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Superior?

LARGE-SIGNAL MODELLING OF
VHF AND MICROWAVE POWER TRANSISTORS

by

Adrian William Alden, B.Sc., M.Eng.

Thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Faculty of Engineering
Carleton University
Ottawa, Ontario

June 1975

⑥ Adrian William Alden, 1975

The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis:

"Large-Signal Modelling of VHF and Microwave Power Transistors"

submitted by Adrian William Alden, B.Sc., M.Eng., in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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August 1, 1975

ABSTRACT

The research reported is concerned mainly with the modelling of VHF and microwave power transistors when operating under saturation and quasi-saturation conditions. It has applications in such fields as the analysis and design of class C and D amplifiers and frequency multipliers, and computer-aided design of integrated circuits.

The models proposed are based upon a 'transport' model for the base region (including large-signal basewidth modulation effects), augmented by a representation of the collector involving a variable-width charge storage region characterized in two-lump form. Implementation of these models for purposes of computer circuit analysis has been carried out and the models have demonstrated their ability to describe device behaviour under d.c., step transient and large-signal sinusoidal drive conditions. Methods are proposed for the evaluation of parameters, particularly with regard to the collector region, effects in which are considered dominant for these devices.

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LIST OF PRINCIPAL SYMBOLS

(arranged by Section in which they first appear)

SECTION	SYMBOL	
2.2	A_C	cross-sectional area of collector-base junction
	A_E	cross-sectional area of emitter-base junction
	N_{epi}	impurity concentration in the (n-type) epitaxial layer
	N_+	impurity concentration in the (n-type) substrate
2.3	W	neutral base width between junction depletion layers
	x_i	injection region width in collector material in quasi-saturation
2.4	r_C	resistance of ohmic part of intrinsic collector (see Fig. 2.9)
	V_o	voltage developed between base contact and ohmic edge of injection region in quasi-saturation (see Fig. 2.9 and Eqn. 2.15)
	x_e	total width of epitaxial layer
2.6	x_{DC}	space charge layer width in collector material
3.2	C_{μ_c}	injection region width modulation capacitance (see Eqn. 4.21)
	C_{μ_o}	injection region width modulation capacitance when the injection region occupies the entire epi layer ($x_i = x_e$) (see Eqn. 4.21)
	K	ratio of widths of two collector charge lumps (see Fig. 3.1)
	q_1	excess charge in first lump (see Fig. 3.1)
	q_2	excess charge in second lump (see Fig. 3.1)
	q_{COL}	total excess charge in collector region (see Fig. 3.1)

SECTION	SYMBOL	
	r_{CO}	resistance of epi layer i.e. $r_C = r_{CO}$ for $x_i = 0$
	τ_1	carrier lifetime in first lump (see Fig. 3.1)
	τ_2	carrier lifetime in second lump (see Fig. 3.1)
	τ_3	carrier 'transit time' across injection region (see Fig. 3.1 and Eqn. 3.8)
	τ_{30}	carrier 'transit time' across epi region for $x_i = x_e$
	τ_4	carrier 'transit time' out of second lump (see Fig. 3.1 and Eqn. 3.11)
	τ_{40}	carrier 'transit time' out of epi region for $x_i = x_e$
	x_1	width of first lump of injection region excess charge (see Fig. 3.1)
	x_2	width of second lump (see Fig. 3.1)
3.3.1	q_{10}	'equilibrium' charge in first lump in high level saturation (see Eqn. 3.26)
	q_{20}	'equilibrium' charge in second lump in high level saturation (see Eqn. 3.28)
	τ_r	recombination velocity lifetime at epi-substrate boundary (see Eqn. 3.37)
3.3.2	τ_p	bulk lifetime in epitaxial material (see Fig. 3.4)
	V_r	recombination velocity at epi-substrate boundary (see Eqn. 3.36)
3.4	x_{DB}	collector depletion layer semiwidth in base material
4.2	β_F	forward active region current gain (see Fig. 4.1)
	β_R	reverse current gain of intrinsic base (see Fig. 4.1)
	C_{μ_b}	basewidth modulation capacitance (see Eqn. 4.10)

	I_{SF}	forward saturation current (see Fig. 4.1)
	I_{SR}	reverse saturation current (see Fig. 4.1)
	n_F	forward injection index (see Fig. 4.1)
	n_R	reverse injection index (see Fig. 4.1)
	τ_F	forward base transit time (see Fig. 4.1)
	τ_R	reverse base transit time (see Fig. 4.1)
4.2.1	N_B	impurity concentration in base region
	q_b	total excess minority carrier charge in base region
	q_e	excess charge of emitter side lump in base (see Fig. 4.5)
	q_c	excess charge of collector side lump in base (see Fig. 4.5)
	τ_c	lifetime in collector side lump (see Fig. 4.5)
	τ_d	transit time across base in basewidth modulation model of Fig. 4.5
	τ_e	lifetime in emitter side lump (see Fig. 4.5)
	x_c	width of collector side lump in base (see Fig. 4.5)
	x_e	width of emitter side lump in base (see Fig. 4.5) Note: this is NOT the x_e used in the collector region models.
4.3.1	C_{JC1}	intrinsic device collector depletion layer capacitance
	C_{JC2}	extrinsic device collector depletion layer capacitance
	q_F	forward excess charge in base region (see Fig. 4.8)
	q_R	reverse excess charge in base region (see Fig. 4.8)
	r_{bb}	extrinsic transverse base resistance (see Fig. 4.11)

	r'_c	extrinsic collector resistance (see Fig. 4.11)
	τ_{BF}	forward base lifetime, $\beta_F = \tau_{BF} / \tau_F$
	τ_{BR}	reverse base lifetime, $\beta_R = \tau_{BR} / \tau_R$
4.3.2	K'	ratio of charge lump widths for extrinsic diode (see Fig. 4.12)
	K^*	ratio of charge lump widths for collector region low level saturation model (see Fig. 4.13)
	q'_1	excess minority carrier charge in first lump of extrinsic diode model (see Fig. 4.12)
	q'_2	excess minority carrier charge in second lump of extrinsic diode model (see Fig. 4.12)
	q_1^*	excess minority carrier charge in first lump (collector region) of low level saturation model (see Fig. 4.13)
	q_2^*	excess charge in second collector region lump of low level saturation model (see Fig. 4.13)
	τ'_1	lifetime in first lump of extrinsic diode model (see Fig. 4.12)
	τ'_2	lifetime in second lump of extrinsic diode model (see Fig. 4.12)
	τ_1^*	lifetime in first lump (collector region) of low level saturation model (see Fig. 4.13)
	τ_2^*	lifetime in second lump (collector region) of low level saturation model (see Fig. 4.13)
	τ'_3	'transit time' of extrinsic diode model (see Fig. 4.12)
	τ_3^*	'transit time' across collector region of low level saturation model (see Fig. 4.13)
	V_{JC1}	intrinsic collector-base applied junction voltage
	V_{JC2}	extrinsic collector-base applied junction voltage

5.2	b	ratio of the excess electron reverse charge in the base to the excess electron charge in the first collector lump (see Eqn. 5.1)
5.2.3	x_{dc2}	extrinsic collector junction semiwidth in epitaxial material (see Eqn. 5.19)
6.2.1	τ_d	total delay time through device (see Eqn. 6.15)
6.2.2.4	r'_{co}	resistance of extrinsic collector region
	x_{dc1}	intrinsic collector junction semiwidth in epitaxial material (see Eqn. 6.10)
6.2.3.1	τ_{COL}	collector charging time (see Eqn. 6.15)
	τ_E	emitter depletion layer charging time (see Eqn. 6.15)
	τ_{JC1}	collector depletion layer transit time (see Eqn. 6.15)
6.3.1	α_I	common base d.c. current gain for Ebers-Moll model in inverse mode (see Fig. 6.12)
	α_N	common base d.c. current gain for Ebers-Moll model in normal mode (see Fig. 6.12)
	I_{SI}	inverse saturation current for Ebers-Moll model (see Fig. 6.12)
	I_{SN}	normal saturation current for Ebers-Moll model (see Fig. 6.12)
	M	ratio of lifetimes in two collector lumps in quasi-saturation (see Eqn. 6.26)
	n_I	inverse injection index for Ebers-Moll model (see Fig. 6.12)
	n_N	normal injection index for Ebers-Moll model (see Fig. 6.12)
	R_c	collector resistance in series with Ebers-Moll model (see Fig. 6.14)
	τ_I	inverse 'transit time' for Ebers-Moll model (see Fig. 6.12)

	τ_N	normal 'transit time' for Ebers-Moll model (see Fig. 6.12)
6.3.2	Δt	integration step size
6.3.3.2	f_{T_b}	neutral base cut-off frequency (see pg. 202)
6.4.1	q'_{10}	equilibrium charge in first lump of extrinsic collector diode (see Table 6.13)
	q'_{20}	equilibrium charge in second lump of extrinsic collector diode (see Table 6.13)
6.4.1.1	I'_{SD}	saturation current of extrinsic collector diode (see Fig. 6.23)
6.4.2.1	α'_I	common base d.c. current gain in inverse mode for low level saturation Ebers-Moll model (see Eqn. 6.64)
	I_{SD}	saturation current of intrinsic collector diode for low-level saturation (see Eqn. 6.61)
	I'_{SI}	inverse saturation current for low level saturation Ebers-Moll model (see Eqn. 6.61)

CHAPTER 1

INTRODUCTION

The purpose of the research is to develop device models suitable for the characterization of VHF and microwave power transistors when operating in large-signal and switching modes (for example, class C and class D amplifiers, and frequency multipliers). Particular attention is paid to operation of the collector region under high current and charge storage conditions (that is, saturation and quasi-saturation operation). Temperature-dependent effects are also considered.

A two-dimensional analysis of the structures studied is not sought, but rather a lumped model of minimum adequate complexity for use in existing computer-aided circuit analysis programs. Satisfactory models in these fields are not yet available, due to inadequate modelling of effects such as quasi-saturation and saturation of the collector junction, dominant in many applications of modern power devices.

The basis of the proposed model is similar in form to the transport models of Gummel and Poon⁽¹⁾ and Logan⁽²⁾. Difficulties in Gummel's 'base push-out model' for switching out of forward injection at the collector are avoided by the use of a two-lump collector model. Detailed modelling of this

quasi-saturation effect, where complex conditions obtain, is at the heart of the work carried out.

For modelling purposes, the devices studied may be considered as divided into two regions: (a) an 'intrinsic'* area under the emitter and (b) an 'extrinsic' region comprising the remainder of the device. The work described then divides into three parts (presented in the order listed): modelling of intrinsic collector effects considered dominant (Chapters 2 and 3); derivation of the 'sub-models' of the remainder of the device, namely the intrinsic base and extrinsic regions (Chapter 4); and combination of the sub-models to form 'total' device models for the three pertinent conditions of operation, namely the quasi-saturation, saturation, and active modes.

Verification of these proposed models is established at each stage by correspondence with 'accepted' device physics, comparison with detailed simulations of device behaviour^(3,4) and by the model's ability to explain measured terminal behaviour of a variety of transistor structures in a number of circuit environments. In Chapter 5, the implementation of the proposed models is considered, and methods are discussed, in Chapter 6,

* This definition of intrinsic is different to that in other literature, i.e. here intrinsic means a projection of the emitter cross-section through the entire device, not just the neutral base region.

for the evaluation of model parameters, suggestions being made for more detailed work in this area. Finally, conclusions are drawn on the basis of certain criteria for a good model, and suggestions for further research are made.

CHAPTER 2

PHYSICAL MODELS OF THE INTRINSIC COLLECTOR REGION

2.1 INTRODUCTION

A description of the devices studied is given in Section 2.2, together with relevant d.c. and transient characteristics typifying the modelling problems involved. From these structures and previous investigations⁽⁵⁾, the complete device is divided into 'sub-devices' or 'regions', each sub-device being identified with major physical effects. The scheme for derivation of the 'total' device circuit models is then described, based upon this semi-intuitive lumping of the transistor regions.

As mentioned in the introductory Chapter, a thesis of this proposal is that intrinsic high current level collector region effects play a dominant role in the behaviour of these modernpower devices, when they operate with a low collector-emitter voltage, owing to saturation or quasi-saturation of the device. The remainder of this Chapter is devoted to a discussion of the physical conditions in the intrinsic collector. In Section 2.3, a set of postulates, describing, in qualitative terms, conditions in the three modes of operation, is drawn up. The set is arrived at on consideration of the one-dimensional simulation studies of Treen⁽³⁾ and Sayeed⁽⁴⁾, and others (Gummel et al⁽⁶⁾, for example). From these postu-

lates, detailed theoretical studies and observed terminal characteristics, physical models of the device in the quasi-saturation, saturation, and active modes can be defined quantitatively (Sections 2.4-2.6, respectively).

2.2 DESCRIPTION OF DEVICES

The devices under study are epitaxial power transistors with an 'alternating' configuration of emitter and base diffusions, as in Fig. 2.1. The chief design guideline in the modern power transistor is to maximize the emitter periphery to the collector area ratio⁽⁷⁾. This rule of thumb comes from the fact that at high current levels the injection of minority carriers tends to take place around the lateral edge of the emitter-base junction. The problem of the edge crowding effect has been solved. Its remedies include transistors of the overlay type, interdigitated, and mesh emitter types, etc.⁽⁸⁾. These geometries all lead to devices having 'intrinsic' and 'extrinsic' regions of charge storage and are well represented by the structure of Fig. 2.1. Measurements throughout were carried out on transistor types 2N1038A, 2N1060A and 2N3375, of f_T 1.1, 1.5GHz and 400MHz, respectively. Negligible emitter crowding for these structures is seen from the 2N3375 d.c. characteristics for which even at the maximum base current $I_{B \text{ max}}$, the ratio of current densities at the edge of the

FIG. 2.1. Structure of typical device.

emitter to that at the centre, $J_C \text{ MAX} / J_C \text{ CENTRE}$ is only 1.2⁽⁹⁾. Quasi-saturation effects come into play at much lower current levels. Thus, a one-dimensional flow of current in the intrinsic region of the device between emitter and collector would appear justified on the basis of the above argument. Two-dimensional effects are considered in the modelling of extrinsic regions. Very recent devices are capable of over 5W output power at 4GHz⁽⁷⁾ but even these latest devices are characterized by an 'extensive' epitaxial collector region, typified by the double-diffused doping profile of Fig. 2.2. Ion implantation techniques⁽⁷⁾ may alter the base profile somewhat; however, the epitaxial region and its dominating effect on device characteristics is unchanged.

Output d.c. characteristics for devices of these types in Fig. 2.3 clearly show the deviations from $I_C \sim V_{CE}$ curves predicted by the Ebers-Moll model⁽¹⁰⁾, and indicate the causes of these observed discrepancies (as identified and modelled in this thesis). Fig. 2.4 shows the large-signal sinusoidal response and again shows effects not explained by the simple model: viz. reduced amplitude and additional delay of the I_C waveform.

From consideration of the device structure, Fig. 2.1, and previous investigations⁽⁵⁾, it appears reasonable to represent the transistor by a combination of models of the following regions of the device:

FIG. 2.2. Typical doping profile.

FIG. 2.3. D.C. characteristics showing deviation from Ebers-Moll predictions.

FIG. 2.4. Measured large-signal sinusoidal drive response waveform compared with that predicted by the Ebers-Moll model.

- i) the intrinsic device, that is the npn transistor structure under the emitter area A_E
- ii) emitter sidewall injection region
- iii) the extrinsic base region. This consists of the total base region excluding that in i). It is charged by collector injection and also to a lesser extent by injection from the emitter sides into the base region between emitters, ii)
- iv) the extensive collector epitaxial storage region, excluding that in i).

Except by very detailed investigation, it is unlikely that 'sub-models' i) and ii) can be distinguished by terminal performance tests. In addition, for the structures concerned, the extrinsic base storage region is charged via ii) much less than by collector injection for a forward biased collector junction. For these reasons, it may be demonstrated⁽⁵⁾ that sub-model ii) may be combined into the intrinsic region model (intrinsic base).

It would thus appear reasonable to divide the modelling problem into two parts, intrinsic device and extrinsic (base and collector) regions. To an extent, the work carried out is focussed upon effects occurring in the intrinsic collector (considered of major importance) and this aspect is examined in depth before the sub-models are incorporated in models for

the complete transistor. The scheme for the derivation and testing of the model, Fig. 2.5, then involves its implementation for purposes of computer circuit analysis, and the demonstration of the model's capability to characterize device behaviour under d.c., step transient, and large-signal sinusoidal drive conditions.

2.3 POSTULATES FOR PHYSICAL MODEL

As mentioned in Section 2.1, the physical model for the intrinsic device is based upon conclusions (postulates) drawn from simulation studies⁽³⁾, the computer acting as a probe for observing behaviour at physically inaccessible locations.

These postulates, deduced in part from the results of the simulation study summarized in Figs. 2.6 to 2.8, are:

- (1) The collector model is required to represent the transistor under three distinct conditions of operation:
 - a) 'Normal' terminal biasing; with both the emitter-base terminals and junction forward biased; and the collector-base terminals and junction reverse biased.
 - b) 'Normal' terminal biasing; as in (a) for both emitter-base and collector-base terminals; however, with the collector junction forward biased. This condition obtains at sufficiently

FIG. 2.5. Scheme for derivation and testing of model.

FIG. 2.6a. Doping profile used in simulation studies.

FIG. 2.6b. Hole concentration as a function of distance.

FIG. 2.7. Electron concentration as a function of distance.

large current levels due to a voltage drop in the collector epitaxial region, and is known as quasi-saturation. It has been noted in the literature, e.g. refs. (11)-(13) and is postulated here to be a major cause of the distinct deviation from calculated Ebers-Moll characteristics observed for these devices, shown in Figs. 2.3 and 2.4, Section 2.2.

c) 'Saturation' terminal biasing with emitter and collector terminal and junction voltages forward biased.

- (2) For conditions 1(b) and 1(c), injection of minority carriers may take place into a large portion of the epitaxial layer, requiring, in theory at least, a distributed representation.

Conclusions (1) and (2) are evident from the distribution of 'minority' (hole) carrier concentration as a function of distance through the epitaxial layer for normal and saturation terminal biasing and over a range of current levels, shown in Fig. 2.6.

- (3) Figs. 2.6 and 2.7 also show that with an n^- collector the ratio of the hole concentration at the collector junction boundary to the epitaxial doping concentration is several orders of magnitude greater than

unity when there is any significant effect of saturation or quasi-saturation. Furthermore, the excess charge of holes stored in the collector is generally much larger than the excess charge in the base due to forward injection. The condition $P/N_{\text{epi}} > 1$ may be used to denote when saturation and quasi-saturation effects are significant, implying that the density of electrons rises substantially ($n = N_{\text{epi}} + P'$) in at least part of the epitaxial collector region, whilst the majority (hole) density in the base remains essentially at its low level value even after the onset of these high level effects in the collector.

- (4) As mentioned, in quasi-saturation, the collector junction is forward biased and a region of the collector epitaxial layer, charged by minority carrier injection, develops adjacent to the junction, with the following conditions:

- (a) very small, or possibly negligible voltage drop across the injection region. This is seen from the plot of normalized electrostatic potential as a function of distance, for various normal active bias conditions, Fig. 2.8. This approximation is considered further in the development

of a quantitative model for quasi-saturation,
Section 2.4.

- (b) the 'quasi-charge neutral' injection region expands towards the n^+ layer as collector current increases, and the 'ohmic' part of the epitaxial region contracts correspondingly.
 - (c) From Fig. 2.7, electron concentration as a function of distance for various emitter bias voltages, it is observed that negligible conductivity modulation of the ohmic part of the epitaxial region occurs.
 - (d) The excess hole charge in the injection region is 'confined' by the strong electric field that develops in the adjacent ohmic region.
- (5) Under saturation conditions, the injection region occupies the entire epitaxial region and is approximately charge neutral. Both p-n and n- n^+ junctions are forward biased (Fig. 2.8), and throughout the region $p, n \gg N_{\text{epi}}$.
- (6) As the collector current is increased under 'normal' bias condition 1(a), with constant collector-base terminal voltage, the collector junction voltage is developed increasingly across the bulk (approximately charge neutral), epitaxial region until, at I_{C1} say,

the collector junction is zero biased. Over this range of currents;

- (a) negligible conductivity modulation of the collector epitaxial region occurs
- (b) the Kirk effect⁽¹⁴⁾ due to mobile charge in the collector space charge region may occur at the larger currents, causing a movement of the base edge of the depletion layer towards the interface and a movement of the collector boundary of the space charge layer further into the collector region⁽¹³⁾. For currents greater than I_{C1} , minority carrier storage in the collector occurs, described by Kirk as an extension of the 'neutral' base region. Kirk's description, taking the 'effective base width' W' as $W + x_i$ (where W is the 'true' neutral base width and x_i the collector injection region width), is a useful way of seeing that, since $x_i \gg W$ in general, the stored charge and device transit time rise dramatically in quasi-saturation and saturation. However, some authors have been misled into making the error of using the 'effective base width' W' to calculate device transit time, by analogy with $W^2/2D$, the forward 'true' base transit

time. The fallacy is that the variation of W' with collector current cannot be neglected due to the movement of the collector injection region boundary x_1 . Using W' may underestimate the fall in f_T by a factor of 5-10⁽¹²⁾. In the work reported, the large carrier concentrations in the collector are described in terms of the forward voltage applied at the collector junction rather than as 'base' carrier densities, since in saturation and quasi-saturation, there is still a collector space charge layer (though small) at the collector-base junction⁽¹³⁾, as with any forward biased junction. The voltage drop across this layer is correspondingly small, implying a forward biased applied junction voltage approximately equal to the contact potential, causing the large increase in minority carrier concentration in the collector from its equilibrium value.

2.4 QUANTITATIVE DESCRIPTION FOR QUASI-SATURATION

The injection of excess holes into the collector region must be accounted for in a circuit model for quasi-saturation operation. The one-dimensional computer simulations give very

detailed information regarding carrier distributions, but since this detail is not required in an equivalent circuit, carrier distributions in the collector region may be approximated using theory developed in this section, based upon the postulates of Section 2.3. The approximate distribution of p and n is shown in Fig. 2.9. From postulate (3), $p/N_{\text{epi}} > 1$ when 'light' saturation effects are significant implying that the density of electrons rises substantially ($n = N_{\text{epi}} + p'$) in part of the collector. The division of the collector into an injection and ohmic region provides a means of quantifying the effect of quasi-saturation, that is, of finding the charge stored in the collector region needed for a charge-controlled representation of this modulated part of the collector.

The conditions valid for operation in this mode have been discussed in Section 2.3 and are outlined briefly here in Fig. 2.9. Due to the 'potential well' existing in the epitaxial region, the 'minority' carriers are constrained so that effectively the boundary between the injection and ohmic regions is 'open-circuited' to holes. Thus, at this boundary $p(x_i) = N_{\text{epi}}$, we certainly have the condition

$$J_p = 0 = q \mu_p p E - q D_p \frac{dp}{dx} \quad (2.1)$$

For the device in the d.c. steady state, we have in the epi region

INJECTION REGION

Conclusions

- Injection Region -
1. 'Neutral' $p-n+N_{epi} \approx 0$
 2. Approximately constant potential
 3. High level injection $n=p \gg N_{epi}$
 4. Holes enter and leave injection region via base
 5. Electrons enter injection region via base
leave injection region via base or collector
- Ohmic Region -
1. $V_{CB} - V_O = I_C r_C$
 2. Negligible conductivity modulation
- Space Charge Region -
1. Junction forward biased, i.e. $V_{JC} > 0$

FIG. 2.9. Conclusions from considerations for quasi-saturation.

$$J = J_n + J_p = \text{constant} \quad (2.2)$$

At $x=x_i$, $J=J_n$ and due to the fact that hole-electron recombination is small in the epitaxial layer ($I_C \gg I_B$), J_n will remain approximately constant over this region. Thus, the hole current J_p will remain very small in comparison to the electron current and we may write:

$$J_p = 0 = q \mu_p p \xi - q D_p \frac{dp}{dx} \quad (2.3)$$

or

$$\xi = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \quad (2.4)$$

On substituting into the drift-diffusion equation for electrons, we have:

$$J_n = q D_n \left(1 + \frac{n}{p} \right) \frac{dn}{dx} \quad (2.5)$$

A 'quasi-neutral transport theory' is assumed for the collector injection region (postulate of 4(b)). This means that the small space charge (small in comparison with the impurity concentration charge) is neglected in spite of the fact that an electric field exists in the 'neutral' region. This implies:

$$p - n + N_{epi} = 0 \quad (2.6)$$

which with the high level injection condition yields

$$p \approx n. \quad (2.7)$$

On substitution in Equation (2.5), we obtain an effective doubling of the electron diffusion constant

$$J_n \approx 2q D_n \frac{dn}{dx} \quad (2.8)$$

and the slope $\left(\frac{dn}{dx}\right)$ of the triangular excess electron distribution is directly related to the electron current which is, approximately, the collector current. To find the charge stored in the collector region, it is now necessary to find the boundary conditions for the injection region.

In quasi-saturation, the collector-base junction is forward biased and the collector space charge layer boundary is assumed to remain fixed in position compared with the movement of the other boundary of the injection region which can vary from $x_i=0$ to $x_i=x_e$. This assumption is borne out by the detailed analysis of Treen.⁽³⁾ It seems reasonable to define the injection-ohmic region boundary as the value of x at which $p=N_{epi}$ because to the left of this boundary, Equation (2.8) is a fair approximation and to the right the injected carrier density can be neglected.

The voltage drop between the boundary of ohmic and charge storage regions of the collector ($x=x_i$) and the base contact with respect to the equilibrium condition is given by the sum of the applied collector junction voltage V_{JC} and the

integral of the electric field (2.4) over the injection region width, as indicated in Fig. 2.9. We have:

$$V_{x_i b} = -V_{JC} + \left\{ - \int_0^{x_i} \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} dx \right\} \quad (2.9)$$

or

$$V_{x_i b} = -V_{JC} - \frac{kT}{q} \ln \frac{p(x_i)}{p(0)} \quad (2.10)$$

or

$$V_{x_i b} = -V_{JC} + \frac{kT}{q} \ln \frac{p(0)}{p_{no}} - \frac{p_{no}}{p(x_i)} \quad (2.11)$$

where p_{no} is the equilibrium hole density, n_i^2/N_{epi} , throughout the epi layer. Assuming low-level conditions in the base, we may relate $p(0)$ to V_{JC} using the well-known junction law:

$$V_{JC} = \frac{kT}{q} \ln \frac{p(0)}{p_{no}} \quad (2.12)$$

By writing the logarithm of products in Equation (2.11) as the sum of individual logarithmic terms, we have:

$$V_{x_i b} = -V_{JC} + \frac{kT}{q} \ln \frac{p(0)}{p_{no}} - \frac{kT}{q} \ln \frac{p(x_i)}{p_{no}} \quad (2.13)$$

By definition of the injection region, we have:

$$p(x_i) = N_{epi} \quad (2.14)$$

and substituting also for $p(0)$ and p_{no} in Equation (2.13) gives:

$$\begin{aligned} V_{x_i b} &= -V_{JC} + V_{JC} - \frac{kT}{q} \ln \left(\frac{N_{epi}}{n_i} \right)^2 \\ &= - \frac{kT}{q} \ln \left(\frac{N_{epi}}{n_i} \right)^2 = V_0 \end{aligned} \quad (2.15)$$

$V_{x_i b}$, the voltage at the boundary x_i relative to the voltage at b , is thus seen to remain a constant value, V_0 (of the order of V_{JC}) irrespective of terminal conditions of voltage and current, provided the device is operating in quasi-saturation. This result may be compared with the computer analysis simulation studies of Treen.^(ref) From Equation (2.15), knowing N_{epi} for the device simulated, we may calculate V_0 .

$$V_0 \text{ (calc.)} = -0.66V \quad (2.16)$$

From the simulation results of Fig. 2.8 for the electrostatic potential as a function of distance, the edge of the injection region (ohmic boundary) is observed to remain at an approximately constant potential with respect to the base contact for increasing current levels in quasi-saturation, that is for x_i varying across the epitaxial region. This constant forward voltage is estimated from the curves to be $-0.64 \pm 0.03V$, in excellent agreement with the theoretical value, $-0.66V$.

