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ISOMERIC RATIOS IN ^{69}Zn , ^{80}Br , ^{81}Se AND ^{165}Dy

by

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A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

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ABSTRACT

Isomeric state production ratios in ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy have been measured for the (n,γ) reaction as a function of neutron energy. A new gamma-ray cascade model for predicting these isomeric ratios has been devised and its results have been compared with those of other models. This new model employed nuclear level densities generated numerically with a computer routine designed by Hillman and Grover. The gamma-ray transition probabilities were estimated from Moszkowski's formulae. The initial spin distribution of the compound nucleus was calculated using Optical model transmission coefficients.

The measured isomeric ratios were found to be in a general sense in accord with the expectation that as the neutron energy increased the production of the high spin state would increase. However, the calculations made with the cascade model did not agree completely with these measurements. It was concluded that while the Optical model transmission coefficients, the level densities of Hillman and Grover, and the use of the transition probability estimates of Moszkowski might be singularly or in combination at fault for this disagreement, the use of level densities in place of experimentally determined low lying levels could be of greater importance. The measurement of the spins and energies of more such levels in ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy would be necessary to make it possible to test the influence of this factor on the isomeric ratio in these nuclei.

It was concluded that despite its present shortcomings, the new cascade model because it is parameter free produced

results which were more easily related conceptually to the forces and other properties of the nuclei that were studied.

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Chapter 1

Introduction

In this work we have measured the ratio of the isomeric state production cross section to the total production cross section, $\sigma_m/(\sigma_m + \sigma_g)$ referred to as the isomeric ratio, for several nuclei. These nuclei were produced by the (n,γ) reaction as a function of neutron energy. The importance of these measurements is that they can give information on nuclear level densities which in turn provides knowledge on nuclear dynamics.

Experimental results, which, either directly or indirectly are a measure of nuclear level densities, may be related to nuclear forces, shapes or other properties through the construction of a nuclear model from which a prediction of the form of the level density is extractable. The experimental and theoretical densities can then be compared and the theory's parameters adjusted until agreement is achieved. These adjusted values of the theoretical level density can then be employed to evaluate the nuclear dynamics of the model used. Experimentally determined isomeric ratios are an example of a type of data that may be analysed in this way. As part of this work, such a theoretical analysis has been carried out for the nuclei whose isomeric ratios we measured.

The isomeric ratios have been studied for the nuclei ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy . These nuclei were selected since they have isomeric states and ground states whose lifetimes are of appropriate length for the experimental procedures used. They were also chosen

because they are readily produced through the (n,γ) reaction from naturally occurring isotopes. The studies carried out in this work were all of isomeric ratios produced by the (n,γ) reaction since this reaction introduces fewer variables to be accounted for in the theoretical analysis. That is, only the level density of a single nucleus is involved which is not the case for reactions involving particle emission. In (n,γ) reactions the influence of the nuclear level density is assumed to be manifest only during the gamma-ray cascade that follows formation of the CN^* . Thus the level density may be thought of as a controlling agency that determines which of the isomeric states are fed during the γ -cascade. The production of an isomeric state of a given spin, however, may not depend simply on the density of levels accessible to the decaying CN , but may also be dependent on the angular momentum present at the formation of the CN . In this work the initial angular momentum distribution of the CN has been determined using transmission coefficients calculated with the Optical Model¹. These initial angular momentum distributions have been calculated with various methods by other authors. For example, Huizenga and Vandenbosch made these calculations in terms of transmission through a parabolic potential barrier.

The calculation of the initial angular momentum distribution when carried out in the manner described above implies that the theoretical analysis of isomeric state production cross sections carried out in this work will include dependence on the potential parameters chosen for the Optical Model calculations as well as on

* CN --Compound Nucleus

the parameters of the model from which the level density formulae are derived. Further elaboration on this point will be made in Chapter 5 which is a discussion of the experimental and theoretical results of this work.

Chapter 2 is a short review of the theories of nuclear level densities and of the manner in which these theoretical level densities have been used in the calculation of isomeric state production cross sections. Also included in the second chapter is a description of the gamma-ray cascade model devised as a part of this work. Chapter 3 is a discussion of the experimental procedures employed in this work for the determination of isomeric state production ratios. Chapter 4 contains a description of the data analysis technique applied to the results of these experiments. The values of the isomeric ratios that result from this analysis are presented in figures 4.3 to 4.6 of chapter 4. As stated above, chapter 5 is a discussion of the experimental and theoretical results obtained in this work.

Chapter 2

Nuclear Level Densities

Experimentally nuclear level densities appear to vary in a manner which is characteristic of a thermodynamic system having a large number of degrees of freedom. As a result, models constructed for the purpose of making predictions of level densities have for the most part described the nucleus as a gas of Fermions whose energy spectrum is quasi-additive. An excellent review of the models devised before 1960 under this philosophy has been presented by Ericson³. He states that the models which can be expressed in terms of elementary excitations of a Fermion system have considered the nucleus as

- 1/ a system of free neutrons and protons confined to the nuclear volume⁴⁻⁷,
- 2/ nucleons moving in a shell model potential⁸⁻¹³, and
- 3/ nucleons with residual pairing interactions in a deformed well potential¹³⁻¹⁶.

The calculation of nuclear level densities in the type of Fermion systems just described is a combinatorial problem; that is, how many combinations of occupied and unoccupied single particle states are possible at a given energy? This is exactly the kind of problem with which statistical mechanics deals. In fact Ericson states that the analogy with statistical mechanics

is so complete that a large part of the vocabulary about level densities has been directly taken over from thermodynamics. He cautions, however, that there are differences between thermodynamic and nuclear systems that must be kept in mind. For example, a major difference is that at low excitation energies a nuclear system has a very small number of excited particles whereas the usual results of statistical mechanics apply to systems where the number of excited particles is large even at low excitation energies.

In statistical theory the level density of a system with eigenvalues E_i is expressed as the Laplace transform of the partition function $Z(\beta)$. Thus

$$\rho(E) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} Z(\beta) e^{\beta E} d\beta \quad 2.1$$

where $Z(\beta) \equiv \sum_j e^{-\beta E_j}$, and β = inverse of temperature.

The integral in equation 2.1 is evaluated by application of the saddle point method¹⁷. This yields

$$\rho(E) = \left(2\pi \frac{d^2 \ln Z(\beta_0)}{d^2 \beta_0} \right)^{-1/2} e^{\ln Z(\beta_0) + \beta_0 E} \quad 2.2$$

Here β_0 is the value of β where the integrand in equation 2.1 has a minimum defined by

$$- \frac{d \ln Z(\beta_0)}{d \beta_0} = E \quad 2.3$$

The exponent in equation 2.2 is defined as the entropy

$$S \equiv \ln Z(\beta_0) + \beta_0 E \quad 2.4$$

Combining equations 2.3 and 2.4 with 2.2 the level density is expressed completely generally as

$$\rho(E) = \left(-2\pi \frac{dE}{d\beta_0}\right)^{-1/2} e^S \quad 2.5$$

This equation shows that knowledge of the entropy or of the partition function is equivalent to knowledge of the level density.

The level density represented by equation 2.5 does not have the discontinuous nature that a summing of the systems eigenstates would produce. Rather, it has become a smooth function as a result of the averaging produced by the application of the saddle point method. The extent of this averaging is determined by the width η of the saddle point

$$\eta = \left(-\frac{dE}{d\beta_0}\right)^{-1/2} \quad 2.6$$

This mathematical smoothing means that small fluctuations and variations in the level density are not describable in this approximation.

The partition function and consequently the level density can now be evaluated for any of the Fermion gas nuclear models listed above. The partition function is evaluated using the formula;

$$\ln Z(\beta) = \int_0^{\infty} g(\epsilon) \ln(1 + e^{-\beta\epsilon}) d\epsilon \quad 2.7$$

The single particle level density $g(\epsilon)$ which appears in this equation is determined from the one Fermion levels of the Fermi gas model for which the level density is being evaluated. That is,

$$g(\epsilon) = \sum_i \delta(\epsilon - \epsilon_i) \quad 2.8$$

For example Bethe's⁴ equidistant spacing model has one Fermion levels at $d, 2d, \dots, nd, \dots$. Thus the single particle level density is $g(\epsilon) = g_0 = 1/d$. The resulting many particle level density for this model after calculating the entropy with this single particle level density is

$$\rho(u) = K \frac{e^{2\sqrt{au}}}{u} \quad 2.9$$

Here K is a numerical factor and $a = \frac{\pi^2}{6} g_0$

When there are two Fermion systems present, as in nuclei where neutron and proton systems can be treated independently and $g(\epsilon) = g_0 = g_n + g_p = 1/d_n + 1/d_p$ the level density is

$$\rho(u) = g_0 \left[\frac{g_0^2}{4g_n g_p} \right]^{1/2} K \frac{e^{2\sqrt{au}}}{(g_0 u)^{5/4}} \quad 2.10$$

where again K is a numerical factor and

$$a = \frac{\pi^2}{6} g_0$$

The distribution of levels as a function of quantum numbers other than the energy could have been included in the above derivation by considering the grand partition function in equation 2.1 rather than the simple partition function. However,

for the purpose of this work this is unnecessary as the only other quantum number of interest is the angular momentum and its influence on the level distribution can be understood from simple physical arguments.

The total angular momentum of an excited nucleus when projected on the Z-axis is M . This is the sum of the Z-axis projections of the angular momenta of the excited particles. The assumption is made that the angular momenta of these particles occur randomly; that is, a particle in state s has a z component of angular momentum $=\hbar m_s$ completely independent of the values of the z components of the angular momenta that the other excited particles have. The total angular momentum is thus a sum of independent random values. Random experimental errors are variables of this type; made up from many independent chance values. The central limit theorem states that such variables have a normal distribution centered about zero. Thus the distribution of M -values is also expected to be normal with its average at zero;

$$f(M) = \frac{1}{(2\pi\sigma^2)} e^{-M^2/2\sigma^2} \quad 2.11$$

where the mean square deviation is given by

$$\sigma^2 = \nu \langle m^2 \rangle. \quad 2.12$$

Since a z -axis projection M can result from states with total angular momentum $J \geq M$;

$$f(M) = \sum_{J=M}^{\infty} \rho(J)$$

Note: In equation 2.12 ν is the number of excited particles and m is the Z -axis component of the angular momentum.

Thus

$$\begin{aligned}\rho(J) &= f(J) - f(J + 1) \\ &\approx \frac{(2J + 1)}{2(2\pi)^{\frac{1}{2}}\sigma^3} e^{-J(J + 1)/2\sigma^2}\end{aligned}$$

Hence the level density as a function of energy and spin is

$$\rho(E, J) \approx \rho(E) \frac{(2J + 1)}{2(2\pi)^{\frac{1}{2}}\sigma^3} \exp\left[-\frac{J(J + 1)}{2\sigma^2}\right] \quad 2.13$$

For small J , $(2J+1)$ is the dominant feature of the spin distribution (J). However as J increases the exponential factor becomes more significant, eventually reducing $\rho(J)$ to zero. The range of J over which this reduction takes place is determined by the value of the mean square deviation, (equation 2.12). This has resulted in the name "spin cutoff parameter" being applied to the mean deviation.

The type of level density formula derived above (equations 2.10 and 2.13) and the derivation technique have been employed, with alterations intended to account for specific characteristics, by many authors for the analysis of experimental data. Table 2.1 lists several of these level density characteristics and some of the changes made in an attempt to account for them. The effects considered include the shell structure influences in terms of Klinkenberg's level scheme¹⁸ and residual interaction influences in terms of the pairing model¹⁵ and the superconducting model.^{19,20}

Table 2.1

Effect accounted for	Alterations made (Reference)
Overall behavior and gross structure of level density throughout the periodic table (Free gas model)	<p>In equidistant spacing model^{3,4,6} (EDS) use: $g_0 = (3/2)A/\epsilon_0$ where A = mass number ϵ_0 = Fermi energy = $\left(\frac{3}{\pi}\right)^{2/3} \left(\frac{9}{4}\right) \frac{\hbar^2}{2Mr_0^2}$ M = nucleon mass r_0 = nucleon radius</p>
Shell Effects	<p>In EDS³ use: $g_0 = \overline{g(\epsilon)}$ where average is over small region in neighbourhood of Fermi energy. Newton⁸ averages $g(\epsilon) \approx 2\alpha A^{2/3} (j_n + j_z + 1)$ where α = adjustable parameter j_n and j_z are the spins of the neutron and proton shell model states (from Klinkenberg's shell model level scheme)¹⁸ Cameron¹⁰ averages $g(\epsilon) \approx 1/d(Z) + 1/d(N)$ where $d(Z)$ and $d(N)$ are the proton and neutron spacings derived from Cameron's empirical mass formula.</p>
Rosenzweig Effect (fewer rearrangements possible for a nearly full or a nearly empty shell than for a half filled shell)	<p>Alteration of excitation energy to:^{3,7} $U' = U + \frac{n^2 d_n}{12} - \frac{d_n}{2} \left(k_n - \frac{n}{2}\right)^2 + \frac{p^2 d_p}{12} - \frac{d_p}{2} \left(k_p - \frac{p}{2}\right)^2$ where d_n and d_p are neutron and proton spacings k's are numbers of particles in shells n and p are the number of neutrons and protons in those shells.</p>
Residual interaction and odd-even effects (pairing model)	<p>Alter excitation energy $U' = U - 2\mu\delta$^{3,15} where 2δ = pairing energy μ = number of broken pairs Calculation of energy and spin made with condition that μ pairs are broken. Then, ρ odd (U) = ρ even ($U + \delta$) ρ odd-odd (U) = ρ even ($U + 2\delta$)</p>

Table 2.1 (Cont'd)

Residual interaction
(Superconducting Model)

"Pairing Energy" appears naturally^{19,20}
as a condensation energy.

