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MUONIC ATOMS:
A TEST OF QUANTUM ELECTRODYNAMICS

by

JAMES SAMUEL WADDEN

A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy.

Department of Physics
Carleton University
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September, 1977

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1977

The undersigned recommend to the Faculty of
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ABSTRACT

Accurate measurements have been made for twelve muonic atom transition energies in five elements. The levels chosen were those which are sensitive to vacuum polarization corrections. The average difference between experiment and theory amounts to $(.18 \pm .17) \%$ of the vacuum polarization corrections.

to betty

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CHAPTER I

INTRODUCTION

The early work with muonic atoms was mainly in using the muon as a probe to study nuclear sizes and shapes. With the advent of high resolution Ge(Li) detectors it became feasible to use transitions in muonic atoms to study other effects, namely the quantum electrodynamical (QED) corrections which must be applied to the muonic energy levels. The necessity of including the lowest order QED corrections in muonic atoms was established by Koslov et al. (1954) using NaI crystals. Anderson (1970) pointed out that higher order QED terms could also be checked with the high resolution detectors available. With this in mind an experiment was done using the cyclotron at the University of Chicago to measure muonic x-ray transitions in a number of elements (Dixit et al. 1971). This experiment showed a systematic discrepancy between the calculated and measured transition energies. In particular, the transitions in Pb and Ba were found to lie several standard deviations below the theoretical values. These results were confirmed by Walter et al. (1972) on measurements of muonic transitions in Hg and Tl. Both these experiments disagreed with an earlier, and lower precision, measurement by Backenstoss et al. (1970) who found agreement between theory and experiment.

If one believes the muon is simply a "heavy electron" the transition energies can be calculated by solving the Dirac equation and including the QED corrections. For the transitions involved the effect of nuclear finite size, electron screening etc. are much smaller than the QED corrections. Thus a large discrepancy between theoretical and experimental energies could

indicate a weakness in QED theory or indicate that the muon was somehow behaving differently than expected. The latter would be somewhat surprising since the muonic $(g-2)$ experiments, Bailey et al. (1968), seem to confirm the heavy electron hypothesis as well as verifying the QED calculations involved. QED seems to have a great deal of success. The Lamb shift in hydrogen and the $(g-2)$ measurements for the electron both agree very well with theory. In high energy physics, experiments on electron-electron and electron-positron scattering as well as electron-electron production provide good verification of QED. The reviews by Pipkin (1970) and Brodsky and Drell (1970) provide a detailed account of the various tests of QED.

In view of the excellent results provided by QED it was somewhat disquieting when the initial muonic atom experiments around 1970 showed possible discrepancies. Since 1971 the situation has improved somewhat. Better theoretical calculations have been made, including the correction of a wrong sign in one of the terms. The paper by Watson and Sundaresan (1974) gives an excellent summary of the situation at that time. Fig. 1.1-1 shows the discrepancy between theory and experiment in 1974.

The present experiment was performed in order to provide better experimental values for the muonic transitions. By using a high resolution intrinsic Ge diode coupled to a high stability electronic system and by doing extensive checks for possible systematic errors we hoped to give a definite answer to whether a serious discrepancy still existed between theory and experiment in muonic atoms.

Figure 1.1-1. Plot of $E(\text{theory}) - E(\text{experiment})$ against energy for several muonic atom transitions as it existed in 1974. Data is taken from Watson and Sundaresan (1974).

CHAPTER II

THEORY

2.1 Muonic Atoms

A muonic atom is formed when a negative muon is captured by the electromagnetic field of a positive nucleus. The muon, once captured, will enter into hydrogen-like orbits about the nucleus of charge Ze . It is the transitions from the high energy to the low energy orbits that are referred to as muonic transitions. Since the muon is approximately 207 times as massive as an electron, the orbits will lie correspondingly closer to the nucleus than electronic orbits and the transition energies will lie in the x-ray region.

If the muon is treated as a heavy electron the energy levels of the muonic atom, assuming a point nucleus, can be calculated from the Dirac equation. The energy levels from the solution to the Dirac equation are given by

$$E_{n,j} = m_{\mu} \left[1 + \left[\frac{Z\alpha}{n - (j + 1/2) + ((j + 1/2)^2 - (Z\alpha)^2)^{1/2}} \right]^2 \right]^{-1/2}$$

where

m_{μ} = mass of muon,

Z = atomic number of nucleus,

α = fine structure constant, and

j = angular momentum of muon.

This expression will be valid for small Z elements and levels which are distant from the nucleus. Otherwise the Dirac equation must be solved numerically to account for the finite size of the nucleus. For example, the $1s$ level in muonic Pb is shifted from a point nucleus value of ≈ 20 MeV to a value of ≈ 10 MeV, a rather dramatic change.

There are other corrections which must be made to the Dirac energies. For the outer levels the effect of the electron cloud can become large. There is, however, an intermediate region where both the finite size and electron screening corrections are fairly small. In this region the largest corrections to the Dirac energy comes from the electron-positron vacuum polarization corrections and it is in this region that the transition energies measured in this work lie.

2.2 Vacuum Polarization

The largest QED correction is the electron-positron vacuum polarization term. This is due to the creation of virtual electron-positron pairs. If there is a large variation in electric field strength in a distance of the order of $\hbar/mc$ an appreciable polarization of the electron-positron pair can result. This polarization causes deviations in the Coulomb law of force between the muon and the nucleus and thus raises the energy levels.

The change in the energy levels may be treated as a small perturbation and we can write

$$\Delta E = \int V(r) |\psi|^2 d\mathbf{r}$$

where $V(r)$ is the deviation from a Coulomb potential. This expression for

ΔE is generally referred to as the Uehling term (Uehling 1935). The Feynman diagram corresponding to the Uehling term is shown in Fig. 2.2-1a. This term has been evaluated by many workers; for details see Watson and Sundaresan (1974). Their evaluation includes the contribution to the second order diagram, Fig. 2.2-1b, and also accounts for the effect of finite nuclear size on the charge distribution producing the vacuum polarization.

Feynman terms of order $\alpha(Z\alpha)^{2n+1}$, Fig. 2.2-(1c-1d), also contribute to the energy shifts. They are important for large Z elements. Their calculation is complicated by the finite size of the high Z nucleus. Terms of order $\alpha(Z\alpha)^{2n}$ vanish by Furry's theorem (Furry 1937) which states that a closed electron loop with an odd number of vertices has a zero matrix element.

Other Feynman diagrams also contribute to the shift in energy levels, for example the $\alpha^2(Z\alpha)$ terms of Fig. 2.2-(1e-1h). The muonic Uehling term, anomalous magnetic moment and Bethe sum are combined into the Lamb shift to provide a further small correction.

2.3 Electron Screening

The innermost electrons in the atom tend to screen the nucleus from the muon. The main screening in heavy atoms is due to the 1s electrons ($\approx 80\%$) and about (5 - 6) % for each 2s electron (Vogel et al. 1977). The principal difficulty in electron screening calculations lies in determining the number and distribution of electrons present. This is due to the ejection of Auger electrons as the muon cascades down from its higher orbits. The 1s electrons will probably be present since Auger ejection is only possible for these electrons for muonic transitions of $n < 7$. At this point radiative transitions are more probable than Auger transitions. Recent measurements (Vogel et al. 1977) indicate a low degree of ionization for the inner shells and tend to support the electron screening calculations of Vogel (1973) which are used in the present work.

Figure 2.2-1. Feynman diagrams of various orders for muonic atom transitions:
 (a) Uehling term of order $\alpha(Z\alpha)$, (b) second-order Uehling contribution, order $\alpha^2(Z\alpha)$, (c, d) terms of order $\alpha(Z\alpha)^2, ^3$,
 (e, f, g, h) terms of order $\alpha^2(Z\alpha)$.

2.4 Finite Size

The overlap between the nucleus and the muonic wave functions causes a reduction in the binding energy of the muon since the charge seen by the muon inside the nucleus is less than that seen outside. For inner orbits this can be quite large as mentioned before. As one moves to larger orbits this effect is reduced. When the overlap is small the energy shift can be calculated as a perturbation using a charge distribution of the Fermi type.

In first order perturbation theory the shift in transition energy due to finite size is given by

$$\Delta E = 4\pi Z \alpha \int \rho(r) [V^i(r) - V^f(r)] r^2 dr$$

where V^i and V^f are the potentials generated by the bound muon in the states i and f and $\rho(r)$ is the Fermi radial charge distribution,

$$\rho(r) = \frac{\rho_0}{1 + \exp[n(r/c - 1)]}$$

The energy shifts have been calculated by Watson and Sundaresan (1974) using values for the parameters c and n in the Fermi distribution from Engfer et al. (1974).

2.5 Other Corrections

Other small corrections to the Dirac transition energies are nuclear polarization and relativistic recoil. The effect of nuclear motion in non-relativistic theory is taken into account by using the reduced mass. This is

only valid when the nucleus is treated as a point. In relativistic theory the nuclear motion is not fully taken into account when the reduced mass is used. Calculations have been done by Fricke (1973), Friar and Negele (1973) and Barrett et al. (1973) for this effect. These calculations are in essential agreement for the levels considered here. The values from Barrett are used in this work.

