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"Linear Memoryless Multiuser Detection of Quasi-Synchronous CDMA"

submitted by **Frank M. van Heeswyk, B.A.Sc., M.A.Sc.**
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Chair, Department of Systems of Computer Engineering
Professor Samy A. Mahmoud

Thesis Co Supervisor
Professor David D. Falconer

Thesis Co Supervisor
Professor Asrar U.H. Sheikh

External Examiner
Professor Elvino Sousa
University of Toronto

Carleton University
September 1996

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Linear Memoryless Multiuser Detection of Quasi-Synchronous CDMA

by

Frank M. van Heeswyk, B.A.Sc., M.A.Sc.

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy**

**Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University
September, 1996**

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Abstract

To date, all spread spectrum code division multiple access (CDMA) schemes have been partitioned into two categories. Either synchronous CDMA, where all users are both chip and bit synchronized, or asynchronous CDMA, where all users are neither chip nor bit synchronized relative to each other. Between these two extremes lies quasi-synchronous CDMA (QS-CDMA) where users are not chip synchronized but are approximately bit synchronized. Quasi-synchronous CDMA arises in indoor radio channels where the combined propagation time and delay spread produces a round trip delay time that is limited to a few chips.

This work examines multiuser detection of QS-CDMA signals at a central base station. The primary goal is to find a low complexity multiuser detector that is near-far resistant. Complexity is an issue since QS-CDMA's primary application is in indoor systems where the quantity of base stations make cost a major consideration. Complexity is kept to a minimum by the use of linear multiuser detectors, by minimizing the amount of side information required, and by memoryless detection.

The detector is separated into three stages. The first stage filters and samples the received signal to produce a set of discrete time samples that are processed by the following stages. Stage two forms a linear combination of the samples according to a selected set of criteria to produce a soft decision estimate of the transmitted data. The criteria are zero-forcing (decorrelating), minimum mean squared error (MMSE), maximum signal to noise ratio (SNR), complexity, and side information dependence. These criteria are applied separately and in combinations to produce ten detectors. Stage three makes hard decisions upon the soft decision variable to form an estimate of the transmitted bits.

The normalized least mean squares (NLMS) algorithm is used to adapt the MMSE detectors to their stationarity point. The convergence rate of the NLMS

algorithm in a multiple user environment under various spreads of the users' powers is estimated and simulated. It is found that the convergence rate is a strong function of the users' power spread.

The asymptotic efficiency of the decorrelating detectors and the bit error rate of the MMSE detectors are used to quantify their performance. Their performance is evaluated for various spreading codes and the best spreading code set is identified.

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Glossary of Terms

AWGN	Additive White Gaussian Noise
BCA	Blank Chip Augmentation
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
CF	Chip Filtering
dB	decibels
DS-CDMA	Direct Sequence Code Division Multiple Access
FDMA	Frequency Division Multiple Access
IBI	Isolation Bit Insertion
ISMAI	Inter-Symbol Multiple Access Interference
LMS	Least Mean Square
MAI	Multiple Access Interference
MF	Matched Filtering
MSE	Mean Squared Error
MMSE	Minimum Mean Squared Error
NLMS	Normalized Least Mean Square
QS-CDMA	Quasi-Synchronous Code Division Multiple Access
SD	Steepest Descent
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access

List of Symbols

Integer Constants

- K Number of users.
- L Degree of oversampling of the CF Stage I.
- Q Degree of quasi-synchronous delay in chips.
- N Number of chips per symbol.

Matrices

- A Matrix containing the spreading codes.
- C Diagonal matrix of complex weights c_k .
- C_p Magnitude of the entries in the matrix C .
- C_ϕ Phase of the entries in the matrix C .
- H $H = AT$.
- I Identity matrix.
- R Correlation matrix of y for the MF Stage I.
- T Matrix of the user delays.
- V General linear combining matrix.
- W MMSE linear combining matrix.
- Ψ Decorrelating linear combining matrix.
- $\Re$ Correlation matrix.
- Υ Covariance matrix of the noise.

Vectors

- $\mathbf{a}$ Length N spreading code vector.
- $\mathbf{b}$ Vector transmitted bits.
- $\mathbf{w}$ Tap weight vector.
- $\mathbf{y}$ Vector of received samples produced by Stage I at the base station.
- $\mathbf{z}$ Vector of soft decision variables.

Signals and Waveforms

- $n(t)$ Gaussian distributed noise signal.
 $p(t)$ Chip waveform.
 $r(t)$ Composite received signal at the base station.
 $s(t)$ Spreading waveform.

Scalars

- c_k Complex weight of user k 's signal.
 J_k Mean squared error of user k .
 P_k BER of user k .
 τ_k Delay of user K .
 η Asymptotic efficiency.
 λ Eigenvalue.
 μ Adaptation algorithm step size.

Scalar Operators

- $\langle x \rangle$ Expected value of x .
 $\lfloor x \rfloor$ Largest integer less than x .
 $\lceil x \rceil$ Smallest integer greater than x .
 $|x|$ Magnitude of x .
 $\bar{x}$ Mean of x .
 $\text{sgn}(x)$ Sign of x . Equals +1 if $x > 0$, and -1 if $x \leq 0$.

Matrix Operators

- X^I Generic inverse of X .
 X^L Left inverse such that $X^L X = I$.
 X^{-1} Inverse of X such that $X^{-1} X = I$ and $XX^{-1} = I$.
 X^+ Moore-Penrose inverse of X .
 X^* Complex conjugate transpose.
 X_{kk} k th diagonal entry X .
 $\text{tr } X$ Trace of X .

I

Introduction

Frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA), are three techniques that are used to allow multiple users to access the wireless medium. Both FDMA and TDMA are orthogonal access techniques; the users occupy either a unique frequency slot or a unique time slot and they do not interfere with each other. CDMA, however, spreads all the users over the same time and frequency slots and all users interfere with each other. A unique code identifies each user and is known by the receiver, and the receiver uses this code to extract the desired user from the ensemble of users.

There are many possible realizations of the CDMA receiver. The conventional matched filter detector that is optimum in FDMA and TDMA systems can be used in a CDMA system, but it is no longer optimum because the undesired users' interference is typically non-Gaussian. This problem prompted research into optimum detectors for CDMA systems. Using the maximum likelihood criterion leads to a detector design that is very complex and requires intimate knowledge of all the signal and communications channel parameters [1, 2]. Detectors whose complexity and performance are between those of the conventional matched filter and the maximum likelihood detector have also been proposed [3, 4].

All of these detectors make one of two assumptions concerning the relative synchronization of the various users. They are either exactly synchronous or completely asynchronous. The area between these extremes, quasi-synchronous CDMA (QS-CDMA), has remained relatively unexamined. Gaudenzi *et al* [5] and DaSilva *et al* [6] examined conventional matched filter detection of QS-CDMA. No work on multiuser detection of QS-CDMA has been published to date.

Quasi-synchronous alignment of the users' signal can occur at a central receiver

or the base station in systems where the users are close to the base station receiver and synchronized to a master clock. The master clock could be derived by the users directly from the base station transmissions or from a pilot channel transmitted by the base station and monitored by all the users. For users sufficiently close to the base station, the round trip propagation time from the base station to the each user and back can be a small fraction of a symbol interval. In this work, this round trip time is assumed to be less than the time interval of a few chips of the CDMA spreading code.

The QS-CDMA assumption opens new opportunities in the design of receivers. Optimum and suboptimum detectors for synchronous CDMA are much less complex than their counterparts for asynchronous CDMA. This work was prompted by asking if the QS-CDMA assumption could be used to simplify the complexity of the receiver relative to the complexity of the receiver for asynchronous CDMA.

Receiver complexity is the sum of the complexity of the detection technique and the complexity of estimating the side information. Side information is that information which is required to detect the transmitted data, but does not contain information about the data. It may include the signal phase, amplitude, and timing. In synchronous CDMA, the users are known to be synchronized at the chip and bit level. In asynchronous CDMA, the users are neither chip nor symbol synchronized. In quasi-synchronous CDMA, the users are chip asynchronous but are symbol aligned to within a few chips. This quasi-alignment is the basis for proposing new receivers that do not require explicit knowledge of the users' timing. This removes the complex task of timing (or delay) estimation at the receiver, thus reducing complexity.

The proposed detectors are members of the linear near-far resistant class of detectors. Linear detectors were chosen because of their moderate complexity. Near-far resistance is desirable because it eliminates the need for power control.

Two types of linear detectors are considered. The decorrelating detector, which separates users into linearly independent spaces and is independent of the users' amplitudes, and the adaptive MMSE detector. It is shown that the adaptive MMSE detector essentially converges to the decorrelating detector for all practical

signal-to-noise ratio conditions. While the direct solution for the MMSE detector does require explicit knowledge of the users' amplitudes, the adaptive version eliminates this requirement.

The asymptotic efficiency and probability of bit error are used as the basis for comparison of the various proposed detectors. The average convergence time of the adaptive detectors under various system parameters is also calculated. It is found that transient behaviour is a sensitive function of the users' power.

The contributions of this work are:

- The introduction of linear multiuser detection of QS-CDMA.
- The proposal of a memoryless detection scheme for QS-CDMA that is based upon the modification of the users' spreading sequences.
- Compares the performance of M-sequences, Gold codes, and Hadamard codes when used as spreading codes for multiuser detected QS-CDMA.
- Proposes and analyzes a delay independent decorrelating detector for QS-CDMA. This is the first delay independent decorrelating detector to be introduced into the research on decorrelating detectors.
- Proposes and analyzes an adaptive linear detector that requires no explicit estimation of side information. Synchronization of the front-end filtering is not required.
- Quantifies the sensitivity of the NLMS algorithm convergence time to the power spread of the users.

II

Spread Spectrum Multiple Access

Spread spectrum communication systems have been studied for a number of decades to date. Early research concentrated on the use of spread spectrum in anti-jamming and low probability of intercept communications for military applications. Recent research has concentrated on spread spectrum as a wireless multiple access technique.

Within the multiple access domain, two types of spread spectrum have been considered; frequency hopping spread spectrum and direct sequence spread spectrum. Frequency hopping spread spectrum randomly hops the narrow band intelligence signal's center frequency in discrete steps over a broad bandwidth using a predetermined pattern. Direct sequence spread spectrum spreads the narrow band intelligence over a broad band using a wide-band code. Both schemes can support multiple users within the same broad bandwidth. This work examines the latter scheme which is commonly called direct sequence spread spectrum code division multiple access (DS-CDMA or just CDMA) when it is used to support multiple users.

Initial research on CDMA concentrated on the system performance as a function of the chosen spreading codes when using a matched filter detector at the receiver [7, 8, 9, 10]. Recent research has been investigating optimal and sub-optimal detection techniques that overcome some of the problems of the initially proposed schemes. Verdu, Varanasi and Aazhang, and Zvonar have been prominent contributors in this area [1, 11, 12]. The paper by Verdu provides an excellent overview of the state of the art.

This work concentrates on the detection of spread spectrum signals in a wireless multiple access environment. This chapter will review the relevant detection schemes in the literature and introduce quasi-synchronous CDMA.

2.1 CDMA Signal Detection Techniques

In a spread spectrum multiple access environment, the received signal is the sum of all the transmitted signals, each modified by the channel characteristics and summed with Gaussian distributed white noise. The lowpass equivalent of the received signal is $r(t) = s(t, \mathbf{b}) + n(t)$ where $n(t)$ is Gaussian noise and $s(t, \mathbf{b})$ is the composite received signal,

$$s(t, \mathbf{b}) = \sum_{i=-M}^M \sum_{k=1}^K c_k(i) b_k(i) s_k(t - iN - \tau_k). \quad (1)$$

Each user's transmitted power, channel attenuation, and phase are contained in the complex coefficient $c_k(i)$. User k 's transmitted symbol in time interval i is $b_k(i)$. All symbols are assumed to be bipolar (i.e., $b_k(i) \in [+1, -1]$). The signature waveform of user k , $s_k(t)$, is real, normalized to unit energy, and is zero outside the range $[0, N]$. The signature waveform provides the spectral spreading and contains the unique identity of each user. The absolute delay of user k relative to an arbitrary time reference is τ_k . The arbitrary time reference is often the base station timing in a cellular environment. The transmitted data has been grouped into a $2M+1$ length block to represent one frame. A unit time interval is one chip interval. A block diagram of the received signal from K users is shown in Figure 1.

The conventional detector for user k consists of a filter that is matched to $s_k(t)$, is phase aligned to $c_k(i)$, and is synchronized to τ_k . Hard decisions are made upon the output to yield an estimate, $\hat{b}_k(i)$, of the transmitted bit $b_k(i)$. The soft decision variable $z_k(i)$ is

$$z_k(i) = \int_{iN+\tau_k}^{(i+1)N+\tau_k} r(t) \frac{c_k^*(i)}{|c_k(i)|} s_k(t - iN - \tau_k) dt. \quad (2)$$

Expanding and separating out the desired user's term yields

$$\begin{aligned} z_k(i) = & |c_k(i)| b_k(i) + \sum_{j \neq i} \sum_{l \neq k} \frac{c_k^*(i)}{|c_k(i)|} c_l(j) b_l(j) \int s_k(t - iN - \tau_k) s_l(t - jN - \tau_l) dt \\ & + \frac{c_k^*(i)}{|c_k(i)|} \int n(t) s_k(t - iN - \tau_k) dt \end{aligned}$$

Figure 1: CDMA transmitter model.

which can be seen to be the sum of the desired bit weighted by the channel attenuation, a multiple access interference term dependent upon the other users' powers and signature waveform cross correlations, and coloured Gaussian noise.

The quality of the hard decision made upon $z_k(i)$ is largely dependent upon the relative size of the multiple access interference compared to the desired signal. This ratio is dependent upon the signature waveforms' cross correlations and the channel attenuation factors. As it is impossible in real systems to choose a set of spreading waveforms with zero cross correlation for all delays, it is impossible to reduce the multiple access interference term to zero. In fact, it is quite possible that the multiple access interference will at times exceed the desired signal power. Such a situation occurs when the desired user is far from the receiver and the interfering users are near. Hence, the well-known near-far problem of spread spectrum systems where the powerful near users obliterate the desired weak far user's signal.

Initial attempts to solve the near-far problem considered controlling the power of the users in a manner to equalize their powers at the receiver [13, 14, 15]. This is done in an open or closed loop control system where the central receiver relays the appropriate transmit power back to the mobile users. This is a simple concept in essence, but suffers from some intrinsic problems. Equalizing the power at the receiver forces the near users to reduce their power to the level of the far users. The bit error performance then becomes dictated by the performance of the worst case far user. In addition, it has been found that the accuracy of the power control must be within $\pm 1\text{dB}$ over a dynamic range of 80 dB to minimize capacity loss [15]. The difficulty in maintaining this accuracy is compounded by the round trip delay and the channel fading. Feedback power control schemes cannot react fast enough to track the rapid changes in the channel and fast fading.

The near-far problem was thought to be an intrinsic characteristic of CDMA and was used to substantiate the inferiority of CDMA compared to FDMA and TDMA. Verdu showed that this is not true. The near-far problem is created by the sub-optimal conventional detector. In a series of papers [16, 17, 18, 19], he developed the multiple user maximum likelihood detector, derived its performance, and proved that it does not suffer from the near-far problem.

The maximum likelihood detector selects a sequence $\hat{\mathbf{b}}$ such that

$$\Omega(\hat{\mathbf{b}}) = \int_{-\infty}^{\infty} |r(t) - s(t, \hat{\mathbf{b}})|^2 dt \quad (3)$$

is minimized. Expanding $s(t, \hat{\mathbf{b}})$ and ignoring the $\int |r(t)|^2 dt$ term since it is independent of the estimated bits $\hat{\mathbf{b}}$ yields

$$\begin{aligned} \Omega(\hat{\mathbf{b}}) = \sum_i \{ & \hat{\mathbf{b}}'(i)C(i)R(-1)C^*(i-1)\hat{\mathbf{b}}(i-1) \\ & + \hat{\mathbf{b}}'(i)C(i)R(0)C^*(i)\hat{\mathbf{b}}(i) \\ & + \hat{\mathbf{b}}'(i)C(i)R(1)C^*(i+1)\hat{\mathbf{b}}(i+1) \\ & - 2\hat{\mathbf{b}}'(i)\text{Re}(C^*(i)y(i)) \} \end{aligned} \quad (4)$$

where

$$\begin{aligned}\hat{\mathbf{b}}(i) &= [\hat{b}_1(i) \hat{b}_2(i) \cdots \hat{b}_K(i)]' \\ C(i) &= \text{diag}[c_1(i) \ c_2(i) \ \cdots \ c_K(i)] \\ \mathbf{y}(i) &= [y_1(i) \ y_2(i) \ \cdots \ y_K(i)]' \\ y_k(i) &= \int r(t) s_k(t - iN - \tau_k) dt \\ R_{kl}(j) &= \int s_k(t - \tau_k) s_l(t - jN - \tau_l) dt.\end{aligned}$$

This equation is very similar to that derived by Zvonar and Brady in [20]. The vector $\mathbf{y}(i)$ is the output from a bank of K filters matched and synchronized to the signature waveforms of the K users. Its entries differ slightly from the conventional detector's $z_k(i)$ in equation (2) in that they are not weighted by the complex conjugate of the channel phase. Vector $\mathbf{y}(i)$ forms a set of sufficient statistics for the detection of the transmitted bit vector $\mathbf{b}$ [16]. Verdu has shown that this receiver may be implemented with a dynamic programming algorithm of the Viterbi type which has a complexity per data bit that is exponential in the number of users.

Expanding the equation for $\mathbf{y}(i)$ yields

$$\begin{aligned}\mathbf{y}(i) &= R(-1)C(i-1)\mathbf{b}(i-1) \\ &\quad + R(0)C(i)\mathbf{b}(i) \\ &\quad + R(1)C(i+1)\mathbf{b}(i+1) \\ &\quad + \mathbf{n}(i)\end{aligned}\tag{5}$$

where $\mathbf{n}(i)$ is a vector of coloured Gaussian noise samples. This equation can be expressed in a more compact form by using the following definitions [2]:

$$\begin{aligned}C &= \text{diag}[c_1(-M) \cdots c_K(-M) \cdots c_1(M) \cdots c_K(M)] \\ \mathbf{y} &= [y_1(-M) \cdots y_K(-M) \cdots y_1(M) \cdots y_K(M)]' \\ \mathbf{b} &= [b_1(-M) \cdots b_K(-M) \cdots b_1(M) \cdots b_K(M)]'\end{aligned}$$

The decorrelating detector is the least complex multiple user linear detector for CDMA. Other linear detectors can be generated by changing the criteria for optimization. The decorrelating detector used the zero forcing criteria. The other common approach is to minimize the mean squared error between the soft decision variable vector, $\mathbf{z}$, and the transmitted bit vector $\mathbf{b}$. This was examined by Xie, Short, and Rushforth [21]. With $\mathbf{z} = \mathbf{V}\mathbf{y}$, they minimized $\langle (\mathbf{b} - \mathbf{z})(\mathbf{b} - \mathbf{z})^T \rangle$, and found the optimum matrix to be

$$\mathbf{V} = (\mathbf{H}\mathbf{C} + N_0/2\mathbf{I})^{-1} \quad (8)$$

where $N_0/2$ is the power spectral density of the additive white Gaussian noise at the receiver input. In the limit as the noise vanishes, this linear detector approaches the decorrelating detector described above.

