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Analysis and Design of a Direct Drive Robot

by

Shoeb Rajguru

B.Eng., Bangalore University, India, 1984.

**A Thesis submitted to the
Faculty of Graduate Studies and Research
In Partial Fulfilment of the Requirements for the Degree of
Master of Engineering
in the
Department of Mechanical and Aeronautical Engineering
Faculty of Engineering,
Carleton University,
Ottawa, Ontario, Canada.**

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Acceptance Sheet

The undersigned recommend to the Faculty of Graduate Studies and Research, acceptance of the thesis entitled "Analysis and Design of a Direct Drive Robot", submitted by Shoeb Rajguru in partial fulfilment of the requirements for the degree of Master of Engineering in Mechanical Engineering.

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Abstract

In direct drive technology, the motor shaft is directly coupled to the input link. Since no gearing is involved, mechanical inaccuracies in terms of backlash, friction and reduced mechanical stiffness are eliminated. The motors used in direct drive robots are designed to provide high torque at low speeds in a compact size. As such, direct drive robots have speeds, accelerations, and precision beyond the reach of conventional robots.

As the motor is directly coupled to the input link, it is very sensitive to the configuration varying inertial effects. In serial robots, the motor size increases from the distal link to the proximal link. This results in a heavy arm design and reduced payload capacity.

In this thesis, the disadvantages of direct drive technology were successfully eliminated and the advantages utilized, by using a five bar linkage. A manipulator was designed to be dynamically decoupled and configuration invariant. Two designs are presented, the first one being dynamically nonlinear and the latter one, linear. This robot will serve as a research tool for graduate and undergraduate students in the area of robotics at Carleton University.

Dedication

To my Parents

Acknowledgements

I would like to express my deepest appreciation and gratitude to Dr H. M. Schwartz, my thesis advisor, for all his guidance and encouragement throughout the course of this work.

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List of Symbols

G	m/sec^2
K_i, K_v, K_p	gain terms of the PID controller
V_o	output voltage
ω_s	input angular velocity
K_t	constant of proportionality
τ_i	torque of the i^{th} joint
θ_i	joint displacement of the i^{th} joint
H_{ij}	the $i - j$ element of the manipulator inertia matrix
θ_{gi}	torque due to gravity
l_1, l_2, l_3	link lengths (in, m, mm)
$\theta_1, \theta_2, \phi_1, \phi_2$	joint angles
J	the Jacobian matrix
F_e	environmental force
m_o, m_1, m_2	link masses (kgs)
L	the Lagrangian
K	kinetic energy
P	potential energy
$l_{1c}, l_{2c}, l_{3c}, l_{4c}$	mass centers of the links
$\dot{\theta}_1, \dot{\theta}_2$	joint rates

$\ddot{\theta}_1, \ddot{\theta}_2$	joint accelerations
v	deflection
E	Youngs Modulus
I	area moment of inertia
S_v	shear strength
S_y	yield strength

Chapter 1

Introduction

1.1 Background

A robot according to the Robot Institute of America, is a programmable, multifunction manipulator, designed to move material, parts, tools or special devices through variable, programmed motion for the performance of a wide variety of tasks.

Robots are used in industries to increase productivity, reduce costs, improve product quality, overcome labour shortages (for example in welding and spray painting operations), and free humans from boring repetitive tasks and to work in hostile environment [1].

Commercially available electromechanical robots employ gear trains, harmonic drives, leadscrew mechanism and chain drives as a means of transmission, and for amplification of the motor torque. Usage of the above methods of transmission causes

inherent mechanical problems in the system in terms of backlash, friction, and reduced mechanical stiffness. The above mentioned methods of transmission degrade the dynamic response of the manipulator. Furthermore, due to backlash and friction, fine movements and pure torque control becomes difficult [2]. The dynamics of a manipulator is configuration dependent. The varying inertias and nonlinear effects such as Coriolis and centrifugal forces further degrade the performance of the manipulator in terms of accuracy, speed and control [3].

Most of the commercially available robots which employ the above stated electromechanical drives have a maximum acceleration of under 0.1G to 0.5G at the arm tip. Furthermore, their speeds are quite low [4]. High speed and high accelerations are essential requirements for a high production rate. For example, in high speed laser cutting applications, the speed and acceleration requirements are on the order of 3m/sec and 3 to 5G's [5]. Thus conventional robots cannot meet the demands of any task that requires high speed, high acceleration and high accuracy.

1.2 Direct Drive Technology

Direct drive technology was pioneered in the United States by Haruhiko Asada [7]. The first version called the Carnegie Mellon University Direct Drive Arm was developed at Carnegie Mellon University. In direct drive robots, the input link is directly coupled to the the direct drive motor shaft.

Since gearing is eliminated, the mechanical inaccuracies in the system (backlash,

friction, and reduced mechanical stiffness) are eliminated. Due to this the dynamic response of the direct drive manipulator is improved. Fine movement and pure torque control is possible. Furthermore, direct drive robots are shown to have maximum acceleration of the arm tip of over 5G and speeds over 10m/sec [4]. Direct drive robots are particularly useful where a high productivity rate is required, and also in applications where high speed and high accuracy is a major concern. Direct drive robots have been effectively used in high speed laser cutting applications (for example, the Shin Meiwa robot), which is beyond the reach of conventional robots.

Though direct drive technology shows distinctive advantages over the conventional electromechanical drives, it does have a few drawbacks. Since the motor shaft is directly coupled to the input link, the complex arm dynamics reflect directly on the motor axes. In other words the configuration varying inertias have a more pronounced effect on direct drive robots [2]. Furthermore, in serial manipulators, the direct drive motors are directly coupled to their respective input links. Therefore the weight of the motor is a load for the next motor down the serial linkage [2,4]. Thus, the motor torques required vary from the distal end to the proximal end of the serial manipulator. Rapid increase in the motor size, results in heavy arm weight and low payload capacity [4].

Extensive research was done at M.I.T by Asada and Youcef- Toumi to overcome this problem [4,5,6]. A parallel drive robot was developed to overcome the inherent difficulties of the direct drive method [6]. In the new approach, the motors were fixed

on the base, and the output link was remotely driven with help of transmission links. Due to this, the weight of one motor is not a load on the other. Thus the reaction torque is eliminated [8]. Furthermore, it was shown by Asada and Youcef- Toumi [4,5,6,7,8], that by following proper design procedure, it is possible to eliminate the varying inertia and nonlinear effects like Coriolis and centrifugal force terms for a five bar parallelogram mechanism.

Based on the above procedure, a dynamically decoupled and configuration invariant SCARA (Selective Compliance Assembly Robot Arm) kinematics robot (Model 4) was developed by Asada and Paul Ro [9].

1.3 Thesis Overview

Commercially available robots which use conventional electromechanical drives pose research limitations and cannot be used as experimental testbeds. This is due to limitations in terms of accuracy, speed, limitations of the host computer, programming language and sketchy or no details provided by the manufacturer for the mechanical parts or the servosystem. The situation gets further aggravated, if the manipulator design prohibits experiments in robot control [7].

Due to the reasons stated above, and the advantages of direct drive technology, it was decided to build an experimental robot (SCARA kinematics) at Carleton University. This robot will be used in the future as a research tool by graduate and undergraduate students. Simulation results can be compared with experimental re-

control systems. Research can be done in areas of control (optimal and adaptive), sensing, vision, and path planning.

Chapter 2 is an overview of the robot, direct drive motors, transducers, drive amplifiers and the controller. Chapter 3 deals with the kinematic structure of the robot, forward kinematics, inverse kinematics, the Jacobian, static forces, dynamic equation of motion and design condition for obtaining a dynamically decoupled and configuration invariant manipulator. Chapter 4 details the design process. The design of the motor housings, links, joint coupling and the end effector are described. Finally, Chapter 5 gives the summary and conclusions.

Chapter 2

A Computer Based Direct Drive Robotic Manipulator Cell

2.1 Introduction

The robotic cell at Carleton University was built under the supervision of Dr H.M. Schwartz of the Systems and Computer Engineering Department. This robotic cell will serve as a research facility in the area of robotics for graduate and undergraduate students.

The present robotic cell consists of a microcomputer (I.B.M 80286), a direct drive robot (SCARA configuration), and two Servo Control Modules (drive amplifiers) purchased from Motion Control Systems. Research is being carried out using the robotic cell in the areas of control (Optimal and Adaptive), Sensing, Vision, and Path Plan-

ning. Future expansion of the robotic cell will be in terms of more robots (to be manufactured in house) and related equipment.

The direct drive robot, employs brushless D.C motors. Each of the motor housings has a tachometer which is used for velocity feedback and a brushless resolver which is used for sensing the position of the motor shaft. The position information is used for electronic commutation of the brushless d.c motor through the use of the built in commutation section housed in the drive amplifier. The system includes D/A, A/D, and R/D (resolver to digital) converters. Two P.I.D controllers, each controlling a direct drive motor, are used to control the motion of the manipulator. Fig 2.1 gives a brief idea of the system components and interconnections [10].

2.2 The Control Computer

The control computer used at present is a 80286 based MS-DOS computer. This computer is equipped with A/D, D/A and parallel I/O boards. It had a 640K RAM, 12 Mhz speed, and a 20 MB storage capacity. At a later stage this computer will be replaced by a faster more advanced model. On going research will develop a computer environment based on several microprocessors working in parallel.

The computer will be used for processing data, running software routines, motion coordination, path planning, adaptive control, digital control and optimal control.

Figure 2.1: System Components and Interconnections

2.3 The Controller

A Proportional- Integral- Derivative (P.I.D) control system was chosen for the direct drive motor [10]. This controller offered fine and precise control. The design considerations were response time, accuracy and stability. The P.I.D system was implemented as an analog circuit. The gain terms K_i , K_v and K_p were specified by a choice of resistors. This enabled easy adjusting of the three gains, by simply replacing the resistors [10]. Further research in robot controls is ongoing.

2.4 The Robot

The robot was manufactured in house at Carleton University. It is a direct drive robot with two degrees of freedom. It has a SCARA (Selective Compliance Assembly Robot Arm) configuration. It employs a closed loop five bar mechanism. Direct drive robots as compared to the commercial robots have advantages in terms of speed, acceleration and accuracy. The arm tip acceleration of direct drive robots are shown to be in excess of 4-5 G's.

Gears, belts, chains, and harmonic drives are some of the common transmission means used by commercially available robots. Usage of the above causes inherent mechanical problems in the manipulator in terms of backlash, friction, and reduced mechanical stiffness, thus degrading the manipulator's performance.

