1

**PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT**

PRECISIONSM RESOLUTION TARGETS

National Library
of Canada

Acquisitions and
Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale
du Canada

Direction des acquisitions et
des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

For further information contact:

For further information contact:

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

Development of an Interactive Design and Analysis Program for Steel Beam-Columns

by

Petr Srna

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering

Department of Civil and Environmental Engineering
Carleton University
Ottawa, Ontario
March 31st, 1995

The Master of Engineering in the Civil and Environmental Engineering
is a joint program with the University of Ottawa,
administered by the Ottawa-Carleton Institute for Civil Engineering

© 1995 by P. Srna

National Library
of Canada

Acquisitions and
Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale
du Canada

Direction des acquisitions et
des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your file Votre référence

Our file Notre référence

THE AUTHOR HAS GRANTED AN
IRREVOCABLE NON-EXCLUSIVE
LICENCE ALLOWING THE NATIONAL
LIBRARY OF CANADA TO
REPRODUCE, LOAN, DISTRIBUTE OR
SELL COPIES OF HIS/HER THESIS BY
ANY MEANS AND IN ANY FORM OR
FORMAT, MAKING THIS THESIS
AVAILABLE TO INTERESTED
PERSONS.

L'AUTEUR A ACCORDE UNE LICENCE
IRREVOCABLE ET NON EXCLUSIVE
PERMETTANT A LA BIBLIOTHEQUE
NATIONALE DU CANADA DE
REPRODUIRE, PRETER, DISTRIBUER
OU VENDRE DES COPIES DE SA
THESE DE QUELQUE MANIERE ET
SOUS QUELQUE FORME QUE CE SOIT
POUR METTRE DES EXEMPLAIRES DE
CETTE THESE A LA DISPOSITION DES
PERSONNE INTERESSEES.

THE AUTHOR RETAINS OWNERSHIP
OF THE COPYRIGHT IN HIS/HER
THESIS. NEITHER THE THESIS NOR
SUBSTANTIAL EXTRACTS FROM IT
MAY BE PRINTED OR OTHERWISE
REPRODUCED WITHOUT HIS/HER
PERMISSION.

L'AUTEUR CONSERVE LA PROPRIETE
DU DROIT D'AUTEUR QUI PROTEGE
SA THESE. NI LA THESE NI DES
EXTRAITS SUBSTANTIELS DE CELLE-
CI NE DOIVENT ETRE IMPRIMES OU
AUTREMENT REPRODUITS SANS SON
AUTORISATION.

ISBN 0-612-03035-0

Canada

Name Petr SRNA

Dissertation Abstracts International is arranged by broad, general subject categories. Please select the one subject which most nearly describes the content of your dissertation. Enter the corresponding four-digit code in the spaces provided.

0534

0534

U·M·I

SUBJECT TERM

SUBJECT CODE

Subject Categories

THE HUMANITIES AND SOCIAL SCIENCES

COMMUNICATIONS AND THE ARTS

Architecture 0729
Art History 0377
Cinema 0900
Dance 0378
Fine Arts 0357
Information Science 0723
Journalism 0391
Library Science 0399
Mass Communications 0708
Music 0413
Speech Communication 0459
Theater 0465

EDUCATION

General 0515
Administration 0514
Adult and Continuing 0516
Agricultural 0517
Art 0273
Bilingual and Multicultural 0282
Business 0688
Community College 0275
Curriculum and Instruction 0727
Early Childhood 0518
Elementary 0524
Finance 0277
Guidance and Counseling 0519
Health 0680
Higher 0745
History of 0520
Home Economics 0278
Industrial 0521
Language and Literature 0279
Mathematics 0280
Music 0522
Philosophy of 0998
Physical 0523

Psychology 0525
Reading 0535
Religious 0527
Sciences 0714
Secondary 0533
Social Sciences 0534
Sociology of 0340
Special 0529
Teacher Training 0530
Technology 0710
Tests and Measurements 0288
Vocational 0747

LANGUAGE, LITERATURE AND LINGUISTICS

Language 0679
General 0289
Ancient 0290
Linguistics 0291
Modern 0401
Literature 0294
General 0295
Classical 0297
Comparative 0298
Medieval 0316
Modern 0305
African 0352
American 0355
Asian 0593
Canadian (English) 0311
Canadian (French) 0312
English 0313
Germanic 0314
Latin American 0501
Middle Eastern 0503
Romance 0505
Slavic and East European 0508

PHILOSOPHY, RELIGION AND THEOLOGY

Philosophy 0422
Religion 0318
General 0321
Biblical Studies 0319
Clergy 0320
History of 0322
Philosophy of 0469
Theology 0323

SOCIAL SCIENCES

American Studies 0323
Anthropology 0324
Archaeology 0326
Cultural 0327
Physical 0310
Business Administration 0272
General 0770
Accounting 0454
Banking 0338
Management 0385
Marketing 0501
Canadian Studies 0503
Economics 0505
General 0508
Agricultural 0509
Commerce Business 0510
Finance 0511
History 0358
Labor 0366
Theory 0351
Folklore 0358
Geography 0366
Gerontology 0351
History 0578

Ancient 0579
Medieval 0581
Modern 0582
Black 0328
African 0331
Asia, Australia and Oceania 0332
Canadian 0334
European 0335
Latin American 0336
Middle Eastern 0333
United States 0337
History of Science 0585
Law 0398
Political Science 0615
General 0616
International Law and Relations 0617
Public Administration 0814
Recreation 0452
Social Work 0626
Sociology 0627
General 0938
Criminology and Penology 0631
Demography 0628
Ethnic and Racial Studies 0629
Individual and Family Studies 0630
Industrial and Labor Relations 0700
Public and Social Welfare 0344
Social Structure and Development 0709
Theory and Methods 0999
Transportation 0453
Urban and Regional Planning 0453
Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES

Agriculture 0473
General 0285
Agronomy 0475
Animal Culture and Nutrition 0476
Animal Pathology 0359
Food Science and Technology 0478
Forestry and Wildlife 0479
Plant Culture 0480
Plant Pathology 0817
Plant Physiology 0777
Range Management 0746
Wood Technology 0306
Biology 0287
General 0308
Anatomy 0309
Biostatistics 0379
Botany 0329
Cell 0353
Ecology 0369
Entomology 0793
Genetics 0410
Limnology 0307
Microbiology 0317
Molecular 0416
Neuroscience 0433
Oceanography 0821
Physiology 0778
Radiation 0472
Veterinary Science 0786
Zoology 0760
Biophysics 0425
General 0996
Medical 0996

EARTH SCIENCES

Biogeochemistry 0425
Geochemistry 0996

Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
Paleozoology 0985
Palynology 0427
Physical Geography 0368
Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES

Environmental Sciences 0768
Health Sciences 0566
General 0300
Audiology 0992
Chemotherapy 0567
Dentistry 0350
Education 0769
Hospital Management 0758
Human Development 0982
Immunology 0564
Medicine and Surgery 0347
Mental Health 0569
Nursing 0570
Nutrition 0380
Obstetrics and Gynecology 0354
Occupational Health and Therapy 0381
Ophthalmology 0571
Pathology 0419
Pharmacology 0572
Pharmacy 0382
Physical Therapy 0573
Public Health 0574
Radiology 0575
Recreation 0575

Speech Pathology 0460
Toxicology 0383
Home Economics 0386

PHYSICAL SCIENCES

Pure Sciences 0485
Chemistry 0749
General 0486
Agricultural 0487
Analytical 0488
Biochemistry 0738
Inorganic 0490
Nuclear 0491
Organic 0494
Pharmaceutical 0495
Physical 0754
Polymer 0405
Radiation 0605
Mathematics 0986
Physics 0606
General 0608
Acoustics 0748
Astronomy and Astrophysics 0607
Atmospheric Science 0798
Atomic 0759
Electronics and Electricity 0609
Elementary Particles and High Energy 0610
Fluid and Plasma 0752
Molecular 0756
Nuclear 0611
Optics 0463
Radiation 0346
Solid State 0984
Statistics 0984
Applied Sciences 0346
Applied Mechanics 0984
Computer Science 0984

Engineering 0537
General 0538
Aerospace 0539
Agricultural 0540
Automotive 0541
Biomedical 0542
Chemical 0543
Civil 0544
Electronics and Electrical 0348
Heat and Thermodynamics 0545
Hydraulic 0546
Industrial 0547
Marine 0794
Materials Science 0548
Mechanical 0743
Metallurgy 0551
Mining 0552
Nuclear 0549
Packaging 0765
Petroleum 0554
Sanitary and Municipal 0790
System Science 0428
Geotechnology 0796
Operations Research 0795
Plastics Technology 0994
Textile Technology 0994

PSYCHOLOGY

General 0621
Behavioral 0384
Clinical 0622
Developmental 0620
Experimental 0623
Industrial 0624
Personality 0625
Physiological 0989
Psychobiology 0632
Psychometrics 0451
Social 0451

Department of Civil and Environmental Engineering

The Undersigned recommend to the Faculty of Graduate
Studies and Research acceptance of the thesis

**Development of an Interactive Design
and Analysis Program for Steel Beam-Columns**

submitted by

Petr Srna

in partial fulfillment of the requirements for
the degree of Master of Engineering

Chair, Department of Civil and
Environmental Engineering

Thesis Supervisor

Carleton University
Ottawa, Ontario, Canada
March 31, 1995

Abstract

The thesis presented describes the theoretical background, purpose and examples of a graphics-oriented, event-driven program BeCol developed for the analysis and design of steel beam-columns.

As an educational tool, BeCol generates, by Newmark's method the in-plane strength interaction curves for I-shapes (with or without residual stresses), HSS and rectangular sections, to illustrate the effects of a non-uniform moment field and to evaluate the equivalent moment factor ω_1 . The behaviour of concentrically loaded columns is demonstrated by plotting both elastic solutions and inelastic curves for all relevant failure modes for any user-defined doubly symmetric, singly symmetric or asymmetric shape.

As a design tool, BeCol designs any doubly symmetric, prismatic steel beam-column, for any number of load combinations and cross-section types in accordance with Canadian Limit State Design standards, CAN/CSA S16.1-M89, S16.1-94. The design rules calculate the capacity of Class 4 sections in accordance with CAN/CSA-S136-M89 and calculate the appropriate bending moment diagrams and lateral torsional buckling capacity (including interactive buckling) for the specified lateral supports. All applicable interaction curves and an engineering report will be displayed for any section selected from this list.

As a research tool and an advanced design tool, specially developed finite element

analysis was included in BeCol to determine beam-column behaviour, particularly for members with singly symmetric or asymmetric cross-sections and complicated boundary and loading conditions. A graphical interface was developed for problem definition, mesh generation and post-processing of results.

A number of library utilities were developed to define any feasible cross-section, to calculate all geometric and torsional properties, to maintain tables of user-defined sections, to allow for ease of development of the graphics user-interface and to maintain the program. The program BeCol, written in the C language, is designed to run on a UNIX platform, using the HOOPS graphics system to display the graphical interface.

The proposed submodules of the package are intended to be integrated with a comprehensive integrated system for analysis, design, fabrication and erection of steel structures.

Acknowledgement

I would like to thank my thesis supervisor Stephen Kennedy for his guidance, support, ideas, accessibility, unfailing encouragement and desire for quality throughout my work on this thesis. His efforts to provide excellent computing resources and opportunities of consulting with other experts are also greatly appreciated.

I would like to extend my thanks to Neal Holtz who gave me his advice and assistance in key moments, the benefit of his experience, and for providing me with a program for accessing steel section tables.

I sincerely thank Charles Albert for providing his finite element program, suggestions and examples that were valuable in porting the program into Unix environment.

I dedicate this thesis to my wife Judith, for her continued patience, support, encouragement and letting me spend hours chasing after bugs when I should have been doing other things.

Contents

Abstract	i
Acknowledgement	iii
List of Figures	viii
List of Symbols	xii
1 Introduction	1
1.1 Background	1
1.2 Objective	2
1.3 Approach	3
1.3.1 Education	3
1.3.2 Design	3
1.3.3 Research	4
1.3.4 Utilities	4
1.3.5 Interface	4
1.4 Scope	5
2 BeCol as an Educational Tool	7

2.1	In-plane Strength Interaction Curves	7
2.1.1	Theoretical Background	7
2.1.2	Implementation	10
2.2	Column Curves	19
2.2.1	General	19
2.2.2	Elastic Column	20
2.2.3	Inelastic Column - CAN/CSA-S16.1-M89	22
2.2.4	Local Buckling - CAN/CSA-S136-M89	24
2.2.5	Inelastic Column - CAN/CSA-S16.1-94	26
2.2.6	Implementation	27
3	Beam-Column Design	35
3.1	CAN/CSA-S16.1-M89: Beam-Column Design Rules	36
3.2	CAN/CSA-S136-M89: Beam-Column Design – Class 4 Sections	40
3.3	Commentary on Beam-Column Design Rules	43
3.3.1	Uniaxial Strong Axis Bending	43
3.3.2	Uniaxial Weak Axis Bending	46
3.3.3	Biaxial Bending	47
3.3.4	Value of K	48
3.4	Design Strategy	49
3.5	Beam-Column Design Check and Redesign	50
3.5.1	Design Example	57
4	BeCol for Research Using Finite Element Analysis	65

4.1	Overview of the FEA Program	66
4.1.1	Element Description	66
4.1.2	Inelastic Incremental Analysis	68
4.1.3	Elastic Bifurcation Analysis	72
4.2	Problem Definition	75
4.2.1	Cross-Section Node Definition	75
4.2.2	Mesh Generation	76
4.2.3	Material Properties, Residual Stresses	77
4.2.4	Boundary Conditions	77
4.2.5	Load Definition	78
4.3	Problem Analysis and Results	78
4.3.1	Analysis Execution	79
4.3.2	Deformed Geometry	80
4.3.3	Stresses and Strains	80
4.3.4	Load-Deformation Diagrams	80
4.3.5	Load Summary, Reactions	81
4.4	Example	82
5	Library Functions in BeCol	103
5.1	Cross-Section Utilities	103
5.1.1	Cross-Section Definition	104
5.1.2	Torsional Properties	105
5.1.3	User-Defined Cross-Sections	109

5.1.4	Organization of Cross-Section Tables	114
5.2	Interface library functions	118
5.2.1	System Architecture	118
5.2.2	Builders	120
5.2.3	Operators	121
5.2.4	Utilities	122
5.2.5	Notes	126
6	Program Description	127
6.1	General Information	127
6.2	Program Execution	128
6.2.1	Menu Organization and Sequence of Execution	128
6.2.2	Execution of Independent Modules	132
6.3	File Description and Organization	133
6.4	Incorporated existing source codes	137
6.5	Program installation	138
7	Conclusions, Recommendations and Future Work	140
7.1	Conclusions	140
7.2	Recommendations and Future Work	142
	References	145

List of Figures

1.1	Front page of BeCol	5
2.1	M-C- Φ relationships for W-shapes with residual stresses	11
2.2	Definition for calculation of M_{max}	12
2.3	Calculated ultimate moment M_{max}	13
2.4	Record of Nemark's integration	15
2.5	Definition for calculation of interaction curves	16
2.6	In-plane strength interaction curves for varying slendernesses ($\kappa =$ -0.5 and $\kappa = -1.0$)	16
2.7	In-plane strength interaction curves for varying κ values	17
2.8	Column curves of a doubly symmetric section, W200x36	29
2.9	Column curves of a doubly symmetric section, Cruciform	30
2.10	Column curves of a singly symmetric section, C310x37	32
2.11	Column curves of a singly symmetric section, WWT250x153	33
2.12	Column curves of an asymmetric section, L150x100x16	34
3.1	Member and load definition for design check	51
3.2	Design check: interaction curves plotted for a constant M_{fy}	53
3.3	Design check: interaction curves plotted for a constant M_{fx}	53

3.4	Interaction equations input - Class ranges	55
3.5	Interaction equations output - Class ranges	55
3.6	Calculation of K -value	57
3.7	Section type database selection	58
3.8	Storey 2-3: gravity load	59
3.9	Storey 1-2: gravity load	59
3.10	Storey 2-3: gravity + lateral load	61
3.11	Storey 1-2: gravity + lateral load	61
3.12	Table of recommended sections for the frame	62
3.13	Storey 2-3: gravity load	63
3.14	Storey 1-2: gravity load	63
3.15	Storey 2-3: gravity + lateral load	64
3.16	Storey 1-2: gravity + lateral load	64
4.1	Shape definition	83
4.2	Base mesh on the cross-section and its refinement	84
4.3	Mesh definition along the member length	85
4.4	Generated triangular mesh	87
4.5	Material properties	87
4.6	Test-to-predicted ratio for different boundary condition models	88
4.7	Definition of boundary conditions	89
4.8	Load definition	90
4.9	Load increment definition and job execution	91
4.10	Summary of loads and reactions at each step	92

4.11	Deformations of the buckled member	93
4.12	Front view of the buckled member	93
4.13	Distribution of strains parallel with the longitudinal axis	94
4.14	Shear strain distribution at the top surfaces	95
4.15	Shear strain distribution at the bottom surfaces	96
4.16	Stress distribution at the top surfaces	97
4.17	Stress distribution at the bottom surfaces	98
4.18	Euclidian norm of total displacements	100
4.19	Mid-height, web-flange juncture: u_z	100
4.20	Mid-height, web-flange juncture: θ_z	101
4.21	Mid-height, web-flange juncture: u_x	101
4.22	Mid-height, web tip: u_x	102
4.23	Mid-height, flange tip: u_y	102
5.1	Calculated dimensions and geometric properties	105
5.2	Menu of cross-section types	110
5.3	I-shape selection table	111
5.4	Plate selection table	111
5.5	List of elements	112
5.6	List of point coordinates	113
5.7	Table of shape properties	113
5.8	Job definition	114
5.9	Typical user-defined section table	116
5.10	Nomenclature used for built-up shapes	117

5.11 Basic system architecture	119
5.12 Menus of utilities (BeCol and EMAP)	123
5.13 Dialogue box with default values and entry check	124
5.14 A table with a scroll-bar (job selection table)	125
6.1 Main menu of BeCol	129
6.2 File organization of BeCol	134

List of Symbols

A	cross-sectional area
A_e	effective cross-sectional area
A_f	flange area
A_m	area bounded by the mean perimeter of a compartment
A_w	web area
B	effective width ratio of an element in compression (b/t)
b	element width; effective width; plate dimension
C	compressive force, axial load; centroid of a cross-section
C_e	Euler buckling strength
C_f	factored axial load
C_r	factored compressive resistance
C_w	warping torsional stiffness (mm^6)
C_y	axial compressive load at yield stress
c	relative thrust ($c = C/C_y$)
$\{D\}$	Nodal degrees of freedom of a member
d	overall depth of section; correction factor in m-c- ϕ relationships
d_{bot}	x -axis-to-bottom depth
d_{top}	y -axis-to-top depth
E	elastic modulus of steel

