**Behaviour of Webs of WWF Shapes Subjected to
Concentrated In-plane Loads : Analytical
Investigation**

by

K. R. Prabha

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in partial fulfilment of the requirements
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Abstract

The current provisions for yielding and crippling of the webs of beams given in Clause 15.9 of CSA Standard CAN/CSA S16.1-94 are semi-empirical or empirical and are based on test data that generally does not represent the population of existing shapes, realistic load lengths and boundary conditions. The objective of this research was to conduct an analytical study in order to describe the behaviour of webs of welded wide flange (WWF) shapes subjected to concentrated interior in-plane loads and end reactions. Then from the results of these analyses, to develop a rational mathematical model and the corresponding design rules to predict the bearing resistance of webs of W and WWF shapes subjected to concentrated in-plane loads. The equivalent column concept, introduced by Benichou (1994) is developed further for both interior loads and end reactions for W and WWF shapes to determine a buckling coefficient, c , that reflects the buckling conditions of the plate. The coefficient is a function of the aspect ratio of the equivalent column. An equivalent slenderness parameter, λ_e , that incorporates the buckling coefficient is then used in conjunction with the cross-sectional area, defined by the effective width of the equivalent column and web thickness, in Clause 13.3.1 of S16.1, which defines the factored axial compressive resistance of columns, to calculate the bearing resistance of webs subjected to concentrated in-plane loads.

Thus a design clause, based on the equivalent column concept has been written which gives a simple, unified, easily understood design approach to predict the bearing resistances of webs of W and WWF shapes for both interior loads and end reactions for the full range of shapes produced by Canadian mills. It is considered appropriate for use in the Canadian Standard CAN/CSA S16.1-94.

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List of Symbols

a	A positive integer
A	Area of equivalent column
b	Width of the flange
B_r	Factored bearing resistance as defined by CAN/CSA-S16.1-94
c	Buckling coefficient
d	Overall depth of web plate
D	Plate rigidity
E	Modulus of elasticity
E_{st}	Strain hardening modulus
f	Coefficient of elastic critical buckling load
F_y	Yield stress
h	Depth of the web
I	Moment of inertia
k	Distance from outer face of the flange to web toe of fillet and Plate buckling coefficient
K	Effective length factor
l	Width of the equivalent column $N + 10t$
L	Span length of the beam
$LVDT$	Linear variable displacement transducer
m	Constant
n	Number of tests
N	Length of bearing of an applied load
p	A positive integer
P	Direct load on web
P_{cr}	Elastic critical buckling load

$P_{S16.1}$	Ultimate load predicted by Clause 15.9, S16.1-94
P_L	Ultimate load predicted by LUSAS FEA
$P_{\frac{N+4t}{A}}$	Predicted load based on effective width $(N + 4t)$
$P_{\frac{N+5t}{A}}$	Predicted load based on effective width $(N + 5t)$
P_p	Predicted load
P_t	Ultimate load determined experimentally
P_y	Yield load
r	Radius of gyration
s	Number of buckling modes
t	Flange thickness
U	Strain energy
V	Coefficient of variation
V_G	Coefficient of variation of geometric properties
V_m	Coefficient of variation of material properties
V_p	Coefficient of variation of test-to-predicted strength ratios
V_R	Coefficient of variation associated with ρ_R
w	Thickness of the web
W	Potential energy
x, y, z	Rectangular coordinate system
$\bar{x}$	Mean value
y_0	Initial out-of-plane displacement at any point of the web
y_{max}	Maximum measured initial imperfection
α_R	Coefficient of separation
β	Reliability index
β_T	Target reliability index
δ	Maximum displacement of the plate for the classical solution
ϵ_u	Ultimate strain

ϵ_y	Yield strain
λ	Slenderness parameter
λ_e	Equivalent slenderness parameter
ν	Poisson's ratio
ϕ	Resistance factor
ρ	Bias coefficient
ρ_G	Bias coefficient for cross-sectional geometry
ρ_m	Bias coefficient for material property
ρ_p	Bias coefficient for design equation
ρ_R	Bias coefficient for resistance
σ_{max}	Maximum average normal stress
σ_u	Ultimate stress
σ_y	Yield stress
σ_{yy}	Static yield stress

Chapter 1

Introduction

1.1 General

Beam and girder webs are required to resist concentrated point loads, that result from beams, purlins, columns, wheel loads on gantry girders or from roller loads during launching of plate and box girder bridges as illustrated in Figure 1.1 . These loads could cause a local failure of the web by either elastic or inelastic instability (web crippling) or by yielding and thus limit the overall member capacity. Generally, either web stiffeners are added at locations of concentrated point loads or a greater web thickness is specified to prevent this type of failure.

Investigations of the effects of web thickness, flange thickness and width, flange rigidity, material properties, shear span and load eccentricity on the behaviour of rolled steel shapes subjected to concentrated in-plane loads and the development of analytical models to describe the behaviour and ultimate load capacity have continued since

1906 as listed chronologically and summarized according to behaviour by Benichou (1994).

Several empirical and semi-empirical expressions from these works have been adapted and incorporated as design rules in specifications and standards around the world. Those used in North America, (AISC 1994) and (CSA 1994), are based on steel sections with slenderness ratios, h/w , flange plate to web plate thickness ratios, t/w , and geometric properties that lie outside the range of most W and WWF shapes produced by North American mills, Benichou (1994). As well there are no specific design rules for loading of webs of hollow structural sections. To overcome this deficiency, a representative sample of W and WWF shapes that describes the full range of behaviour from web yielding to inelastic web buckling to elastic web buckling was tested by Benichou (1994) and Kirkhope (1995) in order to obtain the experimental data required to develop rational mathematical models and simplified design expressions for possible inclusion in the design standard.

1.2 Behaviour of Webs of WWF Shapes

Kirkhope (1995) tested seven WWF Shapes, that were simply supported, with values of h/w ranging from 15 to 88, to determine for both interior loads and end reactions the behaviour and resistance of the webs. The general behaviour can be best described by the load out-of-plane web displacement curve in Figure 1.2 for the WWF 900x262. The web behaved elastically up to approximately 80% of the ultimate load when the web yielded locally at the root of the fillet under the length of the load plate. The yielding spread outwards with increasing deformation. The ultimate load was generally reached when a second plastic hinge developed at the point of maximum

out-of-plane web displacement. This hinge had the form, in elevation, of a circular arc that extended laterally more than the first hinge and arced upward towards the loaded flange. Similar behaviour was observed for the end reaction tests.

A typical principal stress distribution and orientation at ultimate load for the same section, WWF 900x262, for both sides is illustrated in Figure 1.3. These are determined from strain data using an inelastic strain data interpretation program, Benichou (1994), that was modified and is described in Section 2.2. The three figures at any location are the principal stresses parallel to the flange and stem of the T symbol, and the angle between the stem and the vertical measured positive counterclockwise. These data illustrate the variation of stress along the length and through the depth of the web in the region of the concentrated load. The average normal stress diminishes with distance from the center of the load and gives the required data to verify the corresponding finite element analysis, to determine the width influenced by the local load and to establish average vertical normal stress distribution through the depth of the web. A rational mathematical model, based on an equivalent column, has been developed from these data that describes the behaviour and ultimate load for webs subjected to concentrated in-plane loads.

1.3 Development of the Equivalent Column Model

Benichou (1994) developed an equivalent column model to predict the strength of webs of rolled shapes, subjected to concentrated in-plane loads that reflects the internal force distribution across the width and through the depth of the web, to take into account the rotational and translational restraint provided by the flanges to the web, and the bearing length. Equivalent column dimensions were determined from the

inelastic finite element analyses of selected W and WWF shapes that represent the full range of geometry and classes of sections. These are based on the average normal stress distribution on the assumed column width of $N + 5k$, as shown in Figure 1.4(a), and over the depth of the column. Failure types ranging from elastic web buckling to inelastic web buckling to web yielding. The inelastic finite element analysis was verified against series of tests conducted on W shapes only. The average vertical normal stress varies from a maximum at the root of the fillet on the web near the loaded flange to zero at the unloaded flange as shown in Figure 1.4(b) and is given by

$$\sigma(z) = \frac{\sigma_{max}}{2} \left[1 - \cos \left(\frac{\pi z}{h} \right) \right] \quad (1.1)$$

where, h = depth of the web plate, mm
 z = distance from the unloaded flange web junction along the web to the loaded flange, mm
 $\sigma(z)$ = uniform stress on equivalent column model at z , MPa
 σ_{max} = maximum average normal stress, MPa .

The elastic critical buckling load for a pin-ended column with an assumed buckled shape given by,

$$y = \sum_{i=1}^n a_n \sin \left(\frac{n\pi z}{h} \right) \quad (1.2)$$

and with the internal stress distribution given by Equation 1.1 can be determined by minimizing the total energy (strain energy and potential energy) with respect to a_n . The nontrivial solution with n set equal to 1 gives the lowest elastic critical buckling load of,

$$P_{cr} = 2.74 \frac{\pi^2 EI}{h^2} \quad (1.3)$$

where, P_{cr} = elastic buckling load of equivalent column, N
 E = modulus of elasticity, MPa
 I = moment of inertia of column, mm^4 .

Restraint provided to the web by the surrounding web and the flanges within the equivalent column width produces boundary conditions other than simple pinned supports and can be accommodated by incorporating an effective length factor in Equation 1.3 that reflects the degree of restraint provided.

$$P_{cr} = 2.74 \frac{\pi^2 EI}{(Kh)^2} \quad (1.4)$$

where, K = effective length factor.

Based on parametric studies, Benichou (1994), established that K was a function of $(N + 5k)/h$ and t/w given by

$$K = 0.873 \sqrt{\left(\frac{N + 5k}{h}\right) / \left(\frac{t}{w}\right)} + 0.1246 \quad (\text{flange rotation permitted}) \quad (1.5)$$

$$K = 0.513 \sqrt{\left(\frac{N + 5k}{h}\right) / \left(\frac{t}{w}\right)} + 0.161 \quad (\text{flange rotation restrained}) \quad (1.6)$$

The equivalent slenderness parameter of the equivalent column model, λ_e , is obtained by setting the ratio of P_{cr}/P_y equal to $1/\lambda_e^2$,

where,

$$P_y = wF_y(N + 5k) \quad (1.7)$$

and P_{cr} is found from Equation 1.4. The equivalent slenderness parameter λ_e when expressed as a function of the radius of gyration of the equivalent column is given by,

$$\lambda_e = 0.604 \left(\frac{K h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}} \quad (1.8)$$

where, r = radius of gyration, mm .
 $= w/\sqrt{12}$.

