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Constraint-Based Fracture Mechanics Analysis for Surface Cracked Plates

by

XIAO YU, B. Eng.

a thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Master of Applied Science

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

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ABSTRACT

In this thesis, the T -stress weight functions along the crack front of semi-elliptical surface cracks in finite thickness plates under uniaxial loading conditions are derived from the reference T -stress solutions and the appropriate boundary conditions. The resulting weight functions are verified against finite element data. The weight functions are suitable for the calculation of the T -stress under complex loading conditions for any local points of surface cracks with a wide range of aspect ratio, $0 \leq a/c \leq 1$ and relative depth, $0 \leq a/t \leq 0.8$.

Three dimensional finite element analyses have been conducted to calculate the T -stresses for the semi-elliptical surface crack in finite thickness plates under biaxial loading conditions. The method of calculating the T -stress for semi-elliptical surface cracks under nonlinear biaxial loading conditions using weight function method is derived. The method is verified using the finite element data.

Using the derived T -stress solutions as constraint parameter, the constraint-based failure assessments for surface cracked plates under uniaxial and biaxial tension loads are presented. The results are compared with the experimental data of TWI (The Welding Institute). It is demonstrated that the failure assessment results agree well with the experimental data and the conservatism by conventional failure assessments can be reduced significantly.

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NOMENCLATURE

Γ	- integral contour
α	- material dependant component in constraint-based FAD
β	- normalized constraint parameter
Δa	- virtual crack extension
δ_{ij}	- Kronecker symbol
δU	- change in the total potential energy
$\varepsilon_{ij} \ \varepsilon_{ij}^L \ \gamma_{ij}$	- strain components
$\zeta \ \psi \ \xi$	- constants for the calculation of limit load
η	- the correction factor for the plate width effect
θ	- angle of polar coordinates
$\mu_k(s)$	- defines the direction normal to the crack front and in the plane of the crack at point s
λ	- biaxial ratio
ν	- poisson's ratio
σ_0	- characteristic stress or nominal stress
σ_f	- flow stress
$\sigma_{ij} \ \sigma_{ij}^L$	- stress components
$\sigma_i(x)$	- local stress distribution
σ_L	- limit load
σ_{ri}	- reference stress distribution
σ_Y	- yield stress
σ_u	- ultimate strengths
Φ	- parametric angle of ellipse
a	- crack length for an edge crack or crack depth for a semi-elliptical surface crack
A	- cracked area or deepest point of a semi-elliptical surface crack

$a_i b_i c_i d_i$	- parameters of least square curve fitting
$A_i B_i C_i D_i$	- parameters of least square curve fitting
A_c	- the increase in crack area generated
B	- surface point of surface crack
c	- half crack length of a semi-elliptical surface crack
C	- local point of a semi-elliptical surface crack
d	- reference distance
$D_i D_{ij}$	- weight function parameters for T -stress
E	- Young's modulus
f	- point force
f_ϕ	- an angular function
F	- boundary correction factor
F_0	- normalized stress intensity factor
G	- strain energy release rate
g_0	- function in the curve fitting for stress intensity factor
$G_0 H_0 q_0$	- constants in the curve fitting for stress intensity factor
h	- half plate height
H	- general elastic modulus
I	- domain integral
J	- J -integral
J_{IC}	- material toughness
K	- stress intensity factor
K_{IC}	- material toughness
K_{mat}^C	- fracture resistance
K_r	- stress intensity ratio
L_r	- stress ratio
$L_r^{\max}$	- cut of value in FAD
m	- material dependant component in constraint-based FAD
$m_i(x, a)$	- weight function for stress intensity factors

M_i	- weight function parameters for stress intensity factors
n_i	- normals
P_e	- effective stress
P_L	- limit load solution
P_m	- maximum load carrying stress
P_n	- maximum normalized axial stress
q_k	- smooth function in domain integral
Q	- hydro-static Q stress or shape factor of an ellipse
r	- radius from crack tip
S	- stress field
S_i	- components of the traction vector
t	- plate thickness
$t_i(x, a)$	- weight functions for T -stress
T	- T -stress
T_{ri}	- reference T -stress
u_i	- displacement vector components
U	- potential energy
u_i^L	- displacement field
$V V_i V(\phi)$	- normalized T -stress
$V_{(S)}$	- a domain which encloses the crack front segment
w	- strain energy density
W	- plate width
$x y z$	- axis or nodal coordinates
x', y', z'	- nodal coordinates
Y_0	- boundary correction factor at the deepest point
Z_0	- the boundary correction factor at the surface point

Chapter 1

Introduction

Fracture is a problem faced in all engineering structures. With the development of higher strength materials, it becomes more and more important. Conventional theories of fracture assume that the state of stress and strain in the vicinity of a crack tip, and also the onset of failure, is characterized by a single parameter (stress intensity factor K or J -integral). In conventional linear elastic fracture mechanics (Anderson, 1991), the stress intensity factor, K , is used to determine the behavior of a crack in a brittle material. Under fatigue loading, the cyclic crack growth rate is correlated to the range of K . In conventional elastic-plastic fracture mechanics, either the parameter J (Rice, 1968; Hutchinson, 1968; Rice and Rosengren, 1968) or crack tip opening displacement (CTOD, Shih, 1981) is used to determine the stability limit of a crack, and also used to determine the cyclic crack growth rate.

It is common knowledge that for a given cracked specimen, the fracture resistance (toughness), which is the critical value of the crack driving force (K or J), is dependent on the constraint (stress triaxiality) at the crack tip; and all conventional one-parameter fracture mechanics is based on a lower bound toughness obtained from highly constrained test specimens. This will ensure the fracture assessment is always in the safe side. However, over-conservative assessments may sometimes occur in situations where the crack in a component is under conditions of low constraint. Therefore, the constraint-based (two-parameter) fracture mechanics has been developed to describe the stress and strain fields ahead of a crack tip under various

loading conditions corresponding to different constraint levels (Betegon and Hancock, 1991). Based on the constraint-based fracture mechanics, a more realistic fracture assessment can be conducted.

Analytical and experimental studies have shown that the elastic T -stress, or the second term of Williams' series expansion for linear elastic crack tip fields (Williams, 1957), which represents the stress acting parallel to the crack plane, can be used as the parameter to quantify the constraint effect. Larsson and Carlsson (1973) and Rice (1974) showed that the sign and magnitude of the T -stress substantially change the size and the shape of the plane strain crack tip plastic zone at finite load levels. Bilby et al. (1986) showed that the T -stress can strongly affect the magnitude of hydrostatic triaxiality in the near crack tip elastic-plastic fields. The important feature emerging from these works is that the sign and magnitude of T -stress can substantially alter the level of crack tip stress triaxiality, hence influence crack tip constraint. Positive T -stress strengthens the level of crack tip stress triaxiality and leads to high crack tip constraint; while negative T -stress reduces the level of crack tip stress triaxiality and leads to the loss of the crack tip constraint. The work by Betegon and Hancock (1991), Du and Hancock (1991), O'Dowd and Shih (1991), and Wang (1993) indicated that the T -stress, in addition to the J -integral, provides an effective two-parameter characterization of plane strain elastic-plastic crack tip fields in a variety of crack configurations and loading conditions.

Recently, the application of two-parameter fracture mechanics to include the constraint effect in the failure assessment procedure is becoming more and more established (Ainsworth et al. 2000). In order to apply the two-parameter fracture mechanics methodology, it is important to provide T -stress solutions for the crack configuration under consideration.

In practice, surface cracks are among the most common flaws in engineering structural components such as welded joints, pressure vessels and pipes. A semi-elliptical surface crack is a good idealization for these kinds of cracks. Accurate T -stresses for such cracks are necessary for the constraint-based two-parameter failure assessment. The main objective of the current thesis is to develop T -stress solutions

for surface cracked plates.

Similar to the stress intensity factors, the values of the T -stress for semi-elliptical surface cracks strongly depend on the geometry and the loading type. The geometrical parameters for semi-elliptical surface cracks in finite thickness plates which affect the T -stress are the parametric angle, ϕ , the aspect ratio, a/c , and the relative crack depth, a/t , where a is the crack depth, c is the crack half length, and t is the thickness as shown in Figure 1.1.

Due to the complexity of the problem, exact solutions are not available, and approximate methods must be used. Although there are many approximate methods which can be used to solve this problem, the finite element method is the most popular and effective method.

For a semi-elliptical surface crack in a finite thickness plate, Wang (2003) calculated the T -stresses for aspect ratios, a/c , of 0.2, 0.4, 0.6, 1.0 and relative depths, a/t , of 0.2, 0.4, 0.6 and 0.8 for remote uniaxial tension and bending, Wang and Bell (2003) calculated the T -stresses for semi-elliptical surface cracks in finite thickness plates with aspect ratios, a/c , of 0.2, 0.4, 0.6, 1.0 and relative depths, a/t , of 0.2, 0.4, 0.6 and 0.8 under crack surface pressure loadings (uniform, linear, parabolic and cubic stress distributions).

Note that the T -stress for surface cracks depend on the loading conditions, and the finite element method can only solve one loading condition at a time. The results from finite element calculations are therefore limited to a few simple loading conditions. Although superposition can be used when the loading condition can be decomposed into the sum of simple loading conditions whose T -stress solutions are available, the most efficient method to calculate T -stress for complex stress distributions is the weight function method.

The weight function method is a very powerful well-known tool for calculating stress intensity factors (SIF) under arbitrary stress distributions. In references (Sham, 1991; Fett, 1997; Hooton et al. 1998; Wang, 2002a, b), in analogy to the method used for the SIF, the weight function method for the calculation of T -stress was developed. It is a very powerful technique since it removes restrictions on the stress distributions.

Sham (1991) provided methods for the determinations of weight functions for T -stress numerically; while Fett (1997) and Hooton et al. (1998) demonstrated methods for the derivation of weight functions from reference T -stress solutions. Most recently, Wang (2002a) derived and verified the T -stress weight function for 2-D single edge cracked plate (SECP), double edge cracked plate (DECP) and centre cracked plate (CCP) test specimens.

However, the T -stress weight function for surface cracks is presently not available. One of the objectives of the present research is to generate general T -stress weight functions for a wide range of geometrical parameters for semi-elliptical surface cracks in flat plates. The derived weight functions for T -stress are suitable for the calculation of T -stress for semi-elliptical surface cracks in flat plates under non-uniform uniaxial loading conditions as shown in Figure 1.2.

In practice, many structures are subjected to biaxial stress fields such as those arising from pressure loads, thermal loads, explosions, collisions, or pressurized thermal shock (Bass et al. 1992). Therefore, accurate biaxial T -stresses are necessary for the two-parameter failure assessment of those components. However solutions for surface cracked plates under biaxial loading are not available. In this thesis, the finite element code ABAQUS is used to calculate the T -stress for semi-elliptical surface cracks in finite thickness plates with aspect ratios, a/c , of 0.2, 0.4, 0.6, 1.0 and relative depths, a/t , of 0.2, 0.4, 0.6 and 0.8 under remote biaxial tension and bending. Then the weight function method for T -stress calculations for surface cracked plate under biaxial loading conditions as shown in Figure 1.3 is presented and verified.

Using the T -stress solutions as constraint parameter, two-parameter failure assessments are conducted for surface cracks under uniaxial and biaxial loading conditions. The results are consistent with available experimental observations, and it is demonstrated that the conservatism of conventional failure assessments can be reduced significantly under low constraint conditions.

1.1 Outline of Thesis

The main objective of this thesis is to derive general solutions of T -stress using weight function method for semi-elliptical surface cracks in finite thickness plates for a wide range of a/c and a/t under complex uniaxial and biaxial loading conditions. The weight functions over the entire geometric region of $0 \leq a/c \leq 1.0$ and $0 \leq a/t \leq 0.8$ of semi-elliptical surface cracks are derived and verified using the T -stress solutions of semi-elliptical surface cracks under crack surface pressure loadings and remote tension and bending. Three dimensional finite element analyses have been conducted to calculate the T -stresses for semi-elliptical surface cracks in finite thickness plates under remote biaxial tension and bending. The weight function method is applied to the biaxial loading conditions, and it is verified against finite element data. Finally, using the T -stress results as constraint parameter, constraint-based failure assessments for surface cracks are presented.

This thesis is divided into seven chapters. In chapter 2, an overview of two-parameter fracture mechanics, the calculation of T -stress and the constraint-based failure assessment diagrams are presented. Chapter 3 presents the T -stress weight function solutions for surface cracks in finite thickness plates under uniaxial loading. In chapter 4, three dimensional finite element analyses are used to calculate the T -stresses for semi-elliptical surface cracks in finite thickness plates under remote biaxial tension and bending. Chapter 5 presents the T -stress solutions for surface cracks in finite thickness plates under biaxial loading conditions using weight function method. Chapter 6 presents the constraint-based failure assessment for surface cracks. Conclusions are given in chapter 7 along with some recommendations for future work.

Figure 1.1 Semi-elliptical surface crack

Figure 1.2 Surface cracked plate under uniaxial loading

Figure 1.3 Surface cracked plate under biaxial loading

Chapter 2

Background and Literature Review

In this chapter, an overview on the two-parameter fracture mechanics, the calculation of T -stress and the constraint-based failure assessment diagrams is presented.

A cracked body can be loaded in any one or combinations of the three displacement modes shown in Figure 2.1. Mode I loading (the opening mode), where the principal load is applied normal to the crack plane, tends to open the crack. Mode II (the sliding mode) corresponds to on-plane shear loading and tends to slide one crack face with respect to the other. Mode III (the tearing mode) refers to out-of-plane shear, also involves relative sliding of the crack faces. Most cracking problems of engineering interest involve primarily Mode I and are due to direct stresses. In this thesis, only Mode I will be discussed.

2.1 Two-Parameter Fracture Mechanics

2.1.1 Background

In an isotropic linear elastic body containing a three-dimensional crack subject to symmetric (mode I) loading, the leading terms in a series expansions of the stress

fields very near the crack front are (Williams, 1957):

$$\sigma_{xx} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + T \quad (2.1)$$

$$\sigma_{yy} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (2.2)$$

$$\sigma_{zz} = \frac{K}{\sqrt{2\pi r}} 2\nu \cos \frac{\theta}{2} + E\varepsilon_{33} + \nu T \quad (2.3)$$

$$\sigma_{xy} = \frac{K}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (2.4)$$

$$\sigma_{xz} = \sigma_{yz} = 0 \quad (2.5)$$

where the subscripts x , y and z suggests a local co-ordinate system formed by the plane normal to the crack front and the plane tangential to the crack front points; r and θ are the local polar co-ordinates (Figure 2.2); K is the mode I local stress intensity factor; E is the Young's modulus and ν is the Poisson's ratio. The T in the first equation is the elastic T -stress, representing the tensile/compressive stress acting parallel to the cracked plane. As r approaches 0, the first terms in Equation (2.1-2.4) dominate, while the other terms approach zero or are constants. Mathematically, the stresses are said to have a square root singularity.

The stress intensity factor K defines the amplitude of the crack tip singularity (Anderson, 1991). That is, stresses near the crack tip increase in proportion to K . Moreover, the stress intensity factor defines the crack tip conditions; if K is known, it is possible to solve for all components of stress, strain, and displacement as a function of r and θ at the vicinity of the crack tip. It also may be used in the determination of residual strength (for brittle materials) and fatigue crack growth rate.

In the elastic-plastic fracture mechanics, the J -integral proposed by Rice (1968) characterizes the crack tip stress field. Consider an arbitrary counter-clockwise path (I) around the tip of a crack, as illustrated in Figure 2.3. The J -integral is given by:

$$J = \int_I \left(w dy - S_i \frac{\partial u_i}{\partial x} ds \right) \quad (2.6)$$

where w is the strain energy density, S_i are components of the traction vector, u_i are the displacement vector components, and ds is a length increment along the contour Γ . Rice showed that the value of the J -integral is independent of the path of integration around the crack. Thus J is a path independent integral. He also demonstrated that this integral can be considered as an energy release rate for nonlinear elastic materials. It can be expressed as:

$$J = -\frac{dU}{dA} \quad (2.7)$$

where U is the potential energy stored in the member, and A is the cracked area.

If the material considered is linear elastic, the J -integral is equal to the strain energy release rate G . Under elastic-plastic condition, the J -integral is the single parameter to decide the onset of fracture and the crack growth rate.

This single parameter description of crack tip conditions turns out to be one of the most important concepts in fracture mechanics. However, it does have its limitations.

2.1.2 Two-Parameter Fracture Mechanics

The characterization of crack-tip stress and strain fields is fundamental to fracture mechanics. As discussed in the last section, in classical linear elastic fracture mechanics, the appropriate parameter is the stress intensity factor K , and in classical elastic-plastic fracture mechanics, the appropriate parameter is the J -integral (or CTOD).

Classical defect assessment schemes attempt to ensure structural integrity through the measurement of a single parameter, such as K for elastic and J or $CTOD$ for elastic-plastic fracture mechanics. Material data for the limiting fracture toughness (K_{IC} or J_{IC}) are typically obtained from deeply cracked bending specimens which develop a high level of crack tip constraint (level of crack tip stress triaxiality). This provides a lower bound estimate of toughness, and ensures a conservative approach

when applied to real structural defects. Although this approach is safe, it is often conservative, and may, therefore, lead to the imposition of prohibitive repair and inspection policies when applied to cracks which develop unconstrained crack tip conditions.

Recent analytical and experimental studies have shown that two-parameters J and Q suffice to characterize the full range of near-tip environments at the onset of fracture in elastic-plastic fracture mechanics (Dodds et al. 1993). The hydrostatic Q -stress is defined as follows:

$$Q \equiv \frac{\sigma_{\theta\theta} - (\sigma_{\theta\theta})_{HRR}}{\sigma_Y} \quad (2.8)$$

where $(\sigma_{\theta\theta})_{HRR}$ is the hoop stress from HRR stress field and σ_Y is material yield stress.

As explained by Dodds et. al. (1993), the J -integral sets the scale of deformation at the crack tip while the hydrostatic stress parameter Q scales the near-tip stress level relative to a high triaxiality reference stress state. Specimens fracture at critical J values which depend on Q -stress. Therefore, in order to reduce the conservatism in the classical fracture mechanics, two parameters J and Q should be used.

However, because Q -stress is an elastic-plastic parameter, expensive elastic-plastic stress analysis has to be used to calculate it. But, it was shown later that the Q -stress is related to the elastic T -stress which is an elastic parameter and can be approximated as follows (Ainsworth and O'Dowd, 1995):

$$Q = \frac{T}{\sigma_Y} \quad (2.9)$$

Therefore, either Q or T can be used as the constraint parameter. It is observed that a positive T -stress increases the level of crack tip stress triaxiality and leads to high crack tip constraint; while a negative T -stress reduces the level of crack tip stress triaxiality and leads to the loss of the crack tip constraint. Based on these observations, two parameter, (J, T) or (J, Q) , fracture mechanics has been developed to describe the stress and strain fields ahead of a crack tip under various loading conditions corresponding to different constraint levels (O'Dowd and Shih 1991; Betegon and Hancock, 1991). Based on this, more appropriate fracture assessments can be

conducted by removing the conservatism of the classical one parameter approach under low constraint conditions. Details of the constraint-based fracture assessment will be discussed in section 2.3.

In order to apply the two-parameter fracture mechanics methodology, it is important to provide T -stress solutions for the crack configuration under consideration.

2.2 Calculation of Elastic T -stress

In the present research, the main objective is the calculation of T -stress of semi-elliptical surface cracks in finite thickness plates under complex uniaxial and biaxial loadings.

The finite element method (FEM) can be used to calculate the T -stress. Special numerical techniques of modeling the crack tip singularity and extracting T -stress were developed by different researchers. But the T -stress depends on the geometrical configuration of a cracked body and the loading condition. The finite element method only can calculate one geometry and one loading condition at a time. Therefore, it is time-consuming and expensive to calculate the T -stresses using FEM.

The weight function method is a very powerful tool in the calculation of the stress intensity factors. Recently the weight function method has been used to calculate the T -stress for 2-D cracks by Wang (2002a). The T -stress can be calculated very quickly for general stress distributions using the weight function method.

In this section, the applications of the finite element method and the weight function approach to calculate T -stresses are discussed.

2.2.1 Finite Element Method

The finite element method (FEM) has emerged over the past two decades as one of the most powerful numerical tools for the solution of crack problems in fracture mechanics. Specific techniques are employed to treat such problems. In the finite element method, the structure of interest is subdivided into discrete shapes called elements. Element types include one-dimensional beams, two-dimensional plane stress or plane strain elements, and three-dimensional bricks. The elements are connected at node points where continuity of the displacement fields is enforced.

