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**EXPERIMENTAL VERIFICATION OF STRESS DISTRIBUTIONS
IN NON-UNIFORMLY TENSIONED RECTANGULAR MEMBRANES**

by

PETER M. BORMANN, B.Eng., P.Eng., rmc

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master's of Engineering

Department of Mechanical and Aerospace Engineering

Ottawa - Carleton Institute

for Mechanical and Aerospace Engineering

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"Experimental Verification of Stress Distributions in Non-Uniformly
Tensioned Rectangular Membranes"

submitted by Peter M. Bormann, B.Eng., P.Eng., rmc
in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Co-Supervisor

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15 August 1994

ABSTRACT

This thesis verified the use of the finite element method as a method of determining the stress fields in membrane structures for which closed form analytical solutions are not available.

Two methods of experimentally determining the stress fields were used. The first was electrical resistance strain gages. It was found that the strain gages created a local stiffening effect causing inaccuracies in the magnitude of the indicated strain but not necessarily in the orientation of the principal axes.

The second was a non-contact optical system which measured the displacements of markers on the membrane during loading. From this displacement data the stress fields were determined. It was found that, although the stress results were more accurate than those derived from the strain gages, the best correlation was found while comparing displacements. Comparison at this level eliminated the magnification of the inherent experimental errors caused by the process of differentiation.

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LIST OF PRINCIPAL SYMBOLS

SYMBOL	DESCRIPTION
$[K]$	Overall stiffness matrix used in finite element method
$[\delta]$	Finite element method nodal displacement matrix
$[P]$	Force matrix for finite element model
F	Strain gage factor
F_a	Strain gage axial gage factor
F_t	Strain gage transverse gage factor
K_t	Transverse sensitivity coefficient
n	Incremental error due to Wheatstone bridge non-linearity
R	Strain gage resistance
ν	Poisson's ratio
E	Modulus of elasticity
ϵ	Strain indicated by the measurement device (not corrected for transverse sensitivity or Wheatstone bridge non-linearity)
$\epsilon_x, \epsilon_y, \gamma_{xy}$	Components of strain
$\epsilon_a, \epsilon_b, \epsilon_c$	Corrected strains for the three elements of the strain gage rosette
ϵ_1, ϵ_2	Major and minor principal strains respectively
$\sigma_x, \sigma_y, \tau_{xy}$	Components of stress
θ	Angle from reference axis to the direction of the principal stress/strain

SYMBOL	DESCRIPTION
P	Applied load
I	Moment of inertia
σ_1, σ_2	Major and minor principal stresses
$u(x), v(y)$	Displacements in the x and y directions respectively

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

The Department of National Defence (DND) has been pursuing a technology development program relative to the Space-Based Radar (SBR) system for a number of years. A key area of technology is structural dynamics. Several of the satellites proposed, as well as some of their sub-systems, are classified as Large Space Structures (LSS). Typically, LSS are lightweight, structurally very flexible, have low natural frequencies of vibration, exhibit low damping, and are connected by or possess non-linear joints. The need to develop methods of synthesizing test results as well as methods of testing large sub-systems to predict on-orbit performance is recognized in the space community.

The tensioned membrane is a structure that has already had significant use in space systems. It is of special interest to SBR, since one of the proposed SBR systems involves a membrane-type radar array of approximately football-field size which perhaps represents the ultimate in lightweight structures.

The use of tensioned rectangular membranes as antennae in space applications presents the vibration analyst with a challenging analytical problem. In order to function properly as antennae, these membranes must satisfy certain highly stringent specifications. First, wrinkling of the membrane surface is not acceptable. Wrinkling of membranes is considered to occur when one or more of the in-plane principal stresses becomes negative. Secondly, the lateral vibration must be maintained within prescribed tolerance levels. This

latter consideration dictates that accurate theoretical means must be found to establish in advance the membrane free vibration frequencies and mode shapes. It will be appreciated that these two challenges are not independent, since a knowledge of the initial stress distribution is a necessary prerequisite to performing a free vibration analysis.

An example of a structure utilizing a tensioned rectangular membrane is the test article developed for the Lens Antenna Deployment Demonstration (LADD) program, which is a model designed to simulate a proposed space system [1].

The development of the LADD test article was contracted to Grumman Corporation by the USAF Rome Laboratory and NASA Marshall Space Flight Centre as described by Pugh et al. [1]. The space-qualifiable test structure was built for the Strategic Defence Initiative Organization to demonstrate the viability of the critical structural and mechanical elements of single-axis roll-out membrane deployment for space based radar applications. A line diagram of the test article is shown in Figure 1-1 and a photograph is shown in Figure 1-2. The test structure is a precision space structure which is representative of operational designs for space-fed lens arrays and was originally destined to be deployed as a shuttle flight experiment. Plans for the operational deployment of the test frame were cancelled but a series of ground tests were proposed which included static, dynamic, and thermal measurements in a simulated space environment. An additional important objective of these tests was the verification of the analytical tools used to design and develop large precision space structures. This thesis represent a portion of the work completed to achieve this last objective. The project is part of an internal project at the Canadian Space Agency (CSA) entitled the Structural Dynamics and Testing

Figure 1-1: Illustration of the Lens Antenna Deployment Demonstration (LADD) test article in ground deployed configuration [1, p. 157]

Figure 1-2: A photograph of the LADD test article in its ground deployed configuration

of Large Space Structures. It is done in collaboration with the Department of National Defence, Defence Research Establishment, Ottawa, and is in support of the IADD program.

The experimental portion of the LADD project requires, in addition to modal analysis of large test membranes, a verification of conventional static mathematical models normally used in the design of large space structures. Of interest to the designers of these structures is the determination of the stress distribution, or stress field, within the structure. In the case of the LADD program, the component of the structure which is of interest is the large expanse of membrane-like material used within the lens array. The orientation of the tension loads and gravitational field acting on the membrane portion of the structure, and the resulting stress field, do not readily allow the closed form, analytical solution. Designers must therefore rely upon mathematical models or numerically approximate methods such as the finite element method to satisfy the stringent pre-flight structural analysis required for space systems. It is therefore essential to test membranes with non-uniform loading experimentally to verify the results obtained from these approximate methods.

It was suggested that the stress field of a non-uniformly loaded square membrane, as shown in Figure 1-3, be experimentally determined and compared to the predicted stress field obtained from finite element method. The membrane in question is subject to four equal, concentrated, in-plane forces of 222 N (50 lbf) each acting outward along the diagonals of the membrane. Due to the symmetry of the membrane shape and load pattern this thesis shall concentrate on the results obtained from the upper left quadrant of the

Figure 1-3: Test membrane load pattern for verification of mathematical models

membrane. This focussing of attention controls the costs associated with experimentation while providing data sufficient to satisfy the objectives of the thesis.

The material used for the membrane was Kapton 500HN Polyimide film which is synthesized by a bipolycondensation reaction between an aromatic dianhydride and an aromatic diamine [2]. It has an Aluminum layer vacuum deposited on both sides and was chosen because it has similar physical characteristics to the material used in the LADD test article. The physical and mechanical properties of the Kapton film are outlined in Table 1.

Manufacturer		Industrial Films Division, DuPont Company
Single Point Thickness	Minimum	118 μm (4.65 mils)
	Maximum	136 μm (5.35 mils)
Modulus of Elasticity, E		2500 MPa
Poisson's Ratio		0.34
Density		1425.2 kg/m^3
Thermal Coefficient of Expansion		20 $\text{ppm}/^\circ\text{C}$ (11 $\text{ppm}/^\circ\text{F}$)
Heat Sink Characteristics		Fair to Good

Table 1: Significant physical properties of Kapton 500 HN polyimide film [2]

There are a number of numeric and mathematical methods available to designers and engineers for complex problems in which analytical solutions for the equations governing a body's deformation are not available. The finite element method is one of the

most widely used numerical methods in the field of engineering design and development. The recent work of D. J. Gorman in the area of the superposition method [3] also serves as an analytical tool for the modelling of large tensioned membrane structures [4]. Recently, the Canadian Space Agency completed a study entitled, "Investigation of the Stress Distributions in Corner Tensioned Rectangular Membranes" [5]. The study was made of the stress distributions induced in rectangular membranes by concentrated outward corner loads acting along the diagonals. Two methods of analysis were employed. An analytical study was conducted by means of superimposed Airy stress functions. This study was complemented by a parallel study conducted with a commercially available finite element program. As shown in reference [5], very good agreement is obtained when results of the two studies are compared. However, no experimental verification of the stress field was performed. It was, therefore, deemed necessary that the stress field be measured experimentally and compared with the computed stress field obtained with the finite element method or continuum mechanical method.

The objective of this thesis is to study experimentally the stress field in the square corner loaded membrane (shown in Figure 1-3) and compare the results with those obtained with a commercially available finite element program.

1.2 FINITE ELEMENT METHOD

In the finite element method the body under study is discretized into a number of elements of *finite* dimensions [6]. If the body exists in n -dimensions of space, it is

subdivided into a system of n-dimensional finite elements. Two and three dimensional bodies are discretized into lines, planes, and solids which are interconnected by a lattice of points called *nodes*. The flexibility of the finite element method comes from the fact that the elements within the model can be of unequal size, shape, physical or mechanical properties and can be judiciously chosen to examine areas of stress concentration or discontinuities in the body under study.

The precise manner in which finite elements are attached to the nodes is best understood by considering the nodes to act as pinned joints. The nodes secure the elements together but there is no physical continuity between adjacent finite elements.

After the body under study has been sub-divided into a series of elements and nodes, the next step is to determine the *element stiffness matrix* of the individual elements representing the body. These individual contributions are then combined to form the *overall stiffness matrix* for the entire discretized body by requiring that the continuity of displacements and equilibrium of forces prevail at all nodes in the finite element model of the body. This leads to the matrix equation:

$$[K][\delta] = [P] \quad (1)$$

in which $[K]$ denotes the overall stiffness matrix of the discretized body. The *overall force vector* $[P]$ lists the externally applied forces at all the nodes, while $[\delta]$ lists the displacements of all nodes. For a given set of prescribed boundary conditions the external forces acting on the body can be solved uniquely for the nodal displacements, $[\delta]$, from which the stresses and strains within the body can subsequently be computed using

standard theories of elasticity.

To summarize, the finite element method follows the following sequence of activities to obtain a solution in a typical stress analysis problem [7]:

- a. Discretize the body into a system of finite elements;
- b. Derive the stiffness matrix for each element as well as other physical and mechanical properties for each individual element representing the body;
- c. Assemble the overall stiffness matrix, $[K]$, comprising the effects of the individual element stiffness matrices and determine the overall force vector, $[P]$;
- d. Solve the above matrix equation with prescribed boundary conditions to determine the nodal displacements, $[\delta]$; and
- e. Calculate the stresses and strains within the elements from the computed nodal displacements, $[\delta]$, using theories of elasticity.

1.2.1 FINITE ELEMENT MODEL

The results of the Canadian Space Agency finite element analysis were not reported in their entirety and access to missing data was not readily available. It was decided that the finite element analysis would be repeated. This allowed complete access to the results and served as an additional confirmation of both the finite element analysis and the continuum mechanical analysis results.

There are many finite element software applications commercially available. The software program used to generate the finite element results in this case was developed by

the Intergraph Corporation [8] whereas in reference [5] the NASTRAN finite element software package was used.

The Intergraph Finite Element Modeller is a Unix based system which uses its own graphical interface to input the model of the body under study and assist in the assignment of elements and nodes. The system allows the assignment of global or individual physical and mechanical properties, any loading condition as well as any configuration of constraints or boundary conditions required.

Figure 1-4 shows the elements, nodes and load configuration used in the finite element model as well as the boundary conditions that ensure the correct loading of the membrane and zero free body movement. A full diagonal load is specified at each of the lower corner nodes, while at the top right corner node, only the horizontal component of the diagonal load is applied. All of the remaining loads are created by the reactions at the supports. In total, there are 400 elements and 441 nodes used in the approximation of the membrane model. The upper left corner node is constrained in the x, y, and z direction while the upper right corner node is constrained in the y and z direction only. The elements are *plates* with a thickness of 125 μm (0.005 in) and defined with the material properties of the Kapton 500HN film outlined in Table 1.

1.2.2 FINITE ELEMENT METHOD RESULTS

This section contains the detailed results of the finite element analysis which will be used for comparison purposes throughout this thesis. These results were found to duplicate the earlier CSA finite element results found using the NASTRAN software

Figure 1-4: Finite element model showing nodes, elements and boundary conditions

package to the third decimal place which in turn compare favourably with the continuum mechanical results [5].

Due to the symmetry of the problem, the results focus on only the top left hand quarter of the membrane. All of the displacement results are adjusted to be with respect to the reference axes which are located at the centre of the membrane.

The general outline for this section is as follows:

- a. Numerical results for the displacements (in metres) of the nodes with respect to the origin placed at the centre of the membrane in the x (horizontal) and y (vertical) directions are given in Tables 2 and 3;
- b. Graphical representations of the nodal displacements in both the x and y directions are provided in Figures 1-5 and 1-6;
- c. Numerical results are provided for the stresses found in the finite elements shown in Figure 1-4. The stress values (in MPa) are taken at the centre of the element and are presented in the following components:
 - (1) Stress component acting in the x direction (Table 4);
 - (2) Stress component acting in the y direction (Table 5);
 - (3) Shear stress (Table 6);
 - (4) Major principal stress (Table 7); and
 - (5) Minor principal stress (Table 8).
- d. Graphical results of the elemental stresses as outlined above are shown in Figures 1-7 to 1-11.

