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Two Essays on the Real Exchange Rate

by

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Abstract

This thesis consists of two essays that use time-series techniques and recent OECD data to examine the long-run behavior of the real exchange rate within the framework of Balassa-Samuelson model. These essays can be summarized as follows.

Balassa-Samuelson model predicts that rich countries tend to have higher price levels than poor countries. The first essay tests this hypothesis with both cross-section and time series data and confirms the puzzle that although this hypothesis performs well in explaining cross-sectional data, it is relatively poor in tracking time series data (except for the case when Japan is the reference country). To explore this puzzle, this essay uses a simple model which determines both the real exchange rate and the income per capita differential in terms of traded and nontraded productivity differentials under the key assumption that Purchasing Power Parity (PPP) holds for traded goods. This model suggests that the persistence and variation of these productivity differential series are very important in explaining the long run relationship between the real exchange rate and the income per capita differential. This view is supported by test results from both simulation and real data.

The second essay relaxes the assumption that PPP holds for traded goods and extends the Balassa-Samuelson model to allow specialization in traded goods. In this variant, the relative traded productivity can affect the real exchange rate through two channels, the Balassa-Samuelson channel which is positive and the terms of trade

channel which is negative. We conclude that the introduction of specialization in the Balassa-Samuelson model (which leads to the breakdown of PPP for traded goods) can explain weak and sometime reversed relationship between the real exchange rate and the relative productivity differential.

Dedication

To my beautiful wife, Jun Zhou and lovely son, Benjamin Xie.

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Chapter 1

Long-Run Relationship between the Real Exchange Rate and Income Level: Some Evidence from Time Series

1. Introduction

The Balassa-Samuelson Hypothesis was developed originally to explain the empirical phenomenon that when all countries' price levels are expressed in dollars at prevailing nominal exchange rates, rich countries tend to have higher price levels than poor countries. It argues that the relationship between the real exchange rate and the real income per capita arises essentially because rich countries are relatively more productive in the traded goods sector. Most of the evidence in favor of this relationship is based on cross-sectional data, e.g., the well-known Penn World Table for real per capita incomes and price level.¹ The pioneering work by Kravis and Lipsey (1982) has demonstrated that a positive correlation between the price level and the real per capita GDP is robust across numerous cross-sectional specifications (87% of the variation in national price levels of 21 countries in 1975 is explained by per capita GDP).²

The time series data has also been used to explore the effect of income per capita on the real exchange rate, but this evidence has produced less favorable results.

¹ For further evidence, see the survey for emerging economies in Edwards and Savastano (1999).

² See, however, Glick and Taylor (2004), who find that the cross-sectional price-income correlation was not really very strong until the last three or four decades and this relation is essentially a post World War II phenomenon.

For example, Obstfeld and Rogoff (1996) pointed out that time series data generally does not support this relationship except for the case when Japan is the reference country.³ Time series results can differ from cross-sectional results because factors affecting within-country variation (or variation over time) can be quite different from factors affecting the difference across countries. The price-income relation may also perform poorly over time because of cyclical factors. To address this problem, econometric techniques can be used to estimate a long-run relation between the real exchange rate and the income per capita but even the long-run time series relation is not robust. Moreover, it is still a puzzle why, as discussed below, the results for such a long-run relationship vary dramatically among different reference countries.

In this paper, we provide updated results on both cross-section and time series data for 13 OECD countries⁴ with reference to three countries, the US, Japan and the UK. The results from cross-section data confirm a stable relationship between the real exchange rate and the income differential. The existence and the direction of the long-run relationship for time series data are tested using procedures based on Dynamic Ordinary Least Squares (DOLS) as well as the bounds test proposed by Pesaran, Shin and Smith (2001). According to our tests, the results are mixed and weaker for the US and UK than for Japan, suggesting the puzzle that time series results are sensitive to the choice of the reference country.

³ Isard and Symansky (1996) and Faria and Leon-Ledesma (2003) also present some mixed results.

⁴ These 13 countries are Austria, Belgium, Canada, Denmark, Finland, Italy, Japan, Netherlands, Norway, Portugal, Sweden, UK and the US. Germany is dropped from our sample because the OECD industry data for Germany is not comparable before and after the unification. Including emerging economies would make a better “rich” versus “poor” country distinction, but due to data availability, our analysis is restricted to these industrialized countries where the income differences are relatively small.

According to the Balassa-Samuelson model, real exchange rates are mainly determined by the differential between foreign and home relative productivity of traded goods to nontraded goods while income per capita moves in tandem with both traded and nontraded goods productivities. Therefore, it is natural to think that the relationship between the real exchange rate and the income per capita would depend on the productivity behavior of traded and nontraded goods. This relationship may break down if nontraded goods productivity differs strongly among countries. It is puzzling, however, why such a breakdown occurs over time but not across countries.

Long-run time series relationships depend on the persistence and variability of the productivity series and these characteristics can differ from one country to another. The role of these differences in determining the time series relationship has not been adequately explored in the literature. Our empirical analysis of the productivity differential series for these 13 OECD countries with the US, Japan and the UK as the reference countries shows that the persistence and variability of these series vary significantly across countries. We find that the persistence of the traded goods productivity differential is much larger than that of the nontraded goods productivity differential for these countries when Japan is the reference country compared to cases when the US and UK are the reference countries. A key issue examined in this essay is whether these differences can explain why the long-run relationship between the real exchange rate and the income per capita gets mixed results across different pairs of countries.

To explain this puzzle, we first develop a simple model which determines both the real exchange rate and the income per capita differential as functions of traded and nontraded goods productivity differentials between foreign and home countries. In this model, both the real exchange rate and the income per capita differential are positively related to the traded goods productivity differential. However, the nontraded goods productivity differential is negatively related to the real exchange rate but positively related to the income per capita differential. This model suggests that when the traded goods productivity differential is more persistent and variable than the nontraded goods productivity differential, the real exchange rate and the income per capita differential would tend to move together closely and are likely to be positively related in the long run. In contrast, a more persistent and variable nontraded goods productivity differential between foreign and home country would affect the real exchange rate and the income differential in the opposite directions and reverse the long-run relationship.

We first use simulation techniques to explore these effects. We allow for different degrees of persistence and variability for productivity differentials and generate simulated data to examine how these differences affect the long-run relationship between the real exchange rate and the income per capita differential.⁵ The simulation results point to the fact that a more persistent and variable traded productivity differential series is likely to lead to a positive long-run relationship between the real exchange rate and the income differential.

⁵ The differences in persistence and variability we consider for simulations are calibrated to match the productivity differentials observed in the data.

The model suggests that a significant relationship (either positive or negative) between the real exchange rate and the income differential would occur when either traded or nontraded goods productivity differential is very persistent relative to the other. To test this implication with real data, we classify bilateral cases (with respect to three reference countries) into different groups according to persistence in traded goods productivity differentials relative to nontraded goods productivity differentials. Results from the real data present a clear tendency that groups with a higher relative persistence in traded goods productivity differential are more likely to present a positive long-run relationship between the real exchange rate and the income differential, which is in conformity with the simulation results.

Our study shows that the levels of persistence and variability of productivity differential series provide a good indication for the existence of a long-run relationship between the real exchange rate and the income level under the Balassa-Samuelson framework. In the case where an increase in income is mostly a result of advance in traded goods productivity and there is a higher level of persistence and variability in traded goods productivity differentials than in nontraded goods productivity differentials, a strong long-run positive relation exists between the real exchange rate and the per capita income differential. While the persistence and variability of such productivity series are different across countries, they can also change over time, and so will the long-run relationship between the real exchange rate and the income per capita differential for a certain pair of countries.

Our paper's findings offer some insight into why the empirical results based on time series data are less robust than those for cross-section data and are sensitive to the choice of reference countries.

2. Comparing Evidence from Cross-Section and Time Series Data

In this section we reexamine some empirical evidence for the relationship between the real exchange rate and the income level using both cross-section and time series data. We estimate this relationship for 13 OECD countries, for which sectoral productivity series are readily available from 1977 to 2003.

The bilateral CPI-based real exchange rate for foreign country can be expressed as $Q_t = \frac{S_t P_t^*}{P_t}$, where S_t is the nominal exchange rate expressed in units foreign currency per unit domestic currency, and P_t and P_t^* are the CPI's for home and a reference foreign country (a "*" is used to denote variables for the reference country). An increase in Q_t represents a real depreciation of home currency and a real appreciation of the reference currency. The log real exchange rate equals

$$q_t = s_t + p_t^* - p_t, \text{ where } s_t = \log(S_t) \text{ etc.} \quad (1)$$

The real income per capita is simply real GDP divided by population size. We denote per capita income differential between the reference and home countries by m_t .

To estimate the cross-sectional relationship, we use the absolute Purchasing Power Parity values from Penn World Table to convert time series for relative prices

to those for absolute prices. These absolute price indices are then used to calculate the real exchange rates and the real GDP per capita. We choose the US, Japan and the UK as reference countries, and use average values of real exchange rates and income differentials over the 27-year period from 1977 to 2003. Table 1 presents the results for a simple regression, $q_i = \alpha + \beta m_i$, where q_i and m_i stand for the average real exchange rate and the average real per capita income differential for country i relative to a reference country.

Table 1 Regression results for cross-sectional relationship ($q_i = \alpha + \beta m_i$)

	Reference Country		
	US	Japan	UK
β	1.0986	0.8357	0.8207
t	*4.7260	*3.6279	*3.1393
R^2	0.6907	0.7464	0.8682

Note: The 5% critical value for simple t test is 2.06. “*” indicates significance at 5% level.

For all three reference countries, the regression results show a positive slope coefficient on income differential, a significant t statistic and a high R^2 . We also illustrate this relationship with respect to the US, Japan and the UK in Figures 1 to 3. These results give strong support to the view that real exchange rates and income differentials are positively related for cross-sectional data regardless of the choice of reference countries.

Figure 1 Cross-section association between average real exchange rate and average per capita income relative to the US

Figure 2 Cross-section association between average real exchange rate and average per capita income relative to Japan

Figure 3 Cross-section association between average real exchange rate and average per capita income relative to the UK

For time series data, we use two econometric procedures to test for the existence of a long-run relationship between the real exchange rate and the (real) income differential. The first procedure (DOLS test) is based on a Dynamic Ordinary Least Square regression that includes lags and leads of the first differences as well as levels of explanatory variables.⁶ The second procedure (bounds test) involves tests proposed by Pesaran, Shin and Smith (2001) based on estimation of an Error Correction Model (ECM).

For DOLS test, we estimate an equation of the following form,

$$q_t = \alpha + \beta_1 m_t + \beta_2 \Delta m_{t-2} + \beta_3 \Delta m_{t-1} + \beta_4 \Delta m_t + \beta_5 \Delta m_{t+1} + \beta_6 \Delta m_{t+2} + \hat{u}_t \quad (2)$$

with 2 lags and leads of first differences, where q_t is the real exchange rate and m_t is the income differential.⁷ The first difference terms are added to control for possible

⁶ See Hamilton (1994) for a detailed discussion of this test.

⁷ We also tried 1 lag and lead and 3 lags and leads, but found little difference in the results.

correlation between the error term and the explanatory variable. When first difference terms are included in this procedure, the coefficient of the level term represents the long-run effect of the income differential on the real exchange rate. The hypothesis $\beta_1 = 0$ indicates that there is no long run relationship between q_t and m_t . The DOLS methodology is appropriate when both the real exchange rate and the income per capita differential series have a unit root and are cointegrated with each other. In this case, the t statistic on β_1 follows a standard t distribution.⁸ Therefore, we perform unit root and cointegration tests for each real exchange rate and income differential time series before the DOLS estimation.

As in the cross-section tests, we use the US, Japan and the UK as reference countries. Since time series analysis doesn't require absolute comparison among countries, we use relative time series to calculate real exchange rates and income differentials, which are directly obtained from OECD's Economic Outlook. Since the annual time series for real exchange rates and real income differential are short, we apply the Elliot, Rothenberg and Stock (ERS) unit root test because of its relatively high power. The test results are reported in Tables 2 and 3 while the cointegration test⁹ results are listed in Table 4.

⁸ If both traded and nontraded productivity differential series are stationary, β_1 still represents the long-run effect of m_t on q_t . However, the coefficient may be biased in this case because of possible correlation between the explanatory variables and the error term.

⁹ We apply Augmented Engle-Granger (AEG) test for cointegration test in this paper.

Table 2 Unit root test results for real exchange rates

Country	Reference Country					
	US		Japan		UK	
	without trend	with trend	without trend	with trend	without trend	with trend
Austria	*-2.6513	-2.6694	*-2.1114	-3.1025	*-2.8640	-2.9638
Belgium	*-2.9811	-2.9433	-1.5815	-2.2065	*-2.5585	-3.1521
Canada	*-2.1150	-2.6948	-1.5072	*-3.4450	-1.4706	*-4.0287
Denmark	*-2.9537	-2.9456	-1.9050	-2.5504	*-2.8742	-3.1132
Finland	*-2.6152	-2.8310	-1.6024	*-3.9946	-1.7512	-2.7371
Italy	*-2.5457	-2.5410	*-1.9084	*-4.1172	-1.5682	-1.7029
Japan	-1.5702	-2.3895			-1.8711	-2.9009
Netherlands	*-2.8573	-2.8714	-1.6578	-2.3157	*-2.6241	*-3.4108
Norway	*-2.6797	-2.7411	-1.6597	-3.0628	*-2.2322	*-3.4655
Portugal	*-1.9799	-2.3715	*-2.2724	-2.6490	-1.5521	-2.0546
Sweden	*-2.5922	-2.7887	-1.4839	*-4.4265	-0.9476	-1.8336
UK	*-3.2195	*-4.0359	*-2.4323	-2.9009		
USA			-1.5702	-2.3895	*-3.2195	*-4.0359

Notes: Schwartz information criterion is applied to determine the number of lags. The 5% critical value for Elliot, Rothenberg and Stock (ERS) unit root test is -1.9550 without trend and -3.1900 with trend. The symbol “*” denotes rejection of the null hypothesis that there is a unit root at 5% significance level.

