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**AN IMPROVED CORRELATION FOR REPRESENTING
SURFACE ROUGHNESS EFFECTS ON THE LAW OF THE WALL
IN FULLY ROUGH CONDITIONS**

by

David Ross Waigh, B. Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering
in Aerospace Engineering

Ottawa Carleton Institute for Mechanical
and Aerospace Engineering

Department of Mechanical and Aerospace Engineering

Carleton University

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**"AN IMPROVED CORRELATION FOR REPRESENTING SURFACE
ROUGHNESS EFFECTS ON THE LAW OF THE WALL IN FULLY ROUGH
CONDITIONS"**

submitted by **David Ross Waigh, B. Eng.**
in partial fulfilment of the requirements for
the degree of Master of Engineering

Professor R. Bell, Chair

Department of Mechanical And Aerospace Engineering

Professor R. J. Kind, Thesis Supervisor

CARLETON UNIVERSITY

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Abstract

This thesis presents an improved method for prediction of the intercept of the law of the wall for surfaces covered with three-dimensional roughness elements. Previous work has correlated flow behaviour to the relative spacing of the roughness elements that compose the rough surface. When additional geometric parameters of the rough surface are accounted for, it is possible to predict the effects of the roughness more accurately. By examining available experimental data for a variety of rough surfaces, a database of roughness geometries and their corresponding influence on the law-of-the-wall was constructed. Using a Nelder-Mead simplex algorithm, different combinations of geometric parameters were correlated to the flow behaviour. It was found that introduction of a roughness-element bluntness parameter and of a roughness-element aspect ratio led to a significantly improved correlation for the law-of-the-wall intercept. The arrangement of the rows of roughness elements (*i.e.* staggered vs. in-line) was found not to have an important influence on rough-wall flows. Insufficient data were available to determine if the prediction of fully-rough flow over two-dimensional roughness elements could be improved in the same manner.

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Nomenclature

$b(y)$	span of roughness element (normal to the freestream direction) at elevation y
$\bar{b}$	mean span of roughness element (Figure 3.1)
d	zero-plane displacement height; elevation of the effective origin
k	maximum height of roughness element
$\bar{k}$	mean height of roughness on surface (equation 2.13)
k_s	standard sand height (p. 7)
n	number of roughness elements
$s(y)$	streamwise dimension of roughness element at elevation y
$\bar{s}$	mean streamwise dimension of roughness element (Figure 3.1)
u	fluid velocity
u_τ	$\sqrt{\frac{\tau_w}{\rho}}$, shear velocity
$\frac{\Delta u}{u_\tau}$	roughness effect on the logarithmic velocity profile (equation 2.7)
y	elevation from the smooth wall; effective elevation ($y_e - d$) from the rough wall
y_0	absolute elevation from the base of the rough wall (Figure 2.5)
z_0	roughness height
A_f	frontal area of one roughness element, $\bar{b}k$ (Figure 3.1)
A_c	plan area of the wall associated with one roughness element, or roughness cell, (Figure 3.1)
A_F	frontal area of roughness elements perpendicular to the freestream direction, $n\bar{b}k$
A_R	mean cross-sectional area of one roughness element in the plan view, $\bar{b}\bar{s}$

A_s	total plan area of the rough surface
A_v	frontal area of one roughness cell, kM
A_w	total upstream-facing wetted surface area of roughness elements
B	intercept of the logarithmic velocity profile for a smooth-wall
C	change in the logarithmic velocity profile intercept due to roughness; called the roughness constant in fully rough flow
D	Dirling's density parameter, $\sqrt{\frac{A_s}{n}}$
L	spacing or pitch of two-dimensional roughness elements; streamwise dimension of one roughness cell (Figure 3.1)
M	span of one roughness cell
$\frac{dp_e}{dx}$	freestream pressure gradient ($y > \delta$)
U_e	freestream fluid velocity ($y > \delta$)
U_k	fluid velocity at the crest of the roughness element ($y = k$)
Z_b	spanwise aspect ratio, $k/\bar{b}$
Z_r	roughness element bluntness factor, A_w/A_f
Z_s	streamwise aspect ratio, $k/\bar{s}$
β_x	maximum blockage factor (equation 3.4)
β_c	roughness cell aspect ratio, L/M (Table 3.1)
δ	boundary-layer thickness
δ_τ	shear-stress displacement thickness (equation 2.14)
γ	stagger parameter, m/M (Figure 3.1)
κ	Von Kármán constant; $1/\kappa$ is the slope of the logarithmic velocity profile

λ	spacing parameter: Simpson (equation 2.20)
λ_B	spacing parameter: Dirling (equation 2.23)
λ_D	spacing parameter: Dvorak (equation 2.18)
$\lambda_{\bar{D}}$	spacing parameter: Modified Dvorak (equation 3.3)
λ_{DDM}	spacing parameter: Dalle, Donne and Meyer, $\lambda_L - s/k$
λ_{DS}	spacing parameter: Danberg and Sigal (equation 2.25)
λ_L	spacing parameter: Betterman, L/k
μ	dynamic viscosity of fluid
ν	kinematic viscosity of fluid
ρ	fluid density
$\tau(y)$	shear stress in fluid at height y
τ_w	shear stress in fluid at wall

Chapter 1: Introduction

The effect of surface roughness on turbulent boundary-layer development has always been of concern in fluid mechanics. The manufacture of effectively smooth surfaces is not economically feasible for many applications, and is sometimes not even desirable, as is often the case when heat transfer or acoustic damping is required. Smooth surfaces can also deteriorate during their service lifetime to the point where they become roughened. Because of this, surface roughness effects will influence design in almost all fields of applied aerodynamics.

Detailed study of turbulent boundary-layer flow has its foundations in the smooth-wall experiments of Von Kármán and Prandtl (Prandtl, 1960). Preliminary characterization of rough-wall behaviour followed soon afterwards with the work of Nikuradse (1933) and Schlichting (1936). Since that time, many refinements have been made in the prediction of rough-wall flow behaviour. Although it is believed that the physics of turbulent flows can be completely described by the unsteady Navier-Stokes equations, direct numerical solution of even the most simple of turbulent flows is prohibitively demanding on computer resources available at this date (White, 1991). For this reason, semi-empirical modelling is the only practical method currently available that

allows the prediction of rough-wall flows. The present study seeks to develop an improvement in the roughness models used in turbulent boundary-layer analysis.

Mean velocity behaviour in the near wall region ($y \leq \frac{1}{5}\delta$) of turbulent boundary layers is usually well described by the law of the wall, a correlation involving the wall shear stress, effects of roughness, and other parameters. The effects of roughness on the law of the wall can themselves be correlated with the relative spacing of the roughness, but the correlation is rather poor; experimental data show a great deal of scatter when plotted in terms of this spacing parameter. The present study attempts to improve the prediction of the effects of roughness by examining all suitable experimental data for rough surfaces and assessing a variety of additional roughness-configuration parameters, to see if these parameters can improve the collapse of data when introduced to the roughness-effect correlation.

It should be noted that the roughness in the compiled experiments consisted of regular arrays of simply-shaped roughness elements (cubes, spheres, cylinders, *etc.*). Most roughening that occurs through "natural processes", such as surface deterioration, fouling or hoar-frost accretion, as well as the roughness inherent in the originally manufactured surface finish, is more irregular and complex in both its shape and distribution. In order to correctly evaluate the effects of these kinds of roughness, further work is required to characterize the effects of naturally occurring roughness. The most effective approach to this may be to try to determine relations between certain geometric parameters of naturally occurring roughness and the geometric parameters of regular arrays that produce the same aerodynamic effects. Kind and Lawrysyn (1991), and Kind *et al* (1996) had

some success with this approach for hoar frost with relatively isolated frost peaks and for sparsely distributed sand grains, respectively.

The remainder of this thesis is organized as follows. Chapter 2 reviews previous research on the prediction of roughness effects. It also lists and comments on the sources of experimental data used in the present study. The classification of the rough surfaces used in these experiments is described in Chapter 3. The methods used in evaluating the effectiveness of these geometric descriptors are also reviewed in this chapter. Results and discussion of this evaluation follow in Chapter 4. Conclusions from the present work, and recommendations for future work are presented in the final chapter.

Chapter 2: Literature Review

2.1 The Law of the Wall for Smooth Surfaces

The derivation of the logarithmic law-of-the-wall for smooth surfaces can be performed in a variety of ways, using any of a number of initial assumptions. No new insight on this relationship is offered here. The following presentation is quite similar to that given by White (1991); the derivation is included for the sake of completeness, as a foundation for the theory that comes after it.

The time-averaged velocity profile of a turbulent boundary-layer flow can be divided into an inner layer and an outer layer. Prandtl (1926) theorized that in the inner layer, flow behaviour and thus the velocity, u_{inner} , can be scaled to parameters that depend on local conditions only:

$$u_{inner} = u(\tau_w, \rho, \mu, y) \quad (2.1)$$

τ_w is the wall shear stress, ρ is the density of the fluid, μ is the dynamic viscosity of the fluid, and y is the distance from the wall. Under zero-pressure gradient conditions, this layer occupies approximately 20% of the boundary layer (White, 1991). Von Kármán (1930) suggested that viscous effects would not have a significant direct influence on the

flow behaviour in the outer layer, as momentum exchange in the fluid would be mainly due to turbulent eddy motion, which depends principally on inertial effects rather than viscosity. Thus,

$$u_{outer} = u\left(\tau_w, \rho, \delta, y, \frac{dp_e}{dx}\right) \quad (2.2)$$

where δ is the boundary-layer thickness, and $\frac{dp_e}{dx}$ is the freestream pressure-gradient.

In order to have a continuous velocity profile, there must be some overlap of the two layers, where both equations (2.1) and (2.2) must apply:

$$u = u_{inner} = u_{outer} \quad (2.3)$$

This restriction can only be satisfied if the functions in equations (2.1) and (2.2) are logarithmic, yielding the familiar logarithmic law of the wall (White, 1991):

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{yu_\tau}{\nu}\right) + B \quad (2.4)$$

where $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ is known as the shear velocity. Current experimental data indicate $\kappa \approx 0.41$ and $B \approx 5.0$ (White, 1991).

Equation (2.4) is only applicable when pressure gradients are below certain magnitudes. Adverse pressure gradients reduce the relative size of the overlap layer and increase the size of the outer layer (Figure 2.1); the outer layer is visible in Figure 2.1 as a “wake”, as characterized by Coles (1956). In severely adverse pressure gradients, the wake comprises virtually the entire boundary-layer, and there is no logarithmic zone. Conversely, in extremely favourable pressure gradients, re-laminarization of the boundary layer can occur, thus invalidating the turbulent velocity profile model of equation (2.4).

LAW OF THE WALL PLOT NEAR SEPARATION
SERIES I.

LAW OF THE WALL PLOT NEAR SEPARATION
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SERIES I		SERIES II	
x (m)	p^+	x (m)	p^+
1.676	0.0073	1.524	0.0095
1.702	0.024	1.651	0.100
1.727	0.0862	1.702	0.57
1.753	0.456		

FIGURE 2.1: Velocity profile data from Chu and Young (1975) showing the effects of severe adverse pressure gradients on the logarithmic layer. Reproduced from Galbraith *et al* (1977). $u^+ = u/u_\tau$; $y^+ = yu_\tau/\nu$; $p^+ = \nu/(\rho u_\tau^2) dp/dx$.

In moderately favourable pressure gradients, Patel (1965) found inverted wakes in the velocity profile, which also reduced the size of the overlap layer. These wakes were much smaller than those seen in adverse pressure gradients (Figure 2.2).

FIGURE 2.2: Effects of pressure gradients on turbulent boundary-layer velocity profiles: a) favourable pressure gradient; b) adverse pressure gradient. Δ : defect thickness (Clauser, 1954), a non-dimensional measure of the pressure-gradient; H : shape factor, a measure of the velocity profile shape; R_θ : momentum thickness Reynolds number. Reproduced from Patel (1965).

2.2 The Law of the Wall for Rough Walls

As mentioned in the introduction, rough walls must often be accounted for in applied aerodynamics. Fortunately, the effects are relatively simple to characterize theoretically. However, quantifying roughness effects for any particular roughness configuration can be quite difficult.

2.2.1 The Effects of Roughness on the Law of the Wall

With the introduction of roughness to a surface, the boundary-layer velocity profile will be altered. To account for this modification, an arbitrary function $\frac{\Delta u}{u_\tau}$ can be added to the logarithmic velocity profile relation (Hama, 1955):

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{yu_\tau}{\nu}\right) + B + \frac{\Delta u}{u_\tau} \quad (2.5)$$

This new function can be called the roughness effect. Initially, experiments using standard sand roughness were conducted to quantify the roughness effect (Prandtl and Schlichting 1934, Nikuradse 1933). Standard sand roughness requires a surface to be completely covered in sand so the spatial density of the sand grains is fixed at its maximum possible value. Thus, it is possible to describe the geometry of standard sand roughness only in terms of the height of the sand. Prandtl and Schlichting proposed that the roughness effect would then be dependent on the roughness Reynolds number of the rough wall:

$$\frac{\Delta u}{u_\tau} = F\left(\frac{k_s u_\tau}{\nu}\right) \quad (2.6)$$

The effective height of standard sand, k_s , was defined by Nikuradse as the "size of mesh of the coarser of the two sieves through which the sand was sifted" (Prandtl, 1960), which corresponds to the maximum sand grain size.

Prandtl and Schlichting used the roughness Reynolds number of standard sand roughness to characterize the flow over rough walls:

$\frac{k_s u_\tau}{\nu} < 4 :$	hydraulically smooth — roughness has no observable effect on flow field.
$4 \leq \frac{k_s u_\tau}{\nu} \leq 60 :$	transitionally rough — roughness effect is dependent on viscosity.
$60 < \frac{k_s u_\tau}{\nu} :$	fully rough — roughness effect is dependent mainly on the form drag of the roughness elements and is independent of viscosity.

