In compliance with the
Canadian Privacy Legislation
some supporting forms
may have been removed from
this dissertation.

While these forms may be included
in the document page count,
their removal does not represent
any loss of content from the dissertation.

**A Study on the Use of Volterra Integrals for the
Identification of Rotor Blade-Vortex Interactions**

By

**Yuk-Gee Angela Wong
B.Eng. 2000**

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Applied Science

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
Canada

September 2003
© Copyright 2003

National Library
of Canada

Bibliothèque nationale
du Canada

Acquisitions and
Bibliographic Services

Acquisitions et
services bibliographiques

395 Wellington Street
Ottawa ON K1A 0N4
Canada

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

ISBN: 0-612-88899-1

Our file Notre référence

ISBN: 0-612-88899-1

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

Canada

Abstract

The theory of Volterra integral equations for nonlinear system was applied to the prediction of the nonlinear aerodynamic response of an NACA 0012 airfoil during blade-vortex interaction (BVI). The phenomenon was first modelled in two-dimension by an Euler/Navier-Stoke code, and the resulted unsteady aerodynamic data were combined to form a training dataset. The Volterra kernels were identified in time-domain, and the predicted nonlinear aerodynamic responses of the airfoil were compared to the data. The predicted lift time histories of the airfoil were shown to be in good agreement with the aerodynamic data. This study not only have demonstrated that it is feasible for applying the theory of Volterra integral equations for nonlinear system to blade-vortex interaction, but it also have established a simple framework for the development of an engineering level BVI prediction tool.

Table of Contents

List of Tables.....	vi
List of Figures	vii
Nomenclature.....	x
ACKNOWLEDGMENTS	xii
 1.0 Introduction and Historical Review	 1
1.1 Literature Review of Mechanisms of Blade-Vortex Interactions	2
1.2 Literature Review of Volterra Integral Theory for Blade-Vortex Interaction	10
 2.0 Theory	 13
2.1 Vortex Model	13
2.2 Introduction to Volterra Theory for Nonlinear Systems	16
2.3 Nonlinear Volterra Kernel Identification	18
 3.0 Numerical Simulations	 22
3.1 Geometry and Mesh Description.....	22
3.2 Boundary and Initial Conditions	25
3.3 Vortex Characteristics and Initial Conditions	26
3.4 Results and Discussions	28
 4.0 Volterra Kernels Identification for Blade-Vortex Interaction (IDBVI).....	 45
4.1 Methodology Overview.....	46
4.2 Volterra Kernel Identification	48
4.2.1 Basis Functions	48
4.2.2 Discussion	60
4.3 Prediction of Unsteady Aerodynamic System Response	61
4.4 Modelling Time.....	63

5.0	Conclusions	65
5.1	Summary and Conclusions.....	65
5.2	Suggestions for Future Work	67
Reference		68
Appendix: Computer Program Subroutines		70

List of Tables

Table 3.1:	Grid dimensions	25
Table 3.2:	Freestream flow field conditions.....	26
Table 3.3:	Vortex Initial Locations	28
Table 3.4:	Vortex Reference Locations.....	29
Table 3.5:	Normalized Lift Coefficient Values Listing.....	43
Table 4.1:	New Data Vortex Initial Positions.	61

List of Figures

Figure 1.1:	Helical rotor flow field [6]	2
Figure 1.2:	Blade-vortex interaction orientations	3
Figure 1.3:	Schematic of parallel blade vortex interaction	4
Figure 1.4:	Typical vortex induced lift coefficient with instantaneous vortex position on a symmetric NACA 0012 airfoil	6
Figure 1.5:	Schematic of BVI generated pressure wave propagation of an advancing rotor blade [20]	9
Figure 2.1:	Typical Velocity Profile of the Vortex Model	14
Figure 3.1:	Domain outlines for the NACA 0012 airfoil mesh.	23
Figure 3.2:	Grid distribution around the airfoil	24
Figure 3.3:	Schematic of blade vortex interaction	27
Figure 3.4:	BVI Pressure Time History at $t = 0.00705$ sec, $x/c = -0.485$	33
Figure 3.5:	BVI Pressure Time History at $t = 0.00825$ sec, $x/c = -0.0871$	33
Figure 3.6:	BVI Pressure Time History at $t = 0.0084$ sec, $x/c = -0.0400$	34
Figure 3.7:	BVI Pressure Time History at $t = 0.00855$ sec, $x/c = 0.00848$	34
Figure 3.8:	BVI Pressure Time History at $t = 0.00885$ sec, $x/c = 0.108$	35
Figure 3.9:	BVI Pressure Time History at $t = 0.00915$ sec, $x/c = 0.210$	35
Figure 3.10:	BVI Pressure Time History at $t = 0.00945$ sec, $x/c = 0.311$	36
Figure 3.11:	BVI Pressure Time History at $t = 0.00975$ sec, $x/c = 0.411$	36
Figure 3.12:	BVI Pressure Time History at $t = 0.01005$ sec, $x/c = 0.508$	37
Figure 3.13:	BVI Pressure Time History at $t = 0.01035$ sec, $x/c = 0.602$	37
Figure 3.14:	BVI Pressure Time History at $t = 0.01065$ sec, $x/c = 0.696$	38

Figure 3.15:	BVI Pressure Time History at $t = 0.01095$ sec, $x/c = 0.787$	38
Figure 3.16:	BVI Pressure Time History at $t = 0.0114$ sec, $x/c = 0.901$	39
Figure 3.17:	BVI Pressure Time History at $t = 0.01185$ sec, $x/c = 1.03$	39
Figure 3.18:	Lift coefficient time history with instantaneous vortex position for test case 9	41
Figure 3.19:	Lift coefficient time history with instantaneous vortex position for all test cases	41
Figure 3.20:	Normalized Maximum Change in Lift Coefficient vs. Correlation	42
Figure 4.1:	Schematic of the Blade-Vortex Interaction	45
Figure 4.2:	Flow Chart for the BVI Prediction Method	47
Figure 4.3:	Basis Function Spectrum of a 5x5 Set.	50
Figure 4.4:	Typical lift time history of the positive time constants tests.....	52
Figure 4.5:	Typical resulted lift time history of the negative time constants tests	53
Figure 4.6a:	Calculated Lift Response in Comparison with CFD Solution for test case 3, $(y/c)_0 = -0.310$	56
Figure 4.6b:	Calculated Lift Response in Comparison with CFD Solution for test case 4, $(y/c)_0 = -0.175$	56
Figure 4.6d:	Calculated Lift Response in Comparison with CFD Solution for test case 6, $(y/c)_0 = -0.275$	57
Figure 4.6e:	Calculated Lift Response in Comparison with CFD Solution for test case 7, $(y/c)_0 = -0.164$	58
Figure 4.6f:	Calculated Lift Response in Comparison with CFD Solution for test case 8, $(y/c)_0 = -0.145$	58
Figure 4.6g:	Calculated Lift Response in Comparison with CFD Solution for test case 9, $(y/c)_0 = -0.170$	59
Figure 4.7:	Predicted Lift Response in Comparison with CFD Solution for test case 1, $(y/c)_0 = -0.190$	62

Figure 4.8:	Predicted Lift Response in Comparison with CFD Solution for test case 2, $(y/c)_o = -0.080$	62
-------------	---	----

Nomenclature

$[B]$	least squares data vector
c	chord of airfoil
c_j	j -th basis function coefficient
CFD	computational fluid dynamics
C_L	lift coefficient
ΔC_L	maximum change in lift coefficient
C_p	pressure coefficient
d_k	d -th basis function coefficient
i	time step index
j	total number of basis functions used for the first-order kernel
k	total number of basis functions used for the second-order kernel
K_1	first-order Volterra kernel
K_2	second-order Volterra kernel
M	Mach number
$[M]$	least squares motion matrix
r	non-dimensional radial distance
r_c	non-dimensional vortex core radius
SVD	singular value decomposition
t	time
v_θ	tangential velocity of the vortex
U_∞	freestream velocity

x/c	Nondimensional streamwise distance
$\mathbf{x}(\tau)$	system input
$(y/c)_0$	initial vertical distance
$\mathbf{y}(t)$	system response as a function of time
α	angle of attack
τ	time constants for basis functions
τ	integration time variable
ζ	basis function for the first-order kernel
μ	basis function for the second-order kernel
Γ	vortex circulation
$\hat{\Gamma}$	maximum circulation divided by the freestream velocity and the chord of airfoil
$\frac{\Gamma}{U_\infty c}$	non-dimensional circulation
Ψ	azimuth angle

ACKNOWLEDGMENTS

I would like to thank my supervisor Dr. Fred Nitzsche for giving me the opportunity and the resources to work on my thesis. Most importantly I would like to thank Dr. Nitzsche for his trust in me and in the work I have done, even when I am in doubt myself. My gratitude cannot easily be expressed by words. Once again I thank you, Dr. Nitzsche for accepting me as your student.

I would sincerely like to thank Dr. Mahmood Khalid for allowing me to work at his laboratory for my thesis. While I was there, I was treated as an equal member of the CFD group. He provided me with a lot of opportunities to learn from various projects and from other group members. My time there served as an eye-opener to many aspects of life.

I would also like to thank every member of the NRC-IAR CFD Group. I learned a lot during my time there. I am grateful for their guidance and suggestions. I especially would like to thank Dr. Stuart McIlwain, Dr. Francois Fortin and Dr. Mahmoud Mamou for their patience with me and for everything they have taught me.

Lastly, I would like to thank my mother and my family for supporting me throughout these years.

1.0 Introduction and Historical Review

The blade-vortex interaction (BVI) phenomenon is one of the most intriguing subjects in nonlinear unsteady aerodynamics. The physics behind this phenomenon is complex, and it has a critical effect on the performance and stability of a flight vehicle. For over a century, researchers have worked together to better understand the structure and evolution of tip vortices from lifting surfaces. Through their efforts, there are now numerous theories and models available for the dynamics of vortex. With the increasing usage of modern rotorcraft, such as helicopters and tiltrotor aircraft, the focus of most BVI research is on the effects of unsteady impulsive loading on the rotor blades and the associated noise generated during the process. After reviewing the results of many different analyses, it is interesting to note that most studies will arrive to the same trend of results. In the study by Renzoni and Mayle [17], a set of correlations was obtained for the total change in the lift, moment and drag coefficients due to the interaction. This further implies that there is an underlying kernel behind this unsteady three-dimensional phenomenon. The objective of this study is to demonstrate that the Volterra integral theory for nonlinear systems is applicable for the identification of the blade-vortex interaction phenomenon. A simple framework for an engineering tool that could be used by a designer to quickly obtain an accurate prediction of the effects of blade-vortex interactions and enhance the overall design performance can be established.