-0.0017046	-0.000864	-0.00059	-0.000444	-0.000337	-0.000256	-0.00019	-0.000135	-8.65E-05	-4.23E-05	0
-0.00073137	-0.000922	-0.000683	-0.000522	-0.000401	-0.000307	-0.00023	-0.000164	-0.000106	-5.2E-05	0
-0.0005611	-0.000578	-0.000596	-0.000499	-0.000405	-0.00032	-0.000245	-0.000177	-0.000115	-5.67E-05	0
-0.0003798	-0.000428	-0.000438	-0.000419	-0.000363	-0.0003	-0.000236	-0.000174	-0.000115	-5.69E-05	0
-0.000276	-0.000305	-0.000328	-0.000325	-0.000302	-0.00026	-0.000212	-0.00016	-0.000107	-5.35E-05	0
-0.0001929	-0.000219	-0.000239	-0.000247	-0.000238	-0.000215	-0.00018	-0.000139	-9.46E-05	-4.78E-05	0
-0.000132	-0.000153	-0.000173	-0.000184	-0.000184	-0.000171	-0.000148	-0.000117	-8.08E-05	-4.12E-05	0
-8.68E-05	-0.000105	-0.000123	-0.000136	-0.000141	-0.000135	-0.00012	-9.69E-05	-6.79E-05	-3.49E-05	0
-5.59E-05	-7.26E-05	-8.97E-05	-0.000103	-0.00011	-0.000109	-9.87E-05	-8.12E-05	-5.76E-05	-2.99E-05	0
-3.77E-05	-5.35E-05	-6.99E-05	-8.35E-05	-9.16E-05	-9.25E-05	-8.56E-05	-7.14E-05	-5.11E-05	-2.67E-05	0
-3.17E-05	-4.71E-05	-6.33E-05	-7.7E-05	-8.55E-05	-8.71E-05	-8.11E-05	-6.8E-05	-4.89E-05	-2.55E-05	0

Table 2: Numerical results for nodal displacements in the x direction

0.0017346	0.0007314	0.0005612	0.0003799	0.000276	0.0001929	0.000132	8.69E-05	5.59E-05	3.77E-05	3.17E-05
0.0008635	0.0009215	0.0005782	0.0004278	0.0003047	0.0002186	0.0001531	0.0001054	7.27E-05	5.35E-05	4.72E-05
0.0005897	0.0006826	0.0005956	0.0004382	0.0003278	0.0002391	0.0001727	0.0001235	8.97E-05	6.99E-05	6.34E-05
0.0004443	0.0005216	0.0004991	0.0004194	0.0003254	0.0002472	0.0001839	0.0001364	0.0001031	8.35E-05	7.71E-05
0.0003368	0.0004012	0.0004054	0.0003634	0.0003016	0.0002381	0.0001837	0.0001406	0.00011	9.16E-05	8.56E-05
0.0002558	0.0003073	0.00032	0.0003002	0.0002606	0.0002146	0.0001711	0.0001351	0.0001086	9.25E-05	8.71E-05
0.0001901	0.0002303	0.0002447	0.0002362	0.000212	0.0001802	0.0001481	0.00012	9.87E-05	8.56E-05	8.12E-05
0.000135	0.0001645	0.0001772	0.0001744	0.00016	0.0001394	0.0001171	9.69E-05	8.12E-05	7.14E-05	6.8E-05
8.66E-05	0.000106	0.0001152	0.0001148	0.0001069	9.46E-05	8.08E-05	6.79E-05	5.77E-05	5.12E-05	4.89E-05
4.23E-05	5.2E-05	5.68E-05	5.69E-05	5.35E-05	4.78E-05	4.12E-05	3.49E-05	2.99E-05	2.67E-05	2.56E-05
0	0	0	0	0	0	0	0	0	0	0

Table 3: Numerical results for nodal displacements in the y direction

Legend:

Maximum Displacement: $-170.46\text{E-}05$ m

Minimum Displacement: 0 m

Figure 1-5: Graphical representation of nodal displacements in the x direction

Legend:

Maximum Displacement: $170.5\text{E-}05\text{ m}$

Minimum Displacement: 0 m

$0 - 25.0\text{E-}05\text{ m}$		$100.0\text{E-}05 - 125.0\text{E-}05\text{ m}$
$25.0\text{E-}05 - 50.0\text{E-}05\text{ m}$		$125.0\text{E-}05 - 150.0\text{E-}05\text{ m}$
$50.0\text{E-}05 - 75.0\text{E-}05\text{ m}$		$150.0\text{E-}05 - 175.0\text{E-}05\text{ m}$
$75.0\text{E-}05 - 100.0\text{E-}05\text{ m}$		

Figure 1-6: Graphical representation of nodal displacements in the y direction

26.966	13.673	8.7858	6.2433	4.8305	3.9188	3.3236	2.9334	2.6966	2.5843
-1.0163	9.1772	7.6666	6.2698	5.0825	4.2872	3.7277	3.3557	3.126	3.0166
1.2089	1.8474	4.7751	4.8516	4.534	4.1012	3.747	3.4816	3.3099	3.2256
0.025973	1.216	2.2297	3.3035	3.5695	3.5777	3.4758	3.3667	3.2822	3.2378
0.15268	0.53742	1.3859	2.1123	2.6748	2.9424	3.0603	3.0981	3.1051	3.1037
0.052131	0.36394	0.86336	1.4726	1.9881	2.3756	2.6203	2.7678	2.8487	2.8844
0.046818	0.23861	0.61306	1.0719	1.5377	1.9325	2.236	2.4462	2.5768	2.639
0.031693	0.18174	0.46424	0.84014	1.2436	1.6229	1.9399	2.1796	2.3377	2.4159
0.026942	0.14844	0.38538	0.70686	1.0697	1.4272	1.743	1.9926	2.1634	2.2497
0.024088	0.13443	0.34963	0.64671	0.98819	1.3331	1.645	1.897	2.0722	2.1616

Table 4: Numerical results for elemental stress component in the x direction (σ_x)

26.966	-1.0163	1.2089	0.025975	0.15268	0.052132	0.046818	0.031693	0.026942	0.024088
13.673	9.1772	1.8474	1.216	0.53742	0.36394	0.23862	0.18174	0.14844	0.13443
8.7857	7.6665	4.7751	2.2297	1.3859	0.86336	0.61306	0.46425	0.38538	0.34963
6.2432	6.2698	4.8515	3.3035	2.1123	1.4726	1.0719	0.84014	0.70687	0.64671
4.8304	5.0825	4.534	3.5695	2.6748	1.9881	1.5377	1.2436	1.0697	0.9882
3.9187	4.2872	4.1012	3.5777	2.9424	2.3756	1.9325	1.6229	1.4272	1.3331
3.3235	3.7276	3.747	3.4758	3.0603	2.6203	2.236	1.9399	1.743	1.645
2.9334	3.3557	3.4815	3.3666	3.0981	2.7678	2.4462	2.1796	1.9926	1.897
2.6966	3.126	3.3099	3.2822	3.1051	2.8487	2.5768	2.3377	2.1634	2.0722
2.5843	3.0166	3.2256	3.2378	3.1037	2.8844	2.639	2.4159	2.2497	2.1616

Table 5: Numerical results for elemental stress component in the y direction (σ_y)

-18.673	-3.6994	-1.6479	-0.87015	-0.54291	-0.35414	-0.23502	-0.15017	-0.083949	-0.027031
-3.6994	-8.778	-4.257	-2.5685	-1.6107	-1.0696	-0.71189	-0.45621	-0.25526	-0.082239
-1.6479	-4.257	-4.8394	-3.3267	-2.3335	-1.6118	-1.1031	-0.71664	-0.40428	-0.13069
-0.87014	-2.5685	-3.3267	-3.1801	-2.4807	-1.8521	-1.3178	-0.87787	-0.50186	-0.16326
-0.5429	-1.6107	-2.3335	-2.4807	-2.2338	-1.7918	-1.3409	-0.91947	-0.53486	-0.17533
-0.35414	-1.0696	-1.6118	-1.8521	-1.7918	-1.5447	-1.2091	-0.85507	-0.50629	-0.16739
-0.23502	-0.71189	-1.1031	-1.3178	-1.3409	-1.2091	-0.98419	-0.71441	-0.43012	-0.1433
-0.15017	-0.45621	-0.71664	-0.87787	-0.91947	-0.85507	-0.7144	-0.5294	-0.32291	-0.10829
-0.083948	-0.25526	-0.40427	-0.50185	-0.53486	-0.50629	-0.43012	-0.32291	-0.19876	-0.066944
-0.027031	-0.082238	-0.13068	-0.16326	-0.17533	-0.16739	-0.1433	-0.10829	-0.066944	-0.022605

Table 6: Numerical results for shear stress component (τ_{xy})

45.639	14.553	9.1287	6.3628	4.8927	3.951	3.3403	2.9411	2.6993	2.5846
14.552	17.955	9.9133	7.346	5.5954	4.5599	3.8673	3.42	3.1477	3.019
9.1286	9.9133	9.6144	7.1163	5.7747	4.7668	4.0963	3.6431	3.3647	3.2315
6.3627	7.346	7.1163	6.4836	5.4264	4.6555	4.0575	3.6417	3.3766	3.248
4.8926	5.5954	5.7747	5.4264	4.9086	4.3195	3.841	3.4767	3.2371	3.1181
3.9509	4.5599	4.7667	4.6555	4.3195	3.9203	3.5335	3.2243	3.0106	2.9023
3.3403	3.8673	4.0963	4.0575	3.841	3.5334	3.2202	2.951	2.7589	2.6593
2.9411	3.42	3.6431	3.6417	3.4767	3.2243	2.951	2.709	2.5313	2.4376
2.6992	3.1477	3.3647	3.3765	3.2371	3.0106	2.7589	2.5313	2.3622	2.2721
2.5846	3.0189	3.2315	3.248	3.1181	2.9023	2.6593	2.4376	2.2721	2.1842

Table 7: Numerical results for the major principal stresses (σ_1)

8.2924	-1.8953	0.86599	-0.093511	0.090495	0.019964	0.030047	0.023942	0.024305	0.023803
-1.8953	0.39915	-0.39937	0.13981	0.024496	0.09127	0.098953	0.11747	0.12671	0.13209
0.86599	-0.39938	-0.064291	-0.035117	0.14514	0.19781	0.26372	0.30269	0.33053	0.34371
-0.093512	0.13981	-0.035117	0.12334	0.25537	0.39482	0.49024	0.56506	0.61253	0.63647
0.090495	0.024495	0.14514	0.25537	0.44059	0.61105	0.75707	0.86499	0.93773	0.97376
0.019964	0.091269	0.1978	0.39482	0.611	0.83084	1.0194	1.1663	1.2653	1.3153
0.030047	0.098953	0.26372	0.49024	0.75707	1.0194	1.2518	1.4351	1.5609	1.6247
0.023942	0.11746	0.30268	0.56506	0.86499	1.1663	1.4351	1.6502	1.7991	1.8753
0.024304	0.12671	0.33052	0.61252	0.93773	1.2653	1.5609	1.799	1.9646	2.0498
0.023802	0.13208	0.3437	0.63646	0.97376	1.3153	1.6247	1.8753	2.0497	2.139

Table 8: Numerical results for the minor principal stresses (σ_2)

Legend:

Maximum Stress: 26.966 Mpa

Minimum Stress: -1.016 Mpa

0-4 Mpa		16-20 Mpa
4-8 Mpa		20-24 Mpa
8-12 Mpa		24-28 Mpa
12-16 Mpa		

Figure 1-7: Graphical representation of the elemental stress component in the x direction (σ_x)

Legend:**Maximum Stress: 26.966 Mpa****Minimum Stress: -1.0163 Mpa**

■ 0-4 Mpa	□ 16-20 Mpa
■ 4-8 Mpa	■ 20-24 Mpa
■ 8-12 Mpa	□ 24-28 Mpa
■ 12-16 Mpa	

Figure 1-8: Graphical representation of the elemental stress component in the y direction (σ_y)

Legend:

Maximum Stress: 0.023 Mpa

Minimum Stress: -18.673 Mpa

■ -2.5-0 Mpa	□ -12.5- -10.0 Mpa
■ -5.0- -2.5 Mpa	■ -15.0 - -12.5 Mpa
■ -7.5- -5.0 Mpa	□ -17.5- -15.0 Mpa
■ -10.0- -7.5 Mpa	■ -20.0- -17.5 Mpa

Figure 1-9: Graphical representation of the shear stress component (τ_{xy})

Legend:

Maximum Stress: 45.639 Mpa

Minimum Stress: 2.184 Mpa

0-5 Mpa	20-25 Mpa
5-10 Mpa	25-30 Mpa
10-15 Mpa	35-40 Mpa
15-20 Mpa	40-50 Mpa

Figure 1-10: Graphical representation of the major principal stress (σ_1)

Legend:

Maximum Stress: 8.292 Mpa

Minimum Stress: -1.895 Mpa

Figure 1-11: Graphical representation of the minor principal stress (σ_2)

CHAPTER TWO

TECHNIQUES IN EXPERIMENTAL STRESS ANALYSIS

There are many methods for experimentally determining stress distributions in structural members. The possibilities increase if one includes experimental methods which can determine the strain or displacement fields from which the stress distribution can be determined using the theories of elasticity.

The possibilities for ultimately measuring the stress field of a two dimensional test specimen can be grouped by several different means. One possible grouping can be made on the basis of whether the measuring technique requires contact with the test piece or not. Ideally the test method would be non-contact, thus eliminating the possibility of the measuring apparatus interfering with, or affecting, the test.

