Multimode Waveguides and Multimode Ring Resonator Filters on Silicon-on-Insulator

by

Bryce Dorin, B.A.Sc.

A thesis submitted to the
Faculty of Graduate and Postdoctoral Affairs
in partial fulfillment of the requirements for the degree of

Master of Applied Science in Electrical Engineering

Department of Electronics

Carleton University

Ottawa, Ontario

April, 2014

©Copyright

Bryce Dorin, 2014

Abstract

In optical communications the rate of information transfer in a single waveguide can be increased by orders of magnitude by modulating signals onto separate channels. Mode-division multiplexing (MDM) is a new and exciting method that can increase waveguide channel numbers while using only a single laser source. Silicon photonics stands to benefit from MDM, however conventional optical devices fabricated in silicon make use of only one mode, and it is typically non-trivial to make these devices compatible with the large number of modes needed for MDM. The purpose of this thesis is to develop MDM on the silicon-on-insulator (SOI) platform through the numerical simulation and experimental demonstration of multimode waveguides, low-crosstalk waveguide bends, multiplexers, and mode-selective filters.

We first investigate the design criteria for the multimode waveguides needed in MDM communication. Multiplexing, demultiplexing, and cross-talk reduction, which are essential processes in MDM systems, are then demonstrated on the SOI platform. Using these structures multimode ring resonator add/drop filters were developed, allowing the ring resonators to be compatible with MDM systems. These novel devices are utilized as mode-selective filters in a two-mode MDM system. The demonstration of multimode filters is an important step to making MDM viable in on-chip communications.

Acknowledgments

First I would like to acknowledge my supervisor, Dr. Winnie N. Ye, who was a constant source of guidance and motivation throughout this degree. My fellow students, Alreza Aleali, Yule Xiong, Ryan Veenkamp, Shuyo Ding, and Yao Sun provided me with invaluable advice and friendship, which I will always be grateful for. I want to thank Juan Heredia and Elena Duran Valdeiglesias for their time and counsel in the design of the optical test station for this project. I would also like to acknowledge CMC Microsystems, as well as John Bock, Jessica Zhang, and Lukas Chrostoski who organized the SiEPIC Passives Workshop 2013, which gave me the knowledge and tools to transform my ideas into results. Finally, I acknowledge the warmth and support from the students and staff in the Department of Electronics at Carleton University, who made my stay here so enjoyable.

Preface

This integrated thesis includes the journal and conference publications which are cited and described below.

B. A. Dorin and W. N. Ye, “Conditions for Single-Mode and Birefringence-Free Ultrasmall SOI Rib Waveguides at 1310 nm,” J. Light. Technol., vol. 31, no. 21, pp. 3420-3424, 2013.

This article has been peer reviewed and published in the IEEE Journal of Lightwave Technology. It is a journal that covers work done in optical guided-wave science, technology, and engineering. The 2012 impact factor for the journal was 2.555. During the peer review process the article received unanimous support. One reviewer stated that “...such work is important for the design of waveguides and devices.” In reference to the IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of Carleton University’s products or services. Internal or personal use of this material is permitted.

B. A. Dorin and W. N. Ye, “Two-mode division multiplexing in a silicon-insulator ring resonator,” *Opt. Exp.*, vol. 22, no. 4, pp. 4547-4558, Feb. 2014.

This article has been peer reviewed and published in Optics Express. Optics Express is an open-access journal that deals with innovations in optical and photonic technology. The 2012 impact factor for the journal was 3.546. Reviewers stated that the work was “...an interesting study of multi-mode ring resonators and [multiplexers]...” and that the “...idea of looking into the multi-mode operation of an [ring resonator] is novel and worth considering, given the potential offered by [mode-division multiplexing] in interconnects applications.”

B. A. Dorin and W. N. Ye, “A Two-Mode Division Multiplexing Filter Demonstrated Using a SOI Ring Resonator,” *Optical Fiber Communication Conference and Exposition and the National Fiber Optic Engineers Conference (OFC/NFOEC)*, San Francisco, California, USA, 9-13 March, 2014.

This paper has been orally presented and will be published in the proceedings of OFC: Optical Fiber Communication Conference 2014. OFC is a high quality conference that brings together leaders in optical communication and networking. The paper was presented in the spatial-division multiplexing section of the conference.

B. A. Dorin and W. N. Ye, “An Add-Drop Ring Resonator Constructed from Multimode Waveguides,” submitted for oral presentation at *11th International Conference on Group IV Photonics*, Paris, France, 27-29 August, 2014.

This paper has been submitted for oral presentation at Group IV Photonics in 2014, one of the leading international conferences focused on silicon photonics.

Table of Contents

Abstract	ii
Acknowledgments	iii
Preface	iv
Table of Contents	vii
List of Tables	x
List of Figures	xi
Nomenclature	xvi
1 Introduction	1
1.1 Silicon Photonics	1
1.2 Mode-Division Multiplexing	4
1.3 Mode Theory	7
1.4 Ring Resonators	10
1.5 Thesis Overview	16
2 Methods	18
2.1 Numerical Simulation	18

2.1.1	The Finite-Difference Method	19
2.1.2	The Film-Mode Matching Method	20
2.1.3	The Finite-Difference Time-Domain Method	22
2.1.4	The Eigenmode Expansion Method	22
2.1.5	Boundary Conditions	24
2.2	Fabrication	25
2.3	Testing	26
3	Determining the Single-Mode Condition	31
3.1	Abstract	32
3.2	Introduction	33
3.3	Method	36
3.4	Results	37
3.4.1	Large Waveguide Heights	38
3.4.2	Small Waveguide Heights	39
3.5	Conclusions	44
4	The Multimode Ring Resonator Design	49
4.1	Abstract	50
4.2	Introduction	51
4.3	Operation of a multimode ring resonator	53
4.4	Multiplexing and demultiplexing	56
4.5	Ring bends	61
4.6	Experimental Results	65
4.7	Conclusion	72
5	Multiplexer, Demultiplexer, and Bezier Curve Characterization	77

5.1	Abstract	78
5.2	Introduction	79
5.3	Optimization of (de)multiplexers and waveguide bends	80
5.4	Ring resonator performance	83
5.5	Conclusions	84
6	An Asymmetrically Coupled Multimode Ring Resonator	87
6.1	Abstract	88
6.2	Introduction	88
6.3	Design and Demonstration	90
6.4	Conclusions	94
7	Conclusions	96
7.1	Future Work	98
	List of References	101
	Appendix A MATLAB code for simulating the ring resonator	106
	Appendix B Lumerical simulation scripts	109
B.1	Bezier Curves	109
B.2	Multiplexers	114
	Appendix C Test Structure Design	120

List of Tables

1	The determined SMCs for all H ranges considered.	45
2	Theoretical and experimental properties of the TE_{00} and TE_{01} modes in the multimode ring resonator. The wavelengths considered range from $1.54 \mu\text{m}$ to $1.56 \mu\text{m}$	67
3	Properties of the TE_{00} and TE_{01} modes in the multimode ring resonator over the wavelength range $1.558 \mu\text{m}$ to $1.568 \mu\text{m}$	93

List of Figures

1	A MDM system that couples light to a chip, modulates and multiplexes it, then propagates it some distance on a multimode waveguide before demultiplexing it.	5
2	An asymmetric direction coupler acting to multiplex the TE_{00} and TE_{01} modes onto a single bus waveguide.	7
3	The slab waveguide cross section; n_s is the refractive index of the substrate, n_1 is the refractive index of the core, and n_0 is the refractive index of the top cladding.	8
4	A block diagram of a ring resonator with through and drop ports. The section labelled C_{12} is the coupler at the input of the ring, and C_{23} is the coupler at the output. Propagation through one half of the ring circumference is described by the matrix P_2	11
5	The transmission peaks and dips at the through port of a ring resonator demonstrating resonance.	12
6	The E_x field component of the (a) symmetric and (b) asymmetric TE modes in a directional coupler used in supermode analysis. Here, we use an example of 930 x 220 nm Si waveguides with a gap of 200 nm.	14
7	A typical 2-D mesh applied over a waveguide structure being simulated.	20

8	The treatment of a rib waveguide in FIMMWAVE [®] using the FMM method. The 1-D modes of each slice ($N_1(y)$, $N_2(y)$, and $N_3(y)$) are solved for independently in this method.	21
9	The transfer of power from mode set 1 to mode set 2 at an interface between two distinct waveguides.	23
10	An example fiber array containing two TE PM fibers and TM PM fibers. Image obtained from <i>www.PLCconnections.com</i>	27
11	The Micro/NanoPhotonics Laboratory's fiber array testing set-up. . .	29
12	A example of the output loss contour plot from the LabView [™] scanning program. One can see that low losses corresponding to the location of a device are located near the center of the scan.	30
13	The waveguide cross section and design parameters used in the single-mode and birefringence simulations.	34
14	The single-mode cut-off data for large SOI heights. The SMCs from Soref <i>et al.</i> , Pogossian <i>et al.</i> , and Chan <i>et al.</i> (extracted from [6], [7], and [13] respectively) are plotted alongside our simulation results. . .	39
15	The single-mode cut-off data and magic widths for small SOI heights. The SMCs from Soref <i>et al.</i> , Pogossian <i>et al.</i> , and Chan <i>et al.</i> (extracted from [6], [7], and [13] respectively) are plotted alongside our simulation results.	41
16	Ratio of optical power confined in the core to that confined in the rib for the second-order TE (HE_{01}) and TM (EH_{01}) modes for $W/H=1$. A large drop is observable for the TM modes between $H = 0.6$ and $0.8 \mu\text{m}$, consistent with the SMC shift seen in Fig. 14 and 15.	42

17	E_y and E_x field components for the second-order TM (EH_{01}) mode with $H = 0.22 \mu\text{m}$, $D = 0.11 \mu\text{m}$, and $W = 1.2 \mu\text{m}$. While the E_y component appears to be well confined, the E_x field clearly radiates into the rib, indicating loss even at this large waveguide width.	43
18	A ring resonator composed of waveguides supporting two TE modes. In this example, the first-order mode (TE_{01}) is resonant and critically coupled to the drop port of the filter, while the fundamental mode (TE_{00}) is off resonance and passes to the through port.	53
19	The common κ^2 value for TE_{00} and TE_{01} in a directional coupler of length L . The directional coupler gap and waveguide widths are varied to demonstrate that some control over the equal κ^2 values can be achieved. During simulations the waveguide height and wavelength used were 220 nm and 1550 nm, respectively.	56
20	The layout and operation of the ADC multiplexer design. Here W_1 and W_2 are the widths of the ADC input and bus waveguides, respectively; t is the thickness offset of the bus waveguide; g is the gap between the two coupler waveguides.	58
21	The coupling efficiency of tapered and uniform asymmetric directional couplers. FDTD and EME simulations are used to plot coupling fraction against (a) device length, and (b) fabrication errors in W_1 and W_2	60
22	Crosstalk between the TE_{00} and TE_{01} modes in a semi-circular waveguide connected to straight waveguides. Beating is noticed in the crosstalk response, which is attributed to self-imaging in the bend. Data was obtained using the EME method.	62

23	The construction of the 90° Bezier curve. In the image the four points defining the curve, as well as the curve itself are displayed.	63
24	The design of adiabatic Bezier curves: (a) shows the shape of the bends for various values of B, and (b) shows the corresponding level of crosstalk in a 90° bend simulated using FDTD. Crosstalk was measured as the ratio of power in the crosstalk channel to power in the signal channel at the output of the bend. These powers were obtained directly from the FDTD results using an overlap integral.	64
25	The multimode ring resonator mask layout used for fabrication. The different components and dimensions of the device are labelled.	66
26	Normalized optical output for the TE ₀₀ and TE ₀₁ modes from (a) the through port, and (b) the drop port. Crosstalk (XT) results are also presented.	71
27	The mask layout for the fabricated ring resonator device. Asymmetric directional couplers serve as the multiplexer and demultiplexers. Bezier curves are utilized to provide minimal crosstalk in the multimode waveguide bends.	79
28	The measured loss in dB for the TE ₀₁ mode as a function of coupling length and waveguide center width; for the (a) uniform, and (b) tapered ADC. [5]	82
29	Experimental results from our ring resonator for the TE ₀₀ and TE ₀₁ modes. The curves indicate that both modes were able to resonate in the structure with little crosstalk (XT).	84
30	An optical microscope image of the two-mode ring resonator filter. . .	91

31	The through and drop port transmittance for TE_{00} and TE_{01} in the multimode ring resonator filter. The extinction ratios for all channels and output ports are labelled.	92
32	The multiplexer and demultiplexer test structure. The input is multiplexed to the TE_{01} mode of the bus waveguide, which is then demultiplexed after a short distance. This allows for the calculation of (de)multiplexer loss.	121
33	The Bezier curve test structure. The TE_{00} mode propagates through 2, 90° Bezier curves. After the curves the output is demultiplexed to determine the magnitude of the TE_{01} mode that was produced during propagation. This allows for the calculation of curve crosstalk.	122

Nomenclature

List of Abbreviations

- IC: *integrated circuit*
- SOI: *silicon-on-insulator*
- WDM: *wavelength-division multiplexing*
- MDM: *mode-division multiplexing*
- MMI: *multimode interferometer*
- ADC: *asymmetrical directional coupler*
- SMC: *single-mode condition*
- BPM: *beam propagation method*
- FDM: *finite-difference method*
- FSR: *free spectral range*
- FMM: *film-mode matching method*
- TE: *transverse electric*
- TM: *transverse magnetic*

- FDTD: *finite-difference time-domain method*
- EME: *eigenmode expansion method*
- PCL: *perfectly conducting layer*
- PML: *perfectly matched layer*
- PM: *polarization maintaining*
- ZBR: *zero birefringence*
- XT: *crosstalk*
- SXR: *signal-to-crosstalk ratio*

List of Symbols

- n_{eff} : *effective refractive index*
- n_s : *substrate refractive index*
- n_1 : *guiding layer refractive index*
- n_0 : *top cladding refractive index*
- E : *electric field*
- k : *wavenumber*
- β : *wavevector*
- H : *magnetic field*
- a : *thickness of the guiding layer*

- ϕ : phase of a mode
- m : integer related to the mode number
- V : V number of a waveguide
- E_{input} : electric field at the input of a ring resonator
- $E_{through}$: electric field at the through port of a ring resonator
- E_{drop} : electric field at the drop port for a ring resonator
- C_{12} : coupling matrix at the through port for a ring resonator
- C_{23} : coupling matrix at the drop port for a ring resonator
- P_2 : propagation matrix for a ring resonator
- λ : wavelength
- L_c : coupling length
- κ : coupling coefficient for a directional coupler
- t : transmission coefficient for a directional coupler
- θ : phase induced in a ring resonator
- α : attenuation in a ring resonator
- n_{group} : group refractive index
- L_{eff} : effective length of the ring resonator
- $N(y)$: the set of modes used in FMM
- W : waveguide width

- D : waveguide etch depth
- H : SOI height
- r : rib height to SOI height ratio
- Δn_{eff} : birefringence
- Γ : confinement factor
- R : ring radius
- P_{in} : power at the input of the ring resonator
- P_{drop} : power at the drop port of the ring resonator
- T : transmission ratio
- Q : quality factor
- g : directional coupler gap
- L_{π} : beat length
- B : Bezier parameter
- L : directional coupler length
- L_{ADC} : ADC length
- t_a : through port transmission coefficient
- t_b : drop port transmission coefficient

Chapter 1

Introduction

1.1 Silicon Photonics

Photonic devices and technology have been making huge leaps in recent years. The improvements include a higher level of sophistication, smaller feature sizes, and higher yield. At the forefront of this technological surge is silicon photonics, which uses single crystal silicon wafers as the platform for fabricating photonic waveguides. Much of the success experienced in silicon photonics is due to the microelectronics industry, which first developed single crystal silicon as a platform for electronic transistors and integrated circuits (ICs). Many of the crucial fabrication steps needed for realizing silicon photonic devices, including wafer preparation, deep-UV lithography and processing were extracted from decades of research done for the microelectronics industry. Silicon-on-insulator (SOI) wafers, another very important and increasingly common platform for fabricating photonic circuits, were first developed for the fabrication of transistors in the mid 1990s. The overlaps in fabrication processes and required materials make it clear that one could use the existing CMOS foundries to develop SOI photonics. The foundry process, where cost is shared by dividing chips amongst many users, is beginning to emerge for silicon photonics just as it had in

the electronics industry. Belgium's IMEC now offers affordable chip space at state of the art fabrication facilities where researchers and companies can fabricate prototype designs.

Monolithically integrating electronics and photonics on the same chip is a sweeping goal within silicon photonics. Electronics and photonics both have inherent advantages and disadvantages, and luckily the two technologies seem to complement each other. While electronics is clearly unmatched in terms of data storage and computation, it lacks a means of low power and high data rate communication. Through the development of fiber optic networks we have seen how photonics can achieve communication rates that are orders of magnitude above electronics with a low power budget. Methods such as wavelength-division multiplexing (WDM) used to reach these limits in fibers have been implemented on SOI at telecommunications wavelengths. A great example is the demonstration of a 5 cm-long silicon wire waveguide supporting 32 channels, each at 40 Gb/s [1]. However, silicon photonics lacks the ability to store data as well as to manage thermal effects. Integrating silicon waveguides onto an electronic microchip to allow dense communication between microprocessor cores, or from processor to memory seems to be a promising method to increase interconnect capacity for the next generation of computing [2]. IMEC does not currently offer integrated electronics in their process. However, the foundry process at OpSIS allows researchers to develop their integrated electronic and photonic systems, and makes devices such as modulators and photodetectors available on chip.

While the silicon platform seems to meet the needs of both electronics and photonics very well, it does have one inherent drawback as it cannot lase or provide photodetection at telecommunications wavelengths. Unlike III-V semiconductors used for many diode lasers, silicon has an indirect bandgap, and therefore cannot be used for an on-chip laser. There are typically only two options for coupling light into the

on-chip waveguides: use an off-chip source which would have to be carefully aligned to the chip, or integrate a separate material capable of lasing on the silicon chip. Unfortunately lattice mismatch makes it impossible to grow III-V semiconductors on silicon without adding defects to the crystal. Germanium is a semiconductor that can be grown on silicon reasonably well, however it too has an indirect bandgap. It is believed that by straining the germanium crystal and doping it heavily, the bandgap can be made direct. In [3], the authors demonstrate gain in the 1600 - 1608 nm wavelength range at room temperature, which is close to the 1550 nm wavelength commonly used in silicon photonics. Bonding III-V semiconductors to the silicon chip, rather than growing them, is another commonly used method of integrating a lasing material with SOI waveguides [4].

Thus far in silicon photonics the focus has been on single-mode waveguides. These waveguides offer a great deal of advantages as they eliminate modal dispersion and are less lossy than multimode waveguides in certain structures. As well, designing devices for more than one mode can increase the complexity enormously. However, mode-division multiplexing (MDM) presents an advantage to using multimode waveguides. In MDM systems two or more guided modes are multiplexed onto a large bus waveguide, with each mode acting as an individual data channel. Unlike WDM, this technology can theoretically increase channel numbers almost without limits while utilizing only one wavelength of light. This is a tremendous benefit of MDM as coupling light onto the chip is already a difficult and expensive part of a SOI photonic circuit. A single laser could be used to simultaneously excite multiple channels in one multimode waveguide. For this technology to be implemented a good understanding and complete control over the modes in a waveguide is needed. Also, as in any multiple-input multiple-output system, a means of multiplexing, demultiplexing and filtering is needed before complex systems can be constructed. In this thesis I develop

a method for determining the single-mode cut-off in a waveguide, which corresponds to the waveguide dimensions where two modes begin to propagate rather than just one. Understanding this transition between one and two-mode waveguides is important to designing a two-mode division multiplexing system where all intended modes propagate without loss. Using a two-mode waveguide I then design and demonstrate mode multiplexing, demultiplexing, and filtering. Special waveguide bends have also been designed and implemented to reduce inter-modal crosstalk. Simulation and experimental results from these devices are presented. This work is fundamental to the implementation of functional and complex MDM networks for on-chip communications.

1.2 Mode-Division Multiplexing

The investigations into MDM began in optical fibers where they aimed to continue the growth in data transfer capacity. Previous growth was in large part due to developments in WDM, which has now reached a plateau due to a variety of limitations [5]. The demand for internet bandwidth continues to grow exponentially as the number of internet users increases and cloud computing becomes more popular. To continue the growth in fiber optic capacity to match the demand, a new dimension must be exploited to increase the channel numbers on a single fiber. Multimode fibers are capable of accessing the spatial dimension, where channels can be modulated onto not just different wavelengths, but different modes of the fiber as well. In addition, MDM is predicted to have higher energy savings compared to WDM [6]. This has been demonstrated in a few-mode fiber over 96 km [7].