Based on this, Benichou proposed the following modification to the Clause 15.9 of S16.1-M89 to incorporate the equivalent column model using the equivalent slenderness parameter with the column curves of Clause 13.3.1 of CSA 1989.

Clause 15.9 Web Buckling and Yielding 15.9.1

Bearing stiffeners shall be provided where the factored concentrated loads or reactions exceed the factored compressive resistances of webs of rolled beams and welded plate girders. The factored compressive resistance of the web shall be calculated using Clause 13.3.1 with an equivalent slenderness parameter λ_e given by

$$\lambda_e = 0.6 \left(\frac{K h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}} \quad (1.9)$$

where, r = radius of gyration of the equivalent column cross-section = $w/\sqrt{12}$.

15.9.2

The effective length factor for interior loads shall be taken as 0.6 when the loaded flange is restrained against rotation and 0.8 when it is free to rotate. The effective length factor for end reactions shall be taken as 0.6 when both flanges are restrained against lateral displacement and rotation and -- when the unloaded flange is free to displace.

15.9.3

The area of the equivalent column is equal to $w (N + 5 k)$ for interior loads and $w (N + 2.5 k)$ for end reactions, where the length of bearing N shall not be less than k for end reactions.

Based on this proposed form of the clause the mean test-to-predicted value and the coefficient of variation for the six interior load tests and six end reaction tests were 1.082 and 0.077, and 1.043 and 0.118, respectively. Benichou concluded that the equivalent column concept models the internal normal stress distribution and that the corresponding modification to Clause 15.9 accurately predicts the web buckling capacity (web contribution only), incorporates different boundary conditions that reflect the restraint provided by the loaded and unloaded flange; and gives a complete simple description of web instability behaviour for concentrated in-plane loads.

Although it has not been verified, Benichou suggested that it may be possible to design eccentrically loaded webs using the equivalent column and corresponding clauses with the appropriate beam column interaction equations for the strength and in-plane stability only.

1.4 Objectives and Scope

Given the test data by Benichou (1994) and Kirkhope (1995), the limited verification and design value development for W Shapes only, and the lack of test data for very slender webs and stiffened plate girders with longitudinal stiffeners the objectives of this thesis are :

1. to develop finite element predictions based on the LUSAS finite element program using the experimental data on WWF shapes, Kirkhope (1995) and Prowse (1996), and to calibrate the interior and end reaction local web instability model;
2. to investigate the applicability of the equivalent column model developed by Benichou (1994) to WWF shapes and if required formulate the appropriate modifications;
3. to develop a rational design methodology and set of design clauses for possible inclusion in the Standard S16.1, that describe the resistance of webs of WWF shapes when subjected to concentrated in-plane loads.
4. to design a series of test specimens that represent typical bridge girders being built or currently existing that have longitudinal stiffeners and h/w ratios ranging from 175 to 285; and
5. to conduct the preliminary finite element analysis of the test sections of bridge girders to predict the failure load and mode of failure.

1.5 Outline of the Thesis

The first chapter of this thesis presents a general review of the current state of the art of the behaviour of webs of rolled steel shapes subjected to concentrated in-plane loads and includes the description of web behaviour of WWF shapes, the equivalent column model proposed by Benichou (1994) and its application to rolled steel shapes. The second chapter presents the test data for the WWF shapes tested by Kirkhope (1995) and Prowse (1996). It also describes the observations made during the testing of the specimens. Details of the finite element analysis for WWF shapes and bridge

girders along with a comparison with the test results for WWF shapes with interior loads only, are presented in chapter three. The development of design rules that includes both W and WWF shapes is given in chapter four. A summary, conclusions and recommendations are presented in chapter five.

(a) support column load.

(b) transverse beam.

(c) Hillman rollers during launching of a bridge girder.

Figure 1.1: Sources of concentrated in-plane loads.

Figure 1.2: Load versus out-of-plane web displacement.

Figure 1.3: Principal stress distribution at ultimate load, WWF 900x262.

(a) along the length of the beam.

(b) variation through the depth.

(c) assumed deformed shape.

Figure 1.4: Equivalent rectangular stress distribution in the web for the equivalent column model illustrated for interior loads, after Benichou (1994).

Chapter 2

Behaviour of Welded Wide Flanges

2.1 General Behaviour

Nine WWF shapes whose range of geometric parameters represent all possible shapes produced by Canadian mills, all classes and all probable modes of web failure due to concentrated in-plane loads were tested by Kirkhope (1995), and Prowse (1996). Three WWF 500x197 shapes were tested to determine the influence of the additional resistance provided to the web by the flange when the flange capacity was not entirely utilized in carrying the bending moments. Two WWF 985x71 shapes were tested to determine the influence on web capacity of one sided and two sided welding at the flange web junctions, because, for sections of this size one sided welding is commonly used in fabrication to connect the webs to the flanges.

The best description of the web behaviour is given by the applied load versus out-of-plane web displacements illustrated for each beam, for interior loads only, in Figure 2.1(a). The load displacement curves for the WWF 350x212 and the WWF 500x197 series are also illustrated in Figure 2.1(b). The initial slope of each load displacement curve is related to the magnitude of the initial imperfections. The flattest slope is for the WWF 985x71 with the maximum imperfection of 1 in 120 or 0.001 and the steepest slope is for the WWF 350x212 with a maximum imperfection of 1 in 1842 or 0.008. The decreasing stiffness illustrated by the load deflection curves reflects the development of yield bands in the web. The ultimate load is reached when the strain gradient in the yield bands is such that the compressive strains are greater than ϵ_{cr} and the tensile strains are greater than or equal to ϵ_y . With the exception of the WWF 350x212 which failed by web yielding, all others failed by inelastic web buckling, characterized by the formation of two yield bands in the web creating a plastic mechanism.

2.2 Strain Data

First yield of the web is taken as the development of the first yield band which generally occurs in the web at the loaded flange web junction. It is shown on the load deformation curves for each test as a hollow diamond. Beyond first yield the stiffness of the plate is greatly reduced. The ultimate load is reached when the second plastic hinge forms at a distance of $h/8$ to $h/3$ from the loaded flange. The second hinge becomes well defined only after relatively large out-of-plane web displacements of the order of magnitude of web thickness have occurred.

Figure 2.2 shows the locations of the strain rosettes located on both sides of the webs of the WWF shapes. The principal strains and orientation were calculated from

test data, Kirkhope (1995) and Prowse (1996), as part of this work, using Mohr's circle for strains and are illustrated for each shape at the ultimate load for the far side and near side of the beam in Figures 2.3 to 2.10. The general orientation of the principal strains are illustrated by the heavy black 'T' shapes. The three figures for each rosette are the strains parallel to the flange of the T symbol, parallel to the stem, and the orientation of the stem in degrees measured positively counterclockwise from the vertical. Negative strains indicate compression. Insert in each figure is the measured yield strain determined from the tension coupon tests. The extent of the yield at ultimate can be seen by comparing the strains in Figures 2.3 to 2.10 with the measured yield strains. From these the formation of the two plastic yield bands, the one near the loaded flange web junction extending to and beyond the region of $(N + 5k)$ and the second one in the web forming a large circular arc which projects upward and outward to a similar location can be deduced. For example in Figure 2.6 on the 'T' stem on one side near the web the compressive strains are greater than the yield strain and on the other side are very large. This condition extends to the othermost rosette. The extent of yielding shows that the effective width of the equivalent column should be larger than $(N + 5k)$. The strain histories for each rosette, the corresponding principal strains and orientation were subsequently used to determine the elastic and inelastic principal stresses and orientations using the strain gauge data interpretation program, Benichou (1994). These were then used to verify the finite element analyses.

2.3 Material Properties

Kirkhope (1995), and Prowse (1996) tested a statistical sample of tension coupons from the flanges and the web of each WWF shape in accordance with ASTM E8M-88. A summary of the pertinent material properties, mean value and coefficient of

variation, (E , ν , σ_{xy} , σ_u , ϵ_y , ϵ_u) for the tests is given, for each section for the web plates and the flange plates, in Tables 2.1 and 2.2, respectively. A bilinear material model was used for the finite element analysis. The strain gauge data interpretation program models a trilinear curve with a very small slope in the elasto-plastic region. For both the models, E_{xt} was calculated from σ_{xy} and σ_u , and ϵ_y and ϵ_u .

2.4 Geometric Properties

The beam lengths, section depth, flange width and thickness, web height and thickness, distance from the outer face of the flange to the web toe of the fillet weld, load length and all calculated geometric parameters are listed in Tables 2.3 and 2.4. The WWF shapes were also measured to determine the initial out-of-plane web imperfections along the length of the web and over the depth. The latter are summarized for each shape in Table 2.5. All WWF shapes were within specified manufacturing tolerances given in CSA 1987, Kirkhope (1995), and Prowse (1996). The measured values including the initial out-of-plane straightness of the web plate, were used for the definition of the finite element model. The out-of-plane straightness was taken to be of double sinusoidal shape with an amplitude equal to the maximum value given in Table 2.5.

2.5 Test Data Interpretation

2.5.1 Acquisition and Reduction

Data acquisition program, LABCON, Kirkhope (1995), converts voltage readings to strains and displacements and stores the data in text format. This data file is then processed by a series of programs, developed as part of this work, consisting of 'awk'

and 'shell' scripts and C and Fortran programs to generate input files of reduced data that are suitable for plotting, as illustrated by the flowchart shown in Figure 2.11. Descriptions and functions of the programs and the control files required as shown in Figure 2.11 follow:

1. `modinp.awk` : All asterisk (*) symbols that may appear at the end of a value in the LABCON data file as a result of exceeding the precision specified in the data acquisition program are replaced with zeros.
2. `modinp1.c` : A C program that is executed by the following command:
`modinp1.x 'LABCON data file' 'load file' 'strain file' 'LVDT file' 'no. of load cells' 'no. of strain gauges' 'no. of LVDTs'`
 This program divides the data in the LABCON data file into load data, strain data and LVDT data and writes them into separate files. It generates a load step number which is the key to link the data in the three files.

3. `modinp2.c` : A C program that is executed by the following command:
`modinp2.x 'load file' 'strain file' 'LVDT file' 'control file' 'input.dat' 'load-lvdt file' 'lvdt-disp file'`
 This program reads the control file for material properties, type of analysis, number of strain rosettes, load step for which displacement values at all LVDT locations is required, and the number of the LVDT whose load displacement history is to be plotted. A typical control file format is illustrated below. For the number of strain rosettes, the program then reads each line from the control file for the strain gauges that form each rosette to write the measured strain information and orientation into the data file `input.dat`. Measured displacements at all LVDT locations for the required load step are written into the `lvdt-disp` file. Load displacement history for a specified LVDT is written in the `load-lvdt` file.