Subsequent authors have supported this theory of roughness regimes, although somewhat different limits on $\frac{k_s u_\tau}{\nu}$ are often proposed for each regime.

This theory was developed to account for the behaviour of standard sand roughness, and not the behaviour of large, isolated roughness elements on a generally smooth wall. The same smooth, transitional and fully rough flow regimes exist with non-standard sand roughness, however at low roughness element densities these regimes may be defined by quite different roughness Reynolds numbers. Chen and Roberson (1974),

in an investigation of sparsely-distributed hemispherical roughness elements, found that their experimental flow did not become fully rough until the roughness Reynolds number, $\frac{ku_r}{\nu}$, had exceeded a value of 200. Koloseus *et al.* (1962) offer an explanation for this phenomenon: they state that when there are large areas of smooth wall exposed to the flow, which can occur with widely spaced roughness, roughness element form drag will no longer be the dominant source of boundary resistance. Shear forces on the smooth wall can become sufficiently large at low roughness element densities to render the surface transitionally rough or even hydraulically smooth, even when the roughness Reynolds number exceeds the standard sand limits for fully rough flow as given above.

Once the roughness regime has been determined, the roughness effect can be quantified. If the flow is effectively smooth, there is no roughness effect, so $\frac{\Delta u}{u_r} = 0$. In transitionally rough and fully rough regimes, the following equation has been found to hold (White, 1991):

$$\frac{\Delta u}{u_r} = \frac{1}{\kappa} \ln\left(\frac{ku_r}{\nu}\right) + C \quad (2.7)$$

which is consistent with equation (2.6). With standard sand roughness, Nikuradse (1933) found the intercept C to vary with $\frac{ku_r}{\nu}$ in the transitionally rough regime. In the fully rough regime, Nikuradse found C to have a constant value of approximately -3 (Figure 2.3). In his calculations, Nikuradse used values for $1/\kappa$ and B that are no longer in general use, and that differ from the values given with equation (2.4). Re-analysis of Nikuradse's data with the currently accepted values of $1/\kappa$ and B (Grigson, 1984) yielded $C \approx -4$.

FIGURE 2.3: Nikuradse's (1933) results for standard sand roughness showing C , the intercept of the roughness effect approaching a constant value $C \approx -3.0$ in fully rough flow. Reproduced from Schlichting (1968).

Hama (1955) found that non-standard sand roughnesses, where the grains were not closely packed, and wire-screen type roughness behaved in a similar manner to Nikuradse's standard sand roughness, although the intercept, C , approached a different final value in the fully rough regime. Dvorak (1969) compiled more experimental data for a wide variety of roughness configurations in both the fully rough and transitionally rough regimes (Figure 2.4). All experimental data followed the same trends seen in Nikuradse (1933) and Hama (1955).

The effects of roughness on the velocity profile can now be summarized by combining equations (2.5) and (2.7):

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln\left(\frac{v}{k}\right) + B + C \quad (2.8)$$

For the fully rough case, C is a constant for any particular roughness configuration. For this reason, C is hereafter referred to in the fully rough regime as the *roughness constant*.

An alternative method of expressing the roughness constant is given in the concept of the equivalent sand roughness height. Schlichting (1936) characterized C for rough surfaces composed of regular geometric shapes including spheres, cones and flat plates. He expressed his results in terms of standard sand roughness, however, assuming that all his experiments took place in the fully rough regime. Using the roughness constant value of -3 for standard sand roughness in fully rough flow, and experimentally determining the roughness constant for a fully rough surface with non-standard roughness of height k , the non-standard roughness can be related to an equivalent standard sand roughness height, k_s , through equation (2.7). This leads to the following expression after simplification:

$$\frac{k_s}{k} = 10^{\frac{2.98}{5.75} C} \quad (2.9)$$

Equation (2.9) is derived using the value of the roughness constant and the law of the wall coefficients originally used in the standard sand roughness studies of Nikuradse (1933), not those from White (1991) quoted after equation (2.4). Some error will be introduced to equivalent sand roughness height calculations by using these older coefficients if C was calculated using different coefficients. It is, however, necessary to use the given form of equation (2.9) to consistently compare experimental results to those of standard sand roughness. By simply comparing experimental values of the roughness

FIGURE 2.4: Experimental results showing all roughness types to follow the same trends in the transitional and fully rough flow regimes. Reproduced from Dvorak (1969).

constant, this confusion can be avoided.

In atmospheric boundary layer studies, equation (2.8) is usually expressed in terms of the roughness height parameter z_0 . For the fully rough case with no significant heat-transfer effects, or *neutral flow* conditions, this leads to

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{y}{z_0}\right) \quad (2.10)$$

Although equation (2.10) is more compact, equation (2.8) is better suited to the examination of the roughness effect. The intercept, C , is the only part of equation (2.8)

which is dependent on the roughness type; as C is isolated from the rest of the function, it is somewhat easier to mathematically manipulate than z_0 would be.

Despite disagreement on the effects of freestream pressure gradient on smooth wall flows, several experiments have shown it to not have any effect on the roughness constant. Perry and Joubert (1963) and Nigim and Cockrell (1985) showed that adverse pressure gradients did not influence C over two-dimensional roughness elements, while Coleman *et al.* (1977) showed that favourable pressure gradients over spherical roughness also had no significant effect on the roughness constant; suction was applied to a porous wall to control pressure gradients in this last experiment.

The present study focuses on fully rough flow. Transitionally rough flows show C to be dependent on the roughness Reynolds number, and are thus more difficult to analyze. Further studies in the transitional regime would be best conducted after fully rough flow is better understood.

2.2.2 The Displacement Height

Although the preceding sections give both theoretical and experimental justification for the rough-wall law of the wall, it is not immediately obvious how the origin of y in equations (2.8) and (2.10) should be defined. Because a rough surface is inherently uneven, there is no unambiguous way of defining the effective origin of the wall in geometric terms. The effective elevation used in equations (2.8) and (2.10) can be defined

$$y \equiv y_0 - d \quad (2.11)$$

FIGURE 2.5: Illustration of the effects of roughness on the effective origin for the law of the wall, showing the variables of equation (2.12).

where y_0 is the elevation from the wall itself, and d , known as the *displacement height*, is the height of the effective origin above the wall (Figure 2.5).

Some authors have estimated d to be a fixed fraction of the roughness height. This does not take into account any properties of a specific roughness configuration, however. Others (notably Schlichting (1936) and O'Loughlin and MacDonald (1964)), used the average roughness height as d . This too is an unsatisfactory definition, as it relates only to the total volume of roughness, and not to the effects on the flow caused by the roughness (Coleman *et al.*, 1984). A variety of graphical methods have been used since Perry and Joubert (1963) to estimate d from experimental velocity profile data. In these cases, d is chosen to satisfy the logarithmic law of the wall (equation 2.8). Authors usually try to obtain a slope of $1/\kappa$ on a best-fit line through their data; the shear velocity u_τ is often obtained at the same time, requiring iteration. The results for d and u_τ tend

to be very ambiguous. Several authors, including Jackson (1981) and Coleman *et al.* (1984), have stated that this method can lead to inaccurate prediction of both u_τ and d .

Thom (1971), observed that his estimates of d were in very good agreement with the centre of pressure of the roughness elements used in his experiments. This led Jackson (1981) to obtain an explicit definition of the displacement height. Jackson argued that above a certain height, δ_τ , the flow inside the law-of-the-wall region is not affected by individual roughness elements. Above this height, the flow behaviour is governed by the wall shear stress τ_w , but cannot distinguish any details of the surface except the height at which τ_w appears to act, the displacement height, d . Using a control volume analysis, Jackson showed that

$$d - \bar{k} = \delta_\tau \quad (2.12)$$

In equation (2.12), $\bar{k}$ is the average height of the roughness, defined by

$$\bar{k} = \int_0^k \frac{A_R(y)}{A_s} dy \quad (2.13)$$

where $A_R(y)$ is the total plan area of the roughness elements at height y , and A_s is the plan area of the wall. In other words, $\bar{k}$ is the height attained when the total volume of roughness is evenly distributed over a surface (Figure 2.5). δ_τ is the shear-stress displacement thickness, defined as:

$$\delta_\tau = \frac{1}{\tau_w} \int_0^{\delta_\tau} (\tau_w - \tau(y_0)) dy_0 \quad (2.14)$$

where $\tau(y_0)$ is the shear stress in the fluid at the height y_0 . $\tau(y_0)$ is given by:

$$\tau(y_0) = \left(\rho \overline{u'v'} + \frac{1}{\sigma} \iint (\rho uv) d\sigma \right)_{y_0} \quad (2.15)$$

The first term in equation (2.15) is the Reynolds shear stress at height y_0 , and the second term is the effective shear stress in the fluid caused by momentum flux associated with the contorted three-dimensional streamline pattern of the time-averaged flow over the rough surface. Figure 2.6 shows Jackson's interpretation of δ_+ . The term $(\tau_w - \tau)_{y_0}$ can be thought of as the drag-induced stress acting on the roughness elements at height y_0 . Note that $\tau(y_0) = \tau_w$ for $y_0 = \delta_+$. With a smooth wall in zero pressure-gradient, $\tau = \tau_w$ for all y_0 , so $d = 0$ as expected.

Molion and Moore (1983) used mass conservation principles to develop a numerical technique to estimate the displacement height over tall vegetation. Their method uses time-averaged velocity profile measurements from rough and smooth walls

FIGURE 2.6: Jackson's definition of the shear-stress displacement thickness shows the displacement height, d , the height at which the wall shear stress, τ_w , appears to act. Reproduced from Jackson (1981).

with similar freestream flow conditions to estimate d . They obtained very good results which were relatively insensitive to measurement errors, but it is unknown how well their method works over other types of roughness.

2.2.3 Heat Transfer and the Law of the Wall

Heat transfer to or from a wall can substantially affect the near-wall flow. At sufficiently high rates of heat transfer, buoyancy effects can become significant, and the boundary layer will become unstable. Under these conditions, the logarithmic law (equations 2.5 and 2.8) will only be valid over a very small region of the turbulent boundary layer (Monin, 1970). The present study does not deal with experimental data that has significant amounts of heat transfer.

2.2.4 Effects of Roughness Spacing

Although values of the roughness constant C were found for a variety of rough surfaces, there was initially no attempt to correlate these values to the geometry of the roughness. From Schlichting's experiments (1936), it could be seen that C varied with both the shape of the individual roughness elements, and the spacing of the elements. Morris (1955) proposed that the majority of the roughness effect stemmed from vortices and wakes generated by the roughness elements. The vortices and wakes produced by any roughness element would interact with downstream roughness elements and their accompanying wakes and vortices; the magnitude of the roughness effect should depend on the spacing and position of the elements. Morris defined a spacing parameter to characterize the spacing of two-dimensional roughness:

$$\lambda_L = \frac{L}{k} \quad (2.16)$$

where L is the pitch between roughness elements and k is the maximum element height.

Morris also suggested that the flow over rough walls could be divided into three categories, depending on the value of λ_L :

Isolated roughness flow — where roughness spacing is sufficiently sparse to minimize the interaction of roughness element wakes. The roughness effect of a surface is simply the product of the roughness effect of a single element and the number of elements on the surface.

Wake interference flow — where elements are sufficiently close together for their wakes to interact significantly.

Skimming flow — where elements are very close together, resulting in the majority of the flow passing over the elements with stationary vortices being formed in the cavities between elements. Losses come mainly from fluid entrainment into the stationary vortices in the cavities.

Later authors found empirical relations between roughness spacing λ_L and the roughness constant C in fully rough flow. Betterman (1966) studied densely spaced ($\lambda_L \leq 4.2$), two-dimensional square bars and found:

$$C = 28.35 \log_{10} \lambda_L - 17.35 \quad (2.17)$$

Instead of the natural logarithm, a base 10 logarithm is normally used in these correlations to simplify graphical presentation of the data.

FIGURE 2.7: Dvorak's correlation between the roughness spacing parameter λ_D and the roughness constant C . Reproduced from Dvorak (1969).

Dvorak (1969) compiled significantly more experimental data and related C to a slightly different spacing parameter:

$$\lambda_D = \frac{L}{s} \quad (2.18)$$

where s is the streamwise dimension of the roughness element. For many of the roughness shapes examined by Dvorak, $\lambda_D = \lambda_L$ since $s = k$ for elements of square or circular cross-section in the plan view. Dvorak divided the roughness into two groups: for densely spaced roughness ($\lambda_D \leq 4.7$), he restated Betterman's equation (2.17) with the new spacing parameter:

$$C = 28.19 \log_{10} \lambda_D - 17.35 \quad (2.19a)$$

For $\lambda_D > 4.7$, Dvorak found:

$$C = -6.56 \log_{10} \lambda_D + 5.95 \quad (2.19b)$$

Dvorak stated that (2.19a) and (2.19b) were best suited to two-dimensional roughness elements, although several studies using three-dimensional elements were included in his analysis (Figure 2.7). With the exception of Hama's (1955) screen roughness, the span b of every three-dimensional roughness element studied was roughly equal to its streamwise dimension s .

Simpson (1973), introduced a spacing parameter λ , based on the frontal area of the roughness elements, A_F , and the plan surface area of the wall, A_S :

$$\lambda = \frac{A_S}{A_F} \quad (2.20)$$

For two-dimensional roughness elements, this is equivalent to the definition of λ_L given by Morris (1955) and Betterman (1966) in equation (2.16). Using different experimental data from Dvorak, Simpson re-formulated equations (2.19a) and (2.19b) with λ instead of λ_D :

$$C = 28.19 \log_{10} \lambda - 17.35, \lambda \leq 4.7 \quad (2.21a)$$

$$C = -6.56 \log_{10} \lambda + 5.95, \lambda > 4.7 \quad (2.21b)$$

FIGURE 2.8: Simpson's correlation between roughness spacing λ and the roughness constant C . Reproduced from Simpson (1973).