A second grouping can be based on the property which the method actually measures. In the field of elasticity, these properties for possible measurement are displacement, strain or stress. With results in any one of these properties, theoretically, one can find the other two. The following sections outline the most conventional methods for experimentally determining stress distributions in structural members.

2.1 PHOTOELASTICITY

Photoelasticity is a technique to determine stress or strain distributions in structures through the use of plastic models or bonded birefringent coatings [9].

Photoelasticity is particularly useful in situations where analytical solutions are difficult or

impossible due to complex loadings or boundary conditions.

The double refraction of polarized light that is produced when stress is applied to a transparent material is called the photoelastic effect. Epoxies and other polymers are highly photoelastic and can be formed into models of complex structures subjected to complex loading conditions.

The fringes produced when a model is loaded are proportional to the principal stress differences within the structure. With photoelasticity techniques, images of stress or strain distributions for the models are obtained. With proper scaling procedures, these stresses or strains can be translated into quantitative values for the test specimen.

Photoelastic techniques may be applied to both two dimensional and three dimensional models. This method provides advantages such as expedient full-field pictures of stress distributions, measurements of peak stresses even in regions of high stress gradients, and rapid determination of the locations of maximum and minimum stresses. Under complex loading conditions, photoelasticity is valuable as a stress analysis tool, especially in the case where rigorous mathematical analyses to determine the stresses are not feasible.

Photoelasticity is a well documented form of evaluating stress fields both quantitatively and qualitatively. However, a direct photoelastic analysis could not be performed on the Kapton because it was not birefringent. The insistence on using Kapton as the material for the test membrane was based on the fact that it closely represented the LADD test article material characteristics. The next alternative was to use commercially available photoelastic coatings. Flexible birefringent materials could not be used because it

would fundamentally change the characteristics and class of the material being investigated. There are three remaining basic forms for photoelastic coatings [10]:

- a. Solid flat sheets;
- b. Liquids for casting contourable sheets; and
- c. Liquids for spray application.

The solid flat sheets are available in thicknesses ranging from 0.25 mm (0.010 in) to 3.05 mm (0.120 in) but they are brittle. The principle of operation for this form of photoelastic material is to bond the sheet to flat test surfaces and then proceed with the photoelastic analysis. This form was immediately discounted in the case of the present test membrane since it fails to simulate the membrane property of having little or no transverse rigidity.

The second and third forms of photoelastic material suffer from similar problems. The liquid for casting contourable sheets begins in a malleable form but before photoelastic readings can be taken, the material must cure to a brittle form in the shape of the test specimen.

The coating thickness of the spray form of material due to the nature of its application, cannot be controlled and thus the calibration of the material is impossible. This method is, therefore, not suitable for quantitative stress analysis but is rather used for proof testing and other qualitative studies.

It was thus determined that, although the photoelastic method offered many attractive potential features, such as full field determination, the physical requirements of the technique did not readily lend itself to the present study.

2.2 MOIRE METHODS

The Moiré technique is used to measure whole field in-plane and out-of-plane displacements [11]. Moiré methods can thus yield numerical data required to calculate the complete strain or stress fields.

A glass plate ruled with a set of equidistant opaque lines is used as a reference, or *master* screen. A set of equidistant parallel lines is deposited, etched or bonded to the test specimen. In the unstrained condition, the pitch (line-to-line distance) of the specimen and of the master ruling is the same. When subjected to forces, the test specimen is deformed. As a result of the surface strains, the pitch of the specimen ruling changes. In some regions the lines of the master and the specimen will coincide, allowing light to be transmitted, while in another region the lines of the master will face spaces of the specimen ruling, obstructing the passage of light and producing Moiré fringes. The complete strain field at all points on the surface can be determined from measurements of the in-plane displacements.

Photographic reproductions of the deformed specimen grid are recorded for each test condition. Each reproduction is then placed in a fringe multiplication and spacial filtering system to prepare the resulting fringe pattern photographs. These output fringe patterns yield information about the orthogonal displacements (u and v) of the specimen over the x and y coordinate field.

From the displacements over a two-dimensional field, the four displacement derivatives (du/dx , du/dy , dv/dx , dv/dy) can be extracted giving the entire strain field. The stress field is a simple transformation from there.

Complications arose with this method due to the nature of affixing the grid pattern to the test specimen. The requirement to determine the full strain field over a corner area of 45 cm x 45 cm (18 in x 18 in) would require an extensive network of bondable grids which are the most conventional method of depositing the grid pattern on the test surface. Because the grid patterns require a relatively stiff bonding agent in order to accurately transfer strains to the grid pattern, it was anticipated that the stiffening effect of the polyurethane adhesive would certainly alter the physical and mechanical properties as used in the finite element calculations. Since the application of the polyurethane adhesive would not result in a consistent amount of additional stiffening across the entire membrane, the option of measuring the newly formed physical properties and altering the mathematical models was not available.

2.2.1 MOIRE INTERFEROMETRY

An alternate approach to the Moiré method is Moiré interferometry. In the Moiré interferometry, a high-density grating is produced on the surface by a photo reproduction technique and the ruled glass master is replaced by a set of interference bands produced by the intersection of two coherent laser beams [12]. The two beams are caused to interact in space. This interaction generates fringe patterns in the volume of interaction. Normally this method is used in applications involving low strains or high strain gradients where high sensitivity is needed to resolve the strains with acceptable accuracy.

The bonded grating is illuminated by a pair of laser beams. The light diffracted in the direction of the observation generates the display of Moiré fringes, revealing the high

resolution, full-field displacement map. High-resolution film can record this interference pattern. This method suffers from the same constraints as the classical Moiré method in that the requirement to apply grid patterns to the test surface was prohibitive due to equipment cost and practical considerations related to the physical mounting and orientation of several grid pattern sections.

2.3 PIEZOFILM

Piezoelectricity is electric polarization produced by mechanical strain in certain crystals, the polarization being proportional to the amount of strain and changing sign with it. Piezofilm is a flexible, compliant, clear plastic film which can be readily cut and shaped. It is made from a long chain semi-crystalline polymer called polyvinylidene fluoride (PVDF) [13].

When an external force applied to the film results in compressive or tensile strain, the film develops a proportionate open circuit voltage. Exposure to a reciprocating force results in a corresponding alternating electrical signal. The frequency response ranges from 0.001 Hz to gigahertz. Piezofilm is a dynamic material that develops an electrical charge proportional to changes in mechanical stress or strain. The charge developed diminishes with time. Once the deformation stops, no further charge is transferred.

Piezofilm has many attractive characteristics such as being flexible, lightweight and durable. It can easily be fashioned into many forms for custom applications. The input frequency requirement however, precludes it from the essentially static requirements of this test.

2.4 NON CONTACT DISPLACEMENT FOLLOWERS

Non-contact displacement followers are optical systems that track motion of a discontinuity in the image formed by light reflected from or emitted by a moving object [14]. The discontinuity may be an actual edge of the object or a half-dark, half-light target attached to or painted on the object. The deflection current required to re-centre the image within the optical system is a measure of the target displacement as it moves.

This system, however, suffers the same drawback as the piezofilm; this is a dynamic system which requires a frequency input and thus does not conform to the static measurement requirements of this experiment.

2.5 BONDED ELECTRICAL RESISTANCE STRAIN GAGES

Electrical resistance strain gages use the principle that the resistance of a metal filament changes as a function of strain. The combination of the bonded foil strain gage with a Wheatstone bridge circuit has become an extremely popular method of accurately measuring surface strains. Its wide spread use has led to the development of a large body of information which makes strain gages a relatively easy system to employ [9,15]. There are, however, many features of strain gages and their use which must be fully understood in order to ensure accurate results.

One distinct advantage of the strain gage is that, by design, it measures surface strains. This means that the surface stresses of a test specimen are easily determined without extensive data manipulation. The difficulty of strain gages in this application include the fact that the gages are only capable of measuring the average strain over the

gage area at discrete points and accurate, full-field information must be interpolated. Also, since the present study involves the use of a very thin membrane material as the test surface, the possibility is great that the mounting of a measuring device such as a strain gage which is of a similar order of thickness may create significant local stiffening effects.

It was decided that since the use of electrical resistance strain gages is a widely accepted experimental technique for determining surface strains it would be explored in this study. The gage selection, experimental set-up and results are contained in Chapter Three of this thesis.

2.6 EXPERT VISION MOTION ANALYZER

The ExpertVision Motion Analyzer is a system of optical data collection hardware and a Unix based software application [16]. This specialized software provides the ability to acquire digitized video images of moving objects which are then analyzed frame-by-frame to calculate the variables of motion. The system falls into the category of non-contact displacement followers but is capable of following custom, user defined object markers. Depending on the set-up of the data collection hardware, the ExpertVision system is capable of working on just about any scale of magnitude.

This method offered the advantage of being a non-contact method, thus eliminating the possibility that the measurement system could interfere with the experiment. The ExpertVision system, however, measures displacements and not strains or stresses. This means that the resulting numerical data must be manipulated to determine the stress field which is the information of interest. Also, the system measures the

displacements at discrete points requiring the interpolation of the full strain or stress field. The combination of these two points means that a differentiation step must be performed on numerically approximate curves joining the data points which may lead to a magnification of the inherent experimental error contained in the data. Difficulties aside, this method proved to be the most suitable method for the experiments in this thesis and provided the most reliable results. The system requirements and results are contained in Chapter Four.

CHAPTER THREE

MEMBRANE TESTS USING BONDED ELECTRICAL RESISTANCE STRAIN GAGES

This chapter outlines the application of bonded electrical resistance strain gages to the test situation of interest in this thesis. The experimentation included several test items: a piece of membrane material in uniaxial tension (verification membrane), a cantilever beam with an applied point load at its end (calibration beam) and the square, non-uniformly loaded membrane itself.

3.1 STRAIN GAGE SELECTION

The initial and most critical step in preparing for any strain gage installation is the selection of the appropriate gage for the task. The installation and operating characteristics of a strain gage are affected by the following parameters, which may be selected to varying degrees [17]:

- a. Strain sensitive alloy;**
- b. Gage length and pattern;**
- c. Backing material; and**
- d. Self-temperature-compensation number.**

Basically, the gage selection process consists of determining the particular combination of parameters which are most compatible with the operating conditions and best satisfy the installation and operating constraints.

It must be appreciated that the process of gage selection involves compromises. This is because parameter choices which tend to satisfy one of the constraints or requirements may work against satisfying others. Developing an understanding of the effects of selectable parameters is necessary to make the best overall compromise for any particular set of circumstances, and to judge the effects of that compromise and the accuracy and validity of the test data.

3.1.1 STRAIN SENSING ALLOYS

The principal component which determines the operating characteristics of a strain gage is the strain sensitive alloy used in the foil grid. The alloy is not, however, in every case an independently selectable parameter. Manufacturers often design the alloy as part of a complete system, matching the alloy selection with the backing material and other construction features. The most popular alloy materials are as follows [18, pp. 2-3]:

- a. Constantan in self-temperature-compensated form;
- b. Annealed constantan;
- c. Isoelastic; and
- d. Nickel-chromium alloy.

Of all modern strain gage alloys, Constantan is the oldest, and still the most widely used. This situation reflects the fact that Constantan has the best overall combination of properties needed for many strain gage applications. This alloy has, for example, an adequately high strain sensitivity, or gage factor, which is relatively insensitive to strain level and temperature. Its resistivity is high enough to achieve suitable resistance values in

even small grids, and its temperature coefficient of resistance is not excessive. In addition, Constantan is characterized by good fatigue life and relatively high elongation capability. The strain gages used for this application were made from Constantan in self-temperature-compensated form.

3.1.2 GAGE LENGTH AND PATTERN

Gage length and gage pattern are also offered by many manufacturers in combinations which suit the most commonly encountered applications. The physical parameters of the standard strain gage are shown in Figure 3-1 [19]. The gage length is a critical characteristic of the gage and is usually one of the first selection criteria for the gage [17, p. 2]. The nature of the strain gages construction results in the averaging of the strain measurement over the gage's grid area. Figure 3-2 illustrates a representative strain distribution in the vicinity of a stress concentration, and demonstrates the error in peak strain indicated by the gage which is too long with respect to the zone of peak strain. The compromise becomes one of inaccuracy due to strain averaging versus degraded performance. This is because very small strain gages have limited maximum allowable elongations and instability under static strains. Larger gages, if they can be used, offer easier handling during installation and better heat dissipation due to a lower wattage per unit grid area.

The gage pattern refers to the number, orientation and shape of the gage elements as well as the grid resistance. Normally, single gages and two element rosettes are only used when certain information regarding the stress state is known such as the direction of

Figure 3-1: Nomenclature for physical strain gage parameters [19, p. 7]

Figure 3-2: Illustration of strain distribution averaging over grid length [18, p. 15]

the principal stresses. In the general case, when the stress state is unknown, a three element rosette is required. In the present study, the three element rectangular rosette was used to allow the most generalized determination of stresses.

The other aspect of the gage pattern is the grid resistance. The common resistances are 120 ohms and 350 ohms. The higher resistance gage is preferable if it is commercially available because, for a given applied voltage, the heat dissipation is reduced which in turn reduces temperature effects and gives more accurate results. The rosettes chosen for the present investigation had a resistance of 350 ohms.

3.1.3 BACKING MATERIAL

There are two basic types of material used in conventional foils strain gages as the backing for the photoetched metal foil pattern. These materials are polyimide and glass-fibre-reinforced epoxy-phenolic [18, p. 3]. The polyimide backing is a very durable material which is used in most general applications. It is resistant to mechanical damage during installation and is capable of large elongations in excess of 20%. The glass-fibre-reinforced epoxy-phenolic is used in applications involving temperature extremes from -269°C to +400°C. It is limited, however, to elongations in the neighbourhood of 1 to 2%. The Polyimide backing was sufficient for the subject experimentation.