1.1 Literature Review of Mechanisms of Blade-Vortex Interactions

Blade-vortex interaction occurs when a helicopter is in hover or manoeuvring in descending or ascending forward flight. As the rotor rotates, the flow from the lower surface (higher-pressure side) of the blades will travel around the tips to the upper surface (lower pressure side) of the blades to relieve the pressure differences and create strong concentrated tip vortices. Due to the rotation of the rotor and the helicopter's flight conditions, these trailing tip vortex filaments will often result in a distorted interlocking helices formation as shown in Figure 1.1

Figure 1.1: Helical rotor flow field [6]

Under certain flight regimes especially in descending flight, the following blade on the rotor will pass close to (or even collide with) the tip vortex from the preceding blade. The large velocity induced by the vortex on the following blade in turn results in a large change in the blade loading. This vortex induced impulsive loading is a major concern in both the performance and acoustic emissions of a flight vehicle [7].

The actual BVI problem is three-dimensional and unsteady, with the helical vortex filaments intersecting the following blades at different angles. However there are two orientations of the interaction that are of particular concern. First is the case when the vortex axis of rotation is perpendicular to the leading edge of the on-coming blade, as shown in Figure 1.2a. The second is the case when the vortex axis of rotation is parallel to the leading edge of the blade as shown in Figure 1.2b.

a) vortex rotation axis perpendicular to blade

b) vortex rotation axis parallel to blade

Figure 1.2: Blade-vortex interaction orientations

By comparing the two orientations, the area affected by the second (parallel) case is greater than that affected by the first (perpendicular) case. As the vortex travels parallel

to the leading edge of the blade, a large span of the blade will be simultaneously affected by the vortex. In 1971, Widnall, one of the pioneers in blade-vortex interaction noise theoretical studies, by using the unsteady lifting-line and linear acoustic theory has shown that the parallel case is more severe than the perpendicular case. In 1981, Nakamura confirmed Windall's results in an experimental study on the impulsive nature of the noise generated by BVI. In these experiments, the surface pressure fluctuations of an AH-1G helicopter blade during slow descending flight were measured, and was found that the trailing tip vortex was about one and a half revolutions old when parallel BVI occurs. [25] In another experimental study by Martin *et al.* on a model rotor, they have identified the most prominent BVI noise occurs when the blade's azimuth angles, Ψ , are in the range of 70 and 90 degrees, exactly where the trailing vortex is nearly parallel to the blade, as shown in Figure 1.3 [12]. Therefore, the parallel case is considered more crucial than the perpendicular case regarding helicopter BVI noise.

Figure 1.3: Schematic of parallel blade vortex interaction

The helicopter rotor shown in Figure 1.3 is in forward flight with a counter-clockwise rotation. Although helicopters normally travel at a relatively slow speed, however the Mach number at the tips of the blades are usually in the transonic flow regime due to the high aspect ratio of the rotary wing. The complete passage of the vortex over the entire chord of the blade is brief in time but the instantaneous fluctuations in aerodynamic loading over the entire span of the blade are extensive in time. In fact, the blade involved in the interaction will feel the vortex-induced loading even though the core of the vortex is 1 or 2 chords ahead or behind.

The parallel blade vortex interaction can be modelled as a two-dimensional problem because the entire span of the blade undergoes the same impulsive loading induced by the vortex filament. In the 1980s, Srinivasan *et al.* have studied this two-dimensional problem by using both the small-disturbance model and the transonic Navier-Stokes (TNS) flow model to calculate the aerodynamics of the BVI phenomenon [22]. Using the transonic Navier-Stokes equations and adaptive grids, the prescribed vortices were allowed to convect downstream as part of the flow field [3]. Later, in 1993 a study by Baeder and Srinivasan [23], the calculated surface pressures were favourably compared to the surface pressure of an experimental study performed by Caradonna *et al.* using a modified version of the Transonic Unsteady Rotor Navier-Stokes (TURNNS) code originally written by Srinivasan.

A typical profile of the lift coefficient time history from a two-dimensional CFD inviscid model is shown in Figure 1.4. The lift coefficient is plotted against the position

of the vortex in the streamwise direction. The vortex has a clockwise rotation and is travelling towards the airfoil from the left to the right at the freestream velocity.

For a rotor in forward flight, it is typical for the advancing blades to encounter a clockwise rotating vortex while the retreating ones encounter a counter-clockwise rotating vortex. The symmetric NACA 0012 airfoil shown in Figure 1.4 has zero initial lift. Therefore, the recorded lift is solely induced by the vortex.

Figure 1.4: Typical vortex induced lift coefficient with instantaneous vortex position on a symmetric NACA 0012 airfoil [17]

As the vortex approaches, and continues to rotate in the clockwise direction, it forces the stagnation point towards the upper surface. Therefore, the airfoil will experience a downward force, resulting in a momentary loss of lift. The lift coefficient will reach its minimum as the core of the vortex aligns with the leading edge of the airfoil. However, as the vortex passes under the leading edge of the airfoil, the stagnation

point returns to the leading edge while the rotation of the vortex causes the flow to push up against the airfoil. This results in an increase in lift, which eventually reaches its maximum value. As the vortex continues to travel towards the trailing edge, the lift slowly decreases. From Figure 1.4, the induced loading lingers on the airfoil even though the vortex is two chords away from the airfoil. This lingering unsteady effect was seen to slowly fade away only when the vortex has passed several chords downstream as noted in the study conducted by Srinivasan, McCroskey and Baeder [24]. If the airfoil and its surrounding flow field are considered as a system or “plant” in systems terminology, the lift response of the airfoil to the input oncoming vortex is highly nonlinear, as demonstrated in this typical lift coefficient time history.

For a typical parallel BVI, there are a number of characteristic parameters which govern the phenomenon namely: 1.) the strength of the vortex, 2.) the core size of the vortex, 3.) the vertical distance between the core of the vortex and the plane of the blade and, 4.) the blade tip Mach number [12]. In the past decade, various experimental and numerical studies have been conducted to control these parameters and understand the individual effects that they have on the BVI phenomenon.

A lifting blade will produce a tip vortex whose strength is directly related with the lift. The vortex strength is the primary parameter that affects the formation and the movement of the transonic shock wave, (generated by the unsteady flow and locally confined to the supersonic flow region) [8]. Various attempts had been made to control this parameter either by modifying the blade tip shape or by adding winglets, or by an active control technique such as Active Shape Control (ASC). All the former methods have their own advantages, but none have shown a clear success. The vortex strength is a

difficult parameter to control because it involves a battle against the fundamental of physical laws [5].

Similarly, the size of the vortex core, which is typically 5 to 10% of the chord length, has been observed to have strong influence on the velocity gradient induced by the vortex itself [8]. The influence of vortex core size is still a heated topic of debate because the latest optical measurement techniques, like the Laser Doppler Velocimetry (LDV), do not provide reliable data about the core [28].

Another important BVI parameter is the vertical distance between the core of the vortex and the plane of the blade, also known as the vortex miss distance. This parameter is the reason why only under certain conditions BVI becomes a problem. Based on the vertical distance, two types of BVI can occur: 1.) The distance is large enough so that the vortex core passes the entire airfoil without being affected by the airfoil, or 2.) The distance is too small so that as the vortex first contacts the airfoil, the core collides with the leading edge of airfoil and splits itself into two separate vortices, each travelling on one side of the airfoil (upper and lower). Assuming that the vortex passes below and close to the airfoil without splitting, the presence of the vortex causes the stagnation point to move up into the upper surface of the airfoil. A high-pressure region is generated due to the displacement of the stagnation point. This high-pressure region will then propagate upstream and induce shock waves, which are known as “compressibility” waves [10]. From the experimental study on aerodynamic sound generation due to BVI conducted by Lent *et al.* [10], the compressibility waves are shown to get stronger as the vertical distance between the core of the vortex and the airfoil decreases. Therefore, by controlling the flight (or blade) path BVI phenomenon can be alleviated. Based on this

fact active control techniques such as High Harmonic Control (HHC) and Individual Blade Control (IBC) are being investigated to control BVI induced noise in rotorcraft.

The increase of the blade tip Mach number can increase the strength of the compressibility waves. Hence, the blade tip Mach number also plays an important role on the BVI noise associated with high-speed flight. In Figure 1.5, a schematic of the BVI generated pressure wave propagation of an advancing rotor blade is shown. As the blade advances and generates more pressure sources in the wake behind, the wave front of these sources will combine with other nearby waves. At high Mach number, the combination of these waves can form a stronger shock wave. [14]

Figure 1.5: Schematic of BVI generated pressure wave propagation of an advancing rotor blade [20]

1.2 Literature Review of Volterra Integral Theory for Blade Vortex Interaction

The blade-vortex interaction phenomenon is one example of the most complex unsteady aerodynamic problems in rotor flow analysis. Although this phenomenon is unsteady and nonlinear in nature, it is not unpredictable. Renzoni and Mayle [17] have presented a set of correlations among lift, moment, drag and the initial vortex vertical position for parallel blade vortex interaction. In these correlations, the maximum change in lift and moment during the entire interaction has been reduced to a single function of the vortex initial position from the airfoil. The predicted maximum change in lift and moment from these simple equations compared well with experiments. However to a rotor designer who is concerned about the time-dependent and highly impulsive loading caused by the vortex, the maximum change values do not provide enough information. The response of the airfoil to the vortex can be accurately predicted by using computational fluid dynamic (CFD) methods. However, the required computational time is long. Furthermore, a new computation has to be conducted for any change in the parameters. The compromise solution between the complex CFD methods and the simple functional approach proposed by Renzoni and Mayle is the development of an engineering tool that utilizes the theory of Volterra equations to predict the effects of blade-vortex interactions.

The work of the Italian mathematician Vito Volterra on functionals, provided the fundamentals of modelling a physical process with some observable facts that can result with new information about the process [2]. Due to the significant contributions by

Wiener on the development of the Volterra series, this theory is also referred as the Volterra-Wiener theory of nonlinear systems. The Volterra theory of nonlinear systems has long been used as a tool in the field of electrical engineering, mathematical biology, medical imaging and numerous other fields of applications. [16] However, it was only in the past decade that this powerful theory received attention in the field of unsteady aerodynamics. In the modelling of unsteady nonlinear aerodynamics, the focus of interest is on the identification of Volterra kernels in either the time or the frequency domains. The identification of the Volterra kernels is in general difficult; however, the advantages of using Volterra series to model nonlinear systems overweight the difficulties. This modelling technique allows any arbitrary number of degrees of freedom while maintaining relatively simple levels of computations. The Volterra series provide a means to characterize complex nonlinear unsteady aerodynamic systems in terms of polynomial functionals.