Simpson's equations and data are plotted in (Figure 2.8). Certain data did not fit well to the proposed C vs λ relationship. The stone roughness of Mirajgaoker and Charlu (1963) did not agree with equation (2.21a) at low values of λ , but continued to follow the path of equation (2.21b) instead. Simpson hypothesized that this occurred because the shape of the roughness prevented the formation of stationary spanwise vortices, which occur in skimming flow in two-dimensional roughness. Simpson offered no explanation for the poor agreement of various 'short angle' or thin plate roughness elements with equation (2.21b).

2.2.5 Effects of Roughness Element Shape

Wooding *et al.* (1973) compiled the results from a large number of rough-wall experiments, and for a wide variety of two-dimensional and three-dimensional roughness element shapes. They found a relationship for the function, $\sqrt{\frac{C_f}{2}}$ or $\frac{U_e}{u_\tau}$:

$$\frac{U_e}{u_\tau} = 2.87 \ln \left(\frac{\delta}{\frac{k}{\lambda} \phi} \right) - 2.05 \quad \left(30 < \frac{\delta}{\frac{k}{\lambda} \phi} < 2000 \right) \quad (2.22)$$

The boundary layer thickness δ was approximated by $\delta \approx 3.3 + 15 \left(\frac{\phi}{\lambda} \right)^{0.43}$. U_e is the velocity at the outer edge of the boundary layer. The term $\phi = \left(\frac{k}{s} \right)^{0.4}$ defines an aspect-ratio for the roughness elements, where k is the roughness element height and s is the streamwise dimension of the roughness element. Wooding *et al.* pointed out that their database, like Dvorak's (1969), contained few roughness elements where k/s was not equal to k/b , b being the span of the roughness element perpendicular to the freestream direction. While the approximation for δ renders equation (2.22) mathematically inconsistent with the law of the wall, there was a reasonably good collapse of data. From Figure 2.9 it can be seen that neither the data for two-dimensional roughness elements nor the data for Marshall's cylindrical roughness elements (1971) agree with equation (2.22). A noteworthy characteristic of equation (2.22) is its attempt to express the rough-wall velocity profile in a function based completely on the roughness geometry, and not on any flow-dependent variables, such as roughness element drag (although this did require a roughness geometry-based model of the flow-dependent parameter δ).

FIGURE 2.9: Correlation reproduced from Wooding *et al.* (1972).

Dirling (1973) proposed that, in addition to element spacing, the roughness constant depended on the effective bluntness of roughness elements. He defined an effective spacing parameter λ_B to account for both effects:

$$\lambda_B = \left(\frac{D}{k} \right) \left(\frac{A_W}{A_F} \right)^{\frac{1}{3}} \quad (2.23)$$

FIGURE 2.10: Dirling's correlation between λ_B , his effective spacing parameter that accounts for roughness element bluntness, and the roughness constant C . Reprinted from Dirling (1973).

where A_F remains the frontal area of the roughness elements normal to the freestream direction. A_w is defined as the upstream-facing wetted area of the roughness elements — for a cylinder $A_w = \frac{\pi}{2} A_F$, while for a cubic element, $A_w = A_F$. Lastly, the element density D is defined: $D = \sqrt{\frac{A_s}{n}}$, where A_s is the plan surface area of the wall and n is the number of roughness elements on the rough surface.

Using a database similar in size and content to Dvorak (1969) and Simpson (1973), Dirling found:

$$C = 21.74 \log_{10} \lambda_B - 13.26, \quad \lambda_B < 4.93 \quad (2.24a)$$

$$C = -10.93 \log_{10} \lambda_B + 9.32, \quad \lambda_B \geq 4.93 \quad (2.24b)$$

Equations (2.24a) and (2.24b) are plotted in Figure 2.10, and show a good fit to the experimental data. However, Dirling's analysis of two-dimensional roughness elements appears to be incorrect: he assumed $\lambda_B = \lambda_L$, the spacing parameter given by Morris (1955) and Betterman (1966) in equation (2.16). In actuality, because A_s is theoretically infinite for two-dimensional roughness elements while the element height, k , remains finite, the parameter λ_B will be undefined for two-dimensional roughness elements.

Dalle Donne and Meyer (1977) compiled a vast amount of experimental data dealing with the effect of two-dimensional roughness in pipes. For a spacing parameter, they used a modified form of the parameter λ_L given by Morris (1955) by non-dimensionalizing the size of the cavity between roughness elements, instead of the pitch of the elements: $\lambda_{DDM} = \lambda_L - s/k$. However, they only gave a functional relation to C for values of $s/k \approx 1$, so their findings apply to a very limited range of roughness geometries. Much of their data involved significant amounts of heat transfer which was accounted for by use of a non-dimensional temperature parameter.

Sigal and Danberg (1990) differentiated between the roughness effect for two-dimensional and three-dimensional roughness elements, although no theoretical explanation was given for this division. Like Dirling, they incorporated element bluntness into their effective spacing parameter λ_{DS} :

$$\lambda_{DS} = \lambda \left(\frac{A_w}{A_F} \right)^{1.6} \quad (2.25)$$

Since their spacing parameter uses the λ of equation (2.20) instead of Dirling's (1973) D/k , Sigal and Danberg were able to correctly characterize two-dimensional roughness

geometry. For two-dimensional roughness elements, they used three spacing regimes to predict the roughness constant:

$$C = 28.35 \log_{10} \lambda_{DS} - 17.4, \quad 1.4 \leq \lambda_{DS} \leq 4.9 \quad (2.26a)$$

$$C = 2.2, \quad 4.9 < \lambda_{DS} < 13.3 \quad (2.26b)$$

$$C = -6.55 \log_{10} \lambda_{DS} + 9.6, \quad 13.3 \leq \lambda_{DS} \leq 100 \quad (2.26c)$$

For three-dimensional roughness, Danberg and Sigal used data at relatively large element spacings only:

$$C = -7.70 \log_{10} \lambda_{DS} + 9.7, \quad 16 \leq \lambda_{DS} \leq 200 \quad (2.27)$$

The fit to data offered no great improvements on Dirling..

Raupach *et al.* (1991) summarized the dependence of C on roughness shape and spacing, which in fully rough conditions can be expressed:

$$C = F(l_i^*) \quad (2.28)$$

where l_i^* represent the non-dimensionalized forms of all relevant length scales needed to completely characterize the rough surface — the authors said that l_i^* would include a spacing parameter, and parameters based on the roughness element dimensions. The authors also documented possible errors in the measurement techniques of previous experiments. Besides noting the need for “spatial-averaging” of velocity profile measurements, it was also said that “high turbulence intensities encountered near the roughness cause many standard measurement techniques (X-wire anemometry in

particular) to suffer from substantial errors that have often proved difficult to diagnose and correct."

Raupach (1992) proposed that, in isolated roughness flow, the size of the wake generated by individual roughness elements (and thus the magnitude of the drag, according to Morris) was approximately proportional to the velocity U_k at the crest of the element. This velocity however, is also dependent on roughness shape (Sakamoto et. al., 1982), so this relationship does not appreciably simplify the determination of C . For denser roughness spacings, Raupach's analysis showed the roughness constant to be independent of U_k .

2.3 Available Experimental Data

The data compiled in this study to improve the prediction of the roughness constant came from a variety of meteorological and aerodynamic experiments. The data was selected to give a broad range of roughness shapes and spacings. Criteria for the inclusion of data were as follows:

Roughness shape was geometrically well defined and consistent (uniform roughness).

Roughness spacing was regular.

At least the logarithmic layer of the velocity profile was in near equilibrium.

Heat transfer was insufficient to significantly alter the flow or fluid properties.

The experiments providing this data are summarized in Table 2.1 and described in the next section. Absolute precision of the values of the roughness constant found in these experiments is generally ± 1 . Many authors, however, did not provide sufficient information to evaluate the precision of their data, so no quantitative evaluation of the experimental error was possible.

Some data were excluded from this analysis. All data (bluff-bodies) from Hussain and Lee (1980) were excluded on the grounds of poor experimental methodology; spatial averaging was not used in their measurement of velocity profiles. All data (thin rods) from Kawatani and Meroney (1970) were excluded because their assumption of $d=k$ for their displacement thickness led to high levels of uncertainty in their results. Because Sullivan and Greely (1993) assumed $d=0$ in their analysis, results at the closest roughness spacing ($\lambda=10.5$) were rejected. Based on the findings of Raupach *et al* (1980), it was

assumed that the $d=0$ assumption would not adversely affect the remainder of Sullivan and Greeley's results at higher spacings. Finally, some surfaces in Schlichting's (1936) experiments were found by Coleman *et al* (1984) to be transitionally rough, and their data were excluded from this analysis.

2.3.1 Summary of Data

Authors	Type	#	Distrib	Instrument	λ	dP/dx	C_f	d	Note
Antonia and Luxton (1971)	2-D bar	1	N/A	A	4	0	MIE, grp	grp	
Antonia and Wood (1975)	2-D bar	1	N/A	A,P	2	0	grp	grp	
Botterman (1966)	2-D bar	3	N/A	PS	2.7-4.2	0	MIE, grp	grp	
Boisvert et. al. (1977)	2-D bar	1	N/A	A	4	0	MIE, grp	grp	
Chen & Robertson (1974)	hs	1	stag	A,P	51	neg	?	?	1
Coleman, Hodge et. al. (1984)	ca, sp, ss	14	stag, cp	PS	1.1-121	0	MIE, grp	vol	2
Coleman, Moffat et. al. (1977)	sp	2	cp	A	11	0,+	grp	grp	3
Counihan (1971)	block	2	?	?	8.4-113	0	grp	grp	4
Faraya et. al. (1976)	2-D wire	7	N/A	PS	2-64	0	MIE	N/A	5
Gartshore & De Kroos (1977)	plate	1	stag	A	32	0	direct	N/A	
Homi et. al. (1993)	hs	2	stag	A	10-41	0	Rey	N/A	
Lee & Solomon (1977)	cube	5	row	A,P	3.1-25	0	MIE, grp	grp	
Liu et. al. (1966)	2D-bar	4	N/A	A	2-96	0	grp	grp	
Marshall (1971)	cyl, hs	40	row	PS,C	5.7-5000	0	direct	N/A	6
O'Loughlin&MacDonald(1964)	cube	12	row, stag	H,D	1.4-512	0	?	vc:	7
Perry & Joubert (1963)	2-D bar	2	N/A	PS	4	0,-	MIE, grp	grp	8
Raupach,Coppin & Legg(1986)	plate	1	stag	A,E	4.4	0	MIE, Rey	Cp	
Raupach,Thom&Edward(1980)	cyl	5	row, stag	A	5.6-89	0	grp	grp	
Sayre & Albertson (1961)	plate	6	stag	?	4-50	0	?	?	9
Schlichting (1936)	plate	3	stag	PS	17-67	0	MIE, grp	vol	10
Seginer et. al. (1976)	rod	3	stag	A	1-2.7	0,-	MIE:	grp	11
Shen & Logan (1977)	2-D bars	5	N/A	A,PS	2.2-38	0	?	?	12
Sullivan & Greeley (1993)	cyl	6	arc	PS	10.5-167	0	grp	$d=0$	
Thom (1971)	rod	1	stag	A	0.7	0	direct	Cp	

TABLE 2.1: Description of experiments used in the present study.

Explanation of abbreviations and notes in Table 2.1:

Abbreviations

Type: type of roughness element used in experiment

cn: cones **hs:** hemispheres **sp:** spheres **ss:** spherical segments **cyl:** cylinders

plate: flat plate mounted perpendicular to the flow direction

#: number of data points obtained from paper

Distrib: distribution of roughness elements in the experiment

row: roughness elements aligned in rows (flow direction →)

stag: elements aligned in staggered rows (flow direction →)

cp: elements in a close-packed formation (special case of a staggered distribution)

arc: roughness elements arranged in arcs (flow direction →)

?: unknown

Instrument: instrumentation used in the experiment

A: hot-wire anemometer (chiefly X-wire)

C: cup anemometer

E: sonic anemometer

D: dynamic pressure tap

H: depth measurement (to calculate static pressure in water)

P: static pressure tap

PS: Pitot-static tube

?: unknown

λ : range of roughness spacing (from Simpson, 1973) used in the experiment

dP/dx : pressure gradient in experiment

(0): zero or negligible

(-): significantly negative

(+): positive pressure gradient

C_f : method used to determine the skin friction coefficient

MIE: momentum integral equation or momentum balance

grp: graphical (iterative) method

direct: direct measurement of forces

Rey: measurement of the Reynolds stresses ($-\rho \overline{u'v'}$)

?: unknown

d: method used to determine the zero-plane displacement height

d=n: arbitrary value of d was selected

grp: graphical (iterative) method

vol: average roughness height as proposed by Schlichting (1934)

N/A: method used to calculate C did not require calculation of the zero-plane displacement height

Cp: centre of pressure of forces acting on the roughness element

?: unknown

Notes

1. Direct drag measurements on roughness elements showed the proportion of smooth-wall drag to roughness element form drag.

2. Improved analysis of some of Schlichting's (1936) data.
3. Roughness mounted on a porous wall — authors claim this did not effect results. Includes data from Pimenta (1975) for same geometry without a porous wall.
4. Roughness elements were Lego® blocks. Exact distribution was not specified — an aligned row distribution in a square pattern ($\beta=1$, $\gamma=0$) was assumed in the present study.
5. Dvorak's discussion of the paper questions the accuracy of the momentum integral method used to obtain the results.
6. Lack of roughness trip produced a non-equilibrium flow. Wooding *et al.* (1973) suggest that the roughness array was also too close to a wind tunnel contraction. This data was analyzed separately from the rest of the database to qualitatively evaluate Marshall's results, which were believed to be self-consistent.
7. Includes data from Koloseus and Davidian (1960)
8. Includes data from Moore (1951) and Rand (no reference).
9. Authors tested a variety of roughness distributions in addition to spacings.
10. Data was not re-analyzed by Coleman *et al.* (1984).
11. Data for negative pressure gradient experiment may not be in equilibrium due to an abrupt contraction of the wind tunnel.
12. No details of experimental methodology were given.