$[E]$	elastic constitutive matrix
$[E_{ep}]$	elasto-plastic constitutive matrix
F_e	equivalent stress for elastic buckling
F_y	specified minimum yield stress, yield strength
f	calculated stress in compressive element using factored loads and effective section properties; scaling factor used for plotting a deformed member
G	shear modulus of steel
H	constant in solutions of elastic buckling of columns
I_c	moment of inertia of a column section
I_g	moment of inertia of a girder
$I_{\omega x}$	warping product of inertia about x axis
$I_{\omega y}$	warping product of inertia about y axis
I_x	moment of inertia of a section about its strong axis
I_y	moment of inertia of a member about its weak axis
i	index of an end point of an element; index of an element
J	St. Venant torsion constant
j	index of an end point of an element
K	effective length factor
KL	effective length
$[K]$	global structural stiffness matrix
$[K_g]$	global geometric stiffness matrix
$[K_g]_{ref}$	reference global geometric stiffness matrix
$[K_t]$	global tangent-stiffness matrix

k	coefficient for calculation of effective width
$[k_g]$	geometric stiffness matrix of an element
L	length of the member
L_c	unsupported length of a column
L_g	unsupported length of a girder
l_m	length of the mean perimeter
M	bending moment in a member under specified load
M_f	bending moment in a member under factored load
M_{f1}	smaller factored end moment of a beam-column
M_{f2}	larger factored end moment of a beam-column
M_{max}	ultimate in-plane moment strength
M_p	plastic moment ($= ZF_y$)
M_r	factored moment resistance
M'_r	factored moment resistance of slen
M_u	critical elastic moment of a laterally unbraced beam
M_y	yield moment ($= SF_y$)
M_o	larger end moment of a beam-column
m	relative moment ($m = M/M_y$)
N	number of spans
n	number of segments; number of elements
P	elastic buckling load
$\{\mathbf{R}\}$	vector of nodal loads
$\{\mathbf{R}\}_{ref}$	reference load vector

r	radius of gyration about a centroidal principal axes
$\bar{r}_o$	cross-sectional coefficient
S	elastic section modulus of a steel section; shear center
S_e	the governing compressive section modulus based on effective widths of section elements
T	specified torque
t	element thickness; plate thickness
U_1	factor to account for moment gradient and for second-order effects of axial force acting on the deformed member
U_2	amplification factor to account for second-order effects of gravity loads acting on the laterally displaced storey
u	displacement in x -direction
v	displacement in y -direction
W	width ratio (w/t)
W_{lim}	limiting width ratio for fully effective compressive elements
w	element width; displacement in z -direction
w_{yl}	y -axis-to-left section width
w_{yr}	y -axis-to-right section width
x	subscript relating to strong axis of a member
x_o	x -coordinate of shear center with respect to the centroid
y	subscript relating to weak axis of a member
y_o	y -coordinate of shear center with respect to the centroid
Z	plastic section modulus of a steel section

β	cross-sectional constant
$\{\epsilon\}$	strains
δ	eigenvalue measured from a shifted origin
Θ	end rotation; unit angle of twist
κ	ratio of the smaller factored moment to the larger factored moment at opposite ends of the unbraced length, positive for double curvature and negative for single curvature
λ	nondimensional slenderness ratio in column formula
λ_{cr}	eigenvalue
λ_e	equivalent slenderness ratio in column formula
μ	shifted origin of the eigenvalue axis
ν	Poisson's ratio
ρ	perpendicular distance between a point in the cross-section and the centroid
ρ_0	perpendicular distance between a point in the cross-section and the shear center
σ	stress
σ_{cr}	critical stress
σ_y	yield stress
σ_u	ultimate stress
τ	shear stress
Φ	curvature

Φ_y	yield curvature ($\Phi_y = M_y/(EI)$)
ϕ	resistance factor ($\phi = 0.9$); relative curvature ($\phi = \Phi/\Phi_y$)
ω	double sectorial area with respect to the centroid
ω_n	normalized unit warping
ω_0	double sectorial area with respect to the shear center
ω_1	coefficient used to determine equivalent uniform bending effect in beam-columns
ω_2	coefficient to account for increased moment resistance of a laterally unsupported beam segment when subject to a moment gradient

Chapter 1

Introduction

1.1 Background

The rapid development within recent years of computer power with graphics, coupled with its availability and affordability has enabled the development of large, comprehensive and versatile industry-oriented software packages. Structural engineering, known for extensive use of software and programming, cannot lag behind this progress. In order to utilize fully the new potential, one of the initiatives in this area resulted in the commencement of an integrated system for analysis, design, fabrication and erection of steel structures. The proposed system intends to integrate programs for the layout, determination of loads, analysis, design and other modules, based on a common, automatically updated database, to serve the needs of the structural designer, architect, detailer, fabricator and erector to produce, in the final step, the required structural drawings. An important characteristic of this system is an interactive graphical user-interface, which is considered by most users to be indispensable in of any state-of-the-art structural package. This interface imposes new challenges on the software developer. The comprehensiveness, generality and magnitude of this

system, coupled with the graphics-oriented approach, calls for the development of independent modules, that will be linked eventually through a common shell and database. Some of the modules, for geometric modelling; determination of loads; analysis and design of steel beams subjected to bending, bending and torsion; analysis and design of continuous plate girders and various connections have already been or are currently being developed.

One very specific but frequently used element in steel structures is the beam-column. No system would be complete without the capability of analysis and design of this member for doubly symmetric, singly symmetric and asymmetric sections.

1.2 Objective

The objective of this thesis is to develop an interactive design and analysis program for steel beam-columns. The program BeCol must operate as a stand-alone application and be constructed to allow for future integration into a larger system for the analysis and design of steel buildings. Independent modules were developed for analysis and design of columns and beam-columns in accordance with CSA standard CAN/CSA-S16.1-M89, Limit States Design of Steel Structures (CSA 1989); finite element analysis of beam-column members not covered in the present Canadian standard, both coupled with an advanced interactive graphical user-interface tools for determining section properties and tables for built-up members; and for calculating in-plane strength interaction diagrams using Newmark's integration technique. The system does not replace the designer, but rather is a tool for design, education and research.

1.3 Approach

1.3.1 Education

The educational portion of BeCol illustrates some key concepts underlying the beam-column design rules of CAN/CSA-S16.1-M89, 94. The utilization of the equivalent moment factor ω_1 , relating the effects of a non-uniform moment field to a uniform field, is demonstrated on in-plane strength interaction curves for I-shapes bending about either principal axis, with or without predefined residual stresses, and for HSS and rectangular sections. Another important concept in beam-column design is the behaviour of a concentrically axially loaded column in compression with varying restraint and boundary conditions. The program plots both elastic solutions and inelastic curves for all relevant failure modes for any doubly symmetric, singly symmetric or asymmetric shape that can be specified from the section and built-up section tables.

1.3.2 Design

The application BeCol utilizes directly design criteria for doubly symmetric, prismatic beam-columns, as specified in both CAN/CSA-S16.1-M89 (CSA 1989a) and CAN/CSA-S136-M89 (CSA 1989b) for class 4 sections. Additional algorithms have been added to ensure the proper application of the rules for beam-columns for uniaxial bending about either principal axis and for biaxial bending with any combination of bracing. These include: interactive lateral torsional buckling (Schmitke and Kennedy, 1985); three-moment equation to determine the correct bending moment diagram for a beam-column bending in the plane with bracing; other methods for determining ω_1 for complex moment fields (Kirby and Nethercot, 1978); methods for defining

the effective length factor. A design check, design and redesign are implemented for prismatic members loaded by uniaxial or biaxial bending.

1.3.3 Research

There are no design rules in the standard to determine the resistance of beam-columns with sections that are singly symmetric or asymmetric. To enable an analysis of such members and a determination of their resistance, a finite element program (Albert, 1993) based on an 18 degree of freedom shell triangular element with an in-plane twisting degree of freedom at each node, has been incorporated into BeCol. Both elastic bifurcation and inelastic incremental analysis that include initial imperfections, specified residual stresses and material properties are available.

1.3.4 Utilities

Any feasible cross-section can be selected or defined by BeCol. All relevant geometric properties are calculated and listed in a table format. CISC tables of shapes (those produced by Canadian mills and listed in the Handbook of Steel Construction, CISC, 1992), selected shapes and user-defined built-up shapes, together with user-specified price list are maintained in a database accessible by all parts of BeCol.

1.3.5 Interface

The graphical user-interface of BeCol offers interactive operation through menus, selectable tables, icons and dialogue boxes. Results are represented by diagrams, tables, member geometry, finite element mesh or formatted reports. While the program informs the user about the ranges of practical values, an extensive check of errors in

Figure 1.1: Front page of BeCol

entered input values is performed and reported.

1.4 Scope

The application BeCol presented designs a prismatic, doubly symmetric, beam-column under static loads for any number of load or member combinations in accordance with CAN/CSA-S16.1-M89. The elastic solutions and column curves specified in S16.1-M89, 94 can be generated for concentrically loaded columns with sections that are doubly symmetric, singly symmetric and asymmetric. The in-plane strength interaction curves of members with selected shapes can be calculated and illustrated. BeCol provides an algorithm for defining various structural steel cross-sections, and can calculate all relevant geometric properties. BeCol maintains section tables and

prices of selected shapes. A finite element analysis was incorporated for an analysis of beam-columns with complicated loading, boundary conditions and/or for section shapes that are not doubly symmetric. Just as with any other design problem, the software design is open-ended. It is expected that the full advantage of BeCol will accrue after integration of the program with other modules.

Chapter 2

BeCol as an Educational Tool

2.1 In-plane Strength Interaction Curves

2.1.1 Theoretical Background

A comprehensive list of references related to the in-plane strength of beam-columns is compiled in Guide to Stability Design Criteria for Metal Structures (Galambos 1968). The in-plane strength of a beam-column is the resistance of the member to failure by in-plane instability due to the action of the applied axial force and moments with the latter moved due to $P-\delta$ effects i.e., the second-order moments due to the axial force acting on the member deformed between its ends. The limiting strength can be reached when the member is sufficiently braced in the lateral direction, or when the member is bent about its weak axis. As this resistance can be determined only by full-scale testing, or by detailed numerical analysis, design standards make use of simplified and approximate interaction curves, generally obtained by fitting data to available test results. The numerical methods, however, can evaluate the accuracy of the design interaction curves and determine the response of a member to given loading conditions.

Ketter (1961), following an approach originally proposed by von Karman, presented a number of "exact solutions" of ultimate carrying resistance for comparison with approximated interaction curve expressions. It was shown that a uniform moment diagram has the most severe effect on the in-plane strength of a member. The comparisons were made with expressions for calculation of an equivalent moment suggested by Massonet (1959), Horne (1956) and Winter (1960). Another expression, suggested by Austin (1961), is used in CAN/CSA-S16.1-M89. The method used by Ketter, further refined and described for example by Galambos (1968) and Chen and Atsuta (1988), was generally found to be a suitable analytical solution for construction of the ultimate in-plane strength interaction curves and is described below.

In its basic form, this method provides a solution for prismatic, doubly symmetric, homogenous beam-columns, bent in one plane by a pair of end moments M_1 and M_2 . The ratio, $\kappa = M_1/M_2$, of the smaller end moment to the larger end moment is negative if the member is bent in single curvature. The starting point of the analysis is the utilization of the relationship between the combined action of bending moment and axial force on a cross-section and the resulting curvature. The moment-thrust-curvature (M-C- Φ) relationship depends on the yield stress, stress-strain curve, residual stress and shape of the cross-section. Chen and Atsuta (1988) describe in detail a general approach for the derivation of the M-C- Φ curves for beam-column segments of various materials and give parameter functions for the approximate non-dimensionalized m-c- ϕ expressions of the following steel shapes:

- I-shape, strong axis bending, no residual stress, curve 1
- I-shape, strong axis bending, with residual stress, curve 2
- I-shape, weak axis bending, no residual stress, curve 3
- I-shape, weak axis bending, with residual stress, curve 4
- Square tubular section, no residual stress, curve 5
- Solid rectangular section, curve 6

These expressions were obtained using the assumption of elastic-perfectly plastic stress-strain relationship. The expressions for each $m-c-\phi$ relationship are represented by a set of equations, related to the fact that the section can be in one of three regimes: fully elastic regime, yielded in tension or compression or both. For a given cross-section the terms in the equations can be non-dimensionalized by dividing by M_y and C_y appropriately. This leads to negligible loss of accuracy (Chen and Atsuta 1988). The residual stress pattern used for curves 2 and 4 (I-shapes for strong and weak axis bending) is the commonly used simplified pattern for hot-rolled shapes with a compressive stress of $0.3\sigma_y$ at the tips of the flanges.

Having established the $m-c-\phi$ relationship, the maximum strength of a beam-column is determined for a given axial force in the terms of the ultimate magnitude of the larger of the end moments. This means that the full given axial force is applied and held constant, while keeping the ratio of the end moments constant, the bending moments are increased to their maximum values. A particular loading history is therefore prescribed. Evenly spaced points are specified along the length of the

member. Assuming an initial deflection of the member, the total moment at each of these points is calculated (bending moment due to the applied moments and due to $P-\delta$ effects) and the corresponding curvatures are found from the $m-c-\phi$ relationships. Using Newmark's numerical integration technique (1943), the corresponding slopes and deflections can be calculated. This process is repeated until the calculated deflections converge with the updated deflection shape. A solution for this moment field has then been found. The moment values are increased, a new deflected shape assumed and the process repeated. If the deflected shape does not converge, then the bending moments have exceeded the maximum value, which is given by the previous step.

After determining the ultimate moment for a given member, one point of the in-plane strength interaction curve can be plotted. A family of interaction curves then can be constructed by repeating the procedure for values from a range of two selected parameters, such as member length, end moment ratio, C/C_e or the axial force level C/C_v .

2.1.2 Implementation

This portion of the program was implemented primarily for an educational purpose, hence each operation, $M-C-\Phi$ relationships, ultimate moment determination for a given axial force, Newmark's integration and the determination of interaction curves for different K values of slendernesses can be viewed.

Figure 2.1: M-C- Φ relationships for W-shapes with residual stresses

2.1.2.1 M-C- Φ Relationships

The six sets of m - c - ϕ expressions, given by Chen and Atsuta (1988), were chosen as most common examples of M-C- Φ relationships. Initial plots of these expressions revealed insufficient smoothness and it was found necessary to include a correction factor d , provided by Chen and Han (1985). For a selected section type from the icon menu shown in Fig. 2.1, the m - c - ϕ curves are plotted automatically for C/C_y ratios ranging from 0.0 to 0.9 in 0.1 steps. Any value of m or ϕ can be verified and plotted for a pair of remaining parameters.

EDIT PROGRAM	HELP	SECTION TABLES AND DEFINITION	COLUMN ANALYSIS	FRAME ELEMENT ANALYSIS	SELECTIVE PRINT	ENDPRINT	QUIT	
SYSTEM INFO	PLSS				Program By: Anthoniszuk			
Date: Aug 6 1984 Time: 10:45		BEAM-COLUMN S&L CHECK	BEAM-COLUMN S&L DESIGN	GRADE COLUMN	BUILT-UP COLUMN	MULTIPLE BOARD	ADD TEXT	CONTINUE

INPUT FOR MAXIMUM MOMENT Mmax

W8X40
☒
S
M

Section	Available Weights						
W8X40	400	401	391	380	310	287	
W8X35	350	350	310	300	177	150	
W8X30	300	300	110	100	97	87	
W8X25	87		87	81	80	70	
W8X20	82						
W8X16	140	130	130	100	80	71	
W8X10	87	80	81	40	30		

Grades

Properties, Dimensions

A [mm ²]	607
I [mm ⁴]	100
A [inches ²]	0.90
I [inches ⁴]	6.00
Sx [mm ³]	1.00E+09
Zx [mm ³]	1.00E+09
A [mm ²]	607
Iy [mm ⁴]	1.00E+07

Member Length And Load Definition

SLENDerness (L/r): 0 30 ☒ 40 60 80 100 120 140 OTHER LENGTH

C/cy: 0.1 0.2 0.3 ☒ 0.4 0.5 0.6 0.7 0.8 0.9 OTHER

m (M/M1): -1.0 ☒ -0.5 0.0 0.5 1.0 OTHER

M1 = Mmax
0.40 Cy

M2 = 0.50 Mmax
1134 kN

L = 7800 mm U = 40

Solution Parameters

NUMBER OF SEGMENTS: 5 10 ☒ 20 30

Max PRECISION: 0.1 0.01 ☒ 0.001 0.0001

CYCLING LIMIT: 1.0 0.1 ☒ 0.05 0.01

Figure 2.2: Definition for calculation of M_{max}

2.1.2.2 Ultimate Moment Determination

Based on the predefined $m-c-\phi$ curves, the computation of the maximum in-plane moment of a beam-column with simple supports is performed. The member, specified by its cross-section, length, yield stress and plane of bending, is loaded by a specified constant axial force. Refer to Fig. 2.2. The end moments for a specified κ value are applied to the member with decreasing increments. The resulting deflection is tested for convergence. The least number of trials is ensured by selection of increments with an adapted technique of a binary search, which also allows for a required accuracy of the ultimate moment to be achieved. The convergence test compares the absolute

Figure 2.3: Calculated ultimate moment M_{T_u} .

average difference between two successive corrected deflections of each node with a maximum allowable value. After extensive testing of these analytical criteria, suitable values were included in the menu window "Solution parameters", shown in Fig. 2.2.

The first output screen shown in Fig. 2.3, summarizes the member's characteristics and load definition and plots the ascending part of $M-\Theta$ curve up to the ultimate moment M_{max} , where Θ is the rotation of the left end of the member. The selection and order of both successful and unsuccessful increments is presented in a table on the same screen. The elastic deflection, used as the initial estimate, and the inelastic deflection corresponding to M_{max} are plotted in a suitable scale. The initial guess

of the deflection is calculated at each node by the expression for elastic deflection, given by Galambos (1968). The next output screen, illustrated in Figure 2.4, traces Newmark's integration and is represented in tabular form. Newmark's integration is performed as described in Allen and Bulson (1980).

