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***Inelastic Moment Redistribution in Multi-Girder
Prestressed Concrete Simply Supported Bridges***

By

Babak Rostami, B.Sc.

A thesis submitted to the
Faculty of Graduate Studies and Research in partial fulfillment
of the requirement for the degree of

Master of Applied Science

**Department of Civil and Environmental Engineering
Faculty of Engineering
Carleton University
Ottawa, Ontario, Canada
March 2002**

The Master of Applied Science in Civil Engineering is a joint program with the
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Submitted by
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in partial fulfillment
of the requirements for the degree of

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Chair, Department of Civil and Environmental
Engineering

Thesis Supervisor

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ABSTRACT

An extensive analytical investigation was undertaken to study the effect of loading and geometric parameters on transverse moment distribution in simply supported prestressed concrete slab-girder bridges in the entire range of loading up to failure. The load distribution for critical loading cases or geometry were determined and compared with the relevant predictions of the CHBD (1999), and the AASHTO (1994) bridge codes in addition to those of some available methods in the literature.

The analytical tool employed was the finite element program NONLACS and only material nonlinearities were considered. The program was modified so it can handle the nonlinear analysis of prestressed structures built in stages. Subsequently, the results of NONLACS were verified and compared with available experimental results.

The parameters studied included longitudinal truck position, transverse truck location, bridge width, span/width ratio, girder spacing (number of girders), and slab/girder stiffness ratio. This study revealed that although at the ultimate limit state (ULS) all the preceding parameters affect the transverse distribution of bridges, they do not have equal effect. Finally, it was determined that due to inelastic transverse redistribution such bridges could be subjected to a load 15% greater than their design capacity based on existing code methods without affecting their safety.

ACKNOWLEDGEMENT

I would like to express my most sincere gratitude to my supervisor, Prof. A.G. Razaqpur, for his constant guidance, advice, encouragement and patience during the course of this research. It has been a great honor for me to work with him, and this will remain as an unforgettable experience in my memory.

My most sincere acknowledgement is to the National Spiritual Assembly of Canada, for their wonderful spiritual, moral, and financial support during my studies in Canada.

I would like to express my deep and heartfelt appreciation to my parents, Mahboobeh and Ali, my brother Borzou, for their constant support, encouragement and motivation, not only during the course of this project, but also throughout my life. Special thanks to my fiancée, Parisa, for her understanding and never faltering support.

Finally, I would like to thank my numerous friends, colleagues, Carleton University faculty members and staff who provided me with a pleasurable, and peaceful environment during every phase of this work.

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LIST OF IMPORTANT NOTATIONS

A	cross sectional area
$[B]$	strain-displacement matrix
$B_{\tau} \ B_{\nu}$	rotation of the normals to the middle surface of the plane
b	half width of the bridge
C_f	correction factor (CHBD code)
$[D]$	constitutive matrix
D_d	distribution parameter
D_f	distribution factor
D_{τ}	longitudinal flexural rigidity per unit length
D_y	transverse flexural rigidity per unit width
$D_{\tau y}, D_{\nu \tau}$	longitudinal torsional rigidity per unit length
D_1, D_2	transverse torsional rigidity per unit width
$\{d\}, \{\delta\}$	vector of nodal displacements
(dM)	vector of unbalanced moments

$\{d\delta\}_i$	vector of incremental displacement due to incremental load
E	modulus of elasticity
$[E]$	Constitutive matrix
E_c	secant modulus of elasticity of concrete at peak stress
E_0	initial tangent modulus of concrete
E_1, E_2	tangent moduli in the principle material directions
e_g	distance between the centroid of the girder and the middle plane of the slab
E_s	Young's modulus of elasticity of steel
E_s'	strain hardening modulus of steel
F	shear force
F	width dimension factor (CHBD code)
$\{f\}$	load vector
F_m	amplification factor (CHBD code)
$\{F\}_b$	vector of nodal forces due to body forces
$\{F\}_s$	vector of nodal forces due to surface forces
$\{F\}_\epsilon$	vector of nodal forces due to initial strains
$\{F\}_{\bar{\sigma}}$	vector of nodal forces due to initial stresses
$\{F\}_b^\epsilon$	element vector of nodal forces due to body forces
$\{F\}_s^\epsilon$	element vector of nodal forces due to surface forces
$\{F\}_{\epsilon}^\epsilon$	element vector of nodal forces due to initial strains

$\{F\}_\sigma^e$	element vector of nodal forces due to initial stresses
$\{F_i\}$	vector of unbalanced load for the i th iteration
f	yield function
f_c'	compressive strength of concrete
f_t'	uniaxial tensile strength of concrete
f_y	yield strength of steel
G	shear modulus
$\{g\}$	vector of body forces
H	strain hardening function
I_G	flexural moment of inertia of girder cross-section
J_G	torsional moment of inertia of girder cross-section
$[K]$	global structure stiffness matrix
$[K]^e$	local element stiffness matrix
K_σ	longitudinal stiffness parameter
$[K]_{pp}$	inplane element stiffness matrix
$[K]_{bb}$	bending element stiffness matrix
$[K]_{pb}, [K]_{bp}$	coupling stiffness matrices
L	span length
M	total moment on a given section of a bridge
M_{AASHTO}	maximum moment in a single girder due to AASHTO truck loading

M_G	girder moment
M_T	total longitudinal moment on a given section of a bridge
M_y	yield moment of girder
N	No. of trucks
N, N_G	number of girders
N, N_L	number of design lanes
n	modular ratio of girder and deck materials
$[N]$	shape function matrix
NL	number of traffic lanes
P	concentrated load
$\{P\}$	vector of surface forces
$\{R\}$	vector of applied nodal forces
R_L, R_f	modification factor for multiple lane loading
S	girder spacing
$[T]$	transformation matrix
t_s	concrete slab thickness
u, v, w	displacements in x y and z directions
V^e	element volume
W_e	width of a design lane load in meters
α	torsional parameter of bridge
β	cracked shear constant

$\{\Delta f_i\}$	vector of incremental applied load
$\{\Delta \delta_i\}$	vector of total displacements due to incremental load
$\{\delta_i\}$	vector of total displacements due to total applied load
$\{\epsilon\}$	vector of strains
$\left\{\bar{\epsilon}\right\}$	vector of initial strains
$\epsilon_x, \epsilon_y, \gamma_{xy}$	normal and shear strains in plane x-y
ϵ_{cu}	concrete strain at ultimate strength
ν_c	Poisson's ratio of concrete
ν_s	Poisson's ratio of steel
$\{\sigma\}$	vector of stresses
$\left\{\bar{\sigma}\right\}$	vector of initial stresses
$\sigma_x, \sigma_y, \tau_{xy}$	normal and shear stresses in plane x-y
θ_x, θ_y	rotations normal to x-y plane
$\{\chi\}$	vector of curvature due to bending

Chapter 1

INTRODUCTION

1.1 General

Highway bridges are mostly made of steel, reinforced concrete, and prestressed concrete. The latter type is mostly used for its high efficiency and long span coverage capability, consequently, prestressed bridges represent 80% of all long span bridges in North America.

Live loads have been increasing on bridges due to higher permitted loads in new codes. Older bridges designed for smaller live loads are often posted with load limitation, i.e. they are restricted to certain types of traffic. Obviously, the restrictions create detour and increase the time and cost of traveling. The posted loads are, however, obtained from relatively simplified analysis, which do not always reflect the true load carrying capacity of these bridges. The linear elastic method of analysis used to obtain the distribution of internal forces in a member at ultimate limit state do not correspond to the actual forces due to the change in the stiffness of the members caused by cracking and material nonlinearities. The actual ultimate capacity of a bridge depends directly on the extent of

moment and shear redistribution. The consequence of a refined analysis may be a better assessment of the load carrying capacity of a bridge, which may render unnecessary restrictive postings and eliminate the inconvenience and cost associated with detours, or in some cases, with the unnecessary retrofit work.

Existing highway bridge codes, such as Canadian Standards Association (CSA 1998), Canadian Highway Bridge Design Code (CHBD 2000), or the American Association of Highway and Transportation Officials Bridge Code (AASHTO 1994) provide different methods for the prediction of the ultimate load capacity of bridges. This is achieved by multiplying the longitudinal and transverse moments and shears obtained from elastic analysis by certain factors. Such simplifications are generally adequate for design purposes because they are on the safe side; however, they cannot be used to determine the actual distribution of loads in the inelastic and near ultimate limit states. In statically indeterminate bridges, when a part of a bridge section cracks or yields, the intact parts carry a greater proportion of the load compared to the amount shown by elastic analysis. When the redistribution is taken into account, the ultimate load capacity of the bridge increases significantly compared to what one obtains based on elastic analysis. Therefore, in order to realistically evaluate the load carrying capacity of existing bridges, studies are needed to determine the distribution of load beyond the elastic range, particularly at the ultimate limit state. In conducting such studies, it is important that the effects of different parameters at the appropriate stages of loading are considered. This includes the material and geometric properties of the different parts of the bridge. In this thesis we will determine the inelastic load-distribution characteristics of in-situ concrete slab on precast prestressed concrete girder composite bridges. The important parameters

affecting the longitudinal moment redistribution in simple span multigirder bridges will be investigated in detail. The study is motivated by the fact that a large number of existing bridges are of this type and no study has been previously conducted to determine the level and significance of redistribution in such bridges.

1.2 Objectives and Scope

The objective of the work described herein is to determine how at various levels of loading, including ultimate limit state, truck loads are transversely distributed by the concrete slab to the precast prestressed concrete girders in multi-girder simple span bridges. Specifically, the answers to the following questions are sought:

- (i) What is the effect of some key parameters on the inelastic and ultimate strength of composite concrete slab on prestressed concrete girder bridges?
- (ii) What are the magnitude and distribution of moment in each girder and how do these vary with increased loading?
- (iii) Knowing the moment distribution characteristics of these bridges, how do the predictions of the existing codes, such as the Canadian Highway Bridge Design (CHBD 2000) and the American Association of State Highway and Transportation Officials (AASHTO1994) codes, compare to advanced analysis

results and should they be modified to reflect the actual load capacity and distribution of moments in prestressed concrete girder bridges?

In seeking the answer to these questions, the following procedure will be followed.

- (i) The establishment of some key parameters that affect the load distribution characteristics of prestressed concrete composite bridges. In this regard a number of bridges will be designed according to the CHBD code provisions and analyzed to investigate the effect of those parameters on their moment redistribution at different loading stages.
- (ii) In order to capture their full response over the entire range of loading up to failure, the designed bridges will be analyzed by the nonlinear finite element method. The effect of concrete cracking and nonlinearity as well as steel yielding will be considered in the analysis.
- (ii) Based on the result of the parametric study in the above steps, codes predictions of the moment redistribution will be compared with the finite element result and suggestions for possible modifications will be made.

Economic consideration and expediency are the main reasons for choosing analytical versus experimental methodology. Since a number of parameters will be investigated, many bridge models would be required for experimental investigation, which demands a huge amount of money and other resources, while the finite element analysis costs only a fraction of the money needed for experimental investigation. In addition, modelling bridges with realistic dimensions, which is practically impossible in experimental investigations, is another advantage of finite element analysis.

In order to consider the influence of a large number of parameters, the analysis will deal with the behaviour of four, five and six girder bridges. The analysis is limited to material nonlinearities, therefore, large deformations and buckling are not considered. Finally, the scope of this work is restricted to straight, simply supported prismatic members.

Chapter 2

LITERATURE REVIEW

2.1 General

The inelastic behaviour of multi-girder precast prestressed concrete bridges has not been extensively investigated in the past. Past studies have focused mainly on either uncracked sections at the serviceability limit state or on fully cracked sections at ultimate state. However, as far as transverse moment distribution is concerned, previous studies have been almost exclusively focused on linear elastic behaviour of uncracked sections. The purpose of this study is to find the transverse moment redistribution in this kind of bridges at various load levels. Therefore, the moment distribution in prestressed concrete slab-girder bridges at the elastic, inelastic and ultimate states is investigated. The following section will briefly review some of the previous works related to this topic. However, it should be mentioned that very few similar studies are available with regard to transverse moment distribution in prestressed concrete slab-girder bridges.

2.1 Previous work

An analytical procedure for determining transverse shears and moment for both composite and non-composite bridges was developed by Newmark (1948). He analyzed a number of bridges using simplified assumptions based on elastic theory. He

recommended the following relations for transverse distribution of total longitudinal moment at any cross section in multi-girder bridges and presented the result of his work in a series of tables containing the fixed-end moment, distribution factors, and the carry-over factors for both non-composite and composite slab on girder bridges.

$$M_G = D_f M_T \quad (2.1)$$

$$D_f = \frac{S}{K} \quad (2.2)$$

where M_G is the design live load moment of a given girder at the section of interest, M_T is the maximum moment that can be produced in the same girder by a single line of wheels of the design truck, D_f is the distribution factor, S is the girder spacing and K is a constant. Newmark suggested $K = 1.676$ m. It is interesting that the current AASHTO code (1994) uses this formula for girder spacing up to 1.829m (6.0 Ft) in order to determine the design moment for each girder in composite bridges. Some experimental research was carried out by Newmark at the University of Illinois (1948) to verify the above equations.

The Ontario Highway Bridge Design Code (OHBD 1991) and CHBD (2000) codes, too, adopt the basic approach of Newmark for calculating the live load design moment for girders. The maximum live load moment in each girder is obtained by multiplying the distribution factor, D_f , of that girder by the maximum moment caused by the given live load.

In the early fifties, to better assess the live load distribution in slab-girder bridges, more rigorous analyses were performed based on the orthotropic plate theory. In this approach, an equivalent orthotropic plate is analyzed instead of the actual bridge. This transformation, however, in many cases cannot fully simulate the real bridge behaviour.

