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High Strain Rate Tensile Testing

by

Jo Ann Marie Clarke

B. Eng. (Mechanical Engineering)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of

Master of Engineering

in Mechanical Engineering

Department of Mechanical and Aerospace Engineering

Ottawa-Carleton Institute

for Mechanical and Aerospace Engineering

Carleton University

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High Strain Rate Tensile Testing

submitted by

Jo Ann Marie Clarke
B. Eng (Mechanical Engineering)

in partial fulfilment of the requirements
for the degree of Master of Engineering
in Mechanical Engineering

Thesis Supervisor

Chair, Department of Mechanical and Aerospace Engineering

Carleton University
July 1993

Abstract

Ductile fracture is a dominant tensile failure mechanism for ductile metals. This failure mechanism occurs through the nucleation, growth and coalescence of voids to form a crack whose propagation across the material leads to complete failure. The ductile fracture mechanism is strongly influenced by loading rate, triaxiality, strain hardening rate and particle population.

The research for this graduate program comprised the design and commissioning of a Tensile Split Hopkinson Bar facility. The facility was used to investigate high strain rate ductile fracture of brass, which was selected as a model material for the study. Scanning electron and optical microscopy and numerical modelling were incorporated as part of the research.

Two effects of the elevated loading rate on the ductile fracture behaviour of brass were found. At high strain rates, inertial stabilization of void growth occurs which delays the onset of failure by coalescence and increases the ductility. The triaxiality is increased at high strain rates and accelerates the accumulation of damage in the material which decreases ductility. These two effects are competitive; however, inertial stabilization of void growth dominates and therefore the ductility of brass is increased at elevated loading rates.

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Chapter 1

Introduction

Ductile metals undergoing high strain rate tensile loading usually fail by one of two mechanisms: adiabatic shear or ductile fracture. Adiabatic shear is characterized by the formation of narrow bands of intense shear at sites of stress concentration in an otherwise uniformly straining solid. Because of the high deformation rate, the heat generated by the work of plastic deformation is not conducted away from the plastic zone. The deformation is effectively adiabatic and the material temperature increases. The increase in the local temperature leads to thermal softening and encourages further plastic flow. This continuing process leads to the development of a band of intense plastic strain which propagates along planes of maximum shear stress or minimum strength through the material until complete fracture occurs [1]. The other common mechanism, ductile fracture, is characterized by the nucleation and growth of voids and the consolidation or coalescence of these voids to form a central crack whose propagation across the material leads to complete failure [2]. Voids are usually nucleated at second phase particles or inclusions either by the separation of matrix material from the particle or by cracking of the particle [3–5].

The present work is concerned with the latter failure mechanism: ductile fracture. In particular, ductile fracture of brass specimens undergoing high strain rate tensile loading is examined.

To achieve the high loading rates in tension a Tensile Split Hopkinson Bar (TSHB) facility was developed as part of this graduate program. This apparatus is capable of subjecting specimens to tensile loading at gauge length strain rates of $1500s^{-1}$.

Also of interest to the current work are models developed to describe the constitutive response of the plastically deforming material. In particular, the Gurson constitutive model, developed for quasi-static ductile fracture, will be examined for its applicability to the high strain rate, high triaxiality tensile regime characteristic of TSHB testing.

The remainder of the chapter is devoted to a review of technical literature pertaining to the present research covering three main topics:

- The Hopkinson bar apparatus
- Ductile fracture
- Current constitutive models.

1.1 The Hopkinson Bar Apparatus

In 1914, Bertram Hopkinson [6] developed a technique to study the pressures produced by explosive detonation or projectile impact using a long pressure bar. The design was based on the fact that if the pressure bar remained elastic then its displacements would be proportional to the stresses and the wavelength of the stress pulse would be related to the duration of the impact. The magnitudes of the pressures were estimated by means of a ballistic pendulum.

Davies [7] in 1948 and Kolsky [8] in 1949 designed condensers to measure the displacements of the pressure bar. Kolsky [8] introduced the Split Hopkinson Pressure Bar (SHPB) in which a compression specimen is located between two pressure bars, one of which is impacted by a striker. The displacements in the bars are then

used to determine the stress and strain within the specimen. The SHPB, with the introduction of strain gauges to monitor the stress wave behaviour, is now widely used for compression testing of materials at high rates of deformation. The reader is referred to several reviews on this topic [9–12].

A variant of the SHPB technique is the Tensile Split Hopkinson Bar (TSHB), designed to test specimens at elevated strain rates in tension. A number of experimenters [13–21] have developed tensile versions of the SHPB apparatus. The tensile techniques used may be loosely grouped into three main areas:

- Concentric cylinders
- Hat-shaped specimens
- Threads and collars in a compression bar

In the first category, the apparatus [13–16] consists of two concentric cylinders as shown in Figure 1.1. The two inner cylinders, called the input bar and the output bar, are solid and hold the specimen between them. The hollow outer cylinder is attached to the input bar such that when it is impacted, a tensile pulse is induced in the specimen. The main disadvantage to this technique is that the specimen is inaccessible throughout the test.

The second category of apparatus employs a top hat shaped specimen [17] as shown in Figure 1.2. The transmitter bar is a hollow tube which covers the crown and rests on the brim. The incident bar is a solid cylinder which goes inside the hat. When a striker bar impacts the incident bar a tensile pulse is induced in the specimen. The disadvantages of this configuration are that the gauge area of the specimen is actually a tube and the specimen is essentially inaccessible during a test.

In the final category, the specimen is threaded [18–21] between the two bars (Figure 1.3). A collar which contacts both bars is placed around the specimen. The

loading bar is impacted compressively at its end and the collar serves to transmit the compressive pulse across the specimen without yielding the specimen. This compressive pulse reflects off the end of the transmitter bar as a tensile pulse which deforms the specimen in tension. This technique is versatile since it uses the same equipment as a compressive bar. However, unless the collar has holes [20], it provides no access to the specimen.

For the current work an entirely different system was designed as shown in Figure 1.4. The specimen is threaded between two bars. A cylindrical striker rides concentrically on the incident bar and impacts an endcap on the incident bar inducing a tensile pulse. The advantage of this system is that the specimen is readily accessible throughout the test for the purpose of high-speed photography or high temperature experiments.

The TSHB facility developed as part of this thesis is capable of testing metals such as brass, copper and iron. Strain rates of 1500 s^{-1} over the test specimen gauge length can be achieved depending on the specimen configuration and material.

1.1.1 The Hopkinson Bar Equations

In the Hopkinson Bar apparatus, the impact of the striker against the incident bar induces a stress wave in the incident bar which propagates along the incident bar toward the specimen.

Consider Figure 1.5 which depicts the transient response of a striker of length L and initial velocity v_0 impacting a longer bar with the same cross-sectional area. A compressive impact is used for simplicity, however, the propagation characteristics of a tensile pulse are identical except that stress has the opposite sign. Upon impact, at time $t = 0$, a compressive stress wave with magnitude $-\sigma$ begins to propagate through each of the bars. The velocity of propagation of this stress wave is the speed of sound in the bar material, c_0 , which is a function of the modulus of elasticity, E , and the density, ρ , of the material,

$$c_o = \left(\frac{E}{\rho} \right)^{1/2} . \quad (1.1)$$

The time for the wave to propagate the entire length of the striker is equal to L/c_o . The magnitude of the induced stress wave is

$$\sigma = \frac{1}{2} v_o \rho c_o . \quad (1.2)$$

At time L/c_o , the compressive wave will reflect against the free end of the striker as a tensile wave of the same magnitude as the incident compressive wave. The tensile wave serves to unload the striker of the compressive stress. During the time $L/c < t < 2L/c_o$ this unloading wave will travel through the striker and the compressive wave will continue to travel through the long bar. At times exceeding $2L/c_o$, the striker is completely free of stress and a compressive wave of length $2L$ and magnitude $-\sigma$ will continue to propagate through the long bar.

In the Hopkinson Bar apparatus, the induced stress wave propagates along the incident bar towards the specimen. Due to the lower cross-sectional area and lower flow stress of the specimen, an impedance mismatch exists between the bar and specimen. When the stress pulse impinges upon the specimen, the incident stress wave is partially transmitted through the specimen into the transmitter bar and partially reflected back into the incident bar. The test is contrived to provide a stress pulse in the specimen whose magnitude is sufficient to plastically deform it while only elastically deforming the bars.

The incident, reflected, and transmitted components of the stress pulse are recorded by strain gauges at stations midway along each of the bars. The next few paragraphs include the derivation of a set of equations based on elementary wave propagation theory which describe the stress, strain and strain rate in the specimen based on the strain gauge histories. For simplicity, a compressive SHPB

apparatus will be considered for this derivation, however, the equations are identical for the TSHB except that the stress waves have opposite signs.

The derivations of the equations for strain, stress and strain rate in the specimen are given by Nicholas [22]. Figure 1.6 depicts the specimen, bars, and the incident, reflected and transmitted strain pulses, ϵ_i , ϵ_r and ϵ_t . For simplicity the incident end of the specimen is labeled '1' and the transmitter end is labeled '2'. The total strain at the incident end of the specimen, ϵ_1 , is the incident strain, ϵ_i , minus the reflected strain, ϵ_r ,

$$\epsilon_1 = \epsilon_i - \epsilon_r \quad . \quad (1.3)$$

The total strain at the transmitter end of the specimen, ϵ_2 , is simply the transmitted strain, ϵ_t ,

$$\epsilon_2 = \epsilon_t \quad . \quad (1.4)$$

The displacements at the incident end, u_1 , and at the transmitter end, u_2 , are given by

$$u_1 = c_o \int_0^t \epsilon_1 dt \quad , \quad (1.5)$$

$$u_2 = c_o \int_0^t \epsilon_2 dt \quad . \quad (1.6)$$

In terms of the incident, reflected and transmitted strains, the displacements are

$$u_1 = c_o \int_0^t (\epsilon_i - \epsilon_r) dt \quad , \quad (1.7)$$

$$u_2 = c_o \int_0^t \epsilon_t dt \quad . \quad (1.8)$$

The average strain across the gauge length of the specimen, ϵ_s , is the difference in displacements divided by the specimen gauge length, L_s ,

$$\epsilon_s = \frac{u_1 - u_2}{L_s} \quad . \quad (1.9)$$

Substituting for u_1 and u_2 gives

$$\epsilon_s = \frac{c_o}{L_s} \int_0^t (\epsilon_i - \epsilon_r - \epsilon_t) dt \quad . \quad (1.10)$$

The forces, P_1 and P_2 , at each end of the specimen are equal to the strains multiplied by the elastic modulus, E_b , and the cross-sectional area of the bar, A_b ,

$$P_1 = E_b A_b (\epsilon_i + \epsilon_r) \quad , \quad (1.11)$$

$$P_2 = E_b A_b \epsilon_t \quad . \quad (1.12)$$

If we assume that the specimen is deforming uniformly then force equilibrium exists across the specimen, that is

$$P_1 = P_2 \quad , \quad (1.13)$$

and it follows that

$$\epsilon_i + \epsilon_r = \epsilon_t \quad . \quad (1.14)$$

Substituting this equation into Equation 1.10 for the gauge length strain of the specimen gives

$$\epsilon_s = \frac{c_o}{L_s} \int_0^t (\epsilon_t - \epsilon_r - \epsilon_r - \epsilon_t) dt \quad , \quad (1.15)$$

or

$$\epsilon_s = \frac{-2c_o}{L_s} \int_0^t \epsilon_r dt \quad . \quad (1.16)$$

The specimen stress, σ_s , is simply the force divided by the cross-sectional area of the specimen area, A_s ,

$$\sigma_s = P_2/A_s \quad . \quad (1.17)$$

Substituting Equation 1.12 for P_2 gives

$$\sigma_s = E_b \frac{A_b}{A_s} \epsilon_t \quad . \quad (1.18)$$

The strain rate experienced by the specimen is given by the time derivative of the equation for specimen strain

$$\dot{\epsilon}_s = \frac{-2c_o}{L_s} \dot{\epsilon}_r \quad . \quad (1.19)$$

Equations 1.16, 1.18 and 1.19, are known as the Hopkinson bar equations. They depend on the reflected and transmitted strains which are obtained from the strain gauges mounted on the bars. The use of these three equations and the two measured gauge signals permits determination of the stress, strain and strain rate history of

the specimen at strain rates well in excess of those attainable using conventional servo-hydraulic systems.

For the TSHB, the impact of the striker against the endcap imparts a tensile stress pulse in the incident bar as depicted in Figure 1.7. Although at impact a compressive pulse is induced in the endcap, it is almost immediately reflected as a tensile pulse in the incident bar. The striker is designed to have the same linear mass as the incident bar and will induce a tensile pulse in the incident bar with magnitude $+\sigma$ and length $2L$ which will propagate away from the endcap towards the specimen.

The above equations were derived with the assumption the specimen is deforming uniformly. Nonuniform distributions of stress or strain over the specimen may result from frictional restraints imposed on the specimen at the bar interfaces or inertial restraints imposed by radial or axial particle accelerations [11]. Under such circumstances, it is not permissible to simplify the Hopkinson bar equations by assuming that force equilibrium exists across the specimen, thus Equation 1.13 no longer holds. The stress in the specimen may be instead calculated using the average of the forces at either end, that is

$$\sigma_s = \frac{P_1 + P_2}{2A_s} , \quad (1.20)$$

or

$$\sigma_s = \frac{E_b A_b}{2A_s} (\epsilon_i + \epsilon_r + \epsilon_t) , \quad (1.21)$$

The specimen strain is calculated using Equation 1.10 and the specimen strain rate becomes,

$$\dot{\epsilon}_s = \frac{c_o}{L_s} (\epsilon_i - \epsilon_r - \epsilon_t) . \quad (1.22)$$

An example of this usage is small strain SHPB tests [23].

1.1.2 Stress Wave Propagation in the TSHB

The pulse induced in the incident bar is ideally a square wave. However, the pulse disperses as it reflects off the endcap and travels through the bars causing the rise time to lengthen and oscillations to occur. A typical incident stress wave recorded from a TSHB facility is shown in Figure 1.8. This trace shows a main stress pulse and oscillatory components. Since the Hopkinson bar equations used to obtain the stress, strain and strain rate history of the specimen are based on the stress waves travelling through the bar, it is important to understand the nature of these waves.

The stress pulse induced in the incident bar is dispersive in nature because it is composed of a spectrum of frequencies. The higher the impact velocity, the more high frequency components the pulse will contain [24]. The higher frequency components, which appear as oscillations in the main wave, travel more slowly than the lower ones and so the pulse spreads or disperses [24, 25]. Since the strain gauges record the stress pulse at a certain distance from the specimen, it is sometimes necessary to correct for the dispersion that occurs between the gauge stations and the specimen in order to get a more accurate representation of the strain and stress behaviour of the specimen.

The next few sections comprise a short review of stress wave propagation.

Stress Waves in an Extended Elastic Medium

The equations of motion for an isotropic elastic solid in terms of displacements u , v and w in the x , y and z directions are [26]

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x} + \mu \nabla^2 u \quad , \quad (1.23)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial y} + \mu \nabla^2 v \quad (1.24)$$

and

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial z} + \mu \nabla^2 w \quad , \quad (1.25)$$

where ρ is the density of the solid, λ and μ are the Lamé's parameters, $\Delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$ is the dilatation and ∇^2 signifies $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$.

These equations of motion can be shown [26, 27] to correspond to the propagation in the solid of two different types of waves. Considering the x direction for example, we can obtain

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \nabla^2 u \quad , \quad (1.26)$$

which leads to two possible propagation velocities. Longitudinal waves travel with the velocity $[(\lambda + 2\mu)/\rho]^{1/2}$ and the motion is in the direction of propagation of the wave. Transverse or shear waves travel with the velocity $(\mu/\rho)^{1/2}$ and the motion is parallel to the wave front.

Wave Propagation in Long Cylindrical Bars

Stress wave propagation behaviour in bounded isotropic solids, for example a long cylindrical bar, can be derived [26] from the equations of motion with the addition of appropriate boundary conditions. The result is an idealized, one-dimensional description of the wave propagation.

If we assume that plane cross-sections parallel to the bar axis remain plane and that the stress over the cross-sections is uniform, then (for the longitudinal or x

direction) Newton's Second Law gives

$$\rho A \delta x \frac{\partial^2 u}{\partial t^2} = A \frac{\partial \sigma_{xx}}{\partial x} \delta x , \quad (1.27)$$

where A is the cross-sectional area of the bar. Simplification gives

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} , \quad (1.28)$$

where E is the elastic modulus. The solution of this equation is of the form

$$u = f(c_o t - x) + F(c_o t + x) , \quad (1.29)$$

where $c_o = \sqrt{E/\rho}$ is the velocity of propagation and f and F are arbitrary functions that depend upon the initial conditions.

The bar also experiences longitudinal expansions and contractions during loading which result in lateral expansions and contractions due to the Poisson effect. As a result, plane sections do not remain plane and the stress does not act uniformly. This lateral motion is especially important when the operative wavelengths are the same order of magnitude as the bar diameter [26].

For wavelengths exceeding ten times the bar diameter, the assumptions that plane cross-sections remain plane and that stress acts uniformly hold and the stress at point, σ_{xx} , is linearly related to the particle velocity, $\partial u / \partial t$ [26],

$$\sigma_{xx} = \left(\frac{E}{c_o} \right) \frac{\partial u}{\partial t} = \rho c_o \frac{\partial u}{\partial t} , \quad (1.30)$$

where ρc_o is called the characteristic impedance. Since the velocity of propagation, c_o , is independent of frequency, this type of wave is non-dispersive and will reflect

when it reaches either a free or fixed end [26]. In both cases, the reflected pulse is of the same shape and magnitude as the incident pulse, however, reflection from a free end yields a pulse opposite in sign to the incident pulse [26, 28].

The Frequency Equation

Pochhammer (1876) and Chree (1889) independently solved the equations of motion describing the propagation of longitudinal waves in an infinitely long cylindrical bar [26, 27, 29, 30]. Their solution is called the Pochhammer-Chree frequency or dispersion equation and is achieved by converting the equations of motion into cylindrical polar coordinates and incorporating the appropriate boundary conditions. The frequency equation can be solved for three different types of waves: longitudinal, torsional and flexural (transverse). Since the TSHB experiences primarily longitudinal waves, the discussion will be limited to this wave type. The frequency or dispersion equation for longitudinal waves is

$$\left(\frac{\rho p^2}{\mu} - 2k^2 \right) J_1(hR) \left[2 \frac{\partial^2 J_0(gR)}{\partial R^2} - \frac{\lambda \rho p^2 J_0(gR)}{\lambda + 2\mu} \right] + 4k^2 \mu \frac{\partial J_0(gR)}{\partial R} \frac{\partial J_1(hR)}{\partial R} = 0 \quad (1.31)$$

where p is 2π times the wave frequency, k is 2π times the wavelength, J_0 and J_1 are Bessel functions, R is the radius of the cylinder, $h = [(\rho p^2/\mu) - k^2]^{1/2}$ and $g = [\rho p^2/(\lambda + \mu) - k^2]^{1/2}$ [27]. The frequency equation gives values of phase velocity in an infinitely long cylinder. These solutions are not exact for finite bars since the boundary condition that the ends are free from traction is not fulfilled; however, errors are very small when the length of the rod is much greater than the diameter [26].

If the cylinder radius, R , is sufficiently small compared to the wavelength, then we can neglect higher powers of hR and gR when the Bessel functions are expanded

and this leads to approximations of the frequency equation [26]. Expansion of the Bessel function neglecting higher than the first power of hR and gR yields

$$\frac{p^2}{k^2} = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)\rho} , \quad (1.32)$$

which simplifies to a phase velocity equaling $\sqrt{E/\rho}$ as shown in the simple theory (Equation 1.1). Expansion of the Bessel function neglecting higher than the second power of hR and gR leads to

$$\frac{p}{k} = \left(\frac{E}{\rho}\right)^{1/2} \left(1 - \frac{1}{4}\nu^2 k^2 a^2\right) . \quad (1.33)$$

Non-dimensionalization gives

$$\frac{c_p}{c_o} = 1 - \nu^2 \pi^2 \left(\frac{R}{\Lambda}\right)^2 , \quad (1.34)$$

where $c_p = p/k$ is the phase velocity, $c_o = \sqrt{E/\rho}$ is the velocity of infinitely long waves in a bar, Λ is the wavelength and ν is Poisson's ratio [26].

This treatment shows phase velocity decreasing with decreasing wavelength and is only reliable for long wavelengths since it predicts $c_p = 0$ at $\Lambda = \nu\pi R$ [26].

The Pochhammer-Chree solution predicts [26] that the longitudinal stress and displacement will vary across the cross-section especially for small wavelengths. However, for $R/\Lambda < 0.10$ the stress and displacement vary negligibly and since this is the case for the TSHB in the current study, we can consider the strains measured by gauges on the surface as being approximately equal to the centerline strain [7, 25].

Bancroft [31] calculated c_p/c_o for a series of ν values for the first mode of vibration which is the mode excited when longitudinal waves propagate in long cylindrical

bars. Bancroft's results show that for large values of R/Λ the wave velocity is slower than c_0 and approaches an asymptotic value from below.

1.1.3 Correcting for Dispersion in the SHPB

The equations of motion are valid after the effects of the bar end vanishes, which occurs after the wave propagates at least 10 bar diameters from the end [27, 32]. The bar, when impacted, is actually excited by a spectrum of frequencies. The low frequency components travels at c_0 . Higher frequency components travel slower than c_0 since phase velocity decreases with decreasing wavelength [25]. This difference in wave velocity leads to spreading or dispersion of the initial pulse; the rise time lengthens and oscillations occur [25].

Fourier series analysis is often used when studying dispersive stress waves, such as in the SHPB apparatus. This technique resolves any traveling wave into a series of discrete components. These components are simple harmonic motion terms, often sinusoidal. The more terms taken in the series, the more accurate the representation. The Fourier series method can be used to describe an extended duration periodic wave or an aperiodic wave over a limited range. [27]

The Fourier series method has been used extensively in geological applications to study dispersive seismic waves. The use of digital recording devices and the Fast Fourier Transform (FFT) computer algorithm make the use of the Fourier Transform practical in the study of the higher frequency ranges found in stress wave propagation in structural applications. [34]

Many experimenters [24,25,33-36] have incorporated the FFT technique to account for the effects of dispersion in SHPB testing. For example, Hsieh and Kol-sky [36] theoretically predicted the displacement-time curve using Fourier series analysis for the detonation of a small explosive charge at one end of a steel cylinder which agreed very well with the experimental results. Yew and Chen [34, 35] sought to validate the use of the FFT to account for dispersion in stress wave signals in long

slender bars by comparing the Fourier transform of the experimental signals with the analytical wave shapes in the frequency domain. This comparison gave good agreement and was less tedious to perform than comparisons in the time domain.