2.1.2.3 Interaction Curves

The computation of the ultimate moment provides one point of an interaction curve. By repeating of this procedure interaction curves can be generated, for example, to examine the influence of the slenderness of a member for a constant ratio of the end moments. The input screen for this operation is shown in Figure 2.5. For this case, the resulting family of curves for varying slenderness is compared with the corresponding family of curves for the most severe end moment ratio κ equal to -1.0, which is plotted simultaneously and shown in Figure 2.6. If the influence of the value κ on the member resistance is of interest, then a family of interaction curves is plotted for a given slenderness or length of the member as shown in Figure 2.7. The equivalent moment factor ω_1 is calculated, and plotted against κ for different C/C_y values on the same screen. Conceptually, ω_1 defines an equivalent moment field of reduced but uniform intensity $\omega_1 M_o$, having the same effect on the ultimate in-plane member strength as the non-uniform moment field, and has been used for this purpose by various design standards. Based on work of Austin (1961), S16.1 specifies this ratio for members not subjected to transverse loads between supports in Clause 13.8.4(a) as,

$$\omega_1 = 0.6 - 0.4\kappa \leq 0.4 \quad (2.1)$$

RECORD OF THE LAST SUCCESSFUL ITERATION; MAX M-RATIO = 0.721												GO BACK	
OPERATION	0	2	4	6	7	8	9	11	13	15	FACTOR		
1 Elastic deflection [mm]	0.0E+00	13.67	22.08	27.21	27.86	27.47	26.11	20.80	11.74	1.0E-08			
2 Moment due to Mo = 315.2kNm	315.2	264.2	273.2	252.2	241.7	231.2	220.7	199.6	178.8	157.6			
3 Force * deflection [kN]	0.0E+00	15.50	25.72	30.85	31.58	31.16	29.61	23.36	13.32	1.0E-08			
4 Total moment 2 * 3 [kNm]	315.2	308.7	298.6	283.0	273.3	262.3	250.3	223.0	191.9	157.6			
5 Residual M/Ry 4 /My	0.721	0.708	0.684	0.647	0.625	0.600	0.572	0.510	0.439	0.361			
6 Relative curvature	-1.337	-1.191	-1.010	-0.853	-0.778	-0.707	-0.639	-0.516	-0.439	-0.361			
Nemmark's integration													
7 Fictitious point loads	330.0	595.9	505.9	427.2	390.2	354.2	319.9	259.2	219.7	20.61	PHY		
8 Fictitious shear forces	3246.6	2012.4	956.1	63.98	-326.2	-480.3	-1000.3	-1547.5	-2008.5	PHY			
9 New deflections [mm]	0.0E+00	19.24	30.65	35.41	35.82	34.55	32.31	24.76	13.64	0.0E+00			
End of the first cycle.													
New deflections obtained by iteration													
Cycle 2 Difference 2.3681	0.0E+00	21.36	33.92	38.87	38.95	37.64	35.07	28.73	14.82	0.0E+00			
Cycle 6 Difference 0.3600	0.0E+00	24.03	37.66	43.03	42.99	41.26	38.26	28.96	15.96	0.0E+00			
Cycle 10 Difference 0.1244	0.0E+00	24.75	39.04	44.10	43.90	42.17	39.10	29.52	16.24	0.0E+00			
Cycle 14 Difference 0.0593	0.0E+00	25.08	39.30	44.36	44.33	42.56	39.44	29.75	16.36	0.0E+00			
Cycle 18 Difference 0.0324	0.0E+00	25.22	39.74	44.80	44.55	42.77	39.62	29.87	16.42	0.0E+00			
Cycle 22 Difference 0.0191	0.0E+00	25.31	39.88	44.94	44.68	42.88	39.72	29.94	16.46	0.0E+00			
Cycle 27 Difference 0.0104	0.0E+00	25.38	39.96	45.04	44.77	42.96	39.78	29.99	16.48	0.0E+00			
Cycle 28 Difference 0.0093	0.0E+00	25.38	39.99	45.05	44.79	42.98	39.80	30.00	16.48	0.0E+00			
End of iteration, 0.0093 < 0.01													

Figure 2.4: Record of Nemmark's integration

INPUT FOR INTERACTION CURVES

Section Selection

Section: Available Weights:

Section	40	40	40	40	40	40	40
W40X74	40	40	40	40	40	40	40
W40X60	30	30	30	30	30	30	30
W40X50	20	20	20	20	20	20	20
W40X40	10	10	10	10	10	10	10
W40X30	5	5	5	5	5	5	5
W40X20	2	2	2	2	2	2	2
W40X10	1	1	1	1	1	1	1
W40X5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Grades

Properties, Dimensions

Area (in²)	40.7
Depth (in)	40.0
Flange width (in)	10.0
Web thickness (in)	0.475
Flange thickness (in)	0.625
Radius (in)	1.00
Weight (lb/ft)	127.70
Section modulus (in³)	1.00

Plot interaction curves for:

1. a constant ω (M_2/M_1)

2. constant $\omega = -1$ (default)

1. a constant slenderness

2. plot $C/C_y - \omega_1$ curves

ω (M_2/M_1):

W40X74

Solution Parameters

NUMBER OF SEGMENTS:

MINOR PRECISION:

CYCLING LIMIT:

Figure 2.5: Definition for calculation of interaction curves

Figure 2.6: In-plane strength interaction curves for varying slendernesses ($\kappa = -0.5$ and $\kappa = -1.0$)

Figure 2.7: In-plane strength interaction curves for varying κ values

or a value determined by numerical analysis. To illustrate the accuracy of this simplified approximation, Equation 2.1 is plotted on the same diagram. Even though the errors may appear significant and in some ranges unconservative, it was shown, by Kennedy et al. (1990), that they translate into acceptable errors when used in an interaction equation, and in the most unfavourable and unlikely cases, never exceed 5.5% on the conservative side.

2.2 Column Curves

2.2.1 General

A column is defined as a member subjected to a concentric axial compression force. Although true columns seldom occur in real structures, an understanding of their strength is essential to the development of design rules for general compression members. The column strength has been studied extensively for a relatively long time in structural mechanics. The Guide to Stability Design Criteria for Metal Structures (Galambos 1988) provides a list of about one hundred references. The brief discussion which follows is confined to those aspects that are directly reflected in CAN/CSA-S16.1-M89 (CSA 1989a) for prismatic steel columns.

The design rules acknowledge that a column can fail by the instability of the entire member, by local plate element instability or by combination of both. The average stress level, at which the failure is initiated, is crucial for further classification of the failure mode. A short, stocky column will fail by excessive straining with stresses throughout the member equal to yield stress of the material under yield load C_y , where, $C_y = \sigma_y A$. A long, very slender column, will fail by overall buckling, with stresses below the proportional limit. The magnitude of the corresponding elastic buckling load, a stability problem, can be determined by methods of elastic analysis. For intermediate length columns, the failure is initiated when some fibres of a critical section reach stresses above the proportional limit, and are considered to fail by inelastic buckling. The initial out-of-straightness, magnitude and pattern of residual stresses are major factors effecting the inelastic buckling load and assumptions

regarding these factors must be made for an analytical determination of this load.

A numerical analysis to determine the inelastic buckling load for intermediate length columns, the most common in steel buildings, would be too complicated and time consuming for design, hence design standards and specifications generally provide simplified column formulae for doubly symmetric shapes only. These formulae are generally empirical (curve fits to statistically based set of numerical analysis) and differ from standard to specification to code not only in their form, but more importantly, by the underlying philosophy by which they were obtained.

2.2.2 Elastic Column

Elastic buckling solutions are derived in detail for prismatic and initially straight member of a thin-walled and open cross-section, pinned at the ends, with compressive axial forces applied at its ends, Galambos (1968). The material is elastic and homogenous. Assuming small deformations and that the axial compressive forces retain their original position at the centroid, three second-order differential equations are formulated on the deformed member. The three equations given in Galambos (1968, Eq. 4.1, 4.2 and 4.3), describe the equilibrium of any cross-section in the member and in general are mutually dependent. Determination of eigenvalues of these differential equations leads to solutions for critical or buckling loads, under which the equilibrium of the column bifurcates. The buckling loads are:

- For doubly symmetric cross-sections:

$$P_{x,cr} = \frac{\pi^2 EI_x}{L^2} \quad (2.2)$$

$$P_{y,cr} = \frac{\pi^2 EI_y}{L^2} \quad (2.3)$$

$$P_{z,cr} = \frac{I_x + I_y}{A} \left[\frac{\pi^2 EC_w}{L^2} + GJ \right] \quad (2.4)$$

- For singly-symmetric sections about the y -axis:

$$P_{x,cr} = \frac{\pi^2 EI_x}{L^2} \quad (2.5)$$

$$P_{yz,cr} = \frac{P_y + P_z}{2H} \left[1 - \sqrt{1 - \frac{P_y P_z H}{(P_y + P_z)^2}} \right] \quad (2.6)$$

where,

$$H = 1 - \sqrt{\frac{y_o^2}{\bar{r}_o^2}} \quad (2.7)$$

and,

$$\bar{r}_o^2 = y_o^2 + r_x^2 + r_y^2 \quad (2.8)$$

- For asymmetric cross-sections:

$$(P - P_x)(P - P_y)(P - P_z) - P^2(P - P_x)\frac{y_o^2}{\bar{r}_o^2} - P^2(P - P_y)\frac{x_o^2}{\bar{r}_o^2} = 0 \quad (2.9)$$

where,

$$\bar{r}_o^2 = x_o^2 + y_o^2 + r_x^2 + r_y^2 \quad (2.10)$$

Note, the equation for flexural buckling, Eq. 2.2, was first derived by Euler (1744) and is referred to as "Euler's load".

A similar procedure can be used for columns with boundary conditions other than pinned-pinned, but is usually simplified by making use of the effective length concept. By this concept, used also for inelastic buckling, any column is converted to an equivalent pinned-ended member of lengths $K_x L$, $K_y L$ and $K_z L$, where $K_{x,y,z}$ are the effective length factors for buckling about the x and y axis, and twist buckling, respectively.

2.2.3 Inelastic Column - CAN/CSA-S16.1-M89

The column design rules given in CAN/CSA-S16.1-M89, Limit States Design of Steel Structures, are based on a numerical fit of curves obtained from maximum strength analysis of representative geometrically imperfect columns with residual stresses, provided by Bjorhovde (1972). In this study, 112 column curves were generated for members of measured residual stresses, assuming initial sinusoidal out-of-straightness with maximum amplitude of $0.001L$ and zero end restraints. After a statistical analysis, average column curves were obtained for three types of columns resulting in curves known as the SSRC Column Strength Curves 1, 2 and 3 which specify the buckling stress of a column. Setting this stress σ_u to the elastic buckling stress to give the resistance in terms of a slenderness parameter, λ , and incorporating resistance factor ϕ , the expressions for axial compression given in Clause 13.3 are given as:

$$\lambda = \frac{KL}{r} \sqrt{\frac{F_y}{\pi^2 E}} \quad (2.11)$$

where,

λ - slenderness parameter

Clause 13.3.1 - based on SSRC Curve 2: Applicable for W shapes and for HSS manufactured according to CSA Standard G40.20, Class C (cold-formed non-stress relieved) and conforming to the requirements for Class 1, 2 and 3. These expressions are applicable to the W shapes normally rolled in Canada and can be assumed to be valid for other doubly symmetric Class 1, 2, or 3 sections, except for solid round non-stress-relieved cold-straightened bars greater than 50 mm diameter. Welded H shapes should have flange edges flame-cut. Singly

symmetric, asymmetric, or cruciform sections should be checked as to whether torsional-flexural buckling is critical.

- (a) $0 \leq \lambda \leq 0.15$ $C_r = \phi A F_y$
- (b) $0.15 \leq \lambda \leq 1.0$ $C_r = \phi A F_y (1.035 - 0.202\lambda - 0.222\lambda^2)$
- (c) $1.0 \leq \lambda \leq 2.0$ $C_r = \phi A F_y (-0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2})$
- (d) $2.0 \leq \lambda \leq 3.6$ $C_r = \phi A F_y (0.009 + 0.877\lambda^{-2})$
- (e) $3.6 \leq \lambda$ $C_r = \phi A F_y \lambda^{-2}$

Clause 13.3.2 - based on SSRC Curve 1: Applicable to HSS manufactured according to CSA Standard G40.20, Class H (hot-formed or cold-formed stress relieved) and conforming with requirements for Class 1, 2 or 3.

- (a) $0 \leq \lambda \leq 0.15$ $C_r = \phi A F_y$
- (b) $0.15 \leq \lambda \leq 1.2$ $C_r = \phi A F_y (0.990 + 1.122\lambda - 0.367\lambda^2)$
- (c) $1.2 \leq \lambda \leq 1.8$ $C_r = \phi A F_y (0.051 + 0.801\lambda^{-2})$
- (d) $1.8 \leq \lambda \leq 2.8$ $C_r = \phi A F_y (0.008 + 0.942\lambda^{-2})$
- (e) $2.8 \leq \lambda$ $C_r = \phi A F_y \lambda^{-2}$

SSRC Curve 3: It is further recommended in CISC Commentary on CAN/CSA-S16.1-M89 (CSA 1989a), that for heavy sections and sections from universal mill plate, SSRC Curve 3 be used.

- (a) $0 \leq \lambda \leq 0.15$ $C_r = \phi A F_y$
- (b) $0.15 \leq \lambda \leq 0.8$ $C_r = \phi A F_y (1.093 - 0.622\lambda)$
- (c) $0.80 \leq \lambda \leq 2.2$ $C_r = \phi A F_y (-0.128 + 0.707\lambda^{-1} - 0.102\lambda^{-2})$
- (d) $2.2 \leq \lambda \leq 5.0$ $C_r = \phi A F_y (0.008 + 797\lambda^{-2})$
- (e) $50 \leq \lambda$ $C_r = \phi A F_y \lambda^{-2}$

2.2.4 Local Buckling - CAN/CSA-S136-M89

If a cross-section of a column is composed of relatively thin plate elements, a plate may buckle locally before the member buckles as a whole or before the yield stress of the material is reached. Although elastic solutions of the plate critical load are known, the ultimate strength can be considerably higher due to substantial reserve capacity beyond σ_{cr} (post-buckling strength), as a non-uniform redistribution of axial compressive stresses occurs after buckling. Another favourable factor for plates supported along two edges is a development of transverse tensile membrane stresses, which provide post-buckling support. An extensive list of about 250 related references can be found in Galambos (1988).

Clause 11 and Table 1 of CAN/CSA-S16.1-M89 specify limits of width-to-thickness ratios of plate elements, above which an element is assumed to have local buckling as the limit state of structural capacity. Cross-sections containing such elements are classified as Class 4 and their compression resistance is calculated as specified in Clause 13.3.3. To evaluate this clause, it is necessary to determine an effective area in accordance with CAN/CSA-S136-M89. The determination of the effective area is based on a semiempirical effective width concept. It is assumed that the stresses in

the plate are redistributed to the supported edges and only the portions connecting to another plate element remain effective and capable of reaching the yield stress. The maximum compressive resistance of such plate of width w is replaced by an equivalent plate of width b , over which the stress is uniform and equal to the actual stress at the edge of the plate. To determine the effective width b , the actual stress must be computed using the effective section properties. An iterative procedure is required. The effective area is then used instead of the gross area, and the compression resistance of the column is calculated according to Clause 13.3.1 or 13.3.2, as applicable.

The effective design width of elements in compression is given by Clause 5.6.2, CAN/CSA-S136-M89, Cold Formed Steel Structural Members. The effective width ratio B of an element under uniform compressive stress is determined as follows, when $W \leq W_{lim}$:

$$B = W \quad (2.12)$$

when $W > W_{lim}$,

$$B = 0.95 \sqrt{\frac{kE}{f}} \left[1 - \frac{0.208}{W} \sqrt{\frac{kE}{f}} \right] \quad (2.13)$$

where,

$$W_{lim} = 0.644 \sqrt{\frac{kE}{f}} \quad (2.14)$$

f - calculated stress in compressive element ($\leq F_y$) using factored loads and effective section properties

W = w/t

w = element width

k = 4, for elements stiffened on each edge by a web or flange

$k = 0.43$, for elements supported only along one side

2.2.5 Inelastic Column - CAN/CSA-S16.1-94

The final version of CAN/CSA-S16.1-94 was not available at the time of writing of this thesis, hence the design rules of this standard were implemented as they were presented in draft form and include the following modifications,

SSRC Curve 2: Remains applicable for W shapes and for cold formed HSS shapes conforming to the requirements for Class 1, 2 and 3. The curve is also considered applicable to Class 1, 2 and 3 doubly symmetric shapes failing by flexural buckling.

SSRC Curve 1: Assumed applicable for welded H shapes of Classes 1, 2 and 3 with flame-cut edges, based on Chernenko and Kennedy (1991). Remains applicable for HSS manufactured according to CSA Standard G40.20, Class H (hot-formed or cold-formed stress relieved) and conforming with requirements for Class 1, 2 or 3.

The torsional and torsional-flexural buckling modes are directly evaluated by the new standard. The torsional or flexural-torsional buckling resistance of asymmetric, singly-symmetric and cruciform sections are computed by replacing λ in the equations for factored compression C_r (Clause 13.3 of the CAN/CSA-S16.1-M89) by λ_e given by,

$$\lambda_e = \sqrt{\frac{F_y}{F_e}} \quad (2.15)$$

where the critical torsional elastic buckling stress F_e is defined as follows:

- For cruciform sections:

$$F_e = \left[\frac{\pi^2 EC_w}{(K_z L)^2} + GJ \right] \frac{1}{I_x + I_y} \quad (2.16)$$

- For sections singly-symmetric about the y -axis:

$$F_e = \frac{F_{ey} + F_{ez}}{2H} \left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{F_{ey} + F_{ez}}} \right] \quad (2.17)$$

- For asymmetric sections as the lowest root of:

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2(F_e - F_{ey})\frac{x_o^2}{\bar{r}_o^2} - F_e^2(F_e - F_{ex})\frac{y_o^2}{\bar{r}_o^2} = 0 \quad (2.18)$$

where:

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{K_x L}{r_x}\right)^2} \quad (2.19)$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{K_y L}{r_y}\right)^2} \quad (2.20)$$

$$F_{ez} = \frac{\frac{\pi^2 EC_w}{(K_z L)^2} + GJ}{A\bar{r}_o^2} \quad (2.21)$$

where, H , $\bar{r}_o$ are defined by Equations 2.7, 2.8 and 2.10 where applicable.

2.2.6 Implementation

The elastic critical loads, squash load and applicable factored compressive resistance, determined in accordance with CAN/CSA-S16.1 M89 and 94, are calculated for any prismatic steel column. The calculated compressive loads are plotted in a diagram against column slenderness $K_x L/r_x$, which illustrates levels of different failure modes for various lengths of the member. The column can be checked for a given pair of length L and factored compressive load C_f .

The cross-section of the column is defined or selected from any section tables described in Section 5.1.3. The operating screen for columns combines both the input and output. The member definition menu lists all parameters necessary for the calculation of the column resistance and varies depending on the selected section type. Bracing can be specified for weak axis buckling (y) or torsional buckling (z). The length of the column that is associated with the specified bracing, will be taken as the maximum clear distance between bracing or bracing and the supports. The hollow structural sections must be specified as cold formed non-stress relieved or stress relieved. The fabrication process of I-shapes and structural importance are specified only for anticipated changes of the Ontario Highway Bridge Design Code (not described).

The column curves for the squash load, elastic buckling solutions and compressive resistances as determined from the standard are selected from a menu with buttons and notation for all applicable failure modes. The squash load and elastic solutions are calculated with $\phi = 1.0$ (theoretical solution) regardless of the ϕ value entered in the member definition menu. Standard design curves are cut-off at the point of $KL/r_{min} = 200$. The following examples will demonstrate this section of the program for doubly symmetric, singly symmetric and asymmetric columns.

2.2.6.1 Doubly Symmetric Sections

The elastic solutions and column resistance for all applicable failure modes for W200x36, without bracing are plotted in Figure 2.8. Torsional buckling of doubly symmetric sections such as a cruciform (low torsional stiffness, $C_w = 0$) is illustrated in Fig-

Figure 2.8: Column curves of a doubly symmetric section, W200x36

Figure 2.9: Column curves of a doubly symmetric section, Cruciform

ure 2.9 for a simple cruciform built-up of plates, XP300x73 ($d = 300\text{mm}$, $t = 16\text{mm}$). Curve 12 illustrates that torsional buckling will govern the design for columns with slendernesses less than approximately 50 for this case.

2.2.6.2 Singly Symmetric Sections

The flexural-torsional resistance of sections with single symmetry or no symmetry can only be computed with the equations specified in S16.1-94. The following two examples were selected to demonstrate cases in which the flexural-torsional buckling is the governing failure mode for column with practical lengths. The solution for a channel (C310x37), braced against lateral buckling about the weak axis at the third points of its height are shown in Figure 2.10. The elastic solutions, together with capacities calculated by the S16.1-94 are shown for a WWTx250x153 in Figure 2.11. This is a Class 4 section and its capacity is calculated accordingly.

2.2.6.3 Asymmetric Sections

The elastic solution of an asymmetric section is given by a single curve of flexural-torsional buckling (coupled x,y displacements and is shown for L150x100x16 in Figure 2.12. The corresponding inelastic curve calculated by the S16.1-94 is shown with the resistance factor $\phi = 1.0$ as curve 4.

