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Network Flow Optimization Models For Integrated Flexible Manufacturing Systems

by

Ajit Pardasani

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Science (Information and Systems Science)

School of Computer Science

Carleton University

Ottawa, Ontario

January 15, 1990

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Abstract

The determination of optimal part routing and flow rates is important for the most economic utilization of flexible manufacturing systems. Until now the flow optimization problem had been solved only for flexible machining systems, but not for the more recent flexible assembly systems. In this thesis two models are presented to solve this problem for a flexible manufacturing system consisting of a flexible machining or flexible assembly system or the integration of the two. The modelling approach is to view the FMS as integration of subsystems to develop integrated models. The first model takes a steady state view of the system and uses processing network techniques to find the optimal part routing and flow rates in flexible assembly systems. This model is then integrated with the flexible machining system to find the optimal flows in the integrated system. The second model takes a dynamic view of the system and uses information about the time dynamics of the system and environment to find optimal routing, flows and inventory levels of part types over discrete time periods in an integrated flexible manufacturing system. Multiperiod network flow techniques are used to model the dynamics of the system in discrete time periods.

**Dedicated to
Dheerja**

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Chapter 1

Introduction

Manufacturing creates between 60 and 80% of the real wealth of major industrialized countries. The economic growth of industrialized countries depends upon the manufacturing and consumption of consumer goods. Hence, it is the objective of governments and industries to maximize the efficiency of manufacturing. To achieve increased efficiency in manufacturing the trend has been towards increasing the degree of automation. This trend to mass production began in the late 18th and early 19th century and heralded the industrial revolution.

Mass production involves the use of dedicated machines producing huge quantities of product. Rapidly changing consumer preferences have led to shorter product life cycles and increased the number of product variations, forcing manufacturers to switch from mass production to batch production. Batch manufacturing is suitable for products required in small to medium quantities (200–20,000 parts/year). In 1976, according to a report submitted by Comptroller General of United States [1976] to the Congress, 70% of manufactured output was produced by batch manufacturing methods.

Batch manufacturing has its own limitations. Machine utilization is very low and work in process inventory is high. Flexible Manufacturing Systems (FMS) were introduced in the 1960s to increase the efficiency and productivity of batch

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Chapter 1

Introduction

Manufacturing creates between 60 and 80% of the real wealth of major industrialized countries. The economic growth of industrialized countries depends upon the manufacturing and consumption of consumer goods. Hence, it is the objective of governments and industries to maximize the efficiency of manufacturing. To achieve increased efficiency in manufacturing the trend has been towards increasing the degree of automation. This trend to mass production began in the late 18th and early 19th century and heralded the industrial revolution.

Mass production involves the use of dedicated machines producing huge quantities of product. Rapidly changing consumer preferences have led to shorter product life cycles and increased the number of product variations, forcing manufacturers to switch from mass production to batch production. Batch manufacturing is suitable for products required in small to medium quantities (200–20,000 parts/year). In 1976, according to a report submitted by Comptroller General of United States [1976] to the Congress, 70% of manufactured output was produced by batch manufacturing methods.

Batch manufacturing has its own limitations. Machine utilization is very low and work in process inventory is high. Flexible Manufacturing Systems (FMS) were introduced in the 1960s to increase the efficiency and productivity of batch

production. Since then there has been growing interest in the development and implementation of FMS. In 1981 there were 40 of them installed in Japan, United States, Germany, Russia, Poland and Czechoslovakia [Mill, *et. al.* , 1985].

In 1987 a research forum established by the Institute of Industrial Engineers to identify directions, needs and challenges for research by the industrial engineering community in applying computer and information sciences identified FMS as a challenging and major research direction because of its critical importance to modern industry [Nof, *et. al.* , 1989].

An FMS consists of numerically controlled (NC) machines and/or assembly robots where production operations are performed, linked by a material handling system and integrated through a central computer. The workstations in flexible manufacturing systems are flexible in the sense that each workstation can handle a variety of parts. This flexibility makes FMS very complex systems to analyze and hence creates the major problem of item flow control: the more flexible the system, the more complex the flow. Part or item routing is an important problem which has a fundamental effect on the production rate and utilization of FMS [Kimemia, *et. al.* , 1985] [Nof, *et. al.* , 1979]. The routing of a part is a sequence of workstation visits. A common industrial practice is to route the parts in such a way that the workloads at workstations are equal [Olker, 1978] [Solberg, 1977]. Stecke and Solberg [1982] later argued that this policy is not always optimal.

Since FMS are extremely complex, models have been used to aid in design, production planning and operation of the system. Though the dominant modelling tool of FMS has been simulation [Mill, *et. al.* , 1985], analytical models have been found very useful as initial design tools. Initial design should be done using the analytical tools followed by detailed simulation during the fine tuning of the system.

Flexible machining systems and flexible assembly systems are no longer isolated systems but are integral subsystems of a modern FMS. Therefore there is a need for models which have capability to model integrated FMS rather than treating the flexible machining systems and assembly systems in a piecemeal way. The models developed in this thesis have the capability to model the behaviour of flexible

assembly, flexible machining and material handling systems. hence are referred to as integrated models.

In this manuscript network flow optimization techniques have been used to develop integrated models to find optimal flows in the system. Networks are a popular modelling tool as they are more visually informative and intuitively appealing [Koene, 1982]. Applications of networks can be found in a wide array of problems - to mention a few - production, distribution, financial planning, project selection, facilities location, resource management and budget allocation [Glover, *et. al.* , 1977]. Networks have become an efficient way of modelling due to significant advances in solution methodology, development of efficient data structures and improvements in computer software due to intensive code development efforts.

Most of the analytical models of FMS developed so far concentrate on FMS for machining of prismatic parts [Buzacott, 1982]. No attention has been paid to finding optimal flows in flexible assembly systems using analytical techniques. The first model in this thesis aims at computing the optimal flows of parts through a flexible assembly system by applying processing network techniques. The model is then integrated with the flexible machining system to develop the integrated model. This model is described in chapter 5.

The second model is a dynamic model which aims at determining the optimal routes and flows in an integrated flexible machining, assembly and material handling system in discrete time periods. Multiperiod network flow techniques are applied to model the dynamic behaviour of the system.

1.1 Overview of Thesis

Chapter 2, "Network Flows and FMS: a Primer" is designed to introduce the reader to the network flow modelling techniques and FMS. The chapter consists of four sections. The first section is a quick review of network flow modelling techniques. The second section is an introduction to the relatively new class of network flow models known as processing networks. The third section is an introduction to multiperiod

network flows and a quick survey of efficient algorithms to solve them. The last section is an introduction to flexible manufacturing systems, their characteristics and major components.

Chapter 3, "State of the art in modelling of flexible manufacturing systems" introduces the reader to the need for modelling in FMS and current major modelling practices in FMS. The chapter is divided into three sections. The first section describes the features of simulation and three approaches to simulation modelling. The second section deals with analytical models and its two important classifications: network of queue models and prescriptive models. The last section compares the class of simulation and analytical models.

Chapter 4, "The flow optimization problem" is an introduction to the flow optimization problem in FMS, its complexity and its importance to the real world. The chapter is divided into two sections. First section gives a flavour of current models to solve the problem and their limitations. The second section establishes the objectives of the project.

Chapter 5, "Steady State Network Flow Optimization Model for Integrated FMS", is the detailed description and formulation of an aggregate model to find optimal routes and flows in an integrated FMS. The chapter is divided into four sections. The first is on problem environment which describes the real system to be modelled and underlying assumptions to approximate the behaviour of real system. The second section gives the mathematical formulation of the model. In the third section three illustrative examples and their solutions are been presented. The emphasis has been to describe the results graphically rather than in text. The last section discusses the strengths and weaknesses of the model.

Chapter 6, "Dynamic Network Flow Modelling of Integrated FMS", is the detailed description, formulation and an illustrative example of a multiperiod network flow model for FMS. The chapter is divided into six sections. The first section describes the model and underlying assumptions, the second section presents the formal mathematical model of FMS. Section three describes the micro models for NC machines, assembly robot and transporter. The fourth section deals with the factors

which affect the behaviour of model. Section five is an illustrative example and the last section is discussion on possible applications of this model. and describes some further improvements.

Chapter 7, "Conclusions" has two sections. The first section summarizes the contributions of thesis and second section gives the directions for further research to this thesis.

Chapter 2

Network Flows and FMS: a Primer

2.1 Network Primer

The objective of this section is to provide a quick review of network flow models. Readers wishing to learn more about this modelling technique should refer to Lawler [1976] and Papadimitriou and Steiglitz [1982].

Networks constitute one of the most significant classes of techniques in industrial engineering and management science [Glover, 1977]. A wide array of problems in production planning, distribution, and resource management have been formulated and solved as network flow problems [Charnes, *et. al.*, 1961], [Kimemia, *et. al.*, 1985], [Maxwell, *et. al.*, 1981]. For an extensive survey of applications of network flows to real world problems refer to Glover and Klingman [1977]. Maximum flow models and minimum cost network flow models are used to model the FMS in this research, hence are described below.

A network is a directed graph¹ (V, A) where V is the set of vertices and A is the set of arcs. The network has a source $s \in V$ and a sink $t \in V$. Each arc (u, v) of the network has a lower bound $l(u, v)$ and an upper bound $B(u, v)$. The lower bound represents the minimum amount of flow per unit time which must flow through the arc and the upper bound represents the maximum capacity of the arc.

$x(u, v)$ represents the flow in arc (u, v) . Each arc (u, v) of the flow network has a

¹Directed graph is a graph with directions assigned to its edges

cost coefficient $c(u, v)$ associated with it. The flow value v_o in the network is defined as the flow from the source, s to the sink, t . The flow value v_o in the network is such that the flow in each arc is between the lower bound l and upper bound B

$$0 \leq l(u, v) \leq x(u, v) \leq B(u, v) \quad (2.1)$$

and conservation of flow holds at each node. That is, the total amount of flow going into the node is equal to the total amount of flow going out of the node.

$$\sum_{(u,v) \in A} x(u, v) = \sum_{(v,u) \in A} x(v, u) \text{ for all } v \in \{V - s, t\} \quad (2.2)$$

Flows $x(u, v)$ satisfying equation 2.1 are known as feasible flows and equation 2.2 is known as the flow conservation equation. An example of a network is shown in Figure 2.1

Legend: (Lower bound, upper bound) [Feasible flow]

Figure 2.1: Illustrative example of a pure network

Maximum flow problem: The problem of finding the maximum value flow from s to t is a linear program where the objective function is to

$$\text{Maximize } v_o$$

such that there is flow conservation at each node except for the source and the sink, and the flow in each arc is feasible. In more formal language the flow value q_0 is to be maximized subject to the constraints 2.1 and 2.2. In the example described in Figure 2.1 the maximum flow is 15 units.

Minimum cost flow problem: The minimum cost flow problem is to find a feasible source to sink flow v_0 that has minimum cost. The linear program is:

$$\text{Minimize} \quad \sum_{(u,v) \in A} x(u,v)c(u,v)$$

subject to 2.1 and 2.2.

2.2 Processing Networks

Processing networks are an important type of network flow model. The processing network structure arises in a number of applications including energy modelling [Chinneck, *et. al.*, 1984a, 1984b], manpower planning [Charnes, *et. al.*, 1982, 1984] and assembly models [Steinberg, *et. al.*, 1980]. Processing network terminology was introduced by Koene [1982] in his Ph.D. thesis. Koene described a processing node as a "refining node" with a single inflow and multiple outflows or as a "blending node" with multiple inputs and one output. Chinneck [1989] developed a more general model by removing the restrictions on the number of inflows and outflows. It allows more natural modelling of components having more than one inflow and more than one outflow, for example large industrial boilers.

A processing node is illustrated in Figure 2.2. A processing node is represented by a rectangular box. The point where an arc meets the processing node is called a terminal. A processing node imposes proportional restrictions on flows entering or leaving the terminals. A strictly positive number α_i called a "processing ratio figure" is associated with each terminal of the processing node. As illustrated in Figure 2.2 the processing node has r input terminals and $t - r$ output terminals. In order to ensure flow conservation at the node, the following property must hold:

$$\sum_{i=1}^r \alpha_i = \sum_{i=r+1}^t \alpha_i \quad (2.3)$$

α_i : processing ratio for figure for terminal i .
 x_j : flow variable for edge j .

