Validation of a computational model for predicting fatigue life

by

Nilesh Gaonkar

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Dr. John A. Goldak, Thesis Supervisor

Dr. Ronald Miller, Department Chair

Carleton University

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Abstract

A new computer model to predict fatigue life based on the evolution of damage in a structure is presented. The experimental data used to validate the model is provided by the SAE Fatigue Design and Failure Committee (FDE). They initiated the total life project with the objective to improve fatigue life prediction and they have been working on the experimental data for around 8 years. Damage in a fatigue test is caused by changes at the microstructural level such as microcracks and pores that usually cannot be observed. Damage scales the elasticity tensor($\mathcal{D}$). For an isotropic scalar damage field $0 \leq d(x, t) \leq 1.0$. If $d(x, t) = 0$ then there is no damage and the elasticity tensor $\mathcal{D}$ is unchanged. When damage at a point x increases to $d(x, t) = 1.0$, then the elasticity tensor $\mathcal{D}$ is the zero tensor and the point in the structure is considered to be cracked. The model assumes that damage evolution can be computed as a function of the dissipation of hysteresis loop for a sequence of Ramberg-Osgood equations for a sequence of fatigue load cycles. This model does not use the Paris-Erdogan equation for crack growth. Two model parameters are the coefficients of the Ramberg-Osgood equation. The third model parameter is the total dissipation rate for the damage variable to reach a value of 1.0. Computer

simulations of fatigue tests with block loading are demonstrated. A high resolution plane strain FEM analysis that resolves the strain field near a stress concentration is shown to be necessary to achieve accurate predictions of fatigue crack nucleation and fatigue crack growth. The predicted fatigue crack nucleation and crack growth rates are in close agreement with the SAE experimental data.

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Nomenclature

N	Number of cycles
n	Monotonic strain hardening exponent
n'	Cyclic strain hardening exponent
N_f	Cycles to failure
N_i	Fatigue crack initiation life
N_p	Fatigue crack propagation life
ε	Strain
ε_e	Elastic strain
ε_p	Plastic strain
ε'_f	Fatigue ductility coefficient
ε_{max}^a	Elastic plastic strain
σ'_f	Fatigue strength coefficient
E	Modulus of elasticity
K	Monotonic strength coefficient
K'	Cyclic strength coefficient
V	Poisson's ratio
σ_{max}^a	Actual max stress

ε_{max}^a	Elastic plastic strain
$\Delta\sigma_{peak}$	The pseudo linear elastic stress range
$\sigma_{peak,max}$	Maximum stress value
c	Fatigue ductility exponent
b	Fatigue strength exponent
R	Stress ratio (ratio of minimum stress to maximum stress)

Units

Pure SI units are used, i.e. s, m, Pa, etc unless explicitly specified. For example, MPa is often used which is not pure SI units.

Abbreviation

FEM	Finite Element Method
3D	3 Dimensional
2D	2 Dimensional
FEA	Finite Element Analysis
SAE	Society of Automotive Engineering
FDE	Fatigue Design and Evaluation Committee
IIW	International Institute of Welding
ASME	American Society of Mechanical Engineers
ESED	Equivalent Strain Energy Density method
RPC	Remote Parameter Control
NDT	Nondestructive testing
HCF	High Cycle Fatigue
LCF	Low Cycle Fatigue
SIF	Stress Intensity Factor ^{aa}

Chapter 1

Introduction

1.1 Background and challenges

In the 19th century, the word “fatigue” was first used to define the failure of a structure or structural element exposed to cyclic loading. Fatigue research was initially conducted by August Wohler, who examined train axle failures. He found no damage to structural loading well below its static resistance. Furthermore, it can create structural or structural element failure in the case of repeating a similar load over a long period of time. Fatigue was an unexplained phenomenon in the nineteenth century since fatigue damage could not be observed and failure happened without warning.

It became better known and understood in the 20th century that cyclic (repeated) structural loading starts the mechanism of fatigue crack initiation and propagation of cracks respectively. While this fatigue concept was recognized, a lot of research

has been carried out and huge research has been done in establishing fatigue assessment techniques, understanding the fatigue mechanism of structures and materials and designing fatigue - resistant details. However, further research is required to understand this phenomenon [4].

An interpretation of the mechanism of fatigue is a necessary requirement while considering various factors affecting fatigue life and selecting suitable methods of evaluation. A structure or structural element's fatigue life is computed for the phase of crack initiation and crack propagation. Cracks created by cyclic loading tend to happen on a structural element's surface where fatigue damage occurs in crystallographic slip planes in the form of microscopic cracks. This initial phase is known as the "Crack Initiation Phase". In addition, cracks propagate or grow in a direction perpendicular to the maximum principal tensile stress direction from localized cyclic plastic strain to macroscopic size, during the phase of crack propagation [4]. The crack initiation phase even involves microscopic crack growth; however, the human eye and available NDT instruments is still unable to detect it. Assessing the point between the initiation and propagation of crack phases is very difficult. Fatigue is usually a surface phenomenon in the crack initiation phase and based on material surface characteristics and often atmospheric conditions. Moreover, crack propagation relies on the material characteristics within which the crack propagates. In the 20th century, Forsyth [5] was the first person to identify these two phases of crack initiation and propagation, which is one of the greatest achievements in the field of fatigue of metals.

Due to the addition of sharp notches and detrimental material impacts, welding

has raised the risk of fatigue failure for different structures. All structures, particularly those associated in transport such as ships, railcars, automobiles, trucks and aircraft, are affected as they are subjected to cyclic loading. But other structures, like offshore structures, bridges, cranes, engines, pressure vessels, have also experienced fatigue failures at welds [6].

Thus, fatigue assessment in structural design is an important task. In the 1970s, first standards were explained which are [7] and [8] applied in different branches. International suggestions have also been developed, e.g. the International Institute of Welding's (IIW) first fatigue design recommendations in 1982 [9]. All these standards and recommendations were centred on the nominal stress approach. Local approaches such as structural hot-spot stress and the effective notch stress approach and crack propagation approach were also introduced into practical applications in the subsequent years [10] [11].

1.2 Research Objective

The analysis and design of components using stress-based fatigue life prediction works well for situations in which only elastic stresses and strains are present. However, most components may appear to have nominally cyclic elastic stresses, but notches, welds, or other stress concentrations present in the component may result in local cyclic plastic deformation. Under these conditions, another approach that uses the local strains as the governing fatigue parameter (the local strain-life method) was developed in the late 1950s and has been shown to be more effective in predicting

the fatigue life of a component.

The methodology proposed in this thesis is to utilize an approach that would consider FEM Gauss point stress in the direction of crack opening and estimate fatigue life by a local strain based approach. To solve 3D fine mesh FE model of even a simple weld joint, e.g. double fillet T-joint requires a significant amount of computational time. Hence, detailed stress field information, as required for fatigue life estimation, was obtained using a 2D fine mesh finite element analysis model which has the capability to capture micro-geometrical features i.e., weld toe radius and angle. To model and do such a detailed FE analysis in 3D for large size real welded structures such as the boom of an excavator arm requires a fast FEM solver. If a 3D coarse mesh model with larger size finite elements is used instead of a fine mesh model, stresses at the sharp weld corner become inaccurate due to the singularity and hence cannot be directly used for fatigue life estimation. Advanced FEA modelling tools are able to automatically generate 3D coarse mesh FEA models in very short time. A fast multi-grid solver is being developed for VrWeld that is expected to be able to solve the 3D fine mesh needed to predict the fatigue life of structures. Unfortunately, it was not available for this thesis project.

VrSuite [12], a simulation package developed by Goldak Technologies Inc., is used for finite element analyses. VrWeld, which does welding simulations, uses Goldak's double-ellipsoid model and performs the transient 3-D thermal, microstructure and residual stress analyses of the welding process. VrDesigner imports the residual stress field from the weld analysis and computes the additional stresses and fatigue crack propagation due to in-service loading.

The objective of this thesis is to assess the capability to computationally predict the fatigue lives of welded structures, incorporating stresses due to in-service loading. SAE T-joint test problem is simulated to establish the mesh sensitivity to stress concentrations and stress intensity factors by comparing computed FEM results to known analytic solutions. The prediction of local non-linear peak stresses and fatigue nucleation lives from the Vrsuite are compared to experimental results of the SAE T-joint machined specimen.