Direct drive technology can be effectively used to eliminate mechanical inaccu-

cies (backlash, friction, and reduced mechanical stiffness). The motor shaft of the direct drive robot is directly coupled to the input link, thereby eliminating these inaccuracies. The Carleton Robot consists of links, motor housings, an end effector and a mounting stand. Each motor housing contains a brushless d.c motor, resolver and a tachometer. The brushless d.c motors come with their respective drive amplifiers.

2.5 The Transducers

A transducer as defined in [11] is " An elementary device capable within a given field of measurement of converting a physical non electrical input quantity into an electrical output quantity. The transducer itself does not contain any further processing beyond this energy conversion". Position and velocity feedback, through the use of transducers like resolvers and tachometers are very important for the control of a robotic manipulator.

2.5.1 Tachometer

A tachometer or a tachogenerator is a velocity transducer. It is generally coupled directly to the shaft of the servomotor. Tachometers are generally of brush type or brushless type. The tachometer's main purpose is to continuously sense and feedback the speed of the motor [12].

It transforms the physical velocity (or speed) of the rotating shaft into an electrical signal or voltage V_0 . This output voltage V_0 is proportional to the input angular

velocity of the shaft ω , such that,

$$V_0 = K_t \omega, \quad (2.1)$$

where K_t is the constant of proportionality.

Its linearity is a very important criterion in terms of the selection and performance of the tachometer (usually specified by the manufacturer). The tachometer of the analog type can be interfaced easily to the computer using an A/D converter. Its measurement resolution depends on the A/D number of bits.

2.5.2 Resolver

Resolvers are used to measure the rotational position of the servomotor shaft. They can also be used as a velocity transducer. They are generally coupled directly to the servomotor shaft, or indirectly by means of gearing. Resolvers are available in brush as well as brushless types. Brushless types are preferred because of the absence of frictional contact between the resolver's stator and rotor. Resolvers are rugged in construction, reliable and show high immunity to electrical disturbances.

Referring to fig 2.2, the input voltage in the primary winding (on the rotor of the resolver) produces a change in the magnetic field which induces a voltage in the two secondary windings. The secondary windings are wound 90° apart (on the stator of the resolver) [13]. For example if the input voltage is 15V, and the angle of the rotor is 30° , then the output voltage between S_1 and S_3 is $15 \sin 30^\circ$ and between S_2 and S_4 it is $15 \cos 30^\circ$. The R/D converter, converts the resolver signal to a digital output.

Figure 2.2: Resolver System

2.6 The Servo Control Module

The drive amplifier has five built in sections. They are, the Velocity Control Loop Section, the Commutation Section, the Motor Phase Control Section, the Pulse Width Modulation Section and the Power Stage Section. The drive amplifier is a three phase drive for sinusoidally wound brushless d.c motors [14]. The voltage generated across each phase by turning the rotor at a constant speed will have a sinusoidal waveform. The magnitude of the generated voltage is proportional to the angular velocity of the rotor. Phase B and Phase C voltages lag Phase A by 120 and 240 electrical degrees. To develop constant torque from the motor, sinusoidal currents must be injected into each motor winding in phase with the generated voltages. The angular position of the rotor (absolute rotor position) is fed into the Commutation Section of the drive amplifier through the R/D converter. The angular position gets converted into three phase words in the drive amplifier. These three phase words are multiplied with the velocity error signal (from Velocity Control Loop Section) to generate the three phase current request signals. The current control loop compares the three phase current requests with the actual phase currents. The current error signal gets converted into three pulse trains of a constant frequency (Pulse Width Modulation Section), of magnitude of the current errors. The Power Stage Section converts the three P.W.M pulse trains into motor currents by amplifying the low level signals to large voltage pulses of 300V across the motor phase windings. The motor inductance causes the voltage pulses to produce smooth sinusoidal motor current [14]. The block diagram

The block diagram of the drive amplifier is given in fig 2.3 [5].

Unless the commutation is set properly, the motors will not provide constant torque. To initially set the commutation for the motor during the initial start up procedure and calibration of the resolver, the motor is fed a low voltage (25 V d.c), while the current wave form of Phase A is monitored using an oscilloscope. If the motor turns, a roughly sinusoidal waveform is seen. While monitoring the current on the scope, the resolver's stator is rotated manually until the minimum peak to peak amplitude is achieved. The motor's rotation is very smooth at this point. The power is then turned off. The stator of the resolver is then clamped/ locked, so as to not disturb the setting.

Figure 2.3: Block Diagram of Drive Amplifier

Chapter 3

Kinematics And Dynamics

3.1 Introduction

Dynamic performance and response of a robot is directly related to the kinematic structure and design of its links. Dynamic complexity such as coupling and non linearities are major concerns in the control of the manipulator arm [6].

Gear trains are used in robots for two reasons. First, as a mechanical transmission medium and secondly to amplify the torque put out by the motors to drive the links. The use of gears degrades the control performance of robots because of inherent problems such as backlash [15] (play between mating gears). Preloading the gears to eliminate backlash results in increased wear of the gears and excessive friction. Furthermore, constant adjustment and maintenance is required in robots using geared drives.

Direct drive robots do not have gears, thereby eliminating the mechanical inaccuracies in the system in terms of backlash, friction and reduced mechanical stiffness. Motors used for direct drive robots are designed to provide high initial torque at low operational speeds [8]. In direct drive robots the motor shaft is directly coupled to the input link. Due to this, the complex arm dynamics reflect directly on the motor axes [6]. Thus the varying inertial effects as well as the effects of the coupling and nonlinear torques are more pronounced for direct drive robots [6]. This creates difficult problems in design of the control system.

Robots have two types of kinematic structures. The open loop structure or serial manipulators and the closed loop structure for example the four bar and five bar mechanisms. A serial manipulator with more than one degree of freedom poses a lot of problems in terms of control. This is due to the fact that the drive motors coupled to the joints result in additional weight on the links increasing the overall inertia of the manipulator. Since the motors are mounted on each joint of a serial arm linkage, the weight of the motor of the end joint itself is a load for the next motor down the serial linkage resulting in increase in the drive torque required by the motors from the distal joint to the proximal joint [2].

3.2 A Dynamically Decoupled And Configuration Invariant Manipulator

The equation of motion of a manipulator is configuration dependant. In its more general form it is given by [16,4],

$$\tau_i = H_{ii}\ddot{\theta}_i + \sum_{j \neq i} H_{ij}\ddot{\theta}_j + \sum_j \sum_k \left[\frac{\partial H_{ij}}{\partial \theta_k} - \frac{0.5 \partial H_{jk}}{\partial \theta_i} \right] \dot{\theta}_j \dot{\theta}_k + \tau_{gi} \quad (3.1)$$

where,

τ_i = Torque of the i^{th} joint.

θ_i = Joint displacement of the i^{th} joint.

H_{ij} = The $i - j$ element of the manipulator inertia matrix.

τ_{gi} = Torque due to gravity.

The first term on the right hand side represents the inertia torque, the second term is the interactive torque (linear acceleration torque), and the third term is the nonlinear velocity torque due to Coriolis and centrifugal effects. The second term in equation (3.1) vanishes if the inertia tensor H is made diagonal. This is often referred to as decoupling. Furthermore, due to decoupling of the inertia tensor H , the effects of the nonlinear velocity torques get reduced. Equation (3.1) then reduces to,

$$\tau_i = h_{ii}\ddot{\theta}_i + \sum_k \left[\frac{\partial h_{ii}\dot{\theta}_i\dot{\theta}_k}{\partial \theta_k} - \frac{0.5 \partial h_{kk}\dot{\theta}_k^2}{\partial \theta_i} \right] + \tau_{gi} \quad (3.2)$$

It was shown by Asada and Youcef- Toumi [6] that it is possible to decouple manipulators having up to two degrees of freedom by design. This can either be done by actuator relocation or mass redistribution. If the inertia tensor does not change as a function of the manipulator geometry, then the third term also gets eliminated. This particular condition is achievable by employing a closed loop five bar parallelogram mechanism. Then equation (3.2) reduces to,

$$\tau_i = h_{ii}\ddot{\theta}_i + \tau_{gi} \quad (3.3)$$

The equation of motion of an inertially decoupled five bar parallelogram manipulator moving in the horizontal plane is given by,

$$\tau_i = h_{ii}\ddot{\theta}_i \quad (3.4)$$

3.3 Kinematic Structure Of A Five Bar Mechanism (With The Drive Motors Coaxial)

The planar (horizontal) direct drive robot built at Carleton University (refer to Plate 1) employs a five bar linkage (closed loop) mechanism.

Two coaxial direct drive motors fixed on a common non movable vertical support are used to drive the two input links, l_1 and l_2 . The active links being l_3 and l_4

Plate 1: Normal Mode

(which extends to l_5 or the boom). The gripper is attached to the output end of l_5 . To maintain a parallelogram structure the link lengths $l_1 = l_2$ and $l_3 = l_4$. Refer to Table 1 for details of link lengths, and cross section of each link.

3.4 Kinematic Equations OF A Five Bar Mechanism (With Drive Motors Coaxial)

As shown in Figure 3.1, l_1 and l_2 are the two input links while l_3 and l_4 are the active links of the five bar closed loop manipulator. Point $P(x,y)$ is the arm tip of the manipulator. M_1 is the base direct drive motor coupled directly to link l_1 . M_2 is the top direct drive motor also coupled directly, to link l_2 . The angle link l_1 makes with respect to the X- Axis is θ_1 . The angle link l_2 makes with respect to the X- Axis is θ_2 . The angle link l_4 makes with respect to the X- Axis is ϕ_1 . The angle link l_3 makes with respect to the X- Axis is ϕ_2 .

3.4.1 Forward Kinematics

The problem of finding the end effector's position and orientation for a given set of joint displacements is referred to as the forward kinematic problem [5]. The kinematic equations relating the end effector position $P(x,y)$ to the joint displacements (θ_1, θ_2) are given by,

LINK	CENTRE DISTANCE	OVERALL LENGTH	CROSS SECTION (MM)
LINK 1	12 °	15 °	76.2 X 38.1 X 2.0 THK 9.05
LINK 2	10 °	13 °	60.8 X 38.1 X 1.5 THK 9.05
LINK 3	12 °	15 °	60.8 X 38.1 X 1.0 THK 9.05
LINK 4	10 °	11.62 °	76.2 X 32.3 X 2.0 THK 9.05
BOOM	14 °	14 °	76.2 X 25.4

Table 1: Link Specification Table

Figure 3.1: A Five Bar Link Mechanism

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 - (l_5 - l_4) \cos \phi_1 \\ l_1 \sin \theta_1 - (l_5 - l_4) \sin \phi_1 \end{bmatrix} \quad (3.5)$$

3.4.2 Inverse Kinematics

The problem of finding the joint displacements that lead the end effector to the specified position and orientation is referred to as the inverse kinematics problem.