Dispersion Correction

For non-dispersive waves all the components travel with the velocity c_0 a distance x in time x/c_0 . For dispersive waves, a component, n , will instead travel at a velocity c_n which is slower than c_0 . This component will travel the distance x in a longer time x/c_0 and will therefore have a phase angle, ϕ_n , which is different than that for the non-dispersive components. [24]

To correct for dispersion, the new phase angle, ϕ_n , is added to the Fourier transform of the original signal [24, 25, 33]

$$F(t) = \sum_{n=1}^{\infty} [A_n \cos(2\pi n\omega t) + B_n \sin(2\pi n\omega t)] , \quad (1.35)$$

to give

$$F'(t) = \sum_{n=1}^{\infty} [A_n \cos(2\pi n\omega t + \phi_n) + B_n \sin(2\pi n\omega t + \phi_n)] , \quad (1.36)$$

where A_n and B_n are coefficients produced by the FFT calculation [33].

However, $F'(t)$ must take the form of the original signal to be used in the FFT, that is

$$F'(t) = \sum_{n=1}^{\infty} [A'_n \cos(2\pi n\omega t) + B'_n \sin(2\pi n\omega t)] , \quad (1.37)$$

where

$$A'_n = A \cos \phi + B \sin \phi_n \text{ and } B'_n = B \cos \phi - A \sin \phi_n . \quad (1.38)$$

The phase shift is:

$$\phi_n = 2\pi n \omega z \left(\frac{1}{c_o} - \frac{1}{c_n} \right) , \quad (1.39)$$

where c_n is interpolated from Bancroft's [31] data. The new Fourier constants, A'_n and B'_n can be found and the corrected wave reconstructed using the FFT. A positive value of z indicates that dispersion effects are added to the original pulse. Conversely, a negative value of z will cancel out the dispersion in the signal.

The reconstructed signal may not exactly agree with theory since the theoretical conditions may not exactly match the experimental conditions with respect to end effects, loading distributions and material properties [33].

Applicability of Dispersion Correction to Present Work

The propagation of longitudinal stress waves in cylindrical bars is generally dispersive for wavelengths of the same order of magnitude as the bar diameter [33]. In a Hopkinson bar apparatus of this configuration, dispersion correction must be applied to the strain gauge signals in order to ensure accuracy of the results. For stress pulses in which the operative wavelengths of the pulse are much greater than the diameter of the bar, the velocity of propagation is independent of frequency and the pulse travels along the bar without dispersion [26]. The diameter of the Hopkinson bars in the current study is less than 2 cm and the induced stress pulses have operative wavelengths between 20 and 80 cm long and therefore dispersion correction is not required.

1.2 Ductile Fracture

Ductile metals undergoing the high strain rate tensile loading characteristic of TSHB testing often fail by the ductile fracture mechanism. Ductile fracture is a dominant tensile failure mechanism for ductile metals and is of particular interest since the materials tested as part of this research failed at high strain rate by the ductile fracture mode. Ductile fracture is characterized by the nucleation and growth of voids and the consolidation or coalescence of these voids to form an interior crack whose propagation across the material leads to complete failure [2].

Figure 1.9 is a scanning electron micrograph and serves to demonstrate the mechanisms involved in ductile fracture. The specimen is free-cutting brass containing a globular phase lead and was failed at high strain rate in the TSHB facility. The fracture surface is characterized by a dimpled, fibrous texture resulting from failure by microvoid nucleation at the lead particles and subsequent coalescence of those voids. The centre of each dimple occurs at a lead particle which nucleated a void by tearing when exposed to high local strains.

The following few sections will be devoted to a discussion of ductile fracture by microvoid nucleation, growth and coalescence.

1.2.1 The Nucleation of Microvoids

Voids are usually nucleated at second phase particles or inclusions either by the separation of matrix material from the particle or the cracking of the particle [3, 4, 5]. Nucleation of voids at grain boundary triple points or other irregularities is possible but this mechanism is usually restricted to the intermediate temperature, low strain rate regime [3]. Concentration of vacancies during shock loading can also promote void nucleation [37].

The representation of microvoid nucleation at inclusions and second phase particles is a difficult problem in modelling ductile fracture. Often, the size of the

particles varies enormously. For small particles, the dislocation structure around the particle is the main contribution to flow stress and so a dislocation-based model is normally used to predict nucleation. However, as the particle size increases, the strain hardening rate around the particle approaches that of the matrix and therefore a continuum model may be employed. This critical size is roughly 1-2 μm and may be estimated by substituting dislocation density as a function of particle radius into an expression for the local flow stress. [3, 38, 39].

Nucleation at Submicron Particles

Goods and Brown [3] have developed a dislocation model to describe void nucleation in ductile metals containing a dispersion of hard second phase particles. The model considers contributions of flow stress, local stress and hydrostatic tension to decohesion of the particle/matrix interface. The flow stress, σ_f , is defined as

$$\sigma_f = 3\sigma_o + 3\alpha\mu f_v^{1/2} \left(\frac{\epsilon_c b}{r} \right)^{1/2} + 16.2\alpha\mu f_v^{1/2} \left(\frac{\epsilon_c b}{r} \right)^{1/2} . \quad (1.40)$$

The first term is the Orowan stress. The second term is the forest hardening and the third term is the mean internal stress in the matrix in which $\alpha = 1/7$ is a constant, μ is the shear modulus, and f_v is the volume fraction of particles, ϵ_c is the critical plastic strain at which void nucleation occurs, b is the Burger's vector and r is particle radius. The local stress is

$$\sigma_c = 5.4\alpha\mu \left(\frac{\epsilon_c b}{r} \right)^{1/2} . \quad (1.41)$$

The hydrostatic tension is σ_H and does not include the hydrostatic component of the applied tensile stress itself which is accounted for by the flow stress.

The condition for void nucleation is thus

$$\sigma_c + \sigma_f + \sigma_H > \sigma_I \quad . \quad (1.42)$$

The nucleation criterion expressed in terms of a critical strain to cause cavity nucleation, ϵ_c , is

$$\epsilon_c \geq 1.7 \frac{r}{b} \left(\frac{\sigma_I - \sigma_H - 3\sigma_o}{\mu} \right)^2 \left(1 + 3f_v + \frac{f_v^{1/2}}{1.8} \right)^{-2} \quad . \quad (1.43)$$

Nucleation at Larger Particles

Argon, Im and Safoglu [38] have developed a continuum model for void nucleation at particles larger than 1-2 μm which incorporates some microstructural features. They established upper and lower bounding solutions for the particle-matrix interfacial stresses for pure shear deformation based on two extreme idealized conditions: one for a rigid inclusion in an elastic-ideally plastic, non-hardening continuum and the other for a rigid inclusion in an incompressible linear material. The interfacial stress is estimated by superimposing a negative hydrostatic stress component, σ_{hyd} , on the interfacial tensile stress, σ_{rr} , for a pure shear deformation field. This interfacial stress is bounded by

$$\frac{3}{2}k \leq \sigma_{rr} - \sigma_{hyd} \leq 2k \quad , \quad (1.44)$$

where k is the flow stress in shear. The average of these limits is $1.75k$ which is approximately equal to the flow stress in tension, $Y = \sqrt{3}k = 1.732k$. and so the total interfacial tensile stress can be estimated as

$$\sigma_{rr} \approx Y(\bar{\epsilon}^p) + \sigma_{hyd} \quad , \quad (1.45)$$

where $Y(\bar{\epsilon}^p)$ is the flow stress for the average local plastic strain in the region of

inclusion had there been no inclusion present. This model is independent of particle size which is valid for small volume fractions of inclusions in which the inclusions do not interact.

Nucleation at Non-Ideal Particles

It is important to note that the models described above assume that the particles are hard, spheroidal and strongly bonded to the interface and that the volume fraction is sufficiently low to prevent particle interaction. However, in many engineering materials, these ideal conditions may not exist.

The shape of inclusions may vary from spheroidal to lamellar to irregular angular and when large volume fractions of inclusions are present, the particles may interact to hasten separation [39]. As well, many engineering materials undergo processing such as hot-working, which can cause soft inclusions to become highly elongated or can subject particles to severe damage causing crack-like defects [40]. Barnby [41] found that rod-shaped particles will fracture as a result of concentrated local stresses which are generated by slip bands in the matrix material. Goods and Brown [3] suggest that the larger the particle, the more likely it is to contain flaws which will cause the brittle fracture of the inclusions. The work of Gurland [42] and Liu and Gurland [43] shows that larger particles will crack more readily than smaller particles and that cracking in tension occurs preferentially in particles which are oriented such that their longest axis is parallel to the tensile stress direction.

1.2.2 Void Growth

Once a void is nucleated, either by debonding or particle cracking, a localized stress and strain concentration in the adjacent plastic flow field is caused by the now stress-free surface of the void [40]. The continuing plastic deformation of the matrix induces the volumetric growth and shape change of the void. If the voids are

relatively far apart for the initial stages of growth, the process can be modelled as the growth of a single void in an infinite solid [44].

The Rice-Tracey Model for Void Growth

The Rice-Tracey [44] continuum-plasticity model was developed to describe the ductile growth of a spherical void in an elastically rigid incompressible plastic material subject to remote uniform stress and strain rate fields.

The triaxiality of the stress field is characterized by the constant D , physically interpreted as

$$\frac{\dot{R}_o}{R_o} = \left(\frac{2}{3} \dot{\epsilon}_{ij}^{\infty} \dot{\epsilon}_{ij}^{\infty} \right)^{1/2} D, \quad (1.46)$$

where R_o is the initial radius of the void, $\dot{R}_o$ is the average radial velocity of the void surface and $(\frac{2}{3} \dot{\epsilon}_{ij}^{\infty} \dot{\epsilon}_{ij}^{\infty})$ is the equivalent tensile strain rate. For high triaxiality, of interest in the present work, D is found to be

$$D = C(\nu) \exp \left(\frac{\sqrt{3} \sigma^{\infty}}{2 \tau_o} \right), \quad (1.47)$$

where σ^{∞}/τ_o is the dimensionless mean stress. The constant, $C(\nu)$ depends on the remote strain components through the Lode variable, ν , where

$$\nu = - \frac{3 \dot{\epsilon}_2}{\dot{\epsilon}_1 - \dot{\epsilon}_3}, \quad (1.48)$$

in which $\dot{\epsilon}_1 \geq \dot{\epsilon}_2 \geq \dot{\epsilon}_3$. The Lode variable changes from -1 for simple compression, through 0 for simple shear, to $+1$ for simple extension.

This leads to a general approximate model for void growth rates:

$$\dot{R}_{oK} = \left[\left(\frac{5}{3} \text{ to } 2 \right) \dot{\epsilon}_K^\infty + \left(\frac{2}{3} \dot{\epsilon}_L^\infty \dot{\epsilon}_L^\infty \right)^{1/2} D \right] R_o, \quad K, L = 1, 2, 3. \quad (1.49)$$

LeRoy *et al* [45] have modified the Rice-Tracey void growth model to account for void shape and sample geometry changes with strain. They express the Rice-Tracey model in terms of the principal void radii, R_3 and R_1 :

$$R_3 = R_o \exp(D\bar{\epsilon}) \left(2 \exp \frac{3}{2} \bar{\epsilon} - 1 \right)^{2/3}, \quad (1.50)$$

$$R_1 = R_o \exp(D\bar{\epsilon}) \left(2 \exp \frac{3}{2} \bar{\epsilon} - 1 \right)^{-1/3}, \quad (1.51)$$

where $\bar{\epsilon}$ is the equivalent strain. An expression for D is determined depending on the conditions and substituted into the equations for void radii which can then be evaluated for any value of strain, $\bar{\epsilon}$.

1.2.3 Void Coalescence

As plastic strain increases, the growth of microvoids continues until the condition is reached for local plastic instability. The microvoids coalesce by internal necking of the intervvoid matrix [40]. This coalescence forms a central crack whose strain concentration produces deformed regions in which a sheet of small voids forms [2]. The regions fail, extending the crack which continues to propagate in this manner until complete failure is achieved.

The Thomason Model for Microvoid Coalescence

Thomason [40, 46] developed a two-dimensional plane-strain model of the void coalescence process for a work-hardening plastic/rigid solid. Localized internal necking of the intervvoid matrix across a sheet of microvoids is assumed to occur when the condition for plastic failure of the matrix is reached,

$$\sigma_1 = \sigma_n(1 - \sqrt{V_f}) , \quad (1.52)$$

where σ_1 is the applied stress, σ_n is the critical mean stress for initiation of localized plastic flow of the matrix and V_f is the volume fraction of voids.

For small V_f (≈ 0.01), the yield shear stress of the matrix, k_n , is approximately equal to the macroscopic yield shear stress, k , and the criterion for ductile fracture by microvoid coalescence is

$$\frac{\sigma_n}{2k}(1 - \sqrt{V_f}) = \frac{\sigma_1}{2k} = \frac{1}{2} + \frac{\sigma_m}{2k} , \quad (1.53)$$

where σ_m is the mean normal stress.

For $V_f > 0.01$ and large non-coherent particles, $k = (1 - \sqrt{V_f})k_n$, and the ductile fracture criterion is

$$\frac{\sigma_n}{2k_n}(1 - \sqrt{V_f})^{-1} = \frac{\sigma_1}{2k} = \frac{1}{2} + \frac{\sigma_m}{2k} . \quad (1.54)$$

1.2.4 Gurson Constitutive Model

The Gurson constitutive model [47-50] for porous continuum plastic materials describes the process of ductile fracture by microvoid nucleation, growth and coalescence. The Gurson yield function [47],

$$\begin{aligned} \phi &= \left(\frac{\Sigma_{eq}}{\bar{\sigma}} \right)^2 + 2f \cosh \left(\frac{3\Sigma_{hyd}}{2\bar{\sigma}} \right) - 1 - f^2 \\ &= 0 \quad \text{during plastic flow} , \end{aligned} \quad (1.55)$$

is used to predict the macroscopic stresses, Σ_{ij} , necessary to initiate plastic flow in

porous solids. In this function, Σ_{eq} and Σ_{hyd} are the equivalent stress and hydrostatic stress, respectively, f is the volume fraction of voids and $\bar{\sigma}$ is the flow stress. The yield function was developed based on upper bound solutions for the average stresses, Σ_{ij} , which act on the surface of a rigid plastic sphere with a central void undergoing plastic flow. This yield function predicts constitutive softening behaviour in the material, that is, ϕ will decrease with increasing volume fraction of voids and increasing stress triaxiality.

Tvergaard [51], proposed a modification of the Gurson yield function:

$$\phi = \left(\frac{\Sigma_{eq}}{\bar{\sigma}} \right)^2 + 2f q_1 \cosh \left(q_2 \frac{3\Sigma_{hyd}}{2\bar{\sigma}} \right) - 1 - q_3 f^2 = 0 \quad , \quad (1.56)$$

in which q_1 , q_2 and q_3 , are “calibration coefficients” introduced to better represent void interaction during plastic deformation. Tvergaard suggested that $q_1 = 1.5$, $q_2 = 1.0$ and $q_3 = q_1^2$.

Tvergaard and Needleman [52, 53] have incorporated into the Gurson model a void coalescence criterion based on a critical void volume fraction at which coalescence initiates:

$$\begin{aligned} f^* &= f & \text{for } f \leq f_c \\ f^* &= f_c + \frac{f_u^* - f_c}{f_f - f_c} (f - f_c) & \text{for } f \geq f_c \end{aligned} \quad (1.57)$$

in which f^* replaces f in Gurson yield function, f_c is the critical volume fraction at which coalescence initiates, $f_u^* = 1/q_1$ and f_f is the critical final value of void volume fraction at which coalescence is complete. When $f^* = f_u^*$, the macroscopic stress carrying capacity of the material vanishes.

Using the Gurson yield function and its subsequent modifications as a flow potential, the application of the normality condition gives

$$\dot{\epsilon}_{ij}^{p\infty} = \frac{1}{H} \left(\frac{3\Sigma'_{ij}}{2\bar{\sigma}} + \alpha\delta_{ij} \right) \left(\frac{3\Sigma'_{kl}}{2\bar{\sigma}} + \alpha\delta_{kl} \right) \dot{\Sigma}_{kl} , \quad (1.58)$$

in which

$$\alpha = \frac{f}{2} q_1 q_2 \sinh \left(q_2 \frac{3\Sigma_{hyd}}{2\bar{\sigma}} \right) , \quad (1.59)$$

and

$$H = \frac{h}{1-f} \left[\left(\frac{\Sigma_{eq}}{\bar{\sigma}} \right)^2 + \alpha \frac{3\Sigma_{hyd}}{\bar{\sigma}} \right]^2 - 3\bar{\sigma}\alpha(1-f) \left[q_1 \cosh \left(q_2 \frac{3\Sigma_{hyd}}{2\bar{\sigma}} \right) - q_3 f \right] , \quad (1.60)$$

in which h is the slope of the material flow stress versus equivalent plastic strain curve.

The effective plastic strain rate, $\dot{\bar{\epsilon}}^p$, in the matrix material is obtained using work equivalence,

$$\dot{\bar{\epsilon}}^p = \frac{\Sigma_{ij} \dot{\epsilon}_{ij}^{p\infty}}{(1-f)\bar{\sigma}} . \quad (1.61)$$

The rate of increase in porosity has two components: growth of existing voids and nucleation of new voids,

$$\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation} . \quad (1.62)$$

If void nucleation is controlled by plastic strain and the strain to cause nucleation follows a normal distribution, then the rate of increase in porosity due to void nucleation is

$$\dot{f}_{nucleation} = Ah\dot{\bar{\epsilon}}^p . \quad (1.63)$$

where h is the slope of the material flow stress versus equivalent plastic strain curve and A is selected so that the nucleation follows a normal distribution, ie

$$A = \frac{1}{h} \frac{f_N}{S_N \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\bar{\epsilon}^p - \epsilon_N}{S_N} \right)^2 \right] , \quad (1.64)$$

in which f_N is the volume fraction of void nucleating particles, ϵ_N and S_N are the average and standard deviation of the nucleation strains and $\bar{\epsilon}^p$ is the effective plastic strain in the matrix.

If the matrix material is assumed to be incompressible and $\dot{\epsilon}_{kk}^{p\infty}$, the hydrostatic component of plastic strain rate, is entirely due to void growth then

$$\dot{f}_{growth} = (1 - f) \dot{\epsilon}_{kk}^{p\infty} . \quad (1.65)$$

These equations represent the Gurson constitutive model as modified by Tvergaard and Needleman to describe the porosity and constitutive softening in porous materials. Needleman and Tvergaard [53] found that the Gurson model gave good agreement with experiment for the fracture of uniaxial and notched tensile specimens under quasi-static, axi-symmetric and plane strain conditions. Worswick and Pick [54] found that for quasi-static loading the Gurson model predicted void growth rates well for uniaxial specimens but under-predicted growth rates in notched, high triaxiality specimens.

Void Growth at High Strain Rates

The applicability of the Gurson model to inertial conditions has received limited attention. Johnson and Addessio [55] developed a mathematical model of tensile plasticity and void growth based on the use of the Gurson yield surface to represent plastic flow. Their model was found to be sufficiently general to represent plastic

flow and fracture in smooth and notched tensile specimens and uniaxial strain rate tests.

Recent work by Worswick and Pick [49] and Worswick, Wong and Pick [56] has been performed to compare the Gurson model as implemented in the finite element code DYNA2D to experimental results for Symmetric Taylor cylinder tests at strain rates exceeding 10^4 s^{-1} . In these tests, the plastic mushrooming of the cylinders at the impact face causes the development of large tensile triaxial stresses which promote ductile cracking through void nucleation, growth and coalescence. The Gurson model was found to have qualitatively captured the dynamic fracture behaviour, however, the model overpredicted the void growth for high strain rate, high triaxiality conditions indicating the need for further investigation.

Preliminary work has been initiated by Worswick, Clarke and Pick [57] involving more controlled plate impact and tensile split Hopkinson bar experiments in an attempt to ascertain the applicability of the Gurson model for high strain rate, high triaxiality conditions.

1.3 Constitutive Models for High Strain Rate Plastic Deformation

A constitutive model for the behaviour of a material describes the relationship between variables such as stress, strain, strain rate, temperature and internal state variables for that material [58]. The constitutive model may use Hooke's law for the elastic behaviour but must incorporate an additional law to describe the plastic response of the material if the deformation of the material ventures into the plastic regime. The following sections will discuss four commonly used constitutive models for metals relating the plastic flow stress to plastic strain, strain rate and temperature:

- Johnson-Cook
- Zerilli-Armstrong
- Mechanical Threshold Stress
- Steinberg-Guinan

Finite element computer simulation is a useful technique for studying the behaviour of materials experiencing a high strain rate loading regime. Key to the accuracy of the finite element computer model is the implementation of appropriate equations to describe the constitutive response of the material as it plastically deforms. It is for this reason that the development of accurate constitutive models is so important.

1.3.1 Johnson-Cook Constitutive Model

The Johnson-Cook constitutive model [58–60] is an empirical model which accounts for strain hardening, strain-rate hardening and thermal softening. It is a computationally simple model and easily implemented into computer codes. The model is based on adiabatic stress-strain relationships for varying strain rates created with data from tensile Hopkinson bar tests over a range of temperatures, torsion tests at varying strain rates and quasi-static tensile tests with various notch geometries.

The Johnson-Cook model describes flow stress as

$$\sigma = (A + B\epsilon^n)(1 + C \ln \dot{\epsilon}^*)(1 - T^{*m}) \quad , \quad (1.66)$$

where A , B , n , C and m are material constants, ϵ is the equivalent plastic strain, $\dot{\epsilon}^* = \dot{\epsilon}/\dot{\epsilon}_o$ is the dimensionless plastic strain rate for $\dot{\epsilon}_o = 1.0 \text{ s}^{-1}$ and $T^* = (T - T_{Room})/(T_{Melt} - T_{Room})$ is a form of homologous temperature. The first bracketed

term in Equation 1.66 describes flow stress as a power law function of strain for $\dot{\epsilon}^* = 1.0 \text{ s}^{-1}$ and room temperature. The second term predicts a logarithmic effect of strain rate and the third term accounts for the effect of temperature and thermal softening in elevating flow stress.