Figure 2.2: A processing node

Let x_i be the flow variable representing the flow in edge i . The following processing ratio equation is used in the LP formulation to model the processing node.

$$\alpha_j x_i - \alpha_i x_j = 0 \quad (2.4)$$

A processing node having t terminals is represented by a set of $(t - 1)$ independent ratio equations having the form of equation (2.4). Each terminal must appear in at least one equation and exactly $(t - 1)$ equations are required. A processing network model may have regular nodes which are modelled as described in section 2.1.

A processing network in which all processing nodes follow the flow conservation equation (2.3) is called a pure processing network. Some of the real world processing operations when modelled as processing nodes do not obey the flow conservation equation. For example, in manufacturing environments, assembly of several parts to form a single product when represented by a processing node does not follow the flow conservation equation. These processing nodes are called non-conserving processing nodes and a processing network having such nodes is called a non-conserving processing network.

Figure 2.3 models the assembly of three part types to produce a product type. Processing node 1 represents the assembly of one unit of part type 1 with two units of part type 2 to create one unit of subassembly 1. The ratio of flows over the arcs

entering and leaving processing node 1 are the same as the ratio of quantities of part types required to assemble a unit of product. The processing ratio figures are shown on the terminals of processing node 1.

Minimum cost formulation for processing networks: Let c_i be the cost per unit of flow for the i th arc. The objective function is:

$$\text{Minimize } \sum_{i \in A} x_i c_i \quad (2.5)$$

subject to

$$0 \leq l_i \leq x_i \leq B_i$$

and (2.4) for the processing nodes, and flow conservation on regular nodes.

Consider the processing network example in Figure 2.3. There is no explicit source or sink in this example. The network is treated as a system and anything outside the network is the environment. Flows come in from the environment and go out to the environment. The processing nodes in the diagram represent assembly operations. There are two arcs going in each processing node and one arc coming

Figure 2.3: Illustrative example of an processing network

out. The number shown besides the terminals represent the ratio of flows between

the arcs entering or leaving the terminals. For an example the ratio of flows on arc (1,4), (2,4) and (4,6) are 1:2:1 respectively. It implies that for each unit of flow coming out on arc (4,6) there will be two units of flow on the arc (2,4) and one unit of flow on arc (1,4). The linear programming constraints corresponding to the example are as follows:

$$\text{Processing Node 1: } X_1 - X_5 = 0$$

$$X_2 - 2X_5 = 0$$

$$\text{Processing Node 2: } X_3 - 3X_6 = 0$$

$$X_4 - 4X_6 = 0$$

$$\text{Processing Node 3: } X_5 - X_6 = 0$$

$$X_6 - X_7 = 0$$

$$\text{Regular Node 1: } X_8 - X_2 - X_3 = 0$$

2.2.1 Algorithms for Solving Pure and Processing Networks

A network which can be modelled by only following equations is called a pure network or conserving network:

1. Flow conservation equation (2.2).
2. Lower and upper bound constraints (2.1).

A pure network problem can be formulated as a Linear Program (LP) and can be solved by any standard LP solver, but highly efficient special codes have been developed for solving pure networks by Bennington [1972], Barr, Glover and Klingman [1974], Glover Klingman and Napier [1972]. A comparative study of computational performances of codes for min-cost flow network problems is covered in Glover, Karney and Klingman [1974]. The important advantage of network codes over general LP codes is that network codes can solve the pure network problem 150-200 times faster than the general purpose LP code [Chen, *et. al.*, 1986] and typically require

very little central computer memory. Hence very large problems can be solved by network solvers which are considered impossible to solve with general purpose LP codes.

In a network all other equations in addition to the form of (2.1) and (2.2) are called side constraints and such a network is known as a network with side constraints. Algorithms developed by Klingman and Russel [1975], Chen and Saigal [1977] can be used for efficient solution of network with side constraints.

In processing networks the processing ratio equations of the form (2.4) can be considered as side constraints so the algorithms for solving networks with side constraints, as mentioned above, can be used for efficient solution. The efficiency of solution is dependent upon the embedded network structure so as the number of side constraints increases the solution time approaches that of a solution obtained by a standard LP.

2.3 Dynamic Networks : a Primer

The dynamic network model, also known as the minimum cost multiperiod network flow model, is frequently used to assist decision makers in the areas of resource scheduling, planning and distribution. This model describes the network structure over time. Such problems arise in the area of production systems [Zangwill, 1969], and material handling systems [Maxwell, *et. al.* , 1981]. A comprehensive survey of applications is covered by Aronson *et. al.* [1985, 1986].

The minimum cost multiperiod network flow problem (MNP) is a version of the network flow problem where some of the arcs represent commodity flow over time. The class of problems defined by MNP is divided into two subclasses [Aronson, *et. al.* , 1986]:

1. **The maximal dynamic network flow problem** was introduced in 1958 by Ford and Fulkerson [1958, 1962] to determine the maximum flow of a commodity from a single source to a single sink over T discrete time periods. Important applications of the maximal dynamic network flow model are in the area of

building evacuation, freight systems and material handling systems.

2. **The multiperiod trans-shipment problem** has found wide application in production planning problems, traffic flows, freight movement, economics, and energy production.

In Figure 2.4 a single period production planning model is described. The management has the choice of manufacturing the items in the manufacturing department or buying from a contractor to meet the fixed demand of that time period. The objective is to satisfy the demand at minimum total cost. The manufacturing department

Figure 2.4: Single period production planning model for deciding the quantities to make and buy. Cost of production = \$10 per unit and cost of subcontracting = \$12 per unit.

can only produce 100 units in a fixed time period. Since it is cheaper to manufacture the items within the factory, the aim of management is to meet the demand from in-house production as much as possible. It is easy to solve the problem for a single time period but no longer simple when the decisions have to be made for many time periods and the demand and the capacity of the manufacturing department and the contractor may change in every time periods.

The same problem is described over five time periods in Figure 2.5. The demand changes in every time period, but for the sake of simplicity the capacity of the manufacturing department and the contractor are assumed to be constant. The

horizontal arcs going from one time period to the next represent the flow of inventory carried over to the next time period.

Figure 2.5: Multiperiod production planning problem for deciding the quantities to make and buy. Numbers in parenthesis are the optimal quantities.

2.3.1 Algorithms for Solving the MNP Problems

The node-arc structure in an MNP remains the same over all time periods. Efficient algorithms have been developed which exploit the natural decomposition of the MNP.

The **Forward Simplex Algorithm** of Aronson, Morton, and Thompson [1985] solves large multiperiod problems, formulated as LPs, in linear time. Here linear time means that the solution time is of the order of T , where T is the number of time periods. Standard linear programming implementations require solution times of the order of approximately T^2 [Aronson, 1985]. By using the forward simplex algorithm many-period problems can be solved. Aronson [1985] reports the solution of a 100,000 period personnel planning problem having 1.1 million rows and 1.2

million columns on a DEC 2060 by a forward simplex implementation in 2.4 CPU hours at a total cost of \$255.91. Theoretically MINOS would have taken 2.72 CPU years to solve this large problem on a CDC 6600, but practically it could not have solved a problem of this type having 200 or more time periods due to memory limitations.

The Forward Convex-Simplex Method developed by Stanley [1984] is the extension of the forward simplex method to solve problems having a nonlinear objective function.

The Forward Network Simplex Method has been developed and implemented by Aronson and Chen [1986]. It exploits the network and multiperiod structure of the problem. The solution time is linear in the number of time periods whereas for standard network optimization methods the solution time is approximately quadratic.

2.4 Flexible Manufacturing Systems : a Primer

Definition of a Flexible Manufacturing System (FMS): "A number of workstations, comprising computer-controlled machine tools and allied machines, which are capable of automatically carrying out required manufacturing and processing operations on a number of different workpieces, with the workstations being linked by a work-handling system under the control of a computer that schedules the production and the movement of parts both between the workstations and between the workstations and system load/unload stations" [Talavage, 1988]. For a survey on flexible manufacturing systems refer to Gatelmand [1982] and for details on system components refer to Talavage [1988].

FMS have the following characteristics.

1. Each workstation is capable of processing² a variety of part types in any sequence.

²The term processing has been used to include operations like assembly, welding, and painting, etc. as well as machining.

2. Several kind of parts can be manufactured simultaneously by the system.
3. The path followed through the manufacturing system by the material is called a manufacturing path or manufacturing strategy. Any entering workpiece has a choice of several different routes or manufacturing strategies. A manufacturing strategy for a part type assigns each operation to a workstation and also specifies the sequence of workstation visits.
4. A variety of alternative production paths can be followed through the network, depending on the process plan or upon the availability of production units or work stations to process the work order.
5. The machines are supplied automatically with workpieces and tooling.

2.4.1 Components of Flexible Manufacturing Systems

(i) Numerically controlled machines (NC Machines):

NC machines form the basis of FMS. There are two types of control systems employed in NC machines.

1. **Computer Numerical Control (CNC):** The machine tool is controlled by a minicomputer which stores programmed instructions to specify the part geometry and other machine tool operations.
2. **Distributed Numerical Control (DNC):** In DNC several computer numerical controllers are coupled to a main computer which has a large memory to store programs.

NC machines are capable of storing many tools in the **automatic tool changer (ATC)**. Thus the machine can quickly access a tool when needed. The machines are equipped with **changeable worktables**. These enable the next part to be set up on one worktable while another is being machined. At the end of the machining cycle the worktables are swapped and changeover times between parts are thus reduced to a few seconds. ATC and changeable worktables have greatly reduced set up times.

(ii) Assembly Robots:

Like NC machines, assembly robots too have CNC controllers, so that they can easily store a number of programs to vary the tasks they have to perform. Programming can include instructions to change the grippers or tools in addition to changing products. The flexibility and capability of the assembly robot is limited by the flexibility and number of grippers it has available. An example of a flexible assembly system is provided by the Leyland assembly plant at Leyland, England. The complete range of trucks is produced on just two assembly systems, one for larger and one for smaller vehicles. Every truck on the line is invariably different from its neighbour, making it a truly flexible assembly line.

(iii) Material Handling System (MHS):

The material handling system has two important components:

(1) Pallets: The parts are mounted on pallets for easy presentation to the machine. Pallets form the interface between the workpiece and the machines. All the machines in the system accept the same form of pallet. It is possible to mount more than one workpiece on the pallet.

(2) Material Handling Equipment: The primary purpose of the material handling equipment is to transport pallets and workpieces between load/unload stations and to provide storage for in-process inventory.

(i) Machine Shuttle: The objective of the machine shuttle is to decouple the operation of the machines from the scheduling of the material handling system. This leads to maximum utilization of the machines and the material handling system without one significantly interfering with the others.

The machine shuttle accommodates two pallets, one space being kept empty to receive the pallet being machined and the other loaded with the next pallet so that it is immediately available. On completion of the machining of a part, it is transferred to the machine shuttle and thereafter to the localized storage of the workstation.

(ii) **In-Process Inventory Storage**³ Local storage is provided at the NC machines. The transporter loads/unloads the pallets at the local storage. The parts are then transferred to the machine by the machine shuttle.

2.4.2 Recent Developments in FMS

FMS were used earlier only for machining of parts. Systems which are capable only of machining parts are known as flexible machining systems. Flexible machining systems consist of several NC machines linked by a central computer and a material handling system. In fact, the term flexible manufacturing system was originally used to describe systems capable of machining only. The concepts of flexible manufacturing are now finding many applications in the area of robotized assembly. Robots now regularly assemble machine parts [Graf, 1988] and this might be the greatest area for future robot applications [Groover, 1982]. Flexible Assembly Systems may be adopted more rapidly as they are comparatively cheaper than flexible machining systems [Buzacott, 1982].

What is a flexible assembly system?: A flexible assembly system is a collection of assembly robots connected by an automatic material handling system, where each assembly robot is capable of performing a subset or all of the required operations on a variety of parts. The technical capabilities of an assembly robot are such that any or all of the operations for the product type may be performed by any assembly robot. The time to perform an assembly operation will differ from robot to robot [Donath, 1988]. Physical layout of a flexible assembly is described in Figure 2.6.