The nuclear polarization corrections are taken from Ericson and Hufner (1972).

2.6 Theoretical Values

Table 2.6-1 gives the theoretical energies for the twelve lines measured in this work. The energies are taken from Vuilleumier et al. (1976). The energies of the interfering transitions tabulated in Chapter VI are taken from a program by R.J. McKee (private communication) and are not as accurate as the energies listed in Table 2.6-1.

TABLE 2.6-1. Corrections to the Dirac energies and calculated energies for the muonic atom transitions measured in this work. Numbers are taken from Vuilleumier et al. (1976).

All energies are in eV. DP: Dirac point nucleus; EN: extended nucleus; LS: Lamb shift and vacuum polarization due to $\mu^+\mu^-$ pair; RM: relativistic correction of the reduced mass; NP: nuclear polarization; ES: electron screening; E_{th} : calculated energy including all corrections.

TRANSITION	DP	EN	VACUUM POLARIZATION		LS	RM	NP	ES	E_{th}
			$\alpha(Z\alpha)$	$\alpha^2(Z\alpha)$	$\alpha(Z\alpha)$				
Ca $3d_{3/2} \rightarrow 2p_{1/2}$	157518(1)	-78(2)	734(2)	5	-1	1	1 3(2)	-1	158182(4)
$3d_{5/2} \rightarrow 2p_{3/2}$	156152(1)	-28(1)	716(2)	5	-1	-1	1 2(1)	-1	156845(3)
Sn $4f_{5/2} \rightarrow 3d_{3/2}$	348234(2)	-50(1)	1759(3)	12(1)	-14(1)	6	2 6(3)	-13	349978(5)
$4f_{7/2} \rightarrow 3d_{5/2}$	343554(2)	-19	1731(3)	12(1)	-14(1)	-4	2 5(3)	-13	345254(5)
Ba $4f_{5/2} \rightarrow 3d_{3/2}$	439068(2)	-140(3)	2435(4)	17(1)	-20(1)	10(1)	3-9(4)	-17(1)	441365(7)
$4f_{7/2} \rightarrow 3d_{5/2}$	431652(2)	-53(1)	2328(4)	16(1)	-20(1)	-7(1)	3 8(4)	-17(1)	433910(7)
$5g_{7/2} \rightarrow 4f_{5/2}$	200543(1)	0	762(2)	5	-9(1)	2	1 1(1)	-31(1)	201274(3)
$5g_{9/2} \rightarrow 4f_{7/2}$	199193(1)	0	748(2)	5	-9(1)	-1	1 1(1)	-31(1)	199907(3)
Tl $5g_{7/2} \rightarrow 4f_{5/2}$	424849(2)	-9(1)	2116(3)	14(1)	-42(2)	9(1)	2 6(3)	-78(3)	426867(7)
$5g_{9/2} \rightarrow 4f_{7/2}$	418835(2)	-3	2038(3)	14(1)	-41(2)	-6(1)	2 6(3)	-80(3)	420765(7)
Pb $5g_{7/2} \rightarrow 4f_{5/2}$	435664(2)	-10(1)	2190(3)	15(1)	-44(2)	9(1)	2 6(3)	-81(3)	437751(7)
$5g_{9/2} \rightarrow 4f_{7/2}$	429343(2)	-4	2106(3)	15(1)	-43(2)	-6(1)	2 6(3)	-83(3)	431336(7)

CHAPTER III

EXPERIMENTAL METHOD .

3.1 General

The muons were produced by the 600 MeV synchrocyclotron at the Space Radiation Effects Laboratory (SREL) in Newport News, Virginia. The muon beam was directed onto the targets after being degraded. The muon beam was time structured, muons appearing in bursts of about 10 ms duration. Two targets were used simultaneously, these will be referred to as the UP and DOWN targets. Typically each target had about 20,000 muon stops per second. There was negligible feed through between the UP and DOWN targets.

Radioactive sources were used to calibrate the apparatus. The source events were collected simultaneously with the muonic x rays. The source events were stored under different conditions to test for possible systematic errors. These will be described in detail later.

A pulser spectrum was also accumulated. The pulser voltage was set by a computer controlled digital to analogue converter (DAC). This was used to test the linearity of the system. Full details of this method can be found in McKee et al. (1971).

Time information between the muons in the beam and x rays in the detector was measured by a time to amplitude converter. This allowed the separation of events into "prompt" and "delayed" spectra. The prompt events consist of the x rays emitted by the muon as it cascades down through its levels while the delayed events are typically γ rays emitted by the nucleus after it has absorbed

the muon. The windows on the timing spectra were constantly checked to ensure there was no drift.

3.2 Muon Telescope

The detector array used to define the muon beam and provide target information, referred to as the muon telescope, is shown in Fig. 3.2-1. The beam of muons and pions passes through the analyzing magnet, AM. The analyzing magnet is adjusted to select particles with momentum of about 100 MeV/c. These are primarily muons produced by the decay of pions in the backward direction in the pion rest frame.

The detectors C1 and C2 are plastic scintillators mounted on RCA 8575 photomultipliers. A coincidence between C1 and C2 signals the passage of a muon through the telescope. A polyethylene degrader, D, slows the muons down. The thickness of D is adjusted to maximize the rate of muons stopping in the targets, T. A side view of the target assembly is shown in Fig. 3.2-1b. Detectors C3, 4, and 5 are similar to C1 and C2. C3 signals the passage of a muon through the UP target and C4 signals the passage through the DOWN target. C5 is a VETO detector which indicates the muon passed through the targets. The target assembly was set at 45° to the beam direction. The Ge detector faced the targets and was placed just outside the beam. The radioactive sources, RS, used for calibration were placed in front of the detector and just behind C5. The detector C π was placed in the separated pion beam.

Figure 3.2-1. (a) Top view of the experimental arrangement. C1 through C5 and $C\pi$ are plastic scintillator detectors, D is the polyethylene degrader, T the target assembly, AM the analyzing magnet, and RS the radioactive sources.

(b) Side view of the target assembly showing the UP and DOWN targets and their tagging counters.

Sizes and distances are not to scale.

3.3 Ge Detector

The Ge detector was manufactured by the Ge Detector Division of the Lawrence Berkeley Laboratory at the University of California. The intrinsic Ge diode was 2 cm in diameter and 1 cm thick giving a surface area of 3.1 cm^2 . The face of the diode was covered with a Be window approximately .025 cm thick. The output from the detector was fed into a pulsed optical feedback preamplifier before going to the main amplifier.

The resolution was 870 eV at 316 keV.

3.4 Linear Electronics

The Ge detector and all the linear electronics were housed inside a copper hut. This provided RF shielding for the detector and electronics as well as providing an enclosure which could be temperature controlled. The hut and contents were grounded separately from other components in the experiment. Power to the hut was provided by an isolation transformer. The digital signals from the hut were transmitted to the outside by an optical coupler.

The signal from the detector's preamplifier was amplified by a Tennelec 205 linear amplifier (TC-205) and digitized by a Northern Scientific 8192 channel ADC (NS-263). The ADC was zero and gain stabilized. The 122 keV γ ray from ^{57}Co was used for zero stabilization and the 477 keV γ ray from ^7Be was used for gain stabilization. The precision pulser and DAC were also housed in the hut.

3.5 Logic

Several types of spectra could be stored based on the signals from the Ge detector and the plastic scintillator counters.

- 1) $C1 \cdot C2 \cdot C3 \cdot \overline{C5}$ - muonic x ray from the UP target,
- 2) $C1 \cdot C2 \cdot C4 \cdot \overline{C5}$ - muonic x ray from the DOWN target,
- 3) $C1 \cdot C2 \cdot (C3 + C4 + C5)$ - radioactive source in coincidence with a muon which did not stop in a target,
- 4) $\overline{\text{telescope}} \cdot C_{\pi}$ - radioactive source in coincidence with a pion in the beam and nothing in the telescope, and
- 5) a detector event in coincidence with an oscillator which did not meet any of the above conditions. This could be divided into a beam on and a beam off spectrum.

A simplified diagram of the logic used to define the different spectra is shown in Figs. 3.5-1 and 3.5-2. C1 through C5 are signals from the counters shown in Fig. 3.2-1. A STOP event is defined by

$$\text{STOP} = C1 \cdot C2 \cdot (UP + DOWN) \cdot \overline{C5}.$$

This indicates a muon has passed through the beam defining counters, C1 and C2, passed through either the UP tag counter, C3, or the DOWN tag counter, C4, and has stopped in a target as indicated by the absence of a signal from the VETO counter, C5.

A GO signal interrupts the computer when a valid event has occurred. A further requirement for a STOP GO is a coincidence between STOP and GE. The GE origin is shown in Fig. 3.5-1b. This indicates there has been a signal from the detector in coincidence with the beam being on. There is also

pileup protection built into the GE logic. Finally the GO flip flop (F/F) must indicate the computer is in a state of readiness. The GO F/F is reset by the computer after it has processed the event from the previous GO.

