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**The Development of Numeracy: The Relations Between Children's Counting
and Simple Arithmetic**

by

Karen E. Hubbard, B.A.

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of**

Master of Arts

Department of Psychology

Carleton University

Ottawa, Ontario

1995 September 7

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**THE DEVELOPMENT OF NUMERACY: RELATIONS BETWEEN CHILDREN'S
COUNTING AND SIMPLE ARITHMETIC**

submitted by Karen E Hubbard, B A

in partial fulfillment of the requirements for the degree of

Master of Arts

Thesis Supervisor

A handwritten signature in black ink, appearing to read 'Paul R.', is written over a horizontal line.

Chairperson, Department of Psychology

Carleton University

September, 1995

Abstract

A great deal of research has been conducted to examine young children's knowledge and skill related to the domains of counting and simple arithmetic. However, the skills and concepts have been studied in isolation. Because counting is intrinsically related to simple arithmetic (e.g., counting procedures are used by children learning addition), I assessed the component skills and concepts of counting and simple arithmetic over time. Junior and Senior Kindergartners (4- and 5-year-olds) were tested because critical developments in numeracy occur during this age range. Overall, performance on counting improved with time and grade. The best predictor of Senior Kindergartners' performance was their counting limit, presumably because counting limit reflects children's numerical fluency.

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items and remember the number once the items were hidden from view (conservation of cardinality), and make magnitude comparisons (compare-sets). Several of these tasks have become standard measures of cardinality (e.g., Codd & Bialystok, 1994; Saxe, 1977; Sophian, 1987, 1988; Wynn, 1992). Schaeffer et al. found that 3-year-olds could count items correctly but could not conserve cardinality. However, 4-year-olds could count items correctly and could conserve cardinality. Sophian (1987, 1988) tested 3- and 4-year-old children's ability to use counting to determine number of items and to make magnitude comparisons. Her participants were more successful at knowing when to use counting to solve a problem than they were for the counting task itself. Sophian's results imply that children had a conceptual understanding of certain aspects of cardinality but that their procedural skills had not been perfected.

Unfortunately, the researchers who studied children's knowledge of quantification through cardinality neglected to study its relation to quantification through arithmetic (cf. Geary, Bow-Thomas, & Yao, 1992). Thus, although many researchers have studied different aspects of counting knowledge and skill, no single study has examined both children's procedural and conceptual knowledge of counting limit, one-to-one, stable order, order-irrelevance, cardinality, and conservation of cardinality.

In summary, Gelman and Gallistel (1986) singled out five counting principles (abstraction, one-to-one, stable order, order-irrelevance, and cardinality) that capture much of children's factual, procedural, and conceptual knowledge of counting. Children's counting knowledge and skill develop from about 2 to 5 years of age. Children's ability to quantify through counting (i.e., ability to use the cardinality principle) seems to improve rapidly from 4- to 5-years-old. Thus, there is ample evidence that critical developments in children's counting occur at about age 4.

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Introduction

The conceptual and procedural knowledge that form the basis of numeracy develop rapidly between the ages of 2 and 5. Children as young as 2- and 3-years-old exhibit counting knowledge when they recite the number words and use counting to determine the number of items in a set (Sophian, 1988; Wynn, 1990). However, at this young age, children's counting range is quite small, usually below 6 or 7. Children's counting skills improve with age and practice and the number of items that they are capable of counting increases. Furthermore, their vocabulary of number words increases and allows their counting limits to increase. By age 5 or 6, children are highly skilled counters and they have learned to count to quantify.

The next major occurrence in the development of children's numeracy is the emergence, at age 5 or 6, of simple arithmetic skills (e.g., Siegler & Crowley, 1991). Initially, children use a combination of memorized facts (e.g., $1+1=2$) and simple counting-based procedures to solve small addition problems. With age and practice, children invent and use increasingly sophisticated solution procedures. By age 6 or 7, children learn subtraction and display some understanding of arithmetic principles such as commutativity and associativity. Accuracy improves as children become more practiced and older children and adults rely mainly on retrieval to solve arithmetic problems. However, even adults rely somewhat on non-retrieval procedures to solve arithmetic problems (LeFevre, Sadesky, & Bisanz, 1996). Thus, children as young as age 5 show a profile of arithmetic and quantification skills that is similar to that of adults, although the relative use and sophistication of procedures increases with age and practice.

Counting and simple arithmetic are procedurally and conceptually related (e.g.,

Siegler, 1987a, 1987b; Siegler & Jenkins, 1989; Bisanz, Morrison & Dunn, 1995). For example, using the count-all procedure to solve $2+3$, a child would count, "1, 2 [addend]. 1, 2, 3 [augend]. 1, 2, 3, 4, 5. 5 [solution]." The count-all procedure is relatively cumbersome because it requires the maximum number of increments. Other early addition procedures also involve counting, such as count-on (e.g., for $2+3$, "1, 2, 3, 4, 5."), and min. Count-on is less cumbersome than the count-all procedure, but however, it is less efficient than the min procedure. Solving a problem with the min procedure entails counting up from the larger addend the number of times indicated by the smaller addend. For example, using min to solve $2+3$, a child would count, "4, 5." Thus, min requires the minimum number of increments for counting. The use of min requires the knowledge that the counting string can be broken (i.e., counting can start at a number other than 1). Using min also necessitates the cardinal knowledge that the amount of an addend is equal to its cardinal quantity. Moreover, the use of min implicates the commutative principle (that $m+n$ equals $n+m$; Baroody & Gannon, 1984; Resnick, 1992). For these reasons, the use of min implies that children have a relatively sophisticated understanding of counting and arithmetic. Siegler (1982, 1987b) showed that children also use counting-based procedures to solve subtraction problems. Thus, it appears that counting knowledge and skill play critical roles in the emergence of addition skills. Despite the wealth of research on counting and arithmetic, there is a paucity of research investigating the relations between them. Therefore the goal of the present research was to study children's numeracy in the contexts of counting and simple arithmetic and to examine the developmental relations between them.

Counting

Three types of knowledge describe children's understanding of counting: factual, procedural, and conceptual (Bisanz & LeFevre, 1990). Factual knowledge consists of memorized information about counting and numbers. For example, children must remember the words in the counting string and the decade names. Wynn (1992) found that children as young as 2-years-old knew that number words referred to numerosity and some could recite a counting string as high as 6 or 7. Furthermore, Gelman and Gallistel (1986) reported that children rarely used words other than number words to count. Siegler and Robinson (1982) found that many 3-year-olds could count to 19. Moreover, they found that 4- and 5-year-old children were learning the decade names and discovering and using the rules underlying the counting string beyond 20 (i.e., combining the decade name and one, two, three, etc., until the next decade). Some of these 4- and 5-year-old children were capable of counting to 100 and beyond. However, Siegler and Robinson tested only ten 5-year-olds; therefore, there is little normative information about how high most children of this age can count (cf., Fuson, Richards, & Briars, 1982).

The second type of counting knowledge is procedural. Bisanz and LeFevre (1990) defined procedures as mental activities that occur over time, can be stored in memory, and that accomplish a goal. Examples of children's procedural knowledge of counting include two of the counting elements described by Gelman and Gallistel (1986). Gelman and Gallistel described how children must use a stable order of counting words, that is, a consistent counting string. Children must also apply the one-to-one rule and increment the count by one for each item counted. Children typically demonstrate evidence for these two principles before age 4 although

performance is not perfect (e.g., Briars & Siegler, 1984; Chiahara, 1985; Gelman & Gallistel, 1986). Thus, children's procedural knowledge about counting can be assessed by studying what they do while counting.

The third type of counting knowledge is conceptual. Conceptual knowledge that reflects the underlying structure of counting and that can be inferred from the flexible and selective use of appropriate procedures (Bisanz & LeFevre, 1990). Because children's conceptual knowledge cannot be directly tested, it must be inferred from what children do when they count. For example, flexibility of procedure use could reflect a conceptual understanding of additivity. Or, with respect to the counting string, children must understand that the sequence is an unbreakable, enumerable, bidirectional chain of number words (Fuson, 1991). Children who break the number word sequence or who cannot count in either direction would not have a flexible understanding of the counting string. Thus, conceptual knowledge underlying counting must be inferred to the extent that children use procedures flexibly.

The study of procedural and conceptual knowledge is difficult because the two types of knowledge are intertwined. For example, if a child begins counting in the middle of an array, does the child understand the concepts underlying the order-irrelevance principle, is the child performing by rote the steps of the count, or is the child using some combination of knowledge of the procedures and concepts involved? Similarly, for addition, if a child uses the min procedure, does the child understand commutativity, is the child performing a rote addition procedure, or is the child using conceptual knowledge about addition to develop a short-cut procedure?

Gelman and Gallistel (1986) identified several principles that embody conceptual knowledge. First, children must demonstrate that they know that anything

may be counted (the abstraction principle). Conceptual knowledge of the abstraction principle is manifested whenever a child counts stimuli of different types, such as pictures of balloons and dogs. Wynn (1992) demonstrated that children as young as 2-years-old have mastered this principle. Children must also acquire the concept that items in a set can be enumerated in any order (Gelman and Gallistel's order-irrelevance principle). Baroody (1992) showed that 4-year-olds demonstrated knowledge of this principle. Briars and Siegler (1984) modelled several correct and unusual-but-correct counts to test children's understanding of the order-irrelevance concept. The authors found that 3-year-olds were inconsistent in their responses whereas 4- and 5-year-olds were more accurate. Even the older children, however, had false beliefs that counting adjacent items and starting at an end were essential to correct counting. Unfortunately, Briars and Siegler did not ask children to explain their judgments about the counts, thus valuable information about what children know about the order of enumeration may have been lost.

Perhaps the most important principle described by Gelman and Gallistel (1986) was cardinality, manifested when a child states the last number of the count as the amount of a set. Cardinality may reflect knowledge about the relation between counting procedures and understanding of counting's quantitative nature. A great deal of research has been devoted to studying children's knowledge of cardinality. Wynn (1992) studied 2- and 3-year-olds' knowledge of number words and found that these young children learned all the cardinal meanings of the number words within their recitation range (up to seven) simultaneously. Schaeffer, Eggleston, and Scott (1974) gave 2- to 6-year-olds several tasks to assess their knowledge of cardinality. Children had to count drum taps (how-many), give the experimenter some items (give-x), count

items and remember the number once the items were hidden from view (conservation of cardinality), and make magnitude comparisons (compare-sets). Several of these tasks have become standard measures of cardinality (e.g., Codd & Bialystok, 1994; Saxe, 1977; Sophian, 1987, 1988; Wynn, 1992). Schaeffer et al. found that 3-year-olds could count items correctly but could not conserve cardinality. However, 4-year-olds could count items correctly and could conserve cardinality. Sophian (1987, 1988) tested 3- and 4-year-old children's ability to use counting to determine number of items and to make magnitude comparisons. Her participants were more successful at knowing when to use counting to solve a problem than they were for the counting task itself. Sophian's results imply that children had a conceptual understanding of certain aspects of cardinality but that their procedural skills had not been perfected.

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Simple Arithmetic

At about age 4 or 5, children's simple arithmetic skills emerge (e.g., Bisanz et al., 1995; Siegler & Crowley, 1991). Several researchers have found that children of these ages are quite accurate for problems with addends of five or less (e.g., Bisanz et al.; Siegler, 1987a). In general, research has shown that children can add small numbers by about age 5 and learn to subtract by age 6 or 7 (e.g., Carpenter & Moser, 1984; Siegler, 1987a, 1987b, 1988; Steinberg, 1985). For both addition and subtraction, when children do not retrieve a solution (i.e., quickly recall the answer from memory), they invent and use a wide variety of solution procedures, such as count-on, count-all, min, derived-fact procedures (using known arithmetic facts to derive the solution to an unknown problem), or decomposition (e.g., decomposing $12+2$ into $10+2+2$; Bisanz et al., 1995; Carpenter & Moser, 1984; Hubbard & Bisanz, 1995; Siegler, 1987a; Siegler & Crowley, 1991).