Table 3 Unit root test results for real income differentials

Country	Reference Country					
	US		Japan		UK	
	without trend	With trend	without trend	with trend	without trend	With trend
Austria	*-2.4130	*-3.7341	-1.2473	-2.1809	-1.6627	*-4.5250
Belgium	*-3.4785	*-3.3798	-1.3368	-1.0958	*-2.4270	*-3.5494
Canada	-1.4675	-2.6673	-1.0800	-1.8608	*-1.9874	-2.8590
Denmark	-1.6699	-2.4809	-1.3090	-1.8670	-0.6183	-2.7237
Finland	*-2.6111	-2.7946	*-2.3966	-2.5285	*-2.7473	*-3.2898
Italy	-1.7910	*-3.7068	-0.8396	-2.6632	-1.3108	*-4.1359
Japan	-1.8346	-2.2726			-1.5110	-1.7957
Netherlands	-1.5053	-1.7239	-1.2239	-1.7199	-0.7706	-2.7455
Norway	-0.8269	-2.1401	*-2.1240	-2.3047	-1.4376	-2.1964
Portugal	*-2.1649	*-3.5205	*-1.9842	-2.3199	*-3.3437	*-4.6560
Sweden	-1.2885	-1.6572	-1.3022	-1.8104	-0.4352	-1.7785
UK	-1.1831	-2.8818	-1.5110	-1.7957		
US			-1.8346	-2.2726	-0.6209	-1.5393

See notes for Table 2.

Table 4 Results for the cointegration and DOLS tests

Country	Reference Country					
	US		Japan		UK	
	<i>t</i> Stat for Cointegration	<i>t</i> Stat for DOLS	<i>t</i> Stat for Cointegration	<i>t</i> Stat for DOLS	<i>t</i> Stat for Cointegration	<i>t</i> Stat for DOLS
Austria		*2.21	*-4.2	*3.54	-3.36	0.43
Belgium		*7.78	-3.38	*9.56	*-3.74	*5.39
Canada	-2.83	*1.78	-3.18	*13.02	-2.02	-0.35
Denmark	-3.13	-10.13	*-3.55	*4.87	*-3.69	*8.01
Finland	-2.68	-3.37	*-4.49	*15.58	-2.33	*14.93
Italy	-2.64	*8.36	*-3.57	*3.83	-2.47	*10.01
Japan	-2.97	*18.72			-2.76	*4.79
Netherlands	-3.43	*6.93	-3.45	*5.12		*4.13
Norway	-2.89	-10.83	-1.7	-6.81		-14.78
Portugal		*25.71	-3.17	-0.27	*-3.67	*12.29
Sweden	-2.89	*2.18	*-4.65	*7.63	-2.88	*17.7
UK		*4.51	-2.76	*4.79		
USA			-2.97	*18.72		*4.51

Notes: If unit root hypothesis is rejected for either real exchange rate or income differential series (for both the cases without and with a trend) for a pair of countries, we skip the cointegration test for them as they cannot be cointegrated. Bayesian information criterion (BIC) is applied to determine the lag length and the 5% critical value for the Augmented Engle-Granger Cointegration Test is -3.50. Statistic for DOLS test is the t statistic on β_1 in equation (2). The 5% one-sided critical value for the DOLS t test is 1.72.

We see that in many cases, the unit root hypothesis is rejected¹⁰ or the cointegration tests fail to reject the null that there is no cointegration. Note that both cointegration and DOLS tests require the underlying time series to have a unit root. Nevertheless, we still present estimates of DOLS regressions, recognizing that there might be some simultaneous equation bias as the two variables could be jointly

¹⁰ The US real exchange rates seem to be stationary when there is no trend but when a trend is included, we are no longer able to reject the null that there is a unit root. The remaining results from the unit root test for the real exchange rates and income differentials seem to be mixed and not sensitive to whether there is a trend.

independent. In estimating the DOLS regression, we calculate the adjusted t statistics on β_1 , the coefficient of the current level of income per capita in (2), using a procedure suggested by Hamilton (1994) to account for autocorrelation of the error term. The t statistic on β_1 are also presented in Table 4. Significant statistics for both the cointegration and DOLS tests indicate more conclusive evidence in support of the long-run relationship between the real exchange rate and the income differential.

The t statistics for cointegration and DOLS tests suggest that the results are mixed and weaker for the US and UK than for Japan (cointegration is not indicated in many cases, especially for the US). Hence, the DOLS-based evidence in support for a long-run relationship between the real exchange rate and the income level suggested by Balassa-Samuelson Hypothesis is weak and sensitive to the choice of reference countries.

We next use the bounds test to examine the long-run relationship between the real exchange rate and the income differential. The advantage of the bounds test is that it does not require pre-testing the series for the order of integration. Since the unit root test results for real exchange rates and per capita income differentials in table 2 and 3 are mixed, the bounds test provides an appealing procedure to test for the existence of a long-run relation between these variables. An important assumption for the bounds test is that the long-run causality is from the independent variable to the

dependent variable.¹¹ To explore the sensitivity of the relation to the direction of causality, we run the bounds test both with income differential as an independent variable and real exchange rate as an independent variable.

Following Pesaran et al (2001), we estimate the following conditional error correction model (ECM),

$$\Delta q_t = \alpha_0 + \alpha_1 t + \varphi q_{t-1} + \psi m_{t-1} + \sum_{j=1}^{p-1} \beta_j \Delta q_{t-j} + \sum_{j=1}^{q-1} \gamma_j \Delta m_{t-j} + \omega \Delta m_t + u_t, \quad (3)$$

where m_t is the independent variable and q_t is the dependent variable. We consider values from 1 to 4 for p and q and report results for those specifications that minimize the Akaike information criterion (AIC) and Schwartz Bayesian information criterion (BIC) values. The equation is estimated by ordinary least squares, and the absence of a long-run relationship between q_t and m_t is tested by calculating the F statistic for the null hypothesis that $\varphi = \psi = 0$. According to Pesaran et al (2001), when both $\varphi = 0$ and $\psi = 0$, q_t follows a unit root and there are no level effects in the conditional ECM with no possibility of any level relationship between q_t and m_t . The distribution of the test statistic under the null depends on the order of integration of q_t and m_t , and on whether there is an intercept and a time trend in the relation. They provide bounds for asymptotic critical F values at different levels of significance for all these cases.¹² The lower bound is the critical value when all variables are $I(0)$ while the upper bound is the critical

¹¹ According to Pesaran, Shin and Smith (2001), however, this restriction does not preclude the dependent variable being Granger-causal for the independent variable in the short run. Therefore, a Granger causality test may not be appropriate in this case.

¹² See Pesaran et al (2001, Table CI.i, CI.ii, CI.iii and CI.v)

value when all variables are $I(1)$. When the variables concerned are of different order of integration (0 or 1), the critical value will fall between the bounds.

If the null hypothesis is rejected according to the F test, there is a stable long-run relationship between the real exchange rate and the income differential, which is described by

$$q_t = \theta_0 + \theta_1 t + \theta_2 m_t + v_t, \quad (4)$$

where $\theta_0 = -\alpha_0/\varphi$, $\theta_1 = -\alpha_1/\varphi$, $\theta_2 = -\psi/\varphi$ and v_t is the error term.¹³ We can then apply standard t test on the coefficient of the long-run forcing variable (θ_2) to verify whether a stable positive long-run relationship exists between q_t and m_t . A significant and positive t statistic indicates the presence of positive relationship between them.

Although equations (3) and (4) contain a time trend, we also conduct the tests without a trend and report results separately for the bounds test as we are not sure whether the real Data Generating Process (DGP) contains a trend or not. The results presented in Tables 5 to 7 are based on AIC criterion for lag length with the income differential as the independent variable.¹⁴ We also performed bounds test with the real exchange rate as the independent variable and found that this variation does not make much difference to the results.

¹³ See, for example, Coe and Apostolos (2002).

¹⁴ We also used the BIC criterion and found that results were very similar.

Table 5 Bounds test results: US as the reference country

Country	Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t
Austria	[2,1]	3.74	-0.74	[2,1]	3.60	-0.57
Belgium	[2,1]	4.50	-0.27	[2,1]	4.25	-0.22
Canada	[2,1]	2.92	*1.72	[1,1]	5.27	0.74
Denmark	[2,1]	4.33	-1.51	[3,2]	*8.11	-2.96
Finland	[2,3]	2.72	0.96	[2,3]	2.36	0.83
Italy	[1,3]	0.52	0.36	[1,3]	0.02	0.05
Japan	[2,3]	3.70	*2.46	[2,1]	1.90	1.38
Netherlands	[2,1]	5.51	-0.72	[2,1]	5.43	-0.74
Norway	[2,1]	2.66	-0.99	[2,2]	3.79	-2.19
Portugal	[3,3]	1.65	*2.00	[3,3]	1.58	1.43
Sweden	[2,3]	2.86	1.12	[2,3]	2.23	0.54
UK	[3,3]	*8.50	*2.83	[3,3]	*11.40	*3.48

Notes: the lag orders [p,q] of the underlying ECM were selected using the Akaike information criterion (AIC). We

list the F statistic for the null hypothesis that $\phi = \psi = 0$ and t statistic for the null hypothesis that $\theta_2 = 0$.

The symbol “*” denotes significance at 5% level. The critical bounds of the F statistic at 5% for bivariate case are [4.94, 5.73] without trend (Pesaran et al (2001, Table CI.iii)), and [6.56, 7.30] with trend (Pesaran et al (2001, Table CI.v)). One-sided significance level is also applied to the t test.

Table 6 Bounds test results: Japan as the reference country

Country	Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t
Austria	[1,3]	*7.92	*1.89	[1,3]	4.25	1.11
Belgium	[1,3]	*6.44	*2.11	[1,3]	3.00	1.07
Canada	[2,1]	*6.37	*2.82	[3,1]	*8.47	*1.81
Denmark	[2,3]	*8.62	*2.81	[2,3]	*7.29	1.62
Finland	[1,1]	1.76	1.53	[3,1]	*9.59	*1.96
Italy	[3,3]	*25.19	*6.57	[3,3]	*24.40	*2.90
Netherlands	[1,3]	*7.33	*2.12	[1,3]	4.92	*1.86
Norway	[1,2]	1.79	0.44	[3,3]	*9.70	*1.74
Portugal	[3,3]	*7.83	*3.74	[3,3]	*9.40	*4.47
Sweden	[1,1]	2.23	1.24	[3,1]	*7.76	*1.93
UK	[2,2]	*9.55	*2.90	[2,2]	*8.10	*2.73
US	[2,3]	3.70	*2.46	[2,1]	1.90	1.38

See notes for Table 5.

Table 7 Bounds test results: UK as the reference country

Country	Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t
Austria	[2,1]	2.71	-0.15	[2,1]	2.54	0.06
Belgium	[2,1]	2.91	0.49	[2,1]	2.75	0.25
Canada	[3,3]	*13.12	*6.85	[3,3]	*13.17	*2.77
Denmark	[2,1]	4.97	-0.92	[2,2]	*8.10	*2.73
Finland	[3,3]	3.40	*1.77	[2,3]	4.44	1.17
Italy	[1,1]	1.24	0.78	[1,1]	0.55	-0.05
Japan	[2,2]	*9.55	*2.90	[2,2]	5.05	-1.52
Netherlands	[2,1]	4.59	-0.50	[2,1]	4.58	-0.83
Norway	[2,2]	2.72	-1.58	[2,1]	2.73	-0.33
Portugal	[3,3]	*5.95	*3.92	[3,3]	5.87	*2.50
Sweden	[1,3]	4.84	*2.13	[1,3]	4.25	0.51
US	[3,3]	*8.50	*2.83	[3,3]	*11.40	*3.48

See notes for Table 5.

The bounds test results confirm our key results from the DOLS estimation that the long-run relationship is generally weak with time series than cross-section data. Among the three reference countries, the results are weaker for the US (only 1 out of 12 countries shows a significant and positive relationship either with or without a trend) and the UK (5 out of 12 countries significant) compared to those for Japan (11 out of 12 countries significant). The results also reveal that the presence of a long-run relationship between the real exchange rate and the income differential is very sensitive to the choice of the reference country.

In the next section, we develop a model to explore why the time series results differ across different reference countries. In particular this model shows how the results of this section can be explained by differences in the underlying productivity series of the countries being studied.

3. A Simple Model

This section explores how the characteristics of DGP for productivity series can influence the relation between the real exchange rate and the per capita income differential. For this purpose we set up a simple model which shows how the real exchange rate and the per capita income differential are related to the productivity series. We assume all goods are homogeneous and produced under perfect competition so that PPP holds for traded goods. We also assume a multi-country world, diversified production, i.e., all goods are produced in each country under

perfect competition and labor mobility between traded and nontraded sectors within each country, but not between countries.

We use Cobb-Douglas production function for both traded and nontraded sectors and assume for simplicity that labor shares are the same in traded sector and in nontraded sector,

$$Y_{T,t} = A_{T,t} K_{T,t}^{\alpha} L_{T,t}^{\beta}, \quad (5)$$

$$Y_{N,t} = A_{N,t} K_{N,t}^{\alpha} L_{N,t}^{\beta}, \quad (6)$$

where $\alpha + \beta = 1$ and so that constant return to scale holds. These production functions imply that marginal productivity of labor is proportional to average productivity of labor in both sectors,

$$\frac{\partial Y_{T,t}}{\partial L_{T,t}} = \beta \frac{Y_{T,t}}{L_{T,t}}, \quad (7)$$

$$\frac{\partial Y_{N,t}}{\partial L_{N,t}} = \beta \frac{Y_{N,t}}{L_{N,t}}. \quad (8)$$

We follow Canzoneri, Cumby and Diba (1999) and use average products of labor as a measure of productivity in both traded and nontraded sectors.¹⁵ This is helpful in our empirical analysis later as reliable data on sectoral capital stocks (needed to estimate total factor productivity) is difficult to obtain.

Under perfect competition and labour mobility, labour are paid the value of its marginal product and nominal wage rate, W_t , is equalized between the traded and nontraded sectors, i.e.,

¹⁵ One limitation of this analysis by using labor productivity instead of total factor productivity is that labor productivity may be subject to both supply and demand shocks. See Canzoneri, Cumby and Diba (1999) and De Gregorio and Wolf (1994) for further discussion about this issue.

$$W_t = \frac{\partial Y_{T,t}}{\partial L_{T,t}} P_{T,t} = \frac{\partial Y_{N,t}}{\partial L_{N,t}} P_{N,t}, \quad (9)$$

where $P_{T,t}$ and $P_{N,t}$ are price indices for traded and nontraded goods. Therefore, the relative price of nontraded goods to traded goods is equal to the productivity in traded sector relative to nontraded sector.

$$\frac{P_{N,t}}{P_{T,t}} = \frac{Y_{T,t} / L_{T,t}}{Y_{N,t} / L_{N,t}}. \quad (10)$$

Using lower case letters to denote values in logs, we can express (10) as

$$p_{N,t} - p_{T,t} = (y_{T,t} - l_{T,t}) - (y_{N,t} - l_{N,t}). \quad (11)$$

It is generally assumed that overall price index is a geometric weighted average of traded-goods and nontraded-goods prices. Under this assumption,

$$p_t = (1 - \gamma) p_{T,t} + \gamma p_{N,t}, \quad (12)$$

$$p_t^* = (1 - \gamma) p_{T,t}^* + \gamma p_{N,t}^*, \quad (13)$$

where γ is the share of nontraded goods in the price index. Now the log of real exchange rate can be written as

$$q_t = s_t + p_{T,t}^* - p_{T,t} + \gamma^* (p_{N,t}^* - p_{T,t}^*) - \gamma (p_{N,t} - p_{T,t}). \quad (14)$$

As PPP holds for traded goods, i.e., $p_{T,t} = s_t + p_{T,t}^*$, then from (11) and (14) we

have

$$q_t = \gamma^* [(y_{T,t}^* - l_{T,t}^*) - (y_{N,t}^* - l_{N,t}^*)] - \gamma [(y_{T,t} - l_{T,t}) - (y_{N,t} - l_{N,t})] \quad (15)$$

If the shares of nontraded goods in the price index are the same for home and foreign, the real exchange rate becomes

$$q_t = \gamma (tad_t - nad_t) \quad (16)$$

where $tad_t = (y_{T,t}^* - l_{T,t}^*) - (y_{T,t} - l_{T,t})$ and $nad_t = (y_{N,t}^* - l_{N,t}^*) - (y_{N,t} - l_{N,t})$, which are traded and nontraded goods productivity differentials between foreign and home. Equation (16) represents the Balassa-Samuelson relation, according to which the real exchange rate depends on the difference between foreign-home productivity differentials for traded and nontraded goods (hereafter referred as tad_t and nad_t).

