: Observed (Predicted)		0.1607 (0.1609)
Technical Change		-0.010
Negatively Sloped Labour Demand		52
Negatively Sloped Equipment Demand		40
Negatively Sloped Structures Demand		43
Negatively Sloped Fuel Demand		17
Negatively Sloped Materials Demand		52
Sample Mean of Passenger Cost Elasticity	Lower 95%	0.215
	Estimate	0.270
	Upper 95%	0.325
Sample Mean of Freight Cost Elasticity	Lower 95%	0.163
	Estimate	0.335
	Upper 95%	0.505
Sample Mean of Returns to Scale	Lower 95%	1.340
	Estimate	1.664
	Upper 95%	2.495

Table 4.2

Results from the Non-Concave Translog Cost Function: Model A

<u>Parameter</u>	<u>Estimate</u>	<u>Bootstrap Standard Error</u>	<u>Asymptotic Standard Error</u>	<u>Asymptotic t-Ratio</u>
α_0	0.134	0.029	0.029	4.654
α_L	0.401	0.005	0.005	74.356
α_E	0.187	0.003	0.003	56.273
α_S	0.264	0.005	0.005	58.495
α_I	0.037	0.002	0.002	15.385
α_M	0.110	0.003	0.003	33.653
β_P	0.263	0.044	0.042	6.203
β_Q	0.317	0.104	0.100	3.165
α_T	-0.010	0.004	0.004	-2.832
$\gamma_{LL}/2$	0.066	0.011	0.010	6.416
$\gamma_{EE}/2$	0.045	0.003	0.003	13.257
$\gamma_{SS}/2$	0.055	0.004	0.004	15.516
$\gamma_{II}/2$	0.029	0.002	0.002	14.752
$\gamma_{MM}/2$	-0.015	0.012	0.011	-1.321
γ_{LE}	-0.032	0.009	0.009	-3.759
γ_{LS}	-0.075	0.007	0.007	-10.454
γ_{LI}	-0.074	0.007	0.007	-10.199
γ_{LM}	0.050	0.019	0.018	2.751
γ_{ES}	-0.013	0.006	0.005	-2.478
γ_{EF}	-0.006	0.003	0.003	-1.695
γ_{EM}	-0.039	0.008	0.007	-5.208
γ_{IF}	-0.010	0.003	0.003	-3.869
γ_{SM}	-0.012	0.005	0.005	-2.326
γ_{FM}	0.031	0.008	0.008	4.079
$\beta_{PP}/2$	0.012	0.027	0.026	0.453
$\beta_{QQ}/2$	0.062	0.081	0.081	0.770
β_{PQ}	0.012	0.085	0.078	0.156
δ_{QL}	-0.033	0.022	0.021	-1.601
δ_{QE}	-0.019	0.012	0.012	-1.599
δ_{QS}	0.041	0.015	0.015	2.805
δ_{QI}	0.035	0.008	0.007	4.734
δ_{QM}	-0.025	0.015	0.015	-1.673
δ_{PL}	0.042	0.005	0.005	8.463
δ_{PE}	-0.009	0.003	0.003	-2.961
δ_{PS}	0.008	0.004	0.004	2.091
δ_{PI}	-0.022	0.002	0.002	-9.354
δ_{PM}	-0.019	0.003	0.003	-5.544

Log of the Likelihood Function: 733.261

Table 4.3

**Output Cost Elasticities for CN:
Estimates and 95% Confidence Intervals,
Non-Concave Translog Model A**

<u>Year</u>	<u>Passenger Cost Elasticity</u>			<u>Freight Cost Elasticity</u>		
	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>
1956	0.183	0.257	0.337	0.153	0.332	0.501
1957	0.172	0.252	0.338	0.087	0.296	0.484
1958	0.180	0.256	0.336	0.109	0.311	0.493
1959	0.184	0.259	0.339	0.115	0.315	0.496
1960	0.177	0.256	0.342	0.097	0.300	0.487
1961	0.181	0.259	0.345	0.109	0.307	0.489
1962	0.186	0.262	0.343	0.116	0.311	0.490
1963	0.198	0.266	0.335	0.140	0.321	0.490
1964	0.189	0.275	0.366	0.156	0.335	0.511
1965	0.185	0.279	0.378	0.153	0.339	0.519
1966	0.173	0.284	0.391	0.139	0.340	0.530
1967	0.161	0.293	0.419	0.111	0.340	0.556
1968	0.178	0.291	0.401	0.130	0.337	0.530
1969	0.184	0.291	0.393	0.140	0.334	0.523
1970	0.185	0.294	0.399	0.149	0.346	0.535
1971	0.183	0.298	0.415	0.159	0.365	0.563
1972	0.181	0.296	0.412	0.160	0.360	0.564
1973	0.200	0.293	0.386	0.166	0.362	0.564
1974	0.174	0.287	0.402	0.160	0.375	0.592
1975	0.178	0.288	0.401	0.153	0.367	0.576
1976	0.169	0.285	0.407	0.117	0.348	0.565
1977	0.165	0.285	0.408	0.137	0.365	0.584
1978	0.163	0.291	0.423	0.167	0.401	0.633
1979	0.160	0.293	0.429	0.163	0.415	0.658
1980	0.153	0.289	0.431	0.163	0.420	0.673
1981	0.152	0.284	0.423	0.176	0.429	0.680
Mean	0.189	0.279	0.365	0.166	0.349	0.529
Sample Mean	0.215	0.270	0.325	0.163	0.335	0.505

Table 4.4

**Output Cost Elasticities for CP:
Estimates and 95% Confidence Intervals,
Non-Concave Translog Model A**

<u>Year</u>	<u>Passenger Cost Elasticity</u>			<u>Freight Cost Elasticity</u>		
	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>
1956	0.179	0.263	0.349	0.109	0.317	0.509
1957	0.165	0.261	0.359	0.061	0.293	0.510
1958	0.164	0.262	0.364	0.066	0.298	0.509
1959	0.161	0.263	0.370	0.057	0.297	0.516
1960	0.153	0.260	0.370	0.052	0.286	0.505
1961	0.155	0.261	0.374	0.058	0.289	0.507
1962	0.153	0.261	0.376	0.056	0.291	0.509
1963	0.166	0.262	0.361	0.090	0.300	0.496
1964	0.198	0.269	0.346	0.142	0.320	0.487
1965	0.193	0.267	0.345	0.134	0.316	0.491
1966	0.175	0.258	0.341	0.122	0.316	0.509
1967	0.179	0.263	0.349	0.115	0.307	0.496
1968	0.156	0.259	0.363	0.088	0.297	0.501
1969	0.162	0.261	0.360	0.087	0.296	0.502
1970	0.168	0.262	0.354	0.083	0.308	0.529
1971	0.163	0.262	0.356	0.078	0.322	0.557
1972	0.154	0.260	0.355	0.066	0.327	0.578
1973	0.129	0.257	0.374	0.034	0.335	0.623
1974	0.163	0.258	0.344	0.085	0.345	0.596
1975	0.139	0.255	0.360	0.046	0.332	0.601
1976	0.119	0.249	0.369	0.011	0.315	0.606
1977	0.124	0.248	0.363	0.021	0.327	0.619
1978	0.141	0.255	0.360	0.055	0.357	0.647
1979	0.172	0.262	0.349	0.092	0.371	0.641
1980	0.187	0.261	0.334	0.117	0.370	0.618
1981	0.183	0.255	0.325	0.139	0.394	0.646
Mean	0.177	0.260	0.343	0.126	0.320	0.516
Sample Mean	0.215	0.270	0.325	0.163	0.335	0.505

Table 4.5

**Returns to Scale for CN & CP:
Estimates and 95% Confidence Intervals,
Non-Concave Translog Model A**

	<u>Canadian National</u>			<u>Canadian Pacific</u>		
<u>Year</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>
1956	1.347	1.698	2.352	1.334	1.724	2.492
1957	1.389	1.825	2.667	1.349	1.806	2.768
1958	1.351	1.764	2.519	1.319	1.788	2.770
1959	1.354	1.742	2.455	1.306	1.788	2.874
1960	1.363	1.797	2.637	1.315	1.834	3.136
1961	1.338	1.765	2.593	1.281	1.819	3.301
1962	1.343	1.746	2.526	1.275	1.811	3.298
1963	1.355	1.704	2.332	1.302	1.778	2.884
1964	1.286	1.640	2.269	1.336	1.699	2.384
1965	1.263	1.618	2.305	1.328	1.716	2.496
1966	1.201	1.603	2.426	1.261	1.740	3.006
1967	1.141	1.578	2.623	1.279	1.755	2.964
1968	1.189	1.593	2.424	1.223	1.798	3.853
1969	1.205	1.601	2.380	1.226	1.795	3.777
1970	1.169	1.562	2.316	1.202	1.755	3.501
1971	1.124	1.509	2.324	1.163	1.711	3.423
1972	1.137	1.526	2.453	1.142	1.703	3.523
1973	1.175	1.526	2.215	1.054	1.690	4.219
1974	1.132	1.512	2.331	1.135	1.658	3.242
1975	1.151	1.526	2.372	1.102	1.705	4.026
1976	1.162	1.580	2.575	1.075	1.770	4.976
1977	1.129	1.539	2.487	1.085	1.739	4.663
1978	1.068	1.444	2.313	1.065	1.633	3.753
1979	1.042	1.411	2.280	1.094	1.580	2.962
1980	1.034	1.412	2.356	1.143	1.583	2.658
1981	1.034	1.401	2.292	1.126	1.540	2.514
Mean	1.271	1.601	2.303	1.267	1.728	3.194
Sample Mean	1.340	1.664	2.495	1.340	1.664	2.495

Table 4.6

**Allen-Uzawa Elasticities of Substitution:
Estimates and Confidence Intervals,
Non-Concave Translog Model A**

<u>Elasticity</u>	<u>Lower 95%</u>	<u>Estimate</u>	<u>Upper 95%</u>
σ_{LE}	0.215	0.484	0.747
σ_{LS}	-0.022	0.137	0.294
σ_{LF}	-2.752	-2.098	-1.511
σ_{LM}	1.129	1.653	2.118
σ_{ES}	0.011	0.471	0.913
σ_{EF}	-0.792	0.157	1.121
σ_{EM}	-1.531	-0.816	-0.098
σ_{SF}	-0.622	-0.074	0.476
σ_{SM}	0.228	0.589	0.960
σ_{FM}	3.028	4.883	6.977

Table 4.7

**Morishima Elasticities of Substitution:
Estimates and Confidence Intervals,
Non-Concave Translog Model A**

<u>Elasticity</u>	<u>Lower 95%</u>	<u>Estimate</u>	<u>Upper 95%</u>
ω_{LE}	0.137	0.256	0.376
ω_{LS}	0.155	0.244	0.341
ω_{LF}	-0.492	-0.303	-0.120
ω_{LM}	0.934	1.291	1.626
ω_{EL}	0.285	0.479	0.670
ω_{ES}	0.177	0.305	0.430
ω_{EF}	-0.357	-0.189	-0.022
ω_{EM}	0.545	0.894	1.230
ω_{SL}	0.186	0.315	0.432
ω_{SE}	0.107	0.255	0.398
ω_{SF}	-0.392	-0.201	-0.020
ω_{SM}	0.845	1.120	1.395
ω_{FL}	-1.082	-0.740	-0.433
ω_{FE}	0.066	0.213	0.360
ω_{FS}	0.065	0.205	0.343
ω_{FM}	1.272	1.811	2.385
ω_{ML}	0.710	1.030	1.316
ω_{ME}	-0.070	0.083	0.240
ω_{MS}	0.222	0.326	0.442
ω_{MF}	-0.199	0.049	0.287

Table 4.8

Own and Cross Price Elasticities of Factor Demand:
Estimates and Confidence Intervals,
Non-Concave Translog Model A

<u>Elasticity</u>	<u>Lower 95%</u>	<u>Estimate</u>	<u>Upper 95%</u>
e_{LL}	-0.336	-0.250	-0.156
e_{LE}	0.029	0.065	0.101
e_{LS}	-0.004	0.025	0.054
e_{LF}	-0.137	-0.106	-0.077
e_{LM}	0.182	0.266	0.341
e_{EL}	0.103	0.229	0.354
e_{EE}	-0.289	-0.192	-0.089
e_{ES}	0.002	0.086	0.168
e_{EF}	-0.040	0.008	0.055
e_{EM}	-0.247	-0.131	-0.016
e_{SL}	-0.011	0.065	0.139
e_{SE}	0.001	0.063	0.122
e_{SS}	-0.294	-0.219	-0.144
e_{SF}	-0.031	-0.004	0.024
e_{SM}	0.037	0.095	0.154
e_{FL}	-1.297	-0.990	-0.716
e_{FE}	-0.106	0.021	0.149
e_{FS}	-0.113	-0.013	0.088
e_{FF}	0.019	0.197	0.381
e_{FM}	0.484	0.786	1.129
e_{ML}	0.532	0.780	0.997
e_{ME}	-0.204	-0.109	-0.013
e_{MS}	0.041	0.108	0.176
e_{MF}	0.153	0.246	0.347
e_{MM}	-1.301	-1.025	-0.744

R^2 is in excess of 0.9 for all but one equation in the system; this is typical for such models. Due to the large number of independent variables, R^2 is particularly high for the cost function⁴. It is encouraging to note that the 95% confidence bounds of the output cost elasticities do not include negative values. This model produces robust increasing returns to scale at all points in the sample; Table 4.5 shows that none of the 95% confidence intervals include constant returns to scale. The local estimates of returns to scale are quite high and the 95% bounds admit values as high as 4 for Canadian Pacific. This is primarily due to the fact that second-order and interaction terms of the two outputs have not been estimated with sufficient precision. This is evident from Table 4.2⁵. This lack of precision is encountered in all the alternative specifications of the cost function. In models that include rented capital and in those that utilize the conventional user cost of capital, the standard errors of these three parameter estimates are much lower.

Model A is a non-concave model. As a consequence, discussion of the elasticities of substitution may not be meaningful. Table 4.8 shows that the own price elasticity of the demand for fuel has a robust offending sign. Elasticities of substitution are presented in Tables 4.6 and 4.7. Consider the two inputs equipment and materials. Both have negative own price elasticities of demand according to

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⁴ Berndt (1991) suggests the use of the generalized R^2 for systems of equations, but this is not calculated here.

⁵ Those interested in comparing bootstrap to asymptotic standard errors will note from Table 4.2 that there is little difference between the two.

Table 4.8. This assures that utilization will fall as prices increase. Table 4.6 suggests that the two inputs are complements in production ($\sigma_{EM} = \sigma_{ME} = -0.816$), whereas the Morishima elasticity associated with an equipment price change is positive ($\omega_{ME} = 0.083$). This can be interpreted as follows: As the price of equipment increases, the demand for materials declines, but due to concavity (or a negatively sloped demand curve for equipment), so does the usage of equipment at such a rate that the ratio M/E actually falls.

4.3 Introduction of a Measure of Network Size

All transportation firms have a geographic network. In the case of airlines, this is often measured by the number of points served. Research on railways utilizes route-miles as a measure of network size. These are only second best measures since typical networks have nodes and links whereas these measures are 'linear'. An airline could serve ten points in a linear fashion, it could also serve ten points using a hub and spoke configuration. Using points served as a network measure does not incorporate this distinction.

Based on this notion of introducing a network variable in the general cost function, Caves, Christensen and Tretheway (1984) and Caves, Christensen, Tretheway and Windle (1985) draw a distinction between economies of density and economies of scale. They suggest that transportation firms have two options before them, they could

use the existing network more intensively (density), hence increasing output while holding network size constant or, they could increase both output and network size (scale). Research on U.S. railways suggests that there are substantial returns to density but returns to scale are more or less constant.

Following the literature, route-miles is introduced as a measure of network size. The network variable enters the cost function in Equation (4.1) above as an additional output. The number of unknown parameters in the model increases by eight. The linear and squared terms of the network variable add two parameters. The interaction of the network variable with the two outputs adds another two and the interaction with $(N-1)$ input prices adds the last four. The assumption of neutral technical change is maintained. Over the entire sample, CN's route-miles are approximately twice those of CP, therefore the network variable may have the same effect as a cross-sectional dummy variable. Summary results can be found in Tables A3.8 through A3.11 in Appendix 3.

The behaviour of the slopes of the input demand functions and the own price elasticities of demand is similar to that in the specification with neutral technical change. All models indicate technical progress but not to the extent encountered in previous specifications. The coefficient of the time trend is similar in sign and magnitude in models A, AR and BR. In model B the magnitude is halved.

At first glance, returns to scale estimates do not seem unreasonable. In models A, AR and BR, the sample average is around 0.8, which is somewhat lower than that found in U.S. studies. Model B produces sample average returns to scale of around 1.08. These results are misleading. This is evident from the estimates of returns to density. Returns to density are defined as the inverse of the sum of the passenger and freight cost elasticities. Both elasticities are negative for part of the sample in all four models thus producing theoretically inconsistent estimates of economies of density. Returns to scale are defined as the inverse of the sum of output and network cost elasticities. It is coincidental that the negative output cost elasticities are outweighed by the positive network cost elasticity to produce returns to scale estimates that might seem reasonable on average. The behaviour of the output cost elasticities is clearly unacceptable.

4.4 A Restricted Cost Function

An overwhelming majority of studies of the production structure of Canadian and U.S. railways are based on the estimation of a long run cost function. All factors of production are variable and adjustment is instantaneous. One notable exception is the work of Caves, Christensen and Swanson (1981) who estimate a short run or restricted cost function. Way and structures capital is specified as a quasi-fixed factor of production. The railway firm takes the stock of way and structures capital as given

and minimizes variable production costs⁶. This may not be an unreasonable scenario for the problem at hand. Since over the sample period of this thesis, railways were not free to abandon their track at will. In fact, a large majority of the revenues are produced from a small proportion of the track, indicating that the firms have no choice but to treat the track as given. An attempt is made to estimate a variable cost function for Canadian railways assuming way and structures capital is quasi-fixed⁷. Consider the following restricted translog cost function with one quasi-fixed factor of production and neutral technical change:

$$\begin{aligned}
 \ln CV(y, s, p, t) = & \alpha_0 + \sum_{i=1}^N \alpha_i \ln p_i + \sum_{k=1}^M \beta_k \ln y_k + \alpha_s t \\
 & + \alpha_s \ln s + (1/2) \alpha_{ss} (\ln s)^2 \\
 & + (1/2) \sum_{i=1}^N \sum_{j=1}^N \gamma_{ij} \ln p_i \ln p_j \\
 & + (1/2) \sum_{k=1}^M \sum_{l=1}^M \beta_{kl} \ln y_k \ln y_l \\
 & + \sum_{i=1}^N \sum_{k=1}^M \delta_{ik} \ln p_i \ln y_k + \sum_{i=1}^N \alpha_{si} \ln s \ln p_i \\
 & + \sum_{k=1}^M \beta_{sk} \ln s \ln y_k
 \end{aligned} \tag{4.2}$$

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⁶ See Christensen, Cummings and Schoech (1983) for an application to telecommunications.

⁷ It may be argued that equipment capital and perhaps even labour should be treated as quasi-fixed factors, but these possibilities are not examined here. Long term employment contracts may prevent a firm from adjusting its labour force in the short run.

where $CV(.)$ represents variable cost, y is an M -dimensional vector of outputs, p is an N -dimensional vector of prices of variable factors of production, s is the quantity of the quasi-fixed factor of production and t is a measure of technical progress.

Economic theory requires the cost function⁸ in Equation (4.2) to be nondecreasing in p , nondecreasing in y , nonincreasing in s , and positively homogeneous of degree one in p . Further, the price Hessian of the variable cost function is required to be a symmetric negative semidefinite matrix. Some of these properties can be translated in parametric restrictions on the variable cost function. The following restrictions are necessary and sufficient for the above cost function to be linearly homogeneous in input prices.

$$\begin{aligned} \sum_{i=1}^N \alpha_i &= 1 ; \quad \sum_{i=1}^N \alpha_{si} = 0 \\ \sum_{i=1}^N \gamma_{ij} &= 0 \quad j = 1, \dots, N \\ \sum_{i=1}^N \delta_{ik} &= 0 \quad k = 1, \dots, M \end{aligned} \tag{4.3}$$

Symmetry restrictions on the cross-effects imply:

$$\gamma_{ij} = \gamma_{ji} ; \quad \beta_{ki} = \beta_{ik} \tag{4.4}$$

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⁸ These theoretical consistency requirements can be found in Hazilla and Kopp (1986a). For a more detailed treatment see Hazilla and Kopp (1986b).

Shephard's Lemma allows the derivation of the following N factor share equations obtained by partial differentiation of the cost function in (4.2) with respect to $\ln p_i$:

$$S_i = \alpha_i + \sum_{j=1}^N \gamma_{ij} \ln p_j + \sum_{k=1}^M \delta_{ik} \ln y_k + \alpha_{si} \ln s \quad (4.5)$$

The estimation technique is identical to that used for the long run cost function. An additive disturbance term is appended to the variable cost function and the variable factor share equations. The system of equations including the cost function and $(N-1)$ variable factor share equations is estimated using the IZEF estimation technique.

The estimated cost function can be used to derive returns to scale as follows:

$$RTS = \frac{\left(1 - \frac{\partial \ln CV(.)}{\partial \ln s} \right)}{\sum_{k=1}^M \frac{\partial \ln CV(.)}{\partial \ln y_k}} \quad (4.6)$$

Summary results from the estimation are provided in Tables A3.12 through A3.15 in Appendix 3. Monotonicity is the only theoretical consistency requirement satisfied by all four models. All other first and second derivative requirements are violated. As is the case with the long run specification, the models are not concave anywhere. The cost elasticity with respect to the quasi-fixed factor of production or

the structures cost elasticity is positive⁹ and greater than one for 26 of 52 observations, these being observations for Canadian Pacific. This serious violation is encountered in all four models, as is the occurrence of some negative output cost elasticities for both freight and passenger output. Given the poor performance of this model, other comparative static effects are not derived.

4.5 Three Output Models

Both CN and CP transport grain at non-compensatory statutory rates¹⁰. Grain traffic accounts for approximately 17% of total freight ton-miles for CN. Grain traffic is more important for CP, both in relative and absolute terms. It accounts for almost a quarter of the total ton-miles of freight carried by CP. In absolute terms, that is, both in terms of ton-miles and tons, CP carries more grain than CN. The ratio CN ton-miles (tons) to CP ton-miles (tons) is 92%. A three output, five input specification of the translog cost function is estimated in order to identify the cost characteristics of grain movements. Total ton-miles of freight are disaggregated into Grain ton-miles and Non-Grain ton-miles. The third output is passenger-miles. Technical change is restricted to be neutral.

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⁹ The cost function is required to be nonincreasing in the quasi-fixed factor because increase in the latter enlarges the feasibility set, thus opening up new cost minimizing opportunities to the firm.

¹⁰ The so called "Crow Rate".

Summary results are reported in Tables A3.16 through A3.19 in Appendix 3.

The use of this specification does not lead to any improvement in the curvature of the cost function. In model AR, the cost function is concave in input prices at one observation: CN 1956. However, this model also produces one monotonicity violation; for CN, the 1976 share of way and structures capital is negative.

The rate of technical change (around -0.01) in models A and AR is similar to that obtained in the two output translog cost function with neutral technical change. As before, use of the conventional user cost of capital (models B and BR) results in faster technical change (-0.03).

All four models provide disappointing results because the sample average grain freight cost elasticity is negative. All models produce a negative grain freight cost elasticity for the period 1956-72, for both CN and CP. As a consequence estimates of economies of scale are as high as 4.1 in some years. Negative output cost elasticities imply negative marginal costs and are therefore unusable. This violation is quite serious and robust since the estimated coefficient of the first order grain ton-miles term is negative and significant in all four models¹¹.

An alternative three output specification is estimated with the intent to isolate the impact of distance and weight on total costs. The three outputs are: total freight

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¹¹ The estimated coefficient is around -0.2 to -0.3, with an asymptotic t-ratio in excess of 3.

ton-miles, average length of haul, passenger-miles¹². Average length of haul is derived from data on ton-miles of freight and tons of freight; dividing ton-miles by tons yields the requisite measure.

Summary estimation results can be found in Appendix 3, Tables A3.20 through A3.23. All four models (A, AR, B, and BR) are non-concave and the results on technical change are similar to those encountered in the previous three output specification¹³.

Results on economies of scale¹⁴ continue to be disappointing in all four models, with the point estimates ranging from -38 to 533. Increase in average length of haul (holding ton-miles constant) can only be accomplished by reducing tonnage. Decrease in tonnage reduces total costs while shipping the lower tonnage over a longer distance increases total costs. Practically all studies on U.S. railways that include average length of haul as an output find that increase in length of haul causes a net reduction in total costs. The elasticity of total cost with respect to average length of haul is therefore expected to be negative. For a majority of observations, all four models produce a negative point estimate of this elasticity. Unfortunately, they

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¹² Average trip-length is not included primarily due to degrees of freedom problems. Further, due to the VIA passenger-miles allocation problem it is doubtful that a consistent series can be constructed for average trip-length.

¹³ The coefficient of the time trend is around -0.01 in models A and AR. In the other two models it is approximately -0.03.

¹⁴ RTS_2 is used here, so the increase in ton-miles is caused by increase in average length of haul.

produce very large negative values. The sample average is -1.2 in models A and AR and -0.6 in models B and BR. A one percent increase in average length of haul results in a 0.6 to 1.2% reduction in total costs. Studies on U. S. railways find a much smaller reduction in total costs: around 0.02%¹⁵. The unreasonable point estimates of economies of scale are a consequence of the unreasonable estimates of the elasticity of total cost with respect to average length of haul.

4.6 Other Alternatives

Further experiments were conducted with the two output, five input translog with neutral technical change. They alter the components of way and structures, and equipment capital. Caves and Christensen (1978 and 1980b), Caves, Christensen, Swanson and Tretheway (1982), and Freeman et al. (1985a, 1985b and 1987) re-classify investment data to conform to definitions used by U.S. railways¹⁶. Since this is a study of Canadian railways, investment data were used as reported. However, due to the inappropriate curvature of all the specifications estimated above, the investment data were re-classified with the hope that this would improve the behaviour of the demand schedules for the two capital inputs. The results are very similar, and in some respects marginally inferior to those obtained in Section 4.2 above¹⁷.

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¹⁵ For example, see page 134 in Caves, Christensen, Swanson and Tretheway (1982).

¹⁶ Investment in shop machinery and power plant machinery is deducted from total investment in equipment and added to investment in way and structures.

¹⁷ Translog cost function with neutral technical change.