Flexible machining and flexible assembly systems are integrated systems

In real life situations flexible machining and flexible assembly are not isolated systems [Kusiak, 1985]. "The combination of flexible assembly with a flexible machining system is typical of much of today's flexible automated assembly technology" [Tallavage, 1988]. The flexible machining and flexible assembly systems are two major

³It is also referred to as a local buffer

Figure 2.6: A flexible assembly system

Chapter 4

The Flow Optimization Problem

Parts fed into an FMS as raw materials undergo a sequence of operations at workstations and come out as semifinished/finished products. The path followed by a part from entry to exit is called a **manufacturing path or route**. In other words a route is simply a sequence of workstation visits. The flexibility of the system allows for the choice of more than one workstation for each manufacturing operation. So even for a single part there are many alternative routes available. There may be a number of parts of a particular type being produced on the system.

The selection of routes is done by the designer/engineer by taking many factors into consideration. These factors may be the direct and indirect cost of production, economic value of products, availability of workstations, etc. One of the things ignored until now is the demand rate. More than one route may be required to meet the total demand due to bottleneck station(s). There may be several routes to meet the demand rate, workstation capacity constraints and the optimization criteria.

It may be noted that until now we have talked about the routing of a single part type in the system but in real life situations there are a large number of product types being produced simultaneously by the system. The determination of optimum manufacturing paths and flow rates of each part type on these paths subject to the optimization criteria and demand rate of each product type, and capacity constraints on workstations is known as the flow optimization problem in FMS.

The optimal flow problem is a complex problem: Flexibility was introduced in manufacturing systems with the objective of increasing the productivity

Figure 2.8: An integrated FMS

Chapter 3

State of the Art in Modelling of FMS

By 1974, a few FMS were installed in the USA and soon showed the need for improved design and operating procedures. To address the major issues in flexible manufacturing systems design and operation, the National Science Foundation funded research at several institutions. This led to the development of models to aid in the design and operation of FMS.

Figure 3.1 describes the main issues involved in the life cycle of an FMS. Design begins with a survey of the manufacturing requirements of the products to be produced on an FMS. Basic design concepts and detailed design follow. The design procedure is iterative. After the design is finalized the system is installed and is available for operation. The issues involved in the design and operation of FMS are extremely complex due to the versatility of the workstations, hence modelling tools are needed to study the system behaviour. The role of models is to aid the designer/industrial engineer in addressing the issues involved in the design and operation stage with the objective of optimizing the utilization of resources. FMS models can be divided into two major categories, as described in sections 3.1 and 3.2.

3.1 Simulation Models:

Simulation is the dominant modelling tool for FMS [Buzacott, 1986]. In a simulation model the user describes entities such as workpieces and machines as objects.

Figure 3.1: Major issues in design and operation of FMS. The issues in bold indicate areas where the models developed in the thesis can be applied.

Relationships between the objects that describe real world system behaviour are next described. Simulation requires high quality and quantity of data. It provides insight into the detailed dynamic behaviour of the system. The performance data obtained from simulation are credible descriptions of system behaviour provided that a good model has been used. For major characteristics of simulation models, refer to Table 3.1.

The values of the performance measures¹ obtained during the simulation may not be optimal. If the designer is not satisfied with the performance data he/she can identify aspects of design that, if changed, might result in performance improvement. The corresponding changes are made in the model, and performance is again estimated. This iterative process continues until the designer is satisfied.

There are three popular approaches to developing simulation models for manufacturing system. These approaches are described briefly below. Refer to Talavage[1988] for further details.

3.1.1 Network Simulation Modelling

This popular modelling approach, also known as the graphical approach, is robust and easy to use. It requires some programming capability and can be implemented on personal computers. "Network simulation languages have special constructs for representing essentially five major aspects of movement and processing of objects. Thus the objects are modelled as *arriving*, possibly *waiting*, being *processed*, perhaps being *routed* to other processors, and finally *departing* from the system." [Talavage, 1988]

Network simulation procedures that have been widely used for models of manufacturing systems include GPSS, SIMAN, Simulation Language For Alternative Modelling (SLAM), Generalized Manufacturing Simulator (GEMS), and CAPS/ECSL. For details refer to Talavage[1988].

¹Important performance measures are production rate of the system, machine utilizations, average time spent in system

3.1.2 Data Driven Simulation Procedures

Data driven simulation procedures are the easiest form of simulation to use. A model in the form of numerical data is input to the procedure, and there is no need for the user to understand a programming language. They are more restrictive in their modelling capability than the network models. Examples of data driven simulation procedures are General Computerized Manufacturing System (GCMS) developed at Purdue University, MAST, and Modelling and Analysis Program #1 (MAP/1).

3.1.3 Simulation using a Base Programming Language

Simulation models can be written in FORTRAN, Pascal, and C. Other languages which can also be categorized as base languages are SIMSCRIPT and SIMULA. The modelling capabilities of these languages are only limited by the ingenuity of the modeler.

3.2 Analytical Models

Research on the development of analytical models of FMS began as early as 1972. Analytical models of FMS can be divided into two classes as described below.

3.2.1 Network of Queue (n-o-q) Models

These models are based on Jackson network of queues [Jackson, 1963] and have been used for the determination of performance measures. Data requirements for these models are minimal so they can be used during the early design stage, when design details are not known, to help focus on good alternative designs. In a typical manufacturing environment, parts often must visit a number of different workstations before being completely finished. If, at any time, a part encounters a busy workstation, it must wait in a queue, possibly with other parts, before it can be processed by the workstation. Each part encounters a "network of queues" in its journey through the system. This simple fact makes manufacturing systems a suitable application

for n-o-q analysis. Some important characteristics of queuing models are listed in Table 3.1. Several FMS models are based on the n-o-q analysis technique. A survey of n-o-q models is covered in [Buzacott, 1986]. Out of several n-o-q models the CAN-Q and MVAQ models are well known. Major features of these models are described below.

Computer analysis of network of queues (CAN-Q) model:

The CAN-Q model was developed by Solberg [1977] at Purdue University. The FMS system is modelled as a closed queuing network in which the customers are the jobs to be processed by the system and the servers are the machines. It is widely used for the preliminary design of FMS. The major limitation of this model is that it is capable of dealing with only one part type in the system.

Mean value analysis of queues (MVAQ) model:

MVAQ is a computer program, written in FORTRAN, based on mean value analysis of queues [Suri, *et. al.*, 1984]. It models the long term average behaviour of the system and has the capability of dealing with several part types in the system. The performance measures (such as production rate) are obtained on a part-by-part basis.

3.2.2 Prescriptive Models

Also known as optimization or normative models, these models make use of mathematical programming (MP) techniques. They are used where some information regarding the design is available, while some other controllable parameters, e.g. flow, lie within a certain range, e.g. lower bounds and upper bounds on the flows. Prescriptive models find the best (optimal) values of the controllable parameters. Some important characteristics of FMS models are listed in Table 3.1. Models in this class are used mostly for scheduling and operational planning [Buzacott, 1986]. Operational planning refers to the decisions required to operate an FMS facility that is already in production. Decisions are made to resolve various problems such

as selecting part types to be simultaneously produced on the FMS, loading tools in the tool magazines, routing parts through the system and scheduling the workstations. Application of MP techniques in FMS can be found in [Avonts, 1988], [Kusiak, 1985], [Finke, 1985], [Chakravarty, 1987]. The important contributions in this class are flow optimization in FMS [Kimemia, 1985] and tool loading problem [Stecke, 1981]. The models presented in this thesis fall in this class.

3.3 Relative Comparison of Modelling Techniques

The modelling techniques vary in their capability to model real world problems. Only simulation has the capability to model a complete FMS system. On the other hand, each analytical model addresses a subset of issues in FMS. Analytical models are extremely powerful during the initial design stage when limited data is available so detailed simulation studies cannot be performed. Prescriptive models have found major applications in operational planning with some limited application during the detailed design stage. A relative comparison of modelling techniques is given in Table 3.1.

SIMULATION MODELS	ANALYTICAL MODELS	
	Network of queue (n-o-q) models	Prescriptive models
Simulation is based on simulation theory [Winston, 1987].	Based on queuing theory.	Based on mathematical programming.
Few simplifying assumptions are required so models are close representations of real systems	Number of assumptions made are large in comparison to simulation.	More assumptions than that in simulation are required. One of the important assumptions made in MP modelling of all manufacturing systems is that discrete part flows can be approximated by continuous flows. Number of assumptions may be less than in comparison to n-o-q models.
It is not an optimizing technique. Usually many iterations may be required to find near optimal or optimal solution.	It is also not an optimizing technique.	The application of prescriptive models is for finding the optimal feasible solution.
High quality and quantity of data is required.	Minimal data is required.	Data requirements are between simulation and n-o-q models.
Simulation can be costly in terms of computation when number of options tested are large [Kinemias, 1985].	It is least expensive. The CAN-Q model can be implemented on a programmable calculator.	It is more expensive than the n-o-q models but much cheaper than simulation.
It can simulate discrete event and continuous events. Manufacturing systems are discrete systems hence simulation is popular modelling technique in manufacturing.	It has capacity to model only discrete systems.	It can model only continuous systems through linear programming and discrete events through integer programming.
It is a powerful modelling tool for stochastic systems.	It has capability to model both stochastic and deterministic systems.	Most of the applications are for deterministic systems though it is possible to model very limited behavior of stochastic systems.
Usually a single unique model is constructed for the whole system. The construction of model can be lengthy process. So simulation is used at a fairly advanced stage of a project, when fine tuning of system is required. General purpose simulation programs have been developed for use in initial design stage e.g. GCMS, MAP/I, GEMS [Mills, 1985]	These are general purpose models hence the solutions obtained are not very accurate. These models are used in initial design with some applications in detailed design stage phase.	MP models can be both general purpose and tailored to the specific needs of the system. MP models are used mostly for operational planning with some applications in detailed design stage.

Table 3.1 Relative comparison of modelling techniques of FMS

Chapter 4

The Flow Optimization Problem

Parts fed into an FMS as raw materials undergo a sequence of operations at workstations and come out as semifinished/finished products. The path followed by a part from entry to exit is called a **manufacturing path** or **route**. In other words a route is simply a sequence of workstation visits. The flexibility of the system allows for the choice of more than one workstation for each manufacturing operation. So even for a single part there are many alternative routes available. There may be a number of parts of a particular type being produced on the system.

The selection of routes is done by the designer/engineer by taking many factors into consideration. These factors may be the direct and indirect cost of production, economic value of products, availability of workstations, etc. One of the things ignored until now is the demand rate. More than one route may be required to meet the total demand due to bottleneck station(s). There may be several routes to meet the demand rate, workstation capacity constraints and the optimization criteria.

It may be noted that until now we have talked about the routing of a single part type in the system but in real life situations there are a large number of product types being produced simultaneously by the system. The determination of optimum manufacturing paths and flow rates of each part type on these paths subject to the optimization criteria and demand rate of each product type, and capacity constraints on workstations is known as the flow optimization problem in FMS.

The optimal flow problem is a complex problem: Flexibility was introduced in manufacturing systems with the objective of increasing the productivity

of systems producing low to medium quantities, but it created the major problem of part flow control. The flexibility of the system provides for many alternative manufacturing paths for each product. The greater the number of alternative manufacturing paths, the more flexible the system is, so the problem of determining the optimal routes is more complex.

The importance of the optimal flow problem in FMS: An FMS is a substantial capital investment for any company and it is natural that during the operation phase of the system, engineers and managers target for the most economic utilization of the FMS. Economic utilization of the FMS is needed to pay back the capital investment, to support ongoing operating costs and to generate a targeted net rate of return. This can only be achieved by higher production rates, maximum utilization of workstations and reduced total costs (direct and indirect). The production rate and economic utilization of an FMS is determined by the part routing and flows [Kimemia, *et. al.* , 1985] [Nof, *et. al.* , 1979].

4.1 Current Analytical Approaches to Solving the Routing Problem in FMS

The manufacturing paths can either be found in advance or a set of decision rules can be established which allow the sequence of workstation visits for each part to be determined in real time as the part makes its way through the system [Buzacott, 1982].

A heuristic scheduling algorithm developed by Donath and Graves [1988] finds the schedule and routing in a flexible assembly system when due dates are available. The algorithm finds the optimal solution for nearly 30% of test cases depending upon the product structure, number of operations needed, quantity of production and combination of workstation visits.