Energy dependence becomes:¹⁹

$$\rho(U) = \left(\frac{\beta}{\pi g}\right) (A(U))^{-1} \left(-2\pi \frac{\partial U}{\partial \beta}\right)^{-1/2} e^{S(U)}$$

where $\beta = 1/t$, inverse temperature

g = single particle level density

S = entropy

and $A(U) = \int_{-\infty}^{+\infty} \{e^{(x^2 + \beta^2 \theta^2)^{1/2}} / [1 + e^{(x^2 + \beta^2 \theta^2)^{1/2}}]\}^2 dx$

which Vonach et al.²⁰ have evaluated
numerically.

Here $\theta^2 = E_k^2 - \epsilon_k^2$

E_k = single particle excitation
energies in condensed system

ϵ_k = same in Fermi gas system

Effect on spin distribution appears
in the spin cutoff parameter

$$\sigma^2 = ct A(U)$$

c is related to the rigid body moment
of inertia.

Some of the results of the models listed in table 2.1 have been employed in this work in the calculation of isomeric state production ratios.

The solution of the combinatorial problem of occupied and unoccupied single particle states that produces, for example, equation 2.9 is achieved through the application of statistical theory. This results in many mathematically convenient approximations being made so that the level density formulas can be obtained. Any shortcomings of these formulas are usually overcome by the matching of known data through parameter adjustments, for example, Decowski et al.²² The confrontation between nuclear models and level density data is as a consequence somewhat indirect. In an attempt to effect necessary improvements, i.e., to avoid as much as possible purely mathematical approximations, Hillman and Grover²¹ have recently devised a scheme for calculating level densities numerically on a large computer. The elimination of mathematical approximations should produce results that are more directly related to the physical properties of the nucleus, thus implying that the level densities generated numerically by the Hillman and Grover (H&G) method are to be preferred to those determined analytically with the saddle point integration technique. The operation of the H&G computer program is described in the following paragraphs.

Numerical Generation of Nuclear Level Densities:

H&G use an "odometer" to scan the nuclear configurations systematically and the energy and spin distribution of each

configuration is calculated, giving the number of levels of a specific spin at each energy. The number of levels which have this given spin, that is, total angular momentum J , for each configuration of subshells is calculated by finding the number of ways the component j 's can couple to J . The energy of each configuration is obtained by the method of Bardeen, Cooper and Schrieffer²³ (BCS) as described by Seeger²⁴ and Wahlborn²⁵. The energy of configuration β is, by this scheme:

$$E_{\beta} = \sum \epsilon_i^0 + \sum' 2\epsilon_{i\beta}' v_{i\beta}'^2 - \Delta_{\beta}^2/G \quad 2.14$$

where G defines the pairing force parameter (whose value is found by appeal to experiment),

Here $\epsilon' = \epsilon - Gv'^2$

and

$$v_{i\beta}'^2 = \frac{1}{2} \left(1 - \frac{\epsilon_i' - \lambda_{\beta}'}{((\epsilon_{i\beta}' - \lambda_{\beta}')^2 + \Delta_{\beta}^2)^{1/2}} \right).$$

The values of Δ_{β} and λ_{β}' are obtained by solution of the equations

$$\sum' 2v_{i\beta}'^2 = \eta_{\beta}$$

$$\sum' (1/((\epsilon_{i\beta}' - \lambda_{\beta}')^2 + \Delta_{\beta}^2)^{1/2}) = 2/G$$

where $(1/2) \eta_{\beta}$ is the total number of nucleon pairs included in the range of summation.

The superscripted sums are: Σ^0 , over orbitals containing unpaired nucleons

Σ' only over orbitals which do not contain unpaired nucleons.

This summation procedure makes it possible to omit orbital pairs containing unpaired nucleons from the BCS calculation and then add the corresponding single-particle energies to the BCS energy later. In the above equations the ϵ_i are the eigenvalues of the orbitals whose values come from the Nilsson²⁶ single-particle model. The ordering of these single particle levels and their energies were obtained from Seeger²⁷. These values, which are listed in Table 2.2 (taken from H&G²¹) are fed to the computer program.

The number of times a specific spin occurs is summed for all configurations whose energies fall within a predetermined energy interval, (1/50)th of the maximum energy. The proton and neutron spin distributions and corresponding energies are calculated separately and then coupled. When the proton and neutron results are coupled the energies are simply added since a neutron-proton interaction is not considered. The energies of these bins and the number of times each occurs as a function of spin are then output as the calculated level densities.

The level densities derived above are used in models designed to make theoretical predictions of isomeric ratios. Several of these models are described in the next section.

Table 2.2
 Shell Model Used by
 Hillman and Grover²¹

Protons		Neutrons	
Number in Shell	Energy (MeVxA ^{1/3})	Number in Shell	Energy (MeVxA ^{1/3})
2	0.0	2	0.0
4	28.8	4	30.9
2	41.1	2	46.7
6	57.3	6	62.7
4	73.6	2	78.8
2	75.9	4	83.6
8	85.4	8	94.7
6	104.6	4	112.6
4	107.9	6	119.4
10	113.1	2	123.2
2	116.2	10	126.7
8	134.7	6	146.2
6	138.7	8	154.5
12	140.5	2	158.0
4	150.8	12	158.6
2	152.5	4	161.6
10	164.1	8	179.4
14	167.6	10	189.0
8	168.8	14	190.5
6	183.8	4	193.4
4	186.4	6	198.7
2	192.8	2	201.7
12	193.0	10	212.4
16	194.5	16	222.3
10	198.1	12	223.0
8	215.7	6	228.3
6	219.0	8	235.0
18	221.1	2	237.9
14	221.4	4	240.8
12	226.9	12	245.3
4	228.7	18	253.9

THEORETICAL ISOMERIC RATIO CALCULATIONS

Several schemes have been devised to calculate theoretically the isomeric ratios that result following various types of nuclear reactions. All of these schemes are the same in an overall sense. First the distribution of spins and energies is calculated for the CN and then the emission of particles is considered for the cases that lead to the residual nucleus in which the isomer ratio is being sought. This particle emission results in a new distribution of spins and energies that is dependent on the original distribution in the CN and on the level density of available states in the residual nucleus. The final step in these calculations is to consider a gamma-ray cascade that leads to the isomeric or the ground state. The result is a partitioning of the total population between the isomeric state and the ground state.

Such a scheme was devised by Huizenga and Vandenbosch in 1960²⁸ and subsequently employed by several authors²⁹⁻³². Their scheme is a fairly simple and straightforward processing of the described procedure. H&V calculated the cross section for the formation of the compound nucleus of spin J_c at the bombarding energy E using the formula:

$$\sigma(J_c, E) = \pi \lambda^2 \frac{\sum_{S=|I-s|}^{I+s} \sum_{\ell=|J_c-s|}^{J_c+s} T_{\ell}(E) \frac{(2J_c+1)}{(2s+1)(2I+1)}}{2.15} \quad 2.15$$

where λ = de Broglie wavelength of the incoming particle,

I = intrinsic spin of the target nucleus,

S = intrinsic spin of the projectile, and

$T_{\ell}(E)$ = barrier transmission coefficient for a projectile of angular momentum ℓ and energy E . (in this work calculated with the optical model)

Note that the maximum angular momentum of the system is

$J_{c, \max} = \ell_{\max} + S + I$, and is either integral or half integral, depending upon the intrinsic spins.

From the results of the above calculation H&V proceed to a calculation of the probability that the CN, has spin J_c ; that is,

$$P_{J_c} = \frac{\sigma(J_c, E)}{\sum_{J_c=0}^{\infty} \sigma(J_c, E)} \quad 2.16$$

This gives the initial spin distribution of the compound nucleus from which the spin distribution following particle emission is calculated. A particular state of spin J_c can decay by particle emission to a variety of states, and the relative probability of population of state J_f is given by

$$P(J_f) \propto \rho(J_f) \sum_{S=|J_f-s'|}^{J_f+s'} \sum_{\ell'=|J_c-S|}^{J_c+S'} T_{\ell'}'(E) \quad 2.17$$

where s' = intrinsic spin of the emitted particle,

$T_{\ell'}'(E)$ = transmission coefficient of the emitted particle of energy E and angular momentum ℓ'

and $\rho(J_f)$ = the density of levels of spin J_f in the residual nucleus.

Once this new distribution is normalized it is used as the initial distribution for a further particle emission calculation or as the initial distribution for a gamma-ray cascade to the isomeric and/or ground state. The formula above is employed repeatedly until all particles are emitted. For the gamma cascade the probability of decaying from one of the states, spin J_i , populated after the particle emission has stopped to a state of spin J_f is assumed to be simply proportional to the density of final states with spin J_f . This probability is given by

$$P_{J_f} = \frac{F_{J_i} \rho(J_f) \delta(J_i, J_f)}{\sum_{J_f=|J_i-\ell|}^{J_i+\ell} \rho(J_f)} \quad 2.18$$

where $\rho(J_f)$ = the density of levels of spin J_f

$$\delta(J_i, J_f) = 1 \text{ for } |J_i - J_f| \leq \ell \leq |J_i + J_f|$$

= 0 for all other conditions due to selection rules,

ℓ = multipolarity of gamma emission,

F_{J_i} = spin distribution following evaporation of last particle (result of equation 3.17)

This formula is used in a repetitive fashion similar to that for multiple particle emission, until the total number of gamma's in the cascade has been exhausted save for one. From the resulting spin distribution H&V determined the isomeric

ratio by assuming that the last gamma-ray fed either the isomeric state or the ground state depending on which required the smaller spin change.

Since much of the further work on models of this type involves variations of the H&V assumptions, these assumptions are listed here to facilitate discussion of these variations.

Huizenga and Vandenbosch Model Assumptions:

- 1/ No energy dependence in the level density.
- 2/ Only E1 transitions occur in the gamma cascade except for the final transition.
- 3/ The isomeric and the ground state are fed only by the last transition depending on which requires the smaller spin change.
- 4/ The transition probabilities between levels are proportional only to the spin dependent part of the level density of the occupied levels.
- 5/ The multiplicity of the gamma-ray cascade is treated as a free parameter.

In 1961 Troubetzkoy³³ derived from statistical theory a gamma-ray cascade model for an entirely different purpose than that of Vandenbosch and Huizenga. The purpose of his theory was to predict the gross features of the gamma-ray spectra which are produced following intermediate or high energy nuclear reactions. Despite this different goal Troubetzkoy's gamma cascade model was very similar to that of H&V differing only in

assumptions 1/ and 4/. These he replaced with

- 1/ The level spectrum can be considered as composed of a discrete part, with levels at $E=E_i$ ($i=0,1,2,\dots,n$) and of a continuous part with a density $\rho(E)$ such that $\rho(E)dE$ gives the number of levels between E and $E+dE$; i.e. $\rho(E)$ derived from the Fermi-gas model.
- 4/ The transition probability in the continuum region of the level density is proportional to $\rho(E)$
(note; no spin dependence)

This model has been successfully employed for the prediction of gamma-ray spectra but because of the lack of a spin dependence it is not applicable to isomeric ratio calculations.

In 1964 Vonach, Vandenbosch and Huizenga²⁰ (V&V&H) published a paper in which they described a γ -cascade model with a number of changes from the original H&V model described above. These changes consisted of replacing assumptions 1/, 3/ and 4/ listed by the following:

- 1/ The energy dependence of the level density is proportional to

$$\frac{e^{2\sqrt{au}}}{u^2}$$

- 3/ If a gamma transition leads to an energy level whose energy is less than some previously defined lower energy then only the isomeric or ground state are fed by the next transition depending on which requires the smaller spin change. If the transition leads to a level whose energy is above some previously defined upper energy then a new spin distribution is calculated

following the next transition with none of the population going to the isomeric or the ground state.