$\pm \frac{1}{2}$ order of magnitude

Fig. 2.9 summarizes the physical conditions that exist in the device for quasi-saturation operation. The collector-base junction is forward biased and injecting into both base and collector, but only a part of the collector n^- epitaxial region, adjacent to the collector base junction acts as a charge storage region; the remainder is ohmic of width $x_e - x_i$, and supports a voltage drop $(V_{cb} - V_{x_i b})$ due to the flow of I_C . We have, by Ohm's law:

$$I_C = \frac{V_{cb} - V_o}{x_e - x_i} q \mu_n N_{epi} \cdot A_E \quad (2.17)$$

The modelling problem, in quasi-saturation, may be considered as the simultaneous solution of this equation and that for the current crossing the injection region, (2.8), for the width x_i

$$I_C = 2q D_n \frac{dn}{dx} \cdot A_E \quad (2.8)$$

of the injection region.

In Table 2.1, a comparison of computed collector current values from the work of Treen⁽³⁾ and Gummel⁽⁶⁾ and theoretical values based upon Equation (2.17) is made. The agreement is considered good in view of the several sources of error in choosing boundary values from the curves.

For constant V_{cb} in Equation (2.17), increasing the collector current will cause x_i to move towards the substrate

TABLE 2.1. Comparison of Collector Currents from Simulation Studies and Theory of Section 2.4

DEVICE	EPI LAYER RESISTIVITY ρ Ω -cms	OHMIC REGION WIDTH $x_e - x_i \times 10^{-4}$ cms	V_0 (CALCULATED) V	V_0 (MEASURED*) V	COLLECTOR CURRENT (CALCULATED) J_c $\times 10^3$ A/cm ²	COLLECTOR CURRENT (SIMULATED) J_c $\times 10^3$ A/cm ²
ref (3)	1.3	2.7	-0.66	-0.64	4.72	4.69
ref (6)	5.5	3.6	-0.57	not available	0.80	0.70

25°C * from voltage drop across ohmic region $V_{CB} - V_0$ of Fig. 2.8.

boundary x_e (as mentioned in postulate 4(b), Section 2.3). It is also noted that V_{cb} may take a negative (forward biased) value without invalidating Equation (2.17). Thus, the terminals of the device may be forward biased and the device still remain in quasi-saturation, provided the forward bias on the terminals does not exceed V_0 , at which point ~~the~~ injection region occupies the entire epitaxial region x_e and full saturation commences. ^{current reverses} The theoretical developments of this section modify somewhat the definitions for the quasi-saturation and saturation regions of operation, postulates 1(b) and (c), Section 2.3. These modifications are discussed further in the next section.

It should be noted that negligible conductivity modulation of the ohmic region of the device is assumed throughout the work reported, based upon postulate 4(c). If, for certain device types, conductivity modulation did occur, it would be possible to develop similar equations to those of this section with, however, a current dependent mobility term in Equation (2.17). In spite of the increased complexity of the coupling equations, the method of solution, described in Sections 3.2 and 5.2 would still be applicable. *more than that!*

2.5 QUANTITATIVE DESCRIPTION FOR HIGH LEVEL SATURATION

The conditions in high level saturation have been discussed in Section 2.3 and are summarized here in Fig. 2.10. In full saturation (that is, for $V_{cb} < V_0$ (both negative)), there is

Injection Region - Holes enter and leave collector epi region via base.
Electrons enter and leave epi injection region via base or collector.

$$p(x_e) = n(x_e) = N_{epi} \exp q(V_o - V_{cb})/2kT$$

$$V_o < V_{cb}$$

FIG. 2.10. Electron density distribution for high level saturation of the collector.

no problem in defining the boundary of the injection region since this region is seen to fill the entire epitaxial width. However, to quantify the charge in this width, it is necessary to find the hole concentration $p(x_e)$ at this boundary with the n^+ buried layer.

When an n^+-n junction is forward biased so that the n^+ section is negative with respect to the epitaxial region, holes will be drawn towards it, but the hole current will be unable to flow in the n^+ region unless there is a large excess hole density to enable current to flow by diffusion (the field in the high conductivity n^+ layer will be very small). It may be shown from high level theory that the hole concentration on the substrate side of the n^+-n junction, p_+ is related to the electron concentration on the epitaxial side n_{epi} by

$$p_+ = \frac{n_{epi}^2}{N_+} \ll n_{epi} \quad (2.18)$$

where N_+ is the donor doping concentration in the substrate. By analogy to the high injection efficiency at the emitter junction, the hole current will be very small when N_+ is large and the $n-n^+$ junction thus acts as a 'contact' preventing significant minority carrier flow in the n^+ region. Hole current in the epitaxial region thus supports only bulk recombination in this region, which is small for the doping densities considered.

The hole current I_p in the collector may thus be assumed small (in comparison with I_n). This conclusion is supported by the fact that I_p is the current supplied via the base lead to sustain recombination in the collector. It is, at most, equal to the total base current less any current necessary for base recombination and other 'base defect' currents, e.g. emitter depletion layer recombination current, associated with the emitter-base junction. From the typical d.c. characteristics of Fig. 2.3, Section 2.2, except for very close to the origin, the characteristics to be modelled will lie in the region where $I_C \gg I_B$. If we concern ourselves in this high current level region, we may consider $I_p \approx 0$ and a similar expression to Equation (2.4) to hold for the field in the injection region, namely

$$\xi = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \quad (2.4)$$

In an analogous fashion to that for quasi-saturation, the voltage drop across the injection region (i.e. the potential at x_e with respect to the injection boundary) is given by:

$$V_{ox_e} = - \int_0^{x_e} \xi \, dx = \frac{kT}{q} \ln \frac{p(0)}{p(x_e)} \quad (2.19)$$

As before, using the low level (in base) injection law, we obtain:

$$V_{ox_e} = \frac{kT}{q} \ln \frac{p(0)}{p_{no}} \frac{p_{no}}{p(x_e)} = V_{JC} + \frac{kT}{q} \ln \frac{p_{no}}{p(x_e)} \quad (2.20)$$

For high level injection in the collector (postulate 5, Section 2.3), charge neutrality requires that, for $0 \leq x \leq x_e$,

$$p(x) \simeq n(x) \quad (2.21)$$

Substituting in Equation (2.20) gives:

$$V_{ox_e} = V_{JC} - \frac{kT}{q} \ln \frac{n(x_e)}{n_i^2} N_{epi} \quad (2.22)$$

For electron injection across the epi-substrate boundary, we may write:

$$n(x_e) = N_{epi} \exp \frac{q V_{x_e c}}{kT} \quad (2.23)$$

analogous to the injection law at the epi-base boundary.

Combining Equations (2.22) and (2.23) gives:

$$V_{ox_e} = V_{JC} - \frac{kT}{q} \ln \left\{ \frac{N_{epi}^2}{n_i^2} \exp \frac{q V_{x_e c}}{kT} \right\} \quad (2.24)$$

or

$$V_{ox_e} = V_{JC} - V_{x_e c} - \frac{kT}{q} \ln \left(\frac{N_{epi}}{n_i} \right)^2 = V_{JC} - V_{x_e c} + V_0 \quad (2.25)$$

Summing the voltage drops across the three regions of the collector gives:

$$V_{cb} = -V_{JC} - V_{x_e c} + V_{ox_e} \quad (2.26)$$

V_{ox_e} may be eliminated, using Equations (2.25) and (2.26), and we find:

$$V_{JC} - V_{x_e c} + V_0 = V_{cb} + V_{JC} + V_{x_e c} \quad (2.27)$$

or

$$V_{x_e c} = \frac{V_0 - V_{cb}}{2} \quad (2.28)$$

From Equation (2.23), we have:

$$p(x_e) = n(x_e) = N_{epi} \exp \frac{q(V_0 - V_{cb})}{2kT} \quad (2.29)$$

Insufficient information is available in the saturation results of Sayeed⁽⁴⁾ to 'verify' the above theoretical development, in particular Equation (2.29). However, it is noted that, from this equation $V_{cb} = V_0$ is the boundary for saturation operation since throughout the epitaxial region $p(x)$ must be larger than N_{epi} , satisfied when V_0 is larger than V_{cb} . This onset of saturation operation is in agreement with the limit for operation in quasi-saturation detailed in the previous section.

For this mode of operation, as mentioned, no scheme for solution of instantaneous injection region width is necessary, and the electron current through the region is given by the gradient $(n(0)-n(x_e))/x_e$, Fig. 2.10, together with a doubling of the diffusion constant to $2D_n$.

2.6 QUALITATIVE DESCRIPTION FOR THE ACTIVE MODE

The physical model used for this region of operation is shown in Fig. 2.11. As before, it is built largely upon the postulates of Section 2.3 (postulates 1(a), 6 in particular). There is seen to be a 'coupling' relationship between the semi-width of the collector space-charge layer and the width of the epitaxial quasi-neutral region, consistent with the lengths of the two regions (the sum of which is constant in collector material); that is:

$$x_{DC}(V_{JC}, I_C) + \frac{V_{CB} + V_{JC}}{I_C} q\mu_n N_{epi} \cdot A_E = x_e \quad (2.30)$$

where x_{DC} is the space-charge region width in collector material, corresponding to junction voltage V_{JC} and V_{CB} , the potential applied at the collector terminal (with respect to the base region), is shared between the ohmic part of the n-epitaxial region and the space-charge region of the reverse-biased collector junction.

$$x_{DC}(I_C, V_{JC}) + \frac{V_{CB} + V_{JC}}{q \mu_n N_{epi} I_C} A_E = x_e$$

Kirk effect accommodated via $x_{dc}(I_C)$
 No conductivity modulation of $x_e - x_{dc}$ region
 Current carried by electrons
 $V_{CB} > 0, V_{JC} < 0$

FIG. 2.11. Conditions of operation in the active mode.

Careful modelling of the collector space-charge width dependency on junction voltage and collector current (Kirk effect) is necessary in order to express this coupling relationship adequately. It is seen, from Equation (2.30), that for a constant V_{CB} and increasing I_C , the reverse bias value of V_{JC} will decrease. This causes a shrinking of the space-charge layer and a widening of the ohmic region. This is counteracted to a small extent by the increase in x_{DC} with increasing collector current.

In the next Chapter, circuit models are derived, based upon these physical models for the collector region.

CHAPTER 3

CIRCUIT MODELS OF THE INTRINSIC COLLECTOR REGION

3.1 INTRODUCTION

The process of establishing the collector model is, at this stage, one of solving the set of equations describing the physical models of Chapter 2. The circuit concept and its analysis techniques provide strong tools in the analysis of transistor models. Transistor circuit models are composed of three parts:

- (i) a circuit of discrete components loosely called an equivalent circuit (the topology of which is dependent in part upon the physical model from which it is derived);
- (ii) a set of controlling equations which determines the values of the equivalent circuit elements, and
- (iii) a set of device parameters which represent the coefficients in the controlling equations.

These equations must be in a form compatible with the digital computer language, making the models useful in analysis and design of electronic circuits containing these devices.

To obtain models which describe the transient performance of the device use is made of the steady-state properties developed in Chapter 2. In using this knowledge, the approximation is made that conditions applying in the steady state apply also, perhaps in 'suitably extrapolated forms', during transients. The motivation for such an approximation is that many aspects of steady state behaviour are much easier to determine than their transient counterparts. The resulting models will match steady state conditions; how well they match transient performance depends on how rapidly transient disturbances relax towards steady state conditions. The duration of these disturbances should be at least an order of magnitude larger than t_r , the 'charge relaxation time' of a base region of width W and minority carrier diffusion constant D , where

$$t_r \approx 0.2 \times W^2/2D$$

Thus, for a model based upon the 'slow variation of steady state' to be valid, frequencies should be of the order of f_T (internal) or less⁽¹⁶⁾, where the base transit time $W^2/2D$ is given by

$$\frac{1}{2\pi f_{T \text{ int}}} = W^2/2D.$$

As noted, the three forms of circuit model, for quasi-saturation, high level saturation, and active operation will depend largely upon their respective physical models. In particular, the topology will be based upon the division of the intrinsic collector into space charge, injection, and ohmic regions.

In this Chapter, these three circuit models are derived, and a discussion of a low-level saturation model is included in Section 3.3 for completeness.

3.2 INTRINSIC COLLECTOR REGION MODEL FOR QUASI-SATURATION

To make a detailed analysis of the large-signal behaviour of the collector region, it is, as mentioned in Section 2.3, necessary to use at least a partially distributed equivalent circuit. However, this approach leads to some problems of analysis, and before considering how to tackle them, it should be decided what effects are expected. We may thus obtain some feel for the minimum complexity of circuit required to account, in an 'accurate' fashion, for the observed behaviour. A physical model for quasi-saturation has been given in some detail in Section 2.4 dividing the collector region into injection and ohmic parts. Since the unmodulated region of the collector has 'uniform' properties across its width (the conduction of majority (electrons) current being ohmic), a one-lump description

of this region is possible. That is, the properties of the device in this region are assumed constant, and may be modelled as in Fig. 3.1b.

Injection into the modulated region requires, in theory at least, a many-lump representation due to the large extent of this region of varying carrier density. This representation, however, may give greater detail of carrier distributions than is required, if we are concerned primarily with terminal behaviour. Parameter evaluation and analysis of the model will be prohibitively complex, and it may be stated that the smallest number of lumps that allows a 'sufficiently accurate' description of the terminal behaviour should be used.

A charge-controlled model, Fig. 3.1b, for the injection region is proposed since the injection of charge into the collector has been postulated as a dominant feature for the devices under study. A charge-control type collector model is also attractive because computer simulations give detailed information regarding carrier distributions for derivation and validation of the model. In this model, the stored 'charge' associated with each lump is the independent variable, currents in the model being proportional to these charges.

The large-signal dynamic model of the collector adopted here involves the representation of the stored charge in the injection region by means of a two-lump model. The charge stores

(a) Approximate steady state distribution used for quasi-saturation analysis.

(b) Collector region model for quasi-saturation.

FIG. 3.1. Charge distribution and model for collector region in quasi-saturation.

involved are respectively adjacent to the collector-base junction and the injection boundary at x_1 . The need for modelling of remote charge storage in the collector has been demonstrated by collector device tests⁽⁵⁾ in which the fall transient after completion of storage delay was considerably slower than predicted on the basis of charging of collector depletion capacitance, using a one-lump collector model. A two-lump approximation is chosen to describe the carrier distribution due to past success in describing diode and collector diode responses^(5,16,17), for example.

The division of the charge into its components is shown in Fig. 3.1a. It is seen that the total charge is conserved by the lumped approximation. That is, the charge q_{COL} , of triangular distribution, is equal to that in the two-lump model.

$$q_{COL} = q_1 + q_2 \quad (3.1)$$

The centre of the charge sections is chosen as the reference for the carrier density in each lump. This will lead to some inevitable complication in describing boundary conditions, but is essential to prevent zero charge q_2 in the second lump (when $x=0$ and $x=x_1$ are used as the carrier density reference points and $K=1$). With this lumping approximation, charge is maintained correct in each region of width x_1 , x_2 and, as will be shown in this section, the junction voltage V_{JC} is now a function of only

that part of the total charge which is close to the junction. The model will then simulate 'recovery tails' and permit the simulation of variations in recombination time along the x -axis, if necessary. The ratio of the widths of the lumps $K=x_1/x_2$ remains a constant in this model and its value will depend on the relative quantity of charge stored close to and far from the collector junction. Its value is thus dependent on device structure but for modelling from terminal characteristics the value $K=1$ may typically be assigned to it, or it may be deduced from terminal measurements (see Section 6.4).

From Fig. 3.1a, based upon the physical model of Section 2.3, three mechanisms of dynamic charge flow are possible:

- (a) through the junction boundary, into (and out of) the adjacent charge store;
- (b) through the boundary at x_1 , into (and out of) the adjacent charge store;
- (c) between the two stores.

Fig. 3.1b shows the current elements modelling the mechanisms of flow (a), (b) and (c). 'Charge injection' via (a) is modelled by the 'current continuity' boundary condition set up by the junction, namely that electron and hole currents, reaching the depletion layer from the base, both flow into (and out of) charge lump q_1 , except for depletion layer charging current $C_{JC} dV_{JC}/dt$. Current transport between charge stores

is by 'diffusion' (with an effective doubling of the diffusion constant), and its instantaneous value is given from the geometry of Fig. 3.1a as:

$$I_{\text{trans}} = A_E \frac{n_1' - n_2'}{x_i/2} q 2 D_n \quad (3.2)$$

where A_E is the emitter cross-sectional area defined in Section 2.2. Now x_i is divided into x_1, x_2 where

$$\frac{x_1}{x_2} = K \quad (3.3)$$

We have, then:

$$\frac{x_1}{x_i} = \frac{K}{1+K} \quad (3.4)$$

$$\frac{x_2}{x_i} = \frac{1}{1+K} \quad (3.5)$$

and the charges contained in the lumps will be given by:

$$q_1 = x_1 n_1' q A_E \quad (3.6)$$

$$q_2 = x_2 n_2' q A_E \quad (3.7)$$

The transport current, representing charge redistribution within the injection region, thus becomes:

$$I_{\text{trans}} = \frac{1+K}{x_i} \frac{2}{x_i} \left(\frac{q_1}{K} - q_2 \right) 2D_n = (q_1 - K q_2) / \tau_3 \quad (3.8)$$

where

$$\tau_3 = \frac{K}{1+K} \frac{x_i^2}{4D_n} \quad (3.9)$$

Due to the boundary condition $n'(x_i)=0$, we may readily evaluate the current transported out of the second store into the ohmic region. This is given by:

$$I = A_E \frac{n_2' - 0}{x_2/2} q_2 D_n \quad (3.10)$$

or

$$I = \frac{q_2(1+K)}{x_i} \frac{2(1+K)}{x_i} D_n = \frac{q_2}{\tau_4} \quad (3.11)$$

where

$$\tau_4 = \tau_3 / K(1+K) \quad (3.12)$$

Except in the steady state, Fig. 3.1a, the current transported between the two stores and that flowing into the ohmic region will not be the same, allowing for the charging or discharging of q_2 . Flow mechanism (b) also includes a charging effect analogous to basewidth modulation in the base of the transistor, in addition to the transport term q_2/τ_4 through the boundary at x_i . The charging current $C_{uc} dV_{cb}/dt$ represents the charge swept out of the injection region by an incremental change in injection region width x_i , and contributes to the total collector

current. Its derivation is considered in detail in the discussion on 'basewidth modulation' in Section 4.2.1.

Current flow between the injection region boundary at x_1 and the N^+ 'contact' at x_e is ohmic and represented by r_c where

$$r_c = \frac{(x_e - x_i)}{q \mu_n N_{epi} A_E} = \frac{x_e}{q \mu_n N_{epi} A_E} (1 - x_i/x_e) \\ = r_{co} (1 - x_i/x_e) \quad (3.13)$$

and r_{co} is the resistance of the total epitaxial width. From the boundary conditions of Section 2.4, we must have:

$$r_c = r_{co} \left(1 - \frac{x_i}{x_e}\right) = \frac{V_{CB} - V_0}{I_C} \quad (3.14)$$

Adding recombination currents to the injection region yields the model of Fig. 3.1b. When recombination is introduced (see Section 3.3 for further discussion), the value of I_{trans} , Equation (3.8), must be modified slightly to allow for the supply of recombination current to the second lump. That is, in the steady state, q_1 must be increased slightly with respect to q_2 giving the excess electron density profile of Fig. 3.2. These modifications are accounted for automatically by the definitions of current terms in Fig. 3.1b. It is noted that 4 parameters are variable, namely, τ_3 , τ_4 , $C_{\mu c}$ and r_c ; and must be specified in terms of x_i . It is also seen that the

FIG. 3.2. Excess electron density profile in quasi-saturation showing the effect of recombination.

usual 'injection' boundary condition relating q_1 to V_{JC} is not used but rather charge q_2 is specified by the voltage at the injection-ohmic regions boundary V_0 , and the terminal conditions V_{CB} , I_C ; that is,

$$I_C = \frac{q_2}{\tau_4} = \frac{V_{CB} - V_0}{r_C} \quad (3.15)$$

in the steady state. The relation between V_{JC} and q_1 is required, however, re base region modelling, and evaluation of C_{JC} , as detailed in Section 5.2.3. From the Boltzmann law relating $p_n(0)$ to V_{JC} , Equation 2.12, Section 2.4

$$V_{JC} = \frac{kT}{q} \ln \left(1 + \frac{p'(0)}{p_{no}} \right) \quad (2.12)$$

and the triangular electron density distribution of Fig. 3.1a

$$\frac{n'(0)}{x_i} = \frac{n_1}{x_{1/2} + x_2} \quad (3.16)$$

it may be shown that

$$V_{JC} = \frac{kT}{q} \ln \left\{ 1 + \frac{q_1}{\frac{q A_E n_i^2}{N_{epi} \frac{K(K+2)}{2(K+1)^2} x_i}} \right\} \quad (3.17)$$

The collector region model will thus involve the simultaneous solution of the current flow equations across the injection and ohmic regions subject to the variable parameter x_i , the width of the injection layer.

3.3 INTRINSIC COLLECTOR REGION MODEL FOR SATURATION

3.3.1 High Level Saturation

Fig. 3.3a shows the approximate electron distribution for high level saturation. As before, a two-lump model is chosen to describe this distribution owing to the inability of the one-lump model to account adequately for observed switching responses⁽⁵⁾. Fig. 3.3a also indicates the division of the excess 'majority' carrier (electron) charge into its components q_1, q_2 in the 'neutral' N-type epitaxial collector region, extending from the boundary of the collector-base junction space charge layer to the N^+ collector region boundary. From Section 2.5, we have $n'(0)$, given by the junction injection law as,

$$n'(0) = \frac{n_i^2}{N_{\text{epi}}} \left\{ \exp \left(\frac{q V_{JC}}{kT} \right) - 1 \right\} \quad (3.18)$$

and at the NN^+ boundary of the collector, at x_e , we have:

$$n'(x_e) = N_{\text{epi}} \left[\exp \left\{ q(V_0 - V_{cb}) / 2kT \right\} - 1 \right] \quad (2.29)$$

Based upon these equations, the outer boundaries in each lump are chosen as the references for the carrier densities in each lump, and K the ratio of the lump widths is taken as 1 to equate $(q_1 + q_2)$ to the actual total stored charge.

(a) Approximation to electron distribution in high level saturation.

(b) Collector region model for high-level saturation.

FIG. 3.3. Charge distribution and model for collector region in high-level saturation.

The current transport element between the charge stores may be evaluated using the geometry of Fig. 3.3a (neglecting recombination), and the effectively doubled diffusion constant.

$$I_n = A_E \left(\frac{n'(0) - n'(x_e)}{x_e} \right) q 2 D_n \quad (3.19)$$

or

$$I_n = \frac{2q A_E D_n}{x_e} \left(\frac{q_1}{q A_E x_1} - \frac{q_2}{q A_E x_2} \right) \quad (3.20)$$

where A_E is the cross-sectional area of the intrinsic region stores, and other terms are defined in the figure.

In a similar fashion to that for quasi-saturation, we have:

$$x_1 = \frac{K}{1+K} x_e \quad (3.21)$$

$$x_2 = \frac{x_e}{1+K} \quad (3.22)$$

Substituting in Equation (3.20), we obtain:

$$I_n = \frac{2q A_E D_n}{x_e} \frac{(1+K)}{q A_E K x_e} \{q_1 - K q_2\} = \frac{q_1 - K q_2}{\tau_3} \quad (3.23)$$

where

$$\tau_3 = \frac{K}{1+K} \frac{x_e^2}{2D_n} \quad (3.24)$$

In practice, the existence of a 'base' current to support recombination in the epitaxial layer will result in a slight curvature of the excess electron density profile of Fig. 3.3a, so that the electron current entering the collector from the base is larger (by an amount equal to the base current component for recombination in the collector) than the collector current. Strictly, the integral of $p'(x) = n'(x)$ from 0 to x_e determines this component of I_B , but for the two-lump approximation used in this analysis, two current components q_1/τ_1 , q_2/τ_2 may model this effect of recombination upon terminal currents, where

$$I_B \text{ (collector component)} = \frac{q_1}{\tau_1} + \frac{q_2}{\tau_2} \quad (3.25)$$

in a similar fashion to that for quasi-saturation (Section 3.2).

Adding these recombination terms gives the model of Fig. 3.3b

where τ_1 , τ_2 are 'bulk lifetimes' in the epi layer. τ_2 also represents the small hole flow from the epitaxial collector into the N^+ substrate (or this can be thought of as represented by a 'surface recombination velocity' at the interface, Section 3.3.2, since all such carriers recombine in the substrate).

It may be shown that for the high levels considered, this description of hole current flow into the substrate by a term q_2/τ_r , where τ_r is a constant, is not strictly correct; but

from the very small magnitude of this current (Section 2.5), the error involved in this representation is negligible.

τ_β is given by Equation (3.24) and q_1, q_2 are obtained from Equations (3.18) and (2.29) as

$$q_1 = q_{10} \left\{ \exp \frac{q V_{JC}}{kT} - 1 \right\} \quad (3.26)$$

where

$$q_{10} = \frac{q A_E K x_e}{1+K} \frac{n_i^2}{N_{epi}} \quad (3.27)$$

and

$$q_2 = q_{20} \left\{ \exp q(V_0 - V_{cb})/2kT - 1 \right\} \quad (3.28)$$

where

$$q_{20} = \frac{q A_E x_e}{1+K} N_{epi} \quad (3.29)$$

It is seen that the model implementation will not involve the simultaneous solution of two current flow equations for x_i as in quasi-saturation since the injection region width is fixed at x_e . The implementation involves the determination of the charge variables q_1, q_2 from boundary conditions.

3.3.2 Low Level Saturation

For 'small' collector currents, less than the base current, the high level saturation model of Sections 2.5 and

to use the criterion for LLS?

3.3.1 may not be applicable. For the low level case, the 'standard' approach of considering injected minority carriers as the charge variable is taken^(5,16), the model being summarized in Figs. 3.4a to 3.4c. In this model, D_n is used instead of the doubled diffusion coefficient of Section 3.3.1. Bulk recombination in the collector is, as for high level saturation, considered using:

$$I_B \text{ (collector component)} = \frac{q_1}{\tau_p} + \frac{q_2}{\tau_p} = \frac{q_{coll}}{\tau_p} \quad (3.30)$$

where τ_p is the bulk lifetime in the epitaxial layer. Lifetime τ_2 varies from τ_p due to the following considerations.

The Boltzmann relationship between the hole density P_+ on the substrate side of the $N_{epi}-N^+$ interface, and the hole density on the lightly doped side, P_{epi} , gives:

$$\frac{P_+}{P_{epi}} = \frac{P_{+,0}}{P_{epi,0}} \exp \frac{q V_{xec}}{kT} \quad (3.31)$$

where V_{xec} is the 'applied' voltage across this interface and the subscript 0 refers to equilibrium values. From the non-equilibrium law of mass action

$$p_+ n_+ = n_{epi} p_{epi} \quad (3.32)$$

and from the assumption of low level conditions (i.e. the excess

EXCESS MINORITY
CARRIER DENSITY

(a) Low-level minority carrier distribution.

ELECTRON CURRENT

(b) Two-lump model for low-level saturation.

(c) Simplified two-lump model for low-level saturation.

FIG. 3.4. Charge distribution and model for collector region in low-level saturation.

$$\frac{1}{\tau_2} = \frac{1}{\tau_r} + \frac{1}{\tau_p}$$

$$\frac{1}{\tau_1} = \frac{1}{\tau_p}$$

$$\tau_3 = \frac{K}{1+K} \frac{x_e^2}{D_n}$$

hole density in the substrate $p'_+ \ll N_+$), we may obtain the expression:

$$\frac{p_+}{p_{epi}} \approx \frac{N_{epi}}{N_+} \approx \frac{p'_+}{p'_{epi}} \quad (3.33)$$

By comparison of Equations (3.31) and (3.33), we see that a negligible 'applied' voltage appears across the $N_{epi}N^+$ junction, i.e.

$$V_{x_{ec}} \approx 0 \quad (3.34)$$

Thus, the entire applied voltage V_{BC} appears across the collector-base space-charge layer

$$V_{JC} = V_{BC}$$

implicit in the model of Fig. 3.4b.