The procedures that children use to solve arithmetic problems can provide insight into their knowledge of arithmetic principles. For example, Baroody and Gannon (1984) and Resnick (1992) suggested that children's use of the min procedure implies some knowledge of the commutativity principle (that $m+n$ equals $n+m$) because using min disregards addend order. Children's use of derived-fact procedures can reflect their understanding of decomposition and associativity; for example, children might solve the problem $2+3$ by adding $2+2+1$.

Despite the breadth and quantity of research on children's counting (e.g., Briars & Siegler, 1984; Chiahara, 1985; Fuson, 1988, 1991; Fuson, Richards, & Briars, 1982; Geary, Bow-Thomas, & Yao, 1992; Gelman & Gallistel, 1986; Saxe, 1977; Schaeffer et al., 1974; Siegler & Robinson, 1982; Sophian, 1987, 1988; Sophian, Harley &

Martin, 1995; Wynn, 1992) and simple arithmetic (e.g., Baroody, 1987; Baroody & Gannon, 1984; Bermejo & Rodrigues, 1993; Bisanz et al., 1995; Carpenter & Moser, 1984; Leinhardt, Putnam & Hattrup, 1992; Siegler, 1987a, 1987b, 1988; Siegler & Jenkins, 1989; Siegler & Crowley, 1991; Siegler & Shipley, 1995; Sophian & McCorgray, 1994; Steinberg, 1985) and the necessary relation between counting and arithmetic, little research has been conducted to determine the developmental relation between counting and addition.

One study, however, did examine the relation between counting ability and addition skill (Geary et al., 1992). Geary et al. presented 7-year-old normal and math-disabled children with counting tasks (involving one-to-one, stable order, and cardinality knowledge) where correct, correct-but-unusual, and incorrect counts were modelled and children were asked to evaluate the counts. Performance for normal children was close to ceiling levels. The children were also presented with single-digit addition problems; procedure reports revealed that 7-year-olds used mainly counting-related procedures and retrieval. With respect to the relation between counting and addition, Geary et al. found that the frequency of correct identifications for unusual counts correlated reliably with the use of the min algorithm ($r[35]=.47, p<.01$). The specific implications of this correlation are unclear. However, the general implication is that flexible counting (as indexed by recognizing that unusual counts can be correct) may be related to the use of a relatively sophisticated counting-based addition procedure (min). Unfortunately, Geary et al. did not examine any other relations between counting and simple arithmetic. Moreover, the authors neither asked children to count by themselves (to tap their procedural knowledge), nor explain the judgments they gave (to tap their conceptual knowledge). Also because Geary et al. tested 7-

year-olds, who performed at ceiling levels, information about how younger normal children might perform is missing from the study. The goal of Geary et al.'s research was to determine which aspects of counting (as indexed by judgments) math-disabled children do not have and which might lead to their poor computational skills. Information about the relations between counting and addition for normal children would also be useful.

Present Research

To illuminate the developmental relations between counting knowledge and skill and addition performance, I studied 4- and 5-year-olds (Junior and Senior Kindergarten) because children in these age groups are proficient, but not perfect, counters and have not yet received formal instruction in simple arithmetic. My goal was to tap emergent arithmetic skills and study their relations to counting. Furthermore, because children's counting abilities develop rapidly from age four to five, and addition skill emerges at about age five, I examined 4- and 5-year olds. To trace the development of numeracy, the present study had a short-term longitudinal design. Junior Kindergartners were tested early in the year after minimal formal instruction in counting and again late in the year after 5 months of schooling. Senior Kindergartners had presumably received formal counting instruction in Junior Kindergarten but had not yet received formal instruction in arithmetic. I hypothesized that counting abilities would appear before arithmetic skill because counting precedes arithmetic in the development of quantification. In informal discussions with the children's teachers, they reported that Junior Kindergartners focus on one number a month, in sequence, for the school year. The Senior Kindergartners work more extensively with numbers and counting but are given little or no instruction in

arithmetic. Children in both grades sang counting-related songs each week and counted to the date on the calendar each morning.

The tasks I gave the children assessed the skills and concepts related to counting and simple arithmetic. I examined children's understanding in these two domains using the three assessment contexts for assessing conceptual and procedural knowledge described by Bisanz and LeFevre (1992): application, justification, and evaluation. The first context is the application of appropriate procedures to problems. For example, children are asked to solve addition problems and the solution procedures they use are recorded. The limitation of examining only the application context is that children may understand a principle but be unable to or have no need to apply it themselves. For example, if a child uses retrieval to solve an addition problem, the child would have no need to use min but may very well understand that min is a good shortcut procedure. Therefore, the second context that Bisanz and LeFevre recommended is evaluation of the correctness of a solution procedure to examine knowledge about the conditions that govern the applicability of concepts. Solution procedures, such as min, are modelled for children who are required to say whether the procedures are appropriate. The limitation of this context is that children may respond with a bias, such as always answering positively or negatively. The third context, justification, may overcome this limitation. Assessing justification, or provision of a rationale for procedures, can tap children's explicit knowledge of underlying concepts. Although the latter two contexts require a level of articulatory skill that young children may not possess, some indication of children's conceptual understanding may be tapped.

All counting tasks were assessed within the application context. Children's

performance on four cardinality tasks was assessed. The four tasks were modifications of those developed by Schaeffer et al. (1974) and used by other researchers (e.g., Wynn, 1992). The how-many task tested whether children knew that the last number in the count represented the total number of items. To determine that children knew that cardinality entailed more than simply repeating the last number in the count (as in the how-many task), children were given three other cardinality tasks. For the give-x task, the experimenter asked children to give her x number of items. For the point-to-x task, children were asked to point to one array of several that contained x items. For the compare-sets task, children were asked to determine the cardinality of two sets and make a magnitude comparison. The order-inelevance principle was also assessed within the evaluation and justification contexts because Briars and Siegler (1984) observed interesting differences in performance on tasks related to order-irrelevance. For the conservation of cardinality task, children were asked to respond then to give an explanation to assess a different aspect of their cardinality knowledge than was assessed by the how-many task, for example.

For simple arithmetic, children's understanding of changes in amount was assessed in the application context by changing the number of items in a basket (e.g., removing or adding items) and asking children to say if there were more, less, or the same number of items. Children's knowledge of commutativity was measured in the contexts of application, evaluation, and justification. For addition and subtraction, children were asked to solve problems and to report their solution procedures.

It is clear that children's counting improves greatly from age 3 to age 5 (Briars & Siegler, 1984; Gelman & Gallistel, 1986; Siegler & Robinson, 1982); therefore, I expected that Senior Kindergartners (age 5) would be better counters than Junior

Kindergartners (age 4) and that Junior Kindergartners would show large improvements in counting knowledge and skill from fall to spring. I expected the biggest age differences to appear on the cardinality tasks because the cardinality principle reflects both procedural and conceptual knowledge.

With respect to the arithmetic tasks, I expected the older children to have a better understanding of manipulations of quantity and thus perform better than the younger children on the changes-in-amount task. I also expected the older children to be near ceiling-levels for the number recognition task because number recognition is taught in Junior Kindergarten. Similarly, I expected Junior Kindergartners to improve from fall to spring on the number recognition task as a result of instruction. For addition, I expected skills to emerge at about age 5, replicating findings in the literature. I also expected children in this study to use a variety of solution procedures. I expected understanding of the commutativity principle to coincide with or follow addition skill. Other researchers (e.g., Baroody & Gannon, 1984; Bermejo & Rodrigues, 1993) found similar results, that different aspects of commutativity knowledge appear from ages 5 to 8.

Because counting is a central procedure in early addition, I hypothesized that arithmetic skills would be strongly dependent on counting knowledge and skills. Specifically, both counting and addition are intrinsically additive (termed additive composition by Resnick, 1992). Moreover, quantification is the common goal of counting (cardinality) and arithmetic (sum or difference). In general, I expected that good counters would be good adders because the counting-based procedures would be less error-prone for good counters than for poor counters. Specifically, because counting skills appear to coalesce and reach ceiling levels at about age 5 or 6, I

hypothesized that there would be a good predictor of addition performance among the counting tasks. Two likely candidates were apparent. The first was counting limit because a high limit may reflect a knowledge about the additive nature of counting and therefore predict addition. The second likely candidate was cardinality because cardinality reflects the quantification aspect of counting, which may predict addition, which is quantification-based.

Method

Participants

At the first testing session the 31 Junior Kindergarten children ranged in age from 3:11 to 5:10 (years:months) with a median age of 4:5. Seventeen males and 14 females participated; however, one female student was away during the second testing block. One child's score on the counting limit task was an outlier (i.e., greater than three standard deviations above the mean) so the score was replaced with a score one higher than the next highest data point (Tabachnick & Fidell, 1989). The replacement score was used for all analyses; however, the subject's original responses are presented in Figure 1 and Appendix E, which summarize performance on the counting limit task.

The 39 Senior Kindergarten children ranged in age from 4:11 to 6:0 with a median age of 5:5. Nineteen males and 20 females participated; however, one female moved to another school between the first and second testing blocks. I obtained children's participation in the study by soliciting written permission from the parents or guardians and requesting children's verbal consent.

Materials

The tasks which were designed to assess counting, numerical reasoning, and simple arithmetic are summarized in Table 1 and shown in Appendices A and B.

Table 1. Summary of tasks used to measure counting, numerical reasoning, and arithmetic.

Task	Description
<i>Counting</i>	
counting limit	Children asked to count as high as they could ^a .
cardinality	Four tasks assessed knowledge of this principle:
(a) how-many	Reported how many items were in an array ^b .
(b) give-x	Gave experimenter requested number of items ^b .
(c) compare-sets	Counted two sets and compared magnitudes ^b .
(d) point-to-x	Counted four sets and pointed to array with requested number of items ^b .
(e) conservation of cardinality	Asked to determine that experimenter and child have equal number of items ^b .
	Conservation: Asked how many items E has after items were hidden ^c .
One-to-one	Counted two arrays and got 0 if disobeyed principle and 1 if obeyed it ^b .
Stable order	Counted two arrays and got 0 if did not use a stable string and 1 if they did ^b .
Order-irrelevance	Application: Asked to start counting at various points along an array ^b .
	Evaluation: Shown a modelled count and asked if it was a good way or not ^d .
	Justification: Asked to explain their evaluation ^c .

Numerical Reasoning

conservation of number Conservation: Asked which line had more items after one line was lengthened^b.

Justification: Asked to explain their answer to the conservation task^c.

Arithmetic

changes-in-amount Shown various manipulations of quantity and asked to say if there were more, less, or the same number of items as before^b.

number recognition Shown cards with the numbers 0 to 10 on them and asked to read the cards^b.

addition Shown addition problems and asked to solve them^b and report their solution procedures^c.

subtraction Shown subtraction problems and asked to solve them^b and report their solution procedures^c.

Note. ^aChildren's maximum count without errors. ^bResponses were scored as percentage of correct trials. ^cResponses were coded according to coding protocol in Appendix D. ^dResponses were scored as correct (0) or incorrect (1).

To tap children's spatial and visual problem-solving skills, children were given the Puzzle-Solving Subtest from the Perceptual-Performance Scale of the McCarthy Scales of Children's Abilities (McCarthy, 1972).

Procedure

Testing blocks occurred in November and March. Within each testing block, children were interviewed individually on two different days with no more than two

days between the first and second interviews. Interviews were conducted by the author and a 26-year-old female research assistant. To develop rapport with the participants, the experimenters spent a week volunteering in the Junior and Senior Kindergarten classrooms prior to the first testing block. To facilitate children's cooperation during the interviews, the experimenters used a puppet to demonstrate the tasks and give instructions. Interviews occurred either in an unoccupied storage room or an empty classroom. Participants' responses were audiotaped, transcribed, and scored. Nonverbal responses were scored during the interview. During both interviews, children were given the opportunity to rest and do a puzzle. An age-appropriate puzzle (a wooden teddybear or a colourful rubber snail) was given either midway through the interview if the child was restless or fatigued or at the end as a special treat. This procedure was repeated during the spring testing block. However, during the spring testing block, the puzzles children completed were the Puzzle-Solving Subtest (McCarthy, 1972) and the snail.