In 1990, Tromp and Jenkins [26] applied a Volterra kernel identification scheme to model nonlinear aerodynamic reactions of a two dimensional airfoil in a subsonic flow subjected to the input angle of attack. The identification was performed in the frequency domain. The model was built using aerodynamic data from an unsteady Navier-Stokes solver. The first order (linear) kernel was identified, and the second order (non-linear) kernel was identified for a sample problem.

Silva [21] in 1993 also used the output from a CFD solver to identify the Volterra kernels. The method used the system responses to single- and double-impulse input functions to identify the first and second order kernels. After the Volterra kernels were identified, an airfoil in transonic flow undergoing a plunging motion was modelled in a

bilinear state space form. The results demonstrated that the modern control theory could be applied to a nonlinear aerodynamic response model defined in bilinear state space form.

Reisenthel [16] has developed an alternative method that identifies the Volterra kernels from unsteady aerodynamic data obtained from wind tunnel models or flight tests. According to this method, the nonlinear system under consideration is not required to undergo impulsive excitations such as in Silva's method. Reisenthel [16] applied this method to a rigid NACA0015 airfoil that was dynamically pitched about its quarter chord. The nonlinear unsteady aerodynamic data was acquired from wind tunnel testing. The results demonstrated that this method is feasible for unsteady aerodynamic systems extraction of both the first and second order Volterra kernels from experimental data.

2.0 Theory

2.1 Vortex Model

For the analytical vortex to have a structure that is similar to those that were measured in experiments, the Lamb-Oseen style compressible vortex model with Scully's expression was used in all computations. In 1975, Scully suggested the use of the factor $r^2/(r^2+r_c^2)$ to produce a distributed vorticity instead of the conventional concentrated vorticity inside a core radius (i.e. solid body rotation). [7] The vortex model is expressed as

$$\frac{v_\theta}{U_\infty} = \frac{\Gamma}{2\pi r} \left(\frac{r^2}{r^2 + r_c^2} \right) \quad (2.1.1)$$

where v_θ is the tangential velocity of the vortex, r_c and r are the non-dimensional core radius and the non-dimensional radial distance from the centre of the vortex. Both r_c and r are normalized against the chord length of the airfoil which generated the tip vortex. The non-dimensional circulation, Γ , is defined by the maximum circulation divided by the freestream velocity, U_∞ , and the chord length, c . This vortex model has been used in numerous studies such as Lee and Bershader [9], Petrini *et al.* [15], and Straus [25]. The typical tangential velocity profile represented by the above model is shown in Figure 2.1.

Figure 2.1: Typical Velocity Profile of the Vortex Model

In this mathematical representation of the vortex, the singularity of the potential vortex term, $\frac{\hat{\Gamma}}{2\pi r}$, was smoothed out by the $\left(\frac{r^2}{r^2 + r_c^2}\right)$ term. The two peaks in the velocity profile define the size of the vortex core. By following the work of Hoffman and Joubert, Nielsen and Schwind, and Lee and Bershader [9], the vortex structure contains four distinct regions similar to the turbulent boundary layer: 1.) the viscous vortex core region where the effect of viscosity dominates, 2.) the turbulent diffusion governed logarithmic region, 3.) the transition region between the inner viscous region and the outer inviscid region, and 4.) the external inviscid, irrotational region where the

circulation is constant. Based on experimental vortex profiles, the curve-fit normalized

circulations, $\frac{\Gamma}{\Gamma_{\max}}$, with respect to the centre of the vortex are as follows [9]:

Viscous core region:

$$\frac{\Gamma}{\Gamma_{\max}} = 0.80 \left(\frac{r}{r_c} \right)^2 \quad 0 \leq r/r_c \leq 0.62 \quad (2.1.2)$$

Logarithmic region:

$$\frac{\Gamma}{\Gamma_{\max}} = 0.51 + 0.43 \ln \left(\frac{r}{r_c} \right) \quad 0.62 \leq r/r_c \leq 1.8 \quad (2.1.3)$$

Transition region:

$$\frac{\Gamma}{\Gamma_{\max}} = 1 - 0.80 \exp \left[-0.65 \left(\frac{r}{r_c} \right) \right] \quad 1.8 < r/r_c \quad (2.1.4)$$

During the simulations, the vortex is convected with the freestream velocity. The pressure and density fields of the vortex can be determined from the radial momentum equation as follows:

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{v_\theta^2}{r} - \frac{2}{\rho} \left(\frac{\partial}{\partial r} \mu \frac{\partial v_r}{\partial r} + \frac{\mu}{r} \frac{\partial v_r}{\partial r} - \frac{\mu v_r}{r^2} \right) = 0 \quad (2.1.5)$$

Since the radial velocity component, v_r , is much smaller in magnitude when compared to the tangential velocity component, v_θ , all of the radial velocity related terms could be neglected. The reduced form of the radial momentum equation is as follows [9]:

$$\frac{1}{\rho(r)} \frac{\partial p(r)}{\partial r} = \frac{v_\theta(r)^2}{r} \quad (2.1.6)$$

The density, ρ , can be obtained by assuming constant enthalpy throughout the flow and is given by

$$\frac{\gamma}{\gamma-1} \frac{p}{\rho} + \frac{v_\theta^2}{2} = \frac{\gamma}{\gamma-1} \frac{p_\infty}{\rho_\infty} \quad (2.1.7)$$

2.2 Introduction to Volterra Theory for Nonlinear Systems

The Volterra theory states that any nonlinear system can be modelled as an infinite sum of multidimensional convolution integrals. The modelling of a system with nonlinear Volterra integral equations follows the classical method of successive approximations. [4] The order of each term in the series can be regarded as a measurement of the nonlinearity of the system response. For a weakly nonlinear system, only the first few Volterra kernels in the series will be required to define the system. The magnitudes of the higher order kernels will quickly fall off and these terms will become negligible. [21]

The output of a weakly nonlinear time invariant system can be expressed by using the summation of the Volterra functionals as follows: [1]

$$y(t) = y_0 + y_1(t) + y_2(t) + y_3(t) + \dots \quad (2.2.1)$$

where y_0 is a constant, $y_1(t)$ is the linear output, $y_2(t)$ is the bilinear output and $y_3(t)$ is the trilinear output. The system is assumed to be causal, in which the system response at the present time will not be affected by the future inputs.

In most aerodynamic phenomena, a nonlinear unsteady system will exhibit an after-effect, which is often referred as a “nonlinear system with memory”. Examples of such systems are unsteady flows past an oscillating airfoil and blade vortex interactions. In both of these cases, the present time history of the flow will affect the system response in the future. For such cases, the appropriate form for the integral equation is of convolution type with a difference kernel [4]. Therefore, Volterra functionals in Equation (2.2.1) are given by

$$y_1(t) = \int_0^t K_1(t - \tau_1) x(\tau_1) d\tau_1 \quad (2.2.2)$$

$$y_2(t) = \int_0^t \int_0^t K_2(t - \tau_1, t - \tau_2) x(\tau_1) x(\tau_2) d\tau_1 d\tau_2 \quad (2.2.3)$$

$$y_3(t) = \int_0^t \int_0^t \int_0^t K_3(t - \tau_1, t - \tau_2, t - \tau_3) x(\tau_1) x(\tau_2) x(\tau_3) d\tau_1 d\tau_2 d\tau_3 \quad (2.2.4)$$

where t is the upper time limit of the integration, τ is the integration variable, $x(\tau)$ is the system input, and the Volterra kernel $K(\sigma) = 0$ for $\sigma < 0$. [4] For aerodynamic applications, $y(t)$, which is the system response to the input $x(t)$, will be the aerodynamic quantity of interest such as lift coefficient, drag coefficient, pressure coefficient, and etc.

2.3 Nonlinear Volterra Kernel Identification

The prediction of the nonlinear aerodynamic response of a system to an arbitrary input begins from the extraction of the nonlinear kernels of the Volterra Series. The procedure for identifying any order of the Volterra kernels is essentially the same, for simplicity, the truncated Volterra series will be assumed to be of second-order. The fundamental equation for a two terms (i.e. 2nd order) Volterra series expansion has the form:

$$y(t) = \int_0^t K_1(t - \tau_1)x(\tau_1)d\tau_1 + \int_0^t \int_0^t K_2(t - \tau_1, t - \tau_2)x(\tau_1)x(\tau_2)d\tau_1d\tau_2 \quad (2.3.5)$$

The only unknowns in this equation are the kernels, K. The system response due to an arbitrary input can be obtained either from experimental datasets or from the output of a CFD code. The extraction of these kernels is the most critical part of the prediction process because these kernels hold the characteristics of the physical system.

The first task in the extraction process is to discretize the unknown kernels in time and expand them into some basis function space:

$$K_1(t) = \sum_j c_j \zeta_j(t) \quad \text{and} \quad K_2(t_1, t_2) = \sum_k d_k \mu_k(t_1, t_2) \quad (2.3.6)$$

The basis functions ζ_j and μ_k are known functions of time, while $\{c_j\}$ and $\{d_k\}$ are the corresponding unknown coefficients vectors. The integers, j and k, define the total number of basis functions used for the kernel. Thus Equation (2.3.5) can be rewritten as,

$$y(t_i) = \sum_j c_j \int_0^{t_i} \zeta_j(t_i - \tau_1)x(\tau_1)d\tau_1 + \sum_k d_k \int_0^{t_i} \int_0^{t_i} \mu_k(t_i - \tau_1, t_i - \tau_2)x(\tau_1)x(\tau_2)d\tau_1d\tau_2 \quad (2.3.7)$$

where i is the time step index. Since the only unknowns are the coefficient vectors $\{c_j\}$ and $\{d_k\}$, all of the integrals in Equation (2.3.7) can be evaluated and rewritten as:

$$y(t_i) = \sum_j c_j a_{ij}^{(1)} + \sum_k d_k a_{ik}^{(2)} \quad (2.3.8)$$

The $a_{ij}^{(1)}$ and $a_{ik}^{(2)}$ contain the solutions from the one- and two-dimensional integrals.