Control file format :

```

1  3      1      1      253      5      243
2  348.S 202400 0.299 0.1      0.0 1      2
3  1320. 0.002080 0.012435 0.218750
4  3
5  1      11      12      0      0      0      0
6  2      13      14      0      0      0      0
7  3      15      16      0      0      0      0
8  4      17      18      19      0      45      90
9  5      20      21      22      0      45      90

```

where each line translates as

```

1  Rows in      1      1      readings  rosettes  loadstep
   transfor-
   mation
   matrix
2   $\sigma_{xy}$        $E$        $\nu$       slope in  friction  plane      metal
                                elasto-
                                plastic
                                region
3   $E_{st}$        $\epsilon_y$        $\epsilon_{st}$        $\epsilon_u$ 
4  LVDT no.
5  Rosette no.  SG 1  SG 2  0      0      0      0
6  Rosette no.  SG 1  SG 2  0      0      0      0
7  Rosette no.  SG 1  SG 2  0      0      0      0
8  Rosette no.  SG 1  SG 2  SG 3  Incl. 1  Incl. 2  Incl. 3
9  Rosette no.  SG 1  SG 2  SG 3  Incl. 1  Incl. 2  Incl. 3

```

Note: SG = strain gauge, Incl. = inclination

4. `strain.f` : A Fortran program that is executed by the following command:
`strain.x`

It reads the file `input.dat` and writes the output data file `output.dat` which contains the history of principal stresses and strains and their orientation at all strain rosette locations. This strain data interpretation program developed by Benichou (1994) determines the principal stresses and their orientations from the measured strains (elastic and inelastic) for steel that is subjected to plane stress, plane strain, axisymmetric and three dimensional state of stress which are uniquely evaluated. The uniaxial stress-strain curve (modelled as either a bilinear or trilinear curve), the strain histories for each gauge and the identification of the gauges for each rosette and their orientation are required as input to the program. The program then uses plasticity algorithms developed for finite element analysis to determine the corresponding elastic and inelastic principal stress and orientation. This program was used to interpret the strain gauge data for this series of tests. Note, the program was verified by comparing experimental data from tests on biaxially loaded cylinders conducted by Kennedy and Cheng (1987).

5. `modstrdat.sh` : A shell script that extracts the stresses, strains and their orientation from the data file `output.dat` for the load specified in kN and writes them in the stress file specified. It is executed by the following command:

`modstrdat.sh 'Load in kN' 'output.dat' 'stress file'`

(a) WWF 350x212, WWF 500x197 series, WWF 900x262, WWF 1400x405, WWF 1800x510, WWF 985x71.

Figure 2.1: Load versus out-of-plane web displacements for interior loads, WWF shapes, Kirkhope (1995), and Prowse (1996).

(b) WWF 350x212, WWF 500x197 series.

Figure 2.1: Load versus out-of-plane web displacements for interior loads, WWF shapes, Kirkhope (1995), and Prowse (1996) cont'd.

(a) WWF 350x212.

(b) WWF 500x197, WWF 900x262, WWF 1400x405, WWF 1800x510, WWF 985x71.

Figure 2.2: Strain rosette layout for WWF shapes, after Kirkhope (1995) and Prowse (1996).

Figure 2.3: Strain distribution at ultimate load, WWF 350x212.

Figure 2.4: Strain distribution at ultimate load, WWF 500x197, 1700 mm span.

Figure 2.5: Strain distribution at ultimate load, WWF 500x197, 2000 mm span.

Figure 2.6: Strain distribution at ultimate load, WWF 500x197, 3000 mm span.

Figure 2.7: Strain distribution at ultimate load, WWF 900x262.

Figure 2.8: Strain distribution at ultimate load, WWF 1400x405.

Figure 2.9: Strain distribution at ultimate load, WWF 1800x510.

Figure 2.10: Strain distribution at ultimate load, WWF 985x71.

Figure 2.11: Flowchart of experimental data interpretation module.

Table 2.1: Measured material properties of web plates.

WWF Designation		E (MPa)	ν	σ_{yy} (MPa)	σ_u (MPa)	ϵ_y $\times 10^{-6}$	ϵ_u $\times 10^{-6}$
WWF 350X212	$\bar{x}$	196930	0.283	308.5	522.4	1770	174760
	V	0.058	-	0.012	0.007	0.105	0.098
WWF 500X197	$\bar{x}$	203390	0.260	413.2	527.6	2466	151270
	V	0.009	-	0.022	0.006	0.069	0.050
WWF 900X262	$\bar{x}$	196030	0.262	373.0	496.6	2268	162400
	V	0.018	-	0.014	0.024	0.031	0.056
WWF 1400X405	$\bar{x}$	198960	0.291	280.5	474.7	1616	188220
	V	0.012	-	0.018	0.027	0.071	0.068
WWF 1800X510	$\bar{x}$	202400	0.299	348.8	558.7	2080	161000
	V	0.027	-	0.043	0.027	0.036	0.056
WWF 985X71	$\bar{x}$	216600	0.260	378.7	530.3	3270	139409
	V	0.045	-	0.032	0.037	0.076	0.125

Table 2.2: Measured material properties of flange plates.

WWF Designation		E (MPa)	ν^1	σ_{xy} (MPa)	σ_x (MPa)	ϵ_y $\times 10^{-6}$	ϵ_x $\times 10^{-6}$
WWF 350X212	$\bar{z}$	194740	0.30	350.7	566.7	2343	174070
	V	0.010	-	0.026	0.013	0.091	0.0224
WWF 500X197	$\bar{z}$	201670	0.30	350.6	527.5	2090	175340
	V	0.006	-	0.041	0.025	0.101	0.025
WWF 900X262	$\bar{z}$	202650	0.30	354.6	577.5	2120	212840
	V	0.013	-	0.014	0.024	0.031	0.055
WWF 1400X405	$\bar{z}$	204480	0.30	348.9	573.6	2200	198670
	V	0.012	-	0.018	0.027	0.071	0.068
WWF 1800X510	$\bar{z}$	207580	0.30	370.6	586.0	2375	236250
	V	0.027	-	0.043	0.027	0.043	0.036
WWF 985X71	$\bar{z}$	214920	0.30	388.5	556.0	2787	136435
	V	0.020	-	0.050	0.042	0.366	0.143

¹: the value for ν was assumed to be 0.3 as this data was not given.

Table 2.3: Characteristic geometric properties and other parameters.

WWF Designation		L (mm)	d (mm)	h (mm)	b (mm)	t (mm)	w (mm)	k (mm)
WWF 350x212	$\bar{x}$	1500	355.53	291.55	347.52	31.99	18.91	43.80
	V	-	-	-	0.002	0.003	0.004	0.039
WWF 500x197	$\bar{x}$	1700	499.98	461.08	499.67	19.45	12.36	30.34
	V	-	-	-	0.0	0.004	0.01	0.049
WWF 900x262	$\bar{x}$	4000	903.00	839.16	398.22	31.92	12.66	41.94
	V	-	-	-	0.001	0.003	0.013	0.043
WWF 1400x405	$\bar{x}$	5800	1401.00	1336.92	500.80	32.04	16.24	42.75
	V	-	-	-	0.001	0.004	0.009	0.04
WWF 1800x510	$\bar{x}$	6000	1800.00	1735.42	500.30	32.29	19.69	42.63
	V	-	-	-	0.001	0.005	0.019	0.022
WWF 985x71	$\bar{x}$	3200	984.32	965.04	200.00	9.64	5.30	14.64
	V	-	0.004	-	0.007	0.009	0.040	-

Table 2.4: Load length and calculated geometric parameters.

WWF Designation	N (mm)	$\frac{h}{w}$	$\frac{N}{h}$	$N + 5k$ (mm)	$\frac{(N+5k)}{h}$	$N + 10t$ (mm)	$\frac{(N+10t)}{h}$
WWF 350x212	100	15	0.343	315.40	1.08	419.90	1.44
WWF 500x197	200	37	0.507	351.70	0.76	394.50	0.86
WWF 900x262	300	68	0.348	509.70	0.59	619.20	0.72
WWF 1400x405	300	82	0.224	513.75	0.38	620.40	0.46
WWF 1800x510	300	88	0.173	513.15	0.30	622.90	0.36
WWF 985x71	100	182	0.104	173.20	0.18	196.40	0.20

Table 2.5: Measured out-of-straightness for web plates.

WWF Designation	Maximum out of straightness	
	$\Delta_{max}(\text{mm})$	Δ_{max}/d
WWF 350x212	0.19	0.001
WWF 500x197 (L=1700 mm)	1.11	0.002
WWF 500x197 (L=2000 mm)	0.98	0.002
WWF 500x197 (L=3000 mm)	1.02	0.002
WWF 900x262	3.54	0.004
WWF 1400x405	3.15	0.002
WWF 1800x510	1.81	0.001
WWF 985x71	8.20	0.008

Chapter 3

Finite Element Analysis

3.1 Model and Solution Strategy

A primary objective of this work was to develop a finite element solution for comparison with and verification against the interior load tests for WWF shapes conducted by Kirkhope (1995), and Prowse (1996). Subsequently the results of these analyses, describing completely the displacements, strains and stresses in the area of interest of local web failure, are used to develop further the equivalent column concept.

Benichou (1994) used the commercial finite element analysis program LUSAS to make analytical comparisons with tests of web stability for both interior loads and end reactions for W shapes. These tests and analyses included initial imperfections, and geometric and material nonlinearity. The material was assumed to be perfectly elastoplastic with a bilinear stress-strain curve. The material nonlinear behaviour was modelled using an implicit backward Euler von Mises Huber-Hencky associative flow rule

with an isotropic hardening law and an explicit integration scheme for the determination of plastic strains. The geometric nonlinearity was accounted for by using a Total Lagrangian formulation. The Newton-Raphson iteration procedure was used to achieve equilibrium on the deformed configuration and is valid up to ultimate load. Table 3.1 lists the convergence criteria used in the analysis. Post buckling behaviour was not modelled as it is not relevant to the study.