The scope of this thesis is restricted to validation of computational damage model using experimental fatigue test data of SAE FDE T-Joint Machined specimen [13].

1.3 Outline of the thesis

This thesis has been organized in the following manner.

Chapter 1 provides an introduction to the need for accurate fatigue life prediction of welded joints and states the objective and overview of the proposed methodology.

Chapter 2 presents a review of the literature pertinent to this thesis and provides an overview of the principles of both the fatigue analysis and the fracture mechanics.

Chapter 3 explains the problem statement of the proposed research in detail and the experimental setup which is used to validate the computational model.

Chapter 4 presents the proposed methodology.

Chapter 5 presents the experimental fatigue life test results, its comparison with the estimated fatigue life using numerical methods and details of the forward looking approach on ‘total fatigue life’ concept only based on damage mechanics.

Chapter 6 presents the conclusions, which serve as the basis of the recommendation for future works that can be explored to further improve the readiness level of the proposed technology.

Chapter 2

Background and Literature Review

2.1 Overview

Fatigue life prediction of structures is a challenging problem with a epistemological uncertainty that is not yet completely understood. The material damage begins in the crystalline structure and becomes noticeable in a later stage after cyclic plastic deformation. The increase in damage leads to micro-cracks generated on slip bands and merge together which results in growth of the main crack. The subject becomes complicated by several influencing factors. Many experiments have been done and are being conducted to understand the behaviour of several materials and the effect of these influencing factors.

Fatigue of welded structures is even more complicated. The heating and subsequent cooling process and also the fusion process with additional filler material affect the material which results in inhomogeneous and distinct materials. In addition,

common imperfections in welding consists of inclusions, pores, cavities, undercuts, etc. With widely differing geometry parameters, the shape of the weld specimen and unwelded root gaps generate high stress concentrations. Finally, the fatigue behavior is influenced by residual stresses and distortions because of the welding process [14].

As a result, fatigue failures occur often at the welds instead of in the base metal in welded structures, even though the latter contain notches such as openings or re-entrant corners. For this reason, analyzes of fatigue are important to all cyclic loaded welded structures such as ships, offshore structures, cranes, bridges, vehicles, railcars, etc. [14].

The significance of this topic is evident because other than independent researchers, different organizations around the world such as IIW, ASME, and SAE etc. worked across several joint research programs to develop standard guidelines for the design of welded joints and predicting the fatigue life of welded structures [15].

2.2 Local approaches for fatigue assessment

Strength evaluations are called ‘local approaches’ if they are based on parameters of local stress or strain. Local damage processes for the failure of a structure consider various phases which are nucleation of cyclic crack, propagation of cyclic crack and final fracture. The nucleation of a fatigue crack is discussed by the ‘notch stress approach’ or ‘notch strain approach’ depends on the stresses or strains at the notch root (or comparable stress concentration regions). On the contrary, crack propagation and final fracture are defined by the ‘crack propagation approach’ from an

existing incipient crack. Furthermore, the strength analysis as per the complete local approach consists of the notch stress or notch strain approach and the crack propagation approach.[10]

The IIW proposes various approaches for welded joints fatigue analysis [16]. Most commonly utilized methods are the Nominal Stress approach, the structural hot spot stress or strain approach, and the effective notch stress or strain approach, explained in detail by Radaaj [10].

In this Master Thesis only the Strain Life Method for medium and high cycle fatigue at $10^4 < N_f < 10^7$ will be considered.

2.3 Palmgren–Miner’s rule

The rule of Palmgren–Miner [17] may be the oldest rule of cumulative damage equations for failures caused by fatigue. Linear damage evolution is predicted by a damage rule, with the number of cycles and consequently a linear damage accumulation. This damage rule is still widely applied even today for design purposes because of its simplicity.

There are some important limitations in the rule of the Palmgren–Miner. Its main limitation was found to be a lack of consideration of the loading order and its effect; that is the effect of interaction between higher to lower stress levels or vice versa. Depending on the load history, the rule of the Palmgren – Miner may underestimate or overestimate the fatigue damage resulting in conservative or unsafe predictions leading to material waste or unexpected failures, respectively.

There have been several attempts to propose more reliable rules on fatigue damage. Manson and Halford [18], Fatemi and Yang [19] and Schijve [20] describe an comprehensive review of models of fatigue damage. Moreover, the new proposals are limited to very specific conditions (e.g. certain load sequences, materials) that prevent those rules from being adopted in current design practices. In order to demonstrate their feasibility and adoption in design codes, more research on damage accumulation models supported by actual material damage behavior is required.

2.4 Strain Life Method

A significant development of the strain life method was done throughout the 1960s. It was based on the assumption that the fatigue life is controlled by local stresses and strains around a stress concentration.

Local geometry changes in engineering elements and structures induce stress concentrations and result to localized plastic strains. Fatigue cracks are initiated in these areas of local plastic deformation [21]. The fatigue life of an element is determined in the strain-life method based on the material response in these local deformation areas. The fatigue life for crack nucleation is the number of cycles till crack initiation. It uses the history of the local cyclic strain as the grounds for calculating life [22].

This method was initially developed for non-welded components and was later extended to weld joints as the weld toe usually has local plasticity. This concept is focused on crack nucleation life whereas the nominal or the hot spot stress methods estimate the fatigue life to final fracture. The fatigue life expressed as the number

of load cycles to the technical crack nucleation life and the subsequent growth of a long-crack life up to final fracture. The technical crack nucleation life comprises the microstructural crack nucleation life and the short-crack growth life up to the technical crack size. In the un-notched specimens, most of the total life may be consumed in crack nucleation. In sharply notched specimens, on the other hand, the crack initiation life may be very short. In some cases initiated cracks can be arrested to some extent.

Langer [23] studied fatigue under variable amplitude loading and separated the fatigue life into the crack nucleation and crack growth phases and suggested a damage sum of 1.0 for both phases. He also wrote that the application of his hypothesis required determination of the fatigue curves analogous to the nominal stress-life curves. Neuber [24] investigated the notch effect on the monotonic and cyclic deformation of engineering materials. The idea that cyclic plastic strain amplitude was responsible for the fatigue damage was formulated by Coffin [25] and Manson [26]. Working independently on problems associated with fatigue, Coffin and Manson proposed an empirical relationship between the number of stress reversals to the fatigue crack nucleation and the plastic strain amplitude. This idea was promoted later by Topper [27] and Morrow [28] and, along with the development of Neuber's rule [29] and rainflow cycle counting method by Matsuishi and Endo [30], form the basis for the current notch strain-life fatigue analysis. As an alternative to Neuber's rule, Glinka [31] proposed equivalent strain energy density (ESED) approach for estimating elasto-plastic stresses and strains (localized yielding) at the notches and cracks. For an elasto-plastic notch strain analysis, cyclic stress-strain curves are used which

are determined from tests on small smooth specimens.

The actual stress σ_{max}^a and the elastic-plastic strain ε_{max}^a can be calculated with the pseudo linear elastic stress range $\Delta\sigma_{peak}$ and the maximum stress value $\sigma_{peak,max}$ at the weld toe according to strain life approach. The Ramberg-Osgood material cyclic stress-strain curve can be used to find the actual stress and on elastic strain with the Neuber [24] or the ESED rule [32].

$$\frac{(\sigma_{peak,max})^2}{E} = \sigma_{max}^a \varepsilon_{max}^a \text{ - The Neuber Rule} \quad (2.1)$$

$$\varepsilon_{max}^a = \frac{\sigma_{max}^a}{E} + \left(\frac{\sigma_{max}^a}{K'} \right)^{\frac{1}{n'}} \text{ - The Ramberg-Osgood material cyclic stress-strain curve} \quad (2.2)$$

The cyclic stress strain curve defines the material's behavior after several load cycles have been performed during service. Cyclic strength coefficient, K' , cyclic strain hardening exponent, n' , and elastic modulus, E are the material parameters that are used.

$$\frac{\Delta\varepsilon^a}{2} = \frac{\Delta\sigma^a}{2E} + \left(\frac{\Delta\sigma^a}{2K'} \right)^{\frac{1}{n'}} \text{ - The expanded material stress strain curve} \quad (2.3)$$

$$\frac{(\Delta\sigma_{peak})^2}{E} = \Delta\sigma_a \varepsilon_a \text{ - The Neuber Rule} \quad (2.4)$$