Figure 2.10: Column curves of a singly symmetric section, C310x37

Figure 2.11: Column curves of a singly symmetric section, WWT250x153

Chapter 3

Beam-Column Design

The exact prediction of the ultimate resistance of beam-columns requires iterative numerical techniques based on models that are relatively sophisticated, especially in the inelastic range where most beam-columns fail. One such procedure is described in Section 2.1. The solution becomes complex for biaxially loaded members that may exist as parts of frames. Different cross-sections, end restraints and residual stress patterns and magnitudes have to be considered. A simplified approach to define design rules is generally represented by linear interaction curves. The design rules given in CAN/CSA-S16.1-M89 (CSA 1989a), check the beam-column resistance for three potential failure modes: cross-sectional strength, in-plane bending strength and out-of-plane buckling. In CAN/CSA-S136-M89 (CSA 1989b), the effective cross-sectional properties are determined for Class 4 sections by accounting for local buckling of section elements. Based on CAN/CSA-S16.1-M89 and CAN/CSA-S136-M89, BeCol designs prismatic, doubly symmetric, biaxially loaded beam-columns for selected cross-section types (for example W, HSS any built-up shape), and for any number of load combinations. All the applicable interaction curves are plotted along with a summary report to illustrate the results.

3.1 CAN/CSA-S16.1-M89: Beam-Column Design Rules

The beam-column design rules of CAN/CSA-S16.1-M89 (CSA 1989a) are in the form of interaction curves:

- All classes of sections except Class 1 sections of I-shaped members:

$$\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}} \leq 1.0 \quad (3.1)$$

- Class 1 Sections of I-shaped members:

$$\frac{C_f}{C_r} + \frac{0.85U_{1x}M_{fx}}{M_{rx}} + \frac{0.60U_{1y}M_{fy}}{M_{ry}} \leq 1.0 \quad (3.2)$$

$$\frac{M_{fx}}{M_{rx}} + \frac{M_{fy}}{M_{ry}} \leq 1.0 \quad (3.3)$$

The resistance of the member is evaluated for:

1. Cross-sectional strength, in which case:

- C_r is taken as compressive resistance of the section
- M_r is taken as cross-sectional moment resistance of the section
- U_{1x} and U_{1y} are taken as 1.0

2. Overall member strength, in which case:

- C_r is taken as compressive resistance of the member with the value $K = 1.0$ and is based on the maximum slenderness ratio for biaxial bending. For uniaxial strong axis bending, $C_r = C_{rx}$

- M_r is taken as cross-sectional moment resistance of the section
- U_{1x} and U_{1y} are as defined by Eq. 3.4

3. Lateral torsional buckling, when applicable, in which case:

- C_r is taken as compressive resistance of the member with the value $K = 1.0$ and is based on weak-axis buckling.
- M_{rx} is taken as moment resistance of the member against lateral-torsional buckling
- M_{ry} is taken as cross-sectional moment resistance of the section
- U_{1x} is as defined by Eq. 3.4, but not less than 1.0
- U_{1y} is as defined by Eq. 3.4

Although all terms for the three cases corresponding to distinctive failure modes are qualified, for some loading conditions and lateral support arrangements, the definitions are not explicit and a refined commentary is necessary (Section 3.3).

The value of U_1 , a factor to account for moment gradient and for second-order effects, is defined by (Clause 13.8.3):

$$U_1 = \frac{\omega_1}{1 - \frac{C_f}{C_e}} \quad (3.4)$$

The coefficient used to determine equivalent uniform bending effect is given by (Clause 13.8.4):

- For members not subjected to transverse loads between supports:

$$\omega_1 = 0.6 - 0.4\kappa \geq 0.4 \quad (3.5)$$

- For members subjected to distributed loads or a series of point loads between supports: $\omega_1 = 1.0$
- For members subjected to a concentrated load or moment between supports: $\omega_1 = 0.85$

The computation of the factored compressive resistance is carried out according to Clause 13.3, CAN/CSA-S16.1-M89. The applicable factored moment resistance is evaluated according to Clauses 13.5 and 13.6 of CAN/CSA-S16.1-M89:

1. Cross-sectional moment strength:

- For Class 1 and Class 2 sections:

$$M_r = \phi Z F_y = \phi M_p \quad (3.6)$$

- For Class 3 sections:

$$M_r = \phi S F_y = \phi M_y \quad (3.7)$$

- For Class 4 sections: The cross-sectional modulus is reduced, refer to Section 3.2.

2. Lateral-torsional buckling resistance:

- For doubly symmetric Class 1 and Class 2 sections:

when $M_u > 0.67M_p$

$$M_r = 1.15\phi M_p \left(1 - \frac{0.28M_p}{M_u}\right) \leq M_p \quad (3.8)$$

when $M_u \leq 0.67M_p$

$$M_r = \phi M_u \quad (3.9)$$

- For doubly symmetric Class 3 and Class 4 sections:

when $M_u > 0.67M_y$

$$M_r = 1.15\phi M_y \left(1 - \frac{0.28M_y}{M_u}\right) \quad (3.10)$$

$M_r < \phi M_y$ for Class 3 sections and $M_r < \text{cross-sectional moment strength}$ determined according to Section 3.2 for Class 4 sections;

when $M_u \leq 0.67M_y$

$$M_r = \phi M_u \quad (3.11)$$

The critical elastic moment of the unbraced member:

$$M_u = \frac{\omega_2 \pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_u} \quad (3.12)$$

The coefficient ω_2 accounts for the increased moment resistance of a laterally unsupported segment when subject to a moment gradient and

- when subject to end moments:

$$\omega_2 = 1.75 + 1.05\kappa + 0.3\kappa^2 \leq 2.5 \quad (3.13)$$

- when the bending moment at any point within the unbraced length L is larger than the larger end moment:

$$\omega_2 = 1.0 \quad (3.14)$$

The ratio κ is defined as the ratio of the smaller factored moment to the larger factored moment at opposite ends of the unbraced length, and has a positive value for double curvature and a negative value for single curvature.

3.2 CAN/CSA-S136-M89: Beam-Column Design – Class 4 Sections

The class of a beam-column section in general depends on the relative magnitude of the applied factored axial force and is determined by the rules given in Clause 11 and Table 1 of CAN/CSA-S16.1-M89 (CSA 1989a). Class 4 sections reach their maximum resistance when a plate element buckles locally. To account for this mode of failure, only a portion of section is assumed to remain effective. The effective geometric properties must be calculated prior to the use of the limit states design interaction equations. In either case of column and beam, the effective geometric properties are applied directly to cross-sectional strength whereas stability effects are evaluated for the gross section. The column resistance C_r is based on the effective area A_e determined by the iterative process outlined in Section 2.2.4. The cross-sectional moment resistance M_r of Class 4 sections referred to in Clause 13.5, CAN/CSA-S16.1-M89 is given:

- for I-shapes whose flanges meet the requirements of Class 3 and whose webs exceed the limit for Class 3 as

$$M'_r = M_r [1 - 0.0005 \frac{A_w}{A_f} (\frac{h}{w} - 1900 \sqrt{\frac{M_f}{\phi S}})] \quad (3.15)$$

- and for all other cases, the value of M_r must be determined in accordance with CAN/CSA-S136-M89, Cold Formed Steel Structural Members.

The applicable factored cross-sectional resistance is given by:

$$M_r = \phi S_e F_y \quad (3.16)$$

The effective section modulus S_e is calculated for the stress distribution on the cross-section with effective geometric properties and requires an iterative solution procedure. The effective width of elements under uniform stresses is calculated as for columns (Section 2.2.4). For elements stiffened on each edge by a web or flange, $b/2$ is assumed on each end of the element. For elements supported only at one edge, the effective width b is located on the side adjacent to the supporting element.

For webs and stiffened elements under a stress gradient:

1. for webs (f_1 in compression and f_2 in tension),

$$b_1 = \frac{Bt}{3 + q} \quad (3.17)$$

$$b_2 = \frac{Bt}{1 - q} - b_1 \quad (3.18)$$

- when $0.0 \leq q \leq 1.0$

$$k = 4 + 2(1 + q)^3 + 2(1 + q) \quad (3.19)$$

- when $1.0 < q \leq 3.0$

$$k = 6(1 + q) \quad (3.20)$$

2. for other stiffened elements (f_1 and f_2 in compression),

$$b_1 = \frac{Bt}{3 - q} \quad (3.21)$$

$$b_2 = Bt - b_1 \quad (3.22)$$

$$k = 4 + 2(1 - q)^3 + 2(1 - q) \quad (3.23)$$

where,

- b_1, b_2 - effective widths of the element,
(the effective portions adjacent to the connecting elements)
- B - calculated by Eq. 2.12 and 2.13 with $f = f_1$
- q = $|f_2/f_1|$
- f_1, f_2 - stresses calculated at the edges of the element
- W = w/t
- t - thickness of the element
- w - width of the element

For unstiffened elements under stress gradient:

- b - effective width of the element, taken from the end supported
by another element
- B - calculated by Eq. 2.12, 2.13
- k = 0.43
- f - compressive stress calculated at the edge connected
to another element

3.3 Commentary on Beam-Column Design Rules

The design rules given in Clause 13.8, CAN/CSA-S16.1-M89, Limit States Design of Steel Structures, are straightforward for a design of a beam-column under uniaxial bending about either axis or for biaxial bending with no lateral supports. Additional refinement of the design rules, as they were implemented into BeCol, particularly with respect to overall member strength and lateral torsional buckling strength, will be described for specific loading and lateral support arrangements.

3.3.1 Uniaxial Strong Axis Bending

3.3.1.1 No Bracing

With no moments applied about the minor axis of the member, it is assumed, that the end restraints against rotation remain effective. Restricting the value of K_y to 1.0 for the lateral-torsional buckling strength would be too conservative and the C_{ry} is calculated with $K_y \leq 1.0$ depending on specified end conditions in the $x - x$ plane.

3.3.1.2 Weak Axis Bracing

With no moments applied about the minor axis of a beam-column that has a number of lateral supports, the value of C_{ry} is based on the governing clear distance between supports with an applicable value of K_y . If the member fails by lateral-torsional buckling, it is assumed that the spans will buckle interactively. The ultimate moment M_{rx} of a beam-column with discrete lateral supports against weak-axis buckling is analysed by the *equivalent beam method*, proposed by Schmitke and Kennedy (1985) that specifically recognizes that the member buckles laterally as a unit. Restricting

the resistance of the member to the lowest buckling moment of one of the spans without considering interactive buckling would be too conservative. The member that is subject to nonuniform moments is replaced by a shorter member of the same resistance but subject to a uniform moment by the following steps:

1. For each segment between points of lateral supports, the equivalent moment factor $\omega_{2,i}$ is calculated by Eq. 3.5.

2. The critical moment $M_{u,i}$ is calculated for each segment L_i by:

$$M_{u,i} = \frac{\omega_{2,i}\pi}{L_i} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_i}\right)^2 I_y C_w} \quad (3.24)$$

3. An equivalent length $L_{eq,i}$ of each segment is computed. A member of this length would have the same critical moment when subject to a uniform moment $M_{u,i}$:

$$L_{eq,i} = \frac{\pi \sqrt{EI_y GJ}}{\sqrt{2} M_{u,i}} \sqrt{1 + \sqrt{1 + \frac{4M_{u,i}^2 C_w}{I_y (GJ)^2}}} \quad (3.25)$$

4. The member is replaced by an equivalent member with a uniform moment of length L_{eq} :

$$L_{eq} = \sum_{i=1}^n L_{eq,i} \quad (3.26)$$

5. Assuming the interactive buckling of the spans, the effective length of each of the n segments of the equivalent member is:

$$L' = \frac{L_{eq}}{n} \quad (3.27)$$

6. The M_u of the member is computed by the Eq. 3.12, for the effective segment length L' , using $\omega_2 = 1.0$.

7. The M_u is checked against cross-sectional moment resistance and adjusted according to Eq. 3.8 through 3.11 to obtain M_{rx} .

3.3.1.3 Strong Axis Bracing

If discrete lateral supports are provided in the plane of bending, the member acts in this plane as a continuous beam over a number of supports n . The bending moment diagram (BMD) is determined by *three-moment equations*, writing one equation for each intermediate support, (Gere & Timoshenko, 1984). The n equations are solved simultaneously for the n internal moments over the supports. For a member loaded only with the end moments $M_{fx,1}$, $M_{fx,2}$, the BMD remains linear between the points of supports, having its maximum over the end support with the larger of $M_{fx,1}$, $M_{fx,2}$. The overall member strength of a member braced in the plane of bending is calculated for a governing segment with the length equal to the clear distance between supports and with a value of $K = 1.0$. In the evaluation of the lateral-torsional buckling strength, the C_{ry} is calculated for the full member length, with the value of $K \leq 1.0$, as specified. A good estimate of the critical moment for an arbitrary BMD as suggested by Kirby and Nethercot (1978), is to use an equivalent moment factor ω_2 given by,

$$\omega_2 = \frac{3M_2 + 4M_3 + 3M_4 + 2M_{max}}{12M_{max}} \quad (3.28)$$

where,

M_2, M_3, M_4 - absolute values of the BMD at successive quarter points
of the member

M_{max} - absolute maximum value of the BMD

The value of ω_2 determined by this procedure is used to compute M_{rx} for lateral-torsional buckling strength, instead of the definition in Eq. 3.13.

3.3.2 Uniaxial Weak Axis Bending

If no moments are applied about the major axis, lateral-torsional buckling does not occur. This is similar to box sections which are inheritantly stable against this type of failure.

3.3.2.1 No Bracing

The C_r in the evaluation of overall buckling strength is taken as C_{ry} and is based on $K_y = 1.0$.

3.3.2.2 Weak Axis Bracing

A beam-column loaded with moments only about the minor axis and braced laterally at discrete locations about this plane, may fail by

- (a) reaching its full cross-sectional strength due to axial load and moment,
- (b) overall member strength about the weak axis or by
- (c) buckling about the strong axis due to the axial load.

To evaluate the overall member strength in the plane of bending, the BMD is computed by solving the three-moment equations, as discussed in Section 3.3.1.3. The

values of ω_1 , determined by Eq. 3.5, U_{1y} and C_{ry} , based on $K_y = 1.0$, are evaluated for each segment. Using these values, the overall member strength is given by the segment producing the maximum sum of the interaction equation. In the case of out-of-plane buckling strength about the strong axis it is assumed that the bending moments about the weak axis do not reduce the compression resistance C_{rx} of the member. As a result, the bending moment and compression effects are not combined and the out-of-plane buckling resistance of the member is equal to C_{rx} . The end rotation restraints do not induce bending moments and the value of K_x is therefore not restricted to 1.0.

3.3.2.3 Strong Axis Bracing

Bracing the member against strong axis buckling will not increase the member capacity since cross-sectional strength or overall member strength for buckling about the weak axis will always govern.

3.3.3 Biaxial Bending

3.3.3.1 No Bracing

The evaluation follows directly the rules given in Clause 13.8, CAN/CSA-S16.1-M89.

The values of C_{ex} , C_{ey} , C_{rx} and C_{ry} are based on $K = 1.0$.

3.3.3.2 Weak Axis Bracing

The BMD about the weak axis is determined by solving the three-moment equations and values of U_{1y} are calculated for each segment. The equations of the overall

member strength combine $C_r = C_{rx}$ with a maximum value of U_{1y} determined for the found-governing segment. To evaluate the lateral-torsional buckling strength, M_{rx} is computed by the *equivalent beam method*, Schmitke and Kennedy (1985). The values of $C_r = C_{ry}$ and U_{1y} are based on the governing segment.

3.3.3.3 Strong Axis Bracing

The BMD about the strong axis is calculated by solving the three-moment equations. The values of C_{rx} and U_{1x} are calculated for each segment and the governing segment is determined. The value of ω_2 is determined by Eq. 3.28 (Kirby and Nethercot, 1978).

3.3.4 Value of K

The value of the effective length factor K is restricted to 1.0 in the planes of applied moments. A value smaller than 1.0 may be taken for the out-of-plane buckling strength if adequate end conditions are specified. The factor K in this case can be taken as a theoretical or recommended design value, listed in figure B1, Appendix B of CAN/CSA-S16.1-M89. For continuous frames with a translational displacements of the ends prevented, the value of K can be estimated according to Appendix C, CAN/CSA-S16.1-M89. This method assumes that all columns in the portion of the framework considered reach their individual critical load simultaneously. The effective length factor is calculated from:

$$\frac{G_u G_L}{4} \left(\frac{\pi}{K} \right)^2 + \frac{G_u + G_L}{2} \left(1 - \frac{\frac{\pi}{K}}{\tan \frac{\pi}{K}} \right) + 2 \left[\frac{\tan \frac{\pi}{2K}}{\frac{\pi}{K}} \right] = 1.0 \quad (3.29)$$

$$G = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{L_g}} \quad (3.30)$$

where:

U, L - subscripts refer to the joints at the two ends of the column

c, g - subscripts refer to columns or girders

$G_L = 10$ for column ends supported by but not rigidly connected to a foundation

3.4 Design Strategy

The design of a beam-column is performed in accordance with the limit states design interaction equations, CAN/CSA-S16.1-M89, Limit States Design of Steel Structures and the rules described in Section 3.3. The implementation of the rules determines the member resistance for all possible failure modes. The designer must determine the appropriate design loads including the second-order moments due to the relative translational displacements of the beam-column ends. The second-order moments are related to the behaviour of the structure and must be evaluated outside the beam-column module. Therefore it is assumed that the supplied factored moments already account for the effects due to sway as determined from a second-order analysis. Or alternatively, the translational moments may be obtained from a first-order analysis and amplified by a factor U_2 , Clause 8.6, or determined by the method given in Appendix J, CAN/CSA-S16.1-M89.

The design procedure directly evaluates the equations specified by Clause 13.8 and related clauses in CAN/CSA-S16.1-M89, by applying them successively to all

listed sections of specified types in selected database. If a section meets all the strength criteria, it will be listed in the table of recommended shapes, otherwise the section is eliminated when it first fails to meet the requirements. To minimize the total computational time, the sequence of the rules was determined considering the probable severity and computational time of each rule. The time can be reduced by specifying the minimum and maximum dimensions of the cross-section and by selecting a user-defined database of shapes as opposed to the complete CISC section database. One major advantage of this process of elimination is the relatively easy maintenance of the program. The rules for a design check, design and generation of the interaction curves are not duplicated in this technique and their compact format enables a future updating without revising any other selective routines. The compressive resistance C_r of a section or column segment is determined by the column module and is currently linked with the design rules of CAN/CSA-S16.1-M89.

Shear strength checks and serviceability checks have not been implemented as they do not govern in real structures.