In SOI waveguide circuits a MDM system has the same inherent advantages as it does in fibers, and would have the general architecture proposed by [8]. This

architecture is shown in Figure 1. A single laser source is coupled to the chip, which is then split into N waveguides via a power splitter. In each waveguide the light is modulated independently by integrated electro-optic modulators. Then the light is multiplexed onto the modes of a large bus waveguide supporting N modes, and propagated over a distance likely on the order of cm. After propagation the channels are demultiplexed into separate waveguides once again, and converted back to an electronic signal by N photodetectors.

Figure 1: A MDM system that couples light to a chip, modulates and multiplexes it, then propagates it some distance on a multimode waveguide before demultiplexing it.

The $1 \times N$ splitter, modulators, and photodetectors shown in the Figure 1 are silicon photonic devices that have been developed for many years, and have many applications outside of MDM. For example, the simplest form of power splitter is the multimode interferometer (MMI), which uses interference in a very large slab waveguide to split power between outputs [9]. Silicon rib modulators have been studied extensively in silicon waveguides ever since the discovery that the presence of free carriers in silicon causes changes to the refractive index [10, 11]. Finally, germanium photodetectors have been implemented in silicon and exhibit excellent performance [12–14].

Much of the work done in MDM on SOI has focused on the design of the multiplexing and demultiplexing components. One of the first studies used a series of MMIs [15]. This device was large and very complex, rendering it impractical. Other MMI (de)multiplexers have been proposed [16], as well as similar devices using Y-junctions [17], yet both concepts suffer from the inability to multiplex more than two modes. Grating-assisted couplers provide an interesting alternative for mode multiplexing as one is not limited to two-mode systems [18]. However, the grating couplers are inherently very wavelength dependant, rendering them unsuitable for many applications. Asymmetric directional couplers (ADCs) are perhaps conceptually the simplest design, but in many ways are the most robust [8, 19]. A schematic of an ADC is shown in Fig 2. The operation of ADCs is to match the effective refractive index (n_{eff}) of the fundamental mode in the input waveguide to the desired bus waveguide mode, such that $n_{eff}^{input} = n_{eff}^{bus}$. According to directional coupler principles if this is achieved then efficient coupling can occur between the two waveguides by bringing them into close proximity with each other [20]. As well, ADCs can be cascaded to multiplex any number of modes, and are not exceedingly wavelength dependant. Four channel multiplexing and demultiplexing using ADCs has recently been demonstrated with 12-13 dB of insertion loss and -23 dB of crosstalk over a 20 nm wavelength range [8].

Figure 2: An asymmetric direction coupler acting to multiplex the TE_{00} and TE_{01} modes onto a single bus waveguide.

1.3 Mode Theory

This section uses one dimensional waveguide theory to provide an analytical distinction between single-mode and multimode waveguides. The design of waveguides for MDM systems requires a thorough understanding of when modes begin to propagate in the system. For N desired channels one will want to design their waveguide at dimensions to support N guided modes, and ensure the modes are not leaky. As well, it is undesirable to design a waveguide that supports more modes than is required as the waveguide will then be wider, leading to larger device footprints. Incidental coupling to these extra modes could also be an additional source of loss. In the slab waveguide an analytical equation is known for the number of modes supported by the waveguide, which is expressed by the variable V [21]. A cross section of the slab waveguide is shown in Figure 3, where n_s is the refractive index of the substrate, n_1 is the refractive index of the silicon core, and n_0 is the refractive index of the top cladding.

The derivation of V for transverse-electric (TE) modes begins with the wave equation for E_y , the electric field in the y-direction, shown in (1). Here k is the wavenumber

Figure 3: The slab waveguide cross section; n_s is the refractive index of the substrate, n_1 is the refractive index of the core, and n_0 is the refractive index of the top cladding.

defined as $k = 2\pi/\lambda$, where λ is the wavelength of light. The variable n is the refractive index of the medium where the electric field propagates. The wavevector of the mode, β , is equivalent to $2\pi n_{eff}/\lambda$.

$$\frac{d^2 E_y}{dx^2} + (k^2 n^2 - \beta^2) E_y = 0 \quad (1)$$

The general solution for E_y in each of the three layers is presented in (2). By taking the derivative of E_y with respect to x , one can find the general solutions for H_z as well.

$$E_y = \begin{cases} A \cos(\kappa a - \phi) e^{-\sigma(x-a)} & x > a \\ A \cos(\kappa a - \phi) & -a \leq x \leq a \\ A \cos(\kappa a + \phi) e^{\xi(x-a)} & x < -a \end{cases} \quad (2)$$

In the previous expression A is a variable related to the magnitude of the electric fields, and κ , σ and ξ are defined in (3), (4) and (5), respectively.

$$\kappa = \sqrt{k^2 n_1^2 - \beta^2} \quad (3)$$

$$\sigma = \sqrt{\beta^2 - k^2 n_0^2} \quad (4)$$

$$\xi = \sqrt{\beta^2 - k^2 n_s^2} \quad (5)$$

Boundary conditions for this problem dictate that E_y and H_z must be continuous at the boundary. Imposing these conditions on (2) yields the two transcendental equations of (6) and (7) [21].

$$\kappa a = \frac{m\pi}{2} + \frac{1}{2} \tan^{-1} \left(\frac{\xi}{\kappa} \right) + \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\kappa} \right) \quad (6)$$

$$\phi = \frac{m\pi}{2} + \frac{1}{2} \tan^{-1} \left(\frac{\xi}{\kappa} \right) - \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\kappa} \right) \quad (7)$$

for $m = 0, 1, 2, \dots$

Assuming that the refractive index of the substrate is greater than or equal to the index of the top cladding (which is often the case for SOI), we can choose the cut-off point to be where the effective index of the mode is equal to n_s . One can then define the V number which is related to this cut-off point, as shown in (8).

$$V = ka(n_1^2 - n_s^2)^{1/2} \quad (8)$$

The variable m used in the transcendental equations above is the measure of how many guided modes the waveguide supports. For example, if $m = 3$ to satisfy (6), then the waveguide would support 4 modes. A more useful way to approach the problem from a MDM design perspective is to define the number of channels desired,

therefore defining m for the waveguide. The V number can then be calculated using the inequality in (9).

$$\frac{m\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{n_s^2 - n_0^2}{n_1^2 - n_s^2}\right) < V < \frac{(m+1)\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{n_s^2 - n_0^2}{n_1^2 - n_s^2}\right) \quad (9)$$

While this derivation provides good insight into the nature of multimode waveguides and their single-mode condition (SMC), it is not useful for real world applications as all waveguides must be 2D in nature to completely confine the light. There exists no analytical equation for modes in 2D waveguides, therefore other methods must be employed to determine the number of guided modes. Unfortunately these methods often cannot return a V number to easily define the cutoff point, so other criteria have been adapted. Soref *et. al.* was one of the first to explore this problem and used an approximation to the V number for rib waveguides [22]. The effective-index method used in [23] was the next attempt to refine the equation. Other attempts made use of numerical methods such as the beam propagation method (BPM) [24] and the finite difference method (FDM) [25] to determine the SMC. These methods are explored in more detail in Chapters 2 and 3. Ultimately there seems to be no condition that can determine the cut-off of higher-order modes for all waveguide dimensions and wavelengths, therefore work is continually done to determine the SMC on a case-by-case basis when applications call for it.

1.4 Ring Resonators

The ring resonator switches and filters light in a highly wavelength dependant manner. In the two-port ring resonator this means that the output intensity spikes at specific wavelengths corresponding to resonance. This property has been exploited

to create useful devices such as the WDM multiplexer, optical switch, and the on-chip modulator [26–28]. The basic structure of a two-port ring resonator is shown in Figure 4.

Figure 4: A block diagram of a ring resonator with through and drop ports. The section labelled C_{12} is the coupler at the input of the ring, and C_{23} is the coupler at the output. Propagation through one half of the ring circumference is described by the matrix P_2 .

In the ring resonator there are three waveguide electric fields that are of interest, those at the input port (E_{input}), the through port ($E_{through}$), and the drop port (E_{drop}). The resonance condition determines when light is coupled to the drop port ($E_{drop} \simeq E_{input}$), and is entirely dependant on the length of the ring and the β of the mode. When the phase accumulation of the light in the ring is a multiple of 2π , that wavelength is resonant. When off resonance there will be no constructive interference in the coupler between the input waveguide and the ring, therefore little coupling will occur here and light will pass to the through port. During resonance the light from the input will constructively interfere within the ring, causing intensity to build up. The light will then ultimately be coupled to the drop port. To demonstrate this

behaviour transmission at the through port of a ring resonator is shown in Fig. 5 over a wavelength range where resonance dips can be observed. Numerical simulations are usually very difficult to perform on ring resonators as the devices are typically quite large and the resonance in the ring requires long simulation times. Matrix methods provide a good alternative to complete numerical simulations as they are extremely quick and simple to run, and can be calibrated with data from numerical solvers to reach impressive accuracy [29].

Figure 5: The transmission peaks and dips at the through port of a ring resonator demonstrating resonance.

Three matrices can be used to describe the device shown in Figure 4. They are defined as C_{12} , the coupling between input and ring waveguides, C_{23} , the coupling between ring and drop port waveguides, and P_2 , the phase accumulation and attenuation of the electric field in the ring. These matrices assume that either the ring

resonator is single-mode, or that the modes of a multimode ring resonator are completely independent of each other. The complete operation of the ring for one mode can be found through the multiplication shown in (10).

$$\begin{bmatrix} E_{input} \\ E_{through} \end{bmatrix} = C_{12}P_2C_{23} \begin{bmatrix} 0 \\ E_{drop} \end{bmatrix} \quad (10)$$

Each coupler in the system is defined by two parameters, κ and t , which correspond to the fraction of electric field that is coupled, and the fraction that is transmitted, respectively. In a lossless coupler these two values are related such that $\kappa^2 + t^2 = 1$. In this application the coupler used is a simple directional coupler, which exhibits coupling between two waveguides simply by bringing them into close proximity such that their evanescent fields begin to overlap. The values κ and t for the directional coupler can be calculated accurately from numerical models using supermodes analysis [30]. Supermodes analysis requires solving for the first two modes in a system of two weakly coupled waveguides. One of these modes is symmetric, and the other asymmetric, as shown in Figure 6. As these modes propagate along there will be certain lengths where light constructively interferes in the output waveguide and destructively interferes in the input, resulting in complete coupling. By solving for the wavelength dependant effective indices of these modes, called $n_s(\lambda)$ and $n_a(\lambda)$, respectively, the wavelength dependant coupling length (L_c) for the directional coupler can be determined using (11). The coupling coefficients can then be found using (12) and (13), where L is the length of the directional coupler.

$$L_c(\lambda) = \frac{\lambda}{2(n_s(\lambda) - n_a(\lambda))} \quad (11)$$

Figure 6: The E_x field component of the (a) symmetric and (b) asymmetric TE modes in a directional coupler used in supermode analysis. Here, we use an example of 930 x 220 nm Si waveguides with a gap of 200 nm.

$$\kappa = \sin\left(\frac{\pi}{2} \frac{L}{L_c(\lambda)}\right) \quad (12)$$

$$t = \cos\left(\frac{\pi}{2} \frac{L}{L_c(\lambda)}\right) \quad (13)$$

Using these values the coupling matrices for the ring can be defined as (14) and (15). The propagation matrix is defined in (16), where θ is the phase induced, and α is the fraction of light intensity remaining after one revolution of the ring. The variable α is not possible to predict through simulations. The phase change θ however can be found quite easily by simply finding β for the mode and multiplying it by the circumference of the ring.

$$C_{12} = \frac{1}{\kappa_{12}} \begin{bmatrix} -t_{12} & 1 \\ -1 & t_{12} \end{bmatrix} \quad (14)$$

$$C_{23} = \frac{1}{\kappa_{23}} \begin{bmatrix} t_{23} & -1 \\ 1 & -t_{23} \end{bmatrix} \quad (15)$$

$$P_2 = \begin{bmatrix} 0 & \alpha^{1/2} \exp(-j\theta/2) \\ \alpha^{-1/2} \exp(j\theta/2) & 0 \end{bmatrix} \quad (16)$$

After solving for all necessary constants (10) can be solved for E_{drop} , which is usually an interesting port to consider. Assuming that E_{input} has been normalized to unity, the power at the drop port (P_{drop}) can be expressed as the equation shown in (17), this quantity is also known as the extinction ratio. In this equation it becomes clear that the mode is resonant in the ring if θ is a multiple of 2π . The range in wavelengths between resonance conditions is called the free spectral range (FSR). This parameter can be written as $\text{FSR} = \lambda^2/n_{group}L_{eff}$, where n_{group} is the group index of the mode, L_{eff} is the effective length of the ring, and λ is the center wavelength. The attenuation α also plays a large role in equation (17), as it ultimately determines the maximum power that can be coupled to the drop port during resonance. When $\alpha = 1$ a condition known as critical coupling is reached where the power entering the coupler C_{12} from the input is the same as the power entering from the ring, allowing for complete constructive interference. Under this condition, $E_{drop} = E_{input}$ at resonance. In real world situations $\alpha < 1$, yet critical coupling can still be achieved by making the two couplers asymmetric, such that $t_{12} = \alpha t_{23}$ [27]. The full matrix simulation of the ring resonator can be found in Appendix A.

$$P_{drop} = \frac{-\alpha \kappa_{12}^2 \kappa_{23}^2}{2\alpha t_{12} t_{23} \cos(\theta) - (\alpha^2 t_{12}^2 t_{23}^2 + 1)} \quad (17)$$

1.5 Thesis Overview

The goal of this thesis is to study and understand multimode waveguides fabricated on SOI, and to develop the devices necessary for the realization of MDM in silicon photonics. The methods employed in the design, fabrication, and testing of the devices are summarized in Chapter 2. Here the specific roles of different design software, fabrication facilities, and testing equipment used in the process is explained.

In Chapter 3 a method for determining the SMC of a waveguide using numerical methods is proposed. Determining the SMC dictates the minimum width one can use for a two-mode waveguide. This is an important step in making two-mode division multiplexing systems as one must design waveguides large enough to support these modes without loss, but no larger than is necessary to minimize space on the wafer. Using the proposed method the SMC is determined for a wide range of rib waveguide geometries at the 1310 nm wavelength. The SMC for this wavelength range has not been studied in depth before, which presented a good opportunity to demonstrate the method.

Chapter 4 presents the design and initial results from our two-mode division multiplexing ring resonator. In-depth simulations were used to optimize structures such as the ring bends and multiplexers for the best loss and crosstalk performance. A method for achieving equal quality factors for each mode in the ring using long coupling lengths is proposed and demonstrated. The results from the fabricated ring resonator are presented, where the device is proven to work as hypothesized.

Detailed experimental analysis of the ring bends and multiplexers is presented in Chapter 5. These experimental measurements build on the numerical simulations presented in Chapter 4. The operation of the full ring resonator is compared to the results from the ring bends and multiplexers, allowing us to determine the largest

contributors of loss and crosstalk within the structure.

An alternative multimode ring resonator design is developed in Chapter 6. In this device we remove the long coupling regions used in the ring resonator presented in Chapters 4 and 5. Instead we used short coupling regions for a more versatile and space-efficient design. To ensure the extinction ratio of each mode in the ring was sufficient, coupling at the drop port was reduced compared to coupling at the through port, making the device asymmetrical. The fabrication of this distinct design is presented.

Chapter 7 presents conclusions on the work, and suggests future work in the pursuit of MDM on SOI.

Chapter 2

Methods

2.1 Numerical Simulation

Numerical methods used for characterizing devices have become essential to optical design. In the past numerical simulations were exclusive to those who had access to powerful computing servers, but recently computing power has reached the point where rigorous solvers can be run on personal computers. Simulation algorithms are all founded on numerical approximations to Maxwell's equations. The fundamental nature of Maxwell's equations to all electromagnetic problems makes the results from rigorous simulations remarkably accurate in most situations. A variety of numerical methods have been developed for simulating optical devices that approximate Maxwell's equations in slightly different ways. Each method has unique advantages and disadvantages. Often a combination of two or three methods is necessary for designing a device and ensuring its accuracy. The finite-difference, film-mode matching, finite-difference time-domain, and eigenmode expansion method were all used in this thesis project. In all simulations convergence was ensured by doubling the mesh density until negligible changes in results were observed.

2.1.1 The Finite-Difference Method

The finite-difference method (FDM) is available through the commercial software packages Lumerical MODE Solutions[®] and FIMMWAVE[®]. This method allows one to find the full-vectorial solutions to an arbitrary waveguide structure, meaning that hybrid modes can be found. The output includes the β for the mode and the distribution of optical power. This is very useful in the analysis of modes in a single waveguide (both guided and lossy), and supermodes in a directional coupler.

A problem is initially defined by drawing the waveguide structure, with the dielectric permittivity (ε) for each material defined. A 2-D mesh is then defined by specifying a grid over the computational domain. An example of this mesh is shown in Fig. 7. In the case of FIMMWAVE, this mesh must be defined by the user. Lumerical uses a conformal mesh generated by the software, which adjusts the spacing between grid points [31]. This can increase the density of grid points at important boundaries while decreasing the density away from the mode power. The overall effect is a higher accuracy for less computational time. Boundary conditions can be defined in both solvers as electric or magnetic walls.

Using the mesh of ε values, Maxwell's curl equations are approximated by finite-difference formulations. These equations are then written into matrix form. By organizing the equations into a linear system, one can solve for the eigenvalues (or modes) of the problem. Once the transverse $\mathbf{E}$ or $\mathbf{H}$ fields are found, all other field components are easily solved for, producing the final mode profile. While FDM is very robust, it can often be unnecessarily computationally intensive in the case of simple waveguide geometries.

Figure 7: A typical 2-D mesh applied over a waveguide structure being simulated.

2.1.2 The Film-Mode Matching Method

The film-mode matching method (FMM) is available only in the FIMMWAVE[®] software package. Similar to FDM, FMM also aims to provide accurate mode propagation constants and profiles. The difference between the two methods is that FMM does not use a 2-D mesh over the computational domain. Instead, the waveguiding structure is broken up into a series of 1-D slices. A typical rib waveguide and its corresponding slices is shown in Figure 8.

For each 1-D slice the exact transverse-electric (TE) and transverse-magnetic (TM) modes can be solved for analytically, as shown in Section 1.3. These modes are described as $N(y)$. A β for the 2-D mode is then guessed, and used to form the 1-D modes in every slice. The overlap of the modes at the interface between slices is formed into a matrix. The method can now define an error function based on the continuity of the transverse $\mathbf{E}$ and $\mathbf{H}$ fields at the interface, and by the satisfaction of boundary conditions (electric or magnetic walls). By varying the β , this error

Figure 8: The treatment of a rib waveguide in FIMMWAVE® using the FMM method. The 1-D modes of each slice ($N_1(y)$, $N_2(y)$, and $N_3(y)$) are solved for independently in this method.

function can be minimized. The resolution of the solver is determined by how many modes are solved for in each slice. The more modes that must be matched at the interface means more computational time but a higher degree of accuracy.

The advantages of FMM over FDM are most drastic in the case of waveguides with highly homogeneous regions (few slices) and well defined interfaces [32]. This is typically the case for rib and wire waveguides, making it an efficient simulation tool in silicon photonics. Under these conditions the computational time for FMM is much less for a similar accuracy to FDM as the solution is analytic along one axis. However, the method is less accurate when waveguide structures have slanted interfaces as it introduces artificial corners into the waveguide, which is undesirable. As well, the eigenvalue problem in FMM is non-linear, unlike the linear problem in FDM, which is much slower to solve and makes the method less efficient when many modes must be found.

2.1.3 The Finite-Difference Time-Domain Method

The finite-difference time-domain method (FDTD) is a popular numerical method used to simulate the propagation of light through any 2-D or 3-D medium. It works by breaking Maxwell's time-domain curl equations into expressions for all six field components (E_x , E_y , E_z , H_x , H_y and H_z). The derivatives in the curl equations are then evaluated using finite-difference approximations on a 3-D mesh similar to the one presented in Fig. 7. A special mesh known as the Yee scheme is used to define the field components throughout the computational domain [33]. This method is available in Lumerical FDTD Solutions[®], where they offer a conformal Yee mesh that can increase the accuracy of the method at important boundaries.

