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**HANDLING OF MULTIAXLE,
ALL-WHEEL-DRIVE OFF-ROAD VEHICLES**

by

YEUN-CHUNG WU, M.S.Eng.

A thesis submitted to
the Faculty of Graduate Studies
in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Department of Mechanical and Aerospace Engineering

Ottawa-Carleton Institute

for Mechanical and Aerospace Engineering

Carleton University
Ottawa, Ontario
April 10, 2000
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ALL-WHEEL-DRIVE OFF-ROAD VEHICLES

Submitted by YEUN-CHUNG WU, M.S.Eng.
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Chair

Mechanical and Aerospace Engineering

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Thesis Supervisor

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Carleton University
April, 2000

Abstract

While the handling characteristics of road vehicles have been studied extensively in recent years, the handling behaviour of off-road vehicles on deformable terrain has not yet been given sufficient attention. With the increasing use of multi-axle, all-wheel-drive vehicles in cross-country transport, in resource industry and in military operations, there is a need for the development of a realistic simulation model for the handling behaviour of off-road vehicles. This would provide practicing engineers with an analytical framework for the further improvements of the handling of these vehicles.

The forces and moments that control the motion of a ground vehicle, hence its handling, are generated by tire-ground interaction. This research investigated the longitudinal and lateral forces developed by a tire on deformable terrain. A new off-road tire test facility at the Transport Technology Research Laboratory, Carleton University has been designed and constructed. Experiments have been performed to study the general relations between the longitudinal and lateral forces and the longitudinal slip-slip angle for an off-road tire. Tire forces have been measured on a deformable soil under various normal loads and inflation pressures. Furthermore, based on the shear stress-shear displacement relations of soil, a new off-road tire model for predicting the longitudinal and lateral forces under combined longitudinal slip and slip angle has been developed.

The tire model with the data obtained using the tire test facility has been integrated into an off-road vehicle handling simulation model. The handling model has been developed based on the computer code AUTOSIM. It takes into account non-linear characteristics of the steering and suspension systems of the vehicle. To demonstrate the utility of the model, the steady-state handling characteristics of a six-wheel-drive, front-wheel steering vehicle on paved road and on deformable terrain, as well as its behaviour in a lane-change manoeuvre have been evaluated and presented as examples. Furthermore, an animation program for displaying the lane-change response of the vehicle is included.

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With love I dedicate this thesis to my father, my parents-in-law, sisters, wife Wendy Ching-kuang, daughter Belle Ti-yueh and the memory of my mother, whose love has been the foundation for my career.

Contents

	Page
Acceptance	i
Abstract	ii
Acknowledgements.....	iii
List of Figures	vi
List of Tables	xii
List of Symbols.....	xiii
Chapter 1 Introduction	1
1.1 Background.....	1
1.2 Overview of This Research.....	2
Chapter 2 Review of the State of the Art.....	4
2.1 Vehicle Handling	4
2.2 Vehicle Handling Models	8
2.2.1 Three Degree-of-Freedom Model.....	8
2.2.2 Multibody Dynamics Models	12
2.3 Tire Models.....	13
2.3.1 Empirical Models.....	14
2.3.2 Analytical Models.....	17
Chapter 3 Objectives of the Current Research.....	23
3.1 Work Statement	23
3.2 Objective	25
Chapter 4 Development of a Laboratory Tire Test Facility.....	26
4.1 Review of Existing Tire Test Facilities	26
4.2 Development of TTF97.....	29
4.3 Instrumentation	42
4.4 Identification of System Characteristics	48

Chapter 5	Experimental Study of Tire Characteristics	
	Under Combined Longitudinal Slip and Slip Angle.....	53
5.1	Tire Force Measurements	53
5.2	Soil Property measurements	69
Chapter 6	Development of an Off-Road Tire Model.....	76
6.1	Tire Lateral Behaviour on Soft Terrain	76
6.2	Rigid Operating Mode	80
6.3	Elastic Operating Mode	105
6.4	Validation.....	120
Chapter 7	Handling Simulation	127
7.1	Simulation Models Using AUTOSIM	127
7.2	Steering System	130
7.3	Suspension System.....	132
7.4	Vehicle Inertial quantities	144
7.5	Application of Tire Model	149
7.6	Handling of 6WD Vehicle	151
7.7	Animation	167
Chapter 8	Conclusions and Contributions	169
8.1	Conclusions.....	169
8.2	Future Research	173
References		175
Appendix A	Three-DOF Handling Model for 6WD Vehicle	181
Appendix B	Drop Test and Impact Test	189
Appendix C	Determination of Tire Operating Mode.....	195
Appendix D	Determination of Tire Lateral Stiffness	198
Appendix E	Roll and Pitch Frequencies of 6WD Vehicle.....	203

List of Figures

Figure 2.1-1	<i>X-Y</i> plane of earth-fixed axis system (SAE J670e, 1976)
Figure 2.1-2	Vehicle free-body diagram, earth-fixed axis system <i>XYZ</i>
Figure 2.1-3	Vehicle axis system (SAE J670e, 1976)
Figure 2.1-4	Vehicle free-body diagram, vehicle axis system <i>xyz</i>
Figure 2.1-5	Development of lateral force F_y due to slip angle α , top view
Figure 2.2.1-1	Two-mass system, sprung mass m_s and unsprung mass m_u
Figure 2.2.1-2	Tandem bicycle model for the 6WD vehicle
Figure 2.3.1-1	Tire lateral force F_y versus slip angle α (Milliken and Milliken, 1995)
Figure 2.3.1-2	Magic formula, (a) Sine, (b) Cosine version (Pacejka and Besselink, 1997)
Figure 2.3.2-1	Lateral shear displacement and tire deformation, Schwanghart's model
Figure 2.3.2-2	Shear displacement, Grecenko's model
Figure 2.3.2-3	Shear displacement and tire deformation, Crolla and El-Razaz's model
Figure 4.1-1	Mobile tire test rig, CAL (Milliken and Milliken, 1995)
Figure 4.1-2	Tire Research Facility (TRF), Calspan (Milliken and Milliken, 1995)
Figure 4.1-3	Delft-Tire test trailer, TNO
Figure 4.1-4	Schwanghart's off-road tire tester (Schwanghart, 1968)
Figure 4.2-1	Off-road Tire Test Facility, Carleton University, 1987
Figure 4.2-2	Forced-slip mechanism (Chen, 1993)
Figure 4.2-3	Schematic of forced-slip mechanism, for slipping tire
Figure 4.2-4	Schematic of forced-slip mechanism, for skidding tire
Figure 4.2-5	Explosion view of tire frame assembly
Figure 4.2-6	Tire frame
Figure 4.2-7	Wheel-shaft mounted on tire frame at $\alpha = -20^\circ$, top view
Figure 4.2-8	Pulley frame
Figure 4.2-9	Adjustable pulley frame, for implementing forced-slip
Figure 4.2-10	Payload frame
Figure 4.2-11	Side force support
Figure 4.2-12	Wheel-shaft assembly
Figure 4.2-13	Sensor assembly

Figure 4.2-14	TTF97 in operation, at -20° slip angle
Figure 4.3-1	Schematic diagram of instrumentation
Figure 4.3-2	DAQ scanning loop
Figure 4.3-3	Dominant frequency due to tire lugs engaging soil
Figure 4.3-4	Examination of crosstalk, x-y sensor
Figure 4.3-5	Crosstalk effect, x-channel of x-y sensor
Figure 4.3-6	Crosstalk effect, y-channel of x-y sensor
Figure 4.4-1	(a) Free-body diagram, tire frame assembly, top view, (b) Free-body diagram, wheel-shaft assembly, top view
Figure 4.4-2	Experimental data for identifying chain effect, and fitted curve
Figure 4.4-3	Experimental setup for identifying sprocket/chain efficiency
Figure 4.4-4	Measurement data for sprocket/chain efficiency test
Figure 5.1-1	Drawbar pull at 10, 20, and 35% slips
Figure 5.1-2	F_{dbp} / F_z vs. longitudinal slip and slip angle, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation
Figure 5.1-3	F_{dbp} / F_z vs. longitudinal slip, $p_i = 1$ bar, 2 bar and 3 bar
Figure 5.1-4	F_y / F_z vs. slip angle and longitudinal slip, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation
Figure 5.1-5	F_y / F_z vs. slip angle, $p_i = 1$ bar, 2 bar and 3 bar
Figure 5.1-6	F_y / F_z vs. F_{dbp} / F_z and slip angle, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation
Figure 5.1-7	F_y / F_z vs. F_{dbp} / F_z , $p_i = 1$ bar, 2 bar and 3 bar
Figure 5.1-8	Sinkage vs. slip angle and longitudinal slip, $p_i = 1$ bar, three-dimensional representation
Figure 5.1-9	Sinkage vs. slip angle and longitudinal slip, $p_i = 2$ bar, three-dimensional representation
Figure 5.1-10	Sinkage vs. slip angle and longitudinal slip, $p_i = 3$ bar, three-dimensional representation
Figure 5.2-1	Schematic of Bevameter (Chen, 1993)
Figure 5.2-2	(a) Pressure-sinkage test, (b) Shear stress-displacement test
Figure 5.2-3	Fitted curve compared with six penetration tests
Figure 5.2-4	Shear strength versus normal pressure
Figure 5.2-5	Fitted curve compared with three shear tests

Figure 5.2-6	Shear strength versus normal pressure, rubber-sand
Figure 5.2-7	Fitted curve compared with three shear tests, rubber-sand
Figure 6.1-1	Tire lateral deformation at slip angle α , on hard surface (Clark, 1981)
Figure 6.1-2	Tire lateral deformation at slip angle α , on soft terrain
Figure 6.2-1	Tire slip angle induced by tire frame
Figure 6.2-2	Displacement of tire center in $\bar{v}_0$ -direction and in wheel plane
Figure 6.2-3	Slip velocities on x - z plane and in lateral direction
Figure 6.2-4	Slip angle resulting from tire lateral deformation and lateral shear displacement in contact region, rigid mode
Figure 6.2-5	An example demonstrating tire lateral deformation and lateral shear displacement in contact region
Figure 6.2-6	Direction of resultant slip velocity v_r
Figure 6.2-7	Coordinate system for defining contact trajectory
Figure 6.2-8	Trajectory of a point rolling from $\theta = \pi/2$ to $\theta = 0$, on a combined slip tire, with longitudinal slip $i = 10\%$
Figure 6.2-9	Trajectories of point p with respect to ground, for $\alpha = -20^\circ$ and $\beta_r = -6^\circ$, and $i = 0, 10, 20$, and 35% , rigid mode
Figure 6.2-10	Direction of relative motion of circumferential point p on laterally deformed tire relative to terrain
Figure 6.2-11	Comparison between angles ψ'_p and ψ_p
Figure 6.2-12	Bulldozing force acting on tire sidewall
Figure 6.2-13	Sunken sidewall area
Figure 6.2-14	(a) $\alpha = 0^\circ$, no lateral bulldozing force, $F_{yb} = 0$, (b) $\alpha > 0^\circ$, F_{yb} increases with respect to α , (c) $\alpha = 90^\circ$, F_{yb} reaches maximum, $F_{yb} = F_{yb\perp}$
Figure 6.2-15	Position of instantaneous center of slipping tire, rigid mode
Figure 6.2-16	Slip velocity vector $\bar{v}_p$ of an element at point p on sidewall
Figure 6.2-17	Elementary shear force dF_p on positive side of x -axis
Figure 6.2-18	Elementary shear force dF_p on negative side of x -axis
Figure 6.3-1	Tire deformation along wheel plane, and lateral deformation due to carcass and tread elasticity
Figure 6.3-2	Slip angle resulting from tire lateral deformation and lateral shear displacement in contact region, elastic mode

Figure 6.3-3	Trajectories of point p with respect to ground, for $\alpha = -20^\circ$ and $\beta_i = -6^\circ$, and $i = 0, 10, 20$, and 35% , elastic mode
Figure 6.3-4	Longitudinal slip velocity on flat portion BA
Figure 6.3-5	Sidewall area of an elastic tire subject to lateral bulldozing force
Figure 6.4-1	F_y / F_z vs. F_{dbp} / F_z , $p_i = 1$ bar, 2 bar and 3 bar, predictions comparing with measurements
Figure 6.4-2	(a) Modeling of tire contact profile in elastic operating mode, (b) Tire contact profile represented by a continuous curve
Figure 6.4-3	(a) Modeling of tire cross section in this study, (b) Actual cross section of the test tire
Figure 6.4-4	Tread pattern of Michelin XL
Figure 6.4-5	Lateral deformation of sideslip tire, (a) Hard ground, (b) Soft terrain, rigid mode, (c) Soft terrain, elastic mode
Figure 7.1-1	Formulation of equations of motion for rigid bodies, algorithm used by AUTOSIM
Figure 7.2-1	Layout of steering system, top view
Figure 7.2-2	Steered angles of left and right wheel versus steering wheel input
Figure 7.3-1	Layout of suspension, right wheel station, front steered axle, rear view
Figure 7.3-2	Damper characteristic curves, mean and deviation curves
Figure 7.3-3	Modeled damper characteristics comparing with original damper characteristics
Figure 7.3-4	Stiffness at wheel center (wheel rate), by simulation
Figure 7.3-5	Reaction forces of suspension components upon a $2.0g$ step input upwards to wheel center
Figure 7.3-6	Time response of wheel center upon a $2.0g$ step input upwards to wheel center
Figure 7.3-7	Reaction forces of suspension components upon a $5.0g$ step input downwards to wheel center
Figure 7.3-8	Time response of wheel center upon a $5.0g$ step input downwards to wheel center
Figure 7.3-9	Bump camber of wheel-hub assembly
Figure 7.3-10	Bump camber analysis by sketch, rear view
Figure 7.3-11	Bump steer of wheel-hub assembly

Figure 7.4-1	Configuration of sprung mass, 6WD vehicle
Figure 7.5-1	Algorithm of using the proposed off-road tire model, elastic mode
Figure 7.6-1	Vehicle handling simulation
Figure 7.6-2	Step steering input with rise time
Figure 7.6-3	Vehicle trajectories simulated by proposed 17-DOF handling model and Segel's model, on hard ground
Figure 7.6-4	Double-triangular steering input
Figure 7.6-5	Lane-change manoeuvre predicted by 3-DOF handling model, with tire lag and without tire lag
Figure 7.6-6	Time response of yaw velocity, under lane-change manoeuvre
Figure 7.6-7	Time response of lateral acceleration, under lane-change manoeuvre
Figure 7.6-8	Time response of roll angle, sprung mass, under lane-change manoeuvre
Figure 7.6-9	Time response of pitch angle, sprung mass, under lane-change manoeuvre
Figure 7.6-10	Vehicle heading angle, under lane-change manoeuvre
Figure 7.6-11	Vehicle trajectories on XY earth-fixed axis system, under lane-change manoeuvre
Figure 7.6-12	Vehicle trajectories on XY earth-fixed axis system, step steering input with rise time
Figure 7.6-13	Yaw velocity gain characteristics
Figure 7.6-14	Lateral acceleration gain characteristics
Figure 7.7-1	Animation of lane-change manoeuvre, 6WD vehicle
Figure A-1	Tandem bicycle model for 6WD vehicle
Figure A-2	Two-mass system, sprung mass m_s and unsprung mass m_u
Figure B.1-1	Time response and frequency response, $p_i = 138$ kPa
Figure B.1-2	Time response and frequency response, $p_i = 345$ kPa
Figure B.2-1	Accelerometer and charge amplifier, impact test
Figure B.2-2	PSD graphs of acceleration in x -axis
Figure B.2-3	PSD graphs of acceleration in y -axis
Figure B.2-4	PSD graphs of acceleration in z -axis
Figure C-1	Average ground pressure versus inflation pressure, at normal loads
Figure D-1	Tire frame assembly operating on hard sandy surface

Figure D-2	Variation of tire relaxation length, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation
Figure D-3	Variation of tire lateral stiffness, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation
Figure D-4	Mean value of tire lateral stiffness, $p_i = 1$ bar, 2 bar and 3 bar
Figure E.1-1	Roll oscillation of sprung mass
Figure E.2-1	Pitch plane model of sprung mass
Figure E.2-2	Bounce vibration mode of sprung mass, pitch plane model
Figure E.2-3	Pitch vibration mode of sprung mass, pitch plane model

List of Tables

Table 4.3-1	Eight channels of analog signal
Table 5.1-1	Tire characteristics (Michelin, 1997)
Table 5.1-2	Variations of parameter setting
Table 5.2-1	Parameters for pressure-sinkage relationship
Table 5.2-2	Parameters for shear stress-displacement
Table 5.2-3	Parameters for shear stress-displacement, rubber-soil
Table 6.4-1	Goodness-of-fit ε , predictions to measurements
Table 7.3-1	Coefficients of mean and deviation polynomials
Table 7.4-1	Input parameters for calculation of inertial quantities, 6WD vehicle
Table 7.4-2	Calculation output of inertial quantities
Table 7.4-3	Error analysis of inertial quantities
Table 7.6-1	Output data of selected variables, from the 17-DOF handling model
Table B.2-1	Natural frequencies of tire frame structure
Table C-1	Values of p_{gr} for different F_z and p_i
Table C-2	Values of p_{gr} for different F_z
Table C-3	Modes of operation in wheel plane, Michelin 12.5R20 XL tire on Ottawa sand

List of Symbols

α	slip angle [<i>rad</i>]
α_i	slip angle, of the <i>i</i> -th axle [<i>rad</i>]
β	sideslip angle of vehicle mass center [<i>rad</i>]
β_i	tire lateral deformation angle [<i>rad</i>]
γ	camber angle [<i>rad</i>]
γ_i	camber angle, of the <i>i</i> -th axle [<i>rad</i>]
γ_s	soil bulk density [<i>kg / m</i> ³]
δ	steer angle [<i>rad</i>]
δ_i	roll steer, of the <i>i</i> -th axle [<i>rad</i>]
ε	goodness-of-fit
ζ	damping ratio
η_s	sprocket/chain efficiency
η_x	tread longitudinal displacement [<i>m</i>]
η_y	tread lateral displacement [<i>m</i>]
θ	tire-terrain contact angle [<i>rad</i>]
θ	pitch angle [<i>rad</i>]
θ_B	angle at contact point <i>B</i> [<i>rad</i>]
θ_p	angle from point <i>p'</i> to point <i>p</i> [<i>rad</i>]
θ_r	angle of resultant slip velocity <i>v_r</i> [<i>rad</i>]
θ_R	roll axis downward angle [<i>rad</i>]
θ_{IC}	angle at initial contact point [<i>rad</i>]
λ	horizontal coordinate on flat portion <i>BA</i> [<i>m</i>]
ξ	coordinate along tire circumference outside contact region [<i>m</i>]
ξ_p	sprocket gear ratio
ρ	radius of gyration [<i>m</i>]
ρ_i	density of the <i>i</i> -th mass block [<i>kg / m</i> ³]
$\bar{\rho}_p$	position vector of point <i>p</i> [<i>m</i>]
$\bar{\rho}_q$	position vector of point <i>q</i> [<i>m</i>]
τ	shear stress [<i>kPa</i>]
τ_p	shear stress due to shear displacement <i>j_p</i> [<i>kPa</i>]

τ_q	shear stress due to shear displacement j_q [kPa]
τ_r	shear stress due to resultant shear displacement j_r [kPa]
$\tau_{\max}$	shear strength of terrain [kPa]
ϕ	roll angle [rad]
ϕ_s	angle of internal shearing resistance [deg]
ϕ_{rs}	angle of rubber-soil friction [deg]
φ_p	orientation angle of position vector $\bar{\rho}_p$ [rad]
φ_q	orientation angle of position vector $\bar{\rho}_q$ [rad]
ψ	vehicle heading angle [rad]
ψ_p	angle of relative motion, contact trajectory analysis [rad]
ψ'_p	angle of relative motion, slip velocity analysis [rad]
ω_d	frequency of damped oscillation [rad/sec]
ω_n	natural frequency [rad/sec]
ω_t	tire angular speed [rad/sec]
Θ	amplitude of pitch [rad]
a	distance from vehicle mass center to 1st axle [m]
b	distance from vehicle mass center to 2nd axle [m]
b	width of rectangular plate, or radius of circular plate [m]
b_t	tire tread width [m]
c	apparent cohesion of terrain [kPa]
c	longitudinal distance between m_s and m [m]
c	damping coefficient for left or right side of vehicle [$N \cdot s/m$]
c_i	coefficients of polynomial
c_{rs}	tangential adhesion between tire rubber and soil [kPa]
C_α	cornering stiffness, or initial slope of F_{ya} versus α [N/rad]
C_{ai}	cornering stiffness, of the i -th axle [N/rad]
C'_{ai}	cornering stiffness, of a single tire in the i -th axle [N/rad]
$C_{z\alpha}$	aligning torque stiffness, or initial slope of M_{za} vs. α [$N \cdot m/rad$]
C_{zai}	aligning torque stiffness, of the i -th axle [$N \cdot m/rad$]
C'_{zai}	aligning torque stiffness, of a single tire in the i -th axle [$N \cdot m/rad$]
$C_{\gamma i}$	camber stiffness, of the i -th axle [N/rad]
d	distance between point p and p' [m]
d_t	tire diameter [m]

D	derivative operator, $D = d / dt$
e	longitudinal distance between m_U and m [m]
E_{wpi}	linear damper velocity induced by vehicle roll velocity, of the i -th axle [m / rad]
$E_{\delta\dot{\phi}i}$	roll steer coefficient, of the i -th axle
$E_{\gamma\dot{\phi}i}$	roll camber coefficient, of the i -th axle
f_{DCi}	compression damper force per unit stroke velocity, of the i -th axle [$N \cdot s / m$]
f_{DEi}	extension damper force per unit stroke velocity, of the i -th axle [$N \cdot s / m$]
F_c	slip/skid cable tension force [N]
F_{ct}	horizontal component of chain force [N]
F_{dbp}	drawbar pull [N]
F_s	side force [N]
F_t	pulling force on tire frame [N]
F_x	tire longitudinal force [N]
F_{x1}	longitudinal force measured by x-y sensor, left [N]
F_{x2}	longitudinal force measured by x-y sensor, right [N]
F_{xb}	braking force [N]
F_{xp}	longitudinal shear force on tire sidewall, positive x -axis [N]
F_{xq}	longitudinal shear force on tire sidewall, negative x -axis [N]
F_{xt}	tractive force [N]
F_{xsw}	longitudinal shear force on tire sidewall [N]
F_y	tire lateral force [N]
F_{yb}	lateral bulldozing force [N]
$F_{yb\perp}$	lateral bulldozing force, perpendicular to tire sidewall [N]
F_{yi}	tire lateral force, of the i -th axle [N]
F_{ym}	measured tire lateral force [N]
F_{yp}	predicted tire lateral force [N]
F_{ysh}	tire lateral force due to shearing [N]
$F_{y,ss}$	tire lateral force, steady state [N]
F_{yt}	tire lateral force due to k_y [N]
$F_{y\alpha}$	tire lateral force due to α [N]
F_z	tire normal force [N]
F_{zp}	vertical shear force on tire sidewall, positive x -axis [N]
F_{zq}	vertical shear force on tire sidewall, negative x -axis [N]

F_{sw}	vertical shear force on tire sidewall [N]
F_C	centripetal force for cornering vehicle [N]
F_D	deviation damping force [N]
F_E	damper force [N]
F_M	mean damping force [N]
F_T	tangential force for cornering vehicle [N]
g	gravity [m/sec^2]
G_{ay}	lateral acceleration gain [g/deg]
G_r	yaw velocity gain [$rad/sec/deg$]
h	height of m_s above x -axis [m]
h_I	height of I measured from bottom-dead-center [m]
i	tire longitudinal slip [%]
i_e	tire effective longitudinal slip [%]
i_s	tire longitudinal skid [%]
I	instantaneous center of zero velocity
j	shear displacement [m]
j_p	resultant shear displacement on tire sidewall, positive x -axis [m]
j_q	resultant shear displacement on tire sidewall, negative x -axis [m]
j_r	resultant shear displacement [m]
j_x	longitudinal shear displacement [m]
j_{xz}	circumferential shear displacement, on the x - z plane [m]
j_y	lateral shear displacement [m]
Δj_{xBA}	increment of longitudinal shear displacement on flat portion BA [m]
Δj_{yBA}	increment of lateral displacement on flat portion BA [m]
k	ride rate for left or right side of vehicle [N/m]
k_ϕ	pressure-sinkage parameter [kN/m^{n+2}]
k_c	pressure-sinkage parameter [kN/m^{n+1}]
k_b	stiffness of bump stop [N/m]
k_i	ride rate, of the i -th axle [N/m]
k_r	stiffness of rebound stop [N/m]
k_{si}	spring stiffness per single wheelstation, of the i -th axle [N/m]
k_{sp}	stiffness of spring [N/m]
k_y	tire lateral stiffness [kPa]

K	shear deformation modulus [m]
K_{rs}	shear deformation modulus, rubber-soil [m]
K_c	soil resistance coefficient, cohesion
K_q	soil resistance coefficient, surcharge
K_γ	soil resistance coefficient, density
l	coordinate along contact region, elastic mode [m]
l_f	half of contact length on flat portion BA [m]
l_r	relaxation length [m]
l_{si}	lateral separation between springs, of the i -th axle [m]
l_t	total contact length along wheel plane [m]
L	tire circumferential length outside contact region [m]
L_2	distance between 2nd and 3rd axle [m]
m	vehicle mass [kg]
m_i	mass of the i -th hull plate [kg]
m_s	sprung mass [kg]
m_u	unsprung mass [kg]
M_{ai}	aligning torque due to α , of the i -th axle [$N \cdot m$]
M_x	rolling moment [$N \cdot m$]
M_{xp}	damper reaction moment [$N \cdot m$]
$M_{x\phi}$	stiffness reaction moment [$N \cdot m$]
M_z	yawing moment [$N \cdot m$]
n	pressure-sinkage exponent
n	number of axles of vehicle
N	number of data points
p	roll velocity [rad / sec]
p_b	bulldozing pressure [kPa]
p_c	pressure due to carcass stiffness [kPa]
p_{scr}	critical pressure [kPa]
p_{gr}	average ground pressure [kPa]
p_i	tire inflation pressure [kPa] or [bar]
p_n	normal pressure [kPa]
$p_{y\max}$	point of maximum tire lateral deformation
P	frictional soil resistance per unit width of interface [N / m]

q	pitch velocity [rad / sec]
q_s	uniform surcharge pressure [kPa]
r	yaw velocity [rad / sec]
r	distance between tire center and point p [m]
r_1	radius of sprocket on tire axle [m]
r_2	radius of sprocket on slip pulley shaft [m]
r_s	slip pulley radius [m]
r_t	tire rolling radius [m]
R_c	compaction resistance [N]
R^2	coefficient of determination
s	coordinate along contact arc [m]
s_0	displacement along $x_{tire\ frame}$ -direction [m]
s_l	actual displacement of tire center along wheel plane [m]
s_t	theoretical displacement of tire center along wheel plane [m]
s_B	coordinate s at point B [m]
s_D	coordinate s at point D [m]
s_{Di}	radius arm, roll center $\perp$ damper line of action, of the i -th axle [m]
t_c	time constant [sec]
t_p	pneumatic trail [m]
t_s	time span of element in contact with terrain [sec]
T	track of vehicle [m]
u	longitudinal velocity [m / sec]
u_d	soil displacement gradient
v	side velocity [m / sec]
v	stroke velocity of damper [m / sec]
v_0	actual forward speed of tire center [m / sec]
$\bar{v}_0$	velocity vector of tire center [m / sec]
v_l	actual velocity of tire center along wheel plane [m / sec]
v_{jxz}	circumferential slip velocity, on x - z plane [m / sec]
v_{jy}	lateral slip velocity [m / sec]
v_r	resultant slip velocity [m / sec]
v_s	actual velocity of tire center perpendicular to wheel plane [m / sec]
v_t	theoretical velocity of tire center along wheel plane [m / sec]

$\bar{v}_p$	velocity vector of point p [m/sec]
$\bar{v}_{p,x}$	absolute velocity of point p along wheel plane [m/sec]
$\bar{v}_{p,y}$	absolute lateral velocity of point p [m/sec]
V_i	volume of the i -th mass block [m^3]
w	normal velocity [m/sec]
w_i	linear damper velocity, of the i -th axle [m/sec]
$\bar{x}$	position vector along x -axis, center of sprung mass [m]
$\bar{x}_i$	position vector along x -axis, of the i -th hull plate or mass block [m]
y_ξ	tire lateral deformation outside contact region [m]
y_s	tire lateral deformation in contact region [m]
z	bounce [m]
z	sinkage [m]
z_d	tire vertical deflection [m]
z_{el}	maximum tire sinkage in elastic operating mode [m]
z_i	observed value, or measured data
z_{max}	maximum tire sinkage [m]
z_{rg}	maximum tire sinkage in rigid operating mode [m]
z_t	coordinate for tire sinkage [m]
$\bar{z}$	mean value, of measured data
$\bar{z}$	position vector along z -axis, center of sprung mass [m]
$\bar{z}_i$	position vector along z -axis, of the i -th hull plate or mass block [m]
$\hat{z}_i$	regression value, from fitted surface
Z	amplitude of bounce [m]

Chapter 1

Introduction

1.1 Background

This thesis focuses on the investigation of the handling behaviour of multi-axle, all-wheel-drive vehicles. The principal applications of this type of vehicle can be found in off-road transport and in military operations.

The behaviour of a ground vehicle may be characterized by its *performance*, *ride* and *handling*. Performance mainly relates to the characteristics of the engine, power train, tire-ground interaction, and control systems. Performance indicators include acceleration, maximum speed, stopping distance, and gradability of the vehicle. Road vehicles of current design normally have adequate performance with acceptable fuel consumption. For off-road applications, all-wheel-drive vehicles offer better performance, such as higher traction and tractive efficiency, and improved mobility (Wong, 1970; Dick, 1995; Wong et al., 1998). Serious mobility problems may still arise on soft terrain where vehicle sinkage and motion resistance are excessive and the tractive force is relative low. Considerable effort has been expended over the years in solving this problem. Vehicle

mobility on unprepared terrain, however, still poses a challenge to researchers and engineers (Bekker, 1956, 1960, and 1969; Larminie, 1988; Wong, 1989, 1993).

Ride quality affects passenger comfort. With improvements in technology and design, most suspension systems currently in use can provide a reasonable level of ride comfort for road vehicles. To further improve ride quality, semi-active and active suspensions are being considered (Chalasani, 1986a, 1986b; Crolla et al., 1989). On the other hand, in off-road operations, rough surfaces may be encountered. Providing adequate ride quality for off-road vehicles remains a goal to be achieved. The deformability of unprepared terrain further complicates this issue.

Handling is primarily concerned with the response of a vehicle to the driver's steering commands. For road vehicles with single-axle-drive (such as passenger cars) or with tandem-axle-drive (such as tractors in tractor-semitrailer combinations), their handling behaviour have been extensively investigated (Segel, 1956; Wong and El-Gindy, 1986). However, for multi-axle, all-wheel-drive vehicles, relatively few systematic studies have been published regarding their handling characteristics on deformable terrain. As the driving torque affects the lateral behaviour of pneumatic tires, it is anticipated that the handling response of these vehicles may be different from those of single-axle-drive or tandem-axle-drive vehicles. Furthermore, vehicle sideslip and yaw motions are dependent not only on vehicle design parameters and tire characteristics, but also on the mechanical properties of the terrain. The study of the handling characteristics of a multi-axle, all-wheel-drive off-road vehicle on deformable terrain is therefore a significant technological challenge and is the major topic of this thesis.

1.2 Overview of This Research

The research presented in this thesis focuses on the handling behaviour of a multi-axle, all-wheel-drive, off-road vehicle. As the motion of an off-road vehicle is primarily controlled by the forces and moments developed on the tire-terrain interface, an understanding of the mechanics of tire-ground interaction is of importance. While a great deal of research has been carried out in recent years on the tractive performance of tires

on unprepared surfaces, the mechanics of tires under combined longitudinal slip and slip angle on deformable terrain has not yet been adequately elucidated. To reach a better understanding of the relationship between longitudinal and lateral forces and the combined longitudinal slip and slip angle of a tire on deformable terrain, a tire test facility was designed, constructed, and instrumented at the Transport Technology Research Laboratory, Carleton University. This facility was able to measure both longitudinal and lateral forces when a tire is subject to a combination of longitudinal slip/skid and slip angle. The experimental data obtained provide a better understanding of the tire behaviour on a deformable terrain. In addition, this set of data also gives the basic information for the development of a new off-road tire model.

The tire model developed takes into account the effects of combined longitudinal slip and slip angle, tire lateral deformation, direction of slip velocity, contact trajectory, lateral bulldozing force, sidewall shearing, and terrain properties. The correlation between experimental data and predictions by the proposed tire model was found to be acceptable. The tire model was then integrated into a vehicle-handling model for predicting the manoeuvrability of off-road vehicles on deformable terrain.

The handling model developed has 17 degrees of freedom (DOF), including the motion of the *sprung* mass (vehicle body), and relative motions of six *unsprung* masses (wheel assemblies and associated suspension linkages) to the sprung mass. The characteristics of the steering system and suspension system were fully taken into consideration. The model was formulated using the computer code known as AUTOSIM, developed by Mechanical Simulation Corporation, Ann Arbor, Michigan.

To demonstrate the application of the model, the handling characteristics of a six-wheel-drive (6WD), front-wheel-steering military vehicle on hard ground and on deformable surface were simulated and compared. The steering inputs used include a step function with a particular rise time to simulate constant-radius turns, and a double-triangular function to simulate lane-change manoeuvres. In addition, an animation program was developed to show a lane-change manoeuvre of the 6WD vehicle.

Chapter 2

Review of the State of the Art

2.1 Vehicle Handling

Vehicle handling is concerned with vehicle response to steering input. Handling is often regarded as equivalent to *cornering* since a steering input causes a vehicle to turn with respect to a turning center. Among the factors that influence vehicle handling, the lateral behaviour of tires is one of the most important. While the lateral characteristics of tires on hard surfaces have been investigated extensively, the lateral behaviour of tires on deformable terrain has not yet been adequately studied.

Vehicle cornering behaviour can be illustrated using a proper coordinate system. The earth-fixed axis system XYZ (SAE J670e, 1976) is an inertial reference frame. Figure 2.1-1 shows the X - Y plane of the fixed XYZ axis. To form a right-hand coordinate system, the Z -axis is vertically downward. The origin of XYZ axis may be chosen at any convenient location. An ideal choice is the point of initial vehicle mass center projected on the X - Y plane (road surface). The figure also gives definitions and sign conventions of steer angle of wheels, and heading angle and sideslip angle of the vehicle mass center.

Figure 2.1-1 X-Y plane of earth-fixed axis system (SAE J670e, 1976)

Figure 2.1-2 shows the free-body diagram of a cornering vehicle on a horizontal plane, viewed in the XYZ axis system. The vehicle in cornering has a tangential force F_T that controls the acceleration component along the curve path. Laterally, the vehicle is subject to a centripetal force F_C . The force gives a centripetal acceleration that makes the vehicle follow a given curve.

Figure 2.1-2 Vehicle free-body diagram, earth-fixed axis system XYZ

Figure 2.1-3 shows the vehicle axis system xyz (SAE J670e, 1976) and the terms used in vehicle dynamics. The xyz axis system, terminology, and symbols were adopted from the fuselage-fixed axis system in aircraft dynamics (Padfield, 1995).

Figure 2.1-3 Vehicle axis system (SAE J670e, 1976)

The xyz axis system is a non-inertial reference frame, and fixed to and moving with the vehicle. D'Alembert forces, or inertial forces are usually used in the analysis of vehicle handling. These inertial forces are equal, in magnitude, to the vehicle mass times its accelerations seen in the earth-fixed XYZ system, and act in the opposite direction of the accelerations. This is shown in Figure 2.1-4. For handling studies, the d'Alembert force $-m\ddot{y}$ is the centrifugal force due to vehicle cornering. The advantage of using vehicle axis system is that all sensors to measure vehicle cornering responses are mounted onboard. For example, the centrifugal acceleration $-\ddot{y}$ can be obtained directly from an accelerometer aligned to the vehicle y -axis. The other benefit of using the xyz axis system is that the mass moments of inertia and product moments of inertia of the vehicle with respect to this set of axes remain constant. Otherwise as the vehicle moves, its inertial quantities change.

Figure 2.1-4 Vehicle free-body diagram, vehicle axis system xyz

The centrifugal force $-m\ddot{y}$ caused by vehicle cornering is transmitted to axles, through spindles, to the tires. When a tire is subject to a side force F_s at its center, a lateral force F_y is developed at the tire-ground interface, and located behind the projection of the tire center by a distance of t_p , the pneumatic trail. Instead of moving along the wheel plane direction, the tire center moves along a path at an angle α to its wheel plane. This angle α is called *slip angle*. Figure 2.1-5 shows tire lateral force F_y due to a slip angle α . The lateral force may be thought of as the result of the slip angle, and vice versa. The relationship between tire lateral force and slip angle is of importance, and to a great extent influences the handling behaviour of the vehicle. It can be realized that the sum of tire lateral forces from all wheels is equal to the centripetal force F_C , viewed in the earth-fixed axis system, as shown in Figure 2.1-2. The force F_C turns, or yaws the vehicle to follow the cornering curve.

Figure 2.1-5 Development of lateral force F_y due to slip angle α , top view

2.2 Vehicle Handling Models

Although the research presented in this thesis focuses on the study of handling behaviour of off-road vehicles, handling models for both road and off-road vehicles share some basic characteristics. Consequently, a review of the handling models for road vehicles is included and given below. It should be noted that the major difference in handling between road and off-road vehicles lies in the response of the tire to lateral force. In other words, the relations between lateral force with slip angle of a tire on hard surfaces and on deformable terrain differ considerably.

