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**KALMAN FILTERING
OF
NARROWBAND INTERFERENCE
FROM
DIRECT SEQUENCE SPREAD SPECTRUM
COMMUNICATIONS SYSTEMS**

**By
Brian William Kozminchuk, B.Sc.E.E., M.Sc.E.E.**

**A THESIS SUBMITTED TO THE FACULTY OF
GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY**

**OTTAWA-CARLETON INSTITUTE FOR ELECTRICAL ENGINEERING
FACULTY OF ENGINEERING
DEPARTMENT OF SYSTEMS AND COMPUTER ENGINEERING
CARLETON UNIVERSITY**

August 1992

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**KALMAN FILTERING OF NARROWBAND INTERFERENCE
FROM DIRECT SEQUENCE SPREAD SPECTRUM
COMMUNICATIONS SYSTEMS**

submitted by Brian William Kozminchuk, B.Sc.E.E., M.Sc.E.E., in partial fulfilment
of the requirements for the degree of Doctor of Philosophy.

External Examiner

Thesis Supervisor

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August 1992

Abstract

Direct sequence spread spectrum communications systems have an inherent processing gain which is the ratio of the spread bandwidth to the underlying information bandwidth. One purpose of spreading the bandwidth is, through the processing gain, to suppress the effects of interference, resulting in improved signal-to-noise ratio at the output of the pseudo-noise correlator in the spread-spectrum receiver. Sometimes the processing gain is insufficient to neutralize the negative effects of interference and an interruption in communications, as manifested by a sudden increase in bit error rate, will occur. A great deal of research has been devoted to enhancing system performance by temporally or spatially nulling (or excising as it is sometimes referred to) the interference prior to the despreading operation in the receiver.

This thesis is concerned with temporal filtering, utilizing digital techniques. Of importance with the temporal approach are the relative bandwidths of the spread spectrum signal and interference—the larger the bandwidth of the null or notch, the more the signal distortion at the output of the excisor even though interference suppression may be excellent. This latter effect will produce a lower limit on performance. However, this lower limit depends on the type of interference and the filtering algorithm.

Interferers can be extremely narrowband such as that arising from a single tone, or of larger bandwidth such as Gaussian coloured noise, and those of nonlinear modulations. The algorithms can, for example, be based on the fast Fourier transform or least squares methods.

This thesis presents a novel and viable interference excisor based on a Kalman filtering technique. The thesis first examines the behaviour and performance of block

and recursive least squares algorithms for the case of single-tone and swept-tone interferers. For the latter case, the results of the recursive least squares algorithm that utilizes the lattice filter structure with and without its matched filter, poses the question of whether or not an alternative algorithm based on nonlinear concepts could be developed to cope better with nonlinear types of modulations. An extended Kalman filtering technique is suggested. The basic theory of the Kalman filtering approach is presented along with its modified versions, the latter of which can be useful for specific applications.

The Kalman filtering technique is compared to the lattice recursive least squares algorithm from the perspective of bit error rate performance. In the trial cases presented, the Kalman filtering approach performed better than its least squares counterpart by 1 to 2 dB.

A theoretical bit error rate formula is developed for FM interference generated by a state space model. For this case, it is shown that phase-noise at the output of the Kalman filter will set the limiting performance level along with signal distortion. It is furthermore shown that the Kalman filter has a significantly more difficult time with narrowband Gaussian noise because of the added complexity created by a time-varying envelope. This latter characteristic leads to sudden phase changes incapable of being tracked adequately by the Kalman filter and adds to the amount of phase-noise and therefore residual interference at the output of the excisor.

Ways and means of counteracting these effects are presented, the most important of which is the addition of phase-smoothing on the Kalman filter's phase estimate to dampen the additional phase-noise. This results in improved performance by as much as 4 to 5 dB over the case without phase-smoothing.

In an environment in which the interference characteristics change over time, it is desirable to have an adaptive filtering structure. An adaptive scheme utilizing the Kalman filter approach is presented, in which one of its parameters is varied over time while monitoring a mean-squared error function in order to locate the optimum setting for the filter. This is shown to be a reasonable and viable technique. Its performance for narrowband Gaussian noise is shown to be very similar to that of the non-adaptive case in which the parameter settings have been optimized.

The thesis concludes with a presentation of areas requiring further research. Additional architectures are detailed and discussed which incorporate decision-feedback for the purpose of improving bit error rate performance. It is suggested that these structures will be effective if signal distortion is considered to be a significant problem in so far as limiting system performance.

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Contents

Abstract	iii
Acknowledgements	vi
1 Introduction	1
2 Review of Excision Techniques	6
2.1 Introduction	6
2.2 Basic Signal Definitions	6
2.3 Analog Excision Techniques	7
2.4 Digital Excision Techniques	12
2.4.1 Preliminaries	13
2.4.2 Block Least Squares Algorithms	19
2.4.3 Recursive Algorithms	26
2.4.4 A Review of Research Work in Digital Excision	35
2.5 Chapter Summary	50
3 Least Squares Adaptive Excision	52
3.1 Introduction	52
3.2 Communications System Model	52
3.2.1 Stable Tone Performance	54
3.2.2 Swept Tone Performance	79

4	The Kalman Filter Digital Phase-Locked Loop	97
4.1	Introduction	97
4.2	Model of the Excision System	98
4.3	Kalman Filter Algorithm	100
4.3.1	State Space Model of the Interference	101
4.3.2	Extended Kalman Filter Equations	103
4.4	The Digital Phase-Locked Loop (DPLL)	104
4.5	Performance Characteristics of the DPLL	112
4.5.1	Phase Noise Performance	112
4.5.2	Step and Ramp Responses of the DPLL	113
4.6	The Interference Estimator	125
5	Excision Performance of the Kalman Filter	127
5.1	Introduction	127
5.2	Swept Tone Performance	128
5.2.1	Optimum d Determination	129
5.2.2	Bit Error Rate Performance	137
5.3	Theoretical Bit Error Performance	139
5.4	Narrowband Gaussian Noise	162
5.4.1	Filter Lag Effects	162
5.4.2	Phase-Noise Effects	164
5.5	An Adaptive Kalman Filter Excisor	183
5.6	Concluding Remarks	188
6	Conclusions	206
A	A List of Principle Symbols	217
B	A List of Acronymns	226
	Bibliography	229

List of Tables

2.1	Levinson-Durbin Algorithm.	17
2.2	Algorithm Computational Effort.	34
2.3	Summary of Digital Excision.	51
3.1	Unnormalized Pre-Windowed Least Squares Lattice Algorithm.	56
3.2	Computing Predictor Coefficients from the Lattice Parameters.	57
3.3	Attenuation performance of the lattice RLS algorithm.	58
4.1	The Kalman Filter Algorithm.	105
4.2	Parameters for the Step and Ramp Response Tests.	119

List of Figures

1.1	Spread Spectrum Signal of Bandwidth B_s , and Narrowband Jammer of Bandwidth B_j	2
1.2	Two conceptual approaches to interference excision: (a) Analog excisor; (b) Digital excisor.	3
2.1	An analog excisor implemented with a phase-locked loop.	8
2.2	An analog excisor implemented with SAW devices: (a) Conceptual block diagram; (b) Implementation.	10
2.3	Whitening filter for a stationary, p^{th} order autoregressive process: (a) Transfer function; (b) Transversal filter implementation.	14
2.4	Whitening filter for a stationary, p^{th} order autoregressive process, implemented as a lattice structure.	16
2.5	The results of three window types which can be imposed on the data.	20
2.6	Least squares lattice structure.	22
2.7	Two-sided or interpolation error transversal filter which produces the interpolation error $e_{p,n}$	39
2.8	Whitening filter followed by its matched filter.	41
2.9	Maximum likelihood receiver for the case of signal in coloured noise.	45
2.10	Excision filter employing decision feedback, which approximates the maximum likelihood receiver.	46
2.11	Two alternative decision-feedback structures based on: (a) Chip decisions; and, (b) Improving bit decisions.	47
2.12	A decision-feedback structure employing a matched filter arrangement.	49

3.1	Communications model used in the computer simulations.	53
3.2	Recursive least squares lattice structure followed by its matched filter.	54
3.3	Root locations of whitening filter when the interference is a single tone: $p = 2, \lambda = 1.0$	59
3.4	Magnified version of the second root in Fig. 3.3.	60
3.5	Root locations of whitening filter when the interference is a single tone: $p = 2, \lambda = 0.9$	61
3.6	Magnified version of the second root in Fig. 3.5.	62
3.7	Root locations of whitening filter when the interference is a single tone: $p = 4, \lambda = 1.0$	63
3.8	Root locations of whitening filter when the interference is a single tone: $p = 4, \lambda = 0.9$	64
3.9	Root locations of whitening filter when the interference is a single tone: $p = 6, \lambda = 1.0$	65
3.10	Root locations of whitening filter when the interference is a single tone: $p = 6, \lambda = 0.9$	66
3.11	Root locations of whitening filter when the interference is a single tone: $p = 8, \lambda = 1.0$	67
3.12	Root locations of whitening filter when the interference is a single tone: $p = 8, \lambda = 0.9$	68
3.13	Model for a least squares excision filter for the case when $\lambda < 1.0$	70
3.14	Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 1.0$. The theoretical curves are based on Eq. (3.9).	73
3.15	Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.99$. The theoretical curves are based on Eq. (3.9)	74
3.16	Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.98$. The theoretical curves are based on Eq. (3.9)	75

3.17	Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.97$. The theoretical curves are based on Eq. (3.9)	76
3.18	Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.96$. The theoretical curves are based on Eq. (3.9)	77
3.19	Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.95$. The theoretical curves are based on Eq. (3.9)	78
3.20	Modulating signal $m(\tau)$ in Eq. (3.12).	81
3.21	Tracking results for the BLS algorithm for $p = 4$ and block size $N = 100$	82
3.22	Tracking results for the BLS algorithm for $p = 4$ and block size $N = 50$	83
3.23	Tracking results for the BLS algorithm for $p = 4$ and block size $N = 10$	84
3.24	Tracking results for the RLS algorithm for $p = 2$ and $\lambda = 0.90$	85
3.25	Tracking results for the RLS algorithm for $p = 4$ and $\lambda = 0.90$	86
3.26	Tracking results for the RLS algorithm, $p = 2$, $\lambda = 0.95$	87
3.27	Tracking results for the RLS algorithm, $p = 4$, $\lambda = 0.99$	88
3.28	Tracking results for the RLS algorithm, $p = 4$, $\lambda = 1.0$	89
3.29	Bit error rate performance of the BLS algorithm for filter order $p = 2$ and several block sizes N when the interference was a swept tone; SIR/chip = -20 dB.	93
3.30	Bit error rate performance of the BLS algorithm for filter order $p = 4$ and several block sizes N when the interference was a swept tone; SIR/chip = -20 dB.	94
3.31	Bit error rate performance of the RLS algorithm for $\lambda = 0.90$ and filter orders $p = 2$ and $p = 4$ when the interference was a swept tone; the SIR/chip was -20 dB.	95
3.32	Bit error rate performance of the RLS algorithm for filter order $p = 2$ and $\lambda = 0.90$ and 0.95 when the interference was a swept tone; the SIR/chip was -20 dB.	96

4.1	Spread spectrum communications model.	99
4.2	Block diagram of the state space model.	102
4.3	Discrete form of the interference model.	103
4.4	Illustration of the sampling rate requirements: (a) Hypothetical interference spectrum of $q_1(t)$; (b) Convolution of $Q_1(f)$ with $Q_2(f)$; (c) Sampled spectrum of the product $q_1(t)q_2(t)$	107
4.5	Analog and sampled spectra of $q_3(t)$ (see Eq. (4.25)).	108
4.6	State space representation of the DPLL.	109
4.7	Nonlinear baseband model of the DPLL.	110
4.8	Transfer function model representation of the DPLL.	111
4.9	Linear transfer function model representation of the DPLL.	111
4.10	Performance curves using the model in [1] for the case when the Kalman filter is in the filter mode.	114
4.11	Performance curves using the modified Kalman filter in the filter mode.	115
4.12	Performance curves using the model in [1] for the case when the Kalman filter is in the predictor mode.	116
4.13	Performance curves using the model in [1] for the case when the Kalman filter is in the predictor mode.	117
4.14	Error response of the Kalman filter in [1] for a unit step input and $\sigma_v^2 = 0.1$	120
4.15	Error response of the Kalman filter in [1] for a unit step input and $\sigma_v^2 = 0.025$	121
4.16	Error response of the Kalman filter in [1] for $\delta f = 0.09$ Hz, $m_2 = 0.0$ and two values of σ_v^2 (i.e., 0.1 and .025).	122
4.17	Error response of the modified Kalman filter for $\delta f = 0.09$ Hz, $m_2 = 0.0$ and two values of σ_v^2 (i.e., 0.1 and .025).	123
4.18	Error response of the modified Kalman filter for $\delta f = 0.19$ Hz, $m_2 = 0.0$ and 0.1 Hz, for $\sigma_v^2=0.025$ and $d = 0.196$	124
4.19	Block diagram of the interference estimator.	125

5.1	Suppression level $\mathcal{S}$ from Eq. (5.9) for several values of d , and $B_{LPF} = 0.1$ Hz.	132
5.2	An example of the phase-tracking ability of the Kalman filter for $d = 0.20$ rad./sec./volt, and $B_{LPF} = 0.10$ Hz.	133
5.3	An example of the phase error $\theta_n - \hat{\theta}_{n n-1}$ for $d = 0.20$ rad./sec./volt, and $B_{LPF} = 0.10$ Hz.	134
5.4	An example of the envelope estimator for $d = 0.20$ rad./sec./volt, and $B_{LPF} = 0.10$ and 0.025 Hz.	135
5.5	Power spectrum of the interference and its estimate for $d = 0.2$ rad./sec./volt, and $B_{LPF} = 0.1$ Hz.	136
5.6	Comparison of the bit error rate performance between the RLS algorithm for $\lambda = 0.90$, and the Kalman filter algorithm for $d = 0.20$ rad./sec./volt and several values of low pass filter bandwidth (LPF BW) B_{LPF} . The acronyms NMF and MF respectively refer to “No Matched Filter” and “Matched Filter”.	138
5.7	Histogram of residual interference for the case when the interference is FM, generated by the state-space model in Fig. 4.3.	142
5.8	Histogram of phase error ϵ_n	144
5.9	Histogram of $n'_{2,n}/(\sqrt{2}I)$ from Eq. (5.29) before Kalman filtering.	149
5.10	Histogram of $n'_{2,n}/(\sqrt{2}I) * h_{Kal,n}$ from Eq. (5.29).	150
5.11	Histogram of the signal term $s_n \cos(\theta_n)$ from Eq. (5.30) before Kalman filtering.	151
5.12	Histogram of the signal term $s_n \cos(\theta_n) * h_{Kal,n}$ from Eq. (5.30).	152
5.13	Power spectral density of the signal term $s_n \cos(\hat{\theta}_{n n-1})$ and noise term $n_{B,n}/\sqrt{2}$ for the case $E_b/N_0 = 12$ dB, ($E_b = L = 20$).	153
5.14	Power spectral density of $s_n + n_n + i_n$ for $E_b/N_0 = 0$ dB and $E_b/N_0 = 12$ dB.	154
5.15	Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.1$ Hz.	155
5.16	Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.2$ Hz.	156

5.17	Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.3$ Hz.	157
5.18	Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.4$ Hz.	158
5.19	Power spectral density of signal s_n , the filtered signal term $\{[s_n \cos(\hat{\theta}_{n n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n n-1})$ and the residual signal term $s_n - \{[s_n \cos(\hat{\theta}_{n n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n n-1})$. The bandwidth of the lowpass filter is $B_{LPF} = 0.1$ Hz.	159
5.20	Power spectral density of signal s_n , the filtered signal term $\{[s_n \cos(\hat{\theta}_{n n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n n-1})$, and the residual signal term $s_n - \{[s_n \cos(\hat{\theta}_{n n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n n-1})$. The bandwidth of the lowpass filter is $B_{LPF} = 0.4$ Hz.	160
5.21	Power spectral density of interference i_n , the interference estimate $\{[I \cos(\epsilon_n)] * h_{LPF,n}\} \cos(\theta_{n n-1})$, and the residual term $i_n - \{[I \cos(\epsilon_n)] * h_{LPF,n}\} \cos(\theta_{n n-1})$. The bandwidth of the lowpass filter is $B_{LPF} = 0.1$ Hz.	161
5.22	Level of interference suppression $\mathcal{S}$ as a function of d for several lowpass filter bandwidths. The interference is narrowband Gaussian noise of bandwidth 0.02 Hz.	166
5.23	An example of a portion of the envelope I_n and its estimate $\hat{I}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.20$ Hz with $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.	167
5.24	An example of a portion of the interference i_n and its estimate $\hat{i}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.20$ Hz, $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.	168
5.25	An example of a portion of the envelope I_n and its estimate $\hat{I}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.40$ Hz with $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.	169
5.26	An example of a portion of the interference i_n and its estimate $\hat{i}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.40$ Hz, $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.	170

5.27	Modified interference estimator of Fig. 4.19 when the low pass filter is a linear phase FIR filter.	171
5.28	Comparison of the interference suppression levels $\mathcal{S}$ produced by a 7-tap FIR filter and 4 th order Butterworth filter when $B_i = 0.02$ Hz, $B_{LPF} = 0.20$ Hz, and $E_b/N_0 = 12$ dB.	172
5.29	An example of a portion of the envelope I_{n-D} and its estimate $\hat{I}_{n-D}$ when the filter is of the linear phase FIR type of bandwidth $B_{LPF} = 0.20$ Hz with $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.	173
5.30	An example of a portion of the interference i_{n-D} and its estimate $\hat{i}_{n-D}$ when the filter is of the FIR type of bandwidth $B_{LPF} = 0.20$ Hz, $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz. . .	174
5.31	Interference suppression level $\mathcal{S}$ as a function of d for several interference bandwidths, B_i , ranging from 0.01 Hz to 0.04 Hz, with $B_{LPF} = 0.20$ Hz.	175
5.32	Interference estimator when phase-smoothing is being used.	176
5.33	An example of the phase θ_n and its Kalman filter estimate $\hat{\theta}_{n n-1}$, with interference bandwidth $B_i = 0.02$ Hz and $d = 0.30$ rad./sec./volt. . .	177
5.34	An example of the phase θ_n and a 7-point smoothed version $\hat{\theta}'_{n-D}$ of its Kalman filter estimate $\hat{\theta}_{n n-1}$, with interference bandwidth $B_i = 0.02$ Hz and $d = 0.30$ rad./sec./volt.	178
5.35	An example of a segment of interference with and without phase smoothing; interference bandwidth $B_i = 0.02$ Hz and $d = 0.30$ rad./sec./volt. .	179
5.36	Interference suppression level $\mathcal{S}$ as a function d for several interference bandwidths, B_i , ranging from 0.01 Hz to 0.05 Hz, with $B_{LPF} = 0.20$ Hz.	180
5.37	Bit error rate for the case of narrowband Gaussian noise with bandwidths B_i ranging from 0.01 Hz to 0.05 Hz, using the optimum values of d obtained from Fig. 5.36, and an FIR filter with no averaging on the phase estimate $\hat{\theta}_{n n-1}$	181

5.38	Bit error rate for the case of narrowband Gaussian noise with bandwidths B_i ranging from 0.01 Hz to 0.05 Hz, using the optimum values of d obtained from Fig. 5.36, and an FIR filter with 7-point averaging on the phase estimate $\hat{\theta}_{n n-1}$	182
5.39	Profile of the frequency deviation constant d used in the phase- and envelope-tracking tests.	183
5.40	An example of the phase-tracking capability of the Kalman filter when the frequency deviation constant d was varied according to the profile in Fig. 5.39.	190
5.41	An example of the envelope-tracking capability of the Kalman filter when the frequency deviation constant d was varied according to the profile in Fig. 5.39.	191
5.42	Adaptive architecture, which is based on minimizing the squared error of the error signal e_n , while varying d	192
5.43	Illustration of the adaptive Kalman filter process.	193
5.44	An example of the profile of the frequency deviation constant d used in the adaptive algorithm, $B_i = 0.01$ Hz.	194
5.45	An example of the error signal e_{n-D} for the d profile in Fig. 5.44 used in the adaptive algorithm, with $B_i = 0.01$ Hz.	195
5.46	An example of the estimated power in u_{n-D} calculated from Eq. (5.45) for the d profile in Fig. 5.44 used in the adaptive algorithm, with $B_i = 0.01$ Hz.	196
5.47	An example of the ratio of $r_{mse} = \overline{e_{n-D}^2} / \overline{u_{n-D}^2}$ for the d profile in Fig. 5.44 used in the adaptive algorithm, with $B_i = 0.01$ Hz.	197
5.48	An example of the residual interference $\Delta i_{n-D} = i_{n-D} - \hat{i}_{n-D}$ before the despreader in the adaptive algorithm, with $B_i = 0.01$ Hz.	198
5.49	An example illustrating the degree of interference suppression $10 \log(\overline{\Delta i_{n-D}^2} / P_i)$, where $P_i = 100$ is the power in the interferer of bandwidth $B_i = 0.01$ Hz.	199
5.50	An example of the profile of the frequency deviation constant d used in the adaptive algorithm, $B_i = 0.05$ Hz.	200

5.51	An example of the estimated power in u_{n-D} calculated from Eq. (5.45) for the d profile in Fig. 5.50 used in the adaptive algorithm, with $B_i = 0.05$ Hz.	201
5.52	An example of the ratio of $r_{max} = \overline{e_{n-D}^2}/\overline{u_{n-D}^2}$ for the d profile in Fig. 5.50 used in the adaptive algorithm, with $B_i = 0.05$ Hz.	202
5.53	An example of the residual interference $\Delta i_{n-D} = i_{n-D} - \hat{i}_{n-D}$ before the despreader in the adaptive algorithm, with $B_i = 0.05$ Hz.	203
5.54	An example illustrating the degree of interference suppression $10 \log(\overline{\Delta i_{n-D}^2}/P_i)$, where $P_i = 100$ is the power in the interferer of bandwidth $B_i = 0.05$ Hz.	204
5.55	Bit error performance of the adaptive scheme for several interference bandwidths and $E_b/N_0 = 12$ dB using the interference estimator in Fig. 5.32.	205
6.1	An adaptive architecture in which a reference signal I_{ref} is used by the Kalman algorithm: (a) System level diagram; (b) Interference estimator, error processor and controller.	213
6.2	A decision feedback excisor.	214
6.3	The interference estimator of the decision feedback excisor in Fig. 6.2.	215
6.4	A decision feedback excisor that uses an initial training sequence.	216

Chapter 1

Introduction

Direct sequence spread spectrum communication systems have an inherent processing gain which reduces the harmful effects of interference. When the interference has a power advantage over the spread spectrum system, a severe degradation in communications can result as manifested by a sudden increase in the bit error rate. The effects of the interference can be mitigated somewhat by filtering it out, particularly if it is narrowband. This technique is sometimes referred to as interference excision.

The basic concept is illustrated in Fig. 1.1. Consider the received spread spectrum signal of bandwidth $B_{ss} = 2R_c$, centered at an intermediate frequency (IF) of f_0 Hz, where R_c is the chip rate of the spread spectrum signal. Also present in the signal bandwidth is an interferer of bandwidth $B_i \ll B_{ss}$, and centered at $f_0 + \delta f$. The interference can be filtered out either at the IF or at baseband. There are several possible configurations which can effect this filtering. Two concepts are illustrated in Figs. 1.2(a) and (b), which will not necessarily produce equivalent results because of the way in which the excision filters are implemented, i.e., whether or not an optimum criterion is being met. In these figures $u(t)$ is defined as

$$u(t) = s(t) + n(t) + i(t), \quad (1.1)$$

where $s(t)$ is the spread spectrum signal, $n(t)$ is bandpass Gaussian noise and $i(t)$ is the narrowband interfering signal.

In Fig. 1.2(a), $u(t)$ is applied to an excision filter first. The output is despread by correlating it with the pseudo-noise (PN) sequence, mixed down to baseband and

Figure 1.1: Spread Spectrum Signal of Bandwidth B_s , and Narrowband Jammer of Bandwidth B_i .

bit-detected. In Fig. 1.2(b), $u(t)$ is mixed down to baseband first and then sampled at the chip rate. The assumption is made that carrier and chip synchronization have been achieved, which will not be the case initially. The sampled sequence, u_n , is processed by an algorithm which determines a set of filter coefficients for the excision filter using some optimization criterion. The output of the filter, which is, say, of the finite impulse response (FIR) type of order p , is the error signal $e_{p,n}^f$. This error signal is then correlated with the PN sequence and applied to the bit detector.

This thesis presents a novel digital excisor which uses an extended Kalman filter algorithm. The motivation for developing this approach was the desire to filter out interferers having nonlinear modulations, such as those of the FM-type. This idea was spawned by the fact that: (a) most of the research, as the literature survey in the next chapter will attest to, had been devoted to filtering stable tones and coloured noise; and (b) the desire to improve on the results based on the initial research conducted by the author into the use of least-squares filtering algorithms to suppressing interferers having nonlinear modulations. In fact, the results from the least-squares work begged the question as to whether or not an alternative (non-) linear algorithm/approach could be more appropriate. An added requirement was that the new algorithm would

Figure 1.2: Two conceptual approaches to interference excision: (a) Analog excisor; (b) Digital excisor.

have to filter out narrowband Gaussian noise too - a more general type of interference.

The thesis outline is as follows. Chapter 2 summarizes the research conducted by various people in the area of analog and digital excision techniques, with emphasis on the latter. A large portion of the chapter is devoted to highlighting the algorithms that can be used to estimate the filter coefficients in Fig. 1.2(b) when the sampled signal, noise and interference (i.e., u_n) are modeled as an autoregressive process. These algorithms can be of the block type, which estimate the filter coefficients periodically from blocks of data, or of the recursive type, which estimate the filter coefficients from data sample to data sample. In both cases the objective is to obtain practical solutions to the Wiener-Hopf equation resulting from the time domain (autoregressive) model.

Chapter 3 presents work carried out by the author using the autoregressive model. This includes the performance of the recursive least squares algorithm in the presence of a stable tone interferer for various filter orders. A model for the recursive least squares algorithm is then used to produce theoretical bit error rate curves for a stable tone interferer, which are then compared to computer simulations. Following this, the performance of the block and recursive least squares algorithms are compared to one another when the interference is of the FM-type, i.e., an interferer in which the modulating signal is a linearly swept tone. The results of each approach are compared to one another in terms of their tracking capabilities and bit error rate performance. The results suggest that perhaps an alternate excision algorithm may be better, i.e., one suited to interferers with nonlinear modulations. This leads to a new algorithm also based on time-modelling techniques.

Chapter 4 presents the framework for such an algorithm. The interference in this case is considered to be generated by a nonlinear state-space model, to which the extended Kalman filter equations are applied. This results in what has been termed the digital phase-locked loop, which has performance characteristics similar to an analog phase-locked loop. The basic DPLL, which estimates the phase of the interference, is used in an algorithm that estimates the interference. The chapter presents phase-noise performance curves (theoretical and simulated) for filter and predictor algorithms of two forms of the Kalman filter: one form in which the loop filter consists of a lag network (which leads to steady-state phase errors for ramp

inputs), and one form in which the loop filter consists of proportional-plus-integral control (which removes the steady-state phase errors for a ramp input). In fact, this latter Kalman filter is a limiting case of the former, as one of the model parameters of the Kalman filter approaches zero. It should be noted that either the filter or predictor algorithms can be used in the interference excisor; however, the latter is required if the excisor is used in a decision-feedback scheme, which will be alluded to in Chapter 6. The chapter concludes with examples of the step and ramp responses of the two forms of the Kalman filter.

Chapter 5 is devoted to the performance characteristics of the filter form of the Kalman algorithm based on proportional-plus-integral control. The non-adaptive algorithm is considered first. By appropriate selection of the Kalman filter parameters, it is shown through simulation that better performance can be achieved than the RLS algorithm for the case of a swept tone interferer. Following this, the theoretical performance of the excisor for general FM interference (produced by the state-space model for interference in Chapter 4) is presented and compared with simulated results. Next, the performance of the excisor is analyzed against narrowband Gaussian noise interference. This is more difficult to deal with because both the envelope and phase are time-variant. However, ways and means of circumventing this problem are presented. Finally, the chapter concludes with an adaptive algorithm applied to one of a few possible excision architectures.

Chapter 6 concludes the thesis, suggesting areas for further research. Here, three additional architectures are presented, two of which involve decision-feedback.

Chapter 2

Review of Excision Techniques

2.1 Introduction

Several methods of interference excision are presented in this chapter. First a summary of some of the analog techniques to implement excision filters are discussed, which may involve the use of simple notch filters, phase-locked loops (PLL's), or surface acoustic wave (SAW) devices.

Most of the emphasis in this chapter is on adaptive digital filtering methods based on time-domain modeling techniques. There are a number of algorithms associated with these models, and many of these will be highlighted. Following this, the work of numerous researchers in the area of digital excision will be summarized.

2.2 Basic Signal Definitions

In Chapter 1, the received signal $u(t)$ was defined in Eq. (1.1) as the sum of spread spectrum signal, Gaussian noise, and narrowband interference, i.e.,

$$u(t) = s(t) + n(t) + i(t). \quad (2.1)$$

These terms will be further defined here, for later reference.

The spread spectrum signal is defined as

$$s(t) = a(t) \cos(\omega_0 t) \quad (2.2)$$

where

$$a(t) = \sum_k D_k b_k(t - kT_b). \quad (2.3)$$

In Eq. (2.3), D_k is a sequence of data bits of amplitude (± 1) and duration T_b seconds, and $b_k(t - kT_b)$ is the PN sequence pattern for the k^{th} bit, i.e.,

$$b_k(t) = \sum_{j=1}^L c_{kj} q(t - jT_c) \quad (2.4)$$

with L being the number of pseudo random chips per bit, or the processing gain, c_{kj} is the code sequence for the bit, and $q(t)$ represents the basic chip pulse of energy E_c .

The bandpass noise is defined as

$$n(t) = n_1(t) \cos(\omega_0 t) - n_2(t) \sin(\omega_0 t) \quad (2.5)$$

where $n_1(t)$ and $n_2(t)$ are the baseband in-phase and quadrature Gaussian noise components of variance $N_0 B_{ss}$. Finally, the band of interference is defined as

$$i(t) = I(t) \cos(\omega_0 t + \theta(t)) \quad (2.6)$$

where $I(t)$ is the interference amplitude and $\theta(t)$ is the phase modulation. The objective of the excision system is to filter $i(t)$ from $u(t)$.

2.3 Analog Excision Techniques

Several analog techniques for suppressing interference in spread spectrum systems have been proposed [2, 3, 4, 5, 6]. Four methods will now be discussed.

The simplest method [2] is to apply the received signal to a tunable notch filter of bandwidth sufficiently large to attenuate the interference. The output of the filter will consist of residual interference, noise and a somewhat distorted signal due to the filtering operation. This latter effect degrades the correlation of the signal with the PN sequence. The degree of degradation depends on the desired attenuation and the filter order and type.

Experimental performance curves of these tradeoffs were presented in [2]. The interference considered was a double-sideband suppressed carrier signal occupying

a bandwidth equal to 3% of the chip rate and centered at the carrier of the spread spectrum signal. A second order Butterworth filter with a 10 dB bandwidth 2.5 times larger than the interferer's 10 dB bandwidth achieved at least 25 dB interference suppression. The penalty in this case was a 1 dB drop in the PN correlation peak at the output of the PN correlator shown in Fig. 1.2(a). For the same interference conditions and a fourth order Butterworth filter, the degradation in the correlation peak was 1.75 dB. Results also showed that the amount of degradation lessened as the interference and notch filter were offset from the carrier. Finally, theoretical curves indicated that Tchebycheff filters introduced slightly more degradation in the correlation peak than the Butterworth filters of the same order.