The total time to administer all the tasks ranged from 40 to 50 minutes. The tasks were given in two interviews because 20- to 25-minute sessions are reasonable durations for 4- and 5-year-olds. The tasks were ordered such that (a) tasks that were potentially difficult were followed by tasks expected to be easy; (b) each session was expected to last no longer than 20 minutes; and (c) if application and justification tasks were given for the same concept, the application task was always given first. The order of the tasks and their estimated times are presented in Appendix A.

Reliability of Coding

Responses were coded according to the protocol presented in Appendix D. Interrater reliability was obtained using 10% of the data set (three Junior

Kindergartners and four Senior Kindergartners) chosen at random from the sample. Two independent raters (the experimenter and another graduate student) coded the data. Reliability was 97.1% for the counting tasks and 97.3% for the arithmetic tasks.

Results

All results are significant at the $\alpha = .05$ level unless otherwise noted. For multivariate tests, Wilk's lambda criterion was used (Tabachnick & Fidell, 1989). Because I examined children's counting and simple arithmetic, I will review performance in each of these domains, and the different contexts for assessment (i.e., application, evaluation, and justification) separately. I will then discuss the developmental relations between counting and arithmetic. Because sex was not significantly correlated with performance on any of the tasks, the variable gender was not used in any analyses.

Performance-based Results

McCarthy puzzle-solving subtest

Scores were determined by the number of puzzles assembled correctly. Children were given points for each join correctly made as well as bonus points for speed. The minimum score obtainable is 0 (no pieces joined correctly) and the maximum score obtainable is 26 (all puzzles correctly assembled and bonus points given for speed). Junior Kindergartners' performance was lower than that of Senior Kindergarten children's (mean raw scores 11.3 and 19.5, standard deviations of 6.5 and 5.8 respectively), $t(64) = -5.39$. Range of scores was similar for children in grades (3 to 25 vs. 5 to 27). Because not all subtests from the Perceptual-Performance Scale of the McCarthy Scales of Children's Abilities were given, age-related normative data are not available for the Puzzle-Solving Subtest.

Counting

Maximum count. Distributions of maximum count responses are presented in Figure 1 and Appendix E. Overall, older children could count higher than younger children. The mean counting limits in the fall and spring for Junior Kindergartners were 13 and 16 respectively; mean counting limits for Senior Kindergartners in the fall and spring were 44 and 57. The distribution of stopping points for both Junior and Senior Kindergartners was similar to the pattern reported by Siegler and Robinson (1982). Younger children tended not to be able to count to 20. In the fall, the median and modal counting limit was 12. In the spring, the median was 14 and the modal stopping points were 12 and 14. Older children tended to be able to count to 30 and beyond. In the fall, the median limit was 33 and the modal stopping point was 39. The length of Senior Kindergartner's counting string increased somewhat from fall to spring and the median counting point was 39 and the modal scores were 43 and 100. Approximately half of the Junior Kindergartners and two-thirds of the Senior Kindergartners had counting limits that were on or near a decade boundary.

Insert Figure 1 about here

The distributions of counting limits obtained may reflect two kinds of children's knowledge of the counting string. First, children may not have discovered the regularities in the counting string and thus have not generalized the string across decades. This is likely the case for 4-year-olds whose median counting limits were 12 (fall) and 14 (spring). At both sessions, many children seemed to be trying to learn the irregular number string between ten and twenty (see Figure 1a and 1b). Because

most children could not count beyond 20, it seems unlikely that they had discovered the rules for counting the decades.

Alternatively, children may have discovered the regularities in the counting string and have generalized across decades. Thus, stopping points would be determined by fatigue, boredom, or forgetting a decade name. It is likely that this description applies to the 5-year-olds, whose median counting limits were 33 (fall) and 39 (spring). Senior Kindergartners' counting limits were similar to those found by Siegler and Robinson (1982) in that stopping points were centred on or near decades (73% of stopping points in the fall and 68% in the spring). However, whereas Junior Kindergartners tended to stop at the tens or twenties decade, Senior Kindergartners tended to stop at the thirties, forties, seventies, and nineties decades. Moreover, several 5-year-olds were able to count to 100, indicating that they had mastered the rules governing counting the decades. An interesting finding at a future testing session would be if the children who had mastered counting the decades could also master counting the centuries. Several children also reported being able to count as high as they wanted to if they had enough time. In sum, older children could count higher than younger children. Furthermore, the older children appeared to have learned the decade structure of the counting string (many could count to 40 or 100) whereas the younger children appeared to be learning the irregular numbers before 20.

Counting principles. I was also interested in children's performance on four counting principles (measured in percent correct): cardinality, one-to-one, stable order, and order-irrelevance. In general, I expected children in both grades to improve over time on the counting principles. I expected the Junior Kindergartners to make greater gains, however, because the Senior Kindergartners' counting would be close to ceiling

levels.

A summary of frequencies of responses for the four cardinality tasks is shown in Appendix E and descriptive statistics are presented in Table 2.

I examined the relations among the four cardinality tasks to construct a composite measure of cardinality. All four tasks were significantly correlated at the fall testing session; similar, but lower, correlations were evident at the spring session. Because the tasks were related and showed similar patterns of improvement, I averaged the percent correct across the four tasks and used the average as an index of cardinality performance.

Table 2. Means and standard deviations for the four cardinality tasks at each testing session for both grades.

	Junior Kindergarten				Senior Kindergarten			
	Fall		Spring		Fall		Spring	
Task	Mean	s	Mean	s	Mean	s	Mean	s
how-many	67	32	76	25	88	16	94	9
give-x	46	32	59	24	88	17	90	13
compare-sets	40	31	51	33	79	27	83	20
point-to-x	39	38	59	35	81	22	93	11

Table 3 shows performance on the four counting tasks (cardinality, one-to-one, stable order, and order-irrelevance) and conservation of cardinality, including means and standard deviations. Frequencies of responses on these tasks are presented in Appendix E.

To examine age differences over time, I conducted a 2x2 multivariate analysis of variance on percentage correct for the five counting measures (i.e., cardinality,

stable order, one-to-one, order-irrelevance, and conservation of cardinality). The within-subjects independent variable was time (fall vs. spring); the between-subjects independent variable was grade (Junior and Senior Kindergarten).

Table 3. Means and standard deviations for the four counting tasks at each testing session for both grades.

Task	Junior Kindergarten				Senior Kindergarten			
	Fall		Spring		Fall		Spring	
	Mean	s	Mean	s	Mean	s	Mean	s
cardinality	48	28	62	23	84	16	90	8
stable order	60	50	83	38	86	35	97	16
one-to-one	67	44	73	37	81	32	84	31
order-irrelevance	72	20	76	23	86	18	77	21
conservation of cardinality	50	46	76	35	90	29	97	11

The total N of 67 was reduced to 63 because four children had missing scores on one or more tasks. Overall, performance improved with time (74 vs. 80%), $F(5,57)=7.93$, and grade (68 vs. 86%), $F(5,57)=11.75$. The interaction between time and grade approached significance, $F(5,57)=2.20$, $p=.08$. Junior Kindergartners' scores improved more (63 vs. 84%) from fall to spring sessions than did those of Senior Kindergarten children (74 vs. 87%).

Because patterns of performance varied across tasks, I investigated the impact of time and grade on the different counting measures, univariate F tests were performed. Examining the dependent variables separately with univariate tests was appropriate because the measures were uncorrelated. Performance on the cardinality

task improved from fall to spring (66 vs. 76%), $F(1,61)=14.03$, $MSe=213$.

Performance also improved with grade (55 vs. 87%), $F(1,61)=58.99$, $MSe=521$. The interaction between time and grade, however, was not significant. Thus, children in both grades improved on the cardinality tasks and the older children improved to near-ceiling levels. The rates of change for both grades did not differ substantially.

Children's performance on the conservation of cardinality task improved with time (70 vs. 87%), $F(1,61)=16.24$, $MSe=513$, and grade (63 vs. 94%), $F(1,61)=21.15$, $MSe=1383$. The interaction between time and grade was significant such that the Junior Kindergartners improved substantially from fall to spring sessions (50 vs. 76%) whereas the Senior Kindergartners had nearly perfect scores at both testing sessions (90 vs. 97%), $F(1,61)=5.42$, $MSe=513$. Children in both grades improved; however, Junior Kindergartners appeared to improve more than the Senior Kindergartners because the older children's performance was at near-ceiling levels at both testing sessions. Thus, it seems that children in both grades could conserve the number of items in a set despite being prevented from recounting the items.

Children's achievement on the stable order task also improved across sessions, (76 vs. 92%), $F(1,61)=12.02$, $MSe=880$. Junior Kindergartners performed somewhat worse than Senior Kindergartners (76 vs. 93%), $F(1,61)=5.26$, $MSe=1454$. The interaction between time and grade was not significant, however. Thus, both Junior and Senior Kindergartners improved from fall to spring but the rates of improvement did not differ significantly. By the spring session, most Junior Kindergartners and nearly all Senior Kindergartners used a stable counting string.

For the order-irrelevance tasks, the change from fall to spring was not significant, nor did the two grades differ in performance. However, the interaction

between the effects of grade and time on order-irrelevance performance was significant, $F(1,61)=4.59$, $MSe=227$. The most striking differences in performance occurred between Junior and Senior Kindergartners' scores in the fall; however, this difference between the grades was attenuated in the spring, suggesting that most children were reaching ceiling levels. As shown in Appendix E, performance in the fall was at near-ceiling levels for Senior Kindergartners when starting from the beginning or end of an array; however performance was poorer when starting from the middle. Junior Kindergartners had a similar profile in that they performed perfectly when counting from the left, more poorly when counting from the right, and very poorly when starting from the middle. By the spring session, when counting from the middle, Junior Kindergartners' performance had improved whereas Senior Kindergartners' performance decreased slightly. This result may account for the attenuation of differences between the grades. Thus, neither Junior nor Senior Kindergartners had mastered counting from the middle of an array.

With respect to performance on the one-to-one principle, performance did not differ by time or grade nor was the interaction significant. However, the mean percentages correct (see Table 3) show a trend towards improvements with time and grade.

In sum, children in both grades improved from fall to spring on most counting tasks and Senior Kindergartners were better counters than Junior Kindergartners, probably because the older children had more practice and received more instruction on counting than the younger children. Generally, performance improved and became less variable with time and grade. Children's performance on the order-irrelevance task was such that children in both grades performed extremely well when counting

from an edge but neither group of children had mastered counting from the middle. Counting from the middle was the only task that presented children with an unusual manner of counting. Presenting children with tasks involving unusual-but-correct counts and incorrect counts can provide useful information about children's knowledge of counting principles (e.g., Geary et al., 1992; Briars & Siegler, 1984). For example, Briars and Siegler found that young children think that starting to count from an edge is an essential counting feature.

It appears that ages 4 and 5 are critical for the development of several aspects of learning to count. Specifically, 4-year-olds made substantial improvements and 5-year-olds appeared to master many aspects of counting skill, for example, cardinality, conservation of cardinality, and stable order. Because cardinality reflects the quantitative aspect of counting, one implication of this result is that the older children's understanding of quantification (i.e., counting to quantify) is better than the younger children's. This improved understanding of cardinality, and quantification, may help the older children generalize their knowledge to addition, which is also a method of quantification. Children in neither grade reached ceiling levels on the one-to-one and order-irrelevance tasks indicating that both Junior and Senior Kindergartners needed practice matching counting increments to items and needed to improve their ability to count from unusual starting points. By the spring session, children in both grades showed similar levels of procedural knowledge (as indexed by the order-irrelevance, one-to-one, and stable order tasks). However, older children seemed to have a conceptual knowledge advantage over the younger children with respect to an understanding of cardinality.

Arithmetic

Table 4 presents the number of children who successfully solved at least one addition or subtraction problem. Very few Junior Kindergartners could add and none could subtract. Some Senior Kindergartners could add at the first testing session and several more could add by the second. This result implies that age 5 is the critical time period for the emergence of addition abilities. Very few of the older children could subtract. This pattern of performance is likely the result of lack of formal arithmetic instruction. Other researchers (e.g., Carpenter & Moser, 1984; Steinberg, 1985) found that addition ability precedes subtraction.

Table 4. Number of children who successfully solved at least one arithmetic problem.