2.2.1 Three Degree-of-Freedom Model

A basic model for investigating vehicle handling, or directional response, is the three degree-of-freedom (3-DOF) model developed by Segel (1956). A complete derivation can be found in publications by Segel (1956, 1988), and by Ellis (1969, 1989, and 1994). The model has been experimentally validated. It has a number of unique features. First, the model is a linear model, and an analytical solution is available. Secondly, it only requires a limited number of input parameters. One of the shortcomings of this model is its linearization of suspension characteristics, vehicle sideslip angle, and the relation between lateral force and slip angle of the tire. Thus the model is useful for low lateral acceleration handling. The mathematical form involves the solution of two first-order differential equations associated with sideslip and yaw velocity, and a second-order differential equation for roll displacement. Many researchers have made use of Segel's model to study various handling problems.

Bundorf and Leffert (1976) introduced *cornering compliance*, which was derived from the steady-state response of the 3-DOF model. With computed cornering compliance for front and rear axles, the steady-state and transient directional behaviour can be predicted. Nalecz (1987) applied the 3-DOF model as a basis to investigate various suspension designs of light commercial vehicles. He concluded that suspension configurations should be considered in the study of vehicle stability. Metz et al. (1992) conducted a vehicle rollover analysis of a sample of 43 vehicles by comparing their dominant pole locations. The characteristic equation was derived from the 3-DOF handling model.

To apply the model to the 6WD vehicle, the vehicle is treated as a two-mass system, moving at a constant speed u along its longitudinal axis, as shown in Figure 2.2.1-1. One of the masses, the nonrolling unsprung mass m_U (with axes $x_U y_U z_U$) is constrained to translate on the horizontal x - y plane, and to rotate with respect to the z -axis perpendicular to this plane. Thus two degrees of freedom exist. They are the yaw velocity r and the ratio of side velocity v to longitudinal velocity u of the vehicle mass center. The velocity ratio is also defined as the vehicle sideslip angle β , that is $\beta = v/u$, shown in both Figures 2.1-1 and 2.2.1-2. The other rolling sprung mass m_S (with axes $x_S y_S z_S$), shown in Figure 2.2.1-1, is constrained such that it is limited to a roll degree of freedom with a roll angle ϕ about the x -axis fixed in the nonrolling unsprung mass. The actual roll axis is normally inclined downwards to the front axle by an angle of θ_R measured from the x -axis.

ψ = vehicle heading angle, defined in Figure 2.1-1

Figure 2.2.1-1 Two-mass system, sprung mass m_S and unsprung mass m_U

Yaw velocity r is a response variable, which describes how a vehicle follows a given cornering command. The rate of change of sideslip angle β is one of the variables that can be sensed by the driver. The roll motion ϕ , on the other hand, reflects certain degrees

of passenger comfort and stability of a cornering vehicle. Figure 2.2.1-2 shows the *tandem bicycle model* for the 6WD vehicle in plane view.

Figure 2.2.1-2 Tandem bicycle model for the 6WD vehicle

The derivation of equations of motion involves the resultant forces and moments acting on the vehicle, and the translational and rotational inertial quantities of the vehicle. The complete derivation of the 3-DOF handling model for the 6WD vehicle is given in Appendix A. After simplification, equations of motion can be expressed in a matrix form. Using the derivative operator $D = d/dt$, the matrix equation is shown below.

$$\begin{bmatrix} muD - Y_\beta & mu - Y_r & m_s h D^2 - Y_\phi \\ -N_\beta & I_z D - N_r & I_x D^2 - N_\phi \\ m_s h u D & I_x D + m_s h u & I_x D^2 - L_p D - L_\phi \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \phi \end{bmatrix} = \begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix} \delta \quad (2.2.1-1)$$

where $m = m_s + m_u$

$$I_x = I_{xss} + m_s h^2 - 2\theta_R I_{xss} + \theta_R^2 I_{xss}$$

$$I_x = m_s h c - I_{xss} + \theta_R I_{xss}$$

$$I_z = I_{zss} + I_{zu} + m_s c^2 + m_u e^2$$

$$Y_\beta = C_{\alpha 1} + C_{\alpha 2} + C_{\alpha 3}$$

$$Y_r = [a C_{\alpha 1} - b C_{\alpha 2} - (b + L_2) C_{\alpha 3}] / u$$

$$Y_\phi = E_{\gamma\phi 1} C_{\gamma 1} + E_{\gamma\phi 2} C_{\gamma 2} + E_{\gamma\phi 3} C_{\gamma 3} - E_{\delta\phi 1} C_{\alpha 1} - E_{\delta\phi 2} C_{\alpha 2} - E_{\delta\phi 3} C_{\alpha 3}$$

$$Y_\delta = -C_{\alpha 1}$$

$$\begin{aligned}
N_\beta &= aC_{\alpha 1} - bC_{\alpha 2} - (b + L_2)C_{\alpha 3} + C_{z\alpha 1} + C_{z\alpha 2} + C_{z\alpha 3} \\
N_r &= [a^2C_{\alpha 1} + b^2C_{\alpha 2} + (b + L_2)^2C_{\alpha 3} + aC_{z\alpha 1} - bC_{z\alpha 2} - (b + L_2)C_{z\alpha 3}] / u \\
N_\phi &= -aE_{\delta\phi 1}C_{\alpha 1} + aE_{\gamma\phi 1}C_{\gamma 1} + bE_{\delta\phi 2}C_{\alpha 2} - bE_{\gamma\phi 2}C_{\gamma 2} + (b + L_2)E_{\delta\phi 3}C_{\alpha 3} \\
&\quad - (b + L_2)E_{\gamma\phi 3}C_{\gamma 3} - E_{\delta\phi 1}C_{z\alpha 1} - E_{\delta\phi 2}C_{z\alpha 2} - E_{\delta\phi 3}C_{z\alpha 3} \\
N_\delta &= -aC_{\alpha 1} - C_{z\alpha 1} \\
L_p &= \sum_{i=1}^n s_{Di} E_{wpi} (f_{DEi} + f_{DCi}) \\
L_\phi &= m_s gh + \sum_{i=1}^n \frac{1}{2} k_{si} l_{si}^2 \\
I_{xxS} &= \text{mass moment of inertia with respect to } x_S \text{-axis, for } m_s \\
I_{zzS} &= \text{mass moment of inertia with respect to } z_S \text{-axis, for } m_s \\
I_{zzU} &= \text{mass moment of inertia with respect to } z_U \text{-axis, for } m_U \\
I_{xzS} &= \text{product moment of inertia relative to } x_S \text{ and } z_S \text{ axes, for } m_s \\
C_{\alpha i} &= \text{cornering stiffness, of the } i\text{-th axle} \\
C_{\gamma i} &= \text{camber stiffness, of the } i\text{-th axle} \\
C_{z\alpha i} &= \text{aligning moment stiffness, of the } i\text{-th axle} \\
E_{\gamma\phi i} &= \text{roll camber coefficient, of the } i\text{-th axle} \\
E_{\delta\phi i} &= \text{roll steer coefficient, of the } i\text{-th axle} \\
s_{Di} &= \text{radius arm, roll center perpendicular to damper line of action,} \\
&\quad \text{of the } i\text{-th axle} \\
E_{wpi} &= \text{linear damper velocity induced by vehicle roll velocity,} \\
&\quad \text{of the } i\text{-th axle} \\
f_{DEi} &= \text{extension damper force per unit stroke velocity, of the } i\text{-th axle} \\
f_{DCi} &= \text{compression damper force per unit stroke velocity,} \\
&\quad \text{of the } i\text{-th axle} \\
n &= \text{number of axles of the vehicle, } n = 3 \text{ for 6WD vehicle} \\
k_{si} &= \text{spring stiffness per single wheelstation, of the } i\text{-th axle} \\
l_{si} &= \text{lateral separation between springs, of the } i\text{-th axle}
\end{aligned}$$

Matrix (2.2.1-1) is the mathematical basis for the 3-DOF handling model with variables of sideslip angle β , yaw velocity r , and roll angle ϕ . By setting all derivative terms in matrix (2.2.1-1) to zero, the steady-state response of sideslip angle β , yaw velocity r , and roll angle ϕ to a steering input δ can be derived. Accordingly, handling characteristics such as understeer/oversteer, static margin, and characteristic speed are

obtained. Transient response can be predicted by solving the matrix equation. A step function for steering input is commonly used to test the handling characteristics of a vehicle. For a sinusoidal steering input, frequency response is the preferable approach.

In addition to the classic 3-DOF model described above, there are other models with explicit equations of motion. Such models are the sideslip/yaw/steering model for steering system dynamics, and the sideslip/yaw/tire lateral stiffness for tire lateral dynamics (Ellis, 1989, 1994). The 3-DOF model can be further simplified to the 2-DOF sideslip/yaw model by removing the roll degree of freedom. By neglecting roll, this model assumes that the suspension effect on the vehicle cornering behaviour is small. Therefore, roll camber, roll steer, and the reaction moment by springs and dampers are ignored. The 2-DOF sideslip/yaw model was used by many researchers such as Abe (1989), and Nagai (1989). Lund and Bernard (1992) studied vehicle directional responses using both the 2-DOF (sideslip and yaw) and the 3-DOF models. They indicated that for a typical passenger car, step responses predicted by the two models were almost the same, and frequency responses were very close. Xia and Law (1992) developed an 8-DOF handling model to study control strategies of a four-wheel-steering automobile. This model includes the classic 3-DOF, one DOF for vehicle longitudinal velocity, and four DOFs for individual wheel spin. In short, all of these models are either simplified from the existing 3-DOF model or slightly modified from the 3-DOF model for their special needs. Since vehicle handling responses are essentially low frequency phenomena, simple models as reviewed above are normally satisfactory in the linear range, that is, at low lateral accelerations (Dixon, 1996).

2.2.2 Multibody Dynamics Models

For many studies related to vehicle dynamics, simple models with a few degrees of freedom are sufficient for obtaining desired information. Examples are the quarter car model for suspension design and ride quality, and the bicycle model for vehicle handling. However, to model vehicle behaviour in detail, multiple degree-of-freedom models are required. For a complicated model with many degrees of freedom, deriving the equations of motion by hand, transforming them into computer codes and solving them numerically have proven to be a lengthy and error prone tasks. Thus, if a more sophisticated model is

required, *multibody dynamics* software packages are of interest (Kortum and Sharp, 1993). The development of these software packages is also stimulated by the desire of vehicle manufacturers to shorten product development cycle, and to reduce the associated cost for hardware prototyping. Software packages such as ADAMS and DADS provide vehicle designers with the necessary tools to construct “virtual prototypes” for rigid-body systems in a realistic manner, therefore lessening the dependence on the lengthy and expensive hardware building and testing.

A multibody dynamics model is able to simulate vehicle behaviour in great detail and with high accuracy. For example, suspension effects in the classic 3-DOF handling model are represented by parameters such as roll steer and roll camber coefficients, without taking into account the design details. In contrast, a multibody dynamics program is capable of modeling the characteristics of a particular suspension system from the geometrical dimensions of linkages and the non-linear behaviour of springs and dampers. Sayers and Mousseau (1990) demonstrated an application of a multibody dynamics software, known as AUTOSIM which models a passenger car with 10 degrees of freedom, six DOFs for vehicle body and one DOF for vertical motion of each wheel. Wong and El-Gindy (1986) carried out a simulation study of the handling of commercial articulated vehicles using the Phase 4 model, which was developed by the University of Michigan Transportation Research Institute (UMTRI) in 1980s. This multibody dynamics model incorporates up to 71 DOFs, dependent upon tractor-trailer configurations. These multiple degree-of-freedom models, of course, must be solved numerically.

2.3 Tire Models

As noted previously, the motion of a ground vehicle is mainly controlled by the forces and moments on the tire-ground interface. When a vehicle is in straight-line motion, the relation between the longitudinal force and slip of the tire, to a great extent, determines the acceleration and braking performance of the vehicle. In a turning manoeuvre of an all-wheel-drive vehicle, its behaviour is primarily influenced by the longitudinal and lateral forces developed by tires under combined longitudinal slip and slip angle.

While the mechanics of pneumatic tires on hard surfaces has been investigated extensively over the years, the interaction of the tire with deformable terrain has been subjected to systematic study only in recent years. In the following, the current state of the art of the mechanics of tires on road and off-road will be reviewed.

2.3.1 Empirical Models

For a tire on a hard surface, a typical relationship between tire lateral force F_y and slip angle α is shown in Figure 2.3.1-1. A curve like this is normally obtained from curve fitting of measured raw data. Similar to the characteristics of longitudinal force F_x versus slip i , initially the lateral force increases with respect to slip angle, and gradually approaches the friction limit of the tire-ground interface.

Figure 2.3.1-1 Tire lateral force F_y versus slip angle α
(Milliken and Milliken, 1995)

For simulation purpose, it is useful to normalize tire lateral force by tire normal force F_z , so that the ordinate becomes F_y/F_z and the maximum value is equivalent to the coefficient of friction between the tire and the ground. Usually the curve shown in Figure 2.3.1-1 may be represented by a polynomial given below:

$$F_y = c_0 + c_1\alpha + c_2\alpha^2 + c_3\alpha^3 \quad (2.3.1-1)$$

where c_i = coefficients of polynomial, some of which may be expressed as functions of tire normal force

Equation (2.3.1-1) is easy to fit and to use. However, it is an empirical equation and does not provide a correlation between tire lateral force and physical parameters of the tire and the ground. In other words, the polynomial is only useful for a particular test condition, and important parameters such as tire inflation pressure, and frictional coefficient of tire-ground interface are not included.

Figure 2.3.1-2 Magic formula, (a) Sine version, (b) Cosine version
(Pacejka and Besselink, 1997)

A popular empirical model is the so-called Magic Formula. This model was initially developed by Pacejka and Bakker (1991), and its development is still continuing. Based

partly on physical insight into the tire force generating mechanism and partly on experimental data, this model is capable of describing steady-state tire force and aligning torque under pure braking, driving, or sideslip. The latest version includes the prediction of tire forces and moments under a combined longitudinal slip and slip angle, such as braking in a turn. The model, according to the 1997 version (Pacejka and Besselink, 1997), includes two fitting equations. They are sine version and cosine version as shown in Figure 2.3.1-2. The former fits data of tire longitudinal or lateral force whereas the latter is for pneumatic trail or aligning torque. Mathematically, they are given by

Sine version:

$$y = D \sin\{C \arctan[Bx - E(Bx - \arctan Bx)]\} \quad (2.3.1-2)$$

$$Y(X) = y(x) + S_v \quad (2.3.1-3)$$

$$x = X + S_h \quad (2.3.1-4)$$

where Y = output variable, longitudinal force or lateral force

X = input variable, longitudinal slip or slip angle

B = stiffness factor

C = shape factor

D = peak value

E = curvature factor

S_h = horizontal shift

S_v = vertical shift

Cosine version:

For pneumatic trail t_p ,

$$y = D \cos\{C \arctan[B\alpha_t - E(B\alpha_t - \arctan B\alpha_t)]\} \cdot c_\alpha \quad (2.3.1-5)$$

$$Y = y = t_p \quad (2.3.1-6)$$

$$\alpha_t = \alpha + S_h \quad (2.3.1-7)$$

where Y = output of pneumatic trail

c_α = cosine of α (protected against singularity)

For aligning torque M_α ,

$$y = D \cos[\arctan B\alpha_r] \cdot c_\alpha \quad (2.3.1-8)$$

$$Y = y = M_{\alpha} \quad (2.3.1-9)$$

$$\alpha_r = \alpha + S_h \quad (2.3.1-10)$$

where Y = output of aligning torque

Magic Formula is a curve-fitting technique. Although it was developed for characterizing tire performance on paved surfaces, it appears that the same approach may be followed to characterize tire performance on deformable terrain. However, Magic Formula can not be used to predict forces and moments for new tires without experimental data.

For tires on deformable terrain, Metz (1993) proposed a semi-empirical model for lateral behaviour of off-road tires. Based on a review of available experimental data, Metz indicated that *cornering stiffness* and lateral force capability of a tire in off-road operations are approximately half of those of the same tire on paved surfaces. The cornering stiffness C_{α} is defined as the initial slope of the curve of tire lateral force $F_{y\alpha}$ with respect to slip angle α , that is $C_{\alpha} = dF_{y\alpha}/d\alpha$ at $\alpha = 0$. In off-road operations, the magnitude of C_{α} is reduced because in addition to tire lateral deformation, lateral shear displacement takes place. Furthermore, because the shear strength of soil is usually less than the friction between a tire and a dry pavement, the maximum lateral force is reduced. Since the relation between shear stress and shear displacement for deformable terrain is usually of an exponential form, Metz postulated that the relationship between lateral force and slip angle is also an exponential function. This relation was given in a non-dimensional form, with tire lateral force normalized by tire normal force. The combination of the exponential equation and tire lateral deformation provides a means for estimating tire response delay. Metz did not explicitly include terrain properties and tire longitudinal force in his model, and the model was essentially based on experimental data of agricultural tires.

2.3.2 Analytical Models

On a hard ground, the tire force generating mechanism depends on the available coefficient of friction between the tire and the ground. The tire-ground interaction is either adhesive or sliding. However, the situation becomes complicated when a tire is operated on a soft, deformable terrain. Terrain properties have significant effects on tire

behaviour, including tire sinkage, resistance force, longitudinal force and slip, and lateral force and slip angle. There are models developed in recent years describing tire behaviour on deformable terrain based on the assumption of soil properties being homogeneous.

The shear stress-shear displacement equation widely acceptable for predicting the shear stress on a tire-terrain interface takes the following form:

$$\tau = [c + p_n \tan \phi_s] (1 - e^{-j/K}) \quad (2.3.2-1)$$

where τ = shear stress
 c = apparent cohesion of terrain
 p_n = normal pressure
 ϕ_s = angle of internal shearing resistance
 j = shear displacement
 K = shear deformation modulus

This equation was used since the 1950's by Bekker (1956, 1960, and 1969) in the studies of off-road vehicle mobility. More recently, based on the shear stress-shear displacement relationship, Wong and Preston-Thomas (1988) investigated the tractive performance of tracked vehicles with independent suspensions. Chen (1993) and Jastremski (1994) applied the shear stress equation to predict tractive force for different sizes of off-road tires on deformable terrain. Equation (2.3.2-1) can also be used to predict the lateral force and slip angle relationship for tires on deformable terrain. The difference between many tire models is the method of computing the shear displacement j , and the normal pressure distribution p_n . It is noted that the shear displacement j is generated by longitudinal slip i , or slip angle α , or both.

Schwanghart (1968) considered an undriven off-road tire with deep treads rolling on soft terrain. He postulated that the total lateral displacement, composed of a component of tread wall deformation η , and a component j , for soil deformation and shearing, equals the product of the tangent of slip angle α and the longitudinal coordinate x along the contact region, as shown in Figure 2.3.2-1. The equation governing this relationship can be written as

$$j_y + \eta_y = x \tan \alpha \quad (2.3.2-2)$$

Figure 2.3.2-1 Lateral shear displacement and tire deformation, Schwanghart's model

Grecenko (1969), however, included both longitudinal slip i and slip angle α to model the behaviour of an off-road tire subject to a combined longitudinal and lateral force. The longitudinal shear displacement j_x is represented by an expression similar to that for a rigid track. That is, $j_x = xi$ where x is the longitudinal coordinate along the contact region. Because of the existence of longitudinal slip, the lateral shear displacement j_y is given by the following equation.

$$j_y = (x - j_x) \tan \alpha \quad (2.3.2-3)$$

Figure 2.3.2-2 Shear displacement, Grecenko's model

Grecenko introduced the soil displacement gradient u_d , which is a constant along the contact length. It is defined as $u_d = dj/dx$ and is given by

$$u_d = \sqrt{\left(1 - \frac{j_x}{x}\right)^2 \frac{j_y^2}{(x - j_x)^2} + \frac{j_x^2}{x^2}} \quad (2.3.2-4)$$

The resultant shear displacement j is calculated by $j = u_d x$, and is illustrated in Figure 2.3.2-2. Grecenko assumed that the direction of resultant shear force is opposite to the direction of resultant shear displacement j .

Figure 2.3.2-3 Shear displacement and tire deformation,
Crolla and El-Razaz's model

Crolla and El-Razaz (1987) confirmed Grecenko's theory that the phenomenon of tire-terrain shearing is isotropic. In other words, the shearing behaviour in longitudinal direction is equivalent to that in the lateral direction. Separating the longitudinal and lateral components, shear displacement and tire tread deformation terms are expressed as (j_x, j_y) and (η_x, η_y) , respectively. Figure 2.3.2-3 shows the resultant shear displacement j and the resultant tire deformation η with respect to contact coordinate x . By formulating two shear stress-shear displacement equations, and two equations for the sum of shear displacement and tire deformation in the x -axis and y -axis, variables (j_x, j_y) and (η_x, η_y) are solved. They further assumed that the resultant shear displacement j is the vector sum of shear displacements j_x and j_y .

$$j = \sqrt{j_x^2 + j_y^2} \quad (2.3.2-5)$$

In addition to consider the shear stress, Schwanghart's model included the bulldozing effect for tires with deep sinkage. This bulldozing force was the lateral force acting on tire sidewall from the soil. He predicted this bulldozing force by applying the Reece equation (Hettiaratchi et al., 1966). Since in the model tire sinkage was assumed to be significant, the normal pressure distribution on the tire-terrain interface was modeled by two distinct parabolic functions along the wheel plane. On the contrary, Grecenko's model assumed tires with shallow sinkage, rectangular contact area, and constant normal pressure. Del Rosario (1984), according to Crolla et al. (1987), improved Grecenko's model by assuming the normal pressure distribution to follow Bekker's pressure-sinkage equation given below.

$$p_n = \left(\frac{k_c}{b} + k_\phi \right) z^n \quad (2.3.2-6)$$

where p_n = normal pressure
 k_c = pressure-sinkage parameter
 k_ϕ = pressure-sinkage parameter
 n = pressure-sinkage exponent
 b = width of rectangular plate, or radius of circular plate
 z = sinkage

Del Rosario's (1980) model included three force mechanisms to describe the tire-terrain interaction: rubber-terrain shear stress, internal soil shear stress, and bulldozing force. Nevertheless, an important variable, longitudinal slip i was ignored.

In summary, both Schwanghart's and Del Rosario's models did not consider tire longitudinal slip i . In other words, they ignored the effect of tractive (or braking) force on the development of tire lateral force, regardless the truth that the tractive (or braking) force exists in an all-wheel-drive vehicle. Although lateral bulldozing force was taken into account, both models are only suitable for an undriven deep-lug tire operating on soft terrain with significant sinkage. In reality, due to tire sinkage and compaction resistance,

a vehicle may be immobilized on soft terrain if some of the wheels are not driven, or tire sinkage exceeds 30% of the wheel radius (Bekker, 1985). Since most off-road vehicles are all-wheel-drive, and tires are subject to longitudinal slip at all times, Schwanghart's and Del Rosario's tire models therefore are not suitable for the handling study of an all-wheel-drive, off-road vehicle. On the other hand, Grecenko's model assumed that tire-terrain contact region is a horizontal, rectangular area. This implied that tire inflation pressure is relatively low, and the terrain is fairly firm. In this case, the tire is operated in elastic mode according to the criteria presented by Bekker (1960), or Wong (1989, 1993). However, for a low-pressure tire subject to a lateral force, tire lateral deformation due to tire elasticity is considerable, and should be taken into account. Moreover, the direction of shear stress as well as the induced shear force may not coincide with that of shear displacement. The direction can be determined according to the principle of Coulomb friction. That is, the resultant frictional force is in a direction opposite to the relative motion between the tire and the terrain. Both Grecenko's, and Crolla and El-Razaz's models exclude the lateral bulldozing force that imply both models are suitable for tires with shallow sinkage. Based on the above review, the development of a new model for tire-terrain interaction under combined longitudinal slip and slip angle is of importance, and hence forms part of the research presented in this thesis.

Chapter 3

Objectives of the Current Research

3.1 Work Statement

As was shown in Chapter 2, to study the handling behaviour of off-road vehicles on unprepared terrain, an adequate understanding of the mechanics of tire-terrain interaction under combined longitudinal slip and slip angle is of prime importance. To provide a physical basis for the development of a realistic model for the cornering behaviour of tires on unprepared terrain, experimental data on longitudinal and lateral forces for tires under combined slip conditions are required.

Schwanghart (1968) validated his model using a set of tire lateral force data, obtained from agricultural tires operating on sandy clay soil. Grecenko (1969), and Crolla et al. (1987) used the same set of data to verify their tire models. Schwanghart (1981) then published a second set of tire lateral force and rolling resistance data for different types of terrain. These data were obtained with a mobile tire tester, for 7.50-18 and 6.50-20 tractor tires. Metz (1993) presented the handling characteristics of a 4WD front-end loader using this set of lateral force data. However, all Schwanghart's data were obtained from undriven or free rolling tractor tires with deep lugs. Kutzbach (1994) also obtained data

on the vertical dynamic behaviour, as well as the longitudinal and lateral forces under combined slip conditions for agricultural tractor tires. Since different patterns of tires have different lateral characteristics, data for agricultural tires are not necessarily applicable to off-road tires (French, 1985) used in high-speed, military armoured wheeled vehicles, for instance. Consequently, a set of experimental data for off-road tires used in military vehicles is desirable for the studies of the handling of multi-axle, all-wheel-drive, off-road vehicles.

In Section 2.3, various off-road tire models for predicting the lateral behaviour of tires were discussed. Among them, were empirical models entirely based on the curve fitting of experimental data. Empirical models are simple to use and would be useful in estimating the performance of tires with design features similar to those that have been tested under similar operating conditions. It should be noted, however, that empirical relations could not be normally extrapolated beyond the conditions upon which they were derived. Therefore, empirical models have inherent limitations. The review of analytical models showed that the methods for determining the resultant shear displacement j could be further improved. The proposed method for computing j will consider a number of factors such as tire-terrain contact configuration, tire lateral deformation, contact point trajectory, and the analysis of j in a three-dimensional space.

For vehicle handling studies, it is desirable to develop a more general off-road tire model, which is capable of predicting tire behaviour over a wide range of operating conditions. The model should take into account the basic design features of the tire, such as its dimensions, normal load, and inflation pressure, operating conditions such as longitudinal slip and slip angle, as well as terrain behaviour such as the pressure-sinkage relation, and tire-terrain shearing characteristics.

Many studies of vehicle handling in the past few decades focused on road vehicles, including passenger cars, utility vehicles, and articulated vehicles such as tractor-trailer combinations. Handling problems of vehicles with multi-axle, all-wheel-drive operating on deformable terrain has not yet been systematically and thoroughly studied. Consequently, the development of a laboratory test facility for measuring tire forces under combined slip conditions and on deformable soils, the formulation of a model for predicting the lateral behaviour of driven tires on deformable terrain, and the

establishment of a simulation model for the handling of multi-axle, all-wheel-drive, off-road vehicles, are of practical significance. It is believed that the proposed off-road tire model and the handling simulation model described later in this thesis will provide vehicle designers with a realistic engineering tool to evaluate off-road vehicle handling behaviour and possibly to improve its handling performance.

3.2 Objective

The objective of this research is to develop an engineering framework for the study of the handling characteristics of multi-axle, all-wheel-drive, off-road vehicles, particularly with respect to armoured wheeled vehicles operating on soft terrain.

The major tasks of the research are given below:

- 1) To develop a laboratory test facility to measure tire longitudinal and lateral forces under combined longitudinal slip and slip angle in a soil bin.
- 2) To test an off-road tire designed for armoured wheeled vehicles under various combinations of longitudinal slip, slip angle, normal load, and inflation pressure.
- 3) To develop a mathematical off-road tire model to predict the longitudinal and lateral forces based on the design features of the tire, combined slip, rigid or elastic operating mode, as well as terrain properties.
- 4) To develop a multibody dynamics vehicle model having many DOFs, with the proposed tire model as part of the input, to simulate and animate the handling characteristics of multi-axle, all-wheel-drive, off-road vehicles. To demonstrate the utility of the off-road vehicle handling model, a six-wheel-drive, front-wheel-steering armoured wheeled vehicle will be used as an example.

Chapter 4

Development of a Laboratory Tire Test Facility

4.1 Review of Existing Tire Test Facilities

There are existing tire test facilities for measuring the lateral behaviour of tires. A mobile tire tester operating on paved roads was developed by Cornell Aeronautical Laboratory, Inc. (CAL) and commissioned in 1954 (Milliken and Milliken, 1995). Figure 4.1-1 shows the test rig mounted at the rear of a truck. A test tire can be loaded up to 13.34 kN (3,000 lbs), and camber and slip angles can be implemented up to $\pm 30^\circ$. On-road machines operate under actual road conditions; nevertheless, the data obtained usually show a considerable variation, owing to the operating environment not being closely controlled.

To provide a controlled test environment and to improve test repeatability, laboratory tire test facilities have been developed. Figure 4.1-2 shows the Tire Research Facility (TIRF) of Calspan Corporation (formerly CAL). The facility consists of a high speed, flat belt that simulates road surface, and a tire frame on which the test tire was mounted (Milliken and Milliken, 1995). The belt moves as the drums underneath rotate. The tire can be set to various camber and slip angles. This facility has been operated since 1973, and was used for measuring tire lateral force for both passenger car tires and truck tires. Based on

similar technology, MTS System, Inc. developed a flat surface roadway simulator (Langer and Ballard, 1993). Four flat belts were used so that a full-scale automobile can be tested. Each belt was controlled individually to simulate different wheel spin, steer, and slip. It is noted that both Calspan and MTS flat belt systems have a common feature. The road surface is movable, whereas the wheel center is stationary.

Figure 4.1-1 Mobile tire test rig, CAL (Milliken and Milliken, 1995)

Figure 4.1-2 Tire Research Facility (TRF), Calspan (Milliken and Milliken, 1995)

More recently, the TNO Road-Vehicles Research Institute of the Netherlands developed a tire test trailer (TNO, 1997). The test tire was mounted inside a trailer, which was towed by a truck. Tire forces in x , y , and z directions as well as aligning torque were measured. The slip angle can be varied while the trailer was moving. The acquired data were then fitted by Magic Formula for simulation purposes. Figure 4.1-3 shows the operation of the TNO Delft-Tire test trailer on a wet surface.

Figure 4.1-3 Delft-Tire test trailer, TNO

For the general off-road tire force measurements, many soil bins with dynamometric carriages were developed (Bekker, 1969). Some of them were small-scale facilities for educational and research purposes. Other sophisticated machines were capable of testing full-size wheel assemblies in complex loading conditions. However, for apparatuses specifically designed for measuring tire lateral characteristics, relatively little information on the design has been published. Schwanghart (1968) set up a test rig, as shown in Figure 4.1-4, to measure tire lateral force and rolling resistance under different operating conditions. Slip angles up to 42° can be implemented. Tire longitudinal and lateral forces were measured using the rig, and tire sinkage was recorded. There was no mechanism to

provide longitudinal tractive or braking force, so that the tire was free rolling. The test tire was operated in a soil bin, which contained moisture-controlled sandy clay. The tire tester was later installed in a trailer frame, and towed by a truck for field tests (Schwanghart, 1981). Two tractor tires were used for measurements on various terrains such as gravel, dirt road, meadow, and sandy loam. Tire lateral force and rolling resistance were measured at slip angles up to 30° .

Figure 4.1-4 Schwanghart's off-road tire tester (Schwanghart, 1968)

4.2 Development of TTF97

The off-road Tire Test Facility in the Transport Technology Research Laboratory, Carleton University, referred to as TTF87, was first developed and built in 1987. It was originally designed for measuring the relationship between tractive force and longitudinal slip of off-road tires. Figure 4.2-1 shows the general layout of the complete facility. The facility includes a soil bin, a structural frame, a moving carriage, and a tire frame assembly. The carriage was moved by a roller chain, which was driven by a speed-controlled dc motor. The moving carriage towed the tire frame. A load cell was installed between the tire frame and the carriage so that towing force could be measured. The tire frame was the member on which a test tire assembly was mounted.

Figure 4.2-1 Off-road Tire Test Facility, Carleton University, 1987

Figure 4.2-2 Forced-slip mechanism (Chen, 1993)

A forced-slip mechanism, consisting of a cable (wire rope), a slip pulley, a roller chain, and a set of sprockets was installed, as shown in Figure 4.2-2. This mechanism forces the test tire to develop a predetermined longitudinal slip, and generates an associated tractive

force. One of the important indicators for vehicle mobility studies, the drawbar pull, was obtained by subtracting the towing force from the rear cable force.

The operating principle of the forced-slip mechanism is illustrated in Figure 4.2-3. As shown in the figure, a sprocket with pitch radius r_1 and the tire with rolling radius r_t were mounted on the same shaft, while a second sprocket with pitch radius r_2 and a slip pulley with radius r_s were assembled on the other shaft. Torque was transmitted between the two sprockets through a chain. A cable was wound on the pulley and one end of the cable was anchored on the fixed structural frame. As the tire frame, pulled by the carriage, moves forward, a certain length of the cable was unwound from the pulley. The distance the tire center traveled was equal to the length of the cable released from the pulley. By properly selecting the radii of the two sprockets and the radius of the slip pulley in relation to the rolling radius of the tire, a predetermined tire slip was realized.

Figure 4.2-3 Schematic of forced-slip mechanism, for slipping tire

It is well known, the longitudinal slip i of a tire (wheel) with zero slip angle is defined by

$$i = 1 - \frac{\text{actual forward displacement of tire center}}{\text{theoretical forward displacement of tire center}}$$

The slip i can also be defined in terms of the actual forward speed of the tire center, v_0 , and the theoretical forward speed of the tire center, $r_t \omega_t$ (Wong, 1993). For this special case of zero slip angle, both v_0 and $r_t \omega_t$ are developed along the wheel plane.

$$i = 1 - \frac{v_0}{r_t \omega_t} \quad (4.2-1)$$

where v_0 = actual forward speed of tire center
 r_t = tire rolling radius
 ω_t = tire angular speed

Since the cable is assumed to be inextensible, the tire frame forward speed v_0 is equal to the cable release rate, which is $r_s (r_1 \omega_t / r_2)$.

$$\begin{aligned} i &= 1 - \frac{r_s (r_1 \omega_t / r_2)}{r_t \omega_t} \\ &= 1 - \frac{r_s r_1}{r_t r_2} \end{aligned} \quad (4.2-2)$$

where r_s = slip pulley radius
 r_1 = radius of sprocket on tire axle
 r_2 = radius of sprocket on slip pulley shaft

From equation (4.2-2), it can be seen that slip occurs if $r_s r_1 < r_t r_2$. Since the tire rolling radius r_t was assumed to be a constant, various slip values can be obtained by changing the radii of the sprockets or of the slip pulley, r_1 and r_2 , or r_s . Longitudinal slip values of 0, 2, 5, 10, 20, and 35% were used for this research. When $r_s r_1 = r_t r_2$, slip is zero, and the tire is theoretically in zero slip condition. On the other hand, if $r_s r_1 > r_t r_2$, the longitudinal slip becomes negative. This negative slip can be converted to a longitudinal skid i_s according to the definition (Wong, 1993).

$$\begin{aligned} i_s &= 1 - \frac{r_t \omega_t}{v_0} \\ &= \frac{i}{i - 1} \end{aligned} \quad (4.2-3)$$

Figure 4.2-4 shows the tire in a skidding condition. A skidding tire is subject to a braking force F_{xb} from the ground. The newly developed tire test facility known as TTF97 was capable of accommodating the performance tests of a tire under skidding conditions, if a slight modification of the machine is conducted.

Figure 4.2-4 Schematic of forced-slip mechanism, for skidding tire

The TTF87 was used for measuring the tractive performance of a number of military off-road tires. A set of measurements of an 11.00R16 XL tire was conducted by Chen (1993) to verify tractive force predictions from a finite element tire model. Jastremski (1994) carried out a similar set of measurements for a 12.5R20 XL tire to prove his proposed tire traction model.

Since the TTF87 can not accommodate tests of an off-road tire with a slip angle, it was necessary, in this study, to design a new tire frame assembly so that the longitudinal tractive (or braking) force and lateral force under combined slip conditions can be measured at the same time. The longitudinal force is along the wheel plane, and the lateral force perpendicular to the wheel plane. The longitudinal force is generated by the forced-slip mechanism described above, and the lateral force is developed from a preset slip angle. The new tire frame assembly is subject to an external lateral force owing to the effect of slip angle, and thus requires a side force support. Without this support, the complete tire frame assembly will be twisted when the test tire operates at a slip angle and develops tire lateral force. Moreover, the new tire test facility was fully instrumented.

A computer controlled, high-speed data acquisition system was planned from the beginning of the development.

Figure 4.2-5 Explosion view of tire frame assembly

The specifications of the new TTF97 are given below.

- 1) Adjustable slip angle, up to 30°
- 2) Adjustable longitudinal slip, up to 35 %
- 3) Adjustable longitudinal skid, up to 35 %

- 4) Adjustable normal force, up to 10 kN
- 5) No interference of normal force on horizontal force measurement
- 6) Capable of measuring both longitudinal and lateral forces simultaneously
- 7) Capable of measuring longitudinal displacement of tire center
- 8) Capable of measuring tire sinkage
- 9) Various tire sizes, up to 14.00R20
- 10) Computer controlled data acquisition and data logging

The tire frame assembly was composed of a tire frame, a pulley frame, a payload frame, a side force support, and a wheel-shaft assembly. The complete assembly was towed by the moving carriage along the longitudinal direction of the soil bin. Figure 4.2-5 shows an isometric explosion view of the assembly. The tire frame was the central part of the assembly. The pulley frame, which creates longitudinal slip, was located at the rear of tire frame. The payload frame was fastened securely on top of the tire frame. The total structural weight loaded on the wheel-shaft assembly. Since a sideslip tire develops a lateral force as it rolls along the soil bin, a side force support was mounted to the right hand side of the tire frame to transmit this force to a fixed H-beam.