Hole flow into the substrate may be described in terms of a constant recombination velocity V_r at the $N_{epi}-N^+$ interface, on the basis of Equation (3.33) and the following considerations. Minority carrier transport in the substrate is by diffusion (see Section 2.5) and is given by:

$$I_p = q A_E \frac{D_{p^+}}{L_{p^+}} p'_+ \quad (3.35)$$

where D_{p^+}, L_{p^+} are the minority carrier diffusion constant and diffusion length respectively, in the N^+ substrate. Using

Equation (3.33), we obtain for the minority carrier current

$$I_p = q A_E \frac{D_{p+}}{L_{p+}} \frac{N_{epi}}{N_+} p'_{epi} = q A_E V_r p'_{epi} \quad (3.36)$$

Thus, the hole flow at the $N_{epi} - N_+$ boundary is described by means of a recombination velocity term q_2/τ_r where

$$I_p = q A_E V_r p'_{epi} = \frac{q A_E V_r q_2(1+K)}{q A_E x_e} = \frac{q_2}{\tau_r} \quad (3.37)$$

as shown in Fig. 3.4b. This is averaged with recombination in the bulk material of the second store into the usual q_2/τ_2 term, Fig. 3.4c.

Thus, in the low level case, the collector current will be given by a superposition of electron flow (from the intrinsic base region) and a minority carrier recombination current in the two collector lumps, $q_1/\tau_1 + q_2/\tau_2$. From the geometry of Fig. 3.3a, it is easily verified that:

$$\tau_3 = \frac{K}{1+K} \frac{x_e^2}{D_n} = 2\tau_3(\text{high level}) \quad (3.38)$$

$$\frac{1}{\tau_2} = \frac{1}{\tau_p} + \frac{1}{\tau_r} \quad (3.39)$$

where

$$\tau_r = \frac{x_e}{(1+K)V_r} \quad (3.40)$$

in the model of Fig. 3.4c.

3.4 INTRINSIC COLLECTOR REGION MODEL FOR ACTIVE OPERATION

Modelling in this regime of operation follows along conventional lines. The division of the collector into a space charge and ohmic region, Section 2.6, is modelled by a series combination of a junction capacitance and a single lumped resistance. The junction capacitance $C_{JC}(V_{JC}, I_C)$, a non-linear lumped element, describes the variation in space charge layer width related to charging of the collector junction, and is coupled to the series resistance r_C by:

$$x_{DC} + x_{DB} = \frac{A_E \epsilon}{C_{JC}} \quad (3.41)$$

where x_{DB} is the semi-width in base material, and ϵ is the dielectric constant, for epi material. We then have:

$$r_C = (x_e - x_{DC}) \frac{\rho_c}{A_E} = \frac{V_{CB} + V_{JC}}{I_C} \quad (3.42)$$

as shown in Fig. 3.5.

In the sections on saturation and quasi-saturation, the width of the space charge layer in collector material is assumed constant for a forward biased collector junction, the effect of injection across the junction being represented by 'diffusion' charge stores. In the active mode, C_{JC} may not, in general, be constant and various models^(18,19) for example, could be used for its voltage and current dependencies, accord-

$$r_c = \frac{x_e - x_{DC}(V_{JC}, I_C)}{q \mu_n N_{epi} A_E} = \frac{V_{CB} + V_{JC}}{I_C}$$

FIG. 3.5. Intrinsic collector model for active operation.

ing to the degree of accuracy required for modelling these terms.

In the next Chapter, modelling of the remainder of the device, namely the intrinsic and extrinsic base, and extrinsic collector region is carried out, based to an extent on circuit model concepts developed in this Chapter.

CHAPTER 4

CIRCUIT MODELS FOR THE COMPLETE DEVICE

4.1 INTRODUCTION

The regions of the device other than the intrinsic collector are considered at this stage, in accordance with the scheme for derivation of the complete device model discussed in Section 2.2. These device regions (sub-models), namely the intrinsic emitter and base regions, and the extrinsic collector and base are modelled for operation in the quasi-saturation, saturation and active modes. The postulates used for the derivation of these models are not discussed in depth except when new concepts are introduced. In other cases, references are given of detailed discussion of similar type models.

In Section 4.2, the model for the emitter and intrinsic base regions, applicable to all three modes of operation, is introduced. In Section 4.3, extrinsic base and collector effects are considered, leading to complete device models. The rest of the Chapter is devoted to a discussion of the differences in modelling between discrete and integrated devices, including package parasitic effects.

4.2 EMITTER AND INTRINSIC BASE CIRCUIT MODELS

The model adopted for this intrinsic 'device' is indicated in Fig. 4.1. This is a 'transport model' with charge control representation of 'neutral' region charge storage effects. It is based upon the success of the two-lump base region model in describing the behaviour of alloy-junction-type devices (which are of 'intrinsic' type geometry)⁽⁵⁾, use in previous studies^(16,17,20) for example, and intuition gained from certain terminal characteristics of the device in the active region (where decoupling of the total model parameters is such that a one-to-one correspondence between these terminal measurements and intrinsic base parameters is possible). The model is assumed to have the same form under a large range of physical conditions, but the parameter values are allowed to vary to account for the effect of the physical processes occurring. That is, an attempt has been made at the systematic enumeration of the physical processes occurring, by use of the functions f , g and h , as shown. The function dependence ' f ' describes 'static' base-width modulation effects (the current generator $C_{\mu_b} \frac{dV_{cb}}{dt}$ representing base-width modulation effects under large-signal dynamic conditions being treated in Section 4.2.1). The function ' g ' expresses the temperature dependency of all parameters, whilst ' h ' models current-dependent effects, at

$$I_{CE} = I_{SF} (\exp q V_{EB} / n_F kT - 1)$$

$$I_{EC} = I_{SR} (\exp q V_{BC} / n_R kT - 1)$$

$$I_{SF} = I_{SFO} f_1(V_{CB}) g_1(T)$$

V_{EB}, V_{BC} refer to junction voltages

$$I_{SR} = I_{SRO} f_2(V_{EB}) g_2(T)$$

$$\beta_F = \beta_{FO} f_3(V_{CB}) g_3(T) h_3(I_{CE})$$

$$\beta_R = \beta_{RO} f_4(V_{EB}) g_4(T) h_4(I_{EC})$$

$$\tau_F = \tau_{FO} f_5(V_{CB}) g_5(T) h_5(I_{CE})$$

$$\tau_R = \tau_{RO} f_6(V_{EB}) g_6(T) h_6(I_{EC})$$

Functions f, g, h may be expressed in physical terms by 'curve-fitting' approximations (e.g. polynomials), or by tabulations - or a combination of these - as appropriate. For example, a good approximation to the current dependence of β_F is (Section 6.2.2.2):

$$\beta_F = [C_{3F} + C_{4F} I_{CE}^{-1/2} + C_{5F} I_{CE}]^{-1} h_3(I_{CE})$$

FIG. 4.1. 'Transport Model' of intrinsic emitter and base regions.

high levels for example. These dependencies may be expressed in various forms (see Fig. 4.1 and Section 6.2), the parameters involved in their definition being deduced from measured terminal characteristics of the device, and may range from expressions directly drawn from device physics to essentially 'curve fitting' relationships. The measurements required for this purpose must be made under defined temperature conditions, and a test method is described in Section 6.2.1 showing how this may be achieved.

This 'functional dependency method' has the considerable advantage of 'accuracy' over other more analytical approaches⁽¹⁾ as the dependencies must, of course, correspond to measured data, taken at selected points over a large range of conditions. This approach, however, is not feasible in a region of variable width, such as the injection region of the collector in quasi-saturation owing to the intractably large amount of data that would have to be stored describing parameter variations in the collector model.

Based upon previous work and the one-dimensional simulations of Treen⁽³⁾, the intrinsic region, excluding the collector, may be divided into neutral emitter and base regions surrounded by space charge layers at the emitter-base and collector-base junctions, as in Fig. 4.1. Due to the high doping of the emitter, minority charge injection into the

emitter may be neglected, or combined with q_F , the charge store adjacent to the emitter space charge layer on the base side⁽²⁰⁾. Base currents, described by β_F and β_R terms model recombination in the 'neutral' base, both depletion layers, and surface regions, since β_F, β_R , through their functional dependencies, model total base currents rather than just bulk recombination, as in the collector model.

For both the emitter and collector junctions, modelling of the space charge capacitance may be by interpretation of bridge measurements made under reverse bias conditions, and forward bias values of C_j from f_T measurements, as discussed in Section 6.2.

This 'transport-type' model is thus to be preferred to the injection (Ebers-Moll) type since non-ideal components of current do not occur in the equations relating input voltage and output current, but are included in β_F and β_R . The model developed permits an ordered description of physical processes and allows the introduction of other sources of parameter change, for example, neutron radiation or the statistical variation of parameters by the inclusions of further functions, other than f , g and h .

The model could use temperature as a 'terminal' variable (that is, to be computed) rather than a 'parameter' variable;

such a model finds use in high precision circuits, etc. However, a circuit analysis program capable of using temperature as a variable would need to be developed and is not considered here. The interaction of transient thermal effects upon the electrical characteristics has not been considered since for microwave transistors the thermal time constants will, in general, be larger than 'electrical' times considered and we may assume the device to be at a constant temperature. This will not hold necessarily for slower devices, possibly leading to parallel lumped electrical and thermal models. *Good point* +

4.2.1 Basewidth Modulation under Large-Signal Conditions

The effect of 'neutral' region width variations caused by changes in emitter-base and collector-base terminal voltages is discussed in this section. The inclusion of this effect in the device model differs from previous work in the literature by being a large-signal treatment. That is, voltage variations causing large changes in region widths and hence model parameter values are allowed in this analysis. Basewidth modulation is considered initially and the analysis developed in terms of a uniform base, for illustrative purposes. The approach, however, is general and, after the uniform base treatment, the implications of the analysis for a device of general impurity density profile are discussed. The analogous variation of collector

injection region width (in quasi-saturation) with V_{CB} is then considered. This effect is of considerable importance in the quasi-saturation model (of major interest) and is implemented in Chapter 5 to show the validity of the analysis.

The width of the intrinsic base region of modern narrow-base devices operating in the active mode is a significant parameter which influences many of the electrical characteristics of the transistor. Because the active base region is bounded on both sides by junction space-charge layers, its width changes as the dimensions of the space-charge layers change.

Under 'normal' operating conditions, the emitter junction is forward biased and the collector is reverse biased. Consequently, the penetration of the collector space-charge layer into the base is normally much greater than the penetration of the emitter depletion layer. Attention is thus focussed on basewidth changes caused by changes in the collector junction width. If, however, the emitter junction were to become reverse biased in some applications, considerations similar to those investigated in this section would hold true at the emitter junction. Fig. 4.2a shows the distribution of minority carriers in the base of a transistor when a steady direct current is flowing through it. For this device, current in the narrow base region is assumed to flow by diffusion and hole-electron

(a) Steady-State minority carrier distribution in the active region.

(b) Transmission line analogue for the one-dimensional base region.

FIG. 4.2. Minority carrier distribution in the base and its transmission line analogue.

recombination may be neglected. The rate of propagation of the minority carriers at a given point is thus directly proportional to the density gradient, and in circuit terms, we may represent the process of diffusion by a transmission line having series resistance and shunt capacitance⁽¹⁶⁾. Such a line permits a variable charge density to exist along its length, and it is easily shown that the current flowing at a given point is proportional to the charge density gradient at that point. Recombination (assumed of small effect) is represented in the transmission line of Fig. 4.2b by the shunt conductance G .

Upon increasing the reverse bias V_{JC} across the collector junction, the voltage will modulate the base width which is equivalent to changing the length of the transmission line of Fig. 4.2b by sequentially removing RC lumped elements from the right. The work of this section will show that a two-lump charge model, of variable width, is the minimum adequate method of representing large-signal basewidth modulation (or the analogous discharging of a many-lumped transmission line).

The problem is treated in the large-signal range by making use of the concept of the stored minority carrier charge in the base. Consider the distribution of minority carriers in the base under d.c. conditions; that is, for fixed emitter-base and collector-base voltages, as shown by the line ab in

Fig. 4.3a. Let a change in collector voltage from V_{JC} to V_{JC}' occur. The steady state charge pattern will move from ab to ac . Initially minority (electron) carriers are crowded at the collector end of the base, supporting the increased collector current at this boundary associated with the change in collector voltage. This increased gradient will 'spread' through the base region as a function of time, as shown, until the new steady state conditions ' ac ' are reached. This redistribution is approximated by modelling the charge pattern curves from ' ab ' to ' ac ' as a series of triangles, in Fig. 4.3b. Some of the fine detail of the transient behaviour has been lost, but the problem is now simplified because straightforward relationships now exist between the decrease in base charge dq_b in time dt and the change in basewidth dW associated with it. The portion of Fig. 4.3b showing these changes in time dt is re-drawn in Fig. 4.4. For a change in the collector voltage, the associated variation in basewidth dW causes a change dq_b in the minority carrier charge in the base where, for constant $n'(0)$, i.e. constant V_{JE} during dt , we have, for the uniform base case considered:

$$dq_b = q A_E n'(0) \frac{dW}{2} \quad (4.1)$$

The excess 'control' charge dq_b will be swept across the collector depletion layer. Consider the incremental charge

(a) Redistribution of minority carriers due to a change in V_{JC} .

(b) Approximate redistribution patterns used in the analysis.

FIG. 4.3. 'Actual' and approximate redistribution of carriers in the base.

FIG. 4.4. Redistribution of charge in time dt .

dq_c leaving via the collector:

$$dq_c = dq_b \quad (4.2)$$

This removal of charge in time dt constitutes a time dependent component of current, i.e.

$$I_c = I_C + \frac{dq_c}{dt} \quad (4.3)$$

where I_C is the 'instantaneous dc' component of the total terminal current I_c and dq_c/dt is the time dependent component of I_c due to basewidth modulation. In this treatment, none of the excess 'control' charge dq_b leaves via the forward biased emitter junction, i.e. dq_e/dt (analogous to dq_c/dt in Equation (4.3)) is zero. In general, however, the total dq_e , dq_c are not equal to the values discussed in this section, as indicated below.

A two-lump active region base model, Fig. 4.5a and Fig. 4.5b, is specified since a one-lump model, derived from Fig. 4.1 for $q_R=0$, is incapable of representing the removal of dq_b simultaneously through both junctions (via its dq_F/dt charging current).

The derivation of the model equations from the carrier distribution follows along lines similar to those used previously

2 3
OF/DE

$$q_b = q_e + q_c$$

$$x_e/x_c = K = 1 \text{ (arbitrary)}$$

(a) Two-store lumping of base charge. $i_{\text{charge}} = C_{\mu_b} \frac{dV_{cb}}{dt}$

where
$$V_{JE} = \frac{kT}{q} \ln \left\{ 1 + \frac{1}{q_0} (q_e (2+1/K) - Kq_c) \right\}$$

$$q_0 = qA_E W n_1^2 / N_B$$

$$V_{JC} = \frac{kT}{q} \ln \left\{ 1 + \frac{1}{q_0} ((2+K)q_c - q_e/K) \right\}$$

$$C_{\mu_b} = \frac{C_{JC}}{2} \frac{1}{qA_E W N_B} \left\{ q_e (2+1/K) - Kq_c \right\}$$

$W = f(V_{JE}, V_{JC})$ determined by the space-charge region modelling.

$$\tau_d = \frac{W^2}{2D_n} \frac{K}{1+K}$$

τ_e, τ_c constant

(b) Two-lump model including basewidth modulation.

FIG. 4.3. Charge distribution and model for intrinsic base including basewidth modulation.

for collector models. It may be noted from Figs. 4.5a and 4.5b that V_{JE} , V_{JC} define q_e, q_c (for a given K) for a reverse biased V_{JC} . From Fig. 4.5a, q_c is such that $n'(W)$ is negative. The base transit time τ_d is a function of the basewidth W , needed in a large-signal model. The total charging current dq_c/dt in general is not only due to a basewidth modulation term but also due to differences in current transported into and out of the store. The basewidth modulation component i_{charge} must therefore be specified. This is accomplished in terms of the incremental collector voltage. From Equation (4.2), we have:

$$i_{\text{charge}} = \frac{dq_c}{dt} = \frac{dq_b}{dt} \quad (4.4)$$

or

$$i_{\text{charge}} = \frac{dq_b}{dV_{cb}} \frac{dV_{cb}}{dt} = C_{\mu_b} \frac{dV_{cb}}{dt} \quad (4.5)$$

Thus, when the charge dq_c is removed through the collector terminals by an incremental collector voltage dV_{cb} , the effective 'capacitance' of this current component is C_{μ_b} given by Equation (4.5). A more useful form for this capacitance can be obtained by considering the variation of q_b with V_{cb} via the basewidth W . Thus,

$$C_{\mu_b} = \frac{dq_b}{dV_{cb}} = \frac{dq_b}{dW} \frac{dW}{dV_{cb}} \quad (4.6)$$

From Equation (4.1), we have:

$$\frac{dq_b}{dW} = q A_E \frac{n'(0)}{2} \quad (4.7)$$

and from the definition of depletion capacitance, for a uniform base doping N_B : *with many collector doping*

$$C_{JC} = \frac{dq_{JC}}{dV_{cb}} = q A_E N_B \frac{dW}{dV_{cb}} \quad (4.8)$$

From Equations (4.7) and (4.8), we have, for the effective capacitance,

$$C_{\mu_b} = q A_E \frac{n'(0)}{2} \frac{C_{JC}}{q A_E N_B} = C_{JC} \frac{n'(0)}{2N_B} \quad (4.9)$$

or from Fig. 4.5a, for a two-lump approximation to the profile,

$$C_{\mu_b} = \frac{C_{JC}}{2} \frac{1}{q A_E W N_B} \left\{ q_e \left(2 + \frac{1}{K} \right) - K q_c \right\} \quad (4.10)$$

Modern power devices do not have the uniformly doped bases used in the above analysis. A typical minority carrier concentration as a function of distance through the base is shown in Fig. 4.6 (obtained from the simulation studies of Treen⁽³⁾) for an npn double-diffused epitaxial transistor. By comparison of this profile with that for a uniform doping, Fig. 4.4, it may be seen that, for the graded base, the change

FIG. 4.6. Minority carrier concentration in the case as a function of distance for a double-diffused device.

dq_b in the minority carrier charge caused by a variation dW in basewidth will differ from Equation (4.1) since the profile is not linear with distance. In fact, for a general doping distribution, we may write, instead of Equation (4.1):

$$dq_b = q A_E n'(0) \frac{dW}{2} m \quad (4.11)$$

where m is a 'suitably chosen constant'.

From Fig. 4.6 for devices of the type studied, the profile does not vary drastically from the triangular distribution used in the initial analysis and a value of m equal or close to unity is expected to give good results. It should be noted that, even if m were determined for a specific doping profile at a certain neutral basewidth W , m varies with bias point, i.e. with neutral basewidth since, in general, the 'shape' of the minority carrier distribution will change with W . However, this 'shape' is not expected to vary excessively for the basewidth changes likely to be encountered in the normal active operation of the device and, due to the complexities involved in obtaining m as a function of W , the choice of a constant m , equal to unity, is made.

The analysis then follows along identical lines to that for the uniform base except that the constant value N_B must be replaced by the value of the base doping density $N_B(W)$ at

the collector edge of the neutral base. For example, Equation (4.9) must be replaced by:

$$C_{\mu_b} = \frac{C_{JC} n'(0)}{2 N_B(W)} \quad (4.12)$$

where $N_B(W)$ is known from the grading approximation used (in the region of the collector junction) to evaluate C_{JC} from the applied collector-base voltage.

If the basewidth is decreased, a smaller excess minority carrier base charge must exist, setting up the charge gradient corresponding to the new collector current. It is necessary not only to remove majority carrier charge corresponding to this dq_b but also to remove majority carriers to 'uncover' the impurity centres needed for the widened collector depletion layer. These two incremental charges, dq_b and dq_{JC} , are modelled by the discharge of the 'collector side' store q_c through C_{μ_b} and the depletion capacitance C_{JC} .

An analogy exists between variations with V_{CB} of the two-lump base model width W and the injection region width x_i of the quasi-saturation collector model (Fig. 3.1b, Section 3.2 repeated here as Fig. 4.7). As with the base model, if an externally applied step in collector current occurs, in the absence of width modulation charging currents, these step current changes can only be accommodated by instantaneous (often

FIG. 7.7. Collector region model for quasi-saturation.

large) changes in 'neutral' region widths: For the collector model, the effective capacitance $C_{\mu c}$ follows along similar lines, using the theory of Section 3.2.

$$C_{\mu c} = \frac{dq_2}{dV_{cb}} = \frac{dq_2}{dx_i} \frac{dx_i}{dV_{cb}} \quad (4.13)$$

or

$$C_{\mu c} = \frac{dq_{COL}}{dx_i} \frac{dx_i}{dV_{cb}} \quad (4.14)$$

where q_{COL} is the excess electron charge in the collector injection region width x_i , analogous to q_b , and dq_2 is the incremental charge swept into the ohmic region by a change in collector base voltage:

$$dq_2 = dq_{COL} \quad (4.15)$$

From Equations (3.2) and (3.14), we have by equating currents in the injection and ohmic regions:

$$\frac{2 q D_n A_E n'(0)}{x_i} = \frac{(V_{cb} - V_0) x_e}{(x_e - x_i) r_{co}} \quad (4.16)$$

Upon differentiating with respect to V_{cb} , we obtain:

$$\frac{dx_i}{dV_{cb}} = \frac{2 q D_n A_E n'(0) \frac{x_e^2}{r_{co}}}{\left\{ 2 q D_n A_E n'(0) + (V_{cb} - V_0) \frac{x_e}{r_{co}} \right\}^2} \quad (4.17)$$

In an identical fashion to Equation (4.7),

$$\frac{dq_{COL}}{dx_i} = q A_E \frac{n'(0)}{2} \quad (4.18)$$

Combining Equations (4.17) and (4.18) gives, in Equation (4.14):

$$C_{\mu_c} = \frac{-q A_E}{2} n'(0) \frac{dx_i}{dV_{CB}} = \left[\frac{2q D_n A_E n'(0) x_e}{2q D_n A_E n'(0) + (V_{cb} - V_0) \frac{x_e}{r_{co}}} \right]^2 \frac{1}{4D_n r_{co}} \quad (4.19)$$

but from Equation (4.16), we have:

$$x_i = \frac{2q D_n A_E n'(0) x_e}{2q D_n A_E n'(0) + (V_{cb} - V_0) \frac{x_e}{r_{co}}} \quad (4.20)$$

giving

$$C_{\mu_c} = \frac{x_i^2}{4D_n r_{co}} = C_{\mu_0} \left(\frac{x_i}{x_e} \right)^2 \quad (4.21)$$

and C_{μ_c} , like τ_3 , τ_4 , r_c , must be specified in terms of the width x_i .

4.3 COMPLETE DEVICE MODELS

The intrinsic collector region models of Chapter 3 are combined with the 'transport' model for the intrinsic base region (and its adjacent junctions) and the model for the total

intrinsic device takes the form of Figs. 4.8 to 4.10 for quasi-saturation, high-level saturation, and active operation, respectively. Extrinsic effects associated with the base and collector regions are not included at this stage and the effective area for such models can be regarded as being the emitter junction area A_E .

In order to model the actual device structure, two considerations have to be dealt with:

- (1) the collector-base junction area A_C , as mentioned in the description of the devices under study (Section 2.2) is greater than A_E . Thus, the collector consists of a region of area $\sim A_E$, opposite the emitter, together with an external region opposite only the base. Figs. 4.8 to 4.10 show only the model for the former region of the collector.
- (2) extrinsic resistance effects and junction capacitances need to be considered.

4.3.1 Quasi-Saturation

When the modelling of the complete collector is considered, it is realized that for quasi-saturation operation, due to the resistance of extrinsic epitaxial material, and the large width of the extrinsic region compared with the dimensions of the intrinsic base, the collector current is largely restricted to the region under the emitter. This current flow

FIG. 4.8. Complete device (area A_g) model for quasi-saturation.

FIG. 4.9. Complete device (area A_E) model for high-level saturation.

FIG. 4.10. Complete device (area A_E) model for active operation.

through the ohmic collector material will cause the collector junction to forward bias. The injection region extends little beyond the area of the collector junction under the emitter, and in this case, the model of Fig. 4.8 is adequate.

The modelling of extrinsic effects follows along conventional lines. Distributed base resistance effects are represented by a single equivalent base resistance r_{bb} . Under these quasi-saturation conditions, as noted, the region of the collector outside the (quasi-saturated) volume is simply ohmic, with a reverse biased junction boundary. This region can be represented by the C_{JC2} , r_c circuit shown in the complete device model of Fig. 4.11, analogous to the intrinsic collector region in the active mode, Section 3.4.

4.3.2 Saturation

A considerably different situation arises under saturation conditions. The substrate (low resistance N^+ material) extends right across the extrinsic collector region and the NN^+ boundary at x_e causes the region of charge storage to extend far outside the region opposite the emitter. This boundary causes high densities of $n=p \gg N_{epi}$ at x_e . In modelling of saturation operation, the 'outer' collector region must be represented by an additional two-lump sub-model, as shown in

FIG. 4.11. Complete device model including extrinsic elements for quasi-saturation region.

Fig. 4.12. This collector-base diode is developed in terms of the excess minority carrier charge in the collector since in the extrinsic region hole current is no longer negligible and minority carrier injection into the highly doped extrinsic base region may be neglected. The applied terminal voltage V_{cb} is shared between the base-collector junction and the NN^+ boundary. Due to the injecting nature of the PN junction right across the device, the two charges adjacent to the collector-base junction, namely q_1 and q_1' , will be proportional to each other, being both related by the law of the junction to V_{JC} , constant across the device.

The 'outer' sub-model for low-level saturation will take the same form. However, in this case, due to the identical topology of the sub-models in the intrinsic and extrinsic collector regions, these sub-models may be directly combined to take the form of Fig. 4.13.

In the saturation case, the entire collector-base junction area is forward biased; thus, the inclusion of the ohmic r_c representation does not occur and the junction capacitance C_{JC2} is connected across the junction boundary node-pair of the collector sub-model. Similarly, the extrinsic resistance r_{bb} is conductivity modulated and regarded as negligible. Width modulation effects are negligible in the base and collector, as per Fig. 4.9.

FIG. 4.12. High level saturation model with 'outer' collector region included.

FIG. 4.13. Complete low level saturation model combining 'inner' and 'outer' regions.

4.3.3 Active Mode

Modelling of the distributed base resistance and ohmic collector effects follows precisely on the lines outlined for the quasi-saturation model of Section 4.3.1. The total model is shown in Fig. 4.14. If required, the extrinsic base could be divided into more than one lump, each associated with its r_{bb} , and series r_c , C_{JC2} representation.

4.4 INTEGRATED CIRCUIT DEVICE

The cross-section of a typical integrated circuit transistor is shown in Fig. 4.15a, where it is seen that the main difference between the integrated circuit device model and its discrete counterpart relates to the parasitic elements in the junction isolation environment. From structure considerations, the model of the integrated device takes the 'two-transistor' form of Fig. 4.15b, the substrate becoming the 'collector' of a poor transistor (pnp).

In saturation, the substrate may behave like a reverse-biased collector, the forward-biased PN junction injecting minority carriers across the NN^+ region to the P^- substrate. This becomes a difficult modelling problem with regards to the choice of device model for the 'parasitic transistor', and the difficulty in identifying currents in the two transistors, (to obtain the parameters for each device). Steps are usually

(a) Typical IC transistor cross-section.