In general, the behavior of the displacement field in common finite elements is at most quadratic or cubic, so that the stress field is at most linear or quadratic (Zienkiewicz and Taylor, 1991). On the other hand, the stresses in the neighborhood of a crack tip in a linear elastic material are singular. Therefore, in order to model the square root singularity at the crack tip, special techniques were developed. Barsoum (1977) demonstrated that when three-dimensional prism elements with four mid-side nodes at the quarter points (a degenerate cube with one face collapsed) were used and the separate crack tip nodal points were constrained to have the same displacement the resulting stress field would have the $r^{-1/2}$ singularity.

After obtaining the stress and displacement fields in a cracked body using FEM, it is possible to determine the T -stress. Among the methods of extracting T -stress, the interaction integral method introduced by Cardew et al. (1984), Kfoury (1986) and Nakamura and Parks (1992) is the most effective one. The method is summarized here.

Two-Dimensional Crack Problems

For a plane strain problem, assume a load force of magnitude f applied at the crack tip in the direction parallel to the crack as illustrated in Figure 2.4. Using superscript ' L ' to designate the stress and displacement fields, the analytical solution for this problem gives (Timoshenko and Goodier, 1970):

$$\begin{aligned}
\sigma_{11}^L &= -\frac{f}{\pi r} \cos^3 \theta \\
\sigma_{22}^L &= -\frac{f}{\pi r} \cos \theta \sin^2 \theta \\
\sigma_{12}^L &= -\frac{f}{\pi r} \cos^2 \theta \sin \theta
\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}
u_1^L &= -\frac{1-\nu^2}{E} \frac{f}{\pi} \left\{ \ln\left(\frac{r}{d}\right) + \frac{\sin^2 \theta}{2(1-\nu)} \right\} \\
u_2^L &= -\frac{1+\nu^2}{2E} \frac{f}{\pi} \{(1-2\nu)\theta - \cos \theta \sin \theta\}
\end{aligned} \tag{2.11}$$

where the subscripts 1, 2 suggests a local Cartesian co-ordinate system; r and θ are the local polar coordinates; d is a reference distance; E and ν are the Young's modulus and Poisson's ratio of the material, respectively.

Employing the above solution as an auxiliary field, Cardew et al. (1984) and Kfour (1986) extracted the T -stress for 2D crack problems by introducing an interaction J -integral based on an Eshelby's theorem.

For 2D crack problems, the interaction integral can be written in the domain integral form:

$$I = \frac{1}{\Delta a} \int_S \left\{ \left(\sigma_{ij} \frac{\partial u_i^L}{\partial x_k} + \sigma_{ij}^L \frac{\partial u_i}{\partial x_k} \right) \frac{\partial q_k}{\partial x_j} - \sigma_{ij}^L \varepsilon_{ij} \frac{\partial q_k}{\partial x_k} \right\} dS \tag{2.12}$$

where Δa is the virtual crack advance, S is an area which encloses the crack tip q_k defines the virtual extension of the crack tip; σ_{ij} , ε_{ij} and u_i are the stress, strain and displacement components of the 2D crack problem; σ_{ij}^L , ε_{ij}^L and u_i^L are the corresponding components in the point load auxiliary solution given by equations (2.10) and (2.11).

The crack tip T -stress is related to I by (Kfour, 1986):

$$T = \frac{E}{1-\nu^2} \frac{I}{f} \quad (2.13)$$

The computation of the domain integral, equation (2.12), is readily compatible with finite element formulation (Nakamura and Parks, 1992; Delerenz, 1985). Once I is determined, the T -stress can be obtained from equation (2.13) directly.

Three-Dimensional Crack Problems

Nakamura and Parks (1992) extended the method presented in the above section to 3D crack problems and provided the domain integral formulations.

Consider a three-dimensional crack front with a continuously turning tangent as shown in Figure 2.5.a. Assume a line-load of magnitude $f_k = f\mu_k(s)$ to be applied along the crack front as illustrated in Figure 2.5.b. In the figure, $\mu_k(s)$ defines the direction normal to the crack front and in the plane of the crack at point s . The solution for this problem is the case of a plane strain semi-infinite crack with a point force f applied at the crack tip in the direction parallel to the crack. Using superscript ' L ' to designate the stress and displacement fields, the analytical solution are from (Timoshenko and Goodier, 1970), note that they are the same as the auxiliary field used for 2D, with terms involving the third direction added:

$$\begin{aligned} \sigma_{11}^L &= -\frac{f}{\pi r} \cos^3 \theta \\ \sigma_{22}^L &= -\frac{f}{\pi r} \cos \theta \sin^2 \theta \\ \sigma_{33}^L &= -\frac{f}{\pi r} \nu \cos \theta \\ \sigma_{12}^L &= -\frac{f}{\pi r} \cos^2 \theta \sin \theta \\ \sigma_{13}^L &= \sigma_{23}^L = 0 \end{aligned} \quad (2.14)$$

and

$$\begin{aligned}
u_1^L &= -\frac{1-\nu^2}{E} \frac{f}{\pi} \left\{ \ln\left(\frac{r}{d}\right) + \frac{\sin^2 \theta}{2(1-\nu)} \right\} \\
u_2^L &= -\frac{1+\nu^2}{2E} \frac{f}{\pi} \{ (1-2\nu)\theta - \cos \theta \sin \theta \} \\
u_3^L &= 0
\end{aligned} \tag{2.15}$$

where the subscripts 1, 2 and 3 suggests a local Cartesian co-ordinate system formed by the plane normal to the crack front and the plane tangential to the crack front points; r and θ are the local polar coordinates; d is a reference distance.

The interaction integral for 3D crack problems is introduced in the following domain integral form:

$$I(s) = \frac{1}{A_c} \int_{V(s)} \left\{ \left(\sigma_{ij} \frac{\partial u_i^L}{\partial x_k} + \sigma_{ij}^L \frac{\partial u_i}{\partial x_k} \right) \frac{\partial q_k}{\partial x_j} - \sigma_{ij}^L \varepsilon_{ij} \frac{\partial q_k}{\partial x_k} \right\} dV \tag{2.16}$$

where $V(s)$ is a domain with encloses the crack front segment between $s-\varepsilon$ and $s+\varepsilon$ (see Figure 2.5.c), $q_k(s)$ defines the virtual extension of the crack front segment $s-\varepsilon \leq s \leq s+\varepsilon$ and A_c is the increase in crack area generated by the virtual crack advance; σ_{ij} , ε_{ij} and u_i are the stress, strain and displacement components of the 3D crack problem under consideration; σ_{ij}^L and u_i^L are the corresponding stresses and displacement components in the line-load auxiliary solution given by equations (2.15) and (2.16) with ε_{ij}^L being the corresponding strain components.

The crack tip T -stress is related to $I(s)$, the interaction integral, by the following equation (Nakamura and Park, 1992):

$$T(s) = \frac{E}{1-\nu^2} \left\{ \frac{I(s)}{f} + \nu \varepsilon_{33}(s) \right\} \tag{2.17}$$

where $\varepsilon_{33}(s)$ is the extensional strain at point s in the direction tangential to the crack front.

The computation of the domain integral, equation (2.16), is readily compatible with finite element formulation (Nakamura and Parks, 1992). Once $I(s)$ is determined,

the T -stress at point S can be obtained from equation (2.17) directly.

2.2.2 Weight Function Method

In this thesis, the weight function approach is employed to determine the T -stresses under uniaxial and biaxial loading conditions.

Theoretical Background

The concept of weight function was first introduced by Bueckner (1970) and Rice (1972) for the calculation of the stress intensity factors. Bueckner first demonstrated that for a cracked body as shown in Figure 2.6(a), loaded by a stress field S , the stress intensity factor is the same as that for the same cracked body loaded by a crack surface pressure, $\sigma(x)$, induced by the remote stress field S in the uncracked body, as shown in Figure 2.6(b). Therefore, the calculation of the stress intensity factor for a given crack geometry under any applied load is equivalent to the calculation of the stress intensity factor for the same crack geometry with the corresponding crack pressure $\sigma(x)$ acting on the crack surface. The stress intensity factor for a cracked body with loading applied to the crack surface (Figure 2.6 (b)) can be calculated by integrating the product of the weight function, $m(x, a)$, and the stress distribution, $\sigma(x)$, in the crack plane:

$$K = \int_0^a \sigma(x) m(x, a) dx \quad (2.18)$$

Here the weight function $m(x, a)$ depends only on the geometry of the cracked body. Once the weight function has been determined, the stress intensity factor for this geometry can be obtained from equation (2.18) for any stress distribution $\sigma(x)$. The physical meaning of the weight function $m(x, a)$ at point x is the stress intensity factor at the crack tip caused by a pair of unit loads F applied at that point (Figure 2.6

(c)).

The determination of the weight function $m(x, a)$, can be simplified by using the following relationship derived by Rice (1972):

$$m(x, a) = \frac{H}{K_r} \frac{\partial u_r(x, a)}{\partial a} \quad (2.19)$$

where H is the generalized elastic modulus, which equals E for plane stress or $E/(1-\nu^2)$ for plane strain. In order to determine the weight function from equation (2.19), a reference stress intensity factor, K_r , for a specific geometry and applied stress system has to be known in addition to the corresponding crack opening displacement field, $u_r(x, a)$. Based on this theory, many weight functions (for the calculation of stress intensity factor) were developed for different geometries.

For the elastic T -stress, Wang (2002a) demonstrated that for a cracked body as shown in Figure 2.7(a), loaded by a stress field S , the T -stress is the superposition of T -stress for two cases. The first is the T -stress for the same cracked body loaded by a crack face pressure, $\sigma(x)$, induced by the remote stress field S in the uncracked body, as shown in Figure 2.7(b), the second case is the T -stress in the uncracked body under the remote stress field S (Figure 2.7(c)). Therefore, the T -stress for the problem shown in Figure 2.7(a) can be calculated from:

$$T = T_{crack\ pressure} + T_{uncrack} \quad (2.20)$$

The T -stress for a cracked body with loading applied to the crack surface can be calculated by integrating the product of the weight function $t(x, a)$ and the stress distribution, $\sigma(x)$, in the crack plane:

$$T_{crack\ pressure} = \int_0^a \sigma(x) t(x, a) dx \quad (2.21)$$

where $\sigma(x)$ is the stress distribution on the crack face and $t(x, a)$ is the weight function for T -stress (it is alternatively called second order weight function by Sham (1991), or

Green's function for T -stress by Fett (1997)). The weight function for T -stress depends on the crack geometry and is independent to the loading condition. Mathematically, the weight function, $t(x, a)$, is the Green's function for the T -stress. It represents the T -stress at the crack tip for a pair of unit point loads acting on the crack face at the location x as shown in Figure 2.7(d).

The T -stress for an uncracked body with applied load system S , as shown in Figure 2.7(c), can be found from the stresses at the crack tip location in the uncracked body (Wang, 2002a):

$$T_{uncrack} = (\sigma_x - \sigma_y)_{x=a, y=0} \quad (2.22)$$

Note that the $T_{uncrack}$ here represents the T -stress at the crack tip under load system S when the crack is completely closed (Figure 2.7(c)). It depends on the crack length a and the uncracked stress distribution as demonstrated in equation (2.22).

Substituting equations (2.21) and (2.22) into equation (2.20), the T -stress for cracked body as shown in Figure 2.7(a), loaded by a stress field S , is obtained:

$$T = \int_0^a \sigma(x) t(x, a) dx + (\sigma_x - \sigma_y)_{x=a, y=0} \quad (2.23)$$

Equation (2.23) provides a very efficient way for calculating T -stress. Once the weight function for T -stress $t(x, a)$ is determined, and a stress analysis of uncracked body is conducted to obtain $\sigma(x)$ and $(\sigma_x - \sigma_y)_{x=a}$, the corresponding T -stress can be calculated for any loading condition.

Approximate Weight Functions

Sham (1991) extended Bueckner's (1970) theory of fundamental fields to the determination of T -stress. It was demonstrated that $t(x, a)$ represents a combination of displacement from a fundamental field and displacement from a regular field (Sham, 1991), and so $t(x, a)$ is a function of displacement field $u(x, a)$.

$$t(x, a) = f(u(x, a)) \quad (2.24)$$

Using a unified finite element method, Sham (1991) obtained a numerical solution for $t(x, a)$.

Based on the analogy to the weight function for SIF's, Fett (1997) proposed a closed-form approximation of $t(x, a)$ for edge cracks:

$$t(x, a) = \frac{2}{\pi a} \left[D_1 \left(1 - \frac{x}{a} \right)^{1/2} + D_2 \left(1 - \frac{x}{a} \right)^{3/2} + D_3 \left(1 - \frac{x}{a} \right)^{5/2} + \dots + D_n \left(1 - \frac{x}{a} \right)^{n-1/2} \right] \quad (2.25)$$

where the crack tip is at $x=a$, $D_1, D_2, \dots$ and D_n are constants and a is the crack length for given crack geometry. Fett (1997) then used the following two-term expression to approximate the weight functions for T -stress for edge cracks:

$$t(x, a) = \frac{2}{\pi a} \left[D_1 \left(1 - \frac{x}{a} \right)^{1/2} + D_2 \left(1 - \frac{x}{a} \right)^{3/2} \right] \quad (2.26)$$

where D_1 and D_2 can be determined from two reference T -stress solutions.

Wang (2002a, b) demonstrated that equation (2.25) can be used to approximate weight functions for both edge cracks and internal cracks. T -stress weight functions for 2D SECP, DECP and CCP specimens were derived.

Note that up to now, the weight function method has only been applied to calculate T -stress for 2D crack problems. In this thesis, this approach is extended to calculate T -stress for a surface crack which is a 3D crack problem.

2.3 Constraint-Based Failure Assessment Diagram

2.3.1 Background

Methods for assessing the integrity of structures containing defects with a failure assessment diagram (FAD) were introduced in the so-called R6 procedure in 1976 (Harrison et al. 1976). These methods have subsequently been developed so that R6 is now in its third revision (R6, 1997), and failure assessment diagrams have been incorporated in a British Standards Published Document PD6493 (1991) and a Code Case to the ASME Boiler and Pressure Vessel Code (1993).

The loss of crack tip constraint leads to enhanced resistance to both cleavage and ductile tearing. However, all these conventional failure assessment schemes use lower bound toughness obtained from highly constrained test specimens. Over-conservative assessments may occur in situations where the crack in a component is under conditions of low constraint, and may lead to the imposition of prohibitive repair and inspection policies. Constraint effects on fracture have, therefore, received considerable attention recently in efforts to reduce conservatism. Based on the two-parameter fracture mechanics, as discussed in section 2.1, the framework for including the constraint effect in the failure assessment diagram approach were proposed (Ainsworth and O'Dowd, 1995). The recent version of R6 (1997), and the newly developed structural integrity assessment procedures for European industry (SINTAP, 2000), have suggested procedures for failure assessments that include the constraint effect.

2.3.2 Conventional Failure Assessment Diagrams

In this section, the conventional Failure Assessment Diagram procedure is outlined.

The conventional failure assessment diagram methods embedded in R6 (1997) or PD6493 (1991) are essentially single parameter procedures in that fracture is assumed to be governed by a single value of toughness or crack tip opening displacement. The assessment of the potential of failure is determined by the two calculated parameters, (K_r, L_r) determined from

$$K_r = \frac{K_I}{K_{IC}} \quad (2.27)$$

and

$$L_r = \frac{\sigma}{\sigma_L} \quad (2.28)$$

where K_r is the stress intensity ratio with K_I being the stress intensity factor for the crack and K_{IC} the toughness of the material. The parameter L_r is the stress ratio, which is the ratio of the applied stress to the limit stress solution calculated based on the yield stress. Failure is avoided if this point (K_r, L_r) lies within a failure assessment diagram, represented by a curve, $K_r = f(L_r)$ and at the same time L_r is less than a cut-off value $L_r^{\max}$ (Figure 2.8).

2.3.3 Constraint-Based Failure Assessment Diagrams

Allowance for constraint effects in the failure assessment diagram approach was first introduced into the R6 procedure and recently included in SINTAP procedures. Here, the procedure is outlined following R6 recommendations (1997).

Based on two parameter fracture mechanics, R6 recognizes that a fracture

resistance K_{mat}^C relevant to conditions of low constraint, may exceed the conventional fracture resistance K_{IC} measured under conditions of high constraint. To include this effect, the failure assessment diagram is modified to:

$$K_r = f(L_r) \left(\frac{K_{mat}^C}{K_{IC}} \right) \quad (2.29)$$

Following detailed theoretical and experimental analysis (Betegon and Hancock, 1991; O'Dowd and Shih, 1991; Ainsworth and O'Dowd, 1995), this increase in fracture toughness can be represented by:

$$\begin{aligned} K_{mat}^C &= K_{IC} [1 + \alpha(-\beta L_r)^m] & \text{for } \beta < 0 \\ K_{mat}^C &= K_{IC} & \text{for } \beta \geq 0 \end{aligned} \quad (2.30)$$

where α , and m are material dependent constants, which define the dependence of fracture toughness on constraint, and β is a normalized constraint parameter. Substituting (2.30) into (2.29), we have:

$$\begin{aligned} K_r &= f(L_r) [1 + \alpha(-\beta L_r)^m] & \text{for } \beta < 0 \\ K_r &= f(L_r) & \text{for } \beta \geq 0 \end{aligned} \quad (2.31)$$

The first equation of Eq. (2.31) is the constraint-based failure assessment diagram. Failure is avoided if the assessment point (K_r, L_r) (calculated from Eqs. (2.27) and (2.28)) lies within a failure assessment diagram, represented by a curve (equation (2.31)), and L_r is less than a cut-off value $L_r^{\max}$ (Figure 2.8).

The normalized constraint parameter β is defined from the hydrostatic Q -stress or the elastic T -stress:

$$\beta = \frac{Q}{L_r} = \frac{T}{\sigma_Y L_r} \quad (2.32)$$

where σ_Y is the yield stress of the material. Note that here the estimation of the hydrostatic Q stress in terms of T -stress (Equation 2.9) is used.

To conduct a constraint-based failure assessment, the constraint parameter T -stress or the Q -stress is required in addition to the knowledge of the stress intensity factor and the limit load for cracked components.

In the present research, a combination of finite element calculations and the

weight function approach was applied to calculate the T -stresses for semi-elliptical surface cracks in finite thickness plates for a wide range of crack configurations, $0 \leq a/c \leq 1.0$, $0 \leq a/t \leq 0.8$, under complex uniaxial and biaxial loading conditions. The weight functions for semi-elliptical surface cracks in flat plates under uniaxial loading conditions are derived in chapter 3. The finite element calculations for the T -stress of semi-elliptical surface crack under biaxial loading conditions are presented in chapter 4. The T -stress calculation using the weight function method for semi-elliptical surface cracks in flat plates under biaxial loading conditions is presented in chapter 5. Constraint-based failure assessments for semi-elliptical surface cracks in finite thickness plates are performed in chapter 6.

Figure 2.1 Three basic modes of crack surface displacement

Figure 2.2 Three dimensional coordinate system for the region at the crack tip

Figure 2.3 Arbitrary contour around the tip of a crack

Figure 2.4 A point force f applied at the crack tip, the auxiliary field for the interaction integral

Figure 2.5 A schematic of element used in the definition of the interaction integral. (a), (b), (c).

Figure 2.6 Weight function for one dimensional cracks

Figure 2.7 Weight function for T -stress

Figure 2.8 Example failure assessment diagrams

Chapter 3

Weight Functions for T -Stress of Semi-Elliptical Surface Cracks in Finite Thickness Plate under Uniaxial Loading

In this chapter, the T -stress weight function theory for 3D crack problems is presented. First, the general forms of T -stress weight functions for the deepest and local points of semi-elliptical surface cracks and the appropriate boundary conditions applicable to T -stress weight functions are discussed. Then, based on the finite element results (Wang and Bell, 2003), the T -stress weight functions for the deepest point and any local points ($5^\circ \leq \phi \leq 90^\circ$, Figure 3.1) of surface cracks are derived over a wide range of aspect ratio, $0 \leq a/c \leq 1$ and relative depth, $0 \leq a/t \leq 0.8$. The weight functions are verified using finite element data for other nonlinear stress distributions (Wang and Bell, 2003) and remote tension and bending cases (Wang, 2003).

3.1 *T*-Stress Weight Function Theory for 3D Cracks

3.1.1 Theoretical Background

The weight function method is widely used in the calculation of stress intensity factors. Recently, it has been extended by several researchers to the *T*-stress calculations.

As presented in chapter 2, Wang (2002a) showed that for a cracked body as shown in Figure 2.7(a), loaded by a stress field S , the *T*-stress can be calculated by the following equation:

$$T = \int_0^a \sigma(x)t(x,a)dx + (\sigma_x - \sigma_y)|_{x=a,y=0} \quad (2.23)$$

where $\sigma(x)$ is the stress distribution from a stress analysis of uncracked plate, $t(x, a)$ is the weight function for *T*-stress and σ_x, σ_y are the stresses for the uncracked body at the point $x=a$ and $y=0$ in x and y direction. The first term corresponds to $T_{crack\ pressure}$ and second term is $T_{uncrack}$.