Giencke (1955) introduced an exact theory of the so-called orthogonally stiffened plate, which resulted in an eight-order differential equation. The constants of this equation are function of the bridge geometric and material properties. Unfortunately, obtaining a closed-form solution of such a high order equation is nearly impossible. Therefore, in order to simplify the equation, different assumptions need to be made so that the resulting solution is applicable to bridges with specific loading and boundary conditions.

With the introduction of computers into engineering calculations, bridge engineers could use refined numerical techniques to study load distribution problems. Different bridge responses were analyzed and evaluated for various loading conditions. For example, Heins and Loony (1968) and Heins (1976) developed a computer program, based on the slope-deflection method. They modeled the bridge as a continuous orthotropic plate resting on flexible supports (bridge girders). They recommended that this method be used for analyzing bridges with torsionally stiff girders.

Other researchers developed simplified methods for moment distribution in slab-girder bridges. Bakht et al. (1978) used the concept of orthotropic plate and the grillage analogy method for this purpose. In their research, they conducted extensive parametric studies, which led them to find out that the distribution factor of bridges is related to a

torsional parameter α and a flexural parameter θ , which are function of geometry and material properties of the bridge. These parameters are given by:

$$\alpha = \frac{D_{xy} + D_{yx} + D_1 + D_2}{2(D_x D_y)^{0.5}} \quad (2.3)$$

$$\theta = \frac{b}{L} \left(\frac{D_x}{D_y} \right)^{0.25} \quad (2.4)$$

where b is half-width of the bridge, L is the span of the bridge and the various rigidities are given by:

$$D_x = \frac{E_G I_G}{S} + \frac{E_c t^3}{12} \quad (2.5)$$

$$D_y = \frac{E_c t^3}{12(1 - \nu_c^2)} \quad (2.6)$$

$$D_{xy} = \frac{G_G J_G}{S} + \frac{G_c t^3}{6} \quad (2.7)$$

$$D_{yx} = \frac{G_c t^3}{6} \quad (2.8)$$

$$D_1 = D_2 = \nu_c D_y \quad (2.9)$$

in which E , G and ν are the Young's modulus, the shear modulus and the Poisson's ratio respectively, t is the concrete slab thickness, S is the girder spacing, I_G and J_G are the

flexural and torsional moment of inertia of the girder cross section. The subscript G refers to girder and c refers to the concrete slab.

It should be mentioned that this method gives better results than the AASHTO recommendations which assume that the girder spacing S is the only parameter that affects load distribution in slab on girder bridges. This method is the basis of the OHBD (1991) code as well as the CHBD (2000) code provision.

The introduction of limit state design encouraged some researchers to employ limit state design philosophy for evaluating the behaviour of slab-on-girder bridges. Heins and Kuo (1975) used the finite difference method in conjunction with bilinear stress-strain relationship for bridge materials (concrete and steel) to analyze composite bridges at inelastic and ultimate limit states. They suggested a load distribution factor for ultimate limit state. Unfortunately, their study had a narrow scope and the effects of very few parameters were investigated.

Ko stem (1984) used linear finite element analysis to determine the transverse distribution of moments in composite bridges. He proposed the following equation for load distribution factor in concrete slab-on-girder bridges:

$$D_f = \left(2 \frac{N_L}{N_B} \right) + K_1 \left(\frac{S}{L} \right)^n + K_2 \quad (2.10)$$

where N_L , N_B , S , and L are the number of traffic lanes, number of girders, girder spacing and span of the bridge. K_1 and K_2 are geometric constants that depend on bridge width, number of girders, and number of traffic lanes, and n is a constant.

Other investigations were carried out by Heins and Jin (1984) and Roger (1986) to study the response of curved composite bridges in overload cases. Heins and Jin (1984) used finite difference method while Roger (1986) used the finite element method. They introduced different distribution factors for determining the moment carried by each girder.

Fan (1974) proposed the following equation, based on experimental results, for moment distribution factor at ultimate:

$$D_f = \frac{2 \times \text{Number of traffic lanes}}{\text{Number of Girders}} \quad (2.11)$$

A verification of the above equation was carried out by Fu and Heins (1976). They carried out inelastic analysis similar to Heins and Kuo (1975), but due to the limitation of the number of bridges studied and the range of the parameters, they could not reach a definite conclusion.

Regarding continuous composite bridges, some experiments have been carried out by Kennedy and Grace (1983). They came to the conclusion that use of diaphragms in such bridges enhances the distribution as well as the effectiveness of the plate theory in predicting the elastic response of a continuous composite bridge.

Although many experimental studies on different aspects of composite structures have been conducted, one experimental work specifically related to load distribution in slab-on-girder bridges was carried out by Chung et al (1986). They tested a simply supported bridge model, which consisted of six steel girders with steel end diaphragms and a cast-in-place concrete deck slab under static loading. They verified the results of

the finite element program ANSYS by achieving good comparison between the experimental and the analytical results. They analyzed a number of bridges and concluded that the distribution parameter given by OHBD (1983) are 15% to 17% conservative and the distribution of moment at the ultimate state is different from the elastic stage and that the diaphragms have no significant effect on the transverse distribution of moments.

A new elastic load distribution factor for bridges was introduced by Zokaie et al (1991). This new distribution factor is the basis of the AASHTO (1994) code provisions, and is based on experimental and analytical research. According to this study, the transverse distribution of longitudinal bending moment for girder bridges is given by the following equation:

$$D_f = 0.15 + \left(\frac{S}{3}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left[\frac{n(I + Ae^2)}{12Lt^3} \right]^{0.1} \quad (2.12)$$

Using the finite element method, Tarhini and Fredrick (1992) conducted a parametric study of elastic moment distribution in composite bridges and they managed to develop the following equation for the distribution factor:

$$D_f = .00013L^2 - 0.021L + 1.25\sqrt{S} - \frac{(S + 7)}{16} \quad (2.13)$$

Sennah and Kennedy (1997) performed a theoretical and experimental investigation to determine the effect of several variables on the moment and shear distribution in simply supported, straight, composite, concrete deck-steel multicell box girder bridges. They deduced some expressions to compute the moment distribution factors for each girder in bridges with different number of lanes:

For the outer girder of one-lane bridges:

$$D_{MS} = N_C^{-0.7} \quad (2.14)$$

For the outer girder of two-, three-, and four-lane bridges:

$$D_{MS} = 1.15(N_L)^{0.5}(N_C)^{-0.7}(L)^{0.05} \quad (2.15)$$

For the inner girders of one-lane bridges:

$$D_{MS} = 1.6(N_C)^{-0.8}(L)^{-0.04} \quad (2.16)$$

For the inner girder of two-, three-, and four-lane bridges:

$$D_{MS} = 2.3(N_L)^{0.65}(N_C)^{-0.9}(L)^{0.04} \quad (2.17)$$

where D_{MS} is the moment distribution, N_C number of cells, N_L number of lanes, and L is the bridge span.

A theoretical expression for the percentage of redistribution of moment in two span continuous prestressed concrete beam was suggested by Kodur and Campbell (1996). Considering two types of loadings, they suggested:

For beams with concentrated load:

$$x = 60[1 - e^{-\left(\frac{MR}{0.7}\right)}] \quad 0 \leq x \leq 60 \quad (2.18)$$

For beams with uniform dead load:

$$x = 45[1 - e - (\frac{MR}{0.7})] - 10 \quad 0 \leq x \leq 30 \quad (2.19)$$

where x is the percentage redistribution of moment, e is the cable eccentricity, and MR is the moment ratio which is defined by :

$$MR = \left(\frac{M_c + M_{sec}}{M_c} \right) \left(\frac{T_f (M_B + aM_c)}{a(1-a)S_1 (M_c + M_{sec})} - 1 \right) \quad (2.20)$$

where M_B and M_c are the ultimate moment capacities of the span and support critical sections, a is the ratio of the distance of the span critical section from the end support to the span length, S_1 is a factor used in defining the bending moment at the central support, $T_f = 1.0$ for a concentrated load and 2.0 for a uniformly distributed load in each span, M_{sec} is the secondary moment due to prestressing. Equation 2.21 is actually for longitudinal redistribution which is not the subject of this study.

Shahawy and Dongzhou (2001) based on extensive theoretical analysis, together with field testing, proposed some modifications to the load distribution formulas of the AASHTO code in the case of concrete slab-girder bridges. According to their investigation, the AASHTO code provisions are accurate enough for one design lane load but for two or more design lanes loaded some modification should be applied as follows:

Interior girder:

$$g_{interior} = g_{LRFD} (1 - \phi) \quad (2.21)$$

Exterior girder:

$$g_{exterior} = e \times g_{interior} \quad (2.22)$$

where g_{LRFD} is calculated based on AASHTO formula and $\varphi = \varphi_1 \times \varphi_2$. Parameters φ_1 and φ_2 were derived by them and are given in charts, $e = 0.77 + d_e / 2800 \geq 1.0$, where d_e is the overhang distance, i.e. is the distance between the centre of exterior girder and interior edge of curb or traffic barrier.

Razaqpur and Nofal (1988) carried out an extensive parametric study and stated that for simply supported composite steel girder bridges, 6% redistribution of moment can be allowed at ultimate state. Razaqpur and Esfandiari (2001) carried out another parametric study to determine the longitudinal moment redistribution in continuous composite bridges and concluded that the longitudinal moment redistribution at ultimate is significant and varies from 23% to 45% at the maximum negative moment section.

Ebeido and Kennedy (1996) used the finite element method to study continuous composite skew bridges. The results from tests on three continuous composite bridges were used to verify the finite element program. According to these results, they recommended an empirical formula for the distribution factor at mid-span and at the supports.

2.3 Methods of Analysis:

Basically two methods of analysis have been used to determine the redistribution of moments: elastic method and plastic method of analysis. Elastic analyses consist of all sorts of classical and numerical methods. This includes the idealization of bridges by

assumed simplified structures, such as simple beams and grillage, orthotropic plate, finite strip, and finite difference methods.

More refined methods such as the plastic method includes the plastic hinge concept, yield line theory and elasto-plastic numerical methods. In the latter method, the load is incrementally applied and the state of stresses is checked at each increment. In this regard, there is no difference between the finite difference and finite element method. We will not describe all the above methods here. Only a more detailed description of the incremental finite element solution technique will be given in the next chapter.

2.4 Ultimate distribution factor based on nonlinear analysis

Heins and Kuo (1975) used the finite difference method to determine the load distribution factor of slab-girder bridges at ultimate state. They used an elasto-perfectly plastic finite difference model for this purpose. They used two methods to calculate the distribution factor at the ultimate limit state:

Method 1: In this method, based on the fraction of the total moment on the entire section, the distribution factor is given by the following equation:

$$D_f = \frac{2 \left(\text{Max. } \frac{M_y}{M} \right) M_y}{N \times M_{AASHTO}} \quad (2.23)$$

where

M = total longitudinal moment on a given section of the bridge

M_y = Yielding moment of girder

N = No. of trucks / No. of traffic lanes

M_{AASHTO} = Maximum moment in a single girder due to AASHTO truck loading

Method 2: In this method D_f is given based on the fraction of a line of truck wheels:

$$D_f = \frac{\left(\text{Max. } \frac{M_y}{M} \right) (2 \times NL)}{\sum \frac{M}{M_y}} \quad (2.24)$$

where

NL = No. of traffic lanes

$\sum \frac{M}{M_y}$ = Sum of moment ratios for all girders across the bridge

2.5 Code Approaches

There are two important codes for designing bridges in North America: Canadian Highway Bridge Design (CHBD/2000) and the American Association of State Highway and Transportation Officials (AASHTO/1994) codes. In both of these codes the distribution of longitudinal moment in the transverse direction is determined using:

$$M_G = D_f M_T \quad (2.25)$$

where M_G is the design live load moment for a girder, M_T is the maximum moment that can be produced in the same girder by a single truck load, and D_f is the distribution factor. In the CHBD code, D_f is determined as follows:

$$D_f = \frac{F_m n R_L}{N} \quad (2.26)$$

where

n = Number of design lanes

R_L = Modification factor for multiple lane loading

N = Number of girders

F_m = Amplification factor given by

$$F_m = \frac{S N}{F \left(1 + \frac{\mu C_f}{100} \right)} \geq 1.05 \quad (2.27)$$

in which

$$\mu = \frac{w_e - 3.3}{0.6} \leq 1 \quad (2.28)$$

w_e = Width of design lane in meters

S = Center-to center girder spacing in meters

C_f = Correction factor in %

F = width dimension factor

F and C_f are obtained from the specified tables in the code. For bridges with more than four design lanes, F is calculated from the following.

$$F = F_4 \frac{n R_L}{2.80} \quad (2.29)$$

where F_4 is the value of F for a four-design lane bridge.

In the AASHTO code the distribution factor is calculated as follows:

If only one design lane is loaded, D_f is given by

$$D_f = 0.06 + \left(\frac{S}{4300} \right)^{0.4} \left(\frac{S}{L} \right)^{0.3} \left(\frac{K_g}{L t_s^3} \right)^{0.1} \quad (2.30)$$

If two or more design lanes are loaded, D_f is given by

$$D_f = 0.075 + \left(\frac{S}{2900} \right)^{0.6} \left(\frac{S}{L} \right)^{0.2} \left(\frac{K_g}{L t_s^3} \right)^{0.1} \quad (2.31)$$

In the above equations S is the spacing of the girders (mm), L is the span of the bridge (mm), t_s is the depth of concrete slab (mm), and K_g is the longitudinal stiffness parameter given by

$$K_g = n (I + A e_g^2) \quad (2.32)$$

in which A is the area of the girder cross-section (mm^2), n is the modular ratio of girder and deck materials, I is the moment of inertia of the girder (mm^4), and e_g is the distance between the centroid of the girder and middle plane of the slab (mm). It is evident that the parameters in Eqs. 2.31 and 2.32 pertain to the properties of the cross section, but these properties do not have the same significances in the elastic and inelastic states.