	Fall	Spring
Junior Kindergarten (n=30)		
Addition	2	7
Subtraction	0	0
Senior Kindergarten (n=37)		
Addition	18	28
Subtraction	2	9

Table 5 contains descriptive statistics for the number recognition, changes-in-amount, addition, and subtraction tasks, which were used to assess children's proficiency in different aspects of arithmetic. Performance was lower on the pre-arithmetic (changes-in-amount) and arithmetic tasks than on counting or number recognition. Frequency data on these tasks are presented in Appendix E.

Table 5. Means (percentage correct) and standard deviations for the four arithmetic-related tasks at each testing session for both grades.

Task	Junior Kindergarten				Senior Kindergarten			
	Fall		Spring		Fall		Spring	
	Mean	s	Mean	s	Mean	s	Mean	s
number rec.	57	39	75	29	95	11	99	3
changes-am't	40	36	51	31	68	32	77	29
addition	1	4	4	9	22	28	34	30
subtraction	0	0	0	0	1	6	12	27

The Senior Kindergartners performed at near-ceiling levels on the number recognition task; therefore, their data was excluded from further analysis. I conducted a one-way analysis of variance to assess the effects of time on Junior Kindergartners' accuracy (percent correct) for number recognition; children's performance improved significantly over sessions (57 to 75 %), $F(1,29)=18.80$, $MSe=4795$, presumably because they were taught numbers during the year.

I also conducted a 2x2 analysis of variance to determine the effects of time and grade on performance for the changes-in-amount task. Overall, children improved significantly across sessions (54 vs. 64 %), $F(1,65)=5.96$, $MSe=3374$. Children's performance for changes-in-amount also improved with grade (46 vs. 72 %), $F(1,65)=15.81$, $MSe=1478$. Time and grade did not interact, $F(1,65)<1$, $MSe=34$. Thus, performance improved with time and grade but the rates of improvement did not differ substantially. Junior Kindergartners appeared to have some difficulty with this task, perhaps because of a poor understanding of words describing amount (e.g., Cowan, 1991).

The Junior Kindergartners performed at floor-levels on the addition and subtraction tasks; thus, they were excluded from further analyses. The Senior Kindergartners also performed at near-floor levels on the subtraction task. Therefore, I assessed the effects of time on Senior Kindergartners' percent correct on the addition task. A one-way analysis of variance was conducted on thirty-seven cases to determine how performance varied with time: Senior Kindergartners improved significantly from fall to spring (22% vs. 34%), $F(1,36)=12.96$, $MSe=2557$. Senior Kindergartners' accuracy appeared to be lower than commonly reported (e.g., Siegler, 1987a). However, this result is probably a consequence of the problem sets used in the different studies. Other researchers tended to use all the problems that summed to five or less. However, in the present research, children received only five such problems and fifteen problems with sums greater than five. Children in this study who could add tended to be able to solve the smaller problems but not the larger ones, resulting in misleadingly poor accuracy scores.

As well as being interested in children's accuracy on addition, I asked participants to report the procedures they used to solve problems. Examining children's procedure use is of value because it can reveal what children may or may not know about arithmetic principles. For example, the use of the min procedure (counting up from the larger addend) can reflect children's knowledge that counting from the largest addend is advantageous because fewer counts are needed than when both addends are enumerated. Table 6 presents the frequencies of reported procedures. No visible procedure (NVP) refers to situations where children produced an answer and reported that they remembered the answer or did not calculate the solution. Counting refers to situations where children reported counting in their heads

overt counting was observed. Min procedure refers to situations where children reported counting up from the largest addend. Derived-fact procedures refer to situations where children used a known arithmetic fact to derive the solution to an unknown problem. Uninformative reports refer to situations where children gave an ambiguous response, reported not knowing how they arrived at a solution, or guessed.

Table 6. Percentage of Senior Kindergartners' reported procedures for correctly solved addition problems.

<u>NVP</u>	<u>Counting</u>	<u>Min</u>	<u>Derived-fact</u>	<u>Uninformative</u>
4.3	32.7	9.6	3.0	50.3

Half of the responses were uninformative. However, children reported counting on a substantial number of problems. Perhaps the most interesting result is that such young children occasionally reported quite sophisticated procedures (retrieval [NVP], min, and derived-fact procedures), similar to findings by other researchers (e.g., Bisanz et al., 1995, Siegler & Crowley, 1991).

As reported by other researchers (e.g., Siegler & Crowley, 1991), children in this study reported using a variety of solution procedures. Table 7 summarizes percentage of students who solved at least one addition problem and who used different numbers of solution procedures.

Performance was similar for fall and spring sessions. The drop from 11% to 0 for the use of five solution procedures reflects the performance of one child who used the min procedure once in the fall but not in the spring. Children in this sample who used at least two solution procedures tended to rely on counting and retrieval. Of the children who could use more sophisticated procedures, children tended to use both

Table 7. Percentage of Senior Kindergartners using different numbers of solution procedures for correctly solved addition problems.

	Number of Solution Procedures Reported				
	One	Two	Three	Four	Five
Fall (n=19)	16	42	21	11	11
Spring (n=28)	21	46	25	11	0

min and derived-fact procedures; the correlations between percentage use of each procedure at fall and spring sessions were $r(34)=.32$, $p=.06$ and $r(34)=.82$, $p<.01$ respectively. Thus, it appears that most children who could add used both counting and retrieval. However, children who had discovered a sophisticated solution procedure tended to use more than one (i.e., both min and derived-facts), providing evidence that these children had a better understanding of addition than their peers who did not use either procedure.

In summary, the older children were better at recognizing numbers than the younger children. However, the younger children made great improvements on the number recognition task, probably due to instruction. Performance on the changes-in-amount-task was poor for the younger children and slightly better for the older children. These results are similar to those reported by Cowan (1991), where he found that children less than 7 years-of-age are likely to have trouble with the meanings of same, different, and more and to have trouble judging relative numbers of items. With respect to the simple arithmetic tasks, older children were better at addition than the younger children; children in both grades could not solve subtraction problems. The older children's improved performance was likely to be the result of instruction at home or at school on how to solve arithmetic problems. Informal questioning of the

children as to where they learned to add yielded some interesting anecdotes. One child reported having known how to add since he was a baby and another child (who used the multiplication operation to solve subtraction problems) reported having a multiplication table in his pocket and he was picturing it in his head to solve the problem. This same child also used his toes to count when problems summed to an answer greater than ten. Other children reported practicing adding at home with their parents. Children also tended to use a variety of solution procedures and if they used a sophisticated procedure such as min, they were likely to use another sophisticated procedure such as derived-facts. It appears as if children discover short-cut procedures, they are likely to invent other helpful solution procedures as well. Thus, these results imply that as early as Senior Kindergarten, and before formal arithmetic instruction, children are learning and practicing addition problems and inventing helpful short-cut procedures.

Explanation-Based Results

Along with assessing what children do (performance-based measures), I assessed children's evaluations and justifications for four tasks (order-irrelevance, conservation of cardinality, conservation of number [see Appendix C] and commutativity) to get a more complete picture of what the children knew about the concepts underlying these tasks (Giaquinto, 1995). The reason these tasks were chosen was that more than procedural knowledge may be involved in applying the order-irrelevance principle, for example.

Order-irrelevance principle

To investigate children's understanding of the order-irrelevance principle, I modelled counts that began at different positions on an array. Children then responded

whether counting that way was a good way or not and explained their evaluation.

Uninformative justifications refer to situations where children provided an ambiguous or no justification. Incorrect procedure or demonstrations refer to when children reported that counting from a certain point is incorrect; it may also refer to when children demonstrated a count incorrectly. Correct procedure or demonstrations refer to when children reported that counting from a certain point is correct or they demonstrated the count themselves. Logical justifications occurred when children explained the order-irrelevance principle. Table 8 summarizes children's justifications.

Table 8. Percent of children's justification of modelled counts beginning at various places (percent of justifications on which children were accurate on the application task in parentheses).

Justification	Starting place for count		
	Beginning	Middle	End
Uninformative	62 (100)	55 (100)	59 (100)
Incorrect procedure	6 (100)	24 (19)	3 (100)
Correct procedure	26 (100)	12 (59)	24 (90)
Logical	6 (100)	9 (50)	15 (100)

Uninformative justifications were given with similar frequency for each type of count. Incorrect procedures or demonstrations, however, were concentrated at the middle position. Moreover, nearly all children who reported that counting from the middle is wrong could not count from the middle themselves. Correct procedures and demonstrations were given primarily for counts starting at the beginning or end of the array. Logical explanations were given with approximately equal frequency for each starting place. Children who gave logical explanations were largely accurate with their

own counts, with the exception of some counts starting at the middle. Considering the data presented in Table 8, children seemed more likely to be able to apply the order-irrelevance principle themselves than to provide an explanation about its use. What remains unclear is whether children's infrequent use of informative explanations is a result of limited verbal skills or metacognitive awareness or reflects a lack of understanding of the order-irrelevance principle.

Conservation of cardinality

On the conservation of cardinality task, children were asked to report the number of items that the experimenter had hidden. Their explanations for their answers tended to fall into three categories. Uninformative explanations included the response, "I don't know," and no explanation. Visual explanations were based on children's reports that they had seen the array before but included no mention of conservation. Logical explanations included a statement that the children had counted the items and remembered the number. Children's justifications are summarized in Table 9.

Table 9. Percentage of justification given by children who could conserve cardinality.

Grade	Type of Justification		
	Uninformative	Visual	Logical
Junior Kindergarten	28	14	58
Senior Kindergarten	16	14	70

When children were able to give an explanation, they tended to give logical explanations (e.g., "There are 6 because I counted them before and you just covered them up."). Thus, it seems that children of both ages could conserve the number of

items in a set despite being prevented from counting the items. Performance on the conservation of cardinality task was correlated with performance on the cardinality task (see Appendix F).

Commutativity

The number of children who evaluated at least one commutativity problem by stating whether applying the commutative principle was a good way to solve the two problems are presented in Table 10. The Junior Kindergartners performed at floor levels so their data will be excluded from future analyses. The number of Senior Kindergartners who evaluated at least one commutativity problem is similar to the number who responded to at least one arithmetic problem.

Senior Kindergartners' justifications of modelled correct commutativity application may be of more interest than their evaluations because their evaluations could have been biased in some way (e.g., always responding positively).

Table 10. Number of children who evaluated at least one commutativity problem.

	Fall	Spring
Junior Kindergarten (n=30)	2	3
Senior Kindergarten (n=37)	16	25

Uninformative justifications occurred when children's responses were uninterpretable (e.g., "I don't know why it's a good way.") Incorrect application or explanations occurred when children misapplied the commutative principle (e.g., applying the commutative principle to a subtraction problem). Correct application or explanations

occurred when children correctly applied the commutative principle (e.g., spontaneously applying commutativity to two addition problems). Equivalent answers were those that included a statement that the two problems were the same (e.g., "You wouldn't need to solve them both because $2+3$ is the same as $3+2$.") Rule related answers were those in which children stated the commutativity rule (e.g., "You only need to solve one because they're the same problem and they have the same answer.") The percentages of each type of justification given for the modelled procedure are shown in Table 11.

Table 11. The percentage of each type of justification given for modelled commutativity principle.

	Justification Types				
	Uninformative	Incorrect	Correct	Equivalent	Rule-based
Fall	15	15	18	18	33
Spring	22	12	24	12	29

It appears as if children gave more uninformative or incorrect justifications and less equivalent or rule-based justifications in the spring than in the fall. However, this result may reflect the large increase in number of children attempting the commutativity task (see Table 10) in that the new responders in the spring may not have been able to give informative explanations (similar to the 30% of invalid justifications given in the fall). Interestingly, the majority of responses were informative explanations (i.e., correct, equivalent, or rule-based) at both sessions so it appears that once children discover the commutativity principle, they are generally able to give a valid justification for its use.