The rate of technical progress in all four models (A, AR, B, and BR) is not affected at all by the re-classification of capital. A model by model comparison¹⁸ shows that mean returns to scale are marginally higher or identical to those obtained in Section 4.1. Tables A3.24 through A3.27 in Appendix 3 show that the re-classification does not yield more observations at which the input demand functions have a negative slope¹⁹. There is an improvement in the curvature of the cost function in Model AR; concavity is satisfied at four observations. Regrettably, this model also produces high returns to scale. Returns to scale are in excess of 2 (the maximum is 2.8) for the first five to six CN observations, and the first fifteen CP observations. Clearly, the re-classification of investment data does not provide any improvement over the results obtained earlier.

4.7 Conclusions

The major conclusion that can be drawn from the above results is that the cost function for Canadian railways is very sensitive to changes in the data and specification. In all cases, the second-order conditions for cost minimization are not satisfied²⁰. Model A with neutral technical change produces the most acceptable but

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¹⁸ For example, model A estimated in this section is compared to model A in Section 4.1, as are the other models (AR, B, and BR).

¹⁹ The only exception is model A, where the demand way and structures has the appropriate slope at 45, rather than 43 observations.

²⁰With the exception of the minor improvement found in model AR in the preceding section.

nonetheless high returns to scale estimates. Violation of curvature is not uncommon in applied production analysis, however it seems that the problem is particularly severe for the model and data at hand. This problem may be rectified by changing the functional form to one that is globally concave. Inequality constrained estimation of the translog specification is possible, but as shown by Diewert and Wales (1987), this destroys the flexibility of the translog approximation. These authors introduce some new functional forms which permit the imposition of concavity without sacrificing flexibility. This avenue is explored in the next chapter.

CHAPTER 5

A GLOBALLY CONCAVE FLEXIBLE COST FUNCTION

Chapter 4 provided results for various models, all based on the translog specification. None of the models satisfied the second-order conditions for cost minimization. This problem is encountered often in applied production analysis. Fortunately, over the last five to seven years increasing attention is being paid to what Diewert and Wales (1987)¹ describe as "one of the most vexing problems applied economists have encountered in estimating flexible functional forms..." They provide a solution to this problem by proposing two new functional forms; the symmetric generalized McFadden² and the symmetric generalized Barnett³. Both the functional forms are flexible, single output, multiple input specifications and allow the imposition of curvature without sacrificing flexibility⁴.

Since the problem at hand involves more than one output, the cost functions proposed by Diewert and Wales cannot be utilized. In a recent paper, Kumbhakar (1990) extends the work of Diewert and Wales and proposes a multiple output version of the symmetric generalized McFadden cost function. Like the single

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¹ See page 43.

² For a recent application of this see Hazilla (1991).

³ These are based on the original work of McFadden (1978a) and Barnett (1983a).

⁴ The main problem with imposing concavity on the translog using sampling theoretic methods is that it results in a loss of flexibility of the translog approximation.

output variant, this is a flexible cost function and permits the imposition of concavity in the manner described by Diewert and Wales. The cost function is introduced in Section 5.1 below, followed by a discussion of the estimation results.

5.1 The Symmetric Generalized McFadden Cost Function

The following is a symmetric generalized McFadden (SGM) cost function with general technical change, where y is an M -dimensional vector of outputs, p is an N -dimensional vector of factor prices and t is a measure of technical change:

$$\begin{aligned}
 C(y, p, t) = & g(p) \left(\sum_{k=1}^M \beta_k y_k \right) + \sum_{i=1}^N b_i p_i + \sum_{i=1}^N b_{ii} p_i \left(\sum_{k=1}^M \beta_k y_k \right) \\
 & + \sum_{i=1}^N \sum_{k=1}^M a_{ik} p_i y_k t + a_t \left(\sum_{i=1}^N \alpha_i p_i \right) t \\
 & + (1/2) \sum_{k=1}^M \sum_{l=1}^M d_{kl} \left(\sum_{i=1}^N \lambda_i p_i \right) y_k y_l \\
 & + a_{tt} t^2 \left(\sum_{i=1}^N \delta_i p_i \right) \left(\sum_{k=1}^M \beta_k y_k \right)
 \end{aligned} \tag{5.1}$$

The function $g(p)$ is defined as follows:

$$g(p) = (1/2) p' S p / \theta' p \tag{5.2}$$

The matrix $S=S'$ is an N by N symmetric negative semidefinite matrix and $\theta' > 0'_N$ is a vector of nonnegative constants, not all equal to zero. The vector θ_N can be selected

by the researcher⁵. In order to identify all the parameters in the model, some additional restrictions are required. The matrix S has $N(N+1)/2$ unknown parameters. Diewert and Wales impose the following N restrictions on this matrix, which reduce the number of unknowns to $N(N-1)/2$. For some $p^* \geq 0_N$:

$$Sp^* = 0_N \quad (5.3)$$

Further, the vectors α , λ , δ and θ have to be selected by the researcher, they are required to satisfy the following restrictions⁶:

$$\theta'p^* > 0 ; \quad \alpha'p^* = 0 ; \quad \lambda'p^* = 0 ; \quad \delta'p^* = 0 \quad (5.4)$$

Kumbhakar suggests that in the multi-output case, one of parameters in the vector β be normalized to unity. Alternatively, the following normalization may be utilized:

$$\sum_{k=1}^M \beta_k = 1 \quad (5.5)$$

With these restrictions in place, the SGM contains $N(N+1)/2 + M(M+1)/2 + MN + M + N + 1$ parameters. This is exactly the number of parameters required for it to be a flexible functional form.

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⁵ Diewert and Wales suggest that θ_i be measured in units of input in order to ensure invariant elasticity estimates. They set θ_i equal to the sample mean of input i .

⁶ p^* can be a unit vector.

Shephard's Lemma allows the derivation of the following N factor demand equations⁷:

$$\begin{aligned}
 x_i = & \left(\sum_{k=1}^M \beta_k y_k \right) \left[\frac{\sum_{j=1}^N s_{ij} p_j}{\theta' p} - \left(\frac{\theta_i}{2} \right) \frac{\sum_{j=1}^N \sum_{l=1}^N s_{ij} p_i p_j}{(\theta' p)^2} \right] \\
 & + b_i + b_{ii} \left(\sum_{k=1}^M \beta_k y_k \right) + \sum_{k=1}^M a_{ik} y_k t + a_i \alpha_i t \\
 & + (1/2) \sum_{k=1}^M \sum_{l=1}^M d_{kl} \lambda_l y_k y_l + a_{ii} \delta_i t^2 \left(\sum_{k=1}^M \beta_k y_k \right)
 \end{aligned} \tag{5.6}$$

The researcher may wish to impart more flexibility to the cost function by estimating the α 's, δ 's and λ 's. Normalizing a_i and a_{ii} to unity permits the estimation of the α 's and δ 's. The λ 's can be estimated by normalizing one of them to unity.

Prior to estimation, an additive stochastic disturbance term is appended to each factor demand equation. The N factor demand equations can be estimated using non-linear IZEF or some other non-linear maximum likelihood procedure⁸. The cost function is not included in the estimation since it does not contain any additional information.

⁷ Note that these are factor demand equations, in the case of the translog, logarithmic partial differentiation yields factor share equations.

⁸ The single output SGM does not require non-linear estimation, the multiple output variant contains both within and across equation non-linear restrictions.

If $\theta' > 0'_N$ and $p \gg 0_N$ and the estimated matrix $\hat{S}$ is negative semidefinite, then the SGM cost function is globally concave in input prices⁹. If $\hat{S}$ is not NSD then, as Diewert and Wales demonstrate, concavity can be imposed on the SGM without destroying its flexibility. The matrix S can be re-parameterized such that $S = -AA'$ where A' is an upper triangular matrix and $A'p^* = 0_N$. This factorization¹⁰ of the S matrix results in a one for one exchange of parameters, the number of parameters does not change and hence the flexibility of the function remains intact¹¹. The re-parameterized factor demand equations are non-linear, even in the case of the single output SGM, thus requiring a non-linear maximum likelihood estimation method.

The estimated cost function can be used to derive measures of returns to scale, technical change and elasticities of factor demand and factor substitution in the manner discussed in Chapter 2. These expressions no longer take the simple elegant forms encountered in the use of the translog specification. One of the major shortcomings of the translog is that it does not permit zero values of outputs and therefore it is not possible to derive a measure of economies of scope. The SGM cost function is not plagued by similar problems and its use permits the derivation of the degrees of

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⁹ See proof in Appendix, Diewert and Wales (1987).

¹⁰ This is called the Cholesky decomposition by some, see Lau (1978) for a discussion. This technique was used to impose positive definiteness by Wiley, Schmidt and Bramble (1973).

¹¹ In a recent paper, Diane Dupont has applied this technique to the normalized quadratic restricted profit function due to Diewert and Ostensoe.

economies of scope¹² in railway operations. In the two output case, the degree of economies of scope are defined as follows¹³:

$$SC = \frac{[C(Q,0) + C(0,P) - C(Q,P)]}{C(Q,P)} \quad (5.7)$$

where $C(Q,0)$ is the stand alone cost of providing freight service, $C(0,P)$ is the stand alone cost of providing passenger transportation service and $C(Q,P)$ is the joint cost of providing both services. SC measures the relative cost increase that would result from fragmentation of the railways into two specialized firms. Such a division increases, decreases or leaves total costs unchanged as SC is greater than, less than or equal to zero respectively. A positive value of SC indicates the presence of economies of scope, or cost savings from the joint provision of both freight and passenger service by Canadian railways. It is important to note that cost savings from joint production are inadequate to guarantee subadditivity of the cost function. In fact the presence of both economies of scale and scope do not imply subadditivity¹⁴ and therefore natural monopoly¹⁵.

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¹² Economies of scope are a restricted form of subadditivity of the cost function. Economies of scope can be interpreted as being identical to orthogonal subadditivity. Baumol, Panzar and Willig (1988) indicate that economies of scope are a necessary and sufficient condition for the existence of multiproduct firms in contestable markets.

¹³ See page 73, Baumol, Panzar and Willig (1988).

¹⁴ See Proposition 7C3 on page 173 in Baumol, Panzar and Willig (1988).

¹⁵ Examples of cost functions that exhibit both ray subadditivity (scale) and orthogonal subadditivity (scope), but are not subadditive can be found in Sharkey (1982), pages 68-69, and Baumol, Panzar and Willig (1988), pages 173-174.

The issue of subadditivity is not pursued further for two reasons. Firstly, the measure of output used here prevents a theoretically legitimate test of natural monopoly. Jara-Diaz and Winston provide the following comments in this context¹⁶:

"... an aggregate version of transportation output, like ton-miles per unit time, represents a composite commodity whose components do not vary proportionally across observations ... In other words, neither ray nor transray behaviour ... can be appropriately studied from a ton-miles approach..."

Secondly, all models estimated here assume that the production technology is identical for both firms CN and CP. In fact, they are being treated as one firm. Given this assumption, it is not meaningful to conduct a test for natural monopoly.

5.2 Implementation and Results

A two output, five input SGM is estimated here. The elements of the vector θ_N are set equal to the approximate median values of the observed quantities of the five inputs. Since the data are normalized to equal unity at the approximate median of the sample, θ_N is a unit vector. The α 's, δ 's and λ 's are normalized to equal unity and the normalization in Equation (5.5) above is also used. The matrix S is a 5 by 5 symmetric matrix and the fact that it has rank $(N-1)$ is incorporated. No restrictions are put on the general nature of technical change¹⁷. These assumptions yield a SGM cost function that has exactly the number of parameters required for it to be a flexible functional form. The system of five factor demand equations is estimated using a

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¹⁶ See page 441 in Jara-Diaz and Winston (1981).

¹⁷ There is no theoretical guidance on restricting technical change to be neutral.

non-linear estimation method for seemingly unrelated regression equations. The typical factor demand equation takes the following form:

$$\begin{aligned}
 x_i = & (\beta_Q Q + \beta_P P) \left[\frac{\sum_{j=1}^s s_{ij} p_j}{\sum_{i=1}^s p_i} - \left(\frac{1}{2} \right) \frac{\sum_{i=1}^s \sum_{j=1}^s s_{ij} p_i p_j}{\left(\sum_{i=1}^s p_i \right)^2} \right] \\
 & + b_i + b_{ii} (\beta_Q Q + \beta_P P) + a_{iQ} Q t + a_{iP} P t + a_{it} t \\
 & + (d_{QQ}/2) Q^2 + (d_{PP}/2) P^2 + d_{PQ} P Q + a_{it} t^2 (\beta_Q Q + \beta_P P)
 \end{aligned} \tag{5.8}$$

where P is passenger-miles, Q is ton-miles, t is a time trend, p_i is the price of input i , and x_i is the demand for input i .

Initially, models A, AR, B and BR are estimated without concavity restrictions and summary results can be found in Tables A3.28 through A3.31 in Appendix 3. The estimated S matrix is not negative semidefinite in all four models. Like the translog, all models are non-concave. The predicted factor demands are positive in all models, suggesting that the cost function is increasing in prices or monotonic. The rate of technical change in model A (-0.013) is not very different from that in the translog model with general technical change (-0.017). Technical change is lower in models B and BR. In the case of the translog the opposite was true. The inclusion of rented capital results in more inputs having demand schedules with the appropriate slope. The improvement is more noticeable in model AR. In models A and AR, Canadian National displays diseconomies of scope in 1956. For the remaining 51 out

of 52 observations, there are economies from the joint provision of freight and passenger service. In models B and BR, there are economies of scope at every observation in the sample.

So far as returns to scale are concerned, the performance of this specification is disappointing. In model B, the local estimates of freight cost elasticity are negative in 1980 and 1981, for Canadian Pacific. This problem does not occur in the other three models, but the returns to scale estimates are too high. All the high estimates relate to CP for some years over the period 1967-1981. The estimates of returns to scale for CN are considerably superior, in model B for example, they range from 1.04 to 1.79.

The four models are estimated again after re-parameterization of the S matrix in order to impose global concavity on the cost function. Summary results for models AR, B and BR are reported in Tables A3.32 through A3.34 in Appendix 3. Detailed results for model A are presented below¹⁸ in Tables 5.1 through 5.6. The rate of technical change is much higher in models AR and B, compared to their non-concave counterparts. In models A and BR, the average technical change over the sample is unaffected by the imposition of concavity. The mean values of the factor demand elasticities¹⁹ are quite sensitive to the method of constructing the user cost of capital,

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¹⁸ Note that standard errors are not available for parameters that are derived from restrictions. Due to the non-linearity of the model it is not practical to derive bootstrap standard errors.

¹⁹ The own and cross price elasticities of factor demand and the elasticities of substitution are derived using the numeric differentiation routine in SHAZAM. The results reported here are sample averages of the elasticities.

and to the inclusion of rented capital. This also true for non-capital inputs such as labour. The sample average value of the elasticity of demand for labour is -0.022, -0.009, -0.029 and -0.022 in models A, AR, B and BR respectively. On average, the mean degree of economies of scope is somewhat higher in the concave models. This is more noticeable in models (B and BR) that use the conventional user cost of capital. Imposing concavity increases the sample average degree of scope economies from about 0.6 to 0.9²⁰. As before, there are diseconomies of scope in some models for the first or first two observation on CN.

The returns to scale estimates continue to be disappointing. In models B and BR, the freight cost elasticity is negative for four CP observations, this trade-off between positive cost elasticities and concavity is not encountered in models that do not utilize the conventional user cost of capital. Nonetheless, CP continues to show unacceptably high increasing returns.

The SGM cost function provides the first estimates of economies of scope for Canadian railways derived from a well-behaved cost function. The results obtained here suggest a very substantial and unrealistic cost saving from the joint provision of freight and passenger service. This cost function performs poorly on two counts; irrespective of curvature, returns to scale are very high for CP, and so are the estimates of economies of scope. It is not unreasonable to argue that the existence of

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²⁰ A 60 to 90 percent cost saving from the joint provision of the two outputs is highly unlikely.

economies of scope is a robust result; this result is not sensitive either to the various data alternatives or to the curvature of the cost function²¹. Roy and Cofsky (1985) derived estimates of economies of scope for Canadian railways from a non-concave generalized translog cost function. The cost function was estimated over two periods, 1956-78 and 1956-81, resulting in estimates of economies of scope of 0.86 and 4.36 respectively. Kim (1987) estimates a similar cost function for U.S. railroads and finds diseconomies of scope, the estimate being -0.41 for the year 1963²².

In the next chapter, attention is shifted back to the translog cost function with neutral technical change and an attempt is made to impose additional structure on the model.

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²¹ Note that all models indicate the presence of economies (rather than diseconomies) of scope. It is acknowledged that the extent of cost savings is too optimistic.

²² The cost function is estimated using data on 56 firms for the year 1963.

Table 5.1
Results from the Concave SGM Cost Function

<u>Parameter</u>	<u>Estimate</u>	<u>Asymptotic Standard Error</u>	<u>Asymptotic t-Ratio</u>
s_{LL}	-0.034	-	-
s_{LE}	0.076	-	-
s_{LS}	-0.027	-	-
s_{LF}	0.178	-	-
s_{LM}	-0.193	-	-
s_{EE}	-0.172	-	-
s_{ES}	0.115	-	-
s_{EF}	-0.403	-	-
s_{EM}	0.384	-	-
s_{SS}	-0.715	-	-
s_{SF}	0.245	-	-
s_{SM}	0.382	-	-
s_{FF}	-0.946	-	-
s_{FM}	0.926	-	-
s_{MM}	-1.498	-	-
β_1	0.273	-	-
β_2	0.727	0.160	4.547
b_L	-0.093	0.156	-0.598
b_E	0.451	0.193	2.336
b_S	0.239	0.221	1.084
b_F	-0.334	0.172	-1.940
b_M	-0.035	0.189	-0.186
b_{LL}	1.133	0.233	4.873
b_{EE}	0.697	0.267	2.605
b_{SS}	0.996	0.329	3.026
b_{FF}	1.036	0.248	4.170
b_{MM}	1.281	0.279	4.597
a_{LQ}	-0.073	0.012	-5.919
a_{EQ}	-0.071	0.013	-5.428
a_{SQ}	-0.070	0.014	-5.217
a_{FQ}	-0.057	0.012	-4.657
a_{MQ}	-0.050	0.013	-3.966
a_{LP}	-0.056	0.021	-2.723
a_{EP}	-0.032	0.020	-1.563
a_{SP}	-0.013	0.020	-0.659
a_{FP}	-0.074	0.021	-3.469
a_{MP}	-0.065	0.022	-2.917
d_{QQ}	0.500	0.183	2.736
d_{PP}	-0.225	0.066	-3.403
d_{PQ}	0.064	0.141	0.453
a_t	0.065	0.021	3.075
a_{tt}	0.002	0.001	3.500

Log of the Likelihood Function: 355.892

Table 5.2

Technical Change, Output Cost Elasticities,
Economies of Scale and Scope for CN:
Concave SGM Cost Function, Model A

<u>Year</u>	<u>Passenger Cost Elasticity</u>	<u>Freight Cost Elasticity</u>	<u>Returns to Scale</u>	<u>Economies of Scope</u>	<u>Technical Change</u>
1956	0.817	0.426	0.805	-0.058	-0.049
1957	0.717	0.440	0.865	-0.015	-0.047
1958	0.688	0.413	0.908	0.074	-0.037
1959	0.645	0.382	0.974	0.154	-0.033
1960	0.581	0.357	1.066	0.245	-0.027
1961	0.567	0.327	1.118	0.328	-0.022
1962	0.525	0.307	1.202	0.393	-0.020
1963	0.591	0.277	1.153	0.418	-0.025
1964	0.623	0.241	1.158	0.401	-0.032
1965	0.607	0.211	1.221	0.444	-0.031
1966	0.656	0.153	1.235	0.461	-0.034
1967	0.598	0.038	1.570	0.497	-0.033
1968	0.543	0.123	1.503	0.603	-0.024
1969	0.540	0.144	1.461	0.682	-0.020
1970	0.612	0.156	1.302	0.714	-0.020
1971	0.692	0.173	1.155	0.728	-0.019
1972	0.803	0.164	1.034	0.775	-0.021
1973	0.737	0.187	1.082	0.922	-0.017
1974	0.956	0.145	0.908	0.891	-0.020
1975	0.909	0.121	0.971	0.991	-0.015
1976	0.967	0.104	0.934	1.040	-0.014
1977	0.971	0.128	0.910	1.068	-0.010
1978	1.023	0.138	0.861	1.064	-0.007
1979	1.096	0.159	0.797	1.064	-0.005
1980	1.171	0.154	0.755	1.089	-0.003
1981	1.112	0.146	0.795	1.193	0.004
Mean	0.759	0.216	1.067	0.622	-0.022
Min	0.525	0.038	0.755	-0.058	-0.049
Max	1.171	0.440	1.570	1.193	0.004
Sample Mean	0.537	0.215	1.665	0.819	-0.013

Table 5.3

**Technical Change, Output Cost Elasticities,
Economies of Scale and Scope for CP:
Concave SGM Cost Function, Model A**

<u>Year</u>	<u>Passenger Cost Elasticity</u>	<u>Freight Cost Elasticity</u>	<u>Returns to Scale</u>	<u>Economies of Scope</u>	<u>Technical Change</u>
1956	0.635	0.439	0.931	0.038	-0.035
1957	0.536	0.440	1.024	0.099	-0.031
1958	0.516	0.414	1.076	0.177	-0.023
1959	0.441	0.383	1.213	0.278	-0.015
1960	0.401	0.351	1.330	0.369	-0.010
1961	0.384	0.315	1.432	0.464	-0.004
1962	0.310	0.296	1.650	0.548	0.001
1963	0.337	0.275	1.636	0.590	-0.002
1964	0.389	0.268	1.522	0.584	-0.010
1965	0.335	0.248	1.714	0.686	-0.004
1966	0.405	0.188	1.684	0.820	-0.006
1967	0.263	0.198	2.172	0.893	0.001
1968	0.171	0.168	2.947	1.024	0.007
1969	0.145	0.161	3.263	1.092	0.007
1970	0.257	0.145	2.486	1.150	-0.001
1971	0.293	0.136	2.330	1.227	-0.004
1972	0.337	0.125	2.165	1.308	-0.008
1973	0.326	0.103	2.328	1.439	-0.009
1974	0.323	0.127	2.222	1.444	-0.007
1975	0.248	0.100	2.875	1.563	-0.001
1976	0.122	0.085	4.837	1.692	0.004
1977	0.145	0.091	4.231	1.760	0.003
1978	0.228	0.102	3.022	1.773	0.002
1979	0.240	0.129	2.706	1.771	0.004
1980	0.161	0.144	3.273	1.790	0.010
1981	0.218	0.144	2.761	1.841	0.011
Mean	0.314	0.215	2.263	1.016	-0.005
Min	0.122	0.085	0.931	0.038	-0.035
Max	0.635	0.440	4.837	1.841	0.011
Sample Mean	0.537	0.215	1.665	0.819	-0.013

Table 5.4

**Own and Cross Price Elasticities of Factor Demand:
Concave SGM Cost Function, Model A**

	Lab.	Eqp.	Str.	Fuel	Mat.
Labour	-0.022	0.011	-0.011	0.068	-0.045
Equipment	0.018	-0.018	0.006	-0.076	0.069
Structures	-0.020	0.006	-0.061	0.032	0.044
Fuel	0.089	-0.083	0.037	-0.350	0.308
Materials	-0.035	0.040	0.031	0.169	-0.204

Table 5.5

**Morishima Elasticities of Substitution:
Concave SGM Cost Function, Model A**

	Lab.	Eqp.	Str.	Fuel	Mat.
Labour	-	0.029	0.050	0.418	0.159
Equipment	0.041	-	0.067	0.274	0.274
Structures	0.002	0.024	-	0.382	0.248
Fuel	0.111	-0.065	0.098	-	0.513
Materials	-0.013	0.058	0.092	0.519	-

Table 5.6

**Allen-Uzawa Elasticities of Substitution:
Concave SGM Cost Function, Model A**

	Lab.	Eqp.	Str.	Fuel	Mat.
Labour	-	0.080	-0.086	0.384	-0.157
Equipment	-	-	0.030	-0.508	0.249
Structures	-	-	-	0.202	0.164
Fuel	-	-	-	-	1.124
Materials	-	-	-	-	-

Note: All of the above are sample average values of elasticities.

CHAPTER 6

IMPOSING CONCAVITY ON THE TRANSLOG

Chapter 5 clearly indicates that changing the functional form of the cost function does not alleviate all problems. The globally concave SGM cost function yields unreasonable estimates of economies of scale and scope.

The results presented in Chapter 4 demonstrated the robust non-concavity of the translog specification. The various alternatives discussed in that chapter failed to improve the curvature of the cost function. In many instances there was a further deterioration in the theoretical consistency of the cost function. Some local estimates of output cost elasticities were found to have offending signs and in one case a minor monotonicity violation was also encountered. The translog cost function with neutral technical change was found to perform the best therefore this is the only specification used in further investigation.

In this chapter, an attempt is made to impose more structure on the translog cost function by constraining it to be concave in factor prices. Concavity constrained estimation requires the imposition of inequality constraints on the cost function. Imposition of inequality restrictions serves to truncate the parameter space rather than reduce its dimensionality. Two very different methods are used to accomplish this

goal. The first method was recently implemented by Diewert and Wales (1987)¹, and is similar to that used by Jorgenson and Fraumeni (1981). This method imposes global concavity on the translog using Classical estimation techniques². The second is a Bayesian method and has recently been applied to systems of demand equations and cost functions by Chalfant and White (1988), Chalfant, Gray and White (1991), and Chalfant and Wallace (1991).

6.1 Concavity Constrained Estimation: The Classical Approach

Consider the twice differentiable cost function $C(y, p, t)$, where $C(\cdot)$ is total cost, y is an M -dimensional vector of outputs, p is an N -dimensional vector of input prices and t is a scalar measure of technical change. The necessary and sufficient conditions for concavity of $C(\cdot)$ require the matrix³ $\nabla_{pp}C(y, p, t)$ to be NSD⁴, for $y > 0_M$; $p \gg 0_N$. For the translog specification, the matrix $\nabla_{pp}C(y, p, t)$ will be NSD if and only if the matrix $\Gamma - [s^* - ss']$ is NSD⁵. The matrix Γ is an N by N symmetric matrix of γ_{ij} 's, the coefficients of the squared and cross price terms in the translog cost

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¹ This is originally due to Wiley, Schmidt and Bramble.

² See Gallant and Golub (1984) for the inequality constrained estimation of the Fourier flexible form.

