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THE MEASUREMENT OF CONTACT RESISTIVITY

by

Stephen S. Winterton, B.Eng., M.Eng.

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Doctor of Philosophy**

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
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Carleton University
November 1989
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ABSTRACT

In this study, three new approaches to the measurement of contact resistivity, ρ_c , are proposed. These methods have the advantage of a theoretically sounder basis than, and eliminate some of the practical problems associated with, methods previously reported in the literature.

A review of existing techniques suggests that the assumption of a governing Helmholtz equation, which is currently the most sophisticated treatment of devices on thin layers, has significant limitations.

The new methods should prove generally useful where one is willing to use thick, uniformly-doped substrates rather than thin diffused or implanted layers — i.e., where the problem of testing for ρ_c is separated from that of the measurement of other device parameters.

The first new technique uses contacts in the form of two parallel stripes as in the Berger structure. Unlike the Berger structure, the “device well” (here, the substrate) is made infinitely thick to minimize bulk resistance effects. A second method makes use of the true asymptotic behaviour of the resistance between any pair of contacts in an approach to measuring ρ_c using graphical techniques. In the third method, two small and closely spaced square contacts on a thick substrate represent an optimized device layout for the measurement of ρ_c .

Two general approaches to analysis of three-dimensional Laplace problems are used in this thesis. The first, which bears some resemblance to variational calculus, has been used to develop bounds on resistance and ρ_c , and to allow direct computation of contact and bulk *resistance* from measurements of total resistance and device geometry. The second approach mixes a numerical implementation of Green's function solutions (boundary elements) in the semiconducting bulk with finite differences applied to metal-covered regions to allow true three-dimensional modelling of an important class of device structures.

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LIST OF SYMBOLS

ρ_c	contact resistivity
ρ_B	bulk resistivity
σ_B	bulk conductivity
R	resistance
R_s	sheet resistance
R_f	front resistance
R_e	end resistance
R_D	"Dirichlet resistance"
R_N	"Neumann resistance"
R_m	metal sheet resistance
R_{2-D}	resistance of a 2-D sheet (in Ωcm)
TLM	transmission-line model
P	power dissipation
I	total current flowing
V	total potential difference between a pair of electrodes
i	current flowing at a point (e.g., in a one-dimensional transmission-line)
v	voltage drop at a point
ϕ	potential function
j	current density
E	electric field

j_n	current density normal to a surface
T	thickness
W	width
L	length
A_c	contact area
L_{eff}	square root of effective contact area
S	surface area
D	distance between voltage and current probes
s	spacing between a pair of contacts
$ \mathbf{r} - \mathbf{r}' $	distance between source and observation point
x, y, z	Cartesian coordinates
B, m, c	as subscripts: bulk, metal, contact
α	$\rho_c / \rho_B L$
β	spacing to length ratio
BEM	boundary element method
∇^2	Laplacian
∇	gradient
$\nabla \cdot$	divergence
$\frac{\partial}{\partial n}$	component of gradient normal to a surface
Δ	change in or difference
δ	change in

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Chapter 1

INTRODUCTION

The aim of this thesis was the development of an accurate technique for contact resistivity measurement.

The concept of contact resistivity, ρ_c , as defined by Berger [1] is a simple one analogous to bulk resistivity, ρ_B . Figure 1.1 illustrates ρ_c and ρ_B and other parameters discussed below. The concept of bulk resistivity will be discussed first. In the case of bulk samples in the shape of a bar of constant cross-sectional area A , it is found that a resistance is measured between the ends of the form

$$R = \frac{\rho_B \ell}{A} \quad (1.1)$$

where ρ_B is independent of the length and cross-sectional area of the bar. In the case of more general shapes, Laplace's equation must be used

$$\nabla^2 \phi = 0 \quad (1.2)$$

to determine the potential distribution ϕ in the sample. Provided that the contacts are equipotentials and that the potential is known at the contacts one can solve for

Bar-shaped sample

Planar structure

Figure 1.1: Definition of the Electrical Symbols in the Contact Resistance Problem.

the potential everywhere in the bulk. Given a ρ_B which is constant everywhere, it is possible to determine the current

$$I = \frac{1}{\rho_B} \int_S \nabla \phi \cdot dS \quad (1.3)$$

at the electrode, from which a resistance may be calculated.¹

Since bulk resistivity is a constant independent of the sample shape, a single number, ρ_B , completely characterizes a wide range of situations. This makes it possible in some cases to calculate the resistance of structures² analytically, without the need for further experiment. However, the original simplicity of the form of solution (1.1) is lost in the case of general shapes, while the calculations involving (1.2) and (1.3) are usually much more difficult as well.

Much of this carries over to the case of contact resistivity, ρ_c . For a bar of constant cross-section and with two non-ideal contacts at its ends, as pictured in Figure 1.1, the resistance is

$$R = \frac{\rho_B \ell + 2\rho_c}{A} \quad (1.4)$$

However, for more general shapes, the contact merely provides a boundary condition to the problem of (1.2), namely

¹In the case of the most commonly used approaches however, the contacts are formed on a thin layer of non-uniform doping. In the general case of non-uniform conductivity, $\sigma_B(x, y, z)$,

$$\nabla \cdot (\sigma_B(x, y, z) \nabla \phi) = 0$$

holds rather than (1.2). In addition, the metal cannot always be treated as lossless and is therefore also subject to Laplace's equation.

²like the four-point probe and van der Pauw structures used for ρ_B measurement

$$j = \frac{\phi_m - \phi_B}{\rho_c} \quad (1.5)$$

The measurement of small values of ρ_c is made difficult by several facts. As (1.4) suggests, contact resistance is inevitably accompanied by bulk resistance. The difficulty of manufacturing a simple bar-shaped sample of short enough length ℓ that ρ_c can be accurately determined demands the use of planar-technology-type structures (i.e., made on one face of a wafer) with small contacts placed in close proximity to each other. As a result, mathematical problems of considerable difficulty arise. The situation is even worse than the case of bulk resistance considered earlier in that the measurement of ρ_c involves the extraction of what is generally a small contact resistance component from a large bulk resistance "contamination". Contact resistivity is therefore necessarily determined from two independent measurements on differently shaped or sized samples, and this leads to an ill-conditioned problem. Consequently approximate modelling of device structures introduces unacceptable additional uncertainty into the problem, and as a result considerable emphasis has been placed on enhancing model accuracy.

It seems generally recognized that the underlying equations (1.2) and (1.5) form a sound basis for accurately modelling ohmic contacts to silicon, provided the potential drop applied across the contact interface is small [2]. Also, it is obvious in principle even without direct experimental evidence that measurement structures are truly three-dimensional. Since one- [2,3] and two-dimensional [4] models have in turn been proven necessary, it seems justified to attempt to obtain valid three-dimensional results. The poor agreement reported recently [5] between predictions of the two-dimensional Helmholtz equation and laboratory observations would seem likely due in large part to neglect of the effect of the finite thickness of the semiconductor (i.e., its three-dimensional nature).

A fairly large number of structures and models have been proposed in the

interest of measuring ρ_c . It is usually apparent that the structure has been devised so that it will favour some elegant or simple approximate analysis. The approach of this thesis and the main point of departure from earlier approaches has been to optimize the test structure for the measurement of ρ_c and then to model it exactly using analytical bounds or numerical methods.

1.1 Importance of ρ_c in Si Technology

1.1.1 Series Resistance Effects

Pulfrey and Tarr [6] have pointed out that a satisfactory "ohmic" contact for use in a VLSI integrated circuit should be linear and of very low resistance, with a contact resistivity of no more than about $10^{-6}\Omega\text{cm}^2$. The boundary condition (1.5) suggests that this rule of thumb might be relaxed for other semiconductor devices such as solar cells, in which the contact current densities are small and a voltage drop of several millivolts across the contact may be tolerable³ [7-9]. The most immediate and obvious role of ρ_c is therefore to give rise to a series resistance which degrades the current-voltage characteristics of semiconductor devices.

It has been pointed out that the importance of this series resistance effect will increase in future VLSI integrated circuits. For example, Ning and Tang [10]

³An instance from the writer's past provides a concrete example of the fact that contact resistance can occasionally be a dominant effect in a semiconductor device [9]. A few years ago, there was some interest in making solar cell electrodes with thick-film silver pastes. The advantage of this technique was that it was an "open-air" process which could ultimately be carried out in an assembly-line fashion, and it therefore promised low cost. The dominant features of the problem were:

1. an enormous contact resistance following furnace treatment, and
2. contact resistance could effectively be reduced only by dipping the devices in a dilute hydrofluoric acid solution leading to reliability concerns.

have suggested that contact resistivities in the range of $10^{-7} - 7 \times 10^{-7} \Omega cm^2$ are of concern in advanced bipolar transistors with polysilicon emitters, since with this latter technique there is an increase of current gain at the expense of added series resistance. Ning and Tang [10] have also proposed that this may require, in applications calling for higher currents, an increase in the area of the emitter of the transistor.

Naem and Smith [11] have suggested that the effect of contact series resistance will become increasingly significant in scaled-down MOSFETs, because as the transistors are scaled down in size, the contact resistance will increase while the channel resistance falls. Wright, Loh and Saraswat [5] have pointed out that in MOSFETs with channel lengths below $0.5 \mu m$, the contact resistance will become a major component of parasitic resistance. Further to this, Pimbley [12] has stated that series resistance is emerging as one of several factors limiting the performance of submicrometer MOSFETs. In view of these observations, and given the fact that ρ_c values measured in the laboratory are now almost routinely reported to be in the range $10^{-8} - 10^{-6} \Omega cm^2$ [4], there is some interest in accurate measurement of values of ρ_c below about $10^{-7} - 10^{-6} \Omega cm^2$.

1.1.2 Reliability Issues

In analyzing a test structure for ρ_c measurement it is something of a paradox that one must solve for the potential everywhere in order to calculate a single number, the resistance between a pair of contacts. In the approaches developed in this thesis, if the current density were known across the contact windows, one could immediately solve for the resistance or for the potential distribution everywhere. Therefore an immediate by-product of any such analysis of a test structure leading to knowledge of the relationship between resistance and ρ_c , is knowledge also of the contact window current distribution. This may prove useful in studying reliability

issues.

Black [13-14] and Chern, Oldham and Cheung [15-16] have reported on a pair of physical mechanisms termed electromigration which can lead to semiconductor device destruction at high currents and temperatures. One of these mechanisms leads to failure to open-circuit of aluminum conducting lines, while the second leads to failure through short-circuit of n^+/p junctions, in which the silicon is gradually "eroded" when aluminum is used as the contact material. This second mode of failure is similar to a purely thermal reaction which occurs between silicon and pure aluminum [3], which has made pure aluminum an unsatisfactory contact material for modern VLSI circuits [17-18]. The use of Al-Si alloys has proven an effective counter-measure to this problem [3,18], but interestingly does nothing to mitigate the silicon erosion taking place in electromigration [13]. At any rate, this failure mechanism is a strong function of current density [13-14] and silicon pitting takes place preferentially at the contact window edges [13-16]. The computations of Overmeyer [19] and Berger [1-2] have shown the current density to be highest in these regions. Thus an accurate approach to computing the contact window current density distribution may considerably clarify the early stage of failure prior to the onset of the gross physical changes described by Black [13-14]. Electromigration has been pointed out as one of the serious challenges in further scaling down of device size [20].

1.1.3 Current Flow Mechanisms

A third area of interest is in quantitative understanding of the current flow mechanisms operating in ohmic contacts. Previous attempts to model contacts in terms of thermionic emission, tunnelling or thermionic-field emission have been only qualitatively successful [21-25]. It may be that such physical pictures are incomplete since Naguib and Hobbs have given some evidence to suggest that a thin p^+ layer deposits

on the silicon when aluminum contacts to silicon are sintered [17]. However, lack of quantitative understanding may also be due to the absence of completely reliable ρ_c data, as has been suggested recently by Swirhun [25].

1.2 The Absence of a Reliable, Standard Technique

There is to date no one method for measuring contact resistivity which is completely reliable. In the low ranges, the measurements tend to confuse the roles played by bulk, contact and geometry.

Two methods which tend to be used more often than others are the Berger [1,2] and Kelvin [3] techniques. These are described in Chapter 2. The ingenuity underlying these techniques is unquestioned. The first uses a roughly one-dimensional structure in which semiconductor bulk and contact effects are represented by a transmission-line model. The second is arranged so that current flow in the bulk is roughly at right angles to the path along which voltage is detected (to "first order"). However, the analyses on which these methods were originally based are one-dimensional. For low ρ_c , three-dimensional bulk effects operate to dominate the measurement. Two-dimensional effects in the plane of the interface have been studied by a number of workers [4,22,26-30] who have realized a "two-dimensional transmission-line model" in the form of the Helmholtz equation. Others have studied two-dimensional effects arising from the finite thickness of the semiconductor layer [12,19,31-34], largely using conformal-mapping techniques. These studies have shown that the methods in question become completely unreliable in the range 10^{-8} to $10^{-7} \Omega\text{cm}^2$ [29] for $4 \mu\text{m}$ contact windows, the size of device currently employed in the literature in comparisons of ρ_c measurement techniques.

The appeal of these techniques arises from the simplicity of the original analyses

and wears off when one considers the complexity of more satisfactory models and some defects in the experimental methods. ⁴

The principal drawback of both methods is the relative largeness of the parasitic bulk resistance, and the fact that it is fully three-dimensional and has never been analyzed as such. Subsidiary considerations are the neglect of metal resistance and the non-uniform conductivity of the substrate [19,35], and the effect of misalignment of contact holes [3] and related problems originating with photolithography or lateral spreading of diffused areas.

Chapter 2 studies alternative approaches to modelling such structures which may have a limited usefulness for special device geometries, but are not regarded as a solution to the problems raised.

1.3 Contributions to a More Reliable Technique

In accurately measuring any physical property one generally concentrates on asymptotic behaviour, in which case competing effects play a much diminished role, or contrives a situation in which all operating effects can be analyzed in detail. An example of the first approach is the four-point probe method for measurement of the resistivity of wafers and of thin diffused or implanted layers. The probe diameter, probe spacing and thickness of the semiconductor wafer or diffused layer are usually such, or are selected so that a very simple relationship between a measured resistance and an inferred sheet resistance⁵ will hold [21,36].

An example of the second approach is the Haynes-Shockley experiment [37]

⁴A model here refers to a particular solution of the governing equations with a set of fixed assumptions and approximations made, rather than to the general problem of analysis described earlier, to which a solution only exists in principle.

⁵The unit $\Omega/\square$ is the resistance between the ends of one square of material where the thickness is always constant and can be factored out of expressions. That is, ρ_B/T gives units of $\Omega/\square$.

where separation of the competing effects of drift, diffusion and recombination is completed by analysis. Another principle of good measurement practice is to contrive the simplest possible physical arrangement requiring the least analysis to interpret. This is illustrated by the use of guard-rings to minimize fringing fields in devices (e.g., early standard capacitance [38]).

The simplest arrangement possible for ρ_c measurement would be to have two contacts facing each other across a wafer, or a sample diced from a wafer. However, in this case the contact effects would represent only a few percent of the total measured resistance. Other, easy-to-analyze shapes (spheres, cylinders) can not be fabricated. Consequently, it is necessary to follow one trend in the literature which is to place both contacts on one face of the wafer. It is also essential to use “four-point” electrical measurements.

One feature of this thesis has been to place the contacts close together on a thick, uniformly-doped substrate. This has not specifically appeared in the literature and several advantages accrue:

1. bulk resistance is minimized, ⁶
2. ρ_c can be referenced to a specific bulk resistivity on which it strongly depends [1],⁷ and
3. disturbance of the resistivity of the underlying bulk by “sintering” the contact is no longer a serious issue as it has been in previous approaches to measurement [39].

The first method to be developed in this thesis employing this strategy was the parallel stripes technique of Chapter 3. The analysis of a pair of equipotential

⁶Experimentally, the Berger structure [1,2] is especially unsound because of the large electrical paths in the bulk.

⁷Unlike a non-uniformly doped sample where the position of the interface plane may be difficult to identify.

metal stripes by conformal mapping was based upon and extended on some previous studies of the Berger structure by Overmeyer [19] and Schuldt [31]. The new method was only made practical by analyzing the metal resistance by transmission-line methods on the assumption of a slowly varying potential [40,41].

A discovery that this example had mathematically bounded contact resistance gave rise to a proof that this was always the case, as described in Chapter 4. This turned out to be independent rather than original. A proof was first advanced by Lewis [42] in the context of heat flow, and has since been applied to spreading resistance problems [43,44]. The writer has here extended the proof to include a number of cases of interest: non-uniform doping, real (lossy) metal and “end resistance” measurements. These bounds correctly describe the asymptotic behaviour of true solutions to the contact resistance problem and could indicate whether proposed approximate solutions are realistic.

A method for measurement of ρ_c based on asymptotic behaviour has been proposed. This is believed to be an original contribution, and constitutes the first of two apparently reliable approaches to ρ_c measurement developed here.

The dominance of metal resistance in the two stripe method suggested minimizing the structure’s aspect ratio. This gives rise to the method of two dots described in Chapter 5. This was analyzed by three-dimensional Green’s functions, and since metal resistance is generally expected to play a very small role, it represents the second apparently reliable approach to ρ_c measurement developed in this thesis. A similar structure has been independently studied in the literature:

1. to measure bulk resistivity — a different context [45] and
2. to measure ρ_c , without either proper analysis or optimization of the test structure through minimizing contact spacing [46].

Given these differences, this is believed to represent an original contribution. The new method may allow measurement of ρ_c to $10^{-9} - 10^{-8} \Omega\text{cm}^2$ on emitter or source/drain material.

A fourth measurement technique, called the current attenuation method, is one in which every factor can be analyzed. This is covered in Chapter 6. It has no precursors in the literature but, despite being original, it is impractical because dominating bulk effects would generally obscure ρ_c .

Chapter 7 rounds out the discussion with conclusions and recommendations.

1.4 Summary and Closing Remarks

The aim of this thesis was the development of a high-precision technique for ρ_c measurement. There is considerable interest in this area in the literature even if the identifiable need is generally only a modest one. Existing techniques tend to be ingenious rather than experimentally sound. Two new techniques have been developed which have a considerably more rigorous basis, and which also seem quite practical: the method of two dots and the method of bounds. Using these techniques may increase measurement precision 10- or 100-fold. To the criticism that these do not answer practical needs as well as the Berger and Kelvin methods, the reply is either to measure ρ_c separately from other factors or to calibrate the inferior techniques.

No comparably rigorous study of the Kelvin or Berger structures was undertaken, as this would have involved complicated numerical methods. The method of boundary elements [47,48] was used in this thesis because of the possibility of treating open domains and reduction of dimension by one. This might still be a credible option but applied to these other structures would be greatly more involved.

The writer was initially reluctant to use numerical methods and was only gradually convinced of their necessity because the system of differential equations governing the problem at hand was in effect non-linear, and therefore analytically intractable.⁸ The emphasis throughout was instead on simplicity, and numerical modelling of existing techniques is far from simple. A great deal of work has been done on modelling these structures already, and so the potential of a further study would lie in answering questions of detail rather than in the discovery of anything fundamental. Consequently, effort was directed instead at developing new measurement strategies based upon fundamental principles.

A few practical issues have also been neglected. First, there is the question of contact inhomogeneity and interface area indeterminacy (pitting) [1,49,50]. Throughout it has been assumed that ρ_c is a global contact property and that the interface area is equal to the nominal or measured value. This is reasonable since violations of these assumptions are usually reported for pure aluminum contacts which are themselves only of theoretical interest. Secondly, what about issues of reliability, other important electrical properties and manufacturability [51,52]? Does the metal adhere well to oxide and silicon? Is it highly conductive? Does it suffer from electromigration or thermomigration? These have not been studied here, but it can be reasonably hoped that there will be "spin-off" benefits from the present study to these other areas of concern.

⁸The Laplace problem appears to be solvable only when one type of linear boundary condition is given over each whole coordinate surface (i.e., not mixed conditions). In the problems of this thesis the mixed conditions on the top surface of the wafer can be transformed into a consistent Neumann condition on the same surface, but in the process the Laplace equation becomes a nonlinear PDE

$$\nabla^2 e^u = 0$$

Chapter 2

CRITICISM OF THE MAINSTREAM APPROACHES TO ρ_c MEASUREMENT

Many investigations have aimed at improving the theoretical understanding of the Berger and Kelvin structures, which are the most popular approaches to measurement in the literature. The effect of a finite substrate thickness has been studied by several techniques. Two-dimensional effects in the plane of the interface have also been modelled by several workers [4,22,26-30]; a consistent formalism has emerged for this case - the Helmholtz equation - unlike the case for finite substrate thickness effects, which have been studied by conformal mapping [19,30-32,34], dual-layer transmission-lines [12] and analog computation [33].

At first glance, it seems more likely that two-dimensional effects in the interface plane should be the dominant source of parasitic resistance. Is this true? Studies [29] have shown that the parasitic bulk resistance amounts to $10^{-8} - 10^{-7} \Omega cm^2$ in the Kelvin structure with present geometries.¹ A rough calculation would assign a value of

¹e.g., $4\mu m$ contact windows

$$R_s T^2 \quad (2.1)$$

to thickness effects.² This therefore lies in the same range³ and is of comparable importance; in other words, typical devices are fully three-dimensional.

That devices are three-dimensional is not in itself a drawback. One must, however, consider how experimentally sound the methods are, and whether all the difficulties attending a more accurate and much more complicated model, are justified.