Chapter 3: The Characterization of Roughness and the Methodology of Analysis

Using the experimental roughness arrays described in section 2.3, the goal of this work was to correlate the roughness geometry of those arrays to the roughness constants associated with those arrays. The correlations were found with the aid of computerized sorting and optimization routines.

3.1 Geometric Characterization of the Rough-Wall

To examine the effects of roughness geometry on the roughness constant, the roughness arrays in the experimental database had to be accurately and consistently defined in geometric terms. To find the best correlation between roughness shape and the roughness constant, it is necessary to use all geometric parameters that have a significant impact on the flow over the roughness. Conversely, in the interest of simplicity, it is desirable to produce a correlation using a minimum number of parameters. Work described in the previous chapter specified some of the required parameters, while insight into the likely influence of other geometric parameters was obtained from fluid dynamic theory and the examination of previous experimental results.

3.1.1 Parameters Used in the Analysis

Seven non-dimensional geometric parameters were used to characterize the compiled experimental rough surfaces of Table 2.1. The parameters describe either the geometry of individual roughness elements, or the layout of the roughness elements, using a combination of length scales obtained from the roughness array (Figure 3.1). These length scales were defined in such a way as to make them relatively simple to calculate, but also to be physically meaningful for all the roughness shapes studied.

Roughness element height k was defined as the maximum element height, in keeping with the standard convention for rough-walls. The displacement thickness d was not considered in this definition, as it is a flow-dependent parameter; k was measured from the wall. The width and streamwise dimension of roughness elements were defined in terms of their mean values, $\bar{b}$ and $\bar{s}$ respectively:

$$\bar{b} = \frac{1}{k} \int_0^k b(y) dy \quad (3.1)$$

$$\bar{s} = \frac{1}{k} \int_0^k s(y) dy \quad (3.2)$$

When combined with the roughness height, k , this was believed to better represent the element geometry than maximum values of width and depth, b_{max} and s_{max} . The use of b_{max} and s_{max} to describe roughness geometry assumes *a priori* that varying cross-sectional area, as occurs in spherical and conical elements, has no impact on the roughness

FIGURE 3.1: Illustration of the geometric parameters of rough surfaces: a) roughness cell geometry; b) roughness element geometry; c) mean values of roughness element geometry.

constant. As Dirling (1973) showed that element shape influenced the roughness constant, this assumption would likely be incorrect. The frontal area, A_F , of roughness elements with varying cross-sectional area is also correctly calculated by the product of the roughness height, k , and $\bar{b}$, the mean element width; this would not be the case with k and b_{max} .

3.1.2 Roughness Distribution Parameters

Spacing parameters have long been used, beginning with the early attempts to predict roughness effects from roughness geometry. However as seen in Chapter Two, a variety of definitions have been proposed for spacing parameters. As a first step in inspecting the experimental database, plots were constructed based on Simpson's (1973) spacing parameter, λ (equation 2.20), and Dvorak's (1969) spacing parameter, λ_D (equation 2.18). In the latter case, a variant of λ_D based on the mean element plan area was introduced:

$$\lambda_{\bar{D}} = \frac{\frac{1}{k} \int_0^k s(y) b(y) dy}{A_c} = \frac{\bar{s}\bar{b}}{A_c} \quad (3.3)$$

where A_c is the plan area associated with a single roughness cell (Figure 3.1). Again, mean element dimensions were thought to be more theoretically representative of the roughness element geometry. Dirling's (1973) parameter λ_B (equation 2.23), was not examined until the effects of roughness shape were investigated.

In Figure 3.2, the values of the roughness constant from the experimental database show a definite dependence on Simpson's parameter λ , albeit with a wide band of scatter.

FIGURE 3.2: Plot of roughness constant C vs Simpson's (1973) spacing parameter λ (equation 2.18) for the entire experimental database. Data grouping is described in Table 3.2.

In Figure 3.3, little relationship appears evident between $\lambda_{\bar{D}}$ and C at first. Upon closer examination however, the roughness constant values show some dependence on $\lambda_{\bar{D}}$, but only for close roughness spacings (Figure 3.4). Contrary to Dvorak's results (for a rather limited number of data points), no dependence is evident at higher values of $\lambda_{\bar{D}}$. Because of this, λ as defined by equation (2.20), was selected as the principal spacing parameter, although $\lambda_{\bar{D}}$ was later found to serve some role in the analytical process.

Work by several authors, including Marshall (1971) and Sayre and Albertson

FIGURE 3.3: Plot of experimental roughness constant vs λ_D (equation 3.3), a modification of Dvorak's spacing parameter λ_D , for the entire database.

(1961) has shown that the distribution of roughness elements on a surface has some effect on the flow, and thus on the roughness constant. From their observations, it was theorized that the flow around roughness elements would be more severely obstructed when the elements presented a continuous barrier to the flow, as opposed to the elements being evenly distributed over the surface (Figure 3.5). The ultimate extension of this scenario results in a surface covered in two-dimensional roughness elements. To account for element distribution, a maximum blockage parameter β_x was introduced:

FIGURE 3.4: Plot of experimental roughness constant vs λ_D for $\lambda_D = 20$.

$$\beta_x = \frac{A_f}{A_v} \quad (3.4)$$

where $A_v = kM$ is the frontal area of the roughness element cell. β_x increases as the flow around (as opposed to over) the roughness elements is restricted, reaching a maximum value of unity with two-dimensional elements. This parameter is, however, dependent on λ , as the average spacing between elements is proportional to λ . To eliminate this

FIGURE 3.5: Illustration of possible differences in flow likely with variations in roughness element distribution. Element spacing and shape are identical for the two cases.

dependence, β_x was divided by the blockage parameter for a random element distribution at the same roughness spacing, $\beta_{x,rand}$. Analysis showed $\beta_{x,rand} = \sqrt{\frac{1}{\lambda Z_b}}$, where Z_b is the spanwise roughness element aspect ratio (Table 3.1). Thus, the new non-dimensional blockage parameter, β_c is defined as:

$$\beta_c = \frac{\beta_x}{\beta_{x,rand}} = \beta_x \sqrt{Z_b \lambda} = \frac{L}{M} \quad (3.5)$$

L and M are the streamwise and spanwise dimensions, respectively, of the roughness cell (Figure 3.1a). Thus, β_c is also the roughness cell aspect ratio. This parameter does not account for the staggering of the rows of roughness elements.

To account for row staggering, a separate stagger parameter was defined:

$$\gamma = \frac{m}{M} \quad (3.6)$$

where m is the spanwise offset of consecutive rows (Figure 3.1). Values for γ are usually either 0 or 0.5, as an asymmetrical stagger pattern where $m \neq 0.5$ is not commonly investigated. Instead of using this parameter in correlations then, it was decided to simply identify the stagger parameter in plots of other correlations, and see if any differences could be attributed to the stagger pattern.

3.1.3 Parameters of Roughness Element Geometry

Geometric parameters for individual roughness elements (Table 3.1) were defined to describe the aspect ratios of the element's projected area normal (Z_n) and parallel (Z_s) to the freestream direction. For some of the experiments which used flat plate roughness elements, however, the thickness of the plate, s , was not given. Therefore, to calculate the streamwise aspect ratio, Z_s , a reasonable value had to be assumed for the plate thickness.

Since the ratio of surface area to enclosed volume will depend on the shape and surface curvature of an element, it was felt that some parameter could use this ratio to distinguish between roughness element shapes (cubes, cylinders, *etc.*). Originally, this parameter was defined as the ratio of the total roughness element wetted surface area to the frontal area of the roughness element, A_F . However, this parameter was cumbersome to calculate, as it depended on the aspect ratios of the element in many cases. Instead, Dirling's (1973) bluntness parameter, $Z_r = A_w/A_F$, was adopted, where A_w is the wetted area of only the upstream side of the roughness element.

Type	Variable	Comments
Element Geometry	$Z_b = \frac{k}{b}$	
	$Z_s = \frac{k}{s}$	Used by Wooding <i>et al.</i> (1973).
	$Z_r = \frac{A_w}{A_f}$	Bluntness parameter proposed by Dirling (1973).
Element Distribution	λ	Simpson's (1973) spacing parameter
	λ_D	Similar to Dvorak's (1969) spacing parameter
	$\beta_C = \frac{L}{M}$	Roughness cell aspect ratio
	$\gamma = \frac{m}{M}$	Stagger parameter

TABLE 3.1: Parameters used to characterize roughness geometry.

3.1.4 Characterization of Spacing Regimes

Previous authors (Dvorak, 1969, Simpson, 1973, Dirling, 1973) had suggested that, dependent on the value of a particular spacing parameter, the roughness constant could be fit to one of two regimes; these regimes corresponded to Morris' (1955) 'skimming flow' and 'isolated roughness flow'. To avoid confusion with the term *roughness regime* introduced in section 2.2.1 to distinguish between smooth, transitional and fully rough flows, the classifications introduced here will be referred to as *spacing regimes*.

As seen in the previous section, Simpson's spacing parameter λ is a better predictor of the roughness constant C for isolated roughness than Dvorak's parameter λ_D . Consequently, it was initially thought that the boundary between spacing regimes would

FIGURE 3.6: Illustration of the likely effects of roughness element spanwise aspect ratio, $k/\bar{b}$, on rough wall flows. The values of the roughness spacing parameter λ are identical for the two cases.

also be better predicted by λ . Upon more detailed examination however, it was noted that in a plot of C vs λ (Figure 3.2), the boundary between regimes shows some dependence on the shape of the roughness element. Of particular note were the experiments using thin rod roughness (Thom, 1971 and Seginer *et al.*, 1976). In these cases, the value of the roughness constant did not entirely conform to the skimming flow regime at extremely low values of λ , where all other roughness element shapes were well within the skimming flow regime; the results for thin rods appear to fall somewhere between the two regimes. It was concluded that some other means of spacing regime delineation would be necessary.

It was required to provide an explanation for the differences in the flow around thin rod elements and the flow around other roughness element shapes that produced less

atypical results. It was hoped this would lead to a more general theory of spacing regimes. It was realized that the geometry of thin rods inherently produces a deceptively low value of the spacing parameter λ — the height of a thin rod results in a large frontal area even with large distances between the elements. With cylinders of a lower aspect ratio, an equivalent frontal area would require a much closer spacing of the roughness elements (Figure 3.6). It seems reasonable that the flow around the two different configurations of Figure 3.6 should be quite different, even though they have similar values of λ ; the densely arranged short cylinders would present more of a barrier to flow passing between roughness elements, and would promote skimming flow at values of λ where the thin rods would still be sufficiently far apart to allow flow between the elements.

To account for this, it was proposed that skimming flow was more dependent on the *spatial* density of roughness elements, rather than a function of their frontal area; in skimming flow, the bulk of the flow passes over the roughness elements, so the geometric properties of individual roughness elements should have little influence on the flow. λ_D , introduced in equation (3.3), seemed to be a good descriptor of the spatial density of roughness elements, and was therefore adopted for this task. With an appropriate value of λ_D chosen to distinguish between dense (skimming flow) and sparse (isolated roughness) spacing regimes, data in each flow regime can be plotted separately and seen to conform to relatively contiguous groups (Figures 3.7 and 3.8), including the thin rod data. However, it appears that the roughness constant value is dependent on the roughness element shape in these figures. This confirms that improvements can be made

FIGURE 3.7: Plot of C vs λ for the dense spacing regime.

to the correlation if element shape can be accounted for, and supports the theory that roughness element geometry influences the roughness constant significantly. It was felt that the sparse spacing regime begins at the point in Figure 3.4 where C ceases to correlate well with λ_D : at $\lambda_D \geq 6$. With the present experimental dataset, this value cannot be exactly determined, as there is a substantial gap in spatial density between the two flow regimes.

As explained above, skimming flow drag losses would be dependent primarily on

FIGURE 3.8: Plot of C vs λ in the sparse spacing regime.

the spatial density of roughness elements, which is measured by $\lambda_{\bar{D}}$. Drag in the case of isolated roughness flow, however, would *also* be significantly affected by roughness element form drag, and thus element shape. Note that while $\lambda_{\bar{D}}$ is used to determine the spacing regime of roughness (*i.e.* dense or sparse), Simpson's spacing parameter λ is still used as the abscissa in Figures 3.7 and 3.8 and figures prominently in the correlations developed below in Section 3.3. With the roughness geometry well defined, the effect of roughness element shape could begin to be investigated.

3.2 Computer Tools

To allow computerized analysis of the data, a master array containing various parameters from the studied experiments was constructed (Appendix A). This array was effectively a list of records, with each record containing the relevant geometric data and experimental results for a given experimental configuration. In addition to the roughness constant, and the geometric parameters described in Table 3.1, each record held two other variables: a definition of the roughness *type* (spherical, cylindrical, bluff-bodied, *etc.*) in an experiment, and a weighting variable, which could be used to emphasize different data. The "roughness type" variable facilitated sorting of the records; the weighting parameter was only used as a Boolean variable, to exclude certain data.

Processing of this data was done with MATLAB version 4.2c on Silicon Graphics Workstations. Compared to traditional programming languages like FORTRAN, MATLAB can express mathematical problems in a more natural way; matrix operations in particular are much easier to perform. MATLAB programs generally run slower than FORTRAN programs, but in this case the programs used were relatively small and computational speed had little impact on the project. The graphical tools provided by MATLAB increased the speed of the analysis of the results, and allowed more theories to be examined. The MATLAB interface can be less flexible than a low-level language, as it relies heavily on internal functions that are not directly accessible to the programmer, such as array, matrix and vector operations. Conversely, in FORTRAN the programmer would have to write new code to perform a similar task to the internal function of MATLAB — this imposes more of a programming burden, but does provide the analyst

with more programming flexibility.

3.3 Method of Data Analysis

The approach used to correlate geometric parameters of roughness to the roughness constant is summarized in Figure 3.9. To begin with, a MATLAB data file was created from the master array shown in Appendix A; the master array was not directly accessible to the data analysis programs. Two data-conditioning processes could then be performed on the new data file: file modification and file sorting, which yielded various subsets of records. Following these procedures, optimized correlations could be found for each subset.