Figure 4.2-6 Tire frame

The setting of a slip angle is explained as follows. On the tire frame as shown in Figure 4.2-6, there were two horizontal plates each with 16 holes. These predetermined holes when matching with the corresponding mounting studs on the sensor assemblies attached to the wheel-shaft assembly set the slip angle to a desired value. When the shaft was mounted at one particular position, for example at an angle of 20° clockwise from the tire frame y -axis, the tire still rolls along the wheel plane direction as shown in Figure 4.2-7. However, the tire center O was forced to move along the tire frame x -axis. The angle measured from the wheel plane to the x -axis was the slip angle α created by tire frame to the test tire, and $\alpha = -20^\circ$ according to the sign convention (SAE J670e, 1976). The top view of the wheel-shaft mounted on tire frame at $\alpha = -20^\circ$ is shown in Figure 4.2-7.

Figure 4.2-7 Wheel-shaft mounted on tire frame at $\alpha = -20^\circ$, top view

In order to maintain the proper functioning of the forced-slip mechanism when the tire was set at a particular slip angle, an adjustable pulley frame containing a shaft with a slip pulley and a sprocket was constructed, as shown in Figure 4.2-8. The pulley frame was fixed on top of the tire frame, but rotated clockwise at an angle equivalent to tire slip angle. Figure 4.2-9 shows the schematic for testing a tire at a nonzero slip angle with a predetermined longitudinal slip. Before a test operation, the pulley frame was properly adjusted or orientated to match the slip angle setting, so that both sprockets were aligned.

Figure 4.2-8 Pulley frame

Figure 4.2-9 Adjustable pulley frame, for implementing forced-slip

To be able to vary the normal load on the tire, a payload frame for housing dead weights was constructed, and fastened on top of the tire frame. The mass center of the payload frame was shifted forward to balance the overhang weight at the rear end of the tire frame assembly. A wire rope hoisting device was installed, so that the complete assembly, together with the payload frame, could be lifted after each test for soil preparation. Figure 4.2-10 shows the layout of the payload frame.

Figure 4.2-10 Payload frame

As noted previously, to keep the tire frame assembly running smoothly, it was required to design a mechanism to support the lateral force developed by the sideslip tire. The side force support, shown in Figure 4.2-11, includes two moving units, and two stationary ones that connected with each other by a plate. A moving unit consists of a precision-machined circular bar, and two pillow blocks for supporting the bar. This unit was fastened to the right hand side of the tire frame. Each stationary unit was a bracket assembly with a group of three rollers. These rollers support lateral force in either direction, and transmit the force to a W5×16 wide-flange beam. The design has taken into account the slip sinkage effect of tires. A bushing was installed in each stationary unit to allow the bar move vertically through it without interference. As the test tire together with tire frame moves forward, tire sinkage gradually increase. The two moving units, the bars go down with the assembly, but the stationary units remain at the same vertical position, and the rollers keep in contact with the beam. In other words, the assembly supports the lateral force developing from the sideslip tire and acting on the tire frame, while providing no resistance to tire slip sinkage. Furthermore, since the complete

side force support has two units of bar-roller devices, the support assembly can also provide reactions to external moments generated by the tire on the horizontal plane.

Figure 4.2-11 Side force support

Figure 4.2-12 Wheel-shaft assembly

The wheel-shaft assembly, on which the test tire was mounted, connects with two sensor assemblies on both ends, as shown in Figure 4.2-12. These two x-y load cells were used to measure the longitudinal and lateral forces acting on the tire. The tire with rim was mounted on the hub of the shaft. A 54-tooth sprocket, to the right of the tire, was fastened to the same shaft. This sprocket takes the input torque from the chain on the forced-slip mechanism of the pulley frame described previously.

One of the design challenges of the TTF97 was to isolate tire longitudinal and lateral forces on the horizontal plane from the vertical (normal) force. The problem was that the two sensor assemblies of the x-y load cells must support the total weight of the tire frame assembly, and at the same time, each load cell mounted inside the sensor assembly must measure the tire horizontal force. The problem was overcome by introducing an inverted washer of a thrust bearing assembly. The inverted bearing washer has a ground surface with hardness HRC 58. As the tire was loaded vertically, the flat surface of the washer contacts with bearing balls thus provides negligible constraint to frictional force in the horizontal direction. This allows the sensing pins of the wheel-shaft apply horizontal force to the x-y load cells without interference. Figure 4.2-13 shows the sensor assembly fastened to the tire frame.

Figure 4.2-13 Sensor assembly

The complete assembly of the newly designed TTF97 is shown in Figure 4.2-14. The photograph depicts the relative position of each subsystem, shows the wheel-shaft at a slip angle of -20° , and the pulley frame orientates 20° clockwise in order to properly align the sprocket/chain drive. The photo also shows that the forced-slip mechanism was in action. A cable, guided by three rollers, winds around the 20% slip pulley. As the tire frame assembly moves forward, the tensioned cable creates a torque to the pulley shaft, drives the sprocket to pull the roller chain. The chain force provides a driving torque to the test tire so that the tire subjects to a tractive force at 20% slip.

Figure 4.2-14 TTF97 in operation, at -20° slip angle

4.3 Instrumentation

The TTF97 was fully instrumented. There were six sensors with a total of eight channels of signals. All channels were analog; thus the A/D conversion was required. The channel numbers and models of sensors are listed in Table 4.3-1.

The sinkage of the tire was monitored by a rotary potentiometer driven by a timing belt vertically attached to the tire frame. The signal was fed into channel 0 through a specially designed circuit. The circuit was an input buffer made of two operational amplifiers and precision resistors. The function of this circuit was to prevent the sinkage signal from oscillation. A strain gauge amplifier conditions load-cell signals of channels 1 to 6. The horizontal displacement of the moving carriage, or the tire along the tire frame x-axis, was monitored by the second potentiometer, driven by a timing belt fixed to the carriage. This signal was fed into channel 7. All signals from the eight channels were then input, through a connection box, to a data acquisition (DAQ) board. The DAQ board was installed in one of the ISA slots of a Pentium 133 MHz computer. Each sensor together with the complete DAQ system was calibrated before assembling to the TTF97. Figure 4.3-1 is a schematic diagram showing the key elements of the instrumentation.

Table 4.3-1 Eight channels of analog signal

Channel	Description	Capacity	Model	Manufacturer
0	Potentiometer, sinkage	10 turns	3540S-1-503	Bourns, Riverside, CA
1	Load cell, towing force	8,896 N	3132-2K	Lebow, Troy, MI
2	Load cell, cable force	8,896 N	2K	Aries, S. Korea
3	Load cell, x-axis left	4,448 N	6443	Lebow, Troy, MI
4	Load cell, x-axis right	4,448 N	6443	Lebow, Troy, MI
5	Load cell, y-axis left	2,224 N	6443	Lebow, Troy, MI
6	Load cell, y-axis right	2,224 N	6443	Lebow, Troy, MI
7	Potentiometer, x-displ.	10 turns	3540S-1-502	Bourns, Riverside, CA

Figure 4.3-1 Schematic diagram of instrumentation

The data acquisition from the eight channels was by a means of continuous scanning process, starting from channel 0, and ending at channel 7. A rotary encoder with 2,000 pulses per revolution, fixed to the center of the wheel-shaft triggers each scan, and also acts as a tire angular displacement sensor. Once the tire rolls, the encoder generates digital pulses to change the onboard counter reading and to control the DAQ board to begin a scan. After the scan was completed, the DAQ control program compares the new counter reading from the board with the previous one. If these two readings were different, then the tire has rotated to a new position. Consequently, the next scan was performed. Figure 4.3-2 shows the flow chart of the scanning loop. In addition to multiple-channel scan, the DAQ program, written in C language, was capable of carrying out single-channel DAQ, sensor calibrations, daily shunt calibrations, and crosstalk effect measurements.

Figure 4.3-2 DAQ scanning loop

The DAQ board was a multifunction, input/output board, with 12-bit ADC resolution and the maximum data acquisition rate of 50,000 samples per second. The selection of the sampling rate depended on the highest frequency component existing in the system, and the time duration to complete a scanning loop from channel 0 to 7. It was found that at a high longitudinal slip such as 35%, the drawbar pull fluctuated. The frequency of this signal can not be clearly identified unless the DAQ rate was set to 48,000 samples per second. Figure 4.3-3 shows the drawbar pull developed by the tire-terrain interaction under a given condition. It can be seen that even at steady state, the drawbar pull fluctuates considerably, and two frequencies of approximate 3.6 and 15 Hz were identified. The dominant frequency of 3.6 Hz was due to tire lugs engaging the soil.

Figure 4.3-3 Dominant frequency due to tire lugs engaging soil

Figure 4.3-4 Examination of crosstalk, x-y sensor

The calibration of all load cells was conducted on a tensile test machine. That was equivalent to the method of using two transducers connected in series (Dally et al., 1993). The reference load cell on the machine was calibrated regularly by standard dead weights.

The x-y sensor used in this experiment has two separate channels, the radial (x-axis) and side (y-axis) directions, but the two strain gauges were installed on the same piece of steel ring. Therefore, there was a possibility that loading in one direction may affect the reading from the other channel. It was the so-called *crosstalk* effect between two channels, and must be examined in advance. The crosstalk was examined by calibrating one of the x-y sensor channels by tensile loading, while applying the shunt resistor to the other. The shunt resistor provided by the manufacturer commonly corresponds to 70% transducer capacity. The output variation of the channel with shunt was regarded as the crosstalk effect and was closely inspected. Figure 4.3-4 shows that applying and recording a tensile force to one direction, while simultaneously measuring the signal variation from the other channel with shunt resistor activated.

Figure 4.3-5 Crosstalk effect, x-channel of x-y sensor

Figure 4.3-5 demonstrates the crosstalk effect of x-channel. The figure shows the result of loading y-channel starting from zero, and measuring shunt variation of x-channel. The loading on y-channel was conducted three times to the positive direction, and three times to the negative. The measurements show that the maximum variation in x-channel shunt voltage occurs in “(+) Loading#2”. The variation is computed as below.

At y-channel loading = 0.0 kN, x-channel shunt voltage = 3.029 volts
 At y-channel loading = 2.19 kN, x-channel shunt voltage = 3.016 volts

The maximum shunt variation in x-channel is 0.013 (3.029-3.016) volt, and corresponds to 0.43% (0.013/3.029) of the initial shunt voltage. Considering the scale factor 1,042 N/volt of x-channel, this 0.013 volt equals a force of 13.5 N (3.04 lbs). The crosstalk specification is “less than or equal to 1%”.

Similarly, Figure 4.3-6 shows the crosstalk of y-channel on the same x-y sensor. The maximum shunt variation in y-channel occurs in “(+) Loading#2”. The following data were identified from the test.

At x-channel loading = 0.46 kN, y-channel shunt voltage = 3.308 volts
 At x-channel loading = 4.34 kN, y-channel shunt voltage = 3.334 volts

The maximum value of shunt variation in y-channel due to crosstalk is 0.026 volt, which corresponds to 0.785% of the initial shunt voltage. Since the scale factor of y-channel is 522 N/volt, the force variation in y-channel is 13.6 N (3.05 lbs) due to crosstalk effect.

Figure 4.3-6 Crosstalk effect, y-channel of x-y sensor

4.4 Identification of System Characteristics

Before conducting experiments, there were several uncertainties to be clarified. Among them, the horizontal component of chain force on longitudinal force readings from x-y sensors, and the sprocket/chain efficiency must be identified.

For tire force measurements with nonzero slip angles, the readings of the longitudinal force from x-y sensors were the sum of the drawbar pull (tractive force– motion resistance), and the horizontal component of chain force. Because the sprocket/chain setting for the test tire at a nonzero slip angle and at zero slip angle were the same, the chain effect could be established from the zero slip angle condition. Figure 4.4-1 (a) shows a schematic diagram of the tire frame with the tire at the zero slip angle position.

Figure 4.4-1 (a) Free-body diagram, tire frame assembly, top view,
(b) Free-body diagram, wheel-shaft assembly, top view

For the tire frame assembly at steady state, the force equilibrium is given below.

$$F_t + F_{dbp} - F_c = 0 \quad (4.4-1)$$

where F_t = pulling force on tire frame
 F_{dbp} = drawbar pull
 F_c = slip/skid cable tension force

However, from the free-body diagram of the wheel-shaft assembly as shown in Figure 4.4-1 (b), the force balance can be expressed as the following form.

$$F_{dbp} - F_{x1} - F_{x2} - F_{ct} = 0 \quad (4.4-2)$$

where F_{x1} = longitudinal force measured by x-y sensor, left
 F_{x2} = longitudinal force measured by x-y sensor, right
 F_{ct} = horizontal component of chain force

From equations (4.4-1) and (4.4-2), the horizontal force component F_{ct} due to chain tension is derived.

$$F_{ct} = F_c - F_t - F_{x1} - F_{x2} \quad (4.4-3)$$

Forces F_c , F_t , F_{x1} , and F_{x2} were measured by load cells, the chain effect was thus determined. However, for nonzero slip angle cases, equation (4.4-3) was not applicable because forces F_c and F_t were not on the wheel plane. To overcome this problem, a number of measurements from the zero slip angle condition were conducted. The input torque to the slip pulley equals the cable force F_c multiplied by the radius of the slip pulley r_s . Simultaneously, the force component F_{ct} was computed from measured data, according to equation (4.4-3). By changing different sizes of slip pulley, an empirical equation was established. The equation defines the relationship between the input torque to the slip pulley shaft and the horizontal chain force F_{ct} .

$$y = 0.407 + 0.007x \quad (4.4-4)$$

where x = input torque to slip pulley shaft
 y = horizontal component of chain force, $y = F_{ct}$

For tire tests with nonzero slip angle, the input torque was known since the cable force F_c was measured by a load cell, and the slip pulley radius was known. Using equation (4.4-4), the horizontal component of chain force F_{ct} was obtained accordingly. Figure 4.4-2 shows the experimental data and the curve-fit straight line.

Figure 4.4-2 Experimental data for identifying chain effect, and fitted curve

The major energy loss of the tire frame assembly was attributed to the sprocket and chain system; roller and bearing frictions were less significant. Since the torque transmitted to the tire was through a roller chain and two sprockets, the sprocket/chain efficiency η_s , was important in computing tire tractive force. The relation between the tractive force and the sprocket/chain system is given below.

$$F_x = \eta_s \frac{\xi_p F_c r_s}{r_t} \quad (4.4-5)$$

where F_x = tractive force
 η_s = sprocket/chain efficiency
 ξ_p = sprocket gear ratio (r_1 / r_2)
 r_s = slip pulley radius
 r_t = tire rolling radius

An experiment was setup to investigate the sprocket/chain efficiency. The tire frame assembly was lifted in the air so that the tire had no contact with the soil. Along the wheel plane, a cable was fastened around tire circumference, and standard calibration weights were applied to the cable's loose end. The sprocket on the wheel-shaft was driven by the

roller chain, which was connected to the sprocket on the slip pulley shaft. On the pulley shaft, a cable was wound around the slip pulley. This cable was guided by three rollers, and pulled by a winch. A load cell was implemented near the winch to measure the slip cable tension. The experimental scheme is shown in Figure 4.4-3.

Figure 4.4-3 Experimental setup for identifying sprocket/chain efficiency

Figure 4.4-4 Measurement data for sprocket/chain efficiency test

The cable force was recorded when the winch was rotated to lift the standard weights. The sprocket/chain efficiency was thus defined as the ratio of the weight-induced moment divided by the input torque to the tire. This input torque was the torque applied to the slip pulley shaft multiplied by the appropriate sprocket gear ratio. In fact, the efficiency also included frictional effects from three guiding rollers and all bearings. Figure 4.4-4 shows the relationship between the input torque and the output moment. The slope of the straight line is the sprocket/chain efficiency η_s .

Some dynamic characteristics of the test tire and the tire frame assembly were also investigated. The nonrolling dynamics of the tire was examined by drop tests (Wong, 1993). A number of drop tests were conducted for tire inflation pressures of 137.9 kPa (20 psi) and 344.7 kPa (50 psi), respectively. These tests determined that the frequency of damped oscillation, in the given conditions, was approximately 1 Hz. For the tire frame structure, an impulse excitation was given to the structure in the x , y , and z -axis, respectively. The time histories of the accelerations of the structural response in three directions were recorded, and then were analyzed by Fast Fourier Transform. Frequencies for each direction were identified by viewing the Power Spectral Density, and for the x , y , and z -direction the lowest frequencies from the impact tests were 324, 104, and 368 Hz, respectively. The natural frequency of a low profile (shear-web type) load cell, for example the one with a capacity of 8,896 N (2,000 lbs) used to measure the towing force, was in the range of 20 to 25 kHz. An error analysis shows that if the natural frequency of a transducer is 5 to 10 times higher than the frequency of the forcing function, the error is within reasonable limits (Dally et al., 1993). This concludes that even the structure resonance in high frequencies in the x - or y -axis has negligible effects to the measurements of tire force on ground plane. Appendix C describes the details of drop tests and impact tests.

Chapter 5

Experimental Study of Tire Characteristics Under Combined Longitudinal Slip and Slip Angle

5.1 Tire Force Measurements

The objective of the experimental study is to measure and characterize the relationships between tire longitudinal and lateral forces, and longitudinal slip and slip angle. Tire data are presented at various longitudinal slips and slip angles under different tire inflation pressures. These tire data include normalized drawbar pull versus longitudinal slip, normalized lateral force versus slip angle, normalized drawbar pull versus normalized lateral force, and tire sinkage versus slip angle and longitudinal slip at different normal forces. The complete set of experimental data gives fundamental features of off-road tire behaviour operating on dry sand. The data also provide the necessary information for the development and validation of a model for the lateral behaviour of off-road tires on deformable terrain. Since soil properties were necessary as part of the input parameters for tire force predictions, coefficients of the pressure-sinkage and the shear stress-shear displacement relationships for such a terrain were also measured using a Bevameter.

Tire force measurements were conducted using the newly designed laboratory tire test facility TTF97 installed in the Transport Technology Research Laboratory, Carleton

University. Tire lateral force was directly measured by two x-y load cells. The drawbar pull F_{dbp} is computed from equation (4.4-2).

$$F_{dbp} = F_{x1} + F_{x2} + F_{ct} \quad (5.1-1)$$

where F_{x1} and F_{x2} were the measured longitudinal forces from two x-y sensors, and F_{ct} was obtained from equation (4.4-4). The tractive force, if desirable, was computed from the recorded data of cable force F_c and the sprocket/chain efficiency η_s , according to equation (4.4-5).

Table 5.1-1 Tire characteristics (Michelin, 1997)

Physical Parameters	Units	Values
Free Radius	[m]	0.517
Free Section Width	[m]	0.333
Tread Width	[m]	0.235
Revolutions / km		319.45
Rolling Circumference	[m]	3.130
Nominal Mass	[kg]	42.5
Mass Moment of Inertia	[kg · m ²]	7.8
Inflation Pressure, Road	[bar]	3.5
Inflation Pressure, Sand	[bar]	1.1

The test tire was a Michelin 12.5R20 XL. Its characteristics are given in Table 5.1-1. One difficulty was the determination of tire rolling radius r_r under zero slip condition. A number of tests were conducted with tire operating under a free-rolling condition. That was, disengaging the forced-slip mechanism, and setting the tire at zero slip angle. However, the test results confirmed that a towed, free-rolling tire operated on a soft terrain was in fact skidding. The idea to measure tire travel distance, and then divide the distance by angular displacement proves to be incorrect for tires operating on soft terrain. It was therefore to accept the definition of rolling radius from ANSI/SAE Standards (SAE, 1995). In the standard, the rolling radius is defined as “The distance advanced per revolution of the driving axle of a traction device under the specified zero condition,

divided by 2π ". The zero condition refers to a tire operating on a nondeformable surface, and with zero net traction. Based on the data provided by the manufacturer, the rolling radius of the test tire 12.5R20 XL was taken to be 0.498 m.

In order to obtain a set of comprehensive tire force data, a number of parameters were varied. Table 5.1-2 lists the parameter settings for the test tire. For each combination, three tests were carried out to ensure the reliability and repeatability of the test data. Before each test run, the soil was plowed, leveled, and smoothed.

Table 5.1-2 Variations of parameter setting

Parameters	Units	Settings						
Normal Force	[kN]	7.01	8.12	9.23				
Inflation Pressure	[bar]	1	2	3				
Slip Angle	[deg]	0	2	4	6.5	10	15	20
Longitudinal Slip	[%]	0	2	5	10	20	35	

For each tire force measurement, the initial part of the recorded data indicates that the tire was in transient. The objective of this experiment was to obtain the combined tire force at steady state. Thus, for each test, it was required to select steady-state data from the complete time history record. Figure 5.1-1 shows typical time history of the resulting drawbar pulls for three measurements, at 10, 20, and 35% slips. In each measurement, the tire frame was controlled to move from the starting point to the end of the soil bin. Data scanning process was triggered once the encoder rotates. The total number of scan was between 1,000 and 1,200, dependent on the choice of longitudinal slip. The continuous scanning triggered by the encoder was completed within 18 seconds.

For data processing, steady-state data was selected in the range between 41% and 81% of the total scan number. Viewed from the time history in Figure 5.1-1, data within 41% to 81% of the total scan correspond to data within 7.4 to 14.6 seconds. In this range, the data fluctuation was less than 15% of the mean value. Thus the data was considered to represent the steady-state value. The figure shows that the data before the 41% (7.4 seconds) was regarded as transient. On the other hand, the data of 81% (14.6 seconds)

was selected as the upper bound, beyond which the data representing those close to the end of each test. The tire frame assembly was equipped with a limit switch on each end. The limit switch was activated to stop the dc motor when the assembly reaches the end position. However, the tire frame still moves, or even rebounds from the end position for a certain distance due to its massive inertia, and meanwhile the data acquisition system keep logging data. Consequently, the final 19% of the scanned data was discarded.

Figure 5.1-1 Drawbar pull at 10, 20, and 35% slips

The steady-state experimental data are presented in four groups. These four groups are:

- 1) Normalized drawbar pull versus longitudinal slip and slip angles
- 2) Normalized lateral force versus slip angle and longitudinal slips
- 3) Normalized lateral force versus normalized drawbar pull and slip angles
- 4) Sinkage versus slip angle and longitudinal slips at different normal forces

For the first three groups, each group includes two figures, one in three-dimensional (3D) surface plot and the other in two-dimensional (2D) plot. Each figure consists of three graphs for tire inflation pressure p_i of 1.0, 2.0, and 3.0 bar. The fourth group for tire sinkage includes three figures for different p_i . Each figure shows three graphs for normal force F_z of 7.01, 8.12, and 9.23 kN.

To interpret the physical phenomenon of each group of measured data, a regression surface is presented along with the raw data. The reason of choosing a surface instead of

a curve is that a measured quantity, for example the normalized drawbar pull F_{dbp} / F_z is both a function of longitudinal slip i , and a function of slip angle α . Thus the variation of $F_{dbp} / F_z = f(i, \alpha)$ is represented by a continuous surface. The surface is fitted by a quadratic polynomial $z = c_0 + c_1x + c_2y + c_3x^2 + c_4xy + c_5y^2$ (c_i : coefficients, $i = 0 \sim 5$) to the measured data points. On the other hand, curves in each 2D graph are plotted by substituting a specific value of the quantity on the y-axis. In other words, a 2D graph can be regarded as the intersection of a plane at a given value of y and the 3D surface.

A commonly used indicator of how well a regression curve or a surface fits the measured data is R^2 , the *coefficient of determination* (Norusis, 1990). It is the square of the *correlation coefficient* R between z , the observed value of the dependent variable, and $\hat{z}$, the prediction value of z from the fitted surface. Partitioning the total sum of squares into the residual and the regression sum of squares gives an interpretation of R^2 . It is the proportion of the variation in the dependent variable “explained” by the fitted model.

$$\begin{aligned}
 R^2 &= 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}} = \frac{\text{regression sum of squares}}{\text{total sum of squares}} \\
 &= 1 - \frac{\sum_{i=1}^N (z_i - \hat{z}_i)^2}{\sum_{i=1}^N (z_i - \bar{z})^2} = \frac{\sum_{i=1}^N (\hat{z}_i - \bar{z})^2}{\sum_{i=1}^N (z_i - \bar{z})^2} \quad (5.1-1)
 \end{aligned}$$

where z_i = observed value, or measured data
 $\bar{z}$ = mean value, of measured data
 $\hat{z}_i$ = regression value, from fitted surface
 N = number of data points

If all the measured data points fall exactly on the regression surface, R^2 is 1. All 3D surface graphs presented below also indicate the number of data used and the coefficient of determination R^2 from the fitting of the regression surface. The complete set of experimental data is presented in Figure 5.1-2 to Figure 5.1-10. Each data point represents the steady-state value of a single test run. Scales for drawbar pull and lateral force are normalized by tire normal force, respectively. In summary, these figures are organized as below:

- F_{dbp} / F_z vs. i and α , 3D surface graphs, Figure 5.1-2
- F_{dbp} / F_z vs. i , for $\alpha = 0^\circ \sim 20^\circ$, 2D graphs, Figure 5.1-3
- F_y / F_z vs. α and i , 3D surface graphs, Figure 5.1-4
- F_y / F_z vs. α , for $i = 2\% \sim 35\%$, 2D graphs, Figure 5.1-5
- F_y / F_z vs. F_{dbp} / F_z and α , 3D surface graphs, Figure 5.1-6
- F_y / F_z vs. F_{dbp} / F_z , for $\alpha = 0^\circ \sim 20^\circ$, 2D graphs, Figure 5.1-7
- z_{max} vs. α and i , $p_i = 1.0$ bar, 3D surface graphs, Figure 5.1-8
- z_{max} vs. α and i , $p_i = 2.0$ bar, 3D surface graphs, Figure 5.1-9
- z_{max} vs. α and i , $p_i = 3.0$ bar, 3D surface graphs, Figure 5.1-10

where F_{dbp} / F_z = drawbar pull normalized by normal force

F_y / F_z = lateral force normalized by normal force

z_{max} = maximum tire sinkage

α = slip angle

i = longitudinal slip

p_i = tire inflation pressure

The relationship between normalized drawbar pull F_{dbp} / F_z and longitudinal slip i , shown in Figures 5.1-2 and 5.1-3 is typical. As i increases, F_{dbp} / F_z increases and approaches a limit due to tire-terrain friction. The quadratic model may not have been the best choice of models to fit the measured data since the fitted surfaces show decreased F_{dbp} / F_z at very high slips. At low slips, drawbar pull is negative because the tire develops less tractive force than the motion resistance. The drawbar pull roughly remains negative until $i = 10 \sim 15\%$ depends on slip angle and tire inflation pressure. The slip angle α , on the other hand, has an impact on the relationship of F_{dbp} / F_z versus i . Contrary to the concept of friction ellipse (Wong, 1993; Milliken and Milliken, 1995), as slip angle increases, the normalized drawbar pull generally increases except at tire inflation pressure of 1.0 bar. Moreover, the high mobility performance of low pressure tires is confirmed from the figures that at the condition of $i = 35\%$ and $\alpha = 0^\circ$, F_{dbp} / F_z reaches 0.1 at $p_i = 1.0$ bar, 0.025 at $p_i = 2.0$ bar, and 0.02 at $p_i = 3.0$ bar. This is a supporting evidence that the tire behaves differently in longitudinal direction at $p_i = 1.0$ bar, and $p_i = 2.0$ or 3.0 bar. The relative low values of F_{dbp} / F_z are probably attributed to the very loose and dry sand.

Inflation Pressure [bar]: 1

No. of Data: 324
 $R^2 = 0.925$

Inflation Pressure [bar]: 2

No. of Data: 322
 $R^2 = 0.940$

Inflation Pressure [bar]: 3

No. of Data: 322
 $R^2 = 0.939$

Figure 5.1-2 F_{dbp} / F_z vs. longitudinal slip and slip angle, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation

Figure 5.1-3 F_{dbp} / F_z vs. longitudinal slip, $p_i = 1$ bar, 2 bar and 3 bar

Inflation Pressure [bar]: 1

No. of Data: 324
 $R^2 = 0.984$

Inflation Pressure [bar]: 2

No. of Data: 322
 $R^2 = 0.986$

Inflation Pressure [bar]: 3

No. of Data: 322
 $R^2 = 0.985$

Figure 5.1-4 F_y / F_z vs. slip angle and longitudinal slip, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation

Figure 5.1-5 F_y/F_z vs. slip angle, $p_i = 1$ bar, 2 bar and 3 bar

Inflation Pressure [bar]: 1

No. of Data: 324
 $R^2 = 0.982$

Inflation Pressure [bar]: 2

No. of Data: 322
 $R^2 = 0.984$

Inflation Pressure [bar]: 3

No. of Data: 322
 $R^2 = 0.984$

Figure 5.1-6 F_y/F_z vs. F_{dbp}/F_z and slip angle, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation

Figure 5.1-7 F_y/F_z vs. F_{dbp}/F_z , $p_i = 1$ bar, 2 bar and 3 bar

Inflation Pressure [bar]: 1
Normal Force [kN]: 7.01

No. of Data: 128
 $R^2 = 0.803$

Inflation Pressure [bar]: 1
Normal Force [kN]: 8.12

No. of Data: 128
 $R^2 = 0.761$

Inflation Pressure [bar]: 1
Normal Force [kN]: 9.23

No. of Data: 131
 $R^2 = 0.738$

Figure 5.1-8 Sinkage vs. slip angle and longitudinal slip, $p_i = 1$ bar, three-dimensional representation

Inflation Pressure [bar]: 2
Normal Force [kN]: 7.01

No. of Data: 128
 $R^2 = 0.794$

Inflation Pressure [bar]: 2
Normal Force [kN]: 8.12

No. of Data: 129
 $R^2 = 0.788$

Inflation Pressure [bar]: 2
Normal Force [kN]: 9.23

No. of Data: 128
 $R^2 = 0.537$

Figure 5.1-9 Sinkage vs. slip angle and longitudinal slip, $p_i = 2$ bar, three-dimensional representation

Inflation Pressure [bar]: 3
Normal Force [kN]: 7.01

No. of Data: 129
 $R^2 = 0.758$

Inflation Pressure [bar]: 3
Normal Force [kN]: 8.12

No. of Data: 127
 $R^2 = 0.774$

Inflation Pressure [bar]: 3
Normal Force [kN]: 9.23

No. of Data: 132
 $R^2 = 0.792$

Figure 5.1-10 Sinkage vs. slip angle and longitudinal slip, $p_i = 3$ bar, three-dimensional representation

In Figures 5.1-4 and 5.1-5, the normalized lateral force F_y / F_z versus slip angle α do not show a saturation. It appears to be due to the gradually increased lateral bulldozing force of the sinking tire although the shear force within the tire-terrain interface may approach constant. However, the frictional ellipse phenomenon can be realized from the figures since for a given α , F_y / F_z decreases with an increase of i . The magnitude of F_y / F_z includes two components, the shear force at the contact region and the bulldozing force acting on tire sidewall. The latter constitutes no more than 20% of F_y / F_z , according to the tire model predictions described in the next chapter. The maximum F_y / F_z for $p_i = 1.0, 2.0$ and 3.0 bar are very close indicating that the mechanism for developing tire lateral force are similar for various tire inflation pressures. In other words, the majority of F_y / F_z is contributed from shear force. The shear force mechanism and tire lateral deformation for all three inflation pressures may be comparable.

Figures 5.1-6 and 5.1-7 show that the normalized lateral force F_y / F_z varies with respect to the normalized drawbar pull F_{dbp} / F_z and slip angle α . For a particular α , as F_{dbp} / F_z increases from negative to positive, F_y / F_z decreases. The trend is maintained as slip angle increases. The maximum F_y / F_z occurs at the lowest F_{dbp} / F_z which corresponds to zero longitudinal slip i . It can be seen from Figure 5.1-7 that the three graphs display similar behaviour in both experimental data and fitted curves. A minor distinction among the three graphs is the distribution of raw data between the case of $p_i = 1.0$ bar, and cases of $p_i = 2.0$ and 3.0 bar. On the graph of $p_i = 1.0$ bar, the data points to the right, corresponding to $i = 35\%$, give F_{dbp} / F_z approximately of 0.08~0.10. However, for graphs of $p_i = 2.0$ and 3.0 bar, the corresponding data points of $i = 35\%$ give F_{dbp} / F_z about 0.05~0.07.

The maximum tire sinkage z_{max} at steady state is a function of slip angle α and longitudinal slip i . More directly, z_{max} is affected by tire normal force F_z . A total number of nine graphs in Figures 5.1-8, 5.1-9, and 5.1-10 show the variation of z_{max} with respect to the changes of $p_i = 1.0, 2.0$, and 3.0 bar and $F_z = 7.01, 8.12$, and 9.23 kN. The sinkage increases when slip angle, or longitudinal slip, or both slip angle and longitudinal slip increase. However, all graphs show saddle surfaces. The curves between z_{max} and α are concave down, and the curves of z_{max} versus i are concave up. The test tire is operated at slip angle α within the range of $0^\circ \sim 20^\circ$. As the angle α increases, tire lateral slip increases. Because of the effect of slip sinkage, the maximum tire sinkage

$z_{\max}$ increases. Furthermore, the trend of $z_{\max}$ versus α curves reflects the physical mechanism for the relationship of F_y / F_z versus α , shown in Figures 5.1-4 and 5.1-5. This explains why the shape of $z_{\max}$ versus α is concave down. The concave-up of the relation of $z_{\max}$ and i confirms a previously experimental work done by Chen (1993). Chen showed similar concave-up relations of $z_{\max}$ versus i for a Michelin 11.00R16 XL off-road tire operated on Ottawa sand with inflation pressures of 69 kPa (10 psi), 138 kPa (20 psi), and 207 kPa (30 psi). The longitudinal slip setting in this experiment ranges from 0% to 35%, shown in Table 5.1-2. However at 0% slip, the tire does not show free rolling, instead it shows skid. A skidding tire, like a bulldozer, has a tendency of digging into the soil so that developing significant skid sinkage. On the other hand, tire slip sinkage reaches the minimum at low slips around $i = 2 \sim 5\%$, and then increases exponentially with further increases of i . As a consequence, the concave-up relationship of $z_{\max}$ versus i is realized.

5.2 Soil Property Measurements

Tire tests were conducted in a soil bin at laboratory temperature of $21 \sim 22^\circ\text{C}$ and atmospheric pressure of 1.013 bar. The soil used was dry uniform sand, known as the Ottawa sand with ASTM designation C190 and a bulk density of $\gamma_s = 1,755 \text{ kg/m}^3$. To identify the soil conditions under which the performance of the tire was tested and to provide terrain data for the development of an off-road tire model, the mechanical properties of the Ottawa sand were measured using a Bevameter. The schematic of the Bevameter is shown in Figure 5.2-1. Using the apparatus, the pressure-sinkage, shear stress-shear displacement relationships, and shear strength (Mohr-Coulomb failure criterion) of the soil can be measured.

In the pressure-sinkage test, a sinkage plate connected with a load cell was used to simulate the contact patch of a tire. By applying normal force (pressure) to the plate, the resulting plate sinkage was recorded. Plates of different size were used so that the pressure-sinkage parameters can be determined (Bekker, 1960; Wong, 1989, 1993). Figure 5.2-2 (a) shows a pressure-sinkage test being conducted using the Bevameter. For homogeneous soil, the following equation proposed by Bekker (1960) is widely used.

$$p_n = \left(\frac{k_c}{b} + k_\phi \right) z^n \quad (5.2-1)$$

where p_n = normal pressure
 k_c = pressure-sinkage parameter
 k_ϕ = pressure-sinkage parameter
 n = pressure-sinkage exponent
 b = width of rectangular plate, or radius of circular plate
 z = sinkage

Figure 5.2-1 Schematic of Bevameter (Chen, 1993)

In the shear test, a circular shear ring was rotated by a hydraulic ram through a chain. The torque applied and the corresponding angular displacement were monitored by a torque cell and a rotary potentiometer, respectively. Figure 5.2-2 (b) shows a shear test being conducted using the Bevameter. With the mean diameter and the contact area of the shear ring known, the relationship of the measured torque versus angular displacement can be converted to the shear stress-shear displacement relationships (Wong, 1989, 1993). For

homogeneous soil, the equation that widely used for characterizing the shear stress-shear displacement is given below.

$$\begin{aligned}\tau &= \tau_{\max} (1 - e^{-j/K}) \\ &= (c + p_n \tan \phi_s) (1 - e^{-j/K})\end{aligned}\quad (5.2-2)$$

- where τ = shear stress
 $\tau_{\max}$ = shear strength of terrain
 c = apparent cohesion of terrain
 p_n = normal pressure
 ϕ_s = angle of internal shearing resistance
 j = shear displacement
 K = shear deformation modulus

(a)

(b)

Figure 5.2-2 (a) Pressure-sinkage test, (b) Shear stress-shear displacement test

For both soil property tests, the experimental procedures, data processing, and computing of parameters can be found in the works of Wong (1980, 1989, and 1993), Chen (1993), and Jastremski (1994).