In the Berger structure, front resistance measurements (Figure 2.1) inevitably include a large bulk resistance component in the total resistance measured between contacts. Subtraction (roughly speaking) of two such resistances is needed to extract the contact resistance. This requires that there be extremely close matching of bulk, contact and geometry properties between two devices.⁴ It is doubtful that a small contact resistance can accurately survive such treatment [17]. Clearly it would help to bring the contacts close together, to push the boundaries of the diffusion out further from the contacts, and to thicken the device well. This leaves us with something like the “two dots” structure to be discussed later.

Berger [1] proposed an alternative approach called the “end resistance” measurement (Figure 2.2), which diminishes the parasitic bulk resistance. The approach provides a direct computation for ρ_c if R_s is known. It turns out that if R_s , as obtained from front resistance measurements, is used, it often gives a pair of inconsistent values of ρ_c from the two configurations. This is attributed to a sheet resistance change in the bulk under the contact, and is treated on this basis in computations [22,39]. Such ad hoc treatment should be viewed with suspicion in

²Considering that the portion of the current path which is directly normal to the interface plane is common to the separate current application and voltage measurement “circuits”.

³For $R_s \sim 30\Omega/\square$ and $T \sim 0.3\mu m$.

⁴Or usually, two regions of the same device.

side view

top view

Figure 2.1: Front Resistance in the Berger Structure.

the absence of detailed study of this question.⁵ However, there is little doubt that sintering the contacts does affect the nature of the underlying bulk, and that the effect would have a lessened impact if the substrate were made thick.

The Kelvin structure (Figure 2.3) also decreases the amount of parasitic bulk resistance in the measurement. However, issues of misalignment of the contact hole and lateral spreading of diffused areas play an uncomfortably large role in actual measurements [3]. These effects could in principle be removed by analysis, but the measurement tends to be sensitive to small changes in geometry [29] and might therefore be regarded as a poor experimental technique.

In view of all this, the only appropriate solution is to develop experimentally sounder methods for measurement, the course taken in this thesis. The poorer techniques, if practically needed for quality control purposes, for example, might then be calibrated.

The analysis presented in this chapter is expected to accomplish three things:

1. to upset confidence in the transmission-line/Helmholtz approaches,
2. to be useful for cases of special device geometry,⁶ and
3. to provide a qualitative understanding of how a finite substrate thickness may affect a measurement.

2.1 The Transmission-Line Approach

In Figure 2.4 the contact is modelled according to the transmission-line approach.

⁵Because a cynical view of the treatment is that it simply increases the number of unknowns to accommodate the number of measurements.

⁶i.e. when the width of the electrodes roughly equals the width of the diffused well to which they make contact (see Figure 2.1).

$$R_e = \frac{V}{I}$$

Figure 2.2: End Resistance in the Berger Structure.

Figure 2.3: Kelvin Structure.

Figure 2.4: Illustration of the Transmission-line Model.

Accordingly,

$$\frac{dv}{dx} = -\frac{R_s}{W}i \quad (2.2)$$

where R_s is the sheet resistance and W is the electrical width of the contact. Also,

$$\frac{di}{dx} = -\frac{Wv}{\rho_c} \quad (2.3)$$

Then

$$\frac{d^2v}{dx^2} - \frac{R_s}{\rho_c}v = 0 \quad (2.4)$$

which can be solved

$$v = Ae^{-\alpha x} + Be^{\alpha x} \quad (2.5)$$

where the boundary conditions are $i(0) = I$, $v(0) = V_f$ and $i(L) = 0$, so that

$$I = \alpha \frac{WA}{R_s} (1 - e^{-2\alpha L}) \quad (2.6)$$

and

$$V_f = A (1 + e^{-2\alpha L}) \quad (2.7)$$

From this it is concluded that the effective resistance of the contact plus the bulk under the contact is

$$R_f = V_f/I = \frac{\sqrt{R_s \rho_c}}{W} \coth \alpha L \quad (2.8)$$

Similarly

$$V_e = 2Ae^{-\alpha L} \quad (2.9)$$

so that the end resistance defined in Figure 2.2 is

$$R_e = V_e/I = \frac{\sqrt{R_s \rho_c}}{W} \operatorname{csch} \alpha L \quad (2.10)$$

To $2R_f$ we must add a quantity R_B representing bulk resistance between the stripes, to obtain the total measured resistance.

It is apparently not known in the literature that a very simple relationship exists between R_f and R_e . Multiplying (2.2) and (2.3) yields

$$\frac{d}{dx} \left(v^2 - i^2 \frac{R_s \rho_c}{W^2} \right) = 0 \quad (2.11a)$$

or on integration

$$v^2 = C + i^2 \frac{R_s \rho_c}{W^2} \quad (2.11b)$$

Hence

$$R_f^2 = R_e^2 + \frac{R_s \rho_c}{W^2} \quad (2.12a)$$

Thus ρ_c and R_s may be given in closed-form. Eliminating R_s

$$\rho_c = \frac{\sqrt{R_f^2 - R_c^2} W L}{\coth^{-1} (R_f / \sqrt{R_f^2 - R_c^2})} \quad (2.12b)$$

and R_c is then given by (2.12a).

In recent years a numerical approach to modelling the effect of finite contact width has been explored. This employs the 2-D Laplace equation in regions not covered by metal, with the condition of no current flux at the physical boundaries of the test structure. Under the metal, the 2-D Helmholtz equation is used

$$\nabla^2 \phi + \frac{R_s}{\rho_c} (V - \phi) = 0$$

The potentials must be matched at the boundary of "non-contacted" and "contacted" regions. The Helmholtz equation can be imagined as arising from the 3-D Laplace equation where

$$\frac{\frac{\rho_B(V-\phi)}{\rho_c} - 0}{T}$$

is used as an approximation to

$$\frac{\partial^2 \phi}{\partial z^2}$$

2.2 Calculations Based Upon Approximate Stream-Tubes

A three-dimensional situation can often be satisfactorily represented by a two- or even one-dimensional model. The thinness of the diffused or implanted layer and

the largely one-dimensional flow of current in the Berger and Kelvin structures invites such simplification. Therefore the treatment by transmission-line techniques seems so natural that it tends to dominate thinking about these devices [53]. This chapter considers different views of the situation involved. The principle merit of these new approaches is simplicity and the fact that they yield different predictions for R_f and R_c .

Figure 2.5 illustrates one approach to be taken, where current is assumed to flow in laminae in analogy to fluid flow. Writing the resistance contribution from each branch

$$R = \frac{\rho_c}{W\Delta x} + \frac{\rho_B x}{W\Delta y} + \frac{\rho_B y}{W\Delta x} \quad (2.13)$$

Without some assumption of a relationship between the thickness Δy of a lamina and the length Δx of the corresponding region of the contact where the current is collected from the lamina, little progress can be made.

The simplest of possible assumptions is that

$$y = \frac{xT}{L} \quad (2.14)$$

This is not physically realistic, but it allows closed-form solutions to be determined, and illustrates clearly the ideas involved. Adding up the contributions to R_f in parallel

$$\frac{1}{R_f} = \int_0^L \frac{W dx}{\rho_c + \left(\frac{\rho_B L}{T} + \frac{\rho_B T}{L} \right) x} \quad (2.15)$$

or

W = depth into page

Figure 2.5: Approximate Stream-Tubes Model.

$$R_f = \frac{\rho_c}{WL} \frac{u}{\ln(1+u)} \quad (2.16a)$$

where

$$u = \frac{R_s}{\rho_c} (L^2 + T^2) \quad (2.16b)$$

The portion of the voltage drop carried across the trailing end of the contact is

$$\frac{\rho_c/W \Delta x + \frac{1}{2} \rho_{By}/W \Delta x}{\rho_c/W \Delta x + \rho_{Bx}/W \Delta y + \rho_{By}/W \Delta x} \quad (2.16c)$$

and therefore

$$R_e = \left(1 + \frac{1}{2} R_s T^2 / \rho_c\right) \frac{\rho_c}{WL} \cdot \frac{u}{(1+u) \ln(1+u)} \quad (2.16d)$$

If one explores the consequences of assumption (2.14) further, however, one finds that the current is completely drawn into the contact over a distance much shorter than the physical length of the contact. The distributions of potential and current density obtained in this way are not physically realistic. To achieve a reasonably quantitative electrical model of the contact seems to require that these ideas be implemented numerically.

A more physical assumption than (2.14), in keeping with the qualitative behaviour of transmission line solutions, is

$$y = T (1 - e^{-kx}) \quad (2.17)$$

Maximizing the conductance (i.e., using 2.13 and a conductance integral like 2.15) by choosing k optimally leads to reasonable predictions for R_f and R_e . These are

contained in Table 2.1 with $R_s = 20\Omega/\square$, $L = 7.5\mu m$, $W = 30\mu m$ and $T = 0.5\mu m$, along with the transmission-line model predictions. Results are quite different for small ρ_c , yet both approaches appear to be reasonable. A conclusion which can be drawn from the tabulated results and which is clearly correct is that for small values of ρ_c , R_f is essentially a bulk effect.

2.3 Fourier Series Approximation

Let us consider now an approximation with a somewhat more rigorous basis as reported recently by Winterton and Tarr [54]. Assuming the metal at zero potential

$$\frac{\partial \phi}{\partial y} = -\frac{\rho_B}{\rho_c} \phi \quad (2.18)$$

and assuming a potential consisting of only the first two terms of an infinite series⁷

$$\phi = \cos ky(A \cosh kx + B \sinh kx) \quad (2.19)$$

one arrives (without going into detail) at

$$R_f = \frac{R_s L}{W} \frac{\coth kL}{kL} \quad (2.20)$$

and

$$R_e = \frac{R_s L}{W} \frac{\operatorname{csch} kL}{kL} \quad (2.21)$$

⁷The results obtained neatly dovetail with transmission line results and this is taken as justification of the step.

Table 2.1: Comparison of Transmission-Line Model and Approximate Stream-Tubes Model. $R_s = 20\Omega/\square$, $L = 7.5\mu\text{m}$, $W = 30\mu\text{m}$, $T = 0.5\mu\text{m}$. Based on (2.8), (2.10), (2.13), (2.16c) and (2.17).

	Transmission Line		Stream-Tubes	
ρ_c	R_f	R_e	R_f	R_e
Ωcm^2	Ω	Ω	Ω	Ω
10^{-9}	.0471	-	.142	-
3×10^{-9}	.0816	-	.183	-
10^{-8}	.149	-	.260	-
3×10^{-8}	.258	-	.386	-
10^{-7}	.471	-	.644	-
3×10^{-7}	.817	.0036	1.08	.0012
10^{-6}	1.49	.104	1.98	.0252
3×10^{-6}	2.69	.761	3.60	.216
10^{-5}	6.00	3.71	7.65	1.64
3×10^{-5}	15.0	12.5	17.3	8.13
10^{-4}	46.1	43.6	48.9	36.9

where

$$kT \tan kT = \frac{R_s T^2}{\rho_c} \quad (2.22)$$

as demanded by the boundary condition.

Since this model depends on thickness and has a different form from the models considered already, one can expect yet a different set of results corresponding to the conditions of the example of Table 2.1. General conclusions are that when ρ_c is nearly zero, the contact potential is reasonably nearly zero and

$$kT \sim \pi/2 \quad (2.23)$$

while

$$R_f = \frac{2 R_s T}{\pi W} \quad (2.24)$$

whereas if ρ_c is large

$$k \sim \sqrt{\frac{R_s}{\rho_c}} \quad (2.25)$$

while

$$R_f = \frac{\rho_c}{WL} \quad (2.26)$$

Table 2.2 shows typical results with $T = 0.5\mu m$. This clearly illustrates the dominance of bulk resistance when ρ_c is nearly zero. In the transmission-line model, the predicted value of R_f vanishes with ρ_c .

Table 2.2: Comparison of Transmission-Line Model with the Approximate Fourier Series Model. $R_s = 20\Omega/\square$, $L = 7.5\mu\text{m}$, $W = 30\mu\text{m}$, $T = 0.5\mu\text{m}$. Based on (2.8), (2.10), (2.20), (2.21) and (2.22).

	Transmission Line		Fourier Series		
ρ_c	R_f	R_e	kT	R_f	R_e
Ωcm^2	Ω	Ω		Ω	Ω
10^{-9}	.0471	-	1.5400059	.216	-
3×10^{-9}	.0816	-	1.4821034	.225	-
10^{-8}	.149	-	1.3138377	.254	-
3×10^{-8}	.258	-	1.0211139	.326	-
10^{-7}	.471	-	.6532712	.510	.0001
3×10^{-7}	.817	.0036	.397248	.839	.0043
10^{-6}	1.49	.104	.2217604	1.51	.108
3×10^{-6}	2.69	.761	.1287417	2.70	.767
10^{-5}	6.00	3.71	.0706518	6.01	3.72
3×10^{-5}	15.0	12.5	.0408135	15.0	12.5
10^{-4}	46.1	43.6	.0223588	46.1	43.6

2.4 Bounds for R_f

As will be shown in Chapter 4, the resistance between two electrodes is bounded by

$$R_D + \frac{2\rho_c}{A_c} < R < R_N + \frac{2\rho_c}{A_c} \quad (2.27)$$

where R_D and R_N are bulk resistances for the case of constant potential or constant current density in the semiconductor at the interface. Considering the TLM model, the lower and upper bounds are

$$\frac{\sqrt{R_s \rho_c}}{W} \quad (2.28)$$

and

$$\frac{1}{3} \frac{R_s L}{W} + \frac{\rho_c}{WL} \quad (2.29)$$

Thus the lower bound has the wrong functional form. The upper bound also is slightly less (by a factor of $R_s L/6W$) than would be expected, but the first conclusion clearly has the more important implications for measurement.

Table 2.3 contrasts a realistic lower and upper bound for R_f with that predicted by the transmission-line model to illustrate this point - note that for extremes of ρ_c , the resistance must take on the value of the bounds. The lower bound is

$$\frac{R_s T}{2W} + \frac{\rho_c}{WL} \quad (2.30)$$

and is based on Overmeyer's computations [19], while the upper bound follows the form

Table 2.3: Comparison of Transmission-line Model with Lower and Upper Bounds for R_f for extremes of ρ_c . $R_s = 20\Omega/\square$, $L = 7.5\mu\text{m}$, $W = 30\mu\text{m}$, $T = 0.5\mu\text{m}$. Based on (2.8), (2.10), (2.30) and (2.31).

Lower Bound		
	TLM	Bound
ρ_c Ωcm^2	R_f Ω	R_f Ω
10^{-9}	.0471	.167
10^{-8}	.149	.171
10^{-7}	.471	.211
10^{-6}	1.49	.611

Upper Bound		
	TLM	Bound
ρ_c Ωcm^2	R_f Ω	R_f Ω
10^{-7}	.471	2.54
10^{-6}	1.49	2.94
10^{-5}	6.00	6.94
10^{-4}	46.1	46.

$$\frac{R_s L}{2W} + \frac{\rho_c}{WL} \quad (2.31)$$

It can be concluded that the error introduced by the TLM approximation in the contact resistance is of the order of

$$\frac{R_s T}{W} \quad (2.32)$$

In the worked example of the preceding tables ρ_c can not be determined particularly accurately by the TLM method unless it is above about $10^{-7} - 10^{-6} \Omega \text{cm}^2$. This is the central problem if ρ_c is small: improper analysis can lead without question to assignment of bulk effects to ρ_c .

2.5 End Resistance Concepts

Let us consider TLM analysis with regard to end resistance. Figure 2.6 shows an alternative view of things. If the floating contact lies between those to which current is applied, then with it will be associated half the bulk resistance between the same contacts. The same conclusion arises if we move the contact off to 'infinity' (the back of the substrate). Consequently end resistance could contain some bulk resistance - not essentially none as the TLM method suggests. How much obviously depends on its placement.

End resistance does not suggest 1-D transmission line calculations so much as 2-D conformal mapping techniques, especially for closely-spaced and short contacts. In all the approaches considered so far, the current flowing in the semiconductor is rapidly attenuated as one passes from the leading to the trailing edge of the contact, where the voltage is sensed. However, it is also clear that the TLM model is not the

Figure 2.6: End Resistance — Sensitivity to the Location of the Floating Contact.

only way of looking at this process, and calculations based on different principles have been presented which give different quantitative results.

2.6 Numerical Simulations

Finite difference techniques were used to simulate a Berger structure containing a floating contact for measuring end resistance. About one hundred points were used to give fair two-dimensional representation. Table 2.4 illustrates the existence of a minimum bulk resistance component of both R_f and R_e in the simulation. Computed results display some interesting qualitative behaviour as well. Notable features are the existence of peak current densities near the contact edges (leading and trailing). What the calculations suggest which was not expected is that the semiconductor is an equipotential under the floating contact. Also, the potential in the semiconductor rises linearly between the "active" contacts and becomes roughly constant underneath and beyond them, while the contact potential is still higher than this constant semiconductor potential. In view of the behaviour of Fourier series solutions of analytically tractable potential problems this is reasonable. As one moves from the edge of the active contact towards the floating contact and beyond, it is reasonable to expect that potential variations in the bulk will slowly be "averaged out". Another way of putting this is that the average potential at the end of the active contact (averaged over thickness) is equal to the potential of the floating contact.

2.7 Conclusions

Beginning with the transmission-line model, the accumulated evidence has tended to support the usefulness of the concept of contact resistivity. However agreement between theory and data has not always proven satisfactory. The one-dimensional

Table 2.4: Resistance in the Front Configuration and End Resistance Calculated by Finite Differences for the Berger Structure. Three $1\ \mu m$ Long Contacts Spaced at $1\ \mu m$ Intervals on a $0.001\ \Omega\ cm$ Substrate. Resistance normalized to width and resistivity.

$T(\mu m)$	$\rho_c(\Omega cm^2)$	R	R_e
1	4×10^{-10}	1.9	.14
	4×10^{-9}	2.0	.16
	4×10^{-8}	2.8	.49
	4×10^{-7}	10	4.1
3	4×10^{-10}	1.0	.18
	4×10^{-9}	1.1	.22
0.3	4×10^{-10}	4.7	.0083
	4×10^{-8}	6.0	.21

transmission line model discussed in this chapter omits both of the effects of finite thickness and width in the Berger and Kelvin structures. The Helmholtz equation does help to remedy some of the problems since it accounts for finite width effects, and solutions to it can be readily determined [4,22]. It is moreover probably the best of available models for lossy contacts to thin slabs of bulk semiconductor. However, this chapter has presented arguments to motivate the search for more "convincing" measurement techniques. Since the Helmholtz equation neglects thickness effects which are practically significant, this neglect must then introduce uncertainty in measurement techniques employing the model for interpretation of data.

The resistance between a pair of contacts always contains a bulk component. In the case of the Berger structure this amounts to at least

$$R_{TL} \quad (2.33)$$

which can be of the order of $10^{-6} \Omega \text{cm}^2$. This represents a fundamental uncertainty in ρ_c measurement by the Berger technique. For the Kelvin structure the situation is probably much better, perhaps amounting to

$$R_{T^2} \quad (2.34)$$

or about $10^{-8} - 10^{-7} \Omega \text{cm}^2$. An accurate measurement would require ρ_c to be significantly larger than these figures, but it would still be unlikely because of experimental defects and neglect of essential parasitic effects that measurements of ρ_c on source-drain material would be as accurate as these idealized estimates. Finally, the transmission-line model is not the only or most obvious approach to describing devices on thin layers, as this chapter has attempted to demonstrate by providing alternate approximate calculation techniques.

Chapter 3

MEASUREMENT OF ρ_c BY A METHOD USING PARALLEL STRIPE CONTACTS

Work was carried out on a structure resembling the Berger structure except that the thickness of the substrate extends to "infinity", the stripes are made very wide and the bulk has uniform doping. This is to overcome problems of bulk-contact interaction, modelling uncertainties, and importantly, to minimize bulk resistance and thus enhance measurement accuracy.

Initially the structure was held to be roughly two-dimensional owing to the large metal conductivity and the large width of the stripes. Bulk and interface effects were accounted for by conformal mapping as reported elsewhere [40,41]. A solution by Green's functions based on sources and sinks of current rather than positive and negative charges was also developed.¹ These calculations confirmed

¹There is a complete analogy between dielectric and resistive problems. In effect, no resistive problem is governed by Laplace's equation because the external sources and sinks of current are not embodied in this equation. Therefore, rather than Laplace's equation, the general governing equation is

$$\nabla^2 \phi = -\rho_B i_v \quad (3.1)$$

those obtained by the conformal mapping approach and lend a higher degree of confidence to the numerical results.

Later, the finite metal conductivity, which is a dominant effect, was accounted for by transmission-line techniques. Thus, the overall model had an approximate character, and was finally abandoned in favour of more exact techniques. However, it is interesting that the method is perhaps the first to be reported in which all operating power loss mechanisms have been subjected to analysis.

This chapter presents the Green's function method employed in this thesis, its computational implementation² and the results of numerical analysis, the idea of simultaneous measurement of ρ_B and ρ_c based on dimensional analysis concepts, the treatment of metal resistance by transmission-line techniques, and experimental support for the overall theory.

3.1 The Green's Function Method

The origin for the writer of this method lay in the superposition principle applied to Coulomb's law in problems of charge distribution.