FIGURE 3.9: Algorithm of the analytical process used to correlate roughness geometry to the roughness constant.

3.3.1 Data Conditioning

The program *dfix.m* (Appendix B) was used to modify data files. This program detects the type of roughness elements used in each experiment and then alters the values of geometric parameters based on the detected element type. This allowed variations in the selected geometric parameters to be evaluated on a temporary basis, while avoiding manual re-calculation of the parameters, which can be both tedious and error-prone. Once the parameters were finalized, any modifications could be permanently copied into the master array.

To examine different subsets of the master array, sorting routines were created (Appendix C). These routines used a shell-sort algorithm (Sedgewick, 1983) to sort the records of the file by the values of a specified parameter that is in each record. The sorting programs produced a new file containing the sorted records. By selecting the appropriate geometric parameter as the sorting criterion, data of interest could be readily isolated and saved in separate files. For example, the parameter $\lambda_{\bar{D}}$ was used to distinguish between roughness spacing regimes (section 3.1.4). After sorting the records by their values of $\lambda_{\bar{D}}$, files above or below a given value of $\lambda_{\bar{D}}$ could be easily discarded, and the remainder of the records could be saved. The actual deletion of unwanted records was a manual process. Using a similar approach, the sorting routines could be used in preparing data for graphical presentation.

Dense	All records with $\lambda_{\bar{D}} \leq 6.0$ and 3-D roughness
Sparse	All records with $\lambda_{\bar{D}} > 6.0$ and 3-D roughness
Modified Sparse	All records from the Sparse group, excluding thin rods.
Cylinders	All records with cylindrical roughness elements
Marshall's Cylinders	Marshall's (1971) records with cylindrical elements — Marshall's data was excluded from all other analyses.
Dense Bluff-Bodies	All records with bluff-bodied roughness and $\lambda_{\bar{D}} < 5.0$
Sparse Bluff-Bodies	All records with bluff-bodied roughness and $\lambda_{\bar{D}} > 5.0$
Dense Round	All records with round type roughness (spheres, hemispheres, spherical segments and cones) and $\lambda_{\bar{D}} \leq 6.0$
Sparse Round	All records with round type roughness and $\lambda_{\bar{D}} > 6.0$
Dense 2-D	All records with 2-dimensional roughness and $\lambda_{\bar{D}} \leq 6.0$
Sparse 2-D	All records with 2-dimensional roughness and $\lambda_{\bar{D}} > 6.0$

TABLE 3.2: Subsets of the master array, created to evaluate roughness elements in the two spacing regimes, and to examine the behaviour of individual roughness types.

In addition to sparse and dense spacing regimes, the data were also sorted by roughness element type to see if any flow characteristics were not accounted for by the parameters given in Table 3.1. The subgroups studied in this project are summarized in Table 3.2. Because of the limited number of experiments conducted with conical and spherical segment roughness types, data for these experiments were combined with spherical and hemispherical roughness element data into the *round* roughness type. It was felt that the conical elements were closer in shape to spherical elements than they were to cylindrical elements; Hosni *et al* (1992) found the roughness effects of conical and hemispherical roughness elements to be indistinguishable from each other within the

bounds of experimental error. From Table 3.2, the boundary between the dense and sparse spacing regimes cannot be seen to vary substantially between different element types. This further supports the use of the spacing parameter $\lambda_{\bar{D}}$ to define that boundary.

3.3.2 Optimization of the Roughness-Constant Model

Coefficient optimization involved the fitting of a multi-parameter model of the roughness constant to the studied experimental data. The roughness-constant model used geometric parameters from the data set and up to five adjustable coefficients in the form

$$C = a \log_{10}(\lambda Z_1^c Z_2^d Z_3^e) + b \quad (3.7)$$

where C is the predicted value of the roughness constant, Z_i represent geometric parameters and a , b , c , d and e are numerical coefficients for the model. λ represents Simpson's (1973) spacing parameter. The parameter $\lambda_{\bar{D}}$ was not investigated independently as a spacing parameter, as it can be expressed in terms of λ and Z_s :

$$\lambda_{\bar{D}} = \frac{A_s}{\bar{b}\bar{s}} = \frac{A_s}{\bar{b}\bar{k}} \frac{\bar{b}\bar{k}}{\bar{b}\bar{s}} = \lambda Z_s \quad (3.8)$$

MATLAB code for a sample roughness model is contained in appendix D.

To optimize the numerical coefficients, the program *corrhunt.m* was developed. This program loaded data files and operated *fminsmo.m*, a variant of the MATLAB function *fmins.m* where the maximum allowable number of iterations in the program was increased to improve the probability of convergence on a correct result. The *fmins* subroutines use a Nelder-Mead simplex algorithm (Nelder-Mead, 1965) to find the minimum of a function, starting from an initial estimate supplied by *corrhunt.m*. In this

case, the functions to be minimized were defined as the sum of the squared errors, q^2 , between the experimental values of the roughness constant, C_{exp} , and the predicted values of the roughness constant, C . These predicted values of C were obtained from roughness-constant models in the form of equation (3.7). That is:

$$q^2 = \sum_{i=1}^n (C_i - C_{exp,i})^2 \quad (3.9)$$

where n is the number of records in the file, and C_i and $C_{exp,i}$ pertain to the i th record in the file. By minimizing functions of the form of equation (3.9), it was possible to find coefficients for a roughness-constant model that would minimize the total error associated with that model. The fit of the data was expressed in terms of the statistical coefficient of determination, r^2 :

$$r^2 = 1 - \frac{q^2}{\sum_{i=1}^n (C_{exp,i} - \bar{C}_{exp})^2} \quad (3.10)$$

where $\bar{C}_{exp}$ is the average value of the roughness constant for the group of experimental data being examined.

Several different roughness-constant models were examined, using various combinations of the parameters given in Table 3.1; these models are summarized in Table 3.3.

The operation of *fmins* was not unconditionally stable. Occasionally it was unable to converge on a unique solution for the numerical coefficients. This only occurred when trying to optimize roughness models with large numbers of geometric parameters; none of the models given in Table 3.3 had this difficulty. In general, the stability of *fmins* was

Roughness Constant Model	Coefficients for the general equation					
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$					
	a	b	c	d	e	f
errlin	✓	✓				
e1	✓	✓	✓			
e2	✓	✓		✓		
e3	✓	✓			✓	
e4	✓	✓				✓
e5	✓	✓	✓	✓		
e6	✓	✓			✓	✓
e7	✓	✓	✓		✓	
e8	✓	✓	✓			✓
e9	✓	✓		✓		✓
e10	✓	✓		✓		✓
e11	✓	✓	✓	✓	✓	

TABLE 3.3: Geometric Parameters Used in Each Roughness-Constant Model. The symbol ✓ indicates that the parameter associated with the coefficient is used in the model (a blank space indicates a value of zero for that coefficient). The model *errlin* is equivalent to the model proposed by Simpson (1973).

insensitive to the initial estimate supplied by *corrhunt.m* — the same initial estimate was used for virtually all data groups with no difficulties, even though sparse and dense spacing regimes show markedly different slopes and intercepts.

Chapter 4: Results and Discussion

Optimized numerical coefficients and the corresponding coefficient of determination (r^2) for each roughness-constant model are summarized in Tables 4.1–4.12. The effect of these models on the experimental data is best shown graphically, however, and the majority of the discussion will focus on Figures 4.1–4.15. Only those models that were thought to be most significant are presented graphically, but the coefficients given in the tables, in conjunction with the database of experimental results given in Appendix A, provide sufficient information to reconstruct all of the roughness-constant models. Because data in the sparse and dense spacing regimes were optimized separately, they have different effective spacing parameters, and cannot be plotted together, as previous authors have done.

The spanwise aspect ratio, $Z_b = k/\bar{b}$, of two-dimensional roughness elements is zero. Consequently, neither this parameter nor the roughness cell aspect ratio β_c (equation 3.5), which is a function of the spanwise aspect ratio, could be used to predict the roughness constant for two-dimensional roughness elements, as zero-values in the logarithmic roughness models would have resulted in an undefined solution. Two-dimensional and three-dimensional roughness elements were therefore analyzed separately, to allow the impact of the spanwise aspect ratio on the roughness constant to be evaluated for three-dimensional elements

FIGURE 4.1: Plot of the roughness constant C vs Simpson's spacing parameter λ in the dense spacing regime.

4.1 Three-Dimensional Roughness Elements

Results for three-dimensional roughness elements generally indicated the importance of the aspect ratio $k/\bar{b}$ (abbreviated as Z_b in the roughness models) and the roughness element bluntness parameter, $Z_r = A_w/A_F$, in improving roughness-constant models. The roughness cell aspect ratio $\beta_c = L/M$ proved to be ineffective in improving predictions of the roughness constant. The difference between staggered and non-staggered roughness arrangements, characterized in the stagger parameter $\gamma = m/M$, was not apparent in any grouping of the experimental data.

FIGURE 4.2: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.81}$ in the dense spacing regime. This model accounts for the spanwise aspect ratio of the roughness elements.

4.1.1 Dense Spacing Regime

Roughness-constant model coefficients are given in Table 4.1. The model based on the spacing parameter λ only, following Simpson (1973), shows considerable scatter of the experimental data (Figure 4.1); the coefficient of determination was found to be $r^2=0.74$. The model is described by the equation:

$$C = 6.64 \log_{10} \lambda - 5.66 \quad (4.1)$$

FIGURE 4.3: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_r^{0.38}$ in the dense spacing regime. This accounts for the bluntness of roughness elements.

which is very similar to the relation given by Simpson (1973) in equation (2.21a). This indicates that the data used in this portion of the study are consistent with the data used in previous work. The two roughness element types present in this group of data, bluff-bodied and round, follow distinctly different trends in Figure 4.1, with the round-type elements having consistently higher roughness constant values than the bluff-bodied elements at the same roughness spacing.

Including the spanwise aspect ratio $k/\bar{h}$ in the roughness model leads to a substantial reduction in scatter, with $r^2=0.86$ (Figure 4.2). The data for the two roughness

FIGURE 4.4: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.87} Z_r^{0.44}$ in the dense spacing regime. This parameter accounts for both the spanwise aspect ratio and bluntness of the roughness elements.

types remain segregated, suggesting that the element bluntness parameter, Z_r , which was introduced to account for differences in element type, can be used to further improve results. By itself, Z_r does not greatly reduce data scatter (Figure 4.3), although, as predicted, the differences previously seen in the behaviour of the two roughness types (Figure 4.1) appear to be reduced. The effects of the streamwise aspect ratio $Z_s = k/\bar{s}$ were not studied in this case, as this aspect ratio was equal to the spanwise aspect ratio $k/\bar{b}$ for all data in the dense spacing regime.

When both the spanwise aspect ratio and the element bluntness parameter are

FIGURE 4.5: Plot of the roughness constant C vs the effective roughness spacing parameter $\lambda Z_b^{0.87} Z_r^{0.44}$ in the dense spacing regime, comparing the behaviour of staggered rows of roughness elements to non-staggered rows.

included in the roughness model, the fit is excellent, with $r^2=0.95$ (Figure 4.4). The optimized correlation is expressed:

$$C = 10.56 \log_{10}(\lambda Z_b^{0.87} Z_r^{0.44}) - 7.59 \quad (4.2)$$

As mentioned earlier, element arrangement (staggered vs in-line) does not have a visible effect on the roughness constant. Figure 4.5 shows that roughness arrays with staggered rows of roughness elements do not behave in a significantly different manner

from non-staggered roughness arrays at any given roughness spacing, once the geometry of the roughness elements has been accounted for. O'Loughlin and MacDonald (1964), one of the sources of data in this analysis (Appendix A), concluded from their results that roughness arrangement did influence the roughness effect. The magnitude of the effect reported by O'Loughlin and MacDonald, however, is within the typical ± 1 bounds of experimental uncertainty. Their results also showed that some staggered roughness arrays had roughness constant values that were higher than the non-staggered arrays, while others had lower values, so no definitive trend on the effects of staggered roughness arrays can be said to exist.

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin	6.64	-5.66	0	0	0	0	0.74
e1	8.82	-6.42	0.81	0	0	0	0.86
e2	7.30	-6.23	0	0.38	0	0	0.78
e4	6.57	-5.49	0	0	0	0.14	0.76
e5	10.56	-7.59	0.87	0.44	0	0	0.95
e8	8.74	-6.26	0.82	0	0	0.11	0.87
e10	7.21	-6.10	0	0.35	0	0.06	0.78

TABLE 4.1: Results for three-dimensional roughness elements in the dense spacing regime. Bold-faced roughness models are presented graphically in Section 4.1.1.

FIGURE 4.7: Plot of the roughness constant C vs the spacing parameter λ in the sparse spacing regime, with the data for thin rods omitted.

lower values of the roughness constant C at a given roughness spacing.

The thin rod data at the far left of Figure 4.6 skew the best-fit line that defines the roughness-constant model:

$$C = 4.02 \log_{10} \lambda + 2.33 \quad (4.3)$$

These outlying points also reduce the coefficient of determination to $r^2=0.49$, which is not representative of the overall scatter of data. If the data for thin rod roughness elements is eliminated (Figure 4.7), a better representation of the scatter for the majority of the data

FIGURE 4.8: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.62}$ in the sparse roughness regime.

is obtained, with $r^2=0.71$. This model is expressed:

$$C = -6.16 \log_{10} \lambda + 5.70 \quad (4.4)$$

which is very similar to Simpson's (1973) relation for sparsely distributed roughness (equation 2.21b). This again shows that for similar roughness element geometries, the experimental data used in this study were consistent with the data used in previous correlation attempts. While roughness constant values for thin rods deviated from the trend seen in the rest of the data, it was not possible to say if this deviation was due to the extremely high aspect ratio seen in the thin rod elements ($Z_b \gg 1$), or due to some systemic error with the thin rod experiments.

FIGURE 4.9: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.55} Z_r^{1.38}$ in the sparse spacing regime.

