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**A DYNAMIC ECONOMIC MODEL OF GLOBAL WARMING:
FOSSIL FUEL DEPLETION AND CO₂ ACCUMULATION**

by

ANGELA KELLER-HERZOG

**A thesis submitted to
the faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of**

Master of Arts

Department of Economics

**Carleton University
Ottawa, Ontario**

October 6, 1993

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Fossil Fuel Depletion and Co₂ Accumulation"**

submitted by Angela Keller-Herzog, B.A.

**in partial fulfilment of the requirement for
the degree of Master of Arts**

Thesis Supervisor

Chair, Department of Economics

**Carleton University
24 November 1993**

ABSTRACT

A general model of global warming is developed and solved using dynamic optimization techniques. Building on a three factor general equilibrium model, socially optimal and market outcomes are contrasted. Fossil fuels are modelled as an essential, exhaustible resource; the extraction and use of fossil fuels in production in turn contributes to global warming. Global warming is modelled as a cumulative, negative externality on production, thus underlining the interdependence between environment and economy. It is demonstrated that the combination of the dynamics of resource extraction and CO_2 accumulation dictate that optimal tax-based market intervention implies 'carbon taxes' falling over time -- rising tax rates will accelerate global warming. Further it is shown that for global warming severe enough, the market equilibrium solution -- under perfect foresight -- will result in a finite horizon solution to an optimally infinite horizon problem.

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The mind that invents a delusion of power over Nature in order to feel safe is afraid of fear itself. And the more this mind learns to rely on delusion, the less tolerance this mind has for any betrayal of that delusion. For we must remember that this mind has denied that it itself is a thing of Nature. It has begun to identify not only its own survival, but its own existence with culture. The mind believes that it exists because what it thinks is true. Therefore to contradict delusion is to threaten the mind's very existence. And the ideas, words, numbers, concepts have become more real to this mind than material reality.

Thus when this mind is threatened by a material danger, it does not respond rationally. For this mind has lost touch with material reality. It is a mind possessed by madness, by a hallucinated idea of its own power.

Susan Griffin

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List of Latin Symbols

A	arbitrary constant of integration
a	assets
B	total below-ground fossil fuel reserves (stock)
b	per capita fossil fuel reserves (stock)
b₀	base level of per capita fossil fuel reserves at time zero
C	total consumption
c	per capita consumption
D	inverse global warming damage function
d	derivative
E	total extraction of fossil fuels (flow)
e	natural number
F	production function with technology requiring capital and fossil fuel inputs
f	production function, function of capital/fossil fuel ratio
G	production function with technology requiring capital and substitute energy inputs
g	production function, function of capital/substitute energy ratio
H	Hamiltonian function
I	total investment (flow)
i	per capita investment (flow)
j	as in the jth firm
K	total capital (stock)
k	per capita capital (stock)
k₀	base level of per capita capital stock at time zero
$\mathcal{L}$	Lagrangian function
M	total number of firms
m	fixed cost of the substitute energy
N	size of the population (fixed)
n	as in the nth individual
p	price
q	current valued costate variable on capital (technology G, NGW)
q	price of fossil fuel [this will be changed to z, to avoid confusion]
R	rent, rental price of capital
r	market rate of interest
s	substitute energy (flow)
S	Supply
T	the time that the switch from F to G technology is made (variable)
t	time
t*	the time that the discount rate equals the productivity of capital (variable)
t₁	the time that the price of fossil fuel equals the price of substitute energy (variable)
U	utility function
V	present value of profits
W	welfare function
Y	total output
y	per capita output

List of Greek Symbols

β	current valued costate variable on reserves (technology F, GW)
δ	current valued costate variable on reserves (technology F, NGW)
Δ	a small change in
∂	partial derivative
ϵ	per capita extraction of fossil fuels (flow)
η	elasticity of substitution between capital and substitute energy
θ	discount rate
κ	capital/substitute energy ratio
λ	current valued costate variable on capital (technology F, NGW)
λ_0	current valued costate variable on assets
λ_1	current valued costate variable on capital
λ_2	current valued costate variable on reserves
λ_3	λ_2/q
μ_1	present valued costate variable on capital
μ_2	present valued costate variable on reserves
π	profits
σ	elasticity of marginal utility
τ	tax
ϕ	capital/fossil fuel ratio
Ψ	anthropogenic CO ₂ atmospheric concentration (stock)
Ψ_0	base level of CO ₂ concentration at time zero

List of Abbreviations

btu	British thermal unit
CFC	Chloroflourocarbons
CO₂	Carbon dioxide
CRS	Constant returns to scale
CVH	Current valued Hamiltonian
FOC	First order condition
GHG	Greenhouse gas
GW	Global warming
IPCC	Intergovernmental Panel on Climate Change
LHS	Left hand side
MC	Marginal Cost
NGW	No global warming
OECD	Organization for Economic Cooperation and Development
OPEC	Organization of Petroleum Exporting Countries
P	Private
PVH	Present valued Hamiltonian
S	Social
ss	steady state
UNCED	United Nations Conference on Environment and Development

1. Introduction

Environment and Economy

Environmental economics has recently found a new prominence due to rising public concern and consciousness of environmental issues. The question arises whether there is an incipient new sub-discipline called 'environmental economics', or whether the environmental economics agenda is a re-packaging of university calendars to meet the demand of the day. Classic economic treatment of scarcity, natural resource economics, micro-economic treatments of common resources, pollution externalities and regulation, as well as parts of the public choice literature all bring forward their respective contributions to environmental issues. This scholarship is well established.

Historically economists and environmentalists have had much to discuss but for want of a common language precious little public or academic communication has taken place. One suspects that underlying politics are to blame in part. While the environmental movement has a diverse constituency, it has commonly put wholesale blame for environmental degradation on the economic system (i.e. capitalism and consumerism) or at least on unfettered industry, corporate culture and power. Environmentalists have thus argued for ethical, technological and political solutions. Economists, generally working in the realm of given consumer preferences and an existing market system, found little common ground. And marxist or neo-marxist economists, while challenging the efficacy of the market system, made no alternative efforts towards a new analysis of man-nature relations of production and co-existence.

The lack of interdisciplinary cooperation has therefore been essentially methodological. With the rise of the environment onto the mainstream agenda and the global trend towards market based systems, a new pragmatism is evident in the economist-environmentalist dialogue. Environmentalists are seeking solutions within the market system and economists are looking for alternative perspectives broad enough to encompass the environment.

The basic notion underlying this approach is the idea of systemic interaction: the economy affects the environment and the environment affects the economy. While the now classic economic contributions remain valid, they are in a sense micro-economic partial equilibrium exercises. What is currently developing is a 'macro-environmental' economics approach that encompasses a broader general equilibrium as well as addressing questions of dynamics, growth and, perhaps most importantly, stability.

Such a systemic approach provides a natural common language for economists and environmentalists. The modelling of economic systems and ecological systems shares much common mathematical and conceptual ground. This then is also the overall context of this thesis. I have attempted to develop a simple model of global warming -- showing the dynamic two-way interaction between the economy and the environment.

Economic Approaches: Scarcity and Neo-Classical Treatments

The concept of scarcity has a time-honoured place in the discipline of economics. In particular, the Malthusian notion of an absolute scarcity or a finite physical constraint to growth has appeared and reappeared.

As is well known, Malthus was concerned with nature's fixed supply of land, a growing population and diminishing returns, and predicted a gloomy future. In the same vein, some time thereafter, Jevons announced that, "the momentous repeal of the Corn Laws throws us from corn upon coal," and projected, "should the consumption multiply for rather more than a century at the same rate, the average depth of our coal-mines would be 4,000 feet, and the average price of coal much higher than the highest price now paid for the finest kinds of coal...we cannot long continue our rate of progress."¹

In the wake of the OPEC-induced oil crises, we are told, "Energy turns out to be the essential economic problem of this era. It will determine not only the performance of the industrial economies, it will determine the survivability of the political institutions in these Western democracies."²

Neoclassical economics, since the time of Marshall, has had little time for these predictions of the sure decline of our affluence. Students are introduced to Malthus under

¹ Jevons (1909). p.195 and p.11.

² Schlesinger (1990). p.21.

the banner, "The False Prophecies of Malthus"³, and are told that his views were oversimplifications that did not anticipate the miracles of the industrial revolution and technological innovations leading to rapid outward shifts of production possibility frontiers.

In general, the free economic system is held to contain the seeds of its own survival; increasing relative scarcity will result in price appreciation. This will prevent over-exploitation by 'curbing' demand and lead to substitution away from relatively scarce goods, and in itself stimulate the process of innovation. The focus of neoclassical economics has therefore been on allocative efficiency in the context of the relative scarcity of all economic goods. Scarcity is not just seen as a constraint to growth but as economic reality. Depletion of natural exhaustible resources is then treated as a relative scarcity problem -- the problem being to find the optimal depletion profile. Pollution, on the other hand, is not treated as a natural resource phenomenon, rather it is approached under the general heading of market failure and externality.

Substitution and technical progress did provide solutions to the constraints of land, coal and oil supply, and in general the neoclassical mechanics have served well. It does not follow however, that technology will always be a panacea. Logically, diminishing returns should also be attributed to the factor technology -- not only in the short run but especially in the long run. Further, there are likely to be different externalities associated

³ Samuelson (1980), p.28.

with new technology -- technology is rarely a 'free lunch'. These issues are generally not considered in the literature.

Environmental Approaches: Conservationism and Ecology

While monetary values are commonly calculated for the value of a human life, and measures are devised for the amenity value of parklands or conservation areas, the idea that nature has an *inherent* value as a whole is not tractable to the economist. The conservationists' concept of value is more holistic. Not only are social and cultural values attached to 'nature', but humankind is seen as part of an inseparable nature-Man continuum. The dynamic equilibrium of ecological balance of the globe as a whole and its interlinked eco-systems in their diversity and complexity, provide the environment for all human and non-human activity, therefore cannot be measured in static categories like 'the value of agricultural land'.

Leaving aside considerations of rights (as in humankind's right to exploit and dominate nature and extinguish species) and the anthropocentrism inherent to economic methodology, the issue of an overall ecological equilibrium remains. Logically again, nature's absorptive capacity of disequilibrating shocks cannot be infinite. Desertification, soil erosion, salt water intrusion into over-exploited aquifers are all instances of re-equilibration to a lower, less diverse level of biological activity -- unable in some cases

to support human life.⁴

A new absolute scarcity constraint

The impact of economic activity on the activity of the ecosystem is thus complex. One *macro* approach to the economy-environment nexus is the materials balance approach. A convenient conceptual simplification is made to make the problem more tractable: in this methodology, the economic process extracts matter and energy, and releases wastes and byproducts. Both these activities exert stress on the environment and necessitate adjustment by the supporting ecosystem(s). What distinguishes even the simple macro approach from micro and partial exercises is that the possibility of a conceptual break between the process of resource depletion and pollution is not permitted.

Further, a new and more general resource-scarcity problem is made explicit. Increasing environmental degradation will threaten ecological stability and sustainability of the status quo equilibrium level of biological activity. Put in more static terms, increasing use of 'the environment', causes 'the environment' to become increasingly scarce and one is faced with another absolute scarcity constraint.

This paper will model both types of resource scarcity constraints discussed above. Using

⁴ Some alternative approaches have been developed since the 1960s to address these issues. Among them are the materials-energy balance approach referencing itself to the first law of thermodynamics, the throughput concept and discussions of environmental entropy, modelled on the second law of thermodynamics. All of these are more dynamic (and thus less tractable) treatments of environment-economy interaction than the recently much-discussed national resource accounting frameworks.

a conventional model of non-monetary general equilibrium and the technique of dynamic optimization (popular with both growth and natural resource economists), I examine the depletion of an exhaustible resource both with and without a technological innovation-substitute and a general environmental constraint arising out of this activity. Optimal and competitive outcomes are contrasted.

Global Warming

While the above discussion provides the general context of the model, the model itself addresses the 'case' of global warming. Economic modelling habitually engages in largescale abstraction and simplification in order to address qualitative relationships of underlying economic factors. Thus all capital is aggregated, all labour is made homogeneous, the economy is closed, etc., despite the availability of micro-data. Interestingly, climate modellers and in particular global warming experts, also engage in 'rough sketch' modelling -- due to the overwhelming complexity of micro-data -- and similarly seek to address the qualitative relationships of climatic factors⁵. A general functional form model is therefore well suited to model the overall environment-economy interaction of global warming.

In comparing market and planned (or optimal) outcomes in the dynamic general equilibrium model under the global warming cumulative externality, consistent and

⁵ It is interesting to note that the first attempts at mathematical climate modelling were made under the leadership of John von Neumann at Princeton in the 1950s. Just as economics has developed complex CGEs, whose predictions are open to parametric disputes, climatology has developed GCMs (global circulation models) that are similarly uncertain in their prediction and open to challenge.

potentially highly significant market failure is demonstrated. In deriving the optimal corrective policy, it is shown that this requires complete knowledge of future states, and that hence application of a 'carbon tax', or price-based, corrective instrument is likely to have excessive information requirements. Arguments are presented in favour of quantity-based corrective policy instruments (unpopularly known as 'command and control'). Further is shown that in order for a 'carbon tax' to achieve its policy objective, of critical import is the rate of change of taxation over time. Only a declining tax rate will meet the objective of internalizing the costs of global warming; a rising tax rate -- which might have been expected to match the rising costs of global warming over time -- is shown to be directly counter-productive, resulting in greater global warming.

Structure of the Paper

Chapter 2 contextualizes the model by providing background information on global warming as currently understood by climatologists for those readers less familiar with the topic. Beginning with the most simple possible dynamic modelling framework, Chapters 3 to 7 then gradually build an environment-economy model that captures both resource scarcity and an associated negative cumulative externality.

Specifically, Chapter 3 specifies and sets out the most general dynamic equilibrium optimal growth model for an economy with an essential, exhaustible resource. While technology is fixed in Chapter 3, Chapter 4 introduces a backstop technology, permitting substitution away from the scarce resource. Chapter 5 demonstrates the decentralized

equilibrium solution path for an economy with an exhaustible resource depleted in finite time.

In Chapter 6 the global warming externality is then formally specified and a set of optimally adjusted solution paths derived. This solution path set is then contrasted with the market outcome in Chapter 7. Chapter 8 discusses optimal corrective policies and market intervention.

In Chapter 9 market solutions are examined under global warming ‘scenarios’ of differing severity, and results are seen to vary significantly. It is shown that some usually unstated assumptions critically constrain the solution, that a commonly made ‘assumption of convenience’ can preclude the existence of a meaningful equilibrium and that a finite horizon solution may be the market outcome for a potentially infinite horizon problem. Chapter 10 then concludes the dissertation.

Relation to the Literature

This paper is positioned in tangency to a number of well-researched areas; given the general approach taken in the paper, it can therefore easily be extended to incorporate questions raised and explored in these sub-literatures. The energy crises of the 1970s gave birth to a large ‘*energy literature*’. Questions regarding general labour, capital, energy factor substitution and complementarity, as well as inter-fuel substitution possibilities are treated theoretically and empirically for instance. Theoretical *dynamic general*

equilibrium modelling now includes treatments of asset markets and technology under uncertainty which could be incorporated. The *natural resource literature* addresses potential scarcity of exhaustible resources and has (by necessity) shown a long-standing recognition of the importance of dynamic factors, and also, due to increased computational opportunities, now makes use of large scale simulation exercises. The *taxation of natural and mineral resources* has also received extensive study as have *externalities*. Further, the global warming literature is large and growing as briefly discussed below.

From this vast pool of theoretical and empirical enquiry, a number of papers are more closely related to the paper at hand. Dasgupta and Heal (1974) it turns out, present a model essentially identical to the starting framework of Chapter 3 on the general equilibrium model with an exhaustible essential factor of production. The model is also found in Hartwick (1986). Hartwick then uses the Cobb-Douglas version of the general form model to run a simulation exercise. These authors do not however extend the model to incorporate externalities.

Formal presentations of *cumulative externalities* in production are scarce, and I am aware of only one paper addressing both the extraction of exhaustible resources in the presence of an associated cumulative externality. This is the paper by Schulze (1974) whose *firm-based analysis* of internalizing the externality suggests a tax similar to the one derived below. Schulze however does not dwell on this result, going on to discuss declining

numbers of firms, resource recycling and technical change in a multifaceted paper.

The existing *literature on global warming* is vast and includes a number of highly complex calculable and simulation models. These large models are the 'off-spring' of the generation of energy-economy models engendered by the oil crisis. These planning models were then gradually modified to incorporate dynamic interactions and environmental considerations. Models in this literature are generally classified as 'top down' or 'bottom up' models. Bottom-up models rest on engineering and process models and allow for a high degree of specificity in characterizing technologies and technological options; they are weak however in linking the energy system/sector with other sectors of the economy. Top down models, on the other hand are characterized by a far higher degree of aggregation of processes, feature general functional relationships and broader economic variables.

The top down literature includes a number of dynamically optimizing models⁶ but in general no account is taken of the fundamental depletion/scarcity effects that the economic literature predicts for exhaustible resources⁷. The one exception here is a paper

⁶ The best known is Manne and Richels (1990).

⁷ Relevant to making this connection between fossil fuel exhaustion and global warming are empirical studies on whether the Hotelling rule/condition or some modification thereof (equation 27) is actually observed in the price paths (or components thereof) of exhaustible resources. No consensus exists, but there is some suggestion that resource prices do rise once technical improvements in extraction processes encounter diminishing returns and relatively easily accessed deposits have been discovered. I have assumed that the Hotelling rule continues to hold a central in the economic analysis of exhaustible resources, if only because it is presented as the cornerstone of economic inquiry and analysis in all natural resource text books.

by Sinclair (1992); his approach however differs greatly from the one presented in this paper in that he does not take explicit account of the cumulative nature of the global warming externality.

2. Global Warming

Nature of the Problem

99.9% of the atmosphere is made up of a stable mix of nitrogen, oxygen and argon, and a variable amount of water vapour fluctuating according to changes in temperature. The next most abundant gas is carbon dioxide, chief among the so-called greenhouse gases (GHGs). Other GHGs identified so far are methane, nitrous oxide, ground-level ozone (smog) and chloroflourocarbons (CFCs). In normal concentrations, GHGs are essential to survival since they "trap" warmth released by the earth (see **Figure 1**). GHGs are relatively transparent to sunshine but capture heat by absorbing the longer-wavelength infrared radiation released by the earth -- hence the potential for a 'greenhouse effect' should the concentration of theses gases in the atmosphere rise. Without the normal level greenhouse effect, the earth's average surface temperature would drop to about 18 degrees celsius *below zero*.

Atmospheric scientists are able to track historic GHG levels by examination of the composition of Antarctic ice which has accumulated through the ages. Direct evidence links GHG concentrations with the dramatic climate changes of the ice ages; see **Figure 2** for the high correlation between variations in global temperature and CO₂. Current concentrations of atmospheric CO₂ have reached their highest values in more than 150,000 years.

RADIATION CYCLE

INCOMING
SOLAR
RADIATION
(SHORT-WAVE)

LONG-WAVE
RADIATION

(HEAT)
FROM
THE
EARTH

ATMOSPHERIC
"BLANKET"
CONTAINING
GREENHOUSE GASES

EARTH

Variation Of Global Temperature And CO₂ Over Last 160,000 Years

Source: IPCC, 1991.

Figure 2

Findings indicate that over the past 200 years, concentrations have increased more rapidly than at any other time in history and emission rates are still rising rapidly relative to historic rates. Since the industrial revolution, the atmospheric content of carbon dioxide has increased by about 25%. Moreover, half the anthropogenic carbon dioxide added to the atmosphere throughout human history has been emitted in the past 30 years. Methane levels have doubled in the past 100 years and concentrations of CFCs were doubling every 10 to 20 years. Although accelerated implementation of the Montreal Protocol should lessen this last threat, substantial stocks of CFCs remain in use (refrigerators, airconditioners, etc.) ready to be released unless recaptured and safely disposed. Currently CO₂'s share of global GHG emissions is 55%. This share is expected to rise as the contribution of CFCs falls.

It should be noted that carbon has a natural cycle whose magnitudes far outweigh those of anthropogenic emissions; however it is on the margin that the natural balance of the cycle is disturbed and the stock of GHGs thus accumulates more rapidly than reabsorbed through physico-chemical diffusion and photosynthesis (see illustration in **Figure 3**). When discussing 'CO₂ emissions' below, we will be referring to this anthropogenic component of total emissions.

Concern about rising concentrations of GHGs led to the establishment of the Intergovernmental Panel on Climate Change (IPCC) in 1988. Given the ample data linking GHG's and global temperature, there is virtually no debate among scientists about

THE CARBON CYCLE

Source: Energy Mines and Resources, Canada, 1992.

Figure 3

the greenhouse effect as a scientific proposition. There is general agreement that climate change will occur and that this change will be the effect of human activity.

Uncertainty

Uncertainty and substantial disagreement *do* exist as to expected GHG emissions, future atmospheric concentration levels, temperature ranges, climatic adjustment and the expected resulting impact on the ecosystem and the human habitat. I will touch on each of these concerns quickly in turn.

Of the greenhouse gases, Carbon Dioxide accounts for the greatest share (about 50 to 55%), most of which is released in the burning of fossil fuels in modern energy use and production⁸. Since there is no practical technology for 'scrubbing' exhaust gases for CO₂, emissions of CO₂ are linearly related to the combustion of fossil fuels. It should be noted further that the carbon content per thermal unit (Btu) of energy varies significantly over fossil fuels with brown, black coal, oil, natural gas having progressively lower carbon contents.⁹

While CO₂ emissions are a flow, atmospheric CO₂ is a stock. Given expected time path of anthropogenic CO₂ emissions, the stock will increase i.e. the first derivative is positive.

⁸ Not insignificantly, about 20% of current CO₂ emissions are related to rapid deforestation.

⁹ Interfuel substitution and respective elasticities of substitution as well as energy conversion from primary to secondary and tertiary forms of energy thus present formidable problems at the more disaggregated level of modelling.

However, it is uncertain by how much atmospheric levels of CO₂ will increase -- this is the problem of the 'uptake function'. The rate of CO₂ uptake is likely to change as concentration changes and feedback processes enter the equation. Increasing atmospheric concentration may increase plant intake and oceanic absorption and thus lower the accumulation rate of the stock for a given level of emissions. On the other hand, rising temperatures may cause the release of carbon and methane held in the ground as dead organic matter. The sign of this second derivative is therefore uncertain.

As to temperature estimates, according to the 1990 IPCC reference case, average temperature will rise by about 0.3°C per decade over the next century, which represents a rate of change significantly greater than that which occurred naturally over the last 10,000 years. This implies that the average temperature would be about 1°C warmer by 2025 and 3 to 4°C higher by the end of next century. As shown in Figure 4 temperatures will be rising at an increasing rate.