Figure 3.1 illustrates the analogy between the Coulomb's law problem and the problem of distributed current sources and sinks. For a current entering a resistive sheet

$$\mathbf{E} = \frac{\rho_B I_\ell}{2\pi r} \hat{\mathbf{r}} \quad (3.2)$$

where I_ℓ is the current leaving a transverse conductor per unit length, which is where i_v is the volumetric density of current entering the resistive body from the "environment". These sources and sinks of current are perfectly analogous to free charge in dielectric media.

²Boundary integral equation (BIE) or boundary element method (BEM).

supplying the sheet with current. Then

$$\phi = -\frac{\rho_B I_\ell}{2\pi} \ln r + C \quad (3.3)$$

where r is distance from the source. This clearly satisfies the 2-D Laplace equation except at $r = 0$.

The ability to write down the simple expression (3.3) derives from Green's theorem in the plane, and later in Chapter 5, in the volume. The latter allows one to write an explicit solution to the 3-D Poisson's equation in the form

$$\phi = \int (\phi \nabla g - g \nabla \phi) \cdot dS + \int g \nabla^2 \phi dV$$

where the first term corresponds to boundary conditions on the prescribed volume, and the second to volume distributions of current sources and sinks. In this equation, g is the Green's function determined using

$$\nabla^2 g = \delta$$

where δ is the Dirac delta function. On integrating over the prescribed volume

$$\int \delta dV = 1$$

In the problems considered in this thesis the surface integral vanishes at infinity and

$$\phi = \int g \nabla^2 \phi dV$$

This approach is discussed in depth by Butkov [55] and in an informal manner by Kraus and Carver [56].

Figure 3.2 allows us to apply these results to two stripe contacts on an infinitely thick substrate. Let the right hand contact be at potential $+V$,

$$\frac{\partial \phi}{\partial n} = \rho_B \frac{V - \phi}{\rho_c} \quad (3.4)$$

and the left hand contact be at potential $-V$,

$$\frac{\partial \phi}{\partial n} = -\rho_B \frac{V + \phi}{\rho_c} \quad (3.5)$$

Along the x -axis, since we are dealing with a half-space only, (3.3) may be written

$$\phi(x) = -\frac{1}{\pi} \int_{-(s+L)}^{-s} \frac{\partial \phi}{\partial n}(x') \ln |x - x'| dx' - \frac{1}{\pi} \int_s^{s+L} \frac{\partial \phi}{\partial n}(x') \ln |x - x'| dx' \quad (3.6)$$

where x' is the source point and x is the observation point. Using (3.4) and (3.5) in (3.6)

$$\phi(x) = \frac{\rho_B}{\pi \rho_c} \int_s^{s+L} (V - \phi(x')) \ln \left| \frac{x + x'}{x - x'} \right| dx' \quad (3.7)$$

This is a Fredholm integral equation of the second kind, which is (probably) unsolvable analytically, but can provide a basis for numerical computation.

The following sub-section is concerned with approximate treatment of the integral equation (3.7) and should be regarded as an aside to the main discussion.

Figure 3.1: Analogy Between Dielectric and Resistive Problems.

Figure 3.2: Transverse View of Parallel Stripe Structure.

3.1.1 Approximate Solution of the Integral Equation

Before considering exact solutions, some approximations will be considered briefly. In view of numerical results to be described, most of the current flow is confined to the contact edges. Therefore one can conceive of approximating the true situation by means of two current spikes at the contact edges

$$\frac{\partial \phi}{\partial n}(x') = A_s \delta(x' - s) + A_{s+L} \delta(s + L - x') \quad (3.8)$$

where A_s and A_{s+L} are constants to be determined. This gives, on substitution in (3.7)

$$\phi(x) = \frac{A_s}{\pi} \ln \frac{x+s}{x-s} + \frac{A_{s+L}}{\pi} \ln \frac{x+s+L}{s+L-x} \quad (3.9)$$

between the contact edges. One can directly determine a resistance based on the average contact potential and the total current flowing in response to a single current spike sited at the inside edge of the contact.³

Alternatively, one can assume that most of the contribution to the integral giving potential comes from the site of the logarithmic singularity. This is not true since the area under the logarithmic kernel is spread diffusely across the contact; however, calculations based on this assumption give rise to a fair approximation. Assuming then that the logarithm behaves like a Dirac delta function at $x' = x$,

$$\phi(x) \sim \frac{1}{\pi} \frac{\partial \phi}{\partial n}(x) \int_s^{s+L} \ln \left| \frac{x+x'}{x-x'} \right| dx' \quad (3.10)$$

³Presumably one could achieve a good approximation by assuming as well a uniform current density component since the current is not zero between the contact edges in reality. Finally, because current spikes are not physically real, appropriately positioned, single current spikes give rise to resistances which violate the bounds to be developed later. The advantage of such an approach is of course that closed-form bounds on resistance can be determined.

Positive features are that the computed results lie between the exact bounds for the problem and are asymptotic to the upper bound, rising monotonically with ρ_c as is desirable. This is illustrated in Figure 3.3.

We turn now to exact treatment of the problem.

3.2 The Boundary Element Method

The method of employing Green's function formulations in numerical methods is termed the boundary element method, and has been the subject of a number of books [47,48]. Principal advantages of the method are:

1. a reduction in the dimension of the problem by one, and
2. the possibility of treating "open" domains.

These are obvious from the preceding discussion and they represent accuracy and time advantages over the competing finite elements approach.

Of principal interest to us in the present discussion is the treatment of singularities, and the type of polynomial approximation to ϕ used on the elements.

First of all we break up the boundary of the problem into piecewise elements, taking care to avoid representing the contact corners where either cusps or singularities occur.⁴ Secondly, we approximate the integral for the piecewise constant ϕ approximation, by a sum over piecewise elements of

$$\frac{1}{\pi} \frac{\partial \phi}{\partial n} \left(\frac{x'_1 + x'_2}{2} \right) \int_{x'_1}^{x'_2} \ln \left| \frac{x + x'}{x - x'} \right| dx' \quad (3.11)$$

⁴i.e., none of the points at which ϕ is represented is located exactly at the corners.

Figure 3.3: Approximate "Bulk Resistance" ($R - 2\rho_c/A_c$) Versus Dimensionless Contact Resistivity. Based upon (3.10).

where the integration of the kernel can be carried out exactly, or approximated as

$$\frac{1}{\pi} \frac{\partial \phi}{\partial n}(x') \ln \left| \frac{x + x'}{x - x'} \right| \Delta x' \quad (3.12)$$

with no difference to the results for a sufficiently large number of points.

The point $x = x'$ is singular but integration of the kernel around the point is straightforward. The contribution to the integral in (3.11), if the length of the segment is ϵ and the point $x = x'$ is at the center of the segment, is

$$\epsilon + \epsilon \ln \frac{4x}{\epsilon} \quad (3.13)$$

It is important to use at least one much smaller segment for representing the ϕ distribution at the edges of the contact,⁵ in order to achieve accurate results. Finally, the use of higher order polynomial interpolation along the segment has been found not very useful, and even counterproductive in this study. This makes some sense in view of the fact that polynomials cannot represent "nearly singular" functions (i.e., the current density distribution) well [57].

One idea to improve accuracy which occurred to the writer has not been explored, but is only suggested here. This is the use of a "natural" interpolant for the unknown $\phi(x')$ in the integral of (3.7). The idea that some interpolants might be better than others has an analogy in numerical quadrature techniques. That is the equation for $\phi(x)$ can be written

$$\phi(x) = \frac{\rho_B}{\pi \rho_c} \int_s^{s+L} \left[V - \frac{\rho_B}{\pi \rho_c} \int_s^{s+L} (V - \phi(x'')) \ln \left| \frac{x' + x''}{x' - x''} \right| dx'' \right] \ln \left| \frac{x + x'}{x - x'} \right| dx' \quad (3.14)$$

⁵i.e., with the point nearest to the contact corner at which ϕ is represented, displaced slightly from the corner.

by iteration of the integral. If we assume $\phi(x'')$ piecewise constant but unknown, the term in square brackets representing the unknown function $V - \phi(x')$ in (3.7) will now be a smooth interpolating curve. The drawback is that a considerable number of numerical integrals may have to be evaluated.

Finally one is left with a set of linear equations to be solved. Early works [58,59] on numerical methods favoured the use of iterative techniques, but modern texts [48,57] prefer direct methods such as Gaussian elimination. The writer has experimented with both indirect and direct methods and has found direct methods more useful. Use of iterative methods like Jacobi or Gauss-Seidel iteration apparently do not have a great deal to recommend them currently [57]. Modern techniques like gradient methods [57] might be considered seriously since they are guaranteed not to be slower than direct methods in the absence of roundoff error. The writer has experimented with a variant of Jacobi iteration/successive over-relaxation, in which the relaxation parameter is picked optimally to maximize the reduction in the error at each step. However, roundoff errors owing to subtraction (particularly with small ρ_c) can prohibit convergence. On the other hand, use of a least-squares technique of matrix solution has been found helpful to accuracy in this study.

3.3 Numerical Results

The results of calculations carried out as described by Winterton and Tarr in [40,41] by conformal mapping/Fourier series methods, have been reported in Tables 3.1 and 3.2. The attention of those interested in the exact results is directed there, where the resistance between two stripes spaced as far apart as their length, and twice as far apart as their length, are reported. Resistance has been normalized as RW/ρ_B .

3.3.1 The Method of [40,41]

The approach of [40,41] was carried out by the conformal mapping technique described in section 4.3, carrying the two stripes geometry of Figure 4.3(a) into the rectangular geometry of Figure 4.3(b). Since the boundary conditions (3.4) and (3.5) transform according to equations (4.43) and (4.44), the net effect is much like having a rectangular geometry (Figure 4.3(b)) with a variable ρ_c across the contact. One can now represent the solution for potential (and hence solve ultimately for resistance) with a Fourier series

$$\phi = c_0 \frac{u}{b} + \sum_{n=1}^{\infty} c_n \sinh \frac{n\pi u}{c} \cos \frac{n\pi v}{c}$$

where u is the axis directed from (0,0) to (b,0) and v is the axis directed from (0,0) to (0,c) in Figure 4.3(b). The coefficients are to be determined by imposing the transformed boundary condition

$$\frac{\partial \phi(b, v)}{\partial u} = \frac{V - \phi(b, v)}{\alpha |f'(z)|}$$

This clearly cannot be carried out in closed form because of the factor $|f'(z)|$.⁶ Since along the contact $|f'(z)|$ is $\frac{dv}{dx}$, multiplying by $\cos \frac{n\pi v}{c}$ and integrating to make use in the usual way of the orthogonality of trigonometric functions gives rise to integrals of the form

⁶The problem here, and in any attempt to apply orthogonal series solutions to test structures for ρ_c measurement is that the principle of finality [60] which usually allows one to compute the Fourier series coefficients independently of each other, does not apply. Instead of the usual case in which a sequence of independent corrections leading to an increasingly better solution can be determined, the coefficients must be computed all at once in the form of a matrix equation. Only one of the coefficients (c_0) is actually needed to determine the resistance.

$$\int \cos \frac{\ell \pi v(x)}{c} dx$$

where $v(x)$ and c are elliptic integrals. These can be numerically computed with good accuracy by writing x itself as a Fourier series in $v(x)$; the integrals above are then simply related to the coefficients of this series. On evaluating the integrals and solving the resulting matrix equation, the current, which only depends on c_0 , is determined.

For evident reasons, this approach would seem to be more reliable than the Green's function approach when ρ_c tends to 0, while the Green's function approach would appear to be superior when ρ_c becomes large (but conformal mapping cannot be applied to the test structure of Chapter 5). The results of [40,41] were believed to be accurate to the number of digits indicated in the table based on variation of the number of Fourier series coefficients used in representing potential.

3.3.2 The Boundary Element Method

The boundary element method, which is perhaps the best approach to describing the test structure of Chapter 5, and which has been discussed at length in previous sections, was first applied to the parallel stripes technique in order to gain some idea of the possible problems arising on implementing the method. The comparison between the two different computation methods was favourable and lends confidence to the tabulated results. In runs where a small corner segment was included among about 50 boundary elements, results of the two methods appeared to agree to about four digits (Table 3.2(b)).

Table 3.1: Parallel Stripes Technique: Resistance vs Dimensionless ρ_c for Small ρ_c . Computed by the Method of [40,41]. Resistance normalized to W and ρ_B , $\alpha = \rho_c/\rho_B L$. Stripes spaced as far apart and twice as far apart as their length.

α	$R(s = 0.5L)$	$R(s = L)$
10^{-7}	1.279262	1.563402
3×10^{-7}	1.279263	1.563403
10^{-6}	1.279265	1.563405
3×10^{-6}	1.279274	1.563413
10^{-5}	1.279302	1.563440
3×10^{-5}	1.279383	1.563515
10^{-4}	1.27967	1.56378
2	1.28007	1.56415
3	1.28047	1.56452
4	1.28086	1.56489
5	1.28126	1.56526
6	1.28165	1.56562
7	1.28204	1.56598
8	1.28242	1.56635
9	1.28281	1.56671
10^{-3}	1.2832	1.5671
2	1.2869	1.5706
3	1.290	1.574
4	1.294	1.577
5	1.297	1.580
6	1.300	1.583
7	1.304	1.586
8	1.307	1.589
9	1.310	1.592
10^{-2}	1.313	1.595
2	1.341	1.623
3	1.368	1.648
4	1.393	1.672
5	1.417	1.696
6	1.441	1.719
7	1.464	1.742
8	1.487	1.764
9	1.509	1.786
10^{-1}	1.532	1.808

Table 3.2: (a) Parallel Stripes Technique: Resistance vs Dimensionless ρ_c for Large ρ_c . Computed by the Method of [40,41]. Resistance normalized to W and ρ_B , $\alpha = \rho_c/\rho_B L$. Stripes spaced as far apart and twice as far apart as their length.

α	$R(s = 0.5L)$	$R(s = L)$
2×10^{-1}	1.747	2.021
3	1.955	2.227
4	2.160	2.431
5	2.363	2.634
6	2.565	2.836
7	2.768	3.037
8	2.969	3.238
9	3.170	3.439
10^0	3.372	3.640
2	5.376	5.644
3	7.378	7.645
4	9.379	9.646
5	11.38	11.65
6	13.38	13.65
7	15.38	15.65
8	17.38	17.65
9	19.38	19.65
10^1	21.38	21.65
2	41.38	41.65
3	61.38	61.65
4	81.38	81.65
5	101.4	101.6
6	121.4	121.6
7	141.4	141.6
8	161.4	161.6
9	181.4	181.6
10^2	201.4	201.6
3×10^2	601.4	601.6
10^3	2001	2002
3×10^3	6001	6002
10^4	20000	20000
3×10^4	60000	60000
10^5	200000	200000
3×10^5	600000	600000
10^6	2000000	2000000

Table 3.2: (b) Method of Boundary Elements. 50 elements with a small corner segment.

α	$R(s = 0.5L)$
10^{-7}	1.2788
10^{-6}	1.2788
10^{-5}	1.2789
10^{-4}	1.2800
10^{-3}	1.2851
10^{-2}	1.3133
10^{-1}	1.5305
1	3.3699
10^1	21.379
10^2	201.38
10^3	2001.4
10^4	20001
10^5	200000
10^6	2000000

3.3.3 Discussion

Figure 3.4 shows the resistance versus α curves for the two structures, where $\alpha = \rho_c/\rho_B L$, while Figure 3.5 shows the existence of lower and upper bounds which will be discussed in the next chapter. The upper bound bounds all resistance points from above and is a good model for large values of ρ_c , while similarly, the lower bound is less than actual points but is a good model for low values of ρ_c . Figure 3.6 shows the current distribution in the contact window as obtained by the boundary element method. Note that the current density becomes enormous for small values of ρ_c at the edges of the contact window (cusps). For ρ_c equal to zero it actually produces singularities there.

Finally, Figure 3.7 shows the resistance between a pair of stripes with no contact resistance on substrates with varying thickness. This illustrates that the choice of larger thickness will increase the ratio of contact resistance to total resistance, demonstrating an advantage of the new structure over the Berger structure. From the same figure the bulk can be seen to be "infinitely thick" when the criterion

$$k(2s + 2L) \leq T \quad (3.15)$$

where k is roughly 5, is met, which sets limits on the dimensions of the stripes and their spacing.

At this point, a practical method for measurement of ρ_c and ρ_B will be outlined and subjected to the test of experiment.

Figure 3.4: Parallel Stripes Technique: Resistance Versus Dimensionless ρ_c . Resistance Normalized to W and ρ_B , $\alpha = \rho_c / \rho_B L$. Stripes Spaced as Far Apart and Twice as Far Apart as Their Length.

Figure 3.5: Parallel Stripes Technique: Lower and Upper Bounds for "Bulk Resistance" ($R - 2\rho_c/A_c$). Bulk Resistance Normalized to W and ρ_B , $\alpha = \rho_c/\rho_B L$. Stripes Spaced as Far Apart and Twice as Far Apart as Their Length.

Figure 3.6: Cusps in the Current Density Profiled Along the Length of the Contact.
 (a) $\alpha = 1 \times 10^{-7}$, (b) $\alpha = 0.1$.

Figure 3.7: Resistance Versus Ratio of Substrate Thickness to the Factor a With No Contact Resistance. Based Upon (4.55).

3.4 The Simultaneous Measurement of ρ_B and ρ_c

In this section it will be explained how ρ_B and ρ_c are separated in a measurement. The following section will show how the "contamination" of metal resistance is removed from the measured results before this can take place.

Explanation is made by way of Figures 3.8 and 3.9 and the following text. Note that by virtue of dimensional analysis, with $\alpha = \rho_c / \rho_B L$

$$R = \frac{\rho_B}{W} f\left(\alpha, \frac{s}{L}\right) \quad (3.16)$$

If we have R_1 and R_2 corresponding to two different values of spacing s then

$$\frac{R_2}{R_1} = g(\alpha) \quad (3.17)$$

which eliminates ρ_B/W .⁷ Thus if one measures R_1^* and R_2^* ⁸ and takes their ratio, one can obtain an experimental value of α , α^* , from the curve in Figure 3.8. This can then be used in Figure 3.9, the resistance versus α curve for a given spacing

$$\frac{R_1 W}{\rho_B} = h\left(\frac{\rho_c}{\rho_B L}\right) \quad (3.18)$$

to gain an experimental value

$$\left(\frac{R_1 W}{\rho_B}\right)^*$$

From this ρ_B^* may be determined. Finally

⁷A functional dependence on s_1/L and s_2/L is hidden in the notation.

⁸i.e., where the asterisk denotes a measured value.

$$\rho_c^* = \rho_B^* L \alpha^* \quad (3.19)$$

may be determined.

As an example suppose $R_1 W = 1.1 \Omega cm$ and $R_2 W = 1.276 \Omega cm$. Then $R_2/R_1 = 1.16$ as shown in Figure 3.8, and $\alpha \approx 0.2$ from the same graph. This gives $R_1 W/\rho_B = 1.747$ to which value corresponds $\rho_B = 0.630 \Omega cm$. Finally, if $L = 5 \mu m$, $\rho_c = 63 \mu \Omega cm^2$.

An alternative experimental practice would be to measure ρ_B by the four-point-probe or Van der Pauw techniques. Figure 3.9 could then be used to determine α immediately from the known value of RW/ρ_B , and ρ_c could be calculated from (3.19). However, the variability of ρ_B across a wafer would suggest that a "local" value would be preferable to a value not correlated with specific position on the wafer.

3.5 The Treatment of Metal Resistance

The theory above was developed before it was known how metal resistance⁹ would be treated. If metal resistance were negligible, the resistance between a pair of stripes would be simply

$$R = \frac{R_{2-D}}{W} \quad (3.20)$$

where R_{2-D} is the resistance per unit width determined in Sections 3.1-3.3, and

⁹Metal resistance can be determined by four-point probe or by the inclusion of a bridge resistor (over oxide) on the mask. In addition, measurement of dimensions is necessary; a travelling microscope can be used to determine s , L and the metal "length" or any other dimensions entering the equations. Alternatively for greater accuracy a SEM may be used.

Figure 3.8: Ratio of Resistances (i.e., $R(s = L)/R(s = 0.5L)$) Versus Dimensionless ρ_c , $\alpha = \rho_c/\rho_B L$.

Figure 3.9: Resistance Versus α for $s=0.5L$, $\alpha = \rho_c/\rho_B L$.

where R is presented in normalized form in Tables 3.1 and 3.2. However, it was clear that the metal would not generally behave as an equipotential in practice. That is, if current were injected at the middle of a long stripe, it would attenuate along the stripe and there would be important three-dimensional effects. In general, one would have to determine an end correction if the current were not completely attenuated in the metal, or else the samples would have to be diced.

If the attenuation rate is sufficiently slow transmission line methods may be used as suggested by Winterton and Tarr [40,41]. Figure 3.10 shows a typical structure, while Figure 3.11 shows the electrical model. Then

$$\frac{dv}{dx} = -\frac{2R_m}{L}i \quad (3.21)$$

$$\frac{di}{dx} = -\frac{v}{R_{2-D}} \quad (3.22)$$

or

$$\frac{d^2v}{dx^2} - \frac{2R_m}{LR_{2-D}}v = 0 \quad (3.23)$$

to which a solution may be given as

$$v(x) = A_1e^{-\gamma x} + B_1e^{\gamma x} \quad (3.24)$$

where

$$\gamma = \sqrt{\frac{2R_m}{LR_{2-D}}} \quad (3.25)$$

Figure 3.10: Experimental Parallel Stripes Structure for Diced Samples.