Define Y_t as aggregate output. National income is then the product of P_t and Y_t , which is equal to the sum of total value of output in traded and nontraded sectors,

$$P_t Y_t = P_{T,t} Y_{T,t} + P_{N,t} Y_{N,t}. \quad (17)$$

Divide both sides by $P_t L_t$ (where $L_t = L_{T,t} + L_{N,t}$ under full employment), and we obtain the income per capita as

$$\frac{Y_t}{L_t} = \frac{P_{T,t} Y_{T,t} L_{T,t}}{P_t L_{T,t} L_t} + \frac{P_{N,t} Y_{N,t} L_{N,t}}{P_t L_{N,t} L_t}. \quad (18)$$

Since $\frac{P_{T,t} Y_{T,t}}{L_{T,t}} = \frac{P_{N,t} Y_{N,t}}{L_{N,t}}$ and $\frac{L_{T,t}}{L_t} + \frac{L_{N,t}}{L_t} = 1$, the above equation can be simplified as

$$\frac{Y_t}{L_t} = \left(\frac{P_{T,t}}{P_{N,t}} \right)^\gamma \frac{Y_{T,t}}{L_{T,t}} = \left(\frac{Y_{T,t}}{L_{T,t}} \right)^{1-\gamma} \left(\frac{Y_{N,t}}{L_{N,t}} \right)^\gamma. \quad (19)$$

Taking logs, (19) becomes

$$\ln \frac{Y_t}{L_t} = (1-\gamma) \ln \frac{Y_{T,t}}{L_{T,t}} + \gamma \ln \frac{Y_{N,t}}{L_{N,t}}. \quad (20)$$

The above equation holds for the foreign country too, so the income per capita differential between foreign and home country can be rewritten as

$$\ln \frac{Y_t^*}{L_t^*} - \ln \frac{Y_t}{L_t} = (1-\gamma^*) \ln \frac{Y_{T,t}^*}{L_{T,t}^*} + \gamma^* \ln \frac{Y_{N,t}^*}{L_{N,t}^*} - (1-\gamma) \ln \frac{Y_{T,t}}{L_{T,t}} - \gamma \ln \frac{Y_{N,t}}{L_{N,t}} \quad (21)$$

or

$$m_t = (1-\gamma) tad_t + \gamma nad_t \text{ as } \gamma = \gamma^* \text{ and } m_t = \ln \frac{Y_t^*}{L_t^*} - \ln \frac{Y_t}{L_t}. \quad (22)$$

Equation (22) suggests that the per capita income differential between foreign and home countries is a weighted average of tad_t and nad_t . Therefore, an increase in either tad_t or nad_t will lead to a larger income differential between foreign and home.

Now that both the real exchange rate and the per capita income differential between the foreign and home countries are expressed as functions of tad_t and nad_t , we can examine how a change in tad_t or nad_t will affect the relationship between the real exchange rate and the income differential. If tad_t increases, both real exchange rate and income differential will increase, but if nad_t increases, real exchange rate will depreciate but income differential will still increase.

One would expect that the persistence and variation in the tad_t and nad_t processes will have a large impact on the relationship between the real exchange rate and the income differential. When tad_t is very persistent and has a higher variability compared to nad_t , both the real exchange rate and the income differential will be affected more by the fluctuations in tad_t and they will tend to move in the same direction. Therefore, a long-run relationship between them is more likely to be detected in this case. However, if nad_t is more persistent and has a higher variability, then the real exchange rate and the income differential will move in opposite direction and it is less likely for them to be positively related in the long run.¹⁶

¹⁶ Please refer to the Appendix for a discussion of cointegration between the real exchange rate and the income per capita differential.

We next assume that tad_t and nad_t are described by an $AR(1)$ process as follows:

$$tad_t = \rho^T tad_{t-1} + u_{T,t}, \quad \text{var}(u_{T,t}) = (\sigma^T)^2 \quad (23)$$

$$nad_t = \rho^N nad_{t-1} + u_{N,t}, \quad \text{var}(u_{N,t}) = (\sigma^N)^2. \quad (24)$$

Given this process and assuming certain values for parameters, ρ^T , ρ^N , σ^T and σ^N , we generate artificial data for q_t and m_t . We then apply both DOLS and bounds tests to these data to explore how different assumptions about parameter values affect the existence of a long-run relationship between the real exchange rate and the income differential. In particular, we investigate whether a higher persistence and variability in the tad_t relative to nad_t will lead to a higher probability of finding a long-run relationship.

Before undertaking simulations (in section 5), the next section examines how the DGP for productivity differential time series differ across the reference countries.

4. The Behavior of Productivity Differential Series

To obtain productivity series for the two sectors, we define traded sector to include agriculture and manufacturing and nontraded sector to cover all other industries, such as utility supplies, construction and total services. It is well recognized, however, that this classification of traded and nontraded sectors is crude because, for example, agriculture and manufacturing sectors contain nontraded component such as distribution while some services such as financial service are

actually traded. Sectoral productivities are defined as sectoral output divided by sectoral employment. We use OECD industry data¹⁷ to calculate the traded and nontraded goods productivity differentials relative to the US, Japan and the UK. We then use an $AR(1)$ process to estimate the persistence and variation of these productivity differential series and present the results in Table 8, 9 and 10.

Table 8 $AR(1)$ estimation of productivity differentials with respect to the US

Reference:	tad_t			nad_t			$\hat{\rho}^T - \hat{\rho}^N$	$\hat{\sigma}^T - \hat{\sigma}^N$
US	$\hat{\alpha}^T$	$\hat{\rho}^T$	$\hat{\sigma}^T$	$\hat{\alpha}^N$	$\hat{\rho}^N$	$\hat{\sigma}^N$		
Austria	-0.2723	0.5355	0.0625	-0.1423	0.9056	0.0261	-0.3701	0.0364
Belgium	-0.6035	0.4215	0.0565	-0.2102	0.8351	0.0259	-0.4136	0.0306
Canada	-0.2218	0.8984	0.0678	-0.4410	0.8200	0.0266	0.0784	0.0412
Denmark	-0.5921	0.5983	0.0633	-0.3448	0.8141	0.0375	-0.2158	0.0258
Finland	-0.6150	0.5085	0.0670	-0.3598	0.8220	0.0247	-0.3135	0.0422
Italy	-0.0405	0.9598	0.0731	-0.4454	0.7911	0.0228	0.1687	0.0502
Japan	0.0142	0.9895	0.0586	-0.2174	0.7861	0.0248	0.2033	0.0338
Netherlands	-0.1690	0.7236	0.0661	-0.0169	0.9767	0.0267	-0.2531	0.0393
Norway	-0.2381	0.8639	0.0680	-0.7398	0.6298	0.0273	0.2341	0.0407
Portugal	-0.2490	0.5055	0.0679	-0.2453	0.8339	0.0362	-0.3284	0.0317
Sweden	-0.5794	0.3722	0.0571	-0.2249	0.8316	0.0203	-0.4594	0.0368
UK	-0.5529	0.6519	0.0540	-0.2598	0.8399	0.0317	-0.1880	0.0223

Notes: We run the regression $tad_t = \alpha^T + \rho^T tad_{t-1}$ with $\hat{\sigma}^T$ as the standard deviation of the residuals from traded goods productivity differential autoregression and $nad_t = \alpha^N + \rho^N nad_{t-1}$ with $\hat{\sigma}^N$ as the standard deviation of the residuals from nontraded goods productivity differential autoregression.

¹⁷ Data source is from OECD Stan database, which includes industry output and employment for all OECD countries.

Table 9 *AR(1)* estimation for traded and nontraded goods productivity differentials with respect to Japan

Reference: Japan	tad_t			nad_t			$\hat{\rho}^T - \hat{\rho}^N$	$\hat{\sigma}^T - \hat{\sigma}^N$
	$\hat{\alpha}^T$	$\hat{\rho}^T$	$\hat{\sigma}^T$	$\hat{\alpha}^N$	$\hat{\rho}^N$	$\hat{\sigma}^N$		
Austria	-0.0144	0.9713	0.0311	-0.0949	0.7793	0.0303	0.1921	0.0008
Belgium	-0.0744	0.9083	0.0429	-0.0457	0.8137	0.0245	0.0946	0.0184
Canada	0.6727	0.6518	0.0419	-0.1733	0.8758	0.0240	-0.2240	0.0179
Denmark	-0.0761	0.9386	0.0434	-0.2909	0.6558	0.0419	0.2828	0.0015
Finland	-0.0503	0.9571	0.0496	-0.2616	0.7373	0.0243	0.2198	0.0253
Italy	-0.1038	0.8920	0.0387	-0.1343	0.8750	0.0255	0.0170	0.0132
Netherlands	-0.0341	0.8713	0.0358	0.0220	0.9404	0.0217	-0.0691	0.0141
Norway	-0.1548	0.8916	0.0416	-0.2221	0.7764	0.0340	0.1152	0.0076
Portugal	-0.0199	0.8826	0.0421	-0.0777	0.8250	0.0362	0.0575	0.0060
Sweden	-0.0182	0.9965	0.0386	-0.0833	0.7336	0.0270	0.2629	0.0116
UK	-0.1049	0.9214	0.0436	-0.1315	0.7764	0.0312	0.1450	0.0123
US	-0.0142	0.9895	0.0586	0.2174	0.7861	0.0248	0.2033	0.0338

Notes: see notes for Table 8.

Table 10 *AR(1)* estimation for traded and nontraded goods productivity differentials with respect to the UK

Reference: UK	tad_t			nad_t			$\hat{\rho}^T - \hat{\rho}^N$	$\hat{\sigma}^T - \hat{\sigma}^N$
	$\hat{\alpha}^T$	$\hat{\rho}^T$	$\hat{\sigma}^T$	$\hat{\alpha}^N$	$\hat{\rho}^N$	$\hat{\sigma}^N$		
Austria	0.1970	0.8072	0.0414	0.0761	0.5110	0.0215	0.2962	0.0200
Belgium	0.1178	0.7785	0.0307	0.0858	0.7434	0.0316	0.0351	-0.0008
Canada	-0.1054	0.8459	0.0367	-0.0350	0.9488	0.0237	-0.1029	0.0130
Denmark	0.0239	0.8057	0.0443	-0.1479	0.4365	0.0327	0.3693	0.0116
Finland	0.4529	0.5837	0.0515	0.0756	0.4968	0.0211	0.0869	0.0304
Italy	0.0362	0.8825	0.0492	-0.0227	0.9406	0.0238	-0.0581	0.0254
Japan	0.1049	0.9214	0.0436	0.1315	0.7764	0.0312	0.1450	0.0123
Netherlands	0.2453	0.7487	0.0369	0.0094	1.0064	0.0265	-0.2577	0.0104
Norway	-0.0272	0.8510	0.0308	-0.0585	0.8506	0.0335	0.0003	-0.0026
Portugal	0.0691	0.7960	0.0588	-0.1751	0.5717	0.0246	0.2243	0.0342
Sweden	0.1061	0.8411	0.0379	0.0600	0.7848	0.0292	0.0563	0.0087
US	0.5529	0.6519	0.0540	0.2598	0.8399	0.0317	-0.1880	0.0223

Notes: see notes for Table 8.

Table 8 shows that when the US is the reference country, tad_t tends to be less persistent than nad_t ($\hat{\rho}^T < \hat{\rho}^N$ for 8 out of 12 countries). In the case of Japan (Table 9), the results on the persistence of the two series are reversed, i.e., for most countries, the traded goods productivity differentials are more persistent than the nontraded goods productivity differentials (only 2 countries with $\hat{\rho}^T < \hat{\rho}^N$). When the UK is the reference country (Table 10), we have more cases of larger persistence in traded goods productivity differentials compared to the US but fewer such cases compared to Japan. Please recall that results in Table 4 do show the strongest

relationship between the real exchange rate and the income level when Japan is the reference country.

For all the three cases, the standard deviations of the estimated residuals from the traded goods productivity differential autoregression seem to be larger than those from the nontraded goods productivity differential autoregression (except two countries in the UK case), indicating a higher variability for traded goods productivity differential series.¹⁸ However, these differences in variations are relatively small.

The fact that some productivity series are persistent while others are not is not well recognized by the literature. As discussed earlier, the degree of the persistence in these productivity series could be very important in determining the long-run relationship between the real exchange rate and the income per capita differential. The large persistence difference between the traded and nontraded productivity differentials for different reference countries provides us a good opportunity to cross check how the real data fare in terms of the prediction of our model. In the following section, we use simulation to generate real exchange rates and income differentials from productivity differential series with different persistence and variation that are suggested by the real data.

5. Simulations

5.1 Model and Calibration

¹⁸ The higher volatility in the traded productivity differentials may be due to the fact that traded sector is exposed to more technical and global supply shocks.