A more sophisticated analog technique [3], illustrated in Fig. 2.1, uses a PLL. In this application, the received signal is applied to a PLL of bandwidth B_ϕ . For

Figure 2.1: An analog excisor implemented with a phase-locked loop.

the condition of large (> 10 dB) interference-power-to-noise-plus-signal-power ratios, the PLL locks on to the interference. The voltage-controlled oscillator (VCO) signal, which is in quadrature to the interference, is phase-shifted by $\pi/2$ radians and applied to two mixers $M1$ and $M2$. $M1$ mixes the received signal $u(t)$ down to baseband. The baseband signal $y(t)$ is applied to a low pass filter of bandwidth B_{LPF} . The output is an estimate of the interference amplitude, $\hat{I}(t)$, contaminated by noise and spread spectrum signal. The second mixer $M2$ mixes the same $\pi/2$ phase-shifted VCO signal from the PLL with $\hat{I}(t)$ to form an estimate of the interference, i.e.,

$$\hat{i}(t) = \hat{I}(t) \cos(\omega_0 t + \hat{\theta}(t)) \quad (2.7)$$

which is subtracted from $u(t)$. An analysis carried out in [3] determined that the residual interference-to-signal ratio at the input to the spread spectrum processor is $(B_\phi + B_{LPF})/B_{ss}$. This result suggests that, in theory, a significant amount of suppression of the interferer can be achieved, especially for stable tones in which B_ϕ and B_{LPF} can be quite small in comparison with the spread spectrum bandwidth, B_{ss} .

The third analog approach is effective for both narrowband and broadband interference [4]. The signal, noise and interference are applied concurrently to three devices: a hardlimiter, an amplifier, and a gain estimator. The gain estimator is an automatic gain control device for setting the gain of the amplifier so that its output has the same power level as that of the limiter output which is dominated by the interference. The outputs from the hardlimiter and amplifier are applied to a difference circuit which has the effect of suppressing the interference. This approach performs well for input signal-to-interference levels less than -10 dB.

The fourth analog approach to be discussed implements excision with SAW devices. This method is referred to as transform domain filtering. A significant amount of research has been conducted in this area [7, 8, 5, 9, 10, 11]. The concept is illustrated in Figs. 2.2(a) and (b). The received signal is first windowed to T_i seconds by a window function $\mathcal{W}(t)$. Several types of windows have been used, e.g., rectangular, raised cosine, and Hamming windows. The windowed signal is Fourier transformed by the first SAW device. The transform evolves over time, becoming valid when the

Figure 2.2: An analog excisor implemented with SAW devices: (a) Conceptual block diagram; (b) Implementation.

windowed signal is fully contained in the device. The transform, therefore, is valid over the time interval T_b to T_1 , where T_1 is the interaction time of the SAW device.

The output of the first device is multiplied by a transfer function, $H_c(\omega)$, which is related to a notch filter with transfer function $H_R(\omega)$, i.e.,

$$H_c(\omega) = 4H_R(\omega) \cos(\omega T_1) \quad (2.8)$$

where the cosine term adds a time delay of T_1 seconds to the input signal (since the inverse transform calculated by the second SAW device starts to become valid at T_1 seconds). The frequency variable ω is a function of time and is equal to $2\zeta t$, where ζ , which is defined below, is a parameter of the SAW device. As the spectrum evolves over time, the portion containing the interference is "blanked out" by a gating function represented by $H_c(\omega)$ in Eq. (2.8). The shape of $H_c(\omega)$ is rectangular. The notched spectrum is inverse transformed by the second SAW device. This transform produces a time domain signal of duration T_b seconds valid over the range $[T_1, T_1 + T_b]$. This time domain signal consists of a sequence of chips which are then correlated with the appropriate section of the PN sequence, shown in Fig. 2.2(a) as the spread spectrum processor. The output of this processor is then bit-detected.

A detailed representation of the above process is illustrated in Fig. 2.2(b). Within the dashed lines is the way in which the Fourier transforms are implemented with the SAW devices. The windowed waveform is mixed with a chirp signal defined as $\cos(\omega_a t - \zeta t^2)$ where ω_a is the carrier frequency in radians per second and ζ is the phase sweep rate in radians/second² of the chirp. The product is applied to a tapped delay line, constructed on a SAW device, with an impulse response $\cos(\omega_a t + \zeta t^2)$. The inverse Fourier transform is similarly constructed. The PN correlation process at the output of the second SAW device is represented in Fig. 2.2(b) as a filter with impulse response $b_k(kT_b - t)$. This is the matched filter (code sequence) for the particular bit, and is an alternate representation for the correlation process which was shown in Fig. 1.2(a) [12].

Milstein and Das [9] have used the model in Fig. 2.2(b) to develop a test statistic [12] at the output of the bit detector. This test statistic is defined as

$$\mathcal{U} = \int_0^{T_b} u(t)[h_R(t) * b_k(t)]dt \quad (2.9)$$

where $h_R(t)$ is the inverse Fourier transform of $H_R(\omega)$ in Eq. (2.8) and $*$ refers to the convolution operator. From the mean and variance of $\mathcal{U}$ for a given interference and filtering condition, a theoretical bit error rate expression can be determined. Milstein *et al.* presented results for two such cases, i.e., a tone interferer and Gaussian noise jammer. They also presented an adaptive scheme with supportive experimental results [13].

As a perspective on the performance of such an excision system, and the parameter values which are achievable, one can refer to the experimental system in [14]. The SAW devices used therein had center frequencies of $\omega_c = 2\pi(15)\text{MHz}$, bandwidths of 7 MHz, interaction times of $T_1 = 117\mu\text{sec.}$, and chirp rates of $\zeta = 2\pi(3 \times 10^{10}) \text{ rad./sec.}^2$. The spread spectrum code used for the experiment was a PN sequence of length equal to 63 chips. Furthermore, all 63 chips were contained within an information bit, thus providing a processing gain of 63. The chip rate of the PN sequence was 1.875 MHz.

For a single tone interferer which was offset from the carrier by 180 kHz, and an interference-to-signal (I/S) ratio of 20 dB, the bit error rate (BER) for $E_b/N_0 = 10 \text{ dB}$ was 8×10^{-5} when the excision filter was present. Without the filter the BER was 0.5. The theoretical BER for BPSK systems without interference is 4×10^{-6} . The above results suggest that the excision technique improved the system performance considerably.

2.4 Digital Excision Techniques

In Chapter 1 a digital approach to interference removal was depicted in Fig. 1.2(b). With respect to this approach, this section focuses on two areas: (a) the time-domain algorithms that can be used to calculate the filter parameters; and, (b) a review of the research which has been conducted in digital excision. The signals are all considered to be real.

2.4.1 Preliminaries

The time-domain algorithms referred to in Fig. 1.2(b) are based on time-modeling of the dynamics of stochastic processes. A well-known model [15, 16] is one in which the sampled signal u_n is modelled as a p^{th} order stationary autoregressive (AR) process defined as

$$u_n = w_n - a_1^{(p)}u_{n-1} - \dots - a_p^{(p)}u_{n-p} \quad (2.10)$$

where w_n is a white Gaussian noise process of variance σ_w^2 . An AR process can be generated by passing samples of white Gaussian noise through a digital filter with a transfer function defined as

$$H(z) = \frac{1}{1 + a_1^{(p)}z^{-1} + \dots + a_p^{(p)}z^{-p}} = \frac{1}{A_p(z)} \quad (2.11)$$

where p is the order of the filter.

For the moment, assume the objective is to whiten or, alternatively, to decorrelate a received set of samples obtained from a zero mean, stationary, p^{th} order AR process. In addition, assume that the second order statistics of the process are known. To whiten the signal, the set of parameters $\{a_i^{(p)}, i = 1, 2, \dots, p\}$ must be determined, and a transversal filter with transfer function $A_p(z)$ constructed with the $a_i^{(p)}$ as tap weights. The data u_n could then be passed through this filter whose output would be a white noise sequence $e_{p,n}^f$. This is illustrated in Figs. 2.3(a) and (b). The parameters of the whitening filter $A_p(z)$ can be determined in a straightforward manner by employing linear prediction concepts and the orthogonality principle [17].

The one-step linear predictor is defined as

$$\hat{u}_n = a_1^{(p)}u_{n-1} + a_2^{(p)}u_{n-2} + \dots + a_p^{(p)}u_{n-p}. \quad (2.12)$$

The objective is to find those parameters $a_i^{(p)}$ which will minimize the mean-squared error, $E\{e_{p,n}^{f2}\}$, where $e_{p,n}^f$ is the forward prediction error between u_n and $\hat{u}_n$. The best estimate in a minimum mean-squared error (mmse) sense is the one producing an error orthogonal to the data used in estimating u_n , i.e.,

$$E\{e_{p,n}^f u_{n-i}\} = 0, i = 1, 2, \dots, p. \quad (2.13)$$

Figure 2.3: Whitening filter for a stationary, p^{th} order autoregressive process: (a) Transfer function; (b) Transversal filter implementation.

Substituting $e_{p,n}^f = u_n - \hat{u}_n$ into Eq. (2.13) leads to the following p equations known as the Yule-Walker equations, i.e.,

$$r_i = a_1^{(p)} r_{i-1} + a_2^{(p)} r_{i-2} + \dots + a_p^{(p)} r_{i-p}, \quad i = 1, 2, \dots, p \quad (2.14)$$

where $r_i = E\{u_n u_{n-i}\}$ is the i^{th} lag from the discrete autocorrelation function of the AR process u_n . This series of equations can be written in matrix form as

$$R\mathbf{a} = \mathbf{r} \quad (2.15)$$

where R is $p \times p$ Toeplitz matrix whose elements are constant along the diagonals, i.e.,

$$R = \begin{bmatrix} r_0 & r_1 & \cdot & \cdot & \cdot & r_{p-1} \\ r_1 & r_0 & r_1 & \cdot & \cdot & r_{p-2} \\ \cdot & r_1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ r_{p-1} & \cdot & \cdot & \cdot & \cdot & r_0 \end{bmatrix}. \quad (2.16)$$

Also

$$\mathbf{a} = \begin{bmatrix} a_1^{(p)} & a_2^{(p)} & \cdot & \cdot & \cdot & a_p^{(p)} \end{bmatrix}^t \quad (2.17)$$

and

$$\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \cdot & \cdot & \cdot & r_p \end{bmatrix}^t \quad (2.18)$$

where the superscript t refers to the transpose of a matrix. Equation (2.15) is known as the Wiener-Hopf equation.

The solution to the Wiener-Hopf equation involves the inversion of the matrix R . It can be solved by several standard techniques, such as Gaussian reduction or Cholesky decomposition. The number of mathematical operations required to obtain the inverse by these methods is $p^3/3 + \mathcal{O}(p^2)$ and $p^3/6 + \mathcal{O}(p^2)$, respectively. The term $\mathcal{O}(\cdot)$ refers to "in the order of" ($\cdot$) operations, where an operation is defined as a multiplication or division plus an addition or subtraction.

Alternatively, taking advantage of the Toeplitz structure of R , Eq. (2.15) can be solved most efficiently by means of the Levinson-Durbin (LD) algorithm [18] in $\mathcal{O}(p^2)$

operations. This algorithm not only solves for the p^{th} order equation, but also the lower order ones as well, i.e., for $\{i = 0, 1, \dots, p-1\}$. It is therefore an order recursive algorithm. It also solves for the minimum mean-squared error, $\mathcal{E}_{\min}^{(i)}$, (defined also as σ_i^2) for the i^{th} order. The orthogonality principle yields

$$\begin{aligned}\mathcal{E}_{\min}^{(i)} &= E\{e_{p,n}^f u_n\} \\ &= r_0 - \sum_{k=1}^i a_k^{(p)} r_k \\ &= \sigma_i^2.\end{aligned}\tag{2.19}$$

The LD algorithm is listed in Table 2.1. It proceeds recursively to compute the parameter sets $\{a_1^{(p)}, \sigma_1^2\}, \{a_1^{(2)}, \sigma_2^2\}, \dots, \{a_1^{(p)}, a_2^{(p)}, \dots, a_p^{(p)}, \sigma_p^2\}$. The final set is the desired one and is used in the whitening filter in Fig. 2.3(b). The AR sequence u_n can then be applied to the filter to yield a white sequence, $e_{p,n}^f$, with variance $\sigma_p^2 = \sigma_w^2$.

The LD algorithm also implicitly calculates the parameters of a lattice filter [19, 20] as shown in Fig. 2.4. The coefficients of the lattice structure are the parameters

Figure 2.4: Whitening filter for a stationary, p^{th} order autoregressive process, implemented as a lattice structure.

$\{a_i^{(i)} = K_i, i = 1, 2, \dots, p\}$. It can be shown that these parameters are less than one for a stationary process [21], and that this property immediately indicates that the roots of $A_i(z)$ lie within the unit circle, which alternatively means $H(z)$ in Eq. (2.11) is a stable filter.

The lattice filter is an interesting structure in that it contains the transfer functions of all lower order filters, i.e., $A_1(z), A_2(z), \dots, A_p(z)$, as well as the forward prediction error sequences $\{e_{i,n}^f, i = 1, 2, \dots, p\}$ for each of these sections. For the p^{th} order AR

Table 2.1: Levinson-Durbin Algorithm.

Initialization

$$a_1^1 = r_1/r_0$$

$$\sigma_1^2 = [1 - (a_1^{(1)})^2] r_0$$

Repeat for $k = 2, 3, \dots, p$

$$a_k^{(k)} = 1/\sigma_{k-1}^2 [r_k - \sum_{i=1}^{k-1} a_i^{(k-1)} r_{k-i}]$$

$$a_i^{(k)} = a_i^{(k-1)} - a_k^{(k)} a_{k-i}^{k-1}, \quad i = 1, 2, \dots, k-1$$

$$\sigma_k^2 = [1 - (a_k^{(k)})^2] \sigma_{k-1}^2$$

process under consideration, only the output sequence of the p^{th} stage (i.e., $e_{p,n}^f$) will correspond to the minimum mean-squared error sequence. Another important point is that the lattice filter provides as a by-product a set of backward prediction errors denoted by $\{e_{i,n}^b, i = 1, 2, \dots, p\}$ and defined as

$$e_{i,n}^b = u_{n-i} - \sum_{k=1}^i a_k^{(i)} u_{n-i+k}, \quad i = 1, 2, \dots, p. \quad (2.20)$$

The backward prediction error is the difference between u_{n-i} and a weighted sum of i future samples $\{u_{n-i+1}, u_{n-i+2}, \dots, u_n\}$. For completeness, the forward prediction errors are written as

$$e_{i,n}^f = u_n - \sum_{k=1}^i a_k^{(i)} u_{n-k}, \quad i = 1, 2, \dots, p. \quad (2.21)$$

From the lattice structure, another definition for the forward and backward prediction errors is

$$e_{i,n}^f = e_{i-1,n}^f - K_i e_{i-1,n-1}^b \quad (2.22)$$

$$e_{i,n}^b = e_{i-1,n-1}^b - K_i e_{i-1,n}^f. \quad (2.23)$$

The backward prediction errors from each section of the lattice structure are orthogonal to one another. This is because the lattice can be viewed as carrying out a Gram-Schmidt orthogonalization process [22] on the data u_n .

This latter property of the lattice filter can be appreciated by considering Eq. (2.23). Assume $e_{i-1,n-1}^b$ and $e_{i-1,n}^f$, $i = 1, 2, \dots, p$ are two vectors with an arbitrary angle between them (which implies they are correlated). The objective is to produce a new vector $e_{i,n}^b$ which is orthogonal to $e_{i-1,n-1}^b$ by linearly combining $e_{i-1,n-1}^b$ and $e_{i-1,n}^f$ (this implies the removal of the correlated portion of $e_{i-1,n}^f$ from $e_{i-1,n-1}^b$). The parameter K_i is the cosine of the angle between $e_{i-1,n}^f$ and $e_{i-1,n-1}^b$ and is equal to [23]

$$K_i = \frac{E\{e_{i-1,n}^f e_{i-1,n-1}^b\}}{\sqrt{E\{e_{i-1,n}^f{}^2\} E\{e_{i-1,n-1}^b{}^2\}}}. \quad (2.24)$$

Finally, the lattice parameters K_i are referred to as reflection coefficients because of the application of lattice filters to speech processing. The lattice is a model for the vocal tract, where each section of the lattice filter represents one tubular section

of a certain cross sectional area of the tube [24]. As the acoustic wave propagates along the tract, portions of the wave are reflected at boundaries having different cross sectional areas. The amount of reflection at these boundaries is measured by K_i . Thus, in analyzing speech waveforms, the K_i can be used to characterize a particular speaker.

To this point, the solution to the Wiener-Hopf equation (Eq. (2.15)) has been discussed for the case when the second order statistics of the process u_n are known. In practice, these statistics are unknown. The next two sections discuss practical solutions.

2.4.2 Block Least Squares Algorithms

In the practical situation, only a sample realization of u_n is available. The sequence of samples may have been generated by an AR process, or from any unknown process which had been assumed to be AR. Furthermore, only a finite set of samples are available. The problem of determining an optimum set of coefficients $\{a_i^{(p)}, i = 1, 2, \dots, p\}$ resides in the area of linear least squares theory [25, 26, 27]. These coefficients are obtained by solving a series of equations that arise from a model which is similar to the AR process discussed earlier, i.e., each sample within a block of received samples $\{u_n, n = 0, 1, \dots, T\}$ is equal to a linear combination of the p most recent samples plus some error, i.e.,

$$u_n = a_1^{(p)}u_{n-1} + a_2^{(p)}u_{n-2} + \dots + a_p^{(p)}u_{n-p} + e_{p,n}^f. \quad (2.25)$$

The error $e_{p,n}^f$ is usually assumed to be white noise with Gaussian statistics. In Eq. (2.25), it has been assumed that the data has been fully windowed, i.e., $u_n = 0$, $n < 0$ and $n > T$, as shown in Fig. 2.5. Pre-windowed and non-windowed cases are also shown. These latter two cases will be discussed later. Equation (2.25) is closely related to the AR process discussed earlier, except that the samples are deterministic values. Given the data, the objective is to determine the set $\{a_i^{(p)}, i = 1, 2, \dots, p\}$ which will minimize the sum of squared errors between the estimate based on a weighted sum of past values, i.e.,

$$\hat{u}_n = a_1^{(p)}u_{n-1} + \dots + a_p^{(p)}u_{n-p}, \quad (2.26)$$

Figure 2.5: The results of three window types which can be imposed on the data.

and u_n . For the fully windowed case, the following equations are obtained, written in matrix form as

$$\begin{bmatrix} e_{p,0}^f \\ e_{p,1}^f \\ e_{p,2}^f \\ \vdots \\ \vdots \\ e_{p,T}^f \\ e_{p,T+1}^f \\ \vdots \\ \vdots \\ e_{p,T+p}^f \end{bmatrix} = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ \vdots \\ u_T \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ u_0 & 0 & \cdot & \cdot & \cdot & 0 \\ u_1 & u_0 & \cdot & & & \cdot \\ \cdot & u_1 & u_0 & & & \cdot \\ \cdot & & u_1 & \cdot & & \cdot \\ u_{T-1} & & & \cdot & & u_{T-p} \\ u_T & u_{T-1} & & & & u_{T-p+1} \\ \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & \cdot & \cdot & & \\ 0 & 0 & \cdot & \cdot & \cdot & u_T \end{bmatrix} \begin{bmatrix} a_1^{(p)} \\ a_2^{(p)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_p^{(p)} \end{bmatrix}. \quad (2.27)$$

In matrix notation, this becomes

$$e^f = u - Ua \quad (2.28)$$

where U has a Toeplitz structure. The optimum solution in a least squares sense is the one producing a vector e^f orthogonal to the column space of U [25]. In other words, if the columns of U are considered as vectors in an N -dimensional Euclidean space ($N = T + p + 1$), and these p vectors form a sub-space within this N -dimensional space, then the optimum solution is that vector which minimizes the norm $e^{fT}e^f$. For this to be true, e^f must lie in the sub-space orthogonal to the sub-space created by the columns of U . Therefore, the optimum solution can be obtained from

$$U^t e^f = 0. \quad (2.29)$$

Substituting for e^f in this equation, we obtain the following:

$$U^t [u - Ua] = 0 \quad (2.30)$$

$$U^t U a = U^t u. \quad (2.31)$$

This is equivalent to $\hat{R}a = \hat{r}$ where $\hat{R} = U^t U$ and $\hat{r} = U^t u$ and are, respectively, estimates of R and r in Eqs. (2.15) and (2.18). For the fully windowed case, $\hat{R}$ is a $p \times p$ Toeplitz matrix, so that the LD algorithm can be used to solve for a . (Note, however, that for an overdetermined set of N equations in p unknowns, forming $\hat{R}$ and $\hat{r}$ requires $\mathcal{O}(Np^2/2)$ operations, which can consume most of the computational effort for large N .)

If the data are not windowed (i.e., the nonwindowed case, as shown in Fig. 2.5), there are several approaches one can take to solve the problem. Which approach depends on the assumptions that are made about the data (stationary or non-stationary), whether or not the roots of $A_1(z)$ are required to lie within the unit circle, and general performance requirements. Three approaches will be discussed.

Approach 1

This is the simplest of the three approaches. The problem involves minimizing the sum of squared forward prediction errors. The matrix equation of interest is

$$\begin{bmatrix} e_{p,p}^f \\ e_{p,p+1}^f \\ \vdots \\ e_{p,T}^f \end{bmatrix} = \begin{bmatrix} u_p \\ u_{p+1} \\ \vdots \\ u_T \end{bmatrix} - \begin{bmatrix} u_{p-1} & u_{p-2} & \cdot & \cdot & \cdot & u_0 \\ u_p & u_{p-1} & \cdot & \cdot & \cdot & u_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{T-1} & u_{T-2} & \cdot & \cdot & \cdot & u_{T-p} \end{bmatrix} \begin{bmatrix} a_1^{(p)} \\ a_2^{(p)} \\ \vdots \\ a_p^{(p)} \end{bmatrix}. \quad (2.32)$$

Applying the same reasoning that was used in determining the optimum solution for Eq. (2.27), one obtains a matrix $\hat{R}$ which is no longer Toeplitz, so that the LD algorithm cannot be applied. Furthermore, the solution may not yield a filter with roots inside the unit circle. Although the efficient LD algorithm cannot be applied here, efficient algorithms have been developed [28, 29] to provide a solution requiring $O(p^2)$ operations.

One can derive a lattice structure (shown in Fig. 2.6) from the algorithm described in [29]. The reflection coefficients in the forward and backward branches of each sec-

Figure 2.6: Least squares lattice structure.

tion (K_i^f and K_i^b) are not necessarily equal, and their magnitudes are not necessarily less than 1. This latter property can be used to determine whether or not the roots of $A_p(z)$ are within the unit circle. As shown in [30], these parameters are bounded according to the relation $0 < K_i^f K_i^b < 1$. Finally, the degree of disparity between K_i^f and K_i^b is a measure of the amount of non-stationarity in the data [31].

Approach 2 (Burg's Algorithm)

One can refer to [32, 33] for this approach. The assumption is made that the process is stationary. Furthermore, it is required that the roots of $A_i(z)$ be within the unit circle and that only the available data be used (i.e., the nonwindowed case). With these requirements, Burg developed an algorithm in which he suggested that the sum of squares of the forward and backward prediction errors be minimized with respect to the reflection coefficients $\{a_i^{(i)} = K_i, i = 1, 2, \dots, p\}$, i.e., minimize

$$\mathcal{E}_i = \sum_{n=i}^T (e_{i,n}^f{}^2 + e_{i,n}^b{}^2), \quad i = 1, 2, \dots, p. \quad (2.33)$$

The choice of Eq. (2.33) is based on the stationarity assumption for which the statistics of the forward and backward prediction errors are equal. Also, minimization of this equation with respect to the i^{th} stage reflection coefficients is equivalent to a local minimization of each stage of the lattice. A local minimization of each section leads to a global minimization of the entire lattice [19]. If one substitutes Eqs. (2.22) and (2.23) into Eq. (2.33), one obtains $\mathcal{E}_i$ as a function of K_i . Taking its derivative with respect to K_i , and setting it equal to 0, yields the optimum solution for K_i , i.e.,

$$K_i = \frac{2 \sum_{k=i}^T e_{i-1,k}^f e_{i-1,k-1}^b}{\sum_{k=i}^T (e_{i-1,k}^2 + e_{i-1,k-1}^2)}, \quad i = 1, 2, \dots, p. \quad (2.34)$$

Equation (2.34) is the deterministic form of Eq. (2.24) for stationary processes. The Burg algorithm can be viewed as a solution to a constrained least squares problem, where the constraint is the imposition of the lattice structure in Fig. 2.4 as opposed to the least squares structure in Fig. 2.6.

The initial conditions for the algorithm are $e_{0,n}^f = e_{0,n}^b = u_n$. With this initial condition, $K_1 = a_1^{(1)}$ can be calculated from Eq. (2.34). The block of data is passed through the first order lattice filter to produce the sequences $e_{1,k}^f$ and $e_{1,k-1}^b$, from which $a_2^{(2)}$ is determined, and so forth. After K_p is calculated, one can determine the coefficients $\{a_i^{(p)}, i = 1, 2, \dots, p\}$ using the LD recursion if the power spectrum of the sequence u_n is required. An analysis of the computational complexity of the Burg algorithm [23] indicates that $3Np - p^2 - 2N - p$ adds, $3Np - p^2 - N + 3p$

multiplications, and p divisions are required where $N = T + 1$. Storage of $3N + p + 2$ values is also necessary.

Approach 3

The constraint imposed on the data in the Burg algorithm has led to certain problems in its application to spectral estimation, such as spectral line splitting and biases in the frequency estimate [23, 34, 35]. These problems can be mitigated if $\mathcal{E}_i$ in Eq. (2.33) is minimized with respect to all the coefficients $\{a_k^{(p)}, k = 1, 2, \dots, p\}$ [33, 34, 36, 37]. For the derivation of the solution, the forward and backward prediction errors are required:

$$\begin{bmatrix} e_{p,p}^f \\ e_{p,p+1}^f \\ \cdot \\ \cdot \\ \cdot \\ e_{p,T}^f \end{bmatrix} = \begin{bmatrix} u_p \\ u_{p+1} \\ \cdot \\ \cdot \\ \cdot \\ u_T \end{bmatrix} - \begin{bmatrix} u_{p-1} & u_{p-2} & \cdot & \cdot & \cdot & u_0 \\ u_p & u_{p-1} & \cdot & \cdot & \cdot & u_1 \\ \cdot & \cdot & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ u_{T-1} & u_{T-2} & \cdot & \cdot & \cdot & u_{T-p} \end{bmatrix} \begin{bmatrix} a_1^{(p)} \\ a_2^{(p)} \\ \cdot \\ \cdot \\ \cdot \\ a_p^{(p)} \end{bmatrix} \quad (2.35)$$

$$\begin{bmatrix} e_{p,0}^f \\ e_{p,1}^f \\ \cdot \\ \cdot \\ \cdot \\ e_{p,T-p}^f \end{bmatrix} = \begin{bmatrix} u_0 \\ u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_{T-p} \end{bmatrix} - \begin{bmatrix} u_1 & u_2 & \cdot & \cdot & \cdot & u_p \\ u_2 & u_3 & \cdot & \cdot & \cdot & u_{p+1} \\ \cdot & \cdot & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ u_{T-p+1} & u_{T-p+2} & \cdot & \cdot & \cdot & u_T \end{bmatrix} \begin{bmatrix} a_1^{(p)} \\ a_2^{(p)} \\ \cdot \\ \cdot \\ \cdot \\ a_p^{(p)} \end{bmatrix}. \quad (2.36)$$

These equations can be rewritten as

$$e^f = u^f - U^f a \quad (2.37)$$

$$e^b = u^b - U^b a. \quad (2.38)$$

The objective is to minimize

$$\mathcal{E}_p = e^{Jt} e^J + e^{bt} e^b \quad (2.39)$$

with respect to the set $\{a_k^{(p)}, k = 1, 2, \dots, p\}$. This can be achieved by rewriting the prediction error equations as $e = u - Ua$, where

$$e = \begin{bmatrix} e^J \\ e^b \end{bmatrix} \quad (2.40)$$

$$u = \begin{bmatrix} u^J \\ u^b \end{bmatrix} \quad (2.41)$$

$$U = \begin{bmatrix} U^J \\ U^b \end{bmatrix}. \quad (2.42)$$

With these definitions, the solution is obtained from

$$U^t U a = U^t u. \quad (2.43)$$

An order recursive algorithm has been developed by Barrodale and Erickson [36], which utilized the Cholesky decomposition method and certain shift invariant properties associated with the matrices $U^t U$ and $U^t u$. The approach has good numerical qualities. Marple [34] has also developed an efficient algorithm. The computational complexity of this latter algorithm for $N = T + 1$ is $Np + 8p^2 + N + 7p - 8$ additions, $Np + 9p^2 + 2N + 25p - 3$ multiplications, and $5p + 3$ divisions. The algorithm needs $N + 4p + 15$ computer memory locations.

Remark

In approaches 1 and 3, it has been tacitly assumed that the inverse of $U^t U$ exists, i.e., it is of full rank. In many applications this will not be the case, particularly if there are fewer equations than unknowns (the underdetermined case). For this case an infinite number of solutions for a exist which will minimize the mean-squared error. Of this set of solutions, the one with minimum norm is typically selected. To obtain the minimum norm solution, the Moore-Penrose pseudo-inverse [25] of $U^t U$ must be determined.

2.4.3 Recursive Algorithms

In the previous section a number of block structured algorithms were discussed regarding the solution to the Wiener-Hopf equation, i.e., Eq. (2.15). These algorithms led to transversal or lattice filter structures. With deterministic data, the problem was casted into a least squares one, in which the Wiener-Hopf equation was comprised of sample values of the process.

In this section, (time-)recursive solutions to the Wiener-Hopf equation will be considered. In particular, stochastic gradient and recursive least squares algorithms for both the transversal and lattice structures will be discussed.

Stochastic Gradient Algorithms

The objective of the stochastic gradient approach is to make small adjustments to the filter coefficients in such a manner as to minimize the mean-squared error between u_n and $\hat{u}_n$. The adjustments to the filter coefficients are a function of the gradient of the mean squared error, which can be viewed as a multi-dimensional paraboloid with a unique minimum. This minimum point is sought by the algorithm by adjusting the coefficients in a direction opposite to the gradient.

Gradient algorithms have been developed for transversal and lattice structures. Both structures will be discussed, with the former structure being used to present the basic concept behind the gradient technique.

Transversal Structure

If the statistics of the process are known, the recursion for updating the filter parameters, now a function of time, is the following [38]:

$$\begin{aligned} \mathbf{a}_{T+1} &= \mathbf{a}_T - \frac{1}{2}\mu(\nabla E\{e_{p,T}^2\}) \\ &= \mathbf{a}_T + \mu(\mathbf{r} - R\mathbf{a}_T) \end{aligned} \quad (2.44)$$

where $\mathbf{a}_T = [a_{1,T}^{(p)} \ a_{2,T}^{(p)} \ \dots \ a_{p,T}^{(p)}]^t$, T is the time index, μ is a small positive constant which determines the rate of convergence of $\mathbf{a}_T$ to its optimum value $\mathbf{a}_{opt}$, and ∇ is

the gradient operator, i.e.,

$$\nabla = \begin{bmatrix} \partial/\partial a_1^{(p)} \\ \partial/\partial a_2^{(p)} \\ \vdots \\ \partial/\partial a_p^{(p)} \end{bmatrix}. \quad (2.45)$$

The constant μ must be selected within certain bounds to ensure convergence of the algorithm to the optimum solution. The bound is defined by $0 < \mu < 2/\Gamma_{\max}$ where $\Gamma_{\max}$ is the maximum eigenvalue of the autocorrelation matrix $\mathbf{R}$ of the process.