In this study, we will investigate whether the code provision can be applied to inelastic transverse moment redistribution of prestressed concrete slab-girder bridges, particularly at the ultimate state.

Chapter 3

FINITE ELEMENT PROGRAM

3.1 General

It was mentioned in the beginning of this thesis that the nonlinear finite element method will be used for the analyses in this study. In this chapter the finite element program utilized and its modifications will be described.

The finite element program modified for the purpose of the current study is called NONLACS (NONLinear Analysis of Concrete and Steel). The origin of this program goes back to the program FELARC (Finite Element Layered Analysis of Reinforced Concrete) which was originally developed by Ghoneim (1978) and Ghoneim and Ghali (1982) at the university of Calgary. The latest version of NONLACS was developed by Razaqpur & Nofal (1988). They added the capability of analyzing steel and composite structures to FELARC and they introduced new and improved plate and shell elements. NONLACS is chosen in this research due to the fact that this program includes all the elements and material models necessary for analyzing composite concrete slab on prestressed girder bridges, and as will be shown in the next chapter, is fully capable of capturing the detailed behavior of this kind of bridges. NONLACS uses the basic structure of FELARC and it follows the same material model for concrete, reinforcing

steel and prestressed cable bars. It can be used to analyze the full nonlinear behavior of any reinforced or prestressed concrete structure which can be modeled as an assemblage of thin shell elements. This includes beams, plates, shells, box girders, and shear walls, etc.

Time dependent phenomena such as creep and shrinkage and cyclic loads can be also included in the analysis. Large displacements have been included by Razaqpur and Vasilescu (2000) and finally shear connector modification have been added to this program by Razaqpur and Esfandiari (2001) .

In this chapter, the basic finite element equations, finite elements and material models in addition to nonlinear procedure employed in NONLACS will be discussed.

3.2 Basic Equations of Finite Element

Some of the basic equations of the finite element method, as used in this program, are presented in this section. Using the principle of minimization of the total potential energy, the equations of equilibrium can be written in matrix form:

$$[K]\{d\} = \{R\} + \{F\}_b + \{F\}_s + \{F\}_e + \{F\}_o \quad (3.1)$$

where

$[K]$ = Structure stiffness matrix

$\{d\}$ = Vector of nodal displacements

$\{R\}$ = Vector of applied loads

$\{F\}_b$ = Vector of nodal forces due to body forces

$\{F\}_s$ = Vector of nodal forces due to surface forces

$\{F\}_\varepsilon$ = Vector of nodal forces due to initial strains

$\{F\}_\sigma$ = Vector of nodal forces due to initial stresses

The displacement $\{\delta\}$ inside an element is related to the nodal displacement $\{d\}^e$ as

$$\{\delta\} = [N]\{d\}^e \quad (3.2)$$

where $[N]$ is the shape functions matrix. Note that the superscript e is used to denote nodal displacements at the element level. The strain $\{\varepsilon\}$ and nodal displacements $\{d\}^e$ are related by the strain-displacement matrix $[B]$ as:

$$\{\varepsilon\} = [B]\{d\}^e \quad (3.2)$$

Using the constitutive matrix $[D]$, the element stresses can be computed from its strains as follow:

$$\{\sigma\} = [D](\{\varepsilon\} - \{\bar{\varepsilon}\}) + \{\bar{\sigma}\} \quad (3.3)$$

where $\{\bar{\varepsilon}\}$ = vector of initial strains

and $\{\bar{\sigma}\}$ = vector of initial stresses.

Elements of equation (3.1) can be derived from

$$[K] = \sum_e [T]^T [K]^e [T] \quad (3.4)$$

$$\{F\} = \sum_e [T]^T \{F\}^e \quad (3.5)$$

$$[K]^e = \int [B]^T [D] [B] dV^e \quad (3.6)$$

$$\{F\}_b^e = \int [N]^T \{g\} dV^e \quad (3.7)$$

$$\{F\}_s^e = \int [N]^T \{p\} dS^e \quad (3.8)$$

$$\{F\}_\epsilon^e = \int [B]^T [D]^T \left\{ \bar{\epsilon} \right\} dV^e \quad (3.9)$$

$$\{F\}_\sigma^e = - \int [B]^T \left\{ \bar{\sigma} \right\} dV^e \quad (3.10)$$

in the equations above $\{g\}$ and $\{p\}$ are vectors of body and surface forces,

$\left\{ \bar{\epsilon} \right\}$ and $\left\{ \bar{\sigma} \right\}$ are vectors of initial strains and initial stresses and $[N]$ is the shape

function of the element. Matrix $[T]$ is a transformation matrix, which relates components of a vector in the local coordinate system to its components in the global coordinate system.

3.3 The Finite Elements Employed

NONLACS contains five basic elements: bar element, four node plane quadrilateral element, four node plate bending quadrilateral element, four node facet shell element and shear connector element.

3.3.1 Bar Element

Bar element is the standard truss element. It has two end nodes with a linear displacement field. Two local degrees of freedom are considered for each node: u_1 and u_2 . Since this element is well-established, it will not be discussed any more. It must, however, be mentioned that we are using this element to model prestressed cables in the bridge girders, therefore, it will have an important role in all our analyses. The shape function, strain-displacement and stiffness matrices are as follows:

$$[N] = [N_1 \quad N_2] \quad (3.11)$$

$$[B] = \frac{1}{L} \begin{bmatrix} -1 & 1 \end{bmatrix} \quad (3.12)$$

$$[K] = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \quad (3.13)$$

where:

$$K_1 = \frac{EA}{L} \quad (3.14)$$

$$N_1 = 1 - \frac{x}{L} \quad (3.15a)$$

$$N_2 = \frac{x}{L} \quad (3.15b)$$

and E is the modulus of elasticity, A is area of the cross section, x is distance measured along the bar from the first node to the second node, and L is the length of the element.

Fig. 3.1: Bar element local degrees of freedom

3.3.2 Refined Plane Quadrilateral Element (RQUAD4)

This is a four-node quadrilateral element and was developed by Razaqpur for NONLACS and Razaqpur and Nofal (1988) carried out extensive tests to verify its implementation and performance. RQUAD4 is a four-node membrane finite element with three degrees of freedom (DOF) per node as illustrated in Fig. 3.2. The DOF of this element consist of the two components of inplane displacement and a rotation normal to the plane of the element. The displacement functions of this element can be derived from the eight-node plane quadrilateral element QUAD8, (Fig. 3.3). Using the side shape function given in Fig. 3.4 for four sides of RQUAD4 element, the displacement of these two elements can be related as follows.

$$\{d\} = [T] \{d^*\} \quad (3.16)$$

where:

$\{d\}$ = the displacement vector of the degrees of freedom of QUAD8

$[T]$ = 16×12 transformation matrix

$\{d^*\}$ = the displacement vector of the degrees of freedom of RQUAD4

Eq. 3.16 can be rewritten in the expanded form as:

Fig. 3.2: Four node refined plane quadrilateral element

Fig. 3.3: Eight node plane quadrilateral element

Fig. 3.4: Side shape function of RQUAD4

Fig. 3.5: Quadrilateral plate bending element (IDKQ)

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \\ u_6 \\ v_6 \\ u_7 \\ v_7 \\ u_8 \\ v_8 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.5 & 0 & b_1 & 0.5 & 0 & -b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & a_1 & 0 & 0.5 & -a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & b_2 & 0.5 & 0 & -b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & a_2 & 0 & 0.5 & -a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & b_3 & 0.5 & 0 & -b_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & a_3 & 0 & 0.5 & -a_3 & 0 \\ 0.5 & 0 & -b_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & b_4 & 0 \\ 0 & 0.5 & -a_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & a_4 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{Bmatrix} \quad (3.17)$$

where

$$a_i = \frac{(x_i - x_{i+1})}{8} \text{ and } b_i = \frac{(y_i - y_{i+1})}{8} \text{ for } i=1,2,3$$

$$a_4 = \frac{(x_1 - x_4)}{8} \text{ and } b_4 = \frac{(y_4 - y_1)}{8}.$$

x_i and y_i are coordinates of node i of the RQUAD4 element.

The stiffness matrix can be derived from the principle of virtual work. The stiffness matrix of RQUAD4, $[K^*]$, and its equivalent nodal force matrix, $[F^*]$, can be obtained from the following equations:

$$[K^*] = [T]^T [K] [T] \quad (3.18)$$

$$\{F^*\} = [T]^T \{F\} \quad (3.19)$$

where:

$\{F\}$ = the equivalent nodal force matrix of QUAD8

$[K]$ = the stiffness matrix of QUAD8, which can be obtained as

$$[K] = \int [B]^T [D] [B] dV \quad (3.20)$$

In the above equation:

$[B]$ = strain-displacement matrix

$[D]$ = constitutive matrix

V = volume of QUAD8 element .

Substituting $[K]$ from Eq. (3.20) in Eq. (3.18), Eq. (3.18) can be rewritten as

$$[K^*] = \int [B^*]^T [D] [B^*] dV \quad (3.21)$$

where

$$[B^*] = [B][T] \quad (3.22)$$

Further details about this element and its performance can be found in Razaqpur and Nofal (1988).

3.3.3 Improved Discrete Kirchhoff Quadrilateral Plate Bending Element (IDKQ)

This four-node element has 12 degrees of freedom. The degrees of freedom consist of two rotations normal to the plane of the element as illustrated in Fig. 3.5. Chinniah (1985) developed this element. He used the shape functions of nine node Lagrange

element, N_i , and replaced them with the substitute shape functions, $\bar{N}_i$. These shape functions are as follow:

$$N_i = a_1 + a_2\xi + a_3\eta + a_4\xi^2 + a_5\xi\eta + a_6\eta^2 + a_7\xi^2\eta + a_8\xi\eta^2 + a_9\xi^2\eta^2 \quad (3.23)$$

$$\bar{N}_i = b_1 + b_2\xi + b_3\eta + b_4\xi^2 + b_5\xi\eta + b_6\eta^2 + b_7\xi^2\eta + b_8\xi\eta^2 \quad (3.24)$$

where ξ and η are the natural coordinates of a given point of the element, a_i are the Lagrange shape function coefficients, and b_i are the substitute shape function coefficients.

Using the following equations of constraint, Chinniah developed the stiffness matrix IDKQ:

- 1) Transverse shear strains γ_{xz} and γ_{zy} vanish at the corner nodes:

$$\theta_{xi} = w_{,xi} \quad , \quad \theta_{yi} = w_{,yi} \quad \text{at nodes } i=1,2,3,4,9 \quad (3.25)$$

- 2) Transverse shear strains γ_{xz} (expressed in edge tangent coordinate s) vanish at the mid-side nodes so that:

$$\theta_{,sk} = w_{,sk} \quad \text{at nodes } k = 5,6,7,8 \quad (3.26)$$

- 3) Normal slopes $w_{,n} = \theta_n$ of the cubic displacement function w along the element sides are assumed to vary linearly:

$$\theta_{nk} = \frac{1}{2}(\theta_{ni} + \theta_{nj}) \quad \text{at nodes } k = 5,6,7,8 \quad \text{for } i=1,2,3,4 \text{ and } j=2,3,4,1 \quad (3.27)$$

More details about the finite element formulation of the IDKQ element can be found in Aziz (1988) and Chinninah (1985).

3.3.4 Quadrilateral Facet Shell Element (QFSE)

Element QFSE is obtained by combining elements RQUAD4 and IDKQ. The element has four nodes with six degrees of freedom at each node. Six degrees of freedom consist of 3 displacements and 3 rotations. Considering material nonlinearity, the shell element is an anisotropic shell in which the inplane and bending actions are assumed to be coupled. From basic finite element procedures, the strain $\{\varepsilon\}$ at a general point within the element can be written as:

$$\{\varepsilon\} = \{\varepsilon_0\} - z\{\chi\} \quad (3.28)$$

In order to derive the above relation for $\{\varepsilon\}$, the following procedure can be applied:

$$w(x, y) = w_0(x, y) \quad (3.29)$$

$$u(x, y) = u_0(x, y) - z \frac{\partial w}{\partial x} \quad (3.30)$$

$$v(x, y) = v_0(x, y) - z \frac{\partial w}{\partial y} \quad (3.31)$$

where u, v, w are the displacements in x, y, z axis directions at a given point, while u_0, v_0, w_0 are the displacements of the projection of the same point on the middle or reference plane of the shell. The strain components are obtained by differentiating the above equations :

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (3.32)$$

or

$$\{\epsilon\} = \{\epsilon_0\} - z\{\chi\} \quad (3.33)$$

where the inplane strain $\{\epsilon_0\} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}$, the curvature $\{\chi\} = \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$, and z is the

distance of the point at which strains are calculated to the reference plane.

From basic theory we have ,

$$\{\epsilon\} = [B]_p \{d\}_p \quad (3.34)$$

$$\{\chi\} = [B]_b \{d\}_b \quad (3.35)$$

where $[B]_p$ and $[B]_b$ are the inplane and the bending strain-displacement matrices, respectively, and $\{d\}_p$ and $\{d\}_b$ are inplane and bending displacement vectors.

Considering Equations 3.33, 3.34 and 3.35, we can write:

$$\{\epsilon\} = ([B]_p - z[B]_b) \begin{Bmatrix} d_p \\ d_b \end{Bmatrix} = [B] \{d\} \quad (3.36)$$

For the shell element:

$$[B] = ([B]_p - z[B]_b) \quad (3.37)$$

The stiffness matrix of the element is given by

$$[K] = \int [B]^T [E] [B] dV \quad (3.38)$$

where $[E]$ is the constitutive matrix.