To examine the supposition given by Baroody and Gannon (1984) and Resnick

(1992) that the use of min implies an understanding of commutativity, I examined the relation between the use of min and the type of response given on the commutativity problems. I collapsed the response categories previously described in the following way: uninformative and incorrect, equivalence and rule-based, and correct (spontaneous application). The rationale for combining responses in this manner was to portray a hierarchical picture of children's understanding of commutativity. Uninformative responses are least revealing, equivalence or rule-based responses are more revealing, and correct-application responses are most revealing because children spontaneously applied commutativity without being prompted. I categorized children by the type of response they gave. To be categorized as a certain response user, a child had to use a particular response type for at least one-third of the six problems. I used a lenient cut off because very few children attempted all six problems. Table 12 summarizes the percentage of children using min and each type of response.

Table 12. Percentage of children using min and each type of commutativity response.

	Uninformative	Equivalence or Rule-based	Spontaneous Application
Fall	0	5	20
Spring	0	7	11

As Baroody and Gannon and Resnick suggested, the use of min is related to a good understanding of commutativity. Children who could not explain the use of commutativity never used min. Children who could explain commutativity when it was modelled used min only occasionally. However, children who spontaneously applied commutativity were most likely to use the min procedure.

In summary, Senior Kindergartners tended to have discovered the rules

underlying the counting string and could count up to 40, 70, or 100. Senior Kindergartners also had a better understanding of counting principles such as cardinality, which may have helped them to learn addition. Junior Kindergartners improved from fall to spring on counting tasks and also learned to recognize numbers. Older children were better at addition than younger children. Moreover, the older children were more accurate with small problems than large and they used a wide variety of solution procedures. The use of the min procedure was related to the use of logical justifications of the commutativity principle. Thus, it appears that children in Junior Kindergarten made great gains in their counting skills and resembled their highly skilled Senior Kindergarten counterparts by the spring session. It seemed that Senior Kindergartners were able to utilize their greater fluency and experience with numbers to invent and use solution procedures for addition and to discover the commutativity principle.

Relations between counting and arithmetic.

To investigate the relations between counting and arithmetic, I used hierarchical regressions to assess the relative contributions of the various counting principles to addition performance. I examined only the Senior Kindergartners because the Junior Kindergartners performed at floor-levels on the simple arithmetic problems. Performance was predicted only for addition because accuracy on subtraction was near floor levels. The order of entry of the predictor variables was based on the model, which predicts that counting principles will culminate in a high counting limit, which then leads to addition performance. Therefore, after controlling for age, I entered counting limit, then the counting principles as a group to predict addition performance. Correlations among the variables are shown in Appendix F.

The first regression of counting limit and counting principles on addition performance for Senior Kindergartners is shown in Table 13.

Table 13. Predicting addition from counting limit and counting principles for Senior Kindergartners.

<u>Addition</u> in fall (1)	<u>B</u>	<u>β</u>	<u>R^2</u>	<u>R^2</u> change	p of <u>R^2</u> change
age	2.52	.30	.09	.09	.08
McCarthy	1.34	.27	.16	.07	.10
counting limit (1)	.57	.62	.51	.35	.00
principles (1)			.54	.03	.77
cardinality	.27	.15			
one-to-one	.13	.14			
stable order	-.08	-.09			
order-irrel.	.04	.02			
<u>Addition</u> in spring (2)					
age	1.78	.20	.04	.04	.24
addition (1)	.82	.78	.59	.55	.00
McCarthy	.83	.16	.62	.02	.16
counting limit (1)	.40	.42	.71	.09	.00
principles (1)			.74	.03	.58
cardinality	-.23	-.12			
one-to-one	.07	.08			
stable order	.18	.21			
order-irrel.	-.12	-.07			
<u>Addition</u> in spring (2)					
age	1.78	.20	.04	.04	.24
addition (1)	.82	.78	.59	.55	.00

McCarthy	.83	.16	.62	.02	.16
counting limit (2)	.28	.31	.68	.06	.02
principles (2)			.71	.04	.50
cardinality	-.38	-.10			
one-to-one	.17	.18			
stable order	.09	.05			
order-irrel.	-.02	-.01			

Note. Number in parentheses after outcome and predictor variables indicates testing session (1=fall, 2=spring).

Counting limit was the best predictor of addition performance for Senior Kindergartners. One explanation of the influence of counting limit is that a relatively high counting limit reflects numerical fluency. In other words, counting skill improves with time and practice understanding increases with improvements in performance, and understanding improves performance. Thus, developments in counting skill and knowledge lead to a fluency with numbers, which in turn, promotes adding ability.

The question remains as to which component of counting skill is most important? Of the four counting principles tested here, several could play a critical role. For example, stable order could be important in adding because using a consistent counting string is necessary to count up to the right answer. The one-to-one principle could also be important for counting in addition because to get the right answer, each addend must be counted correctly. Both stable-order and one-to-one principles are procedure-related; in other words, they are important for the "how-to" part of counting. Conversely, the cardinality principle reflects conceptual knowledge, the knowledge that counting a set results in knowing its quantity. Thus, cardinality's

potential role in addition is conceptual, in other words, the understanding that "4" means a distinct quantity and 4+4 is another distinct quantity is required. Indeed, Baroody and Gannon (1984) suggested that it is an understanding of cardinality that allows the development of the min procedure and the commutativity principle. For these reasons, I expected that cardinality would be the counting principle most important for predicting counting limit. To test this hypothesis I regressed cardinality, conservation of cardinality, and then the other counting principles (as a group) on counting limit, after controlling for grade. I included both grades in the analyses because: (a) age did not predict counting limit and (b) using all the participants would permit inclusion of more variables of interest. Appendix F displays the correlations among the variables and Table 14 summarizes the regression of counting limit and principles on addition performance for Senior Kindergartners in the fall and in the spring.

Table 14. Predicting counting limit from counting principles for children in both grades.

<u>Counting limit</u> in fall (1)	<u>B</u>	<u>β</u>	<u>R^2</u>	<u>R^2</u> change	p of <u>R^2</u> change
grade	31.4	.56	.31	.31	.00
cardinality (1)	.37	.38	.39	.08	.00
conservation of cardinality (1)	-1.67	-.03	.39	.00	.85
stable order (1)	.11	.17	.41	.02	.14
one-to-one (1)	.06	.09	.42	.01	.43
order-irrelevance (1)	-.05	-.04	.42	.00	.76

<u>Counting limit in spring (2)</u>					
grade	40.89	.63	.39	.39	.00
counting limit (1)	.86	.75	.78	.38	.00
cardinality (1)	.13	.11	.78	.01	.19
conservation of	.89	.01	.78	.00	.88
cardinality (1)					
stable order (1)	.06	.08	.79	.01	.27
one-to-one (1)	.06	.07	.79	.00	.28
order-irrelevance (1)	.02	.01	.79	.00	.86
<u>Counting limit in spring (2)</u>					
grade	40.89	.63	.39	.39	.00
counting limit (1)	.86	.75	.78	.38	.00
cardinality (2)	.10	.07	.78	.00	.42
conservation of	-.93	-.01	.78	.00	.92
cardinality (2)					
stable order (2)	.02	.02	.78	.00	.76
one-to-one (2)	-.08	-.08	.79	.01	.24
order-irrelevance (2)	.05	.03	.79	.00	.63

As expected, cardinality best predicted counting limit, whereas the other principles did not. However, this relation did not hold by the spring session, where counting limit in the fall was the best predictor. In other words, previous counting limit predicted counting limit in the spring. This result is not surprising because, presumably, good counters should continue to be good counters.

One explanation of the relation between counting limit and cardinality concerns the types of counting knowledge involved. Specifically, stable order and one-to-one

principles reflect procedural knowledge. On the other hand, cardinality is a multifaceted concept (Sophian, 1988; Wynn, 1992) that reflects conceptual knowledge about counting, for example, that counting gives you quantity. This knowledge may need more time to develop than procedural knowledge and is thus still important at age four or five.

Counting limit, aside from its critical relation to accuracy in addition, had an important role in procedure use for addition. I correlated the percentages of procedure use with the counting principles (with the effects of age partialled out). The important relations at the fall session were between counting limit and percentage use of derived-fact procedures ($r[34]=.32$, $p=.06$), retrieval ($r[34]=.45$), and min ($r[34]=.44$). A plausible reason for this relation is that a greater fluency with numbers and the number system results in both a high counting limit and a tendency to invent and use sophisticated solution procedures. In other words, counting limit reflects an overall growth in numerical knowledge. The relations among counting limit and solution procedures were not significant by the spring session, presumably because the relation between counting and procedure use did not change from fall to spring. The question remains as to what aspects of a high counting limit help children use sophisticated procedures. One possibility is that a good understanding of cardinality (the best counting principle to predict counting limit) may lead to selection of sophisticated procedures. For example for the problem $2+3$, a child who knows the cardinality of 2 and 3, can make a magnitude comparison, and wants an efficient way to solve addition problems, may discover and utilize the min procedure.

Summary of Results

Children in both grades made great gains in their development of numeracy.

Junior Kindergartners developed stable counting strings, improved their ability to match items and increments in counting (one-to-one knowledge), could start their counts at several different places, counted to determine amount, and conserved cardinality. Senior Kindergartners were good counters in the fall and became expert counters by the spring session. They learned the rules for counting decades and became fluent with numbers and the counting string; in other words, they learned to generate a counting string, not just to recite the counting string. This fluency appeared to be related to the development of arithmetic skills (counting limit was the best predictor of addition performance). Moreover, the development of good arithmetic skills (i.e., use of short-cut procedures) was related to the development of commutativity knowledge.

In general, children's procedural knowledge (e.g., one-to-one or stable order knowledge) is not as critical to the development of numerical fluency as conceptual knowledge such as cardinality. This was the case for both counting limit (where cardinality predicted but procedural knowledge did not) and addition (where numerical fluency or counting limit predicted but other counting variables did not). One implication of this result for the development of numeracy is that if conceptual knowledge is more important than procedural knowledge, then instruction should emphasize the conceptual aspects of counting. For example, children should be encouraged to learn the different facets of cardinality as a way to teach them to quantify.

General Discussion

Children in Junior and Senior Kindergarten made dramatic improvements in the development of numeracy, particularly in the domains of counting and arithmetic.

Two aspects of counting seemed critical to children's numeracy. The first was counting limit, which appears to index both procedural and conceptual knowledge. Procedural knowledge of counting includes such abilities as recitation of the conventional number string. Conceptual knowledge includes understanding the underlying base 10 structure of the string and knowing that the incremental nature of counting is an additive operation. Resnick (1992) termed this incremental property of both counting and addition additive composition; in other words, counting and arithmetic are intrinsically additive operations. For example, counting up is essentially additive (e.g., adding by ones, 1, 2, 3, 4; or by fives, 5, 10, 15, 20). Resnick also described how arithmetic principles reflect additive composition. For example, associativity means that numbers can be decomposed (e.g., $2+2$ is $1+1+1+1$).

The second critical aspect of counting was also the best predictor of counting limit: cardinality. Cardinality reflects aspects of procedural knowledge such as stating the last number in the count as the number of items. Conceptual knowledge of cardinality is reflected in the knowledge that the last number in the count represents the amount of the set. In other words, cardinality captures the underlying purpose of counting, which is to determine quantity. In sum, cardinality and counting limit capture numerical fluency related to counting.

Similarly, counting limit and cardinality are critical aspects of numerical fluency related to addition. For example, counting limit was the best predictor of addition. Furthermore, counting limit and addition share additivity as a common property. In my view, additive composition represents one of the developmental continuities between counting and arithmetic. In other words, once children have discovered the additive nature of counting and learned to count to quantify, they may

generalize this knowledge to addition. The other important developmental continuity between counting and arithmetic is the underlying goal of quantification, either cardinality or sum. Just as children might generalize their knowledge of counting to addition, they might generalize their knowledge of cardinality as amount to addition. For example, when children use the min procedure, we can infer features of children's counting knowledge. To use min, children must know that an addend is equal to a cardinal number which is equal to an amount. In other words, to solve the problem $12+3$ using min, children must know that the number 12 represents the same quantity as they would get if they started counting up from 1.