This is done by solving the forward kinematic equations to get θ_1 and ϕ_1 . Due to the geometry of the structure (i.e. $l_1 = l_3$ and $l_2 = l_4$) $\phi_1 = \theta_2$ and $\phi_2 = \theta_1$. The derivation of the inverse kinematics of this manipulator is given in Appendix D. The results are,

$$\phi_1 = \sin^{-1} \left[\frac{l_1^2 - [x^2 + y^2 + (l_5 - l_4)^2]}{2(l_5 - l_4)\sqrt{x^2 + y^2}} \right] - \sin^{-1} \left[\frac{x}{\sqrt{x^2 + y^2}} \right] \quad (3.6)$$

$$\theta_1 = \cos^{-1} \left[\frac{x^2 + y^2 - l_1^2 - (l_5 - l_4)^2}{-2l_1(l_5 - l_4)} \right] + \phi_1 \quad (3.7)$$

From the inverse kinematic equations (3.6) and (3.7), it can be seen that there is more than one solution for the angles ϕ_1 and θ_1 . There are four possible modes to reach the same end point (refer to Figure 3.2), the constraining factor being the vertical support column that holds the two motors. Each of these modes cover a different workspace, adding to the versatility of the manipulator.

Figure 3.2: Different Modes of the Carleton Robot

3.5 The Jacobian

The Jacobian matrix represents the transformation between link and cartesian velocities [7].

$$dx = Jd\theta \quad (3.8)$$

where the Jacobian matrix J is given by,

$$J = \begin{bmatrix} J_{11} & J_{21} \\ J_{12} & J_{22} \end{bmatrix} \quad (3.9)$$

The Jacobian also relates the end point forces with the corresponding joint torques [8].

$$\tau = J^T F_e \quad (3.10)$$

where J^T is the transpose of J and F_e is the environmental force.

In order to maintain the closed kinematic structure, the two input angles θ_1 and θ_2 (refer to Figure 3.1), and the two output angles ϕ_1 and ϕ_2 must satisfy the following constraint equations,

$$l_1 \cos \theta_1 + l_4 \cos \phi_1 = l_2 \cos \theta_2 + l_3 \cos \phi_2 \quad (3.11)$$

$$l_1 \sin \theta_1 + l_4 \sin \phi_1 = l_2 \sin \theta_2 + l_3 \sin \phi_2 \quad (3.12)$$

To obtain the Jacobian, equations (3.5), (3.11), and (3.12) have to be differentiated in terms of θ_1 , θ_2 , ϕ_1 , and ϕ_2 and then eliminating $\partial \phi_1$ and $\partial \phi_2$ from the differential equations to get the relation between $dX = (dx \ dy)^T$ and $d\theta = (d\theta_1 \ d\phi_1)^T$. The results obtained (refer to Appendix B for detailed derivation) are,

$$J_{11} = -l_1 \sin \theta_1 - \frac{(l_5 - l_4) \sin \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \quad (3.13)$$

$$J_{12} = \frac{(l_5 - l_4) \sin \phi_1 l_2 \sin(\phi_2 - \theta_2)}{l_4 \sin(\phi_2 - \phi_1)} \quad (3.14)$$

$$J_{21} = l_1 \cos \theta_1 + \frac{(l_5 - l_4) \cos \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \quad (3.15)$$

$$J_{22} = \frac{-(l_5 - l_4) \cos \phi_1 l_2 \sin(\phi_2 - \theta_2)}{l_4 \sin(\phi_2 - \phi_1)} \quad (3.16)$$

Each mode of the manipulator is associated with a different Jacobian, which relates to different force velocity characteristics. Thus this manipulator is more versatile as compared to conventional manipulators. It can be shown[9] that the force velocity

characteristics of a five bar parallelogram mechanism (with drive motors coaxial) are poor as compared to the same manipulator with its drive motors not coaxial. But the workspace of the latter is reduced.

The following conclusions can be drawn from the Jacobian,

1. The Jacobian does not exist when $\sin(\phi_2 - \phi_1) = 0$. When this condition is achieved all the movable links of the manipulator get aligned. At this particular instant the manipulator loses one of its degrees of freedom. This is known as a singularity. Singularity is the boundary of the manipulator's workspace. To switch to another mode the manipulator has to go through a singularity.
2. The Jacobian matrix becomes singular (i.e. its determinant is equal to zero) when any of these two conditions are satisfied. This happens when,

$$(a) \sin(\phi_2 - \phi_1) = 0$$

$$(b) \cos \phi_1 = 90^\circ \text{ or } 270^\circ \text{ and } \cos \theta_1 = 90^\circ \text{ or } 270^\circ$$

The Jacobian as said earlier, also relates the end point forces with the corresponding joint torques therefore,

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} F_{ex} \\ F_{ey} \end{bmatrix} \quad (3.17)$$

where,

$$\tau_1 = J_{11}F'_{cx} + J_{12}F'_{cy} \quad (3.18)$$

and

$$\tau_2 = J_{21}F_{cx} + J_{22}F_{cy} \quad (3.19)$$

The equations (3.18) and (3.19) in their full form are

$$\begin{aligned} \tau_1 = & \left[-l_1 \sin \theta_1 - \frac{(l_5 - l_4) \sin \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] F_{cx} \\ & + \left[\frac{(l_5 - l_4) \sin \phi_1 l_2 \sin(\phi_2 - \theta_2)}{l_4 \sin(\phi_2 - \phi_1)} \right] F_{cy} \end{aligned} \quad (3.20)$$

$$\begin{aligned} \tau_2 = & \left[l_1 \cos \theta_1 + \frac{(l_5 - l_4) \cos \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] F_{cx} \\ & + \left[\frac{-(l_5 - l_4) \cos \phi_1 l_2 \sin(\phi_2 - \theta_2)}{l_4 \sin(\phi_2 - \phi_1)} \right] F_{cy} \end{aligned} \quad (3.21)$$

3.6 Link Motion

Due to the structural geometry of the five bar (closed loop) mechanism manipulator, rotation of the base motor M_1 (refer to Figure 3.3), which is directly coupled to link l_1 will cause only translational motion of link l_5 . Rotation of the top motor M_2 which is directly coupled to link l_2 will cause angular motion of link l_5 . Cross drive is also possible (refer to Plate 2).

Figure 3.3: Link Motion

Plate 2: Cross Drive Mode

3.7 Dynamic Equations Of A Five Bar (Closed Loop) Manipulator Moving In A Horizontal Plane

To derive the equations of motion using Lagrange equations, the kinetic energy **K** for each link has to be written. The potential energy **P** is zero since the manipulator moves in the horizontal plane.

By basic definition, the lagrangian **L** is defined as the difference between the kinetic and the potential energies of the system or,

$$\mathbf{L} = \mathbf{K} - \mathbf{P} \quad (3.22)$$

In this case $\mathbf{L} = \mathbf{K}$ as $\mathbf{P} = 0$

Furthermore since the link lengths (centre to centre) $l_1 = l_3$ and $l_2 = l_4$, Figure 3.1 reduces to figure 3.4, where l_4 is the boom length and l_{1c} , l_{2c} , l_{3c} and l_{4c} are the mass centres of the respective links. The angle that the input link l_1 , makes with the Y Axis is θ_1 . The angle that the input link l_2 , makes with the Y Axis is θ_2 .

The equations of motion for a five bar structure were derived by Rivin [22] for a robot moving in the vertical plane. The same equations can be adapted to the present case by simply equating the potential energies to zero (i.e. $g = 0$).

The equations of motion of the structure (refer to Appendix C) were found out

Figure 3.4: Modified Figure 3.1

to be,

$$\begin{aligned}\tau_1 = & (m_1 l_{1c}^2 + I_1 + m_3 l_{3c}^2 + I_3 + m_4 l_1^2 + m_0 l_1^2) \ddot{\theta}_1 \\ & + (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c}) \cos(\theta_1 - \theta_2) \ddot{\theta}_2 \\ & + (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c}) \sin(\theta_1 - \theta_2) \dot{\theta}_2^2\end{aligned}\quad (3.23)$$

$$\begin{aligned}\tau_2 = & (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c}) \cos(\theta_1 - \theta_2) \ddot{\theta}_1 \\ & + (m_2 l_{2c}^2 + I_2 + m_3 l_2^2 + m_4 l_{4c}^2 + I_4 + m_0 l_{4c}^2 + I_0) \ddot{\theta}_2 \\ & - (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c}) \sin(\theta_1 - \theta_2) \dot{\theta}_1^2\end{aligned}\quad (3.24)$$

The above equations can be written in a better form as

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & H_{122} \\ H_{211} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix}\quad (3.25)$$

or as,

$$\tau_1 = H_{11} \ddot{\theta}_1 + H_{12} \ddot{\theta}_2 + H_{122} \dot{\theta}_2^2$$

$$\tau_2 = H_{21} \ddot{\theta}_1 + H_{22} \ddot{\theta}_2 + H_{211} \dot{\theta}_1^2$$

such that,

1. $H_{11} \ddot{\theta}_1$, and $H_{22} \ddot{\theta}_2$ are the effective inertia terms.

2. $H_{12}\ddot{\theta}_2$, and $H_{21}\ddot{\theta}_1$ are the coupling inertia terms.

3. $H_{122}\dot{\theta}_2^2$, and $H_{211}\dot{\theta}_1^2$ are the centrifugal force terms.

where,

$$H_{11} = m_1 l_{1c}^2 + I_1 + m_3 l_{3c}^2 + I_3 + m_4 l_1^2 + m_0 l_1^2$$

$$H_{22} = m_2 l_{2c}^2 + I_2 + m_3 l_2^2 + m_4 l_{4c}^2 + I_4 + m_0 l_{4c}^2 + I_0$$

$$H_{12} = H_{21} = m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c} \cos(\theta_1 - \theta_2)$$

and,

$$H_{122} = H_{211} = m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c} \sin(\theta_1 - \theta_2)$$

From equations (3.23) and (3.24) the following conclusions can be drawn.

1. The Coriolis terms are absent i.e. the combination of terms of the form $H_{iij}\dot{\theta}_1\dot{\theta}_2 + H_{iji}\dot{\theta}_j\dot{\theta}_i$ are not present.
2. Also the number of nonlinear terms in the equation of motion for a five bar (closed loop) manipulator with two degrees of freedom moving in the horizontal plane are substantially less as compared to an open loop manipulator with two degrees of freedom also moving in the horizontal plane.

3.7.1 Decoupling Conditions

As seen from the above equations of motion, the terms $m_2 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_{4c}$ are common to both the coupling inertia terms and the centrifugal force terms.