If the transition leads to an energy E between the previously defined upper, E_U , and lower E_L , energies, then there is a probability $E_U - E$ that the last and deciding gamma-ray is emitted and a probability $E - E_L$ that one gamma-ray precedes the deciding gamma-ray.

- 4/ The transition probabilities between levels are energy dependent through assumption 1/ and through the dependence of the spin cutoff parameter σ on the nuclear temperature;

$$\sigma^2 = ct \quad \text{Fermi Gas}$$

$$\sigma^2 = A(U)ct \quad \text{Superconductor model.}$$

Assumption 5/ was effectively eliminated in this work, having been replaced by:

- 5/ The average gamma-ray transition energy is

$$\bar{E}_\gamma(U_0) = \frac{\int_0^{U_0} \rho(U_0 - E_\gamma) E_\gamma^4 dE_\gamma}{\int_0^{U_0} \rho(U_0 - E_\gamma) E_\gamma^3 dE_\gamma} \quad 2.19$$

V&V&H through numerical calculations found this to be equivalent to the empirical formula

$$\bar{E}_\gamma(U_0) = 4 \left[(U_0/a) - (5/a^2) \right] \text{ Mev} \quad 2.20$$

where a is the single particle level density.

The energy of each gamma-ray in the cascade is calculated using this empirical formula after the excitation energy that remains following the previous gamma-ray has been computed. This assumption combined with the new assumption 3/ regarding the gamma cascade cutoff uniquely determines the number of gammas in the cascade.

Poenitz³⁴ in 1966 devised a gamma-ray cascade model which was a combination and extension of the models of Troubetzkoy, and Huizenga and Vandebosch. His model changed assumption 2/ of the original H&V model as well as the changes in assumptions 1/ and 4/ made by Troubetzkoy. Poenitz altered assumption 2/ to read

- 2/ transitions between levels inside the continuum and transitions from the levels in the continuum range to discrete low-lying levels are of the type E1, E2, M1, M2. Transitions between the discrete levels are established by the experimental values for the transition probability.

This inclusion of transition probabilities in the cascade model made it possible to consider gamma transitions to various levels from any given level, a feat not possible with the original H&V model nor with the V&VH variation of the H&V model. Figure 2.1 shows pictorially what this means.

Sperber³⁵ has described a model very similar to that of Poenitz with two exceptions. First, Sperber's method of evaluating level occupation probabilities included integration

Vandenbosch and Huizenga
Cascade

Poenitz and Sperber
Cascade

FIGURE 2.1

Types of γ -ray Cascades

over their time dependence. Poenitz did not consider this parameter. Second, Sperber did not include in his model the possibility of using experimentally determined low lying levels.

Since both Sperber and Poenitz included the possibility of higher gamma-ray multipolarities in their models, they were able to show that the isomeric ratio has some dependence on the amount of quadrupole radiation in the cascade.

All of the γ -ray cascade models described above employ analytic level density formulae whose derivations depend on mathematically convenient approximations. In order to avoid the use of such formulae we have designed a new gamma-ray cascade model in which the level densities generated numerically by the H&G computer routine may be employed.

Mathematically convenient approximations have also been made in the above described cascade models themselves. The H&V model, for example, deals only with one continuous chain of γ -ray transitions, no overlapping transitions are considered. In Sperber's model the integrations over the continuous region of levels are executed with a successive approximations procedure. The employment of a large fast computer has made it possible to design the new cascade model in such a way that many of these approximations are also avoided. A description of this new cascade model is given in the next section.

A New Gamma-ray Cascade Model:

The initial spin distribution of the CN is calculated in the same way as it was for the H&V model; that is, by use of

transmission coefficients from the Optical model and equations 2.15 and 2.16. The decay of the lowest spin state (0 for an even CN and 1/2 for an odd CN) at the CN formation energy is then considered. The γ -decay transition probabilities, $T(E, J \rightarrow E', J')$, to all levels of lower energy whose spins differ by ≤ 5 units are estimated. These estimated probabilities are multiplied by the number of states at the destination spin and energy, that is by $\rho(E', J')$. The resulting factors are normalized as follows:

$$T(E, J \rightarrow E', J') = \frac{T(E, J \rightarrow E', J') \rho(E', J')}{\sum_{E'' J''} T(E, J \rightarrow E'', J'') \rho(E'', J'')} \quad 2.21$$

The population of the decaying state multiplied by the appropriate normalized transition probability is assumed to be the contribution to the population of the destination state made by this decay.

: The decay of the state of next higher spin (1 for an even CN and 3/2 for an odd CN) at the initial CN energy is then treated in the same manner, the contributions made by its decay to the population of lower energy states being added to the contributions made by the previous state's decay. This process is repeated for each higher spin state until all the populated spin states at the initial CN energy have been exhausted.

The energy level which lies immediately below the one whose decay has just been considered is then treated in an identical way. The populations of its spin states, which are to be the input to the decay, are taken to be those populations that have

been built up by the decay of the energy level above. The contributions made by the decay of each spin state of the present energy level to the populations of all lower energy states are calculated as described above and then added to the populations that have been built up by the decay of the previously treated higher energy level.

The procedure just described is repeated for each succeeding lower energy level until the decay of all levels except the isomeric and ground states has been considered. The total initial population of the CN (normalized to unity; equation 2.16) is now divided between the isomeric state and the ground state; in fact the isomeric ratio is given by the population of the isomeric state since the total population is unity.

The computer routine, written to carry out calculations of gamma-ray cascades as described, has been designed so that at any point in the cascade experimentally determined levels can be introduced. Thus the i^{th} level will have energy E_{ex} and $\rho(E_{\text{ex}}, J) = 0$ except for the case $J = J_{\text{ex}}$, then $\rho(E_{\text{ex}}, J_{\text{ex}}) = 1$. The isomeric and ground states are introduced into the cascade calculations in this way.

The estimates of the gamma-ray transition probabilities that are required in the above calculations are made using the estimation formulae of Moszkowski³⁶. These formulae are listed in Table 2.3. He derived these formulae from the single particle model transition rates for a single proton. This implies that in the above cascade model it is assumed that all transitions are

of single protons. However, little generality will be lost because of this assumption*since single neutron transition rates are probably qualitatively very similar to those of single protons³⁶ and the multiplication by the level densities and subsequent normalization makes only the relative magnitudes of these rates significant. Also, even for the maximum excitations considered in this work, 2 MeV above the neutron binding energies, multiple particle transitions will be relatively insignificant.

Table 2.3

Estimates for the transition probability
of a single proton

Multipolarity	$T(E, J \rightarrow E', J')$ in sec^{-1}			
E1	1.0×10^{14}	$A^{2/3}$	E^3	S
M1	2.9×10^{13}		E^3	S
E2	7.4×10^7	$A^{4/3}$	E^5	S
M2	8.4×10^7	$A^{2/3}$	E^5	S
E3	3.4×10^1	A^2	E^7	S
M3	8.7×10^1	$A^{4/3}$	E^7	S
E4	1.1×10^{-5}	$A^{8/3}$	E^9	S
M4	4.8×10^{-5}	A^2	E^9	S
E5	2.5×10^{-12}	$A^{10/3}$	E^{11}	S
M5	1.7×10^{-11}	$A^{8/3}$	E^{11}	S

Figure 2.2 is a flow diagram for the new cascade model computer routine.

*The validity of the assumption will be discussed further in the Conclusion portion of this thesis.

Flow Diagram for New Cascade Model

Chapter 3

Experimental Procedures

The Li(p,n) Reaction:

Neutrons of variable energy were produced from the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction. The neutrons produced by this reaction have been studied extensively and their energies and angular distributions are well tabulated³⁷. These studies indicate that this reaction is almost ideal for the production of variable energy neutrons. A number of problems do, however, exist:

- 1) there are energy regions in which the reaction produces two neutron energies at a given angle (referred to here as double energy regions)
- 2) there is a large resonance in the neutron production cross section at 2.25 MeV proton energy.

The first of the double neutron energy regions occurs just above the neutron production threshold, arising in this region because the neutrons coming off at 180° in the "center-of-mass" system still have a forward velocity in the lab system. The low energy component disappears when the proton energy is about 35 keV above the threshold, that is, it is present until the neutrons of the high energy component have an energy of about 120 keV. Since only a few of the measurements in this work were made in this energy region the effects of the low energy neutrons

will be deferred for discussion with the experimental results.

The second double energy region occurs at proton energies greater than 2.378 MeV. It is due to the presence of the ${}^7\text{Li}(p,n){}^7\text{Be}^*$ reaction, whose threshold is at 2.378 MeV. The neutrons produced by this reaction may be neglected or, at most, treated as a background effect since this reaction is much less prolific than the ground state reaction ${}^7\text{Li}(p,n){}^7\text{Be}$. The low energy neutron intensity is less than 3% of the high energy component up to a proton energy of 2.5 MeV and is thus considered negligible. Above this energy the intensity is the order of 11% and is treated as a background in the discussion of experimental results.

The proton accelerators employed in this work were the Dynamitron* of the Universities of Carleton and Ottawa and the Van de Graaff** of the National Research Council***.

Calibration:

The calibration system of the NRC Van de Graaff consists of a calibrated bending magnet and slit set. A single point such as the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction threshold is sufficient to relate the magnet and slit system calibration to proton energies. Consequently a threshold curve was taken before the NRC experiments for this purpose.

*Trade name of Radiation Dynamics Corp. for their high current, parallel fed accelerators.

**Trade name of High Voltage Engineering Corp. for their electrostatic fed accelerators.

***I would like to thank Dr. K.W. Geiger and the NRC staff for their gracious hospitality.

The Dynamitron was calibrated by (p,γ) and $(p,\alpha\gamma)$ reaction resonances as well as the lithium reaction neutron threshold and second forward peak. The (p,γ) and $(p,\alpha\gamma)$ resonances employed are listed in table 3.1. The $\text{Li}(p,n)$ reaction threshold was taken to be $1.88036^{+0.00022}$ MeV from Lauritsen³⁸ and the energy of the second peak in the forward yield to be 2.278 MeV from Gibbons³⁹.

Table 3.1

Reaction energies used to calibrate the Dynamitron

Target	Energy
^{27}Al	.991
^{88}Y	1.78
^{13}C	1.748
^{15}N	1.640 1.979

Stability:

Stability and reproducibility tests were carried out on the Dynamitron by Walker and Weaver.⁴⁰ By setting the machine energy half way up the rapidly rising edge of the $\text{Li}(p,n)$ yield curve near threshold any energy change can be observed as a counting rate change. They found that if the Dynamitron was left undisturbed, then the beam energy remained constant to about ± 300 eV. The irradiations carried out in this work were all performed under these undisturbed conditions so that an energy drift of ± 1 keV is a reasonably conservative estimate (three standard deviations according to W&W). W&W and others found that

the correspondence between the indicated machine voltage and the true energy could vary by several tens of keV when the machine was shut down and then restarted. However, these tests also showed that the ratio of indicated machine energy units to real energy units remained constant. This meant that a one point calibration each time the machine was restarted sufficed to establish the relation between the indicated and real energy units. A ${}^7\text{Li}(p,n){}^7\text{Be}$ threshold measurement was made at the beginning of each run in order to make this calibration.

Lithium Targets:

Several facilities⁴² employ the $\text{Li}(p,n)$ reaction to produce neutrons of variable energy for further studies of neutron reactions. The lithium target materials used at most of these facilities are lithium metal and/or LiF which are evaporated onto some backing material. The production and handling of lithium metal targets is complicated and so this type of target was left for later consideration. LiF targets were manufactured and employed for some time in the early stages of this work. However, this work was being done on a facility where other groups were doing (p,γ) reaction studies. Because the $\text{F}(p,\alpha\gamma)$ cross section is large and any stray fluorine would therefore cause a large background effect in the proton capture studies, a search was made for a suitable material to replace the lithium fluoride. It was found that Li_2CO_3 was sufficiently stable that it could be evaporated onto gold plated target backings and would remain there, apparently indefinitely.

Experience showed, however, that the lithium carbonate was removed very easily by water and as a consequence a great deal of care had to be taken to ensure that the targets remained dry. Experience also showed, through employment of a technique described below, that the lithium carbonate targets did not deteriorate appreciably due to bombardment with the proton beam. As a result Li_2CO_3 was employed in all the later part of this work as well as that of Walker and Weaver⁴⁰.

An empirical technique for estimating the thickness of the lithium carbonate targets resulted from the work of W&W. This technique also made it possible to determine whether any deterioration of the targets was taking place as a result of the proton bombardment. Walker and Weaver found that they could reproduce the forward yield peak near threshold from theoretical considerations. During the course of these studies they found that the width of this peak at 90% of its height gave a good numerical estimate of the target thickness. No account can easily be taken of the variation in thickness of the target due to non-uniform deposition of the lithium bearing material with this technique for determining target thicknesses. As a result it has been assumed that the effects of such a non-uniformity are negligible.