For pressure-sinkage, or penetration tests, three circular plates of diameter 76.3, 100, and 200 mm were used. For each plate, six penetration tests were carried out. Soil parameters n , k_c , and k_ϕ in equation (5.2-1) can be obtained from a pair of tests. The pair can be chosen from any combination of two different sizes of plate such as plates of diameter of 76.3 and 200 mm. Table 5.2-1 lists the values of the pressure-sinkage parameters derived from measured data. These values were the mean values derived from 36 pairs of data. Figure 5.2-3 shows the fitted curve using the values of the parameters derived in comparison with the results of six penetration tests with a plate of diameter of 100 mm.

Table 5.2-1 Parameters for pressure-sinkage relationship

Parameters	n	k_c	k_ϕ
Units		$[kN/m^{n+1}]$	$[kN/m^{n+2}]$
Values	1.02	12.25	2427.4

Figure 5.2-3 Fitted curve compared with six penetration tests

For shear stress-shear displacement tests, five different normal pressures were applied to the shear ring. For each normal pressure, three or six measurements were conducted. The mean diameter of the rubber shear ring was 304.8 mm (12 inches), and the grouser height was 3 mm. Parameter K of equation (5.2-2) was computed by minimizing the overall fitting error. Other shear parameters c and ϕ_s in equation (5.2-2) were identified by plotting shear strength $\tau_{\max}$ versus normal pressure p_n . Figure 5.2-4 shows a straight line fitted by regression, from which c and ϕ_s were obtained. Table 5.2-2 lists the magnitudes of all shear parameters derived from these tests.

Figure 5.2-4 Shear strength versus normal pressure

Table 5.2-2 Parameters for shear stress-shear displacement

Parameters	c	ϕ_s	K
Units	[kPa]	[deg]	[mm]
Values	1.37	31.45	47.3

Equation (5.2-2) with parameters given in Table 5.2-2 represents the shear stress-shear displacement relationship of the Ottawa sand used in tire tests. Figure 5.2-5 shows the

fitted shear stress-shear displacement curve in comparison with three measured curves at the normal pressure of 16.58 kPa.

Figure 5.2-5 Fitted curve compared with three shear tests

The shear stress-shear displacement relationship within tire rubber-soil interface also exhibits similar exponential characteristics. Equation (5.2-2) is applicable to interpret this relationship. A set of 18 measurements was conducted using a rubber ring with a mean diameter of 304.8 mm (12 inches). The same iteration process for minimizing the overall fitting error was used for each measurement data to obtain the shear deformation modulus for rubber-soil shearing K_{rs} . Similarly, coefficients c_{rs} and ϕ_{rs} can be identified by plotting the individual shear strength τ_{max} with respect to the applied normal pressure p_n . The measured data of τ_{max} versus p_n as well as the regression straight line are shown in Figure 5.2-6, and the values of c_{rs} , ϕ_{rs} and K_{rs} are given in Table 5.2-3.

Table 5.2-3 Parameters for shear stress-shear displacement , rubber-soil

Parameters	c_{rs}	ϕ_{rs}	K_{rs}
Units	[kPa]	[deg]	[mm]
Values	0.48	29.82	13.9

Figure 5.2-6 Shear strength versus normal pressure, rubber-sand

The shear stress-shear displacement relationship between the tire rubber and the Ottawa sand was characterized by equation (5.2-2) with parameters in Table 5.2-3. Figure 5.2-7 shows the fitted shear stress-shear displacement curve of rubber-sand in comparison with three measured curves at one of the applied normal pressures.

Figure 5.2-7 Fitted curve compared with three shear tests, rubber-sand

Chapter 6

Development of an Off-Road Tire Model

6.1 Tire Lateral Behaviour on Soft Terrain

To develop a realistic handling model for multi-axle, all-wheel-drive, off-road vehicles on deformable terrain, it is essential to have an adequate understanding of the mechanics of tires under a combined longitudinal slip and slip angle condition.

For a sideslip tire, tire lateral deformation is usually represented by the distortion of the *equatorial line*, which is the intersection of the tire tread band with the wheel plane (Segel, 1988; Wong, 1993). Near the forward portion of the contact region on a hard surface, the equatorial line represents the direction of motion of the tire (wheel) center, with a slip angle α to the wheel plane, as shown in Figure 6.1-1. At a small slip angle, the tire-ground interface is in adhesion. No sliding between the tire and ground takes place, and the slip angle is due to tire lateral elasticity. At a large slip angle, however, sliding initiates in the rear contact region and extends towards the front part.

For an off-road tire operating on a soft terrain, shear displacement develops within the tire-terrain interface in order to generate shear stress. Lateral shear displacement initiates

from the initial point of contact, and gradually increases towards the rear. The direction of the equatorial line at the front contact point is no longer at a slip angle α to the wheel plane. Instead, the equatorial line together with lateral shear displacement constitutes the slip angle. This is illustrated in Figure 6.1-2 for a general case of a tire operating on soft terrain. Tire lateral deformation increases along the contact length until the maximum lateral shear displacement or shear stress is reached. Because of the circumferential tension, tire lateral deformation represented by the equatorial line drops off in an exponential fashion for that part of the tire tread not in contact with terrain. Detailed descriptions of tire lateral deformation, equatorial line, and lateral shear displacement for a tire operating on deformable terrain in rigid and in elastic modes are schematically shown in Figures 6.2-3 and 6.3-2, respectively.

Figure 6.1-1 Tire lateral deformation at slip angle α , on hard surface (Clark, 1981)

Figure 6.1-2 Tire lateral deformation at slip angle α , on soft terrain

As noted above, on soft terrain, the shear stress-shear displacement equation (5.2-2) has been widely used for computing the shear stress on the tire-terrain interface. The parameters characterizing this equation include shear parameters c , ϕ , K , normal pressure p_n , and tire-terrain shear displacement j . The shear force developed within the interface is then obtained by integrating the shear stress τ over the entire contact area. However, to evaluate the shear stress using equation (5.2-2), the expression of shear displacement j has to be determined first.

The determination of shear displacement j is dependent upon the contact geometry of the tire-terrain interface. As pointed out by Bekker (1960), and Wong (1989, 1993), if tire inflation pressure is high and terrain is soft, then tire deformation in the wheel plane in relation to terrain deformation will be small. Consequently, the tire will behave like a rigid rim, and the tire is referred to as operating in a *rigid mode*. On the other hand, if tire inflation pressure is low and terrain is firm, then tire deformation in the wheel plane in relation to terrain deformation will be significant. In this case, the bottom of the tire in contact with terrain will be flattened and the tire is referred to as operating in an *elastic mode*. It should be noted, however, that in the following analysis, it is assumed that the tire carcass together with treads will deform laterally, whether the tire in the wheel plane is in rigid mode or in elastic mode. This is because that normally tire lateral stiffness is considerably lower than that of the vertical. Tire lateral deformation will vary with inflation pressure, which will be discussed later. To determine whether a tire with a given normal force and inflation pressure on a particular terrain is operating in rigid or elastic mode, a so-called critical pressure p_{gr} is first calculated (Wong, 1989, 1993).

$$p_{gr} = \left[\frac{k_c}{b_t} + k_\phi \right]^{\frac{1}{2n+1}} \left[\frac{3F_z}{(3-n)b_t\sqrt{d_t}} \right]^{\frac{2n}{2n+1}} \quad (6.1-1)$$

where k_c = pressure-sinkage parameter
 k_ϕ = pressure-sinkage parameter
 n = pressure-sinkage exponent
 b_t = tire tread width
 d_t = tire diameter
 F_z = tire normal force

If the sum of tire inflation pressure p_i and the contact pressure due to carcass stiffness p_c is greater than p_{gr} , then the tire is considered to be in rigid mode of operation, otherwise it is considered to be in an elastic mode. It should be noted that in practice, the pressure p_c exerted on terrain due to carcass stiffness is difficult to determine. As an alternative, the average ground pressure p_{gr} of a tire on hard ground derived from tire normal load and contact area is used to represent the sum of p_i and p_c . The relations of tire normal force and tire-ground contact area under different inflation pressures can be obtained from the so-called generalized deflection chart usually available from tire manufacturers. The determination of operating mode for the test tire at inflation pressures of 1.0, 2.0 and 3.0 bar is given in Appendix C. The results show that the tire is in elastic mode for 1.0 bar, and in rigid mode for 2.0 and 3.0 bar.

The prediction of longitudinal shear displacement for a tire operating in rigid mode on a deformable terrain was presented by Wong and Reece (1967). Shear displacement along the contact arc varies with respect to tire-terrain contact angle and reaches the maximum at the bottom-dead-center. For an off-road tire operating in elastic mode, instead of an arc, the contact contour in the wheel plane consists of an arc and a flat portion. The prediction of shear displacement for the arc is the same as that of the tire operating in rigid mode. However for the flat contact portion, one should use the method for computing shear displacement of a rigid track (Wong, 1989, 1993). This tire model for predicting tractive force was validated experimentally by Chen (1993) and by Jastremski (1994). The test tires were Michelin 11.00R16 XL and 12.5R20 XL, respectively for off-road applications. Furthermore, the model was coded into a computer simulation program by Wong (1997). The program has been successfully used for predicting the tractive performance versus longitudinal slip of several off-road wheeled vehicles and validated by available experimental data.

Due to this reason, the proposed off-road tire model for the combined longitudinal slip and slip angle was developed from the basis of the rigid/elastic tire traction model mentioned above. The derivation of this tire model also includes tires operating in a rigid, and in an elastic mode.

6.2 Rigid Operating Mode

When a vehicle negotiates a turn, owing to the centrifugal effect, a side force is applied to the tire center. As a result, a lateral force is developed on the tire-ground contact patch. Because of the lateral flexibility of the tire and/or the lateral sliding of the tire on the ground, the lateral force causes the tire to move along a path with an angle α to the wheel plane, the angle is usually referred to as the slip angle.

The apparatus for testing the tire at a slip angle α with a particular combination of tire, slip pulley, and sprockets is schematically shown in Figure 6.2-1. The wheel-shaft is fixed on the tire frame so that the tire center moves with the tire frame. As the tire frame moves along the $x_{tire\ frame}$ -direction, the tire is forced to develop a slip angle α . The tire center travels with a velocity vector of $\bar{v}_0$, shown in the figure.

Figure 6.2-1 Tire slip angle induced by tire frame

When the tire frame, as well as the tire center, moves a distance of s_0 along the $x_{tire\ frame}$ -direction, a section of the cable with length of s_0 must be released from the slip pulley. While releasing the cable, both the slip pulley of radius r_s and the sprocket mounted on the pulley shaft rotate an angle of s_0/r_s . Because of the gear reduction ratio of the chain drive, the tire with radius r_t rotates an angle of $s_0 r_2 / r_s r_1$, where r_1 is the radius of the sprocket mounted on tire shaft and r_2 is the radius of the sprocket mounted on pulley shaft. Consequently, the theoretical distance the tire center travels along the wheel plane, s_t , becomes

$$s_t = s_0 \frac{r_t r_2}{r_s r_1} \quad (6.2-1)$$

Owing to the constraint of the tire on the tire frame, the actual distance the tire center travels along the wheel plane, s_t , shown in Figure 6.2-2, is given by

$$s_t = s_0 \cos \alpha \quad (6.2-2)$$

Figure 6.2-2 Displacement of tire center in $\bar{v}_0$ -direction and in wheel plane

For a tire with a nonzero slip angle α , the effective longitudinal slip i_e is defined as

$$i_e = 1 - \frac{\text{actual forward displacement of tire center along wheel plane}}{\text{theoretical forward displacement of tire center along wheel plane}}$$

With this definition, the slip i_e is expressed by

$$\begin{aligned} i_e &= 1 - \frac{s_t}{s_0} \\ &= 1 - \frac{s_0 \cos \alpha}{s_0 \frac{r_t r_2}{r_s r_1}} \\ &= 1 - \frac{r_s r_1}{r_t r_2} \cos \alpha \end{aligned} \quad (6.2-3)$$

Since the ratio of $r_s r_1 / r_t r_2$ was previously defined in equation (4.2-2) as $1 - i$, where i is the longitudinal slip of a tire with zero slip angle, equation (6.2-3) can be rewritten as

$$i_e = 1 - (1 - i) \cos \alpha \quad (6.2-4)$$

Equation (6.2-4) is consistent with equation (4.2-2), which defines the longitudinal slip a tire with zero slip angle. In other words, when $\alpha = 0$, $\cos \alpha = 1$ and $i_e = i$. The effective longitudinal slip i_e can also be expressed in terms of velocities. That is, $i_e = 1 - (v_l / v_t)$, where v_l ($= v_0 \cos \alpha$) is the longitudinal component of the velocity of the tire center, and directs along the wheel plane, and v_t ($= r_t \omega_t$) is the theoretical velocity of the tire center along the wheel plane. Therefore,

$$\begin{aligned} v_l &= v_0 \cos \alpha \\ &= v_t (1 - i_e) \\ &= r_t \omega_t (1 - i_e) \\ &= r_t \omega_t (1 - i) \cos \alpha \end{aligned} \quad (6.2-5)$$

On the other hand, v_s is the lateral component of the velocity of the tire center, which is perpendicular to the wheel plane, and is expressed by the equation given below. Both v_l and v_s are shown in Figure 6.2-3.

$$\begin{aligned} v_s &= v_0 \sin \alpha \\ &= \frac{v_l}{\cos \alpha} \sin \alpha \\ &= r_t \omega_t (1 - i) \sin \alpha \end{aligned} \quad (6.2-6)$$

Considering the circumferential shearing along the wheel plane, the absolute velocity $\bar{v}_{p,x}$ of an arbitrary point p defined by an angle θ on the tire and within the tire-terrain interface is obtained by applying the principle of relative velocity. This is illustrated in Figure 6.2-3.

$$\bar{v}_{p,x} = \bar{v}_0 \cos \alpha + \bar{v}_{p/0,x} \quad (6.2-7)$$

$$\text{where } v_{p/0,x} = r_t \omega_t$$

On the x - z plane, the projection of the absolute velocity $\bar{v}_{p,x}$ onto the tangent of point p is the slip velocity v_{jz} of point p relative to terrain.

$$\begin{aligned} v_{jz} &= r_t \omega_t - r_t \omega_t (1-i) \cos \alpha \cos \theta \\ &= r_t \omega_t [1 - (1-i) \cos \alpha \cos \theta] \end{aligned} \quad (6.2-8)$$

Figure 6.2-3 Slip velocities on x - z plane and in lateral direction

The shear displacement j_z developed at point p is obtained by integrating slip velocity v_{jz} over the time span of point p in contact with terrain.

$$\begin{aligned} j_z &= \int_0^t v_{jz} dt \\ &= \int_{\theta}^{\theta_{ic}} r_t \omega_t [1 - (1-i) \cos \alpha \cos \theta] \frac{d\theta}{\omega_t} \\ &= r_t [(\theta_{ic} - \theta) - (1-i) \cos \alpha (\sin \theta_{ic} - \sin \theta)] \end{aligned} \quad (6.2-9)$$

where θ_{ic} = angle at initial contact point

As noted previously, even though in the wheel plane the tire behaves like a rigid rim, it is assumed that in the lateral direction the tire can deform when subject to a lateral force. The direction of motion at a given slip angle partly results from tire lateral deformation, and partly from lateral shear (sliding) displacement on the tire-terrain interface, as shown in Figure 6.2-4. The figure is obtained by projecting the contact arc CD of Figure 6.2-3 onto the ground plane.

Figure 6.2-4 Slip angle resulting from tire lateral deformation and lateral shear displacement in contact region, rigid mode

Tire lateral deformation is modeled by a simplified relation given by Dugoff et al. (1970), and by Wong (1993). The relation assumes that tire lateral deformation increases from the front contact point to the rear. Although the shear displacement and the associated shear stress on the tire-terrain interface is zero at the front contact point, tire lateral deformation at this point, y_c has a finite value. By assuming an initial *tire lateral deformation angle* β_t as shown in Figure 6.2-4, in the contact region tire lateral deformation, y_s , due to tire elasticity is modeled by the following equation.

$$y_s(s) = y_c + s \tan \beta_t \quad (6.2-10)$$

$$\begin{aligned} \text{where } y_c &= \text{tire lateral deformation at point C, } y_c = l_r \tan \beta_t \\ l_r &= \text{relaxation length} \end{aligned} \quad (6.2-11)$$

$$s = \text{coordinate along contact arc, starting from point } C \text{ to bottom-dead-center, point } D, s = r_t(\theta_{IC} - \theta) \quad (6.2-12)$$

It is noted that y_s varies linearly with respect to both s and θ . Since s is a coordinate along the contact arc, the lateral deformation y_s , view from the top as shown in Figure 6.2-4, shows a nonlinear distribution on the ground plane. The figure also shows the distributions of lateral shear displacement $j_s(\theta)$ and tire lateral deformation $y_s(\theta)$.

For a sideslip tire operating on a deformable terrain, the lateral force equilibrium due to tire-terrain shearing and tire elasticity is analogous to the case of applying a force to an object through a spring. The magnitude of the force acting on the object equals the spring force. Thus, when the tire operates at a steady-state condition, the lateral shear force developed in the tire-terrain interface is equal to tire lateral or internal force that causes tire lateral deformation. The latter is computed by multiplying tire lateral deformation with tire lateral stiffness k_y (with units of kPa) and then integrating around tire circumference. Tire lateral deformation includes y_s in the contact region and y_ξ in the free region. The resulting tire lateral force F_{yt} is given by

$$F_{yt} = k_y \int_0^{s_D} y_s(s) ds + k_y \int_0^L y_\xi(\xi) d\xi \quad (6.2-13)$$

where s_D = coordinate s at point D , $s_D = r_t \theta_{IC}$
 L = tire circumferential length outside contact region
 ξ = coordinate along tire circumference outside contact region, from point D to point C

The tire lateral stiffness k_y was determined from a special experiment described in Appendix D. For tire inflation pressures $p_i = 2.0$ and 3.0 bar, lateral stiffness k_y are 95.58 and 99.95 kPa, respectively. It is noted that for $p_i = 2.0$ and 3.0 bar, the tire is in rigid operating mode. While in the wheel plane the shape of the tire remains round, the tire is assumed to be elastic in lateral direction. The lateral force F_{yt} described by equation (6.2-13) represents the resultant tire internal force due to lateral elasticity.

In the lateral direction, the velocity component of tire center perpendicular to the wheel plane is $\bar{v}_0 \sin \alpha$. However for an arbitrary point p within the contact region and defined by an angle θ , the effect of tire lateral deformation angle β_t should be considered. The angle β_t contributes part of the slip angle α , shown in Figure 6.2-4. The remaining part of the slip angle is due to lateral shearing, or sliding between the tire and terrain. The lateral shear velocity of point p becomes

$$\bar{v}_{p,y} = \bar{v}_0 \sin(\alpha - \beta_t) \quad (6.2-14)$$

Equation (6.2-14) explains the fact that tire-terrain shear velocity becomes zero if $\alpha = \beta_t$. That is the case when tire-ground interface is completely in adhesion, and the slip angle is solely due to tire lateral flexibility, shown in Figure 6.1-1. For a given longitudinal slip i , slip angle α , and tire lateral deformation angle β_t , the lateral slip velocity v_{jy} for any point within the contact interface can be written as

$$v_{jy} = r_t \omega_t (1 - i) \sin(\alpha - \beta_t) \quad (6.2-15)$$

The lateral shear displacement of a point p defined by an angle θ is obtained by integrating v_{jy} / ω_t from θ to θ_{IC} .

$$\begin{aligned} j_y &= \int_0^t v_{jy} dt \\ &= \int_{\theta}^{\theta_{IC}} r_t \omega_t (1 - i) \sin(\alpha - \beta_t) \frac{d\theta}{\omega_t} \\ &= r_t (1 - i) \sin(\alpha - \beta_t) (\theta_{IC} - \theta) \end{aligned} \quad (6.2-16)$$

The shear displacement j_y , derived from lateral slip velocity v_{jy} , and the tire lateral deformation y_t , given by equation (6.2-10) can be demonstrated with an example of typical parameters as below.

$$\begin{aligned} y_C &= 0.01m \\ r_t &= 0.498m \\ \theta_{IC} &= 40^\circ \\ \beta_t &= -6^\circ \end{aligned}$$

$$\begin{aligned}\alpha &= -20^\circ \\ i &= 0\%\end{aligned}$$

According to the plane-view coordinates (d, y) for tire-terrain contact region shown in Figure 6.2-4, both distributions of y_s and j_y are computed with a range of angle θ from 40° to 0° . The lower curve in Figure 6.2-5 represents the distribution of tire lateral deformation y_s , whereas the upper curve is the result of $y_s + j_y$. The region in between y_s and $y_s + j_y$ is the lateral shear displacement j_y . The angle between d -coordinate and the curve $y_s + j_y$ is 20° .

Figure 6.2-5 An example demonstrating tire lateral deformation and lateral shear displacement in contact region

As a starting point, the initial contact angle θ_{IC} can be identified from geometry. However, the angle θ_{IC} may require adjustment to fulfill the equilibrium of tire vertical force during the iteration process.

$$\sin \theta_{IC} = \frac{\sqrt{z_{rg}(2r_t - z_{rg})}}{r_t} \quad (6.2-17)$$

where z_{rg} = maximum tire sinkage in rigid operating mode

Along the tire-terrain interface, the circumferential shear displacement j_x occurs on the wheel plane and tangents to tire circumference. The lateral shear displacement j_y , on the other hand, is perpendicular to the wheel plane. Physically at this particular position defined by the angle θ , there is only one resultant shear displacement between the tire and terrain. This resultant j_r is the vector sum of circumferential shear displacement j_x and lateral shear displacement j_y . The resultant j_r occurs on the tangential plane formed by j_x and j_y . The plane inclines at the angle θ to the tire horizontal plane.

$$j_r = \sqrt{j_x^2 + j_y^2} \quad (6.2-18)$$

Using Bekker's pressure-sinkage equation, the normal pressure distribution p_n can be predicted in terms of tire sinkage z_t , or tire-terrain contact angle θ .

$$\begin{aligned} p_n &= \left(\frac{k_c}{b_t} + k_\phi \right) z_t^n \\ &= \left(\frac{k_c}{b_t} + k_\phi \right) [r_t (\cos \theta - \cos \theta_{tc})]^n \end{aligned} \quad (6.2-19)$$

where k_c = pressure-sinkage parameter
 k_ϕ = pressure-sinkage parameter
 n = pressure-sinkage exponent
 b_t = tire tread width
 z_t = coordinate for tire sinkage

Both the resultant shear displacement j_r and normal pressure distribution p_n are functions of angle θ . According to equation (5.2-2), the shear stress τ_r at a position defined by the angle θ is written as

$$\tau_r = [c + p_n(\theta) \tan \phi_s] (1 - e^{-j_r(\theta)/K}) \quad (6.2-20)$$

For a tire circumferential element of area $b_t r_t d\theta$, the shear force tangent to this element is $dF_r = \tau_r b_t r_t d\theta$. The principle of Coulomb friction indicates that the direction of frictional force is always acting in the direction opposing relative motion. According to

the principle, the elementary shear force dF_r has an opposite direction to that of the resultant slip velocity v_r with a magnitude of $v_r = (v_{jx}^2 + v_{jy}^2)^{1/2}$. The direction of v_r with respect to tire horizontal plane can be identified from Figure 6.2-6. It is noted that the slip velocity v_{jx} tangents to tire circumference. The projection of v_{jx} to the horizontal plane is mn , and is equal to $v_{jx} \cos \theta$. Since the angle $\angle mnp$ is a right angle, the magnitude of mp is thus $(v_{jx}^2 \cos^2 \theta + v_{jy}^2)^{1/2}$. Nevertheless, segment mp is also the projection of the resultant slip velocity v_r onto the horizontal plane. Thus, the angle θ_r , which defines the direction of v_r , or dF_r relative to tire horizontal plane is determined.

$$\cos \theta_r = \frac{\sqrt{v_{jx}^2 \cos^2 \theta + v_{jy}^2}}{v_r} \quad (6.2-21)$$

Figure 6.2-6 Direction of resultant slip velocity v_r

Since the elementary shear force dF_r acts in a direction described by the angle θ_r to the horizontal plane, its horizontal and vertical components are as follows:

$$dF_r \cos \theta_r = \tau_r b_r r_i d\theta \cos \theta_r \quad (6.2-22)$$

$$dF_r \sin \theta_r = \tau_r b_r r_i d\theta \sin \theta_r \quad (6.2-23)$$

The shear force component $dF_r \cos \theta_r$, given by equation (6.2-22) contributes to the combined tire longitudinal and lateral force developed on the horizontal plane, whereas $dF_r \sin \theta_r$, from equation (6.2-23) acts in the vertical direction and forms part of soil reaction to support tire normal force.

It is required to determine how the horizontal component of the elementary shear force $dF_r \cos \theta_r$, is distributed between longitudinal x -axis (on wheel plane) and lateral y -axis (perpendicular to wheel plane), although the magnitude of the total horizontal force can be obtained by integration of $dF_r \cos \theta_r$, from $\theta = 0$ to $\theta = \theta_{lc}$. Because the tire lateral deformation angle β_r is considered in the formulation of lateral slip velocity v_{ly} , the elementary force $dF_r \cos \theta_r$, can be separated into components in the x - and y -direction from the analysis involving slip velocities. In Figure 6.2-6, the angle ψ'_p between tire longitudinal direction and the projection of v_r (or dF_r) on horizontal plane determines the x - and y -axis force components.

$$\cos \psi'_p = \frac{v_{jx} \cos \theta}{\sqrt{v_{jx}^2 \cos^2 \theta + v_{jy}^2}} \quad (6.2-24)$$

Figure 6.2-7 Coordinate system for defining contact trajectory

An alternative approach to determine tire horizontal force in the x - and y -axis is to examine the trajectory of a point on tire circumference with respect to the terrain surface including the effect of tire lateral deformation y_r . To study the contact trajectories, the coordinate (x', y') is chosen. The $x'y'$ axis system, shown in Figure 6.2-7, is a fixed reference frame on the ground, parallels to the tire xy axis, and the origin is at the projection of the initial position of tire center onto the ground surface.

The mathematical expressions for the trajectory of a point on the tire circumference under combined longitudinal slip i and slip angle α can be found from Schwanghart (1968). Further included the deformation term y_s , the trajectory for a point p defined by an angle θ , as shown in Figure 6.2-8, can be expressed by

$$x' = r_i \left[\left(\frac{\pi}{2} - \theta \right) (1 - i) + \cos \left(\frac{\pi}{2} - \theta \right) \right] \quad (6.2-25)$$

$$y' = r_i \left(\frac{\pi}{2} - \theta \right) (1 - i) \tan \alpha - y, \quad (6.2-26)$$

where θ = tire-terrain contact angle

y_s = tire lateral deformation, equation (6.2-10)

Figure 6.2-8 Trajectory of a point rolling from $\theta = \pi/2$ to $\theta = 0$, on a combined slip tire, with longitudinal slip $i = 10\%$

A schematic trajectory is shown in Figure 6.2-8 for a tire with a radius of 0.498 m, an initial contact angle θ_{ic} of 40° , a slip angle α of -20° , a lateral deformation angle β_t of -6° , and a longitudinal slip i of 10%. On tire circumference, a point p is initially

fixed at $\theta = \pi/2$. As the combined-slip tire rolls, the point moves in the direction of $\bar{v}_0$, the velocity vector of tire center. Starting from $x' = 0.498$ m on the fixed x' - y' plane, the trajectory the point p traced from the position at $\theta = \pi/2$ to the bottom-dead-center at $\theta = 0$ is shown in the figure and highlighted in a boldface curve. The coordinates x' and y' describing the trajectory are given by equations (6.2-25) and (6.2-26), respectively. From the figure, it is noted that not the complete trajectory provides useful information. Instead, only the section of the trajectory within the tire-terrain contact zone is useful. The contact zone is the zone between line C_1C_2 and line D_1D_2 . The zone is also the locus of the moving tire-terrain contact region in the direction of $\bar{v}_0$.

Figure 6.2-9 Trajectories of point p with respect to ground, for $\alpha = -20^\circ$ and $\beta_i = -6^\circ$, and $i = 0, 10, 20$, and 35% , rigid mode

The trajectory $x'y'$ are functions of slip angle α and longitudinal slip i . The variations of trajectory with respect to i when $\alpha = -20^\circ$ and $\beta_i = -6^\circ$ are shown in Figure 6.2-9 which includes trajectories for longitudinal slips of 0, 10, 20, and 35%. The dotted section of each curve represents the trajectory of the circumferential point p not in contact with terrain, whereas the solid section of the curve is the trajectory of p in contact with terrain.

A trajectory within the tire-terrain contact zone (solid curve) is of interest. For example the curve of 35% longitudinal slip in Figure 6.2-9, the solid section represents the trajectory of point p on tire circumference relative to ground when the point rotates within the contact zone. At a point within the solid curve such as the point t_p , the slope dy'/dx' at this point defines the direction of relative motion, or the direction of sliding velocity between the tire and terrain of the corresponding point p on the tire circumference, shown in Figure 6.2-10. Thus, the angle ψ_p given below represents the angle of the tire-terrain relative motion at the circumferential point p . The angle is measured with respect to the x -axis or the wheel plane, and on the horizontal plane.

$$\tan \psi_p = \left(\frac{dy'}{dx'} \right)_p \quad (6.2-27)$$

Figure 6.2-10 Direction of relative motion of circumferential point p on laterally deformed tire relative to terrain

The consistency of the two methods for determining the orientation of elementary shear force $dF_r \cos \theta_r$ on the horizontal plane is examined. Using the same example as mentioned above (the tire with $\alpha = -20^\circ$, $\beta_i = -6^\circ$, and $i = 35\%$), angles ψ'_p (from slip velocity analysis) and ψ_p (from contact trajectory analysis) at point p within a range of θ in tire-terrain contact zone is computed. The comparison between the two results is presented in Figure 6.2-11, in which angles ψ'_p and ψ_p are designated as Pspip' and Pspip , respectively. Although both curves are not coincident, they are sufficiently close. The difference shown in the figure between the results obtained using two different

methods is primarily due to numerical errors. Therefore, the angle ψ_p is chosen for the following analysis.

Figure 6.2-11 Comparison between angles ψ'_p and ψ_p

The direction of an elementary shear force acting on the tire from terrain is opposite to the direction of relative motion described by the angle ψ_p . On the horizontal plane, the longitudinal and lateral components of the elementary shear force $dF_r \cos \theta$, become

$$dF_r \cos \theta, \cos \psi_p = \tau_r b_r r_t d\theta \cos \theta, \cos \psi_p \quad (6.2-28)$$

$$dF_r \cos \theta, \sin \psi_p = \tau_r b_r r_t d\theta \cos \theta, \sin \psi_p \quad (6.2-29)$$

Consequently, tractive force (with longitudinal slip $i > 0$) along the wheel plane F_x , and lateral shear force perpendicular to the wheel plane F_{ysh} are obtained by integrating equations (6.2-28) and (6.2-29), respectively over the contact area of tire-terrain interface from $\theta = 0$ to $\theta = \theta_{IC}$.

$$F_x = b_r r_t \int_0^{\theta_{IC}} \tau_r(\theta) \cos \theta, \cos \psi_p d\theta \quad (6.2-30)$$

$$F_{ysh} = b_i r_i \int_0^{\theta_{ic}} \tau_r(\theta) \cos \theta_r \sin \psi_p d\theta \quad (6.2-31)$$

Figure 6.2-12 Bulldozing force acting on tire sidewall

The lateral bulldozing force acting on the tire sidewall also contributes to part of the tire lateral force from the terrain, as shown in Figure 6.2-12. The bulldozing force, developed by vehicle running gears generally causes a passive failure of terrain (Wong, 1993). Examining Table 5.2-2 and 5.2-3, it can be seen that the angle of rubber-soil friction ϕ_{rs} (29.82°) almost equals the angle of internal shearing resistance of the soil ϕ_s (31.46°). This implies that the tire sidewall surface is considered to be rough relative to the soil. For a bulldozing gear with a rough surface, the frictional soil resistance per unit width of interface P , acting at an angle of ϕ_{rs} to the normal direction of the interface area can be computed by the general soil resistance equation given by Hettiaratchi and Reece (1974).

$$P = \gamma_s z_i^2 K_\gamma + cz_i K_c + q_s z_i K_q \quad (6.2-32)$$

- where γ_s = soil bulk density
 c = apparent cohesion of terrain
 q_s = uniform surcharge pressure
 z_i = coordinate of tire sinkage
 K_γ = soil resistance coefficient, density
 K_c = soil resistance coefficient, cohesion
 K_q = soil resistance coefficient, surcharge

In equation (6.2-32), soil resistance coefficients K_γ , K_c , and K_q are constants and are functions of ϕ_s and ϕ_{rs} , but are independent of z_t and γ_s . The present calculation is based on charts provided by Hettiaratchi and Reece (1974). These coefficients are

$$K_\gamma = 4.20$$

$$K_c = 8.53$$

$$K_q = 6.43$$

Terzaghi et al. (1995) indicated that a slope formed by clean dry sand is stable regardless of its height, provided that the angle between the slope and the horizontal ground surface is equal to or smaller than the angle of soil internal shearing resistance ϕ_s for the sand in a loose state, shown in Figure 6.2-12. The concept is applicable to the proposed tire model since the Ottawa sand in this experimental study is dry and clean, and the sand displaced by the tire is in a loose state.

Figure 6.2-13 Sunken sidewall area

It is assumed that the volume of the surcharge equals the volume of the displaced sand by the sunken portion of the tire. The surcharge volume takes a triangular shape with the angle of the slope equivalent to ϕ_s . Thus the uniform surcharge pressure q_s can be estimated. If the tire moves laterally like a bulldozer blade, the bulldozing force F_{ybl} acting perpendicularly on tire sidewall can be obtained by integration of P over the width of interface, as shown in Figure 6.2-13.

$$\begin{aligned}
F_{yb\perp} &= \cos \phi_{rs} \int_{-r_t \sin \theta_{IC}}^{r_t \sin \theta_{IC}} P dx \\
&= \cos \phi_{rs} \int_{-r_t \sin \theta_{IC}}^{r_t \sin \theta_{IC}} (\gamma_s z_t^2 K_\gamma + c z_t K_c + q_s z_t K_q) dx
\end{aligned} \tag{6.2-33}$$

where $z_t = \sqrt{r_t^2 - x^2} - r_t \cos \theta_{IC}$

Figure 6.2-14 (a) $\alpha = 0^\circ$, no lateral bulldozing force, $F_{yb} = 0$,
(b) $\alpha > 0^\circ$, F_{yb} increases with respect to α ,
(c) $\alpha = 90^\circ$, F_{yb} reaches maximum, $F_{yb} = F_{yb\perp}$

When a rolling tire moves laterally, the tire is actually at a slip angle of $\alpha = 90^\circ$. In this case, as shown in Figure 6.2-14 (c), the perpendicular bulldozing force $F_{yb\perp}$ is fully developed on the tire sidewall. However, as the slip angle reduces, the effective lateral bulldozing force F_{yb} drops and entirely vanishes at $\alpha = 0^\circ$, as shown in Figure 6.2-14 (a). For the slip angle less than 90° , as illustrated in Figure 6.2-14 (b), only part of the perpendicular bulldozing force $F_{yb\perp}$ effectively acts on the tire sidewall. Therefore, the general form of the lateral bulldozing force F_{yb} acting on a sideslip tire can be approximated by

$$F_{yb} = F_{yb\perp} \sin \alpha \tag{6.2-34}$$

The total lateral force F_y when the tire operated in rigid mode on a soft terrain includes the lateral force component due to tire-terrain shearing action and the component from bulldozing effect.

$$F_y = F_{ysh} + F_{yb} \quad (6.2-35)$$

Tire lateral force developed outside the contact region, expressed by the second term of equation (6.2-13) can be computed by knowing the distribution of tire lateral deformation $y_z(\xi)$ in this region. Considering tire circumferential tension, boundary conditions of $y_z(\xi) = y_c = y_s(0)$ at $\xi = L$ and $y_z(\xi) = y_D = y_s(s_D)$ at $\xi = 0$, and for normal cases of $L/l_r = 4.5 \sim 6$, the function of $y_z(\xi)$ has a decayed exponential form on front and on rear free regions (Segel, 1988; Wong, 1993). Tire lateral deformation at the front contact point, y_c has a magnitude of $l_r \tan \beta_t$. This is because the slope of the equatorial line is continuous from the free region to the contact region CD , and both region must have a common tangent at point C . At point D , lateral deformation y_D is obtained from equation (6.2-10), and $y_D = y_c + s_D \tan \beta_t$. For the test tire, the criterion of $L/l_r = 4.5 \sim 6$ is satisfied since the average relaxation length is $l_r = 0.55m$, and a typical tire circumferential length outside the contact region is about $L = 2.8m$. The experiment for determining tire relaxation length is presented in Appendix D. Applying equation (6.2-13), tire internal force F_{yt} due to lateral elasticity is

$$\begin{aligned} F_{yt} &= k_y \int_0^{s_D} (y_c + s \tan \beta_t) ds + k_y \int_0^L (y_c e^{(\xi-L)/l_r} + y_D e^{-\xi/l_r}) d\xi \\ &= k_y \left(l_r s_D + \frac{s_D^2}{2} \right) \tan \beta_t + k_y (2l_r + s_D) \tan \beta_t [l_r (1 - e^{-L/l_r})] \\ &= k_y \tan \beta_t \left(\frac{s_D^2}{2} + 2l_r^2 + 2l_r s_D \right) \end{aligned} \quad (6.2-36)$$

where $s_D = r_t \theta_{tC}$

Next, tire internal force F_{yt} from equation (6.2-36) is compared with tire lateral force F_y obtained from equation (6.2-35). If the following equilibrium equation is satisfied,

$$F_{yt} = F_y \quad (6.2-37)$$

the tire lateral deformation angle β_t is a true value. Otherwise, adjustment on β_t must be made to fulfill the equilibrium. In practice, an iteration (inner loop) is required and a preset error must be achieved before the iteration is terminated. As a consequence, the

magnitudes of F_{xt} and F_{ysh} , expressed by equations (6.2-30) and (6.2-31), respectively, are finalized.