3.5 Beam-Column Design Check and Redesign

To perform a **design check** of a beam-column, the required cross-section is defined or selected from section tables and a job is created by the procedure described later (Section 5.1.3). After selecting **design check** from the pull-down menu that is invoked, the job is retrieved and highlighted in a table listing all stored jobs. When the job is confirmed, the input screen for member and load definition appears, as shown in Fig. 3.1 and all input parameters can be entered. The factored bending moments

<div style="display: flex; justify-content: space-between;"> EXIT PROGRAM HELP </div>		<div style="display: flex; justify-content: space-between;"> INTERACTION CURVES SECTION TABLES AND DEFINITION </div>		<div style="display: flex; justify-content: space-between;"> COLUMN ANALYSIS FINITE ELEMENT ANALYSIS </div>		<div style="display: flex; justify-content: space-between;"> SELECTIVE PHOTO SNAPSHOT QUIT </div>	
<div style="display: flex; justify-content: space-between;"> SYSTEM INFO FILES </div>		<div style="display: flex; justify-content: space-between;"> BEAM-COLUMN DESIGN CRANE COLUMN </div>		<div style="display: flex; justify-content: space-between;"> SUBTAP COLLISIONS MULTIPLE SCREEN </div>		<div style="display: flex; justify-content: space-between;"> ADD TEXT CONTINUE </div>	
<div style="display: flex; justify-content: space-between;"> Date: Aug 10, 1994 Time: 10:00 </div>							

Member Length

6000 mm

Fy

250 MPa

E

200000 MPa

Residual stress

—

Resistance Factor

0.90

Loads Between Supports

None

Fu

450 MPa

Strong Axis View

AXIAL FORCE
MOMENT - TOP
MOMENT - BOTTOM
BRACING
CLEAR BRACING

K_x = 1.00

End Restraint Definition

Weak Axis View

MOMENT - TOP
MOMENT - BOTTOM
BRACING
CLEAR BRACING

K_y = 1.00

W410X66

	Bending	Bracing	K value
x direction	yes	no	Not Required
y direction	yes	yes	Not Required

Figure 3.1: Member and load definition for design check

at the end of the beam-column and the lateral supports about the strong (x) axis are entered in the upper field of the screen, while the lower field is allocated for the configuration about the weak (y) axis. Any number of lateral supports at any height can be specified and will be displayed in the corresponding field. Lateral continuous support can also be specified. The end restraint definition is optional and values of $K \leq 1.0$ are applied only in the plane in which no bending moments are applied (out-of-plane buckling). The table in the lower right-corner displays whether a K -value should be specified or taken as 1.0. The input data is passed to the beam-column design module with a request for design check, and the resulting output is fetched and processed.

All applicable interaction curves with a brief summary report are displayed on the output screen as shown in Figures 3.2 and 3.3. The applicable interaction curves are plotted automatically and marked with the following acronyms:

- CSS - Cross-sectional strength
- OMS - Overall member strength
- LTB - Lateral-torsional buckling strength
- OOFB - Out-of-plane buckling strength

The specified factored design load (i.e. dead, live, wind, earthquake or a specified combination) is plotted on the same diagram. The location of the factored axial load factored moment point clearly indicates whether or not the design is satisfactory. The diagram can be non-dimensionalized or actual load values can be displayed by clicking on the axis notation. In the case of biaxial bending, the diagram can be plotted for either a constant M_{fy} or M_{fx} as shown in Figures 3.2 and 3.3, respectively. The

Figure 3.2: Design check: interaction curves plotted for a constant M_{fy}

Figure 3.3: Design check: interaction curves plotted for a constant M_{fx}

section class ranges are marked on the right-hand border of the diagram as seen in Fig. 3.4-3.5.

The summary report lists all the specified data, the Class of the section for the specified factored design load, yield loads and factored moments calculated at the interior supports. The report is concluded by the evaluation of all interaction equations, expressing the ratio of each term for axial load and moment contribution about both principal axes. Another load combination can be checked by entering the corresponding loads. In general the interaction curves must be recalculated.

K -values other than 1.0 for the case of out-of-plane buckling can be specified in the end restraint definition window. The required axis and the appropriate boundary conditions can be selected from the first three icons; i.e., fixed-fixed, fixed-pinned and pinned-pinned (both theoretical and recommended values are displayed) or from the forth icon which invokes the window shown in Fig. 3.6 for the calculation of K by the procedure described in Appendix C, CAN/CSA-S16.1-M89. All required lengths and moments of inertia must be entered and the K value is returned to the main input window.

If the designed member is found unsatisfactory, the **redesign** button can be selected. The member and load definition are stored and used for default values in the design definition screen. The procedure is identical to that of the design procedure.

The beam-column **design** procedure starts by creating a "design job" or by invoking a previously saved job. The first input screen, Figure 3.7, extracts all doubly symmetric, open and closed section types predefined in the cross-section module where a multiple selection of cross-sectional databases is allowed. A database of sections

Member Length
4800 mm

Fy
300 MPa

Ft
300 MPa

Residual stress
—

Radius of Gyration
0.00

Leads Between Supports
None

Fu
450 MPa

Strong Axis View

AXIAL FORCE

MOMENT - TOP

MOMENT - BOTTOM

BRACING

CLEAR BRACING

$R_x = 1.00$

End Restraint Definition

Weak Axis View

MOMENT - TOP

MOMENT - BOTTOM

BRACING

CLEAR BRACING

$R_y = 1.00$

X

W80X125

	Bending	Bracing	K values
x direction	yes	no	Not Required
y direction	no	no	Required

Figure 3.4: Interaction equations input - Class ranges

Figure 3.5: Interaction equations output - Class ranges

must be specified. Built-up shapes will be automatically searched from the user-defined database. Design loads for number of load cases or a number of members is entered. Currently the design procedure does not differentiate between a load case or a definition of another beam-column member. The design loads must be calculated by the engineer. Alternatively, these could be calculated by another analysis/design module, operating between BeCol and a frame analysis program. The next input screen is similar to the member and load definition for the design check. A difference is the option to limit the maximum and minimum dimensions of the section. Different yield stresses can be specified for I-shapes, HSS and other built-up shapes. The input parameters must be entered for each load case and beam-column storey height in the lift (as seen in Figures 3.8–3.11 inclusive).

BeCol preprocesses all the input data and passes the information to the design module. The design module determines the sections conforming with the requirements of Clause 13.8 and sorts them by the weight regardless of the section type. In the next step, BeCol passes the selected sections with their steel grade to another module to determine the cost of each section according to the mill price database.

The design output screen of BeCol displays a table of the sections sorted according to cost and weight as shown in Fig. 3.12. The entry for each section consists of the section designation, its depth, width, area, cost and the value of the governing interaction equation. As with the “design check” procedure, each of the sections listed can now be checked. The interaction curves can be displayed for each load case with the associated summary report by toggling between the table and the reports.

Figure 3.6: Calculation of K -value

3.5.1 Design Example

The use of the design procedure is illustrated by designing the column of the rigid portion of a two-storey frame described by Kennedy, Picard and Beaulieu (1990). The column lift is composed of two beam-columns with the first storey level 1-2 of height $L = 5000 \text{ mm}$ and the second storey level 2-3 of height $L = 6000 \text{ mm}$. Two load combinations, gravity and gravity + lateral load, have been calculated for the frame.

For a comparison, two of cross-section databases, W and HSS (square and rectangular), were selected as shown in Figure 3.7.

The loads for the four cases (2 beam-columns each with 2 load combinations) as described in the paper were defined in the member and load definition screen. Refer

Section Type Definition For Beam-Column Design

WWF

W

S

M

HP

Recommended Section Tables

Number of Combinations: 4

Selected Section Tables

Figure 3.7: Section type database selection

to Figures 3.8 to 3.11 inclusive. It is assumed, that the bending moments are specified about the strong axis and that bending about the weak axis of the member is zero (uniaxial strong axis bending). The yield stress for W shapes is given $F_y = 300 \text{ MPa}$ and for HSS $F_y = 350 \text{ MPa}$. To eliminate excessively large sections, the maximum depth of the sections was specified as 600 mm.

The design procedure was executed and all recommended sections are listed in the table shown in Fig. 3.12. The rectangular section HS305x203x11 with a mass of 82.4 kg/m was determined as the lightest section, with the sum of the governing interaction equation equal to 0.88. The W310x86 section, selected in the paper, was found the lightest W section with the sum of the governing interaction equation 0.99. The section was checked with the values calculated in the paper, and the results were

Figure 3.8: Storey 2-3: gravity load

Figure 3.9: Storey 1-2: gravity load

found to be identical as shown in Fig. 3.13 to 3.16. The missing term for the axial force for a Class 1 section in Fig. 3.13 indicates that the moment interaction equation limit given in Equation 3.3 governs over the axial load moment interaction equation given in Equation 3.2.

Figure 3.10: Storey 2-3: gravity + lateral load

Figure 3.11: Storey 1-2: gravity + lateral load

Figure 3.12: Table of recommended sections for the frame

Figure 3.15: Storey 2-3: gravity + lateral load

Figure 3.16: Storey 1-2: gravity + lateral load

Chapter 4

BeCol for Research Using Finite Element Analysis

The beam-column design rules given in CAN/CSA-S16.1-M89 do not explicitly cover members with cross-sections other than doubly symmetric. To provide a tool for analysing thin-wall prismatic steel members of an arbitrary open or box cross-section, a finite element analysis program was incorporated into the system BeCol. The original finite element program, written by Albert, CISC (1993) in BASIC, was translated to the C language and modified to run in a UNIX environment. The program performs both a non-linear incremental analysis computing load-deflection curves, stress and strain distributions under externally applied static loads and an elastic bifurcation analysis computing the elastic buckling load. A shell triangular element with an in-plane twisting degree of freedom at each node is used for both analyses. A graphical user-interface was created for problem definition and for graphical representation of the results. The extension and flexibility of the interface developed corresponds to the range of problems applicable to the incorporated program. While the interface is independent of the program and can be linked to other types of finite element

analysis, the interface capabilities would have to be expanded for solving problems falling into more complex categories (such as nonprismatic members, starred angles or battened columns).

4.1 Overview of the FEA Program

The primary finite element analysis is represented by a non-linear inelastic incremental analysis. The tangential global stiffness matrix is adjusted for each user-defined load increment, and accounts for the displaced geometry of the member and for non-linear material behaviour.

A secondary type of finite element analysis allows the determination of the elastic buckling load. The purpose of this analysis is to determine the upper bound of the solution, which can assist in estimating the range of the ultimate load for the incremental analysis, and to help in verification of the problem definition.

An overview of the finite element analysis is given in the following sections to indicate the capabilities and the limits of the program.

4.1.1 Element Description

The flat triangular shell finite element utilized was obtained by a superposition of in-plane and plate elements, thereby decoupling in-plane (membrane) and bending actions.

The in-plane triangular element was developed by Lee and Yoo (1988) by introducing a kinematic degree of freedom associated with rotation about the surface normal at each node. This in-plane rotation or 'drilling' degree of freedom elimi-

nates problems caused by zero stiffness corresponding to the in-plane rotation in the shell element. While other concepts were used to avoid the resulting singularity of the structural stiffness matrix, this element was developed by imposing constraints on the degrees of freedom at the initial stage of the formulation of shape functions. The shape functions are derived explicitly for the in-plane rotation $\theta_{x,i}$ and the two usual displacements $u_i, v_i, i = 1, 2, 3$, leading to a membrane element with 9 degrees of freedom. The 9×9 element stiffness matrix is obtained by reduced integration (four-point formula) to reduce the effects of membrane locking, caused by quadratic variation of strains within the element. It was demonstrated by Lee and Yoo that the element does not pass the patch test, however, the discrepancies are insignificant and the element is capable of representing bending behaviour better than the constant strain triangle. Its ability to model a concentrated in-plane torque was found by Lee and Yoo superior to other available elements and sufficiently accurate results can be expected in practical engineering applications.

To formulate a shell element, the membrane element is combined with a triangular plate bending element. The bending element with nine degrees of freedom in the corner nodes in the corner nodes $w_i, \theta_{x,i}, \theta_{y,i}, i = 1, 2, 3$, is based on the classical plate theory assumptions with shape functions derived by Zienkiewicz (1979). The shape functions are non-conforming and may violate slope continuity between elements. Nevertheless, the element gives for most practical applications, results superior to those attainable with equivalent conforming triangles. A patch test by the author for constant curvature which was applied to this element showed that the non-conforming triangle will not converge to the exact solution but will converge to a solution in which

the errors are of the order of errors of the corresponding displacements.

The two 9×9 stiffness matrices are combined into an 18×18 shell element stiffness matrix for the 18 degrees of freedom $u_i, v_i, w_i, \theta_{x,i}, \theta_{y,i}, \theta_{z,i}$, $i=1,2,3$. Having both membrane and bending stiffnesses, the element is capable of modelling a distortion of a cross-section and can be used for an analysis to predict flexural or torsional buckling coupled with plate buckling.

4.1.2 Inelastic Incremental Analysis

The inelastic incremental analysis implemented, incorporates geometric and material non-linear behaviour and calculates load-displacement curves of a steel member for a specified load increment history. The geometric non-linearity, associated with changes in member geometry, is expressed by a dependence of the stiffness matrix on the displacements. The material non-linearity, resulting in a variable stiffness, is accounted for by a prescribed elasto-plastic stress-strain relationship, given by E , E_{st} and F_y . The incremental method makes use of the fact that the solution is known for the initial state and the behaviour can be studied as the displacements are increased as a result of incremented loads. A new tangent-stiffness matrix is calculated in each computational cycle and is used for the next load increment. Convergence can always be obtained by using sufficiently small load increments.

The incremental equation is given by:

$$[K_t]_{i-1} \{\Delta D\}_i = \{\Delta R\}_i \quad (4.1)$$

where:

$[K_t]$ - global tangent-stiffness matrix

$\{\Delta \mathbf{D}\}$ - incremental displacement vector

$\{\Delta \mathbf{R}\}$ - incremental load vector

After each step $\{\Delta \mathbf{D}\}_i$, as the total displacements are accumulated, the nodal coordinates are adjusted and the tangent-stiffness matrix is updated. In the sense of updating the reference configuration at the end of each increment, the "Updated Lagrangian" formulation is used. The tangent-stiffness matrix takes into account the effects of membrane forces, which significantly reduce the bending stiffness of elements in compression and must be included in a buckling problem analysis. The conventional stiffness matrix is therefore augmented by a geometric stiffness matrix, as in Cook, Malkus, Plesha (1989):

$$[\mathbf{K}_t] = [\mathbf{K}] + [\mathbf{K}_g] \quad (4.2)$$

where,

$[\mathbf{K}]$ - global structural stiffness matrix

$[\mathbf{K}_g]$ - global geometric stiffness matrix

The global geometric stiffness matrix $[\mathbf{K}_g]$ (or stress stiffness matrix) is defined by the geometry, displacement field and state of stress of an element and is independent of elastic properties. For a bending element, the element geometric stiffness matrix is found by integrating over the element area:

$$[\mathbf{k}_g] = \iint [\mathbf{G}]^T \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} [\mathbf{G}] dx dy \quad (4.3)$$

where,

- $[k_g]$ - element geometric stiffness matrix
- $[G]$ - matrix of displacement field functions
- N - membrane forces

The $[K_g]$ matrix is based on average nodal membrane stresses across the element thickness calculated in the previous step. The displacement field functions in $[G]$ are taken as linear approximations of the displacements, given by Akkay, Johnson and Will (1977).

The global structural stiffness matrix $[K]$ is recalculated in each computational cycle, based on the deformed geometry calculated in the previous step. To model the elasto-plastic material behaviour, the von Mises criterion and its associated flow rule are used to define the yield surface and the elasto-plastic constituent matrices $[E_{ep}]$. The von Mises isotropic yield surface for the biaxial state of stresses is given by (Zienkiewicz, 1979) as,

$$F = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\sigma_{xy}^2} - F_y \quad (4.4)$$

where, F_y is the yield strength in a uniaxial tensile test and σ_x , σ_y are σ_{xy} are the normal and shear stress components. If the point corresponding to the stress state lies below the yield surface ($F < 0$), then the conventional elastic constitutive matrix is used to obtain the element stiffness matrix $[k]$:

$$[k] = \int_V [B]^T [E] [B] dV \quad (4.5)$$

Since steel unloads elastically, the same form is used for the case of unloading, indicated by $dF < 0$. The elasto-plastic constitutive matrix $[E_{ep}]$ takes the place of the

elasticity matrix $[\mathbf{E}]$ if the element has yielded ($F > 0$):

$$[\mathbf{k}] = \int_V [\mathbf{B}]^T [\mathbf{E}_{ep}] [\mathbf{B}] dV \quad (4.6)$$

Using the associated flow rule, the matrix $[\mathbf{E}_{ep}]$ is given by:

$$[\mathbf{E}_{ep}] = [\mathbf{E}] - \frac{[\mathbf{E}] \left\{ \frac{\partial F}{\partial \sigma} \right\} \left\{ \frac{\partial F}{\partial \sigma} \right\}^T [\mathbf{E}]}{E_{st} + \left\{ \frac{\partial F}{\partial \sigma} \right\}^T [\mathbf{E}] \left\{ \frac{\partial F}{\partial \sigma} \right\}} \quad (4.7)$$

where,

E_{st} - strain hardening modulus

$\left\{ \frac{\partial F}{\partial \sigma} \right\}$ - vector of yield surface derivatives

The matrix is valid even if the material is elastic-perfectly plastic ($E_{st} = 0$). If an element makes the elastic-to-plastic transition, the strain-hardening modulus E_{st} is used instead of E without adjustment for 'rounding the corner' (i.e. bilinear curve).

The procedure for the tangent-stiffness method used can be summarized as follows,

1. The estimated ultimate load $\{\mathbf{R}_0\}$ is applied by incrementing a scaling factor f , so that $\{\mathbf{R}\}_i = f_i \{\mathbf{R}_0\} = \sum_1^i \{\Delta \mathbf{R}\}_i$. The first load increment can be specified to virtually yield the highest stressed element. Small steps are necessary after the onset of yield.
2. At the first cycle, $\{\epsilon\} = \{0\}$, $\{\sigma\} = \{\sigma_{res}\}$ and $[\mathbf{E}_{ep}] = [\mathbf{E}]$ for all elements. The residual stresses are assumed to be longitudinal (along the axis of the member). The first load increment $\{\Delta \mathbf{R}\}_1$ is applied.
3. $[\mathbf{E}_{ep}]_{i-1}$ is evaluated for each element. $[\mathbf{E}_{ep}]_{i-1} = [\mathbf{E}]$ if $F < 0$ or if $dF < 0$. The element tangent-stiffness is calculated for each of n elements and after transfor-

mations, the global tangent-structure stiffness matrix is assembled: $[\mathbf{K}_t]_{i-1} = \sum^n [\mathbf{k}_t]$.

4. The element geometric stiffness matrix is calculated for each element from the membrane stresses $\{\sigma\}_{i-1}$ and added to the tangent-structure stiffness: $[\mathbf{K}_t]_{i-1} = [\mathbf{K}_t]_{i-1} + \sum^n [\mathbf{k}_g]_{i-1}$.
5. The determinant of $[\mathbf{K}_t]_{i-1}$ is calculated. A negative determinant of the tangent stiffness matrix generally means that the maximum load has been attained or that unloading has begun. To unload the member, after ultimate, negative load increments should be used.
6. The incremental equation $[\mathbf{K}_t]_{i-1} \{\Delta \mathbf{D}\}_i = \{\Delta \mathbf{R}\}_i$ is solved by Crout reduction for structure displacement increments $\{\Delta \mathbf{D}\}_i$.
7. The solutions are updated:
 $\{\mathbf{D}\}_i = \{\mathbf{D}\}_{i-1} + \{\Delta \mathbf{D}\}_i$ and $\{\sigma\}_i = \{\sigma\}_{i-1} + \{\Delta \sigma\}_i$ (The increments are calculated without correcting the unbalanced load vector at each load step).
8. The next load increment is applied and the procedure is restarted from the step 3.