From Equation (2.3.8), the problem can be treated as a matrix problem of the type

$$[A][C] = [Y] \quad (2.3.9)$$

For multiple data sets, the linear Equation (8) becomes

$$\sum_n [A^{(n)}][C] = \sum_n [Y^{(n)}] \quad (2.3.10)$$

where $[A^{(n)}] = [a_{ih}^{(n)}]$, $h = 1, \dots, j+k$, $[Y^{(n)}]$ is the system output vector matrix, n is the number of data sets, and $[C]$ is the coefficients vector. Since there will be more than one set of data, the coefficients vector $[C]$ must simultaneously satisfy all available data sets at all times [16]. The basic approach to determine these coefficients is to apply a linear least square fit. When the general linear least squares formulae are implemented in Equation (2.3.10) the following equations are obtained:

$$[M][C] = [B] \quad (2.3.11)$$

The normal equations of the least square problem in matrix form are defined as:

$$[M] = \sum_n [A^{T(n)}][A^{(n)}] \quad (2.3.12)$$

$$[B] = \sum_n [A^{T(n)}][Y^{(n)}] \quad (2.3.13)$$

[13] In Equation (2.3.12), [M] is also known as the least squares motion matrix while [B] in Equation (2.3.13) is known as the least squares data vector. [16] However it is known that the solution of the least square problem using normal equations is highly sensitive to round off errors and that the normal equations can be near singular; therefore the singular value decomposition (SVD) and pseudo-inverse need to be used to avoid the aforementioned problems. [16]

Once all the coefficients have been determined, they can be reinserted into Equation (2.3.6) along with the already known basis functions to yield the Volterra kernels. With the Volterra kernels identified, the model can predict reactions from a nonlinear system due to arbitrary inputs.

In the present study, the interaction was first modelled numerically by solving the Euler equations as the governing equations in the CFD code, WIND [27]. The solutions obtained from the numerical modelling formed a dataset that was used in the kernels identification process. The identification of the kernels is of fundamental importance to the prediction process because it contains the characteristics of the blade-vortex interaction phenomenon. The extraction of the kernel and the prediction of the interaction

effects were accomplished by utilizing nonlinear Volterra theory. The present study will adopt the identification method developed by Reisenthel [16], and the identification will be performed in time-domain.

3.0 Numerical Simulations

Computational fluid dynamics is the science for modern unsteady aerodynamic predictions. Parallel blade-vortex interaction can be simulated in two-dimensions either with the full Navier-Stokes equations set or the Euler equations set. The choice of the governing equations will determine the level of physics being modelled and the total required computational time. Since the purpose of the simulations is to create a dataset for the Volterra kernel identification process, the Euler equations were used in order to save computational time. By choosing the Euler equations as the governing equations, the flow is assumed to be inviscid and that the blade-vortex interaction will not cause flow separation. In all simulations, the vortex is initialized at a distance below the airfoil where the resulting vortex path does not lead to a direct head-on collision with the airfoil, ensuring that the above assumptions are valid.

3.1 Geometry and Mesh Description

The accuracy of the simulation results is strongly dependent on the details of the blade geometry and the mesh. The only geometry used in the simulation was the symmetric NACA 0012 airfoil with a unit chord length. The airfoil was set at a position with a 0° angle of attack; therefore, any lift observed in the simulations is a direct result from the blade vortex interaction. A C-type multi-zone two-dimensional structured grid was constructed around the airfoil. The outer boundaries were located 25 chords away

from the nose of the airfoil. The computational domain was divided into 6 zones as shown in Figure 3.1.

Figure 3.1: Domain outlines for the NACA 0012 airfoil mesh.

The division of the computational domain into smaller sub-zones allows different grid densities to be used in different areas around the airfoil. During the simulation of blade vortex interactions, a very fine grid is required on the path of the vortex in order to preserve its characteristics. Throughout the study, the vortex is set so that it will pass under the target airfoil; therefore the zones directly in front of and under the airfoil

contain the highest grid density as shown in Figure 3.2. The normal distance from the wall for the highest grid density is 5×10^{-4} chords. The grid was constructed using the units of feet, and in such a way that the near field of the airfoil contained a very fine grid distribution that gradually coarsened as it proceeded outward to the far field.

a.) Leading edge of airfoil

b.) Trailing edge of airfoil

Figure 3.2: Grid distribution around the airfoil

The same grid was used for all of the computations. The total number of grid points was 587,799. The normal distance defined for this grid was considered very fine for Euler computations. Using the same grid topology but with a larger value for the normal distance (fewer grid points), the dissipation of the minimum pressure inside the

core was too great; therefore, after the experience was acquired through trial and error the fine grid was selected. The grid size for each of the sub-zones was as follows:

Zone	Dimensions
1	20 x 341
2	20 x 1021
3	120 x 1021
4	502 x 511
5	299 x 511
6	168 x 171

Table 3.1: Grid dimensions

3.2 Boundary and Initial Conditions

At the solid boundaries (*i.e.* the upper and lower surfaces of the airfoil), inviscid (slip) and adiabatic wall conditions were applied. The normal pressure gradient at the wall was assumed to be zero. The ideal gas model was used in the simulations. The freestream flow field conditions were used as reference conditions for normalizing the flow field variables (as shown in Table 3.2). At the downstream (outflow boundary), the pressure for all the points on the boundary was extrapolated from the adjacent points.

Mach Number, M	0.300
Speed of Sound, a	1116.289 ft/s
Flow Velocity, V_∞	334.887 ft/s
Angle of attack, α	0.000 degree
Total pressure	15.643 psia
Static pressure	14.696 psia
Total temperature	528.026 degree Rankine
Static temperature	518.690 degree Rankine

Table 3.2: Freestream flow field conditions

For the flow field initialization, the flow properties of the grid points that were located inside and nearby the vortex were specified according to the vortex model described in the theory section 2.1. All of the remaining grid points in the computational flow field were assumed to have freestream conditions.

The Roe fifth-order upwind-biased spatial integration scheme was used by the finite volume solver, WIND [27], to solve the time-dependent Euler equations. The time step was set at 2.5×10^{-6} seconds, with 20 maximum cycles per time step.

3.3 Vortex Characteristics and Initial Conditions

A vortex with a non-dimensional circulation, $\frac{\Gamma}{U_\infty c} = 0.166$ with a clockwise rotation was convected at the freestream velocity in all simulations. Since the literature shows that for a typical rotor blade the trailing tip vortex has a core diameter of about the

thickness of the blade [24], the core size (or its radius) was set as 6% of the chord length to match the NACA 0012 airfoil used in the simulations. Both the circulation and the core size of the vortex were non-dimensionalized by the chord length of the airfoil. The internal structure of the vortex was constructed according to the vortex model described in section 2.1. The leading edge of the airfoil was positioned at the origin. For each of the simulations, the vortex was initially placed at 2.5 chords ahead of the airfoil with different vertical distances below the airfoil. A schematic of the interaction is shown in Figure 2.3. The vertical distances, y_0 , were measured from the chord line of the airfoil to the core of the vortex.

Figure 3.3: Schematic of blade vortex interaction

An array of test cases with different initial vertical distances was simulated in the present study. The initial locations for each of the test cases are listed in Table 3.3. This set of test cases was selected to cover a narrow range of vertical distances so that the extraction of kernels and the prediction processes could be performed within a minimum number of test cases.

Case #	x/c	y/c
1	-2.53480	-0.245929
2	-2.54205	-0.155371
3	-2.52202	-0.353106
4	-2.53898	-0.19865
5	-2.52461	-0.334044
6	-2.53028	-0.288505
7	-2.54062	-0.176521
8	-2.54161	-0.162314
9	-2.54012	-0.183789

Table 3.3: Vortex Initial Locations

3.4 Results and Discussions

At the beginning of each simulation, after the vortex was initialized in the flow field, the vortex fluctuated in the vertical direction as it convected downstream. After the vortex travelled about 1 chord length from its initial position, this fluctuation in the vortex trajectory ended and the vortex settled down to a vertical distance that was close to its initial value. In general, the vertical distance between the vortex and the airfoil, y/c ,

decreased by approximately $0.03c$. Hereafter, the initial vertical distance, $(y/c)_o$, for each test case will be referred to the levelled values given in Table 3.4:

Case #	x/c	$(y/c)_o$
1	-2.5348	-0.190
2	-2.54205	-0.080
3	-2.52202	-0.310
4	-2.53898	-0.175
5	-2.52461	-0.335
6	-2.53028	-0.275
7	-2.54062	-0.164
8	-2.54161	-0.145
9	-2.54012	-0.170

Table 3.4: Vortex Reference Locations

Since the airfoil had a zero angle of attack, any changes in the pressure distribution or lift were due to the blade-vortex interaction effect. Since the only difference among all the test cases was the vertical distance, y_o , the complete description of the interaction process will only be shown for one of the test cases in the present work. The instantaneous pressure coefficient contours in Figures 3.4 to 3.17 along with the lift coefficient time history in Figure 3.18 for test case 9 will now be examined.

Beginning with Figure 3.4, the vortex is located at 0.485 chords ahead of the airfoil at 0.00705 sec. Even though the strength of the vortex is relatively weak, the induced loading on the airfoil can already be seen from the pressure coefficient distribution.

In Figures 3.5, 3.6, and 3.7, the vortex continues to advance towards the leading edge of the airfoil. The approach of this clockwise rotating, low pressure vortex causes the lower surface of leading edge of the airfoil to become a low pressure region and this effect is indicated by the increasing magnitude of the negative pressure coefficient peaks. At 0.00825 sec, when the vortex was located 0.0871c ahead of the airfoil (*i.e.* Figure 3.5), the minimum pressure coefficient over the airfoil was about -0.6. However at 0.00855 sec, when the vortex was located at 0.00848c and the core of the vortex has just passed the nose of the airfoil, as shown in Figure 3.7, the minimum pressure coefficient over the airfoil jumped to a high of -1. As a result of this rapid decrease in pressure over the lower surface, the lift coefficient over the airfoil also experienced a plunge from zero lift to -0.15, as shown in Figure 3.18. It is interesting to note that once the vortex passes the nose of the airfoil, the lift induced by the vortex rapidly increases and reaches the maximum lift coefficient of 0.047. This rapid increase in lift is associated with the magnitude of the negative pressure peak induced by the vortex on the lower surface of the airfoil decreasing as the vortex moves aft towards the trailing edge.