Typical estimates are for CO₂ levels of 600 parts per million (double the 1900 level) by between 2030 and 2080. A doubling of CO₂ concentration or an equivalent increase in other GHGs, is expected to warm the earth's average surface temperature by 3.0 to 5.5 degrees celsius. Such an increase would be unprecedented in human history. It would match the five-degree warming since the peak of the last ice age 18,000 years ago but -- significant for adjustment -- it would take effect between 10 to 100 times faster.

Just as the doubling of CO₂ levels is a useful benchmark, so is the measure of 'average

GLOBAL TEMPERATURE CHANGE

Figure 4

surface temperature'. Climate models agree that temperature change will be unevenly distributed, with polar regions undergoing greater increases than the tropics. Specific geographical effects are difficult to predict due to the large number of climatic factors involved; generally hotter and drier climates are predicted for mid-continental regions.

Precipitation, run-off patterns, soil moisture and fresh water supplies will be affected significantly. The breadbaskets of the world may move; one estimate, using an anticipated 3 degree warming combined with a 10% drop in precipitation, predicted the crop production loss of the Western states and Great Plains to be nearly a third¹⁰. Other predictions, incorporating positive effects of CO₂ on plant growth, are less alarming. Issues of biodiversity and species extinction also arise, since many species will not be able to migrate or adapt fast enough to cope with anticipated average changes. Sea-level rise is expected to lead to salt water intrusion into fresh water bodies and aquifers as well as increasing frequency of high tides and inundation of low-lying coastal areas. Marine life will clearly be affected. Humans, being extremely adaptable, live in a wide variety of temperature zones, the *direct impact of temperatures* is thus not a concern. *More important* are indirect effects, effects on sectors of human activity that have significant interaction with natural ecosystems: agriculture, forestry and coastal activity.

While changes in mean temperature have significant effects on the ecosystem and human habitat, *variance in climate* threatens the greatest damage to the status quo. Greater

¹⁰ F. Peterson Jr. and A. Keller, as cited by Schneider (1990).

incidence of natural disasters (floods, hurricanes, droughts, heat waves) could potentially severely disrupt the infrastructure on which all economic sectors depend. Significant also is the possibility that dramatic climate change (a reverse ice age) can take place quickly once certain tolerance levels are surpassed -- climate adjustment does not have to be smooth.

Modelling of thermal equilibrium i.e. a temperature-steady state, is difficult due to the lags involved. Different GHGs have different lifetimes, CO₂ has an 'atmospheric residence time' of between 200 to 500 years. CO₂ emissions would have to be reduced *by* 75% to stabilize CO₂ concentrations at present levels -- a highly unlikely scenario.

Of policy discussions, the international proposal that gained the most wide-spread support was the Toronto target of reducing emissions to 80% of the 1988 level by the year 2005. Concerted international action however is not likely in the near future as demonstrated by the failure of the 1992 UNCED conference to secure quantitative commitments. Rising emissions and rising concentration levels far into the future (given the lag effects of present actions) are therefore near-certain -- and attainment of thermal equilibrium very remote.

The key factor determining the economic impact of global warming then, is the climate adjustment process. The climate is always in a natural state of fluctuation. It is likely that the amplitude of these fluctuations will be greater, as temperatures rise and concentrations

of GHGs in the next century will exceed previously observed levels. Nordhaus, though very moderate in his assessment of the effects of changes in mean temperature, is less sanguine about the implications of variance in climate,¹¹

Virtually all observers agree that the uncertainties of climate change are asymmetrical; we are likely to be increasingly adverse to climate change as the change becomes larger. To go from a 2 to a 4 degree warming is much more alarming than to move from a 0 to a 2 degree warming. The greater the warming, the further we move from our current climate and the greater the potential for unforeseen events. Moreover, it is the extreme events -- droughts and hurricanes, heat waves and freezes, river flooding and lake freezing -- that produce major economic losses. As probability distributions shift, the frequency of extreme events increases (or decreases) proportionately more than the change in mean.

Realistically, global warming and climate change present Shackle-like uncertainties or "unforeseeable events", making the calculation of a certainty equivalent, even based on subjective probabilities, a speculative undertaking. For the purpose of our model we do however develop a damage function, positively related and increasing in increasing CO₂ levels. Further, the damage will be modelled not as entering the utility function -- signifying a vague decrease in amenity values caused by a warmer climate -- but as impacting on the general production function -- signifying real economic damage not only in agriculture as mean temperature considerations imply, but damage in all economic sectors as the 'unforeseen events' possibilities imply. The circle of a general environment-economy relationship is presented in a 'strong form'.

¹¹ Nordhaus (1991). p.58.

3. The Basic Framework: Optimal Growth and Scarcity

When an economy is endowed with a finite stock of a non-reproducible, exhaustible factor of production, the existence and depletion path of this factor will affect the optimal consumption profile of the economy. The familiar 'reproducible' capital model, with its associated steady-state solution, and variety of natural rates, neutral rates, golden even platinum states must change. In the presence of an exhaustible but essential factor of production, a factor that cannot grow at any rate save negatively, the very concept of a steady state becomes questionable. We therefore begin with a basic model and investigate the nature of the optimal time-path solutions in the presence of a factor resource constraint.

Expectations

As is customary in basic dynamic equilibrium modelling we assume perfect foresight so that agents and planners act on the basis of a complete vision of future prices, quantities and production conditions. In the absence of perfect future markets stretching to infinity and in the absence of certainty regarding future technical production possibilities which would eliminate dependence on the limited-stock factor, this assumption is more than suspect. We make the assumption nevertheless, if only to establish a benchmark against which outcomes of more realistic knowledge sets could be compared.

Technical Progress

We assume for now that technology is constant for all $t \geq 0$ ¹².

Population/Labour

Demographics and an optimal population policy will be of concern to the social planner. In practice economists tend to follow Solow by assuming an exogenous rate of growth¹³. However, the usual Solow assumption of an exponentially growing population is even more ominous in the explicit presence of finite resources. For this reason, and reasons of convenience, I hold population constant over all time. This is to say that the supply of labour, $N(t) = N$ is perfectly inelastic and constant for $t \geq 0$.

Preferences

Identical infinitely lived individuals plan over an infinite horizon. Their identical preferences are those of the representative agent. Instantaneous utility is assumed to be a time separable function of consumption only and social welfare, W , is taken to be of utilitarian form. The social planner then seeks to maximize at time $t=0$,

$$W(0) = \int_0^{\infty} e^{-\rho t} u(c(t)) N dt$$

¹² Declaring technological change to cease at t is a rather bizarre proposition - especially when combined with a perfect foresight assumption. As the above mention of Malthus showed, this omission has historically already had serious consequences for the validity of model predictions. Nevertheless we begin from this most simple assumption and then in the next chapter allow for knowledge of and potential use of a second technology.

¹³ Solow (1956).

where θ , the 'natural' rate of time preference, is exogenously given and strictly positive, and $c(t)$ is consumption at time t . Normalizing the size of the population, N , to equal one, gives,

$$W(0) = \int_0^{\infty} e^{-\theta t} U(c(t)) dt \quad (1)$$

$U(\cdot)$ is monotonically increasing, strictly concave and twice differentiable everywhere with,

$$U'(c) > 0, \quad U''(c) < 0, \quad \lim_{c \rightarrow 0} U'(c) = +\infty, \quad \text{and} \quad \lim_{c \rightarrow \infty} U'(c) = 0 \quad (2)$$

Further, let $\sigma(c(t))$ be the elasticity of marginal utility, where

$$\sigma(c) = - \frac{c U''(c)}{U'(c)}, \quad \text{and} \quad +\infty > \lim_{c \rightarrow 0} \sigma(c) = \sigma > 0. \quad (3)$$

Capital and Energy

There are two types of goods, fossil fuel energy and a composite good that can be either consumed or invested. Capital, $K(t)$, can be consumed, is durable and can be increased. Capital does not depreciate with time or use and we are assumed to hold an initial positive and known capital stock K_0 . Aggregate output $Y(t)$ in composite good units can thus be consumed or invested so that,

$$Y(t) = C(t) + I(t) = C(t) + \dot{K}(t) \quad (4)$$

or using lower case letters for per capita measures,

$$y(t) = c(t) + i(t) = c(t) + \dot{k}(t) \quad (4')$$

'Energy' is an aggregate of fossil fuels and serves as an essential factor in the production of the composite good. Fossil fuels are assumed to be homogeneous in the model but can be thought of as aggregated by efficiency units (Btu equivalents). This implies that fuels can also be measured in carbon emission units since Btu/carbon ratios are constant per fuel type. We continue to assume that there are no potential efficiency improvements and either that there is no inter-fossil fuel substitution possible or that this change in fuel composition is accounted for in the translation into carbon units.¹⁴

The flow of fossil fuels at t is given by $E(t)$. The initial stock of remaining below-ground reserves is B_0 . There is no depreciation and there are no new discoveries so that B_0 is fixed, positive and known. We assume zero extraction costs.

Production

Again normalizing the size of the population to equal one, per capita output is specified to be a function of two variable factors of production,

$$\frac{Y(t)}{N} = y(t) = F\left(\frac{K(t)}{N}, \frac{E(t)}{N}\right) = F(k(t), \epsilon(t)) \quad (5)$$

¹⁴ Other physical energy stocks such as hydro or nuclear are aggregated with capital. This need not produce perverse results: as fossil fuels become increasingly scarce, 'energy' is increasingly substituted for by capital. Some of this substitute capital may be in the form of alternative forms of energy generation. We limit this possibility however by postulating that capital may substitute for some of the productivity of fossil fuel energy, but can never completely replace it under technology F .

where F is assumed to be homogenous of degree one in k and ϵ , and strictly concave in both factors separately. This allows us to write,

$$F_{k\epsilon} > 0, F_{kk} < 0, F_{\epsilon\epsilon} < 0, F_{k\epsilon} = F_{\epsilon k} > 0,$$

where subscripts are used to represent partial derivatives.

Both factors are essential to production, so that,

$$F(0, \epsilon) = F(k, 0) = 0 \quad ^{15}.$$

By definition the productivity of each factor depends on the availability and use of both factors, i.e. $F_k(k(t), \epsilon(t))$ and $F_\epsilon(k(t), \epsilon(t))$, and we specify that,

$$\lim_{\epsilon \rightarrow 0} F_k = 0 \quad \text{and} \quad \lim_{k \rightarrow 0} F_\epsilon = 0.$$

Optimal Growth

To maximize welfare then, the most general social planning problem is to maximize at time zero, the discounted sum of future utility streams subject to constraints on the supplies of the two variable factors of production over time. Mathematically this can be stated as,

¹⁵ We define 'essential' by this condition.

$$\text{Max } W(0) = \max \int_0^{\infty} e^{-\rho t} U(c(t)) dt \quad (6.0)$$

$$\text{s.t. } \dot{k} = F(k(t), \epsilon(t)) - c(t) \quad (6.1)$$

$$\text{s.t. } \int_0^{\infty} \epsilon(t) dt \leq b_0 \quad (6.2)$$

$$c(t), k(t), \epsilon(t) \geq 0 \quad (6.3)$$

$$\text{where } k_0 > 0, b_0 > 0 \text{ given.} \quad (6.4)$$

This is a classic control problem that can be solved by using the optimum principle. In formulating the Hamiltonian for this problem, it is useful to note a number of things. First, the constraint (6.2) may be handled by defining a new stock variable $b(t)$, remaining (per capita) reserves at t ¹⁶. That is, let

$$b(t) = b_0 - \int_0^t \epsilon, dz, \text{ where } b(t=0) = b_0 > 0 \text{ given.} \quad (7)$$

and we can then replace constraint (6.2) by

$$\epsilon(t) = -db(t)/dt \quad (6.2')$$

where $db(t)/dt \leq 0$, i.e. below ground fossil fuel stocks decline over time.

Secondly, note that due to the assumptions made on the utility and production functions, the inequality constraints on consumption, capital stock and energy flow in (6.3) will not be binding. We therefore need not introduce (6.3) as separate constraints nor do we need to assign them multipliers.

¹⁶ The reader may find the following mnemonic useful: I have chosen 'b' for below-ground energy reserves. 'b' is also popular in natural resource economics where for renewable resources, b will be the stock of biomass.

Thirdly, we assign state and control variables: let the remaining stock of fuel $b(t)$ and the stock of capital $k(t)$ describe the current state of the system and so become the state variables, and let the flow of investment, $dk/dt = i(t)$, and the flow of energy extracted/used $\epsilon(t)$ take the roles of control variables. Consumption is then endogenously determined, depending on the state of the capital stock and the chosen levels of extraction and investment. We may thus write (per capita) consumption as a function c ,

$$c = c(k(t), \epsilon(t), i(t)) = F(k(t), \epsilon(t)) - i(t)$$

and differentiate this function as needed¹⁷.

We may now write the present value Hamiltonian as,

$$PVH = U[C(k(t), i(t), \epsilon(t))] e^{-\theta t} + \mu_1 i(t) + \mu_2 (-\epsilon(t)) \quad (8)$$

where costate variables μ_1 and μ_2 measure the sensitivity of the optimal solution to variations in the state variables. We can also interpret μ_1 as the shadow price of investment (in utility numeraire) and μ_2 as the shadow price of remaining reserves (in utility numeraire).

There are several types of necessary conditions for Hamiltonians, which together with

¹⁷ In solving problems of this type, it is conventional to let capital be the state variable and consumption the control. However, given the identity $Y = C + I$, we may equivalently assign investment as the control variable. The Hamiltonian of the rewritten problem has the basic structure

$$H(k, dk/dt, \mu) = J(k, dk/dt) + \mu I(k, dk/dt)$$

where the control variable is the rate of change of the state variable. Posed in this way the problem gains in mathematical elegance and the essential simplicity of the problem is revealed.

initial conditions define the solution to the problem. In order to interpret the economic meaning of first order conditions it is useful to group them by type and to review them conceptually.

The first type derives from the *maximum principle* and requires that the Hamiltonian must be optimized with respect to each control variable *at each point in time*.

The second set of first order conditions are *balance conditions* requiring that the state variables satisfy optimal conditions *across adjacent periods in time*.

Thirdly we must observe *structural dynamic constraints* which hold *over all time* and are given by the structure of the problem. A problem may also include *inequality constraints*; being structural they must also hold over all time and typically have a 'choice' depending if the inequality binds or not.

The fourth type of condition relates to *initial and terminal conditions* on the each state variable. These serve to 'anchor' the problem in time and may be expressed in terms of the value of the state variable or its respective costate shadow. Terminal conditions are also referred to as transversality conditions.

First Order Conditions then, for the Hamiltonian in (8) are,

from the maximum principle, with respect to control variables $i(t)$ and $\epsilon(t)$,

$$H_i = 0$$

$$\rightarrow e^{-\theta t} \frac{\partial U}{\partial c} \frac{\partial c}{\partial i} + \mu_1 = 0$$

$$\rightarrow e^{-\theta t} \frac{\partial U}{\partial c} = \mu_1$$

$$\rightarrow e^{\theta t} \mu_1 = U'(c) \quad (9)$$

$$H_\epsilon = 0$$

$$\rightarrow e^{-\theta t} \frac{\partial U}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial \epsilon} + \mu_2 (-1) = 0$$

$$\rightarrow \mu_2 e^{\theta t} = U'(c) F_\epsilon \quad (10)$$

from the balance condition for state variables $k(t)$ and $b(t)$,

$$-H_k = \dot{\mu}_1$$

$$\rightarrow \dot{\mu}_1 = -e^{-\theta t} \frac{\partial U}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial k}$$

$$\rightarrow \dot{\mu}_1 = -e^{-\theta t} U'(c) F_k \quad (11)$$

$$-H_b = \dot{\mu}_2$$

$$\rightarrow \dot{\mu}_2 = 0 \quad (12)$$

recovering the dynamic constraints on $k(t)$ and $b(t)$,

$$H_{\mu_1} = \dot{k}$$

$$\rightarrow \dot{k} = i \quad (13)$$

$$H_{\mu_1} = \dot{b}$$

$$\Rightarrow \dot{b} = -\epsilon \quad (14)$$

initial and terminal (transversality) conditions on state variable $k(t)$,

$$k_0 > 0 \text{ and given} \quad (15a)$$

$$\lim_{t \rightarrow \infty} k(t) \mu_1(t) = 0 \quad (15b)$$

initial and terminal (a more elaborately written transversality) conditions on state variable $b(t)$,

$$b_0 > 0 \text{ and given} \quad (16a)$$

$$\mu_2 \geq 0 \text{ and } \mu_2 (b_0 - \int_0^\infty \epsilon dt) = 0 \quad (16b)$$

Interpretation of the first order conditions is facilitated by defining a current value

Hamiltonian (CVH) where costate variables are defined in terms of their contemporaneous or current values.

$$CVH = \{ U[c(t)(k(t), \epsilon(t), \dot{k}(t))] + \lambda(t)(i(t)) + \delta(t)(-\epsilon(t)) \} e^{-\rho t} \quad (17)$$

The relationships of the multipliers of the PVH with those of the CVH, and the time derivatives of the CVH multipliers are given by¹⁸,

$$\lambda = \mu_1 e^{\rho t} \quad (18)$$

¹⁸ We have begin to omit timescripts where they are understood and clutter the presentation. In particular, it is understood that costate variables are generally variable over time, thus could be written with timescripts.

$$\rightarrow \dot{\lambda} = \dot{\mu}_1 e^{\theta t} + \theta \lambda \quad (19)$$

$$\delta = \mu_2 e^{\theta t} \quad (20)$$

$$\rightarrow \dot{\delta} = \dot{\mu}_2 e^{\theta t} + \theta \delta \quad (21)$$

We may now derive the following relationships using the current valued variables, from (9) and (18),

$$U'(c) = \lambda \quad (22)$$

where λ is the marginal value of an additional unit of capital at t , or expressed differently, the contemporaneous shadow price of marginal investment. If the shadow price were less than marginal utility, less investment would be (optimally) undertaken; conversely if λ is greater than marginal utility more capital would be accumulated; hence at the optimal level of capital accumulation, λ equals $U'(c)$.

To see how this utility maximizing condition on investment, which must hold at any point t , relates to the balance condition on capital accumulation which connects periods, we substitute (11) into (19) to get current values,

$$\dot{\lambda} = (-e^{-\theta t} U'(c) F_k) e^{\theta t} + \theta \lambda$$

using (22),

$$\begin{aligned}
\dot{\lambda} &= (-e^{-\theta t} \lambda F_k) e^{\theta t} + \theta \lambda \\
\rightarrow \dot{\lambda} &= -\lambda F_k + \theta \lambda \\
\rightarrow \dot{\lambda} &= \lambda (\theta - F_k)
\end{aligned} \tag{23}$$

where we see that the proportional rise of the shadow price of investment depends on the relative magnitudes of the productivity of capital (its rate of return) and the rate of time discount.

If we are interested in not in shadow value paths but in consumption, we may differentiate (22),

$$\dot{\lambda} = U''(c) \dot{c}$$

substitute into (23) to obtain,

$$U''(c) \dot{c} = U'(c) (\theta - F_k)$$

divide both sides by c , yielding,

$$\frac{\dot{c}}{c} = \frac{U'(c)}{U''(c) c} (\theta - F_k) \tag{24}$$

which gives information on the growth rate of consumption. Finally, substituting in (3) for the inverse of the negative of the instantaneous elasticity of marginal utility, we obtain,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma(c)} (F_k - \theta) \quad (24')$$

which is a frequently encountered rule stating that consumption increases if the marginal product of capital is greater than the rate of time preference (and vice versa).¹⁹

The Optimal Energy Regime

Returning to the hamiltonian first order conditions, we may now investigate the nature of the energy regime. The questions we would like to answer are,

- a) will b , the stock of below-ground reserves, be depleted in finite time?
- b) what does the extraction profile $\epsilon(t)$ look like? Is it rising or falling over time?
- c) what does the shadow value profile of energy look like? What will reserves be worth over time?

In neoclassical economics, in a world with well-defined, twice differentiable continuous functions, optimality occurs at points of indifference at the margin. In our problem there are two 'cases of indifference'. Firstly the utility maximizing rational planner can be indifferent (on the margin) between extracting²⁰ or leaving fossil fuels underground *at*

¹⁹ We observe that while (24) provides interesting information on the optimal behaviour of consumption, we have no basis for claiming that utility is maximized on a steady state path where $dc/dt = 0$, so that $F_k = \theta$ can be read as an equilibrium condition. Proper interpretation of (24) merely observes that, if $F_k = \theta$ at some point in time t , then dc/dt must be zero at that t .

²⁰ For clarity, we assume in our use of language that 'extracted' means 'used in production in that period', so that there are no above-ground stored stocks of fossil fuel.

any one point in time. Secondly, the planner can be indifferent between shifting marginal depletion *between adjacent periods in time*. Recalling our generic interpretation of the hamiltonian FOCs, we are not surprised that the maximum principle condition on $\epsilon(t)$ will give us the first case of indifference and the balance condition on $b(t)$ the second. Recall (10),

$$\mu_2 = e^{-\theta t} \left(U'(c) \frac{\partial F}{\partial \epsilon} \right) \quad (10)$$

This reads, μ_2 (the present value shadow price of marginal below-ground energy stocks, in utility numeraire, at any point in time t), must equal the time discounted marginal utility derived from using that same marginal energy unit in production at t . The first indifference condition is therefore met perfectly.

Turning to the second 'indifference' we examine the balance condition for depletion of reserves in equation (12). We find that along an optimal path, μ_2 , the present valued shadow price of marginal below-ground reserves $b(t)$ does not rise over time, that is to say, it is constant in terms of utility numeraire over all t . This is again reasonable, for if the PV of the marginal reserve unit at time t were say higher than at $t+\Delta t$, then we should shift some extraction from $t+\Delta t$ to t since this would increase the sum of discounted utilities.

If the present value shadow price is constant then the contemporaneous shadow price must be rising to 'keep up' with our (constant) time discounting factor. This is confirmed

in (21) which reduces to,

$$\dot{\delta} = \theta \delta \quad (25)$$

We may translate (10) into current values,

$$\delta = U'(C) F_e \quad (26)$$

Differentiating (26) with respect to time, we obtain,

$$\dot{\delta} = U''(c) \dot{c} F_e + U'(c) \dot{F}_e \quad (27)$$

substituting into (25), so that we have now combined the two indifference conditions, yields,

$$\begin{aligned} U''(c) \dot{c} F_e + U'(c) \dot{F}_e &= \theta [U'(c) F_e] \\ \rightarrow U''(c) \dot{c} F_e &= U'(c) [\theta F_e - \dot{F}_e] \\ \rightarrow \frac{U''(c) \dot{c}}{U'(c)} &= \frac{\theta F_e - \dot{F}_e}{F_e} \\ \rightarrow \frac{U''(c) \dot{c}}{U'(c)} &= \theta - \frac{\dot{F}_e}{F_e} \end{aligned} \quad (28)$$

which is the overall efficiency condition for 'energy management' while supplies last.