If the mass of the payload m_0 is not considered, then strictly by proper mechanical design process this structure can be decoupled. The decoupling condition being,

$$m_3 l_2 l_{3c} - m_4 l_1 l_{4c} = 0 \quad (3.26)$$

Also due to the above condition, the centrifugal force gets eliminated. The equations of motion then reduce to,

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} \quad (3.27)$$

or as,

$$\tau_1 = H_{11} \ddot{\theta}_1$$

and,

$$\tau_2 = H_{22} \ddot{\theta}_2$$

In other words the manipulator is dynamically decoupled and configuration invariant. Thus the manipulator can perform high speed tasks more effectively without losing its accuracy and speed.

Chapter 4

Link, Motor Housing And Gripper Design

4.1 Introduction

As seen in the previous chapter, the equation of motion of a manipulator is configuration dependent. It was also shown [3,4,5,6] that it is possible to eliminate the nonlinear effects such as the Coriolis, centrifugal and the coupling inertia terms for a closed loop manipulator (five bar linkage). The condition obtained for a dynamically decoupled and configuration invariant manipulator (five bar linkage) is,

$$m_3 l_2 l_{3c} - m_4 l_1 l_{4c} = 0$$

It is possible to satisfy the above condition strictly by the design approach.

The mechanical aspect of the robot can be broken into three categories. They are, design of the motor housings, design of the links, and the design of the end effector. The overall design considerations were cost, availability, rigidity, ease of assembly and disassembly (because the manipulator will be used as a research tool by graduate and undergraduate students). The secondary considerations were weight and decoupling.

4.2 Motor Housing Design

4.2.1 Introduction

Mechanical inaccuracies in a robot in terms of backlash, friction, and reduced mechanical stiffness degrade its performance. Gear backlash can be eliminated by using antibacklash gears, preloaded gears, and harmonic drives. Though antibacklash gears, and preloading gears does eliminate backlash, the friction between the mating gears is increased. Due to this, the gears wear out faster. Constant adjustment and maintenance is required. Harmonic drives also eliminate backlash. However, reduced mechanical stiffness and fluctuation in torque transmitted reduces the efficiency of the system [4].

In direct drive technology, the motor (usually brushless d.c. or variable reluctance motors are used) shaft is directly coupled to the input link of the manipulator. There is no gearing involved, therefore the mechanical inaccuracies in the system (backlash, friction, and reduced mechanical stiffness) are eliminated. Thus the performance of

the manipulator is improved. Each one of the motor housings for the Carleton Robot contains a brushless d.c motor, a tachometer, and a resolver.

4.2.2 Brushless D.C Motor

The brushless d.c motor consists of a stator and a rotor. The rotor carries the permanent magnets and the stator carries the windings. Commutation is done electronically. The resolver is directly coupled to the rotor shaft. It transmits the rotor position at every instant to the drive amplifier which supplies the appropriate current to the appropriate stator windings. Since the brushes and commutators are eliminated it is possible for the servomotor to operate at higher speeds.

The brushless d.c. motors were purchased from Motion Control Systems. These motors (B09 - 51) provide a Peak Torque of 210 lb-in (24 N-m), and a Constant Stall Torque of 36 lb-in (4 N-m) [14]. The physical dimensions of the stator and the rotor of the motor are given in Table 2. The air gap between the stator and rotor is 1.15 mm. Therefore the machining of the motor housing parts has to be precise. Any deviation in the specified tolerance of the parts will result in mechanical interference between the stator and rotor.

4.2.3 Choice Of Mounting Method

There are numerous methods for securing frameless motor stators and rotors (listed in Table 3). Each method has its own advantages and disadvantages. Factors like

	STATOR	ROTOR
INTERNAL DIA (mm)	53.2	40.6
EXTERNAL DIA (mm)	88.9	60.9
LENGTH (mm)	70.9	61.6
AIR GAP (mm)	1.16	

Table 2: B.D.C Motor Dimensions

the frame size, the type of cooling to be used, ease of assembly and disassembly, differential thermal expansion and reversal torque levels [14] play an important role in deciding the type of mounting to be used (refer Figure 4.1).

Axial Clamping was chosen as a mounting means for securing the frameless stator. Axial clamping provides high strength and it is easy to assemble and disassemble. The latter is the main reason for choosing this type of mounting method. In this method the stator is clamped between two annular shoulders. One of these two shoulders is an integral part of the motor housing (refer to the motor assembly drawing - part no 17). The other shoulder is a hollow cylindrical motor clamp (refer to the Motor Assembly Drawing - part no 18). The fit between the stator and the main housing

TYPE OF MOUNTING		ADVANTAGES	DISADVANTAGES
STATOR	AXIAL	1. HIGH STRENGTH 2. EASE OF ASSEMBLY AND DISASSEMBLY	1. COSTLY TO MFG 2. PRECISE MACHINING REQUIRED
	RADIAL	1. HIGH STRENGTH 2. EASY DISASSEMBLY	1. EXCESSIVE WEIGHT 2. LARGE SIZE
	CHEMICAL BONDING	1. SIMPLICITY 2. COMPACT/ LIGHTWEIGHT 3. EASE OF ASSEMBLY	1. DESTRUCTIVE DIS-ASSEMBLY 2. NOT RELIABLE AT HIGH TEMPERATURE
	AXIAL CLAMPING	1. EASE OF ASSEMBLY AND DISASSEMBLY 2. HIGH STRENGTH	1. COSTLY TO MFG
	KEYING	1. EASE OF ASSEMBLY AND DISASSEMBLY 2. COMPACT/ LIGHTWEIGHT	1. NOT RECOMMENDED FOR SHOCK LOADING
ROTOR	CHEMICAL BONDING	1. LOW COST 2. EASE OF ASSEMBLY 3. COMPACT SIZE	1. DESTRUCTIVE DIS-ASSEMBLY
	BOLTING FLANGE	1. LOW COST 2. EASE OF ASSEMBLY AND DISASSEMBLY	1. ACCURATE LOCATING HOLES REQUIRED

Table 3: Motor Mounting Methods

Figure 4.1: Motor Mounting Chart

(part no - 17) is a sliding fit. The motor clamp is forced against the stator's shoulder with help of the bearing housing (part no - 19). As an added measure of safety, the motor clamp is further tightened against the stator by using four setscrews 90° apart. This is done to make sure that the stator will stay stationary under vibrations, or if by accident one of the input links impacts against the vertical support column. The stator is located radially by the inner bore of the housing. Thus the inner bore of the main housing has to be precisely machined.

The rotor of the brushless d.c motor is axially clamped on the motor shaft (refer to Figure 4.2 - part nos 5 and 3). The reasons for choosing this sort of mounting method are, high strength and ease of assembly and disassembly. In the axial method for rotor mounting, one end of the rotor rests against the shoulder of the shaft. A locknut is used to secure the other end. Furthermore, the locknut is locked onto the shaft as an added measure of safety. This ensures that the locknut does not loosen.

4.2.4 Choice Of Bearings

To hold the shaft, two S.K.F double row deep groove bearings (No - 4206 A) were chosen. These bearings are cheap and are readily available as compared to high precision bearings of the same kind. These bearings can carry axial as well as radial loads [19]. Furthermore, these bearings have a low noise level, low frictional resistance ($\mu = 0.0015$) and low internal heat generation at high rotational speeds.

The shaft is directly coupled to the input link. Graduate and undergraduate

Figure 4.2: Motor Assembly Drawing

students will be running experiments to control the manipulator motion. There is a high probability that the input link will impact against the vertical column support. Thus the shaft diameter dictated the bearing size.

4.2.5 Shaft - Link Coupling

The shaft of the motor housing is made of M.S - 1020. This material was cheap and readily available in the workshop. The material has a yield strength of 43 kpsi and a tensile strength of 65 kpsi [20]. Internal boring was one way of reducing the weight of the shaft. However due to the cost and time limitations this idea was dropped.

There are many ways to couple the shaft and the hub (the input link in this specific case). The most commonly used methods are either using key or splines. A key connection was chosen because it was cost effective and met the specific design requirements. In this particular case the shaft is directly coupled to the input link (l_1 or l_2) by means of a square end milled key of dimensions 5mm x 5mm x 25mm. The key was made of M.S - 1020. As shown in Appendix E, the key is strong enough to withstand shear and crushing.

4.2.6 Tachometer And Resolver Mounting

The tachometer used is an analog brush tachometer. The tachometer is composed of a stator and a rotor. The only contact between the stator and rotor are four spring loaded brushes. These brushes pick up the voltage generated due to rotation of the

shaft. The generated voltage is directly related to the velocity of the shaft. The physical dimensions of the tachometer are given in Table 4. The air gap between the stator and rotor of the tachometer is 0.45 mm. Refer to the motor assembly drawing (part nos 8,9 and 21) for attachment details.

The resolver used is an analog brushless d.c resolver. The resolver is composed of a stator and a rotor. The air gap between the stator and rotor of the resolver is 0.325 mm (refer to table 4 for physical dimensions). As was mentioned earlier, commutation is brought about in a brushless d.c motor electronically by means of an external resolver. Each of the brushless d.c motors purchased from Motion Control Systems, comes with a drive amplifier (S.C.M - 150/300). This servo control module has a built in commutation section. The S.C.M 150/300 is a three phase drive for the b.d.c. motors [14].

The robot will be used for research, therefore the resolver will be calibrated frequently to ensure the validity of the experimental result. As such it was important that researchers have easy access to the resolver. For ease of access, the resolver was strategically placed at the extreme end of the motor assembly (refer to the motor assembly drawing - part nos 10, 12, 22 and 11). Furthermore, the resolver housing that holds the stator of the resolver is designed in such a way that the stator can be rotated manually by 180° . This facilitates in the process of calibration of the resolver (electronic commutation process) as the zero point can be set at random. The fit between the resolver housing and the stator's external diameter is a sliding fit. Once

TACHOMETER DIMENSIONS

	STATOR	ROTOR
INTERNAL DIA (mm)	53.2	40.6
EXTERNAL DIA (mm)	88.9	50.9
LENGTH (mm)	70.9	51.5
AIR GAP (mm)	1.15	

RESOLVER DIMENSIONS

	STATOR	ROTOR
INTERNAL DIA (mm)	33.45	15.90
EXTERNAL DIA (mm)	52.3	32.7
LENGTH (mm)	27.2	32.2
AIR GAP (mm)	0.325	

Table 4: Tachometer and Resolver Dimensions

the commutation is set the end cover (refer to the motor assembly drawing - pc no 11) is clamped on, so as to not disturb the setting.

4.3 Link Design

4.3.1 Introduction

Robots move at high speeds, especially direct drive robots. This is due to the fact that the input link is coupled directly to the motor shaft. Most of the industrial robots available on the market have an arm tip acceleration up to 0.5G [4]. Direct drive robots are shown to have a maximum acceleration at the arm tip of over 5G. For positional accuracy of the the arm tip, the links must be rigid. It was mentioned earlier, the secondary design considerations were weight and decoupling.