There are three other conditions which may cause a GO, signals S1, S2, and S3 in Fig. 3.5-1. These are the source events referred to in Section 3.1. The muon-source logic is shown in Fig. 3.5-2a. The conditions for an S1 event are

$$S1 = (C1 \cdot C2 \cdot (C3 + C4 + C5)) \cdot \overline{GE \cdot STOP}.$$

Thus we have a muon-source event when the telescope fires in coincidence with a valid GE signal. S1 is, however, inhibited if there was a STOP type event. The window between the telescope and GE was set at 200 ns.

The logic for the pion-source, signal S2, is shown in Fig. 3.5-2b. S2 is generated by a coincidence between the pion counter, $C\pi$, and a GE signal. $C1 \cdot C2$ vetos S2 if there is a particle in the muon telescope.

The oscillator-source logic is shown in Fig. 3.5-2c. Any Ge event in coincidence with the oscillator is considered a potential S3 event. If the coincidence buffer indicates that the GO was not caused by a STOP, S1, or S2 event then the GO is treated as an S3 event. The event is then stored in either a BEAM ON or a BEAM OFF spectrum depending upon the beam condition.

3.6 Computer System

The on-line computer was a DATA GENERAL NOVA 820 with 32 k 16-bit words.

(a)

17.

(b)

Figure 3.5-1. (a) Simplified diagram of telescope logic.
(b) GE logic.

The circles labeled with a c are signals to a coincidence buffer which is strobed by the computer.

(a)

(b)

(c)

Figure 3.5-2. (a) Muon-source logic.
 (b) Pion-source logic.
 (c) oscillator-source logic.

External devices consisted of two disk units, a magnetic tape unit, a display oscilloscope, a teletype and a video console. The computer was operated in the interrupt mode. Upon receiving an interrupt the computer read the data from the energy ADC, the timing ADC and a coincidence buffer. The coincidence buffer contained the information allowing the program to decide upon the type of event being processed, muonic x ray, source event etc. The data buffers were then incremented and a clear signal returned to the logic. After a thousand events had been processed the data were dumped onto disk in the appropriate histograms. The computer handled about 200 interrupts per second with an average processing time of ≈ 300 μ s per interrupt. During the computer's free time the program was available for data display and elementary peak fitting. Data from previous runs could also be examined.

3.7 Data Collection

The data were collected in eight hour runs. At the completion of each run the data were stored on disk, and eventually on magnetic tape, and a new run started. The raw data were also dumped on tape. This allowed data to be reprocessed at a later time with different timing conditions etc. Targets such as Pb and Ba required several runs due to the low efficiency of the Ge detector for the high energy (≈ 400 keV) x rays. Other targets with lower energy x rays required only one or two runs. All the runs for a particular target were summed. In addition the source spectra from all the runs were summed to provide good statistics for calibration.

CHAPTER IV

ANALYSIS

4.1 Fitting Function

The x-ray and γ -ray peaks were fitted with a mathematical function which consisted of a Gaussian with exponential tails added to the upper and lower sides of the main peak. The exponentials were added in a smooth continuous fashion. A quadratic background and a step in the background under the peak were also used. Up to five peaks could be simultaneously fitted over a two hundred channel region. The fitting program was written in Fortran IV and run on the NOVA computer.

The fitting function is given by

$$D(i) = B(i) + \sum_{k=1}^5 [G_k(i)L_k(i) + E_k(i) + f_k/100 \sum_{j=0}^i (G_k(j)L_k(j) + E_k(j))]$$

where the index k runs over five possible peaks in the fit range. The terms in $D(i)$ are given by

a) Background

The background term is quadratic in i

$$B(i) = B_0 + B_1 i + B_2 i^2.$$

b) Gaussian

The response of the detector was essentially Gaussian and $G_k(i)$ has the standard form

$$G_k(i) = N_k / (\sigma_k \sqrt{2\pi}) \cdot \exp[-(i - C_k)^2 / 2\sigma_k^2]$$

where N_k is the intensity, C_k is the centroid and σ_k is the standard deviation of the Gaussian.

c) Exponential Tails

The slight non-Gaussian shape of the x-ray and γ -ray peaks was taken into account by adding exponential tails on the upper and lower sides of the main peak. The form of $E_k(i)$ is

$$\begin{aligned}
 E_k(i) &= N_k g_k^u \cdot \exp[-\lambda_k^u (i - x_k^u)] \cdot [1 - 1/((i - x_k^u)^2 + 1)], \quad i > x_k^u \\
 &= N_k g_k^l \exp[-\lambda_k^l (x_k^l - i)] \cdot [1 - 1/((i - x_k^l)^2 + 1)], \quad i < x_k^l \\
 &= 0, \quad x_k^l < i < x_k^u
 \end{aligned}$$

where $g^{u,l}$ is a fractional intensity, $\lambda^{u,l}$ a decay constant and $x^{u,l}$ a cut-off channel for the upper and lower tails. The term in square brackets ensures the exponential is added in a smooth continuous fashion at the cut-off points. In practice only the lower tail was needed in this work.

d) Background Step

The term $f_k/100 \sum (G_k(j)L_k(j) + E_k(j))$ provided a step in the background underneath the peak and is proportional to the area of the peak up to channel i .

e) Natural Width

The short lifetime of the muonic x rays lead to a broadening of the x-ray peak. The line shape of the x rays is given by the Lorentzian shape

$$l(i) = \Gamma/2 \frac{1}{(i - i_0)^2 + (\Gamma/2)^2}$$

where Γ is the width of the distribution and i_0 the centroid. If this is viewed through a detector having a Gaussian response, the result is the convolution integral

$$C(i) = \int l(i') G(i - i') di'.$$

This integral may be evaluated in terms of the complex error function (Abramowitz and Stegun 1967).

For values of $\Gamma/2\sqrt{2}\sigma$ of $<.2$ an approximation accurate to 1 % or better has been derived by Wilkinson (1971). This expression is given by

$$C(i) = G(i)L(i)$$

where $G(i)$ is the regular Gaussian and

$$L(i) = 1 - \Gamma/\sqrt{2}\pi\sigma \left[1 - \frac{(i - i_0)}{4\sigma^2\sqrt{\pi}} + 2/\sqrt{\pi} \sum ((i - i_0)/\sqrt{2}\sigma) \right] + \Gamma^2/8\sigma^2 \cdot (1 - (i - i_0)^2/\sigma^2).$$

In this expression $\Sigma(x)$ is given by

$$\Sigma(x) = \sum_{n=1}^{\infty} [[\exp(-n^2/4)/n^2] \cdot [1 - \cosh(nx)]].$$

If the detector response is non-Gaussian the above expression will not be valid. However since the peaks analyzed in this work only had a small non-Gaussian component it was felt the Wilkinson approximation could be used.

A χ^2 minimization routine was used to establish the best values of the parameters. A value of χ^2 was calculated for P , $P+dP$, and $P-dP$ for each parameter P . $\chi^2(P)$ was assumed to be a parabolic function and its minimum was calculated. The increments dP were calculated as the change in P necessary to change χ^2 by 1. The fitting program cycled through the various parameters until χ^2 ceased to improve. There was also provision to hold parameters fixed or to constrain parameters to be linear functions of other parameters.

4.2 γ -ray Analysis

The γ -ray peaks from the calibration sources were fitted in two passes. In the first pass all the parameters were allowed to vary. The shape parameters (σ , λ , g , x and f) were then fitted to a linear function of energy. The second pass used the calculated shape parameters and only the background, intensity and centroid parameters were allowed to vary. This caused χ^2/F to increase somewhat but resulted in a consistent set of parameters being used. The centroids of the source peaks from the second pass were used to determine the energy calibration.

4.3 Calibration

The system was calibrated using the radioactive sources shown in Table 4.3-1. The 411 keV γ ray from ^{198}Au is used as a standard and all other energies are

quoted relative to this line.

TABLE 4.3-1. Calibration sources. The 411 keV line in ^{198}Au is used as the standard.

SOURCE	ENERGY (eV)	REFERENCE
^{192}Ir	$468072.3 \pm .4$	a
^{198}Au	$411805.2 \pm \text{xx}$	a
^{192}Ir	$316508.7 \pm .2$	a
^{192}Ir	$308457.1 \pm .3$	a
^{192}Ir	$295958.6 \pm .3$	a
^{228}Th	238631 ± 3	b
^{192}Ir	$205795.8 \pm .2$	a
^{139}Ce	165857 ± 6	c
^{57}Co	136476.7 ± 2	c

a) R.D. Deslattes, private communications

b) R.L. Graham et al. (1965)

c) R.G. Helmer, private communication

The calibration spectra were accumulated simultaneously with the x-ray spectra. There may be energy shifts which are dependent upon the timing conditions under which the x rays or γ rays are accumulated. For example, the muonic x rays appear in bursts related to the cyclotron frequency while the sources are decaying at a uniform rate. The measured energies of the sources may depend upon whether the γ rays were accumulated at a uniform rate or only during those times that muonic x rays were also being produced.