Figure 3.11: Electrical Model Incorporating Metal Resistance Effects for the Experimental Parallel Stripe Structure.

If the sample is terminated by dicing as indicated in Figure 3.10, the resistance is

$$R = \frac{A_1 e^{-\gamma D}}{I} + \frac{B_1 e^{\gamma D}}{I} \quad (3.26)$$

where

$$I = \frac{L}{R_m} (\gamma A_1 - \gamma B_1) \quad (3.27)$$

since the current that is injected sees two "branches". Application of the obvious boundary condition gives

$$-A_1 e^{-\gamma W/2} + B_1 e^{\gamma W/2} = 0 \quad (3.28)$$

so that

$$A_1 = B_1 e^{\gamma W} \quad (3.29)$$

and

$$R = \sqrt{\frac{R_m R_{2-D}}{2L}} \frac{\cosh \gamma \left(\frac{W}{2} - D \right)}{\sinh \frac{\gamma W}{2}} \quad (3.30)$$

Alternatively one may wish to leave the wafers whole. Suppose three measurements of voltage are made giving resistances

$$R_1 = \frac{A_1 e^{-\gamma D}}{I} + \frac{B_1 e^{\gamma D}}{I} \quad (3.31a)$$

$$R_2 = \frac{A_1 e^{-2\gamma D}}{I} + \frac{B_1 e^{2\gamma D}}{I} \quad (3.31b)$$

and

$$R_3 = \frac{A_1 e^{-3\gamma D}}{I} + \frac{B_1 e^{3\gamma D}}{I} \quad (3.31c)$$

where again (3.27) holds. Then eliminating unknowns

$$R_2 e^{-4\gamma D} - (R_1 + R_3) e^{-3\gamma D} + (R_1 + R_3) e^{-\gamma D} - R_2 = 0 \quad (3.32)$$

Note that this equation is expressed solely in terms of the attenuation constant and measurables and therefore gives an interesting approach to measurement. The terminals where voltage is measured are spaced at D , $2D$ and $3D$ from the current probe pads.

Alternatively, equations (3.27), (3.31a) and (3.31b) may be used to determine

$$R_1 \cosh 2\gamma D - R_2 \cosh \gamma D - \frac{R_m}{\gamma L} \sinh \gamma D = 0 \quad (3.33)$$

which provides for a six terminal measurement of resistance, rather than the open-circuit or eight terminal approaches outlined already. Straightforward numerical methods can be used to solve (3.32) or (3.33).

If γ is large and $W \gg D$ it follows that for the open-circuit termination approach

$$R_{2-D} \sim \frac{2LR^2}{R_m} \quad (3.34)$$

This equation is very simple and may occasionally be a good model.

3.6 Experimental Results

To test the ideas described above, pure aluminum contacts on moderately doped (100) p-type silicon ($0.08 \Omega \text{ cm}$ or $3-4 \times 10^{17} \text{ cm}^{-3}$) were deposited through windows in a $1600 \text{ \AA}$ thick oxide¹⁰. The oxide was grown in a low HCl-O_2 ambient at 1100°C and annealed in nitrogen. The aluminum was deposited after the wafers were etched in 10% HF and spun dry. Electron-beam evaporation of aluminum to $1 \mu\text{m}$ followed by hydrogen annealing at 450°C for 20 minutes completed the sample preparation.¹¹

Table 3.3 shows "significant statistics" about the samples: bulk resistivities determined by four-point-probe, thickness measured with calipers, metal resistance determined with a bridge resistor of ten squares and measured device dimensions. Note that the wafer thickness when compared with the maximum overall size of the device ($80 \mu\text{m}$) roughly meets the criterion for the substrate to be regarded as infinite.

Initially diced samples were found to lead to somewhat unreliable measurements, but significant data was later gathered. This early problem may have been due to difficulty in probing the samples reliably, and as a consequence to mechanical damage of the samples in some cases.

Samples were found to fail by two modes: by "burnout" around one of the current probes, and by the appearance of melting of the metal along long runs. Note that the current densities used were at least of the order 10^4 A/cm^2 so this

¹⁰MIS contacts were also briefly investigated but are not reported on here since the purpose is comparison with data in the literature.

¹¹Note that the experimental ratio of spacing to contact cut length is not 1 and 2 here but approximately 1.1 and 2.2. This can be expected to introduce some error into the calculations. Principally, the real value of α would be somewhat higher due to the larger resistance ratio, but as the proportion of bulk resistance would also be higher, ρ_c might be largely unaffected.

Table 3.3: Statistics About the Samples Used.

CR-1	.070-.090
CR-2	.061-.073
CS-1	.081-.082
CS-2	.082-.084

Bulk Resistivity (Nominally .06-.12 Ω cm)

CR-1	310-315
CR-2	335
CS-1	330
CS-2	330

Wafer Thickness (μm)

CR-1	22.8-26.1
CR-2	23.8-26.4
CS-1	20.9-23.5
CS-2	21.4-24.0

Metal Sheet Resistance ($m\Omega/\square$)

Oxide cut length L (Figure 3.2)	18.3-20.1
Metal length L (Figure 3.10)	27.2-30.0
Spacing of narrow devices 2s (Figure 3.2)	21.1 - 21.6
Spacing of wide devices 2s (Figure 3.2)	41.7-43.4

Significant Dimensions (in μm , $\pm 0.5\mu m$)

perhaps should not be greeted with much surprise.

Contact resistance was generally found to be linear as given in Figure 3.12 so that contact resistivity is a meaningful parameter. Of concern now is whether the transmission-line approach and Green's function/conformal mapping solutions represent a good model of the measured data.

Figure 3.13 shows the fit of the transmission-line model to data for a diced sample. As can be seen, the fit is excellent considering we are free to choose only two constants. Apart from this, the calculated ρ_c values also support the use of the transmission-line model.

Figure 3.14 shows the simultaneously calculated $\rho_c - \rho_B$ pairs. Also shown is a best straight line reflecting the data of Berger [1]. In a few cases the ratio of resistances was too high for both ρ_c and ρ_B to be determined, but ρ_B was still usually found to lie in the nominal (manufacturer's) range. The large scatter, of about an order of magnitude in ρ_c , is found also in Berger's $\rho_c - \rho_B$ data [1]. As an alternative, Table 3.4 shows ρ_c determinations when ρ_B is that measured by means of a four-point-probe. These results are consistent with those data presented graphically. Finally, Berger gives [1]

$$\rho_c = 1 - 7 \times 10^{-5} \Omega cm^2$$

for $\rho_B = 0.08 \Omega cm$. In the present study -

$$\rho_c = 0.5 - 10 \times 10^{-5} \Omega cm^2$$

the larger range perhaps reflecting the greater number of measurements made. At any rate the comparison is a favourable one.

Figure 3.12: Linearity of the Contact IV Characteristics. Voltage (μV) versus Current (mA) for Two Samples Tested.

Figure 3.13: How the Transmission Line Model Works Experimentally. Voltage (μV) versus Distance Between Voltage and Current Probes (cm).

Figure 3.14: Simultaneous Measurement of ρ_B (Ωcm) and ρ_c ($\mu\Omega\text{cm}^2$). Scatter is Aggravated Because ρ_c and ρ_B are Simultaneously Extracted From a Pair of Measurements From Two, Only Slightly Different, Test Structure Geometries.

Table 3.4: ρ_c Values Calculated Using ρ_B As Measured By Four-Point Probe Prior to Fabrication. The Voltage Probe Pads Are Spaced at D, 2D and 3D From the Current Probe Pads (Figure 3.10). Interpretation is Based Upon (3.33), (3.32), and (3.30).

CR-1	5-9
CR-2	1-4
CS-1	.8-1
CS-2	.6-1

Six Terminal ($\times 10^{-5} \Omega cm^2$)

CS-1	.9-3
CS-2	1-5

Eight Terminal ($\times 10^{-5} \Omega cm^2$)

CS-1	5-10
------	------

Diced Samples ($\times 10^{-5} \Omega cm^2$)

3.7 Examining the Underlying Assumptions of the Model

Contact resistivity in these measurements represents generally a substantial fraction of the measured resistance - perhaps a quarter or a third. Therefore it is difficult to conceive that a real factor is not being measured. On the other hand, it was long ago thought that the interface between dielectrics always contained free charge [38]. Later it was found that the phenomenon could often be explained as an artifact of analysis which disappeared when matching of the normal electric flux densities and tangential electric fields was carried out. In the same way it may be considered that if metal and bulk effects were properly analyzed the need for the concept of ρ_c might vanish.

Were this true in measurements with the two stripes, the ratio of resistances should have taken on one value in the absence of error, namely 1.222. The measured result was 1.179 with a standard deviation in the mean of .0175. ¹² Therefore there is less than 1% probability that the contact is interface resistance free. This conclusion is altogether likely but not certain since the resistance ratios are only determined after eliminating the strong effects of metal resistance by an approximate calculation technique.

3.7.1 A Minor Difficulty With ρ_c

The definition of ρ_c gives rise to a physical picture in which finite potential drops can take place across an infinitesimal distance. This may be a necessary abstraction, but it also seems physically absurd. The linear relationship found in experiment between ρ_c and ρ_B is in itself somewhat suspicious.

¹²But occasionally values higher than 1.222 were found.

Figure 3.15 shows a possible distribution of ϕ with distance across the interface layer

$$\frac{\partial \phi_I}{\partial n} = \frac{V - \phi}{z_o} = \frac{\rho_I}{\rho_B} \frac{\partial \phi_B}{\partial n} \quad (3.35)$$

which leads to

$$\frac{1}{\rho_B} \frac{\partial \phi_B}{\partial n} = \frac{V - \phi}{\rho_c} \quad (3.36)$$

Thus if the interface were very resistive compared to the bulk, it would explain why the distance $\frac{\rho_c}{\rho_B}$ is so large, when the interface itself must be about 30 Å thick.

3.8 Conclusions

The method of parallel stripes falls back upon approximations which are at first glance reasonable but not known to be accurate — namely, it solves a 2-D problem in the semiconductor relying on the metal potential to be slowly varying along the width of the stripe. Metal resistance effects are then accounted for by transmission-line methods. Interestingly, three methods based on this analysis have been introduced and confronted by experiment: (1) the eight terminal method, in which the transmission-line attenuation constant is sensed directly, (2) the six terminal method, in which a transcendental equation involving several quantities is solved for the attenuation constant, and (3) the four terminal method, in which the samples must be diced. The overall method is found to be applicable either to structures on complete wafers, or to samples diced from the wafer.

Experimentally, the method performs satisfactorily and most significant is perhaps the first approach in which all regions of the structure: interface, bulk and

Figure 3.15: Potential Variation Across the Interface.

metal, have been analyzed. Simultaneous measurement of ρ_B and ρ_c has proven possible experimentally and the transmission-line model has been confirmed to be a good one. Contact resistivity values comparable to those reported by Berger [1] have been produced here; ρ_B values generally in the nominal, manufacturer's range are determined. On the negative side, there is considerable scatter in the ρ_c data, as is found for other methods, owing most likely to the fact that no attempt was made to correlate measured factors, particularly geometry, with specific devices under test.

The method reported here is considered to be a practical one, although it calls for multiple probes or wafer dicing, (generally speaking the Berger or Kelvin methods call for four probes). Furthermore, it represents an attempt to minimize bulk effects, which is essential if ρ_c is to be measured accurately. It is expected to be of particular interest in situations where a shallow doped layer is difficult to manufacture, or in the case of lightly doped substrates, although it is by no means limited to these cases. Silicon wafers doped from $0.001\Omega\text{ cm}$ to $100\Omega\text{ cm}$ are fairly readily obtainable, as is well known. With this method, ρ_c values in the range from $10^{-9} - 10^{-8}\Omega\text{ cm}^2$ might in principle be determined, if metal resistance were an insignificant effect.

One aspect of the structure which has been noted is the enormous rise of current density near the contact edges. In the literature, one finds that contact failure, occurring at large currents, is principally by two modes: thermomigration and electromigration [51]. In both cases some physical degradation takes place: thermomigration is associated with degradation at the boundaries of contact windows and electromigration with physical breaks in long runs of metal. These modes of failure [51] do not appear to be well understood at the present time, since they are sensitively related to current, geometry, temperature and time. Considering the near singularities of current density at the contact edges found in idealized calculations, it is not surprising that degradation also takes place at the contact edges.

Thus it may be that such calculations can enhance understanding and possibly control of failure modes.

We will now look at more rigorous analyses of the contact resistance problem which lead to even better methods for measurement.

12

1

Chapter 4

BOUNDS ON RESISTANCE AND ρ_c

Computer simulation of the two stripe structure gave evidence that the resistance, R , between two identical electrodes was approximately given by

$$R \sim R_0 + 2\rho_c/A_c \quad (4.1)$$

without a great deal of error. This fact and various plots of the computed results suggested that the resistance was bounded from below and above as follows

$$R_1 + 2\frac{\rho_c}{A_c} < R < R_2 + 2\frac{\rho_c}{A_c} \quad (4.2)$$

The writer independently proved this, although later discovering that it had been proved, if not originally, then early on in the context of heat flow by Lewis [42]. He [43] and others [44] have also considered the problem of spreading resistance measurements, but the fact has not received much attention among workers on contact resistivity. The two subjects, although similar, represent diverging interests. Generally speaking, the bounds obtained from any structure will be within several

percent (perhaps ten) of each other.

More recently the writer has extended the proof to non-uniform substrates (i.e., Berger and Kelvin techniques) and to a situation of high but not infinite metal conductivity, which are believed to be original contributions. These proofs have a practical end, in that they, together with dimensional analysis arguments, lead to a new measurement approach. They also set a constraint on the behaviour of approximate solutions as we have already seen, and provide part of a framework within which to understand the overall problem.

4.1 Uniform Substrates

To maintain generality in the arguments to follow we will assume the two contacts to be of arbitrary shape and size on a uniformly doped substrate of arbitrary shape and size. For one contact¹ on surface S_1 , and at potential V_1 , current flows into the semiconductor

$$\frac{1}{\rho_B} \frac{\partial \phi}{\partial n} = \frac{V_1 - \phi}{\rho_c} \quad (4.3)$$

while for the other on surface S_2 , and at potential $-V_2$, current flows out of the semiconductor

$$\frac{1}{\rho_B} \frac{\partial \phi}{\partial n} = - \left(\frac{V_2 + \phi}{\rho_c} \right) \quad (4.4)$$

In the bulk

$$\nabla^2 \phi = 0 \quad (4.5)$$

¹ Assumed an equipotential: an assumption which will be removed later.

while over the remainder of the bounding surface Γ of which S_1 and S_2 are a part, there is no current flow

$$-\frac{1}{\rho_B} \frac{\partial \phi}{\partial n} = 0 \quad (4.6)$$

Consequently, the arguments apply closely to any pair of highly conductive electrodes on any type of substrate.²

The power dissipation in the bulk is

$$P = \frac{1}{\rho_B} \int_{S_1+S_2} \phi \frac{\partial \phi}{\partial n} dS + \rho_c \int_{S_1+S_2} j_n^2 dS \quad (4.7)$$

since by Green's theorem, considering bulk effects only

$$\frac{1}{\rho_B} \int_{\tau} |\nabla \phi|^2 d\tau = \frac{1}{\rho_B} \int_{S_1+S_2} \phi \frac{\partial \phi}{\partial n} dS \quad (4.8)$$

where the left hand term is recognized as a definition of the power dissipation. Also if one substitutes (4.3) and (4.4) into (4.7), one obtains

$$P = (V_1 + V_2) I \quad (4.9)$$

where V_1 and V_2 are both positive, and their sum represents the potential difference between the terminals. Equation (4.9) is also a definition of power dissipation in the overall structure.

Now suppose there is a small change in ρ_c , $\delta \rho_c$; correspondingly, one may expect a change in the distribution of potential $\delta \phi$, or current density δj_n under the contacts. By the laws of differential calculus,

²e.g., thin or infinitely thick.

$$P = - \int_{S_1+S_2} j_n \phi dS + \rho_c \int_{S_1+S_2} j_n^2 dS \quad (4.10)$$

becomes

$$\delta P = - \int_{S_1+S_2} \delta j_n \phi dS - \int_{S_1+S_2} j_n \delta \phi dS + \delta \rho_c \int_{S_1+S_2} j_n^2 dS + 2\rho_c \int_{S_1+S_2} j_n \delta j_n dS \quad (4.11)$$

Now suppose

$$\int_{S_1} \delta j_n dS = 0 \quad (4.12)$$

since this means only that measurements are carried out at constant current.³ Also from

$$\nabla \cdot \delta j = 0 \quad (4.13)$$

and

$$\nabla^2 \delta \phi = 0 \quad (4.14)$$

which are physically essential, it follows that ∇ and δ are commutative.⁴ Now

³Or equally

$$\int_{S_2} \delta j_n dS = 0$$

⁴Since ∇ and δ are differential operators, it is actually more to the point to say that they commute for the reason that $\nabla \delta \phi = \nabla (\phi_1) - \nabla (\phi_2) = (\nabla \phi)_1 - (\nabla \phi)_2 = \delta \nabla \phi$.

$$\nabla \cdot (j\delta\phi) = \nabla \cdot (\phi\delta j) \quad (4.15a)$$

since on expansion, (4.15a) becomes

$$j \cdot \nabla \delta\phi + \delta\phi \nabla \cdot j = \nabla \phi \cdot \delta j + \phi \nabla \cdot \delta j \quad (4.15b)$$

Given the rule of commutation (4.15b) can be seen to be an identity.

Consequently

$$\int_{S_1+S_2} j_n \delta\phi dS = \int_{S_1+S_2} \phi \delta j_n dS \quad (4.16)$$

Returning to (4.11), and before using (4.16) it follows that

$$\delta P = - \int_{S_1+S_2} j_n \delta\phi dS + \delta\rho_c \int_{S_1+S_2} j_n^2 dS + \rho_c \int_{S_1+S_2} j_n \delta j_n dS \quad (4.17)$$

because of immediate application of (4.3) and (4.4). Then by (4.16)

$$\delta P = - \int_{S_1+S_2} \phi \delta j_n dS + \delta\rho_c \int_{S_1+S_2} j_n^2 dS + \rho_c \int_{S_1+S_2} j_n \delta j_n dS \quad (4.18)$$

The first and last terms on the right hand side can then be eliminated as in (4.17)⁵

$$\delta P = \delta\rho_c \int_{S_1+S_2} j_n^2 dS \quad (4.19)$$

⁵If $\delta\rho_c = 0$ in (4.11), and if $\delta\phi$ and δj are regarded as arbitrary changes in the distribution of potential and current density, then ultimately $\delta P = 0$. This means that the distributions of current density and potential giving rise to the correct solution to the problem also give rise to an extremum in power dissipation.

It follows automatically that power dissipation and resistance rise monotonically with ρ_c as we would expect intuitively. Furthermore

$$\int j_n^2 dS \geq \frac{1}{A_c} \left(\int j_n dS \right)^2 \quad (4.20)$$

is seen to be true by writing

$$\int j_n^2 dS = \int \left(j_n - \frac{1}{A_c} \int j_n dS \right)^2 dS + \frac{1}{A_c} \left(\int j_n dS \right)^2 \quad (4.21)$$

Consequently

$$\delta R \geq \frac{\delta \rho_c}{A_{c1}} + \frac{\delta \rho_c}{A_{c2}} \quad (4.22)$$

since

$$R = \frac{P}{I^2} \quad (4.23)$$

With this result, one can now show that the resistance is bounded from below and above, treating (4.22) as a statement in ordinary differentials

$$\int_0^{\rho_c} dR(\rho'_c) \geq \int_0^{\rho_c} d\rho'_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.24a)$$

and therefore

$$R(\rho_c) \geq R(0) + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.24b)$$

The resistance with no contact resistivity will be denoted R_D , with the D standing for Dirichlet, since as ρ_c goes to zero

$$\phi \approx V_1 \text{ or } -V_2 \quad (4.25)$$

Thus

$$R \geq R_D + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.26)$$

Similarly,

$$\lim_{\rho_c^* \rightarrow \infty} \left(\int_{\rho_c}^{\rho_c^*} dR(\rho_c') - \int_{\rho_c}^{\rho_c^*} d\rho_c' \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \right) \geq 0 \quad (4.27)$$

$$\lim_{\rho_c^* \rightarrow \infty} \left(R(\rho_c^*) - R(\rho_c) - (\rho_c^* - \rho_c) \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \right) \geq 0 \quad (4.28)$$

Now as ρ_c goes to infinity, the boundary conditions tend to

$$\frac{1}{\rho_B} \frac{\partial \phi}{\partial n} \sim \frac{V_1}{\rho_c} \text{ or } -\frac{V_2}{\rho_c} \quad (4.29)$$

and so the bulk resistance tends to a fixed value, called the Neumann resistance, R_N . Consequently, as $\rho_c^* \rightarrow \infty$

$$R(\rho_c^*) \rightarrow \left(R_N + \rho_c^* \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \right) \quad (4.30)$$

and by (4.28)

$$R(\rho_c) \leq R_N + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.31)$$

This completes the proof,⁶ which seems more physical than that of Lewis [42].

4.1.1 Illustration of the Ideas of Section 4.1

Figure 4.1 shows a plot of computed resistance for the two stripes along with lower and upper bounds on the resistance. At very small values of ρ_c , $R(\rho_c)$ must be closer to the lower bound, while at large values of ρ_c it must be asymptotic to the upper bound.