3.1.4 SELF-TEMPERATURE-COMPENSATION NUMBER

Self-temperature-compensation is a controllable property of the alloys used in most electrical resistance strain gages. The self-temperature-compensation number is the

approximate thermal expansion coefficient in ppm/°F of the material on which the gage will display minimum thermal output over the temperature range -45° to +200°C [18, p. 2]. The Kapton polyimide film which was used as the test surface has a thermal expansion coefficient of 20 ppm/°C (11 ppm/°F) [2, p. 4]. The strain gages selected for this application had a self-temperature-compensation number of 13 which corresponds to 13 ppm/°F. The thermal output curve for the type and lot number of gages used is shown in Figure 3-3 [20]. The slight positive mismatch in the self-temperature-compensation numbers causes the thermal output curve to rotate slightly clockwise about the room temperature reference point. Since this testing was conducted near the room temperature reference point any effects of this slight offset are negligible.

3.2 DESCRIPTION OF THE STRAIN GAGES ROSETTES USED FOR PRESENT TESTING

Table 9 summarizes the properties of the strain gages which were used in the testing of the calibration beam, verification membrane and test membrane.

3.3 SOURCES AND REDUCTION OF ERRORS

Once the strain gages are selected the emphasis becomes ensuring that the experimental set-up reduces the noise and error caused by the environment and measurement equipment.

Figure 3-3: Thermal output curve for CEA-13-125UR-350 strain gages, lot R-A48AF124 [20]

Gage Type	CEA-13-125UR-350		
General Description	General purpose 45° single-plane rosette, cast polyimide backing and encapsulation, Constantan alloy in self-temperature-compensated form, exposed solder tabs (2.0 x 1.5 mm)		
Lot Number	R-A48AF124		
Resistance (at 24°C)	350.0 ± 0.4 % Ω		
Gage Length	3.18 mm (0.125 in)		
Gage Width	7.62 mm (0.300 in)		
Overall Pattern Length	7.62 mm (0.300 in)		
Overall Pattern Width	14.22 mm (0.560 in)		
Matrix Size	10.7 mm Long x 15.7 mm Wide (0.42 in Long x 0.62 in Wide)		
Self-temperature-compensation Number	13		
	Element 1	Element 2	Element 3
Gage Factor (at 24°C)	2.125 ± 0.5 %	2.155 ± 0.5 %	2.125 ± 0.5 %
Transverse Sensitivity (at 24°C)	(+0.5 ± 0.2) %	(+0.6 ± 0.2) %	(+0.5 ± 0.2) %

Table 9: Characteristics of strain gages used [19, 20]

3.3.1 NOISE CONTROL

Strain measurements must often be made in the presence of electric and magnetic fields which can superimpose electrical noise on the measurement signals. If not controlled, the noise can lead to inaccurate results and incorrect interpretation of the strain signals and, in severe cases, can obscure the strain signals altogether. In order to control the noise level, and maximize the signal-to-noise ratio, it is necessary first to understand

the types and characteristics of electrical noise, as well as the sources of such noise. With this understanding, it is then possible to apply the most effective noise-reduction measures to any particular instrumentation problem.

Virtually every electrical device which generates, consumes, or transmits power is a potential source for causing noise in strain gage circuits. And, in general, the higher the voltage or current level, and the closer the strain gage to the electrical device, the greater will be the induced noise.

Electrical noise from these sources can be categorized into two basic types: electrostatic and magnetic. The two types of noise are fundamentally different, and thus require different noise reduction measures [21]. Unfortunately, most of the common noise sources produce combinations of the two noise types, which can complicate the noise reduction problem.

Electrostatic fields are generated by the presence of voltage with or without current flow. Alternating electrical fields inject noise into strain gage systems through the phenomenon of captive coupling, by which charges of correspondingly alternating sign are developed on any electrical conductors subjected to the field.

Magnetic fields are ordinarily created either by the flow of electric current or by the presence of permanent magnetism. In order for noise voltage to be developed in a conductor, magnetic lines of flux must be cut by the conductor.

3.3.2 ELECTROSTATIC AND ELECTROMAGNETIC NOISE REDUCTION

The most effective forms of electrostatic shielding are braided or conductive foil shielding for the lead wires used in the circuit [21]. These shielding techniques, however, were not employed, primarily due to cost, but also because the testing environment was controllable and the lead wires were only approximately one metre in length. Efforts to reduce the potential effects of electrostatic noise were limited to reducing the number and proximity of likely noise sources in the work space.

The most effective method of reducing electromagnetic noise is to achieve noise cancellation at the indicator's amplifier by ensuring that the lead wires are equally exposed to any magnetically generated noise [21]. This is achieved by using three conductor twisted cable (in the case of a quarter bridge circuit) which ensures that the leads are of equal length and equally exposed to magnetic field gradients. The lead wire which was used in this operation was 30 gage, twisted cable, three-conductor, stranded, silver-plated, copper wire.

3.3.3 WHEATSTONE BRIDGE NON-LINEARITY

Commercial static strain indicators vary in the manner in which they employ the Wheatstone bridge circuit to measure the change in the strain gage's resistance. When strain measurements are made with an *unbalanced* Wheatstone bridge circuit there are certain conditions under which the output of the bridge circuit is a non-linear function of the resistance change producing that output [22].

The classical employment of the Wheatstone bridge circuit for the measuring of an unknown resistance involves the balancing of the bridge at the time of measuring by adjusting the resistances of the other arms. In the standard strain indicator, however, the bridge is *balanced* when the specimen is in the unloaded condition and is *unbalanced* at the time of measurement by the changing resistance of the strain gage.

The error due to non-linearity is normally quite small and only becomes significant at relatively large strains. In this application, a quarter bridge set-up was employed as shown in Figure 3-4.

The output equation for this circuit may be written as:

$$\frac{E_o}{E} = \frac{F \times \epsilon \times 10^{-3}}{4} \left[\frac{2}{2 + F \times \epsilon \times 10^{-6}} \right] \quad (2)$$

where:

E_o/E = Dimensionless bridge output, mV/V

E_o = Output voltage, mV

E = Bridge supply voltage, V = 2.00 Vdc $\pm$ 0.1 %

F = Gage factor of the strain gage = $2.125 \pm 0.5\%$ or $2.155 \pm 0.5\%$

ϵ = strain, $\mu\epsilon$ (microstrains)

In this equation the term in the parentheses represents the non-linearity. From this we can determine the incremental error which must be added to the measured strain to produce the actual strain corrected for non-linearity [22]. This incremental error is given by,

Figure 3-4: Quarter bridge circuit arrangement for strain indication

$$n = \frac{F (\epsilon')^2 \times 10^{-6}}{2 - F \times \epsilon' \times 10^{-6}} \quad (3)$$

here:

ϵ' = Indicated strain as given by the strain indicator, $\mu\epsilon$

n = Incremental error in the indicated strain.

As a means of demonstrating that the effects of errors due to non-linearity can safely be ignored in the present study one can perform the following sample calculation. During the experimentation on the test membrane, the maximum indicated strain was found to be 1854 $\mu\epsilon$ from a gage element with a gage factor of 2.125. The incremental error due to non-linearity can then be calculated as

$$\frac{2.125 \times (1854)^2 \times 10^{-6}}{2 - 2.125 \times 1854 \times 10^{-6}} = 3.66 \mu\epsilon \quad (4)$$

This represents less than a 0.2% error on the largest of the measured strains.

3.3.4 EXCITATION LEVELS

The voltage applied to the Wheatstone bridge circuit results in a power loss in each arm which is dissipated in the form of heat energy. This generation of heat in the strain gage means that the gage is operating at a higher temperature than the test surface. There is also a localized heat gradient on the test specimen in the area of the strain gage due to the conduction of heat through the bonding agent. This heat transfer is a function of the heat sink capacity of the test material and the strain gage power level. If the temperature

rise or differential is excessive, the gage begins to lose its self-temperature-compensation characteristics and the stability of the bridge circuit becomes difficult to maintain [23].

The strain indicator used in this experimentation was a Model P-3500 manufactured by Measurements Group Inc. which had a fixed bridge excitation voltage of $2.00\text{Vdc} \pm 0.1\%$. Other significant technical characteristics of the strain indicator are contained in Table 10. The manufacturer of the strain gages provided an optimum gage excitation chart which is reproduced in Figure 3-5. The gages used in this application were CEA-13-125UR-350 rosettes. To determine the consequences of utilizing this particular excitation level, first enter the chart on the horizontal axis at the 125UR arrow (grid area of approximately 5.0 mm^2) and determine the intersection of this line with the bridge excitation value of 2.00 volts from the left vertical axis. This intersection point falls between the 1.6 and 3.1 kW/m^2 sloped power density lines. The heat sink capacity of the Aluminum coated Kapton 500HN Polyimide Film can be considered to be fair to good [2, p. 4]. According to the guidelines provided by Measurements Group Inc., under these conditions, an excitation voltage of 2.00 V will yield moderate (3% to 5% error) to high accuracy (1% to 2% error) in the strain gage measurements [23].

3.3.5 ERRORS DUE TO TRANSVERSE SENSITIVITY

Transverse sensitivity in a strain gage refers to the response of the gage to strains which are perpendicular to the primary sensing axis of the gage [15, pp. 291-336]. Ideally,

**Figure 3-5: Optimum gage excitation chart for 350 Ω Measurements
Group Inc. strain gages [23]**

Resolution	1 $\mu\epsilon$
Accuracy	$\pm 0.05\%$ reading $\pm 3 \mu\epsilon$ for Gage Factor ≥ 1.000
Circuit Balance	Balance is accomplished by electronic voltage injection at the amplifier input. Balance controls do not load the bridge circuit or compromise the measurement range.
Gage Factor	Range 0.500 to 9.900 with resolution of 0.001, Accuracy of $\pm 0.02\%$ max for Gage Factor > 1.000
Bridge Excitation	2.00 Vdc $\pm 0.1\%$ with temperature stability of $\pm 0.02\%/^{\circ}\text{C}$
Calibration	Internal 350 ohm shunt resistor simulates 5000 $\mu\epsilon \pm 0.05\%$

Table 10: Model P-3500 strain indicator technical specifications [24]

it would be preferable if strain gages were completely insensitive to transverse strains but, in practice, most gages exhibit some degree of transverse sensitivity. The effect is controlled in the strain gage design process and is ordinarily quite small, and of the order of several percent of the axial sensitivity [15, p. 291].

In foil strain gages the transverse sensitivity arises from almost every aspect of grid design and gage construction. In addition to end loop effects in which a portion of the gage lies in the transverse direction, the foil grid lines, having a large ratio of width-to-thickness, are strained significantly by transverse strains. The magnitude of the transverse strain transmission into the grid lines is determined by the relative thickness and elastic moduli of the backing and foil, by the width-to-thickness ratio of the foil grid lines, and, to

a lesser degree, by several other parameters, including the presence or lack of a protective encapsulating layer over the grid. While the transverse sensitivity of a foil gage is subject to a certain degree of control in the design of the gage, the compromises necessary to optimize all aspects of gage performance generally limit the attainable reduction in transverse sensitivity.

Errors in strain indication due to transverse sensitivity are generally quite small since the transverse sensitivity itself is small. In strain fields characterized by extreme ratios between principal strains, however, the percentage error in the smaller strain can be great if not corrected for transverse sensitivity. It is important to note that when a strain gage is used under any conditions other than those employed in the gage-factor calibration, there is always some degree of error due to transverse sensitivity.

In general, a strain gage actually has two gage factors, F_a and F_t , which refer to the gage factors as determined in a uniaxial strain field (not uniaxial stress) with, respectively, the gage axes aligned parallel to and perpendicular to the strain field. For any strain field, the output of the strain gage can be expressed as:

$$\frac{\Delta R}{R} = F_a \epsilon_a + F_t \epsilon_t \quad (5)$$

where:

ΔR = change in the strain gage resistance due to loading

R = strain gage resistance

ϵ_a, ϵ_t = strains parallel to and perpendicular to the gage axis respectively

F_a = axial gage factor

F_t = transverse gage factor.

Or,

$$\frac{\Delta R}{R} = F(\epsilon_a + K_t \epsilon_t) \quad (6)$$

where:

K_t = transverse sensitivity coefficient, or just *transverse sensitivity*

When the gage is calibrated for gage factor in a uniaxial stress field on a material with Poisson's ratio, ν ,

$$\epsilon_t = -\nu \epsilon_a \quad (7)$$

Therefore,

$$\frac{\Delta R}{R} = F_a(1 - \nu K_t) \epsilon_a \quad (8)$$

The strain gage manufacturers commonly write this as:

$$\frac{\Delta R}{R} = F \epsilon \quad (9)$$

where: F = manufacturer's gage number

whereas, in reality:

$$F = F_a(1 - \nu K_t) \quad (10)$$

Furthermore, ϵ is actually ϵ_a , the strain along the gage axis (and only one of two strains sensed by the gage during calibration) when the gage is aligned with the maximum principal stress axis in a uniaxial stress (not uniaxial strain) field, on a material with $\nu =$

0.285. Errors and confusion occur through failure to fully comprehend and always account for the real meanings of F and e as used by the manufacturers. These values generally mask their inclusion of transverse sensitivity during the calibration process.

It is imperative to realize that for any strain field there is always an error in strain indication if the transverse sensitivity of the strain gage is other than zero. In some instances the error is small enough to be neglected while in others it may be necessary to adjust for the error.