The evaporated lithium fluoride or carbonate targets were mounted in a chamber designed to provide both quick target change capability and direct water cooling. Figure 3.1 is a diagram of this target chamber.

FIGURE 3.1 TARGET CHAMBER

The sources irradiated with neutrons in this work were mounted so that they could be counted in the 4π beta counter described below. The mountings for the selenium and bromine sources consisted of an aluminium ring, on both surfaces of which was glued aluminium coated mylar sheets. The selenium metal powder, or the potassium bromide powder, was packed between the mylar sheets. For the zinc sources the mylar sheet was replaced with zinc foil with small pieces of zinc metal placed between these foils. For the dysprosium sources the aluminium ring was replaced with one made of plexiglass so that a 2.3 minute aluminium activity would not be produced, as this would mask the 1.255 minute dysprosium isomeric state activity. The dysprosium oxide powder was packed between mylar sheets in a manner similar to the Se and Br samples.

Sample Irradiation:

For the purpose of irradiation these sources were in most cases mounted directly on the target chamber. Although this arrangement produced an effective spread in the neutron energy, up to 50 keV, due to the large angular acceptance involved, it was necessary in order to produce the maximum amount of activity. Some of the irradiations were, however, made with the sources mounted at a distance of one foot from the target chamber and at various angles to the incident proton beam. This type of arrangement made it possible to produce activities with several different neutron energies using a single proton energy. The resulting activities were found to be so significantly reduced

from those of the directly mounted cases, that this procedure was employed sparingly, being employed only for a few of the bromine irradiations. All the samples irradiated were wrapped in cadmium foil in order to avoid the effect of thermalized neutrons.

During the irradiation of all the sources the neutron flux was monitored by a system consisting of a conventional long counter of the type described by Allen.⁴³ The stability of this monitoring system was checked by counting the neutrons from a ^{252}Cf fission source for a period of ten days. A long term drift of +1.5% was observed in the total counts collected over eight hour periods. The stability of the monitoring system was also checked by W&W for two separate 100 hour periods by counting an Am-Be source. They found that there were no apparent counting rate shifts due to gain changes during the whole of this period greater than the counting statistics which were at the 1% level (1 standard deviation). Since the longest periods over which the neutron flux was monitored were less than ten hours, the stability of the long counter system was considered to be adequate for this purpose. The calibration of the system was re-established before each run with an Am-Be source.

The output of the long counter was fed through a pre-amplifier, main amplifier and discriminator to either a multi-scaler or scaler-timer plus scaler. The multi-scaler system consisted of a Victoreen PIP-400 analyser set in the multi-scale mode with a digital timer* as an external regulator of the time

* I wish to thank Mr. G. S. Black who designed and built this

period of each channel. When this system was not available a simple scaler-timer scaler combination was used to make one minute counts of the neutron flux every five or ten minutes.

The purpose of this monitoring of the neutron flux was twofold:

- 1) to aid the machine operator in maintaining a constant neutron flux
- 2) to collect data necessary for the determination of the isomeric ratio (see discussion of equations 4.2-4.5).

The periods over which the irradiations were made were determined by several factors. Under ideal circumstances the irradiations were carried on for a period of time of at least two and if possible three half lives of the long lived state. This meant that the activity was very nearly saturated. In many cases however, the irradiation periods were shorter than this ideal because:

- 1) the accelerator failed (especially significant at higher energies)
- 2) the proton or neutron flux showed a marked and irretrievable deterioration
- 3) the fact that the long half life was simply too long to make an irradiation for two such periods practical.

These shortened irradiation periods (saturation not reached) were taken account of in the data analysis computer routine by including a formation function calculation. This function is described in the section on data analysis.

Once a sample was activated its decay was followed for

at least several half lives of the short lived state and usually for one or two half lives of the long lived state. For this purpose, a 4π beta counter of the Chalk River design was employed.⁴⁴ The counter was connected through a pre-amplifier, amplifier and discriminator to a PDP-9 computer which was programmed to function as a multi-scaler. The discriminator was necessary for two reasons: First, the computer interface required the three volt positive going pulses which the discriminator provided; second, the discriminator level was used to eliminate background noise, which came principally from the accelerator.

The energy deposited in the 4π counter by the beta particles was in general determined by the path length traversed in the methane gas, the longest possible being ~ 1.5 inches. Thus the greatest amount of energy deposited in the counter was ~ 10 keV. This means that many of the betas, whose maximum energies for all the nuclei studied are ~ 1 MeV, completely traverse the counter. On the other hand low energy betas and many of the conversion electrons were absorbed in the samples. The conversion electrons which escaped from the samples were accounted for in the data analysis as described in chapter 4.

A computer routine was written which added the pulses entering from the 4π counter into a single channel. This addition continued going into the same channel until a predetermined time had passed according to the computer's internal clock, at which time the routine began adding the

was made to function as a multi-scaler. The computer routine was arranged so that the period of the multi-scaler channel was alterable from the computer keyboard.

Compilation of the beta counter output by the computer multi-scaler system was started at the instant the irradiation of the sample was stopped. The sample was then transported by hand as quickly as possible from the target room to the counting room and mounted in the beta counter. This procedure ensured that the lower edge of the first multi-scaler channel was time zero for the decay of the sample. The faster the transport of the sample was accomplished the fewer were the blank channels at the beginning of the decay spectrum, a factor especially significant for ^{165}Dy (1.255 minute half life) but not very important for the other samples (all other half lives greater than 17 minutes).

The decay curves which resulted from the above procedure were transcribed for use in the Sigma-7 Fortran data analysis routine described in the next chapter.

At the NRC Van de Graaff facility a similar procedure of irradiation, neutron monitoring and decay curve compilation was followed except that a Northern Scientific* ADC complete with memory unit was employed in place of the computer. The ADC had a built in multi-scaler option with the possibility of advancing the channel externally. The multi-scaler channels were advanced in this case by the digital timer employed with the neutron monitoring system described above. The neutron

* Northern Scientific Inc.

monitor counting system consisted of a scaler which was regulated by a timer-scaler. The scaler and timer-scaler were interfaced to a teletype so that the compiled counts could be printed out, the scalers then cleared themselves and the counting cycle was then repeated.

The decay curves accumulated at NRC were also transcribed for analysis with the Sigma-7.

The decay curves accumulated by the above described procedures were not influenced by dead times in the 4π counter and multi-scaling systems since at no time were the counting rates greater than ~10,000 counts per minute (~300 counts per second). As a result no corrections for these type of effects were necessary in the data analysis discussed in Chapter 4.

In the $\text{Li}(p,n)$ threshold measurements made to calibrate the energy scales of the Dynamitron and Van de Graaff accelerators, the long counter employed as a neutron monitor was used to measure the 0° neutron flux as a function of the indicated proton energy. Since the long counter subtended a half-angle of 40° the neutron flux measured was the total production up to a neutron energy ~15 keV. It was therefore possible to extrapolate the neutron production back from an energy ~10 keV above threshold to determine the threshold itself. The indicated proton energies were thus related to the energy of the $\text{Li}(p,n)$ threshold, 1.8804 MeV. The effect of this calibration procedure on the determination of the neutron energies at which irradiations were made is discussed under error estimates in the next chapter.

Chapter 4

Data Analysis

The determination of isomeric ratios from the experimental multi-scaled decay curves was accomplished with a computer program written to do a least squares fit of a growth and/or decay function to these spectra. The function programmed for this purpose was taken from Wing;⁴⁶

$$A_m + A_g = \left| \frac{p\alpha'_m}{1+\alpha'_m} + b_m + \frac{g_m\alpha_m}{1+\alpha_m} \right| A_m^0 e^{-\lambda_m t} + \left| b_g + \frac{g_g\alpha_g}{1+\alpha_g} \right| \left\{ A_g^0 e^{-\lambda_g t} + \frac{pA_m^0\lambda_g}{\lambda_g - \lambda_m} (e^{-\lambda_m t} - e^{-\lambda_g t}) \right\} \quad 4.1$$

where A_m and A_g are the observed decay rates for the isomeric and ground state decays respectively. A_m^0 and A_g^0 are the time zero decay rates of these states, time zero here refers to the time at which activation was stopped and counting begun. λ_m and λ_g are the inverse mean-lives of the isomeric and ground states respectively. Figure 4.1, which comes from Wing's paper, explains the other terms in this function. For the purposes of evaluating the data, background terms were added to this function, one a constant to account for the background due to the counter's environment and a second with a half life dependence to account for an undesired activity produced in the samples. The value

FIGURE 4.1
A Generalized Decay Scheme of Isomers

- b- percentage of beta emission in the total disintegration
- c- percentage of electron capture in the total disintegration
- p- percentage of isomeric transition in the total disintegration
- g- percentage of gamma transition in the total disintegration
- α - internal conversion coefficient, the ratio of the number of converted electrons to the number of unconverted photons

of the constant background term was evaluated by counting an unirradiated sample.

The central problem in analysis is determination of the time zero decay rates, A_m^0 and A_g^0 , since the ratio of these values is related to the isomeric ratio. This relationship is discussed below. Two methods were employed in this work for calculating these initial decay rates from the data. The first of these methods was the technique used by Bishop³² to calculate isomeric ratios from his experimental data. He assumed the values of the half-lives of the isomeric and the ground states to be those published in the literature, a basic assumption made in this work as well. With these lifetime values Bishop calculated the value of the time zero decay rate for the long lived state, either the isomeric or the ground state. This was accomplished by considering the experimental counting rate at a time of the order of several times the half-life of the short lived state, thus reducing the function to one in which the only unknown was the initial decay rate of the long lived state. Once this value was calculated, it was used to reduce the function at a time short compared with both half-lives to one with a single unknown, the initial decay rate of the short lived state. This, Bishop evaluated from the experimental counting rate at the earliest possible time.

When Bishop's technique was employed in this work to calculate the initial decay rates the results were found to be inconsistent. Generally the decay rate of the long lived state was overestimated.

The second technique employed in this work to calculate the initial decay rates was to make a linear least squares fit of equation 4.1 to the data. The data were smoothed before the linear least squares fitting but the χ^2 of the fits were calculated with the raw data. The smoothing process was based on the method developed by Savitsky and Golay⁴⁸ and as applied by Yule⁴⁹ who used it for gamma ray spectra. The method of calculation is to compute a new value of each datum in the spectrum based on a least squares fit of a power function to a small neighbouring region of the raw data. If the spectrum was defined as the array C, the number of counts in channel i was C_i . The smoothed value of the datum C_i was then D_i computed from;

$$D_i = (1/N_n) \sum_{j=-n}^{j=+n} a_{n,j} C_{i+j} \quad 4.2$$

The constants $a_{n,j}$ and the normalizing factor N_n were taken from tables presented by Savitsky and Golay.⁴⁸

In the least squares calculations the normal equations were solved using the Crout method⁵⁰ of matrix solving. The standard deviation of each of the parameters in the resulting fit may be calculated from the diagonal elements of the solution matrix.⁵¹ Two of the parameters in this fit of equation 4.1 to the data, A_m^0 and A_g^0 , are used to calculate the isomeric ratio. The standard deviation of this ratio was calculated from the standard deviation of these parameters. The resulting deviation

accounts only for the random error, that is the counting statistics, and for the goodness of the fit. The inclusion of other factors in the errors is discussed later. Figures 4.2, 4.3, 4.4 and 4.5 are examples of fits to the decay spectra for ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy respectively.

The values of the isomeric ratios were calculated from the results of the decay curve fits in a manner which takes account of the fact that the source irradiations were not of infinite duration and that the neutron flux was not a constant throughout the irradiations. This was accomplished in the following way. The number of nuclei produced in the isomeric state at the end of irradiation period t_n is given by;

$$\begin{aligned}
 N_m(t_n) &= \frac{\sigma_m N_0}{\lambda_m} e^{-\lambda_m t_n} \sum_{i=1}^n \phi_i (e^{\lambda_m t_i} - e^{\lambda_m t_{i-1}}) \\
 &= \frac{\sigma_m N_0}{\lambda_m} e^{-\lambda_m t_n} S_m
 \end{aligned} \tag{4.3}$$

where ϕ_i is the neutron flux during time period $(t_i - t_{i-1})$,

N_0 is the number of target nuclei in the beam path,

σ_m is the production cross section for the isomeric state,

and λ_m is the inverse mean life of the isomeric state.

In equations 4.3, 4.4 and 4.5 S_m and S_g and S_{gm} are used as short forms for the sums appearing in those equations.

