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Practical Square Cross-Section Helical Antennas

by

Michael C. Britton B. Eng.

**A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of**

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
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Ottawa, Ontario, Canada**

February 1999

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submitted by

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Master of Engineering

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Abstract

The design and implementation of square and other cross section helical antennas is discussed in this thesis. These designs have several advantages over conventional circular cross section helical antennas used in satellite communication and other applications. The linear segments which make up a square cross section helical antenna can be easily supported on a structure which can be cheaply designed and manufactured even for variable cross section or pitch helical antennas. This structure has the additional advantage of having minimal conductor-dielectric proximity, thus virtually eliminating unwanted frequency shift and loss due to dielectric loading. The linear elements are also well suited to simulation using the method of moments, in which cylindrical conductors are nearly always standard elements. Other polygonal cross sections allow some improvement in the axial ratio performance of the helical antennas.

The theory and history of helical antennas is discussed, and axial-mode helical antenna models are designed to illustrate the relative performance of circular and square helical antennas. These models and others with other cross sections are simulated using WIPL, a method-of-moments simulation software package. Circular and square helical antennas are constructed and measured, and the effects of dielectric loading immediately within the helical winding are measured. These simulation and measurement results illustrate the performance advantages of square and other non-circular cross sections for helical antennas.

Acknowledgements

I would like to acknowledge the support of the Telecommunications Research Institute of Ontario (TRIO), the Center for Innovation and Technology of Ontario (CITO), the Communications Research Centre, and EMS Technologies Limited for their assistance in providing the facilities for this research alongside Carleton University. The assistance of Jacques Lemieux, Nagui Mikhail, Aldo Petosa, and the staff of the Science and Technology Centre have also been invaluable.

The financial support of the Cable Telecommunications Research Foundation and of Carleton University is gratefully acknowledged.

The individuals whose personal contributions have been vital to this work are many: my supervisor, Professor Jim Wight, whose guidance and reassurance have been the steady hand behind this work; Professor Peter Strickland, who provided the initial inspiration and direction for it; my parents, whose support in all dimensions has been unceasing; and Suzanne, my fiancée, who has been there to provide comfort and new energy, even when things seemed worst; and the innumerable department staff, professors, colleagues, students, and friends who have contributed in all their different ways. To all of them, I offer my deepest thanks.

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List of Symbols and Abbreviations

T	Turn length of a helical winding
C	Cross sectional circumference or perimeter of helix
α	Pitch angle of helix
d	Edge length of square cross section
θ	Elevation angle, measured from the positive z axis
ϕ	Azimuth angle, measured from the positive x axis, positive towards the positive y axis
E	Electric field magnitude
E_ϕ	Electric field component in the ϕ direction
E_θ	Electric field component in the θ direction
H	Magnetic field magnitude
H_ϕ	Magnetic field component in the ϕ direction
H_θ	Magnetic field component in the θ direction
ϵ_r	Relative permittivity (i.e., dielectric constant) of a material
π	Pi, the ratio of the diameter of a circle to its circumference
r_{loop}	Radius of a circular wire loop
I_0	Current in an antenna
e	Base of natural logarithms
j	The square root of negative one
k	Wave number

r	Radius vector length from the origin to a point in space
λ	Wavelength
d	Side length of a square wire loop
AF	Array factor
N	Number of turns in a helix
β	Phase difference between successive array elements
s	Helix pitch (i.e., separation between winding turns)
ϵ_r	Relative permittivity of a dielectric to that of free space
FDTD	Finite difference time domain simulation technique
MoM	Method of moments simulation technique
$\mathbf{T}_i$	i^{th} basis function
i	Basis function index
N	Number of basis functions used in a MoM simulation
I_i	Amplitude of i^{th} basis function
Z	Impedance
Z_{ij}	Mutual impedance between elements i and j
RHCP	Right hand circularly polarized (electromagnetic radiation)
LHCP	Left hand circularly polarized (electromagnetic radiation)
G	Antenna gain
A_e	Effective aperture of antenna
WIPL	A software package for MoM simulation; the name is taken from the words “Wires” and “PLates,” which are the types of elements it considers.

AUT	Antenna Under Test
s_{11}	Input reflection coefficient of a network

Chapter 1

Introduction

1.1 Introductory Remarks

Helical antennas have been in use for over fifty years, and will likely continue to find application for many more. The fundamental idea of square cross-section helical antennas, which is the focus of this thesis, has been known for over four decades. The idea was never much used when it was first conceived, however, and so the present work aims to bring it into the corpus of engineering practice by demonstrating its advantages over conventional circular cross-section helical antennas: square helical antennas can be significantly less costly to build than circular ones, and they also are easier to design using simulation tools due to the absence of dielectric material near the conductor for most of its length. In addition, some modifications to the conventional circular cross section helix design are further extended to cover square cross-section helical antennas.

1.2 Basic Definitions

An antenna is a device used to convert electromagnetic energy in a circuit into electromagnetic waves travelling in a medium and vice versa. Helical antennas are a particular subset of this vast collection of devices, and are generally classed among the wider grouping of wire antennas. They have been known for fifty years, and are still in use today in numerous applications.

Helical antennas typically consist of one or more wires wrapped around a form (which may be a solid object or an abstract shape) which varies comparatively little in cross-section along its principal axis (for example, a cylinder, rectangular prism, or truncated steep-sided pyramid). The wire wrapping starts at a point on the perimeter of the cross section at one end of the form's major axis and ends at a point on the perimeter of the cross section at the other end of the form, making some number of turns about the form. The turns are usually, but need not be, evenly spaced. The degenerate cases of a helical antenna correspond to a straight-wire mono- or dipole (for a point cross section) and a loop antenna (for a zero height antenna).

For brevity's sake, helical antennas with a particular shape of cross-section may be referred to as helical antennas of that shape; for example, a square cross-section helical antenna will often be called a square helical antenna.

1.3 Purpose

While a more detailed examination of helical antennas's general properties is deferred to Chapter 2, a summary of the general characteristics of helical antennas which make them useful are:

- ☐ Large relative bandwidth
- ☐ Circular polarization
- ☐ Simple broadband impedance matching in axial mode
- ☐ Significant gain with small cross-section in axial mode
- ☐ Suitability for arrays
- ☐ Omnidirectional pattern in normal mode
- ☐ Low sensitivity to manufacturing tolerances

With all this, we may be led to wonder what more could be needed. One of the principal challenges faced when building helical antennas for many applications is the design and construction of the form around which the conductor will be wound. Particularly in the case of conical helical antennas, this form must be manufactured by expensive molding processes. Additionally, the contact between the form and the conductor requires very well-controlled dielectric properties in the form material, which is costly and requires complex simulations for design calculations. In order to significantly reduce these problems, square cross-section helical antennas and variations on them are proposed as a solution. They offer considerably lower construction costs and do not require the structural

support to be in contact with the wire for significant portions of its length, all without significantly altering the size or performance of the helical antenna in any other way. Finally, they are more rapidly simulated in many software systems using the method of moments due to the need for few elements to emulate structures constructed from thin linear conductors.

1.4 Thesis Outline

Following accepted practice, Chapter 2 provides an overview of helical antenna design and applications. The specific terminology used in later chapters is also defined there.

Chapter 3 focuses more closely on analytical models of helical antennas. A summary of the theoretical differences and similarities between circular and square helical antennas is presented.

Simulation tools, models and results are discussed in Chapter 4. In this section, several variations on the theme of helical antennas are explored to verify the general applicability of the proposed parallel between circular and square helical antennas and to suggest further practical helical antenna designs with different cross sections.

In Chapter 5, the design, construction, and testing of actual helical antennas are

presented. The results of these tests are then compared to those predicted in simulations as a means of validating the results advanced in the preceding chapter.

Finally, conclusions from the results of the work, as well as a few suggestions for further research, are offered in Chapter 6.