FDTD does not require any preprocessing of Maxwell's equations, and propagates all the field components within the medium. For this reason it is an incredibly easy to use and versatile method. With no additional complexity or computation time it can handle complex shapes, many boundaries, inhomogeneous materials, anisotropy, and non-linear materials. The price of this convenience is computational time. At each time step the method must compute new field components at every point in the mesh. For large simulations, those requiring a fine mesh, or some that need a long propagation time, this creates a large numerical expense in terms of memory requirements and CPU run time. As a result FDTD is only practical on modern computers, and often large computing servers are needed to run simulations in any reasonable time.

2.1.4 The Eigenmode Expansion Method

The eigenmode expansion method (EME) presents a considerably cheaper alternative to FDTD when propagating light along one axis through long, uniform structures.

This method is available in FIMMPROP® [34]. The method begins by identifying the different waveguide sections in the structure. For every section a basis set of modes is computed, which is defined by the user. The mode solver can be selected based on what is appropriate for each waveguide (FDM or FMM). Once the basis set of modes is computed and an excitation is applied, the software can propagate the light through that section analytically with essentially no computing power. At the interfaces between different waveguide sections, each with a distinct mode basis set, the algorithm must compute the transfer of power from the modes in the first section to the modes of the second section. An example of this power transfer at an interface between a wide waveguide with mode set 1 and a narrow waveguide with mode set 2 is shown in Fig. 9. This process is repeated until light reaches the end of the structure. The MMI is an example of a device with long and uniform waveguiding sections, and is one that can be modelled easily using EME.

Figure 9: The transfer of power from mode set 1 to mode set 2 at an interface between two distinct waveguides.

There are many advantages to this method. For example if one wanted to scan the length of a MMI to determine the best coupling, no additional numerical analysis would be needed after all basis sets had been calculated. Another advantage of EME over FDTD is the power in each optical mode is already determined by the solver. This information is often important in optical design. In FDTD, post-processing must be applied to the results to retrieve this information. This can be done by solving for

the mode profile of interest and performing an overlap integral of the mode profile with the output electric field from the FDTD simulation.

When one is studying small devices with many different sections, the advantages of EME begin to disappear. A large number of sections means that many basis sets must be computed and stored in memory, adding to computational requirements. As well, structures that do not contain a uniform set of modes, such as tapers and S-curves, must be broken up into many sections where modes are solved for. In many cases designers can limit themselves to looking at only the few guided modes in a waveguide, and disregard the radiation modes to improve the efficiency of EME. However, if radiation modes are important, often > 50 modes may need to be computed at each step to ensure no important modes are missed. Finally, the FDTD method is one that does not require the operator to make many important optimizations of the method to achieve accurate results. EME is very different in this respect as a user needs to ensure that each section of a device has a basis set of modes that is accurate and large enough to effectively model the device.

2.1.5 Boundary Conditions

Boundary conditions in numerical simulations are needed to specify the electromagnetic field amplitude along the boundary of the computational domain. These are essential for convergence in mode-solving methods. Boundary conditions are also important in propagation simulations to limit the extent of the computation domain that must be simulated. Common boundary conditions include perfectly conducting layers (PCLs). These come in two forms, perfect electrical conductors and perfect magnetic conductors. In the former all tangential electric fields are set to zero along the boundary, and in the later all tangential magnetic fields are set to zero. In most mode-solvers the boundary conditions are placed sufficiently far away from the center

of the mode such that they have a negligible effect on the solutions. However in propagation simulations this is not always possible, and non-physical reflections off PCLs can confuse simulation results. Therefore, absorbing boundary conditions are often used in place of PCLs for FDTD simulations.

The perfectly matched layer (PML) is an absorbing boundary condition that can be applied to electromagnetic problems. These are essential to FDTD simulations to avoid reflections off the electric and magnetic walls at the boundaries of the computation domain that would drastically confuse the results of the simulation. PMLs can also be applied to the mode solvers to determine the loss of radiation modes into the cladding of a waveguide. This information is important when studying the cut-off of higher order modes in a waveguide.

Physical absorbing materials are defined by an electric and magnetic conductivity, which attenuate the light entering the medium. However, there does not exist a real material that can completely absorb light at a boundary with no reflection at any incident angle. Therefore, the PMLs described in [35] add a non-physical quality to the absorbing material to eliminate reflection at any angle. In the PML the H_z field is split into two components, H_{zx} and H_{zy} , each with its own magnetic conductivity. By optimizing the magnitudes of the conductivities, and the thickness of the PML layer, reflection can be nearly eliminated at all angles.

2.2 Fabrication

Initial designs were fabricated through an electron beam lithography run organized by the University of British Columbia and CMC Microsystems. Design rule decks for Mentor Graphics were provided by the group, allowing the participants to prepare their mask. Two etches on 220 nm thick SOI were available in the fabrication run. A

full etch allowed one to fabricate typical wire waveguides, and a 70 nm etch allowed one to fabricate grating couplers. The minimum feature size allowed through this process was under 100 nm. Grating couplers for coupling light to and from the chip were designed at the University of British Columbia and were supplied with the design rule deck. A detailed study of the grating coupler performance is provided in [36]. The layouts from each designer were compiled into one file and sent to the University of Washington Microfabrication/Nanotechnology User Facility, where they were fabricated using electron beam lithography. CMC Microsystems helped subsidize the cost of this fabrication run.

A second fabrication run was done using the IMEC foundry process through CMC Microsystems. This run used deep-UV lithography to write the patterns on 220 nm SOI. The same full and 70 nm etches were available. The minimum feature size in this process was set to 150 nm. The Mentor Graphics rule deck was supplied by IMEC through CMC Microsystems. The same grating couplers described in [36] were used for coupling light to and from the chip.

2.3 Testing

To avoid dicing the chip, automated testing was done at the University of British Columbia for the electron beam run. Light was supplied and collected from the grating couplers using a fiber array supplied by PLC Connections. The array holds polarization maintaining (PM) fibers oriented to supply/collect either TE or TM light. The fibers are positioned in a silica casing 127 μm apart. A cross section of the array is shown in Figure 10. The array used had 3 TE fibers, of which one was used as an input, and 2 were used as outputs. The use of the fiber array simplifies testing as alignment only has to be performed once per device, rather than once for every

input and output fiber. In the layout of the electron beam mask all grating couplers were oriented in the same direction, with input and output grating couplers spaced $127\ \mu\text{m}$ apart. This allowed for the quick automated testing of the chip.

Figure 10: An example fiber array containing two TE PM fibers and TM PM fibers. Image obtained from *www.PLCconnections.com*.

We constructed a testing set-up utilizing the same fiber array at Carleton University in the Micro/NanoPhotonics Lab. The device under test was placed on a piezoelectric stage with x, y and z control. Manual stages also allowed for tip, tilt and rotation of the chip. A separate stage mounted the fiber array, which was held in place with a fiber array holder. A manual goniometer angled the fiber array at 10.8° to the vertical, which allowed for the designed 20° coupling to the grating couplers. Manual x, y, and z alignment was also available on the fiber array stage. An optical microscope and fiber light source were used for rough alignment of the fiber array to the devices. An image of the optical testing set-up is shown in Figure 11. A Luna Optical Vector AnalyzerTM with polarization control performed the wavelength scans on the device.

Precise alignment to individual devices was achieved using a scan which measured the transmittance over an array of points on the chip. This scan was automated using LabViewTM. Before starting the scan, the chip would be placed on the stage

and manually aligned to the fiber array using the rotation and tip/tilt stages. Next, the manual x, y, and z stages would move the fiber array into a position approximately above the grating couplers of interest using the microscope. The operator can then specify the number of steps and step size for the scan. At the beginning of each step the program would communicate with the piezoelectric stage drivers to move them to the next position. The program then opens a remote interface with the laser module, telling it to perform a wavelength scan over a range and center wavelength specified by the operator before the scan is started. From the string of transmittance values the laser returns the lowest loss point is stored in a 2D array corresponding to that point on the chip. Each step takes approximately 5 s for the program to complete. After the scan is completed a contour plot of the loss at each point of the scan range is filled with data from the scan, from which it is easy to determine the position of one or more devices. An example of a contour plot from a fine scan over a device is shown in Figure 12, where one can see that the lowest loss, and therefore the location of the device, is near the center of the scan range.

Figure 11: The Micro/NanoPhotonics Laboratory's fiber array testing set-up.

Figure 12: A example of the output loss contour plot from the LabViewTM scanning program. One can see that low losses corresponding to the location of a device are located near the center of the scan.

Chapter 3

Determining the Single-Mode Condition

The following article, *Conditions for Single-Mode and Birefringence-Free Ultra-Small SOI Rib Waveguides at 1310 nm*, is a summary of our proposed technique for determining the single-mode to multimode transition in SOI waveguides as the waveguide width is increased. This technique is utilized in later work to determine when higher-order modes become non-lossy. In this work the technique is applied to rib waveguides at the 1310 nm wavelength. This was chosen as it is an interesting and relatively unstudied realm of waveguide design. The techniques used in this chapter, as well as the trends we found, also apply to the 1550 nm wavelength used in later chapters.

Bryce Dorin performed all simulations, analysis of the data, and wrote the article. Reprinted, with permission from Bryce A. Dorin and Winnie N. Ye, *Conditions for Single-Mode and Birefringence-Free Ultra-Small SOI Rib Waveguides at 1310 nm*, Journal of Lightwave Technology, volume 31, number 21, pages 3420-3424, in 2013.

Conditions for Single-Mode and Birefringence-Free Ultra-Small SOI Rib Waveguides at 1310 nm

Bryce A. Dorin and Winnie N. Ye

Department of Electronics, Carleton University

1125 Colonel By Drive

Ottawa, Ontario, Canada K1S 5B6

[B. A. Dorin and W. N. Ye, “Conditions for Single-Mode and Birefringence-Free Ultrasmall SOI Rib Waveguides at 1310 nm,” J. Light. Technol., vol. 31, no. 21, pp. 3420-3424, 2013.]

©IEEE

3.1 Abstract

The design of silicon-on-insulator (SOI) rib waveguide structures requires careful selection of dimensions for single-mode and birefringence-free operation, which is often necessary in optical devices. At large SOI heights, the single-mode condition (SMC) is well understood. Here, we present a SMC study extending to ultra-small waveguides, from slab heights on the order of a wavelength down to 220 nm at the 1310

nm wavelength. In addition, the condition for achieving zero-birefringence (ZBR) is presented alongside the SMC for a complete design guideline when both conditions must be met simultaneously. Simulation results show that at small SOI heights (< 800 nm), previous SMCs no longer apply, and birefringence-free operation becomes difficult to achieve. New expressions are proposed that define the single-mode cut-off condition at these dimensions.

3.2 Introduction

Silicon is an attractive platform for achieving photonic systems on chip as the thermal, optical, and electrical properties of the material are all known to be excellent. Silicon-on-insulator (SOI) devices have been growing in complexity while improving performance as nanometer-scale dimensions are realized by advanced fabrication technology. Most SOI waveguides are designed for single-mode and polarization independent operation. Exceptions to this include mode-division multiplexing (MDM) systems, where multimode waveguides are utilized to increase channel numbers [1–3]. Also, it has been demonstrated that polarization independence can be achieved through polarization diversity technology which neglects birefringence effects by separating the two polarizations and handling them independently [4, 5]. However, simultaneous fulfilment of single-mode and birefringence-free operation still remains crucial to many waveguide applications.

Several previous studies have focused on the single-mode and zero-birefringence (ZBR) design criteria in the rib waveguide [6–15]. Work first done by Soref *et al.* [6], and then later modified by Pogossian *et al.* [7], produced the single-mode condition (SMC) shown in (18) for a large rib waveguide with shallow etches at a wavelength of 1300 nm.

Figure 13: The waveguide cross section and design parameters used in the single-mode and birefringence simulations.

$$W = \alpha H + \frac{rH}{\sqrt{1-r^2}}$$

$$\text{for } 0.5 \leq r < 1 \tag{18}$$

$$\text{where } r = \frac{H-D}{H}$$

Parameters width (W), height (H), and etch depth (D) are illustrated in Fig. 13. In [6], $\alpha = 0.3$ was proposed and then confirmed by beam propagation method (BPM) simulations. The effective-index method used in [7] imposed more stringent constraints on the single-mode behaviour, and determined $\alpha = 0$.

Lousteau *et al.* [9] used the finite difference method (FDM) with perfectly matched layers (PMLs) on the boundaries to predict the loss of higher-order modes in large rib waveguides at 1523 nm and 1150 nm wavelengths. This study concluded that a simple relation is not sufficient to determine the SMC of a large rib waveguide, as the loss of higher-order modes did not follow a consistent trend with varying waveguide dimensions. Instead, the authors recommend that each design should be modelled on a case-by-case basis to ensure single-mode operation.

To calculate the necessary waveguide parameters for ZBR, the film mode matching

(FMM) method has been used [12]. Here, birefringence (Δn_{eff}) is defined as the difference in effective refractive indices between the fundamental quasi-transverse-electric (TE) and quasi-transverse-magnetic (TM) modes, that is $\Delta n_{\text{eff}} = n_{\text{eff}}^{\text{TE}} - n_{\text{eff}}^{\text{TM}}$. Ye *et al.* later demonstrated stress-engineering as a viable mechanism for polarization control and achieving ZBR in rib waveguides [16].

Soon after, Chan *et al.* [13] combined ZBR and SMC studies for small rib waveguides, down to $H = 1 \mu\text{m}$ at the 1550 nm wavelength. This work was also unique in that deeply etched structures ($0.3 < r < 0.5$) were considered exclusively. BPM simulations were used to determine the SMCs, and showed that it is indeed possible to meet the requirement for ZBR and single-mode operation simultaneously in a rib waveguide. The modified SMC found is presented below in (19).

$$W = 0.05H + \frac{(0.94H + 0.25H^2)r}{\sqrt{1 - r^2}} \quad (19)$$

for $0.3 \leq r \leq 0.5$

The birefringence and single-mode requirements for SOI heights of 250 - 500 nm are investigated in [15], for 1550 nm. A FDM mode solver, with PML boundary conditions, was used in the analysis. From the calculated mode properties, the authors used two criteria for determining the SMC. The first stated that a guided second-order mode must have a loss $< 200 \text{ dB/mm}$, and the second that it must have $\geq 5\%$ of its optical power confined to the silicon rib region. Although the authors did not compare their SMC to the analytical SMCs of [6, 7, 13], it is clear that single-mode waveguides with relatively large W/H ratios may be achieved at small H values.

Our study differs from the previous work in that the wavelength used is 1310

nm. This wavelength has not been used previously in SMC studies at small waveguide dimensions due to the ultra-low propagation loss in conventional single-mode fibers at 1550 nm. However, the 1310 nm band is commonly used for medium range data transmission [17, 18]. Furthermore, the band presents advantages in terms of germanium detector responsivity for both hybrid and monolithically integrated systems [19, 20]. Intuitively, one expects the single-mode cut-off width at 1310 nm to be less than that for 1550 nm used in previous work [13], therefore their conditions would not apply to devices designed at this wavelength. Our analyses confirm that the SMC of a waveguide is wavelength dependant, and provide design guidelines for operating at 1310 nm. As recent waveguide designs most often make use of deeply etched ribs, etch depths such that $0.3 \leq r \leq 0.5$ will be considered here.

3.3 Method

All simulations were performed using the full-vectorial mode solvers in FIMMWAVE®. For the birefringence data the FMM method was used. This method has the advantage of solving the problem numerically in only one dimension, rather than the two dimensions necessary for other methods. The benefit of the FMM method is a higher accuracy for less computational time. The loss of waveguide modes was determined using the FDM to solve for the modes with PMLs defined on each boundary. The PMLs impose a lossy yet reflectionless layer on the boundaries. The size of the computational domain was chosen to be large enough such that guided modes would have negligible overlap with the PMLs and therefore experience no artificial loss due to the boundaries. The loss of the higher-order modes was then computed based on the imaginary component of their propagation constants. From this data the SMC could be determined for a certain H and D value by scanning W.

The cut-off points were chosen at the roll-off in the mode loss curve, indicating the mode was transitioning from lossy to lossless. An additional requirement was that the loss at cut-off must be greater than a pre-set value to ensure higher-order modes are sufficiently attenuated in real devices. A minimum loss of 150 dB/cm was chosen for waveguides with $H \geq 0.8 \mu\text{m}$. At small SOI heights ($H < 0.8 \mu\text{m}$) device lengths are typically shorter, therefore a minimum loss of 300 dB/cm was selected.

The computed second-order modes are broken up into two categories. The horizontal second-order TE and TM modes, referred to as HE_{01} and EH_{01} respectively, were often found to become guiding at the smallest widths, and therefore define the SMC. The second category, vertical second-order modes (HE_{10} and EH_{10}), were found to become guiding at larger widths, and were rarely valid for the SMC determination. Results were consistent with the BPM results of [13] at 1550 nm. The method has additional advantages over the BPM as the SMC can be determined directly from the mode properties, without any need for propagation.

3.4 Results

Our ZBR and SMC results for six different SOI heights are displayed in Fig. 14 ($H = 1.3, 1, \text{ and } 0.8 \mu\text{m}$) and Fig. 15 ($H = 0.6, 0.34, \text{ and } 0.22 \mu\text{m}$). The plot format is consistent with those presented in [13]. Although the SMCs provided by previous work ([6, 7, 13]) do not consider 1310 nm light, or extend to the waveguides dimensions we present here, their single-mode cut-off equations have been extrapolated and plotted for comparison. Achieving a single-mode and polarization independent waveguide requires designing at dimensions along the ZBR curve, and below the lowest single-mode cut-off width. We see that ZBR and single-mode operation is attainable at $H = 1.3, 1, \text{ and } 0.6 \mu\text{m}$, and possibly so at $0.8 \mu\text{m}$. However, at smaller heights such as

$H = 0.34$ and $0.22 \mu\text{m}$, the ZBR condition vanishes.

3.4.1 Large Waveguide Heights

We first consider the SMCs for waveguide structures where H is $0.8 \mu\text{m}$ and larger, as is presented in Fig. 14. It is clear that the EH_{01} mode imposes the most restrictive conditions on waveguide dimensions for single-mode behaviour at these SOI heights, which agrees with the conclusion reached in [13]. In addition, our single-mode cut-off data agrees better with the SMC of Chan *et al.* [13], compared to those predicted by Soref *et al.* [6] and Pogossian *et al.* [7]. When closely comparing our results to those in [13], where the wavelength used was 1550 nm , different trends in the SMC become apparent. At lower etch depths, the SMC for 1310 nm is much lower than that for 1550 nm . However, as etch depth increases the SMCs begin to merge. Therefore, the wavelength dependence of the SMC near 1310 nm is much stronger at high r values. We found that (18) matched the trend of our simulation results for this range of H quite well, with only minor corrections needed to improve the fit to the data, which are presented in (20). An $\alpha = 0.08$ was found to be ideal, which lies in between the values predicted by Soref *et al.* and Pogossian *et al.*

$$\begin{aligned}
 W &= 0.08H + \frac{1.12Hr}{\sqrt{1-r^2}} \\
 \text{for } 0.8\mu\text{m} \leq H \leq 1.3\mu\text{m} \\
 \text{and } 0.3 \leq r \leq 0.5
 \end{aligned} \tag{20}$$

Figure 14: The single-mode cut-off data for large SOI heights. The SMCs from Soref *et al.*, Pogossian *et al.*, and Chan *et al.* (extracted from [6], [7], and [13] respectively) are plotted alongside our simulation results.

3.4.2 Small Waveguide Heights

When the SOI height is lowered below $0.8 \mu\text{m}$, we notice dramatic changes in the single-mode cut-off behaviour, as evident in Fig. 15. The single-mode cut-off points for these SOI heights deviate from the previous SMCs for large H values, such that a single-mode waveguide can be much wider than is predicted by extrapolating the previous work of [6], [7], and [13]. When comparing these results to previous work on small waveguide heights at 1550 nm [15], the SMC results seem to agree well.

However, the wavelength dependence of the SMC yields a ± 100 nm variation in cut-off width.

The slope of the TM SMC abruptly changes from negative to slightly positive in Fig. 15(a). The slope TE SMC however, is less affected by the shrinking H values. We rationalize this abrupt change in the TM SMC by examining the confinement factor of the horizontal second-order modes in the core (Γ_{core}) and rib (Γ_{rib}) for a constant width-to-height ratio ($W/H = 1$). The ratio of these confinement factors, represented as $R = \Gamma_{\text{core}} / \Gamma_{\text{rib}}$, gives a clear indication of where the optical power lies in the waveguide. Obviously, a high R value corresponds to a strongly guided mode, while a low value indicates a slab-like mode. The R data is presented in Fig. 16 for both TE (HE_{01}) and TM (EH_{01}) modes, with $r = 0.3, 0.4$, and 0.5 .