Finally we may ask for a relationship between capital and energy over time. Recalling (24), which may be rewritten as,

$$\frac{U''(c) \dot{c}}{U'(c)} = \theta - F_k \quad (24)$$

and we see that for $\epsilon(t) > 0$, the fundamental efficiency condition is,

$$F_k = \frac{\dot{F}_e}{F_e} \quad (29)$$

Although (29) may seem at first surprising, since it equates a rate of return and a rate of change, it is actually reassuring when interpreted as an efficiency condition. Investment in stocks of capital and maintenance of stocks of fossil fuels (b), are undertaken such that the return on the two factors of production is equalized at the margin. The return from capital is given by its productivity (or marginal product) which is variable over time²¹. The implied marginal productivity of below-ground reserves is reflected in appreciation over time in value, akin to a rate of "capital" gains or, expressed differently, a reward for waiting to extract b, before realizing the marginal product of energy F_e in production.

We may now address the three questions we posed earlier. With regard to the value profile of energy, the present value shadow price of reserves is constant, and consequently, the contemporaneous shadow value of b rises at the constant discount rate θ . The marginal product of energy in production rises at a proportional rate equal to the rate of interest or the marginal product of capital. These conditions apply when there is

²¹ Overfamiliarity with steady-state models may lead us astray. There is no presumption that $dc/dt = 0$ in the solution, hence the optimal F_k in (24) will not solve to be a steady state constant, but rather is variable over time as the capital stock evolves.

a flow of extraction greater than zero no matter how small. Extraction will always remain positive i.e. the non-negativity condition never binds under an optimal regime which requires a positive marginal product of capital and energy.

The exhaustible resource will therefore not be depleted in finite time and

$$b(t) > 0, \epsilon(t) > 0 \quad \text{for all } t \geq 0.$$

We know however that remaining reserves are declining since $\epsilon_t = -db/dt$ and can also determine whether the rate of extraction over time $d\epsilon/dt = -d^2b/dt^2$ is increasing or decreasing:

Because $F_k > 0$ for all $t \geq 0$,

then $dF_k/dt > 0$ from (29).

Knowing however, that $dF_k/d\epsilon < 0$

implies that $-d^2b/dt^2 = d\epsilon/dt < 0$. (30)

This demonstrates that the rate of extraction must decline over time as stocks decline.

Capital and the Productivity of Capital

From the above results it follows that the productivity of capital will be falling over all time. While this is intuitively obvious since the productivity of k is a function of both k and ϵ , and ϵ approaches zero in the limit of time, mathematically the argument can be made as follows,

$$\frac{dF(k,\epsilon)}{dt} = F_k \dot{k} + F_\epsilon \dot{\epsilon} \quad (31)$$

for a production function under constant returns to scale, it must also be true that,

$$F(k,\epsilon) = F_k k + F_\epsilon \epsilon \quad (32)$$

at any time t . Differentiating again,

$$\frac{d(F_k k + F_\epsilon \epsilon)}{dt} = \dot{F}_k k + F_k \dot{k} + \dot{F}_\epsilon \epsilon + F_\epsilon \dot{\epsilon} \quad (33)$$

Equating (31) and (33) yields,

$$\begin{aligned} F_k \dot{k} + F_\epsilon \dot{\epsilon} &= \dot{F}_k k + F_k \dot{k} + \dot{F}_\epsilon \epsilon + F_\epsilon \dot{\epsilon} \\ \Rightarrow 0 &= \dot{F}_k k + \dot{F}_\epsilon \epsilon ; \end{aligned} \quad (34)$$

since $\dot{F}_\epsilon > 0$, $k > 0$, $\epsilon > 0 \quad \forall t$,

$$\Rightarrow \dot{F}_k < 0 \quad \forall t. \quad (35)$$

The timepath of the marginal product of capital therefore declines over all t *regardless* of the initial level of capital. Further we know by specification that F_k will decline to approach zero as ϵ approaches zero.

Characteristics of the Solution

Recall that our per capita production function is $F(k(t), \epsilon(t))$ where F is characterized by constant returns to scale. Thus we are free to define $\epsilon F(k/\epsilon, 1) = \epsilon f(k/\epsilon) = \epsilon f(\phi)$ where $k/\epsilon = \phi$. Working out the basic relationships, we derive,

$$F_k = \epsilon f'\left(\frac{k}{\epsilon}\right) \frac{1}{\epsilon} = f'(\phi) \quad (36)$$

$$F_\epsilon = f\left(\frac{k}{\epsilon}\right) + \epsilon f'\left(\frac{k}{\epsilon}\right) k (-1) \epsilon^{-2} = f(\phi) - \phi f'(\phi) \quad (37)$$

and where,

$$f'(\phi) > 0, \quad f''(\phi) < 0 \quad . \quad (38)$$

The elasticity of substitution between k and ϵ must clearly play a large role in our problem, so we derive an expression for this in terms of our 'new' production function.

$$\eta_{k\epsilon} = - \frac{\frac{d\left(\frac{k}{\epsilon}\right)}{d\left(\frac{F_k}{F_\epsilon}\right)}}{\frac{\frac{k}{\epsilon}}{\frac{F_k}{F_\epsilon}}} = - \frac{1}{\eta(\phi)} = \frac{d\left(\frac{F_k}{F_\epsilon}\right)}{d\left(\frac{k}{\epsilon}\right)} \frac{\frac{k}{\epsilon}}{\frac{F_k}{F_\epsilon}} = \frac{d\left(\frac{F_k}{F_\epsilon}\right)}{d\phi} \frac{F_\epsilon}{F_k} \phi \quad (39)$$

aside,

$$\frac{d\left(\frac{F_k}{F_\epsilon}\right)}{d\phi} = \frac{\frac{dF_k}{d\phi} F_\epsilon - \frac{dF_\epsilon}{d\phi} F_k}{F_\epsilon^2}$$

RHS terms of which,

$$F_e \frac{dF_k}{d\phi} = F_e f''(\phi) = [f(\phi) - \phi f'(\phi)] f''(\phi)$$

$$F_k \frac{dF_e}{d\phi} = f'(\phi) [f'(\phi) - [f''(\phi)\phi + f'(\phi)]] = -\phi f'(\phi) f''(\phi)$$

substituting back,

$$-\frac{1}{\eta(\phi)} = \frac{[[f(\phi) - \phi f'(\phi)] f''(\phi)] - [-\phi f'(\phi) f''(\phi)]}{F_e^2} \frac{F_e \phi}{F_k} = \frac{f(\phi) f''(\phi) \phi}{F_e F_k}$$

so that,

$$\eta(\phi) = \frac{F_e F_k}{-\phi f(\phi) f''(\phi)} = \frac{f'(\phi) [f(\phi) - \phi f'(\phi)]}{-\phi f(\phi) f''(\phi)} \quad (40)$$

where $\eta(\phi) > 0$. Using (40) we can read (29) as,

$$\begin{aligned} \frac{d[f(\phi) - \phi f'(\phi)]}{d\phi} &= F_e F_k \\ -f'(\phi) \frac{d\phi}{d\tau} - \frac{d\phi}{d\tau} f'(\phi) - \phi f''(\phi) \frac{d\phi}{d\tau} &= [f(\phi) - \phi f'(\phi)] f'(\phi) \\ -\frac{d(\phi)}{d\tau} &= \phi = \frac{[f(\phi) - \phi f'(\phi)] f'(\phi)}{-f''(\phi) \phi} \end{aligned} \quad (41)$$

which implies that $d\phi/d\tau > 0$ i.e. that the capital-energy ratio must rise over all time.

Using (40) and (41) we may in turn write,

$$\frac{\dot{\phi}}{\phi} = \frac{\eta(\phi) f(\phi)}{\phi} \quad (42)$$

which depends only on the production function. To summarize, we now have two equations of motion,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma(c)} (F_k - \theta) \quad (24')$$

and

$$\frac{\dot{\phi}}{\phi} = \frac{\eta(\phi) f(\phi)}{\phi} \quad (43)$$

which govern our optimal program over all $t \geq 0$. The equations are by no means independent however, since consumption is a function of energy, capital and investment. For diagrammatic interpretation, for initial k_0 small enough and e_0, b_0 large enough, see **Diagrams 1 (a) to (e).**²²

²² With respect to **Diagram 10 (b)** we note that the marginal product of energy does not go to infinity. We have derived the optimal condition,

$$F_k = \frac{\dot{F}_e}{F_e}$$

which solves to,

$$F_e(t) = F_e(0) e^{\int_0^t F_k(u) du}$$

We know that F_k consistently falls over time, approaching zero in the limit. The proportional growth rate is therefore falling over time, the integral approaches a finite value, $F_e(0)$ is compounding at a lesser and lesser rate, and $F_e(t)$ asymptotically approaches a constant level in the limit (not infinity).

Diagrams 1 (a) to (e)

Appendix to Chapter 3: Investment Over Time and the Transversality Condition on Capital

Above we note that both capital accumulation and decumulation are permissible under the specification and first order conditions governing the model. It therefore becomes of interest under which conditions it will be optimal to invest (accumulate more capital) and how the terminal condition on capital will be met. To show this, it is convenient to first undertake a small review of the optimizing technique we have employed to solve our dynamic problem. In solving the problem²³,

$$\text{Max } W_0 = \text{Max} \int_0^{\infty} e^{-\rho t} U(c(t)) dt$$

$$\{\dot{k}(t)\}$$

$$\text{s.t. } F(k, \bar{\epsilon}) - c - \dot{k} = 0 \quad ,$$

we define μ as a costate variable, a dynamic equivalent of the familiar Lagrange multiplier. At the most general level $\mu(t)$ is a non-zero function of time. Since, in this case, $U(\cdot)$ is increasing, we know that $\mu(t)$ must be positive. We then define the Lagrangian function,

$$\mathcal{L} = W_0 + \int_0^{\infty} \mu [F(k, \bar{\epsilon}) - c - \dot{k}] dt$$

$$\rightarrow \mathcal{L} = \int_0^{\infty} \{ e^{-\rho t} U(c) + \mu [F(k, \bar{\epsilon}) - c - \dot{k}] \} dt$$

²³ We may for the time being hold ϵ constant as it does not change the nature of the problem.

The optimal value of the objective functional is the value of the Lagrangian function at the saddle point²⁴. The saddle point is in the space of functions where $(\{\dot{k}^*(t)\}, \{\mu^*(t)\})$ represents a saddle point if,

$$\mathcal{L}(\{\dot{k}(t)\}, \{\mu^*(t)\}) \leq \mathcal{L}(\{\dot{k}^*(t)\}, \{\mu^*(t)\}) \leq \mathcal{L}(\{\dot{k}^*(t)\}, \{\mu(t)\})$$

The key property of $\mathcal{L}$ is that it is concave-convex; when μ is fixed, the function $\mathcal{L}_\mu(\dot{k})$ that sends $\dot{k}$ to $\mathcal{L}(\dot{k}, \mu)$ is concave; when $\dot{k}$ is fixed, the function $\mathcal{L}_\dot{k}(\mu)$ that sends μ to $\mathcal{L}(\dot{k}, \mu)$ is convex. The expression above therefore says that for fixed μ^* , $\dot{k}^*$ maximizes $\mathcal{L}_{\mu^*}(\dot{k})$, and for fixed $\dot{k}^*$, μ^* minimizes $\mathcal{L}_{\dot{k}^*}(\mu)$.

We now define the Hamiltonian function as,

$$H(k, \mu, t) = \underset{\{\dot{k}\}}{\text{Max}} U(c) e^{-\theta t} + \mu (F(k, \bar{c}) - c)$$

so that,

$$\mathcal{L}(k, \dot{k}, \mu, t) = \int_0^\infty [H(k, \mu, t) - \mu \dot{k}] dt$$

We can therefore clearly discern the derivation of the first order conditions which we routinely apply directly to the Hamiltonian ($H_\mu = \dot{k}$ and $H_{\dot{k}} = 0$). We may also now ask,

²⁴ The proof for this assertion is provided in Intrilligator (1971), 14.1.5 to 14.1.9 .

under which condition would (more) investment be optimal? This would be the case when in the optimizing process with respect to the level of capital, the Lagrangian, (holding μ fixed), has not yet achieved its optimal or maximal value; mathematically, additional investment is optimal where,

$$\begin{aligned} \frac{\partial \mathcal{L}(k, \dot{k}, \mu^*)}{\partial k(t)} &< \frac{\partial H^*}{\partial k(t)} \\ \Rightarrow \frac{\partial \int_0^\infty H(k, \mu^*, t) dt}{\partial k(t)} &< \mu^* - e^{-\theta t} U'(c)^* \\ \Rightarrow - \int_t^\infty U'(c) e^{-\theta s} F_k ds + \mu^*(t) &< \mu^*(t) - e^{-\theta t} U'(c)^* \\ \Rightarrow e^{\theta t} \int_t^\infty U'(c) e^{-\theta s} F_k ds &> U'(c)^*(t) \end{aligned}$$

Or in words, more investment is required when the current value of the discounted sum of utilities accruing due to a marginal change in capital is greater than the current (optimized) utility of consuming a marginal unit of the composite capital/consumption good. Optimally of course, the two sides will equal. We see then that investment is a positive function of future marginal utility and a positive function of the future marginal productivity of capital. Over time, in our model, marginal utility rises (except possibly initially), and marginal productivity of capital falls.

The terminal condition on capital is,

$$\lim_{t \rightarrow \infty} k(t) \mu(t) = 0$$

Further, from above we see that,

$$\mu^*(t) = \int_t^\infty U'(c(s)) e^{-\theta s} F_k(s) ds$$

so that as F_k go to zero in the limit (as we have shown that it will), the value of the integral must approach zero as t approaches infinity, and the value of μ^* will optimally approach zero in the limit. The transversality condition on state variable k is therefore met without requiring that capital 'be eaten away' to approach a zero level in the limit i.e. without requiring a period of asymptotic divestment.

4. Optimal Growth with a Technology Option

The Malthusian assumption of an absolute resource constraint in the previous section seems rather farfetched, given the possible existence of alternative forms of energy generation. This section therefore considers the possibility that when fossil fuels become too expensive/scarc, a technological switch will be made to production without them. While the Chapter 3 model solution requires extraction to be positive over all t , implying that positive below-ground reserves remain over all t and that fossil fuels will not be depleted in finite time, the possibility of a technical change now also admits the possibility that fossil fuel reserves may be completely exhausted at some finite point in time or that partial reserves remain unused underground forever.

Technology

We now assume that fossil fuels are not essential to production in the sense that there exists an alternative production technology G , using an alternative energy source ' s '²⁵. In the literature this has become known as a 'backstop' technology²⁶. More specifically we assume that s is in perfect elastic supply (by nature), but can become usable only at a relatively 'high' fixed cost m . The potential supply curve of energy thus is a combination of two perfect substitutes ϵ and s . See the supply and demand **Diagram 2**.

²⁵ The mnemonic ' s ' for solar is convenient, although in reality of course solar energy is not the 'first in line' substitute to fossil fuels on a conventional cost basis.

²⁶ Nordhaus has popularized the usage of this term.

Diagram 2

The diagram is static but over time, the supply of energy, $S_\epsilon(t)$ will shift up, as ϵ becomes scarcer. $S_\epsilon(t)$ will shift up until at $S_\epsilon(T)$, the equilibrium price reaches m where the switch to substitute energy S will take place. Natural resource economics frequently refers to m as the 'choke price'²⁷.

Decentralized market equilibration is achieved through adjustment of *prices* and

²⁷ For a clear exposition of the basics of natural resource economics see Neher (1990).

quantities; in factor markets through adjustment of prices and quantities resulting from relative marginal factor productivities. The planner's socially optimal equilibrium, on the other hand, is derived on the basis of *marginal utilities* and *marginal productivities*. To introduce the choke price condition, it is therefore useful to make use of the implicit shadow price of energy, as given by the dynamic Lagrangian current multiplier δ . Supply and demand Diagram 2 then represents interpretation in shadow values. 'm', the price at which s is supplied perfectly elastically, is equal to $U'(c)G_s$ (in utility numeraire) and the supply of fossil fuel is determined by $\delta(t)$, which is rising with increasing scarcity. Demand is a function of the relative productivity of energy in utility numeraire.

We therefore return to our previous model but now add the restriction that,

$$\lim_{t \rightarrow T} \delta = \lim_{s \rightarrow 0} U'(c)F_s = m \quad \text{where} \quad m < +\infty \quad (44)$$

and add the existence of production possibility G , such that

$$y(t) = G(k(t), s(t)) \quad (45)$$

where G is strictly concave, $G_k > 0$, $G_s > 0$, $G_{kk} < 0$, $G_{ss} < 0$, and G is characterized by constant returns to scale.

We avoid the difficulties caused by switch points in linear programming by postulating a '*perfect substitute technology*' based on perfect substitutability of the two energy factors ϵ and s . In addition we assume that the technology switching costs are negligible. In the neighbourhood of the switch point therefore, the marginal product of capital and the

optimal stock of capital remain defined and continuous.

We assume that the economy begins with a stock of fossil fuels sufficiently large that production begins using fossil fuels. The date T is then defined to be the date at which production switches to substitute energy.

Utility

The overall utility maximization problem can be separated into two phases. Since we have assumed time separability in the utility function, this cut at T is not a problem in terms of summing present value utility streams so that we may write,

$$W_0 = W_0^1 + W_0^2 \quad (46)$$

or equivalently,

$$\int_0^\infty e^{-\delta t} U(c(t)) dt = \int_0^T e^{-\delta t} U(c(t)) dt + e^{-\delta T} \int_T^\infty e^{-\delta(t-T)} U(c(t)) dt . \quad (47)$$

Phase I

The solution of Chapter 3 suitably modified to incorporate restriction (44) will now become phase I of the problem. Since in the Chapter 3 solution F_t approached a positive limit, and m is specified to be a finite positive number, we are not guaranteed that T exists. We therefore add the condition that $\lim_{t \rightarrow \infty} U'(c)F_t > m$, for all t and k , so that we

are assured that a switch will occur at some point in time T^{28} . We turn to see how the solution path is affected²⁹.

Consider the rate of extraction $\epsilon(t)$: If $U'(c)F_c$ is bounded from above by m , and if this constraint begins to bind at $t=T$, then for $t \geq T$, $dF_c/dt = 0$. By equation (29) this implies that $F_k=0$ which cannot be. Equation (29) must hold if and only if $\epsilon(t)>0$. Therefore $\epsilon(t=T) = \epsilon_T = 0$ ³⁰ and the two energy sources will never be used jointly.

While we know that $\epsilon_T=0$, we do not yet know the *level* of ϵ just prior to T . In order to determine the complete path of $\epsilon(t)$, we need to fully specify the problem. Thus while it is true that $\epsilon(t)$ terminates at T , the overall problem is still 'hanging' because we have not specified a terminal condition for state variable k . Once we specify initial and terminal conditions for all state variables in the problem, as well as boundary conditions for state variables that 'cross' switch points, the paths of control variables such as $\epsilon(t)$ will be determinate.

To illustrate the (present) indeterminacy of the $\epsilon(t)$ path, we can see that there is no presumption that the time path of ϵ must smoothly approach level zero directly. From (30)

²⁸ Note that T is a variable not a fixed value.

²⁹ To be rigorous, in order to be able to carry out this discussion of how the Chapter 3 hamiltonian solution is affected, we must also assume that $F(k,0) > 0$, otherwise the integrand of the objective function collapses.

³⁰ $\epsilon(t)$ cannot be negative by constraint 6.3.

we have some qualitative information, but can see that there is a 'family' of solution paths that $\epsilon(t)$ can obey to achieve this.

Graphically, below all curves are consistent with,

$$b_0 > 0, \text{ given.}$$

$$db/dt < 0$$

$$d^2b/dt^2 < 0$$

$$b_0 - \int_0^T \epsilon(t) dt = b_T$$

$$-\epsilon(t) = db/dt$$

$$-d\epsilon/dt = d^2b/dt^2$$

ϵ_0 is not fixed but optimally chosen.

Diagram 3

Consider the level of remaining below-ground reserves $b(t)$: the problem requires $\epsilon(T) = 0$, so that $db/dt = 0$ at T . The question then is whether the stock of b_0 will have been depleted completely or whether extraction could have stopped with a positive stock of fossil fuel remaining.

We can answer this by examination of shadow price paths. If $U'(c)F_\lambda(t)$, (the price of $\epsilon(t)$), is rising no longer and is expected never to rise again, then the shadow price of remaining below-ground energy in its contemporaneous values $\delta(t)$, will be constant for

all $t > T$. If $\delta(t)$ is constant then the present value shadow price of $b(t)$, $\mu_2(t)$, must be falling over time. But this cannot be since by first order condition (12), $d\mu_2/dt = 0$. So we have a proof by contradiction: if there remained $b(T) > 0$, then its present value marginal shadow value in utility units is falling. But this clearly contradicts optimality conditions, therefore $b(T)=0$.

Therefore we must also have satisfied the terminal condition on state variable b with,

$$\begin{aligned} \mu_2 (b_0 - \int_0^T \epsilon(t) dt) &= 0, \quad \dot{\mu}_2 = 0, \quad \mu_2 > 0 \\ \rightarrow \mu_2 (b(T)) &= 0 \\ \rightarrow \delta(t) e^{-\theta t} b(T) &= 0 \\ \therefore \text{since } \delta(t) > 0 \quad \forall t, \\ \rightarrow b(T) &= 0 \end{aligned}$$

so that optimally fossil fuels are completely exhausted by the end of phase 1, i.e. at the switch point.

Switch point

In order to make the transition in energy regimes we also need information on state variable k in the neighbourhood of T . Parallel to our ignorance as to the level of ϵ at the limit, we do not know the level of k at the limit. Due to our convenient assumption of constant returns to scale, we can however make statements on ϕ in the limit. Using (37),

$$m(T) = \lim_{\epsilon \rightarrow 0} U'(c) F_{\epsilon}(t) = \lim_{\epsilon \rightarrow 0} U'(c) [f(\phi)(t) - \phi(t) f'(\phi)(t)] \quad (48)$$

Since G becomes instantaneously operational at T we hasten to define,

$$G\left(\frac{k}{j}, 1\right) = g\left(\frac{k}{j}\right) = g(\kappa) \quad (49)$$

where,

$$m = U'(c) G_x(T) = U'(c) [g(\kappa)(T) - \kappa(T) g'(\kappa)(T)] \quad (50)$$

and with marginal utilities terms falling out,

$$\lim_{t \rightarrow T} f(\phi)(t) = g(\kappa)(T) \quad (51)$$

$$\lim_{t \rightarrow T} \phi(t) = \kappa(T) \quad (52)$$

$$\lim_{t \rightarrow T} f_k(t) = g_k(T) \quad (53)$$

So that given m , we can solve for the limiting capital-energy ratio, which is equal to the capital-substitute energy ratio at $t=T$.