The validity of the Johnson-Cook model was estimated by simulating Taylor cylinder impact tests and torsion-tension tests. Since the parameters A , B and n differ depending on whether the model was calibrated with torsion or tension data, average values were used thus introducing a certain discrepancy. Also, the effect of grain size, which is known to influence strength and ductility of a material is not accounted for in the Johnson-Cook model. As a result, the model failed to accurately reproduce the final shape of the impacted cylinders. However, the Johnson-Cook model is advantageous in its computational simplicity, since its main variables are computed in most finite element codes.

Cumulative-Damage Fracture Model

The Johnson-Cook strength model has been enhanced [60] with a fracture model which incorporates the effects of pressure, strain, strain-rate and temperature, and accounts for the cumulative damage path dependency. The damage, D , is defined as

$$D = \sum \frac{\Delta \epsilon}{\epsilon^f} , \quad (1.67)$$

where $\Delta \epsilon$ is the increment of equivalent plastic strain occurring during one strain step. Fracture occurs when $D = 1$. The equivalent strain to fracture, ϵ^f , is

$$\epsilon^f = [D_1 + D_2 \exp D_3 \sigma^*][1 + D_4 \ln \dot{\epsilon}^*][1 + D_5 T^*] , \quad (1.68)$$

in which $D_1 \dots D_5$ are constants. The equivalent pressure-stress ratio, σ^* , (also

called the triaxiality) is defined as $\sigma^* = \frac{\sigma_m}{\bar{\sigma}}$ where σ_m is the hydrostatic stress and $\bar{\sigma}$ is the von Mises equivalent stress. T^* and $\dot{\epsilon}^*$ are the same as in the strength model. The first bracketed term describes the strain to fracture as a decreasing function of hydrostatic tension. The second bracketed term represents the effect of strain rate and the third term the effect of temperature.

This enhanced model requires little additional computational time and gives reasonably good agreement with experimental data [60].

1.3.2 Zerilli-Armstrong Constitutive Model

The Zerilli-Armstrong constitutive model [58, 61] is an approximate physical model based on dislocation mechanics. This model considers the effect of strain, strain rate, temperature and grain size on flow stress. It considers body-centered cubic and face-centered cubic microstructures separately. Since damage and fracture depend on local values of stress and strain, dislocation models offer the possibility of more accurate prediction of metals behaviour compared to models which are based on average values.

For BCC structures, yield stress has a stronger dependence on temperature and strain rate than is exhibited for FCC structures. For BCC metals, it is thought that the Peierls stress is the controlling mechanism for the thermal activation analysis and that Gibbs free energy is essentially independent of strain. The uniaxial stress, σ_{th} , for BCC metals is

$$\sigma_{th} = c_1 \exp(-c_3 T + c_4 T \ln \dot{\epsilon}) , \quad (1.69)$$

where T is the absolute temperature, $\dot{\epsilon}$ is the strain rate and c_i are experimental material constants. The contribution of plastic strain-hardening to flow stress in the BCC case is included separately and is assumed to have a power law dependence on

strain,

$$\Delta\sigma_G = c_5\epsilon^n , \quad (1.70)$$

where c_5 and n are experimental constants.

For FCC structures, there is a strong dependence of thermal activation analysis behaviour on strain and the intersection of dislocations is thought to be the controlling mechanism. The uniaxial stress, σ_{th} , for FCC metals is

$$\sigma_{th} = c_2\epsilon^{1/2} \exp(-c_3T + c_4T \ln \dot{\epsilon}) . \quad (1.71)$$

The influence of solute and original dislocation density on yield stress in both FCC and BCC structures is accounted for by an additional term $\Delta\sigma'_G$.

Also, at lower temperatures the polycrystal flow stress is higher to accommodate the movement of slip across grain boundaries to propagate plastic flow through the structure and the stress must be raised by an increment

$$\Delta\sigma = kl^{-1/2} , \quad (1.72)$$

where k is the microstructural stress intensity and l is the average grain diameter.

The resulting complete constitutive relations are as follows.

For FCC structures:

$$\sigma_{th} = \Delta\sigma'_G + c_2\epsilon^{1/2} \exp(-c_3T + c_4T \ln \dot{\epsilon}) + kl^{-1/2} . \quad (1.73)$$

For BCC structures:

$$\sigma_{th} = \Delta\sigma'_G + c_1 \exp(-c_3T + c_4T \ln \dot{\epsilon}) + c_5\epsilon^n + kl^{-1/2} . \quad (1.74)$$

When used to model Taylor tests, the Zerilli-Armstrong model gives excellent results for copper; however, for iron, the results show no improvement over those from the Johnson-Cook model.

Deformation Twinning

Failure of the model to accurately describe Taylor tests of iron led to speculation [62] that plastic flow by slip is replaced by deformation twinning at high strain rates and large grain size; in fact micrographs of iron impact specimens support this theory. The problem was not encountered with copper since the stress level at which slip is replaced by twinning is much higher for copper.

Twinning is believed to raise the flow stress of a material by refining the grain size. In shock wave loading with stress intensities above the Hugoniot elastic limit, an elastic stress wave is induced followed by a plastic wave [28]. Twinning occurs at the elastic wave front and propagates with it causing a refinement of the grain size. Thus, the following plastic wave must travel through an effectively finer grained, stronger material [62].

Twinning is assumed [62] to occur when a critical von Mises stress is exceeded. Recall for BCC metals that the flow stress is

$$\sigma_{th} = \Delta\sigma'_G + c_1 \exp(-c_3T + c_4T \ln \dot{\epsilon}) + c_5\epsilon^n + kl^{-1/2} . \quad (1.75)$$

The grain size effect is accounted for by

$$\Delta\sigma = kl^{-1/2} . \quad (1.76)$$

This contribution to flow stress is further increased when grain refinement occurs due to twinning. If N twins occur in a grain, the corresponding increase in the grain size contribution to flow stress would be

$$\Delta(\Delta\sigma) = kl^{-1/2}[(N + 1)^{1/2} - 1] \quad . \quad (1.77)$$

The twinning stress as a function of grain size can be predicted, however, the number of twins per grain must be determined experimentally or predicted based on twinning strain.

This twinning model is easily implemented into the existing model with only three additional lines of code. The Taylor test simulations gave excellent results for iron cylinder impact test when this twinning model was included [62].

Dislocation Drag

A further enhancement [63] of the Zerilli-Armstrong model includes the effect of dislocation drag on the stress-strain behaviour of FCC metals. The incorporation of the effect of dislocation drag predicts a significant difference in FCC stress-strain behaviour. However, no difference was noted for BCC metals since thermal activation for these metals is independent of strain.

1.3.3 The Mechanical Threshold Stress Constitutive Model

The Mechanical Threshold Stress (MTS) constitutive model is a detailed theoretical model due to Follansbee [58, 64]. It uses internal state variables to represent the current state of the material. Expressions are developed for the variation of yield stress with strain rate and temperature and the evolution of state variables with strain, strain rate and temperature. The model can account for history dependence

also.

The MTS model assumes that the kinetics of thermally activated interactions between dislocations and obstacles controls the deformation. For any given microstructural condition or at any instant in time

$$\frac{\sigma}{\mu} = \frac{\sigma_a}{\mu} + \sum_{i=1}^n s_i(\dot{\epsilon}, T) \frac{\sigma_i}{\mu} , \quad (1.78)$$

where σ is applied stress, μ is the temperature dependent shear modulus, σ_a is the athermal stress, σ_i is the mechanical threshold stress required to overcome obstacle i in the absence of any thermal activation energy contribution and s_i characterizes the ratio of applied stress (excluding athermal stresses) to mechanical threshold stress. The stress contributions from individual obstacles are assumed to be linearly additive which is an approximation. The parameter s_i can be expressed for thermally activated deformation as

$$s_i = \left[1 - \left(\frac{kT}{g_{oi}\mu b^3} \log \frac{\dot{\epsilon}_{oi}}{\dot{\epsilon}} \right)^{\frac{1}{q_i}} \right]^{\frac{1}{p_i}} , \quad (1.79)$$

where k is the Boltzmann constant, g_{oi} is the normalized total activation energy, b is the Burger's vector and ϵ_{oi} , q_i and p_i are constants. This equation can be modified [65] to account for dislocation drag at strain rates exceeding 10^6 s^{-1} .

The density of defects increases during strain hardening, therefore the mechanical threshold stress for obstacle D evolves with strain. The MTS also evolves with strain rate and temperature and the hardening, θ , can be expressed as

$$\theta = \frac{d\sigma_D}{d\epsilon} = \theta_o \left[1 - F \left(\frac{\sigma_D}{\sigma_{D*}(\dot{\epsilon}, T)} \right) \right] , \quad (1.80)$$

where θ_o is usually a constant, however in copper it is a weak function of strain rate

at low rates and linear for strain rates exceeding 10^3 s^{-1} [66]. F is a function which is chosen so that the equation gives a successful description of the hardening, for example $F = \tanh(2x)/\tanh(2)$ works well for copper [66]. The limiting mechanical threshold stress, σ_{Ds} , is a function of strain rate and temperature except for very high stacking fault energy materials such as aluminum and pure BCC metals. For FCC metals

$$\log \frac{\dot{\epsilon}}{\dot{\epsilon}_{Ds}} = \frac{A}{kT} \log \frac{\sigma_{Ds}}{\sigma_{Dso}} , \quad (1.81)$$

where $\dot{\epsilon}_{Ds}$, σ_{Dso} and A are constants. A and $\dot{\epsilon}_{Ds}$ scale with stacking fault energy for FCC metals.

Experimental methods are required to measure the parameters for the MTS constitutive relations. Many tests [66] incorporating a prestrain-unload-reload cycle were performed for varying temperatures, strains and strain rates. As many as 600 tests were performed for copper alone, which represents a serious disadvantage of the MTS model.

This model is especially suited for modeling strain rate sensitivity of materials since it can distinguish between strain rate sensitivity for constant and evolving structures. The model is also capable of accounting for history dependencies such as changes in temperature and strain rate. The MTS model is yet to be implemented in readily available computer codes [64].

Deformation Twinning

Evidence of twinning is found in many materials such as low stacking fault energy FCC metals (austenitic stainless steels, for example) and BCC metals in which high temperature and strain rate sensitivities of yield stress allow for stresses high enough to nucleate and grow twins [67].

To account for deformation twinning [68], the MTS model defines the total mechanical threshold stress as

$$\sigma = \sigma_a + s_t \sigma_t + s_I \sigma_I + s_e \sigma_e , \quad (1.82)$$

where σ_a is the mechanical threshold stress for athermal dislocation interactions, σ_t is the mechanical threshold stress for deformation by twinning, σ_I is the initial mechanical threshold stress due to the Peierls barrier and σ_e is the mechanical threshold stress due to the evolving dislocation density.

The model is currently being improved to account for the effect of multiple strengthening mechanisms [64].

Copper has been tested extensively [64, 69] in developing the model because it has strong strain rate sensitivity at high rates and it is a model FCC metal [64]. Preliminary studies in iron and 4340 steel [67] and titanium alloy [70] are in progress. Gourdin and Lassila [71] have applied the MTS model and found it successfully describes the behaviour of copper over a wide range of grain sizes, strain rates and temperatures.

1.3.4 Steinberg-Guinan Constitutive Model

The Steinberg-Guinan constitutive model [72] attempts to describe shear modulus and yield strength as functions of plastic strain, pressure and temperature (or internal energy). This model is an extension of the elastic-perfectly plastic model to account for the effect of these variables and is only applicable at extremely high strain rates, i.e. greater than 10^5 s^{-1} . The model has been implemented for fourteen different metals [72].

The Steinberg-Guinan model is primarily used for simulating shock-wave phenomena. Although this model is not directly applicable to the current work, it is

included here for completeness.

The model is strain rate independent. This is based on the assumption that yield stress becomes rate-independent at very high strain rates. Experimental work [72] confirms this assumption for strain rates beyond 10^5 s^{-1} , and the model successfully reproduced the experimental data in this region.

Shear modulus, G , and yield strength, Y , are expressed as functions of strain, ϵ , pressure, P , and temperature T , at high strain rate as

$$G = G_o \left[1 + \left(\frac{G'_p}{G_o} \right) \frac{P}{\eta^{1/3}} + \left(\frac{G'_T}{G_o} \right) (T - 300) \right] , \quad (1.83)$$

$$Y = Y_o [1 + \beta(\epsilon + \epsilon_i)]^n \left[1 + \left(\frac{Y'_p}{Y_o} \right) \frac{P}{\eta^{1/3}} + \left(\frac{Y'_T}{Y_o} \right) (T - 300) \right] , \quad (1.84)$$

subject to the work hardening limit

$$Y_o [1 + \beta(\epsilon + \epsilon_i)]^n \leq Y_{max} , \quad (1.85)$$

where η is initial specific volume divided by current specific volume, β and n are work-hardening parameters, and ϵ_i is the initial equivalent plastic strain. The subscript o indicates reference state at $T = 300K$, $P = 0$ and $\epsilon = 0$. The prime refers to the derivative of the parameter with respect to the variable indicated by the associated subscript at the reference state.

The work-hardening expression

$$[1 + \beta(\epsilon + \epsilon_i)]^n , \quad (1.86)$$

is a semi-empirical best fit for all the fourteen metals studied.

The Steinberg-Guinan rate-independent model has been enhanced [73] to include the effect of strain rate for strain-rate sensitive materials such as beryllium and uranium and includes the effect of spall.

This model has been further enhanced [74] to extend its validity from $10^{-4} < \dot{\epsilon} < 10^6 s^{-1}$ by separating yield strength into an athermal component which incorporates work-hardening and a thermal component which is a function of strain rate and temperature.

1.4 Present Research

The present research focuses on the development of the TSHB facility and its use for investigating the high strain rate tensile behaviour of brass. Scanning electron and optical microscopy complement the testing, providing a qualitative view of the failure mechanisms. Numerical models of the apparatus are used to investigate the wave propagation characteristics of the apparatus and to support the experimental work with respect to specimen behaviour and engineering stress-strain to true stress-strain correlation.

The remainder of this thesis is organized as follows. Chapter 2 describes the mechanical components and the associated instrumentation of the TSHB facility. The work in numerical modelling is covered in Chapter 3. Chapter 4 describes the experimental procedure, details the experiments performed and discusses experimental results. Finally, Chapter 5 contains concluding remarks.

Chapter 2

The Tensile Split Hopkinson Bar Apparatus

The Tensile Split Hopkinson Bar (TSHB) apparatus is designed to test specimens at elevated strain rates in tension. The facility developed as part of this research is capable of testing metals, such as brass, copper and iron, at strain rates based on gauge length of up to 1500 s^{-1} , depending on the test specimen configuration and material.

Figure 2.1 is a photograph of the TSHB facility and Figure 2.2 is a schematic of the facility in which the components are labeled. A specimen is threaded between two long, horizontal, high strength steel bars, one termed the incident bar and the other the transmitter bar. The incident bar is machined so that a shoulder, called the endcap, is located at the end of the bar away from the specimen. A cylindrical striker is located concentrically about the incident bar and slides along the bar on two teflon bushings press-fit into each end. A compressed gas gun, also positioned concentrically around the incident bar, propels the striker along the incident bar towards the endcap at velocities up to 20 m/s. When the striker cylinder impacts the endcap, it imparts a tensile wave to the incident bar. This stress wave travels down the incident bar towards the specimen which is deformed in tension at a high rate. The forward motion of the incident bar is arrested with a momentum trap.

Strain gauges are mounted on the incident and transmitter bars, equidistant from the specimen, and are used to record the magnitudes and durations of the stress pulses as they propagate through the bars. By use of the Hopkinson bar equations given in Chapter 1, the strain, strain rate, and stress history of the specimen can be determined from the strain gauge histories.

The mechanical components and instrumentation that comprise the TSHB facility are described in the following sections.

2.1 Mechanical Components

2.1.1 Hopkinson Bars and Striker

The basic design parameters for the TSHB apparatus were obtained by selecting target values for cross-sectional area, strain and strain rate of the specimen. The material selected for the bars and striker was AISI 4340 steel because of its superior strength, toughness and ready availability.

The target value of strain rate for the specimen, $\dot{\epsilon}_s$, was selected as 2000 s^{-1} . Based on this target value, the reflected strain, ϵ_r , was estimated by rearranging Equation 1.19 to give

$$\epsilon_r = -\dot{\epsilon}_s \frac{L_s}{2c_o} \quad , \quad (2.1)$$

in which L_s is the specimen gauge length and c_o is the wave propagation velocity in the bar. By limiting the stress in the incident bar to 80% of the yield strength, σ_y , of the bar material to provide a margin of error against yielding the bar, the maximum incident strain, ϵ_i , may be determined as

$$\epsilon_i = \frac{0.8\sigma_y}{E_b} \quad , \quad (2.2)$$

in which E_b is the modulus of elasticity of the bar material. The transmitted strain, ϵ_t , was then determined by substituting these values for ϵ_i and ϵ_r into Equation 1.14,

$$\epsilon_t = \epsilon_i + \epsilon_r \quad .$$

Equation 1.18 was rearranged to give

$$A_b = \frac{\sigma_s A_s}{E_b \epsilon_t} \quad , \quad (2.3)$$

from which the minimum area, A_b , of the Hopkinson bars required to prevent plastic deformation of the bars was determined given an arbitrary specimen cross-sectional area, A_s .

Figure 2.3 is a plot of the positions of the various stress waves in the Hopkinson bars as a function of time. The leading and trailing edges of each stress pulse are plotted. To prevent overlapping of the stress pulses as they pass through the strain gauges, the distance between the strain gauge and a given end of the bar must exceed the pulse length. The pulse duration, Δt , was selected to obtain a total specimen strain of 40%. This duration may be calculated by integrating Equation 1.16 and rearranging to give

$$\Delta t = -\frac{\epsilon_s L_s}{2c_o \epsilon_r} \quad . \quad (2.4)$$

The striker length, L_{st} , necessary to induce this pulse may be determined from

$$L_{st} = \frac{c_o \Delta t}{2} \quad . \quad (2.5)$$

The impact velocity of the striker, v_o , can be calculated by rearranging Equation 1.2 to give

$$v_o = 2 \frac{\sigma_b}{\rho_b c_o} \quad (2.6)$$

The strikers are cylinders with an inner diameter of 2.2 cm, large enough to accomodate the incident bar and press-fit teflon bushings. An outer diameter of 2.7 cm was selected so that the linear mass of the striker would be the same as that of the bar, as required by the Hopkinson bar equations. A variety of strikers ranging from 10 to 40 cm in length were manufactured to give a variety of pulse durations ranging from 40 to 160 μs .

The diameter of the endcap on the incident bar was chosen to be the same as the outer diameter of the stiker to accomodate the impact. The length of the endcap was arbitrailly chosen as 2.5 cm.

The design calculations are given in Appendix A and the final design parameters are as follows:

- Striker lengths = 10, 20, 30, 40 cm;
- Diameter of Hopkinson bars = 1.6 cm;
- Length of incident bar = 204 cm;
- Length of transmitter bar = 190 cm;

The strain rates achieved during testing were lower than the target value of 2000 s^{-1} , as the design calculations were based on a longer gauge length

2.1.2 Gas Gun

Figure 2.4 is a schematic of the gas gun. The gas gun is positioned concentrically around the incident bar and serves to propel the striker against the endcap with the

necessary impact velocity.

The gas gun consists of two concentric, thick-walled cylinders with endcaps. The endcaps are keyed in order to locate the cylinders and o-rings are used to ensure a seal. The hole in the endcap nearest the specimen is lined with a brass bushing within which the incident bar rides. The hole in the other endcap is slightly larger than the diameter of the bore of the inner cylinder. The annular volume between the two cylinders is the pressure chamber of the gun. The bore of the inner cylinder is the barrel of the gas gun and houses the striker cylinder and the pusher.

The pressure chamber is pressurized with nitrogen through a supply tube. Holes drilled through the inner cylinder connect the pressure chamber to the barrel as shown in Figure 2.4. The pusher, outfitted with two o-rings, is inserted in the barrel so that it seals these holes. The striker is inserted into the pusher as shown in Figure 2.5. To fire the gun: (i) the pressure chamber is charged with nitrogen at the desired pressure and (ii) nitrogen is bled in behind the pusher through a secondary supply inlet causing the pusher to edge forward. As the pusher moves past the holes, the gas in the pressure chamber rushes into the barrel behind the pusher, propelling it and the striker toward the endcap of the incident bar.

Figure 2.6 is a photograph of the plumbing associated with the gas gun. The gas gun is equipped with a manual valve system which controls the supply of gas into the pressure chamber and the inlet behind the pusher for firing. The valve controlling the pusher inlet is a three-way valve which allows any gas leaking behind the pusher during charging of the pressure chamber to flow to atmosphere to prevent a miss-fire of the striker. If necessary, a main discharge valve can be used to empty the pressure chamber.

The required pressure chamber size was estimated using energy considerations. Neglecting losses, the kinetic energy needed to impart the necessary impact velocity to the striker is equal to the energy released by the gas as it expands. The expansion of the gas was assumed to be an isentropic or reversible adiabatic process. The

kinetic energy of the striker, K_s , is given by

$$K_s = \frac{1}{2} m_{st} v_o^2 , \quad (2.7)$$

where v_o is the striker velocity and m_{st} is the mass of the striker.

An expression in terms of volume for the energy, U_g , released by the isentropic expansion of the gas may be obtained by integrating

$$U_g = \int_{V_o}^{V_f} p dV , \quad (2.8)$$

where p is pressure and V is volume. The initial volume, V_o , is the volume of the pressure chamber with the pusher in the pre-launch position and the final volume, V_f , is the volume of the pressure chamber plus the barrel with the pusher at the exit.

The force exerted by the gas on the striker must be maintained until the striker has accelerated to the required velocity. This implies that the striker must remain housed by the barrel until the acceleration is complete. The distance required can be easily calculated which gives an estimate of 50 cm as the required barrel length.