Unlike the decorrelating detector, the minimum mean squared error (MMSE) linear detector requires knowledge of the users' powers and the noise power spectral density. This makes the MMSE receiver more complex than the decorrelating receiver since these parameters must be estimated.

2.2 Side Information

Side information is information that is required for the detection of the data bits, but is not directly related to those bits. For the system in question this encompasses the user spreading codes, user delays, transmitted powers, channel attenuation, and channel phase. The three latter parameters are lumped into the complex channel coefficient $c_k(i)$.

The maximum likelihood detector requires all of this side information for detection. This increases the receiver complexity because these parameters must be estimated at the receiver before detection can proceed. The linear decorrelating detector eliminates the need for an estimate of the received power.

Differential detection can be used to circumvent the requirement of the knowledge of the phase of $c_k(i)$. This is particularly useful in fast fading channels where it is difficult to track the phase. Varanasi and Aazhang [22, 23] derived a class of differentially coherent multiple user CDMA receivers and showed that these

receivers are near-far resistant. Differential detection can be applied to maximum likelihood, decorrelating, and linear MMSE detectors by simply differentially encoding the data bits at all transmitters and differentially decoding the soft decision variable of each user at the base station. This results in a slight bit error rate performance degradation compared to coherent detection schemes.

All multiple user detection schemes considered to this day require knowledge of the user delay, τ_k . The set of user delays $\{\tau_k, k = 1, 2, \dots, K\}$ are used to synchronize the bank of matched filters at the receiver front-end and to calculate the soft decision variables. For the maximum likelihood detector, the delays are required to calculate the metric to be minimized. For the decorrelating and linear MMSE detectors, the delays are required to calculate the correlation matrix.

2.3 Performance Measures

The bit error rate of user k , F_k , is typically used to characterize the performance of a detection scheme. For CDMA, the bit error rate is a strong function of the desired user's signal-to-noise ratio (SNR) and the power of the interfering users. In a heavily loaded CDMA system, the total interference from the other users is typically much larger than the background Gaussian noise. By considering the performance as the background noise vanishes, the detection scheme can be evaluated on the basis of its ability to withstand interference, or the near-far problem.

Verdu proposed the asymptotic efficiency as a measure to characterize performance as the background noise vanishes [16]. It is

$$\eta_k = \sup \left\{ 0 \leq \alpha \leq 1; \lim_{\sigma \rightarrow 0} \frac{P_k(\sigma)}{Q(\sqrt{\alpha}|c_k|/\sigma)} < +\infty \right\} \quad (9)$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

and $P_k(\sigma)$ is the error rate of user k for the detection scheme being tested. The asymptotic efficiency is equivalent to the ratio of the effective energy, $\epsilon_k(\sigma)$, required to achieve the bit error rate $P_k(\sigma)$ in a single user system to the actual

energy required to achieve that bit error rate in the tested detection scheme as the background noise approaches zero. Thus

$$\eta_k = \lim_{\sigma \rightarrow 0} \frac{\epsilon_k(\sigma)}{|c_k|^2}. \quad (10)$$

Since the power required in the single user system is always less than or equal to the power required in the multiple user system, the asymptotic efficiency is bounded by zero and unity. Even though the asymptotic efficiency is calculated as the background noise approaches zero, it remains very close to the efficiency for most SNR's used in CDMA applications [1]. The asymptotic efficiency was proposed as a performance measure because it is easier to calculate than the bit error rate.

The asymptotic efficiency is a function of the power of the interfering users. Lupas and Verdu defined the near-far resistance as the minimum asymptotic efficiency over all possible powers of the interfering users [4]

$$\tilde{\eta}_k = \inf_{|c_l| > 0, l \neq k} \eta_k. \quad (11)$$

It represents the worst case performance of the CDMA system in the presence of other users.

The near-far resistance of the conventional matched filter detector is zero [1]. There exists at least one set of interfering users' powers that will not allow error free detection of the desired user. Lupas and Verdu have shown that the near-far resistance of both the maximum likelihood and the linear decorrelating detector is not zero provided the users have linearly independent signals [2]. These detectors are called near-far resistant because they allow error free detection in a high SNR environment independent of the power of the interfering signals.

2.4 Synchronization

The multiple user detection schemes considered to date make one of two possible assumptions about the relative synchronization of the users. They are either all exactly synchronized or they are completely asynchronous. The synchronous case is unrealistic in practical systems and is mainly used to highlight some of

the characteristics of multiple user detection schemes in a simplified operating environment. The asynchronous case covers all real systems and requires more complex detectors than the synchronous case. The asynchronous assumption may be too broad in some circumstances. The user's signals may be approximately symbol aligned while still being completely asynchronous at the chip level. The phrase quasi-synchronous CDMA (QS-CDMA) was introduced to cover this case. Quasi-synchronous CDMA lies between synchronous and asynchronous CDMA and it can be applied to some microcellular systems. The analysis and application of QS-CDMA are the primary motivators of this work.

Techniques for synchronizing the base station receiver to the phasing and timing of the users are not being considered in this work. Where necessary, this synchronization is assumed.

III

Quasi-Synchronous CDMA

The term quasi-synchronous CDMA is used to describe the timing relationship between the various users' signals arriving at a central receiver (base station). Figure 2 illustrates the synchronization of a given bit of three users, each with a spreading gain of five, for three degrees of synchronization as seen at the receiver. The left case illustrates synchronous CDMA where all users are both chip and bit synchronized. The center case illustrates asynchronous CDMA where neither chip nor bit synchronization exists. The right case illustrates single chip ($Q=1$) QS-CDMA where the users are bit synchronized to within a single chip.

Figure 2: Synchronous, asynchronous, and quasi-synchronous CDMA timing relationships.

This interpretation of QS-CDMA is slightly different than that defined by Gaudenzi *et al* [5]. Their system is slightly quasi-synchronous because of jitter between the transmitter clock and the receiver clock. The clocks are synchronized on average and the timing jitter is restricted to a few tenths of a chip. The definition of QS-CDMA used in this work is akin to that of DaSilva *et al* [6]. It includes the clock jitter case but is also broad enough to cover multiple chip

misalignments. It is primarily intended to model systems that are naturally quasi-synchronous due to their structure, and not solely due to their timing jitter.

3.1 QS-CDMA Application

In a small cellular system where the users are close to the base station and slaved to the base station transmissions, the reverse link is naturally quasi-synchronous as seen by the base station receiver. The propagation delay from the base station to the near user and back (round trip delay) is approximately zero while the far user's round trip delay is $2T_d$, where T_d is the one way propagation delay to the cell boundary as depicted in Figure 3. This system can be made quasi-synchronous by restricting the round trip delay to a few chips. Single chip quasi-synchronous CDMA sets $2T_d = T_c$, the chip interval. Multiple chip QS-CDMA sets the round trip delay equal to multiples of a chip interval.

The quasi-synchronous assumption applies only to users within the cell of interest. Users from other cells will not be quasi-synchronous since they are not synchronized to the base station in question and their delays are larger than T_d . In multiple user detection schemes, the users within a given cell are jointly detected, and those outside the cell are considered to be interference. Multiple user detection solves the intracell interference problem, but usually ignores the intercell interference.

Restricting the round trip delay to Q chips sets the maximum cell diameter to

$$D = 300(2T_d) = 300 Q T_c \text{ meters} \quad (12)$$

where T_c is in microseconds. A T_c equal to one microsecond corresponds to a maximum cell diameter of 300 meters for single chip QS-CDMA. This size encompasses most indoor wireless systems and some outdoor microcell systems.

If, in addition to the propagation time, the delay spread is taken into account, the cell diameter must be reduced to account for the arrival of multipath components. Assuming the far user locks onto a delayed multipath component from the base station and the base station locks onto a delayed multipath component returned by the user, the round trip delay time is increased by $2\tau_{rms}$ on average,

Figure 3: Near and far users in a single cell.

where τ_{rms} is the root mean square (rms) delay spread in microseconds. The excess delay reduces the diameter of the cell by $300(2\tau_{rms})$. The rms delay spread is dependent upon the type of indoor channel considered. It can range from approximately 10 nanoseconds in small single rooms to as much as 300 nanoseconds in large offices and factories [24]. Using a typical value of 150 nanoseconds results in a cell size reduction of 90 meters, leaving a cell diameter on the order of 200 meters for single chip QS-CDMA. This still encompasses many microcell applications.

In addition to round trip propagation delay, each user's transceiver processing delay must be specified to maintain quasi-synchronous alignment at the base station. Processing delay is the time required for a user's transceiver to process the received signal and produce a transmitted response. It is added to the propagation delay to get the total round trip delay seen by the base station. By specifying all transceivers to have the same processing delay, all signals received at the base station will arrive within Q chips of each other as determined by the maximum cell diameter. The processing delay has to be specified to allow interoperability of transceivers from different manufacturers.

In the remaining text, Q chip QS-CDMA is defined to encompass systems in which the users' bits are time aligned to within Q chips as seen by the base station. It is assumed that the users' transceiver turn around delays are all equal and equal

to zero. This assumption is not restrictive provided that the turn around delay is known a priori at the base station in a real system. Given these assumptions, and remembering that the user's transceiver bit timing is slaved to the base station bit timing, the maximum cell diameter is given by equation (12).

3.2 Why Use QS-CDMA?

While the above definition of QS-CDMA naturally fits between synchronous and asynchronous CDMA, and the microcell application is realistic, the most important question has yet to be addressed. Why consider QS-CDMA ?

The primary motivation for using QS-CDMA is to reduce the complexity of the base station design by exploiting the quasi-synchronous nature of the users.

In large cellular systems, the base stations are generally large, expensive, and support a large number of users from a single location. In microcellular systems, however, many small and inexpensive base stations are deployed in small cells, each supporting far fewer users than the base station in a large cellular system. A microcellular base station must be simple to keep the system costs low.

The primary goal of this thesis is to propose and analyze low complexity near-far resistant base station receiver designs for QS-CDMA. Near-far resistance will be achieved through the use of multiuser detectors.

3.3 A QS-CDMA System Design

The complexity of the base station receiver is determined by the type of detection scheme used and the required side information. The conventional matched filter detector has relatively low complexity, but it is not near-far resistant. The maximum likelihood detector is near-far resistant, but it is the most complex receiver and requires the most side information. Linear detection schemes will be examined since they lie in the middle complexity ground. They are near-far resistant, and they do not necessarily require as much side information as the maximum likelihood detector.

Detector complexity is minimized in a memoryless system. When a system has memory, the received signal must be observed over multiple symbols before a good

decision can be made. This results in a processing delay and requires memory at the receiver. Memoryless detection keeps the processing delay to a minimum and does not require extra memory.

3.3.1 Truncating Memory

In a real system, the users' bits do not arrive in isolation as depicted in Figure 2. Instead, they arrive strung together as illustrated in Figure 4. Each numbered block in this figure is a transmitted bit and each bit consists of N chips. The chips are not shown. Synchronizing the receiver to bit 0 of user k will introduce interference from bits -1, 0, and 1 of all the other users. The interference from bit 0 of the other users is called multiple access interference (MAI), and the interference from the adjacent bits of the other users is called intersymbol multiple access interference (ISMAI). The ISMAI introduces memory into the received signal.

Detection in the presence of memory can be accomplished by processing the data in blocks of $2M+1$ bits (from bit $-M$ to bit M). There are edge effects due to ISMAI from the bits just outside of the block, but this is minimal for long blocks. Zheng and Barton [25] proposed a scheme called isolation bit insertion to eliminate these edge effects. Isolation bit insertion (IBI) inserts a zero bit following the last bit in the block at the users' transmitters as illustrated in Figure 5. With IBI, the block length can be reduced without edge effects dominating the interference. This allows a reduction in detector complexity since blocks can be made as short as one bit. At a block length of one bit however, the capacity is cut by 50% since every second bit is zero. This is undesirable in most systems and unnecessary in a QS-CDMA system.

A QS-CDMA system suffers from less ISMAI than an asynchronous CDMA system since the bits overlap by only a few chips. Kajiwara [26] proposed the first memoryless bit-by-bit receiver to take advantage of this limited overlap. The receiver observes only that portion of bit 0 that contains no ISMAI, as indicated in Figure 6. This interval spans the time from the beginning of the bit from the user with the longest relative delay to the end of the bit from the user with the least relative delay, thus discarding a portion of each user's transmitted power.

Instead of changing the receiver, a scheme similar to IBI can be used at the chip level to remove ISMAI in a QS-CDMA system. This is accomplished by setting the last Q chips of Q chip QS-CDMA to zero at the users' transmitters, as illustrated in Figure 7. The observation interval for the desired bit spans both the zero and non-zero chips of the user with the least relative delay. Unlike Kajiwara's scheme, this scheme does not discard a portion of the user's transmitted power and it removes memory at the users' transmitters, not at the base station. This scheme will be called blank chip augmentation (BCA) [27, 28] since it augments the spreading code with Q blank (zero) chips. BCA is a logical extension of IBI to QS-CDMA, and it eliminates the detection delay inherent in IBI [29].

Figure 4: Multiple users' bit streams in asynchronous CDMA.

Figure 5: Isolation bit insertion (IBI) in asynchronous CDMA.

Figure 6: Multiple users' bit streams in QS-CDMA.

Figure 7: Blank chip augmentation (BCA) in QS-CDMA.

3.3.2 Proposed Receiver Structure

The proposed receiver has three major building blocks as depicted in Figure 8. Stage I filters and samples the continuous time received signal to produce a discrete time vector y . Stage II linearly combines the elements in y to produce a soft decision variable vector z that has K entries, one for each user. The weights of the linear combiner in Stage II are stored in matrix form. Stage III makes hard decisions on the vector z to yield an estimated bit vector $\hat{b}$ that contains K bits. Two Stage I processors and multiple Stage II processors will be considered. The Stage II matrices are partitioned into two families; decorrelating matrices and MMSE matrices. This receiver processes the signal on a bit-by-bit basis and it relies upon BCA to ensure memoryless detection.

Figure 8: Block diagram of the proposed QS-CDMA receiver.

3.3.3 Thesis Format

The balance of this thesis derives and analyzes several combinations of Stage I and Stage II. Two versions of Stage I are proposed. The first is the standard matched filter bank, and the second is a single oversampled chip filter. The second version has the advantage of not requiring the users' timing. Several implementations of the decorrelating and MMSE matrices in Stage II are proposed. Most of the decorrelating matrix implementations require explicit side information estimation, and the MMSE matrices are adapted to the side information using the NLMS algorithm. The performance of the various receiver combinations is evaluated for QS-CDMA systems with a moderate spreading gain (around 32), and with three spreading code sets.

IV

Stage I: Filtering

In this section, two Stage I's are proposed and expressions for the vector $\mathbf{y}$ in terms of the spreading codes, signal powers, phases, and delays are derived. It is also shown how BCA allows memoryless bit-by-bit detection at the base station.

The received signal at the base station is the sum of all the active users transmitted signals modified by the channel and summed with Gaussian distributed white noise. It is written as $r(t) = s(t, \mathbf{b}) + n(t)$ where $n(t)$ is Gaussian white noise and $s(t, \mathbf{b})$ is the composite received signal given by

$$s(t, \mathbf{b}) = \sum_{i=-M}^M \sum_{k=1}^K c_k(i) b_k(i) s_k(t - iN - \tau_k).$$

The parameters in this equation were defined in Chapter II. The first Stage I considered consists of a bank of K matched filters where filter k is matched and synchronized to the spreading waveform of user k . This Stage I is called a waveform matched filter bank or matched filtering (MF). The second Stage I consists of a single integrate-and-dump filter with integration time equal to $1/L$ where L is the oversampling factor. The filter is synchronized to the base station transmissions and is followed by a NL tap delay line. When $L = 1$ and rectangular chip waveforms are used, the integrate-and-dump filter is equivalent to a chip matched filter. This scheme is called chip matched filtering or chip filtering (CF), even if $L > 1$.

4.1 Waveform Matched Filter Bank

This detector first processes the received signal with a bank of K filters where filter k is matched and synchronized to the spreading waveform of user k . The

sampled output of the k th filter in bit interval i is

$$\begin{aligned} y_k(i) &= \int_{-\infty}^{\infty} r(t) s_k(t - iN - \tau_k) dt \\ &= \sum_{j=-M}^M \sum_{l=1}^K c_l(j) b_l(j) R_{kl}(j - i) + n_k \quad k = 1, \dots, K \end{aligned} \quad (13)$$

where

$$R_{kl}(m) = \int_0^N s_k(t) s_l(t - mN - \tau_{kl}) dt \quad m \in [-1, 0, 1] \quad (14)$$

and

$$n_k = \int_0^N n(t) s_k(t) dt. \quad (15)$$

There are three signature waveform correlation matrices, $R(m)$, $m \in [-1, 0, 1]$. The relative delay of user l with respect to user k is $\tau_{kl} = \tau_l - \tau_k$, and n_k is the Gaussian noise sample from filter k . A block diagram of the matched filter bank Stage I is illustrated in Figure 9.

Figure 9: Waveform matched filter bank Stage I block diagram.

Since $s_k(t)$ is strictly time limited to $[0, N]$, $R_{kl}(m) = 0$ for $|m| > 1$. Thus

$$\begin{aligned} y_k(i) &= \sum_l c_l(i) b_l(i) R_{kl}(0) \\ &+ \sum_l c_l(i-1) b_l(i-1) R_{kl}(-1) \\ &+ \sum_l c_l(i+1) b_l(i+1) R_{kl}(1) + n_k \end{aligned} \quad (16)$$

The sampled matched filter outputs $\{y_k(i) \forall k, i\}$ form a set of sufficient statistics for the detection of all transmitted bits. The first term is due to the desired transmitted bit vector, and the next two terms represent intersymbol multiple access interference (ISMAI) from the adjacent transmitted bits of other users.