³ This is the N -dimensional square matrix of the second order partial derivatives of the cost function with respect to the elements of the input price vector p .

⁴ Either the Hessian of the cost function or the matrix of Allen-Uzawa elasticities of substitution can be used to check for concavity. If all the nonzero eigenvalues of either matrix are negative, then the matrix in question is NSD. An alternative procedure can be found on page 493, Berndt (1991). A detailed general treatment of concavity and convexity can be found in Morey (1986).

⁵ See Diewert and Wales (1987) page 48, Equation (15).

function in Equation (4.1) above. The vector s is an N -dimensional column vector of factor shares and s' is an N by N diagonal matrix with the vector s on the main diagonal.

Concavity in input prices does not require the matrix Γ to be NSD. If the share vector s is nonnegative, the matrix $-(s's')$ will be NSD. Nevertheless, if the matrix Γ is NSD, the cost function will be concave, hence a sufficient condition for concavity is that Γ be a NSD matrix. This will ensure that the translog is globally concave.

Thus in order to implement the sampling theoretic method of imposing concavity on the translog, all that is required is a re-parameterization⁶ of the cost function such that $\Gamma = -AA'$, where A is a lower triangular matrix⁷. Note that this results in a one for one exchange of parameters, but the estimation problem becomes non-linear. The cost function and $N-1$ factor share equations are estimated using a Maximum Likelihood estimation method for non-linear seemingly unrelated regression equations.

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⁶ Hazilla and Kopp (1986a and 1986b) re-parameterize the Cholesky decomposition in terms of the parameters of the restricted translog cost function. This turns the problem into a non-linear optimization problem subject to non-linear inequality constraints. The method used by Diewert and Wales (1987) solves the problem in an easier way.

⁷ This decomposition is similar to that used by Jorgenson and Fraumeni (1981), Figure 2.1, page 24.

Diewert and Wales (1987) implement this procedure, but they do not recommend its use because it can potentially destroy the flexibility of the cost function since the estimated values of some elements of Γ will be near zero. Further, this procedure fixes the problem too well. It makes the matrix of Allen-Uzawa elasticities of substitution "too negative semidefinite" thereby causing an upward bias in the degree of substitutability and the absolute values of the price elasticities of factor demand.

Their concerns regarding this approach can be demonstrated by considering the following:

$$\epsilon_1 = -1 + s_1 + \frac{\gamma_{11}}{s_1} \quad (6.1)$$

where ϵ_1 is the own price elasticity of demand for input 1, s_1 is the share of factor 1 in total cost and γ_{11} is the first element on the main diagonal of the Γ matrix. Suppose the cost function from which this measure is derived is not concave so that this elasticity is positive. This results from (γ_{11}/s_1) being positive and large enough to outweigh the negative⁸ term $(-1+s_1)$. γ_{11} need not be negative in order for the elasticity to be negative, it can be a small positive number. Re-parameterization of Γ as discussed above, forces γ_{11} to be negative⁹ thereby causing an upward bias in the

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⁸ This term is negative because s_1 , being a factor share is less than one.

⁹ This is because γ_{11} is replaced by $-(a_{11})^2$, where a_{11} is the (1,1)th element of the lower triangular matrix A .

absolute value of the elasticity¹⁰. The bias would remain if the estimated value of γ_{11} were effectively zero. The translog cost function in Equation (2.1) has exactly the number of parameters required for it to be a flexible functional form. If some elements of the Γ matrix take on zero values there is a loss in the number of parameters and the flexibility of the translog approximation is compromised, that is, it no longer provides an approximation to an arbitrary functional form.

Tables 6.1 through 6.8 above provide estimation results for model A¹¹. Summary results for models AR, B and BR can be found in Tables A3.35 through A3.37 in Appendix 3. All models are globally concave, therefore all input demand functions are have the appropriate slope at every observation. All models are also found to satisfy monotonicity, the predicted factor shares in total cost are positive at every observation. Models B and BR have negative passenger cost elasticities for 18 and 13 observations, respectively. For CN, negative passenger cost elasticities are encountered over the period 1958-62, the remainder relate to the initial observations on CP. In models A and AR, this problem does not exist, but the passenger cost elasticities have small positive values for the initial few observations on both firms. Tables 6.2 and 6.3 provide the 95% confidence interval for local estimates of passenger cost elasticities, this interval includes negative numbers.

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¹⁰ A negative γ_{11} also causes the Hessian and the matrix of elasticities of substitution to be "too negative semidefinite".

¹¹ In Table 6.1 asymptotic standard errors are not available for parameters derived using linear homogeneity restrictions.

Table 6.1

Results from the Globally Concave Translog Cost Function

<u>Parameter</u>	<u>Estimate</u>	<u>Bootstrap Standard Error</u>	<u>Asymptotic Standard Error</u>	<u>Asymptotic t-Ratio</u>
α_0	0.226	0.025	0.027	8.274
α_L	0.453	0.009	0.009	49.118
α_E	0.167	0.007	0.007	22.811
α_S	0.217	0.008	0.009	23.380
α_F	0.036	0.004	0.004	8.919
α_M	0.126	0.005	-	-
β_P	0.031	0.047	0.054	0.575
β_Q	0.784	0.108	0.124	6.328
α_T	-0.034	0.004	0.004	-7.635
γ_{LL}	-0.026	0.024	-	-
γ_{EE}	-0.001	0.005	-	-
γ_{SS}	0.000	0.008	-	-
γ_{FF}	0.000	0.006	-	-
γ_{MM}	-0.023	0.032	-	-
γ_{LE}	0.005	0.009	-	-
γ_{LS}	-0.004	0.008	-	-
γ_{LF}	0.000	0.010	-	-
γ_{LM}	0.024	0.021	-	-
γ_{ES}	0.001	0.004	-	-
γ_{EF}	0.000	0.004	-	-
γ_{EM}	-0.005	0.009	-	-
γ_{SF}	0.000	0.006	-	-
γ_{SM}	0.003	0.012	-	-
γ_{FM}	0.000	0.011	-	-
β_{PP}	-0.035	0.054	0.061	-0.563
β_{QQ}	-0.441	0.181	0.194	-2.271
β_{PQ}	0.186	0.080	0.106	1.752
δ_{QL}	0.143	0.028	0.024	5.904
δ_{QE}	-0.123	0.017	0.014	-8.636
δ_{QS}	-0.109	0.021	0.018	-6.132
δ_{QF}	0.038	0.011	0.009	4.386
δ_{QM}	0.052	0.016	-	-
δ_{PL}	0.050	0.008	0.008	6.193
δ_{PE}	-0.022	0.007	0.007	-3.216
δ_{PS}	-0.013	0.008	0.008	-1.632
δ_{PF}	-0.006	0.004	0.004	-1.555
δ_{PM}	-0.010	0.005	-	-

Log of the Likelihood Function: 626.424

Table 6.2

**Passenger Cost Elasticities for CN:
Estimates and Confidence Intervals,
Globally Concave Translog Model A**

<u>Year</u>	<u>Lower 95%</u>	<u>Lower 10%</u>	<u>Estimate</u>	<u>Upper 10%</u>	<u>Upper 95%</u>
1956	0.008	0.088	0.092	0.099	0.178
1957	-0.018	0.069	0.074	0.084	0.176
1958	-0.022	0.064	0.070	0.077	0.166
1959	-0.020	0.061	0.067	0.075	0.159
1960	-0.032	0.056	0.062	0.069	0.155
1961	-0.015	0.070	0.074	0.081	0.165
1962	-0.013	0.070	0.076	0.083	0.166
1963	0.026	0.098	0.102	0.108	0.180
1964	0.022	0.106	0.111	0.117	0.200
1965	0.016	0.109	0.115	0.121	0.209
1966	0.013	0.122	0.129	0.136	0.238
1967	-0.019	0.110	0.120	0.127	0.250
1968	0.014	0.123	0.131	0.137	0.243
1969	0.039	0.139	0.146	0.153	0.248
1970	0.061	0.162	0.168	0.175	0.268
1971	0.076	0.179	0.185	0.190	0.289
1972	0.104	0.205	0.209	0.218	0.316
1973	0.138	0.218	0.221	0.229	0.304
1974	0.137	0.237	0.240	0.250	0.346
1975	0.152	0.256	0.257	0.268	0.364
1976	0.161	0.275	0.275	0.289	0.397
1977	0.154	0.268	0.268	0.281	0.388
1978	0.158	0.275	0.278	0.289	0.401
1979	0.165	0.284	0.289	0.301	0.416
1980	0.165	0.287	0.293	0.304	0.421
1981	0.163	0.283	0.288	0.300	0.415
Mean	0.078	0.165	0.167	0.173	0.255
Sample Mean	0.092	0.149	0.151	0.156	0.210

Table 6.3

**Passenger Cost Elasticities for CP:
Estimates and Confidence Intervals,
Globally Concave Translog Model A**

<u>Year</u>	<u>Lower 95%</u>	<u>Lower 10%</u>	<u>Estimate</u>	<u>Upper 10%</u>	<u>Upper 95%</u>
1956	-0.060	0.024	0.031	0.037	0.121
1957	-0.092	0.007	0.013	0.020	0.115
1958	-0.087	0.013	0.018	0.027	0.120
1959	-0.099	0.001	0.007	0.015	0.110
1960	-0.097	0.005	0.011	0.018	0.115
1961	-0.082	0.021	0.027	0.034	0.131
1962	-0.089	0.013	0.020	0.026	0.125
1963	-0.051	0.040	0.046	0.051	0.136
1964	-0.009	0.061	0.067	0.071	0.141
1965	-0.002	0.068	0.073	0.078	0.148
1966	0.036	0.109	0.115	0.119	0.192
1967	0.023	0.095	0.102	0.106	0.182
1968	0.011	0.102	0.108	0.114	0.207
1969	0.026	0.114	0.120	0.127	0.216
1970	0.071	0.158	0.161	0.167	0.250
1971	0.091	0.178	0.181	0.188	0.271
1972	0.105	0.197	0.200	0.208	0.295
1973	0.098	0.216	0.220	0.230	0.340
1974	0.126	0.211	0.215	0.223	0.303
1975	0.129	0.235	0.238	0.248	0.351
1976	0.121	0.242	0.245	0.257	0.376
1977	0.127	0.247	0.249	0.260	0.373
1978	0.151	0.259	0.263	0.274	0.374
1979	0.176	0.261	0.262	0.271	0.356
1980	0.181	0.251	0.252	0.260	0.330
1981	0.183	0.250	0.252	0.260	0.329
Mean	0.056	0.130	0.134	0.140	0.213
Sample Mean	0.092	0.149	0.151	0.156	0.210

Table 6.4

**Freight Cost Elasticities for CN & CP:
Estimates and 95% Confidence Intervals,
Globally Concave Translog Model A**

	<u>Canadian National</u>			<u>Canadian Pacific</u>		
<u>Year</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>
1956	0.693	0.891	1.088	0.559	0.784	0.989
1957	0.783	1.004	1.242	0.613	0.861	1.099
1958	0.706	0.933	1.159	0.589	0.840	1.077
1959	0.671	0.893	1.109	0.547	0.803	1.050
1960	0.677	0.901	1.126	0.545	0.800	1.040
1961	0.661	0.874	1.091	0.516	0.765	1.000
1962	0.662	0.872	1.082	0.498	0.747	0.982
1963	0.656	0.851	1.048	0.496	0.723	0.930
1964	0.665	0.861	1.061	0.500	0.689	0.871
1965	0.673	0.868	1.069	0.479	0.674	0.858
1966	0.685	0.884	1.095	0.350	0.556	0.738
1967	0.708	0.931	1.164	0.416	0.622	0.804
1968	0.680	0.881	1.091	0.370	0.592	0.794
1969	0.652	0.854	1.055	0.365	0.583	0.781
1970	0.606	0.816	1.016	0.324	0.544	0.750
1971	0.526	0.744	0.951	0.239	0.476	0.684
1972	0.522	0.752	0.969	0.175	0.431	0.660
1973	0.512	0.722	0.935	0.048	0.348	0.614
1974	0.529	0.767	1.003	0.172	0.439	0.669
1975	0.690	0.908	1.153	0.280	0.551	0.816
1976	0.760	0.989	1.248	0.299	0.579	0.878
1977	0.629	0.874	1.126	0.221	0.513	0.795
1978	0.608	0.860	1.119	0.227	0.520	0.802
1979	0.548	0.821	1.088	0.281	0.555	0.809
1980	0.506	0.792	1.068	0.350	0.603	0.837
1981	0.507	0.788	1.062	0.274	0.533	0.772
Mean	0.667	0.859	1.058	0.411	0.620	0.809
Sample Mean	0.555	0.740	0.915	0.555	0.740	0.915

Table 6.5

Returns to Scale for CN & CP:
Estimates and 95% Confidence Intervals,
Globally Concave Translog Model A

<u>Year</u>	<u>Canadian National</u>			<u>Canadian Pacific</u>		
	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>
1956	0.864	1.017	1.221	1.014	1.227	1.586
1957	0.775	0.928	1.122	0.925	1.144	1.490
1958	0.831	0.997	1.225	0.929	1.165	1.548
1959	0.872	1.042	1.285	0.971	1.234	1.696
1960	0.858	1.039	1.295	0.957	1.234	1.713
1961	0.869	1.055	1.328	0.961	1.263	1.840
1962	0.877	1.055	1.313	0.989	1.304	1.924
1963	0.892	1.049	1.261	1.017	1.301	1.855
1964	0.869	1.029	1.238	1.097	1.323	1.701
1965	0.852	1.017	1.244	1.087	1.339	1.781
1966	0.811	0.987	1.246	1.136	1.491	2.225
1967	0.759	0.951	1.251	1.076	1.382	1.991
1968	0.808	0.989	1.257	1.057	1.430	2.230
1969	0.828	1.000	1.259	1.058	1.423	2.208
1970	0.837	1.017	1.301	1.049	1.418	2.237
1971	0.866	1.076	1.441	1.096	1.523	2.613
1972	0.840	1.040	1.392	1.120	1.584	2.976
1973	0.873	1.060	1.370	1.138	1.763	4.786
1974	0.805	0.992	1.323	1.108	1.531	2.707
1975	0.707	0.859	1.071	0.901	1.267	2.090
1976	0.646	0.791	0.972	0.839	1.214	2.121
1977	0.708	0.876	1.133	0.909	1.311	2.391
1978	0.709	0.878	1.161	0.912	1.278	2.165
1979	0.720	0.901	1.222	0.930	1.224	1.848
1980	0.733	0.922	1.277	0.925	1.169	1.638
1981	0.740	0.929	1.284	0.997	1.273	1.862
Mean	0.830	0.981	1.187	1.039	1.339	2.009
Sample Mean	0.979	1.160	1.551	0.979	1.160	1.551

Table 6.6

Allen-Uzawa Elasticities of Substitution:
Estimates and 95% Confidence Intervals,
Globally Concave Translog Model A

<u>Elasticity</u>	<u>Lower</u>	<u>Estimate</u>	<u>Upper</u>
σ_{LE}	0.848	1.079	1.456
σ_{LS}	0.879	0.958	1.256
σ_{LF}	0.275	1.000	1.994
σ_{LM}	0.970	1.319	2.033
σ_{ES}	0.612	1.028	1.366
σ_{EF}	-0.421	1.000	2.125
σ_{EM}	0.325	0.782	2.022
σ_{SF}	-0.397	1.000	2.286
σ_{SM}	0.672	1.115	2.259
σ_{FM}	-0.274	1.000	5.274

Table 6.7

Morishima Elasticities of Substitution:
Estimates and Confidence Intervals,
Globally Concave Translog Model A

<u>Elasticity</u>	<u>Lower 95%</u>	<u>Estimate</u>	<u>Upper 95%</u>
ω_{LE}	1.000	1.018	1.187
ω_{LS}	1.000	0.995	1.190
ω_{LF}	0.999	1.000	1.477
ω_{LM}	1.021	1.194	1.892
ω_{EL}	0.994	1.092	1.390
ω_{ES}	0.984	1.008	1.188
ω_{EF}	0.998	1.000	1.437
ω_{EM}	1.000	1.108	1.857
ω_{SL}	0.991	1.035	1.316
ω_{SE}	0.994	1.011	1.165
ω_{SF}	1.000	1.000	1.449
ω_{SM}	1.021	1.161	1.897
ω_{FL}	0.783	1.054	1.623
ω_{FE}	0.884	1.007	1.237
ω_{FS}	0.831	1.003	1.352
ω_{FM}	0.938	1.143	2.329
ω_{ML}	1.011	1.205	1.671
ω_{ME}	0.950	0.978	1.261
ω_{MS}	0.973	1.024	1.377
ω_{MF}	0.987	1.000	1.597

Table 6.8

Own and Cross Price Elasticities of Factor Demand:
Estimates and Confidence Intervals,
Globally Concave Translog Model A

<u>Elasticity</u>	<u>Lower 95%</u>	<u>Estimate</u>	<u>Upper 95%</u>
e_{LL}	-0.738	-0.582	-0.540
e_{LE}	0.113	0.144	0.195
e_{LS}	0.160	0.175	0.233
e_{LF}	0.014	0.051	0.099
e_{LM}	0.154	0.213	0.326
e_{EL}	0.401	0.510	0.686
e_{EE}	-1.019	-0.874	-0.866
e_{ES}	0.115	0.188	0.250
e_{EF}	-0.021	0.051	0.106
e_{EM}	0.054	0.126	0.332
e_{SL}	0.412	0.453	0.591
e_{SE}	0.083	0.137	0.184
e_{SS}	-0.983	-0.820	-0.817
e_{SF}	-0.021	0.051	0.114
e_{SM}	0.107	0.180	0.361
e_{FL}	0.127	0.472	0.944
e_{FE}	-0.056	0.133	0.283
e_{FS}	-0.072	0.183	0.423
e_{FF}	-1.401	-0.949	-0.950
e_{FM}	-0.045	0.161	0.839
e_{ML}	0.453	0.623	0.957
e_{ME}	0.043	0.104	0.273
e_{MS}	0.125	0.204	0.411
e_{MF}	-0.014	0.051	0.263
e_{MM}	-1.597	-0.982	-0.861

In an attempt to determine the severity of the problem, the width of the confidence interval was reduced to the point that excludes negative values at each observation. This results in the 10% confidence interval shown in Tables 6.2 and 6.3.

Though confidence bands are not derived for model AR, it is expected that the results are qualitatively similar to those of model A. In the initial years of the sample, passenger cost elasticities for CN and CP are of the same order of magnitude in both models A and AR. Since the precision of the estimates of first and second-order terms relating to passenger output is only slightly superior in model AR¹², it is not unreasonable to expect that the 95% confidence interval of passenger cost elasticities will admit negative values. The trade-off between concavity and positive marginal costs is not affected by the inclusion of rented capital.

The non-concave translog cost function with neutral technical change produced statistically significant positive output cost elasticities, the globally concave models produce the opposite results. These results show some similarity to those derived in Chapter 5, using the SGM. Models B and BR indicated a trade-off between positive marginal costs and concavity, however, the problem was not as severe as that encountered with the globally concave translog. Further, the parameter relating to the interaction term between passenger and freight output was not estimated with sufficient precision in the SGM cost function. It is not unreasonable to speculate that

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¹² For the first-order term, the precision is only marginally superior, the t-ratio being 0.7 in model AR. For the squared term, the t-ratio is -1.1.

had confidence intervals been constructed for local estimates of output cost elasticities using the SGM cost function, they would have admitted negative values of output cost elasticities for observations where the point estimates are small positive numbers.

The results of Chapter 5 and those presented here suggest that models that utilize the conventional user cost of capital are inferior since imposing appropriate curvature on the cost function results in negative estimates of output cost elasticities. Of course, this cannot be statistically rejected in models A and AR even though all local estimates of output cost elasticities are positive.

Sample average returns to scale are 1.16 in model A and 1.2 in model AR. The mean returns to scale for CP are approximately 1.4 in both models. Model A suggests that on average CN displays moderately decreasing returns whereas model AR predicts marginally increasing returns. Unfortunately, as indicated by the 95 % confidence bounds, these results lack precision.

Consider now the parameter estimates provided in Table 6.1. They help demonstrate the concerns expressed by Diewert and Wales with regard to imposing global concavity on the translog. Many of the elements of the Γ matrix take on zero values thereby destroying the flexibility of the translog cost function. If interest lies in the output effects, it may be argued that loss of flexibility on the input side is not of much concern. If, however, there is some interest in the elasticities of factor demand

and factor substitution, then the loss of flexibility has serious ramifications. Tables 6.6 through 6.8 indicate that the imposition of global concavity causes all pairs of inputs to be both Allen-Uzawa and Morishima substitutes. All the cross price elasticities of factor demand are positive. Table 6.1 shows that γ_{LP} , γ_{EF} and γ_{SF} are all equal to zero; this causes the Allen-Uzawa elasticities of substitution σ_{LP} , σ_{EF} and σ_{SF} to equal unity. Further, given that these are all equal to one, the cross price elasticities of demand e_{LP} , e_{EF} and e_{SF} will all be equal and in this case they will take on a value equal to the mean of the predicted share of fuel in total cost¹³ which is approximately 0.05. These biases in the own and cross price elasticities of factor demand will in turn be transferred to the Morishima elasticities of substitution. The own price elasticity of the demand for fuel provides another example of the biases. For the globally concave translog, all four models produce a value of approximately -0.9¹⁴. In the case of the SGM cost function, which is both globally concave and flexible, all four models produce a value of around -0.3. A similar order of magnitude difference can be found in the own price elasticity of demand for materials.

The classical approach to inequality constrained estimation of the translog yields unsatisfactory results. In general, the flexibility of the cost function is compromised, and the elasticities of substitution are biased. In this particular case, imposing global concavity also leads to negative marginal costs. At a more general

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¹³ Note that the elasticities are calculated using the mean values of predicted factor shares.

¹⁴ The point estimate of γ_{FF} is zero in model A and very close to zero in models AR, B and BR.

level, there is an absence of distribution theory for inequality constrained models and in some cases the constraint becomes binding, thus yielding uninteresting results that lie at one end of the distribution.

Bayesian methods provide a solution to some of these problems. In particular, they provide a way to impose regularity on the translog without compromising flexibility. Given the trade-off between positive marginal costs and global concavity, an attempt is made to impose minimum curvature on the cost function. Concavity is imposed using only the mean values of the predicted factor shares. This method is implemented using model A only, and is explained in the following section.

6.2 Concavity Constrained Estimation: The Bayesian Approach

Theoretical consistency requirements such as concavity of a cost function can be treated as prior information and be incorporated into empirical analysis using Bayesian methods¹⁵. The starting point of Bayesian analysis is the specification of prior information by means of a prior probability density function $g(\theta)$, where θ is a vector of parameters. Probability is defined in terms of the degree of belief, and can be based on quantitative or qualitative information. Since the probability is subjective, it can potentially vary from researcher to researcher¹⁶.

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¹⁵ This introductory discussion is drawn from Griffiths (1988) and Judge et al. (1988).

¹⁶ This is one of the major criticisms of the Bayesian approach.

The next step is to obtain sample information which can be represented by a joint density function $f(y|\theta)$, where y is a vector of sample observations. $f(y|\theta)$ is regarded as a conditional density function for y . The function $f(y|\theta)$ is algebraically identical to the likelihood function for θ and can be written as $l(\theta|y)$. Combining the prior information and the likelihood function yields a posterior distribution of θ which summarizes all the information about θ after y has been observed. This can be represented by $g(\theta|y)$. The procedure that permits the derivation of the posterior distribution from the prior distribution and sample information is known as *Bayes' Theorem*.

More formally, let the joint density of θ and y be represented by $h(\theta, y)$, then:

$$h(\theta, y) = f(y|\theta)g(\theta) = g(\theta|y)f(y) \quad (6.2)$$

Rearranging the above yields the following expression, which is Bayes' Theorem:

$$g(\theta|y) = \frac{f(y|\theta)g(\theta)}{f(y)} \quad (6.3)$$

$f(y|\theta)$ can be written as the likelihood function and $f(y)$ can be regarded as a constant with respect to θ . The above equation can be rewritten as:

$$g(\theta|y) \propto l(\theta|y)g(\theta) \quad (6.4)$$

which states that the posterior information is proportional to the product of the prior and sample information.

The derivation of the posterior distribution can be regarded as the concluding point of any Bayesian exercise. The posterior distribution can then be used as prior information relating to some future sample. In many instances however, a point estimate of the parameter of interest may be required. The researcher will incur a loss from choosing one specific point estimate θ^* for all possible true values of θ . The consequences of using a point estimate can be represented by a loss function. The following is a quadratic loss function, where κ is some constant:

$$L(\theta, \theta^*) = \kappa(\theta^* - \theta)^2 \quad (6.5)$$

Under quadratic loss, the mean of the posterior distribution minimizes expected posterior loss and is the point estimate of interest¹⁷.

Prior information regarding the parameter vector θ can take various forms. Judge et al. (1988) provide an example of an *informative* prior where the researcher has prior information on the mean and variance of θ . Further, if prior beliefs can be

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¹⁷ See Judge et al. (1988) pages 134-136 for a derivation.

adequately represented by a normal distribution, then the prior density will take the following form¹⁸, which is that of a normal distribution:

$$g(\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left[-\frac{1}{2\sigma_\theta^2}(\theta - \bar{\theta})^2\right] \quad (6.6)$$

Alternatively, there may be lack of knowledge, or prior information may be *vague* or *diffuse*. If the prior is completely non-informative, then the prior density takes the form of a uniform density and can be written as follows¹⁹:

$$g(\theta) \propto \tau \quad -\infty < \theta < +\infty \quad (6.7)$$

where τ is some constant. In this case, the posterior density is completely dominated by the likelihood function or the sample information.

In many instances, diffuse prior information may take the form of inequality restrictions. For example, θ could be restricted to lie between zero and one. In this case the prior density can be written as follows:

$$g(\theta) = \begin{cases} 1 & \text{if } 0 < \theta < 1 \\ 0 & \text{otherwise} \end{cases} \quad (6.8)$$

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¹⁸ θ is a scalar here.

¹⁹ This is an improper density, but the posterior density derived from this will be proper.

Since inequality restrictions serve to truncate the parameter space, the posterior distribution for the above example will be truncated at 0 and 1. Imposing concavity on the translog cost function also involves inequality restrictions, the particulars are discussed in the following section.