Practically this sets a limit to the detection of small values of ρ_c , as roughly

$$\sim 0.01 R_D A_c \quad (4.32)$$

which amounts to

$$\sim .0064 \rho_B L \quad (4.33)$$

for the two stripes, or $3 \times 10^{-9} \Omega \text{cm}^2$ for reasonable resistivity/geometry.⁷ If instead of calculating $R(\rho_c)$ at many values of ρ_c , one only determines lower and upper bounds, then one can only compute ρ_c to

⁶It may appear possible for $\rho_c \frac{\partial \phi}{\partial n}$ to take on a finite value when $\rho_c \rightarrow 0$ or for ϕ to do so when $\rho_c \rightarrow \infty$. This is not the case. Any structure for measurement may be modelled in principle as a sufficiently fine regular rectangular array of resistors. The resistor values in the bulk will be of the order ρ_B/L while those at the contact will be of order ρ_c/L^2 where L is the distance between resistor nodes. If the grid size is fixed but arbitrary, we can always find a ρ_c sufficiently small such that the semiconductor potential tends to V_1 or $-V_2$. Similarly, we can find a ρ_c sufficiently large that $V_1 \gg \phi$ and $V_2 \gg -\phi$.

⁷i.e., with $\rho_B \sim 0.001 \Omega \text{cm}$ and $L = 4 \mu\text{m}$.

Figure 4.1: Bounds for the Parallel Stripes ($s = 0.5L$), $\alpha = \rho_c/\rho_B L$. Computed Resistances Lie Between the Theoretical Bounds.

$$(R_N - R_D)A_c \quad (4.34)$$

or about $2 \times 10^{-8} \Omega \text{cm}^2$ for the two stripes. The complete solution therefore seems worth determining.

4.1.2 End Resistance

Now consider the case where a third contact is used to sense potential in an end resistance type of configuration. Here under S_3

$$\frac{1}{\rho_B} \frac{\partial \phi}{\partial n} = \frac{\langle \phi \rangle - \phi}{\rho_c} \quad (4.35)$$

where $\langle \phi \rangle$ denotes the average of ϕ over the contact area. It seems intuitively reasonable that the current density under some portions of the floating contact can be positive outwards, and elsewhere positive inwards. Consider, for example, if the floating contact were at the centre of the pair of active contacts, very long and placed on a very thin layer. In this case, current would "prefer" to flow in the metal rather than the bulk, and this leads to the conclusion just given. A sketch (Figure 4.2) clarifies this point of view.

It is fairly clear that the "front" resistance rises monotonely with ρ_c , but the end resistance requires a more detailed investigation. In particular, if armed only with a real meter with finite input impedance, the onset of large values of ρ_c might ultimately lead to fluctuating readings of the voltage indicated in the diagram.

It follows automatically from previous considerations that for the present case, the increase in total power dissipation corresponding to a rise in ρ_c is given by

Figure 4.2: Sketch of Current Streamlines in an End Resistance Configuration.

$$\delta P = \delta \rho_c \int_{S_1+S_2+S_3} j_n^2 dS \quad (4.36a)$$

Then

$$\delta P = \delta \rho_c \int_{S_1} j_n^2 dS + \delta \rho_c \int_{S_2} j_n^2 dS + \delta \rho_c \int_{S_3} j_n^2 dS \quad (4.36b)$$

$$\geq \frac{\delta \rho_c}{A_{c1}} I^2 + \frac{\delta \rho_c}{A_{c2}} I^2 + \frac{\delta \rho_c}{A_{c3}} (0)^2 \quad (4.36c)$$

since the current flowing to or from S_3 is zero. Thus^b

$$R(\rho_c) \geq R_D + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.37a)$$

and

$$R(\rho_c) \leq R_N + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.37b)$$

This proves that the resistance between a pair of contacts in a front resistance configuration is bounded even if there is a third floating contact located nearby.

Before considering end resistance, suppose we have only two symmetrically situated contacts such that a fixed equipotential surface (regardless of ρ_c) obtains in the body of the semiconductor. Then on a per contact basis

$$\delta P = \delta \rho_c \int_S j_n^2 dS \quad (4.38a)$$

^b R_D would be measured with S_1 , S_2 and S_3 equipotential surfaces, as the potential difference between S_1 and S_2 divided by the current I . R_N is the average potential difference between S_1 and S_2 divided by the current I with all three contacts having constant current density distributions — i.e., S_3 becomes indistinguishable from other parts of the free surface.

$$\geq \frac{\delta \rho_c}{A_c} I^2 \quad (4.38b)$$

or

$$\delta R \geq \frac{\delta \rho_c}{A_c} \quad (4.38c)$$

It is therefore natural to suppose that the resistance between any pair of contacts will contain at least a pure contact resistance component

$$\frac{\delta \rho_c}{A_c}$$

from each contact, or in general,

$$\frac{\delta \rho_c}{A_c} \frac{(\int j_n dS)^2}{I^2} \quad (4.39)$$

This last equation is included since we must distinguish cases where the contact does not carry current (floats) or does (active) carry the full current I . This amounts to a plausible demonstration that end resistance is bounded but is somewhat less than a proof.

Returning to the situation with three contacts, we note first that any network of resistances with four terminals leading to the outside world is governed by (i.e., two-port)

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad (4.40a)$$

$$V_2 = Z_{12}I_1 + Z_{22}I_2 \quad (4.40b)$$

where the Z_{ij} are pure resistances and where matrix symmetry derives from reciprocity. If the resistance between S_i and S_j is R_{ij} (Figure 4.2), we have therefore

$$R_e = \frac{1}{2}(R_{12} + R_{23} - R_{13}) \quad (4.41)$$

This form has appeared in the work of Reeves and Harrison [39]. As we have already seen, the R_{ij} values are bounded, so that R_e is also bounded.

These results may amount to a strong criticism of transmission line analysis in practical cases, since R_f is a weak function of ρ_c while R_e is (in the TLM model) an extremely strong function of ρ_c . In other words, as R_f violated the exact bounds, R_e may be expected to violate them even more strongly.

4.2 Advantage of the Upper Bound for Calculation Purposes

The upper bound is unique among all cases in that it can readily be determined analytically for a variety of cases. In particular the two stripes, the two dots (to be discussed in Chapter 5) and the Berger structure with finite width contacts on a non-uniformly doped substrate can all be handled by treating the bulk as a rectangular prism of suitably large size and solving by Fourier series methods. Analytical calculations of the upper bound are usually possible even in three-dimensional cases and the prediction is a value of resistance actually taken on under appropriate circumstances — i.e., provided ρ_c is sufficiently large.

In view of the certain indeterminacy in real situations stemming from the fact that ρ_c , ρ_B and geometry vary at different sites on a sample, and because of the possibility of undesirable bulk-contact interactions during sintering and the presence of metal resistance, it is perhaps reasonable to search for some appropriate

alternative definition of ρ_c .⁹ Since R_D and R_N are generally close together, the error of using one bound may often be tolerable. One obvious asset of defining ρ_c based on the upper bound would be that the latter corresponds with worst-case power loss, a situation of interest to the engineer. Values of ρ_c as low as $3 \times 10^{-8} \Omega \text{cm}^2$ ($\alpha = .06$) could be determined with the two stripes to within a factor of two by adopting this alternative definition.

For some applications, use of the upper bound as an approximation to the true resistance may have considerable merit.¹⁰

4.3 Bounds for the Parallel Stripes

Calculation of the lower bound is readily carried out using the approach of conformal mapping, while the upper bound can be determined by the Green's function method. Other methods, like Fourier series can and have been used, but the present methods yield compact analytic solutions, so they will be described here.

The lower bound can be found by conformally mapping the structure of Figure 4.3a into that of Figure 4.3b. The basis of the approach is the Schwarz-Christoffel transformation used by Overmeyer [19]

$$w = f(z) = \int_0^z \frac{d\zeta}{\sqrt{(\zeta^2 - s^2)(\zeta^2 - (s + L)^2)}} \quad (4.43)$$

⁹It is clear that ρ_c is practically and in effect defined for the Berger and Kelvin structures differently than in this report.

¹⁰Note that if the bulk resistance in any measurement is ignored, a crude upper bound can be computed

$$\rho_c \leq RA_c/2 \quad (4.42)$$

which may occasionally have use.

(a) parallel stripes, z -domain

(b) parallel stripes, w -domain

Figure 4.3: Conformal Mapping of the Parallel Stripes Geometry.

By application of the chain rule and the Cauchy-Riemann equations, potentials are identically the same in the z - and w -planes, while gradients are transformed as

$$\nabla_w = \frac{\nabla_z}{|f'(z)|} \quad (4.44)$$

Thus the rectangle has $+V$ and $-V$ applied to its ends and its sides are insulated.

Now

$$b = \int_0^s \frac{dx}{\sqrt{(s^2 - x^2)((s + L)^2 - x^2)}} \quad (4.45)$$

which becomes under the transformation

$$x = s \sin \theta \quad (4.46)$$

$$b = \frac{1}{s + L} \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad (4.47a)$$

where

$$k = \frac{s}{s + L} \quad (4.47b)$$

This is the standard form for the Legendre complete elliptic integral of the first kind

$$b = \frac{1}{s + L} F\left(\frac{s}{s + L}, \frac{\pi}{2}\right) \quad (4.47c)$$

Similarly

$$c = \int_s^{s+L} \frac{dy}{\sqrt{(y^2 - s^2)((s+L)^2 - y^2)}} \quad (4.48)$$

Transforming first by

$$y = s \sec \phi \quad (4.49a)$$

and then by

$$\sin \theta = \frac{s+L}{\sqrt{(s+L)^2 - s^2}} \sin \phi \quad (4.49b)$$

$$c = \frac{1}{s+L} \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad (4.50a)$$

$$k = \frac{\sqrt{(s+L)^2 - s^2}}{s+L} \quad (4.50b)$$

or in standard form

$$c = \frac{1}{s+L} F \left(\frac{\sqrt{(s+L)^2 - s^2}}{s+L}, \frac{\pi}{2} \right) \quad (4.51)$$

The resistance is

$$2b/c \quad (4.52)$$

Elliptic integrals can be readily evaluated using Landen's transformation [61],

$$F(k, \phi) = \sqrt{\frac{k_1 k_2 \dots}{k}} \ln \tan \left(\frac{\pi}{4} + \frac{\Phi}{2} \right) \quad (4.53a)$$

where

$$k_1 = \frac{2\sqrt{k}}{1+k}, \quad k_2 = \frac{2\sqrt{k_1}}{1+k_1} \quad (4.53b)$$

and

$$\tan \phi_{i-1} = \frac{\sin 2\phi_i}{k_{i-1} + \cos 2\phi_i} \quad (4.53c)$$

$$\Phi = \lim_{i \rightarrow \infty} \phi_i \quad (4.53d)$$

Despite a complicated appearance, the process converges extremely rapidly and it can be implemented with ease on a digital computer. The result for a pair of stripes of spacing equal to their length is 1.2793, and for a spacing twice the length, the result is 1.5634.

Considering now a substrate of finite thickness, T , the transformation from one of infinite thickness is given by Overmeyer [19] as

$$w = \frac{T}{\pi} \ln \frac{z - \frac{\pi}{2T}}{z + \frac{\pi}{2T}} - jT \quad (4.54)$$

Consequently the ratio

$$\left(\frac{s+L}{s} \right)' = \frac{\tanh \frac{\pi}{2T}(s+L)}{\tanh \frac{\pi}{2T}s} \quad (4.55)$$

describes the spacing-length ratio for the infinite thickness case in terms of that for the finite thickness case. Resistance may be found for this transformed case as before. These calculated results have already appeared in this report and illustrate the usefulness of conformal mapping techniques for some problems.

The upper bound can be found by using the equation derived in the previous chapter

$$\phi = \frac{1}{\pi} \int_s^{s+L} \frac{\partial \phi}{\partial n}(x') \ln \left| \frac{x+x'}{x-x'} \right| dx' \quad (4.56)$$

where $\frac{\partial \phi}{\partial n}(x')$ is taken as constant. This is a straightforward integration which can be completed to give a few closed form terms. The resistance is

$$\begin{aligned} -\frac{2\rho_B}{\pi L^2} \left(L^2 \left(\ln(L) - \frac{1}{2} \right) + (2s+L)^2 \left(\ln(2s+L) - \frac{1}{2} \right) \right. \\ \left. - 2(s+L)^2 \left(\ln(2(s+L)) - \frac{1}{2} \right) \right. \\ \left. - 2s^2 \left(\ln(2s) - \frac{1}{2} \right) \right) \quad (4.57) \end{aligned}$$

The result for a spacing to length ratio of one is 1.3822, and for double this value of spacing to length ratio the result is 1.6483.

In this section the bounds for the parallel stripes have been computed by appropriate analytic methods. Note that they are within about 10% of each other. This has been found to be the usual case by the writer and by others [43] in other examples. Attention will now be turned to developing bounds for other cases of practical concern.

4.4 Non-Uniform Substrate Conductivities

The equation governing the potential in the bulk is now

$$\nabla \cdot (\sigma \nabla \phi) = 0 \quad (4.58)$$

The power dissipation per unit volume in the bulk is

$$\sigma_B |\nabla \phi|^2 = \nabla \cdot (\sigma_B \phi \nabla \phi) \quad (4.59)$$

and therefore the total power dissipation in the bulk is

$$\int \sigma_B \phi \frac{\partial \phi}{\partial n} dS \quad (4.60)$$

If the conductivity is uniform on the surface under the electrode this same quantity is

$$\frac{1}{\rho_B} \int \phi \frac{\partial \phi}{\partial n} dS \quad (4.61)$$

Now as before

$$\nabla \cdot \delta j = 0 \quad (4.62a)$$

and

$$\nabla \cdot (\sigma \nabla \delta \phi) = 0 \quad (4.62b)$$

as it is required that the continuity equation be satisfied, so that δ and ∇ are interchangeable. Finally

$$\nabla \cdot (\sigma \delta \phi \nabla \phi) = \nabla \cdot (\sigma \phi \nabla \delta \phi) \quad (4.63a)$$

since by expansion and use of commutativity (4.62), (4.63a) leads to an identity. Equation (4.63a) may also be written

$$\nabla \cdot (j \delta \phi) = \nabla \cdot (\phi \delta j) \quad (4.63b)$$

Consequently all the conclusions made for the uniform doping case apply to that of non-uniform doping. In particular, resistance rises monotonely with ρ_c , and is bounded

$$R_D + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \leq R(\rho_c) \leq R_N + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.64)$$

although R_D and R_N may now be somewhat more difficult to compute. Thus, the bounds proof is applicable to the Berger and Kelvin structures in a practical measurement.

4.5 Realistic Treatment of Contact Metal

Consider first the power dissipated in the bulk and interface over the bounding surface (which only includes the contacts)

$$- \int_{\Gamma_B} \phi_B j_B dS + \rho_c \int_{\Gamma_B} j_B^2 dS \quad (4.65)$$

where the boundary conditions are now

$$j_B = -\frac{1}{\rho_B} \frac{\partial \phi_B}{\partial n} = -\frac{\phi_m - \phi_B}{\rho_c} \quad (4.66)$$

The change in power dissipation is, considering

$$\int_{\Gamma_B} \phi_B \delta j_B dS = \int_{\Gamma_B} j_B \delta \phi_B dS \quad (4.67)$$

therefore

$$-2 \int_{\Gamma_B} \phi_m \delta j_B dS + \delta \rho_c \int_{\Gamma_B} j_B^2 dS \quad (4.68)$$

where Γ_B consists of both of the electrode surfaces within the bulk. Now if we define

$$j_m = -\frac{1}{\rho_m} \frac{\partial \phi_m}{\partial n_m} \quad (4.69)$$

with n_m outwards from the metal, it follows from current continuity that

$$\frac{1}{\rho_m} \frac{\partial \phi_m}{\partial n_m} = -\frac{1}{\rho_B} \frac{\partial \phi_B}{\partial n_B} \quad (4.70)$$

Thus the bulk dissipation is

$$2 \int_{\Gamma_B} \phi_m \delta j_m dS + \delta \rho_c \int_{\Gamma_B} j_m^2 dS \quad (4.71)$$

Consider now the power dissipation in the metal

$$- \int_{\Gamma_m} \phi_m j_m dS \quad (4.72)$$

The change of power dissipation in the metal by virtue of

$$\int_{\Gamma_m} \phi_m \delta j_m dS = \int_{\Gamma_m} \delta \phi_m j_m dS \quad (4.73)$$

is

$$- 2 \int_{\Gamma_m} \phi_m \delta j_m dS \quad (4.74)$$

Thus the total change in power dissipation, since $|j| = |j_m| = |j_B|$, is

$$\delta \rho_c \int_{\Gamma_B} j^2 dS \quad (4.75)$$

In deriving (4.75) from (4.71) and (4.74), we have omitted a small point. Note that Γ_m includes the region around the current probes as well as the electrode surface within the metal. However, we may take ϕ_m as fixed across the current probe, and thus since

$$\int_{\Gamma_m} \delta j_m dS = 0 \quad (4.76)$$

the conclusions stand.

The power dissipation rises monotonically with ρ_c , and the resistance remains bounded. When there is no ρ_c ,

$$\phi_m = \phi_B \quad (4.77)$$

so that there is a bulk contribution from the metal as well as the semiconductor body,

$$R \geq R_D + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.78)$$

Finally as ρ_c tends to a very large value, both the bulk and metal will begin to look like equipotential bodies. Consequently

$$R \leq R_N + \rho_c \left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}} \right) \quad (4.79)$$

where the electrodes have constant current distributions when determining R_N .

4.6 Scaling Rules

Dimensional analysis has proven fairly useful in treating the parallel stripe structure, and will shortly be presented as the basis of a second measurement technique. Two speculative possibilities will now be briefly considered — again, for the example of the parallel stripes method. Among other dimensionless forms appearing is

$$\frac{R_m L}{\rho_B} \quad (4.80)$$

Thus if different resistivity substrates are used and all the dimensions are simultaneously scaled, then to keep the metal effects consistent from sample to sample the metal thickness should be scaled as

$$T_m \propto \frac{L}{\rho_B} \quad (4.81)$$

Metal thickness can be varied over about two orders of magnitude so this may not be as impractical as it seems at first glance.

Secondly the thickness and dose of an implanted layer can be closely controlled. Therefore scaling the thickness of the layer when altering the dimensions of the contacts would seem appropriate. In this case, only

$$\frac{\rho_c}{\rho_B L} \quad (4.82)$$

will vary. It is central to good experimental practice that only one factor be varied at a time in an investigation.

4.7 Experimental Approach to ρ_c Measurement

If one scales the structure with

$$L_{eff} = \frac{1}{\left(\frac{1}{A_{c1}} + \frac{1}{A_{c2}}\right)^{1/2}} \quad (4.83)$$

then for low values of L_{eff}

$$R \sim R_N + \frac{\rho_c}{L_{eff}^2} \quad (4.84)$$

or

$$\frac{RL_{eff}}{\rho_B} \sim \frac{R_N L_{eff}}{\rho_B} + \frac{\rho_c}{\rho_B L_{eff}} \quad (4.85)$$

The first term on the right hand side is constant. For large values of L_{eff}

$$R \frac{L_{eff}}{\rho_B} \sim R_D \frac{L_{eff}}{\rho_B} + \frac{\rho_c}{\rho_B L_{eff}} \quad (4.86)$$

Thus if one plots $\frac{RL_{eff}}{\rho_B}$ against $(\rho_B L_{eff})^{-1}$, a pair of straight line asymptotes at low or high values of the abscissa will emerge, giving R_D and ρ_C , respectively. Also, the ordinate where the asymptote for large ρ_c values crosses the axis can be used to determine the bulk resistivity fairly readily (if metal resistance effects are small, which demands small contact windows). Such a procedure applied to the Berger or Kelvin structures would probably demand that the junction depth be scaled with the geometry in the interface plane.

In practice only one asymptote would likely be detected by the method, given that the geometry could only be scaled practically by about two orders of magnitude. However, this may often be sufficient to determine ρ_c with reasonable accuracy.

4.8 A Geometry Independent Result?

The following idea arose from dimensional analysis concepts. In general, the algebraic relationships relating physical variables must all, by Buckingham's Π -theorem, be expressible in the form of dimensionless parameters. In standard notation, if π is the dependent and π_i are the independent dimensionless variables, then

$$\pi = \phi(\pi_1, \pi_2, \dots, \pi_m) \quad (4.87)$$

expresses the required form that any "true" physical relation must take on. A host of situations may therefore be simple equivalents of each other provided that the dimensionless π_i values remain invariant while the dimensional physical parameters change. Such conditions are called similarity conditions, and among these the requirement for similar shape seems to have a unique status [62]. It will be assumed here that this requirement of geometric similarity can sometimes be relaxed. In particular, the quantities R_N , R_D and L_{eff} have been identified which have no

reference to shape and are interrelated by a function

$$\frac{RL_{eff}}{\rho_B} = \phi \left(\frac{R_D L_{eff}}{\rho_B}, \frac{R_N L_{eff}}{\rho_B}, \frac{\rho_c}{\rho_B L_{eff}} \right) \quad (4.88)$$

for structures falling in a specified class of shapes. This relationship totally determines the resistance between two stripes or two dots, and by virtue of the fact that one is obtained from the other by a geometric dilation, the same function governs both situations.¹¹

Note further that a universal function applies to any structures satisfying

$$R_N = R_D \quad (4.89)$$

— i.e., 1 - D situations of facing contacts, or any radial section of spherical or cylindrical symmetry. Asymptotically, the resistance is given for small area contacts by

$$R = R_N + \frac{\rho_c}{L_{eff}^2} \quad (4.90)$$

or for large area contacts by

$$R = R_D + \frac{\rho_c}{L_{eff}^2} \quad (4.91)$$

so in these cases, universal shape invariant functions of the form (4.88) arise.