4.1.3 Elastic Bifurcation Analysis

The elastic bifurcation analysis computes the bifurcation elastic load of a member based on the original member geometry configuration by solving the following equation for a non-trivial solution (Cook, Malkus, Plesha 1989):

$$[\mathbf{K}]\{d\mathbf{D}\} + \lambda_{cr}[\mathbf{K}_g]_{ref}\{d\mathbf{D}\} = \{0\} \quad (4.8)$$

where,

- $[\mathbf{K}]$ - global stiffness matrix
- $\{d\mathbf{D}\}$ - eigenvector
- λ_{cr} - eigenvalue
- $[\mathbf{K}_g]_{ref}$ - global geometric stiffness matrix

The element stiffness matrices assembled into $[\mathbf{K}]$ use the same formulation of the combined in-plane and bending triangular elements, the elastic constitutive matrix $[\mathbf{E}]$ and perfectly straight geometry of the member. A static analysis is carried out to obtain the membrane stresses in the elements due to the reference load $\{\mathbf{R}\}_{ref}$. These stresses are then used to generate a reference geometric stiffness matrix $[\mathbf{K}_g]_{ref}$, corresponding to $\{\mathbf{R}\}_{ref}$, by the linear approximation described by the Eq. 4.3. Pre-buckling non-linearities that impose a dependence of $[\mathbf{K}]$ and $[\mathbf{K}_g]$ on the geometric configuration just before buckling are ignored.

The inverse iteration with shifts is used as a numerical method to determine the eigenvalue λ and the eigenvector $\{d\mathbf{D}\}$. The eigenvalue is given by (Humar, 1990):

$$\lambda = \mu + \delta \quad (4.9)$$

where, μ is the shifted origin and δ is the modified eigenvalue measured from μ . Substituting equation 4.9 into equation 4.8, gives:

$$([\mathbf{K}] + \mu[\mathbf{K}_g]_{ref})\{d\mathbf{D}\} + \delta[\mathbf{K}_g]_{ref}\{d\mathbf{D}\} = \{0\} \quad (4.10)$$

The Eq. 4.10 is solved for δ . The eigenvalue shift μ is supplied by the user who must ensure, that the value of λ determined is the minimum lambda λ_{cr} . The associated eigenvector $\{dD\}$ corresponds to the governing buckling mode. The eigenvector $\{dD\}$ that identifies the buckling shape is normalized and is plotted with an arbitrary amplitude entered by the user. Finally, the elastic buckling load is given by:

$$\{\mathbf{R}\}_{cr} = \lambda_{cr} \{\mathbf{R}\}_{ref} \quad (4.11)$$

4.2 Problem Definition

A graphical user-interface was developed to preprocess the problem definition and to postprocess the results of the finite element analysis program. The preprocessing procedure results in a formatting of input files as prescribed by the translated program. The generated input text files contain all the necessary data together with the rules of their format. If required, further changes can be made directly to these files.

4.2.1 Cross-Section Node Definition

A mesh generator tailored to a prismatic structural steel member has been implemented to define easily a 3-dimensional mesh composed of triangular elements. The cross-section of the member is selected from section tables or a new cross-section is defined as described in Section 5.1.3. The cross-section with its dimensions is retrieved by the cross-section mesh definition procedure and a base mesh is automatically created. The procedure locates nodes at the junctures and ends of section plates. Additional nodes are automatically defined within those plates if they are more than 1.5 times longer than the shortest plate. The default base mesh is displayed and can be further adjusted by the user. The mesh can be made finer or coarser by specifying a factor relating the base coarseness to the new required mesh coarseness. Any node located within a plate can be deleted. If necessary, the cross-section can be rotated by an arbitrary angle about its centroid with respect to the original principal x , y -axes. An index is automatically assigned to each node by a method analogous to the integration over the cross-section starting from an arbitrary node, as described in Section 5.1.2. The node definition on the cross-section remains constant in all planes

2 of/de 2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

perpendicular to the longitudinal z -axis of the member.

4.2.2 Mesh Generation

The 3-dimensional finite element mesh is automatically created for a specified length of the member. The average distance between two adjacent nodes defined for the cross-section is used as a default z -dimension of generated triangles to keep their aspect ratio close to 1.0. This optimal element length is displayed and can be overridden by specifying a range for the element z -dimension. The nodes are generated automatically at equidistant transverse planes between the ends of the member. New planes can be inserted if a lateral support or load has to be accommodated at a particular location. Subsequently, the transverse planes are regenerated at equidistant intervals between the additional planes. A sway in the x -direction and/or camber in the y -direction can be applied to the coordinates of the member by specifying the amplitude at midspan of a half sinewave. The nodes and element indices are updated with every change of the mesh, with the numbering starting from the end band of elements and proceeding through subsequent bands along the length of the member. The side of a triangle running parallel with the global axis z is assigned node indices i, j where, $z_i < z_j$ to accommodate an orientation for residual stresses.

The half bandwidth of the global stiffness matrix is equal to $6 \times (d_n + 1)$ where, d_n is defined as the greatest difference between 2 node indices in any element. The half-band width is determined for the numerical reduction of the stiffness matrix.

The mesh that has been created can be viewed in a 3-dimensional projection. This mesh can be checked, the coordinates of nodes listed and each element can be

inspected for its thickness and associated nodes. A "camera", available for viewing, allows zooming in or out and rotation or translation of the image. The indices of nodes and elements are turned off by default to speed up the plotting but can be turned on and magnified if required. The same viewing tools were integrated with other interactive utilities to assist in identification of a specified location on the member.

4.2.3 Material Properties, Residual Stresses

The elasto-plastic stress-strain curve is defined by the values of E , F_y and the strain hardening modulus E_{st} , which can be specified as zero. A value of Poisson's ratio μ is entered to define the constitutive matrices $[E]$ and $[E_{cp}]$. Residual stresses, parallel to the longitudinal axis are defined for each element of the cross-section and are kept constant along the length of the member. A value specified for an element can be automatically assigned to all elements symmetric with respect to the principal axes of the section. Currently, a check is not provided to ensure that equilibrium of the residual stresses exists.

4.2.4 Boundary Conditions

To assist in specifying boundary conditions, the member can be displayed either in a 3-D projection or more conveniently viewed, by scrolling through the transverse planes of the mesh nodes. When in the scrolling mode, any transverse plane can be selected from a sideview of the member and the corresponding section displayed. The node numbers and the z -coordinate of the transverse plane are also displayed. In either projection, the boundary conditions can be defined by one of the following three ways:

View/Edit Boundary conditions for a selected node are listed and can be altered by scrolling through a menu, which lists ‘free’, ‘fixed’ or gives a spring stiffness option for the six degrees of freedom u , v , w , θ_x , θ_y and θ_z .

Node Template A template of boundary conditions can be defined and applied to selected nodes.

Array Template A template of boundary conditions can be defined and applied to a consequent array of nodes, defined by start and end node indices.

Nodes with at least one degree of freedom defined as not ‘free’ are highlighted and can be easily identified for inspection.

4.2.5 Load Definition

In the current version, loading can be applied only as a series of nodal forces F_x , F_y , F_z , M_x , M_y and M_z . The two types of projections of the member are provided as described for the definition of boundary conditions. The nodal forces are listed for a selected node in a table given in the View/Edit mode. The total axial force given at either end of the member will be assigned to the end nodes in a form of nodal F_x forces and distributed according to the tributary areas of the nodes. Nodes with at least one non-zero nodal force are highlighted and can be easily identified for inspection.

4.3 Problem Analysis and Results

Additional parameters pertinent to a specific type of analysis are supplied to complete the input for the analysis. BeCol postprocesses the output files produced by the

translated finite element program and represents the results in a graphical form. The output files are formatted standard text files and can be inspected without restarting BeCol.

4.3.1 Analysis Execution

The bifurcation analysis represents one step with respect to the execution time of the incremental analysis and should be executed first to verify the problem definition. The elastic buckling load can be used as the limiting load for the incremental analysis. As the inverse iteration method will converge to the lowest (dominant) eigenvalue λ , the value of λ can become negative, indicating critical loads acting in the opposite direction. In those cases an eigenvalue shift can be specified other than the default zero value.

To execute the inelastic incremental analysis, the user must supply the increments of the loads in form of fractions of the scaling factor f of the load vector $\{\mathbf{R}_0\}$ into a table that is provided. This method implies that the load is applied in each step in constant proportions of its components and that no individual load can be kept constant during any part of the loading. This is consistent with the fact that iterations are not performed for an applied load level. The determinant of the global tangent-stiffness matrix is displayed against the corresponding load increment in the same table and must be monitored for a negative sign. As the execution proceeds, only the available values of determinants for steps already calculated are displayed.

Either type of analysis (bifurcation or incremental) can be started immediately or a script can be created which will start the execution at any required time.

4.3.2 Deformed Geometry

The deformed geometry of the member can be viewed in a 3-dimensional projection for each load increment. The scale of deformations can be specified, where 1.0 will display the actual computed deformations for the incremental analysis, while only the shape of the buckled member is valid in the case of the bifurcation analysis. For a selected node, the magnitudes of the six deformations are listed in a table for a given load step.

4.3.3 Stresses and Strains

The level of stresses and strains can be examined for each step on the deformed mesh geometry. Each element is painted with a colour corresponding to a range of stresses or strains. The ranges are automatically scaled to a given value of F_y and are correlated to the palette of provided colours. The actual stress or strain can be displayed for a selected element.

The strains are evaluated at the centroid for three locations across the thickness of each element: bottom, middle and top surfaces. The location of the top surface of an element can be determined by the right-hand rule applied to the local x, y -coordinate system of an element.

4.3.4 Load–Deformation Diagrams

Load–deformation relationships can be plotted for any degree of freedom of any node. If an axial load is applied to the member, its level will be used as the y -coordinate of the diagram, otherwise the magnitude of the scaling factor f will be used for this purpose. The nodes to be monitored can be stored in a provided table. The load–

deformation diagram for any other node can be selected and viewed at any time. To identify quickly the maximum deformations, nodes can be sorted on deflections or rotations in a descending order for a given step and listed in a table which can also be used to specify the required node for plotting the curves.

To depict the overall level of the deformation process, the Euclidian norm of total displacements can be plotted against the axial load or load factor as applicable.

4.3.5 Load Summary, Reactions

A table is provided to review the actual applied loads at each step and the resulting reactions for which the calculation has been completed.

4.4 Example

The functionality of the interface developed is demonstrated by a simulation of test ST5B2 by Kennedy and Madugula (1972). This test was performed to determine the compressive capacity of a hinged T-strut, failing by the torsional-flexural buckling. The values given represent the average of three identical tests and in SI-units are:

Dimensions of T:	$d = 126 \text{ mm}$
	$b_f = 100 \text{ mm}$
	$w = 4.7 \text{ mm}$
	$t_f = 5.45 \text{ mm}$
	$A = 1110 \text{ mm}^2$
Length:	1794 mm
F_y :	342 MPa
E :	200000 MPa
E_{st} :	not reported
camber:	not reported
ν :	0.29
σ_{cr} :	168.25 MPa
P_{cr} :	186.7 kN

The given shape is defined in Figure 4.1 by using the procedure described in Section 5.1.3 and the automatically-generated base mesh on the cross-section is retrieved. To refine the mesh, the coarseness factor is defined as 0.5 and the newly created nodes are shown in Fig. 4.2. Using the new nodes on the cross-section, the triangular mesh

PROPERTIES

A	111.2	mm ²
Dist. top	0.00	mm
ix	1.70	10 ⁻³ mm ²
ixx	10.0	10 ⁻³ mm ²
ixy	40.1	mm ³
ixx	30.0	10 ⁻³ mm ²
iy	0.400	10 ⁻³ mm ²
ixy	0.10	10 ⁻³ mm ²
iyx	20.2	mm ³
ixx	14.2	10 ⁻³ mm ²
J	0.07	10 ⁻³ mm ²
Cxx	0.000	10 ⁻³ mm ²
ix	0.0	mm
yx	22.0	mm
Base x	-0.1	mm
Base y	-0	mm
r, u	60.2	mm
Depth	100.0	mm

RIGHT: 100 x 0

Diagram showing a cross-section of a member with a horizontal x-axis and a vertical y-axis. The cross-section is a thin vertical rectangle centered on the y-axis.

Buttons:

- List Properties
- Modify Computational Model
- Display N.A. For Plastic Model
- List Point Coordinates
- Display Shear center
- Create Job
- List Elements
- Display Scheme
- Go Back to Section Table

Figure 4.1: Shape definition

is assembled for the length of the member with the z -dimension of the triangles corresponding to the optimal aspect ratio as shown in Fig. 4.3. Because the camber and sweep of the member were not reported, the specified maximum value of $1/1000L$ was used. To eliminate high local deformations resulting from concentrated loads applied to the nodes at the ends of a member, four bands of arbitrarily thickened elements were inserted at each end of the member with the “add node” function. The generated triangular mesh can be plotted as shown in Fig. 4.4 and can be visually inspected. The given material properties are entered on the screen shown in Fig. 4.5. Note, E_{st} was not reported and was arbitrarily set at 6000 MPa (approximately $E/30$). The boundary conditions of a monosymmetric hinged member can be only approximated with respect to the expected mode of failure. In the experiment, the ends of the strut

Figure 4.2: Base mesh on the cross-section and its refinement

DEFINITION OF MESH ALONG MEMBER LENGTH

Length of Member:

1794 mm

Calculated Optimal Element Length:

28 mm

Specified Minimum Element Length:

14 mm

Specified Maximum Element Length:

42 mm

Camber:

1.8 mm

Sweep:

1.8 mm

Note: Additional transverse planes of nodes can be specified.
Other planes of nodes are generated automatically.

Z - coordinates of Node Planes

Section #:	Z [mm]	Added:
1	0	
2	28	X
3	56	X
4	84	X
5	112	X
6	140	
7	168	
8	196	
9	224	
10	252	
11	280	
12	308	
13	336	
14	364	
15	392	

Add Node

Delete Node

Save

Figure 4.3: Mesh definition along the member length

were lubricated to minimize friction. As the inelastic analysis is very sensitive to the specification of boundary conditions, six configurations in total were examined for the analysis to model the hinges. Three models of the hinges were combined with two schemes of thickened end bands of the elements as detailed in Fig. 4.6 to try to simulate the boundary conditions definitions in the test. These are described below:

Model 1-a,b: Fork supports at either end. Only the web of the section was supported against longitudinal displacements (solid node marks represent longitudinal displacements prevented).

Model 2-a,b: As in model 1 with the addition that the flange was supported against longitudinal displacements.

Model 3-a,b: As in model 2 with the addition that the nodes of the web were prevented from displacing laterally.

The two schemes of thickened end elements were defined as follows:

a: Two bands of elements were thickened at either end (to 12 mm and 18 mm).

b: Four bands of elements were thickened at either end (to 9 mm, 12 mm, 15 mm and 18 mm).

The critical axial load P_{cr} determined by the test was used as a total reference load applied at the top ends of the member. The axial load was divided into nodal forces with magnitudes proportional to the tributary areas of the individual nodes, calculated automatically by the program, as shown in Fig. 4.8.

Figure 4.4: Generated triangular mesh

Figure 4.5: Material properties

Specified boundary conditions at the bottom	Specified boundary conditions at the top	(Model) n *	$\frac{P}{P_f}$
(1a) 2	1.19
		(1b) 4	1.15
(2a) 2	1.06
		(2b) 4	1.05
(3a) 2	1.06
		(3b) 4	1.00

* Number of bands of thickened elements at one end

Figure 4.6: Test-to-predicted ratio for different boundary condition models

Figure 4.7: Definition of boundary conditions

Figure 4.9: Load increment definition and job execution

Approximately 30 increments were defined for each analysis with these increments being refined to $0.005 P_{cr}$ as the critical load was approached, as shown in Figures 4.9 and 4.10. Two or three trial runs were necessary in each model to achieve this accuracy. The last loads with a positive determinant of the global stiffness matrix are summarized as test-to-predicted ratios given in Fig. 4.6. The applied loads together with computed reactions in the restrained nodes at each load step are listed in a table shown in Fig. 4.10.

As an example, detailed results are shown for the case with two thickened end element bands combined with the fork supports and flange prevented from longitudinal displacements (boundary condition model 3a), for which the test-to-predicted ratio was found 1.06. The deformed configuration of the member geometry determined for

Figure 4.11: Deformations of the buckled member

Figure 4.12: Front view of the buckled member

Figure 4.13: Distribution of strains parallel with the longitudinal axis

Figure 4.14: Shear strain distribution at the top surfaces

Figure 4.15: Shear strain distribution at the bottom surfaces

Figure 4.16: Stress distribution at the top surfaces

Figure 4.17: Stress distribution at the bottom surfaces

Web-flange junction: Node 291, rotation about the longitudinal axis θ_z , refer to Fig. 4.20

Web-flange junction: Node 291, displacement perpendicular to the weak axis u_x , refer to Fig. 4.21

Web tip: Node 297, displacement perpendicular to the weak axis u_x , refer to Fig. 4.22

Flange tip: Node 293, displacement perpendicular to the strong axis u_y , refer to Fig. 4.23

In all cases the load displacement or rotation curves exhibit classical characteristics of inelastic buckling. The ability to readily select nodes to plot the load-displacement curves for comparison with experimental data shows the versatility of the program.

Figure 4.18: Euclidian norm of total displacements

Figure 4.19: Mid-height, web-flange juncture: u_z

Figure 4.22: Mid-height, web tip: u_x

Figure 4.23: Mid-height, flange tip: u_y

Chapter 5

Library Functions in BeCol

5.1 Cross-Section Utilities

To enhance designer's options of cross-section selection and to deliver geometric data necessary to other modules of the system, library functions were written for a cross-section definition of a large assortment of built-up shapes, calculation of geometric properties and management of user-defined section tables. Geometric properties of shapes produced by Canadian mills are directly retrieved from files provided by the Canadian Institute of Steel Construction (CISC). Built-up shapes can be defined, or selected from user-defined tables, and used for a member definition, employed by other parts of the program. Geometric properties are calculated assuming that the shapes are composed of straight plate elements, and if applicable, standard shapes listed in the CISC files. Welds and roundings are neglected. Torsional properties of open and hollow sections are calculated by equations based on theories for thin-wall sections. Calculation of torsional properties of sections with intermittent connectors have not been included, as it is outside the scope of the thesis. Both the graphical interface and computational functions can be integrated with other programs.

5.1.1 Cross-Section Definition

The definition of a general shape contains all information necessary for:

- calculation of geometric properties (effective geometric properties for Class 4 sections)
- classification of section (Class 1, 2, 3 ,4)
- generation of a basic cross-section mesh for finite element analysis
- shape type identification (open, hollow or intermittently connected)
- symmetry type determination (asymmetric, singly or doubly symmetric)

Each shape is replaced by a number of straight plate elements in x,y -coordinate system. Each element is defined by its thickness and the end points which coincide with junctures of the mid-line of the section. The end points of each shape are defined by a general expression for the determination of their x,y -coordinates. To eliminate overlapping elements in the junctures and to ensure a continuity of the mid-line required for numerical integration, additional dummy elements of zero thickness were defined. The cross-section is aligned with its principal axes and the basic geometric properties listed in Fig. 5.1.1.

This definition is further used as a starting point for calculation of torsional properties, as discussed in Sec. 5.1.2, and for the generation of the basic cross-section finite element mesh described in Sec.4.2.1.

Another set of elements is used to define the cross-section for classification of the section and to determine effective geometric properties under compressive stress.