Substituting $[B]$ from Eq. 3.37 in Eq. 3.38, we get

$$[K] = \int \begin{bmatrix} [B]_p^T [E] [B]_b & -z [B]_p^T [E] [B]_b \\ -z [B]_b^T [E] [B]_p & z^2 [B]_b^T [E] [B]_b \end{bmatrix} dV \quad (3.39)$$

or

$$[K] = \begin{bmatrix} [K]_{pp} & [K]_{pb} \\ [K]_{bp} & [K]_{bb} \end{bmatrix} = \begin{bmatrix} [K]_{11} & [K]_{12} \\ [K]_{21} & [K]_{22} \end{bmatrix} \quad (3.40)$$

where the submatrices can be determined by

$$[K]_{ij} = \int [B]_i^T [D]_{ij} [B]_j dA \quad (3.41)$$

in which indices i, j stand for p, b , respectively, ($i = 1, 2$ and $j = 1, 2$ but no summation is implied). Dividing the thickness of element into n layers, $[D]_{ij}$ can be obtained as follows:

$$[D]_{11} = [D]_{pp} = \int [E] dz = \sum_{i=1}^n (z_{i+1} - z_i) [E]_i \quad (3.42)$$

$$[D]_{12} = [D]_{21} = [D]_{pb} = [D]_{bp} = - \int z [E] dz = - \frac{1}{2} \sum_{i=1}^n (z_{i+1}^2 - z_i^2) [E]_i \quad (3.43)$$

$$[D]_{22} = [D]_{bb} = \int z^2 [E] dz = \frac{1}{3} \sum_{i=1}^n (z_{i+1}^3 - z_i^3) [E]_i \quad (3.44)$$

where $[D]_i$ is the constitutive matrix of layer i , and z_{i+1}, z_i are the z coordinates of the top and bottom surfaces of layer i . Of course, the constitutive matrix for each layer will depend upon whether the layer is concrete, smeared reinforcement or steel plate. This will be further explained in the sections dealing with material modeling.

In order to derive the equivalent nodal forces due to initial strain $\{F\}_\varepsilon$ and initial stress $\{F\}_\sigma$, we use Eqs. 3.9 and 3.10 in conjunction with Eq. 3.37,

$$\{F\}_\varepsilon = \iint \begin{bmatrix} [B]_p^T \{\bar{\sigma}_\varepsilon\}_p \\ [B]_b^T \{\bar{\sigma}_\varepsilon\}_b \end{bmatrix} dx dy \quad (3.45)$$

$$\{F\}_\sigma = - \iint \begin{bmatrix} [B]_p^T \{\bar{\sigma}_\sigma\}_p \\ [B]_b^T \{\bar{\sigma}_\sigma\}_b \end{bmatrix} dx dy \quad (3.46)$$

where $\{\bar{\sigma}_\varepsilon\}$ and $\{\bar{\sigma}_\sigma\}$ are the vectors of stress resultants due to initial strain and initial stress, i.e.

$$\{\bar{\sigma}_\varepsilon\}_p = \int [E] \{\bar{\varepsilon}\} dz = \sum_{i=1}^n (z_{i+1} - z_i) [E]_i \{\bar{\varepsilon}\}_i \quad (3.47)$$

$$\{\bar{\sigma}_\varepsilon\}_b = \int -z [E] \{\bar{\varepsilon}\} dz = -\frac{1}{2} \sum_{i=1}^n (z_{i+1}^2 - z_i^2) [E]_i \{\bar{\varepsilon}\}_i \quad (3.48)$$

$$\{\bar{\sigma}_\sigma\}_p = \int [E] \{\bar{\sigma}\} dz = \sum_{i=1}^n (z_{i+1} - z_i) [E]_i \{\bar{\sigma}\}_i \quad (3.49)$$

$$\{\bar{\sigma}_\sigma\}_b = \int -z [E] \{\bar{\sigma}\} dz = -\frac{1}{2} \sum_{i=1}^n (z_{i+1}^2 - z_i^2) [E]_i \{\bar{\sigma}\}_i \quad (3.50)$$

In which $\{\bar{\varepsilon}\}_i$, $\{\bar{\sigma}\}_i$ are vectors of initial strain and initial stress for layer i .

3.3.5 Shear Connector Elements

Razaqpur and Nofal (1998) first added this type of element to NONLACS. Esfandiari (2001) fully modified this element for all types of composite bridges. This kind of element is mostly used in modeling composite slab-on-girder constructions

because the concrete slab is connected to the steel girders by means of shear connectors. Since the connectors are deformable, under load there will be relative deformation, or slip, at the interface of the concrete slab and the steel girders. This element is not utilized in the current study, therefore, the reader is referred to Esfandiari (2001) for more details.

3.4 Material Modeling

In this study, three types of materials are involved: concrete, reinforcing steel, and prestressing steel. Therefore, we will restrict our description to the modelling of these materials only. As stated earlier, NOLACS can model steel and composite structures too, and the interested readers may refer to the work of Nofal (1988) and Esfandiari (2001).

3.4.1 Concrete Modeling

The stress-strain relationship for concrete is shown in Fig.3.6. The first part is represented by Saenz's (1964) proposed equation:

$$\sigma = \frac{E_0 \varepsilon}{1 + \left(\frac{E_0}{E_{sc}} - 2\right) \frac{\varepsilon}{\varepsilon_{cu}} + \left(\frac{\varepsilon}{\varepsilon_{cu}}\right)^2} \quad 0 \leq \varepsilon \leq \varepsilon_{cu} \quad (3.51)$$

and the second part is based on Smith-Young's (1955) equation:

$$\sigma = \sigma_c \left(\frac{\varepsilon}{\varepsilon_{cu}}\right) e^{\left(1 - \frac{\varepsilon}{\varepsilon_{cu}}\right)} \quad \varepsilon_{cu} \leq \varepsilon \leq \varepsilon_{max} \quad (3.52)$$

where:

E_{sc} = secant modulus of elasticity at peak stress

E_0 = initial tangent modulus

σ = stress

σ_c = compressive strength

ε = strain

ε_{cu} = strain at peak stress

Under biaxial and combined stress fields, concrete is assumed to behave as a stress-induced orthotropic material. This means that the initial tangent moduli in the two principal directions are different except when $\frac{\sigma_1}{\sigma_2} = 1$, where σ_1 and σ_2 are the major and minor principal stresses. The procedure used to model concrete under a state of mixed inplane stress by an equivalent biaxially loaded orthotropic material is based on Darwin and Pecknold (1977) work. In this procedure, increments of stress are related to the increments of strain as:

$$\{\Delta\sigma\} = [E] \{\Delta\varepsilon\} \quad (3.53)$$

where $\{\Delta\sigma\}$ is a vector of incremental plane stresses, $\{\Delta\varepsilon\}$ is the corresponding vector of incremental strains and $[E]$ is the constitutive matrix :

Fig. 3.6: Uniaxial stress strain relationship of concrete

Fig. 3.7: Tensile stress-strain relationship of concrete

$$[E] = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1 E_2} & 0 \\ \nu\sqrt{E_1 E_2} & E_2 & 0 \\ 0 & 0 & \frac{1}{4}(E_1 - E_2 - 2\nu\sqrt{E_1 E_2}) \end{bmatrix} \quad (3.54)$$

where ν = Poisson's ratio; and E_1 and E_2 are the tangent moduli in the two principal material directions.

For uncracked concrete, the principal material directions coincide with the principal stress directions while for cracked concrete, these directions are parallel and normal to the cracks. It is assumed that the values of E_1 and E_2 are function of the stress level. Therefore, failure in a particular direction occurs if σ_1 or σ_2 exceeds the concrete strength. On the other hand, if the compressive strain in one of these directions exceeds the ultimate concrete strain, concrete is regarded as crushed. When this happens, the $[E]$ matrix becomes null.

The magnitudes of σ_1 and σ_2 govern the cracking situation. The concrete strength under biaxial and combined stress fields is determined from the failure envelope of Kupfer and Gerstle (1973) as shown in Fig. 3.8. If the principal tension at a point equals or exceeds the tensile strength of concrete, cracking is assumed perpendicular to the particular direction. At any integration point in an element if tensile strength for instance in direction 1, exceeds the tensile strength f_t' , the constitutive matrix, $[E]$ is reduced to:

$$[E] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & \beta G \end{bmatrix} \quad (3.55)$$

where G =elastic shear modulus, and β =shear retention factor with a value less than 1 .

The factor β is used to account in an approximate fashion for the shear stiffness due to aggregate interlock and dowel action.

Concrete is able to transfer tension between the cracks after formation of cracks. This phenomena is called tension-stiffening which has a significant effect on the magnitude of deflection after cracking. In NONLACS this phenomena is considered by assuming a descending branch for concrete in tension. It means after formation of cracks at a given point, concrete still can carry some tension but is assumed to have zero stiffness normal to the direction of cracks.

3.4.2 Modeling reinforcing bars and prestressing steel

Reinforcing bars and prestressing steel are modeled as elasto-plastic, strain-hardening material. The bilinear stress-strain relationship is shown in Fig.3.9. The material parameters f_y, E_s, E'_s , and ϵ_{su} shown in this figure, are respectively the yield stress; the Young's modulus; the strain-hardening modulus and the ultimate strain. Notice that the reduction in yield strength due to cyclic loading, known as the Bauschinger effect, is approximately modeled.

3.4.3 Steel Plate Modelling

For modeling steel plates, the theory of incremental plasticity is used in NONLACS. This theory is well established [Hill (1950) and Owen and Hinton (1980)] and will not be discussed in detail. Like concrete structures, for steel structures a layered

Fig. 3.8: Biaxial failure envelope of concrete [Kupfer and Gerstle (1973)]

Fig. 3.9: Stress-strain relationship of steel

model is used. Since steel plate modelling is not utilized in the current study, we refer the readers to Razaqpur and Nofal (1998) for more details.

3.5 Nonlinear Analysis Method

NONLACS uses an incremental-iterative procedure, Fig. 3.10, to solve the nonlinear system of equations governing the problem. The nonlinearity dealt with here is due to the nonlinear stress-strain relationship of the materials involved. From the basic finite element:

$$[K]\{d\} = \{f\} \quad (3.56)$$

In incremental methods, the load vector $\{f\}$ is subdivided into a number of increments, N , such that:

$$\{f\} = \sum_{i=1}^N \{\Delta f_i\} \quad (3.57)$$

These load increments are applied one after another and their effects are added together to find the final response of the structure. At each load step, the equations of equilibrium are solved by an iterative procedure. The unbalanced load of the k th iteration in load step i is computed using:

$$\{F_k^i\} = \sum_{j=1}^i \{\Delta f_j\} - \{F_{k-1}^i\} \quad (3.58)$$

where $\{\Delta f_j\}$ is the j th incremental applied load ($j = 1$ to i) and $\{F_{k-1}^e\}$ is the equilibrated load after the previous iteration ($k - 1$). The incremental displacement $\{d\delta_k^i\}$ corresponding to $\{F_k^i\}$ is then computed by :

$$\{d\delta_k^i\} = [K_{k-1}]^{-1} \{F_k^i\} \quad (3.59)$$

where $[K_{k-1}]$ is the stiffness matrix corresponding to the results of previous iteration.

Fig. 3.10: Incremental-iterative nonlinear analysis method

The total displacement is then:

$$\{\Delta \delta_i\} = \sum_{j=1}^k \{d\delta_j^i\} \quad (3.60)$$

All the above procedure is in a loop and continues until the difference of the norms of the unbalanced load vector, $\{F_k'\}$, between iteration (k) th and iteration $(k + 1)$ th becomes less than the maximum value specified as input value. The total displacement $\{\delta_i\}$ due to the total applied load at the end of i th load increment is computed as:

$$\{\delta_i\} = \sum_{j=1}^i \{\Delta \delta_j\} \quad (3.61)$$

The convergence/divergence criteria may be specified in terms of forces or displacements.

3.6 Modification of NONLACS for Concrete Structures Constructed in Stages.

Most major structures are built in stages due to either construction condition or simplicity. This is particularly the case in precast-prestressed bridges. The analysis of structures built in stages is complex, particularly when structural nonlinearity needs to be considered. Since the principle of superposition cannot be applied to nonlinear problems, the analysis requires the initial geometry, stresses, deformations, and stiffness of the structure as input at each new stage of construction. In the finite element analysis context, a new definition of the structure topology, which may involve renumbering of nodes and elements, is often required. These changes are complex to implement and extensive book-keeping is needed. In the so-called frontal solution technique, (Irons 1980), the addition of new nodes and elements does not lead to as much complexity as in other methods of finite element solutions, nevertheless tackling this problem is not trivial.

In this study, an alternative approach to the traditional methods of analysis is proposed. The method is relatively simple and does not require a change in the topology of the structure. The final or completed structure is discretized in the usual manner, but the parts added at each new stage are assigned a stage number or identifier. The finite elements belonging to any stage can be activated or deactivated by predefined activation indices. Each group of finite elements belonging to a given stage is assigned an activation/deactivation index at each load step, where the load steps span over all the construction stages.

Once a group of elements are assigned a deactivation index, they are assigned negligibly small stiffness and their initial stresses and deformations are set equal to zero or at their existing level. However, the nodal deformations common to active and deactive (inactive) elements are kept active. The existing stresses of deactive elements due to the load at the current stage are reversed in sign and applied as equivalent unbalanced forces.

In the context of simply supported prestressed concrete bridges, there are essentially two stages. The first stage consists of individual girders subjected to self-weight, prestressing, and the weight of the cast-in situ slab. The second stage involves the composite slab-girder bridge subjected to superimposed dead and live loads. Although in design we need to check a precast member stresses at other intermediate stages, in the finite element analysis, we do not require these checks.

In the current analysis the complete bridge is discretized as usual. In the first stage, all elements belonging to the cast-in situ concrete slab are deactivated by assigning very

small elastic moduli to them. Then the prestress and the self-weight of the prestressed girder and the slab are applied in a number of load steps. The stresses and deformations of the deactivated elements are made zero at the end of each iteration and load step.