4.3.2 Choice Of Link Material

The basic factors that governed the choice of material for the links of the robot were, ease of availability and cost. The secondary factors were weight and strength. Aluminum - 6061 - T4 was chosen as the material for the links of the manipulator. This material is readily available in the market. Furthermore, this material can be obtained in sheet and plate form. The chosen material offers a good range of mechanical properties. Its tensile strength is 240 MPa (35 ksi), its yield strength is 145 MPa (21 ksi), and its Youngs Modulus is 70,000 MPa [21]. Since the density of

the material is approximately 2.79 gms/cm^3 , its weight is $1/3$ as compared M.S -1020. Also this particular material is used in a wide variety of products ranging from screw machine parts to structural components. High strength steel is another alternative.

4.3.3 Link Details

As mentioned in Chapter 3, there are four links, such that l_1, l_2, l_3 and l_4 (link l_4 extends to the arm tip). A choice had to be made in the type of cross section to be used for the link. There were two possibilities in the type of cross section to be used. The first one was to have the links made of a hollow circular or a hollow rectangular cross section. Generally the links in serial manipulators have one of the above two cross sections. The internal hollow area can be used to provide conduits for electric power, communication cables, hoses, power transmitting components and control rods [22]. Due to the structural geometry of the five bar linkage, it would be difficult to route cables or control rods through the hollow cross section. Furthermore, bearing mounts at the either ends of the link would have to be welded. Constraints from the workshop in terms of fabrication and time were the limiting factors. Also aesthetic appeal would be lost.

The other alternative, which was finally chosen, was to fabricate the links out of plates. This gave flexibility in terms of deciding the shape of the links, type of cross section, machining and aesthetic appeal. Also the links could be overdesigned and then if required, the excess material could be removed. All the links l_1, l_2, l_3 and l_4

were given a H cross section. Refer to Appendix A for detailed link drawings. Table 5 gives their individual masses and approximate centers of gravity.

The connections between the motors and links are given in Figure 4.3. Since the structure is a parallelogram ($l_1 = l_3, l_2 = l_4$), the center distances of the pivot points of the links have to be maintained. Failure in maintenance of the center distances will cause inherent mechanical errors, as the end point actually reached by the arm tip will be different from the one specified. To eliminate this problem, the workshop was told to mark and machine the links in pairs so that the center distances would be maintained.

4.3.4 Approximate Deflection Analysis Of The Input Links

Since the direct drive robot is capable of achieving acceleration in excess of 4G, the links must be rigid. An approximate analysis is done to check the elastic deflection of the input links. In this analysis the two input links were considered to have a uniform H cross section over their entire length (center to center). Refer to Figure 4.4 for details. The Servo Control Module is capable of handling upto 10 *amps_{rms}* of current. The torque put out by the motor is given by [14],

$$\tau = i_{rms} \times \text{Torque const}$$

$$\tau = 10 \times 0.442 = 4.42 \text{ N.m}$$

also,

$$\tau = I_z \times \ddot{\theta}$$

LINK NOS	APPROX MASS (KGS)	APPROX C.G. (CMS)
LINK 1	2.17	$\bar{X} = 19.05$ $\bar{Y} = 0$ $\bar{Z} = 3.81$
LINK 2	1.37	$\bar{X} = 16.61$ $\bar{Y} = 0$ $\bar{Z} = 2.54$
LINK 3	1.40	$\bar{X} = 19.05$ $\bar{Y} = 0$ $\bar{Z} = 2.54$
LINK 4	2.69	$\bar{X} = 16.87$ $\bar{Y} = 0$ $\bar{Z} = 3.81$
BOOM	2.21	$\bar{X} = 16.71$ $\bar{Y} = 0$ $\bar{Z} = 3.81$

Table 5: Link Masses and C.G's

Figure 4.3: Coupling Chart

Link 1	Link 2
$c = 0.02\text{m}$	$c = 0.020\text{m}$
$h = 0.076\text{m}$	$h = 0.059\text{m}$
$b = 0.0381\text{m}$	$b = 0.0381\text{m}$
$d = 0.0281\text{m}$	$d = 0.0179\text{m}$
$L = 0.304\text{m}$	$L = 0.254\text{m}$

Figure 4.4: Link Details for Approximate Deflection Analysis

Consider the input link l_2 to be directly coupled to the motor shaft. The mass moment of inertia about the z Axis through the pivot point of the input link l_2 is 0.018 kg.m^2 . Therefore $\ddot{\theta}$ is 245 rad/sec^2 .

Deflection of a revolute link is given by [22],

$$y = \frac{7\gamma\ddot{\theta}L^5}{6EI} \quad (4.1)$$

where,

$$\gamma = \text{mass/unit length} = 0.339 \text{ kg/m}$$

$$E = \text{Youngs Modulus for Aluminum- 6061} = 7 \times 10^9 \text{ Pa}$$

$$\ddot{\theta} = \text{angular acceleration} = 245 \text{ rad/sec}^2$$

$$I = \text{area moment of inertia of the link C.S} = 2.1 \times 10^{-7} \text{ m}^4$$

The deflection of the input link l_2 is 0.69 mm . Similar analysis can also be done for input link l_1 . The deflection of link l_1 is found out to be 0.79 mm . This approximate analysis shows that the links are quite rigid even at the maximum achievable acceleration.

4.3.5 Link Coupling

As was mentioned earlier, graduate and undergraduate students will be using the manipulator as a research tool. The design considerations here were minimal friction, and ease of assembly and disassembly of the link joints. The links are coupled to each other using spacer and bearing joints. Refer to Figure 4.5 for link assembly sectional

details. Single row deep groove bearings (No 6005) were chosen. These bearings were cost effective and were easily available. Also, they are capable of carrying axial as well as radial loads. Furthermore, they are suitable for high speed operation and require little attention in service [19]. The bearings are preloaded (removal of play between the balls and the race) by tightening the spacer nut. Preloading improves the accuracy of rotation. A spacer was introduced between the bearings to prevent excessive tightening of the nut, which may damage the bearings.

4.3.6 Decoupling

As was mentioned earlier, decoupling was a secondary design consideration. The decoupling condition is,

$$m_3 l_2 l_{3c} - m_4 l_1 l_{4c} = 0$$

where,

m_3 = Mass of link l_3

l_2 = Length of link l_2 (center to center)

l_{3c} = Mass center of link l_3

l_1 = Length of link l_1 (center to center)

l_{4c} = Mass center of the boom.

The robot in its present state is not decoupled. Graduate and undergraduate students will test experimental control algorithms on this nonlinear system.

Figure 4.5: Link Assembly Drawing

To decouple the present manipulator, link l_3 will have to be fabricated as per the specifications in figure 4.10 (Drawing A). The mass of this link is approximately 1.7 kgs. Therefore,

$$1.7 \times 25.4 \times 15.24 = m_4 \times 30.48 \times 17.78$$

$$m_4 = 1.21 \text{ kgs.}$$

The present mass of the boom is 2.21 kgs. Therefore for decoupling 1 kg of material must be removed from the boom. This can be done by removing 1.1" of material from the height of the boom.

4.4 Gripper Design

4.4.1 Introduction

The term end effector is a generic word for all systems mounted at the end of the robot whose task is to grip objects, or tools and/ or transfer them from place to place [23]. The tool may be either held by a gripper or mounted directly on the wrist mounting surface. Supporting action, Mechanical Clamping, Vacuum cups and Electromagnets are the most common means employed to pick and hold objects [13].

Mechanical grippers like the jaw type and the finger type are the most common means employed to pick and hold objects. In either case the gripper exerts a clamping force on the object, either by expanding within the inside of the object or closing on

Figure 4.6: Drawing A

it from the outside. The frictional force required to hold the object must be greater than the total of all the forces being applied [24]. The relation between frictional force and clamping force is given by $F_f = \mu \times F_{bc}$, where μ is the coefficient of friction. Limitations of the clamping force is required for safety reasons [24]. Tactile sensors can be used to limit the clamping force [24] so that the jaws or fingers can hold and not crush the object. Limit switches or strain gauges can be used for this purpose.

4.4.2 Design Considerations

While designing the gripper the following points have to be taken into consideration [24].

1. **Overall length of the jaws.** This is decided by the type of object to be handled. The jaws have to be short and wide to handle rectangular objects, long and slender for objects like a wine glass.
2. **Shape of the jaws.** Depends upon the shape of the object. The jaws have to be contoured if it is to hold circular objects (to obtain more contact area) to enhance its grasping capability.
3. **Material.** The chosen material must have low density, and a high strength to weight ratio. Aluminum alloys are most commonly used. Also, it must be cost effective and readily available.

4. **Surface contact area.** A heavy object can be handled well if the area of surface contact of the jaws is large. Also, it is always better to grip the object close to its center of gravity as possible to reduce slippage while the manipulator is in motion [24].

4.4.3 Design Requirements

It was specified that the jaws of the gripper are to have a maximum opening of 3". No external gearing was needed because the provided bidirectional d.c. motor had a built in speed reducer of ratio 20:1. Furthermore, the gripper is required to hold the object without crushing or damaging it. Undergraduate students are presently working on the aspect of tactile sensing and control of the jaw motion.

4.4.4 Actual Design

After some experimentations a graph of r.p.m versus motor input voltage was plotted (refer to Figure 4.7, Graph 1). The graph indicated that the maximum operating speed within the specified voltage (± 38 V) of the motor was 120 r.p.m.

With this data it was felt that the simplest mechanism for obtaining the open and close action of the jaws would be a lead screw mechanism. Linkage mechanism employing gears is another alternative. Lead screw mechanisms are basically used for converting rotary motion into linear travel of its follower nut, by preventing the follower nut from rotating by means of a stabilizer rod. Ballscrews can be used instead

Figure 4.7: Graph 1

of leadscrews to reduce the friction and improve the response. Due to cost limitations the idea of using ballscrews was dropped.

In order to adapt the leadscrew mechanism to obtain the open and close motion of the jaws of the gripper, one of the jaws had to be fixed while the other could traverse the leadscrew. The follower nut made of brass (to reduce friction) was press fitted into the movable jaw. To restrain the rotation of the movable jaw in order to obtain linear travel a stabilizer rod, made of M.S - 1020 was used (refer to Figure 4.9). Another press fitted brass bush located in the movable jaw (part no 10) reduces the friction between the stabilizer rod and the jaw. A linear bearing is a good alternative.

The bidirectional d.c. motor is directly coupled to the leadscrew. Two single row deep groove bearings are used to facilitate the leadscrew rotation with minimum friction. The two jaws of the gripper are flat and have a contact area of 1.236 in^2 or 800 mm^2 (refer to pc - 14). Rubber bumpers can be used to provide a better gripping surface.