3.3.5.1 CORRECTION FOR TRANSVERSE SENSITIVITY

In our case, the transverse sensitivities of the gage elements in the rosette are other than zero and the individual strain readings will need to be corrected for transverse sensitivity.

Correction for the effects of transverse sensitivity can be made either on the individual strain readings or on the principal strains or principal stresses calculated from these data. Numbering the gage elements consecutively, the correction equations for all three gages are as follows [15, pp. 298-299]:

$$\epsilon_1 = \frac{1 - \nu K_t}{1 - K_t^2} (\epsilon'_1 - K_t \epsilon'_3) \quad (11)$$

$$\epsilon_2 = \frac{1 - \nu K_t}{1 - K_t^2} [\epsilon'_2 - K_t (\epsilon'_1 + \epsilon'_3 - \epsilon'_2)] \quad (12)$$

$$e_3 = \frac{1 - \nu K_t}{1 - K_t^2} (\epsilon'_3 - K_t \epsilon'_1) \quad (13)$$

where:

$\epsilon'_1, \epsilon'_2, \epsilon'_3$ = indicated strains from the respective gage elements

e_1, e_2, e_3 = corrected strains along the gage axes

K_t = transverse sensitivity

It should be noted that the above equations are based upon the assumption that the transverse sensitivities of the gage elements are the same. This is not true for planar foil rosettes since the individual gage elements do not have the same orientation with respect to the direction in which the foil was rolled. It is common practice, however, for manufacturers to etch the rosettes in a position of symmetry about the foil rolling direction, and therefore the transverse sensitivities of the left (1) and right (3) gage elements will normally be the same, while that of the centre (2) element may differ. This is the case for the gages used in this application for which the transverse sensitivity for elements (1) and (3) is $(+0.5 \pm 0.2)\%$ and for element (2) is $(+0.6 \pm 0.2)\%$ [20]. The correction equations for this case are as follows:

Since the transverse sensitivities of the orthogonal gages (1) and (3) are the same, let

$$K_{11} = K_{33} = K_{13} \quad (14)$$

Then:

$$\epsilon_1 = \frac{1 - \nu K_{13}}{1 - K_{13}^2} (\epsilon'_1 - K_{13} \epsilon'_3) \quad (15)$$

$$\epsilon_2 = \frac{(1 - \nu K_{12})(1 + K_{13})\epsilon'_2 - K_{12}(1 - \nu K_{13})(\epsilon'_1 + \epsilon'_3)}{(1 + K_{13})(1 - K_{12})} \quad (16)$$

$$\epsilon_3 = \frac{1 - \nu K_{13}}{1 - K_{13}^2} (\epsilon'_3 - K_{13} \epsilon'_1) \quad (17)$$

where:

$\epsilon'_1, \epsilon'_2, \epsilon'_3$ = indicated strains from gages (1), (2), and (3), uncorrected
 K_{11}, K_{12}, K_{13} = transverse sensitivities of gages (1), (2), and (3)
 K_{113} = transverse sensitivity of orthogonal gages (1) and (3)
 $\epsilon_1, \epsilon_2, \epsilon_3$ = actual strains along gage axes (1), (2), and (3)

3.4 STRAIN GAGE DATA REDUCTION METHOD

In order to specify the state of stress on a free surface of a test specimen it is necessary to know three independent quantities. These quantities are the magnitudes of the two principal stresses, σ_1 and σ_2 , and a direction, θ or $\theta + 90^\circ$, with respect to some reference.

For isotropic elastic materials these values can be calculated from strains measured on the surface at the point in question, and since three independent quantities are to be determined, in general, it is necessary to make three independent measurements of strain. It should be reiterated that the physical measurement of strain gives an average over a

small distance rather than the condition at a point. Therefore the results of strain rosettes will give the average over some small distance. This method is valid provided the strain gradient is not too great. This error can be minimized by the proper selection of strain gages.

In general, the principal experiment at hand is an example of plane stress. The determination of plane stress at a point in any direction relative to a chosen coordinate system is well developed [25] and will be used here to outline the data reduction techniques for strain rosettes.

In the case of simple tension or compression, one knows that the directions of the principal stress axes will be parallel and perpendicular to the direction of the applied force and that the magnitude of the principal stress whose direction is at right angles to the load will be zero. This means that two of the three quantities are known from the prevailing physical conditions. In this situation, it is therefore only necessary to make a single observation of the strain along the direction of the load in order to determine the one remaining unknown quantity. For an isotropic, homogeneous, elastic body, the stress is related to strain according to

$$\sigma = E \epsilon \quad (18)$$

where:

σ = the stress intensity

E = the modulus of elasticity of the material

ϵ = the measured strain (positive for tension and negative for compression)

In the general case, neither the magnitudes of the principal stresses nor the

directions of their axes are known. This means that for a complete description of the state of stress, at any particular point, three independent quantities must be found. Therefore, it will be necessary to make three measurements of linear strain in different directions to compute the two principal stress magnitudes and their directions.

Figure 3-6 illustrates a pair of orthogonal reference axes, OX and OY, and three other axes, OA, OB and OC, making angles θ_a , θ_b and θ_c , respectively, with respect to the reference axis OX. The axes OA, OB, and OC represent the element directions of the rosette, and if corresponding linear strains, e_a , e_b , and e_c , are measured in their respective directions, the strains, e_x , e_y , and γ_{xy} , corresponding to the OX and OY axes of reference, can be calculated [15, pp. 253-288].

The values e_x , e_y , and γ_{xy} are calculated in terms of the measured strains, e_a , e_b , and e_c using the following:

$$e_x = \frac{e_a + e_b}{2} + \frac{e_a - e_b}{2} \cos 2\theta_a + \frac{\gamma_{xy}}{2} \sin 2\theta_a \quad (19)$$

$$e_b = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2} \cos 2\theta_b + \frac{\gamma_{xy}}{2} \sin 2\theta_b \quad (20)$$

$$e_c = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2} \cos 2\theta_c + \frac{\gamma_{xy}}{2} \sin 2\theta_c \quad (21)$$

This gives three independent equations that can be solved simultaneously for e_x , e_y , and γ_{xy} . When e_x , e_y , and γ_{xy} have been determined the principal strains, e_1 and e_2 , may be found by using

Figure 3-6: Orientation of rosette element directions with reference axe.

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad (22)$$

where,

$\epsilon_{1,2}$ = maximum and minimum principal strains respectively

The orientation of the principal axes may be obtained from Mohr's circle for strain, or by using,

$$\tan 2\theta = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \quad (23)$$

with either

$$\sin 2\theta = \frac{\gamma_{xy}}{\sqrt{(\epsilon_x - \epsilon_y)^2 + (\gamma_{xy})^2}} \quad (24)$$

or

$$\cos 2\theta = \frac{\epsilon_x - \epsilon_y}{\sqrt{(\epsilon_x - \epsilon_y)^2 + (\gamma_{xy})^2}} \quad (25)$$

The relative directions of strain measurement, angles θ_a , θ_b , and θ_c , can take on any value but certain orientations permit a much simpler reduction of the strains into terms of stress. There are two basic arrangements, the rectangular and the delta rosette orientations. The rectangular rosette orientation was used for this application in which the

three gage axes are laid out at 45° and 90° to each other. The next section outlines the data reduction techniques for the rectangular rosette.

3.4.1 DATA REDUCTION FOR THE RECTANGULAR ROSETTE

This analysis is for the three-element rectangular rosette, and is started by taking the OA axis of the rosette in Figure 3-6 as the reference axis and making it coincident with the OX axis. The respective directions of the measured linear strains are then given by:

$$\theta_a = 0^\circ, \quad \theta_b = 45^\circ, \quad \theta_c = 90^\circ$$

These values can be substituted into equations (19)-(21) to give the three simultaneous equations necessary to determine e_x , e_y , and γ_{xy} from which it becomes apparent that,

$$e_x = e_a \tag{26}$$

and

$$e_y = e_c \tag{27}$$

The shearing term can be determined in terms of the strain measurements by substituting the values of $e_x = e_a$ and $e_y = e_c$ into equation (20). Solving for γ_{xy} , this gives,

$$\gamma_{xy} = 2e_b - (e_a + e_c) \tag{28}$$

One can then use the values of e_x , e_y , and γ_{xy} to determine the principal strains or they can be calculated directly from the measured strain readings using,

$$\epsilon_{1,2} = \frac{\epsilon_a + \epsilon_c}{2} \pm \frac{1}{\sqrt{2}} \sqrt{(\epsilon_a - \epsilon_b)^2 + (\epsilon_b - \epsilon_c)^2} \quad (29)$$

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\epsilon_b - \epsilon_a - \epsilon_c}{\epsilon_a - \epsilon_c} \right) \quad (30)$$

where:

- $\epsilon_{1,2}$ = the maximum and minimum principal strains respectively, and
 θ = the acute angle from the reference grid of the rosette to the principal axis.
 Its direction is counterclockwise if positive and clockwise if negative.

Since $\tan 2\theta = \tan 2(\theta + 90^\circ)$, the angle calculated above can refer to either of the principal axes. For the unambiguous determination of the principal angle the following rules, which are derived from the Mohr's circles for each case, must be applied to the calculated result [15, pp. 261-264]:

- a. If $\epsilon_a > \epsilon_c$ then θ represents the acute angle from the reference grid of the rosette to the algebraically maximum principal axis;
- b. If $\epsilon_a < \epsilon_c$ then θ represents the acute angle from the reference grid of the rosette to the algebraically minimum principal axis;
- c. If $\epsilon_a = \epsilon_c$ and $\epsilon_b < \epsilon_a$ then θ represents the acute angle from the reference grid of the rosette to the algebraically maximum principal axis and its magnitude is -45° ;
- d. If $\epsilon_a = \epsilon_c$ and $\epsilon_b > \epsilon_a$ then θ represents the acute angle from the reference grid of the rosette to the algebraically maximum principal axis and its magnitude is $+45^\circ$; and
- e. If $\epsilon_a = \epsilon_b = \epsilon_c$ then θ is indeterminate and a state of biaxial strain exists.

Since the principal strains are seldom the values of interest, the principal stresses can also be calculated directly from the measured strains. The angles of the principal stresses are the same as for the principal strains and their magnitudes in terms of the measured strains are given by:

$$\sigma_{1,2} = \frac{E}{2} \left[\frac{\epsilon_a + \epsilon_c}{1-\nu} \pm \frac{\sqrt{2}}{1+\nu} \sqrt{(\epsilon_a - \epsilon_b)^2 + (\epsilon_b - \epsilon_c)^2} \right] \quad (31)$$

3.5 STRAIN GAGE RESULTS

3.5.1 VERIFICATION TESTS

3.5.1.1 MEMBRANE IN UNIAXIAL TENSION

In order to verify the use of the strain rosettes in this application, the method was first tried in the well defined case of a rectangular section of membrane in uniaxial tension. Figure 3-7 shows the location of the strain rosette on the verification membrane. A photograph of the actual test set-up is given in Figure 3-8. The bar shown in Figure 3-8 was clamped to the lower edge of the membrane and secured with 11 machine screws. A similar method was used to secure the membrane to the support structure. At the time, it was deemed that such an arrangement would ensure that the applied load would result in a uniform uniaxial tension across the width of the test specimen. Subsequent results proved this to not be exactly true (see p. 99). However, proceeding with the assumption of a uniform uniaxial tension, the expected values for the principal strains can be determined from the theories of elasticity. This is a simple uniaxial normal stress distribution giving:

Figure 3-7: Strain gage rosette verification membrane set-up

Figure 3-8: A photograph of test frame for strain gage verification membrane

$$\sigma_y = \frac{LOAD}{AREA} = \frac{183.25 \text{ N}}{(220 \text{ mm})(0.125 \text{ mm})} = 6.664 \text{ Mpa (or 966.5 psi)} \quad (32)$$

$$\sigma_x = 0, \quad \sigma_z = 0 \quad (33)$$

The expected principal strains can be calculated in terms of the orientation outlined in Figure 3-7 as follows,

$$\epsilon_1 = \frac{\sigma_y}{E} = \frac{6.664 \text{ MPa}}{2500 \text{ MPa}} = 2665 \text{ } \mu\epsilon \quad (34)$$

$$\epsilon_2 = -\nu \epsilon_1 = -(2665 \text{ } \mu\epsilon)(0.34) = -906 \text{ } \mu\epsilon \quad (35)$$

$$\theta = 45^\circ \quad (36)$$

where,

E = Modulus of Elasticity for Kapton 500 HN Polyimide Film = 2500 MPa
 ν = Poisson's Ratio for Kapton 500 HN Polyimide Film = 0.34

The actual results of the strain element readings for the verification membrane were

$$\epsilon_a = 363 \text{ } \mu\epsilon, \quad \epsilon_b = 1088 \text{ } \mu\epsilon, \quad \epsilon_c = 388 \text{ } \mu\epsilon \quad (37)$$

The next step is to correct for transverse sensitivity. Using the above readings and the data provided by the manufacturer for the strain rosette, the corrected elemental strains may be calculated using equations (15-17). The results are,

$$\epsilon_a = 339 \text{ } \mu\epsilon, \quad \epsilon_b = 1089 \text{ } \mu\epsilon, \quad \epsilon_c = 364 \text{ } \mu\epsilon \quad (38)$$

Then, using the corrected elemental strains, equations (22-25) may be employed to calculate the principal strains and the direction of their axes. These turn out to be

$$\epsilon_1 = 1087 \mu\epsilon, \quad \epsilon_2 = -386 \mu\epsilon, \quad \theta = 44.50^\circ \quad (39)$$

It can be seen that these results fall far short of those calculated from the theories of elasticity. The direction of the principal axes are correct within a reasonable degree of error but the magnitudes are approximately 40% of those predicted. It was hypothesized at this point that the bonding of the strain rosette to the membrane material added a significant amount of local stiffening and, in effect, increased the local modulus of elasticity in such a manner that the magnitude of the strain field was altered but not its orientation.