Four types of calibration spectra were obtained using the signals from the telescope logic, Section 3.5. An event in the Ge detector was

considered to be from a source and was stored in the appropriate spectrum, (SPn), if it was in coincidence with;

- 1) a muon passing through and activating the telescope logic but not stopping in the target - SP4,
- 2) a pion from the beam and nothing through the telescope - SP6,
- 3) a BEAM ON signal but no muon or pion in the telescope - SP8, or
- 4) a BEAM OFF signal - SP9.

The latter two also required a coincidence between an oscillator and the detector. This allowed an adjustment of the rate at which data were stored. A comparison of the source peak positions and the calibration factors obtained for each type of calibration spectra may indicate possible sources of error.

Fig. 4.3-1 shows the relative positions of three of the source peaks for the different spectra from one of the runs. Table 4.3-2 gives the calibration factor obtained and the calculated energy of an x ray centered at channel 4000. There is a definite shift for SP8 and SP9 relative to SP4 for the high energy γ rays. This clearly indicates the need to accumulate calibration data under conditions similar to that under which the muonic x rays are stored. The difference between calibrations from SP4 and SP9 at channel 4000 is 22 eV. There is a negligible difference between SP4 and SP6.

TABLE 4.3-2. Calibration factors from the different calibration spectra. The calibration is given as $E = a_0(\text{Ch})^2 + a_1(\text{Ch}) + a_2$ where Ch is the channel number.

SPECTRUM	TYPE	$a_0(\text{eV}/\text{Ch}^2)$ $\times 10^{-5}$	$a_1(\text{eV}/\text{Ch})^*$	$a_2(\text{eV})$	$E(4000)$ eV
SP4	muon-src	1.568	78.1216	117420.5	430158.0
SP6	pion-src	1.663	78.1167	117430.2	430159.2
SP8	BEAM ON	1.704	78.1191	117424.4	430173.4
SP9	BEAM OFF	1.779	78.1179	117423.6	430179.8

SP4 - MUON-SOURCE
 SP6 - PION-SOURCE
 SP8 - OSC.-SOURCE
 SP9 - BEAM OFF

Th (239 keV)

Ir (316 keV)

SPECTRUM NUMBER

Figure 4.3-1. The differences between measured energies in SP4 and SPn for three of the calibration sources.

The final calibration was obtained by fitting the summed source peaks. Initially the relation between energy and channel number was taken as a linear function. The deviations between actual energy and calculated energy from the weighted least squares fit are shown in Fig. 4.3-2. These deviations are quadratic with energy as can be seen from the quadratic fit done to the deviations, Fig. 4.3-2. Based on this and the fact that an examination of the pulser spectrum showed a similar behaviour the final calibration was done assuming a quadratic relation between energy and channel number.

The parameters were determined by a weighted least squares fit. Two such fits were done, one using all nine lines and the other leaving out the 166 keV line of ^{139}Ce , the 136 keV line of ^{57}Co and the 308 keV line of ^{192}Ir . Fig. 4.3-3 shows a plot of the deviation between the actual and calculated energies for the two fits. The justification for leaving out the three lines is as follows;

- 1) ^{57}Co - the 136.477 keV line is contaminated by a γ ray from ^{192}Ir at about 136.3 keV.
- 2) ^{139}Ce - there is an interfering line at about 167 keV, Fig. 4.3-4. This is attributed to the K α transition in muonic oxygen. It is definitely associated with muons since it appears in the muon related calibration spectrum and only very weakly in the others.
- 3) ^{192}Ir - the 308 keV line has a background problem, Section 5.1. Fig. 5.1-1 indicates a small shift in this line possibly due to background.

Fig 4.3-5 shows the energy difference as a function of channel number between the two calibrations. There is a negligible difference except for channels below 2000.

Table 4.3-3 gives the calibration parameters for the two fits. The six-source parameters were used to determine the energies of the muonic x rays.

TABLE 4.3-3. Calibration parameters for the six- and nine-source fits. The parameters from the six-source fit were used for the final x-ray energies.

No. of SOURCES	a_0 eV/Ch ² $\times 10^{-5}$	a_1 eV/Ch	a_2 eV	RMS DEVIATION eV
6	1.500302	78.1269	117413.5	0.84
9	1.537764	78.1244	117417.9	2.44

Figure 4.3-2. The differences between actual and calculated energies using a linear relation between energy and channel number is plotted against energy (triangles). The squares show the quadratic fit to these differences.

□ - NINE SOURCES RMS = 2.44 eV

⊗ - SIX SOURCES RMS = 0.84 eV

Figure 4.3-3. The differences between actual and calculated energies plotted against energy for the six- and nine-source fits. A quadratic relation between energy and channel number is assumed.

Figure 4.3-4. Plot of the ^{139}Ce γ ray showing the interfering x ray.

Figure 4.3-5. The differences between the energies obtained using the nine-and six-source calibration plotted against channel number

CHAPTER V

SYSTEMATIC ERROR SEARCH

5.1 Background

The background underneath a peak is always difficult to accurately measure. Each γ -ray and x-ray peak has its own background and any fitting procedure should be able to provide centroids which are essentially background independent.

The ^{192}Ir source was encapsulated in a 1 cm cube of plastic scintillator attached to a RCA photomultiplier. This allowed the Ir γ rays to be tagged by the β^- emitted with each decay. The Ir decays which were not tagged (β^- below threshold) were stored in the normal source spectrum and those which were tagged went into a different spectrum. The background of the normal and tagged spectra are quite different, in particular the 308 keV line which sits near the Compton edge of the 477 keV line from ^7Be used for stabilization.

Fig. 5.1-1 shows the shift between the normal and tagged spectra taken from the summed data. The shifts are all consistent with a zero shift. The largest shift is the -3.9 eV shift for the 308 keV line. The average shift for the four lines is (-0.5 ± 1.1) eV. We conclude that the fitting procedure seems to handle the background quite well and no serious errors are introduced because of background. An error of ± 1 eV was folded into the final error for the x-ray energies because of background effects.

SP4 - NORMAL SPECTRUM

SP5 - TAGGED SPECTRUM

Figure 5.1-1. The difference in energy between the normal and tagged spectra for the Ir γ rays plotted against energy.

5.2 Angular Effect

The electric field in the diode of the detector may cause an energy shift which is dependent upon the direction in which the photoelectrons are moving and the energy of the x ray or γ ray (Gunnink et al. 1968). The photoelectrons will have an angular distribution which, for the energies involved in this work, will peak within 25° of the forward direction relative to the incident photon direction. The acceleration of the electron and hence the number of ion pairs formed, will depend upon the relative orientation of the electric field and the photon direction.

An example of this effect is shown in Fig. 5.2-1 where the shift in energy between sources measured at 0° and 90° to a Ge(Li) detector is plotted. The measurements were made prior to the muonic x-ray experiment using an Ortec model 8113-101905 Low Energy Photon Spectrometer operated at -1500 volts. The active area was 80 mm^2 with a sensitive depth of 5 mm. This may not be a strictly valid measurement in the sense that at 90° the photons had to pass through the sides of the detector housing where small angle Compton scattering may cause an apparent lowering of the photon energy. Fig. 5.2-1 would then be an upper bound on the angle effect.

Assuming a $(1 - \cos \theta)$ dependence and Fig. 5.2-1 we see that a 400 keV photon incident at 30° would only have a shift of about 1.7 eV. Since 30° was larger than any of the angles involved in this experiment it was felt that the angle effect would not play any significant role.

However since time did not permit the same type of measurement to be carried out on the detector used in the muonic x-ray measurements, other types of tests were made which involved the actual geometry used in the experiment.

Figure 5.2-1. The energy shift between sources measured at 0° and 90° relative to a Ge(Li) detector is plotted against energy. Ir and Ba sources were alternated between 0° and 90° .

- 1) Tagged ^{192}Ir sources were used to measure the angular effect as a function of energy and position. One source was placed on the center of the target, one at the upper right hand corner and another at the lower left hand corner of the target. This provided a set of angles covering the range of interest, the angular size of the target as seen by the detector. The results from the tagged Ir data are shown in Table 5.2-1. The numbers given are the difference in eV between the center source and the ones at the far corners of the target.

TABLE 5.2-1. Angular effect measurements using tagged Ir sources at the corners of the target. The numbers shown are the energy differences in eV between the center source and the corner sources.

ENERGY keV	UPPER RIGHT eV	LOWER LEFT eV
468	0.0 ± 3.6	4.8 ± 4.1
316	0.8 ± 1.6	2.0 ± 1.9
308	-0.8 ± 2.7	0.3 ± 3.1
296	-1.0 ± 1.2	-4.3 ± 1.4
avg.	0.1 ± 1.2	0.8 ± 1.4

- 2) Two targets were used for each run. One of the runs had a Pb target in the UP position while another run had the Pb target in the DOWN position. A comparison of the Pb muonic x-ray positions from the UP and DOWN targets will show any UP-DOWN asymmetry which would include an angular effect, if any. The shifts between the Pb lines measured in the UP and

and DOWN target positions were (1.5 ± 7.6) eV (431 keV line) and (4.2 ± 9.4) eV (437 keV line).