From Figure 3.8 to 3.17, the vortex had travelled a distance of 0.922c in 0.003 sec. Each of the figures shown were taken at a time intervals of approximately 0.0003 sec and the average distance travelled by the vortex was about 0.1c. By comparing the pressure coefficient distributions in Figures 3.8 to 3.16, it can be seen that the propagation of the pressure peak towards the trailing edge corresponds to the convection of the vortex. As the negative pressure coefficient peak decreases, the overall pressure on the lower surface of the airfoil also becomes less negative. This is due to the clockwise rotating vortex and its instantaneous position, the velocity (in the normal direction)

induced by the vortex has increased the effective angle of attack of airfoil. The lower surface becomes the higher-pressure region and, therefore, lift has been gained in the process. At 0.01185 sec, the vortex has reached the trailing edge of the airfoil as shown in Figure 3.17. The core of the vortex is now located at $1.03c$. Therefore only a small part of the vortex induced the negative pressure coefficient peak that can be seen in the pressure coefficient distribution. In Figure 3.17d, the velocity vectors are shown as the flow leaves the trailing edge. This is an example where the flow on the lower surface near the vortex had a larger velocity magnitude while the flow on the upper surface has been shielded by the airfoil during the entire interaction and, therefore, remained unaffected by the vortex. Due to this increase in velocity, vortices are known to have the capability of delaying separation. With a negative pressure coefficient peak located at the trailing edge, the airfoil will experience an increase in lift similar to an increase in its effective angle. This small increase in lift is shown as a small bump in the lift coefficient time history in Figure 3.18. Although the vortex has passed the airfoil, its effect on the airfoil continues to linger until the vortex is a few chords downstream.

The similar trends in the airfoil lift responses due to the vortex are best observed when the test cases' lift history distributions are plotted together in Figure 3.19.

In the studies of Renzoni and Mayle [17], a set of simple correlations for expressing the overall behaviour of the maximum change in lift and moment were presented and shown to compare well against the experiment. The maximum change in lift coefficient, ΔC_L , is defined, as shown in Figure 3.18, and the simple equation for the lift coefficient is given by [17]:

$$\Delta C_L = 0.7 \frac{\Gamma}{U_\infty c \sqrt{|(y/c)_o|}} \quad \text{where } (y/c)_o \neq 0 \quad (3.4.1)$$

Likewise, the maximum change in lift coefficient for the test cases were plotted against the initial vortex vertical distance in the present study for validation of the above equation. This is shown in Figure 3.20.

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.4: BVI Pressure Time History at $t = 0.00705$ sec, $x/c = -0.485$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.5: BVI Pressure Time History at $t = 0.00825$ sec, $x/c = -0.0871$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.6: BVI Pressure Time History
at $t = 0.0084\text{sec}$, $x/c = -0.0400$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.7: BVI Pressure Time History
at $t = 0.00855\text{sec}$, $x/c = 0.00848$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.8: BVI Pressure Time History at $t = 0.00885$ sec, $x/c = 0.108$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.9: BVI Pressure Time History at $t = 0.00915$ sec, $x/c = 0.210$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.10: BVI Pressure Time History
at $t = 0.00945$ sec, $x/c = 0.311$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.11: BVI Pressure Time History
at $t = 0.00975$ sec, $x/c = 0.411$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.12: BVI Pressure Time History
at $t = 0.01005$ sec, $x/c = 0.508$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.13: BVI Pressure Time History
at $t = 0.01035$ sec, $x/c = 0.602$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.14: BVI Pressure Time History
at $t = 0.01065$ sec, $x/c = 0.696$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.15: BVI Pressure Time History
at $t = 0.01095$ sec, $x/c = 0.787$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.16: BVI Pressure Time History
at $t = 0.0114$ sec, $x/c = 0.901$

a.) Pressure Coefficient Contours

b.) Stream lines

c.) Pressure Coefficient Distribution

Figure 3.17: BVI Pressure Time History
at $t = 0.01185$ sec, $x/c = 1.03$

Figure 3.17 d.) Velocity Vectors, At $t = 0.01185$ sec, $x/c = 1.03$

Figure 3.18: Lift coefficient time history with instantaneous vortex position for test case 9

Figure 3.19: Lift coefficient time history with instantaneous vortex position for all test cases

Figure 3.20: Normalized Maximum Change in Lift Coefficient vs. Correlation

The normalized maximum changes in lift coefficient for all test cases from the simulations shown in Table 3.5 are compared to the correlation in Figure 3.20. The change in lift is expected to increase as the initial vertical distance between the airfoil and the core of the vortex decreases. During the simulations, some cases were found to reach convergence slower. The test cases which converged within the preset 20 maximum cycles per time step are shown to conform very well with the correlation. While the test cases which has an initial vertical distance within the range of -0.15 to -0.20 , except test case 4 with initial vertical distance of -0.175 , did not converge had shown poor agreement to the correlation. Since all test cases used the same grid and settings, hence, the difficulties in convergence for these cases are of the code. The difference between the converged simulation solutions and the correlation values varies between 2.5 to 15

percent. It is unfair to compare the cases that did not converge to the correlation, but the values were added to Figure 3.20 for completeness. Table 3.5 presents a listing of the normalized lift coefficient values for both the correlation and the simulation solutions. It was decided due to time constraints to keep all simulations in the dataset although it is expected that the results could be improved if all cases presented good convergence characteristics. Since the lift responses from the numerical simulations will serve as datasets or constraints to the Volterra identification process, the prediction by the Volterra theory is expected to have the same level of accuracy as the simulations.

Case #	$(y/c)_0$	$\Delta C_L U_\infty c / \Gamma$		% Difference	Convergence
		Correlation	CFD		
1	-0.190	1.231	1.606	23.329	NO
2	-0.080	2.425	2.475	2.014	YES
3	-0.310	1.446	1.257	15.022	YES
4	-0.175	1.458	1.673	12.860	YES
5	-0.335	1.101	1.209	8.969	YES
6	-0.275	1.450	1.335	8.601	YES
7	-0.164	1.254	1.729	27.434	NO
8	-0.145	2.468	1.838	34.244	NO
9	-0.170	1.263	1.698	25.599	NO

Table 3.5: Normalized Lift Coefficient Values Listing

By comparing test case 2 and 5, the simulated solutions for both of these test cases are seen to be in agreement with the correlation. Although the change in vertical distance, $(y/c)_0$, between these two test cases is only 0.255c (or 76 %), the resulting change in lift response is more than double (120.27 %). Based on these results it can be concluded that the lift response for a given vortex strength is highly sensitive to the

vertical distance between the core of the vortex to the airfoil. With the simulations completed, all lift responses were combined in matrix form and used as dataset in the subsequent identification of the Volterra kernels and the prediction processes.

On average, the INTEL PENTIUM IV 2.2 GHz single CPU took 3.86×10^{-4} sec/node per iteration. The version of the solver WIND used in the present simulations is relatively slow and the grid refinement at high gradient areas, necessary to capture the vortex, further downgraded WIND performance. In order to simulate the complete interaction process (*i.e.* a vortex travel from a distance $-2.5c$ ahead of the airfoil to a distance $2c$ aft the airfoil), approximately 10 to 14 days of computation were required. If every change of a parameter during the design process requires a different simulation, the time and computational resources required are expected to significantly impact the duration of the design process. This exercise demonstrates the improvements on the design process that the Volterra kernel approach can accomplish.

CFD is currently the most powerful method for predicting complicated unsteady aerodynamics. Even though CFD numerical simulations can accurately predict the BVI phenomenon and provide a number of details, due to the massive computational power and time required, the CFD approach is considered too expensive for industrial applications. Until there is a major improvement in CFD methods, a tool such as the Volterra kernel identification for blade-vortex interaction is expected to serve as an attractive intermediate step in the design of rotor blades.

4.0 Volterra Kernels Identification for Blade-Vortex Interaction (IDBVI)

Nonlinearities are present in most unsteady aerodynamic systems. When the airfoil and its surrounding flow field (the system) are subjected to the input vortex as shown in the previous section Figure 3.4 to 3.17, the system responses are seen to fluctuate in time as shown in Figure 3.19. As to the magnitude of the fluctuation, it is seen to be highly sensitive to any change in its vertical distance, $(y/c)_0$. The unsteady behaviour of the flow field can be formulated as an infinite sum of multidimensional convolution integrals. Silva demonstrated that the Volterra theory for nonlinear systems can be applied in the identification of the air loads of an airfoil in transonic flow undergoing a plunging motion [21]. The two-dimensional problem of a concentrated tip vortex interacting with an airfoil has been represented schematically in Figure 4.1. The airfoil and its surrounding flow field compose the nonlinear aerodynamic system or “plant” in the following developments.

Figure 4.1: Schematic of the Blade-Vortex Interaction

4.1 Methodology Overview

The approach of the developed methodology on predicting nonlinear aerodynamic response of the airfoil to a vortex from the simulated solutions is depicted in the flow chart shown in Figure 4.2. Beginning from the numerical solutions obtained from the simulations, the unsteady aerodynamic data provides information regarding the nonlinear unsteady aerodynamic response of the airfoil at the given freestream conditions due to the prescribed vortex. In order to predict these nonlinear aerodynamic responses, the method must first identify the nonlinear Volterra kernels. Based on the given vortex initial parameters, and the numerically simulated unsteady aerodynamic data, the kernel identification process described in the theory of Section 2.3 can be applied. The input to the system, $\mathbf{x}(\tau)$, in Equation (2.3.5) is the effective angle of attack of the vortex. Once the kernels have been extracted from the aerodynamic data, the prediction process can begin. Both the extraction process and the prediction process were implemented in Matlab©.

During the numerical simulations, the nine test cases were selected so that the dataset will cover a narrow range of initial vertical distances between the core of the vortex and the airfoil. For an actual prediction methodology that can be used in the design process, nine test cases are not enough to provide all the information required for the kernel extraction process. However, the objective of this study is to demonstrate that the nonlinear Volterra kernel identification is feasible for blade-vortex interactions. Therefore, with this selected narrow range of vertical distances, a minimum number of test cases could be used.

The purpose of this method is to provide a quick and accurate prediction of the effects of blade-vortex interactions to enhance the design turn-around time. In order to validate the method, new aerodynamic data (i.e. dataset that was not used in the identification process) will be used to verify the accuracy of the prediction by the extracted Volterra kernels.