Chapter 2

Helical Antenna Design and Applications

2.1 Specific Definitions

Before discussing the particulars of helical antennas, it is necessary first to define the terms which are particular to them. These terms relate to the parameters which define the helical structure.

The *helix length* is the distance along the helix axis from one end of the helix to the other. The antenna feed location is usually, but not always, at one end of the helix. In these cases, the end at which the feed is located will sometimes be called the *proximal* or *feed end* of the helix, and the other end may be called the *distal* or *free end* of the helix.

A helix may have one or more conductors wound about it. Typically, these *windings* are spaced at equal angles about the helix axis. A helix with more than one such winding (usually two or four) is called *multifilar* (or, in the special cases mentioned, *bifilar* or *quadrifilar*).

The *cross-sectional area* and *circumference* C are self-explanatory. It is, however, important to note that these are not necessarily constant over the entire helix. The *pitch angle* α , defined as the angle between the conductor and a plane perpendicular to the helix axis at a particular point, can also vary over the helix. The *turn length* T is also variable, and represents the length of conductor required to make a single full winding around the helix. It is related to the circumference and the pitch angle by

$$T = \frac{C}{\cos \alpha} \quad (2.1)$$

The *total conductor length* is simply the length along the path followed by the helix conductor from one end of the helix to the other. In a multifilar helix, each conductor will generally have the same length.

The conductor cross-section is also significant. A cylindrical conductor is characterized by a *conductor radius* or *diameter*, while a tape-wound helix's conductor is characterized by a *conductor thickness* and *conductor width*, where the thickness is measured perpendicular to the form's surface, and the width is measured perpendicular to both the thickness and the edge of the tape.

2.2 **General Types of Helical Antennas**

Helical antennas are principally used in two different manners, corresponding to two different modes of charge movement and electromagnetic wave distribution on and around the helix. These modes are often called the *normal* and *axial modes*, since the maximum-gain direction is normal to the helix axis or along it, respectively. Even though a particular antenna may be usable in both modes, it is generally designed for one or the other, and this distinction is the most fundamental between different types of helical antennas. An intermediate mode, called the conical mode, is sometimes employed, in which the greatest antenna gain directions lie on a cone defined by some angle from the helix axis.

The two principal modes are best considered separately, but one common feature is worth mentioning: antennas can be designed for each mode to radiate nearly circular polarization over a wide band of frequencies. This feature makes them useful in a wide variety of applications, such as mobile and satellite communications, and the degree of circularity of the radiated wave's polarization is an important performance measure for these antennas.

2.2.1 Normal-Mode Helical Antennas

The normal mode was the first to be discovered [Whee47], and is very closely related to the modes existing on electrically small linear and loop dipoles. It is character-

ized by nearly uniform current phase on the helix, and a magnitude which decays to zero at the end of the helix away from the feed. The helix must be electrically small (total conductor length less than half of a wavelength) in order for a helix to radiate in this mode, similarly to an electrically small dipole.

The radiation pattern of such a helix is nearly identical to that of a theoretical dipole, and its polarization is a function of the pitch angle of the helix. It is possible, by an appropriate choice of this pitch angle, to obtain nearly perfectly circular polarization over the entire beam (see Chapter 3). These antennas are not uncommon in mobile communication systems, where circular polarization can reduce multipath fading due to change in polarization on reflection.

2.2.2 Axial-Mode Helical Antennas

Most of the helical antenna designs which are created are of the axial-mode variety, invented by John Kraus [Krau47]. This radiation mode is effective over a relatively wide band of frequencies (more than half an octave in uniform helical antennas), and can provide considerable gain with good polarization circularity.

These antennas are used as single elements, as array elements, and as reflector feeds. A single helical element can be treated as a linear array operating in the endfire condition, and the direction of radiation along the helix axis (towards or away from the

feed) can be determined by appropriate design of the feed and inclusion of a ground plane. For reflector feeds, a backfire helix with no ground plane provides a very low-profile broadband circularly-polarized feed with good front-to-back ratio performance.

The choice between a single element and an array of helices is often based on structural considerations balanced against feed network simplicity. Multiple helical antennas are easier to support and have a much lower profile than a single long helix, but require a larger surface for mounting. They also require a more complex feed network to allow distribution of power to the elements, and the mutual coupling between elements must be taken into consideration. The maximum obtainable gain is a function of the total number of helical turns in either case.

A common modification to axial-mode helical antennas is the tapering of the form. This can be done for two reasons: first, a gradual taper over the entire helix length can increase the bandwidth significantly at the expense of only a small amount of gain; and second, a steeper inward taper in the two or three turns nearest the helix's free end reduces the energy reflected along the helix which would radiate mainly backwards and degrade polarization performance. Figure 2.1 illustrates the profile of a typical dual-tapered antenna compared to that of a uniform helix.

Figure 2.1:
Profiles of helical antennas
(free end of helix at top)
(a): Uniform helix
(b): Double-tapered helix

2.3 Helical Antenna Construction

Circular helical antennas are usually constructed in one of two ways: either a sufficiently rigid conductor is chosen to allow it to be self-supporting, or else the conductor is fixed to a dielectric form around which it is wrapped. The first of these methods results in a rather heavy antenna which is subject to mechanical distortion under loads, and is also difficult to produce due to the tight but precise bend radius required in a thick conductor (typically either very heavy gauge wire, or copper pipe). The second method requires fabrication of a thin, rigid dielectric form with good dielectric properties (the definition of which forms part of the purpose of this research) to which the wire can be attached. The form is thus usually quite expensive, since it must be specially molded for all but uniform antennas with convenient dimensions for which commercially available pipe can be found.

Square helical antennas offer the option of construction using supports which are in contact with the wire only at the corners. A possible design for such supports which allows non-uniform helical structures to be mounted is shown in Figure 2.2, contrasted against the second mounting structure discussed above for a circular helical antenna. Not only is this design easily constructed by cutting sheets of dielectric material, but it has the added advantage of having little dielectric material in close proximity to the helical winding. If dielectric material near the helical winding is desired in order to alter antenna performance, it can be chosen without having to consider its structural rigidity, and thus allows greater freedom in design.

Figure 2.2: Top view of circular and square helical antenna supports

- (a): Circular helical antenna
(b): Square helical antenna

Dielectric material
Conductor

Chapter 3

Analysis of Helical Antenna Operation

3.1 Basic Assumptions

The behaviour of a helical antenna can be treated by analytical methods when certain assumptions are made. Two principal modes of operation can be treated this way: normal-mode radiation and axial-mode radiation. In each case, the helix is considered as an array of individual turns, on each of which the currents are similar except for a difference in phase for axial-mode radiation, plus a fractional turn in the case of helices with a non-integral number of winding turns. The currents on these conductors are assumed to be oriented only along the conductor axis, and are considered to be carried on infinitely thin, perfectly conductive wires. The validity of these assumptions will be discussed later in the chapter, but we will begin by examining in detail the behaviour of a single helix turn as a radiating element carrying some ideal currents.

3.2 Normal-Mode Helical Antennas

The normal mode of radiation is the simplest to analyze, and relies on the fact that the dimensions of the helix are very small compared to the wavelength of radiated energy. This allows the assumption of a constant-phase current along the entire helix, and if sufficiently many turns are present, the amplitude along each winding is approximately constant as well. Kraus [Krau88] performs this analysis for a helical winding of circular cross section beginning with a winding of square cross section, and his analysis has been recognized to match experimental results quite well. His analysis will not be duplicated here; we will examine only the differences between circular and square cross-section helices to determine whether any significant difference in radiated pattern exists.

For the purposes of this analysis, a single helix turn will be treated as a combination of two fundamental elements whose behaviour is already well-understood: an electrically small linear dipole element, and an electrically small current loop, as shown in Figure 3.1.

The square cross section of a small helix affects only the shape of the loop, and the linear dipole element is unaffected. The fields from the circular loop and for the square loop can be readily derived when the loop is electrically small: for a circular loop of radius r_{loop} carrying a uniform time-varying current I_0 with wave number k and wavelength λ , Kraus shows the radiated (i.e., far-field) electric field in the ϕ direction, E_ϕ , at a

Figure 3.1: Model of an electrically small helical turn as a combination of magnetic and electric dipoles.