(b) Two-transistor model for integrated circuit device.

FIG. 4.15. Integrated circuit device and its model.

taken in the processing to minimize the effect of the parasitic pnp transistor. In many applications, only the capacitive effects of the collector-isolation junction need be considered.

Another characteristic of the integrated circuit environment is the top collector contact which results in a distributed resistance between the collector terminal and the extrinsic device. It may, to a first approximation, be regarded either as a single lumped value, modelled in the same way as the substrate resistance in the discrete device, or as a component of the lumped resistance modelling the voltage drop from the collector contact to the buried layer, and along the buried layer to the extrinsic collector resistance r_c , depending upon the device geometry, Fig. 4.15a.

4.5 PACKAGE PARASITICS

To be considered in the total modelling process is the effect of package parasitics, due to the necessity of communicating electrical signals to the transistor chip. The actual parasitic network required depends upon the form of package used ('can' devices, stripline or coaxial packages), and may be derived from the topological pattern shown in Fig. 4.16⁽²¹⁾ which displays all possible parasitic dependencies of the package and wiring. Some of these connections may be unnecessary due to the form of package used and the consideration of only

FIG. 4.16. Topological pattern for parasitics.

the most important parasitics. For example, up to high frequencies (about 100MHz), it is often sufficient to take into account only the header capacitance, whilst at higher frequencies, the equivalent circuit may take a form similar to that in Fig. 4.17b evaluated for the 2N3375 device⁽²²⁾, showing consideration of lead inductances. The package structure of this device is shown in Fig. 4.17a.

In general, evaluation of package parameters will involve postulating the least complex equivalent circuit necessary to describe parasitic effects. Capacitance and inductance measurements are then carried out on the terminal leads, with the chip removed or shorted out. From these measurements, approximate values for the equivalent circuit parameters are estimated and these values may then be used as initial values for an optimization routine for measured and calculated S-parameters (without the chip and with physical constraints on circuit values). If insufficient accuracy is obtained, the parasitics model must be modified.

After evaluation by these techniques, the effect of the parasitics on the intrinsic and extrinsic parameters should be analyzed.

(a) Package structure of the 2N3375.

(b) Package parasitics for the 2N3375.

FIG. 4.17. Effect of packaging the 2N3375.

CHAPTER 5

MODEL IMPLEMENTATION

5.1 INTRODUCTION

At this stage, the models derived above must be considered with respect to the ease with which they can be applied by the circuit engineer. The models have been implemented in terms of the CIRC circuit analysis program on the Carleton University Sigma 9 computer. As it happens, this particular circuit analysis program is rather unsuitable for the purpose of setting up device models of the complexity required in this work, especially for quasi-saturation. It is necessary to treat the model as a circuit, using the available elements provided in CIRC, and to write sets of special equations in order to define the relationships involved between these elements. Considerable difficulty was experienced in setting up the model in this way. A circuit analysis program in which the device model could take the form of a separate sub-routine (that is, written in software form rather than in the form of an equivalent electrical circuit) would have been more appropriate for the task. However, the CIRC program was considered adequate for the task of showing the models' potential in computer-aided circuit analysis and design in spite of the inefficient form of implementation.

Implementation of the overall model of the transistor involves the representation for four possible distinct regions of operation (considered in Sections 5.2 to 5.5) namely:

1. Quasi-saturation
2. High level saturation
3. Low level saturation
4. Active and cut-off.

As seen in Chapter 4, the form of the model is different in each of these regimes. In the model implementation, it is necessary to define the boundary conditions between the possible regimes. These conditions have to be tested during the course of a circuit analysis computation, and the model specified accordingly. This process is readily carried out in a software realization of the model in compact sub-routine form, different 'chapters' of the subroutine being used to the different regime models, with routing through them defined by logical conditions specified by the boundary conditions. There is, however, no such simple implementation possible in the CIRC realization of the model; changing the form of the model in mid-computation is very difficult to achieve. For this reason, owing to the limitations of CIRC, investigations into model implementation have considered operation in each particular regime separately.

High level saturation and (particularly) quasi-saturation are covered in greater depth than the other regions of operation as the modelling of these regimes is more complex and is the main objective of the work undertaken.

5.2 SIMPLIFICATION OF THE MODEL FOR QUASI-SATURATION

At an early stage, the model equations for quasi-saturation, Fig. 4.8, Section 4.3, were examined using the 'analogue' techniques of SIMUL8, a Carleton University Sigma 9 in-house program designed to simulate the performance of an analogue computer by digital means. These investigations were carried out to gain experience in the manipulation of the equations prior to their circuit analysis program implementation. The studies using SIMUL8 confirmed the ability of the model equations to give unique solutions, provided an iterative scheme for the injection region width x_i is employed, based upon the simultaneous solution of Equations (2.17) and (2.8):

$$I_C \approx I_n \approx (V_{CB} - V_0)/r_c \quad (2.17)$$

where

$$r_c = (x_e - x_i)/q \mu_n N_{epi} A_E \quad (2.8)$$

and

$$I_C \approx I_n = 2q D_n \frac{dn}{dx} A_E$$

In the analogue studies and as the basis of the main task of CIRC implementation, the model employed is that of

Fig. 5.1. It is derived from the total device model of Section 4.3 on the basis of certain simplifying assumptions listed below.

1) The dependence of the intrinsic base parameters on voltage, temperature, and current by means of the functions f , g and h , Fig. 4.1 is neglected. This approximation and the others to be discussed are based upon one of the main theses of this study; that the terminal behaviour in quasi-saturation is dominated by charge injection effects in the intrinsic collector region, due to the low doping density of this region compared to the base. In the CIRC implementation, the equivalent circuit configuration is similar to that of the model itself, with additional circuit elements whose function is to set up, in circuit form, associated functional relationships between currents and voltages in the model itself. Since high level effects manifest themselves in the collector before the base, and due to the already complex nature of the equivalent circuit, it was considered advantageous to neglect effects of secondary importance. Thus, work was concentrated on modelling the collector injection region (of variable width) by using as simple a model as possible for the rest of the device. Variation of collector region parameters (for example, lifetime variations with current level and temperature) is also neglected.

(2) From terminal measurements of emitter current response⁽⁵⁾, it is seen that reverse transport into the emitter, q_R/τ_R , is very small at the frequencies of operation and device geometries encountered and may be neglected in comparison to the forward injection term q_F/τ_F .

(3) Strictly speaking, 'reverse' recombination in the base, q_R/τ_{BR} , should be included.

The forward biased collector junction causes a larger increase in collector excess minority carrier charge $p_n'(0)$ than in excess base charge $n_p'(W)$ at the collector edge of the 'neutral' base region. Due also to the much greater width of the collector epitaxial layer, the excess charge of holes stored in the collector

$$q_{COL} = q_1 + q_2 \quad (3.1)$$

is generally much larger than the excess electron charge in the base, q_R , that is

$$q_R = b q_1 \quad (5.1)$$

where

$$b \ll 1 \quad (5.2)$$

For charge lump boundaries of constant width, the junction injection law predicts direct proportionality between q_R and q_1 ⁽⁵⁾.

However, for quasi-saturation x_i is variable and hence so is b in Equation (5.1). From Fig. 4.11, we have, under d.c. conditions, with q_R/τ_R neglected:

$$\frac{q_F}{\tau_F} = \frac{q_R}{\tau_{BR}} + \frac{q_1}{\tau_1} + \frac{q_1^{-K} q_2}{\tau_3} \quad (5.3)$$

or from Equation (5.1)

$$\frac{q_F}{\tau_F} = \frac{q_1}{\tau_1} \left\{ 1 + b \frac{\tau_1}{\tau_{BR}} \right\} + \frac{q_1^{-K} q_2}{\tau_3} = \frac{q_1}{\tau_1} + \frac{q_1^{-K} q_2}{\tau_3} \quad (5.4)$$

Since τ_1 is of the order of τ_{BR} for these devices, neglect of the term $b \tau_1/\tau_{BR}$ will lead to insignificant error. Thus, the base recombination term q_R/τ_{BR} may be neglected, or if necessary included by means of small modification to τ_1 , Equation (5.4).

(4) In quasi-saturation, both junctions in the intrinsic device are forward biased, and the depletion layer charging currents are neglected in comparison with the 'diffusion capacitance' currents charging the neutral base and collector regions (15,18). This assumption may not be valid at low current levels, especially for the large C_{JE} value (the emitter diffusion capacitance is proportional to I_E). However, at the high current levels used in practice to obtain maximum f_T , the assumption, Equations (5.5) and (5.6), is justified.

$$C_{JE} \, dV_{JE}/dt \ll dq_F/dt \quad (5.5)$$

$$C_{JC1} \, dV_{JC}/dt \ll dq_1/dt \quad (5.6)$$

(5) The extrinsic elements of Fig. 4.11, $r_{bb'}$, C_{JC2} and r_c , are excluded for the same reasons as used in assumptions (1)-(4), namely the dominance of charge injection, and the considerable difficulties in the CIRC environment caused by adding further circuit elements to the basic essential circuit model. These problems are due to the method necessary in CIRC for finding an iterative solution for x_1 (described later in this Chapter).

(6) Forward bias conditions at both junctions lead to small variations in depletion capacitance widths under transient conditions. For this reason, basewidth modulation is considered small at the two junctions, and neglected here (see Section 4.2.1).

Consideration of model implementation, without invoking approximations (1)-(6), is made in Section 5.2.3 with respect to CIRC and more efficient analysis programs.

5.2.1 Equivalent Circuit Model Used in CIRC

In the CIRC implementation, Fig. 5.2, the circuit may be regarded as consisting of two parts:

(a) the circuit equivalent of the symbolic model of Fig. 5.1, within the dashed lines. This section of the circuit

FIG. 5.2. Equivalent circuit used in CIRC analysis.

is of similar form to that of the model itself. Parameters τ_F , τ_{BF} in the base region and τ_1, τ_2 in the epitaxial injection region are constants and may be described by conventional circuit elements, namely resistances for the 'combinances'⁽¹⁶⁾ and a voltage dependent current generator for the transported current q_F/τ_F . The voltages across these resistances are to be regarded as the circuit analogues of the excess minority carrier charges q_F , q_1 and q_2 ; and capacitances placed across these resistances model these charge stores, carrying the charging currents for these lumps. These circuit elements and their analogues are summarized in Table 5.1.

The remaining elements of the equivalent circuit, modelling currents $(q_1 - Kq_2)/\tau_3$, q_2/τ_4 , $C_{\mu_g} dv_{cb}/dt$ and $(V_{cb} - V_0)/r_c$ are functions of (x_i/x_e) , Fig. 5.1, and must be described to CIRC in terms of the instantaneous value of x_i . They are included in a 'special equations' subroutine in CIRC, described later in this section. These special elements are denoted by the dotted lines in Fig. 5.2.

(b) The second part of the equivalent circuit consists of additional circuitry whose purpose is to set up, in circuit form, associated functional relationships between currents and voltages in the model itself. As indicated in Section 2.4, the quasi-saturation model depends upon the simultaneous solution

SYMBOLIC MODEL

EXCESS 'CONTROL' CHARGE q_x CHARGING CURRENT dq_x/dt RECOMBINATION CURRENT q_x/τ_{REC} TRANSPORTED CURRENT q_x/τ

EQUIVALENT CIRCUIT

VOLTAGE V $\frac{dV}{dt}$ = CAPACITOR CHARGING CURRENT
FOR $C=1$ V/R CURRENT THROUGH RESISTORSVOLTAGE CONTROLLED CURRENT
GENERATOR $g_T V$

TABLE 5.1. Equivalent circuit elements and model analogues.

of two current relationships, Equations (2.17) and (2.8) in order to define the instantaneous value of collector injection region width x_i . In a software subroutine, this can be done by an iterative procedure; in the CIRC realization of the model, the solution is achieved by defining an error current in a circuit branch, resistor R1 (the error being dependent essentially upon the two current relationships) and requiring this current to be zero.

The solution of the simultaneous equations using the 'error' branch concept is clarified by considering a specific example. For a given operating point on the output characteristics of Fig. 2.3, Section 2.2, that is for V_{CE} , I_B given, we initially choose an arbitrary value of x_i/x_e . This gives the parameters r_3 , r_4 , r_e , $C_{\mu c}$ of the device model that vary with x_i/x_e . An initial choice of V_{EB} is also made giving q_F from the junction law:

$$q_F = q_{F0} \left\{ \exp \frac{qV_{EB}}{kT} - 1 \right\} \quad (5.7)$$

and collector current I_C from (for an equivalent pnp device)

$$I_C = \frac{V_0 - (V_{CE} - V_{EB})}{r_c} \quad (5.8)$$

But in the d.c. steady state, summing currents at the second charge lump q_2 , Fig. 5.1:

$$I_C = \frac{q_2}{\tau_4} = \frac{q_1 - K q_2}{\tau_3} - \frac{q_2}{\tau_2} \quad (5.9)$$

hence q_2 and also q_1 , ~~are known~~. Similarly, for currents into the collector

$$\frac{q_F}{\tau_F} = \frac{q_1 - K q_2}{\tau_3} + \frac{q_1}{\tau_1} \quad (5.10)$$

and hence q_F from q_1 and q_2 . However, q_F was defined by an arbitrary choice of V_{EB} , Equation (5.7). Thus, V_{EB} must be iterated until Equations (5.7) and (5.10) are simultaneously satisfied. A further constraint is that of base current. An arbitrary choice of (x_i/x_e) leads, via iteration on V_{EB} , to values of q_1 , q_2 , q_F giving an 'erroneous' value for 'base current'; that is, unless x_i/x_e is 'chosen' correctly, the total base current will not agree with the constrained value, and an error current will flow in a suitably positioned branch out of the base node of the circuit model, given by:

$$I_{\text{Error}} = \frac{q_F}{\tau_F} + \frac{q_1}{\tau_1} + \frac{q_2}{\tau_2} - I_B \quad (5.11)$$

For the CIRC program to iterate on non-linear operation points (V_{EB} in this case) and x_i/x_e , a current path must 'physically' exist to carry this 'error' current. The operating point is found when this error current, through resistor $R1$ in Fig. 5.2, is zero. The capacitance $C4$ across $R1$ prevents any step in

base current drive from flowing through the error resistance R_1 at the instant of application. After the step, if any, is applied, C_4 is removed by use of a table of C_4 values as a function of time. The other elements in the error current path namely the voltage source V_0 and the current dependent current generator CUR_1 set up the correct voltage at the edge of the injection region, namely $V_{EB} + V_0$, and provide an error current path to ground, respectively.

In a software subroutine, the functional relationship between the charge variable q_F and the emitter-base voltage V_{EB} may be described simply using Equation (5.7). In CIRC, interfacing between the 'internal' voltage between nodes 6 and 7 representing the analogue of q_F and the 'external' applied voltage V_{EB} is accomplished using the diode $DI01$ and associated circuitry in Fig. 5.2. This circuit satisfies the requirements of the circuit model for the depletion region discussed in Section 4.2, namely current continuity across the layer, and minority carrier densities given by the Boltzmann injection law.

We then obtain

$$I_{DI01(DC)} = I_{s1} \left\{ \exp \frac{q V_{EB}}{kT} - 1 \right\} = \frac{q_F}{\tau_F} + \frac{q_F}{\tau_{BF}} \quad (5.12)$$

$$\begin{aligned}
 I_e &= I_{DIO1(TOT)} = I_{DIO1(DC)} + \tau_D \frac{dI_{DIO1(DC)}}{dt} \\
 &= \frac{q_F}{\tau_F} + \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt}
 \end{aligned} \tag{5.13}$$

where

$$\tau_D = \frac{1}{(1/\tau_F + 1/\tau_{BF})}$$

and

$$\begin{aligned}
 q_F &= \frac{I_{S1}}{1/\tau_F + 1/\tau_{BF}} \left\{ \exp \frac{q V_{EB}}{kT} - 1 \right\} \\
 &= q_{F0} \left\{ \exp \frac{q V_{EB}}{kT} - 1 \right\}
 \end{aligned} \tag{5.14}$$

If the depletion capacitance C_{JC} is neglected, the injection relationships at the collector junction need not be considered explicitly. Current continuity at the injection-ohmic boundary is used instead of expressions relating q_1 to V_{JC} via x_i .

The additional circuit elements, namely C_5 , $CUR2$, and $CUR3$ allow the effect of 'width modulation' in the collector injection region to be quantified by defining a current proportional to dV_{cb}/dt . Once this current is specified, the 'width modulation' current $C_{\mu_c} dV_{cb}/dt$, Fig. 5.2, may be implemented.

5.2.2 Iterative Procedure for the Operating Point

In Section 5.2.1, the need for iteration on x_i and the non-linear 'diode' operating point (V_{EB} in this case) was pointed out. Preliminary investigations showed that as the iterated value of V_{EB} approached its 'correct' value (for a specified injection region width x_i), subsequent variation of x_i for V_{EB} fixed near but not at its 'operating point' resulted in a change of sign of the error current. This error current was found to be monotonically decreasing with decreasing x_i passing through zero near the final device operating point. Thus, a sequential procedure of iterating first on V_{EB} then on x_i and so on was found to lead to an error current below a specified minimum criterion at a required operating point. Thus, the CIRC implementation basically involves the interplay between the CIRC routine for the improvement to non-linear device operating point estimates and the additional iteration on x_i . The flowchart used for this means is shown in Fig. 5.3, and its salient features described below.

Step 1. At time $t=0$, x_i/x_e is set to some prescribed value, used for the initial iteration of V_{EB} towards its operating point. At subsequent times t_n , x_i/x_e is initialized to the 'correct' value of x_i/x_e evaluated at time t_{n-1} . A more efficient procedure would involve extrapolation from past times t_{n-1} , t_{n-2} , ... etc. to estimated initial values of x_i/x_e at time t_n .

FIG. 5.3. Flowchart for implementation in CIRC.

Step 2. The program writes Kirchoff's current law equations in the form of a nodal admittance matrix for the 'standard' branch elements of the circuit of Fig. 5.2, namely

$$[Y][V] = [I] \quad (5.15)$$

Step 3. Modifications are made to the nodal admittance matrix (using the specified x_i/x_e) to include currents dependent upon x_i/x_e , as indicated by the dotted lines between nodes in Fig. 5.2.

Step 4. A preliminary run may be necessary to determine how far convergence on V_{EB} can run before a minimum of $|I_{Error}|$ is obtainable. For simplicity at this stage, a count of the number of 'loops' on V_{EB} made is taken, and the looping temporarily stopped according to experience gained in the preliminary run. Typically, after 12 loops of V_{EB} convergence, iteration on x_i/x_e is possible. After the x_i/x_e convergence routine, control is returned to the V_{EB} loop, and then the x_i/x_e routine is implemented automatically for each pass through the estimation procedure for V_{EB} .

Step 5. In the user subroutine for convergence of I_{Error} , for each value of x_i/x_e , the circuit must be analyzed for the magnitude of I_{Error} . This involves inversion of the nodal admittance matrix. Double precision Gaussian elimination was found satisfactory, changes in pivotal

elements of small magnitude being made.

Step 7. A half-interval search⁽²³⁾ was used for convergence on x_i/x_e . This method is possible due to the change in sign of I_{Error} with x_i , and is chosen on account of its simplicity and past success in its use. The intervals of search were modified somewhat depending upon whether the initial condition or a transient time step is being considered (see Step 1). The intervals for the most efficient computation may be determined during a short preliminary run.

5.2.3 Additional Effects

In the discussion above, the model involved certain simplifying approximations which allow solution of the problem of injection region width variation. The implementation of a model in which these approximations are not invoked is considered briefly here with special attention to CIRC-oriented models. No attempt has been made to implement this total model, though the implementation of the sub-models representing the various effects has been considered individually. As throughout, methods involved in a CIRC-oriented solution give insight into solutions for more efficient circuit analysis programs.

Description of the dependence of complete model parameters on current, voltage and temperature in terms of the functions f , g , h is complicated, both on account of the large amount of data required for its determination and the difficulty of setting up this dependence as sets of equations in the subroutine. For quasi-saturation, high level effects in the base tend to be masked by collector injection, and the 'first order' model assumes constant base parameters and variation of collector parameters only as described by the model of Section 5.2.1: namely with x_i . A hierarchy of progressively more complicated models is possible. This might start with the inclusion of temperature dependence of important base parameters; temperature dependence of the collector model perhaps being adequately dealt with by changes in V_0 , as given by:

$$V_0 = -\frac{kT}{q} \ln \left(\frac{N_{epi}}{n_i} \right)^2 \quad (2.15)$$

and in r_c where

$$r_c(T) = r_c(T_0) \{1 + \alpha(T - T_0)\} \quad (5.16)$$

and culminating in functional dependency on V , T , I described for all parameters.

By comparison of the CIRC equivalent circuit of Fig. 5.2 with the total device model of Fig. 4.11, it is seen that

reverse injection and recombination terms in the base, q_R/τ_R and q_R/τ_{BR} must be connected between nodes 4 and 6, and nodes 4 and 7 respectively to give the same topology as the symbolic device model. However, a parallel RC combination modelling q_R/τ_{BR} , dq_R/dt , and a current generator describing q_R/τ_R are unable (unlike forward (q_F) terms) to describe these currents. This is due to the fact that the voltage across nodes 4 and 7 is the analogue of q_1 and because of the variable injection width, a direct proportionality between charge lumps on either side of the collector junction does not exist. In fact, from the charge distribution of Fig. 3.1a and high level theory, it is readily shown that:

$$q_R = \text{const.} \left(\frac{q_1}{x_i/x_e} \right)^2 \quad (5.17)$$

Since x_i/x_e is specified in the user subroutine (Section 5.2.2), upon describing currents proportional to $(q_1/x_i/x_e)^2$ to the 'auxiliary circuit' (1) in Fig. 5.4, base currents q_R/τ_{BR} , q_R/τ_R , dq_R/dt may be obtained. V_{JC} is also available, using the injection law relating q_R to V_{JC} , modelled using diode DI02 in auxiliary circuit(2). We may then model the collector depletion layer current using the Gummel-type capacitances⁽¹⁸⁾ incorporated in diodes 2 and 3. This definition of V_{JC} may also be used to yield a base width modulation current $C_{\mu_b} dv_{JC}/dt$ if necessary.

FIG. 5.4. Possible implementation of additional effects in quasi-saturation.

The emitter depletion capacitance C_{JE} is simulated by a diode across the emitter junction, diode DI04 in Fig. 5.4. This diode could have a very small saturation current giving total 'diode' current $C_{JE} \frac{dV_{EB}}{dt}$, or its current-voltage ($I_{D4} - V_{EB}$) relationship could model variations in β_F with I_C .

Extrinsic elements, r_{bb} , C_{JC2} , r_c , are included in Fig. 5.4 and it may be possible to neglect coupling between C_{JC2} and r_c in a first order representation of extrinsic elements in quasi-saturation. If coupling is to be considered, a rigorous solution would involve the simultaneous solution of the current $I_{EXTRINSIC}$ through the depletion and ohmic regions, for the depletion semiwidth x_{dc2} on the collector side.

$$I_{EXTRINSIC} = C_{JC2}(V_{JC2}) \frac{dV_{JC2}}{dt} \quad (5.18)$$

$$I_{EXTRINSIC} = \frac{(V_{CB} - V_{JC2})(A_C - A_E)q\mu_n N_{epi}}{(x_e - x_{dc2})} \quad (5.19)$$

Upon writing a linear approximation for the derivative of Equation (5.18), we obtain:

$$\frac{(V_{CB}^n - V_{JC2}^n)(A_C - A_E)q\mu_n N_{epi}}{(x_e - x_{dc2}^n)} = \frac{C_{JC2}^n}{\Delta T} (V_{JC2}^n - V_{JC2}^{n-1}) \quad (5.20)$$

where the superscripts refer to present time (n), and one time step back ($n-1$). A choice of x_{dc}^n defines both C_{JC2}^n and V_{JC2}^n and, since V_{CB}^n is known and V_{JC2}^{n-1} evaluated at the time step prior to that considered, x_{dc2}^n must be iterated until both sides of Equation (5.20) agree. This could be accomplished fairly readily in an efficient analysis program with subroutine implementation of models.

5.3 HIGH LEVEL SATURATION

The saturation model of Fig. 4.12, Section 4.3.2, represents the situation where the injection region has:

- i) expanded to encompass the full width x_e of the epitaxial collector material
- ii) expanded beyond the emitter area to the full collector area.

The model implementation thus does not involve the simultaneous solution of two current flow equations, but does incorporate the outer collector region sub-model. For both the 'inner' and 'outer' collector models, the charge storage region representation is almost identical in form to that of the base region when in the saturation state. It is therefore possible to use similar 'transistor' models for each of the three regions as in Fig. 5.5 where 'Ebers-Moll' configurations are used. The Ebers-Moll transistor model is equivalent to the two-lump

FIG. 5.5. Complete high level saturation model - Ebers-Moll implementation.

description used for each of the three regions of the device, namely intrinsic base and collector, and extrinsic collector, Fig. 4.12. Transistors T_1 and T_2 , in series, model the intrinsic (base + collector) device while transistor T_3 , with its collector open, represents the extrinsic region collector-base diode. Thus, in Fig. 5.5, node C_1 represents the 'internal' collector point (C^* in Fig. 4.12) and node B_2 is the device base contact. Space charge capacitance C_{JC1} is developed between these two nodes. Extrinsic diode current flows to the device collector via the base terminal of T_3 , for which nodes B_3 - B_2 represent the extrinsic region collector-base junction. As mentioned below, the voltage at B_3 must equal that at C_1 . Node C_3 is open-circuited reflecting the fact that no hole current flows to the contact. This node (C_3) will then float to a potential analogous to the excess minority carrier charge q'_2 of the extrinsic diode. The actual CIRC implementation, Fig. 5.6, is complicated somewhat by the boundary conditions imposed on these models, Fig. 4.12, namely:

$$\text{Transistor 3} \quad q'_1 = q_1 \quad (5.21)$$

$$\text{Transistor 2} \quad q_2 = q_{20} \left\{ \exp q \frac{V_{cb} - V_0}{2kT} - 1 \right\} \quad (5.22)$$

As mentioned in Section 4.3.2, Equation (5.21) reflects the injecting nature of the PN (collector-base) junction right across

FIG. 5.6. CIRC implementation of high level saturation model.

the device, and Equation (5.22) describes the excess electron density at the NN^+ boundary of the intrinsic device.

The voltage source V_0 and an injection index of 2 for the reverse diode (q_2) of transistor T_2 satisfy Equation (5.22) for charge q_2 . The additional circuit elements ensure the voltage at the base of transistor T_3 equals that at the collector of transistor T_1 , that is satisfying Equation (5.21). A large resistor $R(10^6 \text{ ohms})$ between node C_1 and ground allows the voltage at C_1 to be monitored without drawing significant current from the Ebers-Moll models. Current generators CUR2 and CUR3 carry a current proportional to that through resistor $R(1)$, such that the current through these generators is much larger than the base current drawn by transistor T_3 . The voltage drop across R_1 then closely approximates that across R . By using a very small value (10^{-6} ohms) for R_2 , which carries the base current of T_3 , one side of R_1 will be approximately at ground potential, and the other (the base of T_3) will have the same voltage as node C_1 , as required by Equation (5.21). Current through the outer sub-model is routed to the collector of the device via current generator CUR1. Junction capacitances in the Ebers-Moll models may be used to model directly C_{JE} , C_{JC1} and C_{JC2} .

This high level model is valid for applied V_{CB} greater than V_0 and for intrinsic collector recombination current small compared to I_C (high level condition). Similar comments as for quasi-saturation may be made regarding functional (f,g,h) dependence of model parameters (see Fig. 3.1). The model has been verified to work, typical responses being discussed in Section 6.4.1.

5.4 LOW LEVEL SATURATION

In a similar fashion to the high level case, the low level saturation model of Fig. 4.13, Section 4.3.2, may be implemented by the interconnection of similar device models⁽²⁴⁾ in this case 'pnp' and 'nnp' Ebers-Moll transistor equivalent circuits, Fig. 5.7. It is of interest that this model, unlike that for high level operation, is valid also in the active region reducing to the usual one lump (q_F) active regime model.