Endogenous Solution for T

The change of phase takes place at time T , the value of which is itself endogenously determined by the shape of the chosen extraction and consumption profiles. Recall (29):

$$\frac{\dot{F}_k}{F_k} = F_k(t)$$

so that for some $t = h$: $h < T$,

$$F_k(h) = F_k(0) e^{\int_0^h F_k(t) dt}$$

From (44) we have that the highest obtainable value ³¹ (or choke price) is,

$$m = \lim_{c \rightarrow 0} U'(c) F_c(t)$$

which implies,

$$m = \lim_{c \rightarrow 0} U'(c) [F_c(0) e^{\int_0^t F_k(t) dt}]$$

so that because the productivity of energy is *continuous* across the switch point (though not smooth),

$$m = U'(c) F_c(0) e^{\int_0^T F_k(t) dt}$$

$$\rightarrow \ln \left(\frac{m}{U'(c) F_c(0)} \right) = \int_0^T F_k(t) dt \quad (54)$$

To solve this for T requires specific functional form for the function $F_k(t)$.

At this point the reader may wonder what happened to the conventional wisdom that 'high discount rates lead to rapid exploitation of resource stocks'. We therefore briefly turn to find the relationship between θ and T. Once again we use $\delta(t)$, the contemporary shadow price of the marginal b unit to illustrate.

Recall (44),

$$\lim_{t \rightarrow T} \delta = m$$

³¹ we note that m is constant in current values; this means that its present value equivalent in utility numeraire will be falling over time due to the discounting process. This is in contrast to μ_2 which stayed constant over time during Phase One.

and recall from (25) that this utility numeraired shadow price must rise at the constant rate of time preference θ . This allows us to write,

$$\begin{aligned}
 \delta(0) e^{\theta T} &= \lim_{t \rightarrow T} \delta(t) \\
 \rightarrow e^{\theta T} &= \frac{\lim_{t \rightarrow T} \delta(t)}{\delta(0)} \\
 \rightarrow \theta T &= \ln \left(\frac{\lim_{t \rightarrow T} \delta(t)}{\delta(0)} \right) \\
 \rightarrow T &= \frac{\ln \left(\frac{\lim_{t \rightarrow T} \delta(t)}{\delta(0)} \right)}{\theta} \\
 \rightarrow T &= \frac{\ln \left(\frac{m}{U'(c(0)) F_0(0)} \right)}{\theta} \tag{55}
 \end{aligned}$$

Substituting from (54) then gives,

$$T = \frac{\int_0^T F_k(t) dt}{\theta} \tag{56}$$

So that in our model conventional wisdom is preserved with $\partial T / \partial \theta < 0$ ³².

Phase II

Phase Two in itself is a familiar exercise in maximizing a discounted utility stream for

³² This 'conventional wisdom' is challenged by Farzin [1984] whose model demonstrates that a reduction in the discount rate could result in a faster extraction of resources. His model includes a potential substitute for the resource as well as capital costs associated with the extraction of the exhaustible resource and the production of its substitute. A lowering of the rate of discount then has two counteracting effects: the conservation effect which our model demonstrates above and a resource 'disinvestment effect'. The latter is obtained to the extent that the discount rate reflects the interest rate or the cost of capital services, and whereby a lower discount rate lowers unit costs in the resource sector and induces faster depletion. His argument, which then hinges on the capital-intensity of resource extraction, is valid but far easier to carry through in his partial equilibrium framework. Our analysis stays at the more abstract disaggregation level of factors of production, rather than the more applied sectors of production and relative factor intensities. In our model a lower marginal product of capital unambiguously implies a later depletion date.

$t=[T,\infty]$. We can argue that the substitute energy is produced at a cost, thus some consumption must be sacrificed to obtain it in every period. Output may therefore now be divided between consumption (c), consumption foregone to add to the capital stock (i) and consumption foregone to produce the substitute energy (ms), where m is a fixed cost price in composite good units per energy unit. We can write,

$$\begin{aligned}\frac{dk}{dt} &= \dot{k} = i \\ G(k,s) &= C(k,s,i) + i + ms\end{aligned}\tag{57}$$

The current value hamiltonian with state variable k , control variables s and i , costate variable q , is³³:

$$CVH = e^{-\alpha t} [U(C(k,s,i)) + q(i)]\tag{58}$$

with first order conditions,

$$\begin{aligned}\frac{\partial H}{\partial \alpha} &= 0 \\ \rightarrow U'(C) &= q\end{aligned}\tag{59}$$

$$\begin{aligned}\frac{\partial H}{\partial s} &= 0 \\ \rightarrow e^{-\alpha t} U'(C) [G_s - m] &= 0 \\ \rightarrow G_s - m &= 0\end{aligned}\tag{60}$$

meaning that at the margin the benefit from producing more substitute energy must just

³³ The problem essentially is a two sector heterogeneous capital model. However we cannot set up a dynamic problem which chooses optimal investment rates into the k and s sectors and solves for an optimal k/s ratio since this approach would necessitate two state variables k and s . S does not lend itself to being a state variable since we cannot determine the initial level of s a priori.

offset the cost in every period. The balance condition is,

$$-\frac{\partial H}{\partial k} = \frac{d(e^{-\theta t} q)}{dt} \quad (61)$$

$$\rightarrow \dot{q} = \theta q - U'(C) G_k$$

which together with (59), yields the familiar result that,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma(c)} [G_k - \theta]. \quad (62)$$

Further we require a transversality condition on terminal k which will close the program,

$$\lim_{t \rightarrow \infty} k(t) e^{-\theta t} q(t) = 0 \quad (63)$$

Optimal Growth and Adjustment to Steady State

We have so far separately discussed conditions characterizing the optimal solution for phase I, phase II, the switch point and the endogenous solution for T . We now characterize the overall solution.

We note that we have determined that state variable b terminates at T since its transversality condition is satisfied. For state variable k however, a boundary condition must hold. We can state the problem as follows,

$$W_0 = \int_0^T e^{-\theta t} U(c(t)) dt + W_0^2(T, k(T))$$

where W^2 is equivalent to the *salvage function* of finite horizon optimization problems.

The appropriate boundary condition on state variable k is then,

$$\frac{\partial W^2}{\partial k(T)} = \lambda(T) .$$

For phase II the costate variable q is defined,

$$\frac{\partial W^2}{\partial k(t)} = q(t)$$

so that at the initial point of phase II we must have,

$$\frac{\partial W^2}{\partial k(T)} = q(T)$$

therefore $\lambda(T) = q(T)$, and this condition preserves continuity in the state variable k and its shadow.

We can arrive at the same result more directly, using intuition. Consider the overall problem of utility maximization: it is clear that for a dynamically efficient choice of T and $k(T)$, welfare cannot be increased by a different feasible $k(T+\Delta)$. Letting W_0^1 be the overall maximized present value utility stream of phase I, and W_0^2 the maximized discounted welfare from phase II, this can be expressed as,

$$\frac{\partial W_0^1}{\partial k_T} = \frac{\partial W_0^2}{\partial k_T} . \quad (64)$$

Since the costate variable is defined as the change in the integrand of the maximized objective function for a marginal change in the respective state variable, (64) is precisely equivalent to,

$$\lambda(T) = q(T) . \quad (65)$$

The phase II optimum (since the economy is no longer constrained by a factor in fixed supply over time), is the optimal k/s ratio κ in steady state. Further, since labour supply is fixed, not only will the ratio be constant, but the *levels* of k and s will be constant -- the optimum is a stationary state. Given the exogenous fixity of m , we therefore have,

$$U'(c) G_s = \bar{m} \quad \dot{s} = 0$$

$$G_k = \bar{\theta} \quad \dot{k} = 0$$

$$\frac{k}{s} = \bar{\kappa} \quad \dot{c} = 0$$

for the optimized phase II program. Working 'backwards' in a sense from this optimal state, and recalling that we have defined $t^* = \{ t: F_k = \theta \}$ in Chapter 3, we therefore additionally know that for the overall optimal path, $T = t^*$.³⁴

Overall then phase I is a period of smooth adjustment towards the stationary state attained in phase II.

³⁴ If 'the social planner' miscalculates the optimal T , a discontinuous adjustment path will result. The above result is an illustration of '*the principle of optimality*' as derived in Richard Bellman's *dynamic programming approach*. This principle, broadly stated is as follows, "an optimal path has the property that whatever the initial conditions and control values over some initial period, the control (or decision variables) over the remaining period must be optimal for the remaining problem, with the state resulting from the early decisions considered as the initial conditions." (Kamien and Schwartz (1981), p.238).

5. Decentralized Equilibrium

Having derived the optimal solution, we now turn to the decentralized economy to see whether the centralized planner solution can be attained by a system of markets where all agents maximize subject only to technology and prices. There are four markets; factor markets for capital and energy, the market for the composite consumption good and an asset market where households may also borrow and lend across time. The composite output is used as numeraire.

The Firm

There are M identical firms³⁵ each with a choice of technology F or G as described in the last section. Firms will choose F or G depending on whether e or s is less costly in that period. Firms rent capital at the rental price $R(t)$, extract fossil fuel energy for a royalty of $q(t)$ per unit energy or acquire substitute energy at a cost of m per unit energy. Firms are price takers, taking the prices of all factors and output as given.

Firms demand factor inputs from households and supply output to households. Households in turn, supply inputs and demand output. The competitive input and output markets then clear for respective demands and supplies, providing the equilibrium prices which agents take as given.

³⁵ The CRS assumption implies that the number of firms does not affect the solution provided the firms behave competitively. We therefore let the number of firms at any time t be $M(t)$, where M is large enough to imply atomistic behaviour.

The problem of the firms is simple since the household as owner of all assets makes all intertemporal decisions, leaving the firm as a period by period profit maximizer.³⁶ For convenience we drop the subscript denoting the j th firm, and may state the firms' problem as,

$$\text{Max}_{(k,\epsilon,s)} V(0) = \int_0^{\infty} e^{-\int_0^t r(u) du} \pi(t) dt \quad (65)$$

$$\text{where } \pi(t) = F(k,\epsilon) - Rk - q\epsilon \quad \text{if } q < m$$

$$\text{where } \pi(t) = G(k,s) - Rk - ms \quad \text{if } q \geq m$$

Given that fossil fuel is in overall fixed supply, we know that q , the contemporaneous price, will rise with increasing scarcity. We then mark the time that q will have risen to choke price m as t_1 , and can then write first order conditions as, for $t < t_1$,

$$F_k^* = R \quad (66)$$

$$F_\epsilon^* = q \quad (67)$$

which must hold simultaneously, giving the familiar,

$$\frac{F_k^*}{F_\epsilon^*} = \frac{R}{q} \quad (68)$$

which together with $\pi^*(t)$, and household intertemporal decisions on investment

³⁶ Equivalently one could model the firm owning capital and making the investment decision to maximize its net present value. Note also that because technology is not owned and all resources are owned outside the firm, free entry also will impose $\pi(t) = 0$ so that V_0 is maximized at $V_0 = 0$ in the final equilibrium.

and extraction (see below), will solve in (65) for $k^*(t)$ and $e^*(t)$ for $t < t_1$.

For $t \geq t_1$,

$$G_k^* = R \quad (69)$$

$$G_s^* = m \quad (70)$$

which give,

$$\frac{G_k^*}{G_s^*} = \frac{R}{m} \quad (71)$$

which, when combined with household decisions, will solve in (65) for $k^*(t)$ and $s^*(t)$ for $t \geq t_1$. We may also note that since m is a constant, the marginal product of s will also be constant on an equilibrium path.

The Household

The N infinitely lived households identically have the utilitarian welfare function given in (1), have perfect foresight³⁷, and own all assets. Initial asset endowments may vary across households but no household wields enough economic power to significantly affect

³⁷ Agents are identical, rational and have perfect foresight. 'Rational' here refers to optimizing behaviour subject to known and foreseen prices and price paths, taking the behaviour of *other agents* as given. Rational behaviour does not incorporate strategic behaviour with regard to the identity (i.e. identicalness, pun intended) of agents. That is to say, free market agent 'j' cannot reason as follows: if I choose (cooperative) action 'i', so must everyone else, because being identical, they are also represented by me. If agents were aware of, and acted on knowledge of, their identity, then they would act as an identity, i.e. as a social planner. The decentralized solution would therefore always be the socially optimal solution, and no externalities could logically exist in this most general of models. Following the literature we do use the general model to model an externality but make the usually implicit assumption explicit: we 'bound' rationality to exclude this type of strategic behaviour. The equilibrium concept then involves explicit noncooperative behaviour and is often denoted as a Nash equilibrium.

market outcomes -- period by period or intertemporally -- agents are strictly price-takers. This is the meaning of the *competitive condition*³⁸. Assets come in the form of below-ground energy holdings or physical capital. The household then, must decide at each point in time how much to save or consume. Further, with respect to its savings, the household must decide on a portfolio balance: how much to lend to other households at interest rate $r(t)$, how much to invest in capital stock, how much to conserve below-ground as natural resource stock.³⁹

At the most general level the i th household's problem looks as follows:

$$\text{Max } u_0 = \int_0^{\infty} e^{-\theta t} u(c(t)) dt \quad (72)$$

with budget constraint,

$$\dot{a}(t) = \pi(t) + r(t)a(t) - c(t) \quad \forall t \quad (73)$$

so that the change in the value of assets 'a' owned by the i th household at any point in time must equal the value of inflows minus the value of outflows. Inflows are profits ' π ', plus returns on assets ' ra '. Outflows are the value of consumption.

We may simplify (73) immediately by noting that π must be zero in a competitive

³⁸ Implicit in the literature, we explicitly rule out collusive behaviour by groups of households -- no OPECs possible! This, although usually taken for granted, is an extension (i.e. further restriction) of the rationality assumed in of this model (as made explicit in footnote 37).

³⁹ We are assuming the existence of asset markets that work perfectly with no transactions costs involved.

equilibrium, since the product is exhausted by factor payments. To solve the households' problem we write the current value Hamiltonian, where consumption is the choice variable, the value of assets the state variable and $\lambda_0(t)$ the current costate variable⁴⁰,

$$CVH = e^{-\theta t} \{ u(c(t)) + \lambda_0(t) [(r(t)a(t) - c(t))] \} \quad (74)$$

which we shall refer to as the general asset hamiltonian. Initial period assets $a(0) = a_0$ are assumed given and positive, and we stipulate that

$$\lim_{t \rightarrow \infty} a(t) \lambda_0 e^{-\theta t} = 0 \quad (75)$$

implying, reasonably enough, that the value of assets which households choose to hold at the end of their horizon is zero.

Dropping timescripts we may manipulate the remaining first order conditions as follows,

$$\begin{aligned} \frac{\partial H}{\partial c} &= 0 \\ \rightarrow e^{-\theta t} [u'(c) - \lambda_0] &= 0 \\ \rightarrow u'(c) &= \lambda_0 \end{aligned} \quad (76)$$

differentiating with respect to time,

$$\dot{\lambda}_0 = u''(c) \dot{c} \quad (77)$$

using the balance condition,

⁴⁰ Note that the subscript on λ_0 does not denote time or a fixed value (the costate variable varies over time). The subscript is for identification only and will be useful later on.

$$\begin{aligned}
-\frac{\partial H}{\partial a} &= (e^{-\alpha} \dot{\lambda}_0) \\
\rightarrow -(e^{-\alpha} \lambda_0 r) &= -\theta e^{-\alpha} \lambda_0 + \dot{\lambda}_0 e^{-\alpha} \\
\rightarrow -\theta \lambda_0 + \dot{\lambda}_0 &= -\lambda_0 r \\
\rightarrow \dot{\lambda}_0 &= \lambda_0 (\theta - r)
\end{aligned} \tag{78}$$

substituting (77) into (78),

$$u''(c)\dot{c} = \lambda_0 (\theta - r) \tag{79}$$

substituting in (76),

$$u''(c)\dot{c} = u'(c)(\theta - r) \tag{80}$$

so that,

$$\frac{\dot{c}}{c} = \frac{u'(c)}{u''(c)c} (\theta - r) \tag{81}$$

We note that we have derived (81) under necessary conditions which must hold over all time. Leaving (81) for the moment, we turn to the asset market.

The Asset Market

By Walras Law we know that if three of four markets are clearing, the fourth market must also be in equilibrium. We can therefore assume asset market equilibrium. This implies that aggregate debt must be equal to zero and that the rate of return to holding any asset must be equal across all assets.⁴¹

⁴¹ If this were not so, households would change their portfolio balance towards the higher earning opportunity. In equilibrium the return from physical capital in production, from lending composite goods to others, and from resource investment, must equal.

Implicitly aggregating across all identical households, the value of all assets (or wealth)⁴², at any one time is then,

$$a = p_k k + q b \quad \forall t \quad (82)$$

where p_k is the price of buying the asset capital and q is the capital price of natural resources i.e. the cost to the firm of using a unit of fossil fuels in production this period.

Differentiating with respect to time this gives,

$$\dot{a} = \dot{k} p_k + k \dot{p}_k + \dot{q} b + q \dot{b} \quad \forall t \quad (83)$$

Since capital and the consumption good are identical as a composite good, the price of which is numeraire, $p_k = 1$ and, since the model is real, $dp_k/dt = 0$. Equation (83) therefore reduces to,

$$\dot{a} = \dot{k} + \dot{q} b + q \dot{b} \quad \forall t. \quad (84)$$

Turning then, to rates of return, the real interest rate $r(t)$ (which is also the borrowing and lending rate on a , see footnote 41) must equal the return from holding capital or the resource. At the most general level the rate of return 'r' to holding the asset capital will be,

$$r = \frac{R_k + \frac{dp_k}{dt} + \delta p_k}{p_k}$$

where R_k is the rental price per period; δ is depreciation which we have set to zero; and

⁴² The substitute energy is never 'held' as wealth, it is only instantaneously acquired by firms on demand.

$dp_k/dt/p_k$ is the rate of change of capital prices relative to the unit in which assets are measured. Hence if assets are measured in the same units as capital (which is our case), this capital gains rate once again reduces to zero, and our model reduces (85) to,

$$r = R$$

The return on holding asset b on the other hand, features no rental return (the factor energy cannot be rented as it is literally 'burned' in production), it does not depreciate with time, its price however can appreciate relative to the numeraire. The return to b, is thus pure capital gains and we can write,

$$r = R = \frac{\dot{p}_b}{p_b} \quad (85)$$

telling us that the real rate of interest is equal to the rental rate on capital which is equal to capital gains in energy in equilibrium.

Competitive Equilibrium

Since identities (82) and (83) and equilibrium condition (85) from the asset market must hold for all t , we may substitute them directly into the full budget constraint (73), aggregated across the economy. With (82) and (83),

$$\dot{a} = \dot{k} + q\dot{b} + \dot{q}b = \pi + r[k + qb] - c \quad \forall t \quad (86)$$

with (85),

$$\begin{aligned} \dot{k} + q\dot{b} + \dot{q}b &= \pi + Rk + \frac{\dot{q}}{q}qb - c \\ \Rightarrow \dot{k} + q\dot{b} &= \pi + Rk - c \quad \forall t \end{aligned} \quad (87)$$

Next, recalling necessary conditions (66) to (70) from the firms' problem, we substitute in equilibrium marginal products,

$$\dot{k} + F_{\epsilon}^* \dot{b} = \pi + F_k^* k + G_k^* k - c \quad (88)$$

Further⁴³ with the condition $\pi=0$, and the physical constraint $db/dt = -c(t)$,

$$\dot{k} - F_{\epsilon}^* \epsilon = F_k^* k + G_k^* k - c \quad \forall t \quad (89)$$

Lastly we impose the time restrictions from the firms' FOCs.

FOC's for $t < t_1$

where technology F is used and $s = 0$, from (89),

$$\begin{aligned} \dot{k} &= F_k^* k + F_{\epsilon}^* \epsilon - c \\ \Rightarrow \dot{k} &= F(k, \epsilon) - c \end{aligned} \quad (90)$$

Letting $F(k, \epsilon) = \epsilon f(k/\epsilon)$, and letting $k/\epsilon = \phi$, we can again derive,

$$\frac{\dot{\phi}}{\phi} = \eta(\phi) \frac{f(\phi)}{\phi} \quad (91)$$

Recalling (85), we have $r = R_k = F_k^*$ and can therefore write (81) as,

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{1}{\sigma(c)} (F_k^* - \theta) \quad (92)$$

as well as noting that since $R_k = dq/dt / dq$, it must be that,

⁴³ It is understood that F_k or G_k will apply depending on t .

$$F_k^* = \frac{\dot{F}_t^*}{F_t^*} \quad (93)$$

Conditions (90), (91), (81), (93), together with the given physical constraint $-\varepsilon = db/dt$, characterize the behaviour of the decentralized economy for $t < t_1$ and we note that they are *identical* to the conditions governing the centrally planned economy. The two solutions over the first phase are thus apparently identically shaped: consumption is falling over time while the capital/energy ratio and the marginal product of energy are rising.

Depletion Point

In comparing the centrally planned solution to the decentralized outcome, we can also ascertain whether depletion point $t_1 = T$. Households know the complete path of the royalty price for fossil fuels to be,

$$q(t) = q(0) e^{\int_0^t r(s) ds} \quad (94)$$

with limit,

$$\lim_{t \rightarrow t_1} q(t) = m \quad (95)$$

so that,

$$\begin{aligned}
m &= q(0) e^{\int_0^T r(t) dt} \\
\rightarrow \frac{m}{q(0)} &= e^{\int_0^T r(t) dt} \\
\rightarrow \ln \left(\frac{m}{q(0)} \right) &= \int_0^T r(t) dt
\end{aligned} \tag{96}$$

Since the shadow price of the optimal solution equals the market determined price in the absence of externalities, $U'(c)F_k(0) = q(0)^{44}$, and the respective productivities equal, $F_k(t) = r(t)$, from equation (54) we can therefore verify that indeed $t_1 = T$.

FOC's for $t \geq t_1$

To complete the decentralized equilibrium path it remains to be shown that the economy will be in optimal stationary state for all $t \geq t_1$. The following first order conditions apply over $t \geq t_1$:

From households' decisions,

$$\frac{\dot{c}}{c} = \frac{u'(c)}{u''(c)c} (\theta - r) \quad \forall t$$

From the asset market equilibrium,

$$r(t) = R_k \quad \forall t$$

⁴⁴ To show that this is in fact the case requires a change of units. $U'(c)F_k(0)$ is in utility numeraire (i.e. utils per energy unit at $t=0$); $q(0)$ is in composite good numeraire (i.e. capital units per energy unit at $t=0$). To compare them it is therefore necessary to find the 'exchange rate for k's to u's at $t=0$ '. Since we also know that λ is the shadow price of investment (in utils per k), we can divide $U'(c)F_k(0)$ by $\lambda(0)$ to change units. Since from FOCs $\lambda = U'(c)$ for all t , we therefore have $[U'(c)F_k(0)]/U'(c) = F_k(0) = q(0)$ which is what we set out to show.