Koenigsberg and Mamer [1982] developed an analytical model, based on queuing theory, for a flexible assembly system consisting of a work transporter to feed the

workstations and a carousel conveyor¹. The system of workstations is a cyclic queue. They examined the production subsystem by decomposing it into its major parts (carousel conveyor, work transporter, and workstations) and then used the results to compute overall system performance. The results obtained by the model are approximate.

The network of queue analysis models CAN-Q and MVAQ have been used to study the effect of routing on throughput and in-process inventory of a Flexible Machining System.

Linear Programming, a prescriptive modelling technique, was used by Kimemia and Gershwin [1985] to develop a flow optimization model for a Flexible Machining System. The model generated routes for each part type and the rate at which parts should be dispatched on the routes. The Dantzig-Wolfe decomposition algorithm was applied to decompose the problem into small linear programmes thereby reducing the solution time of the problem. The solution techniques can be used in real time or in advance to determine the manufacturing paths.

4.1.1 Limitations of Existing Models

Limitations of queuing models: Flexible machining systems were introduced in the 1970s and flexible assembly systems are being implemented now. So, most of the research has been on flexible machining systems. CAN-Q and MVAQ can only model flexible machining systems. One of the major limitations of CAN-Q is that it deals with FMS in which only one part type is being produced. It is possible to model multiple part types by aggregating the operational requirements of several part types into a single part type. Performance measures thus obtained are highly aggregate in nature. The method of calculating an equivalent part type from multiple part types is described by Talavage [1988].

MVAQ has the capability of modelling an FMS producing several part types. But MVAQ, CAN-Q and the flexible assembly system model developed by Koenigsberg and Mamer [1982], all are based on several assumptions which are usually not

¹Also known as linking conveyor

satisfied by an FMS. These assumptions are:

1. The processing time of a part has exponential probability distribution, whereas in an FMS the processing time of a part is known quite accurately.
2. The routing of a part to the next machine is chosen probabilistically whereas in an FMS the next operation and machine is predetermined.

All the above mentioned queuing models calculate aggregate performance measures, hence are good only as preliminary analysis tool. All these models lack the capability to find the minimum cost routing through the system.

The optimization model for FMS developed by Kimemia and Gershwin [1985] for finding the minimum cost manufacturing path is applicable only to flexible machining systems. There is no flow optimization model for flexible assembly systems to date. Table 4.1 describes the current status of flow optimization models for FMS. The problem of determining the optimal routes and flows can be solved by consid-

Target system for modelling	Steady state	Dynamic
Flexible Machining	Kimemia and Gershwin [1985].	Model presented in chapter 6.
Flexible assembly	Model presented in chapter 5.	Model presented in chapter 6.
Flexible machining + Flexible Assembly	Model presented in chapter 5.	Model presented in chapter 6.
Flexible machining + Flexible Assembly + Material handling system	The model presented in chapter 5 describes only limited details of material handling system in integrated FMS.	Model presented in chapter 6.

Table 4.1 Status of current models for flow optimization in FMS.

Table 4.1: Scenario of models for flow optimization in FMS

ering the system as steady state or dynamic. If system parameters are considered to be steady state then demand, supply and other system parameters are considered to be constant hence time invariant. The flow optimization model by Kimemia and Gershwin [1985] is a steady state model. Steady state models provide valuable insight into the behaviour of the system but they ignore the dynamic nature of the environment and system.

The dynamic model uses advance knowledge of the time varying behaviour of the system. The dynamic system can be modelled either as continuously time variant or discrete time variant² systems. Until now simulation was the only way to model the dynamic behaviour of dynamic FMS. An analytical dynamic optimization model would permit the modeller to determine optimal routes by taking varying demand and other system parameters into consideration.

Current modelling practices in production planning take a modular view of the problem. The modular view treats the subsystems in a piecemeal way, thereby neglecting major linkages between them. This is considered to be a major gap between modelling practice and real world problems [Saad, 1982]. Most of the production planning models, in isolation, are well formulated and have a rich theory to support their solution. In suggesting ways to develop more realistic models. Saad [1982] has emphasised that models, as much as possible, should take an integrated view of the system instead of a modular view. This has motivated the development of the integrated models in this thesis. An integrated view of the system allows us to treat the FMS as consisting of three subsystems: flexible machining system, flexible assembly system and material handling system.

4.2 Objectives of the Project

Considering the limitations of existing models discussed above, the objectives of this thesis are:

1. To develop a steady state network flow model to find the optimal manufacturing paths and flow in a flexible assembly system, and to integrate it with a model of a flexible machining system. This model is described in chapter 5.
2. To develop a dynamic network flow model to find the optimal manufacturing paths, flows and inventory levels of part types in an integrated FMS to aid in the scheduling of parts. The model should have the capability to model the

²Details in chapter 6

time varying nature of:

- (i) Environmental parameters such as demand of final products and supply of raw materials, and
- (ii) System parameters such as capacity of workstations, availability of workstations, cost of manufacturing operations and costs of raw materials

This necessitates the development of micro models for NC machines and assembly robots. The model is described in chapter 6.

Chapter 5

Steady State Network Flow Optimization Model for Integrated FMS

Due to the complex nature of FMS it is very important to determine how to operate it to maximize the overall effectiveness. Since a part type can be produced by several alternative strategies it is important to determine the minimum cost manufacturing paths for each part type through the system and its flow rate through each workstation. Solution techniques for finding the optimal routing in flexible machining system were developed by Kimemia and Gershwin [1985] using a network flow optimization approach. However, flexible manufacturing is not limited only to machining of parts. As mentioned in chapter 2 the concepts of flexible manufacturing are finding wide application in flexible assembly systems. Like flexible machining systems these are also complex to analyze. Very little research has focused on this area [Buzacott, 1986] and there is no network flow optimization model to find the optimal manufacturing paths for the assemblies.

In this chapter the capability and ease of modelling assembly operations through processing networks has been used to develop a steady state model for finding the optimal flows in flexible assembly systems. An actual FMS reaches steady state when time scales are of the order of days/weeks/months. Steady state is obtained when performance measures are constant on average. The steady state model developed in this chapter will be useful to find optimal manufacturing paths in an FMS when the system runs uninterrupted for days/weeks/months for the same part mix ratio.

A processing network formulation is presented for finding the optimal flows in an integrated flexible assembly and flexible machining system. Like all the linear programming models for production planning, this model also assumes that discrete part movements in manufacturing environments can be approximated through continuous flows.

5.1 Problem Environment

Consider a multi-product firm which produces medium quantities of products using an FMS. The bill of material of a final product is represented by a product structure diagram as shown in Figure 5.1. Components exist at different levels in the product structure. A component may either be machined or it may be assembled with other components to form a new product. A component may be used in the production of more than one finished product. Each component type is identified by a unique identification number. The number in the parentheses adjacent to each component represents the quantity required for producing the next higher level item. For example, shown in dashed box, one unit of component type 6 is machined to become one unit of component type 16 which in turn is assembled with two units of part type 5 to form an unit of component type 9. Component types 1, 2, 3, 4 and 6 are used in both the assemblies. Any component entering the flexible manufacturing system is raw material. Components entering the system from the environment go through a sequence of machining and/or assembly operations. The machining operations are carried out in the flexible machining system and assembly operations are carried out on the flexible assembly system. Both the subsystems are linked through a circular conveyor which also acts as a buffer (Figure 2.8). The flexible machining system consists of M NC machines and the flexible assembly system consists of N robotized or manual or manually assisted robotized work stations.

Drilling, tapping and boring of holes are treated as machining operations. The assembly operation is the merging (joining) of two or more components through fasteners or other means to form a single component. A particular machining operation

CHAPTER 5. STEADY STATE FLOW OPTIMIZATION MODEL FOR FMS37

Figure 5.1: Product structure of assemblies 1 and 2.

can be done at one or more machining centres and similarly a particular assembly can be done at one or more assembly stations. The components representing the raw materials may have to go through the following operations (not necessarily in this order) to become a finished product. The sequence of operations is determined by the process plan of each assembly.

1. Components entering the system from the environment may be machined to make new components.
2. Components may be assembled to form subassemblies.
3. The subassemblies may need to be machined. Instead of machining the individual components and then assembling them, the assembled part is machined to allow for proper matching of machined surfaces.
4. The subassemblies or subassemblies and components may be assembled to form the final assembly.

A manufacturing (machining or assembly) operation is a set of simple processes (such as drilling, tapping, matching of parts, fixing fasteners, etc.) that are com-

CHAPTER 5. STEADY STATE FLOW OPTIMIZATION MODEL FOR FMS38

pleted during a single work station visit. The flexibility of the system allows for the possible choice of more than one work station for each operation. The time to complete a machining operation on a workstation is the time elapsed between picking up a component from the buffer and returning it to the buffer after performing the necessary operations. The time required to complete an assembly operation is the time elapsed between picking up the first component from the input buffer and returning the assembly to the output buffer after assembling the required parts. The rate of production of component type i on workstation j is defined as X_{ij} . The time to complete an operation on component type i on work station j is a random variable with mean t_{ij} . Variations in machining times of a component type on the same machine may occur due to adaptive control. Variations in assembly times may occur due to the human factor in manual or manually assisted robotized assembly systems.

The cost of performing an operation on component i on workstation j is represented by c_{ij} . Where the objective is to minimize the total transit time in the system (i.e. maximize throughput) c_{ij} is taken to be t_{ij} . The transportation of components between the assembly stations and between central storage and assembly stations is by means of a circular conveyor and loading/unloading robots. The components are dispatched on the circular conveyor and are picked up by a loading/unloading robot to be delivered to the input bins of the assembly robots (Figure 2.6). The assembled components are picked up by the loading/unloading robot from the output bins of the assembly robots and delivered to the circular conveyor for the next operation. It is assumed that during steady state operation of the system, there are always components available in the input bins and there is always space in the output bin for a part. This assumption implies that there is no blocking in the system. It means that during steady state operation the transportation times of components can be ignored.

The manufacturing process is represented by a processing network of flows of components. Machining operations are represented by regular nodes and assembly operations are represented by processing nodes. The processing node is denoted by

a double index (ij) where i is the part type assembled at workstation j . The flow of raw material is represented by arcs from the environment to the system and the flow of final products is represented by arcs from the system to the environment. The network in Figure 5.3 represents the logical flow of components for producing assembly 1 shown in Figure 5.2. The logical network need not resemble the physical

Figure 5.2: Product structure diagram for assembly 1. It can be assembled on both the workstations k and m.

Figure 5.3: Logical network flow diagram neglecting transportation times.

layout. Since a particular component can be produced on more than one workstation, components of the same type are merged on a distribution node and also distributed for the next operation to the nodes representing the next manufacturing

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operation in the sequence. This regular node is known as a distribution node and is labelled as D_i where i is the component type. The flow rate of component type i from distribution node D_i to manufacturing node jk is $X_{d,jk}$ and the processing ratio attached to this arc is $\alpha_{d,jk}$. Similarly the flow rate of component type j from manufacturing node jk to distribution node D_j is X_{jkd} and the processing ratio figure attached to it is α_{jkd} .

5.2 Problem Formulation

The total demand for products is known to be Q . Let the total production rate be equal to the total demand Q . The product mix ratio is known. It is the ratio of production quantities of each product to total production. Let the rate of production of the i th product be r_i where r_i is the flow variable representing the flow of the i th product to the environment.

$$r_i = \beta_i Q \quad (5.1)$$

where β_i is the quantitative ratio of component i to total production quantity:

$$\sum_i \beta_i = 1, \quad (5.2)$$

Figure 5.3 may be referred to understand the formulation.