Sham (1991), Fett (1997), Hooton et al. (1998) and Wang (2002a, b) developed the theory and derived weight functions for the *T*-stress $t(x, a)$ in 2D cracked geometries. No *T*-stress weight functions are available in the literature for surface cracks, which are 3D crack geometries.

Now, consider the surface crack in a flat plate as shown in Figure 3.2. The *T*-stress is changing along the crack front. For a general loading condition, which is varying depending on x , the *T*-stress at a point C along the crack front (Figure 3.1) can be calculated as follows (Wang and Bell, 2003):

$$T(C) = T(C)_{crack\ pressure} + T(C)_{uncrack} = \int_0^a \sigma(x)t(x,a;C)dx + (\sigma_r - \sigma_z)|_C \quad (3.1)$$

Here $\sigma(x)$ is the stress distribution from a stress analysis of uncracked plate, $t(x, a; C)$ is the weight function for T -stress at point C and σ_r, σ_z are the stresses for the uncracked body at the point C in radial and normal directions. Once the weight function $t(x, a; C)$ is determined, Equation (3.1) can be used to calculate T -stresses for any loading conditions. Equation (3.1) is the counterpart of Equation (2.23) for 3D crack problems.

3.1.2 Approximate T -stress Weight Functions for Surface Cracks

Now, let us discuss how to determine weight functions $t(x, a; C)$. For the sake of brevity, we use $t_C(x, a)$ to represent $t(x, a; C)$ from now on. In present research, Wang's (2002a, b) approach is extended to 3D cracks to derive the T -stress weight functions for semi-elliptical surface cracks in finite thickness plates. The weight function at the deepest point A is discussed first, then the weight function at general point C is presented.

Weight Function Forms for the Deepest Point of Surface Cracks

Wang (2002a, b) has demonstrated that the following equation can be used to approximate T -stress weight functions for a wide range of edge cracks and internal cracks for 2D crack problems:

$$t(x, a) = \frac{2}{\pi a} \left[D_1 \left(1 - \frac{x}{a} \right)^{1/2} + D_2 \left(1 - \frac{x}{a} \right)^{3/2} + D_3 \left(1 - \frac{x}{a} \right)^{5/2} + \dots + D_n \left(1 - \frac{x}{a} \right)^{n-1/2} \right] \quad (2.25)$$

In the thesis, this approach is extended to 3D cases. Note that here the stresses only vary in the x direction (Figure 3.2). The following equation is used to approximate the weight functions for T -stress for the deepest point of semi-elliptical surface cracks:

$$t_A(x, a) = \frac{2}{\pi a} \left[D_{A1} \left(1 - \frac{x}{a} \right)^{1/2} + D_{A2} \left(1 - \frac{x}{a} \right)^{3/2} + D_{A3} \left(1 - \frac{x}{a} \right)^{5/2} + \dots + D_{An} \left(1 - \frac{x}{a} \right)^{n-1/2} \right] \quad (3.2)$$

Both two-term and three-term weight functions of Equation (3.2) were tested. More accurate results can be obtained using the following three-term weight function:

$$t_A(x, a) = \frac{2}{\pi a} \left[D_{A1} \left(1 - \frac{x}{a} \right)^{1/2} + D_{A2} \left(1 - \frac{x}{a} \right)^{3/2} + D_{A3} \left(1 - \frac{x}{a} \right)^{5/2} \right] \quad (3.3)$$

where D_{A1} , D_{A2} and D_{A3} are constants and need to be determined for given crack geometry under consideration.

In order to use the weight functions to calculate the T -stresses, all the constants D_{A1} , D_{A2} , and D_{A3} must be determined. They can be obtained from equation (3.1) providing that three T -stress expressions T_{r1} , T_{r2} and T_{r3} , corresponding respectively to three different stress distributions σ_{r1} , σ_{r2} and σ_{r3} , are known. These stress distribution expressions and the weight function of (3.3) can be substituted for $\sigma(x)$ and $t(x, a)$ in equation (3.1). This leads to three equations sufficient for the determination of the three unknowns D_{A1} , D_{A2} , and D_{A3} .

Forms of the Local Weight Functions of the Surface Cracks and the Boundary Conditions

For cracks subjected to nonlinear stress distributions, fracture may occur at any point along crack front of surface cracks. Therefore the calculation of T -stress at any point along the crack front may be required. In this section, approximate closed form weight functions for any point, C , along the crack front of a semi-elliptical surface crack in a finite thickness plate, are derived.

The local weight functions of stress intensity factors for semi-elliptical surface cracks in finite thickness plates were derived by Wang and Lambert (1995). Based on the analogy of their work, the weight functions of T -stress for semi-elliptical surface cracks in finite thickness plates are derived here.

Wang and Lambert (1995) found that the local weight functions of the stress intensity factors for semi-elliptical surface cracks at point C were:

for $0 < x \leq a \sin(\phi)$,

$$m_C(x, a) = \frac{\sqrt{\sin \phi + 1}}{\sqrt{\pi(a \sin \phi - x)}} \left[1 + M_{1C} \left(1 - \frac{x}{a \sin \phi} \right) + M_{2C} \left(1 - \frac{x}{a \sin \phi} \right)^2 \right] \quad (3.4)$$

for $a \sin(\phi) \leq x < a$,

$$m_C(x, a) = \frac{\sqrt{1 - \sin \phi}}{\sqrt{\pi(x - a \sin \phi)}} \left[1 + M_{3C} \left(\frac{x}{a \sin \phi} - 1 \right)^{\frac{1}{2}} + M_{4C} \left(\frac{x}{a \sin \phi} - 1 \right) \right] \quad (3.5)$$

By analyzing the closed form weight functions of stress intensity factors in a finite thickness plate for a local point (Equation 3.4 and 3.5), C , in the range $0 < \phi < \pi/2$, along the crack front in a semi-elliptical surface crack as shown in Figure 3.2, the following forms are proposed to approximate the weight function of T -stress at any point C , $t_C(x, a)$:

for $0 < x \leq a \sin(\phi)$,

$$t_C(x, a) = \frac{2}{\pi a \sin(\phi)} \left[D_{C1} \left(1 - \frac{x}{a \sin(\phi)} \right)^{\frac{1}{2}} + D_{C2} \left(1 - \frac{x}{a \sin(\phi)} \right)^{\frac{3}{2}} \right] \quad (3.6)$$

for $a \sin(\phi) \leq x < a$,

$$t_C(x, a) = \frac{2}{\pi a \sin(\phi)} \left[D_{C3} \left(\frac{x}{a \sin(\phi)} - 1 \right)^{\frac{1}{2}} + D_{C4} \left(\frac{x}{a \sin(\phi)} - 1 \right)^{\frac{3}{2}} \right] \quad (3.7)$$

This functional form depends not only on x and a , but also on the position of the local point C , indicated by the angle ϕ . Both parts of the weight function, $t_C(x, a)$ in equations (3.6) and (3.7), must be used to perform the integration of equation (3.1) to calculate the T -stress of cracked body, i.e:

$$T(C) = \int_0^{a \sin(\phi)} \sigma(x) t_C(x, a) dx + \int_{a \sin(\phi)}^a \sigma(x) t_C(x, a) dx + (\sigma_r - \sigma_z)|_C \quad (3.8)$$

Without additional information, four reference T -stresses are required to determine the four parameters, D_{C1} , D_{C2} , D_{C3} and D_{C4} in Equations (3.6) and (3.7).

To simplify the derivation of weight functions for stress intensity factors, a set of

boundary conditions was used to decrease the number of the reference stress intensity factors required (Fett, et al. 1987). However no boundary condition for the T -stress weight functions is available in the literature. In order to reduce the number of the reference T -stress solutions required, this thesis introduces boundary conditions for the T -stress weight functions.

For a semi-elliptical surface crack in a finite thickness plate as shown in Figure 3.1, the surfaces near the crack mouth are stress free, so both the slope and the curvature of the crack surface at the crack mouth are zero (Fett, et al. 1987), i.e.

$$\frac{\partial u(x, a)}{\partial x} = 0; \quad \frac{\partial^2 u(x, a)}{\partial x^2} \Big|_{x=0} = 0 \quad (3.9)$$

where $u(x, a)$ is the crack mouth opening displacement.

Sham (1991) demonstrated that the T -stress weight function is a function of the crack mouth opening displacement:

$$t(x, a) = f(u(x, a)) \quad (3.10)$$

Applying the relationship between $u(x, a)$ and $t(x, a)$ in equation (3.10), the second derivatives of the T -stress weight function are zero at $x=0$:

$$\frac{\partial^2 t(x, a)}{\partial x^2} \Big|_{x=0} = 0 \quad (3.11)$$

So, for point C:

$$\frac{\partial^2 t_c(x, a)}{\partial x^2} \Big|_{x=0} = 0 \quad (3.12)$$

Equation (3.12) is used as the first boundary condition for the local T -stress weight functions in the present research.

The second boundary condition comes from the physical meaning of the T -stress weight function. The weight function of the T -stress $t_c(x, a)$ at point x has the following physical meaning: the T -stress at point C caused by a pair of unit loads applied at point x . For a semi-elliptical surface crack in a finite thickness plate as shown in Figure 3.2, with a pair of unit loads applied at the deepest point A , the T -stress at any point C is zero. This means the weight function (Equation 3.7) should

be zero at the point $x=a$:

at $x=a$:

$$t_c(x, a)|_{x=a} = 0 \quad (3.13)$$

Equation (3.13) is the second boundary condition used in the present research. These two boundary conditions are verified using the reference T -stress solutions.

By using the two boundary conditions (3.12 and 3.13) and two reference T -stresses, the constants D_{C1} , D_{C2} , D_{C3} and D_{C4} can be determined.

All these weight functions (equations (3.3), (3.6) and (3.7)) and the boundary conditions (equations (3.12) and (3.13)) will be used and verified later in this chapter and in chapter 5.

3.2 Derivation and Validation of Weight Functions at the Deepest Point

Weight functions for the deepest point of a semi-elliptical surface crack, $t_A(x, a)$, are derived in this section. The three-term form is used.

3.2.1 Reference T -Stress Solutions

Wang and Bell (2003) calculated the local T -stresses ($5^\circ \leq \phi \leq 90^\circ$) for semi-elliptical surface cracks in finite thickness plates under crack face pressure using the finite element method. Constant, linear, parabolic and cubic stress loading distributions were used. The local T -stress ($5^\circ \leq \phi \leq 90^\circ$) results were available for aspect ratios, a/c , of 0.2, 0.4, 0.6 and 1.0, and relative crack depths, a/t , of 0.2, 0.4, 0.6 and 0.8 under those four stress distributions.

Wang (2003) also calculated the local T -stresses ($5^\circ \leq \phi \leq 90^\circ$) for semi-elliptical surface cracks in finite thickness plates under remote tension and bending using the finite element method. The local T -stress results ($5^\circ \leq \phi \leq 90^\circ$) were presented for aspect ratios, a/c , of 0.2, 0.4, 0.6 and 1.0, and relative crack depths, a/t , of 0.2, 0.4, 0.6 and 0.8 under remote tension and bending.

The results in (Wang and Bell, 2003) and (Wang, 2003) are normalized as follows:

$$V = \frac{T}{\sigma_0} \quad (3.14)$$

where V is the normalized T -stress and σ_0 is the nominal stress for each loading condition.

Based on these finite element results, weight functions at the deepest point are derived and verified in this thesis for the entire range of $0 \leq a/c \leq 1$ and $0 \leq a/t \leq 0.8$. Three reference T -stress solutions are used.

At the deepest point of the surface crack, the numerical solutions for constant, linear and parabolic crack face loadings for $a/c=0.2, 0.4, 0.6, 1.0$ and $a/t=0.2, 0.4, 0.6, 0.8$ are approximated by empirical formulas fitted with an accuracy of 1% or better using least squares curve fitting. The choice of equations is based on engineering judgement.

The normalized T -stresses (V_0) for constant pressure loading,

$$\sigma(x) = \sigma_0 \quad (3.15)$$

are:

$$V_0 = \frac{T}{\sigma_0} = A_0 + B_0 \left(\frac{a}{t} \right)^2 + C_0 \left(\frac{a}{t} \right)^4 + D_0 \left(\frac{a}{t} \right)^6 \quad (3.16)$$

where:

$$A_0 = 0.4270 + 0.3874 \left(\frac{a}{c} \right) - 0.6029 \left(\frac{a}{c} \right)^2 + 0.2560 \left(\frac{a}{c} \right)^3$$

$$B_0 = 0.8038 - 6.5775 \left(\frac{a}{c} \right) + 12.1927 \left(\frac{a}{c} \right)^2 - 6.5200 \left(\frac{a}{c} \right)^3$$

$$C_0 = -4.7310 + 28.6236\left(\frac{a}{c}\right) - 51.4616\left(\frac{a}{c}\right)^2 + 27.5210\left(\frac{a}{c}\right)^3$$

$$D_0 = 6.1838 - 40.3138\left(\frac{a}{c}\right) + 68.9988\left(\frac{a}{c}\right)^2 - 35.6426\left(\frac{a}{c}\right)^3$$

The normalized T -stresses (V_1) for linear stress distribution,

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right) \quad (3.17)$$

are:

$$V_1 = \frac{T}{\sigma_0} = A_1 + B_1\left(\frac{a}{t}\right)^2 + C_1\left(\frac{a}{t}\right)^4 + D_1\left(\frac{a}{t}\right)^6 \quad (3.18)$$

where:

$$A_1 = 0.2925 + 0.2545\left(\frac{a}{c}\right) - 0.4412\left(\frac{a}{c}\right)^2 + 0.1721\left(\frac{a}{c}\right)^3$$

$$B_1 = 0.6449 - 4.7443\left(\frac{a}{c}\right) + 8.6739\left(\frac{a}{c}\right)^2 - 4.6155\left(\frac{a}{c}\right)^3$$

$$C_1 = -3.6043 + 20.1719\left(\frac{a}{c}\right) - 36.1024\left(\frac{a}{c}\right)^2 + 19.3016\left(\frac{a}{c}\right)^3$$

$$D_1 = 5.6907 - 30.6801\left(\frac{a}{c}\right) + 51.1455\left(\frac{a}{c}\right)^2 - 26.2225\left(\frac{a}{c}\right)^3$$

The normalized T -stresses (V_2) for parabolic stress distribution,

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right)^2 \quad (3.19)$$

are:

$$V_2 = \frac{T}{\sigma_0} = A_2 + B_2\left(\frac{a}{t}\right)^2 + C_2\left(\frac{a}{t}\right)^4 + D_2\left(\frac{a}{t}\right)^6 \quad (3.20)$$

where:

$$A_2 = 0.2259 + 0.1881\left(\frac{a}{c}\right) - 0.3299\left(\frac{a}{c}\right)^2 + 0.1245\left(\frac{a}{c}\right)^3$$

$$B_2 = 0.4884 - 3.5425\left(\frac{a}{c}\right) + 6.5025\left(\frac{a}{c}\right)^2 - 3.4720\left(\frac{a}{c}\right)^3$$

$$C_2 = -2.6602 + 14.7120\left(\frac{a}{c}\right) - 26.6363\left(\frac{a}{c}\right)^2 + 14.3500\left(\frac{a}{c}\right)^3$$

$$D_2 = 4.4978 - 23.1027\left(\frac{a}{c}\right) + 38.4967\left(\frac{a}{c}\right)^2 - 19.8025\left(\frac{a}{c}\right)^3$$

The curve fitting results and the finite element data on which they were based are plotted in Figures 3.3-3.6 for $a/c=0.2, 0.4, 0.6$ and 1.0 ($n=0, 1, 2$ corresponding to constant, linear and parabolic stress distributions, respectively).

3.2.2 Weight Functions

In section 3.1, a three-term weight function for the deepest point of a semi-elliptical surface crack was presented (Equation 3.3):

$$t_A(x, a) = \frac{2}{\pi a} \left[D_{A1} \left(1 - \frac{x}{a}\right)^{1/2} + D_{A2} \left(1 - \frac{x}{a}\right)^{3/2} + D_{A3} \left(1 - \frac{x}{a}\right)^{5/2} \right] \quad (3.3)$$

Three reference T -stresses were used here, corresponding to constant, linear and parabolic stress distributions acting on the crack surface, to determine the constants D_{A1} , D_{A2} and D_{A3} .

From equation (3.1), the total T -stress equals to the T -stress of crack pressure plus the T -stress of uncracked body. In the present case, the load is applied at crack face, so the T -stress of uncracked body is zero, i.e.

$$T = T_{crack \text{ pressure}} + T_{uncrack} = T_{crack \text{ pressure}} = \int_0^a \sigma(x) t_A(x, a) dx \quad (3.21)$$

By substituting Equations (3.15-3.20) into Equation (3.21), the three-term weight function constants can be determined:

$$\begin{aligned}
D_{A1} &= \pi \left(\frac{35}{64} \cdot \alpha \cdot V_0 - \frac{21}{32} \cdot \beta \cdot V_1 + \frac{15}{64} \cdot \gamma \cdot V_2 \right) \\
D_{A2} &= \pi \left(-\frac{63}{32} \cdot \alpha \cdot V_0 + \frac{45}{16} \cdot \beta \cdot V_1 - \frac{35}{32} \cdot \gamma \cdot V_2 \right) \\
D_{A3} &= \pi \left(\frac{99}{64} \cdot \alpha \cdot V_0 - \frac{77}{32} \cdot \beta \cdot V_1 + \frac{63}{64} \cdot \gamma \cdot V_2 \right)
\end{aligned} \tag{3.22}$$

where α , β and γ are constants:

$$\alpha = \frac{105}{4}, \quad \beta = \frac{315}{4}, \quad \gamma = \frac{693}{4}$$

The three-term weight function for the deepest point of a semi-elliptical surface crack can then be determined directly in the form of Equation (3.3).

3.2.3 Validations

Using Nonlinear Stress Distributions

Using this weight function $t_A(x, a)$ and Equation (3.21), the normalized T -stresses for the following nonlinear crack surface loading distribution are predicted:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a} \right)^n \tag{3.23}$$

where $n=0, 1, 2 \dots 5$.

The weight function is verified using finite element results for the cubic crack surface loading distribution ($n=3$). The normalized T -stress calculated from the derived weight function and from the finite element calculations for the cubic stress distribution are shown in Figures 3.3-3.6. The differences between the normalized T -stress values are less than 2.16%. Note that $n=0, 1, 2$ are reference cases; for the purpose of comparison, the results for $n=4$ and 5 are also plotted.

Using Remote Tension and Bending

In this section, the T -stresses for the deepest point of the surface crack in a finite thickness plate under remote tension and bending (as shown in Figure 3.2) are predicted using the weight function $t_A(x, a)$ derived above. The results are compared with FEM data (Wang, 2003).

For a surface crack in a flat plate (Figure 3.2) under remote tension, the stresses at the crack tip of the deepest point for the uncracked body is:

$$\begin{aligned}\sigma_x &= \sigma_y = 0 \\ \sigma_z(x) &= \sigma_0\end{aligned}\tag{3.24}$$

So, using Equation (3.1), the T -stress for the uncracked body of the deepest point under remote tension is ($\sigma_r = \sigma_x$ at deepest point):

$$T_{uncracked} = \sigma_r - \sigma_z \Big|_{x=a, z=0} = -\sigma_0\tag{3.25}$$

For remote bending, the stresses of the deepest point at the crack tip (as shown in Figure 3.1) for the uncracked body is:

$$\begin{aligned}\sigma_x &= \sigma_y = 0 \\ \sigma_z(x) &= \sigma_0 \left(1 - \frac{2a}{t}\right)\end{aligned}\tag{3.26}$$

So, using Equation (3.1), the T -stress for the uncracked body at deepest point under remote bending is ($\sigma_r = \sigma_x$ at deepest point):

$$T_{uncracked} = \sigma_r - \sigma_z \Big|_{x=a, z=0} = -\sigma_0 \left(1 - \frac{2x}{t}\right) = -\sigma_0 \left(1 - \frac{2a}{t}\right)\tag{3.27}$$

Equations (3.25) and (3.27) are the T -stress for the uncracked body under remote uniaxial tension and bending.

The total T -stress equals to the T -stress for crack face loading plus the T -stress of the uncracked body Equation (3.1), so the T -stress for the deepest point under remote tension is:

$$T = T_{crack\ pressure} + T_{uncrack} = T_{crack\ pressure} = \int_0^a \sigma(x) t_A(x, a) dx - \sigma_0 \quad (3.28)$$

and the T -stress for the deepest point under remote bending is:

$$T = T_{crack\ pressure} + T_{uncrack} = T_{crack\ pressure} = \int_0^a \sigma(x) t_A(x, a) dx - \sigma_0 \left(1 - \frac{2a}{t}\right) \quad (3.29)$$

The derived weight function $t_A(x, a)$ is used to predict the T -stress results. The results are verified using finite element results for the remote tension and bending cases (Wang, 2003). The normalized T -stress calculated from the derived weight functions and from the finite element calculations are shown in Figures 3.7 and 3.8 for remote tension and bending, respectively. The differences between the normalized T -stress values are less than 0.5% for remote tension and less than 1% for remote bending.