It can be shown that Neuber's rule is actually a particular case of the ESED

method, namely when the elastic SED is equal to the plastic work done in ESED method. The reason for the overestimation of the local strains using Neuber's rule is thus explained essentially and the physical meaning of ESED method in both monotonic and cyclic form is further defined [33].

$$\frac{(\sigma_{peak,max})^2}{2E} = \frac{(\sigma_{max}^a)^2}{2E} + \frac{\sigma_{max}^a}{n' + 1} \left(\frac{\sigma_{max}^a}{K'} \right)^{\frac{1}{n'}} - \text{The ESED rule} \quad (2.5)$$

$$\varepsilon_{max}^a = \frac{\sigma_{max}^a}{E} + \left(\frac{\sigma_{max}^a}{K'} \right)^{\frac{1}{n'}} - \text{Ramberg Osgood material stress strain curve} \quad (2.6)$$

Similarly the elasto-plastic strain range and associated stress range are calculated with the stress-strain curve expanded by the factor of 2.

$$\frac{(\Delta\sigma_{peak})^2}{2E} = \frac{(\Delta\sigma^a)^2}{2E} + \frac{2\Delta\sigma^a}{n' + 1} \left(\frac{\Delta\sigma^a}{2K'} \right)^{\frac{1}{n'}} - \text{The ESED rule} \quad (2.7)$$

$$\Delta\varepsilon^a = \frac{\Delta\sigma^a}{E} + 2 \left(\frac{\Delta\sigma^a}{2K'} \right)^{\frac{1}{n'}} - \text{The expanded material stress strain curve} \quad (2.8)$$

2.5 Ramberg-Osgood Equation

A strain-life approach is a comprehensive approach which can be applied for the treatment of both low cycle and high cycle fatigue. In the low cycle region, the anelastic strain component is dominant, whereas in the high cycle region the elastic

strain component is dominant. Therefore, at large strain better fatigue resistance depends more on ductility, while at smaller strains it depends more on the yield stress.

Following equations used to explain the cyclic stress versus strain curve. The stress range ($\Delta\sigma$) is calculated by the maximum and minimum stress in which the maximum stress is σ_{max} and the minimum stress in the hysteresis loop is σ_{min} .

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \quad (2.9)$$

Mean stress (σ_m) in the hysteresis loop is the average of maximum and minimum stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad (2.10)$$

The total strain range ($\Delta\varepsilon_t$) is defined as the sum of elastic ($\Delta\varepsilon_e$) and anelastic ($\Delta\varepsilon_p$) strain ranges,

$$\Delta\varepsilon_t = \Delta\varepsilon_e + \Delta\varepsilon_p \quad (2.11)$$

The stress amplitude (σ_a) and strain amplitude (ε_a) is half the stress range ($\Delta\sigma$) and strain range ($\Delta\varepsilon$) are defined as,

$$\varepsilon_a = \frac{\Delta\varepsilon}{2} \quad (2.12)$$

$$\sigma_a = \frac{\Delta\sigma}{2} \quad (2.13)$$

The stress ratio (R) also explains the test conditions. The stress ratio is the ratio of the maximum to minimum stress,

$$R = \frac{\sigma_{max}}{\sigma_{min}} \quad (2.14)$$

Strain hardening is the increase in stress over time during constant strain-controlled testing as shown in figure 2.5.1. Over time the magnitude of the maximum and minimum stress could increase due to strain hardening in a hysteresis loop. Stress could decrease over time during constant strain testing due to strain softening as shown in figure 2.5.1. The magnitude of the hysteresis loop's maximum and minimum stress is reduced over time.

Figure 2.5.1: Strain hardening and softening [1].

Linear regression analysis is used to interpolate the cyclic stress-strain and strain life data at each temperature, as shown in figure 2.5.2.

Figure 2.5.2: Cyclic stress-strain and strain-life curves

The cyclic stress-strain curve is made up of linear-elastic portion (ε_e) and the anelastic stress-strain (ε_p). Equation 2.15 gives the linear-elastic strain for a uni-axial stress state as a function of the elastic modulus (E).

$$\varepsilon_e = \frac{\sigma}{E} \quad (2.15)$$

Equation 2.16 gives the anelastic strain (ε_p) which is a function of the cyclic strain hardening coefficient (K') and the cyclic strain hardening exponent (n').

$$\varepsilon_p = \frac{\sigma}{K'}^{\frac{1}{n'}} \quad (2.16)$$

Therefore, the total strain (ε_t) is given by combining elastic and anelastic strains,

$$\varepsilon_t = \frac{\sigma}{E} + \frac{\sigma}{K'}^{\frac{1}{n'}} \quad (2.17)$$

This is the Ramberg-Osgood equation.

2.6 Total Strain-Life Equation

Similarly, the equation of strain - life consists of an elastic and anelastic term. The elastic strain-life term is known as Basquin's equation [34],

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_f}{E} (2N_f)^b \quad (2.18)$$

where σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, E is the elastic modulus.

The Coffin-Manson Equation gives anelastic strain-life [34],

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f (2N_f)^c \quad (2.19)$$

Where ε'_f is the strain ductility coefficient and c is the strain ductility exponent.

The combined total strain equation is known as the strain-life equation,

$$\frac{\Delta \varepsilon_t}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (2.20)$$

Where N_f or $2N_f$ is the number of cycles or reversals to failure respectively.

2.7 Stress intensity factor (SIF)

The fracture mechanics approach to the prediction of fatigue life is a method based on the analysis of propagation of fatigue crack. In the case of monotonic load, the combination of load / stress and geometry is defined by the stress intensity factor. For cyclic loading, the range of the stress intensity factor is used. The stress intensity factor range is the most important parameter that governs the growth rate of a fatigue crack. Irwin [35] introduced the stress intensity factor and he showed that the instant rapid fracture occurs is when the stress intensity factor reaches a certain critical magnitude that exceeds the material's "fracture toughness" for this crack.

Rakesh [36] gives details of the methodology developed to determine the peak stress and associated non-linear through-thickness stress distribution on the critical weld toe crack plane using only the geometry-dependent stress concentration factors along with the appropriate, unique reference stress calculated in an efficient manner.

The peak stress at the weld toe can then be used to estimate the initiation life of the fatigue crack. The non-linear through-thickness stress distribution and weight function method can be used to determine stress intensity factors and to analyze subsequent growth of fatigue crack. Precise peak stress estimation involves models of a fine mesh finite element (FE) with micro-geometric characteristics such as the angle of the weld toe and the radius of the weld toe.

In this thesis a fine mesh finite element model approach is considered for accurate estimation of peak stress.

2.8 Initial Crack Size for Fracture Mechanics Analysis

The definition of the initial crack dimensions for fracture mechanics analyses is a problem on its own. Zerbst give a detailed discussion of this subject [37]. Generally the detection limit of non-destructive testing provides the initial crack size in the context of damage tolerant design. But, this is not true with regard to the total lifetime of an analysis of the S - N curve where the initial size of the crack is normally well below this detection limit [38].

A comment about the current practice is due at this place. The “equivalent initial flaw size” (EIFS) approach is a widely used method for defining an initial crack size in the context of fatigue life and strength. A brief review is given by [39]. The initial size of the crack is recalculated from S-N data or adapted to it. Generally, the calculations are based on the linear elastic ΔK concept, that is the long crack $\frac{da}{dN}-K$

curve and only one crack propagation is assumed. If anything, crack closure and residual stress are considered in a very generalised manner. Radaj [10] and Schork [40] explained a detailed description of the depth and aspect ratios of certain fixed crack. Clearly, this initial crack size is not a physically meaningful quantity, but a model parameter. When transferability issues (e.g., between various component geometries) come into effect, it has its limitations.

As is often done in the design stage in industry, crack growth is assumed to start with a crack 0.5 mm deep and 3.0 mm long in this thesis.

Chapter 3

Design of Experiment and Experimental Results

T-joint Steel structures are extensively used in the construction, automobile, appliances and other infrastructure. Strength, hardness, and fatigue properties of the material are important to the designer of a structure. These properties give the designer important data required to predict how a part will perform in service. Hardness and strength properties can be predefined with hardness tests and tensile tests. Fatigue properties can be produced utilizing a few distinct techniques relying upon the design technique used. Prediction of fatigue life is one of the most important problems in the design stage to improve reliability and quality of structures.