(a): Sketch of single helical turn (b): Sketch of dipole combination

distance r to be

$$E_{\phi} = \frac{120\pi^3 r_{loop}^2 I_0 e^{-jkr}}{r\lambda^2} \sin\theta \quad (3.1)$$

and for a square loop of side length d , a similar calculation yields

$$E_{\phi} = \frac{120\pi^2 d^2 I_0 e^{-jkr}}{r\lambda} \left[\sqrt{1-X^2} \sin\left(\frac{\pi d}{\lambda} Y\right) + \sqrt{1-Y^2} \sin\left(\frac{\pi d}{\lambda} X\right) \right] \quad (3.2)$$

where

$$X = \sin\theta \sin\phi \quad Y = \sin\theta \cos\phi \quad (3.3)$$

For small loops relative to the wavelength, we can use the approximation

$$\sin\left(\frac{\pi d}{\lambda} X\right) \approx \frac{\pi d}{\lambda} X \quad \sin\left(\frac{\pi d}{\lambda} Y\right) \approx \frac{\pi d}{\lambda} Y \quad (3.4)$$

since the magnitudes of X and Y are bounded by 1. Application of trigonometric identities to the resulting approximate equation allows us to show that the field magnitude is

$$E_{\phi} \approx \frac{120\pi^2 d^2 I_0 e^{-jkr}}{r\lambda^2} \sin\theta \quad (3.5)$$

which is independent of azimuth. If we set the cross sectional areas of the circular and square loops to be equal, the fields are also equal, in keeping with the principle that a current loop's far field is independent of loop shape so long as the area is kept constant. The field intensities from the loops are proportional to their areas, so from this derivation, we can conclude that a square helical antenna can be constructed which is functionally identical (at least in the far field) to a circular one by making the turn spacings and cross sectional areas identical.

It is worth mentioning here that since the polarizations of the radiation from the magnetic and electric dipoles are linear and perpendicular to each other everywhere, and since the field strengths of both radiated components vary identically in elevation (and, as already shown, not at all in azimuth), it is possible to make the radiated wave circularly polarized in all directions by choosing the correct ratio of antenna pitch and cross sectional area [Krau88],[Bala82].

3.3 Axial-Mode Helical Antennas

The analysis for the axial mode of radiation is more complex, since the phase of

the current varies along the helix winding. Typically, the phase progression is assumed to be uniform, and the amplitude is assumed to be constant with the same justification as for the normal-mode antenna described above. This assumption is weaker here, for reasons which will be discussed in Section 3.5. There has been a standard practice of approximating a circular helix by a square one in classical analysis due to the greater tractability of linear elements, before the advent of sophisticated numerical simulation software (q.v.) to analyze complex current distributions. This led to questions about the validity of approximating a circular helix by a square one in analytical work, and a comparative theoretical analysis of circular and square helical elements in the axial mode was published by Knudsen [Knud50], showing the validity of that assumption, but simultaneously making it unnecessary, since the exact expression for a circular helical element was then available. Thereafter, square helices have received little attention beyond their historical interest.

3.4 Helical Antennas as Arrays of Single Turns

The treatment of a helix as a linear array of single turns with similar current distributions is not generally problematic, except where the helix is so short that end effects cover nearly the entire length of the helix. This model results in the total antenna pattern being given by the product of the individual turn pattern with an array factor determined by the number of elements (in this case, each element being a single turn), their relative position (in this case, linearly displaced along the antenna axis by the turn spacing), and the phase difference between successive elements. The array factor AF does not depend

on individual element patterns, and can be given for a linear array of N identical elements with the same current amplitude as:

$$AF = \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \quad (3.6)$$

where

$$\psi = \frac{s}{\lambda} \cos \theta + \beta \quad (3.7)$$

and s is the separation between turns, λ is the wavelength, θ is the angle between the direction of interest and the positive z axis, and β is the phase difference between the currents on successive array elements [Bala82]. For an axial-mode helical antenna, the desired phase progression between turns is

$$\beta = -\frac{s}{\lambda} \quad (3.8)$$

so that in the boresight direction,

$$\psi = 0 \quad (3.9)$$

and therefore,

$$AF = N \quad (3.10)$$

which is the maximum possible gain from the array factor. Because of the resonant nature of the helical antenna, this condition is approximated over a fairly wide bandwidth by variation of the phase velocity of the currents on the conductor.

3.5 **Analysis of Assumptions**

The models presented in the preceding sections are not strictly exact, and so it is necessary to examine here the deviations between these approximations and the actual behaviour of the currents and fields. For a circular helix, these deviations relate almost entirely to end effects (effects caused by the initial and terminal discontinuities of the helix winding), and for a square helix, corner effects are added to these. The radius of the helix conductor (or, in the case of a tape-wound helix, its width) also contributes to minor changes in behaviour.

As discussed in the beginning of this chapter, a normal-mode helical antenna behaves like a monopole (or, in the case of a balanced helix, a dipole), and like a monopole, has a current distribution which tapers to zero at its free end. The performance of the helical antenna as a linear array of identical single-turn helical elements must thus be modified to take into account the lesser magnitude of current on windings near the end of the helix. In addition, the current on each winding is non-uniform, and so a slight asymmetry in azimuthal pattern is produced. The phase variation along the helix may contribute some difference between the theoretical result described above and the actual radiated pattern when the total current path length is an appreciable fraction of a wavelength or more. As can be shown using the expressions for array factor (Equation 3.6), the result in this case is a slight squinting of the beam away from the plane normal to the antenna axis toward a conical beam as shown in Figure 3.2.

Figure 3.2: General cross sections of normal-mode helical antenna patterns.

- a) sketch of ideal pattern (equivalent to ideal dipole pattern)
- b) sketch of pattern accounting for phase lag along wire (slightly conical pattern)

For an axial-mode helical antenna, the differences between theoretical performance and actual performance are best described by discussing the modes of waves propagating along the helix, following Kraus' discussion in [Krau88]. In general, four significant modes are present when the helix circumference and the wavelength of radiated energy are similar. Two of these modes originate at the input (feed) end of the helix, and two at the free end, resulting from a reflection of energy back along the helix from the discontinuity at the helix extremity. The first two modes are an exponentially decaying "zero-order" mode corresponding to the normal-mode radiation mode which contributes little to radiated energy, and then only to sidelobes, and a relatively constant-amplitude or linearly decaying "first-order" mode (which is the only mode assumed to be present in the theoretical model) in which the phase of the current progresses linearly along the conductor. The latter two modes are similar to the first two, but the direction of phase change along the conductor is, of course, reversed for the first-order mode from this end. This reflected first-order mode is the most important factor in causing differences between the-

oretical and actual performance: it radiates a pattern similar to the regular first-order mode, but with opposite polarization, and thus increases the axial ratio of the mainlobe radiation undesirably. It also contributes reactance to the antenna's input impedance, making matching more difficult. Fortunately, it has been shown that this reflection from the free end of the helix can be significantly reduced by tapering the end of the helix inwards as previously shown in Figure 2.1 or by end-loading the helix with a plate as shown in Figure 3.3.

The conductor cross section, as was mentioned above, has a minor effect on antenna performance. In general, a larger cross sectional perimeter appears to achieve a slight improvement in antenna bandwidth, probably due to more degrees of freedom for matching the current path length to the propagating wave (i.e., a significant difference in path length can be realized between the shortest and longest helical path lengths possible on a helix with a large conductor cross sectional perimeter). In the theoretical analysis,

Figure 3.3: Helical antenna with plate for end-loading.

the assumption is that currents exist only in the direction of the helix conductor axis, and that no circumferential current component exists.