- 3) The 412 keV line in ^{198}Au was measured using a point Au source placed at the target center and an extended Au source. The extended source was the same size as the target and placed in the target position. The difference between the point source and the extended source was (3 ± 2) eV.

We conclude that for the small angles involved in this experiment that the angular effect will be negligible.

5.3 Source events in x-ray spectra

Some of the x-ray events defined in Section 3.2 (1) and 2)) were, in fact, due to radioactive sources. These occurred when the γ ray from the source was in accidental coincidence with the telescope logic and the x ray was not detected. These events were used to check for shifts between the x-ray and source spectra since these γ rays were measured under the exact same conditions as the x rays. Table 5.3-1 gives the energy differences between the muon-source energies (SP4) and the energies measured in the prompt and delayed x-ray spectra.

Due to interfering x rays not all the sources could be measured in the x-ray spectra. The delayed spectra have quite good statistics and the average shifts are less than 1 eV. The poorer statistics in the prompt spectra make any conclusions less certain but overall there appears to be no significant shift in energies measured in the calibration spectrum and the x-ray spectrum.

TABLE 5.3-1. Energy differences, in eV, between source energies measured in the muon-source spectrum and the prompt and delayed x-ray spectra.

SOURCE	PROMPT	UP eV	DELAYED	PROMPT	DOWN eV	DELAYED
$^{192}\text{Ir}(468)$	-2.4 ± 17.7		10.1 ± 5.0	-7.3 ± 18.5		0.3 ± 5.0
$^{198}\text{Au}(412)$	-12.9 ± 10.2		-1.3 ± 2.9	8.1 ± 9.9		2.9 ± 2.9
$^{192}\text{Ir}(316)$	-13.6 ± 6.4		1.6 ± 2.0	-6.9 ± 6.3		0.9 ± 2.0
$^{192}\text{Ir}(308)$	-5.4 ± 14.8		-4.8 ± 3.9	--		-4.1 ± 3.9
$^{192}\text{Ir}(296)$	-23.5 ± 20.7		0.8 ± 3.7	-5.5 ± 18.5		3.7 ± 3.7
$^{228}\text{Th}(239)$	--		0.5 ± 3.9	--		-3.4 ± 3.9
$^{192}\text{Ir}(206)$	--		-1.2 ± 9.5	--		-0.7 ± 9.0
average	-12.3 ± 4.8		0.7 ± 1.3	-3.1 ± 4.9		0.6 ± 1.3

5.4 ^{137}Cs capture γ ray

The 455.5 keV γ ray from ^{137}Cs can be used as a test for any shifts in the muonic x-ray spectrum. ^{137}Cs is produced when the muon is absorbed by the Ba nucleus,

This γ ray will appear in both the prompt and delayed spectra. The energy of the Cs γ ray has been measured by Fransson et al. (1975). They reported a value of (455490 ± 3) eV based on the ^{198}Au standard of 411794 eV. Using the latest value of 411805.2 eV for the standard, the value of the ^{137}Cs γ ray becomes (455503 ± 3) eV.

The measured value from the delayed spectrum was (455507.4 ± 8.9) eV. The difference between this and the value of Fransson is (4.4 ± 9) eV. The

value from the prompt spectrum was (455461 ± 31) eV which gives a difference of (-42 ± 31) eV. It appears that energies measured in the x-ray spectra give accurate results.

5.5 ^{203}Tl capture γ ray

A further test of the system is provided by the capture γ ray produced by muon absorption in the ^{204}Pb nucleus,

The γ ray from ^{203}Tl has an energy of 279.1 keV and appears in the prompt and delayed spectra. ^{203}Tl can be produced from a ^{203}Hg source,

A comparison can be made of the position of the 279.1 keV line in the x-ray spectra and the source spectrum using a ^{203}Hg source. The results from such a comparison are given in Table 5.5-1.

TABLE 5.5-1. Comparison of the energies measured for the 279 keV γ ray of ^{203}Tl . The quoted errors include only the statistical error.

PROMPT eV	DELAYED eV	SOURCE eV
279192.2 ± 15.3	297197.8 ± 3.7	279199.6 ± 1.8

The agreement is excellent and again indicates there is no significant shift between energies measured in the source spectrum and those measured in the x-ray spectra.

CHAPTER VI

MUONIC X-RAY ANALYSIS

The muonic x rays were fitted using the shape parameters established from the source fits. In addition, the Lorentzian width parameter Γ was used for the high energy transitions. Each of the x-ray fits is discussed in detail in the following sections.

6.1 Ca

The transitions of interest in Ca are the ($3d_{5/2} \rightarrow 2p_{3/2}$) transition at 156.8 keV and the ($3d_{3/2} \rightarrow 2p_{1/2}$) transition at 158.2 keV. There is only one interfering line, the ($3d_{3/2} \rightarrow 2p_{3/2}$) transition at 156.7 keV. This transition has an intensity of 11.1 % of the ($3d_{5/2} \rightarrow 2p_{3/2}$) transition and sits 140 eV below it. Fig. 6.1-1 shows the Ca peaks and the fit obtained. The results from the fit are given in Table 6.1-1.

TABLE 6.1-1. Results for the ($3 \rightarrow 2$) transitions in Ca. Only statistical errors are shown.

TRANSITION	ENERGY eV	AREA counts	χ^2/F
$3d_{5/2} \rightarrow 2p_{3/2}$	156837.9 ± 2.5	28281 ± 230	
$3d_{3/2} \rightarrow 2p_{1/2}$	158180.1 ± 3.5	16308 ± 190	1.64
$3d_{3/2} \rightarrow 2p_{3/2}^*$	$156698 \pm -$	$3139 \pm -$	

* interfering line

(a)

Ca (3-2) LINES
SQRT PLOT
SPECTRUM NO. 2

43.

(b)

Ca (3-2) LINES
LINEAR PLOT
SPECTRUM NO. 2

Figure 6.1-1. (a) Square root plot of the (3 → 2) transitions in Ca and the fit obtained.

(b) Linear plot of the individual components.

6.2 Ba

The transitions of interest in Ba are the
 $(5g_{9/2} \rightarrow 4f_{7/2})$ transition at 199.9 keV, the
 $(5g_{7/2} \rightarrow 4f_{5/2})$ transition at 201.3 keV, the
 $(4f_{7/2} \rightarrow 3d_{5/2})$ transition at 433.9 keV, and the
 $(4f_{5/2} \rightarrow 3d_{3/2})$ transition at 441.4 keV.

For the low energy transitions, Fig. 6.2-1, there are three interfering lines. The energies and intensities of the two triplets involved are given in Table 6.2-1. The energies are taken from Table 2.6-1 and the relative intensities from Condon and Shortley (1963).

TABLE 6.2-1. Calculated energies and intensities for the $(5 \rightarrow 4)$ transitions in muonic Ba.

TRANSITION	ENERGY eV	RELATIVE INTENSITY
$5f_{5/2} \rightarrow 4d_{3/2}$	204330	70
$5f_{7/2} \rightarrow 4d_{5/2}^*$	201367	100
$5f_{5/2} \rightarrow 4d_{5/2}^*$	200357	5
$5g_{7/2} \rightarrow 4f_{5/2}$	201274	77.1
$5g_{9/2} \rightarrow 4f_{7/2}$	199907	100
$5g_{7/2} \rightarrow 4f_{7/2}^*$	199317	2.9

* interfering line

By measuring the intensity of the $(5f_{5/2} \rightarrow 4d_{3/2})$ transition and using the relative intensities from Table 6.2-1, the intensities of the two $(5f \rightarrow 4d)$ interfering lines were determined. The $(5g \rightarrow 4f)$ interfering line intensity was

fixed at 2.9 % of the ($5g_{9/2} \rightarrow 4f_{7/2}$) transition. Fig. 6.2-2 shows the fitted data and Table 6.2-2 gives the results of the fits.

TABLE 6.2-2. Results for the ($5 \rightarrow 4$) transitions in Ba. Only statistical errors are shown.

TRANSITION	ENERGY eV	AREA counts	χ^2/F
$5g_{9/2} \rightarrow 4f_{7/2}$	199910.6 ± 2.2	65474 ± 395	2.05
$5g_{7/2} \rightarrow 4f_{5/2}$	201269.8 ± 2.7	50477 ± 372	
$5g_{7/2} \rightarrow 4f_{7/2}^*$	$199320 \pm -$	$1899 \pm -$	
$5f_{7/2} \rightarrow 4d_{5/2}^*$	$201371 \pm -$	$5375 \pm -$	
$5f_{5/2} \rightarrow 4d_{5/2}^*$	$200361 \pm -$	$269 \pm -$	

* interfering line

The calculated energies and intensities for the high energy, ($4 \rightarrow 3$), Ba transitions are given in Table 6.2-3. The region is shown in Fig. 6.2-3.

TABLE 6.2-3. Calculated energies and intensities for the ($4 \rightarrow 3$) transitions in muonic Ba.