In this section, we first generate productivity differential series for traded and nontraded goods based on different assumptions for values of ρ^T , ρ^N , σ^T and σ^N and then generate time series for the real exchange rate and the income differential according to our simple model. The $AR(1)$ autocorrelation coefficients for equations (23) and (24) in Table 8, 9 and 10 range between 0.5 to 1 and the residuals' standard deviations range between 0.01 and 0.05. First, we consider the cases when both ρ^T and ρ^N have a unit root or near unit root ($\rho^T = \rho^N = 1$ or $\rho^T = \rho^N = 0.9$) as many productivity differential series from real data suggest a high persistence where the autocorrelation coefficient is close to unity but not necessarily equal to one,

$$tad_t = tad_{t-1} + u_{T,t}, \quad \sigma^T = 0.05 \quad \text{or} \quad \sigma^T = 0.01 \quad (25)$$

$$tad_t = 0.9tad_{t-1} + u_{T,t}, \quad \sigma^T = 0.05 \quad \text{or} \quad \sigma^T = 0.01 \quad (26)$$

$$nad_t = nad_{t-1} + u_{N,t}, \quad \sigma^N = 0.05 \quad \text{or} \quad \sigma^N = 0.01 \quad (27)$$

$$nad_t = 0.9nad_{t-1} + u_{N,t}, \quad \sigma^N = 0.05 \quad \text{or} \quad \sigma^N = 0.01. \quad (28)$$

We then run simulations assuming both tad_t and nad_t are stationary ($\rho^T = \rho^N = 0.5$ and $\rho^T = \rho^N = 0.6$) to see how the long-run relationship between the real exchange rate and the income differential is affected.

$$tad_t = 0.5tad_{t-1} + u_{T,t}, \quad \sigma^T = 0.05 \quad \text{or} \quad \sigma^T = 0.01 \quad (29)$$

$$tad_t = 0.6tad_{t-1} + u_{T,t}, \quad \sigma^T = 0.05 \quad \text{or} \quad \sigma^T = 0.01 \quad (30)$$

$$nad_t = 0.5nad_{t-1} + u_{N,t}, \quad \sigma^N = 0.05 \quad \text{or} \quad \sigma^N = 0.01 \quad (31)$$

$$nad_t = 0.6nad_{t-1} + u_{N,t}, \quad \sigma^N = 0.05 \quad \text{or} \quad \sigma^N = 0.01. \quad (32)$$

Most importantly, we experiment the cases where one productivity differential has a unit root or near unit root and the other productivity differential is stationary to highlight the role of persistence differences. As we discussed in section 4, the important difference between traded goods and nontraded goods productivities lies in persistence. However, in order to explore the impact of different variations in tad_t and nad_t on the long-run relationship given certain persistence in the productivity differential time series, we generate a shock with standard deviations of either 0.05 or 0.01 for productivity differential series with regard to each set of values of ρ^T and ρ^N .

We estimate $\gamma(\gamma^*)$, the share of nontraded goods, by calculating the share of value added of nontraded goods in total value added of GDP with data for 13 OECD countries. The estimated shares of nontraded goods are very similar for these countries and we obtain an average about 0.6, therefore, we assume $\gamma = \gamma^* = 0.6$ in our simulations.¹⁹ We then calculate the real exchange rate and the income per capita differentials as follows,

$$q_t = 0.6tad_t - 0.6nad_t \quad (33)$$

$$m_t = 0.4tad_t + 0.6nad_t. \quad (34)$$

Given the DGP described above, a productivity improvement in the traded goods sector for foreign country relative to home country will obviously raise both the real exchange rate and the income differential, while a productivity improvement in

¹⁹ De Gregorio et al (1994)'s classification of traded and nontraded goods leads to a share of 50-60 percent of nontraded goods in GDP, which is consistent with our estimation.

the nontraded sector for foreign relative to home will decrease the real exchange rate but still increase the income differential.²⁰

Since the simulation could be sensitive to the unknown initial value, we generate 230 observations and only include the last 30 in our estimation to ensure the results valid for different initial values.²¹

5.2 DOLS Tests Based on Simulations

We run 10,000 regressions for each set of ρ^T , ρ^N , σ^T and σ^N , calculate the adjusted t statistic on the current level of income per capita in equation (2), and then obtain the percentage of significant and positive t statistics among the 10,000 simulations.

As in the estimation based on actual data in section 2, we use 2 lags and leads for the first difference of the independent variable indicated in equation (22) for the following simulations.

²⁰ Given the Data Generating Processes, we can easily find out whether a cointegration relation exists between the real exchange rate and the income differential and derive the cointegration vector explicitly. Please refer to the Appendix for a discussion which distinguishes cases where cointegration is imposed a priori and where it is not.

²¹ We also try sample size for 60 and 120 observations and obtain slightly better test results.

Table 11 Simulation results from DOLS without cointegration test

$\rho^T=0.9, \rho^N=0.9$			$\rho^T=1, \rho^N=1$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	90%	0.05	0.01	87%
0.05	0.05	30%	0.05	0.05	36%
0.01	0.05	1%	0.01	0.05	3%
0.01	0.01	29%	0.01	0.01	39%
$\rho^T=0.5, \rho^N=0.5$			$\rho^T=0.6, \rho^N=0.6$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	88%	0.05	0.01	91%
0.05	0.05	10%	0.05	0.05	15%
0.01	0.05	0%	0.01	0.05	0%
0.01	0.01	14%	0.01	0.01	15%
$\rho^T=0.9, \rho^N=0.5$			$\rho^T=1, \rho^N=0.6$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	99%	0.05	0.01	99%
0.05	0.05	67%	0.05	0.05	76%
0.01	0.05	55%	0.01	0.05	48%
0.01	0.01	68%	0.01	0.01	74%
$\rho^T=0.5, \rho^N=0.9$			$\rho^T=0.6, \rho^N=1$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	6%	0.05	0.01	7%
0.05	0.05	1%	0.05	0.05	2%
0.01	0.05	0%	0.01	0.05	0%
0.01	0.01	2%	0.01	0.01	3%

Notes: The one-sided 5% critical value for DOLS t statistic is 1.72.

Table 11 provides the percentage of regressions for which we detect significant positive long-run relationship between the real exchange rate and the income differential (we call this percentage the success rate) using DOLS test but without pre-testing for cointegration. The simulation results clearly show the importance of the relative level of persistence in the traded goods productivity differentials, i.e., given the variations of traded and nontraded goods productivity differentials, higher persistence in traded goods productivity differentials relative to nontraded goods productivity will lead to a higher success rate (percentage of a significant and positive t). For the cases of a more persistent traded goods productivity differential than the nontraded goods productivity differential, the success rates range from 48% to 99%. Note that there is not much difference between the cases when one productivity differential has a unit root and when it has a near unit root while the other productivity is stationary ($\rho^T=1$, $\rho^N=0.6$ and $\rho^T=0.9$, $\rho^N=0.5$, and $\rho^T=0.6$, $\rho^N=1$ and $\rho^T=0.5$, $\rho^N=0.9$). We also find that given the degree of persistence for traded and nontraded goods productivity differentials, the higher the variation of traded goods productivity differentials relative to that of nontraded goods productivity differentials, the higher the success rates. When the persistence is the same for traded and nontraded productivity differentials, the success rates become solely dependent on the variations of the two productivity differential series and they reach over 80% when the traded productivity differential is more variable ($\sigma^T=0.5$, $\sigma^N=0.1$) and drop to below 3% when the nontraded productivity differential is more variable ($\sigma^T=0.1$, $\sigma^N=0.5$). The success rate is the highest (99%) when both the

persistence and variation of the traded goods productivity differentials are higher than those of nontraded goods productivity differentials. These results are consistent with what our model predicts.

Since the productivity differential series are generated both with and without unit root, we also run DOLS with cointegration test first and count the success cases only when the real exchange rate and the income differential are cointegrated. The test results are presented in Table 12. The extra cointegration test leads to lower success rates because it excludes the cases when the real exchange rate and the income differential are not cointegrated but DOLS t statistic is still significant. It's worth mentioning that when the traded productivity differential has a unit root and the nontraded productivity differential is stationary ($\rho^T = 1$, $\rho^N = 0.6$), q_t and m_t are in fact cointegrated and positively related in the long run,²² but the cointegration test seems to fail to reject the null that there is no cointegration in a lot of cases and result in significantly lower success rates (ranging between 3% to 29% given different combinations of variations in tad_t and nad_t). This is probably due to the low power of the cointegration test (type II error). Unlike the results without pre-testing cointegration, the success rates for the cases with equal ρ^T and ρ^N are also very low, especially when both productivity differential series have a unit root or near unit root (below 15%).²³ However, the following results are in conformity with those in Table 11 in that higher persistence and variation in traded goods productivity

²² Please refer to the discussion in appendix.

²³ As discussed in appendix, when both productivity differential series have a unit root (or near unit root) or are stationary, the real exchange rate and the income differential should not be cointegrated.

differential relative to nontraded goods productivity differential tend to strengthen the positive long-run relationship between the real exchange rate and the income differential. We also experimented with one more and one less lead and lag and found the results are not sensitive to the number (between 1 and 3) of leads and lags included in the regression.

Table 12 Simulation results from DOLS with cointegration test

$\rho^T=0.9, \rho^N=0.9$			$\rho^T=1, \rho^N=1$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	15%	0.05	0.01	11%
0.05	0.05	5%	0.05	0.05	5%
0.01	0.05	0%	0.01	0.05	1%
0.01	0.01	5%	0.01	0.01	5%
$\rho^T=0.5, \rho^N=0.5$			$\rho^T=0.6, \rho^N=0.6$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	34%	0.05	0.01	27%
0.05	0.05	4%	0.05	0.05	5%
0.01	0.05	0%	0.01	0.05	0%
0.01	0.01	4%	0.01	0.01	6%
$\rho^T=0.9, \rho^N=0.5$			$\rho^T=1, \rho^N=0.6$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	37%	0.05	0.01	29%
0.05	0.05	15%	0.05	0.05	16%
0.01	0.05	1%	0.01	0.05	3%
0.01	0.01	17%	0.01	0.01	16%
$\rho^T=0.5, \rho^N=0.9$			$\rho^T=0.6, \rho^N=1$		
$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t	$\hat{\sigma}^T$	$\hat{\sigma}^N$	% of significant t
0.05	0.01	12%	0.05	0.01	8%
0.05	0.05	1%	0.05	0.05	1%
0.01	0.05	0%	0.01	0.05	0%
0.01	0.01	0%	0.01	0.01	1%

5.3 Bounds Tests Based on Simulations

We then use the bounds test to examine the long-run relationship between simulated real exchange rate and income per capita differential series.²⁴ Table 13 presents the simulation results from the bounds test when income differential is the independent variable and we let ρ^T and ρ^N equal 0.9 or 0.5. We also let ρ^T and ρ^N equal 1 or 0.5 and list the results in Table 14. Bounds test results have a very similar pattern as those from the DOLS estimation in Table 11 and 12, i.e. higher success rates are associated with higher ρ^T , higher σ^T or both. Again, we find that the difference between the case when the productivity differential series have a unit root and the case when they have a near unit root is small.

²⁴ Refer to equations (3) and (4). We also use the same procedures applied to real data to determine the lag length, but present the results using both AIC and BIC.

Table 13 Simulation results from the bounds test when income differential is the independent variable ($\rho^T, \rho^N = 0.9$ or 0.5)

$\rho^T = 0.9, \rho^N = 0.9$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	12%	15%	11%	14%
0.05	0.05	3%	3%	2%	2%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	4%	4%	3%	4%
$\rho^T = 0.5, \rho^N = 0.5$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	42%	35%	40%	34%
0.05	0.05	2%	3%	2%	3%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	3%	3%	3%	3%
$\rho^T = 0.9, \rho^N = 0.5$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	50%	45%	48%	41%
0.05	0.05	11%	7%	11%	6%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	13%	9%	12%	8%
$\rho^T = 0.5, \rho^N = 0.9$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	7%	11%	6%	10%
0.05	0.05	1%	1%	1%	1%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	0%	0%	0%	0%

Table 14 Simulation results from the bounds test when income differential is the independent variable ($\rho^T, \rho^N = 1$ or 0.6)

$\rho^T = 1, \rho^N = 1$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	12%	13%	10%	12%
0.05	0.05	3%	2%	3%	2%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	3%	3%	3%	2%
$\rho^T = 0.6, \rho^N = 0.6$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	33%	29%	30%	27%
0.05	0.05	2%	3%	2%	2%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	2%	3%	2%	3%
$\rho^T = 1, \rho^N = 0.6$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	40%	35%	36%	32%
0.05	0.05	12%	6%	12%	6%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	12%	6%	10%	6%
$\rho^T = 0.6, \rho^N = 1$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	3%	8%	3%	8%
0.05	0.05	0%	1%	0%	0%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	0%	1%	0%	1%

As we mentioned earlier, a very important assumption for the bounds test is that the causality is from the independent variable to the dependent variable. Since in our model, the productivity series are exogenous and the real exchange rate and the income per capita differential are both determined by the traded and nontraded goods productivities, the presumption of causality from independent variable to dependent variable needed for the bounds test may not be satisfied. We apply bounds tests also for the case when the real exchange rate is the independent variable and find that the results on success rates are almost identical.²⁵ Therefore, the results on the existence of the long-run relationship between the real exchange rate and the income differential obtained from the bounds test do not appear to be sensitive to the direction of causality.

²⁵ Please see Table 15 and 16 for these results.

Table 15 Simulation results from the bounds test when real exchange rate is the independent variable ($\rho^T, \rho^N = 0.9$ or 0.5)

$\rho^T = 0.9, \rho^N = 0.9$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	13%	18%	11%	15%
0.05	0.05	3%	3%	2%	2%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	2%	3%	2%	2%
$\rho^T = 0.5, \rho^N = 0.5$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	40%	38%	38%	36%
0.05	0.05	1%	1%	1%	1%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	2%	2%	2%	2%
$\rho^T = 0.9, \rho^N = 0.5$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	51%	43%	47%	40%
0.05	0.05	13%	9%	12%	8%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	14%	9%	13%	9%
$\rho^T = 0.5, \rho^N = 0.9$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	6%	9%	6%	8%
0.05	0.05	0%	0%	0%	0%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	0%	1%	0%	1%

Table 16 Simulation results from the bounds test when real exchange rate is the independent variable ($\rho^T, \rho^N = 1$ or 0.6)

$\rho^T = 1, \rho^N = 1$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	15%	21%	13%	18%
0.05	0.05	4%	2%	3%	2%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	3%	3%	3%	3%
$\rho^T = 0.6, \rho^N = 0.6$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	34%	31%	30%	28%
0.05	0.05	3%	3%	3%	3%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	3%	4%	3%	3%
$\rho^T = 1, \rho^N = 0.6$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	39%	33%	35%	30%
0.05	0.05	12%	7%	12%	7%
0.01	0.05	1%	0%	1%	0%
0.01	0.01	12%	7%	12%	7%
$\rho^T = 0.6, \rho^N = 1$					
$\hat{\sigma}^T$	$\hat{\sigma}^N$	AIC without trend	AIC with trend	BIC without trend	BIC with trend
0.05	0.01	4%	10%	4%	9%
0.05	0.05	0%	1%	0%	1%
0.01	0.05	0%	0%	0%	0%
0.01	0.01	0%	1%	0%	1%

6. Interpreting the Evidence from Real Data

Both our basic Balassa-Samuelson model and simulation results suggest that relative persistence and variation of the traded goods productivity differential series play an important role in determining the significance and the sign of the long-run relationship between the real exchange rate and the income levels. This section examines whether the results based on real data can be explained by the features of the productivity differential series.