In a CDMA system, the signature waveforms are given by

$$s_k(t) = \frac{1}{N} \sum_{i=0}^{N-1} a_k(i) p(t-i) \quad (17)$$

where the chip waveform, $p(t)$, is zero outside the range $[0, 1]$ and $a_k(i)$ is the i th chip of the k th user's spreading code. Substituting this equation into equation (14) for R_{kl} yields

$$\begin{aligned} R_{kl}(j) &= \frac{1}{N^2} \sum_{u,v=0}^{N-1} a_k(u) a_l(v) \int_0^1 p(t) p(t+u-v-Nj-\tau_{kl}) dt \\ &= \frac{1}{N^2} \sum_{i=0}^{N-1} a_k(i) a_l(i-Nj-\lfloor \tau_{kl} \rfloor) \int_0^1 p(t) p(t-\tau_{kl} + \lfloor \tau_{kl} \rfloor) dt \\ &\quad + \frac{1}{N^2} \sum_{i=0}^{N-1} a_k(i) a_l(i-Nj-\lfloor \tau_{kl} \rfloor-1) \int_0^1 p(t) p(t-\tau_{kl} + \lfloor \tau_{kl} \rfloor + 1) dt. \end{aligned} \quad (18)$$

Each term in this equation is the product of two components. The first is the spreading code correlation and the second is the chip waveform correlation. The function $\lfloor x \rfloor$ selects the largest integer less than or equal to x .

The single chip ($Q = 1$) quasi-synchronous delay criterion is equivalent to restricting $|\tau_{kl}| < 1$. Upon applying this assumption, the various terms in the

summations of equation (19) become

$$\begin{array}{rcl}
 j = -1 & j = 0 & j = 1 \\
 \sum a_k(i)a_l(i-Nj-1) = & a_k(0)a_l(N-1) & \sum a_k(i)a_l(i-1) & 0 \\
 \sum a_k(i)a_l(i-Nj) = & 0 & \sum a_k(i)a_l(i) & 0 \\
 \sum a_k(i)a_l(i-Nj+1) = & 0 & \sum a_k(i)a_l(i+1) & a_k(N-1)a_l(0).
 \end{array} \quad (19)$$

The two chip ($Q=2$) quasi-synchronous delay criterion is equivalent to restricting $|\tau_{kl}| < 2$. For this case, the various terms in the summations are

$$\begin{array}{rcl}
 j = -1 & j = 0 & j = 1 \\
 \sum a_k(i)a_l(i-Nj-2) = & a_k(0)a_l(N-2) & \sum a_k(i)a_l(i-2) & 0 \\
 & +a_k(1)a_l(N-1) & & \\
 \sum a_k(i)a_l(i-Nj-1) = & a_k(0)a_l(N-1) & \sum a_k(i)a_l(i-1) & 0 \\
 \sum a_k(i)a_l(i-Nj) = & 0 & \sum a_k(i)a_l(i) & 0 \\
 \sum a_k(i)a_l(i-Nj+1) = & 0 & \sum a_k(i)a_l(i+1) & a_k(N-1)a_l(0) \\
 \sum a_k(i)a_l(i-Nj+2) = & 0 & \sum a_k(i)a_l(i+2) & a_k(N-2)a_l(0) \\
 & & & +a_k(N-1)a_l(1).
 \end{array} \quad (20)$$

Similar equations can be listed for $Q > 2$ chip quasi-synchronous systems.

From these expansions it can be seen that for single chip QS-CDMA the ISMAI is introduced by the correlation of the first and last chips of the spreading sequence. Similarly, for two chip QS-CDMA, the ISMAI is introduced by the first two and last two chips of the spreading sequences. In general, for Q chip QS-CDMA, the ISMAI is introduced by the correlation of the first Q and last Q chips of the spreading sequences. Thus, the ISMAI can be removed by simply setting the last Q chips in Q chip QS-CDMA to zero, as was proposed in the previous chapter with the introduction of BCA.

By setting the last Q chips to zero, the signature waveform correlation matrices $R(\pm 1)$ vanish, and the output of the k th matched filter is

$$y_k(i) = \sum_l c_l(i)b_l(i)R_{kl}(0) + n_k \quad (21)$$

or, in matrix form,

$$\mathbf{y} = R\mathbf{C}\mathbf{b} + \mathbf{n} \quad (22)$$

where

$$R = \{R_{kl}(0) \forall k, l\}$$

$$C = \text{diag}[c_1 \ c_2 \ \cdots \ c_K]$$

$$\mathbf{b} = [b_1 \ b_2 \ \cdots \ b_K]'$$

$$\mathbf{n} = [n_1 \ n_2 \ \cdots \ n_K]'$$

and the reference to a specific bit i has been dropped since these equations apply to any bit.

The notable characteristics of the MF Stage I are that it requires knowledge of the users' spreading codes and delays; it uses multiple matched filters; and that the correlation matrix R is a complicated function of both the users' spreading codes and delays. These characteristics make the MF Stage I more complex than the CF Stage I described in the next section.

4.2 Chip Filter

The MF Stage I requires K matched filters, each synchronized to the respective user. The chip filtering (CF) Stage I proposed in this section does not require multiple filters or synchronization to the individual users.

The composite received signal at the base station is the sum of all the individual user transmissions and can be written as $r(t) = s(t, \mathbf{d}) + n(t)$ where $n(t)$ is Gaussian white noise and $s(t, \mathbf{d})$ is the composite received signal.

$$s(t, \mathbf{d}) = \sum_{i=-N(M+1/2)}^{N(M+1/2)} \sum_{k=1}^K d_k(i) p(t - \tau_k - i) \quad (23)$$

where

$$d_k(i) = a_k(i - N\lfloor i/N \rfloor) b_k(\lfloor i/N \rfloor) c_k(\lfloor i/N \rfloor) \quad (24)$$

is the combination of the spreading code a_k , the transmitted bit b_k of user k , and the complex channel weighting c_k during the i th chip. Since there are N chips per

bit, the limits on the first summation expand to cover the $N(2M+1)$ chips in the block of $2M+1$ bits. The chip waveform, $p(t)$, has unity duration. Throughout it is assumed that the chip waveform is a unit energy rectangular waveform, and that BCA is applied to all the users' transmitted signals.

At the base station the signal is first processed by an integrate-and-dump filter whose integration time is $1/L$ th of the chip interval. This is equivalent to L times oversampling of the received signal. The sampling instance at the output of this filter is synchronized to the base station transmissions and not to any individual user's signal. Later it will be shown that the lack of exact synchronization to a specific user results in a loss of performance when sampling at the chip rate. Sampling at multiples of the chip rate can recover all of the lost performance.

With no oversampling ($L = 1$), the integrate-and-dump filter output in the j th chip is

$$\begin{aligned}
 y(j) &= \int_j^{j+1} r(t) dt \\
 &= \sum_{i=-N(M+1/2)}^{N(M+1/2)} \sum_{k=1}^K d_k(i) \int_j^{j+1} p(t - \tau_k - i) dt + n \\
 &= \sum_{k=1}^K (d_k(l_k) \gamma_k + d_k(l_k + 1)(1 - \gamma_k)) + n
 \end{aligned} \tag{25}$$

where

$$l_k = j - \lceil \tau_k \rceil \tag{26}$$

$$\gamma_k = \tau_k - \lfloor \tau_k \rfloor. \tag{27}$$

The weight, γ_k , lies in the range $[0, 1]$ and is only dependent upon the delay of user k . It determines the contribution of the two adjoining chips that span the integrate-and-dump filter's integration time.

When $L > 1$, the receiver is oversampling the signal. Instead of developing a new equation for the filter output for this case, equation (25) will be used with a small modification to the spreading code $\mathbf{a}_k = [a_k(0) a_k(1) \cdots a_k(N-1)]'$. For L times oversampling, simply repeat the chips of the spreading code L times to produce a new spreading code with LN chips. To maintain Q chip quasi-synchronous

CDMA, also multiply the delay time, τ_K , by L . Then simply use equation (25) to yield the oversampled output. Now $y(j)$ represents the integrate-and-dump filter output for the j th subchip, where there are L subchips per transmitted chip and LN subchips per bit. The unit time interval is now a subchip. A block diagram of the chip filter Stage I with L times oversampling is shown in Figure 10.

Figure 10: Chip filter Stage I block diagram

For example, consider a single chip QS-CDMA system with BCA where the base spreading codes are four chips long. If the first user's spreading code is $\mathbf{a}_1^{(1)} = [+ - + - 0]'$, the new spreading code for two times oversampling ($L = 2$) is $\mathbf{a}_1^{(2)} = [+ + - - + + - - + + 00]'$ where the superscript indicates the degree of oversampling, a + indicates a +1, and a - indicates a -1. The delay for the first case is limited to $0 \leq \tau_1^{(1)} \leq 1$, and for the second case it is expanded to $0 \leq \tau_1^{(2)} \leq 2$ to maintain single chip QS-CDMA. Each entry in $\mathbf{a}_1^{(2)}$ is considered to be a subchip by the receiver. The user's transmitter still sends N chips per bit, but at the oversampling receiver, it looks like LN subchips have been sent.

All of the equations developed in this section are for the case with no oversampling. The L times oversampling equations can be produced by using the procedure described above.

In a $2M+1$ bit transmission to the base station, a user will send $N(2M+1)$ chips and the base station CF will produce an equally long string of samples. This string can be processed using a variety of techniques to recover the $2M+1$ transmitted bits. As discussed in Chapter III, we are interested in techniques that allow memoryless bit-by-bit detection. This is accomplished through the use

Figure 11: User k 's timing alignment relative to the base station timing for $L = 1$ and $Q = 2$. CF outputs 0 to $N - 1$ only span the non-zero chips of the desired bit for all delays τ_k up to two chips.

of BC. at the user's transmitter and by windowing the string of samples from the CF into bit-sized vectors. Since the exact arrival time of user k 's bits is unknown, the window chosen must encompass the desired bit for all possible delays. By considering a single bit of the transmission in Q chip QS-CDMA and referring to Figure 11, it can be seen that the window can be constructed in three ways:

- Select a window that encompasses all the chips of the desired bit. Samples 0 to $N - 1 + Q$ in the example figure.
- Select a window that encompasses as many chips of the desired bit as possible without including chips from adjacent bits. Samples Q to $N - 1$ in the example figure.
- Set Q of the end chips of the user's spreading code at the mobile station equal to zero (i.e., BCA), and choose a window that encompasses all of the desired bit chips. Samples 0 to $N - 1$ in the example figure.

The first window includes all of the user's transmitted chips but introduces up to Q chips of intersymbol interference (ISI) from both of the adjacent bits. The second window introduces no ISI but discards up to a fraction Q/N of the received energy. The third window introduces no ISI and discards none of the

received energy of the desired bit. It will be used for the remainder of the model development.

Partitioning the integrate-and-dump filter output (equation (25)) according to the third window yields a length N vector $\mathbf{y}$ given by

$$\mathbf{y} = \sum_{k=1}^K (\gamma_k \mathbf{d}_k(-\lceil \tau_k \rceil) + (1 - \gamma_k) \mathbf{d}_k(-\lceil \tau_k \rceil + 1)) \quad (28)$$

where the additive Gaussian noise term has been dropped for now to clarify the equation development. The vector $\mathbf{y}$ contains all the information required to detect a single transmitted bit from each of the K users. With no oversampling, the vector $\mathbf{d}_k(j)$ contains N consecutive samples of $d_k(i)$ (equation (24)) starting at the point j where the samples $d_k(i)$ are considered to be cyclic modulo N . Thus

$$\mathbf{d}_k(j) = [d_k(j) \ d_k(j+1) \ \cdots \ d_k(N-1) \ d_k(0) \ \cdots \ d_k(j-1)]'.$$

The vector $\mathbf{y}$ can be expressed in matrix form as

$$\begin{aligned} \mathbf{y} = & [\mathbf{d}_1(-\lceil \tau_1 \rceil) \ \mathbf{d}_2(-\lceil \tau_2 \rceil) \ \cdots \ \mathbf{d}_K(-\lceil \tau_K \rceil)] \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_K \end{bmatrix} \\ & + [\mathbf{d}_1(-\lceil \tau_1 \rceil + 1) \ \mathbf{d}_2(-\lceil \tau_2 \rceil + 1) \ \cdots \ \mathbf{d}_K(-\lceil \tau_K \rceil + 1)] \begin{bmatrix} 1 - \gamma_1 \\ \vdots \\ 1 - \gamma_K \end{bmatrix}. \end{aligned}$$

Using equation (24) to expand $\mathbf{d}_k$ into the product of the spreading code $\mathbf{a}_k$, the channel characteristic c_k , and the transmitted bit b_k yields

$$\mathbf{y} = [\mathbf{a}_1 \ \mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_2 \ \cdots \ \mathbf{a}_K \ \mathbf{a}_K] \begin{bmatrix} \begin{pmatrix} \gamma_1 \\ 1 - \gamma_1 \end{pmatrix} & \mathbf{0} & & \\ \mathbf{0} & \begin{pmatrix} \gamma_2 \\ 1 - \gamma_2 \end{pmatrix} & & \\ & & \ddots & \\ & & & \begin{pmatrix} \gamma_K \\ 1 - \gamma_K \end{pmatrix} \end{bmatrix} C \mathbf{b}$$

or

$$\mathbf{y} = \Lambda \Gamma C \mathbf{b} \quad (29)$$

where

$$\Lambda = [\dot{\mathbf{a}}_1 \ddot{\mathbf{a}}_1 \dot{\mathbf{a}}_2 \ddot{\mathbf{a}}_2 \cdots \dot{\mathbf{a}}_K \ddot{\mathbf{a}}_K]$$

$$\dot{\mathbf{a}}_k = \mathbf{a}_k(-\lceil \tau_k \rceil)$$

$$\ddot{\mathbf{a}}_k = \mathbf{a}_k(-\lceil \tau_k \rceil + 1).$$

Vector $\mathbf{a}_k$ is the length N binary spreading code of the k th user. It contains bipolar values in the first $N-Q$ entries and zeros in the last Q entries (i.e., BCA). The first $N-Q$ entries are called the base spreading code since these entries achieve the spectrum spreading. The vector $\mathbf{a}_k(j)$ is the j chip modulo N cyclic shift of the vector $\mathbf{a}_k$, i.e., $\mathbf{a}_k(j) = [a_k(j) a_k(j+1) \cdots a_k(N-1) a_k(0) \cdots a_k(j-1)]'$. The vector $\mathbf{y}$ is the product of a spreading code matrix Λ , a weight matrix Γ whose entries are a function of the users' delays only, a diagonal channel matrix C , and the users' transmitted bits $\mathbf{b}$. The spreading code matrix is a function of the chosen spreading codes and the set of user delays. The characteristics of this matrix will be examined in the following paragraphs with the goal being to factor out the dependence upon the user delays.

For single chip QS-CDMA, the delay is restricted to $0 \leq \tau_k \leq 1$. Then $\dot{\mathbf{a}}_k = \mathbf{a}_k(-1)$ and $\ddot{\mathbf{a}}_k = \mathbf{a}_k(0)$ have the form

$$\begin{aligned} \dot{\mathbf{a}}_k &= [0 \quad a_k(1) \quad a_k(2) \quad a_k(3) \quad \cdots \quad a_k(N-1)]' \\ \ddot{\mathbf{a}}_k &= [a_k(1) \quad a_k(2) \quad a_k(3) \quad \cdots \quad a_k(N-1) \quad 0]' \end{aligned}$$

where $a_k(i)$ are the individual elements of the base spreading code. Since $\dot{\mathbf{a}}_k$ and $\ddot{\mathbf{a}}_k$ are linearly independent vectors, they form a two dimensional subspace in N dimensional space. User k 's signal is restricted to this subspace under all delays up to a single chip independent of the channel attenuation, channel phase, or the transmitted bits. The location of the signal vector within this subspace is determined by the value of the delay τ_k .

For double chip QS-CDMA, the delay is restricted to $0 \leq \tau_k \leq 2$. Now, two scenarios can arise. The delay is either less than unity or greater than unity. The resulting entries in Λ are

$$\begin{aligned}
& 0 \leq \tau_k \leq 1 & 1 \leq \tau_k \leq 2 \\
\dot{\mathbf{a}}_k = & [0 \ a_k(1) \ a_k(2) \ \cdots \ a_k(N-2) \ 0]' & [0 \ 0 \ a_k(1) \ a_k(2) \ \cdots \ a_k(N-2)]' \\
\ddot{\mathbf{a}}_k = & [a_k(1) \ a_k(2) \ \cdots \ a_k(N-2) \ 0 \ 0]' & [0 \ a_k(1) \ a_k(2) \ \cdots \ a_k(N-2) \ 0]'.
\end{aligned}$$

Three linearly independent vectors have been created for each user under all possible delays. Each user occupies a three dimensional subspace in N dimensional space. This pattern can be extended to Q chip delays where each user will occupy a $Q+1$ dimensional subspace in N dimensional space.

This relationship between the maximum delay and the size of the subspace occupied by a user can be used to construct a new code matrix A to replace Λ that is independent of the user delays. The format of A is

$$A = [\mathbf{a}_1(0) \cdots \mathbf{a}_1(Q) \ \mathbf{a}_2(0) \cdots \mathbf{a}_2(Q) \cdots \cdots \mathbf{a}_K(0) \cdots \mathbf{a}_K(Q)] \quad (30)$$

where $\mathbf{a}_k(q)$ is the q chip cyclically shifted version of $\mathbf{a}_k$.

The weight matrix Γ must be replaced with a new matrix T that properly weights the vectors in A that span the subspace of each user such that $AT = \Lambda\Gamma$. Matrix T has the format

$$T = \begin{bmatrix} \mathbf{t}(\tau_1) & & & \\ & \mathbf{t}(\tau_2) & & \\ & & \ddots & \\ & & & \mathbf{t}(\tau_K) \end{bmatrix} \quad (31)$$

where

$$\mathbf{t}(\tau) = [t_0(\tau) \ t_1(\tau) \ \cdots \ t_Q(\tau)]' \quad (32)$$

is a $Q+1$ element vector whose entries are a function of a user's delay. All other entries in the matrix T are zero. The functions $t_q(\tau)$ are

$$t_0(\tau) = \begin{cases} 1 - \tau & 0 \leq \tau \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

$$t_q(\tau) = \begin{cases} \tau - q + 1 & q - 1 \leq \tau \leq q \\ q + 1 - \tau & q \leq \tau \leq q + 1 \\ 0 & \text{otherwise} \end{cases} \quad q = 1, 2, \dots, Q-1 \quad (34)$$

$$t_Q(\tau) = \begin{cases} \tau - Q + 1 & Q - 1 \leq \tau \leq Q \\ 0 & \text{otherwise} \end{cases} \quad (35)$$

and are sketched in Figure 12. The received vector is now given by

$$\mathbf{y} = \mathbf{A} \mathbf{T} \mathbf{C} \mathbf{b} + \mathbf{n} \quad (36)$$

where $\mathbf{C}$ and $\mathbf{b}$ remain as previously defined, and the Gaussian noise vector $\mathbf{n}$ has been included again.