Figure 6.2-15 Position of instantaneous center of slipping tire, rigid mode

The bulldozing force acting on tire sidewall provides a frictional force due to the sliding between the sidewall of rotating tire and the soil. The frictional force includes a vertical and a longitudinal component. The vertical force, if sufficiently large, may affect the equilibrium of tire normal force. The longitudinal force, on the other hand, may give the tire an additional tractive force or resistance. The problem is thus to examine the shearing force between a vertical rotating surface and the soil. To analyze the sidewall frictional force, the instantaneous center of zero velocity I on a slipping tire is first identified. The position of I on a tire subject to a longitudinal slip i is located within the height between the tire center and the lowest contact point (bottom-dead-center), shown in Figure 6.2-15.

The longitudinal slip i , previously given in equation (4.2-1) has the following form.

$$i = 1 - \frac{v_0}{r_t \omega_t} \quad (6.2-38)$$

With respect to the instantaneous center I , the linear velocity of tire center can be expressed as $v_0 = (r_t - h_t) \omega_t$. Thus, the position of I measured from the bottom-dead-center h_t can be obtained from equation (6.2-38).

$$h_t = r_t i \quad (6.2-39)$$

Once the instantaneous center is located, the direction of the instantaneous velocity, or slip velocity $\bar{v}_p$ of an element at an arbitrary point p within the sunken sidewall area is readily found since it must be perpendicular to the line joining point p with I . With the position vector $\bar{\rho}_p$ of point p shown in Figure 6.2-16, the magnitude of the velocity vector $\bar{v}_p$ is written by

$$v_p = \rho_p \omega_t \quad (6.2-40)$$

$$\text{where } \rho_p = \sqrt{x^2 + (z_t - z_{rg} + r_t i)^2} \quad (6.2-41)$$

Figure 6.2-16 Slip velocity vector $\bar{v}_p$ of an element at point p on sidewall

The resultant shear displacement j_p is obtained by integrating slip velocity v_p over the time span t_s of the element coming into contact with the terrain.

$$j_p = \int_0^{t_s} v_p dt \quad (6.2-42)$$

The time span t_s is the finite time required for the element slipping from point p' , the beginning of contact with the terrain to point p .

$$t_s = \frac{\theta_p}{\omega_t} \quad (6.2-43)$$

$$\text{where } \theta_p = \cos^{-1} \frac{2r^2 - d^2}{2r^2} \quad (6.2-44)$$

r = distance between tire center and point p or p' ,

$$r = \sqrt{x^2 + (z_i - z_{rg} + r_i)^2} \quad (6.2-45)$$

$$d = \text{distance between point } p \text{ and } p', \quad d = \sqrt{(x - x')^2 + z_i^2} \quad (6.2-46)$$

$$x' = \sqrt{r^2 - (r_i - z_{rg})^2} \quad (6.2-47)$$

The normal component of the passive earth pressure, or bulldozing pressure p_b at a depth z_i below soil surface is expressed by $(2\gamma_i z_i K_\gamma + cK_c + q_i K_q) \cos \phi_{rs}$, which is obtained by differentiating the normal component of P , given by equation (6.2-32) with respect to z_i . For a sideslip tire with a slip angle α , the effective component of bulldozing pressure normal to the sidewall becomes $p_b \sin \alpha$. The shear stress τ_p developed on the element at point p can be obtained from the shear stress-shear displacement equation shown in equation (5.2-2). It is noted that τ_p is a function of coordinates x and z_i . Parameters c_{rs} , ϕ_{rs} and K_{rs} for tire rubber-soil interface are given in Table 5.2-3.

$$\tau_p = [c_{rs} + (2\gamma_i z_i K_\gamma + cK_c + q_i K_q) \sin \phi_{rs} \sin \alpha] (1 - e^{-j_p/K_{rs}}) \quad (6.2-48)$$

Figure 6.2-17 Elementary shear force dF_p on positive side of x -axis

The elementary shear force dF_p developed on the sidewall element is $\tau_p dx dz_i$, with a direction opposite to that of $\bar{v}_p$ as shown in Figure 6.2-17. The force can be separated into two components, the longitudinal force $dF_p \sin \phi_p$, and the vertical force $dF_p \cos \phi_p$. The angle ϕ_p is computed from the orientation of position vector $\bar{r}_p$ of the element relative to I . For the sidewall region on the positive side of the x -axis, angle ϕ_p is computed by

$$\tan \varphi_p = \frac{z_t - z_{rg} + r_t i}{x} \quad (6.2-49)$$

If point p is above the instantaneous center I shown in the figure, the numerator becomes negative and so does the angle φ_p . The elementary force component $dF_p \sin \varphi_p$ is thus a resistance force. Otherwise, $dF_p \sin \varphi_p$ is a tractive force. However for component $dF_p \cos \varphi_p$, it is a supporting force whether φ_p is positive or negative. To integrate both force components over the sidewall area on the positive side of the x -axis, the resultant longitudinal force F_{xp} and vertical force F_{zp} are

$$F_{xp} = \int_0^{z_{rg}} \int_0^{x(z_i)} \tau_p \sin \varphi_p dx dz_i \quad (6.2-50)$$

$$F_{zp} = \int_0^{z_{rg}} \int_0^{x(z_i)} \tau_p \cos \varphi_p dx dz_i \quad (6.2-51)$$

$$\text{where } x(z_i) = \sqrt{r_t^2 - (z_t - z_{rg} + r_t)^2} \quad (6.2-52)$$

Figure 6.2-18 Elementary shear force dF_q on negative side of x -axis

For an element at a point q located on the negative side of the x -axis as shown in Figure 6.2-18, the same procedure can be followed to compute the shear displacement j_q and the induced shear stress τ_q . The computed elementary shear force component $-dF_q \cos \varphi_q$ is always downwards. The longitudinal component $-dF_q \sin \varphi_q$ is a

resistance when φ_q is positive, or a tractive force when φ_q is negative. Similar to equation (6.2-49), the angle φ_q is given as below.

$$\tan \varphi_q = \frac{z_t - z_{rg} + r_t i}{x} \quad (6.2-53)$$

The resultant longitudinal force F_{xq} and vertical force F_{zq} acting on the rear half of the sidewall region is obtained by integrating the two elementary force components over the sidewall area on the negative side of the x -axis. They have the following forms.

$$F_{xq} = - \int_0^{z_{rg}} \int_{-x(z_i)}^0 \tau_q \sin \varphi_q dx dz_i, \quad (6.2-54)$$

$$F_{zq} = - \int_0^{z_{rg}} \int_{-x(z_i)}^0 \tau_q \cos \varphi_q dx dz_i, \quad (6.2-55)$$

where τ_q = shear stress due to shear displacement j_q

The total shear forces acting on tire sidewall due to bulldozing pressure are F_{xsw} and F_{zsw} for longitudinal and vertical directions, respectively.

$$F_{xsw} = F_{xp} + F_{xq} \quad (6.2-56)$$

$$F_{zsw} = F_{zp} + F_{zq} \quad (6.2-57)$$

Forces F_{xsw} and F_{zsw} are functions of slip angle α and longitudinal slip i , but mainly depend on the latter if other parameters of the tire and soil are fixed. For the case of maximum bulldozing pressure (tire sliding laterally and $\alpha = 90^\circ$) with i varying from 2% to 35%, F_{xsw} is negative, that is, a resistance. The magnitude of F_{xsw} ranges from 1% to 3% of tire normal force F_z . On the other hand, the magnitude of F_{zsw} is nearly zero and barely affects the equilibrium of the tire in vertical direction. In addition, tire slip angles are much less than 90° under normal operations and the effective bulldozing pressure drops with the decrease of slip angle so that F_{zsw} is even smaller. As a consequence, the force F_{zsw} can be neglected in the analysis. Without considering the

effect of F_{xsw} , the contact angle θ_{IC} is determined from satisfying the equilibrium of vertical forces acting on the tire given below.

$$F_z = b_t r_t \left[\int_0^{\theta_{IC}} p_n(\theta) \cos \theta d\theta + \int_0^{\theta_{IC}} \tau_r(\theta) \sin \theta d\theta \right] \quad (6.2-58)$$

The first term in the bracket represents the contribution of the vertical component of the normal pressure given by equation (6.2-19), whereas the second term refers to the contribution of the vertical component of the shear stress expressed by equation (6.2-23). An iteration scheme (outer loop) is used to solve the above equilibrium equation, and to determine the actual tire sinkage z_{rg} or tire contact angle θ_{IC} . The compaction resistance R_c is subsequently computed by integrating the horizontal component of the normal pressure over the entire contact region.

$$\begin{aligned} R_c &= b_t r_t \int_0^{\theta_{IC}} p_n(\theta) \sin \theta d\theta \\ &= b_t \left[\left(\frac{k_c}{b_t} + k_\phi \right) \frac{z_{rg}^{n+1}}{n+1} \right] \end{aligned} \quad (6.2-59)$$

Tire longitudinal tractive force F_x in the horizontal direction can not be measured directly using the tire test facility TTF97. This is because there are three external forces exerted on the tire-terrain interface. One is the tractive force F_x created by the forced-slip mechanism, the second is the motion resistance R_c from the terrain, and the third is the longitudinal component of shear force F_{xsw} acting on tire sidewall. These three forces are combined and inseparable. Fortunately, the resultant of the tractive force, motion resistance, and longitudinal component of shear force on sidewall is the drawbar pull F_{dbp} , which is a measurable quantity. Therefore, the validation of the off-road tire model relies on the comparison of the relationship of normalized F_y versus normalized F_{dbp} between predicted results and experimental data. From equations (6.2-30), (6.2-59), and (6.2-56), the drawbar pull can be written as

$$F_{dbp} = b_t r_t \int_0^{\theta_{IC}} \tau_r(\theta) \cos \theta_r \cos \psi_p d\theta - b_t r_t \int_0^{\theta_{IC}} p_n(\theta) \sin \theta d\theta - F_{xsw} \quad (6.2-60)$$

6.3 Elastic Operating Mode

Referring to Figure 6.3-1, when a tire encounters a soft terrain in elastic operating mode, two different deformation profiles are formed along the tire circumference in the wheel plane. These portions are the front arc CB , and the flat portion BA . In order to predict longitudinal and lateral forces of a tire under combined longitudinal slip and slip angle, it is desired to compute the resultant shear force developed on each contact portion using different methods. The total shear force is the sum of the resultant shear force from the arc CB , and the flat portion BA .

Figure 6.3-1 Tire deformation along wheel plane, and lateral deformation due to carcass and tread elasticity

Similar to that of the rigid mode, tire lateral deformation y_t in the contact region and y_s in the free region must be taken into account for sideslip tires, and the y_s constitutes part of the slip angle α , shown in Figure 6.3-2. By assuming an initial tire lateral deformation angle β_t , in the contact region tire lateral deformation y_t due to lateral elasticity is modeled by the following equation.

$$y_s(l) = y_c + l \tan \beta_t \quad (6.3-1)$$

where l = coordinate representing s , along the contact arc for front arc CB ,
and representing λ , a linear coordinate for flat portion BA

Figure 6.3-2 Slip angle resulting from tire lateral deformation and lateral shear displacement in contact region, elastic mode

It is noted that tire lateral deformation y_s increasing with the coordinate l till the rearmost contact point A and reaching the maximum at point A are assumptions. Beyond point A , the lateral deformation y_s reduces due to tire circumferential or hoop tension, and the equatorial line gradually returns to the wheel plane as shown in Figure 6.3-2. Similar to equation (6.2-13), the resulting tire lateral force F_{yt} due to lateral deformation around the complete circumference is given by

$$F_{yt} = k_y \int_0^{s_B} y_{sCB}(s) ds + k_y \int_0^{2l_f} y_{sBA}(\lambda) d\lambda + k_y \int_0^L y_s(\xi) d\xi \quad (6.3-2)$$

where k_y = tire lateral stiffness

y_s = tire lateral deformation,

subscripts CB and BA refer to contact portions

s_B = coordinate s at point B

l_f = half of contact length on flat portion BA

L = tire circumferential length outside contact region

ξ = coordinate along tire circumference outside contact region, starting from point A to point C

The first term on the right-hand side of equation (6.3-2) is for the front contact arc CB , the second for the flat portion BA , and the third term refers to tire circumference outside the contact region. Tire lateral stiffness k_y was determined from an experiment described in Appendix D. For tire inflation pressure $p_i = 1.0$ bar, lateral stiffness k_y is 90.80 kPa. The force F_{yt} described by equation (6.3-2) represents the resultant tire internal force due to lateral elasticity.

The trajectory of a point on the circumference of a tire in elastic operating mode can be obtained in a similar way as that of a tire in rigid operating mode. Tire lateral deformation y_s is included, and the tire is subjected to a longitudinal slip i and a slip angle α . The trajectory for each portion is given below. The (x', y') coordinate system, as shown in Figure 6.2-7, is chosen to describe the contact trajectories.

From point C to point B , $\theta_B \leq \theta \leq \theta_{IC}$

$$x'_{CB} = r_t \left[\left(\frac{\pi}{2} - \theta \right) (1 - i) + \cos \left(\frac{\pi}{2} - \theta \right) \right] \quad (6.3-3)$$

$$y'_{CB} = r_t \left(\frac{\pi}{2} - \theta \right) (1 - i) \tan \alpha - y_{sCB} \quad (6.3-4)$$

where $(x'_{CB}, y'_{CB}) =$ trajectory for front contact arc CB

$$y_{sCB} = y_C + s \tan \beta_t \quad (6.3-5)$$

$$s = r_t (\theta_{IC} - \theta) \quad (6.3-6)$$

From point B to point A , $0 < \lambda \leq 2l_f$

$$x'_{BA} = x'_B - i\lambda \quad (6.3-7)$$

$$y'_{BA} = y'_B + (1 - i)\lambda \tan \alpha - y_{sBA} \quad (6.3-8)$$

where $(x'_{BA}, y'_{BA}) =$ trajectory for flat portion BA

$(x'_B, y'_B) =$ trajectory for point B , obtained by substituting $\theta = \theta_B$
into equations (6.3-3) and (6.3-4)

$l_f =$ half of contact length on flat portion BA ,

$$l_f = \sqrt{z_d(2r_t - z_d)} \quad (6.3-9)$$

$$y_{sBA} = y_B + \lambda \tan \beta_t \quad (6.3-10)$$

λ = horizontal coordinate, point B to point A ,

$$\lambda = r_t \cos \theta_B (\tan \theta_B - \tan \theta) \quad (6.3-11)$$

Tire vertical deflection z_d , or the flat contact length $2l_f$ is determined from the vertical force equilibrium of the tire using an iteration scheme, which will be discussed later. For initializing the iteration process, the starting value of z_d may be taken as the difference between the maximum tire sinkage in rigid mode z_{rg} and that in elastic mode z_{el} . The latter is mainly determined by the average ground pressure p_{gr} , which was defined previously in Section 6.1. The tire sinkage in either rigid or elastic operating mode can be predicted by methods developed by Bekker (1960), and by Wong (1989, 1993). The deflection z_d can be written as

$$\begin{aligned} z_d &= z_{rg} - z_{el} \\ &= \left[\frac{3F_z}{b_t(3-n)(k_c/b_t + k_\phi)\sqrt{d_t}} \right]^{\frac{2}{2n+1}} - \left(\frac{p_{gr}}{k_c/b_t + k_\phi} \right)^{\frac{1}{n}} \end{aligned} \quad (6.3-12)$$

The contact angle at point B , θ_B and the initial contact angle θ_{IC} are determined from tire-terrain geometrical relationships.

$$\sin \theta_B = \frac{\sqrt{z_d(2r_t - z_d)}}{r_t} \quad (6.3-13)$$

$$\cos \theta_{IC} = \frac{r_t - z_d - z_{el}}{r_t} \quad (6.3-14)$$

Similar to that of the rigid mode, Figure 6.3-3 shows the plane view of trajectories of a point p rotating from the position at $\theta = \pi/2$ to the rearmost contact point at $\theta = -\theta_B$ (or $\lambda = 2l_f$) while the tire rolls at a given slip angle, and various longitudinal slips. Figure 6.3-3 is plotted based on the following assumed parameters.

$$r_t = 0.498m$$

$$\theta_{IC} = 32^\circ$$

$$\begin{aligned}
\theta_B &= 19^\circ \\
\beta_i &= -6^\circ \\
\alpha &= -20^\circ \\
i &= 0, 10, 20, \text{ and } 35\%
\end{aligned}$$

Figure 6.3-3 Trajectories of point p with respect to ground, for $\alpha = -20^\circ$ and $\beta_i = -6^\circ$, and $i = 0, 10, 20$, and 35% , elastic mode

Referring to Figure 6.3-1, the derivation of tire force is carried out separately for each contact region. They are 1) front arc CB , and 2) flat portion BA .

1) Front Arc CB

The method for computing shear displacements for this portion is the same as that for a tire in rigid operating mode. The shear displacement j_{xCB} along the tire circumference and in the x - z plane is calculated by

$$j_{xCB} = r_t [(\theta_{IC} - \theta) - (1 - i) \cos \alpha (\sin \theta_{IC} - \sin \theta)] \quad (6.3-15)$$

It is noted that equation (6.3-15) is valid within the arc CB , from $\theta = \theta_B$ to $\theta = \theta_{IC}$. The lateral shear displacement starts from the initial tire-terrain contact point C and increases along the arc to point B , and then to the rearmost point A . It is reasonable to assume that tire lateral deformation is also varied in the same pattern. Considering the tire lateral deformation angle β_i and the lateral slip velocity v_{jyCB} the same as those in rigid mode, the lateral shear displacement in arc CB , j_{yCB} is obtained by integrating v_{jyCB}/ω_i from θ to θ_{IC} . Both v_{jyCB} and j_{yCB} are given below.

$$v_{jyCB} = r_i \omega_i (1 - i) \sin(\alpha - \beta_i) \quad (6.3-16)$$

$$j_{yCB} = r_i (1 - i) \sin(\alpha - \beta_i) (\theta_{IC} - \theta) \quad (6.3-17)$$

Thus, the resultant shear displacement j_{rCB} which is located on the plane formed by j_{xCB} and j_{yCB} , is expressed by

$$j_{rCB} = \sqrt{j_{xCB}^2 + j_{yCB}^2} \quad (6.3-18)$$

The normal pressure distribution on the tire-terrain interface is assumed to follow Bekker's pressure-sinkage relationship.

$$p_{nCB} = \left(\frac{k_c}{b_i} + k_\phi \right) [r_i (\cos \theta - \cos \theta_{IC})]^n \quad (6.3-19)$$

The magnitude of shear stress at a position described by an angle θ is computed by inserting j_{rCB} into the shear stress-shear displacement equation.

$$\tau_{rCB} = [c + p_{nCB}(\theta) \tan \phi_s] (1 - e^{-j_{rCB}(\theta)/K}) \quad (6.3-20)$$

On an arbitrary element of area $b_i r_i d\theta$, an elementary shear force dF_{rCB} is developed. Similar to the case of tires in rigid operating mode, the direction of dF_{rCB} is determined by considering slip velocities v_{jxCB} and v_{jyCB} in a three-dimensional space. The angle θ_{rCB} between the resultant slip velocity vector v_{rCB} and the horizontal plane is given by

$$\cos \theta_{rCB} = \frac{\sqrt{v_{jxCB}^2 \cos^2 \theta + v_{jyCB}^2}}{v_{rCB}} \quad (6.3-21)$$

$$\text{where } v_{jxCB} = r_i \omega_i [1 - (1 - i) \cos \alpha \cos \theta] \quad (6.3-22)$$

$$v_{rCB} = (v_{jxCB}^2 + v_{jyCB}^2)^{1/2} \quad (6.3-23)$$

The horizontal and vertical components of the elementary shear force dF_{rCB} can be written as

$$dF_{rCB} \cos \theta_{rCB} = \tau_{rCB} b_i r_i d\theta \cos \theta_{rCB} \quad (6.3-24)$$

$$dF_{rCB} \sin \theta_{rCB} = \tau_{rCB} b_i r_i d\theta \sin \theta_{rCB} \quad (6.3-25)$$

Referring to Figure 6.3-3, for example the solid curve of $i = 20\%$, the slope dy'_{CB}/dx'_{CB} at an arbitrary trajectory point t_{pCB} defines the direction of sliding velocity between the point p within the arc CB and the terrain. Coordinates x'_{CB} and y'_{CB} are computed from equations (6.3-3) and (6.3-4), respectively. Thus, the angle ψ_{pCB} defining the direction of sliding velocity of tire circumferential point p becomes

$$\tan \psi_{pCB} = \left. \frac{dy'_{CB}}{dx'_{CB}} \right)_p \quad (6.3-26)$$

The longitudinal and lateral components of the horizontal shear force $dF_{rCB} \cos \theta_{rCB}$ are thus given below.

$$dF_{rCB} \cos \theta_{rCB} \cos \psi_{pCB} = \tau_{rCB} b_i r_i d\theta \cos \theta_{rCB} \cos \psi_{pCB} \quad (6.3-27)$$

$$dF_{rCB} \cos \theta_{rCB} \sin \psi_{pCB} = \tau_{rCB} b_i r_i d\theta \cos \theta_{rCB} \sin \psi_{pCB} \quad (6.3-28)$$

Consequently, tractive force (with longitudinal slip $i > 0$) F_{xCB} and lateral force F_{yCB} are obtained by integrating equations (6.3-27) and (6.3-28), respectively along the contact arc of the tire-terrain interface from $\theta = \theta_B$ to $\theta = \theta_{IC}$.

$$F_{x_{CB}} = b_t r_t \int_{\theta_B}^{\theta_C} \tau_{r_{CB}}(\theta) \cos \theta_{r_{CB}} \cos \psi_{p_{CB}} d\theta \quad (6.3-29)$$

$$F_{y_{CB}} = b_t r_t \int_{\theta_B}^{\theta_C} \tau_{r_{CB}}(\theta) \cos \theta_{r_{CB}} \sin \psi_{p_{CB}} d\theta \quad (6.3-30)$$

2) Flat Portion BA

For the flat portion BA of the tire-terrain contact region, the calculation of longitudinal shear displacement is different from that of a rigid wheel. From Figure 6.3-4, the velocity of tire center in the wheel plane is $\bar{v}_0 \cos \alpha$. At steady state, the tire rolls at a constant angular velocity ω_t along the wheel plane. Since the tire circumference may be regarded as a continuous elastic band, at any point p within the flat portion BA , its velocity relative to the tire center $v_{p/0,x}$ may be assumed to be $r_t \omega_t$. Based on the principle of relative motion, the absolute velocity $\bar{v}_{p,x}$ of point p relative to terrain is then

$$\bar{v}_{p,x} = \bar{v}_0 \cos \alpha + \bar{v}_{p/0,x} \quad (6.3-31)$$

Figure 6.3-4 Longitudinal slip velocity on flat portion BA

The absolute velocity $\bar{v}_{p,x}$ is designated as the slip velocity v_{jxBA} of an elementary area $b_i d\lambda$ at the tire circumferential point p , with respect to terrain. When the tire slips longitudinally, v_{jxBA} is developed on the ground plane and in the direction opposite to that of tire motion. The magnitude of v_{jxBA} is

$$\begin{aligned} v_{jxBA} &= r_i \omega_i - r_i \omega_i (1-i) \cos \alpha \\ &= r_i \omega_i [1 - (1-i) \cos \alpha] \\ &= v_i [1 - (1-i) \cos \alpha] \end{aligned} \quad (6.3-32)$$

where v_i = theoretical velocity of tire center along wheel plane,

$$v_i = r_i \omega_i \quad (6.3-33)$$

Within the flat portion from B to A , the increment of longitudinal shear displacement Δj_{xBA} of a point p is defined in terms of coordinate λ , the origin of which is at point B .

$$\begin{aligned} \Delta j_{xBA} &= \int_0^{\lambda} v_{jxBA} dt \\ &= \int_0^{\lambda} v_i [1 - (1-i) \cos \alpha] \frac{d\lambda}{v_i} \\ &= \lambda [1 - (1-i) \cos \alpha] \end{aligned} \quad (6.3-34)$$

Since the shearing action starts from point C defined by the angle θ_{iC} , the cumulative longitudinal shear displacement j_{xBA} is therefore the sum of shear displacement at point B and the increment Δj_{xBA} .

$$j_{xBA} = j_{xB} + \Delta j_{xBA} \quad (6.3-35)$$

Referring to equation (6.3-15), the shear displacement j_{xB} at point B is found by substituting θ in the equation with θ_B .

$$j_{xB} = r_i [(\theta_{iC} - \theta_B) - (1-i) \cos \alpha (\sin \theta_{iC} - \sin \theta_B)] \quad (6.3-36)$$

The lateral shear displacement within the flat portion BA , j_{yBA} is the sum of lateral shear displacement j_{yB} at point B and the increment of lateral shear displacement Δj_{yBA} developed on this portion.

$$j_{yBA} = j_{yB} + \Delta j_{yBA} \quad (6.3-37)$$

The lateral shear displacement j_{yB} is obtained by inserting $\theta = \theta_B$ into equation (6.3-17).

$$j_{yB} = r_i(1-i)\sin(\alpha - \beta_i)(\theta_{IC} - \theta_B) \quad (6.3-38)$$

The absolute lateral velocity, or lateral slip velocity v_{jyBA} for a point p on the flat portion BA and defined by coordinate λ , is given below. The increment in lateral displacement Δj_{yBA} from point B to point A is obtained by integrating v_{jyBA}/v_i from 0 to λ .

$$\begin{aligned} v_{jyBA} &= r_i \omega_i (1-i) \sin(\alpha - \beta_i) \\ &= v_i (1-i) \sin(\alpha - \beta_i) \end{aligned} \quad (6.3-39)$$

$$\begin{aligned} \Delta j_{yBA} &= \int_0^i v_{jyBA} dt \\ &= \int_0^\lambda v_i (1-i) \sin(\alpha - \beta_i) \frac{d\lambda}{v_i} \\ &= \lambda (1-i) \sin(\alpha - \beta_i) \end{aligned} \quad (6.3-40)$$

After the computation of j_{xBA} and j_{yBA} , the resultant shear displacement j_{rBA} , a function of λ is obtained from the following equation.

$$j_{rBA} = \sqrt{j_{xBA}^2 + j_{yBA}^2} \quad (6.3-41)$$

Since the point p , in this case is an arbitrary point on the flat contact portion BA , the resultant j_{rBA} is on the horizontal plane. For a flat contact region, the normal pressure distribution p_n becomes a constant according to Bekker's pressure-sinkage concept, and equals the average ground pressure p_{gr} . The shear stress developed at point p is therefore written in the form

$$\tau_{rBA} = [c + p_{gr} \tan \phi_s] (1 - e^{-j_{rBA}(\lambda)/K}) \quad (6.3-42)$$

Referring to the contact trajectory of, for example $i = 20\%$ as shown in Figure 6.3-3, the slope dy'_{BA}/dx'_{BA} at an arbitrary trajectory point t_{pBA} defines the direction of sliding velocity between the tire element at point p and the terrain. Trajectory coordinates x'_{BA} and y'_{BA} are obtained from equations (6.3-7) and (6.3-8), respectively. The angle ψ_{pBA} defining the tangential direction of the sliding velocity for point p on the horizontal plane is given by

$$\tan \psi_{pBA} = \left. \frac{dy'_{BA}}{dx'_{BA}} \right)_p \quad (6.3-43)$$

For point p defined by coordinate λ , the angle ψ_{pBA} also determines the direction of the resultant shear force at this point. For the element with area $b_t d\lambda$ at this position shown in Figure 6.3-4, the horizontal shear force dF_{rBA} has a magnitude of $\tau_{rBA} b_t d\lambda$. The longitudinal and lateral components of this elementary shear force can be written as

$$dF_{rBA} \cos \psi_{pBA} = \tau_{rBA} b_t d\lambda \cos \psi_{pBA} \quad (6.3-44)$$

$$dF_{rBA} \sin \psi_{pBA} = \tau_{rBA} b_t d\lambda \sin \psi_{pBA} \quad (6.3-45)$$

Tractive force (with longitudinal slip $i > 0$) along the wheel plane F_{xBA} , and lateral force perpendicular to the wheel plane F_{yBA} are obtained by integrating equations (6.3-44) and (6.3-45), respectively along the flat portion from $\lambda = 0$ at point B to $\lambda = 2l_f$ at point A .

$$F_{xBA} = b_t \int_0^{2l_f} \tau_{rBA}(\lambda) \cos \psi_{pBA} d\lambda \quad (6.3-46)$$

$$F_{yBA} = b_t \int_0^{2l_f} \tau_{rBA}(\lambda) \sin \psi_{pBA} d\lambda \quad (6.3-47)$$

For a tire operating on soft terrain in elastic operating mode, the total tractive (longitudinal) force F_x is the sum of longitudinal forces developed on arc CB and on flat

portion BA . Similarly, the total lateral force due to shearing F_{ysh} , is the sum of lateral shear forces generated on these two portions. Consequently, tire shear forces developed in directions of x -axis and y -axis, respectively, are as follows:

$$F_{xt} = F_{xCB} + F_{xBA} \quad (6.3-48)$$

$$F_{ysh} = F_{yCB} + F_{yBA} \quad (6.3-49)$$

Figure 6.3-5 Sidewall area of an elastic tire subject to lateral bulldozing force

For a tire operating in elastic mode, the area on which the soil against tire sidewall can be modeled by the difference of sidewall projection areas between two rigid wheels with different sinkage, as illustrated in Figure 6.3-5. The lateral force acting perpendicularly on tire sidewall $F_{yb\perp}$ due to bulldozing can be obtained from the bulldozing force on the large area A_1 subtracting that on the small area A_2 .

$$F_{yb\perp} = \cos \phi_{rs} \left(\int_{-r_t \sin \theta_{IC}}^{r_t \sin \theta_{IC}} P dx - \int_{-r_t \sin \theta_B}^{r_t \sin \theta_B} P dx \right) \quad (6.3-50)$$

$$= \cos \phi_{rs} \left[\int_{-r_t \sin \theta_{IC}}^{r_t \sin \theta_{IC}} (\gamma_s z_t^2 K_\gamma + c z_t K_c + q_s z_t K_q) dx - \int_{-r_t \sin \theta_B}^{r_t \sin \theta_B} (\gamma_s z_t^2 K_\gamma + c z_t K_c + q_s z_t K_q) dx \right]$$

where $z_t = \sqrt{r_t^2 - x^2} - r_t \cos \theta_{IC}$

Similar to the case for a tire operating in rigid mode discussed previously, the effective bulldozing force F_{yb} contributing to the lateral force for a tire in elastic mode with a slip angle $0^\circ < \alpha < 90^\circ$ is

$$F_{yb} = F_{yb\perp} \sin \alpha \quad (6.3-51)$$

The total lateral force F_y when the tire operating in elastic mode on a soft terrain includes the lateral force component due to tire-terrain shearing action and the component from bulldozing effect.

$$F_y = F_{ysh} + F_{yb} \quad (6.3-52)$$

The total tire-terrain contact length l_t along tire circumference is the sum of curve length s_B of the front arc CB , and longitudinal length $2l_f$ of the flat portion BA .

$$l_t = s_B + 2l_f \quad (6.3-53)$$

$$\text{where } s_B = r_t(\theta_{tC} - \theta_B) \quad (6.3-54)$$

Tire lateral force developed outside the contact region, expressed by the third term of equation (6.3-2) can be computed by a similar procedure as that used in tire rigid mode. By identifying the two boundary conditions $y_\xi(\xi) = y_C = y_{sCB}(0)$ at $\xi = L$ (front contact point) and $y_\xi(\xi) = y_A = y_{sBA}(2l_f)$ at $\xi = 0$ (rearmost contact point), tire internal force F_{yt} due to lateral elasticity is obtained by applying equation (6.3-2).

$$F_{yt} = k_y \int_0^{s_B} (y_C + s \tan \beta_t) ds + k_y \int_0^{2l_f} (y_B + \lambda \tan \beta_t) d\lambda + k_y \int_0^L (y_C e^{(\xi-L)/l_r} + y_A e^{-\xi/l_r}) d\xi \quad (6.3-55)$$

Tire lateral deformation at contact points C is the same as equation (6.2-11), for points B and A are calculated from equations (6.3-5), and (6.3-10), respectively. They are:

$$y_B = y_C + s_B \tan \beta_t = (l_r + s_B) \tan \beta_t \quad (6.3-56)$$

$$y_A = y_B + 2l_f \tan \beta_t = (l_r + s_B + 2l_f) \tan \beta_t \quad (6.3-57)$$

After integration, equation (6.3-55) can be simplified to

$$\begin{aligned} F_{yt} &= k_y \left(l_r s_B + \frac{s_B^2}{2} \right) \tan \beta_t + k_y (2l_r l_f + 2s_B l_f + 2l_f^2) \tan \beta_t \\ &\quad + k_y (2l_r + s_B + 2l_f) \tan \beta_t \left[l_r (1 - e^{-L/l_r}) \right] \\ &= k_y \tan \beta_t \left[\frac{(s_B + 2l_f)^2}{2} + 2l_r^2 + 2l_r (s_B + 2l_f) \right] \\ &= k_y \tan \beta_t \left(\frac{l_r^2}{2} + 2l_r^2 + 2l_r l_t \right) \end{aligned} \quad (6.3-58)$$

At steady state, tire internal force F_{yt} due to lateral tread and carcass deformation, obtained from equation (6.3-58) must equal tire lateral force F_y that developed in the tire-terrain interface because of shearing, and on the tire sidewall due to bulldozing effect, given by equation (6.3-52).

$$F_{yt} = F_y \quad (6.3-59)$$

The exact value of the tire lateral deformation angle β_t , shown in equation (6.3-58) is determined by an iteration (inner loop) to fulfill the above lateral force equilibrium equation. Accordingly, shear forces F_{xt} and F_{ysh} , given by equations (6.3-48) and (6.3-49), respectively, are determined.

The sidewall shear force can be computed by the net shear force acting on the sidewall area A_1 , and on area A_2 . For area A_1 , the longitudinal and vertical shear forces F_{xswA1} and F_{zswA1} can be obtained from the analysis of rigid operating mode. The procedure includes equations from (6.2-40) to (6.2-57) by substituting z_{rg} with $z_d + z_d$. Similarly for area A_2 , the longitudinal and vertical shear forces F_{xswA2} and F_{zswA2} can be computed from the same equations by substituting z_{rg} with z_d .

The longitudinal and vertical components F_{xsw} and F_{zsw} of the total shear force acting on tire sidewall due to bulldozing pressure are:

$$F_{xsw} = F_{xswA1} - F_{xswA2} \quad (6.3-60)$$

$$F_{zsw} = F_{zswA1} - F_{zswA2} \quad (6.3-61)$$

Similar to those of the rigid mode, both F_{xsw} and F_{zsw} are small. The vertical force F_{zsw} is nearly zero so that it is ignored. However, the longitudinal component F_{xsw} should be included in the formulation of the resultant tire longitudinal force.

The equilibrium of tire vertical force described in the following equation determines the contact geometry of a tire in elastic operating mode.

$$F_z = b_t r_t \left[\int_{\theta_B}^{\theta_{IC}} p_{nCB}(\theta) \cos \theta d\theta + \int_{\theta_B}^{\theta_{IC}} \tau_{rCB}(\theta) \sin \theta_{rCB} d\theta \right] + 2b_t l_f p_g \quad (6.3-62)$$

The first term in the bracket represents the resultant vertical reaction due to the normal pressure acting on the front arc CB , whereas the second term in the bracket refers to the resultant vertical reaction because of the shear stress acting on CB . The last term of the equation represents the vertical reaction owing to the average ground pressure acting on the flat portion BA . A second iteration (outer loop) is employed to determine l_f , the half of the flat contact length in the above equation. Accordingly, tire vertical deflection z_d is calculated from equation (6.3-9), angle θ_B from equation (6.3-13), and angle θ_{IC} from equation (6.3-14).

The compaction resistance R_c is subsequently computed from the horizontal component of normal pressure (Wong, 1993). In terms of the maximum tire sinkage in elastic mode z_{el} , that is

$$R_c = b_t \left[\left(\frac{k_c}{b_t} + k_\phi \right) \frac{z_{el}^{n+1}}{n+1} \right] \quad (6.3-63)$$

The validation of the tire model for tires operating in elastic mode is conducted by comparing the relationship of normalized F_y versus normalized drawbar pull F_{dbp} between predicted results and experimental data. The drawbar pull is calculated from

$$F_{dbp} = F_{xt} - R_c + F_{xsw} \quad (6.3-64)$$

In equation (6.3-64), F_{xt} , given by equation (6.3-48) is the tractive (longitudinal) force developed on the tire-terrain interface. The compaction resistance R_c and the longitudinal shear force on tire sidewall F_{xsw} are expressed by equations (6.3-63) and (6.3-60), respectively. All forces are developed along tire longitudinal direction.