The linear array assumption is valid except near the helix ends, where the exponentially-decaying current components are significant. This generally is a problem only for helices of fewer than a half-dozen turns. Even then, this contributes only a minor reduction in gain and a corresponding increase in sidelobe level. For such short helices, numerically computed solutions are the usual means of analysis, and work quite well. A non-integral number of turns also limits this assumption, since an additional unique element must be considered. In practice, the fractional turn contributes slightly to the gain (in proportion to its size relative to the total size of the antenna) and causes a slight increase in ellipticity due to the loss of symmetry.

These analyses apply to uniform helices, where the helix cross section and pitch are invariant along its length. Unless the helix is at least piecewise uniform, it is not tractable to purely analytical treatment, and is best considered using numerical simulation, since the symmetries relied on for analysis no longer exist and the linear array simplification cannot be justified since successive turns are not identical. Such non-uniform helices are nevertheless quite useful, and several examples will be discussed in greater detail in the next chapter.

Chapter 4

Numerical Modeling of Helical Antennas

4.1 Introductory Remarks

Numerical simulation is now the method of choice for developing and optimizing an antenna design for construction. The cost of simulation software and of the hardware platform for it is often minimal when measured against the cost of building a succession of physical prototypes in a “cut and try” approach. The simulations’ reliability is somewhat sensitive to implementation details, but several software packages, among them WIPL [Kolu95][Kolu96], are available which have been amply validated for particular classes of antennas.

In most cases, the result of this software simulation is a design for an antenna which, when constructed, behaves similarly but not identically to the predictions of the

software analysis. Differences are attributable to many things, including manufacturing imperfections and surrounding structures not included in the model. Experienced designers can predict these effects somewhat, and either adapt the model in the simulation to account for it or modify the design between simulation and construction.

4.2 Overview of Numerical Electromagnetic Simulation

Two distinct methods for numerical electromagnetic simulation are commonly used: finite difference time domain (FDTD) methods, and the method of moments (MoM). FDTD simulation is commonly used where structures include volumes of different electrical and magnetic properties, while MoM is popular when structures can be easily approximated as consisting only of conductive wires and plates, although some dielectric volume simulation tools have become available very recently.

Helical antennas without large volumes of dielectric material included in the structure (e.g., a solid core of dielectric) are good subjects for MoM simulation, since they consist primarily of a long bent wire. FDTD will therefore be only briefly surveyed here; MoM will be treated in greater detail.

FDTD methods rely on the conversion of the differential Maxwell's equations to a set of difference equations and solving them for a four-dimensional matrix in space and time with a resolution sufficient to ensure accuracy of the solution. Spatial resolution is

dependent on the size of the structural elements and on the wavelength in the materials present in the model at the frequency of interest. Temporal resolution is dependent on frequency, and is usually expressed in fractions of a full- or half-period at the frequency of interest. For calculation of far-field antenna characteristics, a Green's function (usually free space or free space bounded by an infinite ground plane) is chosen and applied to the near-field distribution determined from simulation.

The MoM is based on solving for current magnitude and phase in the various parts of a structure, then deriving total fields from the contributions of these current flows. The analysis is conducted for a particular frequency, and conductors are generally assumed to be perfect, although this is not intrinsically necessary for the use of the MoM.

The currents are expressed as a sum of products of basis functions $\mathbf{T}_i$ ($i=1, \dots, N$) and amplitudes I_i ($i=1, \dots, N$). The choice of basis functions varies, but they generally are chosen to be non-zero only over a finite domain [Moor84]. Weighting functions are then chosen, and the well-established Galerkin procedure [Harr67] is commonly applied, setting the weighting functions to be identical to the basis functions. These weighting functions are then multiplied in succession by the *potential integro-differential equation* [Moor84] to create a set of equations relating voltages and currents to an *impedance matrix* whose coefficients can be calculated directly from the geometry of the structure. Galerkin's method ensures that this matrix is symmetrical, that is,

$$Z_{ij} = Z_{ji} \quad (4.1)$$

This property ensures the reciprocity of the solution. The excitation of the structure is then applied, and the resulting currents can be determined directly. Using a Green's function, the fields can be determined for any desired location in space.

4.3 **Construction of Antenna Models for Simulation**

The antenna models were designed to operate in the axial mode with the main beam towards the free end of the helix, since this is the most common configuration for helical antennas. They consist of a set of cylindrical wire segments which connect at their endpoints, a ground plane (or cavity) with a node at the antenna feed point, and in a few cases, a plate at the free end of the helix serving as an end load [Foo97]. These elements can be modeled in WIPL quite readily, being specific cases of the basic building blocks (right-truncated cones and bilinear surfaces) from which all structures defined in it must be constructed.

Little attention was paid to the antenna feed, since simulations are seldom accurate in determining input impedance of an antenna, and this is the only parameter which feed details influence significantly. This inaccuracy is due to the sensitivity of input impedance to small details of construction in the vicinity of the feed. Additionally, the input impedance is usually set by shaping the first quarter-turn of the helix above the ground plane

when it is used in a forward endfire mode. (For other feed arrangements used in backfire and other modes, refer to [Krau48].) As such, the feed model consists of a voltage source connected across a very small gap between the base of the helix and the ground plane, as shown in Figure 4.1.

Figure 4.1: Detail view of antenna feed simulation model.

The antennas simulated were generally similar to those constructed for measurement, although other designs were surveyed to ensure that these were representative. In general, however, little variation in form beyond those examined here is possible, since a simple scaling of the antenna dimensions is sufficient to allow radiation in any frequency band, subject to practical considerations of antenna construction.

Models were generated using software written by the author expressly for this purpose, since non-uniform helices of arbitrary cross-section are not easily generated using the tools available in WIPL. This software enables easy generation of helical antenna structures with circular cross-section conductors (not to be confused with the forms around which the conductors are wrapped!) with the following variations:

- ☐ regular polygonal cross sections of helical forms with up to 100 sides
- ☐ tapered-cross section helical forms
- ☐ multiple segments representing complex tapers and variations in winding spacing
- ☐ regular polygonal ground planes with up to 100 sides
- ☐ arbitrary rotation about the helix axis of the ground plane with respect to the helix
- ☐ variable-height and -angle border around the ground plane allowing creation of a cavity-backed helix
- ☐ end-loading plate at the free end of the helix
- ☐ multiple helical windings
- ☐ conductor bend radius modeling segments at corners
- ☐ rectangular and rhomboid cross section forms
- ☐ conductor diameter

Circular helical antennas were modeled using thirty-sided polygonal cross sections. As can be seen from Figure 4.2, results vary minimally with number of sides used beyond approximately twenty sides, so this value provides a good level of reliability while maintaining reasonable simulation running times, which vary approximately with the square of the number of segments.

Related to this last point, the simulation time required for square cross section helical antennas for which relatively few basis functions are required for accurate simulation, is significantly less (typically by a factor of approximately 10) than that for an equivalent circular helical antenna.

Figure 4.2: RHCP gain versus frequency plot for approximations to a circular helical antenna.

1: 10 sides 2: 20 sides 3: 30 sides 4: 40 sides

4.4 Simulation Results

4.4.1 Uniform Circular Helical Antenna

The antenna model on which we will focus here is shown in Figure 4.3. It will serve as a basis of comparison for the other models which will be discussed later. Its dimensions are:

☐	Helix length:	32 cm
☐	Cross-section circumference:	16.24 cm
☐	Number of turns:	8
☐	Conductor diameter:	1 mm

Figure 4.3: Simulation model of circular helical antenna.

- ☐ Ground plane size and shape: 14.7 cm square
- ☐ Number of windings: 1
- ☐ Source length: 0.5 mm

The results for this model, shown in Figure 4.4, are quite close to those predicted by referring to the design curves in [King91], which formed the basis for the design. (In part (f) of the figure, the scale must be estimated using the plots in part (e).) The input impedance, as discussed earlier, is not accurate due to the inaccuracy of the feed design; it is included for completeness, and will be referenced only briefly in comparisons with results from other models. The formation of slight azimuthal lobes with approximate fourfold symmetry is mainly attributable to the ground plane shape, as will be discussed further in Section 4.4.4.