In order to verify this notion, the strain gage mounting procedure, circuit design and electronics had to be validated. This was done with the mounting of two strain rosettes on a cantilever beam to which a point end load was applied. Since the beam is much thicker than the membrane material and its modulus of elasticity much greater, the potential for the strain gage application to significantly affect the strain readings due to local stiffening should be eliminated. The next step was to verify that the difficulties encountered were not particular to this one single mounting of a strain rosette. It was decided that a series of 15 rosettes would be mounted to the test membrane and subjected to non-uniform loading. This would provide significantly more data from which the effects of mounting the gages on the membrane material could be ascertained with greater confidence. The following two sections outline the results of these additional tests.

3.5.1.2 CANTILEVER BEAM VERIFICATION TEST

As stated above, the purpose of this verification test was to ensure that the procedures employed during the mounting of the strain gage and the circuit set-up and instrumentation were valid. The set-up for the test is shown in Figure 3-9 and a photograph of the instrumented beam is shown in Figure 3-10.

The beam that was used was a piece of T6061 aluminum of the dimensions shown in Figure 3-9. It was clamped to a rigid, right-angled surface with three C-clamps, the footprints of which are outlined in the plan view. The classical solution to this problem is well understood. The expected strains for a given location and loading condition are given by the following

$$\epsilon_z = \frac{-Fyz}{EI} \quad (40)$$

$$\epsilon_y = \frac{\nu Pyz}{EI} \quad (41)$$

$$\gamma_{xy} = \frac{-P}{2IG} (c^2 - z^2) \quad (42)$$

where:

E = Modulus of Elasticity = 68.9 GPa (10×10^6 psi)

P = Load, N

ν = Poisson's Ratio

I = Moment of Inertia = $bh^3/12$, mm^4

G = $E/2(1+\nu)$, GPa

c = one half of the beam thickness

b = width of the beam

h = beam thickness

Figure 3-9: Cantilever beam verification test set-up

Figure 3-10: A photograph of the instrumented cantilever beam

The strains of interest in this experiment are ϵ_x and ϵ_y which are being measured with the strain rosettes at two locations. These locations were $y_1 = 130$ mm, $z_1 = -1.6$ mm, and, $y_2 = 220$ mm, $z_2 = -1.6$ mm. The beam was also subjected to three loading conditions, 0.75 kg, 1.41 kg, and 2.13 kg. The expected and actual results are listed in Tables 11 and 12. The actual results were calculated in the same manner as that outlined in the section on data reduction for rectangular strain rosettes. It can be seen that the actual results are within an acceptable tolerance of the expected strain values. The conclusion which may then be drawn from this is that the processes of mounting the strain gages and gathering the data were valid.

3.5.2 TEST MEMBRANE RESULTS

The next test was to place 15 strain rosettes on the square test membrane and subject the membrane to the non-uniform loading condition of interest in this application. The locations of the strain rosettes on the test membrane are identified in Figure 3-11 and a photograph of the test stand for the non-uniform loading of the rectangular test membrane is shown in Figure 3-12. In order to control the expense of strain gaging the membrane, the gages were placed in a triangular pattern covering one eighth of the membrane surface. Due to the symmetry of the expected strain and stress field this was the minimum coverage required to allow for the interpretation of the results. The data was

	Strain Gage Location No. 1 $y_1 = 130 \text{ mm (5.118 in)}$ $z_1 = -1.6 \text{ mm (-0.0625 in)}$		Strain Gage Location No. 2 $y_2 = 220 \text{ mm (8.661 in)}$ $z_2 = -1.6 \text{ mm (-0.0625 in)}$	
	Expected Strain, $\mu\epsilon$	Actual Strain, $\mu\epsilon$	Expected Strain, $\mu\epsilon$	Actual Strain, $\mu\epsilon$
Load Condition 1 $P = 0.75 \text{ kg (1.65 lb)}$	66	68	112	113
Load Condition 2 $P = 1.41 \text{ kg (3.10 lb)}$	124	126	210	212
Load Condition 3 $P = 2.13 \text{ kg (4.70 lb)}$	188	191	318	320

Table 11: ϵ_y values for the cantilever beam verification tests

	Strain Gage Location No. 1 $y_1 = 130 \text{ mm (5.118 in)}$ $z_1 = -1.6 \text{ mm (-0.0625 in)}$		Strain Gage Location No. 2 $y_2 = 220 \text{ mm (8.661 in)}$ $z_2 = -1.6 \text{ mm (-0.0625 in)}$	
	Expected Strain, $\mu\epsilon$	Actual Strain, $\mu\epsilon$	Expected Strain, $\mu\epsilon$	Actual Strain, $\mu\epsilon$
Load Condition 1 $P = 0.75 \text{ kg (1.65 lb)}$	-22	-21	-38	-39
Load Condition 2 $P = 1.41 \text{ kg (3.10 lb)}$	-42	-48	-71	-76
Load Condition 3 $P = 2.13 \text{ kg (4.70 lb)}$	-64	-67	-108	-110

Table 12: ϵ_x values for the cantilever beam verification tests

reduced in the same manner as for the verification membrane. The results are contained in Figures 3-13 to 3-17 where the orientations of the stresses are depicted graphically on the left (not to scale) and the corresponding numerical results are tabulated on the right.

3.6 CONCLUSION

From Figures 3-13 to 3-17, it is obvious that the stress values computed by finite element analysis are much higher than the corresponding measured stress values using strain gages. These results are similar in nature to those found for the verification membrane. By comparing the measured results with the finite element results it is found that the range of percentage errors for σ_1 is 58.8% to 76.4%, for σ_2 it is 34.1% to 331.4% but in the case of θ it is only 0.4% to 31.4%. It may be concluded that since the mounting and data reduction procedures have been shown to be sound, the strain gages are producing a local stiffening effect which is significantly affecting the magnitude of the results but not necessarily the indicated orientation of the strain field. The significance of these invasive effects upon the experimental results precludes the use of conventional strain gaging methods in membrane applications such as this one. Thus, due to the physical characteristics of the test membrane material, a more appropriate measurement system must be employed. It was, therefore, considered appropriate to carry out the membrane test using the ExpertVision.

Figure 3-11: Locations of the strain rosettes on the square test membrane

Figure 3-12: A photograph of the test frame used for loading the square test membrane

Gage No.1		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	17.955	4.719
	σ_2	0.399	-0.051
Angle to Major Principal Stress (Degrees)	θ	90	91.49

Gage No.2		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	7.346	2.148
	σ_2	0.140	-0.324
Angle to Major Principal Stress (Degrees)	θ	112.19	107.96

Gage No.3		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	4.560	1.197
	σ_2	0.091	-0.115
Angle to Major Principal Stress (Degrees)	θ	120.66	128.19

Legend: $\longleftrightarrow$ Finite Element Results $\longleftrightarrow$ Strain Gage Results

Figure 3-13: Comparison of the orientation of the strain gage results with the finite element results for gage locations 1, 2 and 3

Gage No. 4		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	3.420	1.147
	σ_2	0.117	-0.144
Angle to Major Principal Stress (Degrees)	θ	126.97	129.06

Gage No. 5		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	3.019	1.244
	σ_2	0.132	0.087
Angle to Major Principal Stress (Degrees)	θ	133.36	136.17

Gage No. 6		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	6.484	2.010
	σ_2	0.123	0.003
Angle to Major Principal Stress (Degrees)	θ	90	88.85

Legend: $\longleftrightarrow$ Finite Element Results $\longleftrightarrow$ Strain Gage Results

Figure 3-14: Comparison of the orientation of the strain gage results with the finite element results for gage locations 4, 5 and 6

Gage No. 7		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	4.656	1.428
	σ_2	0.395	0.074
Angle to Major Principal Stress (Degrees)	θ	104.75	104.37

Gage No. 8		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	3.642	1.022
	σ_2	0.565	0.063
Angle to Major Principal Stress (Degrees)	θ	117.57	119.69

Gage No. 9		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	3.248	1.204
	σ_2	0.636	0.208
Angle to Major Principal Stress (Degrees)	θ	131.40	138.98

Legend: $\longleftrightarrow$ Finite Element Results $\longleftrightarrow$ Strain Gage Results

Figure 3-15: Comparison of the orientation of the strain gage results with the finite element results for gage locations 7, 8 and 9

Gage No. 10		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	3.920	0.925
	σ_2	0.831	0.128
Angle to Major Principal Stress (Degrees)	θ	90	87.49

Gage No. 11		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	3.224	0.953
	σ_2	1.166	0.496
Angle to Major Principal Stress (Degrees)	θ	106.89	122.78

Gage No. 12		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	2.902	1.028
	σ_2	1.315	0.398
Angle to Major Principal Stress (Degrees)	θ	128.91	148.80

Legend: $\longleftrightarrow$ Finite Element Results $\longleftrightarrow$ Strain Gage Results

Figure 3-16: Comparison of the orientation of the strain gage results with the finite element results for gage locations 10, 11 and 12

Gage No. 13		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	2.709	0.709
	σ_2	1.650	0.616
Angle to Major Principal Stress (Degrees)	θ	90	79.24

Gage No. 14		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	2.438	0.942
	σ_2	1.875	0.516
Angle to Major Principal Stress (Degrees)	θ	123.67	84.87

Gage No. 15		Finite Element Results	Strain Gage Results
Principal Stresses (MPa)	σ_1	2.184	0.722
	σ_2	2.139	0.429
Angle to Major Principal Stress (Degrees)	θ	90	84.56

Legend: $\longleftrightarrow$ Finite Element Results $\longleftrightarrow$ Strain Gage Results

Figure 3-17: Comparison of the orientation of the strain gage results with the finite element results for gage locations 13, 14 and 15

2 of/de 2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

CHAPTER FOUR

MEMBRANE TESTS USING EXPERTVISION MOTION ANALYZER

ExpertVision™ is an integrated computer-video system which falls into the category of non-contact displacement followers [16]. This system is one of only a handful of such systems operating in Canada and is owned and operated by the Photographic Support Section of the Land Engineering Test Establishment, Department of National Defence, Ottawa.

The ExpertVision system hardware, shown in Figure 4-1, consists of:

- a. A closed circuit television system consisting of a video camera, recorder, and monitor;**
- b. The VP320 Video Processor which is a state-of-the-art video processor that can accept synchronized simultaneous video input from up to four video sources at either 60, 180 or 200 frames-per-second (f/s). The data can be digitized and stored for all four cameras simultaneously. These data are time-synchronized and time-matched and thus suitable for use in both 2-D and 3-D motion analysis. The unit has filter and mask controls which allow the manipulation of contrast settings and field discrimination respectively; and**
- c. A Sun Unix computer/graphics workstation.**

Figure 4-1: A photograph of the ExpertVision Motion Analyzer hardware

The ExpertVision software runs as an application program under the Unix operating system. This specialized ExpertVision software provides the ability to acquire digitized video images of moving objects processed by the video processor. The software then enables automatic frame-by-frame analysis of the resulting image sequences. Under the ExpertVision system, variables of motion are computed, statistically analyzed, and displayed.

The general sequence of events is as follows:

- a. Objects in motion are recorded with a video camera;
- b. Video data are directly input into the video processor or are recorded on video tape for later processing;
- c. The video processor converts video images to digital outlines. The processing of the video image reduces the amount of data that has to be analyzed;
- d. Using the video processor, both the raw video and the digitized images are simultaneously monitored. The video image is monitored to obtain proper illumination and image clarity while the digitized image is monitored so that proper threshold and filter settings can be obtained; and
- e. The digitized image is then captured by the computer and processed through a series of software operations to gather the information required for motion analysis.

The ExpertVision Motion Analyzer is a generalized tool for the analysis of motion in many different disciplines. It was originally developed primarily for scientific

investigations of simultaneous translational movements of multiple subjects in two dimensions. For simple translation, the object is represented by its centroid. Rotation and flexion information may be gathered from the analysis of the centroid motions as they vary with respect to each other.

The task at hand uses only the most basic of the available ExpertVision operations for data collection, manipulation and analysis. In a two dimensional translational problem only the most cursory motion processors will be required. The chief benefit to the use of this system is the ability, with appropriate lighting and magnification, to accurately measure displacements while not contacting the test surface. By interfacing the camera with optical equipment from microscopes to telescopes one can cover the full range of possible displacements.

4.1 EXPERTVISION IMAGE PROCESSING

The final step in the process outlined above requires the application of several software modules on the digitized video data. The processing sequence can be traced by the data files that are generated. The following sections outline the steps required for this test [16].

4.1.1 VIDEO FILE

The first, or most basic, of the file formats is a video file. Video files contain a series of frames ordered in time in which each data record represents a single video frame analogous to a single frame in a motion picture. Each frame consists of a variable number

of pixels extracted by the VP320 Video Processor during the execution of the input of video data.

The field of view within each video frame is represented as a rectangular grid 256 pixels wide and 240 pixels high, with the origin in the extreme upper left hand corner of the field. In this coordinate system the values of the vertical component increase as one moves down the image. This system was used because the coordinate values are based on the digitized video signal scan lines from the video processor. Each standard video frame is scanned from left to right and from top to bottom.

4.1.2 CENTROID FILES

The first stage in the processing of the raw video data captured in the video file is the creation of a centroid file. Centroid files contain centroid data which, like the video data from which they are computed, consist of a series of frames ordered in time. Each data record represents a single video frame. The analysis of translational movement requires that objects represented by sets of pixels within each frame be tracked through space-time. The centroid operator performs the process of segmentation whereby sets of pixels within each frame are recognized as belonging to the same object marker for tracking.