Practical implementation of the algorithm involves approximating $\mathbf{r}$ and $\mathbf{R}$ in Eq. (2.44) by $\hat{\mathbf{r}}_T$ and $\hat{\mathbf{R}}_{T-1}$ at time T , where

$$\hat{\mathbf{r}}_T = \mathbf{u}_T \begin{bmatrix} u_{T-1} \\ u_{T-2} \\ \vdots \\ u_{T-p} \end{bmatrix} = \mathbf{u}_T \mathbf{u}_{T-1}, \quad (2.46)$$

and $\hat{\mathbf{R}}_{T-1} = \mathbf{u}_{T-1} \mathbf{u}_{T-1}^t$. After substituting $\hat{\mathbf{r}}_T$ and $\hat{\mathbf{R}}_{T-1}$ into Eq. (2.44), the update equation for the taps of the transversal filter becomes

$$\mathbf{a}_{T+1} = \mathbf{a}_T + \mu e_{p,T}^f \mathbf{u}_{T-1} \quad (2.47)$$

where

$$e_{p,T}^f = u_T - \mathbf{a}_T^t \mathbf{u}_{T-1}. \quad (2.48)$$

It should be noted that the term $e_{p,T}^f \mathbf{u}_{T-1}$ in Eq. (2.47) is a noisy, unbiased estimate of the gradient $E\{e_{p,T}^f \mathbf{u}_{T-1}\}$. Equation (2.47) is referred to as the Widrow-Hoff LMS algorithm.

Remarks

- (1) With the above approximation, the gradient is noisy; however, $E\{\mathbf{a}_T\} \rightarrow \mathbf{a}_{opt}$

if the condition $0 < \mu < 2/\Gamma_{\max}$ is met.

- (2) The tap weights fluctuate about a_{opt} . The degree of fluctuation is a function of the step size μ . Small μ implies averaging over more past data and therefore the variance of the fluctuations is smaller. The cost is a slower convergence rate.
- (3) Tap weight fluctuations lead to an excess error in steady state. Thus the mean-squared level of the output error sequence $e_{p,T}^f$ is larger than the minimum achievable. This leads to an excess mean-squared error.
- (4) The average mean-squared error converges to its steady state value if and only if the step size parameter μ satisfies the condition

$$0 < \mu < \frac{2}{\sum_{i=1}^p \Gamma_i} = \frac{2}{pr_0} \quad (2.49)$$

for a stationary process where r_0 is the power in the signal. If this condition is met, the LMS algorithm is convergent in the mean-square. This imposes a tighter bound on μ .

- (5) In certain applications such as speech processing, where power levels can fluctuate significantly, the LMS algorithm can become unstable. In these cases the step size μ is usually made a function of time [39]. One possibility is [40]

$$\mu_T = \frac{\nu}{p\hat{r}_{0,T}}. \quad (2.50)$$

In Eq. (2.50) $0 < \nu < 2$ and $\hat{r}_{0,T}$ is an unbiased estimate of the power in the sequence u_T , i.e.,

$$\hat{r}_{0,T} = (1 - \xi)\hat{r}_{0,T-1} + \xi u_T^2 \quad (2.51)$$

where $0 < \xi < 1$ and $\hat{r}_{0,-1}$ is some reasonable estimate of the initial condition for r_0 at $T = -1$.

- (6) The rate at which the LMS algorithm converges to the steady state value is dependent on the eigenvalue spread. It has been determined [41] that the overall time constant τ_a for any tap weight $a_i^{(p)}$ is bounded by the relation

$$\frac{-1}{\ln(1 - \mu\Gamma_{\max})} \leq \tau_a \leq \frac{-1}{\ln(1 - \mu\Gamma_{\min})}. \quad (2.52)$$

- (7) The LMS algorithm can track processes which are non-stationary. The degree of success in its tracking capability is a function of the amount of non-stationarity in the process, the eigenvalue disparity and the step size μ .

Lattice Structure

For the lattice structure, one can refer to [31, 39, 42].

The derivation of the gradient adaptive lattice algorithms is similar to the method used to derive the LMS algorithm, except that the gradient is applied to each reflection coefficient. There are two approaches one can take. If the data are assumed to be nonstationary, then the forward and backward reflection coefficients K_i^f and K_i^b are chosen to minimize, respectively, the mean-squared errors $E\{e_{i,T}^{f2}\}$ and $E\{e_{i,T}^{b2}\}$ of each lattice section. The statistical update equations are

$$K_{i,T+1}^f = K_{i,T}^f - \frac{1}{2}\mu_i^f \nabla^f E\{e_{i,T}^{b2}\} \quad (2.53)$$

$$K_{i,T+1}^b = K_{i,T}^b - \frac{1}{2}\mu_i^b \nabla^b E\{e_{i,T}^{f2}\} \quad (2.54)$$

where μ_i^f and μ_i^b are the step sizes for the i^{th} section, and ∇^f and ∇^b are the one-dimensional gradient operators, $\partial/\partial K_i^f$ and $\partial/\partial K_i^b$, respectively. After substituting for $e_{i,T}^f$ and $e_{i,T}^b$ from Eqs. (2.22) and (2.23) into Eqs. (2.53) and (2.54), taking the derivative and approximating the expectation operator by the instantaneous values, the following approximate gradient equations are obtained:

$$K_{i,T+1}^f = K_{i,T}^f + \mu_i^f e_{i-1,T}^f e_{i,T}^b \quad (2.55)$$

$$K_{i,T+1}^b = K_{i,T}^b + \mu_i^b e_{i,T}^f e_{i-1,T-1}^b. \quad (2.56)$$

The second approach assumes the data are stationary, implying that $K_i^f = K_i^b = K_i$. The objective is to find that K_i which minimizes the sum $E\{e_{i,T}^{f2}\} + E\{e_{i,T}^{b2}\}$. The approximate gradient equation for this case is

$$K_{i,T+1} = K_{i,T} + \mu_i [e_{i,T}^f e_{i-1,T-1}^b + e_{i-1,T}^f e_{i,T}^b]. \quad (2.57)$$

Remarks

- (1) The gradient lattice algorithms converge to the steady state faster than the LMS transversal algorithm [39, 42, 43] for the case in which there is a large spread in the eigenvalues. This is because the lattice algorithm has more degrees of freedom in adjusting its parameters; it has at least one convergence parameter μ for each section. This should be compared to the LMS algorithm, in which there is only one parameter for the entire filter.
- (2) The step size can be power normalized, in a manner similar to that for the LMS algorithm [39, 42]. For example, in Eq. (2.57) normalization is of the form

$$\mu_{i,T} = 1/\hat{\sigma}_{i,T}^2 \quad (2.58)$$

where

$$\hat{\sigma}_{i,T}^2 = (1 - \xi)\hat{\sigma}_{i,T-1}^2 + \xi [e_{i,T}^{f2} + e_{i,T-1}^{b2}] \quad (2.59)$$

and $0 < \xi < 1$. Equation (2.59) is similar to Eq. (2.51), except that the error power from each lattice stage i is estimated.

- (3) The gradient lattice algorithm is considerably more difficult to analyze than the gradient transversal algorithm because of the highly nonlinear nature of the adaptation (i.e., each section converges at a different rate). It has been found empirically [39] that for the power-normalized case and the stationary form of the lattice structure, stabilization will be maintained if $0 < 1/\hat{\sigma}^2 < 1$.

Recursive Least Squares Algorithms

In this section the pre-windowed (Fig. 2.5) recursive least squares algorithm [44] will be discussed. The non-windowed case has a recursive solution as well, albeit the algorithm is more complex and requires a more sophisticated start-up procedure [43, 45, 46, 47, 48].

The matrix equation for the pre-windowed case at time T is

$$\begin{bmatrix} e_{p,0}^f \\ e_{p,1}^f \\ \vdots \\ e_{p,T}^f \end{bmatrix} = \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_T \end{bmatrix} - \begin{bmatrix} 0 & 0 & \cdots & 0 \\ u_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ u_{T-1} & u_{T-2} & \cdots & u_{T-p} \end{bmatrix} \begin{bmatrix} a_1^{(p)} \\ a_2^{(p)} \\ \vdots \\ a_p^{(p)} \end{bmatrix}. \quad (2.60)$$

The optimum solution for $\mathbf{a}$ at time T for a block of data where $T \geq p$ can be solved using, for example, the generalized Levinson recursions in [29], modified for pre-windowed signals.

The major computational burden associated with block methods of computing the optimum coefficients is the matrix $\hat{\mathbf{R}} = \mathbf{U}^t \mathbf{U}$, as noted earlier, when the number of samples is large. Furthermore, $\hat{\mathbf{R}}$ must be recomputed when the least squares estimate is to be updated for every new data sample, i.e., as T increases. A number of algorithms over the years has evolved which address these problems [44, 49, 50, 51, 52, 53, 54].

The first algorithms were recursive in time for a specific filter order. The original one [52], based on a Kalman filter model of the least squares problem, required $\mathcal{O}(p^2)$ operations per time update. A more efficient method [51, 54] known as the fast Kalman algorithm, was developed requiring $\mathcal{O}(8p)$ operations per update for AR modeling. The number of operations was reduced to $\mathcal{O}(5p)$ by the FAEST algorithm [44, 49], i.e., fast *a posteriori* error sequential technique, and FTF algorithm [50], i.e., Fast Transversal Filters algorithm. The fast Kalman and FAEST algorithms will now be discussed.

The optimum solution at time T for a p^{th} order system, derived from Eq. (2.60), is obtained from $\hat{\mathbf{R}}_{T-1} \mathbf{a}_T = \hat{\mathbf{r}}_T$ where

$$\hat{\mathbf{R}}_{T-1} = \sum_{i=0}^{T-1} \mathbf{u}_i \mathbf{u}_i^t \quad (2.61)$$

$$\hat{\mathbf{r}}_T = \sum_{i=0}^T \mathbf{u}_i \mathbf{u}_{i-1}^t \quad (2.62)$$

$$\mathbf{u}_i = [u_i \ u_{i-1} \ \dots \ u_{i-p+1}]^t. \quad (2.63)$$

The problem is to determine $\mathbf{a}_{T+1}$ when the sample u_{T+1} arrives. The key to the development of a sequential algorithm is the existence of simple time-updating formulas for $\hat{\mathbf{R}}_T$ and $\hat{\mathbf{r}}_{T+1}$ in terms of $\hat{\mathbf{R}}_{T-1}$ and $\hat{\mathbf{r}}_T$, respectively, and the most recent data $\{u_{T+1}, \mathbf{u}_T\}$. These update formulas arise from the Toeplitz structure of the data matrix $\mathbf{U}$ in Eq. (2.60). Using these updating formulas [29], it can be shown that the linear prediction update equation is of the form

$$\mathbf{a}_{T+1} = \mathbf{a}_T + \mathbf{R}_T^{-1} \mathbf{u}_T e_{p,T+1}^{fp} \quad (2.64)$$

$$= \mathbf{a}_T + \mathbf{K}_{G,T} e_{p,T+1}^{fp} \quad (2.65)$$

where $\mathbf{K}_{G,T}$ is referred to as the Kalman gain vector and $e_{p,T+1}^{fp}$ is the *a priori* error defined as

$$e_{p,T+1}^{fp} = u_{T+1} - \mathbf{a}_T^t \mathbf{u}_T. \quad (2.66)$$

It is sometimes referred to as the predicted error (which is the reason for the superscript p) since the prediction of u_{T+1} , namely $\mathbf{a}_T^t \mathbf{u}_T$, is based on the tap weight vector at time T , not $T+1$. The fast Kalman algorithm [51, 54] is a series of operations per time step concerned with updating the Kalman gain vector $\mathbf{K}_{G,T}$.

With modifications to the fast Kalman algorithm, the FAEST algorithm [44, 49] was developed using an alternative formulation which had the effect of reducing the number of operations required to update the Kalman gain vector. The update equation in this case is

$$\begin{aligned} \mathbf{a}_{T+1} &= \mathbf{a}_T + \mathbf{R}_{T+1}^{-1} \mathbf{u}_T e_{p,T+1}^f \\ &= \mathbf{a}_T + \mathbf{K}'_{G,T} e_{p,T+1}^f \end{aligned} \quad (2.67)$$

where $\mathbf{K}'_{G,T}$ is the alternative Kalman gain and

$$e_{p,T+1}^f = u_{T+1} - \mathbf{a}_{T+1}^t \mathbf{u}_T \quad (2.68)$$

is the *a posteriori* error.

When either of these algorithms is started at $T = 0$, the rank of the autocorrelation matrix $\hat{\mathbf{R}}_T$ is less than the filter order p . To prevent a singularity from occurring in $\hat{\mathbf{R}}_T$

for $T < p$, the initial minimum mean-squared error is set equal to a small constant ϵ . This is equivalent to initializing the algorithms's autocorrelation matrix to ϵI where I is the identity matrix.

Remarks

- (1) The fast least squares algorithms are less sensitive to eigenvalue spread than the LMS algorithm. The smaller the eigenvalue spread, the less the difference between the LMS and fast algorithms.
- (2) If the right hand side of Eq. (2.60) is premultiplied by $\Lambda^{1/2}$ where

$$\Lambda = \begin{bmatrix} \lambda^T & 0 & \cdot & \cdot & 0 \\ 0 & \lambda^{T-1} & \cdot & \cdot & 0 \\ \cdot & \cdot & & & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & & \cdot \\ 0 & 0 & \cdot & \cdot & 1 \end{bmatrix} \quad (2.69)$$

and $\lambda < 1$, then one is concerned with a weighted least squares problem in which the old data is weighted out. This provides the capability of tracking time-varying processes. The memory of the algorithm in this case is approximately $1/(1 - \lambda)$ samples.

- (3) The fast algorithms have a problem with numerical stability due to round-off error accumulation [47, 50, 55, 56]. Rescue techniques have been developed [47, 50] to deal with this problem. Cioffi [50] also developed normalized algorithms which increase the dynamic range performance of finite precision algorithms, leading to a reduction in the number of rescues that are required. The choice of the initial value ϵ can also lead to instability [50]. To remove the requirement for this initial condition, Cioffi developed a fast start-up procedure for the FTF algorithm based on the minimum-norm least squares solution [25]. This procedure produces the exact least squares solution from $T = 0$.

- (4) Both unnormalized and normalized recursive least squares algorithms have been developed for the lattice structures [43, 45, 46, 57, 58, 59]. Though more stable than their transversal counterpart, they can still develop numerical problems [50] and eventually diverge after continuous operation. One advantage of the lattice structure is the availability of all filter orders up to some upper limit. This may be useful in some applications in which the correct order to use is unknown. The disadvantage is the increase in the number of computations/iteration. Table 2.2 summarizes the computational effort per iteration required by several of the recursive algorithms [50].

Table 2.2: Algorithm Computational Effort.

Algorithm	Computations for Linear Prediction
LMS	2p
Fast Kalman	8p, 2 div.
FAEST	5p, 2 div.
FTF	5p, 2 div.
Normalized FTF	9p, 2 square roots, 2 div.
Unnormalized Lattice	11p, 3p div.
Normalized Lattice	18p, 3p square roots, 4p div.

2.4.4 A Review of Research Work in Digital Excision

In the previous section, several adaptive algorithms and their characteristics were discussed. With the advent of high-speed digital technology, these algorithms are becoming feasible in interference excisors. Many of these algorithms have been evaluated by researchers either analytically, under computer simulation, or through implementation.

The earliest work revealed that Wiener filter theory could indeed be applied to the suppression of interference in DS/SS systems, thus providing significant improvement in SNR. This revelation spawned a great deal of research in three areas:

- (a) algorithm performance under various interference conditions;
- (b) the development of theoretical performance models; and,
- (c) filter structures (prediction and interpolation error filters and filters employing decision-feedback).

This section reviews a large portion of this work in somewhat chronological order starting with the basic principle.

The filtering of narrowband interference from DS/SS systems is based on the property that the signal and receiver noise have flat spectra and, therefore are unpredictable, whereas the narrowband interference has a peaked spectrum relative to the signal and noise and hence is predictable. With the linear prediction algorithms of Section 2.4, this interference component can be estimated and removed from u_n in Fig. 1.2(b) using transversal or lattice filters. The output of the filter is an error signal consisting of the sum of the filtered versions of the desired spread spectrum signal, thermal noise and residual interference.

Hsu and Giordano [60] appear to have been the first to suggest this method of interference removal. Their input signal model was a sequence $u_n = s_n + n_n + i_n$ where s_n was a chip sample of amplitude ± 1 , n_n was a sample of white Gaussian noise of variance $\sigma_{n,n}^2$, and i_n was a sample of narrowband interference with autocorrelation function ρ_i , $i = 0, \pm 1, \pm 2, \dots$. The assumption was made that the signal, noise and

interference were independent from one another and the signal sequence was white with autocorrelation function

$$\begin{aligned} R_{ss} &= \delta(n) \quad \text{for } n = 0 \\ &= 0 \quad \text{for } n \neq 0. \end{aligned} \quad (2.70)$$

With these assumptions, the interference becomes the predictable component as now shown.

Consider a p^{th} order linear predictor as in Eq. (2.12). Its coefficients are determined from Eq. (2.15). The elements of the autocorrelation matrix, $\mathbf{R}$, are given by

$$\begin{aligned} r_0 &= E\{u_n^2\} \\ &= R_{ss} + \sigma_{nse}^2 + \rho_0 \end{aligned} \quad (2.71)$$

where ρ_0 is the interference power, and

$$\begin{aligned} r_i &= E\{u_n u_{n-i}\} \\ &= E\{i_n i_{n-i}\} \\ &= \rho_i, \quad i = \pm 1, \pm 2, \dots, \pm p. \end{aligned} \quad (2.72)$$

Since the autocorrelation function r_i for $|i| > 0$ in Eq. (2.72) depends only on ρ_i and not the autocorrelation functions of the noise and signal, linear prediction methods can be used to predict the interference component.

The output of the excision filter is the error sequence $e_{p,n}^f$ consisting of three terms, i.e.,

$$e_{p,n}^f = u_n - \sum_{j=1}^p a_j^{(p)} u_{n-j} \quad (2.73)$$

$$= \left(s_n - \sum_{j=1}^p a_j^{(p)} s_{n-j} \right) + \left(n_n - \sum_{j=1}^p a_j^{(p)} n_{n-j} \right) + \left(i_n - \sum_{j=1}^p a_j^{(p)} i_{n-j} \right) \quad (2.74)$$

The first two terms are non-white signal and noise sequences, and the third is the residual interference. The direct sequence despreader correlates the error signal with the PN sequence, as shown in Fig. 1.2(b).

Hsu and Giordano studied the performance of prediction (i.e., single-sided) transversal filters with respect to the suppression of interference modelled as the sum of sinusoids of equal amplitude and random phases. Furthermore, the interference spanned a bandwidth equal to 20% of the chip rate. In their analysis, the filter coefficients were calculated by means of the LD and Burg algorithms.

The receiver's performance was measured in terms of a signal-to-noise (SNR) ratio improvement parameter at the output of the PN correlator, i.e., the ratio of the SNR with whitening, to the SNR without whitening. If the output of the bit detector is the test statistic $\mathcal{U}$, then the SNR when the excision filter is present can be shown to be equal to

$$SNR_o = \frac{E\{\mathcal{U}\}^2}{\text{var}\{\mathcal{U}\}} \quad (2.75)$$

$$= \frac{L}{\sum_{j=1}^p a_j^{(p)2} + \left(\rho_0 - 2 \sum_{j=1}^p a_j^{(p)} \rho_j + \sum_{j=1}^p \sum_{k=1}^p a_j^{(p)} a_k^{(p)} \rho_{j-k} \right) + \sigma_{nse}^2 \left(1 + \sum_{j=1}^p a_j^{(p)2} \right)} \quad (2.76)$$

$$= \frac{L}{\sigma^2} \quad (2.77)$$

where L is the number of chips/bit, σ^2 is the overall noise power and the chip rate is 1 Hz. (It should be noted that for chips of amplitude v_c and duration T_c , the energy per chip E_c is $v_c^2 T_c$. The energy per bit E_b is therefore LE_c . For chips of unit energy, $E\{\mathcal{U}\} = L$.) This noise power consists of three parts, as noted, in the denominator of Eq. (2.76), i.e., a signal distortion term due to the excision filter, residual interference, and Gaussian noise.

Without the filter, the SNR is simply

$$SNR_{no} = \frac{L}{\rho_0 + \sigma_{nse}^2}. \quad (2.78)$$

Thus, the figure of merit for the excisor is the ratio $\eta = SNR_o / SNR_{no}$, i.e.,

$$\eta = \frac{\rho_0 + \sigma_{nse}^2}{\sigma^2} \quad (2.79)$$

$$\approx \frac{\rho_0}{\sigma^2}, \quad \text{for } \rho_0 \gg \sigma_{nse}^2. \quad (2.80)$$

Both the LD and Burg algorithms yielded substantial SNR improvement for large I/S ratios. For example, for $L = 10$, $\sigma_{nse}^2 = .01$, 10 tones and $SNR_{no} = -16\text{dB}$

(large ρ_0), η was approximately 20 dB for the Burg algorithm and 12 dB for the LD algorithm. The LD algorithm demonstrated a leveling off in η for large I/S , whereas the Burg algorithm did not. The inferior performance exhibited by the LD algorithm can be attributed to its characteristic tendency of providing a larger bandwidth notch compared to the non-windowed least squares algorithms, resulting in more residual interference and signal distortion. Finally, performance did not vary much with filter order for high I/S conditions, i.e., low ($p = 4$) and high order ($p = 29$) filters produced very similar results for the case of 10 tones. For 100 tones the difference started to manifest itself, with the high order filter ($p = 29$) providing better performance, indicating the removal of more interference. In contrast, for low I/S conditions, the performance of the high order filter was worse than the low order one. This difference in performance can be attributed to the additional signal distortion introduced by the extra zeroes in the larger order whitening filter.

Li and Milstein [61] compared the performance of the single-sided (prediction error) filter to the two-sided (interpolation error) linear phase transversal filter with the same number of taps as the prediction filter. The two-sided filter configuration is shown in Fig. 2.7. Single- and multiple-tone interferers were considered.

For the case of a single-tone interferer, an expression for η from Eq. (2.80) was derived using the filter coefficients obtained from the exact solution to the Wiener-Hopf equation. The results showed that when the tone was in close proximity to the carrier (approximately within 0.05 Hz for a normalized sampling rate of 1 Hz) and the noise variance was zero (the ideal case), the two-sided filter provided up to 6 dB additional SNR improvement over the prediction filter. This was due to the more accurate estimate of the interference produced by the interpolator. For frequencies further away from the carrier, the performance advantage exhibited by the interpolator over the predictor diminished. This phenomenon is likely due to the decrease in correlation between interference samples for increasing frequency offsets.

Analytical expressions were not developed for the multi-tone case because of the difficult nature of the problem. However, simulations were conducted with the Widrow-Hoff LMS algorithm. In general, the two-sided filter performed better than the prediction filter by 3 to 10 dB for I/S 's ranging from 2 to 30 dB.

Figure 2.7: Two-sided or interpolation error transversal filter which produces the interpolation error $e_{p,n}$.

Li and Milstein [62] also analyzed the performance of a direct sequence QPSK spread-spectrum system using complex prediction and interpolation error filters in the presence of pulsed CW interference. They examined the convergence behaviour of the LMS algorithm during the “on” and “off” portions of the interference, and found that for the case when the product of the number of taps times the jammer power was much greater than the sum of the signal and noise powers, i.e., $p\rho_0 \gg R_{ss} + \sigma_{nsc}^2$, the taps changed much slower during the “off” portion of the pulse, thereby maintaining good rejection capability when the “on” portion of the interference returned. Furthermore, by deriving an autocorrelation function for the pulsed signal, they were able to calculate the optimum weights and, therefore, obtain an expression for η for the worst case situation, i.e., when the start of the pulsed interference coincides with the leading edge of a symbol. This situation generates large interference spikes at the output of the filter, which gradually diminish as the interference cancellation begins to take effect. For both filter types, η demonstrated a large degree of sensitivity to filter order for a small duty factor (1%), and almost no sensitivity for larger duty factors

(10%). This behaviour suggests a requirement for higher filter orders to attenuate interferers of increasing bandwidths.

Ketchum and Proakis [63] based their work on that of Hsu and Giordano [60]. They considered:

- (a) single-band and multiple-band interference; and,
- (b) two additional algorithms for estimating the filter coefficients.

The interference consisted of either narrowband Gaussian noise or a sum of closely spaced sinusoids of equal amplitude and random phase, spanning a bandwidth equal to 20% of the chip rate in a single-band segment or in multiple-band segments. Besides the LD and Burg algorithms, they also assessed the performance of the non-windowed, unconstrained least squares algorithm (Eq. (2.43)) as well as a frequency domain algorithm based on the FFT. The next two paragraphs provide a brief description of this latter algorithm.

The frequency domain method relies on the characteristic that the power spectral density of the PN sequence is much broader compared to the spectrum of the more highly peaked narrowband interference. The first step in this method is to estimate the power spectral density of the received signal. The spectral estimate can be obtained, for example, by the Welch algorithm [64]. Once the power spectral density of the received signal is estimated, the interference suppression filter can be designed.

The transfer function of the suppression filter for a p -tap filter is defined as the reciprocal of the square root of the power spectral density at equally spaced frequencies $(0, R_c/p, 2R_c/p, \dots, (p-1)R_c/p)$, where R_c is the chip rate. This transfer function is

$$\mathcal{A}(k) = \frac{1}{\sqrt{P(kR_c/p)}} e^{-j\frac{2\pi}{p}(\frac{p-1}{2})k}, \quad k = 0, 1, \dots, p-1 \quad (2.81)$$

where $P(\cdot)$ is the power spectral density function. The term $e^{-j\frac{2\pi}{p}(\frac{p-1}{2})k}$ produces a realizable filter with delay $(p-1)/2$ samples. Using the inverse DFT on $\mathcal{A}(k)$, the impulse response (tap weights) of the filter can be calculated. The result is a linear phase finite impulse response (FIR) filter, which yields an interpolation error filter as discussed earlier.

Theoretical and simulated performance curves were presented for both the time domain and frequency domain algorithms.

For the time domain algorithms, SNR improvement for single-band interference of a specified bandwidth was very similar for 4th order and 15th order filters, suggesting that accurate modeling of the interference was not critical. In contrast, for multiple-band interference, the filter order that was required depended on the number of interfering bands; at a minimum, one pair of zeroes was recommended per band for BPSK signalling (or one complex zero for QPSK signaling). Finally, the suppression filters exhibited little difference in performance between the two models representing the interference, i.e., whether it was modelled as narrowband Gaussian noise or as a sum of sinusoids.

They also considered the prediction error filter $A_p(z)$ in cascade with its matched filter $A_p^*(1/z^*)$, as shown in Fig. 2.8. This arrangement produces a linear phase

Figure 2.8: Whitening filter followed by its matched filter.

filter like the interpolation error filter, but whose center tap is not equal to unity. This structure also resembles a maximum likelihood (ML) receiver (to be discussed later), in which the received signal is whitened by the error filter $A_p(z)$ and the result is applied to its matched filter. As an example of its performance, this structure exhibited approximately 5 dB of additional improvement in performance over its prediction filter counterpart for the case of $I/S = 20$ dB.

The frequency domain algorithm also proved to be a viable means of interference suppression. However, there were two main differences between it and the time-modeling approaches. First, the notch of the frequency domain filter was not as deep, leading to a degradation in η of a few dB for the example considered. Second, the time domain modeling techniques required much fewer data samples than the frequency domain approach to provide good interference suppression. For example,

the time-domain method provided suppression approaching that predicted by theory with as few as 50 data samples, whereas the Welch algorithm required 992 points to obtain similar performance. This latter result may prove to be unacceptable in an interference environment which changes quickly. As a final point, Ketchum *et al.* derived BER expressions based on the assumption that the noise at the output of the adaptive filter was Gaussian. (Recall from Eq. (2.76) that the noise consisted of three terms, two of which were non-Gaussian.) The resulting BER curves compared favourably to those obtained from a Monte Carlo simulation, confirming the validity of the assumption. Furthermore, considerable improvement in BER performance could be achieved, as noted earlier, by passing the output of the prediction error filter through its matched filter. For a processing gain of 60, $I/S = 20$ dB, 10 sinusoids spanning a bandwidth equal to 20% of the chip rate, and $E_b/N_0 = 10$ dB, the BER improved from 2×10^{-2} (unmatched) to 1×10^{-4} (matched).

Saulnier *et al.* [65, 66, 67, 68, 69] emphasized the implementation of the LMS algorithm as applied to interference suppression. They considered both charge transfer devices (CTD's) and digital hardware. Since direct implementation of the LMS algorithm requires two multipliers per filter tap (i.e., one for the tap weight update increment, and a second to calculate the signal sample), a burst processing structure was used in order to reduce the hardware multiplier count. This approach allows implementation of the adaptive algorithm with only two multipliers regardless of filter order. The cost, however, is a reduction in bandwidth processing capability by a factor equal to the number of taps.

In the CCD implementation [65], a stable tone jammer was considered for cases where $I/S = 10$ and 15 dB. A 7 chip PN sequence with a 26.3 kHz chip rate (sample rate) was used along with a 16 tap interpolation filter (8 taps on each side of the center tap for a total of 17 taps). Empirical BER curves were compared with a theoretical curve based on SNR_o in Eq. (2.75) at the output of the PN correlator. The results were interesting. As an example, for $E_b/N_0 = 10$ dB, the theoretical result yielded a BER of 2×10^{-5} for both interference conditions, whereas the experimental results were 2×10^{-3} ($I/S = 10$ dB) and 1×10^{-2} ($I/S = 15$ dB). The main reason for the discrepancy between the theoretical and experimental results appears to be the use

of a 7 chip PN sequence relative to the number of taps in the filter. This leads to a certain degree of code predictability for cases in which adjacent bits are equal, resulting in some code cancellation. Other reasons for the discrepancy would be due to:

- (a) the use of the LMS algorithm, which only approximates the Wiener-Hopf equation;
- (b) inaccuracies due to implementation; and,
- (c) charge transfer inefficiencies in the device.

A digital implementation of the LMS algorithm using the same burst configuration was assessed [66, 68]. The sampling rate in this case was increased to 177 kHz. In addition, the input was quantized to 8 bits and both 7 chip and 31 chip PN sequences were used; the full PN code was contained in one symbol. Three interferer types were examined, i.e., a 30 kHz stable tone, narrowband Gaussian noise of bandwidths of 1 and 2 kHz centered at 30 kHz, and a swept tone [68].

With respect to the stable interferers (i.e., the tone and narrowband Gaussian noise), the effect of interference bandwidth was quite evident. For the case of $I/S = 15$ dB and $E_b/N_0 = 10$ dB, the tone jammer yielded a BER of 2×10^{-4} ; the 1 and 2 kHz noise jammers yielded BER's of 1×10^{-3} and 5×10^{-3} , respectively. This performance degradation stems from two phenomena:

- (a) the wider notches created by the excision filter, which has the effect of introducing more signal distortion; and
- (b) a reduction in the predictability of the interference as the interference bandwidth increases.

In addition, for the single tone case, the effect of the level of interference power for fixed signal power with and without the excision filter was examined. Without the filter, and I/S ranging from 0 dB to 20 dB, the BER ranged from 2×10^{-4} to 0.5. With the filter present, the BER ranged from 1×10^{-4} to 5×10^{-4} , indicating the sensitivity of the system without the filter to different power levels.