Figure 4.4: Simulation results for circular helical antenna model.
 (a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain
 (continued on next page)

Figure 4.4 (cont'd.) (e): Elevation RHCP gain profiles
 (f): RHCP logarithmic gain versus elevation (θ) and azimuth (ϕ) 3D plots
 Plot numbers indicate frequency as follows:
 1: 1.5 GHz 2: 2.0 GHz 3: 2.5 GHz 4: 3.0 GHz

4.4.2 Uniform Square Helical Antennas

A sample model of a uniform square helical antenna is shown in Figure 4.5. Variations in cross section perimeter were investigated here; other parameters were identical to those for the circular helix above, including the ground plane orientation with respect to the antenna feed position. The antenna feed coincides with a corner of the cross section in this model; variations in this feed position are considered in Section 4.4.7.

When the square helical antenna's cross section perimeter was set equal to that of the circular helical antenna's, three significant differences in performance were observed, and are illustrated in Figure 4.6:

- ☐ higher-frequency operation
- ☐ lower gain
- ☐ lower axial ratio

Figure 4.5: Simulation model of square helical antenna.

Figure 4.6: Simulation results for square and circular helical antenna models. (continued on next page)

(a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain
(e): Elevation RHCP gain profiles A: circular helix B: square helix

Plot numbers indicate frequency as follows:

1: 1.5 GHz 2: 2.0 GHz 3: 2.5 GHz 4: 3.0 GHz

Figure 4.6 (cont'd.) (f): RHCP logarithmic gain versus elevation (θ) and azimuth (ϕ) 3D plots (square helical antenna only)

Plot numbers indicate frequency as follows:

1: 1.5 GHz 2: 2.0 GHz 3: 2.5 GHz 4: 3.0 GHz

The first of these requires an analysis of currents on the helical winding before being explainable, and so further discussion of this effect is deferred to Section 4.4.10.

The third effect is most likely attributable to an end effect, but the detailed mechanism by which it arises was not determinable with the results available.

The second observation, however, can be easily explained qualitatively: the areas of a square and a circle with the same perimeter differ by a factor of 1.27, which would represent about 1 dB of gain at the same frequency if effective aperture is directly related to cross sectional area. Here, the peak gain occurs at a frequency approximately 200 MHz

higher, resulting in a predicted peak gain approximately 0.29 dB lower than that of the circular helical antenna when the difference in effective aperture is considered, using the expression [Bala82]

$$G = \frac{4\pi A_e}{\lambda^2} \quad (4.2)$$

In order to achieve performance nearly identical to that of a given circular helical antenna, a square helical antenna must be constructed so that its cross section perimeter is 1.05 times that of the circular helical antenna. Results for square helical antennas with cross section perimeters 1.025, 1.050, and 1.075 that of the circular helical antenna above,

Figure 4.7: Cross section perimeter variation effects in square helical antennas. 1: Circular reference 2: 1.025 reference perimeter 3: 1.050 reference perimeter 4: 1.075 reference perimeter (a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

and for the circular helical antenna, are compared in Figure 4.7. The performance of the second of these square helical antennas is as close to that of the circular helical antenna as can be achieved by varying only a single parameter; a more exact duplication of performance would require multidimensional optimization (of number of turns, helix length, and cross section perimeter) which is very time-consuming without computational tools to automate the process. Note that here again, the slight four-lobed azimuthal radiation pattern appears, but it is again related to the ground plane shape, not the winding cross section.

4.4.3 Non-Uniform Circular and Square Helical Antennas

It is often desirable to taper a helical antenna slightly over its length in order to increase its operating bandwidth, and also to taper its distal end more steeply in order to reduce reflected currents along the winding, which degrade the antenna's gain and axial ratio. For tapered helical antennas and helical antennas with non-uniform pitch, simulations rapidly demonstrated the correspondence between circular helical antennas and square helical antennas with cross section perimeters 1.05 times that of their circular counterparts. A sample of such a result for dual-tapered antennas shown in Figure 4.8 with different pitches on each tapered section is shown in Figure 4.9. The ground plane and conductor diameter have been kept the same as they have been up to date in these models.

Figure 4.8: Dual-tapered helical antenna models.
 (a): circular cross section (b): square cross section

Figure 4.9: Performance of circular and square dual-tapered helical antennas. 1: Circular cross section 2: Square cross section
 (a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

4.4.4 Multifilar Helical Antennas

Multiple helical windings are usable to improve gain and axial ratio performance for a given size of helical antenna. The different windings are fed with relative phase shifts corresponding to the offset angle of the feed location about the helix axis. Here again, simulations showed correspondences between circular and square helical antennas with cross section perimeter ratios of 1.05. Models of bifilar and quadrifilar helical antennas are shown in Figure 4.10, and simulation results for them are shown in Figure 4.11.

Figure 4.10: Multifilar helical antenna models.

- (a): Circular bifilar helical antenna (b): Square bifilar helical antenna
(c): Circular quadrifilar helical antenna (d): Square quadrifilar helical antenna

4.4.5 Ground Plane Shape Variations

It has previously been stated that the variation in azimuth of the radiation patterns of the helical antennas were attributable to the ground plane's square shape, chosen for ease of construction of the actual antenna prototypes which are the subject of the next chapter. While it is easy to see how a square ground plane can lead to a four-lobed azimuthal pattern, results for antennas with circular ground planes, shown in Figure 4.12, are more perplexing. (The groundplane radius here is 8 cm.) The circular helix with circular ground plane configuration, which would intuitively appear to offer the greatest degree of azimuthal symmetry, has strong variations in azimuthal radiation pattern compared to the

Figure 4.12: Helical antennas with circular ground planes.
(a): Circular helical antenna (b): Square helical antenna

corresponding square helical antenna with an identical ground plane. Azimuthal radiation pattern cuts at 60 degrees from the boresight direction are shown in figure 4.13. The boresight gain performance is nevertheless similar for the two antennas, and so investigation of this azimuthal variation was not conducted in significant depth.

4.4.6 Corner Radius Effects for Square Helical Antennas

When the first square helical antenna prototype was constructed, the bend radius of the wire at the corners was quite large (approximately 3 mm), and the effect of this difference between the actual implementation and the ideal model discussed above was deemed worth investigating. As shall be seen, the effect is significant enough that a new model was constructed with a substantially tighter bend radius.

Figure 4.13: Effect of ground plane shape on azimuthal pattern at $\theta=60^\circ$.

1 and 3: Circular helical antenna 2 and 4: Square helical antenna

1 and 2: Square ground plane 3 and 4: Circular ground plane

(a): 1.5 GHz (b): 2.0 GHz (c): 2.5 GHz (d): 3.0 GHz

Two different models of bending at the corners were considered, based on the physical construction method for these antennas. Both use a single wire segment at an intermediate angle to approximate the bending curve, which is valid so long as the radius is small compared to the side length of the cross section and not large compared to the wire radius. Both models also modify the cross section as shown in Figure 4.14, with the card width being a parameter of the model.

The first model has flat corner segments (i.e., they lie in planes perpendicular to the helix axis), which reflects the shape of a slot cut in a relatively thick support card. The second model maintains a constant pitch angle for the conductor, which is likely to be

mostly valid for thin support card structures where the wire's stiffness prevents the more complex and tighter bend structure implied by the first model. This second model also would reflect a structure with angled slots cut into thicker card edges, but a process to cut such angled slots would be significantly more costly than that to cut perpendicular ones. Detail views of these two models are shown in Figure 4.15.

Figure 4.14: Effect of dielectric support card thickness on square helical antenna cross section

- (a): Cross section with zero-thickness cards and zero bend radius
(b): Cross section of model with non-zero card thickness

Figure 4.15: Detail views of corner radius models
(a): Flat corner segments (b): Sloped corner segments

Results from these simulations were, as mentioned, useful in designing the final antenna prototypes. Due to the increase in cross section perimeter, the frequency band of operation was lowered in both models. Both models exhibited slight loss in gain and increase in axial ratio which worsened as the corner segments were made larger, as can be seen in Figures 4.16 and 4.17. From these results, it is plain that thick card structures, especially ones with perpendicular-cut slots to support the conductors should be avoided when building square helical antennas.