6.4 Validation

The proposed off-road tire model described in Section 6.2 and 6.3 predicts forces for a tire subject to a combined longitudinal slip and slip angle. Elastic operating mode of the tire model was developed for tire inflation pressure p_i of 1.0 bar, whereas rigid operating mode for 2.0 and 3.0 bar, according to the criteria in Appendix C. The tire model was substantiated by comparing the predicted results with experimental data presented in Chapter 5. In particular, the relationships between the normalized lateral force F_y/F_z and the normalized drawbar pull F_{dbp}/F_z at various slip angles α were examined. The correlation between the predicted and measured drawbar pull F_{dbp} indirectly substantiates the validity of the prediction of tractive force F_{xt} , compaction resistance R_c , and longitudinal shear force on tire sidewall F_{xsw} . It should be noted that the relationship between tire lateral force F_y and drawbar pull F_{dbp} at various slip angles is an important input for simulating the handling behaviour of an all-wheel-drive, off-road vehicle.

Figure 6.4-1 shows the comparisons of the predicted and measured relations for F_y/F_z versus F_{dbp}/F_z at $\alpha = 0^\circ \sim 20^\circ$ with tire inflation pressures $p_i = 1.0, 2.0$, and 3.0 bar. The experimental data, shown in solid lines were the curves presented in Figure 5.1-7. The predicted results, in dash curves, were obtained from equations (6.3-52) and (6.3-64) for $p_i = 1.0$ bar, and from equations (6.2-35) and (6.2-60) for $p_i = 2.0$ and 3.0 bar.

Figure 6.4-1 F_y / F_z vs. F_{dbp} / F_z , $p_i = 1$ bar, 2 bar and 3 bar, predictions comparing with measurements

The figures show that the trends of predicted curves at slip angles α above 10° demonstrate a better agreement with those measured. However, for slip angles lower than 10° , the trends of predicted curves show less agreement with those measured. In general, the agreement improves as F_{dbp}/F_z increases from -0.10 to 0.10 . The reason of this is probably that at lower or negative F_{dbp}/F_z , which indicates low longitudinal slips, instead of slipping, tire skidding occurs. It is also observed that the group of predicted curves of $p_i = 1.0$ bar shows different characteristics from those of $p_i = 2.0$ and 3.0 bar, and the latter two are quite similar. This is because at both 2.0 and 3.0 bar, the tire is predicted in the rigid operating mode. The only difference is the magnitudes of the tire lateral stiffness, 95.58 kPa for $p_i = 2.0$ bar and 99.95 kPa for $p_i = 3.0$ bar. Therefore substantiates the assumptions of different tire operating modes in the wheel plane, in relation to the terrain characteristics.

The correlation between the measurement data and the predictions by the proposed off-road tire model may be examined by a statistical means. For problems of vehicle running gears operating on unprepared terrain, a typical method for calculating the *goodness-of-fit* can be found from publications by Wong (1980, 1989). The goodness-of-fit ε was calculated according to the following definition.

$$\varepsilon = 1 - \frac{\sqrt{[\sum (F_{ym} - F_{yp})^2] / (N - 2)}}{\sum F_{ym} / N} \quad (6.4-1)$$

where F_{ym} = measured tire lateral force

F_{yp} = predicted tire lateral force

N = number of data points

The numerator of equation (6.4-1) is the *standard error of estimate*, or the *standard deviation of the residuals* (Norusis, 1990). It computes the variability of the measured or observed F_{ym} values from the predicted F_{yp} values. The denominator is the mean of all measured F_{ym} . Thus equation (6.4-1) gives the goodness-of-fit which equals 1 minus the ratio of standard error of estimate to the mean. For a perfect fit, $\varepsilon = 1$, when the standard error of estimate becomes zero. Table 6.4-1 lists the values of ε corresponding to all curves shown in Figure 6.4-1.

Table 6.4-1 Goodness-of-fit ε , predictions to measurements

Tire Inflation Pressure [bar]	Slip Angle						
	0°	2°	4°	6.5°	10°	15°	20°
1.0	-	0.287	0.591	0.720	0.817	0.930	0.935
2.0	-	0.324	0.628	0.709	0.760	0.827	0.903
3.0	-	0.347	0.643	0.718	0.766	0.831	0.905

The range of the goodness-of-fit increases from approximately 0.3 to 0.9 as slip angle increases from 2° to 20°, although discrepancies between predictions and measurements for different combinations of inflation pressure and slip angle are similar. This can be realized by examining equation (6.4-1). For most combinations, the absolute values of deviation do not change dramatically so that the standard errors of estimate have similar values. Therefore, if the mean value of a set of data increases, the goodness-of-fit increases. Additionally, it can be seen that all values of ε for $p_i = 3.0$ bar are slightly higher than those for $p_i = 2.0$ bar. This indicates that when tire inflation pressure increases, the rigid-mode tire model becomes more applicable.

The discrepancies between predictions and measurements may be attributed to the modeling of the terrain, tire, and tire-terrain interaction. There were six soil parameters (n , k_c , k_ϕ , c , ϕ , and K) used as input to the proposed tire model. All of these parameters were obtained from curve fitting of soil measurements. The curve-fitting process incurs errors, as can be seen in Figure 5.2-3. The figure shows that at sinkage of 150 mm, the measured normal pressure from different tests varied from 290 to 380 kPa. Similar variation was found for shear stress tests as shown in Figure 5.2-5. Therefore, it was unavoidable that parameters derived from curve fitting of experimental data have inherent errors. Furthermore, the soil was never homogeneous and its density may vary.

The discrepancies may also be due to the geometrical modeling of the tire. It is noted that, in the elastic operating mode, the contact profile of the tire-terrain interface in the wheel plane was modeled by a front arc and a flat portion, as shown in Figure 6.4-2 (a). However, in reality, a reasonable pattern of contact profile in the wheel plane may be a continuous curve as shown in Figure 6.4-2 (b).

Figure 6.4-2 (a) Modeling of tire contact profile in elastic operating mode,
(b) Tire contact profile represented by a continuous curve

Similarly, the geometry of the tire cross section under normal load was approximated by straight lines and right angles that may differ from the cross-sectional shape of a real loaded tire, as shown in Figure 6.4-3 (a) and (b), respectively. The geometry of the curved sidewall was not taken into consideration in the tire model.

Figure 6.4-3 (a) Modeling of tire cross section in this study,
(b) Actual cross section of the test tire

The modeling of tread pattern was excluded in the tire model. The test tire, designated as XL and used mainly for military vehicles, has short lugs or shallow treads, as shown in Figure 6.4-4. The depth of the treads is 16 mm. However, in the proposed tire model, it was assumed that the tread surface is smooth, and the width of the tire is constant and equals the tread width.

Figure 6.4-4 Tread pattern of Michelin XL

Figure 6.4-5 Lateral deformation of sideslip tire, (a) Hard ground, (b) Soft terrain, rigid mode, (c) Soft terrain, elastic mode

The assumption of tire lateral deformation also requires further elaboration. For a tire rolling with a slip angle α on- or off-road, the tire deforms laterally. The distribution of tire lateral deformation represented by the equatorial line is shown in Figure 6.4-5 (a) for a tire operating on a hard surface. At the instant of contact with the surface, it is observed that the equatorial line already has a finite lateral deformation off the wheel plane. The deformation increases along the adhesion region until the local friction limit is reached at point $p_{y,max}$. Beyond the point, the deformed tire element begins to recover towards the wheel plane because of tire circumferential tension. Unless the slip angle is fairly small, the longitudinal location of $p_{y,max}$ is generally believed to be within the projection of tire center on the ground and the rearmost contact point. Because the exact position of $p_{y,max}$ was unknown, it was assumed, in the proposed tire model that $p_{y,max}$ occurs at the rearmost contact point for tires operating on deformable terrain. Unlike the adhesion and

sliding regions on a hard surface, tire elements within the entire contact region slide relative to terrain surface and develop shear displacement. Thus the maximum shear displacement was assumed to take place at the rearmost contact point. It was therefore reasonable to assume that the position of $p_{y,max}$ also occurs at the rearmost contact point. For both rigid and elastic modes, it was also assumed that tire lateral deformation gradually drops to zero, as the tire element is no longer in contact with the terrain, as shown in Figure 6.4-5 (b) and (c).

It was inevitable that modeling of the highly sophisticated tire-terrain interaction by an approximation method results in certain errors. These errors were due to modeling of soil properties as well as the tire. From the experimental point of view, errors also result from the non-homogeneity of soil, measuring instruments, and the data acquisition system. These factors include the crosstalk effect of the x-y sensors, the assumed value of the horizontal component of the chain force, and the chain drive efficiency, as presented in Sections 4.3 and 4.4.

For the test tire operating on Ottawa sand, the correlation between predictions and measurements at higher slip angles may be considered reasonable. This was shown by the statistical evidence, and presented in Table 6.4-1. The proposed off-road tire model was, therefore, integrated with a vehicle model for predicting the handling behaviour of a multi-axle, all-wheel-drive vehicle on deformable terrain. The handling simulation will be discussed in the next chapter.

Chapter 7

Handling Simulation

7.1 Simulation Models Using AUTOSIM

As mentioned previously, to shorten the development and design cycle of new products, vehicle manufacturers rely more and more on computer simulation as a means to evaluate various design options and to optimize vehicle design parameters. To provide a realistic tool for assessing the handling behaviour of multi-axle, all-wheel-drive, off-road vehicles, particularly 6WD vehicles, a computer simulation model has been developed. It takes into account the design features of the vehicle, and the mechanics of tire-terrain interaction under combined longitudinal slip and slip angle described in previous chapters. The model was based on a software package for multibody dynamics analysis, known as AUTOSIM, developed by Mechanical Simulation Corporation (Sayers, 1989; Sayers and Mousseau, 1990; Sayers and Mink, 1995; Sayers and Han, 1996).

To demonstrate the basic features of the simulation model developed, a three-axle, six-wheel-drive, front-wheel-steering off-road vehicle was used as an example. Its design parameters are similar to those of an existing armoured fighting vehicle.

For the handling study of the 6WD vehicle, the vehicle was divided into seven rigid bodies, a sprung mass (vehicle body) and six unsprung masses (wheel assemblies). All components within the sprung mass were assumed securely fastened, and therefore there was no relative movement between each on-board component and the body structure. The effect of steering system dynamics was ignored because the mass of steering linkages is small compared to the masses of other rigid bodies, and these linkages move at a relatively slow speed.

In this study, a 17-DOF model was used to describe the 6WD vehicle. The sprung mass was assumed to be a rigid body that has six DOFs, fore-aft, lateral, and vertical translations, and roll, pitch, and yaw rotations. However, because of the assumption of constant longitudinal velocity, the sprung mass actually has five DOFs. An unsprung mass is a wheel assembly. The assembly includes a hub, a wheel rim, a tire, a run-flat insert (Ogorkiewick, 1991), and half of the mass of drive shaft. Each unsprung mass has two DOFs, wheel spin, and roll motion with respect to the sprung mass.

The 17-DOF handling model was formulated by AUTOSIM, a multibody dynamics software package developed mainly for simulating ground vehicles. Based on an analytical method with symbolic manipulation techniques, AUTOSIM provides users with a realistic tool to model and analyze the motions of rigid bodies with given constraints. Generally, analysts use AUTOSIM to create system layout, describe forces, moments, and constraints, apply external subroutines, and set output variables of interest. Modelers were not required to derive equations of motion by hand. Thus the potential errors associated with deriving equations and coding them were largely eliminated.

In AUTOSIM language, a multibody system is composed of many jointed objects. The macro *body* represents each rigid object. A body has its own body coordinate system $x_i y_i z_i$, a mass, a position of mass center, an inertia matrix, a *joint*, a *parent* body, and a number of *children* bodies. Each body is linked kinematically to its parent coordinate by a directional cosine matrix. Thus the complete multibody system can be arranged in a tree topology. A generalized coordinate for each DOF of the joint is introduced. However, the resulting number of DOF is determined by the total number of generalized coordinates subtracting the number of constraints.

Figure 7.1-1 Formulation of equations of motion for rigid bodies, algorithm used by AUTOSIM

Figure 7.1-1 shows the AUTOSIM method to formulate the equations of motion for a multibody dynamic system. The procedures are based on the application of d'Alembert's principle for a rigid-body system which is subject to a number of *nonholonomic* constraints (Kane and Levinson, 1985; Greenwood, 1988). These non-linear equations of motion are derived in a symbolic form. Differing from other multibody dynamics codes such as DADS and ADAMS, AUTOSIM generates independent, self-contained source codes in Fortran or C. Thus, it facilitates end users to execute the dynamic simulations in an efficient way. Similar to other codes, the solution procedure in handling studies is to numerically integrate the equations of motion to obtain the time domain response of the vehicle subject to a disturbance or to a steering input. The integration method can be chosen between constant time step, and variable time step integration.

For the 6WD vehicle, each subsystem was modeled by AUTOSIM to simulate kinematics and/or dynamics behaviour. The complete 6WD vehicle handling simulation model was formed by assembling subsystem models with the vehicle body model, and including the on- and off-road tire model. The sprung mass with vehicle axis coordinate xyz , shown in Figure 2.1-3, was regarded as the parent for other subsystems. Each unsprung mass was connected to the parent through suspension joints. The following sections illustrate the modeling of each subsystem and of the complete vehicle.

7.2 Steering System

The steering system of the 6WD vehicle used in the simulation model is shown in Figure 7.2-1. The system consists of a steering wheel, a power-assisted steering gear, a Pitman arm, a drag link, a steering rack, and two track rods. This double-idler-arm linkage system is supported horizontally by two idler arms, which pivot to the vehicle hull and rotate with respect to the z -axis. The rotational input from the steering wheel, through steering gear, transmits to Pitman arm. As the Pitman arm rotates, the drag link and the steering rack move accordingly. A track rod is the member, which connects the steering rack to one of the wheel-hub assemblies. The connection is by means of two ball joints that allow the unsprung mass to move up and down freely.

Since the dynamics effect of steering system was ignored, only the kinematics was simulated. The system was modeled by one rotational DOF with respect to the z -axis (downwards) of the Pitman arm. The rotations of wheel assemblies as well as all steering members were controlled by the movement of Pitman arm. The steering kinematics was simulated within the range of steering wheel rotating between a full-lock position on one side and the full-lock position on the other side. With the steering gear ratio of 17.7, the simulation result gives the relationship between the steering wheel input and the steered angles of the left and right wheels, as shown in Figure 7.2-2.

Figure 7.2-1 Layout of steering system, top view

When the steering wheel is turned to the full-lock position in either direction, the steered angle of the inside wheel should be greater than that of the outside wheel, according to Ackerman steering geometry. However, for this particular steering system simulated, Figure 7.2-2 shows the opposite trend. That is, the steered angle of the outside wheel is larger than that of the inside wheel. This phenomenon is mainly due to the forward-placed trapezoidal steering geometry, and is unfavorable because of substantial tire scrub during low-speed steering or parking. However, when the vehicle is operating at high speed, the steered angles of front wheels are usually less than ten degrees. Within this range, the steered angles of both inside and outside wheels are almost linear with respect to the steering wheel input. In other words, the kinematics behaviour of the two steered

wheels approaches the so-called parallel steering. This is primarily due to the double-idler arm configuration of steering linkages.

Figure 7.2-2 Steered angles of left and right wheel versus steering wheel input

7.3 Suspension System

The 6WD vehicle used in the simulation was equipped with double-wishbone (or Short-Long Arm, SLA) independent suspension on all wheel stations. With the exception of the spring stiffness of the front wheels being 19% higher than those of the intermediate and rear wheels, other components were exactly the same. The system, shown in Figure 7.3-1, has an upper control arm, a lower wishbone, a telescopic damper, dual coil springs, a bump stop, and a restraining cable. The wheel assembly is mounted on the hub final drive. The roll-plane (y - z plane) motion of the wheel-hub assembly is governed by the control arm and wishbone through two kingpins. Coil springs and damper are pivoted on lower wishbone, and reacted by the sprung mass (vehicle body). As the wheel-hub assembly moves up to a certain position, the rubber bump stop is engaged. Similarly, when the assembly approaches rebound, it is restrained by a cable. The functions of bump stop and restraining cable are chiefly to protect coil springs and damper.

Figure 7.3-1 Layout of suspension, right wheel station, front steered axle, rear view

Kinematically, the motion of control arm and wishbone in the y - z plane (roll plane) causes the wheel-hub assembly move not only in the roll plane resulting *bump camber*, but also rotate with respect to an axis passing through two kingpins (kingpin inclination axis) causing *bump steer*. The change in camber due to a suspension movement causes a change in tire lateral force. On the other hand, the bump steer induces an extra steer angle as the wheel-hub bumps or rebounds relative to the vehicle body. Moreover, the suspension can not be interpreted by a common mass-spring-damper system. Modeling of damper, and the equivalent spring were treated separately. The overall suspension system was composed of all the components noted above. The system was tested dynamically by input a vertical step force with rise time at the tire center, and examined kinematically for the bump camber and bump steer.

1) Damper

The characteristics of the damper used in the 6WD vehicle are given in Figure 7.3-2. The figure shows nonlinear behaviour for extension, curve A and compression, curve B. Similar to a common passive damper, the damping force developed during extension

(wheel rebound) is significantly higher than that during compression (wheel bump). The difference is due to the fact that the wheel bump velocity is considerably higher than the wheel rebound velocity. For maintaining an even damping force in either direction, it is desired to have unequal damping constants for extension and compression strokes (Milliken and Milliken, 1995). Nevertheless, Dixon (1996) explained the unequal damping characteristics in an alternative way. He concluded that a higher damping constant at rebound helps to limit the wheel from dropping into potholes.

Figure 7.3-2 Damper characteristic curves, mean and deviation curves

The unequal characteristics of the damper were modeled by a superposition of two third-order polynomials. One of the polynomials, curve C, models the mean value of extension and compression damping force with respect to stroke velocity. The second polynomial, curve D, represents the deviation of damping force from either extension or compression curve to the mean value curve C. Consequently, both extension and compression curves can be reproduced by the combination of the mean and deviation polynomials. By regression, curves C and D have the following mathematical forms. Coefficients of the two polynomials are listed in Table 7.3-1.

$$F_M = a_1 v + a_2 v^2 + a_3 v^3 \quad (7.3-1)$$

$$F_D = d_1 v + d_2 v^2 + d_3 v^3 \quad (7.3-2)$$

where F_M = mean damping force
 F_D = deviation damping force
 v = stroke velocity of damper

Table 7.3-1 Coefficients of mean and deviation polynomials

Coefficients	a_1	a_2	a_3	d_1	d_2	d_3
Values	18.10	10.66	-31.32	3.73	40.50	-57.79

Figure 7.3-3 Modeled damper characteristics comparing with original damper characteristics

Since the sign of the stroke velocity v changes from wheel rebound to wheel bump or vice versa, different damper characteristics must match the direction of damper velocity. In other words, damper extension curve A is used when v is positive, and compression

curve B is employed when v is negative. Therefore, the damper force F_E can be modeled by a single polynomial for both extension and compression, if a sign function is included to take into account the direction of stroke velocity.

$$\begin{aligned} F_E &= -\text{sgn}(v) \cdot (F_M + F_D) \\ &= -\text{sgn}(v) \cdot (m_1|v| + m_2v^2 + m_3|v|^3 + (d_1v + \text{sgn}(v)d_2v^2 + d_3v^3)) \end{aligned} \quad (7.3-3)$$

where $\text{sgn}(v)$ = sign function, returns 1 or -1 dependent upon the direction of stroke velocity v

Figure 7.3-3 shows the damper characteristics modeled by equation (7.3-3) comparing with the original damper curves A and B.

2) Equivalent Suspension Stiffness

The suspension stiffness is a combined stiffness from two coil springs in parallel, a bump stop, and a restraining cable. The coil spring has a uniform wire diameter, a uniform pitch, and a uniform outside diameter. The two springs give the wheel-hub assembly nearly a constant stiffness k_{sp} . The bump stop is a cylindrical, hollow member made of rubber. The stop is an elastic device, which provides the wheel-hub an extra stiffness k_b . The third member is a restraining cable made of a 12-mm diameter wire rope. It functions as a rebound stop. The cable also provides the wheel assembly an additional stiffness k_r .

Bump stop and restraining cable function only when the wheel-hub assembly approaches bump (extremely upward), or rebound (extremely downward) positions. For the 6WD vehicle used in this simulation, bump stop is engaged as the wheel center moves 0.115 m upwards. Beyond this limit, the wheel stiffness becomes a superposition of coil spring stiffness and bump stop stiffness, $k_{sp} + k_b$. On the other hand, restraining cable is activated when the wheel drops to -0.167 m. Below this position, the wheel stiffness reaches an extensively high value. This is due to the combined stiffness of restraining cable and two coil springs, $k_{sp} + k_r$. It is anticipated that within the range from -0.167 m up to 0.115 m, the wheel stiffness should be nearly a constant since neither bump stop nor restraining cable is in operation.

Figure 7.3-4 Stiffness at wheel center (wheel rate), by simulation

To examine the characteristics of the equivalent stiffness, the suspension system was modeled as if it was mounted on an inertial reference frame or on the stationary sprung mass. Since the motion is constrained on y - z plane (roll plane), as shown in Figure 7.3-1, the suspension linkages behave like a four-bar-linkage. Therefore, the system has only one DOF. That is the rotation of wishbone with respect to an axis through the wishbone pivots and parallel to the vehicle x -axis.

The equivalent suspension stiffness at the wheel center can be identified by simulation. By setting the damping constant of the viscous damper to zero, the relationship between the vertical (z -axis) displacement of the wheel center and the applied vertical force on the wheel center can be obtained, as shown in Figure 7.3-4. The slope of the curve is the equivalent spring stiffness at the wheel center, defined as the *wheel rate* (SAE J670e, 1976). The figure shows that within the vertical position range of -0.167 m to 0.115 m, only coil springs are activated. Above 0.115 m, the effect of bump stop is added, and below -0.167 m, the restraining cable is in operation. The total suspension travel at the wheel center is 0.389 m.

3) Dynamics Test

The objective of dynamic test was to examine whether each spring and damper component responds properly upon given an input force. Specifically, the modeling of bump stop and restraining cable were of most challenging since they are effective only at extreme positions of suspension travel. If an input force is applied to the wheel center vertically, the force must be sufficiently large in order to activate these two components.

Figure 7.3-5 Reaction forces of suspension components upon a 2.0g step input upwards to wheel center

A bump test was conducted by input a force at the wheel center in vertical z-axis direction. The input function was a step force with 0.2 seconds rise time. Figure 7.3-5 shows the result of a bump test when the input force is 9,712 N (2.0g in acceleration) upwards. The simulation was executed from 0 to 2.0 second with 0.005 seconds time increment. The figure shows the reaction forces of the springs, damper, and bump stop with the 2.0g step input to the wheel center. As expected, the spring stores the majority of the total energy for later release. The damper force shows an uneven distribution with respect to the reference line of zero force. This is because the positive damper force is due to damper compression with negative stroke velocity (wheel bump), whereas the

negative damper force corresponds to damper extension with positive stroke velocity (wheel rebound). This phenomenon can be realized from the damper characteristics shown in Figure 7.3-3. On the other hand, the reaction force of the bump stop shows only one-sided distribution. Unlike springs and damper, there is no physical connection between the wheel-hub assembly and the bump stop. As the wheel-hub oscillates up and down, bump stop is reacted only when the assembly moves upwards to or beyond a certain position. Otherwise, the bump stop force is simply zero.

Figure 7.3-6 Time response of wheel center upon a 2.0g step input upwards to wheel center

The time response of the wheel center is shown in Figure 7.3-6. The vertical wheel position starts from 0.5 m at 0 second since the z-axis coordinate of the wheel center is 0.5 m relative to the origin of the inertial reference frame. The wheel center response is very similar to the spring force reaction, shown in Figure 7.3-5. This is mainly due to the fact that coil springs are vertically mounted in a double-wishbone suspension. For reference purpose, the suspension damping ratio $\zeta = 0.193$ was determined by the method of logarithmic decrement although the curve is not the one of free oscillation. The damping ratio provided by automotive passive dampers is usually in the range of 0.2 to 0.4, according to Ogorkiewick (1991a), Gillespie (1992), and Wong (1993), respectively.

Figure 7.3-7 Reaction forces of suspension components upon a 5.0g step input downwards to wheel center

Figure 7.3-8 Time response of wheel center upon a 5.0g step input downwards to wheel center

For the rebound test, a step input with 0.2 seconds rise time was applied to the wheel center vertically downwards. Figure 7.3-7 shows the reaction force with time for each suspension component. The figure shows an inverted impulse-like force of the restraining

cable when the wheel center is subject to a 5.0g step input in the downward z-axis direction. This indicates that the cable is only activated in a short period of time from 0.25 to 0.29 second. During the rest of time history, the wheel-hub can not reach so low as to tension the cable. Further tests indicate that for lower input forces such as the force causing 4.0g acceleration downward, the cable will not be activated at all. Time response of the wheel-hub assembly is shown in Figure 7.3-8.

4) Kinematics Test

Bump camber and bump steer are the variations of wheel camber and wheel steer angles when the suspension moves up and down. Bump camber is mainly influenced by suspension linkage designs. For example, a conventional MacPherson strut suspension has a significant bump camber. On the other hand, an equal-length parallel double-wishbone suspension, or a trailing arm suspension has zero bump camber. The camber change of a wheel directly contributes to a change in tire lateral force. Bump steer is the other important effect caused by suspension movement. However, bump steer is a result of improper location and length of the steering trackrod, instead of suspension linkage error. Further details regarding to bump camber and bump steer can be found from the literature such as Gillespie (1992), Bastow and Howard (1993), and Dixon (1996).

For ordinary vehicles, strict zero bump camber and zero bump steer are not attempted. Although these effects were automatically taken into account in the vehicle simulation model, it was desirable to learn these phenomena explicitly from the design point of view. For front wheels, bump steer simulation is also a method to test the modeling of proper connection between suspension and steering.

Investigation of bump camber and bump steer effects in a three-dimensional space is a kinematics problem. In the model, coil springs, damper, bump stop, and restraining cable are all ignored. The model consists of four rigid bodies. The wishbone, the control arm, and the trackrod each with one rotational DOF on the y - z plane (roll plane), are jointed individually to the inertial reference frame. The wheel-hub assembly with two DOFs of rotations about its longitudinal axis and about the kingpin inclination axis, is connected to the wishbone by the lower kingpin. The wheel-hub is further constrained by the control

arm and by the track rod, thus eliminates one DOF for each member. The remaining two DOFs are the wishbone rotation with respect to its longitudinal axis, and the wheel-hub rotation relative to the kingpin inclination axis.

The simulation was executed within a 2.0 seconds period of time. By fixing a pseudo point on the wheel center, and by letting this point to move from rebound position (-0.179 m) to bump position (0.210 m), the locus of suspension travel was determined. The bump camber of the wheel-hub is the sum of the absolute angular displacement of wishbone and the relative angular displacement of wheel-hub to wishbone. The bump steer is the output of the absolute angular displacement of wheel-hub with respect to the kingpin inclination axis.

Figure 7.3-9 Bump camber of wheel-hub assembly

Figure 7.3-9 shows the variation of bump camber with the vertical displacement of wheel-hub from the kinematics simulation. The figure shows that camber angle is zero at static equilibrium position, and increases at a rate nearly constant to -5.47° at bump position (0.210 m). In the opposite direction, bump camber increases to positive 1.41° at the wheel-hub position of -0.155 m near rebound. The accuracy of this kinematics simulation was considered to be satisfactory by comparing the results with a traditional

hand-drawing analysis, shown in Figure 7.3-10. It shows that bump camber reaches -4.5° at the wheel-hub position of 0.205 m, whereas bump camber becomes 1.5° at the position of -0.212 m.

Figure 7.3-10 Bump camber analysis by sketch, rear view

Figure 7.3-11 Bump steer of wheel-hub assembly

Similarly, Figure 7.3-11 shows simulated bump steer of the wheel-hub assembly. The bump steer angle is zero at static equilibrium position, and gradually increases towards the toe-in direction when the wheel-hub moves vertically away from the zero position. The maximum bump steer angle is 1.02° when the assembly is at full bump. At the rebound position, bump steer reaches to 0.28° .

7.4 Vehicle Inertial quantities

In the analysis of a rigid body motion such as a vehicle body that has a rotational acceleration about an axis, the information of mass moments of inertia and product moments of inertia are required as input.

In general, there are two methods to find inertial quantities of vehicles. The first is experimental methods. Pendulum method (Dixon, 1996) is a popular one for obtaining inertial quantities for small vehicles. A vehicle is mounted on a cable-hoisted pendulum, by rotating the pendulum and recording the oscillatory motion, the natural frequencies of the assembly as well as the vehicle's inertia can be deduced. For heavier vehicles such as railroad cars, the hinge-supported method is preferred (Dukkipati and Amyot, 1987). Using this method, a vehicle is supported with a hinge located below the center of mass, and with linear springs on both ends along the pitch y -axis or the roll x -axis. The vehicle is excited at one end, and the oscillatory motion is recorded. The resonant frequency of roll or pitch oscillation is used to obtain the mass moment of inertia with respect to the x -axis or y -axis for the vehicle. The above mentioned measurements require sophisticated testing facilities so that only limited organizations are capable to do so.

The other method is to compute inertial quantities by knowing the detailed configuration (such as mass, center of mass, and mass distribution) of all vehicle components and their precise locations within the vehicle. Nowadays, most vehicle manufacturers create detailed vehicle models in computer systems. Therefore it is possible to use commercially available software such as a solid modeling code to calculate inertial quantities of each component and of the modeled vehicle.

However, for a third party, such as a researcher, none of the above methods is practical because of the complexities of experimental setups, and the lack of information regarding to inertial quantities of all components and their precise locations. As a consequence, an alternative approach is to develop an approximate method to estimate inertial quantities, particularly for armoured vehicles. This proposed method does not require users to generate all lumped masses constituting the vehicle, instead only a finite number of parameters are necessary.

The baseline vehicle simulated has an all-welded hull with three axles. Inside the steel hull, there are powerplant, driver, and crew compartments. The steel hull is a closed-shell structure without longitudinal beams or lateral crossmembers. The mass of armour plates constitutes up to 35% of the gross vehicle mass. The modeling approach was to divide the vehicle mass into seven hull plates and four interior mass blocks, to compute individual inertial quantities and to sum them up.

Figure 7.4-1 Configuration of sprung mass, 6WD vehicle

Almost all wheeled armoured vehicles in service today have very similar design configuration. The 6WD vehicle simulated in this study has a body configuration shown in Figure 7.4-1. Several parameters of the vehicle hull are easy to identify. They are the gross vehicle mass, center of mass, width of the vehicle, thickness of each plate, and the coordinates of five corner points on the vehicle side view. With these information, the inertial quantities for each plate was computed with respect to its centroidal axes. By axes rotation and parallel translation, these inertial quantities were then transformed to the

inertia with respect to the origin of an arbitrary chosen axes fixed to the vehicle, and were finally transformed to the one about the center of sprung mass. Table 7.4-1 lists the input parameters for computing inertial quantities of the 6WD vehicle.

Table 7.4-1 Input parameters for calculation of inertial quantities, 6WD vehicle

DESCRIPTIONS	PARAMETERS	UNITS	VALUES
Gross vehicle mass	m	[kg]	15000
Sprung mass	ms	[kg]	12030
Overall vehicle width	vw	[m]	2.67
Tire diameter	tireDia	[m]	1.034
Tire width	tireW	[m]	0.333
X-coordinate axle#1	xaxle1	[m]	1.44
X-coordinate axle#2	xaxle2	[m]	-0.95
X-coordinate axle#3	xaxle3	[m]	-2.36
Material density plates	dnty	[kg/m ³]	7830
Material density factor plates	Dfactor	[]	1.1
Thickness plate#1	th1	[m]	0.012
Thickness plate#2	th2	[m]	0.01
Thickness plate#3	th3	[m]	0.01
Thickness plate#4	th4	[m]	0.01
Thickness plate#5	th5	[m]	0.012
Thickness plate#6	th6	[m]	0.01
Thickness plate#7	th7	[m]	0.01
Coordinate of the origin [x,y,z]	ox,oy,oz	[m]	0 0 0
Coordinate sprung mass center [x,y,z]	msx,msy,msz	[m]	-0.13 0 -0.22
Coordinate point#1 [x,z]	p1x,p1z	[m]	3.16 -0.23
Coordinate point#2 [x,z]	p2x,p2z	[m]	1.24 -1.17
Coordinate point#3 [x,z]	p3x,p3z	[m]	-3.54 -1.17
Coordinate point#4 [x,z]	p4x,p4z	[m]	-3.18 0.54
Coordinate point#5 [x,z]	p5x,p5z	[m]	2.17 0.54

The interior space was modeled by four mass blocks, as illustrated in Figure 7.4-1. Block 8 accounts for heavier components including transmission, axles, propeller shafts, and part of suspension masses. Block 9a is for engine, radiator, fan, and accessories. Block 9b is for axles, hydraulic components, and fuel tanks. Block 10 is mainly for occupants, personal equipment, and logistics. The mass for each block was unknown although the block volume can be calculated. Densities for each mass block ρ_8 , ρ_9 , and ρ_{10} were found by solving the following three simultaneous equations. The first two account for

the location of sprung mass center on the x - z plane. The third is the mass consistency of the sprung mass.

$$\sum_{i=8}^{10} \rho_i V_i \bar{x}_i + \sum_{i=1}^7 m_i \bar{x}_i = m_s \bar{x} \quad (7.4-1)$$

$$\sum_{i=8}^{10} \rho_i V_i \bar{z}_i + \sum_{i=1}^7 m_i \bar{z}_i = m_s \bar{z} \quad (7.4-2)$$

$$\sum_{i=8}^{10} \rho_i V_i + \sum_{i=1}^7 m_i = m_s \quad (7.4-3)$$

- where V_i = volume of the i -th mass block
 m_i = mass of the i -th hull plate
 m_s = sprung mass
 $\bar{x}_i$ = position vector along x -axis, of the i -th hull plate or mass block
 $\bar{z}_i$ = position vector along z -axis, of the i -th hull plate or mass block
 $\bar{x}$ = position vector along x -axis, center of sprung mass
 $\bar{z}$ = position vector along z -axis, center of sprung mass

Each block mass was computed by its density multiplied by its volume. Likewise, the inertial quantities can be calculated with respect to its centroidal axes, and were transformed to the inertia with respect to axes passing through the center of sprung mass. Moreover, the center of sprung mass is located on the x - z plane (pitch plane). All products of inertia involving the y -axis are zero, that is, $I_{xy} = 0$ and $I_{yz} = 0$, because of the symmetry of sprung mass.

Table 7.4-2 shows the calculated results of the inertial quantities for the sprung mass. The computation was validated by examining the consistency of the combined mass contributed from each plate and block to the sprung mass.

Table 7.4-2 Calculation output of inertial quantities

Mass No.		lxx	lyy	lzz	lxy	lxz	lyz
		[kg-m ²]	[kg-m ²]	[kg-m ²]	[kg-m ²]	[kg-m ²]	[kg-m ²]
Plate#1	Centroidal	350.5	224.7	577.2	0	0	0
	Rotation	393.9	224.7	531.7	0	-88.7	0
	Translation	683	3369	3387	0	819.8	0
Plate#2	Centroidal	653	2093	2746	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	2157.8	5051.5	4199.8	0	-1479	0
Plate#3	Centroidal	238.7	102.3	341	0	0	0
	Rotation	337.7	102.3	243.1	0	-20.6	0
	Translation	377.6	4679	4780	0	-447.9	0
Plate#4	Centroidal	730.9	2934.6	3667.6	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	1089.7	3607.1	3979.2	0	337.6	0
Plate#5	Centroidal	207.7	47.5	251	0	0	0
	Rotation	222.7	47.5	233.9	0	22	0
	Translation	231	2511.8	2692	0	-121	0
Plate#6	Centroidal	207.1	2777.5	2570.5	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	1807.8	2964.5	4187.5	-394.2	-93	-357.3
Plate#7	Centroidal	207.1	2777.5	2570.5	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	1807.8	2964.5	4187.5	394.2	-93	357.3
Block#8	Centroidal	954.4	951	1830.4	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	1067.9	3803.9	4570.8	0	-557.4	0
Block#9a	Centroidal	1627.3	1189	2292	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	2413.7	4757.9	5072.5	0	1478.7	0
Block#9b	Centroidal	1138.8	1597.7	2647.2	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	1273.1	6387.9	7302	0	790.6	0
Block#10	Centroidal	694	997.8	1468.2	0	0	0
	Rotation	0	0	0	0	0	0
	Translation	1029.4	3991.1	4127.2	0	-944.2	0
Total_Iner	Origin	13932.7	44087.4	48484.4	0	-307.8	0
Total_Iner	MassCtr	14515	44870.8	48687.7	0	-650.8	0
Mass	Plate#1	Plate#2	Plate#3	Plate#4	Plate#5	Plate#6	Plate#7
[kg]	589.94	1099.24	401.86	1230.32	347.21	849.73	849.73
Volume	Block#8	Block#9a	Block#9b	Block#10			
[m ³]	3.84	7.87	4.58	10.29			
Density	Rho8	Rho9	Rho10				
[kg/m ³]	401.68	334.36	97.35				
Mass	Block#8	Block#9a	Block#9b	Block#10			
[kg]	1543.42	2297.95	1841.68	980.03			
SpruMass	[kg]:	12030					

The accuracy of this method relies on the assumption that a large percentage of the sprung mass is distributed on the external shell structure, the steel hull. An error analysis is given in Table 7.4-3. It shows the sum of the mass moments of inertia I_{xx} , I_{yy} , and I_{zz} for the seven plates, and the sum of those of the four mass blocks, respectively. It was realized that the inertial quantities calculated from the plates are accurate since the thickness of steel plates and their coordinates are easy to obtain. Suppose there is a 25% error of I_{xx} , I_{yy} , and I_{zz} from the estimation of the four mass blocks, this error constitutes a total error less than 11% of the combined inertia, of the sprung mass.