The total life project was started in 2012 by the SAE (Society of Automotive Engineers) Fatigue Design and Evaluation Committee (FDE) with the goal to improve the prediction of fatigue performance in ground vehicle welded structures. Produc-

tion of such structures must be closure to reduce overdesign and/or undersign of a structure. Fatigue design is challenging since there isn't any exact science with the uncertain fatigue. Both the initiation and the crack growth life must be taken into account to predict the fatigue life of the structure. Fatigue life is determined by the FDE as the total number of loading cycles including crack initiation and crack growth before the component breaks. Defining a crack initiation is the most challenging. A36 Steel was selected for the experiments which has low strength and high ductility with significant local plasticity is generally utilized in the Ground Vehicle Industry. Constant amplitude and variable amplitude of loading applied with both tensile and compressive mean stresses because the choice of material and loading conditions is the most difficult combination in the ground vehicle industry for crack growth propagation.

3.1 Specimen Geometry

The simplified T-Joint geometry is shown in Figure 3.1.1. The T-Joint geometry is made with a 101.6mm x 101.6mm x 50.8mm base block, and a 101.6mm x 25.4mm x 203.2mm vertical plate [13].

The width and thickness of the vertical plate was chosen so a significant portion of the fatigue life would be in crack propagation. The machined T-Joint was cut from a solid block of A36 steel. The design intent of both the welded and machined specimen types was to have their geometry and base material match as close as possible. It can be observed that the machined fillet and radii resemble the welded

Figure 3.1.1: Simplified T-Joint specimen geometry [2].

joint in Figure 3.1.2. The final leg dimension of the fillet was 16.5 mm for the welded and 14.3 mm for the machined specimen.

Figure 3.1.2: Comparison of welded and machined specimen geometry [2].

For simplicity in this thesis the computational damage model is validated using experimental data of the SAE FDE T-Joint Machined specimen provided by C. Gales.

3.2 Test Rig: Loading Mechanism/Fixture

The test rig loading mechanism and fixturing that was used to induce cyclic loads on the specimens, consisted of a servo-hydraulic structural actuator with ± 50 kN

force capacity and a stroke of 152.4 mm. The actuator was built with a base end and rod end swivel as shown in figure 3.2.1a. The load cell measurement range of ± 50 kN was assembled in-line between the actuator rod and rod end swivel. The actuator was equipped with an LVDT which was used to set stroke limits to ± 63.5 mm, at which point the specimen was not able to apply the programmed load and would result in the end of the test. The test setup is pictured in figure 3.2.1.

The test fixture included of the top and bottom clamping systems shown in Figure 3.2.1a. The top clamp was made from three plates of steel that were designed to clamshell clamp over the top of the specimen. The front-fixture plate was drilled with through holes to allow four M24 bolts to pass through the front-fixture plate and the top part of the specimen then thread into the back-fixture plate. The bottom portion of the fixture was designed to clamp the specimen base into place making sure the specimen was perpendicular and square to the direction of loading. The specimen setup was performed with care such that minimal to no load was applied while clamping the specimen into the fixtures. The overall applied load line measured from the center axis of the actuator rod to the toe radius was 222.25mm. Figure 3.2.1b shows a close-up view of the fixtured specimen with the measured applied load line.

All tests were run by controlling the applied load to the specimen. Before each type of load history was run, testing was performed using a setup specimen for tuning the rig. A load was programmed to the setup specimen and gains were adjusted until the feedback loop matched the prescribed load. The programmed loads and feedback loops were verified routinely throughout the test program. Testing was controlled

(a)

(b)

Figure 3.2.1: (a) Test setup for T-Joint configuration specimen (b) Close-up of fixed specimen with applied load line [2].

with MTS FlexTest IIM controllers using MTS Basic Testware for constant amplitude loading, Multipurpose Testware for block loading, and RPC Pro software for iterating and controlling the variable amplitude loading. Displacement was measured for each load cycle.

3.3 Loading Histories

The testing was performed using three forms of loading time histories: constant amplitude loading, block cycle loading, and variable amplitude loading. All tests were run at a frequency of 5.1 Hz in load control until failure, at which time the crack grew through the specimen to a large enough size that the load level could not be maintained. The different forms of loadings were selected to generate a range of fatigue lives (10,000 to 1,000,000 cycles) using various load range and mean load level sequences. Example of the constant amplitude and block loading time histories are shown in figure 3.3.1a and figure 3.3.2a. Example of the specimen fracture surfaces generated by the different constant amplitude and block loadings are shown in Figure 3.3.1b and figure 3.3.2b. It can be observed that the fracture surface shapes vary due to the different loading range and mean.

Figure 3.3.1: (a) Constant amplitude load history data from 24kN R=0.3 loading
(b) Fracture surface of machined specimen under 24kN R=0.3 loading [2].

(a)

(b)

Figure 3.3.2: (a) Block cycle loading history data for 24kN R=0.1(5k cycles)/ R=0.5 (40k cycles) (b) Fracture surface of machined specimen under block loading [2].

3.4 Findings

Successful fatigue testing of thirteen machined T-Joint specimens was completed by SAE FDE. Constant amplitude tests were run at three load levels and three R ratios. The T-Joint was also tested for block loading at one load level. The experimental test results which are considered for validation of computational damage model is shown in Table 3.1 [13]. The recorded test cycles represent the total fatigue life where the specimen under test was unable to carry the load. Displacement reached allowed limits.

Table 3.1: Experimental test results of Machined T-Joint specimens [2].

Case	Max Load (kN)	R Ratio Dimensionless	Experimental Test Cycles
Case 1	24	0.1	58,481
	24	0.1	70,011
Case 2	24**	0.3	266,012
	24	0.3	218,671
	24	0.3	200,464
Case 3	24*	0.5	2,471,943
Case 4	18****	0.1	424,431
	18	0.1	411,745
Case 5	14*	0.1	3,495,011
Case 6	24	****Block Load: 0.1/0.5	326,135
	24	****Block Load: 0.1/0.5	301,938
Note *Run-out **Tested after R=0.5 run-out ***Tested after R=0.1 run-out ****5k 24kN R=0.1 Cycles followed by 40k 24kN R=0.5 Cycles			

Chapter 4

Proposed Methodology

4.1 Ramberg - Osgood Approach

The total strain at a point is the sum of the elastic and anelastic components:

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad (4.1)$$

The stress-strain behaviour of ductile solids can be described by the Ramberg-Osgood relationship [41], a non-linear elastic constitutive equation [42], which is given by

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K} \right)^{\frac{1}{n}} \quad (4.2)$$

where K and n are the monotonic strength coefficient and hardening exponent, respectively.

The Ramberg-Osgood relationship is useful at stress concentration to approxi-

mate the local elastic-anelastic strain. But it does not represent actual anelasticity because the stress and strain will return along the same path if the load is removed. The stabilized stress-strain response of the cyclic material can be approximated to the cyclic Ramberg-Osgood relationship, which takes the same form as equation 4.2. Each curve is a different Ramberg-Osgood equation with a different coordinate system. Engineering materials can cyclically harden or soften under cyclical loading until equilibrium or cyclical stability is achieved. Through various test procedures, the cyclic stress-strain curve can be obtained, all of which cycle the material at different strain amplitudes, generating several hysteresis loops. For the cyclic stress-strain response [43] as seen in Figure 4.1.1, a curve is fitted to the peaks of each hysteresis loop.

The cyclic Ramberg-Osgood equation is

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'} \right)^{\frac{1}{n'}} \quad (4.3)$$

where k and n' are the cyclic strength coefficient and hardening exponent, respectively.

If the cyclic strain amplitude is:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \quad (4.4)$$

Then, according to the Massing hypothesis, the stabilized hysteresis behavior of the material can be defined by doubling the cyclic stress-strain curve.