From the firms' decisions,

$$G_k = R_k, \quad G_s = \bar{m}.$$

Since $\varepsilon = 0$, equation (89) now solves to the following dynamic condition on capital,

$$\dot{k} = G_k k - c$$

Further, certain conditions arise from the technology employed. For a CRS production function, the factor ratio will uniquely determine marginal products. Given that $G_s = \bar{m}$, is now fixed, we may conclude that $\kappa = k/s$ is constant, G_k is constant and that optimally,

$$\dot{k} = \dot{s}$$

Combining the above conditions we therefore obtain,

$$\frac{\dot{c}}{c} = \frac{u'(c)}{u''(c)c} (\theta - G_k) \quad (97)$$

$$\dot{s} = \dot{k} = G_k k - c \quad (98)$$

These conditions are the necessary conditions for motion *toward* a stationary state.⁴⁵ To show mathematically that the system will already *be* at stationary state at t_1 , is a little more complicated.

⁴⁵ Notice that we can rewrite the latter two parts of (98) using product exhaustion under CRS as,

$$\dot{k} + ms = G(k,s) - c$$

There is thus a clear trade-off between 'investments' in productive factors and consumption. This gives rise to the saddlepoint characteristics of the solution.

Adjustment to Steady State

We begin by establishing continuity in certain variables across the switch point: recall the general asset Hamiltonian (74) where $\lambda_0(t)$ denotes the current valued multiplier on the budget constraint. First order conditions generated by the households' problem and its associated general asset Hamiltonian must hold over all time and $-\partial H/\partial a = \lambda_0(t)$ is therefore defined and continuous for all t including t_1 .

Since we can disaggregate assets s.t. $a(t) = k(t) + q(t)b(t)$ (82), clearly we can then also write an asset-disaggregated Hamiltonian with costate variables λ_1 and λ_2 serving as the multipliers on the temporal constraints on k and b :

$$CVH = e^{-\theta t} \{ u(c) + \lambda_1(rk + rbq - c - \dot{b}q - \dot{q}b) + \lambda_2(\frac{1}{q}(rk + rbq - c - \dot{k} - \dot{q}b)) \} \quad (99)$$

Let $\lambda_2/q = \lambda_3$ for notational convenience. FOCs are; from the maximum principle,

$$u'(c) = \lambda_1 + \lambda_3 = \lambda_0 \quad (100)$$

from the balance conditions,

$$\begin{aligned} -\frac{\partial H}{\partial t} &= (\lambda_1 e^{-\theta t}) \\ \Rightarrow -(\lambda_1(r) + \lambda_3(r)) &= -\theta \lambda_1 + \dot{\lambda}_1 \\ \Rightarrow \dot{\lambda}_1 &= \lambda_1(\theta - r) - \lambda_3 r \end{aligned} \quad (101)$$

$$\begin{aligned}
-\frac{\partial H}{\partial b} &= (\lambda_2 e^{-\theta t}) \\
&= -(\lambda_1(rq - q) + \lambda_3(rq - q)) = -\theta \lambda_2 + \dot{\lambda}_2 \\
\text{since } r - \frac{q}{q} &= 0 \\
\rightarrow 0 &= \frac{-\theta \lambda_2 + \dot{\lambda}_2}{q} \\
\rightarrow \dot{\lambda}_2 &= \theta \lambda_2
\end{aligned} \tag{102}$$

We also know that $q(t_1)b(t_1) = 0$ and that λ_3 must therefore disappear at t_1 . From the maximum principle conditions from the two Hamiltonians we then have,

$$\begin{aligned}
\lambda_0 &= \lambda_1 + \lambda_3 & t < t_1 \\
\lambda_0 &= \lambda_1 & t \geq t_1
\end{aligned} \tag{103}$$

so that at the limit as t goes to t_1 ,

$$\begin{aligned}
\lim_{t \rightarrow t_1} (\lambda_1 + \lambda_3) &= \lambda_0(t_1) \\
\rightarrow \lim_{t \rightarrow t_1} (\dot{\lambda}_1 + \dot{\lambda}_3) &= \dot{\lambda}_0(t_1) .
\end{aligned} \tag{104}$$

Next we note that the final steady state is such that,

$$\lim_{t \rightarrow \infty} (\dot{\lambda}_0) = 0$$

$$\rightarrow \lim_{t \rightarrow \infty} (\dot{\lambda}_1 + \dot{\lambda}_3) = 0$$

$$\rightarrow \lim_{t \rightarrow \infty} [(\lambda_1(\theta - r(t)) - \lambda_3 r(t)) + \theta \lambda_3] = 0$$

$$\text{since } \lim_{t \rightarrow t_1} \lambda_3 = 0$$

$$\rightarrow \lim_{t \rightarrow \infty} [\lambda_1(\theta - r(t))] = 0$$

$$\text{since } \lambda_1 = U'(c) > 0 \quad \forall t$$

$$\rightarrow \lim_{t \rightarrow \infty} (\theta - r(t)) = 0$$

since the marginal product of capital is constant after the switch, and must equal the interest rate,

$$\rightarrow \overline{r(t)} = r(t) : t_1 \leq t < \infty$$

which implies that we can 'move' the limit backwards in time (since a constant cannot approach another constant over time smoothly) i.e.,

$$\rightarrow \lim_{t \rightarrow t_1} (\theta - r(t)) = 0$$

$$\rightarrow (\theta - r(t_1)) = 0$$

(105)

$$\rightarrow r(t_1) = \theta$$

$$\rightarrow G_k(t) = \theta \quad \forall t \geq t_1$$

which is what we set out to prove. We have therefore shown that the dynamic paths of the centrally planned and the decentralized economy are equivalent, both coming to rest at an optimal stationary state with $dc/dt = 0$ at $t=T=t_1$.

6. Environment and Economy: Optimal Trade-Offs

The global warming effect will now be modelled as a negative externality associated with the use of fossil fuels by the economy. We are concerned with the anthropogenic emissions of CO_2 which are additions to the naturally existing atmospheric concentration of CO_2 (i.e. additions to the long run equilibrium concentration that would exist in the absence of human fossil fuel use). As discussed in Chapter 2, we choose to model the impact of global warming not through the utility function, but as directly impacting on the production function. The fact that CO_2 is *accumulating* in the atmosphere, complicates the modelling of the externality, but at the same time, illustrates the usefulness of dynamic optimization techniques by showing how this problem becomes tractable.

We now build on the basic model set out in Chapter 3 with the representative consumer maximizing consumption-based utility over an infinite horizon. With the utility function unaltered, the demand side of the analysis in Chapter 3 is unchanged and environmental effects enter the supply side of the problem through the production function.

We focus and build on the most basic structure of the extraction problem, that is to say there is no switching option to a backstop technology permitted in this Chapter. Leaving alternative energy technologies aside⁴⁶, the economy's production possibility is now

⁴⁶ So that for the moment we revert to $F(k,0) = F(0,\varepsilon) = 0$.

described by the following technology:

$$y(t) = D(\Psi(t)) F(k(t), \epsilon(t)) \quad (106)$$

where output, DF, is now determined by the interaction of the production function F, and a damage function D. We continue to assume that F is constant returns to scale (CRS), but will find out that DF is not⁴⁷. Ψ is defined as the anthropogenic CO₂ atmospheric concentration, a stock. Ψ is positive and the initial level is known:

$$\Psi \geq 0, \quad \Psi(0) = \Psi_0. \quad (107)$$

We note carefully that $\Psi(0)$ denotes the level of CO₂ at time zero (i.e. the present), whereas $\Psi(t) = 0$ denotes a long run natural equilibrium level of CO₂ without human intervention (i.e. a pre-industrial revolution average)⁴⁸. The time derivative of the CO₂ stock variable then gives us the CO₂ accumulation function which is determined by the economy's fossil fuel use,

$$\dot{\Psi} = \epsilon(t), \quad (108)$$

and we therefore have,

$$\Psi(t) = \int_0^t \epsilon(t) dt + \Psi(0) \quad (109)$$

$$= b_0 - b(t) + \Psi_0 \quad (103)$$

⁴⁷ $y(t)$ would be CRS if k and ϵ could be increased holding Ψ constant. But since an increase in extraction directly implies an increase in the rate of CO₂ accumulation, this is not possible.

⁴⁸ Clearly the natural level itself is subject to massive changes over the millennia, along with natural climate change. Over a shorter time horizon however -- say one thousand years -- positing a natural long run equilibrium average is defensible, especially in light of the conceptually clear distinction between natural and human-induced CO₂ emissions.

where,

$$\frac{d\Psi}{db} = -1$$

so that CO₂ can either remain below ground, in a 'carbon sink', or be emitted above ground into the atmosphere. With a view to formalizing our problem, we have the choice of specifying Ψ or b (but not both) as the state variable, since one stock implies the other.

For convenience, we further specify that,

$$\frac{\partial \Psi(t)}{\partial a(t)} = 0 \quad (111)$$

so that the current fossil fuel use at t is 'in transit' so to speak, and has not 'reached' the atmosphere yet. Since we are working in continuous time, each t is infinitely small, emission at a single point t can thus be neglected.

Next we establish a global warming relationship between Ψ (CO₂) and output $y(t)$ i.e. the damage function. The $D(\Psi)$ function in (106) is assumed to compress the economy's production possibilities. We normalize the D function so that it takes the value of 1 when $\Psi=0$ (i.e. in natural equilibrium), and becomes a decreasing positive fraction as Ψ increases⁴⁹. That is,

$$D(\Psi=0) = 1, \quad 1 \geq D(\Psi) \geq 0 \quad \text{with} \quad (112)$$

⁴⁹ The reader will note that by defining D in this way, D is not really a damage function -- since a higher D means more output -- but rather an *inverse* damage function. It is convenient for exposition, however, to talk about $D(\cdot)F(\cdot)$ as 'damaged production'.

$$D'(\Psi) < 0 \quad , \quad D''(\Psi) < 0 \quad . \quad (113)$$

By modelling global warming so that higher levels of CO₂ result in less output (i.e. more production is lost due to climate change), global warming becomes an increasingly serious or *cumulative* problem. While we have specified this in terms of average production damage, more damage can reflect a higher risk (a cost) of climate adjustment, i.e. that there will be greater variance about the average as more CO₂ increases the likelihood of more extreme events.⁵⁰

Graphically $D(\Psi)$ can be represented as a smooth, well behaved concave function (see Diagram 4) that allows us to derive results more easily⁵¹. For reference we mark the

⁵⁰ While we have characterized D as a simple direct function of Ψ , it should be interpreted as an expected value. This allows incorporation of the risk that climate adjustment will not be smooth, that greater climatic variance (i.e. extreme events) are more likely to occur and damage production. $D(\Psi)$'s concavity can therefore be interpreted as a shortcut to modelling $D(E(\Psi), \text{Var}(\Psi))$.

In the part of GW literature concerned with monetizing damage, this is in fact a bone of contention. Most estimates are based on 'best guesses' rather than expected values. According to the stochastic model developed by Frankhauser (1993), the probability distribution of greenhouse damage is significantly skewed and underlines the importance of low probability/high impact events.

⁵¹ As is common in modelling the use of concavity involves a compromise with reality. An alternate and perhaps superior shape for $D(\Psi)$ would be declining to some constant at an increasing, then decreasing rate, and asymptotically approaching a positive constant. This would then imply that environmental damage can (seriously) compress the output of economic activity, but never completely. This seems reasonable -- radical climate change will seriously impair production, but given human adaptability, will never 'kill off' economic activity altogether.

(Perhaps the strictly concave model should be applied to the nuclear industry. Modelling to infinity, radiation emissions from an increasing stockpile of contaminated nuclear waste cause increasing levels of genetic damage to all biological life forms, including 'workers'. Once certain levels are exceeded workers fail to reproduce healthy new workers. And/or Ψ can reflect the increasing risk of (globally serious) nuclear plant accidents over time.)

At the more general level of interpretation of the model also, if Ψ is taken as the level of environmental stress globally -- including such factors as toxin levels in the food supply, contamination and depletion of fresh water supplies, radiation and uv levels etc. -- it would still be appropriate that the system would devolve to an equilibrium lower limit level of activity. This equilibrium level of activity would be in turn subject to shocks stemming from development of new technology and/or new human-induced stresses on the environment.

level of CO_2 concentration where $D(\Psi) = 0$ as Ψ^c (for Ψ critical).

Diagram 4

Having specified global warming formally, we return to state the enhanced social planning problem as:

This conceptual discussion leads naturally to the re-examination of the concept of equilibrium in complex systems -- whether economic or physical/ecological. As witnessed by current scientific discussions on 'the will to complexity' in the context of theories of evolution, the notion of *tendencies* of motion toward (temporarily) stable, existing and possibly unique equilibria is highly attractive to anyone seeking to explain phenomena. Because the concept of equilibrium lends tractability to the modelling effort, one may suspect an inherent bias towards the use of the concept of equilibrium in explanations of imperfectly understood complex phenomena. Notable, by way of an aside, is that the scientist's concept of equilibrium differs from the economist's. Roughly speaking, the climatologist's equilibrium corresponds to the economic steady state; to an economist however, the dynamic model must be in equilibrium at every instant t , then coming to rest in the steady state equilibrium.

All this having been said however, the modelling of such environmental damage functions falls to the future research agenda. We make the usual 'concession to concavity' and employ the well-behaved D function as specified above.

$$\begin{aligned}
\text{Max } W_0 &= \text{Max} \int_0^{\infty} e^{-\delta t} U(c(t)) dt \\
&= \text{Max} \int_0^{\infty} e^{-\delta t} U[D(\Psi(t)) F(k(t), \epsilon(t)) - i(t)] dt \\
\text{s.t. } D(\Psi(t)) F(k(t), \epsilon(t)) - c(t) &= i(t) \\
\text{s.t. } \dot{k}(t) &= i(t) \\
\text{s.t. } \dot{b}(t) &= -\epsilon(t) \\
\text{s.t. } \dot{\Psi}(t) &= -\dot{b}(t) \\
\text{where } c, k, \epsilon &> 0 \\
\text{where } 0 \leq D &\leq 1 \\
\text{where } k_0, b_0, \Psi_0 &> 0 \quad \text{given constant.}
\end{aligned} \tag{114}$$

Our control variables are the flows ϵ and i , and noting the dependency between b and Ψ , we choose our state variables to be the stocks k and b . Alternatively one could have equivalently chosen c (rather than i) as a control variable and Ψ (rather than b) as a state variable. However because the choice of one will determine the other, the 'variable not chosen' is also endogenously determined as a function of the other variables. For later convenience we can note the derivatives of c in the utility function,

from $c = D(\Psi)F(k, \epsilon) - i$,

$$\begin{aligned}
\frac{\partial c}{\partial k} &= D(\Psi) \frac{\partial F}{\partial k} = D(\Psi) F_k & \text{where } \frac{\partial c}{\partial k} &> 0 \\
\frac{\partial c}{\partial \epsilon} &= D(\Psi) \frac{\partial F}{\partial \epsilon} = D(\Psi) F_\epsilon & \text{where } \frac{\partial c}{\partial \epsilon} &> 0 \\
\frac{\partial c}{\partial k} &= \frac{\partial c}{\partial i} = -1 & \text{where } \frac{\partial c}{\partial k} &< 0 \\
\frac{\partial c}{\partial \Psi} &= D'(\Psi) F(k, \epsilon) & \text{where } \frac{\partial c}{\partial \Psi} &< 0 \\
\frac{\partial c}{\partial b} &= D'(\Psi) \frac{\partial \Psi}{\partial b} F(k, \epsilon) = -D'(\Psi) F(k, \epsilon) & \text{where } \frac{\partial c}{\partial b} &> 0 .
\end{aligned} \tag{115}$$

Resolution Test Chart Labels:

- 1.0
- 1.1
- 1.25
- 1.4
- 1.6
- 1.8
- 2.0
- 2.2
- 2.5
- 2.8
- 3.0
- 3.2
- 3.6
- 4.0

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The present value Hamiltonian is,

$$PVH = U(c)e^{-\theta t} + \mu_1(i) + \mu_2(-\epsilon) \quad (116)$$

Because it is more convenient to work with the current value Hamiltonian, we write the problem as,

$$CVH = [U(c) + \lambda(i) + \beta(-\epsilon)] e^{-\theta t} \quad (117)$$

denoting the current valued shadow price of below ground reserves as β , and the shadow price of investment as λ . From the maximum principle, first order conditions on the control variables investment and energy extraction are⁵²,

$$\begin{aligned} H_i &= 0 \\ \rightarrow e^{-\theta t} \left(\frac{\partial U}{\partial c} \frac{\partial c}{\partial i} + \lambda \right) &= 0 \\ \rightarrow U'(c) &= \lambda \end{aligned} \quad (118)$$

$$\begin{aligned} H_\epsilon &= 0 \\ \rightarrow e^{-\theta t} \left(\frac{\partial U}{\partial c} \frac{\partial c}{\partial \epsilon} - \beta \right) &= 0 \\ \rightarrow U'(c) D(\Psi) F_\epsilon &= \beta \end{aligned} \quad (119)$$

From the balance conditions for the state variable, capital,

⁵² The following derivations will employ substitutions from (115) where convenient.

$$\begin{aligned}
-H_k &= \dot{(\lambda e^{-\theta t})} \\
\rightarrow -e^{-\theta t} \frac{\partial U}{\partial c} \frac{\partial c}{\partial k} &= \dot{\lambda} e^{-\theta t} - \theta e^{-\theta t} \lambda \\
\rightarrow -U'(c) D(\Psi) F_k &= \dot{\lambda} - U'(c) \theta \\
\rightarrow \dot{\lambda} &= U'(c) [\theta - D(\Psi) F_k]
\end{aligned} \tag{120}$$

which can also be written as,

$$\frac{\dot{\lambda}}{\lambda} = \theta - D(\Psi) F_k \tag{121}$$

so that once again the proportional rise of the current shadow price of investment depends on the productivity of capital relative to the rate of time discount. Clearly, comparing the above result to the equivalent line in Chapter 3, equation (23), capital can be seen to be less productive due to the global warming effect i.e. $F_k > D(\Psi) F_k$ for given levels of k and ϵ (and $\Psi > 0$).

The across-period optimum for energy reserves,

$$\begin{aligned}
-H_b &= \dot{(\beta e^{-\theta t})} \\
\rightarrow -e^{-\theta t} \frac{\partial U}{\partial c} \frac{\partial c}{\partial b} &= \dot{\beta} e^{-\theta t} - \theta e^{-\theta t} \beta \\
\rightarrow U'(c) D'(\Psi) F(k, \epsilon) &= \dot{\beta} - \theta U'(c) D(\Psi) F_\epsilon \\
\rightarrow \dot{\beta} &= U'(c) [D'(\Psi) F(k, \epsilon) + \theta D(\Psi) F_\epsilon]
\end{aligned} \tag{122}$$

which can also be written as,

$$\frac{\dot{\beta}}{\beta} = \frac{D'(\Psi)F(k,\epsilon)}{D(\Psi)F_k} + \theta$$

Again, we may compare this result with the pre-global warming scenario by recalling equation (25) from Chapter 3,

$$\frac{\dot{\beta}}{\beta} = \theta \quad . \quad (25)$$

Note there is now a *key* difference in the structure of the solution. Whereas before the present value shadow price of below ground reserves was rising merely to keep up with time discounting, now a new term has entered the equation. This term is negative. The rise in marginal value is then *slowed* by the harmful side effects of extraction due to global warming. The social planner accounts for the harmful side effects of ϵ by decreasing the rate at which energy is drawn from underground fossil fuel sinks and released into the atmosphere.

Turning to the third set of FOCs, we recover the dynamic constraints on k and b , which yield the equations of motion,

$$\begin{aligned} H_{(\lambda, \cdot)} &= \dot{k} \\ \rightarrow i &= \dot{k} = D(\Psi)F(k, \epsilon) - c \end{aligned} \quad (123)$$

$$\begin{aligned} H_{(b, \cdot)} &= \dot{b} \\ \rightarrow -\epsilon &= \dot{b} \end{aligned} \quad (124)$$

And lastly, the initial and transversality conditions on capital and energy reserves are,

$$\begin{aligned} k(0) &= k_0, \quad k_0 > 0 \\ \lim_{t \rightarrow \infty} k(t) \lambda e^{-\theta t} &= 0 \end{aligned} \quad (125)$$

$$\begin{aligned} b(0) &= b_0, \quad b_0 > 0 \\ \lim_{t \rightarrow \infty} b(t) \beta e^{-\theta t} &= 0. \end{aligned} \quad (126)$$

Working with the FOCs, we take the time derivatives of the costate variables in (118) and (119), respectively:

$$\dot{\lambda} = U''(c)\dot{c} \quad (127)$$

$$\dot{\beta} = U''(c)\dot{c} D(\Psi)F_k + U'(c)D'(\Psi)\dot{\Psi} F_c + U'(c)D(\Psi)\dot{F}_c \quad (128)$$

Substituting (120) into (127) we obtain,

$$\begin{aligned} U''(c)\dot{c} &= U'(c)[\theta - D(\Psi)F_k] \\ \Rightarrow \dot{c} &= \frac{U'(c)}{U''(c)} [\theta - D(\Psi)F_k] \\ \Rightarrow \frac{\dot{c}}{c} &= \frac{1}{\sigma(c)} [D(\Psi)F_k - \theta] \end{aligned} \quad (129)$$

which corresponds to (25) of the simple solution and essentially remains the same, the difference reflecting only the compression of productivity due to global warming in the marginal product of capital variable.

In turn substituting (124) into (128), yields,

$$\begin{aligned}
 U''(c)\dot{c} D(\Psi)F_k + U'(c)D'(\Psi)\dot{\Psi}F_k + U'(c)D(\Psi)\dot{F}_k \\
 = U'(c)[D'(\Psi)F(k,\epsilon) + \theta D(\Psi)F_k] \\
 \Rightarrow \dot{c} = \frac{U'(c)}{U''(c)} \left[\theta + \frac{D'(\Psi)F(k,\epsilon)}{D(\Psi)F_k} - \frac{\dot{(D(\Psi)F_k)}}{D(\Psi)F_k} \right] \\
 \Rightarrow \frac{\dot{c}}{c} = \frac{1}{\sigma(c)} \left[\frac{\dot{(D(\Psi)F_k)}}{D(\Psi)F_k} - \frac{D'(\Psi)F(k,\epsilon)}{D(\Psi)F_k} - \theta \right] .
 \end{aligned} \tag{130}$$

which is the new overall efficiency condition on energy use -- incorporating the proportional growth in productivity of energy over time, the adverse effect of increasing atmospheric concentration of CO₂, as well as the effect of the chosen discount rate. As can be seen by comparing (130) with the previous energy efficiency equation (28), the rate of growth of consumption has been substantially modified by the externality. Not only does the marginal product of energy reflect global warming directly (the D(Ψ)'s), but a new marginal damage term has entered the optimality condition as the second last term on the RHS of (130).