In the application at hand we are primarily concerned with translational movement which does not require detailed knowledge of the objects structure. The translational movement can then be simply defined as the displacement of the object's centroid from its position before loading to its final position after loading. The x -coordinate of a centroid is

the average of the x-coordinates of the pixels comprising the object's outline. Similarly, a centroid's y-coordinate is the average of the y-coordinates of the pixels comprising the object's outline.

The centroid operator calls for the definition of a neighbourhood size by which accumulations of pixels are judged to be elements of a single object or not. Within each frame, the centroid operator centres the neighbourhood over an arbitrary pixel and collects all pixels within the area enclosed by the neighbourhood as part of a single object. The operator, in turn, centres the neighbourhood over each of the collected pixels to accumulate all pixels within the projections of the neighbourhood. The operator repeats this process until it can find no other pixels in the frame within a neighbourhood size of the pixels it has accumulated. If the number of pixels accumulated is greater or equal to the minimum required for a valid centroid, it calculates the x-coordinate average and the y-coordinate average, and stores the x and y average as the coordinates of the centroid. Then if the frame still contains pixels which have not been used as centres to position the neighbourhood, the process is started again with one of the remaining pixels. When all pixels within a frame have been utilized to position the neighbourhood, the operator moves on to the next frame. The neighbourhood size used in the present study was 10 pixels.

Centroids are represented using the same system of coordinates used to localize pixels in video data. However, since the coordinates of a centroid are generally estimated from multiple pixel coordinates, centroids may be determined with slightly greater precision.

4.1.3 OTHER FILES

One other extremely important ExpertVision operator is the mask function. This is an interactive editing tool which allows the video or centroid files to be cleansed of unwanted background noise which could be interpreted by the software as object markers. As mentioned earlier, the ExpertVision system also offers many sophisticated options and tools for the analysis of objects in motion. The ExpertVision system allows further manipulation of the data after the creation of the centroid file with files such as path and trajectory, time series, statistical parameter, and amplitude and phase spectra files.

4.1.4 IMAGE GUIDELINES

This section discusses aspects of the video imagery needed to produce a digitized picture for the ExpertVision system. Generally speaking, the image necessary to produce a digitized picture for computer analysis is not the same image that is appealing to the eye. Therefore, the standard lighting, camera angle, and other factors normally used for video imagery often needs to be changed to produce the best image for the ExpertVision system. Each application, regardless of complexity, has its own particular set of requirements relating to environmental conditions, illumination levels and picture resolution.

4.1.5 MARKERS

Successful ExpertVision analysis rests on the development of a proper video image and the selection of an appropriate data sampling frequency. A proper image is one that has high contrast meaning the subject to be tracked is either the lightest or darkest element

in the image field. The sampling frequency must be high enough to capture the desired event characteristics with confidence.

In this application, since the aim was to measure displacements in a manner which minimized interference with the membrane, cooperative passive markers were used as opposed to active light sources. The surface of the Aluminium coated Kapton was highly reflective which dictated that the markers had to be dark in order to generate the required contrast. The markers which were used were the "." portions of the "i" characters from a 10 point Univers 56 Letraset™ Instant Lettering set. The markers were easily applied with a small amount of pressure. They measured approximately 0.2 mm square.

4.1.6 ILLUMINATION

The illumination guideline is that the subject must be well isolated from the surroundings in terms of the gray scale. Very light or very dark objects of interest are the goal. It was found through trial and error, that the best lighting set-up was a light ring surrounding the camera lens. Other lighting sources had a tendency to create shadows which digitized and created a confusing environment for the computer to distinguish object markers from background noise.

4.2 RESULTS

4.2.1 VERIFICATION TEST MEMBRANE

In order to verify the ExpertVision system for use in this application, the method was first used on a well defined case. The verification experiment involved a section of

membrane 20 cm by 40 cm in uniaxial tension. Figure 4-2 shows the set up of the experiment. The same test frame as for the strain gage verification membrane was used in this experiment.

From the theories of elasticity, the expected results can be calculated as follows. Assuming that the applied load is equally distributed at the end of the test specimen, the applied stress is given by:

$$\sigma_y = \frac{LOAD}{AREA} = \frac{183.25 N}{(20 cm) (0.0125 cm)} = 7.33 MPa \quad (43)$$

$$\sigma_x = 0, \quad \sigma_z = 0 \quad (44)$$

Since the ExpertVision system is a non-contact displacement follower it would be preferable to reduce the expected stress results to displacements rather than process the experimental data which would magnify any imbedded experimental error. Using Hooke's law and the above results, the strain field in the test membrane may be determined from the following equations

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y), \quad \epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) \quad (45)$$

where:

E = Modulus of Elasticity = 2500 MPa

ν = Poisson's Ratio = 0.34

Figure 4-2: ExpertVision verification membrane set-up

The resulting strains are

$$\epsilon_x = -9.97 \times 10^{-4} , \quad \epsilon_y = 2.93 \times 10^{-3} \quad (46)$$

The last step integrates the results above to arrive at the predicted displacements.

$$\epsilon_x = \frac{\partial u}{\partial x} , \quad \epsilon_y = \frac{\partial v}{\partial y} \quad (47)$$

Therefore

$$u(x) = (-9.97 \times 10^{-4})(x) + C_1 , \quad v(y) = (2.93 \times 10^{-3})(y) + C_2 \quad (48)$$

where:

C_1, C_2 = Constants of integration

x, y = Coordinates of point of interest on the unloaded membrane, metres

$u(x)$ = Displacement of point of interest in the x-direction, metres

$v(y)$ = Displacement of point of interest in the y-direction, metres

The markers used for this verification experiment were the same as those mentioned in section 4.1.5. They were approximately 0.2 mm square and were located as outlined in Figure 4-2. In view of Saint Venant's principle [25, pp. 39-40], the marker locations were placed away from the applied and reaction loads.

Based on the set-up, marker location and axis orientation as outlined above the expected displacements are given in Tables 13 and 14.

The first step in the application of the ExpertVision Motion Analyzer was to capture the motion of the membrane from the unloaded to the loaded condition on video and to download the data to the computer via the video processor. The threshold and contrast controls were adjusted during this process to produce the best digitized image of the event. The display of the entire video file for dot number 7 (Figure 4-2) of the

0.0798	0.0399	0	-0.0399	-0.0798
0.0798	0.0399	0	-0.0399	-0.0798
0.0798	0.0399	0	-0.0399	-0.0798
0.0798	0.0399	0	-0.0399	-0.0798
0.0798	0.0399	0	-0.0399	-0.0798

Table 13: Expected $u(x)$ displacements, mm

-0.3518	-0.3518	-0.3518	-0.3518	-0.3518
-0.4691	-0.4691	-0.4691	-0.4691	-0.4691
-0.5864	-0.5864	-0.5864	-0.5864	-0.5864
-0.7037	-0.7037	-0.7037	-0.7037	-0.7037
-0.821	-0.821	-0.821	-0.821	-0.821

Table 14: Expected $v(y)$ displacements, mm

verification membrane is shown in Figure 4-3. It can be seen that due to the highly reflective surface and the sensitivity of the ExpertVision hardware, noise is introduced into the digitized image despite efforts to limit it through manipulation of the threshold control during the raw data processing stage.

The next step is to eliminate the obvious background noise from the digitized video image through the use of the *mask* operator. Figure 4-4 displays the ExpertVision screen and 50 frames of the mask file for dot number 7 of the verification membrane. Once the background noise had been removed from the masked image, the data was sent to the *centroid* operator which calculated the centroid locations of the marker image for each frame in the masked video file. Then, returning to the mask operator which allows the easy movement between frames and display of pixel coordinates, the pre-load and post-load marker centroid coordinates were determined. The displacement created by the loading of the membrane was then the algebraic difference between these two sets of pixel coordinates.

The conversion between pixel displacements and millimetres was accomplished by placing a machinist's ruler into the video image and then measuring the pixel count for the known centreline to centreline distance on the ruler scale. This process was carried out for each dot. On average, the conversion turned out to be 52 pixels/mm which gave a precision of approximately 0.01 mm on the centroid calculations. In order to increase the ExpertVision's sensitivity, one would have to either magnify the image or move the camera closer to the event. In either case, the larger displacements experienced would have exceeded the field of view.

Figure 4-3: ExpertVision display of video file for dot number 7 of the verification membrane

Figure 4-4: ExpertVision mask screen showing 50 frames of mask file for dot number 7 of the verification membrane

The actual measured result, expressed in millimetres (pixels), are shown in Tables

15 and 16.

0.07 (3.5)	0.02 (1.0)	0 (0)	-0.04 (-2.0)	-0.07 (-3.5)
0.07 (3.5)	0.04 (2.0)	0.01 (0.5)	-0.03 (-1.5)	-0.07 (-3.5)
0.10 (5.0)	0.05 (2.5)	0 (0)	-0.03 (-1.5)	-0.09 (-4.5)
0.07 (3.5)	0.05 (2.5)	0.01 (0.5)	-0.05 (-2.5)	-0.09 (-4.5)
0.06 (3.0)	0.03 (1.5)	0 (0)	-0.03 (-1.5)	-0.08 (-4.0)

Table 15: Actual $u(x)$ displacements in mm (pixels)

-0.32 (-16.0)	-0.36 (18.0)	-0.40 (20.0)	-0.40 (20.0)	-0.37 (18.5)
-0.46 (-23.0)	-0.46 (23.0)	-0.52 (26.0)	-0.47 (23.5)	-0.49 (24.5)
-0.59 (29.5)	-0.58 (29.0)	-0.59 (29.5)	-0.61 (30.5)	-0.59 (29.5)
-0.70 (35.0)	-0.68 (34.0)	-0.70 (35.0)	-0.73 (36.5)	-0.74 (37.0)
-0.84 (42.0)	-0.83 (41.5)	-0.81 (40.5)	-0.81 (40.5)	-0.89 (44.5)

Table 16: Actual $v(y)$ displacements in mm (pixels)

By comparing Table 13 with Table 15, and Table 14 with Table 16 respectively, it can be readily seen that the ExpertVision results are much more in line with those predicted by the established theories of elasticity than were the strain gage results. As an illustration, one may consider the results for the bottom row of dots (dots 21-25, Figure 4-2) for which the deviations from the expected values of $v(y)$ are most severe. By comparing the ExpertVision results with the expected results at these points, one finds that the percentage errors for $u(x)$ range from 0% to 24.8% and 1.1% to 8.4% for $v(y)$. These deviations may be indicative of not only experimental error, but also indicate that the uniaxial tension across the width of the test membrane was not exactly uniform. However, compared to the strain gage results, the advantage and promise of the ExpertVision approach as a non-contact method of testing become quite obvious. The next logical step was to conduct an ExpertVision analysis on the non-uniformly loaded test membrane.

4.2.2 TEST MEMBRANE RESULTS

The same data collection and processing methods as for the verification membrane were used on the square test membrane under non-uniform loading conditions. The markers were placed in a nine by nine grid as shown in Figure 4-5. Again, due to the symmetry of the problem, the top left quarter of the test membrane represents the area of interest. The reference axes are located at the centre of the membrane and all displacements were adjusted to be with respect to this point. This allows for the direct comparison of these results with those of the finite element method. The marker locations

correspond to the centres of the elements used in the finite element model from which the expected stress values are obtained and purposely avoid the boundary areas. A photograph of the test frame used to load the membrane is shown in Figure 4-6. Weights are diagonally applied to the top corners and reaction forces are created at the bottom corners.

Figures 4-7 to 4-11 then show graphical comparisons between the stress results predicted by the finite element method and those found using the ExpertVision Motion Analyzer over the area defined by the 81 marker locations. The five components of stress that are shown are the stress component in the x direction, the stress component in the y direction, the shear stress, the major principal stress and the minor principal stress.

Since the ExpertVision system measures displacements at discrete locations it is necessary to perform the differentiation and transformation prescribed by the theories of elasticity on the data to generate stress results. This could be achieved by determining a high order polynomial curve or surface which passes through the data points derived by the ExpertVision system and then performing the necessary mathematics on this curve or surface. It was decided, however, to take advantage of an option offered by the Intergraph Finite Element Modeller [8]. This software application allows the user to specify displacements on nodes as boundary conditions for the model. The motivation for using this method of deriving the stress values was to eliminate the possibility of deviations between the expected and measured results due to differing processes of mathematical manipulation of displacement data.

Figure 4-5: Marker locations for square test membrane

Figure 4-6: A photograph of the test frame for the square membrane

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend:

Figure 4-7: Graphical comparison of finite element and ExpertVision results for the stress component in the x direction (σ_x)

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend:

-2 - 0 Mpa	4 - 6 Mpa
0 - 2 Mpa	6 - 8 Mpa
2 - 4 Mpa	8 - 10 Mpa

Figure 4-8: Graphical comparison of finite element and ExpertVision results for the stress component in the y direction (σ_y)

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend:

Figure 4-9: Graphical comparison of finite element and ExpertVision results for the shear stress component (τ_{xy})

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend:

Figure 4-10: Graphical comparison of finite element and ExpertVision results for the major principal stress (σ_1)

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend:

Figure 4-11: Graphical comparison of finite element and ExpertVision results for the minor principal stress (σ_2)

The validity of employing this software feature as a means of calculating stress values from discrete displacement data first had to be verified. The boundary conditions in the finite element model used to generate the expected values were limited to two pinned locations, to prevent translational movement of the model, and the corner loads required to generate the in-plane, non-uniform loading condition which is of interest. As a normal by-product of the finite element method one may generate displacement field data for the model although the stress field is usually the condition of interest. It was important to ensure that the finite element software's mathematical manipulation of displacement data to yield a stress field was consistent regardless of whether the displacements were imposed on the nodes or calculated as a result of applied forces on the membrane. The original calculated displacement field (obtained as a result of the applied forces) was imposed on the nodes and the corner forces were removed. The application software was run again for this new model and the stress results generated with imposed nodal displacements were identical to those obtained in the beginning with applied forces. It can be concluded from this result that, regardless of whether the model has force boundary conditions or imposed nodal displacements, the algorithm for the calculation of stress components is the same. Thus, running the Intergraph finite element software with the experimental data imposed on the model's nodal locations would eliminate errors due to varying methods of calculating the components of stress.