At high H values, R is significantly larger in waveguides with deep etches, representing well confined modes for $r = 0.3$ for example. For decreasing waveguide heights, a large drop in R between $H = 0.8$ and $0.6 \mu\text{m}$ is clear, and is more dramatic at small r values. This drop in R represents the confinement of the optical power moving from the core to the rib, which explains the abrupt modification in the SMC trends for deeply etched structures as illustrated in Fig. 14(c) and 15(a). The change in R is considerably larger for TM modes compared to that for TE, supporting the appreciably different observed change in their cut-off behaviour. Where the TM R value at $r = 0.3$ drops dramatically between $H = 0.6$ and $0.8 \mu\text{m}$, the drop in R for TE is much more gradual. Although the confinement factor analysis does not provide insight into the fundamental reason for the observed change in SMC trends at low SOI heights, it does confirm the change and help illustrate it in a more intuitive way.

When the height of the silicon layer is reduced to $0.34 \mu\text{m}$ and less, as seen in Fig. 15(b) and 15(c), a special situation arises in which the EH_{01} mode ceases to become

Figure 15: The single-mode cut-off data and magic widths for small SOI heights. The SMCs from Soref *et al.*, Pogossian *et al.*, and Chan *et al.* (extracted from [6], [7], and [13] respectively) are plotted alongside our simulation results.

fully guiding at low etch depths. Instead the mode couples to the lossy TE slab mode. This is demonstrated in Fig. 17, where $D = 0.11 \mu\text{m}$ and $H = 0.22 \mu\text{m}$. Here it can be seen that even at the relatively large width of $1.2 \mu\text{m}$, the E_x field still radiates into the rib causing loss, while the E_y component appears well confined. The E_x radiation is produced by the conversion of the guided TM mode to the radiative TE slab mode at the rib interface, producing both a transmitted and reflected TE wave. The phase matching condition is possible during this conversion as the effective index of the

TM mode is relatively low compared to the slab TE mode. At small rib heights the index of the TE slab mode may drop below that of the TM mode, therefore avoiding loss by eliminating the phase matching condition. As well, specific waveguide widths exist (referred to as magic widths) such that the reflected and transmitted TE waves produced at multiple points along the rib interface will destructively interfere [21, 22]. At these widths the radiation loss significantly diminishes.

Figure 16: Ratio of optical power confined in the core to that confined in the rib for the second-order TE (HE_{01}) and TM (EH_{01}) modes for $W/H=1$. A large drop is observable for the TM modes between $H = 0.6$ and $0.8 \mu\text{m}$, consistent with the SMC shift seen in Fig. 14 and 15.

In Fig. 15(b) and 15(c), both the SMC and magic widths are plotted for the EH_{01} modes. Unlike the SMC, designing a waveguide with a width greater than the magic width does not guarantee that the EH_{01} mode will be non-lossy. The magic widths only indicate specific points where the waveguide dimensions result in minimal loss for the mode. As well, the loss at the magic widths does not reduce to zero, which can be explained using a ray optics view of the modes. At these small waveguide heights the modes are weakly guided, which is equivalent to a large propagation angle near that of the critical angle. At small propagation angles the

magnitudes of the transmitted and reflected TE waves are nearly equal, which allows for near perfect destructive interference. However at large angles the two waves are different in magnitude, resulting in incomplete destructive interference [21]. Finally, it is important to note that the magic widths are periodic, so more low-loss EH_{01} modes will exist at higher widths.

For H values of $0.6 \mu\text{m}$, $0.34 \mu\text{m}$, and $0.22 \mu\text{m}$, (20) no longer suitably defines the SMC. For $H = 0.6 \mu\text{m}$ and $D < 0.34 \mu\text{m}$, the EH_{01} mode still determines the SMC, as seen before with large H values. But at larger etch depths the HE_{01} mode begins to limit the width for single-mode behaviour. In the case of $H = 0.34 \mu\text{m}$, only the HE_{01} mode determines the SMC. Although the EH_{01} magic widths do lie below this line at times, they are not considered to be single-mode cut-off points as single-mode waveguides still exist at higher widths. A new equation is proposed to fit the TE SMC data for $0.34 \mu\text{m} \leq H \leq 0.6 \mu\text{m}$, displayed in (21). Due to the complexity of the TM modes in the small SOI height regime, it is advised that the single-mode behaviour be studied on a case-by-case basis. For $H = 0.22 \mu\text{m}$, fluctuations in the cut-off widths leads to a SMC without a clear trend. As a result, for $H < 0.34 \mu\text{m}$, we recommend once again that the second-order modes be investigated on a case-by-case basis.

Figure 17: E_y and E_x field components for the second-order TM (EH_{01}) mode with $H = 0.22 \mu\text{m}$, $D = 0.11 \mu\text{m}$, and $W = 1.2 \mu\text{m}$. While the E_y component appears to be well confined, the E_x field clearly radiates into the rib, indicating loss even at this large waveguide width.

$$\begin{aligned}
W &= 0.34\sqrt{H} + \frac{(0.97H + \frac{0.07}{H})r}{\sqrt{1-r^2}} \\
&\text{for HE}_{01} \text{ modes} \\
&\text{with } 0.34\mu m \leq H \leq 0.6\mu m \\
&\text{and } 0.3 \leq r \leq 0.5
\end{aligned} \tag{21}$$

3.5 Conclusions

Our simulation results provide a detailed guideline for designing waveguides for single-mode and/or zero-birefringence (ZBR) operation at 1310 nm. A wide range of silicon-on-insulator (SOI) heights and etch depths has been studied, and the single-mode condition (SMC) and ZBR results have been summarized in Table 1. We observe that the SMC has a strong dependence on both the SOI height and operating wavelength. The ZBR condition is also found to have a strong H dependence, only being attainable in waveguides with $H \geq 0.6 \mu m$. This analysis could also be applied to the design of multimode systems for mode-division multiplexing (MDM) applications. In these systems the waveguide dimensions should be well above the SMC, in the multimode regime, to ensure negligible loss of the desired higher-order modes.

Table 1: The determined SMCs for all H ranges considered.

W	for H
$0.08H + \frac{1.12Hr}{\sqrt{1-r^2}}$	$0.8\mu\text{m} \leq H \leq 1.3\mu\text{m};$
$0.34\sqrt{H} + \frac{(0.97H + \frac{0.07}{H})r}{\sqrt{1-r^2}}$	$0.34\mu\text{m} \leq H \leq 0.6\mu\text{m},$ for HE ₀₁ modes only;
case-by-case	$H < 0.34\mu\text{m}$

List of References

- [1] M. Greenberg and M. Orenstein, “Simultaneous dual mode add/drop multiplexers for optical interconnects buses,” *Optics Communications*, vol. 266, no. 2, pp. 527–531, Oct. 2006.
- [2] J. Park, D.-M. Yeo, and S.-Y. Shin, “Variable Optical Mode Generator in a Multimode Waveguide,” *IEEE Photonics Technology Letters*, vol. 18, no. 20, pp. 2084–2086, Oct. 2006.
- [3] S. Bagheri and W. M. J. Green, “Silicon-on-insulator mode-selective add-drop unit for on-chip mode-division multiplexing,” in *2009 6th IEEE International Conference on Group IV Photonics*. IEEE, Sep. 2009, pp. 166–168.
- [4] D. Dai, L. Liu, S. Gao, D. Xu, and S. He, “Polarization management for silicon photonic integrated circuits,” *Laser & Photonics Reviews*, vol. 1, no. 26, Oct. 2012.
- [5] T. Barwicz, M. R. Watts, M. Popović, P. T. Rakich, L. Socci, F. X. Kärtner, E. P. Ippen, and H. I. Smith, “Polarization-transparent microphotonic devices in the strong confinement limit,” *Nature Photonics*, vol. 1, no. 1, pp. 57–60, Jan. 2007.
- [6] R. A. Soref, J. Schmidtchen, and K. Petermann, “Large Single-Mode Rib Waveguides in GeSi-Si and Si-on-SO₂,” *IEEE Journal of Quantum Electronics*, vol. 27, no. 8, pp. 1971–1974, 1991.
- [7] S. Pogossian, L. Vescan, and a. Vonsovici, “The single-mode condition for semiconductor rib waveguides with large cross section,” *Journal of Lightwave Technology*, vol. 16, no. 10, pp. 1851–1853, 1998.
- [8] O. Powell, “Single-mode condition for silicon rib waveguides,” *Journal of Lightwave Technology*, vol. 20, no. 10, pp. 1851–1855, Oct. 2002.

- [9] J. Lousteau, D. Furniss, A. B. Seddon, T. M. Benson, A. Vukovic, and P. Sewell, "The Single-Mode Condition for Silicon-on-Insulator Optical Rib Waveguides With Large Cross Section," *Journal of Lightwave Technology*, vol. 22, no. 8, pp. 1923–1929, 2004.
- [10] J. Xia and J. Yu, "Single-mode condition for silicon rib waveguides with trapezoidal cross-section," *Optics Communications*, vol. 230, no. 4-6, pp. 253–257, Feb. 2004.
- [11] J. Schmidtcehn, A. Splett, B. Schuppert, and K. Petermann, "Low Loss Singlemode Optical Waveguides with Large Cross-Section in Silicon-On-Insulator," *Electronics Letters*, vol. 27, no. 16, pp. 1486–1488, 1991.
- [12] L. Vivien, S. Laval, B. Dumont, S. Lardenois, A. Koster, and E. Cassan, "Polarization-independent single-mode rib waveguides on silicon-on-insulator for telecommunication wavelengths," *Optics Communications*, vol. 210, no. 2002, pp. 43–49, 2002.
- [13] S. P. Chan, C. E. Png, S. T. Lim, G. T. Reed, and V. M. N. Passaro, "Single-Mode and Polarization-Independent Silicon-on-Insulator Waveguides With Small Cross Section," *Journal of Lightwave Technology*, vol. 23, no. 6, pp. 2103–2111, 2005.
- [14] D. Dai and S. He, "Analysis of the birefringence of a silicon-on-insulator rib waveguide," *Applied Optics*, vol. 43, no. 5, pp. 1156–1161, 2004.
- [15] D. Dai and Z. Sheng, "Numerical analysis of silicon-on-insulator ridge nanowires by using a full-vectorial finite-difference method mode solver," *Journal of the Optical Society of America B*, vol. 24, no. 11, p. 2853, 2007.
- [16] W. N. Ye, D.-X. Xu, S. Janz, P. Cheben, M.-J. Picard, B. Lamontagne, and N. G. Tarr, "Birefringence Control Using Stress Engineering in Silicon-on-Insulator (SOI) Waveguides," *Journal of Lightwave Technology*, vol. 23, no. 3, pp. 1308–1318, 2005.
- [17] L. Chen, K. Preston, M. Lipson, C. R. Doerr, and Y.-k. Chen, "Silicon photonics for on-chip interconnects and telecommunications," in *Optoelectric Integrated Circuits XII*, L. A. Eldada and E.-H. Lee, Eds., vol. 7605, Feb. 2010, pp. 760 509–760 509–11.

- [18] P. Herve and S. Ovadia, “Optical Technologies for Enterprise Networks,” *Intel Technology Journal*, vol. 08, no. 02, 2004.
- [19] L. Colace, M. Balbi, G. Masini, G. Assanto, H.-C. Luan, and L. C. Kimerling, “Si-based near infrared photodetectors operating at 10Gbit/s,” *Journal of Luminescence*, vol. 121, no. 2, pp. 413–416, Dec. 2006.
- [20] J. Liu, D. D. Cannon, K. Wada, Y. Ishikawa, S. Jongthammanurak, D. T. Danielson, J. Michel, and L. C. Kimerling, “Tensile strained Ge p-i-n photodetectors on Si platform for C and L band telecommunications,” *Applied Physics Letters*, vol. 87, no. 1, p. 011110, 2005.
- [21] M. A. Webster, R. M. Pafchek, A. Mitchell, and T. L. Koch, “Width Dependence of Inherent TM-Mode Lateral Leakage Loss in Silicon-On-Insulator Ridge Waveguides,” *IEEE Photonics Technology Letters*, vol. 19, no. 6, pp. 429–431, 2007.
- [22] T. G. Nguyen, R. S. Tummididi, T. L. Koch, and A. Mitchell, “Rigorous Modeling of Lateral Leakage Loss in SOI Thin-Ridge Waveguides and Couplers,” *IEEE Photonics Technology Letters*, vol. 21, no. 7, pp. 486–488, 2009.

Chapter 4

The Multimode Ring Resonator Design

The following article, *Two-mode division multiplexing in a silicon-on-insulator ring resonator*, provides details on the design of the ring resonators. Simulations of the (de)multiplexers and ring bends are focused on in this article. The Lumerical script files used to generate and simulate the structures can be found in Appendix B. The experimental operation of the ring resonator is also presented.

Bryce Dorin performed all simulations, device design, analysis of the data, and wrote the article. The device was fabricated at the University of Washington and tested using the automated testing station at the University of British Columbia. The article was published in Optics Express, volume 22, issue 4, on pages 4547-4558 in 2014.

Two-mode division multiplexing in a silicon-on-insulator ring resonator

Bryce Dorin and Winnie N. Ye

Department of Electronics, Carleton University

1125 Colonel By Drive

Ottawa, Ontario, Canada K1S 5B6

[B. A. Dorin and W. N. Ye, “Two-mode division multiplexing in a silicon-on-insulator ring resonator,” *Opt. Exp.*, vol. 22, no. 4, pp. 4547-4558, Feb. 2014.]

©OSA

4.1 Abstract

Mode-division multiplexing (MDM) is an emerging multiple-input multiple-output method, utilizing multimode waveguides to increase channel numbers. In the past, silicon-on-insulator (SOI) devices have been primarily focused on single-mode waveguides. We present the design and fabrication of a two-mode SOI ring resonator for MDM systems. By optimizing the device parameters, we have ensured that each mode is treated equally within the ring. Using adiabatic Bezier curves in the ring bends, our ring demonstrated a signal-to-crosstalk ratio above 18 dB for both modes at the through and drop ports. We conclude that the ring resonator has the potential

for filtering and switching for MDM systems on SOI.

4.2 Introduction

Silicon-on-insulator (SOI) wire waveguides have been identified to potentially replace copper wires for intra-chip communication. Developments in the fabrication of photonic integrated circuits are leading the way to making this vision a reality. To take advantage of the wide bandwidth of SOI waveguides, wavelength-division multiplexing (WDM) has been implemented, drastically increasing data transfer rates [1]. However, in SOI systems WDM does have inherent drawbacks, as it requires the operation and alignment of many individual laser sources. Another form of multiple-input multiple-output transmission is mode-division multiplexing (MDM), where channels are separated spatially by multiplexing them onto the different modes of a waveguide. The advantage of a MDM system is that it requires only one laser source to be coupled to the chip, which is then divided into N waveguides via a power splitter, and modulated using integrated silicon modulators [2]. As well, mode multiplexing in SOI has a huge advantage in terms of the utilization and manipulation of all supported modes for added functionalities without sacrificing device footprint. This is demonstrated in [2] where multiplexers are presented with lengths less than $100\text{ }\mu\text{m}$ for four modal channels. MDM is a viable alternative to WDM in multiple-input multiple-output photonic circuits on SOI, and presents advantages as it requires only one carrier wavelength to operate. MDM is also being aggressively pursued for fiber-optic telecommunications, where WDM has been tremendously successful but is beginning to reach spectral efficiency limits [3]. Recently, three modal channels were demonstrated in a fiber over a transmission distance of 96 km [4].

Single-mode operation has previously been regarded as an important design requirement for SOI waveguide-based devices [5]. Therefore, most optical devices designed in SOI are incompatible with MDM systems which require multimode waveguides. The ring resonator is an important device used for multiplexing, switching, and modulating in on-chip communications [6–8]. In [9] a multimode bus waveguide is coupled to a single-mode ring resonator, where both modes exhibited a resonance response. However, this device is not suitable for MDM applications as the fundamental mode of the single-mode ring resonator was excited regardless of input mode, therefore significant crosstalk between modes was found during resonance. In the past wide waveguides have been used in ring resonators for their ability to reduce propagation and bending loss for the fundamental mode [10]. The waveguide in these structures was designed wide enough to be multimode when straight, but the authors ensured that the higher-order modes were lossy in the ring bends to avoid resonance of any mode but the TE_{00} for single-mode operation in the device. Such a device is also incompatible with MDM systems.

For the first time we demonstrate a ring resonator designed to support two modes, TE_{00} and TE_{01} , for MDM applications. Such a device could drastically increase the complexity and functionality of on-chip MDM communications by allowing for mode selective add/drop filtering. By adopting an active multimode ring resonator, mode selective switching and modulating could also be achieved [7, 8]. In this paper we outline the operation of a ring resonator composed of multimode waveguides, and present experimental results from fabricated devices. The design and optimization of the (de)multiplexers and ring bends are also reported.

4.3 Operation of a multimode ring resonator

A schematic detailing the proposed multimode ring resonator structure is shown in Fig. 18. The input waveguide in the diagram follows a multiplexing stage where two input channels are added by coupling them to the TE_{00} and TE_{01} modes. The two modes then interact with the ring resonator via a directional coupler. If the mode is resonant in the ring it will be coupled to the drop port, if not the signal will pass to the through port. A demultiplexing stage is included after both the through and drop ports to recover the two channels separately for analysis. In the example provided in Fig. 18 the TE_{01} mode is resonant in the ring, while the TE_{00} mode is not. The transfer function at the drop port of the resonator (T_{drop}) can be described using (22) for any mode in the ring [11].

$$T_{drop} = \frac{P_{drop}}{P_{in}} = \frac{-\alpha\kappa^4}{2\alpha t^2 \cos(\theta) - (\alpha^2 t^4 + 1)} \quad (22)$$

Figure 18: A ring resonator composed of waveguides supporting two TE modes. In this example, the first-order mode (TE_{01}) is resonant and critically coupled to the drop port of the filter, while the fundamental mode (TE_{00}) is off resonance and passes to the through port.

Here, α is the attenuation factor of the mode after one revolution of the ring

(where $\alpha = 1$ represents no attenuation), and θ represents the phase induced by the revolution of the ring. P_{drop} is defined as the power measured at the drop port, and P_{in} is the power at the input. The directional couplers between the ring and the add/drop ports are described by κ and t , the coupling and transmission coefficients of the electric field, respectively. In a lossless directional coupler these are related such that $t^2 + \kappa^2 = 1$. It can be seen in (22), when $\theta = 2\pi n$, where n is an integer, the mode is on resonance and transmission at the drop port approaches a maximum determined by the value of α . The modes TE_{00} and TE_{01} have different effective refractive indices (n_{eff}), and therefore will become resonant in the ring at different wavelengths. This produces slightly different periodicities in the wavelength response of the filter. Another difference in the response of the two modes is seen in the directional coupler. The evanescent field of the TE_{01} mode extends much further into the cladding than the TE_{00} evanescent field. The coupling coefficients κ and t are heavily dependant on the overlap of the evanescent fields between the two waveguides, therefore the lengths for complete coupling (L_c) for each mode can differ by nearly an order of magnitude depending on the waveguide separation. The result of this difference has the largest effect on the quality factor (Q) of the resonator defined in (23), where L_{eff} is the effective length of the ring, and λ_0 is the center wavelength which is set to 1550 nm here [11].

$$Q = \frac{\pi L_{eff} n_{eff}}{\lambda_0 \arccos\left(\frac{1+t^4\alpha^2-4t^2\alpha}{-2t^2\alpha}\right)} \quad (23)$$

It is often desirable for the individual channels on a waveguide to have a similar response in devices such as filters. In many cases it would be ideal for the two modes to have near equal Q factors. Therefore, the κ and t values should be matched for TE_{00} and TE_{01} . Although Q is also dependant on n_{eff} and α , these values differ much

less between modes. By considering directional coupler lengths (L) greater than the coupling length for TE₀₁, one can achieve equal κ values. The general expressions for calculating κ for each mode are presented in (24) and (25).

$$\kappa^{TE00} = \sin \left[\frac{\pi}{2} \left(\frac{L}{L_c^{TE00}} \right) \right] \quad (24)$$

$$\kappa^{TE01} = \sin \left[\frac{\pi}{2} \left(\frac{L}{L_c^{TE01}} \right) \right] \quad (25)$$

L_c^{TE00} and L_c^{TE01} are the coupling lengths for the TE₀₀ and TE₀₁ modes, respectively. In (26), κ^{TE00} is set equal to κ^{TE01} , where the range of L considered is $L_c^{TE01} < L < L_c^{TE00}$.

$$\sin \left[\frac{\pi}{2} \left(\frac{L}{L_c^{TE00}} \right) \right] = \sin \left[\pi - \frac{\pi}{2} \left(\frac{L}{L_c^{TE01}} \right) \right] \quad (26)$$

Solving (26) we find an analytic solution for L, which is displayed in (27).

$$L = \frac{2L_c^{TE01}L_c^{TE00}}{L_c^{TE00} + L_c^{TE01}} \quad (27)$$

Using this expression, a single length of coupler (L) can be found resulting in equal κ , with similar Q for the two modes. A disadvantage of this method is that for a fixed directional coupler geometry only one κ value is possible, which may be suitable for specific applications. However, it is possible to adjust the geometry of the directional couplers by tuning the waveguide widths or the gap between waveguides, which changes where κ is equal for both modes. In Fig. 19 the common coupling value for the two modes ($[\kappa^{TE00}(L)]^2$ and $[\kappa^{TE01}(L)]^2$) is plotted as a function of directional coupler gap for two waveguide widths. The values are calculated using (24) and (25) for a directional coupler of length L, which was calculated using (27). From this plot

it is clear that only modest control over the coupling in the directional coupler is possible by adjusting these two parameters. For two-mode waveguides 980 nm wide, with a gap of 200 nm between them, 95 μm long directional couplers were found to produce 13.4% coupling (κ^2) for each mode.