Since the analysis of productivity differential series with different reference countries reveals that there is significant difference in terms of persistence but not in terms of variability, we focus on persistence differences in productivity differentials.

We consider an autocorrelation coefficient greater than 0.9²⁶ as high persistence and an autocorrelation coefficient smaller than 0.75 as low persistence. We then group all bilateral cases (with three reference countries) into four groups. The first group has high persistence in traded goods productivity differential and low persistence in nontraded goods productivity differential. The second group has the opposite features, i.e., low persistence in traded goods productivity differential and high persistence in nontraded goods productivity differential. For the remaining cases, the difference between the persistences of traded and nontraded goods productivity differentials are not as pronounced as the first two groups. Nevertheless, we divide

²⁶ As shown by the simulation results in section 5, the difference of success rates between a unit root and a near unit root (autocorrelation coefficient is equal to 0.9) is very small, so we don't distinguish them while testing with the real data.

these into two groups, group 3 with higher persistence in traded goods productivity differentials and group 4 with higher persistence in nontraded goods productivity differentials. The DOLS and bounds test results for these groups are presented in Table 17 through 20.

Table 17 DOLS and Bounds test results for group 1 ($\hat{\rho}^T \geq 0.9$ and $\hat{\rho}^N < 0.75$)

Country	Reference	t Stat for Cointegration	t Stat for DOLS	Bounds without trend		Bounds with trend	
				F	t	F	t
Denmark	Japan	*-3.55	*4.87	*8.62	*2.81	*7.29	1.62
Finland	Japan	*-4.49	*15.58	1.76	1.53	*9.59	*1.96
Sweden	Japan	*-4.65	*7.63	2.23	1.24	*7.76	*1.93

Table 18 DOLS and Bounds test results for group 2 ($\hat{\rho}^T < 0.75$ and $\hat{\rho}^N \geq 0.9$)

Country	Reference	t Stat for Cointegration	t Stat for DOLS	Bounds without trend		Bounds with trend	
				F	t	F	t
Austria	US	-2.52	2.21	3.74	-0.74	3.60	-0.57
Netherlands	US	-3.43	*6.93	5.51	-0.72	5.43	-0.74
Netherlands	UK		*4.13	4.59	-0.50	4.58	-0.83

Table 19 DOLS and Bounds test results for group 3

Country	Reference	t Stat for	t Stat for	Bounds without trend		Bounds with trend	
		Cointegration	DOLS	F	t	F	t
Canada	US	-2.83	*1.78	2.92	1.72	5.27	0.74
Italy	US	-2.64	*8.36	0.52	0.36	0.02	0.05
Japan	US	-2.97	*18.72	3.7	*2.46	1.9	1.38
Norway	US	-2.89	-10.83	2.66	-0.99	3.79	-2.19
Austria	Japan	*-4.2	*3.54	*7.92	*1.89	4.25	1.11
Belgium	Japan	-3.38	*9.56	*6.44	*2.11	3	1.07
Italy	Japan	*-3.57	*3.83	*25.19	*6.57	*24.40	*2.90
Norway	Japan	-1.7	-6.81	1.79	0.44	*9.70	*1.74
Portugal	Japan	-3.17	-0.27	*7.83	*3.74	*9.40	*4.47
UK	Japan	-2.76	*4.79	*9.55	*2.90	*8.10	*2.73
US	Japan	-2.97	*18.72	3.7	2.46	1.9	1.38
Austria	UK	-3.36	0.43	2.71	-0.15	2.54	0.06
Belgium	UK	*-3.74	*5.39	2.91	0.49	2.75	0.25
Denmark	UK	*-3.69	*8.01	4.97	-0.92	*8.10	*2.73
Finland	UK	-2.33	*14.93	3.4	*1.77	4.44	1.17
Japan	UK	-2.76	*4.79	*9.55	*2.90	5.05	-1.52
Norway	UK		-14.78	2.72	-1.58	2.73	-0.33
Portugal	UK	*-3.67	*12.29	*5.95	*3.92	5.87	*2.50
Sweden	UK	-2.88	*17.7	4.84	*2.13	4.25	0.51

Notes: see text for description of group 3.

Table 20 DOLS and Bounds test results for group 4 (lower persistence in traded goods productivity differential but the differences are inconclusive)

Country	Reference	t Stat for Cointegration	t Stat for DOLS	Bounds without trend		Bounds with trend	
				F	t	F	t
Belgium	US	-3.43	*7.78	4.50	-0.27	4.25	-0.22
Denmark	US	-3.13	-10.13	4.33	-1.51	*8.11	-2.96
Finland	US	-2.68	-3.37	2.72	0.96	2.36	0.83
Portugal	US	-2.94	*25.71	1.65	*2.00	1.58	1.43
Sweden	US	-2.89	*2.18	2.86	1.12	2.23	0.54
UK	US		*4.51	*8.50	*2.83	*11.40	*3.48
Canada	Japan	-3.18	*13.02	*6.37	*2.82	*8.47	*1.81
Netherlands	Japan	-3.45	*5.12	*7.33	*2.12	4.92	*1.86
Canada	UK	-2.02	-0.35	*13.12	*6.85	*13.17	*2.77
Italy	UK	-2.47	*10.01	1.24	0.78	0.55	-0.05
US	UK		*4.51	*8.50	*2.83	*11.40	*3.48

Notes: see text for description of group 4.

In Table 17, we have 3 cases for a high persistence in traded goods productivity differential and a low persistence in nontraded goods productivity differential, all with reference to Japan. We find a significant long-run relationship between the real exchange rate and the income differential indicated by both DOLS (with pre-testing for cointegration) and Bounds tests (with or without trend) for each case. Table 18 lists 3 cases with a low persistence in tad_t and a low persistence in nad_t , for which we don't record a long-run relationship between the real exchange rate and the income differential by either DOLS (with pre-testing for cointegration) or bounds tests. The third group contains the 19 cases for which the persistence for traded goods productivity differential is higher but the difference is not marked.

Among them, we find the long-run relationship exists for 5 cases according to the DOLS test (with pre-testing for cointegration) and 9 cases according to the bounds test (either with or without trend). 11 cases with a higher persistence in nontraded goods productivity are listed in Table 20 and we find 5 cases for which a long-run relationship between the real exchange rate and the income differential is recorded according to the bounds test (either with or without trend) but none of them are significant according to the DOLS tests (with cointegration). The differences in results between groups from above exercise for real data are remarkable, i.e., a significantly higher persistence in traded goods productivity differential is more likely to lead to a significant long-run relationship between the real exchange rate and the income differential, which at least show support partly to what our theory suggests.

The analysis conducted here on real data is not complete because we do not take the variation of productivity differential series into consideration. Nonetheless, we are able to explain the difference in the long-run relationship for different countries by looking at the persistence of productivity differential series, despite the simplification of our model that assumes PPP holds for traded goods.

7. Conclusion

The Balassa-Samuelson model suggests that rich countries should have higher relative price levels. However, conflicting evidence from cross-sectional and time series studies has puzzled economists for many years. To explore the puzzling relationship between the real exchange rate and the income differential, this essay

uses a simple model to relate the two variables to traded and nontraded goods productivity differentials. Our model suggests that a significant long-run relation tends to exist when the traded goods productivity differentials dominate the behavior of productivity.

Our simulation results show that the persistence and variability features of productivity differentials have an important influence on the long-run relationship between the real exchange rate and the income differential. When the traded goods productivity differentials have a higher persistence and variation than the nontraded goods productivity differentials, the real exchange rate and the income differential tend to move together in the long run. On the other hand, when the nontraded goods productivity differentials have a higher persistence and variation than the traded goods productivity differentials, the real exchange rate and the income differential are less likely to have a positive long-run relationship. Empirical based on real data supports these implications.

Although we used a fairly simple model, it is remarkable that the model can still explain much of the actual behavior of the real exchange rate and the income differential based on differences in the persistences of traded and nontraded productivity differential series. Half of the countries in our sample are members of the European Union with fixed nominal exchange rates between member countries. Thus, the variation of the bilateral real exchange rates between these countries is due only to changes in relative price levels. Our results suggest that if productivities in these countries advance along different trajectories, then they will experience painful

adjustment in the short run if prices are sticky since nominal exchange rates can not adjust.

In the second essay, we examine the implications of the Balassa-Samuelson model for the direct relationship between the real exchange rate and the relative productivity. By relaxing the assumption that PPP holds for traded goods and endogenizing the terms of trade in the model, we consider an extended model to address further puzzles about the relationship between the real exchange rate and the relative traded goods productivity.

Appendix

As shown in section 3, both the real exchange rate and the income differential can be expressed in terms of traded goods and nontraded goods productivity differentials,

$$q_t = \gamma(tad_t - nad_t),$$

$$m_t = (1 - \gamma)tad_t + \gamma nad_t.$$

We derive the cointegrating vector in the context of

$$q_t = \alpha + \beta m_t + u_t,$$

where u_t is the error term.

If both tad_t and nad_t are stationary, then q_t and m_t are also stationary.

If tad_t is stationary and nad_t is not, we can show

$$q_t + m_t = tad_t,$$

which is stationary, so we conclude the cointegrating vector is $(1, 1)$, or $\beta = -1$. If

nad_t is stationary and tad_t is not, we can show

$$q_t - \frac{\gamma}{(1 - \gamma)} m_t = (-\gamma - \frac{\gamma^2}{1 - \gamma}) nad_t,$$

which is stationary, so the cointegrating vector is $(1, -\frac{\gamma}{1 - \gamma})$, $\beta = \frac{\gamma}{1 - \gamma}$. If both

tad_t and nad_t are nonstationary, then both q_t and m_t are nonstationary and there is no way that q_t and m_t can be cointegrated (related in the long run).

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Chapter 2

Long-Run Determinants of the Real Exchange Rate: Balassa-Samuelson Model Reexamined

1. Introduction

The real exchange rate is an important relative price in an open economy and there has been much interest in understanding its behavior. One explanation of the real exchange rate has focused on the purchasing power parity (PPP) doctrine, i.e., the hypothesis that the real exchange rate equals unity (in its absolute version) or equals a constant (in its relative version). The “law of one price” (LOP) is a fundamental building block of the PPP relation. This law is based on the idea of frictionless goods arbitrage. There is now considerable evidence that the LOP and PPP break down in the short run. Isard (1977)’s study, for example, provides evidence of striking violations of the LOP. The evidence also shows (e.g., see Rogoff (1996)) that short-run deviations from PPP are large, and they dampen out at a very slow rate implying a half life of three to five years.

On the other hand, the issue of whether PPP holds in the long run is not yet settled. Taylor (2000) uses data for a group of 20 countries over one hundred years to present favorable evidence for the existence of long-run PPP. Based on post-war panel data, however, Pedroni (2001) shows overwhelming rejection of the standard

PPP hypothesis.¹ Imbs et al (2005) suggest that deviations from PPP could be due to an aggregation bias. However, it is not generally agreed whether this bias can explain the magnitude of the deviations. Chen and Engel (2004) argue, for example, that after correcting the small sample bias in the half-life estimates, heterogeneity and aggregation bias do not fully resolve the PPP puzzle.

The Balassa-Samuelson model provides a popular explanation of why PPP may not hold in the long run. The standard version of the model assumes that PPP holds for traded goods and predicts that the long-run real exchange rate depends on relative productivity trends of traded and nontraded goods in the home and foreign countries.² If relative productivity is governed by a stochastic trend, the real exchange rate will similarly be driven and will not exhibit any mean-reverting behavior, thus providing theoretical justification for PPP to be violated even in the long run. The dependence of the real exchange rate on relative productivity is based on two relations: first the link between relative productivity and relative price and then the link between relative price and the real exchange rate. Asea and Mendoza (1994) utilize the Hodrick-Prescott filter to extract long-run components from a panel database for 14 OECD countries. Their evidence indicates that relative labour productivities do explain low frequency movements in relative prices but the predicted relative prices are only partially successful in explaining long-run deviations from PPP. Similarly, Canzoneri, Cumby and Diba (1999) use data for a panel of

¹ He does find supporting evidence for weak PPP, which allows the coefficients on home and foreign price level to be different from 1, but note that in this case the real exchange rate is not a constant in the long run.

² Another relation predicated by Balassa-Samuelson model is between the real exchange rate and the income level. We studied that relation in the first essay.

OECD countries to test the Balassa-Samuelson model. They show that relative prices generally reflect relative labour productivities, but there is less conclusive evidence on the relationship between the real exchange rate and relative prices. Further support for the Balassa-Samuelson model is provided by Lane and Milesi-Ferretti (2002, 2004), who detect a direct long-run relationship between the real exchange rate and the relative productivity using data from OECD economies³.

The real exchange rate can be decomposed into traded goods and nontraded goods components where the traded goods component is the real exchange rate for traded goods and the nontraded goods component is the residual component that reflects the effect of the relative prices of nontraded to traded goods. According to the standard Balassa-Samuelson theory, the traded goods component does not vary and the relative productivity effect works through the nontraded goods component. Thus the nontraded goods component is the key determinant of the real exchange rate. However, Engel (1999) reports the surprising finding that this component accounts for almost none of the movements of the real exchange rate even over fairly long periods. Chari, Kehoe and McGrattan (2002) use data for the US and an aggregate of Europe to show that only about 2% of the variance of real exchange rates is due to fluctuation in the relative prices of nontraded to traded goods.

It is puzzling that even though there is evidence linking the real exchange rate or the relative price of nontraded to traded goods with relative productivity in the long-run, the nontraded goods component explains little of the variation in the real

³ Their tests also include additional variables such as terms of trade and trade balance.

exchange rate. One possible explanation is that the traded goods PPP does not hold and variations in the traded goods component are a dominant source of real exchange rate fluctuations. Our tests show that the traded goods component of real exchange rate is not stationary. We also conduct the variance decomposition for the real exchange rate and similar to Engel (1999), and find that the traded goods component, x_t , accounts for most of the variation in the real exchange rate. Although there has been considerable discussion of why there are deviations from traded goods PPP, the role of these deviations in the context of Balassa-Samuelson model has not been adequately explored.

A key objective of this essay is to reexamine the Balassa-Samuelson hypothesis and develop tests which take account of the role of departures from the traded goods PPP emphasized in the literature. These departures could be explained by modifying the Balassa-Samuelson model to let the home country be specialized in production. In this modified version, the effect of the relative productivity differential does not have to operate only through the nontraded goods component, but also through the traded goods component via changes in the terms of trade.