Leadscrews of different nominal diameters have different standard threads / inch (t.p.i). Acme thread form is generally used in leadscrews as it offers high strength and low frictional resistance [26]. For a single start thread, the lead is equal to the pitch. Furthermore, the pitch is equal to $1 / \text{t.p.i}$. Calculations were done for maximum attainable speed using different single start acme threads for a fixed motor r.p.m (100 r.p.m, refer Table 6). It was found that the $5/8'' - 8$ ACME thread gave fast linear travel of the jaw without being too bulky (refer to Figure 4.8, Graph 2).

ACME AND STUB THREADS
(ANSI B1.5-1977)

NOMINAL DIA (in)	THREADS/INCH	PITCH (in)	LINEAR TRAVEL (IN/MIN) AT 100 R.P.M
1/4	16	.062	6.2
5/16	14	.07	7
3/8	12	.08	8
7/16	12	.08	8
1/2	10	0.1	10
5/8	8	0.125	12.5
3/4	6	.16	16

Table 6: Leadscrew Selection Table

Figure 4.8: Graph 2

For a $5/8" - 8$ ACME leadscrew, the pitch = $0.125"$ or 3.175 mm. Therefore the movable jaw will linearly travel at a maximum rate of $12.5" / \text{min}$ or $0.2" / \text{sec}$. The minimum opening or closing speed of the gripper is approximately $0.002" / \text{sec}$ (at 1 r.p.m). Thus the gripper speed can be manipulated to obtain very good force control in picking an object, without deforming or crushing it.

4.4.5 Wrist Roll

Since this gripper has been designed for a direct drive robot with two degrees of freedom, both being in the horizontal plane it becomes imperative that the wrist be given at least one degree of freedom.

Roll was chosen as this degree of freedom (rotation about the vertical axis). This d.o.f gives the robot the ability to grasp the object and rotate it. As an added advantage, trajectory planning becomes more simplified as the angle of attacking the object is increased by 360° (refer to Figure 4.9. Gripper Assembly Drawing for details).

The total mass of the gripper with two d.o.f is approximately 1.6 kgs, the two d.c. motors weighing a total of 600 gms being the major weight contributors.

Figure 4.9: Gripper Assembly Drawing

Chapter 5

Summary and Conclusions

This thesis had two main objectives. The first objective was to design and manufacture a robot at Carleton University. This robot would serve as a research tool for graduate and undergraduate students in the area of robotics. The second objective was to design a dynamically decoupled and configuration invariant direct drive robot. Both of the above objectives were successfully achieved.

Robots are used in industries to increase the productivity rate. Direct drive robots have speeds and accelerations beyond the reach of conventional robots. Also, some of the main advantages of using direct drive technology are, elimination of backlash, elimination of friction, and high mechanical stiffness. These inherent mechanical inaccuracies if present, degrade the performance of the robot.

The main disadvantage of direct drive technology is that the complex arm dynamics of the manipulator reflect directly on the motor axes. Thus, for effective use

of direct drive technology, the manipulator must be designed to be dynamically decoupled and configuration invariant. Also, as in the case of serial drive manipulators, the motor size increases from the distal link to the proximal link. Due to this, the manipulator's arm becomes heavy and the payload capacity is reduced.

The elimination of the nonlinearities in terms of coupling torques, Coriolis, and centrifugal force terms is important especially in direct drive robots where speed and accuracy are prime concerns. In the Carleton robot, a five bar linkage mechanism was employed. Based on the dynamics of the manipulator, the equations of motion were derived. By following proper mechanical design procedure, the configuration variance terms, and the interactive coupling torque terms were eliminated. In other words the nonlinearities in the arm dynamics were eliminated. Thus the control performance of the manipulator is vastly improved.

Further work can be suggested in the area of deflection analysis of the links using the finite element method, and comparing the validity of these results with actual experimental results. The two motor assemblies can be utilized in the design of a dynamically decoupled configuration variant serial (SCARA configuration) manipulator.

Appendix A

SPACER-A (MILD STEEL)
QTY-INOS

SPACER-B (MILD STEEL)
QTY-INOS

NOTE

- 1 ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
- 2 ALL CORNERS TO BE CHAMFERED TO 0.25 X 45
- 3 ALL TOLERANCES ARE ± 0.1 UNLESS SPECIFIED

SPACER-A / SPACER B

DATE: 10/10/10
BY: S. RAJGURU
DIRECT DRIVE ROBOT
02

SPACER-C (MS) QTY-1NOS

FLANGE (MS) QTY-2NOS

SPACER-F (MS)
QTY-1NOS NOTE

1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
2. ALL CORNERS TO BE CHAMFERED TO 0.25X45.
3. ALL TOLERANCES ARE ± 0.1 UNLESS SPECIFIED.

SPACER-C/SPACER-F / FLANGE	
REV.	1
DATE	11-2-81
BY	J. A. ...
CHKD BY	S. R. ...
QTY	05

MAIN HOUSING - ALUMINUM 6061-QTY.2 NOS

WU

1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
2. ALL TOLERANCES * CHINA UNLESS STATED OTHERWISE.
3. ALL CORNERS TO BE CHAMFERED TO 05 X 45.

MAIN HOUSING

LOBO DRIVE ROBOT

DATA BY: S BAZOVEN

RESOLVER HOUSING-ALUMINUM 6061-QTY-2NOS.

FRONT VIEW

SECTION B-B

SECTION A-A

NOTE

1. ALL DIMENSIONS ARE IN MM.
2. ALL TOLERANCES ARE ± 0.1 MM UNLESS STATED OTHERWISE.
3. ALL CORNERS TO BE CHAMFERED TO $0.5 \times 45^\circ$.

RESOLVER HOUSING

DIRECT DRIVE ROBOT

DATE: 10/10/80

TACH HOUSING - ALUMINUM 6061-QTY-2 NOS.

SECTION B-B

END COVER - ALUMINUM 6061-QTY-2 NOS.

NO 16 DRILL-0216-24UNC-28-3 NOS-120 APART ON- $\phi 102$ BC.

NOTE:

1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
2. ALL CORNERS TO BE CHAMFERED TO 1X45°.
3. ALL TOLERANCES ARE ±0.1MM UNLESS SPECIFIED.

TACH HOUSING/END COVER

DIRECT DRIVE ROBOT

DRN BY S. B. A. J. G. M. W. T. T.

PART D-MS-1020] QTY-1 NOS

PART E-MS-1020] QTY-1 NOS

PART F-ALUMINUM 6061-QTY-1 NOS

FRONT VIEW

SECTION A-A

PART G-ALUMINUM 6061-QTY-1 NOS

SECTION B-B

FRONT VIEW

NOTE

1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
2. ALL TOLERANCES ARE 0.1 MM UNLESS STATED OTHERWISE.
3. ALL CORNERS TO BE CHAMFERED TO 0.5 X 45°.

PART-D/PART-E/PART-F/PART-G

QTY

GRIPPER

DATE

BY

15

2

OF/DE

2

MOTOR SEAT- ALUMINUM 6061-QTY-2 NOS

SECTION A-A

NO 7 DRILL-1/4-20 UNC-28
8 NOS-45° APART ON $\phi 147$ BC

TOP VIEW

SECTION B-B

NOTE

1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
2. ALL TOLERANCES ARE ± 0.1 MM UNLESS STATED OTHERWISE.
3. ALL CORNERS TO BE CHAMFERED TO $0.5 \times 45^\circ$

MOTOR SEAT

DESIGNED BY: S. R. S. S. S.	DATE: 18
DRAWN BY: S. R. S. S. S.	
CHECKED BY: S. R. S. S. S.	
APPROVED BY: S. R. S. S. S.	

DIRECT DRIVE ROBOT

SIDE PLATES - ALUMINUM 6061 - QTY 2 NOS

NOTE

1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
2. ALL TOLERANCES ARE ± 0.1 MM UNLESS STATED OTHERWISE.

SIDE PLATES

DATE	11/11/11
BY	11/11/11
CHECKED	11/11/11
APPROVED	11/11/11
DESIGNED	11/11/11
DRAWN	11/11/11
DATE	11/11/11
BY	11/11/11
CHECKED	11/11/11
APPROVED	11/11/11
DESIGNED	11/11/11
DRAWN	11/11/11

TCP SUPPORT-ALUMINUM 6061-QTY-2 NOS

- NOTE.
1. ALL DIMENSIONS ARE IN MM UNLESS STATED OTHERWISE.
 2. ALL TOLERANCES ARE $\pm 0.1 \text{ MM}$ UNLESS STATED OTHERWISE.
 3. ALL CORNERS TO BE CHAMFERED TO 05×45

TOP SUPPORT

DIRECT DRIVE ROBOT

DWG 6115.8430.00 20

Appendix B

Derivation Of the Jacobian

Referring to Figure 3.1, by geometry,

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 - (l_5 - l_4) \cos \phi_1 \\ l_1 \sin \theta_1 - (l_5 - l_4) \sin \phi_1 \end{bmatrix} \quad (\text{B.1})$$

also,

$$l_1 \cos \theta_1 + l_4 \cos \phi_1 = l_2 \cos \theta_2 + l_3 \cos \phi_2 \quad (\text{B.2})$$

$$l_1 \sin \theta_1 + l_4 \sin \phi_1 = l_2 \sin \theta_2 + l_3 \sin \phi_2 \quad (\text{B.3})$$

Equations (B.1), (B.2) and (B.3) represent the constraint equations that must be satisfied to maintain the closed kinematic chain.