Figure 19: The common κ^2 value for TE_{00} and TE_{01} in a directional coupler of length L . The directional coupler gap and waveguide widths are varied to demonstrate that some control over the equal κ^2 values can be achieved. During simulations the waveguide height and wavelength used were 220 nm and 1550 nm, respectively.

4.4 Multiplexing and demultiplexing

In order to demonstrate the multimode ring resonator, a reliable device for multiplexing and demultiplexing the TE_{01} mode is required. Many designs have been proposed to achieve this, including Y-junctions [12] and multimode interferometers (MMIs) [13]. Ultimately these designs were avoided as they cannot be easily extrapolated to multiplexing more than two channels, which would likely be a requirement in

future MDM systems. Recently grating-assisted couplers have been proposed, which can multiplex several channels by adjusting the period of the grating between the input and bus waveguides [14]. The disadvantages of this design is a small bandwidth and relatively long coupling lengths. Asymmetrical directional couplers (ADCs) provide a good compromise for multiplexing, as they are easily cascaded for any number of modes [2], and are not strongly wavelength dependant. In wire waveguides ADCs have been shown to achieve (de)multiplexing on-chip while only occupying small fractions of the valuable chip space [2, 15]. The ADC concept requires a single-mode input waveguide that is phase matched to the desired higher-order mode in the bus waveguide [16]. In our case the phase matching we desired occurred between TE_{00} of the input waveguide and TE_{01} of the bus waveguide, which meant designing at widths that ensured the n_{eff} of each mode were equal. It is generally accepted that directional couplers exhibit poor fabrication sensitivity, but this has been overcome in many ways by introducing a linear taper in the bus waveguide [15]. A schematic of the tapered ADC design is shown in Fig. 20, where the width of bus waveguide increases linearly along the length of the ADC. For $W_1 = 450$ nm, it was found that W_2 must be 930 nm to satisfy the phase matching condition. The gap between waveguides, g , is maintained at 200 nm. The value of t is 50 nm for the tapered design, and reduced to 0 nm for the uniform case. The variable L_{ADC} represents the length of device for maximum coupling, which changes with the value of t .

Both the tapered and uniform ADC multiplexer designs have been simulated, and the results are shown in Fig. 21. The eigenmode expansion (EME) method available in FIMMWAVE[®], and the finite-difference time-domain (FDTD) method available in Lumerical[®] were both used to simulate the device and agree quite well. Fig. 21(a) predicts the tapered ADC design will require a longer length and produce a lower

Figure 20: The layout and operation of the ADC multiplexer design. Here W_1 and W_2 are the widths of the ADC input and bus waveguides, respectively; t is the thickness offset of the bus waveguide; g is the gap between the two coupler waveguides.

peak coupling in return for a much broader coupling peak with respect to length. Theoretically perfect coupling is not possible in the tapered ADC, as it is in the uniform ADC, because the phase matching condition is only met locally along the device. Note that the ADC input and output bends were not simulated with the EME method, which is why it appears to predict slightly longer lengths than FDTD. The FDTD results are therefore taken to be more accurate. The tapered design is also found to achieve reduced sensitivity to fabrication errors, as demonstrated in Fig. 21(b). Error is simulated by increasing the waveguide widths (W_1 and W_2) by the error value, and by decreasing g by the error value. This method of defining error is meant to simulate how fabrication imperfections would affect the performance of a real ADC device. It can be seen that the taper allows approximately ± 50 nm in error while maintaining 3 dB coupling, where the uniform design allows for only ± 20 nm. As the fabrication of the ring resonator would require a total of three (de)multiplexers, the tapered ADC design was chosen to improve consistency.

After the demultiplexing stages of the device, the bus waveguides were tapered down to 500 nm where they became single-mode. The waveguide was also bent

through 90° several times with a radius of $5\ \mu\text{m}$. This ensured that any residual TE_{01} mode not coupled out of the bus waveguide in the demultiplexer would be radiated out of the waveguide, further reducing crosstalk.

(a)

(b)

Figure 21: The coupling efficiency of tapered and uniform asymmetric directional couplers. FDTD and EME simulations are used to plot coupling fraction against (a) device length, and (b) fabrication errors in W_1 and W_2 .

4.5 Ring bends

To ensure the free spectral range (FSR) of the filter is sufficiently large for filtering applications, a ring bend radius (R) of $20\ \mu\text{m}$ or less was desired. However, the mode profiles in a bent waveguide of this radius differ significantly from those in a straight waveguide. The result is that when the two waveguides are connected mode mismatch at the interface will occur, leading to crosstalk between channels. The magnitude of this crosstalk has been calculated using the EME method, and is presented in Fig. 22 for radii of curvature up to $20\ \mu\text{m}$. Here, periodic dips in crosstalk are noticed as a function of radius, which is due to self-imaging within the bend [17]. At specific radii the length of the bend will be equal to $2L_\pi$, where L_π is the beat length of the two confined modes in the waveguide [18]. This produces the exact optical profile that was initially incident on the interface, and therefore greatly reduces crosstalk. The periodicity of the dips in Fig. 22 was found to be $\sim 1.5\ \mu\text{m}$, which agreed with the theoretical periodicity of $1.45\ \mu\text{m}$ calculated from the beat lengths. While it is tempting to reduce crosstalk by simply choosing the ring resonator bend radius at one of the crosstalk dips, this was not pursued as the self-imaging condition is too wavelength dependant. It can be seen in Fig. 22 that away from the self-imaging dips the crosstalk levels are above the standard -20 dB required for many commercial applications.

Another approach to reducing crosstalk in the ring bend is to remove the mode mismatch at the interface. This was achieved using transformation optics to design a bent waveguide with mode profiles that matched that of the straight waveguide [19]. Grayscale lithography was required to fabricate these waveguides, making them unsuitable for most applications. To keep fabrication relatively simple, we chose to replace the abrupt straight-to-bend transition using circular curves with an adiabatic

Figure 22: Crosstalk between the TE_{00} and TE_{01} modes in a semi-circular waveguide connected to straight waveguides. Beating is noticed in the crosstalk response, which is attributed to self-imaging in the bend. Data was obtained using the EME method.

curve. The adiabatic curve uses a gradual change in radius which acts to minimize conversion between modes. This is analogous to the adiabatic taper in fibers, which avoids modal conversion by narrowing the waveguide over a relatively long distance [20]. Adiabatic performance has been shown to exist in Bezier curves, which have been previously implemented in SOI waveguide devices [21]. Original work on Bezier curves focused only on reducing loss in single-mode waveguide bends [21]. In this work we study the reduction of crosstalk when using the adiabatic Bezier curves in our two-mode waveguide bends.

Bezier curves are typically defined by 4 points. For the 90° bends in [21], the start and end points are set to be $(0, 0)$ and (R, R) , respectively. The next two points then characterize how the bend travels from the start to the end, and were defined as $(R(1 - B), 0)$ and (R, B) . The mathematical construction is demonstrated in Fig. 23 [22]. The unitless parameter B was optimized to reduce crosstalk for our MDM

Figure 23: The construction of the 90° Bezier curve. In the image the four points defining the curve, as well as the curve itself are displayed.

application. A selection of the Bezier curves simulated are shown in Fig. 24(a). It can be seen that when $B = 0.45$, the curve closely resembles a circle where the transition between straight and bent waveguides is abrupt. In contrast, at $B = 0.05$ there is a very gradual change in radius at the interface, but a small radius near the knee of the bend. The crosstalk levels after propagating through a 90° Bezier curve were found using FDTD simulations, and are shown in Fig. 24(b). To remove the effects of self-imaging in the results of Fig. 24(b), the crosstalk levels were chosen from the peak value over a wide wavelength range of 100 nm, from 1500 to 1600 nm, over which two self-imaging instances are observed. We notice that the ideal value of B is consistently less than 0.45 regardless of the radius considered. We can therefore conclude that Bezier curves effectively mitigate crosstalk compared with circular curves. For our desired radius of $20\ \mu\text{m}$, $B = 0.15$ returned a crosstalk value of -41.9 dB, which is considered sufficiently low for communication applications. We note that the Bezier curve design has a great deal of potential for reducing crosstalk in any bend within a MDM system.

(a)

(b)

Figure 24: The design of adiabatic Bezier curves: (a) shows the shape of the bends for various values of B , and (b) shows the corresponding level of crosstalk in a 90° bend simulated using FDTD. Crosstalk was measured as the ratio of power in the crosstalk channel to power in the signal channel at the output of the bend. These powers were obtained directly from the FDTD results using an overlap integral.

4.6 Experimental Results

Our multimode ring resonators were fabricated onto 220 nm thick SOI wafers. Light was coupled to and from the chip using grating couplers [23]. In Fig. 25 a layout of the fabricated device is displayed. The input and output grating couplers are shown, along with all important device components and dimensions. The single-mode waveguides connecting the grating couplers to the (de)multiplexers were 500 nm wide, while the multimode waveguide sections were 980 nm wide. A 200 nm gap was used in all directional coupler configurations. The testing set-up employed for the measurements made use of a 3-channel fiber array with one input and two outputs. The fiber array coupled to the grating couplers displayed in Fig. 25 for light injection and collection. The middle fiber channel served as the input, while the other two fiber channels served as the through and drop port outputs. One measurement was done for the TE_{01} mode using the grating coupler array on the far left, and one was done for the TE_{00} mode using the grating coupler array to the right. Ideally, the measurements would have been performed with two, 3-channel fiber arrays, using one for each mode. Crosstalk measurements could then be performed by exciting either the TE_{00} or TE_{01} channel and observing the outputs of the other channel. However, such measurements would increase the complexity of the testing set-up and increase the footprint of the device. For the ease of measurement, crosstalk levels were obtained from two identically designed rings fabricated on the same wafer as the original device. The grating couplers in these devices were rearranged so that input for the TE_{00} mode was aligned with the TE_{01} mode outputs, and likewise for the TE_{01} mode input. To demonstrate the resonance response of the filter a wavelength sweep over 20 nm (from 1540 to 1560 nm) was performed during testing, however in actual device operation a single operating wavelength would be used.

Figure 25: The multimode ring resonator mask layout used for fabrication. The different components and dimensions of the device are labelled.

The optical output from the through and drop ports is presented in Fig. 26. The data here clearly displays unique resonances for the TE_{00} and TE_{01} modes. A second identically designed ring was fabricated on the same wafer. It was found that the resonance peaks of this device were shifted by ~ 0.3 nm compared to the peaks presented in Fig. 26 for each mode, indicating that fabrication defects will have a modest effect on resonant wavelengths. The crosstalk results from one device are also shown in Fig. 26. Here, TE_{01} XT is the optical power measured from the TE_{01} output when a TE_{00} mode is input into the ring, and likewise for TE_{00} XT. We summarize the properties of both modes within the ring resonator device in Table 2 over the 20 nm wavelength range displayed in Fig. 26.

When comparing the fiber-to-fiber loss of the TE_{00} and TE_{01} modes from Table 2, we notice that the trends are consistent with both theory and previous work. Since

Table 2: Theoretical and experimental properties of the TE_{00} and TE_{01} modes in the multimode ring resonator. The wavelengths considered range from $1.54 \mu\text{m}$ to $1.56 \mu\text{m}$.

Property	TE_{00}	TE_{01}
Through port loss (exp.)	12 dB	15 dB
Drop port loss (exp.)	15 dB	19 dB
Minimum SXR at the through port (exp.)	24 dB	22 dB
Minimum SXR at the drop port (exp.)	18 dB	23 dB
Through port extinction ratio (exp.)	13 dB	12 dB
Drop port extinction ratio (exp.)	25 dB	20 dB
FSR (simulated)	1.896 nm	1.692 nm
FSR (exp.)	1.90 nm	1.68 nm
Q (exp.)	22 000	26 000

coupling inefficiencies in the grating couplers account for approximately 10.5 dB of loss [23], we can conclude that the loss of the TE_{00} mode at the through port is largely attributed to the grating couplers. The TE_{01} mode experienced about 3 dB more loss than the fundamental mode from two additional sources. First, incomplete coupling at the multiplexers and demultiplexers reduces the power of only the TE_{01} mode. From separate measurements of the ADC multiplexers and demultiplexers, we calculated an average loss of 1.25 dB per (de)multiplexer. Therefore, the ADCs account for approximately 2.5 dB of the additional loss observed in the TE_{01} mode. The second source of loss is increased sidewall scattering, as the optical power for this mode extends further in the horizontal direction compared to TE_{00} , resulting in a larger overlap with the waveguide sidewalls. Assuming 10.5 dB of loss in the grating couplers, and 2.5 dB of loss in the ADCs for the TE_{01} mode, we calculate the propagation losses of the TE_{00} and TE_{01} modes at the through port to be 1.5 dB and 2 dB, respectively. In the case of the drop port, the loss of the modes is 3 and 4 dB

higher than at the through port, for the TE_{00} and TE_{01} modes, respectively. The ring resonator has not satisfied the condition for critical coupling for either mode, which accounts for some reduced output power at the drop port during resonance [7]. To achieve critical coupling in the add/drop ring resonator, the coupling at the through port must match the loss in the ring plus the coupling at the drop port, such that $\kappa_{through} = (1 - \alpha) + \kappa_{drop}$, for each mode. This condition for single-mode rings is well known for directional coupler gaps of $g_{through} < g_{drop}$. In our case the critical coupling condition would need to be satisfied for both modes, making it much more difficult to achieve. Propagation within the ring is another expected source of the increased loss. The loss should be higher for the TE_{01} mode once again due to additional sidewall scattering.

We see that the signal-to-crosstalk ratio (SXR) at the drop port of the ring resonator device is larger than what we expected from mode mismatch in the Bezier curves of Fig. 24(b). However, the result is not surprising as fabrication defects such as waveguide sidewall roughness present an additional source of crosstalk not included in simulations. Imperfections at the waveguide edges can couple the two modes regardless of how the waveguide is designed. Although simulations predict very low levels of crosstalk from the ADC (de)multiplexers ($SXR > 30$ dB), fabrication defects decreased the measured SXR from six ADC devices tested separately to an average of 23.9 dB. Additional ring resonator crosstalk measurements, which are not displayed in Fig. 26, also demonstrated a SXR above 18 dB for both modes and all output ports. These results agree with the results presented in Fig. 26 and Table 2, indicating there are no significant fluctuations in overall crosstalk performance between the two devices. By omitting that single isolated spike in crosstalk for the TE_{00} mode at the drop port in Fig. 26, we can see that the SXR is roughly equal at both the through and the drop ports. Theoretically, the ring bends will have very little effect

on the SXR of the through port, as the light does not resonate and build up intensity in the ring. Therefore, we can conclude that the Bezier curves were not the largest contributor to crosstalk in the system. It is likely instead that the fabrication defects were the dominating source of mode conversion, as it would affect the SXR of the through and drop ports almost equally.

The distinct values of n_{eff} for each mode explain the slightly different FSRs we notice in both simulations and experiments. The excellent agreement between simulated and experimental FSRs confirms that we have witnessed the resonance response of the TE_{00} and TE_{01} modes as intended. From (23) we expect TE_{00} to have a higher Q value than TE_{01} based on its higher n_{eff} , assuming κ is equal for both modes. As well, TE_{01} has a higher loss which would also reduce the Q value for this mode. However, this is not the case in our experimental results. Therefore we speculate that the TE_{00} mode had slightly stronger coupling in the directional coupler compared to TE_{01} , reducing the Q value. Table 2 also compares the extinction ratios for both modes at the through and the drop port. The low values for extinction ratio (<15 dB) at the through port again indicates that the critical coupling condition has not been met for either mode. At the drop port however, the filter exhibits much larger extinction ratios. We therefore conclude that the coupling and loss within the ring is better optimized at this port near the 1550 nm wavelength.

By inspecting the results of Fig. 26, a wide range of applications become clear for this device. The slightly different FSR for each mode allows one to design the ring resonator to selectively filter one or more specified modes. From the wavelength response of the filter it is clear that filtering operation can be optimized for one specific mode, both modes, or no mode at all. For example, in a MDM system using this ring resonator at $1.549 \mu\text{m}$, both modes are coupled to the drop port. This is a unique function of the ring resonator in a MDM system, as the number of modal

channels that are filtered can be designed for by adjusting the radius of the ring or the operating wavelength. The number of applications for the filter grow further when considering an active ring resonator design, such as those demonstrated in the past [7, 8]. Channel selective switching then becomes possible simply by adjusting the refractive index of the ring. As well, the switching would be in no way limited to a single channel at a time. Furthermore, the two-mode ring can be extended to support additional modes for MDM systems with larger channel numbers. There are no theoretical limits to the number of modes a MDM system can support, however practical limitations such as large waveguide widths leading to weak coupling for the fundamental mode will likely limit this kind of system to 5-10 modes. Matching the values of κ for many modes would add to the design challenge, however similar Q values may not be essential for every application, which would increase the level of freedom one has in the ring resonator design.

(a)

(b)

Figure 26: Normalized optical output for the TE_{00} and TE_{01} modes from (a) the through port, and (b) the drop port. Crosstalk (XT) results are also presented.

4.7 Conclusion

We have successfully demonstrated a two-mode division multiplexing ring resonator. From our results we conclude that each mode resonates independent of the other within the ring. Optimization of the ring parameters was performed so that similar responses were observed for each mode. We have compared two variations of the asymmetric directional coupler for use as the multiplexer and demultiplexer for the device. In the fabricated device a tapered design was employed as it demonstrated a resilience to fabrication errors. The interface between straight and bent waveguide sections was identified as a potential source of crosstalk in the design. Simulations and experiments both showed that the introduction of Bezier curves in the ring bends could mitigate the crosstalk. The signal-to-crosstalk ratio in the ring stayed above 18 dB for both modes at the through and drop ports. We conclude that the ring resonator is an excellent channel selective filter for mode-division multiplexing (MDM) systems. Switching is also possible using rings by integrating refractive index tuning in the device.

Acknowledgments

The authors would like to thank CMC Microsystems and the University of British Columbia for their support in fabrication, which was done at the University of Washington Microfabrication/Nanotechnology User Facility. As well we thank CMC Microsystems for providing access to the simulation and design software. We acknowledge the inspiring work of L. Chrostowski and J. Flueckiger in the Bezier bend design, and Y. Wang for the fiber grating coupler design. Finally, we are grateful to V. Donzella who prepared the mask, R. Bojko for the fabrication, J. Flueckiger and C. Lin for the designing and constructing the experimental set-up, and Wyichen Wu for

conducting the measurements.