Assembly Node Constraints

$$\frac{\alpha_{jkd_j}}{\alpha_{d,jk}} X_{d,jk} - X_{jkd_j} = 0, \quad j \in \text{family of assembled components} \quad (5.3)$$

$k = 1, 2, \dots, N$, where there are N assembly stations
 $i = \text{component of assembly } j$

Machining Node Constraints

$$X_{d,jk} - X_{jkd_j} = 0, \quad j \in \text{family of machined parts} \quad (5.4)$$

$k = 1, 2, \dots, M$, where there are M NC machines

Distribution Node Constraints

$$\sum_j X_{jkd} = \sum_j X_{d,jm} \quad (5.5)$$

Nonnegativity Constraints

Since the production quantities can not be negative,

$$r_i \geq 0, \quad X_{d,jk} \geq 0, \quad \beta_i \geq 0 \quad (5.6)$$

Machine Utilization Constraints

In manufacturing systems one of the most important performance measures is the workstation utilization. It is defined to be the proportion of time that a workstation is operating on a part. Let X_{ij} be the flow rate in units/hour of component type i through workstation j . t_{ij} is the processing time, defined earlier, in hours. The machine utilization constraint is

$$\sum_i X_{ij} t_{ij} \leq 1, \quad j = 1, 2, \dots, (M + N) \quad (5.7)$$

5.2.1 Flexible Assembly System Model

The flexible assembly system optimization model can be stated as

$$\text{Maximize/Minimize } f(X)$$

subject to the constraints 5.1 to 5.3 and 5.5 to 5.7 where $f(X)$ is a linear function dependent on the vector of flows X_{ij} . The objective function may be the maximization of throughput or minimization of the cost of production. Where throughput is to be maximized the objective function is:

$$\text{Maximize } \sum_i r_i$$

For minimization of total cost of production the objective function will be

$$\text{Minimize } \sum_{i,j} X_{ij} c_{ij}$$

5.2.2 Flexible Machining System Model

The flow optimization model for the flexible machining system is

$$\text{Maximize/Minimize } f(X)$$

subject to the constraints 5.1, 5.2, 5.4 to 5.7

5.2.3 Integrated Flexible Machining and Flexible Assembly Model

$$\text{Maximize/Minimize } f(X)$$

subject to the constraints 5.1 to 5.7

Equations 5.4 and 5.5 are pure network constraints arising out of flow conservation at regular nodes. Equation 5.3 is the processing network constraint also known as a side constraint. It represents the flow ratios at processing nodes. Equation 5.7 is an extra side constraint. The number of extra side constraints is equal to the number of workstations in the system.

5.3 Examples

Example 1: Numerical results for two assemblies and two assembly stations

The objective of the first example is to explain the basic ideas in min cost flow formulation of a very simple flexible assembly system using two robotized assembly work stations and a circular conveyor. The system is described in Figure 2.6. Two different types of assemblies are produced by the system. The product structures of both the assemblies are described in Figure 5.4. Assembly 1 is assembled from component types 1 and 2, assembly 2 is assembled from component types 3 and 4. The capability of a robot arm to assemble a product depends upon the number and flexibility of tooling and grippers attached to it. The range of grippers and tooling is restricted by economics and space limitations, which is a separate problem. The

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Figure 5.4: Product structure diagram for assembly 1 and 2.

flexibility of tooling and grippers will determine the available routes through the system. The times to assemble the products by each robotized arm are shown in Table 5.1. Assembly 1 can be assembled only at workstation 1 and assembly 2 is assembled at both the workstations.

Part/Assembly	Assembled from	Assembly time on workstation	
		Work station 1	Workstation 2
Assembly 1	P1 and P2	1.2	-
Assembly 2	P3 and P4	2.5	2.0

Table 5.1: Operational requirements for assemblies 1 and 2 in minutes.

The problem has been described as a network of flows in Figure 5.5. The assembly operations carried out on assembly workstation 1 are grouped in a dashed box to enhance the visual clarity of the problem. Total demand has been assumed to be 60 units per hour and the product mix of assembly 1 and 2 is 40% and 60%. The linear program corresponding to the example was solved on the microcomputer package 'microsolve'. The results obtained are illustrated in Tables 5.2 and 5.3. The total demand for assembly 2 is 36 units/hr. Since it is cheaper to produce assembly 2 on workstation 2, the maximum possible flow of 30.3 units/hr. of assembly 2 is handled by workstation 2. The utilization of workstation 2 is 100%. The remaining production of 5.7 units/hr. of assembly 2 is at workstation 1. It is obvious from

Figure 5.5: Processing network for two assemblies and two assembly station example. The values on the terminals of processing nodes are the processing ratios.

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Assembly #	Strategy	Flow rate units/hr.	split
Assembly 1		24.0	1.0
Assembly 2		5.7	0.16
	30.3	0.84

Total production = 60.0 units/hr.

Table 5.2: Optimal routes and flow rates for example 1. Dual index ij within the processing node represents the assembly of part i on workstation j .

workstation 1	workstation 2
72%	100%

Table 5.3: Workstation utilizations for example 1

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the solution of this problem that a network flow model (eqns 5.1 to 5.3 and 5.5 to 5.7) leads to minimum cost paths in flexible assembly systems. The cost coefficients for a particular operation, in this example and all other subsequent examples, are taken equal to the time to perform the operation. If the real data on cost of each operation is available then cost coefficients can be taken equal to the actual costs.

Example 2: Three products, three assembly workstations problem

In the next problem, which is more complex than the first, a production system having three flexible assembly workstations is considered. There are three products to be assembled. The product structure is described in Figure 5.6. Table 5.4 shows

Figure 5.6: Product structure of three assemblies in example 2.

the operational requirements for three products to be produced on the three workstations. There are 12, 6 and 18 strategies available for producing assemblies 1, 2 and 3 respectively. The total number of possible paths for all the assemblies is 36. It is very difficult to enumerate the paths in advance for bigger problems.

The network flow diagram corresponding to this problem is shown in Figure 5.7. The corresponding linear program has 58 variables. There are 47 equality constraints arising from conservation of flow and 3 inequality constraints from workstation utilization. In the optimal solution 24 flow variables are reduced to zero. Out of 36

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Assembly/ Subassy.	Assembled from following parts or sub assemblies	Assembly time on work station in minutes		
		1	2	3
a	Part 1 & 2	2.0	-	0.9
b	Part 3 & 4	2.5	1.2	1.0
c	Sub b & part 2	1.5	1.8	-
Assy. 1	Sub a & sub b	1.8	2.0	-
Assy. 2	Sub a, part 3 & 4	2.2	2.0	1.7
Assy. 3	Sub c & part 6	1.2	1.5	1.7

Table 5.4: Operational requirements for three assemblies in example 2.

possible paths only 9 are needed to satisfy the demand at minimum cost. The proportion of the time required to produce a product on each workstation is shown in Figure 5.8. Though each workstation has the capability of assembling most of the products, only a few are produced on each workstation to get the optimal result.

Example 3: Three products, two flexible assembly stations and two NC machines problem

In this problem an integrated flexible machining and flexible assembly system is considered. It is assumed that the system has to produce three products which require machining as well as assembly operations.

The product structure of the assemblies is described in Figure 5.9. Table 5.5 depicts the operational requirements for the assemblies. The network flow diagram is shown in Figure 5.10. The machining operations are shown by two concentric circles. The linear programming formulation of the problem has 45 equality and 4

Figure 5.7: Network flow diagram for three assemblies, three workstation example.

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Figure 5.8: Utilization of workstations by the assemblies in example 2. The bar graph describes the proportion of time each workstation is busy processing each product

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inequality constraints. The results are tabulated in Table 5.6. Out of 28 possible

Figure 5.9: Product structure of three assemblies in example 3.

manufacturing paths only 4 are needed to meet the total demand of 16 units/hr with the product mix of Assembly 1, 2 and 3 being 50, 20 and 30%. The machine utilization are 100% and 92% for the machine 1 and 2 respectively, and 13%, 45% for the assembly stations 1 and 2 respectively. The reason for very low utilization of assembly workstations is that assembly times are very small in comparison to machining times. So two assembly stations can do assembly of components faster than the two machines. Moreover, for example if three parts are machined and then assembled then all the three parts are to be machined separately but while assembling these three parts only one set of operation is involved.

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Part/ Assy.	Operation	Assembled/ machined from	Time on work station in minutes			
			NC Machine		Assembly Station	
			1	2	3	4
P3	Machining	P1	3.0	3.0		
P4	Machining	P2	3.5	5.0		
Sub a	Assembly	P3 & P4			1.0	1.2
Sub b	Assembly	P5 & P6			0.70	1.0
Assy. 1	Assembly	Sub a & sub b			2.2	1.70
Sub c	Assembly	P1 & P6			1.9	1.2
P8	Machining	P5	3.9	3.5		
Assy. 3	Assembly	P7 & P8			-	2.5
Assy. 2	Machining	Sub c	4.0	2.0		
P7	Machining	Sub b	3.5	1.5		

Table 5.5: Operational requirements of assemblies in example 3

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Figure 5.10: Network flow diagram for example 3.

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Assembly	Strategies	Flow rate	split
Assembly 1		1.44	0.18
		6.5 6	0.82
Assembly 2		4.8	1.0
Assembly 3		3.2	1.0

Production Rate 16.0

Legend:

Machine utilizations: #1: 100%
#2: 92%

Assembly workstation utilizations: #1: 13%
#2: 45%

Table 5.6: Optimal routes and flow rates for example 3.

5.4 Discussion

In the flexible assembly system flow optimization model it is assumed that the components to be assembled are always available to the assembly robot. This is accomplished by a loading/unloading robot performing the task of picking up and delivering the parts to bins (input buffer) of the assembly robot. This results in better utilization and flexibility of the assembly robot than where the assembly robot has to itself pick up the parts from the circular conveyor. Another arrangement where the parts are available to the assembly robot is where the various parts are collected on a pallet so the parts are fed along with the main object. With this layout more parts can be assembled each time [Johnson, 1986].

One strength of the flow optimization FAS model is that it is a multiproduct model and allows a component or subassembly to be used for the assembly of more than one product. Most often it will be suitable to find the optimal manufacturing paths and flow rates for simultaneous production of products having a number of components or subassemblies in common. The effect of increasing/decreasing the production of one product type can be studied by sensitivity analysis.

There will be certain conditions in which the optimization models presented in this chapter will not work very well. This is when there is a significant congestion in material transport and/or material storage (buffer) systems, leading to a significant amount of "blocking" of workpiece movement. The problem arises because of the assumption that, as soon as a component completes processing at a workstation it can move out in the output buffer and there are always components available in the input buffer for the next operation to be performed on that workstation. If there is blocking on a workstation for a significant portion of time, the flow optimization model may seriously overestimate the production rate.

Of all the assumptions made in this model blocking is the most severe. The assumption that transporter scheduling does not affect the performance of workstations is less severe. The last assumption of modelling discrete part flows with continuous flows is least severe. The more severe the assumption, the more distortion it can create in the results if modelled situation is not close to real life situation.

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For the reasons mentioned above the solutions obtained by LP model are too optimistic because of the assumption that machines can be fully utilized. To arrive at a realistic solution through LP, Avonts and Wassenhove [1988] suggested integration of LP models with queuing models. Their ideas can be used to find realistic optimal manufacturing paths and flows of part types, and utilizations of workstations by following steps:

1. Solve the LP model with maximum workstation utilizations of 100%. The output of LP model describes the manufacturing paths and flows of part types.
2. Avonts and Wassenhove [1988] used the CAN-Q queuing model for flexible machining systems to compute various performance measures given the manufacturing paths and flows obtained from the LP solution. The results from the CAN-Q are workstation utilizations and size of the queues in front of workstations. Similarly, the model by Koenigsberg and Mamer [1982] is to be used to find these performance measures in a flexible assembly system.
3. Adjust the maximum workstation utilizations in LP model as obtained by queuing models.
4. Repeat steps 2 and 3 until the manufacturing paths and flows stabilize.

The material handling system has been completely neglected in the models presented so far. It is possible to extend the model to include limited details of the material handling system. This can be accomplished by modifying the logical flow of components within the system as described in Figure 5.11. In the flexible assembly model (eqns 5.1 to 5.3 and 5.5 to 5.7) components of the same type were merged on a distribution node and then passed on to the next manufacturing operation. To model the material handling robot, each assembly operation has a node before and after. The node before the assembly operation represents the collector and the node after the assembly operation represents the distributor. The distribution node of a manufacturing operation is joined through arcs to the collector nodes of all the subsequent manufacturing operations. It may be recalled that a manufacturing operation can be performed on more than one possible workstation.

Figure 6.3: Aggregate dynamic network corresponding to the flows at logical level.