6.2.1 Inequality Constrained Estimation of the Translog

Let θ be a J dimensional vector of parameters of the translog cost function²⁰ and let D be a proper subset of $\mathbb{R}^J$, where $\mathbb{R}$ is the set of real numbers. The prior density for concavity restrictions can be represented as follows:

$$g(\theta) \propto \kappa \quad \forall \theta \in D \quad (6.9)$$

where κ is some constant. The prior density states that θ is contained in some region D with probability one. The region D is specified by economic theory. In this particular case it is the region of concavity for the translog cost function and all values of θ in this region are considered to be equally likely. Having specified the prior density, this can be combined with sample information via Bayes' Theorem to obtain the posterior density. Once the posterior density is obtained, under quadratic loss, all that is required is the calculation of the mean of the posterior distribution.

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²⁰ Note that reference is being made to the cost function restricted to be linear homogenous in factor prices.

For the problem at hand, that is, a translog cost function with two outputs, five inputs and neutral technical change, the parameter vector θ has 29 elements.

Calculating the mean of the posterior requires the evaluation of 29 integrals. Even if the posterior were a familiar density like the multivariate normal²¹, it would not be feasible to rely on analytical methods. Instead, integration by Monte Carlo methods is utilized to derive the mean of the posterior distribution²².

The use of Monte Carlo methods in Bayesian estimation is conceptually different from the use of Monte Carlo methods in classical estimation. Traditionally, the focus is on the properties of the finite sample distributions of estimators and test statistics. In this exercise, the integration is carried out over the data space by drawing repeated samples of artificial data. The exercise is also conducted in the context of a given model and a given point in the parameter space. Monte Carlo in the Bayesian context does not generate artificial economic data. The integration is carried out over the parameter space and the values of the parameters of interest are not known.

The estimation of the cost function and (N-1) factor share equations using IZEF yields the location and scale parameters, that is, an estimate of θ and the

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²¹ The posterior would be truncated due to the inequality restrictions.

²² This is based on the work of Klok and van Dijk (1978), van Dijk and Klok (1980), and Geweke (1986, 1988 and 1989). The particular application to flexible cost functions and systems of demand equations is due to Chalfant and White (1988), Chalfant, Gray and White (1991) and Chalfant and Wallace (1991). This method has also been applied to the estimation of consumer surplus by Adamowicz, Graham-Tomasi and Fletcher (1989).

associated contemporaneous covariance matrix Σ . Chalfant and White (1988) show that when Σ is known, the posterior distribution of θ is multivariate normal.

Unfortunately, this is not the case when Σ is not known. The posterior distribution of θ is a marginal distribution $g(\theta|y)$, which can be obtained by from the joint posterior density of θ and Σ by integrating over the posterior density of Σ . Zellner (1971) suggests the following diffuse prior for Σ as being suitable²³:

$$g(\Sigma) \propto |\Sigma|^{-\frac{G+1}{2}} \quad (6.10)$$

where G is the number of equations in the system. Given the above equation and that the prior on θ is also diffuse, the marginal density of θ is tractable and takes the following form:

$$g(\theta|y) \propto |E|^{-\frac{T}{2}} \quad (6.11)$$

where T is the number of observations and E is the following G by G residual cross-products matrix:

$$E = [\Omega'\Omega] \quad (6.12)$$

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²³ See Zellner (1971) pages 240-243 or Judge et al. (1985) pages 478-479 or Srivastava and Giles (1987) pages 316-318.

Ω is a T by G matrix of residuals evaluated using a value of θ where the posterior is defined. Due to the inequality restrictions, the posterior will be truncated. It is defined over the region D and will take the following form:

$$g^D(\theta|y) = I(\theta) \cdot g(\theta|y) \text{ where } I(\theta) = \begin{cases} 1 & \text{if } \theta \in D \\ 0 & \text{otherwise} \end{cases} \quad (6.13)$$

where $I(\theta)$ is an indicator function. The mean of the posterior is derived as follows:

$$E(\theta|y) = \int_{\theta \in D} \theta \tau g^D(\theta|y) d\theta \quad (6.14)$$

where τ is a constant of proportionality. The problem is that the posterior density is not a familiar density.

The use of *importance sampling* as suggested by van Dijk and Kloek (1980) provides a solution to the problem of finding the mean of an unfamiliar density. They suggest use of the following:

$$E(\theta|y) = \int_{\theta \in D} \theta \frac{\tau g^D(\theta|y)}{w^D(\theta|y)} w^D(\theta|y) d\theta \quad (6.15)$$

where $w^D(\theta|y)$ is the *importance function*, which is a density function defined over the parameter space or a subset thereof. It should be an approximation of the posterior density with the same support. $g^D(\theta|y)/w^D(\theta|y)$ are the *importance weights*, they

correct for the fact that the drawings are not from the posterior itself. If the drawings were from the posterior distribution, these weights would equal one for each draw. In the calculation of the mean of the posterior distribution of θ , those draws for which the importance weights are closer to one will receive a larger weight than those for which the importance weights are closer to zero. Van Dijk and Kloek (1980) suggest the multivariate-t family and the multivariate Normal as obvious candidates for the importance density. Following Chalfant and White (1988), Chalfant, Gray and White (1991) and Chalfant and Wallace (1991) the multivariate-t with 4 degrees of freedom is used as the importance function.

For the multivariate-t with λ degrees of freedom, the importance function takes the following form²⁴:

$$w(\theta|y) \propto [\lambda + \mathbf{a}'V^{-1}\mathbf{a}]^{-\frac{(\lambda+J)}{2}} \quad (6.16)$$

where:

$$\mathbf{a} = \frac{Pz}{h}; z \sim N(0, I); h \sim \chi^2_{\lambda} \quad (6.17)$$

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²⁴ This is discussed in Chalfant, Gray and White (1991) and in van Dijk and Kloek (1980) page 315.

In the above two equations, I is an identity matrix of order J . P can be obtained from the eigenvalues and eigenvectors of V or the Cholesky decomposition can be used to obtain a lower triangular matrix P such that $V=PP'$.

The initial step in implementation is the estimation of the cost function and $(N-1)$ factor share equations using the IZEF method. This yields the location and scale²⁵ parameters θ^* , and $V(\theta^*)$. Replications from the multivariate t can then be generated such that $\theta^A = \theta^* + a$. Following Geweke (1988), antithetic replications are also utilized. In a more recent paper Geweke (1989) suggests that some attention must be paid to the thickness of the tails of the importance function. In particular, they should be at least as thick as the tails of the posterior. Further, large variance in the importance weights is not desirable. Chalfant, Gray and White (1991) and Chalfant and Wallace (1991) suggest²⁶ that the thickness of the tails can be checked by plotting the replications of each element of θ against $g^D(\theta|y)/w^D(\theta|y)$. If the weights approach zero in the tails (i.e. for extreme values of θ_i) then one has covered the tails of the posterior adequately.

Chalfant and White (1987) calculate the probability p_D that the concavity restrictions hold as the proportion of trials that satisfy concavity. The standard error of the proportion is obtained using the formula in Equation (6.18). Geweke (1989)

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²⁵ Van Dijk and Klock (1980) suggest that V may be multiplied by some ϕ to enlarge the region of integration. $\phi=3.5$ is used below.

²⁶ Also see Barnett, Geweke and Yue (1991).

suggests the use of the numerical standard error (nse) as a measure of the numerical accuracy of each estimated expectation. The standard deviation (sd) of the posterior can also be obtained. The formulae are reproduced below, where N is the total number of replications:

$$s.e.(p_D) = \sqrt{p_D(1-p_D)/N} \quad (6.18)$$

$$nse(\theta) = \frac{\left[\sum_{k=1}^N (\theta_k - \bar{\theta})^2 \left[\frac{g^D(\theta_k|y)}{w^D(\theta_k|y)} \right]^2 \right]^{1/2}}{\left[\sum_{k=1}^N \frac{g^D(\theta_k|y)}{w^D(\theta_k|y)} \right]^2} \quad (6.19)$$

$$sd(\theta) = \frac{\left[\sum_{k=1}^N (\theta_k - \bar{\theta})^2 \left[\frac{g^D(\theta_k|y)}{w^D(\theta_k|y)} \right] \right]^{1/2}}{\sum_{k=1}^N \frac{g^D(\theta_k|y)}{w^D(\theta_k|y)}} \quad (6.20)$$

Results from the implementation of the Bayesian approach are provided below in Tables 6.9 through 6.13. Table 6.9 shows that none of the elements of the Γ matrix take on zero values, therefore flexibility is maintained. The model is monotonic and all the output cost elasticities are positive. Though the probability of local concavity is very low for the model and data at hand, there is some improvement in the local behaviour of the input demand functions.

TABLE 6.9

Results from Bayesian Inference: Locally Concave Translog

<u>Parameter</u>	<u>Estimate</u>	<u>Numerical Standard Error</u>	<u>Standard Deviation</u>
α_0	0.1721	0.0073	0.0169
α_L	0.4067	0.0001	0.0002
α_E	0.1866	0.0003	0.0006
α_S	0.2643	0.0001	0.0002
α_F	0.0387	0.0004	0.0008
α_M	0.1037	-	-
β_P	0.2704	0.0036	0.0084
β_Q	0.5531	0.0414	0.0965
α_T	-0.0209	0.0018	0.0043
$\gamma_{LL}/2$	0.0475	0.0033	0.0076
$\gamma_{EE}/2$	0.0401	0.0001	0.0003
$\gamma_{SS}/2$	0.0554	0.0009	0.0021
$\gamma_{FF}/2$	0.0208	0.0000	0.0000
γ_{MM}	-0.0024	-	-
γ_{LE}	-0.0189	0.0012	0.0027
γ_{LS}	-0.0731	0.0026	0.0062
γ_{LF}	-0.0541	0.0005	0.0011
γ_{LM}	0.0511	-	-
γ_{ES}	-0.0094	0.0011	0.0025
γ_{EF}	-0.0047	0.0002	0.0004
γ_{EM}	-0.0473	-	-
γ_{SF}	-0.0061	0.0003	0.0008
γ_{SM}	-0.0221	-	-
γ_{FM}	0.0232	-	-
$\beta_{PP}/2$	-0.0461	0.0040	0.0093
$\beta_{QQ}/2$	0.0459	0.0120	0.0280
β_{PQ}	-0.2161	0.0127	0.0297
δ_{LQ}	-0.0027	0.0077	0.0180
δ_{EQ}	-0.0389	0.0009	0.0022
δ_{SQ}	0.0460	0.0066	0.0154
δ_{FQ}	0.0376	0.0011	0.0025
δ_{MQ}	-0.0420	-	-
δ_{LP}	0.0468	0.0016	0.0037
δ_{EP}	-0.0126	0.0000	0.0000
δ_{SP}	0.0131	0.0006	0.0014
δ_{FP}	-0.0211	0.0008	0.0018
δ_{MP}	-0.0263	-	-
Probability of Concavity		0.003	
Standard Error of the Probability		0.0004	

Table 6.10

**Output Cost Elasticities and Returns to Scale for CN & CP:
Locally Concave Translog Model A**

<u>Year</u>	<u>Canadian National</u>			<u>Canadian Pacific</u>		
	<u>Passenger Cost Elasticity</u>	<u>Freight Cost Elasticity</u>	<u>Returns to Scale</u>	<u>Passenger Cost Elasticity</u>	<u>Freight Cost Elasticity</u>	<u>Returns to Scale</u>
1956	0.208	0.559	1.304	0.270	0.553	1.214
1957	0.230	0.523	1.328	0.289	0.527	1.225
1958	0.265	0.584	1.178	0.313	0.571	1.131
1959	0.265	0.585	1.176	0.326	0.577	1.107
1960	0.278	0.582	1.164	0.335	0.588	1.083
1961	0.288	0.617	1.104	0.353	0.637	1.010
1962	0.280	0.609	1.124	0.356	0.636	1.008
1963	0.248	0.603	1.174	0.332	0.644	1.024
1964	0.196	0.539	1.362	0.281	0.613	1.119
1965	0.183	0.524	1.414	0.297	0.646	1.060
1966	0.153	0.490	1.554	0.314	0.746	0.943
1967	0.141	0.441	1.718	0.323	0.715	0.964
1968	0.159	0.483	1.558	0.351	0.760	0.900
1969	0.161	0.505	1.503	0.344	0.763	0.903
1970	0.150	0.528	1.474	0.326	0.803	0.886
1971	0.135	0.549	1.461	0.323	0.846	0.856
1972	0.123	0.560	1.466	0.316	0.872	0.842
1973	0.153	0.627	1.283	0.340	0.954	0.773
1974	0.110	0.610	1.388	0.295	0.885	0.848
1975	0.117	0.616	1.365	0.320	0.929	0.801
1976	0.100	0.590	1.451	0.337	0.950	0.777
1977	0.092	0.602	1.441	0.322	0.954	0.784
1978	0.092	0.646	1.356	0.307	0.972	0.782
1979	0.085	0.667	1.328	0.278	0.930	0.828
1980	0.071	0.667	1.354	0.254	0.882	0.880
1981	0.072	0.681	1.328	0.240	0.906	0.873
Min	0.071	0.441	1.104	0.240	0.527	0.773
Max	0.288	0.681	1.718	0.356	0.972	1.225
Mean	0.167	0.576	1.360	0.313	0.764	0.947
Sample Mean	0.240	0.670	1.153	0.240	0.670	1.153

Table 6.11

**Allen-Uzawa Elasticities of Substitution:
Locally Concave Translog Model A**

	Lab.	Eqp.	Str.	Fuel	Mat.
Labour	-	0.702	0.140	-1.132	1.679
Equipment	-	-	0.619	0.363	-1.180
Structures	-	-	-	0.374	0.240
Fuel	-	-	-	-	3.674
Materials	-	-	-	-	-

Table 6.12

**Own and Cross Price Elasticities of Factor Demand:
Locally Concave Translog Model A**

	Lab.	Eqp.	Str.	Fuel	Mat.
Labour	-0.329	0.095	0.025	-0.061	0.269
Equipment	0.329	-0.272	0.112	0.020	-0.189
Structures	0.066	0.084	-0.208	0.020	0.038
Fuel	-0.530	0.049	0.068	-0.176	0.589
Materials	0.787	-0.160	0.043	0.199	-0.870

Table 6.13

**Morishima Elasticities of Substitution:
Locally Concave Translog Model A**

	Lab.	Eqp.	Str.	Fuel	Mat.
Labour	-	0.367	0.233	0.115	1.139
Equipment	0.658	-	0.320	0.196	0.680
Structures	0.394	0.355	-	0.196	0.908
Fuel	-0.202	0.321	0.276	-	1.459
Materials	1.116	0.112	0.252	0.375	-

In the non-concave translog, the input demand functions for labour, equipment, structures, fuel, and materials were negatively sloped at 52, 40, 43, 17, and 52 out of 52 observations, respectively. For the locally concave model, the respective results are 52, 44, 45, 32, and 45 out of 52.

In the non-concave case, mean returns to scale were 1.6 and 1.7 for CN and CP, respectively; the sample average being 1.66. Imposing minimum regularity on the cost function modifies these results considerably. CN continues to exhibit increasing returns at about 1.36, whereas CP displays moderately decreasing returns at 0.95. The sample average returns to scale are 1.15. The probability of sample average increasing returns to scale²⁷, conditional on concavity is 0.76, the standard error of this probability being 0.07. These results are the reverse of those obtained in the globally concave translog. CN exhibited moderately decreasing returns on average (0.98) and CP displayed increasing returns (1.33), the sample average being 1.16. The trend in the local estimates of scale economies may be of concern to some, but such results have been obtained elsewhere in the literature. De Borger (1991) provides estimates of returns to scale for Belgian railways for the period 1950-86. The earlier part of the sample displays increasing returns to scale, while the latter shows decreasing returns. The values are 1.65 for the period 1950-62, 1.35 for the period 1963-74 and 0.62 for the remaining years²⁸.

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²⁷ 31 out of 41 trials produce sample average increasing returns to scale.

²⁸ See Table 11.3 in Gillen, Oum and Tretheway (1985) for another example in the context of airlines.

The own and cross price elasticities of factor demand and the elasticities of substitution are quite different from the those obtained using the SGM cost function. The SGM specification suggests that labour and fuel, and equipment and materials are Allen-Uzawa substitutes, the locally concave translog predicts the opposite. There is also a substantial difference in the magnitude of the own price elasticities of demand. Use of the SGM resulted in values of -0.022, -0.018, -0.061, -0.350, and -0.204 for labour, equipment, structures, fuel, and materials, respectively. For four out of five inputs, these are considerably lower than the values reported in Table 6.12.

Model A, is retained as the preferred model. In models B and BR, there was a trade-off between concavity and positive marginal costs. The globally concave model AR was not that inferior to model A however, in the non-concave case model AR produced very high estimates of returns to scale. Since the imposition of local concavity is only a slight departure from the non-concave situation, similarly high returns to scale are encountered in model AR²⁹. The globally concave translog models either have negative output cost elasticities at some observations or negative values are admitted by the 95% confidence bounds. The estimates of returns to scale from the SGM specification were clearly too high to be of any use. The locally concave or Bayesian version of model A is the most acceptable, but the globally concave version is also retained in the examination of productivity in Canadian railways. Chapter 7 provides parametric estimates of efficiency for Canadian railways.

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²⁹ These results are not reported here since they are qualitatively similar to those obtained in the non-concave case.

This measure is derived from a cost function that allows the technology of the two firms to differ.

CHAPTER 7

MEASURING DIFFERENCES IN EFFICIENCY

All models discussed in Chapters 4 through 6 assume that the production technology of Canadian National and Canadian Pacific is identical. Efficiency differences between firms are ruled out by definition. In this chapter, an attempt is made to allow both the level of total cost, and the rate of cost diminution to vary across firms.

The discussion begins in Section 7.1 with the five input specification of the cost function. This specification was used in earlier chapters. The introduction of firm specific variables leads to a marked deterioration in results. The concavity violations encountered in earlier chapters persist, while a new problem of negative output cost elasticities arises. With a view to improving the curvature of the cost function, and to reducing the dimensionality of the problem, a three input specification is used in Section 7.2. All four data alternatives, that is, models A, AR, B and BR are retained throughout. Both Classical and Bayesian methods of imposing inequality constraints on the translog cost function are used in order to arrive at a theoretically consistent model¹. The last section is devoted to using the information from the cost function to decompose the conventional Divisia index of total factor productivity.

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¹ These were discussed in Chapter 6.

7.1 Five Input Specification

In order to allow for cost differences between firms, the following terms are added to the translog cost function in Equation (4.1):

$$(\alpha_D + \alpha_{TD}T)D \quad (7.1)$$

where T is the measure of technical change, or a time trend. D is a dummy variable, and is defined as:

$$D = \begin{cases} 1 & \text{for CN} \\ 0 & \text{for CP} \end{cases} \quad (7.2)$$

This specification allows for heterogeneous intercepts, and allows the rate of cost diminution to vary across firms. Two degrees of freedom are lost due to the introduction of additional parameters in the cost function.

Summary results from the estimation are reported in Tables A3.38 through A3.41 in Appendix 3. In all models (A, AR, B and BR) there is a considerable deterioration in theoretical consistency. The coefficient of the natural logarithm of freight ton-miles is negative in models A, B, and BR. In the latter two, the estimated value of the coefficient is around -0.3 and this is statistically significant. In model A, the value is a small negative number and not significantly different from zero. Model AR is the only model where this coefficient has the appropriate sign, but it is not

significantly different from zero. All models produce negative output cost elasticities. On this account, model A performs the best with 10 of 104 output cost elasticities being negative. In model AR, 20 of 104 elasticities are negative, and in models B and BR both passenger and freight cost elasticities are negative at every observation in the sample². The reader may recall from the discussion in Chapter 4 that the non-concave version of model A produced positive output cost elasticities at every observation and the 95% confidence interval did not admit negative values. The introduction of two additional parameters results in negative output cost elasticities at some or all observations. Those that are positive have very small positive values therefore it is not unreasonable to speculate that the 95% confidence intervals would include negative values. Given the poor local estimates of output cost elasticities, the unreasonable estimates of returns to scale are not surprising.

There is some improvement in the curvature of the cost function. The number of observations at which input demand functions have the appropriate slope is much higher than encountered earlier. Further, at the mean of the predicted shares, own price elasticities of factor demand for all five inputs have the appropriate sign in all models. Despite this improvement, the cost functions remain non-concave³.

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² This also results in negative returns to scale at all observations in models B and BR.

³ As noted in Table A3.38, model A is now concave at one observation.

The estimate of α_D has a positive sign, and is statistically significant in all four models. The positive sign would be expected if the property rights hypothesis on efficiency differentials between government-owned and privately owned firms were correct. α_{TD} has a positive sign in all four models indicating that CP has a faster rate of technical change than CN. This coefficient is not significantly different from zero in models A and AR. These models indicate that there is a difference between the cost levels of the two firms for the same output levels and identical factor prices, but there is no difference across firms in the rate of cost diminution. In model A, α_D has a value of 0.37 indicating that the ratio of CN to CP total costs is about 1.45 at identical output levels and factor prices⁴.

These results are quite sensitive to the choice of the user cost of capital. The magnitude of α_D is slightly higher, while that of α_{TD} is substantially higher in models that use the conventional user cost of capital (models B and BR). α_{TD} is positive, statistically significant, and higher in magnitude than α_T ; this implies that CN exhibits technical regress.

In summary, the specification that allows firms to differ is theoretically inconsistent on two counts; the cost function is not concave, and there are negative marginal costs at some or all observations in the sample. Restricting the technology to be identical has some benefits in that the cost function only violates the second-order

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⁴ The logarithm of total costs differs by 0.37, so the ratio of costs is the exponent of 0.37, or 1.45.

conditions for cost minimization; it also results in more reasonable estimates of economies of scale. While the hypothesis that the two firms have a different production technology cannot be rejected, the results on cost differences between firms do not have much credence because they are based on cost functions that do not conform to economic theory.

No attempt is made to impose curvature on this specification by using either Classical or Bayesian methods since it exhibits inferior performance on two fronts. In earlier chapters, such models were summarily rejected⁵. An alternative approach is pursued in the following section. The five input model is abandoned, and is replaced by a three input specification of the cost function.

7.2 Three Input Specification

Prior to commencing empirical analysis, researchers often choose the level of aggregation at which they wish to conduct their analysis. In many instances this choice is dictated by sample size. Following Braeutigam et al. (1984) the simple correlation between pairs of prices is used to make the aggregation decision. The prices of labour, fuel, and materials are highly correlated, therefore, these are

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⁵ For example, in Chapter 4, all four models in the three output specification (with grain ton-miles) were non-concave and had negative output cost elasticities.

aggregated to form a single Divisia price index⁶. Aggregation on this basis may be justified by the composite commodity theorem of Hicks which asserts that a group of commodities can be treated as a single good if their prices move in parallel⁷. This aggregation results in a three input specification of the cost function. The three inputs are equipment, structures and a composite input comprised of labour, materials, and fuel.

Aggregation of inputs in this manner implies imposition of the *a priori* restriction of separability on the five input cost function⁸. The cost function takes the following form:

$$C = g(w_E, w_S, h(w_L, w_M, w_F), Y_{PM}, Y_{TM}, t) \quad (7.3)$$

where w_E , w_S , w_L , w_M , and w_F represent prices of equipment, structures, labour, materials, and fuel, respectively. Y_{PM} and Y_{TM} represent passenger-miles and freight ton-miles, respectively, and t is a measure of technical change. If prices w_i and w_j are separable from w_k then:

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⁶ The correlation coefficients between the prices (or the log of prices) of labour and materials, labour and fuel, and materials and fuel lie between 0.95 and 0.99. Braeutigam et al. (1984) aggregate those prices that have a correlation coefficient in excess of 0.95.

⁷ See Section 5.1 in Deaton and Muellbauer (1980).

⁸ In Chapter 3, Chambers (1988) refers to this form of separability as weak separability in the extended partition or weak homothetic separability since $h(\cdot)$ is independent of outputs. Also see Berndt and Christensen (1973). For empirical tests of separability see Berndt and Christensen (1974), and Denny and Fuss (1977).

$$\epsilon_{ik} = \epsilon_{jk} \quad (7.4)$$

where ϵ_{ik} and ϵ_{jk} are cross-price elasticities of factor demand. Since the degree of Allen-Uzawa substitutability is defined by elasticities of factor demand, separability also restricts the degree and direction of Allen-Uzawa substitution relationships. If w_i and w_j are separable from w_k then inputs x_i and x_j can be Allen-Uzawa substitutes only if x_i and x_k are also Allen-Uzawa substitutes. Inputs within a group relate in a similar manner to those outside the group.

All four data alternatives (models A, AR, B and BR) are retained in the empirical implementation of this specification. The restriction of neutral technical change is also maintained, as are the two additional parameters in Equation (7.1). The only difference between the present specification and the one used in the previous section is that the number of inputs is reduced from five to three.

Summary results from the IZEF estimation are provided in Table A3.42, and Tables A3.44 through A3.46 in Appendix 3. The coefficient of the natural logarithm of freight output is negative in all four models; it is significantly different from zero in models A, B, and BR. Similar to the five input specification discussed in the previous section, the magnitude of this coefficient is larger in models B, and BR. All four models have negative output cost elasticities. In models A and AR about 20 of 104 output cost elasticities are negative, the number being at least 40 of 104 in models B and BR. In the five input specification, models B and BR produced negative output

cost elasticities at every observation; clearly aggregation has led to an improvement on this account. Nonetheless, models that use the conventional user cost of capital (models B and BR) result in more observations at which output cost elasticities are negative. In other words, irrespective of input aggregation, models that use the Boadway, Bruce and Mintz formulation of the user cost of capital exhibit superior performance to those that use the conventional cost of capital.

Table 7.1 provides additional results; it shows the significant favourable impact of input aggregation on curvature⁹. All four cost functions are concave at a majority of observations. Models that use the Boadway, Bruce and Mintz cost of capital are concave at more observations, as are those that include rented capital. Since all models have negative output cost elasticities, they violate theoretical consistency and produce unreasonable estimates of returns to scale¹⁰.

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⁹ Three input models with identical technology for both firms were also estimated. In models A and AR, concavity was satisfied at 19, and 41 observations, respectively, but there were negative output cost elasticities. Models B and BR were concave at 0, and 4 observations, respectively, and marginal costs were positive. The more significant the improvement in curvature, the more observations at which output cost elasticities are negative. All four non-concave, five input models discussed in Section 4.2 had positive marginal costs, therefore, aggregation is not costless. Introducing firm dummies in the three input model leads to a further improvement in curvature and a further deterioration with respect to negative output cost elasticities. The coefficient of the log of freight ton-miles also has a negative sign in some models; this was not encountered in models where firms were identical. These results are similar to the five input case, where introduction of firm dummies improved the slopes of the input demand functions but led to negative marginal costs.