At the end of the first stage, all elements are activated and the second stage is commenced. At this stage the superimposed dead and live loads are applied to the composite structures in a number of load steps or the load is constantly increased until failure is reached.

3.6.1 Theoretical verification of the proposed method of analysis

To demonstrate this procedure, consider the simply supported rectangular prestressed concrete beam and the associated cast-in situ slab as shown in Fig. 3.11. Imagine that the rectangular girder is precast and prestressed initially. Subsequently, the concrete slab is cast, with the prestressed girder being unshored during construction. Upon curing of the slab, the composite member is subjected to the central point load shown in Fig. 3.11.

The beam is simply supported on a 5.0 m span and consists of a 300×600 mm concrete girder attached to a 125 mm thick and 700 mm wide slab. The material properties for the bridge are assumed as follows: concrete strength 40 MPa for the slab and 60 MPa for prestressed girder, initial strength for concrete in the girder 25 MPa.

(a): Elevation

(b): Cross section

Fig. 3.11: A composite prestressed girder constructed in two stages

concrete density 2400 Kg/m^3 , Young's modulus for concrete 31.6 GPa . For prestressing cables, yield stress is 1800 MPa and elastic modulus is 190 GPa . For the reinforcing steel, yield stress is 400 MPa , Young's modulus is 200 GPa , and no strain hardening is considered for either the prestressing or the reinforcing steel. The initial applied prestress

is assumed equal to 70% of the yield capacity of cables and the prestress loss is assumed to be 20% of the initial tendon force.

The finite elements belonging to the two stages of this structure are designated as (a) and (b) in Figure 3.12, while the elements in the complete composite structure are shown in (c). We began by discretizing the complete composite girder as shown in Fig.3.12 (c). In the first stage of the analysis, the elements in Fig. 3.12(b), which represent the slab, are deactivated and considering the self-weight of the whole member and the prestress, the analysis is performed by dividing the load into a number of steps. After the application of the entire load in the first stage, the second stage is commenced, the elements in Fig. 3.12(b) are activated and the full structure in Fig. 3.12(c) is subjected to the applied point load. Once again the analysis is performed in several load steps.

Fig. 3.12: (a) Finite element mesh of the girder (b) Finite element mesh of the slab (c) Finite element mesh of the composite girder

Results of the above-mentioned process for this example are shown in Figure 3.13. In the figure, the longitudinal normal stress distributions acting on the section at midspan are shown for the various stages of construction. Stress distributions (b), (c), and (d) are for the naked prestressed girder and are, respectively, due to axial effect of prestressing, bending effect of prestressing and self-weight of the naked girder and the slab, where negative concrete stress represents compression. The stress distribution (e) represents the stresses due to an applied service load of 450 *KN* acting on the composite section at stage 2. The net or final stresses on the composite section are given by Fig 3.13 (f). Observe that the section will be uncracked at all stages of loading as the tensile strength of concrete in the girder is assumed to be 4 MPa.

In Fig. 3.13 for each stage, the stress values from hand-calculation, based on elastic analysis and conventional beam theory, are shown between parentheses. We observe that the results of finite element closely match those from hand-calculations.

Additional results are shown for this beam. The variation of stress in the extreme bottom and top fibres at midspan with the applied concentrated load are plotted in Figs. 3.14 and 3.15, respectively. Figure 3.16 shows the variations of stress in the prestressed cables at midspan with the applied load. Notice that the cable stress, as expected, increases marginally until the concentrated load reaches 650 *KN* , at which point the beam cracks and the stress in the steel increases rapidly. The advent of cracking is also signalled by the change in the stiffness of the beam in the load-midspan deflection curve in Fig. 3.17. Observe that the negative deflection in the latter figure signifies camber due to prestressing. The results of this analysis conform to the expected values based on hand calculations according to conventional beam theory. Accordingly, they verify the

proposed finite element analysis strategy for prestressed concrete structures built in stages.

Fig. 3.13: At midspan (a) Cross section (b) Prestress force (c) Moment due to prestressing (d) Stress after application of slab dead load (f) Stress due to applied live load

Fig. 3.14: Tensile stress at the bottom of the girder at midspan

Fig. 3.15: Compressive stress at the top of the slab at midspan

Fig. 3.16: Average stress in prestressed cables at midspan

Fig. 3.17: Girder midspan deflection versus applied load

3.6.2 Experimental verification of the proposed method of analysis

Experimental results for structures built in stages are scarce in the literature. Roller and Russell (1995) performed a physical test on a simply supported prestressed concrete slab-girder specimen. The experimental results of this test will be compared with the computed results of NONLACS. As shown in Fig. 3.18, the beam was simply supported on a 70 ft (21.9m) span and consisted of a 54 in (1372mm) deep bulb-tee girder attached to a 9.5 in (241.3mm) thick and 10 ft (3.13m) wide cast in situ slab. As the original dimensions were given in U.S. customary units, we shall use the same unit system here. The material properties are shown in Table 3.1 and 3.2. Values for a number of properties were not, however, specified in their work and had to be assumed as indicated in those tables. The only material properties given were the 28 day compressive strength of

(a) Elevation

(b) Cross section

Fig. 3.18: Geometry and loading of the precast prestressed girder tested by Roller and Russell (1995)

Table 3.1 Concrete properties in Roller and Russell (1995) test girder

Material		Compressive strength (MPa)	Tensile strength (MPa)	Modulus of Elasticity (GPa)	ϵ_c Strain corresponding to compressive strength	ϵ_t Ultimate strain in tension	ϵ_{max} Ultimate strain
Concrete	Girder	69(10000psi)	5.3	41.7	0.002	0.0002	0.0030
Concrete	Slab	41(6000psi)	4.1	32.4	0.002	0.0002	0.0030

Table 3.2 Prestressed and reinforcing steel properties in Roller and Russell (1995) test girder

Material		Tensile Strength (MPa)	Strain Hardening (MPa)	Modulus of Elasticity (GPa)	ϵ_y Yield strain	ϵ_{max} Ultimate strain
Steel	Prestressed cables	1861(270000psi)	1600	200	0.0093	0.038
Steel	Reinforcing steel	480(70000psi)	0	200	0.0024	0.018

concrete and the yield strength of the reinforcing steel and the ultimate strength of the prestressed steel. The assumed values are reasonable based on general information in the literature. The finite element idealization of the naked bulb T-girder is shown in Fig.3.19. The girder is divided into 38 equal facet shell elements longitudinally as indicated in Fig. 3.19(a) and (c). The cross-section is modeled by 10 elements, 4 elements in each flange and two elements in the web. An isometric view of the finite element mesh is shown in Fig.3.19(c).

The finite element mesh for the full composite girder is shown in Fig.3.20. Notice that the in situ slab is transversally idealized by 8 elements. In addition, as indicated in

(a) Elevation

(b) Prestressed girder

(c) Isometric view of the finite element mesh

Fig. 3.19: Finite element discretization of the prestressed girder

(a) Isometric view of entire cross section

(b) Cross-section discretization

Fig. 3.20: Finite element mesh and layer system of the full composite girder

Fig.3.20(b), the slab is divided into 4 equal concrete layers, while the top flange of the prestressed girder is modeled by four 60 mm thickness concrete layers of width 182 mm and five layers of width 364 mm. Similarly, the bulb at the bottom is idealized by 4 layers of width 152 mm and 5 layers of width 153 mm as shown in the figure. The reinforcing and prestressed steel are discretized as smeared layers as indicated in Fig. 3.20.

As described earlier, the elements in the slab will be ignored in the first stage of the analysis and the naked girder will be subjected to prestress, self-weight and the weight of the slab. In the next stage the ignored elements are activated and the external load effect is calculated in the composite girder. Figure 3.21 shows the variation of longitudinal normal stress at midspan at different stages of loading. As can be seen, the stress At the bottom of the girder starts from -15.7 MPa compression and then continues to drop until it goes into tension. When the applied load reached 50% of its full value, the section had already cracked and the stress at bottom had dropped to zero. The stress distribution at ultimate, (Fig. 3.21 (f)), shows a large portion of the section to be cracked. This behavior is expected although no experimental data exists to verify the current stress values. However, Roller and Russell have provided the applied load-midspan deflection curve for this beam as shown in Fig. 3.22. As can be noticed, there is close agreement between the experimental and finite element results. The figure indicates that the cracking occurred at approximately 900 kN, i.e. at the point on the curve when the slope for the first time changes noticeably. Yielding of prestress steel occurred at approximately 1550 kN. The latter observation agrees with the calculated stress in the prestressed steel as indicated in Fig. 3.23, which indicates more rapid increase in stress after 900 kN.

Based on the favorable agreement between the hand calculation and finite element results on the one hand and the experimental data and the corresponding finite element results on the other, it can be stated that the finite element program, NONLACS, using the elements and material models described in Chapter 3, can capture the detailed behavior of prestressed precast concrete girder bridges in the entire range of loading up to failure, and can be used for finding the ultimate moment distribution in continuous multi-girder composite bridges. Finally, it is important to mention that Razaqpur et. al. (1990) have previously verified the applicability of NONLACS to other types of prestressed concrete structures that were not built in stages.

Fig. 3.21: Variation of composite girder stresses at section of maximum moment (a) At the end of stage 1 (b) After application of slab dead load (c) At cracking stage (d) After application of half the live load in stage 2 (e) At ultimate stage

Fig. 3.22: Load-midspan deflection

Fig. 3.23: Stress in cables from finite element analysis

Chapter 4

PARAMETRIC STUDY

4.1 Selection of parameters

Based on the investigations of Heins and Kuo (1975), Bakht and Jaeger (1985), Razaqpur and Nofal (1988), and Kennedy and Soliman (1992), some of the main parameters that influence moment redistribution in slab-on-girder bridges are as follows:

1. Longitudinal truck position.
2. Transverse location of trucks.
3. Number of girders (Girder spacing)
4. Number of lanes loaded
5. Bridge width /span ratio
6. Relative rigidity of slab to girders

These parameters have been found relevant in the elastic state, but their influence in the post-cracking and ultimate states in prestressed concrete bridges has not been previously investigated.

In this study the effect of the above parameters on the distribution of loads and moments in precast-prestressed slab-girder bridges over the complete loading range up to failure will be investigated. Note that the degree of interaction, i.e. of the composite action between the slab and the girder, may also have some influence on load distribution in this kind of bridges. Here we will not consider the latter effect because it is assumed that there are always sufficient stirrups crossing the interface of girders and slab to provide complete composite action. The geometry of the analyzed bridges has been selected using the CHBD (2000) code provision. For example, to use the CHBD empirical method of design for composite bridges, slab thickness should be at least 225 mm. As a result in all cases slab thickness is 225 mm unless the effect of slab thickness is being investigated.

In selecting the range of parameters, generally the CPCI design handbook (1996) was used as a guide. For example, in selecting the maximum feasible span for slab-on-prestressed girder bridges for typical cross sections, the CPCI recommendations were followed. In some cases in this study extreme values of certain parameters beyond standard recommendations have been used mainly to establish the bounds for the effects of a particular parameter.

4.2 Selection and Design of Bridges

The bridges analyzed were designed according to the recommendation of CHBD (2000). Based on this recommendation, longitudinal bending moment diagrams shall be obtained by treating the bridge as a beam, subjected to two types of loading: the first load case consists of one truck with two lines of wheels, and the second load case comprises the lane load, equal to 70% of the above truck load, plus 9 kN/m uniformly distributed load per lane. The load in each case is multiplied by the factor $(1+DLA)$, where DLA is dynamic load allowance factor. The details of calculating the distribution factors suggested by the CHBD code are described in Chapter 2. In most of the designed bridges in this study, the governing load case is load case 2, i.e. 70% of the CHBD truck that produce maximum bending moment in the span plus 9 kN/m uniformly distributed load per lane.

In designing the prestressed girders, the first step is choosing the prestressed system. Two types of prestressing systems exist: pretension and post-tension. A pretensioned beam is a prestressed beam in which the prestressing tendon is tensioned prior to casting the section, while a post-tensioned beam is one in which the prestressing tendon is tensioned after the beam has been cast and has achieved the major portion of its concrete strength. Theoretically, the prestressing system does not have much effect on ultimate load capacity of the section.

Composite slab-girder sections normally consist of a precast prestressed concrete girder with a precast or cast in place concrete slab. The concrete slab is cast on either shored or unshored girders. In case of shored construction, the slab weight acts on the

composite section, which has a substantially larger section modulus than the precast section alone. Hence, the concrete stress calculations have to take this situation into account in the design. On the other hand, the precast girders carry the slab weight in unshored construction. Therefore, the girder should be designed in a manner to carry the slab dead load before erection when concrete is still wet and after erection. In both cases the composite section of slab-girder carries the live load and the superimposed load. Figure 4.1 shows the relative position of the neutral axis in the precast and composite cross sections. Figure 4.2 illustrates the flexural stress distribution in composite beams under different cases of loading, where W_T , W_D , W_{SD} and W_L denote the total, the dead, the superimposed dead and the live load, respectively.