In my view, counting limit, additive composition, and cardinality are critical elements of numerical fluency. What remains to be determined, however, is the nature of children's mental representations of these constructs. Wynn (1995) suggested that children have an innate, internal system called an accumulator for representing number and amount. The accumulator monitors input of items and makes one increment for each item encountered. Thus, the accumulator contains information about number and amount. However, the accumulator is an internal representation so children must learn the appropriate counting words, which Wynn termed linguistic counting, and map them onto the accumulator's contents. Thus, counting limit reflects the extent to which children have mapped the counting words and their ordinal positions onto the internal, magnitudinal representation. However, Wynn did not explain how the mapping process occurs, how children translate the amount in the accumulator into a cardinal quantity, or the nature of the mental representation once linguistic counting has been mapped onto the accumulator.

An alternative view of children's mental representation is that children actively

construct a representation of their numerical knowledge. Huttenlocher, Jordan and Levine (1994) explored one such mental model. They described a mental model as representations that preserve the relevant features of a situation in that attributes and relations are analogous to the physical parts and relations in the situation that the model represents. Moreover, models are used to produce simulations of changes in these situations. Thus, children might solve an arithmetic problem using symbolic representations to simulate the manipulations of quantity. The main problem with a mental-model representation is that a great many cognitive capabilities (e.g., working memory, symbolic thought, and manipulation of amount) are attributed to young children that they may not possess.

In my view, a combination of these two approaches to mental representations is appropriate. Specifically, children's initial representation of numerical information may take the form of an accumulator. However, with experience and practice with counting, children begin to map the number words onto the contents of the accumulator. The extent to which children are successful may determine their counting limits and understanding of cardinality. For example, children who are successful at mapping conventional number words onto the contents of the accumulator may have many experiences with the word three mapping onto the quantity "three" in the third ordinal position. Thus, these children would be likely to develop a notion of three as a quantity. Moreover, these children would likely be successful counters and it is likely that their counting limits would increase rapidly. With many such experiences over time, in concert with the development of symbolic thought, these children may come to create their own mental models of counting. In other words, they would use experiences with their internal representations of numbers

to create a mental model of the number system. For example, children might discover the additive nature of counting and generalize it to addition. Then, they may be more likely to generate their own mental model of the additive nature of the number system and learn arithmetic. They may be more likely than children with a limited mental model of addition to invent and use sophisticated short-cut procedures such as min and derived-facts. It is also likely that they would discover and apply addition principles such as commutativity.

My data support this view of development. Specifically, Junior Kindergartners had not yet discovered how to generate the number string as they were still trying to remember the irregular numbers between 10 and 20. This could have been a result of incomplete maps of linguistic counting onto the mental representation. Moreover, their performance on the cardinality tasks was poor in the fall and improved to an average success rate of 60% by the spring. Junior Kindergartners had also not learned addition. These results imply the lack of mental models of the number system. On the other hand, many Senior Kindergartners had learned to generate the counting string up to 39 in the fall and to 100 in the spring. These children may have successfully mapped linguistic counting onto their internal representations of number. Senior Kindergartners had also mastered the cardinality tasks by the spring session. Many Senior Kindergartners had also learned to add. Perhaps they had generated accurate mental models of the additive relations among numbers in counting and addition. Furthermore, the older children's understanding of addition seemed to be generalized. In other words, if they had discovered one sophisticated solution procedure, they tended to have discovered others (e.g., min, derived-facts). Moreover, the use of min was related to the spontaneous application of commutativity, perhaps the consequence

of a good mental model of the numerical relations in addition. It appears that even without formal instruction in arithmetic, children as young as those in Senior Kindergarten discovered the regularities in the number system and used them to invent and apply sophisticated procedures (e.g., min and derived-facts) and principles (e.g., commutativity). In my view, it is an understanding of the additive, quantitative nature of counting that underlies both counting and addition and once children have discovered it, they can create mental models to capture these relations. In my view, this is the essential accomplishment in the development of numeracy.

Educational Implications

Because counting limit, additive composition, and cardinality are the critical components of numeracy, educational curricula should emphasize these concepts. If children have difficulty learning addition, coaching on cardinality and additivity should help. If mapping linguistic counting onto mental representations is critical, then counting drills and exercises that emphasize the additive nature of counting may be useful. For example, instructors could stress the conceptual similarity between counting "1, 2, 3" and adding " $2+1$ ". This could help children develop reliable counting strings and assist them to generalize their understanding of counting to arithmetic. The goal of educators should be to enable children to create sound mental models of numbers and numerical relations.

Directions for Future Research

To extend the present study, I will test the participants of the present study in January 1996 after the older children have had formal arithmetic instruction in Grade 1. I expect to find a similar pattern of results for the younger children as shown by the Senior Kindergartners in this study because the younger children will have moved

up to Senior Kindergarten and had practice and instruction in counting. I predict that the older children will have mastered counting (e.g., unusual starting points and one-to-one correspondence). With respect to simple arithmetic, I expect the older children's performance to improve, and the range of numerosities they can add to increase. I also expect to see a similar diversification of procedure use and an increased reliance on retrieval by some children as they memorize the number facts. I also expect subtraction performance to be better than in the present research because children will have received instruction on subtraction. With respect to commutativity, I expect more children to apply it to addition problems. I also predict that some children will overgeneralize (incorrectly) and apply commutativity to subtraction problems, just as children tend to overextend grammatical structures (e.g., "I seed the puppy yesterday."; Rescorla, 1980). I expect the same relations between counting limit and addition to hold as children's counting limits should continue to increase. I expect that the changes-in-amount task should predict performance in subtraction because the changes-in-amount task presumably taps children's knowledge of how adding increases amount and removing items decreases amount.

Conclusions

In conclusion, Junior and Senior Kindergarten children make remarkable improvements in the development of numeracy. In my view, the contribution of this research to the field of developmental mathematical cognition is the discovery of a predictor of arithmetic performance: counting limit. This finding parallels the widespread finding in research on reading acquisition that the best predictor of reading performance is fluency with the alphabet. Specifically, children must not merely recite the letters of the alphabet, they must acquire the conceptual knowledge that letters

represent different sounds and those sounds combine to make words (Chall, 1983).

Similarly, for counting limit, it is not merely reciting the counting string that is important, it is the underlying knowledge that the numbers represent amounts and that there are constant additive relations among the numbers. Thus, essential to the development of numeracy is deciphering the numerical relations of counting and generalizing them to addition.

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Figure 1a. Junior Kindergartners' counting limits in the fall.

Figure 1b. Junior Kindergartners' counting limits in the spring.

Figure 1c. Senior Kindergartners' counting limits in the fall.

Figure 1d. Senior Kindergartners' counting limits in the spring.

Appendix A

Summary of Tasks

Skill/Concept Assessed	Task	No. of Items	No. of Trials	Est. Time (min)	Description and Example
1. Counting					
a. counting ability	counting	n/a	1	2	Count as high as they can.
	limit				e.g., 1, 2, 3, 4,....19
b. one-to-one	count-the-	20	2	2	e.g., Here are some dogs. Can you count
	pictures				them for me?
c. stable order	count-the-	as above			
	pictures				
d. order-irrelevance	application	10	3	5	e.g., Here are some butterflies. Can you show me
					middle, and end)?

Skill/Concept	Task	Items	Trials	Time	Description
	justification and evaluation	10	3	5	e.g., Puppet counts starting at the beginning. middle, and end of array. "Is this a good way to how you would start counting here (beginning. to do it?" the puppet asks.
e. cardinality	how-many	3, 4 7, 8 12, 13	6 (2x3)	5	Shown an array. Asked how many items there are. Given 2 of each level of difficulty until get 2 wrong.
	give-x	3, 4 7, 8 12, 13	6 (2x3)	5	Shown pile of small toys. Asked to give experimenter x toys. Given 2 of each level of difficulty until get 2 wrong.
	point-to-x	3, 4, <u>5</u> , 6 6, <u>7</u> , 8, 9 10, <u>11</u> , <u>12</u> , 13	6 (2x3)	5	Shown 4 arrays. Asked to point to array of x . Given each level of difficulty.

Skill/Concept	Task	Items	Trials	Time	Description
	compare-sets	3, 4	6 (2x3)	5	Shown pairs of arrays. Asked to say which is the biggest or smallest. Given 2 of each level
		7, 8			until get 2 wrong.
	conservation of cardinality	6, 11	2	5	Puppet counts out equal numbers of items for experimenter and child. Cover E's items and ask child how many there were.
2. Arithmetic					
a. changes in amount		n/a	3	5	e.g., "See my box of bears? I want to put some in (take some out, not put any in or take any out). I think I might have the same number of buttons, but I'm not sure. What do you think?"
b. number recognition		digits 0-10	11	3	e.g., Shown a card with "1" written on it. Puppet asks child what the card says.

Skill/Concept	Task	Items	Trials	Time	Description
c. addition		n/a	20	10	e.g., "How much do you get if you add 2 and 3 together?" Given easy, medium, hard, and challenge problems until they get 3 wrong.
d. subtraction		n/a	20	10	e.g., "How much do you get when you take 1 away from 2?" Given easy, medium, hard, and challenge problems until they get 3 wrong.
e. commutativity	application	4	8 (4x2)	10	Shown two commuted problems (e.g., $2+3$, $3+2$) and asked to solve them (1 pair each of small and large addition and subtraction problems).
	evaluation and justification	4	8 (4x2)	10	e.g., "If I get two problems like this, then I only have to do one. What do you think about that?" 2 easy, medium, and challenge addition and subtraction problems.

Skill/Concept	Task	Items	Trials	Time	Description
3. Conservation of number		6 and 11	2	5	Shown 2 arrays of equal number and asked which has more. One array is rearranged to make a longer line and the question is repeated.

Notes.

1. The estimated times for each tasks appear to exceed the 20 allotted for each session. However, not all tasks were given to all children (e.g., many 4-year-olds could not do the arithmetic problems) and some tasks were performed quickly by some children (e.g., many 5-year-olds could count very quickly). Thus, each session lasted for a maximum of 20 minutes.

2. The tasks were ordered in the following manner:

Session 1: Counting limit, compare-sets, order-irrelevance (application then evaluation/justification), changes in amount, number recognition, and addition and subtraction.

Session 2: One-to-one and stable order, how-many, conservation of number, give-x, conservation of cardinality, point-to-x, and commutativity.

Appendix B

Simple Arithmetic Problems

Problems are shown in the order that they were presented to the children.

Small	Medium	Large	Challenge
1+1	3+3	4+8	7+7
5-2	8-2	12-5	18-9
1+3	2+6	5+6	8+7
3-1	6-3	10-4	14-7
2+2	3+4	5+7	9+7
4-1	9-4	11-6	16-8
1+2	3+5	6+7	9+9
2-1	7-3	12-4	16-9
5-1	4+5	4+6	8+8
2+3	8-5	13-6	18-8

Commutativity

Small	Medium	Challenge
2+3, 3+2	7+2, 2+7	35+96, 96+35
6-4, 6-2	9-6, 9-3	142-89, 142-53
2+4, 4+2	3+6, 6+3	53+89, 89+53
5-3, 5-2	9-7, 9-2	131-96, 131-35

Appendix C

Performance on the Number Conservation Task

Descriptive statistics for conservation of number and conservation of cardinality are shown below. Because performance on this task was similar to results widely reported in the literature and because conservation performance was unrelated to other tasks in the study, I will not discuss the conservation of number task further.

Means (percentage correct) and standard deviations for the conservation of number task at each testing session for both grades.

Task	Junior Kindergarten				Senior Kindergarten			
	Fall		Spring		Fall		Spring	
	Mean	s	Mean	s	Mean	s	Mean	s
conserv. of	32	48	20	37	26	42	57	46
number								

Appendix D

Coding Protocol

The coding protocol is summarized below.

Counting

Counting limit. Both the maximum count with and without errors were recorded.

One-to-one. Children performed correctly on this task if they incremented their count by one for each item. Incorrect performance included missing items or counting them twice. Performance was coded as percent correct (0, 50, or 100).

Stable order. This task was performed at the same time as the one-to-one task.

Correct performance required the exact counting string to be consistent from the first trial to the second. Correct performance was coded 1, incorrect coded 0. Because not all children could count all 20 items, the maximum count (error-free) was recorded for both trials.