TRANSITION	ENERGY eV	RELATIVE INTENSITY
$4f_{5/2} \rightarrow 3d_{3/2}$	441365	70
$4f_{7/2} \rightarrow 3d_{5/2}$	433910	100
$4f_{5/2} \rightarrow 3d_{5/2}^*$	431940	5
$4d_{3/2} \rightarrow 3p_{1/2}$	446840	55.6
$4d_{5/2} \rightarrow 3p_{3/2}^*$	432660	100
$4d_{3/2} \rightarrow 3p_{3/2}$	428690	11.1

* interfering line

Ba (5-4) LINES
LINEAR PLOT
SPECTRUM NO. 2

Figure 6.2-1. Plot of the two (5 $\rightarrow$ 4) triplets in Ba. The weak component of the (5f $\rightarrow$ 4d) triplet is too weak to be seen.

(a)

Ba (5-4) LINES
SQRT PLOT
SPECTRUM NO. 2

47.

(b)

Ba (5-4) LINES
*LINEAR PLOT
SPECTRUM NO. 2

Figure 6.2-2. (a) Square root plot of the (5 + 4) transitions in Ba and the fit obtained.

(b) Linear plot of the individual components. The (5f + 4d) line at channel 1061 is too weak to see in this plot.

Ba (4-3)* LINES

LINEAR PLOT

SPECTRUM NO. 2

48.

1000 COUNTS PER TICK

- Figure 6.2-3. Plot of the two (4 → 3) triplets in Ba. The weak component of the (4d → 3p) triplet lies below channel 4000 and is not visible.

The ($4f_{5/2} \rightarrow 3d_{5/2}$) and the ($4d_{5/2} \rightarrow 3p_{3/2}$) transitions interfere with the ($4f_{7/2} \rightarrow 3d_{5/2}$) transition. The intensity of the ($4d_{5/2} \rightarrow 3p_{3/2}$) transition was determined by measuring the intensity of the ($4d_{3/2} \rightarrow 3p_{1/2}$) transition and using Table 6.2-3. The intensity of the ($4f_{5/2} \rightarrow 3d_{5/2}$) interfering line was constrained to be 5% of the intensity of the ($4f_{7/2} \rightarrow 3d_{5/2}$) line. The ($4f_{5/2} \rightarrow 3d_{3/2}$) line is uncontaminated. The results of the fits are given in Table 6.2-4 and shown in Figs. 6.2-4 and 6.2-5.

TABLE 6.2-4. Results for the (4 → 3) transitions in Ba. Only statistical errors are shown.

TRANSITION	ENERGY eV	Γ eV	AREA counts	χ^2/F
$4f_{5/2} \rightarrow 3d_{3/2}$	441363.3 ± 4.5	146 ± 12	21660 ± 220	1.36
$4f_{7/2} \rightarrow 3d_{5/2}$	433908.9 ± 3.8	134 ± 10	31326 ± 263	
$4d_{5/2} \rightarrow 3p_{3/2}^*$	$432657 \pm -$	$134 \pm -$	$977 \pm -$	1.49
$4f_{5/2} \rightarrow 3d_{5/2}^*$	$431936 \pm -$	$134 \pm -$	$1566 \pm -$	

* interfering line

Figure 6.2-4. Square root plot of the ($4f_{5/2} \rightarrow 3d_{3/2}$) transition in Ba and the fit obtained.

(a)

Ba ($4f_{7/2} - 3d_{5/2}$)
SQRT PLOT
SPECTRUM NO. 2

51.

(b)

Ba ($4f_{7/2} - 3d_{5/2}$)
LINEAR PLOT
SPECTRUM NO. 2

Figure 6.2-5. (a) Square root plot of the ($4f_{7/2} + 3d_{5/2}$) transition in Ba and the fit obtained.

(b) Linear plot of the individual components.

6.3 Sn

The transitions of interest in Sn are the $(4f_{7/2} \rightarrow 3d_{5/2})$ transition at 345.3 keV and the $(4f_{5/2} \rightarrow 3d_{3/2})$ transition at 350.0 keV. The interfering transitions are the $(4d_{5/2} \rightarrow 3p_{3/2})$ and the $(4f_{5/2} \rightarrow 3d_{5/2})$ transitions at 346 and 344 keV. The calculated energies and intensities of the two triplets are given in Table 6.3-1. Fig. 6.3-1 shows the region of interest.

TABLE 6.3-1. Calculated energies and intensities of the $(4 \rightarrow 3)$ transitions in muonic Sn.

TRANSITION	ENERGY eV	RELATIVE INTENSITY
$4f_{5/2} \rightarrow 3d_{3/2}$	349978	70
$4f_{7/2} \rightarrow 3d_{5/2}$	345254	100
$4f_{5/2} \rightarrow 3d_{5/2}^*$	344000	5
$4d_{3/2} \rightarrow 3p_{1/2}$	356370	55.6
$4d_{5/2} \rightarrow 3p_{3/2}^*$	345984	100
$4d_{3/2} \rightarrow 3p_{3/2}$	343470	11.1

* interfering line

The $4f \rightarrow 3d$ interfering line at 344 keV stands out far enough below the 345 keV main line that it can be fitted with free parameters. The $4d \rightarrow 3p$ interfering line had its parameters fixed from Table 6.3-1 and using the measured intensity of the $4d_{3/2} \rightarrow 3p_{1/2}$ line at 356 keV. The intensity of the $4d \rightarrow 3p$ interfering line was calculated to be (450 ± 110) counts. The effect of this line on the main line at 345.25 keV was determined by doing three fits with the interfering intensity

Sn (4-3) LINES
LINEAR PLOT
SPECTRUM NO. 0

53.

Figure 6.3-1. Plot of the two (4 → 3) triplets in Sn.

set at 450, 560 and 340 counts (I , $I+\sigma$, $I-\sigma$). The three results are shown in Table 6.3-2.

There is also an interfering line at ≈ 346.7 keV. This line shows up in all the muonic x-ray spectra but not in the BEAM OFF spectrum. It was also included in the fit and its centroid and intensity allowed to vary.

An error of ± 3 eV was folded into the uncertainty of the $(4f_{7/2} \rightarrow 3d_{5/2})$ transition energy to account for the interfering lines. The final results are given in Table 6.3-3 and the fit is shown in Fig. 6.3-2.

TABLE 6.3-2. Results of varying the intensity of the interfering line on the main Sn line.

INTERFERING INTENSITY counts	ENERGY OF MAIN LINE eV	χ^2/F_0
450	345257.6 ± 4.1	1.31
560	345255.9 ± 4.1	1.32
340	345259.1 ± 4.0	1.31

TABLE 6.3-3. Results for the $(4f \rightarrow 3d)$ transitions in muonic Sn. Only the statistical errors are shown.

TRANSITION	ENERGY eV	Γ eV	AREA counts	χ^2/F
$4f_{5/2} \rightarrow 3d_{3/2}$	349970.9 ± 5.0	74 ± 12	15743 ± 191	1.22
$4f_{7/2} \rightarrow 3d_{5/2}$	345257.6 ± 4.1	81 ± 10	23343 ± 233	
$4d_{5/2} \rightarrow 3p_{3/2}^*$	346719 ± 76	$81 \pm -$	$450 \pm -$	
$4f_{5/2} \rightarrow 3d_{5/2}^*$	344003 ± 40	$81 \pm -$	1388 ± 100	

* interfering line

(a)

Sn (4-3) LINES
SQRT PLOT
SPECTRUM NO. 0

55

(b)

Sn (4-3) LINES
LINEAR PLOT
SPECTRUM NO. 0

Figure 6.3-2. (a) Square root plot of the (4+3) transitions in Sn and the fit obtained.

(b) Linear plot of the individual components.

6.4 Pb

The transitions of interest in Pb are the ($5g_{7/2} \rightarrow 4d_{5/2}$) transition at 437.8 keV and the ($5g_{9/2} \rightarrow 4f_{7/2}$) transition at 431.3 keV. The calculated energies and intensities of the two triplets are given in Table 6.4-1 and the region of interest is shown in Fig. 6.4-1.

TABLE 6.4-1. Calculated energies and intensities of the ($5 \rightarrow 4$) transitions in muonic Pb.

TRANSITION	ENERGY eV	RELATIVE INTENSITY
$5g_{7/2} \rightarrow 4f_{5/2}$	437751	77.1
$5g_{9/2} \rightarrow 4f_{7/2}$	431336	100
$5g_{7/2} \rightarrow 4f_{7/2}^*$	428590	2.9
$5f_{5/2} \rightarrow 4d_{3/2}$	450050	70
$5f_{7/2} \rightarrow 4d_{5/2}^*$	437010	100
$5f_{5/2} \rightarrow 4d_{5/2}$	432290	5

* interfering line

The ($5g_{7/2} \rightarrow 4f_{7/2}$) interfering line stands well below the ($5g_{9/2} \rightarrow 4f_{7/2}$) line and can be fitted with position and intensity free. The intensity of the ($5f_{7/2} \rightarrow 4d_{5/2}$) interfering line is found by measuring the intensity of the ($5f_{5/2} \rightarrow 4d_{3/2}$) line at 450 keV and using Table 6.4-1. Its position was initially fixed 720 eV below the ($5g_{7/2} \rightarrow 4f_{5/2}$) line.