Figure 4.1.1: Monotonic and cyclic stress-strain curve

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'} \right)^{\frac{1}{n'}} \quad (4.5)$$

Because the Ramberg-Osgood equation demonstrates non-linear elasticity rather than plasticity, equation 4.5 will not generate the hysteresis loop of stress-strain for cyclic loading alone. Each load cycle has two Ramberg-Osgood equation that are assembled to approximate the hysteresis loop for cyclic dissipation. If experimental data necessary to define the hysteresis loop is not available, the cyclic hardening exponent, n' , and the cyclic strength coefficient, k' , can be approximated from the strain-life coefficients as [44].

$$n' = \frac{b}{c} \quad (4.6)$$

$$k' = \frac{\sigma'_f}{(\varepsilon'_f)^{n'}} \quad (4.7)$$

4.2 Strain-Life Approach

Basquin [45] observed that a linear relationship exists between the logarithm of the amplitude of stress and the number of reversals of load, $2N_f$, to failure for constant amplitude loading.

The Basquin relationship is given by,

$$\frac{\Delta\sigma_e}{2} = \sigma'_f (2N_f)^b \quad (4.8)$$

where σ'_f is the fatigue strength coefficient, b is the fatigue strength (Basquin) exponent.

Since the elastic strain amplitude is:

$$\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E} \quad (4.9)$$

The relationship between elastic strain amplitude and number of load reversals to failure is:

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_f}{E} (2N_f)^b \quad (4.10)$$

Likewise, Coffin and Manson [46] [47] noticed that there is a linear relationship between the logarithm of plastic strain amplitude and the number of load reversals to fatigue crack nucleation.

The Coffin-Manson equation is given by

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2N_f)^c \quad (4.11)$$

where ε'_f is the fatigue ductility coefficient and c is the fatigue ductility exponent.

The total strain amplitude, $\Delta\varepsilon$, is the linear superposition of the elastic and plastic components. This provides the equation of strain - life, which relates the total strain amplitude to the number of failure reversals.

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (4.12)$$

Also, $\Delta d = 1/N_f$ is the maximum increment in damage at any Gauss point in one load cycle.

Therefore the equation to compute damage is given by:

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2/\Delta d)^b + \varepsilon'_f (2/\Delta d)^c \quad (4.13)$$

Where σ'_f , ε'_f , b and c are the parameters set in the design file and E is the elastic modulus set in the material library of the design file.

This increment in Δd damage due to one load cycle is multiplied by n cycles to compute the increment in damage due to n additional load cycles.

If d_n is the damage at a Gauss point after n load cycles, then d_{n+m} is the damage at a Gauss point after $n+m$ load cycles.

$$d_{n+m} = d_n + m\Delta d$$

4.3 Geometry and Boundary Conditions

The simplified T-Joint geometry is already shown and explained in figure 3.1.1. The two-dimensional geometry was developed in Solidworks and the geometry was exported in .stl format to VrSuite. The effects of various weld toe radii on stress concentration factors are conducted in [48]. The smaller the radius of weld toe, the higher the concentration of stress. The SAE T-Joint specimen has a weld toe radius of 0.008m and hence its effect on stress concentration factor needs to be accurately predicted. The T-Joint was precisely modelled and a fine mesh (30 micron element size) near the weld toe was used.

For fatigue testing the T-Joint geometry was constrained at base block (101.6mm x 50.8mm) and vertical plate (25.4mm x 203.2mm) is loaded by applying in-plane nodal load of 24KN with R ratio of 0.1, 0.3 and 0.5 at distance of 222.25 mm from the toe radius as per SAE experimental setup conditions.

Figure 4.3.1: SAE T- joint geometry and constraints

4.4 Material Property

The welded and machined specimens were manufactured from the same heat of A36 steel. The steel selected for specimen fabrication is a common structural steel that closely matched grain structure, chemistry, and strength of the A36 material that was previously used in other weld test/analysis efforts [49]. A36 steel was the base

material under investigation. A36 steel is an AISI standard specification for carbon structural steel used in bridge and building construction and in general structural applications. Its welding properties are excellent. A36 is a steel with carbon content (less than 0.3 weight % carbon), allows easy machining, welding and forming of A36 steel, which makes it useful as a general purpose steel. The mechanical properties and the fatigue properties are listed in Table 4.1 and Table 4.2 respectively and a typical chemical composition of A36 Steel is listed in Table 4.3.

Table 4.1: A36 steel mechanical properties [3].

Mechanical Properties	Value(Units)
Young's Modulus (E)	190 GPa
Yield Strength (0.2% offset) (σ_0)	224.07967 Mpa

Table 4.2: A36 steel fatigue properties [3].

Fatigue Properties	Value(Units)
Cyclic Strength Coefficient (K')	1096 Mpa
Cyclic Strain Hardening Exponent (n')	0.249
Fatigue Strength Coefficient (σ'_f)	1013 Mpa
Fatigue Strength Exponent (b)	-0.132
Fatigue Ductility Coefficient (ϵ'_f)	0.271
Fatigue Ductility Exponent (c)	-0.451

Table 4.3: Typical chemical composition for A36 carbon structural steel (wt. %) [3].

C	Mn	P	S	Si	Ni	Cr	Mo	Cu	Fe
0.21	1.1	0.12	0.021	0.08	0.08	0.05	0.08	0.10	98.159

4.5 Mesh Generation

The meshes I, II and III consisting of structured quadrilateral mesh was created using VrSuite PQ mesher. Mesh independent study was carried out and mesh III with 130,526 four node quadrilateral and 131,138 nodes was finalized.

Figure 4.5.1: Mesh I: Coarse mesh for weld analysis

Table 4.4: Comparing nodes and elements of different 20 mesh

Mesh	Number of nodes	Number of elements
Mesh I	1112	1032
Mesh II	9558	9320
Mesh III	131138	130526

Figure 4.5.2: Mesh II: Fine Mesh for fatigue analysis

Figure 4.5.3 shows Mesh 3 the very fine mesh near crack nucleation area. Number of nodes were declared on the radial section of the geometry to capture 112,138 accurately the mechanics of crack nucleation.

Figure 4.5.3: Very Fine Mesh in Crack Prorogation Area

The transition of fine mesh and coarse mesh was purposefully made in the way shown in figure 4.5.3. Ideally, the smooth transition of mesh yields the appropriate predictions. In this study, this issue was dealt with using the merge with Cmat option in VrSuite which merges master slave and slave nodes.

The transition of coarse and fine mesh can be clearly seen in figure 4.5.4.

The complete mesh can be shown in the figure 4.5.5.

Figure 4.5.4: Very Fine to Coarse Mesh Transition

Figure 4.5.5: Very Fine Mesh of the 2D geometry - Complete view

Chapter 5

Results & Discussions

5.1 Fatigue analysis

A computational damage model which uses the Ramberg – Osgood equation was implemented in VrSuite. Several cases from SAE T-Joint machined specimen experiments were analysed to validate the computational finite element fatigue model.

The cases considered for validating the model were loaded through constant amplitude and block cycle load histories. The constant amplitude analysis was run at three load levels and three R ratios. The block loading was analysed at one load level.

The predicted fatigue cycles and the experimental test results for the cases can be found in table 5.1. When SAE experimental tests were performed, the machined specimens that experienced run out were re-tested at different fatigue load levels [13]. Whereas the computational fatigue model predicts the number of fatigue cycles to

failure for such cases.

The analysis was carried out using three different meshes and the case-specific mesh was finalised considering computational time as the cases with low loads require comparatively high computational time for a very fine mesh.

Table 5.1: Experimental test results and predicted fatigue cycles of Machined T-Joint specimens

Case	Max Load (kN)	R Ratio Dimensionless	Experimental Test Cycles	Mesh	Predicted Fatigue Cycles to Failure
Case 1	24	0.1	58,481	Mesh III	60,428
	24	0.1	70,011		
Case 2	24**	0.3	266,012	Mesh II	402,772
	24	0.3	218,671		
	24	0.3	200,464		
Case 3	24*	0.5	2,471,943	Mesh II	3,420,890
Case 4	18****	0.1	424,431	Mesh II	407,016
	18	0.1	411,745		
Case 5	14*	0.1	3,495,011	Mesh II	7,187,780
Case 6	24	****Block Load: 0.1/0.5	326,135	Mesh II	540,387
	24	****Block Load: 0.1/0.5	301,938		
Note: *Run-out **Tested after R=0.5 run-out ***Tested after R=0.1 run-out ****5k 24kN R=0.1 Cycles followed by 40k 24kN R=0.5 Cycles					

The predicted fatigue cycles to failure are in good agreement with the SAE ex-

perimental data.