The cross section of prestressed concrete girders can be selected based on standard sections and cable profiles in the CPCI handbook (CPCI 1996). The maximum span and the shape of tendons in the span as well as the limitations of stresses for both concrete and steel at different construction stages are specified in this handbook based on the recommendations of the Canadian Highway Bridge Design Code (2000). Based on these recommendations, the following limitations must be checked to ensure safety and serviceability of the structure:

- i) At the initial load stage when only the prestressing force and self-weight is applied to the girder, the stress at the top tensile stress fibre and bottom compressive stress fibre should not exceed $0.2\sqrt{f_{ci}}$ and $-0.6f_{ci}$, respectively. The stress at the top and the bottom is defined as follow:

Fig. 4.1: Position of neutral axis in (a) Precast section (b) Composite section

Fig. 4.2: Flexural stress distribution in composite beams (a) Cross-section (b) Stress distribution in girder in unshored construction (c) Live load stress distribution (d) Elastic distribution due to total load

$$\text{at the top : } f'_t = -\frac{F_i}{A_g} + \frac{F_i e_p}{S_{ip}} - \frac{M_D}{S_{ip}} \leq 0.2\sqrt{f_{ci}} \quad (4.1)$$

$$\text{at the bottom : } f'_b = -\frac{F_i}{A_g} - \frac{F_i e_p}{S_{bp}} + \frac{M_D}{S_{bp}} \geq -0.6f_{ci} \quad (4.2)$$

- ii) At the serviceability load stage, depending on shored or unshored construction, the stress at bottom tensile stress fibre and the top compressive stress fibre should not exceed $0.4\sqrt{f_c}$ and $-0.6f_c$, where stress at top and bottom is defined as follow:

For shored construction:

$$\text{at top : } f' = -\frac{F_c}{A_g} + \frac{F_c e_p}{S_{ip}} - \frac{M_D}{S_{ip}} - \frac{M_{DS} + M_{SP} + M_L}{S_{tc}} \geq -0.6f_c \quad (4.3)$$

$$\text{at bottom : } f^b = -\frac{F_c}{A_g} - \frac{F_c e_p}{S_{bp}} + \frac{M_D}{S_{bp}} + \frac{M_{DS} + M_{SP} + M_L}{S_{bc}} \leq 0.4\sqrt{f_c} \quad (4.4)$$

For unshored construction:

$$\text{at top: } f' = -\frac{F_c}{A_g} + \frac{F_c e_p}{S_{ip}} - \frac{M_D + M_{DS}}{S_{ip}} - \frac{M_{SP} + M_L}{S_{tc}} \geq -0.6f_c \quad (4.5)$$

$$\text{at bottom : } f^b = -\frac{F_c}{A_g} - \frac{F_c e_p}{S_{bp}} + \frac{M_D + M_{DS}}{S_{bp}} + \frac{M_{SP} + M_L}{S_{bc}} \leq 0.4\sqrt{f_c} \quad (4.6)$$

where f'_t and f'_b are the initial stress, after applying the prestressing force, at the top and bottom of the beam, respectively, F_i is the initial force at jacking, A_g is the gross

section of girder, e_p is the eccentricity of F_i from the centroid of the section, S_{ip} , S_{bp} , S_{ic} and S_{bc} are the section moduli of top and bottom of the girder and the slab-girder cross sections, respectively. The quantity f_{ci} is the compression stress of concrete 7 days after casting or when the jacks are released, F_e is the effective force of prestressed tendons after all losses, f_c' is the compressive strength of concrete 28 days after casting, M_D , M_{DS} , M_{SP} and M_L are the moments of dead load of beam, dead load of slab, superimposed load and live load, respectively.

In addition to the above, two other stresses should be controlled and kept smaller than their permissible values. Stress at the top of slab under live load:

$$(f_t)_{slab} = -\frac{M_L}{S_{ic}} \geq -0.6f_c' \quad (4.7)$$

and stress under sustained loads :

$$f_{cs} = -\frac{F_e}{A_g} + \frac{F_e e_p}{S_{ic}} - \frac{(M_D + M_{DS})}{S_{ic}} \geq -0.45f_c' \quad (4.8)$$

According to CPCI handbook, the maximum allowable stress that can be applied at jacking on the cables is 70% of yield stress. The following assumptions are made in defining the behaviour of the section at ultimate load. The values of α_1 and β_1 are the rectangular stress block parameter (CPCI 1996) as shown in Figure 4.3.

1. The strain distribution is assumed to be linear. This assumption is based on the usual hypothesis that plane sections remain plane before bending and perpendicular to neutral axis after bending.
2. The strain in the steel and the surrounding concrete is always the same.
3. Concrete is weak in tension, and it cracks at an early stage of loading at about 10 percent of its compressive strength limit. Consequently, concrete in the tension zone of the section is neglected in the flexural analysis and design computations, and the tension reinforcement is assumed to carry the total tensile force.

Fig. 4.3: Stress and strain distribution across beam depth (a) Beam cross section. (b)

Strains (c) Assumed equivalent stress block at ultimate.

The calculated moment capacity of the section, including the material strength factors, should equal or exceed the factored applied moment. The material strength and load factors are taken from CHBD code (2000).

Shear resistance of the section is defined by the following equations:

$$V_r = V_c + V_s \quad (4.9)$$

where V_r is the shear resistance of the section and V_c is the resisting portion of concrete and V_s is the contribution of shear reinforcement. According to the CHBD code provisions, V_c is chosen from the following equations:

$$V_c = 2.5\beta\phi_c f_{cr} b_v d_v \quad (4.10)$$

where β is a factor used to account for the shear resistance of cracked concrete, $\phi_c=0.75$ is the resistance factor for concrete, f_{cr} is the cracking strength of concrete, b_v and d_v are the effective web and shear depth respectively.

The contribution of reinforcement to shear resistance is calculated as:

$$V_{\text{Reinf.}} = V_s + \phi_p V_p \quad (4.11)$$

where V_p is the component in the direction of the applied shear of all the effective prestressing forces crossing the critical section, to be taken as positive if resisting the applied shear, $\phi_p=0.95$ is the resistance factor for tendons, V_s is the factored shear resistance provided by shear reinforcement and is defined as follow:

$$V_s = \frac{\phi_s f_y A_v d_v (\cot \theta + \cot \alpha) \sin \alpha}{s} \quad (4.12)$$

where $\phi_s = 0.90$ is the resistance factor for shear reinforcement, f_y is the specified yield strength of shear reinforcement, A_v is the area of transverse shear reinforcement perpendicular to the axis of a member within a distance s , θ is the angle of inclination of the principal diagonal compressive stress to the longitudinal axis of the member, α is the angle of inclination of transverse reinforcement to the longitudinal axis of the member and s is the maximum centre-to-centre spacing of transverse reinforcement.

The nominal shear stress, v , shall be computed from:

$$v = \frac{V_f - \phi_p V_p}{b_v d_v} \quad (4.13)$$

The section shall be proportioned so that the nominal shear stress, v , does not exceed $0.25\phi_c f'_c$.

In calculating the stresses, one must consider prestress losses. The total loss depends on parameters such as steel relaxation, creep, shrinkage, elastic shortening, friction, etc. It is well established that the initial prestressing force applied to the concrete elements undergoes a process of reduction over a period of approximately five years. The reduction in the prestressing force can be grouped into two categories:

- Immediate elastic loss during the fabrication or construction process, including elastic shortening of concrete, anchorage losses, and frictional losses.

- Time-dependent losses such as creep, shrinkage, and those due to temperature effect and steel relaxation, all of which is determinable at the service-load limit state of stress in the prestressed concrete element.

For the purpose of this study, the exact calculation of these losses is not feasible. Generally, a very high degree of refinement of loss estimation is neither desirable nor warranted because many factors, which are difficult to evaluate precisely, affect the actual losses. Consequently, a lump-sum estimate of losses was used in this study, and the total loss was assumed to be 20% of the initial jacking force.

It should be mentioned that in designing the bridges in this study, the design method was incorporated into a computer program to find the design moment and select the best cross section and area of prestressed tendons and finally to check the ultimate resistance of the cross section. Typical cross-sections of girders used for the bridges in this study are shown in Figure 4.4. These are the standard CPCI 1600 and CPCI 1900 sections, with the dimensions as indicated in the figure.

The properties of the bridges analysed in this study are given in Table 4.1. To identify each bridge, a mnemonic system is used, consisting of two letters ending with a number. As indicated in column 1 of Table 4.1, the letters denote the relevant parameter being considered and the number represents the order of a bridge in each set or group of bridges. For instance, BW3 is a bridge in the group of bridges analyzed for the effect of bridge width and is the third bridge in this group.

The material properties for all the bridges were assumed as follows: concrete strength 40 MPa for the slab and 50 MPa for prestressed girders, initial strength of

CPCI 1600

CPCI 1900

Fig. 4.4: Typical girders cross-sections used in the analyzed bridges

Table 4.1: Geometry of analyzed bridges

Bridges	No. of Girders	Girder type	Slab Thickness (mm)	Girder Spacing (mm)	Number of Prestressed Tendons	Slab Width (m)	Span length (m)
LP1	4	CPCI 1600	225	3700	35 #13	15	30
LP2	5	CPCI 1600	225	3000	30 #13	15	30
BW1	4	CPCI 1600	225	3000	30 #13	13	30
BW2	4	CPCI 1600	225	3700	35 #13	17	30
BW3	4	CPCI 1600	225	4500	39 #13	21	30
SW1	4	CPCI 1600	225	3700	24 #13	15	24
SW2	4	CPCI 1600	225	3700	35 #13	15	30
SW3	4	CPCI 1900	225	3700	42 #13	15	36
GS1	4	CPCI 1600	225	3700	35 #13	15	30
GS2	5	CPCI 1600	225	3000	30 #13	15	30
GS3	6	CPCI 1600	225	2500	26 #13	15	30
ST1	4	CPCI 1600	180	3700	34 #13	15	30
ST2	4	CPCI 1600	225	3700	35 #13	15	30
ST3	4	CPCI 1600	260	3700	36 #13	15	30

concrete at jacking 25 MPa, concrete density 2400 kg/m^3 , Young's modulus 28 GPa for concrete of slab and 32 GPa for concrete of girders. Poisson's ratio is considered 0.15 for all types of concrete. For prestressing cables, yield stress 1750 MPa, ultimate stress 1860 MPa, Young's modulus 190 GPa, strain-hardening modulus 1090 MPa. For the reinforcing steel, yield stress 400 MPa, Young's modulus 200 GPa, and zero strain hardening. The jacking stress in the tendons is assumed equal to $0.7 f_y$.

4.3 Calculation procedure for distribution parameter and presentation of results

The bridges were analyzed by NONLACS using the finite element models described in Chapter 3. The program outputs stresses, strains, curvatures, displacement and rotations at selected points in the bridge. We will limit ourselves to basically the stresses. Actually, the longitudinal normal stresses are the main output parameter that is necessary to consider here. These stresses are used to calculate the resultant moment acting on each section. Two types of diagrams are plotted: moment ratio versus truck load, which illustrates the portion of total moment resisted by each girder at various load levels, and distribution parameter versus truck load, which will be described shortly. Note that girders at each section are numbered sequentially as G1, G2, G3, etc., with G1 being the left-most girder in the cross section.

The distribution parameter, D_{di} , is a parameter which is used to determine the design moment, M_G , for any girder. In the CHBD code M_G is determined by the following equation:

$$M_G = \frac{S}{D_d} \times \left[\frac{M_T}{N \times R_f} \right] \quad (4.14)$$

where M_T is the maximum total moment due to the truck load acting on the bridge, S is the girder spacing, N is the number of wheels on the cross-section of the bridge and R_f is a reduction factor due to number of loaded lanes according to CHBD code. Equation 4.14 can be rewritten as:

$$D_d = S \times \left[\frac{M_T}{M_G \times N \times R_f} \right] \quad (4.15)$$

In order to calculate the distribution parameter, one must first determine the maximum longitudinal moment acting on the bridge based on the stresses computed by the finite element. Moment is calculated by summing the moment contribution of all the concrete and steel layers in the cross-section. For each layer, the moment is calculated by multiplying the stress in the layer by the area of the layer and by the distance of the middle line of the layer from the centroidal axis of the transformed cross-section. This entails a tremendous amount of calculation and is best handled by another computer program. Hence, a separate program was written to process the finite element results.

To relate the findings of this study to the provisions of the code, the distribution parameter D_d for each girder will be plotted. Subsequently, the distribution parameters of the critical interior and exterior girders will be compared to their corresponding values based on the AASHTO (1994) and the CHBD (2000) codes.

We will also compare the prediction of a modified form of Fan's (1974) equation with the finite element results. As mentioned in Chapter 2, Fan's modified equation for

distribution parameter, which considers the effect of reduction factor of multilane loading, can be rewritten as follows:

$$D_d = \frac{S \times \text{Number of Girders}}{2 \times \text{Number of Lanes}} \quad (4.16)$$

where D_d is the distribution parameter, and S is the spacing of the girders. Razaqpur and Nofal (1988) showed that for composite concrete-steel slab-girder bridges good results could be obtained from this equation for moment redistribution parameter at ultimate state.

4.4 Parametric study and discussion of the results

In this section the effect of various parameters on the transverse distribution of longitudinal moment in prestressed precast slab-girder bridges will be presented. For each parameter the distribution characteristics will be studied through the D_d diagrams. The transverse section of interest is the section of maximum positive moment based on elastic analysis, which is located under the third axle from the front of the CHBD truck. Although the distribution parameters for all load stages will be shown, in fact there are three important load stages that are of interest in design: Elastic stage, post-cracking stage and the ultimate or failure stage. Later these stages will be more precisely defined in terms of the level of applied live load, and the distribution parameter at each stage for each girder will be calculated.

4.4.1 Truck position

The study of truck position involves finding the appropriate longitudinal and transverse locations of the truck and the number of loaded lanes. The critical loading configuration is the smallest total load that would cause failure of the bridge designated as position 1.