Property	Unit	Property	Unit	Property	Unit
A	mm^2	I_x	mm^4	$S_{x,top}$	mm^3
$Dead\ load$	kN/m	S_x	mm^3	$S_{x,bot}$	mm^3
$Depth$	mm	r_x	mm	$S_{y,lft}$	mm^3
$Width$	mm	Z_x	mm^3	$S_{y,rgt}$	mm^3
d_{top}	mm	I_y	mm^4	$Z_{x,top}$	mm^3
d_{bot}	mm	S_y	mm^3	$Z_{x,bot}$	mm^3
w_{lft}	mm	r_y	mm	$Z_{y,lft}$	mm^3
w_{rgt}	mm	Z_y	mm^3	$Z_{y,rgt}$	mm^3

Figure 5.1: Calculated dimensions and geometric properties

Each element for each shape is identified according to Table 1 of CAN/CSA-S16.1-M89 (CSA 1989a). The widths and thicknesses corresponding to the element are prescribed according to Clause 11.3. After the yield strength, type and magnitude of applied loads have been specified by the user, other modules determine the Class and effective properties of the section.

5.1.2 Torsional Properties

Torsional properties of open sections are calculated from Equations 5.1 to 5.8 reproduced from Table 2.3 of Structural Members and Frames, Galambos (1968). The equations are derived on the assumptions that the thickness of its elements is an order of magnitude smaller than the other dimensions of the section and that the mid-line of the section does not contain a closed loop (i.e. open thin-wall section).

$$\omega_j = \sum_{i=0}^i \rho_{ij} b_{ij} \quad (5.1)$$

$$\omega_{0j} = \sum_{i=0}^i \rho_{0i,j} b_{i,j} \quad (5.2)$$

$$\omega_{nj} = \frac{1}{A} \left[\frac{1}{2} \sum_{i=0}^n (\omega_{0i} + \omega_{0j}) t_{i,j} b_{i,j} \right] - \omega_{0j} \quad (5.3)$$

$$I_{\omega x} = \frac{1}{3} \sum_{i=0}^n (\omega_j x_i + \omega_j x_j) t_{i,j} b_{i,j} + \frac{1}{6} \sum_{i=0}^n (\omega_j x_j + \omega_j x_i) t_{i,j} b_{i,j} \quad (5.4)$$

$$I_{\omega y} = \frac{1}{3} \sum_{i=0}^n (\omega_j y_i + \omega_j y_j) t_{i,j} b_{i,j} + \frac{1}{6} \sum_{i=0}^n (\omega_j y_j + \omega_j y_i) t_{i,j} b_{i,j} \quad (5.5)$$

$$C_w = \frac{1}{3} \sum_{i=0}^n (\omega_{ni}^2 + \omega_{ni} \omega_{nj} + \omega_{nj}^2) t_{i,j} b_{i,j} \quad (5.6)$$

$$J = \frac{1}{3} \sum_{i=0}^n t_{i,j}^3 b_{i,j} \quad (5.7)$$

$$x_o = \frac{I_{\omega y}}{I_x} \quad y_o = -\frac{I_{\omega x}}{I_x} \quad (5.8)$$

where,

i, j = indices of end points of an element

$C(0, 0)$ = centroid

$S(x_o, y_o)$ = shear center

ρ = perpendicular distance between a point in the cross-section
and the centroid

ρ_0 = perpendicular distance between a point in the cross-section

and the shear center

ω	=	double sectorial area with respect to the centroid
ω_0	=	double sectorial area with respect to the shear center
ω_n	=	normalized unit warping
b, t	=	element dimensions
$I_{\omega x}$	=	warping product of inertia about the x axis
$I_{\omega y}$	=	warping product of inertia about the y axis
C_w	=	warping torsional stiffness
J	=	St. Venant torsion constant

The summation in equations 5.1 to 5.3 represents a numerical integration for straight elements of constant thicknesses. To carry out the integration, it is necessary to determine the integration line. The starting point of the line, must be one of free ends of the section, where the shear flow $\tau t = 0$. Although the integration can proceed in an arbitrary direction, the integration line must only go through each element once. Each branch of the line must be terminated by a free end of the section. The integration scheme is identical to a tree-shaped structure and the shape is conveniently traversed by an algorithm of recursive technique.

The equations for torsion for closed sections with one or more compartments, given by Eq. 5.9 and 5.10, are based on Prandtl elastic-membrane (soap-film) analogy. The unit angle of twist Θ is assumed to be the same for each compartment.

$$T = 2 \sum_{i=1}^N A_{m,i} \tau_i t_i \quad (5.9)$$

$$\Theta = \frac{1}{2GA_{m,i}} \oint_{l_{m,i}} \frac{\tau_i t_i - \tau'_i t'_i}{t} dl \quad i = 1, 2, \dots, N \quad (5.10)$$

where,

T - specified torque

Θ - unit angle of twist

$A_{m,i}$ - area bounded by the mean perimeter for the i -th compartment

$\tau' t'$ - shear flow for the compartment adjacent to the i -th compartment
where dl is located

$l_{m,i}$ - length of the mean perimeter for the i -th compartment

Sections having several compartments represent a statically indeterminate problem as the starting point for integration, a point of shear flow $\tau t = 0$, is not known. The solution of the $N + 1$ equations given by Equations 5.9 and 5.10, for sections with N compartments gives the unit angle of twist for a specified torque, Boresi (1985). Based on this solution, the St. Venant torsion constant can be calculated by:

$$J = \frac{T}{G\Theta} \quad (5.11)$$

For hollow sections with a single compartment the solution simply becomes:

$$J = \frac{4A_m}{\oint_0^{l_m} t dl} \quad (5.12)$$

The geometric constants β_x and β_y , used in equations of member buckling (Galam-bos 1968), are calculated from Equations 5.13 and 5.14, respectively.

$$\beta_x = \frac{1}{I_x} \int_A y(x^2 + y^2) dA - 2y_o \quad (5.13)$$

$$\beta_y = \frac{1}{I_y} \int_A x(x^2 + y^2) dA - 2x_o \quad (5.14)$$

where $\bar{r}_o$ is given by,

$$\bar{r}_o = \sqrt{x_o^2 + y_o^2 + \frac{I_x^2 + I_y^2}{A}} \quad (5.15)$$

The current version of BeCol does not calculate torsional properties of sections built-up of shapes that are connected intermittently (i.e. starred-angles or battened column sections). The properties can only be calculated only after the spacing of the connectors is specified. The calculation of geometric properties for these shapes is beyond the scope of this thesis.

Within the described scope, the geometric property algorithms are general and can be applied to other cross-sectional types that could be added to the set of predefined single or built-up shapes.

5.1.3 User-Defined Cross-Sections

An icon menu of all predefined cross-section types shown in Fig. 5.2 can be invoked from the main menu of BeCol. Constituent simple shapes are highlighted by a black solid pattern and optional plates are marked by dashed borders. In some cases, a plate can be disregarded by defining its thickness as zero. For example, a welded T-shape or a simple plate can be defined instead of a welded I-shape. The program prompts the user to specify the individual parts of the built-up section. The individual constituent

Figure 5.2: Menu of cross-section types

GET PROBLEM	HELP	INTERACTION CURVES	COLLAPSE ANALYSIS	PLATE BUCKLING ANALYSIS	SELECTIVE PHOTO	SHAPES	QUIT
SYSTEM INFO	FILE	BEAM-COLLAPSE S101 CHECK	BEAM-COLLAPSE S101 DESIGN	CRANE COLLAPSE	BUILT-UP COLLAPSE	Path: C:\Program Files\path\path.p	
Date: Jul 25, 1999 Time: 14:10						MULTIPLE DESIGN	ADD TEST
						CONTINUE

Please Select Member 1:

Section Selection

Section	Available Weights
W8	35
W10	45
W12	55
W14	65
W16	75
W18	85
W20	95
W22	105
W24	115
W26	125
W28	135
W30	145
W32	155
W34	165
W36	175
W38	185
W40	195
W42	205
W44	215
W46	225
W48	235
W50	245
W52	255
W54	265
W56	275
W58	285
W60	295
W62	305
W64	315
W66	325
W68	335
W70	345
W72	355
W74	365
W76	375
W78	385
W80	395
W82	405
W84	415
W86	425
W88	435
W90	445
W92	455
W94	465
W96	475
W98	485
W100	495

Section	Plate Dimensions	Dimension Y'

Figure 5.3: I-shape selection table

GET PROBLEM	HELP	INTERACTION CURVES	COLLAPSE ANALYSIS	PLATE BUCKLING ANALYSIS	SELECTIVE PHOTO	SHAPES	QUIT
SYSTEM INFO	FILE	BEAM-COLLAPSE S101 CHECK	BEAM-COLLAPSE S101 DESIGN	CRANE COLLAPSE	BUILT-UP COLLAPSE	Path: C:\Program Files\path\path.p	
Date: Jul 25, 1999 Time: 14:10						MULTIPLE DESIGN	ADD TEST
						CONTINUE

Please Enter Dimensions of Plate:

Nominal Thickness (mm)		Width (mm)
First Preference	Second Preference	
6.5	6.5	100
8.0	8.0	150
10	10	200
11	11	250
12	12	300
14	14	350
16	16	400
18	18	450
20	20	500
22	22	550
24	24	600
26	26	650
28	28	700
30	30	750
32	32	800
34	34	850
36	36	900
38	38	950
40	40	1000

Section	Plate Dimensions	Dimension Y'
W10x10		

Figure 5.4: Plate selection table

Figure 5.5: List of elements

shapes are selected from tables in which the shapes are organized by size and weight. Refer to Figure 5.3. Plate dimensions are defined from a table, illustrated in Fig. 5.4, which lists the thicknesses for first and second preference as specified in the Handbook of Steel Construction for flat metal products, Sheet & Plate, CISC (1992). Depending on the section type, a distance or length may need to be specified. The program will check the constructability of the specified section, calculate all geometric properties, draw the cross-section in scale and assign the proper shape designation (e.g. TWPW 491x389). The user can view the geometric properties listed in table format, as shown in Fig. 5.7, and by activating the appropriate button, display the shear center and neutral axes for plastic moduli. To allow an inspection of the shape definition, all elements and the coordinates of their end points are tabulated. Refer to Figures 5.5

Job Definition

Job Name:

Directory:

Title:

Description:

Note:

To add a new section, click on the corresponding section

Section	Plate Dimensions	Dimension "x"
WeldK113	750 x 14	400 0 mm

Figure 5.8: Job definition

and 5.6. By saving the specified section, the section is added to a table maintained for that particular cross-section type. To make the section available to other parts of the system, a separate file must be created by a procedure termed “create job”. Notes can be attached to the section definition as illustrated in Fig. 5.8.

5.1.4 Organization of Cross-Section Tables

Three databases of sections are controlled by BeCol. The first database consists of CISC files (section selection tables files distributed by Canadian Institute of Steel Construction) for rolled shapes (W, S, M, HP, WT, HSS, channels, angles) and welded shapes (WWF, WRF, WWT). This database can be updated only with a revised version of these files. These files list nominal dimensions of the shapes and geometric properties calculated on the basis of minimum dimensions specified by mill tolerances.

The original general access functions for retrieval of this data, written by Holtz, Carleton University (1991) were linked with BeCol. These shapes are internally defined by BeCol as one built-up shape for generating a basic cross-section finite element mesh, classification of the section and for determining effective geometric properties for Class 4 sections. The geometric properties are not recalculated.

The second database is built from sections selected from CISC tables. Based on preference and availability of sections in Canadian mills, shapes can be copied to or deleted from this database and beam-column design can be performed using this reduced database only.

The third database contains all user-defined built-up shapes and is fully maintained by the user. To management this database, each built-up shape type was given a shape designation, an acronym based on the shape description, followed by *name depth x mass (kg/m)* of that shape. Refer to Figure 5.10 for shape designations.

All three databases are accessible from the main section icon menu. The stored shapes are sorted by depth and weight and are presented in tables, listing relevant geometric properties and components of the built-up shapes. Any section can be retrieved from these tables, drawn to scale, inspected or used for "create a job", as shown in Fig. 5.8.

A separate database is maintained for confidential mill prices of selected sections for various grades of steel. A program was written to search this database for a specified section to retrieve the price and then to pass the information to other modules as required.

Figure 5.9: Typical user-defined section table

Figure 5.10: Nomenclature used for built-up shapes

5.2 Interface library functions

HOOPS is a system for creating graphics applications for a wide range of purposes. Principally, HOOPS Graphics System (1992) creates a database storing information about which geometric objects to draw, where they should be displayed and how they should be rendered. The HOOPS database is organized in a tree-like hierarchy of segments, where each segment consists of geometrical primitives, cameras, lights, rendering and modelling attributes. If the attributes are not prescribed, the child segment inherits the attributes from its parent segment. Routines are provided for modifying, querying, searching and displaying the database. The programmer's task is to specify the contents of the database, sequences of its modification and requests for the updating of the image on the screen. An application written in a directive language, such as C or Fortran, can include routine calls to HOOPS libraries anywhere throughout the program. The HOOPS routine calls are compiled as any other part of the application by the compiler of the language used. The program must be linked with the HOOPS system libraries. The programming language selected for BeCol is C.

5.2.1 System Architecture

Although the code of HOOPS and C must work together, it was necessary to separate the graphics-oriented portions of the application associated with the user-interface from the components executing the actual engineering calculations to achieve a modular architecture of the system. Related functions were grouped together and maintained in the form of shared libraries.

Figure 5.11: Basic system architecture

The package BeCol was designed with an overall architecture outlined in the Fig. 5.11. The individual components perform the following tasks:

System top Determines the environment and mode in which the system should be executed.

System overhead Depending on the environment and mode, it disconnects the interface or connects the interface in a requested mode and controls the overall flow of the execution of the system.

Interface overhead Called by the system overhead, it controls the flow of the interface tasks; collects information from the user and returns it to the system overhead; accepts and displays results.

Interface libraries Collection of application independent libraries maintained by the programmer.

C modules Program modules written in C and compiled as a part of the system.

These can be compiled with other systems or other graphic interfaces which have to be created for them.

C programs Application independent programs using input-output files with their execution.

It was desirable to keep the size of the system and the interface overheads to a minimum. All reusable interface functions are stored in the interface libraries which are grouped according to their purpose. Depending on the size and frequency of calls, C programs are directly compiled with the system or built as stand-alone programs (preferable option). The preprocessing tasks of the system include operations that quite often have to be duplicated in the stand-alone program.

Three types of interface functions — builders, operators and utilities — were developed and are stored in HOOPS and C interface libraries. The source code together with its compiled version “EMAP” (empty application) are generated from BeCol as described in Sec. 6.5. EMAP provides the same interface as BeCol as shown in Fig. 5.12, and can be quickly connected to other applications.

5.2.2 Builders

In a development of a structural engineering software, the image is mostly required to be treated as a collection of geometric objects rather than as bit maps. In this respect,

HOOPS offers routines for creating a number of geometric primitives, but does not provide library functions for building composed objects such as menus, dialogue boxes or graphs. Routines that build such objects were written and stored in the libraries. If these library objects are unsuitable for a given application, they can be used as templates for more general or more specific functions. The objects constructed by builders include:

Buttons Various number, horizontal or vertical series, various types can be specified.

Icons and a display can be included.

Menus Main menu, utility menu, pull-down menus can be defined.

Dialogue boxes Integer, float numbers or strings can be entered through a dialog box which may offer a default value or buttons for a series of recommended values. Refer to Fig. 5.13. The entry can be checked to determine its validity and an appropriate error be displayed.

Tables Tables with a slider, header and titles of any number of columns and rows can be defined. An example is shown in Fig. 5.14, the job selection table.

Graphs Creates a box with markers at suitable intervals annotated along x and y axes. An appropriate coordinate transformation is performed and passed to other builders that construct points and curves in the created coordinate box.

5.2.3 Operators

The size of the interface overhead increases with the number of possible events to which the program must respond. The process of analysing an event and reacting

to it cannot be made fully automatic. Many events must be handled exclusively. Nevertheless, groups of "standard" events can be identified and the process of testing for these events with an adequate response can be defined. These functions are applied on objects built by builders, communicate information and modify the image on the screen accordingly. Typical tasks performed by these routines are represented by pressing or highlighting a button, scrolling a table or processing entered data to a dialogue box. The functions operating on objects created by the bulders include:

Object	Operator
Group of buttons	Press, release, highlight, supply data
Menus	Select an item, delete, supply data
Dialogue boxes	Enable editing, entry check, display an error message for illegal entries (such as out of range), supply data
Tables	Scroll data, select an entry, highlight, supply data
Display	Empty display, show data

5.2.4 Utilities

The utilities are procedures that are called frequently, that are independent of the content of the on-the-screen-display and accessible at any time. These routines provide the capacity to dump the screen or a portion into a required graphics format file, to display information about the content of the screen display, to provide help information, to display the date or to add notes on the screen display. The functions are connected to pull-down menus and are activated by the buttons from the main menu as illustarted in (Fig. 5.12).

Figure 5.12: Menus of utilities (BeCol and EMAP)

EXIT PROGRAM SYSTEM INFO Date: Aug 12, 1994 Time: 10:46	INTERACTION CURVES BEAM-COLUMN DESIGN	SECTION TABLES AND DEFINITION BEAM-COLUMN S16.1 DESIGN	COLUMN ANALYSIS CRANE COLUMN	FINITE ELEMENT ANALYSIS BUILT-UP COLUMNS	SELECTIVE PHOTO MULTIPLE SCREEN ADD TEXT CONTINUE	SHAPENOT CLUT
---	--	---	---------------------------------	---	--	------------------

Strong Axis View

300.0 kN, 5.0 kNm, 4.0 kNm, 300.0 kN

End Restraint Definition

Y, X

Weak Axis View

300.0 kN, 2.0 kNm, 1.0 kNm, 300.0 kN

Material Properties

Member Length	6000 mm
P_y	300 MPa
E	200000 MPa
Not applicable	---
Residual Factor	0.00
Leads Between Supports	None
F_u	460 MPa

Design Parameters

Please enter specified minimum Yield Stress:	
240	300
300	360
400	420
3000 F_y (MPa)	

Design Checks

Value is off the allowable range.	
Values accepted only from the range < 150.0 ; 800.0>	

Bracing

BENDING	CLEAR BRACING	$K_y = 1.00$
yes	yes	

K values

Bending	Bracing	K values
x direction	no	Not Required
y direction	no	Not Required

Other Data

XP000073

Figure 5.13: Dialogue box with default values and entry check

metafile (a picture capture redisplayable by HOOPS), or a file in HPGL format (plot file). A segment can be stored in a temporary database of HOOPS and displayed by "Multiple screen".

Snapshot Creates a PostScript file of a predefined name in landscape orientation.

Add text Provides the capacity to edit a note on the display screen.

Help Internally provides a label marking the position within the execution of the system. This label is used to call a viewer of an associated help file. Cross-referencing depends on the type of the help files.

5.2.5 Notes

Another group of functions suitable for inclusion into the interface libraries is represented by functions for assistance in sizing various interface objects directly from the screen display. The most frequently used interface building functions could be made available from menus and a screen layout could be built (draft) by dragging and editing on the screen without intermediate program recompiling. The image could then be saved in the metafile format and with minor modifications included into the application code and compiled. A considerable amount of programming of these functions would be necessary and is outside the scope of this thesis.

Interactive builders of graphical user-interfaces are now available in the market and should be considered for development of similar applications.

Chapter 6

Program Description

6.1 General Information

The current version of the package BeCol was developed on SUN workstations with the SPARC microprocessor architecture under Sun Operating System SunOS 4.1.3, a version of UNIX operating system used by Sun Microsystems. The source code was written in the C programming language and compiled with *cc* and *gcc* compilers.