¹⁰ Note that negative returns to scale are a result of negative output cost elasticities. Negative output cost elasticities imply negative marginal costs. Conclusions on relative efficiency are drawn only from theoretically consistent models.

In all cases the estimate of α_{TD} is not significantly different from zero therefore all models indicate that there is no significant difference in the rate of technical change of CN and CP¹¹. The magnitude, sign and statistical significance of α_D is insensitive to aggregation. The results suggest that at the same output levels and identical factor prices, the ratio of CN to CP total costs is at least around 1.45.

Using the Bayesian methods discussed in Chapter 6, theoretical consistency requirements are imposed on model A¹². Summary results are reported in Table A3.43 in Appendix 3. The probability of concavity using mean values of predicted factor shares is found to be 80%, that of monotonicity at every observation is 65%, while that of positive output cost elasticities at every observation is only 1.7%. The probability that all three theoretical consistency requirements are satisfied is 0.7%¹³. Table 7.1 shows that the IZEF estimation and the Bayesian model produce an identical number of observations (39) at which the cost function is concave. The magnitude of

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¹¹ Three input models with non-neutral technical change were also estimated. In cases where the technology is identical, and in those which include firm dummies (interacting with all time-interaction terms), the results are similar to those obtained with neutral technical change. That is, the curvature of the cost function improves (due to aggregation) but marginal costs are negative; introduction of firm dummies improves curvature further, but the problem of negative marginal costs worsens. As in the case of neutral technical change, if firm dummies are included, the coefficient of the log of freight ton-miles has a negative sign. The problem is more severe because it also has a larger magnitude. Input augmentation terms and the intercept are significantly different across firms. If global concavity is imposed on models with firm dummies, only the intercept is significantly different across firms. Flexibility is compromised because all elements of the Hessian take on zero values, and negative marginal costs persist. Using Bayesian techniques on model A does not resolve the problem of negative marginal costs; only 2/15000 observations satisfy all theoretical consistency requirements. This number is insufficient to obtain a robust estimate of the mean of the posterior distribution of the parameters.

¹² The importance function and the number of trials are identical to that used in Chapter 6.

¹³ 107 of 15000 trials satisfy theoretical consistency. This number is adequate to obtain a robust estimate of the first moment of the posterior distribution of the parameters.

α_D is marginally lower in the Bayesian version of model A and the rate of technical change (α_T) is faster. Mean returns to scale are very high but this is not unexpected since output cost elasticities in three input models are found to have lower magnitudes. This is another cost of aggregation, the locally concave version of the five input model resulted in more reasonable estimates of returns to scale.

Table 7.1					
Results from Locally Concave Translog Models with Three Inputs					
Model	Number (%) of observ. at which Concave	Mean RTS	α_D	α_T	α_{TD}
A	39 (75%)	-8.31	0.39 (26.27)	-0.007 (-5.52)	-0.0002 (-0.097)
A-Bayes	39 (75%)	5.72	0.36	-0.010	-0.0006
AR	42 (81%)	8.62	0.36 (21.36)	-0.007 (-5.39)	0.0007 (0.370)
B	28 (54%)	-1.75	0.44 (27.09)	-0.005 (-2.88)	0.0017 (0.834)
BR	36 (69%)	1.58	0.41 (21.64)	-0.006 (-2.97)	0.0021 (0.90)
Note: Asymptotic t-ratios are provided in parentheses. All models are concave at the mean. All models satisfy monotonicity at each observation. All models except Model A (Bayesian) have negative output cost elasticities at some observations; Models B and BR have more observations at which output cost elasticities are negative.					

The Bayesian version of Model A is the only theoretically consistent model available at this juncture¹⁴. It does not compromise the flexibility of the translog approximation and also allows for firms to differ¹⁵. It indicates that at identical output levels and factor prices, the ratio of CN to CP total costs is 1.44, the difference between firms in the rate of cost diminution is not statistically significant. This indicates that there is no convergence, and the privately owned firm remains more efficient than the government-owned firm. This is consistent with the property rights hypothesis.

The sensitivity of this conclusion to global concavity is also examined. Global concavity is imposed on all four models using the methods discussed in Chapter 6. Summary results are reported in Tables A3.47 through A3.50 in Appendix 3. In all cases, there is a very significant loss in flexibility, all elements of the Hessian of the cost function are zero. In other words, all the squared and cross-price terms are eliminated from the cost function; the factor share equations depend only on a constant and outputs. Compared to the five input model in Chapter 6, this is a very serious loss of flexibility. The imposition of global concavity results in the coefficient of the natural logarithm of freight output to be positive in all models. In all cases it is not statistically significantly different from zero. This resolves the problem of negative

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¹⁴ In Chapter 5 the SGM cost function was estimated with the intent of improving the curvature of the cost function. It is not used here because the translog is concave at a majority of observations.

¹⁵ Note that in model A there is no significant difference in the rate of technical change between firms.

output cost elasticities encountered earlier, but since the elasticities have small positive values, estimates of returns to scale continue to be high. These and other results are reported in Table 7.2.

Compared to the locally concave models, the magnitude of α_D is lower in all cases, and more so in models B and BR. α_{TD} has switched signs in all models except model A. In models A and AR α_{TD} is not significantly different from zero whereas in models B and BR, not only are the signs reversed, but α_{TD} is significant. Thus, models B and BR indicate that CN has a faster rate of technical change than CP so there is convergence, but catch-up occurs in the next century. Model B predicts that CN will catch up with CP in the year 2011 and model BR indicates that this will occur in 2018. These conclusions are drawn from cost functions that are theoretically consistent but not flexible.

Table 7.2				
Results from Globally Concave Translog Models with Three Inputs				
Model	Mean Returns to Scale	α_D	α_T	α_{TD}
A	3.50	0.34 (14.67)	-0.013 (-8.44)	-0.003 (-1.57)
AR	2.94	0.30 (12.81)	-0.014 (-9.03)	-0.002 (-1.00)
B	3.24	0.33 (11.54)	-0.014 (-5.11)	-0.006 (-2.27)
BR	3.64	0.32 (10.88)	-0.012 (-4.65)	-0.005 (-1.91)
Note: Asymptotic t-ratios are provided in parentheses. All models satisfy monotonicity at each observation. All models have positive output cost elasticities at each observation.				

7.3 Decomposition of TFP Growth

Tinbergen (1942)¹⁶ and Solow (1957) developed the foundations of what has now become the dominant approach to TFP measurement. The rate of growth of total factor productivity can be derived quite simply by subtracting the rate of growth of aggregate input from the rate of growth of aggregate output. This is a measure of the rate of shift in a production function¹⁷. The work of Shephard (1953 and 1970), Uzawa (1964), and McFadden (1978b) has demonstrated, that under the assumption of cost minimization, the cost function is an equally fundamental and equivalent characterization of the underlying production technology. The contributions of Diewert have shown the unique correspondence between the underlying production technology and the type of index number used to form measures of aggregate inputs and outputs.

In this section, information from the cost function is used to decompose the conventional Divisia Index of TFP. The decomposition is implemented only for model A since the Bayesian version of model A maintains both theoretical consistency and flexibility. The method is based entirely on Denny, Fuss and Waverman (1981).

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¹⁶ It seems that this very insightful work is not quoted often enough in the present day productivity literature (with the exception of Diewert). Tinbergen introduces the production function as the first element of the "theory of trends". Tinbergen utilizes a Cobb-Douglas production function since it is more general than a linear one. He then adds the term ϵ' to the production function. This term is meant to capture "technical development". On page 193, Tinbergen clarifies the meaning of technical development, he writes "we can ... obtain a higher volume of production with the same volume of labour and capital".

¹⁷ If there are constant returns to scale and firms engage in marginal cost pricing.

They provide an excellent treatment of this subject in the context of single and multiple output firms.

The conventional index number approach to TFP measurement defines the rate of growth of TFP as follows:

$$TFP = \sum_i \frac{p_i y_i}{R} \dot{y}_i - \sum_j \frac{w_j x_j}{C} \dot{x}_j \quad (7.5)$$

where y_i is output i , p_i is the price of output i , R is total revenue, C is total cost, x_j is the quantity of input j , and w_j is the corresponding factor price. Equation (7.5) can be written as:

$$TFP = \dot{Y}^P - \dot{F} \quad (7.6)$$

which defines the rate of growth of TFP as the difference between the rate of growth of output and the rate of growth of input.

Freeman et al. (1987) use (7.6) to study productivity differences between CN and CP¹⁸. Denny, Fuss and Waverman (1981) show that Equation (7.6) is the correct measure of the rate of cost diminution if there are constant returns to scale and marginal cost pricing. Equation (7.6) permits comparisons of productivity over time

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¹⁸ Freeman et al. (1987) use the multilateral version of Equation (7.6). The multilateral index was proposed by Caves, Christensen and Diewert (1982).

for any particular firm. In order to make comparisons across firms, the following translog bilateral productivity index¹⁹ due to Jorgenson and Nishimizu (1978) and Caves, Christensen and Diewert (1982) is used:

$$\ln\left(\frac{TFP_l}{TFP_k}\right) = \sum_{i=1}^N \left(\frac{r_{il} + r_{ik}}{2}\right) \ln\left(\frac{y_{il}}{y_{ik}}\right) - \sum_{j=1}^M \left(\frac{s_{jl} + s_{jk}}{2}\right) \ln\left(\frac{x_{jl}}{x_{jk}}\right) \quad (7.7)$$

where the subscripts k and l represent cross-sections, r_i is the share of output i in total revenue and s_j is the share of input j in total costs. This index is base firm invariant, but its use requires choice of a base year²⁰ to make comparisons across cross-sections²¹. This relative comparison can then be extended to other time periods using Equation (7.6). Quantities of inputs and outputs, and factor shares in total cost from model A are used to construct the rate of growth and relative TFP. Revenue shares reported in Freeman et al. (1987) are used, these are reproduced in Tables A3.52 and A3.53 in Appendix 3. The TFP results are reported in Table A3.51. They indicate that relative TFP (CN/CP) ranges from 0.85 to 0.98 over the period 1956-81. The average annual rate of growth of TFP over the same period is 3.5% for CN and 3.3% for CP. These results are similar, but not identical to those obtained by Freeman et al.

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¹⁹ Since there are only two firms, the multilateral index is not used here. If there were more than two firms, then the bilateral index is not appropriate since sets of bilateral comparisons are not transitive.

²⁰ CP 1956=1 is used here.

²¹ Equation (7.7) is used to make comparisons of TFP levels in studies that use the conventional index number approach. In this study information on relative costs is obtained directly from the cost function.

(1987); the difference arises due to differences in data manipulation²². Freeman et al. (1987) conclude that CP was more productive than CN for every year in the sample since relative TFP (CN/CP) ranges from 0.8 to 0.95. They find that the average annual rate of growth of TFP is 3.4% and 3.5%, for CN and CP, respectively. The relative TFP results are similar to those obtained here, but this study finds the rate of growth of TFP of CN to exceed that of CP.

Information from the cost function can be used to decompose the rate of growth of TFP as calculated using the conventional index number approach. The following is twice differentiable cost function where C represents total cost, y is an M -dimensional vector of outputs, w is an N -dimensional vector of factor prices and t is a scalar measure of technical change.

$$C = g(y, w, t) \quad (7.8)$$

Totally differentiating the cost function with respect to time yields the following expression:

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²² As mentioned in Chapter 3, all input prices and quantities are derived in a different manner. There is a large disparity across studies in the rate of growth of fuel input. The average annual rate of growth of fuel input is -1.2% for CN and -1.47% for CP. Freeman et al. find the respective rates to be -3.9% and -4%. For CN, the average annual rate of growth of aggregate input is -0.69% in this study and -0.3% in Freeman et al. This is partly due to the differences in the rate of growth of equipment input (-0.06% compared to 3.7% in Freeman et al.). Further, the share of labour input in total cost is about 52% in both studies, but the rate of growth of labour input is -2.5% in this study and -2.1% in Freeman et al. For CP, the average annual rate of growth of aggregate input is -0.91% in this study and -1.2% in Freeman et al. Structures input for CP declines over the sample in Freeman et al. (-0.02%) and increases in this study (0.5%). Labour input growth rates are similar (-3.3%) but the mean share of labour is higher by 3 percentage points in Freeman et al.

$$\frac{dC}{dt} = \sum_{i=1}^M \frac{\partial g}{\partial y_i} \frac{dy_i}{dt} + \sum_{j=1}^N \frac{\partial g}{\partial w_j} \frac{dw_j}{dt} + \frac{\partial g}{\partial t} \quad (7.9)$$

The use of Shephard's Lemma to derive the cost minimizing factor demands, x_j and some simple manipulation results in the following:

$$\frac{1}{C} \frac{dC}{dt} = \sum_{i=1}^M \frac{\partial g}{\partial y_i} \frac{y_i}{C} \frac{dy_i}{dt} \frac{1}{y_i} + \sum_{j=1}^N \frac{w_j x_j}{C} \frac{dw_j}{dt} \frac{1}{w_j} + \frac{\partial g}{\partial t} \frac{1}{C} \quad (7.10)$$

Equation (7.10) can be rewritten as:

$$\dot{C} = \sum_{i=1}^M \epsilon_i^y \dot{y}_i + \sum_{j=1}^N s_j \dot{w}_j + \dot{B} \quad (7.11)$$

where ϵ_i^y is the elasticity of total cost with respect to output i , and s_j is the share of input j in total cost. Total cost can alternatively be expressed as shown below:

$$C = \sum_{j=1}^N w_j x_j \quad (7.12)$$

Totally differentiation of the above equation with respect to time, the use of Shephard's Lemma and some simple manipulation permits the derivation of Equation (7.13) below:

$$\dot{C} = \sum_{j=1}^N s_j \dot{w}_j + \sum_{j=1}^N s_j \dot{x}_j \quad (7.13)$$

Combining Equations (7.11), and (7.13) provides an expression for the downward shift in the cost function over time due to technical change:

$$-\dot{B} = \sum_{i=1}^M e_i' \dot{y}_i - \sum_{j=1}^N s_j \dot{x}_j \quad (7.14)$$

Caves, Christensen and Swanson (1980) suggest that Equation (7.14) is a general measure of productivity in that it does not assume marginal cost pricing or constant returns to scale. The study of Caves and Christensen (1980b) on Canadian railways is an example of this approach. Caves and Christensen (1980b) use Equation (7.14) to calculate three alternative measures of TFP. The primary productivity estimates are based on two outputs: freight ton-miles and passenger-miles, the first alternative productivity estimates include two additional outputs: average length of haul and average trip-length. The second alternative productivity estimates utilize the two output specification of the primary productivity estimates but the two outputs are constructed from very disaggregated data which classifies various types of freight by many mileage bands²³.

All three measures of TFP indicate that CN exhibits a higher rate of growth of TFP than CP. The primary productivity estimates show that the average annual rate of

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²³ In principle, this approach of starting at a very disaggregated level is preferable, but as mentioned by Caves and Christensen, the disaggregated data is based on the Waybill analysis which is not reliable.

growth of TFP over the period 1956-75 was 3.1% for CN and 2.5% for CP.

Comparisons of relative TFP indicate that CN was less productive than CP prior to 1967. For the remainder of the sample, relative TFP (CN/CP) lies between 0.99 and 1.12. Thus catch-up occurred after the passage of the *National Transportation Act*²⁴.

Caves and Christensen (1980b) utilize output cost elasticities derived from a U.S. cost function to construct their index of TFP. In the region of output levels of Canadian railways, the cost function for U.S. railways does not reject the hypothesis of constant returns to scale. The sum of the output cost elasticities is therefore not significantly different from unity. The ton-miles cost elasticity lies between 0.76 and 0.82, and the passenger-miles elasticity lies between 0.14 and 0.24. Caves and Christensen (1980b) do not utilize all the information from the cost function since they use observed rather than predicted factor shares to construct their measure of aggregate input²⁵. Their TFP index includes pure shifts in the cost function and a residual.

The results obtained in this thesis are based on Canadian data, and indicate that railways exhibit increasing returns to scale; increasing returns to scale are particularly strong in three input models. This is contrary to the assertion of Caves and Christensen. Thus, studies by Freeman et al. (1987) and Caves and Christensen

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²⁴ Caves, Christensen, Swanson and Tretheway (1982) extend the sample size of the first alternative productivity estimates of Caves and Christensen (1980b). This does not alter the basic conclusions of Caves and Christensen.

²⁵ In this context, see page 210 in Denny, Fuss and Waverman (1981).

(1980b) are based on restrictive assumptions. This thesis derives a parametric measure of pure technical efficiency.

Following Denny, Fuss and Waverman (1981) an index of aggregate output growth using cost elasticity shares is defined as:

$$\dot{Y}^C = \sum_i \frac{e_i}{\sum_i e_i} \dot{y}_i = \left[\sum_i e_i \right]^{-1} \left[\sum_i e_i \dot{y}_i \right] \quad (7.15)$$

Substituting Equation (7.15) in Equation (7.14), and using Equation (7.6) yields:

$$TFP = \left[-\dot{B}_R \right] + \left[1 - \sum_i e_i \right] \dot{Y}^C + \left[\dot{Y}^P - \dot{Y}^C \right] \quad (7.16)$$

The first term on the right hand side of the above equation is a residual measure of technical change, the second term captures movements along the cost function or economies of scale, and the last term capture the impact of deviations from marginal cost pricing. The left hand side of the above equation is the rate of growth of TFP using revenue shares to aggregate output as was done by Freeman et al. (1987). Clearly this measure contains more than just the downward shift in the cost function. The residual measure of technical efficiency can be decomposed further into a parametric measure of technical efficiency as is derived in this thesis, and a residual from regression errors, as follows:

$$[-\dot{B}_R] = [-\dot{B}_P] + R \quad (7.17)$$

Table 7.3 provides a comparison of the results obtained here and in previous studies.

As discussed above, the measure of efficiency differs across studies²⁶.

Table 7.3		
Comparison of Average Annual Percentage Rate of Technical Change with Previous Studies		
Study	CN	CP
Caves and Christensen (1980b) 1956-75 (Primary)	3.1	2.5
Caves and Christensen (1980b) 1956-75 (1st. Alternative)	2.9	2.4
Caves and Christensen (1980b) 1956-75 (2nd. Alternative)	3.6	2.6
Caves, Christensen, Swanson and Tretheway (1982) 1956-79	3.0	2.2
Freeman et al. (1987) 1956-81	3.1	3.5
This Study (Using Revenue Shares) 1956-81	3.5	3.3
This Study (Parametric: Bayesian Model A) 1956-81	1.1	1.0
This Study (Parametric: Globally Concave Model A) 1956-81	1.7	1.3

Using Equations (7.16) and (7.17) the rate of growth of TFP is decomposed in Table 7.4²⁷. Table 7.5 provides a comparison between the parametric and residual

²⁶ The first four studies in Table 7.3 use Equation (7.14) on page 171 and the next two use Equation (7.6) on page 167. The parametric estimates are the negative of the partial derivative of the natural logarithm of cost with respect to the time trend, or, $-(\alpha_T + \alpha_{TD})$ for CN and $-\alpha_T$ for CP (see Equation (7.1) on page 155).

²⁷ The last four columns of Table 7.4 sum to 100%.

measures of technical change²⁸. Table 7.4 provides a complete accounting for TFP growth²⁹. A comparison of the contribution of non-marginal cost pricing and non-constant returns indicates that the former makes a larger contribution to TFP growth of CP whereas the latter is a more important explanatory factor of the TFP growth of CN. This result does not change if the cost function is restricted to be globally concave. The significant contribution of non-marginal cost pricing to the TFP growth of CP over the period 1956-67 can be explained with the help of Tables A3.52 and A3.53 in Appendix 3. These tables present output cost elasticities and revenue shares for freight and passenger outputs. Table A3.53 shows that the revenue share of passenger service for CP over the period 1956-67 lies between 4% and 12%. If output cost elasticity shares (from either the locally, or globally concave model) are used to form a measure of aggregate output, then passenger service is given a weight in excess of 50% for most years over the period 1956-67. Over the same period, the average annual rate of growth of passenger-miles for CP was -6.73%³⁰. The measure of aggregate output that uses output cost elasticity shares as weights gives much more

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²⁸ Though the parametric estimates of technical change obtained in this study (as reported in Tables 7.3 and 7.5) indicate that CN and CP have a different rate of technical change, these differences are not statistically significant. If the globally concave version of model A is re-estimated allowing only the intercept to differ across firms, the rate of technical change for CN and CP is 1.39%.

²⁹ The residual is a regression error; though it may form a large component of TFP in certain years, it is small when viewed from the perspective of the cost function. For further discussion see page 208 in Denny, Fuss and Waverman (1981).

³⁰ Over the same period, freight output increased at an annual rate of 1.37%. Over the period 1956-67, both passenger and freight outputs for CN increased at an annual rate of 5.11% and 1.58% respectively.

importance to this decline than one that uses revenue shares³¹. Use of the latter understates the decline in output and overstates productivity.

For the period 1956-81 the imposition of global concavity increases the contribution of non-marginal cost pricing to the TFP growth of CN, while that of non-constant returns decreases. The opposite holds for CP. Shifts in the cost function make a higher percentage contribution to TFP of both firms prior to 1967. Imposing global concavity on the cost function does not alter this result. Imposing global concavity does however increase the importance of technical change as an explanatory factor. On average, technical change accounts for about 33% of the rate of growth of TFP.

It is important to emphasize that the approach taken in this thesis yields a parametric measure of technical change or efficiency directly from the cost function. Since Caves and Christensen (1980b) did not estimate a cost function for Canadian railways, they could not obtain such a measure. Under certain restrictive assumptions, their measure of TFP approximates the one obtained here. Table 7.4 demonstrates that the conventional revenue share weighted index of TFP is a gross measure that includes

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³¹ The difference between the sample mean values of the output cost elasticities of CN and CP was decomposed in order to determine whether the differences were driven by factor prices or outputs. The results are mixed. For example, in the locally concave model, differences in (the log of) passenger-miles, freight ton-miles, price of equipment, and price of structures explain 77%, 32%, -17%, and 7% of the differences in passenger cost elasticities. In the globally concave model, the respective contributions are 124%, 162%, -101% and -82%. In either case, the price of labour+fuel+materials plays a minor role (1% and -3%).

effects other than just efficiency. Caves and Christensen (1980b) have assumed away the impact of deviations from marginal cost pricing and that of non-constant returns to scale since they use output cost elasticities as weights in their measure of aggregate output³². As mentioned earlier, their measure of TFP growth is comprised of a measure of efficiency (shifts in the cost function) and a residual; it is in this sense that their measure of TFP growth can be interpreted as a measure of efficiency.

The measure of TFP growth in Table 7.4 contains both behavioural (non-marginal cost pricing) and structural (variable returns to scale and shifts in the cost function) components. One may wish to assume away the behavioral component by defining a new measure of TFP growth that is based on an output index that uses output cost elasticity *shares* as weights. This measure can be decomposed into movements along the cost function (scale effects), shifts in the cost function (efficiency) and a residual. The construction of this measure highlights the significant role played by increasing returns to scale, particularly for Canadian Pacific. Consider the globally concave version of Model A³³. The cost elasticity *share* weighted index indicates that the average annual rate of growth of TFP was 3.2% for CN, and 0.8% for CP over the period 1956-81³⁴. For CN, the contribution of non-constant returns to

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³² The output cost elasticities sum to unity due to the assumption of constant returns to scale.

³³ The locally concave model yields similar results.

³⁴ The relative TFP comparison from this measure indicates the TFP of CN exceeds that of CP over the period 1965-81.

scale is 50.7% while that of shifts in the cost function is 51.7%³⁵. For CP, the contribution of non-constant returns to scale is -52.5% while that of shifts in the cost function is 158.6%. Compared to the revenue share weighted TFP index, the cost elasticity share weighted index over emphasizes the importance of passenger output, particularly for CP³⁶. As mentioned earlier, over the period 1956-67, the rate of growth of passenger output was -6.73% for CP and 5.11% for CN. Thus, though both railways exhibit significant increasing returns to scale, CN is moving down the scale curve and therefore movements along the cost curve make a positive contribution to TFP growth whereas CP is moving in the opposite direction. In Chapter 6, where both firms had an identical technology by definition, the locally concave version of Model A indicated that conditional on concavity, the probability of increasing returns to scale is 76%. Table 7.2 showed that aggregating to three inputs and allowing firms to differ also yields high increasing returns to scale. Thus increasing returns to scale are an inherent characteristic of the production technology of Canadian railways; this casts doubt on the validity of the assumptions made by Caves and Christensen³⁷.

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³⁵ The residual being approximately -2.5%.

³⁶ For CN, the cost elasticity share of passenger output lies between 0.1 and 0.3 while that for CP ranges from 0.7 in 1956 to 0.1 in 1981.

³⁷ It is important to reiterate that the weights used in the construction of the conventional index number measure of TFP have little bearing on the efficiency results obtained here since estimation of the cost function yields a direct measure of efficiency. Certainly, if one concentrates purely on the index number measure of TFP, the rate of growth of TFP and relative TFP will differ depending on whether revenue shares, cost elasticity shares or cost elasticities are used to form a measure of aggregate output.

In summary, the results obtained here indicate that at identical factor prices and for the same output levels, total costs of CN are about 45% higher than those of CP. The rate of cost diminution does not differ across firms. If the conventional user cost of capital is used, and the cost function is restricted to be globally concave, CN has a higher rate of technical change than CP but the advantage is insufficient to ensure convergence in the sample period. In fact, convergence occurs in the next century. These results are at odds with those of Caves and Christensen (1980b), and support the predictions of the property rights theory that the private firm is more efficient than the government-owned firm.

Results from the conventional index number approach to TFP measurement indicate that CN is less productive than CP over the period 1956-81. CN exhibits a higher rate of growth of TFP than CP, but is unable to catch up with CP in the sample period. Freeman et al. (1987) also find that CN is less productive than CP over the same time period. Contrary to the results obtained here, they find that CP has a higher rate of growth of TFP than CN.