4.4.1.1 Longitudinal truck position

For simply supported bridges, the longitudinal location of the truck that produces the maximum moment is found by trial and error. Obviously, the maximum moment will be under one of the truck axles, probably one of the two heavier axles. To find the proper location of the truck, assume that A, B, C and D in Fig. 4.5 represent the truck axles. The support reaction, A_v , and the moment at B, M_B , can be determined as follows:

$$A_v = \frac{R (L/2 + x - d)}{L} = \frac{R}{2} + \frac{Rx}{L} - \frac{Rd}{L}$$

$$M_B = A_v \left(\frac{L}{2} - x \right) - Aa = \left(\frac{R}{2} + \frac{Rx}{L} - \frac{Rd}{L} \right) \left(\frac{L}{2} - x \right) - Aa = \frac{RL}{4} - \frac{Rd}{2} - \frac{Rx^2}{L} + \frac{Rxd}{L} - Aa$$

The location of maximum moment can be found from:

$$\frac{dM_B}{dx} = -\frac{2Rx}{L} + \frac{Rd}{L} = 0 \quad (4.17)$$

where R is the resultant of the axle loads and d is the distance between the point of application of the resultant R and the axle B and x is the distance of axle B from the centre of the beam. Solving Eq. 4.17, results in $x = \frac{d}{2}$. Hence, the maximum moment

will occur when the resultant R and the axle B are acting at equal distance from the mid span of the bridge, and this is designated as position 1.

Fig. 4.5: Truck axles on simply supported beam

A similar approach can be used to find the longitudinal truck position for maximum shear, which is designated as position 2. However, the interest in this study is mainly in the bending action. These two truck positions are referred to as longitudinal position 1 and position 2, respectively in this thesis. For each of these positions, several transverse truck configurations are considered as described in the next section. The outcome of these loading configurations will be establishment of the critical loading case for the study of the effects of other parameters.

4.4.1.2 Transverse truck position

It is mentioned in the CHBD code that in order to produce the maximum longitudinal moment in a bridge, first the bridge must be idealized as a single span beam

and then the truck must be placed on the beam in such a way that it will produce the maximum longitudinal moment in this idealized beam. The assumption of the code method is that the number of girders, the transverse position of the truck and other bridge parameters do not influence this moment. This assumption seems to be reasonable for narrow to medium width bridges, but for wider bridges, it will overestimate the girder moment. In the case of reinforced and prestressed girder bridges, redistribution of forces and moments may occur due to inelasticity, or because all the girders may not reach cracking and ultimate moment simultaneously. In this study, we will investigate the effect of number and transverse positions of trucks on load redistribution. Hence, while maintaining their longitudinal positions the same, the trucks are placed in various transverse configurations in order to find the maximum moment and shear in the bridge. The critical load configuration is the one that yields the smallest failure load.

To find the critical load configuration, two bridges were analyzed, each for many possible transverse positions of CHBD truck as indicated in Figs. 4.6 to 4.9. The trucks illustrated in these figures represent the CHBD truck which has the axle configuration shown in Fig.4.10. Nine loading cases were considered for each bridge; load case 9, in which all the lanes are loaded, is not shown in these figures. The bridges have been designated as LP1 and LP2, where LP1 refers to the four-girder bridge and LP2 to the five-girder one. The specifications for these bridges are given in Table 4.1.

Figures 4.11 (a) and (b) show the failure or maximum load that the two bridges could carry in each loading case. Observe that the two longitudinal positions of the truck

Fig.4.6. Transverse truck positions for bridge LP1 (cases 1 to 4)

Fig.4.7. Transverse truck positions for bridge LP1 (cases 5 to 8)

Fig.4.8. Transverse truck positions for bridge LP2 (cases 1 to 4)

Fig.4.9. Transverse truck positions for bridge LP2 (cases 5 to 8)

Fig.4.10: CHBD Truck configuration

lead to practically identical results. Therefore, in subsequent analysis only positions 1, as described in section 5.4.1.1, will be considered. From the latter figures, the critical loading case for the four girder and five girder bridges can be determined. As can be seen, load case 7 for both the four and five girder bridge is the critical case because it corresponds to the smallest load that would cause failure. Accordingly, in the parametric study to follow, all the bridges will be subjected to longitudinal position 1 and transverse load case 7.

Results of the analysis will be displayed as moment ratio and distribution parameter versus load diagrams. Using the procedure described in Section 4.3 and using Eq. 4.15, Figs. 4.12 (a) and (b), respectively, show typical variation of moment ratio (M_{girder} / M_{total}) and the computed distribution parameter D_d with increased loading. Although Figs. 4.12 (a) and (b) essentially show the variation of the same basic parameter, the M_{girder} / M_{total} diagram is shown for clarity. It clearly indicates for the section of maximum moment, the change in the girders moment compared to the change in the total moment, caused by the increase in the live load. Based on elastic analysis, this ratio would be constant, irrespective of the load level, but as Fig. 4.12(a) shows, it actually changes with level of load. The figure also shows that the girder moments are initially quite different in magnitude, but with increased loading they tend to become closer in value. Note, however, that the initiation of cracking and the yielding of reinforcement are the two important factors that influence redistribution. The load at which the marked ascension or descension in the moment ratio diagrams appear correspond to the cracking of the girder under maximum moment. The beginning of the plateau at the end of the

(a) The failure load in each load case for 4-girder bridge

(b) The failure load in each load case for 5-girder bridge

Fig. 4.11: Failure load for four and five girder bridges

(a)

(b)

Fig. 4.12: (a) Variation of moment ratio (b) Variation of distribution parameter

ascending/descending portions generally marks initiation of yield in the heavily loaded girder. As can be seen in Fig. 4.12, at the elastic stage, the moment resulting from the applied load is distributed highly unequally among the girders. Increasing the load leads to cracking of the heaviest loaded girder, and the stiffness of this girder is reduced.

Subsequently, the cracked girder cannot carry the same proportion of the total moment and thus the other girders absorb a higher percentage of the total moment until they reach the cracking stage. Further redistribution occurs as the remaining uncracked girders absorb a greater portion of the total moment. The extent of redistribution after yielding does not appear to be very high in Fig. 4.11. This is not entirely unexpected because prestress steel lacks the yield plateau that is characteristic of reinforcing steel. Nevertheless, as can be seen in the Fig. 4.12, the ultimate moments of the girders are much closer to each other compared to their elastic values.

The results of the analysis for all the loading cases are plotted in Appendix A. Similarly to Fig. 4.12, in the appendix, part (a) and (b), respectively, show the variation of moment ratio and distribution parameter D_d with increasing load.

Although the graphs in Fig. 4.12 and Appendix A show the variation of D_d over the entire loading range, in practice, distribution parameters of three states are of interest; namely, the elastic state, post-cracking state, i.e. at approximately 60% of ultimate load, and the ultimate state. Figures 4.13 and 4.14, respectively, show the values of the distribution parameters for the critical external and internal girders at the foregoing three states. The figures also show the corresponding values according to the AASHTO (1994),

CHBD (2000) and Fan's equation. The charts in these figures are based on the results of the critical loading as described earlier.

The charts indicate over 28% moment redistribution in the critical external girder in going from the elastic state to the cracked state. This is true for both the 4 and 5 girder bridges. On the other hand, the extent of redistribution does not seem to be appreciable in going from the post-cracking state to the ultimate state. As for the critical internal girder, the same observation applies, but the extent of redistribution in going from the elastic (uncracked) to the post-cracking state is somewhat smaller, up to 19% and 21% for the 4 and the 5 girder bridge, respectively.

Fig. 4.13: Variation of distribution parameter for the critical external girder

Fig.4.14. Variation of distribution parameter for the critical internal girders

The charts show that Fan's equation does not apply to prestressed concrete bridges and its results would be on the unsafe side. The AASHTO method appears to overestimate the moment in the external girder but its results are close to the finite element results for the internal girders, while the CHBD code recommendations agree closely with the current results for elastic analysis. Both the AASHTO and CHBD methods are conservative for predicting the ultimate moment distribution in these bridges.

4.4.2 Bridge Width

As mentioned in the literature review, at elastic stage bridge width has an important effect on the load distribution characteristics of slab-girder bridges. Bakht et al

(1978) reported that the distribution parameters of bridges are related to their torsional and flexural parameters α and θ as described in Chapter 2. According to Eq. 2.4, θ is proportional to bridge width; therefore, it affects the distribution characteristics of bridges in the elastic state. However, its effect in the inelastic stage may not be the same and needs to be investigated for prestressed concrete slab-girder bridges. Thus, three bridges were analyzed to investigate the effect of this parameter upon load distribution throughout the entire loading range. The bridges have width of 13.0 m, 17 m and 21 m, with a constant span length of 30 m. These bridges were named as BW1, BW2 and BW3. Note that the girder cross-section and concrete slab thickness are held constant in these bridges.

Figures 4.15 and 4.16 illustrate the values of the distribution parameters for the critical external and internal girders, respectively. The corresponding values according to the AASHTO (1994) and CHBD (2000) codes, as well as Fan's equation, are shown in these graphs. The finite element values are based on the results of the critical loading as described earlier.

The charts indicate 29%, 26% and 22% moment redistribution in the critical external girder for BW1, BW2 and BW3, respectively, in going from the elastic state to the ultimate state. As for the critical internal girder, the same observation applies, and the percent of redistribution in going from the elastic (uncracked) to the ultimate state for the three bridge widths are, respectively, 30%, 25% and 21%. The charts show that the results of Fan's equation are on the unsafe side and the AASHTO method overestimate the moment of the external girder but predicts a moment close to finite element analysis results for the internal girder, while the CHBD code recommendations agree closely with

Fig.4.15: Distribution parameter, D_a , with bridge width for critical external girder

Fig.4.16: Distribution parameter, D_a , with bridge width for critical internal girder

the elastic analysis results for these bridges. Both the AASHTO and CHBD methods are conservative for predicting the ultimate moment distribution in these bridges.

From these charts we can see that the widest bridge experiences the least redistribution. For the widest bridge, the extent of redistribution is basically small in going from the post-cracking, which is considered to be 60% of ultimate load, to the ultimate state, whereas in the narrow bridges, some redistribution occurs between the previous two states. In conclusion, it can be stated that bridge width is an important parameter that can have noticeable effect on load distribution.

4.4.3 Span / Width Ratio

In this section we will determine the effect of span/width ratio on prestressed concrete slab-girder bridges. Three bridges are modeled with span lengths of 24 m , 30 m and 36 m , with the width being kept constant at 15 m . These bridges are designated as SW1, SW2 and SW3, where bridge SW2 is the same as bridge LP1. The geometry and dimensions of these bridges are given in Table 4.1.

Bakht et al (1978) reported that in the elastic state, if the slab width is kept constant, the variation of span within reasonable limits would not significantly influence the load distribution of bridges, but the effect of this parameter is unknown for prestressed concrete slab-girder bridges. Therefore, we performed the present analyses.

The distribution parameters, D_u , for all the girders in these bridges are plotted in Fig A.40 to A.42 in Appendix A. Figures 4.17 and 4.18 show the values of the distribution parameters at these states for the critical external and internal girders.

Fig. 4.17: Distribution parameter variation, D_d with span/width ratio, for the critical external girder

Fig. 4.18: Distribution parameter variation, D_d with span/width ratio, for critical internal girder

respectively. The corresponding values according to the AASHTO (1994) and CHBD (2000) codes, as well as Fan's equation, are also shown in these figures.

Figure 4.17 indicates almost 40%, 30% and 20% moment redistribution in the critical external girder for bridges SW1, SW2 and SW3, respectively, in going from the elastic state to the cracked state. As for the critical internal girder, 30%, 21% and 18% moment redistribution occurred as shown in Fig.4.18. It can be observed that the smaller the span/width ratio, the higher the moment redistribution in going from the elastic to the ultimate state, but in going from the post-cracking to the ultimate state, this parameter does not have a strong effect on redistribution.

The charts show that Fan's equation again does not apply to prestressed concrete bridges and its results would be on the unsafe side. The AASHTO method seems to overestimate the distribution parameter for external girders but it yields reasonably close agreement with the finite element results for internal girders, while the CHBD code recommendations always agree closely with the current results for elastic analysis. Both the CHBD and AASHTO methods are conservative for predicting ultimate moment distribution in these bridges. The reason is that all of the equations of the CHBD and AASHTO are based on constant stiffness of the system at the elastic state while for prestressed concrete slab-girder bridges cracking noticeably changes the stiffness and leads to appreciable moment redistribution compared to the elastic state.

It is important to point out that the results in Fig. 4.17 and 4.18 reflect the change in the moment of the critical external and internal girders, respectively. Clearly, as one can observe in Fig.A.40 to A.42 in Appendix A, the moment in the other girders are also

affected by the change in stiffness and therefore experience some redistribution. Finally, from the above results, we reach the conclusion that the effect of width/span on the variation of D_d is important and should be considered when calculating the distribution parameters of prestressed concrete girder bridges.

4.4.4 Girder Spacing (Number of Girders)

It is assumed that this parameter has a significant effect on load distribution in prestressed concrete girder-slab bridges. Both CHBD and AASHTO consider this parameter in the calculation of the distribution factor. According to the CHBD code, there is a linear relationship between S and D_d . It should be mentioned that the code provisions are based on linear elasticity with constant stiffness over the entire loading range. It remains to be investigated whether it has the same significant effect in the inelastic states as in the elastic state.

In the current study the influence of girder spacing, S , was studied by analyzing four, five and six girder bridges, each with three different girder spacing. The span length, and the width and thickness of the slab were kept constant, and only the spacing between girders was varied as 3.7, 3 and 2.5 m, respectively. These bridges are denoted as GS1, GS2 and GS3 whose dimensions and structural details are shown in Table 4.1.

In Fig. A.43 to A.45 in Appendix A, the variation of moment ratio and distribution parameter is shown for these bridges over their complete loading range. The values of the distribution parameters for the critical external and internal girders are shown in Figures 4.19 and 4.20. The figures also show the corresponding values according to the AASHTO (1994) and CHBD (2000) codes as well as Fan's equation. The charts indicate 17%, 20%

and 26% moment redistribution in the critical external girder for GS1, GS2 and GS3, respectively. In GS1, the extent of redistribution does not seem to be appreciable in going from the post-cracking to the ultimate state while in GS2 and GS3, there appears to be almost equal redistribution from the elastic to the cracked stages on the one hand and from the cracked to the ultimate states on the other. As for the critical internal girders, redistribution over 18%, 20% and 23% can be observed. The above comments with respect to the extent of redistribution of different stages also apply to the critical internal girder.