Detailed design calculations are given in Appendix A. Note that the calculations are simplistic and neglect losses. The approach was considered adequate for design purposes, however, since the volume and maximum pressure eventually selected for the gas gun greatly exceed the requirements for the current experimental work.

2.1.3 Momentum Trap

After impact of the striker with the endcap and fracture of the specimen, the forward motion of the incident bar is arrested by means of a momentum trap which is shown in Figures 2.7 and 2.8. This apparatus is a heavy steel unit in two parts:

a base with a rectangular groove along its entire length and a lid with a matching groove that is bolted to the base. The grooves form a rectangular hole that houses a rubber brake. A steel pin, having the same diameter as the incident bar endcap, is seated in the rubber brake. The rubber brake is tightened around the pin by securing the lid of the momentum trap to the base. The momentum trap also has lead blocks positioned in the hole behind the rubber brake and an end plate bolted to the back of the unit. Upon impact, the incident bar endcap contacts the pin which drives the pin partially through the brake and dissipates the kinetic energy of the incident bar. This energy must be absorbed to prevent the incident bar from rebounding and causing damage to the fracture surfaces of the specimen.

2.1.4 Stripper

It is desirable that the stress pulse induced in the incident bar by the striker resemble a square wave as closely as possible. Dispersion of the stress pulse as it reflects in the endcap and travels through the bars is inevitable, so it is necessary to impart a wave with the striker which is as square as possible. Comparison of the stress pulses generated within the bars by a striker propelled by an aluminum pusher versus a teflon pusher indicated that the pusher contributed significantly to the stress pulse in the bars. This contribution was more noticeable for the aluminum pusher which has a greater mass than the teflon pusher.

A stripper was fabricated and positioned after the gas gun to strip the pusher from the striker prior to the impact of the striker on the endcap. This device, shown in Figure 2.9, was designed to arrest the motion of the pusher while still allowing the unimpeded passage of the striker.

2.1.5 Assembly and Alignment

The mechanical components comprising the TSHB apparatus are bolted to heavy steel plates which are, in turn, mounted on a hollow structural section 203.4 x 203.4 x 9.5 mm and 4.3 m long. The gas gun is strapped into a cradle which is bolted to the HSS. The bars are held in position by a series of adjustable shaft supports. Preliminary alignment of the components with respect to each other was achieved by shimming of the plates as they were bolted to the HSS.

A final alignment was achieved using a low-powered laser. With the bars removed, the laser was mounted at the momentum trap end of the assembly and directed along the center-line of the gas gun barrel. A plug with a small axial hole was inserted into the exit of the barrel. A second hollow plug having a piece of mylar with scribed cross-hairs glued over one end was inserted into the brass bushings at the other end of the barrel. The laser was adjusted until the beam shone through the first plug and onto the center of the cross-hairs and was thus aligned with the gas gun barrel as a reference.

Once the laser was aligned with the barrel, progressive alignment of the shaft supports with the laser beam was performed. Initially, a plug having a mirror with scribed cross-hairs was inserted into the shaft support to give a rough adjustment since the shaft supports have several translational and rotational adjustments. After this initial adjustment, a plug with an axial hole was inserted into the shaft supports and adjustments were made until the laser beam shone through the hole. Once the laser beam shone through the the plugs along the entire axis of the apparatus, the alignment was said to be complete. The accuracy of the alignment was checked by the ease of glide of the bars through the supports.

Realignment of the assembly using this technique is only necessary after moving or replacing any of the components.

2.2 Specimen Design

Selection of the final specimen configurations to be used in the experimental work was an evolutionary process in which a variety of specimen geometries were investigated. The specimen is threaded between the incident and transmitter bars as shown in Figure 2.10. The incident bar has a right-hand thread and the transmitter bar had a left-hand thread so that the specimen can be inserted without turning the bars and twisting the strain gauge lead wires. Two grooving tools were specially fabricated to machine the specimen notches, one with 0.635 cm diameter and a second with 0.409 cm diameter. The material used to fabricate the specimens was UNS C36000 free cutting brass with 3% nominal lead content added to improve machinability. The brass is predominantly α brass with small regions of β brass. Some of the specimen material was annealed at 850°C for two hours before machining to promote spheroidization of the lead phase.

The effect of the notch in a specimen is to elevate the triaxiality at the center of the notch. Triaxiality is defined [75] as $\sigma_m/\bar{\sigma}$, where $\sigma_m = \frac{1}{3}\sigma_{kk}$ is the hydrostatic or mean stress and $\bar{\sigma}$ is the flow stress. An expression for triaxiality for a given specimen notch configuration under quasi-static loading can be obtained using a Bridgman analysis [75, 76]. The triaxiality, expressed in terms of the radius of the notch, R , and the gauge radius of the specimen, r , shown in Figure 2.11, is

$$\left(\frac{\sigma_m}{\bar{\sigma}}\right)_{max} = \frac{1}{3} + \ln\left(\frac{r}{2R} + 1\right) . \quad (2.9)$$

The value of triaxiality for a TSHB specimen gives an indication of the severity of the stress state which will be experienced by the specimen during loading. During high rate deformation, the radial inertia further elevates the triaxiality of the specimen. The increase in triaxiality increases the rate of void growth as described by the Rice-Tracey equations [44]. The smaller the radius of the notch compared to the gauge radius (ie the sharper the notch), the higher the induced triaxiality,

as given by Equation 2.9. In order to compare the effect of different notch configurations/triaxiality on the specimen behaviour, the objective of the specimen design process was to arrive at two specimen configurations that behaved well under realistic loading conditions with notches to give different triaxialities. The various specimen designs considered are summarized in Table 2.1 and are discussed in the following paragraphs.

The earliest specimen design, designated 'S1', had both notch radius and gauge radius of 0.318 and a shoulder diameter the same as the major diameter of the threads, that is, 0.953 cm. The specimens were manufactured from non-annealed brass. The gauge radius was chosen so that the minimum diameter of the specimen would be less than the shoulder and minor thread diameter to promote failure at the notch. These specimens did not fail readily, even at high striker impact velocities since the small difference between the minimum diameter and the shoulder/minor thread diameter did not sufficiently focus the deformation within the notch.

The specimen design was altered by adding shoulders with the intent of increasing the localization of strain at the notch center during testing. Specimens were fabricated from both non-annealed and annealed brass with gauge radius of 0.318 cm and new shoulder diameter of 1.59 cm equal to the diameter of the incident and transmitter bars. Two notch radii and shoulder widths were considered; the 'S2' specimen had a notch radius of 0.318 cm and a shoulder width of 0.635 cm and the 'S3' specimen had a notch radius of 0.204 cm and a shoulder width of 0.318 cm. Both types of specimens failed only under very high loading velocities. Also, plastic deformation occurred where the threads meet the shoulder as well as at the notch center. Plasticity at the thread throat was especially prevalent in the annealed brass specimens in which the yield strength is significantly lower than that of the as-received brass. This plastic deformation in the threads caused the specimen to loosen and made the tests invalid. These unwanted deformation characteristics were attributed to two aspects of the specimen design: (i) the large mass of the

shoulders and (ii) the closeness of the gauge diameter to the minor diameter of the threads. These two facets combined to induce a high stress in the threads which caused yielding.

A new series of specimen geometries, 'S4' through 'S7' was created and fabricated from annealed brass. These specimens have the same notch radii as the two above, 0.318 and 0.204 cm, and the same shoulder width as the 'S3' specimen, 0.318 cm. The new shoulder diameter was selected as 1.27 cm and new gauge radii of 0.254 cm and 0.191 cm were considered. It was intended that these new configurations would focus the deformation an amount sufficient for the specimen to fail readily at realistic applied loads and to ensure that the threads remained elastic. The specimens with the 0.254 cm gauge diameter failed readily but experienced some plasticity at the thread throat. The 0.191 cm gauge diameter specimens behaved well with no plasticity at the throat.

Based on these results concerning the effect of specimen geometry, a final pair of specimens were selected. The shoulder diameter of 1.27 cm, shoulder width of 0.318 cm and the two notch radii of 0.318 cm and 0.204 cm were kept the same as the previous series. A new gauge radius of 0.204 cm was selected. The specimen designated as 'S8' has gauge radius of 0.204 cm and notch radius of 0.204 cm which conforms to the notch configuration used by Worswick [50] in quasi-static testing and is designated A-notched by Hancock and Mackenzie [75]. The 'S9' specimen has gauge radius of 0.204 cm and notch radius of 0.318 cm. These two new specimen configurations performed very satisfactorily and were selected as the specimen geometries for all of the experimental work to follow. The triaxialities for these two specimens as calculated from Equation 2.9 are 0.74 for the S8 specimen and 0.61 for the S9 specimen.

2.3 Instrumentation

2.3.1 Strain Gauges

The strains associated with the stress pulses propagating through the bars are measured by means of semi-conductor strain gauges located on the incident and transmitter bars. A pair of gauges is mounted on each bar 90 cm from the specimen. Each pair of strain gauges is connected into its own Wheatstone bridge. The strain gauges are placed on opposite arms of the bridge to compensate for bending and to double the effective gauge factor of the system. The use of a second pair of strain gauges mounted transverse to the original pair would form a temperature-compensating Wheatstone bridge configuration, however this was not considered for the current work since the short durations of the tests and small anticipated temperature rise negate the need to compensate for temperature change.

Each bridge is excited by a 1 volt supply. A variable resistance potentiometer is included in one arm of each bridge and a voltmeter is attached across each bridge, so that the bridge may be manually balanced or zeroed. The voltmeter remains attached for the purpose of balancing the bridge, however, its response time is too slow to permit data acquisition during a test.

Semi-conductor strain gauges were chosen for their high gauge factor of approximately 140 compared to a range of 2 to 3 for standard foil gauges. This high gauge factor substantially increases the resolution of the signal and improves signal-to-noise ratio, eliminating the need for a high speed amplifier. The gauges consist of a strain sensitive piezoresistive crystal. As well as having a high sensitivity, these gauges are also very small in size, have a larger resistance, higher fatigue life and low hysteresis [77].

In an effort to reduce noise in the strain gauge systems and eliminate cross-talk between channels, the wheatstone bridges are each contained in their own box and each powered with their own voltage source. Twisted pair and shielded cable

replaced ordinary wire wherever practical.

2.3.2 Data Acquisition

A Polar Instruments DS102 digital acquisition/storage device was initially used to record the strain gauge signals during the test. The gauge signal from each wheatstone bridge was carried to the DS102 by a coaxial cable connected across it. The sampling rate was 2048 points per millisecond per channel. The stored data is transferred to a PC-compatible computer via an RS232 interface. A B+K Precision model 2520 digital oscilloscope was connected to the DS102 to provide a visual display of the signals. Data from the DS102 was converted into specimen stress, strain, strain rate using the Hopkinson bar equations as implemented within custom programs on a Unix workstation.

During the main experimental program, the digitizer system was replaced by a Nicolet Pro 30 digital storage oscilloscope shown in Figure 2.12. The sampling rate usually used for experimental data acquisition was 5000 points per millisecond per channel, though occasionally 10 000 points per millisecond were used. An input filter was employed to reduce noise on the signal by blocking signals with frequency exceeding 100KHz. A fast Fourier transform analysis of the signal without the input filter indicated very little frequency content exceeding this value, thereby justifying the usage of the input filter without fear of eliminating important signal content. The voltage-time bridge data is stored on disk by the oscilloscope. These data files are then used to calculate specimen stress, strain and strain rate using the Hopkinson bar equations as implemented within custom programs on a Unix workstation.

Appendix B contains listings of all computer programs used for acquisition and conversion of experimental data.

2.3.3 Striker Velocity

The impact velocity of the striker is recorded using a velocity detector positioned near the endcap support as shown in Figure 2.13. This device consists of a pair of reflective infrared sensors, separated by a known distance of 2.5 cm. When the striker passes the first sensor it triggers a counter which is stopped when the striker passes the second sensor. The detector displays the recorded time and dividing the separation distance of the sensors by this time gives the striker impact velocity.

2.4 Strain Gauge Calibration

The strain gauge system was calibrated by means of in-situ static tension tests in which a number of additional fixtures are used to apply known tensile loads to the bars. Figure 2.14 is a schematic of the calibration setup. The endcap of the incident bar is braced against the breech by means of the longest striker and a split ring. A threaded tie made from AISI 4340 steel connects the two bars in the same manner as a specimen. The end of the transmitter bar away from the specimen is threaded and is connected to a load cell. A 4340 steel tie rod, connected to the other side of the load cell, is passed through a cradle which is bolted to the HSS. The end of the tie rod is threaded to accommodate a nut which, when tightened against the cradle, induces a tensile load in the two bars. Details of this system are shown in Figure 2.15. A voltmeter connected to the load cell permits the applied load to be recorded. Each bridge voltage is measured by voltmeters and recorded. A plot of applied load versus bridge voltage will yield the strain gauge factor. This calibration is routinely performed for gauges both in pairs and individually.

As a check on the entire data acquisition system, the gauge factor calibration is occasionally performed with a long time acquisition active on the oscilloscope or digitizer. The output is converted to values of strain and compared with values of strain calculated from the bridge voltage recorded at each load interval and with

values of strain calculated from the applied loads.

The strain gauge calibration is also checked dynamically by comparing the recorded level of the incident pulse induced by the striker with that predicted based on the measured striker impact velocity. This comparison was made for the range of impact velocities typically used during experimental testing.

Chapter 3

Numerical Model

Finite element analysis was incorporated in the current work to study wave propagation in the TSHB apparatus and to support the experimental work. Two levels of numerical analyses were performed: a model of the entire apparatus with various specimens and strikers, and a model that just considers the specimen for a range of specimen configurations.

The finite element code DYNA2D [78, 79] was used for the numerical simulations. The geometry and meshes for all of the models were prepared using PATRAN [80] and translated into DYNA2D input with PATDY2 [81]. Post-processing of the analyses was performed using PATRAN [80] and ORION [82].

3.1 Wave Propagation in the TSHB

For the study of wave propagation in the TSHB a model of the entire apparatus was used. This model includes the incident and transmitter bars, specimen and striker. A range of specimens and strikers were prepared to reflect the experimental work.

The apparatus is axisymmetric and therefore only half of the cross-section is discretized. The model is constrained along the entire center line against displacement

in the radial direction. The specimen is modelled as fully bonded to the incident and transmitter bars. The striker bar is assigned an initial velocity at which it will impact the endcap. Figure 3.1 is a schematic of the model geometry showing the applied boundary conditions.

Figure 3.2 shows the mesh details for the incident bar and 40 cm striker and Figure 3.3 shows a typical specimen mesh. Each component of the numerical model, except for the specimen, is meshed without grading since large deformations are not expected. The mesh of the specimen is very finely graded at the center of the notch where large strains and gradients are expected. Since DYNA2D bases its calculations on the lumped mass formulation, a rapid increase in size from one element to another may cause a reflection of an impinging stress pulse. This would give rise to an inaccurate representation of the constitutive behaviour of the material. As a result, care was taken to create fine, well-graded meshes of the specimens.

The bars and striker are made of AISI 4340 steel and this material was modelled by a simple elastic-plastic constitutive model with isotropic hardening. This constitutive model was used for its simplicity since these components are expected to remain elastic throughout the loading. It was necessary to have a model capable of plasticity, however, so that if plastic deformation developed at the endcap or striker impact surface at high striker velocities it would be detected.

Oxygen-Free High Conductivity copper was selected as the specimen material for this study since it is a well characterized material. The Johnson-Cook constitutive model [60] was employed to describe the plastic response of the deforming material since constitutive data for copper was readily available.

Figure 3.4 shows the predicted propagation of the stress pulse in the apparatus for the duration of a test. Upon impact, the striker induces a tensile pulse in the incident bar. The striker used for this analysis was 30 cm and so the pulse is 60 cm long. The impact velocity of the striker was 20 m/s giving an incident stress wave with an approximate magnitude of 404 MPa. At time 0.2 ms, this tensile

pulse is propagating through the incident bar towards the specimen, as shown in the figure. At 0.5 ms, shown in the figure, the pulse impinges upon the specimen, and the impedance mismatch between the specimen and bar due to the lower cross-section and lower strength of the specimen material causes the pulse to be partially transmitted and partially reflected. The incident pulse at this time has only partially impinged upon the specimen and so the transmitted and reflected pulses have not yet reached their full length. The transmitted pulse is sufficient to cause plastic deformation in the specimen, however, its magnitude is much smaller than that of the incident and reflected pulses due to the low load-carrying capacity of the specimen compared to that of the Hopkinson bars. At time 0.7 ms, the full reflected and transmitted pulses propagate through the bars. In the experimental apparatus, these incident, reflected and transmitted pulses would be recorded by strain gauges at stations indicated in the figure.

Figure 3.5 shows the numerical prediction of the strain gauges histories. The strain histories are acquired by monitoring the strain in the elements that are located in the same position as the strain gauges on the experimental apparatus. The striker used for this analysis was 30 cm and so the pulse duration is 120 μ s. The impact velocity of the striker was 20 m/s giving an incident strain wave of approximate magnitude 0.0020. As the figure indicates, the magnitude of the transmitted pulse is much smaller than that of the incident and reflected pulses due to the low load-carrying capacity of the specimen compared to that of the Hopkinson bars.

3.2 Specimen Behaviour

The numerical model provided support for the experimental work in two respects: (i) to help interpret the experimental results; and (ii) to develop a correlation between engineering stress and strain and true stress and strain. Also of interest to the current work is the applicability of the Gurson model, developed to model

quasi-static ductile fracture by microvoid coalescence, to the high strain rate, high triaxiality regime. Comparison of the numerical predictions with experimental behaviour will be discussed in Chapter 4 as part of the experimental results.

For the purpose of these studies, a simplified specimen-only model was developed. The specimen-only model offers the advantage of being considerably less CPU-intensive than the full model; reducing computing time from over 35 hours to less than 1 hour.

These models were prepared for the various specimen configurations used in the experimental work. Figure 3.6 shows a specimen-only model. The model includes the specimen as well as the parts of the incident and transmitter bars that radially surround the threads of the specimen which are outlined in the figure. The rationale for including these small parts of the bars is the assumption that they would provide a more realistic constraint on the specimen.

Figure 3.7 is a schematic of the specimen-only model indicating the applied boundary conditions. The specimen is fixed along the center line against radial displacement and fixed along the transmitter end against displacement in the axial direction. The specimen is deformed by applying prescribed velocity history load curves to the incident nodes. The load curve is an idealized square wave. The temporal duration of the load curves is selected to emulate the pulse duration of the various strikers used in the experimentation, calculated from

$$\Delta t = 2L_{st}/c \quad , \quad (3.1)$$

where L_{st} is the striker length and c is the wave propagation speed in the bar material. The load curve velocity magnitude, V , is estimated based on strain rate

$$V = \dot{\epsilon} * L_{spec} \quad , \quad (3.2)$$

where $\dot{\epsilon}$ is the strain rate either obtained from experiment or selected arbitrarily, and L_{spec} is the gauge length of the specimen measured from the outer edge of one shoulder to the outer edge of the other.

The Gurson constitutive model [47, 48], described in Chapter 1, was employed to describe the response of UNS C36000 brass specimens. This model was implemented in DYNA2D by Worswick [83]. Values for the Tvergaard coefficients suggested by Worswick and Pick [84] of $q_1 = 1.25$, $q_2 = 0.95$, and $q_3 = q_1^2$ were adopted. The stress-strain behaviour of the brass is described by a polynomial fit to quasi-static uniaxial tensile data [54]:

$$\bar{\sigma} = (94.4 + 1594\bar{\epsilon}^p - 1693(\bar{\epsilon}^p)^2 + 645(\bar{\epsilon}^p)^3)(1 + 0.0009\ln(\dot{\epsilon})) \text{ (MPa)} , \quad (3.3)$$

where $\bar{\sigma}$ is the material flow stress, $\bar{\epsilon}^p$ is the effective plastic strain and $\dot{\epsilon}$ is the effective strain rate. The strain rate sensitivity of the brass is described by the constant 0.0009 suggested by Wong [85].

Also specified in the model are the nucleation strain (0.13), the nucleation void volume fraction (0.025), the coalescence void volume fraction (0.15) and the void volume fraction at which complete loss of strength occurs (0.25).

3.2.1 Numerical Results

Numerical modelling permits a detailed study of the development of strain, strain rate, stress state and damage within the deforming specimen which is not possible during an actual experiment. In this manner, the model was used to examine in detail the high strain rate deformation behaviour of the specimen in order to better interpret the experimental results.

For this work, the specimen-only model incorporating the Gurson nucleation-strain constitutive model for the free cutting brass was used. Two specimen configurations, shown in Figure 3.8 and 3.9, were prepared based on the S8 and S9

specimens used for the experimental work, as described in Chapter 2. Load curves applied to the specimens were calculated to give a range of strain rates which reflect those rates achieved during experimental testing for each specimen configuration. The duration of the load curves was equal to that induced by a 40 cm striker which was the length used for the experimentation on these specimen types.

Development of Plastic Strain

Figure 3.10 shows the plastic strain history for two elements in the center of an S8-type specimen, one on the surface and one on the axis as indicated in the figure. The nominal strain rate based on gauge length is 1210 s^{-1} , where the gauge length is defined as the length between the outer edges of the two shoulders. The figure shows that the plastic strain increases throughout the duration of a test and remains constant after the pulse has passed. The figure also indicates that the element on the surface experiences a slightly higher level of plastic strain than the axial element.

Figure 3.11 shows the plastic strain history of the surface element for a series of S8 specimen tests having different nominal strain rates, in the range 270 to 1210 s^{-1} . The figure shows that for all strain rates, the plastic strain increases monotonically during the test and remains constant after the pulse has passed. The amount of plastic strain experienced by the element is larger for the higher strain rate tests since the loading is more intense for these tests.

Figure 3.12 shows the development of plastic strain in an S8 specimen at various times throughout the duration of a test. The nominal strain rate based on gauge length is 1210 s^{-1} . The duration of the test is $180 \mu\text{s}$. The series of contour plots in Figure 3.12 indicate that the development of the plastic strain field begins at the surface of the specimen and spreads to the central region as the magnitude of strain increases.