5.5 ACTIVE MODE

The full device model for the active regime is described in Section 4.3.3 and shown in Fig. 4.14. As for quasi-saturation, a rigorous solution would involve simultaneous solution of two equations for the current flow in the ohmic and depletion regions (cf injection region in quasi-saturation):

FIG. 5.7. Low level saturation model - Ebers-Moll implementation.

$$I_C = \frac{(V_{CB} - V_{JC1}) q_n N_{epi} A_E}{x_e - x_{dc}} \quad (5.23)$$

$$I_C = \frac{q_F}{\tau_F} - \frac{q_R}{\tau_R} - \frac{q_R}{\tau_{BR}} - \frac{dq_R}{dt} - C_{JC1} \frac{dV_{JC1}}{dt} \quad (5.24)$$

subject to another boundary constraint (base current I_B , for example). The solution would involve iteration on the collector junction semiwidth x_{dc} and the non-linear diode of voltage V_{EB} for zero 'error' current. Implementation of the active region model was not the primary objective of the study and has only been carried out for the device sub-models individually. These sub-models correspond to specific terminal measurements in the forward and inverse modes, and are considered in detail in Chapter 6, where the ability of the model to describe terminal characteristics is discussed.

CHAPTER 6

COMPARISON OF MODEL AND DEVICE BEHAVIOUR

6.1 INTRODUCTION

The process of modelling may be divided into three stages.

1. The first stage involves the development of the model equations, starting with the physical postulates of Chapter 2, and implemented as described in the previous Chapter. This step, namely the selection of the basic assumptions of form and interaction, will be referred to as the 'model equations' stage in the discussion below.

2. From Stage 1 (Chapters 2 to 5), we obtain a number of different component equations or relationships; and the second stage involves the fit of the 'best' values of the parameters for each relationship independently, by using relevant subsets of available device data on structure and performance. Naturally, different measures of quality of fit may be adopted, and each will lead to particular sets of parameter values, thereby exerting a considerable influence on the quality of the final model. Having fitted all the 'individual' parameters, there will still remain some parameters characteristic of the interconnections between the different

relationships that form the full model. The process of finding the 'best' values for these parameters can only be effected by 'running' the whole model and altering these parameters until good agreement is obtained between overall behaviour of the device and its model.

Ideally, one would wish to find the data necessary for definition of the model parameter values on the manufacturer's specification sheet for the device. Unfortunately, most data sheets do not contain sufficient information to define the entire model, and the designer (even for parameters of existing models such as the Ebers-Moll representation) must make additional measurements. It is often of comparable difficulty in modelling to determine what measurements to make and how to make them as it is to formulate the equations and topology, Stage 1. The cost of generating transistor data in the form of the models of Chapter 4 is very largely the cost of the measurements. There is thus a premium on simplicity and efficiency in measurement method.

Just enough experimental data points could be taken so that the defining model equations can be solved explicitly; however, a small error in just one data point could be reflected as large errors in the model parameters. Alternatively, a larger number of data points could be used to curve fit the

functional dependence of parameter values over a wide range. In the method chosen, a large number of data points is taken, encompassing more fully the performance of the devices. This enables the relationships postulated in Stage 1 to be tested with data not used in the model equations and parameter fitting stages. This is discussed further in Stage 3 below concerning the 'validation' of the models.

Fitting thus involves data collection and then data reduction or fitting, in this case, of data to relevant mathematical functions. It is perhaps of interest to note that, if it is impractical to obtain real data, one may be able to generate it 'synthetically' by, for example, the computer-implemented mathematical model of Treen⁽³⁾.

3. This final stage involves the validation of the model implementations of Chapter 5. That is, the model must be capable of characterizing the transistor in the three regions of operation. As we move up the hierarchy from 1 'model equations' to 3 'validation', each stage will tend to compensate for any errors in formulation or fit in the previous stage, as the criteria applied to obtain a best result at each step have a strong effect on the operation and interpretation of the next. That is, if the model proves unsatisfactory when used to forecast behaviour, it may be unclear whether it is the fitting that is at fault or whether the

basic structure is to blame. It becomes apparent that the model must be validated, by testing directly against data not previously used, at each stage.

This Chapter is concerned with the testing of the model equations implementation (Steps 2 and 3) for the three regions of operation. The parameters, intimately associated with this testing, are listed according to the measurements from which they are extracted. Computer analysis (and terminal measurement) of device behaviour has been carried out for:

- (a) d.c. conditions of operation
- (b) step transient response with voltage or current drive applied to the base or collector
- (c) small- or large-signal sinusoidal drive conditions.

In cases where the evaluation of certain parameters involves new or involved methods of determination, these methods are described in detail. Other parameters, such as current gain in the base region model, and junction depletion capacitance, do not require special measurement procedures, and in these cases, the characterization of the bias dependence of the parameter is given primary emphasis.

6.2 ACTIVE REGION MODEL

Extensive measurement studies have been made (mainly on the 1060A, 1038A and 3375 devices) to verify the sufficiency

of the active region model of Fig. 4.14, Section 4.3.3. These have involved measurement of d.c. relationships ($I_C \sim V_{BE}$, $I_E \sim V_{BC}$, β_F , β_R), of junction capacitances, and of small-signal S-parameters. Where necessary, the pulse method described below has been used to control device temperature.

6.2.1 Pulse Measurement Techniques

Because of the strong temperature dependence of the active region model parameters, especially under high current conditions, it is essential to be able to carry out measurement of device characteristics and parameters under controlled, defined temperature conditions. For this purpose, the device temperature needs to be monitored (it is not adequate to hold a transistor can or substrate at constant temperature and assume that the junction remains at constant temperature even under pulsed conditions). For any (elevated) operating temperature, the junction temperature can be found by briefly adjusting the emitter (or collector) current to a low 'reference' value and measuring V_{BE} , this voltage having been previously calibrated against temperature in an oven under d.c. conditions (the reference I_E is so low that, during the calibration measurement, dissipation in the device is negligible so that the junction temperature = that of the external ambient). V_{BE} can be used in this manner as a basis of either

measuring or controlling junction temperature, as an alternative to 'controlling' the junction temperature by supplying heat to (or removing it from) the device by means of a probe attached to the can⁽²⁾ (the probe being preset to a specified temperature). A versatile method using pulse-testing techniques and standard laboratory equipment (with V_{BE} as the 'controlled' quantity) is described below⁽²⁵⁾.

The method is indicated in Fig. 6.1. The emitter current waveform consists of three separately defined portions: (a) 'heat', (b) 'measure', (c) 'temperature check'. The purpose of (a) is to enable the mean dissipation in the device to be adjusted, by varying I_E during (a) or the duration of (a) itself. The value of V_{BE} during (c) with 'standard' I_E (e.g. 0.1mA) flowing gives the junction temperature at the instant of measurement. The measurement of device characteristics is carried out during (b) under chosen bias conditions, using sampling techniques. In practice, because of the thermal time constant of the transistor itself, it is necessary to carry out measurements and temperature checking at times close to the transition between (b) and (c) to ensure that there is negligible temperature change. For example, in Fig. 6.1, temperature change during (b) and (c) is evident from the V_{BE} waveform, but is of no consequence if the

FIG. 6.1. Pulse-testing method.

measurement and temperature checking are at the times specified. For the 2N1038A transistor used for the measurements reported in Figs. 6.2 and 6.4, the thermal time constant of the device was approximately 100 μ s. For the d.c. results, Fig. 6.2, Section 6.2.2, measurements were made within 1 μ s of the (b)-(c) transition time, and the duration of (b) was 5 μ s. Although the testing method and results are presented here (largely for simplicity) in terms of I_E , with some small elaboration of instrumentation, I_C can equally be used as the current-bias parameter.

With this method, both d.c. characteristics and S-parameters may be measured under isothermal conditions. S-parameters, measured on an H.P. scattering parameter measurement system, have, by computer conversion, given h_{fe} , and hence the device transit time $t_d = 1/\omega_T = 1/(\omega |h_{fe}|)$ as in Fig. 6.4, Section 6.2.3.1. Owing to the limited output bandwidth of the network analyzer (10kHz), some elaboration of procedure is required because of the need to make the 'measurement' period (b) sufficiently large (>100 μ s) for a reliable analogue output quantity to be obtained for the S-parameter under measurement. The junction temperature during (b) is adjusted using the repetition frequency as a fine control to bring V_{BE} to the same value as read from the isothermal curves of the d.c. characteristics (previously taken) for a given emitter current

and operating temperature.

It should be noted that, if the transistor is cooled by any convenient, uncalibrated means, the method may also be used below room temperature. An automated setup using this technique would greatly simplify the evaluation of the d.c. and a.c. characteristics at working temperatures and of the temperature dependence of transistor model parameters.

6.2.2 D.C. Test Methods

6.2.2.1 Saturation Currents

~~D.C. measurements of 'transported' current~~ $I_C \sim V_{BE}$ with V_{CB} , T as parameters and $I_E \sim V_{CB}$ with V_{EB} , T as parameters have been made to test the model sufficiency, specifically the relations for forward and inverse current transport in Fig. 4.1, Section 4.2.

$$I_C = I_{CE} = I_{SF} \left\{ \exp \frac{q V_{JE}}{n_F k T} - 1 \right\} \quad (6.1)$$

$$I_E = I_{EC} = I_{SR} \left\{ \exp \frac{q V_{JC1}}{n_R k T} - 1 \right\} \quad (6.2)$$

Equations (6.1) and (6.2) hold for reverse bias conditions at the collector-base and emitter-base junctions, respectively. Typical results are shown in Fig. 6.2 for a non-zero collector voltage bias and emphasize the need for temperature control.

FIG. 6.2. Isothermal I_C - V_{BE} characteristics taken by pulse-testing method.

The 'isothermal' results were obtained by the pulse testing method of Section 6.2.1 while, for the non-isothermal ones, obtained without application of temperature control (actually, simple d.c. biasing without a heat sink), each measured point corresponds to a different temperature.

In summary, the main findings of these measurements are:

1. The 'transport' equations (6.1) and (6.2) are capable of accurate representation of device characteristics over an exceedingly wide range of bias conditions, i.e. constant slope on a log-linear plot over typically 5 decades of current.
2. Variation of saturation currents I_{SF}, I_{SR} with the parameters V_{CB} or V_{EB} , and temperature is significant and should be represented by suitable functions 'f' and 'g', discussed below.
3. The emitter and collector 'injection indices', n_F, n_R respectively, are equal and close to the 'expected' ⁽¹⁾ value of unity (except, perhaps, at very high current levels). This property holds over a wide temperature range. For example, for the 1060A,

40°C	$n_F = 1.05$	$n_R = 1.05$
115°C	$n_F = 1.10$	$n_R = 1.10$

4. I_{SF} is equal to I_{SR} for the 2N1038A and 2N1060A devices, as expected from one-dimensional theory⁽¹⁾. This conclusion and that of 3 does not apply for all high frequency devices⁽²⁶⁾, for example, for the 2N3375 at 25°C,
 $I_{SF}=2.1 \times 10^{-14}$ A; $I_{SR}=9.6 \times 10^{-12}$ A.

Ebers and Moll⁽¹⁰⁾ showed theoretically that these two saturation currents were equal even for a two-dimensional geometry. However, in their paper, they assumed negligible drift currents compared to diffusion, which is often not the case for modern high frequency devices. Inclusion of drift fields (caused by non-uniform doping) can lead to inequalities in measured values of I_{SF}, I_{SR} for the two-dimensional structures considered.

Saturation currents I_{SF}, I_{SR} in the active region model of Fig. 4.14, Section 4.3.3, refer to current components in the intrinsic device, and since current flow from the emitter to the collector is very closely one-dimensional (see Section 2.2), it is this measured value I_{SF} that is used for both forward and reverse saturation currents in the intrinsic region model, that is:

$$I_{SF(\text{intrinsic})} = I_{SR(\text{intrinsic})} = I_{SF(\text{measured})} \quad (6.3)$$

These findings showed the need to express accurately the voltage and temperature dependencies of the saturation currents.

The expression proposed is:

$$I_{SF} = \frac{I_{SFO} \frac{n_i^2(T) D_n(T)}{n_i^2(T_o) D_n(T_o)}}{1 - b_F \int_0^{V_{JC1}} C_{JC1} dV_{JC1}} = I_{SFO} f_1(V_{JC1}) g_1(T) \quad (6.4)$$

The temperature dependence of the junction capacitance C_{JC1} results predominantly from the reduction in contact potential (see Appendix 2), which implies that temperature changes in capacitance are significant, if at all, only at forward and low reverse bias voltages. Temperature dependence of C_{JC1} has been neglected here, but may be significant for some devices.

I_{SFO} is the value of I_{SF} measured for $V_{JC1}=0$ at $T=T_o$ the reference temperature. This expression has been derived from Gummel's expression⁽¹⁾ for the reverse saturation current in terms of base charge. The integral in Equation (6.4) may be evaluated upon making a grading approximation for the collector junction. For example, assuming a linear junction approximation for C_{JC1} yields:

$$I_{SF} = \frac{I_{SFO} \frac{n_i^2(T) D_n(T)}{n_i^2(T_o) D_n(T_o)}}{1 - \frac{3}{2} b_F \phi_{C1} C_{JC1}(0) \left[\left(1 + \frac{V_{JC1}}{\phi_{C1}} \right)^{2/3} - 1 \right]} \quad (6.5)$$

Thus, assuming $C_{JC1}(0)$, ϕ_{C1} , the 'built-in potential', can be found from capacitance measurements, and analytic expressions exist for the temperature dependent terms (see Appendix 2) only I_{SFO} , b_F have to be found by curve fitting to measured $I_{SF} - V_{CB}$ data in order to define Equation (6.4). This has been carried out and the expression (6.4) with $D_n(T)$ dependence neglected has been shown to accurately model saturation currents over a wide range of bias conditions and temperature. To avoid errors due to resistance elements r_{bb} , and r_C , actual evaluation was carried out at current levels low enough to neglect differences between junction and terminal voltages. In a similar fashion, I_{SR} may be proposed.

$$I_{SR} = \frac{I_{SRO} \frac{n_i^2(T) D_n(T)}{n_i^2(T_0) D_n(T_0)}}{1 - b_R \int_0^{V_{JE}} C_{JE} dV_{JE}} = I_{SRO} f_2(V_{JE}) g_2(T) \quad (6.6)$$

6.2.2.2 Current Gain

The proposed dependency for the forward gain term is:

$$\beta_F = \beta_{FO} \left[1 + C_{1F} V_{JC1} \right] \left[1 + C_{2F} (T - T_0) \right] \left[C_{3F} + C_{4F} I_{CE}^{-1/2} + C_{5F} I_{CE} \right]^{-1} \quad (6.7)$$

$$= \beta_{FO} f_3(V_{JC1}) g_3(T) h_3(I_{CE}) \quad (6.8)$$

where the coefficients C_{1F} to C_{5F} are represented by simple expressions, for example, constants or linear terms in the three variables V_{JC1} , T , I_{CE} . The current dependence function $h_3(I_{CE})$ was developed⁽²⁷⁾ on the basis of device physics which predicts an $I_{CE}^{1/2}$ term at low current levels due to recombination in the emitter depletion region and an I_{CE}^{-1} fall-off of gain due to high level effects in the base. For the other functions, 'f' and 'g', the recombination properties of the devices are not known to the detail required for theoretical calculation of β_F and curve fitting techniques must be used. However, the general form of the dependencies may be predicted by a qualitative understanding of the physical processes involved. For example, in silicon, most of the recombination takes place at imperfections in the crystal structure, located near surfaces within the emitter space-charge layer, and in the base. As the temperature increases, the rapid thermal motion of the carriers is thought to make their 'capture' by such imperfections less probable, thus decreasing the recombination rate and increasing the current gain.

Extensive measurements of current gain (β_F and β_R) variations with voltage, temperature and current have been made and have shown the ability of the proposed dependency, Equation (6.7) to model these variations. A typical curve for the current dependency of β_F is shown in Fig. 6.3 for the 1060A.

FIG. 6.3. Typical forward and reverse current gain variations with current level.

A good fit (less than 5% difference) is noted between the measured β_F and the function $h_3(I_{CE})$ over many decades of current, except at high current levels. At these levels, the device enters quasi-saturation (as observed on the I_C - V_{CE} curves) and forward injection at the collector reduces the measured value of 'forward gain'. The function $h_3(I_{CE})$ is thus fitted to measured β_F at lower current levels, quasi-saturation 'swamping' the proposed high level fall-off of β_F . The temperature dependence of β_F varies from that reported by others⁽²⁾. In particular, although β_F is linearly dependent on temperature for a given collector current bias, this dependency increases as the bias current increases.

A similar expression to Equation (6.7) may be derived for β_R , but due to recombination in the collector, 'intrinsic base' current gain dependencies will be more difficult to extract. This is shown by the degradation in fit (up to 30% difference) of the current dependent term for β_R over the entire current range (compared with β_F), in Fig. 6.3 and emphasizes the need for modelling of the collector intrinsic and extrinsic regions.

6.2.2.3 Extrinsic Base Resistance

In Fig. 4.14 for the active regime model, transverse voltage drops in the base are modelled using a single resistance r_{bb}' . Numerical values for r_{bb}' as a function of V_{JC2} , T , I_B have been found from the fall-off of I_C - V_{EB} characteristics from the ideal exponential law, as indicated in Fig. A2.2, Appendix 2. This fall-off is caused by differences in the applied and junction emitter-base voltages due to r_{bb}' and allows its evaluation.

$$r_{bb}' = (V_{EB} - V_{JE}) / I_B \quad (6.9)$$

These measurements, taken at low to moderate current levels where quasi-saturation effects are not called into play,

show that conductivity modulation at these current levels is small and thus the current level dependency of r_{bb}' may be neglected (except in saturation where r_{bb}' is taken as zero). However, r_{bb}' dependence upon T and V_{JC2} (extrinsic base width modulation) is significant and should be included in a complete model. The base resistance r_{bb}' is an approximate representation, and intended to account for the transverse voltage drop across the ohmic base material due to the flow of various components of the base current to the base contact. It is therefore quite possible to obtain different results for

r_{bb}' from practical measurements under various conditions of base current flow. Various test methods exist⁽²⁸⁾ for the evaluation of r_{bb}' and the method most closely approximating the intended mode of operation of the model should be chosen. By the d.c. test method described above, for the 2N3375, a value of $r_{bb}' = 3.1$ ohms (at room temperature and $V_{JC2} = 0$) was found. For comparison, from structural considerations, the extrinsic base region resistance is calculated as 1.6 ohms.

6.2.2.4 Collector Resistances

In the active region model, coupling relations between depletion and ohmic regions require knowledge of the resistance of the total intrinsic collector region, r_{co} and that of the extrinsic collector r_{co}' :

$$r_c = r_{co} \left(1 - \frac{x_{dc1}}{x_e} \right) \quad (6.10)$$

$$r_c' = r_{co}' \left(1 - \frac{x_{dc2}}{x_e} \right) \quad (6.11)$$

where

$$r_{co} = \frac{x_e}{q \mu_n N_{epi} A_E} \quad (6.12)$$

$$r_{co}' = \frac{x_e}{q \mu_n N_{epi} (A_C - A_E)} \quad (6.13)$$

and x_{dc1}, x_{dc2} are the intrinsic and extrinsic junction semi-widths in collector material.

This reference resistance r_{co} is well approximated by the intrinsic collector resistance for $V_{JC1}=0$, for all coupling relations; and may be evaluated using two techniques described in Sections 6.3.1 and 6.3.2 on quasi-saturation. The extrinsic reference r_{co}' is then readily obtained from the ratio (at a reverse biased collector-base terminal voltage) of the intrinsic to extrinsic collector capacitances, Section 6.2.3.2.

$$\frac{C_{JC1}}{C_{JC2}} = \frac{A_E}{A_C - A_E} = \frac{r_{co}'}{r_{co}} \quad (6.14)$$

6.2.3 A.C. Test Methods

6.2.3.1 Transit Time

In an attempt to evaluate the forward transit time, τ_F and other parameters of the intrinsic model, S-parameter measurements on the devices studied were carried out. Frequency domain measurements involve the small-signal linearization of device behaviour about some bias point. In this manner, a 'hybrid- π ' small-signal equivalent circuit may be derived from the two charge-store intrinsic region model. It can be shown from this hybrid- π model that the transistor

cut-off frequency f_T is related to the collector current I_C in the active region by:

$$t_d = \frac{1}{2\pi f_T} = \tau_F + \frac{kT}{q I_C} (C_{JE} + C_{JC1}) + \frac{x_{dcl} + x_{dB}}{2V_d} + r_c C_{JC1} \quad (6.15)$$

where t_d is the total emitter-collector delay time. Its first component, τ_F , is the forward transit time τ_F defined in the intrinsic base model of Fig. 4.1. The second term represents the emitter depletion layer charging time, with C_{JE} , C_{JC1} the emitter and intrinsic collector space charge capacitances, respectively. The third component is the collector depletion layer transit time where $x_{dcl} + x_{dB}$ is the width of the transition layer and V_d is the field-dependent velocity of the injected carriers through this junction. Under collector reverse bias conditions, V_d is usually the scatter-limited value of 10^7 cm/sec. The last term describes the collector charging time, where r_c is the collector series resistance.

Early attempts at the non-isothermal evaluation of t_d showed a large deviation from predicted results and the need for careful isothermal evaluation of the S-parameters. Fig. 6.4 shows typical experimental results for the variation of t_d with $1/I_C$ (with V_{CB} as a parameter) under isothermal and non-isothermal conditions of test for the 2N1038A. These results showed a significant temperature dependence not pre-

FIG. 6.4. Experimental transit time results.

viously investigated in the literature. The 'non-isothermal' results are carried out with the transistor operating without a heat sink: the junction temperature is different at each bias point as indicated by the plot of T_j . The transit time is reduced below that at the lower controlled temperature of 25°C to an extent that increases as the device temperature is increased. Theoretical computations of t_d^{-1}/I_C were carried out for several doping profiles* and epi width dimensions to substantiate experimental variation of f_T with temperature and to gain insight into the various dependencies of the model parameters. This method of analysis, detailed in Appendix 2, could be used as an aid in high frequency design considerations with specific f_T requirements at working temperatures. Representative results are shown in Figs. 6.5-6.8. The same qualitative variations are observed in the experimental and computed results, but it should be noted that great flexibility in the variations with temperature is afforded by changes in epitaxial layer width. In particular, for the 2N1038A device, t_d is seen to decrease with increasing temperature, Fig. 6.4. It has been verified theoretically that, for this particular device structure, the observed temperature dependence can be attributed to a reduction in the

* The major differences between the two doping profiles used are the epitaxial layer doping density and, to a lesser extent, the maximum doping level in the base. The first factor influences the collector charging time τ_{COL} , whilst the base transit time τ_F is increased for the device with the higher doping density in the base (profile 1), as expected.

FIG. 6.5. Computed transit times.

FIG. 6.6. Computed transit times continued.

FIG. 6.7. Computed transit times continued.

FIG. 6.8. Computed transit times continued.

intrinsic forward base transit time with increasing temperature (due to increase of the carrier-diffusion coefficient); this is partially offset by an increase with temperature in the emitter time constant $\tau_E = (C_{JE} + C_{JC1})kT/qI_C$ to an extent that diminishes as I_C is increased. The theoretical studies have also shown that the temperature dependence of t_d is structure-dependent and may not be as reported here for the 2N1038A device. For example, effects associated with a wide collector epitaxial region may cause t_d to increase, rather than decrease, with increase of temperature, Fig. 6.7, or t_d may decrease with increasing temperature (at a particular bias point) over a certain temperature range and then start increasing as the temperature is raised further, as noted for the 2N3375 device⁽²⁹⁾.

From the above discussion and the analysis of Appendix 2, a theoretical evaluation of τ_F requires the actual doping profile of the devices studied. Because this is not usually available, curve fitting approximations to τ_F dependencies are used, ensuring that the predicted behaviour will match actual device responses. A detailed fit of t_d from Equation (6.15) with measured values is complex, if all parametric dependencies and coupling relationships are considered, and requires computer solution. However, from the plot of $1/\omega_T$ against $1/I_C$, Fig. 6.4, in the non-saturated region, the resulting straight

line will give, on extrapolation to $1/I_C=0$ and for negligibly small collector junction transit times:

$$\frac{1}{\omega_T(0)} = \tau_F + r_c C_{JC1} \quad (6.16)$$

With the low-level values of r_c , C_{JC1} known, τ_F can be calculated as a function of V_{JC1} (basewidth modulation) and temperature T . Degradation of f_T with current level occurs at the onset of quasi-saturation and a constant value of τ_F may be used at the lower current levels, in the active regime. A fully automated pulsed S-parameter set-up would simplify the evaluation of τ_F and other parameters greatly.

It has been shown⁽⁵⁾ that for the frequencies of interest, for the devices studied, the reverse transport current q_R/τ_R may be neglected. For lower frequency devices or for devices of different structure where this term cannot be neglected, a similar method to that for τ_F could be used but here complications arise due to minority carrier storage in the collector invalidating the analytical expression for τ_d (reverse), analogous to Equation (6.15). Other methods for τ_R have been suggested⁽²⁸⁾ but all involve difficulties on the addition of a two-lump collector model.

6.2.3.2 Junction Capacitances

Linear or exponential grading approximations are used for the emitter and collector junctions. A linear grading approximation is useful for the emitter junction, but a poor approximation for the collector. The exponential grading approximation is capable of representing either junction over a wide range of conditions and is a very good model for the collector junction of the devices concerned here, with a lightly doped collector epitaxial region⁽¹⁹⁾. Difficulties occur with these models as the applied forward bias approaches the 'contact potential'. Various solutions exist to this problem. For analysis programs considering complete depletion, the 'built-in' potential may be chosen 'artificially high', exceeding the maximum forward voltage anticipated in a circuit environment. However, with such a choice, the quality of the representation of the junction capacitance for low forward bias is degraded. The 'blow-up' difficulty may be resolved using Gummel's modification to the linear approximation for junction capacitance⁽¹⁸⁾ (used in CIRC). However, with this approach, degradation in fit is possible for applied voltages greater than the built-in voltage. Yet, another solution is to interpolate between values in the reverse bias and forward bias regions, Fig. 6.9. This technique can be used for any

FIG. 6.9. Parabolic fit to forward biased emitter capacitance values.

grading approximation and may, for this reason, be preferred. In this method, the depletion capacitances C_{JE} and C_{JC1} , C_{JC2} are measured using a.c. bridges for curve-fitting to the above grading models, for reverse bias. Interpolations are made between these values and a high forward bias value of C_{JE} (and C_{JC1} , hence C_{JC2}) obtained from the slope of the forward (and reverse) delay time against $1/I_C$, see Equation (6.15). In general, the form of interpolation (and extrapolation into the high forward bias region) should be such that the capacitance peaks as the applied voltage reaches the 'true' contact potential⁽³⁰⁾, Fig. 6.9. A parabolic fit, using a high forward bias capacitance value and a low forward bias value (where the diffusion capacitance contribution to the measured capacitance value may be considered negligible, the voltage being = contact potential/2⁽³¹⁾, say) as the data points, peaking when the applied voltage equals the 'diffusion' voltage ~0.7V for silicon, should be satisfactory. The collector capacitance, C_{JC} , measured using an a.c. admittance bridge, is the combined intrinsic and extrinsic region capacitances:

$$C_{JC} = C_{JC1} + C_{JC2} \quad (6.17)$$

A method exists⁽³²⁾ for the separation of these two capacitances, once r_{bb} is known. The ratio of these values

leads to the ratio $(A_C - A_E)/A_E$, Equation (6.14) needed for the extrinsic collector region reference resistance r_{co} '.

As mentioned in Section 6.2.2.1, there may exist significant temperature dependence of junction capacitance at low forward bias. Logan⁽²⁾ has noted a 10% change over the range 40-80°C and this temperature dependence of the junction capacitances may be modelled by the temperature variations of the contact potential, and the dielectric constant (see Appendix 2).