Order-irrelevance: Application. Performance was scored either correct (1) or incorrect (0) for the order-irrelevance application task. A correct score indicated that a child began counting at the beginning, middle, or end of a line and counted all the items. One-to-one or number-string accuracy were not assessed here because the concept of interest was order-irrelevance. In other words, children could have made counting errors other than order-irrelevance-errors and still have gotten a score of 1.

Order-irrelevance: Justification. Performance on this task was coded according to the following classifications.

- 0 No explanation or an uninformative explanation, e.g., "I don't know."
 "Because."
- 1 Ambiguous, e.g., "Because it's easy." "It's hard."

- 2 Incorrect procedure, e.g., "You'll miss some if you start there."
- 3 Incorrect demonstration, or repeated experimenter's count incorrectly, e.g., "Can't do it because it's wrong."
- 4 Demonstration, repeated the count or pointed in the correct manner to count, some statement of direction for count or counting procedure may have been demonstrated, e.g., "You start counting here (left) to here (right) because it's the start."
- 5 Logical explanation of order-irrelevance or referred to cardinality, e.g., "You can count any way." "You can start counting anywhere." "Counting that way still gives you 10." "You count them all and don't skip any."

After reliability was established and all data were coded, the coding protocol for the order-irrelevance justification task was collapsed according to the following scheme because there were too few data points per category in the old scheme.

Old Code	New Code
0, 1	1, uninformative or ambiguous
2, 3	2, incorrect procedure or incorrect demonstration
4	3, correct procedure or correct demonstration
5	4, logical explanation

How-many. To be scored as correct on this task, children had to report accurately the number of items in the set. Number and percent correct (0, 17, 33, 50, 67, 83, or 100) were also recorded.

Give-x. Children had to give the experimenter the exact number of items requested to be counted as correct (1=correct, 0=incorrect or no response). Number and percent correct (0, 17, 33, 50, 67, 83, 100) were also recorded.

Point-to-x. Children were required to point to the section of the display containing the requested number of items. Number and percent correct (0, 17, 33, 50, 67, 83, 100) were also recorded.

Compare-sets (strict). Correct performance required participants to count both sets correctly and choose the larger or smaller of the two as instructed. Percent correct (0, 33, 67, 100) was also recorded.

Compare-sets (lenient). Correct performance did not require children to count the sets accurately but did require children to select the larger or smaller set as instructed. Percent correct (0, 33, 67, or 100) was also recorded.

Conservation-of-cardinality. First, a child had to agree or state that he or she and the experimenter had the same number of items. Unless children did not do the task, they scored correctly because they needed to complete this task to continue to the second part of the task (1=correct, 0=incorrect). For the second part of the task, children received a score of 1 if they conserved the cardinality of the set and 0 if they did not or if they did not respond. For the third part of the task, children's justifications were coded according to the following scheme.

- 10 No explanation, uninformative.
e.g., "Because."
- 20 Visual reference. Explanation based on what children can see.
- 21 Incorrect, e.g., "There are 0 because you buried them."
- 22 Correct, e.g., "You just covered them up and I saw them before."
- 30 Conserved cardinality, or referred to the cardinality of the set.
e.g., "There were 6 before, you just covered them up." "Because I counted mine." "I counted them before."

Arithmetic

Changes in amount. Participants performed correctly on this task if they said there were more items if items were added, less if items were removed or same if no items were removed or added. Percent correct (0, 33, 67, or 100) was also recorded.

Number recognition. Participants' scores were number correct from total of 11 (all digits 0 through 10).

Simple Arithmetic. Based on children's verbal self-reports and observed behaviours (such as counting on their fingers), codes were assigned to the following scheme:

- 10 No visible procedure (NVP), presumably retrieval
e.g., Retrieved answers. "I just knew it."
- 20 Counting. Counted either mentally or on fingers or toes. In cases where it was impossible to determine if children used the count-all (e.g., for $2+3$, 1, 2, 3, 4, 5," or count-on procedures, coders assigned the general code of 20.
- 21 Min, started to add with the largest addend.
e.g., for $2+3$, "3, 4, 5."
- 22 Count-on, counted on from one of the addends.
e.g., for $2+3$, "2, 3, 4, 5."
- 30 Derived-fact procedures. Used a known arithmetic fact to derive the solution.
e.g., for $3+4$, "4+2 is 6 so one more makes 7."
- 31 Derived-fact procedure with 10 root. Used a number fact involving 10 to derive the solution.
e.g., for $9+6$, "10+6 is 16 so one less is 15."
- 32 Derived-fact procedure with tie root. Used a number fact involving a tie (e.g., $2+2$).

e.g., for $4+3$, "3+3 is 6 so one more is 7."

33 Derived-fact procedure with other root. Used a root fact other than a 10 fact or a tie.

40 Other, idiosyncratic strategy.

e.g., for $2+3$, "23."

50 Guessed.

e.g., "I guessed."

80 Indiscernible. Unable to determine the procedure used.

After reliability was established and all data were coded, the coding protocol for the simple arithmetic problems was collapsed according to the following scheme because there were too few data points per category in the old scheme.

Old Code	New Code
40-80	10, uninformative or ambiguous
20, 22	20, counting procedure
21	21, <u>min</u> procedure
30-33	30, derived-fact procedure
10	40, no visible procedure

Commutativity. Children's responses were coded according to the following scheme.

In the case of multiple explanations, the response that revealed children's highest level of knowledge. For example, if a child stated that the equations were the same (40) and then goes on to give the commutativity rule (60), the child received a code for the rule (60).

10 No explanation or action.

e.g., "I don't know."

- 20 Ambiguous.
 e.g., "It's better."
- 30 Solve both, incorrect (for addition). This code could be assigned for either the action of adding for both problems or for a verbal explanation.
 e.g., Solved both problems or stated, "Would need to solve both."
- 31 Solve both, correct (subtraction). This code could be assigned for either the action of subtracting for both problems or for a verbal explanation.
 e.g., Solved both problems or stated, "Would need to solve both."
- 40 Equivalence. Statement of sameness with no further elaboration.
 e.g., "They're the same numbers." "The answer is the same."
- 50 Spontaneous application of commutativity principle, correct. Child applies the commutativity rule without being prompted (for addition).
- 51 Spontaneous application of commutativity principle, incorrect. Child applies the commutativity rule without being prompted (subtraction).
- 60 Commutativity rule. Child explains commutativity.
 e.g., "2+3 is the same as 3+2 so they both have the same answer so you don't need to solve both." "If you figure out one, you get the answer to both because they both mean the same thing."

After reliability was established and all data were coded, the coding protocol for the commutativity task was collapsed according to the following scheme because there were too few data points per category in the old scheme.

Old Code	New Code
10, 20	1, uninformative or ambiguous
30, 32, 51	2, incorrect application or explanation of commutativity for the

operation

- 31, 50 3, correct application or explanation of commutativity for the operation
- 40 4, equivalence-related explanation
- 60 5, commutativity rule

McCarthy Puzzle-Solving Subtest. The McCarthy Puzzle-Solving Subtest was scored at the time of testing according to instructions from the testing manual.

Conservation. Codes for children's responses on the conservation task were taken from Bisanz, Morrison, and Dunn (1995), with some modifications. Codes for explanations were assigned according to the following scheme.

- 10 One-to-one matching
 - 11 Accurate matching. Verbal or nonverbal indication that each object in one row corresponds to an object in the other row.
e.g., "They line up two by two."
 - 12 Inaccurate matching. An unsuccessful attempt to match the corresponding objects in each row. "They don't match up at the ends."
- 20 Counting
 - 21 Accurate counting. Verbal or nonverbal indication of counting one row of objects at a time to justify that the two rows have the same number of objects. OR no indication (verbal or nonverbal) that counting occurred, but assertion that both rows contain n objects.
e.g., The child counted items in each row aloud or by pointing to each item in succession and then comparing the two totals.
 - 22 Inaccurate counting. Verbal or nonverbal indication of counting one row of objects at a time, but counting errors occur.

e.g., "They are different because this one has five and this one has six."

30 Logic

- 31 Compensation. Statement implying that even though one row has been compressed or lengthened, change is irrelevant and the two rows contain the same number.

e.g., "You just moved some out."

- 32 Inversion. Statement implying that the experimenter has neither added to nor taken away from the two rows of objects; they still have the same number.

e.g., "You didn't take any away."

- 33 Inversion: "Same as before." Statement without further elaboration that the two rows are the same as before.

e.g., "Last time they were the same; this time they're still the same."

- 34 Inversion: "No change." Statement, without further elaboration that no change has occurred despite the transformation.

e.g., The child copies the experimenter's gestures and says, "I saw you go like that."

- 40 Length. Indication that the two rows are the same length and thus are the same, or indication that the longest row has more.

e.g., "The ends are even." "It's longer than these that are squished together."

"This row has more because it's more spread out."

- 50 Density. Indication that the compressed row has more.

e.g., "These are more because they are more squished together."

- 60 Appearance. Indication, without elaboration, that the rows are the same

because they look the same.

e.g., "They look the same." "You can see it." "I can tell by looking."

- 70 Ambiguous response. Vague reference to quantity or similarity without justification of the response in terms of any of the aforementioned categories. Also, responses irrelevant to quantity and responses in which observers could not clearly ascertain the child's comments.

e.g., "You made them the same."

- 80 No response. No justification given.

e.g., "I don't know." "I just know."

After reliability was established and all data were coded, the coding protocol for the conservation task was collapsed according to the following scheme because there were too few data points per category in the old scheme.

Old Code	New Code
70-80	1, uninformative or ambiguous
40-60	2, appearance-related explanation
10-12, 20-22	3, counting or matching
30-34	4, logical explanation

Appendix E

Frequencies of Responses

All frequencies are in percentage correct unless otherwise indicated. Percents that do not sum to 100 are due to rounding error.

Counting Limit

Junior Kindergarten

Senior Kindergarten

Fall (n=30)

Spring (n=30)

Fall (n=37)

Spring (n=37)

limit n

limit n

limit n

limit n

2 1

2 1

7 1

12 1

3 1

4 2

10 1

15 1

4 3

5 1

12 2

16 2

5 1

10 2

14 1

18 1

7 2

11 3

15 1

19 1

10 3

12 5

20 2

20 1

11 1

13 1

24 3

24 1

12 5

14 5

25 1

28 1

13 2

16 1

28 1

33 1

14 4

19 2

29 3

39 8

15 1

20 2

30 1

43 1

17 1

26 1

33 2

49 4

20 2

29 1

38 2

59 1

26 1

30 1

39 6

75 1

29 1

40 1

40 1

99 1

120 1

69 1

77 1

100 10

78	1	109	1
99	2		
100	5		
103	1		

One-to-one (percentage of children in each grade)

	0	50	100
Fall			
Junior Kindergarten (n=31)	26	13	61
Senior Kindergarten (n=38)	8	21	71
Spring			
Junior Kindergarten (n=30)	13	27	60
Senior Kindergarten (n=36)	8	17	75

Stable order (percentage of children in each grade)

	0	100
Fall		
Junior Kindergarten (n=31)	39	61
Senior Kindergarten (n=38)	11	82
Spring		
Junior Kindergarten (n=30)	17	83
Senior Kindergarten (n=36)	3	97

Order-Irrelevance (Application, percentage of children in each grade)

	Beginning		Middle		End	
	0	100	0	100	0	100
Fall						

Junior Kindergarten (n=31)	0	100	71	29	10	90
Senior Kindergarten (n=38)	0	100	42	58	0	100
Spring						
Junior Kindergarten (n=30)	0	100	60	40	13	87
Senior Kindergarten (n=37)	0	100	59	41	8	92

Order-Irrelevance (Evaluation, percentage of children in each grade)

	Beginning		Middle		End	
	0	100	0	100	0	100
Fall						
Junior Kindergarten (n=31)	17	83	27	73	17	80
Senior Kindergarten (n=38)	0	100	30	70	11	89
Spring						
Junior Kindergarten (n=29)	14	86	24	76	7	93
Senior Kindergarten (n=37)	0	100	32	38	5	95

Order-Irrelevance (Justification, percentage of children in each grade, percent justification)