To derive the Jacobian, the equations (B.1) (B.2) and (B.3) must be differentiated in terms of θ_1 , θ_2 , ϕ_1 and ϕ_2 and then eliminating $\partial\phi_1$ and $\partial\phi_2$ from them to obtain infinitesimal displacements between $dX = (dxdy)^T$ and $d\theta = (d\theta_1 d\phi_1)^T$

$$dX = Jd\theta \quad (B.4)$$

where the jacobian matrix is given by

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \quad (B.5)$$

i.e. differentiating eqn (B.1) w.r.t θ_1 we get,

$$\frac{\partial X}{\partial \theta_1} = -l_1 \sin \theta_1 + (l_5 - l_4) \sin \phi_1 \frac{\partial \phi_1}{\partial \theta_1} \quad (B.6)$$

$$\frac{\partial Y}{\partial \theta_1} = l_1 \cos \theta_1 - (l_5 - l_4) \cos \phi_1 \frac{\partial \phi_1}{\partial \theta_1} \quad (B.7)$$

differentiating equations (B.2) and (B.3) w.r.t θ_1 , we get

$$-l_1 \sin \theta_1 - l_4 \sin \phi_1 \frac{\partial \phi_1}{\partial \theta_1} = -l_2 \sin \theta_2 \frac{\partial \theta_2}{\partial \theta_1} - l_3 \sin \phi_2 \frac{\partial \phi_2}{\partial \theta_1} \quad (B.8)$$

$$l_1 \cos \theta_1 + l_4 \cos \phi_1 \frac{\partial \phi_1}{\partial \theta_1} = l_2 \cos \theta_2 \frac{\partial \theta_2}{\partial \theta_1} + l_3 \cos \phi_2 \frac{\partial \phi_2}{\partial \theta_1} \quad (\text{B.9})$$

multiplying equation (B.8) by $\cos \phi_2$ and equation (B.9) by $\sin \phi_2$, we get,

$$\begin{aligned} & -l_1 \sin \theta_1 \cos \phi_2 - l_4 \sin \phi_1 \cos \phi_2 \frac{\partial \phi_1}{\partial \theta_1} = \\ & -l_2 \sin \theta_2 \cos \phi_2 \frac{\partial \theta_2}{\partial \theta_1} - l_3 \sin \phi_2 \cos \phi_2 \frac{\partial \phi_2}{\partial \theta_1} \end{aligned} \quad (\text{B.10})$$

and,

$$\begin{aligned} & l_1 \cos \theta_1 \sin \phi_2 + l_4 \cos \phi_1 \sin \phi_2 \frac{\partial \phi_1}{\partial \theta_1} = \\ & l_2 \cos \theta_2 \sin \phi_2 \frac{\partial \theta_2}{\partial \theta_1} + l_3 \cos \phi_2 \sin \phi_2 \frac{\partial \phi_2}{\partial \theta_1} \end{aligned} \quad (\text{B.11})$$

adding equations (B.10) and (B.11), we get

$$\begin{aligned} & l_1 \cos \theta_1 \sin \phi_2 - l_1 \sin \theta_1 \cos \phi_2 + l_4 \cos \phi_1 \sin \phi_2 \frac{\partial \phi_1}{\partial \theta_1} \\ & - l_4 \sin \phi_1 \cos \phi_2 \frac{\partial \phi_1}{\partial \theta_1} = l_2 \cos \theta_2 \sin \phi_2 \frac{\partial \theta_2}{\partial \theta_1} - l_2 \sin \theta_2 \cos \phi_2 \frac{\partial \theta_2}{\partial \theta_1} \end{aligned} \quad (\text{B.12})$$

therefore

$$l_1 \sin(\phi_2 - \theta_1) + l_4 \sin(\phi_2 - \phi_1) \frac{\partial \phi_1}{\partial \theta_1} = l_2 \sin(\phi_2 - \theta_2) \frac{\partial \theta_2}{\partial \theta_1} \quad (\text{B.13})$$

solving equation (B.13) for $\frac{\partial \phi_1}{\partial \theta_1}$, we get

$$\frac{\partial \phi_1}{\partial \theta_1} = \frac{l_2 \sin(\phi_2 - \theta_2) \frac{\partial \theta_2}{\partial \theta_1} - l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \quad (\text{B.14})$$

substituting equation (B.14) in equation (B.6), we get

$$\begin{aligned} \frac{\partial X}{\partial \theta_1} = & -l_1 \sin \theta_1 + (l_5 - l_4) \sin \phi_1 \\ & \left[\frac{l_2 \sin(\phi_2 - \theta_2) \frac{\partial \theta_2}{\partial \theta_1} - l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] \end{aligned} \quad (\text{B.15})$$

therefore

$$\begin{aligned} \frac{\partial X}{\partial \theta_1} = & -l_1 \sin \theta_1 - \left[\frac{(l_5 - l_4) \sin \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] \\ & + \left[\frac{(l_5 - l_4) \sin \phi_1 l_2 \sin(\phi_2 - \theta_2) \frac{\partial \theta_2}{\partial \theta_1}}{l_4 \sin(\phi_2 - \phi_1)} \right] \end{aligned} \quad (\text{B.16})$$

therefore,

$$J_{11} = -l_1 \sin \theta_1 - \left[\frac{(l_5 - l_4) \sin \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] \quad (\text{B.17})$$

$$J_{12} = \left[\frac{(l_5 - l_4) \sin \phi_1 l_2 \sin(\phi_2 - \theta_2)}{l_4 \sin(\phi_2 - \phi_1)} \right] \quad (\text{B.18})$$

Also substituting equation (B.14) in equation (B.7), we get

$$\frac{\partial Y}{\partial \theta_1} = l_1 \cos \theta_1 - (l_5 - l_4) \cos \phi_1 \left[\frac{l_2 \sin(\phi_2 - \theta_2) \frac{\partial \theta_2}{\partial \theta_1} - l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] \quad (\text{B.19})$$

therefore,

$$\begin{aligned} \frac{\partial Y}{\partial \theta_1} = l_1 \cos \theta_1 + & \left[\frac{(l_5 - l_4) \cos \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] \\ & - \left[\frac{(l_5 - l_4) \cos \phi_1 l_2 \sin(\phi_2 - \theta_2) \frac{\partial \theta_2}{\partial \theta_1}}{l_4 \sin(\phi_2 - \phi_1)} \right] \end{aligned} \quad (\text{B.20})$$

therefore

$$J_{21} = l_1 \cos \theta_1 + \left[\frac{(l_5 - l_4) \cos \phi_1 l_1 \sin(\phi_2 - \theta_1)}{l_4 \sin(\phi_2 - \phi_1)} \right] \quad (\text{B.21})$$

and,

$$J_{22} = \left[\frac{-(l_5 - l_4) \cos \phi_1 l_2 \sin(\phi_2 - \theta_2)}{l_4 \sin(\phi_2 - \phi_1)} \right] \quad (\text{B.22})$$

Appendix C

Dynamic Equations Of A Five Bar Linkage Moving In A Horizontal Plane

The dynamic equations of a five bar linkage moving in the vertical were derived by Rivin [22]. Since the Carleton robot which has the same structural geometry as in [22] except that it moves in the horizontal plane, the equations derived in [22] can be modified to apply for a five bar linkage manipulator moving in the horizontal plane.

The popular methods for deriving the dynamic equations of motion of manipulators are the Lagrangian method and the recursive Newton-Euler method. The Lagrangian method is used in this thesis for the derivation of the dynamic equations of motion.

$$L = K - P \quad (C.1)$$

where K is the total kinetic energy of the system, and P is the total potential energy of the system. Since the manipulator moves in the horizontal plane the potential energy of the system is zero.

From the Lagrangian relation for the forces,

$$F_i = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} \quad (C.2)$$

where F_i is the force applied to the i^{th} joint and x_i is the generalized coordinate of the i^{th} link.

Since the manipulator has only revolute joints, the actuator force F_i can be replaced by torque τ_i , and the generalized space coordinate x_i by the joint angle θ_i .

Thus

$$\tau_i = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i} \quad (C.3)$$

Links l_1 and l_2 participate only in revolute motions associated with direct drive motors M_1 , and M_2 (or coordinates θ_1 and θ_2). The partial motions of links l_3 and l_4 associated with each coordinates, such that θ_1 and θ_2 are calculated separately and then the resultant velocities are found.

Link 1:

$$K_1 = 0.5m_1 \left[l_{1c}\dot{\theta}_1 \right]^2 + 0.5I_1\dot{\theta}_1^2 \quad (\text{C.4})$$

Link 2:

$$K_2 = 0.5m_2 \left[l_{1c}\dot{\theta}_2 \right]^2 + 0.5I_2\dot{\theta}_2^2 \quad (\text{C.5})$$

Link 3:

a When link l_2 is stationary ($\dot{\theta}_2 = 0$), then

1. $\omega_3 = \omega_1 = \dot{\theta}_1$ and $v_{cg,3} = l_{3c}\dot{\theta}_1$
2. $[v_{cg,3}]_x = -v_{cg,3}\cos\theta_1 = -l_{3c}\dot{\theta}_1\cos\theta_1$
3. $[v_{cg,3}]_y = -v_{cg,3}\sin\theta_1 = -l_{3c}\dot{\theta}_1\sin\theta_1$

b When link l_1 is stationary ($\dot{\theta}_1 = 0$), then $\omega_1 = \omega_3 = 0$

1. $v_{cg,3} = v_{o,3} = l_2\dot{\theta}_2$ and $[v_{cg,3}]_x = -l_2\dot{\theta}_2\cos\theta_2$
2. $[v_{cg,3}]_y = -l_2\dot{\theta}_2\sin\theta_2$

The resultant velocities, $\omega_2 = \omega_3 = \ddot{\theta}_1$ and

$$\begin{aligned} v_{cg,3}^2 &= \left[-l_{3c}\dot{\theta}_1\cos\theta_1 - l_2\dot{\theta}_2\cos\theta_2 \right]^2 + \left[-l_{3c}\dot{\theta}_1\sin\theta_1 - l_2\dot{\theta}_2\sin\theta_2 \right]^2 \\ &= l_2^2\dot{\theta}_2^2 + l_{3c}^2\dot{\theta}_1^2 + 2l_2l_{3c}\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2) \end{aligned} \quad (\text{C.6})$$

$$K_3 = 0.5m_3 \left[l_{3c}^2 \dot{\theta}_1^2 + I_2^2 \dot{\theta}_2^2 + 2l_2 l_{3c} \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \right] + 0.5 I_3 \dot{\theta}_1^2 \quad (C.7)$$

Link 4:

a. When link l_2 is stationary ($\dot{\theta}_2 = 0$), then

$$1. \omega_4 = \omega_2 = 0$$

$$2. v_{cg,4} = v_{0,2} = l_1 \dot{\theta}_1$$

$$3. [v_{cg,4}]_x = -v_{cg,4} \cos \theta_1$$

$$4. [v_{cg,4}]_y = -v_{cg,4} \sin \theta_1$$

b. When link l_1 is stationary ($\dot{\theta}_1 = 0$), then

$$1. \omega_4 = \omega_2 = \dot{\theta}_2$$

$$2. v_{cg,4} = l_{4c} \dot{\theta}_2$$

$$3. [v_{cg,4}]_x = v_{cg,4} \cos \theta_2$$

$$4. [v_{cg,4}]_y = -[v_{cg,4}] \sin \theta_2$$

The resultant velocities $\omega_4 = \omega_2 = \dot{\theta}_2$ and

$$\begin{aligned} v_{cg,4}^2 &= [-l_1 \dot{\theta}_1 \cos \theta_1 + l_{4c} \dot{\theta}_2 \cos \theta_2]^2 + [-l_1 \dot{\theta}_1 \sin \theta_1 + l_{4c} \dot{\theta}_2 \sin \theta_2]^2 \\ &= l_1^2 \dot{\theta}_1^2 + l_{4c}^2 \dot{\theta}_2^2 - 2l_1 l_{4c} \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \end{aligned} \quad (C.8)$$

$$K_4 = 0.5m_4 \left[l_1^2 \dot{\theta}_1^2 + l_{4c}^2 \dot{\theta}_2^2 - 2l_1 l_{4c} \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \right] + 0.5I_4 \dot{\theta}_2^2 \quad (C.9)$$