On the negative side is the fact that if a curve $R(\alpha)$ is completely determined by the "endpoints" $R(0) = R_D$ and $R(\alpha) \sim R_N + 2\alpha$ then it would seem it must

¹¹i.e., the two stripes and the two dots are special cases of a general class of objects.

satisfy a second order ordinary differential equation. It appears very unlikely that the governing partial differential equation and boundary conditions would simplify in this way.

Also, the familiar form of the Laplace equation in 3-D arises only in Cartesian coordinate systems (rotated, translated, scaled) unlike the 2-D case. There, a host of transformations can be identified carrying one shape into others while leaving a governing Laplace equation invariant (i.e., conformal mapping). However, this in itself does not rule out the possibility of shape invariance for 3-D cases, but it does make it difficult to demonstrate.

The idea of shape invariance must have occurred to anyone who has had to face communicating the results of an experimental study. If one has odd-shaped samples, it is more natural to present the dependent variable as a function of perimeter or area, with the unstated belief that these results might actually apply to other shapes. The question is whether this belief is ever really justified in cases other than trivial ones.

4.9 Miscellaneous Results

It has been shown in 4.1 that the resistance R between two contacts can be expressed

$$\frac{\partial R}{\partial \rho_c} = \frac{R_c}{\rho_c} \quad (4.92a)$$

if only ρ_c varies, where R_c is the contact resistance defined in terms of power dissipation

$$I^2 R_c = \rho_c \int_S j_n^2 dS \quad (4.92b)$$

If only ρ_B changes, it follows analogously that

$$\frac{\partial R}{\partial \rho_B} = \frac{R_B}{\rho_B} \quad (4.93a)$$

where R_B is the bulk resistance

$$I^2 R_B = - \int_S \phi j dS \quad (4.93b)$$

Therefore, if both ρ_B and ρ_c vary

$$\rho_B \frac{\partial R}{\partial \rho_B} + \rho_c \frac{\partial R}{\partial \rho_c} = R \quad (4.94)$$

However, this gives no additional information over what we already have, which would be needed in order to solve the general problem completely.

4.9.1 Ideas Having a Direct Application to Measurement

If we write

$$R = \frac{\rho_B}{L} f \left(\frac{\rho_c}{\rho_B L} \right) \quad (4.95)$$

and using (4.92a) and (4.92b), then if only L and R vary

$$R_c = -R - L \frac{dR}{dL} \quad (4.96a)$$

and

$$R_B = R - R_c \quad (4.96b)$$

Hence we can experimentally determine the bulk and contact resistances as defined above, which does go part of the way towards solving the problem in the general case.

Finally, provided we have similar shaped structures then again

$$R = \frac{\rho_B}{L} f \left(\frac{\rho_c}{\rho_B L} \right)$$

We will further assume the existence of a large amount of R-L data for each of a number of wafers in each of which ρ_B and ρ_c are fixed but unknown. If ρ_B and ρ_c have been accurately determined by any method for even one sample, we could in principle determine ρ_B , ρ_c and f experimentally merely by shifting the experimental curves along the RL (vertical) and $1/L$ (horizontal) axes of a plot. Thus an extensive experimental study could be carried out initially and used to interpret all future measurements. New sets of measurements could furthermore be used to refine knowledge of f as they became available. This would make the modelling problem largely empirical, eliminating to an extent the need for numerical analysis.

4.10 Conclusions

The identification of the bounds on resistance provides part of a framework for understanding contact resistivity problems. Moreover, the bounds demonstrate that the practical measurement of ρ_c can never completely eliminate, in principle, a contribution to resistance from the substrate. Even the Kelvin structure, despite its ingenuity, is seen to suffer from this limitation, (see also [29]).

The method of transmission-line analysis is fundamentally in error for the small values of ρ_c measured in practice, owing to the inaccurate behaviour of the lower bound obtained from such an analysis.

Upper and lower bounds may often be useful in bracketing measured ρ_c values since measured resistances are subject to considerable scatter anyway. That is, theoretical error arising from this approximate treatment may be tolerable if experimental error is sufficiently large.

The bounds apply to a variety of realistic situations and can even be used together with scaling rules as a basis for measuring ρ_c without detailed analysis; a dimensionless plot of $\frac{RL_{eff}}{\rho_B}$ against $(\rho_B L_{eff})^{-1}$ could give ρ_c directly.

Chapter 5

AN OPTIMIZED TECHNIQUE FOR ρ_c MEASUREMENT

After developing the method of parallel stripes it was clear that it had aspects which were not wholly satisfactory. The idea arose of using an arrangement consisting of a central circular contact with surrounding annular contacts, but it had limited appeal since, based upon rough calculations, metal resistance effects would be large. The bulk effects could be treated by the 3-D Laplace equation in cylindrical coordinates, with symmetry about the axis of the cylinder assumed

$$\frac{\partial^2}{\partial r^2}U + \frac{\partial^2}{\partial z^2}U = -\frac{U}{4r^2} \quad (5.1a)$$

$$U = \sqrt{r}\phi \quad (5.1b)$$

The metal resistance effects could then be treated by transmission-line methods provided the bulk were of fairly high resistivity.¹ Unfortunately, conformal mapping, usefully applied to the parallel stripes, would not be applicable here. Moreover,

¹Say 0.01 Ω cm.

provision would have to be made for large contact pads ($100 \times 100 \mu m^2$) for "communication with the outside world", so some double layer metallization scheme would doubtless be needed.

An alternative idea was to let the contacts be two small and closely spaced square or circular dots. A possible experimental advantage should be noted for the proposed structure over the Berger structure which itself calls for wide contacts placed far apart. Unfortunately, analysis promised to be difficult for this new approach. Bipolar coordinates [63] are not much use since separability does not obtain for a three-dimensional situation. This and the fact that there is no effort made to constrain the current flow to definite pathways, also problematic for analysis, makes the structure appear initially to be a poor choice for an experimental method. However, the structure does five things to improve on other techniques: (1) it minimizes bulk resistance in the measurement while enhancing contact effects; (2) it raises resistance through the use of small contact windows; (3) it minimizes metal resistance through the use of a small aspect ratio; (4) it minimizes contact window area so that ρ_c , which could be variable from point to point in a large contact, is sampled over a small area, and (5) like the parallel stripe method, simultaneous measurement of ρ_B and ρ_c is possible. Parenthetically, the use of a small contact area also suggests that the upper bound will be a good approximation. In the end square contacts were simulated; it may be that some rounding of the contact windows could occur, but with modern lithographic techniques this should be a small effect provided the contact windows are not too small.

Very recently, workers at the National Bureau of Standards [46] have explored a similar structure but with circular contact windows placed far apart. Their work reflects an interest in principles in the use of thick, uniformly doped substrates, which have been little emphasized by others. However, their treatment remains approximate, because bulk and contact resistances are modelled as lumped elements. Their proposed measurement technique also uses only two probes, and the effect of

the parasitic probe resistance is substantial for highly conductive wafers. Others have very recently carried out an approximate numerical analysis of bulk losses in structures similar to the one here [45] which also omits the effects of interface resistance.

5.1 The Green's Function Method

According to the procedure outlined in Chapter 3, the electric field can be written for the 3-D case as

$$\mathbf{E}(\mathbf{r}) = \frac{\rho_B I}{4\pi r^2} \hat{\mathbf{r}} \quad (5.2)$$

where I is the current entering the volume from a point source. Consequently,

$$\phi(r) = \frac{\rho_B I}{4\pi r} \quad (5.3)$$

Superposition of an infinite number of such sources gives (see Figure 5.1)

$$\begin{aligned} \phi(x, z) = & \frac{1}{2\pi} \int_{-L/2}^{L/2} \int_{-(s+L)}^{-s} \frac{\partial \phi}{\partial n}(x', z') \frac{dx' dz'}{((x - x')^2 + (z - z')^2)^{1/2}} \\ & + \frac{1}{2\pi} \int_{-L/2}^{L/2} \int_s^{s+L} \frac{\partial \phi}{\partial n}(x', z') \frac{dx' dz'}{((x - x')^2 + (z - z')^2)^{1/2}} \end{aligned} \quad (5.4)$$

on the plane $y = 0$, since we are dealing with a half space.

For the right hand contact

Figure 5.1: Two Dots Structure.

$$\frac{\partial \phi}{\partial n} = \rho_B \frac{V - \phi}{\rho_c} \quad (5.5)$$

while for the left hand contact

$$\frac{\partial \phi}{\partial n} = -\rho_B \frac{V + \phi}{\rho_c} \quad (5.6)$$

Thus

$$\phi(x, z) = \frac{\rho_B}{2\pi\rho_c} \int_{-L/2}^{L/2} \int_s^{s+L} (V - \phi(x', z')) \left(\frac{1}{((x - x')^2 + (z - z')^2)^{1/2}} - \frac{1}{((x + x')^2 + (z - z')^2)^{1/2}} \right) dx' dz' \quad (5.7)$$

This is an integral equation which can be solved numerically for $\phi(x, z)$.

5.2 The Boundary Element Method

As for the two stripes problem, it was found more accurate to model the potential distribution by many piecewise constant elements rather than by any polynomial of higher order. The reason for this is presumed again to have to do with the presence of "near singularities", found here at the vertices of the contact windows.

Thus over all elements

$$\phi(\mathbf{r}) = \Sigma \frac{1}{2\pi} \frac{\partial \phi}{\partial n}(\mathbf{r}') \int_{\Delta S'} \frac{dS'}{|\mathbf{r} - \mathbf{r}'|} \quad (5.8)$$

As before, $r = r'$ represents a singularity, and the integral of (5.8) must be explicitly evaluated around this point. That is, for a small square region $\Delta S'$, ϵ in length along the side, about the singular point, the integral in (5.8) becomes

$$\int_{\Delta S'} \int \frac{dx' dz'}{((x - x')^2 + (z - z')^2)^{1/2}} \quad (5.9a)$$

$$= \int_{-\epsilon/2}^{\epsilon/2} \int_{-\epsilon/2}^{\epsilon/2} \frac{du dv}{(u^2 + v^2)^{1/2}} \quad (5.9b)$$

$$= 8 \int_0^{\pi/4} \int_0^{\frac{\epsilon/2}{\cos \theta}} dr d\theta \quad (5.9c)$$

$$= 4\epsilon \int_0^{\pi/4} \frac{d\theta}{\cos \theta} \quad (5.9d)$$

$$= 4\epsilon \ln \tan \frac{3\pi}{8} \quad (5.9e)$$

Two axes of symmetry exist which divide the problem into quarters, reducing the storage requirements by four.

Finally the use of smaller "corner elements" was unproductive, unlike the case of the two stripes, perhaps because very little contribution to the integral of (5.7) is made by the singularities or cusps occurring there.

5.3 Numerical Results

Since the results were only determined by one method, it is necessary to examine the question of convergence. It will be seen in Table 5.1 and Figure 5.2 that at

the largest number of boundary elements used, numerical results appear to have converged with fair (graphical) accuracy. Table 5.2 gives the computed resistances for a spacing equal to the length of the dot and for double this ratio of spacing to length, for those interested in exact quantitative results. In the latter results, 196 elements were used to represent each contact window. It will be seen that R_D and R_N differ by a small though significant amount (this can be seen by subtracting 2α from the normalized resistance to obtain the bulk resistance when $\alpha \sim 10^{-7}$ and $\alpha \sim 1 - 10$), as appears to be usually the case. Clearly using the numbers in Table 5.2, one may determine ρ_B and ρ_c simultaneously as in the two stripe case.

Figure 5.3 shows the distribution of current density over the contacts for $\alpha = 10^{-7}$ and 1.0. These plots suggest that the corners of the windows will be the likely site where reliability problems will make themselves evident.

5.4 An Empirical Equation Fitted to These Numerical Results

In attempting to interpret large quantities of experimental data, it would be more convenient to use an equation than tabulated or graphical results. The non-dimensional resistance RL/ρ_B is approximately given by

$$\frac{RL}{\rho_B} = R_D \frac{\sigma\alpha + 1}{\tau\alpha + 1} + 2\alpha \quad (5.10)$$

This reduces the computed results in Table 5.2 to an empirical equation. For $s = 0.5L$, $R_D = 0.717$, $\sigma = 10.88$ and $\tau = 10$, while for $s = L$, $R_D = 0.775$, $\sigma = 10.71$ and $\tau = 10$. The fit to the computed results is good to about three digits and should be adequate for any practical purpose.

Note that if α is small

Figure 5.2: Convergence in the Calculated Resistance Values: (a) Resistance vs Logarithm of α as a Function of the Number of Boundary Elements, (b) Resistance vs the Number of Boundary Elements for $\alpha = 10^{-7}$ and 10^{-6} .

Table 5.1: Resistance Between Two Dots for $s = 0.5L$ Versus Number of Elements per Window. Resistance Normalized to Contact Window Length and ρ_B .

N	$\alpha = 10^{-7}$	1	10^6
16	.762	2.79	2×10^6
25	.749	2.78	
36	.740		
49	.734		
64	.730		
81	.727	2.77	
100	.724		
144	.720		
196	.717		

Table 5.2: Resistance Versus Dimensionless ρ_c for 196 Boundary Elements. Resistance Normalized to Contact Window Length and ρ_B .

α	$\beta = 1$	$\beta = 2$
10^{-7}	.717	.775
10^{-6}	.717	.775
10^{-5}	.717	.775
10^{-4}	.717	.775
.0003	.718	.776
.001	.720	.777
.003	.725	.783
.01	.743	.801
.03	.792	.849
.1	.948	1.01
.3	1.36	1.42
1	2.77	2.83
3	6.78	6.83
10	20.8	20.8
30	60.8	-
100	201	201
1000	2000	2000
10^4	2×10^4	2×10^4
10^5	2×10^5	2×10^5
10^6	2×10^6	2×10^6

(a)

(b)

Figure 5.3: Three-Dimensional Current Distribution in the Contact Window: (a) $\alpha = 10^{-7}$, (b) $\alpha = 1.0$.

$$\frac{RL}{\rho_B} = R_D + (2 + (\sigma - \tau) R_D) \alpha \quad (5.11)$$

so that a lumped resistance approach will tend to overestimate α (or ρ_c).

5.5 Metal Resistance: Rough Calculations

The effect of metal resistance on the results is anticipated to be quite small and is neglected in the calculations given already.

Supposing a dipole composed of a current source and a current sink on a thin metal sheet of sheet resistance R_m ,² the potential at any point is given by

$$\phi = \frac{R_m I}{2\pi} \ln \frac{r_2}{r_1} \quad (5.12)$$

where r_2 is the distance from the observation point to the current sink, and r_1 the distance to the current source. Given r_1 and r_2 are of similar magnitude, as is a reasonable estimate, the parasitic metal resistance is therefore of the order of a few milliohms. Metal resistance effects would therefore only become an important issue in the event the samples were doped to the solid solubility limit and the contact windows were made large. Under expected conditions of measurement, for typical emitter resistivities and contact windows 20 μm long, the effects are anticipated to be modest.

However a calculation technique has been devised for the general case and is described below.

²i.e., effects in the metal are essentially two-dimensional and may be calculated roughly independently of conditions in the bulk

5.6 Numerical Model Incorporating Metal Resistance Effects

The two dots and parallel stripes have been modelled in a strategy incorporating metal resistance effects in a FORTRAN 77 program called CONRES (for contact resistance). The model of the metal potential in the contact windows was

$$\nabla^2 \phi_m + \frac{R_m}{\rho_c} (\phi_B - \phi_m) = 0 \quad (5.13)$$

where ϕ_B is the potential of the bulk just under the metal. Elsewhere

$$\nabla^2 \phi_m = 0 \quad (5.14)$$

The error in treating the metal as two- rather than three-dimensional should be very small (of order $0.1 \text{ n}\Omega \text{ cm}^2$).

The semiconductor potential in the contact windows was treated by Green's function methods, and was taken to be piecewise constant over small square boundary elements. As before, a set of linear equations can be used to represent the solution.

The 2-D distribution of metal potential was approximated using bi-quadratic formulas. This amounts to a simple finite difference approach. However, a formal approach to finite differences using quadratic formulas helps to resolve the usual uncertainty as to how boundary conditions are to be implemented.³

The program could be readily modified to employ the appropriate Green's

³i.e., suppose we have a rectangular grid of points. At a free surface we could write the surface potential as equal to that of the interior point nearest to the surface. Alternatively we could set it equal to the average of the potentials of the two adjacent surface points and the nearest interior point.

function kernel to take advantage of symmetries in the problem.⁴ Finally, only as many equations are needed as there are points in the metal. That is, one can eliminate the terms in ϕ_B common to both sets of equations; i.e., those describing the metal and those describing the semiconductor. A considerable improvement in run time can be realized by prior elimination of some of the linear equations in this way.

Figure 5.4 shows the dimensionless "bulk resistance"

$$\frac{RL}{\rho_B} - 2\alpha \quad (5.15)$$

versus the dimensionless metal sheet conductance

$$\gamma = \frac{\rho_c}{R_m L^2} \quad (5.16)$$

based on numerical simulations using the new approach. The diagram indicates that metal resistance effects can often be neglected (e.g., if $\rho_c/\rho_B = 2\mu m$, $L = 10\mu m$, $T_m = 1\mu m$, and $\rho_m = 2.8 \times 10^{-6} \Omega cm$, then for $\gamma \geq 30$, or negligible metal resistance effects in the diagram, this requires $\rho_B > 0.004 \Omega cm$).

5.7 Experimental Results

5.7.1 Experiment 1

Samples used here have been described in Section 3.6.

The test structure used is pictured in Figure 5.5. This design had the draw-

⁴i.e., the two dots problem has one axis of symmetry and the two stripes problem has two axes of symmetry.

Figure 5.4: Dimensionless Bulk Resistance ($RL/\rho_B - 2\alpha$) Versus Dimensionless Metal Sheet Conductance ($\gamma = \rho_c/R_m L^2$) for $\alpha = 0.2$. Metal Resistance is Significant for Wafers Doped Near the Solid Solubility.

back that the contact window could be damaged by the current probe. On the other hand, the window showed very little photolithographic distortion at $20 \times 20 \mu\text{m}^2$. In general, there was poor correlation between ρ_c and ρ_B as in the parallel stripe case, with ρ_c showing a scatter of an order of magnitude. It was not always possible to determine ρ_c and ρ_B simultaneously in the manner outlined in Section 3.4; problems occurred when ρ_c became apparently unusually large or small. However, ρ_B generally fell in the nominal, or manufacturer's range, with the experimental result here being 0.06-0.11 $\Omega \text{ cm}$. The range of ρ_c found was $0.2 - 3 \times 10^{-5} \Omega \text{ cm}^2$, corresponding to a ratio of ρ_c to ρ_B of $2 \mu\text{m}$. The latter is about half the value reported long ago by Berger [1] and is more consistent with recent findings [46] of about $1 \mu\text{m}$ for the Al-p-Si system.

On this basis, the method appeared feasible. The weak point of the scheme in experiment was that much of the data had to be rejected or treated in an ad hoc way because of substantial scatter in the measured resistances (i.e., the estimated ρ_B frequently fell outside of the manufacturer's range).

5.7.2 Experiment 2

A more detailed experiment was carried out to collect data for a range of resistivities for comparison with Berger [1] and with Carver et al. [46]. Again it was found, even when device dimensions and resistance measurements were correlated, that it was not always possible to extract ρ_c and ρ_B at the same time. This is because the difference in electrical behaviour between the two device structures used is very small, and this, coupled with errors in the measured parameters (e.g., 10% in resistance), gives rise to an ill-conditioned problem.⁵ Improvement in the experimental

⁵The measurements were also used to estimate variation in feature sizes, i.e., contact window lengths, from one device to another. Assuming reasonable ρ_B and ρ_c values at the outset, it was then only necessary to estimate one parameter, L , from one datum, R , using one equation, (5.10). This problem is not ill-conditioned and the results therefore had small error. Ill-conditioning arises when two parameters are estimated by solving two coupled algebraic equations simultaneously.

results could therefore be had by using two devices with equal s/L ratios but much different L values, since changes in L have a much greater impact on resistance than does s/L .

Table 5.3 shows the best results for ρ_c/ρ_B with ρ_B measured by four-point-probe prior to the experiment. Note that Berger reported 1-10 (about 4) μm , while Carver reported 0.3-3 (about 1) μm for ρ_c/ρ_B . SUPREM simulations showed that ρ_B would be disturbed by about 5-10% at the surface by the oxidation/anneal procedure, and that this disturbance would propagate only a small depth (about 1 μm) into the bulk. Initial FPP measurements also indicated that point-to-point variation in ρ_B was modest. Consequently, use of pre-measured FPP bulk resistivity values should be adequate. The table also shows that SUPREM qualitatively predicts the doping-enhanced-oxidation observed for degenerately doped samples using surface profiling measurements.

Measured metal resistivities agreeing with published data are also shown in the table. Note that patterns made in deposited aluminum deviate from mask dimensions due to etchant undercutting, as was confirmed during measurement.