	Type of Justification					
	0	1	2	3	4	5
Fall						
Junior Kindergarten (n=31)	37	20	3	3	37	0
Senior Kindergarten (n=38)	46	11	3	0	32	8
Spring						
Junior Kindergarten (n=27)	37	33	7	7	11	4
Senior Kindergarten (n=37)	35	24	3	0	24	14

Cardinality - How-many (percentage of children in each grade)

	0	17	33	50	67	83	100
Fall							
Junior Kindergarten (n=31)	6	6	6	19	10	19	32
Senior Kindergarten (n=38)	0	0	3	5	3	37	53
Spring							
Junior Kindergarten (n=30)	0	3	7	17	17	17	40
Senior Kindergarten (n=37)	3	0	0	0	3	32	62

Give-x (Percentage of children in each grade)

	0	17	33	50	67	83	100
Fall							
Junior Kindergarten (n=31)	13	16	19	13	19	10	10
Senior Kindergarten (n=37)	0	3	5	0	16	35	43
Spring							
Junior Kindergarten (n=30)	3	3	20	17	23	30	3
Senior Kindergarten (n=36)	0	0	0	3	11	33	53

Point-to-x (Percentage of children in each grade)

	0	17	33	50	67	83	100
Fall							
Junior Kindergarten (n=31)	39	3	10	13	13	10	13
Senior Kindergarten (n=37)	0	3	5	0	16	32	43
Spring							
Junior Kindergarten (n=28)	7	18	14	7	7	21	25
Senior Kindergarten (n=37)	0	0	3	0	8	32	65

Compare-sets (Percentage of children in each grade)

	0	33	67	100
Fall				
Junior Kindergarten (n=31)	26	32	35	6
Senior Kindergarten (n=38)	0	18	24	58
Spring				
Junior Kindergarten (n=30)	13	40	27	20
Senior Kindergarten (n=37)	0	5	41	54

Conservation of Cardinality (Conserved cardinality, yes or no, in percent)

	Small		Large	
	Yes	No	Yes	No
Fall				
Junior Kindergarten (n=29)	7	93	14	86
Senior Kindergarten (n=38)	11	89	0	100
Spring				
Junior Kindergarten (n=30)	17	83	54	43
Senior Kindergarten (n=36)	0	100	14	86

Conservation of Cardinality (Percentage of each type of justification)

	Small				Large			
	10	21	22	30	10	21	22	30
Fall								
Junior Kindergarten (n=29)	45	17	10	28	25	20	55	0
Senior Kindergarten (n=38)	21	5	18	55	24	8	11	57
Spring								

Junior Kindergarten (n=25)	44	0	20	36	44	0	2036
Senior Kindergarten (n=36)	14	0	17	69	8	0	2269

Arithmetic

Changes in amount (Percentage of children in each grade)

	0	33	67	100
Fall				
Junior Kindergarten (n=31)	35	26	26	13
Senior Kindergarten (n=38)	3	26	29	42
Spring				
Junior Kindergarten (n=30)	13	37	33	17
Senior Kindergarten (n=37)	8	11	30	51

Number recognition (Percentage of children in each grade)

	0	18	27	36	45	55	64	73	82	91	100
Fall											
JK ^a	7	13	13	3	3	3	7	10	7	7	27
(n=30)											
SK ^b	0	0	0	0	3	0	0	5	8	5	78
(n=37)											
Spring											
JK ^a	3	3	0	10	3	7	10	10	7	0	47
(n=30)											
SK ^b	0	0	0	0	0	0	0	0	3	3	95
(n=37)											

^aJunior Kindergarten, ^bSenior Kindergarten

Appendix F

Correlation Matrices of Dependent Measures

All correlations marked "*" are significant at $\alpha = .05$. A list of abbreviations is presented below. The suffix ".1" or ".2" denotes testing session (1=fall, 2=spring).

Abbreviation Measure

card.	cardinality
cardcon	conservation of cardinality
one-one	one-to-one
order	order-irrelevance
stable	stable order
limit	counting limit
am't.	changes-in-amount
recog.	number recognition
add.	addition
subtract.	subtraction
conserv.	conservation of number
McCarthy	McCarthy Puzzle-Solving Subtest
sex	sex

Junior Kindergarten (age partialled out)

	1	2	3	4	5	6	7	8	9	10
1. card.1	1.0									
2. card.2	.40*	1.0								
3. cardcon.1	.60*	.56*	1.0							
4. cardcon.2	.64*	.25	.58*	1.0						
5. one-one.1	.28	.44*	.01	-.13	1.0					
6. one-one.2	.34	.48*	.27	.07	.59*	1.0				
7. order.1	.19	.21	.27	.37	.13	.16	1.0			
8. order.2	.07	-.07	.37	.25	-.10	-.12	.40*	1.0		
9. stable.1	.25	.15	.11	.24	.017	-.13	-.12	-.08	1.0	
10. stable.2	.11	.26	.45*	.26	-.07	.14	-.01	.17	.19	1.0

Junior Kindergarten, continued

	1	2	3	4	5	6	7	8	9	10
11. limit.1	.55*	.21	.37	.43*	-.07	-.03	.04	.01	.31	-.03
12. limit.2	.53*	.31	.40*	.23	.10	-.15	-.21	-.04	.34	.06
13. am't.1	.46*	.25	.44*	.36	.19	.02	-.03	.06	.36	.24
14. am't.2	.39*	.51	.21	.29	.35	.16	.04	-.23	.48*	-.04
15. recog.1	.79*	.40	.62*	.71*	.02	.06	.37*	.26	.16	.20
16. recog.2	.60*	.19	.44*	.68*	-.03	.05	.32	.26	.03	.22
17. add.1	.30	.20	.15	-.02	.20	-.02	-.14	.06	.22	.12
18. add.2	.24	.31	.44*	.25	.11	.25	.23	.19	-.14	.16
19. conserv.1	-.13	-.40	-.17	.05	-.22	-.07	-.16	-.14	.20	.11
20. conserv.2	.24	-.02	.37	.33	-.09	.22	.23	.05	-.05	.07
21. McCarthy	.26	.36	.20	.23	.26	.08	.57*	.04	-.10	.12
22. sex	.11	.20	.12	-.03	.02	.16	.24	-.15	.17	.17

Junior Kindergarten, continued

	11	12	13	14	15	16	17	18	19	20
11. limit.1	1.0									
12. limit.2	.65*	1.0								
13. am't.1	.22	.29	1.0							
14. am't.2	.14	.23	.47*	1.0						
15. recog.1	.46*	.40*	.35	.19	1.0					
16. recog.2	.27	.29	.26	.11	.81*	1.0				
17. add.1	.65*	.46*	.16	.09	.30	.23	1.0			
18. add.2	.07	.12	.31	.15	.37	.32	.02	1.0		
19. conserv.1	.01	-.05	.04	-.13	-.18	-.11	-.19	-.24	1.0	
20. conserv.2	.39*	.03	.26	-.03	.17	.27	.17	-.01	.26	1.0
21. McCarthy	.18	.06	.22	.38	.37	.34	.28	.19	-.09	.13
22. sex	.05	.09	.01	.22	-.14	-.27	-.24	-.28	.17	.10
	21	22								
21. McCarthy	1.0									
22. sex	-.06	1.0								

Senior Kindergarten (age partialled out)

	1	2	3	4	5	6	7	8	9	10
1. card.1	1.0									
2. card.2	.13	1.0								
3. cardcon.1	.22	.37*	1.0							
4. cardcon.2	.27	.13	.34	1.0						
5. one-one.1	.27	.27	-.19	.16	1.0					
6. one-one.2	-.04	.15	-.22	-.13	.43*	1.0				
7. order.1	.18	-.14	.30	.25	-.24	-.09	1.0			
8. order.2	.39*	-.05	.29	.32	-.33	-.38*	.55*	1.0		
9. stable.1	.78*	.11	.12	.25	.11	.03	.13	.34*	1.0	
10. stable.2	.18	-.20	-.04	-.04	-.08	-.08	-.14	.07	.44*	1.0

2 of / de 2

PM-1 3 1/2" x 4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Senior Kindergarten, continued

	1	2	3	4	5	6	7	8	9	10
11. limit.1	.46*	.22	.14	.16	.28	.08	-.01	.14	.41*	.18
12. limit.2	.49*	.24	.16	.20	.32	.00	.09	.21	.46*	.19
13. am't.1	-.06	-.13	-.02	-.14	.10	-.12	-.30	-.33*	-.12	.03
14. am't.2	.12	.25	.00	-.20	.35*	.02	-.13	-.11	.03	.06
15. recog.1	.54*	.32	.28	.77*	.19	.17	.14	.24	.52*	-.06
16. recog.2	.07	.20	-.13	-.06	.20	.10	.18	-.12	.11	-.02
17. add.1	.40*	.17	.04	-.12	.35*	.16	.00	.00	.29	.17
18. add.2	.41	.16	.18	-.08	.35*	.24	-.09	.00	.40	.21
19. subtract.1	.17	.00	.04	.04	.10	-.18	-.15	-.08	.06	.04
20. subtract.2	.24	.14	.14	.11	.21	.04	-.20	.05	.17	.09
21. conserv.1	.10	.08	.17	-.15	.21	-.07	-.43*	-.07	.03	.13
22. conserv.2	.16	.19	.23	-.19	.09	.05	-.22	.00	.00	.
23. McCarthy	.21	.23	.00	-.04	.34*	.28	-.02	-.09	.18	.19
24. sex	-.14	-.03	-.09	-.24	.07	-.01	.23	.14	.07	.19

Senior Kindergarten, continued

	11	12	13	14	15	16	17	18	19	20
11. limit.1	1.0									
12. limit.2	.79*	1.0								
13. am't.1	.30	.06	1.0							
14. am't.2	.38*	.18	.42*	1.0						
15. recog.1	.30	.38*	-.17	-.15	1.0					
16. recog.2	.07	.14	.03	-.11	.36*	1.0				
17. add.1	.67*	.55*	.29	.34*	.13	-.01	1.0			
18. add.2	.74*	.65	.24	.37*	.15	.07	.76*	1.0		
19. subtract.1	-.05	.19	-.20	-.04	.07	.03	.17	.19	1.0	
20. subtract.2	.58*	.48*	.16	.30	.10	-.12	.60*	.70*	.32	1.0
21. conserv.1	.19	.08	.30	.25	.02	.10	.21	.40*	.27	.56*
22. conserv.2	.36*	.36*	.17	.38*	-.07	.06	.38*	.53*	.11	.36
23. McCarthy	.22	.09	-.12	.34	.02	-.13	.22	.23	.26	.21
24. sex	-.05	-.07	-.08	.13	-.20	.05	-.11	.08	-.21	-.08
	21	22	23	24						
21. conserv.1	1.0									
22. conserv.2	.36	1.0								
23. McCarthy	.16	.01	1.0							
24. sex	.00	.04	.20	1.0						

Correlation matrix containing the variables entered into the regression equation predicting counting limit from counting principles.

Junior and Senior Kindergarten

	1	2	3	4	5	6	7	8	9	10
1. card.1	1.0									
2. card.2	.67*	1.0								
3. cardcon.1	.64*	.66*	1.0							
4. cardcon.2	.66*	.48*	.61*	1.0						
5. limit.1	.57*	.45*	.38*	.34*	1.0					
6. limit.2	.62*	.51*	.43*	.35*	.86*	1.0				
7. order.1	.30*	.23	.35*	.38*	.16	.20	1.0			
8. order.2	.18	-.03	.32*	.31*	.09	.12	.46*	1.0		
9. stable.1	.49*	.29*	.22	.26*	.39*	.43*	.07	.13	1.0	
10. stable.2	.14	.18	.30*	.14	.14	.17	-.04	.21	.31*	1.0
11. one-one.1	.37	.39*	.06	.03	.26*	.29*	-.01	-.23	.13	-.08
12. one-one.2	.22	.33*	.10	.05	.13	.07	.05	-.25	.00	-.06
13. grade	.63*	.64*	.48*	.43*	.56*	.61*	.22*	.04	.27*	.17
	11	12	13							
11. one-one.1	1.0									
12. one-one.2	.54*	1.0								
13. grade	.16	.11	1.0							

END

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FIN