Development of Triaxiality

Figure 3.13 shows the triaxiality history for a surface element and an axial element for a test with nominal strain rate 1210 s^{-1} . The axial element experiences an initial spike and then a gradual increase in triaxiality. The surface element experiences lower triaxiality than the axial element and a slightly decreasing trend. The large drop in triaxiality in the axial element occurs after failure initiates at the axis of the specimen and relieves the local stress. At this time, the triaxiality increases nearer to the surface. The triaxiality for this S8 notch geometry based on a Bridgman analysis (Equation 2.9) is 0.74 which is approximately the average of the initial values predicted for the surface and axis elements. After the initial stages of the test, the triaxiality at the axis increases to approximately the Bridgman value.

Figure 3.14 shows the triaxiality history for the axial element for a series of tests with nominal strain rates as indicated. The figure shows that an initial spike in triaxiality occurs for all strain rates. At the lowest strain rate, 270 s^{-1} , the initial spike decreases less rapidly than at the higher strain rates and then remains fairly constant. At the higher strain rates, the triaxiality demonstrates a roughly linear increase with superimposed oscillations due to wave reflection through the specimen. Larger increases in triaxiality are experienced by specimens at higher strain rates.

Figure 3.15 shows the development of triaxiality with true diametral strain for the same tests. The figure again demonstrates an initial spike and a roughly linear trend with oscillations. As the figure indicates, triaxiality increases slightly with strain at the same rate for all tests, however, the higher strain rate test, experiencing a higher strain also experiences a more severe triaxial stress state.

Figure 3.16 is a series of contour plots showing the development of triaxiality for the S8 specimen tested at 1210 s^{-1} . The first contour plot at $8 \mu\text{s}$ shows that a region of high triaxiality develops initially at the center of the specimen. This central region then splits and two high triaxiality regions migrate away from the center. This behaviour gives rise to the initial spike in triaxiality experienced by the

axial element shown in Figures 3.13 to 3.15. The migration of the regions of high triaxiality occur as the local hardening rates decrease. At $140\mu s$ a high triaxiality region once again develops in the central region. When failure initiates, as shown in the fifth plot, the triaxial stress conditions are relieved in the vicinity of the failure. However, the triaxiality is still high at the tip of the failed region and will therefore maintain the progress of the failure towards the outer surface of the specimen.

Figure 3.17 is a plot of triaxiality versus true diametral strain for both S8 and S9 specimens tested at $1195s^{-1}$. The S8 specimen has a higher triaxiality than the S9 specimen for the same strain throughout the test until failure. As well, failure initiated earlier in the S8 specimen. This behaviour is expected since the S8 specimen has a sharper notch which increases the severity of the triaxial stress state.

Development of Strain Rate

The specimens are assumed to experience a nominal strain rate through the duration of a test which depends on the loading intensity. However, the nominal strain rate is calculated based on the gauge length of the specimen and as such represents an average of the strain rates felt by the specimen. The strain rate experienced locally is much higher than this nominal rate. Figure 3.18 shows how the development of strain rate in the specimen occurs and is useful since it indicates both how the strain rate develops in the specimen and what rates are reached at various locations in the specimen. The first and second contour plots show that the entire central region of the notch from axis to edge is deforming at a rate between 10^3 and 10^4s^{-1} . This region of intense deformation rate localizes, as shown in the third plot, and a small area of very high strain rate is experienced at the center of the specimen where the failure will initiate. In the fourth contour plot, the strain rate local to the failure is much lower in intensity, however, a very high local rate still exists at the tip of the failure. As the failure advances further, shown in the last

plot, the local strain rate subsides to give a fairly uniform rate throughout the whole central region, except for the area right at the tip of the failure since the pulse has passed.

Development of Porosity

Figure 3.19 shows the porosity history for a surface element and an axial element for a test with nominal strain rate 1210 s^{-1} . As the figure indicates, the porosity development at the axis is approximately exponential and reaches a much higher level than the porosity at the surface. The porosity levels off late in the test since the pulse has passed.

Figure 3.20 shows the porosity history for the axial element for tests with a range of strain rates, as indicated. As shown in the figure, the development of porosity increases throughout the duration of a test but the increase is much more pronounced at the higher strain rates.

Figure 3.21 is a plot of porosity versus true diametral strain and indicates that the development of porosity with increasing strain is the same for all strain rates. The increase in slope with larger strains is due to increased triaxiality as seen in Figure 3.15.

Figure 3.22 is a series of contour plots showing the development of porosity in an S8 specimen tested at a nominal strain rate of 1210 s^{-1} . The first plot indicates a more intense development of porosity at the surface of the notch. Since this is the same region as initial plastic strain development, the development of porosity at the early stages may be promoted by strain. The second plot in the series indicates that the porosity development has equalized throughout the specimen gauge region. It may be surmised that at this stage of the test, the triaxiality which is more intense at the central region and the plastic strain field which is more intense at the outer edge of the specimen, both act to promote porosity development throughout the specimen gauge region. In the last three contour plots, it is apparent that the high

triaxiality at the specimen center dominates as the driving mechanism behind the development of porosity, which intensifies further with the failure initiation at the center of the specimen.

Figure 3.23 is a plot of porosity versus true diametral strain for both S8 and S9 specimens tested at the same strain rate. The figure indicates that the S8 specimen reached a higher level of porosity as compared to the S9 specimen and reached it at a lower strain. As discussed in the previous section, the S8 specimen will experience a more severe triaxial stress state than the S9 specimen. Since triaxiality enhances the growth of voids, it is expected that the higher triaxial stress state would produce a higher rate of void growth. The final porosity reached by the S8 specimen is also larger than that of the S9 specimen. This is because, according to the Gurson model, the rate of increase in porosity due to void growth depends on the hydrostatic component of the plastic strain rate which, in turn, depends on the triaxiality. Therefore, increasing the triaxiality will increase the plastic strain rate, necessarily increasing the hydrostatic component of plastic strain rate, which increases the level of porosity reached by the specimen.

3.3 Correlating Engineering Stress-Strain and True Stress-Strain

The specimen-only models were also used to establish a correlation between engineering stress-strain and true stress-strain that is used to interpret the experimental data. This correlation is important since only engineering stress and strain based on specimen gauge length and initial notch diameter are available from experimental data. High speed photography could be used to record the true diameter of the specimen throughout a test, however this type of system was not available for the current work.

To establish the correlation, a series of specimen-only simulations was performed.

The S8 and S9 free-cutting brass specimens were modelled employing the Gurson constitutive model for the plastic response of the material. A range of strain rates was selected based on the experimental values.

From the numerical results, a plot of engineering axial strain, ϵ_{axial} , versus true diametral strain, ϵ_{diam} , was created where

$$\epsilon_{axial} = \frac{l - l_o}{l_o} , \quad (3.4)$$

where l_o and l are the initial and current gauge lengths respectively, and

$$\epsilon_{diam} = 2 \ln \left(\frac{d_o}{d} \right) , \quad (3.5)$$

where d_o and d are the initial and current gauge diameters respectively.

A cubic polynomial was fit to the data for each of the notch types, as shown in Figure 3.24 and these equations are applied to convert the measured engineering axial strain values to obtain true diametral strain values.

The true diametral strain, ϵ_{diam} , is also defined as

$$\epsilon_{diam} = \ln \left(\frac{A_o}{A} \right) , \quad (3.6)$$

where A_o and A are the initial and current gauge areas, respectively. Equating Equation 3.5 for the experimental values and Equation 3.6 allows for the calculation of current true area of the specimen. The experimental values of engineering stress, S_{exp} , are based on the initial gauge area of the specimen and so values of true stress can be obtained by

$$\sigma_{true} = S_{exp} \frac{A_o}{A} . \quad (3.7)$$

The implementation of these equations in a simple computer program allows conversion of the experimental engineering stress-strain curve into a true stress-strain curve.

To check the accuracy of the correlation, plots of engineering axial strain versus true diametral strain were determined based on the deformed specimen and compared with the corresponding data from the numerical model in Figure 3.25. For the experimental points, the true dimensions were obtained by measuring the diameter of the failed specimens with a Vernier caliper. As the figure indicates, the experimental data lies only in a limited range, however, the numerical curve gives an adequate prediction of the values. Specimens experiencing lower strains than those indicated in the figure are not included since they did not fail on the incident stress pulse and were reloaded by subsequent incident pulses created by reflections of the initial pulse. The reloading experienced by these specimens causes strain and so measurements of the final diameter are not indicative of the strain produced by the incident pulse.

Chapter 4

Experimental Work

4.1 TSHB Experimentation

Two specimen configurations were considered for the experimental testing based on the results of the specimen design study described in Chapter 2. The two specimens are shown in Figures 4.1 and 4.2. The gauge diameter for the two specimens is the same, however, they have different notch radii selected to give two different values of nominal triaxiality so that the effect of triaxiality could be examined.

In total, over 100 experimental tests were performed. Many of these tests comprised the commissioning of the facility and specimen design. With respect to experimental results discussed in this chapter, 28 tests will be examined. Out of the total, 19 were S9 specimens and 9 were S8 specimens. Unfortunately, a machining accident decreased the availability of more S8 specimens. The specimens were tested at striker velocities ranging from 4 to 16 m/s, to give a range of nominal strain rates of 310 to 1430 s^{-1} .

The experiments performed are summarized in Tables 4.1a and 4.1b which indicate specimen type, material stock, anneal condition and strain rate. The material designations 'Primary' and 'Secondary' refer to the brass stock used to fabricate the specimen. The strain rates depicted in the table are nominal rates reached during

the test and were calculated using the Hopkinson bar equation for strain rate.

The specimens were fabricated from UNS C36000 free cutting brass with a nominal composition of 61.5% copper, 35.5% zinc and 3% lead [86] added to improve machinability. The brass is predominantly α phase with small regions of β phase. For many specimens, the as-received brass was annealed for two hours at 850°C to promote spheroidization of the lead phase. Testing a material with a spheroidal particulate phase was desired so that comparisons could be made between the experimental results and the numerical results involving the Gurson constitutive model which assumes spherical voids. All of the specimens except for two were made from the same primary brass rod stock. The other two were made from a secondary brass rod stock and the differences between the materials will be discussed subsequently.

The experimental investigations focus on characterizing the ductile fracture and stress-strain behaviour of brass for high strain rate loading conditions. Scanning electron and optical microscopy are incorporated to provide a metallographic examination of the ductile fracture mechanisms.

The effect of the high strain rate loading, triaxiality, hardening rate, lead particle population and alloy content will be examined. Comparisons of ductility between the high strain rate tests and quasi-static tests will be performed. Predictions of ductility from the numerical model incorporating the Gurson constitutive model will be evaluated.

The experimental data obtained from the TSHB tests are engineering stress-strain values. The numerical model, described in Chapter 3, was used to establish a correlation between engineering stress-strain and true stress-strain values and this correlation was applied to the experimental data to obtain the true stress-strain curves which are presented in this chapter.

4.2 Metallography

Figures 4.3 and 4.4 are optical micrographs of the as-received and the annealed brass. As the figures indicate, the annealing process promotes the development of larger and more spherical lead particles.

The non-annealed brass specimens failed primarily in a cup-cone fashion characterized by a fibrous central region and a shear lip along the perimeter of one of the halves of the failed specimen. The central fibrous region of the fracture surface is shown in Figure 4.5 which is a scanning electron micrograph. The dimples in the surface which give it the fibrous texture are a clear indication of ductile fracture by microvoid coalescence. Each dimple represents one half of a void which was nucleated by the bursting of the lead particle within it. The subsequent coalescence of these voids leads to the development of a central crack which extends outwards until the condition for plastic instability is sufficient for the perimeter to shear off.

The annealed brass specimens failed without the characteristic shear lip; the fibrous texture covers the entire fracture surface. Figure 4.6 is a scanning electron micrograph which shows the dimpled fracture surface, again indicating failure by microvoid coalescence. Comparison of the micrographs in Figures 4.5 and 4.6 reveals that the dimples are smaller in the non-annealed brass since those voids were nucleated by larger numbers of smaller lead particles than in the annealed brass. Although not seen in Figure 4.6, the shear lip is absent; the entire fracture surface for the annealed brass specimens is dimpled. One effect of annealing is to increase the hardening rate over the non-annealed material; therefore, the annealed specimens harden more and are less susceptible to shear.

Further examination of Figure 4.6 reveals that several regions of much finer dimples, indicative of nucleation and coalescence, exist on the fracture surface as indicated by arrows. The nucleating medium for these voids is necessarily much smaller than the lead particles. These finer dimples are thought to occur in regions

where the intervold spacing is larger than average. The fine dimples likely form during the necking of a larger intervold ligament during coalescence of two widely spaced voids.

The annealed brass specimens demonstrate a difference in shape of each half of the fracture surface depending on the notch configuration. As shown in Figure 4.7, the sharper notched S8 specimen (having higher triaxiality) has one convex surface and one concave surface whereas each half of the less sharply notched S9 specimen (slightly lower triaxiality) has a roughly flat surface. The coalescence of microvoids usually occurs by necking of the intervold matrix through the development of bands of intense local shear. The highest intensity of shear will always occur at 45° to a tensile load. In the S8 specimens, coalescence initiates in the center of the specimen where the triaxiality is the highest and acts to open up voids. Then, the coalescence proceeds from this central crack at roughly a 45° angle until it progresses to the edge of the specimen producing the characteristic convex-concave fracture surface. In the S9 specimen coalescence initiates in the center of the specimen where the triaxiality is the highest and acts to open up voids. However, the S9 specimen, having a less sharp notch than the S8 specimen, does not experience as severe a triaxiality. Therefore the strain at the surface of the S9 specimens also acts to open up voids. Thus damage progresses more uniformly across the S9 minimum section. In this manner, the internal crack does not proceed in a convex-concave configuration, but rather has a roughly flat surface. This theory is supported by the numerical model, which demonstrated in Chapter 3 that the S8 specimen experiences a more severe triaxial stress state than the S9 loaded at the same nominal strain rate. In order to simulate the development of the shape of the fracture surface, it would be necessary to use a model with an extremely fine mesh in the central region. Such a model was considered beyond the scope of the present work.

A typical section through a failed S9 specimen that was tested in the annealed condition is shown Figure 4.8. The dimpling at the fracture surface is again clearly

shown and the level of porosity is highest in the vicinity of the fracture surface. It should also be noted that the lead particles below the fracture surface are elongated in the direction of loading whereas the aspect ratio of the lead particles in the annealed brass before testing (Figure 4.4) as well as those well below the fracture surface is roughly unity. Some local coalescence of voids is also apparent in the area just under the fracture surface.

Higher magnification SEM micrographs, in Figures 4.9 and 4.10, serve to further illustrate the failure mechanisms. These micrographs, taken near the fracture surface, show lead particles which have burst to nucleate voids and also show instances of void coalescence through localized shearing. Figure 4.11 is a high-magnification optical micrograph which shows the elongation of voids and the coalescence of several closely spaced voids.

Figure 4.12, taken with the SEM, indicates a large region of microvoid coalescence leading to the development of a large secondary fracture beneath the primary fracture surface for a non annealed brass specimen. It is thought that the development of a secondary fracture surface is possible due to the inertial effects associated with the high loading rates; that is, the rate of the deformation is so high that the localization of the fracture is not confined to one region. Large secondary cracks were not observed in the annealed specimens. It is believed that the larger inter-void spacing in the annealed material delays the coalescence process until a global instability occurs. The smaller particle spacing in the non-annealed brass allows the simultaneous formation of a number of coalescence zones which eventually form cracks. The determination of which coalesced region eventually propagates to form the final fracture depends upon the statistical distribution of voids in close proximity to the coalescence zone which are available to permit further crack propagation. Also, the non-annealed specimens exhibited lower ductilities compared to the annealed specimens as will be discussed in a later section. This lower ductility is associated with the lower strain hardening available to the non-annealed specimens.

These two effects, that is lower strain hardening and lower intervoid spacing, may combine to enhance the coalescence of voids through shear.

4.3 Mechanical Testing

The experimental results are summarized in Tables 4.2a and 4.2b which indicate the specimen configuration, anneal condition, material stock, strain rate, failure/unloading strain and measured failure strains. The material designations 'Primary' and 'Secondary' refer to the brass stock used to fabricate the specimen. The strain rates depicted in the table are the nominal rates reached during the test which were calculated from the Hopkinson bar equation for strain rate. For specimens that failed on the incident stress pulse, the failure strain is indicated. For specimens that did not fail but underwent elastic unloading, the unloading strain is indicated. The failure strains and unloading strains are values of true diametral strain taken from the true stress-strain curves at the point where the material strength diminishes or elastic unloading occurs, respectively. The measured failure strains were obtained from measuring the final diameter of the failed specimens with a Vernier caliper. The inherent difficulties in measuring the specimens with a Vernier caliper are responsible for the discrepancy between the measured values of failure strain and the values obtained from the stress-strain curves.

Examination of Tables 4.2a and 4.2b reveals several trends:

- For the specimens that did not fail on the incident pulse, the unloading strain increases with strain rate for both specimen types of annealed primary stock brass.
- The more sharply notched S8 type specimens failed at lower strains than the S9 specimens for the same strain rates.

- The non-annealed specimens failed at significantly lower strains than the annealed specimens for similar strain rates.
- The specimens made from the secondary stock brass failed at lower strains than the primary stock specimens for similar strain rates.

To summarize, the ductility demonstrated by the material is higher for the annealed specimens compared with the non-annealed specimens. The two material stocks used also exhibited markedly different ductilities. The more sharply notched S8 type specimens failed at lower strains than did the S9 type specimens for the same strain rates. The results of the experimental testing and the causes of these trends will be discussed in detail in the following sections.

4.3.1 Wave Propagation Measurements

Prior to discussing the mechanical testing results, a discussion of the wave propagation measurements is warranted. The incident strain pulse in the TSHB facility depends on the striker impact velocity and the density and sound speed of the bar material. The wave always has an approximately square shape with a finite rise and fall time. Figure 4.13 is an example of a typical incident pulse from the TSHB facility for a 40 cm striker with an impact velocity of 10.7 m/s. Also shown is an idealized pulse for the same striker length and impact velocity. The ideal strain pulse has a length of 160 μ s and a magnitude of 0.00104. The discrepancy between the magnitudes of the experimental and ideal pulses is likely due to error in the velocity detecting device and wave dispersions associated with the endcap. The durations of the experimental and ideal pulses are similar, however the experimental pulse has distinct rise and fall times. The higher frequency oscillations present in the experimental pulse are caused by dispersion of the pulse as it travels to the strain gauge where its shape and magnitude are recorded. As mentioned in Chapter

1, the effects of this dispersion are not significant since the bar diameter is so much smaller than the wavelength and so a correction for dispersion will not be made.

The reflected and transmitted strain pulses recorded during testing demonstrate different features depending upon whether the specimen failed or not. Since the stress-strain and strain rate history of the specimens are evaluated by means of these pulses, these curves take on characteristic shapes and will be discussed in the next section.

Specimens which failed on the incident stress wave show a reflected strain pulse which has a jump in magnitude towards the end of the wave as shown in Figure 4.14. This jump occurs after the specimen has failed and can no longer transmit any of the pulse. As a consequence, all of the incident pulse is reflected from the failure point onwards thereby increasing the magnitude of the reflected pulse to near that of the incident pulse. Oscillations which appear in the reflected pulse are again due to dispersive effects during the wave propagation. As well, the larger oscillatory components in the higher magnitude part of the reflected wave after the specimen failure are a consequence of the unloading pulse created by the failure and the impedance mismatch at the specimen-bar interface.

Since the specimen ceases to transmit any load after failing, the transmitted pulse, shown in Figure 4.15, is shorter in duration than the incident pulse because it is reduced to zero magnitude following the specimen failure. The transmitted pulse shows a characteristic hardening curve since it represents the load transmitted by the specimen during the test. The transmitted strain pulse also shows oscillations following the fracture.

The peak magnitude of the transmitted pulse is much smaller than that of the incident and reflected pulses due to the low load-carrying capacity of the specimen compared to that of the Hopkinson bars. The strain pulses are shown together in Figure 4.16 to give an idea of relative magnitude. Note that the pulses in Figures 4.14 through 4.16 have been offset in time so that they start at the same origin for

the sake of comparison.

Figure 4.17 shows the strain pulses for a specimen which did not fail. As the figure indicates, the reflected pulse does not have a jump in magnitude towards the end since the specimen continued to transmit the load throughout the duration of the initial pulse. As a result, the transmitted pulse is the same length as the incident pulse. The transmitted pulse also shows a characteristic elastic unloading curve which is markedly different from the unloading curve in Figure 4.15 for a specimen which fractured.

In some of the lower strain rate tests, the specimen does not fail on the first incident pulse but since the pulse then reflects off of the free ends of the bars, the specimen may fail on a subsequent incident pulse. This specimen appears physically identical to a specimen failing on the first incident pulse. Consequently, the shape of the strain pulses must always be examined to confirm whether the specimen failed on the first incident pulse or on a subsequent incident pulse. For simplicity, the specimens that failed on the first incident stress wave will be termed incident-pulse-failed (IPF) specimens and the specimens that did not fail or failed on a subsequent incident pulse will be termed non-failed (NF) specimens.

The NF specimens may experience several compressive and tensile reflections of the initial induced stress wave. Consequently, the final diameter of these specimens may not be indicative of the diametral strain caused by the incident pulse. As such, the failure strain of these specimens cannot be accurately calculated based on the final diameter of the specimen.

4.3.2 Stress and Strain Response

The incident, reflected and transmitted strain pulse behaviour is very significant with respect to the application of the Hopkinson bar equations. As mentioned in Chapter 1, if force equilibrium is assumed to exist across the specimen, it follows that the transmitted strain pulse, ϵ_t , is equal to the sum of the incident strain pulse,

ϵ_i , and the reflected strain pulse, ϵ_r . Figure 4.18 depicts the transmitted strain pulse and the 'incident + reflected' strain pulse for a typical TSHB test on brass. As demonstrated by the figure, the equality $\epsilon_t = \epsilon_i + \epsilon_r$ does not hold true since the two curves do not coincide. Examination of a large number of TSHB tests indicated that $\epsilon_t \neq \epsilon_i + \epsilon_r$ in general; indicating that force equilibrium does not exist across the specimen. Nonuniform distributions of stress or strain over the specimen may result from inertial restraints imposed by radial or axial particle accelerations [11].