Figure 4.16: Performance of square helical antennas with flat corner segments. Corner segment length:
 1: 0.0 mm 2: 1.5 mm 3: 3.0 mm 4: 5.0 mm
 (a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

Figure 4.17: Performance of square helical antennas with sloped corner segments. Corner segment length:
 1: 0.0 mm 2: mm 3: mm 4: mm
 (a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

4.4.7 Square Helical Antennas with Non-Corner Feed Points

A square helical antenna's feed need not be located at a corner of its cross section, and an antenna illustrating this is shown in Figure 4.18. The minor change in performance due to changing the location of the feed (and consequently, of the free end as well) with respect to the cross section is shown in Figure 4.19.

Figure 4.18: Square helical antenna model with start point of helix winding offset along the cross section edge by a factor of 0.25.

Figure 4.19: Performance of square helical antennas with winding start points offset along the cross section edge.

Offset from corner: 1: 0.00 2: 0.25 3: 0.50 4: 0.75

(a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

4.4.8 Other Regular Polygons as Cross Sections for Helical Antennas

After extensive work with square helical antennas, the idea of considering other polygons, such as triangles, pentagons, and hexagons, was conceived. Models, a few of which are shown in Figure 4.20, were constructed with different numbers of sides in the cross section, all having the same cross section perimeter and other characteristics; their performance is compared to those of the circular and square helical antennas in Figure 4.21.

The results of these simulations follow the same pattern as those for the square helical antenna model in Section 4.4.2: higher-frequency operating band, lower gain, and lower axial ratio as the number of sides of the cross section polygon is reduced. Beyond 20 sides, the effect is so small as to be imperceptible, but between 3 and 8 sides, the effects

Figure 4.20: Sample helical antenna models with different cross sections.

(a): Triangular cross section (b): Pentagonal cross section
(c): Octagonal cross section

are quite large. An interesting and potentially useful note here is that the triangular cross section helical antenna has very low axial ratio on boresight, which could find applications in communications systems with polarization-division multiplexing to double usable bandwidth.

Figure 4.21: Performance of polygonal cross section helical antennas. Numeric labels on curves denote number of sides in cross section polygons.
(a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

4.4.9 Rhomboid and Rectangular Cross Section Helical Antennas

The idea of elliptical cross section helical antennas was first proposed by Kraus as a means of reducing the axial ratio of a helical antenna [Krau48], and was solved analytically by Wong and Loh [Wong59]. Rhomboid and rectangular cross section helical antennas are analogous to elliptical cross section helical antennas as square helical antennas are analogous to circular helical antennas. Sample rectangular and rhomboid helical antenna structures use in simulations are shown in Figure 4.22.

Since the cross section is no longer symmetrical, the orientation of the rectangle or rhombus with respect to the feed point is a variable which affects performance signifi-

Figure 4.22: Sample rectangular and rhomboid cross section helical antenna models Aspect ratio 0.8, start point offset 0.25
(a): Rectangular cross section (b): Rhomboid cross section

cantly. Additionally, the aspect ratio of the rhombus or rectangle has a significant effect. The results of simulations showing these effects are displayed in Figure 4.23. As can be seen from these results, the reduction in cross sectional area results in a slightly lower gain for these antennas, but the slight asymmetry of the structure balances the linear polarization components, resulting in a decreased polarization axial ratio.

It is worth mentioning here that elliptical helical antennas are even more difficult to design and build than circular ones, but these rectangular and rhomboid cross section helices can be constructed in almost exactly the same way as the square helical antennas.

Figure 4.23: Performance of rectangular and rhomboid cross section helical antennas. 1: Square helical antenna 2: Rectangular helical antenna with side ratio 1.20 and start point offset 0.75 3: Rhomboid helical antenna with diagonal ratio 0.80 and start point offset 0.25

(a): RHCP gain (b): LHCP gain (c): Axial ratio (d): Total gain

This results in low costs, making them excellent candidates for practical applications requiring beam antennas with low polarization axial ratios.

4.4.10 Current Distributions on Square Helical Antennas

These simulation results are quite different from the others cited above. The model used is the basic uniform square helical antenna introduced in Section 4.4.2, but here, the current magnitude and phase along a sample turn in the middle region of the helix is examined in detail.

Current distributions for the fifth turn (of eight) are shown for various frequencies in Figure 4.24. Considering first the magnitude of the current, it is apparent that some degree of a standing wave pattern exists, attributable to reflected current from the free end of the helix. The irregularity of the standing wave pattern may be due to the two reflected modes discussed by Kraus and explained in the previous chapter. In any event, this pattern is not unlike that found on a circular helical conductor.

The phase progression of the current, however, is more important in revealing the details of operation of the axial mode square helical antenna. From the figure, it is evident that the rate of current phase progression along the conductor is not uniform on a square helix, but is faster in the central part of the straight segments than it is in the corners, rather like a race car decelerating into and accelerating out of sharp corners. This leads to the

Figure 4.24: Current phase on a square helical antenna winding.
 Current phases are shown for four successive segments making up one full turn, with the left side of the graphs being the proximal end of each wire segment. Frequencies: (a): 2.0 GHz (b): 2.2 GHz (c): 2.4 GHz
 Vertical scale: -180 degrees to +180 degrees.

observed current phase along the main part of each straight conductor being more nearly uniform. The contributions of the differential elements of current along the main part of the wire, being aligned in space and only slightly different in position along the helix's principal axis, are thus more strongly reinforcing of each other than they would be if the phase difference between them were larger.

At the corners, however, the resonance of the currents and the waves present in and around the structure imposes a large change in phase angle in order that the radiated

energy from the next straight segment (which is polarized along the opposite axis from that of the first segment when considered at boresight of the far field) be slightly more than 90 degrees out of phase from that of the previous segment, providing circularly-polarized radiation. (For a more detailed discussion of this resonance behaviour, which is the source of the large high-gain bandwidth of helical antennas, refer to [Krau48].) This change in direction of current also leads to significant near-field intensities near the corners due to the acceleration imposed on the current carriers by the bend in the wire. (This concept of corner radiators can also be expressed in terms of boundary conditions on Maxwell's equations.)

4.5 Summary of Numerical Simulations

The simulations conducted using WIPL suggest a number of useful results. While by no means exhaustive, the variety of antennas simulated should provide a sufficient range of examples to demonstrate the general relationship between circular and square helical antenna performance. It is worth restating the fact that circular helical antennas, being represented by a large number of small linear elements, required significantly more time to simulate, making the square helical antennas more attractive for rapid computer-based design. Additionally, the simulations of antennas with other cross sections offer potentially fruitful design options for antennas destined for specific applications. In the following chapter, the validity of these particular simulations will be verified experimentally.

Chapter 5

Construction and Measurement of Antenna Prototypes

5.1 Introductory Remarks

While the results discussed in the previous two chapters show considerable promise for square antenna performance, the work described in this chapter is the most crucial in demonstrating the validity of the thesis. The proof of the pudding, as it were, is to be found here.

The design and construction of the antennas is discussed first, followed by the measurement apparatus and technique. The results of the measurements are then summarized, and finally, the results are analyzed and compared to the predictions made by

numerical simulations.

5.2 Design and Construction of Antennas

The antennas were designed with ease of construction and measurement in mind. The centre frequency was chosen to be approximately 2.2 GHz, since the physical antenna size at that frequency is not unmanageably large but still large enough not to require very fine and precise construction work. Additionally, thin extruded polyvinyl-chloride (PVC) tubing was readily available in diameters suitable for constructing a cylindrical form for such an antenna.