Part Type 1

	W.S.#1	W.S.#2
Operation 1	2.0	3.0
Operation 2	2.0	1.0

Part Type 2

	W.S. #1	W.S. #2
Operation 1	1.0	3.0
Operation 2	3.0	2.0

Table 6.1: Operational requirements on part type 1 and 2 in minutes.

CHAPTER 5. STEADY STATE FLOW OPTIMIZATION MODEL FOR FMS'S

The system modeller may assign a single representative travel time by averaging the times to make all the moves on all of the transportation arcs. Although an approximation, this is a particularly convenient option for the user, since only one representative data item needs to be specified. If the single aggregate travel time is thought to be too crude an estimate, each transportation arc may be assigned a different estimated time.

The inclusion of transportation times will facilitate the determination of the utilization factor of the material handling robot. It may be crucial to ensure that this robot does not become a bottleneck. The transportation times are treated as costs and are included in the cost minimization objective function. It may be noted that the transportation cost of a component/subassembly is zero if it remains at the same work station for the next manufacturing operation.

The part loading schedules can be easily obtained from optimal flow rates by using a very simple deterministic scheduling algorithm developed by Kimemia and Gershwin [1985]. Let X_{ij}^* parts/hour be the optimal flow rate of part type i on workstation j . The part type i is dispatched to workstation j at the regular interval of $\frac{1}{X_{ij}^*}$ hour.

Chapter 6

Dynamic Network Flow Modelling of Integrated FMS

The deterministic model for integrated FMS proposed in chapter 5 assumes that the demand and supply rates and system parameters (capacities of the equipment) do not change over time and therefore have fixed values. The supply, demand flows and capacities of equipments are the steady state values for the period of interest. If the period of interest changes then the problem has to be solved again with new parameters. Each solution is treated separately. This calls for a new approach which takes the dynamic natures of the environment and the system into account. Dynamism in the environment is introduced by the fact that the supply of raw materials and demand for the final products changes over time in real life situations. Maxwell and Wilson [1982] developed a dynamic network flow model for fixed path material handling systems. Manufacturing systems are dynamic in the sense that the capacities of the equipment may depend upon the operators training and the time period of the day when used. Some of the equipment may not be utilized to full capacity or not used at all during the night shift. It is also possible that equipment may be taken out of service for maintenance for some time. All these factors call for a model which can take into account the dynamic nature of the FMS. The importance of the need is increased by the fact that the objective of the FMS is to produce low to medium quantities of products to meet time varying demand profiles caused by rapidly changing consumer interests.

Apart from simulation and heuristics, there are two approaches to solving cost minimization problems in time varying systems. The first is the classical approach where demand for the products is assumed to occur continuously over time and uses differential calculus to determine the optimal solution. The second approach, known as the dynamic approach, divides time into discrete periods and postulates that the demand in each period is constant. Mathematical programming is then used to determine the optimal production schedule.

In this chapter, the dynamic approach is used to develop a model to determine the optimal manufacturing paths and flows and in-process inventory levels of part types in an integrated FMS. The optimal part routing and flows obtained over the discretized time intervals are used to aid in determining optimal schedules for part types. The model is further extended to include determination of optimal inventory levels of final products. The major underlying assumption behind the model is that discrete part movement in a manufacturing environment can be approximated by continuous flows. The modelling tool used is networks of flows.

6.1 Model Description

Dynamic network flow models are a class of network flow models where lower and upper bounds on the arcs, supply and demand change over time. Figure 6.1 describes a simple dynamic network. The flow from source, s , to sink, t , has three possible paths: $[(s-1), (1-t)]$, $[(s-1), (1-2), (2-t)]$ and $[(s-2), (2-t)]$. Each arc has a triple (l, B, Δ) attached to it where l is the lower bound and B is the upper bound on the amount of flow through the arc. Δ represents the travel or transit time over the arc which represents the time for a unit to travel from the tail to the head of the arc. The dynamic network in Figure 6.1 is transformed into an equivalent time expanded static network as shown in Figure 6.2. In the time expanded network, time intervals are represented horizontally and nodes of the original network are represented vertically. The arcs in Figure 6.2 have their tail in a time period and their head in a later time period. The number of time intervals spanned by an arc is equal to

Figure 6.1: A simple dynamic network

the transit time. At each node the units can either be transshipped immediately to another node or held over for later shipment. The horizontal arcs represent the hold over of units from one time period to the next and are known as inventory or holding or accumulation arcs.

Consider a flexible manufacturing system consisting of two NC machines and a transporter. A transporter connects both the machines and the loading/unloading station. The FMS is represented by a network of flows between workstations, represented as nodes. An arc represents the potential flow of pallets from one node to another. In order to model the dynamic behaviour of the pallets flowing through the network of directed arcs and nodes, a travel time is attached to each arc. The selection of the travel time is described later in the chapter.

The flow of pallets between workstations at a logical level is described by the preliminary dynamic network in Figure 6.3. It is referred to as an aggregate dynamic network as it takes only the logical connections of workstations into account. As explained above, a triple (l, B, Δ) is attached to each arc.

A pallet undergoes a sequence of machining operations before being delivered to the unloading station. The machining times and sequence of machining operations are known as operational requirements, as given in Table 6.1. The operational requirement data for each part type is then combined with the aggregate dynamic

Figure 6.2: Time expanded static network for dynamic network in Figure 6.1. Numbers in parenthesis are lower and upper bounds on the flows over the arcs. Only the flow paths completed within seven time slices are shown.

Figure 6.3: Aggregate dynamic network corresponding to the flows at logical level.

Part Type 1

	W.S.#1	W.S.#2
Operation 1	2.0	3.0
Operation 2	2.0	1.0

Part Type 2

	W.S. #1	W.S. #2
Operation 1	1.0	3.0
Operation 2	3.0	2.0

Table 6.1: Operational requirements on part type 1 and 2 in minutes.

network (Figure 6.3) to produce a set of detailed dynamic network flow (DDNF) diagrams. The number of DDNF in the set is equal to the number of part types selected for simultaneous production on the FMS. The DDNF corresponding to part type 1 is shown in Figure 6.4. The thick arc between two concentric circles shows a machining operation and a regular weight arc represents transportation. The dashed arcs simply represent the continuation of the next operation on the same workstation. The DDNF corresponding to each part type is then transformed into

Figure 6.4: DDNF based on aggregate dynamic network and operational requirements.

the equivalent time expanded static network model. The time expanded network flow layers for the production of the two part types mentioned previously are given in Figure 6.5. The number of time expanded networks will be equal to the number

of part types. Time expanded network flow diagrams can be visualized as lying in unconnected separate layers. Each layer of the network represents the flow of a part type through a series of machining, assembly and material handling operations over discrete time periods. These separate layers of network flows are interrelated through bundle constraints which represent the capacity constraints on workstations.

Each network has an underlying grid. The columns represent the discrete time periods ($t = 1, 2, 3, \dots, T$) and the rows represent the machining, transportation or assembly operations. The flexibility of the system permits the choice of more than one workstation for a particular operation. Therefore, the number of rows will be dependent on the flexibility of the system. The greater the flexibility in producing part types, the larger the number of rows.

6.1.1 Model Assumptions

1. Machining, assembly and material handling system flows can be approximated by discretizing time and by discretizing space. Discretization in space implies that the manufacturing path of a part type can be divided into a small number of discrete but connected sections. Time is discretized by assuming that the time to move from one point in space to another is an integer multiple of a basic time unit. Let θ represent the length of the basic time interval. Let t index a time slice where $t = 1, 2, 3, \dots, T$. For example if $\theta = 5$ minutes then $t = 10$ corresponds to an elapsed time of 50 minutes.
2. Each layer of the network represents the potential flow of part(s) for manufacturing a product type through the alternative sequences of machining, assembly and transportation operations.
3. Each node in the network is a loading, unloading, transfer or merge point for the machines, assembly or transportation devices. A node may represent one or any combination of all of these points.
4. Machining and transportation operations are represented by arcs from one node in the network to another node in the same network. Unless and until

Part Type 1, layer 1 **Part type 2, layer 2**

Figure 6.5: Multiperiod network flows for part type 1 and 2.

specified only one pallet can be loaded on a NC machine at a time. An assembly operation is represented by a processing node.

5. Parts are mounted on pallets and the number of parts loaded on a pallet will depend upon the design of the pallet. All the parts loaded on a pallet are of the same type. Pallet type is identified by the types of parts mounted on it.
6. Only one pallet can be loaded at a time on a NC Machine. The next pallet can be loaded only after the serviced (machined) pallet is moved to the buffer.
7. Parts are depalletized (unmounted from the pallets) before proceeding to assembly.

6.2 Model formulation

6.2.1 Definitions

1. Let there be M NC machines in the system simultaneously producing N part types.
2. Let θ be the length of time interval in minutes.
3. The transit time is defined as the number of time intervals required for a unit to travel along an arc from origin node to destination node. The transit time between nodes p and q is defined as Δ_{pq} time intervals or $\theta\Delta_{pq}$ minutes.
4. There are two types of arcs in multitime network problems (MNP). An arc which has its origin and destination nodes in the same time interval is called a *local arc*. An arc which has its origin node in time interval t and destination node in period $t + \Delta$ is a *linking arc*.
5. With each node is associated a requirement, positive for a supply node, negative for a demand node, and zero for a trans-shipment node. The demand for product i at time interval t is r_{it} .

6. An arc on the time expanded static network can represent one of the four types of operations: machining, assembly, inventory and transportation. The flow over the arc may be the flow of parts undergoing machining on a NC machine, or the flow of parts through an assembly robot or transportation flow or flow through an inventory space.
7. A quadruple (i, p, q, t) represents an arc in layer i originating at node p at time t and terminating at node q at time $t + \Delta_{pq}$.
8. X_{ipqt} represents the unknown amount of flow over the arc (i, p, q, t) .
9. c_{ipqt} is the cost per unit of flow over the arc (i, p, q, t) . The cost associated with an arc may be any one of the machining, inventory, transportation or assembly costs per unit of flow.
10. l_{ipqt} is the lower bound on the flow over arc (i, p, q, t) .
11. B_{ipqt} is the upper bound on the flow over arc (i, p, q, t) .
12. α_{ipqt} is the processing ratio figure attached to the terminal of processing node p where arc (i, p, q, t) exits the processing node p . Similarly α_{iqpt} is the processing ratio figure attached to the terminal of processing node p where arc (i, q, p, t) enters node p .
13. Let PN be the set of processing nodes and RN be the set of regular nodes in the time expanded layered network.
14. Let $t' = t - \Delta_{qp}$

Objective Function:

$$\text{Minimize } \sum_{i=1}^N \sum_{p,q,t} X_{ipqt} c_{ipqt} \quad (6.1)$$

Constraints:

$$\sum_q X_{iqpt'} - \sum_q X_{ipqt} = 0 \quad p \in PN; \text{ for all } i, p, t \quad (6.2)$$

$$\alpha_{ipqt} X_{iqpt'} - \alpha_{iqpt'} X_{ipqt} = 0 \quad p \in PN; \text{ for all } i, p, t \quad (6.3)$$

$$0 \leq l_{ipqt} \leq X_{ipqt} \leq B_{ipqt}$$

6.3 Model Components

In this section micro models for NC machine, assembly robot and transporter are described. Micro models approximate the flow characteristics of these equipments.

6.3.1 Micro Model for Numerically Controlled Machine

The machining of parts mounted on a pallet is characterized by the residence of the pallet on the machine for a fixed time period equal to the machining time. There may be more than one workpiece mounted on a pallet as NC machines are capable of machining several workpieces mounted on a single pallet. Machining operation k on a palletized workpiece(s) on workstation j is identified by the double index jk . The machining operation is represented by an arc with tail in time slice t and head in time slice $t + \Delta$. Here Δ is the transit time which is defined as the number of time slices needed to machine the workpiece(s) mounted on a pallet.