Figure 4.2: Flow Chart for the BVI Prediction Method

4.2 Volterra Kernel Identification

4.2.1 Basis Functions

The basic form and the number of basis functions used in the Volterra kernel identification can only be determined by a trial and error exercise. After many trials, a second-order truncated Volterra series was found to be more suitable to model the current system. The identification process used below adopted the simultaneous extraction method of the first- and second-order Volterra kernels method developed by Reisenhel [16]. Following the work by Reisenhel, the first- and second-order Volterra kernel are assumed to take the following initial forms:

$$K_1(t) = \sum_{j=1}^N c_j e^{-t/\tau_j} \quad (4.2.1)$$

$$K_2(t_1, t_2) = \sum_{k=1}^N d_k \left[e^{(-t_1/\tau_{1,k} - t_2/\tau_{2,k})} + e^{(-t_2/\tau_{1,k} - t_1/\tau_{2,k})} \right] \quad (4.2.2)$$

where j and k are the number of basis functions used for the first- and second-order kernel respectively. $\{c_j\}$ and $\{d_k\}$ are the unknown coefficients vectors to be determined. In other words, the basis functions, $\zeta_j(t)$ and $\mu_k(t)$ are assumed to be an expansion of a series of exponentials with various time constants, τ , as follows:

$$\zeta_j(t) = e^{-t/\tau_j} \quad (4.2.3)$$

$$\mu_k(t_1, t_2) = e^{(-t_1/\tau_{1,k} - t_2/\tau_{2,k})} + e^{(-t_2/\tau_{1,k} - t_1/\tau_{2,k})} \quad (4.2.4)$$

The selection of the time constants is critical to the extraction process. The values of the time constants were chosen to cover a range of time scales. At the beginning of the kernel

extraction process, very little is known about the system. Therefore a range of time scales has to be tested in an algebraic sequence in order to determine the proper values. For the first-order kernel, the time constants, τ_i , were kept in a one-dimensional array. Because the total number of the first-order time constants compared to the total set of the time constants is relatively small, once selected these values were left unchanged while identifying the second-order kernel.

The impact of choosing the correct range of time scales for the higher order kernel time constants is very noticeable. This is because the basis functions are constructed based on the symmetric matrix which is formed by these time constants. By imposing symmetry on the time constants matrix, one also imposes symmetry on the kernel. A set of three time constants of the second-order kernel will result in a set of nine two-dimensional basis functions. The 3x3 symmetric matrix of time constants is defined as follows:

$$\tau_k = \begin{bmatrix} \tau_1 & \tau_2 & \tau_3 \\ \tau_2 & \tau_3 & \tau_1 \\ \tau_3 & \tau_1 & \tau_2 \end{bmatrix} \quad (4.2.5)$$

When choosing the time constants, the range of the time scales is controlled by the minimum and maximum values of the set, while the individual values provide control over the resolution or spectrum of the basis functions [16]. For example, for a 5x5 symmetric matrix of time constants with the following constants:

$$\begin{bmatrix} 0.25 & 0.5 & 1.5 & 3.0 & 5.0 \\ 0.5 & 1.5 & 3.0 & 5.0 & 0.25 \\ 1.5 & 3.0 & 5.0 & 0.25 & 0.5 \\ 3.0 & 5.0 & 0.25 & 0.5 & 1.5 \\ 5.0 & 0.25 & 0.5 & 1.5 & 3.0 \end{bmatrix} \quad (4.2.6)$$

Beginning from 0.25, the intervals between the current values to the next expected value in the matrix slowly coarsen. As a result, finer intervals are generated in the area of interest. The flooded contour plot of the spectrum represented by the above time constants is shown in Figure 4.3. The area that is represented by a certain (τ_1, τ_2) pair is shown in a different colour. In the Figure, the colour pattern is set for visualization purposes only.

Figure 4.3: Basis Function Spectrum of a 5x5 Set.

As mentioned before, choosing of basic form and the number of basis functions for the Volterra kernel identification is not straight forward. It is because even after determining the desired range of time scales, there are still many different possible combinations of time constants (τ_1, τ_2) pairing. Although the final results (i.e. the predicted aerodynamics response) shall be insensitive to the details of the chosen basis functions, having a *correct* basic form with the minimal number of terms as opposed to any arbitrary form with hundreds of terms will make a significant improvement on the computational resources required.

At first, when nothing was known about the present system, many series of positive time constants sets were used in the testing. The first-order kernel was chosen to have six terms and was kept constant throughout the exercise. The corresponding six time constants were 0.3, 0.2402, 0.1804, 0.0907, 0.0309, and 0.001. The approach taken was to have the positive constants spanning over a range of time scales and, because of the chosen exponential form for the basis functions, positive constants would provide the required stability to the Volterra kernel in consideration. The basis functions constructed using these time constants were integrated, and the unknown coefficients vectors were solved as described in the section 2.3. After conducting series of tests, it was found that the basis functions formed by these positive constants alone could not capture the rapid changes in the lift time history. In Figure 4.4, some of the tests results are shown with their corresponding time constants. The resulting lift time history was compared to the lift time history of Test Case 3. Even though variations can be seen as the time constants change, the resulting lift time history appeared to be locked within the range of 0.05 to -0.05.

Figure 4.4: Typical lift time history of the positive time constants tests.

Since the positive time constants proved that alone are not sufficient to model the highly nonlinear lift response of the airfoil, negative time constants were added and combined with the positive constants to account for the rapid grow characteristics of the lift time history. During these second tests, the six time constants for the first-order kernel were unchanged. Some of the second test results are shown in Figure 4.5 with their

corresponding constants. A comparison with the corresponding lift time histories of the Test Case 3 is also provided in the Figure.

Figure 4.5: Typical resulted lift time history of the negative time constants tests

As before, the range of time scales using the negative time constants was tested in an algebraic sequence, as shown in Figure 4.5 a and b. It was noted that only when the time constants are very small (i.e. of the order of one-thousandth), the resulted lift

responses started to capture the minima and maxima of the actual lift response. The negative constants of Figure 4.5c are -0.002 , -0.004 and -0.008 . Even with such small magnitudes, the resulted lift response was unable to capture the rapid fluctuation of the maxima and minima of the lift. Therefore, the values of the negative constants were once again reduced. In Figure 4.5d, the negative constants have their values reduced in the range between 0 and -0.003 . At the beginning of time history, the resulted lift curve was smooth and conformed well with the test case curve. However, the anticipated instability problem was encountered as the integration proceeded forward in time. It was found that the lift response of the airfoil due to the BVI is particularly not easy to model because the time between which the minima and the maxima occur is very short. The curves resemble two delta functions having opposite signs that are placed together. Based on this observation, the basic forms that were originally selected were modified. An extra term, which resembles the Gaussian function, was added to Equation (4.2.4), which was rewritten as follows:

$$\mu_k(t) = e^{(-t_1/\tau_{1,k} - t_2/\tau_{2,k})} + e^{(-t_2^2/\tau_{1,k}^2 - t_1^2/\tau_{2,k}^2)} + \frac{1}{\sqrt{\pi}} e^{(-t_2^2/\tau_{2,k}^2)} \quad (4.2.7)$$

It is worthwhile to point out that the form of the basis functions for the first-order Volterra kernel was unchanged in the latter approach.

In the Volterra kernel identification process, only Test Cases 3 to 9 were used for the basis functions testing. Test Case 1 and 2 were left out of this process so that they can be used as new data for later validation of the method. The training dataset formed by

these seven test cases covered a range of initial vertical distances, $(y/c)_0$, from $-0.145c$ to $-0.335c$.

The same set of time constants was used for the first-order kernel, where the values were 0.3, 0.2402, 0.1804, 0.0907, 0.0309, and 0.001. Immediate improvements were observed from the calculated lift responses obtained from the above modified basis function for the second-order kernel. The calculated lift responses for each of the test cases are shown in Figure 4.6a to 4.6g, and in comparison with their corresponding CFD solutions. The eight time constants used for the second-order kernel were 0.10, 0.01, 0.005, -0.0015, -0.0018, -0.0022, -0.0025 and -0.003, which are the same used for the solution shown in Figure 4.5d.

Comparing Figure 4.5d and Figure 4.6a, referred to Test Case 3 using the same set of time constants, it becomes clear the effect of the adopted modified second-order kernel. The instability problem in Figure 4.5d is no longer present in Figure 4.6a. With only eight time constants, even though the minima and the maxima of the lift responses were not perfectly captured, the kernels have shown to reproduce the general characteristics of the time-dependent unsteady responses.

Figure 4.6a: Calculated Lift Response in Comparison with CFD Solution for test case 3, $(y/c)_o = -0.310$

Figure 4.6b: Calculated Lift Response in Comparison with CFD Solution for test case 4, $(y/c)_o = -0.175$

Figure 4.6c: Calculated Lift Response in Comparison with CFD Solution for test case 5, $(y/c)_0 = -0.335$

Figure 4.6d: Calculated Lift Response in Comparison with CFD Solution for test case 6, $(y/c)_0 = -0.275$

Figure 4.6e: Calculated Lift Response in Comparison with CFD Solution for test case 7, $(y/c)_0 = -0.164$

Figure 4.6f: Calculated Lift Response in Comparison with CFD Solution for test case 8, $(y/c)_0 = -0.145$

Figure 4.6g: Calculated Lift Response in Comparison with CFD Solution for test case 9, $(y/c)_0 = -0.170$

4.2.2 Discussion

The present Volterra kernel identification methodology has demonstrated to be both powerful and simple. The kernel identification process is a technique, which progressively improves a simple model of a certain physical phenomenon [2]. Beginning with very little knowledge of the system and a few assumed initial forms of the first- and second-order Volterra kernels, one can by adjusting the total number of basis functions and their time constants to obtain significant improvements in the original model, as it was demonstrated in Figure 4.4 and Figure 4.5. Next, by simply modifying the basis function of the second-order kernel, based on a better understanding of the physical phenomenon, the calculated lift responses were able to reproduce most of the general characteristics of the system. This implies that the kernels formed by the selected time constants and the modified basic forms were able to capture the general characteristics of the unsteady nonlinear aerodynamics associated with BVI.

Based on the present investigations, it was possible to verify that the Volterra kernels can model the BVI phenomenon with accuracy. However, as mentioned before, the kernel identification is a progressive improvement process. In order to determine the kernels, it would require more computational resources, which is beyond the scope of this initial study. At the present stage, the technique has only demonstrated its feasibility to reproduce the blade-vortex interaction phenomenon. Next, the extracted kernels will be used in prediction process involving new inputs for the purpose of validation of the new methodology.