Similarly the number of nuclei produced in the ground state is given by;

$$\begin{aligned}
 N_g(t_n) &= \frac{p\sigma_m + \sigma_g}{\lambda_g} N_o e^{-\lambda_g t_n} \sum_{i=1}^n \phi_i (e^{\lambda_g t_i} - e^{\lambda_g t_{i-1}}) \\
 &\quad + \frac{p\sigma_m N_o}{\lambda_g - \lambda_m} e^{-\lambda_g t_n} \sum_{i=1}^n \phi_i (e^{(\lambda_g - \lambda_m)t_{i-1}} - e^{(\lambda_g - \lambda_m)t_i}) \\
 &= \frac{p\sigma_m + \sigma_g}{\lambda_g} N_o e^{-\lambda_g t_n} S_g + \frac{p\sigma_m N_o}{\lambda_g - \lambda_m} e^{-\lambda_g t_n} S_{gm} \quad 4.4
 \end{aligned}$$

where ϕ_i , N_o , λ_m and σ_m are as above. In equation 4.4 λ_g is the inverse mean life of the ground state, p is the fraction of the decay of the isomeric state that feeds the ground state, and σ_g is the production cross section for the ground state. Taking the ratio of $N_g(t_n)$ to $N_m(t_n)$ and rearranging gives;

$$\frac{p\sigma_m + \sigma_g}{\sigma_m} = \frac{\lambda_g N_g(t_n)}{\lambda_m N_m(t_n)} e^{-(\lambda_m - \lambda_g)t_n} \frac{S_m}{S_g} + \frac{p\lambda_g}{(\lambda_m - \lambda_g)} \frac{S_{gm}}{S_g} \quad 4.5$$

If t_n is the final period of the irradiation the decay rates at time t_n are equivalent to the decay rates at time zero for the decay curves since the decay curve counting was begun the instant the irradiation was stopped. Thus $(\lambda_g N_g(t_n) / \lambda_m N_m(t_n)) = (A_g^0 / A_m^0)$. Calculation of the isomeric ratio, $\sigma_m / (\sigma_m + \sigma_g)$, was then made by using in equation 4.5 the values of A_g^0 and A_m^0 found by the fitting routine described above and the values of p , λ_m and λ_g taken from the literature, and the neutron flux values taken from the neutron monitoring system. The use of these values for the neutron flux is valid here since in equation 4.5 the factors

S_m , S_g and S_{gm} , which contain the dependence on the neutron flux, all appear as ratios. Hence any normalization factor applied to the neutron flux cancels out. Table 4.1 is a list of the decay scheme constants whose values were taken from the literature. As can be seen from this table, some of the internal conversion coefficients were set equal to zero rather than to the value which appears in the literature. This was necessary in the cases of ^{80}Br , ^{81}Se and ^{165}Dy where the conversion electrons have energies of ~ 100 keV or less since most of these electrons will be absorbed in the samples which were generally .04 to .07 inches thick. The use of a discriminator to eliminate background noise is considered to have also eliminated the small signals which result from the conversion electrons which escape the samples. In the case of ^{69}Zn , a large fraction of the conversion electrons which have an energy of 439 keV escape from the samples so the discriminator level was kept as low as possible in order to count as many as possible of these electrons. Thus, for zinc, the conversion coefficient value used was that taken from the literature.

The isomeric ratios calculated from the experimental data for the ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy are plotted in figures 5.2, 5.3, 5.4 and 5.5 respectively. The error bars on the data points in these figures were derived as discussed in the following paragraphs.

For errors in the values of the isomeric ratios the factors considered were;

- 1) Random errors (counting statistics).

These were accounted for in the data fitting routine

Table 4.1

Constant	^{69}Zn	^{80}Br	^{81}Se	^{165}Dy
α'_m	0.054 ^a	0.0 ^c	0.0 ^c	0.0 ^c
α_m	0.0 ^a	0.0 ^b	0.0 ^b	0.0 ^c
p	1.0 ^a	1.0 ^b	1.0 ^b	.975 ^b
b_m	0.0 ^a	0.0 ^b	0.0 ^b	.025 ^b
α_g	0.0 ^a	0.0 ^b	0.0 ^b	0.0 ^c
b_g	1.0 ^a	0.95 ^b	1.0 ^b	1.0 ^b
g_m	0.0 ^a	0.0 ^b	0.0 ^b	0.0 ^c
g_g	0.0 ^a	0.0 ^b	0.0 ^b	0.0 ^c
c_g	0.0 ^a	0.05 ^b	0.0 ^b	0.0 ^b
t_m	834.0 min	263.0 min	57.0 min	1.255 min
t_g	58.0 min	17.6 min	18.6 min	139.2 min

(a) Nuclear Data Sheets

(b) C.M. Lederer, J.M. Hollander and I. Perlman
Table of Isotopes, Sixth Edition
John Wiley and Sons (1968)

(c) These values have been set to zero rather than to the number which appears in the literature since the conversion electrons are considered to be absorbed in the samples. This is discussed in the error estimates section of Chapter 4.

2) Systematic errors.

a) Due to the 4π beta counter

i) The stability of this system was checked using a ^{90}Sr source and found to be better than 1% over a counting period of 24 hours.

ii) The time required for the beta counter to recover following insertion of a source was tested with a ^{90}Sr source and found to be less than ten seconds, and hence negligible.

iii) The efficiency with which different energy betas are detected is not of importance for the ^{69}Zn , ^{80}Br and ^{81}Se sources since in these cases only the ground state decays by beta emission. In the case of the ^{165}Dy source however, the isomeric state decays through beta emission in 2.5% of the cases.

The manner in which the existence of both beta groups is accounted for in the data fitting function assumes that both beta groups are detected with equal efficiency. In fact, the energies of the betas emitted in the decay of the isomeric state are almost the same as the energies of the betas resulting from the decay of the ground state. This implies that the detection efficiencies for the two groups of betas are not significantly different and since the percentage of isomeric state beta emissions is small, it is therefore concluded that this decay

mode has been adequately accounted for in equation 4.5 in the determination of the isomeric ratio.

b) Due to errors in the irradiation techniques.

i) Timing the irradiation period was difficult because of machine breakdowns and the fact that the neutron flux took some time to build up after the proton beam was turned onto the Li target. All these effects were accounted for through the use of equation 4.5 to calculate the isomeric ratio. To test the importance that the non-constant neutron flux might have had, the isomeric ratios were calculated assuming the neutron flux to be constant over the time of the irradiation. The results of these calculations were compared with the results found by using equation 4.5; the differences were found to be less than 2% even in the cases where the neutron flux went to zero for short periods of time. Consequently an error of 1% was considered adequate to account uncertainties in the estimation of these effects.

ii) In order to test the effects of the empty channels that occur in the decay curves due to the transportation delay between the irradiation room and the counting room, a decay curve was obtained which had no empty channels. This was accomplished by irradiating the source already mounted in the 4π

counter, thus making it possible to start the counting the instant the irradiation ended. The resulting decay curve was repeatedly fitted using the technique described above but starting the fit at later and later channels. The effect on the estimation of the initial decay rate was found to be less than 2% provided the delay was less than one half life of the source. Delays much shorter than this were achieved for all the samples with the exception of the ^{165}Dy samples. In this case the delay was usually of the order of one half life of the isomeric state and inclusion of a 2% error due to this effect was felt to be necessary in the ^{165}Dy isomeric ratios.

- c) Due to the presence in the target materials of nuclei other than the intended target nucleus:
 - i) Zinc; the target material was zinc metal and natural zinc is made up of the isotopes listed in Table 4.2. Also shown in this table are the isotopes of zinc produced by the irradiation with neutrons. Of these, only the production of ^{71}Zn will cause errors in the calculation of the isomeric ratio in ^{69}Zn . In Table 4.2, it is calculated that the unwanted ^{71}Zn may cause as much as a 3.34% error. Thus 3.34% has been added to error estimates in order to account for this effect.
 - ii) Bromine; the target material was KBr. The

Table 4.2

Zinc Isotopes

Isotope	Percentage	Half life	Comments
^{64}Zn	48.49%		
^{65}Zn		$t_g = 245$ days	Production will be negligible.
^{66}Zn	27.81%		
^{67}Zn	4.11%		
^{68}Zn	18.57%		
^{69}Zn		$t_m = 834$ minutes $t_g = 58$ minutes	To be studied.
^{70}Zn	.62%		
^{71}Zn		$t_m = 3.9$ hours $t_g = 2.5$ minutes	$(.62/18.57) \times 100$ = 3.34% effect.

bromine isotopes are listed in Table 4.3. Of the isotopes of bromine produced by the neutron irradiation only the 6.1 minute isomeric state in ^{82}Br will introduce a background problem in the determination of the isomeric ratio in ^{80}Br . This state beta decays only 1% of the time and thus can be adequately accounted for by the addition of 1% to the error estimate. The isotopes of potassium produced by neutron activation are long lived and therefore will be only produced in small quantities which may be accounted for in the background terms.

iii) Selenium; the target material used was selenium metal powder. The isotopes of selenium are listed in Table 4.4. The production of the 25 minute ground state in ^{83}Se requires, as explained in the column for comments in Table 4.4, the inclusion of a 10% factor in the error estimates for isomeric ratio determined for ^{81}Se .

iv) Dysprosium; the target material used was DyO . Natural oxygen has no nuclei with which neutrons can react to produce a significant amount of background beta decay. The isotopes of dysprosium are listed in Table 4.5 and as shown there, only the isotope to be studied is produced.

- d) Due to the presence of decay modes of the states being studied that are not properly accounted for

Table 4.3

Bromine Isotopes

Isotope	Percentage	Half life	Comments
^{79}Br	50.54%		
^{80}Br		$t_m = 263$ minutes $t_g = 17.6$ minutes	To be studied.
^{81}Br	49.46%		
^{82}Br		$t_m = 6.1$ minutes $t_g = 35.3$ hours	Isomeric state decays via beta emission 1% of the time and therefore may produce a 1% back-ground effect. The production of the long lived ground state will be negligible.

Table 4.4

Selenium Isotopes

Isotope	Percentage	Half life	Comments
^{74}Se	0.87%		
^{75}Se		$t_g=120$ days	A negligible amount will be produced.
^{76}Se	9.02%		
^{77}Se	7.58%		
^{78}Se	23.52%		
^{79}Se		$t_m=3.9$ minutes $t_g=7 \times 10^4$ years	Short half life's effect can be eliminated by extension of start channel in fit to decay curve, production of long lived state will be negligible.
^{80}Se	49.82%		
^{81}Se		$t_m=57$ minutes $t_g=18.6$ minutes	To be studied.
^{82}Se	9.19%		
^{83}Se		$t_m=69$ seconds $t_g=25$ minutes	Short lived state lost in beginning. Long lived state; $(9.19/49.82) \times 100 = 18.45\%$. Production cross section at thermal energies half that for ^{81}Se , therefore estimate production of ^{83}Se will cause 10% error in isomeric ratio determination.

Table 4.5

Dysprosium Isotopes

Isotope	Percentage	Half life	Comments
^{160}Dy	2.29%		
^{161}Dy	18.88%		
^{162}Dy	25.53%		
^{163}Dy	24.97%		
^{164}Dy	28.18%		
^{165}Dy		$t_m=1.255$ minutes $t_g=139.2$ minutes	To be studied.

by equation 4.1, and the decay constants of Table 4.1. As discussed above the conversion electrons in ^{80}Br , ^{81}Se and ^{165}Dy are considered to be excluded from the spectra through absorption in the samples and electronic discrimination. If, however, some of these conversion electrons are counted then the use of conversion coefficients equal to zero in the calculations will result in the isomeric ratio estimate being high. The good agreement achieved between the results of this work and that of previous authors for bromine and selenium indicates that this effect may be considered as negligible. In the case of zinc the conversion electrons are considered to have been counted principally because of their high energy and also because the zinc samples consisted of thin foils of zinc metal with no mylar foil coverings. The conversion coefficient in this case was taken from the literature. If some of the conversion electrons are not counted the isomeric ratio will be underestimated. However, since the electron conversion only takes place in 5% of the isomeric state decays, the loss of as many as 10% of the conversion electrons will result in only a 0.5% error in the isomeric ratio. This effect is therefore considered to be negligible. The number of internal conversion electrons that

occur in the gamma cascades that follow the beta decays are negligible for all the samples studied. All of the other modes of decay are considered to be adequately accounted for by the decay constants appearing in Table 4.1.

- e) Due to double energy regions in the neutron production reaction $\text{Li}(p,n)$.
 - i) In the low energy region, near threshold, the difference between the energies of the two neutron groups is the order of the energy spread that is discussed below and is therefore considered to be of negligible importance.
 - ii) In the high energy region, above $E_p = 2.5$ MeV. For example, the situation at $E_p = 3$ MeV was analysed as follows.