Finally one may combine the conditions for efficient resource depletion (130) and optimal capital accumulation (129) to obtain the fundamental efficiency condition governing this system,

$$D(\Psi)F_k = \frac{\dot{(D(\Psi)F_k)}}{D(\Psi)F_k} - \frac{D'(\Psi)F(k,\epsilon)}{D(\Psi)F_k} . \tag{131}$$

Once again the fundamental efficiency relationship over time relates the returns from the stock of capital and the stock of fossil fuel energy reserves at the margin. The first two terms in the equation are equivalent to the result of Chapter 3 (see equation (29)), modified only to reflect the ‘damaged productivity’ aspect of the global warming scenario. The effective marginal productivity of capital is equal to the proportional change in value over time of the marginal b unit. The last term, is new. Since the sign of the term is negative, its addition reflects a positive return earned by b (i.e. a reduction in damage earned by keeping reserves in the ground). Recalling the partial derivative of output with respect to below ground reserves, i.e.,

$$\frac{\partial [D(\Psi)F(k,\epsilon)]}{\partial b} = \frac{\partial D}{\partial \Psi} \frac{\partial \Psi}{\partial b} F(k,\epsilon) = - D'(\Psi)F(k,\epsilon) > 0.$$

The last term in (131) can be seen to represent the proportional benefit⁵³ of not-extracting (i.e. conserving), the marginal b unit -- thereby preventing further harmful additions to the stock of Ψ , and thereby preventing further damage to economic capacity.

Rewriting (131) slightly by expanding the first term on the RHS, yields a further intuitive interpretation:

$$D(\Psi)F_k = \frac{D'(\Psi)\dot{\Psi}}{D(\Psi)} + \frac{\dot{F}_\epsilon}{F_\epsilon} - \frac{D'(\Psi)F(k,\epsilon)}{D(\Psi)F_\epsilon} \quad (131')$$

⁵³ It is a ‘proportional benefit’ since it is the marginal gain of not using a b unit in production *per* gain of using the same last unit of b in production ($D(\Psi)F_\epsilon$).

$$= \frac{\dot{F}_\epsilon}{F_\epsilon} + \frac{D'(\Psi)\epsilon}{D(\Psi)} + \left[-\frac{D'(\Psi)F(k,\epsilon)}{D(\Psi)F_\epsilon} \right] \quad (131'')$$

The level of socially efficient use of fossil fuel reserves is then affected by three factors on the margin.

The first term on the RHS is a pure *scarcity* or *exhaustion benefit* from the proportionally rising value of reserves. This effect, which optimally spreads the use of the resource over time, was the centrepiece of the dynamics derived in Chapter 3.

The second term on the RHS is a direct *global warming cost*. This is due to the fact that drawing down reserves and using them in production, results in accumulation of CO₂ i.e. $-db/dt(t) = \epsilon(t) = d\Psi/dt$. This (proportional) increase in global warming then is the production damage caused by using up b .

The third term is a *conservation benefit*, in the sense of costs foregone. This term captures the proportional benefit of preventing damage to the economy's production possibility i.e. preventing further additions to the stock of Ψ . Incorporation of this term results in fossil fuel stocks being depleted less rapidly.

Given the number of endogenous variables, a phase diagram of the dynamics of the solution is not possible -- only 'partial' diagrams can be drawn. We do, however, wish

to get a thorough understanding of the relative movement of variables as the system moves through time. In particular the following questions are of interest:

1. Will output go to zero and the system collapse? We have modelled global warming as cumulative, where the marginal damage is increasingly negative -- $D''(\Psi) < 0$. Will this constraint act as a 'natural physical law', increasingly and inevitably compressing output to zero in finite time? Have we constructed an environmental doomsday scenario?
2. What is happening to capital accumulation? In particular how is the capital/energy ratio moving over time?
3. What is the profile of consumption? How does it differ from the no-global warming scenario?

We answer each of these questions in turn. Thereafter we will return to the fundamental dynamic efficiency equation and will solve it for the timepath of the marginal product of energy. Focusing on specific timepaths of variables, we will then graphically analyze the movements of variables of interest in the remainder of the chapter, thereby gaining better insight into the dynamics of the system as a whole.

Economic and Environmental Collapse

If we keep using positive amounts of an essential exhaustible resource, it would seem that eventually the resource must run out. If we keep increasing the damage to the economic-environmental system, eventually it must collapse. Fortunately this logic is not complete, and we are saved by Xeno's paradox. If we keep moving forward toward a fixed point, but each step we take is a fraction of the size of the previous, then we will never get there.

In our analysis this scenario first arose in Chapter 3 where the increasing scarcity of an essential factor of production induced lower and lower levels of use, until only infinitely small amounts of the resource were used. Now, as increasing damage threatens doomsday, the marginal value of conservation rises, and only infinitesimal amounts of the damage-producing (but essential) resource are used to keep economic production and consumption continuing. Global warming thus *reinforces* the conservation incentive and decreases the use of fossil fuels in production. Output levels will be increasingly compressed but again never collapse completely. The overall solution to the system then is asymptotic devolution to a new lower level of economic activity.

Production Relations and the Capital/Energy Ratio

In Chapter 3 we solved for the elasticity of substitution between capital and energy making use of the CRS properties of the production function. While the production function under global warming, $D(\Psi)F(k, \epsilon)$, is not CRS, the previous CRS properties are

still 'embedded'. We may therefore manipulate as follows,

$$D(\Psi)F(k, \epsilon) = D(\Psi)\epsilon F\left(\frac{k}{\epsilon}, 1\right) = D(\Psi)\epsilon f(\phi) \quad (132)$$

where, as before, $\phi = k/\epsilon$, $f'(\phi) > 0$, $f''(\phi) < 0$, and,

$$D(\Psi)F_k = D(\Psi)f'(\phi) \quad (133)$$

and ignoring the instantaneous global warming effect of extraction as above,

$$D(\Psi)F_\epsilon = D(\Psi)[f(\phi) - \phi f'(\phi)] \quad (134)$$

In deriving the elasticity of substitution, we find that the damage productivity effects cancel, and that the elasticity of substitution, $\eta(\phi)$, is exactly as before,

$$\eta(\phi) = \frac{f'(\phi)[f(\phi) - \phi f'(\phi)]}{-\phi f(\phi)f''(\phi)} = \frac{F_\epsilon F_k}{-\phi f(\phi)f''(\phi)} \quad (135)$$

Writing (131) as,

$$D(\Psi)\dot{F}_\epsilon = D'(\Psi)F(k, \epsilon) + D(\Psi)F_k D(\Psi)F_\epsilon - \dot{D}(\Psi)F_\epsilon \quad (136)$$

we substitute (134) into (136), to obtain,

$$\begin{aligned} D(\Psi)\frac{f(\phi) - \phi f'(\phi)}{\epsilon} &= D'(\Psi)F(k, \epsilon) + D(\Psi)F_k D(\Psi)F_\epsilon - \dot{D}(\Psi)F_\epsilon \\ \rightarrow D(\Psi)[f'(\phi)\phi - \phi f'(\phi) - \phi f''(\phi)\phi] &= D'(\Psi)F(k, \epsilon) + D(\Psi)F_k D(\Psi)F_\epsilon - \dot{D}(\Psi)F_\epsilon \\ \rightarrow \dot{\phi} &= - \frac{D'(\Psi)F(k, \epsilon) + D(\Psi)F_k D(\Psi)F_\epsilon - \dot{D}(\Psi)F_\epsilon}{D(\Psi)\phi f''(\phi)} \\ \rightarrow \dot{\phi} &= D(\Psi)\eta(\phi)f(\phi) + \frac{\dot{D}(\Psi)F_\epsilon}{D(\Psi)\phi f''(\phi)} - \frac{D'(\Psi)F(k, \epsilon)}{D(\Psi)\phi f''(\phi)} \end{aligned} \quad (137)$$

The signing of this expression is not obvious at first glance: on the RHS the first two terms are positive, but then we subtract a positive term. Fortunately from the second last line in (137) we see that since the denominator is negative, the sign of the change in the capital/energy ratio depends on the sign of the numerator. The expression in the numerator is exactly equal to^{54,55},

$$D(\Psi)\dot{F}_\epsilon = D(\Psi) (F_{\epsilon\epsilon}\dot{\epsilon} + F_{\epsilon k}\dot{k}) > 0 \quad (138)$$

We therefore may infer that the socially optimal capital/energy ratio is rising over time i.e. $d\phi/dt > 0$, but that this rise is slowed by the output compressing effect of global warming.

Consumption

The most convenient way to inquire about the consumption path is to begin with the second time derivative of consumption i.e. to ascertain whether growth in consumption is increasing or decreasing. From the necessary equilibrium condition (129) we see that the sign of $d[dc/dt]/dt$ must coincide with the sign of $d[D(\Psi)F_k]/dt$, and,

$$\frac{d[D(\Psi)F_k]}{dt} = D'(\Psi)\dot{\Psi} F_k + D(\Psi)\dot{F}_k < 0 \quad (139)$$

⁵⁴ The reader will note that the *partial* derivative F_k is a function of given levels of both ϵ and k . The full derivative with respect to time must thus be taken with respect to ϵ and k , as shown above.

⁵⁵ For convenience we list the following signs, which are known either from functional specification at the outset or results derived thus far:

$D(\Psi) > 0$	$D'(\Psi) < 0$	$D''(\Psi) < 0$	$\Psi > 0$	$d\Psi/dt > 0$
$F(k, \epsilon) > 0$	$f(\phi) > 0$	$f'(\phi) > 0$	$f''(\phi) < 0$	
$c > 0$	$k > 0$	$\epsilon > 0$	$d\epsilon/dt < 0$	
$F_k > 0$	$F_\epsilon > 0$	$F_{kk} < 0$	$F_{\epsilon\epsilon} < 0$	$F_{\epsilon k} = F_{k\epsilon} > 0$
$dF_k/dt > 0$	$dF_\epsilon/dt < 0$			

as both terms are negative. The rate of growth of consumption is therefore falling throughout.

Next, again with reference to optimality condition (129) we see that the rate of change of consumption (dc/dt) is again determined by the relative size of the rate of time discounting and the productivity of capital. Initially, if k_0 is sufficiently small, then $D(\Psi)F_k(0) > \theta$ and consumption increases over time. As capital accumulates and less energy is used, the marginal product of capital decreases over time, eventually falling below θ , and asymptotically approaching zero. As in chapter 3, consumption may increase at a decreasing rate for a time, but must eventually peak and decline to zero in the limit.

How then does the consumption path under global warming differ from the 'fair weather' solution in Chapter 3? We note that because $F_{kk} < 0$ and $D(\Psi) < 1$,

$$\begin{aligned} \text{if } \theta &= F_k|_{k_1} = D(\Psi)F_k|_{k_2}, \\ \text{then } k_2 &< k_1. \end{aligned}$$

This implies that consumption will peak at a smaller capital stock under the global warming scenario. Drawing the above information on consumption together, then enables us to graphically demonstrate the timepaths of consumption as shown in **Diagrams 5 (a) to (c)**⁵⁶.

⁵⁶ As drawn the diagrams assume that $D(\Psi)(0)$ is very close to one, and that $DF_{k(now)}(0)$ is only slightly lower than $F_{k(now)}(0)$. The diagrams also assume (as we did in the corresponding Chapter 3 depiction) that the initial stock of capital is relatively low. For k_0 large enough, Df_k would be less than θ throughout and $c(t)$ would continuously decline over time.

Diagrams 5 (a) to (c)

Social Marginal Products Over Time

As we have modelled global warming on the supply side, the externality effects show in the production efficiency relations. Hence the relative marginal productivities and the optimal condition governing their evolution over time, (131), become the crux of the problem. Turning to a more mathematical approach, we note that the optimal pattern of change derived in equation (131), is a first order differential equation. We may therefore solve this equation to derive the time path of $D(\Psi)F_e$, the 'damaged' or 'effective' social marginal product of energy. Since this variable also describes the *social value* of the marginal below-ground fossil fuel unit, it is central to the problem⁵⁷. We rewrite (131),

$$\frac{d(D(\Psi)F_e)}{dt} - D(\Psi)F_k (D(\Psi)F_e) = D'(\Psi)F(k, \epsilon) \quad (131'')$$

to obtain definite integrals, we integrate to some arbitrary point in time T ,

$$DF_e(T) = A e^{-\int_0^T -DF_k dv} + \int_0^T D'F e^{-\int_t^T -DF_k dv} dt \quad (140)$$

where A is some constant. Letting $T=0$, we find,

$$DF_e(0) = A \quad (141)$$

substituting (141) into (140),

⁵⁷ It will be shown that once we have solved for this key timepath, we shall be able to infer the shapes of other (especially linearly related) variables. Graphical analysis will be used extensively towards this purpose; the graphs presented in this section therefore do not merely represent ancillary illustration, but are used as a tool of analysis and form an integral part of the text.

$$DF_{\epsilon}(T) = e^{\int_0^T DF_t dv} DF_{\epsilon}(0) + \int_0^T D'F e^{\int_t^T DF_t dv} dt \quad (142)$$

which must be true for any t , so that one may write,

$$DF_{\epsilon}(t) = e^{\int_0^t DF_t dv} DF_{\epsilon}(0) + \int_0^t D'F(s) e^{\int_s^t DF_t dv} ds \quad (143)$$

which we can call solution (a). Since we are aware that our solution path crucially hinges on its terminal point, we can also derive a more intuitive expression for the solution path of the social marginal product. Rearranging (142) gives,

$$DF_{\epsilon}(0) = e^{-\int_0^T DF_t dt} \left(DF_{\epsilon}(T) - \int_0^T D'F e^{\int_t^T DF_t ds} dt \right) \quad (144)$$

letting T go to infinity,

$$DF_{\epsilon}(0) = e^{-\int_0^{\infty} DF_t dt} \left(\lim_{t \rightarrow \infty} DF_{\epsilon}(t) - \int_0^{\infty} D'F e^{\int_t^{\infty} DF_t ds} dt \right) \quad (145)$$

which must hold not only for $t=0$ but any t ,

$$DF_{\epsilon}(t) = e^{-\int_t^{\infty} DF_t ds} \left(\lim_{t \rightarrow \infty} DF_{\epsilon}(t) - \int_t^{\infty} D'F(s) e^{\int_s^{\infty} DF_t dv} ds \right) \quad (146)$$

which we can call solution (b), which is exactly equivalent to (a) since we have only rearranged the 'reference point'. From inspection of version (b) it is clear that the DF_t path is determined by discounting backwards from the limit value while making allowance for (subtracting) marginal cumulative damage. This accords exactly with the intuition that while there will be a benefit to extraction at t , there will also be an associated stream of costs affecting all future production. The environmental cost is the sum of future marginal damages, each compounded to infinity to reflect the stream of damage caused by permanent additions to the atmospheric CO_2 stock. Equation (146) can then also be written as,

$$DF_t(t) = e^{-\int_t^\infty DF_s ds} \left(\lim_{t \rightarrow \infty} DF_t(t) \right) - \int_t^\infty D'F(s) e^{-\int_t^s DF_v dv} ds \quad (147)$$

These results can be graphed over t as seen in Diagrams 6 (a) to (c). Diagram 6 (a) contains the information in (131)". We begin with the curvature of DF_t as reference. The slope of DF_t ,

$$\frac{dDF_t}{dt} = D'(\Psi) \dot{\Psi} F_k + D(\Psi) (F_{kk} \dot{k} + F_{kt} \dot{t})$$

is decreasingly negative over all t , converging smoothly towards zero at infinity. The socially optimal fundamental efficiency relation is (131),

$$D(\Psi) F_k = \frac{(D(\Psi) F_k)}{D(\Psi) F_k} - \frac{D'(\Psi) F(k, t)}{D(\Psi) F_k}$$

Diagrams 6 (a) to (c)

where $D'F/DF_t$ is negative and U shaped over time. By vertical subtraction we then obtain the curvature of the proportional growth of the marginal product of energy.

Diagram 6 (b) illustrates the increasing curvature of the proportional growth of DF_t for respectively more serious global warming scenarios. If the GW effect is severe, the marginal damage imposed will be larger, hence $dDF_t/dt/DF_t$ will 'dip down' further. No matter how serious the GW threat however, DF_t growth will remain positive, the productivity growth rate approaching zero asymptotically. Were DF_t growth negative, this would imply that,

$$\left| \frac{D'(\Psi)F(k,t)}{D(\Psi)F_t} \right| > D(\Psi)F_k$$

meaning that the marginal proportional benefit of conservation is greater than the marginal benefit of extraction over time. If this were so the optimal strategy would be to conserve even marginal reserves of fossil fuel. But under our current no-options technology, zero extraction means zero output, zero consumption; negative DF_t growth is therefore not a feasible optimal path. **Diagram 6 (c)** then follows, showing that DF_t asymptotically approaches a constant level. Note that a more severe GW threat implies a higher $DF_t(0)$ initial value (less extraction at the outset) and a lower asymptotic limit of $DF_t(t)$.

The latter part of this chapter has contrasted the socially optimal solution under global warming with the Chapter 3 basic general equilibrium extraction problem in the absence of an externality.

We have found that under the optimal consumption program, the effect of global warming is to consistently lower the level of consumption at every point in time and to bring about declining consumption levels earlier in time. The resource- and externality-constrained consumption profile is in dramatic contrast to the usual textbook presentation of never-decreasing consumption and welfare levels.

Alongside lower total consumption, we have found that the productivity of capital is lowered by global warming and that the productivity growth of energy is slowed and that hence the productivity of energy rises less steeply over time.

So far we have contrasted two optimally planned paths, the logical next step in the analysis will be to compare the socially optimal solution with the market outcome under global warming; this will be the task of Chapter 7.

Reference Summary of Key Equations: Optimal Solution/with Global Warming/no Backstop)

$$\dot{k} = D(\Psi)F(k, \epsilon) - c = i \quad (123)$$

$$\dot{\theta} = -\epsilon \quad (124)$$

$$\dot{\Psi} = \epsilon(t) \quad (108)$$

$$\lambda = U'(c) \quad , \quad \frac{\dot{\lambda}}{\lambda} = \theta - D(\Psi)F_k \quad (118),(121)$$

$$\beta = U'(c) D(\Psi)F_\epsilon \quad , \quad \frac{\dot{\beta}}{\beta} = \frac{D'(\Psi)F(k, \epsilon)}{D(\Psi)F_\epsilon} + \theta \quad (119),(122)$$

$$\dot{\phi} = D(\Psi) \eta(\phi) f(\phi) + \frac{D(\Psi)F_\epsilon}{D(\Psi)\phi f''(\phi)} - \frac{D'(\Psi)F(k, \epsilon)}{D(\Psi)\phi f''(\phi)} \quad (137)$$

$$\dot{c} = \frac{U'(c)}{U''(c)} [\theta - D(\Psi)F_k] \quad (129)$$

$$\dot{c} = \frac{U'(c)}{U''(c)} \left[\theta + \frac{D'(\Psi)F(k, \epsilon)}{D(\Psi)F_\epsilon} - \frac{\dot{(D(\Psi)F_\epsilon)}}{D(\Psi)F_\epsilon} \right] \quad (130)$$

$$D(\Psi)F_k = \frac{\dot{(D(\Psi)F_\epsilon)}}{D(\Psi)F_\epsilon} - \frac{D'(\Psi)F(k, \epsilon)}{D(\Psi)F_\epsilon} \quad (131)$$

$$DF_\epsilon(t) = e^{\int_0^t DF_k dv} DF_\epsilon(0) + \int_0^t D'F(s) e^{\int_s^t DF_k dv} ds \quad (143)$$

$$DF_\epsilon(t) = e^{-\int_t^\infty DF_k ds} \left(\lim_{t \rightarrow \infty} DF_\epsilon(t) - \int_t^\infty D'F(s) e^{\int_s^\infty DF_k dv} ds \right) \quad (146)$$

7. Fossil Fuel Dependent Economy under Global Warming: The Market Solution

This chapter models the decentralized solution in the presence of global warming. We use the model we have developed where firms are dependent on fossil fuels. As in Chapter 6, there is therefore no choice of technology. As we have laid out the structure and method of solution quite carefully in Chapter 5, we can now proceed more rapidly, taking earlier definitions of variables and functions, as well as simple derivations, for granted.

The technology employed is such that the effective production function is the composite DF function. It combines global warming production conditions with a standard embedded CRS production function, the latter condition allows us to use the Euler Equation representation of output to write,

$$D(\Psi)F(k, \epsilon) = D(\Psi)(F_k k + F_\epsilon \epsilon) = D(\Psi)F_k k + D(\Psi)F_\epsilon \epsilon \quad (148)$$

The physical extraction and global warming relations are again given as,

$$\epsilon(t) = -\dot{b} \quad , \quad \epsilon(t) = \dot{\Psi} \quad , \quad (149)$$

where we add the further restriction⁵⁸ that,

$$\Psi^c > \Psi_0 + b_0 \quad . \quad (150)$$

⁵⁸ The significance of this restriction will be discussed further on.

The firm's profit maximization problem now becomes,

$$\text{Max}_{(k,\epsilon)} V(0) = \int_0^{\infty} \{ e^{-\int_0^t r(u) du} [D(\Psi)F(k,\epsilon) - Rk - q\epsilon] \} dt \quad (151)$$

FOCs are,

$$D(\Psi)F_k = R \quad (152)$$

$$D(\Psi)F_{\epsilon} = q \quad (153)$$

where effective marginal products now reflect the damage (or compression) due to global warming.

The asset market equilibrium condition on returns is, as before,

$$r = R = \frac{\dot{q}}{q} \quad (154)$$

so that returns from holding reserves of the natural resource are again pure capital gains.

The households' problem is,

$$\text{Max}_{(c)} U(0) = \int_0^{\infty} e^{-\theta t} U(c(t)) dt \quad (155)$$

with wealth budget constraint,

$$\text{s.t. } \dot{a} = ra - c \quad (156)$$

where the value of assets at any one time is the value of the physical capital stock

maintained plus the value of fossil fuel reserves kept at any one t :

$$a = k + qb \quad (157)$$

where the aggregate quantity of b at any time is known to households, and where the aggregate quantity of b at $t=0$ is known and positive.