Table 7.4-3 Error analysis of inertial quantities

	$I_{xx} [kg \cdot m^2]$	$I_{yy} [kg \cdot m^2]$	$I_{zz} [kg \cdot m^2]$
Sum of plate#1 to #7	8149.60	25147.40	27413.00
Sum of all mass blocks	5783.10	18937.80	21071.40
25% error from blocks	1447.78	4734.45	5267.85
Combined inertia	13932.70	44087.40	48484.40
Total Error [%]	10.38	10.74	10.87

7.5 Application of Tire Model

The off-road tire model presented in Chapter 6 was used for handling simulation of the 6WD vehicle operating on a sandy terrain. The model, written in a subroutine, includes three iterations. Two of them were previously discussed in both rigid and elastic modes of tire operation. They were the equilibrium of tire lateral force and internal force due to tire elasticity (equations 6.2-34 and 6.3-59), and the equilibrium of normal force from applied load and from vertical soil reaction (equations 6.2-55 and 6.3-62). The third iteration is the adjustment of longitudinal slip i . For steady-state handling, the required drawbar pull F_{dbp} for each tire is equal to zero. Otherwise, the nonzero drawbar pull acts as an external force in the tire longitudinal direction, and accelerates the vehicle. Since the drawbar pull is developed from tractive force and the tractive force is due to longitudinal slip i , the adjustment of i is a means to control the magnitude of F_{dbp} .

Figure 7.5-1 Algorithm of using the proposed off-road tire model, elastic mode

It is noted that on a soft terrain, the tractive force generated by a tire is heavily dependent on the type of slip-controlled drive system of the vehicle. Since traction control is beyond the scope of this study, it was assumed that the 6WD vehicle equipped with a smart

driving system. The system provides every tire with the optimal traction upon demanding. A typical modeling of the power train for automobiles can be found from literatures such as the one by Lugner and Mittermayr (1989). The algorithm of using the proposed elastic operating mode, off-road tire model in vehicle handling simulation is summarized in Figure 7.5-1.

In addition to the off-road tire model, the tire model subroutine integrated with the vehicle simulation program also includes a model for predicting tire lateral force on hard surface. This tire model for road operations was obtained from data provided by the tire manufacturer (Michelin, 1988).

7.6 Handling of 6WD Vehicle

The simulation program of the proposed 17-DOF handling model for 6WD vehicles was generated by AUTOSIM. Linked together with the subroutines for on- and off-road tire models, the program simulates the handling behaviour of a 6WD vehicle subject to a steering input, and outputs time history of specified handling variables. The flow chart shown in Figure 7.6-1 gives the algorithm of the vehicle handling simulation.

The basic features of the 17-DOF handling model for the 6WD vehicle must be substantiated before the model can be applied with confidence. Ideally, a simulation model should be validated using appropriate experimental data. Because of the lack of test facilities and the unavailability of a test vehicle, it was not possible in this research to conduct field tests to substantiate the handling model proposed. As an alternative, the predictions obtained using the 17-DOF model were compared with those obtained using Segel's 3-DOF handling model, which is widely accepted as a benchmark and has been validated experimentally. It should be noted that such a comparison, strictly speaking, was not a proper means for validating the proposed model. It merely intended to show whether the predictions by the 17-DOF model exhibit similar trends as those obtained using an experimentally validated benchmark model.

Figure 7.6-1 Vehicle handling simulation

In the comparison of the predictions by the proposed 17-DOF model and Segel's model, a step function with rise time of one second was chosen for the steering wheel input, as shown in Figure 7.6-2. At steady state, the vehicle was expected to follow a path with a constant curvature.

Figure 7.6-2 Step steering input with rise time

The two models were examined by comparing the vehicle trajectories with respect to the earth-fixed XY axis system on hard ground. The vehicle was simulated for 17.96 seconds with a constant longitudinal velocity of 11.5 m/s. Figure 7.6-3 shows the trajectories predicted by the proposed 17-DOF, and Segel's models, respectively. It is interesting to observe that the trajectory predicted by the 17-DOF model shows a smaller turning radius than that of the Segel's model. However, the difference is marginal. Since Segel's model is widely accepted and considered as a benchmark model, the similarity of the trajectories predicted by both models appears to confirm that the basic features of the proposed 17-DOF model are reasonable.

Figure 7.6-3 Vehicle trajectories simulated by proposed 17-DOF handling model and Segel's model, on hard ground

Figure 7.6-4 Double-triangular steering input

The handling of the 6WD vehicle on a sandy terrain was simulated by the 17-DOF handling model with the proposed off-road tire model. The transient handling behaviour

of the vehicle was simulated with a forward velocity of 11.5 m/s, and with a double-triangular steering input (Dugoff et al., 1970; El-Gindy and Woodrooffe, 1990), shown in Figure 7.6-4. This simulates a vehicle lane-change manoeuvre with a constant angular speed of the steering wheel input in both directions.

The dynamic response of a tire to an input of slip angle is always delayed. This is because for a sudden change of slip angle, a tire needs a finite response time for its contact patch to take up a new distorted shape. Especially for accident avoidance manoeuvring of road vehicles, tire force data obtained under steady state conditions may not be sufficient to predict the vehicle directional response. The lateral force delay of a sideslip tire is commonly called *tire lag*, and is expressed as a first-order differential equation.

$$\dot{F}_y + \frac{u}{l_r} F_y = \frac{u}{l_r} F_{y,ss} \quad (7.6-1)$$

where F_y = tire lateral force
 $F_{y,ss}$ = tire lateral force, steady state
 u = longitudinal velocity
 l_r = relaxation length

The solution is given by

$$F_y = F_{y,ss} (1 - e^{-t/t_c}) \quad (7.6-2)$$

where t_c = time constant, $t_c = l_r/u$

Tire relaxation length l_r can be obtained from experiments such as the method given in Appendix D. Rough estimations can be found such as $l_r \approx (1/4)$ tire revolution by Clark (1981), $l_r \approx (1/6)$ tire revolution by Lund et al. (1992), $l_r \approx (1/2)$ tire revolution by Sultan et al. (1995), and $l_r \approx$ tire rolling radius by Crolla et al. (1987), Wong (1993), and Dixon (1996). As the relaxation length l_r increases or the vehicle forward velocity u decreases, the effect of tire lag becomes significant. However, for the transient handling response of the 6WD vehicle, the time constant t_c is relative small so that tire lag is negligible. For example, with input parameters of $u = 11.5$ m/s and $l_r = 1.25$ m

($0.4 \times$ tire revolution), and the double-triangular steering input (Figure 7.6-4), the vehicle trajectories predicted by the 3-DOF model with and without considering tire lag are given in Figure 7.6-5. Segel (1956) and Dixon (1996), respectively, gave a criterion that tire lag should be considered for steering frequencies beyond 3 Hz.

Figure 7.6-5 Lane-change manoeuvre predicted by 3-DOF handling model, with tire lag and without tire lag

To demonstrate the handling behaviour of the 6WD vehicle under off-road conditions, the handling characteristics of the same vehicle on hard ground was also simulated for the comparison purpose. The variations of typical indicators for transient handling in a lane-change manoeuvre, such as yaw velocity and lateral acceleration of the vehicle, roll and pitch angles of the sprung mass, and vehicle heading angle and XY trajectory, with respect to time history are shown in Figure 7.6-6 to 7.6-11, respectively.

Figure 7.6-6 shows that there is a significant difference of the time response of yaw velocity in a lane-change, or obstacle avoidance manoeuvre between the vehicle on hard surface and on sandy terrain. On hard surface, the yaw velocity appears to follow closely the pattern of steering input as shown in Figure 7.6-4. However, on sandy terrain, the yaw velocity lag occurs and its magnitude is considerably lower. This is primarily due to the

fact that on sandy terrain, the tire lateral force that can be developed is remarkably lower than that on hard surface. Moreover, the 6WD vehicle is a heavy vehicle and weights 15 tonnes. As the tire lateral force changes direction, or even gradually approaches zero, the vehicle inertia in yaw still dominates yaw motion for a period of time, and causes yaw velocity lag or phase shift with respect to the steering input.

Figure 7.6-6 Time response of yaw velocity, under lane-change manoeuvre

Figure 7.6-7 Time response of lateral acceleration, under lane-change manoeuvre

In contrast to the yaw velocity response, the lateral acceleration shown in Figure 7.6-7 displays many overshoots for the operation on hard ground although the curve is still roughly follow the trend of the double-triangular steering input. The phenomenon can be explained by observing the roll angle response of the sprung mass. It can be seen from Figure 7.6-8 that for on hard ground and on sandy terrain, the response curves of roll angle show frequencies of 1.01 and 1.07 Hz, respectively. Owing to this roll motion, a lateral load transfer takes place so that the normal force on the left and right tires fluctuates. As the tire normal force changes, tire lateral force also varies. Since the sum of these fluctuated tire lateral forces on all wheels is the only source that sustains the vehicle cornering, the vehicle's centrifugal or lateral acceleration also fluctuates as shown in Figure 7.6-7.

To verify the roll frequency of 1.01 and 1.07 Hz, the damped frequency of oscillation in roll was computed analytically. The natural frequency was found to be 1.11 Hz for the vehicle about the static equilibrium position. Details are given in Appendix E.

Figure 7.6-8 Time response of roll angle, sprung mass, under lane-change manoeuvre

The time response for pitch, shown in Figure 7.6-9, also displays a resonant frequency especially for the operation on sandy terrain. The pitch oscillation affects principally the

vehicle ride. The response of pitch has less effect directly on vehicle handling. Indirectly, however, the pitch mode creates a fore and aft load transfer so that tire normal and lateral forces are changed accordingly. The variations of tire lateral forces do affect the vehicle handling. For the vehicle operating on hard ground, the curve shows that the pitch motion decays exponentially with a frequency of 0.86 Hz. However on sandy terrain, in addition to the dominant frequency of 0.83 Hz, a second frequency of 0.275 Hz was involved. To confirm the validity of the simulation, the pitch vibration of the 6WD vehicle was analyzed using the pitch plane model, and a pitch natural frequency of 0.896 Hz was found. Details of the modeling and calculations are given in Appendix E. The frequencies of 0.83 and 0.86 Hz also lie in the range between 0.5 and 2.5 Hz for general motions of sprung mass (Ellis, 1989). An interesting point was discovered from the figure that both curves show the steady-state pitch angle of -0.01875 radian. This corresponds to the 6WD vehicle exhibiting a nose-dive of 1.075° when operating at steady state.

Figure 7.6-9 Time response of pitch angle, sprung mass, under lane-change manoeuvre

For the XYZ earth-fixed coordinate, the resulting vehicle motion due to the double-triangular steering input can be interpreted by the vehicle heading angle ψ and the vehicle trajectory on the XY plane (both are defined in Figure 2.1-1).

Figure 7.6-10 Vehicle heading angle, under lane-change manoeuvre

Figure 7.6-11 Vehicle trajectories on XY earth-fixed axis system, under lane-change manoeuvre

Figure 7.6-10 shows that operating on hard ground the heading angle ψ reaches -0.18 radian at time equals 4 seconds ($t = 4s$). Referring to Figure 7.6-4, this is the instant when the steering wheel completes its first triangular input. However, the second triangular input causes the heading angle to change at a faster rate. The heading angle

reaches zero at $t = 7s$, and continuously increases until $t = 8s$. When the steering input becomes zero, the residual heading angle remains 0.03 radian (1.72°). This effect can be observed from the time response of yaw velocity, shown in Figure 7.6-6. The figure shows that the positive area enclosed by the hard ground curve and the time span 4~8 seconds is slightly larger than the negative area enclosed by the curve and the time span 0~4 seconds. Since the heading angle ψ is defined as the integration of yaw velocity r over the elapse time, that is $\psi = \int r dt$, a positive residual heading angle is expected. The primary reason of this phenomenon may probably be attributed to the fluctuating tire lateral forces due to load transfer from the mixed roll and pitch motion.

The positive residual heading angle explains the reason why the XY trajectory when operating on hard ground shown in Figure 7.6-11 changes orientation after reaches the maximum lateral displacement of 7.9 m. Since the steering input is a lane-change command, the simulation results indicate that the driver has to make a correction in steering input at $t = 7s$, or at X position equals 78.6 m. The residual heading angle and the resulting XY trajectory from the 17-DOF model, when comparing with the trajectory simulated by Segel's model as shown in Figure 7.6-5, shows the lane-change simulation is reasonably realistic.

On soft terrain, vehicle directional response lags. It can be seen that the heading angle ψ does not reach a turning point until $t = 5s$. Afterwards, the heading angle gradually approaches -0.01 radian, close to the initial heading angle ($\psi = 0$) of the vehicle. Unlike the case on hard ground, the XY trajectory for operating on sandy terrain shows that the vehicle moves laterally at a much lower rate. For the transient, or unsteady-state handling of the 6WD vehicle, a conclusion can be made regarding to Figure 7.6-11. The vehicle on sandy terrain shows an insufficient response to the double-triangular steering input so that the driver has to give an additional steering input. However on hard ground, the vehicle shows an excessive response that the driver has to make a correction in steering input in the opposite direction in order to fully execute the lane-change.

Part of the simulation output from the 17-DOF handling model is shown in Table 7.6-1. Data listed in the table correspond to Figure 7.6-4 of steering input, and Figures 7.6-6 to 7.6-11 of vehicle directional responses with respect to time for the 6WD vehicle operating on sandy terrain, at a longitudinal velocity of 11.5 m/s.

Table 7.6-1 Output data of selected variables, from the 17-DOF handling model

Time	RollDsp	PitchDsp	YawVel	LatrAcc	XDispl	YDispl	SWheelA	HeadVB
[sec]	[rad]	[rad]	[rad/s]	[m/s ²]	[m]	[m]	[rad]	[rad]
0.00	0.0000	0.0000	0.0000	0.0000	-4.202	0.000	0.0000	0.0000
0.20	0.0000	-0.0049	-0.0004	-0.0007	-1.902	0.000	-0.0524	0.0000
0.40	0.0001	-0.0219	-0.0010	-0.0051	0.396	0.000	-0.1047	-0.0002
0.60	0.0003	-0.0551	-0.0023	-0.0185	2.697	-0.001	-0.1571	-0.0004
0.80	0.0008	-0.0849	-0.0048	-0.0143	5.013	-0.003	-0.2094	-0.0012
1.00	0.0009	-0.0613	-0.0056	-0.0347	7.328	-0.005	-0.2618	-0.0021
1.20	0.0005	-0.0103	-0.0067	-0.0255	9.630	-0.009	-0.3142	-0.0033
1.40	0.0002	0.0254	-0.0087	-0.0333	11.932	-0.013	-0.3665	-0.0048
1.60	0.0002	0.0053	-0.0116	-0.0526	14.233	-0.019	-0.4189	-0.0068
1.80	0.0006	-0.0471	-0.0152	-0.0754	16.536	-0.027	-0.4712	-0.0095
2.00	0.0012	-0.0704	-0.0192	-0.0932	18.843	-0.038	-0.5236	-0.0130
2.20	0.0022	-0.0541	-0.0233	-0.1005	21.146	-0.053	-0.4713	-0.0172
2.40	0.0022	-0.0118	-0.0266	-0.1115	23.445	-0.072	-0.4189	-0.0221
2.60	0.0004	0.0132	-0.0291	-0.1092	25.745	-0.096	-0.3665	-0.0277
2.80	-0.0001	-0.0035	-0.0299	-0.1157	28.045	-0.125	-0.3142	-0.0336
3.00	0.0015	-0.0323	-0.0298	-0.1486	30.345	-0.158	-0.2618	-0.0397
3.20	0.0033	-0.0471	-0.0298	-0.1775	32.645	-0.198	-0.2095	-0.0457
3.40	0.0024	-0.0393	-0.0303	-0.2209	34.945	-0.246	-0.1571	-0.0516
3.60	-0.0005	-0.0223	-0.0290	-0.1756	37.248	-0.303	-0.1048	-0.0573
3.80	-0.0006	-0.0046	-0.0266	-0.1403	39.550	-0.368	-0.0524	-0.0629
4.00	0.0026	0.0029	-0.0225	-0.1815	41.851	-0.439	-0.0001	-0.0678
4.20	0.0035	-0.0117	-0.0165	-0.2309	44.151	-0.518	0.0523	-0.0717
4.40	0.0001	-0.0406	-0.0127	-0.2018	46.451	-0.607	0.1047	-0.0747
4.60	-0.0029	-0.0512	-0.0106	-0.1529	48.755	-0.704	0.1571	-0.0771
4.80	-0.0015	-0.0312	-0.0073	-0.1389	51.057	-0.807	0.2095	-0.0789
5.00	0.0022	0.0007	-0.0022	-0.1429	53.356	-0.915	0.2619	-0.0798
5.20	0.0028	0.0084	0.0051	-0.1216	55.654	-1.029	0.3142	-0.0795
5.40	-0.0008	-0.0181	0.0107	-0.0730	57.952	-1.148	0.3666	-0.0779
5.60	-0.0038	-0.0436	0.0146	-0.0467	60.251	-1.270	0.4190	-0.0754
5.80	-0.0029	-0.0437	0.0173	-0.0376	62.549	-1.394	0.4714	-0.0722
6.00	-0.0003	-0.0211	0.0192	-0.0158	64.847	-1.519	0.5234	-0.0685

6.20	0.0003	-0.0008	0.0206	0.0036	67.145	-1.646	0.4711	-0.0645
6.40	-0.0013	-0.0026	0.0211	0.0192	69.442	-1.772	0.4187	-0.0603
6.60	-0.0021	-0.0185	0.0216	0.0221	71.740	-1.897	0.3663	-0.0561
6.80	-0.0010	-0.0309	0.0227	0.0108	74.035	-2.022	0.3139	-0.0516
7.00	-0.0010	-0.0331	0.0239	0.0224	76.333	-2.146	0.2615	-0.0470
7.20	-0.0019	-0.0240	0.0243	0.0565	78.631	-2.270	0.2092	-0.0421
7.40	-0.0021	-0.0076	0.0236	0.0816	80.929	-2.391	0.1568	-0.0374
7.60	-0.0012	-0.0010	0.0221	0.0959	83.227	-2.509	0.1044	-0.0328
7.80	-0.0006	-0.0145	0.0201	0.0964	85.525	-2.623	0.0520	-0.0286
8.00	-0.0021	-0.0343	0.0176	0.0956	87.823	-2.733	0.0000	-0.0248
8.20	-0.0032	-0.0371	0.0150	0.1151	90.124	-2.839	0.0000	-0.0215
8.40	-0.0031	-0.0205	0.0128	0.1225	92.424	-2.941	0.0000	-0.0188
8.60	-0.0012	-0.0026	0.0106	0.1249	94.722	-3.037	0.0000	-0.0164
8.80	-0.0002	-0.0046	0.0082	0.1116	97.020	-3.128	0.0000	-0.0145
9.00	-0.0018	-0.0218	0.0058	0.1156	99.318	-3.215	0.0000	-0.0131
9.20	-0.0030	-0.0328	0.0040	0.1335	101.618	-3.297	0.0000	-0.0120
9.40	-0.0023	-0.0277	0.0031	0.1357	103.919	-3.374	0.0000	-0.0113
9.60	-0.0006	-0.0159	0.0022	0.1046	106.220	-3.447	0.0000	-0.0109
9.80	-0.0002	-0.0089	0.0009	0.0812	108.521	-3.516	0.0000	-0.0106
10.00	-0.0015	-0.0110	-0.0010	0.0765	110.820	-3.583	0.0000	-0.0106

The steady-state handling behaviour of the 6WD vehicle was simulated by given a step steering function with rise time, as shown in Figure 7.6-2. The step input theoretically makes the vehicle follow a constant-curvature path at steady state. The steering angle of 0.5236 radian (30°) is maintained up to $t = 30s$. The vehicle is operated at a constant longitudinal velocity of 11.5 m/s, and tire inflation pressure is set to 1.0 bar for operating on sandy terrain and 3.5 bar for operating on hard surface. Figure 7.6-12 shows the XY trajectories under these two operating conditions. Each trajectory depicts the position of the vehicle mass center from its starting point to the position when the vehicle is ready to head back. It can be seen from the figure that the vehicle simulated operating on sandy terrain requires approximately 65% additional time in comparison with that on hard ground to complete one quarter of the turning circle. The turning radius on soft terrain is also larger than that on hard ground by the same percentage.

Figure 7.6-12 Vehicle trajectories on XY earth-fixed axis system, step steering input with rise time

The 17-DOF model was also applicable to simulate other steady-state responses such as the *yaw velocity gain*, and the *lateral acceleration gain*. Yaw velocity gain G_r is defined as the ratio of the steady-state yaw velocity to the input steered wheel angle, and lateral acceleration gain G_{ay} is the ratio of lateral acceleration in steady state to the steered wheel angle. Unlike the 3-DOF model with explicit expressions (Wong, 1993), G_r and G_{ay} can not be calculated or predicted directly by a single execution of the 17-DOF simulation model. Instead the 6WD vehicle was simulated by given a step steering input at various vehicle longitudinal speeds. The step function was similar to the one shown in Figure 7.6-2. The steady-state yaw velocity gain was determined from the steady-state yaw velocity [rad/sec] divided by the angle of steered wheels [deg]. Similar procedure was applied for obtaining the lateral acceleration gain [g/deg]. For the 6WD vehicle operating on sandy terrain and on hard ground, plots of G_r and G_{ay} versus vehicle longitudinal speed are shown in Figure 7.6-13, and Figure 7.6-14, respectively.

For the 6WD vehicle operating on sandy terrain, the dash line in Figure 7.6-13 shows that the yaw velocity gain G_r increases at an almost linear rate with the increase of vehicle longitudinal speed. This indicates that the steady-state handling behaviour of the 6WD

vehicle on sandy terrain is stable and responsive, and approaches *neutral steer*. However on hard ground, the G_r increases at a higher rate than the vehicle speed for speed range below 7 m/s, and thus the vehicle behaves *oversteer*. Beyond this speed range, the rate of increase of G_r drops, and the vehicle exhibits *understeer*.

Figure 7.6-13 Yaw velocity gain characteristics

Figure 7.6-14 Lateral acceleration gain characteristics

Figure 7.6-14 gives the understeer/oversteer properties in terms of the lateral acceleration gain G_{ay} . For the vehicle on sandy terrain, the G_{ay} is nearly proportional to the square of the vehicle longitudinal speed. This confirms that the handling characteristics of the 6WD vehicle operating on sand is neutral steer, according to the criterion from Segel's 3-DOF model (Cole, 1972; Gillespie, 1992; Wong, 1993). For the hard surface condition, the solid curve in the figure demonstrates the similar vehicle behaviour as the curve of yaw velocity gain. That is, for the longitudinal speeds lower than 7 m/s, the vehicle tends to oversteer, and beyond that speed, the vehicle behaves understeer.

The steady-state handling behaviour of the 6WD vehicle operating on sandy terrain and on hard ground are significantly different. On hard ground, the vehicle shows oversteer for longitudinal speeds lower than 7 m/s (25.2 km/h). Fortunately, the operating speeds of the 6WD vehicle on highways are normally within 60 to 100 km/h. For speeds higher than 7 m/s, the vehicle is understeer and stable. The neutral steer behaviour of the vehicle on sandy terrain confirms that the vehicle is stable and responsive within the complete speed range up to 11.5 m/s (41.4 km/h). The 6WD vehicle is generally operated in this speed range on a deformable surface such as the sandy terrain used in the simulation.

Based on the simulation results, the following observations can be made:

- 1) Predictions using the proposed handling model with 17 DOFs bear close resemblance to those obtained using a widely accepted and experimentally validated model. This would indicate that the basic features of the proposed model are realistic.
- 2) There are significant differences in the handling responses of the 6WD vehicle used in the simulation operating on sandy terrain and on hard ground. In a simulated lane-change manoeuvre of the vehicle operating on sandy terrain, time response of yaw velocity shows a 20% time lag, comparing with the steering input. Additionally, the maximum lateral acceleration reaches only 25% of that for operating on hard surface. This is primarily due to the fact that the maximum tire lateral force developed on sandy terrain is much lower than that on hard ground.

- 3) Simulation results also show that with a double-triangular steering input, to fully execute a lane-change manoeuvre, the driver has to make a correction in the opposite direction when operating on hard ground, whereas the driver has to give an additional steering input on sandy terrain. This phenomenon attributes to the combination of two factors. One is the variation of tire normal force due to fore and aft and/or lateral load transfer. The fore and aft load transfer is because of the pitch motion of sprung mass, and the lateral load transfer is due to roll motion. Since tire normal force changed, tire lateral force changed accordingly. The other factor is the non-linear relationship between tire lateral force and slip angle, comparing with that of the linear relationship used in Segel's 3-DOF handling model. Because of the capability of taking into account the load transfer due to the combined roll and pitch, and the non-linear tire model, the 17-DOF handling model can simulate finer details such as the residual heading angle that can not be predicted by Segel's model.
- 4) For steady-state handling, it was shown that with a step steering input, the turning radius on sandy terrain is approximately 65% larger than that on hard ground. Furthermore, the 17-DOF model is able to predict the yaw velocity gain and lateral acceleration gain with a non-linear tire lateral force model. The vehicle was found to behave oversteer at lower longitudinal speeds and understeer at speeds over 25 km/h when operating on hard ground. The vehicle operating on sandy terrain, however, shows neutral steer over the simulated speed range.

7.7 Animation

A picture is worth a thousand words. The best method to demonstrate the handling behaviour of a vehicle is probably by the means of animation. A picture animation program was developed for this study. One of the common guidelines to study the handling, or directional control of a vehicle is to judge its response while following a lane-change manoeuvre (Schieschke and Hiemenz, 1991). Consequently, a lane-change manoeuvre was selected to animate the transient handling response of the 6WD vehicle. For documentation purpose, Figure 7.7-1 shows an animation of only eight pictures during the lane-change manoeuvre. In the program written by visual C++, many pictures

can be serialized to create an image of continuous vehicle motion. The program animates the vehicle handling behaviour in two separate views. The top view shows that the vehicle is changing the lane on a dual-traffic road when given a sinusoidal steering input. This XY trajectory is the result of the effects of yaw velocity, and side velocity. Additionally, vehicle heading angle and steering angle of front wheels are also shown in this view. The other rear view displays mainly the roll motion of the sprung mass (vehicle body) relative to the unsprung mass. From both views, it can be seen that as the vehicle is steered to the left, the sprung mass rolls to the right. The rear view also shows the steered front wheels, and the oscillations of suspension arms.

Figure 7.7-1 Animation of lane-change manoeuvre, 6WD vehicle

Chapter 8

Conclusions and Contributions

8.1 Conclusions

The objectives of this research listed in Section 3.2 have been achieved, and the tasks accomplished and the results obtained may be summarized as follows:

- 1) Development of the laboratory Tire Test Facility (TTF97) with instrumentation and data acquisition system.

An existing tire tester was modified for measuring tire performance under combined longitudinal slip and slip angle in a soil bin. The redesigned tester was equipped with x-y load cells and other instruments to monitor the longitudinal force, lateral force, applied torque (from the measured cable force), drawbar pull, tire sinkage, and linear displacement. A new data acquisition system of eight channels with a continuously scanning algorithm was assembled to acquire and to process the recorded data in an efficient manner. To ensure the validity and accuracy of the experimental work, the sampling rate, crosstalk of x-y sensors, horizontal component of the chain force, chain drive efficiency, tire drop tests, and structure impact tests were examined. The

TTF97 provides a great deal improvement in tire force measurements in comparison with its predecessor, TTF87, and Schwanghart's test rig (Schwanghart, 1968).

- 2) Conducting a series of tire tests to establish the relationships among drawbar pull, lateral force, longitudinal slip, slip angle, and tire sinkage.

Tests were carried out using a full-size, off-road tire under three different inflation pressures and three normal forces. The experimental data collected provides a basic understanding of the development of longitudinal and lateral forces under the combined longitudinal slip and slip angle for an off-road tire. The data also gives the needed information for formulating and validating a model for predicting the combined force of tires on deformable terrain. The experimental data were presented in fitted curves together with raw data. Additionally, soil penetration and shear properties of the Ottawa sand used in the soil bin were measured using Bevameter techniques. Soil parameters were obtained accordingly and incorporated into the new tire model. The experimental data fill the gap in providing force data for tires with combined slips and operation on dry sand. The off-road tire used in this experimental study was a short-lug, high-speed tire for military vehicles. The tire has remarkably distinct characteristics when compared with those of agricultural tires used by Schwanghart (1968, 1981), and by Kutzbach (1994).

- 3) Development of a new tire model for predicting the longitudinal and lateral forces of off-road tires on deformable terrain under different longitudinal slips and slip angles. The model includes all relevant parameters of the tire and of the soil, tire lateral deformation, tire-terrain contact trajectories, lateral bulldozing effects, and sidewall shearing. Furthermore, the rigid or elastic operating mode, relaxation length, and tire lateral stiffness were also taken into account. The predictions based on the model were compared with the experimental data obtained from the laboratory tire test facility TTF97. It was found that there is a reasonable agreement between them, particularly at high slip angles. In comparison with previous work, the new model provides a more thorough representation of tire physical behaviour on deformable terrain, and thus well suitable for the simulations of all-wheel-drive vehicles. In particular, the new model offers significant improvements over Schwanghart's and Del Rosario's (1980) models by including the effects of longitudinal slip, and over

Grechenko's (1969) and Crolla and El-Razaz's (1987) models by taking into account the three-dimensional effect of a tire with deep sinkage on soft terrain.

- 4) Development of a computer simulation model for the handling behaviour of all-wheel-drive, off-road vehicles on deformable terrain.

Based on the software package AUTOSIM, a 17-DOF simulation model was developed, and all important sub-systems such as the steering and suspension systems were considered in detail. Because of the lack of test facilities and the unavailability of a full-size multi-axle, all-wheel-drive, off-road vehicle, it was not possible to conduct field tests to verify the simulation model. The handling behaviour of a 6WD vehicle on a hard surface predicted by the simulation model developed, however, was compared with that predicted by a widely accepted and experimentally validated model, known as Segel's model. It appears that the predicted result bears quite close resemblance to that based on Segel's model. With the new off-road tire model as input, the transient, as well as steady-state handling behaviour of the 6WD vehicle on deformable terrain were predicted. The handling behaviour was examined under a lane-change and a step steering input with rise time manoeuvres. The yaw velocity gain and lateral acceleration gain of the vehicle on paved road and on deformable terrain were evaluated. The 17-DOF model can also simulate some of the factors such as the highly nonlinear effects of pitch, residual heading angle, and roll and pitch frequencies. Since publications on handling simulation of off-road vehicles are few, this research provides a significant insight into the handling behaviour of a vehicle operating on soft terrain in comparison with that on hard ground. Another improvement over Segel's model was that the 17-DOF model is not only for vehicle handling studies but also for simulating vehicle ride comfort if a road profile is used as input. Moreover, a Windows-based two-dimensional animation was developed to show the vehicle's directional response in a lane-change manoeuvre.

Based on the results of the research described in the thesis, the following conclusions and contributions may be drawn:

- 1) A laboratory tire test facility (TTF97) has been successfully developed, with which the longitudinal and lateral tire forces under combined longitudinal slip and slip angle on deformable soil can be measured. Some of the new features of the TTF97

include the abilities of simultaneously measuring tire lateral force, drawbar pull, applied torque (from the measured cable force), sinkage and linear speed of the tire center, and of accommodating tire size up to 14.00R20.

- 2) The longitudinal and lateral behaviour of an off-road tire designed for military vehicles was extensively tested using the laboratory tire test facility on a sandy soil. The test results provide a physical understanding of the tire behaviour, and provide experimental data for the verification of the proposed off-road tire model.
- 3) Based on an analytical study, a new off-road tire model for predicting tire forces under combined longitudinal slip and slip angle on deformable terrain was developed. In comparison with models developed previously, the new model takes into account the effects of longitudinal slip, tire sinkage, and the variations of inflation pressure. In particular, the model predicts the relationship between tire lateral force and drawbar pull at different slip angles and at rigid or elastic operating mode, thus facilitates the simulation of the handling of off-road vehicles with significant tire sinkage and considerable motion resistance on deformable terrain.
- 4) The predictions obtained using the new tire model were compared with measured data. It was found that they exhibit similar trends and acceptable correlation. Thus the new tire model provides a reasonable analytical framework for the development of a realistic and comprehensive vehicle handling model for practical applications.
- 5) A comprehensive, 17-DOF model for the handling behaviour of off-road vehicles, based on the computer code AUTOSIM, has been developed. It was found that the 17-DOF model can be used to simulate off-road vehicle handling behaviour in lane-change and other manoeuvres on both hard ground and deformable terrain.
- 6) In comparison with the handling behaviour on hard ground, the handling of an all-wheel-drive off-road vehicle on deformable terrain demonstrates quite different characteristics. In a lane-change manoeuvre, it shows that due to the low tire lateral force generated on soft terrain, both yaw velocity and lateral acceleration have lower magnitudes, and that considerable time-lag exists in the response with respect to the steering input. To fully execute the course, the driver has to make a correction in the

opposite direction when operating on hard ground, whereas the driver has to give an additional steering input on sandy terrain.

- 7) For a step steering input with rise time, the simulated vehicle on sandy terrain shows that a turning radius is 65% larger than the one on hard ground. Furthermore, the variations of the yaw velocity gain and lateral acceleration gain with respect to forward speed show that the steady-state handling of the vehicle on hard ground changes from oversteer to understeer, as the vehicle speed varies from 0 to 41 km/h. However, the vehicle demonstrates a consistent neutral steer behaviour when operating on soft terrain.

8.2 Future Research

During the course of study, it has been found that certain aspects of the tire test facility, tire model, and off-road vehicle handling model presented in this thesis can be further improved. Some future research topics are suggested and listed below:

- 1) The development of a test facility, or upgrading the existing tire tester TTF97, to study the relationships between lateral force and camber angle of off-road tires under various loading conditions and inflation pressures. This information is of importance since a tire on an off-road vehicle is normally subject to changes in camber with the movement of suspension system. In addition, consider the possibility of controlling tire slip angle during the test so that tire transient characteristics can be measured.
- 2) Conducting a series of tire force measurements with the combined longitudinal skid (when a tire is subject to a braking torque) and slip angle on deformable terrain. This information is important to the simulation of vehicle handling during a turning manoeuvre with brakes applied.
- 3) Conducting tire tests on a variety of terrains, especially the clay soil with different moisture contents. This information will provide additional tire force data to the

further development of the simulation model for off-road vehicle handling over different types of terrain.

- 4) Further development of the analytical off-road tire model to take into account the effects of camber force and aligning torque, and to further improve the modeling of the tire-terrain contact profile, normal pressure distribution, and tire lateral force component due to bulldozing effect.
- 5) Conducting handling tests using a full-size, all-wheel-drive off-road vehicle. To evaluate the accuracy of the 17-DOF vehicle simulation model, it is necessary to conduct vehicle handling tests in the field.
- 6) Incorporating traction control strategies to study handling behaviour of all-wheel-drive vehicles on deformable terrain. A more sophisticated handling model may include modeling of traction control devices. Since the technology of vehicle dynamics control (VDC) can ensure vehicle directional stability under various conditions, using the computer simulation model to evaluate the effects of VDC on off-road vehicle handling behaviour may also be of practical interest.

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Appendix A

Three-DOF Handling Model for 6WD Vehicle

The 3-DOF handling model was developed by Segel (1956) for passenger cars. Based on Segel's approach, the 3-DOF handling model for six-wheel-drive vehicles is derived below. Since the vehicle has three axes, the model becomes a *tandem bicycle model*. For convenience, Figure 2.2.1-2 is replotted as follows.

Figure A-1 Tandem bicycle model for 6WD vehicle

To clarify the interaction among the sprung mass m_s , unsprung mass m_u , and vehicle mass $m (= m_s + m_u)$, Figure A-2 shows the relative positions of sprung mass axes $x_s y_s z_s$, unsprung mass axes $x_u y_u z_u$, entire vehicle axes $x' y' z'$, vehicle axis system xyz , and the earth-fixed axis system XYZ .