The intensity of low energy neutrons is 10% of the intensity of high energy neutrons, the low energy neutrons having an energy of 828 keV, the high energy neutrons 1.305 MeV. For ^{81}Se the measured isomeric ratios at these two energies differ by approximately 20%, therefore, the introduction of an error of 2% should be adequate to account for the existence of the two neutron energies. For ^{80}Br the isomeric ratios differ by approximately 10%, therefore an error of 1% was introduced. No error was introduced into the estimate for the cases

of ^{68}Zn and ^{165}Dy since the former had no measurements made in this region while the latter had an isomeric ratio that was practically constant over the energy region.

For the energy spread the factors considered were:

- 1) The thicknesses of the lithium carbonate targets which were measured by the W&W technique. The targets used in this work all had thicknesses between 15 and 50 keV which gave rise to neutron energy spreads from 8 to 25 keV.
- 2) The proton beam energy spread was measured with a very thin aluminium target using the 992 keV $\text{Al}(p,\gamma)$ resonance. It was found to be about 3 keV.
- 3) The drift in the energy of the proton beam was taken to be less than 1 keV over the duration of all the runs. This point was discussed in Chapter 3.
- 4) The finite angle subtended by the sources was measured to be 45° and the energy spread due to this was found from the tables in Gibbons and Newson³⁹ at each neutron energy. This resulted in neutron energy spreads as large as ± 50 keV.
- 5) The error introduced by the extrapolation procedure used to determine the $\text{Li}(p,n)$ threshold and consequently the calibration of the neutron energies was taken to be ± 2 keV. This is considered to be an adequately conservative estimate since this

extrapolation procedure has been established to be correct to ~ 0.5 keV.⁵²

Some of the isomeric ratios measured in this work that are plotted in Figures 5.1, 5.2, 5.3 and 5.4 do not have error bars. This is due to the fact that the error bar, if plotted, would have been smaller than the symbol used to indicate the value of the datum. The results shown in these figures are discussed in the next chapter and a comparison is made with theoretical predictions.

Chapter 5

Discussions and Conclusions

The energy dependence of the isomeric ratio in ^{80}Br as determined in this work agrees well with the results obtained previously for this nuclide by Cox⁵³ and by Johnsrud et al.⁵⁴ The results obtained in this work for ^{81}Se are lower than the results of Tolstikov et al.⁵⁵ throughout the entire energy region studied. However, figure 5.4 shows that the error bars for the two sets of data do overlap so that it may be concluded that the disagreement is not one of major significance. The energy dependence of the isomeric ratios in ^{69}Zn and in ^{165}Dy have not, as far as we have been able to determine, been measured previous to this work.

From figures 5.2 and 5.3 it can be seen that there is an increase in the isomeric ratio in ^{80}Br and in ^{81}Se as the neutron bombarding energy increases. This can be understood from simple physical considerations. As the bombarding neutron energy is increased the probability of creating a high spin CN is also increased. Since, in the cases of ^{80}Br and ^{81}Se , the isomeric state has a higher spin than the ground state, a greater probability of a high spin CN implies a greater probability of populating the isomeric state. It will be seen later that this simple argument is verified in the theoretical calculations made for bromine and selenium.

Since the isomeric state in ^{68}Zn is the high spin state, the isomeric ratio in this nucleus would be expected to show an increase with increasing neutron energy. There are, however,

not a sufficient number of data points to make such a trend easily perceptible. The data are, however, sufficient to indicate that if such a trend does exist for zinc, it has a small slope. This is in agreement with some of the theoretical predictions to be discussed below.

In ^{165}Dy the situation is the reverse of those described above, the isomeric state has a low spin, $(1/2)$, while the ground state has a high spin, $(7/2)$. From the discussion presented above, the isomeric ratio would in this situation be expected to decrease as a function of neutron bombarding energy. The dysprosium results, however, seem to indicate that the isomeric ratio is a constant independent of the neutron energy. Several types of theoretical calculations have been tried for this case and only after a very drastic physical assumption had been made was good agreement between theory and experiment achieved.

Figures 5.2 to 5.5 show all the experimental data available as well as some of the theoretical results obtained in this work. In these graphs the isomeric ratio curves labeled H&V were calculated using the original model of Huizenga and Vandebosch²⁸ as described in Chapter 2. The curves labeled V&V&H were calculated using the model of Vonach, Vandebosch and Huizenga²⁰ that was also described in Chapter 2. The label H&G appears on curves calculated with the cascade model designed as part of this work which employs the level densities generated by the method of Hillman and Grover²¹.

The H&V model calculations were made using a computer

program described by Hafner, Huizenga and Vandenbosch.⁵⁶ In order to employ this routine the values of a number of parameters had to be put in. These are discussed in Appendix II.

The computer program which was used to make the V&V&H model calculations was basically the same program with which the H&V model calculations were made. The original program was altered where necessary to make the calculations correspond to those described by Vonach, Vandenbosch and Huizenga.²⁰ The values of several parameters were still required. These are discussed in Appendix II.

The parameter GAMAL described in Appendix II was given the value 1 in the H&V and V&V&H calculations with one exception. Sperber³⁵ found, when trying to calculate the isomeric ratio produced by thermal neutrons using his model, that in the case of ^{197}Hg , he had to postulate a ratio of E2 to E1 gamma ray intensities of 30:1 in order to achieve agreement with experiment. As a result, a calculation using the V&V&H model with GAMAL=2 was carried out in this work for ^{165}Dy .

The isomeric ratio calculations made with the new cascade model designed in this work did not require the input of any parameters. The calculations of the level densities with the H&G computer program,* however, did require the value of G, the pairing force parameter. In agreement with the results of H&G²¹, the value of G was set equal to $9/A$ for ^{69}Zn , ^{80}Br

* We would like to thank Drs. Hillman and Grover for kindly sending us a copy of their computer program.

and ^{81}Se and equal to $10/A$ for ^{165}Dy . Here A is the atomic number.

The results of the calculations made with these models are given in tables 5.1 to 5.4. At the head of each column in these tables is the title of the model and the values of its parameters employed to calculate the contents of that column. The superscripts that appear on the isomeric ratio values calculated with the V&V&H model represent the number of gamma rays the model required in the cascade to reach the isomeric and ground states. The results listed in these tables that are in closest agreement with the experimental measurements are plotted in figures 5.2 to 5.5 for comparison with the experimental isomeric ratios.

Zinc

Figure 5.2 shows that the theoretical calculations of the isomeric ratio for ^{69}Zn made with the H&V cascade model and with the new cascade model (columns 1 and 4 of table 5.1) are in reasonable agreement with the measured values. The V&V&H model calculations on the other hand predict an isomeric ratio that is too high, a result which may be due to the large number of gamma rays (11) required for the cascade to reach the isomeric and ground states. Column 2 of table 5.1 indicates that after only four gamma rays have been emitted, the H&V model also predicts a ratio that is too high. However, the V&V&H and new models predict a rate of increase in the isomeric ratio as a function of neutron energy which appears to be in better agreement with the experimental data than that predicted by the H&V model. This inability of the H&V model to yield the correct slope may be due to the fact that it considers so few transitions (3) in the cascade, whereas the other models which predict more correct slopes consider many more transitions. Figure 5.1 shows graphically how the spreading of the spin distribution is dependent on the number of gamma rays in the cascade. The good fit, both magnitude and slope, obtained with the new cascade model indicates that the level densities generated with the H&G computer routine, the transmission coefficients calculated with the Optical model and the Moszkowski estimates of the gamma-ray transition probabilities are all reasonably correct in this case.

FIGURE 5.1

Spreading of Spin Distribution in Gamma Cascade

FIGURE 5.2

Table 5.1
Theoretical Results for ^{69}Zn

$E_n(\text{keV})$	1	2	3	4
	Ratio (H&V) $\sigma=3$ 3E γ 's SD=2.5	Ratio (H&V) $\sigma=3$ 4E γ 's SD=2.5	Ratio (V&V&H) $\sigma=\text{ct}$ E γ 's SD=2.5 a=20	Ratio (H&G)
100	.068	.151	.198 ¹¹	.086
200	.102	.178	.205	.105
300	.125	.197	.210	.119
400	.143	.212	.214	.129
500	.158	.224	.217	.138
600	.170	.235	.221	.146
700	.181	.244	.224	.153
800	.191	.252	.224 ¹²	.159
900	.201	.260	.226	
1000	.210	.268	.228	.170

Bromine

Figure 5.3 shows that the V&V&H model calculations of the isomeric ratio in ^{80}Br produce results in excellent agreement with the experimentally measured values. Comparison of columns 2 and 3 of table 5.2 reveals that this good agreement was only achieved by using a value for the spin cutoff parameter σ which remains constant at a larger value than that predicted by the Fermi gas model for the same energies. This is in agreement with the findings of V&V&H²⁰ for the $^{80}\text{Se}(p,n)^{80}\text{Br}$, ^{80m}Br reaction. The H&V model and the new model predict values for the isomeric ratio which are low. The inclusion of another gamma ray in the H&V model calculations would have improved the results of that model somewhat but the average number of gamma rays in a cascade, referred to as the multiplicity, has been measured by Muehlhause⁵⁷ for thermal neutron capture in natural bromine and found to be 3.4. Thus even the employment of six transitions in the cascade is difficult to justify. The lack of agreement between the experimental results and the values calculated with the new cascade model will be discussed later.

Selenium

In figure 5.4 the results of the H&V model calculations of the isomeric ratio in ^{81}Se are seen to be in excellent agreement with the experimental results. On the other hand the predictions made by the V&V&H and new cascade models, while agreeing closely with each other, do not agree with the experimental measurements, they yield results which are too large.

FIGURE 5.3

Table 5.2
Theoretical Results for ^{80}Br

$E_n(\text{keV})$	1 Ratio (H&V) $\sigma=3.5$ 6El γ 's SD=3.0	2 Ratio (V&V&H) $\sigma=\text{ct}$ El γ 's SD=3.0 a=20	3 Ratio (V&V&H) $\sigma\geq 3.5$ El γ 's SD=3.0 a=20	4 Ratio (H&G)
100	.251	.211 ¹²	.346 ¹²	.263
200	.259	.214	.351	.273
300	.264	.217	.355	
400	.269	.220	.359	.288
500	.274	.223	.363	.295
600	.280	.225	.367	
700	.285	.228	.371	.311
800	.291	.231	.375	.317
900	.298	.235	.379	
1000	.305	.240	.383	.332
1100		.243	.387	.339
1200		.243 ¹³	.390 ¹³	.347
1300		.246	.394	.354
1400		.250	.398	.362
1500			.402	.369
1600			.405	.376
1700			.409	.382
1800			.413	.388
1900			.416	.394
2000			.419	.399

FIGURE 5.4

Table 5.3
Theoretical Results for ^{81}Se

$E_n(\text{keV})$	1	2	3	4	5	6
	Ratio (H&V) $\sigma=3$ 3E γ 's SD=1.5	Ratio (H&V) $\sigma=4$ 3E γ 's SD=1.5	Ratio (H&V) $\sigma=3$ 6E γ 's SD=1.5	Ratio (V&V&H) $\sigma=ct$ E γ 's SD=1.5 a=20	Ratio (V&V&H) $\sigma \geq 3.5$ E γ 's SD=1.5 a=20	Ratio (H&G)
100	.361	.403	.554	.588	.698	.599
200	.379	.422	.560	.593	.703	.611
300	.392	.436	.565	.600	.710	.621
400	.404	.448	.569	.601	.710	.630
500	.416	.461	.575	.604	.712	.639
600	.429	.474		.604	.714	.648
700	.443	.488		.608	.717	.657
800	.457	.503				.667
900	.472	.517		.615	.724	.676
1000				.617	.725	.686
1100				.623	.731	.696
1200				.627	.734	.706
1300				.630	.738	.715
1400				.634	.741	.724
1500				.638	.744	.733
1600				.641	.748	.741
1700				.644	.751	.749
1800					.754	.757
1900					.757	.763
2000					.759	.770

Column 5 of table 5.3 are the results for the V&V&H model calculations made with the same assumption that improved the bromine calculations, that is, that the spin cutoff factor must remain constant and larger than the Fermi gas model prediction. In this case, however, this assumption did not improve the results but increased the disagreement between the models predicted values and the experimental values.

As in the case of the zinc results with this model the employment of too many gamma rays in the cascade (11) may be the factor responsible for the overly large outcome. The lack of agreement between the experimental and new model results will be discussed later.