Table 7.4					
Decomposition of Rate of Growth of Total Factor Productivity: Model A					
Period	Percentage rate of growth of TFP	Percentage contribution to rate of growth of TFP of efficiency gains due to			
		Non- marginal cost pricing $\dot{Y}^P - \dot{Y}^C$	Non- constant returns to scale $(1 - \sum \alpha_i^C) \dot{Y}^C$	Shifts in the cost function $-\dot{B}_P$	Residual R
Canadian National: Globally Concave					
1956-67	2.90	-18.03	63.19	57.24	-2.40
1968-81	3.97	23.41	36.96	41.78	-2.16
1956-81	3.50	8.31	46.52	47.42	-2.24
Canadian Pacific: Globally Concave					
1956-67	2.60	132.33	-84.89	50.94	1.61
1968-81	3.89	44.62	24.45	34.09	-3.16
1956-81	3.32	74.85	-13.23	39.90	-1.52
Canadian National: Locally Concave (Bayesian)					
1956-67	2.90	1.97	52.67	37.58	7.78
1968-81	3.97	5.88	63.01	27.43	3.67
1956-81	3.50	4.46	59.24	31.13	5.17
Canadian Pacific: Locally Concave (Bayesian)					
1956-67	2.60	153.99	-111.48	39.65	17.84
1968-81	3.89	53.17	25.89	26.54	-5.60
1956-81	3.32	87.92	-21.46	31.06	2.48

Table 7.5				
Two Measures of Percentage Rate of Technical Change: Model A				
Period	Canadian National		Canadian Pacific	
	Residual	Parametric	Residual	Parametric
	$-\dot{B}_R$	$-\dot{B}_P$	$-\dot{B}_R$	$-\dot{B}_P$
Globally Concave				
1956-67	1.59	1.66	1.37	1.33
1968-81	1.57	1.66	1.20	1.33
1956-81	1.58	1.66	1.28	1.33
Locally Concave (Bayesian)				
1956-67	1.32	1.09	1.50	1.03
1968-81	1.24	1.09	0.81	1.03
1956-81	1.27	1.09	1.11	1.03

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CHAPTER 8

CONCLUSIONS

The main objective of this thesis is to examine efficiency differentials between Canadian National and Canadian Pacific. In order to achieve this objective, a parametric measure of technical change is derived from a theoretically consistent flexible cost function. Estimation of the cost function also yields estimates of output cost elasticities. These are used to decompose TFP estimates obtained from the conventional Divisia index number approach. The results obtained here are also compared with those obtained in previous research. Since the inappropriate curvature of the cost function is a major problem, Classical and Bayesian methods are used to resolve this problem. Finally, the sensitivity of the results to the data and the extent of curvature of the cost function is also examined.

In models that restrict the production technology of the two firms to be identical, the non-concave long run translog cost function with neutral technical change indicates that on average, Canadian railways exhibit statistically significant increasing returns to scale (1.66). The cost function is quite sensitive to changes in both specification and data. Use of models with general technical change results in a deterioration of the curvature of the cost function. The short run cost function also produces theoretically inconsistent results. Other avenues were also explored. Models

with three outputs, and those with a network variable also result in non-concave cost functions.

Use of the Symmetric Generalized McFadden cost function solves the curvature problem, but the resulting local estimates of economies to scale, and scope are too high. The mean returns to scale are similar to those obtained using the translog (1.66). However, estimates of returns to scale for CP are in excess of 2, over the period 1967-81. The degree of economies of scope is 0.8, indicating a considerable cost saving from the joint provision of freight and passenger transportation service.

Given the sensitivity of the results to choice of functional form, the translog specification was retained, and an attempt was made to impose global concavity on the cost function. As predicted by Diewert and Wales, this compromised the flexibility of the translog approximation and produced upwardly biased estimates of the Allen-Uzawa elasticities of substitution. The globally concave translog produced marginally increasing returns to scale (1.16) on average. This estimate was imprecise and the 95% confidence bounds admitted values ranging from 0.97 to 1.55. Further, the 95% confidence intervals for local estimates of passenger cost elasticity admitted negative values. These results suggest a trade-off between concavity and positive marginal costs. This was also evident from the SGM cost function.

The trade-off was more apparent in models that used the conventional user cost of capital. For some years in the sample, local estimates of output cost elasticities are negative. The fact that the less arbitrary formulation of the user cost of capital provides superior results is a source of some encouragement.

Given the loss of flexibility of the translog due to the imposition of global concavity, Bayesian methods were utilized to impose minimal curvature on the cost function. The probability of concavity was found to be 0.003, and sample average returns to scale were 1.15. Conditional on concavity, the probability of sample average increasing returns to scale was 0.76. Imposing global concavity on the translog also resulted in all inputs being Allen-Uzawa substitutes. The locally concave model suggests complementarity between fuel and labour, and materials and equipment.

In models that allow for efficiency differences between firms, the five input specification was found to perform poorly on two counts; the cost function had inappropriate curvature and output cost elasticities were negative for some or all observations. Three input models were also estimated and aggregation led to a considerable improvement in the curvature of the cost function. Though there is improvement on this front most of the three input models yield negative output cost elasticities. Allowing firms to differ makes it worse.

Bayesian and Classical methods were used to ensure theoretical consistency of the three input specification. The imposition of global concavity by means of Classical methods results in a serious loss of flexibility of the translog cost function; more so than in the five input case. The locally concave (Bayesian) version of the three input specification yields very high estimates of returns to scale; mean returns to scale being 5.7. Mean returns to scale decline to around 3.5 if the cost function is restricted to be globally concave.

Restricting firms to have an identical production technology, as was done in Chapters 4 through 6, has some advantages. The estimates of returns to scale fall within the bounds of those for U.S. railways. This specification is theoretically inconsistent in only one respect, violating second-order conditions for cost minimization. Input aggregation, while restricting the technology to be identical improves curvature of the cost function, but results in negative marginal costs. The more significant the improvement in curvature, the more observations at which output cost elasticities are negative. Allowing differences between firms in the three input model leads to a further improvement in curvature and a further deterioration with respect to negative marginal costs.

The conventional TFP index indicates that the rate of growth of TFP was 3.5% for CN and 3.3% for CP over the period 1956-81. A comparison of TFP levels indicates that CN is less productive than CP for the entire sample. Freeman et al.

(1987) found that CP had an advantage over CN with respect to *both* the level and the rate of growth of TFP.

Parametric measures of technical change indicate that at the same output levels, and at identical factor prices the total costs of CN are about 45% higher than those of CP. This is a very robust result. It is insensitive to the extent of curvature of the cost function, to the level of input aggregation, to the exclusion of rented capital and to the various measures of the user cost of capital. There is no statistically significant difference in the rate of cost diminution across firms; the annual percentage rate varies from 1.03% to 1.66%. If the conventional user cost of capital is used and the cost function is restricted to be globally concave (which also turns out to be inflexible), the rate of technical change for CN lies between 1.7% and 2%, while that for CP ranges from 1.2% to 1.4%. Despite this advantage, CN is unable to catch up with CP prior to the year 2011. The decomposition of the conventional index of TFP growth indicates that downward shifts in the cost function account for about 33% of TFP growth.

The results obtained in this thesis support the predictions of the property rights theory on the relative efficiency of privately owned versus government-owned firms. They contradict the results obtained by Caves and Christensen more than a decade ago. The primary productivity estimates of Caves and Christensen (1980b) indicate that over the period 1956-75, the average annual rate of growth of TFP was higher for

CN (3.1%) than for CP (2.5%). Relative TFP (CN/CP) was 0.87 in 1956, but TFP of CN exceeded that of CP by 1967. This was a result of CN exhibiting a substantially higher rate of growth of TFP than CP, prior to 1967. Over the period 1956-67, the average annual rate of growth of TFP was 3.9% for CN, and only 2.1% for CP. The opposite was true for the remainder of the sample (1968-75). The rate of growth of TFP was 2.1% for CN and 2.9% for CP. Thus while relative TFP (CN/CP) exceeded unity in the late 1960s, it was 0.99 in 1975. The gains made by CN were almost entirely lost by 1975.

Convergence in 1967 was due to the government-owned firm out-performing the privately owned firm prior to "deregulation". This result cannot be attributed to the "competitive environment" ushered in by the *National Transportation Act* of 1967. Since relative TFP (CN/CP) is equal to or in excess of unity after 1967, Caves and Christensen attribute this to a change in the regulatory environment. Their results on the rate of growth of TFP after 1967 should have led them to conclude that "deregulation" had a more favourable impact on the private firm, thus confirming the predictions of the property rights hypothesis.

Since the cost functions estimated in this thesis do not allow firms to differ before, and after 1967, they do not permit one to draw inferences regarding the impact of the *National Transportation Act*. They indicate that over the period 1956-81, the privately owned firm was more efficient than the government-owned firm, and both

firms had an identical rate of neutral technical change over the sample. Thus, there is no convergence.

APPENDIX 1

BIASES IN THE INDEX OF FUEL INPUT

The purpose of this appendix is to elaborate on the nature of biases in the measure of fuel input. As mentioned in Chapter 3, the various types of fuel utilized by the railways are aggregated using the Fisher ideal index. Due to the shift from steam to diesel technology, certain fuels such as, fuel oil, and bituminous coal, were not utilized after the mid 1960s. Data on quantity purchased¹ and expenditure on these fuels obviously take on zero values after the mid 1960's. The presence of these zero observations prevents the use of the Tornqvist index in constructing the aggregate fuel input. A Fisher ideal index is used instead, but as demonstrated by Diewert (1980), the resulting aggregate is biased. The following discussion is based on a hypothetical example constructed for demonstration purposes. The discussion relates to the construction of an explicit price index, but as will become evident, the results can easily be extended to quantity index numbers.

Diewert discusses the biases in the context of the new goods problem². The researcher may have a data set where a new good has been introduced at a time,

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¹ The BTU conversion factors are: Diesel: 0.16142452, Crude Oil: 0.1607431, Fuel Oil: 0.1743161, Gasoline: 0.1446494, the units for fluids being Million BTU per Imperial Gallon. Electricity: 0.003412 Mill. BTU per KWH, Bituminous Coal: 26.2 Mill. BTU per ton. These factors were obtained from The Department of Energy, Mines and Resources, Canada and are marginally different from those used in previous studies.

² See Diewert (1980) Section 8.6.

other than the beginning of the sample. No data is available on the price of the new good for the period preceding its introduction. The quantities purchased will also be zero. In such cases, Diewert suggests that³:

"...we should attempt to estimate the *reservation price* of the new good for the previous period that would rationalize the zero demand for the new good of the previous period."

Having estimated the reservation price, the "theoretically correct" price index⁴ can be constructed using the following:

$$\ln\left(\frac{F_t}{F_{t-1}}\right) = 1/2 \ln\left(\frac{\sum_{j=1}^n P_{it} X_{it}}{\sum_{j=1}^n P_{it-1} X_{it}}\right) + 1/2 \ln\left(\frac{\sum_{j=1}^n P_{it} X_{it-1}}{\sum_{j=1}^n P_{it-1} X_{it-1}}\right) \quad (A1.1)$$

where F_t is the Fisher ideal price index at time t , P_{it} is the price of commodity i at time t and X_{it} is the quantity of commodity i at time t . Assume the new good is introduced in period 1, hence the period 0 quantity is zero. Further, let the theoretically correct aggregate in period 1 be F_1 .

An alternative price index can be constructed assuming both the price and the quantity of the new good to be zero in period 0. Let F_1^* represent this "incorrect" price aggregate in period 1. Diewert identifies the bias in the incorrect index number as:

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³ Diewert (1980) page 499.

⁴ Note that quantities will still be zero.

$$\frac{F_1^*}{F_1} = \left(\frac{1}{1 - S_1^{01}} \right)^{1/2} > 1 \quad (\text{A1.2})$$

where S_1^{01} is the share of the new good using period 0 prices and period 1 quantities. The bias cannot be calculated without knowledge of the reservation price, however, it is clear from the above that the incorrect price index is biased upwards from period 1 onwards.

With the help of a simple example, this argument is extended to obsolete goods, and fashion goods. Fashion goods refers to goods that are 'in' one period and 'out' the other. Consider the following price (P_1, P_2, P_3) and quantity (X_1, X_2, X_3) data on three goods.

Table A1.1						
Time	P1	X1	P2	X2	P3	X3
1	1.1	5.0	2.0	7.0	0.5	3.0
2	0.9	4.5	2.0	8.0	0.7	2.0
3	1.2	4.4	3.0	9.0	0.8	2.0
4	1.3	4.0	3.2	20.0	1.0	1.5
5	1.1	3.0	4.0	11.0	2.0	1.0

Suppose good 3 is introduced in period 3, the incorrect price index F^* can be calculated by setting both P_3 and X_3 equal to zero in the first two time periods. The theoretically correct price index F is calculated assuming price data are available for the first two time periods. In order to derive the bias, the share of good 3 is

calculated using period 2 prices and period 3 quantities. The incorrect index is biased upwards in all periods including, and following the introduction of the new good. The results are presented in the following table:

Table A1.2						
	New Good			Obsolete Good		
Time	F*	F	F*/F	F*	F	F*/F
1	1.000	1.000	1.000	1.000	1.000	1.000
2	0.953	0.953	1.000	0.979	0.979	1.000
3	1.433	1.390	1.031	1.417	1.417	1.000
4	1.543	1.496	1.031	1.479	1.522	0.972
5	1.893	1.836	1.031	1.778	1.828	0.972

Now assume that good 3 becomes obsolete, and is only available in the first three time periods. As before, two alternative price index numbers are calculated, and as is evident from Table A1.2, the incorrect price index is biased downwards in periods 4 and 5. The bias is the inverse of the right hand side of equation (A1.2). In this case, the share of good 3 is calculated using period 3 quantities and period 4 prices.

Consider a third alternative where good 3 is a fashion good and is unavailable in period 3. From the above discussion, it is easy to deduce that the incorrect price index will have a new good (upward) bias in periods 4 and 5, and an obsolete good (downward) bias in periods 3, 4, and 5. For periods 4 and 5, the product of the two

biases yields the net bias. The direction of the net effect cannot be predicted *a priori*.

The results from this exercise are presented in the table below:

Table A1.3					
				Bias	
Time	F'	F	F'/F	New	Obsolete
1	1.000	1.000	1.000	1.000	1.000
2	0.979	0.979	1.000	1.000	1.000
3	1.390	1.427	0.974	1.000	0.974
4	1.516	1.530	0.990	1.017	0.974
5	1.860	1.878	0.990	1.017	0.974

For the Fisher ideal quantity index, the bias is opposite to that of the price index. The case of obsolete goods discussed above resulted in a bias of 0.972 in the price index, if an explicit quantity index were constructed with the same data, the bias would be the inverse of 0.972, or 1.029.

A theoretically correct quantity index for railway fuel could not be constructed due to the inability to obtain appropriate price indices for fuels such as coal. The data suggest that the fuel quantity index for CN and CP is identical to the theoretically correct index for the period 1956-60. The CP index is upward biased in 1961 and the CN index is upward biased over the period 1961-64. For the remainder of the sample, the bias cannot be predicted.

APPENDIX 2

DATA TABLES

The data in the following tables were used in estimating the various models in this thesis. The first eight tables (A2.1 to A2.8) present factor shares in total cost, and total cost, the latter being measured in millions of current dollars. Data are provided for all four models. Models A and AR use capital prices based on the method of Boadway Bruce and Mintz, or the "ARIMA" cost of capital. Models B and BR use the conventional cost of capital. The only difference between the two sets therefore lies in the different price of equipment, and way and structures; prices of all other inputs are identical. Further, models A & B exclude rented capital where as models AR and BR include rented capital.

Tables A2.9 and A2.10 provide input price data for models A and AR, and Table A2.11 provides the prices of equipment, and way and structures for models B and BR. The input price data are in the form of index numbers. The data are normalized at the approximate median of the sample (CP 1956=1). Table A2.12 provides data on output. These are also normalized at the approximate median of the sample.

Table A2.1**Factor Shares in Total Cost and Total Cost****Model A - Canadian National**

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.530	0.101	0.140	0.080	0.149	633.46
1957	0.543	0.121	0.092	0.068	0.176	626.57
1958	0.490	0.117	0.172	0.047	0.174	658.38
1959	0.467	0.136	0.205	0.039	0.153	712.63
1960	0.455	0.156	0.194	0.035	0.159	690.53
1961	0.462	0.130	0.224	0.034	0.151	707.96
1962	0.452	0.127	0.235	0.035	0.151	710.86
1963	0.468	0.120	0.218	0.037	0.158	692.94
1964	0.487	0.120	0.202	0.037	0.154	743.12
1965	0.501	0.117	0.201	0.037	0.145	773.93
1966	0.504	0.124	0.167	0.037	0.168	797.20
1967	0.511	0.122	0.176	0.035	0.156	865.13
1968	0.507	0.138	0.184	0.034	0.137	867.83
1969	0.515	0.137	0.172	0.033	0.144	912.10
1970	0.528	0.123	0.183	0.033	0.133	930.54
1971	0.501	0.119	0.211	0.033	0.136	1055.66
1972	0.551	0.114	0.155	0.037	0.143	1052.85
1973	0.549	0.097	0.169	0.038	0.147	1142.12
1974	0.574	0.083	0.113	0.057	0.173	1347.50
1975	0.607	0.050	0.080	0.062	0.201	1405.73
1976	0.627	0.048	0.044	0.069	0.211	1438.30
1977	0.588	0.066	0.069	0.072	0.206	1626.14
1978	0.562	0.040	0.107	0.076	0.216	1824.21
1979	0.554	0.037	0.126	0.077	0.207	2043.92
1980	0.543	0.043	0.118	0.088	0.207	2327.44
1981	0.497	0.043	0.127	0.103	0.230	2738.47

Table A2.2**Factor Shares in Total Cost and Total Cost****Model A - Canadian Pacific**

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.425	0.174	0.225	0.062	0.115	604.06
1957	0.430	0.204	0.190	0.050	0.126	584.23
1958	0.424	0.184	0.235	0.040	0.117	578.54
1959	0.383	0.206	0.269	0.032	0.110	649.34
1960	0.386	0.228	0.247	0.028	0.110	614.59
1961	0.391	0.201	0.273	0.028	0.106	608.57
1962	0.365	0.204	0.299	0.026	0.106	644.77
1963	0.388	0.190	0.276	0.029	0.117	621.54
1964	0.391	0.193	0.270	0.031	0.115	639.75
1965	0.389	0.194	0.267	0.031	0.119	658.01
1966	0.396	0.209	0.238	0.032	0.126	659.09
1967	0.403	0.202	0.236	0.031	0.128	687.65
1968	0.401	0.216	0.235	0.031	0.117	694.04
1969	0.407	0.218	0.215	0.031	0.129	721.74
1970	0.435	0.191	0.192	0.035	0.147	701.65
1971	0.423	0.179	0.214	0.036	0.148	764.09
1972	0.411	0.191	0.198	0.041	0.160	808.33
1973	0.400	0.176	0.227	0.041	0.155	891.19
1974	0.428	0.165	0.185	0.057	0.164	1028.10
1975	0.491	0.104	0.138	0.068	0.198	957.26
1976	0.511	0.099	0.104	0.070	0.216	1009.22
1977	0.478	0.113	0.114	0.072	0.223	1174.88
1978	0.473	0.072	0.148	0.078	0.228	1291.87
1979	0.464	0.064	0.159	0.083	0.230	1443.66
1980	0.458	0.072	0.145	0.099	0.227	1559.95
1981	0.399	0.079	0.169	0.117	0.236	1952.41

Table A2.3**Factor Shares in Total Cost and Total Cost****Model AR - Canadian National**

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.531	0.103	0.136	0.080	0.150	631.55
1957	0.551	0.113	0.089	0.069	0.179	617.56
1958	0.498	0.108	0.170	0.048	0.177	647.45
1959	0.474	0.128	0.203	0.040	0.155	702.58
1960	0.463	0.147	0.192	0.036	0.162	678.85
1961	0.469	0.121	0.222	0.035	0.153	697.01
1962	0.459	0.121	0.232	0.035	0.153	701.17
1963	0.475	0.113	0.214	0.038	0.161	682.56
1964	0.492	0.117	0.197	0.038	0.156	735.11
1965	0.506	0.113	0.198	0.037	0.146	766.08
1966	0.507	0.124	0.163	0.037	0.169	792.42
1967	0.514	0.122	0.172	0.035	0.157	860.48
1968	0.511	0.136	0.180	0.034	0.138	860.54
1969	0.517	0.138	0.166	0.033	0.145	907.16
1970	0.531	0.125	0.177	0.033	0.134	924.62
1971	0.501	0.126	0.204	0.033	0.136	1054.68
1972	0.540	0.139	0.144	0.036	0.140	1074.32
1973	0.538	0.123	0.158	0.038	0.144	1165.39
1974	0.559	0.113	0.104	0.055	0.168	1382.70
1975	0.586	0.090	0.071	0.060	0.194	1456.40
1976	0.606	0.087	0.036	0.067	0.204	1488.81
1977	0.573	0.098	0.058	0.070	0.201	1667.90
1978	0.550	0.071	0.095	0.074	0.211	1864.17
1979	0.542	0.068	0.112	0.075	0.202	2089.09
1980	0.529	0.078	0.104	0.086	0.202	2389.48
1981	0.485	0.075	0.115	0.101	0.224	2804.93

Table A2.4

Factor Shares in Total Cost and Total Cost

Model AR - Canadian Pacific

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.425	0.172	0.227	0.061	0.115	604.87
1957	0.431	0.201	0.192	0.050	0.126	583.29
1958	0.425	0.180	0.237	0.040	0.118	576.88
1959	0.384	0.202	0.272	0.032	0.110	647.12
1960	0.389	0.221	0.250	0.029	0.111	610.50
1961	0.394	0.195	0.276	0.028	0.107	604.83
1962	0.366	0.201	0.301	0.026	0.107	643.48
1963	0.390	0.186	0.278	0.029	0.118	618.85
1964	0.393	0.189	0.272	0.031	0.115	636.95
1965	0.392	0.188	0.269	0.032	0.120	653.70
1966	0.397	0.205	0.240	0.032	0.126	657.29
1967	0.402	0.201	0.238	0.031	0.128	688.09
1968	0.400	0.213	0.239	0.031	0.117	695.21
1969	0.406	0.217	0.218	0.031	0.128	724.86
1970	0.431	0.193	0.197	0.034	0.145	708.52
1971	0.419	0.180	0.218	0.035	0.147	770.15
1972	0.406	0.196	0.200	0.040	0.158	818.43
1973	0.396	0.180	0.229	0.041	0.154	899.63
1974	0.422	0.173	0.187	0.056	0.162	1042.33
1975	0.482	0.117	0.140	0.067	0.194	976.38
1976	0.500	0.114	0.106	0.069	0.211	1030.38
1977	0.471	0.124	0.115	0.071	0.219	1193.31
1978	0.468	0.080	0.149	0.077	0.225	1306.15
1979	0.455	0.079	0.159	0.081	0.226	1471.56
1980	0.448	0.091	0.143	0.097	0.222	1594.71
1981	0.391	0.097	0.166	0.114	0.231	1994.20

Table A2.5

Factor Shares in Total Cost and Total Cost

Model B - Canadian National

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.567	0.104	0.083	0.086	0.160	591.55
1957	0.536	0.106	0.117	0.067	0.174	634.27
1958	0.513	0.115	0.141	0.049	0.182	629.29
1959	0.506	0.126	0.160	0.042	0.165	657.99
1960	0.481	0.132	0.181	0.037	0.168	653.44
1961	0.478	0.126	0.204	0.036	0.156	683.18
1962	0.464	0.134	0.212	0.035	0.155	693.43
1963	0.465	0.132	0.209	0.037	0.157	697.37
1964	0.486	0.120	0.203	0.037	0.154	744.83
1965	0.507	0.121	0.189	0.037	0.146	764.78
1966	0.491	0.125	0.184	0.036	0.164	817.41
1967	0.503	0.130	0.179	0.035	0.154	879.17
1968	0.502	0.142	0.188	0.033	0.135	877.23
1969	0.508	0.132	0.185	0.032	0.143	924.38
1970	0.489	0.133	0.225	0.030	0.123	1005.59
1971	0.505	0.125	0.200	0.033	0.137	1046.62
1972	0.527	0.111	0.190	0.036	0.137	1101.18
1973	0.540	0.098	0.180	0.038	0.145	1160.39
1974	0.546	0.077	0.158	0.054	0.164	1416.03
1975	0.553	0.073	0.134	0.057	0.183	1542.51
1976	0.557	0.074	0.120	0.061	0.188	1619.40
1977	0.566	0.053	0.113	0.069	0.198	1687.49
1978	0.537	0.047	0.138	0.072	0.206	1908.62
1979	0.512	0.059	0.168	0.071	0.191	2212.99
1980	0.459	0.070	0.221	0.075	0.175	2756.07
1981	0.405	0.066	0.258	0.084	0.187	3361.45

Table A2.6

Factor Shares in Total Cost and Total Cost

Model B - Canadian Pacific

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.530	0.148	0.102	0.077	0.143	484.31
1957	0.496	0.155	0.147	0.058	0.145	506.92
1958	0.484	0.167	0.170	0.045	0.134	506.92
1959	0.471	0.176	0.179	0.039	0.135	528.18
1960	0.452	0.185	0.200	0.033	0.129	525.44
1961	0.445	0.183	0.220	0.031	0.121	535.83
1962	0.430	0.191	0.223	0.031	0.125	546.92
1963	0.440	0.184	0.211	0.033	0.133	548.67
1964	0.450	0.170	0.213	0.036	0.132	556.10
1965	0.457	0.174	0.193	0.037	0.140	560.25
1966	0.443	0.186	0.194	0.036	0.141	588.95
1967	0.436	0.197	0.195	0.034	0.139	635.69
1968	0.428	0.209	0.205	0.033	0.125	650.14
1969	0.428	0.202	0.203	0.032	0.135	687.21
1970	0.390	0.206	0.241	0.031	0.132	782.34
1971	0.415	0.192	0.212	0.035	0.146	778.62
1972	0.422	0.172	0.200	0.042	0.164	787.19
1973	0.446	0.156	0.180	0.046	0.173	799.49
1974	0.437	0.138	0.199	0.058	0.168	1006.60
1975	0.446	0.130	0.182	0.062	0.180	1054.16
1976	0.461	0.122	0.159	0.063	0.195	1118.30
1977	0.489	0.080	0.129	0.074	0.228	1148.30
1978	0.469	0.071	0.157	0.077	0.226	1304.48
1979	0.434	0.087	0.186	0.077	0.216	1542.19
1980	0.375	0.109	0.248	0.081	0.186	1903.16
1981	0.327	0.104	0.279	0.096	0.194	2381.56

Table A2.7**Factor Shares in Total Cost and Total Cost****Model BR - Canadian National**