The ability of the model, using quadrilateral 8-node semi-loof shell elements, (QSL8), which account for both in-plane and out-of-plane bending deformations, and the solution strategy described above, to predict accurately the inelastic behaviour of the tests of this type gives confidence for the same method of analysis to be used for WWF shapes. Benichou also tested the LUSAS program against the results of structural steel members that behave elastically, inelastically and those that fail by inelastic stability, Timoshenko (1955), Massonet and Save '1965), Driver and Kennedy (1987), and Roberts (1981), and found the comparison to be excellent. Based on finite element analyses results of W shapes, Benichou (1994) comparisons of the mean test-to-predicted values and coefficient of variation for interior loads and end reactions are 1.068 and 0.0663, and 0.951 and 0.141, respectively.

By taking advantage of symmetry, only one half of the beam was modelled. Typically, the WWF shapes were modelled as three plates consisting of 38 QSL8 elements for each flange plate and 228 for the web plate. The exception to this was the WWF 985x71 which was modelled with 56 elements for each flange, 112 elements for the web and 16 elements for each pair of stiffeners. The general mesh configurations are illustrated in Figures 3.1 and 3.2, respectively. The mesh is refined in the area of interest to correspond to physical geometry and to model better the expected large strain gradients within the loaded region. An element aspect ratio of 1:3 was used compared to that previously chosen of 1:6 and an area of the smallest element

equal of 80 mm^2 as compared to 640 mm^2 was used. Refining the mesh by doubling the number of elements locally in the region of the load improved the prediction of ultimate load. The test-to-predicted load ratio was 0.962 for the refined mesh as against 1.11 for the original mesh configuration. Based on this, it was deemed that the refined mesh would be adequate for all the other models of WWF shapes.

A typical cross-section showing the centerline definition of the elements is illustrated in Figure 3.3. Flange elements labelled 1, 2 and 15, 16 have a thickness set equal to the measured flange thickness. The thickness of the web element 14 is the top flange width. Web element 13 the average width across the fillet weld as indicated by the dashed line. Web elements 3 to 12 have a thickness equal to the measured thickness of the web. As illustrated in Figure 3.4 the boundary conditions reflect symmetry at the centerline, the restraint provided by the load and reaction devices, and additional lateral and rotational restraint provided to the top flange at the beam end. Due to the modelling, two rows of web elements exist within the top flange and the region of the fillet weld. Based on compatibility and physical observation the same restraint was applied to nodes at these elevations as for nodes along the top flange web junction. The applied load was modelled as nodal loads of the elements at the top flange web junction for a distance of half the length of the load plate from the centerline. Also shown in Figure 3.4 is the location and distribution of the applied load and reaction.

The initial imperfections were input to the model as nodal coordinates with a shape described by two dimensional sine wave given by

$$y_0 = y_{max} \sin\left(\frac{\pi z}{h}\right) \sin\left(\frac{\pi x}{L}\right) \quad (3.1)$$

where, y_{max} = maximum measured out-of-plane initial imperfection in the web

at midspan as listed in Table 2.5, *mm*

y_0 = initial imperfection at any node in the web, *mm*

h = depth of the web, *mm*

L = length of the beam, *mm*

x, z = Rectangular coordinates, *mm*

3.2 Comparison with WWF Test Data

3.2.1 Load Deflection Response

The finite element model with boundary conditions, load description, initial imperfections and solution strategy defined in the previous section was used to analyse the interior load tests conducted on WWF shapes by Kirkhope (1995), and Prowse (1996).

The web behaviour is described by the load versus maximum out-of-plane web displacements as illustrated for the tests on WWF 350x212, WWF 500x197, WWF 900x262, WWF 1400x405, WWF 1800x510 and WWF 985x71 in Figures 3.5 to 3.12, respectively. The location where the maximum out-of-plane web displacement occurred in the test is shown and the finite element analysis displacement is shown for the same location. The out-of-plane web displacements for WWF 1400x405 illustrated in Figure 3.10 have been adjusted to account for displacements related to rotation of the loaded flange during seating of the beam in the initial stages of loading. Two load displacement test curves, for the WWF 985x71 one where the flange to web weld was one-sided and one where the flange to web weld made on both sides are illustrated in Figure 3.12. In general the characteristic curves were modelled well as shown by the

correspondence between the finite element predictions and test data. The ultimate loads and the test to predicted ratios are given in Table 3.2. The mean value and the coefficient of variation are 0.986 and 0.041, respectively. These values indicate excellent correspondence and compare very favourably with the values of 0.99 and 0.062 reported by Albert, Essa, and Kennedy (1992), for tests on lateral torsional buckling of steel beams.

3.2.2 Out-of-plane web displacements

The LVDT layout for the WWF shapes are illustrated in Figure 3.13(a) for WWF 350x212, in Figure 3.13(b) for the WWF 500x197 series, WWF 900x262, WWF 1400x405 and WWF 1800x510 and in Figure 3.13(c) for the WWF 985x71. The measured out-of-plane web displacements at ultimate load for all the LVDT locations, from test data, Kirkhope (1995) and Prowse (1996), recreated as part of this work, and the corresponding displacements from the finite element analysis at ultimate load are shown in round brackets in Figures 3.14 to 3.21. Based on the data of measured out-of-plane web displacements it was determined that there were some small rotations and twisting of the LVDT support structure associated with rotation of the loaded flange to which it was attached. These rotations calculated from displacement data on the web and are relatively small ranging between 0.000037 to 0.01155 radians. The projection of the rigid LVDT support structure leads to corresponding rigid body displacements of the same order of magnitude as the out-of-plane web displacements resulting from the load. Linear adjustments were therefore applied to the finite element displacement farthest away along the centerline from the load point. The adjusted values are given in the square brackets in Figures 3.14 to 3.20. There are no adjusted values for the WWF 985x71, Prowse (1996), because a new

articulating LVDT support structure was made to eliminate rigid body rotational displacements that occurred for the other WWF sections tested by Kirkhope (1995). Aside from problems related to the interpretation of the displacements measured in the tests, it was found from a parametric study that the ultimate displacements of the web were sensitive to the initial imperfections assumed. Any variations between the assumed and the actual imperfections (not measured at all locations along the length) would lead to variances in the displacements between those predicted and the test values. Notwithstanding, the comparisons of the adjusted test values with the finite elements values for displacements, indicate similar displaced shapes.

3.2.3 Principal stresses

A comparison of near and far side principal stresses and orientation computed from measured strains using the strain gauge data interpretation program developed by Benichou (1994), from test data, Kirkhope (1995) and Prowse (1996), calculated as part of this work, with the corresponding stresses from the finite element analysis, at the nearest mesh points are presented in Figure 3.22 to Figure 3.29. A comparison of magnitudes, orientation and distribution of stresses illustrate good correspondence between the principal stresses determined from measured strains and those from the finite element analysis. Differences in the regions of large localized out-of-plane displacements where significant curvature occurred are the result of the spread of plasticity through the web thickness and are dependent upon the displaced configuration corresponding to the selection of the ultimate load. These were not necessarily matched for the tests and finite element analysis. Other differences are likely due to the large strain gradients in the region of measured strains where these are measured over a finite area, approximately 15 mm^2 to 20 mm^2 , while the finite element strains

are determined at a point.

Based on the general good correspondence of loads, deflections and strains there exists the necessary level of confidence to use the finite element results for further model development.

3.3 Finite Element Analysis for Bridge Girders

A preferred method of erecting continuous slender bridge girders is to launch them from shore. In this method the entire length of the girder is assembled on shore with the bottom flange resting on sets of Hillman rollers (see Figure 1.1(c)) and the girder is pushed or pulled across the river, cantilevering outwards until it lands on the next river pier. The web is subject to the reaction of the rollers all along the bottom flange as the bridge is moved forward.

The behaviour of the webs of slender bridge girders with longitudinal stiffeners when subjected to concentrated loads is therefore a central design issue. To this end some preliminary finite element analyses were conducted to predict the mode of failure for a set of typical bridge girders that have longitudinal stiffeners and slenderness ratios h/w ranging from 175 to 285 as given in Table 3.3. Based on this preliminary numerical investigation and current design these shapes were then fabricated and subsequently tested, Prowse (1996). A typical finite element mesh for these girders is illustrated in Figure 3.30. The element type, material model and analysis technique along with the boundary conditions are similar to those for other WWF shapes with additional out-of-plane displacement restraint at the points where Watt mechanisms were provided in the test to prevent lateral torsional buckling. Further analyses of these girders is beyond the scope of this work.

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Figure 3.1: Finite element mesh for WWF 350x212, WWF 500x197, WWF 900x262, WWF 1400x405, 1800x510.

Figure 3.2: Finite element mesh for WWF 985x71.

Figure 3.3: Typical WWF cross-section and centerline definition of the finite elements.

Figure 3.4: Elevation illustrating boundary conditions.

Figure 3.5: Load versus out-of-plane web displacement, WWF 350x212.

Figure 3.6: Load versus out-of-plane web displacement, WWF 500x197, span 1700 mm.

Figure 3.7: Load versus out-of-plane web displacement, WWF 500x197, span 2000 mm.

Figure 3.8: Load versus out-of-plane web displacement, WWF 500x197, span 3000 mm.

Figure 3.9: Load versus out-of-plane web displacement, WWF 900x262.

Figure 3.10: Load versus out-of-plane web displacement, WWF 1400x405.

Figure 3.11: Load versus out-of-plane web displacement, WWF 1800x510.

Figure 3.12: Load versus out-of-plane web displacement, WWF 985x71.

(a) LVDI layout for WWF 350x212.

(b) LVDI layout for WWF 500x197, WWF 900x262, WWF 1400x405, WWF 1800x510.

Figure 3.13: LVDI layout for WWF shapes, after Kirkhope (1995) (a) (b) and Prowse (1996) (c).

(c) LVDT layout for WWF 985x71.

Figure 3.13: LVDT layout for WWF shapes cont'd, after Kirkhope (1995) (a) (b) and Prowse (1996) (c).

Figure 3.14: Out-of-plane web displacements at ultimate load in mm for WWF 350x212.

Figure 3.15: Out-of-plane web displacements at ultimate load in mm for WWF 500x197, span 1700 mm.

Figure 3.16: Out-of-plane web displacements at ultimate load in mm for WWF 500x197, span 2000 mm.

Figure 3.17: Out-of-plane web displacements at ultimate load in mm for WWF 500x197, span 3000 mm.

Figure 3.18: Out-of-plane web displacements at ultimate load in mm for WWF 900x262.

Figure 3.19: Out-of-plane web displacements at ultimate load in mm for WWF 1400x405.

Figure 3.20: Out-of-plane web displacements at ultimate load in mm for WWF 1800x510.