List of References

- [1] B. G. Lee, X. Chen, A. Biberman, X. Liu, I.-W. Hsieh, C.-Y. Chou, J. I. Dadap, F. Xia, W. M. J. Green, L. Sekaric, Y. a. Vlasov, R. M. Osgood, and K. Bergman, “Ultrahigh-Bandwidth Silicon Photonic Nanowire Waveguides for On-Chip Networks,” *IEEE Photon. Technol. Lett.* **20**(6), 398–400 (2008).
- [2] D. Dai, J. Wang, and Y. Shi, “Silicon mode (de)multiplexer enabling high capacity photonic networks-on-chip with a single-wavelength-carrier light,” *Opt. Lett.* **38**(9), 1422–1424 (2013).
- [3] R.-j. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel, “Capacity Limits of Optical Fiber Networks,” *IEEE J. of Lightw. Technol.* **28**(4), 662–701 (2010).
- [4] R. Ryf, S. Randel, A. H. Gnauck, C. Bolle, A. Sierra, S. Mumtaz, M. Esmaeelpour, E. C. Burrows, R.-j. Essiambre, P. J. Winzer, D. W. Peckham, A. H. McCurdy, and R. Lingle, “Mode-Division Multiplexing Over 96 km of Few-Mode Fiber Using Coherent 6 x 6 MIMO Processing,” *IEEE J. of Lightw. Technol.* **30**(4), 521–531 (2012).
- [5] S. P. Chan, C. E. Png, S. T. Lim, G. T. Reed, and V. M. N. Passaro, “Single-Mode and Polarization-Independent Silicon-on-Insulator Waveguides With Small Cross Section,” *IEEE J. of Lightw. Technol.* **23**(6), 2103–2111 (2005).
- [6] P. Dumon, W. Bogaerts, V. Wiaux, J. Wouters, S. Beckx, J. V. Campenhout, D. Taillaert, B. Luyssaert, P. Bienstman, and D. V. Thourhout, “Low-Loss SOI Photonic Wires and Ring Resonators Fabricated With Deep UV Lithography,” *IEEE Photon. Technol. Lett.* **16**(5), 1328–1330 (2004).
- [7] P. Dong, S. F. Preble, and M. Lipson, “All-optical compact silicon comb switch,” *Opt. Exp.* **15**(15), 9600–9605 (2007).

- [8] Q. Xu, B. Schmidt, S. Pradhan, and M. Lipson, “Micrometre-scale silicon electro-optic modulator.” *Nature* **435**, 325–327 (2005).
- [9] A. C. Ruege, S. Member, and R. M. Reano, “Sharp Fano Resonances From a Two-Mode Waveguide Coupled to a Single-Mode Resonator,” *IEEE J. of Lightw. Technol.* **28**(20), 2964–2968 (2010).
- [10] Y. Tanushi and S. Yokoyama, “Compact Multimode Optical Ring Resonators for Interconnection on Silicon Chips,” *Jpn. J. of Appl. Phys.* **46**(4B), 2364–2368 (2007).
- [11] A. Vorckel, M. Monster, W. Henschel, P. H. Bolivar, and H. Kurz, “Asymmetrically Coupled Silicon-On-Insulator Microring Resonators for Compact Add-Drop Multiplexers,” *IEEE Photon. Technol. Lett.* **15**(7), 921–923 (2003).
- [12] J. B. Driscoll, R. R. Grote, B. Souhan, J. I. Dadap, M. Lu, and R. M. Osgood, “Asymmetric Y junctions in silicon waveguides for on-chip mode-division multiplexing,” *Opt. Lett.* **38**(11), 1854–1856 (2013).
- [13] T. Uematsu, Y. Ishizaka, Y. Kawaguchi, K. Saitoh, and M. Koshiba, “Design of a Compact Two-Mode Multi/Demultiplexer Consisting of Multimode Interference Waveguides and a Wavelength-Insensitive Phase Shifter for Mode-Division Multiplexing Transmission,” *IEEE J. of Lightw. Technol.* **30**(15), 2421–2426 (2012).
- [14] H. Qiu, H. Yu, T. Hu, G. Jiang, H. Shao, P. Yu, J. Yang, and X. Jiang, “Silicon mode multi/demultiplexer based on multimode grating-assisted couplers,” *Opt. Exp.* **21**(15), 17904–17911 (2013).
- [15] Y. Ding, J. Xu, F. Da Ros, B. Huang, H. Ou, and C. Peucheret, “On-chip two-mode division multiplexing using tapered directional coupler-based mode multiplexer and demultiplexer,” *Opt. Exp.* **21**(8), 10376 (2013).
- [16] M. Greenberg and M. Orenstein, “Multimode add-drop multiplexing by adiabatic linearly tapered coupling,” *Opt. Exp.* **13**(23), 9381–9387 (2005).
- [17] A. Melloni, P. Monguzzi, R. Costa, and M. Martinelli, “Design of curved waveguides: the matched bend.” *J. Opt. Soc. Am. A* **20**, 130–137 (2003).
- [18] L. Soldano and E. Pennings, “Optical multi-mode interference devices based on self-imaging: principles and applications,” *IEEE J. of Lightw. Technol.* **13**(4), 615–627 (1995).

- [19] L. H. Gabrielli, D. Liu, S. G. Johnson, and M. Lipson, “On-chip transformation optics for multimode waveguide bends.” *Nat. Comm.* **3**, 1217 (2012).
- [20] M. Sumetsky, “Theory of adiabatic optical fiber and microfiber tapers,” in “Optical Fiber Communication Conference and Exposition and The National Fiber Optic Engineers Conference,” Technical Digest (CD) (Optical Society of America, 2006), paper OTuH2.
- [21] H. Yun, W. Shi, Y. Wang, L. Chrostowski, and N. A. F. Jaeger, “2 x 2 Adiabatic 3-dB Coupler on Silicon-on-Insulator Rib Waveguides,” *Proc. SPIE* **8915** (2013).
- [22] D. Lancaster, “The Math Behind Bezier Cubic Splines,” <http://www.tinaja.com/glib/cubemath.pdf>.
- [23] Y. Wang, J. Flueckiger, C. Lin, and L. Chrostowski, “Universal Grating Coupler Design,” *Proc. SPIE* **8915** (2013).

Chapter 5

Multiplexer, Demultiplexer, and Bezier Curve Characterization

The follow paper, *A Two-Mode Division Multiplexing Filter Demonstrated Using a SOI Ring Resonator*, provides details on the experimental performance of the (de)multiplexers and Bezier curves. The layout of the test structures used to obtain the results presented here are provided in Appendix C. The final operation of the multimode ring resonator is also presented.

Bryce Dorin performed all device design, analysis of the data, and wrote the paper. The device was fabricated at the University of Washington and tested using the automated testing station at the University of British Columbia. The paper was presented at the Optical Fiber Communication Conference (OFC) on March 13, 2014.

A Two-Mode Division Multiplexing Filter Demonstrated Using a SOI Ring Resonator

Bryce Dorin and Winnie N. Ye

Department of Electronics, Carleton University

1125 Colonel By Drive

Ottawa, Ontario, Canada K1S 5B6

[B. A. Dorin and W. N. Ye, “A Two-Mode Division Multiplexing Filter Demonstrated Using a SOI Ring Resonator,” *Optical Fiber Communication Conference and Exposition and the National Fiber Optic Engineers Conference (OFC/NFOEC)*, San Francisco, California, USA, 9-13 March, 2014.]

©OSA

5.1 Abstract

We present experimentally a two-mode ring resonator add/drop filter which exhibits distinct resonances for each mode. This device demonstrates strong potential for channel selective filtering in mode-division multiplexing systems.

5.2 Introduction

Silicon-on-insulator (SOI) is a promising platform with which to integrate optical systems for efficient on-chip communications. Wavelength-division multiplexing (WDM) on SOI has been extensively studied due to its capability to maximize data transmission [1]. However, the main drawback of WDM systems is their need to couple many different wavelengths onto the same chip. Alternatively, mode-division multiplexing (MDM) presents an opportunity in silicon photonics to increase the number of channels on a waveguide by increasing the number of guided modes it supports. Each mode of the waveguide is utilized as a separate data channel operating at a fixed wavelength. Most reported SOI devices require single-mode waveguides, and therefore are incompatible with the emerging MDM systems. In this paper we present a SOI ring resonator filter in a two-mode division multiplexing system.

The ring resonator configuration has been widely used for filtering signals in single-mode waveguides [2]. To demonstrate the practicality of ring resonators in MDM systems, an add/drop filter has been optimized for multimode waveguides fabricated on a 220 nm SOI wafer. The mask layout of the device is shown in Fig. 27, where the input, output, and ring waveguides are all designed to support the first two TE modes (TE_{00} and TE_{01}).

Figure 27: The mask layout for the fabricated ring resonator device. Asymmetric directional couplers serve as the multiplexer and demultiplexers. Bezier curves are utilized to provide minimal crosstalk in the multimode waveguide bends.

5.3 Optimization of (de)multiplexers and waveguide bends

Multiplexing and demultiplexing are fundamental operations in any multiple-input multiple-output system. For the demonstration of the multimode ring resonator shown in Fig. 27, there are three of these devices; one multiplexer at the input, and two demultiplexers at the through and drop ports. Asymmetrical directional couplers (ADCs) were determined to be ideal for this application as they could be cascaded to multiplex many modes, and were not prohibitively long or wavelength dependant [3]. These ADCs phase match a single-mode waveguide to the desired higher-order mode in the bus waveguide by optimizing the bus waveguide width. To improve fabrication tolerances, tapers have been introduced along the bus waveguide in the ADC [4], as illustrated in Fig. 28(b). Here the bus waveguide at the beginning of the ADC is 50 nm narrower than is required for phase matching, then is tapered along the length of the ADC, finally ending with a width 50 nm wider than the phase matching width. This allows phase matching to occur even if the waveguide widths differ from what is ideal after fabrication. The disadvantage of this design is that perfect phase matching only occurs instantaneously along the ADC, resulting in less than ideal coupling of the TE_{01} mode to the bus waveguide. Devices consisting of two ADCs that multiplex and demultiplex the TE_{01} mode were fabricated with various coupling lengths and bus waveguide center widths, where the nominal width of the single-mode input waveguide is 450 nm. Fig. 28 presents our experimental measurements of the excess loss of the TE_{01} mode per ADC, for both the uniform and tapered configurations. In each case the results from up to 3 devices were averaged. We conclude that the uniform ADC achieves higher peak coupling (less loss) yet is much more sensitive to waveguide center width, as predicted by theory. Average crosstalk was found to

be -20.9 and -28.3 dB for the tapered and uniform devices, respectively. Since the (de)multiplexers are critical components for the multimode ring resonator, we chose the more fabrication tolerant tapered ADC design, with a target center width of 930 nm and a length of 39 μm .

The transition from a straight to a bent waveguide (or vice versa) in the ring triggers the projection of a single input mode onto many output modes. In a MDM system this is an unacceptable source of crosstalk. To mitigate the crosstalk, the abrupt transition must be replaced by a smooth transition along the length of the curve. This requires a gradual change from the infinite radius of a straight waveguide to the small radius at the corner of the bend. These properties were found to exist in Bezier curves [6], which are displayed in the ring bends of Fig. 27. In these curves the region of transition between straight and bent waveguides is extended over several micrometers, in contrast to the abrupt transition in circular curves. The curves were optimized such that the transition was gradual enough to minimize mode mismatch, therefore achieving negligible crosstalk. Our experimental measurements concluded that the Bezier curves were able to exhibit a noticeable decrease in crosstalk compared to circular curves. The measured average crosstalk for 7 Bezier curves with a radius of 20 μm was -22.4 ± 2.6 dB per 180° bend, compared to -18.9 ± 2.5 dB for circular curves of the same radius.

(a)

(b)

Figure 28: The measured loss in dB for the TE_{01} mode as a function of coupling length and waveguide center width; for the (a) uniform, and (b) tapered ADC. [5]

5.4 Ring resonator performance

The transmittance at the through port of a two-mode ring resonator is shown in Fig. 29. Here it can be seen that both the TE_{00} and TE_{01} modes have clear and distinct resonances within the ring. The free spectral range of each resonance was found to be approximately 2 nm, but varied slightly with mode order due to each mode's distinctive effective index, which agrees well with simulations. Similar Q values of 22 000 and 26 000 were also found for the TE_{00} and TE_{01} modes, respectively. This was achieved by computing a directional coupler length which resulted in equal coupling between the ring and bus waveguides for each mode. Fig. 29 shows that the loss of TE_{01} was ~ 3 dB greater than that of TE_{00} . This was expected as incomplete coupling for the TE_{01} mode occurred at the multiplexing and demultiplexing stages. As well, the propagation loss is higher in the TE_{01} mode because of its larger electric field at the waveguide sidewalls. Due to the limitations of the test set-up, the output signals and crosstalk could not be measured simultaneously on a single device. The crosstalk levels at the through and the drop ports are obtained from a second device designed to be identical. From the crosstalk data it appears that the system effectively obtained a signal-to-crosstalk ratio (SXR) of more than 18 dB, which is greater than the extinction ratio (~ 12 dB), implying that crosstalk will be negligible for most applications. Finally, the resonances in Fig. 29 demonstrate that our two-mode ring resonator filter may drop no modes, one mode, or both modes simply by selecting specific wavelengths. This makes the filter very versatile in a MDM network.

Figure 29: Experimental results from our ring resonator for the TE_{00} and TE_{01} modes. The curves indicate that both modes were able to resonate in the structure with little crosstalk (XT).

5.5 Conclusions

We have experimentally demonstrated a two-mode ring resonator filter to add/drop zero, one, or two modes at specific wavelengths. Q values above 20 000, and a SXR above 18 dB were found for both modes over a 20 nm wavelength range. This work is useful in mode-division multiplexing systems for the selective switching of channels when the index of the multimode ring can be dynamically adjusted.

Acknowledgements

The authors acknowledge the excellent work of L. Chrostowski and J. Flueckiger in the Bezier bend design, and Y. Wang in the grating coupler design. We would also like to thank V. Donzella for the mask preparation, R. Bojko for the fabrication at the University of Washington Microfabrication/Nanotechnology User Facility, J.

Flueckiger and C. Lin for the experimental set-up, Wyichen Wu for conducting the measurements, and CMC Microsystems for their support.

List of References

- [1] B. G. Lee, X. Chen, A. Biberman, X. Liu, I.-W. Hsieh, C.-Y. Chou, J. I. Dadap, F. Xia, W. M. J. Green, L. Sekaric, Y. a. Vlasov, R. M. Osgood, and K. Bergman, “Ultrahigh-Bandwidth Silicon Photonic Nanowire Waveguides for On-Chip Networks,” *IEEE Photonics Technology Letters* **20**, 398–400 (2008).
- [2] P. Dumon, W. Bogaerts, V. Wiaux, J. Wouters, S. Beckx, J. V. Campenhout, D. Taillaert, B. Luyssaert, P. Bienstman, and D. V. Thourhout, “Low-Loss SOI Photonic Wires and Ring Resonators Fabricated With Deep UV Lithography,” *IEEE Photonics Technology Letters* **16**, 1328–1330 (2004).
- [3] D. Dai, J. Wang, and Y. Shi, “Silicon mode (de)multiplexer enabling high capacity photonic networks-on-chip with a single-wavelength-carrier light,” *Optics letters* **38**, 1422–4 (2013).
- [4] Y. Ding, J. Xu, F. Da Ros, B. Huang, H. Ou, and C. Peucheret, “On-chip two-mode division multiplexing using tapered directional coupler-based mode multiplexer and demultiplexer,” *Optics Express* **21**, 10,376 (2013).
- [5] Y. Wang, J. Flueckiger, C. Lin, and L. Chrostowski, “Universal Grating Coupler Design,” in Paper: OPTO-4-P-5, Photonics North, Ottawa, Canada, Jun 3-5 (2013).
- [6] H. Yun, W. Shi, Y. Wang, L. Chrostowski, and N. A. F. Jaeger, “2 x 2 Adiabatic 3-dB Coupler on Silicon-on-Insulator Rib Waveguides,” in Paper: OPTO-4-30-5, Photonics North, Ottawa, Canada, Jun 3-5 (2013).

Chapter 6

An Asymmetrically Coupled Multimode Ring Resonator

The following paper, *An Add-Drop Ring Resonator Constructed from Multimode Waveguides*, presents an alternative multimode ring resonator design that is distinct from the previous design in its short coupling lengths at the add and drop ports. This difference increases the versatility of the filter, as well as reduces device footprint. Asymmetrical coupling was also added to improve the device performance. The operation of the fabricated device is outlined in this paper.

Bryce Dorin performed all device design, testing of the device, analysis of the data, and wrote the paper. The device was fabricated at IMEC and tested using the new testing station in the Micro/NanoPhotonics Lab. The paper has been submitted for oral presentation at the 11th International Conference on Group IV Photonics in Paris, France, during the 27-29th of August, 2014.

An Add-Drop Ring Resonator Constructed from Multimode Waveguides

Bryce Dorin and Winnie N. Ye

Department of Electronics, Carleton University

1125 Colonel By Drive

Ottawa, Ontario, Canada K1S 5B6

[B. A. Dorin and W. N. Ye, “An Add-Drop Ring Resonator Constructed from Multimode Waveguides,” submitted for oral presentation at *11th International Conference on Group IV Photonics*, Paris, France, 27-29 August, 2014.]

6.1 Abstract

Silicon-on-insulator ring resonators supporting two waveguide modes for application in mode-division multiplexing circuits are presented. We demonstrate mode-selective filtering with approximately equal extinction ratios for each channel in the system.

6.2 Introduction

Silicon is a robust platform for both electrical and optical systems. It has been shown that efficient optical communication systems can be shrunk down to the nano-scale

using silicon waveguides [1]. These optical guides are commonly fabricated on the silicon-on-insulator (SOI) platform, which has tremendous advantages in terms of optical confinement and mature fabrication processes. Recent research demonstrated that communication speeds can be increased by using multimode waveguides [2]. Mode-division multiplexing (MDM) systems are able to utilize the higher-order modes as additional channels, effectively increasing the data transfer rate along the waveguide. It has been proposed that MDM systems on SOI could be operated using only a single laser source, which can be coupled to the chip before splitting into the necessary number of channels in separate waveguides. Each channel would then be modulated independently, and multiplexed onto the multimode bus waveguide [2]. This approach presents the designers with a less complicated solution for coupling input channels to the chip.

In this work, we develop a MDM system on the SOI platform to demonstrate a mode-selective filter. Our filter structure is composed of a ring resonator that supports two modes, TE_{00} and TE_{01} . The ring is coupled to add and drop waveguides that also support two modes. The effective index of each mode in the ring is unique, therefore different resonance conditions for each mode are found. Filtering then becomes possible by tuning the resonator such that the desired modes are resonant in the ring and dropped. Previously, we have designed multimode ring resonators for two modes [3], where the coupling for the two modes into the ring was made equal by using long coupling lengths. However, this approach limits potential filter designs in terms of the number of channels that can be supported and their quality factors, as well as the device footprint. We present here an alternative to the previous design by not restricting the coupling length at the through and drop ports. Instead, relatively short coupling lengths on the order of several microns were implemented. By introducing asymmetric coupling at the through and the drop ports, we were able to

increase the extinction ratio for the TE_{00} mode [4,5], and equate it to the extinction ratio of the TE_{01} mode.

6.3 Design and Demonstration

An optical microscope image of our fabricated multimode ring resonator composed of wire waveguides on 220 nm SOI is shown in Fig. 30. The fabrication took place at the IMEC foundry. Testing was performed using grating couplers known to have a 10.5 dB insertion loss [6]. The asymmetrical directional coupler (ADC) multiplexer and two ADC demultiplexers are designed for adding and removing the TE_{01} channel from the bus waveguide [2,3]. The length of the ADCs, labelled L_{ADC} , is set to 30 μm for maximum coupling into the TE_{01} mode. The input/output waveguides connecting the ADCs to the grating couplers are 450 nm wide. To phase match the TE_{00} mode of the input waveguides to the TE_{01} mode of the bus waveguide, a bus waveguide width of 930 nm was necessary. This width was maintained for the ring resonator structure as shown in Fig. 30. The length of directional coupler at the ring's through and drop ports (L_{DC}) was set to 5 μm , which is relatively short compared to previous designs [3]. Simulations have shown the bends of a ring resonator to be a large source of crosstalk. This issue has been addressed through the use of Bezier curves in the ring, where the radius (R) was 20 μm .

The coupling into the ring for the TE_{00} mode was substantially smaller than that of the TE_{01} mode, due to the weaker horizontal evanescent field of the mode. This leads to elevated undercoupling at the through port and weaker extinction ratios for this modal channel. To improve the extinction ratio for TE_{00} , the directional coupler gap at the drop port (g_{drop}) in Fig. 30 was widened, which has been demonstrated before as a viable means for achieving critical coupling [4]. The condition for critical

Figure 30: An optical microscope image of the two-mode ring resonator filter.

coupling dictates that the transmission coefficients at each directional coupler of the ring resonator should be related such that $t_a = \alpha t_b$ [5]. The variable α represents the round trip attenuation of the signal, and t_a and t_b are the transmission coefficients at the through and drop ports, respectively. In this model they are related to the coupling coefficients at the through and drop ports, κ_a and κ_b , through the relation $t^2 + \kappa^2 = 1$. By increasing g_{drop} , the value of t_b can be increased to approach critical coupling when loss is present in the ring. A peak extinction ratio for TE_{00} was found when $g_{through} = 200$ nm and $g_{drop} = 230$ nm. When increasing g_{drop} , it was found that the TE_{01} mode became overcoupled at the through port and therefore its extinction ratio was lowered. At $g_{drop} = 230$ nm the extinction ratios for the two modes became approximately equal.