With respect to the swept tone interferer, the objective was to assess the effect of μ in Eq. (2.47) for several sweep rates. The sweep rates ranged from 0 to approximately 0.001 Hz/sec. (normalized with respect to the sampling rate of 177 kHz), and μ ranged from 0.002 to 0.029. For small μ (i.e., 0.002) and a sweep rate of .001 Hz/sec., the BER was 0.15; decreasing the sweep rate to 0.0001 Hz/sec., resulted in a slightly less BER, even though the sweep rate was reduced by a factor of 10. On the other hand, for larger μ (i.e., 0.029) and the same sweep rates, the BER's were 5×10^{-2} and 8×10^{-3} , respectively, confirming the requirement for a larger convergence parameter in the LMS algorithm in order to better track the tone.

Ittis and Milstein [70] developed BER expressions in terms of the excision filter coefficients for two receiver types. The first receiver consisted of a bandpass RF filter followed by an analogue tapped delay line; the second has already been discussed (Fig. 1.2(b)). Each receiver was subjected to either a tone or narrowband Gaussian jammer and evaluated in terms of its BER performance. Three design criteria were used to calculate the filter coefficients.

The first criterion used an autocorrelation function derived from the combination of signal, noise and jammer. This refers to the familiar case already discussed in which the least squares filter whitens the spread spectrum signal, jammer and noise in a proportional sense, with most of the whitening being applied to the jammer.

The second criterion used only the noise and jammer in the autocorrelation function (it was assumed that somehow the signal could be removed). In fact, the only difference between the autocorrelation matrices of criteria 1 and 2 was the disappearance of the signal power from the diagonal elements of R in Eq. (2.71) when criterion 2 was applied. The second criterion was motivated by the structure of the Maximum Likelihood (ML) receiver [12] for the problem of the optimum detection of a signal in coloured noise n_n^c as shown in Fig. 2.9. The ML receiver consists of a whitening filter for the coloured noise (in this case noise plus jammer) followed by a filter matched to the whitening filter and the signal.

The third criterion used tap weights which would produce zeroes of $A_p(z)$ located on the unit circle. This is a limiting condition of the analytical equation developed by Li and Milstein [61] for a stable tone jammer. As the jammer power approaches

Figure 2.9: Maximum likelihood receiver for the case of signal in coloured noise.

infinity, the zeroes of the transversal filter tend towards the unit circle.

The BER results produced by these three criteria were compared to each other for both prediction and interpolation filters. They were also compared to the performance of ML receivers derived for tone and Gaussian noise jammers. The ML receivers provided a benchmark for the other receivers. Of the three criteria, the second criterion, in which the signal power R_{ss} did not form part of the autocorrelation matrix, yielded the best results, approaching the performance of the ML receiver. The filter consisted of the prediction filter followed by its matched filter. This structure is similar to the one proposed by Ketchum in Fig. 2.8, in which the filter taps were calculated using the first criterion. Achieving the performance level of the ML receiver would be difficult in practice unless the signal could be removed from the input u_n .

It is suggested one such approach [70], employing decision-feedback, as shown in Fig. 2.10. The spread spectrum signal contained in u_n , delayed by one bit (L chips), is removed most of the time for low BER's. The filter coefficients are thus calculated on the basis of the jammer and noise. The basic concept will now be described with the help of Fig. 2.10.

Consider a linear interpolator and stationary processes. The interference i_{n-L} is estimated from p future and p past samples from the sequence y_{n-L} . The mean-squared value of the error (call this error $e'_{2p,n-L}$, which is not shown in Fig. 2.10, but is contained in the box labelled *Adaptive Filter Algorithm*) between y_{n-L} and its estimate $\hat{i}_{n-L}$ is minimized with respect to the set of $2p$ coefficients $\{a_i^{(2p)}, i = -p, \dots, -1, 1, \dots, p\}$. The optimum set of coefficients is then used in the excisor to which u_n is applied. Since it was the mean-squared value of the error sequence

Figure 2.10: Excision filter employing decision feedback, which approximates the maximum likelihood receiver.

$e'_{2p,n-L}$ (and not $e_{2p,n}$, the interpolation error from criterion 1) which was minimized with respect to the coefficients $a_i^{(2p)}$, the signal s_n in the sequence $e_{2p,n}$ will not be as distorted by the filtering operation.

In another paper by Ketchum [71], three decision feedback schemes were suggested. The basic principle is illustrated in Fig. 2.11(a). The concept is somewhat different from Illtis'. A delayed chip decision $\hat{s}_{n-1}$ is fed back and subtracted from a delayed version of the input u_n , producing the sequence $y_{n-1} = i_{n-1} + n_{n-1}$. This sequence is applied to a linear prediction filter $G_p(z) = 1 - A_p(z)$ which forms an estimate of the interference $\hat{i}_n$. The optimum estimate is obtained from the orthogonality of the error $e_{p,n}^f$ to the data set $\{y_{n-1}, i = 1, 2, \dots, p\}$. The performance of this method is sensitive to the chip error rate. Under the condition of no chip errors, the signal distortion term in Eq. (2.76) is zero. For low signal-to-noise-plus-interference conditions, the performance tends to drop off quickly because of the increase in chip errors. Two other techniques, which provide better performance at the expense of system complexity, are described next.

Figure 2.11: Two alternative decision-feedback structures based on: (a) Chip decisions; and, (b) Improving bit decisions.

An improvement to the scheme in Fig. 2.11(a) is shown in Fig. 2.11(b), in which an evolving estimate for the k^{th} bit $\hat{D}_{kn}$ is quantized at each chip interval nT_c and multiplied by the local PN correlator, yielding improved estimates of the signal. This technique produces more chip errors near the beginning of the bit interval, but as the bit evolves at the output of the summer, the chip error rate decreases. The improvement in SNR that one achieves for this case thus becomes a function of the processing gain. The performance of this technique rivals that produced by the whitening filter/matched filter combination discussed earlier (Fig. 2.8).

Better performance is achieved with the more complex configuration shown in Fig. 2.12. The sequence y_{n-1} , with the spread spectrum signal having been “removed” by the technique in Fig. 2.11(b), is applied to an interpolation filter $R_{2p}(z)$. This filter is formed by first combining the error filter $A_p(z) = 1 + G_p(z)$ with its matched filter $A_p^*(1/z^*)$ and then removing the center tap of value $g'_0 u_{n-p-1}$. As an example of the performance of this approach, for $I/S = 20$ dB, an interferer with bandwidth equal to 20% of the chip rate, and a processing gain of 60, this system experienced increases in η of 14 and 7 dB over the systems in Figs. 2.8 and 2.12 respectively.

Other research has been conducted on decision feedback approaches. For more on this topic, one can refer to Takawira and Milstein [72] and Shah and Saulnier [73].

To this stage the emphasis in adaptive digital excision techniques has been on transversal filter structures. Gradient lattice [69, 74] and least squares lattice [75, 76] structures have also been examined. In [74], the gradient lattice was used in filtering a stable tone; in [69] it was implemented on a TMS32010 processor.

In [76] several receiver configurations employing least-squares (LS) lattice filters were assessed with respect to excision and channel equalization, with the interference being coloured noise. Two important issues were addressed:

- (a) utilizing the LS lattice forward predictor in series with the backward predictor (prediction filter/matched filter combination) to remove the interference; and
- (b) demonstrating that the LS lattice could be implemented as a joint process estimator to carry out decision-directed equalization and interference suppression simultaneously, provided the SNR was low (which is often the case in DS/SS

Figure 2.12: A decision-feedback structure employing a matched filter arrangement.

systems) and the signal spectrum was reasonably flat.

It was shown that the joint process estimator approximated a transfer function consisting of two filters in series, i.e., a filter matched to the channel's transfer function and a filter to whiten the coloured noise. When the joint process estimator was used, simulation results for $\lambda = 1.0$ in Eq. (2.69) produced results which were marginally better than the LS lattice whitening filter, even when the LS whitening filter was preceded by the channel's exact matched filter. However, for $\lambda < 1.0$, the use of simultaneous equalization/whitening produced inferior results compared to the LS whitening filter. This result suggests that the joint process estimator is more sensitive to the misadjustment error when $\lambda < 1.0$ (see Section 3.2.1 for the definition of misadjustment error as applied to the recursive least squares algorithm).

2.5 Chapter Summary

This chapter presented a host of analog and digital approaches which can suppress narrowband interference in DS spread spectrum communication systems. The main emphasis was on digital adaptive methods involving, primarily, time-domain modeling techniques based on the theory of least squares. A frequency domain technique was also described.

As the literature review has indicated, different facets of the problem have been examined, including the use of decision-feedback techniques. Furthermore, a large percentage of the effort has been devoted to stationary types of interferers (e.g., stable tones and narrowband coloured noise). The next chapter presents work conducted by the author on the application of least squares techniques to the filtering of stable and FM interference.

Table 2.3: Summary of Digital Excision.

Hsu, Giordano	Stable Tone(s) NBGN	Prediction	LD, Burg
Li, Milstein	Stable Tone(s)	Prediction, Interpolation	One Tone (Anal.) Mult. Tones (LMS)
Li, Milstein	Pulsed Tone	Complex Prediction, Interpolation	Analytical/LMS
Ketchum, Proakis	Single/Multiple Band(s) of Tones and NBGN	Prediction, Interpolation, FFT	LD, Burg, Unconstrained LS Welch Algorithm
Ketchum	NBGN	Prediction, Interpolation with 3 Decision Feedback Schemes	Analytical
Iltis, Milstein	Stable Tone, NBGN	Prediction, Interpolation, ML, Decision Feedback	Analytical
Saulnier <i>et al.</i>	Stable Tone, NBGN, Swept Tone	Prediction	LMS Digital Implementation
Saulnier <i>et al.</i>	Stable Tone	Prediction	Gradient Lattice Implementation
Guilford, Das	NBGN	Prediction	Gradient Lattice Simulation
Kozminchuk, Sheikh	Swept Tone	Prediction, Interpolation	LS Lattice Simulation
Reichman, Scholtz	NBGN	Prediction, Interpolation	LS Lattice Spatial-Temporal Excision, Equalization

Chapter 3

Least Squares Adaptive Excision

3.1 Introduction

This chapter presents theoretical and simulation results of the application of least squares filters to the removal of interference from a spread spectrum system. Some of these results are presented in [75].

The chapter outline is as follows. Section 3.2 describes the communications model used in the simulations. Section 3.2.1 considers the performance of the lattice recursive least squares algorithm applied to filtering a stable tone. The simulation results are compared to a theoretical model of the adaptive filter. Section 3.2.2 is concerned with the performance of block and recursive least squares algorithms as applied to a linearly swept tone. The results pose the question as to whether or not another digital filtering technique, other than that provided by the least squares methods, could be more appropriate for FM-types of interferers. The chapter concludes with a suggestion to use a Kalman filter excisor which has performance characteristics similar to an analog PLL.

3.2 Communications System Model

The communications model used in this chapter and shown in Fig. 3.1 is the low-pass equivalent form of an undistorted BPSK signal with additive white Gaussian noise.

The PN sequence is modelled as rectangular chips of amplitude ± 1 , and of duration $T_b = L/R_c$, where L is the number of chips per bit. The signal is corrupted by white Gaussian noise, n_n , of variance $\sigma_{nse}^2 = N_0/2$, and interference i_n of power ρ_0 . The effect of the integrate-and-dump circuit on the interference as shown in Fig. 1.2(b) has not been included in the model.

Figure 3.1: Communications model used in the computer simulations.

Two adaptive filter algorithms are considered, i.e., the block least squares and recursive least squares algorithms which were discussed in Chapter 2. The block algorithm calculates a set of tap coefficients for a transversal filter using the software in [36]. (An alternative would have been to use the software in [34, 35].) The recursive algorithm listed in Table 3.1 calculates a set of forward and backward reflection coefficients for a lattice least squares filter, as well as the forward and backward prediction errors.

For the BLS algorithm, two FIR filter transfer functions will be considered, i.e., $A(z)$ and $A(z)A^*(1/z^*)$, where $A^*(1/z^*)$ is the matched filter of $A(z)$. This latter configuration causes a delay of p samples, where p is the filter order of $A(z)$. Thus, the PN sequence in the correlator must be delayed by p chips for this case. The filtered data are applied to the PN correlator whose output is summed and sampled every L chips.

As discussed in Section 2.4.3, the RLS algorithm can have associated with it a weighting parameter or “forgetting factor” λ , where $0 < \lambda < 1$. The effect of this parameter on the interference rejection capability of the adaptive filter for stable and

swept tones will be examined. Finally, the matched filter for the lattice RLS algorithm is obtained by feeding the forward prediction error from the i^{th} lattice stage into a second lattice having the same reflection coefficients. The output is the backward prediction error $e_{i,n-i}^b$ of the i^{th} stage of that filter as shown in Fig. 3.2. The output is delayed by i samples. (If one were to use the FTF least squares algorithm, for example, as discussed in Section 2.4.3, then the matched filter would be the backward predictor.)

Figure 3.2: Recursive least squares lattice structure followed by its matched filter.

3.2.1 Stable Tone Performance

The performance of the pre-windowed unnormalized RLS lattice algorithm is examined with respect to a stable tone located at 0.20 Hz. The objective is to evaluate the effect of the filter order p and forgetting factor λ on the amount of suppression that the algorithm can provide.

The results are presented in terms of:

- (a) root locations of the whitening filter on the z -plane;

- (b) the average attenuation at the interference frequency; and,
- (c) bit error rate performance.

To determine the root locations of the whitening filter, the coefficients of $A(z)$ must be calculated from the lattice filter. This can be done using the algorithm in Table 3.2 [43, 45] with the one in Table 3.1. The algorithm in Table 3.2 is a more generalized version of the LD algorithm, developed for the case of pre-windowed signals. All filter orders for $A(z)$ up to and including p are determined. The coefficients of each order are represented as $\{a_0^{(0)}\}$, $\{a_0^{(1)}, a_1^{(1)}\}$, $\{a_0^{(p)}, a_1^{(p)}, \dots, a_p^{(p)}\}$, where $a_0^{(i)} = 1$. The tone is defined as

$$i_n = I \cos(2\pi\delta f n + \theta) \quad (3.1)$$

where $I = 1.0$, $\delta f = 0.20$ Hz, and θ is a uniform random variable between 0 and 2π radians. The interference-to-noise ratio is 20 dB. Table 3.3 lists the average attenuation at the tone frequency achieved for each situation. This average was calculated on the basis of 950 sample points, excluding the initial transient. The same data were used in each test. Figures 3.3 to 3.12 show the distribution of the roots of $A_p(z)$ for each time step or iteration for filter orders ranging from $p = 2$ to $p = 8$. Corresponding to each filter order is a weighting parameter λ of either 1.0 or 0.9. There are several observations that can be made from these figures and the data in Table 3.3:

- (a) for $\lambda = 1.0$, the attenuation improves as the filter order increases; it increases steadily until $p = 6$; at $p = 8$, there is no change. When the least squares algorithm first starts, roots are produced that appear either outside the unit circle or further away from the main pack. After the least squares algorithm has converged, one pair of zeroes is dedicated to removing the tone, whereas the others are contained within the circle and provide little attenuation at the frequencies at which they are located as determined by their distance from the unit circle;
- (b) for $\lambda = 0.9$, the attenuation is less, and does not improve with increasing order. The roots wander around more, and in many cases stray towards the edge of the

Table 3.1: Unnormalized Pre-Windowed Least Squares Lattice Algorithm.

Input parameters:

p = maximum order of lattice
 u_n = data sequence
 λ = exponential weighting factor

Variables:

$\mathcal{E}_{i,n}^f, \mathcal{E}_{i,n-1}^b$ = sample variances of forward/backward errors
 $\Delta_{i,n}$ = sample partial correlation coefficient
 $\gamma_{i,n}^c$ = likelihood variable
 $e_{i,n}^f, e_{i,n-1}^b$ = forward/backward prediction errors
 $K_{i,n}^f, K_{i,n}^b$ = forward/backward reflection coefficients

Computations performed once for every time step ($n = 0, \dots, T_{max}$).

Initialize:

$$\begin{aligned}
 e_{0,n}^f &= e_{0,n}^b = u_n \\
 \mathcal{E}_{0,n}^f &= \mathcal{E}_{0,n}^b = \lambda \mathcal{E}_{0,n-1}^f + u_n^2 \\
 \gamma_{-1,n}^c &= 1
 \end{aligned}$$

Repeat for $i = 0$ to $\min(p, n)-1$:

$$\begin{aligned}
 \Delta_{i+1,n} &= \lambda \Delta_{i+1,n-1} + e_{i,n}^f e_{i,n-1}^b / \gamma_{i-1,n-1}^c \\
 \gamma_{i,n}^c &= \gamma_{i-1,n}^c - e_{i,n}^{b2} / \mathcal{E}_{i,n}^b \\
 K_{i+1,n}^b &= \Delta_{i+1,n} / \mathcal{E}_{i,n-1}^b \\
 e_{i+1,n}^f &= e_{i,n}^f - K_{i+1,n}^b e_{i,n-1}^b \\
 \mathcal{E}_{i+1,n}^f &= \mathcal{E}_{i,n}^f - K_{i+1,n}^b \Delta_{i+1,n} \\
 K_{i+1,n}^f &= \Delta_{i+1,n} / \mathcal{E}_{i,n}^f \\
 e_{i+1,n}^b &= e_{i,n-1}^b - K_{i+1,n}^f e_{i,n}^f \\
 \mathcal{E}_{i+1,n}^b &= \mathcal{E}_{i,n-1}^b - \Delta_{i+1,n} K_{i+1,n}^f
 \end{aligned}$$

Table 3.2: Computing Predictor Coefficients from the Lattice Parameters.

Initialize:

$$b_i^{-1'} = 0 \text{ for } i = 0, \dots, p-1$$

$$c_0^{(0)} = 0$$

Outer LoopRepeat for $i = 0, \dots, p$

$$a_0^{(i)} = b_0^{(i)} = 1, \quad i = 0$$

$$a_0^{(i)} = b_0^{(i)} = 0, \quad i > 0$$

Inner LoopRepeat for $j = 0, \dots, p-1$

$$b_j^{(i)'} = b_j^{(i)} - e_j^b c_j^{(i)} / \gamma_{j-1}^c$$

$$c_{j+1}^{(i)} = c_j^{(i)} - e_j^b b_j^{(i)} / \varepsilon_j^b$$

$$a_{j+1}^{(i)} = a_j^{(i)} - K_{j+1}^b b_j^{(i-1)'}$$

$$b_{j+1}^{(i)} = b_j^{(i-1)'} - K_{j+1}^f a_j^{(i)}$$

End of Inner LoopEnd of Outer Loop

Table 3.3: Attenuation performance of the lattice RLS algorithm.

λ	Filter Order	Attenuation (dB)
1.0	2	47
0.9	2	42
1.0	4	53
0.9	4	42
1.0	6	57
0.9	6	42
1.0	8	57
0.9	8	41

Figure 3.3: Root locations of whitening filter when the interference is a single tone: $p = 2$, $\lambda = 1.0$.

Figure 3.4: Magnified version of the second root in Fig. 3.3.

Figure 3.5: Root locations of whitening filter when the interference is a single tone: $p = 2$, $\lambda = 0.9$.

Figure 3.6: Magnified version of the second root in Fig. 3.5.

Figure 3.7: Root locations of whitening filter when the interference is a single tone: $p = 4$, $\lambda = 1.0$.

Figure 3.8: Root locations of whitening filter when the interference is a single tone: $p = 4$, $\lambda = 0.9$.

Figure 3.9: Root locations of whitening filter when the interference is a single tone:
 $p = 6$, $\lambda = 1.0$.

Figure 3.10: Root locations of whitening filter when the interference is a single tone: $p = 6$, $\lambda = 0.9$.

Figure 3.11: Root locations of whitening filter when the interference is a single tone: $p = 8$, $\lambda = 1.0$.

Figure 3.12: Root locations of whitening filter when the interference is a single tone:
 $p = 8$, $\lambda = 0.9$.

unit circle, suggesting that deep notches would be created at these extraneous frequencies;

- (c) as p increases, the additional roots tend to distribute themselves around the unit circle.

These observations lead to the conclusion that the RLS algorithm is expected to perform better for the case of $\lambda = 1.0$ and a filter order of 6 or 8. This latter point makes sense, since a sinusoid in noise is best modelled as an autoregressive moving average (ARMA) process [15, 23], which is equivalent to an AR process of infinite order. The effect of the additional roots is to model the background noise (two of the roots are dedicated to the tone). For $p = 2$, the filter must balance its resources to model both interference and noise. Thus its notch will not be as deep. As p increases, more resources are allocated to model the noise.

For values of $\lambda < 1.0$, the effective memory of the adaptive filter is reduced, so that there is not as much data averaging. This exponential window also leads to fluctuations in the filter coefficients, which increase the movement of the roots of the rejection filter.

The above observations for $\lambda = 1.0$ do not necessarily mean that the excision filter will perform better as one goes to higher filter orders. Recall that the objective of the excision filter is to whiten the sum of signal, noise and interference. As the filter order increases, more signal distortion will occur, and at some point will become a significant contributor to noise in Eq. (2.76).

Bit Error Rate Performance

This next part considers the bit error rate performance of the RLS algorithm as applied to the stable tone defined in Eq. (3.1). Both theoretical and simulation results will be presented. The model for calculating the theoretical BER is shown in Fig. 3.13. This model is similar to the one in [77], in which the LMS algorithm performance was assessed. This type of model is required for the RLS algorithm when $\lambda < 1.0$ which, as mentioned above, leads to filter fluctuations and an increase in noise, known as excess noise.

 Figure 3.13: Model for a least squares excision filter for the case when $\lambda < 1.0$.

The average mean-squared value of this excess error is [78]

$$\mathcal{E}_{excess} = E\{\Delta \mathbf{a}^T \mathbf{R} \Delta \mathbf{a}\} \quad (3.2)$$

$$= \frac{1-\lambda}{1+\lambda} \text{Tr}\{\mathbf{I} + E\{\mathbf{P}_n^2\} \mathcal{E}_{opt}\} \quad (3.3)$$

where $\Delta \mathbf{a}$ is the vector of filter misadjustments due to the exponential window, $\mathbf{R}$ is the autocorrelation matrix of the data, Tr refers to the trace operator, $\mathcal{E}_{opt}$ is the optimum mean-squared error obtained from the Wiener-Hopf solution, $\mathbf{I}$ is the identity matrix, and $\mathbf{P}_n$ is a zero mean fluctuation matrix. A simplification [78] of Eq. (3.3) is

$$\mathcal{E}_{excess} = \frac{1-\lambda}{1+\lambda} \left[1 + \frac{1-\lambda}{1+\lambda} \xi \right] p \mathcal{E}_{opt} \quad (3.4)$$

where $\xi = \text{var}\{u_n^2\} / \text{var}^2\{u_n\}$. If u_n is a Gaussian process, then $\xi = 2$.

To determine the BER for the system in Fig. 3.13, the test statistic at the output of the bit detector must be determined. This test statistic is

$$\mathcal{U} = \sum_{n=1}^L c_n (e_{p,n}^f + e_{n,excess}) \quad (3.5)$$

$$= \sum_{n=1}^L c_n u_n - \sum_{n=1}^L c_n \sum_{i=1}^p a_i u_{n-i} + \sum_{n=1}^L c_n e_{n,excess}. \quad (3.6)$$

Letting $u_n = s_n + i_n + n_n$, and assuming that the excess error is zero mean and the chips are independent samples, $E\{\mathcal{U}\} = L$ in Eq. (3.6). The variance is similar to the denominator in Eq. (2.76) except for the term due to the excess error. Thus,

$$\begin{aligned} \text{var}\{\mathcal{U}\} &= L \left[\sum_{j=1}^p a_j^{(p)^2} + \rho_0 - 2 \sum_{j=1}^p a_j^{(p)} \rho_j \right. \\ &\quad + \sum_{j=1}^p \sum_{k=1}^p a_j^{(p)} a_k^{(p)} \rho_{j-k} \\ &\quad \left. + \sigma_{nse}^2 + \sigma_{nse}^2 \sum_{j=1}^p a_j^{(p)^2} + \mathcal{E}_{excess} \right] \end{aligned} \quad (3.7)$$

$$= \sigma^2. \quad (3.8)$$

If the assumption is made that $\mathcal{U}$ is a Gaussian random variable [63], with $\text{var}\{\mathcal{U}\} = \sigma^2$, then the probability of a bit error is defined as

$$P_b = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{SNR_o}}^{\infty} e^{-x^2/2} dz \quad (3.9)$$

where $SNR_o = L^2/\sigma^2$ (Eq. (2.77)). The excess noise variance $\mathcal{E}_{excess}$ can be calculated from Eq. (3.4) where, for the process under consideration, ξ can be calculated from

$$\text{var}^2\{u_n\} = [1 + \sigma_{nse}^2 + \rho_0]^2 \quad (3.10)$$

$$\text{var}\{u_n^2\} = 2\sigma_{nse}^2[\sigma_{nse}^2 + 2] + 4\rho_0[1 + \sigma_{nse}^2] + \frac{1}{2}\rho_0^2. \quad (3.11)$$

The theoretical BER curves as determined from Eq. (3.9) are shown in Figs. 3.14 to 3.16 along with the simulation results. The conditions here correspond to a forward prediction filter without its matched filter, and filter orders of 2,4,6, and 8. The weighting factors were $\lambda = 1.0, 0.95$, and 0.90 . The frequency of the interference was offset from the carrier by 0.2 Hz.

The computer simulations were based on a signal-to-interference ratio of -20 dB, a processing gain of 20, and a sampling rate $1/T_c$ equal to 1 Hz. For these simulations 95000 random bits of information were generated, yielding 1.9 million chips after multiplication by a random PN sequence. To each signal sample (chip) was added a sample of white Gaussian noise n_n and interference i_n . The noise variance was

determined from the E_b/N_0 of interest, i.e., $\sigma_{nae}^2 = L/(2E_b/N_0)$ where N_0 is the single-sided noise power spectral density. The amplitude of the interference was set equal to 14.142 to provide an SIR/chip of -20 dB. The BER was determined by comparing the bits "transmitted" to those "received".

Consider the results illustrated in Fig. 3.14, which correspond to $\lambda = 1.0$. For this particular value of λ , the excess error in Eq. (3.4) is zero. The theoretical curves are, therefore, a function of the mean-squared value of $e_{p,n}^f$ in Fig. 3.13, i.e., $\mathcal{E}_{opt}$ obtained from the Wiener-Hopf equation for the particular order p . The theoretical and simulated results show that performance improves as the filter order increases. For low filter orders ($p = 2$ and 4), the theory predicts worse performance than those obtained from simulation; for $p = 8$ theory predicts better performance than that obtained from simulation. These results suggest that the statistics assumed in Eq. (3.9) for $\sqrt{SNR_o}$ are only approximately Gaussian for $\lambda = 1.0$. This makes intuitive sense since the filter coefficients stabilize to the Wiener-Hopf solution, therefore providing fixed attenuation on the interference. The density function of the residual interference would therefore stray away from the Gaussian density function, particularly in the tails. For example, for filter orders of $p = 2$ and 4, the tails of the true density function would appear to taper off quicker as indicated by the better performance predicted from simulation. For $p = 8$ the tails of the true density function would appear to taper off less quickly than the Gaussian function.

The results for $\lambda < 1.0$ are illustrated in Figs. 3.15 to 3.19. They show the progression of the simulated results toward Gaussian statistics, since the agreement between theory and simulation is good, especially for filter orders greater than 2. This too makes intuitive sense, since the residual interference will have statistics closer to Gaussian due to the fluctuating filter coefficients which would lead to fluctuating attenuation of the interference. For the higher filter orders, the range of values of the residual interference would increase because of the deeper notches due to the movement of the zeroes of the whitening filter toward the edge of the unit circle. For $p = 2$, one needs a smaller λ for the range of values of the residual interference to increase and, therefore, to take on the appearance of a Gaussian density function.

In conclusion, the results indicate that treating the residual interference at the

output of the excisor as a Gaussian random variable is an approximate assumption. This assumption, however, becomes more valid as λ decreases. In addition, the results also show that agreement between theory and simulation depends on the operating conditions but is, nonetheless, accurate to within 1 dB for the filter orders and values of λ considered.

The next section considers a swept tone interferer. The performance of the BLS and RLS algorithms are presented in terms of their tracking ability and the bit error rate at the output of the bit detector. The results were obtained from computer simulation.

Figure 3.14: Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 1.0$. The theoretical curves are based on Eq. (3.9).

Figure 3.15: Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.99$. The theoretical curves are based on Eq. (3.9)

Figure 3.16: Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.98$. The theoretical curves are based on Eq. (3.9)

Figure 3.17: Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.97$. The theoretical curves are based on Eq. (3.9)

Figure 3.18: Theoretical and simulated bit error rate curves for single-tone interference located at 0.20 Hz and $\lambda = 0.96$. The theoretical curves are based on Eq. (3.9)

2

PM-1 3 1/2" x 4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

Figure 3.19: Theoretical and simulated bit error rate curves for single tone interference located at 0.20 Hz and $\lambda = 0.95$. The theoretical curves are based on Eq. (3.9)

3.2.2 Swept Tone Performance

The interference is modelled as a frequency modulated waveform of the type

$$\begin{aligned} i(t) &= I \cos(\theta(t)) \\ &= I \cos(2\pi\delta f t + d \int_0^t m(\tau) d\tau + \varphi) \end{aligned} \quad (3.12)$$

where δf is an offset frequency from the spread spectrum signal carrier, $m(\tau)$ is the modulating signal of the interferer, d is the frequency deviation constant of the modulator in radians/second/volt [79], and φ is an initial random phase.

To demonstrate the tracking behaviour of the BLS and RLS adaptive filters, a swept tone with sweep rates of ± 0.001 Hz/sec. was modelled using Eq. (3.12). A section of the instantaneous frequency $\dot{\theta}(t)/2\pi$ from Eq. (3.12) is shown in Fig. 3.20, sweeping between 0.1 and 0.3 Hz, with $\delta f = 0.2$ Hz and $d = 0.628$ radians/second/volt. The peak frequency deviation Δf for this case was 0.1 Hz. The interference-to-noise ratio was set at 20 dB. The spread spectrum signal was not included in this set of tests.

The BLS algorithm was examined first. For each block of data, the set of autoregressive parameters $\{a_i^{(p)}, i = 1, 2, \dots, p, \sigma_p^2\}$ were obtained along with the roots of $A_p(z)$, which represent the locations of the notches created by the filter.

Figures 3.21, 3.22 and 3.23 show three examples of the tracking capability of a fourth order BLS filter $A_p(z)$. The number of samples per block is, respectively, 100, 50, and 10. Only the positive frequency is plotted. The horizontal lines correspond to the frequencies of two of the roots of $A_p(z)$ in the upper half of the complex z -plane, the other two roots being in the lower half plane. The location of the horizontal lines indicates the position of the notches for the particular block of data processed to produce the adaptive filter coefficients.

For the case of 100 samples per block, Fig. 3.21 shows that two roots appear to be inadequate for spanning the range of frequencies within the 100-second block of data; the two roots are attempting to balance their available resources over the 100-second section for which they are responsible. Given the relatively large space between the notches for a particular data-block, or their relative closeness in others, one would expect a reasonable amount of residual interference. To improve performance, a higher

order filter (perhaps $p=6$ or 8) would probably be required. However, depending on the block size and filter order, one may run the risk of introducing more signal distortion. Reducing the number of samples per block to 50 (Fig. 3.22) improves the tracking capability somewhat, but at times only two out of the four roots follow the tone. Figure 3.23 shows good tracking of the tone when only 10 samples per block are used. In this case, one pair of roots tracks the tone, whereas the other two are located at 0.0 and 0.5 Hz most of the time, i.e., on the real axis of the complex z -plane.

For the RLS lattice algorithm, the samples were processed recursively using the prewindowed lattice algorithm listed in Table 3.1. The filter parameters of $A_p(z)$ for this lattice were obtained from the algorithm listed in Table 3.2.