For the payload K_0 is expressed similar to K_4 , with m_0 instead of m_4 and l_4 instead of l_{4c}

$$K_0 = 0.5m_0 \left[l_1^2 \dot{\theta}_1^2 + l_4^2 \dot{\theta}_2^2 - 2l_1 l_4 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \right] + 0.5I_o \dot{\theta}_2^2 \quad (C.10)$$

Thus the kinetic energies of the five bar linkage moving in the horizontal plane is given by,

$$\begin{aligned} K = & 0.5 \left[m_1 l_{1c}^2 + I_1 \right] \dot{\theta}_1^2 + 0.5 \left[m_2 l_{2c}^2 + I_2 \right] \dot{\theta}_2^2 + 0.5m_3 \\ & \left[l_{3c} \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2l_2 l_{3c} \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \right] + 0.5I_3 \dot{\theta}_1^2 + 0.5m_4 \\ & \left[l_1^2 \dot{\theta}_1^2 + l_{4c}^2 \dot{\theta}_2^2 - 2l_1 l_{4c} \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \right] + 0.5I_4 \dot{\theta}_2^2 + 0.5m_o \\ & \left[l_1^2 \dot{\theta}_1^2 + l_4^2 \dot{\theta}_2^2 - 2l_1 l_4 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \right] + 0.5I_o \dot{\theta}_2^2 \end{aligned} \quad (C.11)$$

The terms of the lagrange equations develops as

$$\begin{aligned} \frac{\partial L}{\partial \dot{\theta}_1} = \frac{\partial K - \partial V}{\partial \dot{\theta}_1} = & \left[m_1 l_{1c}^2 + I_1 \right] \dot{\theta}_1 + m_2 l_{3c}^2 \dot{\theta}_1 + m_3 l_2 l_{3c} \cos(\theta_1 - \theta_2) \dot{\theta}_2 \\ & + I_3 \dot{\theta}_1 + m_4 l_1^2 \dot{\theta}_1 - m_4 l_1 l_{4c} \cos(\theta_1 - \theta_2) \dot{\theta}_2 \\ & + m_o l_1^2 \dot{\theta}_1 - m_o l_1 l_4 \cos(\theta_1 - \theta_2) \dot{\theta}_2 \end{aligned} \quad (C.12)$$

$$\begin{aligned}
\frac{d\partial L}{dt\partial\dot{\theta}_1} &= [m_1 l_{1c}^2 + I_1 + m_3 l_{3c}^2 + I_3 + m_4 l_1^2 + m_o l_1^2] \ddot{\theta}_1 \\
&\quad + [m_2 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \cos(\theta_1 - \theta_2) \ddot{\theta}_2 \\
&\quad - [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \sin(\theta_1 - \theta_2) \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2)
\end{aligned} \tag{C.13}$$

$$\begin{aligned}
\frac{\partial L}{\partial \theta_1} &= -m_3 l_2 l_{3c} \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_4 l_1 l_{4c} \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) \\
&\quad + m_o l_1 l_4 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)
\end{aligned} \tag{C.14}$$

$$\begin{aligned}
\frac{\partial L}{\partial \dot{\theta}_2} &= [m_2 l_{2c}^2 + I_2] \dot{\theta}_2 + m_3 l_2^2 \dot{\theta}_2 + m_3 l_2 l_{3c} \cos(\theta_1 - \theta_2) \dot{\theta}_1 + I_4 \dot{\theta}_2 \\
&\quad + m_o l_{4c}^2 \dot{\theta}_2 - m_4 l_1 l_{4c} \cos(\theta_1 - \theta_2) \dot{\theta}_1 + I_4 \dot{\theta}_2 + m_o l_4^2 \dot{\theta}_2 \\
&\quad - m_o l_1 l_4 \cos(\theta_1 - \theta_2) \dot{\theta}_1 + I_o \dot{\theta}_2
\end{aligned} \tag{C.15}$$

$$\begin{aligned}
\frac{d\partial L}{dt\partial\dot{\theta}_2} &= [m_2 l_{2c}^2 + I_2 + m_3 l_2^2 + m_4 l_{4c}^2 + I_4 + m_o l_4^2 + I_o] \ddot{\theta}_2 \\
&\quad + [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \cos(\theta_1 - \theta_2) \ddot{\theta}_1 \\
&\quad - [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \sin(\theta_1 - \theta_2) \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2)
\end{aligned} \tag{C.16}$$

$$\frac{\partial L}{\partial \theta_2} = [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \sin(\theta_1 - \theta_2) \dot{\theta}_1 \dot{\theta}_2 \tag{C.17}$$

The equations of motion are

$$\begin{aligned}
 \tau_1 = & \left[m_1 l_{1c}^2 + I_1 + m_3 l_{3c}^2 + I_3 + m_4 l_1^2 + m_o l_1^2 \right] \ddot{\theta}_1 \\
 & + [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \cos(\theta_1 - \theta_2) \ddot{\theta}_2 \\
 & + [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \sin(\theta_1 - \theta_2) \dot{\theta}_2^2
 \end{aligned} \tag{C.18}$$

$$\begin{aligned}
 \tau_2 = & [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \cos(\theta_1 - \theta_2) \ddot{\theta}_1 \\
 & + [m_2 l_{2c}^2 + I_2 + m_3 l_2^2 + m_4 l_{4c}^2 + I_4 + m_o l_4^2 + I_o] \ddot{\theta}_2 \\
 & - [m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_o l_1 l_4] \sin(\theta_1 - \theta_2) \dot{\theta}_1^2
 \end{aligned} \tag{C.19}$$

Appendix D

Inverse Kinematics

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 - (l_5 - l_4) \cos \phi_1 \\ l_1 \sin \theta_1 - (l_5 - l_4) \sin \phi_1 \end{bmatrix} \quad (\text{D.1})$$

therefore,

$$x + (l_5 - l_4) \cos \phi_1 = l_1 \cos \theta_1 \quad (\text{D.2})$$

and

$$y + (l_5 - l_4) \sin \phi_1 = l_1 \sin \theta_1 \quad (\text{D.3})$$

squaring equations (D.2) and (D.3) we get

$$[x + (l_5 - l_4)\cos\phi_1]^2 = [l_1\cos\theta_1]^2 \quad (\text{D.4})$$

and

$$[y + (l_5 - l_4)\sin\phi_1]^2 = [l_1\sin\theta_1]^2 \quad (\text{D.5})$$

adding equations D.4 and D.5 we get

$$\begin{aligned} x^2 + y^2 + (l_5 - l_4)^2 + 2x(l_5 - l_4)\cos\phi_1 + 2y(l_5 - l_4)\sin\phi_1 \\ = l_1^2(\sin^2\theta_1 + \cos^2\theta_1) \end{aligned} \quad (\text{D.6})$$

therefore

$$x\cos\phi_1 + y\sin\phi_1 = \frac{l_1^2 - [x^2 + y^2 + (l_5 - l_4)^2]}{2(l_5 - l_4)} \quad (\text{D.7})$$

therefore

$$\begin{aligned} \sqrt{x^2 + y^2} \left[\frac{x\cos\phi_1}{\sqrt{x^2 + y^2}} + \frac{y\sin\phi_1}{\sqrt{x^2 + y^2}} \right] \\ = \frac{l_1^2 - [x^2 + y^2 + (l_5 - l_4)^2]}{2(l_5 - l_4)} \end{aligned} \quad (\text{D.8})$$

let $x/\sqrt{x^2 + y^2} = \sin\alpha$

therefore

$$\begin{aligned} & \sqrt{x^2 + y^2} [\sin\alpha \cos\phi_1 + \cos\alpha \sin\phi_1] \\ &= \frac{l_1^2 - [x^2 + y^2 + (l_5 - l_4)^2]}{2(l_5 - l_4)} \end{aligned} \quad (D.9)$$

therefore

$$\sin(\alpha + \phi_1) = \frac{l_1^2 - [x^2 + y^2 + (l_5 - l_4)^2]}{2(l_5 - l_4)\sqrt{x^2 + y^2}} \quad (D.10)$$

therefore

$$\phi_1 = \sin^{-1} \left[\frac{l_1^2 - [x^2 + y^2 + (l_5 - l_4)^2]}{2(l_5 - l_4)\sqrt{x^2 + y^2}} \right] - \alpha \quad (D.11)$$

Similarly solving for θ_1 gives

$$\cos(\theta_1 - \phi_1) = \frac{x^2 + y^2 - l_1^2 - (l_5 - l_4)^2}{-2l_1(l_5 - l_4)} \quad (D.12)$$

therefore

$$\theta_1 = \cos^{-1} \frac{x^2 + y^2 - l_1^2 - (l_5 - l_4)^2}{-2l_1(l_5 - l_4)} + \phi_1 \quad (D.13)$$

Appendix E

Check For Key Strength

The material used for the rotor shaft and the key is M.S. 1020. The yield strength S_y of M.S. 1020 is 43 kpsi. The rotor shaft dia is 25mm (0.984"). The key's dimensions are 5x 5x 25 mm³ (0.196" x 0.196" x 0.984"). As per the data supplied by M.C.S (refer table 1, chapter 4), the B.D.C motor has a peak torque of 210 lbs-in.

The force F at the shaft

$$F = \left(\frac{\tau}{r} \right) \quad (E.1)$$

i.e. $F = 426.82$ lbs.

By the distortion theory, the shear strength

$$S_{sy} = 0.577S_y \quad (E.2)$$

$$S_{sy} = 0.577 \times 43$$

$$S_{sy} = 24.81 \text{ k.p.s.i}$$

Failure by shear across the cross sectional area of the key will create a stress of

$$\tau = \left(\frac{F}{tl} \right) \quad (\text{E.3})$$

where, l is the key length

Substituting the strength divided by a factor of safety of 3 for τ gives

$$\left(\frac{S_{sy}}{f.o.s} \right) = \left(\frac{F}{tl} \right) \quad (\text{E.4})$$

Therefore $l = 0.26'' = 6.67 \text{ mm}$

i.e. to prevent failure by shear the key length should be $\geq 6.67 \text{ mm}$.

To resist crushing, the area of one half the key face is used.

$$\left(\frac{S_{sy}}{f.o.s} \right) = \left(\frac{F}{0.5tl} \right) \quad (\text{E.5})$$

Therefore $l = 0.30'' = 7.70 \text{ mm}$.

The key selected has a length of 25 mm, therefore it can resist crushing, and will not fail in shear.

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