The low κ values for the TE_{00} mode in the ring makes the extinction ratio for the channel very sensitive to α . Increasing losses within the ring can quickly degrade the extinction ratio, therefore low-loss propagation within the ring was desired. The use

of the adiabatic Bezier curves in the corners of the ring removes the abrupt straight-to-bend transition that can result in bending loss, hence leading to a more optimized performance.

Figure 31: The through and drop port transmittance for TE_{00} and TE_{01} in the multimode ring resonator filter. The extinction ratios for all channels and output ports are labelled.

In Fig. 31 the transmittance at the through and drop ports for both modal channels is presented. The potential for mode filtering is demonstrated in this device. The choice of which mode is filtered can be made simply by adjusting the system operating wavelength to the resonance of that mode, or alternatively by adjusting the effective ring length to match the mode resonance with the operating wavelength. Both modes exhibited extinction ratios of at least 10 dB at the through port, and over 22 dB extinction ratios at the drop port, making the filter suitable for either mode.

Other filter parameters extracted from Fig. 31 are summarized in Table 3. The round trip loss, α , for each mode in the resonator was estimated from their extinction ratios and quality factors. For the fundamental mode the loss was limited to just 0.01 dB, allowing for the relatively high extinction ratio that was observed. The signal-to-crosstalk ratio (SXR) within the ring was found to be above 20 dB across the wavelength range considered, except for one anomalous spike in TE_{01} SXR at the drop port which reduced the level to 14 dB. The simulated and experimental free spectral range (FSR) values agree well with each other for both modes, confirming we are measuring the intended modal responses. Finally, one can see the quality factor (Q) of the TE_{00} mode is much larger than that of the TE_{01} mode. This is because the TE_{00} mode experienced much smaller coupling into the ring, reducing the full-width half-max of the peak.

Table 3: Properties of the TE_{00} and TE_{01} modes in the multimode ring resonator over the wavelength range 1.558 μm to 1.568 μm .

Property	TE_{00}	TE_{01}
Through port loss (exp.)	16 dB	18 dB
Drop port loss (exp.)	25 dB	24 dB
Through port extinction ratio (exp.)	10 dB	13 dB
Drop port extinction ratio (exp.)	28 dB	22 dB
Minimum SXR at the through port (exp.)	23 dB	21 dB
Minimum SXR at the drop port (exp.)	21 dB	14 dB
FSR (simulated)	4.124 nm	3.626 nm
FSR (exp.)	4.157 nm	3.666 nm
Q (exp.)	400 000	8 000
α (exp.)	-0.01 dB	-0.02 dB

6.4 Conclusions

A two-mode add-drop ring resonator demonstrated filtering for both modes using a short coupling length. Asymmetrical coupling and low loss propagation allowed for extinction ratios of at least 10 dB for both modes. The signal-to-crosstalk levels were above this value for all modes and output ports. This device has potential for passive mode-selective filtering in mode-division multiplexing systems.

Acknowledgments

The authors would like to thank CMC Microsystems for their support in fabrication. We are grateful to M. Vachon for helping with the microscope image.

List of References

- [1] B. G. Lee, X. Chen, A. Biberman, X. Liu, I.-W. Hsieh, C.-Y. Chou, J. I. Dadap, F. Xia, W. M. J. Green, L. Sekaric, Y. a. Vlasov, R. M. Osgood, and K. Bergman, “Ultrahigh-Bandwidth Silicon Photonic Nanowire Waveguides for On-Chip Networks,” *IEEE Photon. Technol. Lett.*, vol. 20, no. 6, pp. 398–400, 2008.
- [2] D. Dai, J. Wang, and Y. Shi, “Silicon mode (de)multiplexer enabling high capacity photonic networks-on-chip with a single-wavelength-carrier light,” *Opt. Lett.*, vol. 38, no. 9, pp. 1422–1424, May 2013.
- [3] B. A. Dorin and W. N. Ye, “Two-mode division multiplexing in a silicon-on-insulator ring resonator,” *Opt. Exp.*, vol. 22, no. 4, pp. 4547–4558, Feb. 2014.
- [4] P. Dong, S. F. Preble, and M. Lipson, “All-optical compact silicon comb switch,” *Opt. Exp.*, vol. 15, no. 15, pp. 9600–9605, Jul. 2007.
- [5] A. Vorckel, M. Monster, W. Henschel, P. H. Bolivar, and H. Kurz, “Asymmetrically coupled silicon-on-insulator microring resonators for compact add-drop multiplexer,” *IEEE Photon. Technol. Lett.*, vol. 15, no. 7, pp. 921–923, 2003.
- [6] Y. Wang, J. Flueckiger, C. Lin, and L. Chrostowski, “Universal Grating Coupler Design,” *Proc. SPIE Photonics North 2013*, SPIE 8915, vol. 8915, pp. 89150Y-1 – 89150Y-6.

Chapter 7

Conclusions

This thesis covers the detailed design and analysis of multimode waveguides, and applies them in ring resonator structures on the silicon-on-insulator (SOI) platform. Ultimately, we demonstrate that mode-selective filtering can be achieved using these rings for mode-division multiplexing (MDM) systems, just as wavelength-selective filtering can be achieved in wavelength-division multiplexing (WDM) systems. This is a large step towards making MDM a viable alternative to WDM in silicon photonics. MDM also presents advantages compared to WDM as it can be operated using only a single laser source. This is possible as channels with the same wavelength of light can be separated into different modes on a multimode waveguide. This allows one source to be coupled to the chip, where it is split into the different channels.

Numerical methods such as the finite-difference method, mode-film matching method, finite-difference time-domain method, and the eigen-mode expansion method played a large role in optimizing the waveguide and device dimensions. A method for determining the single-mode condition (SMC) was proposed and demonstrated using these simulations. Simulations were also performed for the multiplexers, demultiplexers, Bezier curves, and ring resonators. The results of these simulations were confirmed using fabricated devices from foundry services.

The method for determining when a waveguide geometry supports only one mode, or many modes, was shown for a variety of rib waveguide geometries at the 1310 nm wavelength range. This method used the magnitude of the higher-order mode loss, and the trends in mode loss to determine the point where a waveguide began to support more than one mode. We found that for large SOI heights a simple and convenient SMC could be determined. However, at small SOI heights more complex phenomena such as magic widths began to appear. This lead to the conclusion that waveguides with small SOI heights would need to be evaluated on a case-by-case basis to determine their SMC.

The design and in-depth simulations of the multiplexers and Bezier curves has been presented. Asymmetric directional couplers (ADCs) were used for multiplexing, which are known to be sensitive to fabrication defects. Therefore a resilience to fabrication defects was the focus of the ADC design through simulations, which lead to the use of a taper along the length of the bus waveguide in the ADC. Bezier curves were introduced in the ring resonator bends to minimize crosstalk between modes while maintaining a small bend radius. A variety of Bezier curves at different radii were simulated to determine the ideal parameters for reducing crosstalk.

Through the fabrication of multiplexer/demultiplexer and Bezier curve test structures, we were able to confirm the results seen in simulations. Simulations and experiments both found that the tapered ADC would not be able to achieve the same low loss for the TE_{01} mode in the multiplexer as the uniform ADC could. However, both sets of results conclude that the tapered ADC design is much less sensitive to variation in waveguide width. For this reason it was chosen for use in the final ring resonator structure. Bezier curves showed drastic (20 dB) improvement in crosstalk levels through simulations. Experimentally, the improvement is a more modest 3.5 dB. This is due to the fact that fabrication defects will also lead to coupling between

modes regardless of how the curve is designed. These fabrication defects could not be included in simulations. However, experiments did confirm the Bezier curves to be superior to circular curves in terms of suppressing crosstalk, and were used in the fabricated ring resonators.

The fabricated ring resonators demonstrated the desired ability to filter modes selectively. In initial designs the quality factors for each mode were made nearly equal by matching their level of coupling into the ring. The loss of the TE_{01} mode was found to be 3-4 dB higher than the fundamental mode due to loss in the multiplexer and demultiplexers, as well as higher propagation losses. The signal-to-crosstalk ratio for both modes and both output ports stayed above 18 dB, which is reasonable for SOI devices. Later designs removed the condition that the quality factors of each mode must be equal in the filter. The advantage of this approach is that the ring resonator can use any coupling length at the through and drop ports, allowing for more design freedom and smaller device footprints. We found that the extinction ratio of the fundamental mode initially suffered using this design approach. However, using asymmetrical coupling in the ring, we were able to boost the extinction ratio of this channel up to 10 dB. The crosstalk in the device stayed above 14 dB.

7.1 Future Work

Overall this thesis provides a significant contribution to the study of MDM on SOI. Using the devices and methods we propose, a complex multiple-input multiple-output communication system could be implemented on silicon chips for communication between cores, or from core to memory. Future work on the current ring resonator design could include optimizing the fabrication process to reduce the loss and crosstalk for each mode. To do this the quality of the waveguide sidewalls must be improved by

reducing the surface roughness that leads to scattering. Less surface roughness may indeed become possible through foundry services as the technology and processes improve with time. One could also use different waveguide designs to improve these characteristics. For example, using wider waveguides so that the modes have less overlap with the roughness at the sidewalls may reduce crosstalk and loss. A challenge associated with this approach is that as the waveguide is widened more modes will be supported. The designer will then need to ensure that these extra modes do not increase crosstalk levels.

More guided modes can also be an advantage, as these modes could then be used as additional channels in a MDM system. In future work it would be interesting to see if there are any limitations to the number of modal channels the MDM system we have proposed could support. For example, are there any limitations to the number of modes that can be multiplexed and demultiplexed using ADCs? As well, one should investigate if the Bezier curve will remain a good option for reducing crosstalk in ring bends as waveguides get wider. Finally, the maximum number of modal channels the ring resonator can support while still providing good filter characteristics is not clear at this time. These are important questions to be answered if MDM is to be a viable alternative to WDM in SOI circuits, as we have already seen WDM reach 32 channels experimentally [1].

In the future, the ring resonators designed here could be extended to switching applications, rather than being limited to passively filtering modes. This has been demonstrated before in single-mode ring resonators by adding active refractive index modulation in the ring waveguides [27, 28]. Switching is important in large communication networks, and would become necessary when MDM networks on SOI begin using many bus waveguides communicating between a large number of nodes, where each bus waveguide carries several modal channels.

Finally, I would like to propose improvements to the fiber array testing set-up at the Carleton Micro/NanoPhotonics Lab. Currently, the set-up can achieve accurate alignment and losses comparable to the testing set-up at the University of British Columbia. However, the alignment program is limited in that it requires an operator to perform alignment and measurement for each device independently. A much more convenient solution would be to have the operator align the fiber array to one device, then using the relative coordinates of the other devices extracted from the mask layout, have the stages move the array automatically to each device on the chip, measuring and saving the data at each point. This would significantly reduce the testing time required for a chip. A proposal has been prepared for a probe station that would not only allow for automated testing, but also for simultaneous optical and electrical probing of devices.

List of References

- [1] B. G. Lee, X. Chen, A. Biberman, X. Liu, I.-W. Hsieh, C.-Y. Chou, J. I. Dadap, F. Xia, W. M. J. Green, L. Sekaric, Y. a. Vlasov, R. M. Osgood, and K. Bergman, “Ultrahigh-Bandwidth Silicon Photonic Nanowire Waveguides for On-Chip Networks,” *IEEE Photonics Technology Letters*, vol. 20, no. 6, pp. 398–400, 2008. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4451195>
- [2] D. Miller, “Device Requirements for Optical Interconnects to Silicon Chips,” *Proceedings of the IEEE*, vol. 97, no. 7, pp. 1166–1185, Jul. 2009. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=5071309>
- [3] J. Liu, X. Sun, L. C. Kimerling, and J. Michel, “Direct-gap optical gain of Ge on Si at room temperature.” *Optics letters*, vol. 34, no. 11, pp. 1738–40, Jun. 2009. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/19488166>
- [4] A. W. Fang, H. Park, O. Cohen, R. Jones, M. J. Paniccia, and J. E. Bowers, “Electrically pumped hybrid AlGaInAs-silicon evanescent laser.” *Optics express*, vol. 14, no. 20, pp. 9203–10, Oct. 2006. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/19529301>
- [5] R.-j. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel, “Capacity Limits of Optical Fiber Networks,” *Journal of Lightwave Technology*, vol. 28, no. 4, pp. 662–701, 2010.
- [6] P. J. Winzer, “Energy-Efficient Optical Transport Capacity Scaling Through Spatial Multiplexing,” *IEEE Photonics Technology Letters*, vol. 23, no. 13, pp. 851–853, Jul. 2011. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=5752818>

- [7] R. Ryf, S. Randel, A. H. Gnauck, C. Bolle, A. Sierra, S. Mumtaz, M. Esmaelpour, E. C. Burrows, R.-j. Essiambre, P. J. Winzer, D. W. Peckham, A. H. Mccurdy, and R. Lingle, "Mode-Division Multiplexing Over 96 km of Few-Mode Fiber Using Coherent 6 x 6 MIMO Processing," *Journal of Lightwave Technology*, vol. 30, no. 4, pp. 521–531, 2012.
- [8] D. Dai, J. Wang, and Y. Shi, "Silicon mode (de)multiplexer enabling high capacity photonic networks-on-chip with a single-wavelength-carrier light," *Optics letters*, vol. 38, no. 9, pp. 1422–4, May 2013. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/23632505>
- [9] L. Soldano and E. Pennings, "Optical multi-mode interference devices based on self-imaging: principles and applications," *Journal of Lightwave Technology*, vol. 13, no. 4, pp. 615–627, Apr. 1995. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=372474>
- [10] R. Soref and B. Bennett, "Electrooptical effects in silicon," *IEEE Journal of Quantum Electronics*, vol. 23, no. 1, pp. 123–129, Jan. 1987. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1073206>
- [11] P. D. Hewitt and G. T. Reed, "Improving the Response of Optical Phase Modulators in SOI by Computer Simulation," *Journal of Lightwave Technology*, vol. 18, no. 3, pp. 443–450, 2000.
- [12] L. Colace, G. Masini, and G. Assanto, "Ge-on-Si Approaches to the Detection of Near-Infrared Light," *IEEE Journal of Quantum Electronics*, vol. 35, no. 12, pp. 1843–1852, 1999.
- [13] J. Liu, D. D. Cannon, K. Wada, Y. Ishikawa, S. Jongthammanurak, D. T. Danielson, J. Michel, and L. C. Kimerling, "Tensile strained Ge p-i-n photodetectors on Si platform for C and L band telecommunications," *Applied Physics Letters*, vol. 87, no. 1, p. 011110, 2005. [Online]. Available: <http://link.aip.org/link/APPLAB/v87/i1/p011110/s1&Agg=doi>
- [14] J. Mathews, R. Roucka, C. Weng, J. Tolle, J. Menéndez, and J. Kouvetakis, "Germanium p-i-n photodiode on silicon for integrated photonic applications," in *Silicon Photonics V*, J. A. Kubby and G. T. Reed, Eds., vol. 7606, Feb. 2010, pp. 76 061L–76 061L–8. [Online]. Available: <http://proceedings.spiedigitallibrary.org/proceeding.aspx?doi=10.1117/12.855384>

- [15] B.-T. Lee and S.-Y. Shin, "Mode-order converter in a multimode waveguide." *Optics letters*, vol. 28, no. 18, pp. 1660–2, Sep. 2003. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/13677528>
- [16] T. Uematsu, Y. Ishizaka, Y. Kawaguchi, K. Saitoh, and M. Koshiba, "Design of a Compact Two-Mode Multi/Demultiplexer Consisting of Multimode Interference Waveguides and a Wavelength-Insensitive Phase Shifter for Mode-Division Multiplexing Transmission," *Journal of Lightwave Technology*, vol. 30, no. 15, pp. 2421–2426, Aug. 2012. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6202311>
- [17] J. B. Driscoll, R. R. Grote, B. Souhan, J. I. Dadap, M. Lu, and R. M. Osgood, "Asymmetric Y junctions in silicon waveguides for on-chip mode-division multiplexing." *Optics letters*, vol. 38, no. 11, pp. 1854–6, Jun. 2013. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/23722767>
- [18] H. Qiu, H. Yu, T. Hu, G. Jiang, H. Shao, P. Yu, J. Yang, and X. Jiang, "Silicon mode multi/demultiplexer based on multimode grating-assisted couplers," *Optics Express*, vol. 21, no. 15, pp. 17 904–17 911, 2013.
- [19] Y. Ding, J. Xu, F. Da Ros, B. Huang, H. Ou, and C. Peucheret, "On-chip two-mode division multiplexing using tapered directional coupler-based mode multiplexer and demultiplexer," *Optics Express*, vol. 21, no. 8, p. 10376, Apr. 2013. [Online]. Available: <http://www.opticsinfobase.org/abstract.cfm?URI=oe-21-8-10376>
- [20] M. Greenberg and M. Orenstein, "Simultaneous dual mode add/drop multiplexers for optical interconnects buses," *Optics Communications*, vol. 266, no. 2, pp. 527–531, Oct. 2006. [Online]. Available: <http://linkinghub.elsevier.com/retrieve/pii/S0030401806005530>
- [21] K. Okamoto, *Fundamental of Optical Waveguides*, 2nd ed. Elsevier, 2006.
- [22] R. A. Soref, J. Schmidtchen, and K. Petermann, "Large Single-Mode Rib Waveguides in GeSi-Si and Si-on-SO₂," *IEEE Journal of Quantum Electronics*, vol. 27, no. 8, pp. 1971–1974, 1991.
- [23] S. Pogossian, L. Vescan, and a. Vonsovici, "The single-mode condition for semiconductor rib waveguides with large cross section," *Journal of Lightwave Technology*, vol. 16, no. 10, pp. 1851–1853, 1998. [Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=721072>

- [24] S. P. Chan, C. E. Png, S. T. Lim, G. T. Reed, and V. M. N. Passaro, "Single-Mode and Polarization-Independent Silicon-on-Insulator Waveguides With Small Cross Section," *Journal of Lightwave Technology*, vol. 23, no. 6, pp. 2103–2111, 2005.
- [25] D. Dai and Z. Sheng, "Numerical analysis of silicon-on-insulator ridge nanowires by using a full-vectorial finite-difference method mode solver," *Journal of the Optical Society of America B*, vol. 24, no. 11, p. 2853, 2007. [Online]. Available: <http://www.opticsinfobase.org/abstract.cfm?URI=josab-24-11-2853>
- [26] P. Dumon, W. Bogaerts, V. Wiaux, J. Wouters, S. Beckx, J. V. Campenhout, D. Taillaert, B. Luyssaert, P. Bienstman, and D. V. Thourhout, "Low-Loss SOI Photonic Wires and Ring Resonators Fabricated With Deep UV Lithography," *IEEE Photonics Technology Letters*, vol. 16, no. 5, pp. 1328–1330, 2004.
- [27] P. Dong, S. F. Preble, and M. Lipson, "All-optical compact silicon comb switch," *Optics express*, vol. 15, no. 15, pp. 9600–5, Jul. 2007. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/19547308>
- [28] Q. Xu, B. Schmidt, S. Pradhan, and M. Lipson, "Micrometre-scale silicon electro-optic modulator." *Nature*, vol. 435, no. 7040, pp. 325–7, May 2005. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/15902253>
- [29] A. Yariv, "Universal relations for coupling of optical power between microresonators and dielectric waveguides," *Electronics Letters*, vol. 36, no. 4, pp. 321–322, 2000.
- [30] G. T. Reed, *Silicon Photonics: state of the art*. Sussex, England: John Wiley & Sons Ltd, 2008.
- [31] Z. Zhu and T. Brown, "Full-vectorial finite-difference analysis of microstructured optical fibers." *Optics Express*, vol. 10, no. 17, pp. 853–64, Aug. 2002. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/19451938>
- [32] A. S. Sudbo, "Film mode matching : a versatile numerical method for vector mode field calculations in dielectric waveguides," *Pure Applied Optics*, vol. 2, pp. 211–233, 1993.
- [33] K. S. Yee, "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," *IEEE Transactions on Antennas and Propagation*, vol. 14, no. 3, pp. 302–307, 1966.