As a consequence, Equation 1.18

$$\sigma_s = E_b \frac{A_b}{A_s} \epsilon_t ,$$

which was simplified by the assumption $\epsilon_t = \epsilon_i + \epsilon_r$, would lead to unrealistically high values of stress in the specimen. This was verified by applying Equation 1.18 to the experimental data which lead to values of stress in the specimen of almost twice the strength of brass. Similarly, Equation 1.16 for specimen strain

$$\epsilon_s = \frac{-2c_o}{L_s} \int_0^t \epsilon_r dt ,$$

and Equation 1.19 for strain rate

$$\dot{\epsilon}_s = \frac{-2c_o}{L_s} \dot{\epsilon}_r ,$$

also simplified by the assumption $\epsilon_t = \epsilon_i + \epsilon_r$, would lead to erroneous results.

Strain and strain rate for the specimen must be calculated using Equation 1.10 for strain

$$\epsilon_s = \frac{c}{L_s} \int_0^t (\epsilon_i - \epsilon_r - \epsilon_t) dt ,$$

and Equation 1.22 for strain rate.

$$\dot{\epsilon}_s = \frac{c}{L_s}(\epsilon_i - \epsilon_r - \epsilon_t) \quad ,$$

which were derived without the assumption of force equilibrium across the specimen.

In order to derive an equation for stress in the specimen, in cases where the forces at either end of the specimen are not equal, the force distribution in the specimen must be obtained or assumed. This equation for stress would then be a function of some combination of the incident, reflected and transmitted strain pulses. The actual manner in which to treat this data had not yet been determined. Equation 1.21 for stress

$$\sigma_s = \frac{E_b A_b}{2A_s}(\epsilon_i + \epsilon_r + \epsilon_t) \quad ,$$

which assumes a linear stress distribution across the specimen was used by Gary *et al* [23], as described in Chapter 1. Application of Equation 1.21 to the experimental data lead to the stress-strain curves contained in Figure 4.19. As the figure indicates, the magnitudes of stress vary greatly in a negative strain rate dependent way. Such a wide spread of strengths and such a distinctive negative strain rate effect is not possible for the brass material under study. This behaviour indicates that the stress distribution across the specimen is strain rate dependent.

Instead, the transmitted pulse was treated as capturing the specimen's load transmitting capacity. Its shape is indicative of the work hardening and constitutive softening behaviour of the material; however, it is also subject to inertial effects. Thus, to permit the trends in the material behaviour to be exhibited, the calculation of stress for the specimens will be performed using Equation 1.18.

$$\sigma_s = E_b \frac{A_b}{A_s} \epsilon_t \quad ,$$

in which only the transmitted pulse is used. Although this approach leads to stress magnitudes which are artificially high, its usage is acceptable here since the plots will be examined for behavioural trends only.

Effect of Strain Rate

Figure 4.20 is a plot of true stress versus true diametral strain for a number of S8 annealed brass specimens. Figures 4.21 and 4.22 are true stress versus true diametral strain curves for S9 annealed brass specimens. Figure 4.21 pertains to S9 specimen tests in which the data was acquired with the 8-bit DS102 system. The data reported in Figure 4.22 was collected with the 12-bit Nicolet oscilloscope. Also, the notch in these specimens was machined with a refined cutting tool. As such, it was thought that the two groups of S9 specimens were not directly comparable. The nominal strain rates achieved during each test are indicated and range between 310 and 1430 s^{-1} for the S8 specimens and between 400 and 1195 s^{-1} for the S9 specimens. The noise and aliasing apparent in Figures 4.20 and 4.21 are due to the lower resolution DS102 system and are greatly reduced in Figure 4.22 since the Nicolet oscilloscope is capable of filtering out higher frequency noise.

As the figures indicate, elastic unloading is apparent in the lower strain rate NF specimens of both specimen types at the lower strains. At the higher strain rates, the specimens failed on the incident pulse and therefore do not demonstrate any unloading. Some variation in failure strain was observed although not necessarily in a manner related to strain rate.

The pulse duration of 160 μs was the same in all the tests determined by the striker length of 40 cm. Consequently, the specimens loaded at higher strain rates

experience greater nominal strains. These higher rate specimens also experience higher triaxialities as seen in the numerically predicted triaxialities in Figures 3.14 and 3.15. The numerical model indicates that the triaxiality increases with time but the increase is considerably larger for the higher strain rate tests. Increases in triaxiality occur at high strain rates since the inertia of the material inhibits radial displacement of the particles and so the specimen experiences a greater radial constraint. At higher strain rates a more pronounced inertial effect is expected.

High triaxiality enhances the damage accumulation in the specimen since it results in a more severe stress state imposed on the material. The numerical work indicates that the level of porosity in the specimen increases exponentially with time and with strain, as seen in Figures 3.20 and 3.21. The increase in slope with larger strains is due to increased triaxiality. This accumulation of damage acts to weaken the specimen and promote failure at lower strains for the higher strain rate tests.

The numerical work shows that a region of extremely high strain rate develops at the center of the specimen where the failure later initiates. This localization of intense strain rate will also contribute to accelerating the failure at the higher strain rates.

Effect of Notch Configuration

The notch configurations considered here have values of triaxiality of 0.74 for the S8 specimen and 0.61 for the S9 specimen based on a Bridgman analysis described in Chapter 2. The numerical work indicates that the triaxiality increases during a dynamic test, especially at the higher strain rates and that the initial triaxiality at the surface of the specimen is almost twice that predicted analytically. This behaviour is expected since the Bridgman analysis is based on quasi-static conditions, however, it does indicate that triaxiality depends on notch geometry. To evaluate the effect of notch configuration on specimen behaviour, two pairs of S8 and S9 tests with the same nominal strain rates are compared in Figure 4.23.

As the figure indicates, for the same nominal strain rate, the S8 specimens failed at a lower strain and also at slightly lower stresses than the S9 specimens. As discussed in the previous section, a high triaxial stress state will act to promote earlier failure by increasing the void growth or rate of damage accumulation. Thus, the higher triaxiality of the S8 configuration causes this specimen type to fail at lower strains.

Effect of Annealing

Figure 4.24 is a comparison of true stress-strain behaviour for annealed and non-annealed S9 specimens. The figure indicates several behavioural differences due to the annealing process: (i) the annealed material has a lower yield strength, (ii) the annealed material has a steeper strain hardening curve and (iii) the annealed material is considerably more ductile.

The effect of annealing is to promote recovery in the material which increases the strain hardening rate and decreases the yield strength. Annealing also changes the particle morphology; the small lead particles diffuse together to form larger spheroidal particles having a more favourable surface energy. Thus the size and spacing of the particles is increased. Grain growth does occur, but is limited by the lead particles which restrict grain boundary migration. The high strain hardening rate and large intervoid spacing of the annealed material promotes stable void growth which delays the transition to unstable coalescence resulting in increased ductility. The non-annealed material has a lower strain hardening rate since it did not undergo recovery and is therefore more susceptible to shear. In addition, the voids are closely spaced in the non-annealed material. These two effects combine to promote coalescence of voids along bands of localized shear causing a global instability to occur which leads to lower ductility. These experimental results are supported by the metallography, as described earlier in the chapter, in which the non-annealed brass demonstrated the development of secondary fracture surfaces.

To isolate the two effects of particle population and hardening rate, it would be useful to anneal the brass at a temperature sufficient to cause recovery but lower than that required for rapid diffusion of the lead phase. This material would then have a hardening rate similar to that of the annealed specimens used in the experiments and the same size and spacing of lead particles as the non-annealed material.

Of the three non-annealed specimens, the one tested at the highest strain rate demonstrates a higher strength than the others, as indicated in Figure 4.24. Several sources of this anomaly are possible including experimental error, strain rate effect or differences in material. It is difficult to deduce the actual cause given the small number of experiments performed with non-annealed material.

Effect of Alloy Content

To maintain a consistency in the material used to fabricate the specimens, the majority of the specimens tested were machined from the same (primary) rod stock and annealed with the same procedure. Two S9 specimens were also prepared from a different (secondary) rod stock but annealed in the same manner. The stress-strain curves for specimens fabricated from the two materials indicate a markedly different behaviour as shown in Figure 4.25. Specimens fabricated from the secondary rod have significantly higher strength and lower ductility than those fabricated from the primary stock.

Both scanning electron and optical microscopy were employed to investigate the difference between the materials. The SEM uses x-ray spectrometry to determine the atomic weights of elements present in a sample and to identify the percentages of each type of element present. A comparison of elemental content for samples taken from the two different materials indicates a variation from the nominal composition for UNS C36000 free cutting brass of 61.5% copper, 35.5% zinc, and 3% lead [86]. The average composition given by x-ray spectrometry for the primary bar stock is 62.91% copper, 34.51% zinc, and 2.58% lead and for the secondary bar stock is

61.58% copper, 36.17% zinc, and 2.25% lead.

To explain why each of the materials differ with respect to phase content, it is necessary to examine a binary phase diagram for brass, shown in Figure 4.26. Using a zinc equivalence to account for the lead [87], and recalling that the anneal temperature for both materials is 850°C , the composition of both materials places them in the $\alpha + \beta$ zone. Since the zinc content of the secondary bar is slightly higher than that of the primary bar, it lies closer to the single phase β region and thus contains a higher percentage of β [88]. The samples were quenched after annealing and it is possible that the phase compositions present at the anneal temperature were trapped, thus causing a difference in phase content.

The optical micrographs in Figures 4.27 and 4.28 are sections of the primary and secondary rod, respectively. The sections are etched with a potassium dichromate solution to emphasize the phases. These figures indicate a significant difference in structure between the two materials. The lighter regions are α brass and are predominant in the primary material. The darker regions are β brass and are predominant in the secondary material. The α phase is a face-centered cubic structure and has good strength and ductility. The β phase, a body centered cubic structure, is less ductile. Since the secondary material is predominantly β brass, it would be expected to be less ductile than the α -dominated primary material [89] as observed in the experiments.

Comparison of High Strain Rate and Quasi-Static Behaviour

Quasi-static tension test data for free-cutting brass S8 specimens [50] was available for comparison with the S8 TSHB specimens. Table 4.3 depicts the effect of strain rate on ductility by comparing the failure strains of quasi-static and high strain rate tests for IPF specimens of S8 type. As indicated by the table, the failure strain is significantly lower for the quasi-static test.

Coalescence of voids requires the development of an intense shear zone between

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the voids causing the failure of the intervoid ligaments. The failure of an intervoid ligament occurs by necking induced by the intense shear. This intervoid necking requires a transition of the local velocity field from that of stable void growth to localized shearing. Under quasi-static tensile loading, the velocities are very low and the changes in the velocity field to cause void coalescence appear sudden. At high strain rates, however, the inertia of the material inhibits the transition of the local velocity fields and so necking of the intervoid matrix is delayed. As a consequence, stable void growth can continue at higher strains and a pronounced increase in ductility is realized for specimens tested at high strain rates compared with specimens loaded quasi-statically.

Ductility

As revealed by the experimental work discussed above, the ductility of the specimens is influenced by several factors: (i) loading rate, (ii) triaxiality, (iii) hardening rate and (iv) particle population. The effects of each of these factors on the ductility of the material will be summarized here.

The effect of high strain rate is to increase the ductility of brass since it causes inertial stabilization of void growth. The transition to coalescence is therefore delayed and failure occurs at higher strains.

Triaxiality increases throughout the duration of a test due to the increasing radial constraint imposed by the specimen's inertia at the high strain rates. These large triaxialities enhance void growth and serve to exacerbate the accumulation of damage.

Annealing spheroidizes the lead particles into larger, more widely spaced particles and reduces dislocation density thereby increasing the rate of strain hardening. The effect is the promotion of stable void growth and delay of the transition to unstable coalescence in the annealed brass which increases the level of strain it undergoes before failure. The non-annealed brass, on the other hand, has a lower hardening

rate and closely spaced voids which combine to promote earlier coalescence and lower ductility at failure.

4.3.3 Comparison of Numerical and Experimental Results

Table 4.4 compares the numerical predictions for failure strains with the experimental values for the same specimen type and strain rate. The numerically predicted failure strains are actually the strains at which failure initiated in the model. Numerical tests in which the failure did not initiate are denoted by 'unload'. Complete separation of the specimen in the numerical model did not occur for any of the strain rates analysed.

The Gurson constitutive model failed to accurately predict the ductility at failure. In the Gurson model, the critical values of void volume fraction at which coalescence initiates and at which complete loss of strength occurs are specified. It is possible that the values used in the model do not accurately reflect those experienced by the experimental specimens.

As well, it is possible that the rate of void growth in the material due to the pronounced increases in triaxiality is not being predicted accurately by the model, as discussed by Worswick *et al* [49]. Consequently, the damage accumulated in the experiments is greater than that predicted by the numerical model and so the measured failure strains are lower than predicted.

Finally, since DYNA2D used quadratic elements with only one integration point in the center of the element, it is possible that the mesh used for the specimens is insufficiently fine to capture the fracture behaviour of the material correctly. In general, predictions of crack initiation and propagation are sensitive to mesh refinement.

Chapter 5

Conclusions

The objective of the current work was to develop a facility for investigating the high strain rate tensile behaviour of materials and to study the high strain rate ductile fracture of brass, selected as a model material for the study. To fulfil this goal, the research for this graduate program encompassed several areas:

- The design and commissioning of a TSHB facility.
- The use of the TSHB facility to investigate the high strain rate tensile behaviour of brass, involving both mechanical testing and metallography.
- The development of several numerical models (i) to study wave propagation in the apparatus, (ii) to predict specimen behaviour, (iii) to convert experimental data into true stress-strain curves and (iv) to evaluate the applicability of the Gurson constitutive model.

This chapter summarizes the major trends revealed during the experimental and numerical work and outlines areas requiring further work.

5.1 Experimental Facility

The THSB facility, at present, is capable of testing specimens at strain rates based on gauge length between 300 and 1400 s^{-1} depending on the specimen material and configuration. The facility provided an excellent tool to investigate the influences of loading rate on the ductile fracture process.

To enhance the facility's capability with respect to strain rate, it would be necessary to fabricate the incident and transmitter bars from a specialty steel of superior strength to AISI 4340, such as a 300M maraging steel. The higher strength of these bars would permit a higher striker impact velocity, inducing a more intense stress pulse and therefore higher strain rates within the specimen.

In designing the facility, an effort was made to maintain the same linear mass in the striker as in the bars. However, the endcap violates this continuity and is thought to induce some oscillations and lower the rise time of the incident pulse. Redesigning the endcap to have the same linear mass as the striker and incident bar is not strictly possible; however, it should be possible to tailor the endcap profile to improve the wave propagation behaviour by reducing oscillations and sharpening the rise time of the pulse so that it more closely resembles an ideal square wave.

It was necessary to employ the numerical model to establish a correlation between engineering stress-strain obtained from the experiments and true stress-strain. Although measurements of the failed specimens support the use of this approach, the introduction of high speed photography to record true diameter of specimens during testing would provide a more direct method of obtaining true stress-strain data.

At present, specimens that do not fail on the incident pulse are necessarily reloaded by subsequent reflected pulses. Reloading of these specimens negates the validity of the metallurgical results and the values of true diametral strain at failure obtained by measuring the final diameter of the specimen. Trapping the transmitted

and reflected strain pulses after they have passed the strain gauges would prevent reloading of the non-failed specimens.

The experimental work for this thesis revealed that force equilibrium does not exist across the TSHB specimen. Furthermore, the stress distribution in the specimen is suspected to be strain rate dependent. Further work is required in this area to (i) establish the true nature of the stress distribution in the specimen through the means of a numerical model and (ii) based on such a model, develop a method of treating the incident, reflected and transmitted pulses so that the true stress-strain history within the notch of the deforming specimen can be determined.

5.2 Ductile Fracture at High Strain Rates

The experimental tests, metallography and numerical modelling performed for this thesis focussed on determining the effect of high strain rate and associated triaxiality increases on the ductile fracture process. The influence of hardening rate and particle population on the ductile fracture mechanism at elevated strain rates was also examined.

Effect of Strain Rate and Triaxiality

Elevating the loading rate has two major effects on the ductile fracture process: (i) inertial stabilization of void growth which increases ductility and (ii) increase of triaxiality which enhances damage accumulation. These two effects are competitive; inertial stabilization enhances ductility, while increasing triaxiality should decrease ductility.

The inertia of the material inhibits or delays the transition from stable void growth to coalescence through intervvoid necking. As a consequence, stable void growth can continue to higher strains and an increase in failure strain is realized for specimens tested at high strain rates.

At elevated loading rates, the inertia of the material also inhibits radial displacement of the particles. The resultant radial constraint on flow within the specimen increases the triaxiality and enhances the damage accumulation. This accumulation of damage promotes failure at lower strains for the higher strain rate tests; however, inertial stabilization of void growth dominates and so the overall effect of high strain rate loading is to increase the ductility of the material. This result was seen in the comparison between the present work and previous quasi-static tests [50] which indicated a pronounced increase in ductility of the high strain rate specimens over the quasi-static specimens.

The higher triaxiality S8 specimens failed at lower strains than the S9 specimens since they accumulated more damage. Future work concerning the effect of triaxiality is recommended. The present work studied only two different notch geometries or values of nominal triaxiality. It would be useful to study a broader range of triaxialities by both increasing and decreasing the sharpness of the notch.

Effect of Hardening Rate and Particle Population

The annealing process contributes three major effects to the material; (i) increasing the size of the lead particles, (ii) spheroidizing the lead particles and (iii) increasing the strain hardening rate of the material. The high strain hardening rate and larger intervoid spacing of the annealed material promoted stable void growth, delaying the transition to unstable coalescence and increasing ductility. The lower strain hardening rate and closely spaced particles in the non-annealed material combine to promote coalescence of voids along bands of localized shear; the achievement of a global instability occurs earlier and failure occurs at a lower strain. The two effects of particle population and hardening rate could be isolated by annealing the brass at a temperature sufficient for recovery but insufficient for diffusion of the lead phase and should be considered in future work.

The metallography revealed that for annealed brass, coalescence extended across

the entire fracture surface. For the non-annealed brass, coalescence formed the central region but a shear lip was present around the perimeter. The susceptibility of non-annealed brass to shear due to its lower strain hardening rate allows this failure mechanism to take over from the coalescence mechanism as the internal crack nears the perimeter of the specimen.

The low hardening rate and close intervaid spacing of the non-annealed brass also lead to the development of secondary fracture surfaces which are regions of local void coalescence along a shear band. The development of this secondary fracture surface is possible due to the inertial effects at the high loading rates which limit the time for the localization of coalescence at one surface. This phenomenon was not demonstrated to the same extent in annealed brass which has larger spacing between particles and a higher hardening rate.

5.3 Numerical Predictions of Ductility

The Gurson constitutive model over-predicted the failure strains at the high strain rates. The Gurson model uses critical values of void volume fraction at which coalescence initiates and at which coalescence is complete. A detailed metallographic study may be able to more accurately determine what these critical void volume fractions are and once implemented into the Gurson constitutive model may improve the predictions for ductility at the higher rates.

It would also be useful to compare the predicted void growth with that measured using quantitative metallographic techniques. Such work is in progress [91].

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Tables

Tables

Table 2.1: Summary of Specimen Geometries

Specimen	Shoulder Diameter d (cm)	Shoulder Width w (cm)	Notch Radius R (cm)	Gauge Radius r (cm)
S1	0.953	0.635	0.318	0.318
S2	1.59	0.635	0.318	0.318
S3	1.59	0.318	0.204	0.318
S4	1.27	0.318	0.318	0.254
S5	1.27	0.318	0.204	0.254
S6	1.27	0.318	0.318	0.191
S7	1.27	0.318	0.204	0.191
S8	1.27	0.318	0.204	0.204
S9	1.27	0.318	0.204	0.204

R = Notch Radius
 r = Gauge Radius

w = Shoulder Diameter
 d = Shoulder Diameter

Table 4.1a: Summary of Experimental Tests for S8 Specimens

Specimen Type	Material	Anneal	Strain Rate s^{-1}	Striker Velocity m/s	Firing Pressure psi
S8	Primary	Yes	310	3.8	25
S8	Primary	Yes	500	5.7	30
S8	Primary	Yes	700	7.1	30
S8	Primary	Yes	740	7.8	40
S8	Primary	Yes	970	9.8	40
S8	Primary	Yes	1100	13.6	50
S8	Primary	Yes	1195	15.9	50
S8	Primary	Yes	1210	12.2	50
S8	Primary	Yes	1430	15.5	60

Table 4.1b: Summary of Experimental Tests for S9 Specimens

Specimen Type	Material	Anneal	Strain Rate s^{-1}	Striker Velocity m/s	Firing Pressure psi
S9	Primary	Yes	400	8.5	35
S9	Primary	Yes	450	6.1	30
S9	Primary	Yes	530	6.8	30
S9	Primary	Yes	730	9.3	40
S9	Primary	Yes	730	9.9	40
S9	Primary	Yes	790	9.9	40
S9	Primary	Yes	900	11.3	50
S9	Primary	Yes	970	11.5	50
S9	Primary	Yes	1000	12.2	50
S9	Primary	Yes	1050	14.6	50
S9	Primary	Yes	1110	13.2	55
S9	Primary	Yes	1110	14.3	60
S9	Primary	Yes	1130	15.2	60
S9	Primary	Yes	1195	16.4	60
S9	Secondary	Yes	920	13.0	50
S9	Secondary	Yes	1030	14.6	60
S9	Primary	No	290	3.7	40
S9	Primary	No	480	12.0	50
S9	Primary	No	940	12.6	60

Table 4.2a: Summary of Experimental Results for S8 Specimens

Specimen Type	Anneal	Material	Strain Rate s^{-1}	Failure/Unloading Strain		Measured Failure Strain
S8	Yes	Primary	310	0.19	unloading	
S8	Yes	Primary	500	0.29	unloading	
S8	Yes	Primary	700	0.41	unloading	
S8	Yes	Primary	740	0.45	unloading	
S8	Yes	Primary	970	0.52	failure	0.57
S8	Yes	Primary	1100	0.44	failure	0.58
S8	Yes	Primary	1195	0.46	failure	0.53
S8	Yes	Primary	1210	0.47	failure	0.54
S8	Yes	Primary	1430	0.45	failure	0.53

Table 4.2b: Summary of Experimental Results for S9 Specimens

Specimen Type	Anneal	Material	Strain Rate s^{-1}	Failure/Unloading Strain		Measured Failure Strain
S9	Yes	Primary	400	0.24	unloading	
S9	Yes	Primary	450	0.26	unloading	
S9	Yes	Primary	530	0.30	unloading	
S9	Yes	Primary	730	0.43	unloading	
S9	Yes	Primary	730	0.45	unloading	
S9	Yes	Primary	790	0.49	unloading	
S9	Yes	Primary	900	0.58	unloading	
S9	Yes	Primary	970	0.63	unloading	
S9	Yes	Primary	1000	0.61	failure	0.54
S9	Yes	Primary	1050	0.52	failure	0.53
S9	Yes	Primary	1110	0.68	failure	0.52
S9	Yes	Primary	1110	0.61	failure	0.55
S9	Yes	Primary	1130	0.55	failure	0.55
S9	Yes	Primary	1195	0.62	failure	0.48
S9	Yes	Secondary	920	0.37	failure	0.38
S9	Yes	Secondary	1030	0.35	failure	0.39
S9	No	Primary	290	0.18	unloading	
S9	No	Primary	480	0.24	failure	0.40
S9	No	Primary	940	0.26	failure	0.50

Table 4.3: Effect of Strain Rate on Ductility

Strain Rate	Failure Strain
quasi-static	0.39
970	0.52
1100	0.44
1195	0.46
1210	0.47
1430	0.45

Quasi-static test data was obtained from [50].