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.569	0.105	0.079	0.086	0.160	589.65
1957	0.544	0.097	0.114	0.068	0.177	625.26
1958	0.522	0.105	0.138	0.050	0.185	618.37
1959	0.514	0.118	0.157	0.043	0.168	647.94
1960	0.490	0.122	0.179	0.038	0.171	641.75
1961	0.486	0.117	0.202	0.036	0.159	672.23
1962	0.470	0.127	0.210	0.036	0.157	683.74
1963	0.472	0.125	0.206	0.037	0.160	686.99
1964	0.491	0.117	0.199	0.038	0.155	736.81
1965	0.512	0.117	0.186	0.038	0.148	756.94
1966	0.494	0.125	0.180	0.036	0.165	812.63
1967	0.506	0.130	0.175	0.035	0.155	874.52
1968	0.506	0.140	0.184	0.034	0.137	869.93
1969	0.510	0.133	0.180	0.032	0.143	919.44
1970	0.492	0.135	0.219	0.030	0.124	999.67
1971	0.505	0.132	0.193	0.033	0.137	1045.65
1972	0.517	0.135	0.179	0.035	0.134	1122.65
1973	0.529	0.124	0.168	0.037	0.142	1183.66
1974	0.533	0.105	0.148	0.053	0.160	1451.22
1975	0.535	0.109	0.124	0.055	0.177	1593.18
1976	0.540	0.108	0.110	0.060	0.182	1669.90
1977	0.553	0.084	0.102	0.068	0.193	1729.25
1978	0.526	0.076	0.126	0.071	0.202	1948.58
1979	0.501	0.087	0.154	0.070	0.187	2258.16
1980	0.449	0.100	0.207	0.073	0.171	2818.11
1981	0.397	0.092	0.245	0.083	0.184	3427.90

Table A2.8

Factor Shares in Total Cost and Total Cost

Model BR - Canadian Pacific

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>	<u>Cost (10⁶)</u>
1956	0.529	0.147	0.104	0.077	0.143	485.13
1957	0.496	0.152	0.149	0.058	0.145	505.98
1958	0.485	0.162	0.172	0.045	0.135	505.26
1959	0.473	0.170	0.182	0.040	0.135	525.96
1960	0.455	0.178	0.203	0.034	0.130	521.35
1961	0.448	0.176	0.223	0.031	0.122	532.08
1962	0.431	0.187	0.225	0.031	0.126	545.62
1963	0.442	0.179	0.213	0.033	0.133	545.98
1964	0.452	0.165	0.214	0.036	0.132	553.29
1965	0.460	0.167	0.194	0.037	0.141	555.93
1966	0.445	0.182	0.196	0.036	0.141	587.15
1967	0.435	0.195	0.198	0.034	0.139	636.13
1968	0.427	0.206	0.209	0.033	0.125	651.31
1969	0.426	0.201	0.207	0.032	0.134	690.32
1970	0.387	0.207	0.245	0.031	0.131	789.21
1971	0.412	0.193	0.216	0.035	0.145	784.68
1972	0.417	0.178	0.202	0.041	0.162	797.29
1973	0.441	0.160	0.182	0.045	0.171	807.94
1974	0.431	0.146	0.200	0.057	0.165	1020.84
1975	0.438	0.141	0.183	0.061	0.177	1073.27
1976	0.452	0.135	0.159	0.062	0.191	1139.46
1977	0.481	0.091	0.130	0.073	0.224	1166.73
1978	0.464	0.079	0.158	0.076	0.223	1318.77
1979	0.426	0.101	0.185	0.076	0.212	1570.09
1980	0.368	0.124	0.245	0.080	0.183	1937.93
1981	0.321	0.119	0.275	0.094	0.190	2423.36

Table A2.9

Factor Prices

Models A and AR - Canadian National

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>
1956	0.957	0.495	0.407	1.365	1.000
1957	1.010	0.521	0.258	1.431	1.022
1958	1.064	0.498	0.470	1.355	1.037
1959	1.135	0.603	0.587	1.352	1.058
1960	1.169	0.687	0.522	1.334	1.071
1961	1.246	0.622	0.606	1.308	1.076
1962	1.275	0.632	0.633	1.317	1.088
1963	1.325	0.607	0.559	1.310	1.098
1964	1.369	0.655	0.554	1.299	1.119
1965	1.436	0.652	0.578	1.305	1.158
1966	1.561	0.692	0.496	1.318	1.193
1967	1.713	0.717	0.560	1.303	1.239
1968	1.867	0.803	0.590	1.303	1.270
1969	2.024	0.846	0.559	1.303	1.319
1970	2.163	0.791	0.606	1.313	1.366
1971	2.371	0.883	0.788	1.388	1.426
1972	2.571	0.850	0.577	1.481	1.495
1973	2.931	0.812	0.683	1.701	1.587
1974	3.314	0.824	0.534	2.707	1.808
1975	3.809	0.491	0.388	3.252	2.044
1976	4.272	0.476	0.217	3.691	2.193
1977	4.674	0.770	0.373	4.171	2.411
1978	4.972	0.533	0.648	4.890	2.607
1979	5.574	0.566	0.839	5.513	2.865
1980	6.270	0.770	0.877	7.214	3.138
1981	7.235	0.923	1.094	10.305	3.479

Table A2.10

Factor Prices

Models A and AR - Canadian Pacific

<u>Year</u>	<u>Lab.</u>	<u>Eqp.</u>	<u>Str.</u>	<u>Fuel</u>	<u>Mat.</u>
1956	1.000	1.000	1.000	1.000	1.000
1957	1.050	1.065	0.811	1.039	1.022
1958	1.099	0.935	0.992	1.029	1.037
1959	1.167	1.181	1.274	1.037	1.058
1960	1.202	1.263	1.109	1.025	1.071
1961	1.289	1.143	1.211	1.005	1.076
1962	1.302	1.267	1.403	1.012	1.088
1963	1.339	1.188	1.233	1.021	1.098
1964	1.380	1.276	1.244	1.047	1.119
1965	1.457	1.258	1.261	1.067	1.158
1966	1.563	1.316	1.120	1.064	1.193
1967	1.713	1.308	1.149	1.075	1.239
1968	1.874	1.432	1.154	1.109	1.270
1969	2.028	1.499	1.091	1.134	1.319
1970	2.182	1.257	0.944	1.154	1.366
1971	2.381	1.312	1.137	1.238	1.426
1972	2.614	1.465	1.113	1.379	1.495
1973	2.935	1.522	1.408	1.527	1.587
1974	3.385	1.677	1.323	2.314	1.808
1975	3.897	0.991	0.911	2.700	2.044
1976	4.337	1.014	0.723	3.029	2.193
1977	4.767	1.387	0.918	3.536	2.411
1978	5.059	0.997	1.303	4.007	2.607
1979	5.615	1.005	1.543	4.591	2.865
1980	6.185	1.234	1.489	5.953	3.138
1981	6.834	1.691	2.156	8.864	3.479

Table A2.11

Prices of Capital Inputs

Models B and BR - Canadian National and Canadian Pacific

<u>Year</u>	<u>Canadian National</u>		<u>Canadian Pacific</u>	
	<u>Eqp.</u>	<u>Str.</u>	<u>Eqp.</u>	<u>Str.</u>
1956	0.692	0.627	1.000	1.000
1957	0.674	0.917	1.030	1.495
1958	0.683	1.020	1.088	1.734
1959	0.758	1.165	1.197	1.904
1960	0.803	1.275	1.286	2.122
1961	0.854	1.471	1.341	2.370
1962	0.948	1.542	1.470	2.451
1963	0.986	1.495	1.486	2.297
1964	0.959	1.546	1.431	2.344
1965	0.975	1.482	1.407	2.138
1966	1.044	1.543	1.534	2.259
1967	1.137	1.600	1.718	2.433
1968	1.221	1.676	1.901	2.600
1969	1.211	1.689	1.930	2.707
1970	1.353	2.217	2.211	3.640
1971	1.344	2.037	2.096	3.173
1972	1.269	2.042	1.886	3.020
1973	1.223	2.032	1.765	2.753
1974	1.176	2.162	2.000	3.839
1975	1.155	1.964	1.989	3.652
1976	1.207	1.823	2.024	3.375
1977	0.937	1.763	1.400	2.806
1978	0.960	2.406	1.451	3.842
1979	1.441	3.340	2.140	5.304
1980	2.183	5.375	3.343	8.618
1981	2.554	7.516	3.963	12.015

Table A2.12
Output and Network Variables
Canadian National and Canadian Pacific

<u>Year</u>	<u>Canadian National</u>			<u>Canadian Pacific</u>		
	<u>Frt.</u>	<u>Pass.</u>	<u>Rt. Miles</u>	<u>Frt.</u>	<u>Pass.</u>	<u>Rt. Miles</u>
1956	1.235	1.055	1.295	1.000	1.000	1.000
1957	1.085	1.062	1.256	0.896	1.022	1.001
1958	1.048	0.895	1.283	0.883	0.878	0.999
1959	1.058	0.900	1.289	0.853	0.849	0.995
1960	1.015	0.851	1.286	0.846	0.770	0.992
1961	1.042	0.764	1.285	0.869	0.639	0.978
1962	1.063	0.803	1.282	0.856	0.645	0.971
1963	1.207	0.852	1.276	0.957	0.641	0.967
1964	1.343	1.175	1.273	1.115	0.782	0.967
1965	1.386	1.279	1.277	1.110	0.671	0.963
1966	1.495	1.475	1.274	1.247	0.426	0.963
1967	1.469	1.851	1.281	1.163	0.477	0.968
1968	1.493	1.512	1.285	1.136	0.377	0.970
1969	1.559	1.354	1.307	1.189	0.370	0.967
1970	1.707	1.289	1.322	1.180	0.326	0.969
1971	1.881	1.265	1.316	1.499	0.287	0.966
1972	2.029	1.176	1.328	1.616	0.258	0.967
1973	2.013	0.908	1.329	1.684	0.189	0.965
1974	2.222	1.019	1.326	1.730	0.267	0.967
1975	2.175	1.015	1.325	1.685	0.220	0.958
1976	2.258	1.043	1.319	1.644	0.188	0.956
1977	2.349	1.034	1.305	1.746	0.189	0.941
1978	2.450	1.043	1.294	1.862	0.205	0.930
1979	2.588	1.012	1.291	1.933	0.263	0.919
1980	2.693	1.010	1.281	1.918	0.322	0.911
1981	2.660	0.997	1.316	2.017	0.316	0.900

APPENDIX 3

SUPPLEMENTARY RESULTS

Presented below, are the results from alternative specifications of the cost function discussed in Chapters 4 through 7. Summary results are provided here to complement the discussion in the above-mentioned chapters. With one possible exception, results from these specifications are not used in the measurement of Total Factor Productivity. The information provided, varies from specification to specification, but in each case, the intent is to highlight the lack of theoretical consistency of these models.

Tables A3.1 through A3.27 pertain to Chapter 4, and are based on the translog cost function. If no information is provided on the curvature of the cost function, the reader is to assume that the model in question is non-concave. If no information is provided on monotonicity violations, the reader is to assume that the model in question satisfies monotonicity. Sample mean, minimum, and maximum values are provided for economies of scale, economies of density, output cost elasticities and the cost elasticity of the quasi-fixed factor of production. The frequency of a negatively sloped input demand function is also provided for the various inputs, this is a number less than or equal to the sample size of 52. Own price elasticities of factor demand are also reported for some models; these are calculated using the sample mean values of the predicted factor shares in total cost.

For models with a general specification of technical change, sample mean values of the measure of technical change are reported. In models with neutral technical change, the estimated coefficient of the time trend is reported.

Results from the SGM cost function used in Chapter 5, are reported in Tables A3.28 through A3.34. Sample mean values of technical change and own price elasticities of factor demand are reported for this specification.

Tables A3.35 through A3.37 provide supplementary results for the globally concave version of the translog cost function with neutral technical change. This specification was used in Chapter 6. Own price elasticities of factor demand are calculated using the mean values of predicted factor shares in total cost. Under technical change, the coefficient of the time trend is reported.

Tables A3.38 through A3.50 provide supplementary results for models that allow firms to differ. Both five and three input models with firm dummies are estimated in Chapter 7. In these models, the estimated coefficient of the dummy variable is reported with the asymptotic t-ratio in parentheses. Tables A3.51 through A3.53 relate to TFP measurement.

Table A3.1		
Specification: Model A - General Translog		
Economies of Scale	Minimum	0.708
	Maximum	23.198
	Mean	2.778
Passenger Cost Elasticity	Minimum	0.176
	Maximum	0.383
	Mean	0.253
Freight Cost Elasticity	Minimum	-0.235
	Maximum	1.201
	Mean	0.464
Negatively sloped Labour demand		0
Negatively sloped Equipment demand		35
Negatively sloped Structures demand		43
Negatively sloped Fuel demand		24
Negatively sloped Materials demand		0
Own price elasticity of Labour demand		0.108
Own price elasticity of Equipment demand		-0.084
Own price elasticity of Structures demand		-0.203
Own price elasticity of Fuel demand		-0.047
Own price elasticity of Materials demand		0.281
Technical change		-0.017

Table A3.2		
Specification: Model AR - General Translog		
Economies of Scale	Minimum	0.705
	Maximum	5.842
	Mean	1.744
Passenger Cost Elasticity	Minimum	0.185
	Maximum	0.342
	Mean	0.247
Freight Cost Elasticity	Minimum	-0.080
	Maximum	1.186
	Mean	0.522
Negatively sloped Labour demand		0
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		41
Negatively sloped Fuel demand		24
Negatively sloped Materials demand		16
Own price elasticity of Labour demand		0.039
Own price elasticity of Equipment demand		-0.167
Own price elasticity of Structures demand		-0.191
Own price elasticity of Fuel demand		-0.057
Own price elasticity of Materials demand		0.040
Technical change		-0.017
Monotonicity: predicted share of structures is negative for CN (1976)		

Table A3.3		
Specification: Model B - General Translog		
Economies of Scale	Minimum	0.803
	Maximum	3.067
	Mean	1.324
Passenger Cost Elasticity	Minimum	0.132
	Maximum	0.342
	Mean	0.223
Freight Cost Elasticity	Minimum	0.104
	Maximum	1.012
	Mean	0.643
Negatively sloped Labour demand		0
Negatively sloped Equipment demand		20
Negatively sloped Structures demand		23
Negatively sloped Fuel demand		28
Negatively sloped Materials demand		2
Own price elasticity of Labour demand		0.046
Own price elasticity of Equipment demand		0.045
Own price elasticity of Structures demand		0.029
Own price elasticity of Fuel demand		-0.112
Own price elasticity of Materials demand		0.227
Technical change		-0.027

Table A3.4		
Specification: Model BR - General Translog		
Economies of Scale	Minimum	0.760
	Maximum	2.394
	Mean	1.189
Passenger Cost Elasticity	Minimum	0.124
	Maximum	0.394
	Mean	0.222
Freight Cost Elasticity	Minimum	0.178
	Maximum	1.096
	Mean	0.705
Negatively sloped Labour demand		50
Negatively sloped Equipment demand		29
Negatively sloped Structures demand		23
Negatively sloped Fuel demand		25
Negatively sloped Materials demand		14
Own price elasticity of Labour demand		-0.020
Own price elasticity of Equipment demand		-0.007
Own price elasticity of Structures demand		0.022
Own price elasticity of Fuel demand		-0.039
Own price elasticity of Materials demand		0.046
Technical change		-0.027

Table A3.5		
Specification: Model AR - Neutral Technical Change		
Economies of Scale	Minimum	1.064
	Maximum	2.818
	Mean	1.796
Passenger Cost Elasticity	Minimum	0.188
	Maximum	0.305
	Mean	0.246
Freight Cost Elasticity	Minimum	0.159
	Maximum	0.642
	Mean	0.354
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		46
Negatively sloped Structures demand		41
Negatively sloped Fuel demand		19
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.296
Own price elasticity of Equipment demand		-0.299
Own price elasticity of Structures demand		-0.202
Own price elasticity of Fuel demand		0.137
Own price elasticity of Materials demand		-1.122
Technical change		-0.010

Table A3.6		
Specification: Model B - Neutral Technical Change		
Economies of Scale	Minimum	0.683
	Maximum	1.134
	Mean	0.910
Passenger Cost Elasticity	Minimum	0.049
	Maximum	0.221
	Mean	0.150
Freight Cost Elasticity	Minimum	0.709
	Maximum	1.307
	Mean	0.975
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		40
Negatively sloped Structures demand		18
Negatively sloped Fuel demand		25
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.170
Own price elasticity of Equipment demand		-0.106
Own price elasticity of Structures demand		0.048
Own price elasticity of Fuel demand		-0.052
Own price elasticity of Materials demand		-0.450
Technical change		-0.041

Table A3.7		
Specification: Model BR - Neutral Technical Change		
Economies of Scale	Minimum	0.699
	Maximum	1.328
	Mean	0.978
Passenger Cost Elasticity	Minimum	0.076
	Maximum	0.225
	Mean	0.153
Freight Cost Elasticity	Minimum	0.590
	Maximum	1.285
	Mean	0.909
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		12
Negatively sloped Fuel demand		25
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.186
Own price elasticity of Equipment demand		-0.188
Own price elasticity of Structures demand		0.099
Own price elasticity of Fuel demand		0.025
Own price elasticity of Materials demand		-0.495
Technical change		-0.036

Table A3.8		
Specification: Model A - Network		
Economies of Scale	Minimum	0.600
	Maximum	0.986
	Mean	0.796
Economies of Density	Minimum	-403.16
	Maximum	52.747
	Mean	-10.108
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		40
Negatively sloped Structures demand		45
Negatively sloped Fuel demand		18
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.290
Own price elasticity of Equipment demand		-0.187
Own price elasticity of Structures demand		-0.231
Own price elasticity of Fuel demand		0.072
Own price elasticity of Materials demand		-1.207
Technical change		-0.006

Table A3.9		
Specification: Model AR - Network		
Economies of Scale	Minimum	0.633
	Maximum	0.915
	Mean	0.754
Economies of Density	Minimum	-50.092
	Maximum	316.23
	Mean	7.886
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		46
Negatively sloped Structures demand		44
Negatively sloped Fuel demand		18
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.333
Own price elasticity of Equipment demand		-0.282
Own price elasticity of Structures demand		-0.236
Own price elasticity of Fuel demand		0.083
Own price elasticity of Materials demand		-1.285
Technical change		-0.007

Table A3.10		
Specification: Model B - Network		
Economies of Scale	Minimum	0.509
	Maximum	2.258
	Mean	1.084
Economies of Density	Minimum	-92.597
	Maximum	100.100
	Mean	-0.881
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		38
Negatively sloped Structures demand		5
Negatively sloped Fuel demand		23
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.165
Own price elasticity of Equipment demand		-0.095
Own price elasticity of Structures demand		0.122
Own price elasticity of Fuel demand		-0.083
Own price elasticity of Materials demand		-0.686
Technical change		-0.003

Table A3.11		
Specification: Model BR - Network		
Economies of Scale	Minimum	0.634
	Maximum	0.989
	Mean	0.791
Economies of Density	Minimum	-158.900
	Maximum	140.050
	Mean	7.200
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		4
Negatively sloped Fuel demand		19
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.166
Own price elasticity of Equipment demand		-0.144
Own price elasticity of Structures demand		0.156
Own price elasticity of Fuel demand		0.047
Own price elasticity of Materials demand		-0.474
Technical change		-0.006

Table A3.12		
Specification: Model A - Quasi-Fixed Way and Structures		
Economies of Scale	Minimum	-465.13
	Maximum	10.759
	Mean	-8.473
Passenger Cost Elasticity	Minimum	-0.035
	Maximum	0.080
	Mean	0.027
Freight Cost Elasticity	Minimum	-0.110
	Maximum	0.924
	Mean	0.332
Structures Cost Elasticity	Minimum	-0.712
	Maximum	1.392
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		43
Negatively sloped Fuel demand		19
Negatively sloped Materials demand		52

Table A3.13		
Specification: Model AR - Quasi-Fixed Way and Structures		
Economies of Scale	Minimum	-44.526
	Maximum	36.700
	Mean	1.802
Passenger Cost Elasticity	Minimum	-0.012
	Maximum	0.153
	Mean	0.046
Freight Cost Elasticity	Minimum	-0.161
	Maximum	0.916
	Mean	0.316
Structures Cost Elasticity	Minimum	-0.888
	Maximum	1.798
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		45
Negatively sloped Fuel demand		20
Negatively sloped Materials demand		52

Table A3.14		
Specification: Model B - Quasi-Fixed Way and Structures		
Economies of Scale	Minimum	-4.537
	Maximum	5.977
	Mean	0.347
Passenger Cost Elasticity	Minimum	-0.085
	Maximum	0.164
	Mean	0.016
Freight Cost Elasticity	Minimum	-0.010
	Maximum	0.887
	Mean	0.374
Structures Cost Elasticity	Minimum	-0.884
	Maximum	1.811
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		20
Negatively sloped Fuel demand		30
Negatively sloped Materials demand		52

Table A3.15		
Specification: Model BR - Quasi-Fixed Way and Structures		
Economies of Scale	Minimum	-39.000
	Maximum	64.964
	Mean	-1.108
Passenger Cost Elasticity	Minimum	-0.067
	Maximum	0.182
	Mean	0.023
Freight Cost Elasticity	Minimum	-0.153
	Maximum	0.852
	Mean	0.270
Structures Cost Elasticity	Minimum	-1.098
	Maximum	2.185
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		42
Negatively sloped Fuel		19
Negatively sloped Materials demand		52

Table A3.16		
Specification: Model A - Grain		
Economies of Scale	Minimum	1.168
	Maximum	4.078
	Mean	1.983
Passenger Cost Elasticity	Minimum	0.163
	Maximum	0.276
	Mean	0.215
Other Freight Cost Elasticity	Minimum	0.262
	Maximum	0.766
	Mean	0.489
Grain Freight Cost Elasticity	Minimum	-0.453
	Maximum	0.149
	Mean	-0.147
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		40
Negatively sloped Structures demand		42
Negatively sloped Fuel demand		17
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.296
Own price elasticity of Equipment demand		-0.194
Own price elasticity of Structures demand		-0.203
Own price elasticity of Fuel demand		0.518
Own price elasticity of Materials demand		-1.045
Technical change		-0.014

Table A3.17		
Specification: Model AR - Grain		
Economies of Scale	Minimum	1.126
	Maximum	3.929
	Mean	2.023
Passenger Cost Elasticity	Minimum	0.141
	Maximum	0.286
	Mean	0.202
Other Freight Cost Elasticity	Minimum	0.309
	Maximum	0.737
	Mean	0.514
Grain Freight Cost Elasticity	Minimum	-0.439
	Maximum	0.111
	Mean	-0.153
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		46
Negatively sloped Structures demand		41
Negatively sloped Fuel demand		17
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.348
Own price elasticity of Equipment demand		-0.230
Own price elasticity of Structures demand		-0.187
Own price elasticity of Fuel demand		0.140
Own price elasticity of Materials demand		-1.231
Technical change		-0.013
Monotonicity violation: CN 1976, way and structures		
Concavity satisfied: CN 1956		

Table A3.18		
Specification: Model B - Grain		
Economies of Scale	Minimum	0.820
	Maximum	1.559
	Mean	1.173
Passenger Cost Elasticity	Minimum	0.072
	Maximum	0.221
	Mean	0.133
Other Freight Cost Elasticity	Minimum	0.631
	Maximum	0.956
	Mean	0.799
Grain Freight Cost Elasticity	Minimum	-0.249
	Maximum	0.132
	Mean	-0.045
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		41
Negatively sloped Structures demand		16
Negatively sloped Fuel demand		25
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.171
Own price elasticity of Equipment demand		-0.151
Own price elasticity of Structures demand		0.062
Own price elasticity of Fuel demand		-0.001
Own price elasticity of Materials demand		-0.590
Technical change		-0.035

Table A3.19		
Specification: Model BR - Grain		
Economies of Scale	Minimum	0.813
	Maximum	1.776
	Mean	1.221
Passenger Cost Elasticity	Minimum	0.055
	Maximum	0.231
	Mean	0.129
Other Freight Cost Elasticity	Minimum	0.586
	Maximum	1.045
	Mean	0.801
Grain Freight Cost Elasticity	Minimum	-0.255
	Maximum	0.089
	Mean	-0.064
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		46
Negatively sloped Structures demand		12
Negatively sloped Fuel demand		22
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.201
Own price elasticity of Equipment demand		-0.212
Own price elasticity of Structures demand		0.105
Own price elasticity of Fuel demand		0.053
Own price elasticity of Materials demand		-0.613
Technical change		-0.032

Table A3.20		
Specification: Model A - Average Haul		
Economies of Scale	Minimum	-6.673
	Maximum	66.669
	Mean	1.303
Passenger Cost Elasticity	Minimum	0.107
	Maximum	0.262
	Mean	0.194
Freight Cost Elasticity	Minimum	0.501
	Maximum	1.074
	Mean	0.824
Average Haul Cost Elasticity	Minimum	-2.480
	Maximum	0.490
	Mean	-1.265
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		34
Negatively sloped Structures demand		41
Negatively sloped Fuel demand		24
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.545
Own price elasticity of Equipment demand		-0.578
Own price elasticity of Structures demand		-0.938
Own price elasticity of Fuel demand		-1.129
Own price elasticity of Materials demand		-5.051
Technical change		-0.01

Table A3.21		
Specification: Model AR - Average Haul		
Economies of Scale	Minimum	-13.168
	Maximum	16.347
	Mean	0.127
Passenger Cost Elasticity	Minimum	0.088
	Maximum	0.270
	Mean	0.197
Freight Cost Elasticity	Minimum	0.528
	Maximum	1.055
	Mean	0.820
Average Haul Cost Elasticity	Minimum	-2.333
	Maximum	0.174
	Mean	-1.281
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		33
Negatively sloped Structures demand		41
Negatively sloped Fuel demand		24
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.307
Own price elasticity of Equipment demand		-0.107
Own price elasticity of Structures demand		-0.153
Own price elasticity of Fuel demand		-0.027
Own price elasticity of Materials demand		-1.027
Technical change		-0.007
Monotonocity violation: CN 1976, way and structures		

Table A3.22		
Specification: Model B - Average Haul		
Economies of Scale	Minimum	-73.076
	Maximum	533.23
	Mean	8.178
Passenger Cost Elasticity	Minimum	-0.041
	Maximum	0.247
	Mean	0.128
Freight Cost Elasticity	Minimum	0.884
	Maximum	1.288
	Mean	1.092
Average Haul Cost Elasticity	Minimum	-1.848
	Maximum	0.640
	Mean	-0.624
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		27
Negatively sloped Structures demand		12
Negatively sloped Fuel demand		28
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.283
Own price elasticity of Equipment demand		0.007
Own price elasticity of Structures demand		0.478
Own price elasticity of Fuel demand		-2.530
Own price elasticity of Materials demand		-2.542
Technical change		-0.035