4.3 Prediction of Unsteady Aerodynamic System Response

Once the coefficient vectors $\{c_j\}$ and $\{d_k\}$ of Equation (2.3.7) have been determined and the kernels identified, the system response due to new inputs can be predicted. Two of the nine test cases were not used in the extraction process. They were purposely left out so that they can be used as new data to test the performance of the extracted kernels. The vortex initial positions of the two test cases are as follows:

Case #	x/c	$(y/c)_o$
1	-2.5348	-0.190
2	-2.5421	-0.080

Table 4.1: New Data Vortex Initial Positions.

The initial vertical position of Test Case 1 is well within the range covered by the training dataset, while the initial vertical position of test case 2 is outside of the same range. The effective angles of attacks for each test case were used as the system inputs. The lift responses predicted by the extracted kernels are shown in Figure 4.7 and Figure 4.8.

Figure 4.7: Predicted Lift Response in Comparison with CFD Solution for test case 1, $(y/c)_0 = -0.190$

Figure 4.8: Predicted Lift Response in Comparison with CFD Solution for test case 2, $(y/c)_0 = -0.080$

Test Case 1 can be regarded as the case with less nonlinearity from the two. The predicted lift response shown in Figure 4.7 compared extremely well with the numerical simulation solution. Even though the minima was slightly over predicted, the predicted maxima was in very good agreement with the numerical simulation solution using CFD. As for the more severe case of the two, Test Case 2, the initial vertical distance is outside of the range covered by the training dataset. Although the minima and maxima were not very well captured, the predicted lift response did show the general characterises of the simulation solution as shown in Figure 4.8. Based on the above comparisons, it can be concluded that the extracted Volterra kernels can be improved by using additional datasets but they prove that the proposed methodology for predicting BVI is consistent.

4.4 Modelling Time

The main purpose of this methodology is not to replace CFD simulations, but to provide means for the designers to get the specific information they need within the shortest time. Once all the unsteady aerodynamic data from the simulations had been stored into the dataset matrix, the Volterra kernel identification process can begin. On an INTEL PENTIUM III 935 MHz single CPU with 256 MB RAM, the time required to extract the kernels using the CFD simulations is about one to two days (depending on the number of basis functions used). The time required to predict the unsteady aerodynamic response from new data using the known kernels is about one to two hours, which also depends on the number of basis functions used. Since the extraction of the kernels only

needs to be performed once, the user will only need to wait for one to two hours to obtain the required information. Even though the proposed methodology does not provide as much information as a simulation would, it is very time efficient if the user is only seeking for a specific set of information such as the airfoil lift distribution.

5.0 Conclusions

5.1 Summary and Conclusions

The theory of Volterra integral equations for nonlinear systems was applied to model the nonlinear aerodynamic responses of an NACA 0012 airfoil in rotor blade-vortex interaction phenomena. The airfoil and its surrounding flow field composed the unsteady aerodynamic system. The prescribed vortex was considered as the system input and the nonlinear aerodynamic responses (i.e. the lift coefficient) of the airfoil were regarded as the system output. The theory relies on the identification of the Volterra kernels. Unsteady aerodynamic data from a Euler/Navier-Stokes code was used to build a training dataset. Since the objective of this study is to demonstrate that it is feasible to apply the theory of Volterra integral equations for nonlinear system to model blade-vortex interaction (BVI), assumptions were made to keep this pioneer study simple and doable within the available time and CFD resources.

The Euler equations were solved for the numerical simulations; i.e. the flow was assumed to be inviscid so the blade-vortex interaction could not induce flow separation. A total of nine test cases were simulated, and the prescribed clockwise rotating vortex in all test cases was initialized at a vertical distance below the airfoil, where its passage would not lead to a head-on collision with the airfoil. The non-dimensional circulation of the vortex was set to be 0.166 for all cases, and it was allowed to convect at the freestream velocity at Mach 0.3. A NACA 0012 airfoil with a unit chord length and 0° angle of attack was used to define the problem geometry. Therefore, any lift observed on the airfoil was a direct result of the BVI. As the vertical distance between the vortex and

the airfoil increases, the total change in lift was seen to decrease and vice versa. The unsteady lift responses of the airfoil were determined and the maximum change lift coefficient was compared to the correlation found in the literature. The lift responses from the converged test cases were shown to be in good agreement with the correlation. These CFD simulated solutions were then combined to form the unsteady aerodynamic dataset used in the Volterra kernel identification process.

After a long series of trial and error exercises, the type of the Volterra kernels for the present BVI system was identified. The classic form of the second-order kernel was modified with the addition of an extra term to better represent the impulsive characteristics of the system. Following the time-domain numerical technique developed by Reischel for the simultaneous extraction of first- and second-order kernels, a set of six time constants was used for the first-order kernel, while a set of eight time constants was used for the second-order kernel. The predicted lift responses due to new system inputs were shown to be in good agreement with the CFD solutions as shown in Figure 4.7 and Figure 4.8.

The computational time for obtaining a prediction using the nonlinear systems theory was approximately one to two hours. In comparison, ten to fourteen days were required for a new CFD simulation to produce the comparable lift responses, suggesting that the proposed method is very time efficient.

5.2 Suggestions for Future Work

The proposed methodology has shown promising results even at this preliminary stage of study. The extracted kernels are acceptable for this study but still can be improved. The most obvious way for improvement will be to continue the trial and error exercise for the first- and second-order kernels basis functions. The current first-order kernel uses the same set of time constants in all test cases. However, the total number of constants and their values can be changed. Also, more time constants can be added to the current set of constants for the second-order kernel. Different integration schemes should also be tested to see the effects on the program accuracy and computation time.

The above developed method is only a small demonstration of the capability of this powerful theory. To truly develop an engineering level blade-vortex interaction prediction tool, a much larger and more thorough set of training dataset will be required. If CFD computational resources permit, the parallel blade-vortex interaction could be simulated in two- or even three-dimensions with the full Navier-Stokes equations set. Therefore, the boundary layer, and the near wake effects could be properly incorporated into the identification of the kernel.

Reference

- [1] Bendat, J.S., *Nonlinear System Analysis and Identification From Random Data*, John Wiley & Sons, Inc. New York, 1990.
- [2] Burton, T.A., *Volterra Integral and Differential Equations*, Academic Press, Inc., New York, 1983.
- [3] Caradonna, F.X., *The Application of CFD to Rotary Wing Flow Problems*, AGARD -R-781, 1990.
- [4] Cochran, J.A., *The Analysis of Linear Integral Equations*, Bell Telephone Laboratories, Inc., New York, 1972.
- [5] Hardin, J.C., and Lamkin, S.L., *Concepts for Reduction of Blade/Vortex Interaction Noise*, J. Aircraft, Vol. 24, No. 2, 1987
- [6] Fortin, F., *An Overview of Computational Fluid Dynamics at IAR*, NRC-IAR, 2003
- [7] Johnson, W., *Helicopter Theory*, Dover Publications, Inc., New York, 1980.
- [8] Korber, S., and Ballmann, J., *Mechanisms and Acoustics of Blade-Vortex Interactions*, Springer-Verlag, 1995
- [9] Lee, S., and Bershader, D., *Head-On Parallel Blade-Vortex Interaction*, AIAA Journal, Vol. 32, No. 1, January 1994
- [10] Lent, H.M. et al., *Mechanisms of Transonic Blade-Vortex Interaction Noise*, Journal of Aircraft, Vol. 30, No. 1, January 1993
- [11] Martin, R. M. and Splettstoesser, W.R., *Blade-Vortex Interaction Acoustic Results from a 40% Model Rotor in the DNW*. Journal of the American Helicopter Society, Vol. 33, No. 1, Jan. 1988
- [12] McCluer, M. S., *Helicopter Blade-Vortex Interaction Noise with Comparisons to CFD Calculations*, NASA Technical Memorandum 110423, 1996
- [13] *Numerical Recipes In Fortran 77: The Art of Scientific Computing*, Cambridge University Press, England, 1992
- [14] O'Neill, C., *Helicopter Rotor Unsteady Aerodynamics*, Oklahoma State University, USA, 2002

- [15] Petrini, E., Efraimsson, G., and Nordstrom, J., *A Numerical Study of the Introduction and Propagation of a 2-D Vortex*, FFA, Sweden, 1998
- [16] Reisenhel, P.H., *A Nonlinear Volterra Kernel Identification System for Aeroelastic Applications*, Nielsen Engineering & Research, Inc., California, 1999.
- [17] Renzoni P., and Mayle R.E., *Incremental Force and Moment Coefficients for a Parallel Blade-Vortex Interaction*, AIAA Journal, Vol. 29, No. 1, 1991
- [18] Rugh, W.J., *Nonlinear System Theory: The Volterra / Wiener Approach*, Johns Hopkins Press Ltd., London, 1981.
- [19] Schetzen, M., *The Volterra and Wiener Theories of Nonlinear Systems*, John Wiley & Sons, Inc. New York, 1980.
- [20] Schmitz F.H., and Sim B.W.-C., *Acoustic Phasing, Directionality and Amplification Effects of Helicopter Blade-Vortex Interactions*, Journal of the American Helicopter Society, Vol. 46, Oct. 2001.
- [21] Silva, W.A., *Application of Nonlinear Systems Theory to Transonic Unsteady Aerodynamic Responses*, Journal of Aircraft, Vol. 30, No. 5, Sept.-Oct. 1993
- [22] Srinivasan G.R., McCroskey W.J., Baeder J.D., and Edwards T.A., *Numerical Simulation of Tip Vortices of Wings in Subsonic and Transonic Flows*, AIAA Journal, Vol. 26, No. 10 Oct. 1988.
- [23] Srinivasan, G.R. and Baeder, J.D., *TURNS: A Free-Wake Euler/Navier-Stokes Numerical Method for Helicopter Rotors*, AIAA Journal, Vol. 31, No. 5, May 1993
- [24] Srinivasan G.R., McCroskey W.J., and Baeder J.D., *Aerodynamics of Two-Dimensional Blade-Vortex Interaction*, AIAA Journal, Vol. 24, No. 10, October 1986
- [25] Straus, J., *Pressure and Velocity Measurements About An Airfoil During a Parallel Blade-Vortex Interaction* (Ph.D. Thesis), Rensselaer Polytechnic Institute, New York, 1991.
- [26] Tromp, J.C., and Jenkins, J.E., *A Volterra Kernel Identification Scheme Applied to Aerodynamic Reactions*, AIAA PAPER 90-2803, 1990
- [27] *WIND User's Guide*, NPARC Alliance, 2001
- [28] www.ae.gatech.edu/research/windtunnel/vortorgn/vortex.htm