The number of load cycles to failure predicted by the computational damage model agrees very well with most of the experimentally measured load cycles to failure. The error percentage is -5.94% for case 1 (24kN) and -2.64% for case 4 (18kN) both with R 0.1. The observed predicted cycles to failure of the damage model agree more closely with measured cycles to failure for R ratio of 0.1 for which the Ramberg-Osgood dissipation and the damage increment is larger than that for R ratios of 0.3 and 0.5. This is because the dissipation seen in a hysteresis plot is less for R of 0.5 and the number of predicted load cycles is greater.

The damage model accounts for weld toe stress concentration by using a very fine mesh and hence there is no need to compute the stress intensity factor or use ideas from fracture mechanics. The predicted number of load cycles tend to be on higher side for a finer mesh. The design parameters such as cyclic fatigue parameters are set once for all cases as shown in Table 4.2. The parameters are with respect to A36 steel and they are not fitted to the experimental measurements of the SAE FDE committee. Therefore, the predicted cycles are in fair agreement with the experimental cycles for certain cases such as case 2.

The damage model computes dissipation at each Gauss point from the two Ramberg-Osgood curves defined for each load cycle. The increment in damage for every load cycle at each Gauss point is a function of the dissipation for that load cycle at that Gauss point. Hence the damage model predicts crack nucleation and crack propagation life with good agreement to the experimental measurement of crack nucleation and crack growth. The Paris law for the rate of crack growth is not used. In

fact one could argue that the Paris law is predicted by the damage model. The crack nucleation point and crack propagation path are predicted and not specified. The model predicts the crack location and the crack growth purely based on computed stress distribution and evolution of damage in the structure. The model predicts approximately 80% of fatigue life in nucleation and the remaining life in crack growth and final failure. The estimate of 80% fatigue life in nucleation and 20% in crack propagation is in good agreement to the FDE experimental data.

Fatigue life strongly depends on surface roughness when the average asperity heights exceed the intrinsic defects in the material [50]. The damage model in this thesis assumes the specimen to be defect free with a smooth surface and hence predicted number of cycles to failure are on the higher side. In practice, the surface is never smooth and there exist surface roughness and abnormalities.

5.2 Constant amplitude

Case 1: 24kN $R = 0.1$

The fatigue analysis was performed using the implemented computational damage model and various plots and images were generated to study the performance of the damage model. The Nodal Damage Image 5.2.1 shows the position of crack which is predicted by the damage model. The damage model accurately predicts the crack location at the weld toe and also simulates the crack growth. The simulated displacement Image 5.2.2 represents the displacement gradient where the maximum displacement is at the overall point of applied load and it gradually decreases to

zero at the base block where the specimen is constrained. The simulated Min-Max displacement vs timestep plot depicts constant displacement until crack nucleation and then gradually increased displacement during the crack growth. The simulated Gauss point yy-stress distribution is visualized for different timestep to study the stress concentration profile during the crack nucleation and crack growth. It is observed that the yy-stress of a Gauss point reaches a very high peak before it fails. i.e. when the damage value of that Gauss point reaches 1.0.

(a)

(b)

Figure 5.2.1: Shows nodal damage that depicts crack growth through the thickness.
a) full view of specimen with damage b) Zoomed near the crack tip

(b)

Figure 5.2.2: a) Simulated displacement of specimen during loading cycle. b) Min-Max displacement plot

(a)

(b)

(c)

(d)

Figure 5.2.3: Gauss point stress near the crack tip at various numbers of load which shows high stress during crack growth. a) Stress distribution during crack nucleation, b), c) and d) shows stress during crack growth with max stress at crack during specified time step.

(a)

(b)

Figure 5.2.4: YY- Stress distribution at a line $y=69.64\text{mm}$ using Mesh III for first load cycle. Case 1: 24KN $R=0.1$. a) Max load b) Min load

In order to carry out the analysis the yy-stress distribution (figure 5.2.4) in the critical weld toe area was determined using the VrSuite's FEM model. The yy-stress field shown in figure 5.2.4 was obtained at a line $y=69.64\text{mm}$. This stress distribution shows yy-stress concentration at weld toe which is captured by the very fine mesh.

Figure 5.2.5: Case 1: Damage distribution when damage in the first element reached a value of 1.0 i.e. the crack nucleated

Figure 5.2.6: Fatigue crack length vs time step (TS 1 = 57663 cycles, TS 200 = 59114 cycles)

Gauss points are used to compute stress from the displacement gradient as well as strain and the damage at each Gauss point. The finite element equation is based on a displacement model and displacement within the volume element is calculated using FEM shape function. Displacements are calculated at the nodes. At all other points, displacements are interpolated from nodes using the FEM shape functions. The stress (and resulting strain) is found by taking the derivative of the displacement field w.r.t (x, y, z) using the FEM shape functions. Hence stress and strain are calculated from these derivatives and the nodal displacements. The stress happens to be most accurately computed at the Gauss points in the isoparametric FEM formulation. Nodal stresses and strains in a FEM analysis are actually an averaged (smoothed) value of the stresses/strains computed at Gauss points of the elements to which the node is connected. Figure 5.2.7a shows stress at a Gauss point near the crack tip. This stress is based on the Ramberg-Osgood model. The stress remains constant for the initial few timesteps until the damage value at that Gauss point increment from $D = 0$. Gauss point is completely damaged when the damage value becomes $D=1$ and at that point the stress level decreases because the elasticity tensor eventually approaches zero. The yy-stress Plot 5.2.8 for a Gauss point located in the crack nucleation region indicates that for the first few load cycles the stress remains constant and then the stress concentration of the weld toe radius decays as the crack starts to nucleate. For the predicted yy-stress Plot 5.2.7 of a Gauss point which is just in front of the crack tip, the max stress for first few load cycles is less than the stress concentration at the weld toe. This is because the Gauss point is away from the stress gradient and the stress intensity of the weld toe radius decays exponentially.

(a)

(b)

(c)

Figure 5.2.7: Gauss point located near the crack tip: a) YY Stress b) Global YY Strain and c) RambergOsgood hysteresis loop for Gauss point to reach damage = 1 at TS 60 crack length = 0.646mm and 59077 cycles.

Figure 5.2.8: YY-Stress for one Gauss point located near the crack nucleation region.

(a)

(b)

Figure 5.2.9: a) Gauss point stress and b) Gauss point strain(Global) for Time step=60 (Crack Length = 0.646mm, No of Cycles: 59114)

Figure 5.2.10: Comparing stress distribution through the thickness for Mesh I, Mesh II and III at a line $y=69.64\text{mm}$.

Figure 5.2.11: Comparing fatigue crack length vs time step for case 1, case 2 and case 3.

Figure 5.2.12: Comparing YY - Stress distribution in case 1, case 2 and case 3 for first load cycle for max load.

Figure 5.2.13: Comparing YY - Stress distribution in case 1, case 2 and case 3 for first load cycle for min load.

5.3 Variable amplitude - Block loading

The block loading consists of two distinct blocks controlled by the constant amplitude load. A predefined number (5000) of loading cycles is applied to the first block, followed by a second block that is applied for 40,000 cycles. The sequence is repeated until failure occurs.

The block loading demonstrates that fatigue damage evolves nonlinearly as a function of the amplitude of stress with the number of load cycles. These observations suggest a nonlinear damage accumulation rule with the effects of the load sequence.

The next step is to obtain the hysteresis loop and recognize the maximum and minimum stress and stress points for each loop in each block so that these data can be entered into a calculation of damage.

Figure 5.3.1: Fatigue crack growth vs no of cycles to failure

Figure 5.3.2: Total Displacement Magnitude for first few time steps of block loading

Figure 5.3.3: YY-Stress Distribution for Max load of block loading (24KN) in first few load cycles.

Figure 5.3.4: YY-Stress Distribution for Min load of block loading (24KN R0.1, R0.5 i.e 2.4KN and 12KN) for first few load cycles.

Figure 5.3.5: YY-Stress vs time steps for block loading (24KN) at one gauss point

Figure 5.3.6: Mathematica plot of the Ramberg - Osgood Min-Max stress - strain hysteresis loop for a Gauss point near the crack tip - Block loading 24KN, R 0.1 and R 0.5 (first two block loads)

Figure 5.3.7: Mathematica plot of the Ramberg - Osgood equation stress - strain hysteresis loop for a Gauss point near the crack tip to reach a damage level of 1.0 - Block loading 24KN, R 0.1 and R 0.5 (several block loading cycles)

Chapter 6

Conclusions and Future work

6.1 Conclusion

The SAE Experimental Fatigue tests were successfully run to provide data to help understand how well our damage mechanics for fatigue life predictions correlate to fatigue life data for machined specimens with the same geometry.