The households' FOCs solve to the familiar,

$$\frac{\dot{c}}{c} = \frac{U'(c)}{U''(c)c} (\theta - r) = \frac{1}{\sigma(c)} (r - \theta) \quad (158)$$

Substituting in asset and firms' FOCs, (154), (152) and (153), we obtain two differential equations for choice variable c :

$$\dot{c} = \frac{U'(c)}{U''(c)} (\theta - D(\Psi)F_k) \quad (159)$$

$$\dot{c} = \frac{U'(c)}{U''(c)} \left(\theta - \frac{\dot{(D(\Psi)F_e)}}{D(\Psi)F_e} \right) \quad (160)$$

which can be consolidated to give the fundamental private efficiency condition, a differential equation in the marginal product of energy,

$$D(\Psi)F_k = \frac{\dot{(D(\Psi)F_e)}}{D(\Psi)F_e} \quad (161)$$

This in turn solves to,

$$DF_{\epsilon}(t) = e^{\int_0^t DF_{\epsilon}(s) ds} DF_{\epsilon}(0) \quad (162)$$

which can equivalently be expressed as,

$$DF_{\epsilon}(t) = e^{-\int_t^{\infty} DF_{\epsilon}(s) ds} \lim_{t \rightarrow \infty} DF_{\epsilon}(t) \quad (163)$$

so that $DF_{\epsilon}(t)$'s path is derived by discounting backwards from the terminal limit. Once this has been done and $DF_{\epsilon}(0)$ has been determined, the path can equivalently be shown by compounding forwards in time as in (162).

Of the other two required equations of motion for endogenous variables k and ϵ , one is parametrically given in (149), the other is derived as follows. We take wealth i.e. the value of assets as given in (157), and differentiate with respect to time,

$$\dot{a} = \dot{k} + \dot{q}b + \dot{b}q \quad (164)$$

set this equal to the growth of wealth as permitted by the wealth budget constraint,

$$\dot{k} + \dot{q}b + \dot{b}q = ra - c \quad (165)$$

expand the first term on the RHS using (154),

$$\dot{k} + \dot{q}b + \dot{b}q = Rk + \frac{1}{q}qb - c \quad (166)$$

$$\Rightarrow \dot{k} + \dot{b}q = Rk - c$$

substituting in the firms' FOCs and physical condition (149),

$$\Rightarrow \dot{k} - \epsilon D(\Psi)F_{\epsilon} = D(\Psi)F_k k - c \quad (167)$$

$$\Rightarrow \dot{k} = D(\Psi) [F_k k + F_{\epsilon} \epsilon] - c$$

which, given (148), solves to the optimal dynamic condition on capital accumulation for the market economy,

$$\dot{k} = D(\Psi) F(k, \epsilon) - c \quad (168)$$

Lastly, we inquire as to the movement of the capital/energy ratio over time;

$$\eta(\phi) = \frac{F_{\epsilon} F_k}{-\phi f(\phi) f''(\phi)}$$

as before; we take the fundamental efficiency condition (161) and substitute in,

$$\dot{F}_{\epsilon} = \frac{d(f(\phi) - \phi f'(\phi))}{d\epsilon}$$

solving obtains,

$$\dot{\phi} = D(\Psi) \eta(\phi) f(\phi) + \frac{D(\Psi) F_{\epsilon}}{D(\Psi) \phi f''(\phi)} \quad (169)$$

This completes the decentralized solution. We now turn to compare the optimal and market solutions under global warming. Direct comparison of key conditions reveals a number of similarities and differences³⁹.

Productivity of factors of production has been compressed by global warming in both solutions; effective marginal products thus are preceded by $D(\Psi)$ throughout. On the 'demand' side the two problems remain the same, and the condition underlying the consumption/investment choice (equations (129) in the social and (159) in the private solution) are structurally equivalent.

Significant structural differences enter in the equations relating to the supply of energy i.e. extraction. The condition governing the consumption/extraction choice or trade-off is found in equation (130) in the social solution and contains an extra term not found in the corresponding equation (160) in the private solution. We recall that this term originates (in terms of our solution methodology) in equation (122), the intertemporal optimality condition on the shadow price of fossil fuel reserves, b . This term, earlier referred to as the global warming conservation effect, will turn out to be key -- it represents the internalization of the externality in the socially optimal solution. Accounting for the externality, the social solution's fundamental efficiency condition (131) then differs from the efficiency condition governing the valuation of relative productivities in the market

³⁹ See summary of key equations found at the end of Chapter 6.

solution (161) (again by the same term). Solutions of the dynamic equation for the timepath of $D(\Psi)F_e(t)$ are then correspondingly different.

While it is relatively easy to point out the structural differences in corresponding equations of the two solutions, it is more difficult -- but more insightful -- to present the different dynamics achieved by these structural differences. This is what we turn to in the next section of this chapter.

Comparison of the Market and Socially Optimal Solutions

In comparing the 'evolution' of the two dynamic systems of simultaneous equations, the socially optimal and market solutions to the most general possible global warming problem, it is apparent that they both asymptotically describe a steady state solution -- approaching a constant (zero) level of economic activity at infinity. As such, both systems are extremely stable from the modeller's perspective.

If the solutions are asymptotic to infinity and equivalent in this respect, then it is apparent that the usual 'economics comparison' procedure of assuming steady state (ss) and then performing a series of comparative static (ss) shocks⁶⁰ will not be possible. By the structure of the problem, if we assume steady state, this is equivalent to assuming we are 'at infinity'.

⁶⁰ See Neher, Chapter 11 on exercises of this nature.

The alternative then is to examine the time paths of endogenous variables of particular interest, and obtain an understanding of the differences in the internal motion of the system over time⁶¹. The method is to solve for some specific time paths, that can then be used together with knowledge of curvatures to 'mark' certain points in time. This knowledge then allows us to compare different evolution paths such as those of the social and private solutions above. Before proceeding we should note that appearances can be deceptive. For instance, equations (123) and (168) on market driven and socially planned capital accumulation appear identical. This however does not imply that the two solution time paths are identical -- the endogenous variables involved may be taking on different values at comparable times -- the similarity in the equations thus merely implies that they are shaped similarly.

The easiest way to present and interpret the results for the two systems is in graphical form. We assume that initial values for state variables b_0 , k_0 , and Ψ_0 are identical; it is important to note however that the initial values for the endogenously solved variables, such as $c(0)$, $DF_x(0)$ and $DF_t(0)$ will differ for the social (S) and private (P) solution paths⁶².

⁶¹ The phase diagram is usually a convenient avenue (short cut) to this, however also unavailable to the interpretation of this model due to the large number of endogenous variables.

⁶² Regarding the notation used to annotate the graphs: 'S' refers to the socially optimal paths and 'P' (for private) refers to the decentralized or market solution. 'D' is 'damaged' for short or 'inverse damage index'.

For the comparative productivity of capital *qua* interest rate, the growth of consumption and the consumption level, we see from Diagram 7 (a) to (c) that the private solution initially begins with a higher capital productivity. This is due to the fact that a higher initial level of energy is employed, thus relatively pushing up the productivity of the complementary factor capital. The higher private activity level then also causes Ψ to accumulate faster, and then the global warming direct effect [$PD(\Psi)$] cuts into productivity more rapidly than in the social solution⁶³. The PDF_t path falls more rapidly, approaching the asymptotic base level of activity sooner throughout Diagrams 7 (a) to (c). The overall effect of this results in a far greater overall consumption area for the social solution (irrespective of discounting).

This finding coincides with ones general intuition that the market solution in the presence of an un-internalized negative externality will lead to initial 'over-consumption' in the short run and result in less consumption overall as compared to what would be accomplished in the social optimum.

In Diagrams 7 (a) to (c), it is assumed that the private rate of time preference across households, happens to coincide with the social discount rate θ , as divined by the social planner. The effect illustrated is therefore entirely due to the externality and not due to different private and social rates of discounting. The choice of discount rate is a topic of fundamental importance in environmental economics and a sub-literature exists which (a)

⁶³ This accounts for the more pronounced convexity of the latter part of the PDF_t path.

Diagrams 7 (a) to (d)

empirically shows that private discount rates are significantly higher than public discount rates and (b) provides a number of theoretical explanations for this observed behaviour.

The importance of the choice of discount rate can be illustrated in our model. Should the private θ be higher than the socially optimal rate -- as it is widely acknowledged to be -- then the trade-off between current (or near-by) consumption and future consumption will be accentuated. This is illustrated in Diagram 7 (d), by respective shifts in the consumption profile. We see that total consumption decreases with the use of higher private discount rates.

Overall then the social or optimal path is that of an *initially more moderate but over time more sustained growth scenario*⁶⁴. To complete the diagrammatic interpretation of results, the relative socially optimal and market-solution-generated suboptimal time paths of extraction rates, fossil fuel reserve levels, and CO₂ accumulation levels remain to be shown. These paths are illustrated in Diagrams 8 (a) to (c), and it is seen that private extraction levels initially exceed optimal energy use, but subsequently must fall below the optimum. Correspondingly the depletion Diagram 8 (b) shows consistently higher remaining reserves optimally i.e. excessively rapid depletion if the market outcome is not corrected. Finally Diagram 8 (c) on the accumulated concentration of CO₂, Ψ , shows that the sub-optimal market outcome leads to consistently higher levels of global warming.

⁶⁴ The relative productivity paths of capital and energy are illustrated in an Appendix to this chapter.

Diagrams 8 (a) to (c)

The systematic exposition of the comparative suboptimality of the market solution, then also directly suggests the *optimal corrective policy* that should be undertaken to correct the market failure. Such market correction or intervention will be discussed in the next chapter.

Appendix to Chapter 7: Optimal and Market-generated Productivity Trends

Examining fossil fuel paths, one may map the information contained in the fundamental efficiency equations and the time path solutions for DF_t . Beginning with a known path we graph SDF_t and PDF_t over time. Due to more prudent fossil fuel use, $SDF_t(0) < PDF_t(0)$. The fundamental efficiency equations are,

$$PD(\Psi)F_t = \frac{\dot{(D(\Psi)F_t)}}{D(\Psi)F_t}$$

$$SD(\Psi)F_t = \frac{\dot{(D(\Psi)F_t)}}{D(\Psi)F_t} - \frac{D'(\Psi)F(k, \epsilon)}{D(\Psi)F_t}$$

These relationships are given in **Diagram 9 (a)**.

From this, using the simple private solution as reference, the growth in relative marginal productivity of fossil fuel over time can then be shown as in **Diagram 9 (b)**.

In **Diagram 9 (c)**, the path of DF_t is then such that it converges to a constant value for both the private and social cases, with,

$$SD(\Psi)F_t(0) > PD(\Psi)F_t(0) \quad .$$

Diagrams 9 (a) to (c)

8. Optimal Market Intervention

In static problems, once the cost due to an externality -- the difference between private marginal cost and social marginal cost -- has been identified, optimal policy is readily apparent. The economist's advice is that a Pigovian tax, equivalent to the externality cost, be levied, thereby efficiently forcing the market to internalize the externality at the production margin⁶⁵. For the complex dynamic problem that this paper has modelled -- incorporating stock exhaustion effects and a cumulative externality -- the tax solution to internalizing the externality is not quite as straightforward as in the static case.

With regard to global warming, the common understanding that taxes may be used to offset missing incentives to the market in cases of negative externalities, has led to calls for the imposition of a '*carbon tax*'. This advocacy has included calls for a *rising* carbon tax which would match the increasingly costly externality.

⁶⁵ Direct regulation (i.e. quantitative restrictions on output across an industry) is shown to be less efficient on the margin, and is recommended as second best in cases where marginal cost schedules cannot be estimated but an overall desired ceiling on permissible pollution level has been determined. Tradeable emission permit systems are then seen to combine the virtues of efficiency (trading creates a price at the margin) and lesser data requirements (emission quantity is directly controlled through restricting the total number of permits issued) although they may have higher administrative costs.

We briefly turn to review the basic principles of the taxation of exhaustible resources⁴⁶ and relate them to our model.

Recall that we have found the optimal intertemporal supply condition for fossil fuel energy (an exhaustible resource) to be such that the real resource price of fossil fuels, q , must be equal to the expected opportunity cost of depletion. Admitting positive marginal extraction costs, under perfect foresight, this is equal to the marginal cost of extraction (a function of b) plus rent, R . The rent is a return for waiting, or can equivalently be interpreted as the pure factor earning of below-ground resource units. Using discrete time and a partial equilibrium framework for simplicity, one can write,

$$\begin{aligned} q_t &= MC_t + R \\ q_t &= MC_t + (q_{t+1} - MC_{t+1}) \left(\frac{1}{1+r_t} \right) \end{aligned} \tag{170}$$

When extraction costs are zero as in our model, a *once and for all time increase in the level of tax* has no intertemporal effect on this equilibrium condition, since for,

$$(1-\tau) q_t = (1-\tau) (q_{t+1}) \left(\frac{1}{1+r_t} \right) \tag{171}$$

⁴⁶ Exhaustible resources, particularly energy and minerals, are taxed and subsidized variously in practice, once the field is further opened to theoretically feasible taxes, the possibilities and combinations of possibilities are virtually endless. There are profits taxes (also referred to as rent taxes), Brown taxes, taxes on per unit output, ad valorem taxes, severance taxes, and property taxes, then there are depletion allowances, depreciation allowances, exploration subsidies, tax holidays and other favours. A large (dynamic) partial equilibrium literature exists on the (distortionary) effects these various taxes will have on extraction rates, depletion dates and recoverable reserves under various assumptions regarding costs and market structure. For our purposes we will concentrate on simple profits taxes alone.

we can 'cancel' the taxes i.e. energy owners cannot avoid the tax by shifting production from one period to another. While inter-temporal allocation of extraction is thus not directly affected, the tax will however result in a general equilibrium distortionary effect. By making fossil fuel more expensive relative to capital, the tax effects a shift away from energy use. Therefore, (with reference to Diagram 8), $\epsilon(0)^* < \epsilon(0)$ and $DF_A(0)^* > DF_A(0)$, thereby shifting all timepaths of the private solution above in the desired direction.

Graphic representation of this result in a *static* framework is made difficult by the inter-temporal nature of the determination of supply. The overall stock of the resource is fixed. For a non-exhaustible resource such as land, this implies a vertical supply curve. For energy however, a vertical supply curve would represent the aggregation of amounts supplied over all time; it therefore is not relevant to a 'period' optimization diagram except in the sense of an extreme limit.

What we can do, is conceptually derive an '*effective*' supply curve from the intertemporal equilibrium condition⁶⁷. From the view of an energy owner/supplier: for prices offered above the opportunity cost of the resource, she is willing to sell her entire stock. For prices below the opportunity cost of waiting, she prefers to wait and offers nothing for sale; when market price is equal to her opportunity cost she is indifferent to the quantity sold. This maps out a horizontal individual '*effective*' supply schedule, aggregating

⁶⁷ Intuitively, deriving a supply curve *from* an equilibrium condition, seems illegitimate. It is however effective towards illustrating the tax incidence; and since everything is determined simultaneously, mathematically it seems fine.

individual supplies horizontally gives an equally horizontal industry supply curve (bounded only on the extreme right of the quantity axis by the total stock remaining).

With a horizontal effective supply curve, tax incidence falls entirely on the consumer, in this case the firm demanding the factor input. The conventional textbook notion that the taxation of natural resources is a 'pure taxation of rents', that has no distortionary effects on production, should therefore not be assumed to hold for all cases.

In the presence of positive marginal costs, we can distinguish between a rent tax and an excise tax or royalty that is levied per energy unit extracted. The *rent tax* has the same effect as above; in that in

$$(1 - \tau)(q_t - MC_t) = (1 - \tau)(q_{t+1} - MC_{t+1})\left(\frac{1}{1+r_t}\right) \quad (172)$$

we can cancel the tax showing the absence of suppliers intertemporal reallocation. In the presence of MC the supply schedule will now be positively sloped, and tax incidence will therefore be shared by suppliers and users. The desired intertemporal distortion effect of the tax with respect to capital and energy use (to offset the externality) will therefore be less.

The *royalty tax* on the other hand will also *directly* shift extraction to the future since in,

$$(1 - \tau)(q_t) - MC_t = [(1 - \tau)q_{t+1} - MC_{t+1}]\left(\frac{1}{1+r_t}\right) \quad (173)$$

the tax does not affect the two periods equally. Initial extraction levels therefore fall and the depletion path is more gradual over time. In the overall model this then again represents a shift of the private paths towards the optimal paths.

We therefore find that various taxes are beneficial in reducing the externality associated with energy use; in the absence of a specific global warming or carbon tax, desirable in particular are those taxes which effect a *high degree of distortion* away from fossil fuel use. And clearly the converse also holds with respect to energy and exploration subsidies, tax breaks and tax reform which increases the neutrality of energy taxes.

On a more general level, the model illustrates quite clearly how taxation *policies designed for economic objectives will inevitably have environmental impacts*. Of particular note of course are those economic policies which are designed for short term objectives (eg. granting rainforest concessions and tax holidays due to balance of payments problems in Latin America, Hibernia subsidies for short term employment creation, exploration cost write-offs due to OPEC fears etc.) but have direct long-term and irreversible environmental and associated future economic welfare consequences.

Since global warming costs are increasing over time, and taxation has been shown to be beneficial in off-setting the externality, one might initially be led to think that corrective taxes should also be imposed at a rising rate over time, or generally that rising energy taxes are desirable in the absence of specific carbon taxes. A rising tax level on

exhaustible resources, it turns out however, serves to worsen the associated externality.

A rising tax directly affects the intertemporal supply decision where for $\tau_t < \tau_{t+1}$,

$$(1 - \tau_t) (q_t - MC_t) = (1 - \tau_{t+1}) (q_{t+1} - MC_{t+1}) \left(\frac{1}{1+r_t} \right) \quad (174)$$

it is apparent that the supplier will attempt to shift extraction towards the present, in order to avoid the larger taxes in the future. This would serve to aggravate the global warming externality and the corrective policy would turn out to be extremely counter-productive⁶⁸.

We now turn to see how this is borne out in our model. Comparison of the private and socially optimal solution paths of the model show marginal externality costs at any time t to be the difference between equations (147) and (163), so that at any time t , the Pigovian tax $\tau(t)$ to internalize the cumulative externality is,

$$\tau(t) = - \int_t^{\infty} D'(\Psi) F(k, \epsilon) e^{-\int_t^s D F_s ds} ds \quad (175)$$

where we see that the level of the tax is given by the sum of discounted future damages caused by the marginal fossil fuel unit extracted and used in production at time t . The tax expression underlines the cumulative nature of the externality once again. The externality cost is a stream of costs stretching into the infinite future. In our perfect model, if this

⁶⁸ The overall effect on actual extraction rates will then also depend on the price elasticity of the demand curve over the relevant portion.

optimal Pigovian tax is applied, this will bring the market solution exactly in line with the socially optimal.

With respect to the tax rate over time (as opposed to the level of taxation), with $(-D'F)$ greater than zero for all t , as t approaches infinity, the value of the integral in (175) (a sum) is monotonically decreasing in size, implying a *falling tax rate over time* to offset the cumulative global warming externality. The dynamic model developed in this dissertation therefore shows that optimal intervention consists of a falling tax rate over time. And conversely, rising energy taxes (whether applied with environmental objectives or not) will have an adverse environmental impact.

The difficulties and large information requirements associated with calculating optimal environmental taxes in a static framework are discussed in most textbooks on the subject, and are often formidable. These difficulties pale in comparison to the calculation involved above. By making the corrective tax explicit mathematically, our most simple model shows that computation of τ involves computing the present value of all future marginal cumulative external costs of a unit increase in extraction. This involves knowledge of all other economic and environmental variable paths to the infinite future. It is the cumulative nature of the externality, which in turn requires this foresight into the economic future, and that complicates the estimation of the optimal global warming 'carbon tax'.

This finding makes a case for direct emission (or quantity based) regulation of CO₂ emissions levels in favour of tax-based (or price mechanism based) strategies⁶⁹. Given an uncertain economic environment, risks of long term welfare costs due to economists' miscalculation are lowered by direct measures. While still formidable, due to the uncertainty regarding potential global warming impacts, the information requirements for direct regulation are much lower -- focusing on the timepath of global Ψ . If the concentration level of CO₂ that is deemed critical as regards climate stability can be estimated based on physical impact data and associated costs to the habitat, then a notional limit (or target limit) can underlie policy, and a physical path of maximally allowable increases in emissions calculated. This calculation would be independent of estimates of future economic growth rates and interest/discount rates. A quantitative policy targeted directly at reducing the current level of CO₂ emissions (or, more realistically, at reducing the current global growth in CO₂ emissions) furthermore could be utilized in a tradeable system -- a system trading CO₂ emissions in one location for similar emissions in another and -- a system potentially efficient 'on the margin'. The 'trades' likely to result from the imposition of carbon taxes on the other hand, are likely to be trades (or switches) between different taxes to assuage energy stakeholders -- eventual carbon taxes would have to compete against a myriad of other current energy policy instruments whose net effect on fossil fuel depletion internationally is near impossible to determine. International monitoring and verification of compliance concerns therefore also speak in favour of quantitatively, rather than price/tax based regulation.

⁶⁹ This could be extended to an advocacy for a system of tradeable emissions permits, which also essentially has a 'CO₂ level' based information requirement.

Quantitative restrictions (or CO₂ quota) would also meet the criteria "one policy objective per policy instrument" and are more likely to be understood by the public and its administrators. Worldwide, depending on nation's fossil fuel endowments and level of development, energy policies and their objectives differ widely. Given however that a common global policy objective -- countering the global warming threat -- exists, agreement on distinct CO₂-targeted quantitative instruments is likely to be far easier, than agreements on the use of price and taxation rate schedule based energy policies.

Summarizing the above in a nutshell, the prescription is for targeted direct command and control regulation accompanied by high, distortionary and falling taxes in general.

9. Global Warming: Warm, Warmer, Hot?

We can now turn to consider three global warming scenarios that are drawn in the broadest of brushstrokes. The scenarios consider three possible levels of global warming threat.

Scenario 1: Global warming is expected to impose damage, but on balance presents only a moderate threat. Although it is recognized that a negative externality exists, the loss of discounted future utility associated with unregulated fossil fuel use is not of an alarming magnitude. The (so far unconsidered) costs, inefficiencies and externalities incurred by governments' intervention, together with the transactions costs of eventual inter-governmental cooperation, with respect to world energy use are estimated to outweigh benefits.

Scenario 2: Global warming is expected to impose severe damage, presents a serious threat to future welfare. The benefits of slowing global warming are estimated to clearly outweigh the costs of doing so. However, the concern closer on the time horizon is the energy crisis caused by the exhaustion of essential fossil fuels. Depletion is the dominating concern. This case implies that even if all currently existing fossil fuel sinks' CO₂ contents

were to be transferred into the atmosphere, the resulting global warming damage would not outweigh the resource scarcity concern over time.

Scenario 3: Fossil fuel reserves are relatively vast; the dominating and critical threat is that of global warming. Partial depletion of total fossil reserves suffices to destabilize the environment to an extent that imposes critical damage on welfare and disrupts orderly economic activity well before depletion appears on the near horizon.⁷⁰

Besides bargaining costs, Scenario 1 implicitly raises the issue of market-failure versus government failure in efficient allocation. While it is known that the status quo is not optimal, since costs of intervention exceed benefits, the problem only has a 'second-best' solution: do nothing, business as usual. The proven existence of a negative externality therefore is not a sufficient condition to warrant intervention.