Table 4.4: Numerical Predictions of Ductility

Specimen Type	Strain Rate s^{-1}	Failure Strain	
		Experimental	Numerical
S8	970	0.52	0.68 unload
S8	1195	0.46	0.89
S8	1210	0.47	0.88
S9	1050	0.52	0.82 unload
S9	1130	0.55	0.95 unload
S9	1195	0.61	0.95

Figures

Figure 1.1: Schematic of Concentric Cylinder TSHB (after [14])

Figure 1.2: Schematic of Hat Shaped Specimen (after [17])

Figure 1.3: Schematic of Collar TSHB (after [19])

Figure 1.4: TSHB Configuration for Current Work

Figure 1.5: Transient Response of Striker Impacting Bar (after [28])

Figure 1.6: Incident, Reflected and Transmitted Pulses in Bars

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Figure 1.8: Typical TSHB Incident Pulse

Figure 1.9: Typical Ductile Fracture Surface

Figure 2.1: Photograph of the TSHB Facility

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R = Notch Radius

r = Gauge Radius

Figure 2.11: Schematic of Specimen

Figure 2.12: Photograph of the Oscilloscope and Wheatstone Bridges

Figure 2.13: Photograph of the Velocity Detector

Figure 2.14: Schematic of the Strain Gauge Calibration Setup

Figure 2.15: Details of Strain Gauge Calibration Setup

Figure 3.1: Schematic of Model Geometry and Boundary Conditions

Figure 3.2: Mesh Details for Incident Bar and Striker

Specimen (Shoulder Radius = 0.635 cm, Notch Radius = 0.204 cm)

Transmitter Bar (Radius = 0.79 cm)

Incident Bar (Radius = 0.79 cm)

Figure 3.3: Mesh Details for Specimen

Figure 3.4: Stress Wave Propagation

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Figure 3.6: Specimen-Only Model

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Figure 3.10: Plastic Strain History

Figure 3.11: Plastic Strain History

Figure 3.12: Development of Plastic Strain

Figure 3.13: Triaxiality History

Figure 3.14: Triaxiality History

Figure 3.15: Triaxiality versus True Diametral Strain

Figure 3.16: Development of Triaxiality

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Figure 3.18: Development of Strain Rate

Figure 3.19: Porosity History

Figure 3.20: Porosity History

Figure 3.21: Porosity versus True Diametral Strain

Figure 3.22: Development of Porosity

Figure 3.23: Porosity versus True Diametral Strain

Figure 3.24: Engineering Strain versus True Strain

Figure 3.25: Engineering Strain versus True Strain

Figure 4.1: S8 Specimen Configuration

Figure 4.2: S9 Specimen Configuration

Figure 4.3: Optical Micrograph of As-Received Brass

Figure 4.4: Optical Micrograph of Annealed Brass

**Figure 4.5: Scanning Electron Micrograph of Fracture Surface
for Non-Annealed Brass**

**Figure 4.6: Scanning Electron Micrograph of Fracture Surface
for Annealed Brass**

**Figure 4.7: Fractured S8 Specimen (below) and
Fractured S9 Specimen (above)**

**Figure 4.8: Section through the Fracture Surface of an
Annealed Brass Specimen**

Figure 4.9: Scanning Electron Micrograph of Region Near Fracture Surface

Fracture surface is just off to the right side of the micrograph

Figure 4.10: Scanning Electron Micrograph of Region Near Fracture Surface
Fracture surface is just off to the right side of the micrograph

**Figure 4.11: Optical Micrograph Showing Elongation and
Coalescence of Voids**

**Figure 4.12: Secondary Fracture Surface Development in a
Non-Annealed Brass Specimen**

Tensile Straining is in the Vertical Direction

Figure 4.13: Typical Incident Pulse

Striker Length = 40 cm, Striker Velocity = 10.7 m/s

Figure 4.14: Typical Reflected Pulse

Striker Length = 40 cm, Striker Velocity = 10.7 m/s

Figure 4.15: Typical Transmitted Pulse

Striker Length = 40 cm, Striker Velocity = 10.7 m/s

Figure 4.16: Typical Strain Pulses

Striker Length = 40 cm, Striker Velocity = 10.7 m/s

Figure 4.17: Typical Strain Pulses

Striker Length = 40 cm, Striker Velocity = 9.9 m/s

Figure 4.18: Comparison of Pulses

Striker Length = 40 cm, Striker Velocity = 10.7 m/s

Figure 4.19: True Stress versus True Strain

Figure 4.20: True Stress versus True Strain

Figure 4.21: True Stress versus True Strain

3 of/de 3

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Figure 4.22: True Stress versus True Strain

Figure 4.23: True Stress versus True Strain

Figure 4.24: True Stress versus True Strain

Figure 4.25: True Stress versus True Strain

Figure 4.26: Detail of Copper-Zinc Binary Phase Diagram (after [90])

Figure 4.27: Optical Micrograph of Primary Brass

Figure 4.28: Optical Micrograph of Secondary Brass

Appendices

Appendix A

Design Calculations

Hopkinson Bars and Striker

The basic design parameters for the TSHB apparatus were obtained by selecting target values for cross-sectional area, strain and strain rate of the specimen. The material selected for the bars and striker was AISI 4340 steel.

The speed of wave propagation, c_o , in the steel bars is

$$\begin{aligned} c_o &= \sqrt{\frac{E_b}{\rho_b}} \\ &= \sqrt{\frac{2.0685 \times 10^{11} \text{ N/m}^2}{7886.9 \text{ kg/m}^3}} \\ &= 5121.2 \text{ m/s} , \end{aligned} \tag{A.1}$$

in which E_b and ρ_b are the elastic modulus and density of the bar material, respectively.

The target value of strain rate for the specimen, $\dot{\epsilon}_s$, was selected as 1000 s^{-1} . Specimen length was assumed to be 5 mm. Based on these values, the reflected strain, ϵ_r , was estimated by

$$\begin{aligned}
 \epsilon_r &= -\dot{\epsilon}_s \frac{L_s}{2c_o} & (A.2) \\
 &= -1000 \text{ s}^{-1} \frac{0.005 \text{ m}}{2 * 5121.2 \text{ m/s}} \\
 &= -0.00097 .
 \end{aligned}$$

By limiting the stress in the incident bar to 80% of the yield strength, σ_y , of the bar material to provide a margin of error against yielding the bar, the maximum incident strain, ϵ_i , may be determined as

$$\begin{aligned}
 \epsilon_i &= \frac{0.8\sigma_y}{E_b} & (A.3) \\
 &= \frac{0.8 * 6.895 * 10^8 \text{ N/m}^2}{2.0685 * 10^{11} \text{ N/m}^2} \\
 &= 0.00267 .
 \end{aligned}$$

The transmitted strain, ϵ_t , was then determined by

$$\begin{aligned}
 \epsilon_t &= \epsilon_i + \epsilon_r & (A.4) \\
 &= 0.00267 - 0.00097 \\
 &= 0.00170 .
 \end{aligned}$$

The specimen stress, σ , must exceed the tensile strength of the specimen material. Iron was selected for the calculations since it was anticipated to be one of the stronger materials tested and its tensile strength is 648 Mpa. The specimen diameter was chosen as 0.635 cm to give a cross-sectional area of 0.317 cm². The required bar area to prevent plastic deformation is

$$\begin{aligned}
A_b &= \frac{\sigma_s A_s}{E_b \epsilon_t} \\
&= \frac{6.48 * 10^8 \text{ N/m}^2 * 0.317 \text{ cm}^2}{2.0685 * 10^{11} \text{ N/m}^2 * 0.00170} \\
&= 0.584 \text{ cm}^2 ,
\end{aligned} \tag{A.5}$$

from which the minimum bar diameter to prevent plasticity is 0.862 cm. The actual bar diameter was chosen as 1.59 cm.

The maximum impact velocity of the striker, v_o , must not produce a stress wave exceeding the strength of the bars and can be calculated by

$$\begin{aligned}
v_o &= 2 \frac{\sigma_b}{\rho_b c_o} \\
&= 2 \frac{0.8 * 6.895 * 10^8 \text{ N/m}^2}{7886.9 \text{ kg/m}^3 * 5121.2 \text{ m/s}} \\
&= 27.3 \text{ m/s}
\end{aligned} \tag{A.6}$$

The pulse duration, Δt , to obtain a total specimen strain of 40% may be calculated by

$$\begin{aligned}
\Delta t &= -\frac{\epsilon_s L_s}{2c_o \epsilon_r} \\
&= \frac{\epsilon_s}{\dot{\epsilon}_s} \\
&= \frac{0.4}{2000 \text{ s}^{-1}} \\
&= 0.0002 \text{ s} .
\end{aligned} \tag{A.7}$$

The striker length, L_{st} , necessary to induce this pulse may be determined from

$$\begin{aligned}
L_{st} &= \frac{c_o \Delta t}{2} & (A.8) \\
&= \frac{5121.2 \text{ m/s} * 0.0002 \text{ s}}{2} \\
&= 0.512 \text{ m}
\end{aligned}$$

The strain and strain rate within the specimen can be varied by changing the length of the striker and so strikers ranging from 10 to 40 cm in length were manufactured to give a variety of pulse durations ranging from 40 to 160 μs . To prevent overlapping of the stress pulses as they pass through the strain gauges, the distance between the strain gauge and a given end of the bar must exceed the pulse length or twice the striker length. The length of the bars were chosen as 204 cm for the incident bar and 190 cm for the transmitter bar. The incident bar is longer since it must accommodate the gas gun.

The inner diameter of the strikers was selected as 2.22 cm, large enough to accommodate teflon bushings that are press-fit into them. The outer diameter of the striker was selected as 2.73 cm so that the linear mass of the striker would be the same as that of the bar, as required by the Hopkinson bar equations. A variety of strikers

The diameter of the endcap on the incident bar was chosen to be the same as the outer diameter of the stiker to accommodate the impact. The length of the endcap was arbitrarily chosen as 25 mm.

The final design parameters are as follows:

- Striker lengths = 10, 20, 30, 40 cm;
- Diameter of Hopkinson bars = 1.59 cm;
- Length of incident bar = 204 cm;

- Length of transmitter bar $\approx$ 190 cm;

Gas Gun

The energy required by the 40 cm striker to accelerate it to 27.3 m/s (the maximum permissible velocity) is calculated by

$$\begin{aligned}
 K_s &= \frac{1}{2} m_s v_o^2 & (A.9) \\
 &= \frac{1}{2} * 7886.9 \text{ kg/m}^3 * \frac{\pi}{4} * (0.0273^2 - 0.0222^2) * 0.4 \text{ m}^3 * (27.3 \text{ m/s})^2 \\
 &= 233.1 \text{ J}
 \end{aligned}$$

The energy produced by the expansion of gas in the gas gun must exceed this value. Several assumptions were made to perform this calculation: (i) the pusher acts as a piston such that the initial volume, V_o , is the volume of the gas gun chamber and the final volume, V_f , is the volume of the chamber plus the volume of the barrel, (ii) the gas expands isentropically. (iii) losses due to the flow of gas through the holes into the barrel are negligible. An expression in terms of volume for the energy, U_g , released by the isentropic expansion of the gas may be obtained by integrating

$$U_g = \int_{V_o}^{V_f} p dV , \quad (A.10)$$

where V is volume and p is pressure. Assuming isentropic expansion of the gas gives $pV^\gamma = K$ where K is a constant and $\gamma = 1.4$ for nitrogen. Therefore, $K = p_o V_o^{1.4}$, where 'o' denotes the initial states. And so

$$U_g = \int_{V_o}^{V_f} \frac{K}{V^{1.4}} dV \quad (A.11)$$

$$= -\frac{p_o V_o^{1.4}}{0.4} \left(\frac{1}{V_f^{0.4}} - \frac{1}{V_o^{0.4}} \right) \geq K_s . \quad (\text{A.12})$$

This equation is satisfied for $V_o = 3.39 * 10^{-3}$ and $V_f = 3.77 * 10^{-3}$ which gives $U_s = 2460 J$. The capacity of the gas gun greatly exceeds the requirements. However, if new bars are fabricated from a high strength specialty steel, then the striker velocity can be increased and a large capacity gas gun would be required.

Appendix B

Computer Programs

B.1 Data Reduction Programs for Nicolet

Calculation of Specimen Stress, Strain and Strain Rate

PROGRAM Nicholet

```
C CONVERTS NICHOLET BRIDGE VOLTAGE DATA INTO AXIAL STRAINS
C THEN DETERMINES SPECIMEN STRESS, STRAIN RATE, AND STRAIN.
C CALCULATES AVERAGE SPECIMEN STRESS BASED ON TRANSMITTED
C PULSE ONLY

      CHARACTER*20 NAME1,NAME2
      CHARACTER*72 JUNK
      INTEGER I,INDEXI,INDEXR
      REAL CH1(2,5000), CH2(2,5000)
      REAL L,D,ASPEC,PI,FINC,FTRAN,V1,V2
      REAL EZINC(1500),EZREFL(1500),EZTRAN(1500),SUM
      REAL ERATE,STRESS(1500),STRAIN(1500),TIME(1500),INCREFL(1500)
      REAL T1,T2,T3,START,STOP,CUTOFF,X,Y,XY,XX,TOTAL,SLOPE,INTER
      REAL C,E,ABAR
      REAL ESTRESS(1500),EE(1500),TSTRAIN(1500)
```

```

PARAMETER (PI=3.141592654, E=2.0685E+05, C=5121.2)
PARAMETER (ABAR=197.9326)
REAL TRUEDE,TRUEA
REAL A1,A2,A3,A4,A5,J1,J2,J3,J4,J5
CHARACTER*5 N
PARAMETER (A1=109.429, A2=-18.6547, A3=4.65053, A4=0.000644222)
PARAMETER (J1=127.648, J2=-19.6583, J3=4.59068, J4=0.0000905171)

C  ABAR IS IN mm^2
C  STRESSES ARE IN MPa
C  DATA FILES MUST HAVE 5000 SAMPLES
C  UPPER TRIGGER LEVEL MUST BE 20 mV
C  READ DATA FROM FILES INTO ARRAYS
      PRINT *, 'ENTER FILE NAME FOR CHANNEL 1: '
      READ (*,900), NAME1
      PRINT *, 'ENTER FILE NAME FOR CHANNEL 2: '
      READ (*,900), NAME2
      OPEN (UNIT=43, FILE=NAME1, STATUS = 'OLD')
      OPEN (UNIT=44, FILE=NAME2, STATUS = 'OLD')
      DO 10 I=1,13
          READ (43,FMT=900) JUNK
          READ (44,FMT=900) JUNK
10      CONTINUE
          DO 20 I=1,5000
              READ (43,FMT=910) CH1(1,I),CH1(2,I)
              READ (44,FMT=910) CH2(1,I),CH2(2,I)
20      CONTINUE
          CLOSE (43)
          CLOSE (44)

```

C **AQUIRE SPECIMEN STATISTICS**

```
PRINT *, 'ENTER GAUGE LENGTH OF SPECIMEN (mm): '  
READ *, L  
PRINT *, 'ENTER GAUGE DIAMETER OF SPECIMEN (mm): '  
READ *, D  
ASPEC = PI/4.0*D*D
```

C **AQUIRE GAUGE FACTOR**

```
PRINT *, 'ENTER GAUGE FACTOR FOR INCIDENT GAUGES: '  
READ *, FINC  
PRINT *, 'ENTER GAUGE FACTOR FOR TRANSMITTER GAUGES: '  
READ *, FTRAN
```

C **AQUIRE TEST RUN EXCITATION VOLTAGES**

```
PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 1 (mV) : '  
READ *, V1  
PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 2 (mV) : '  
READ *, V2  
OPEN (UNIT=299, FILE='erate.exp', STATUS = 'NEW')
```

C **GET STARTING INDICES FOR PULSES**

```
DO 30 I=1,5000  
    IF (CH1(2,I) .GT. 2.0000E-2) GOTO 40  
30  CONTINUE  
40  INDEXI = I-100  
    DO 50 I=1,5000  
        IF (CH1(2,I) .LT. -2.0000E-2) GOTO 60  
50  CONTINUE  
60  INDEXR = I-100
```

C **CALCULATE SPECIMEN STRAIN RATE**

```
DO 80 I=1,1500
```

```

EZINC(I) = 4.0*CH1(2,INDEXI+I)*1000.0/V1/FINC
EZREFL(I) = 4.0*CH1(2,INDEXR+I)*1000.0/V1/FINC
EZTRAN(I) = 4.0*CH2(2,INDEXR+I)*1000.0/V2/FTRAN
ERATE = C*(EZINC(I)-EZREFL(I)-EZTRAN(I))/L*1000.0
STRAIN(I) = EZINC(I)-EZREFL(I)-EZTRAN(I)
TIME(I) = CH1(1,INDEXI+I)*1000.0
WRITE (299,FMT=920) TIME(I),ERATE
80  CONTINUE
    CLOSE (299)
C  GET CUTOFF TIME FOR STRESS CURVE
    PRINT *, 'ENTER CUTOFF TIME FOR STRESS CURVE : '
    READ *, T3
    DO 125 I=1,1500
        IF (TIME(I) .GT. T3) GOTO 126
125  CONTINUE
126  CUTOFF=I
C  CALCULATE THE VALUES OF STRESS AND WRITE TO FILE
    OPEN (UNIT=300, FILE='stress.exp', STATUS = 'NEW')
    DO 150 I=1,CUTOFF
        STRESS(I) = E*ABAR/ASPEC*EZTRAN(I)
        WRITE (300,FMT=920) CH1(1,INDEXI+I)*1000.0,STRESS(I)
150  CONTINUE
    CLOSE (300)
C  CALCULATE ENGINEERING STRESS AND ENGINEERING STRAIN
    OPEN (UNIT=303, FILE='s-e.exp', STATUS = 'NEW')
    SUM = 0.0
    DO 160 I=1,CUTOFF-1
        ESTRESS(I) = (STRESS(I)+STRESS(I+1))/2.0

```



```

        SUM = SUM + (STRAIN(I)+STRAIN(I+1))/2.0*0.2E-6
        EE(I) = C*SUM/L*1000.0
        WRITE (303,FMT=920) EE(I),ESTRESS(I)
160    CONTINUE
        CLOSE (303)
C    CALCULATE TRUE STRESS-STRAIN CURVES
        OPEN (UNIT=306, FILE='ts-e.exp', STATUS = 'NEW')
C    ACQUIRE SPECIMEN STATISTICS
        PRINT *, 'J-notch or A-notch specimen? (J/A): '
        READ (*,900), N
C    CONVERT EXPERIMENTAL STRESS-STRAIN DATA TO TRUE STRESS-STRAIN
        DO 170 I=1,CUTOFF-1
            IF (N .EQ. 'A') TRUEDE=A1*EE(I)**3 + A2*EE(I)**2 + A3*EE(I) + A4
            IF (N .EQ. 'J') TRUEDE=J1*EE(I)**3 + J2*EE(I)**2 + J3*EE(I) + J4
            TRUEA=ASPEC/EXP(TRUEDE)
            TSTRESS=ESTRESS(I)*ASPEC/TRUEA
            WRITE (306,FMT=920) TRUEDE,TSTRESS
170    CONTINUE
C    CLOSE FILES
        CLOSE (306)
        PRINT *, 'DONE'
900    FORMAT (A)
910    FORMAT (F14.7,F14.7)
920    FORMAT (F20.10,F20.10)
        END

```

Calculation of Strain Pulse Summations

PROGRAM Strain

C CONVERTS NICHOLET BRIDGE VOLTAGE DATA INTO AXIAL STRAINS
C AND COMPARES VARIOUS SUMMATIONS OF THE PULSES

CHARACTER*20 NAME1,NAME2

CHARACTER*72 JUNK

INTEGER I,INDEXI,INDEXR

REAL CH1(2,5000), CH2(2,5000)

REAL FINC,FTRAN,V1,V2

REAL EZINC,EZREFL,EZTRAN,INCREFL,IRTPLUS,IRTMINUS

C DATA FILES MUST HAVE 5000 SAMPLES

C UPPER TRIGGER LEVEL MUST BE 20 mV

C READ DATA FROM FILES INTO ARRAYS

PRINT *, 'ENTER FILE NAME FOR CHANNEL 1: '

READ (*,900), NAME1

PRINT *, 'ENTER FILE NAME FOR CHANNEL 2: '

READ (*,900), NAME2

OPEN (UNIT=43, FILE=NAME1, STATUS = 'OLD')

OPEN (UNIT=44, FILE=NAME2, STATUS = 'OLD')

DO 10 I=1,13

READ (43,FMT=900) JUNK

READ (44,FMT=900) JUNK

10 CONTINUE

DO 20 I=1,5000

READ (43,FMT=910) CH1(1,I),CH1(2,I)

READ (44,FMT=910) CH2(1,I),CH2(2,I)