Accounting for the spanwise aspect ratio, Z_b , in the roughness-constant model leads to better prediction of the roughness constant (Figure 4.8). The thin rod data points now follow the trend of the rest of the data, and overall scatter is reduced to $r^2=0.82$. The roughness constant is still dependent on the roughness type, however. When both Z_b and the element bluntness parameter, Z_r , are incorporated into the roughness-constant model, this dependence appears to be eliminated, and the data follow the trend line very closely (Figure 4.9), with $r^2=0.91$. This model can be expressed:

$$C = 5.75 \log_{10}(\lambda Z_b^{0.55} Z_r^{1.38}) + 5.78 \quad (4.5)$$

FIGURE 4.10: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.55} Z_r^{1.38}$ in the sparse spacing regime, showing the effect of staggered rows of roughness elements.

This is somewhat similar to Dirling's findings (equation 2.23b), which can be rewritten in terms of the parameters used in the present study:

$$C = 5.47 \log_{10}(\lambda Z_b^{0.5} Z_r^{1.33}) + 8.84 \quad (4.6)$$

Although the intercept in equation (4.6) differs substantially from the intercept in equation (4.5), the magnitude of the coefficients for the geometric parameters in each equation are quite similar; this may be only a coincidence, however, since there were significant flaws in Dirling's methodology, as pointed out in Chapter 2.

Significantly, when the thin rod data are eliminated, the optimized coefficients of the roughness-constant model are not dramatically affected (Table 4.3). This indicates that the peculiar positioning of the thin rod data in Figure 4.6 is probably not due to experimental error as was feared, but is instead the result of the extremely high value of the spanwise aspect ratio, Z_b , associated with thin rod roughness elements.

As in the dense spacing regime, the data in the sparse regime show no dependence on the arrangement of roughness elements, once roughness element geometry is accounted for (Figure 4.10). The streamwise aspect ratio Z_s does not aid in the prediction of the roughness constant with the sparse regime data (Table 4.2) either, whether it is used by itself, or in concert with other parameters. Numerically, this is due to both thin rods and flat plates having high values of Z_s while following opposite trends from each other in terms of their roughness constant values. This is seen in Figure 4.6, where roughness constant values for bluff-bodied elements (which includes flat plates) are underpredicted by the roughness-constant model; thin rod values of the roughness constant have already been described as being overpredicted by the model of Figure 4.6. Thus, these two groups lie on opposite sides of the main body of data, even though both groups have values of Z_s that are much higher than average. Any mathematical process that would move one group closer to the main body of data would then move the other group farther away from the main body, so no significant reductions in data scatter are possible. The likely physical explanation for this behaviour is that the streamwise aspect ratio Z_s is considerably less important than the spanwise aspect ratio Z_b in predicting the roughness constant. The range of Z_s values present in the database is limited, however, so further

FIGURE 4.11: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.74} Z_s^{-0.23}$ for the sparse spacing regime.

investigation of this parameter may be necessary. When the thin rod data are excluded, the numerical constraints described above are removed, and Z_s becomes a much better predictor of the roughness constant (Table 4.3). This is likely due to the bulk of the data having $Z_b = Z_s$, and the apparent effects of the Z_s parameter are in fact being caused by the spanwise aspect ratio, Z_b . This illustrates the statistical phenomenon of *collinearity*.

Including the thin rods again in the calculations reveals some interesting results, however. Although the streamwise aspect ratio does not correlate well with the roughness constant by itself, when the streamwise aspect ratio is combined with the spanwise aspect ratio, Z_b , in a roughness-constant model (Figure 4.11), there is a small but noticeable

reduction in scatter ($r^2=0.87$) over the roughness-constant model that only uses the spanwise aspect ratio. This reduction in scatter is almost as good as that seen in Figure 4.9, where a roughness-constant model employing the spanwise aspect ratio and the element bluntness parameter, Z_r , was used. Were the two roughness element aspect ratios entirely collinear, no differences would be evident between the roughness-constant model that included both Z_s and Z_b , and a model that used only one of these aspect ratios; this seems to indicate that the streamwise aspect ratio has some influence on the roughness constant, although the roughness constant is affected more by the two other roughness element descriptors, Z_b and Z_r .

When all three parameters are combined, (Table 4.2), no significant improvement is made on the model presented in equation (4.5), and the coefficients for Z_b and Z_r are virtually unaffected. This may mean that the streamwise aspect ratio may be collinear with some combination of the two other parameters. Given the limited range of values for the streamwise aspect ratio in the experimental database, this parameter likely requires further investigation.

The theory presented in Chapter 3 predicted that the geometric properties of roughness elements would have little influence on the roughness constant in the dense spacing regime. The results do not fully support this hypothesis. Roughness element geometry seems to be important in both the sparse and dense spacing regimes. However, for the best roughness-constant model (equations 4.2 and 4.5), the coefficients of the geometric parameters are significantly larger in the sparse spacing regime, (approximately twice as large), indicating that element geometry has more influence in that regime. As

a cautionary note, the ability of the roughness element density parameter $\lambda_{\bar{D}}$ (equation 3.3) to distinguish between spacing regimes may be due to the experimental database having data for only near-symmetrical shapes ($Z_b \approx Z_s \approx 1$) at low values of $\lambda_{\bar{D}}$. This could mean that the apparent dependence of C on $\lambda_{\bar{D}}$ that defined the dense spacing regime is only the result of the database having roughness elements of similar shape at low values of $\lambda_{\bar{D}}$. To investigate this, experiments with closely spaced thin rods or similar roughness elements will need to be performed.

Error Function	Coefficients for the general equation $C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						Coefficient of Determination r^2
	a	b	c	d	e	f	
errlin	-4.02	2.33	0	0	0	0	0.49
e1	-6.19	5.33	0.62	0	0	0	0.82
e2	-3.91	3.74	0	2.97	0	0	0.70
e3	-3.56	1.42	0	0	-0.20	0	0.50
e4	-4.00	2.30	0	0	0	0.04	0.49
e5	-5.75	5.78	0.55	1.38	0	0	0.91
e6	-3.56	1.43	0	0	-0.20	-0.01	0.50
e7	-5.51	3.90	0.74	0	-0.23	0	0.87
e8	-6.29	5.56	0.61	0	0	-0.13	0.82
e9	-4.13	4.27	0	2.97	0.09	0	0.70
e10	-4.07	4.25	0	3.01	0	-0.41	0.71
e11	-5.50	5.01	0.62	1.19	-0.11	0	0.92

TABLE 4.2: Results for three-dimensional roughness elements in the sparse spacing regime. Bold face roughness models are presented graphically in Section 4.1.2.

Roughness Model	Coefficients for the general equation $C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						Coefficient of Determination r^2
	a	b	c	d	e	f	
errlin	-6.16	5.70	0	0	0	0	0.71
e1	-6.11	5.16	0.68	0	0	0	0.83
e2	-5.70	6.27	0	1.67	0	0	0.85
e3	-5.49	4.12	0	0	-0.38	0	0.83
e4	-6.15	5.68	0	0	0	0.01	0.71
e5	-5.74	5.75	0.57	1.37	0	0	0.91
e6	-5.55	4.26	0	0	-0.38	-0.10	0.83
e7	-5.63	4.12	0.54	0	-0.28	0	0.89
e8	-6.20	5.36	0.69	0	0	-0.12	0.83
e9	-5.44	5.16	0	1.23	-0.23	0	0.88
e10	-5.83	6.70	0	1.73	0	-0.25	0.86
e11	-5.57	5.10	0.52	1.12	-0.14	0	0.93

TABLE 4.3: Results for sparse spacing regime data, excluding thin rod roughness elements. The roughness model *errlin* is plotted in Figure 4.7.

4.1.3 Roughness Types and the Roughness Constant

To ensure that different roughness shapes did not correlate to radically different parameters, each roughness type was also investigated separately. Because these sub-groupings are relatively small, more care must be taken to ensure that perceived geometric relationships are not due to variations in experimental results instead. In other words, because most sources of experimental data used only one type of roughness element, any systemic error in the experimental results of one source could be interpreted by the optimization program as an effect of the roughness geometry. With the main database, there is a large number of experimental sources, so the effect of errors from one experiment would be mitigated by the results from the other experiments. Thus, significant “false correlations” would be much less likely in the results of the main database than they would be with the following sub-groups of that data, since each sub-group contain fewer independent sources of experimental data.

4.1.3.1 Bluff-Bodied Elements

Tables 4.4 and 4.5 present the results for bluff-bodied roughness elements. As all elements in this group had the same value for the element bluntness parameter, Z_r , this parameter served no useful purpose in roughness constant modelling, and was not investigated.

In the dense spacing regime, the base C vs λ roughness-constant model proposed by Simpson (1973) fit the data very well ($r^2=0.94$). This model appears to be accurate within the bounds of experimental uncertainty. All roughness elements in this group were cubes, and thus showed no variation in either the streamwise or spanwise aspect ratios,

FIGURE 4.12: Plot of the roughness constant C vs the effective spacing parameter $\lambda Z_b^{0.67}$ for bluff-bodied roughness elements in the sparse spacing regime.

Z_s and Z_b respectively, so only variables describing the arrangement of roughness, such as the cell aspect ratio, β_c , and the stagger parameter, γ , could be used in roughness-constant models. Neither β_c nor γ were seen to affect the scatter of the data, although, given the excellent fit of the experimental data to the simple C vs λ model, it is unlikely that any additional parameter could give significant improvements. These results appear to reinforce the findings of the previous section, where the geometric parameters of individual roughness elements, and not the parameters of roughness arrangement, affect the value of the roughness constant — as there were no differences in element geometry

in this group, no improvements could be made to the roughness-constant model.

The roughness elements present in the sparse spacing regime sub-group were cubes, Lego® blocks and flat plates. The fit to Simpson's model was less impressive ($r^2=0.73$) than was the case in the dense spacing regime. Using the spanwise aspect ratio, Z_b , in the roughness-constant model substantially reduced the scatter ($r^2=0.91$), following the trend of the main dataset. Additionally, little difference could be seen between the behaviour of flat-plate and block-shaped roughness elements (Figure 4.12); when both element aspect ratios were accounted for in the model, scatter was further reduced only slightly ($r^2=0.94$), showing that the streamwise aspect ratio is substantially less influential on the roughness constant than is the spanwise aspect ratio in this sub-group. The roughness-constant model that included both β_c and Z_b also reduced scatter only marginally ($r^2=0.93$) over that seen in the model that used the aspect ratio Z_b alone, showing again that the arrangement of roughness elements at a given spacing does not appear to affect the roughness constant.

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						r^2
	a	b	c	d	e	f	
errlin	10.42	-7.55	0	0	0	0	0.94
e4	10.46	-7.60	0	0	0	-.01	0.94

TABLE 4.4: Results for bluff-bodied roughness elements — dense spacing regime.

Roughness Model	Coefficients for the general equation $C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_x^f) + b$						Coefficient of Determination
	a	b	c	d	e	f	r ²
errlin	-5.20	5.62	0	0	0	0	0.73
e1	-5.30	5.07	0.67	0	0	0	0.91
e3	-4.92	4.49	0	0	-0.26	0	0.82
e4	-5.33	5.97	0	0	0	-0.21	0.74
e6	-5.06	4.87	0	0	-0.26	-0.24	0.83
e7	-5.11	4.43	0.61	0	-0.16	0	0.94
e8	-5.50	5.55	0.67	0	0	-0.28	0.93

TABLE 4.5: Results for bluff-bodied roughness elements — sparse spacing regime. Roughness model *e1* is shown in Figure 4.12

4.1.3.2 Round Elements

Results for the sparse spacing regime are given in Table 4.6. Only five data points were available in the dense regime, so no meaningful results could be obtained for this subgroup. In the sparse spacing regime, results generally supported the trends seen in the main database, although the element bluntness parameter, Z_r , appears to play an even more important role in the roughness-constant models. As all roughness arrays in this group were arranged in staggered element patterns, the effects of staggered vs non-staggered roughness arrays could not be investigated.

4.1.3.3 Cylindrical Elements

Results are summarized in Table 4.7. All data points in this group fell into the sparse spacing regime. Due to symmetry, the spanwise and streamwise aspect ratios of cylindrical roughness elements are equal, so it is not possible to distinguish between these

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin	-6.99	5.33	0	0	0	0	0.86
e1	-6.96	5.32	-0.08	0	0	0	0.86
e2	-6.79	14.5	0	4.41	0	0	0.93
e4	-6.94	5.29	0	0	0	-0.11	0.86
e5	-6.87	18.27	0.47	6.15	0	0	0.95
e8	-6.93	5.30	-0.07	0	0	-0.07	0.86
e10	-6.62	14.88	0	4.80	0	-0.40	0.93

TABLE 4.6: Results for round-type roughness elements (spheres, hemispheres, cones and spherical segments) – sparse spacing regime.

two parameters. As with bluff-bodied roughness elements, the element bluntness parameter, Z_r , is constant for all elements in this group. The results for the cylindrical subgroup support those of the main database: the roughness aspect ratio leads to better prediction of the roughness constant.

The roughness elements in this group could be divided into only two geometric groups: thin rods with $Z_b \gg 1$, and square cylinders with $Z_b \approx 1$. This limited variety in the geometry of the data is significant when discussing the results for Marshall's experiment (1970), which included many different arrays of cylindrical roughness elements in a range from $2 \geq Z_b \geq 0.2$.

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin	-2.89	-0.85	0	0	0	0	0.67
e1	-5.67	3.99	0.53	0	0	0	0.87
e4	-2.96	-0.66	0	0	0	-0.27	0.67
e8	-5.84	4.42	0.52	0	0	-0.26	0.88

TABLE 4.7: Results for cylindrical roughness elements — sparse spacing regime.