Figure 3.21: Out-of-plane web displacements at ultimate load in mm for WWF 985x71.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.22: Comparison of stresses in MPa at ultimate load, WWF 350x212.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.24: Comparison of stresses in MPa at ultimate load, WWF 500x197, span = 2000 mm.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.25: Comparison of stresses in MPa at ultimate load, WWF 500x197, span = 3000 mm.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.26: Comparison of stresses in MPa at ultimate load, WWF 900x262.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.27: Comparison of stresses in MPa at ultimate load, WWF 1400x405.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.28: Comparison of stresses in MPa at ultimate, WWF 1800x510.

(a) Stresses from experimental data.

(b) Stresses from finite element analysis.

Figure 3.29: Comparison of stresses in MPa at ultimate, WWF 985x71.

Figure 3.30: Finite element mesh for bridge girders.

Table 3.1: Convergence criteria.

Convergence Criterion Type	Limits Suggested by LUSAS	Limits Used in Analysis
Maximum absolute residual	Depending on units used	$< 10^{-1}$
Root mean square of the residual	Depending on units used	$< 10^{-2}$
Euclidian displacement norm	Between 0.1 and 1.0 (reasonable) Between $1E-3$ and 0.1 (tight)	$< 10^{-1}$
Euclidian residual norm (most versatile)	Between 1.0 and 5.0 (M.N.L.) ¹ less than 0.1 (G.N.L.) ²	$< 10^{-1}$
Work norm	Between $1E-3$ and $1E-1$ (slack) Between $1E-6$ and $1E-3$ (reasonable) Between $1E-9$ and $1E-6$ (tight)	$< 10^{-6}$

¹ Material nonlinearity.² Geometric nonlinearity.

Table 3.2: Test-to-predicted ratios for finite element analyses, WWF shapes, interior load case.

WWF Designation	Test Load (kN)	Predicted Load (kN)	P_t/P_p
WWF 350x212	2246.0	2282.0	0.984
WWF 500x197 ($L^1 = 1700$ mm)	2157.3	2212.0	0.975
WWF 500x197 ($L = 2000$ mm)	2126.5	2099.0	1.013
WWF 500x197 ($L = 3000$ mm)	1917.0	2059.0	0.931
WWF 900x262	2183.2	2265.0	0.964
WWF 1400x405	2582.2	2489.0	1.037
WWF 1800x510	4010.1	3910.0	1.026
WWF 985x71 ²	297.0	292.0	1.017
WWF 985x71 ³	271.0	292.0	0.928
Mean $\bar{x}$			0.986
Coeff. of variation V			0.041

¹ : span length of the beam.

² : welded one side.

³ : welded both sides.

Table 3.3: Nominal geometric properties and elastic buckling load for bridge girders with longitudinal stiffeners.

WWF Designation	L (mm)	b (mm)	t (mm)	h (mm)	w (mm)	a (mm)	P_L^1 (kN)
WWF 1450x247 ²	5500	400	25.4	1400	8	1700	1397
WWF 1850x273 ³	6000	400	25.4	1800	8	1700	1146
WWF 1850x257 ⁴	7000	420	25.4	1800	6.4	1700	549

¹ : Elastic buckling load, kN

² : Longituinal stiffener plate 10x100, mm^2
transverse stiffener plate 10x100, mm^2 .

³ : Longituinal stiffener plate 10x100, mm^2
transverse stiffener plate 12x100, mm^2 .

⁴ : Longituinal stiffener plate 12.7x100, mm^2
transverse stiffener plate 16x120, mm^2 .

Chapter 4

Development of Design Rules

4.1 Equivalent Column Model

The elastic critical buckling load developed by Benichou (1994) for the equivalent column concept described in Chapter 1, was determined by the energy method, (Timoshenko and Gere 1961), for columns with cross-sectional width of $(N + 5k)$ and thickness w for interior loads, with a sinusoidal varying average normal stress distribution. The assumed buckled shape with single curvature is given by,

$$y = \sum_{i=1}^n a_n \sin\left(\frac{n\pi z}{h}\right) \quad (4.1)$$

When investigating the applicability of this solution to WWF shapes, it was determined that the width of $(N + 5k)$ did not represent the width of influence of the applied load. The average normal stress distribution was different, the energy method solution was based on an isolated column and not on a continuous plate and thus the assumed buckled shape is inconsistent with the boundary conditions for the continuous web plate.

Based on inelastic finite element analysis of stocky W and WWF shapes for interior loads only, Benichou (1994), found that the effective width of the column, calculated by dividing the integral of the normal stresses at the critical section at the ultimate load by the yield stress of the web plate and web thickness was better defined as $(N + 10t)$ rather than $(N + 5k)$. A summary of these results is reproduced and shown in Table 4.1. The table gives a comparison of the calculated values of the effective width of the equivalent column with the values of $(N + 5k)$ and $(N + 10t)$ for sections that failed by yielding. The effective column width should be consistent for all failure modes and is based on data for sections that failed by yielding at $(N + 10t)$ for interior loads.

Starting with the assumed equivalent column width of $(N + 10t)$, the average normal stresses at ultimate load for all the WWF shapes tested were determined at various levels through the depth of the web from the inelastic finite element analysis results. Figures 4.1 to 4.6 illustrate the variation of the average normal stress at ultimate for the WWF shapes and the corresponding curve that fits the results as given by :

$$\sigma(z) = \sigma_{max} \left(\frac{z}{h} \right)^p \quad (4.2)$$

where, $\sigma(z)$ = average normal stress at any distance z measured upward
from the bottom flange, MPa ,
 σ_{max} = maximum average normal stress on the web at root of fillet weld
under the loaded flange, MPa ,
 p = a positive integer.

The fitted curves are listed as types A, B, C and D with values of the integer p of 1, 2, 3 and 7, respectively. In Figures 4.1 through 4.6 the depth through the web is

plotted in the reverse direction to z as defined in Equation 4.2 and is measured from the root of the fillet weld on the weld at the top flange web junction to the bottom flange web junction. The negative sign for the average normal stress values indicates compression.

The boundary conditions for the plate section for the interior load case are illustrated in Figure 4.7. The corresponding assumed buckled shape is given by Equation 4.3

$$y = \frac{\delta}{4} \left[\left(1 - \cos \frac{2\pi z}{h} \right) \left(1 - \cos \frac{2\pi x}{l} \right) \right] \quad (4.3)$$

where, $l = N + 10t$,

δ = maximum out-of-plane displacement of the plate,

and is illustrated by Figure 4.8 for the two cases of fixed and simply supported top and bottom, and left and right edges.

The elastic critical buckling loads per millimeter length of plate for the buckling of thin plates, based on the energy method described by Timoshenko and Gere (1961), for plates with an equivalent width of $(N + 10t)$, the average normal stress distributions that fitted those determined from finite element results and the assumed buckled shape given by Equation 4.3, were determined for all the WWF shapes tested. The solution for each integer value of p and any aspect ratio is given in Table 4.2 for all edges fixed and in Table 4.3 for all edges pinned while for the sections tested are given in Table 4.4 for both assumptions of all edges fixed and all edges pinned. A detailed sample calculation for the WWF 350x212, curve type A, with $p = 1$ follows.

The distribution of $\sigma(z)$ as a function of σ_{max} is given as

$$\sigma(z) = \sigma_{max} \left(\frac{z}{h} \right)^1 \quad (4.4)$$

where, $\sigma(z)$ is related to $P_{\sigma}(z)$ as,

$$\begin{aligned} P_{\sigma}(z) &= w \int_0^z \sigma(z) dz \\ P_{\sigma}(z) &= w \int_0^z \sigma_{max} \left(\frac{z}{h} \right)^1 dz \\ P_{\sigma}(z) &= w \frac{\sigma_{max}}{h} \left(\frac{z^2}{2} \right) \end{aligned} \quad (4.5)$$

and the maximum value of $P_{\sigma}(z)$, P_{σ} , then becomes,

$$\begin{aligned} P_{\sigma} &= w \int_0^h \sigma(z) dz \\ P_{\sigma} &= w \sigma_{max} \left(\frac{h}{2} \right) \end{aligned} \quad (4.6)$$

The load $P_{\sigma}(z)$, as a function of P_{σ} , is given as

$$P_{\sigma}(z) = \frac{P_{\sigma}}{h^2} (z^2) \quad (4.7)$$

The strain energy and potential energy of the system are defined by Equations 4.8 and 4.9, respectively.

$$U = \int_0^l \int_0^h \frac{D}{2} \left[\left(\frac{d^2 y}{dz^2} \right) + \left(\frac{d^2 y}{dx^2} \right) \right]^2 dz dx \quad (4.8)$$

where, D = plate rigidity given by $(E w^3)/12(1 - \nu^2)$.

$$W = \int_0^l \int_0^h \frac{P_{\sigma}(z)}{2} \left(\frac{dy}{dz} \right)^2 dz dx \quad (4.9)$$

Assuming that the buckled shape of the column can be expressed as

$$y = \frac{\delta}{4} \left[\left(1 - \cos \frac{2\pi z}{h} \right) \left(1 - \cos \frac{2\pi x}{l} \right) \right] \quad (4.10)$$

The first and second derivatives with respect to z and x , respectively are,

$$\begin{aligned}\frac{dy}{dz} &= \frac{\delta}{2} \left(\frac{\pi}{h} \right) \sin \frac{2\pi z}{h} \left(1 - \cos \frac{2\pi x}{l} \right) \\ \frac{d^2y}{dz^2} &= \delta \left(\frac{\pi}{h} \right)^2 \cos \frac{2\pi z}{h} \left(1 - \cos \frac{2\pi x}{l} \right) \\ \frac{dy}{dx} &= \frac{\delta}{2} \left(\frac{\pi}{l} \right) \left(1 - \cos \frac{2\pi z}{h} \right) \sin \left(\frac{2\pi x}{l} \right) \\ \frac{d^2y}{dx^2} &= \delta \left(\frac{\pi}{l} \right)^2 \left(1 - \cos \frac{2\pi z}{h} \right) \cos \left(\frac{2\pi x}{l} \right)\end{aligned}$$

Evaluation of the strain energy yields,

$$\begin{aligned}U &= \int_0^l \int_0^h \frac{D}{2} \left[\left(\frac{d^2y}{dz^2} \right)^2 + \left(\frac{d^2y}{dx^2} \right)^2 \right] dz dx \\ U &= \frac{\pi^4 \delta^2 D}{8} (hl) \left(\frac{3}{h^4} + \frac{3}{l^4} + \frac{2}{h^2 l^2} \right)\end{aligned}\quad (4.11)$$