The length and pitch angle of the antennas were then selected. A 32 cm long, 8-turn helix was considered ideal for several reasons: the length was not so great that weight would cause a significant stress, the number of turns was sufficient to make end effects minor, the pitch angle of 14.0 degrees for the circular helical antenna provided good bandwidth and polarization, and the measurements for wire placement were easily calculated. Based on this, the square helical antenna was constructed so that the cross-section perimeter was 105% that of the circular helix. Allowing 0.5 mm between the support and the center of the conductor, this made the square support's cross section's diagonal length 5.84 cm, for a pitch angle of 13.4 degrees.

A ground plane 28 cm square was chosen in order to provide near-maximal gain

and high front-to-back ratio, which is important for accurate measurements in indoor far-field antenna test facilities. A printed circuit board with solid copper on one side (mounted forward) was used for the purpose, since the dielectric behind the copper provided good structural rigidity for mounting.

The support structure was constructed differently for the circular and square helical antennas. For the circular helical antenna, a PVC tube 1/16" thick with an outside diameter of 2" was used, with a 3/8" acrylic insert in the base into which nylon bolts were fixed in order to secure the antenna rigidly to the ground plane. For the square helical antenna, 1/16" acrylic plastic sheets were used in an "X" configuration, with slots cut into their edges for the conductor, small triangular tabs to provide rigidity, and a 3/8" acrylic base with nylon bolts to secure it to the ground plane.

The conductor, 14-gauge tinned copper wire (approximately 1 mm diameter) was selected for strength, small bending radius, ease of soldering, and sufficient diameter to provide good performance [Tice49]. To secure it to the cylindrical form, small holes were drilled in the form and small droplets of epoxy were applied every quarter-turn. For the square form, the corner bends in the notches provide sufficient wire localization; only the free end of the wire required epoxy to remain secure.

The feed for each antenna is based on a "pin-thru" SMA connector with mounting bolts soldered to the ground plane for good electrical contact, and the conductor soldered

to the pin. Matching was achieved by modifying the first quarter-turn of each helix to form an exponential taper. Instead of having a height of 1 cm, it covers only 0.5 cm of height, and the conductor is bent into an approximately exponential curve rising from near-parallel to the ground plane to the pitch angle. The complete helical antennas are shown in Figure 5.1 to illustrate the construction method.

Figure 5.1: Completed helical antennas.
(a): Circular helical antenna. (b): Square helical antenna.

Dielectric tests were performed using the square helical antenna. Sheets of dielectric of known permittivities were inserted behind the conductor (i.e., inside the coil) so that the conductor was in contact or extremely close proximity to the dielectric over most of its length, except for the corners where the acrylic support precluded dielectric placement. The square helical antenna with a dielectric (here, the 0.031" RT/5880) inserted is shown in Figure 5.2, and the dielectrics used are listed in Table 5.1.

Dielectric Type	Thickness	ϵ_r
RT/5880	0.020"	2.20
RT/5880	0.031"	2.20
RT/5880	0.062"	2.20
RO4003	0.020"	3.38
RO4003	0.032"	3.38
RO4003	0.060"	3.38
RT/6006	0.025"	6.15
RT/6010	0.025"	10.2

Table 5.1: Dielectrics used in helical antenna measurements. [Roge98]

Figure 5.2: Square helical antenna with dielectric insert.

5.3 Antenna Measurement Apparatus and Methodology

The measurement of the electrical parameters of the antennas was done in two stages. The first stage was the measurement of input reflection coefficients from the antennas in order to characterize impedance matching performance. The second and more complex stage was the measurement of the antenna radiation characteristics to determine gain, beam shape, and axial ratio.

Reflection coefficient measurements were conducted using a network analyzer. This instrument allows measurement of reflection and transmission s-parameters of general electrical networks over wide bands of frequencies. In the strict sense of the word,

antennas are two-port devices, but from the perspective of a circuit, they are usually treated as one-port networks. The measurement obtained is valid so long as no significant input occurs at the second (free-space) port of the antenna (i.e., no large reflections or major interference are present). To verify this, the measurement was conducted with the antenna pointing in several directions, so that any reflection or interference would be visible as a variation in measured reflection coefficient. (No significant variation was observed.)

Antenna pattern measurements were conducted at the Communications Research Centre indoor far-field antenna test range. After gain calibration using a pair of fully-characterized antennas, the helical antennas were mounted (one at a time) and measured. A small intermediate cable was required to mount the antennas, and its insertion loss in dB (plotted as negative gain) is shown in Figure 5.3. This insertion loss also accounts for connector losses from the additional connector pair required.

Figure 5.3: Insertion loss of semi-rigid coaxial cable used for antenna mounting.

(Scale: -0.1 dB per vertical division; reference (0 dB) level at top of graph.)

Antenna pattern measurements were made using a spinning linear transmit antenna. In this form of test, a calibrated linearly-polarized antenna is made to rotate so that the polarization vector changes as the measurements are made, resulting in a periodic variation in measured gain whose magnitude is the axial ratio of the antenna under test (AUT). The AUT is meanwhile rotated in order to measure gain in different directions, using a synchronized positioner. The positioner data and the gain are then plotted to provide a *pattern cut*. This pattern cut shows the periodic variation in gain due to the spinning linear element as a periodic function of azimuth or elevation angle. From this data,

axial ratio and circular-polarization gains can be computed.

Samples of gain were taken every degree of AUT orientation change for each of 16 equally spaced frequencies ranging from 1.5 to 3.0 GHz. The spin rate of the linear element was carefully set so as to provide an easily discerned variation in amplitude while preserving reasonable pattern resolution. A sample plot of the data obtained from this measurement is shown in Figure 5.4. (The data is for elevation plots of the circular and square helical antennas at a frequency of 1.90 GHz.)

Figure 5.4: Typical spinning-linear antenna measurement data plot.

solid curve: Circular helical antenna.
dashed curve: Square helical antenna.

For the circular helical antenna and the square helical antenna without dielectric inserts, azimuthal pattern cuts at elevations of 10 and 30 degrees from the helix axis, or *boresight*, were made in order to verify azimuthal symmetry. Elevation cuts were made for both of these antennas, and for the square helix with each dielectric inserted. These elevation cuts extend through 240 degrees (120 degrees each way from boresight), which is the maximum achievable at the range in question due to the effect of the antenna mounting tower.

5.4 Measurement Results

5.4.1 Input Impedance

Reflection coefficient measurement demonstrated the effectiveness of the exponentially-tapered matching quarter-turn for both the circular and square helical antennas. Input impedance (normalized to 50 ohms) and reflection coefficient magnitude for the circular helical antenna are plotted on a Smith chart in Figure 5.5. It is clear that the circular helical antenna shows significant resonances near the upper edge of its useful frequency band. The minimum reflection loss for this antenna is 6.5 dB, resulting in a 1.1 dB maximum insertion loss due to mismatch.

Figure 5.5: Input impedance (s_{11}) of circular helical antenna.

(a): Smith chart (50 Ω). (b): Magnitude.

The square helical antenna's input impedance is plotted in Figure 5.6 (solid lines). Again, significant resonances arise near the upper band edge. The worst-case match (occurring at the extreme high-frequency end of the band) has a return loss of approximately 4.8 dB, leading to an insertion loss of 1.75 dB. It is worth noting, however, that the matching in the main part of the frequency band is much better, with a return loss of at least 8 dB (0.75 dB insertion matching loss).

Also plotted in Figure 5.6 is the return loss (plotted as s_{11} magnitude) for the square helical antenna with the RT/6010 dielectric inserts in place (dashed line). By comparing this plot to the result without the dielectric insert, it is clear that the effect of the dielectric is to shift the input impedance characteristic downward in frequency and slightly alter the resonances, altering the null spacing and depth.

5.4.2 Azimuthal Pattern Symmetry

The right-hand circularly-polarized gain and the axial ratio have been calculated for evaluating performance over the frequency band and for comparing the antennas. Azimuthal symmetry is good within the central portion of the band for both circular and square helical antennas.