In performing these tests, one interesting econometric issue is how to measure or estimate the long-run relationship. Most of the previous studies of Balassa-Samuelson effects test for cointegrating relations that require the underlying series to be $I(1)$, or have a unit root. However, as variables in these tests are differentials of productivity and prices between two sectors or two countries, it is not

clear that these variables are likely to be $I(1)$.⁴ Furthermore, unit root tests have low power when the samples are short. To avoid this problem, we apply bounds test, a new econometric technique developed by Pesaran, Shin and Smith (2001),⁵ which is capable of testing for the existence of a long-run relationship regardless of whether the underlying time series are individually $I(1)$, $I(0)$ or sometimes cointegrated. This is another important contribution of our work.

We reexamine the Balassa-Samuelson theory with updated OECD data. To verify previous results, we first look at the standard form of Balassa-Samuelson model, i.e., the long-run relationship between the real exchange rate and the relative productivity differential. We focus on two key bilateral exchange rates, one with the US and the other with the UK.⁶ Our results suggest that this relationship is absent in a number of cases and when we do find a long-run relationship, it tends to be negative, especially for the case when the UK is the reference country. The presence of a negative relationship is a new finding which cannot be explained by the standard version and this problem has not been emphasized by the literature.⁷

To understand this negative relationship, we apply the extended model for the specialization case which endogenizes the terms of trade⁸ and explore the effect of

⁴ As discussed below, we test real exchange rates and relative productivity differentials for unit root and find mixed evidence on whether there is a unit root or not.

⁵ Pesaran, M. H., Shin, Y., Smith, R. (2001), "Bounds testing approaches to the analysis of level relationships", *Journal of Applied Econometrics* 16.

⁶ We also examined the case for Japan and the results are not that different from the US, so we dropped Japan as a reference country to save space.

⁷ Benigno and Thoenissen (2003) use a stochastic general equilibrium model and find a similar relation.

⁸ Previous studies have also used terms of trade as a determinant of the real exchange rate, but treated it more as an exogenous variable.

the relative productivity differential on both traded goods component and nontraded goods component. We are able to show that the relative productivity differential and terms of trade are negatively related so the relative productivity differential can affect both components through terms of trade and exert a negative impact on the real exchange rate. The test results largely confirm this negative relationship between the relative productivity differential and terms of trade. We then further explore the direct impact of the relative productivity differential on the two components separately with bounds test. As we expected, the long-run relation between the relative productivity and the traded goods component is negative. As for the nontraded goods component which is a weighted difference of the relative price of nontraded to traded goods prices in each country, it is actually positively related to relative productivity differential according to our results. This is because the Balassa-Samuelson effects are still there and working in the opposite direction. We also test for the key component of the Balassa-Samuelson theory and find that the long-run price-productivity relation holds better for each individual country, implying that an improvement in relative productivity of traded goods sector will lead to a higher relative nontraded goods price.

Since the traded goods component is negatively related to the relative productivity differential and it accounts for most of the variation in the real exchange rate, it is not surprising for us to observe a negative relation between the real exchange rate and the relative productivity differential.

The structure of the rest of this paper is organized as follows. Section 2 briefly outlines the analytical framework and derives testable relations. Section 3 introduces the econometric methodology and data set we are going to use and presents the empirical results. Section 4 provides some conclusions.

2. Analytical framework

This section derives some testable key long-run relations. We start with the standard Balassa-Samuelson version, where all goods are homogeneous and produced under perfect competition. We assume a multi-country world. The standard version also assumes diversified production, i.e., all goods are produced in each country under perfect competition. Since our focus is on the long run, all variables below denote long-run values.

The bilateral CPI-based real exchange rate for foreign country can be expressed as $Q_t = \frac{S_t P_t^*}{P_t}$, where S_t is the nominal exchange rate, and P_t and P_t^* are the CPI's for home and a reference foreign country (an “*” is used to denote variables for the reference country). An increase in Q_t represents a real depreciation of home currency and a real appreciation of the reference currency. Using lower case letters to denote values in logs, the log real exchange rate equals

$$q_t = s_t + p_t^* - p_t. \quad (1)$$

It is generally assumed that overall price index is a geometric weighted average of traded-goods and nontraded-goods prices. Under this assumption,

$$p_t = (1 - \gamma)p_{T,t} + \gamma p_{N,t}, \quad (2)$$

where $p_{T,t}$ and $p_{N,t}$ are logs of price indices for tradables and nontradables, and γ is the share of nontraded goods in the price index.

Following Engel (1999)'s decomposition of the real exchange rate, q_t can be written as the sum of a traded-goods component (x_t) and a nontraded-goods component (y_t):

$$q_t = x_t + y_t, \quad (3)$$

$$x_t = s_t + p_{T,t}^* - p_{T,t}, \quad (4)$$

$$y_t = \gamma^* (p_{N,t}^* - p_{T,t}^*) - \gamma (p_{N,t} - p_{T,t}). \quad (5)$$

Assuming that the "Law of One Price" holds in the long run for traded goods, we have

$$s_t = p_{T,t} - p_{T,t}^*. \quad (6)$$

Relation (6) represents the PPP for traded goods. If (6) holds, then the traded goods component equals zero and the real exchange rate is solely determined by the weighted differential between relative prices of nontraded goods in the two countries.

Under perfect competition, labour is paid the value of its marginal product, and labour mobility in the long run implies that the nominal wage rate, W_t , is equalized between the traded and nontraded sectors. Therefore,

$$W_t = \frac{\partial Y_{T,t}}{\partial L_{T,t}} P_{T,t} = \frac{\partial Y_{N,t}}{\partial L_{N,t}} P_{N,t}, \quad (7)$$

where $Y_{T,t}$ and $Y_{N,t}$ are outputs in the traded and nontraded sectors, and $L_{T,t}$ and $L_{N,t}$ are employment levels in the two sectors. A similar relation holds for the foreign country.

For simplicity, we assume that technology can be represented by a Ricardian model with only one factor (labour),⁹ so that

$$Y_{T,t} = A_{T,t}L_{T,t}, \quad Y_{N,t} = A_{N,t}L_{N,t}, \quad (8)$$

where $A_{T,t}$ and $A_{N,t}$ are the productivity indices in the two sectors. Production functions in (8) imply that

$$\frac{\partial Y_{T,t}}{\partial L_{T,t}} = \frac{Y_{T,t}}{L_{T,t}} = A_{T,t}, \quad \frac{\partial Y_{N,t}}{\partial L_{N,t}} = \frac{Y_{N,t}}{L_{N,t}} = A_{N,t},$$

i.e. the marginal product of labour is the same as the average product of labour in both traded and nontraded sectors. Here we follow Canzoneri, Cumby and Diba (1999) and use average products of labour as a measure of productivity in both traded and non-traded sectors.

We can then use (7) to express the relative nontraded goods price in logs as,

$$rp_t = ra_t, \quad (9)$$

where $rp_t = \ln\left(\frac{P_{N,t}}{P_{T,t}}\right)$ is the log relative price of nontraded goods and

$ra_t = \ln\left(\frac{Y_{T,t}/L_{T,t}}{Y_{N,t}/L_{N,t}}\right)$ is the productivity for traded sector relative to nontraded sector

in logs. Relation (9) can be expressed in the form of a country-differential version as follows,

$$rpd_t = rad_t, \quad (10)$$

where $rpd_t = rp_t^* - rp_t$ and $rad_t = ra_t^* - ra_t$.

⁹ However, the key relations can be easily generalized to two-factor case.

If we assume further that the shares of nontraded goods in the price index are the same for home and foreign countries ($\gamma = \gamma^*$), then from equation (3), (6) and (10), we can simply express q_t as

$$q_t = \gamma a d_t. \quad (11)$$

This is the standard relation for the one-factor version of the Balassa-Samuelson model, where only the relative labour productivity differential between foreign and home has an effect on the real exchange rate.

Relation (11) can be divided into two parts to derive a two-equation version of the Balassa-Samuelson model. The first equation is represented by (10) which links the relative price differential with the relative productivity differential between the two countries. Note that equation (10) is based on equation (9) for the differential between two countries. The second equation states that the real exchange rate is determined by the differential of relative nontraded-goods prices between the home and reference countries, i.e.,

$$q_t = \gamma p d_t. \quad (12)$$

Even under perfect competition, PPP may not hold for traded goods in the case of specialization where traded goods produced at home are not produced abroad. Under specialization, traded goods price can be expressed as a weighted average of price for export goods and price for import goods,

$$p_{T,t} = (1 - \theta) p_{X,t} + \theta p_{M,t}, \quad (13)$$

where $p_{X,t}$ and $p_{M,t}$ are price indices for export goods and import goods, and θ is the share of import goods in traded goods price index and we assume it is the same

for each country. Relation (9) and its foreign counterpart should still be valid in a modified version with $p_{T,t}$ replaced by the price of national traded goods or the export goods, $p_{X,t}$,

$$rp'_t = ra_t, \quad (14)$$

where $rp'_t = p_{N,t} - p_{X,t}$.

Using equation (13), the traded goods price component, x_t , can be expressed as

$$x_t = (1 - \theta)tt_t^* - (1 - \theta)tt_t + s_t + p_{M,t}^* - p_{M,t} \quad (15)$$

where $tt_t = px_t - pm_t$ is the terms of trade for home country. Letting tt_d_t denote the terms of trade differential between the foreign and home country, we can restate equation (15) as

$$x_t = (1 - \theta)tt_d_t + s_t + p_{M,t}^* - p_{M,t} \quad (16)$$

Clearly, the terms of trade differential has a positive impact on x_t , i.e. an improvement in the terms of trade for foreign country will raise x_t and appreciate the real exchange rate. The second term, $s_t + p_{M,t}^* - p_{M,t}$, is simply the difference between the import goods prices of foreign and home countries in same currency. For our multi-country model, this term should be close to zero if countries have similar tariff restrictions for import goods and similar shares of import goods in traded goods price index.¹⁰

In the presence of specialization, the nontraded goods component, y_t , can be written as

¹⁰ In a two country model, however, foreign import goods are the same as home export goods and this term might weaken the terms of trade effect on the real exchange rate.

$$y_t = \pi p d'_t + \gamma \theta t d_t \quad (17)$$

where $rp d'_t = (p_{N,t}^* - p_{X,t}^*) - (p_{N,t} - p_{X,t})$, the differential of nontraded goods price relative to export goods price between the foreign and home countries. The coefficient of the terms of trade differential will have the same sign in x_t as in y_t relation.

Now that each country's traded sector only produces export goods,

$$W_t = P_{X,t} \frac{Y_{T,t}}{L_{T,t}} = P_{N,t} \frac{Y_{N,t}}{L_{N,t}}. \quad (18)$$

Therefore, for the specialization case, the terms of trade and the relative productivity for home country are related as follows.

$$t t_t = p_{X,t} - p_{M,t} = p_{N,t} + \ln\left(\frac{Y_{N,t}}{L_{N,t}}\right) - \ln\left(\frac{Y_{T,t}}{L_{T,t}}\right) - p_{M,t} \quad (19)$$

Using (2) and (18), we can easily show that the log real wage rate w_t is a weighted average of productivities in both sectors,

$$w_t - p_t = (1 - \gamma) \ln \frac{Y_{T,t}}{L_{T,t}} + \gamma \ln \frac{Y_{N,t}}{L_{N,t}}, \quad (20)$$

which implies if there is a productivity advance in the traded (export) goods sector, real wage in this sector will also increase but not as much as the productivity because it is tied with real wage in nontraded sector.

Equations (19) holds for the foreign country also, so the terms of trade differential can then be expressed as

$$t t d_t = -r a d_t + (p_{N,t}^* - p_{N,t}) - (p_{M,t}^* - p_{M,t}). \quad (21)$$

Equation (21) suggests a negative relation between the terms of trade differential and the relative traded goods productivity differential.¹¹

In the specialization case, therefore, x_t and y_t become

$$x_t = -(1-\theta)rad_t + (1-\theta)(p_N^* - p_N) + s_t + \theta(p_{M,t}^* - p_{M,t}) \quad (22)$$

$$y_t = \gamma(1-\theta)rad_t + \gamma\theta(p_N^* - p_N) - \gamma\theta(p_{M,t}^* - p_{M,t}). \quad (23)$$

In a complete specialization case, an increase in output and higher profitability will probably put downward pressure on the export goods price, therefore the terms of trade will worsen for that country and this will tend to depreciate the real exchange rate as shown in equation (19).¹² On the other hand, the wage increase in nontraded sector will drive up the nontraded goods price and this causes the real exchange rate to appreciate, partly offsetting the downward pressure from traded goods productivity advance through terms of trade. So in the case of specialization, the relative productivity differential basically works to effect the real exchange rate through two channels, one is through Balassa-Samuelson which is positive and the other is through the terms of trade which is negative. We show in (22) and (23) that the relative productivity differential is negatively related to x_t and positively related to y_t and as a consequence, the positive relation between relative productivity differential and the real exchange rate predicted by Balassa-Samuelson model can be weak or even reversed.

¹¹ The nontraded goods price may also be affected by relative productivity change but this impact is relatively small compared to that on the terms of trade differential.

¹² The negative relation between the terms of trade and the relative productivity derived here is mainly based on the analysis of supply side. Some studies focused on the demand side suggest that this relation depends on the elasticity of substitution between the home and foreign consumption bundles. See, for example, Corsetti, Dedola and Leduc (2004) and Choudhri and Schembri (2007).

3. Empirical Analysis

Since sectoral data is not readily available for most developing countries, we limit our empirical analysis to 13 OECD countries with annual data from 1977 to 2003. Nominal exchange rates, CPIs and import/export prices are from OECD's Economic Outlook. Following other studies, we define traded sector to include agriculture and manufacturing and nontraded sector to cover all other industries, such as utility supplies, construction and total services. Sectoral prices are implicit deflators derived from data on value-added in current local currency units and in volume. Sectoral productivities are defined as sectoral output divided by sectoral employment.