Evaluation of the potential energy yields,

$$W = \int_0^l \int_0^h \frac{\sigma(z)(w)}{2} \left(\frac{dy}{dz} \right)^2 dz dx \quad (4.12)$$

$$\begin{aligned}W &= \int_0^l \int_0^h \frac{P_{cr}(z)}{2} \left[\frac{\delta}{2} \left(\frac{\pi}{h} \right) \sin \frac{2\pi z}{h} \left(1 - \cos \frac{2\pi x}{l} \right) \right]^2 dz dx \\ W &= \frac{\delta^2 P_{cr} l}{3072 h} (-36 + 96\pi^2)\end{aligned}\quad (4.13)$$

Equating the potential energy to the strain energy of bending to calculate the value of the critical elastic buckling load

$$\delta^2 \frac{\pi^4 D}{8} (hl) \left(\frac{3}{h^4} + \frac{3}{l^4} + \frac{2}{h^2 l^2} \right) = \delta^2 \frac{P_{cr} l}{3072 h} (-36 + 96\pi^2) \quad (4.14)$$

The elastic critical buckling load for a fixed-ended plate with the load distribution described by Equation 4.5 is,

$$P_{cr} = \left(\frac{384\pi^2}{-36 + 96\pi^2} \right) \left(\frac{3h^4 + 3l^4 + 2h^2 l^2}{l^4} \right) \frac{\pi^2 D}{h^2} \quad (4.15)$$

or for the width of the equivalent column as

$$P_{cr} = f(N + 10t) \frac{\pi^2 D}{h^2} \quad (4.16)$$

where,

$$f = \left(\frac{384\pi^2}{-36 + 96\pi^2} \right) \left(\frac{3h^4 + 3l^4 + 2h^2l^2}{l^4} \right)$$

To use a column curve, as given in S16.1 in terms of the slenderness parameter, λ , an equivalent slenderness parameter must be determined. The ratio of the elastic critical buckling load to the yield load is $(1/\lambda_c^2)$, i.e.,

$$\frac{P_{cr}}{P_y} = \frac{1}{\lambda_c^2} \quad (4.17)$$

where

$$P_y = w (N + 10t) F_y \quad (4.18)$$

Thus by substituting Equation 4.16 and Equation 4.18 into Equation 4.17 the equivalent slenderness parameter, λ_c , is

$$\lambda_c = \sqrt{\frac{12}{f}} \left(\frac{h}{w} \right) \sqrt{\frac{F_y}{\pi^2 E}} \quad (4.19)$$

or

$$\lambda_c = c \left(\frac{h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}} \quad (4.20)$$

where, r = radius of gyration of the equivalent column cross-section and is $= w/\sqrt{12}$,

c = a plate buckling coefficient which is a function of the boundary conditions, the displaced shape, the average normal stress distributions and aspect ratio of the plate, which is equal to $\sqrt{1/f}$.

Values of c are calculated from the test data as follows :

$$P_t = B_r = \phi A F_y (1 + \lambda_c^{2n})^{-\frac{1}{n}}$$

which then gives

$$\lambda_c = \left[\left(\frac{P_t}{\phi A F_y} \right)^{-n} - 1 \right]^{\frac{1}{2n}}$$

where λ_c can be expressed as,

$$\lambda_c = c \left(\frac{h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}}$$

and by rearranging the equation c is given by

$$c = \lambda_c / \left(\frac{h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}}$$

For the classical solutions the value of c is calculated from the expression $\sqrt{1/f}$ where f is the coefficient associated with the elastic critical buckling load, P_{cr} . The values of c for the WWF shapes are shown in Figure 4.9 as a function of the aspect ratio of the plate, $(N + at)/h$.

For the test data the relationship

$$c = 0.4 \frac{(N + at)}{h} \quad (4.21)$$

fits the data well. For the theoretical or classical analyses, when the edges of the column are considered to be simply supported, the relationship is

$$c = 0.329 \frac{(N + at)}{h} - 0.045, \quad (4.22)$$

and when the edges are considered to be fixed the relationship

$$c = 0.183 \frac{(N + at)}{h} - 0.031 \quad (4.23)$$

is the best fit solution. Clearly, within the other assumptions made, the value of c determined experimentally is better modelled by the simply supported boundary

conditions. This fact indicates that the deflected shapes from the test exhibit characteristics that make the web plate behaviour even more flexible than the classical theoretical simply supported case. This is illustrated in Figure 4.10 for the WWF 1400x405 which shows that at the boundary of $(N + 10t)$ there exists a lateral displacement of the edge of the equivalent column. Hence the equivalent column exhibits more flexibility than the theoretical one with simply supported edges. Taking the values of c to be that given by Equation 4.21, the value of the equivalent slenderness parameter is computed from Equation 4.20. The value of a is equal to 10 for all shapes that are Class 3 or better. The three sets of data, one set of test data and two sets of predicted data, give a strong correlation between c and the aspect ratio of the plate.

The bearing resistance of the web is then calculated from the column curve given in Clause 13.3.1 of S16.1-94,

$$B_r = C_r = \phi A F_y (1 + \lambda_c^{2n})^{-\frac{1}{n}} \quad (4.24)$$

where, ϕ = resistance factor,

A = area of the equivalent column, $w(N + 10t)$, mm^2 ,

F_y = yield stress of the material, MPa ,

$n = 2.24$.

This value of n , the highest given in Clause 13.3, is used because the effect of residual stresses on the buckling of the web plate is negligible.

A similar analytical investigation was conducted for end reaction tests for both W and WWF shapes. Following the current logic of S16.1-94 in determining the critical

length for web yielding it would be reasonable to define the effective width of the equivalent column as $(N + 5t)$. This length is likely an upper limit as the continuity of the flange and web for interior loads has a tendency to distribute the load in the web over a larger length. As a result shorter lengths of $(N + 4t)$ or less were investigated. The values of the buckling coefficient determined from test data for end reaction tests for equivalent widths of $(N + 4t)$ and $(N + 5t)$ are shown in Figures 4.11 and 4.12, respectively. The test-to-predicted ratio and the associated coefficient of variation was superior when an effective width of the column of $(N + 4t)$ was used as shown in Tables 4.7 and 4.8. This is discussed further in Section 4.3. The buckling coefficient, c , is then $1.1(N + 4t)/h$ appropriate. The coefficient of 1.1 as opposed to 0.4 results from the increased flexibility of the web at the end of the beam, i.e., continuity along the length of the beam which provides the possibility of a biaxial stress field exists only to one side of the load point, not both. There is no lateral or rotational restraint along the free edge.

For Class 4 shapes with slenderness values, h/w , greater than $(m/\sqrt{F_y})$ it is suggested that the effective column width be calculated in accordance with CSA S136 with a plate buckling coefficient, k , equal to 2. The determination of the m and the verification of the value k will require additional study.

4.2 Proposed Clause 15.9

Based on the application of the equivalent column concept to determine the bearing resistance of webs due to concentrated in-plane loads, the effective column width and buckling coefficients as evaluated from test data for both interior and end reaction tests the following is proposed in place of the existing Clause 15.9 in CSA Standard

S16.1.

Clause 15.9 Web Buckling and Yielding

15.9.1

Bearing stiffeners shall be provided where the factored concentrated loads or reactions exceed the factored bearing resistances of webs of rolled beams and welded plate girders. The factored bearing resistance of the web shall be taken as

$$B_r = \phi A F_y (1 + \lambda_e^{2n})^{-\frac{1}{n}}$$

where $n = 2.24$, and λ_e is the equivalent slenderness parameter and shall be taken as

$$\lambda_e = c \left(\frac{h}{r} \right) \sqrt{\frac{F_y}{\pi^2 E}}$$

where, r = radius of gyration of the equivalent column cross-section
 $= w/\sqrt{12}$,
 $c = 0.4 [(N + 10t)/h]$ for interior loads and
 $= 1.1 [(N + 4t)/h]$ for end reactions.

15.9.2

The area of the equivalent column is equal to $w(N + 10t)$ for interior loads and $w(N + 4t)$ for end reactions, where the length of bearing shall not be less than $2t$, for webs with a slenderness ratio less than $m^1/\sqrt{F_y}$.

15.9.3

For webs with slenderness greater than $m^1/\sqrt{F_y}$ the value for c and the area of the

equivalent column shall be based on the effective width of the equivalent column calculated in accordance with CSA S136 with a plate buckling coefficient equal to 2.

¹ The value of m is in the range of 1900 to 2200 but is still to be finalized.

4.3 Statistical Evaluation of Test Data for Limit States Design

The test-to-predicted ratios based on prediction using Clause 15.9, S16.1-94 and the proposed replacement clause are given for interior loads and end reaction tests for WWF and W shapes in Tables 4.5, 4.6, 4.7 (a),(b), and 4.8 (a),(b) respectively. The test-to-predicted ratios for the proposed clause are also illustrated for WWF and W shapes on a column curve for interior loads and also for end reaction where for the latter the effective column width is defined as $(N + 4t)$ and $(N + 5t)$ in Figures 4.13, 4.14 and 4.15, respectively.

For interior loads, the mean value and the coefficient of variation for test-to-predicted values for WWF shapes based on the proposed clause and the existing standard S16.1-94, are 1.064 and 0.073, and 1.09 and 0.132, respectively. Corresponding values for W shapes are 1.002 and 0.029, and 0.983 and 0.026, respectively. The proposed clause gives a consistent mean test-to-predicted ratio with a small coefficient of variation for both WWF and W shapes, for the full range of anticipated behaviour.

The coefficient of variation of the test-to-predicted values for S16.1-94 for WWF shapes illustrates a lack of correspondence with the observed behaviour. The proposed clause provides a simple, consistent, rational portrayal of the behaviour of the webs

under load that is readily understood by the practising engineer. The same cannot be said for the cumbersome, empirically based expressions that currently exist.

For end reactions, the mean value and coefficient of variation for test to predicted values for WWF shapes based on the proposed clause for equivalent width of $(N + 4t)$ and S16.1-94 are 1.012 and 0.053, and 1.116 and 0.053, respectively. Corresponding values for W shapes are 1.171 and 0.205, and 1.102 and 0.101, respectively. Of these the value of 0.205 is somewhat troublesome.

With the exception of the W 310x31 and the W 250x24 the proposed clause gives consistent test to predicted ratios with a small coefficient of variation for both WWF and W shapes for the full range of anticipated behaviour. The data for the W 310x31 and W 250x24 needs reinvestigation. Similar comments for the test to predicted values of S16.1-94 and interpretation of the clauses for interior loads can be made.