20 CONTINUE

CLOSE (43)

```

        CLOSE (44)

C   ACQUIRE GAUGE FACTOR
        PRINT *, 'ENTER GAUGE FACTOR FOR INCIDENT GAUGES: '
        READ *, FINC
        PRINT *, 'ENTER GAUGE FACTOR FOR TRANSMITTER GAUGES: '
        READ *, FTRAN

C   ACQUIRE TEST RUN EXCITATION VOLTAGES
        PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 1 (mV) : '
        READ *, V1
        PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 2 (mV) : '
        READ *, V2

C   OPEN OUTPUT FILES
        OPEN (UNIT=299, FILE='ez.inc.exp', STATUS = 'NEW')
        OPEN (UNIT=300, FILE='ez.refl.exp', STATUS = 'NEW')
        OPEN (UNIT=301, FILE='ez.tran.exp', STATUS = 'NEW')
        OPEN (UNIT=302, FILE='ez.inc+refl.exp', STATUS = 'NEW')
        OPEN (UNIT=303, FILE='ez.inc+refl+tran.exp', STATUS = 'NEW')
        OPEN (UNIT=304, FILE='ez.inc-refl-tran.exp', STATUS = 'NEW')

C   GET STARTING INDICES FOR PULSES
        DO 30 I=1,5000
            IF (CH1(2,I) .GT. 2.0000E-2) GOTO 40
30      CONTINUE
        40      INDEXI = I-100
            DO 50 I=1,5000
                IF (CH1(2,I) .LT. -2.0000E-2) GOTO 60
50      CONTINUE
        60      INDEXR = I-100

C   CALCULATE SPECIMEN STRAIN

```

```

DO 80 I=1,2000
    EZINC = 4.0*CH1(2,INDEXI+I)*1000.0/V1/FINC
    EZREFL = 4.0*CH1(2,INDEXR+I)*1000.0/V1/FINC
    EZTRAN = 4.0*CH2(2,INDEXR+I)*1000.0/V2/FTRAN
    INCREFL = EZINC + EZREFL
    IRTPLUS = EZINC + EZREFL + EZTRAN
    IRTMINUS = EZINC - EZREFL - EZTRAN
    WRITE (299,FMT=920) CH1(1,INDEXI+I)*1000.0,EZINC
    WRITE (300,FMT=920) CH1(1,INDEXI+I)*1000.0,EZREFL
    WRITE (301,FMT=920) CH1(1,INDEXI+I)*1000.0,EZTRAN
    WRITE (302,FMT=920) CH1(1,INDEXI+I)*1000.0,INCREFL
    WRITE (303,FMT=920) CH1(1,INDEXI+I)*1000.0,IRTPLUS
    WRITE (304,FMT=920) CH1(1,INDEXI+I)*1000.0,IRTMINUS
80  CONTINUE
    CLOSE (299)
    CLOSE (300)
    CLOSE (301)
    CLOSE (302)
    CLOSE (303)
    CLOSE (304)
    PRINT *, 'DONE'
900  FORMAT (A)
910  FORMAT (F14.7,F14.7)
920  FORMAT (F20.10,F20.10)
    END

```

Calculation of Incident Stress

```
PROGRAM IncStress
C CALCULATES THE INCIDENT STRESS PULSE FROM THE BRIDGE VOLTAGES AND
C COMPARES WITH INCIDENT STRESS CALCULATED WITH STRIKER VELOCITY
  CHARACTER*6 NAME
  CHARACTER*9 NAMEIN,NAMEOUT
  REAL FINC,VEXC,TIME,VOLT,SINC,E,STIME,VSTRESS,P,C
  CHARACTER*72 JUNK
  INTEGER J
  PARAMETER (E=2.0685E+05,P=7886.9,C=5121.2)
C STRESSES ARE IN MPa
C OPEN FILES
  PRINT *, 'ENTER FILE NAME (EXTENSION .ad): '
  READ (*,900), NAME
  WRITE (NAMEIN,905) NAME, '.ad'
  WRITE (NAMEOUT,905) NAME, '.st'
  OPEN (UNIT=43, FILE=NAMEIN, STATUS = 'OLD')
  OPEN (UNIT=297, FILE=NAMEOUT, STATUS = 'NEW')
C CALCULATE STRESS BASED ON STRIKER VELOCITY
  PRINT *, 'ENTER STRIKER IMPACT TIME (MICROSECONDS): '
  READ *, STIME
  VSTRESS = 0.0254/STIME/2*P*C
  WRITE (297,FMT=890) '# Stress from striker velocity: ',VSTRESS
C ACQUIRE GAUGE FACTOR
  PRINT *, 'ENTER GAUGE FACTOR FOR INCIDENT GAUGES: '
  READ *, FINC
C ACQUIRE TEST RUN EXCITATION VOLTAGES
  PRINT *, 'EXCITATION VOLTAGE (mV) : '
```

```

      READ *, VEXC
      PRINT *, 'WAIT...'
C    READ NICHOLET BRIDGE VOLTAGES FROM FILES
      DO 5 J=1,13
        READ (43,FMT=900) JUNK
5      CONTINUE
10     READ (43,FMT=910, END=100) TIME,VOLT
      IF (TIME .GT. 0.0002) GOTO 100
C    CONVERT BRIDGE VOLTAGE TO STRESS AND WRITE TO FILE
      SINC = 4.0*VOLT*1000.0/VEXC/FINC*E
      WRITE (297,FMT=920) TIME*1000.0,SINC
      GOTO 10
C    CLOSE FILES
100    CLOSE (43)
      CLOSE (297)
890    FORMAT (A32,F6.2)
900    FORMAT (A)
905    FORMAT (A6,A3)
910    FORMAT (F14.7,F14.7)
920    FORMAT (F20.10,F20.10)
      END

```

B.2 Programs for Reduction of Data from DS102

Data Acquisition from DS102

```
      REM DS102/RS232 PROGRAM FOR PC
      DIM REAL(2048),TEMPR(2048),TIME(2050)
      CH=0 : TIMEBASE=0.1
      $COM1 2048
      SCREEN 2
      :
10    CLS
      PRINT ""
      PRINT "HOPKINSON BAR DATA AQUISITION"
      PRINT ""
      PRINT "(1) ACQUIRE CHANNEL 1 & CHANNEL 2 DATA"
      PRINT "(2) ACQUIRE CHANNEL 1 DATA ONLY"
      PRINT "(3) ACQUIRE CHANNEL 2 DATA ONLY"
      PRINT "(4) RESET DEFAULT TIME BASE (CURRENT VALUE 0.1 ms/div)"
      PRINT "(5) END"
      PRINT ""
      PRINT "CHANNEL 1 DATA STORED IN A:CH1.DAT"
      PRINT "CHANNEL 2 DATA STORED IN A:CH2.DAT"
      PRINT ""
      INPUT "PLEASE SELECT: ",CHOICE
      DELAY 1
      CLS
      ON CHOICE GOTO 40,40,50,20,120
      GOTO 10
      :
```

```

20      INPUT "ENTER NEW TIME BASE (ms/div): ",TIMEBASE
      GOSUB TBASE
      GOTO 10
      :
40      CH=1
      GOSUB TBASE
      GOSUB CH1
      GOTO 70
      :
50      CH=2
      GOSUB TBASE
      GOSUB CH2
      CHOICE=0
      :
70      OPEN "COM1:2400,N,8,1,CS,DS" AS #3  'OPEN COM FILE
      ON COM(1) GOSUB TRAP  'SET TRAP FOR INCOMING SERIAL DATA
      :
      LOCATE 2,20 : PRINT "PRESS PLOT TO VIEW WAVEFORM. PRESS SPACEBAR WHEN
      WHILE NOT INSTAT
      :
      :
      COM(1) ON          'ENABLE TRAPPING OF COM DATA.
      :
      WEND
      CLOSE #3
      CLS
      IF CHOICE=1 GOTO 50
      GOTO 10

```



```
120    END
      :
TBASE:
      CLS
      PRINT ""
      PRINT "ASSIGNING TIME ARRAY"
      DELTAT=TIMEBASE*10/2048
      TIME(1)=0.0
      FOR X=2 TO 2048
      TIME(X)=TIME(X-1)+DELTAT
      NEXT
      RETURN
      :
CH1:
      CLS
      PRINT ""
      PRINT "READY TO ACQUIRE CHANNEL 1 DATA"
      PRINT ""
      PRINT "ENSURE SWITCH IS IN CHANNEL 1 POSITION"
      INPUT "AND PRESS RETURN ",IN
      OPEN "C:CH1.DAT" FOR OUTPUT AS #1
      CLS
      RETURN
      :
CH2:
      CLS
      PRINT ""
      PRINT "READY TO ACQUIRE CHANNEL 2 DATA"
```

```

PRINT ""
PRINT "ENSURE SWITCH IS IN CHANNEL 2 POSITION"
INPUT "AND PRESS RETURN ",IN
OPEN "C:CH2.DAT" FOR OUTPUT AS #2
CLS
RETURN
:
TRAP:
CLS
COM(1) OFF : X=0  'DISABLE TRAPPING & RESET COUNT.
:
LOCATE 1,30 : PRINT "PLOTING WAVEFORM NOW."
LOCATE 2,35 : PRINT "OR PRESS ANY KEY TO CONTINUE."
WHILE NOT EOF(3)  'WHILE THERE IS DATA IN THE INPUT BUFFER
A$ = INPUT$(LOC(3),#3)
:
FOR I=1 TO LEN(A$)
:
REAL(X)=ASC(MID$(A$,I,1))
IF CH=2 GOTO 140
LOCATE 4,30 : PRINT "CH 1"
PRINT #1,USING "%.#####^---- ###";TIME(X),REAL(X)
GOTO 150
140 LOCATE 4,30 : PRINT "CH 2"
PRINT #2,USING "%.#####^---- ###";TIME(X),REAL(X)
150 TEMPR(X)=REAL(X)/.65
PSET(X/3,400-TEMPR(X))
X=X+1

```

```

      :
      :
      :
    NEXT I
    DELAY .1
      :
    WEND
    CNT=X+1
    IF CH=2 GOTO 160
    WRITE #1,TIME(CNT)
    CLOSE #1
    GOTO 170
160   WRITE #2,TIME(CNT)
      CLOSE #2
      :
170   RETURN

```

Calculation of Specimen Stress, Strain and Strain Rate

```

PROGRAM Polar
C  CONVERTS POLAR UNIT DATA INTO AXIAL STRAINS
C  THEN DETERMINES SPECIMEN STRESS, STRAIN RATE, AND STRAIN.
C  CALCULATES AVERAGE SPECIMEN STRESS BASED ON TRANSMITTED
C  PULSE ONLY
      CHARACTER*20 NAME1,NAME2
      CHARACTER*72 JUNK
      REAL M1,B1,M2,B2,DATA1,DATA2
      INTEGER I,INDEXI,INDEXR

```

```

REAL CH1(2,2048), CH2(2,2048)
REAL L,D,ASPEC,PI,FINC,FTRAN,V1,V2
REAL EZINC(500),EZREFL(500),EZTRAN(500),SUM
REAL ERATE,STRESS(500),STRAIN(500),TIME(500),INCREFL(500)
REAL T1,T2,T3,START,STOP,CUTOFF,X,Y,XY,XX,TOTAL,SLOPE,INTER
REAL C,E,ABAR
REAL ESTRESS(500),EE(500),TSTRAIN(500)
PARAMETER (PI=3.141592654, E=2.0685E+05, C=5121.2)
PARAMETER (ABAR=197.9326)
REAL TRUEDE,TRUEA
REAL A1,A2,A3,A4,A5,J1,J2,J3,J4,J5
CHARACTER*5 N
PARAMETER (A1=109.429, A2=-18.6547, A3=4.65053, A4=0.000644222)
PARAMETER (J1=127.648, J2=-19.6583, J3=4.59068, J4=0.0000905171)
C  ABAR IS IN mm2
C  STRESSES ARE IN MPa
C  READ DATA FROM FILES INTO ARRAYS
    PRINT *, 'ENTER FILE NAME FOR CHANNEL 1: '
    READ (*,900), NAME1
    PRINT *, 'ENTER FILE NAME FOR CHANNEL 2: '
    READ (*,900), NAME2
    OPEN (UNIT=43, FILE=NAME1, STATUS = 'OLD')
    OPEN (UNIT=44, FILE=NAME2, STATUS = 'OLD')
C  ACQUIRE POLAR UNIT TO BRIDGE VOLTAGE CALIBRATION
    PRINT *, 'ENTER SLOPE AND INTERCEPT OF POLAR UNIT DATA'
    PRINT *, 'VERSUS BRIDGE VOLTAGE (mV) PLOT FOR CHANNEL 1: '
    READ *, M1,B1
    PRINT *, 'ENTER SLOPE AND INTERCEPT OF POLAR UNIT DATA'

```

```

        PRINT *, 'VERSUS BRIDGE VOLTAGE (mV) PLOT FOR CHANNEL 2: '
        READ *, M2,B2
C   CONVERT POLAR UNIT DATA TO BRIDGE VOLTAGES AND PUT INTO ARRAYS
        DO 10 I=1,2048
            READ (43,FMT=905) CH1(1,I),DATA1
            READ (44,FMT=905) CH2(1,I),DATA2
            CH1(2,I) = (DATA1 - B1)/M1
            CH2(2,I) = (DATA2 - B2)/M2
10    CONTINUE
        CLOSE (43)
        CLOSE (44)
C   ACQUIRE SPECIMEN STATISTICS
        PRINT *, 'ENTER GAUGE LENGTH OF SPECIMEN (mm): '
        READ *, L
        PRINT *, 'ENTER GAUGE DIAMETER OF SPECIMEN (mm): '
        READ *, D
        ASPEC = PI/4.0*D*D
C   ACQUIRE GAUGE FACTOR
        PRINT *, 'ENTER GAUGE FACTOR FOR INCIDENT GAUGES: '
        READ *, FINC
        PRINT *, 'ENTER GAUGE FACTOR FOR TRANSMITTER GAUGES: '
        READ *, FTRAN
C   ACQUIRE TEST RUN EXCITATION VOLTAGES
        PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 1 (mV) : '
        READ *, V1
        PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 2 (mV) : '
        READ *, V2
C   OPEN OUTPUT FILES

```

```

        OPEN (UNIT=299, FILE='erate.exp', STATUS = 'NEW')
C   GET STARTING INDICES FOR PULSES
        DO 30 I=1,2048
            IF (CH1(2,I) .GT. 40.0) GOTO 40
30    CONTINUE
40    INDEXI = I
        DO 50 I=1,2048
            IF (CH1(2,I) .LT. -40.0) GOTO 60
50    CONTINUE
60    INDEXR = I-INDEXI
C   CALCULATE SPECIMEN STRAIN RATE
        DO 80 I=1,500
            EZINC(I) = 4.0*CH1(2,I)/V1/FINC
            EZREFL(I) = 4.0*CH1(2,INDEXR+I)/V1/FINC
            EZTRAN(I) = 4.0*CH2(2,INDEXR+I)/V2/FTRAN
            ERATE = C*(EZINC(I)-EZREFL(I)-EZTRAN(I))/L*1000.0
            STRAIN(I) = EZINC(I)-EZREFL(I)-EZTRAN(I)
            TIME(I) = CH1(1,INDEXI+I)
            WRITE (299,FMT=920) TIME(I),ERATE
80    CONTINUE
        CLOSE (299)
C   CALCULATE SPECIMEN STRESS
        PRINT *, 'ENTER CUTOFF TIME FOR STRESS CURVE : '
        READ *, T3
C   FIND THE INDEX CORRESPONDING TO T3
        DO 125 I=1,500
            IF (TIME(I) .GT. T3) GOTO 126
125    CONTINUE

```

```

126  CUTOFF=I
C    CALCULATE THE VALUES OF STRESS AND WRITE TO FILE
      OPEN (UNIT=300, FILE='stress.exp', STATUS = 'NEW')
      DO 150 I=1,CUTOFF
          STRESS(I) = E*ABAR/ASPEC*EZTRAN(I)
          WRITE (300,FMT=920) CH1(1,INDEXI+I),STRESS(I)
150  CONTINUE
      CLOSE (300)
C    CALCULATE ENGINEERING STRESS-STRAIN CURVES
      OPEN (UNIT=303, FILE='s-e.exp', STATUS = 'NEW')
C    CALCULATE ENGINEERING STRESS AND ENGINEERING STRAIN
      SUM = 0.0
      DO 160 I=1,CUTOFF-1
          ESTRESS(I) = (STRESS(I)+STRESS(I+1))/2.0
          SUM = SUM + (STRAIN(I)+STRAIN(I+1))/2.0*0.00048828/1000.0
          EE(I) = C*SUM/L*1000.0
          WRITE (303,FMT=920) EE(I),ESTRESS(I)
160  CONTINUE
      CLOSE (303)
C    CALCULATE TRUE STRESS-STRAIN CURVES
      OPEN (UNIT=306, FILE='ts-e.exp', STATUS = 'NEW')
C    ACQUIRE SPECIMEN STATISTICS
      PRINT *, 'J-notch or A-notch specimen? (J/A): '
      READ (*,900), N
C    CONVERT EXPERIMENTAL STRESS-STRAIN DATA TO TRUE STRESS-STRAIN
      DO 170 I=1,CUTOFF-1
          IF (N .EQ. 'A') TRUEDE=A1*EE(I)**3 + A2*EE(I)**2 + A3*EE(I) + A4
          IF (N .EQ. 'J') TRUEDE=J1*EE(I)**3 + J2*EE(I)**2 + J3*EE(I) + J4

```

```

        TRUEA=ASPEC/EXP(TRUEDE)
        TSTRESS=ESTRESS(I)*ASPEC/TRUEA
        WRITE (306,FMT=920) TRUEDE,TSTRESS
170    CONTINUE
C    CLOSE FILES
        CLOSE (306)
        PRINT *, 'DONE'
900    FORMAT (A)
905    FORMAT (F11.5,F4)
910    FORMAT (F14.7,F14.7)
920    FORMAT (F20.10,F20.10)
        END

```

Calculation of Strain Pulses

```

        PROGRAM Strain
C    CONVERTS POLAR UNIT DATA INTO AXIAL STRAINS
        CHARACTER*20 NAME1,NAME2
        CHARACTER*72 JUNK
        REAL M1,B1,M2,B2,DATA1,DATA2
        INTEGER I,INDEXI,INDEXR
        REAL CH1(2,2048), CH2(2,2048)
        REAL FINC,FTRAN,V1,V2
        REAL EZINC,EZREFL,EZTRAN,INCREFL
C    READ DATA FROM FILES INTO ARRAYS
        PRINT *, 'ENTER FILE NAME FOR CHANNEL 1: '
        READ (*,900), NAME1
        PRINT *, 'ENTER FILE NAME FOR CHANNEL 2: '

```



```

      READ (*,900), NAME2
      OPEN (UNIT=43, FILE=NAME1, STATUS = 'OLD')
      OPEN (UNIT=44, FILE=NAME2, STATUS = 'OLD')
C   ACQUIRE POLAR UNIT TO BRIDGE VOLTAGE CALIBRATION
      PRINT *, 'ENTER SLOPE AND INTERCEPT OF POLAR UNIT DATA'
      PRINT *, 'VERSUS BRIDGE VOLTAGE (mV) PLOT FOR CHANNEL 1: '
      READ *, M1,B1
      PRINT *, 'ENTER SLOPE AND INTERCEPT OF POLAR UNIT DATA'
      PRINT *, 'VERSUS BRIDGE VOLTAGE (mV) PLOT FOR CHANNEL 2: '
      READ *, M2,B2
C   CONVERT POLAR UNIT DATA TO BRIDGE VOLTAGES AND PUT INTO ARRAYS
      DO 10 I=1,2048
          READ (43,FMT=905) CH1(1,I),DATA1
          READ (44,FMT=905) CH2(1,I),DATA2
          CH1(2,I) = (DATA1 - B1)/M1
          CH2(2,I) = (DATA2 - B2)/M2
10   CONTINUE
      CLOSE (43)
      CLOSE (44)
C   ACQUIRE GAUGE FACTOR
      PRINT *, 'ENTER GAUGE FACTOR FOR INCIDENT GAUGES: '
      READ *, FINC
      PRINT *, 'ENTER GAUGE FACTOR FOR TRANSMITTER GAUGES: '
      READ *, FTRAN
C   ACQUIRE TEST RUN EXCITATION VOLTAGES
      PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 1 (mV) : '
      READ *, V1
      PRINT *, 'EXCITATION VOLTAGE FOR CHANNEL 2 (mV) : '

```

```

      READ *, V2
C   OPEN OUTPUT FILES
      OPEN (UNIT=299, FILE='ez.inc.exp', STATUS = 'NEW')
      OPEN (UNIT=300, FILE='ez.refl.exp', STATUS = 'NEW')
      OPEN (UNIT=301, FILE='ez.tran.exp', STATUS = 'NEW')
      OPEN (UNIT=302, FILE='ez.inc+refl.exp', STATUS = 'NEW')
C   GET STARTING INDICES FOR PULSES
      DO 30 I=1,2048
        IF (CH1(2,I) .GT. 40.0) GOTO 40
30    CONTINUE
40    INDEXI = I
      DO 50 I=1,2048
        IF (CH1(2,I) .LT. -40.0) GOTO 60
50    CONTINUE
60    INDEXR = I-INDEXI
C   CALCULATE STRAIN
      DO 80 I=1,500
        EZINC = 4.0*CH1(2,I)/V1/FINC
        EZREFL = 4.0*CH1(2,INDEXR+I)/V1/FINC
        EZTRAN = 4.0*CH2(2,INDEXR+I)/V2/FTRAN
        INCREFL = EZINC + EZREFL
        WRITE (299,FMT=920) CH1(1,INDEXI+I),EZINC
        WRITE (300,FMT=920) CH1(1,INDEXI+I),EZREFL
        WRITE (301,FMT=920) CH1(1,INDEXI+I),EZTRAN
        WRITE (302,FMT=920) CH1(1,INDEXI+I),INCREFL
80    CONTINUE
      CLOSE (299)
      CLOSE (300)

```

```
CLOSE (301)
CLOSE (302)
PRINT *, 'DONE'
900  FORMAT (A)
905  FORMAT (F11.5,F4)
910  FORMAT (F14.7,F14.7)
920  FORMAT (F20.10,F20.10)
END
```

END

2 0 1 0 7 - 9 4

FIN