The accumulation or inventory of pallets in the buffer or local storage is represented by horizontal arcs pointing to and from the head and tail nodes of the arc representing the machining operation. The length of an inventory arc is only one time slice. The inventory arc pointing to the machining node represents the carry over of inventory flow from the previous time slice. Similarly the inventory arc pointing away from the machining node represents the flow of inventory to be carried to the next time slice. Figure 6.6 shows the micro model for a NC machine. Production of a part type requires a number of machining operations. So in each

Figure 6.6: Micro model for NC machine.

time interval there may be several arcs representing machining operations on workstation j . All the arcs representing machining operations on workstation j in all layers are bundled together into a **bundle constraint**, also known as a binding constraint. The bundle constraint ensures that the capacity of the workstation is not violated. Let $M(i, j, t)$ be the set of arcs representing machining operations on machine j during the time interval t in network layer i . The bundle constraint for the NC machine is:

$$\sum_{i=1}^N \sum_{M(i,j,t)} \frac{X_{ipqt}}{B_{ipqt}} \leq 1 \quad j = 1, 2, \dots, M$$

$$t = 1, 2, \dots, T$$

In some manufacturing plants a number of identical NC machines are grouped together. Let the number of identical machines in a group be R then the group can be represented as an aggregated workstation by a micro model similar to a NC machine with bundle constraint:

$$\sum_{i=1}^N \sum_{M(i,j,t)} \frac{X_{ipqt}}{B_{ipqt}} \leq R_j \quad j = 1, 2, \dots, \text{Number of machine groups}$$

$$t = 1, 2, \dots, T$$

Flows on accumulation arcs represent the flow of accumulated parts in the buffer. The inventory is represented before and after an operation. As for the NC Machine, bundle constraints are required to enforce the limited capacity of the buffer. Let the capacity of the buffer of workstation j be C_j per unit time interval. Let $I_{i,j,t}$ be the set of arcs representing the local inventory on workstation j during the time interval t in network layer i . The inventory bundle constraint is:

$$\sum_{i=1}^N \sum_{I(i,j,t)} X_{ipqt} \leq C_j, \quad j = 1, 2, \dots, M$$

$$t = 1, 2, \dots, T$$

An example to illustrate the formulation of side constraints

Figure 6.7 shows sections of two layers of networks. The arcs represent machining operations carried out on the same workstation on two part types. The arcs of

network layer 1 represent the same type of machining operation but beginning in different time intervals. The same applies for network layer 2. The bundle constraint corresponding to the time interval five is:

$$\frac{X_{11}}{B_{11}} + \frac{X_{12}}{B_{12}} + \frac{X_{13}}{B_{13}} + \frac{X_{21}}{B_{21}} + \frac{X_{22}}{B_{22}} \leq 1 \quad (6.1)$$

Figure 6.7: Sections from two network layers to illustrate the formulation of side constraints

6.3.2 Micro Model for Assembly Robot

The assembly of a product is characterized by the joining of parts in given fixed ratios. The assembly operation is carried out by an assembly robot. The parts to be assembled are stored in bins which act as buffers. The assembly robot picks up the parts from the bins for assembly. The assembly time of a product is the time elapsed between picking up the first part and placing the completed assembly in the output bin. The task of providing parts to the buffer of the assembly robot is performed by a material handling robot.

The micro model for the assembly robot is described in Figure 6.8. The major building block for the micro model is the processing node which enforces the merging of flows in given proportions. Like the micro model of the NC machine the horizontal arcs represent the flow of parts through local inventory or buffer. Other details of this model are similar to the NC machine.

Figure 6.8: Micro model for assembly robot

Let $A(i, j, t)$ represent the arcs incident on the processing node for assembly

robot j , falling in time slice t , producing product i . The bundle constraint is given by:

$$\sum_{i=1}^N \sum_{A(i,j,t)} \frac{X_{ipqt}}{B_{ipqt}} \leq 1, \quad j = 1, 2, \dots, \text{Number of assembly robots} \\ t = 1, 2, \dots, T \quad (6.5)$$

6.3.3 Micro Model for Transporter

The transportation operation between two workstations is represented by an arc with transit time value equal to the travel time between the two as shown in Figure 6.9.

Figure 6.9: Micro model for transporter.

The transportation arc represents the potential flow of pallets from one workstation to another. One important assumption made in this representation is that the transporter never travels empty between two workstations. The micro model described in this section is an approximate representation for transporter vehicles which can access the workstations in any sequence. Robot arms and automated guided vehicles are good examples in this category. Network flow micro models for continuous roller conveyors and accumulation roller conveyors have been described by Maxwell *et. al.* [1981].

Let $T(i, j, t)$ be the set of arcs representing transporter j in the i th layer during the time interval t . Let the flow capacity of the arc representing the transporter be

B_{ipqt} . The bundle constraint for the transporter is:

$$\sum_{i=1}^N \sum_{T(i,j,t)} \frac{X_{ipqt}}{B_{ipqt}} \leq 1 \quad \begin{matrix} j = 1, 2, \dots, \text{Number of transporters} \\ t = 1, 2, \dots, T \end{matrix}$$

6.4 Selecting Parameter Values

6.4.1 Length of Time Slice

The continuous flow of parts in the model is approximated by discretizing time into intervals of length θ . The ideal value of θ is the maximum common divisor of the processing times on workstations. The processing time of part type i on workstation j is t_{ij} . Let $\dot{A}$ represent the set of processing times t_{ij} . The time interval θ is the largest time which divides t_{ij} evenly for all ij in $\dot{A}$.

$$\theta = m.c.d[t_{ij}] \quad \forall ij \in \dot{A}$$

Smaller values of θ increase the precision of the dynamics of the model and larger values lead to less precision. Moreover, as the time interval gets smaller the number of nodes and arcs in a fixed modelling period gets larger. This causes the problem to increase in size.

6.4.2 Cost Coefficients

The cost for each unit of flow on an arc is chosen by the user. In the absence of real cost data, the cost coefficients on machining arcs can be taken proportional to the machining times. For the transportation and inventory arcs the cost coefficients may either be the actual costs or may be chosen to enforce other objectives described in this paragraph. The user can adjust cost coefficients of transportation and inventory arcs to arrive at a flow distribution of particular interest. A large positive cost coefficient on transporter arcs will lead to less utilization of transporters, hence less movement of pallets in the system. This may lead to situations where even if

the choice is available to perform a subsequent operation at less cost on another machine, the high transportation cost coefficient may force the solution to perform the expensive operation on the same machine. Negative cost coefficients are not recommended for the transporter arcs as this will lead to undesirable increased movement of parts between workstations.

The choice of cost coefficients for the inventory arcs may depend upon the policy of inventory management of the organization. Some organizations may want to minimize the work in process inventory while others may try to maintain a certain level for safety. A large positive cost coefficient on the inventory arc tends to minimize the units stored on the arc during the time slice. A large negative cost tends to maximize the flows into the inventory arcs. You can study the impact of inventory policies on workstation flows and utilization by changing the cost coefficients associated with inventory arcs. By heuristic selection of cost coefficients, the user can find a solution to meet the demands which tends to minimize inventory and machine costs. If the objective function is minimization of machining costs, it is recommended that the cost coefficients on the inventory arcs and transportation arcs should be much less than the smallest cost coefficient of machining operations. It is also possible to assign high priority to any particular workstation by assigning low costs (even negative) to the flows on that workstation.

6.4.3 Upper Bounds For Non-Inventory Arcs:

Let θ be the duration of a time interval in minutes.

$$\text{Number of time intervals, } \Delta = \frac{\text{Transit time}}{\theta} \quad (6.6)$$

$$\text{Transit Time} = \Delta\theta \text{ minutes} \quad (6.7)$$

Let the processing time for a particular operation on part type i at workstation j be t_{ij} . Let upper bound on the arc representing this operation be B_{ij} .

$$\begin{aligned} \text{maximum flow capacity of workstation, } B_{ij} &= \frac{1}{t_{ij}} \text{ units/minute} \\ &= \frac{\Delta\theta}{t_{ij}} \text{ units} \end{aligned} \quad (6.8)$$

Steps to compute the bounds:

The following steps have been formalized to compute the upper bounds on the arcs:

1. Choose the duration of a time interval, θ . Though the best suggested value is the maximum common divisor, the engineer can choose any other value depending upon the level of detail required.
2. Find the transit time, Δ , in terms of time intervals by the following equation, where 'round' is the function to round up the values to the nearest integer.

$$\Delta = \max \left[1, \text{round} \left(\frac{t_{ij}}{\theta} \right) \right] \quad (6.9)$$

The transit time in terms of time intervals is an integer value and its minimum value is one.

3. Compute transit time = $\Delta\theta$ minutes.
4. Compute bounds from Equation 6.8.

Let us consider an example where the set of manufacturing times on a workstation is [2, 4, 6] minutes for part types 1, 2 and 3 respectively. Let the upper bounds on the arcs representing the corresponding manufacturing operations be B_1 , B_2 , and B_3 respectively. Three cases are considered:

Case 1: $\theta < \text{Min}[t_{ij}]$. The value of time interval is chosen as one minute which is less than the minimum value in the set of manufacturing times. The transit time for the representative arc of part type 1, as computed by the above mentioned step 2, is 2 time intervals. The transit time as computed by the third step is 2 minutes. The upper bound on the arc representing this operation is 1 unit as computed in the fourth step. Similarly $B_2 = 1$ and $B_3 = 1$

Case 2: $\theta > \text{Max}[t_{ij}]$. Let $\theta = 60$ minutes. Following steps 2 to 4 we obtain $B_1 = 30$, $B_2 = 15$, $B_3 = 10$.

Case 3: $\theta = \text{m.c.d.}$ Let $\theta = 2$ minutes, which is equal to the maximum common divisor. The value of bounds are: $B_1 = B_2 = B_3 = 1$.

6.4.4 Upper bounds for inventory arcs:

Let the maximum accumulation capacity of the local inventory space be P_j pallets on the workstation j .

The upper bound on the flow over the arc representing the local inventory space on workstation $j = P_j$.

6.5 Example

The dynamic network flow modelling of a 2 products and 2 workstations flexible machining system is illustrated in this example. The problem is a simplified version where two NC machines produce two part types. The machines and loading/unloading stations are physically integrated through a robot arm or automated guided vehicle. Each part type needs two machining operations. Both machines are capable of performing both the operations on both part types. There are in all 8 strategies of manufacturing available. The operational requirements for both the part types are shown in Table 6.1. Both the NC machines have their individual local storage which can hold a maximum of two pallets each. For the purpose of simplicity it is assumed for this example that there is only one part mounted on a pallet. Palletization and depalletization operations are ignored. The dynamic network flow corresponding to the physical layout is depicted in Figure 6.3. The DDNF diagram corresponding to part type 1 is shown in Figure 6.4. The time expanded static network flow diagrams corresponding to part types 1 and 2 are in Figure 6.5

The duration of each time interval is assumed to be 1 minute which is also the maximum common divisor of operation times shown in Table 6.1. It is assumed that all the workstations are available to their full capacity in all time intervals. All lower bounds are assumed to be zero. Upper bounds on the arcs were computed by the Equation 6.8.

The network flow and bundle constraints corresponding to time $t = 1$ were formulated. The basic structure of the problem is the same in all time slices. This fact was exploited by time expanding the linear constraints corresponding to time $t = 1$

over 19 time slices using a program written in Turbo Pascal. The constraints defining the demand at various time slices and initial conditions of the system are added to the time expanded network constraints. There are in all 409 constraints and 686 variables. The average inventory cost is \$ 0.25/unit and the average transportation cost is \$ 0.40/unit. The machining costs are taken as \$1/minute of machining time. The objective is to minimize the total production costs subject to meeting the demands shown in Table 6.2. The linear program is then converted to the format required for the MINOS 5.0. using a second program written in Turbo Pascal. The problem was solved on a Honeywell CP-6 in 8:04.09 CPU Minutes. The objective

Product Type	Time Interval						
	9	12	13	15	16	19	21
1	1	1	0	1	0	1	0
2	0	0	1	0	1	0	1

Table 6.2: Demand for product types 1 and 2 in various time intervals. The demand for both product types for time intervals not shown is zero. Total demand for both the product types 1 and 2 is 4 and 3 respectively.

of the model is to gain insights into the following issues:

1. Is it possible to meet the demand pattern? If so, then:
2. What are the optimal flows of pallets on the machines over time?
3. What are the work in process inventory levels over time?
4. What are the flows on the transporter over time?
5. What is the utilization of equipment over time?