Through these analysis, it is demonstrated that the current weight function form, Equation (3.3), gives very accurate weight functions for the deepest point of the surface crack.

3.3 Derivation and Validation of Weight Functions at Local Points

The form of T -stress weight functions of local points and the boundary conditions for semi-elliptical surface cracks were discussed in section 3.1. The weight function forms for the local points (Equation 3.6 and 3.7) and the boundary conditions (Equation 3.12 and 3.13) are used in this section to derive weight functions:

For $0 \leq x \leq a \sin(\phi)$,

$$t_c(x, a) = \frac{2}{\pi a \sin(\phi)} \left[D_{c1} \left(1 - \frac{x}{a \sin(\phi)} \right)^{\frac{1}{2}} + D_{c2} \left(1 - \frac{x}{a \sin(\phi)} \right)^{\frac{3}{2}} \right] \quad (3.6)$$

For $a \sin(\phi) \leq x \leq a$,

$$t_c(x, a) = \frac{2}{\pi a \sin(\phi)} \left[D_{c3} \left(\frac{x}{a \sin(\phi)} - 1 \right)^{\frac{1}{2}} + D_{c4} \left(\frac{x}{a \sin(\phi)} - 1 \right)^{\frac{3}{2}} \right] \quad (3.7)$$

At $x=0$:

$$\frac{\partial^2 t_c(x, a)}{\partial x^2} \Big|_{x=0} = 0 \quad (3.12)$$

At $x=a$:

$$t_c(x, a) \Big|_{x=a} = 0 \quad (3.13)$$

3.3.1 Reference T -stress solutions

There are four constants in the weight functions for the local points of surface cracks. Two reference T -stresses combined with two boundary conditions are used to determine the four constants.

For the local points of the surface crack, the numerical solutions from Wang and Bell (2003) for $5^\circ \leq \phi \leq 90^\circ$, $a/c=0.2, 0.4, 0.6, 1.0$ and $a/t=0.2, 0.4, 0.6, 0.8$ are approximated by empirical formulas using least squares curve fitting. The choice of equations is based on engineering judgement.

The normalized T -stress results for constant crack face pressure:

$$\sigma(x) = \sigma_0 \quad (3.15)$$

are:

$$V_0 = \frac{T}{\sigma_0} = A_0 + B_0 \left(\frac{a}{t} \right)^2 + C_0 \left(\frac{a}{t} \right)^4 + D_0 \left(\frac{a}{t} \right)^6 \quad (3.30)$$

where:

$$A_0 = a_1 + a_2 \left(\frac{a}{c} \right) + a_3 \left(\frac{a}{c} \right)^2 + a_4 \left(\frac{a}{c} \right)^3$$

$$B_0 = b_1 + b_2 \left(\frac{a}{c} \right) + b_3 \left(\frac{a}{c} \right)^2 + b_4 \left(\frac{a}{c} \right)^3$$

$$C_0 = c_1 + c_2 \left(\frac{a}{c} \right) + c_3 \left(\frac{a}{c} \right)^2 + c_4 \left(\frac{a}{c} \right)^3$$

$$D_0 = d_1 + d_2 \left(\frac{a}{c} \right) + d_3 \left(\frac{a}{c} \right)^2 + d_4 \left(\frac{a}{c} \right)^3$$

and:

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} = [M_{a0}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix},$$

$$\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix} = [M_{b0}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = [M_{c0}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix},$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} = [M_{d0}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix}$$

where:

$$[M_{d0}]^T = \begin{bmatrix} -5.08886881\text{E}-9 & 3.01096065\text{E}-8 & -5.02716792\text{E}-8 & 2.54398340\text{E}-8 \\ 1.43489555\text{E}-6 & -8.35512205\text{E}-6 & 1.37704979\text{E}-5 & -6.90969571\text{E}-6 \\ -1.52575269\text{E}-4 & 8.67082286\text{E}-4 & -1.399960661\text{E}-3 & 6.92620116\text{E}-4 \\ 0.0074953452 & -0.040850848 & 0.063624399 & -0.030686639 \\ -0.16482198 & 0.82403371 & -1.19371420 & 0.54679867 \\ 1.67668521 & -4.56596578 & 5.34762748 & -2.15467603 \end{bmatrix}$$

$$[M_{b0}]^T = \begin{bmatrix} 9.27974701\text{E}-10 & -2.25669403\text{E}-8 & 7.05351897\text{E}-8 & -4.90522595\text{E}-8 \\ -3.62506948\text{E}-7 & 6.80383616\text{E}-6 & -2.035607867\text{E}-5 & 1.39599636\text{E}-5 \\ 4.35592957\text{E}-5 & -7.14716856\text{E}-4 & 0.0021009347 & -0.0014343365 \\ -0.0022361381 & 0.031591722 & -0.092882803 & 0.063716582 \\ 0.057820146 & -0.59036920 & 1.69159122 & -1.16360786 \\ 0.19910538 & -1.14309334 & -0.67694155 & 1.67533559 \end{bmatrix}$$

$$[M_{c0}]^T = \begin{bmatrix} -3.93762552\text{E}-8 & 3.69843413\text{E}-7 & -8.60495438\text{E}-7 & 5.30870358\text{E}-7 \\ 1.24741792\text{E}-5 & -1.07354261\text{E}-4 & 2.42853451\text{E}-4 & -1.48156363\text{E}-4 \\ -0.0014701843 & 0.011604698 & -0.025425362 & 0.015309457 \\ 0.080664766 & -0.57609751 & 1.20715683 & -0.71309956 \\ -2.01270899 & 12.61209665 & -24.64557544 & 14.10530794 \\ 9.29267011 & -42.19196851 & 73.98376136 & -41.84090587 \end{bmatrix}$$

$$[M_{d0}]^T = \begin{bmatrix} -1.26393878\text{E}-7 & 3.60155083\text{E}-7 & -1.69224465\text{E}-7 & -6.42914082\text{E}-8 \\ 3.06328454\text{E}-5 & -8.24119248\text{E}-5 & 2.47706083\text{E}-5 & 2.69813876\text{E}-5 \\ -0.0025084272 & 0.0059470390 & 6.81732241\text{E}-4 & -0.0041238454 \\ 0.066699390 & -0.078097245 & -0.28732051 & 0.29946059 \\ 0.48497133 & -5.74818974 & 15.16234897 & -9.95207950 \\ -13.24543233 & 57.88935612 & -96.15292736 & 52.36792687 \end{bmatrix}$$

The normalized T -stress results for a linear stress distribution (applied at the

crack surface):

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right) \quad (3.17)$$

are:

$$V_1 = \frac{T}{\sigma_0} = A_1 + B_1 \left(\frac{a}{t}\right)^2 + C_1 \left(\frac{a}{t}\right)^4 + D_1 \left(\frac{a}{t}\right)^6 \quad (3.31)$$

where:

$$A_1 = a_5 + a_6 \left(\frac{a}{c}\right) + a_7 \left(\frac{a}{c}\right)^2 + a_8 \left(\frac{a}{c}\right)^3$$

$$B_1 = b_5 + b_6 \left(\frac{a}{c}\right) + b_7 \left(\frac{a}{c}\right)^2 + b_8 \left(\frac{a}{c}\right)^3$$

$$C_1 = c_5 + c_6 \left(\frac{a}{c}\right) + c_7 \left(\frac{a}{c}\right)^2 + c_8 \left(\frac{a}{c}\right)^3$$

$$D_1 = d_5 + d_6 \left(\frac{a}{c}\right) + d_7 \left(\frac{a}{c}\right)^2 + d_8 \left(\frac{a}{c}\right)^3$$

and:

$$\begin{Bmatrix} a_5 \\ a_6 \\ a_7 \\ a_8 \end{Bmatrix} = [M_{a1}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix},$$

$$\begin{Bmatrix} b_5 \\ b_6 \\ b_7 \\ b_8 \end{Bmatrix} = [M_{bl}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} c_5 \\ c_6 \\ c_7 \\ c_8 \end{Bmatrix} = [M_{cl}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix},$$

$$\begin{Bmatrix} d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix} = [M_{dl}] \begin{Bmatrix} \phi^5 \\ \phi^4 \\ \phi^3 \\ \phi^2 \\ \phi^1 \\ 1 \end{Bmatrix}$$

where:

$$[M_{al}]^T = \begin{bmatrix} -4.04481390E-9 & 2.37825505E-8 & -3.95773904E-8 & 1.99920658E-8 \\ 1.13296517E-6 & -6.57297392E-6 & 1.08054813E-5 & -5.41349755E-6 \\ -1.196749727E-4 & 6.80414669E-4 & -1.09786757E-3 & 5.42930533E-4 \\ 0.00587428378 & -0.0321552110 & 0.050260874 & -0.024301612 \\ -0.13314116 & 0.663341957 & -0.97307998 & 0.44975284 \\ 1.48287050 & -4.16543678 & 5.07301701 & -2.10503127 \end{bmatrix}$$

$$[M_{bl}]^T = \begin{bmatrix} -1.75032807E-10 & -1.30292791E-8 & 4.71921133E-8 & -3.42326392E-8 \\ -1.29072164E-7 & 4.39737905E-6 & -1.41910610E-5 & 9.98242025E-6 \\ 2.98148344E-5 & -5.12073600E-4 & 0.0015343393 & -0.0010572361 \\ -0.0021103820 & 0.025111771 & -0.071521306 & 0.048715915 \\ 0.059459504 & -0.50455270 & 1.35714356 & -0.91633952 \\ 0.10582091 & -0.77463304 & -0.62078275 & 1.33622833 \end{bmatrix}$$

$$[M_{c1}]^T = \begin{bmatrix} -1.74760944E-8 & 2.13049303E-7 & -5.39275148E-7 & 3.45203936E-7 \\ 6.67094307E-6 & -6.56565581E-5 & 1.57412992E-4 & -9.87455165E-5 \\ -9.03228711E-4 & 0.0075306470 & -0.017093251 & 0.010492221 \\ 0.054926462 & -0.39535809 & 0.84258567 & -0.50353649 \\ -1.45392248 & 9.00390936 & -17.71173338 & 10.20944199 \\ 6.63742273 & -29.43346700 & 52.61931860 & -30.38246251 \end{bmatrix}$$

$$[M_{d1}]^T = \begin{bmatrix} -1.10028115E-7 & 3.53265538E-7 & -2.69931710E-7 & 2.52836742E-8 \\ 2.59396222E-5 & -7.94475545E-5 & 4.97933823E-5 & 4.01684491E-6 \\ -0.0020531018 & 0.0056796422 & -0.0017536276 & -0.0018969764 \\ 0.052580612 & -0.088935083 & -0.15385315 & 0.19135958 \\ 0.37522180 & -4.04833440 & 10.78199958 & -7.15317300 \\ -10.00460814 & 42.76565617 & -71.55910944 & 39.42529303 \end{bmatrix}$$

The curve fitting results and the finite element data on which they were based ($n=0$ and 1) are plotted in Figures 3.9-3.24, for $a/c=0.2, 0.4, 0.6, 1.0$; $a/t=0.2, 0.4, 0.6, 0.8$.

3.3.2 Weight Functions

For the plate under crack pressure, the T -stress for the uncracked body is zero. So, from Equation (3.1), following equation can be used to calculate the T -stresses:

$$T = T_{crack\ pressure} = \int_0^{a \sin(\phi)} \sigma(x) t_c(x, a) dx + \int_{a \sin(\phi)}^a \sigma(x) t_c(x, a) dx \quad (3.32)$$

where $t_c(x, a)$ is the weight function (Equation 3.6 and 3.7).

By substituting Equations (3.15, 3.17, 3.30 and 3.31) into Equation (3.32), and applying the boundary conditions represented by Equations (3.12) and (3.13), the weight function constants can be determined:

$$D_{C1} = \frac{15\pi(-1 + \sin(\phi))}{2(4 \sin(\phi) + 9)} V_0 + \frac{105\pi}{8(4 \sin(\phi) + 9)} V_1 \quad (3.33)$$

$$D_{c2} = \frac{5\pi(-1 + \sin(\phi))}{2(4\sin(\phi) + 9)}V_0 + \frac{35\pi}{8(4\sin(\phi) + 9)}V_1 \quad (3.34)$$

$$D_{c3} = + \frac{15\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{3}{2}} (8\sin(\phi) - 21)}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14\sin(\phi) + 9 + 4\sin^3(\phi))}V_0 \quad (3.35)$$

$$+ \frac{315\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{3}{2}}}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14\sin(\phi) + 9 + 4\sin^3(\phi))}V_1$$

$$D_{c4} = - \frac{15\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{1}{2}} (8\sin(\phi) - 21)}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14\sin(\phi) + 9 + 4\sin^3(\phi))}V_0 \quad (3.36)$$

$$- \frac{315\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{1}{2}}}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14\sin(\phi) + 9 + 4\sin^3(\phi))}V_1$$

The weight function for the local points of a semi-elliptical surface crack can then be determined directly from Equations (3.6) and (3.7).

3.3.3 Validations

Using Nonlinear Stress Distributions

Using these weight functions, the normalized local T -stresses for $n=0-5$ (Equation 3.23) loading distributions are predicted. The normalized T -stresses calculated from the derived weight functions and from the finite element calculations for the above stress distributions are shown in Figures 3.9-3.24. The results for $n=2$ and 3 are used to verify the derived weight function. The differences between the

normalized T -stress values are less than 10% (less than 5% for most points) except $a/t=0.8$ and $80^\circ < \phi < 90^\circ$. When the relative depth $a/t=0.8$ and $80^\circ < \phi < 90^\circ$, the differences is more than 20%. However, for those cases, the absolute T -stress values are close to zero. Generally the results for high aspect ratio cracks are better than for low aspect ratio cracks.

Note that results for $n=0$ and 1 are reference cases, and results for $n=4$ and 5 shown in the figures are for the purpose of comparison.

Using Remote Tension and Bending

In this section, the local T -stresses of the surface crack in a finite thickness plate under remote tension and bending (as shown in Figure 3.2) are predicted using the weight functions derived above. The results were compared with the FEM data (Wang 2003).

For a surface crack in a flat plate (Figure 3.2) under remote tension, the stresses at a general point C , represented by ϕ , of the crack front for the uncracked body is:

$$\begin{aligned} \sigma_x = \sigma_y = 0; \quad \text{then} \quad \sigma_r = 0 \\ \sigma_z(x) = \sigma_0 \end{aligned} \quad (3.37)$$

So, using Equation (3.1), the T -stress for the uncracked body at point C is:

$$T_{uncracked} = \sigma_r - \sigma_z \Big|_{x=a \sin(\phi), z=0} = -\sigma_0 \quad (3.38)$$

For a surface crack in a flat plate (Figure 3.2) under remote bending, the stresses of the point C at the crack front (as shown in Figure 3.1) for the uncracked body is:

$$\begin{aligned} \sigma_x = \sigma_y = 0; \quad \text{then} \quad \sigma_r = 0 \\ \sigma_z(x) = \sigma_0 \left(1 - \frac{2x}{t} \right) \end{aligned} \quad (3.39)$$

So, using Equation (3.1), the T -stress for the uncracked body at point C is:

$$T_{uncracked} = \sigma_r - \sigma_z \Big|_{x=a \sin(\phi), z=0} = -\sigma_0 \left(1 - \frac{2x}{t}\right) = -\sigma_0 \left(1 - \frac{2a \sin(\phi)}{t}\right) \quad (3.40)$$

Equations (3.38) and (3.40) are the T -stress for the local points of uncracked body under remote tension and bending.

So for a local point C the total T -stress under remote tension is (Equation (3.1)):

$$T = T_{crack \ pressure} + T_{uncracked} \quad (3.41)$$

$$= \int_0^{a \sin(\phi)} \sigma_z(x) t_C(x, a) dx + \int_{a \sin(\phi)}^a \sigma_z(x) t_C(x, a) dx - \sigma_0$$

and the total T -stress at point C for remote bending is:

$$T = T_{crack \ pressure} + T_{uncracked} \quad (3.42)$$

$$= \int_0^{a \sin(\phi)} \sigma_z(x) t_C(x, a) dx + \int_{a \sin(\phi)}^a \sigma_z(x) t_C(x, a) dx - \sigma_0 \left(1 - \frac{2a \sin(\phi)}{t}\right)$$

The term $t_C(x, a)$ in Equation (3.41) and (3.42) is the weight function as shown in Equation 3.6 and 3.7. The constants D_{C1} , D_{C2} , D_{C3} and D_{C4} have been determined and summarized by Equations (3.33-3.36).

The local normalized T -stresses ($5^\circ \leq \phi \leq 90^\circ$, $a/c=0.2, 0.4, 0.6, 1.0$ and $a/t=0.2, 0.4, 0.6, 0.8$) are predicted using Equation (3.41) and (3.42). The results are compared with the finite element results for the remote tension and bending cases (Wang, 2003). The normalized T -stress calculated from the derived weight functions and from the finite element calculations are shown in Figures 3.25-3.28 for remote tension and 3.29-3.32 for remote bending. The differences between the normalized T -stress values

are less than 5% (less than 1% for most points) for remote tension and bending except the aspect ratio $a/c=0.2$. When $a/c=0.2$, the accuracy is within 10% (within 5% for most points) for both cases.

Through the validations, it can be concluded that the general forms of weight functions at point C provides appropriate weight functions.

3.4 Discussions and Conclusions

The T -stress weight functions for the deepest point and local points of semi-elliptical surface cracks in finite thickness plates were derived. The weight functions were verified using finite element results for various linear and non-linear loading cases. The accuracy is generally very good along the crack front.

Note that the normalized T -stress, $V=0$ line is plotted in all the figures. The T -stresses are positive for all the crack surface pressure cases, this means the plate is in high constraint. However, for the remote tension case, all the T -stresses are negative, so the plate is in low constraint condition under remote tension and conventional failure assessment methods would be conservative. When the plate under remote bending, the constraint is high for the points near the deepest point and is low near the surface point.

These derived weight functions are suitable for T -stress calculations for any complex uniaxial loading conditions.

Figure 3.1 Semi-elliptical surface crack

Figure 3.2 Surface cracked plate under uniaxial load

Figure 3.3 Comparison of three-term weight function based normalized T -stress V and FEM data for the deepest point $a/c=0.2$ under stress distribution of Equation (3.23)

Figure 3.4 Comparison of three-term weight function based normalized T -stress V and FEM data for the deepest point $a/c=0.4$ under stress distribution of Equation (3.23)

Figure 3.5 Comparison of three-term weight function based normalized T -stress V and FEM data for the deepest point $a/c=0.6$ under stress distribution of Equation (3.23)

Figure 3.6 Comparison of three-term weight function based normalized T -stress V and FEM data for the deepest point $a/c=1.0$ under stress distribution of Equation (3.23)

Figure 3.7 Comparison of three term weight function based normalized T -stress V and FEM data for the deepest point under remote tension

Figure 3.8 Comparison of three term weight function based normalized T -stress V and FEM data for the deepest point under remote bending

Figure 3.9 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.2$ $a/t=0.2$ under stress distribution of Equation (3.23)

Figure 3.10 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.2$ $a/t=0.4$ under stress distribution of Equation (3.23)

Figure 3.11 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.2$ $a/t=0.6$ under stress distribution of Equation (3.23)

Figure 3.12 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.2$ $a/t=0.8$ under stress distribution of Equation (3.23)

Figure 3.13 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.4$ $a/t=0.2$ under stress distribution of Equation (3.23)

Figure 3.14 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.4$ $a/t=0.4$ under stress distribution of Equation (3.23)

Figure 3.15 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.4$ $a/t=0.6$ under stress distribution of Equation (3.23)

Figure 3.16 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.4$ $a/t=0.8$ under stress distribution of Equation (3.23)

Figure 3.17 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.6$ $a/t=0.2$ under stress distribution of Equation (3.23)

Figure 3.18 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.6$ $a/t=0.4$ under stress distribution of Equation (3.23)

Figure 3.19 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.6$ $a/t=0.6$ under stress distribution of Equation (3.23)

Figure 3.20 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=0.6$ $a/t=0.8$ under stress distribution of Equation (3.23)

Figure 3.21 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=1.0$ $a/t=0.2$ under stress distribution of Equation (3.23)

Figure 3.22 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=1.0$ $a/t=0.4$ under stress distribution of Equation (3.23)

Figure 3.23 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=1.0$ $a/t=0.6$ under stress distribution of Equation (3.23)

Figure 3.24 Comparison of weight function based normalized T -stress V and FEM data for local points $a/c=1.0$ $a/t=0.8$ under stress distribution of Equation (3.23)

Figure 3.25 Comparison of weight function based normalized T -stress V and FEM data for local points under remote tension $a/c=0.2$

Figure 3.26 Comparison of weight function based normalized T -stress V and FEM data for local points under remote tension $a/c=0.4$

Figure 3.27 Comparison of weight function based normalized T -stress V and FEM data for local points under remote tension $a/c=0.6$

Figure 3.28 Comparison of weight function based normalized T -stress V and FEM data for local points under remote tension $a/c=1.0$

Figure 3.29 Comparison of weight function based normalized T -stress V and FEM data for local points under remote bending $a/c=0.2$

Figure 3.30 Comparison of weight function based normalized T -stress V and FEM data for local points under remote bending $a/c=0.4$

Figure 3.31 Comparison of weight function based normalized T -stress V and FEM data for local points under remote bending $a/c=0.6$

Figure 3.32 Comparison of weight function based normalized T -stress V and FEM data for local points under remote bending $a/c=1.0$

Chapter 4

Finite Element Calculations for Surface Cracked Plate under Biaxial Loading

In chapter 3, weight functions suitable for T -stress calculations of semi-elliptical surface cracks in finite thickness flat plates were derived. The loading conditions considered are uniaxial cases (along z -direction shown in Figure 3.2). In practice, many engineering components, such as pressure vessels, are under biaxial loading. Accurate T -stress solutions for surface cracks under biaxial loading are therefore needed for a constraint-based failure assessment.