As discussed above, we apply the bounds test suggested by Pesaran et al (2001) to test for the long-run relationship between the real exchange rate and the relative traded goods productivity differential. Their approach is based on the estimation of a dynamic error correction representation and tests whether the lagged levels of the variables in the relationship are significant or not. The test assumes that the long-run causality is from the independent variable to the dependent variable. To test for the long-run relationship between the real exchange rate q_t and the relative productivity differential between foreign and home, rad_t , given by equation (11) of section 2, we estimate the following conditional error correction model (ECM),

$$\Delta q_t = \alpha_0 + \alpha_1 t + \phi q_{t-1} + \psi rad_{t-1} + \sum_{j=1}^{p-1} \beta_j \Delta q_{t-j} + \sum_{j=1}^{q-1} \gamma_j \Delta rad_{t-j} + \omega \Delta rad_t + u_t, \quad (24)$$

where q_t is the dependent variable and rad_t is the independent variable. According to standard theory, relative productivity is determined exogenously and thus the causality assumption of the test is satisfied. In our tests based on (24) as well as other long-run relations discussed below, we consider values from 1 to 4 for p and q and report results for those specifications that minimize the Akaike information criterion (AIC) value.¹³ The equation is estimated by ordinary least squares, and the absence of a long-run relationship between q_t and rad_t is tested by calculating the F statistic for the null hypothesis that $\phi = \psi = 0$. According to Pesaran et al (2001), when both $\phi = 0$ and $\psi = 0$, q_t follows a unit root and there are no level effects in the conditional ECM and the null hypothesis of no level relationship between q_t and rad_t is not rejected. The distribution of the test statistic under the null depends on the order of integration of q_t and rad_t , and on whether there is an intercept and a time trend in the relation. They provide bounds for asymptotic critical F values at a certain level of significance for all these cases¹⁴. The lower bound is the critical value when all variables are $I(0)$ while the upper bound is the critical value when all variables are $I(1)$. When the variables concerned are of different order of integration (0 or 1), the critical value will fall between the bounds.

If the null hypothesis is rejected according to the F test, there is a stable long-run relationship between the real exchange rate and the relative traded goods productivity, which is described by

¹³ We also calculated results based on the Schwartz Bayesian information criterion (BIC) and found they were very similar to those based on AIC.

¹⁴ See Pesaran et al (2001, Table CI.i, CI.ii, CI.iii and CI.v)

$$q_t = \theta_0 + \theta_1 t + \theta_2 rad_t + v_t, \quad (25)$$

where $\theta_0 = -\alpha_0/\varphi$, $\theta_1 = -\alpha_1/\varphi$, $\theta_2 = -\psi/\varphi$ and v_t is the error term. We can then apply standard t test on the coefficient of the long-run forcing variable (θ_2) to verify whether a stable positive long-run relationship exists between the dependent variable q_t and the long-run forcing variable rad_t . If we can reject the null that $\varphi = \psi = 0$ according to the F statistic, but fail to reject $\theta_2 = 0$ by the t test, then we find the invalid long-run relationship which seems to involve only one of the variables.

As noted earlier, the advantage of bounds test is that it does not require pre-testing for order of integration of underlying variables. This is especially useful when the evidence on the unit root is not clear. For example, Table 1 and Table 2 show the unit root test¹⁵ results for the real exchange rates and one of its key determinants, the relative productivity differential. It can be seen that the results provide mixed evidence on whether the two series contain a unit root. The real exchange rates with respect to the US seem to be $I(1)$ without a trend but stationary when a trend is included. But the unit root test results of the real exchange rates for UK are mixed both for with and without trend. As for the relative productivity differential series for US and UK, they tend to be stationary. Therefore, the bounds test is an appealing procedure for exploring the long-run relationship between these series.

¹⁵ Elliot, Rothenberg and Stock (ERS) unit root test is applied due to its relatively higher power.

Table 1 Unit root test for the US and UK real exchange rates

Country	US		UK	
	without trend	with trend	without trend	with trend
Austria	*-2.6513	-2.6694	*-2.8640	-2.9638
Belgium	*-2.9811	-2.9433	*-2.5585	-3.1521
Canada	*-2.1150	-2.6948	-1.4706	*-4.0287
Denmark	*-2.9537	-2.9456	*-2.8742	-3.1132
Finland	*-2.6152	-2.8310	-1.7512	-2.7371
Italy	*-2.5457	-2.5410	-1.5682	-1.7029
Japan	-1.5702	-2.3895	-1.8711	-2.9009
Netherlands	*-2.8573	-2.8714	*-2.6241	*-3.4108
Norway	*-2.6797	-2.7411	*-2.2322	*-3.4655
Portugal	*-1.9799	-2.3715	-1.5521	-2.0546
Sweden	*-2.5922	-2.7887	-0.9476	-1.8336
UK	*-3.2195	*-4.0359		
USA			*-3.2195	*-4.0359

Notes: The 5% critical value for Elliot, Rothenberg and Stock (ERS) unit root test is -1.9550 without trend and

-3.1900 with trend. The symbol “*” denotes significance at 5% level.

Table 2 Unit root test for the US and UK relative productivity differentials

Country	US		UK	
	without trend	with trend	without trend	with trend
Austria	-0.4289	-2.9860	-1.8562	-1.9560
Belgium	*-2.0137	-2.9199	-1.4107	-2.3452
Canada	-0.7638	-2.3290	-1.7189	-2.0979
Denmark	-1.3118	-2.6775	*-2.1127	-2.1484
Finland	-1.4361	-2.3395	-1.5610	-1.6884
Italy	-0.6141	-1.7187	-1.6261	-1.7620
Japan	-0.2973	*-3.4926	-1.1228	-1.9275
Netherlands	*-2.5775	-2.6969	-0.9603	-1.9425
Norway	-0.8485	-2.8082	*-2.1605	-2.5841
Portugal	-1.2962	-2.6979	*-2.6886	-2.8156
Sweden	-1.6120	-2.1405	-1.1555	-1.6117
UK	-0.7911	*-3.2079		
USA			-0.7911	*-3.2079

See notes for Table 1.

We now discuss the results of the standard form of the Balassa-Samuelson relation: the relationship between the real exchange rate and the relative productivity differential. We test the null hypothesis that the real exchange rate and the relative productivity differential are not related in the long run (i.e., $\theta_2 = 0$) against the Balassa-Samuelson alternative that $\theta_2 > 0$.

As shown in Table 3, when the US is the reference country, we barely observe any long-run relationship between the real exchange rate and the relative productivity differential (only Portugal has both a significant F and t statistic when there is no trend; but the sign of the t statistic indicates a negative relationship). Canzoneri et al

(1999) also find that the Balassa-Samuelson hypothesis seems to fare poorly for the US real exchange rate.

A striking piece of evidence here is that when the UK is the reference country, real exchange rates and relative productivity differentials are significantly and negatively related for most countries (9 out of 12 countries, with or without trend) in the long run as t statistics of these countries are negative. Even for countries whose F and t statistics are not significant, the t statistics are still negative in most cases. This evidence is clearly inconsistent with the prediction of Balassa-Samuelson model.

Table 3 Bounds F and t statistics for long-run relationship between q_t and rad_t

Country	US						UK					
	Without trend			With trend			Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t
Austria	[2,2]	4.26	-1.26	[2,1]	5.97	-1.21	[1,3]	*25.53	*-9.29	[1,3]	*22.89	*-8.99
Belgium	[2,1]	5.18	-0.35	[2,1]	4.49	0.05	[2,2]	*8.23	-0.95	[2,2]	*7.62	-0.93
Canada	[2,1]	2.31	-0.02	[3,1]	5.49	-0.65	[2,1]	3.09	*-2.18	[3,3]	*18.85	*-5.53
Denmark	[2,1]	5.69	-1.44	[1,1]	*7.53	-1.55	[2,3]	*11.86	*-4.18	[2,3]	*12.12	*-4.29
Finland	[2,1]	3.22	-0.21	[2,3]	5.03	-1.78	[2,2]	5.16	*-3.48	[2,3]	*10.99	*-4.94
Italy	[2,1]	2.78	-0.29	[2,1]	2.57	-0.18	[1,1]	3.4	-1.31	[1,2]	*7.93	*-1.88
Japan	[2,1]	3.12	-1.6	[2,1]	3.71	-1.22	[2,2]	*6.95	*-2.92	[1,2]	*8.78	*-2.9
Netherlands	[2,1]	*6.45	-0.87	[2,1]	5.81	-0.87	[3,2]	*18.88	*-3.55	[2,2]	*22.78	*-2.38
Norway	[2,1]	3.41	-0.12	[2,1]	3.14	-0.31	[2,1]	*7.46	*-2.06	[2,3]	*10.29	*-3.07
Portugal	[2,3]	*6.6	*-2.03	[2,3]	6.17	0.12	[1,1]	*6.78	-0.95	[1,1]	6.73	-0.99
Sweden	[2,1]	3.54	-0.4	[2,1]	3.36	-0.64	[1,1]	*8.04	*-5.02	[1,1]	*7.47	*-3.62
UK	[3,3]	*5.86	-1.33	[3,2]	*7.98	1.53						
US							[3,3]	*5.86	-1.33	[3,2]	*7.98	1.53

Notes: the lag orders [p,q] of the underlying ECM were selected using the Akaike information criterion (AIC). The

symbol “*” denotes significance at 5% level. The critical bounds of the F statistic at 5% for bivariate case are

[4.94, 5.73] without trend (Pesaran et al (2001, Table CI.iii)), and [6.56, 7.30] with trend (Pesaran et al (2001,

Table CI.v)). One-sided significance level is also applied to the t test.

One possible reason for the above surprising result is that the Balassa-Samuelson assumption that $x_t = 0$ does not hold. So we examine the behavior of the two components of the real exchange rate, the traded goods price differential component x_t and the relative nontraded goods price differential component y_t to explore what factors may account for the lack of a long-run

relationship, and sometimes, even a negative relationship between the real exchange rate and the relative productivity differential.

A lot of studies have tested the relative PPP in traded goods and there is mounting evidence suggesting that the traded goods component is not stationary or shows a very high degree of persistence. As shown below in Table 4, our data confirms this evidence.

Table 4 Unit root test for x_t with respect to U.S and UK

Country	US		UK	
	without trend	with trend	without trend	with trend
Austria	-1.5713	-2.8373	*-2.5070	-2.5900
Belgium	-1.6945	-2.6280	-1.8747	-2.5986
Canada	-0.4292	-2.2823	*-2.7797	-2.9961
Denmark	-1.5823	-2.7243	-1.8676	-3.0489
Finland	-1.4890	-2.7185	-1.8995	-1.9302
Italy	-1.3430	-1.9773	-1.4991	-1.5095
Japan	-0.8620	-1.9634	-1.3741	-1.8101
Netherlands	*-2.5845	-2.8303	-1.6026	-3.0628
Norway	-1.1958	-2.6962	*-1.9571	-2.2866
Portugal	-0.7656	-2.6428	-1.5778	-2.3855
Sweden	*-2.8501	-2.9368	-1.4748	-2.3967
UK	-1.9379	*-5.0661		
USA			-1.9379	*-5.0661

See notes for Table 1.

When the US is the reference country, we can only reject the null that there is a unit root in x_t for 2 out of 12 countries without a trend and 1 out of 12 countries with a trend. For the case of the UK, unit root can be rejected for 3 out of 12 countries

without a trend and 1 out a 12 countries with a trend. Overall, the majority of evidence is consistent with a unit root. Even if we could reject the unit-root null, the series of x_t could be highly persistent, which would still be inconsistent with the standard theory's assumption that x_t is 0. To provide evidence on the degree of persistence, we use a simple measure of persistence based on a one-lag autoregression.

Table 5 Persistence in x_t with respect to the US and UK, $x_t = \alpha + \rho x_{t-1}$

Country	US		UK	
	$\hat{\alpha}$	$\hat{\rho}$	$\hat{\alpha}$	$\hat{\rho}$
Austria	0.5131	0.8582 (0.1111)	0.4303	0.7495 (0.1318)
Belgium	0.7846	0.7779 (0.1277)	0.4069	0.7411 (0.0830)
Canada	0.2221	0.9108 (0.1254)	0.1924	0.7163 (0.1419)
Denmark	0.8478	0.7951 (0.1325)	0.6747	0.6908 (0.1261)
Finland	0.7408	0.8085 (0.1172)	0.5035	0.7390 (0.1361)
Italy	0.3831	0.7942 (0.1186)	-0.0141	0.8303 (0.1149)
Japan	0.7040	0.9040 (0.0617)	0.8838	0.8411 (0.0993)
Netherlands	0.7800	0.7582 (0.1376)	0.3495	0.7228 (0.0926)
Norway	0.6654	0.8499 (0.1276)	0.9547	0.6190 (0.1305)
Portugal	0.3194	0.9255 (0.0970)	0.5575	0.7789 (0.1288)
Sweden	1.0665	0.6886 (0.1486)	0.3760	0.7430 (0.1131)
UK	0.3868	0.8003 (0.1129)		
USA			-0.3868	0.8003 (0.1129)

Notes: standard deviations for the autocorrelation estimates are in the parentheses.

The results in Table 5 show that the x_t series tend to be very persistent and we have slightly higher persistence in x_t when US is the reference country than

when UK is the reference country. Therefore, the evidence clearly rejects the basic assumption underlying the Balassa-Samuelson that PPP holds for traded goods. This behavior of x_t is probably the key reason why the long-run relationship between the real exchange rate and the relative productivity differential breaks down.

Another problem suggested by Engel (1999) is that y_t accounts for little of the movement in the real exchange rate. We use our data to examine the extent to which variations in x_t and y_t account for the variation in the real exchange rate in the long run. We first obtain the series of the 2-year differences for q_t , x_t and y_t , and then calculate the variances for the three series and the covariance between the differenced x_t and y_t series. We attribute the covariance between differenced x_t and y_t first to the variance of differenced x_t and then to the variance of differenced y_t , so that we obtain upper limit and lower limits for the proportion of the variation in q_t that x_t and y_t can account for¹⁶. Table 6 lists such intervals based on both the US and UK.

¹⁶ Engel (1999) reports a single number for the decomposition with two procedures, one attributing half of the comovements to each component and the other ignoring the comovements. Similar to Engel (1999), we also find the comovements are negative.

Table 6 Intervals of 2-lag difference variance for x_t and y_t

Country	US		UK	
	x_t	y_t	x_t	y_t
Austria	[0.78, 1.00]	[0.00, 0.22]	[0.54, 0.78]	[0.22, 0.46]
Belgium	[0.81, 1.18]	[-0.18, 0.19]	[0.68, 0.91]	[0.09, 0.32]
Canada	[-0.29, 1.32]	[-0.32, 1.29]	[0.68, 1.00]	[0.00, 0.32]
Denmark	[0.66, 1.06]	[-0.06, 0.34]	[0.44, 1.12]	[-0.12, 0.56]
Finland	[0.82, 1.31]	[-0.31, 0.18]	[0.59, 1.40]	[-0.40, 0.41]
Italy	[0.78, 1.17]	[-0.17, 0.22]	[0.31, 1.19]	[-0.19, 0.69]
Japan	[0.71, 0.98]	[0.02, 0.29]	[0.71, 0.86]	[0.14, 0.29]
Netherlands	[0.78, 1.06]	[-0.06, 0.22]	[0.73, 0.94]	[0.06, 0.27]
Norway	[0.67, 1.13]	[-0.13, 0.33]	[0.57, 0.82]	[0.18, 0.43]
Portugal	[0.85, 1.22]	[-0.22, 0.15]	[0.67, 1.11]	[-0.11, 0.33]
Sweden	[0.70, 1.29]	[-0.29, 0.30]	[0.50, 1.27]	[-0.27, 0.50]
UK	[0.75, 1.34]	[-0.34, 0.25]		
USA			[0.75, 1.34]	[-0.34, 0.25]

The results in Table 6 indicate first that x_t accounts for a much larger portion of the variation in q_t than y_t . This is very similar to the results presented in section IV of Engel (1999) where the overall PPI is used as a proxy of traded-goods prices.¹⁷ The implicit deflators we use to calculate the price index for traded and nontraded goods price is also a measure of Producer's Price Index (PPI). The negative correlation between x_t and y_t is probably due to that $p_{T,t}^* - p_{T,t}$ appears with a positive sign in x_t but with a negative sign in y_t .