Dysprosium

In figure 5.5 it is seen that the values of the isomeric ratio calculated for ^{165}Dy with the new cascade model and with the V&V&H model are approaching agreement with the experimental results at neutron energies greater than 1 MeV. However, below these energies neither model, employing the usual assumptions, is able to produce any reasonable agreement with the experimental results. Comparison of columns 3 and 4 of table 5.4 shows that the results of the V&V&H model have some dependence on the choice of the energies E_U and E_L , but a change in their values will not produce the required corrections. The V&V&H calculations were, however, brought into very good agreement with the experimental data at all energies only after the cascade was assumed to consist purely of E2 transitions. The results calculated with the H&V

Table 5.4

Theoretical Results for ^{165}Dy

E_n (keV)	1	2	3	4	5	6
	Ratio (H&V) $\sigma=3.5$ 6E γ 's SD=1.5	Ratio (H&V) $\sigma=3.5$ 6E γ 's SD=2.5	Ratio (V&V&H) $\sigma=ct$ E γ 's SD=1.5 a=20	Ratio (V&V&H) $\sigma=ct$ E γ 's SD=1.5 E $_U$ =2.5 E $_L$ =1.5	Ratio (V&V&H) $\sigma=ct$ E2 γ 's SD=1.5 a=20	Ratio (H&G)
100	.405	.718	.248	.277	.1127	.235
200	.376	.683	.231	.253	.1112	.213
300	.358	.662	.220	.237	.1095	.199
400	.347	.649	.213	.227	.108	.191
500	.340	.640	.208	.227	.107	.184
600	.335	.633	.204	.222	.106	.179
700	.330	.628	.201	.218	.105	.175
800	.326	.622	.197	.213	.104	.171
900						
1000			.190		.103	.163
1100			.186		.102	.160
1200			.183		.101	.156
1300			.179		.100	.152
1400			.178		.099	.151
1500			.172			
1600			.170		.098	.141
1700			.167			
1800			.164			
1900						
2000						

model disagree so appreciably with the measured values that they could not be plotted in figure 5.5. As was the case in bromine, these calculations could have been brought into better agreement with the data by the use of more gamma rays in the cascade. The multiplicity has been measured for this nuclide by Muehlhause⁵⁷ and by Draper and Springer⁵⁸, their results were 3.7 and 3.1 respectively. It is therefore difficult to justify the use of the six gamma rays that yield the present result (column 1, table 5.4) and justification becomes impossible if several more transitions are included to improve the result. The disagreement between the experimental values and those resulting from the new cascade model will be discussed in the next section.

General Discussion

In the above discussions of the isomeric ratio calculations the results achieved through use of the H&V and the V&V&H models were seen to be dependent on parameter values. Some of these parameters have little or no relation to the physics of the nuclei being studied but are factors which must be adjusted in order to compensate for the limited nature of the cascade models themselves. The cascade cutoff parameters, E_U and E_L , employed in the V&V&H model, and the number gamma rays in a cascade, N_{GE} , used in the H&V model, are examples of such parameters. The values of E_U and E_L determine how many transitions are considered in the cascade and how the spin distribution that exists as a result of these transitions is to be divided between the isomeric

and ground states. V&V&H²⁰ found, as we have in this work for the dysprosium case, that the model's results are dependent on the values chosen for E_U and E_L . However, there is no physically intuitive reasoning that would help in selecting the values to be used for these cascade cutoff factors. The values 2 and 1 respectively were used throughout this work, with a single exception for test purposes, in order to maintain a consistent relation between all of the results calculated with this model. The opposite situation occurs in the H&V model where physical intuition would require NGE to have a value close to the average number of gamma rays in a cascade. However, the results in this work for bromine and dysprosium indicate that values of NGE much greater than the experimentally measured multiplicities would be required to achieve agreement between the calculated and experimental isomeric ratios. These large values of NGE are probably necessary as a compensation for some inadequacy of the cascade model; perhaps the lack of transition types other than E_L .

The existence of factors such as E_U and E_L and NGE in the theoretical calculations decreases the significance of the values determined for other parameters of these models. For example, the spin cutoff parameter σ , which occurs in the level density formula that is employed in both the V&V&H and the H&V models, is physically related to the rigid body moment of inertia, I , through $\sigma^2 = cI$ where $c = I/\hbar$. A determination of σ would therefore give answers to physical questions such as

does the moment of inertia of the nucleus being studied agree with the rigid body estimate or not? However, the value of the spin cutoff parameter determined through the study of isomeric ratios with the H&V and the V&V&H models is not completely independent of the values employed in these models for factors such as N_{GE} and E_U and E_L . The physical information is therefore somewhat concealed by the limitations of the cascade models.

The new cascade model which has been designed as part of this work does not contain any parameters whose values must be adjusted to compensate for a limitation inherent in the model itself. Therefore if the model does not yield correct isomeric ratios, as is the case for the ^{80}Br , ^{81}Se and ^{165}Dy calculations made in this work, it must be concluded that some of the physical assumptions upon which the model is constructed are in error.

These assumptions are;

- 1) that all the assumptions made by Hillman and Grover²¹ for the derivation of the level densities are valid,
- 2) that use of the Optical model (the values of its parameters were taken from Rosen⁵⁹) is appropriate for calculation of the neutron transmission coefficients used to estimate the initial spin distribution of the compound nucleus, and
- 3) that the Moszkowski³⁶ estimates of the single proton transition probabilities are an adequate measure of the relative magnitude of all transition probabilities, i.e., of the proton, neutron and many particle transition probabilities.

These assumptions were accepted as reasonably correct in the above discussion of the zinc results, since in that case there appeared to be good agreement between the experimental and calculated values of the isomeric ratio. However, the results of the new cascade model calculations for the other three nuclei studied are not in such good agreement with the experimental data. The fact that the calculated values for bromine are low while those for selenium are high would seem to indicate that the procedure for calculating the initial spin distribution is not responsible for this disparity. This conclusion is arrived at by considering that in both of these nuclei the isomeric states are the high spin states when compared to the ground states. Thus if the Optical model were over or underestimating the high spin transmission coefficients, the effect on the results would be the same for both nuclei, the calculated isomeric ratios being either high or low but the same in both cases. Notwithstanding this argument for trusting the Optical model calculations, there may be some inadequacies in its use. The inability of the new cascade model to predict the correct energy dependence for the isomeric ratio may indicate that the rate of change of the initial spin distribution as a function of the neutron energy is not being properly estimated. This may be due to the omission of details in the Optical model calculations. For example, the presence of the 2^+ states at 666 keV and at 1451 keV in the target nucleus ^{80}Se are not accounted for in the Optical model calculations. These may be significant in that they leave open the possibil-

ity of (n,n') reactions which would change the initial spin distribution of the ^{81}Se that is produced. That is, once the bombarding neutron energy is above 666 keV, the emission of a neutron with orbital angular momentum $\ell=0$ leading to the 666 keV 2^+ state becomes possible. The occurrence of an emission such as this would deplete the population of the $3/2$ and $5/2$ states of the ^{81}Se compound nucleus. It may be possible to account for this effect by including the results of a Hauser-Feshbach theory calculation in the estimation of the initial population distribution.

The comparisons made by Hillman and Grover²¹ between the level densities generated by their computer routine and those measured experimentally indicated that the two sets of values agreed reasonably well. Thus it may be concluded that the H&G calculation procedure is correct. This argument for the correctness of the H&G calculated level densities does not preclude the objection that ideally the γ -ray cascade should be calculated using experimentally determined levels. However, at high excitation energies this latter procedure would not only be impractical but probably would not influence the results significantly. This conclusion follows from the fact that the number of levels that exist at these high energies is very large and consequently their effect on the γ -cascade would be principally through distribution, the same type of effect that the calculated level densities have. At low excitation energies on the other hand, the number of levels is small and generally they are not well represented by the calculated level densities. It is therefore

of primary importance that in this low energy region as many experimentally determined levels be employed as is possible. This last statement is further strengthened by the fact that the spins and energies of the levels in this region are very influential in determining the final division of the population between the isomeric and ground states. In this work the new model calculations were made using as many experimentally measured levels as could be found in the literature. The number of these levels for which the spin and energy are known is not large for any of the nuclei studied in this work. It was therefore, not possible to determine how high up the excitation energy scale levels, whose parameters are known from experiment, must be used in place of the level densities. Nor was it possible to determine how large an influence the use of the limited number of known levels had on the results.

: Another factor that can be blamed for the incorrect results of the new cascade model is the assumption that all transition probabilities can be estimated with the single proton transition probabilities of Moszkowski. There are several reasons for the use and the manner of use of these estimates being suspect. First, the assumption of the value one for Moszkowski's statistical factor S , while being reasonably correct, has eliminated from consideration any effects that the radial part of the matrix element might have on the transition probabilities. Calculation of this part of the matrix element cannot be made for the transitions treated by the present model since this model deals with transitions for which only the energy and angular momentum changes

are defined. Determination of the effects of the radial dependence would require much more detailed information about the transitions. For example, it would be necessary to know that a transition is that of a neutron in state $2d_{5/2}$ going to state $1g_{9/2}$. Such a detailed calculation would not be executable in terms of level densities.

A second reason for use of the Moszkowski estimates being suspect is that in the present calculations the possibility of favoured transitions is completely ignored. For example, if correlated states such as exist in a rotational band are present then a substantial fraction of the cascade could be funneled through these states. Accounting for this type of effect would require that the relative significance of the various multipolarities be changed from that predicted by the Moszkowski estimates.

The third problem with using the Moszkowski estimates is that there is the possibility that the neutron and proton transition probabilities are quite different in regard to the relationship between the various multipolarities. As with the first problem described above, this possibility could be accounted for only if more details than the present model requires about the transitions were made available. In this case it would be necessary to know whether the transition was that of a proton or that of a neutron.

Conclusions

In this work, isomeric ratios have been measured for several nuclei using (n,γ) reactions. The determination of such ratios does not require knowledge of the difficult to obtain absolute neutron flux while providing results that are useful for testing theories of nuclear level densities in combination with theories of compound nucleus formation and theories of gamma ray cascades.

The isomeric ratios that have been measured were in ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy , and it can be concluded that these results are in a general sense in accord with the expectation that as the neutron energy increased the production of the high spin state, whether it is the isomeric state or the ground state, would increase. Several gamma ray cascade models have been employed in this work for the purpose of reproducing theoretically the details of this rise and the value of the isomeric ratio itself. Two of the models, the H&V and the V&V&H models, although relatively successful in some instances, were found to contain too many parameters to make definite physical statements about their results possible. A new gamma ray cascade model was designed as a part of this work in order to make use of the level densities generated by the Hillman and Grover computer routine. The fact that it does not contain any parameters has made possible a discussion of the physical assumptions upon which its construction is based. From these discussions, it is concluded that the model has some defects that

could be corrected by employing a much more complex set of calculations. It must be stated, however, that the results achieved with this model are reasonably good when it is considered that they were obtained without adjustable parameters. Thus the conclusion is reached that the physical basis of this model is credible, that is, that the employment of the level densities of Hillman and Grover, the transmission coefficients from the Optical model and the transition probability estimates of Moszkowski is a reasonably correct procedure. It may be possible to determine which of these factors, the level densities, the transmission coefficients or the transition probabilities has the most influence on the new model's results by designing a "theoretical experiment" for this purpose. However, before this test is carried out, the measurement of the spins and energies of many more low lying levels in ^{69}Zn , ^{80}Br , ^{81}Se and ^{165}Dy should be made. This is necessary in order to decide at what energy the number of levels per energy interval becomes sufficiently large so that representation by the H&G level densities is adequate.

Determination of the average number of gamma rays that occur in a γ -cascade would be useful for comparison with the predictions of cascade models or to eliminate a degree of freedom in some of these models. During this work an attempt was made to measure this number, the multiplicity, but the background radiation in the presence of the neutron beam was much too high to make such a measurement possible.

The effects of states in the target nucleus, such as the 2^+ states in ^{80}Se , and the effects of preferred transitions in the γ -cascade, such as occur for a rotation band, may appear as sharp changes in the isomeric ratio as a function of neutron energy. For example, the existence of the 2^+ 666 keV state in ^{80}Se would be expected to begin influencing the isomeric ratio soon after the neutron energy becomes greater than 666 keV. It was found in the course of this work that an experiment, with better energy resolution than that of the present one, would be necessary to determine if such "fine structure" exists.

The calculations by Hillman and Grover of nuclear level densities directly from a nuclear model without the necessity of mathematical approximations and the calculations made in this work using these level densities in a parameter free γ -cascade model are examples that are only practical when made with large fast computers. The use of such procedures makes possible a direct testing of physical theories without the interference of computationally convenient approximations. It is concluded, therefore, that the most useful aspect of the γ -ray cascade model designed as part of this work is the production of results that are conceptually relatable to the forces and other parameters of the nuclei being studied.

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APPENDIX I

Computer Routine for New Cascade Model Calculations

Input 1 - Control Data File

- Line 1 Atomic Number of CN
 Projectile Spin
 Target Spin
- Line 2 IBCS 1 for H&G, and 0 for continuous level densities..
 Number of H&G or continuous levels to be used.
 Number of experimental levels to be read in
- Line 3-N Energies and Spins of experimental levels.
- Line N+1 Energies and Spins of isomeric and ground states.