In this chapter, three dimensional finite element analyses for semi-elliptical surface cracks in finite thickness plates under biaxial loadings have been conducted. The T -stresses are determined for local points of surface cracks ($5^\circ \leq \phi \leq 90^\circ$) with aspect ratios, $a/c=0.2, 0.4, 0.6$ and 1.0 , and relative depth, $a/t=0.2, 0.4, 0.6$ and 0.8 , under remote biaxial tension and bending. Two different biaxial ratios ($\lambda=0.5$ and 1) are used.

4.1 Finite Element Model

Three-dimensional finite elements were used to model the symmetric quarter of a plate containing a semi-elliptical surface crack. Figure 1.3 shows the geometry and

co-ordinate system used. The finite element analyses were made using ABAQUS, version 6.2 (ABAQUS, 2001), with 20-noded isoparametric three-dimensional solid elements and reduced integration.

In order to model the square root singularity at the crack tip, three-dimensional prism elements with four mid-side nodes at the quarter points (a degenerate cube with one face collapsed) were used and the separate crack tip nodal points were constrained to have the same displacement (Barsoum, 1977).

The T -stress was calculated from the interaction integral I which was calculated using domain integral method (section 2.2.1). The analyses were made with a linear elastic material model with a Young's modulus, E , of 2.07GPa and Poisson's ratio, ν of 0.3. The relationship between I and T was used to calculate T :

$$I(s) = \frac{1}{A_c} \int_{V(s)} \left\{ \left(\sigma_{ij}^L \frac{\partial u_i^L}{\partial x_k} + \sigma_{ij}^L \frac{\partial u_i^L}{\partial x_k} \right) \frac{\partial q_k}{\partial x_j} - \sigma_{ij}^L \varepsilon_{ij} \frac{\partial q_k}{\partial x_k} \right\} dV \quad (2.16)$$

$$T(s) = \frac{E}{1-\nu^2} \left\{ \frac{I(s)}{f} + \nu \varepsilon_{33}(s) \right\} \quad (2.17)$$

Semi-elliptical surface cracks with aspect ratios, a/c , of 0.2, 0.4, 0.6 or 1.0 and relative depths, a/t , of 0.2, 0.4, 0.6 or 0.8 were analyzed. The loads were applied to both sides (y direction and z direction in Figure 1.3) of the cracked plate. The biaxial ratio λ was defined as follows:

$$\lambda = \frac{\sigma_y(x)}{\sigma_z(x)} \quad (4.1)$$

where $\sigma_z(x)$ is the load applied in the z direction and $\sigma_y(x)$ is the load applied in the y direction.

Four types of loading were applied to each crack geometry, with the following stress distributions

1. Biaxial tension $\lambda=0.5$:

$$\sigma_z(x) = 2\sigma_y(x) = \sigma_0 \quad (4.2)$$

2. Biaxial tension $\lambda=1.0$:

$$\sigma_z(x) = \sigma_y(x) = \sigma_0 \quad (4.3)$$

3. Biaxial bending $\lambda=0.5$:

$$\sigma_z(x) = 2\sigma_y(x) = \sigma_0 \left(1 - 2\frac{x}{t}\right) \quad (4.4)$$

4. Biaxial bending $\lambda=1.0$:

$$\sigma_z(x) = \sigma_y(x) = \sigma_0 \left(1 - 2\frac{x}{t}\right) \quad (4.5)$$

where σ_0 is the nominal stress for tension or bending loading conditions.

A mesh generator was used (Wang, 1997) to generate all required input files for the analysis. The finite element model for the semi-elliptical surface crack was obtained from the finite element model for a circular crack by using an elliptical transformation (Raju and Newman, 1979). This transformation is needed because the T -stresses must be evaluated along the normals to the crack front. If (x, y, z) are the Cartesian coordinates of a node in the circular-crack model and (x', y', z') are the coordinates of that same node in the elliptical crack model, then the transformation is:

$$x' = x \sqrt{1 + \frac{c^2 - a^2}{x^2 + z^2}}, \quad y' = y, \quad z' = z \quad (4.6)$$

for x and z not both at the origin. Figure 4.1 shows how circular arcs and radial lines in the x - z plane of the elliptical model are transformed into ellipse and hyperbolas, respectively, in the x' - z' plane of the elliptical model using Equation (4.6).

In order to model the sharpest curvature of the low aspect ratio elliptical cracks,

the mesh was refined extensively. A typical model for the present analysis used about 15,000 degrees-of-freedom. A typical mesh was plotted in Figure 4.2. Sufficient elements were added in the width direction to eliminate the influence of plane width.

4.2 Verification of Finite Element Model

The current finite element model has been extensively verified in (Wang, 2003) for the T -stress calculation. The verifications are summarized as follows.

4.2.1 Verification with Exact Solutions

In order to verify the finite element model and the procedure of extracting T -stress, T -stresses along the front of an embedded circular crack in an infinite body under far field tension were calculated by Wang (2003). A Poisson's ratio ν of 0.5 was used in the calculation. In the finite element models, w and t were large enough (w/c and $t/a \geq 10$) so that the free boundary had a negligible effect (Raju and Newman, 1981). The number of elements used was similar to that used in the current surface crack models. Comparisons were made between the T -stresses calculated from the finite element analysis and from the exact solution. For the specific case of $\nu = 0.5$, the exact T -stress solution was derived for an embedded circular crack in an infinite body under tensile load, and the T -stress along the crack front was found to be equal to:

$$T(\phi) = -\sigma_0 \quad (4.7)$$

where σ_0 is the tensile stress applied at the far ends.

The results along the crack front from the finite element calculation were presented in (Wang, 2003), and the maximum difference from the exact solution was less than 0.78%. The present method is therefore considered suitable for the analysis of T -stress for three dimensional crack configurations.

4.2.2 Verification with Approximate Solutions

In order to verify the finite element method's ability to handle cracks in a finite thickness plate, T -stresses for edge cracks in finite thickness plate were calculated (Wang, 2003). Two-dimensional finite element models degenerated from the three-dimensional finite element models in the x - z plane (see Figure 1.1 and 1.2) were used. Remote tension loads were applied, and cracks with $a/t=0.2, 0.4, 0.6$ and 0.8 were analyzed. Comparisons were made with T -stress results from finite element analysis (Wang, 2003) and from Sham (1991). The results were presented in (Wang, 2003). The maximum difference was within 3.62%.

Based on these results, the present finite element model is considered suitable for the T -stress analysis of semi-elliptical surface cracks in finite thickness plates.

4.3 T -Stress Results

The T -stress distributions along the crack front for semi-elliptical surface cracks ($a/t=0.2, 0.4, 0.6$ and 0.8) in a finite thickness plate with aspect ratios, a/c , of $0.2, 0.4, 0.6$ and 1.0 under remote biaxial tension and bending loads have been determined. The Poisson's ratio used in the current analysis is 0.3 .

The T -stress results have been normalized as follows:

$$V(\phi) = \frac{T(\phi)}{\sigma_0} \quad (4.8)$$

where $V(\phi)$ is the normalized T -stress at specific angle ϕ , and σ_0 is the nominal tensile or bending stress. The results are summarized in Tables 4.1-4.16 (tables 4.1-4.8 for biaxial tension ($\lambda=0.5$ and 1.0) and 4.9-4.16 for biaxial bending ($\lambda=0.5$ and 1.0)).

It is well known that at the intersection of the crack front and the free surface, the $r^{-1/2}$ singularity vanishes (Sih and Lee, 1989). That is, the $r^{-1/2}$ singularity as described in Equation 2.1-2.5, occurs only near crack front points embedded entirely in the material.

$$\sigma_{11} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + T \quad (2.1)$$

$$\sigma_{22} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (2.2)$$

$$\sigma_{33} = \frac{K}{\sqrt{2\pi r}} 2\nu \cos \frac{\theta}{2} + E\varepsilon_{33} + \nu T \quad (2.3)$$

$$\sigma_{12} = \frac{K}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (2.4)$$

$$\sigma_{13} = \sigma_{23} = 0 \quad (2.5)$$

Due to this complication, the numerical results near the free surface are not reliable. The same observation was obtained by Nakamura and Parks (1992). The values of the T -stress for $\phi < 5^\circ$ are therefore neglected in this thesis.

Discussions on these finite element results will be conducted in chapter 5.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.244891	0.266986	0.247944	-0.24761
6.25	0.141024	0.153457	0.122938	-0.41521
7.5	0.090693	0.102485	0.070684	-0.51293
8.75	0.009574	0.015113	-0.03718	-0.63653
10	-0.04421	-0.04075	-0.09555	-0.69576
11.25	-0.10566	-0.10615	-0.17453	-0.75269
12.5	-0.15024	-0.15237	-0.2206	-0.76041
13.75	-0.19428	-0.1985	-0.27203	-0.74978
15	-0.22812	-0.23316	-0.30366	-0.70925
16.25	-0.25992	-0.26587	-0.3364	-0.65429
17.5	-0.28498	-0.29109	-0.35701	-0.57894
18.75	-0.30848	-0.3148	-0.37742	-0.51362
20	-0.32741	-0.33349	-0.39009	-0.44171
21.25	-0.34536	-0.35128	-0.40331	-0.38422
22.5	-0.37489	-0.38606	-0.44754	-0.37671
26.25	-0.39881	-0.40279	-0.42006	-0.1417
30	-0.42237	-0.4238	-0.40834	0.038978
33.75	-0.44396	-0.44366	-0.40017	0.106195
37.5	-0.45806	-0.45584	-0.38672	0.180445
41.25	-0.47039	-0.46841	-0.39945	0.177866
45	-0.47832	-0.47618	-0.4115	0.149124
48.75	-0.48576	-0.48622	-0.44675	-0.02434
52.5	-0.49062	-0.49308	-0.47615	-0.1764
56.25	-0.49637	-0.50265	-0.50955	-0.28103
60	-0.50052	-0.50995	-0.53526	-0.35425
63.75	-0.50583	-0.51913	-0.55722	-0.43318
67.5	-0.51	-0.52646	-0.57602	-0.50677
71.25	-0.51465	-0.53438	-0.59286	-0.58051
75	-0.51835	-0.54064	-0.60614	-0.64648
78.75	-0.52163	-0.54619	-0.61755	-0.6949
82.5	-0.5241	-0.55019	-0.62537	-0.73501
86.25	-0.5255	-0.55263	-0.63048	-0.75272
90	-0.52608	-0.55354	-0.63239	-0.76426

Table 4.1. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 0.2$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.085473	0.09121	0.121951	0.04027
6.25	0.066164	0.072478	0.088387	0.052223
7.5	0.06789	0.077665	0.115021	0.089456
8.75	0.051861	0.062995	0.097073	0.071504
10	0.040174	0.053267	0.09927	0.094921
11.25	0.019542	0.033519	0.078612	0.070645
12.5	0.00067	0.015735	0.06799	0.083929
13.75	-0.02251	-0.00694	0.045747	0.060783
15	-0.04404	-0.02782	0.028872	0.065956
16.25	-0.06813	-0.0517	0.005299	0.04375
17.5	-0.09042	-0.07365	-0.01434	0.042551
18.75	-0.11368	-0.09685	-0.03803	0.019078
20	-0.13529	-0.1183	-0.05866	0.011644
21.25	-0.15757	-0.14073	-0.0816	-0.01339
22.5	-0.19508	-0.18124	-0.12765	-0.06793
26.25	-0.23678	-0.22077	-0.16315	-0.06969
30	-0.28221	-0.26641	-0.20804	-0.08066
33.75	-0.32556	-0.31185	-0.25708	-0.12617
37.5	-0.35783	-0.34524	-0.29104	-0.15147
41.25	-0.38798	-0.37848	-0.33213	-0.22291
45	-0.41032	-0.40303	-0.36224	-0.2821
48.75	-0.43133	-0.42821	-0.40061	-0.36947
52.5	-0.44683	-0.44699	-0.43096	-0.44119
56.25	-0.46156	-0.46663	-0.46711	-0.52624
60	-0.47224	-0.48137	-0.49534	-0.5944
63.75	-0.48252	-0.49672	-0.5257	-0.66109
67.5	-0.48969	-0.50812	-0.5503	-0.71788
71.25	-0.49667	-0.51955	-0.57397	-0.76999
75	-0.50112	-0.52771	-0.59223	-0.81269
78.75	-0.50546	-0.53518	-0.60771	-0.8476
82.5	-0.50762	-0.5398	-0.61834	-0.87358
86.25	-0.5097	-0.54318	-0.62507	-0.88846
90	-0.50961	-0.54371	-0.62714	-0.89546

Table 4.2. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 0.4$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	-0.04275	-0.04683	-0.0353	-0.02721
6.25	-0.03169	-0.03337	-0.04124	-0.00849
7.5	-0.02307	-0.02253	-0.01862	0.035948
8.75	-0.02021	-0.01787	-0.01232	0.0351
10	-0.01887	-0.01485	-0.00228	0.065745
11.25	-0.02128	-0.01583	-0.00117	0.059329
12.5	-0.02506	-0.0183	0.000995	0.078052
13.75	-0.0317	-0.02384	-0.00205	0.07048
15	-0.03903	-0.03013	-0.00473	0.079028
16.25	-0.04884	-0.03911	-0.01152	0.068968
17.5	-0.05881	-0.04827	-0.01779	0.069791
18.75	-0.07061	-0.05946	-0.02742	0.056839
20	-0.0822	-0.07044	-0.03642	0.050777
21.25	-0.09576	-0.08362	-0.04812	0.034444
22.5	-0.11847	-0.10671	-0.07088	0.008009
26.25	-0.15237	-0.13944	-0.10099	-0.02751
30	-0.19176	-0.17845	-0.13776	-0.06831
33.75	-0.23603	-0.22406	-0.18714	-0.13344
37.5	-0.27299	-0.26195	-0.22667	-0.18244
41.25	-0.31111	-0.3026	-0.27429	-0.25372
45	-0.34222	-0.33574	-0.31245	-0.31111
48.75	-0.3731	-0.37004	-0.35741	-0.38504
52.5	-0.39776	-0.39756	-0.39418	-0.4482
56.25	-0.42175	-0.42546	-0.43517	-0.52356
60	-0.44037	-0.44742	-0.46796	-0.58503
63.75	-0.45826	-0.46925	-0.50254	-0.65142
67.5	-0.47153	-0.48585	-0.53028	-0.7078
71.25	-0.48416	-0.50188	-0.55753	-0.76099
75	-0.4927	-0.51326	-0.57823	-0.80427
78.75	-0.5007	-0.52363	-0.59631	-0.84027
82.5	-0.50497	-0.52975	-0.60839	-0.86644
86.25	-0.50876	-0.53453	-0.61646	-0.8821
90	-0.50882	-0.53503	-0.61841	-0.88843

Table 4.3. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 0.6$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	-0.14689	-0.152	-0.17054	-0.16745
6.25	-0.1416	-0.14602	-0.15778	-0.15174
7.5	-0.13264	-0.13624	-0.14863	-0.13237
8.75	-0.12667	-0.12953	-0.13747	-0.12617
10	-0.12323	-0.12551	-0.13225	-0.11461
11.25	-0.1198	-0.12145	-0.12608	-0.11211
12.5	-0.11917	-0.12036	-0.12344	-0.10605
13.75	-0.11842	-0.11909	-0.12095	-0.10545
15	-0.11975	-0.12003	-0.12038	-0.10319
16.25	-0.12107	-0.12093	-0.12039	-0.10468
17.5	-0.12412	-0.12364	-0.12167	-0.1052
18.75	-0.12692	-0.12609	-0.12363	-0.10878
20	-0.13064	-0.12951	-0.12626	-0.11191
21.25	-0.13535	-0.13396	-0.12981	-0.11688
22.5	-0.14102	-0.13922	-0.13415	-0.12198
26.25	-0.15989	-0.15779	-0.15188	-0.14529
30	-0.18093	-0.17844	-0.17091	-0.16807
33.75	-0.20766	-0.20534	-0.19922	-0.20525
37.5	-0.23437	-0.23224	-0.22621	-0.23764
41.25	-0.26513	-0.26383	-0.26076	-0.28356
45	-0.29446	-0.29396	-0.29287	-0.32401
48.75	-0.32655	-0.32744	-0.3316	-0.37845
52.5	-0.35602	-0.35827	-0.36675	-0.42813
56.25	-0.38688	-0.39097	-0.40628	-0.48813
60	-0.4141	-0.41993	-0.44092	-0.53912
63.75	-0.44148	-0.44933	-0.47765	-0.59662
67.5	-0.46434	-0.47404	-0.50848	-0.64594
71.25	-0.48631	-0.49786	-0.53921	-0.69663
75	-0.50307	-0.51625	-0.56305	-0.73665
78.75	-0.51813	-0.53265	-0.58452	-0.77313
82.5	-0.52748	-0.54307	-0.59842	-0.79772
86.25	-0.53464	-0.55081	-0.60854	-0.81479
90	-0.53567	-0.55209	-0.6103	-0.81896

Table 4.4. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 1.0$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.676231	0.698423	0.679497	0.183942
6.25	0.533445	0.54597	0.515636	-0.02245
7.5	0.445404	0.457286	0.425709	-0.15783
8.75	0.327102	0.332724	0.280668	-0.31859
10	0.239366	0.242908	0.188331	-0.41179
11.25	0.146878	0.146454	0.078285	-0.4998
12.5	0.074644	0.072586	0.004531	-0.53522
13.75	0.006138	0.001991	-0.07139	-0.54908
15	-0.04933	-0.05431	-0.12467	-0.5302
16.25	-0.10002	-0.10592	-0.17631	-0.49414
17.5	-0.1418	-0.14785	-0.21367	-0.4355
18.75	-0.17985	-0.18613	-0.24866	-0.38475
20	-0.21174	-0.21777	-0.27429	-0.32579
21.25	-0.24095	-0.24683	-0.29879	-0.27959
22.5	-0.28811	-0.29926	-0.36068	-0.28975
26.25	-0.32871	-0.33266	-0.34986	-0.07136
30	-0.36944	-0.37084	-0.35533	0.09217
33.75	-0.40306	-0.40274	-0.35919	0.147358
37.5	-0.42657	-0.42433	-0.35515	0.212216
41.25	-0.44578	-0.44377	-0.37475	0.202772
45	-0.45925	-0.45707	-0.39234	0.1685
48.75	-0.47087	-0.4713	-0.43178	-0.00919
52.5	-0.47917	-0.4816	-0.46462	-0.16471
56.25	-0.48756	-0.49383	-0.50067	-0.27199
60	-0.4939	-0.50331	-0.52857	-0.3474
63.75	-0.50091	-0.51419	-0.55223	-0.42803
67.5	-0.50646	-0.52292	-0.57241	-0.50299
71.25	-0.51218	-0.5319	-0.59032	-0.5778
75	-0.51673	-0.539	-0.60444	-0.6446
78.75	-0.52062	-0.54517	-0.61645	-0.69363
82.5	-0.52352	-0.54959	-0.62471	-0.73416
86.25	-0.52516	-0.55227	-0.63007	-0.75212
90	-0.52579	-0.55324	-0.63203	-0.76371