Figure 12: Weighting functions in the matrix $\mathbf{T}$.

There are a few notable characteristics of the CF Stage I that are in contrast to the MF Stage I. The CF Stage I does not require knowledge of the users' spreading codes or delays; it requires only one integrate-and-dump filter instead of K matched filters; and the expression for the discrete time vector $\mathbf{y}$ can be factored into a product of the spreading codes, delays, channel characteristics, and transmitted bits. The last property will be used later to implement a family of reduced complexity detectors that require very little side information.

V

Stage II: Linear Combining

Stage II produces the soft decision vector $\mathbf{z}$ as a weighted linear combination of the elements in vector $\mathbf{y}$. If the combining matrix is V , then $\mathbf{z} = V\mathbf{y}$. Two classes of linear combiners are considered; decorrelating matrices ($V = \Psi$) and MMSE matrices ($V = W$). The equations for these two classes of linear combiners and their performance measures are derived in the first three sections, and specific implementations of the two classes are derived in the remaining sections of this chapter.

5.1 Decorrelating Matrices

The decorrelating detector's bit estimates are given by

$$\hat{\mathbf{b}} = \text{sgn}(\mathbf{z}) = \text{sgn}(\Psi\mathbf{y}) \quad (37)$$

where $\mathbf{z} = \Psi\mathbf{y}$ and Ψ is the decorrelating matrix. Vector $\mathbf{y}$ is

$$\mathbf{y} = H\mathbf{C}_\phi\mathbf{C}_\rho\mathbf{b} + \mathbf{n} \quad (38)$$

where $H = R$ for the MF Stage I or $H = AT$ for the CF Stage I as defined in equations (22) and (36) respectively. Diagonal matrices C_ϕ and C_ρ contain the phase and magnitude information respectively from diagonal matrix C such that $C = C_\phi C_\rho$. Vector $\mathbf{n}$ contains samples of Gaussian distributed noise. With the MF Stage I, the noise covariance matrix is $\mathbf{T} = \langle \mathbf{nn}^* \rangle = \sigma^2 R$, and with the CF Stage I, the noise covariance matrix is $\mathbf{T} = \sigma^2 I$. The decorrelating matrix for this system is

$$\Psi = C_\phi^* H^I \quad (39)$$

where H^I is the inverse of H and is defined by $H^I H = I$ where I is the identity matrix. The decorrelating detector is the multiuser version of the zero forcing de-

tector used to mitigate intersymbol interference in a single user scenario. It inverts the channel assuming that the noise is negligible. In high SNR environments, this works well, but at low SNRs, it can produce noise enhancement.

Let $C_\phi C_\phi \mathbf{b} = \mathbf{g}$ and the noise be zero for the moment. Now $\mathbf{y} = H\mathbf{g}$. Vector $\mathbf{y}$ is known by the receiver, and decorrelating detection is equivalent to solving this system of equations for vector $\mathbf{g}$. There are two scenarios to consider. Matrix H has linearly independent columns (i.e., it has full column rank), or it does not. Both cases will be considered below.

When H has full column rank, the system of equations has a unique solution and a unique bit vector can be found. If H is square, the unique solution is $\mathbf{g} = H^{-1}\mathbf{y}$ where H^{-1} is the regular matrix inverse of H . If H is not square, it must have linearly independent columns to have full column rank, which implies that H has more rows than columns. Then the unique solution is $\mathbf{g} = H^L\mathbf{y}$ where H^L is the left inverse of H [30]. The left inverse differs from the regular inverse in that it is not a square matrix and it is not unique. One possible left inverse is $H^L = (H^*H)^{-1}H^*$, but many can exist. All left inverses will, however, yield the same unique solution for $\mathbf{g}$. Note that when H is square, $H^L = H^{-1}$.

When H does not have full column rank, the system of equations does not have a unique solution. Since a unique bit vector cannot be found, this case is of no interest.

For the cases of interest then, the decorrelating matrix is

$$\Psi = C_\phi^* H^L \quad (40)$$

and encompasses both square and rectangular full column rank matrices.

With the CF Stage I, matrix H can be factored into a product $H = AT$. There are two approaches to forming the decorrelating matrix in this case. Either the product can be inverted (i.e., $(AT)^L$), or the individual terms can be inverted (i.e., $T^L A^L$) where it has been assumed that both A and T have full column rank. In general $(AT)^L \neq T^L A^L$ and the two cases produce two different decorrelating detectors.

Assuming the A has full column rank, the left inverse is $A^L = (A'A)^{-1}A'$. It can be calculated a priori and remains static during receiver operation. For A to

have linearly independent columns, it must have no more rows than columns, or $K(Q+1) \leq N$.

Multiplying y by A^L separates the K users into K subspaces, each with dimension $Q+1$. The k th group of $Q+1$ entries in the vector $A^L y$ contains all the information required to detect the k th user, and no interference from the other users. The distribution of the energy of the k th user's bit over these $Q+1$ entries is determined by the delay matrix T .

The spreading codes must be selected a priori such that that matrix A has full column rank. This creates an upper limit on the number of users that can be supported for a given spreading gain, N , and degree of delay, Q . The limit is

$$K_{max} = \left\lfloor \frac{N}{Q+1} \right\rfloor. \quad (41)$$

Thus, for single chip QS-CDMA ($Q = 1$), the capacity is approximately equal to half the spreading gain. The capacity continues to shrink as the amount of asynchronism increases.

It is possible to increase the capacity by oversampling the chip filter in Stage I. With L times oversampling, the capacity becomes

$$K_{max} = \left\lfloor \frac{N}{Q+L^{-1}} \right\rfloor. \quad (42)$$

For single chip QS-CDMA, the capacity can be increased by approximately 33% by doubling the sampling rate of Stage I. By tripling the rate, the capacity can be increased by approximately 50%. Diminishing returns set in beyond three times oversampling. The capacity increase is less dramatic for $Q > 1$. The increase in capacity assumes that one can find $K(LQ+1)$ linearly independent codes to form the matrix A , which can be a difficult task in practice.

Specific implementations of the left inverse of the delay matrix T are dependent upon the desired design goals. Several implementations of the left inverse are outlined in later sections.

In effect, the chip matched filtering approach eliminates the need to estimate delays at the expense of increasing the effective number of interferers by $Q+1$.

5.2 MMSE Matrices

The MMSE detector's bit estimates are given by

$$\hat{\mathbf{b}} = \text{sgn}(\mathbf{z}) = \text{sgn}(W\mathbf{y}) \quad (43)$$

where $\mathbf{z} = W\mathbf{y}$ and W is the MMSE matrix. Vector $\mathbf{y}$ is

$$\mathbf{y} = HC_\rho C_\rho^* \mathbf{b} + \mathbf{n} \quad (44)$$

where $H = R$ for the MF Stage I or $H = AT$ for the CF Stage I, and $\mathbf{n}$ is Gaussian distributed noise. With the MF Stage I, the noise covariance matrix is $\Upsilon = \langle \mathbf{nn}^* \rangle = \sigma^2 R$, and with the CF Stage I, the noise covariance matrix is $\Upsilon = \sigma^2 I$.

The MMSE solution minimizes the expected value of the squared difference between the transmitted bit, b_k , and its soft estimate, $\mathbf{w}_k^* \mathbf{y}$, for all k where $\mathbf{w}_k$ is the weight vector for user k . Weight vector $\mathbf{w}_k^*$ is the k th row of the matrix W . The expected value of the squared difference is called the error and is denoted as J_k for user k . It is given by

$$J_k = \langle (b_k - \mathbf{w}_k^* \mathbf{y})^2 \rangle. \quad (45)$$

The vector $\mathbf{w}_k$ that minimizes this error is a solution to the Wiener-Hopf equations [31]

$$\Re \mathbf{w}_k = \mathbf{p}_k \quad (46)$$

where the correlation matrix $\Re$ is

$$\Re = \langle \mathbf{y}\mathbf{y}^* \rangle = HCC^*H^* + \langle \mathbf{nn}^* \rangle = HC_\rho^2 H^* + \Upsilon \quad (47)$$

and Υ is the noise covariance matrix. The vector $\mathbf{p}_k$ is

$$\mathbf{p}_k = \langle \mathbf{y}b_k \rangle = HC\mathbf{1}_k \quad (48)$$

where the vector $\mathbf{1}_k$ contains all zeros except for a one in its k th position.

Substituting these expressions into the Wiener-Hopf equations and solving for $\mathbf{w}_k$ yields

$$\mathbf{w}_k = (HC_\rho^2 H^* + \Upsilon)^{-1} HC\mathbf{1}_k. \quad (49)$$

This solution raises two questions. First, does the above matrix inverse exist, and second, what happens in the limit as the noise vanishes? The answers depend upon the matrices H and Υ which are dependent upon the selection of Stage I. These questions will be addressed later when the details of the various receivers are derived.

The MMSE matrix is formed by combining the weight vectors of the K users into the matrix W ,

$$W = \begin{bmatrix} \mathbf{w}_1^* \\ \vdots \\ \mathbf{w}_K^* \end{bmatrix} = C^* H^* (H C_\rho^2 H^* + \Upsilon)^{-1}. \quad (50)$$

When using the CF Stage I and preprocessing the vector $\mathbf{y}$ by A^L , the error function becomes

$$J_k = \langle (b_k - \mathbf{w}_k^* A^L \mathbf{y})^2 \rangle. \quad (51)$$

The correlation matrix $\mathfrak{R}$ is

$$\begin{aligned} \mathfrak{R} &= \langle A^L \mathbf{y} \mathbf{y}^* (A^L)^* \rangle \\ &= T C_\rho^2 T' + A^L \Upsilon (A^L)^* \\ &= T C_\rho^2 T' + \sigma^2 (A^L A)^{-1} \end{aligned} \quad (52)$$

and the vector $\mathbf{p}_k$ is

$$\mathbf{p}_k = \langle A^L \mathbf{y} b_k \rangle = T C \mathbf{1}_k. \quad (53)$$

Solving the Wiener-Hopf equations yields

$$\mathbf{w}_k = (T C_\rho^2 T' + \sigma^2 (A^L A)^{-1})^{-1} T C \mathbf{1}_k \quad (54)$$

In this case, the MMSE matrix is the combination of all the vectors $\mathbf{w}_k$ and the left inverse of A .

$$W = C^* T' (T C_\rho^2 T' + \sigma^2 (A^L A)^{-1})^{-1} A^L. \quad (55)$$

The two versions W in equations (50) and (55) will be contrasted when the details of the various receivers are derived later.

5.3 Performance Measures

The bit error rate of user k , P_k , is typically used to characterize the performance of a detection scheme. For the system $\mathbf{y} = \mathbf{H}\mathbf{C}\mathbf{b} + \mathbf{n}$ and the combining matrix $\mathbf{V}$ in Stage II, the bit error probability (or BER) is [4]

$$P_k = 2^{1-K} \sum_{\substack{\mathbf{b} \in \{-1, 1\}^K \\ b_k = -1}} Q \left(\frac{(VHC)_{kk} - \sum_{j \neq k} (VHC)_{kj} b_j}{\sqrt{(V\Upsilon V^*)_{kk}}} \right) \quad (56)$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-(v^2/2)} dv$$

where $(\cdot)_{jk}$ refers to the matrix element in the j th row and k th column. This equation can be applied to both decorrelating and MMSE detectors. Calculation of the BER is computationally intensive because of the large number of interfering users' bit combinations that even a modest number of users produce. For example, with 11 users, there are 1024 combinations of the interfering user's bits per user. Monte Carlo simulations were used instead of the above equation to find the BER which resulted in a significant computational savings.

For CDMA, the bit error rate is a strong function of the desired user's signal to background noise ratio (SNR) and the power of the interfering users. In a heavily loaded CDMA system the interference from the other users is typically much larger than the background Gaussian noise. By considering the performance as the background noise vanishes, the system can be evaluated on the basis of its ability to withstand interference, or the near-far problem.

Verdu proposed the asymptotic efficiency as a measure to characterize performance as the background noise vanishes. The asymptotic efficiency was introduced in an earlier chapter and its definition is

$$\eta_k = \sup \left\{ 0 \leq \alpha \leq 1; \lim_{\sigma \rightarrow 0} \frac{P_k(\sigma)}{Q(\sqrt{\alpha}|c_k|/\sigma)} < +\infty \right\}. \quad (57)$$

Substituting P_k from equation (56) yields [4]

$$\eta_k = \max^2 \left\{ 0, \frac{\sigma}{|c_k|} \frac{(VHC)_{kk} - \sum_{j \neq k} |(VHC)_{kj}|}{\sqrt{(V\Upsilon V^*)_{kk}}} \right\}. \quad (58)$$

This measure is far less complex to calculate than the BER since it need not be evaluated over all interfering bit combinations.

Using the fact that $V = \Psi = C_o^* H^L$ for decorrelating detectors, the asymptotic efficiency is

$$\eta_k = \max \left\{ 0, \frac{1}{(H^L \hat{\Upsilon} (H^L)^*)_{kk}} \right\} \quad (59)$$

where $\hat{\Upsilon} = \Upsilon/\sigma^2$ is the normalized noise covariance matrix. This equation applies to all implementations of the left inverse H^L . The resulting probability of a bit error is

$$P_k = Q \left(\sqrt{\eta_k} \frac{|c_k|}{\sigma} \right). \quad (60)$$

Since the asymptotic efficiency of the decorrelating detector is independent of the users' powers, it is equal to the near-far resistance. The asymptotic efficiency will be used to quantify the performance of all the decorrelating detectors since it fully characterizes their bit error performance.

Three performance measures will be considered for the MMSE detectors. The minimum mean squared error (MMSE), the bit error rate (BER), the asymptotic efficiency. With the tap weight vector $\mathbf{w}_k$ for user k given by the Wiener solution, the mean squared error achieves its minimum value $J_k(\min)$ given by [31]

$$J_k(\min) = 1 - \mathbf{p}_k^* \mathbf{w}_k. \quad (61)$$

Combining all K values of $J_k(\min)$ into a vector yields

$$\mathbf{J}(\min) = \text{diag}[\mathbf{I} - \mathbf{W} \mathbf{H} \mathbf{C}]. \quad (62)$$

The bit error rate can be calculated using equation (56) with $V = W$. The asymptotic efficiency is calculated using equation (58) with $V = \lim_{\sigma^2 \rightarrow 0} W$, which is the Wiener solution with vanishing noise. The details of these calculations are shown in the next section.

5.4 Implementations

There are two implementations of Stage I and many implementations of Stage II, each with differing complexity and performance. This section delineates several combinations of these two stages. The combinations are grouped into two

categories, with each category using one of the two implementations of Stage I. Several Stage II combining matrices are defined for each Stage I, and the detector performance measure is derived for each combination.

5.4.1 MF Stage I

MF detectors use the waveform matched filter bank in Stage I and either the decorrelating matrix Ψ or the MMSE matrix W in Stage II. In this case, $H = R$, R is square, real, symmetric, and non-singular, and $\Upsilon = \sigma^2 R$. Substituting this into equations (40) and (50) yields

$$\Psi^1 = C_\phi^* R^{-1} \quad (63)$$

$$\begin{aligned} W^1 &= C^* R^* (R C_\phi^2 R^* + \sigma^2 R)^{-1} \\ &= C_\phi^{-1} C_\phi^* (R + \sigma^2 C_\phi^{-2})^{-1}. \end{aligned} \quad (64)$$

Matrix R is singular only in pathological cases such as when all users have the same delay and the same signature waveform. Under normal circumstances, where users have unique spreading codes with low cross correlation and independent delays, R is non-singular [2].

With the MMSE matrix, taking the limit $\lim_{\sigma^2 \rightarrow 0} W^1 = C_\phi^{-1} C_\phi^* R^{-1} = C_\phi^{-1} \Psi^1$, it is readily seen that the MMSE matrix approaches the decorrelating matrix to within a constant multiplier. The constant multiplier can be dropped since it does not affect the hard decision made upon the soft decision vector $\mathbf{z}$. Thus, both the decorrelating detector and the MMSE detector have the same asymptotic efficiency with the MF Stage I. The asymptotic efficiency is (equation (59))

$$\eta_k^1 = 1/R_{kk}^{-1}. \quad (65)$$

The MMSE is $\mathbf{J}(\min) = \text{diag}[I - WRC]$ and the BER is evaluated using Monte Carlo simulations in Chapter VII.

5.4.2 CF Stage I

In general, matrix H is not square. It has dimensions N by K where $N \geq K$. It will have full column rank if linearly independent spreading codes are chosen.

Then a decorrelating matrix is

$$\Psi^2 = C_\phi^*(H'H)^{-1}H' \quad (66)$$

where H^* has been replaced by H' since H is real. Since the normalized noise covariance matrix is the identity matrix, the asymptotic efficiency of this detector is

$$\eta_k^2 = 1/(H'H)_{kk}^{-1}. \quad (67)$$

The MMSE matrix is

$$W^2 = C_\phi^* C_\rho H' (H C_\rho^2 H' + \sigma^2 I)^{-1}. \quad (68)$$

Since $H C_\rho^2 H'$ is singular if H is not square, taking the limit as the noise approaches zero is more complicated than it was with the MF Stage I. The limit does exist, however, and is given by [32]

$$\lim_{\sigma^2 \rightarrow 0} E^*(E E^* + \sigma^2 I)^{-1} = E^+ \quad (69)$$

where E^+ is the Moore-Penrose inverse of H [32]. The Moore-Penrose inverse satisfies the four equations

$$E F E = E \quad (70)$$

$$F E F = F \quad (71)$$

$$(E F)^* = E F \quad (72)$$

$$(F E)^* = F E \quad (73)$$

where $F = E^+$. When E has full column rank, its Moore-Penrose inverse is equal to its left inverse. Thus

$$\begin{aligned} \lim_{\sigma^2 \rightarrow 0} W^2 &= C_\phi^* C_\rho^{-1} H^+ \\ &= C_\phi^* C_\rho^{-1} (H'H)^{-1} H' \end{aligned} \quad (74)$$

$$= C_\rho^{-1} \Psi^2. \quad (75)$$

This is equivalent to the decorrelating matrix Ψ^2 except for a scaling factor C_ρ^{-1} . The scaling factor can be dropped since it does not affect the hard decisions made

upon $\mathbf{z}$. Therefore, as the noise approaches zero, the MMSE matrix converges to the decorrelating matrix, and both detectors have the same asymptotic efficiency.

This decorrelating detector requires knowledge of the user delays and phases to form the matrix $H = AT$ and C_ϕ respectively. Matrix A is known a priori and is static. The matrices T and C_ϕ however, depend upon the physical channel characteristics, and must be estimated at the receiver before detection can proceed.