In the electrical measurements, a constant current of 1 mA was supplied via a 10 V supply and series 10 k Ω resistor. Illumination effects were negligible in these samples. Voltage and current were measured with high precision (μV) digital multimeters giving a repeatable four digits. Linearity of the sample IV characteristics is confirmed in Table 5.3. Geometry was measured by optical and SEM techniques and gave spacing to length ratios of 1.1 and 2.2. An error of about 1 μm in optical measurements is unavoidable due to the diffraction limit of optical microscopes.

Electrical models of the interface for heavy doping would be quite difficult to obtain, but must clearly begin with a correct metallurgical picture. It should be noted that silicon diffuses extremely rapidly and is very soluble in aluminum at the sintering temperature, rather than the reverse situation [21,64-66]. In forming an

Table 5.3: Experimental Data Gathered in Experiment 2.

$\rho_B(\Omega\text{-cm})$	$\rho_c/\rho_B(\mu m)$
.00083	.5
.00079	.7
.0092	3.5
.0089	3.6
.081	3.6
.099	2.9

(a) contact/bulk resistivity data

doping	ellipsometric	SUPREM
moderate	950-1000	900
heavy	1200-1250	1000

(b) observed and predicted oxide thickness ($\text{\AA}$)

substrate	measured	accepted	accepted TCR
oxide/silicon	2.83 ± 0.21	$2.75 \pm 7.5\%$	$0.0042 \pm 7.5\%$
silicon		3.1	

(c) observed (23°C) and accepted (20°C) metal resistivities ($\mu\Omega\text{-cm}$)

current multiple	resistance
1/10	0.377
1	0.372
10	0.367
100	0.377

(d) resistance (Ω) observed for one device when varying the test current around 1 mA

ohmic contact therefore, consumption of some 50-60 Å of silicon takes place, rather than doping of the silicon.⁶

5.7.3 Experiment 3

In this final experiment the test pattern was redesigned as shown in Figure 5.6 to overcome difficulties already pointed out. A range of contact window sizes from 5-50 μm with s/L fixed, and a range of wafer resistivities was used here. A lower temperature of 1000°C was used to prepare the insulating oxide layer. On some samples, a nitride layer was deposited over this oxide by LPCVD. Openings were made in these insulating layers defining contact windows, and metal was then deposited and patterned. Finally, a phosphorus doped CVD oxide was deposited at low temperature over the metal on those samples having only a pure oxide insulator. Openings were then made only to the current and voltage test pads in Figure 5.6. All of these steps were introduced, possibly unnecessarily, to ensure that the devices were free to the extent possible of anomalous effects observed in poorly fabricated MOS devices.

Measurements were carried out keeping the voltage drop across the contact interface at about 1 mV. This was based on the idea that difficulties in Experiments 1 and 2 could be due to some nonlinearity in the contact IV characteristics. For the sake of illustration, the current-voltage characteristics of a diode

$$j = j_0 (e^{V/nT} - 1) \quad (5.17)$$

could appear linear around zero bias for large enough saturation current density. Two such diodes back to back would appear even more linear

⁶It may also be that "pits" in the silicon accompany the formation of "beads" of aluminum arising from surface tension effects.

Figure 5.5: Test Structure of Experiments 1 and 2.

Figure 5.6: Two Dots Structure in Experiment 3.

$$V = IR_D + 2v_T \tanh^{-1} \left(\frac{I}{I_0} \right)$$

for small enough currents. For the low current regime, (5.17) becomes

$$j \approx j_0 V / v_T \quad (5.18)$$

which corresponds exactly to the behaviour we associate with ρ_c . For larger currents

$$j \approx \frac{j_0 V}{v_T} \left(1 + \frac{1}{2} \left(\frac{V}{v_T} \right) \right) \quad (5.19)$$

and so " ρ_c " could be expected to vary from point to point across the contact window, unless $V \ll v_T$. Since $v_T \approx 25$ mV, the test voltage was chosen to be 1 mV. The ideas here are qualitative and basically wrong, since the contacts are almost linear over a range much larger than ± 25 mV around zero bias. However, there are small nonlinearities in the IV characteristics, and v_T would likely figure in any model of this behaviour, so that having restriction of some kind on the measurement conditions seems sensible.

Samples were sintered at successively higher temperatures between 450°C and 525°C, and the data fitted to (5.10) and to

$$R \approx K \frac{\rho_B}{L} + \frac{2\rho_c}{L^2} \quad (5.20)$$

A reasonable fit could be achieved by using either model, but by using (5.10) the parameters R_D and σ could be estimated from the data. Ideas used in fitting theory to data are discussed in Appendix 7. These ideas can be applied directly to (5.20)

but call for a numerical minimization in application to (5.10).⁷ The experimental values found were $R_D \approx 0.7 \pm 0.1$ and $\sigma \approx 11 \pm 2$, in agreement with numerical results. In addition, the error of the fit was substantially reduced for (5.10) compared to (5.20). On the average the sum of squared errors was halved.

Samples sintered at temperatures near 450°C showed anomalous values of ρ_B , possibly due to nonlinear effects, with ρ_B higher than FPP measured values. However, this anomaly was eliminated after continued heat treatment. Values of ρ_B and ρ_c obtained after highest temperature sintering are reported in Table 5.4 with FPP ρ_B values for comparison. Each pair of ρ_B and ρ_c values is based on 50-100 measurements of resistance. Finally, Figure 5.7 shows typical results with R plotted against L and with data fitted to the theoretical equation, (5.10). Each point in the graph is an average of about 10 measured resistances.

An attempt was made to explain scatter in measured R values based upon (5.20). Assuming a standard deviation σ_L in L as the source of σ_R (as claimed to be the case in [1,46])

$$\sigma_R \approx \left(\frac{K\rho_B}{L^2} + \frac{4\rho_c}{L^3} \right) \sigma_L \quad (5.21)$$

or assuming scatter in ρ_B and ρ_c as the source of σ_R

$$\rho_c \propto \rho_B \quad (5.22)$$

$$\sigma_R \approx \left(\frac{K\rho_B}{\rho_c L} + \frac{2}{L^2} \right) \sigma_{\rho_c} \quad (5.23)$$

but each failed to give a complete explanation of observations. Generally these two

⁷Fitting rational functions like (5.10) to data is known to be problematic and there are no reliable general methods [57].

Figure 5.7: Resistance Versus Contact Window Length for Samples ROC-7 ($\rho_B = 0.0008\Omega cm$) and ROC-8 ($\rho_B = 0.0084\Omega cm$) Fitted to (5.10). ρ_B and ρ_c Are Extracted From the Data Yielding A Fitted Curve To the Same Data.

Table 5.4: Results from Experiment 3.

Sample	FPP	Two Dots		Comments
	$\rho_B(\Omega \text{ cm})$	$\rho_B(\Omega \text{ cm})$	$\rho_c(\Omega \text{ cm}^2)$	
ROC-1B	.0008	.0011	1.6×10^{-7}	Oxide insulator. No passivation layer.
-2	.0082	.012	4.3×10^{-6}	Oxide insulator. Passivation layer.
-3	.077	.083	2.7×10^{-5}	
-4	1.97	2.2	2.3×10^{-4}	
-5	8-10	8.5	8.2×10^{-4}	
-6	15-35	22	2.6×10^{-3}	
-7	.00075	.00093	9.7×10^{-8}	Oxide/nitride insulator. No passivation layer.
-8	.0084	.010	3.1×10^{-6}	
-9	.077	.089	1.9×10^{-5}	
-10	2.54	2.9	2.4×10^{-4}	
-11	8-10	8.9	8.9×10^{-4}	
-12	15-35	24	3.0×10^{-3}	

approaches appeared to be complementary — e.g., about half of the samples (or about 17 considering the combinations of resistivities and heat treatments tried) were more accurately described by (5.21) than by (5.23) and the reverse was true for the other half of the samples. In summary, the source(s) of observed errors has not been correctly identified either here or in the literature. Typical measured standard deviations are shown in Table 5.5 corresponding to the samples of Figure 5.7.

Finally, pitting was observed in the contact windows of some of the samples at the highest sintering temperatures. It appeared to take place where there was a concentrated mass of aluminum nearby, but interestingly was completely suppressed in those samples having a “passivation layer”. Pitting was not found to lead to any model-violating anomaly in the electrical measurements such as has been claimed by others [49].

5.8 Conclusions

The two dots have been analyzed and the results presented in a form similar to that for the parallel stripes. With the computed results, ρ_B and ρ_c may be determined simultaneously with less concern about metal resistance affecting the results, and with bulk effects minimized to enhance the interface effects. The use of small contact windows will raise the measured resistance, which is also essential to measurement accuracy.

These calculations underscore the usefulness of Green's functions and the boundary element method in numerical computations. In both the case of the parallel stripes and the two dots, calculations would require very large numbers of points to attain a similarly high degree of accuracy if conducted by any of the other commonly used methods, i.e., finite differences or elements.

Table 5.5: Measured Scatter in Resistance Data for ROC-7 ($\rho_B = 0.0008\Omega cm$) and ROC-8 ($\rho_B = 0.0084\Omega cm$).

ROC-7			ROC-8	
$L(\mu m)$	Resistance (Ω)	Standard Deviation (Ω)	Resistance (Ω)	Standard Deviation (Ω)
5.5	1.84	.073	34.4	2.6
11.8	0.814	.028	11.2	.38
18.2	0.467	.018	6.47	.19
24.4	0.347	.009	4.54	.12
30.9	0.258	.0059	3.31	.075
37.2	0.199	.0067	2.59	.051
49.9	0.139	.0051	1.76	.041
10.4	1.07	.040	16.2	.76
21.4	0.462	.048	5.8	.12
32.4	0.253	.0083	3.39	.052

Numerical results have been obtained with graphical accuracy for contact windows of spacing to length ratios of one or two. The number of boundary elements per window used in the simulations was about two hundred. Three-dimensional curves of current density over the interface plane show large current densities at the vertices of the contact windows: a potential source of reliability problems.

Experimental results obtained with this new method show the need to revise downwards the accepted values of ρ_c for Al-p-Si contacts [1], a fact which has also been reported by others recently [4,46].

Chapter 6

CURRENT ATTENUATION METHOD

In the approaches described so far, it has not proven possible to determine resistance by means of closed-form expressions. It is clearly desirable to do so, but not possible generally owing either to complicated geometry, or mixed conditions at the boundaries.¹ This chapter describes one sample structure in which analysis can be completed in closed-form, but suffers from the practical limitation that the substrate must be thinned substantially to measure ρ_c accurately.

6.1 Analysis of the Method

Figure 6.1 shows the structure to be modelled. This structure continues out to "infinity" in the radial direction and is governed by

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (6.1)$$

¹Leading at some point to a matrix formulation in all attacks made on the problem.

in the semiconductor or the metal. Carrying out a separation of variables

$$\phi = R(r)Z(z) \quad (6.2a)$$

where

$$r^2 R'' + rR' - \mu^2 r^2 R = 0 \quad (6.2b)$$

and

$$Z'' + \mu^2 Z = 0 \quad (6.2c)$$

These forms are chosen to yield smooth functions of radius. Since the potential must decay to zero as $r \rightarrow \infty$ and letting $\phi = 0$ at the back contact

$$\phi = AK_0(\mu r) \sin \mu z \quad (6.3a)$$

in the semiconductor and

$$\phi = BK_0(\mu r) \cos \mu(z - T_m) \quad (6.3b)$$

in the metal, where K_0 is a modified Bessel function of second kind and zeroth order. There are three unknowns μ , A and B , to be furnished by the three conditions that: (1) the total current injected at $r = 0$ is I , (2) the normal current density is continuous at the interface and (3) the normal current density is governed by

$$\frac{\partial \phi_B}{\partial n} = \rho_B \frac{\phi_m - \phi_B}{\rho_c} \quad (6.4)$$

Thus, applying the boundary conditions

$$A(\mu \cos \mu T + \frac{\rho_B}{\rho_c} \sin \mu T) = B \frac{\rho_B}{\rho_c} \cos \mu T_m \quad (6.5a)$$

and

$$A\mu \cos \mu T = \frac{\rho_B}{\rho_m} B\mu \sin \mu T_m \quad (6.5b)$$

yielding

$$\tan \mu T_m = \frac{\rho_m / \rho_c}{\mu + \rho_B / \rho_c} \tan \mu T \quad (6.5c)$$

as a condition for selecting μ .

Finally,

$$I = \frac{1}{\rho_B} \int_0^\infty AK_0(\mu r) \mu \pi r dr \quad (6.6a)$$

so that

$$A = \rho_B \frac{\mu I}{\pi} \quad (6.6b)$$

Thus the potential is given by

$$\phi = \rho_B \frac{\mu I}{\pi} K_0(\mu r) \sin \mu z \quad (6.7)$$

in the bulk and

$$\phi = \rho_m \frac{\mu I}{\pi} \frac{\cos \mu T}{\sin \mu T_m} K_0(\mu r) \cos \mu(z - T_m) \quad (6.8)$$

in the metal.

There is no intrinsic difficulty in introducing a finite contact resistivity at the back contact, although practically a "zero" resistivity contact could be made there by doping the back of the substrate heavily prior to making contact. Finally, if the sample were actually a small rectangular prism sectioned from a wafer, finite instead of infinite in extent, the approach above could be applied to an infinite rectangular array of image current sources (probes) without undue difficulty (double sum).

6.2 Computed Results

Equation (6.5c) is the condition which determines the practicality of the method.² Note that in this equation, it must be true that

$$0 \leq \mu \leq \frac{\pi}{2T} \quad (6.9)$$

Table 6.1 gives some particular results. Clearly the substrate must be thinned to a few micrometers in order for the method to discriminate among small values of ρ_c sufficiently. This gives another way of looking at ρ_c measurement methods; the relative importance of interface and bulk effects can be evaluated by considering the form

$$\rho_c + \rho_B \ell \quad (6.10)$$

²i.e., what substrate thickness should be chosen in order to make ρ_c measurable.

Table 6.1: Attenuation Coefficient (μ) for Different Sample Structures and Material Properties.

ρ_B	ρ_c	μ
.0001	4×10^{-9}	44.98
	4×10^{-8}	44.96
	4×10^{-7}	44.73
100	0.004	.08160
	0.04	.08111
	0.4	.07670

(a) $T = 300\mu m$, $T_m = 1\mu m$, $\rho_m = 2 \times 10^{-6}\Omega - cm$

T_m	ρ_B	ρ_c	μ
10^{-4}	.0001	4×10^{-8}	44.96
10^{-5}		4×10^{-8}	51.50
10^{-5}		0	51.50
10^{-4}	100	0.04	0.08111
10^{-5}		0.04	0.2565
10^{-5}		0	0.2582

(b) $T = 300\mu m$, $\rho_m = 2 \times 10^{-6}\Omega - cm$

ρ_B	ρ_c	μ
.0001	4×10^{-8}	275.7
	0	296.6
100	0.04	.2887
	0	.3162

(c) $T = 20\mu m$, $T_m = 1\mu m$, $\rho_m = 2 \times 10^{-6}\Omega - cm$

ρ_B	ρ_c	μ
.0001	4×10^{-8}	632.0
	0	1405
100	0.04	0.6325
	0	1.4142

(d) $T = 1\mu m$, $T_m = 1\mu m$, $\rho_m = 2 \times 10^{-6}\Omega - cm$

where ℓ is an "effective length" or typical current path length. When $\ell \sim 300\mu m$ in the computed results, μ varies only by about one percent with ρ_c , but when $\ell \sim 1\mu m$, μ varies by nearly 100% (for $\rho_c/\rho_B = 4 \times 10^{-4}$ cm). This is confirmed by the criterion above. In the two dots and two stripes methods with $L = 20\mu m$, interface resistance is about 1/3 or 1/2 of the total resistance. In view of all this, the criterion (6.10) above is sensible.

6.3 Conclusions

Clearly the method outlined in this chapter would not furnish a practical approach to ρ_c measurement since substrates are not easy to thin to the required levels. However, the fact that it is possible to analyze the case exactly is interesting and adds to insight about the general problem.

Chapter 7

CONCLUSIONS

The aim of this thesis was to develop a more accurate technique for contact resistivity measurement.

Chapter 2 presented a motivation for this study. There it was stated that the Helmholtz equation probably was the best of available models for test structures on thin layers of semiconductor. On the other hand, the Helmholtz equation neglects thickness effects. These would seem to be of practical significance considering both the small values of ρ_c often reported and the experimental conditions for which they are reported. A variety of calculation techniques [54] suggested that the role of "substrate" thickness in the Berger structure was to introduce an additive parasitic bulk term to the contact resistivity of the order of

$$R_s TL \quad (7.1)$$

Similar considerations for the Kelvin structure gave

$$R_s T^2 \quad (7.2)$$

as a result in that case. Therefore the range $10^{-7} - 10^{-6} \Omega \text{cm}^2$ is the dividing line between more or less meaningful and more or less meaningless results as measured by these two methods. One of the main advantages of the Berger and Kelvin structures is that they lend themselves to needs such as monitoring contact quality when used in test-inserts in integrated circuit manufacture.

7.1 Need for a More Accurate Technique

In situations other than monitoring contact quality, a more accurate measurement technique would be of direct value, since:

1. The role of ρ_c in future devices will be greater than it is now because the effect scales above other parasitic resistance effects. An accurate method would be of use in making confident projections of this exact role.
2. One can use it in studying and comparing different metals and alloys and deposition techniques to select those suitable for practical IC manufacture on the basis of the value of ρ_c . Such studies would benefit from freedom from anomalous effects associated purely with measurements carried out on thin diffused or implanted layers (Chapter 2).
3. One could also study the nature of contact reliability under controlled conditions, in which a great deal is now known about the quantitative distribution of current across the contact window. This would probably clarify understanding considerably given the important role played by current density in reliability issues [51].
4. One can also use accurately measured ρ_c values in order to perfect quantitative understanding of the current flow mechanisms operating in ohmic contacts.

5. If the chosen method is found to perform satisfactorily, study of the statistics of feature size (contact window) variation from device to device may yield knowledge of practical benefit in improving fabrication control, and
6. Anomalous behaviour observed when using the Berger and Kelvin structures could be studied and clarified under controlled conditions of measurement, in which the effect of incomplete analysis can be distinguished from real physical effects (such as pitting).

7.2 Contributions of This Thesis

The general objective of this thesis in developing new measurement techniques was to *choose device structures to optimize measurement conditions* rather than choosing them to make valid some approximate analysis. The latter characterizes the contributions to this area of all other workers.

Three new methods of measurement have been proposed here and explored in some depth.

The *parallel stripes technique* has been analyzed in two dimensions by two methods: conformal mapping and the boundary element method. Using this as a model of the electrical behaviour of a cross-section of the device, a quasi-three-dimensional solution incorporating the effects of the metal was then established using transmission-line methods. *This was the first approach to measuring ρ_c known in which bulk, contact, metal and geometry effects could all be subjected to scrutiny* [40-41]. The method was found to behave satisfactorily in an experimental test. Several possible experimental implementations have been analyzed and verified experimentally: namely, *six and eight terminal methods using whole wafers, and a four terminal method using diced samples*. Despite the obvious drawback that ρ_c is buried in significant metal resistance effects, this method retains the advantage of

a thorough analysis.

The existence and nature of *bounds on the values of R or ρ_c* was established. Although this had been done much earlier by Lewis [42] in a different context, the writer only became aware of the fact after developing an independent approach to the problem. The proof was extended here to several cases, all of practical interest to ρ_c measurement. In addition, *a technique for measuring ρ_c based on the asymptotic behaviour of the resistance between two lossy contacts on a slab of semiconductor has been proposed*. It has not been experimentally tested since it was felt that the new method was amply verified by experimental tests of the two other techniques developed in this thesis.

Finally, the *two dots approach* of Chapter 5 was analyzed by Green's functions in three dimensions in the semiconductor, and in two dimensions by finite differences in the metal. This is therefore felt to represent a *valid three-dimensional solution of a ρ_c measurement technique, the first of its kind ever reported*. Convincing experimental confirmation of the method has been obtained and reported here. The *demonstrated contribution is considered to be two-fold: a valid measurement method has been developed and a technique of analysis for a class of useful device geometries shown to work*.

As well, it is possible that resistance and geometry data can be used to produce dimensionless $R-\alpha$ curves directly, doing away with the need for numerical analysis, although the evidence for this is not as yet convincing (see Section 4.9.1).

7.3 Recommendations

The parallel stripes and the two dots could be considered as two examples of a class of structures applicable and useful to accurate ρ_c measurement. A technique of numerical analysis yielding three-dimensional electrical models of these structures

has been demonstrated here. With some careful thought given to the presentation of results and the aid of a more powerful computer these structures could be accurately analyzed in a way that would eliminate the need for any further study of the analysis problem (see Section 5.6). It might be that standard layouts for these structures can be defined and fixed on a meaningful basis, eliminating one difficulty for those most interested in using the methods in experiment.

An important issue to be resolved is identification of the source of scatter in experimental results. It has proven impossible in this thesis to confirm that this owes to variation in contact window size from device to device as Berger proposed [1] or to ρ_B/ρ_c variation from one device to another. It may be that it owes to a combination of these effects or possibly partly to a residual nonlinearity in the contact IV characteristics. Without a clear attribution of errors to causes, it is of course impossible to say what confidence attaches to measured ρ_c values.

There is scope for studying other issues as well; a number of these have been touched on in various places in the thesis (see Section 7.2, for example), and do not need repeating here.