The Kirk effect, namely the variation of C_{JX} with transported minority carrier current for reverse-biased junctions, was found to be negligible for the devices under test, though this condition will not hold for all devices.

6.3 QUASI-SATURATION MODEL

A model is of little use unless acquisition of model parameters can be accomplished in some logical and effective manner. Four methods are considered by which these parameter values may be chosen; that is, the parameters are either:

- (a) those obtained 'directly' from device terminal characteristics
- (b) those parameters from known device structural data
- (c) those chosen 'arbitrarily' where this is justified
- (d) those evaluated via some logical 'curve fitting' procedure for determining the remaining model parameters by comparison with measured data.

Based upon this scheme, it is shown in this section that simple d.c. and transient measurements of the device response (2N3375 here) are sufficient 'terminal information' for the extraction of the model parameters.

The parameters are discussed in the sequence in which they are obtained and with regards to which method ((a)-(d)) is involved in their determination. Section 6.3.1 deals largely with the evaluation of parameters from d.c. measurements by method (a), while Section 6.3.2 is concerned, for the greater part, in curve-fitting techniques (method (d)) to measured transient responses.

Following the principles of validation put forward in Section 6.1, Sections 6.3.1 and 6.3.2 include verification of the model under similar conditions of operation as those utilized in setting up the parameter values, but over a wider range of operation. That is, the device measurements taken are used for both parameter evaluation and model verification.

In Section 6.3.3, the validity of the model is tested under conditions different to those used to obtain parameter values. This final section deals only with the 'performance' testing of the proposed model. The model's ability to accurately predict observed responses would constitute strong general evidence in favour of the model's validity.

6.3.1 D.C. Characteristics

The quasi-saturation model, whose parameter values are to be determined, is that of Fig. 5.1, Section 5.2. The intrinsic base parameters τ_F , τ_{BF} , I_{SF} , n_F are found using the techniques discussed for the active region model, Section 6.2. This section (6.3.1) is concerned with the evaluation of the collector region parameters, these parameters being listed, together with a brief description, in Table 6.1.

TABLE 6.1. Collector Region Parameters for Quasi-Saturation

<u>Collector Region Parameter</u>	<u>Description.</u>
V_0	Constant voltage between intrinsic base and ohmic edge of injection region
r_{co}	Resistance of total intrinsic epitaxial collector region
K	Ratio of widths of excess charge lump q_1 to charge lump q_2
τ_{40}	'Diffusion' time constant for current flow out of the injection region
τ_1, τ_2	Lifetimes in first and second excess charge regions respectively
$C_{\mu 0}$	Collector width modulation constant
τ_{30}	'Diffusion' time constant for current transport between charge lumps q_1 and q_2

As previously mentioned, these parameters are discussed in the order in which they are determined.

Fig. 6.10 shows measured collector characteristics (in common emitter) for a 2N3375 transistor, displaying the proposed loci of the boundaries of the three regions of operation. The position of these boundaries is evident from the inflections on these characteristics. At the saturation-quasi-saturation boundary (points a,b,c) we have, utilizing the analysis of Section 2.4:

$$V_{CB'} = V_{CE} + V_{EB} + I_b r_{bb'} = V_0 \quad (6.18)$$

That is, at this boundary, the injection region occupies the entire intrinsic epitaxial region, r_c thus approaches zero and the 'applied' collector-internal base voltage $V_{CB'}$ equals V_0 . This boundary condition may be used to evaluate V_0 from the I_C - V_{CE} curves. If a distinct inflection is visible in these curves at this boundary, the value of $V_{CB'}$ evaluated from Equation (6.18) will give V_0 . Values for V_0 were computed for several curves (base current as parameter) and shown below in Table 6.2. Excellent agreement (less than 5% variation) was obtained between these values.

TABLE 6.2. Values of V_0 from Common Emitter Curves.

25°C $r_{bb'} = 3.1\Omega$ (Section 6.2.2.3)	I_B mA	V_{CE}	V_{EB}^*	V_0
	1	0.10	-0.72	-0.62
	2	0.16	-0.78	-0.61
	3	0.20	-0.80	-0.59
	4	0.22	-0.81	-0.58
	5	0.24	-0.82	-0.57

* from I_C - V_{EB} curve (Section 6.2.2.1)

FIG. 6.10. Measured collector characteristics.

As an alternative to this 'direct' measurement technique for parameter evaluation, V_0 may be computed at the temperature in question, if the doping profile is accurately known since (Section 2.4)

$$V_0 = -\frac{kT}{q} \ln \left(\frac{N_{\text{epi}}}{n_i} \right)^2 \quad (6.19)$$

The doping profile of the 3375 is available⁽⁹⁾ and allows comparison of experimental and theoretical values of V_0 . For this device, $N_{\text{epi}} = 1.6 \times 10^{15} / \text{cm}^3$, and at 25°C, we have from Equation (6.19):

$$V_0 = -0.60\text{V} \quad (6.20)$$

compared with an average value of $V_0 = -0.59\text{V}$ from Table 6.2.

If the profile is not known, the experimentally obtained value of V_0 can be used to estimate N_{epi} . This information is useful in expressing $r_{\text{co}}(T)$ dependence from mobility-temperature curves if required, and for estimating the product $D_n(N_{\text{epi}})\rho_n(N_{\text{epi}})$ (required for q_{10} in Table 6.13, Section 6.4.1).

Once V_0 is found, the collector resistance r_c at any bias point I_C, V_{CB} , (in quasi-saturation) may be evaluated using Equation (3.14):

$$r_c = r_{\text{co}} \left(1 - \frac{x_i}{x_e} \right) = \frac{V_{\text{CB}} - V_0}{I_C} \quad (3.14)$$

The collector resistance r_c has been determined by this technique for several collector currents, and values are shown in column 4 of Table 6.3 below. A value of $V_0 = -0.60V$ has been used for this purpose.

TABLE 6.3. Measured and Computed Values of r_c .

I_C	I_B	V_{CE}	r_c (from d.c. characteristics)	r_c (from transient characteristics)	r_c (computed from d.c. model)
mA	mA	V	Ω	Ω	Ω
136	3	1.0	5.90	5.49	6.16
115	3	0.6	4.04	3.89	3.60
66	3	0.3	1.64	1.78	1.84

If distinct inflection points (points 1,2,3 in Fig. 6.10) exist at the active-quasi-saturation regions boundary, r_{co} may also be evaluated in this fashion. Difficulties in determining the boundary point may be resolved by measuring current gain β_F for the device at the required V_{CE} . I_B is increased until $\beta_F = I_C/I_B$ starts to fall drastically, as in Fig. 6.3. This point we may consider the onset of quasi-saturation and then obtain r_{co} from Equation (3.14). Values of r_{co} from the inflection points using Equation (3.14) are shown in Table 6.4 for various I_C - V_{CE} curves (base current as parameter). Results for r_{co} from these curves agree to within 10%. This is deemed satisfactory, considering the difficulty in precise determination of the quasi-saturation boundary.

TABLE 6.4. Values of r_{co} from $I_C \sim V_{CE}$ Curves.

$V_0 = -0.60V$	I_C	I_B	V_{CE}	V_{EB}	r_{co}	$r_{co}(AVE)$
$25^\circ C$	mA	mA	V	V	Ω	Ω
$r_{bb'} = 3.1\Omega$						
	233	5	2.20	0.83	8.5	8.5
	181	4	1.60	0.83	7.7	
	138	3	1.50	0.82	9.3	
	88	2	0.90	0.80	8.0	
	39	1	0.52	0.77	9.1	

The average value, 8.5Ω , is chosen for r_{co} . This value compares favourably with $r_{co} = 9\Omega$ determined by Anderson et al⁽⁹⁾ using an admittance bridge method for the 2N3375 device. r_{co} may also be evaluated from the device geometry. For $A_E = (12.7\mu m)^2$ and $x_e \approx 10\mu m$, we have $r_{co} \approx 11.6\Omega$ compared with the 'direct' measurement value of 8.5Ω . This approximate agreement supports the thesis of one-dimensional flow under the emitter. It may further be noted that, since r_c at any bias point and r_{co} are known, x_i/x_e is defined at that operating point, Equation (3.14). If the other parameters of the model (Fig. 5.1, Section 5.2) are known, the current flowing in any branch of the model may be found, at the bias in question, using:

$$\tau_3 = \tau_{30} \left(\frac{x_i}{x_e} \right)^2 \quad (6.21)$$

$$\tau_4 = \tau_{40} \left(\frac{x_i}{x_e} \right)^2 = \frac{\tau_3}{K(1+K)} \quad (6.22)$$

$$C_{\mu c} = C_{\mu o} \left(\frac{x_i}{x_e} \right)^2 \quad (6.23)$$

$$r_c = r_{co} \left(1 - \frac{x_i}{x_e} \right) \quad (6.24)$$

Hence, with this method for x_i/x_e , the small-signal (fixed width) model can be determined at each d.c. bias point of interest.

K, the ratio of the widths of the two lumps q_1, q_2 , which describe the injected charge in the epitaxial region, is given by (Section 3.2):

$$K = \frac{x_1}{x_2} \quad (3.3)$$

The current transport equations through and out of the injection region are set up, based upon an unspecified value of $0 < K < \infty$, and for convenience, a value of $K=1$ is arbitrarily chosen. Once K is fixed, time constant τ_{40} describing current flow out of the injection region, is given by Equation (6.22) in terms of τ_{30} (to be evaluated in Section 6.3.2).

As a first step towards the evaluation of τ_{30} and the bulk lifetimes τ_1, τ_2 in the injection region, let us consider the ratio of the d.c. collector current to the component of base current supporting recombination in the collector injection region; that is, from Fig. 5.1:

3

OF/DE

3

$$\frac{I_C}{I_{B(\text{coll.})}} = \frac{q_2/\tau_4}{\frac{q_1}{\tau_1} + \frac{q_2}{\tau_2}} \quad (6.25)$$

By using the triangular distribution of charge in the injection region (of relative width x_1/x_e), Equation (6.22) and

$$\tau_2^s = M \tau_1 \quad (6.26)$$

we may readily show that

$$\frac{I_C}{I_{B(\text{coll.})}} = \frac{K(K+1)}{K(K+2) + \frac{1}{M}} \frac{\tau_1}{\tau_{30} \left(\frac{x_1}{x_e} \right)^2} \quad (6.27)$$

Parameters τ_1, τ_2 are the lifetimes (in material of doping N_{epi}) under large injection conditions, since in this injection region

$$n, p \gg N_{\text{epi}} \quad (6.28)$$

We may thus expect τ_1 and τ_2 to approach the same high-injection level asymptotic value⁽³³⁾. As far as the model is concerned, q_R is included (by 'transference through the junction' as per the earlier discussion of Section 5.2) with q_1 . Because of the large volume of epi and the lower doping of this region compared with the base, the effect of τ_{BR} is very small, except for conditions where x_1 tends to 0; therefore, it is reasonable to choose M equal to 1 in Equation (6.26).

Then, in Equation (6.27), if a value of $I_C/I_{B(\text{coll.})}$ can be found experimentally, the ratio τ_1/τ_{30} (and τ_2/τ_{30}) may be estimated. For a point (I_C, V_{CE}) in quasi-saturation on the common emitter characteristics of Fig. 6.10, we have:

$$I_{B(\text{coll.})} = I_B - q_F/\tau_{BF} \quad (6.29)$$

where I_B is the total base current, τ_{BF} is the (predetermined) active base region 'forward' lifetime (Fig. 5.1, Section 5.2) of the forward charge lump q_F . This equation is rigorously correct due to the inclusion of τ_{BR} with τ_1 . Now, q_F/τ_{BF} is given by:

$$\frac{q_F}{\tau_{BF}} = \frac{q_F}{\tau_F} \cdot \left(\frac{\tau_F}{\tau_{BF}} \right) = (I_C + I_{B(\text{coll.})}) \cdot \frac{\tau_F}{\tau_{BF}} \quad (6.30)$$

where τ_{BF}/τ_F is the intrinsic base region current gain β_F , evaluated from active region measurements. On substituting Equation (6.30) in Equation (6.29), we obtain:

$$I_{B(\text{coll.})} = I_B - (I_C + I_{B(\text{coll.})}) \frac{1}{\beta_F} \quad (6.31)$$

or

$$I_{B(\text{coll.})} = (I_B - I_C/\beta_F)/(1 + 1/\beta_F) \quad (6.32)$$

or

$$\frac{I_C}{I_{B(\text{coll.})}} = \frac{I_C(1 + 1/\beta_F)}{I_B - I_C/\beta_F} \quad (6.33)$$

For a set of operating points in quasi-saturation, the ratio $I_C/I_{B(\text{coll.})}$ is determined using Equation (6.33) and these values are used in Equation (6.27) to provide estimates for the ratio τ_1/τ_{30} . The results of this procedure are displayed in Table 6.5.

TABLE 6.5. Estimates for τ_1/τ_{30} .

$V_0 = -0.60V$ $r_{co} = 8.5\Omega$ $r_{bb'} = 3.1\Omega$ $K=1$ $M=1$ $\beta_F=50$	I_C mA	I_B mA	V_{CE} V	V_{EB} V	τ_1/τ_{30}
	205.0	5	1.0	0.83	77.2
	175.0	4	1.0	0.822	83.7
	120.0	3	0.6	0.81	80.7
	76.5	2	0.4	0.795	80.8
	31.0	1	0.2	0.78	70.8

An average value, 78.6, is chosen for this ratio, actual values varying less than $\pm 10\%$ from this average. Again, the excellent results support the validity of the various approximations made.

It is noted at this stage that, for the d.c. quasi-saturation model, only the ratio of τ_1 and τ_{30} is required, not their 'actual' values (in the same manner as only the ratio $\tau_{BF}/\tau_F = \beta_F$ is required for the intrinsic base region d.c. model). Thus, providing these ratios are maintained correct, the choice of τ_{30} and τ_F (say) is somewhat arbitrary. They are limited in possible value only by accuracy considerations for the circuit analysis program used. Collector width modulation is not involved in the computation of d.c. characteristics and an arbitrary choice of $C_{\mu 0}$ may be made.

Table 6.6 is a summary of the methods of evaluation and values for the parameters of the d.c. model. D.C. common emitter characteristics in quasi-saturation are computed for this parameter set and are shown in Fig. 6.11 together with

TABLE 6.6. Model Parameter Values for D.C. Computations.

intrinsic base region	Parameter	Value	Method of Determination
	I_{SF}	$.21 \times 10^{-13} \text{ A}$	$I_C - V_{EB}$ intercept
	n_F	1	$I_C - V_{EB}$ slope
	τ_F	arbitrary	
	τ_{BF}	$50 \times \tau_F$	current gain in active region
	V_0	-0.60V 0	'Knee' on $I_C - V_{CE}$ curve (or from doping profile)
	r_{co}	8.5 Ω	'Knee' on $I_C - V_{CE}$ curve (or from structural data)
	K	1	arbitrary
	τ_{40}	$\tau_{30}/K(K+1)$	
	τ_{30}	arbitrary	
	τ_1	$78.6 \times \tau_{30}$	from ratio of $I_C/I_B(\text{coll.})$
	τ_2	$M \tau_1 = \tau_2$ $M=1$	theoretical prediction
	$C_{\mu o}$	arbitrary

FIG. 6.11. Measured and computed quasi-saturation characteristics.

measured $I_C \sim V_{CE}$ values. These curves demonstrate the ability of the model to give good (less than 6% variation between computed and measured curves) results over the entire quasi-saturation region of operation not just at the bias points used to evaluate the model parameters. This agreement supports the method of d.c. parameter evaluation proposed. This method is recommended as the most effective approach to model parameter evaluation due to the great difficulty encountered in matching curves with 'trial and error' variation of an initially arbitrarily chosen set of collector region parameters.

From the model computations, it is noted that the relative width x_i/x_e of the injection region decreases from 1 at the quasi-saturation-saturation boundary (dashed lines) to 0 at the quasi-saturation-active region boundary (dashed lines), as expected. This is illustrated in column 6 of Table 6.3 where computed values of r_c are listed (for variation of bias point along the $I_B = 3\text{mA}$ characteristic) and compared with measured r_c values (column 4).

Small variations in turn on each collector region parameter were made in an attempt to improve the correlation between measured and 'model' curves but with no significant success. Simultaneous display of computed and measured characteristics on a curve tracer would greatly facilitate

the 'improvement' of these parameters. This could be accomplished by imbedding the transistor model in a circuit environment similar to that used in the curve tracer (for the actual device response) and sweeping out the collector characteristics in an identical fashion. The model parameters could be varied at will, the effect of these changes would be noted immediately, and a feel could be obtained for the 'best' parameter set to match the measured curves. This method could prove a good alternative to the involved task of computer optimization of the parameters.

The extent of quasi-saturation in devices of this type is seen by comparison of measured responses with those computed from the Ebers-Moll model of Fig. 6.12. These characteristics are displayed in Fig. 6.13 and comparison with the computations of Fig. 6.11 shows the need for a model describing effects caused by quasi-saturation.

By comparison of the Ebers-Moll 'injection type' model with the 'transport type' intrinsic base region representation (used in the quasi-saturation model of Fig. 5.1, Section 5.2), we may derive a correspondence between the parameters of each model.

$$I_N = I_{SN} \left(\exp \frac{qV_{EB}}{n_N kT} - 1 \right)$$

$$I_I = I_{SI} \left(\exp \frac{qV_{CB}}{n_I kT} - 1 \right)$$

FIG. 6.12. Ebers-Moll model.

FIG. 6.13. Ebers-Moll and measured DC characteristics.

Ebers-Moll

Quasi-Saturation

$$I_N = I_{SN} \left(\exp \frac{q V_{EB}}{n_N k T} - 1 \right) = \frac{q_N}{\tau_N} \equiv \frac{q_F}{\tau_F} + \frac{q_F}{\tau_{BF}} = (1 + 1/\beta_F) I_{SF} \left(\exp \frac{q V_{EB}}{n_F k T} - 1 \right) \quad (6.34)$$

Hence,

$$I_{SN} = I_{SF} (1 + 1/\beta_F) \quad (6.35)$$

$$\frac{1}{\tau_N} = \frac{1}{\tau_F} + \frac{1}{\tau_{BF}} \quad (6.36)$$

$$n_N = n_F \quad (6.37)$$

$$\beta_N = \beta_F \quad (6.38)$$

The measurements of Fig. 6.3 show β_R varying between 0.3 and 4, while β_F varies from 5 to 50, over the same range of transported currents I_{EC} and I_{CE} , respectively. These results are for the same range of V_{JC} and V_{JE} respectively, since the saturation current I_S is the same for forward and reverse transport. In the quasi-saturation mode, however, $|V_{JE}|$ should be much greater than $|V_{JC}|$ over most of the region (starting at the active/quasi-saturation boundary with $V_{JC}=0$, and with V_{JC} nearing V_{JE} as saturation is approached). Thus, it seems reasonable to assume $\beta_R \ll \beta_F$ (β_R at low end of above range when β_F is at high end). Under these conditions, for quasi-

saturation, the assumption of $\beta_R \approx 0$ seems appropriate for devices of the type and rating for which quasi-saturation is a significant effect. On the other hand, in saturation, this assumption would appear not to be valid.

Thus, for the quasi-saturation model, we have:

$$\beta_I = \beta_R = 0 \quad (6.39)$$

and for one-dimensional flow

$$I_{SI} = I_{SR} = I_{SF} \quad (6.40)$$

$$n_I = n_R = n_F \quad (6.41)$$

For a single lifetime representation of recombination in the collector

$$\tau_I = \tau_1 \quad (6.42)$$

The actual d.c. parameter values used are listed below in Table 6.7.

TABLE 6.7. Parameter Values for DC Ebers-Moll Model.

	Parameter	Value
'normal' parameters	I_{SN}	$.214 \times 10^{-13} \text{ A}$
	n_N	1
	β_N	50
'inverse' parameters	I_{SI}	$.21 \times 10^{-13} \text{ A}$
	n_I	1
	β_I	0

From Fig. 6.13, the Ebers-Moll model is seen to poorly predict behaviour at the low collector-emitter voltages of interest. Even if β_I is increased from the low value expected in quasi-saturation (≈ 0) to its maximum measured value (≈ 3 , expected in full-saturation), the computed curve (dashed lines for $I_B = 3\text{mA}$) is changed only slightly and shows very poor agreement with the measured behaviour. The inclusion of a resistance in series with the collector fails to remedy the situation to any large extent (Fig. 6.14 $\beta_I = 0$ solid line, $\beta_I = 3$ dashed line). The value $R_C = 6\Omega$ was chosen to give agreement between measured and computed I_C values at one bias point ($V_{CE} = 1.4\text{V}$, $I_B = 3\text{mA}$ in this case).

6.3.2 Collector Transient Response

It may be shown, from the large-signal width-modulation analysis of Section 4.2.1, that the collector width-modulation parameter $C_{\mu 0}$ is related to τ_{30} by:

$$C_{\mu 0} = \frac{1+K}{K} \frac{\tau_{30}}{\tau_{c0}} \quad (6.43)$$

and thus, from the considerations of the previous section, only the 'absolute' value of τ_{30} need be determined for the set of model parameters to be complete; enabling the calculation of large-signal model responses. Just as the 'absolute'

FIG. 6.14. Effect of collector series resistance on Ebers-Moll characteristics.

value of the base transit time τ_F must be determined by some dynamic test method (Section 6.2.3.1), so must the injection region 'transit time' constant τ_{30} .

The method used is to switch the transistor from one quasi-saturation state to another by applying step voltage drive to the collector (with the transistor in common emitter configuration, base current defined constant). It is evident that by applying a voltage stimulus to the collector, the storage region width x_i and hence τ_3 is directly controlled. This test method is thus to be preferred to other more 'indirect' tests such as an applied step current drive to the base (Section 6.3.3) for the evaluation of τ_{30} . Due to the approximate nature of the collector width-modulation analysis of Section 4.2.1, the integration step size, used for the model computations, is not arbitrary but must be chosen according to the procedure outlined below.

By applying this voltage step, the transistor may be taken to a state less deeply in quasi-saturation (say); x_i is therefore reduced and the charge stored in the collector is also reduced. The current response represents the dynamic behaviour of the charge storage region of the collector and is the source of data from which τ_{30} may be deduced. Using the test circuit of Fig. 6.15a, current responses to collector

(a) Test circuit for 'transient' evaluation of r_c .

(b) Typical response for 'transient' determination of r_c .

FIG. 6.15. 'Transient' evaluation of r_c .

voltage steps were measured at various bias points, a typical response being shown in Fig. 6.15b. From the discussion on collector width-modulation, when the collector voltage undergoes a step change, the width of the injection region x_i cannot change instantaneously. We have:

$$I_C = \frac{V_{CB} - V_0}{r_c} \quad (2.17)$$

and thus may write:

$$\Delta I_C = \frac{\Delta V_{cb}}{r_c} \quad (6.44)$$

where ΔV_{CB} is the applied step change in V_{CB} and ΔI_C is the initial step change in I_C . The collector resistance r_c can be found immediately from this test, upon measuring ΔI_C and ΔV_{cb} and then utilizing Equation (6.44). Values of r_c determined by this method are shown below in Table 6.8 and are compared in Table 6.3 with values of r_c found at the same bias points by the d.c. method proposed in Section 6.3.1.

TABLE 6.8. Collector Resistance Values from Transient Test.

I_C mA	V_{CE} V	ΔI_C mA	ΔV_{CE} mV	r_c Ω
136	1.0	9.1	50	5.49
115	0.6	15.4	60	3.89
66	0.3	28.0	50	1.78

Agreement is within $\pm 10\%$ and is considered reasonable due to the uncertainty in measuring ΔI_C because of initial 'ringing' in the collector circuit, Fig. 6.15b. This transient method for r_c may be used for an ill-defined boundary between saturation and quasi-saturation; that is, when direct estimates of V_0 are difficult to make. By substituting r_c (from the ΔV_{CB} test) in Equation (2.17), estimates of V_0 may be made at any operating point in quasi-saturation.

This same test method is used to determine the injection region 'transit time' constant τ_{30} by comparison of computed collector current waveforms with those experimentally determined. The delay time t_s to go from the initial to the final quasi-saturation state, Fig. 6.16a, is used for this purpose. The magnitude of τ_{30} (and hence also $\tau_1, \tau_2 = 78.6\tau_{30}$ here, and $C_{\mu 0}$ from Equation (6.43)) is varied until the experimental and computed t_s values agree (for the same drive conditions). The results are shown in Fig. 6.16a for τ_{30} decreasing from 60ns to the final chosen value of 4.3ns. For this latter value, agreement ($t_s = 0.9\mu s$ cf $1.0\mu s$) is considered good, as is the general form of the responses, considering the simplifications made in the quasi-saturation model to make the problem tractable for the CIRC analysis program. Parameters used are summarized in Table 6.9. Several operating points, other than that chosen

(MAGNIFIED)

(b) Applied V_{CB} waveform in transient test. (a) Matching of transient responses to give τ_{30} .FIG. 6.16. Response of collector to step change in V_{CB} .

for parameter evaluation, were selected and current responses using this same test method, but with different ΔV_{CB} , were observed. Correlation between measured and computed current waveforms was of the same order as that shown in Fig. 6.16a and gave support to the choice of parameter values made and the validity of the model. As for the d.c. case, simultaneous display of measured and computed transient responses would greatly ease the evaluation of τ_{30} and comparison of curves for a wide range of drive conditions.

As previously stated, at the 'instant' of application of the collector voltage 'step', the injection region width x_i cannot change. Thus, in the model of Fig. 5.1, Section 5.2, we require that the step increase in collector current (passing through the ohmic region) flows through the collector width modulation 'capacitance' $C_{\mu c}$ as a capacitive charging current

$$\Delta I_C = \frac{\Delta V_{CB}}{r_c} = C_{\mu o} \left(\frac{x_i}{x_e} \right)^2 \frac{\Delta V_{CB}}{\Delta t} \quad (6.45)$$

where Δt is the model integration time step to be chosen, and $C_{\mu o}$ is given by Equation (6.43). The step size Δt is thus given by:

$$\Delta t = C_{\mu o} r_{co} \left(1 - \frac{x_i}{x_e} \right) \left(\frac{x_i}{x_e} \right)^2 \quad (6.46)$$

TABLE 6.9. 'Complete' Model Parameter Values for Quasi-Saturation.

Parameter	Value	Method of Determination
I_{SF}	$.21 \times 10^{-13} \text{ A}$	$I_C \sim V_{EB}$ intercept
n_F	1	$I_C \sim V_{EB}$ slope
τ_F	.28ns	$1/f_T$ intercept
τ_{BF}	$50\tau_F = 14\text{ns}$	current gain in active region
V_0	-0.60V	'Knee' on $I_C \sim V_{CE}$ curve
τ_{co}	8.5n	'Knee' on $I_C \sim V_{CE}$ curve
K	1	arbitrary
τ_{40}	$\frac{\tau_{30}}{K(K+1)} = 2.15\text{ns}$	from excess charge triangular distribution
τ_{30}	4.3ns	'curve fit' to ΔV_{CE} response
τ_1	$78.6\tau_{30} = .34\mu\text{s}$	from ratio of $I_C/I_{B(\text{coll.})}$
τ_2	$M = \tau_1/\tau_2 = 1$	theoretical prediction
$C_{\mu 0}$	1012pF	from τ_{30} , τ_{co} via Eq. (6.43)

That is, in simulation, for a 'sudden' change of V_{CB} by ΔV_{CB} , for the initial current to be correctly simulated, it is necessary for Δt to be as given by Equation (6.46). The applied V_{CB} waveform is then as shown in Fig. 6.16b. According to the theoretical discussion of Section 4.2.1, the correct charge is then removed from the injection region (at this stage mainly from lump 2) to agree with the new V_{CB} in the steady

state, and this charge redistributes during the subsequent transient. If some other Δt is chosen, then the relationships of Equations (6.45) and (6.43) do not agree. This is a consequence of the simple model used to represent the distributed width modulation effect. Time step Δt is seen from Equation (6.46) to be a function of operating point (x_i/x_e). It is, however, a slowly varying function over nearly all the region of quasi-saturation operation, and a constant value of Δt , evaluated at a 'representative' value of x_i/x_e , will lead to only small errors in the current conditions required at the instant of application of the step. For $x_i/x_e = 0.4$ and $C_{\mu 0} = 1012 \text{ pF}$, we have $\Delta t = 8.25 \times 10^{-10} \text{ secs}$. This choice of Δt was verified by measuring the initial current change ΔI_C on the computed waveform of Fig. 6.16a. The value $\Delta t = 8.25 \times 10^{-10} \text{ secs}$ lead to an initial current step of $\Delta V_{CB}/r_{C_{\text{initial}}}$ as required. For other choices of Δt , x_i/x_e changes and ΔI_C is not as predicted by Equation (6.45).