Figure 5.6: Input impedance (s_{11}) of square helical antenna.

(a): Smith chart (50 Ω) (b): Magnitude
solid curve: No dielectric dashed curve: RT/6010 dielectric

Antenna azimuth measurements at 10 degrees from boresight show minimal (less than 2 dB) azimuthal variation in the low- to mid-frequency regions of the band, with typical plots resembling Figure 5.7. At higher frequencies, a “lopsidedness” becomes apparent, appearing in the circular helix above approximately 2.25 GHz and in the square helix above approximately 2.55 GHz, as shown in Figure 5.8. At yet higher frequencies, the measured data does not follow a consistent pattern.

At 30 degrees from boresight, the azimuthal pattern loses symmetry without as identifiable a pattern as frequency is increased past the center frequency. Again, the loss of symmetry occurs at a slightly lower frequency for the circular helix than for the square one.

Figure 5.7: Typical mid-band azimuth pattern cuts at 10 degrees from boresight.

solid curve: Circular helical antenna.
dashed curve: Square helical antenna.

Figure 5.8: Typical above-band azimuth pattern cuts at 10 degrees from boresight.

solid curve: Circular helical antenna.
dashed curve: Square helical antenna.
(a): 2.4 GHz (b): 2.7 GHz (c): 3.0 GHz

5.4.3 Beam Shape and Sidelobe Structure

Antenna gain measurements match simulation results in general form when gain versus elevation plots are compared, although the measured gains are slightly lower than those predicted by simulations. Elevation cuts show progressively narrower beams as the frequency approaches midband from the low end, with sidelobes becoming progressively more numerous and complex, as can be seen in Figure 5.9. Past the maximum gain point, the main lobe of the pattern disintegrates rapidly as frequency is increased, eventually becoming a null in an asymmetrical conical pattern, as can be seen in Figure 5.10.

5.4.4 Boresight Gain

The boresight gain profiles for the circular and square helices are plotted in Figure 5.11. The frequency response of the circular helical antenna is lower than predicted by simulation, which can be attributed to the PVC dielectric immediately under the antenna winding. Gain levels are approximately 1 dB below those predicted by simulation (plotted to the same scale for convenience in Figure 5.12), but this can be easily explained by cable losses (0.2 dB) and mismatch losses (0.8 dB) as described above.

Figure 5.9: Elevation pattern cuts of helical antennas in low- to mid-band operation.

(a): 1.5 GHz (b): 2.0 GHz (c): 2.1 GHz
solid line: circular helical antenna. dashed line: square helical antenna.

Figure 5.10: Elevation pattern cuts of helical antennas in upper- to above-band operation.

(a): 2.4 GHz (b): 2.6 GHz (c): 2.7 GHz

solid line: circular helical antenna. dashed line: square helical antenna.

Figure 5.11: Bore sight RHCP gain of circular and square helical antennas.

Figure 5.12: Bore sight RHCP gain of circular and square helical antennas from WIPL simulations.
1: circular helical antenna. 2: square helical antenna.

5.4.5 Boresight Axial Ratio

The axial ratio measurements are plotted in Figure 5.13. Again, the circular helical antenna's response is similar in form to the square helical antenna's, although at a lower frequency. Interestingly, the axial ratio performance of the square helical antenna is slightly better than that of the circular one. The presence of the dielectric is the most likely explanation for this, as discussed in the following section.

Figure 5.13: Axial ratio of helical antennas at boresight.

5.4.6 Dielectric Effects

As mentioned briefly above, the overall effects of a layer of dielectric material under the helix winding are to:

- ☐ lower the frequency band of operation
- ☐ reduce the boresight gain
- ☐ increase the axial ratio.

The first two of these are illustrated in Figures 5.6, 5.14 and 5.15, which show the effects of higher dielectric constants and of larger thicknesses of dielectric, respectively. The last is shown in Figure 5.16.

The latter two of these three are small effects and are probably related: a small proportion of the energy that is radiated in the desired polarization towards boresight from an antenna without dielectric material under the conductor is instead radiated in the inverse polarization towards boresight due to distortion of the field lines of the desired mode discussed in Chapter 2.

The beam shape and sidelobe structure for antennas with dielectric resemble those of the antenna without dielectric inserts, save for the lower frequency at which characteristics appear. The general scaling in frequency is reasonably accurately reflected by the gain profiles, and so no additional details are discussed here.

Figure 5.14: Effect of dielectric constant on antenna gain.

(a): 0.031" dielectric thickness

(b): 0.025" dielectric thickness

Figure 5.15: Effect of dielectric thickness on antenna gain.

(a): $\epsilon_r=2.20$ (b): $\epsilon_r=3.38$

Figure 5.16: Boresight axial ratio of square helical antenna with 0.031" dielectric inserts

The frequency shift, however, is a major influence on the antenna's performance. The plots of Figures 5.14 and 5.15 are potentially useful as providing a basis for predicting the difference between simulated and actual performance of helical antennas with dielectric material under the winding, since simulations of antennas which include dielectric materials are either unreliable or very time-consuming (or even both!).

Chapter 6

Conclusion

6.1 Summary of Research Conducted

The behaviour of square helical antennas has been investigated in some detail. The principal focus has been on determining their performance relative to classical circular helical antennas operating in the axial mode, and on effective techniques for the design of a square helical antenna. To this end, a number of different types of circular and square helical antennas were compared in simulation, and the simulation results were validated by comparison to both previously published and directly measured results.

In addition to this primary thrust, effort has also been directed towards investigating the operation of helical antennas with other cross sections, and towards determining the effect of dielectric material under the helical winding of these antennas. The first of

these was examined using numerical simulation, and the latter, using direct measurements.

6.2 Summary of Results

The principal conclusion of this research has been to demonstrate that a given circular helical antenna's performance can be approximated by a square helical antenna with a cross section circumference 1.05 times that of the circular helical antenna's. Since a square helical antenna does not require a molded or otherwise hard-to-machine form to support it, this result may realize significant cost savings for producers of antennas for mobile and satellite communications. They also require considerably less simulation time in many MoM-based simulation packages due to the small number of elements required to define their structure.

Other cross sections for helical antennas were also found to have potential applications: rectangular and rhomboid cross section helical antennas, as well as triangular cross section helical antennas, may be useful in applications where axial ratio performance of the antenna is paramount. The former two of these share the square helical antenna's ease of manufacture, making them more practical design options than the hard-to-design elliptical cross section helical antennas on which they are based.

Finally, dielectric materials were shown to have a significant effect on the frequency band of operation of a helical antenna, and a slight effect on gain and axial ratio.

Quantitative estimates may be interpolated from the experimental results offered here, facilitating both the design of helical antennas in which the form is in contact with the conductor over its entire length and that of antennas in which the dielectric in contact the wire is intended as a deliberate dielectric loading (independent of structural considerations) in order to reduce the size of an antenna operating in a given frequency band.

6.3 Areas for Additional Research

The behaviour of square cross section helical antennas operating in the normal mode has not been investigated in detail here, although a brief analytical treatment was presented in Chapter 3. Of particular interest for this application would be the difference in performance with frequency between antennas with circular and square cross sections, since the square helical antenna's conductor length is greater than that of the circular helical antenna for antennas of equal cross sectional areas.

In addition, the coupling between square helical elements in array configurations has not been investigated here. A basis for such work would be the experiments conducted with circular helical elements discussed briefly by Kraus [Krau88]. The oft-cited result of this work is that the coupling between two circular helical elements is minimal when the antenna feeds are as far apart as possible. For square helical antennas, the near-field effects at the corners may prove to be dominant factors in element coupling, and thus the orientations of not only the feed points but also of the corners of the squares must be

considered.

Finally, the effect of ground plane shape on azimuthal pattern remains to be determined completely. Analysis of the currents on the ground plane is quite difficult since the currents are not generally expressed in convenient forms by the simulation packages available. A great deal of data analysis would thus be required to convert the simulations' current results into a form from which current modes on the ground plane could be determined.

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