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APPENDIX 1

A PROCEDURE FOR SMOOTH INTERPOLATION

To the problem of interpolation between discrete points, one answer is provided by the use of splines. Suppose there are m points. Between each pair of points a unique n th order polynomial is to be fitted by matching the polynomials of adjoining regions as well as their first $n - 1$ derivatives. This problem therefore has $(m - 1)(n + 1)$ unknowns furnished by $n(m - 2) + m$ conditions, leaving $(n - 1)$ constraints to be arbitrarily established. The unknowns are to be determined by solving a matrix. The necessary arbitrariness of this procedure suggests we seek a simpler route avoiding the need for solving a matrix [57].

The lowest order interpolant we can consider using is a cubic; quadratics do not exhibit points of inflection and tend to "scallop" as a result. Suppose we wish to interpolate with a cubic between the leftmost two points. If we supply an estimated slope at the leftmost point then there are three conditions to be furnished to determine the unknowns. Using the leftmost three points (i.e., "smelling" trouble ahead) as the data in question, the four pieces of information give us an interpolating curve between the leftmost two points. Calculating the slope at the rightmost of these two gives a starting point for interpolating between the next pair of points, (x_2, y_2) and (x_3, y_3) . To express this mathematically, we wish to fit a cubic between (x_1, y_1) and (x_2, y_2) .

$$y = ax^3 + bx^2 + cx + d \quad (A1.1)$$

If we know or estimate the slope at (x_1, y_1) , s_1 ,

$$s_1 = 3ax_1^2 + 2bx_1 + c \quad (A1.2)$$

Fitting the curve to (x_1, y_1) , (x_2, y_2) and (x_3, y_3)

$$y_1 = ax_1^3 + bx_1^2 + cx_1 + d$$

$$y_2 = ax_2^3 + bx_2^2 + cx_2 + d \quad (A1.3)$$

$$y_3 = ax_3^3 + bx_3^2 + cx_3 + d$$

this gives us our interpolating curve,

$$a = \left(\frac{y_2 - y_1 - s_1(x_2 - x_1)}{(x_2 - x_1)^2} - \frac{y_3 - y_1 - s_1(x_3 - x_1)}{(x_3 - x_1)^2} \right) \frac{1}{x_2 - x_3} \quad (A1.4)$$

$$b = \frac{y_2 - y_1 - s_1(x_2 - x_1)}{(x_2 - x_1)^2} - a(2x_1 + x_2) \quad (A1.5)$$

$$c = s_1 - 3ax_1^2 - 2bx_1 \quad (A1.6)$$

and

$$d = y_1 - ax_1^3 - bx_1^2 - cx_1 \quad (A1.7)$$

To determine a curve between (x_2, y_2) and (x_3, y_3) , we calculate the slope at (x_2, y_2)

$$s_1 = 3ax_2^2 + 2bx_2 + c \quad (A1.8)$$

and then proceed in an analogous manner. Since the first derivative is continuous, the curve is smooth to the eye and has quite an acceptable appearance.

Although it has not been tried, one could eliminate the need for a preliminary slope estimation by fitting the first cubic curve to the leftmost four points (only using it to interpolate between the leftmost two points).

Alternatively, the first fitted curve could be a quadratic fitted to the leftmost three points.

APPENDIX 2

SCATTER IN CONTACT RESISTANCE DATA

There are several components to the scatter in contact resistance data, but the most important are bulk resistivity (and therefore ρ_c) and geometry irregularities within a device, and variations in ρ_B , ρ_c , and geometry from one device to another. A simple model is developed here and applied to the explanation of measured results for the two dots structure.

Bulk resistivity and contact resistivity are interdependent and vary from one point to another on the wafer. A simple assumption will be made that this variation follows a random walk model. If we start at any point on the wafer and move in a straight line to another point, profiling the resistivity as we move in infinitesimal steps Δx , it is proposed that ρ_B changes by $\pm \Delta \rho$ at each step.¹ After N such steps, the variance in the resistivity will be

$$\sigma_N^2 = N \Delta \rho^2 \quad (A2.1)$$

Supposing we move a distance m times still larger, then the variance is

¹Therefore in the limit as $\Delta x \rightarrow 0$, ρ_B and ρ_c will be described by continuous functions of distance along any line traced on the wafer, as seems physically necessary.

$$\sigma_{Nm}^2 = Nm\Delta\rho^2 = m\sigma_N^2 \quad (A2.2)$$

Thus if the device size is D , the wafer diameter W , the variance in resistivity across a wafer is σ_W^2 and the variance in resistivity across a device σ_D^2 ,

$$\sigma_D \sim \sqrt{\frac{D}{W}} \sigma_W \quad (A2.3)$$

No special model seems appropriate or will be invoked for the geometry irregularities.

Since the contact resistance is approximately

$$R \sim K \frac{\rho_B}{L} + \frac{2\rho_c}{L^2} \quad (A2.4)$$

with K constant, then with the scatter in each factor denoted by δ

$$\delta R \sim K \frac{\delta\rho_B}{L} + K \frac{\rho_B \delta L}{L^2} + \frac{2\delta\rho_c}{L^2} + 4 \frac{\rho_c \delta L}{L^3} \quad (A2.5)$$

Since ρ_c and ρ_B are related for aluminum contacts by

$$\rho_c = \lambda\rho_B \quad (A2.6)$$

therefore

$$\delta R = \left(K + \frac{2\lambda}{L}\right) \frac{\delta\rho_B}{L} + \left(K + \frac{4\lambda}{L}\right) \rho_B \frac{\delta L}{L^2} \quad (A2.7)$$

The scatter in L , δL , was measured to be $\pm 1.64 \mu m$. The scatter in ρ_B is by (A2.3)

$$\sqrt{\frac{0.03}{2 \times 2.54}} \times \pm 0.03 = \pm 0.00231 \Omega cm$$

The remaining parameters are $K = 0.754$, $\lambda = 2 \times 10^{-4} cm$, $L = 19.7 \mu m$ and $\rho_B = 0.08 \Omega cm$. Therefore

$$\delta R = \pm 1.122 \pm 3.922 \Omega$$

The resistance may therefore be estimated as

$$39 \pm 5 \Omega$$

The measured result was

$$45 \pm 8 \Omega$$

so that a reasonable prediction has been obtained by means of a theoretical model.

The advantage of smaller, scaled-down devices of enhancing the contact resistance, is offset by the considerable error which would be incurred. At $4 \mu m$ this amounts to about

$$360 \pm 230 \Omega$$

so that there must be an optimum device size, or at least, moderate size is preferable to either extreme.

APPENDIX 3

COMPUTER LIMITATIONS IN SIMULATING A MEASUREMENT STRUCTURE

On the Carleton Honeywell CP-6 computer system, it was found that a 100×100 matrix representation of the integral equation governing the two stripes or dots could be set up and solved in about 10 minutes. For the two dots this represents an accuracy of roughly $1/7$ or 15%. Roughly speaking, if a is accuracy and t is time

$$t = \frac{K}{a^6} \quad (\text{A3.1})$$

Improvements in accuracy are possible by using a different machine. On a CRAY super-computer, capable of solving for 1000-1500 unknowns in one minute (ignoring the substantial time needed to set up the matrix), an accuracy of 2% would be possible in a comparable run time. However, increasing the run time on a given machine would never in a practical case do more than to marginally improve the accuracy.

Consequently, without a means of representing the electrical behaviour of general device structures using very small matrices (e.g., using some interpolation scheme), they are, except in simple cases having infinite domains and some degree of geometric symmetry, impossible to solve with reasonable accuracy on the

university computer system. Only the most crude representation of the Berger or Kelvin structures would be possible with 100 surface elements, leading to answers good to $\pm 100\%$, if that.

Although considerable progress could be made towards modelling general device structures by using a CRAY super-computer (apparently accessible), it was felt that the search for a better solution method was of greater interest.

APPENDIX 4

SOME IDEAS ON THE INTEGRAL EQUATIONS ARISING IN THIS THESIS

(a) KERNEL SIMPLIFICATION

It is possible to find lower and upper bounds for the potential by direct manipulation of the kernel in the integral equation. It is not known whether these bounding solutions have a practical or merely a theoretical interest.

If two contacts at potentials $\pm V$ are on an infinitely thick substrate, the potential is given by

$$\phi(\mathbf{r}) = \frac{\rho_B}{2\pi\rho_c} \int_{S_1} \frac{V - \phi(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dS' - \frac{\rho_B}{2\pi\rho_c} \int_{S_2} \frac{V + \phi(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dS' \quad (\text{A4.1})$$

Considering only the first term, it is bounded, since

$$\begin{aligned} \frac{1}{r+r'} &< \frac{1}{|\mathbf{r}-\mathbf{r}'|} < \frac{1}{r-r'} & r > r' \\ \frac{1}{r+r'} &< \frac{1}{|\mathbf{r}-\mathbf{r}'|} < \frac{1}{r'-r} & r' > r \end{aligned} \quad (\text{A4.2})$$

As a result, the potential is bounded.

(b) DISCRETE TRANSFORM

The integral equation can be subjected to a discrete Fourier transform

$$q_b = \sum p_a e^{\frac{i2\pi ab}{N}} \quad (A4.3a)$$

$$p_a = \frac{1}{N} \sum q_b e^{-\frac{i2\pi ab}{N}} \quad (A4.3b)$$

which turns the convolution integrals (the Green's function operating on a potential problem always produces convolution-like integrals) into products. Hence, at first glance, it would appear easy to solve all such integral equation problems. Unfortunately, it is not clear that the difficulties encountered in trying to implement this approach can be resolved. Considering the integral equation for the two stripe problem, one of these difficulties is that $\ln(x + x')$ must be represented over a period $2L$ and not L , which is the period required to represent the potential in the contact window. This difficulty owes to a fundamental difference between integral and discrete sum convolutions.

(c) MEAN-VALUE THEOREM

By means of the mean-value theorem, the integral equation for the two stripe problem may be written

$$\phi(x) = \frac{\rho_B L}{\pi \rho_c} (V - \phi(\epsilon)) \ln \left| \frac{x + \epsilon}{x - \epsilon} \right| \quad (A4.4)$$

for some ϵ . It is not known whether this can be turned to practical use even though it would appear to be a promising direction in which to search for a solution.

APPENDIX 5

RESISTANCE BETWEEN A PAIR OF CONTACTS: FURTHER DIFFERENTIAL RELATIONS

(a) VARIATION WITH ρ_B

As before

$$P = - \int_S \phi j dS + \rho_c \int_S j^2 dS \quad (A5.1)$$

The change in P in response to a change in ρ_B , $\delta\rho_B$, is

$$\delta P = - \int_S \delta\phi j dS - \int_S \phi \delta j dS + \rho_c \int_S 2j \delta j dS \quad (A5.2)$$

$$= - \int_S \delta\phi j dS + \int_S \phi \delta j dS \quad (A5.3)$$

Similarly to results in Chapter 4, $\delta(\nabla\phi) = \nabla(\delta\phi)$,

$$\nabla \cdot (\phi \nabla \delta\phi) = \nabla \cdot (\delta\phi \nabla \phi) \quad (A5.4)$$

and letting

$$f = \frac{\partial \phi}{\partial n} \quad (A5.5)$$

$$\int_S \phi \delta f dS = \int_S \delta \phi f dS \quad (A5.6)$$

Since

$$f = -\rho_B j \quad (A5.7)$$

(A5.6) becomes

$$\rho_B \int_S \phi \delta j dS + \delta \rho_B \int_S \phi j dS = \rho_B \int_S \delta \phi j dS \quad (A5.8)$$

Substituting in (A5.3)

$$\delta P = -\frac{\delta \rho_B}{\rho_B} \int_S \phi j dS \quad (A5.9)$$

According to the definition for bulk resistance adopted in the thesis

$$\frac{\partial R}{\partial \rho_B} = \frac{R_B}{\rho_B} \quad (A5.10)$$

(b) VARIATION WITH L

We will imagine that the structure is uniformly scaled up in size by a small amount. If we further imagine superimposing this new structure on the original one, the original and final contacts will have an area in common, S , and there will be an area found only in the original contact, A , and an area found only in the final contact, B . Denote

$$S' = S \cup A \cup B \quad (\text{A5.11})$$

$$S_i = S \cup A \quad (\text{A5.12})$$

and

$$S_f = S \cup B \quad (\text{A5.13})$$

We can define $\delta\phi$ and δj meaningfully throughout the structure. The change in power dissipation is

$$\delta P = - \int_{S'} \phi \delta j dS - \int_{S'} \delta \phi j dS + \rho_c \int_{S'} 2j \delta j dS \quad (\text{A5.14})$$

and as before

$$\nabla \cdot (j \delta \phi) = \nabla \cdot (\phi \delta j) \quad (\text{A5.15})$$

so that

$$\int_{S'} \delta \phi j dS = \int_{S'} \phi \delta j dS \quad (\text{A5.16})$$

Consequently

$$\delta P = - \int_{S'} 2\phi \delta j dS + \rho_c \int_{S'} 2j \delta j dS \quad (\text{A5.17})$$

$$= - \int_{S_i} 2\phi \delta j dS + \rho_c \int_{S_i} 2j \delta j dS - \int_B 2\phi \delta j dS + \rho_c \int_B 2j \delta j dS \quad (A5.18)$$

Since initially the current is zero on B

$$\delta P = - \int_{S_i} 2V \delta j dS - \int_B 2\phi \delta j dS \quad (A5.19)$$

$$= - \int_{S'} 2V \delta j dS + \int_B 2V \delta j dS - \int_B 2\phi \delta j dS \quad (A5.20)$$

The change, δj , in the current on B is

$$\delta j = - \frac{V - \phi}{\rho_c} \quad (A5.21)$$

Consequently,

$$\delta P = - \int_B 2 \frac{(V - \phi)^2}{\rho_c} dS \quad (A5.22)$$

$$= -2\rho_c \int_B j^2 dS \quad (A5.23)$$

Similar reasoning gives

$$\delta P = -2\rho_c \int_A j^2 dS \quad (A5.24)$$

or

$$\delta P = -\rho_c \int_{A \cup B} j^2 dS \quad (A5.25)$$

This formula may appear strange, but it should be borne in mind that $A \cup B$ corresponds to a differential area around the perimeter of the contact. If p is perimeter

$$\frac{\partial R}{\partial L} = -\frac{c\rho_c}{I^2} \int_{A \cup B} j^2 dp \quad (A5.26)$$

The constant c is introduced because L is an arbitrarily, though conveniently, chosen feature size in the problem.

APPENDIX 6

THE $\rho_c - \rho_B$ RELATIONSHIP ACCORDING TO A THERMIONIC-EMISSION MODEL

The following calculations are meant only to give an illustration of how a linear $\rho_c - \rho_B$ relationship might arise in practice.

The well known relation between current and voltage in a rectifying Schottky barrier may be written in the form

$$J = qp v_{th} P (e^{qV/kT} - 1) \quad (A6.1)$$

q = electronic charge,

p = hole density,

$v_{th} = \sqrt{\frac{kT}{2\pi m_h^*}}$ = expected carrier velocity given motion only to the left or right of a plane,

P = probability of a carrier being emitted from the semiconductor over the barrier into the metal,

k = Boltzmann's constant, and

T = temperature.

Taking

$$P = 1 \quad (A6.2)$$

$$J_{\max} = qp v_{th} (e^{qV/kT} - 1) \quad (A6.3)$$

which is equal around zero bias to

$$J = \frac{q^2 p v_{th}}{kT} V \quad (A6.4)$$

Thus

$$\rho_c = \frac{kT}{q^2 p v_{th}} \quad (A6.5)$$

Since

$$\rho_B = \frac{1}{q \mu_h p} \quad (A6.6)$$

μ_h = hole mobility,

therefore

$$\rho_c \approx \left(\frac{kT \mu_h}{q v_{th}} \right) \rho_B \quad (A6.7)$$

This estimate is wrong by several orders of magnitude, but it could serve as a basis for a proper model. Note that real contacts do not exhibit the marked

rectification anticipated by (A6.3), but this may be due to field emission at the contact edges occurring even at low voltages in real contacts.

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APPENDIX 7

SOME RESULTS FOUND USEFUL IN THE TREATMENT OF DATA

(a) CALCULATION OF CONTACT WINDOW LENGTHS USING RESISTANCE DATA

Suppose we approximate the $R - L$ relationship by

$$R = \frac{a}{L} + \frac{b}{L^2} \quad (A7.1)$$

After fitting $R - L$ data to this equation we may take a and b as known. It may then be assumed that small deviations of the data from the "theoretical" curve could be due to inaccurate length measurements. On this assumption, L may be estimated from

$$L = \frac{a + \sqrt{a^2 + 4bR}}{2R} \quad (A7.2)$$

(b) DEVIATION OF CONTACT WINDOWS FROM THEIR IDEAL SHAPE

Both square and circular contact windows were used in this study. The actual window shapes realized in practice were found to be accurately described as a square

with rounded corners. If the diagonal of this rounded square is D and its "length" is a , the area is

$$a^2 - \frac{4 - \pi}{(\sqrt{2} - 1)^2} \left(\frac{a}{\sqrt{2}} - \frac{D}{2} \right)^2 \quad (A7.3)$$

In measurements made with a travelling microscope, low a values and high D values were believed to be better estimators of the true values of a and D than the population means. This is because alignment of the wafer on the moving microscope stage can not be performed perfectly and this can be expected to introduce a bias in the measurements (especially of D).

An error of $\pm 0.3 \mu m$ was estimated as applicable to measured contact window sizes in the range from 5 to 50 μm .

It should be noted that SEM measurements carried out in this study indicated that contact window sizes do not vary significantly from one device to another. The scatter was found to be of the order of a few hundred nanometers while optical measurements indicated a pessimistic value of $\approx 1\text{-}2 \mu m$. This result contradicts the assumptions underlying previous treatments of resistance scatter in the literature [1,44].

(c) FITTING A STRAIGHT LINE TO DATA

Equation (A7.1) amounts to a straight line if R and L are known,

$$RL = a + \frac{b}{L} \quad (A7.4)$$

or

$$y = mx + b \quad (\text{A7.5})$$

where

$$y = RL \quad (\text{A7.6})$$

and

$$x = \frac{1}{L} \quad (\text{A7.7})$$

Almost everyone is familiar with the method of least-squares for fitting a straight line to data. In this method, equation (A7.5) is fitted to the observations (x, y) by choosing m and b to minimize

$$\Sigma (y - (mx + b))^2 \quad (\text{A7.8})$$

This leads to

$$m = \frac{N \Sigma xy - \Sigma x \Sigma y}{N \Sigma x^2 - (\Sigma x)^2} \quad (\text{A7.9})$$

$$b = \frac{\Sigma y - m \Sigma x}{N} \quad (\text{A7.10})$$

Two other methods have occurred to the writer. The first is to minimize the "relative error"

$$\Sigma \left(\frac{y_p - y_m}{y_m} \right)^2 \quad (A7.11)$$

y_p = predicted y ,

y_m = measured y .

Then

$$m = \frac{\Sigma x/y \Sigma 1/y^2 - \Sigma 1/y \Sigma x/y^2}{\Sigma x^2/y^2 \Sigma 1/y^2 - (\Sigma x/y^2)^2} \quad (A7.12)$$

$$b = \frac{\Sigma 1/y - m \Sigma x/y^2}{\Sigma 1/y^2} \quad (A7.13)$$

This approach has an advantage if the y -data is characterized by a relative rather than an absolute error, which often seems to be the case. A measure of the goodness of fit is

$$\sqrt{\frac{1}{N} \Sigma \left(\frac{y_p - y_m}{y_p} \right)^2} \quad (A7.14)$$

which is characteristic of the relative error with which the data deviates from the predicted straight line.

The second approach is called here parameter-ordering. By ordering the parameters:

$$m = \frac{y - b}{x} \quad (A7.15)$$

and averaging over the data

$$m = \frac{1}{N} (\Sigma y/x - b \Sigma 1/x) \quad (\text{A7.16})$$

$$b = \frac{1}{N} (\Sigma y - m \Sigma x) \quad (\text{A7.17})$$

We can solve simultaneously for m and b

$$m = \frac{\frac{1}{N} \Sigma y/x - \frac{1}{N^2} \Sigma y \Sigma 1/x}{1 - \frac{1}{N^2} \Sigma x \Sigma 1/x} \quad (\text{A7.18})$$

The idea behind this is that the estimate of m using (A7.15) has the same merit regardless of the point at which the estimate is made. Hence the mean of all such estimates should be a good predictor of the true value of m , m^* . The mean is a good choice for m^* because

$$\Sigma (m - m^*)^2 \quad (\text{A7.19})$$

where m is an observation and m^* is the estimate of the true value, is a minimum if m^* is chosen to be the mean.

Finally, it should be noted that treatment of (A7.4) by any of these methods does not necessarily give a good fit of (A7.1) to data. To illustrate the distinction between a (A7.1) and (A7.4), suppose

$$R \approx \frac{a}{L} \quad (\text{A7.20})$$

is to be fitted to data. Re-writing this as $RL = a$ we can estimate by least squares that

$$a = \frac{\Sigma RL}{N} \quad (A7.21)$$

but using (A7.20) directly

$$a = \frac{\Sigma R/L}{\Sigma 1/L^2} \quad (A7.22)$$

In fact, however, direct treatment of (A7.1) is no more difficult than that of (A7.4) and (A7.5).

In cases other than a straight line and especially for equations in more than one parameter, it is expected that numerical methods will usually be necessary to implement any of these approaches.

END

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FIN