Figures 3.24 and 3.25 show the tracking capability of the RLS lattice filter for the case of $\lambda = 0.90$ and filter orders of 2 and 4, respectively. As in the BLS case, for a fourth order filter, two of the roots in Fig. 3.25, track the tone, whereas the other pair wander around, similar to the case for the stable tone.

These results lead to several conclusions concerning the BLS and RLS filters. First, in the case of the BLS filter and a fairly fast sweeping tone covering a large percentage of the spread bandwidth (in this example, the swept tone covered a bandwidth equal to 20% of the chip rate), as few samples as possible per block should be used to achieve reasonable tracking and filtering. Second, for both the BLS and RLS algorithms, a second order filter appears to be more appropriate, since the additional roots from higher order filters have a tendency either to locate themselves on the real axis or elsewhere. These extra roots will introduce additional spectral distortion in the spread spectrum signal. The degree of distortion would depend on the depth and quantity of the notches. As demonstrated for the stable tone interferer, for decreasing values of λ , the roots were less confined to one spot in the interior of the unit circle and had a tendency to wander quite significantly. The notch depths would be enhanced if the higher order filter was cascaded by its matched filter $A_p^*(1/z^*)$. For larger values of λ , the tracking capability of the filter would degrade, i.e., a larger lag would be introduced (Fig. 3.26). Thus, the amount of interference attenuation achieved would be less, thereby increasing the BER. As λ approaches 1.0, all the roots contribute to the filtering; they also exhibit much less mobility (Fig. 3.27). In the limit, when

Figure 3.20: Modulating signal $m(\tau)$ in Eq. (3.12).

Figure 3.21: Tracking results for the BLS algorithm for $p = 4$ and block size $N = 100$.

Figure 3.22: Tracking results for the BLS algorithm for $p = 4$ and block size $N = 50$.

Figure 3.23: Tracking results for the BLS algorithm for $p = 4$ and block size $N = 10$.

Figure 3.24: Tracking results for the RLS algorithm for $p = 2$ and $\lambda = 0.90$.

Figure 3.25: Tracking results for the RLS algorithm for $p = 4$ and $\lambda = 0.90$.

$\lambda = 1.0$ the roots tend to become stationary (Fig. 3.28). In other words, the RLS algorithm approaches the Wiener-Hopf solution.

Mobility of the roots is not essential for the RLS algorithm to provide interference suppression. As shown in Fig. 3.28, the roots appear to settle down to stable locations. The two notches as represented by the bold lines will suppress as much interference as they are capable of, given that there are only two of them. There would probably be a larger amount of residual interference compared to the case where $p = 2$ and $\lambda = 0.90$ as illustrated in Fig. 3.24. To reduce this residual interference, a larger filter order would be required but, as pointed out for the BLS case involving large block sizes, signal distortion would then become an important factor since a larger percentage of signal would be notched out as well.

Figure 3.26: Tracking results for the RLS algorithm, $p = 2$, $\lambda = 0.95$.

Figure 3.27: Tracking results for the RLS algorithm, $p = 4$, $\lambda = 0.99$.

Figure 3.28: Tracking results for the RLS algorithm, $p = 4$, $\lambda = 1.0$.

Bit Error Rate Performance

The simulation conditions were the same as those used for the stable tone. Figures 3.29 and 3.30 show the BER's for the BLS method for second order ($p = 2$) and fourth order ($p = 4$) filters $A_p(z)$, respectively; the matched filter results are also included here. In general, the BER improves as the block size decreases, confirming what was observed previously when the BLS algorithm's tracking capability was explored (Figs. 3.21 to 3.23). First consider the results for $N = 100$. When the matched filter was not used, there was a slight improvement in performance in going from $p = 2$ to $p = 4$ (Figs. 3.29 and 3.30). This suggests that the higher order filter is sharper, thus suppressing more of the interference (note that from Fig. 3.21(b) all the roots took part in the filtering process). When the matched filter was included, the performance improved significantly, particularly when the filter order was increased to $p = 4$, indicating a much larger reduction in the interference for this case.

For $N = 50$ and no matched filter, the BER results reveal very little difference between the second order and fourth order filters. Referring to Fig. 3.22(a), we see that for $p = 4$ there is a tendency for two of the roots to track the tone and the other two to sometimes wander around. Furthermore, because of the almost imperceptible difference between the results for $p = 2$ and $p = 4$ in Figs. 3.29 and 3.30, one comes to the conclusion that the wandering roots do not introduce much signal distortion, that the noise in Eq. (2.76) is dominated by the residual interference. However, the matched filter results indicate slightly inferior performance for the fourth order filter, suggesting that perhaps signal distortion is starting to manifest itself. For $N = 10$, this effect is more evident.

Figures 3.31 and 3.32 illustrate the BER performance for the RLS lattice filter. The performance of both the second and fourth order filters for $\lambda = 0.90$ are shown in Fig. 3.31. Consider first the case without the matched filter. The results exhibit little difference in performance for the fourth order filters. When the matched filter was present, the performance between the two cases was also very similar, suggesting two things:

- (a) the y both removed about the same level of interference; and,

- (b) a filter order of $p = 4$ did not appear to increase the other noise terms (signal distortion, Gaussian noise, and the excess noise due to filter coefficient fluctuation).

More tests using higher order filters would have to be conducted to examine this problem in more depth.

Similar comments hold for the results in Fig. 3.32 for the case of a second order filter, and $\lambda = 0.90$ and 0.95 . Recall from Figs. 3.25 and 3.26 that increasing λ resulted in an increase in the lag between the predictor and the interferer's modulating signal. This would result in an increase in the residual interference and, therefore, an increase in BER as shown when the matched filter was not present. With the matched filter, the effect of the lag is not as noticeable.

Remarks

- (1) The results suggest that for a single swept tone, a second order filter will achieve better performance than higher order filters, since the latter may introduce additional signal distortion. This was particularly evident with the BLS algorithm and small block sizes. For the RLS algorithm, it was shown too that a second order filter is adequate enough for the suppression of this type of interference; signal distortion did not appear to present much of a problem for a fourth order filter. One would expect a noticeable degradation in performance for higher filter orders and $\lambda < 1.0$. To be more conclusive, additional work would have to be conducted in this area. Also indicated was the sensitivity of BER to changes in λ , especially when the matched filter was not used. Finally, the results produced by the BLS and RLS algorithms contrast those of the stable tone in which increasing the filter order improved the performance.
- (2) The tracking examples involving the RLS algorithm exhibited a lag between the modulating signal and the estimate. This lag is a function of λ and suggests that an artificial delay could be imposed on the data to compensate for it. One would therefore expect an improvement in performance if this were done.

- (3) From the results presented for the swept tone, the block and recursive least squares second order filters can be viewed as tunable notch filters or, conversely, as frequency demodulators if one calculates the roots of the excision filter over time. This concept opens the door to other possible filtering schemes.
- (4) With this in mind, the author has developed a new digital adaptive algorithm which is based on an extended Kalman filter [80]. Kelly and Gupta [81] and Polk and Gupta [1], using Kalman filtering theory, developed a digital PLL which has performance characteristics similar to that of an analog PLL. This model forms the basis of a new excision algorithm to be described in the next chapter. Their model has been modified, and extended to render it adaptive. In particular, the preliminary performance of the new algorithm is presented for the case of FM and narrowband Gaussian noise interferers.

Figure 3.29: Bit error rate performance of the BLS algorithm for filter order $p = 2$ and several block sizes N when the interference was a swept tone; SIR/chip = -20 dB.

Figure 3.30: Bit error rate performance of the BLS algorithm for filter order $p = 4$ and several block sizes N when the interference was a swept tone; SIR/chip = -20 dB.

Figure 3.31: Bit error rate performance of the RLS algorithm for $\lambda = 0.90$ and filter orders $p = 2$ and $p = 4$ when the interference was a swept tone; the SIR/chip was -20 dB.

Figure 3.32: Bit error rate performance of the RLS algorithm for filter order $p = 2$ and $\lambda = 0.90$ and 0.95 when the interference was a swept tone; the SIR/chip was -20 dB.

Chapter 4

The Kalman Filter Digital Phase-Locked Loop

4.1 Introduction

This chapter presents a different time-modelling approach to interference excision compared to the time-modelling techniques discussed in the previous chapter. A function related to the interference is assumed to have been generated by a second order state-space system to which the extended Kalman filter equations are applied. This results in what has been termed the digital phase-locked loop (DPLL) [1, 81]. There are certain benefits to using the DPLL over its analog counterpart [3]. First, it can be made adaptive, responding to changing interference conditions; and, second, being digital, other DPLL's can be switched in as required to handle more than one interferer. Finally, a digital PLL, as represented by the Kalman filter, can potentially provide better performance than the RLS algorithm in the case of FM interference as shown in the next chapter, and the analog PLL.

As pointed out in Section 2.3 and [3], the analog PLL can theoretically provide a rejection capability which is in the order of a factor equal to $(B_{\phi} + B_{LPF})/B_{ss}$, where B_{ϕ} , B_{LPF} , and B_{ss} are, respectively, bandwidths for the PLL, low pass filter, and spread spectrum signal (Fig. 2.1). However, because of component inaccuracies when implementing the PLL, this performance cannot be achieved. An example

in [3] indicated that for a stable tone interferer and design parameters from the NAVSTAR Global Positioning System ($B_p = 100$ Hz, $B_{LPF} = 100$ Hz, $B_{ss} = 20$ MHz), the theoretical rejection capability was 50 dB. The estimated rejection, after taking hardware component inaccuracies into account, was between 15 and 20 dB. With a digital PLL, once the signal is digitized, component variations are no longer relevant.

The outline of the chapter is as follows. Section 4.2 describes the spread spectrum communication model, which is similar to the one described in Section 3.2. Section 4.3.1 describes the state-space model which produces a function related to the interference. Section 4.3.2 presents the Kalman algorithm used to estimate the phase of the interference; it will be shown that the steady-state model contains a loop filter that is of the lag type. Another model [82] is also presented, which is slightly different from the one in [1], in that its loop filter consists of proportional-plus-integral control. Section 4.5 discusses the phase-noise performance of both the filter and predictor forms of the Kalman algorithm. The predictor form, as will be shown in the final chapter of this thesis, is useful for an excision architecture that incorporates decision-feedback. The phase-noise performance is presented in terms of curves based on theory and simulation. Section 4.5.2 presents examples of the step and ramp performance of two Kalman filters (i.e., the one with the lag network and the other involving proportional-plus-integral control). The chapter concludes with the estimator for the envelope of the interference, which is combined with the Kalman filter to estimate the interference. This leads into the next chapter where the bit error rate performance of the Kalman filter excisor is presented.

4.2 Model of the Excision System

The basic elements of the BPSK PN spread spectrum system are shown in Fig. 4.1. The received waveform $r(t)$, consisting of a spread spectrum signal, additive white Gaussian noise, and narrowband interference is applied to a bandpass filter with the transfer function $H_{bp}(f)$, whose output is defined as

$$u(t) = s(t) + n(t) + i(t). \quad (4.1)$$

Figure 4.1: Spread spectrum communications model.

The bandpass filter $H_{bp}(f)$, for the application considered here, is assumed to be a filter matched to a chip and centered at the carrier frequency ω_0 of the spread spectrum signal, the latter of which was defined in Eqs. (2.2) and (2.3).

The noise $n(t)$ is Gaussian and has a power spectral density

$$S_n(f) = \frac{N_0}{2} |H_{bp}(f)|^2, \quad (4.2)$$

where $N_0/2$ is the power spectral density of the assumed white Gaussian noise from the channel. The interference, defined in Eq. (2.6), is of bandwidth $B_i \ll 2R_c$. It has been assumed that the effect of the bandpass filter $H_{bp}(f)$ is negligible on the interference $i(t)$.

Referring to Fig. 4.1, the output $u(t)$ of the bandpass filter $H_{bp}(f)$ is bandpass sampled and applied to a limiter/bandpass filter and interference estimator.

Consider the bandpass sampler first. The analog signal $u(t)$ from Eq. (4.1) is sampled at $f_s = 2R_c$ ($mf_s = \omega_0/2\pi + R_c/2$ for some integer m). The resultant sampled signal is, therefore,

$$u_n = s_n + n_n + i_n, \quad (4.3)$$

where s_n consists of the sequence $\{\dots, 0, (-1)^n a_n, 0, (-1)^{n+2} a_{n+2}, \dots\}$ where the a_n are of energy E_c and coded according to $c_{kj} D_k$ for the j^{th} chip in the k^{th} transmitted bit, n_n^{-1} are uncorrelated Gaussian noise samples of variance $\sigma_{n_n}^2 = E_c(N_0/2)$, and

¹Coherent bandpass sampling has been assumed, so that the in-phase component is

i_n is the sampled version of Eq. (2.6). The samples u_n are applied to the interference estimator and interference canceller.

Consider now the branch containing the limiter. Here, u_n is applied to a limiter/bandpass filter. The input to the limiter referenced to the interference is redefined as

$$u_n = \sqrt{[I_n + n'_{1,n} + a'_{1,n}]^2 + [n'_{2,n} + a'_{2,n}]^2} \cos[\omega_0 n + \theta_n + \phi_{u,n}] \quad (4.4)$$

where

$$\phi_{u,n} = \arctan \left(\frac{n'_{2,n}/\sqrt{2} + a'_{2,n}}{I_n + n'_{1,n}/\sqrt{2} + a'_{1,n}} \right) \quad (4.5)$$

is a noise-like phase fluctuation on the interferer's phase θ_n , and is due to the noise and spread spectrum signal. The terms $n'_{1,n}$, $n'_{2,n}$, $a'_{1,n}$, and $a'_{2,n}$ are in-phase and quadrature components of the noise and spread spectrum signal with respect to the interference phase θ_n . The output of the limiter/bandpass filter is [83]

$$bp_n = \frac{4A'}{\pi} \cos[\omega_0 n + \theta_n + \phi_{u,n}], \quad (4.6)$$

where A' is the limiter's output level. This signal is redefined as (letting $A' = \sqrt{2}\pi/4$ for convenience)

$$bp_n = \sqrt{2} \cos[\omega_0 n + \theta_n + \phi_{u,n}]. \quad (4.7)$$

It should be noted that for large interference-to-noise ratios in which the interference is of constant envelope, $\phi_{u,n}$ in Eq. (4.7) is approximately Gaussian [83]. The sampled signal in Eq. (4.7) is what is processed by the Kalman filter, which estimates the phase θ_n of the interference.

4.3 Kalman Filter Algorithm

This section presents the analog and digital state-space models of the interference, followed by the extended Kalman algorithm used to estimate the state variables. The development here follows that in [1].

$(n_{1,n}/\sqrt{2}) \cos(n\pi/2)$ and the quadrature component is $(n_{2,n}/\sqrt{2}) \sin(n\pi/2)$

4.3.1 State Space Model of the Interference

A block diagram of the analog state space model which generates the analog version of a function related to bp_n in Eq. (4.7) is shown in Fig. 4.2; this function is represented by $bp(t)$ ². The state-space equations are defined as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & d \\ 0 & -\alpha \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \sqrt{K_w} \end{bmatrix} w(t), \quad (4.8)$$

where $x_1(t)$ and $x_2(t)$ are the phase function and modulating signal state variables of the interference, respectively. In compact form Eq. (4.8) becomes

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{g}w(t). \quad (4.9)$$

The interferer's modulating signal $x_2(t)$ is assumed to be generated by applying Gaussian white noise, $w(t)$, of unit variance to an amplifier with gain $\sqrt{K_w}$ followed by a first order low pass filter of bandwidth $\alpha = 2\pi\alpha_f$. The steady state variance of the coloured noise at the output of the lowpass filter is unity if $K_w = 2\alpha$. The output is multiplied by the frequency deviation constant, d , and then integrated to yield the phase function

$$x_1(t) = d \int_0^t x_2(\tau) d\tau. \quad (4.10)$$

An FM type of signal (interferer) is generated by phase modulating a carrier, ω_0 , which has been set equal to the carrier of the spread spectrum signal, since any offsets from ω_0 can be incorporated into $x_1(t)$, yielding

$$y(t) = \sqrt{2} \cos[\omega_0 t + x_1(t)]. \quad (4.11)$$

This signal is corrupted by white Gaussian noise $v(t)$ with power spectral density $N_{obs}/2$ generally different from $N_0/2$ defined earlier for the communications model in Fig. 4.1. The result is an FM signal related to the interference in Eq. (4.7) except for the fact that the noise term is additive here, i.e.,

$$bp(t)' = y(t) + v(t). \quad (4.12)$$

²In the analog version of Eq. (4.7), the noise is one of the arguments of the cosine function. For the model discussed here, the noise will be additive.

For large interference-to-signal-plus-noise conditions, the final effect of the additive noise in estimating the phase $x_1(t)$ in Eq. (4.11) will be similar to the effect of $\phi_{u,n}$ in Eq. (4.7).

Given the observation process $bp'(t)$ from Eq. (4.12), the objective is to estimate the state $x_1(t)$ and the related state $x_2(t)$, as well as $y(t)$ in Eq. (4.11). One approximate technique reserved for nonlinear estimation problems [80] is the extended Kalman filter algorithm.

Figure 4.2: Block diagram of the state space model.

Before the algorithm is presented, the discrete forms of Eqs. (4.8) and (4.9) will be stated, namely,

$$\mathbf{x}_{n+1} = \phi \mathbf{x}_n + \mathbf{w}_n, \quad (4.13)$$

where n represents discrete time,

$$\phi = \begin{bmatrix} 1 & \beta(1 - e^{-\alpha T_s}) \\ 0 & e^{-\alpha T_s} \end{bmatrix} = \begin{bmatrix} 1 & \phi_{12} \\ 0 & \phi_{22} \end{bmatrix} \quad (4.14)$$

and

$$\mathbf{w}_n = \int_{nT_s}^{(n+1)T_s} \phi \mathbf{g} \mathbf{w}(z) dz. \quad (4.15)$$

In Eq. (4.14), $\beta = d/\alpha$ and is termed the bandwidth expansion ratio in units of volts^{-1} [84]. In Eq. (4.15), $\mathbf{w}_n$ is a stationary zero-mean white Gaussian vector sequence whose covariance, (i.e., $E\{\mathbf{w}_n \mathbf{w}_n^t\}$), is given by

$$\mathbf{V}_w = \int_{nT_s}^{(n+1)T_s} \phi \mathbf{g} \mathbf{g}^t \phi du. \quad (4.16)$$

Finally, Eq. (4.12) becomes,

$$bp'_n = y_n + v_n, \quad (4.17)$$

where y_n is the sampled version of Eq. (4.11), i.e.,

$$y_n = \sqrt{2} \cos(\omega_0 n + x_{1,n}) \quad (4.18)$$

It has been assumed that $bp'(t)$ from Eq. (4.12) has been applied to a bandpass filter of bandwidth B , whose output has been sampled at a rate so as to yield uncorrelated noise samples v_n of variance $\sigma_v^2 = B(N_{obs}/2)$ [81].

The noise covariance matrix, V_w , is obtained by substituting Eq. (4.14) and g from Eq. (4.9) into Eq. (4.16) and integrating each term, yielding

$$V_w = \frac{K_w}{2\alpha} \begin{bmatrix} \beta^2 (4\pi/\gamma - 3 + 4e^{-2\pi/\gamma} - e^{-4\pi/\gamma}) & \beta (1 - e^{-2\pi/\gamma}) \\ \beta (1 - e^{-2\pi/\gamma}) & 1 - e^{-4\pi/\gamma} \end{bmatrix} \quad (4.19)$$

where $\gamma = 2\pi/(\alpha T_s)$, which is the ratio of the sampling rate $f_s = 1/T_s$ to the interferer's modulating signal bandwidth α_f . The discrete form of the interference model is shown in Fig. 4.3.

Figure 4.3: Discrete form of the interference model.

4.3.2 Extended Kalman Filter Equations

The extended Kalman filter equations [80] can now be applied to the discrete model. The resulting algorithm is listed in Table 4.1. In the algorithm, $\hat{x}_n$ is defined as

the estimate of the true state vector $\mathbf{x}_n$, $\hat{\mathbf{x}}_n$ is defined as the post error $\mathbf{x}_n - \hat{\mathbf{x}}_n$. $\mathbf{K}_n$ is the Kalman gain vector, $\mathbf{V}_{\hat{\mathbf{x}}_n} = E\{\hat{\mathbf{x}}_n \hat{\mathbf{x}}_n^t\}$ is the post error covariance and $\mathbf{V}_{\hat{\mathbf{x}}_{n|n-1}} = E\{\hat{\mathbf{x}}_{n|n-1} \hat{\mathbf{x}}_{n|n-1}^t\}$ is the prior error covariance. Furthermore, $f_{n|n-1}^{-1}$ is known as the "coupled equation", since the estimate of the interferer's phase is contained in it. However, this equation can be further approximated as noted in [1]. The approximation is

$$f_{n|n-1}^{-1} \approx \frac{1}{V_{\hat{x}_{11},n|n-1}} \left\{ 1 - \sqrt{\frac{\sigma_v^2}{\sigma_v^2 + V_{\hat{x}_{11},n|n-1}}} \right\}. \quad (4.20)$$

4.4 The Digital Phase-Locked Loop (DPLL)

The Kalman algorithm results in a digital PLL [1]. This fact will be elaborated upon here so as to illustrate sampling rate requirements which are related to the allowable interference bandwidth and for later reference when performance characteristics of the DPLL will be addressed.

Substitution of the Kalman gain $\mathbf{K}_n$ from Table (4.1) into the state estimate equation for $\hat{\mathbf{x}}_n$ yields

$$\begin{aligned} \hat{\mathbf{x}}_n &= \phi \hat{\mathbf{x}}_{n-1} + \mathbf{V}_{\hat{\mathbf{x}}_n} \begin{bmatrix} 1/\sigma_v^2 \\ 0 \end{bmatrix} \left\{ -bp'_n \sqrt{2} \sin[\omega_0 n + \hat{\theta}_{n|n-1}] \right. \\ &\quad \left. + \sin[\omega_0 n + \hat{\theta}_{n|n-1}] \cos[\omega_0 n + \hat{\theta}_{n|n-1}] \right\} \end{aligned} \quad (4.21)$$

Consider the term within braces. Substituting for bp'_n from Eq. (4.17) and noting that $x_{1,n} = \theta_n$, one obtains

$$\begin{aligned} h_n &= \sin(\theta_n - \hat{\theta}_{n|n-1}) - \sin(2\omega_0 n + \theta_n + \hat{\theta}_{n|n-1}) \\ &\quad + \sin(2\omega_0 n + 2\theta_n) - \sqrt{2}v_n \sin(\omega_0 n + \hat{\theta}_{n|n-1}) \end{aligned} \quad (4.22)$$

which consists of four terms. The first refers to the baseband term. The second and third are second harmonic terms. Recalling that v_n is the sampled version of bandpass noise centered at ω_0 , the fourth term turns out to be baseband noise which can be

Table 4.1: The Kalman Filter Algorithm.

Initial Conditions:

$$\hat{\mathbf{x}}_{-1} = E\{\hat{\mathbf{x}}_{-1}\} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}; \quad \mathbf{V}_{\hat{\mathbf{x}}_{-1}} = E\{\hat{\mathbf{x}}_{-1}\hat{\mathbf{x}}_{-1}^t\}$$

Do for $n = 0$ to T :

$$\hat{\theta}_{n|n-1} = \begin{bmatrix} 1 & 0 \end{bmatrix} \phi \hat{\mathbf{x}}_{n-1}$$

$$\mathbf{V}_{\hat{\mathbf{x}}_{n|n-1}} = \phi \mathbf{V}_{\hat{\mathbf{x}}_{n-1}} \phi^t + \mathbf{V}_w$$

$$f_{n|n-1}^{-1} = \frac{1 - \cos(2\omega_0 n + 2\hat{\theta}_{n|n-1})}{\sigma_v^2 + V_{\hat{x}_{11,n|n-1}} [1 - \cos(2\omega_0 n + 2\hat{\theta}_{n|n-1})]}$$

$$\mathbf{V}_{\hat{\mathbf{x}}_n} = \mathbf{V}_{\hat{\mathbf{x}}_{n|n-1}} - \mathbf{V}_{\hat{\mathbf{x}}_{n|n-1}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{V}_{\hat{\mathbf{x}}_{n|n-1}} f_{n|n-1}^{-1}$$

$$\mathbf{K}_n = -\mathbf{V}_{\hat{\mathbf{x}}_n} \begin{bmatrix} 1/\sigma_v^2 \\ 0 \end{bmatrix} \sqrt{2} \sin(\omega_0 n + \hat{\theta}_{n|n-1})$$

$$\hat{\mathbf{x}}_n = \phi \hat{\mathbf{x}}_{n-1} + \mathbf{K}_n [bp'_n - \sqrt{2} \cos(\omega_0 n + \hat{\theta}_{n|n-1})]$$

modeled as a zero mean white Gaussian noise process [84]. The first and fourth terms are the desired ones, since they correspond to their analog PLL counterparts. Also, as in the analog implementation, the second harmonic terms can be neglected, provided they do not alias into the baseband region.

To ensure that no spectral aliasing of these terms occurs, the sampling rate must be selected according to $f_s \geq 4B_i$. This can be seen from the following heuristic argument. As a starting point consider analog spectra. Then assume that no noise is present and $\hat{\theta}(t) \approx \theta(t)$. Also let the terms within braces in Eq. (4.21) be represented as follows:

$$q_1(t) = bp'(t) = \sqrt{2} \cos[\omega_0 t + \theta(t)] \quad (4.23)$$

$$q_2(t) = \sqrt{2} \sin[\omega_0 t + \hat{\theta}(t)] \quad (4.24)$$

$$\begin{aligned} q_3(t) &= \sin[\omega_0 t + \hat{\theta}(t)] \cos[\omega_0 t + \hat{\theta}(t)] \\ &= \frac{1}{2} \sin[2\omega_0 t + 2\hat{\theta}(t)] \end{aligned} \quad (4.25)$$

The amplitude spectrum of $q_1(t)$ is shown in Fig. 4.4(a).

It is assumed that the amplitude spectrum of $q_2(t)$ is approximately equal to it for the case of $\theta(t) \approx \hat{\theta}(t)$. Taking the product of $q_1(t)$ and $q_2(t)$ in Eqs. (4.23) and (4.24) results in

$$q_4(t) = \sin[\theta(t) - \hat{\theta}(t)] + \sin[2\omega_0 t + \theta(t) + \hat{\theta}(t)] \quad (4.26)$$

whose spectrum is shown in Fig. 4.4(b), the convolution of $Q_1(f)$ and $Q_2(f)$. If an integer multiple of the sampling frequency mf_s is chosen as shown in Fig. 4.4(b), then $f_s \geq 4B_i$ is the required sampling rate for no aliasing in the baseband region. Thus, the baseband and second harmonic terms interleave as shown in Fig. 4.4(c). Similarly, the spectrum for $q_3(t)$ and its sampled spectrum are shown in Figs. 4.5(a) and (b).

The second harmonic terms can therefore be neglected and Eq. (4.21) becomes

$$\begin{aligned} \hat{\mathbf{x}}_n &= \phi \hat{\mathbf{x}}_{n-1} + \mathbf{V}_{\hat{\mathbf{x}},n} \begin{bmatrix} 1/\sigma_v^2 \\ 0 \end{bmatrix} \left\{ -bp'_n \sqrt{2} \sin[\omega_0 n + \hat{\theta}_{n|n-1}] \right\} \\ &= \phi \hat{\mathbf{x}}_{n-1} + \mathbf{V}_{\hat{\mathbf{x}},n} \begin{bmatrix} 1/\sigma_v^2 \\ 0 \end{bmatrix} \left\{ \sin[\theta - \hat{\theta}_{n|n-1} + v'_n] \right\} \end{aligned} \quad (4.27)$$

(a)

(b)

(c)

Figure 4.4: Illustration of the sampling rate requirements: (a) Hypothetical interference spectrum of $q_1(t)$; (b) Convolution of $Q_1(f)$ with $Q_2(f)$; (c) Sampled spectrum of the product $q_1(t)q_2(t)$.

Figure 4.5: Analog and sampled spectra of $q_3(t)$ (see Eq. (4.25)).

where

$$v'_n = -\sqrt{2}v_n \sin[\omega_0 n + \hat{\theta}_{n|n-1}] \quad (4.28)$$

and is of variance $(N_{obs}/2)f_s$.

The state-space diagram of the DPLL is illustrated in Fig. 4.6. Observe that the

Figure 4.6: State space representation of the DPLL.

phase estimate $\hat{\theta}_{n|n-1}$ is applied to a phase modulator of frequency ω_0 , resulting in a “VCO” signal defined as

$$VCO_n = -\sqrt{2} \sin(\omega_0 n + \hat{\theta}_{n|n-1}). \quad (4.29)$$

Shifting this signal by $-\pi/2$ radians results in a filtered estimate of y_n in Eq. (4.17). The baseband version of the DPLL is illustrated in Fig. 4.7. This form is useful for simulations. It was this model that was used for the simulations to be presented later.

For high SNR conditions in which $\theta_n \approx \hat{\theta}_{n|n-1}$, Eq. (4.27) becomes the linearized version, i.e.,

$$\hat{x}_n = \phi \hat{x}_{n-1} + V_{\hat{x},n} \begin{bmatrix} 1/\sigma_v^2 \\ 0 \end{bmatrix} \{\theta_n - \hat{\theta}_{n|n-1} + v'_n\}. \quad (4.30)$$

Finally, by rearranging the system in Fig. 4.6 into a transfer function model at steady state, one obtains the nonlinear and linear systems illustrated in Figs. 4.8 and 4.9,

Figure 4.7: Nonlinear baseband model of the DPLL.

respectively where

$$C = \frac{2\phi_{12}V_{\hat{x}_{12,\infty}} + V_{\hat{x}_{12,\infty}}}{\sigma_v^2}, \quad (4.31)$$

$$b = \frac{\phi_{12}V_{\hat{x}_{12,\infty}} + V_{\hat{x}_{12,\infty}}\phi_{22}}{2\phi_{12}V_{\hat{x}_{12,\infty}} + V_{\hat{x}_{12,\infty}}}, \quad (4.32)$$

and

$$a = \phi_{22}. \quad (4.33)$$

The DPLL consists of a gain C , followed by the loop filter, and integrator. These equations indicate that the gain and loop filter parameters (pole and zero) are linked to the steady state phase variance $V_{\hat{x}_{11,\infty}}$, the phase/modulating signal covariance $V_{\hat{x}_{12,\infty}}$, the state transition parameters ϕ_{12} and ϕ_{22} , and the variance of the observation noise σ_v^2 . Steady state performance ultimately depends on the choices of d , σ_v^2 and T_s of the model.

The loop filter shown in Fig. 4.8 will lead to a steady state error if the phase θ_n is a ramp. This loop filter can be changed to one consisting of proportional-plus-integral control by modifying the interference model [82]. The modification involves replacing the low pass filter by an integrator (i.e., by setting $\alpha = 0$ in $\mathbf{A}$ in Eq. (4.9)). With this change, the state transition matrix ϕ and covariance matrix $\mathbf{V}_w$ become,

Figure 4.8: Transfer function model representation of the DPLL.

Figure 4.9: Linear transfer function model representation of the DPLL.

respectively,

$$\phi = \begin{bmatrix} 1 & dT_s \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \phi_{12} \\ 0 & \phi_{22} \end{bmatrix} \quad (4.34)$$

and

$$V_w = \frac{K_w}{2\alpha} \begin{bmatrix} 2\beta^2\alpha^3T_s^3 & 2\beta\alpha^2T_s^2 \\ 2\beta\alpha^2T_s^2 & 2\alpha T_s \end{bmatrix}. \quad (4.35)$$

Using these terms in the Kalman equations results in a slightly modified algorithm. This is the algorithm that has been used in the simulations in the next chapter. The transfer function parameters in Eqs. (4.31) and (4.32) remain unchanged, but $a = 1$ in Eq. (4.33).