4.1.3.4 Marshall's Cylinders

Results are presented in Table 4.8. As mentioned in section 2.3, Marshall's data were excluded from the main database because of errors in his experimental methodology which influenced his final results. It was felt (Wooding *et al*, 1973), however, that these errors would be relatively consistent for all of Marshall's data. Thus, any trends seen in Marshall's results would be valid, even though his numerical results were erroneous. In a plot of C vs λ (Figure 4.13), the roughness constant decreases with the element aspect ratio Z_b , contrary to the findings for the main database. The differences in C between cylinders of different aspect ratios are comparable to the typical ± 1 value of experimental uncertainty, so this trend may not be very important. If Marshall's data is self-consistent, and the trends shown by his data are correct, concerns must be raised about the universality of the findings for the main database. To examine this discrepancy,

FIGURE 4.13: Plot of the roughness constant C vs the spacing parameter λ for Marshall's (1971) experiments on cylindrical roughness elements in the sparse spacing regime.

experiments should be performed to evaluate more low aspect ratio roughness elements.

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin	-6.67	2.93	0	0	0	0	0.92
e1	-6.86	3.84	-0.31	0	0	0	0.94

TABLE 4.8: Results for Marshall's (1971) sparse cylindrical elements. The *errlin* roughness model is plotted in Figure 4.13.

FIGURE 4.14: Plot of the roughness constant C vs the spacing parameter λ for two-dimensional roughness elements in the dense regime. Other roughness models offer little improvement on these results.

4.2 Two-Dimensional Roughness Elements

Results are given in Tables 4.9 and 4.10. The data are plotted in Figures 4.14 and 4.15. Improvements over the simple C vs λ model in the prediction of the roughness constant were very small in both spacing regimes. Unfortunately, the roughness elements in the database of two-dimensional elements were all very similar in shape, so it is unclear if the poor results from multi-parameter roughness-constant models are due to actual flow behaviour or to the limited range in the values of geometric parameters. Experimental uncertainty appears to be higher with these elements: for identical element

FIGURE 4.15: Plot of the roughness constant C vs the spacing parameter λ for two-dimensional roughness elements in the sparse regime. No significant improvements could be made to this model.

geometries, there are often very different results from different experiments. In Figure 4.14 at $\lambda=4$, for example, the experimentally determined values of the roughness constant for square bars vary from +2 to -2; this is far in excess of the typical ± 1 value of experimental uncertainty in the roughness constant for three-dimensional roughness elements. This high imprecision is likely due to flaws in experimental methods. Use of the momentum-integral equation in the processing of experimental results is quite common with two-dimensional roughness elements, and often relies on the assumption of two-dimensional flow over the roughness elements. Three-dimensionality in the flow

can occur, however, as "two-dimensional" roughness arrays are not in fact two-dimensional; roughness elements are of a finite length, bounded by wind-tunnel walls at their ends. Moreover, cavity flows of the type that would occur in the dense spacing regime often exhibit a cellular periodicity in the spanwise direction (Weihs and Katz, 1983), which would also invalidate the two-dimensional flow assumption. If the simplifying assumption of two-dimensionality is invalid, it could contribute to significant errors in results.

On the whole, the incorporation of additional parameters for two-dimensional roughness does not show convincing improvements in the correlation for the roughness constant. The results also indicate a need to re-evaluate this type of roughness with more reliable experimental techniques and with more variety in the roughness element geometry.

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin2D	15.24	-9.63	0	0	0	0	0.74
e22D	14.30	-9.08	0	-0.25	0	0	0.75
e32D	15.11	-9.77	0	0	-2.11	0	0.78
e92D	14.55	-9.43	0	-0.15	-2.09	0	0.78

TABLE 4.9: Results for two-dimensional roughness elements — dense spacing regime. The *errlin2D* roughness model is plotted in Figure 4.14.

Roughness Model	Coefficients for the general equation $C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						Coefficient of Determination r^2
	a	b	c	d	e	f	
errlin2D	-3.53	4.15	0	0	0	0	0.64
e22D	-3.48	3.97	0	-0.40	0	0	0.65
e32D	-3.69	4.20	0	0	1.70	0	0.68
e92D	-3.62	3.82	0	-0.83	2.20	0	0.71

TABLE 4.10: Results for two-dimensional roughness elements – sparse spacing regime. The *errlin2D* roughness model is plotted in Figure 4.11.

4.3 Simultaneous Analysis of Two-Dimensional and Three-Dimensional Roughness Elements

Results are shown in Tables 4.11 and 4.12. Because of the geometric restrictions imposed by two-dimensional roughness elements, this analysis could only investigate the effect of the geometric parameters Z_b and Z_r on roughness constant prediction. The use of the element bluntness parameter Z_s led to somewhat better results, while the streamwise aspect ratio was ineffective. The element bluntness parameter did not reduce the scatter of the main body of data, but instead brought the outlying data points towards the relatively diffuse main body of data points.

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Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin2D	8.54	-6.39	0	0	0	0	0.64
e22D	8.97	-6.69	0	0.18	0	0	0.65
e32D	10.48	-7.16	0	0	0.83	0	0.72
e92D	11.82	-7.98	0	0.30	0.87	0	0.75

TABLE 4.11: Results for all roughness element types in the dense spacing regime.

Roughness Model	Coefficients for the general equation						Coefficient of Determination
	$C = a \log_{10}(\lambda Z_b^c Z_r^d Z_s^e \beta_c^f) + b$						
	a	b	c	d	e	f	r ²
errlin2D	-3.98	2.80	0	0	0	0	0.45
e22D	-3.93	4.09	0	2.81	0	0	0.64
e32D	-3.96	2.75	0	0	-0.01	0	0.45
e92D	-4.34	4.96	0	2.76	0.16	0	0.66

TABLE 4.12: Results for all roughness element types in the sparse spacing regime.

Chapter 5: Conclusions and Recommendations

5.1 Conclusions

From this analysis, the following conclusions can be drawn:

- 1) Significant improvement in the prediction of the roughness constant for three-dimensional roughness elements is possible through the use of secondary geometric parameters describing the shape of individual roughness elements; it appears that at least two geometric parameters have a significant influence on the value of the roughness constant. These parameters are the spanwise aspect ratio, $k/\bar{h}$, and the roughness element bluntness parameter, A_w/A_F . The streamwise aspect ratio $k/\bar{s}$ appeared to have substantially less influence on the roughness constant. Predictive improvements could also be made through re-analysis of old experimental data, which introduced errors into previous models.
- 2) With two-dimensional roughness elements, additional geometric parameters were unable to significantly improve prediction of the roughness constant. The experimental data used in this portion of the study, however, were generally for

very similar geometries, and appeared to be very imprecise. As a result, no definitive conclusions can be made with respect to two-dimensional roughness elements.

- 3) The spatial arrangement of roughness elements does not appear to substantially influence the roughness constant. Neither the roughness cell aspect ratio β (see Figure 3.1), nor the use of staggered rows rather than in-line rows of roughness elements had a consistent effect on the roughness constant.
- 4) Two spacing regimes can be observed for rough-walls. Roughness can be described as either dense or sparse, with the delineation depending on the roughness element spacing parameter, $\lambda_{\bar{h}}$ of equation (3.3), and not the spacing parameter λ of equation (2.20). The boundary between the dense and sparse spacing regimes occurs at about $\lambda_{\bar{h}} = 6.0$.
- 5) The current experimental database, while more extensive than those used by previous authors, is still insufficient to allow a complete evaluation of the effect of roughness geometry on the roughness constant.
- 6) Experimental determination of the roughness constant is difficult and the majority of the experimental data used in the present work were obtained with methods that are somewhat inaccurate. Significant theoretical advances and developments in

experimental techniques have occurred in the past 10-15 years and improved accuracy could be achieved in new experiments.

5.2 Recommendations

Despite the apparent success of this work in improving the prediction capability for the roughness constant for three-dimensional roughness elements, several experiments should be performed to improve the accuracy and reliability of these findings:

- 1) The boundary between rough and sparse spacing regimes needs to be more accurately determined to quantify the dependence of its value on roughness element shape.
- 2) Low aspect ratio cylindrical roughness elements require further study to see if they conform to the new model, as Marshall's (1971) results suggest they do not.
- 3) Two-dimensional roughness elements should be examined for a variety of roughness element geometries, as the available data were generally limited to one shape.
- 4) The effects of the streamwise aspect ratio Z , should be more completely investigated for both three-dimensional and two-dimensional roughness elements.

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Appendix A: MATLAB database of experimental data, *rough.m*

This program listing has been annotated to show the sources of the experimental data; these references must be deleted before the array can be loaded by MATLAB

%rough.m: MATLAB data file containing experimentally determined roughness constant values % for various rough surfaces

%Explanation of Abbreviations -

%C: Experimental Results

%Lam: Spacing=Total Plan Surface Area/Projected Frontal Roughness Area

%Zg: Spanwise Aspect Ratio=k/beff

%W: Weighting Factor used to distinguish 3-D from 2-D elements

%Sh: Shape Type 0:Rod/cylinder;1:Block/plate;2:Sphere;3:Hemisphere

% 4:Spher.segment;5:cone;6:2D bar/wire

%Zs: Streamwise Aspect Ratio=k/seff

%Zr: Bluntness Factor: Frontal Wetted Surface Area/

% Projected Frontal Area

%LamD: Roughness Element Density (Equation 3.3)

%Bc: Roughness Cell Aspect Ratio (L/M)

%G: Stagger Parameter = 0 for non-staggered

% 0.5 for staggered

%

% Lines that have been commented out (%) indicate data that were not included
% in the analysis (See section 2.3 for explanations for each case). Marshall %
% (1971) data for cylindrical roughness elements was analyzed separately for %
% reasons given in the text (section 2.3).

% Sources for the data are given in the far right column. and are not part of the %
% original MATLAB code.

%

% C	Lam	Zg	W	Sh	Zs	Zr	LamD	Bc	G	Source
dat=[										
-8.27	121.2	1.273	1	2	1.273	2	196.4	1	0.5	
-5.21	30.3	1.273	1	2	1.273	2	49.1	1	0.5	
-0.76	7.57	1.273	1	2	1.273	2	12.27	1	0.5	
-0.60	2.73	1.273	1	2	1.273	2	4.42	1	0.5	
-5.41	1.10	1.273	1	2	1.273	2	1.78	0.866	0.5	
-5.09	28.9	1.273	1	2	1.273	2	46.8	1	0.5	
-0.72	7.22	1.273	1	2	1.273	2	11.7	1	0.5	Coleman
-13.01	106.8	0.451	1	4	0.451	2.39		1	0.5	etal
-11.4	60.1	0.451	1	4	0.451	2.39		1	0.5	(1984)
-6.18	26.7	0.451	1	4	0.451	2.39	15.3	1	0.5	
-3.10	3.69	0.451	1	4	0.451	2.39	2.12	0.866	0.5	
-10.67	95.4	0.839	1	5	0.839	2.16		1	0.5	
-8.23	53.7	0.839	1	5	0.839	2.16	32.03	1	0.5	
-4.86	23.9	0.839	1	5	0.839	2.16	14.24	1	0.5	
-4.92	2.7	0	0	6	1	1	2.7	0	0	Butterman
-2.45	3.3	0	0	6	1	1	3.3	0	0	(1966)
-0.08	4.0	0	0	6	1	1	4.0	0	0	
-0.2	4.0	0	0	6	1	1	4.0	0	0	
-0.54	4	0	0	6	1	1	4	0	0	Perry &
+0.68	8.1	0	0	6	1	1	8.1	0	0	Joubert
-2.98	33.7	0	0	6	1	1	33.7	0	0	(1963)

-5.95	1.43	1	1	1	1	1	1.43	1	0.5	
-3.54	2.27	1	1	1	1	1	2.27	1	0.5	
-0.94	4.00	1	1	1	1	1	4.00	1	0.5	Wroughton
-0.30	9.09	1	1	1	1	1	9.09	1	0.5	g
-6.54	1.43	1	1	1	1	1	1.43	0.25	0	MacDonald
-3.01	2.27	1	1	1	1	1	2.27	0.25	0	(1964)
-1.36	4.00	1	1	1	1	1	4.00	0.25	0	
-1.55	9.09	1	1	1	1	1	9.09	0.25	0	
-0.16	8	1	1	1	1	1	8	0.5	0.5	Kotloseus
-3.57	32	1	1	1	1	1	32	0.5	0.5	g
-6.44	128	1	1	1	1	1	128	0.5	0.5	Davidian
-10.01	512	1	1	1	1	1	512	0.5	0.5	(1966)
-3.63	32	2.01	1	1	24	1	14500	1	0.5	W. Antshore
										8DeRoos (1977)
+1.8	4	0	0	6	1	1	4	0	0	Antonia &
										Luxton (1971)
-9.42	167	0.945	1	0	0.945	1.57	201	1	0	
-10.0	167	0.945	1	0	0.945	1.57	201	1	0	Sullivan &
-5.12	42	0.945	1	0	0.945	1.57	50.6	1	0	Greeley (1993)
-3.75	42	0.945	1	0	0.945	1.57	50.6	1	0	
%+1.67	10.5	0.945	1	0	0.945	1.57	12.6	1	0	
%+1.51	10.5	0.945	1	0	0.945	1.57	12.6	1	0	
-6.49	89	1	1	0	1	1.57	113	0.5	0	
-4.1	44	1	1	0	1	1.57	56	1	0.5	
-3.12	22	1	1	0	1	1.57	28	0.5	0	Raupach et al
-1.95	11	1	1	0	1	1.57	14	1	0.5	(1980)
-1.63	5.6	1	1	0	1	1.57	7.13	0.5	0	
-0.68	0.7	143	1	0	143	1.57	127	0.866	0.5	
%-0.29	20	1	1	1	1	1	20	1	0	
%+0.23	16	1	1	1	1	1	16	1	0	
%+0.54	13.3	1	1	1	1	1	13.3	1	0	
%+0.89	10	1	1	1	1	1	10	1	0	
%+0.78	8	1	1	1	1	1	8	1	0	
%+0.62	6.67	1	1	1	1	1	6.67	1	0	Hussain
%+0.92	4	1	1	1	1	1	4	1	0	g
%-0.02	3.33	1	1	1	1	1	3.33	1	0	Lee
%-0.18	40	0.5	1	1	1	1	20	1	0	(1980)
%+1.13	20	0.5	1	1	1	1	10	1	0	
%+1.13	16.0	0.5	1	1	1	1	8	1	0	
%+1.26	12.5	0.5	1	1	1	1	6.25	1	0	
%+1.69	8.89	0.5	1	1	1	1	4.44	1	0	
%+1.06	6.67	0.5	1	1	1	1	3.33	1	0	
%+0.74	5.0	0.5	1	1	1	1	2.5	1	0	
-5.63	2	0	0	6	1	1	2	0	0	
-1.48	4	0	0	6	1	1	4	0	0	Liu et al
-1.08	12	0	0	6	1	1	12	0	0	(1966)
-3.08	96	0	0	6	1	1	96	0	0	
-4.15	1.10	1.273	1	2	1.273	2	1.78	0.866	0.5	Coleman et al
-4.75	1.10	1.273	1	2	1.273	2	1.78	0.866	0.5	(1977)