The present computational evaluation is carried out under cyclic loading (constant amplitude and variable amplitude – block-loading history) only for machined specimens. Based on the various case studies mentioned in this thesis, it can be concluded that in general good agreement has been found between the predicted fatigue lives and the fatigue lives measured by FDE committee for several different T-Joint cases subjected to different loading configurations. Hence, it can be further concluded that the stress analysis data obtained using the proposed methodology can be used for the estimation of fatigue lives of machined joints and structures.

During this work, the stress analysis and the damage mechanics model to predict fatigue life of structures has been developed and implemented in VrSuite. The method is based on a 2D plane strain fine finite element mesh model. The purpose of the method is to avoid very time and labour ntensive accurate modelling of global and local geometrical features of 3D welded structures but still provide sufficient stress information necessary for fatigue analyses. The modelling technique captures both the magnitude and the gradient of the stress near the weld toe which are necessary for capturing stress concentration and the peak stress at critical cross-sections, e.g. at the weld toe.

The calculations with the developed approach could be successfully repeated at different weld geometries with high agreements between experiment data and numerical predicted fatigue life values.

6.2 Fatigue life of welded structure

In future work, our damage mechanics model to predict fatigue life will be validated using SAE T-Joint Welded specimen experimental data by mapping 3D coarse mesh weld analysis result to a 2D plane strain fine mesh domain. Also, a meshing tool to generate localised fine mesh around crack tip is under development. This tool is expected to generate localised fine mesh near the crack tip as crack grows.

Figure 6.2.1: Predicted YY residual stress through thickness near weld toe using a coarse 3D mesh.

Efforts are also made to optimize the results of welding residual stress which is an important factor for accurate prediction of fatigue lives. J. Goldak writes [51]: *A computer model to predict the fatigue life of a welded structure is expected to*

include data from the simulation of the welding process. In particular, each weld joint introduces a fusion zone and Heat-Affected zone (HAZ) with microstructures that have their own thermal and mechanical properties. In addition, the weld introduces residual stress and distortion. Most important, welds introduce defects such as pores, cracks and notches such as at a weld toe. The initial state from manufacturing, including welding, should be the initial state for the in-service loads that are expected to limit fatigue life. One also expects that fatigue loads with negative R values, such as $R = -1.0$ are likely to rapidly relax residual stress while higher R values such as 0.5 are less likely to relax residual stress.

If the coarse-grained HAZ could have reduced yield stress, this could increase plastic strain that increases damage in the coarse-grained HAZ. Combined with the stress concentration from a weld toe radius that is often less than 2 mm, this could lead to fatigue crack nucleation near the weld toe. These are some of the reasons that it is important to develop the capability to predict the fatigue life of a welded structure in the early design stage.

However, computing the state in a welded structure requires a 3D transient analysis. This cannot be done with a FEM mesh that is fine enough to resolve damage evolution. Therefore the state in a welded structure with a coarser 3D mesh must first be mapped to due to in-service loads. Then the areas with a high risk of damage evolution must be identified. These areas must be analyzed with a finer 2D plane strain mesh that is fine enough to resolve the evolution of damage and hence fatigue crack nucleation and fatigue crack growth. The software to do this must be developed.

References

- [1] F. Campbell, *Elements of Metallurgy and Engineering Alloys*. ASM International, 2008. [Online]. Available: <https://books.google.ca/books?id=8zBGmQEACAAJ>
- [2] E. Norton, C. Gales, R. Blodlg, and M. Lister, “Testing of welded and machined a36 steel t-joint configuration specimens,” *SAE Technical Paper*, 2019.
- [3] Y. Higashida, J. Burk, and F. Lawrence, “Strain-controlled fatigue behavior of astm a36 and a514 grade f steels and 5083-0 aluminum weld materials,” *Weld Res Suppl*, vol. 5083, pp. 334s–344s, 1978.
- [4] J. Schijve, *Fatigue of structures and materials*. Springer Science & Business Media, 2001.
- [5] P. Forsyth, *The physical basis of metal fatigue*. American Elsevier Pub. Co., 1969. [Online]. Available: <https://books.google.ca/books?id=VD5kAAAAMAAJ>
- [6] W. Fricke, “Recent developments and future challenges in fatigue strength assessment of welded joints,” *Proceedings of the Institution of Mechanical Engi-*

- neers, Part C: Journal of Mechanical Engineering Science*, vol. 229, no. 7, pp. 1224–1239, 2015.
- [7] “Cranes; principles relating to steel structures; design of cranes on vehicles,” no. DIN15018, 1984.
 - [8] B. S. I. Staff, B. S. Institution, and BSI., *Steel, Concrete and Composite Bridges. Code of Practice for Fatigue*, ser. B.S. (Series). B S I Standards, 1980, no. pt. 10.
 - [9] IIW, “Design recommendations for cyclic loaded welded steel structures,” *Weld World*, pp. 153–165, 1982.
 - [10] D. Radaaj, C. M. Sonsino, and W. Fricke, *Fatigue assessment of welded joints by local approaches*. Woodhead publishing, 2006, no. 59.
 - [11] A. Hobbacher, *Recommendations for fatigue design of welded joints and components*. Springer, 2009.
 - [12] J. Goldak, “Vr Software Suite,” *Goldak Technologies Inc.* [Online]. Available: <http://www.goldaktec.com/>
 - [13] J. Mach and C. Gales, “Fde total life t-sample residual stress analytical predictions and measured results,” *SAE Technical Paper*, 2019.
 - [14] W. Fricke, “Fatigue analysis of welded joints: state of development,” *Marine structures*, vol. 16, no. 3, pp. 185–200, 2003.

- [15] R. K. Goyal, “A stress analysis method for fatigue life prediction of welded structures,” Master’s thesis, University of Waterloo, Canada, 2015.
- [16] J. Schubnell, “Fatigue life prediction of high frequency mechanical impact (hfmi) treated welded joints by numerical simulation and damage mechanics approaches,” Master’s thesis, Offenburg’s University, Germany, 2015.
- [17] M. Miner, “Cumulative damage in fatigue,” *Journal of Applied Mechanics*, vol. 2, no. 12, pp. A159–A164, 1945.
- [18] S. Manson and G. R. Halford, “Re-examination of cumulative fatigue damage analysis—an engineering perspective,” *Engineering Fracture Mechanics*, vol. 25, no. 5-6, pp. 539–571, 1986.
- [19] A. Fatemi and L. Yang, “Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials,” *International journal of fatigue*, vol. 20, no. 1, pp. 9–34, 1998.
- [20] J. Schijve, “Fatigue of structures and materials in the 20th century and the state of the art,” *International Journal of fatigue*, vol. 25, no. 8, pp. 679–702, 2003.
- [21] N. Dowling, *Mechanical Behavior of Materials*, 3rd ed. Virginia Polytechnic Institute & State University: Pearson, 2007.
- [22] D. Hartman, “Robust model for fatigue life estimation from monotonic properties data for steels,” Master’s thesis, University of Waterloo, 2013.
- [23] B. Langer, “Fatigue failure from stress cycles of varying amplitude,” *Journal of Applied Mechanics*, vol. 59, pp. A160–A162, 1937.

- [24] H. Neuber, *Kerbspannungslehre: Grundlagen für Genaue Spannungsrechnung*. Berlin Heidelberg: Springer-Verlag, 1937.
- [25] L. Coffin and J. Tavernelli, “Experimental support for generalized equation predicting low cycle fatigue,” *Transactions of the American Society of Mechanical Engineers (ASME), Journal of Basic Engineering*, vol. 84, pp. 533–537, 1962.
- [26] R. W. Smith, M. H. Hirschberg, and S. Manson, “Fatigue behavior of materials under strain cycling in low and intermediate life range,” NATIONAL AERONAUTICS AND SPACE ADMINISTRATION CLEVELAND OH LEWIS RESEARCH CENTER, Tech. Rep., 1963.
- [27] K. Smith, P. Watson, and T. Topper, “A stress-strain function for the fatigue of metals,” *Journal of Materials*, vol. 5, no. 4, pp. 767–778, 1970.
- [28] T. H. Topper, B. I. Sandor, and J. D. Morrow, “Cumulative fatigue damage under cyclic strain control,” *Journal of Materials*, vol. 4, no. 1, pp. 189–9, Jun. 1967.
- [29] H. Neuber, “Theory of stress concentration for shear-strained prismatical bodies with arbitrary nonlinear stress-strain law,” *Journal of applied mechanics*, vol. 28, no. 4, pp. 544–550, 1961.
- [30] M. Matsuichi and E. Tsutomu, “Fatigue of metals subjected to varying stress,” Fukuoka, Japan, Mar. 1968.