5.4.3 CF Stage I, Split Stage II

The above CF decorrelating and MMSE detectors treated H as a single matrix even though it can be factored into the product AT . The receivers derived in the remaining sections make use of the factorization to reduce complexity by first multiplying the vector $\mathbf{y}$ by the left inverse of A and then finding the decorrelating or MMSE detectors. This two step process is implemented in two substages, Stage IIa and Stage IIb, within Stage II. Stage IIa is always the left inverse of A which is equal to $A^L = (A'A)^{-1}A'$.

Decorrelating Stage IIb. The decorrelating matrix is $\Psi = C_\phi^* T^L A^L$ when Stage IIa is A^L . Since the delay matrix T has full column rank, one possible left inverse of T is $T^L = (T'T)^{-1}T'$. The corresponding decorrelating matrix is $\Psi^3 = C_\phi^* (T'T)^{-1}T'A^L$.

The asymptotic efficiency of this decorrelating detector is

$$\begin{aligned} \eta_k^3 &= \frac{1}{(H^L \hat{\Upsilon} (H^L)^*)_{kk}} \\ &= \frac{1}{(T^L A^L (T^L A^L)')_{kk}} \\ &= \frac{1}{((T'T)^{-1}T'(A'A)^{-1}T(T'T)^{-1})_{kk}}. \end{aligned} \quad (76)$$

SNR Maximizing Stage IIb. Since many left inverses of T exist, a natural question to ask is which one is best? Which left inverse yields the lowest probability of bit error? Since the matrix A^L separates the users into k independent subspaces, each of which is free of interference from the other users, minimizing

the BER is equivalent to maximizing the SNR of the soft decision variable, z_k , of user k . The soft decision variable is a weighted linear combination of the $Q+1$ elements in the vector $A^L y$ that correspond to the k th user's subspace. Assuming the unknown weight vector is w_k , z_k is

$$z_k = w_k^* (t_k + n_k) \quad (77)$$

where $t_k = t(\tau_k)$ is the delay vector of user k (equation (32)), and n_k is a Gaussian distributed random vector with zero mean and covariance matrix Υ_k . The noise covariance matrix is given by

$$\begin{aligned} \Upsilon &= \langle A^L y y^* (A^L)^* \rangle \\ &= A^L (A^L)^* \sigma^2 \\ &= \sigma^2 (A' A)^{-1}, \end{aligned} \quad (78)$$

and Υ_k is the k th square submatrix with dimensions $Q+1$ by $Q+1$ on the diagonal of Υ .

The signal amplitude in z_k is $w_k^* t_k$ and the noise variance is $w_k^* \Upsilon_k w_k$. The SNR is

$$\text{SNR}_k = (w_k^* t_k)^2 / w_k^* \Upsilon_k w_k. \quad (79)$$

Maximizing the SNR is a difficult task since the unknown weights appear in both the numerator and the denominator. The problem can be simplified without loss of generality by fixing the signal amplitude to a constant and minimizing the noise variance subject to the constraint. By setting the signal amplitude to unity, the problem simplifies to minimizing $w_k^* \Upsilon_k w_k$ subject to the constraint $w_k^* t_k = 1$. This can be solved using Lagrange multipliers. The cost function is

$$J = w_k^* \Upsilon_k w_k + \lambda (w_k^* t_k - 1) \quad (80)$$

where λ is a Lagrange multiplier. The solution to this equation is [31]

$$\begin{aligned} w_k &= \frac{\Upsilon_k^{-1} t_k}{t_k^* \Upsilon_k^{-1} t_k} \\ &= \frac{(A' A)_k t_k}{t_k^* (A' A)_k^{-1} t_k} \end{aligned} \quad (81)$$

where $(A'A)_k$ is the k th square submatrix with dimensions $Q+1$ by $Q+1$ on the diagonal of $(A'A)$. The optimum left inverse of T is

$$T^{L4} = \begin{bmatrix} \mathbf{w}_1^* & & \\ & \ddots & \\ & & \mathbf{w}_K^* \end{bmatrix}. \quad (82)$$

This delay dependent left inverse also requires knowledge of the noise covariances. Fortunately the noise covariance matrix is known a priori since it is solely determined by the code matrix A . The decorrelating matrix based upon this left inverse of T is $\Psi^4 = C_\phi^* T^{L4} A^L$.

The asymptotic efficiency of this decorrelating detector is

$$\begin{aligned} \eta_k^4 &= \frac{1}{(T^{L4} A^L (T^{L4} A^L)')_{kk}} \\ &= \frac{1}{(T^{L4} (A'A)^{-1} (T^{L4})')_{kk}}. \end{aligned} \quad (83)$$

SNR Maximizing Stage IIb With Unknown Delays. When the delays are unknown and the expected value of the signal amplitude portion of z_k is constrained to unity, $\langle \mathbf{w}_k^* \mathbf{t}_k \rangle = \mathbf{w}_k^* \langle \mathbf{t}_k \rangle = 1$, the weight vector that maximizes the SNR becomes

$$\mathbf{w}_k = \frac{(A'A)_k \langle \mathbf{t}_k \rangle}{\langle \mathbf{t}_k' \rangle (A'A)_k \langle \mathbf{t}_k \rangle}. \quad (84)$$

If the user delays are uniformly distributed over Q chips, the expected value of the vector $\mathbf{t}_k$ is

$$\langle \mathbf{t}_k \rangle = [0.5 \ 1 \ 1 \ \dots \ 1 \ 0.5]' / Q. \quad (85)$$

It contains $Q+1$ elements of which first and last elements are equal to $1/2Q$, and the middle elements are equal to $1/Q$.

The delay independent SNR maximizing left inverse of T is

$$T^{L5} = \begin{bmatrix} \mathbf{w}_1^* & & \\ & \ddots & \\ & & \mathbf{w}_K^* \end{bmatrix} \quad (86)$$

and the resulting delay independent decorrelating matrix is $\Psi^5 = C_0^* T^{L5} A^L$. The asymptotic efficiency is

$$\begin{aligned}\eta_k^5 &= \frac{1}{(T^{L5} A^L (T^{L5} A^L)')_{kk}} \\ &= \frac{1}{(T^{L5} (A' A)^{-1} (T^{L5})')_{kk}}.\end{aligned}\quad (87)$$

This combination of the CF Stage I, A^L for Stage IIa, and T^{L5} for Stage IIb will be called the delay independent decorrelating detector since all stages do not require knowledge of the users' delays.

Long Vector MMSE Matrix Stage IIb. The MMSE matrix for the processed vector $A^L y$ is (equation (50))

$$W^6 = (TC)^* (TC(TC)^* + \sigma^2 (A' A)^{-1})^{-1} A^L. \quad (88)$$

This is called the long vector MMSE matrix since each weight vector in W^6 contains N non-zero entries. This is in contrast to the short vector MMSE matrix to be derived in the next section that has only $Q + 1$ non-zero entries.

Since $A' A$ is symmetric and positive definite it can be equated to the square of a symmetric non-singular matrix D . Let

$$A' A = D^2. \quad (89)$$

Substituting this definition into W^6 and re-arranging yields

$$W^6 = (DTC)^* (DTC(DTC)^* + \sigma^2 I)^{-1} D A^L. \quad (90)$$

As long as $\sigma^2 \neq 0$ the above inverse exists. Under low noise conditions the solution becomes

$$\lim_{\sigma^2 \rightarrow 0} W^6 = (DTC)^* D A^L \quad (91)$$

where the limit was evaluated using equation (69). In terms of the spreading code matrix A , this expression is

$$\lim_{\sigma^2 \rightarrow 0} W^6 = (DTC)^* D A^L$$

$$\begin{aligned}
&= C^{-1}(DT)^+DA^L \\
&= C^{-1}((DT)'DT)^{-1}(DT)'DA^L \\
&= C^{-1}(T'D^2T)^{-1}T'D^2A^L \\
&= C^{-1}(T'A'AT)^{-1}T'A'A(A'A)^{-1}A' \\
&= C^{-1}(T'A'AT)^{-1}T'A'.
\end{aligned} \tag{92}$$

The Moore-Penrose inverse was replaced by the left inverse in the third line since the matrix in question has full column rank. Let $\Psi^6 = C_\phi^*(T'A'AT)^{-1}T'A'$ be the corresponding decorrelating detector. This is a left inverse of ATC since $\Psi^6ATC_\phi = I$. The asymptotic efficiency of this MMSE detector is

$$\eta_k^6 = 1/(T'A'AT)_{kk}^{-1}. \tag{93}$$

Short Vector MMSE Matrix Stage IIb. An alternate solution exists for the Wiener-Hopf equations. Expanding equation (46) and substituting equations (52) and (53) yields

$$\left[\begin{pmatrix} |c_1|^2 \mathbf{t}_1 \mathbf{t}_1' & & \\ & \ddots & \\ & & |c_K|^2 \mathbf{t}_K \mathbf{t}_K' \end{pmatrix} + \Upsilon \right] \mathbf{w}_k = c_k \begin{bmatrix} 0 \\ \vdots \\ \mathbf{t}_k \\ \vdots \\ 0 \end{bmatrix} \tag{94}$$

where $\Upsilon = \sigma^2(A'A)^{-1}$. A new solution to this equation that minimizes the number of non-zero tap weights in $\mathbf{w}_k$ while still capturing all the energy from the desired user has the form

$$\mathbf{w}_k = \begin{bmatrix} 0 \\ \vdots \\ \tilde{\mathbf{w}}_k \\ \vdots \\ 0 \end{bmatrix} \tag{95}$$

where $\tilde{\mathbf{w}}_k$ is a length $Q + 1$ vector. By substituting this short vector, the above matrix equation can be reduced to

$$(|c_k|^2 \mathbf{t}_k \mathbf{t}_k' + \Upsilon_k) \tilde{\mathbf{w}}_k = c_k \mathbf{t}_k \tag{96}$$

and by forming the solutions for $\hat{\mathbf{w}}_k$ into a matrix, the MMSE matrix can be written as

$$\mathbf{W}^L = \mathbf{W}^L \mathbf{A}^L = \begin{bmatrix} \hat{\mathbf{w}}_1^* & & \\ & \ddots & \\ & & \hat{\mathbf{w}}_K^* \end{bmatrix} \mathbf{A}^L. \quad (97)$$

Since the submatrix $\Upsilon_k = \sigma^2 (\mathbf{A}' \mathbf{A})_k^{-1}$ is symmetric and positive definite, it can be equated to the square of a symmetric non-singular matrix. Let $(\mathbf{A}' \mathbf{A})_k = D_k^2$. Solving for $\hat{\mathbf{w}}_k$ yields

$$\begin{aligned} \hat{\mathbf{w}}_k &= (\mathbf{c}_k \mathbf{t}_k (\mathbf{c}_k \mathbf{t}_k)^* + \sigma^2 D_k^{-2})^{-1} \mathbf{c}_k \mathbf{t}_k \\ &= D_k (D_k \mathbf{c}_k \mathbf{t}_k (D_k \mathbf{c}_k \mathbf{t}_k)^* + \sigma^2 \mathbf{I})^{-1} D_k \mathbf{c}_k \mathbf{t}_k. \end{aligned} \quad (98)$$

Under low noise conditions the solution converges to

$$\begin{aligned} \lim_{\sigma^2 \rightarrow 0} \hat{\mathbf{w}}_k^* &= (D_k \mathbf{c}_k \mathbf{t}_k)^+ D_k \\ &= \mathbf{c}_k^{-1} (\mathbf{t}_k' (\mathbf{A}' \mathbf{A})_k \mathbf{t}_k)^{-1} \mathbf{t}_k' (\mathbf{A}' \mathbf{A})_k \\ &= \frac{\mathbf{t}_k' (\mathbf{A}' \mathbf{A})_k}{\mathbf{c}_k \mathbf{t}_k' (\mathbf{A}' \mathbf{A})_k \mathbf{t}_k}. \end{aligned} \quad (99)$$

Since $\mathbf{t}_k$ has full column rank, its Moore-Penrose inverse was replaced by its left inverse. This weight vector is equivalent to within a constant multiplier to the weight vector for the SNR maximizing Stage IIb derived earlier (equation (80)). Thus, minimizing the MSE is equivalent to maximizing the SNR for the short vector linear combiner.

5.5 Summary of the Receivers

The various combinations of Stage I and Stage II yielded several receivers. Their equations, asymptotic efficiency, and properties are outlined in the following tables.

Stage I	Stage II	
	Decorrelating	MMSE
MF	$\Psi^1 = C_\phi^* R^{-1}$	$W^1 = C_\phi^* (R + \sigma^2 C_\phi^{-2})^{-1}$
CF	$\Psi^2 = C_\phi^* (H' H)^{-1} H'$	$W^2 = C_\phi^* H' (H C_\phi^2 H' + \sigma^2 I)^{-1}$
CF	$\Psi^3 = C_\phi^* (T' T)^{-1} T' A^L$	
CF	$\Psi^4 = C_\phi^* T^{L4} A^L$	$W^4 = W A^L$
CF	$\Psi^5 = C_\phi^* T^{L5} A^L$	
CF	$\Psi^6 = C_\phi^* (T' A' A T)^{-1} T' A'$	$W^6 = C_\phi^* T' (T C (T C)^* + \sigma^2 (A' A)^{-1})^{-1} A^L$

Table 1: Decorrelating and MMSE Detector Equations.

Detector	Asymptotic Efficiency
Ψ^1	$\eta_k^1 = 1/R_{kk}^{-1}$
Ψ^2	$\eta_k^2 = 1/(H' H)_{kk}^{-1}$
Ψ^3	$\eta_k^3 = 1/((T' T)^{-1} T' (A' A)^{-1} T (T' T)^{-1})_{kk}$
Ψ^4	$\eta_k^4 = 1/(T^{L4} (A' A)^{-1} (T^{L4})')_{kk}$
Ψ^5	$\eta_k^5 = 1/(T^{L5} (A' A)^{-1} (T^{L5})')_{kk}$
Ψ^6	$\eta_k^6 = 1/(T' A' A T)_{kk}^{-1}$

Table 2: Detector Asymptotic Efficiency Expressions.

Detector	Properties
Ψ^1, W^1	Matched filter decorrelating and MMSE detectors <ul style="list-style-type: none"> - matched filter bank Stage I, one matched filter per user. - each matched filter synchronized to a specific user. - Stage I and Stage II both require knowledge of the user delays.
Ψ^2, W^2	Chip filter decorrelating and MMSE detectors <ul style="list-style-type: none"> - chip filter Stage I, synchronized to the base station. - only Stage II requires knowledge of the user delays.
Ψ^3	Split Stage II decorrelating detector <ul style="list-style-type: none"> - chip filter Stage I, synchronized to the base station. - split Stage II, Stage IIa is the code matrix left inverse A^L. - Stage IIb is the left inverse of the delay matrix. - only Stage IIb requires knowledge of the user delays.
Ψ^4, W^4	Short vector decorrelating and MMSE detectors <ul style="list-style-type: none"> - chip filter Stage I, synchronized to the base station. - split Stage II, Stage IIa is the code matrix left inverse A^L. - decorrelating detector maximizes SNR. - $Q+1$ tap weights per user (short vector).
Ψ^5	Delay independent decorrelating detector <ul style="list-style-type: none"> - chip filter Stage I, synchronized to the base station. - split Stage II, Stage IIa is the code matrix left inverse A^L. - Stage IIb maximizes SNR without knowledge of the user delays. - $Q+1$ tap weights per user.
Ψ^6, W^6	Long vector decorrelating and MMSE detectors <ul style="list-style-type: none"> - chip filter Stage I, synchronized to the base station. - split Stage II, Stage IIa is the code matrix left inverse A^L. - N tap weights per user (long vector).

Table 3: **Properties of the Decorrelating and MMSE Detectors.**

VI

Adaptive MMSE Detectors

In deriving the MMSE matrices, it was assumed that all of the system parameters were known. In real implementations, these parameters are unknown a priori. In general, there are two approaches to solving this problem. Either directly estimate the unknown parameters or use known data to train the detector before detection proceeds.

Direct estimation suffers from a few basic problems. Accurate estimation in the presence of noise and interference is complex, and the direct solution of the Wiener-Hopf equations using these estimates is complex and prone to numerical errors. The second step can also compound the inaccuracies of the estimation in the first step.

Training the detector before detection with known data is typically more accurate and more stable than the direct estimation and equation solution approach. Detectors operate in two modes when training is used. First, the transmitters send a sequence of bits known to the receiver. The receiver processes this data and compares the result to the known sequence. The difference between the receiver processed sequence and the known sequence is used to adapt the detector to the MMSE solution. Once the MMSE solution is found, the detector switches to its operating and tracking mode, and transmission of the unknown data proceeds. In this mode, decisions on the a priori unknown transmitted data are assumed to be correct and are used to continuously train the detector. The detector will adapt to slow changes in the operating environment in this mode without the need for switching back to the training mode.

Among the simplest algorithms used for adapting the receiver are the Steepest Descent (SD) algorithm, the Least Mean Square (LMS) algorithm, and the Normalized Least Mean Square (NLMS) algorithm. Each will be described briefly

below.

Steepest Descent (SD) Algorithm. The soft decision variable for user k is

$$z_k = \mathbf{w}_k^* \mathbf{y} \quad (100)$$

where $\mathbf{y}$ is the vector produced by Stage I and $\mathbf{w}_k$ is the weight vector that minimizes the mean squared error of user k . The mean squared error is

$$J_k = \langle |b_k - z_k|^2 \rangle \quad (101)$$

where b_k are the bits transmitted by user k . The optimum weight vector is found by solving the Wiener-Hopf equations. Adaptive algorithms find the optimum weight vector by iterating to the solution using an weight vector update equation. Let the weight vector at step i be $\mathbf{w}_k(i)$ and the the initial weight vector be $\mathbf{w}_k(0)$. Let the mean squared error at step i be $J_k(i)$. The steepest descent algorithm calculates the weight vector at step $i+1$ as [31]

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) + \frac{1}{2} \mu (-\nabla(J_k(i))) \quad (102)$$

where $\nabla(J_k(i))$ is the gradient of the error function with respect to the weight vector $\mathbf{w}_k$ and μ a positive constant called the step size. The algorithm is stable as long as the step size satisfies

$$0 < \mu < \frac{2}{\lambda_{max}} \quad (103)$$

where λ_{max} is the largest eigenvalue of the correlation matrix $\mathbf{R} = \langle \mathbf{y} \mathbf{y}^* \rangle$. Adaptation is complete once the mean squared error becomes close to the minimum mean square error, $J_k(min)$. The minimum mean squared error is

$$J_k(min) = 1 - \mathbf{p}_k^* \mathbf{w}_k \quad (104)$$

where $\mathbf{p}_k = \langle \mathbf{y} b_k^* \rangle$ and $\mathbf{w}_k$ is the optimum weight vector.