The measured intensity of the ($5f_{5/2} \rightarrow 4d_{3/2}$) line was (1921 ± 144) counts giving an intensity for the ($5f_{7/2} \rightarrow 4d_{5/2}$) line of (2744 ± 206) counts. A test was made for the effect of the interfering line on the energy of the main line. The intensity of the interfering line was set at 2744, 2950,

Pb (5-4) LINES

LINEAR PLOT

SPECTRUM NO. 0

Figure 6.4-1. Plot of the two (5 $\rightarrow$ 4) triplets in Pb. The weak component of the (5f $\rightarrow$ 4d) triplet is too weak to be seen.

and 2538 (I , $I+\sigma$, and $I-\sigma$). At each of these intensities the position of the interfering line was fixed 720 eV below the main line, and then varied. The results of the six resulting fits are given in Table 6.4-2.

TABLE 6.4-2. Results of varying the parameters of the interfering line on the ($5g_{7/2} \rightarrow 4f_{5/2}$) transition energy.

I(interfer) counts	E(interfer) eV	E($5g \rightarrow 4f$) eV	χ^2/F
2744	437009 $\pm$ -	437730.4 $\pm$ 2.9	1.39
2744	436977 $\pm$ 46	437731.3 $\pm$ 2.9	1.38
2950	437011 $\pm$ -	437732.6 $\pm$ 2.9	1.37
2950	436990 $\pm$ 43	437733.2 $\pm$ 2.9	1.37 used
2538	437007 $\pm$ -	437728.9 $\pm$ 2.9	1.40
2538	436963 $\pm$ 49	437729.9 $\pm$ 2.9	1.39

The fit with the smallest χ^2/F was used in determining the energy of the ($5g_{7/2} \rightarrow 4f_{5/2}$) transition. The largest change in the energy of this transition is, from Table 6.4-2, 4.3 eV. An error of ± 2.2 eV due to the interfering line was folded into the final error for this transition. The final results for the Pb transitions are given in Table 6.4-3. The fit is shown in Fig. 6.4-2.

TABLE 6.4-3. Results for the (5g $\rightarrow$ 4f) transitions in Pb. Only statistical errors are shown.

TRANSITION	ENERGY eV	Γ eV	AREA counts	χ^2/F
5g _{7/2} $\rightarrow$ 4f _{5/2}	437733.2 $\pm$ 2.9	120 $\pm$ 8	60586 $\pm$ 388	1.37
5g _{9/2} $\rightarrow$ 4f _{7/2}	431324.4 $\pm$ 2.4	130 $\pm$ 6	79875 $\pm$ 430	
5f _{7/2} $\rightarrow$ 4d _{5/2} *	436990 $\pm$ 43	120 $\pm$ -	2950 $\pm$ -	
5g _{7/2} $\rightarrow$ 4f _{7/2} *	428518 $\pm$ 40	130 $\pm$ -	2394 $\pm$ 165	

* interfering line

(a)

Pb (5-4) LINES
SQRT PLOT
SPECTRUM NO. 2

60.

(b)

Pb (5-4) LINES
LINEAR PLOT
SPECTRUM NO. 2

Figure 6.4-2. (a) Square root plot of the (5 + 4) transitions in Pb and the fit obtained. (b) Linear plot of the individual components.

6.5 Tl

The transitions of interest in Tl are the ($5g_{7/2} \rightarrow 4f_{5/2}$) transition at 426.9 keV and the ($5g_{9/2} \rightarrow 4f_{7/2}$) transition at 420.8 keV. The calculated energies and intensities are given in Table 6.5-1 and the peaks are shown in Fig. 6.5-1.

TABLE 6.5-1. Calculated energies and intensities of the (5 $\rightarrow$ 4) transitions in muonic Tl.

TRANSITION	ENERGY eV	RELATIVE INTENSITY
$5g_{7/2} \rightarrow 4f_{5/2}$	426867	77.1
$5g_{9/2} \rightarrow 4f_{7/2}$	420765	100
$5g_{7/2} \rightarrow 4f_{7/2}$	418150	2.9
$5f_{5/2} \rightarrow 4d_{3/2}$	438690	70
$5f_{7/2} \rightarrow 4d_{5/2}^*$	426230	100
$5f_{5/2} \rightarrow 4d_{5/2}$	421740	5

* interfering line

The only interfering line is the ($5f_{7/2} \rightarrow 4d_{5/2}$) transition at 426.2 keV. Its intensity can be found from the intensity of the ($5f_{5/2} \rightarrow 4d_{3/2}$) transition at 438.7 keV. This line has a rather serious problem in that there are two γ rays sitting practically on top of it, Fig. 6.5-1. These γ rays are produced by muon absorption by the Tl nucleus. They appear strongly in the delayed spectrum, Fig. 6.5-2a, and weakly in the prompt spectrum where they interfere with the muonic transitions.

The γ rays were fitted in the delayed spectrum to obtain their positions.

Tl (5-4) LINES
LINEAR PLOT
SPECTRUM NO. 0

62.

Figure 6.5-1. Plot of the two (5 $\rightarrow$ 4) triplets in Tl. The weak component of the (5f $\rightarrow$ 4d) triplet is too weak to be seen.

Tl DELAYED
SQRT PLOT
SPECTRUM NO. 1

63.

(a)

1000 COUNTS PER TICK

(b)

Tl PROMPT
SQRT PLOT
SPECTRUM NO. 0

100 COUNTS PER TICK

Figure 6.5-2. (a) Capture γ rays in the delayed spectrum of Tl.

(b) Capture γ rays and the $(5f+4d)$ x ray in the prompt spectrum.

Then a fit was done on the $(5f_{5/2} \rightarrow 4d_{3/2})$ line with the positions of the two γ rays fixed. This fit is shown in Fig. 6.5-2b. This gave an intensity of (1400 ± 120) counts and consequently an intensity of (2004 ± 174) counts for the $(5f_{7/2} \rightarrow 4d_{5/2})$ interfering transition.

As with Pb, the $(5g_{7/2} \rightarrow 4f_{5/2})$ line was fit with the intensity of the interfering line set at 1, $1+\sigma$, and $1-\sigma$ and the position both fixed and varied. The results of these fits are shown in Table 6.5-2.

TABLE 6.5-2. Results of varying the parameters of the interfering line on the $(5g_{7/2} \rightarrow 4f_{5/2})$ transition energy.

I(interfer)	E(interfer) eV	E($5g \rightarrow 4f$) eV	χ^2/F
2004	$426189 \pm -$	426859.6 ± 3.9	1.64
2004	426261 ± 52	426858.0 ± 4.0	1.61
2178	$426191 \pm -$	426862.0 ± 4.0	1.63
2178	426281 ± 50	426859.9 ± 3.8	1.43 used
1830	$426186 \pm -$	426857.1 ± 3.9	1.64
1830	426251 ± 56	426855.7 ± 3.9	1.62

The largest change in the energy of the $(5g_{7/2} \rightarrow 4f_{5/2})$ line is about 6.3 eV. The fit with a χ^2/F of 1.43, Fig. 6.5-3, is best so the energy from this fit will be used and an additional error of ± 3 eV will be folded into the final uncertainty on the energy of the $(5g_{7/2} \rightarrow 4f_{5/2})$ transition.

The $(5g_{9/2} \rightarrow 4f_{7/2})$ transition is clean, the fit to this line is shown in Fig. 6.5-4. The results of the fits are given in Table 6.5-3.

TABLE 6.5-3. Results for the (S → 4) transitions in Tl. Only statistical errors are shown.

TRANSITION	ENERGY eV	Γ eV	AREA counts	χ^2/F
$5g_{7/2} \rightarrow 4f_{5/2}$	426859.9 ± 3.9	105 ± 10	34864 ± 300	1.43
$5f_{7/2} \rightarrow 4d_{5/2}^*$	$426281 \pm -$	$105 \pm -$	$2178 \pm -$	
$5g_{9/2} \rightarrow 4f_{7/2}$	420759.2 ± 3.2	116 ± 8	46292 ± 330	1.22

* interfering line

(a) Tl ($5g_{7/2} - 4f_{5/2}$)
SQRT PLOT
SPECTRUM NO. 0

66.

(b) Tl ($5g_{7/2} - 4f_{5/2}$)
LINEAR PLOT
SPECTRUM NO. 0

Figure 6.5-3. (a) Square root plot of the ($5g_{7/2} + 4f_{5/2}$) transition and the fit obtained.

(b) Linear plot of the individual components.