4.5 Performance Characteristics of the DPLL

This section considers some basic performance characteristics of the DPLL. First, phase-noise performance curves of the filter and predictor forms of the Kalman algorithms will be presented. These curves are represented in terms of the interference-to-noise ratio (INR) in the model's modulating signal bandwidth α_f versus the reciprocal of the phase variance, $1/V_{\hat{x}_{11},\infty}$. These curves will be useful in the next chapter when the theoretical bit error rate performance of the Kalman excisor will be presented. Following this, examples of the step and ramp performance of the two Kalman filters presented in Section 4.4 and differentiated by their state transition matrices (Eqs. (4.14) and (4.34)) and their state covariance matrices (Eqs. (4.19) and (4.35)) will be discussed.

4.5.1 Phase Noise Performance

The theoretical performance curves were obtained using the prior error covariance equation in Table 4.1 (i.e., $V_{\hat{x}_{11},n-1}$). The experimental curves were obtained using the model in Fig. 4.3 for a range of values for d and σ_v^2 ; α_f was fixed at 0.0003125 Hz.

The results are illustrated in Figs. 4.10 to 4.13. Figures 4.10 and 4.11 refer to the *filter* form of the Kalman algorithm. Figures 4.12 and 4.13 refer to the *predictor* form. The difference between these two forms is discussed below.

Filter Model

The algorithm for the filter form corresponds to the equations in Table 4.1. In the case of the modified Kalman algorithm, Eqs. (4.34) and (4.35) replace Eqs. (4.14) and (4.19).

The results are illustrated in Figs. 4.10 and 4.11. They indicate reasonable agreement between theory and simulation. It must be pointed out that the theory does not predict the threshold at which the variance of the phase-noise increases dramatically; this must be determined from experimentation. As cited in [1], an approximate threshold is a phase variance of 0.25 radians²/second², (i.e., $1/V_{\hat{x}_{11},\infty} = 4$). The results for the modified Kalman algorithm as shown in Fig. 4.11 exhibit a higher threshold. Otherwise, from the perspective of phase-noise variance, the two algorithms perform almost identically.

Predictor Model

To obtain the predictor model, two equations must be added to the algorithm in Table 4.1. They are

$$\hat{x}_{n+1|n} = \phi \hat{x}_n, \quad (4.36)$$

and

$$V_{\hat{x}_{n+1|n}} = \phi^t V_{\hat{x}_n} \phi. \quad (4.37)$$

The results are shown in Figs. 4.12 and 4.13 and indicate that performance has degraded slightly. This is more evident with signals having lower FM bandwidths. There is an imperceptible difference between the filter and predictor models for signals of larger FM bandwidths. Finally, the thresholds appear to have shifted to the right for both Kalman filter algorithms. This shift is likely due to the additional prediction error incurred by the prediction equations.

4.5.2 Step and Ramp Responses of the DPLL

This section presents some performance examples of the Kalman filter algorithm using the baseband model in Fig. 4.7, with $v'_n = 0$. In particular, step and ramp responses

Figure 4.10: Performance curves using the model in [1] for the case when the Kalman filter is in the filter mode.

Figure 4.11: Performance curves using the modified Kalman filter in the filter mode.

Figure 4.12: Performance curves using the model in [1] for the case when the Kalman filter is in the predictor mode.

Figure 4.13: Performance curves using the model in [1] for the case when the Kalman filter is in the predictor mode.

of the algorithms will be presented for various parameter values. The results are presented in terms of the phase error $\theta_n - \hat{\theta}_{n|n-1}$. It will be shown that the step response produces zero steady state errors, whereas the ramp response produces some phase errors, the value of which depends on the slope of the ramp. It will also be shown that the steady state errors can be reduced by increasing the gain, but that this can lead to instabilities, or they can be removed by using the modified Kalman algorithm.

Step Response

The input signal condition in this case is $\theta_n = \mathcal{U}_n$ radians, where $\mathcal{U}_n$ is the unit step function. The model parameters of the Kalman filter are shown in Table 4.2. Two values of d were used for two values of σ_v^2 .

The error response for $\sigma_v^2 = 0.1$ for the two values of β (or d) from Table 4.2 is shown in Fig. 4.14. It can be seen that by increasing d , the response is faster at the expense of more overshoot. The effect of increasing d while holding the other parameters constant is twofold: (a) it increases the closed loop bandwidth; and, (b) it increases the loop gain. In fact, for the two tests considered, the gains were $G = 0.34$ ($\beta = 100$) and 1.53 ($\beta = 1000$) as computed from Eq. (4.31) in the steady state.

The error response for the case when $\sigma_v^2 = 0.025$ is shown in Fig. 4.15. The effect of decreasing the noise variance is to increase the gain and closed loop bandwidth. This leads to more overshoot for the case of $d=1.96$ and may be at the verge of becoming unstable. In fact, tests with large values of d led to unstable situations.

The response of the system to ramps of various slopes is presented next.

Ramp Response

The objective here is to illustrate the steady state error performance of the two Kalman filters (i.e., the one in [1] and the modified one) when the input is a ramp. This is accomplished by illustrating the effect of the gain on the steady state errors, and the effect of the initial condition $E\{x_{2,-1}\} = m_2$. The model parameters considered here were the same as those used with the step response. The input signal

Table 4.2: Parameters for the Step and Ramp Response Tests.

Initial Conditions

$$E\{x_{1,-1}\} = 0, E\{x_{2,-1}\} = 0 \text{ or } 0.1 \text{ Hz}$$

$$\mathbf{V}_{\hat{x}_{-1}} = \begin{bmatrix} 0.1 & 0.0 \\ 0.0 & 0.1 \end{bmatrix}$$

Model Parameters

$$\sqrt{K_w} = 2\alpha$$

$$\text{var}\{x_{2,\infty}\} = 1.0 \text{ (for the model in [1])}$$

$$\alpha = 2\pi(0.0003125), \alpha_f = 0.0003125 \text{ Hz}$$

$$d = \beta\alpha = \{0.196, 1.96\}; \beta = 100, 1000$$

$$\sigma_v^2 = 0.1, 0.025$$

Figure 4.14: Error response of the Kalman filter in [1] for a unit step input and $\sigma_v^2 = 0.1$.

Figure 4.15: Error response of the Kalman filter in [1] for a unit step input and $\sigma_v^2 = 0.025$.

conditions for the first two cases were $\theta_n = 2\pi(\delta f)n$ with $\delta f = 0.09$ Hz. For the initial condition test, δf was set at 0.19 Hz.

The results for $m_2 = 0$ Hz, $\delta f = 0.09$ Hz, the model in [1] and two values of σ_v^2 (i.e., 0.1 and 0.025), are shown in Fig. 4.16. The results show that increasing the gain has the effect of reducing the steady state error. Using the modified Kalman filter, the steady state error is reduced to zero as illustrated in Fig. 4.17. Finally, the effect of increasing δf to 0.19 Hz ($\sigma_v^2 = 0.025$ and $d = 0.196$) is illustrated in Fig. 4.18. For $m_2 = 0.0$ Hz, the Kalman filter slips one cycle as indicated by the steady state error of 6.28 radians. However, by increasing the initial condition for m_2 to 0.1 Hz, the single cycle of slippage is removed.

Figure 4.16: Error response of the Kalman filter in [1] for $\delta f = 0.09$ Hz, $m_2 = 0.0$ and two values of σ_v^2 (i.e., 0.1 and .025).

Figure 4.17: Error response of the modified Kalman filter for $\delta f = 0.09$ Hz, $m_2 = 0.0$ and two values of σ_v^2 (i.e., 0.1 and .025).

Figure 4.18: Error response of the modified Kalman filter for $\delta f = 0.19$ Hz, $m_2 = 0.0$ and 0.1 Hz, for $\sigma_v^2 = 0.025$ and $d = 0.196$.

4.6 The Interference Estimator

This section concludes the chapter with a description of the interference estimator which was shown in Fig. 4.1 and which will be used, along with its variants, in the next chapter where theoretical and simulated bit error rate results will be presented for the Kalman filter excisor. As noted earlier, the Kalman algorithm produces a filtered estimate of y_n in Eq. (4.18), which can be used as a basis for estimating a sampled version of the envelope of $i(t)$ (i.e., I_n in Eq. (2.6)). The interference estimator is illustrated in Fig. 4.19. The output of the first multiplier is a baseband term and is, using Eq. (4.3), excluding the $\sqrt{2}$ factor,

$$\begin{aligned}\hat{I}'_n &= I_n \cos(\theta_n - \hat{\theta}_{n|n-1}) \\ &+ n_n \cos(\omega_0 n + \hat{\theta}_{n|n-1}) + a_n \cos(\hat{\theta}_{n|n-1}) \\ &+ a_n \cos(2\omega_0 n + \hat{\theta}_{n|n-1})\end{aligned}\quad (4.38)$$

Equation (4.38) consists of four terms: the first term is related to the desired ampli-

Figure 4.19: Block diagram of the interference estimator.

tude of the interference; the second is approximately Gaussian baseband noise [84]; and the third and fourth terms are noise-like terms emanating from the spread spectrum signal. The fourth term, because of the sampling rate conditions discussed in Section 4.4, is essentially filtered out by the low pass filter of bandwidth $B_{LPF} < 0.50$ Hz and, therefore, will be ignored in the baseband simulations to be discussed in the next chapter. The term $\hat{I}'_n/\sqrt{2}$ is filtered, resulting in the estimate of the interference envelope, $\hat{I}_n/\sqrt{2}$. Combining this with an estimate of y_n in Eq. (4.18), $\hat{y}_n$, shown in

Fig. 4.19 yields the estimate of the interference

$$\hat{i}_n = \hat{I}_n \cos(\omega_0 n + \hat{\theta}_{n|n-1}), \quad (4.39)$$

which is subtracted from u_n as illustrated in Fig. 4.1. Because the input was coherently bandpass sampled at twice the chip rate, the output of the canceller (summer) undergoes decimation and alternate sign reversal, resulting in the error signal e_n , which undergoes correlation with the local PN code and bit detection.

Chapter 5

Excision Performance of the Kalman Filter

5.1 Introduction

The previous chapter presented two forms of a Kalman filter-based digital phase-locked loop with accompanying performance characteristics. Also discussed was how the DPLL could form part of an interference estimator in a PN spread spectrum system corrupted by narrowband interference.

This chapter is concerned with characterizing the bit error rate performance of the Kalman filter-based excisor, building on the results of the previous chapter. Recall from Section 3.2.2, that the motivation for suggesting an alternate excisor to the least squares approach was to develop a technique amenable to interferers with nonlinear modulations. With this in mind, Section 5.2 compares, through computer simulation, the performance of the Kalman filter with the RLS algorithm for the case of a swept tone interferer. Section 5.3 develops a theoretical bit error rate formula for FM interference using the interference model in the previous chapter; this development is based on several assumptions which are elaborated. In Section 5.4, the excisor is extended to the case of narrowband Gaussian noise, which is more difficult to handle because of the time-varying envelope as well. Ways of overcoming some of the problems encountered are presented, which have the effect of enhancing the performance.

Section 5.5 concludes the chapter by focussing on an adaptive Kalman excisor. It is discussed and illustrated that by time-varying one of the Kalman filter parameters, adaptive architectures can be developed. One architecture in particular is presented at length.

5.2 Swept Tone Performance

The objective of this section is to compare the performance of the Kalman filter excisor of the previous chapter to the RLS algorithm for the case of a swept tone. For this comparison, the Kalman filter is not adaptive; an adaptive architecture will be discussed in Section 5.5. It is intended here to illustrate the potential performance of the Kalman filtering approach. The filter-form of the modified Kalman algorithm is the one that will be used.

Recall from Chapter 4 that the performance of the Kalman filter is governed by three parameters. These are the frequency deviation constant d , the variance of the observation noise $\sigma_v^2 = N_{obs}/2$ assumed in the model, and the 3 dB bandwidth, α_f , of a low pass filter for filtering white Gaussian noise to produce the interferer's modulating signal. It is important to note that α_f , which is related to $\gamma = f_s/\alpha_f$, the ratio of the sampling rate to this bandwidth, establishes a family of performance curves such as those in Figs. 4.10 and 4.11. Thus, for a given α_f and f_s , the approach to be taken will be to determine that value of d which minimizes the mean-squared value of the residual interference before the PN correlator in Fig. 4.1. It is assumed here that for small enough B_{LPF} in Fig. 4.19, the main source for residual interference will be the phase-noise.¹ Therefore, using the minimum mean-squared value of the residual interference as the criterion for optimality, the value of d determined would provide as close to the optimum performance achievable and, hence, a reasonable baseline for comparison with the adaptive structure. Finally, the variance parameter for the observation noise, σ_v^2 , will be set according to the noise level in the experiment. This noise level, as shown below, is a function of the power spectral densities of the channel

¹This topic is elaborated upon later when the theoretical bit error rate for constant envelope interference is developed.

noise and spread spectrum signal.

The simulation conditions are the same as those used in Section 3.2.2, i.e., the interference is defined as in Eq. (3.12); the interference-to-signal ratio is 20 dB; the interference is a swept tone, sweeping between 0.1 Hz and 0.3 Hz at a rate of ± 0.001 Hz/sec. as illustrated in Fig. 3.20; the processing gain is $L = 20$; and the energy/chip is $E_c = 1$ unit. The excisor used is the one in Fig. 4.1, with the interference estimator as shown in Fig. 4.19. In the latter figure, the low pass filter is of the 4th order Butterworth type of bandwidth B_{LPF} , and is a variable in the simulations.

The experiment consists of two parts. The first part is concerned with determining from simulation the optimum value of d for a particular σ_v^2 ; this corresponds to moving vertically across the d lines in Fig. 4.11 at a particular INR . The second part is concerned with obtaining the bit error rate curve for the swept tone interferer using this optimum value of d , while modifying σ_v^2 in relation to the particular E_b/N_0 . This corresponds to moving along the optimum d line in Fig. 4.11 while varying INR .

5.2.1 Optimum d Determination

For fixed α_f and f_s , it suffices to determine the value of the power spectral density of the observation noise $\sigma_v^2 = N_{obs}/2$ to be used in the Kalman filter before varying d to obtain the optimum value. This is accomplished as follows.

The interference-to-noise ratio INR in the equivalent noise bandwidth of the filter of 3 dB bandwidth α_f in the interference model in Fig. 4.3 is defined as [1]

$$INR = \frac{2}{\pi \alpha_f N_{obs}}. \quad (5.1)$$

From Chapter 4, Eq. (4.5), the phase-noise at the output of the limiter and bandpass filter was defined as

$$\phi_{u,n} = \arctan \left(\frac{n'_{2,n}/\sqrt{2} + a'_{2,n}}{I + n'_{1,n}/\sqrt{2} + a'_{1,n}} \right) \quad (5.2)$$

where I has replaced I_n , since a constant envelope interferer is being considered. In Eq. (5.2), using the definition for the signal and noise from Eq. (4.3), the noise and

spread spectrum signal referenced to the phase of the interference become

$$n'_{1,n} = n_{1,n} \cos(\theta_n) + n_{2,n} \sin(\theta_n) \quad (5.3)$$

$$n'_{2,n} = n_{2,n} \cos(\theta_n) - n_{1,n} \sin(\theta_n) \quad (5.4)$$

$$a'_{1,n} = a_n \cos(\theta_n) \quad (5.5)$$

$$a'_{2,n} = -a_n \sin(\theta_n). \quad (5.6)$$

For $I \gg (s_n + n_n)$, Eq. (5.2) simplifies to

$$\phi_{u,n} \approx \left(\frac{n'_{2,n}/\sqrt{2} + a'_{2,n}}{I} \right). \quad (5.7)$$

The approximate mean-squared value of the phase-noise is, therefore,

$$\begin{aligned} E\{\phi_{u,n}^2\} &\approx (N_0/2 + 0.5)/I^2 \\ &\approx \sigma_v^2 = N_{obs}/2 \end{aligned} \quad (5.8)$$

where the 0.5 in Eq. (5.8) refers to the mean-squared value of the signal term in Eq. (5.7). It can be seen from Eq. (5.8) that the variance of the observation noise, σ_v^2 , is a function of the channel noise power spectral density N_0 , the power in the spread spectrum signal, and the power in the interference.

The kind of performance that can be achieved with the Kalman filter using particular values for σ_v^2 and α_f while varying d is considered next. For this example N_0 (and therefore σ_v^2 from Eq. (5.8)) was determined from $E_b/N_0 = 12$ dB, α_f was set to a value of 0.0003125 Hz, which corresponds to the value used in Figs. 4.10 and 4.11, d ranged from 0.1 to 2.0 rad./sec./volt, and the bandwidth of the low pass filter designated as B_{LPF} in Fig. 4.19 was either 0.10 or 0.025 Hz. Figure 5.1 shows estimates of the degree of interference suppression at the output of the Kalman filter excisor, where suppression is defined as

$$S = 10 \log(\overline{\Delta i_n^2}/P_i) \quad (5.9)$$

where $\Delta i_n = i_n - \hat{i}_n$ is the residual interference, and P_i is the power in the interference. The number of samples used in determining the estimate of S in Eq. (5.9) was 20,000. Observe that the "optimum" value for d is ≈ 0.20 rad./sec./volt.

From Eq. (5.9) and the results in Fig. 5.1, the efficiency η in Eq. (2.80) can be approximately (why only approximately will be discussed later) determined from

$$\eta = \frac{1}{\sigma_{nse}^2 / \rho_0 + S} \quad (5.10)$$

where, for the case considered here, $\sigma_{nse}^2 = N_0/2$ and $\rho_0 = P_i$. This can be seen by noting that, for the linear prediction approach for which this equation was developed, the overall noise term σ^2 in Eq. (2.77) can also be expressed as

$$\sigma^2 = \sigma_{nse}^2 + E\{\Delta i_n^2\}. \quad (5.11)$$

As an example, for $E_b/N_0 = 12$ dB, $I/S = 20$ dB and $S = -22$ dB at $d = 0.20$ rad./sec./volt in Fig. 5.1, $\eta = 19$ dB.

An example of the phase-tracking capability of the Kalman filter for the swept tone interferer is illustrated in Fig. 5.2, using the optimum value of $d = 0.20$ rad./sec./volt from Fig. 5.1. There is a slight offset of the phase estimate $\hat{\theta}_{n|n-1}$ from the exact phase θ_n . This offset is 2π radians and is due to initial cycle slippage as discussed in Chapter 4. Furthermore, because of the nonlinearly varying form of the phase (it has a "sinusoidal" feature to it superimposed on a ramp of slope $2\pi(0.2)T_s$ radians/sec. — refer to Eq. (3.12) and the modulating signal in Fig. 3.20), the estimate moves closer and further away from the exact phase. This phenomenon is better illustrated in Fig. 5.3 which shows the phase error $\theta_n - \hat{\theta}_{n|n-1}$, with the initial 2π offset removed, and is due to the inability of the modified Kalman filter to track nonlinear phase functions without some residual phase error.

The envelope-tracking capability of the interference estimator of Fig. 4.19 is illustrated in Fig. 5.4 for two cases: $B_{LPF} = 0.10$ and 0.025 Hz. Observe that by using a smaller value of B_{LPF} , the envelope noise has decreased, which is to be expected, but at the expense of a longer time constant. Furthermore, it will be shown later that, for decreasing values of B_{LPF} , the additive noise can be reduced significantly, with the main noise contribution from the phase-noise.

Finally, the power spectra for the interference and its estimate are illustrated in Fig. 5.5. The accuracy of the estimate is excellent in the region of the spectrum where most of the interference power is located; beyond this region one can see the inaccuracies imposed by the noise due to both envelope and phase-noise.

Figure 5.1: Suppression level S from Eq. (5.9) for several values of d , and $B_{L,PF} = 0.1$ Hz.

Figure 5.2: An example of the phase-tracking ability of the Kalman filter for $d = 0.20$ rad./sec./volt, and $B_{LPF} = 0.10$ Hz.

Figure 5.3: An example of the phase error $\theta_n - \hat{\theta}_{n|n-1}$ for $d = 0.20$ rad./sec./volt, and $B_{LPF} = 0.10$ Hz.

Figure 5.4: An example of the envelope estimator for $d = 0.20$ rad./sec./volt, and $B_{LPF} = 0.10$ and 0.025 Hz.

Figure 5.5: Power spectrum of the interference and its estimate for $d = 0.2$ rad./sec./volt, and $B_{LPF} = 0.1$ Hz.

5.2.2 Bit Error Rate Performance

In this section the bit error rate performance of the Kalman filter is compared with the RLS algorithm discussed in Chapter 3. For the simulation, a total of 50,000 bits were transmitted (1,000,000 chips), with E_b/N_0 ranging from 0 to 12 dB. The Kalman filter parameters were set up as follows: $\alpha_f = 0.0003125$ Hz, $d = 0.20$ rad./sec./volt (i.e., the optimum value from Fig. 5.1), and a range of values for σ_v^2 , which were calculated from Eq. 5.8 for the different values of E_b/N_0 . Finally, the lowpass filter B_{LPF} in Fig. 4.19 used to estimate the envelope I of the interference, was 0.10, 0.05 and 0.025 Hz. The parameters of the RLS algorithm were the same as those used in Section 3.2.2.

The results are illustrated in Fig. 5.6. They indicate that, for the values of B_{LPF} used, the Kalman filter performs better than the RLS algorithm with its matched filter. In fact, the improvement in performance gets progressively better as B_{LPF} decreases. However, there is hardly any noticeable difference between $B_{LPF} = 0.05$ Hz and $B_{LPF} = 0.025$ Hz, suggesting that the phase-noise from the Kalman filter, as opposed to the additive noise, and, to some degree, signal distortion, are the main contributors to the lower bound in performance. Considering the nonlinear phase function which the Kalman filter is attempting track, it would be interesting to see if a higher order Kalman filter could provide even better bit error rate performance.

The next section is concerned with the theoretical bit error rate for the case of general FM interference using the model in Fig. 4.3.

Figure 5.6: Comparison of the bit error rate performance between the RLS algorithm for $\lambda = 0.90$, and the Kalman filter algorithm for $d = 0.20$ rad./sec./volt and several values of low pass filter bandwidth (LPF BW) B_{LPF} . The acronyms NMF and MF respectively refer to “No Matched Filter” and “Matched Filter”.

5.3 Theoretical Bit Error Performance

For the development of the theoretical bit error rate expression, one can refer to Figs. 4.1 and 4.19 from the previous chapter. In Fig. 4.1, the error signal e_n prior to despreading (but following decimation and sign reversal) will be used. This error is defined as

$$\begin{aligned} e_n &= u_n - \hat{i}_n \\ &= s_n + n_n + i_n - \hat{i}_n \\ &= s_n + n_n + \Delta i_n. \end{aligned} \quad (5.12)$$

In Eq. (5.12) the signal s_n has been set equal to a_n , the latter of which was defined following Eq. (4.3), and n_n has been set equal to the in-phase-noise term $n_{1,n}$. It is assumed that the interferer is of constant envelope and that the baseband representation of it is used, i.e.,

$$i_n = I \cos(\theta_n). \quad (5.13)$$

Furthermore, it is generated using the interference model in Fig 4.3.

From Fig. 4.19, the interference estimate $\hat{i}_n$ is

$$\hat{i}_n = \hat{I} \cos(\hat{\theta}_{n|n-1}) \quad (5.14)$$

where

$$\hat{I} = \left[I \cos(\epsilon_n) + \sqrt{E_c/2} n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1}) \right] * h_{LPF,n}. \quad (5.15)$$

In Eq. (5.15),

$$\epsilon_n = \theta_n - \hat{\theta}_{n|n-1} \quad (5.16)$$

and $n_{B,n}$ is a baseband noise term defined as

$$n_{B,n} = n_{1,n} \cos(\hat{\theta}_{n|n-1}) + n_{2,n} \sin(\hat{\theta}_{n|n-1}) \quad (5.17)$$

where the terms $n_{1,n}$ and $n_{2,n}$ are the in-phase and quadrature noise terms of n_n prior to decimation. Furthermore, $h_{LPF,n}$ is the impulse response of the low pass filter in

Fig. 4.19 and $*$ is the convolution operator. With E_c of unit energy and Eq. (5.15) substituted into Eq. (5.14), the interference estimate becomes

$$\begin{aligned} \hat{i}_n = & \left\{ [I \cos(\epsilon_n)] * h_{LPF,n} \right\} \cos(\hat{\theta}_{n|n-1}) + \\ & + \left\{ [n_{B,n}/\sqrt{2} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n} \right\} \cos(\hat{\theta}_{n|n-1}). \end{aligned} \quad (5.18)$$

This interference estimate consists of two terms: (a) the first term is closely related to the interference in Eq. (5.13); (b) the second term is low pass filtered noise and signal, modulated by an estimate of the phase variations of the interference.

The first term, because it is a function of $\hat{\theta}_{n|n-1}$, which in turn is a function of the past data, is uncorrelated with the signal and noise, s_n and n_n . The same cannot be said about the second term, since it contains noise and signal terms at time n . However, the degree of cross-correlation will be a function of the bandwidth of the interference characteristics and the low pass filter with impulse response $h_{LPF,n}$.

To illustrate the point, consider a simple example which is close to the worst case situation in which the low pass filter in Fig. 4.19 is of a rectangular shape of bandwidth $B_{LPF} = 0.50$ Hz. Furthermore, assume the interference has a very narrow bandwidth, close to a single tone, offset from the spread spectrum signal by a small amount δf Hz. If, for illustrative purposes, the phase-noise is considered to be negligible, then $\hat{\theta}_{n|n-1} \approx 2\pi\delta f n$, so that the signal term in Eq. (4.38), $a_n \cos \hat{\theta}_{n|n-1}$, after low pass filtering, will have a truncated spectrum offset from 0 Hz by $\pm\delta f$ Hz. This new signal, when mixed by $\hat{y}_n$ in Fig. 4.19, will result in a shifting back to baseband of the truncated spectrum, with significant signal energy in the neighbourhood of 0 Hz, i.e., in the region

$$-B_{LPF} + \delta f < f < B_{LPF} - \delta f.$$

After decimation this baseband term becomes

$$\{[a_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF}\} \cos(\hat{\theta}_{n|n-1})$$

where, as noted earlier, $a_n = s_n$. One can see that the amount of signal energy in the region around 0 Hz diminishes as B_{LPF} decreases from its maximum value of 0.50 Hz and as δf increases. A similar situation occurs with the noise.

Therefore, consider the case where the bandwidth B_{LPF} is sufficiently small so that the second term in Eq. (5.18) is weakly correlated with s_n and n_n . With these simplifications, the signal-to-noise ratio at the output of the despreader can be approximated by

$$\begin{aligned} SNR_o &= \frac{L^2}{LN_0/2 + LE\{\Delta i_n^2\}} \\ &= \frac{L}{N_0/2 + E\{\Delta i_n^2\}} \end{aligned} \quad (5.19)$$

where the derivation of Eq. (5.19) is similar to the derivation of Eq. (2.76) for the linear prediction case. This is why, as indicated earlier, η in Eq. (5.10) was only an approximation for the Kalman filtering case. In deriving Eq. (5.19), it has been assumed that Δi_n is weakly correlated with s_n and n_n for the reasons stated above, that s_n is uncorrelated with n_n , and that the chips of the PN code over a bit interval are uncorrelated. Finally, because the energy per chip was set to unity, and the number of chips per bit is L , the energy per bit is $E_b = L$. Thus, for no residual interference Δi , Eq. (5.19) reduces to the standard equation for BPSK signals in AWGN.

If it is assumed that the noise plus residual interference have Gaussian statistics [71], the bit error rate can be represented by

$$P_b = \frac{1}{2} \operatorname{erfc} \sqrt{SNR_o/2}, \quad (5.20)$$

where $\operatorname{erfc}(x)$ is the complementary error function. As an example of the Gaussian assumption, a histogram of the residual interference resulting from an FM interferer is illustrated in Fig. 5.7 along with a normal curve. As can be seen, the fit is quite good for the most part. In fact, for this example, the number of degrees of freedom was 100, and $\chi^2 = 114$ for a 10% rejection probability. However, as pointed out in Section 3.2.1 in relation to the stable tone for high signal-to-noise ratios at the output of the despreader, performance will be governed by the behaviour of the density function in its tail-region. This behaviour in turn will also be a function of the operating conditions of the excisor.

The next step is to develop an expression for $E\{\Delta i_n^2\}$ in Eq. (5.19) for later substitution into Eq. (5.20). The residual interference is, using Eqs. (5.13) and (5.18),

Figure 5.7: Histogram of residual interference for the case when the interference is FM, generated by the state-space model in Fig. 4.3.

$$\begin{aligned}
 \Delta i_n = & I \left\{ 1 - [\cos(\epsilon_n) * h_{LPF,n}] \cos(\epsilon_n) \right. \\
 & + 1/I \left([1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n} \right) \cos(\epsilon_n) \Big\} \cos(\theta_n) \\
 & + I \left\{ [\cos(\epsilon_n) * h_{LPF,n}] \sin(\epsilon_n) \right. \\
 & + 1/I \left([1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n} \right) \sin(\epsilon_n) \Big\} \sin(\theta_n). \quad (5.21)
 \end{aligned}$$

Equation (5.21) shows that the residual interference consists of two terms, i.e., one related to the in-phase interference component and one related to the quadrature component.

Consider the in-phase term first. If the term $[\cos(\epsilon_n) * h_{LPF,n}] \cos(\epsilon_n)$ is approximated by $\cos^2(\epsilon_n)$ (the worst case situation in which the low pass filter has no effect on the phase-noise term), and if $\epsilon_n \ll 1$ (see Fig. 5.8 for an example), and $\cos(\epsilon_n) \approx 1$, then the in-phase term, as represented below by $\Delta i_{I,n}$, can be approximated by

$$\Delta i_{I,n} \approx \left\{ I\epsilon_n^2 + [1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n} \right\} \cos(\theta_n). \quad (5.22)$$

Equation (5.22) contains two noise terms — one due to the phase-noise (a second order effect) and one due to additive noise and signal which have been low pass filtered.

Now consider the quadrature term in Eq. (5.21). Using the approximation $\sin(\epsilon_n) \approx \epsilon_n$,

$$\Delta i_{Q,n} \approx \left\{ I\epsilon_n + ([1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}) \epsilon_n \right\} \sin(\theta_n). \quad (5.23)$$

Equation (5.23) shows that the residual interference in the quadrature direction consists of a first order phase-noise term and a combination of the additive noise and phase-noise (a second order effect). As a point worthy of noting, considering Eqs. (5.22) and (5.23) one might intuitively conclude that the second order terms can be neglected for low phase-noise cases, and that the residual interference is therefore dominated by the noise and signal terms in Eq. (5.22) and the phase-noise term in Eq. (5.23). Furthermore, for a narrow bandwidth low pass filter of bandwidth B_{LPF} , it is conceivable that the quadrature term in Eq. (5.23) could dominate. This was

Figure 5.8: Histogram of phase error ϵ_n .

in fact pointed out in Fig. 5.6 when little difference in performance was achieved between $B_{LPF} = 0.05$ Hz and 0.025 Hz.

Continuing, if it is assumed that ϵ_n is weakly correlated with

$$\left[1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})\right] * h_{LPF,n}$$

in Eqs. (5.22) and (5.23), the residual interference power in the in-phase direction is

$$E\{\Delta i_{I,n}^2\} \approx (I^2/2)E\{\epsilon_n^4\} + P_{sn}/2 \quad (5.24)$$

where P_{sn} is the power of the signal- and noise-related terms in Eq. (5.22), i.e., the power in

$$[1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}.$$

Similarly, the residual interference power in the quadrature component is

$$\begin{aligned} E\{\Delta i_{Q,n}^2\} &\approx (I^2/2)E\{\epsilon_n^2\} + (1/2)E\{\epsilon_n^2\}P_{sn} \\ &\approx (I^2/2)E\{\epsilon_n^2\} \end{aligned} \quad (5.25)$$

where it has been assumed that $I^2 \gg P_{sn}$. Finally, the residual interference power is, using Eqs. (5.24) and (5.25),

$$\begin{aligned} E\{\Delta i^2\} &\approx E\{\Delta i_{I,n}^2\} + E\{\Delta i_{Q,n}^2\} \\ &\approx (I^2/2)E\{\epsilon_n^2\} + P_{sn}/2 + (I^2/2)E\{\epsilon_n^4\} \\ &\approx (I^2/2)E\{\epsilon_n^2\} + P_{sn}/2 \end{aligned} \quad (5.26)$$

where the term $(I^2/2)E\{\epsilon_n^4\}$ can be neglected under low phase-noise conditions. It should be noted that P_{sn} can also be neglected when the low pass filter bandwidth B_{LPF} is small.