Scenario 2, is the case that this dissertation has addressed so far. The reader will note that the private solution above has been explicitly derived under Scenario 2 conditions; recall restriction (150). The socially optimal and decentralized solution paths for Scenario 2 are such that while fossil fuels are not depleted in finite time, stocks nevertheless asymptotically approach complete depletion. This was the result of the basic scarcity

⁷⁰ Although no supporting evidence is offered due to the necessarily limited scope of this thesis, it is suggested that Scenario 3 is in fact the most likely -- especially if the lives and livelihoods of those living in developing countries are not 'discounted' by a Northern, industrialized country perspective.

framework laid out in Chapters 3 and 5. Relative to these solution paths, imposing the global warming constraint then results in compression of output levels and shifts in extraction profiles over time.

We have shown that the global warming externality presents a distinct constraint in addition to the scarcity constraint. While the market will tend to generate price paths suitable for inter-temporal allocation in production, these scarcity-induced price-rises do not subsume, replace or account for, the externality. Even though scarcity and exhaustion may be the dominating concern, separate policy intervention is warranted to address global warming. Both effects then tend to encourage conservation.

In part the answer to the question of where Scenario 1 ends and Scenario 2 begins, hinges on the discount rate used. Costs of bargaining, negotiation and international cooperation are largely current costs. Benefits of avoided global warming are largely future benefits. Depending then on how lightly or heavily the welfare of future generations is weighted, the range of applicability of the Scenario 1 case description will increase or decrease. If the public discount rate is seen as an *economic institution*, then this can again be seen as the crucial interplay of economic and environmental variables.

Scenario 3, presents a case that we have not addressed thus far; the difference in Scenarios 2 and 3 however, is significant for the solution of the model presented in this paper.

Recall that the critical, or perhaps more appropriately referred to as 'doomsday', level of CO₂ concentration is denoted by Ψ^c where $D(\Psi^c) = 0$. This level corresponds to a critical level of extraction of fossil fuels,

$$\begin{aligned}\Psi^c(t) &= \Psi_0(0) + \left(\int_0^t \epsilon(s) ds\right)^c \\ &= \Psi_0 + (b_0 - b(t))^c \\ &= \Psi_0 + b_0 - b^c\end{aligned}\tag{176}$$

where we have denoted the critical (minimal) level of remaining reserves as b^c . The inequality condition that separates Scenario 2 from potentially catastrophic global warming outcome under Scenario 3 is,

$$\Psi(t) < \Psi^c(t) \quad \forall t \tag{177}$$

which can equivalently be written as,

$$\begin{aligned}\Psi^c &> \Psi_0 + b_0 \\ \Rightarrow b^c &= 0\end{aligned}\tag{178}$$

since b levels cannot be negative.

The socially optimal time paths for $\epsilon(t)$, $b(t)$ and $\Psi(t)$ are now shaped as illustrated in **Diagrams 10 (a) to (c)**. The social program recognizes the limit imposed by the GW constraint and extraction is curtailed to asymptotically approach zero along this (stock)

Diagrams 10 (a) to (c)

limit. *In the optimal solution, positive reserves of fossil fuels will therefore remain underground to infinity.*

The decentralized solution to this variation of the problem is dramatically different.

We have seen that energy owners, in order to determine the current price, determine the terminal or limit price on the last (marginal) unit of fossil fuel and then discount backwards by the respective interest rate. Under Scenario 3, using perfect foresight, energy owners realize that some reserves will remain underground indefinitely. The value of the last unit is therefore zero; discounting zero back through time then results in the collapse of the entire price path and all reserves will be available immediately at zero cost.

If knowledge of the critical/doomsday global warming level is added at a certain time to (· previously incomplete!) knowledge set of energy owners, the future energy price path will implode instantaneously.

The same argument can be made intuitively, looking forward instead. Energy owner 'j' owns her energy stock b_{j0} which is identical to all other reserves held by the other $N-1$ identical energy owners. Owner j wishes to maximize the present value of *her* reserves. She knows that overall some stocks will 'never make it to the market' due to the global

warming constraint⁷¹. To make sure that it will not be her reserves that stay unexploited, she decides to forego a little revenue, in return for a 'certain' sale i.e. she undercuts the market price by a small amount. In the meanwhile all other energy owners act identically, and the price of fossil fuels is immediately competitively bid to zero.

Essentially what happens is that the global warming constraint now dominates the scarcity constraint. Since there will be 'excess reserves' in the limit, fossil fuels are no longer economically 'scarce', and there will be no conserving 'scarcity price'. Depending on the nature of the aggregate production function, Ψ^c will be reached instantaneously (the popular Cobb-Douglas specification will achieve this) or after a number of production periods⁷².

Such behaviour however, is obviously inconsistent with observed behaviour, and we conclude that while the assumption of constant or zero extraction costs under Scenario 2 is merely a *simplifying assumption* (as commonly made in the natural resource literature), under Scenario 3, it becomes a *crucial assumption*, leading to a model outcome inconsistent with even casual empirical evidence. We therefore turn to see how the additional assumption of rising marginal costs will affect the decentralized Scenario 3 solution.

⁷¹ Her actions alone cannot alter the progress of GW, they only account for an infinitesimal amount of the damage. She takes GW as given.

⁷² Optimal intervention by way of the carbon tax derived in the previous chapter or equivalent quantitative regulation of emissions, would obviously be warranted, and would steer the economy onto the optimal paths illustrated in Diagrams 11.

Rising Marginal Costs

Additionally then we assume smoothly rising marginal costs over the total amount of fossil fuel extracted economy-wide (i.e. it costs more to dig deeper into reserves). Note that the assumption of positive constant marginal cost would not be sufficient to our purpose *nor* is the assumption of rising marginal costs over the extraction rate (i.e. it costs more to dig faster). The former condition still will result in price instantaneously falling to the fixed MC; the latter condition implies an intercept of $MC=0$ at $\epsilon=0$, implying again the undesired limit-zero terminal price.

With rising marginal costs, energy owners can now calculate a positive terminal price. Letting T denote the terminal time point where CO_2 concentration effects compress output completely, we have,

$$D(\Psi^c) F_k = 0 = r(T) \quad (179)$$

Fossil fuel rents at T are therefore zero and the efficiency condition reduces to,

$$q(T) = MC(T) \quad (180)$$

reserves with extraction costs above $MC(T)$ are recognized to be effectively valueless and do not affect the problem⁷³. One can therefore derive a stable path for prices, energy rents, and extraction rates to terminal time T . The decentralized market outcome does not

⁷³ We assume that knowledge of global warming also extends into the past. If global warming were suddenly added to an 'almost perfect foresight' information set, the value of these reserves would be capitalized.

internalize the GW-induced future costs of extraction, and (in contrast to the socially optimal path) extraction does not approach zero rate asymptotically to T since,

$$\lim_{t \rightarrow T} q(t) = \lim_{t \rightarrow T} DF_{\epsilon}(t) \quad (181)$$

solves for a positive level of ϵ in the market outcome, Ψ therefore continues to smoothly and directly accumulate to Ψ^c .

Decentralized market agents -- having perfect foresight -- appear therefore to be in the 'interesting' situation of knowing that the collective impact of their individual utility and profit maximizing actions will lead to the termination of economic activity and generation of utility at time T.

To the economist -- as well as any other rational by-stander -- this result is radical and has some strong counter-intuitive components. The finite horizon implies a loss (or cost) of the discounted utility streams to infinity of N (very many) individuals. And marginal utility at the 'survival margin' is infinite to boot. Surely rational agents would not incur a cost of (loosely speaking) N times infinity squared!

We therefore 'double-check' the result *within the behavioural parameters given by the model*, i.e. we ascertain internal consistency.

Individual Action

Given the cost of certain death to the individual -- the loss of marginal utility approaching infinity looms large, even if discounted over long periods -- we consider whether individual action can change the collective outcome. Period-by-period an individual's energy extraction activity is insignificant relative to the (collective) market activity -- this follows from the competitive condition.

But could a (well-endowed) individual not choose to forego all consumption save the barest minimum and 'buy out' global warming? There are two problems with this. Firstly, while utility may be approaching an infinite number of 'utils', the individual is still operating subject to a budget constraint. With perfect capital markets, the individual therefore can borrow up to the net present value of lifetime wealth. To buy out global warming Mr. Rockefeller needs to buy (and conserve) a total volume of fossil fuels of b^c , valued at $(q(t) - MC)^{74}$. Secondly, if b^c is a *significant* amount (relative to b_0), then such a purchase *must be* outside the total budget-constrained reach of an individual -- this again follows from the competitive condition, as applied across time⁷⁵. Were this not so, an individual generally could have influence on price paths over time by her actions alone⁷⁶.

⁷⁴ The conservationist only pays $q(t) - MC$, the energy user/demander pays $q(t)$. The CO₂ conservationist must ensure that he buys the rights to fossil fuel stocks for which the marginal cost of extraction is low enough that they would have been extracted before 'doomsday'.

⁷⁵ This could be phrased as 'individuals are price path takers'.

⁷⁶ An ancillary result of this thought experiment is that monopolists tend to work in favour of conservationists.

One can therefore conclude that individual action can forestall the finite horizon solution of Scenario 3 if and only if a *corner solution* exists such that

a) b^c happens to be very close to zero, and

b) the distribution of wealth is such that there exists at least one individual j such that $PV a_j(t) > b^c (q(t) - MC_k)$ where $MC_k > MC_k(T)$.

Save for the (remote) possibility of this corner solution⁷⁷, the finite horizon can then not be extended by individual action.

Collective Action and Cooperative Solutions

Up to now we have implicitly assumed that collective solutions to the GW externality are ruled out by the very large transactions costs involving N agents bargaining towards a cooperative solution. This is in line with the common notion that the market does not solve public externalities involving common property resources. Since there are no property rights, no private market price mechanism is available. True preference revelation problems therefore arise and further, the greater the number of individuals involved, the greater the free rider problem.

⁷⁷ An aside observation: it is interesting how wealth and optimal conserving behaviour are related here. It brings to mind the conjunction of poverty and environmental degradation worldwide on one hand, and international debt-for-nature swaps undertaken by 'rich' creditors on the other hand.

Two conceptually distinct ways offer themselves to uphold the model result that the market solution has a finite (fatal) horizon and that private optimal arrangements will not be made. Firstly, one may argue that the strategic behaviour and bargaining required for a cooperative solution is not permitted under the bounded rationality behavioural parameter upon which the entire decentralized, competitive model rests (see footnotes 37 and 38).

Secondly, one can argue that the bargaining process and enforcement arrangements required to internalize an externality affecting all N agents is exactly equivalent to that which is more commonly referred to as 'government'⁷⁸. But 'government' clearly has not been defined as an agent existing in the model. If government solves the externality whether by unitizing production or by means of taxes, subsidies, rules and regulations, then by commonly accepted definition, this is *not* a private, decentralized market solution.

The decentralized model then, as defined, provides only for individual utility and profit maximizing behaviour. This model is internally consistent as it stands and solves to a finite doomsday horizon under Scenario 3 (with the addition of marginal costs). Further,

⁷⁸ Democratic government here is only one among many possible bargaining outcomes. Any government will do (for lifting the finite horizon) as long as it is able to enforce its solution.

the model is not so grossly at odds with observed reality as to warrant outright rejection⁷⁹.

⁷⁹ Should one wish to debate in this questionable manner -- relating the feasibility of a theoretical model's equilibrium existence to previously observed real-world phenomena -- the Montreal Protocol on the reduction of CFC use may be raised as a counter. However, the number of agents affected (current stakeholders) was far less than for GW and the uncertainty regarding the ozone depletion is less than that for GW. Witness (as cases in point) on the other hand, the phenomenon of fish stock exhaustion depriving the agents of their livelihood, and the self-imposed radical environmental degradation of Easter Island leading to the extinction of the Island's population. 'Death by externality' is therefore a phenomenon which has been observed, and is therefore a feasible model outcome.

10. Summary and Conclusions

This dissertation has been essentially concerned with the 'theory' of the environment-economy relationship, with particular reference to global warming. It is an exercise conducted at a high degree of abstraction and aggregation, seeking to build on only the most fundamental of physical relationships and the most basic of economic relationships.

A general model of global warming is developed, building on the most orthodox general equilibrium model. The model does not depend on special, suspect assumptions to derive counter-intuitive results. Rather, outcomes are the result of *combining* dynamic welfare optimization, the economics of resource extraction and an environmental cumulative externality. Conclusions and implications therefore are of general interest and relevance.

The approach taken is highly systematic, contrasting socially optimal and market dynamic outcomes step by step. *Chapter 3* begins with the most elementary neoclassical general equilibrium model in general functional form. It assumes perfect foresight, constant technology, identical infinitely lived individuals, a utilitarian time-separable social welfare function, twice differentiable concave production and utility functions in a three factor world. Labour is held in fixed supply with population growth rate zero. Capital is a composite good which may be consumed or invested. Fossil fuel energy is available in fixed supply and is essential to production. Using the technique of dynamic optimization to an infinite horizon to solve, it was found that in the socially optimal outcome, (a) the

energy stock will never be exhausted, the extraction rate declines until it asymptotically approaches zero over time; (b) the productivity of capital falls throughout as more and more capital is used to substitute for declining energy inputs and the capital/energy ratio rises throughout; and (c) consumption may initially rise (depending on the initial stock of capital), but then must decline and asymptotically approach zero. The model has nothing to say about the distribution of income at any one point in time, it does however address the inter-temporal distribution of income. Due to the use of a discount rate, income is 'front-end loaded', i.e. the model does not result in a sustained growth or welfare path.

In *Chapter 4* a new assumption on technology was introduced: a backstop technology which employs a substitute non-fossil fuel energy at a relatively high fixed cost. Again the socially optimal solution was derived. It was found that the effect of the backstop technology is the more rapid depletion and then complete exhaustion of fossil fuels, whereafter a switch to the substitute energy is made. Consumption rises, then stays at a constant level after the switch. A higher discount rate results in faster depletion/sooner exhaustion/earlier switching.

In *Chapter 5* the social planner is replaced by a decentralized market economy, made up of a large number of price-taking firms. Households and firms are individual 'rational' utility and profit optimizers and do not engage in strategic behaviour i.e. the solution is a Nash equilibrium. It was found that the decentralized or market solution is exactly

identical to the socially optimal (Chapter 4) solution. Under the assumptions made therefore, introduction of an exhaustible resource in itself does not lead to an externality or inefficient inter-temporal market allocation in resource use.

Chapter 6 returned to the socially optimal framework, with the additional assumption that the extraction and use of fossil fuels in production results in the global warming effect, a negative externality. Cumulative CO₂ atmospheric concentration exerts an increasingly negative, damaging effect on the production possibilities of the model economy. The backstop technology assumption is withdrawn; fossil fuels are once again essential to production. The characteristics of the optimal solution were found to be that (a) optimal extraction/depletion rates are reduced in accounting for the harmful side-effect of using fossil fuels. The conservation incentive, previously stemming from scarcity, is reinforced by the desire to prevent future damage/future losses in utility; and (b) consumption may rise for a time (depending on the initial capital stock), but must thereafter fall, asymptotically approaching the lower level limit over time. Compared to the Chapter 3 solution, consumption levels are lower throughout, and begin to fall sooner.

Following the systematic progression of analysis, *Chapter 7* then solved for the general equilibrium market outcome with exhaustible fossil fuels generating a cumulative negative externality. In comparing market and planned (or optimal) outcomes of the dynamic general equilibrium model, consistent and potentially highly significant market failure is demonstrated. Agents chose between consumption and investment, and between extraction

and conservation in the presence of global warming. Characteristics of the market solution were (a) initially excessive rates of fossil fuel extraction and use; and (b) initial over-consumption, then sub-optimal levels of consumption and utility with an overall lower level of welfare. Further it was shown that if the private rate of time preference is higher than the social rate of discount (as widely held), then the market solution of a sub-optimally initially high, but then unsustainable growth path, is reinforced, leading to a further reduction in total consumption.

With a view to internalizing the externality, *Chapter 8* examined several aspects of optimal market intervention. It was seen that it should not be assumed that the taxation of resources in fixed supply represents a pure taxation of rents nor that such taxation is non-distorting between factor use or inter-temporally. Different taxes will have different effects on the optimal extraction profile of an exhaustible resource. And by way of significant corollary, taxes intended towards purely economic objectives, can have important environmental consequences.

It was found that in the context of the model presented, the optimal pigouvian carbon tax level is the entire sum of discounted future damages (to infinity) caused by marginal extraction. In deriving the optimal corrective policy, it is shown that this requires complete knowledge of all future states, including growth rates and interest rates. Hence, it was then argued that real-world application of a 'carbon tax', or price-based, corrective instrument is likely to have excessive information requirements in and of itself, and

in view of the myriad of currently used energy instruments and policies which complicate the calculation of the net effect of an additional energy policy instrument. Arguments are presented in favour of quantity-based corrective policy instruments (unpopularly known as 'command and control') -- with a lower information requirement based on threshold measures of maximally acceptable physical global warming damages. Carbon quota could then also be tradeable.

Returning to carbon taxes -- currently popular in global warming fora, and already implemented in a number of Scandinavian countries -- it is shown that in order for a 'carbon tax' to achieve its policy objective in the dynamic framework, the rate of change of taxation over time is of critical importance. Only a declining tax rate will meet the objective of internalizing the costs of global warming. A rising tax rate -- which might have been expected to match the rising costs of global warming over time -- is shown to be directly counter-productive to the policy objective, resulting in greater global warming. Consideration of the dynamic general equilibrium economics of depletion and cumulative externality then, lead to the conclusion that as regards carbon taxes, high, distortionary, but falling taxes are warranted.

Chapter 9 then turned to consider different scenarios of fossil fuel availability and global warming severity. The above analysis has shown that the global warming externality presents a distinct constraint in addition to the scarcity constraint. While the market theoretically will tend to generate price paths suitable for inter-temporal allocation in

production, these scarcity-induced price-rises do not subsume, replace or account for, the externality. Even though scarcity and exhaustion may be the dominating concern, separate policy intervention is warranted to address global warming. Both effects then tend to encourage conservation.

It is noted that the proven existence of a negative externality is in itself not a sufficient condition to warrant government intervention -- the issue of market-failure versus government failure in efficient allocation arises.

Chapter 9 then addressed the scenario where, "Fossil fuel reserves are relatively vast; the dominating and critical threat is that of GW. Partial depletion of total fossil fuel reserves suffices to destabilize the environment to an extent that imposes critical damage on welfare and disrupts orderly economic activity, well before depletion appears on the near horizon" -- the case deemed most likely in practice. It was then shown that the socially optimal economic program recognizes the limit imposed by the GW constraint and curtails extraction to asymptotically approach zero along this (stock) limit. In the optimal solution, positive reserves of fossil fuels will therefore remain underground to infinity.

The decentralized market outcome, on the other hand, does not take account of the potentially detrimental, cumulative effect of CO₂ emissions -- despite knowledge thereof. Atmospheric CO₂ concentration levels therefore rise beyond the critical damage level and

the model solves to a *finite* time horizon⁸⁰. This is to say that decentralized market agents -- despite having perfect foresight and despite their N individual marginal utilities of consumption approaching infinity at the 'survival margin' -- cause the termination of economic activity and generation of utility.

Under the 'strong form' global warming specification then, the thesis shows that the decentralized course can prove 'fatal' if the environmental threat is potentially, cumulatively severe enough. Intertemporal allocations of resource-use, consumption and investment drive these powerful results. Given that global warming is widely held to be a very real threat -- as documented in Chapter 2 -- the model results do have an important message regarding potential welfare losses if environmental factors are not taken into account.

The model's simplicity *reinforces* this result, rather than weakening it. We need not argue about a revision in the estimated temperature in Saskatoon in July 2100 as does the popular press. Rather we show that, given only qualitative knowledge of fundamental relationships, and the direction of change, we can predict that without market intervention there will be social costs. These costs may be substantial and are avoidable. Further, in cases of environmental threats, there exists no 'invisible hand', or safety mechanism in the working of the market itself -- if not corrected by public intervention, the market will

⁸⁰ While the assumption of constant or zero extraction costs is commonly made in simple natural resource models, this simplifying assumption here becomes a crucial assumption, leading to the collapse of the price path of energy. The assumption of rising marginal costs is therefore adopted in the derivation of the above result.

'drive us over the edge'.⁸¹

CO₂ emissions put into the atmosphere today, will stay there for between 200 and 500 years. Irrespective of this wide range estimate (accounting for uncertainty regarding uncertainty in reabsorption rates), this is a relatively long time horizon. Long, relative to the modelling exercises commonly undertaken in the literature (usually to the middle of the next century), and very long relative to the terms of elected (and unelected) policy-makers -- but short, relative to the expected continuation of 'orderly economic activity'. Given the sub-optimal market outcome generated in a model where individuals are infinitely lived, the potential for short-sighted behaviour on the part of short-lived generations of economic agents is even greater -- and currently there is no policy-making (market-correcting) institution that takes 'the long view'.⁸²

Intertemporal trade-offs, intergenerational equity and the rate of time discounting are intimately connected concepts. It is easy to understand how a change in the rate of time discounting will tilt consumption profiles in favour of, or against, future generations. It is more complicated to understand how a seemingly abstract, theoretical variable can crucially affect the future welfare of the planet. Ecologists are loath to hear that a new

⁸¹ This is particularly relevant at a time when the worldwide trend is to believe that 'the market will do it better', and 'the market will fix it'.

⁸² The hope then is that technology will 'save' us, just as technological developments have warded off other scarcity and doomsday threats in the past. Recollecting the results of Chapters 4 and 5 however, this may not be the case. It was demonstrated that the potential of a substitute (fossil fuel-free) energy technology will accelerate fossil fuel extraction rates, and thus accelerate global warming.

sub-discipline of economics is concerned with calculating 'optimal extinction rates' (employing the same mathematical techniques as we have done in this paper). The likely popular and 'good faith' reaction is that optimal extinction rates calculated by these economists apply only to obscure mosses and plants of no medicinal use.

This dissertation has shown however that such extinction will equally apply to human life by economic logic i.e. it is a feasible market equilibrium outcome. If the environment that supports us is pushed to the verge of instability or toxicity, and the public discount rates employed are high enough, this directly implies a conscious choice for bringing about the planet's environmental and economic collapse. While this may seem melodramatic, this dissertation has demonstrated the feasibility of such an outcome.

Notable is that we have modelled this outcome under conditions of perfect foreknowledge, perfect competition and perfect capital mobility. At the most general level therefore the conclusion emerges that *human-created economic institutions* of the *market mechanism* and *discounting* are powerful instruments, and as such must be *managed* responsibly. Blind faith in invisible hands aside, there is no reason to believe that current economic institutions will automatically safeguard future environmentally-based welfare.

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