-6.74	2	0	0	6	1	1.57	1.57	0	0	Furuya et al (1976)
-4.20	2	0	0	6	1	1.57	3.14	0	0	
+0.66	4	0	0	6	1	1.57	6.28	0	0	
+1.69	8	0	0	6	1	1.57	12.56	0	0	
+1.4	16	0	0	6	1	1.57	25.12	0	0	
-0.73	32	0	0	6	1	1.57	50.24	0	0	Mosni et al (1993)
-2.78	64	0	0	6	1	1.57	100.5	0	0	
-3.9	40.7	0.637	1	3	0.637	2	33.1	1	0.5	
-0.5	10.2	0.637	1	3	0.637	2	8.28	1	0.5	
+0.29	4.4	6.0	1	1	60	1	264	0.733	0.5	
										et al (1986)
-2.17	2.7	29.4	1	0	29.4	1.57	62.4	0.5	0.5	Seginer et al (1976)
-3.23	2.7	29.4	1	0	29.4	1.57	62.4	0.5	0.5	
-3.16	1.0	79.8	1	0	79.8	1.57	62.4	0.5	0.5	
+3.98	3.97	0.25	1	1	24	1	95.2	0.253	0.5	
+2.88	11.9	0.25	1	1	24	1	286	0.760	0.5	
+0.43	33.1	0.25	1	1	24	1	793	2.278	0.5	Sayre et al (1961)
+2.02	5.93	0.25	1	1	24	1	142	0.127	0.5	
+1.21	17.8	0.25	1	1	24	1	427	0.380	0.5	
-1.84	49.4	0.25	1	1	24	1	1187	1.139	0.5	
+2.26	0.75	18.75	1	0	18.75	1.57	17.9	0.5	0.5	
+2.78	1.49	18.75	1	0	18.75	1.57	35.6	1	0	Kawatani & Meroney (1970)
+3.93	2.99	18.75	1	0	18.75	1.57	71.4	0.5	0.5	
+1.45	5.97	18.75	1	0	18.75	1.57	143	1	0	
-6.79	51.1	0.637	1	3	0.637	2	41.6	0.507	0.5	
										- Chen et al (1974)
-1.97	4	0	0	6	1	1	4	0	0	Boisvert et al (1977)
+0.23	8.4	0.597	1	1	0.597	1	5	1	0	
-1.41	11.3	0.597	1	1	0.597	1	6.75	1	0	
-1.35	25	1	1	1	1	1	25	1	0	
+0.21	12.5	1	1	1	1	1	12.5	1	0	
-0.35	8	1	1	1	1	1	8	1	0	Lee & Soliman (1977)
-2.03	4	1	1	1	1	1	4	1	0	
-2.42	3.13	1	1	1	1	1	3.13	1	0	
-3.7	2	0	0	6	1	1	2	0	0	
										- Antonia & Wood (1975)
+0.932	17.7	0	0	6	0.904	1	17.7	0	0	Siuru et al (1977)
+1.74	8.85	0	0	6	0.904	1	8.85	0	0	
+0.91	4.42	0	0	6	0.904	1	4.42	0	0	
-2.70	2.21	0	0	6	0.904	1	2.21	0	0	
-0.69	37.7	0	0	6	0.616	1	37.7	0	0	
-3.07	66.7	0.375	1	1	6	1	178	1	0.5	Schlichting (1968)
-1.24	37.5	0.375	1	1	6	1	100	1	0.5	
+0.88	16.7	0.375	1	1	6	1	44.4	1	0.5	
+17.6	5000	2	0	0	2	1.57	12730	1	0	
+17.5	2500	1	0	0	1	1.57	3180	1	0	
+17.3	1592	0.637	0	3	0.637	2	1292	1	0	Marshall (1971)
+17.1	1250	0.5	0	0	0.5	1.57	796	1	0	
-17.2	833	0.333	1	0	0.333	1.57	354	1	0	
-17.0	500	0.2	1	0	0.2	1.57	127	1	0	

-17.2	2450	2	0	0	2	1.57	6240	1	0	Marshall (1971)
-16.8	1225	1	0	0	1	1.57	1560	1	0	
-17.4	780	0.637	1	3	0.637	2	633	1	0	
-16.6	613	0.5	1	0	0.5	1.57	390	1	0	
-16.0	408	0.333	1	0	0.333	1.57	173	1	0	
-15.6	245	0.2	1	0	0.2	1.57	62.4	1	0	
-15.5	1250	2.0	0	0	2	1.57	3180	1	0	
-15.7	800	2.0	1	0	2	1.57	2040	1	0	
-13.5	420.5	2.0	1	0	2	1.57	1070	1	0	
-11.8	242	2.0	1	0	2	1.57	616	1	0	
-11.7	196	1	1	0	1	1.57	250	1	0	
-11.6	125	0.637	1	3	0.637	2	101.4	1	0	
-10.1	98	0.5	1	0	0.5	1.57	62.5	1	0	
-10.6	96.3	0.333	1	0	0.333	1.57	40.9	1	0	
-9.2	65.3	0.333	1	0	0.333	1.57	27.7	1	0	
-10.8	80	0.2	1	0	0.2	1.57	20.4	1	0	
-8.7	98	2.0	1	0	2	1.57	250	1	0	
-7.0	72	2.0	1	0	2	1.57	183	1	0	
-8.1	40.7	0.637	1	3	0.637	2	33.1	1	0	
-7.6	40.3	0.333	1	0	0.333	1.57	17.1	1	0	
-6.3	49	1	1	0	1	1.57	62.4	1	0	
-7.1	32	0.5	1	0	0.5	1.57	20.4	1	0	
-6.4	27	0.333	1	0	0.333	1.57	11.5	1	0	
-4.8	18	2.0	1	0	2	1.57	45.8	1	0	
-6.1	36	1	1	0	1	1.57	45.8	1	0	
-4.4	9	1	1	0	1	1.57	11.5	1	0	
-3.7	5.73	0.637	1	3	0.637	2	4.65	1	0	
-5.9	18	0.5	1	0	0.5	1.57	11.5	1	0	
-4.9	4.5	0.5	0	0	0.5	1.57	2.86	1	0	
-5.8	21.3	0.333	1	0	0.333	1.57	9.05	1	0	
-4.4	12	0.333	1	0	0.333	1.57	5.09	1	0	
-2.3	5.33	0.333	0	0	0.333	1.57	2.26	1	0	
-5.0	20	0.2	1	0	0.2	1.57	5.09	1	0	
-2.5	9.8	0.2	0	0	0.2	1.57	2.50	1	0	

J;

Appendix B: MATLAB program listing for *dfix.m*, data manipulation routine

```

%~/thesis/matdat/dfix.m Matlab routine to temporarily modify values of 'rough.m'
%   currently set up to change LamD from Dvorak's (1969)  $\lambda_D$  to the
%   roughness element density parameter,  $\lambda_{\bar{D}}$  given by equation (3.3).
%   'rough.m' has since been permanently modified to have  $\text{LamD} = \lambda_{\bar{D}}$ , so this
%   routine is no longer needed for that function.
clear

% Loads data from .MAT file; data must be named 'dat' and in the format of 'rough.m'
f1=input('Enter name of .MAT file containing data: ','s');
eval(['load ',f1]);

Zb=dat(:,3);           %Geometric parameter (k/beff)
LamD=dat(:,8);          %Spacing parameter
Zr=dat(:,7);           %Bluntness parameter (see Dirling)
Sh=dat(:,5);           %Element Type (see rough.m)

%intermediary calculations for conic roughness elements
tbo=0.1176;            %tbo=t/bo (cone tip width effects)
con=0.5*(tbo+1);

%Loop modifies values of variable(s) (LamD in this case) in the array, based on their
%shape
for lp=1:size(Sh)
    if (Sh(lp)<=1)                %cylinders or bluff-bodies
        LamD(lp)=LamD(lp);
    elseif Sh(lp)==2              %spheres
        LamD(lp)=LamD(lp).*Zb(lp).^2;
    elseif Sh(lp)==3              %hemispheres
        LamD(lp)=LamD(lp).*(2*Zb(lp)).^2;
    elseif Sh(lp)==4              %spherical segments (Coleman et al, 1984)
        LamD(lp)=LamD(lp).*1.388.^2;
    elseif Sh(lp)==5              %conical elements (Coleman)
        LamD(lp)=LamD(lp)/con;
    elseif Sh(lp)==6              %2-D elements
        LamD(lp)=LamD(lp).*Zr(lp);
    end
end
dat(:,8)=LamD;                % The new Density parameter is stored in the array

%outputs modified 'dat' array to new file
fill1=input('Enter name of .MAT output file: ','s');
eval(['save ',fill1,' dat']);

```

**Appendix C: MATLAB program listing for *shellsort.m*,
sorting routine**

```

clear
% MATLAB Row-oriented Shellsorter for arrays
% D. Waigh 1995
fill1=input('Enter name of .MAT file containing data: ','s');

eval(['load ',fill1]);

N=max(size(dat));
h=1;
col=5;      %Column used as sorting criterion (see rough.m)
while h<N    %establishes consistent increment sequence
    h=h*3+1;
end

while h>1
    h=fix(h/3);
    for I=(h+1):N
        V=dat(I,:);
        J=I;
        while dat(J-h,col)>V(col)
            dat(J,:)=dat(J-h,:);
            J=J-h;
            if J<=h, break, end
        end
        dat(J,:)=V;
    end
end
clear I J h V N col
fill1=input('Enter name of .MAT output file: ','s');
eval(['save ',fill1]);

```

Appendix D: MATLAB program listing for *e5.m*, a roughness constant model

This program listing is included as a sample of the type of function used to optimize a roughness constant model (see Figure 3.3 for a listing of the models).

```
function q=e5(x)
global Lam Zs LamD Cexp W Zr Zb Bx
a=x(1);b=x(2);c=x(3);d=x(4);
q=W.*(a.*log10(Lam.*Zb.^c.*Zr.^d)+b-Cexp);
q=sum(q.^2);
```

**Appendix E: MATLAB program listing for *corrhunt.m*,
coefficient optimization routine**


```

% ~/thesis/matdat/corrhunt.m
% Correlation Hunter
% Optimizes coefficients to multi-variable functions for a given set of data
% David Waigh 1995

%Variable initialization
clear
global Lam LamD Zb Cexp W Zr Zs Sh Bx G

% Reads in data from file specified by user - file must be in the format of rough.m
fill1=input('Enter name of .MAT file containing data: ','s');
fill2=[load fill1];
eval(fill2);
clear coef q2 R2;    %prevents "intruders" from input file that can disrupt
                    %the optimization loop

%Description of Variables
Cexp=dat(:,1);      %Experimental values of the roughness constant
Lam=dat(:,2);       %Spacing parameter used by Simpson(1973)
Zb=dat(:,3);        %Geometric parameter (h/beff)
W=dat(:,4);         %Weighting (could be made proportional to confidence in data
                    % -currently just a switch to include/exclude data)
Sh=dat(:,5);        %Shape classification of roughness element (see rough.m)
Zs=dat(:,6);        %Geometric parameter (h/beff)
Zr=dat(:,7);        %Bluntness parameter used by Dirling(1973)
LamD=dat(:,8);      %Slight variant on Dvorak's (1968) spacing parameter
Bx=dat(:,9);        %Roughness Cell Aspect Ratio
G=dat(:,10);        %Stagger Parameter (non-dim offset of streamwise rows)

%Calculation of intermediary values used in the analysis
Cbar=mean(Cexp).*ones(size(Cexp));    %Average value of Cexp
r=sum((W.*(Cexp-Cbar)).^2);           %Sum-squared-error of (Cexp-Cbar)
%r=sum((Cexp-Cbar).^2);               %(2-D variant)

x0=[-6 6 .1 .1 .1];    %initial vector of coefficients for the
                        %correlation optimization loop

%list of correlations (or models) to be optimized in loop (labour-saving device)
%    'ea' and 'eb' correspond to 'e10' and 'e11', respectively - MATLAB requires
%    all strings in the 'func' matrix to be the same size.
func=['er','e1','e2','e3','e4','e5','e6','e7','e8','e9','ea','eb'];

```

```
%optimization loop - finds optimal coefficients for the function by minimizing the
%    sum-squared error of (C-Cexp) through the Nelder-Mead function minimization
%    process contained in 'fminsmod'.
```

```
for loop = 1:max(size(func))
    coef(loop,:)=fminsmod(func(loop,:),x0); %fmins call-optimizes function coefficients
    q2(loop,:)=feval(func(loop,:),coef(loop,:)); %Calculates the sum-squared error
                                                % of (C-Cexp)
    R2(loop,:)=1-q2(loop,:)/r; %Calculates R^2 value (coefficient of
                                determination)
end
```

```
%output results to ASCII text and MATLAB .MAT files
%.MAT files allow the results to be better formatted for presentation in graphs, etc.
eval(['save ',fill1,'o coef q2 dat R2 /ascii']);
eval(['save ',fill1,'out coef func q2 dat Cexp Lam Zb W Sh Zs Zr LamD Bx G R2']);
```

END

1 9 0 6 - 9 7

FIN