According to the modified version of Balassa-Samuelson model based on specialization in section 2, changes in the relative productivity differential are a key

¹⁷ Engel (1999) uses real exchange rate time series of the United States relative to a number of high-income OECD countries from early 70s to late 90s, which are similar to our sample and time frame.

driving force for movements of both x_t and y_t . The relative productivity differential affects x_t and y_t both through the terms of trade channel and the standard Balassa-Samuelson effect channel.

We first verify the negative relation between the terms of trade differential and the relative productivity differential as indicated in equation (21) and the results are listed in Table 7.

Table 7 Bounds F and t statistics for long-run relationship between tt_d and rad_t

Country	U.S.						U.K.					
	Without trend			With trend			Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t
Austria	[1,1]	3.74	1.19	[1,1]	3.65	0.13	[1,3]	3.51	-0.95	[3,2]	*13.87	-1.59
Belgium	[2,1]	2.61	-0.01	[2,1]	3.42	0.64	[3,3]	5.42	-0.21	[3,3]	5.38	0.43
Canada	[2,1]	*20.74	1.17	[2,1]	*21.22	-0.02	[3,1]	1.44	-1.16	[2,2]	*11.39	*-2.74
Denmark	[1,1]	5.09	-3.11	[1,1]	4.82	-1.40	[2,1]	2.94	-0.68	[2,2]	*16.42	*-3.26
Finland	[2,1]	1.54	0.16	[3,3]	2.30	-1.27	[3,2]	*8.96	*-2.59	[3,2]	*7.99	*-2.18
Italy	[1,1]	1.85	0.31	[1,1]	1.13	0.47	[2,3]	0.89	0.13	[1,2]	5.58	-1.31
Japan	[2,1]	2.14	-0.92	[2,1]	2.52	-1.22	[2,1]	3.67	-1.73	[2,2]	4.92	-2.07
Netherlands	[3,3]	*11.25	-0.88	[3,3]	*29.37	-1.13	[2,2]	5.94	*-3.79	[3,3]	*8.49	0.29
Norway	[2,1]	2.46	1.01	[1,1]	2.20	0.69	[3,3]	4.90	1.36	[3,3]	2.88	0.85
Portugal	[1,3]	4.7	*-3.29	[3,3]	*10.19	1.30	[1,2]	5.50	-1.20	[1,2]	*7.76	*-3.52
Sweden	[2,1]	1.55	-0.36	[2,1]	2.70	-0.64	[1,3]	*7.16	*-4.94	[2,2]	*10.39	*-4.03
UK	[3,2]	*15.69	*-2.51	[3,1]	*28.54	*-2.36						
U.S.							[3,2]	*15.69	*-2.51	[3,1]	*28.54	*-2.36

See notes for Table 3.

Table 7 presents the bounds test results for the long run relationship between the terms of trade differential and the relative productivity differential for the null that the terms of trade differential and the relative productivity differential are not related in the long run against the alternative that they are negatively related. For the US case, we can reject the null that the terms of trade differential and the relative productivity differential are not related in the long-run only for UK and this long-run relationship is negative. We also notice that in the US case more than half of the t statistics are negative though most of them are not significant. When UK is the reference country, there are 6 out of 12 countries (with or without trend) showing significant long-run relationship, all of which are negative. The results for UK are broadly consistent with our theoretical prediction in section 2 that a productivity advance in the traded sector will have a negative impact on the terms of trade if specialization is complete and home producers for exportables have monopoly power. Since the price index for export goods from our data is not strictly the same as that in the traded sector described in our model,¹⁸ there could be some bias in this long-run relationship test and this may be the reason why we do not detect much significant negative relationship between the terms of trade and the relative productivity differential for the US case.

Now we apply bounds test to see how each component of the real exchange rate is related with relative productivity differential in the long run. First we test the hypothesis that x_t and the relative productivity differential are not related in the long

¹⁸ First, the coverage of goods may be different and second, the terms of trade are based on unit value while the sectoral prices are implicit deflators.

run against the alternative suggested by the specialization version that the traded goods price component and the relative productivity differential are negatively related. As shown in Table 8, when the US is the reference country, we do not find much significance in the statistics (only Denmark has both significant F and t when there is a trend) though most t statistics are negative. For the case of UK, we observe more countries show both significant F and t statistics (7 for without trend and 6 for with trend). All countries showing stable long-run relationship between x_t and the relative productivity differential (have both a significant F and t) present a negative t statistic, indicating that this relationship is negative. These results seem to be broadly consistent with the specialization version.

Table 8 F and t statistics for long-run relationship between x_t and rad_t

Country	U.S.						U.K.					
	Without trend			With trend			Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t
Austria	[2,1]	4.14	-2.50	[2,3]	4.99	-1.41	[2,2]	*13.05	*-5.71	[1,2]	*15.19	*-6.99
Belgium	[2,1]	3.96	-0.63	[2,1]	4.50	0.24	[2,2]	5.53	-1.83	[2,2]	4.91	-1.35
Canada	[3,3]	5.64	-1.69	[3,3]	4.60	-0.52	[3,1]	*7.89	*-4.44	[3,1]	*7.80	*-4.07
Denmark	[1,1]	5.01	-1.86	[1,3]	*8.29	*-2.26	[2,3]	*9.44	*-5.94	[2,3]	*8.55	*-5.61
Finland	[2,3]	4.58	-2.91	[2,3]	7.32	-2.12	[2,3]	5.20	-3.94	[2,3]	5.46	-4.70
Italy	[2,2]	2.07	-0.90	[2,2]	2.10	-0.55	[1,1]	3.34	-1.25	[1,2]	4.38	-1.86
Japan	[2,1]	2.09	-3.06	[2,1]	1.91	-0.94	[2,2]	*7.07	*-6.03	[1,2]	*7.88	*-4.05
Netherlands	[2,1]	3.40	-0.91	[2,1]	4.37	-0.97	[3,2]	*20.71	*-7.79	[2,2]	*24.65	*-5.41
Norway	[2,2]	5.19	-2.59	[2,2]	4.56	-0.89	[3,3]	*21.39	*-7.05	[3,2]	*20.61	*-5.84
Portugal	[1,3]	6.40	-3.05	[3,3]	*9.33	0.73	[1,1]	2.82	-1.18	[1,1]	3.10	-1.17
Sweden	[2,2]	5.32	-1.92	[2,2]	4.63	-1.61	[1,1]	*7.43	*-6.47	[1,1]	6.67	-5.04
UK	[2,1]	4.02	-2.60	[3,3]	*11.51	0.82						
U.S.							[2,1]	4.02	-2.6	[3,3]	*11.51	0.82

See notes for Table 3.

Next we test the null that the nontraded goods price component and the relative productivity differential are not related in the long run versus the alternative that the coefficient on relative productivity is significant and positive since both the diversification and specialization cases indicate a positive relationship between them.

Table 9 F and t statistics for long-run relationship between y_t and rad_t

Country	U.S.						U.K.					
	Without trend			With trend			Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t	[p,q]	F	t
Austria	[2,3]	0.64	0.37	[1,3]	5.41	-0.45	[1,1]	5.25	-0.34	[1,3]	*8.07	*-2.27
Belgium	[1,3]	0.94	0.63	[1,2]	6.07	0.53	[2,2]	4.82	-0.73	[2,2]	4.61	-0.30
Canada	[2,2]	0.07	0.48	[2,2]	1.50	0.79	[3,1]	3.37	0.75	[1,1]	2.17	-0.62
Denmark	[1,1]	*8.47	*4.41	[1,1]	*9.69	*2.66	[1,1]	*6.60	-0.19	[1,3]	*9.30	*3.11
Finland	[1,1]	2.04	0.09	[1,1]	6.45	2.81	[1,1]	4.00	-0.75	[1,3]	*10.12	*2.62
Italy	[2,3]	4.12	5.98	[2,3]	*7.71	*6.68	[1,1]	2.05	0.13	[1,1]	1.34	0.10
Japan	[2,3]	*5.73	*12.82	[2,1]	*7.42	-0.09	[1,1]	0.25	0.59	[1,3]	*8.48	0.06
Netherlands	[3,2]	3.29	2.19	[1,1]	2.34	-0.19	[1,2]	*7.38	*7.11	[1,2]	7.13	3.82
Norway	[2,3]	*7.88	*14.74	[3,3]	5.52	12.36	[1,1]	0.80	-0.21	[3,1]	*7.71	*3.48
Portugal	[1,1]	*5.86	*8.12	[1,1]	3.62	3.84	[3,1]	2.42	1.01	[3,1]	2.79	1.06
Sweden	[2,1]	3.60	-0.01	[2,1]	2.98	0.39	[1,1]	*6.08	-0.15	[1,1]	6.06	0.42
UK	[1,1]	2.84	0.31	[1,1]	0.96	0.16						
U.S.							[1,1]	2.84	0.31	[1,1]	0.96	0.16

See notes for Table 3.

In Table 9, both in the case of the US and UK, we see that 5 out of 12 countries (with or without trends) show significant positive relationship between y_t and the relative productivity differential, most of which are positive (in the UK case, Austria shows a negative relationship though). This evidence is consistent with both the diversification and specialization cases. Our evidence shows that in general, the traded goods price component is negatively related to the relative productivity differential while the nontraded goods price component is positively related to the relative productivity differential. The relative productivity affects the real exchange rate through two channels, the terms of trade channel which affects both components,

and the Balassa-Samuelson channel which is embodied in the nontraded goods component. Since x_t is the dominant component as discussed before, it is not surprising that we observe a negative relation between the real exchange rate and the relative productivity differential.

The long-run relationship between y_t and rad_t is based on the assumption that the shares for nontraded goods (γ and γ^*) are the same for the two countries. If this assumption does not hold, an aggregation bias may be introduced in this relation. To explore this possibility, we also test for the relationship between the relative nontraded goods price and the relative productivity in traded sector (equation (9)) separately for each country. These separate relations are the key component in Balassa-Samuelson model.

Table 10 Bounds F and t statistics for the long-run relationship between rp_t and ra_t

Country	Without trend			With trend		
	[p,q]	F	t	[p,q]	F	t
Austria	[1,1]	3.34	0.04	[1,1]	2.20	-0.96
Belgium	[2,1]	*7.76	*15.18	[1,2]	*7.63	*2.99
Canada	[2,1]	*7.42	*3.82	[3,3]	3.54	-0.52
Denmark	[1,1]	*6.03	*14.37	[3,1]	6.69	4.93
Finland	[3,2]	4.38	11.55	[3,2]	4.50	2.58
Italy	[1,1]	2.25	13.08	[1,1]	3.80	3.27
Japan	[2,2]	*8.53	*10.46	[2,2]	*8.06	*6.13
Netherlands	[2,2]	1.76	8.24	[2,1]	2.27	-0.41
Norway	[2,1]	5.57	11.26	[3,1]	5.13	3.98
Portugal	[1,1]	3.43	11.38	[2,1]	*9.35	2.33
Sweden	[2,1]	*5.90	*7.31	[2,1]	4.55	0.97
UK	[1,1]	*8.34	*2.52	[1,3]	*10.76	-0.45
US	[2,3]	*7.3	*26.8	[3,3]	*10.17	*16.43

See notes for Table 3.

The results in Table 10 indicate that 7 out of 13 countries (with or without trend) show a significant long-run relationship between the relative price and the relative productivity. Thus the key Balassa-Samuelson effect seems to hold well when tested for individual countries. These single-country relations have also found support in previous studies, such as Asea and Mendosa (1994) and Canzoneri et al (1999).

4. Concluding remarks

Real exchange rate fluctuations can have important allocative effects and a great amount of work has been done to explain why real exchange rates appear to change over time. The Balassa-Samuelson model predicts that equilibrium real exchange rates are driven by the relative productivity in the traded and nontraded sector. To test for Balassa-Samuelson effects, most studies use cointegration tests which require the underlying variables to be $I(1)$. But the time series we use here may have different behavior and our unit root tests give very mixed results. To circumvent this issue, this paper uses the Pesaran et al (2001)'s bounds test approach which allows time series to be either $I(1)$ or $I(0)$ to test for the existence of a long-run relationship between the real exchange rate and its possible long-run determinants, such as differential of relative productivity in traded sector between the foreign and home countries, traded-goods price and nontraded-goods price component.

We first test the standard version of Balassa-Samuelson model and find that Balassa-Samuelson effects are not significant (the US case) and when the results are significant, the relation seem to be reversed (the UK case).

We then test for PPP in traded goods explicitly and in most cases we cannot reject the null that there is a unit root for the traded goods component. The estimated persistence ($AR(1)$ autocorrelation coefficients) is very high for the traded goods component and this further supports the argument that the traded goods component can affect the real exchange rate in the long run. The descriptive tests for the two components of the real exchange rate show that the traded goods component actually

account for most of the variation in the real exchange rate, so we can confidently conclude that one reason why Balassa-Samuelson effects do not hold is the failure of PPP in traded goods.

If we assume complete specialization, terms of trade may enter both components of the real exchange rate with a positive sign. While the terms of trade and the relative productivity are negatively related, the traded goods component is also negatively related to the relative traded goods productivity. But the nontraded goods component is basically positively related to the relative traded goods productivity according to our final tests due to the Balassa-Samuelson effects based on the positive relationship between the relative nontraded goods price and relative traded goods productivity for individual countries. Since the relative productivity differential works through both the Balassa-Samuelson effects channel (positive) and the terms of trade channel (negative) therefore the net effect of relative productivity differential is no longer predictable. An advance in the traded goods productivity may worsen the terms of trade and then depreciate the real exchange rate for the home country, so it is not unusual for us to obtain the reversed relationship between the real exchange rate and the relative productivity differential.

Our work can shed some light on the long-run effects of some important factors on the real exchange rate, especially the role played by the relative productivity differential and the terms of trade. Furthermore, the evidently different results for the US and UK case suggest that the choice of reference currency is very

important and whether PPP holds for traded goods may have a large impact on the long-run behavior of the real exchange rate.

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