The transient behaviour of the mean-squared error is [31]

$$J_k(i) = J_k(min) + \sum_{j=1}^M \lambda_j (1 - \mu \lambda_j)^{2i} |u_j|^2 \quad (105)$$

where λ_j is the j th eigenvalue of the correlation matrix $\mathfrak{R} = \langle \mathbf{y}\mathbf{y}^* \rangle$ and u_j is the j th entry in the vector $\mathbf{u} = U^*(\mathbf{w}_k(0) - \mathbf{w}_k)$. Matrix U contains the eigenvectors of $\mathfrak{R}$. The transient behaviour is a complicated function of the step size, the eigenvalues and eigenvectors, and the algorithm starting point. In general, for a fixed step size μ , convergence takes longer for a large eigenvalue spread $\lambda^{\max}/\lambda^{\min}$ than for a small eigenvalue spread.

Least Mean Squares (LMS) Algorithm. The SD algorithm must estimate the correlation of the input vector $\mathbf{y}$ to find the gradient of the error function. The LMS algorithm circumvents this step by using an instantaneous estimate of the gradient. The instantaneous estimate is

$$\hat{\nabla}(J_k(i)) = \mathbf{y}(i)e_k^*(i) \quad (106)$$

$$e_k(i) = b_k(i) - \mathbf{w}_k^*(i)\mathbf{y}(i). \quad (107)$$

The tap weight update equation is

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) + \mu \mathbf{y}(i)e_k^*(i). \quad (108)$$

The LMS algorithm converges in the mean to the optimum tap weight vector if the step size parameter μ satisfies [31]

$$0 < \mu < 2/\lambda_{\max} \quad (109)$$

$$\sum_{j=1}^M \frac{\mu \lambda_j}{2 - \mu \lambda_j} < 1. \quad (110)$$

The step size parameter influences the rate of convergence to the optimum solution and the amount of excess error on top of $J_k(\text{min})$ at convergence. Large μ speeds convergence and increases the excess mean squared error, and small μ slows convergence and decreases the excess mean squared error.

Normalized LMS Algorithm. In practice the eigenvalues of the correlation matrix are not known a priori. To ensure that the step size meets the convergence criteria, the eigenvalues could be calculated from the estimated correlation matrix and used to adjust the step size of the LMS algorithm. The Normalized LMS

algorithm (NLMS) effectively incorporates this idea in the update equations by dividing the step size by the norm of the vector $\mathbf{y}$. The update equation is

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) + \hat{\mu} \frac{\mathbf{y}(i)}{|\mathbf{y}(i)|^2} e_k^*(i). \quad (111)$$

The NLMS algorithm converges in the mean to the optimum tap weight vector if the step size parameter satisfies

$$0 < \hat{\mu} < 2 \quad (112)$$

thus eliminating the explicit dependence upon the correlation matrix eigenvalues. For this reason, the NLMS algorithm was chosen to adapt the weight vector.

Both the LMS and the NLMS algorithms transient behaviour analysis is more complicated than that for the SD algorithm. However, the basic parameters remain the same. The convergence time is a function of the correlation matrix eigenvalues and eigenvectors, the starting point, and the step size parameter. Systems with large eigenvalue spreads converge slower than systems with small eigenvalue spreads. A small step size parameter slows convergence and reduces the excess mean square error at convergence when compared to a large step size.

Unlike the SD algorithm, an accurate and simple analytical expression for the transient behaviour of the NLMS algorithm does not exist. Complicated expressions that apply for Gaussian distributed inputs have been derived in [33, 34]. The model derived by Slock [35] is much less complicated and captures the essence of the convergence characteristics of the adaptive systems in this thesis. It will be used to estimate the convergence time of the NLMS algorithm as a function of the QS-CDMA system parameters.

NLMS Transient Behaviour Model. Slock's [35] model makes two assumptions about the input vectors, $\mathbf{y}(i)$, to the adaptive filter. The first is that all input vectors are independent and identically distributed. This is a standard assumption that was also applied in the derivation of the convergence behaviour of the SD algorithm. It is true for the QS-CDMA systems in this thesis since the data bits and noise used to generate $\mathbf{y}(i)$ are assumed to be independent and identically

distributed. The second assumption is that the input vector generator model is

$$\mathbf{y}(i) = s\mathbf{r}\mathbf{u} \quad (113)$$

where

$$\Pr(s = \pm 1) = 1/2 \quad (114)$$

$$r \sim ||\mathbf{y}|| \quad (115)$$

$$\Pr(\mathbf{u} = U_k) = \lambda_k / \text{tr } \mathfrak{R}. \quad (116)$$

In these equations r is a random variable with the same distribution as the norm of $\mathbf{y}$; U_k is the k th eigenvector in the eigenvector matrix U ; U is the eigenvector matrix of the matrix $\mathfrak{R}$; λ_k is the eigenvalue associated with eigenvector U_k ; and $\mathfrak{R}$ is the correlation matrix of the vector $\mathbf{y}$ ($\mathfrak{R} = \langle \mathbf{y}\mathbf{y}^* \rangle$). This model assumes that the vector $\mathbf{y}$ only contains the eigenmodes of the correlation matrix $\mathfrak{R}$ with a frequency that is proportional to the normalized eigenvalues of the modes. The true vector $\mathbf{y}$ actually contains a linear combination of the eigenmodes of $\mathfrak{R}$ at any one time. This simplification produces a transient behaviour model similar to the SD algorithm in which the modes of convergence are decoupled. The mean squared error at step i is

$$J_k(i) = J_k(\infty) + \sum_{j=1}^M \lambda_j \alpha_j^i (\tilde{\lambda}_j - J_k(\infty) \mu / (2 \text{tr } \mathfrak{R})) \quad (117)$$

where

$$J_k(\infty) = J_k(\min) \cdot (1 + \tilde{\mu} / (2 - \tilde{\mu})) \quad (118)$$

$$\alpha_j = 1 - \tilde{\mu}(2 - \tilde{\mu})\lambda_j / \text{tr } \mathfrak{R} \quad (119)$$

$$\tilde{\lambda}_j = (U^* \mathbf{w}_k^* \mathbf{w}_k U)_{jj}. \quad (120)$$

The mean squared error at convergence (i.e., $i = \infty$) is larger than the minimum mean squared error produced by the optimum solution $J_k(\min)$ because of the noise produced by the finite step size $\tilde{\mu}$. If $\tilde{\mu}$ is reduced to zero, then $J_k(\infty) = J_k(\min)$, but the algorithm will never converge.

This transient behaviour equation is very similar to that for the SD algorithm in that each mode, α_j , is dependent upon a single eigenvalue λ_j . The decoupling

of the modes allows a simple solution of the transient behaviour equation for i when a target mean squared error, $J_k(i)$, is set.

This model will be used to find the convergence time to a target mean squared error over various system parameters. The accuracy of this model will be tested at several points against Monte Carlo simulations of the transient behaviour of the NLMS algorithm in Chapter VII.

VII

Performance Evaluation

Several linear multiuser detectors have been derived in the previous chapters. In this chapter, the performance of these detectors is compared to each other and to an orthogonal multiple access scheme that uses BPSK modulation. The orthogonal multiple access scheme represents the upper performance limit of all the multiuser detectors, or equivalently, an asymptotic efficiency of unity.

The following dimensions of performance are investigated:

- The distribution of the asymptotic efficiency for decorrelating detectors with uniformly distributed delays over one and two chips.
- BER vs SNR curves for MMSE detectors with comparisons to selected decorrelating detectors.
- MMSE transient behaviour using Slock's simplified model of the NLMS algorithm and direct simulations of the NLMS algorithm.
- Comparison of the decorrelating and MMSE detectors performance to a BPSK modulated orthogonal multiple access scheme.

7.1 Decorrelating Detectors

The asymptotic efficiency characterizes the performance of decorrelating detectors. It is evaluated for various spreading codes, under full system loading, and with uniformly distributed user delays. Full system loading presents the worst case performance since the interference is maximized.

Ideally, we would like to select a set of spreading codes that maximizes the asymptotic efficiency of a given detector. In reality, code selection is very difficult due to the large number of available spreading codes for a given processing gain.

With a processing gain of 5, there are $2^5 = 32$ possible codes from which a set of 5 must be selected. There are C_5^{32} or just over 200,000 combinations of subsets. The asymptotic efficiency must be evaluated for each combination and compared to that of all other combinations to find the maximum value. Typical processing gains on the order of 16 or 32 will make this task formidable.

In reality, most of the code subsets will have poor asymptotic efficiency. The task of selecting codes can be greatly simplified by heuristically selecting a subset of the exhaustive set. The chosen heuristic is the code cross correlation since codes with low cross correlation should have to high asymptotic efficiency. Three common subsets that meet this criteria are maximal length sequences (M-sequences), Gold codes, and Hadamard codes. Each will be evaluated against the asymptotic efficiency measure.

The asymptotic efficiency is a function of the user delays. Each user's delay is assumed to be uniformly distributed over a Q chip interval for QS-CDMA. This probabilistic description of the delays leads to a probabilistic description of the asymptotic efficiency. The mapping between these distributions is a complicated function usually involving matrix inversion. Since an analytical solution is difficult to develop, Monte Carlo simulations were performed to determine the statistics of the asymptotic efficiency for one ($Q = 1$) and two ($Q = 2$) chip QS-CDMA for each of the three spreading code sets.

A single Monte Carlo trial produces a single asymptotic efficiency measure η_k for user k . The random variables in this trial are the user delays. The allocation of spreading codes to users is held fixed for the entire set of trials associated with each of the three code sets. The asymptotic efficiency is calculated using the equation appropriate for the detector in question from Table 2. Let trial i produce the asymptotic efficiency measure $\eta_k(i)$. The K user delays for trial i are selected randomly from K independent random variables, each uniformly distributed over the interval $[0, Q]$.

The raw data from M Monte Carlo trials was formatted to fulfill two goals. First, to illustrate the performance of the selected spreading code set relative to other spreading code sets, and second, to illustrate the variation in performance

of the individual users within the spreading code set. These goals were met by forming three distributions of the asymptotic efficiency:

1. The composite distribution is formed over all the generated data by $D(\{\eta_k(i) \forall k, i\})$ where the operator D forms the distribution. It illustrates the asymptotic efficiency variation of the entire spreading code set and is used to compare spreading code sets.
2. The distribution of the means is formed by first calculating the mean asymptotic efficiency of each user using $\bar{\eta}_k = \sum_i \eta_k(i)/M$. The distribution of the means is $D(\{\bar{\eta}_k \forall k\})$. It illustrates the variation of the users' mean asymptotic efficiency over the individual users' codes.
3. The overlay of distributions is formed by overlaying the distributions of each of the K users. A single user's distribution is $D(\{\eta_k(i) \forall i\})$. The resultant graph contains K of these distributions. It illustrates the variation of the different users' asymptotic efficiency distributions over the delays.

7.2 MF Decorrelating Detector

The MF decorrelating detector uses a matched filter bank for Stage I and the combining matrix Ψ^1 in Stage II. The length of the tested M-sequences and Gold codes is 31, and the length of the tested Hadamard codes is 32. The M-sequence set was constructed by cyclically shifting a length 31 M-sequence by single chips to produce the set of 31 spreading sequences. Each base spreading code set was augmented with Q zeros for Q chip QS-CDMA where $Q = 1$ or $Q = 2$. Thus, total spreading sequence lengths, N , were 32 or 33 for M-sequences and Gold codes, and 33 or 34 for Hadamard codes. Rectangular chip waveforms were used in all simulations, and one thousand Monte Carlo trials were used to generate the raw data for each code set. The asymptotic efficiency equation is η^1 .

Figures 13 through 15 contain the composite, mean, and overlay distributions of the asymptotic efficiency for M-sequences, Gold codes, and Hadamard codes for single and double chip QS-CDMA. In all cases, the asymptotic efficiency is

expressed in decibels. The mean asymptotic efficiency of the various spreading code sets is listed in Table 4.

Code Set	$Q = 1$	$Q = 2$
M-sequences	-1 dB	-2 dB
Gold codes	-1 dB	-2.3 dB
Hadamard codes	-0.55 dB	-1.4 dB

Table 4: MF decorrelating detector Ψ^1 mean asymptotic efficiency for three spreading code sets.

Based upon the mean of the spreading code set and variance of the composite distribution (Figure 13) Hadamard codes are the best spreading code set choice for the MF decorrelating detector. Examination of the distribution of the mean asymptotic efficiency within a given code set (Figure 14) shows that the performance of individual users has a larger variation for Hadamard codes than for M-sequences or Gold codes. The distribution of the means for M-sequences and Gold codes is tightly bound around the composite distribution's mean, while those for Hadamard codes are not. This is also reflected in the overlay of the individual distributions in Figure 15. The individual distributions for M-sequences are quite similar (tightly grouped horizontal lines in each bar) but those for Hadamard codes vary substantially (widely varying horizontal lines in each bar). The Gold code distributions lie between these two cases.

This characteristic is due to the symmetry of the code set. Since the M-sequence code set is formed by cyclically shifting a given M-sequence, all codes exhibit the same auto and cross correlation properties. The Gold code set is somewhat less symmetric than the M-sequence set, and the Hadamard code set exhibits very little symmetry. In fact, the code sequence consisting of all 1's in the Hadamard code set does not even have the balance of all the other codes in the set.

When using Hadamard codes, the asymmetry of the code set will give some users an inherent performance advantage over the other users. For single chip

QS-CDMA, the advantage is less than 1 dB, and for double chip QS-CDMA, the advantage expands to just over 2 dB on average.

Figure 13: Composite distribution of the asymptotic efficiency of detector Ψ^1 for three spreading code sets and $Q=1,2$. The horizontal scales are the asymptotic efficiency in dB and the vertical scales are the probability of occurrence.

Figure 14: Distribution of the mean asymptotic efficiency of detector Ψ^1 for three spreading code sets and $Q=1,2$. The horizontal scales are the mean asymptotic efficiency in dB and the vertical scales are the frequency of occurrence.

Figure 15: Overlay of the asymptotic efficiency distributions of detector Ψ^1 for three spreading code sets and $Q=1,2$. The horizontal scales are the mean asymptotic efficiency in dB and the vertical scales are the probability of occurrence.

7.3 CF Decorrelating Detector

The CF decorrelating detector uses a chip filter in Stage I and Ψ^2 in Stage II. The same spreading sequences and performance metrics that were used for decorrelating detector Ψ^1 were used for the chip filter decorrelating detector Ψ^2 . In addition, the oversampling rate was varied to determine the effect upon performance.

Figure 16. Distribution of the mean asymptotic efficiency of decorrelating detector Ψ^2 with three degrees of oversampling. The horizontal scale is in dB and the vertical scale is the frequency of occurrence. Parameters: Gold codes, $N=32$, $Q=1$.

The distribution of the mean asymptotic efficiency for various degrees of oversampling is illustrated in Figure 16. It can be seen that the composite mean asymptotic efficiency increases dramatically from $L = 1$ to $L = 2$ times oversampling (i.e., from -11 dB to -2.7 dB). For $L > 2$, the further performance improvement is marginal and is not worth the increased complexity. Similar results were observed when M-sequences and Hadamard codes were used as spreading sequences. Two times oversampling was subsequently used in all performance evaluations.

With two times oversampling, there are two differences in the distributions of the asymptotic efficiency of Ψ^1 and Ψ^2 . First, the mean asymptotic efficiency of Ψ^2 is slightly less than that of Ψ^1 , and second, the variance of all the Ψ^2 distributions is larger. The horizontal scales in Figures 17 through 19 cover twice the range of the horizontal scales in the corresponding Ψ^1 figures. Otherwise, Ψ^2 exhibits all the same characteristics with respect to base spreading code selection as Ψ^1 . Once again, Hadamard codes yield the highest mean asymptotic efficiency.

Code Set	$Q = 1$	$Q = 2$
M-sequences	-2.5 dB	-3.9 dB
Gold codes	-2.7 dB	-4.6 dB
Hadamard codes	-1.7 dB	-2.9 dB

Table 5: CF decorrelating detector Ψ^2 mean asymptotic efficiency for three spreading code sets.

Figure 17: Composite distribution of the asymptotic efficiency of detector Ψ^2 for three spreading code sets and $Q=1, 2$. The horizontal scales are the asymptotic efficiency in dB and the vertical scales are the probability of occurrence.

Figure 18: Distribution of the mean asymptotic efficiency of detector ψ^2 for three spreading code sets and $Q=1,2$. The horizontal scales are the mean asymptotic efficiency in dB and the vertical scales are the frequency of occurrence.

Figure 19: Overlay of the asymptotic efficiency distributions of detector Ψ^2 for three spreading code sets and $Q=1,2$. The horizontal scales are the asymptotic efficiency in dB and the vertical scales are the probability of occurrence.

7.4 Split Stage II Decorrelating Detectors

For a given processing gain N , the CF split Stage II decorrelating detectors can support at most $K = \lfloor N/(Q + L^{-1}) \rfloor$ users. The codes for these K users are selected from a universe of 2^{N-Q} codes. Since evaluating the asymptotic efficiency for all possible code sets is an almost impossible task, the problem was simplified to selecting K codes from a subset of this universe of codes. The subsets chosen were M-sequences, Gold codes, and Hadamard codes as evaluated for the MF and CF decorrelating detectors.

Even with this simplification, there are C_K^{N-Q} different subsets to be tested. For small spreading gains an exhaustive search for the best asymptotic efficiency can be performed. For example, with $N = 8$, $Q = 1$, and $L = 1$ the maximum number of users is 4, and the number of subsets to be tested is just 35. However, for a spreading gain of 32, this number grows to $C_{16}^{31} > 300 \cdot 10^6$. An exhaustive search over all these subsets would take a long time. The maximum spreading gain for practical exhaustive searches is limited to approximately 16.

One way to find good subsets of spreading codes with large processing gains is to extend the patterns observed in code selection for systems with small processing gains. This technique will work for M-sequences and Hadamard codes since they have code sets with low processing gains, but it will not work for Gold codes where the minimum processing gain is 32. We are limited to either a computationally intensive exhaustive search which will guarantee finding the best subset, or a random search which may find a good subset in far less time.

Regardless of how the code search is performed, the performance metric is always the same. We are looking for the subset with the highest composite mean asymptotic efficiency for the delay independent decorrelating detector Ψ^5 . This detector has the lowest implementation complexity and thus is the most interesting case. Only single chip QS-CDMA systems are considered since systems with $Q > 1$ have drastically reduced capacity.

M-Sequences. The first set of codes to be evaluated is M sequence—constructed as described in the MF decorrelating detector section. Exhaustive searches were

performed for processing gains of 8 and 16, and the results were extended to postulate a good subset with a processing gain of 32. The results are tabulated in Table 6. In this table, $\bar{\eta}$ is the composite mean asymptotic efficiency of the subset; η_{min} is the minimum asymptotic efficiency in the subset; η_{max} is the maximum asymptotic efficiency in the subset; and η_k lists the asymptotic efficiencies of all the codes in the subset. The mapping from the code numbers in this table to the actual codes is contained in the Appendix.