- [31] G. Glinka, “Energy density approach to calculation of inelastic strain-stress near notches and cracks,” *Engineering Fracture Mechanics*, vol. 22, no. 3, pp. 485–508, 1985.
- [32] K. Molski and G. Glinka, “A method of elastic-plastic stress and strain calculation at a notch root,” *Materials Science and Engineering*, vol. 50, no. 1, pp. 93–100, 1981.
- [33] D. Ye, S. Matsuoka, N. Suzuki, and Y. Maeda, “Further investigation of neuber’s rule and the equivalent strain energy density (esed) method,” *International journal of fatigue*, vol. 26, no. 5, pp. 447–455, 2004.
- [34] J. Bannantine, J. Comer, and J. Handrock, *Fundamentals of Metal Fatigue Analysis*. Prentice Hall, 1990. [Online]. Available: <https://books.google.ca/books?id=jdZRAAAAMAAJ>
- [35] G. R. Irwin, “Analysis of stresses and strains near the end of a crack traversing a plate,” *J. appl. Mech.*, 1957.
- [36] R. Goyal, M. El-Zein, and G. Glinka, “A robust stress analysis method for fatigue life prediction of welded structures,” *Welding in the World*, vol. 60, no. 2, pp. 299–314, 2016.
- [37] U. Zerbst, M. Madia, and M. Vormwald, “Fatigue strength and fracture mechanics,” *Procedia Structural Integrity*, vol. 5, pp. 745–752, 2017.
- [38] U. Zerbst, M. Madia, B. Schork, J. Hensel, P. Kucharczyk, D. Ngoula, D. Tchuindjang, J. Bernhard, and C. Beckmann, *Fatigue and Fracture of Weld-*

ments: The IBESS Approach for the Determination of the Fatigue Life and Strength of Weldments by Fracture Mechanics Analysis. Springer International Publishing, 2018.

- [39] W. S. Johnson, “The history, logic and uses of the equivalent initial flaw size approach to total fatigue life prediction,” *Procedia Engineering*, vol. 2, no. 1, pp. 47–58, 2010.
- [40] B. Schork, P. Kucharczyk, M. Madia, U. Zerbst, J. Hensel, J. Bernhard, D. Tchuindjang, M. Kaffenberger, and M. Oechsner, “The effect of the local and global weld geometry as well as material defects on crack initiation and fatigue strength,” *Engineering Fracture Mechanics*, vol. 198, pp. 103–122, 2018.
- [41] S. Suresh, *Fatigue of materials.* Cambridge university press, 1998.
- [42] R. Denzer, F. J. Barth, and P. Steinmann, “Studies in elastic fracture mechanics based on the material force method,” *International journal for numerical methods in engineering*, vol. 58, no. 12, pp. 1817–1835, 2003.
- [43] W. Richard, “Deformation and fracture mechanics of engineering materials,” 1976.
- [44] ETBX, “Strain - life fatigue analysis,” *tech. rep.*, 2005.
- [45] O. Basquin, “The exponential law of endurance tests,” *Proc Am Soc Test Mater*, vol. 10, pp. 625–630, 1910.

- [46] L. F. Coffin Jr, “A study of the effects of cyclic thermal stresses on a ductile metal,” *Transactions of the American Society of Mechanical Engineers, New York*, vol. 76, pp. 931–950, 1954.
- [47] S. S. Manson, “Behavior of materials under conditions of thermal stress,” 1954.
- [48] H. Pang, “Analysis of weld toe radius effects on fatigue weld toe cracks,” *International journal of pressure vessels and piping*, vol. 58, no. 2, pp. 171–177, 1994.
- [49] D. Lingenfelser and T. Cordes, “Fatigue case study fde weld challenge 6,” Jun. 2014.
- [50] S. Andrews and H. Sehitoglu, “A computer model for fatigue crack growth from rough surfaces,” *International Journal of fatigue*, vol. 22, no. 7, pp. 619–630, 2000.
- [51] J. Goldak, “Personal communication,” *Carleton University*, May 6, 2019.

Appendix A

Case 2 Load=24kN, R=0.3, Mesh III

(a)

(b)

(c)

Figure A.0.1: a) YY Stress b) Global YY Strain and c) RambergOsgood hysteresis loop for one gauss point to reach damage = 1 at TS 60 crack length = 0.346mm and 566,025 cycles.

(a)

(b)

Figure A.0.2: YY-Stress Distribution for first load cycle. Case 2: 24KN R=0.3 a) Max Load b) Min Load

Figure A.0.3: Fatigue crack length vs time step (TS 1 = 543895 cycles, TS 200 = 565985 cycles)

(a)

(b)

Figure A.0.4: a) Gauss Stress and b) Gauss Strain(Global) for Time step=60 (Crack length = 0.346mm and 566025 cycles)

Figure A.0.5: Nodal Damage that depicts crack growth through thickness Case 2:
24KN R=0.3

Appendix B

Case 3 Load=24kN, R=0.5, Mesh III

(a)

(b)

(c)

Figure B.0.1: a) YY Stress b) Global YY Strain and c) Ramberg-Osgood hysteresis loop for one Gauss point to reach damage = 1 at TS 60 crack length = 0.383mm and 8,556,150 cycles.

(a)

(b)

Figure B.0.2: YY-Stress Distribution for first load cycle. Case 3: 24KN R=0.5 a) Max Load b) Min Load

Figure B.0.3: Fatigue crack length vs time step (TS 1 = 8,219,600 cycles, TS 300 = 8,556,200 cycles)

(a)

(b)

Figure B.0.4: a) Gauss Stress and b) Gauss Strain(Global) for Time step=60 (Crack length = 0.383mm and 8,556,150 cycles)₉₅

Figure B.0.5: Nodal Damage that depicts crack growth through thickness Case 3:
24KN R=0.5

Appendix C

Case 4 Load=18kN, R=0.1, Mesh III

(a)

(b)

(c)

Figure C.0.1: a) YY Stress b) Global YY Strain and c) Ramberg-Osgood hysteresis loop for one Gauss point to reach damage = 1 at TS 60 crack length = 0.472mm and 757,220 cycles.

(a)

(b)

Figure C.0.2: YY-Stress Distribution for first load cycle. Case 4: 18KN R=0.1 a) Max Load b) Min Load

Figure C.0.3: Fatigue crack length vs time step (TS 1 = 737,345 cycles, TS 300 = 757,290 cycles)

(a)

(b)

Figure C.0.4: a) Gauss Stress and b) Gauss Strain(Global) for Time step=60 (Crack length = 0.472mm & 757,220 cycles)

Figure C.0.5: Nodal Damage that depicts crack growth through thickness Case 4:
18KN R=0.1

Appendix D

Case 5 Load=14kN, R=0.1, Mesh III

(a)

(b)

(c)

Figure D.0.1: a) YY Stress b) Global YY Strain and c) Ramberg-Osgood hysteresis loop for one Gauss point to reach damage = 1 at TS 60, crack length = 0.410mm and 5,800,300 cycles.

(a)

(b)

Figure D.0.2: YY-Stress Distribution for first load cycle. Case 5: 14KN R=0.1 a) Max Load b) Min Load

Figure D.0.3: Fatigue crack length vs time step (TS 1 = 5,583,200 cycles, TS 200 = 5,800,350 cycles)

(a)

(b)

Figure D.0.4: a) Gauss Stress and b) Gauss Strain(Global) for Time step=60 (Crack length = 0.410mm and 5800300 cycles) 109

Figure D.0.5: Nodal Damage that depicts crack growth through thickness Case 5:
14KN R=0.1