Expressions for $E\{\epsilon_n^2\}$, $E\{\epsilon_n^4\}$, and P_{sn} in Eq. (5.26) will now be determined. If the Kalman filter has reached steady state and its linear form is assumed (refer to Fig. 4.9), then the filtered phase-noise of Eq. (5.7) is

$$\epsilon_n \approx \phi_{u,n} * h_{Kal,n} \quad (5.27)$$

where $h_{Kal,n}$ is the steady state impulse response of the optimum filter. Expanding Eq. (5.27),

$$\begin{aligned}\epsilon_n &= 1/I \left[(n'_{2,n}/\sqrt{2}) * h_{Kal,n} + [s_n \sin(\theta_n)] * h_{Kal,n} \right] \\ &= N_n/I + S_n/I\end{aligned}\quad (5.28)$$

where

$$N_n = (n'_{2,n}/\sqrt{2}) * h_{Kal,n} \quad (5.29)$$

and

$$S_n = [s_n \sin(\theta_n)] * h_{Kal,n}. \quad (5.30)$$

Since s_n and $n'_{2,n}$ from Eq. (5.4) are uncorrelated, S_n and N_n are uncorrelated. It will be assumed, furthermore, that N_n and S_n are Gaussian random variables. Figure 5.9 illustrates the histogram of the term $n'_{2,n}/(\sqrt{2}I)$ in Eq. (5.29) before Kalman filtering and Fig. 5.10 illustrates the histogram of the same term after filtering. Observe that in both cases the statistics indicate a definite Gaussian feature. Figure 5.11 illustrates the histogram of $s_n \sin(\theta_n)$ in Eq. (5.30) before filtering, and Fig. 5.12 shows the same term after filtering. The input term in Fig. 5.12 is far from Gaussian, whereas the output term has a reasonable Gaussian feature. If the mean-squared values of the output phase-noise terms N_n/I and S_n/I from Eq. (5.28) are combined, they can be represented by $V_{\hat{z}_{11,\infty}}$, the steady state phase-noise variance at the output of the Kalman filter, the performance curves of which were illustrated in Fig. 4.10. With this Gaussian assumption then,

$$E\{\epsilon_n^2\} = V_{\hat{z}_{11,\infty}} \quad (5.31)$$

and

$$E\{\epsilon_n^4\} = 3V_{\hat{z}_{11,\infty}}^2. \quad (5.32)$$

Finally, to complete the derivation for an expression for $E\{\Delta \epsilon_n^2\}$ in Eq. (5.19), an expression for P_{sn} from Eq. (5.26) must be determined. Recall that P_{sn} is the power in the term $[1/\sqrt{2}n_{B,n} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}$ in Eq. (5.21). If the terms $n'_{B,n}/\sqrt{2}$ and $s_n \cos(\hat{\theta}_{n|n-1})$ from Eq. (5.17) are assumed to be flat over the lowpass

filter bandwidth with power spectral densities $N_0/2$ and 0.5 respectively (refer to Fig. 5.13 for an example), then

$$P_{sn} \approx B_{LPF}(N_0/2 + 0.5). \quad (5.33)$$

With these approximations,

$$E\{\Delta i^2\} \approx (I^2/2)(3V_{\hat{x}_{11,\infty}}^2 + V_{\hat{x}_{11,\infty}}) + B_{LPF}(N_0/2 + 0.5)/2 \quad (5.34)$$

and

$$SNR_o = \frac{L}{N_0/2 + (I^2/2)(3V_{\hat{x}_{11,\infty}}^2 + V_{\hat{x}_{11,\infty}}) + B_{LPF}(N_0/2 + 0.5)/2}. \quad (5.35)$$

Examples of the accuracy of the bit error rate formula as represented by Eq. (5.20) using Eq. (5.35), are presented next. The interference considered is of constant envelope and of the FM type generated from the interference model in Fig. 4.3. The interference-to-signal ratio was set at 20 dB and the bandwidth expansion factor β was 400. The processing gain was $L=20$ (also equal to E_b). The values for E_b/N_0 ranged from 0 to 12 dB. The modified Kalman algorithm was used in the simulations. In addition, the performance curves illustrated in Fig. 4.11 for the filter form of the modified Kalman algorithm were used to obtain the values for $V_{\hat{x}_{11,\infty}}$ for use in Eq. (5.35). Finally, the 3 dB bandwidth, B_{LPF} , of a 4-pole low pass Butterworth filter was used in Eq. (5.35).

The power spectrum of the signal, noise, plus interference is illustrated in Fig. 5.14. This was determined using the Welch algorithm for two cases, i.e., $E_b/N_0 = 0$ dB and 12 dB.

The bit error rate performance is illustrated in Figs. 5.15 to 5.18. The low pass filter bandwidth B_{LPF} ranged from 0.1 Hz to 0.4 Hz in steps of 0.1 Hz. For $B_{LPF} = 0.1$ and 0.2 Hz (Figs. 5.15 and 5.16), there is reasonable agreement between theory and the computer simulations. The difference starts to manifest itself for larger values of B_{LPF} , i.e., 0.3 and 0.4 Hz (Figs. 5.17 and 5.18). This is to be expected, as it was pointed out in the development of Eq. (5.35) that the correlation between the low pass filtered signal and noise terms, (i.e., the term

$$\{[n_{B,n}/\sqrt{2} + s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$$

in Eqs. (5.22) and (5.23)) and $s_n + n_n$ would increase with increasing B_{LPF} . This phenomenon is illustrated in Figs. 5.19 and 5.20 for $B_{LPF} = 0.1$ and 0.4 Hz, respectively.

Figure 5.19 shows very little distortion of the signal at the output of the excisor, i.e.,

$$s_n - \{[s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$$

for $B_{LPF} = 0.1$ Hz before the PN correlator in Fig. 4.1. The curve referring to

$$\{[s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1}),$$

is approximately 10 dB lower in power in the vicinity of 0 Hz. Figure 5.20, which corresponds to $B_{LPF} = 0.4$ Hz, shows some distortion of the signal. In fact, the difference in power was 20% or 1 dB. Similar distortion phenomena also occurred with the noise term

$$\{n'_{B,n} * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$$

for the two lowpass filter bandwidths, although these are not illustrated here.

Finally, Fig. 5.21 shows an example of the power spectral density of the interference, i_n , interference estimate,

$$\{[I \cos(\epsilon_n)] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$$

(which excludes the signal and noise term effects), and the residual interference,

$$i_n - \{[I \cos(\epsilon_n)] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1}),$$

for $E_b/N_0 = 12$ dB and $B_{LPF} = 0.1$ Hz. The degree of interference suppression for this example was 17.38 dB, and the mean-squared value of the residual interference was 1.83. For $B_{LPF} = 0.4$ Hz, the degree of interference suppression and mean-squared value of the residual interference were almost identical to that for $B_{LPF} = 0.1$ Hz, i.e., 17.39 dB and 1.82. This leads one to the conclusion that residual interference was not a factor in the reasons for the difference between the bit error rate curves of $B_{LPF} = 0.4$ Hz, that it was due to other factors such as signal and noise distortion and the violation of several assumptions which were made in the development of the

bit error rate formula in Eq. (5.20), the primary assumption being that B_{LPF} be significantly less than 0.50 Hz.

The next section considers a more general type of interference, namely narrowband Gaussian noise.

Figure 5.9: Histogram of $n'_{2,n}/(\sqrt{2}I)$ from Eq. (5.29) before Kalman filtering.

Figure 5.10: Histogram of $n'_{2,n}/(\sqrt{2}I) * h_{Kal,n}$ from Eq. (5.29).

Figure 5.11: Histogram of the signal term $s_n \cos(\theta_n)$ from Eq. (5.30) before Kalman filtering.

Figure 5.12: Histogram of the signal term $s_n \cos(\theta_n) * h_{Kal,n}$ from Eq. (5.30).

Figure 5.13: Power spectral density of the signal term $s_n \cos(\hat{\theta}_{n|n-1})$ and noise term $n_{B,n}/\sqrt{2}$ for the case $E_b/N_0 = 12$ dB, ($E_b = L = 20$).

Figure 5.14: Power spectral density of $s_n + n_n + i_n$ for $E_b/N_0 = 0 \text{ dB}$ and $E_b/N_0 = 12 \text{ dB}$.

Figure 5.15: Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.1$ Hz.

Figure 5.16: Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.2$ Hz.

Figure 5.17: Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.3$ Hz.

Figure 5.18: Comparison of the bit error rate performance of the modified Kalman filter algorithm with Eq. (5.20) for the case of $B_{LPF} = 0.4$ Hz.

Figure 5.19: Power spectral density of signal s_n , the filtered signal term $\{[s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$ and the residual signal term $s_n - \{[s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$. The bandwidth of the lowpass filter is $B_{LPF} = 0.1$ Hz.

Figure 5.20: Power spectral density of signal s_n , the filtered signal term $\{[s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$, and the residual signal term $s_n - \{[s_n \cos(\hat{\theta}_{n|n-1})] * h_{LPF,n}\} \cos(\hat{\theta}_{n|n-1})$. The bandwidth of the lowpass filter is $B_{LPF} = 0.4$ Hz.

Figure 5.21: Power spectral density of interference i_n , the interference estimate $\{[I \cos(\epsilon_n)] * h_{LP,n}\} \cos(\theta_{n|n-1})$, and the residual term $i_n - \{[I \cos(\epsilon_n)] * h_{LP,n}\} \cos(\theta_{n|n-1})$. The bandwidth of the lowpass filter is $B_{LPF} = 0.1$ Hz.

5.4 Narrowband Gaussian Noise

The next interferer that is considered is narrowband Gaussian noise. In the previous two sections, the envelope was constant. For narrowband Gaussian noise, the envelope varies as well as the phase. There are two issues to be concerned with here that are related to the time-varying envelope. First, the low pass filter used in estimating the envelope I_n in Fig. 4.19 will introduce a lag in $\hat{I}_n$ thereby increasing the amount of residual interference at the output of the canceller. Second, a time-varying envelope will have the impact of introducing larger phase errors, particularly when the envelope gets very small, since the Kalman filter will not be able to track the fast phase transitions during these instances. These effects will now be dealt with at length.

5.4.1 Filter Lag Effects

First consider the effect of the lag introduced by the Butterworth filter of bandwidth B_{LPF} in estimating I_n in Fig. 4.19. As an example, Fig. 5.22 illustrates the degree of interference suppression, defined in Eq. (5.9) as a function of d for several values of B_{LPF} when the interference bandwidth is $B_i = 0.02$ Hz. The interference was generated by applying white Gaussian noise to a fourth order low pass Butterworth filter resulting in the baseband interference $i_n = I_n \cos(\theta_n)$. Observe that as B_{LPF} increases, the degree of suppression improves. This phenomenon is due to the reduction in the lag as B_{LPF} increases. It is more clearly illustrated in Figs. 5.23 and 5.25, in which sections of the envelope I_n and its estimate $\hat{I}_n$ have been plotted for $B_{LPF} = 0.20$ and 0.40 Hz for the case when $d = 0.30$ rad./sec/volt which is approximately the minimum in Fig. 5.22. A comparison of Fig. 5.25 with Fig. 5.23 reveals a reduction in the delay but, as to be expected, an increase in envelope noise. The effect of the delay is further illustrated in Figs. 5.24 and 5.26 for the overall interference i_n and $\hat{i}_n$ for the two cases. In Fig. 5.24, one can also see the effect of the phase noise, since the envelope noise for this case was small as indicated in Fig. 5.23.

From these results, and the results of the previous section concerning constant envelope interference, there are conflicting requirements. The results from the constant envelope analysis suggested the use of as small a bandwidth for the lowpass

filter as possible, whereas here, as large a value as possible is required to reduce the lag, but at the expense of introducing more envelope noise and signal distortion. This suggests that one could compensate for the delay or use a linear phase FIR filter with some predefined delay. Either of these approaches will, of course, translate into the requirement for a similar delay in the PN correlator. Except where indicated, the remainder of this chapter will consider an FIR low pass filter with pre-defined delay.

To accomodate this change, the interference esimator of Fig. 4.19 will have to be modified somewhat to account for this type of filter. The modified version of Fig. 4.19 is illustrated in Fig. 5.27. The FIR low pass filter has a delay D , which requires that the $\pi/2$ phase-shifted VCO signal from the Kalman filter of Fig. 4.6

$$\hat{y}_n = \sqrt{2} \cos(\omega_0 n + \hat{\theta}_{n|n-1}) \quad (5.36)$$

must be delayed by D as well to yield

$$\hat{y}_{n-D} = \sqrt{2} \cos[\omega_0(n - D) + \hat{\theta}_{n-D}] \quad (5.37)$$

before combining it with the envelope estimate, $\hat{I}_{n-D}$, to produce the interference estimate $\hat{i}_{n-D}$. This requires too that the input signal $u_n = s_n + n_n + i_n$ be delayed by D samples before removal of the interference estimate. The output of the canceller is

$$\begin{aligned} e_{n-D} &= u_{n-D} - \hat{i}_{n-D} \\ &= s_{n-D} + n_{n-D} + i_{n-D} - \hat{i}_{n-D}. \end{aligned} \quad (5.38)$$

This error signal is then applied to the PN correlator whose PN sequence is delayed by D as well.

A comparison of the suppression performance of the systems in Figs. 4.19 and 5.27 is now presented.

The bandwidth of the FIR and Butterworth filters for this case was $B_{LPF} = 0.20$ Hz. The FIR filter had 7 taps with rectangular weighting, the interference bandwidth was $B_i = 0.02$ Hz and $E_b/N_0 = 12$ dB. The results are illustrated in Fig. 5.28 and indicate that the FIR filter improves the amount of interference suppression by 2.7 dB. Furthermore, a comparison of Fig. 5.22 with 5.28 shows that the FIR filter of bandwidth 0.20 Hz provides an additional 1 dB of interference suppression over the

Butterworth filter of bandwidth 0.40 Hz. This gives one some idea of the degradation in performance produced by the additive envelope noise using the interference estimator in Fig. 4.19 for the larger bandwidth situation.

Examples of the envelope and interference estimates are also illustrated in Figs. 5.29 and 5.30. Notice in Fig. 5.30 between 200 and 300 seconds the additional noise due to phase-noise. Finally, Fig. 5.31 shows the suppression level for several interference bandwidths, with $B_{LPF} = 0.20$ Hz and $E_b/N_0 = 12$ dB. It is important to note here that as the bandwidth of the interference increases, the value of d at which the minimum occurs increases, emphasizing the requirement for a larger Kalman filter bandwidth to cope with the faster rate of change of the phase.

5.4.2 Phase-Noise Effects

The second issue to consider is the tendency of the Kalman filter to temporarily lose track when the phase varies rapidly during those instances when the envelope becomes small. It is conceivable, therefore, that with some additional phase-smoothing apart from that provided by the Kalman filter, the phase estimates can improve, thereby reducing envelope and phase-noise caused by inaccurate phase estimates.

To accomodate this idea, an additional modification must be made to the interference estimator in Fig. 5.27; this change is illustrated in Fig. 5.32. In this figure, the phase estimate $\hat{\theta}_{n|n-1}$ from the Kalman filter in Fig. 4.6 is applied to a smoother that introduces the same delay D as the FIR filter used in filtering the envelope. The result is the smoothed phase $\hat{\theta}'_{n-D}$ which is applied to a phase modulator, yielding the signal

$$\hat{y}'_{n-D} = \sqrt{2} \cos[\omega_0(n-D) + \hat{\theta}'_{n-D}]. \quad (5.39)$$

This signal is combined with the envelope estimate $\hat{I}_{n-D}$ to produce the interference estimate

$$\hat{i}_{n-D} = \hat{I}_{n-D} \cos[\omega_0(n-D) + \hat{\theta}'_{n-D}] \quad (5.40)$$

that is then subtracted from the delayed input u_{n-D} to yield the error signal e_{n-D} .

Figure 5.33 shows a section of the phase θ_n and its Kalman filtered estimate $\hat{\theta}_{n|n-1}$ for the case when the interference bandwidth is $B_i = 0.02$ Hz and $d = 0.30$

rad./sec./volt using the system in Fig. 5.27. This should be compared with Fig. 5.34, which is a 7-point smoothed version of $\hat{\theta}_{n|n-1}$ based on the system in Fig. 5.32; the effect of smoothing the phase estimate is quite apparent. Figure 5.35 shows a magnified version of a segment of interference i_n along with the estimates obtained with and without phase smoothing. Here too, the improvement is quite noticeable.

The level of interference suppression as a function of d is illustrated in Fig. 5.36 for several interference bandwidths. As can be seen, the degree of suppression has improved by 3 to 4 dB in comparison with the results in Fig. 5.31 in which no phase-smoothing was used. In addition, there is a broader range of values for d over which the maximum suppression level can be maintained.

The bit error rate performance of the two interference estimators in Figs. 5.27 and 5.32 is presented next as determined from computer simulation. The test conditions were as follows. A total of 50,000 bits were transmitted with a processing gain of $L = 20$, implying a total of 1,000,000 chips. The interference was narrowband Gaussian noise ranging in bandwidth from $B_i = 0.01$ Hz to 0.05 Hz. The approximated optimum values for d obtained from Figs. 5.31 and 5.36 were used in the Kalman filter. The term α_f was set at 0.0003125 Hz and the spectral density of the observation noise $N_{obs}/2$ was obtained from Eq. (5.8), with I^2 replaced by $\overline{I_n^2}$, the mean-squared value of the envelope. The bandwidth, B_{LPF} , selected for the linear phase FIR filter was 0.20 Hz and introduced a delay of $D = 3$ samples. The phase estimate $\hat{\theta}_{n|n-1}$ in Fig. 5.32 was applied to a 7-point rectangular smoother, which also introduced a delay of $D = 3$ samples.

The results of the two interference estimators are plotted in Figs. 5.37 and 5.38. The bit error rate performance of the interference estimator with phase-smoothing (Fig. 5.38) indicates a significant improvement in performance over its non-smoothed counterpart shown in Fig. 5.37. In fact, the improvement is 4 to 5 dB in each of the cases when bit error rates at $E_b/N_0 = 12$ dB are compared. These results also indicate the role of the phase-noise insofar as its effect on the bit error rate for narrowband Gaussian noise is concerned. This was also exhibited in the amount of interference suppression between the two approaches (Figs. 5.31 and 5.36). There too the improvement in suppression was approximately 4 dB.

It should be noted that only one value of B_{LPF} (i.e., 0.20 Hz) was selected for these comparisons, and only one type of phase-smoother. Other combinations may produce superior performance; however, further work would have to be conducted to assess if indeed this would be possible. Finally, given the significance of the phase-noise at the output of the Kalman filter for narrowband Gaussian noise interference, other architectures perhaps can be developed using the phase-smoothing concept to further improve performance.

The next section presents an adaptive Kalman filtering algorithm which builds on the results so far obtained.

Figure 5.22: Level of interference suppression S as a function of d for several lowpass filter bandwidths. The interference is narrowband Gaussian noise of bandwidth 0.02 Hz.

Figure 5.23: An example of a portion of the envelope I_n and its estimate $\hat{I}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.20$ Hz with $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.

Figure 5.24: An example of a portion of the interference i_n and its estimate $\hat{i}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.20$ Hz, $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.

Figure 5.25: An example of a portion of the envelope I_n and its estimate $\hat{I}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.40$ Hz with $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.

Figure 5.26: An example of a portion of the interference i_n and its estimate $\hat{i}_n$ when the filter is of the Butterworth type of bandwidth $B_{LPF} = 0.40$ Hz, $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.

Figure 5.27: Modified interference estimator of Fig. 4.19 when the low pass filter is a linear phase FIR filter.

Figure 5.28: Comparison of the interference suppression levels S produced by a 7-tap FIR filter and 4th order Butterworth filter when $B_i = 0.02$ Hz, $B_{LPF} = 0.20$ Hz, and $E_b/N_0 = 12$ dB.

Figure 5.29: An example of a portion of the envelope I_{n-D} and its estimate $\hat{I}_{n-D}$ when the filter is of the linear phase FIR type of bandwidth $B_{LPF} = 0.20$ Hz with $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.

Figure 5.30: An example of a portion of the interference i_{n-D} and its estimate $\hat{i}_{n-D}$ when the filter is of the FIR type of bandwidth $B_{LPF} = 0.20$ Hz, $d = 0.30$ rad./sec./volt and interference bandwidth $B_i = 0.02$ Hz.

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Figure 5.31: Interference suppression level S as a function of d for several interference bandwidths, B_i , ranging from 0.01 Hz to 0.04 Hz, with $B_{LPF} = 0.20$ Hz.

Figure 5.32: Interference estimator when phase-smoothing is being used.

Figure 5.33: An example of the phase θ_n and its Kalman filter estimate $\hat{\theta}_{n|n-1}$, with interference bandwidth $B_i = 0.02$ Hz and $d = 0.30$ rad./sec./volt.

Figure 5.34: An example of the phase θ_n and a 7-point smoothed version $\hat{\theta}'_{n-D}$ of its Kalman filter estimate $\hat{\theta}_{n|n-1}$, with interference bandwidth $B_i = 0.02$ Hz and $d = 0.30$ rad./sec./volt.

Figure 5.35: An example of a segment of interference with and without phase smoothing; interference bandwidth $B_i = 0.02$ Hz and $d = 0.30$ rad./sec./volt.

Figure 5.36: Interference suppression level S as a function d for several interference bandwidths, B_i , ranging from 0.01 Hz to 0.05 Hz, with $B_{LPF} = 0.20$ Hz.

Figure 5.37: Bit error rate for the case of narrowband Gaussian noise with bandwidths B_i ranging from 0.01 Hz to 0.05 Hz, using the optimum values of d obtained from Fig. 5.36, and an FIR filter with no averaging on the phase estimate $\hat{\theta}_{n|n-1}$.

Figure 5.38: Bit error rate for the case of narrowband Gaussian noise with bandwidths B_i ranging from 0.01 Hz to 0.05 Hz, using the optimum values of d obtained from Fig. 5.36, and an FIR filter with 7-point averaging on the phase estimate $\hat{\theta}_{n|n-1}$.

5.5 An Adaptive Kalman Filter Excisor

To this point, a non-adaptive Kalman filter has been discussed. It was shown that by varying the frequency deviation constant d , while keeping the variance σ_v^2 of the observation noise and the modulating signal bandwidth α_f of the Kalman filter model for FM interference fixed, a minimum for the mean-squared value of the residual interference, $E\{\Delta i^2\}$, was obtained. These results suggest an adaptive scheme, which this section is concerned with.

The proposed scheme is to vary d over time (with σ_v^2 and α_f fixed) and to monitor an estimate of $E\{e_n^2\}$, the mean-squared value of the excisor's error signal in Fig. 4.1, which is related to $E\{\Delta i^2\}$ as discussed in Section 5.3 and below.

Figure 5.39: Profile of the frequency deviation constant d used in the phase- and envelope-tracking tests.

As an example, Fig. 5.39 illustrates a time-varying frequency deviation constant d . The interference for this case was narrowband Gaussian noise with bandwidth

$B_i = 0.01$ Hz and the interference estimator was the system in Fig. 4.19 with $B_{LPF} = 0.40$ Hz. The phase- and envelope-tracking capabilities of a combination of the Kalman filter and envelope estimator arising from this time-varying function are shown in Figs. 5.40 and 5.41. Referring to Fig. 5.40, for large values of d , the phase-noise is evident until at about 400 iterations, at which point the phase-noise has decreased substantially, indicating that the bandwidth of the Kalman filter has decreased. Observe also that initially, the algorithm exhibits some slippage, taking from 10 to 15 radians before it settles down to a tracking error of 2π radians after approximately 50 iterations; the slippage is, in part, due to d being initially too large. At the other extreme, when d becomes too small, the algorithm eventually loses track because the rate of change of the phase is too fast for it (i.e., the bandwidth of the Kalman filter is too small).

Figure 5.41 illustrates the estimate of the envelope of the interference during the same time period. As d decreases, $\hat{I}_n$ appears to improve, until at approximately 1200 iterations, the estimate degrades substantially due to the inability of the Kalman filter to adequately track the phase θ_n as shown in Fig. 5.40.

The results in Figs. 5.40 and 5.41 suggest that an optimum value of d exists that will lead to accurate tracking of the interferer and minimum mean-squared interference error; it also suggests a strategy for an adaptive algorithm. An architecture that illustrates the concept is shown in Fig. 5.42, which is based on finding that value of d which minimizes the mean-squared value of e_n at the input to the PN correlator. This concept does not include the FIR low pass filter nor phase smoothing as discussed in the previous section; however, the results presented later are based on these additional features.

Minimizing the mean-squared value of e_n is equivalent to determining the minimum of

$$\begin{aligned} E\{e_n^2\} &= E\{(s_n + n_n + \Delta i)^2\} \\ &\approx E\{s_n^2\} + E\{n_n^2\} + E\{\Delta i^2\} \end{aligned} \quad (5.41)$$

where Δi^2 has been defined in Eq. (5.21) and the assumptions used in developing Eq. (5.19) have been applied.

Figure 5.43 conceptually shows the steps of the adaptive process. Figure 5.43(a) illustrates an example of the d profile which is decreased according to the equation

$$d_n = d_0 \exp^{-c_0 n}, \quad (5.42)$$

where d_0 is the initial condition, and c_0 controls the rate of change of d_n . This part of the profile forms the search phase, during which time an estimate of the mean-squared value of the error e_n at the input to the PN correlator is continually being calculated using the relation

$$\overline{e_n^2} = \xi_1 \overline{e_{n-1}^2} + (1 - \xi_1) e_n^2 \quad (5.43)$$

where ξ_1 is a number close to 1.0. As d is decreased according to Eq. (5.42), a scaled version of $\overline{e_n^2}$ (scaled by an estimate of the power in u_n in Fig. 5.42) is monitored. This scaled term is defined as

$$r_{mse} = \overline{e_n^2} / \overline{u_n^2} \quad (5.44)$$

where $\overline{u_n^2}$ is an estimate of the power, i.e.,

$$\overline{u_n^2} = \xi_2 \overline{u_{n-1}^2} + (1 - \xi_2) u_n^2 \quad (5.45)$$

where ξ_2 is a number close to 1.0. Equation (5.45) requires a reasonable initial condition to counter the long time constants resulting from those values of ξ_2 very close to 1.0. One approach that is used in the results to follow is to produce a "quick" estimate based on the relation

$$\overline{u_j^2} = \frac{1}{j} \sum_{i=1}^j u_i^2 \quad (5.46)$$

where j is set to some reasonable value. After j samples, Eq. (5.46) is replaced by Eq. (5.45). Another possibility is to break up the estimate of $\overline{u_n^2}$ into, say, two parts. The first part could correspond to one value of ξ_2 that would produce a quick estimate of $\overline{u_n^2}$, and the second part to another value of ξ_2 closer to 1.0, using the result from the first part as an initial condition.

An example of the type of profile created by Eq. (5.44) is illustrated in Fig. 5.43(b). The search for the minimum commences when r_{mse} first crosses the threshold defined by *thres1* occurring at T_1 seconds. The value of d (call this d_{opt}) in Fig. 5.43(a) at which the minimum of r_{mse} occurs, is retained. The search ceases when r_{mse}

crosses the pre-defined threshold denoted by *thres2* at T_2 seconds. At this time d is immediately changed to the optimum value d_{opt} determined in the search phase, and the Kalman filter is re-initialized.

Two examples of the adaptive algorithm will now be discussed for two narrow-band Gaussian noise interferers of bandwidths $B_i = 0.01$ Hz and 0.05 Hz. For these examples, $E_b/N_0 = 12$ dB, the interference-to-signal ratio is 20 dB, and $L=20$. The interference estimator was the one in Fig. 5.32. The low pass filter had 7 taps with rectangular weighting and was of bandwidth $B_{LPF} = 0.20$ Hz; thus the delay D was 3 samples. The phase was also smoothed over 7 samples.

A few words must be mentioned about the selection of specific values for the constants d_0 and c_0 in Eq. (5.42). In the examples to be considered, d_0 has been set to 1.75 rad./sec./volt, corresponding to a reasonable value obtained from Fig. 5.36 for the interference bandwidths considered there. The value for c_0 is selected according to the user's desired maximum change in d , i.e., Δd_{max} , over the time-constant determined by ξ_1 in Eq. (5.43) which estimates the power in the error signal e_n ; for the examples considered, Δd_{max} was chosen to be 0.10 rad./sec./volt, and $\xi_1 = 0.999$ (a time-constant of 1000 seconds). Therefore, with $c_0 = 0.000065$ and $d_0 = 1.75$ rad./sec./volt, d was guaranteed not to change by more than 0.10 rad./sec./volt over any 1000 second interval during the search phase. This strategy provides some guidance to the user as to the rationale in selecting values for these constants, bearing in mind, however, that several trade-offs involving these parameters must be considered. These tradeoffs include the accuracy in estimating $\bar{e}_n^2$ and $\bar{u}_n^2$, speed of adaptation, the quality of the r_{mse} curve in Fig. 5.43(b), and the threshold levels *thres1* and *thres2*.

Figure 5.44 shows the profile of the frequency deviation constant that resulted when the bandwidth of the interferer was 0.01 Hz. The steady-state value in this case was $d_{opt} = 0.30$ rad./sec./volt which agrees reasonably well with the results in Fig. 5.36. The error signal e_{n-D} , illustrated in Fig. 5.45, shows the effect of the changing deviation constant in Fig. 5.44. Observe that, as d becomes too small, the error signal starts to increase, indicating less interference cancellation. Towards the end, at $\approx T = 70,000$ seconds, when *thres2* is finally crossed, the Kalman filter is re-initialized and set to a value of d_{opt} , resulting in a lower level of e_{n-D} . The

mean-squared value of the reference signal u_{n-D} is illustrated in Fig. 5.46. The initial condition was calculated with $j = 1000$ in Eq. (5.46). After 1000 samples, the result $\overline{u_n^2}$ was inserted into Eq. (5.45) with $\xi_2 = 0.9999$

The mean-squared value of e_{n-D} was calculated using Eq. (5.43) with $\xi_1 = 0.999$ and scaled by $\overline{u_n^2}$ in Fig. 5.46. The resulting ratio r_{mse} from Eq. (5.44) is shown in Fig. 5.47. The two thresholds *thres1* and *thres2*, were set to 0.05 (-13 dB) and 0.07 (-11.6 dB) respectively; these values were determined with the aid of Fig. 5.36 and experimentally. Referring to Figs. 5.43 and 5.47, *thres1* is crossed at $T_1 \approx 4000$ seconds and *thres2* is crossed at $T_2 \approx 71000$ seconds. The residual interference and approximate suppression level for this case are illustrated in Figs. 5.48 and 5.49. The latter figure shows an average interference suppression value of 23 dB, which agrees with the results in Fig. 5.36.