The value of τ_{30} may be compared with that expected from structural data. From Section 2.4, we have:

$$\tau_{30} = \frac{K}{1+K} \frac{x_e^2}{4D_n} \quad (6.47)$$

where x_e is the width of the epi region, and D_n is the electron diffusion constant in the epitaxial layer. From the profile

and device structure⁽⁹⁾, we thus obtain:

$$\begin{aligned}\tau_{30} &= \frac{1}{2} \cdot \frac{(10 \times 10^{-4})^2 \text{ sec}}{4 \times 30} \\ &= 4.17 \text{ ns}\end{aligned}\tag{6.48}$$

in excellent agreement with the experimentally determined value of 4.3 ns.

In practice, the epitaxial region lifetimes τ_1, τ_2 can only be evaluated experimentally since the recombination properties are not known in sufficient detail to allow their prediction (see Section 6.2.2.2). For example, from Table 6.9, τ_{BF} has a value of 14 ns compared with that 'expected' for the bulk base region of the order of 1 μ s. This is because most of the recombination occurs near the surface in the emitter depletion layer rather than in the bulk neutral regions of the base. From Section 5.2, we have for τ_1 :

$$\tau_1 = \frac{\tau_1'}{1 + b \frac{\tau_1'}{\tau_{BR}}}\tag{6.49}$$

where τ_1' is the epitaxial bulk lifetime, b is the ratio of excess reverse base charge q_R to excess charge q_1 in the first lump and τ_{BR} is the reverse base lifetime, which may include significant recombination in the collector depletion layer.

Considering only neutral bulk lifetimes and 'b' less than 0.1 for the 2N3375, τ_1 would normally not vary significantly from τ_1' . However, if τ_{BR} is reduced from this 'bulk' value in a similar fashion to τ_{BF} , τ_1 could differ considerably from τ_1' . For example, if $\tau_{BR} = \tau_{BF}$ we have, from Equation (6.49), using $\tau_1' = 1.45\mu s$ (large injection value):

$$\tau_1 = \frac{1.45}{1 + \frac{0.1 \times 1.45}{0.014}} = 0.12\mu s \quad (6.50)$$

rather less than the experimental value ($0.34\mu s$). Differences in the ratio $M = \tau_1/\tau_2$ from unity should, strictly, then be considered to improve the degree of fit between device and model responses.

As for the d.c. case, the computed response of the 'corresponding' Ebers-Moll model, Fig. 6.12, is evaluated, for comparison purposes. Calculated transient responses are shown in Fig. 6.17 for the parameter set of Table 6.10 and compared to device results.

TABLE 6.10. Large-Signal Ebers-Moll Parameters.

	Parameter	Value
'normal' parameters	I_{SN}	$.214 \times 10^{-13} A$
	n_N	1
	β_N	50
	τ_N	.275ns
'inverse' parameters	I_{SI}	$.21 \times 10^{-13} A$
	n_I	1
	β_I	0
	τ_I	.34 μs

'MODIFIED' EBERS-MOLL

$$R_C = 5.26\Omega, I_B = 3\text{mA}$$

$$V_{CE(\text{applied})} = 0.8\text{V}$$

$$V_{CE(\text{junction})} = 0.14\text{V}$$

$$< 30\text{secs CPU time}$$

$$\Delta V_{CE} = 50\text{mV}$$

FIG. 6.17. Ebers-Moll model transient response.

The response of the Ebers-Moll model when unmodified by a resistance R_C in series with the collector (Section 6.3.1) is first considered. With $R_C=0$, no change in collector current is found for a step change ΔV_{CE} in collector bias. This is expected for an increase in reverse bias upon an already reverse-biased collector-base junction.

If R_C is increased to 5.26Ω to ensure the same d.c. operating point for the Ebers-Moll model as actually observed (for $V_{CE(\text{applied})}=0.8V$, $I_B=3mA$), the response of the Ebers-Moll model is of the 'correct form, with the observed value of initial current step. The model, however, comes to a final steady-state collector current value much higher than that observed, due to the model's inability to match measured d.c. characteristics in the low V_{CE} region. This 'modified' Ebers-Moll model suffers from the fact that, to give increased performance over the 'original' Ebers-Moll formulation, a series collector resistance must be specified. However, this R_C varies in a manner that may be estimated only by curve-fitting of computed and measured responses at every bias point of interest, rendering the model practically unuseable for prediction of response for a wide range of conditions of operation.

Sections 6.3.1 and 6.3.2 present a systematic method for the determination of the collector region parameters using two straightforward experimental techniques. With this parameter set, device and model responses were found by the same test methods but over a wider range of conditions than used for the actual parameter evaluation. The good correspondence between measured and computed characteristics indicates the ability of the model to predict device behaviour.

6.3.3 'Performance' Tests

In this section, two tests are performed upon the 2N3375 device and its proposed model. These tests are of a different nature to those used for the determination of the model parameters, Table 6.9; to support the claims of validity put forward for the quasi-saturation model in Sections 6.3.1 and 6.3.2.

6.3.3.1 Step Base Current Drive

The first 'performance' test proposed is, however, in some respects similar to the transient test of the last section. It consists of switching the device (and its model) from one quasi-saturation state to another by applying a step in base current drive (with the transistor in common emitter). The measured delay time, t_s , to go from the initial to the

final state is then compared with that computed from the quasi-saturation (and Ebers-Moll) model using the parameter values of Tables 6.9 and 6.10. The test circuit used, and typical results, are shown in Figs. 6.18a and 6.18b for the device and its models. The device operating point moves down the (220Ω , $V_{\text{supply}}=13.9\text{V}$) load line from the initial state ($I_B=2\text{mA}$, $V_{CE}=0.3\text{V}$) to its final state ($I_B=1.65\text{mA}$, $V_{CE}=0.39\text{V}$) in approximately $2\mu\text{sec}$ compared with a predicted t_s of $1.7\mu\text{sec}$ from the quasi-saturation model. This order of agreement for t_s is considered good for a test using parameters determined from a different set of driving conditions.

As before, the Ebers-Moll model (with $R_C=0$) gives the same order of agreement for t_s ; however, the initial or final steady state is considerably different to that observed unless an 'artificial' series resistance is chosen, of the correct value ($R_C=3\Omega$ here).

6.3.3.2 Large-Signal Sinusoidal Drive

The second test intended as a verification of the adequacy of the proposed model consists of dynamic operation across the quasi-saturation region. The transistor functions as a large-signal common emitter amplifier, Fig. 6.19a, operating at a high frequency of the order of the base cut-off

(a) Base current step performance test conditions.

(b) Base current step test results.

FIG. 6.18. Response of collector to step change in base current.

(a) Common emitter amplifier for large-signal base drive.

(b) DC characteristics and load line for large-signal drive responses.

FIG. 6.19. Operating conditions for large-signal drive responses.

frequency $f_{T_b} = 1/2\pi\tau_F$ (~500MHz for the 2N3375). Source V_G supplies a large-signal voltage sinusoid to the base capable of driving the device completely through the quasi-saturation region.

Fig. 6.19b shows a part of the measured and computed d.c. characteristics of Fig. 6.11, with superimposed upon them the load line along which the dynamic operation of the device and its quasi-saturation model takes place. The base voltage drive is chosen so that the transistor swings from the edge of quasi-saturation (r_c approaching r_{c0}) into deeper quasi-saturation (r_c less than r_{c0}).

The Ebers-Moll model is also driven under the same conditions for comparison purposes and its d.c. characteristics and 'large-signal load line' are shown in Fig. 6.20.

The adequacy of the models used is determined by their ability to predict the collector current responses observed for the device under a range of voltage drive (V_G) amplitudes. Specifically, the time delay between the input voltage drive and output current maximum, and the magnitude of that response (I_C) are considered for the 2N3375 and its quasi-saturation and Ebers-Moll models. Fig. 6.21 shows these large-signal characteristics for small variations in V_G , namely 40mV pp. The frequency of the source V_G is chosen at 100MHz ($\sim f_{T_b}$).

FIG. 6.20. Ebers-Moll DC characteristics and load line for large-signal drive responses.

FIG. 6.21. Large-signal sinusoidal current responses (smallest voltage drive condition).

giving a period $T=1/10^8$ secs with a maximum at $T/4$, or 25×10^{-10} secs. The three current responses are seen to be approximately sinusoidal, but with the device and quasi-saturation model amplitudes considerably smaller than that computed using the Ebers-Moll model. This ('E-M') model has a collector current maximum at approximately $T/2$ or 50×10^{-10} secs giving a time delay between output and input of $T/4$ secs or $\pi/2$ radians. This is explained by the following considerations. Neglecting depletion layer charging currents, the base current I_B consists of two components (see Fig. 6.12, Section 6.3.2) for the Ebers-Moll model operating in the normal active mode.

$$I_B = \frac{I_N}{\beta_N} + \tau_N \frac{dI_N}{dt} \quad (6.51)$$

and

$$I_C = I_N \frac{\beta_N}{1+\beta_N} \quad (6.52)$$

At the high frequencies considered, the second or base charging current component in Equation (6.51) is generally much larger than the first and we may write:

$$I_B \approx \tau_N \frac{dI_N}{dt} \quad (6.53)$$

or

$$I_N = \frac{1}{\tau_N} \int I_B dt \quad (6.54)$$

From Equations (6.54) and (6.52), the collector current response becomes the integral of the base current drive:

$$I_C = \frac{\beta_N}{1+\beta_N} \frac{1}{\tau_N} \int I_B dt \quad (6.55)$$

For the normal active mode considered, the sinusoidal voltage drive ΔV_G will cause an approximate sinusoid of base current, leading to the observed I_C sinewave with a delay of $\pi/2$.

For the actual device and the quasi-saturation model, because of the charge storage in the collector region, it is expected that the collector response waveforms exhibit delay in addition to the $\pi/2$ due to base charge storage alone. This is evident in the current waveforms of Fig. 6.21. The time delay calculated from the model is seen to be a little less than that measured though agreement is considered good. Model time delays would be further increased by the inclusion of depletion layer capacitances and extrinsic elements $r_{bb'}$ and C_{JC2} as in the total model of Fig. 4.11, Section 4.3.1. The time delays observed are listed in Table 6.11, together with CPU times involved in their determination. The programs were allowed to run for several cycles to allow the initial transients to die down.

TABLE 6.11. Measured and Computed Delay Times.

ΔV_G (pp) mV	T_D (measured) $\times 10^{-10}$ sec	T_D (q.s. model) $\times 10^{-10}$ sec	T_D (Ebers-Moll) $\times 10^{-10}$ sec	CPU Time (q.s.) secs	Ebers-Moll secs
40	36	33	25	120 (for -3 cycles)	<30 (for 4 cycles)
400	37	33.5	-	120 (for -2 cycles)	<30 (for 4 cycles)
4300	38	34	-	120 (for 1 cycle)	<30 (for 1 cycle)

As the voltage drive increases the output current variations along the 50Ω load line increase in amplitude as expected. Fig. 6.22 shows the I_C waveforms for $V_G=400\text{mV pp}$. The device and quasi-saturation model characteristics are basically as before in form and value of output/input time delay, Table 6.11. The Ebers-Moll waveform, however, takes on a very flattened form. Due to the increased drive, the model response is driven up the load line (Fig. 6.20) and the Ebers-Moll model enters the saturation region of operation. The collector current response is then no longer given by the simple integral of Equation (6.55) which was derived for the normal active mode of operation, since the inverse injection components of current are now no longer negligible.

If ΔV_G is increased still further, the quasi-saturation model still predicts a 'sinusoidal' collector current of

FIG. 6:22. Large-signal sinusoidal current responses (drive increased).

approximately the correct amplitude and increase in time delay over the 'reference' value of 25×10^{-10} secs ($\pi/2$ radians), Table 6.11. The Ebers-Moll model response is further 'distorted' by increased operation in saturation.

These 'performance' tests show the quasi-saturation model able to explain features of device operation which the simpler Ebers-Moll model cannot. In particular, the ability of the model to describe increased output time delay when operating as a large-signal common emitter amplifier is, due to the very different test conditions employed (compared with the parameter evaluation stage), seen as strong confirmation of the validity of the model structure and parameter evaluation technique.

6.4 SATURATION MODEL

As seen in Fig. 6.10, Section 6.3.1, at sufficiently reduced values of V_{CE} (for a given I_B), the saturation region is entered. Section 6.4.1 considers the model parameters under high level conditions, which obtain over nearly all this region of operation. For smaller values of I_C (of the order of I_B or less) the low level model of Fig. 5.7, Section 5.4 is used, parameter values being considered in Section 6.4.2. In both cases, the parameters are discussed in the

sequence in which they are obtained and with regards to whether they may be directly found from previous (active and quasi-saturation) considerations or whether additional 'data reduction' schemes must be developed. In a similar fashion to quasi-saturation, measured and computed d.c. and then transient responses are matched, device measurements being carried out over a wider range than is necessary for parameter determination to allow verification of the model.

6.4.1 High Level

The high level model, whose parameter values are to be determined, is that of Fig. 5.5, Section 5.3. The intrinsic base 'forward' parameters τ_F , τ_{BF} , I_{SF} , n_F have the same values as used in the quasi-saturation model, Table 6.9, Section 6.3.2, and are not discussed further. This section deals with the remaining model parameters listed below in Table 6.12. From the active region discussion of Section 6.2.2.1, and the theoretical development of the quasi-saturation and high level models, Sections 3.2 and 3.3.1 respectively, a direct correspondence between certain saturation model parameters and the corresponding parameters of the quasi-saturation model becomes evident. Saturation model parameters which may be given previously determined values are listed in Table 6.13.

TABLE 6.12. High Level Saturation Model Parameters.

	Parameter	Description
intrinsic base reverse injection parameters	I_{SR}	'Reverse' saturation current
	n_R	'Reverse' injection index
	τ_R	'Reverse' base transit time
	τ_{BR}	'Reverse' base lifetime
intrinsic collector region parameters	V_0	Constant voltage between intrinsic base and ohmic edge of injection region in quasi-saturation
	τ_3	'Diffusion' time constant for current transport between charge lumps q_1 and q_2
	τ_1	Lifetime in first excess charge region (q_1)
	τ_2	Lifetime in second excess charge region, including recombination velocity term at $n-n^+$ boundary
	K	Ratio of widths of excess charge lump q_1 to charge lump q_2
	q_{10}	'Equilibrium' charge in first excess charge region (see Fig. 6.23)
extrinsic collector region parameters	q_{20}	'Equilibrium' charge in second excess charge region (see Fig. 6.23)
	τ_3'	Diffusion time constant for current transport
	τ_1'	Lifetime in first excess charge lump
	τ_2'	Lifetime in second excess charge lump
	K'	Ratio of widths of excess charge lump q_1 to charge lump q_2

TABLE 6.12 (cont'd)

q_{10}'	'Equilibrium' charge in lump q_1' (see Fig. 6.23)
q_{20}'	'Equilibrium' charge in lump q_2' (see Fig. 6.23)

TABLE 6.13 Correspondence Between High Level Saturation and Quasi-Saturation Model Parameters.

<u>Parameter</u>	<u>Value Given</u>
I_{SR}	I_{SF} of active base region
n_R	n_F of active base
V_0	V_0 from quasi-saturation results
τ_3	$2\tau_{30}$ where τ_{30} is a quasi-saturation parameter
τ_1	τ_1 of quasi-saturation model
K	K of quasi-saturation model
q_{10}	$4q_D \frac{N_{epi}^2}{n_i} \frac{\tau_{30} n_i^2}{r_{co} N_{epi}}$ <p>(N_{epi} known from V_0 ρ_n is resistivity of epi layer)</p>
q_{20}	$q_{10} \frac{N_{epi}^2}{n_i^2} \frac{1}{K}$
q_{10}'	$\frac{(A_C - A_E)}{A_E} q_{10}$ <p>(Section 6.2.3.2 for ratio $(A_C - A_E)/A_E$)</p>
q_{20}'	arbitrary

The remaining parameters must be determined by techniques based upon d.c. and transient device measurements taken in saturation.

6.4.1.1 D.C. Characteristics

Under static conditions, the high level model of Fig. 5.5 takes the form shown in Fig. 6.23. The two-lump extrinsic collector-base diode has been reduced to a d.c. diode model with only one parameter, namely the saturation current I_{SD}' , where

$$I_{SD}' = q_{10}' \left\{ \frac{1}{\tau_1'} + \frac{1}{\tau_3' + K' \tau_2'} \right\} \quad (6.56)$$

From consideration of Fig. 6.23 and Tables 6.12 and 6.13, only three unknown parameters remain in the d.c. model; they are $\beta_R = \tau_{BR}/\tau_R$, I_{SD}' and τ_2 . These three terms cannot be determined directly from straightforward electrical measurements, and trial values are chosen and varied until agreement is obtained between measured and computed I_C - V_{CE} values. This matching was attempted for one d.c. curve ($I_B = 3\text{mA}$) from the low level model boundary up to the quasi-saturation boundary. Table 6.14 gives the values of the parameters of the d.c. model finally obtained.

TABLE 6.14. D.C. Saturation Model Parameter Values.

<u>Parameter</u>	<u>Value</u>
I_{SR}	$.21 \times 10^{-13} \text{ A}$
n_R	1
$\beta_R = \tau_{BR} / \tau_R$	20
V_0	-0.6V
τ_3	8.6ns
τ_1	.34 μ s
τ_2	51.4ns
K	1
q_{10}	$41 \times 10^{-22} \text{ c}$
q_{20}	$47 \times 10^{-12} \text{ c}$
I_{SD}	$.5 \times 10^{-15} \text{ A}$

From this table, it is seen that the reverse gain β_R has a value 20, considerably larger than the measured 'value' $\beta_I = I_E / I_B$ (with $V_{EB} = 0$) of the order of 3 for the 2N3375. This is due to the fact that I_B includes recombination in the collector region for a forward biased collector-base junction rather than just a ' τ_{BR} ' component. Time constant τ_2 is seen to be smaller than τ_1 due to the inclusion of a recombination velocity term in τ_2 . The value of q_{10} is in good agreement with that estimated from structural data

($28 \times 10^{-22} \text{C}$). D.C. common emitter characteristics in saturation are computed for this parameter set and are shown in Fig. 6.24 together with measured I_C - V_{CE} values. These curves demonstrate the ability of the model to give good (less than 10% variation between computed and measured curves) over the entire high level saturation region of operation and not just at the bias points used to evaluate I_{SD}' , β_R and τ_2 . The good fit is continuous with the equally good matching of characteristics in the quasi-saturation region.

A systematic approach is possible (conceivably) using an as yet unexploited source of data - namely the V_{EB} value at a given bias point. This approach to the exact determination of these three parameters would involve (a) an initial choice of β_R (b) calculation of I_{SD}' from the measured reverse gain value using:

$$\beta I_{\text{meas}} = \frac{\frac{q_R}{\tau_R}}{\frac{q_R}{\tau_{BR}} + \frac{q_1}{\tau_1} + \frac{q_2}{\tau_2} + I_{D'}} = \frac{I_{SR}}{\frac{I_{SR}}{\beta_R} + I_{SD}' \left(1 + \frac{q_{10}}{q_{10}'} \right)} \quad (6.57)$$

(c) variation of τ_2 until the correct value of I_C is obtained at one bias point (d) variation of β_R and hence the process (b) to (c) until the values of I_C and V_{EB} agree with those measured at the single bias point. In the case of the data fit given above in Fig. 6.24, good agreement between measured

FIG. 8.24. Computed and measured DC saturation characteristics.

and computed values of V_{EB} is noted. For example, at the bias point $I_B = 1\text{mA}$, $V_{CE} = 0.1\text{V}$, measured and computed values of V_{EB} are 0.705V and 0.707V respectively.

The inability of the Ebers-Moll model to describe the d.c. saturation characteristics is noted by comparison of Fig. 6.13 with the high level saturation results of Fig. 6.24. Specifically, the Ebers-Moll representation predicts a considerably smaller $V_{CE(\text{sat})}$ than is observed. Previously, this has been 'remedied' by the inclusion of a 'collector resistance' in series with the Ebers-Moll model, with the value of R_C constant or curve fitted as a function of I_C to measured $V_{CE(\text{SAT})}$. This resistance, however, must be larger than any actual resistance path in the collector or base regions, due to the modulation of these regions by 'minority' carriers injected across the collector junction. The high level saturation model is seen to predict correctly the I_C - V_{CE} curves without any recourse to a 'fictitious' series resistance. The model predicts accurately the boundary between saturation and quasi-saturation.

6.4.1.2 Transient Response

Certain parameters of the high level model of Fig. 5.5, Section 5.3, can only be determined by transient measurement techniques as was the case for quasi-saturation. For the

saturation model, these parameters are τ_R , the reverse base transit time, and the elements of the extrinsic collector-base diode, τ_1' , τ_2' , τ_3' and K . The intrinsic collector region is treated as being identical with the extrinsic collector for the mode of operation considered in this section (collector-base diode with the emitter tied to the base), characterized by τ_1' , τ_2' , etc., the junction area now being the total collector junction, A_C .

Difficulty is experienced in the extraction of τ_R from delay time measurements performed upon the device (operating in the inverse mode) due to the large storage of minority carriers in the epitaxial region. For this reason, a default value for $\tau_R = \tau_F$ is chosen, though in general τ_R will be rather greater than τ_F . Owing to the larger delay times encountered due to collector region storage, any error in the value of τ_R chosen in this manner will not cause significant deviations of the model from device performance.

Three methods are investigated for the determination of the parameters of the extrinsic collector region model by terminal measurement. In each case, to simplify the experimental investigation on collector storage effects, the emitter-base junction, as mentioned, was shorted and the collector-base junction treated strictly as a pn junction

diode⁽⁵⁾. The procedure involves the comparison of the device with an analogue version of its internal model, of the form shown in Fig. 6.25. The relationships between the analogue circuit and the collector diode model parameters τ_1' , τ_2' , K' , τ_3' are listed in the figure. These different methods are investigated to determine the degree of correlation that exists between model values for greatly varying conditions of test. It should be noted that in all three methods, the 'collector' diode model includes reverse base injection in its first lump and represents the total collector-intrinsic plus extrinsic regions.

6.4.1.2.1 Forward Diode Impedance Test (a)

The forward impedance (magnitude and phase) of the forward biased collector-base diode is determined by computer conversion from data obtained from a Wayne-Kerr B601 admittance bridge and an HP network analyzer S-parameter set-up, over their respective frequency ranges. A resistance-capacitance network analogue of the two lump model is designed, its initial values being determined from experience derived from previous analogue tests⁽⁵⁾. The element values of the analogue are varied in a systematic fashion until an 'optimal' fit of forward impedance between the analogue and the actual device

'Analogue model' of collector region-collector diode operation.

$$T'_1 = R_1 C_1$$

$$\tau'_2 = R_2 C_2$$

$$\tau'_3 = R_3 C_1$$

$$K' = C_1 / C_2$$

$\tau'_1 \neq T'_1$ due to inclusion of reverse injection terms.

METHODS: (a) Forward diode impedance
(b) Turn-off (t_s, t_r)
(c) Turn-on

RESULTS: 2N3375

	T'_1 μs	τ'_2 μs	τ'_3 μs	K'
(a)	4.5	1.02	0.45	0.37
(b)	4.0	0.15	0.69	1.47
(c)	20.4	0.47	0.30	2.37

FIG. 6.25. Collector model parameters from three methods.

is obtained over the frequency range 0.1MHz to 0.6GHz. Due to the higher frequencies involved in the S-parameter measurements, evaluation of the G-C analogue impedance must be computed at the higher frequencies.

The results of this test, (a), are shown in Fig. 6.26 for the 2N3375 and in Fig. 6.27 for the 2N1060A, giving the measured diode impedance and the calculated network impedance for a best fit. Forward impedance calculations were also performed (not shown here) for the networks found by test methods (b) and (c) (model parameters tabulated in Fig. 6.25), Sections 6.4.1.2.2 and 6.4.1.2.3 below. Correlation between the measured and calculated values of Fig. 6.26 is good (less than 5% difference over 4 decades of frequency) in comparison with values from tests (b) and (c), where the computed magnitude and phase may differ by up to 50% from the transistor diode experimental values. Even better correspondence in test (a) may be achieved by computer optimization techniques, which would allow this fit to be achieved much more quickly and efficiently.

6.4.1.2.2 Turn-off Test Method (b)

In order to measure turn-off times for the collector diode, the emitter diode is shorted out as in method (a).

A fit of actual and analogue turn-off responses requires

FIG. 6.26: Forward impedance test results for the 2N3375.

FIG. 6.27. Forward impedance test results for the 2N1060A.

initially the measurement of device storage delay and rise times. The basic circuit used is shown in Fig. 6.28a where the diode is held under reverse bias except for the pulse of duration t_0 . The current through the diode is found from the difference voltage across R , measured on an oscilloscope. The actual circuit used in these tests was as in Fig. 6.28b, where a second identical diode to the test device is used to 'balance' out the collector capacitance charging current during rise time measurements; otherwise, this component of current would tend to mask the effect of the discharging of the two-lump model⁽⁵⁾. For this purpose, the 'balance' device is maintained under reverse bias at all times, being approximately zero biased during the discharge of the transistor. The turn-off response and definition of its time components are shown in Fig. 6.28c. The measurement of the analogue switching response makes use of a drive circuit developed and described in earlier work⁽⁵⁾ and shown in Fig. 6.29.

Model element values are adjusted so that storage delay time t_s and rise time t_r , in response to turn-off drive, are matched for the device and its analogue, with the latter operating on a conveniently chosen slower time scale. Results are shown in Fig. 6.30 for the 2N3375 for a forward/reverse current ratio I_F/I_R ranging between 1.5 and 9.5. The switching

FIG. 6.28. Circuits and response for turn-off measurements.

FIG. 6.29. Circuit for measurement of switching response of analogue models.

FIG. 6.30. Storage delay and rise time results.

responses of the analogue networks found by the other two test methods (a) and (c) were also determined and are also shown in Fig. 6.30. The poor agreement of these tests with device values shows the effect of variations of conditions of test on measured two-lump values.

6.4.1.2.3 Turn-on Test (c)

A bridge measuring circuit was used for the determination of the parameters of the diode model under turn-on conditions, as shown in Fig. 6.31a. The elements R_1 , C_1 , R_3 , R_2 , C_2 of the base bridge circuit, and R_L , are adjusted until the voltage between emitter and base closely approximates to zero throughout the duration of the collector excitation voltage V , and the emitter current waveforms are observed as the voltage across R_L . The transistor is thus operating in the inverse mode in a condition appropriate for determination of the parameters of the collector diode. Since $V_{JE} \approx 0$ throughout, there is no need to correct results for the change of depletion layer charges due to bias variations at the emitter-base junction.

It may be shown⁽⁵⁾ that the two-lump collector diode model may be referred to either the collector side or the base side of the junction, upon combination of sub-models for the charging of the collector and base regions. Upon excitation

(a) Turn-On test method.

(b) Observed current waveforms.