Table 4.5. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda=1.0$, $V=T/\sigma_0$, $a/c = 0.2$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.568491	0.574349	0.604988	0.523217
6.25	0.536217	0.542646	0.558481	0.522274
7.5	0.52372	0.533609	0.570953	0.545385
8.75	0.491176	0.502424	0.536522	0.510982
10	0.462313	0.475506	0.521582	0.517296
11.25	0.423125	0.437204	0.482395	0.474509
12.5	0.385564	0.400726	0.453116	0.469163
13.75	0.34312	0.358786	0.411611	0.426783
15	0.302603	0.318911	0.375772	0.413004
16.25	0.259531	0.276057	0.333227	0.371834
17.5	0.218839	0.235704	0.295181	0.35224
18.75	0.177567	0.194485	0.253483	0.31076
20	0.138674	0.155743	0.215559	0.286033
21.25	0.099769	0.116686	0.17598	0.244335
22.5	0.034546	0.048454	0.102239	0.162124
26.25	-0.03892	-0.02284	0.034931	0.128535
30	-0.12076	-0.1049	-0.04639	0.081119
33.75	-0.19409	-0.18032	-0.12542	0.005617
37.5	-0.25155	-0.23891	-0.1846	-0.04489
41.25	-0.30215	-0.2926	-0.24614	-0.13681
45	-0.34173	-0.33439	-0.29351	-0.21327
48.75	-0.3767	-0.37354	-0.34586	-0.31463
52.5	-0.40402	-0.40414	-0.38803	-0.39819
56.25	-0.42831	-0.43335	-0.43376	-0.49283
60	-0.44706	-0.45617	-0.47008	-0.56906
63.75	-0.46383	-0.47802	-0.50694	-0.64226
67.5	-0.47641	-0.49483	-0.53695	-0.70445
71.25	-0.48763	-0.51051	-0.56488	-0.76081
75	-0.49549	-0.52207	-0.58653	-0.80691
78.75	-0.50232	-0.53204	-0.60451	-0.84431
82.5	-0.50626	-0.53844	-0.61693	-0.87208
86.25	-0.50935	-0.54284	-0.62468	-0.88796
90	-0.50948	-0.54358	-0.62696	-0.89517

Table 4.6. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 1.0$, $V = T/\sigma_0$, $a/c = 0.4$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.448106	0.444136	0.455607	0.463295
6.25	0.453309	0.45165	0.443716	0.476338
7.5	0.455232	0.455733	0.459608	0.514369
8.75	0.449993	0.452254	0.457775	0.505099
10	0.442475	0.446459	0.459023	0.527097
11.25	0.430145	0.435481	0.450155	0.510474
12.5	0.415806	0.422506	0.441829	0.518839
13.75	0.397772	0.405515	0.427344	0.499657
15	0.378592	0.387412	0.412874	0.496493
16.25	0.356328	0.365931	0.393585	0.473783
17.5	0.333607	0.344049	0.374611	0.461979
18.75	0.30865	0.319671	0.351805	0.435718
20	0.283788	0.295438	0.32956	0.416468
21.25	0.256756	0.268747	0.304349	0.386512
22.5	0.210058	0.221679	0.257673	0.336434
26.25	0.146089	0.158805	0.197383	0.270256
30	0.067745	0.081033	0.121863	0.191195
33.75	-0.01289	-0.00113	0.035945	0.089047
37.5	-0.08353	-0.07255	-0.03712	0.006956
41.25	-0.15173	-0.14342	-0.11496	-0.09492
45	-0.21012	-0.20369	-0.18028	-0.1791
48.75	-0.26468	-0.26179	-0.24904	-0.27711
52.5	-0.31049	-0.31036	-0.30687	-0.36105
56.25	-0.35246	-0.35633	-0.36594	-0.45474
60	-0.38689	-0.39402	-0.41449	-0.53171
63.75	-0.41791	-0.42905	-0.46228	-0.6115
67.5	-0.44247	-0.45688	-0.50125	-0.67894
71.25	-0.4641	-0.48197	-0.53759	-0.74139
75	-0.48007	-0.50074	-0.56567	-0.79188
78.75	-0.49351	-0.5166	-0.58926	-0.83353
82.5	-0.5018	-0.5267	-0.60532	-0.8635
86.25	-0.5078	-0.53375	-0.61567	-0.8816
90	-0.50843	-0.53474	-0.6181	-0.88824

Table 4.7. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 1.0$, $V = T/\sigma_0$, $a/c = 0.6$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.349536	0.344434	0.325798	0.328832
6.25	0.352649	0.348222	0.336372	0.342351
7.5	0.359057	0.35545	0.342975	0.35918
8.75	0.361916	0.359057	0.35103	0.362279
10	0.361894	0.359608	0.352794	0.370385
11.25	0.361313	0.359659	0.354956	0.368868
12.5	0.357591	0.356408	0.353251	0.370595
13.75	0.353462	0.352787	0.350871	0.366313
15	0.34693	0.346662	0.346234	0.363382
16.25	0.339898	0.340044	0.340515	0.356183
17.5	0.330842	0.331313	0.333215	0.349644
18.75	0.321538	0.322373	0.324768	0.339565
20	0.31103	0.31217	0.315348	0.329659
21.25	0.299093	0.300479	0.304557	0.317446
22.5	0.279405	0.281197	0.286241	0.298396
26.25	0.242402	0.244514	0.250363	0.256916
30	0.194485	0.196974	0.204456	0.207271
33.75	0.138062	0.14038	0.146473	0.140407
37.5	0.080616	0.082737	0.088741	0.077297
41.25	0.01749	0.018798	0.021845	-0.00096
45	-0.04433	-0.04382	-0.04274	-0.07389
48.75	-0.10925	-0.11014	-0.1143	-0.16116
52.5	-0.17074	-0.17299	-0.18146	-0.24284
56.25	-0.23269	-0.23677	-0.25208	-0.3339
60	-0.28924	-0.29507	-0.31604	-0.41422
63.75	-0.34385	-0.3517	-0.38	-0.49894
67.5	-0.39137	-0.40107	-0.43548	-0.57291
71.25	-0.43488	-0.44642	-0.48774	-0.64512
75	-0.46992	-0.4831	-0.52985	-0.70342
78.75	-0.49936	-0.51388	-0.5657	-0.75428
82.5	-0.51935	-0.53496	-0.59025	-0.78952
86.25	-0.53278	-0.54895	-0.60662	-0.81284
90	-0.53517	-0.55159	-0.60975	-0.81837

Table 4.8. Normalized T -stress V for semi-elliptical surface cracks under biaxial tension, $\lambda = 1.0$, $V = T/\sigma_0$, $a/c = 1.0$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.216865	0.215544	0.174354	-0.15165
6.25	0.124073	0.124929	0.088232	-0.24771
7.5	0.082099	0.089765	0.060112	-0.29584
8.75	0.01268	0.025554	-0.00764	-0.35902
10	-0.03109	-0.011	-0.03647	-0.37853
11.25	-0.0814	-0.05439	-0.07915	-0.39514
12.5	-0.11608	-0.08127	-0.09644	-0.37915
13.75	-0.14982	-0.10702	-0.11634	-0.35073
15	-0.17431	-0.1233	-0.12143	-0.30295
16.25	-0.19655	-0.13704	-0.12594	-0.24559
17.5	-0.21284	-0.14494	-0.12201	-0.17585
18.75	-0.22726	-0.1507	-0.11624	-0.11088
20	-0.23782	-0.15279	-0.10537	-0.04091
21.25	-0.24731	-0.15372	-0.09415	0.024292
22.5	-0.27028	-0.17393	-0.11216	0.053121
26.25	-0.2683	-0.14098	-0.02517	0.27148
30	-0.27083	-0.12052	0.037519	0.439543
33.75	-0.27001	-0.09614	0.102112	0.55074
37.5	-0.26497	-0.07058	0.163578	0.655312
41.25	-0.25774	-0.0441	0.210073	0.717279
45	-0.24852	-0.01764	0.25103	0.761531
48.75	-0.23905	0.006782	0.278861	0.750958
52.5	-0.22869	0.03094	0.30627	0.746437
56.25	-0.2201	0.051142	0.329615	0.773476
60	-0.21104	0.071231	0.354913	0.809572
63.75	-0.20476	0.086477	0.377837	0.846524
67.5	-0.19798	0.102073	0.401161	0.88262
71.25	-0.19404	0.11246	0.418462	0.912373
75	-0.18949	0.123636	0.437126	0.944129
78.75	-0.1875	0.129443	0.44746	0.965675
82.5	-0.18481	0.136375	0.459731	0.990094
86.25	-0.18443	0.137838	0.462467	0.997671
90	-0.18332	0.140489	0.467104	1.007358

Table 4.9. Normalized T -stress V for semi-elliptical surface cracks under biaxialbending, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 0.2$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.06603	0.053371	0.057078	-0.0161
6.25	0.051004	0.042793	0.038342	0.000139
7.5	0.055636	0.052816	0.065726	0.034766
8.75	0.044518	0.04728	0.061436	0.033925
10	0.03729	0.045875	0.072522	0.061417
11.25	0.022576	0.037436	0.068374	0.059294
12.5	0.0094	0.030636	0.07196	0.082666
13.75	-0.00698	0.021158	0.068963	0.084319
15	-0.02195	0.013109	0.07003	0.10506
16.25	-0.03854	0.003911	0.068136	0.11042
17.5	-0.05361	-0.0038	0.069089	0.129674
18.75	-0.06892	-0.01133	0.068877	0.137333
20	-0.08286	-0.0176	0.070738	0.155522
21.25	-0.09701	-0.02387	0.072223	0.164499
22.5	-0.1257	-0.04596	0.055811	0.153716
26.25	-0.143	-0.03761	0.090651	0.232112
30	-0.16519	-0.03668	0.115483	0.299057
33.75	-0.18335	-0.03132	0.144024	0.355239
37.5	-0.19331	-0.01963	0.17812	0.416284
41.25	-0.20058	-0.00637	0.210181	0.459216
45	-0.20247	0.010666	0.244768	0.50225
48.75	-0.20356	0.026445	0.273556	0.536865
52.5	-0.20106	0.044689	0.30381	0.57365
56.25	-0.19916	0.059654	0.327184	0.604412
60	-0.19462	0.076557	0.352692	0.639006
63.75	-0.19177	0.08889	0.371219	0.671865
67.5	-0.18682	0.103015	0.391517	0.706669
71.25	-0.18431	0.111835	0.404717	0.73627
75	-0.17995	0.122445	0.420131	0.768331
78.75	-0.17851	0.127406	0.427794	0.789332
82.5	-0.17536	0.134204	0.437845	0.812997
86.25	-0.1754	0.135231	0.439615	0.819862
90	-0.17372	0.138128	0.443708	0.828962

Table 4.10. Normalized T -stress V for semi-elliptical surface cracks under biaxialbending, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 0.4$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	-0.05226	-0.06459	-0.06271	-0.07259
6.25	-0.03918	-0.0478	-0.06009	-0.05079
7.5	-0.02846	-0.03338	-0.03555	-0.01065
8.75	-0.02268	-0.02339	-0.02233	-0.00066
10	-0.01848	-0.01506	-0.00641	0.031334
11.25	-0.01721	-0.00911	0.004019	0.039222
12.5	-0.01735	-0.00461	0.015191	0.064718
13.75	-0.01958	-0.00165	0.023857	0.073882
15	-0.02256	0.00052	0.032735	0.094405
16.25	-0.02725	0.001505	0.04006	0.104165
17.5	-0.03221	0.002205	0.047676	0.121546
18.75	-0.03827	0.002301	0.054371	0.131768
20	-0.04425	0.002419	0.061409	0.146851
21.25	-0.0516	0.001485	0.067687	0.156727
22.5	-0.0668	-0.00644	0.067493	0.162293
26.25	-0.08115	-0.00047	0.094765	0.207496
30	-0.09988	0.001929	0.119664	0.25217
33.75	-0.12034	0.004149	0.143544	0.293875
37.5	-0.13503	0.011247	0.17238	0.342351
41.25	-0.14942	0.018623	0.19984	0.385573
45	-0.15849	0.03019	0.230936	0.432148
48.75	-0.16735	0.040742	0.257729	0.474049
52.5	-0.1717	0.054755	0.287068	0.517337
56.25	-0.17652	0.066153	0.31074	0.553512
60	-0.17749	0.080491	0.337054	0.592627
63.75	-0.17982	0.090782	0.356335	0.625493
67.5	-0.1788	0.103712	0.37783	0.659971
71.25	-0.17995	0.111522	0.391546	0.687932
75	-0.17807	0.121821	0.407656	0.717808
78.75	-0.17893	0.126312	0.415573	0.737213
82.5	-0.17697	0.133226	0.425871	0.758679
86.25	-0.1781	0.133964	0.427714	0.76516
90	-0.17647	0.13706	0.431872	0.773157

Table 4.11. Normalized T -stress V for semi-elliptical surface cracks under biaxialbending, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 0.6$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	-0.14655	-0.14975	-0.16027	-0.1556
6.25	-0.13883	-0.13909	-0.14263	-0.13458
7.5	-0.12788	-0.12554	-0.12764	-0.10975
8.75	-0.11939	-0.11398	-0.11062	-0.0944
10	-0.11345	-0.10511	-0.09835	-0.07474
11.25	-0.10719	-0.09557	-0.08474	-0.06115
12.5	-0.10369	-0.08883	-0.07394	-0.04465
13.75	-0.09975	-0.08132	-0.06257	-0.03137
15	-0.09784	-0.07587	-0.05273	-0.01665
16.25	-0.0956	-0.06977	-0.04259	-0.00378
17.5	-0.09505	-0.06536	-0.0335	0.010007
18.75	-0.09394	-0.06008	-0.02405	0.022575
20	-0.09379	-0.05581	-0.01529	0.035607
21.25	-0.09425	-0.05188	-0.00643	0.04825
22.5	-0.09464	-0.04664	0.004724	0.064733
26.25	-0.10006	-0.03839	0.027104	0.099014
30	-0.10585	-0.02866	0.053414	0.139948
33.75	-0.11492	-0.02017	0.078829	0.180595
37.5	-0.12349	-0.01074	0.106584	0.22529
41.25	-0.13428	-0.00216	0.13369	0.269601
45	-0.14337	0.008373	0.163512	0.317308
48.75	-0.15428	0.017602	0.191103	0.36291
52.5	-0.16267	0.02929	0.22135	0.410457
56.25	-0.17266	0.038819	0.248026	0.45455
60	-0.1795	0.051064	0.277322	0.501052
63.75	-0.18799	0.059964	0.301176	0.54225
67.5	-0.19287	0.071666	0.327242	0.584049
71.25	-0.19962	0.078777	0.345922	0.618382
75	-0.20244	0.088681	0.366517	0.652946
78.75	-0.20734	0.092921	0.378331	0.676662
82.5	-0.20812	0.099909	0.391734	0.699848
86.25	-0.21119	0.100509	0.395348	0.708803
90	-0.21009	0.103536	0.400036	0.71606

Table 4.12. Normalized T -stress V for semi-elliptical surface cracks under biaxial bending, $\lambda = 0.5$, $V = T/\sigma_0$, $a/c = 1.0$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.633657	0.615704	0.550247	0.189884
6.25	0.499398	0.481393	0.418853	0.049288
7.5	0.418897	0.40717	0.351698	-0.03605
8.75	0.310929	0.303324	0.243846	-0.13837
10	0.233382	0.233403	0.182663	-0.18837
11.25	0.15148	0.158135	0.108287	-0.23565
12.5	0.089865	0.105251	0.066337	-0.24284
13.75	0.031615	0.055158	0.022727	-0.23744
15	-0.01357	0.019327	-0.00029	-0.20686
16.25	-0.05446	-0.01267	-0.02191	-0.16604
17.5	-0.08647	-0.0351	-0.03079	-0.10812
18.75	-0.11508	-0.05454	-0.03702	-0.05498
20	-0.13757	-0.06744	-0.03478	0.008951
21.25	-0.15795	-0.07875	-0.03204	0.076356
22.5	-0.20435	-0.1281	-0.08324	0.076122
26.25	-0.21081	-0.09541	0.011046	0.301597
30	-0.22761	-0.08632	0.063934	0.452881
33.75	-0.23827	-0.07307	0.116654	0.556626
37.5	-0.24052	-0.05298	0.174702	0.663295
41.25	-0.23959	-0.03244	0.215044	0.714057
45	-0.23443	-0.00866	0.253701	0.756081
48.75	-0.22866	0.012529	0.279253	0.74418
52.5	-0.22053	0.03567	0.306829	0.742003
56.25	-0.21423	0.05397	0.329093	0.77106
60	-0.2064	0.073811	0.355718	0.808592
63.75	-0.20163	0.087769	0.377373	0.844695
67.5	-0.19546	0.103568	0.401814	0.882801
71.25	-0.19254	0.112997	0.418179	0.911408
75	-0.1882	0.124605	0.437845	0.945131
78.75	-0.18692	0.129613	0.447308	0.965457
82.5	-0.18417	0.1371	0.460617	0.991597
86.25	-0.18426	0.137857	0.462424	0.997888
90	-0.18298	0.140869	0.467634	1.008476

Table 4.13. Normalized T -stress V for semi-elliptical surface cracks under biaxial bending, $\lambda = 1.0$, $V = T/\sigma_0$, $a/c = 0.2$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.532337	0.502946	0.48873	0.396713
6.25	0.500682	0.472104	0.446154	0.385595
7.5	0.487794	0.46135	0.449615	0.393142
8.75	0.457192	0.433389	0.420015	0.364086
10	0.430225	0.409724	0.406451	0.364543
11.25	0.394739	0.378324	0.377213	0.335109
12.5	0.361089	0.349274	0.356923	0.332939
13.75	0.323955	0.317583	0.330312	0.309456
15	0.288911	0.288382	0.309303	0.30717
16.25	0.252525	0.258563	0.286001	0.290464
17.5	0.218563	0.231502	0.2673	0.289623
18.75	0.184935	0.205341	0.248164	0.278716
20	0.153774	0.181938	0.232975	0.280639
21.25	0.123039	0.159107	0.21812	0.274173
22.5	0.063024	0.102116	0.163316	0.221727
26.25	0.019728	0.090202	0.183745	0.291597
30	-0.03569	0.06105	0.18172	0.334296
33.75	-0.08116	0.041759	0.187983	0.370254
37.5	-0.11261	0.035613	0.207954	0.421401
41.25	-0.13742	0.034205	0.228316	0.454412
45	-0.15299	0.041072	0.256103	0.493774
48.75	-0.16538	0.048197	0.278795	0.525718
52.5	-0.17154	0.060921	0.306778	0.562714
56.25	-0.17697	0.070756	0.327221	0.592808
60	-0.17785	0.084905	0.352605	0.630457
63.75	-0.17981	0.094129	0.369652	0.663433
67.5	-0.17812	0.107128	0.391016	0.7018
71.25	-0.17872	0.113985	0.403389	0.73156
75	-0.17615	0.124411	0.420283	0.766749
78.75	-0.17663	0.128044	0.427155	0.787511
82.5	-0.17418	0.135195	0.438687	0.813723
86.25	-0.17522	0.135262	0.439478	0.819623
90	-0.17343	0.138588	0.444361	0.829855

Table 4.14. Normalized T -stress V for semi-elliptical surface cracks under biaxialbending, $\lambda = 1.0$, $V = T/\sigma_0$, $a/c = 0.4$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.421509	0.392163	0.376589	0.349122
6.25	0.42463	0.395015	0.361277	0.34902
7.5	0.424717	0.394913	0.367525	0.367097
8.75	0.418737	0.389528	0.361756	0.354485
10	0.410631	0.382104	0.358556	0.364013
11.25	0.398774	0.371778	0.34955	0.349289
12.5	0.385131	0.359797	0.341364	0.352591
13.75	0.368839	0.346067	0.330711	0.339768
15	0.351589	0.331531	0.320515	0.33897
16.25	0.332344	0.315885	0.309136	0.327881
17.5	0.312729	0.300044	0.298411	0.325254
18.75	0.291981	0.283926	0.287337	0.316212
20	0.271451	0.268237	0.277264	0.312961
21.25	0.249521	0.251611	0.266851	0.305247
22.5	0.207489	0.214035	0.234238	0.275573
26.25	0.164136	0.192134	0.234797	0.295116
30	0.107513	0.157446	0.223607	0.304521
33.75	0.052917	0.127929	0.217925	0.318977
37.5	0.008142	0.108357	0.223592	0.347997
41.25	-0.03231	0.093808	0.233287	0.376988
45	-0.06381	0.087646	0.251386	0.414956
48.75	-0.0917	0.083882	0.268382	0.452366
52.5	-0.11206	0.086891	0.291778	0.494993
56.25	-0.1304	0.089313	0.310994	0.530639
60	-0.14239	0.097334	0.335747	0.572903
63.75	-0.15403	0.102157	0.353273	0.607823
67.5	-0.16024	0.111852	0.375595	0.6473
71.25	-0.16757	0.116375	0.388824	0.677634
75	-0.17003	0.125361	0.406713	0.712417
78.75	-0.17464	0.127858	0.414311	0.733171
82.5	-0.17473	0.134619	0.426444	0.758476
86.25	-0.17761	0.13416	0.427533	0.764637
90	-0.17604	0.137612	0.432562	0.774035

Table 4.15. Normalized T -stress V for semi-elliptical surface cracks under biaxial bending, $\lambda=1.0$, $V=T/\sigma_0$, $a/c = 0.6$.