Appendix: Computer Program Subroutines

```
% *****
% Program: TrainVK2D
% *****

% This program is the training module which will simultaneously
% extract both the first- and second-order kernels based on the technique
% developed by Reisenthel [16]. This program assumes both the already
% known system inputs and outputs are already present in the workspace.
%
%
% Parameters:
%
% a -- motion matrix
% j -- basis function index for the single integral
% k -- basis function index for the double integrals
% qj, qk -- time constants (pick so they sort of overlap)
% DT -- delta time step
% INPUTNUM -- the current input number.
% INPUTNUMmax -- the total number of dataset available.
% TOTALpts -- total number of data points
% Tcount -- time index
% Tmax -- maximum time
% time -- time array
%
%
% Program Inputs:
%       alphaRAD -- the effective angle of attack (in radians) due to the
%                   input vortex
%       y -- the known solution vector (matrix)
%
% Program Outputs:
%       C -- coefficient vector
%
% *****

% Physical timestep used in WIND was 0.0000025 for 60 Newton Time Level

DT = 0.0000025 * 60;
TOTALpts = 88
Tmax = DT * TOTALpts
```



```

time = [0.0:DT:Tmax]';

[m,n]=size(time);

% -----
%  TIME CONSTANTS SETTINGS
% -----

% ---
%  Linear Kernel prediction parameters
% ---

j = [1:1:6];
qj = [0.3 0.2402 0.1804 0.0907 0.0309 0.001];

% ---
%  Second order Kernel prediction parameters
%  k begins from 1 but continue after j in matrix a.
% ---

k = [1:1:8];
qk = [0.1 0.01 0.005 -0.0015 -0.0018 -0.0022 -0.0025 -0.003];

% ---
%  Parameter initialization
% ---

INPUTNUMmax = 7

a = zeros(m,max(j) + max(k)^2,INPUTNUMmax);
M = zeros (max(j) + max(k)^2,max(j) + max(k)^2);
B = zeros(max(j) + max(k)^2,1);
tempC = zeros(max(j) + max(k)^2, INPUTNUMmax);
sumC = zeros(max(j) + max(k)^2,1);
tempM = zeros(max(j) + max(k)^2,max(j) + max(k)^2,INPUTNUMmax);
tempB = zeros(max(j) + max(k)^2,1,INPUTNUMmax);

%  Note : x in radians

for INPUTNUM = 1:INPUTNUMmax

%  NOTE: time(1) == 0.0

```

```

% Filling up motion matrix a based on initial parameters
% Assume TWO terms Volterra series expansion
% NOTE: x is now alphaRAD from workspace
% -----
%     First term
% -----
for jcount = 1:max(j),
    qconst = qj(jcount)
    for Tcount = 2:m,
        timenow = time(Tcount);
        a(Tcount,jcount, INPUTNUM) = INTEGRALB(timenow, qconst,
        alphaRAD(Tcount,INPUTNUM));
    end
end

% -----
%     Second term
% -----
for kcount = 1:max(k),
    qconst1 = qk(kcount)
    for count = 1:max(k),
        qconst2 = qk(count)
        for Tcount = 2:m,
            timenow = time(Tcount);
            a(Tcount,max(j)+max(k)*(kcount-1)+count,INPUTNUM) =
            INTEGRALB22(timenow,qconst1,qconst2,alphaRAD(Tcount,
            INPUTNUM));
        end
    end
end

end

% *****
% --- END OF INPUTNUM FOR LOOP ---
% *****

% *****
%           --- SVD ---
% *****

% Note: output y should already be in workspace.

```

```

for xcount = 1:INPUTNUMmax
    tempM(:, :, xcount) = transpose(a(:, :, xcount))*a(:, :, xcount);
    tempB(:, :, xcount) = transpose(a(:, :, xcount))*y(:, :, xcount);
end

```

```

for i = 1:INPUTNUMmax
    Mx = pinv(tempM(:, :, i));
    Bx = tempB(:, :, i);
    tempC(:, i) = Mx*Bx;
    sumC = sumC + tempC(:, i);
end

```

```

C = sumC/7;

```

```

% *****
%      --- END OF TrainVK2D ---
% *****

```

```

% *****
% Function: INTEGRALB
% *****
%
% This function evaluates the single integral with the input
% basis function.
%
% Parameters:
%
% tol -- tolerance
%
% Function Inputs:
%     timenow -- current time value
%     qconst  -- current time constant value
%     x       -- current effective angle of attack
%
% Function Outputs:
%     ib -- solution of the integration
%
% *****

```

```

function ib = INTEGRALB(timenow, qconst, x)

```

```

    tol = 1e-9;

```

```

    ib = quadl(@BASIS, 0, timenow, tol, [], timenow, qconst, x);

% *****
% --- END OF FUNCTION INTEGRALB ---
% *****
% *****
% Function: BASIS
% *****
%
% This function contains the basis form of the first-order basis function.
%
% Parameters:
%
% Function Inputs:
%     timenow -- current time value
%     qconst  -- current time constant value
%     x       -- current effective angle of attack
%
% Function Outputs:
%     BAS -- basis form of the basis function
%
% *****

function BAS = BASIS(tow, timenow, qconst, x)

    BAS = (exp(-(timenow - tow)/qconst)).*x;

% *****
% --- END OF FUNCTION BASIS ---
% *****

% *****
% Function: INTEGRALB22
% *****
%
% This function evaluates a double integral with
% the current time index as the upper limit and zero as the lower limit
% for both integrals.
%
% Parameters:
%
%     tol -- tolerance
%
% Function Inputs:

```

```

%      timenow -- current time value
%      qconst1, qconst2 -- current time constants values
%      x      -- current effective angle of attack
%
% Function Outputs:
%      ib22 -- solution of the integration
%
% *****

function ib22 = INTEGRALB22(timenow, qconst1, qconst2, x)

tol = 1.e-6;

ib22 = dblquad(@BASIS22, 0, timenow, 0, timenow, tol, @quadl, timenow, qconst1,
qconst2, x);

% *****
% --- END OF FUNCTION INTEGRALB2 ---
% *****

% *****
% Function: BASIS22
% *****
%
% This function contains the basis form of the second-order basis function.
%
% Parameters:
%
%      tow1, tow2 -- integration dummy variables
%
% Function Inputs:
%      timenow -- current time value
%      qconst1, qconst2 -- current time constants values
%      x      -- current effective angle of attack
%
% Function Outputs:
%      BAS22 -- basis form of the basis function
%
% *****

function BAS22 = BASIS22(tow1, tow2, timenow, qconst1, qconst2, x)

if ((timenow-tow1) - (timenow-tow2)) > 0

```

```

        BAS22 = exp(-((timenow-tow1) - (timenow-tow2))/qconst1)*exp(-(timenow-
tow2)^2/qconst2^2)*x + exp(-(timenow-tow2)^2/qconst2^2)/(qconst2*sqrt(pi));
elseif ((timenow-tow2) - (timenow-tow1)) > 0
    BAS22 = exp(-((timenow-tow2) - (timenow-tow1))/qconst1).*exp(-(timenow-
tow1)/qconst2)*x + exp(-(timenow-tow2)^2/qconst2^2)/(qconst2*sqrt(pi));
else
    BAS22 = exp(-(timenow-tow1).^2/qconst1^2)/(qconst1*sqrt(pi)) + exp(-(timenow-
tow2)^2/qconst2^2)/(qconst2*sqrt(pi));

```

End

```

% *****
% --- END OF FUNCTION BASIS22 ---
% *****

```

```

% *****
% Program: Predict
% *****

```

```

% This program predict the system response based on the previously
% extracted kernels (which are formed by the selected basis form,
% the time constants and the now known coefficient vector).

```

```

%
% Parameters:
%
% newa -- motion matrix
% j -- basis function index for the single integral
% k -- basis function index for the double integrals
% qj, qk -- time constants (pick so they sort of overlap)
% newDT -- delta time step
% newINPUTNUM -- the current input number.
% newINPUTNUMmax -- the total number of dataset available.
% newTOTALpts -- total number of data points
% Tcount -- time index
% Tmax -- maximum time
% newtime -- time array

```

```

%
% Program Inputs:
% alphaR -- the effective angle of attack (in radians) due to the
% input vortex
% C -- previously determined coefficient vector

```

```

%
% Program Outputs:
%      predy --      the predicted system response vector
%
% *****

% For test case # 1
% Physical timestep used in WIND was 0.000005 for 60 Newton Time Level

newDT = 0.000005 * 60;
newTOTALpts = 49

newTmax = newDT * newTOTALpts
newtime = [0.0:newDT:newTmax]';
[newm,newn]=size(newtime);

% -----
% TIME CONSTANTS SETTINGS
% -----

% ---
% Linear Kernel prediction parameters
% ---
j = [1:1:6];
qj = [0.3 0.2402 0.1804 0.0907 0.0309 0.001];

% ---
% Second order Kernel prediction parameters
% k begins from 1 but continue after j in a matrix.
% ---

k = [1:1:8];
qk = [0.1 0.01 0.005 -0.0015 -0.0018 -0.0022 -0.0025 -0.003];

NEWMAX = 1;
newa = zeros(newm,max(j) + max(k)^2,1);

% Note : x in radians

for INPUTNUM = 1:NEWMAX

```

```

% NOTE: time(1) == 0.0

% Filling up motion matrix a based on initial parameters
% Assume TWO terms Volterra series expansion

% -----
%      First term
% -----
for jcount = 1:max(j),
    qconst = qj(jcount)
    for Tcount = 2:newm,
        timenow = time(Tcount);
        newa(Tcount,jcount, INPUTNUM) = INTEGRALB(timenow, qconst,
            alphaR(Tcount,INPUTNUM));
    end
end

% -----
%      Second term
% -----
for kcount = 1:max(k),
    qconst1 = qk(kcount)
    for count = 1:max(k),
        qconst2 = qk(count)
        for Tcount = 2:newm,
            timenow = newtime(Tcount);
            newa(Tcount,max(j)+max(k)*(kcount-1)+count,INPUTNUM) =
                INTEGRALB22(timenow, qconst1,qconst2,alphaR(Tcount,INPUTNUM));
        end
    end
end

end

predy = newa(:,1)*C;

% *****
% --- END OF PROGRAM PREDICT---
% *****

```