For the shorter codes, it appears that choosing every second code in the M-sequence set yields the best performance. When this pattern was extended to length 32 M-sequences, it yielded a subset with the same average performance and slightly larger variance than the lower spreading gain subsets.

The choice of every second code in the M-sequence set is supported by the underlying construction of the set. Each code is a single chip shifted version of the previous code. When a given code is delayed by one chip, it will be heavily correlated with the next code. This artifact of the M-sequence set can be avoided by including only every second code in the subset, as was found by the exhaustive search.

Gold Codes. Since the minimum Gold code length is 31, an exhaustive search for the best subset would take a very long time. Instead, a random search was performed to find a good subset, and it is listed in Table 6. Gold codes have better performance than the same length M-sequences but do not exhibit the alternating code selection pattern. Experiments with an alternating pattern in selecting the Gold code subset yielded a code matrix A with insufficient rank. Better Gold code subsets may exist, but only an exhaustive search will tell.

Hadamard Codes. Since low processing gain Hadamard codes exist, an exhaustive search can be performed to find the best subset. The best subsets for $N=9$ and $N=17$ are listed in Table 6 and consist of every second code from the Hadamard set. This characteristic was used to postulate the best subset with a spreading gain of 32 that is listed in the table. For nearly equal processing gains, Hadamard codes have a much higher average asymptotic efficiency and much lower

M-sequences

N	K	Code numbers	$\bar{\eta}$	η_{min}	η_{max}	η_k
8	4	2 4 6 7	-8.2	-8.9	-7.2	-7.2 -8.5 -8.9 -8.9
16	8	2 4 6 8 10 11 13 15	-8.2	-11	-7.4	-7.4 -7.7 -8.2 -7.7 -11 -11 -7.7 -7.7
32	16	2 4 6 8 10 12 14 16	-8.2	-14	-7.7	-7.7 -7.7 -7.7 -7.7
		18 20 22 24 26 28				-8.0 -7.7 -8.0 -7.7
		30 31				-7.7 -8.0 -8.0 -7.7
						-8.0 -7.7 -14 -14

Gold Codes

N	K	Code numbers	$\bar{\eta}$	η_{min}	η_{max}	η_k
32	16	1 2 3 4 5 6 8 13 16	-7.2	-9.6	-5.4	-5.4 -8.5 -7.2 -6.6
		17 19 20 22 25 26				-7.7 -9.6 -8.2 -7.4
		28				-7.4 -5.7 -6.6 -5.4
						-9.2 -5.5 -8.0 -8.5

Hadamard Codes

N	K	Code numbers	$\bar{\eta}$	η_{min}	η_{max}	η_k
9	4	1 3 5 7	-2.2	-3.0	-0.5	-0.5 -3.0 -3.0 -3.0
17	8	1 3 5 7 9 11 13 15	-2.5	-3.0	-0.3	-0.3 -3.0 -3.0 -3.0 -3.0 -3.0 -3.0 -3.0
33	16	1 3 5 7 9 11 13 15	-2.8	-3.0	-0.1	-0.1 -3.0 -3.0 -3.0
		17 19 21 23 25 27				-3.0 -3.0 -3.0 -3.0
		29 31				-3.0 -3.0 -3.0 -3.0
						-3.0 -3.0 -3.0 -3.0

Table 6: Delay independent decorrelating detector Ψ^5 asymptotic efficiency of various spreading code sets for single chip QS-CDMA without oversampling. The asymptotic efficiency is expressed in dB.

variance than either M-sequences or Gold codes and are thus the best choice of spreading codes for decorrelating detector Ψ^5 .

Other Stage IIb's. The best spreading code sets were found for the delay independent decorrelating detector Ψ^5 . The distributions of the mean asymptotic efficiency of the users for all possible Stage IIb's (i.e., Ψ^3 to Ψ^6) are illustrated in Figures 20 to 22. These distributions were formed from the raw data generated by one thousand Monte Carlo trials with the user delays uniformly distributed over a single chip interval ($Q = 1$). The distributions are numbered 3, 4, 5, and 6 to correspond to decorrelating detectors Ψ^3 , Ψ^4 , Ψ^5 , and Ψ^6 respectively. The spreading code subsets from Table 6 were used in all cases.

The figures illustrate the following characteristics of the split Stage II decorrelating detectors:

- Detectors that use the delay information (i.e., Ψ^3 , Ψ^4 , and Ψ^6) have higher mean asymptotic efficiency than those that do not use this information (i.e., Ψ^5). This observation agrees with our intuition that better decisions can be made with more information.
- Using all outputs of the code matrix left inverse A^L (i.e., Ψ^6) yields a higher asymptotic efficiency than using only a subset of the outputs (i.e., Ψ^4 and Ψ^5). This performance gain, however, is at the expense of K times greater complexity since Ψ^6 requires $N = K(Q+1)$ combining weights per user, and Ψ^4 and Ψ^5 require only $Q+1$ weights per user.
- Using the standard left inverse of the delay matrix (in Ψ^3) yields poorer performance than using a SNR maximizing left inverse of the delay matrix (in Ψ^6) even though both Ψ^3 and Ψ^6 use all N outputs of the code matrix left inverse. This affirms the use of an additional criterion such as SNR in selecting the left inverse of the combining matrix.

Decorrelating Detector Performance Summary. Table 7 summarizes the average asymptotic efficiency of the six decorrelating detectors for the three code

sets. The data illustrates the performance versus complexity trade-off. The most complex detector has the highest asymptotic efficiency, and the least complex detector has the lowest asymptotic efficiency with the exception of the Hadamard code set. It is obvious that the best spreading codes are Hadamard codes independent of the choice of the detector.

Detector	Gold Codes	M-Sequences	Hadamard Codes	
Ψ^1	-1.0 dB	-1.0 dB	-0.55 dB	Most complex
Ψ^2	-2.7 dB	-2.5 dB	-1.7 dB	2X oversampling
Ψ^3	-6.3 dB	-6.0 dB	-4.4 dB	Delay dependent
Ψ^4	-6.0 dB	-5.9 dB	-2.6 dB	Short vector
Ψ^5	-7.1 dB	-8.2 dB	-2.8 dB	Least complex
Ψ^6	-2.8 dB	-2.6 dB	-1.0 dB	Long vector

Table 7: Code set average asymptotic efficiency for the decorrelating detectors ($Q = 1$).

Figure 20: Mean asymptotic efficiency distributions for detectors Ψ^3 through Ψ^6 with Gold spreading codes.

Figure 21: Mean asymptotic efficiency distributions for detectors Ψ^1 through Ψ^6 with M-sequence spreading codes.

Figure 22: Mean asymptotic efficiency distributions for detectors Ψ^3 through Ψ^6 with Hadamard spreading codes.

Simplified Delay Independent Decorrelating Detector for Hadamard Codes. For M-sequences and Gold codes, the code matrix A is square and has full rank. For Hadamard codes, the code matrix is rectangular with one more row than the number of columns and it has full column rank. This section examines the effect of discarding one row of A to make the code matrix square for Hadamard codes [36].

Consider the system of equations $\mathbf{z} = A\mathbf{x}$ where $\mathbf{x} = TC\mathbf{b}$. Partition the matrix A into a square matrix G of dimension $K(Q+1)$ and a row vector $\mathbf{g}'$, i.e.,

$$A = \begin{bmatrix} G \\ \mathbf{g}' \end{bmatrix} \quad (121)$$

where $\mathbf{g}'$ is the last row of A . Since A was selected to have linearly independent columns, matrix G has full rank and a unique inverse G^{-1} . Vector $\mathbf{g}$ can be expressed as a linear combination of the rows in G by $\mathbf{g} = G'\mathbf{h}$ where $\mathbf{h}$ is only dependent upon the choice of spreading codes. Now $\mathbf{z}$ can be expressed as

$$\mathbf{z} = \begin{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_{N-1} \end{bmatrix} \\ z_N \end{bmatrix} = \begin{bmatrix} G \\ \mathbf{g}' \end{bmatrix} \mathbf{x} \quad (122)$$

Thus

$$z_N = \mathbf{g}'\mathbf{x} = \mathbf{h}'G\mathbf{x} = \mathbf{h}' \begin{bmatrix} z_1 \\ \vdots \\ z_{N-1} \end{bmatrix} \quad (123)$$

which shows that z_N is an a priori known linear combination of the first $N-1$ entries in $\mathbf{z}$. It yields no further information about the transmitted bit vector $\mathbf{b}$ and can be discarded in the detection process. Discarding the last entry in $\mathbf{z}$ reduces the system equations to

$$\begin{bmatrix} z_1 \\ \vdots \\ z_{N-1} \end{bmatrix} = GTC\mathbf{b} \quad (124)$$

The delay independent decorrelating matrix, Ψ^H , for this system is

$$\Psi^H = C_z^* T^{L5} G^{-1} \quad (125)$$

where T^{L5} is the previously defined delay independent left inverse of T .

Choosing to discard the last row in A was arbitrary. The above analysis can be repeated for discarding any row in A , and each case will yield a different matrix G . There are N possible matrices G since A has N rows. These N cases were tested to determine which ones yielded the highest average asymptotic efficiency for the Hadamard code sets in Table 6. Forming G by discarding one odd numbered row in A yielded detectors with the asymptotic efficiency equal to -3 dB for all users. Discarding one even numbered row yielded a slightly lower asymptotic efficiency.

By discarding the last row in A , the product $T^{L5}G^{-1}$ is

$$T^{L5}G^{-1} = \begin{bmatrix} 0 & a_1(2) & 0 & a_1(4) & 0 & \cdots & a_1(N-1) \\ 0 & a_2(2) & 0 & a_2(4) & 0 & \cdots & a_2(N-1) \\ 0 & a_3(2) & 0 & a_3(4) & 0 & \cdots & a_3(N-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & a_K(2) & 0 & a_K(4) & 0 & \cdots & a_K(N-1) \end{bmatrix} \cdot \frac{1}{K} \quad (126)$$

where $a_k(l)$ is the l th chip of the k th user's spreading code and is equal to either +1 or -1. The receiver for the code set is extremely simple as shown in Figure 23. Since multiplication by the bipolar spreading code can be accomplished by a simple sign change, forming the decision variable for each user requires only K additions and one multiplication. This is far less complex than any of the decorrelating detectors for M-sequences or Gold codes consider so far, and is comparable to the complexity of the conventional correlation receiver.

Implementation of this new delay independent decorrelating detector for Hadamard codes entails discarding the last entry from the received vector $\mathbf{y}$. This results in a slight decrease in the average asymptotic efficiency to -3 dB for all the Hadamard codes as listed in Table 6. This slight performance loss is a small price to pay for the large reduction in complexity of the delay independent decorrelating detector for Hadamard codes.

Figure 23: Delay independent decorrelating detector for user k when using the Hadamard code subset.

Oversampling Factor. The oversampling factor is the final dimension to be examined for the split Stage II decorrelating detectors. Previously it was shown that the detector capacity can be improved by increasing the oversampling factor. For single chip QS-CDMA, the capacity can be improved by up to 33% for two times oversampling and by up to 50% for three times oversampling. This potential capacity increase can only be realized if there exists a subset which meets the criteria that the code matrix A has linearly independent columns. In all the simulations performed using M-sequence and Hadamard code sets with a processing gain of 16 or less, it was found that the availability of codes limited the capacity. No subsets could be found that produced a code matrix A with sufficient rank. This does not mean that the potential capacity increase cannot be realized. Expanding the code set from which subsets are selected may bear fruit, but at the expense of increased complexity in the search process. This avenue was not examined at this time.

Figure 28: BER curves for CF detectors Ψ^2 (decorrelating) and W^2 (MMSE) with Gold spreading codes.

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7.5 MMSE Detectors

Unlike the decorrelating matrix, the MMSE matrix is a function of the users' signal-to-noise ratio. The asymptotic efficiency of the MMSE detector cannot be used to quantify the BER performance over all SNR's. Instead, the BER must be calculated for the SNR's of interest.

There are two approaches to calculating the BER. It can be calculated analytically using equation (56), or it may be estimated by Monte Carlo simulations. The former technique is computationally intensive when the number of users is large since the interference is calculated over all interfering bit combinations. For a 32 user system, there are 2^{31} interfering bit combinations, resulting in over 2 billion calculations to get a single BER point. For moderate BER's, a Monte Carlo simulation can yield reliable results using far fewer calculations. As a general guide, the number of Monte Carlo trials performed should be at least ten times the inverse of the BER. Thus, for a BER of 10^{-4} , use 10^5 trials. The Monte Carlo simulation only starts to approach the complexity of the analytical BER calculation for BER's less than 10^{-6} . BER's this low are of little interest since the application is a voice communication system where BER's in the range of 10^{-3} to 10^{-4} are common.

The asymptotic efficiency of the MMSE detector is equal to that of the corresponding decorrelating detector, and thus need not be calculated since it was covered in the previous section.

The SNR is defined as the bit energy to power spectral density ratio (E_b/N_0) where the bit energy is

$$E_b = |c_k|^2 \int_0^N s_k^2(t) dt. \quad (127)$$

In this equation, $|c_k|$ is the amplitude and $s_k(t)$ is the spreading waveform of user k . This SNR definition allows easy comparison of the simulated BER curves to standard BPSK BER curves, which lower bound the BER of all the multiuser detectors examined. The BPSK BER curve is calculated for a single user operating in an AWGN environment with a SNR as indicated on the graphs. There is no interference from other users. This represents the best performance that any multiuser detector can achieve and is equivalent to operation with fully orthogonal

codes with zero relative delays.

7.5.1 MF Stage I MMSE Detector

The BER as a function of the SNR for the MF MMSE receiver W^1 is shown in Figures 24 to 26. The average BER taken over all users is displayed in the figures. The variation of the BER among the users follows the variation of the asymptotic efficiency among the users as shown in previous sections. The bound on the delay is set to one and two chips, and the BER curves for the decorrelating matrix Ψ^1 are superimposed for comparison. In all cases, the MMSE Stage II performs slightly better than the decorrelating Stage II. The MMSE Stage II was expected to have better performance since it uses all the side information, including the user amplitudes. The degree of improvement indicates that the additional knowledge of the user amplitudes provides only a marginal advantage over the decorrelating Stage II.

Gold spreading codes yielded the lowest BER for a given SNR. Hadamard codes yielded a slightly higher BER, and M-sequences yielded the worst BER. This result disagrees with the asymptotic efficiency measures calculated in the previous sections. According to the average asymptotic efficiency, Hadamard codes should yield the lowest BER, Gold codes should be next, and M-sequences should be last. The switched position of Hadamard codes and Gold codes in the BER results is due to the distribution of the asymptotic efficiency. The mean asymptotic efficiency was calculated assuming that each trial has equal weight, but the BER is a non-linear function of the asymptotic efficiency. The occurrence of a low asymptotic efficiency, which produces a high BER, is heavily weighted in the averaging process. Referring back to the distributions of the asymptotic efficiency (Figure 14), it can be seen that the variance of the asymptotic efficiency for Gold codes is much less than the variance of the asymptotic efficiency for Hadamard codes. The Hadamard code trials with low asymptotic efficiency increase the average BER compared to that of Gold codes.

Figure 24: BER curves for MF detectors Ψ^1 (decorrelating) and W^1 (MMSE) with M-sequence spreading codes.

Figure 25: BER curves for MF detectors Ψ^1 (decorrelating) and W^1 (MMSE) with Gold spreading codes.

haviour and average convergence time is calculated over the delays.

- The user phases are assumed to be independent and uniformly distributed over $[0, 2\pi)$. The average transient behaviour and average convergence time is calculated over the phases.
- The minimum SNR is 10 dB. This limit yields a good minimum BER with the spreading codes and detectors evaluated in the previous sections.
- The NLMS algorithm step size is $\mu = 1$. This achieves minimum convergence time while still maintaining stability [35].
- The starting weight vector is $\mathbf{w}_k(0) = \mathbf{0}$ for all users.

The independent variable to be adjusted in the simulations is the power spread of the users, or equivalently, the SNR spread of the users since a user's SNR is directly proportional to his power.

The transient behaviour of the NLMS algorithm is presented in two ways. First, the average time to convergence as a function of the users' SNR is illustrated. These results were calculated using Slock's model. Secondly, the NLMS algorithm is simulated for a few test cases to confirm the validity of Slock's model when applied to QS-CDMA systems.

The transient behaviour was simulated over the three spreading code sets for SNR's between 10 and 30 dB. The data is presented for three combinations of the users' powers:

- All users have equal power. This simulates perfect power control.
- There is one weak user with a SNR of 10 dB, and the remaining users' SNR's vary from 10 to 30 dB.
- There is one strong user with a SNR that varies from 10 to 30 dB, and the remaining users are held at 10 dB SNR.

The second two cases are the extreme operating points of the system. One weak user operating in the presence of many strong users is the strongest interference

7.5.2 CF Stage I MMSE Detectors

There are five implementations of Stage II when Stage I is a chip filter. The BER performance of the five implementations is detailed below.

W^2 Stage II. The BER curves for Stage II equal to W^2 are shown in Figures 27 to 29. Stage I was oversampled at twice the chip rate to improve performance. Oversampling at higher rates yielded only marginal performance improvements over this case. By comparing to the MF detector W^1 , it can be seen that there is a loss in performance. This is mainly due to the larger variance in the asymptotic efficiency when the chip filter is used in place of the matched filter bank. As with the matched filter bank results, Gold spreading codes yielded the lowest BER for a given SNR.

Figure 27: BER curves for CF detectors Ψ^2 (decorrelating) and W^2 (MMSE) with M-sequence spreading codes.

Figure 28: BER curves for CF detectors Ψ^2 (decorrelating) and W^2 (MMSE) with Gold spreading codes.

Figure 29: BER curves for CF detectors Ψ^2 (decorrelating) and W^2 (MMSE) with Hadamard spreading codes.

Split Stage II MMSE Detectors. When the chip filter is used in Stage I, the complexity of Stage II is reduced by splitting it into two parts. Stage IIa is the left inverse of the code matrix, and the entire Stage II is either W^4 (short vector) or W^6 (long vector). The BER performance of these two receivers is illustrated in Figures 30 to 32. The BER curves of the corresponding decorrelating Stage II's (Ψ^4 and Ψ^6) are included in the figures for the purpose of comparison, and the BER curve of the delay independent decorrelating detector Ψ^5 is also plotted.

These receivers have poor BER performance when Gold codes or M-sequences are used for spreading codes. This was also found in the asymptotic efficiency simulations of the corresponding decorrelating detectors analyzed previously. Hadamard codes, however, exhibit respectable performance with the BER less than 10^{-3} at 10 dB SNR for even the short vector detector. It can also be seen that there is practically no performance advantage in using the short vector MMSE detector over the delay independent decorrelating detector for Hadamard codes. For Gold codes and M-sequences, the delay independent decorrelating detector has the worst performance.