Input 2 - H&G Generated Level Densities

Input 3 - Capture Data

- Line 1 Number of Initial Populations to be Read.
 Capture Energy
- Line 2 Initial Populations.
- Line: 3 Same as Line 1 for Different Case, etc.

```

1.000      COMMON IA, I(100,55)
2.000      COMMON E(100), J(55), IEM, EMAX
3.000      COMMON RADI, ASMAL, IBCS, UCJNT, BCS, JJ
4.000      DIMENSION POP(100,55), MAXJS(100), BCS(50,55), JJ(100)
5.000      REAL JP, JI, JCN, J
6.000      INTEGER DJ, BCS
7.000 C     200=CONTROL DATA FILE, 201=INITIAL POP FILE, 250=H+G LEVDEN
FILE
8.000      RADI=1.25
9.000      ASMAL=20.
10.000     DO 310 I=1,100
11.000 310  JJ(I)=0.
12.000      IEN=100
13.000     DO 402 I=1,50
14.000     DO 402 II=1,55
15.000 402  BCS(I,II)=0.
16.000      READ (200,1), IA, JP, JI
17.000      READ(200,5), IBCS, IEM, NOJ
18.000      IEMK=IEM
19.000      NOJK=NOJ
20.000 313  JCN=JP+JI
21.000      JCNT=2.*JCN
22.000      IF(MOD(JCNT,2).LT.1)GO TO 201
23.000      FJ=.5
24.000      GO TO 202
25.000 201  FJ=1.
26.000 202  DO 200 I=1,55
27.000      FI=I
28.000 200  J(I)=FI-FJ
29.000      IEM=IEM+1
30.000      IF(IBC5.61.0)GO TO 302
31.000      EMAX=.2*IEM
32.000      EBIN=.2
33.000      DO 203 I=1, IEM
34.000      FI=I-1
35.000 203  E(I)=EMAX-FI*EBIN
36.000      GO TO 303
37.000 302  READ(250,9)NUCZ,NUCN,NUCA
38.000      IEX=IEM-1
39.000      DO 304 I=1, IEX
40.000      II=IEM-I+1
41.000      READ(250,7)E(II),AL,BL,AS,BLS,MAXJE
42.000 304  READ(250,6), (BCS(II,II), II=1, MAXJE)
43.000      UCJNT=E(IEN)-.00001
44.000 303  IF(NJJ.LT.1)GO TO 307
45.000      IEN=IEM+1
46.000      IEM=IEM+NOJ+1
47.000      DO 306 I=IEN, IEM
48.000 306  READ(250,10)E(I), JJ(I)
49.000      IF(UCJNT.LE.E(IEN))GO TO 400
50.000      GO TO 308
51.000 400  IEN=IEX-1
52.000      IEM=IEX-1
53.000      UCJNT=E(IEX-1)-.0.00001

```

```

54.000      DO 401 I=IEN, IEM
55.000      E(I)=E(I+1)
56.000      401 JJ(I)=JJ(I+1)
57.000      IF (UCONT.LE.E(IEN))GO TO 400
58.000      308 IEM=IEM+2
59.000      IEN=IEM-1
60.000      GO TO 305
61.000      307 IEM=IEM+3
62.000      IEN=IEM+1
63.000      305 READ(200,10)E(IEN),JJ(IEN),E(IEM),JJ(IEM)
64.000      IEN=IEM-1
65.000      IEL=IEM-2
66.000      101 DO 301 I1=1,100
67.000      MAXJS(I1)=0
68.000      DO 301 I2=1,55
69.000      301 POP(I1,I2)=0.
70.000      501 READ (201,11,END=102)MAXJ,CNENG
71.000      READ (201,3),(POP(I,IJ),IJ=1,MAXJ)
72.000      WRITE (108,4),IA,CNENG
73.000      E(I)=CNENG
74.000      MAXJS(I)=MAXJ
75.000      EMAX=CNENG
76.000      602 DO 100 IE=1,IEL
77.000      MAX=MAXJS(IE)
78.000      IES=IE+1
79.000      DO 600 IJ=1,MAX
80.000      POPE=POP(IE,IJ)
81.000      IF(POPE.LE.0.)GO TO 600
82.000      CALL SEJ(IE,IJ)
83.000      MAXJI=IJ+5
84.000      IF(MAXJI.G1.55)MAXJI=55
85.000      DO 601 IEI=IES,IEM
86.000      DE=E(IE)-E(IEI)
87.000      MAXJS(IEI)=MAX0(MAXJS(IEI),MAXJI)
88.000      DO 601 IJI=1,MAXJI
89.000      DJ=ABS(J(IJ)-J(IJI))
90.000      IF(DJ.LT.1.)R=DJ.G1.5)GO TO 601
91.000      S=1/(IEI,IJI)
92.000      POP(IEI,IJI)=POP(IEI,IJI)+S*POPE
93.000      601 CONTINUE
94.000      600 CONTINUE
95.000      100 CONTINUE
96.000      DO 300 I2=1,IEM
97.000      IF(E(I2).GT.EMAX)GO TO 300
98.000      MAX1=MAXJS(I2)
99.000      MAX2=10
100.000      IF(MAX1.LT.10)MAX2=MAX1
101.000      WRITE(108,2),E(I2),(POP(I2,I1),I1=1,MAX2)
102.000      IF(MAX1.LE.10)GO TO 300
103.000      WRITE(108,8),(POP(I2,I1),I1=11,MAX1)
104.000      300 CONTINUE
105.000      603 IEN=IEM
106.000      GO TO 101
107.000      102 CALL EXIT

```

```
108.000 1 FORMAT(I3,2F3.1,F8.4,I3)
109.000 2 FORMAT(1H ,F6.3,1X,10(E9.4,1X))
110.000 3 FORMAT(10F8.6)
111.000 4 FORMAT(1H1,'A=',I3,2X,'CAPTURE ENERGY=',F7.4)
112.000 5 FORMAT(3I3,4I2,F8.4)
113.000 6 FORMAT(10E8.2)
114.000 7 FORMAT(F6.3,2(E8.2,F6.3),I3)
115.000 8 FORMAT(E18.4,9E10.4)
116.000 9 FORMAT(3I5)
117.000 10 FORMAT(10F8.4)
118.000 11 FORMAT(20.0)
119.000 END
```

```

120.000      SUBROUTINE SEJ(IE,IJ)
121.000      COMMON IA,I(100,55)
122.000      COMMON E(100),J(55),IEM,MAX
123.000      COMMON RAD,ASML,IBCS,UCONT,BCS,DJ
124.000      DIMENSION BCS(50,55),JJ(100)
125.000      REAL J
126.000      INTEGER DJ,BCS
127.000      A=IA
128.000      A23=A*(2./3.)
129.000      A43=A23*A23
130.000      A83=A43*A43
131.000      A103=A83*A23
132.000      RIDG=(2./5.)*(931./(1.973*1.973))*A
133.000      RIDG=RIDG*(RAD*ASML*(1./3.))
134.000      RIDG=RIDG*1.E-04
135.000      DJ 200 I1=1,IEM
136.000      DJ 200 I2=1,55
137.000      200 I(I1,I2)=0.
138.000      IES=IE+1
139.000      IJE=IJ+5
140.000      IF(IJE.GT.55) IJE=55
141.000      ITOTL=0.
142.000      DJ 102 IE1=IES,IEM
143.000      DE=E(IE)-E(IE1)
144.000      IF(DE.LE.0.) GO TO 102
145.000      DE2=DE*DE
146.000      DE3=DE*DE2
147.000      DE5=DE3*DE2
148.000      DE7=DE5*DE2
149.000      DE9=DE7*DE2
150.000      DE11=DE9*DE2
151.000      U=E(IE1)
152.000      IF(BCS.GT.0) GO TO 201
153.000      IF(U.LT.UCONT) GO TO 201
154.000      RHJE=(1./(U*U))*EXP(2.*SQRT(ASML*U))
155.000      TTHERM=(1.+SQRT(1.+4.*ASML*U))/(2.*ASML)
156.000      SIGMA2=RIDG*TTHERM
157.000      GO TO 210
158.000      201 RHJE=1.
159.000      210 IE1=1.E+14*A23*DE3
160.000      IM1=2.9E+13*DE3
161.000      IE2=7.4E+07*A43*DE5
162.000      IM2=8.4E+07*A23*DE5
163.000      IE3=34*A*A*DE7
164.000      IM3=87*A43*DE7
165.000      IE4=1.1E-05*A83*DE9
166.000      IM4=4.3E-05*A*A*DE9
167.000      IE5=2.5E-12*A103*DE11
168.000      IM5=1.7E-11*A83*DE11
169.000      DJ 300 IJ1=1,IJE
170.000      IF(U.LT.UCONT) GO TO 203
171.000      IF(BCS.GT.0) GO TO 202
172.000      ARG=-((J(IJ1)+.5)**2.)/(2.*SIGMA2)
173.000      RHUJ=(2.*J(IJ1)+1.)*EXP(ARG)

```

```

174.000      GO TO 211
175.000      202 RHOJ=BOS(IET,IJT)
176.000      GO TO 211
177.000      203 JTEST=J(IET)-J(IJT)
178.000      IF(JTEST.NE.0)GO TO 300
179.000      RHOJ=1.
180.000      211 DJ=ABS(J(IJ)-J(IJT))
181.000      IF(DJ.LT.1.OR.DJ.GT.5)GO TO 300
182.000      RHDEJ=RHOJ*RHOJ
183.000 C      ASSUME STATISTICAL FACTOR = 1
184.000      ST=1.
185.000      ST=ST*RHDEJ
186.000      IF(DJ.EQ.5)GO TO 110
187.000      IF(DJ.EQ.4)GO TO 111
188.000      IF(DJ.EQ.3)GO TO 112
189.000      IF(DJ.EQ.2)GO TO 113
190.000      IF(DJ.EQ.1)GO TO 114
191.000      110 T(IET,IJT)=(TE5+TM5)*ST
192.000      GO TO 103
193.000      111 T(IET,IJT)=(TE4+TM4)*ST
194.000      GO TO 103
195.000      112 T(IET,IJT)=(TE3+TM3)*ST
196.000      GO TO 103
197.000      113 T(IET,IJT)=(TE2+TM2)*ST
198.000      GO TO 103
199.000      114 T(IET,IJT)=(TE1+TM1)*ST
200.000      103 TTOTL=TTOTL+T(IET,IJT)
201.000      300 CONTINUE
202.000      102 CONTINUE
203.000      DO 101 IET=IES, IEM
204.000      DO 101 IJT=1, IJE
205.000      101 T(IET,IJT)=T(IET,IJT)/TTOTL
206.000      RETURN
207.000      END

```

*

APPENDIX II

Cascade Model Parameters

Huizenga and Vandenbosch⁵⁶ model parameters:

1) SIGMA - the spin cutoff parameter that appears in the spin dependent part of the level density; see equation 2.13. Values between 3 and 4 were used for σ in this work in agreement with the analyses of other workers²⁸⁻³².

2) NGE - the number of gamma rays in the cascade. This was treated as a free parameter, where calculations were practical a sufficient number of gamma rays was considered to achieve agreement with experiment.

3) GAMAL - the multipolarity of the gamma rays emitted during the cascade, with all gamma rays considered to have the same multipolarity. Usually only E1 transitions were considered.

4) SD - the spin at which the last gamma-ray transition divided the population distribution produced by the next-to-last transition: For example, in the zinc calculations $SD=2.5=(5/2)$ so that the third (last) transition in the cascade takes place either between states of spin $\leq 5/2$ and the ground state (spin $1/2$) or between states of spin $>5/2$ and the isomeric state (spin $9/2$). Thus SD is the parameter which determines how the populations of the spin distribution that exists following the second (next-to-last) transition will be divided between the isomeric state and the ground state.

Vonach, Vandenbosch and Huizenga model parameters:

- 1) UUP - the upper energy, E_U , at which division of the populations between the isomeric and ground states began.
- 2) ULOW - the energy, E_L , below which only a transition to either the isomeric or the ground state was considered.
- 3) SD - the spin dividing parameter discussed with the H&V model above.
- 4) GAMAL - the multipolarity of the gamma rays in the cascade.
- 5) SMALA - a , the single particle level density.
- 6) SIGMA - the spin cutoff parameter; in this case the minimum value to which $\sigma=ct$ is allowed to go. For example, some of the ^{80}Br and ^{81}Se calculations were made with $\sigma=3.5$ thus effectively introducing a constant temperature behaviour below the energy at which $ct=(3.5)^2$.

The parameters UUP and ULOW, which are described in more detail in Chapter 2 as E_U and E_L , were given the values 2 and 1 Mev respectively after the effects of using different values had been considered.