The charts show that Fan's equation gives unsafe results for 4 girder bridges, is reasonable for 5 girder bridges and becomes conservative for 6 girder bridges. Hence, as the number of girders increase, its results tend to become more conservative. As in the previous charts, the AASHTO method appears to in some cases slightly overestimate the distribution parameter for external girders, but yields reasonable results for internal girders. On the other hand, the CHBD code provisions agree closely with the finite element results. Again both the AASHTO and the CHBD methods are conservative for predicting the ultimate moment distribution in these bridges. It can be concluded from these results that increasing the number of prestressed girders in a constant width bridge, leads to better redistribution of total moment in the cross-section of a bridge, which means that this parameter has significant effect on the distribution parameter. For instance, in increasing the number of girders from four to six, the percent redistribution for the external girder at ultimate increased from 17% to 26%.

Fig.4.19: Distribution parameter variation, D_d , for the critical external girder

Fig.4.20: Distribution parameter variation, D_d , for the critical internal girder

4.4.5 Slab/Girder Stiffness Ratio

From elastic analysis we know that the distribution parameter, D_d , is a function of the torsional and flexural rigidities of the transformed cross-section of the bridge. Flexural rigidity can be determined using:

$$D_{\tau} = (D_{\tau})_s + (D_{\tau})_G$$

Where $(D_{\tau})_s$ is the flexural rigidity of a unit width of the slab, and $(D_{\tau})_G$ is an equivalent rigidity obtained by dividing the transformed girder flexural rigidity by the girder spacing. This equation indicates that for an individual girder, the relative magnitude of $(D_{\tau})_s$ and $(D_{\tau})_G$ is not significant. The important quantity is their sum, D_{τ} . Consequently, as far as the elastic behaviour is concerned, the relative stiffness of the slab to girder has no effect on load distribution characteristics of prestressed concrete girder-slab bridges. But this cannot be assumed in the cracked and inelastic stages because after cracking the girder contribution to the stiffness will decrease relative to the slab contribution.

Therefore, in order to study the effect of slab/girder stiffness ratio, three bridges are analyzed, namely ST1, ST2 and ST3. All the bridges have the same dimensions with the exception of the slab thickness. The details of these bridges are presented in Table 4.1. The thickness of the slabs is 180 mm, 225 mm and 260 mm, for ST1, ST2 and ST3, respectively.

The results of analysis are illustrated in Fig.A.46 to A.48 in Appendix A, where the moment ratio and the distribution parameter diagrams are plotted over the entire loading

Fig.4.21: Variation of distribution parameter, D_d for span/width ratio for critical external girder

Fig.4.22: Variation of distribution parameter, D_d for span/width ratio for critical internal girder

range. Using the latter results, Fig. 4.21 and 4.22 show the values of the distribution parameters for the critical external and internal girders as well as the relevant values according to Fan's equation and the AASHTO (1994) and CHBD (2000) codes. Once again these charts are based on the results of the critical loading case.

The latter figures indicate about 22%, 30% and 40% moment redistribution in the critical external girders of bridges, ST1, ST2 and ST3 respectively. For the critical internal girder, the corresponding values are 23%, 23% and 26%, respectively. The charts show that Fan's equation does not apply to these prestressed concrete bridges and results would be on the unsafe side. The AASHTO method, as before, overestimates the distribution parameter for the external girder but is reasonable for the internal girder. In all the cases the CHBD code recommendations closely agree with the elastic analysis results for these bridges. Both the AASHTO and CHBD code recommendations appear to be conservative for predicting the ultimate moment distribution. Relatively speaking, an increase in slab thickness within the usual range does not lead to any significant change in moment redistribution.

4.4.6 Summary of parametric analysis results

In the previous section a number of bridges were analyzed and their redistribution characteristics were examined. The level and amount of redistribution was quantified by calculating the distribution parameter in the uncracked elastic state, the cracked state and the ultimate state. Current code recommendations are based on results of elastic analysis, assuming uncracked section. We noticed in the previous sections that significant redistribution can occur upon advent of cracking or just before ultimate load. To gauge

the extent of redistribution due to cracking and inelasticity, Table 4.2 summarizes the values of the distribution parameters for all the analyzed bridges. Values based on the results of the nonlinear finite element results and on code recommendations, as well as on Fan's equation, are shown. For the nonlinear analysis results, values for the uncracked elastic state and for the ultimate state are indicated. The last column shows the percentage difference between the elastic and ultimate state values calculated as :

$$\% \text{ Redistribution} = \frac{D_d^u - D_d^e}{D_d^e}$$

where D_d^u and D_d^e are the distribution parameter for the ultimate and elastic states according to finite element results.

Since Fan's equation is intended for the ultimate state, we notice that its predictions differ in most cases substantially from the ultimate state values obtained by finite element analysis. The results of the equation are actually on the unsafe side and therefore not suitable for the type of bridges studied in the current investigation.

The AASHTO results are reasonable and almost always on the conservative side compared to the values obtained by elastic finite element analysis. On the other the CHBD predictions are very close to the elastic analysis results. Therefore, it can be concluded that both codes recommendations are reasonable for design. However, the results of both codes are conservative because when compared to the nonlinear analysis results, they are much more conservative and overestimate the girder moments. The extent of redistribution due to inelasticity varies from 17% to a maximum of 40%. Since in this study no attempt will be made to develop a precise method for predicting the

Table 4.2: Distribution parameter values:

Bridge	Critical girder	Fan's equation	CHBD code	AASHTO code	Elastic stage	Ultimate stage	Percent of redistribution
LP1	External	1.87	1.00	1.13	1.00	1.30	30%
LP1	Internal	1.87	1.06	1.13	1.14	1.38	21%
LP2	External	1.50	1.25	1.33	1.2	1.55	29%
LP2	Internal	1.50	1.33	1.33	1.3	1.60	23%
BW1	External	2.20	1.08	1.15	1.05	1.35	29%
BW1	Internal	2.20	1.10	1.20	1.15	1.50	30%
BW2	External	2.12	0.9	1.02	0.9	1.13	26%
BW2	Internal	2.12	0.96	1.02	1.00	1.25	25%
BW3	External	1.75	0.75	0.9	0.79	0.97	22%
BW3	Internal	1.75	0.77	0.9	0.81	0.98	21%
SW1	External	1.85	0.99	1.07	1.00	1.40	40%
SW1	Internal	1.85	1.04	1.07	1.15	1.50	30%
SW2	External	1.85	1.00	1.13	1.00	1.3	30%
SW2	Internal	1.85	1.07	1.13	1.14	1.38	21%
SW3	External	1.85	1.00	1.14	1.00	1.2	20%
SW3	Internal	1.85	1.075	1.14	1.10	1.30	18%
GS1	External	1.87	1.00	1.13	1.00	1.17	17%
GS1	Internal	1.87	1.06	1.13	1.10	1.3	18%
GS2	External	1.50	1.25	1.33	1.25	1.50	20%
GS2	Internal	1.50	1.33	1.33	1.30	1.57	20%
GS3	External	1.25	1.50	1.52	1.50	1.90	26%
GS3	Internal	1.25	1.56	1.52	1.50	1.84	23%
ST1	External	1.85	1.00	1.19	1.00	1.22	22%
ST1	Internal	1.85	1.10	1.19	1.10	1.35	23%
ST2	External	1.85	1.00	1.16	1.00	1.30	30%
ST2	Internal	1.85	1.10	1.16	1.10	1.35	23%
ST3	External	1.85	1.00	1.13	0.95	1.33	40%
ST3	Internal	1.85	1.10	1.13	1.14	1.44	26%

ultimate distribution parameter, it would be reasonable to recommend that a minimum of 15% redistribution can be assumed in design. This means that one could subject the bridge to a load 15% greater than its design capacity based on existing code methods, without jeopardizing its safety. Further studies are needed to develop a more rigorous design method that would permit higher redistribution in bridges with specific geometry and other relevant properties.

Chapter 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary

In this study extensive nonlinear finite element analysis was undertaken to investigate the effect of different parameters on transverse moment distribution in prestressed concrete slab-girder bridges.

After reviewing available literature, it was shown that with respect to load distribution at ultimate state, there is lack of research and studies need to be carried out to find its significance with respect to the ultimate load capacity of multi-girder prestressed concrete bridges. This led to the work described in this thesis.

The finite element program NONLACS was modified and was used in the analyses in this study because of its nonlinear analysis capabilities, with particular reference to prestressed concrete slab-girder bridges. The modification of the program was necessary

to properly account for the manner in which multi-girder prestressed concrete bridges are constructed. The construction often involves precast prestressed girders attached to a cast-in situ concrete slab. The nonlinear method of analysis proposed here for stage construction is relatively simple and does not require a change in the topology of the structure because the topology of the final, or completed structure, is employed in the usual manner. Subsequently, the modified program was verified by comparing the analytical results of NONLACS with available experimental results for a precast prestressed concrete composite girder.

Thereafter, a detailed parametric study was undertaken to determine the influence of some geometric and loading parameters on moment distribution of this type of bridges in the whole range of loading up to failure. The studied parameters were longitudinal truck position, transverse truck location, bridge width, span/width ratio, girder spacing (number of girders), and slab thickness. Bridges analyzed in this work were 24 *m* to 36 *m* long and 13 *m* to 21 *m* wide, with slab thickness of 180 to 260 *mm* and they had between 4 and 6 girders. All the bridges were assumed to be simply supported. Comparison of the finite element results with available methods, including that of CHBD and AASHTO codes, and with the so-called Fan's equation, revealed that existing methods cannot accurately predict the load distribution properties of prestressed concrete slab-girder bridges at all stages of loading. However, in nearly every case in the current study, the CHBD code method was found to yield accurate results for the elastic state and safe values for the ultimate state. The AASHTO code results were generally more conservative for both the elastic and ultimate states. The results of the study support the following conclusions.

5.2 Conclusions

1. The nonlinear finite element method is an accurate and powerful analytical tool for predicting the behavior of prestressed concrete slab-girder bridges in the entire loading range up to failure.
2. The proposed method of nonlinear analysis for structures constructed in several stages yields reasonable results compared with existing experimental data and closed-form hand calculations.
3. Maintenance of the same topology throughout all the stages of analysis reduces the book-keeping tremendously and makes the computer programming simpler.
4. The load distribution characteristic of prestressed concrete slab-girder bridges is affected by the level of loading due to the change in stiffness brought by cracking of concrete and yielding of reinforcement.
5. When all design lanes of a bridge are loaded, generally, full transverse redistribution of moment occurs in the girders. In other words, under full nonlinear or inelastic condition, all the girders at the section of maximum moment equally participate in resisting the applied load.
6. Among the parameters considered in this study, it was determined that longitudinal truck position does not have any significant effect upon load redistribution in this type of bridges.

7. Span/width ratio has a significant influence upon load redistribution and up to 20% change in the value of the distribution parameter can occur when bridge width is kept constant while the span is increased.
8. The effect of other parameters such as bridge width, girder spacing, slab/girder stiffness ratio is relatively small but they do have some effect on the distribution parameter.
9. The CHBD (2000) code gives good results for distribution parameters of this type of bridges at the elastic state but gives conservative results at the ultimate state.
10. The AASHTO (1994) code results are generally more conservative than the CHBD (2000) code. In a few cases, the AASHTO results appear to be on the unsafe side.
11. Modified Fan's equation is always on the unsafe side for this type of bridges and can not be used for predicting the distribution parameter in prestressed concrete slab-girder bridges.
12. Based on the results of the current study, a minimum 15% redistribution may be permitted for the multi-girder simply supported prestressed concrete bridges of the type investigated in this study.

5.3 Recommendations for Future Work

The current study was restricted to simply supported concrete slab on prestressed girders. Although this type of bridges represents a large number of actual bridges, the

following future work is recommended with regard to ultimate moment distribution in multi-girder prestressed:

- Continuous bridges
- Skew bridges
- Curved bridges
- Bridges subjected to dynamic loading

We restricted our attention in this study to the longitudinal and transverse moments. It may also be of interest to investigate the longitudinal and transverse shear at the interface of prestressed girder with concrete slab. In addition, experimental investigations are required to verify the applicability of the nonlinear finite element program NONLACS to the above cases.

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APPENDIX A

In this appendix the moment ratio (M_{girder} / M_{total}) and distribution parameter,

D_d , diagrams of all the bridges analyzed in this study are shown. The diagrams show the variation of these quantities for each girder with the level of applied live load.

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.1: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig.A.2: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig A.3: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.4: Bridge LP1, Position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.5: Bridge LP1, Position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.6: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.7: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.8: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig.A.9: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A10: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.11: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.12: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.13: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.14: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.15: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.16: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.17: Bridge LP1, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.18: Bridge LP1, position 2

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.19: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig.A.20: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig A.21: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.22: Bridge LP2, Position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.23: Bridge LP2, Position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.24: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.25: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.26: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig.A.27: Bridge LP2. position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A28: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.29: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.30: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.31: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.32: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.33: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.34: Bridge LP2, position 2

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.35: Bridge LP2, position 1

(a): Variation of moment ratio

(b): Variation of distribution parameter

Fig. A.36: Bridge LP2, position 2

Bridge width :

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.37: Bridge BW1

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.38: Bridge BW2

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.39: Bridge BW3

Span/width ratio:

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.40: Bridge SW1

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.41: Bridge SW2

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.42: Bridge SW3

Girder spacing:

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.43: Bridge GS1

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.44: Bridge GS2

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.45: Bridge GS3

Slab/girder stiffness ratio:

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.46: Bridge STI

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.47: Bridge ST2

(a): Variation of moment ratio

(b): Variation of Distribution parameter

Fig.A.48: Bridge ST3