Figure 30: BER curves for CF split Stage II detectors with M-sequence spreading codes.

Figure 31: BER curves for CF split Stage II detectors with Gold spreading codes.

Figure 32: BER curves for CF split Stage II detectors with Hadamard spreading codes.

7.6 MSE Transient Behaviour

Slock's simplified transient behaviour model is used to yield insight into the relationship between the system parameters and the transient behaviour of the NLMS algorithm as applied to the MMSE detector. Monte Carlo simulations of the NLMS algorithm were performed to confirm the results of the model. This approach was used because Monte Carlo simulations of the NLMS algorithm over all system parameters is a very time consuming task. Slock's model greatly simplified the task since it enables a quasi-analytical solution for the convergence time.

Lee [37] noted that the convergence time of the NLMS algorithm is a strong function of the users' power spread. In practical systems, the convergence time is limited to an interval called the training time. This limit imposes a corresponding limit upon the power spread of the users so that they can achieve convergence. Analysis of the transient behaviour is motivated by the need to determine the maximum power spread that can be achieved in a specific time interval.

A definition of convergence is required to determine the time to convergence. The NLMS algorithm will converge to $J(\infty)$, the minimum mean squared error, in an infinite number of iterations. In practice, convergence is assumed once the mean squared error gets close to $J(\infty)$. Close is defined as to within 1 dB in this work.

Even with the quasi-analytic expression for the transient behaviour of the NLMS algorithm, the analysis is complicated by all the combinations of user spreading codes, delays, phases, and powers. To facilitate the analysis, the following simplifications were used:

- Hadamard codes, Gold codes, and M-sequences are used as spreading codes. This follows the analysis of the asymptotic efficiency and BER of the previous sections. For Hadamard codes, $N = 33$, and for Gold codes and M-sequences, $N = 31$. The systems are loaded to full capacity.
- The user delays are assumed to be independent and uniformly distributed over a Q chip interval, and $Q = 1$ in all cases. The average transient be-

haviour and average convergence time is calculated over the delays.

- The user phases are assumed to be independent and uniformly distributed over $[0, 2\pi)$. The average transient behaviour and average convergence time is calculated over the phases.
- The minimum SNR is 10 dB. This limit yields a good minimum BER with the spreading codes and detectors evaluated in the previous sections.
- The NLMS algorithm step size is $\mu = 1$. This achieves minimum convergence time while still maintaining stability [35].
- The starting weight vector is $\mathbf{w}_k(0) = \mathbf{0}$ for all users.

The independent variable to be adjusted in the simulations is the power spread of the users, or equivalently, the SNR spread of the users since a user's SNR is directly proportional to his power.

The transient behaviour of the NLMS algorithm is presented in two ways. First, the average time to convergence as a function of the users' SNR is illustrated. These results were calculated using Slock's model. Secondly, the NLMS algorithm is simulated for a few test cases to confirm the validity of Slock's model when applied to QS-CDMA systems.

The transient behaviour was simulated over the three spreading code sets for SNR's between 10 and 30 dB. The data is presented for three combinations of the users' powers:

- All users have equal power. This simulates perfect power control.
- There is one weak user with a SNR of 10 dB, and the remaining users' SNR's vary from 10 to 30 dB.
- There is one strong user with a SNR that varies from 10 to 30 dB, and the remaining users are held at 10 dB SNR.

The second two cases are the extreme operating points of the system. One weak user operating in the presence of many strong users is the strongest interference

operating condition for the weak user. Under these conditions, adapting to the weak user should take a very long time. All other power combinations will yield shorter convergence times than this worst case

7.6.1 MMSE Detectors W^1 and W^2

Figures 33 and 34 show the average convergence time of equal power users as a function of their SNR and the spreading code set for detectors W^1 and W^2 . Each graph contains K curves, one for each user.

Figures 35 and 36 show the average convergence time of all users as a function of their SNR and the spreading code set with one weak user (fixed SNR=10 dB) and $K-1$ strong users for detectors W^1 and W^2 . The convergence time of the weak user increases exponentially as the power difference between the weak and strong users increases. The strong users' convergence time is substantially less than that of the weak user. These results illustrate the worst case relationship between the convergence time and the power spread of the users.

Placing an upper limit on the convergence time of the weak user limits the maximum power spread between the weak user and the strong users. Table 8 lists this maximum power spread for upper limits of 5000 and 10000 iterations. Some form of loose power control is required to keep the power within this spread of 12 to 18 dB. This is a far wider tolerance on power control than is required by CDMA systems that use the conventional matched filter for a detector which require power control to within a single dB.

Figures 37 and 38 show the average convergence time of all users as a function of their SNR and the spreading code set with one strong and $K-1$ weak users (fixed SNR=10 dB) for detectors W^1 and W^2 . The presence of one strong user does not appreciably affect the convergence time of the weak users when compared to the equal power case.

Figures 39 to 44 show the MSE transient behaviour of the simulated NLMS algorithm and Slock's model for the following cases:

- Equal power users at 10 and 30 dB SNR (Figures 39 and 42).

Detector	Code Set	Maximum Iterations	
		5000	10000
W^1	Hadamard	13.5 dB	16.0 dB
	Gold	13.0 dB	15.0 dB
	M-seq.	12.0 dB	14.0 dB
W^2	Hadamard	15.0 dB	18.5 dB
	Gold	15.0 dB	18.0 dB
	M-seq.	13.5 dB	18.0 dB

Table 8: Maximum power spread in dB between the weak user and the strong users for a given convergence time for the MF and CF MMSE detectors W^1 and W^2 .

- One weak user at 10 dB SNR and $K - 1$ strong users at 30 dB SNR (Figures 40 and 43).
- One strong user at 30 dB SNR and $K - 1$ weak users at 10 dB SNR (Figures 41 and 44).

In all cases, Slock's model either accurately estimates the transient behaviour or slightly over estimates the convergence time. Slock's model thus gives a slightly conservative estimate of the convergence time.

Figure 33: Average convergence time of the MF MMSE detector W^1 with equal power users.

Figure 34: Average convergence time of the CF MMSE detector W^2 with equal power users.

Figure 35: Average convergence time of the MF MMSE detector W^1 for all users when the single weak user's SNR is held constant at 10 dB and the remaining users' SNR is varied.

Figure 36: Average convergence time of the CF MMSE detector W^2 for all users when the single weak user's SNR is held constant at 10 dB and the remaining users' SNR is varied.

Figure 37: Average convergence time of the MF MMSE detector W^1 for all users when the single strong user's SNR is varied and the remaining weak users have a fixed SNR=10 dB.

Figure 38: Average convergence time of the CF MMSE detector W^2 for all users when the single strong user's SNR is varied and the remaining weak users have a fixed SNR=10 dB.

Figure 39: NLMS MSE transient behaviour of the MF MMSE detector W^1 with equal power users.

Figure 40: NLMS MSE transient behaviour of the MF MMSE detector W^1 with one weak user.

Figure 41: NLMS MSE transient behaviour of the MF MMSE detector W^1 with one strong user.

Figure 42: NLMS MSE transient behaviour of the CF MMSE detector W^2 with equal power users.

Figure 43: NLMS MSE transient behaviour of the CF MMSE detector W^2 with one weak user.

Figure 44: NLMS MSE transient behaviour of the CF MMSE detector W^2 with one strong user.

7.6.2 MMSE Detectors W^4 and W^6

Stage II was separated into two parts to simplify the receiver. Stage IIa is a static code matrix, and Stage IIb was designed to achieve the MMSE. Two MMSE matrices were considered. The long vector version W^6 uses all the outputs from Stage IIa to form a decision variable for each user, and the short vector version W^4 uses only a subset of the outputs from Stage IIa. The adaptive algorithm has to adapt N tap weights per user for the long vector version, and only $Q + 1$ tap weights per user for the short vector version.

Figures 45 to 47 contain the NLMS simulations of the transient behaviour of the short and long vector MMSE detectors with equal power users at 10 and 30 dB SNR for the three spreading code sets. All of the long vector examples converge in approximately the same time (150 iterations). The short vector examples converge in far less time (less than 20 iterations). This is expected since reducing the number of adaptive taps reduces the convergence time.

Figure 48 illustrates the transient behaviour when there is one weak user (10 dB SNR) and $K - 1$ strong users (30 dB SNR). This corresponds to the worst case power spread from the previous examples. In all cases, the strong users converge in a time similar to that for equal power users. For the long vector MMSE detector, the weak user takes a very long time to converge, but for the short vector MMSE detector, the weak user converges in approximately the same time as the strong users. This is due to the separation of the users by the inverse code matrix, A^L , into independent subspaces. Each subspace contains no interference from the other users, and thus adaptation is independent of the power spread of the users.

Gold Codes, N=32, Q=1, K=16

Figure 45: NLMS MSE transient behaviour of CF MMSE detectors W^4 (short) and W^6 (long) with equal power users at 10 dB and 30 dB and Gold spreading codes.

M-sequences, $N=32$, $Q=1$, $K=16$

Figure 46: NLMS MSE transient behaviour of CF MMSE detectors W^4 (short) and W^6 (long) with equal power users at 10 dB and 30 dB and M-sequence spreading codes.

Hadamard Codes, $N=33$, $Q=1$, $K=16$

Figure 47: NLMS MSE transient behaviour of CF MMSE detectors W^4 (short) and W^6 (long) with equal power users at 10 dB and 30 dB and Hadamard spreading codes.

Figure 48: NLMS MSE transient behaviour of CF MMSE detectors W^4 (short) and W^6 (long) with one weak user in the presence of many strong users.

VIII

Conclusions

This work examined several aspects of a QS-CDMA system. These aspects are outlined in the following paragraphs.

Multuser Detection of QS-CDMA. Quasi-synchronous CDMA restricts the degree of relative synchronization between the users as seen by the centralized base station receiver. It naturally occurs in indoor applications where the round trip propagation time from the base station transmitter to the user and back is limited to a few spreading chip intervals. This characteristic was used as a means to reduce the complexity of linear multuser base station detectors by eliminating the need for explicit delay estimation of the users.

Spreading Code Selection for Memoryless Detection. To further reduce the complexity of the detection process, the users' spreading codes were modified to allow memoryless detection at the base station. This was achieved by appending Q blank chips to the spreading codes where Q is the degree of quasi-synchronous alignment of the users.

Receiver Design. The proposed receivers consists of three stages. Stage I filters and samples the received signal. Stage II linearly combines the samples to form a soft decision variable. Stage III makes hard decisions upon the soft decision variable to produce an estimate of the transmitted bits. Two types of Stage I were considered; the matched filtering (MF) and the chip filtering (CF) versions. The MF version requires explicit knowledge of the users' delays, and the CF version does not. It was found that the CF version must be oversampled by at least twice the chip rate to approach the performance of the MF version. Multuser decorrelating and MMSE linear combiners were used in Stage II.

Performance of Decorrelating and MMSE Combiners. Six decorrelating and four MMSE combiners were derived and analyzed for Stage II. In general, the greater the complexity of the combined Stages I and II, the better the performance. Detectors that use the delay information have better performance than detectors that do not, and detectors that combine all of the samples from Stage I instead of just a subset have better performance.

Delay Independent Receivers. With QS-CDMA, it is possible to detect the users without explicit knowledge of their delays. The decorrelating detector Ψ^5 is independent of the users' delays, and the MMSE detectors W^4 and W^6 adapt to the unknown delays. The performance of these receivers is inferior to those that do use the delay information, but their complexity is substantially reduced by eliminating the need for explicit delay estimation.

Spreading Code Selection. Three sets of spreading codes were considered; M-sequences, Gold codes, and Hadamard codes. It was consistently found that Hadamard codes have the best mean asymptotic efficiency and M-sequences have the worst. Gold codes lie in the middle of the performance range. Hadamard codes, however, have the highest variance in asymptotic efficiency across the code set.

NLMS Convergence. The transient behaviour of the mean squared error of the MMSE detectors when adapted using the NLMS algorithm was examined. It was found that the number of iterations to convergence is a sensitive function of the power spread of the users. Some form of loose power control to within 12 to 18 dB is required to ensure convergence in a reasonable time frame.

8.1 Future Research

This work introduced the use of linear multiuser detectors for QS-CDMA, proposed several detectors that trade off performance for complexity, and evaluated the performance of the detectors for three spreading code sets. There is a large

potential for more research in this area. A few possible areas are:

- Determine the effect on receiver performance of a user violating the QS-CDMA assumption. This may happen if a user lies outside of the acceptable cell boundary dictated by the QS-CDMA assumption, but still attempts to communicate with the base station.
- What is the effect on performance of power spilling into the blank chip in BCA? This could be caused by imperfect gating of a user's power amplifier during the blank chips.
- Investigate separating W^1 and W^2 into the product of two parts. The first part has real taps that separate the users and the second part is a single complex tap that adapts to the user's phase. This structure is based upon the decorrelating Stage II which consists of two parts; a real decorrelating matrix followed by a complex phase adjustment. Separating the adaptive Stage II into two parts will reduce the complexity and decrease convergence time since the first part has only real weights.
- For the split Stage II decorrelating matrices, find a left inverse of the code matrix A that is independent of the users' delays and maximizes the SNR of the decision variable. This would be used in conjunction with the left inverse of the delay matrix T that maximizes the SNR of the decision variable.
- The QS-CDMA system examined in this work has rectangular chip waveforms which occupy a large bandwidth. What is the effect of using different chip waveforms or filtering the rectangular chip waveforms to limit the bandwidth?

Appendix

Spreading codes

Listed below are the M-sequences, Gold codes, and Hadamard codes used in the performance evaluation of the detectors. Each line consists of the code number followed by the code. A + indicates a +1 and a - indicates a -1.

M-sequences

$N = 7$

1	- - + + + - +	2	- + + + - + -	3	+ + + - + - -
4	+ + - + - - +	5	+ - + - - + +	6	- + - - + + +
7	+ - - + + + -				

$N = 15$

1	- - - + + + + - + - + + - - +	2	- - + + + + - + - + + - - + -
3	- + + + + - + - + + - - + - -	4	+ + + + - + - + + - - + - - -
5	+ + + - + - + + - - + - - - +	6	+ + - + - + + - - + - - - + +
7	+ - + - + + - - + - - - + + +	8	- + - + + - - + - - - + + + +
9	+ - + + - - + - - - + + + + -	10	- + + - - + - - - + + + + - +
11	+ + - - + - - - + + + + - + -	12	+ - - + - - - + + + + - + - +
13	- - + - - - + + + + - + - + +	14	- + - - - + + + + - + - + + -
15	+ - - - + + + + - + - + + - -		

$N = 31$

1	- - - - + - + - + + + - + + - - - + + + + - - + + - + - - +
2	- - - + - + - + + + - + + - - - + + + + - - + + - + - - + -
3	- - + - + - + + + - + + - - - + + + + - - + + - + - - + - -
4	- + - + - + + + - + + - - - + + + + - - + + - + - - + - - -
5	+ - + - + + + - + + - - - + + + + - - + + - + - - + - - - -
6	- + - + + + - + + - - - + + + + - - + + - + - - + - - - +
7	+ - + + + - + + - - - + + + + - - + + - + - - + - - - + -
8	- + + + - + + - - - + + + + - - + + - + - - + - - - + - +
9	+ + + - + + - - - + + + + - - + + - + - - + - - - + - + -

```

10  +-+--+--+++-+--+--+--+--+--+--+
11  +-+--+--+++-+--+--+--+--+--+--+
12  -++--+--+++-+--+--+--+--+--+--+
13  +-+--+--+++-+--+--+--+--+--+--+
14  +-+--+--+++-+--+--+--+--+--+--+
15  ---++--+--+++-+--+--+--+--+--+
16  ---++--+--+++-+--+--+--+--+--+
17  -++--+--+++-+--+--+--+--+--+
18  ++++--+--+++-+--+--+--+--+--+
19  ++++--+--+++-+--+--+--+--+--+
20  ++++--+--+++-+--+--+--+--+--+
21  +-+--+--+++-+--+--+--+--+--+
22  +-+--+--+++-+--+--+--+--+--+
23  ---++--+--+++-+--+--+--+--+--+
24  -++--+--+++-+--+--+--+--+--+
25  +-+--+--+++-+--+--+--+--+--+
26  +-+--+--+++-+--+--+--+--+--+
27  -++--+--+++-+--+--+--+--+--+
28  +-+--+--+++-+--+--+--+--+--+
29  ---++--+--+++-+--+--+--+--+--+
30  -++--+--+++-+--+--+--+--+--+
31  +-+--+--+++-+--+--+--+--+--+

```

Hadamard Codes

$N = 8$

```

1  ++++++++      2  +-+-+-+-+
4  +-+--+--+      5  ++++----
7  ++-----++   8  +-+--+--+

```

$N = 16$

```

1  ++++++++
3  +-+--+--+
5  ++++----
7  ++-----++
9  +++++--+
11 +-+--+--+
13 +++++--+
15 ++-----++
2  +-+-+-+-+
4  +-+--+--+
6  +-+--+--+
8  +-+--+--+
10 +-+--+--+
12 +-+--+--+
14 +-+--+--+
16 +-+--+--+

```


8 --+-++--++++-+-++-++++-+-++-
 9 +- -++- -++++- -++- -++- -++-
 10 ++- - -++- -++- -++- -++- -++-
 11 +++-+++- - -++- -++- -++- -++-
 12 ++++- - -++- -++- -++- -++- -++-
 13 ++++- -+++- -+++- -+++- -+++- -++-
 14 -+++-++- -+++- -+++- -+++- -+++-
 15 +-++-+- -+++- -+++- -+++- -+++-
 16 ++-+-++- -+++- -+++- -+++- -++-
 17 +++- -++- -++- -++- -++- -++- -++-
 18 -+++-++- -+++- -+++- -+++- -+++-
 19 - -++- -++- -++- -++- -++- -++-
 20 - -++- -++- -++- -++- -++- -++-
 21 +- - -++- -++- -++- -++- -++- -++-
 22 -+- -++- -++- -++- -++- -++- -++-
 23 +-+-++- -++- -++- -++- -++- -++-
 24 -+-++- -++- -++- -++- -++- -++-
 25 +-+- -++- -++- -++- -++- -++- -++-
 26 +-+-+++- -++- -++- -++- -++- -++-
 27 -+- -++- -++- -++- -++- -++- -++-
 28 +-+-+++- -++- -++- -++- -++- -++-
 29 -+- -++- -++- -++- -++- -++- -++-
 30 - -++- -++- -++- -++- -++- -++-
 31 - -++- -++- -++- -++- -++- -++-

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