The graphical user-interface was created by HOOPS graphics system, version 3.20-6, Copyright 1986–1992 by Ithaca Software, Alameda, California, and compiled with the *cc* compiler. The HOOPS system libraries can either be linked by static binding, which increases the size of the program, or linked with the HOOPS libraries which must reside on the server and are retrieved during the program's execution. In both cases, the computer must be running a windowing system to open the screen display for the HOOPS graphical interface. The current version requires an X Window System based window management system such as OpenWindows or SunView, but with certain additions and modifications, HOOPS can be combined with Motif, X11 windows or windows running on personal computers.

6.2 Program Execution

The program is started from the directory BeCol by typing at the shell prompt:

```
becol
```

at which time the front page of the program will appear on the display screen as shown in Fig. 1.1. From this point, the program is executed interactively by making selections from menus.

System environment variables are set automatically by the program regardless whether the program is run locally or remotely. The default display type is a colour monitor. The following options are available:

```
becol [-c/m] [-r/p file_name]
```

- c - Force colours for colour monitor.
- m - Force colours for monochrome monitor.
- r - Record all events to `file_name`.
- p - Play back the execution recorded in `file_name`.

6.2.1 Menu Organization and Sequence of Execution

The main menu located at the top of the main window remains displayed throughout the execution of the program. The main application menu only changes when the finite element analysis option is invoked with the buttons further defining the required applications within a given task. The sequence is illustrated in Fig. 6.1. Any application selected from the main menu can be terminated by the QUIT button. The next

predefined interface window is called by the CONTINUE button. The program is terminated by EXIT PROGRAM. Every application described in this thesis is called by selecting its corresponding button from the main menu. The sequence of calls is arbitrary after a job has been defined. The application main menu consists of:

Interaction curves: A pull-down menu offers three options and each is executed independently:

m-c- ϕ curves: Plots m-c- ϕ curves for a selected section type, axis of bending and residual stress pattern.

Mmax: Calculates the ultimate in-plane moment capacity of a member loaded by a constant concentric axial force and a specified ratio of end moments in one plane.

Interaction curves: Generates families of in-plane ultimate strength interaction curves for various slenderness and end moment ratios.

Section tables and definition: Provides the capacity to access section tables of those shapes produced in Canadian Mills and define user-specified built-up shapes. A job is created for a required section and can be retrieved by applications for column analysis, finite element analysis and beam-column S16.1 check or design.

Column analysis: Generates the appropriate column curves for any doubly symmetric, singly symmetric, or asymmetric section.

Finite element analysis: A pull-down menu will display two options:

New job: Creates a new job by copying a cross-section from an existing job defined with the "Section tables and definition" application.

Existing job: Retrieves an existing job.

After selecting of a job, the main menu will change into a problem definition menu for the finite element analysis, as shown in Figure 6.1, where cross-section mesh, longitudinal mesh, material properties, boundary conditions and loads are defined. Once the definition is complete and after selecting the "Analysis" application, the main menu will change to the problem analysis menu, shown in Fig. 6.1, where the type of analysis is defined and the results can be reviewed.

Beam-column S16.1 check: Generates beam-column curves for a selected job in accordance with CAN/CSA-S16.1-M89. Only doubly symmetric sections are considered. Interaction curves and summary reports indicating the member adequacy are displayed.

Beam-column S16.1 design: Displays a pull-down menu with two options:

New job: Defines a new job where the section, loads, boundary conditions and bracing must be specified.

Existing job: Retrieves the definition of an existing job.

Designs prismatic doubly-symmetric beam-columns conforming to CAN/CSA-S16.1-M89. All valid sections from the shape types selected are listed in order of cost and weight. Interaction curves and summary reports can be displayed for each section.

Crane column: Not implemented

Built-up columns: Not implemented

The utilities described in Sec. 5.2.4 can be called at any point during the execution of BeCol. Selections are made from the pull-down menus shown in Fig. 5.12.

6.2.2 Execution of Independent Modules

Three independent programs **becod**, **trefa** and **price** are called by the main program **becol**. The process is hidden from the user. To use the programs without opening the graphical user-interface, execute the following commands:

1. Beam-column design program:

```
becod -c/d [-r] [-f file_name]
```

c	-	Check a beam-column design for a given cross-section.
d	-	Design a beam-column for given loads.
r	-	Print a detailed report of the results.
f	-	Use file_name as the input file instead of the default file bc.in and store results in file_name.out.

2. Finite element analysis program:

```
trefa -i/e job_dir
```

i	-	Perform inelastic incremental analysis.
e	-	Perform elastic bifurcation analysis.
job_dir	-	Directory containing input files and a destination for output files.

Jobs created by BeCol which are intended to be executed later or on another machine are called by scripts with names defined at prompts during BeCol execution. The scripts are stored in Trefa directory.

3. Price searching program:

```
price input_file [output_file]
```

- input_file - Contains any number of section designations and steel grades.
- output_file - If the file name is not specified, then the prices are written against each section into the input_file. If the file name is specified, then the original input_file will remain unchanged and will be copied into the output_file, where the costs are written against each section.

6.3 File Description and Organization

The package BeCol integrates four executable files — becol, becod, trefa and price, and the CISC database cisc of standard shapes. The source code, makefiles, object files and user-defined database are only required for updating and maintenance of the application. The file organization is illustrated in Fig. 6.2 and a description of the individual files follows:

BeCol: Relative top (home) directory of the system

source: Directory containing the source code of the system overhead, integrating all other parts of BeCol. The file prefixes indicate the purpose of the following sub-modules:

Figure 6.2: File organization of BeCol

bc_	beam-column
bs_	built-up shape
c_	C libraries
col	column
fe_	finite element
h_	emap libraries
ic_	in-plane strength interaction curves
job_	job management
k_	effective length factor
sst	tables of simple structural sections

For speed considerations, the sub-modules are linked into a single executable file **becol**. If required, stand-alone programs or libraries can be created by selecting and modifying the appropriate source code files and the associated header files. This process was automated for generating the user-interface libraries resulting in the **emap** application.

Becod: Directory containing the source code of the design module, **becod**, a stand-alone C program, and its makefile.

Xprice: Directory containing the source code of **price**, a stand-alone C program, and its makefile. The confidential database of Mill prices are stored in **price.dat**.

Trefa: Top directory of the finite element program module. Job output files are deleted interactively with **clear_all**, job definition and output files are deleted with **delete_all**. Contains scripts (text files listing a short sequence of commands

executable by Unix) to allow to schedule the execution of finite element analysis jobs.

cisc: Directory contains section tables in files distributed by the CISC. Each gives the geometric data for one section type: wwf.dat, w.dat, s.dat, hp.dat, m.dat, c.dat, hssr.dat, hsss.dat, hssso.dat, wrf.dat, wwf.dat, wt.dat, l.dat, l_.dat, il_.dat, il_.dat. The file, flat.dat, was added to this database and contains standard thicknesses of flat steel products.

sec_tabs: Directory contains tables of built-up shapes and lists of standard shapes specified by the user. BeCol creates, sorts and maintains one file with .ctg suffix for each cross-section type.

xsec0s, xsec1s, xsec2s: Default directories containing jobs created by BeCol for asymmetric, singly symmetric and doubly symmetric sections.

bc_jobs: Default directory which stores jobs for beam-column design.

ps: Default directory for storing PostScript files.

hp: Default directory for storing plot files.

playback: Default directory for storing playback files.

help: Default directory for storing help files.

becol: The top executable file of the system BeCol.

becod: The executable file of beam-column design program.

makefile: The top makefile for the package BeCol; calls all makefiles within BeCol and updates all modules.

get_emap: Script that will extract **emap** libraries.

get_becol: Script that will create a tar file of BeCol.

Trefa/trefa: The executable file of a stand-alone C finite element program.

Trefa/source: Contains the source files and makefile of the finite element program **trefa**.

Trefa/jobs: Contains jobs created for the finite element analysis, each stored in a separate directory. Job definition and job output files can be deleted with **delete-job**; job output files (while preserving the job definition) can be deleted with **clear-job**.

6.4 Incorporated existing source codes

Access functions for retrieval of data from the CISC files of standard structural steel shapes were written by Holtz (1991). The source code of the libraries which provides both sequential and random access is stored in files **sstlib.c** and **sst.h**. The existing routines were expanded to allow access of the following additional shapes: channels, T-shapes and square, rectangular and circular hollow structural shapes. The library is linked with two programs, **becol** and **becod**.

The finite element program **trefa** is a translation of a program written by Albert (1993). The original program **TSHELL**, written in QuickBasic for MS-DOS, was

translated to the C programming language for UNIX operating system. To take advantage of the large virtual memory space offered by workstations and to substantially decrease the execution time, the intermediate step of storing matrices was replaced by dynamic allocations of the memory. The format of input-output files were modified, system calls were adjusted and other changes were introduced as a result of different array indexing by QuickBasic and C. Two problems were tested by the original and translated versions of the program and a detailed comparison of results verified the translation yielding identical results for both elastic bifurcation and the inelastic incremental analysis. The program `trefa` operates as a stand-alone program and is started by a system call from `becol` or by a script created by `beam-column` for a scheduled execution.

6.5 Program installation

The whole package `BeCol` can be wrapped and compressed into one tar file `BeCol.tar.Z` by executing command `get_becol`. The program is then installed into a directory by

```
uncompress BeCol.tar.Z; tar xvf BeCol.tar
```

To make changes to the source code and to debug the program, the C compiler and HOOPS graphics system must be installed on the host machine. To compile and link the modified program, makefiles are provided for each of the four programs and one top makefile for the whole application. To use a debugger, the `-g` option must be specified in the makefiles and a macro `USE_DEBUG_TOOL` must be set to true in `becol.c`.

Similarly, the `emap` libraries can be extracted into one file `emap.tar.Z` by executing command `get_emap` and installed into another directory by

```
uncompress emap.tar.Z; tar xvf emap.tar
```

The `main()` function is located in `emap.c`.

Chapter 7

Conclusions, Recommendations and Future Work

7.1 Conclusions

The interactive system BeCol was developed for design and analysis of steel beam-columns. As an advanced design program it is educational, contains the latest design standards and research related to the behaviour of the beam-columns and was developed in consultation with committee members of the standard to obtain the best interpretation. Limitations of equations given in design standards are specifically spelled out. The theoretical background of these and other rules implemented are described in a detail with examples illustrating their application. The package BeCol is self-contained and ready for use by any structural engineer without need of lengthy user manuals. The graphic displays (commands, menus, dialogue boxes, graphs and images) are self-explanatory, realistic and provide effective control over the required operations of the program. Results are shown in the form of summary reports, diagrams, tables or figures. Wherever possible, graphical representation of the behaviour of steel beam-columns is provided to visually inspect or verify the results. Examples

are used throughout the thesis to demonstrate and verify the capabilities of the package developed.

BeCol contains submodules that will:

- (1) **calculate the in-plane stability strength curves** for any wide flange shape for bending about either principal axis, with or without predefined residual stresses, and for HSS and rectangular sections. Families of these curves can be plotted for varying slenderness or end moment ratios. The equivalent moment factor ω_1 is numerically obtained and plotted for varying ratios of axial load and end moment ratios. The equation given in CAN/CSA-S16.1-M89 for ω_1 is plotted on the same diagram to illustrate its applicability.
- (2) **calculate column curves** for any doubly symmetric, singly symmetric and asymmetric shape with varying restraint and boundary conditions. The curves are generated for both the theoretical elastic buckling solutions and the complete column curve (elastic, inelastic and yielding) calculated according to CAN/CSA-S16.1-M89, 94, for all possible failure modes for the specified shape.
- (3) **design a beam-column or check** according to CAN/CSA-S16.1-M89 (CSA 1989a).
A beam-column for any given member restraints with any number of load combinations, can be designed with the member selected from an intermix of specified doubly symmetric shape data bases. The costing of members is calculated from price list provided by Canadian mills.
- (4) **perform an elastic bifurcation finite element analysis** of a prismatic member. The calculation of the elastic buckling load can be used to verify the mem-

ber definition and to obtain an estimate of the ultimate load for the inelastic incremental finite element analysis.

- (5) perform an inelastic incremental finite element analysis** of a prismatic member. The incorporated FEA program (Albert 1994) uses an 18 degree of freedom triangular shell element with an in-plane twisting degree of freedom at each node that has given excellent predictions of members subject to flexural and/or torsional buckling coupled with plate buckling. An extensive graphical interface was developed to make the input definition (i.e. the finite element mesh creation) easy and to provide graphical representation of the results obtained from the analysis.
- (6) calculate cross-sectional properties** for over 25 thin-wall or built-up cross-sections commonly produced by Canadian mills and fabricators. Tables of user-defined shape tables are maintained by BeCol and are utilized by all submodules for cross-section definition.

7.2 Recommendations and Future Work

Program development is an open-ended task and BeCol does not solve every possible problem related to steel beam-columns. The program has been developed to give the capacity to include future work on built-up columns, crane columns, finite element analysis as a design tool and integration. The following recommendations to maintain, utilize, upgrade or expand the code of this package are made:

- Maintain the current code, correct encountered bugs and record implemented changes. Test the program against well-documented solutions whenever possible. Adjust or improve the interface when needs arise. Maintain libraries of graphics-related subroutines and define general guidelines for a unified graphics appearance. Consider an object-oriented language and interface builders for future development of new modules.
- Implement changes introduced in new editions of CAN/CSA-S16.1, CAN/CSA-S136, CISC steel shape databases and steel mill cost databases.
- Implement built-up members with battens, lacing and tie-plates in accordance with Clause 19 of CAN/CSA-S16.1-M89. The interface icons and diagrams can make use of tables provided in the CISC Commentary for CAN/CSA-S16.1-M89.
- Implement interface, design and design check for a typical crane-column consisting of two segments with the possibility of different flexural stiffness. Proportional and non-proportional loads could be included.
- Integrate BeCol or its submodules with other modules (such as modules for geometric modelling; determination of loads; design of connections; flexural members etc.). The design submodule could be converted into an invisible module with an option to be called when a member is identified by a label. The requested member would be then retrieved together with its geometry, load, all design load combinations and other design data. This would allow a detailed review of the design or redesign of the member by the structural engineer.

Communication routines for intermediate IO-files would have to be written to coordinate the individual modules.

- Perform a comparative study between the finite element analysis and test results to determine the ability of the implemented FE program to model steel beam-column behaviour. Implement more elements that will model shear connectors and curved elements. Other advanced FEA programs may be investigated for their suitability to solve the complex stability problems associated with the behaviour of steel beam-columns. Development of algorithms to assist in preliminary member selection for beam-columns that can only be analysed and designed using finite elements. Upgrade the graphical user-interface for the FEA module to model more general cases with respect to non-prismatic, built-up members and members with stiffeners.

References

- ALBERT, C. 1993. Finite element program TSHELL, QuickBasic. Private communication.
- ALLEN, H.G. AND BULSON, P.S. 1980. Background to buckling. McGraw-Hill Book Company (UK) Limited, pp. 157-168.
- AUSTIN, W.M. 1961. Strength and design of metal beam-column. ASCE Journal of the Structural Division, **87**(4) pp. 1-32.
- BORESİ, A.P. AND SIDEBOTTOM, O.M. 1985. Advanced mechanics of materials. John Wiley & Sons.
- CHEN, W.F. AND ATSUTA, T. 1988. in-plane behaviour and design, Volume 1 of Theory of Beam-Columns. McGraw-Hill, Inc.
- CHEN, W.F. AND HAN, D.J. 1985. Tubular members in offshore structures, pp. 198. Pitman, London, U.K.

- **CHERNENKO, D.E., AND KENNEDY, D.J.L.** 1991. An analysis of the performance of welded wide flange columns. *Canadian Journal of Civil Engineering*, 18 pp. 537-555.
- **CISC.** 1992. Handbook of steel construction. 5th Ed. Canadian Institute of Steel Construction, Willowdale, Ont.
- **COOK, R.D., MALKUS, D.S. AND PLESHA, M.E.** 1989. Concepts and applications of finite element analysis, 3rd Ed. John Wiley & Sons, Inc., USA.
- **CSA.** 1989a. Limit states design of steel structures. CAN/CSA-S16.1-M89, Canadian Standards Association, Rexdale, Ont.
- **CSA.** 1994. Limit states design of steel structures. CSA Standard S16.1-94, (proposed). Canadian Standards Association, Rexdale, Ont.
- **CSA.** 1989a. Cold formed steel structural members. CAN/CSA-S136-M89, Canadian Standard Association, Rexdale, Ont.
- **GALAMBOS, T.V.** 1968. Structural members and frames. Prentice-Hall Inc., Englewood Cliffs, NJ.
- **GALAMBOS, T.V. (Editor)** 1988. Guide to stability design criteria for metal structures. Structural Stability Research Council, 4th Ed. John Wiley & Sons, Inc., New York, NY.
- **GERE, J.M. AND TIMOSHENKO, S.P.** 1984. Mechanics of materials. 2nd Ed. Wadsworth, Inc., Belmont, California 94002.

- **HOOPS Graphics System Reference Manual.** 1992. Ithaca Software, 1301 Marina Village Parkway, Alameda. CA 94501.
- **HOLTZ, N.M.** 1991. Library functions to access CISC SST tables, in C language. Private communication. Carleton University.
- **HORNE, M.R.** 1956. The flexural-torsional buckling of members of symmetrical I-section under combined thrust and unequal terminal moments. Quarterly journal of mechanics and applied mathematics, No. 4.
- **HUMAR, J.L.** 1990. Dynamics of structures. Prentice-Hall, Inc., Division of Simon & Schuster, Englewood Cliffs, New Jersey 07632.
- **KENNEDY, D.J.L.** 1994. Steel Compression Member Design in the 1994 Canadian Standard. Private correspondence.
- **KENNEDY, J.B. AND MADUGULA, K.S.M.** 1972. Buckling of steel angle and tee struts. ASCE Journal of the Structural Division, **98** (ST11) pp. 2507-2521.
- **KENNEDY, D.J.L., PICARD, A. AND BEAULIEU, D.** 1990. New Canadian provisions for the design of steel beam-columns. Canadian Journal of Civil Engineering, **17**(6), pp. 873-893.
- **KETTER, R.L.** 1961. Further studies of the strength of beam-columns. ASCE Journal of the Structural Division, **87**(ST6) pp. 135-152.
- **LEE, S.C., AND YOO, C.H.** 1988. A novel shell element including in-plane torque effect. Computers & Structures. **28**(4) pp. 505-522.

- NETHERCOT, D.A., AND TRAHAIR, N.S. 1976a. Inelastic lateral buckling of determinate beams. ASCE Journal of the Structural Division, 102(ST4) pp. 701-717.
- NETHERCOT, D.A., AND TRAHAIR, N.S. 1976b. Lateral buckling approximations for elastic beams. Structural Engineer, 54 pp. 197-204.
- NETHERCOT, D.A., AND TRAHAIR, N.S. 1977. Lateral buckling calculations for braced beams. Institution of Engineers, Australia, Civil Engineering Transactions, CE19 pp. 211-214.
- PEEK J., O'REILLY T. AND LOUKIDES M. 1993. UNIX power tools. O'Reilly & Associates, Inc., Sebastopol, CA.
- SCHMITKE, C.D., AND KENNEDY, D.J.L. 1985. Effective lengths of laterally continuous, laterally unsupported steel beams. Canadian Journal of Civil Engineering, 12(6) pp. 603-616.
- VALLEY, J. 1992. C Programming for UNIX. 1st Ed. SAMS Publishing, Division of Prentice Hall Computer Publishing, Indiana, USA.
- ZIENKIEWICZ, O.C. 1979. The finite element method. 3rd Ed. McGraw-Hill Book Company (UK) Limited.

END

1 7 - 1 / 1 - 9 5

FIN