4.3 DYNAMICS OF MAGNIFICATION OF EXPERIMENTAL ERROR

It can be seen in Figures 4-7 to 4-11 that, although the general pattern of the measured stress fields are similar to those expected, there are still discrepancies. These discrepancies can be explained by examining the magnification of experimental error caused by the process of differentiating experimental data. It will be shown that this is the source of most of the apparent error depicted in these figures.

As an example of the dynamics of the magnification of experimental error through differentiation, take the simple polynomial equation,

$$y(x) = x^2 + x^3 \quad (49)$$

over the range of $-5 \leq x \leq 5$.

Since in the present study data points were taken at discrete points, one may approximate the exact solution at several regularly spaced points with arbitrarily chosen small deviations added to their value to represent experimental error. Table 17 shows the exact and the *experimental* data used in this example.

Using these *experimental* data, one can approximate the curve of best fit using a high order polynomial equation of the form,

$$Y(x) = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 + Hx^7 \quad (50)$$

The results of this regression calculation are given in Table 18 and the results are plotted against the exact solution in Figure 4-12.

x	Exact Solution $y(x)$	<i>Experimental</i> Solution $Y(x)$
-5	-100	-98
-4	-48	-46.5
-3	-18	-19.2
-2	-4	-6.3
-1	0	1.2
0	0	-0.6
1	2	3.8
2	12	11.4
3	36	38
4	80	78.9
5	150	152

Table 17: Exact and *experimental* data used to demonstrate error magnification

Coefficients	Value
A	0.52760
B	-0.19122
C	0.71642
D	1.30400
E	0.02434
F	-0.03304
G	-0.00043
H	0.00085

Table 18: Values of regression variables

$y(x)$ = Exact curve

$Y(x)$ = Seventh order polynomial curve fit between the arbitrarily chosen
experimental data points

Figure 4-12: Exact and *experimental* data curves

$y'(x)$ = Exact curve after differentiation with respect to x

$Y'(x)$ = Differentiated polynomial curve representing the arbitrarily chosen experimental data

Figure 4-13: Magnification of experimental error following differentiation

The next step is to differentiate the two curves which gives,

$$y'(x) = 2x + 3x^2 \quad (51)$$

$$Y'(x) = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5 + 7Hx^6 \quad (52)$$

The results of these two equations, (51) and (52) are plotted in Figure 4-13. It can be readily seen that the requirement to use high order terms to accurately curve fit experimental data causes peaks and deviations to occur after that curve undergoes differentiation. This is the phenomenon which is at play in Figures 4-7 to 4-11. To avoid this complication it was decided to compare the measured ExpertVision data to the finite element nodal displacement data. The results are contained in the following section.

4.4 DISPLACEMENT RESULTS

By design, the finite element method derives its strain and stress results from nodal displacements. It is true that the stress field within a structure is of ultimate importance to the design engineer. However, if the finite element method can be experimentally verified at the displacement level then confidence can be drawn from the strain and stress values that the finite element analysis calculates on the basis of the well formulated theories of elasticity. As it has been shown in this thesis that if the test structure's strains cannot be examined directly by a suitable measurement technique then the magnification of the inherent experimental error by the differentiation process leads one to the errant conclusion that the model fails to accurately predict how the test structure will react to loading. Comparing the displacement predictions of the mathematical model to the experimental displacement measurements is therefore more valid and indeed preferable for

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend:

Figure 4-14: Graphical comparison of finite element and ExpertVision results for nodal displacements in the x direction

Results Predicted by Finite Element Analysis

Results Obtained from ExpertVision Motion Analyzer

Legend

Figure 4-15: Graphical comparison of finite element and ExpertVision results for nodal displacements in the y direction

0.93/0.922 0.9%	0.71/0.683 4.0%	0.53/0.522 1.5%	0.38/0.401 5.2%	0.28/0.307 8.8%	0.25/0.23 8.7%	0.16/0.164 2.4%	0.10/0.106 5.7%	0.08/0.052 53.8%
0.59/0.578 2.1%	0.61/0.596 2.3%	0.46/0.499 7.8%	0.45/0.405 11.1%	0.30/0.320 6.3%	0.27/0.245 10.2%	0.17/0.177 4.0%	0.09/0.115 21.7%	0.07/0.057 22.8%
0.41/0.428 4.2%	0.41/0.438 6.4%	0.44/0.419 5.0%	0.37/0.363 1.9%	0.27/0.300 10.0%	0.27/0.236 14.4%	0.16/0.174 8.0%	0.08/0.115 30.4%	0.08/0.057 40.4%
0.29/0.305 4.9%	0.34/0.328 3.7%	0.35/0.325 7.7%	0.32/0.302 6.0%	0.27/0.260 3.8%	0.20/0.212 5.7%	0.17/0.160 6.3%	0.11/0.107 2.8%	0.05/0.053 5.7%
0.24/0.219 9.6%	0.25/0.239 4.6%	0.22/0.247 10.9%	0.24/0.238 0.8%	0.21/0.215 2.3%	0.19/0.180 5.6%	0.10/0.139 28.1%	0.10/0.095 5.3%	0.04/0.048 16.7%
0.13/0.153 15.0%	0.18/0.173 4.0%	0.20/0.184 8.7%	0.16/0.184 13.0%	0.18/0.171 5.3%	0.17/0.148 14.9%	0.11/0.117 6.0%	0.09/0.081 11.1%	0.04/0.041 2.4%
0.12/0.105 14.3%	0.13/0.123 5.7%	0.14/0.136 2.9%	0.17/0.141 20.6%	0.12/0.135 11.1%	0.13/0.120 11.0%	0.12/0.097 23.7%	0.08/0.068 17.6%	0.05/0.035 42.9%
0.06/0.073 17.8%	0.08/0.090 11.1%	0.11/0.103 6.8%	0.13/0.110 18.2%	0.14/0.109 28.4%	0.10/0.099 1.0%	0.07/0.081 13.6%	0.06/0.058 3.4%	0.05/0.030 66.7%
0.08/0.054 48.1%	0.06/0.070 14.3%	0.10/0.083 20.5%	0.09/0.092 2.2%	0.07/0.092 23.9%	0.10/0.086 16.3%	0.06/0.071 15.5%	0.05/0.051 2.0%	0.03/0.027 11.1%

Expert Vision Results/Finite Element Results
Percentage Error

Table 19: Comparison between marker displacements (in mm) in the x direction as measured by the ExpertVision system and the finite element results with percentage errors

0.91/0.922 1.3%	0.57/0.578 1.4%	0.41/0.428 4.2%	0.31/0.305 1.6%	0.21/0.219 4.1%	0.15/0.153 2.0%	0.10/0.105 4.8%	0.05/0.073 31.5%	0.05/0.053 5.7%
0.68/0.683 0.4%	0.56/0.596 6.0%	0.43/0.438 1.8%	0.34/0.328 3.7%	0.26/0.239 8.8%	0.19/0.173 9.8%	0.10/0.123 18.7%	0.09/0.09 0.0%	0.08/0.070 14.3%
0.53/0.522 1.5%	0.51/0.499 2.2%	0.45/0.419 7.4%	0.34/0.325 4.6%	0.24/0.247 2.8%	0.19/0.184 3.3%	0.11/0.136 19.1%	0.12/0.103 16.5%	0.06/0.083 27.7%
0.38/0.401 5.2%	0.43/0.405 6.2%	0.37/0.363 1.9%	0.30/0.302 0.7%	0.25/0.238 5.0%	0.16/0.184 13.0%	0.16/0.141 13.5%	0.11/0.110 0.0%	0.12/0.092 30.4%
0.30/0.307 2.3%	0.28/0.32 12.5%	0.31/0.300 3.3%	0.28/0.261 7.3%	0.23/0.215 7.0%	0.21/0.171 22.8%	0.16/0.135 18.5%	0.12/0.109 10.1%	0.13/0.092 41.3%
0.22/0.23 4.3%	0.27/0.245 10.2%	0.24/0.236 1.7%	0.17/0.212 19.8%	0.17/0.180 5.6%	0.12/0.148 18.9%	0.06/0.120 50.0%	0.09/0.099 9.1%	0.10/0.086 16.3%
0.16/0.164 2.4%	0.18/0.177 1.7%	0.20/0.174 14.9%	0.17/0.160 6.3%	0.12/0.139 13.7%	0.09/0.117 23.1%	0.13/0.097 34.0%	0.07/0.081 13.6%	0.05/0.071 29.6%
0.10/0.106 5.7%	0.09/0.115 21.7%	0.10/0.115 13.0%	0.09/0.107 15.9%	0.09/0.095 5.3%	0.07/0.081 13.6%	0.09/0.068 32.4%	0.06/0.058 34.5%	0.08/0.051 56.9%
0.05/0.052 3.8%	0.05/0.057 12.3%	0.05/0.057 12.3%	0.09/0.053 69.8%	0.04/0.048 16.7%	0.03/0.041 26.8%	0.05/0.035 42.9%	0.05/0.030 66.7%	0.07/0.027 159.3%

ExpertVision Results/Finite Element Results Percentage Error

Table 20: Comparison between marker displacements (in mm) in the y direction as measured by the ExpertVision system and the finite element results with percentage errors

the verification of the model's usefulness.

Shown in Figures 4-14 and 4-15 are the graphical comparisons between the finite element nodal displacements and the displacements measured using the ExpertVision Motion Analyzer. Tables 19 and 20 then show the numerical values of the displacements of the ExpertVision system. It can be readily seen that comparisons on this level are much closer than those done after the mathematical manipulation of the experimental data to yield stresses. The average percentage error for displacements in the x-direction was 12.1% and 13.9% for displacements in the y-direction.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

This thesis has investigated the stress distributions in non-uniformly tensioned membrane structures with the aim of verifying experimentally the analytical models of stress field determination such as the finite element method or the superposition method. Following an examination of the available techniques for the determination of the stress and strain fields, it was determined that the most applicable methods were bonded electrical resistance strain gages and the ExpertVision Motion Analyzer. The strain gaging technique was chosen because it represented the classical and most commonly used method for the determination of surface strains. The ExpertVision system was later chosen after it was determined that the mounting of the strain gages on the membrane material was not a viable option.

The results of the strain gage testing led to the conclusion that the strain gages were creating a local stiffening effect which significantly affected the magnitude of the indicated strain but not necessarily the orientation of the principal axes. It can be concluded that the classical method of strain determination through the use of strain gages is not practical in membrane applications such as this.

The ExpertVision Motion Analyzer, which is a non-contact optical system, allowed for the very accurate determination of displacements. The stress distribution, however, was the parameter of interest due to its practical use by engineers and it was

found that in the mathematical manipulation of the experimental data to derive the stress field results exaggerated experimental error creating deviations from the expected results. When the displacements (the parameter which is in fact the more fundamental measure) measured with the ExpertVision system were compared to the displacement matrix formed by the finite element method the results were found to be in agreement. The validity of the mathematically approximate finite element method has thus been demonstrated in terms of displacements. As a result, since the stress and strain results derived from the displacements calculated by the finite element method are arrived at through the use of well established theories of elasticity, this method is an adequate means of determining stress data in cases where analytical solutions are not available.

5.2 RECOMMENDATIONS

It is recommended that, due to the local stiffening effect caused by the mounting of strain gages, alternate strain or displacement measurement techniques be used when determining stress fields in membrane structures. It was found that optical systems that determine displacements while not contacting the surface of the test structure offer accurate results. If, however, such a system is being used to verify analytical models, then it is further recommended that comparisons be made in the more fundamental measurement parameter which in this case was displacement.

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END

3 1 - 0 5 - 9 5

FIN

the average of the x-coordinates of the pixels comprising the object's outline. Similarly, a centroid's y-coordinate is the average of the y-coordinates of the pixels comprising the object's outline.

The centroid operator calls for the definition of a neighbourhood size by which accumulations of pixels are judged to be elements of a single object or not. Within each frame, the centroid operator centres the neighbourhood over an arbitrary pixel and collects all pixels within the area enclosed by the neighbourhood as part of a single object. The operator, in turn, centres the neighbourhood over each of the collected pixels to accumulate all pixels within the projections of the neighbourhood. The operator repeats this process until it can find no other pixels in the frame within a neighbourhood size of the pixels it has accumulated. If the number of pixels accumulated is greater or equal to the minimum required for a valid centroid, it calculates the x-coordinate average and the y-coordinate average, and stores the x and y average as the coordinates of the centroid. Then if the frame still contains pixels which have not been used as centres to position the neighbourhood, the process is started again with one of the remaining pixels. When all pixels within a frame have been utilized to position the neighbourhood, the operator moves on to the next frame. The neighbourhood size used in the present study was 10 pixels.

Centroids are represented using the same system of coordinates used to localize pixels in video data. However, since the coordinates of a centroid are generally estimated from multiple pixel coordinates, centroids may be determined with slightly greater precision.