Table A3.23		
Specification: Model BR - Average Haul		
Economies of Scale	Minimum	-38.497
	Maximum	72.432
	Mean	1.073
Passenger Cost Elasticity	Minimum	-0.068
	Maximum	0.252
	Mean	0.125
Freight Cost Elasticity	Minimum	0.777
	Maximum	1.455
	Mean	1.114
Average Haul Cost Elasticity	Minimum	-1.789
	Maximum	0.514
	Mean	-0.684
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		29
Negatively sloped Structures demand		5
Negatively sloped Fuel demand		26
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.182
Own price elasticity of Equipment demand		-0.018
Own price elasticity of Structures demand		0.141
Own price elasticity of Fuel demand		-0.058
Own price elasticity of Materials demand		-0.448
Technical change		-0.032

Table A3.24		
Specification: Model A - Alternative Capital		
Economies of Scale	Minimum	1.455
	Maximum	1.826
	Mean	1.685
Passenger Cost Elasticity	Minimum	0.252
	Maximum	0.300
	Mean	0.273
Freight Cost Elasticity	Minimum	0.288
	Maximum	0.401
	Mean	0.323
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		40
Negatively sloped Structures demand		45
Negatively sloped Fuel demand		17
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.250
Own price elasticity of Equipment demand		-0.188
Own price elasticity of Structures demand		-0.220
Own price elasticity of Fuel demand		0.203
Own price elasticity of Materials demand		-1.032
Technical change		-0.01

Table A3.25		
Specification: Model AR - Alternative Capital		
Economies of Scale	Minimum	1.086
	Maximum	2.789
	Mean	1.808
Passenger Cost Elasticity	Minimum	0.188
	Maximum	0.308
	Mean	0.248
Freight Cost Elasticity	Minimum	0.163
	Maximum	0.620
	Mean	0.346
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		46
Negatively sloped Structures demand		40
Negatively sloped Fuel demand		18
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.296
Own price elasticity of Equipment demand		-0.299
Own price elasticity of Structures demand		-0.204
Own price elasticity of Fuel demand		0.143
Own price elasticity of Materials demand		-1.125
Technical change		-0.01
Concavity satisfied: CN 1956, CP 1978,1979,1980		

Table A3.26		
Specification: Model B - Alternative Capital		
Economies of Scale	Minimum	0.678
	Maximum	1.125
	Mean	0.906
Passenger Cost Elasticity	Minimum	0.041
	Maximum	0.226
	Mean	0.150
Freight Cost Elasticity	Minimum	0.714
	Maximum	1.318
	Mean	0.981
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		38
Negatively sloped Structures demand		18
Negatively sloped Fuel demand		25
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.169
Own price elasticity of Equipment demand		-0.096
Own price elasticity of Structures demand		0.049
Own price elasticity of Fuel demand		-0.053
Own price elasticity of Materials demand		-0.450
Technical change		-0.041

Table A3.27		
Specification: Model BR - Alternative Capital		
Economies of Scale	Minimum	0.696
	Maximum	1.314
	Mean	0.973
Passenger Cost Elasticity	Minimum	0.072
	Maximum	0.227
	Mean	0.153
Freight Cost Elasticity	Minimum	0.599
	Maximum	1.284
	Mean	0.914
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		12
Negatively sloped Fuel demand		24
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.185
Own price elasticity of Equipment demand		-0.182
Own price elasticity of Structures demand		0.098
Own price elasticity of Fuel demand		0.025
Own price elasticity of Materials demand		-0.496
Technical change		-0.036

Table A3.28		
Specification: Model A - Non Concave SGM Cost Function		
Economies of Scale	Minimum	0.814
	Maximum	3.766
	Mean	1.532
Passenger Cost Elasticity	Minimum	0.081
	Maximum	0.446
	Mean	0.219
Freight Cost Elasticity	Minimum	0.184
	Maximum	1.077
	Mean	0.544
Economies of Scope	Minimum	-0.039
	Maximum	1.733
	Mean	0.790
Own Price Elasticity of Labour demand		0.280
Own Price Elasticity of Equipment demand		0.078
Own Price Elasticity of Structures demand		0.001
Own Price Elasticity of Fuel demand		-0.273
Own Price Elasticity of Materials demand		0.139
Technical Change		-0.013

Table A3.29		
Specification: Model AR - Non Concave SGM Cost Function		
Economies of Scale	Minimum	0.737
	Maximum	3.940
	Mean	1.595
Passenger Cost Elasticity	Minimum	0.073
	Maximum	0.495
	Mean	0.232
Freight Cost Elasticity	Minimum	0.108
	Maximum	1.223
	Mean	0.532
Economies of Scope	Minimum	-0.044
	Maximum	2.050
	Mean	0.965
Own Price Elasticity of Labour demand		0.339
Own Price Elasticity of Equipment demand		-0.467
Own Price Elasticity of Structures demand		-0.005
Own Price Elasticity of Fuel demand		-0.345
Own Price Elasticity of Materials demand		0.067
Technical Change		-0.009

Table A3.30		
Specification: Model B - Non Concave SGM Cost Function		
Economies of Scale	Minimum	1.050
	Maximum	9.613
	Mean	2.557
Passenger Cost Elasticity	Minimum	0.089
	Maximum	0.667
	Mean	0.348
Freight Cost Elasticity	Minimum	-0.012
	Maximum	0.527
	Mean	0.213
Economies of Scope	Minimum	0.223
	Maximum	1.237
	Mean	0.678
Own Price Elasticity of Labour demand		0.198
Own Price Elasticity of Equipment demand		0.047
Own Price Elasticity of Structures demand		0.025
Own Price Elasticity of Fuel demand		-0.178
Own Price Elasticity of Materials demand		0.170
Technical Change		-0.003

Table A3.31		
Specification: Model BR - Non Concave SGM Cost Function		
Economies of Scale	Minimum	0.886
	Maximum	4.340
	Mean	1.806
Passenger Cost Elasticity	Minimum	0.086
	Maximum	0.586
	Mean	0.302
Freight Cost Elasticity	Minimum	0.121
	Maximum	0.829
	Mean	0.364
Economies of Scope	Minimum	0.109
	Maximum	1.217
	Mean	0.611
Own Price Elasticity of Labour demand		0.381
Own Price Elasticity of Equipment demand		-0.500
Own Price Elasticity of Structures demand		0.043
Own Price Elasticity of Fuel demand		-0.150
Own Price Elasticity of Materials demand		0.481
Technical Change		-0.004

Table A3.32		
Specification: Model AR - Concave SGM Cost Function		
Economies of Scale	Minimum	0.704
	Maximum	3.377
	Mean	1.505
Passenger Cost Elasticity	Minimum	0.006
	Maximum	0.456
	Mean	0.211
Freight Cost Elasticity	Minimum	0.149
	Maximum	1.285
	Mean	0.584
Economies of Scope	Minimum	-0.085
	Maximum	1.697
	Mean	0.766
Own Price Elasticity of Labour demand		-0.009
Own Price Elasticity of Equipment demand		-0.434
Own Price Elasticity of Structures demand		-0.014
Own Price Elasticity of Fuel demand		-0.375
Own Price Elasticity of Materials demand		-0.335
Technical Change		-0.012

Table A3.33		
Specification: Model B - Concave SGM Cost Function		
Economies of Scale	Minimum	0.825
	Maximum	29.870
	Mean	2.837
Passenger Cost Elasticity	Minimum	0.119
	Maximum	0.550
	Mean	0.306
Freight Cost Elasticity	Minimum	-0.142
	Maximum	0.764
	Mean	0.346
Economies of Scope	Minimum	0.019
	Maximum	2.012
	Mean	0.935
Own Price Elasticity of Labour demand		-0.029
Own Price Elasticity of Equipment demand		-0.040
Own Price Elasticity of Structures demand		-0.052
Own Price Elasticity of Fuel demand		-0.326
Own Price Elasticity of Materials demand		-0.258
Technical Change		-0.006

Table A3.34		
Specification: Model BR - Concave SGM Cost Function		
Economies of Scale	Minimum	0.813
	Maximum	19.775
	Mean	2.548
Passenger Cost Elasticity	Minimum	0.166
	Maximum	0.595
	Mean	0.308
Freight Cost Elasticity	Minimum	-0.123
	Maximum	0.819
	Mean	0.355
Economies of Scope	Minimum	-0.02
	Maximum	1.988
	Mean	0.911
Own Price Elasticity of Labour demand		-0.022
Own Price Elasticity of Equipment demand		-0.489
Own Price Elasticity of Structures demand		-0.039
Own Price Elasticity of Fuel demand		-0.301
Own Price Elasticity of Materials demand		-0.314
Technical Change		-0.004

Table A3.35				
Specification: Model AR - Globally Concave Translog				
		CN	CP	Sample
Economies of Scale	Minimum	0.760	1.088	0.760
	Maximum	1.227	1.667	1.667
	Mean	1.030	1.421	1.226
Passenger Cost Elasticity	Minimum	0.073	0.018	0.018
	Maximum	0.276	0.295	0.295
	Mean	0.161	0.156	0.158
Freight Cost Elasticity	Minimum	0.729	0.393	0.393
	Maximum	1.047	0.680	1.047
	Mean	0.829	0.561	0.695
Own price elasticity of Labour demand				-0.604
Own price elasticity of Equipment demand				-0.866
Own price elasticity of Structures demand				-0.820
Own price elasticity of Fuel demand				-0.956
Own price elasticity of Materials demand				-1.037
Technical change				-0.029

Table A3.36		
Specification: Model B - Globally Concave Translog		
Economies of Scale	Minimum	0.656
	Maximum	6.185
	Mean	1.495
Passenger Cost Elasticity	Minimum	-0.169
	Maximum	0.493
	Mean	0.125
Freight Cost Elasticity	Minimum	0.053
	Maximum	1.307
	Mean	0.808
Own price elasticity of Labour demand		-0.533
Own price elasticity of Equipment demand		-0.873
Own price elasticity of Structures demand		-0.818
Own price elasticity of Fuel demand		-0.957
Own price elasticity of Materials demand		-0.866
Technical change		-0.039

Table A3.37		
Specification: Model BR - Globally Concave Translog		
Economies of Scale	Minimum	0.752
	Maximum	3.050
	Mean	1.334
Passenger Cost Elasticity	Minimum	-0.122
	Maximum	0.425
	Mean	0.133
Freight Cost Elasticity	Minimum	0.140
	Maximum	1.174
	Mean	0.743
Own price elasticity of Labour demand		-0.563
Own price elasticity of Equipment demand		-0.878
Own price elasticity of Structures demand		-0.820
Own price elasticity of Fuel demand		-0.969
Own price elasticity of Materials demand		-0.941
Technical change		-0.033

Table A3.38		
Specification: Model A - Shift and Interactive CN Dummy		
Economies of Scale	Minimum	-179.79
	Maximum	454.66
	Mean	14.458
Passenger Cost Elasticity	Minimum	-0.002
	Maximum	0.069
	Mean	0.037
Freight Cost Elasticity	Minimum	-0.063
	Maximum	0.255
	Mean	0.084
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		39
Negatively sloped Structures demand		48
Negatively sloped Fuel demand		29
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.261
Own price elasticity of Equipment demand		-0.159
Own price elasticity of Structures demand		-0.287
Own price elasticity of Fuel demand		-0.120
Own price elasticity of Materials demand		-1.055
Technical change (α_T)		-0.009
CN Dummy (α_D)		0.369 (28.6)
CN Interactive Dummy (α_{TD})		0.002 (1.29)
Concavity is satisfied for CN in 1956		

Table A3.39		
Specification: Model AR - Shift and Interactive CN Dummy		
Economies of Scale	Minimum	2.51
	Maximum	33.92
	Mean	8.86
Passenger Cost Elasticity	Minimum	-0.042
	Maximum	0.067
	Mean	0.024
Freight Cost Elasticity	Minimum	-0.014
	Maximum	0.363
	Mean	0.158
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		46
Negatively sloped Fuel demand		24
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.283
Own price elasticity of Equipment demand		-0.151
Own price elasticity of Structures demand		-0.281
Own price elasticity of Fuel demand		-0.044
Own price elasticity of Materials demand		-1.198
Technical change (α_T)		-0.01
CN Dummy (α_D)		0.366 (22.4)
CN Interactive Dummy (α_{TD})		0.002 (1.13)

Table A3.40		
Specification: Model B - Shift and Interactive CN Dummy		
Economies of Scale	Minimum	-8.99
	Maximum	-2.54
	Mean	-5.02
Passenger Cost Elasticity	Minimum	-0.114
	Maximum	-0.003
	Mean	-0.050
Freight Cost Elasticity	Minimum	-0.297
	Maximum	-0.055
	Mean	-0.188
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		38
Negatively sloped Structures demand		33
Negatively sloped Fuel demand		52
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.143
Own price elasticity of Equipment demand		-0.070
Own price elasticity of Structures demand		-0.025
Own price elasticity of Fuel demand		-0.341
Own price elasticity of Materials demand		-0.485
Technical change (α_T)		-0.004
CN Dummy (α_D)		0.457 (39.36)
CN Interactive Dummy (α_{TD})		0.009 (6.86)

Table A3.41		
Specification: Model BR - Shift and Interactive CN Dummy		
Economies of Scale	Minimum	-473.73
	Maximum	-2.513
	Mean	-22.465
Passenger Cost Elasticity	Minimum	-0.139
	Maximum	0.024
	Mean	-0.045
Freight Cost Elasticity	Minimum	-0.291
	Maximum	0.026
	Mean	-0.138
Negatively sloped Labour demand		52
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		34
Negatively sloped Fuel demand		52
Negatively sloped Materials demand		52
Own price elasticity of Labour demand		-0.200
Own price elasticity of Equipment demand		-0.159
Own price elasticity of Structures demand		-0.039
Own price elasticity of Fuel demand		-0.372
Own price elasticity of Materials demand		-0.674
Technical change (α_T)		-0.004
CN Dummy (α_D)		0.438 (30.19)
CN Interactive Dummy (α_{TD})		0.01 (5.87)

Table A3.42		
Specification: Three Input Model A - Shift and Interactive CN Dummy Locally Concave		
Economies of Scale	Minimum	-766.52
	Maximum	189.45
	Mean	-8.31
Passenger Cost Elasticity	Minimum	-0.001
	Maximum	0.082
	Mean	0.047
Freight Cost Elasticity	Minimum	-0.128
	Maximum	0.229
	Mean	0.057
Negatively sloped Labour+Materials+Fuel demand		44
Negatively sloped Equipment demand		39
Negatively sloped Structures demand		48
Own price elasticity of Labour+Materials+Fuel demand		-0.069
Own price elasticity of Equipment demand		-0.122
Own price elasticity of Structures demand		-0.283

Table A3.43		
Specification: Three Input Model A - Shift and Interactive CN Dummy Locally Concave (Bayesian)		
Economies of Scale	Minimum	2.60
	Maximum	13.09
	Mean	5.72
Passenger Cost Elasticity	Minimum	0.014
	Maximum	0.010
	Mean	0.055
Freight Cost Elasticity	Minimum	0.011
	Maximum	0.295
	Mean	0.160
Negatively sloped Labour+Materials+Fuel demand		45
Negatively sloped Equipment demand		39
Negatively sloped Structures demand		48
Own price elasticity of Labour+Materials+Fuel demand		-0.084
Own price elasticity of Equipment demand		-0.146
Own price elasticity of Structures demand		-0.287

Table A3.44		
Specification: Three Input Model AR - Shift and Interactive CN Dummy Locally Concave		
Economies of Scale	Minimum	-267.84
	Maximum	357.82
	Mean	8.62
Passenger Cost Elasticity	Minimum	-0.006
	Maximum	0.096
	Mean	0.048
Freight Cost Elasticity	Minimum	-0.083
	Maximum	0.306
	Mean	0.125
Negatively sloped Labour+Materials+Fuel demand		45
Negatively sloped Equipment demand		43
Negatively sloped Structures demand		47
Own price elasticity of Labour+Materials+Fuel demand		-0.096
Own price elasticity of Equipment demand		-0.200
Own price elasticity of Structures demand		-0.297

Table A3.45		
Specification: Three Input Model B - Shift and Interactive CN Dummy Locally Concave		
Economies of Scale	Minimum	-114.48
	Maximum	162.40
	Mean	-1.75
Passenger Cost Elasticity	Minimum	-0.021
	Maximum	0.040
	Mean	0.014
Freight Cost Elasticity	Minimum	-0.236
	Maximum	0.119
	Mean	-0.072
Negatively sloped Labour+Materials+Fuel demand		37
Negatively sloped Equipment demand		33
Negatively sloped Structures demand		40
Own price elasticity of Labour+Materials+Fuel demand		-0.013
Own price elasticity of Equipment demand		-0.022
Own price elasticity of Structures demand		-0.066

Table A3.46		
Specification: Three Input Model BR - Shift and Interactive CN Dummy Locally Concave		
Economies of Scale	Minimum	-28.47
	Maximum	67.26
	Mean	1.58
Passenger Cost Elasticity	Minimum	-0.039
	Maximum	0.056
	Mean	0.017
Freight Cost Elasticity	Minimum	-0.200
	Maximum	0.240
	Mean	0.008
Negatively sloped Labour+Materials+Fuel demand		40
Negatively sloped Equipment demand		42
Negatively sloped Structures demand		42
Own price elasticity of Labour+Materials+Fuel demand		-0.025
Own price elasticity of Equipment demand		-0.125
Own price elasticity of Structures demand		-0.093

Table A3.47		
Specification: Three Input Model A - Shift and Interactive CN Dummy Globally Concave		
Economies of Scale	Minimum	1.45
	Maximum	9.30
	Mean	3.50
Passenger Cost Elasticity	Minimum	0.044
	Maximum	0.118
	Mean	0.083
Freight Cost Elasticity	Minimum	0.030
	Maximum	0.600
	Mean	0.269
Own price elasticity of Labour+Materials+Fuel demand		-0.319
Own price elasticity of Equipment demand		-0.866
Own price elasticity of Structures demand		-0.815

Table A3.48		
Specification: Three Input Model AR - Shift and Interactive CN Dummy Globally Concave		
Economies of Scale	Minimum	1.28
	Maximum	6.97
	Mean	2.94
Passenger Cost Elasticity	Minimum	0.041
	Maximum	0.139
	Mean	0.084
Freight Cost Elasticity	Minimum	0.072
	Maximum	0.684
	Mean	0.326
Own price elasticity of Labour+Materials+Fuel demand		-0.322
Own price elasticity of Equipment demand		-0.860
Own price elasticity of Structures demand		-0.818

Table A3.49		
Specification: Three Input Model B - Shift and Interactive CN Dummy Globally Concave		
Economies of Scale	Minimum	1.43
	Maximum	7.64
	Mean	3.24
Passenger Cost Elasticity	Minimum	0.075
	Maximum	0.139
	Mean	0.010
Freight Cost Elasticity	Minimum	0.056
	Maximum	0.563
	Mean	0.284
Own price elasticity of Labour+Materials+Fuel demand		-0.317
Own price elasticity of Equipment demand		-0.868
Own price elasticity of Structures demand		-0.815

Table A3.50		
Specification: Three Input Model BR - Shift and Interactive CN Dummy Globally Concave		
Economies of Scale	Minimum	1.40
	Maximum	11.46
	Mean	3.64
Passenger Cost Elasticity	Minimum	0.070
	Maximum	0.151
	Mean	0.098
Freight Cost Elasticity	Minimum	0.002
	Maximum	0.620
	Mean	0.294
Own price elasticity of Labour+Materials+Fuel demand		-0.319
Own price elasticity of Equipment demand		-0.863
Own price elasticity of Structures demand		-0.818

Table A3.51

**Percentage Rate of Growth of Output, Input and
TFP, and Relative TFP for CN and CP: Model A**

(Using Revenue Shares from Freeman et al.)

<u>Year</u>	<u>Canadian National</u>				<u>Canadian Pacific</u>		
	<u>Output</u>	<u>Input</u>	<u>TFP</u>	<u>CN/CP</u>	<u>Output</u>	<u>Input</u>	<u>TFP</u>
1956	-	-	-	0.907	-	-	-
1957	-11.70	-0.09	-11.61	0.863	-9.46	-2.74	-6.72
1958	-4.77	-4.80	0.03	0.849	-3.05	-4.81	1.76
1959	0.91	-2.07	2.99	0.887	-3.46	-1.98	-1.48
1960	-4.21	-4.22	0.01	0.862	-1.75	-4.64	2.89
1961	1.55	-2.13	3.68	0.854	0.83	-3.85	4.68
1962	2.22	-2.07	4.29	0.893	-1.29	-1.02	-0.27
1963	12.17	-1.11	13.28	0.923	10.19	0.14	10.06
1964	12.32	4.43	7.88	0.852	15.59	-0.21	15.80
1965	3.55	0.37	3.18	0.895	-1.54	0.16	-1.70
1966	8.15	0.38	7.76	0.881	8.44	-0.87	9.31
1967	0.77	0.35	0.42	0.937	-6.11	-0.43	-5.68
1968	-0.74	-6.81	6.07	0.978	-3.32	-5.08	1.75
1969	2.95	0.56	2.39	0.965	4.24	0.48	3.76
1970	7.85	-2.53	10.38	0.933	13.86	0.08	13.78
1971	8.80	0.60	8.20	0.932	7.61	-0.65	8.26
1972	6.50	0.70	5.79	0.912	6.99	-1.01	8.00
1973	-2.39	-2.71	0.33	0.868	3.26	-2.02	5.28
1974	9.97	8.64	1.33	0.884	3.42	3.91	-0.48
1975	-2.01	-0.91	-1.09	0.867	-3.00	-3.80	0.80
1976	3.67	-3.40	7.08	0.959	-2.76	0.21	-2.97
1977	3.66	-1.78	5.44	0.968	5.88	1.42	4.46
1978	4.02	2.21	1.81	0.946	6.51	2.37	4.14
1979	4.81	-1.12	5.94	0.964	4.40	0.31	4.09
1980	3.52	0.64	2.88	0.971	0.11	-1.99	2.11
1981	-1.25	-0.35	-0.91	0.949	4.75	3.31	1.44
Mean	2.81	-0.69	3.50		2.41	-0.91	3.32

Table A3.52

**Canadian National
Output Cost Elasticities and Revenue Shares (Freeman et al.)
Globally and Locally Concave Model A**

<u>Year</u>	<u>Global Pass.</u>	<u>Global Frt.</u>	<u>Local Pass.</u>	<u>Local Frt.</u>	<u>Revenue Shares Pass.</u>	<u>Revenue Shares Frt.</u>
1956	0.09	0.22	0.05	0.10	0.09	0.91
1957	0.11	0.29	0.05	0.07	0.09	0.91
1958	0.11	0.25	0.06	0.07	0.09	0.91
1959	0.10	0.21	0.06	0.07	0.08	0.92
1960	0.11	0.22	0.07	0.06	0.09	0.91
1961	0.11	0.23	0.07	0.07	0.08	0.93
1962	0.10	0.23	0.07	0.08	0.07	0.93
1963	0.09	0.24	0.06	0.10	0.07	0.93
1964	0.08	0.19	0.04	0.11	0.08	0.92
1965	0.07	0.19	0.03	0.11	0.08	0.92
1966	0.06	0.20	0.02	0.12	0.09	0.91
1967	0.06	0.17	0.02	0.10	0.12	0.88
1968	0.06	0.19	0.02	0.12	0.10	0.90
1969	0.06	0.22	0.02	0.14	0.09	0.91
1970	0.06	0.23	0.02	0.16	0.08	0.92
1971	0.04	0.20	0.02	0.18	0.07	0.93
1972	0.05	0.26	0.02	0.21	0.07	0.93
1973	0.06	0.30	0.03	0.22	0.06	0.94
1974	0.05	0.35	0.02	0.24	0.07	0.94
1975	0.08	0.50	0.02	0.25	0.06	0.94
1976	0.09	0.60	0.02	0.26	0.06	0.94
1977	0.07	0.49	0.02	0.26	0.06	0.94
1978	0.07	0.48	0.02	0.27	0.06	0.94
1979	0.06	0.47	0.02	0.28	0.10	0.90
1980	0.05	0.45	0.01	0.29	0.12	0.88
1981	0.05	0.45	0.02	0.29	0.13	0.88
Mean	0.07	0.30	0.03	0.16	0.08	0.92

Table A3.53

Canadian Pacific
Output Cost Elasticities and Revenue Shares (Freeman et al.)
Globally and Locally Concave Model A

<u>Year</u>	<u>Global</u> <u>Pass.</u>	<u>Global</u> <u>Frt.</u>	<u>Local</u> <u>Pass.</u>	<u>Local</u> <u>Frt.</u>	<u>Revenue Shares</u> <u>Pass.</u>	<u>Revenue Shares</u> <u>Frt.</u>
1956	0.08	0.03	0.06	0.04	0.11	0.89
1957	0.09	0.06	0.06	0.01	0.12	0.89
1958	0.10	0.08	0.07	0.02	0.11	0.89
1959	0.10	0.04	0.07	0.01	0.10	0.90
1960	0.10	0.06	0.08	0.02	0.10	0.90
1961	0.10	0.10	0.08	0.03	0.08	0.92
1962	0.10	0.07	0.08	0.03	0.09	0.92
1963	0.09	0.09	0.08	0.06	0.08	0.92
1964	0.07	0.06	0.06	0.08	0.08	0.92
1965	0.08	0.08	0.07	0.09	0.07	0.93
1966	0.08	0.15	0.07	0.14	0.04	0.96
1967	0.09	0.16	0.08	0.12	0.06	0.95
1968	0.09	0.19	0.08	0.13	0.04	0.96
1969	0.09	0.21	0.08	0.14	0.04	0.96
1970	0.09	0.27	0.08	0.19	0.04	0.97
1971	0.08	0.27	0.08	0.22	0.03	0.97
1972	0.08	0.28	0.08	0.24	0.03	0.97
1973	0.08	0.30	0.09	0.27	0.02	0.98
1974	0.07	0.30	0.07	0.25	0.03	0.97
1975	0.10	0.48	0.09	0.27	0.02	0.98
1976	0.12	0.55	0.10	0.27	0.02	0.98
1977	0.10	0.50	0.09	0.29	0.02	0.98
1978	0.10	0.50	0.09	0.30	0.02	0.98
1979	0.09	0.48	0.08	0.29	0.04	0.96
1980	0.09	0.46	0.07	0.28	0.04	0.96
1981	0.08	0.41	0.07	0.29	0.04	0.96
Mean	0.09	0.24	0.08	0.16	0.06	0.94

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