Location ϕ°	$a/t = 0.2$	$a/t = 0.4$	$a/t = 0.6$	$a/t = 0.8$
5	0.332525	0.312032	0.284042	0.271321
6.25	0.333868	0.312112	0.286894	0.273338
7.5	0.338026	0.314652	0.286778	0.278984
8.75	0.33939	0.315051	0.288599	0.275087
10	0.337816	0.312358	0.285363	0.275348
11.25	0.33627	0.310298	0.283549	0.269666
12.5	0.331597	0.305022	0.278636	0.266785
13.75	0.327134	0.300617	0.274528	0.260972
15	0.320276	0.293737	0.268621	0.256575
16.25	0.313614	0.287729	0.263389	0.250769
17.5	0.304869	0.279594	0.256821	0.245943
18.75	0.296655	0.272714	0.251205	0.240385
20	0.287213	0.264608	0.244768	0.235486
21.25	0.276974	0.256168	0.238781	0.230755
22.5	0.259485	0.241284	0.22693	0.221386
26.25	0.230878	0.221241	0.215827	0.216909
30	0.193781	0.195232	0.202221	0.213919
33.75	0.153766	0.171538	0.193991	0.219376
37.5	0.114088	0.149507	0.190116	0.23225
41.25	0.07361	0.131041	0.192634	0.254224
45	0.035426	0.115875	0.200334	0.283353
48.75	-0.00249	0.103917	0.212322	0.318962
52.5	-0.03658	0.096178	0.22963	0.360218
56.25	-0.06985	0.090241	0.248353	0.40373
60	-0.09811	0.089014	0.272206	0.452787
63.75	-0.12545	0.087419	0.293628	0.499594
67.5	-0.14687	0.090704	0.319536	0.549572
71.25	-0.16769	0.091205	0.338795	0.591655
75	-0.18192	0.096566	0.361887	0.635776
78.75	-0.19593	0.096987	0.374877	0.665689
82.5	-0.20292	0.102179	0.391103	0.696321
86.25	-0.21004	0.100945	0.394848	0.707388
90	-0.20961	0.103997	0.400508	0.716546

Table 4.16. Normalized T -stress V for semi-elliptical surface cracks under biaxial bending, $\lambda = 1.0$, $V = T/\sigma_0$, $a/c = 1.0$.

Figure 4.1 Elliptical transformation

Figure 4.2 Typical finite element mesh (part), $a/c=0.4$, $a/t=0.2$

Chapter 5

***T*-stress Calculations for Semi-Elliptical Surface Cracks in Finite Thickness Plates under Biaxial Loading Using Weight Function Method**

In chapter 4, a finite element analysis was conducted to calculate the *T*-stress for surface cracks under simple biaxial tension and bending conditions. However, more complex nonlinear biaxial loads occur in engineering structures. In this chapter, the method of calculating the *T*-stress for semi-elliptical surface cracks under nonlinear biaxial loading conditions is presented. The method is verified using the finite element data calculated in chapter 4.

5.1 Calculation of *T*-Stress Using Weight Function Method

The *T*-stress calculation using the weight function method for the cracked plate under nonlinear uniaxial loading conditions was presented in chapter 3. In this section, the plate under nonlinear biaxial loading conditions is discussed.

From Chapter 3, it was presented that the *T*-stress could be calculated as follows:

$$T(C) = T(C)_{crack \ pressure} + T(C)_{uncrack} = \int_0^a \sigma(x) l(x, a; C) dx + (\sigma_r - \sigma_z)|_C \quad (3.1)$$

For a cracked plate under biaxial loading as shown in Figure 1.3, the stresses in the second direction do not change the crack face pressure because the stresses in the second direction (y direction) are parallel to the crack face. Thus, the first part of Equation (3.1), $T(C)_{crack\ pressure}$ (the T -stress for the cracked body under crack face pressure) is the same for both the uniaxial and biaxial loading cases. This means for both uniaxial and biaxial, the weight functions derived in chapter 3 can be used to calculate the T -stresses generated by the crack face pressure on the cracked body.

Therefore, Equation (3.1) can be used to calculate T -stress for any complex biaxial loading cases. The only difference between uniaxial and biaxial cases is the T -stress for the uncracked body, $T(C)_{uncrack}$ (the second part in Equation 3.1).

For a cracked body under biaxial loading, the stress in the second direction is not zero:

$$\sigma_y(x) = \lambda \sigma_z(x); \quad \text{and} \quad \sigma_x = 0 \quad (5.1)$$

So (from Figure 5.1):

$$\begin{aligned} \theta &= \tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \\ \sigma_r &= \sigma_y(x) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) \end{aligned} \quad (5.2)$$

Substitute Equation (5.2) to the second part of Equation (3.1), the T -stress for the uncracked body under biaxial loading is:

$$T(C)_{uncrack} = \left(\sigma_y(x) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma_z(x) \right) \Big|_{x=a \sin \phi, z=0} \quad (5.3)$$

where $\sigma_y(x)$ and $\sigma_z(x)$ are the nominal stresses in the y and z direction, respectively.

For biaxial tension:

$$\sigma_x = 0; \quad \sigma_y(x) = \lambda \sigma_0; \quad \text{and} \quad \sigma_z(x) = \sigma_0 \quad (5.4)$$

So:

$$T(C)_{uncrack} = \left(\lambda \sigma_0 \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma_0 \right) \quad (5.5)$$

where σ_0 is the nominal tensile stress.

For remote biaxial bending:

$$\sigma_x = 0; \quad \sigma_y(x) = \lambda \sigma_0 \left(1 - 2 \frac{a}{t} \sin \phi \right) \quad \text{and} \quad \sigma_z(x) = \sigma_0 \left(1 - 2 \frac{a}{t} \sin \phi \right) \quad (5.6)$$

So:

$$T(C)_{uncrack} = \lambda \sigma_0 \left(1 - 2 \frac{a}{t} \sin \phi \right) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma_0 \left(1 - 2 \frac{a}{t} \sin \phi \right) \quad (5.7)$$

where σ_0 is the nominal bending.

Equations (5.5 and 5.7) are the T -stress of the uncracked body ($T(C)_{uncrack}$) under biaxial tension and bending. Substitute Equations (5.5) and (5.7) into Equation (3.1), and the T -stress for surface cracks under biaxial tension and bending can be calculated. This method will be verified in section 5.2.

5.2 Validations

In this section, the method described in the last section is used to predict the local T -stresses ($5^\circ \leq \phi \leq 90^\circ$) of surface cracks over a wide range of aspect ratio, $0 \leq a/c \leq 1$ and relative depth, $0 \leq a/t \leq 0.8$ of semi-elliptical surface cracks in finite thickness plates. Two types of loads are used: remote biaxial tension and remote biaxial bending. Two different biaxial ratios ($\lambda=0.5$ and 1.0) are considered. The results are verified using the finite element data calculated in chapter 4.

5.2.1 Weight Functions

The local weight functions were derived in chapter 3:

for $0 \leq x \leq a \sin(\phi)$,

$$t_c(x, a) = \frac{2}{\pi a \sin(\phi)} \left[D_{c1} \left(1 - \frac{x}{a \sin(\phi)} \right)^{\frac{1}{2}} + D_{c2} \left(1 - \frac{x}{a \sin(\phi)} \right)^{\frac{3}{2}} \right] \quad (3.6)$$

for $a \sin(\phi) \leq x \leq a$,

$$t_c(x, a) = \frac{2}{\pi a \sin(\phi)} \left[D_{c3} \left(\frac{x}{a \sin(\phi)} - 1 \right)^{\frac{1}{2}} + D_{c4} \left(\frac{x}{a \sin(\phi)} - 1 \right)^{\frac{3}{2}} \right] \quad (3.7)$$

In chapter 3, the constants D_{c1} , D_{c2} , D_{c3} and D_{c4} in the weight functions were determined using two reference T -stresses (V_0 and V_1 , they can be determined using the curve fitting results (Equations 3.30 and 3.31)) and the boundary conditions (Equation 3.12 and 3.13). The constants are:

$$D_{c1} = \frac{15\pi(-1 + \sin(\phi))}{2(4 \sin(\phi) + 9)} V_0 + \frac{105\pi}{8(4 \sin(\phi) + 9)} V_1 \quad (3.33)$$

$$D_{c2} = \frac{5\pi(-1 + \sin(\phi))}{2(4 \sin(\phi) + 9)} V_0 + \frac{35\pi}{8(4 \sin(\phi) + 9)} V_1 \quad (3.34)$$

$$\begin{aligned} D_{c3} = & + \frac{15\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{3}{2}} (8 \sin(\phi) - 21)}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14 \sin(\phi) + 9 + 4 \sin^3(\phi))} V_0 \\ & + \frac{315\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{3}{2}}}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14 \sin(\phi) + 9 + 4 \sin^3(\phi))} V_1 \end{aligned} \quad (3.35)$$

$$D_{C4} = -\frac{15\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{1}{2}} (8 \sin(\phi) - 21)}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14 \sin(\phi) + 9 + 4 \sin^3(\phi))} V_0$$

$$-\frac{315\pi \sin(\phi) \left(-\frac{-1 + \sin(\phi)}{\sin(\phi)} \right)^{\frac{1}{2}}}{8(-1 + \sin(\phi))(\sin^2(\phi) - 14 \sin(\phi) + 9 + 4 \sin^3(\phi))} V_1 \quad (3.36)$$

The T -stress weight functions presented above can be used to calculate the first part of T -stress ($T(C)_{crack \text{ pressure}}$) for surface cracks under biaxial load.

5.2.2 Remote Biaxial Tension

When the cracked plate under remote biaxial tension, the stress distribution in the crack surface is:

$$\sigma(x) = \sigma_0 \quad (5.8)$$

where σ_0 is the tension stress in the z direction of Figure 1.3.

Substitute Equations (5.8) and (5.5) into (3.1), the total T -stress can then be determined by the following equations for different λ ratios:

When $\lambda=0.5$:

$$T(C) = T(C)_{crack \text{ pressure}} + T(C)_{uncrack}$$

$$= \int_0^{a \sin(\phi)} \sigma(x) t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma(x) t_{C2}(x, a) dx$$

$$+ \left(\sigma_y(x) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma_z(x) \right) \Big|_{x=a \sin \phi, z=0}$$

$$= \int_0^{a \sin(\phi)} \sigma_0 t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma_0 t_{C2}(x, a) dx + \sigma_0 \left(\frac{1}{2} \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - 1 \right) \quad (5.9)$$

When $\lambda=1.0$:

$$\begin{aligned}
 T(C) &= T(C)_{\text{crack pressure}} + T(C)_{\text{uncrack}} \\
 &= \int_0^{a \sin(\phi)} \sigma(x) t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma(x) t_{C2}(x, a) dx \\
 &\quad + \left(\sigma_y(x) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma_z(x) \right) \Big|_{x=a \sin \phi, z=0} \\
 &= \int_0^{a \sin(\phi)} \sigma_0 t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma_0 t_{C2}(x, a) dx + \sigma_0 \left(\cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - 1 \right)
 \end{aligned} \tag{5.10}$$

Results for biaxial tension obtained from the current analysis are presented in Figures 5.1-5.8 for $a/c=0.2, 0.4, 0.6$ and 1.0 , $\lambda=0.5$ and 1.0 . Finite element results obtained in chapter 4 are also plotted.

5.2.3 Remote Biaxial Bending

When the cracked plate is subjected to remote biaxial bending, the stress distribution over the crack surface is:

$$\sigma(x) = \sigma_0 \left(1 - 2 \frac{x}{t} \right) \tag{5.11}$$

where σ_0 is the nominal bending stress in the z direction of Figure 1.3.

Substituting Equations (5.11) and (5.7) into Equation (3.1), the total T -stress can be determined by the following equations for different λ ratios:

When $\lambda=0.5$:

$$\begin{aligned}
 T_{uncrack} &= T_{crack \text{ pressure}} + T_{uncrack} \\
 &= \int_0^{a \sin(\phi)} \sigma(x) t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma(x) t_{C2}(x, a) dx \\
 &\quad + \left(\sigma_y(x) \left(1 - 2 \frac{a}{t} \sin \phi \right) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma(x)_z \left(1 - 2 \frac{a}{t} \sin \phi \right) \right) \Bigg|_{x=a \sin \phi, z=0} \\
 &= \int_0^{a \sin(\phi)} \sigma_0 \left(1 - 2 \frac{x}{t} \right) t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma_0 \left(1 - 2 \frac{x}{t} \right) t_{C2}(x, a) dx \\
 &\quad + \sigma_0 \left(\frac{1}{2} \left(1 - 2 \frac{a}{t} \sin \phi \right) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \left(1 - 2 \frac{a}{t} \sin \phi \right) \right)
 \end{aligned} \tag{5.12}$$

When $\lambda=1.0$:

$$\begin{aligned}
 T(C) &= T(C)_{crack \text{ pressure}} + T(C)_{uncrack} \\
 &= \int_0^{a \sin(\phi)} \sigma(x) t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma(x) t_{C2}(x, a) dx \\
 &\quad + \left(\sigma_y(x) \left(1 - 2 \frac{a}{t} \sin \phi \right) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \sigma(x)_z \left(1 - 2 \frac{a}{t} \sin \phi \right) \right) \Bigg|_{x=a \sin \phi, z=0} \\
 &= \int_0^{a \sin(\phi)} \sigma_0 \left(1 - 2 \frac{x}{t} \right) t_{C1}(x, a) dx + \int_{a \sin(\phi)}^a \sigma_0 \left(1 - 2 \frac{x}{t} \right) t_{C2}(x, a) dx \\
 &\quad + \sigma_0 \left(\left(1 - 2 \frac{a}{t} \sin \phi \right) \cos^2 \left(\tan^{-1} \left(-\frac{a}{c \tan \phi} \right) + \frac{\pi}{2} \right) - \left(1 - 2 \frac{a}{t} \sin \phi \right) \right)
 \end{aligned} \tag{5.13}$$

Results for biaxial bending obtained using the weight function method are plotted in Figures 5.9-5.16 for $a/c=0.2, 0.4, 0.6$ and 1.0 , $\lambda=0.5$ and 1.0 . Finite element results obtained in chapter 4 are also plotted.

5.2.4 Comparison and Validation

Comparisons between the normalized T -stresses calculated using the weight

function method and using the finite element method are shown in Figures 5.1-5.16. The difference between the normalized T -stress values was less than 3% for most points except $a/t=0.8$ and $\phi < 22^\circ$ where the difference was 10%. These results indicate that the weight function method can be used to calculate T -stress for surface cracks under general biaxial loading conditions.

5.3 Discussions and Conclusions

From the results shown in Figure 5.1-5.16, it can be seen that for the biaxial tension $\lambda=0.5$, all points along the crack front are under low constraint (T -stresses are negative); but when $\lambda=1.0$, only the points near the deepest point are under low constraint. For the biaxial bending case, all points along the crack front are under high constraint except when the crack has low relative depth.

Comparing biaxial cases with the uniaxial loading cases (results are presented in Figures 3.9-3.24), it is found that at the deepest point the T -stress is the same for the corresponding uniaxial and biaxial cases, however for all other points, the T -stress is larger for biaxial case than for the uniaxial case. Therefore, the constraint is higher for the biaxial case for all points except the deepest point, and the difference becomes bigger with decreasing ϕ , this means the constraint increases with decreasing ϕ . The constraint also will increase with increasing λ .

In conclusion, the method of calculating T -stress for surface cracks in flat plates under biaxial loadings using the weight function method was presented. The method was verified using the finite element results of remote biaxial tension and biaxial bending cases. The accuracy was very good.

Therefore, the method demonstrated is suitable for T -stress calculations for surface cracks in flat plates under general complex biaxial loading conditions.

Figure 5.1 Stress transformation

Figure 5.2 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=0.2$ $\lambda=0.5$.

Figure 5.3 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=0.2$ $\lambda=1.0$.

Figure 5.4 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=0.4$ $\lambda=0.5$.

Figure 5.5 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=0.4$ $\lambda=1.0$.

Figure 5.6 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=0.6$ $\lambda=0.5$.

Figure 5.7 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=0.6$ $\lambda=1.0$.

Figure 5.8 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=1.0$ $\lambda=0.5$.

Figure 5.9 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial tension $a/c=1.0$ $\lambda=1.0$.

Figure 5.10 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=0.2$ $\lambda=0.5$.

Figure 5.11 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=0.2$ $\lambda=1.0$.

Figure 5.12 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=0.4$ $\lambda=0.5$.

Figure 5.13 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=0.4$ $\lambda=1.0$.

Figure 5.14 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=0.6$ $\lambda=0.5$.

Figure 5.15 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=0.6$ $\lambda=1.0$.

Figure 5.16 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=1.0$ $\lambda=0.5$.

Figure 5.17 Comparison of weight function based normalized T -stress V and FEM data for local points under remote biaxial bending $a/c=1.0$ $\lambda=1.0$.

Chapter 6

Constraint-Based Failure Assessment for Surface Cracked Plates

In the previous chapters, T -stress solutions for semi-elliptical surface cracks under uniaxial and biaxial loading conditions were derived and verified. In this chapter, the application of the calculated T -stresses as the constraint parameter to conduct the constraint-based failure assessment for a surface cracked plate is demonstrated. Both uniaxial and biaxial tension cases are considered.

6.1 Constraint-Based Failure Assessment Diagram

As presented in chapter 2, fracture assessment method for cracked components using the failure assessment diagram approach was well established (PD 6493, 1991; R6, 1997). However, the conventional failure assessment schemes use a lower bound toughness obtained from highly constrained test specimens. Over-conservative assessments may sometimes occur in situations where the crack in a component is under conditions of low constraint. Based on two-parameter fracture mechanics, a framework for the failure assessment including constraint effect was proposed (Ainsworth, and O'Dowd, 1995).

In the conventional failure assessment diagram, the assessment of the potential of

failure is determined by the two calculated parameters, (K_r, L_r) , determined from:

$$K_r = \frac{K_I}{K_{IC}} \quad (2.27)$$

and

$$L_r = \frac{\sigma}{\sigma_L} \quad (2.28)$$

where K_r is the stress intensity ratio with K_I being the stress intensity factor for the crack and K_{IC} the toughness of the material. The parameter L_r is the stress ratio, which is the ratio of the applied stress to the limit stress solution calculated based on the yield stress. Failure is avoided if this point (K_r, L_r) lies within a failure assessment diagram, represented by a curve, $K_r = f(L_r)$ and at the same time L_r is less than a cut-off value $L_r^{\max}$. In present thesis, the failure assessment diagram approach of Level 3 of PD6493 (1991) is used. In PD6493 (1991), a lower bound FAD curve that is independent of geometry and the material stress-strain curve is given by:

$$K_r = (1 - 0.14L_r^2) \left(0.3 + 0.7 \exp(-0.65L_r^6) \right) \quad (6.1)$$

In the constraint-based failure assessment, a fracture resistance K_{mat}^C was used to modify the failure assessment diagram.

$$K_r = f(L_r) \left(\frac{K_{mat}^C}{K_{IC}} \right) \quad (2.29)$$

Following detailed theoretical and experimental analysis (Betegon and Hancock, 1991; O'Dowd and Shih, 1991; Ainsworth and O'Dowd, 1995), this increase in fracture toughness can be represented by:

$$\begin{aligned} K_{mat}^C &= K_{IC} [1 + \alpha(-\beta L_r)^m] & \text{for } \beta < 0 \\ K_{mat}^C &= K_{IC} & \text{for } \beta \geq 0 \end{aligned} \quad (2.30)$$

where α , and m are material dependent constants, which define the dependence of fracture toughness on constraint, and β is a normalized constraint parameter. Substituting (2.30) into (6.1), for $\beta < 0$, we have

$$K_r = (1 - 0.14L_r^2) \left(0.3 + 0.7 \exp(-0.65L_r^6) \right) [1 + \alpha(-\beta L_r)^m] \quad (6.2)$$

Failure is avoided if the assessment point (K_r, L_r) (calculated from Equations.

(2.27) and (2.28)) lies within a failure assessment diagram, represented by a curve (Equation (6.2)), and L_r is less than a cut-off value $L_r^{\max}$.

The normalized constraint parameter β is defined from the hydrostatic Q -stress or the elastic T -stress. In the present analysis, T -stress is used:

$$\beta = \frac{T}{\sigma_Y L_r} \quad (2.32)$$

where σ_Y is the yield stress of the material.