A statistical analysis for the determination of the appropriate resistance factor for the proposal clause for S16.1 has been conducted in accordance with the Technical Memorandum No. 10 for the next edition of the guide to Structural Stability Research Council, (Kennedy and Kennedy 1995). The mean values and the coefficients of variation for the geometric property (web thickness for rolled shapes or plates), material property (yield strength) and professional factor for WWF and W shapes are summarized in Table 4.9. Wherever possible the statistical data related to geometric and material properties of the tests were used, Benichou (1994), Kirkhope (1995), and Prowse (1996), and for the WWF shapes these are listed in Table 4.10. The nominal values for yield strength of the web plates for the WWF shapes were assumed to be 350 MPa. Two out of six values were considerably lower than the nominal value giving statistical characteristics for the distribution that are quite different than others reported in literature, Gagnon and Kennedy (1989), Chernenko and Kennedy (1991),

and Kennedy and Baker (1984). As mill reports were not provided and the statistical characteristics were suspect, the statistical data for yield strength of plates, Kennedy and Baker (1984), was used instead.

The resistance factor can be calculated from the following equation, Galambos and Ravindra (1973, 1977, 1978),

$$\phi = \rho_R \exp(-\beta \alpha_R V_R) \quad (4.25)$$

where, ρ_R = the bias coefficient for the resistance, i.e. the mean value of the ratio of the measured resistance to the nominal resistance,
 = the product of $\rho_G \rho_M \rho_P$ where in turn,
 ρ_G = the bias coefficient for the cross-sectional geometry, i.e. the mean value of the ratio of the relevant geometric property, web thickness, to the nominal value as given in steel design handbooks,
 ρ_M = the bias coefficient for the material property, i.e. the mean value of the ratio of the relevant material property, yield strength, to the nominal value as given in published steel material specifications,
 ρ_P = the bias coefficient for the design equation, i.e. the mean value of the ratio of the measured or test strength to that predicted by a design equation,
 V_R = the coefficient of variation associated with ρ_R ,
 = $\sqrt{V_G^2 + V_M^2 + V_P^2}$, when the quantities are not correlated, and V_G , V_M and V_P are the coefficients of variation of the geometric properties, material properties and test/predicted strength ratios,
 β = reliability index, currently taken to have a target value of about 3.0

for structural steel members in buildings and 4.5 for structural connections,

$\alpha_R =$ the coefficient of separation taken as 0.55 (Galambos and Ravindra 1973) (not to be confused with the load factor α_i).

Based on a target reliability index, β_T , equal to 3.0, the resistance factors for WWF and W shapes for both interior loads and end reactions were calculated to be 0.899 and 1.040, respectively. Setting ϕ equal to 0.9 yields the corresponding reliability indexes of 2.99 and 4.35.

Figure 4.1: Average normal stress distribution at ultimate load, WWF 350x212.

Figure 4.2: Average normal stress distribution at ultimate load, WWF 500x197.

Figure 4.3: Average normal stress distribution at ultimate load, WWF 900x262.

Figure 4.4: Average normal stress distribution at ultimate load, WWF 1400x405

Figure 4.5: Average normal stress distribution at ultimate load, WWF 1800x510.

Figure 4.6: Average normal stress distribution at ultimate load, WWF 985x71.

Figure 4.7: Boundary conditions for the equivalent column concept: interior loads.

Figure 4.8: Assumed buckled shapes for classical solution.

Figure 4.9: Variation of coefficient c with $(N + at)/h$: interior load.

Figure 4.10: Predicted displaced shape for WWF 1400x405.

Figure 4.11: Variation of coefficient c with $(N + 4t)/h$ for end reaction.

Figure 4.12: Variation of coefficient c with $(N + 5t)/h$ for end reaction.

Figure 4.13: Test-to-predicted bearing resistances for interior loads.

Figure 4.14: Test-to-predicted bearing resistances for end reactions with $N + 4t$.

Figure 4.15: Test-to-predicted bearing resistances for end reactions with $N + 5t$.

Table 4.1: Stress distribution due to yielding, (after Benichou (1994)).

Designation	$\frac{\int \sigma dA}{\sigma_w w}$ (mm)	$N + 5 k$ (mm)	$N + 10 t$ (mm)
W 310x143	324 ($N + 5.6 k$)	300	329
W 250x131	348 ($N + 5.9 k$)	310	351
WWF 350x212	399 ($N + 7.3 k$)	305	400
W 310x143	326 ($N + 5.7 k$)	300	329

Table 4.2: Elastic critical buckling load for all four edges fixed.

Curve Type	$\sigma(z)$ (MPa)	P_{cr} (kN/mm)
A	$\sigma_{max} \left(\frac{z}{h} \right)$	$\left(\frac{384\pi^2}{-36+96\pi^2} \right) \left(\frac{3h^4+3I^4+2h^2I^2}{I^4} \right) \frac{\pi^2 D}{h^2}$
B	$\sigma_{max} \left(\frac{z}{h} \right)^2$	$\left(\frac{512\pi^2}{-72+96\pi^2} \right) \left(\frac{3h^4+3I^4+2h^2I^2}{I^4} \right) \frac{\pi^2 D}{h^2}$
C	$\sigma_{max} \left(\frac{z}{h} \right)^3$	$\left(\frac{2560\pi^2}{-480+12\pi^2+348\pi^2} \right) \left(\frac{3h^4+3I^4+2h^2I^2}{I^4} \right) \frac{\pi^2 D}{h^2}$
D	$\sigma_{max} \left(\frac{z}{h} \right)^7$	$\left(\frac{12288\pi^2}{-4608+1024\pi^2+\frac{2512}{\pi^2}-\frac{12128}{\pi^4}+\frac{12024}{\pi^2}} \right) \left(\frac{3h^4+3I^4+2h^2I^2}{I^4} \right) \frac{\pi^2 D}{h^2}$

Table 4.3: Elastic critical buckling load for all four edges simply supported.

Curve Type	$\sigma(z)$ (MPa)	P_{cr} (kN/mm)
A	$\sigma_{max} \left(\frac{z}{h}\right)$	$\left(\frac{6\pi^2}{3+2\pi^2}\right) \left(\frac{h^2+l^2}{l^2}\right)^2 \frac{\pi^2 D}{h^2}$
B	$\sigma_{max} \left(\frac{z}{h}\right)^2$	$\left(\frac{4\pi^2}{3+\pi^2}\right) \left(\frac{h^2+l^2}{l^2}\right)^2 \frac{\pi^2 D}{h^2}$
C	$\sigma_{max} \left(\frac{z}{h}\right)^3$	$\left(\frac{10\pi^2}{10-\frac{12}{\pi^2}+2\pi^2}\right) \left(\frac{h^2+l^2}{l^2}\right)^2 \frac{\pi^2 D}{h^2}$
D	$\sigma_{max} \left(\frac{z}{h}\right)^7$	$\left(\frac{144\pi^8}{2\pi^8+36\pi^7-378\pi^5+1890\pi^3-5670\pi}\right) \left(\frac{h^2+l^2}{l^2}\right)^2 \frac{\pi^2 D}{h^2}$

Table 4.4: Elastic critical buckling load for WWF shapes.

WWF Designation	Curve Type	Fixed P_{cr} (kN/mm)	Simply Supported P_{cr} (kN/mm)
WWF 350x212	A	19.38 $\frac{\pi^2 D}{h^2}$	5.72 $\frac{\pi^2 D}{h^2}$
WWF 500x197	A	47.11 $\frac{\pi^2 D}{h^2}$	14.58 $\frac{\pi^2 D}{h^2}$
WWF 900x262	B	104.43 $\frac{\pi^2 D}{h^2}$	26.41 $\frac{\pi^2 D}{h^2}$
WWF 1400x405	B	444.33 $\frac{\pi^2 D}{h^2}$	97.71 $\frac{\pi^2 D}{h^2}$
WWF 1800x510	C	1693.58 $\frac{\pi^2 D}{h^2}$	242.40 $\frac{\pi^2 D}{h^2}$
WWF 985x71	D	132057.80 $\frac{\pi^2 D}{h^2}$	112571.50 $\frac{\pi^2 D}{h^2}$

Table 4.5: Test-to-predicted ratios, WWF shapes, interior loads, $c = 0.4(N + at)/h$

WWF Designation	Test Load (kN)	Predicted Load (kN)	P_t/P_p	S16.1 (kN)	$P_t/P_{S16.1}$
WWF 350x212	2246.0	2431.1	0.924	2021.5	1.111
WWF 500x197	1917.0	1907.30	1.005	1879.1	1.020
WWF 900x262	2183.2	2271.0	0.961	1841.6	1.185
WWF 1400x405	2582.2	2670.2	0.967	2290.7	1.127
WWF 1800x510	4010.1	4122.1	0.973	3444.0	1.164
WWF 985x71	271.0	273.6	0.991	248.7	1.090
$\bar{x}$			1.064		1.090
V			0.073		0.132

Table 4.6: Test-to-predicted ratios, W shapes, interior loads, $c = 0.4(N + at)/h$

W Designation	Test Load (kN)	Predicted Load (kN)	P_t/P_p	S16.1 (kN)	$P_t/P_{S16.1}$
W 530x101	1345.0	1435.0	0.937	1378.9	0.975
W 530x82	1180.0	1239.9	0.952	1174.3	1.005
W 410x39	553.0	585.9	0.944	569.5	0.971
W 360x45	665.0	682.6	0.974	698.7	0.952
W 310x31	497.0	467.7	1.063	486.4	1.022
W 250x24	434.0	379.7	1.143	446.8	0.971
$\bar{x}$			1.002		0.983
V			0.029		0.026

Table 4.7: Test-to-predicted ratios, WWF shapes, end reactions.

WWF Designation	Test Load (kN)	LUSAS Load (kN)	P_t/P_L	S16.1 (kN)	$P_t/P_{S16.1}$
WWF 350x212	967.0	876.0	1.104	1011.0	0.956
WWF 500x197	756.0	732.0	1.033	777.0	0.973
WWF 900x262	885.0	882.0	1.003	834.0	1.061
WWF 1400x405	1196.0	1014.0	1.179	1043.0	1.147
WWF 1800x510	1654.0	1656.6	0.998	1563.0	1.058
WWF 985x71	159.0	125.0	1.272	118.0	1.347
$\bar{x}$			0.970		1.116
V			0.029		0.053

(a) Finite element predictions and S16.1-94.