Figure 6.5-4) Square root plot of the ($5g_{9/2} - 4f_{7/2}$) transition in Tl and the fit obtained.

a) Fitting

Two errors for the peak centroid expressed in channels were calculated by the fitting program. The first was the statistical error, σ_s , assuming a Gaussian distribution, given by

$$\sigma_s = \text{FWHM} / [2.354\sqrt{N}]$$

where FWHM is the Full Width at Half Maximum and N is the total number of counts under the peak. The second error calculated was the variation in the centroid necessary to change the total χ^2 by one, σ_x . For a one-parameter fit, σ_x can be shown to be approximately the standard deviation of the parameter. This has been generalized to a multi-parameter fit where the correlations between parameters has been ignored. The largest of σ_s and σ_x was used for the uncertainty in the centroid. In practice it was found that σ_x was the largest, $\sigma_x/\sigma_s \approx 1.5$.

b) Calibration

The weights used in the least squares fit to the calibration sources were calculated using

$$\sigma_w = [\sigma_e^2 + \sigma_c^2]^{1/2}$$

where σ_e is the uncertainty in the energy and σ_c the uncertainty in the centroid of the source position, both expressed in eV.

The error on the energy of the x-ray peaks, due both to fitting and calibration factor, can be expressed as

$$\sigma_E' = [\sigma_{a_2}^2 + X^2 \sigma_{a_1}^2 + a_0^2 \sigma_x^2]^{1/2}$$

where $E = a_0 X^2 + a_1 X + a_2$. X is the centroid (in channels) of the x-ray peak and the a 's are given in Table 4.3-3. σ_x is the error on the centroid position. The uncertainty in the a_0 term has negligible effect on σ_E' . σ_{a_2} is approximately 1 eV and σ_{a_1} is estimated at about 0.0004 eV/channel.

An additional error is folded in because of the difference in calibration found using all nine sources and using only six sources, Section 4.3. This error, σ_g , was read from Fig. 4.3-5 and reflects the fact that the lower energy lines are more affected by this difference. The calibration error of the energy of an x ray centered at channel X is then given by

$$\sigma_E = [\sigma_{a_2}^2 + X^2 \sigma_{a_1}^2 + a_0^2 \sigma_x^2 + \sigma_g^2]^{1/2}$$

c) Background

An uncertainty of $\sigma_b = \pm 1$ eV was folded into the error calculations for all x-ray peaks, Section 5.1.

d) Interfering Lines

The effect of interfering lines, σ_I , varies from peak to peak and is discussed separately for each x-ray line. σ_I was usually 2 or 3 eV.

e) Drift

The system was very stable during the data collection period. After each run (≈ 8 hours) a check was made on the positions and widths of the source peaks. No shifts or peak broadening were observed.

f) Lorentzian Width

Ideally the addition of a Lorentzian width to the x-ray peaks should not change the measured energies. This was tested by fitting the ($4f_{5/2} \rightarrow 3d_{3/2}$) transition in Ba (Fig. 6.2-4) with and without the Lorentzian width parameter. The results are shown in Table 6.6-1.

TABLE 6.6-1. Results of Lorentzian width test on Ba.

FWHM eV	Γ eV	ENERGY eV	χ^2/F
1024	146 ± 12	441363.3 ± 4.5	1.36
1024	-	441364.6 ± 5.5	2.87
1120 ± 12	-	441362.4 ± 5.9	1.39

In the first two fits the FWHM is fixed at the value determined from the source runs. Clearly the x-ray peak is broader than a γ -ray peak of the same energy. However the measured energy has little dependence upon whether the Lorentzian width Γ is used or the FWHM is allowed to increase.

The effect will increase if there are interfering x rays under the main x-ray peak. Tests on Pb showed a 2 eV shift between fits using a Γ with a fixed FWHM or just an enlarged FWHM. An additional error of $\sigma_L = \pm 1$ eV is

folded into the final error for those peaks where Γ was used.

g) Other

Tests for systematic errors due to angle effect and shifts between source and x-ray spectra in general gave results consistent with zero within experimental error. However an additional error of $\sigma_o = \pm 2$ eV has been folded into the final uncertainty of the x-ray peaks to account for these effects.

The final uncertainty in measured x-ray energies is given by

$$\sigma = [\sigma_E^2 + \sigma_b^2 + \sigma_I^2 + \sigma_L^2 + \sigma_o^2]^{1/2}$$

where

σ_E = error due to fitting and calibration, b),

$\sigma_b = \pm 1$ eV due to background, 5.1,

σ_I = error due to interfering lines, d),

$\sigma_o = \pm 2$ eV due to other effects, g), and

$\sigma_L = \pm 1$ eV due to Γ , f).

CHAPTER VII

RESULTS AND CONCLUSIONS

Table 7.1 gives a summary of the twelve precision measurements and the theoretical energies. Also included is the total vacuum polarization correction for each transition. Fig. 7.1 shows a plot of $E = E(\text{theory}) - E(\text{measured})$ against energy for the twelve transitions.

TABLE 7.1. Comparison of experimental and theoretical energies of the twelve muonic atom transitions measured in this work. Also shown is the total vacuum polarization correction, VP, for each transition.

TRANSITION	EXPERIMENT eV	THEORY eV	VP eV	$E(\text{th}) - E(\text{exp})$ eV
Ca (3d→2p)	156837.9 ± 6.9	156845 ± 3	720	7.1 ± 7.5
	158180.1 ± 7.3	158182 ± 4	738	1.9 ± 8.3
Ba (5g→4f)	199910.6 ± 6.5	199907 ± 3	744	-3.6 ± 7.2
	201269.8 ± 6.8	201274 ± 3	758	4.2 ± 7.4
Sn (4f→3d)	345257.6 ± 7.7	345254 ± 5	1729	-3.6 ± 8.8
	349970.9 ± 7.6	349978 ± 5	1793	7.1 ± 9.1
Tl (5g→4f)	420759.2 ± 6.7	420765 ± 7	2011	5.8 ± 9.7
	426859.8 ± 7.6	426867 ± 7	2088	7.1 ± 10.3
Pb (5g→4f)	431324.4 ± 6.6	431336 ± 7	2078	11.6 ± 9.6
	437733.2 ± 7.2	437751 ± 7	2161	17.8 ± 10.0
Ba (4f→3d)	433908.9 ± 7.2	433910 ± 7	2324	1.1 ± 10.0
	441363.3 ± 7.4	441365 ± 7	2432	1.7 ± 10.2

Figure 7.1. Plot of $\Delta E = E(\text{theory}) - E(\text{experiment})$ against energy for the twelve muonic atom transitions measured in this work.

The agreement between the theoretical numbers and the experimental numbers is quite good. The largest discrepancy is the 437 keV transition in Pb. This line is high by 1.8 standard deviations. It may be that the interfering transition under this peak was not handled properly.

An independent analysis of the data was carried out by H. Mes of NRC. The fitting program used the ratio of two fourth order polynomials. The technique is described in McKee et al. (1971). The results of this analysis (to be published) are in essential agreement with the results quoted here, the largest discrepancy again being the 437 keV line in Pb. Fig. 7.2 shows the difference between the two analyses. The RMS deviation between the two is (excluding the Pb line) 4.9 eV. The differences can be attributed to different methods of handling the background, shape parameters and interfering peaks. There appears to be no systematic trend. An additional error of ± 5 eV has been folded into the final values in Table 7.1.

If we express the values of ΔE in Table 7.1 as a percentage of the vacuum polarization correction, the average deviation amounts to $(.25 \pm .16) \%$ of the vacuum polarization term. Excluding the Pb line at 437 keV gives an average deviation of $(.18 \pm .17) \%$. This difference between experiment and theory removes the discrepancy which existed in muonic atom calculations and leaves QED as one of the best verified theories of physics.

The electron screening correction is the second largest correction for the elements Ba (≈ 30 eV), Tl (≈ 75 eV), and Pb (≈ 80 eV). The overall agreement confirms these calculations to within 10 %.

We have reached the point where the experimental and theoretical

uncertainties are nearly the same. There seems little point in expending a large effort to reduce the experimental errors unless there is a corresponding reduction on the theoretical side. This is not to say the art of x-ray spectroscopy should not be improved, but only that future experiments of this nature should wait until both experimentalists and theoreticians have made advances.

One obvious way to improve the experimental uncertainties is to improve the statistics (without increasing the counting time!). This will be possible with the new meson factories. However the statistical errors in this experiment only amounted to 2-3 eV so this alone will not gain much. The major source of error comes from determining the correct shape of the x-ray and γ -ray peaks and unsorting the interfering transitions. New detectors with better resolution will help considerably. Problems such as angle effect, small angle Compton scattering etc. will have to be studied much more carefully as well if the experimental errors are to be reduced to <5 eV. With very intense muon beams it may be possible to make direct energy measurements with bent crystal spectrometers.

The only comment I can make on the theoretical side is to observe that the uncertainties on the vacuum polarization terms, electron screening and nuclear polarization are about the same size for the high energy transitions. Hence any improvements must be made on several fronts.

Figure 7.2 Plot of the differences between the results obtained in this analysis and those obtained by H. Mes against energy.

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MUONIC ATOMS:
A TEST OF QUANTUM ELECTRODYNAMICS

by

JAMES SAMUEL WADDEN

A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

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