Time Slice	Flows on Machine 1				Flows on Machine 2				Flows on Transporter						Total flows		
	I1	I2	P1	P2	I1	I2	P1	P2	L-1	L-2	1-2	2-1	1-U	2-U	TR	MC1	MC2
1									1.0						1.0		
2				1.0					1.0						1.0	1.0	
3			1.0								1.0				1.0	1.0	
4			1.0			1.0			1.0						1.0	1.0	
5			1.0			1.0					1.0				1.0	1.0	
6			1.0			1.0	1.0		1.0						1.0	1.0	1.0
7	1.0			1.0				1.0						1.0	1.0	1.0	1.0
8		1.0						1.0		1.0					1.0	1.0	1.0
9		1.0	1.0			1.0	1.0		1.0						1.0	1.0	1.0
10		1.0	1.0	1.0		1.0	1.0						1.0		1.0	1.0	1.0
11		1.0		1.0		1.0	1.0							1.0	1.0	1.0	1.0
12		1.0		1.0		1.0	1.0		1.0						1.0	1.0	1.0
13		2.0												1.0	1.0	1.0	
14		1.0	1.0										1.0		1.0	1.0	
15			1.0	1.0							1.0				1.0	1.0	
16							1.0				1.0				1.0		1.0
17								1.0						1.0	1.0	1.0	1.0
18								1.0							0.0		1.0
19														1.0	1.0		

Duration of each time interval = 1 minute

Legends:

Ip = Level of inventory of part type p.

Pq = Amount of flow of part type q over the machining arc.

L-n : Amount of flow from loading station to machine n.

n-m: Amount of flow from machine n to m.

n-U: Amount of flow from machine n to unloading station.

TR: Transporter.

MC n: Machine n.

Table: 6.3 Results for dynamic network modelling of FMS example

The problem was formulated and tested for various demand patterns. The network flow algorithm initially looks for a feasible solution i.e. whether it is feasible to meet the demand pattern under the given set of constraints. If a feasible solution is found then the optimal production schedule is computed. The results for the example are shown in Table 6.3. Some of the values in Table 6.3 are in bold face, indicating the beginning of operation on a new part. The regular face values indicate the continuation of operation on the part from the previous time interval.

The numerical values in the table are the amount of flows through the workstations and inventory levels during a particular time period. For example cell (7.1) of Table 6.3 indicates the inventory level of part type 1 in the buffer of NC machine 1 is 1 unit during the 7th minute of system operation. Cell (3.3) indicates the beginning of the machining operation on part type 1 in the 3rd minute. Cell (4.3) indicates the continuation of the machining operation on the part from the previous time interval. The flows over the workstations can be used to find the workstation utilization over the time intervals. The ratio of the total flow through the workstation to the capacity of workstation (upper bound) will give utilization factor for that particular time interval. The average utilization of a workstation can be computed by averaging the utilization factors during the period of interest. The average utilizations of NC machines 1 and 2 are 73.7% and 52.6% over the 19 minutes of system operation. The transporter is busiest amongst all the workstations. Except in the 18th minute, its utilization is 100% in all other time intervals. The average utilization of the transporter is 94.7%.

The solution in Table 6.3 has all integer values. This is not a general characteristic of the model: results for other demand patterns for the same problem were non-integer. The integer results are presented due to their ease in understanding. Interpretation of non-integer results is discussed later.

The problem was reformulated and tested for various other demand patterns and it was observed that the feasibility of the problem depends on both the total demand and the demand pattern over a fixed time period.

6.6 Discussion

6.6.1 Applications

The dynamic network model for FMS has applications in following areas.

1. The model is most beneficial for manufacturing firms where the demand pattern undergoes a great deal of fluctuation. The model can be used to determine the feasibility of production to meet a particular demand pattern and the corresponding optimal manufacturing flows.
2. Time is discretized in the dynamic network model, so it can be used to determine operating schedules for optimal utilization of resources. By changing the weights on the arcs in the time expanded network the designer can obtain alternative solutions having alternative schedules. A large time slice can be used to develop schedules at the macro level (production quantities for a week, etc.) and a smaller time slice can be used to develop schedules at the micro level (loading, unloading of individual jobs).
3. Just In Time (JIT) is a philosophy of producing parts as they are needed. In JIT production is controlled by demand. The objective of JIT is to reduce work in process inventory. The dynamic network FMS model can be used as an tool for developing schedules to meet JIT objectives.

6.6.2 Algorithms for Efficient Solution

Due to the multiperiod structure of the problem and the flexibility of the manufacturing system being modelled, the formulation results in large number of constraints and variables. The large number of constraints and variables are typical of all multiperiod network flow problems. In the example presented in Section 6.5, there were 409 constraints and 686 variables respectively to model a two part types, two workstation problem. It is a very simple problem in comparison to real world problems. In real world situations there are 2 to 6 NC machines and/or the same range of assembly robots producing up to 10 parts simultaneously. The number of constraints

and variables in real world problems might be of the order of tens of thousands depending upon the number of time intervals considered.

Of all the constraints there is a good proportion of side constraints to deter the use of efficient algorithms exploiting the embedded network structure. The side constraints arise due to the proportionality requirements on the flows over the arcs of processing nodes and due to the bundle constraints to link the network layers corresponding to various part types.

An efficient solution of the problem can be obtained by formulating the problem as a linear program and then using the Forward Simplex Algorithm of Aronson, Morton and Thomson [1985] for solution. Solution time using this algorithm is linear in the number of time intervals T , whereas it is T^2 if a standard LP code is used. The example solved in section 5.5 could have been solved in approximately 25 seconds on the Honeywell CP-6 using the Forward Simplex Algorithm.

6.6.3 Interpretation of Non-Integer Results

In a pure network problem with integer bounds there exists an optimal solution with integer values. As mentioned above, the time expanded network formulation of an FMS is a network with side constraints, hence it is not a pure network problem. The optimal solution of the problem may have non-integers. For example, let the results obtained for the problem shown in Figure 6.7 be:

$$X_{11} = 0.0, X_{12} = 0.5, X_{13} = 0.5, X_{21} = 0.4, \text{ and } X_{22} = 0.0$$

and the upper bounds on the flows over the arc are

$$B_{11} = 2.0, B_{12} = 1.5, B_{13} = 3.0, B_{21} = 2.5, B_{22} = 3.0.$$

Substituting the values of flow variable and bounds in the left hand side of Equation 6.4, the efficiency of the workstation during the fifth time interval is obtained as 66%.

Figure 6.10: Network flow diagram for computing optimal levels of final products.

6.6.4 Nonlinear Objective Functions

In the model we have only considered the linear objective function. In real life situations the total cost of production is usually not linear hence the objective function will be nonlinear for the cost minimization problem. Standard linear programming codes cannot be used to solve the problem with a nonlinear objective function. The Forward Convex-Simplex Method, developed by Stanley [1984] for multiperiod network flows having a convex objective function, may be used in this case.

6.6.5 Optimizing Inventory Levels of Finished Products

Until now, in this model, only the determination of optimal levels of in-process inventory has been considered. Drawing from the ideas of Zangwill [1969] the model can be extended to include the optimization of inventory levels of final products. The optimal levels of final products can be modelled by providing additional arcs in the bottom row of the time expanded network as shown in Figure 6.10.

Each time interval has an arc in the forward direction and an arc in the backward direction. The forward arcs permit the flow of products to later time intervals, whereas the backwards arcs permit the flow of products backward. The forward flow of inventory satisfies the demand in later time periods. The backward flow of

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inventory is known as backlogging of inventory, for when demand is backlogged, the final products produced later in time to satisfy a previous demand. The flows on forward arcs will have inventory costs associated with them, whereas backward flows of inventory will have shortage costs associated with them.

Chapter 7

Conclusions

The models developed in this thesis are not intended to replace simulation, rather to supplement it during the design and operational planning stage. The optimal solutions must be further modelled in detail using simulation.

The first objective of the thesis was to develop a steady state model for determining optimal manufacturing paths and flows for an integrated FMS. The model developed in chapter five achieves this objective. The model relies on well known and robust linear optimization techniques, and will be of great practical use.

The second objective of the thesis as described in section 4.2 has been achieved with the development of a dynamic model in chapter six. The model determines optimal manufacturing paths, flows and inventory levels of part types in an integrated FMS over discrete time intervals. Solved through efficient code, the model will be of great practical use to aid in developing optimal schedules of part types for dynamic FMS in a dynamic environment.

It has been stated in Table 3.1 that the models developed in the thesis can be used to answer four major issues listed below:

1. Number of transporters required.
2. Capacity of local storage.
3. Routing of parts
4. Sequencing and timing of workflow in the system.

The number of transporters needed can be found using the second model by varying the upper bound on the transportation arcs. The minimum number of transporters can be found by determining the smallest upper bound at which the transporter is not a blocking work station.

The capacity of local storage can be found using the second model by changing the upper bounds of the inventory arcs to obtain the desired levels of inventory.

Both the models solve the routing problem for different time scales.

The last issue, sequencing and timing of workflow, can be solved by both the models. The deterministic scheduling algorithm discussed at the end of Chapter 5 generates schedules for parts using the optimal flows from the steady state model. The application of the dynamic model to aid in the scheduling of resources is discussed in Section 6.6.1.

7.1 Contributions

1. Until now simulation has been the only effective technique for dynamic analysis of flexible manufacturing systems but it has no ability to optimize, only to model alternative approaches. The dynamic network flow model presented in chapter six can be used to find the minimum cost inventory levels and manufacturing paths over discrete time by considering time varying system and environment parameters. The results obtained from the model can be used for scheduling of part types. It is the first use of dynamic networks to model an integrated FMS. By changing the cost coefficients on the arcs, the engineer can study alternative manufacturing paths. The model can be used for dynamic analysis of the FMS. By selecting an appropriate time interval the engineer can focus on the desired level of details.
2. The model developed in chapter five is the first optimization model to find optimal manufacturing paths in flexible assembly systems. It is also the first use of processing networks to model flexible assembly systems.
3. Both the models developed in this thesis are the first integrated models which

combine the three important subsystems of an FMS: flexible machining, flexible assembly and material handling systems. The integrated view of the system facilitates the determination of optimal manufacturing paths through all of the equipments in the entire integrated manufacturing system.

7.2 Directions for future research

The directions proposed for future research are natural extension of work presented in this thesis. A number of avenues have been listed which if pursued may lead to new and better models for FMS.

1. It will be profitable to create a solution procedure to find the optimal manufacturing paths in flexible assembly systems in real time. This procedure could be used by the parts-dispatching computer to find optimal routes in real time. In case of equipment breakdown, a new network can be formulated and solved to find new routes, permitting the manufacturing system to respond in real time.
2. Reliability is an important issue in flexible assembly robots. Though most of the assembly operations are correctly performed by the robots, a small number of assemblies may turn out to be bad. The reliability of the assembly robot depends upon the capability of the robot and the complexity of the assembly operation. One particular robot may be good at performing an assembly operation whereas another may not be. Reliability can play an important role in determining the optimal routing which may be worth investigating. The modeller may want to find the manufacturing paths which have a high degree of reliability and minimum total cost of production. After determining the reliability of operations, the cost coefficients on the arcs representing the operation may possibly be adjusted to include the reliability of the operation. Less reliable assembly operation may contribute to a high cost coefficient and vice versa.

3. Micromodels for NC machines and assembly robots are proposed in chapter 6. Micro models for continuous roller conveyors, accumulation conveyors and tow cart systems were proposed by Maxwell [1981] with research still underway on development of other micromodels. It will be worthwhile to explore the integration of other MHS micro models with the micro models of NC machine and assembly robot developed in this thesis.
4. In both the models the possibility of producing defective parts was ignored. In real life situations a proportion of defective parts or assemblies are beyond reclaim, others are reprocessed in the system. This introduces feedback in the system and hence in the network. A processing network having such feedback flows can experience a class of errors called "nonviability" [Chinneck, 1989]. Nonviability testing should be incorporated in the FMS assembly models having feedback.
5. The aggregate model for finding optimal part routing in flexible assembly system and in integrated FMS, and the dynamic model for integrated FMS need to be implemented and tested in real life situations. Real life testing of models will provide insight into the models, leading to possible improvements. Testing of the models will include the comparison of actual performance measures of the system to theoretical performance measures derived through the models.

Chapter 8

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