WWF Designation	Test Load (kN)	$Load_{N+4t}$ (kN)	P_t/P_{N+4t}	$Load_{N+3t}$ (kN)	P_t/P_{N+3t}
WWF 350x212	967.0	1024.0	0.944	1192.0	0.811
WWF 500x197	756.0	797.0	0.949	830.0	0.911
WWF 900x262	885.0	809.0	1.094	773.0	1.145
WWF 1400x405	1196.0	1112.0	1.076	1161.0	1.030
WWF 1800x510	1654.0	1754.0	0.943	1864.0	0.887
WWF 985x71	159.0	149.0	1.067	153.0	1.039
$\bar{x}$			1.012		0.971
V			0.053		0.126

(b) Proposed

Table 4.8: Test-to-predicted ratios, W shapes, end reactions.

W Designation	Test Load (kN)	LUSAS Load (kN)	P_t/P_L	S16.1 (kN)	$P_t/P_{S16.1}$
W 530x101	628.5 ¹	575.0	1.093	571.0	1.102
W 530x82	569.0 ¹	524.0	1.086	472.0	1.206
W 410x39	216.0 ¹	263.0	0.821	218.0	0.991
W 360x45	264.5 ¹	301.0	0.879	266.0	0.992
W 310x31	221.0 ¹	241.0	0.917	181.0	1.221
W 250x24	196.0	215.0	0.912	159.0	1.245
$\bar{x}$			0.951		1.102
V			0.141		0.101

Notes : ¹ these values are the average of two end reaction tests from the same beam.

(a) Finite element predictions and S16.1-94.

W Designation	Test Load (kN)	Load $\frac{N+1}{A}$ (kN)	$P_t/P_{\frac{N+1}{A}}$	Load $\frac{N+31}{A}$ (kN)	$P_t/P_{\frac{N+31}{A}}$
W 530x101	628.5 ¹	601.0	1.046	623.0	1.010
W 530x82	569.0 ¹	516.0	1.103	529.0	1.076
W 410x39	216.0 ¹	226.0	0.956	225.0	0.960
W 360x45	264.5 ¹	263.0	1.006	261.0	1.013
W 310x31	221.0 ¹	166.0	1.331	161.0	1.373
W 250x24	196.0	124.0	1.581	121.0	1.620
$\bar{x}$			1.171		1.175
V			0.205		0.224

Notes : ¹ these values are the average of two end reaction tests from the same beam.

(b) Proposed

Table 4.9: Statistical data for determining resistance factor.

Shape	ρ_G	V_G	ρ_m	V_m	ρ_p	V_p	ρ_R	V_R	ϕ	β
WWF	1.047	0.077	1.101	0.110	1.002	0.070	1.154	0.151	0.899	2.991
W	1.045	0.048	1.262	0.061	1.086	0.178	1.432	0.194	1.040	4.350

Notes : 1. ϕ is based on a target reliability index $\beta_T = 3$.

2. β , reliability index, is for $\phi = 0.9$.

Table 4.10: Test data for statistical analysis of WWF shapes.

WWF Designation	w_{measured} (mm)	w_{nominal} (mm)	$w_{m/n}^1$	$\sigma_{y\text{measured}}$ (MPa)	$\sigma_{y\text{nominal}}^3$ (MPa)	$\sigma_{y m/n}^2$
WWF 350x212	18.91	20.00	0.9455	308.5	350.0 ⁴	0.8814
WWF 500x197	12.36	11.00	1.1236	413.2	350.0	1.1806
WWF 900x262	12.66	11.00	1.1509	373.0	350.0	1.0657
WWF 1400x405	16.24	16.00	1.0150	280.0	350.0 ⁴	0.8000
WWF 1800x510	19.69	20.00	0.9845	348.8	350.0	0.9966
WWF 985x71	5.30	5.00	1.0600	378.8	350.0	1.0822
ρ		$\rho_G =$	1.0466		$\rho_m =$	1.0011
V		$V_G =$	0.0765		$V_m =$	0.1397

¹ : The ratio of measured to nominal values for web thickness.

² : The ratio of measured to nominal values for yield stress.

³ : The nominal strength was assumed to be 350 MPa for all the web plates as mill reports were not provided for the steel.

⁴ : These nominal values are suspect since the variation from the measured yield strength is not typical of statistical data.

Chapter 5

Summary, Conclusions, Recommendations and Future Work

5.1 Summary

The current provisions for web yielding and web crippling given in Clause 15.9 are semi-empirical or empirical and are based on test data that generally does not represent the population of existing shapes, realistic load lengths and boundary conditions. Therefore, the objective of this research was to conduct an analytical study to describe the behaviour of webs of WWF shapes subjected to concentrated interior in-plane loads, representing the full range of anticipated web behaviour and the full range of geometry and class of sections produced in Canadian mills. Then from the results of these analyses, to develop a rational mathematical model and the cor-

responding design rules to predict the bearing resistance of webs of W and WWF shapes subjected to concentrated in-plane loads when applied as interior loads or end reactions, thus replacing the existing provisions given in Clause 15.9 of CSA 1994a.

The strain and displacement data obtained by Kirkhope (1995), and Prowse (1996), from interior load tests on WWF 350x212, WWF 500x197 (spans of 1700 mm, 2000 mm and 3000 mm), WWF 900x262, WWF 1400x405, WWF 1800x510 and WWF 985x71, was processed into a format suitable for direct comparison with the results of finite element analysis of the test beams. The commercial finite element program LUSAS was used to determine the behaviour and inelastic web buckling capacity based on measured geometry, measured initial out-of-plane web imperfections, measured material properties and boundary conditions that modelled those of the test.

Given the complete description of the local stability failure of the webs, the equivalent column concept first introduced by Benichou (1994) was further developed. It considers the web that is defined by the effective column width, $(N + 10t)$ for interior loads and $(N + 4t)$ for end reactions, and the web depth, to act as a column with a modified slenderness parameter which incorporates a buckling coefficient that reflects the buckling conditions of a plate. Clause 13.3.1, that gives the factored axial compressive resistance of columns, is then used with an equivalent slenderness parameter previously described and a cross-sectional area defined by the effective column width and web thickness to calculate the bearing resistance of the web due to concentrated in-plane loads. The buckling coefficient, c , is equal to 0.4 times the aspect ratio, $(N + 10t)/h$, for interior loads and 1.1 times the aspect ratio, $(N + 4t)/h$, for end reactions. For Class 4 sections the effective width of the column can be determined using the effective width concept for plate buckling and corresponding rules defined in CSA 1994b.

The appropriate design clauses for CSA 1994a, for the determination of bearing resistance of the web, based on the equivalent column concept have been written. A statistical evaluation of the test data and the test-to-predicted ratios was used to determine a value of the resistance factor, $\phi = 0.9$ is appropriate and results in a reliability index of 3.0 or more.

5.2 Conclusions

Based on this analytical study and the supporting test data the following conclusions are drawn:

1. Sufficient quantity of test data has been produced to describe the load deflection behaviour, displaced shape configuration, and internal stress distributions for webs subjected to concentrated in-plane loads for interior and end reactions, for the full range of W and WWF shapes produced in Canadian mills, Benichou (1994), Kirkhope (1995), and Prowse (1996);
2. The mean value and coefficient of variation of the test to predicted ratios for the WWF shapes, interior load cases, for predictions resulting from the finite element analysis described within Chapter 3 are 0.986 and 0.041 respectively. The comparison also illustrates close correspondence with the test data for load deflection behaviour, displaced shapes and internal stress distributions.
3. Based on the series of tests as described and the corresponding finite element analysis, an equivalent column model and design equations were developed. The mean test to predicted ratios and coefficients of variation were 1.064 and 0.073, for WWF shapes with interior loads and 1.012 and

0.053, for WWF shapes with end reactions, and 1.002 and 0.029, for W shapes with interior loads and 1.171 and 0.205, for W shapes with end reactions.

The equivalent column concept includes :

- (a) an effective width defined by determining the width of webs with an equivalent rectangular stress distribution equal to yield strength, required to equilibrate the ultimate load determined from finite element analysis of stocky webs. This was verified subsequently by tests;
- (b) based on classical plate buckling solutions with average normal stress distribution over the equivalent column width, for fixed and simply supported boundary conditions on all edges, the elastic critical buckling load and the corresponding plate buckling coefficients were determined. The average normal stress distributions through the depth were obtained from finite element analyses and verified with the test data;
- (c) a strong correlation was found between the buckling coefficient and aspect ratio of the equivalent column with an equivalent column width of $N + 10t$. For fixed edge and simply supported edge cases the coefficients in which the aspect ratio is multiplied to give the buckling coefficient are 0.183 and 0.329, respectively. Based on test load data the same strong correlation was observed with a coefficient of 0.4 for interior load cases. The increased value reflects the greater flexibility of the web plate in the test with respect to the assumed displacement field and boundary conditions used in the classical buckling analysis on the equivalent column;

- (d) for end conditions where there is no continuity at the end of the beam the coefficient was determined from tests to be 1.1 and the equivalent column width to be $(N + 4t)$;
 - (e) from the classical approach the equivalent slenderness parameter was determined from the relation $P_{cr}/P_y = 1/\lambda_e^2$;
 - (f) the equivalent slenderness parameter is then used to calculate the bearing resistance using the column curve in Clause 13.3.1 with the appropriate definition of area.
4. Based on a statistical evaluation of the data and predictions based on the proposed clause, the resistance factor was determined to be 1.040 for W shapes and 0.899 for WWF shapes, for a target reliability index of 3.0. A reliability index calculated for a resistance factor of 0.9 was 4.35 for W shapes and 2.99 for WWF shapes.

The proposed clause gives a simple, unified, easily understood design approach to predict the bearing resistances of webs of W and WWF shapes for both interior loads and end reactions for the full range of shapes produced by Canadian mills.

5.3 Recommendations and Future Work

From this study on the behaviour of W and WWF shapes subjected to concentrated in-plane loads it is apparent that further analytical investigation and experimental verification are required to increase the data base of the equivalent slenderness parameter, λ_e , values for webs of box girder and plate girder bridges with longitudinal stiffeners that are launched. Much of the suggested future work is currently in progress.

1. Analytical study and experimental verification of behaviour of webs of bridge girders with a longitudinal stiffener at $0.2h$ and with $h/w \geq 175$.
2. Further tests and corresponding finite element analysis of Class 4 web sections in which the slenderness limit for h/w equals a $m/\sqrt{F_y}$ and the buckling coefficient, k , required to determine the slenderness limit, $m/\sqrt{F_y}$, and the effective width for use in S136, need to be defined.
3. Reinvestigation of W 310x31 and W 250x24 as indicated in Section 4.3.
4. A literature review, tests and analytical investigation to develop the design rules for hollow structural sections.

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