Figure A-2 Two-mass system, sprung mass m_s and unsprung mass m_u

A.1 Equations of Motion

Regardless of the number of axles, the equations of motion for Segel's 3-DOF handling model are given below.

$$\sum F_y = mu(\dot{\beta} + r) + m_s h \dot{p} \quad (\text{A.1-1})$$

$$\sum M_z = I_z \dot{r} + I_{xz} \dot{p} \quad (\text{A.1-2})$$

$$\sum M_x = m_s h u (\dot{\beta} + r) + I_x \dot{p} + I_{xz} \dot{r} \quad (\text{A.1-3})$$

$$\text{where } I_x = I_{xs} + m_s h^2 - 2\theta_R I_{xs} + \theta_R^2 I_{zs} \quad (\text{A.1-4})$$

$$I_{xs} = m_s h c - I_{xs} + \theta_R I_{zs} \quad (\text{A.1-5})$$

$$I_z = I_{zs} + I_{zu} + m_s c^2 + m_u e^2 \quad (\text{A.1-6})$$

ΣF_y = external lateral force acting in y -axis direction

ΣM_z = external yawing moment about vertical z -axis

ΣM_x = external rolling moment about inclined roll axis

u = longitudinal velocity

v = side velocity

β = sideslip angle of vehicle mass center, $\beta = v/u$

ϕ = roll angle, m_s relative to m_u

p = roll velocity, $p = \dot{\phi}$

r = yaw velocity

θ_R = roll axis downward angle

I_{xs} = mass moment of inertia with respect to x_s -axis, for m_s

I_{zs} = mass moment of inertia with respect to z_s -axis, for m_s

I_{zu} = mass moment of inertia with respect to z_u -axis, for m_u

I_{xs} = product moment of inertia relative to x_s and z_s axes, for m_s

c = longitudinal distance between m_s and m

e = longitudinal distance between m_u and m

h = height of m_s to x -axis

A.2 External lateral force

For the i -th axle, the developed tire lateral force F_{yi} is a combination of lateral forces generated by a slip angle α and a camber angle γ . The camber angle change is due to vehicle roll motion. The combined lateral force can be expressed as the following.

$$F_{yi} = C_{\alpha i} \alpha_i + C_{\gamma i} E_{\gamma i} \phi \quad (\text{A.2-1})$$

where $C_{\alpha i}$ = cornering stiffness, of the i -th axle

$C_{\gamma i}$ = camber stiffness, of the i -th axle

$E_{\gamma i}$ = roll camber coefficient, of the i -th axle, $E_{\gamma i} = \partial \gamma_i / \partial \phi$

The total lateral force generated from road to vehicle is equal to the sum of the developed lateral force on each axle.

$$\sum F_y = F_{y1} + F_{y2} + F_{y3} \quad (\text{A.2-2})$$

Thus the total lateral force can be expressed in terms of all lateral forces due to slip angles, and all camber forces because of vehicle roll.

$$\sum F_y = C_{\alpha 1} \alpha_1 + C_{\gamma 1} E_{\gamma \phi 1} \phi + C_{\alpha 2} \alpha_2 + C_{\gamma 2} E_{\gamma \phi 2} \phi + C_{\alpha 3} \alpha_3 + C_{\gamma 3} E_{\gamma \phi 3} \phi \quad (\text{A.2-3})$$

Slip angles of each axle can be expressed as the followings.

$$\alpha_1 = \beta + \frac{ar}{u} - \delta - E_{\delta \phi 1} \phi \quad (\text{A.2-4})$$

$$\alpha_2 = \beta - \frac{br}{u} - E_{\delta \phi 2} \phi \quad (\text{A.2-5})$$

$$\alpha_3 = \beta - \frac{(b + L_2)r}{u} - E_{\delta \phi 3} \phi \quad (\text{A.2-6})$$

where $E_{\delta \phi i}$ = roll steer coefficient, of the i -th axle, $E_{\delta \phi i} = \partial \delta_i / \partial \phi$
 δ_i = roll steer, of the i -th axle

Substituting these slip angles into equation (A.2-3), the total lateral force can be further derived accordingly.

$$\begin{aligned} \sum F_y &= C_{\alpha 1} (\beta + ar/u - \delta - E_{\delta \phi 1} \phi) + C_{\gamma 1} E_{\gamma \phi 1} \phi + C_{\alpha 2} (\beta - br/u - E_{\delta \phi 2} \phi) \\ &\quad + C_{\gamma 2} E_{\gamma \phi 2} \phi + C_{\alpha 3} [\beta - (b + L_2)r/u - E_{\delta \phi 3} \phi] + C_{\gamma 3} E_{\gamma \phi 3} \phi \\ &= [C_{\alpha 1} + C_{\alpha 2} + C_{\alpha 3}] \beta + [aC_{\alpha 1} - bC_{\alpha 2} - (b + L_2)C_{\alpha 3}] r/u \\ &\quad + [E_{\gamma \phi 1} C_{\gamma 1} + E_{\gamma \phi 2} C_{\gamma 2} + E_{\gamma \phi 3} C_{\gamma 3} - E_{\delta \phi 1} C_{\alpha 1} - E_{\delta \phi 2} C_{\alpha 2} - E_{\delta \phi 3} C_{\alpha 3}] \phi - C_{\alpha 1} \delta \\ &= Y_\beta \beta + Y_r r + Y_\phi \phi + Y_\delta \delta \end{aligned} \quad (\text{A.2-7})$$

where $Y_\beta = C_{\alpha 1} + C_{\alpha 2} + C_{\alpha 3}$
 $Y_r = [aC_{\alpha 1} - bC_{\alpha 2} - (b + L_2)C_{\alpha 3}] / u$

$$\begin{aligned}
Y_\phi &= E_{\gamma\phi 1} C_{\gamma 1} + E_{\gamma\phi 2} C_{\gamma 2} + E_{\gamma\phi 3} C_{\gamma 3} - E_{\delta\phi 1} C_{\alpha 1} - E_{\delta\phi 2} C_{\alpha 2} - E_{\delta\phi 3} C_{\alpha 3} \\
Y_\delta &= -C_{\alpha 1} \\
C_{\alpha i} &= \text{cornering stiffness (negative), of the } i\text{-th axle, } C_{\alpha i} = 2C'_{\alpha i} \\
C'_{\alpha i} &= \text{cornering stiffness (negative), of a single tire in the } i\text{-th axle}
\end{aligned}$$

A.3 External yawing moment

The external yawing moment M_z is the moment generated from road to vehicle with respect to the z -axis. On each axle, the moment is developed due to two mechanisms. One is the moment created by tire lateral force from the axle multiplying the distance from this axle to the vehicle mass center. The other is the aligning torque generated from sideslip tires on this axle. Thus, the total yawing moment has the following form.

$$\sum M_z = aF_{y1} - bF_{y2} - (b + L_2)F_{y3} + M_{\alpha 1} + M_{\alpha 2} + M_{\alpha 3} \quad (\text{A.3-1})$$

For each axle, the aligning torque due to a tire slip angle α can be computed from the following expression.

$$M_{\alpha i} = C_{z\alpha i} \alpha_i \quad (\text{A.3-2})$$

where $C_{z\alpha i}$ = aligning torque stiff. (positive), of the i -th axle, $C_{z\alpha i} = 2C'_{z\alpha i}$
 $C'_{z\alpha i}$ = aligning torque stiff. (positive), of a single tire in the i -th axle

Substituting equations (A.2-1), (A.2-4), (A.2-5), (A.2-6), and (A.3-2) into equation (A.3-1) of the total yawing moment, the following simplification can be obtained.

$$\begin{aligned}
\sum M_z &= a(C_{\alpha 1} \alpha_1 + C_{\gamma 1} E_{\gamma\phi 1} \phi) - b(C_{\alpha 2} \alpha_2 + C_{\gamma 2} E_{\gamma\phi 2} \phi) \\
&\quad - (b + L_2)(C_{\alpha 3} \alpha_3 + C_{\gamma 3} E_{\gamma\phi 3} \phi) + C_{z\alpha 1} \alpha_1 + C_{z\alpha 2} \alpha_2 + C_{z\alpha 3} \alpha_3 \\
&= aC_{\alpha 1} (\beta + ar/u - \delta - E_{\delta\phi 1} \phi) + aC_{\gamma 1} E_{\gamma\phi 1} \phi - bC_{\alpha 2} (\beta - br/u - E_{\delta\phi 2} \phi) \\
&\quad - bC_{\gamma 2} E_{\gamma\phi 2} \phi - (b + L_2)C_{\alpha 3} [\beta - (b + L_2)r/u - E_{\delta\phi 3} \phi] - (b + L_2)C_{\gamma 3} E_{\gamma\phi 3} \phi \\
&\quad + C_{z\alpha 1} (\beta + ar/u - \delta - E_{\delta\phi 1} \phi) + C_{z\alpha 2} (\beta - br/u - E_{\delta\phi 2} \phi) \\
&\quad + C_{z\alpha 3} [\beta - (b + L_2)r/u - E_{\delta\phi 3} \phi]
\end{aligned}$$

$$\begin{aligned}
&= [aC_{\alpha 1} - bC_{\alpha 2} - (b + L_2)C_{\alpha 3} + C_{z\alpha 1} + C_{z\alpha 2} + C_{z\alpha 3}] \beta \\
&\quad + [a^2 C_{\alpha 1} + b^2 C_{\alpha 2} + (b + L_2)^2 C_{\alpha 3} + aC_{z\alpha 1} - bC_{z\alpha 2} - (b + L_2)C_{z\alpha 3}] r / u \\
&\quad + [-aE_{\delta\phi 1} C_{\alpha 1} + aE_{\gamma\phi 1} C_{\gamma 1} + bE_{\delta\phi 2} C_{\alpha 2} - bE_{\gamma\phi 2} C_{\gamma 2} + (b + L_2)E_{\delta\phi 3} C_{\alpha 3} - \\
&\quad (b + L_2)E_{\gamma\phi 3} C_{\gamma 3} - E_{\delta\phi 1} C_{z\alpha 1} - E_{\delta\phi 2} C_{z\alpha 2} - E_{\delta\phi 3} C_{z\alpha 3}] \phi \\
&\quad + [-aC_{\alpha 1} - C_{z\alpha 1}] \delta \\
&= N_\beta \beta + N_r r + N_\phi \phi + N_\delta \delta
\end{aligned} \tag{A.3-3}$$

$$\begin{aligned}
\text{where } N_\beta &= aC_{\alpha 1} - bC_{\alpha 2} - (b + L_2)C_{\alpha 3} + C_{z\alpha 1} + C_{z\alpha 2} + C_{z\alpha 3} \\
N_r &= [a^2 C_{\alpha 1} + b^2 C_{\alpha 2} + (b + L_2)^2 C_{\alpha 3} + aC_{z\alpha 1} - bC_{z\alpha 2} - (b + L_2)C_{z\alpha 3}] / u \\
N_\phi &= -aE_{\delta\phi 1} C_{\alpha 1} + aE_{\gamma\phi 1} C_{\gamma 1} + bE_{\delta\phi 2} C_{\alpha 2} - bE_{\gamma\phi 2} C_{\gamma 2} + (b + L_2)E_{\delta\phi 3} C_{\alpha 3} \\
&\quad - (b + L_2)E_{\gamma\phi 3} C_{\gamma 3} - E_{\delta\phi 1} C_{z\alpha 1} - E_{\delta\phi 2} C_{z\alpha 2} - E_{\delta\phi 3} C_{z\alpha 3} \\
N_\delta &= -aC_{\alpha 1} - C_{z\alpha 1}
\end{aligned}$$

A.4 External rolling moment

Two suspension mechanisms contribute to the external rolling moment about the inclined roll axis. The first is due to damper force created by the relative roll velocity of the non-rolling chassis to the rolling vehicle body. By a linearized analysis, the damper reaction moment M_{xp} can be expressed as

$$M_{xp} = (\partial M_{xp} / \partial p) \cdot p \tag{A.4-1}$$

For a multi-axle vehicle, the total damper reaction moment per unit vehicle roll velocity is the sum of the reaction moment per unit roll velocity from each axle. Thus,

$$\begin{aligned}
\frac{\partial M_{xp}}{\partial p} &= \sum_{i=1}^n s_{Di} \frac{\partial w_i}{\partial p} \left(\frac{\partial F_{DEi}}{\partial w_i} + \frac{\partial F_{DCi}}{\partial w_i} \right) \\
&= \sum_{i=1}^n s_{Di} E_{wpi} (f_{DEi} + f_{DCi})
\end{aligned} \tag{A.4-2}$$

where s_{Di} = radius arm, roll center perpendicular to damper line of action, of the i -th axle

w_i = linear damper velocity, of the i -th axle

- n = number of axles of vehicle, $n = 3$ for 6WD vehicle
 E_{wpi} = linear damper velocity induced by vehicle roll velocity, of the i -th axle, $E_{wpi} = \partial w_i / \partial p$
 f_{DEi} = extension damper force per unit stroke velocity, of the i -th axle, $f_{DEi} = \partial F_{DEi} / \partial w_i$
 f_{DCi} = compression damper force per unit stroke velocity, of the i -th axle, $f_{DCi} = \partial F_{DCi} / \partial w_i$

The second mechanism to react the roll moment induced by the vehicle roll is also given by a linearized expression.

$$M_{x\phi} = (\partial M_{xp} / \partial \phi) \cdot \phi \quad (\text{A.4-3})$$

The reaction roll moment per unit roll angle ϕ of vehicle body is contributed from two components, the weight-induced moment of sprung mass and the reaction moment due to suspension stiffness.

$$\frac{\partial M_{x\phi}}{\partial \phi} = m_s g h + \sum_{i=1}^n \frac{1}{2} k_{si} l_{si}^2 \quad (\text{A.4-4})$$

- where k_{si} = spring stiffness per single wheelstation, of the i -th axle
 l_{si} = lateral separation between springs, of the i -th axle

Therefore, the external rolling moment is equal to

$$\begin{aligned}
 \sum M_x &= M_{xp} + M_{x\phi} \\
 &= \left[\sum_{i=1}^n S_{Di} E_{wpi} (f_{DEi} + f_{DCi}) \right] p + \left[m_s g h + \sum_{i=1}^n \frac{1}{2} k_{si} l_{si}^2 \right] \phi \\
 &= L_p p + L_\phi \phi
 \end{aligned} \quad (\text{A.4-5})$$

where $L_p = \sum_{i=1}^n S_{Di} E_{wpi} (f_{DEi} + f_{DCi})$

$$L_\phi = m_s g h + \sum_{i=1}^n \frac{1}{2} k_{si} l_{si}^2$$

A.5 Equations of motion of the 3-DOF model

The complete set of 3-DOF equations of motion for the 6WD vehicle is obtained by substituting the external lateral force, equation (A.2-7), the external yawing moment, equation (A.3-3), and the external rolling moment, equation (A.4-5) into equations (A.1-1), (A.1-2), and (A.1-3). Consequently, the equations of motion of the 3-DOF vehicle handling model become

$$mu(\dot{\beta} + r) + m_s h \dot{p} = Y_\beta \beta + Y_r r + Y_\phi \dot{\phi} + Y_\delta \delta \quad (\text{A.5-1})$$

$$I_z \dot{r} + I_{xz} \dot{p} = N_\beta \beta + N_r r + N_\phi \dot{\phi} + N_\delta \delta \quad (\text{A.5-2})$$

$$m_s h u (\dot{\beta} + r) + I_x \dot{p} + I_{xz} \dot{r} = L_p p + L_\phi \dot{\phi} \quad (\text{A.5-3})$$

The above three equations of motion can be expressed in a matrix form. Using the derivative operator $D = d / dt$, the matrix is given below.

$$\begin{bmatrix} muD - Y_\beta & mu - Y_r & m_s h D^2 - Y_\phi \\ -N_\beta & I_z D - N_r & I_{xz} D^2 - N_\phi \\ m_s h u D & I_{xz} D + m_s h u & I_x D^2 - L_p D - L_\phi \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \phi \end{bmatrix} = \begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix} \delta \quad (\text{A.5-4})$$

Appendix B

Drop Test and Impact Test

Drop test and *impact test* are methods for identifying dynamic characteristics of the tire frame assembly. In particular, the former one is for the wheel-axle assembly under the loading condition. The latter is mainly for the steel structure. Procedures and results of each test are given below, respectively.

B.1 Drop Test

According to the procedure described by Wong (1993), the non-rolling tire dynamics in the z-axis can be investigated by a drop test. The test tire was mounted on the tire frame. The complete assembly was lifted up so that the tire barely contacts with a hard, flat surface. The tire was then released suddenly to oscillate against the surface. The normal force was set to 9.23 kN (2,075 lbs), and three measurements for tire inflation pressures p_i of 138 kPa (20 psi) and 345 kPa (50 psi), respectively were conducted. The following two figures show the tire responses due to drop tests in time domain and in frequency domain for $p_i = 138$ kPa, and $p_i = 345$ kPa. For each figure, there are three graphs,

which show the displacement in the z-axis versus time for the three measurements. The fourth graph shows the Power Spectral Density (PSD) in frequency domain, and corresponds to one of the three time response graphs.

Figure B.1-1 Time response and frequency response, $p_i = 138$ kPa

Figure B.1-2 Time response and frequency response, $p_i = 345$ kPa

The PSD was obtained by Fast Fourier Transform (FFT). Both PSD graphs show that the dominant frequency component of the loaded tire in the z-direction, is 1 Hz.

B.2 Impact Test

The objective of impact test was to investigate natural frequencies of the tire frame structure. Since the tire generates a combined longitudinal and lateral force at certain frequencies, it was desired to avoid the resonance of the tire frame. Furthermore, knowing the natural frequencies of the structure, the error associated with the application of load cells was realized.

The tire frame is basically a steel structure supported by a tire, which is supported by the deformable soil. The structure is guided vertically by two four-bar-linkages on left and right hand side each, and is also guided laterally by two groups of side force support. The method of this experiment is to mount a piezoelectric accelerometer in a desired direction, for example, lateral y -direction, and at a position away from the structure mass center. Then strike the structure by a hammer in this lateral y -direction. The preferable point of strike usually locates far away from the point of accelerometer mounting. Normally, this method is used for measuring natural frequencies of simple structures such as a free-free beam or a clamped plate, and the hammer is an instrumented impact hammer. For larger structures such as an aircraft or a vehicle body-in-white, a strike from an impact hammer is insufficient to excite the system. Instead three or more shakers will be used in this situation. The tire frame can be regarded as a structure in between a simple and a large system. Therefore, it was worth to try to excite (strike) the system by a regular steel hammer at a number of locations in each direction. It should be noted that, however, such a hammer blow might not have sufficient energy to provide excitation at low frequencies.

An ideal impact from a hammer generates a unit impulse function. By Fourier Transform, theoretically its frequency spectrum is a continuous function. In other words, it excites a structure with numerous discrete frequencies. If the impulse has enough energy and frequency contents, it can excite all natural frequencies of the system. A typically strike from an instrumented impact hammer is able to excite natural frequencies of a simple structure up to 2,000 Hz (James et al., 1993). For the tire frame structure, it was expected to obtain natural frequencies below 500 Hz. Particularly, the structural frequencies in the range of 3 to 20 Hz that is commonly found from a quarter-car model, would be the primary concern.

The response of the structure was measured by an accelerometer. The piezoelectric accelerometer generates electrical charges, which must be conditioned and amplified by a charge amplifier. The position of accelerometer mounting is shown in Figure B.2-1. The selected frequency range of interest was between 2 and 1,000 Hz. The voltage signal was transmitted from the amplifier to the data acquisition board in a computer.

Figure B.2-1 Accelerometer and charge amplifier, impact test

The recorded data from the accelerometer was the acceleration (m/sec^2 or g) response of the structure due to the hammer strike. This time domain response was then transformed to the frequency domain by FFT algorithm. The standard procedure was to view the acceleration signal in PSD versus frequency in Hertz. The PSD can be plotted by either scale in magnitude or in decibels (dB). Since a substantial damping exists in the system, it was difficult to locate resonance from the response curve in dB scale. Therefore, the PSD graphs were interpreted by the scale in magnitude. At least three measurements were conducted for the hammer strike in each direction, and the associated PSD versus frequency graphs are shown in Figures B.2-2, B.2-3, and B.2-4. Natural

frequencies can be identified by observation from either curves or from the output data. These frequencies are natural frequencies of the tire frame structure. As expected, the structural response in the range of 3 to 20 Hz can not be excited by such an impact test.

Figure B.2-2 PSD graphs of acceleration in *x*-axis

Figure B.2-3 PSD graphs of acceleration in *y*-axis

Figure B.2-4 PSD graphs of acceleration in z-axis

In summary, natural frequencies of the tire frame structure identified from this impact test are listed in Table B.2-1.

Table B.2-1 Natural frequencies of tire frame structure

Direction	Lowest Freq.[Hz]	Second Freq.[Hz]	Third Freq.[Hz]	Remarks
<i>x</i>	324 ~ 364	460 ~ 492		Estimated
<i>y</i>	100 ~ 132	208 ~ 220	420 ~ 468	Estimated
<i>z</i>	368			Exact

Appendix C

Determination of Tire Operating Mode

The required raw data for determining the operating mode of the test tire can be found from the generalized deflection table presented by Jastremski (1994). The average ground pressure p_{gr} within the tire-ground interface was obtained from the applied normal force divided by the measured contact area. Figure C-1 shows the variations of average ground pressure with respect to tire inflation pressure at six normal forces of the range from 2.23 kN to 13.35 kN. The magnitudes of p_{gr} for tire normal forces F_z as well as inflation pressures p_i used in this research were obtained by interpolation from curves in Figure C-1. These average ground pressures are listed in Table C-1.

Table C-1 Values of p_{gr} for different F_z and p_i
Units: kPa

	$p_i = 1 \text{ bar}$	$p_i = 2 \text{ bar}$	$p_i = 3 \text{ bar}$
$F_z = 7.01 \text{ kN}$	122	215	272
$F_z = 8.12 \text{ kN}$	121	210	272
$F_z = 9.23 \text{ kN}$	120	200	271

Figure C-1 Average ground pressure versus inflation pressure, at normal loads

According to Wong (1989, 1993), the mode of operation in the wheel plane of an off-road tire operating on a given terrain is determined by comparing p_{gr} with the critical pressure p_{gr} , which is calculated by

$$p_{gr} = \left[\frac{k_c}{b_t} + k_\phi \right]^{\frac{1}{2n+1}} \left[\frac{3F_z}{(3-n)b_t\sqrt{d_t}} \right]^{\frac{2n}{2n+1}} \quad (C-1)$$

With the following parameters,

$$\begin{aligned} n &= 1.02 \\ k_c &= 12.25 \text{ [kN/m}^{n+1}\text{]} \\ k_\phi &= 2427.4 \text{ [kN/m}^{n+2}\text{]} \\ b_t &= 0.235 \text{ [m]} \\ d_t &= 0.996 \text{ [m]} \\ F_z &= 7.01, 8.12, 9.23 \text{ [kN]} \end{aligned}$$

the critical pressures p_{gr} for the test tire operating on Ottawa sand were obtained from equation (C-1), and are presented in Table C-2. From equation (C-1), it is noted that p_{gr} is independent of tire inflation pressure p_i .

Table C-2 Values of p_{gr} for different F_z

Units: kPa

	$p_i = 1 \text{ bar, } 2 \text{ bar, } 3 \text{ bar}$
$F_z = 7.01 \text{ kN}$	168
$F_z = 8.12 \text{ kN}$	186
$F_z = 9.23 \text{ kN}$	203

The criteria to judge whether the tire operating on Ottawa sand is in rigid or in elastic operating mode in the wheel plane is by comparing the average ground pressure p_{gr} listed in Table C-1 with the critical pressure p_{gr} calculated from equation (C-1) and listed in Table C-2. If p_{gr} is greater than the corresponding p_{gr} , the tire will remain round like a rigid wheel, thus the tire is in rigid operating mode in the wheel plane. On the other hand, if p_{gr} is less than p_{gr} , a portion of the circumference of the tire will be flattened, and the tire is operating in elastic mode. By comparing Table C-1 with Table C-2, the operating modes of the test tire on this particular terrain condition were determined and shown in Table C-3.

Table C-3 Modes of operation in wheel plane,
Michelin 12.5R20XL tire on Ottawa sand

	$p_i = 1 \text{ bar}$	$p_i = 2 \text{ bar}$	$p_i = 3 \text{ bar}$
$F_z = 7.01 \text{ kN}$	Elastic	Rigid	Rigid
$F_z = 8.12 \text{ kN}$	Elastic	Rigid	Rigid
$F_z = 9.23 \text{ kN}$	Elastic	Rigid	Rigid

Appendix D

Determination of Tire Lateral Stiffness

On a hard surface coated with sand particles as shown in Figure D-1, a set of experiments was conducted to investigate the relaxation length of the test tire. Since the lateral stiffness k_y will be used for predicting tire lateral force of both rigid and elastic operating modes, tire inflation pressure was set to 1.0, 2.0, and 3.0 bar. Three normal forces of 7.01, 8.12, and 9.23 kN were applied to the tire for the experiment. Without the forced-slip mechanism, the tire was in free rolling at a specified slip angle. The *relaxation length* is defined as the tire travel distance when tire lateral force reaches $1 - e^{-1}$ (or 63.2 %) of the steady-state value upon a slip angle change (Lund et al., 1992; Dixon, 1996). This is because for a sudden change of slip angle, the tire needs a finite response time for its contact patch to take up a new distorted shape. Figure D-2 shows that tire relaxation length varies with respect to slip angle and tire normal force. As the slip angle increases, the relaxation length drops. On the other hand, the relaxation length is also affected by the normal force acting on the tire; however, this effect is marginal.

Based on the *stretched string theory*, tire lateral stiffness k_y for a rolling, sideslip tire can be identified from the relationship between tire lateral force F_y and slip angle α (Segel, 1988; Wong, 1993). This relationship is described by the following equation.

$$\frac{F_y}{\alpha} = 2k_y \left(l_r + \frac{l_t}{2} \right)^2 \quad (D-1)$$

$$l_t = 2\sqrt{z_d(d_t - z_d)} \quad (D-2)$$

where l_r = relaxation length
 l_t = contact length along wheel plane
 z_d = tire vertical deflection
 d_t = tire diameter

On the hard sandy surface, tire lateral forces at slip angles of 2°, 4°, 6.5°, and 10° were measured. For each combination of normal force and slip angle, the corresponding tire relaxation length was obtained. Accordingly, tire lateral stiffness k_y was computed from equation (D-1). The variation of k_y at each tire inflation pressure is shown in Figure D-3, and the mean value of k_y is shown in Figure D-4. The mean values of 90.80, 95.58, and 99.95 kPa for inflation pressures of 1.0, 2.0, and 3.0 bar, respectively, were used for the lateral stiffness k_y in computing tire lateral force due to elasticity.

Figure D-1 Tire frame assembly operating on hard sandy surface

Inflation Pressure [bar]: 1

Inflation Pressure [bar]: 2

Inflation Pressure [bar]: 3

Figure D-2 Variation of tire relaxation length, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation

Inflation Pressure [bar]: 1

Inflation Pressure [bar]: 2

Inflation Pressure [bar]: 3

Figure D-3 Variation of tire lateral stiffness, $p_i = 1$ bar, 2 bar and 3 bar, three-dimensional representation

Figure D-4 Mean value of tire lateral stiffness, $p_i = 1$ bar, 2 bar and 3 bar

Appendix E

Roll and Pitch Frequencies of 6WD Vehicle

A fundamental method to verify the *damped frequency of oscillation* for a dynamic system is by forming the equations of motion according to Newton's second law of motion. For the 6WD vehicle, it was desired to investigate the damped frequency in roll and in pitch for the sprung mass relative to unsprung mass. The following analysis assumes that the vehicle is in static equilibrium position.

E.1 Roll Frequency

Figure E.1-1 shows a schematic diagram of the vehicle subject to a roll motion. With the excitation of a small roll angle ϕ , the spring and damper forces create a reaction moment to balance the roll motion of the sprung mass. The moment equation about the roll center 0 is written as equation (E.1-1), and simplified as (E.1-2).

$$\Sigma M_0 = -2\left(k\frac{T}{2}\phi\right)\frac{T}{2} - 2\left(c\frac{T}{2}\dot{\phi}\right)\frac{T}{2} = I_{xx}\ddot{\phi} \quad (\text{E.1-1})$$

$$\ddot{\phi} + c \frac{T^2}{2I_{xx}} \dot{\phi} + k \frac{T^2}{2I_{xx}} \phi = 0 \quad (\text{E.1-2})$$

- where ϕ = roll angle, $[\text{rad}]$
 c = damping coefficient for left or right side of vehicle, $[\text{N} \cdot \text{s} / \text{m}]$
 k = ride rate for left or right side of vehicle, $[\text{N} / \text{m}]$
 I_{xx} = mass moment of inertia about x -axis, sprung mass, $[\text{kg} \cdot \text{m}^2]$
 T = track of vehicle, $[\text{m}]$

Figure E.1-1 Roll oscillation of sprung mass

The differential equation of this one DOF system can be expressed in terms of damping ratio ζ and natural frequency ω_n .

$$\ddot{\phi} + 2\zeta\omega_n\dot{\phi} + \omega_n^2\phi = 0 \quad (\text{E.1-3})$$

$$\text{where } \omega_n = \sqrt{\frac{k}{2I_{xx}}}T \quad (\text{E.1-4})$$

$$\zeta = \frac{cT}{2\sqrt{2I_{xx}k}} \quad (\text{E.1-5})$$

The homogeneous solution of this second-order differential equation can be found from most textbooks. The solution $\phi(t)$ corresponds physically to that of the *free-damped vibration* in roll. Without solving the equation, the solution is given as below.

$$\phi(t) = e^{-\zeta\omega_n t} \left(\frac{\dot{\phi}(0) + \zeta\omega_n \phi(0)}{\omega_n \sqrt{1-\zeta^2}} \sin \sqrt{1-\zeta^2} \omega_n t + \phi(0) \cos \sqrt{1-\zeta^2} \omega_n t \right) \quad (\text{E.1-6})$$

The solution indicates that the frequency of damped oscillation in roll is equal to

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \quad (\text{E.1-7})$$

For the 6WD vehicle, related parameters are listed below:

$$\begin{aligned} I_{xx} &= 14,515 \text{ [kg} \cdot \text{m}^2\text{]} \\ k &= 288,814 \text{ [N/m]} \\ c &= 24,054 \text{ [N} \cdot \text{s/m]} \\ T &= 2.314 \text{ [m]} \end{aligned}$$

From equations (E.1-4) and (E.1-5), both the fundamental natural frequency and the damping ratio of the vehicle in roll can be computed.

$$\begin{aligned} \omega_n &= 7.30 \text{ [rad/s]} \\ \zeta &= 0.304 \end{aligned}$$

The damped frequency in roll is obtained from equation (E.1-7).

$$\begin{aligned} \omega_d &= 6.95 \text{ [rad/s]} \\ &= 1.11 \text{ [Hz]} \end{aligned}$$

The frequency approaches 1.01 and 1.07 Hz for on hard ground and on sandy terrain, respectively, of the simulation result from the 17-DOF handling model. It is noted that the 1.11 Hz was obtained from the simplified vehicle roll model in static equilibrium position, whereas the 1.01 and 1.07 Hz were from the simulated vehicle in manoeuvring.

E.2 Pitch Frequency

The natural frequency of pitch oscillation for a wheeled vehicle can be obtained from the simplified 2-DOF ride model, the *pitch plane model*. The model for passenger cars can be found from literatures such as Cole (1972), Gillespie (1992) and Thomson (1993). Considering the 6WD, 3-axle vehicle, the pitch plane model is illustrated in Figure E.2-1. Similar to that of the roll model, the sprung mass can be effectively modeled without consideration of the unsprung mass because of the wide separation of natural frequencies.

Figure E.2-1 Pitch plane model of sprung mass

It is noted that the damped natural frequency ω_d relates to the undamped natural frequency ω_n by the expression of $\omega_d = \omega_n \sqrt{1 - \zeta^2}$. Both frequencies are essentially the same in magnitude for many real systems that have damping of less than 20 percent ($\zeta < 0.2$). For this reason, the natural frequencies and the associated mode shapes of multiple-DOF systems are usually determined by considering undamped free vibration, the *normal mode vibration*. The differential equations for describing the bounce z and pitch θ can be written as

$$\ddot{z} + \alpha z + \beta \theta = 0 \quad (\text{E.2-1})$$

$$\ddot{\theta} + \frac{\beta}{\rho^2} z + \gamma \theta = 0 \quad (\text{E.2-2})$$

$$\text{where } \alpha = (k_1 + k_2 + k_3)/m_s \quad (\text{E.2-3})$$

$$\beta = [k_2 b + k_3 (b + L_2) - k_1 a]/m_s \quad (\text{E.2-4})$$

$$\gamma = [k_1 a^2 + k_2 b^2 + k_3 (b + L_2)^2]/m_s \rho^2 \quad (\text{E.2-5})$$

$$\begin{aligned}
k_i &= \text{ride rate, of the } i\text{-th axle, } [N/m] \\
m_s &= \text{sprung mass, } [kg] \\
I_{yy} &= \text{mass moment of inertia about } y\text{-axis, sprung mass, } [kg \cdot m^2] \\
\rho &= \text{radius of gyration, } \rho = \sqrt{I_{yy}/m_s}, [m]
\end{aligned}$$

Examining differential equations (E.2-1) and (E.2-2), it was found that the motions of bounce and pitch are coupled unless $\beta = 0$ or $k_1 a = k_2 b + k_3 (b + L_2)$. The special case of $\beta = 0$ results in inferior ride. Generally, the bounce and pitch occur simultaneously and are not of a simple harmonic nature. Since the two coupled vibration modes are bounce dominated and pitch dominated, the problem can be treated as the normal mode vibration. For the normal mode of oscillation, the sprung mass undergoes harmonic motions in bounce and in pitch at the same frequency ω . The solutions to the differential equations may be expressed in sinusoidal form:

$$z = Z \cos \omega t \quad (E.2-6)$$

$$\theta = \Theta \cos \omega t \quad (E.2-7)$$

$$\begin{aligned}
\text{where } Z &= \text{amplitude of bounce, } [m] \\
\Theta &= \text{amplitude of pitch, } [rad]
\end{aligned}$$

Differentiate both equations (E.2-6) and (E.2-7) twice, substitute z , $\ddot{z}$, θ , and $\ddot{\theta}$ into equations (E.2-1) and (E.2-2), and divide each term by $\cos \omega t$, the differential equations are simplified to

$$\frac{Z}{\Theta} = -\frac{\beta}{\alpha - \omega^2} \quad (E.2-8)$$

$$\frac{Z}{\Theta} = -\frac{\gamma - \omega^2}{\beta / \rho^2} \quad (E.2-9)$$

Equations (E.2-8) and (E.2-9) define conditions under which the coupled motions can occur. In other words, they give the amplitude ratios of bounce to pitch, or normal modes

for two different natural frequencies. The natural frequencies of the two modes can be obtained by equating the right sides of equations (E.2-8) and (E.2-9).

$$\omega^4 - (\alpha + \gamma)\omega^2 + \alpha\gamma - \frac{\beta^2}{\rho^2} = 0 \quad (\text{E.2-10})$$

Two of the four roots from the above equation are imaginary and can be ignored. The other two roots represent the natural frequencies of the vibration modes.

$$\omega_{1,2} = \sqrt{\frac{\alpha + \gamma}{2} \pm \sqrt{\frac{(\alpha - \gamma)^2}{4} + \frac{\beta^2}{\rho^2}}} \quad (\text{E.2-11})$$

The amplitude ratios, or normal modes can be identified by substituting ω_1 and ω_2 into equation (E.2-8) or (E.2-9). When Z/Θ is positive, both Z and Θ must be positive or both negative. Thus the oscillation center locates ahead of the sprung mass center by a distance of $x = Z/\Theta$. For negative Z/Θ , the oscillation center is behind the mass center by a distance x equal to Z/Θ . Furthermore, one distance is large enough that the oscillation center falls outside the vehicle wheelbase, and the other is small so that the center locates within the wheelbase. When the oscillation center is outside the wheelbase, the motion is predominantly bounce, and the associated frequency is called the bounce frequency. For the center within the wheelbase, the motion is predominantly pitch, and the associated frequency is the pitch frequency.

The following parameters of the 6WD vehicle were given for computing the two natural frequencies.

$$\begin{aligned} m_s &= 12,030 \text{ [kg]} \\ I_{yy} &= 44,870 \text{ [kg} \cdot \text{m}^2\text{]} \\ k_1 &= 215,532 \text{ [N/m]} \\ k_2 &= 181,048 \text{ [N/m]} \\ k_3 &= 181,048 \text{ [N/m]} \\ a &= 1.712 \text{ [m]} \\ b &= 0.688 \text{ [m]} \\ L_2 &= 1.41 \text{ [m]} \end{aligned}$$

Accordingly, the computed variables were:

$$\begin{aligned}\alpha &= 48.016 \\ \beta &= 11.256 \\ \gamma &= 33.759 \\ \rho &= 1.931 [m]\end{aligned}$$

Natural frequencies ω_1 and ω_2 were calculated from equation (E.2-11).

$$\begin{aligned}\omega_1 &= 7.078 [rad/s] = 1.126 [Hz] \\ \omega_2 &= 5.628 [rad/s] = 0.896 [Hz]\end{aligned}$$

Each amplitude ratio Z/Θ can be obtained by substituting the associated frequency ω_1 or ω_2 into equation (E.2-8) or (E.2-9). Therefore,

$$\left(\frac{Z}{\Theta}\right)_{\omega_1} = 5.41 [m/rad] = 0.189 [m/deg]$$

$$\left(\frac{Z}{\Theta}\right)_{\omega_2} = -0.689 [m/rad] = -0.024 [m/deg]$$

Figure E.2-2 Bounce vibration mode of sprung mass, pitch plane model

The mode shape corresponding to ω_1 is illustrated in Figure E.2-2. The positive Z/Θ indicates that the sign of z and the sign of θ are always the same. The vibration mode can be imagined as a coupled bounce and pitch motion with respect to a pseudo oscillation center, which locates 5.41 m ahead of the sprung mass center and outside the

wheelbase. When the sprung mass oscillates with 1° positive or negative, it also bounces 0.189 m down or up. Since this coupled mode is dominated by bounce, the oscillation center is called the bounce center.

The negative Z/Θ associated to ω_2 can be explained by Figure E.2-3. The mode shape shows when the sprung mass oscillates 1° positive, the mass bounces 0.024 m negative (upwards), or vice versa. The pitch mode dominates this coupled motion, and the pitch center locates 0.689 m behind the sprung mass center.

Figure E.2-3 Pitch vibration mode of sprung mass, pitch plane model

The analysis was carried out with the assumption of the 6WD vehicle in normal mode vibrations. The natural frequency associated to the pitch dominated mode, $\omega_2 = 0.896$ Hz was found to be fairly close to the pitch frequencies of 0.83 and 0.86 Hz for operations on sandy terrain and on hard ground, from the handling simulation given in Section 7.6.