FIG. 6.31. Turn-On test method and waveforms.

at the collector terminal, to set up and maintain a constant emitter current, the charge control requirements of the 'near' and 'remote' storage regions of the device (here predominantly the collector region) must be met. If the network supplying this charge and its associated recombination current is such that the charge on the first or 'near' lump remains fixed through the excitation, it may be shown from the device model for inverse operation that a step increase in emitter current will result. In the bridge circuit, the G-C analogue of the two-lump charge storage model of the device supplies the charging needs of the transistor, for inverse operation, via the base, its components being adjusted to 'match' those of the model at the bridge null condition, Fig. 6.31b. The resistance-capacitance networks derived by the other methods of test gave poor results under test condition (c), their resulting waveforms being of the type shown in Fig. 6.31b, a distinct 'wiggle' in the waveform being noted, due to the inability of these networks to provide the 'correct' charge requirements of both the 'near' and 'remote' regions of the device.

6.4.1.2.4 Analysis of Test Results

All three tests have been carried out for the slowest device (2N3375) for ease in measurement technique but are applicable to the faster devices. The time constants of the

collector model are given, for each test, in Fig. 6.25.

Correlation between these values is poor and underlines the different conditions of test used. However, the tests do show the ability of the two-lump collector model to accurately describe device behaviour in each case (with the device acting as a diode or in the inverse transistor mode). Differences in test conditions in the three cases may be readily understood. In test (a) the forward small-signal impedance measurements (and elements of the model) will, in practice, depend upon the d.c. bias point, which was arbitrarily fixed at 0.1mA forward current, whereas in test (b) large-signal swings out of saturation are considered. In the third test, the charge on the first lump C_1 represents both the charge in the 'near' neutral regions and the collector depletion layer charge. Attempts to separate these charges for proper physical interpretation of the measurements would appear difficult due to the non-linear dependence of each term on collector current. The conclusion is drawn that the test method used must be selected to be appropriate to the conditions of operation of the transistor in the circuit application of interest. Both methods (a) and (c) are inappropriate where switching processes are involved, the former because of the very high frequencies of impedance measurement for much of the data, and the latter because of the extreme impulsive conditions imposed in the test circuit.

Method (b) is regarded as the most suitable method of deriving the model elements, and it is noted, from the results of Fig. 6.30, that the model gives accurate representation over an extremely wide range of switching conditions.

From these test methods, the parameters of the 'outer' collector region of the high level saturation model of Fig. 5.5 may be deduced. The parameters τ_2' , τ_3' , K' are given directly by these collector diode tests due to the identical nature of the analogue circuit used in the tests, Fig. 6.25, and the extrinsic diode model, Fig. 5.5. Parameter τ_1' varies from the measured value T_1' due to the inclusion of reverse injection terms in T_1' , which leads to the expression:

$$\frac{q_{10}' + q_{10}}{T_1'} - \frac{q_{10}' + q_{10}}{\tau_1'} = \frac{q_{RO}}{\tau_R} + \frac{q_{RO}}{\tau_{BR}} \quad (6.58)$$

where

$$\frac{q_{RO}}{\tau_R} = I_{SR} \quad (6.59)$$

or

$$\frac{1}{T_1'} = \frac{1}{\tau_1'} + \frac{I_{SR}}{q_{10}' + q_{10}} \left(1 + \frac{1}{\beta_R} \right) \quad (6.60)$$

from which τ_1' may be deduced. Upon examination of Equations (6.56), (6.57) and (6.60) a more systematic method for certain

high level saturation parameters emerges. For, given the parameters I_{SR} , q_{10} , q_{10}' and the results of the reverse gain measurements β_{I_m} and the collector diode tests for T_1' , τ_2' , τ_3' and K' , these three equations may, in principle, be solved for the 3 unknowns β_R , τ_1' and I_{SD}' . We need then only iterate upon τ_2 as described previously to obtain all the parameters of the high level transistor saturation model. This procedure was not carried out since the three tests (a)-(c) (performed successfully over a wider range than was necessary to obtain the relevant device parameters), were seen as excellent validation of the transient modelling approach taken, namely two-lump modelling of the intrinsic and extrinsic collector regions. Fit of computed and measured transistor storage delay times would, in practice, involve a formal optimization scheme based upon the initial values derived by the method above. Transistor storage delay 'performance' tests were carried out only qualitatively but showed clearly storage delay and reverse recovery as noted for the device.

6.4.2 Low Level

The low level model of Fig. 5.7, Section 5.4, should be used for small collector currents (of the order of I_B or less) in place of the high-level model of the previous section.

6.4.2.1 D.C. Characteristics

Under d.c. conditions, the low level model reduces to an equivalent Ebers-Moll model of the form shown in Fig. 6.12, Section 6.3.1. The forward parameters remain unaltered but the reverse parameters I_{SI} and β_I must be adjusted to accommodate the increase in base current due to injection by the intrinsic and extrinsic collector-base diode. Thus,

$$I_{SI}' = I_{SI} + I_{SD} + I_{SD}' \quad (6.61)$$

where I_{SI}' , I_{SI} are the modified and 'original' Ebers-Moll reverse saturation currents, and I_{SD} , I_{SD}' are the intrinsic and extrinsic collector diode saturation currents respectively. These last two saturation currents will be in the ratio of their cross-sectional areas.

$$\frac{I_{SD}}{I_{SD}'} = \frac{A_E}{A_C - A_E} \quad (6.62)$$

Thus, Equation (6.61) becomes:

$$I_{SI}' = I_{SI} + I_{SD}' \left(1 + \frac{A_E}{A_C - A_E} \right) \quad (6.63)$$

Since the reverse transport of current to the emitter remains unchanged, using similar notation, we obtain:

$$I_{SI} \alpha_I = I_{SI}' \alpha_I' \quad (6.64)$$

or

$$I_{SI} \frac{\beta_I}{1+\beta_I} = I_{SI}' \frac{\beta_I'}{1+\beta_I'} \quad (6.65)$$

from which an equivalent reverse gain β_I' may be deduced.

Upon substitution of numerical values in Equations (6.63) and (6.65), we obtain:

$$I_{SI}' = .31 \times 10^{-13} \text{ A} \quad \beta_I' = 2.1$$

$$I_{SI} = .28 \times 10^{-13} \text{ A} \quad \beta_I = 3$$

On examination of the Ebers-Moll common emitter characteristics of Fig. 6.13 (with $\beta_I=3$, $I_{SI}=.28 \times 10^{-13} \text{ A}$), it is predicted that good agreement between measured and computed curves is possible over the very small range considered ($0 < I_C \leq I_B$). The values I_{SI}' , β_I' calculated above should only be considered as initial estimates in 'exact' curve fitting to measured responses.

6.4.2.2 Transient Responses

Due to the correspondence between the two-lump collector diode sub-model in the low level model of Fig. 5.7 and the resistance-capacitance analogue used in the three diode tests of Sections 6.4.1.2.1-6.4.1.2.3, the parameters of the collector-sub-model may be obtained directly from these measurements, except for τ_1^* . This parameter must be extracted using the procedure given in Section 6.4.1.2.4 based upon Equation (6.60).

The collector diode tests performed over a wide range of conditions support the choice of low level model made. Further collector diode tests performed by Maldonado et al⁽¹⁷⁾ lend support to the choice of a two-lump representation of effects in the collector region.

CHAPTER 7

CONCLUSIONS AND FURTHER WORK

Each modeller characteristically is identified by technique and methodology. Differences in approach, likes, and dislikes play meaningful roles in the ultimate achievement of an acceptable solution. What follows is a discussion of the direction taken and the original contributions made in the formulation of the model.

The notion of model arises in the scientific sense when we have a candidate for a theory. The model formulation has consisted of four sequential steps:

- (i) observation and classification of transistor phenomena of interest to give a physical model of the interactions between the system variables
- (ii) development of a mathematical model of the physical theory of the device which could be expected to yield mathematical quantities equivalent to available transistor terminal data
- (iii) the translation from an abstract theory to a usable model for the three device regions of operation

(iv) evaluation of the performance of the model.

The main contributions of the work are:

- (a) the development of a model allowing the effects of charge injection in the collector (considered dominant) to be quantified for large-signal operation. This has involved the representation of a variable-width charge storage region characterized in two-lump form.
- (b) the formulation of a base region 'transport' model that systematically includes effects due to parameter variation, such as large-signal basewidth modulation.
- (c) techniques that should allow the successful implementation of these models in most computer-aided circuit analysis programs.
- (d) systematic methods giving the parameters of these models by relatively straightforward measurement techniques, with a minimum of complicated optimization methods.

The proposed models appear satisfactory on the basis of criteria proposed by Sparkes⁽³⁴⁾ and Hamilton et al⁽³⁵⁾. They make use of parameters that exhibit a one-to-one correspondence with physical processes. This 'regional' approach allows characterization of additional physical effects, such as carrier generation due to radiation, in

any region of the device. The models have parameters whose values can be determined from terminal measurements (the explicit morphology of the device is not often known), and which are amenable to statistical design. The models involve a degree of approximation sufficient to enhance qualitative understanding while maintaining desired quantitative accuracy. That is, these models meet the requirement of minimum complexity for representation of the physical effects involved.

Other circuit models are usually restricted by being valid for only d.c. operation, or if they are large-signal models (Ebers-Moll for example), they do not possess sufficient exactness to be generally useful for this class of device. For example, the Gummel-Poon transistor model⁽¹⁾ may be capable of quantifying effects due to collector storage in a d.c. analysis⁽¹²⁾ but difficulties in Gummel's 'base push-out model' for the transient case cannot be resolved without lumped modelling of the collector region.

Certain aspects of the work are worthy of continued research: the most important of these are the implementation of the model in more efficient form than was possible with CIRC; and further 'best fit' parameter determination for the models from device tests. More efficient model implementation is considered briefly in Appendix 1. The models could be a

foundation for the study of basic aspects of amplifier and multiplier design where other models have failed to give completely satisfactory results (references (36-37) for example).

It is concluded that the models developed in the work are capable of characterizing the behaviour of VHF and microwave transistors in the active, quasi-saturation, and saturation regions of operation. It is hoped that the models described here may prove a source of new ideas and that some readers may modify and improve on a model here described, producing in the process something new and satisfying. Black boxes will have been opened and their contents mastered.

APPENDIX 1

CONSIDERATIONS FOR MORE EFFICIENT IMPLEMENTATION

It has been noted in Chapter 5 that the CIRC program is rather unsuitable for the purpose of setting up device models of the complexity required in this work. A circuit analysis program in which the device model could take the form of a separate sub-routine (i.e. written in software form rather than in the form of an equivalent electrical circuit) would be more appropriate for the task.

Modern circuit analysis routines, such as the NANSIM program⁽³⁸⁾ at the Bell-Northern Electric Research Laboratories, Ottawa, are constructed in modular form, each independent module dealing with a specific function; one handles the input of information, a second with numeric analysis, a third with modelling circuit components, a fourth with displaying results on a terminal, and so on. This format allows each module to be changed without affecting the others. Thus, new models, such as those developed here, may be readily introduced in sub-routine form into the appropriate module, Fig. A1.1.

Simulation is usually carried out using the nodal admittance formulation of the circuit equations (as in CIRC);

FIG. A1.1. Model subroutine layout for efficient computation.

d.c. and transient analyses of non-linear circuits (including, for example, transistor models) being performed using the Newton-Raphson method of successive approximations. In this method, the solution of a system of non-linear equations is recalculated a number of times, starting from an initial guess provided by the designer and continuing until the accuracy of the solution is within specified limits. Consider a two terminal non-linear device model, Fig. A1.2a. The current equations describing this model are:

$$I_1[V_1, V_2] + I_2[V_1, V_2] = 0 \quad (A1.1)$$

or

$$I_J[V_1, V_2] = 0 \quad J=1,2 \quad (A1.2)$$

where 'trial' voltages V_1, V_2 are applied to the model, giving terminal currents I_1, I_2 , defined in the figure. Equation (A1.2) may be rewritten in vector notation as:

$$\underline{I}(\underline{V}) = 0 \quad (A1.3)$$

where

$$\underline{I} = (I_1, I_2) \quad \underline{V} = [V_1, V_2] \quad (A1.4)$$

We may then derive, using the Newton-Raphson method⁽³⁹⁾, an iteration equation for the (i+1)th approximation to $\underline{V}$.

$$\underline{V}^{i+1} = \underline{V}^i + \underline{\Delta V}^i \quad (A1.5)$$

(a) General two-terminal device model.

(b) Three-terminal quasi-saturation model.

FIG. A1.2. General two-terminal and quasi-saturation three-terminal models.

The 'change vector' $\underline{\Delta V}^1$ is defined by the solution of

$$\underline{J}(I) [\underline{\Delta V}] = -\underline{I} \quad (\text{A1.6})$$

at the i th step. $\underline{J}$ is known as the Jacobian of $\underline{I}$ and has the form, in this '2-dimensional' example, at the i th iteration, of

$$\underline{J} = \begin{bmatrix} \frac{\partial I_1}{\partial V_1} & \frac{\partial I_1}{\partial V_2} \\ \frac{\partial I_2}{\partial V_1} & \frac{\partial I_2}{\partial V_2} \end{bmatrix}_{\underline{V}=\underline{V}^i} \quad (\text{A1.7})$$

Thus, the introduction of any N-terminal model as a subroutine must include an analytic expression for the N-dimensional Jacobian matrix and (of course) terminal currents. This matrix is seen to be identical to the admittance matrix of the device needed for a small-signal calculation. Thus, the small-signal circuit admittances needed in this kind of analysis are calculated automatically by the program for each non-linear device using the particular bias conditions previously determined in the d.c. simulation. This admittance matrix will be referred to as the $\underline{G}$ -matrix in the following discussion to avoid confusion with the nodal admittance matrix $\underline{Y}$.

Implementation of the overall model involves the representation for four possible regimes of operation (Section 5.1). In the software realization of the model, Fig. A1.1, different 'chapters' of the subroutine are used for the different regime models, with routing through them defined by logical conditions specified by the boundary conditions. For example, the high-level saturation model of Fig. 5.5, Section 5.3, should be used for V_{cb} (given by the trial voltages) greater than V_0 and for $I_C > I_B$.

Evaluation of terminal (and element) currents in these models may themselves require subroutines. For example, the quasi-saturation model of Fig. A1.2b where the reverse transport current q_R/τ_R into the emitter has been neglected, depends upon the simultaneous solution of two current equations, (2.8) and (2.17) in order to define the instantaneous value of collector injection region width x_i . This is done by an iterative procedure and x_i used to give the terminal currents and G-matrix; as shown below.

For the quasi-saturation model of Fig. A1.2b, we may write, at the emitter:

$$I_1 = I_{SF} \left\{ \exp q \frac{V_1 - V_3}{kT} - 1 \right\} + \frac{d}{dt} \left(\tau_F I_{SF} \left\{ \exp q \frac{V_1 - V_3}{kT} - 1 \right\} \right) \quad (A1.8)$$

For constant V_2, V_3

$$\frac{\partial I_1}{\partial V_1} = \frac{q}{kT} (I_1 + I_{SF}) \quad (A1.9)$$

The collector current is given by:

$$I_2 = \frac{V_2 - V_3 - V_0}{r_c} \quad (A1.10)$$

and if we may assume x_i constant for small perturbations in V_1, V_2, V_3

$$\frac{\partial I_2}{\partial V_1} = 0 \quad (A1.11)$$

This result and those to follow are based upon the assumption that the variation of x_i with the small voltage perturbations is of negligible effect. If the Jacobian matrix, evaluated in this way does not lead to convergence, the effect of changes in x_i must be considered.

The three terminal currents, or their increments, must always sum to zero, i.e.

$$\sum_{n=1}^3 \frac{\partial I_n}{\partial V_k} = 0, \quad k=1, 2, \text{ or } 3 \quad (A1.12)$$

Using (A1.10) to (A1.12), we obtain:

$$\frac{\partial I_3}{\partial V_1} = - \frac{\partial I_1}{\partial V_1} = - \frac{q}{kT} (I_1 + I_{SF}) \quad (A1.13)$$

For constant V_1, V_3

From Equation (A1.10)

$$\frac{\partial I_2}{\partial V_2} = \frac{1}{r_c} \quad (\text{A1.14})$$

and since I_1 , Equation (A1.8), is not a function of V_2

$$\frac{\partial I_1}{\partial V_2} = 0 \quad (\text{A1.15})$$

From Equation (A1.12)

$$\frac{\partial I_3}{\partial V_2} = - \frac{\partial I_2}{\partial V_2} = - \frac{1}{r_c} \quad (\text{A1.16})$$

For constant V_1, V_2

We get from Equation (A1.10)

$$\frac{\partial I_2}{\partial V_3} = - \frac{1}{r_c} \quad (\text{A1.17})$$

and from Equation (A1.8)

$$\frac{\partial I_1}{\partial V_3} = - \frac{q}{kT} (I_1 + I_{SF}) \quad (\text{A1.18})$$

Summing these three currents to zero gives:

$$\frac{\partial I_3}{\partial V_3} = \frac{1}{r_c} + \frac{q}{kT} (I_1 + I_{SF}) \quad (\text{A1.19})$$

These equations give directly the G-matrix, Equation (A1.20), needed in the convergence scheme.

$$\tilde{G} = \begin{bmatrix} \frac{\partial I_1}{\partial V_1} & \frac{\partial I_1}{\partial V_2} & \frac{\partial I_1}{\partial V_3} \\ \frac{\partial I_2}{\partial V_1} & \frac{\partial I_2}{\partial V_2} & \frac{\partial I_2}{\partial V_3} \\ \frac{\partial I_3}{\partial V_1} & \frac{\partial I_3}{\partial V_2} & \frac{\partial I_3}{\partial V_3} \end{bmatrix} \quad (\text{A1.20})$$

APPENDIX 2

COMPUTATION OF DEVICE TIME DELAY

In order to understand the reason for the behaviour of the forward transit time (Fig. 6.4, Section 6.2.3.1), and to isolate the corresponding conditions of operation of the device and any implications of these conditions in terms of the device model, analyses have been carried out for a device structure of an impurity density profile and region dimensions similar to that of the 2N1038A studied experimentally. The overall forward transit time was computed as:

$$t_d = \frac{1}{\omega_T} = \tau_F + \frac{kT}{qI_c} (C_{JE} + C_{JC1}) + \frac{x_{dc1} x_{dB}}{2V_d} + r_c C_{JC1} \quad (6.15)$$

or

$$t_d = \tau_F + \tau_E + \tau_{JC1} + \tau_{COL} \quad (A2.1)$$

where

τ_F = base region transit time

τ_E = emitter charging time

τ_{JC1} = intrinsic collector space charge layer transit time

τ_{COL} = collector charging time

In general, each component of t_d in Equation (A2.1) is significant and a careful description of each term is essential.

The flowchart used for this calculation is given in Fig. A2.1 and description of each step given below:

1. Since we only require agreement in the manner in which the computed and experimental t_d vary with temperature, knowledge of the actual doping profile of the 2N1038A transistor used is not needed, and we may use a typical double Gaussian distribution for a microwave transistor of f_T about 1.5GHz shown in Fig. 2.2, Section 2.2, and described by:

$$N(X) = N_B \exp[-0.25(X/h_B)^2] - N_E \exp[-0.25(X/h_E)^2] - N_C \quad (A2.2)$$

where the subscripts B, E, C refer to base, emitter and epitaxial doping parameters respectively. This doping profile is the basis of the model used in this computation, and is essential in the determination of the components of t_d .

2. The first requirement in their evaluation is knowledge of the metallurgical junctions separating emitter-base and base-collector regions, and the slopes of the profiles at these boundaries. This information is readily computed from the distribution of Equation (A2.2). Also required for a later stage in the modelling process is the maximum doping in the base region.

FIG. A2.1. Flow chart for computer solution of emitter-collector delay time.

3. The purpose of this exercise is to evaluate f_T about various defined operating points for a range of temperatures. For this, a set of d.c. (I_C/V_{EB}) characteristics was measured for the device for a range of temperatures, as shown in Fig. A2.2. A number of operating points is chosen on each of these curves, at which the small-signal parameters may be computed.

4. Before the base transit time can be found, the neutral base region must be defined for the bias conditions defined in (3) above. This requires modelling of the emitter and collector semi-widths under applied voltages. This is accomplished by use of complete depletion theory (appropriate for low-level, active normal operation of the device) together with approximations to the double Gaussian distributions at the junctions, shown in Fig. A2.3, namely an exponential doping distribution at the emitter and a composite (linear-abrupt) doping profile for the collector. These distributions are chosen on the basis of past success in their use^(3,40). For the composite model of the collector junction, Poisson's equation reduces to the solution of⁽⁴⁰⁾:

$$V_{\text{total}} = V_{JC1} + \phi_{C1} = V_0 \left\{ \frac{1}{3} \epsilon_2^3 + \frac{1}{2} \epsilon_1^2 + \frac{1}{6} \right\} \quad (\text{A2.3})$$

FIG. A2.2. Pulsed DC characteristics for the 2N1038A.

where $V_0 = N_0 x_0^2 \frac{q}{\epsilon}$ (q is the charge on the electron, ϵ the dielectric constant), V_{JC1} is the applied junction voltage, and ϕ_{C1} the contact potential is given by:

$$\phi_{C1} = \frac{kT}{q} \ln \left(\frac{N(X-d_{B0})N(X+d_{C10})}{n_i^2} \right) \quad (A2.4)$$

(where n_i^2 is the intrinsic carrier concentration squared and other terms are as defined in Fig. A2.3). Evaluation of the semi-widths requires knowledge of V_{JC1} , ϕ_{C1} for the temperatures considered. V_{JC1} is given by:

$$V_{JC1} = V_{CB} - I_C r_C \quad (A2.5)$$

where r_C is temperature dependent and given by:

$$r_C = \frac{\rho(N_{epi}, T)}{A_E} x_e \quad (A2.6)$$

where $\rho(N_{epi}, T)$ is the temperature dependent resistivity of the epi layer and may be found from published data⁽³³⁾, x_e is the width of this layer ($\gg x_{dc1}$) and A_E is a typical emitter area, chosen to give agreement with published values for emitter capacitance. ϕ_{C1} is given by Equation (A2.4) where the temperature dependence of n_i^2 is described by⁽³³⁾:

$$n_i^2(T) = N_C N_V \exp(-E_G/kT) \quad (A2.7)$$

where

(a) Emitter Junction (b) Normalized (c) Collector Junction (d) Normalized

FIG. A2.3. Grading approximations used at junctions.

$$E_G = E_{G0} - \alpha_G kT \quad (A2.8)$$

$$N_C N_V = 4 (2\pi k/h^2)^3 (m_n^* m_p^*)^{3/2} T^3 \quad (A2.9)$$

E_{G0} is the extrapolated energy gap, α_G rate of decrease of energy gap with temperature, N_C, N_V are the effective densities of state at the conduction and valence band edges respectively, h is Planck's constant, and m_p^*, m_n^* are the effective masses of the hole and electron.

For the exponential model of the emitter junction, Poisson's equation reduces⁽⁴⁰⁾ to the solution of:

$$V_{\text{total}} = V_{JE} + \phi_E = V_0(\xi_1 + \xi_2) (B-1) - \frac{1}{2} (\xi_1 - \xi_2) \quad (A2.10)$$

where

$$B = \left(\xi_1 + e^{-\xi_1} \right) = \left(e^{\xi_2} - \xi_2 \right) \quad (A2.11)$$

5. In the computation of t_d , we restrict ourselves to the regime under which $\phi_E > |V_{JE}|$. This justifies the use of the complete depletion approximation for the emitter junction. Using the Newton-Raphson iterative method for both Equations (A2.3) and (A2.10) yields the semi-widths, hence depletion capacitances and neutral base boundaries. This allows evaluation of the emitter charging time and collector charging time components in the expression for t_d .

against $1/I_C$ in Equation (6.15) with temperature as the varying parameter.

To evaluate the collector depletion layer transit time, we make use of the fact that if the field in the depletion region is large enough ($\sim 10^4$ V/cm) V_d reaches a scatter-limited drift velocity V_{SL} . An approximate value for the maximum electric field in the region is given by linear grading theory as:

$$\xi_{\max} = \frac{q a_C d_{Cl}^2}{2\epsilon} \quad (A2.12)$$

where a_C is the slope of the doping profile. For the profiles under consideration,

$$\xi_{\max} = 1.5 \times 10^5 \text{ V/cm.}$$

It would thus appear reasonable to assume that V_d reaches V_{SL} over a large portion of the depletion region. Any errors involved by the assumption of constant V_d throughout this region will be shown, from the magnitude of this component of transit time, to be negligible in comparison with other terms. The temperature dependence of V_{SL} should, in theory, be considered but this dependence has been shown to be small⁽⁴¹⁾ and, for the same reasons as above, may be neglected.

6. The remaining term τ_F is found from the Moll-Ross relationship⁽⁴²⁾:

$$\tau_F = \int_0^W \frac{1}{N(x)} \int_x^W \frac{N(y)}{D(y)} dy dx \quad (A2.13)$$

where the neutral base extends, for simplicity of notation, from $x=0$ to $x=W$, and $N(y), D(y)$ are the position-dependent doping density and carrier diffusion coefficient, respectively. This expression was derived on the basis of low-level injection (assumed throughout) and neglect of recombination effects in the base. This second assumption may be justified by a comparison of transit times calculated with and without recombination for a uniform base, using the expressions of Mostafa⁽⁴³⁾ which show an error of about 0.05% is involved. In the literature, (Moll, Ross⁽⁴²⁾, for example) the expression (A2.13) is simplified by assuming a constant D throughout the base. This leads to an expression:

$$\tau_F = W^2 / 2D\eta \quad (A2.14)$$

where W is the electrical base width, D is taken as an 'average' value of diffusion coefficient in the base, and η is a constant (≥ 1) depending on the doping profile in the base.

Equation (A2.14) is thus seen to be a poorly defined expression especially with regard to the value of n that should be chosen in an individual case. Justification for the use of Equation (A2.14) for modern double-diffused transistors is not available in the literature and hence an attempt has been made at the correlation of the 'exact' expression (A2.13) and its approximation, Equation (A2.14). This involves computation of the double integral of Equation (A2.13) which may be carried out using the trapezoidal rule providing the functional dependence of D on the doping profile and temperature is known. This may be found using the Einstein relation:

$$D(N_B, T) = \frac{kT}{q} \mu(N_B, T) \quad (A2.15)$$

where the total carrier mobility μ is a function of impurity scattering mobility (μ_I) and lattice mobility μ_L :

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} \quad (A2.16)$$

where (44)

$$\mu_{L \text{ ELECTRONS}} = 2.1 \times 10^9 T^{-2.5} \text{ cm}^2/\text{V-sec} \quad (A2.17)$$

$$\mu_I = \frac{4.7 T^{3/2}}{N_B \ln(1 + 4.5 \times 10^8 T^2 / N_B^{2/3})} \text{ cm}^2/\text{V-sec} \quad (A2.18)$$

It should be noted that mobility is determined by the sum of acceptor and donor impurity concentrations and that the impurity concentration N_B here is defined as

$$N_B = N_A(X) + N_D(X)$$

To enable a comparison of expressions for τ_F to be made, the average total doping density in the base is evaluated, its corresponding D for a given temperature evaluated, and the ratio

$$\eta = \frac{w^2 / 2D(\bar{N}_B)}{\int_0^w \frac{1}{N(x)} \int_x^w \frac{N(y)}{D(y)} dy dx} \quad (\text{A2.19})$$

evaluated for the chosen range of bias conditions and temperatures.

The expression

$$\eta = \frac{w^2 / 2D(\bar{N}_B)}{\tau_F \text{ MOLL-ROSS}}$$

is shown in Fig. A2.4 for the range of conditions encountered and demonstrates the ability of Equation (A2.14), repeated here,

$$\tau_F = w^2 / 2D \eta \quad (\text{A2.14})$$

FIG. A2.4. Computed η factor in base transit time τ_F .

to express the transit time for a range of bias currents and temperature, but shows the need for recomputation of η for each doping profile and collector bias variation. This demonstrates that a constant and somewhat arbitrarily chosen η cannot describe the base transit time for varying conditions of bias.

The four components of t_d are now known and, summed, give the variation of $t_d - 1/I_c$ with temperature and V_{CB} as parameters. Discussion of the results of these computations is carried out in Section 6.2.3.1.

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