To conduct a constraint-based failure assessment, the T -stress is required in addition to the knowledge of the stress intensity factor and the limit load for cracked components.

6.2 Calculation of Fracture Parameters/Limit Load Solutions

In this section, the calculation of the fracture parameters: stress intensity factor, elastic T -stress, and the limit load for a surface cracked plate is presented. The loading conditions considered are uniaxial and biaxial tension cases. Since we are dealing with a 3D problem, the stress intensity factor and T -stress varies along the crack front.

6.2.1 Stress Intensity Factor Solution

The stress intensity factor solutions for semi-elliptical surface cracks in finite thickness plates were obtained by many authors using the finite element and weight function methods. In the current analysis, stress intensity factors were calculated using the results from Wang and Lambert (1995). In that paper, the stress intensity

factors for semi-elliptical surface cracks of $0 \leq a/c \leq 1$, $0 \leq a/t \leq 0.8$ and $0^\circ \leq \phi \leq 90^\circ$ in flat plate under general linear/nonlinear loads were presented. The stress intensity factor is normalized as follows,

$$F = \frac{K}{\sigma_0 \sqrt{\pi a / Q}} \quad (6.3)$$

where F is the so-called boundary correction factor, σ_0 is the nominal tensile stress and Q is the shape factor of an ellipse, which depends on the crack aspect ratio a/c and is given by the following equation:

$$Q = 1.0 + 1.464 \left(\frac{a}{c} \right)^{1.65} \quad (6.4)$$

The solutions of the boundary factor F for surface cracks with different a/c , a/t and ϕ in a flat plate under uniform tensile is summarized below (Wang and Lambert, 1995).

The normalized stress intensity factors F_θ for uniform tension:

$$\sigma(x) = \sigma_0$$

are:

$$F_\theta(\phi) = \left[\frac{2\phi}{\pi} Y_0 + \left(1 - \frac{2\phi}{\pi} \right) \frac{Z_0}{\sqrt{a/c}} \right] f_\phi g_0 \quad (6.5)$$

where Y_0 is boundary correction factor at the deepest point corresponding to constant stress distribution:

$$Y_0 = B_0 + B_1 \left(\frac{a}{t} \right)^2 + B_2 \left(\frac{a}{t} \right)^4 + B_3 \left(\frac{a}{t} \right)^6 \quad (6.6)$$

where

$$B_0 = 1.0929 + 0.2581 \left(\frac{a}{c} \right) - 0.7703 \left(\frac{a}{c} \right)^2 + 0.4394 \left(\frac{a}{c} \right)^3$$

$$B_1 = 0.456 - 3.045 \left(\frac{a}{c} \right) + 2.007 \left(\frac{a}{c} \right)^2 + \frac{1.0}{0.147 + \left(\frac{a}{c} \right)^{0.688}}$$

$$B_2 = 0.995 - \frac{1.0}{0.027 + \frac{a}{c}} + 22.000 \left(1.0 - \frac{a}{c} \right)^{9.953}$$

$$B_3 = -1.459 + \frac{1.0}{0.014 + \frac{a}{c}} - 24.211 \left(1.0 - \frac{a}{c}\right)^{8.071}$$

Z_0 is the boundary correction factor at the surface point corresponding to a constant stress distribution.

$$Z_0 = \left[C_0 + C_1 \left(\frac{a}{t}\right)^2 + C_2 \left(\frac{a}{t}\right)^4 \right] \sqrt{\frac{a}{c}} \quad (6.7)$$

where

$$C_0 = 1.2972 - 0.1548 \left(\frac{a}{c}\right) - 0.0185 \left(\frac{a}{c}\right)^2$$

$$C_1 = 1.5083 - 1.3219 \left(\frac{a}{c}\right) + 0.5128 \left(\frac{a}{c}\right)^2$$

$$C_2 = -1.101 + \frac{0.879}{0.157 + \frac{a}{c}}$$

The function f_ϕ , an angular function from the embedded elliptical crack solution (Green and Snedden, 1950), is:

$$f_\phi = \left[\left(\frac{a}{c}\right)^2 \cos^2 \phi + \sin^2 \phi \right]^{1/4} \quad (6.8)$$

g_0 is a function which has the form:

$$g_0 = G_0 + H_0 \left(1 - \frac{a}{c}\right)^{q_0} \quad (6.9)$$

$$G_0 = a_0 + b_0 \frac{a}{t}$$

$$H_0 = c_0 + d_0 \frac{a}{t}$$

where

$$a_0 = 1.000 + 0.013 \sin 2\phi - 0.007 \sin 4\phi + 0.014 \sin 6\phi$$

$$b_0 = 0.122 \sin 2\phi + 0.003 \sin 4\phi - 0.03 \sin 6\phi$$

$$c_0 = -0.092 \sin 2\phi + 0.073 \sin 4\phi - 0.007 \sin 6\phi$$

$$d_0 = 0.041 \sin 2\phi - 0.391 \sin 4\phi - 0.058 \sin 6\phi$$

$$q_0 = -4.598 + 29.282 \left(\frac{a}{t} \right) - 15.55 \left(\frac{a}{t} \right)^2$$

Using the Equations (6.3-6.6), the normalized stress intensity factors for local points of a semi-elliptical surface cracks under remote uniaxial tension can be calculated. Note that the weight function method presented in Wang and Lambert (1995) should be used to calculate the stress intensity factors when the cracked plate is subject to nonlinear stress distributions.

The stress intensity factors for the local points of a semi-elliptical surface crack under remote biaxial tension are the same as for a plate under remote uniaxial tension, so Equations (6.3-6.6) also can be used in the remote biaxial tension case.

6.2.2 Elastic T -stress Solution

The weight function method which can be used to calculate T -stress along the crack front of surface cracks with different a/c and a/t values in flat plates under uniaxial and biaxial loads was presented in chapter 3 and 5. Here the weight function method is used to calculate the T -stresses under uniaxial and biaxial tension.

The T -stress is normalized as follows,

$$V = \frac{T}{\sigma_0} \quad (4.8)$$

where V is the normalized T -stress and σ_0 is the nominal tensile stress. Please see sections 3.2 and 3.3 for solutions under uniaxial tension and section 5.2 for solutions under biaxial tension.

6.2.3 Limit load solution

Different limit load solutions for surface cracked plate under uniaxial tensile loads are presented by different authors. For example, Mattheck et al. (1984) proposed an expression for ligament yielding based on Dugdale model calculations. Miller (1988) proposed the limit load based on the effective load carrying area. Most recently, Kim et al (2002) suggested expressions based on detailed finite element analysis. In the current work, Kim's expressions are used:

For uniaxial tension:

$$P_L = 2wt\sigma_Y\eta\left(\frac{\xi}{\zeta + \sqrt{\zeta^2 + \xi}}\right) \quad (6.10)$$

where:

$$\begin{aligned} \xi &= (1 - \zeta)^2 + 2\zeta(\psi - \zeta) \\ \psi &= \frac{a}{t}, \quad \zeta = \frac{ac}{wt} \end{aligned} \quad (6.11)$$

and η is the correction factor for the plate width effect:

$$\eta = \begin{cases} 1 & \text{for } w/c \leq 4 \\ -0.01(w/c) + 1.04 & \text{for } 4 \leq w/c \leq 9 \\ 0.95 & \text{for } w/c \geq 9 \end{cases} \quad (6.12)$$

Here a , c , t , and w are geometry parameters from Figure 3.2, σ_Y is the yield stress.

For biaxial loads, there are no generally accepted limit load solutions for semi-elliptical surface cracks in flat plates. Miller (1988) presented a biaxial limit load solution for 2D edge cracks. In this thesis, Miller's method is extended to 3D cases. It will be demonstrated later that the results fit well with experimental data (Phaal, et al. 1995).

For a semi-elliptical surface crack under remote biaxial tension, the limit load solution is proposed to be:

$$P_L = 2wt\sigma_Y\eta\left(\frac{\xi}{\zeta + \sqrt{\zeta^2 + \xi}}\right) / \sqrt{(1 - \lambda + \lambda^2)} \quad (6.13)$$

Here P_L is the biaxial limit load and λ is the biaxial ratio given in Equation (4.1).

6.3 Failure Assessment of Surface Cracked Plate

Based on the stress intensity factor, T -stress and limit load solutions presented in the above sections, the failure assessment of surface cracked plate is conducted here. First, a surface cracked plate under uniaxial load is assessed. Then the assessment is repeated for a plate under biaxial load. The biaxiality ratios $\lambda=0.5$ and 1.0 will be used. Because the stress intensity factors and the T -stresses are different along the crack front, here, two typical points $\phi=90^\circ$ (deepest point) and $\phi=5^\circ$ (near the surface point) will be considered. The method can be used to conduct a failure assessment for any other point.

In the current analysis, the material used is A533B (the typical nuclear pressure vessel low alloy steel, ASME, 1992). At 70°C , the tensile yield stress is 471 MPa and the ultimate strength is 591 MPa. The Young's modulus of the material is 196.5×10^3 MPa. The initiation toughness of the material J_{IC} is assumed to be 0.38 MJ/m^2 (Phaal, et al. 1995). The material properties are taken from ASME Boiler & Pressure Vessel Code, Section II, Materials, 1992 Edition. The fracture toughness K_{IC} is obtained by converting the J_{IC} value. The constraint related material constants α and m , which define the dependence of the fracture toughness on constraint, are assumed to be 1.5 and 1, respectively (Ainsworth, and O'Dowd, 1995). The thickness of the plate is assumed to be 95.25 mm.

Since the load ratio is defined in terms of the yield strength, L_r can be greater than 1. The typical cut-off is 1.2 for C-Mn Steel and 1.8 for austenitic stainless steel (PD 6493, 1991). In the current analysis, since the flow stress is known for the material used, the ratio of flow stress to the yield stress, $\sigma_f/\sigma_y=1.12$, is used as the cut-off.

Constraint-Based Failure Assessment Diagrams

Using the method presented above, constraint-based failure assessments are conducted. For a given a/c ratio, a series of cracks with different a/t ratios are assumed. The corresponding FADs are generated. Figures 6.1-6.6 are the FADs for the deepest point with $a/c=0.2$ and 1.0 (6.1-6.2 for uniaxial loading; 6.3-6.4 for biaxial loading with $\lambda=0.5$ and 6.5-6.6 for biaxial loading with $\lambda=1.0$). Figures 6.7-6.12 are the FADs for the 5^0 point with $a/c=0.2$ and 1.0 (6.7-6.8 for uniaxial loading; 6.9-6.10 for biaxial loading with $\lambda=0.5$ and 6.11-6.12 for biaxial loading with $\lambda=1.0$). The figures show that using the constraint based failure assessment presented above, the conservatism can be reduced for low constraint conditions.

At the deepest point, the FADs do not change when the biaxiality ratio λ changes from 0 to 1; that is because the constraint parameter (T -stress) remains unchanged. However, at the 5^0 point, the constraint based FADs converge back to conventional FADs as λ increases from 0 to 1; that is because at the surface point, the constraint parameter (T -stress) increase from negative to positive as λ increases from 0 to 1. The effect of a biaxial load on the constraint will increase with the decrease of ϕ , it has no effect at the deepest point and has the biggest effect at the surface point as we observed in chapter 5.

From the failure assessment diagrams, the maximum load carrying capacity for surface cracks under uniaxial and biaxial loading conditions can be calculated.

The resulting maximum axial stress is normalized as follows:

$$P_n = \frac{\sigma}{\sigma_f} \quad (6.14)$$

where σ_f is the flow stress of the material equals $0.5(\sigma_y + \sigma_u)$, with σ_y and σ_u are the yield and ultimate strengths, respectively. The resulting maximum normalized axial stress P_n for the deepest point for $a/c=0.2$ and 1.0 with different a/t ratios is presented in Figures 6.13-14 (uniaxial tension), Figure 6.15-16 (biaxial tension $\lambda=0.5$) and Figures 6.17-18 (biaxial tension $\lambda=1.0$). From the figures, it also can be seen that the maximum load carrying capacity is increased using the constraint-based failure assessment diagram.

From the above calculations, the low constraint has a significant effect on the allowable tensile stresses, and it can be observed that it is overly conservative to exclude the low constraint effect. By accounting for the effect of low constraint, the increase in the allowable tensile stress can be as much as 16%-38% in the case of uniaxial and biaxial tension cases.

Comparison with the Experimental Results

The Welding Institute (TWI) conducted a series of test programs investigating the effects of biaxial loading on the fracture behavior of A533B pressure vessel steel (). All their tests have been assessed to the conventional R6/PD6493 procedures, showing that the conventional R6/PD6493 procedures are safe and conservative for assessing A533B under the action of a biaxial stress field. They have the result for biaxial ratio $\lambda=0$ (uniaxial), $\lambda=0.5$ and $\lambda=1$. The dimensions of the test specimens as shown in Figure (1.3) and the maximum load carrying stress P_m are summarized in Table 6.1.

In the thesis, constraint-based failure assessments are conducted based on the deepest point for the 12 testing specimens. The resulting FADs are shown in Figures 6.19-21. The testing results, i.e., assessment points (K_r , L_r), are directly obtained from TWI paper (Phaal, et al. 1995).

From the uniaxial tension case (Figure 6.19), it can be seen that the constraint-based failure assessment presented are safe and the conservatism can be reduced. From the biaxial cases (Figures 6.20-21), it is also clear that the conservatism can be reduced using the constraint based failure assessment.

Note that, for the deepest point, the T -stress for the uniaxial load is the same as the biaxial load. So in the constraint-based failure assessment, the only difference for these two loading conditions is the limit load solution. From the biaxial limit load solution presented in this thesis, it can be predicted that for $\lambda=1$, the load capacity is the same as for the uniaxial case, but for $\lambda=0.5$, the load capacity can be 15.5% larger than the uniaxial case. This agrees with the experimental data: for the $\lambda=0$ (uniaxial) and $\lambda=1$, the final stress P_m is around 500 MPa, and for $\lambda=0.5$, is around 600 MPa.

This suggested that the limit load solution proposed in the thesis is a reasonable extension from 2D to 3D.

6.4 Conclusions

Using the T -stress solutions derived in the previous chapters, constraint-based failure assessments were conducted for surface cracks under uniaxial and biaxial tension loadings. It was demonstrated that the conservatism by conventional failure assessments can be reduced significantly under low constraint conditions, and the results agree with experimental observations.

Note that, although only uniaxial and biaxial tension loadings are considered in this chapter, the T -stress results developed in the previous chapters can be used for the constraint-based failure assessment for any complex uniaxial and biaxial loading cases.

Specimen No	λ	T(°C)	t(mm)	W(mm)	a(mm)	2c(mm)	P _m (N/mm ²)
M01-01	0	70	51.5	496	24.9	145.8	476
M01-10	0	-129	51.0	500	25.9	135.0	279
M01-11	1	70	25.0	500	17.0	135.0	460
M01-12	0	70	25.0	500	14.6	135.0	447
M01-13	0.5	70	25.0	500	13.4	148.5	618
M01-20(1)	1	-129	51.0	500	24.2	134.5	287
M01-20(2)	0	-66	25.0	500	10.4	135.3	501
M01-21	1	-70	25.0	500	11.3	143.0	491
M01-22	0.5	-70	25.0	500	11.0	145.0	625
M01-23	0.5	-94	26.3	500	10.6	221.1	547
M01-24	1	-97	27.0	500	11.5	145.0	462
M01-25	1	-103	50.5	500	34.3	135.9	294

Table 6.1 Summary of TWI Test Results (Phaal, et. al., 1995)

Figure 6.1 Failure Assessment Diagram for Deepest Point under Uniaxial Tension $a/c = 0.2$

Figure 6.2 Failure Assessment Diagram for Deepest Point under Uniaxial Tension $a/c = 1.0$

Figure 6.3 Failure Assessment Diagram for Deepest Point under Biaxial Tension $a/c = 0.2$ $\lambda = 0.5$

Figure 6.4 Failure Assessment Diagram for Deepest Point under Biaxial Tension $a/c = 1.0$ $\lambda = 0.5$

Figure 6.5 Failure Assessment Diagram for Deepest Point under Biaxial Tension $a/c = 0.2$ $\lambda = 1.0$

Figure 6.6 Failure Assessment Diagram for Deepest Point under Biaxial Tension $a/c=1.0$ $\lambda=1.0$

Figure 6.7 Failure Assessment Diagram for 5 Degree Point under Uniaxial Tension $a/c = 0.2$

Figure 6.8 Failure Assessment Diagram for 5 Degree Point under Uniaxial Tension $a/c = 1.0$

Figure 6.9 Failure Assessment Diagram for 5 Degree Point under Biaxial Tension $a/c=0.2$ $\lambda=0.5$

Figure 6.10 Failure Assessment Diagram for 5 Degree Point under Biaxial Tension $a/c=1.0$ $\lambda=0.5$

Figure 6.11 Failure Assessment Diagram for 5 Degree Point under Biaxial Tension $a/c=0.2$ $\lambda=1.0$

Figure 6.12 Failure Assessment Diagram for 5 Degree Point under Biaxial Tension $a/c=1.0$ $\lambda=1.0$

Figure 6.13 Comparison of Conventional FAD and Constraint-Based FAD
Uniaxial Tension Deepest Point $a/c=0.2$.

Figure 6.14 Comparison of Conventional FAD and Constraint-Based FAD
Uniaxial Tension Deepest Point $a/c=1.0$.

Figure 6.15 Comparison of Conventional FAD and Constraint-Based FAD
Biaxial Tension Deepest Point $a/c=0.2$ $\lambda=0.5$.

Figure 6.16 Comparison of Conventional FAD and Constraint-Based FAD
Biaxial Tension Deepest Point $a/c=1.0$ $\lambda=0.5$.

Figure 6.17 Comparison of Conventional FAD and Constraint-Based FAD
Biaxial Tension Deepest Point $a/c=0.2$ $\lambda=1.0$.

Figure 6.18 Comparison of Conventional FAD and Constraint-Based FAD
Biaxial Tension Deepest Point $a/c=1.0$ $\lambda=1.0$.

Figure 6.19 Comparison of Experimental Data and Constraint-Based FAD Uniaxial Tension Deepest Point.

Figure 6.20 Comparison of Experimental Data and Constraint-Based FAD
Biaxial Tension Deepest Point $\lambda=0.5$.

Figure 6.21 Comparison of Experimental Data and Constraint-Based FAD
Biaxial Tension Deepest Point $\lambda=1.0$.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

The T -stress weight functions for the deepest point and local points over the entire geometric region, $0 \leq a/c \leq 1.0$ and $0 \leq a/t \leq 0.8$, of semi-elliptical surface cracks under uniaxial load were derived. Reference T -stress solutions and the appropriate boundary conditions on T -stress weight functions were used to derive those weight functions. The weight functions were verified using the T -stress solutions for non-linear crack surface pressure loadings (Wang and Bell, 2003) and remote tension and bending (Wang, 2003). Excellent accuracy was achieved.

Three dimensional finite element analyses for semi-elliptical surface cracks in finite thickness plates under biaxial loadings have been conducted. The T -stresses were determined for surface cracks with aspect ratios, $a/c=0.2, 0.4, 0.6$ and 1.0 , and relative depth, $a/t=0.2, 0.4, 0.6$ and 0.8 , under remote biaxial tension and bending. Two different biaxiality ratios ($\lambda=0.5$ and 1) were considered.

A method of calculating T -stress for surface cracks in flat plates under complex biaxial loadings using the weight function method was presented. The method was verified using the finite element results for remote biaxial tension and biaxial bending cases. Excellent agreements were achieved.

Using the results of the T -stress, the constraint-based failure assessment for surface cracks under uniaxial and biaxial tension was conducted. The results were

compared with experimental data of TWI (Phaal, et al. 1995). It was demonstrated that the constraint-based fracture assessment was consistent with the available experimental observations and the constraint-based approach can reduce the conservatism significantly under low constraint conditions.

7.2 Recommendations

In this thesis, the T -stress and the weight functions were considered for uniaxial and biaxial nonuniform stress distributions along the x axis, i.e., in the crack depth direction (Figure 1.2). In practice, it is possible to encounter stress distributions which change in the crack length direction as well. Further analysis is required to calculate T -stress for this kind of stress distributions.

In the current research, the T -stress and the weight functions were considered for surface cracks with aspect ratio of $0 \leq a/c \leq 1.0$, relative depth of $0 \leq a/t \leq 0.8$ and $5^\circ \leq \phi \leq 90^\circ$. More finite element calculations are required to generate solutions for aspect ratios in the region of $a/c > 1.0$, relative depth $a/t > 0.8$ and $\phi < 5^\circ$. Especially for the points of $\phi < 5^\circ$, a refined finite element model needed.

The limit load solution for surface cracked plate under biaxial tension conditions used in the thesis was a direct extension from a 2D solution. Although the failure assessment results are consistent with experiment data, more finite element calculations should be conducted to verify the accuracy of this limit load solution.

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