Figures 5.50 to 5.54 illustrate the results when the interference bandwidth has been increased to $B_i = 0.05$ Hz. As shown in Fig. 5.50, the final "optimum" value for d is ≈ 0.82 rad./sec./volt. This larger value for d is to be expected, since a larger bandwidth is required for the Kalman filter to track the faster varying phase. Figure 5.51 illustrates the power estimated in the delayed input signal u_{n-D} which, for this particular example, provided a better estimate than the one in Fig. 5.46; the initial condition calculated from Eq. (5.46) was closer to the desired value. The ratio, r_{mse} , as estimated according to Eq. (5.44), is illustrated in Fig. 5.52. For this larger bandwidth interferer, the adaptive system reacts faster as compared to the case where $B_i = 0.01$ Hz, the r_{mse} of which was illustrated in Fig. 5.47. The residual interference, Δi_{n-D} , shown in Fig. 5.53, compared with the results in Fig. 5.48, reveals a significantly higher level of residual interference. Finally, Fig. 5.54 shows the average interference suppression which is approximately 17 dB after adaptation is completed.

The bit error performance of this adaptive algorithm is presented next. For this example, 10,000 bits were transmitted and the bit errors were counted over time, during and after the adaptation period. The conditions were the same as those used in the previous examples except that the interferers ranged in bandwidth from 0.01 Hz to 0.05 Hz in steps of 0.01 Hz. The bit error profiles are illustrated in Fig. 5.55. The profiles consist of four sections having different slopes, which correspond to the high

and low levels of the residual interference results in Figs. 5.48 and 5.53. Furthermore, the times T_2 at which the adaptive algorithm switches to the optimum value for the frequency deviation constant d_{opt} occur in the pattern as expected: as B_i increases T_2 decreases.

5.6 Concluding Remarks

This chapter presented the performance capabilities of the Kalman filter excisor. For the swept tone interferer, it was demonstrated that the excisor has the potential for outperforming the RLS algorithm, at least for the parameters considered. It was argued that the limitation in performance in the case of the Kalman filter was due to the residual phase-noise caused by the nonlinear phase function being tracked and signal distortion, albeit this latter contribution may not have contributed that significantly to the lower bound achieved; to confirm this the residual phase would have to be removed, which could be done by using a higher order Kalman filter.

A theoretical bit error rate formula was developed for constant envelope interference, which was based on several assumptions, an important one of which was the requirement for a bandwidth of the low pass filter used to estimate the envelope, I_n , to be significantly less than 0.5 Hz. The price to be paid for larger bandwidths was signal distortion at the output of the excisor. In fact, the degradation according to the BER curves could be in the order of 2 to 3 dB from $B_{LPF} = 0.10$ Hz to $B_{LPF} = 0.40$.

Although the Kalman excisor is close to optimum for constant envelope, FM types of interferers, its performance was also assessed for narrowband Gaussian noise. Ways and means of circumventing the problems associated with this type of interference—a much more difficult problem to deal with because of the time-varying envelope—were suggested. These included the use of a linear phase FIR filter to estimate the interferer's envelope, and phase-smoothing to dampen some of the additional phase-noise caused by temporary loss of lock of the Kalman filter. These changes improved performance considerably, particularly phase-smoothing. An important point must be noted here. The fact that smoothing improved performance suggests that signal distortion was not a factor and that a higher sampling rate for the same interference

bandwidths could have been employed to improve the phase estimate. Though not attempted as part of this thesis, increasing the sampling rate for larger bandwidth interferers, combined with these architectures, could lead to some interesting results. More on this topic will be discussed in the final chapter.

The chapter concluded with an adaptive scheme based on time-varying the frequency deviation constant of the Kalman filter while holding its other parameters fixed. The scheme was based on finding that value of d which would minimize the mean-squared error at the excisor output. This proved to be a valid technique. One possible drawback was the adaptation time. Further analysis with the various system parameters would have to be conducted in order to establish if the adaptation period could be reduced, bearing in mind, of course, that this would incur penalties in performance because of the requirement to balance several conflicting tradeoffs.

Figure 5.40: An example of the phase-tracking capability of the Kalman filter when the frequency deviation constant d was varied according to the profile in Fig. 5.39.

Figure 5.41: An example of the envelope-tracking capability of the Kalman filter when the frequency deviation constant d was varied according to the profile in Fig. 5.39.

Figure 5.42: Adaptive architecture, which is based on minimizing the squared error of the error signal e_n , while varying d .

Figure 5.43: Illustration of the adaptive Kalman filter process.

Figure 5.44: An example of the profile of the frequency deviation constant d used in the adaptive algorithm, $B_i = 0.01$ Hz.

Figure 5.45: An example of the error signal e_{n-D} for the d profile in Fig. 5.44 used in the adaptive algorithm, with $B_i = 0.01$ Hz.

Figure 5.46: An example of the estimated power in u_{n-D} calculated from Eq. (5.45) for the d profile in Fig. 5.44 used in the adaptive algorithm, with $B_i = 0.01$ Hz.

Figure 5.47: An example of the ratio of $r_{max} = \overline{e_{n-D}^2} / \overline{u_{n-D}^2}$ for the d profile in Fig. 5.44 used in the adaptive algorithm, with $B_i = 0.01$ Hz.

Figure 5.48: An example of the residual interference $\Delta i_{n-D} = i_{n-D} - \hat{i}_{n-D}$ before the despreader in the adaptive algorithm, with $B_i = 0.01$ Hz.

Figure 5.49: An example illustrating the degree of interference suppression $10 \log(\Delta i_{n-D}^2 / P_i)$, where $P_i = 100$ is the power in the interferer of bandwidth $B_i = 0.01$ Hz.

Figure 5.50: An example of the profile of the frequency deviation constant d used in the adaptive algorithm, $B_i = 0.05$ Hz.

Figure 5.51: An example of the estimated power in u_{n-D} calculated from Eq. (5.45) for the d profile in Fig. 5.50 used in the adaptive algorithm, with $B_1 = 0.05$ Hz.

Figure 5.52: An example of the ratio of $r_{mse} = \overline{e_{n-D}^2} / \overline{u_{n-D}^2}$ for the d profile in Fig. 5.50 used in the adaptive algorithm, with $B_i = 0.05$ Hz.

Figure 5.53: An example of the residual interference $\Delta i_{n-D} = i_{n-D} - \hat{i}_{n-D}$ before the despreader in the adaptive algorithm, with $B_i = 0.05$ Hz.

Figure 5.54: An example illustrating the degree of interference suppression $10 \log(\Delta i_{n-D}^2 / P_i)$, where $P_i = 100$ is the power in the interferer of bandwidth $B_i = 0.05$ Hz.

Figure 5.55: Bit error performance of the adaptive scheme for several interference bandwidths and $E_b/N_0 = 12 \text{ dB}$ using the interference estimator in Fig. 5.32.

Chapter 6

Conclusions

This thesis has presented a novel and viable Kalman filtering approach to the excision of narrowband interference from DS/SS communication systems. It has also opened the door to several interesting areas for further research. The purpose of this chapter, therefore, is to highlight the salient points and conclusions of this thesis and to end it with three interesting architectures which warrant further investigation and about which some speculative comments will be made *vis à vis* performance.

As this thesis has pointed out, a significant amount of research has been conducted in the area of suppressing nuisance and hostile signals from DS/SS communications systems, emphasizing the importance of the topic; these areas included a host of analog and digital techniques involving temporal and spatial filtering. It was felt early on in this research that little work had been devoted to interferer's with nonlinear modulations. The swept tone interferer was singled out because of its importance as a jamming signal. The main objective, then, was to develop an excisor that could outperform the least squares approaches for this type of interferer. A nonlinear Kalman filtering technique based on an FM model of the interference was selected.

The thesis demonstrated that with proper parameter selection in the Kalman filter, it could achieve better performance than its RLS counterpart—9 dB for the simple RLS algorithm and between 1 and 2 dB for the RLS algorithm followed by its matched filter. It was argued that phase-noise was the main contributor to the residual interference as opposed to the additive noise, since the latter could be reduced

significantly by the use of a very narrowband filter in the envelope estimation portion of the excisor. Along this vein, because of the nonlinear characteristics of the phase in the swept tone interferer, it was illustrated that some residual phase error between the phase estimate and the true phase existed with the use of the modified Kalman filter, which has the characteristic of being able to track a ramp in phase without any steady-state phase error. This suggests the use, therefore, of a third order Kalman filter that could track interferers with nonlinear phase characteristics, although the structure would be more complex in nature. This could lead to even more improvement.

The capabilities of the Kalman filter in suppressing narrowband Gaussian noise were also assessed. Because of the time-varying envelope, lag introduced by the low pass filter for estimating the envelope, and phase-noise from the Kalman filter estimate were the main contributors to residual interference. Performance was enhanced by approximately 1 dB by replacing the Butterworth filter in the envelope estimator by a linear phase FIR filter. Performance was enhanced even more by another 3 to 4 dB by also incorporating phase-smoothing on the Kalman filter phase estimate. This area warrants further research as there may still be more room for reducing the negative effects of the rapidly varying phase when the envelope approaches zero.

The Kalman filter presented herein is a function of several parameters. The approach taken was to adjust one parameter while fixing the others at predefined values. By so doing, one is essentially confining oneself to a set of performance curves which would be useful for a range of interference bandwidths. As was indicated in Section 4.4 with respect to the sampling rate requirements of the Kalman filter, to achieve better phase-noise performance, one would be forced to use higher sampling rates. Some exploratory work not reported in this thesis had been conducted in this area with the interference estimator in Fig. 4.19 for the case of narrowband Gaussian noise; the results, however, were not encouraging. Revisiting this problem with a linear phase FIR filter replacing the one in Fig. 4.19 accompanied by phase-smoothing may perhaps lead to more encouraging results. One must, however, be wary of the fact that, as B_i increases, so does the amount of signal distortion at the output of the excisor, unless one can somehow remove the signal which would lead to better performance with increasing sampling rate. A proposed structure incorporating decision-feedback that

attempts to remove the signal before estimating the interference will be presented in later paragraphs.

Unless one is concerned with off-line signal processing and analysis, or there is a particular interferer of known characteristics which is causing problems with the spread spectrum communications system, an adaptive architecture is preferred. One structure was proposed and analysed which was based on varying the frequency deviation constant d in a decreasing exponential manner. This approach provided results comparable to those for the non-adaptive or static case. Other schemes not reported in the thesis had been attempted which involved varying d continually in both upward and downward directions, searching for an optimum value while the mean-squared value of the error signal, $E\{e_n^2\}$, prior to the despreader was being monitored. This approach had difficulties because the Kalman filter often had the tendency to lose lock when the direction of adjustment of d was reversed. If it is desirable to adjust d in a continuous fashion in both directions, further research should be conducted on the effects this will have on the gain C in Fig. 4.8, the state transition parameters ϕ_{ij} and the covariance parameters $V_{x_{ij},n}$ and $V_{x_{ij},n|n-1}$. This exercise may determine if a degenerate or runaway condition is occurring in the Kalman filter/transfer function model presented in Section 4.4 and suggest ways of overcoming these problems.

This thesis dealt with a single processing gain. The primary objective was to evaluate algorithms for a range of bandwidths that were a percentage of the chip rate for a fixed processing gain. However, it should be noted that as the processing gain is increased while the interference bandwidth is kept fixed, the bit error rate performance will improve since an information bit for the higher processing gain system will be able to withstand code distortion more so than the one with less processing gain.

To conclude the thesis, three additional architectures which warrant further research will be discussed. The first architecture is concerned with that genre of interferers whose envelopes are constant or vary very slowly. The latter two, which are significantly more complex structures, may lead to improved performance; these architectures incorporate decision feedback, utilizing both filter and predictor forms of the Kalman filter, whose forms were presented in Chapter 4. The third, however, also incorporates a training sequence.

The first architecture is illustrated in Fig. 6.1(a), which is an extension to the one in Fig. 4.1. The main difference is the inclusion of an envelope detector which provides a reference signal for the Kalman filter. If the low pass filter following the envelope detector has a very narrow bandwidth, thus producing an accurate estimate of the envelope, and the interference-to-signal-plus-noise ratio is large, this is a reasonable approach.

The reference signal I_{ref} in Fig. 6.1(a) is used by the interference estimator as illustrated in Fig. 6.1(b). The adaptive algorithm is intended to work as follows. Given that an interferer is present at some unknown frequency relative to the spread spectrum signal carrier, the frequency deviation constant d of the Kalman filter is set to a sufficiently large value as discussed in the previous chapter. The frequency deviation constant d is then decreased at some rate using a profile similar to the one in Eq. (5.42). Before the Kalman filter locks on to the interferer, the error between the reference I_{ref} and $\hat{I}$ will be large. It will remain so until d approaches the vicinity of the optimum value in a mean-squared error context.

If the error is defined as

$$\epsilon_n = I_{ref} - \hat{I}, \quad (6.1)$$

then for a constant envelope interferer, and large interference-to-signal-plus-noise ratios,

$$(\theta_n - \hat{\theta}_{n|n-1})^2 \approx 2\epsilon_n/I_{ref} - 2\frac{\text{noise terms}}{I_{ref}}. \quad (6.2)$$

This can be seen by noting from Eq. (4.38) that the interference envelope prior to the low pass filter in Fig. 4.19 is

$$\hat{I}' = I \cos(\theta_n - \hat{\theta}_{n|n-1}) + \text{noise terms}. \quad (6.3)$$

For low phase-noise conditions, if the first two terms of an approximation to the cosine term are retained, then

$$\hat{I}' = I \left[1 - \frac{(\theta_n - \hat{\theta}_{n|n-1})^2}{2} \right] + \text{noise terms}. \quad (6.4)$$

If it is assumed that $I_{ref} \approx I$, then

$$\frac{2(I_{ref} - \hat{I})}{I_{ref}} \approx (\theta_n - \hat{\theta}_{n|n-1})^2 + 2\left(\frac{\text{noise terms}}{I_{ref}}\right) \quad (6.5)$$

which gives Eq. (6.2). The *Error processor* estimates the mean of Eq. (6.2) resulting in

$$\overline{(\theta_n - \hat{\theta}_{n|n-1})^2} = \overline{2\varepsilon_n/I_{ref}} - 2\frac{\overline{\text{noise terms}}}{I_{ref}} \quad (6.6)$$

$$\approx \overline{2\varepsilon_n/I_{ref}} \quad (6.7)$$

where the noise terms are of zero mean as can be determined from Eq. (4.38). The box labelled *Control* monitors the mean-squared-error. If d becomes too small, then the mean-squared-error starts to increase, implying that the optimum point has passed. Preliminary results comparing the performance of this excisor to the RLS algorithm are reported in [85] for the case of a sinusoidally modulated FM interferer. The results showed that the Kalman filter could provide comparable if not slightly better performance than the RLS algorithm.

Another architecture, which uses decision feedback, is illustrated in Fig. 6.2 with the details of the interference estimator in Fig. 6.3. The intent here is to present a system that may possibly enhance performance by “removing” the signal prior to the interference estimator, the overall reason being to reduce the effects of signal distortion at the output of the excisor if distortion is significant relative to the amount of residual interference. Researchers have examined this problem as indicated in Chapter 2, using transversal structures. The concept will be re-oriented to a Kalman filter structure. These structures are more complex than those considered so far.

Referring to Fig. 6.2, the input $u(t)$ at the output of the bandpass filter $H_{bp}(f)$ is sampled at twice the chip rate resulting in the signal u_n . One branch of this signal is delayed by one sample producing u_{n-1} . From this delayed signal is removed an estimate of the signal $\hat{s}_{n-1}$. If the signal estimate is correct, then the input to the limiter and bandpass filter, defined as u'_{n-1} , will consist of interference and noise only. Notice that the signal estimate $\hat{s}_{n-1}$ is delayed by one chip sample. This is because the bit detector produces bit estimates at the chip rate; this technique was illustrated in Fig. 2.11 and is based on the idea that the bit decisions improve over the course of the bit, thereby improving the accuracy of $\hat{s}_{n-1}$ as the bit evolves at the output of the integrate and dump section. One output of the bit detector is respread by the local PN code and scaled by the energy E_c of a chip to yield the estimate s'_{n-1} . The error

signal e_n at the output of the decimator is delayed by one sample giving the delayed error signal e_{n-1} from which $\hat{s}'_{n-1}$ is removed. The new error signal is, for the case of perfect signal cancellation,

$$\epsilon_{n-1} = n_{n-1} + \Delta i_{n-1}. \quad (6.8)$$

The mean-squared error of ϵ_{n-1} is determined and minimized by adjusting d in the Kalman filter. The other branch of $\hat{s}'_{n-1}$ is oversampled to twice the chip rate and sign-alternated to comply with the input data u_{n-1} ; the result is $\hat{s}_{n-1}$.

Now consider the interference estimator in Fig. 6.3. The input to the interference estimator consists of the two signals bp_{n-1} and u'_{n-1} from Fig. 6.2, both of which, under ideal conditions, have no signal components—only noise and interference. The interference estimator uses both the filter and predictor forms of the Kalman filter which were discussed in Chapter 4; in Fig. 6.3 they are respectively represented by the state-transition matrix ϕ and its squared version ϕ^2 .

The filtering section produces the $\pi/2$ phase-shifted signal

$$\hat{y}_{n-1} = \sqrt{2} \cos[\omega_0(n-1) + \hat{\theta}_{n-1|n-2}] \quad (6.9)$$

which is combined with u'_{n-1} , the result of which is filtered by the low pass filter of bandwidth B_{LPF} , yielding the envelope estimate $\hat{I}_{n-1}/\sqrt{2}$ delayed by one sample.

The predictor portion of the interference estimator produces the signal

$$\hat{y}_n^p = \sqrt{2} \cos(\omega_0 n + \hat{\theta}_{n|n-1}). \quad (6.10)$$

This signal is combined with the envelope estimate $\hat{I}_{n-1}/\sqrt{2}$ to produce the interference estimate

$$\hat{i}_n = \hat{I}_{n-1} \cos(\omega_0 n + \hat{\theta}_{n|n-1}). \quad (6.11)$$

If the assumption is made that $\hat{I}_{n-1} \approx \hat{I}_n$, then

$$\hat{i}_n = \hat{I}_n \cos(\omega_0 n + \hat{\theta}_{n|n-1}). \quad (6.12)$$

The performance of this excisor will be heavily dependent on the initial chip error rate which would expected to be high. As the interference is removed the chip and bit error rates should improve, resulting in better performance. It is expected,

however, that the larger the bandwidth of the interference, the worse the estimate and therefore the more the residual interference, which would have the effect of masking any improvement the decision feedback approach would bring. It is therefore felt that this approach would be more amenable to interferers with fairly narrow bandwidths.

The initial conditions may prove to be important in which case one can resort to “kick-starting” the process by the use of a training sequence long enough for the Kalman filter to stabilize. A variation of the decision feedback system in Fig. 6.2 is illustrated in Fig. 6.4.

This latter structure and the one in Fig. 5.3 could be modified to accommodate sampling the received signal, noise and interference at a higher rate. The signal $u(t)$, sampled at this higher rate, would be applied to a delay element z^{-D} instead of z^{-1} , where D is equal to the number of samples/chip. This would result in the delayed sequences $\hat{s}_{n-D}$, u_{n-D} and u'_{n-D} . During the training sequence, when the signal is known, good interference estimates would be achieved. With the residual interference quite low, the initial bit error rate at the end of the training sequence will be low when the decision feedback mode is entered. The amount of residual interference will be a function of the accuracy of the interference estimate. With the higher sampling rate, the predictor section in Fig. 6.3 would have to be of the multi-step type, predicting the interference in the next chip interval D samples ahead which would degrade performance somewhat. During the initial stages of the decision feedback mode, stability of this approach would depend on the amount of residual interference and the behaviour of the chip error rate.

(a)

(b)

Figure 6.1: An adaptive architecture in which a reference signal I_{ref} is used by the Kalman algorithm: (a) System level diagram; (b) Interference estimator, error processor and controller.

Figure 6.3: The interference estimator of the decision feedback excisor in Fig. 6.2.

Appendix A

A List of Principle Symbols

$\mathbf{a}$	$p \times 1$ vector of filter coefficients $a_i^{(p)}$
$a_k^{(i)}$	k^{th} filter coefficient of an FIR filter of order i
$a_{k,T}^{(i)}$	k^{th} filter coefficient of an FIR filter of order i at time T
$a'_{1,n}, a'_{2,n}$	in-phase and quadrature components of spread spectrum signal relative to θ_n
$A_i(z)$	FIR whitening filter of order i with filter coefficients $a_k^{(i)}$
A'	limiter output level
$bp'(t)$	observation process in state-space model of FM interference
$b_k(t)$	analog PN sequence pattern for the k^{th} bit
bp_n	output of limiter and bandpass filter
bp'_n	discrete form of $bp(t)'$
B_i	interference bandwidth
B_{LPF}	bandwidth of lowpass filter for estimating $I(t)$
B_{ss}	spread spectrum signal bandwidth

B_ϕ	phase-locked loop bandwidth
c_{kj}	discrete code sequence for the k^{th} information bit
C	gain in the transfer function model of the DPLL
d	frequency deviation constant of an FM interferer's modulator (radians/second/volt)
d_n	time-varying form of d in adaptive Kalman filter algorithm
d_{opt}	optimum frequency deviation constant d
D_k	k^{th} transmitted information bit
$\hat{D}_k$	k^{th} detected information bit
e_n	error signal at the output of the Kalman filter excisor
e^f, e^b	vectors of forward and backward prediction errors $e_{i,n}^f$ and $e_{i,n}^b$
$e_{i,n}$	interpolation error from a filter of order i
$e_{n,excess}$	excess prediction error from $\Delta \mathbf{a}$
$e_{i,n}^f, e_{i,n}^b$	forward and backward prediction errors from an i^{th} order filter
$E\{ \}$	statistical expectation operator
E_b, E_c	energy per bit and energy per chip
$\mathcal{E}_{excess}$	average excess mean-squared error due to filter parameter fluctuations in $\mathbf{a}$
$\mathcal{E}_i$	total squared error from an i^{th} ordered prediction filter
$\mathcal{E}_{opt}$	optimum mean-squared error from the Weiner-Hopf solution
$\mathcal{E}_{min}^{(i)}$	mmse for an i^{th} order filter

f_0	carrier frequency of the spread spectrum signal (Hz)
f_s	sampling frequency (Hz)
g	gain vector in state space model
$G_i(z)$	i^{th} order linear prediction filter equal to $1 - A_i(z)$
$h_{Kal,n}$	impulse response of steady-state Kalman filter DPLL
$h_{LPF,n}$	impulse response of lowpass filter for estimating I_n
$h_R(t)$	impulse response of the notch filter in the SAW implementation
$H(z)$	all pole digital filter of order p equal to the reciprocal of $A_p(z)$
$H_{bp}(f)$	receiver bandpass filter
$H_c(\omega)$	transfer function of notch filter with delay T_1 seconds in SAW implementation
$H_R(\omega)$	transfer function of notch filter in SAW implementation
$i(t)$	analog interference of bandwidth B_i
i_n	discrete form of the interference $i(t)$
$\hat{i}(t)$	analog estimate of interference $i(t)$
$I(t)$	envelope of interference $i(t)$
I/S	interference power-to-signal power ratio
I_n	discrete form of $I(t)$
$\hat{I}'_n$	estimate of interference envelope prior to low pass filtering by filter of bandwidth B_{LPF}
$\hat{I}(t)$	analog estimate of interference envelope $I(t)$

K_i	reflection coefficient of i^{th} section of a lattice filter
$K_{i,T}$	reflection coefficients of the i^{th} section of a lattice filter at time T
K_i^f, K_i^b	forward and backward reflection coefficients of the i^{th} section of a lattice filter
$K_{i,T}^f, K_{i,T}^b$	forward and backward reflection coefficients of the i^{th} section of a lattice filter at time T
$K_{G,T}, K'_{G,T}$	Kalman gain vectors in the fast Kalman RLS algorithm and FAEST algorithm
K_n	Kalman gain vector in the Kalman filter excisor
K_w	amplifier gain in the Kalman filter model
L	the number of pseudo random chips per bit or processing gain
$m(\tau)$	modulating signal of an FM interferer
m_1, m_2	initial conditions for state variables $x_{1,n}$ and $x_{2,n}$
$n(t)$	analog bandpass Gaussian noise
$n_{B,n}$	baseband noise related to both $n_{1,n}$ and $n_{2,n}$
n_n	discrete uncorrelated noise samples of variance σ_{nec}^2
n_n^c	coloured noise sequence
$n_1(t), n_2(t)$	baseband in-phase and quadrature components of narrowband Gaussian noise
$n'_{1,n}, n'_{2,n}$	in-phase and quadrature components of noise relative to θ_n
N_0	single-sided noise power spectral density (Watts/Hz)
$\mathcal{O}(\)$	"in the order of" () operations

p	order of transversal or lattice filter
$p(t)$	baseband spread spectrum signal
P_m	power in signal- and noise-related terms at the output of the envelope estimator of bandwidth B_{LPF}
$q(t)$	basic chip pulse of duration T_c and energy E_c
$\mathbf{r}$	$p \times 1$ vector of autocorrelation values r_i
$r(t)$	analog input signal from channel
$\hat{\mathbf{r}}$	estimate of the autocorrelation vector $\mathbf{r}$
r_i	i^{th} lag from the discrete autocorrelation function of an AR process
r_{mac}	ratio of excisor output power $\overline{e_n^2}$ to input power $\overline{u_n^2}$
$\hat{\mathbf{r}}_T$	estimate of $\mathbf{r}$ at time T
$\mathbf{R}$	$p \times p$ autocorrelation matrix with elements r_i
$\hat{\mathbf{R}}$	estimate of the autocorrelation matrix $\mathbf{R}$
$R_i(z)$	i^{th} order interpolation filter
$\hat{\mathbf{R}}_T$	estimate of $\mathbf{R}$ at time T
R_c	chip rate of the spread spectrum signal
R_{ss}	autocorrelation function of spread spectrum signal
$s(t)$	analog spread spectrum signal of bandwidth B_{ss}
s_n	discrete form of the spread spectrum signal $s(t)$
$S_n(f)$	noise power spectral density function at output of $H_{bp}(f)$ (Watts/Hz)
SNR_{no}	signal-to-noise ratio at the input of the excisor

SNR_o	signal-to-noise ratio at the output of the excisor
T	duration of discrete data window
Tr	matrix trace operator
T_b	duration of an information bit
T_c	duration of a chip equal to $1/R_c$
T_1	interaction time of SAW device
$\mathbf{u}$	vector of input data samples $\mathbf{u}^f$ and $\mathbf{u}^b$
$\mathbf{u}^f, \mathbf{u}^b$	vectors of input samples u_n for the forward and backward predictors
u_n	discrete form of $u(t)$
$\hat{u}_n$	predicted estimate of u_n
$u(t)$	input analog waveform consisting of spread spectrum signal, noise and interference
U	matrix of input data samples u_n
U^f, U^b	matrices of input samples u_n for the forward and backward predictors
$\mathcal{U}$	test statistic at the output of the bit detector
$var\{ \}$	statistical variance operator
$v(t)$	observation noise in state-space model
v_n	discrete form of $v(t)$ and of variance σ_v^2
v'_n	baseband noise component in the DPLL with approximate variance $(N_{obs}/2)f_s$
V_w	covariance $E\{\mathbf{w}_n \mathbf{w}_n^t\}$

$V_{\hat{x}_n}$	post error covariance $E\{\hat{\tilde{x}}_n \hat{\tilde{x}}_n^t\}$ with components $V_{\hat{x}_{j,n}}$
$V_{\hat{x}_{n n-1}}$	prior error covariance $E\{\hat{\tilde{x}}_{n n-1} \hat{\tilde{x}}_{n n-1}^t\}$ with components $V_{\hat{x}_{j,n n-1}}$
VCO_n	discrete form of Kalman filter VCO signal
$w(t)$	white Gaussian noise of variance σ_w^2 used in the Kalman filter model of the interference
w_n	discrete form of $w(t)$ of variance σ_w^2
w_n	zero-mean white Gaussian vector sequence of covariance V_w
$\mathcal{W}(t)$	windowing function used in SAW excisor
$\mathbf{x}(t)$	state space vector with elements $x_1(t)$ and $x_2(t)$
$\dot{\mathbf{x}}(t)$	time derivative of $\mathbf{x}(t)$
$\hat{\mathbf{x}}_n$	estimate of the true state vector $\mathbf{x}_n$
$\hat{\tilde{\mathbf{x}}}_n$	post error $\hat{\tilde{\mathbf{x}}}_n = \hat{\mathbf{x}}_n - \mathbf{x}_n$
$\mathbf{x}_n$	discrete form of state-space vector $\mathbf{x}(t)$
$x_1(t), x_2(t)$	phase and modulating signal state variables in the Kalman filter model
$x_{1,n}, x_{2,n}$	discrete forms of the state variables $x_1(t)$ and $x_2(t)$
$\hat{x}_{1,n}, \hat{x}_{2,n}$	estimates of the true states $x_{1,n}$ and $x_{2,n}$
$y(t)$	output of the phase modulator in the state-space model for interference
y_n	discrete form of $y(t)$
α, α_f	bandwidths in radians/second and Hz, respectively, of the first order lowpass filter used to produce the modulating signal $x_2(t)$
β	bandwidth expansion ratio (volts ¹)

γ	ratio of sampling rate f_s to α_f
Γ_i	i^{th} eigenvalue of R
δf	frequency offset from f_0
$\Delta i_{I,n}, \Delta i_{Q,n}$	in-phase and quadrature components of residual interference
Δi	residual interference at excisor output
Δf	peak frequency deviation of an FM interferer (Hz)
Δa	vector of filter misadjustments due to λ
ϵ_n	phase error $\theta_n - \hat{\theta}_{n n-1}$
ζ	sweep rate of chirp signal (radians/second ²)
η	figure of merit of the excisor equal to SNR_o/SNR_{no}
$\hat{\theta}_{n n-1}$	Kalman filter estimate of θ_n
θ_n	discrete form of $\theta(t)$
$\theta(t)$	phase of the interference
λ	weight parameter in the RLS algorithm
μ	constant convergence parameter in the LMS algorithm
μ_T	convergence parameter in the LMS algorithm at time T
μ_i^f, μ_i^b	forward and backward convergence parameters of the i^{th} section of a lattice filter at time T
ρ_k	autocorrelation function of interference at k^{th} lag
σ_i^2	mmse for an i^{th} order filter
σ_{nsc}^2	power of additive Gaussian noise n_n from the channel

σ_w^2	variance of w_n
ϕ	state transition matrix with elements $\phi_{i,j}$
$\phi_{u,n}$	phase-noise on θ_n due to spread spectrum signal and channel noise
ω_a	carrier frequency of chirp signal (radians/second)
ω_0	carrier frequency of spread spectrum signal (radians/sec)
∇	gradient operator as a function of α
∇^f, ∇^b	forward and backward gradient operators as a function of K_i^f and K_i^b

Appendix B

A List of Acronymns

AR	autoregressive
ARMA	autoregressive moving average
AWGN	additive white Gaussian noise
BER	bit error rate
BLS	block least squares
BPSK	binary phase-shift-keyed
BW	bandwidth
CCD	charge-coupled device
CTD	charge-transfer device
dB	decibel
DFT	discrete Fourier transform
DPLL	digital phase-locked loop
DS/SS	direct sequence spread spectrum
FAEST	fast <i>a posteriori</i> error sequential technique

FFT	fast Fourier transform
FIR	finite impulse response
FM	frequency modulation
FTF	fast transversal filter
IF	intermediate frequency
INR	interference power-to-noise power ratio
KF	Kalman filter
LD	Levinson-Durbin
LMS	least mean square
LPF	low pass filter
LS	least squares
mmse	minimum mean-square error
MF	matched filter
ML	maximum likelihood
NBGN	narrow band Gaussian noise
NMF	no matched filter
PLL	phase-locked loop
PN	pseudo noise
QPSK	quadrature phase-shift keyed
RF	radio frequency
RLS	recursive least squares

SAW	surface acoustic wave
SIR	signal power-to-interference power ratio
VCO	voltage-controlled oscillator

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