- [34] D. F. G. Gallagher and T. P. Felici, “Eigenmode Expansion Methods for Photonics - Pros and Cons,” in *Photonics West*, 2003, pp. 1–14.
- [35] J.-P. Berenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *Journal of Computational Physics*, vol. 114, pp. 185–200, Oct. 1994. [Online]. Available: <http://linkinghub.elsevier.com/retrieve/pii/S0021999184711594>
- [36] Y. Wang, J. Flueckiger, C. Lin, and L. Chrostowski, “Universal Grating Coupler Design,” in *Photonics North*, 2013.

Appendix A

MATLAB code for simulating the ring resonator

This MATLAB program runs a simulation of the ring resonator for two modes. The wavelength dependant transmission of the two modes at the through and the drop ports is produced. The code uses the matrix formulation presented in Section 1.4. The wavelength dependant variables κ , t , and n_{eff} are simulated using numerical methods and then approximated in the code using a linear line of best fit. Properties such as ring section length and coupler length are defined in the code as well. An estimate of mode propagation loss in the ring can also be added.

```
function [ lambda,T_TE00 ] = RingResonator_DCCoupled( )
%Simulates the response of a Ring Resonator

%%Input Parameters
loss_TE01=0;      %dB/cm

coupler_length_bend_TE00=8;
coupler_length_bend_TE01=8;

coupler_length=95;      %e-6 m
ring_section_length=95e-6+8e-6+34e-6;

alpha_TE01_ring=2*log((10^(-loss_TE01/10)))*100;      %losses due to propagation in
the ring in /m
alpha_TE00_ring=2*log(1-(1-(10^(-loss_TE01/10)))/5.33)*100;

L=50000;      %number of data points
```

```

lambda_start=1.53;          %um
lambda_stop=1.57;          %um

D_TE00=zeros(L+1,1);
D_TE01=zeros(L+1,1);
T_TE00=zeros(L+1,1);
T_TE01=zeros(L+1,1);
lambda=zeros(L+1,1);

for i=1:L+1
lambda(i)=lambda_start+(i-1)*(lambda_stop-lambda_start)/L;

%%Propagation Constants
n_eff_TE00=3.8344-0.7052*lambda(i);
n_eff_TE01=4.3081-1.2306*lambda(i);

beta_TE00=2*pi*n_eff_TE00/(lambda(i)*1e-6);
beta_TE01=2*pi*n_eff_TE01/(lambda(i)*1e-6);

%%g = 200 nm coupling
Lc_1_TE00=3516.4-1985.5*lambda(i);
Lc_1_TE01=554.35-319.3*lambda(i);

Lc_2_TE00=Lc_1_TE00;
Lc_2_TE01=Lc_1_TE01;

%%Input Coupler
%%TE00
coupler_bar_1_TE00=cos((pi/2)*((coupler_length+coupler_length_bend_TE00)/Lc_1_TE00));
coupler_cross_1_TE00=sin((pi/2)*((coupler_length+coupler_length_bend_TE00)/Lc_1_TE00));

%%TE01
coupler_bar_1_TE01=cos((pi/2)*((coupler_length+coupler_length_bend_TE01)/Lc_1_TE01));
coupler_cross_1_TE01=sin((pi/2)*((coupler_length+coupler_length_bend_TE01)/Lc_1_TE01));

%%Output Coupler
%%TE00
coupler_bar_2_TE00=cos((pi/2)*((coupler_length+coupler_length_bend_TE00)/Lc_2_TE00));
coupler_cross_2_TE00=sin((pi/2)*((coupler_length+coupler_length_bend_TE00)/Lc_2_TE00));

%%TE01
coupler_bar_2_TE01=cos((pi/2)*((coupler_length+coupler_length_bend_TE01)/Lc_2_TE01));
coupler_cross_2_TE01=sin((pi/2)*((coupler_length+coupler_length_bend_TE01)/Lc_2_TE01));

%%Converting to proper units
t_1_TE00=(coupler_bar_1_TE00);
k_1_TE00=(coupler_cross_1_TE00);

t_1_TE01=(coupler_bar_1_TE01);
k_1_TE01=(coupler_cross_1_TE01);

t_2_TE00=(coupler_bar_2_TE00);
k_2_TE00=(coupler_cross_2_TE00);

t_2_TE01=(coupler_bar_2_TE01);
k_2_TE01=(coupler_cross_2_TE01);

```

```

%%%%Calculating Outputs

%%%%First coupler
C_12_TE00= (1/k_1_TE00)*[-t_1_TE00,1;
                        -(k_1_TE00^2+t_1_TE00^2),t_1_TE00];

C_12_TE01= (1/k_1_TE01)*[-t_1_TE01,1;
                        -(k_1_TE01^2+t_1_TE01^2),t_1_TE01];

%%%%Second coupler
C_34_TE00= (1/k_2_TE00)*[t_2_TE00,-1;
                        k_2_TE00^2+t_2_TE00^2,-t_2_TE00];

C_34_TE01= (1/k_2_TE01)*[t_2_TE01,-1;
                        k_2_TE01^2+t_2_TE01^2,-t_2_TE01];

%%%%180 Degree Curve
P_23_TE00= [0, exp(ring_section_length*(-1i*beta_TE00+alpha_TE00_ring/2));
            exp(ring_section_length*(1i*beta_TE00-alpha_TE00_ring/2)),0];

P_23_TE01= [0, exp(ring_section_length*(-1i*beta_TE01+alpha_TE01_ring/2));
            exp(ring_section_length*(1i*beta_TE01-alpha_TE01_ring/2)),0];

%%%%Transfer Matrices
M_TE00=C_12_TE00*P_23_TE00*C_34_TE00;

M_TE01=C_12_TE01*P_23_TE01*C_34_TE01;

%%%%Drop Port
D_TE00(i) = (1/M_TE00(1,2))*conj(1/M_TE00(1,2));

D_TE01(i) = (1/M_TE01(1,2))*conj(1/M_TE01(1,2));

%%%%Through Port
T_TE00(i) = ((1/M_TE00(1,2))*M_TE00(2,2))*conj((1/M_TE00(1,2))*M_TE00(2,2));

T_TE01(i) = ((1/M_TE01(1,2))*M_TE01(2,2))*conj((1/M_TE01(1,2))*M_TE01(2,2));

end

figure
plot(lambda,D_TE00,lambda,D_TE01)
xlabel('Wavelength (um)')
ylabel('Transmission')
title('TE00 Drop Port')
legend('TE00','TE01')

figure
plot(lambda,T_TE00,lambda,T_TE01)
xlabel('Wavelength (um)')
ylabel('Transmission')
title('TE00 Through Port')

end

```

Appendix B

Lumerical simulation scripts

B.1 Bezier Curves

This is the Lumerical script used to draw and simulate the Bezier curves using FDTD. The results from these simulations are presented in Chapter 4. The curve radius (R), waveguide width (W_{wg}), *wavelength*, and Bezier parameter (B) can all be altered in the code. As well, the code allows for two levels of accuracy in the simulation. One is fast and approximate, while the other is slow and accurate.

```
# 180 degree bezair curve simulations simulations (2 bezair curves)

deleteall;

newproject;

wavelength=1550e-9;

Wwg= 980e-9;           #width
R=5e-6;                #curve radius
B=0;                   #bezair parameter

SIMDRAFT = 1; # 1= fast , 0=accurate

materials; # creates a dispersive material model in Lumerical.

if (SIMDRAFT==1) {
    diff=0.01;
    MESH = 1; # test ,
    FREQPTS = 1; # fast test
    FDTD_above=200e-9; # Extra simulation volume added, 0.5 um on top and
                        bottom
```

```

        FDTD_below=200e-9;
        BC = "metal";    # boundary conditions on non-critical sides.
        Autoshutoff=5e-2;
    } else {
        diff=0.005;
        MESH = 3;    # final = 4
        FREQ_PTS = 100;
        FDTD_above=800e-9;    # Extra simulation volume added, 0.5 um on top and
            bottom
        FDTD_below=800e-9;
        BC = "PML";    # boundary conditions on non-critical sides.
        Autoshutoff=1e-5;
    }

    setglobalsource("center frequency",c/wavelength);
    setglobalsource("frequency span",c/wavelength/5);
    Material_Si = "Si (Silicon) - Dispersive & Lossless";
    Thickness_Si=0.22e-6;
    Material_Ox ="SiO2 (Glass) - Const";

    maxvzWAFER=Thickness_Si+FDTD_above;    minvzWAFER=-FDTD_below;

#####Bezair Points
x0=0;    y0=-R;    #start of first curve
x1=(1-B)*R;    y1=-R;
x2=R;    y2=-(1-B)*R;
x3=R;    y3=0;    #end of first curve
x4=R;    y4=(1-B)*R;
x5=(1-B)*R;    y5=R;
x6=0;    y6=R;

#####Equation Space
A1=x3-3*x2+3*x1-x0;    E1=y3-3*y2+3*y1-y0;
B1=3*x2-6*x1+3*x0;    F1=3*y2-6*y1+3*y0;
C1=3*x1-3*x0;    G1=3*y1-3*y0;
D1=x0;    H1=y0;

A2=x3-3*x4+3*x5-x6;    E2=y3-3*y4+3*y5-y6;
B2=3*x4-6*x5+3*x6;    F2=3*y4-6*y5+3*y6;
C2=3*x5-3*x6;    G2=3*y5-3*y6;
D2=x6;    H2=y6;

#####Calculating the curves the curves
xx1=[0:diff:1];
yy1=[0:diff:1];

xx2=[0:diff:1];
yy2=[0:diff:1];

N=length(xx1);

t=0;
for(i=1:N)
{
    xx1(i)=A1*t^3+B1*t^2+C1*t+D1;
    yy1(i)=E1*t^3+F1*t^2+G1*t+H1;

    xx2(i)=A2*t^3+B2*t^2+C2*t+D2;
    yy2(i)=E2*t^3+F2*t^2+G2*t+H2;

    t=t+diff;
}

dxx1=[0:diff:1];

```

```

dyy1=[0:diff:1];

dxx2=[0:diff:1];
dyy2=[0:diff:1];

dt=diff/2;

for (i=2:N-1)
{
    dxx1(i)=(xx1(i+1)-xx1(i-1))/dt;
    dyy1(i)=(yy1(i+1)-yy1(i-1))/dt;

    dxx2(i)=(xx2(i+1)-xx2(i-1))/dt;
    dyy2(i)=(yy2(i+1)-yy2(i-1))/dt;
}
dxx1(1)=dxx1(2);
dxx1(N)=dxx1(N-1);
dyy1(1)=dyy1(2);
dyy1(N)=dyy1(N-1);

dxx2(1)=dxx2(2);
dxx2(N)=dxx2(N-1);
dyy2(1)=dyy2(2);
dyy2(N)=dyy2(N-1);

Normalx1=[0:diff:1];
Normaly1=[0:diff:1];

Curvex1=[0:diff:1];
Curvey1=[0:diff:1];

Normalx2=[0:diff:1];
Normaly2=[0:diff:1];

Curvex2=[0:diff:1];
Curvey2=[0:diff:1];

wg1_x1=[0:diff:1];
wg1_x2=[0:diff:1];

wg1_y1=[0:diff:1];
wg1_y2=[0:diff:1];

wg2_x1=[0:diff:1];
wg2_x2=[0:diff:1];

wg2_y1=[0:diff:1];
wg2_y2=[0:diff:1];

for (i=1:N)
{
    Normaly1(i)=dxx1(i)/(dxx1(i)^2+dyy1(i)^2)^0.5;
    Normalx1(i)=-dyy1(i)/(dxx1(i)^2+dyy1(i)^2)^0.5;

    Normaly2(i)=dxx2(i)/(dxx2(i)^2+dyy2(i)^2)^0.5;
    Normalx2(i)=-dyy2(i)/(dxx2(i)^2+dyy2(i)^2)^0.5;

    wg1_x1(i)=xx1(i)+Normalx1(i)*(Wwg/2);
    wg1_x2(i)=xx1(i)-Normalx1(i)*(Wwg/2);

    wg1_y1(i)=yy1(i)+Normaly1(i)*(Wwg/2);
    wg1_y2(i)=yy1(i)-Normaly1(i)*(Wwg/2);

    wg2_x1(i)=xx2(i)+Normalx2(i)*(Wwg/2);
    wg2_x2(i)=xx2(i)-Normalx2(i)*(Wwg/2);

```

```

        wg2_y1(i)=yy2(i)+Normaly2(i)*(Wwg/2);
        wg2_y2(i)=yy2(i)-Normaly2(i)*(Wwg/2);
    }

#####Drawing the curves

    addgroup; set("name","curve"); groupscope("curve");

    addrect;
    set("y span",Wwg);
    set("x max",0);
    set("x min",-4e-6);
    set("y",-R);

    copy;
    set("y",R);

    V=matrix(2,4);
    V(1,1)=0;
    V(1,2)=0;
    V(1,3)=wg1_x1(1);
    V(1,4)=wg1_x2(1);
    V(2,1)=-R-Wwg/2;
    V(2,2)=-R+Wwg/2;
    V(2,3)=wg1_y1(1);
    V(2,4)=wg1_y2(1);
    addpoly;
    set("vertices",V);

    for(i=1:N-1)
    {
        V=matrix(2,4);
        V(1,1)=wg1_x1(i);
        V(1,2)=wg1_x2(i);
        V(1,3)=wg1_x2(i+1);
        V(1,4)=wg1_x1(i+1);
        V(2,1)=wg1_y1(i);
        V(2,2)=wg1_y2(i);
        V(2,3)=wg1_y2(i+1);
        V(2,4)=wg1_y1(i+1);
        addpoly;
        set("vertices",V);
    }

    V=matrix(2,4);
    V(1,1)=wg1_x1(N);
    V(1,2)=wg1_x2(N);
    V(1,3)=wg2_x1(N);
    V(1,4)=wg2_x2(N);
    V(2,1)=wg1_y1(N);
    V(2,2)=wg1_y2(N);
    V(2,3)=wg2_y1(N);
    V(2,4)=wg2_y2(N);
    addpoly;
    set("vertices",V);

    for(i=1:N-1)
    {
        V=matrix(2,4);
        V(1,1)=wg2_x1(i);
        V(1,2)=wg2_x2(i);
        V(1,3)=wg2_x2(i+1);
        V(1,4)=wg2_x1(i+1);
        V(2,1)=wg2_y1(i);

```

```

V(2,2)=wg2_y2(i);
V(2,3)=wg2_y2(i+1);
V(2,4)=wg2_y1(i+1);
addpoly;
set("vertices",V);
}

V=matrix(2,4);
V(1,1)=0;
V(1,2)=0;
V(1,3)=wg2_x1(1);
V(1,4)=wg2_x2(1);
V(2,1)=R-Wwg/2;
V(2,2)=R+Wwg/2;
V(2,3)=wg2_y1(1);
V(2,4)=wg2_y2(1);
addpoly;
set("vertices",V);

selectall;
set("material",Material_Si);
set("z min",0);
set("z max",Thickness_Si);

groupscope("::model");

minvxWAFER=-3*Wwg;
maxvxWAFER=-minvxWAFER;
minvy=-3*Wwg;
maxvy=-minvy;

# buried Oxide
addrect; set("name", "Oxide");
set("x min", minvxWAFER-3e-6); set("y min", minvy-R);
set("x max", maxvxWAFER+3e-6+R); set("y max", maxvy+R);
set("z min", -2e-6); set("z max", 0);
set("material", Material_Ox);
set("alpha",0.5);

# Cladding oxide
addrect; set("name", "Cladding");
set("x min", minvxWAFER-3e-6); set("y min", minvy-R);
set("x max", maxvxWAFER+3e-6+R); set("y max", maxvy+R);
set("z min", 0); set("z max", 2.3e-6);
set("material", Material_Ox);
set("alpha",0.1);
set("override mesh order from material database", 1);
set("mesh order", 4); # make the cladding the background, i.e., "send to back".

MonSpanY=2e-6+Wwg;
MonSpanX=2e-6+Wwg;
SimMarginX=1e-6;
SimMarginY=1e-6;
SourceMarginX=2e-6;
MonMarginX=2e-6;

#####Simulation
addfdtd;
set("x min", minvxWAFER+SimMarginX); set("x max", maxvxWAFER-SimMarginX+R);
set("y min", minvy+SimMarginY-R); set("y max", maxvy-SimMarginY+R);
set("z min", minvzWAFER); set("z max", maxvzWAFER);
set("mesh accuracy", MESH);
set("z min bc", BC); set("z max bc", BC); set("y min bc", BC); set("y max bc", BC);
set("x max bc", BC);
set("auto shutoff min", Autoshutoff);

```



```

set("simulation time", 2*pi*R/(c/4.5));

addmode; set("name", "simulation-port");
set("injection axis", "x-axis");
set("x", -1e-6); # simulate up to the layout port (wg_width x wg_width box)
set("y span", MonSpanY);
set("y", -R);
set("z min", minvzWAFER); set("z max", maxvzWAFER);

addpower; set("name", "Output");
set('monitor type', '2d X-Normal');
set('y', R);
set("y span", MonSpanY);
set("z min", minvzWAFER);
set("z max", maxvzWAFER);
set('x', -1e-6);
set('override global monitor settings', 1);
set("frequency points", FREQ_PTS);
set("use linear wavelength spacing", 1);

addmodeexpansion; set("name", "TE00");
set('monitor type', '2d X-Normal');
set('y', R);
set("y span", MonSpanY);
set("z min", minvzWAFER);
set("z max", maxvzWAFER);
set('x', -1e-6);
set('override global monitor settings', 1);
set("auto update before analysis", 0);
set("frequency points", FREQ_PTS);
set("use linear wavelength spacing", 1);

addmodeexpansion; set("name", "TE01");
set('monitor type', '2d X-Normal');
set('y', R);
set("y span", MonSpanY);
set("z min", minvzWAFER);
set("z max", maxvzWAFER);
set('x', -1e-6);
set('override global monitor settings', 1);
set("auto update before analysis", 0);
set("frequency points", FREQ_PTS);
set("use linear wavelength spacing", 1);

select("FDTD"); setview("extent");

```

B.2 Multiplexers

This is the Lumerical script used to draw and simulate the multiplexers using FDTD. The results from these simulations are presented in Chapter 4. The gap size (g), input waveguide width (W_1), bus waveguide width (W_{wg}), input waveguide curve radius ($radius$), coupler length (L_c), taper size ($taper$), and *wavelength* can all be altered in the code. As well, the code allows for two levels of accuracy in the simulation. One

is fast and approximate, while the other is slow and accurate.

```
# inputs:
#      gap:          directional coupler gap
#      Lc:           coupler length, 0=point coupling
#      wavelength:   centre wavelength, e.g., 1550e-9
#      wg_width:     width of waveguide, e.g., 500e-9;

deleteall;

newproject;

DCError=0e-9;

wavelength=1550e-9;
gap=200e-9-DCError;
Lc=39e-6;
radius=5e-6;
taper=100e-9;

w1 = 450e-9+DCError;
Wwg = 930e-9+DCError;

Wwg1=Wwg-taper/2;
Wwg2=Wwg+taper/2;

SIM_DRAFT = 0; # 1= fast , 0=accurate

materials; # creates a dispersive material model in Lumerical.

if (SIM_DRAFT==1) {
    MESH = 1; # test ,
    FREQ_PTS = 1; # fast test
    FDTD_above=200e-9; # Extra simulation volume added, 0.5 um on top and
        bottom
    FDTD_below=200e-9;
    BC = "metal"; # boundary conditions on non-critical sides.
    Autoshtutoff=5e-2;
} else {
    MESH = 3; # final = 4
    FREQ_PTS = 100;
    FDTD_above=800e-9; # Extra simulation volume added, 0.5 um on top and
        bottom
    FDTD_below=800e-9;
    BC = "PML"; # boundary conditions on non-critical sides.
    Autoshtutoff=1e-5;
}

setglobalsource("center frequency",c/wavelength);
setglobalsource("frequency span",c/wavelength/15.5);
Material_Si = "Si (Silicon) - Dispersive & Lossless";
Thickness_Si=0.22e-6;

#Etch2=70e-9; #i.e., the slab thickness is 220-70 = 150 nm
Etch2=220e-9; #i.e., the slab thickness is 220-220 = 0 nm

Material_Ox ="SiO2 (Glass) - Const";

maxvzWAFER=Thickness_Si+FDTD_above; minvzWAFER=-FDTD_below;
```

```

# *****
## WAVEGUIDE FOR COUPLER:
# *****
wg_bottom_input_y1=Wwg1/2-gap/2;
wg_bottom_input_y2=Wwg2/2-gap/2;

wg_top_input_y=w1/2+gap/2;

addgroup; set("name","bus"); groupscope("bus");

addrect;
set("y span",Wwg1);
set("y",wg_bottom_input_y1);
set("x min",-Lc/2-3e-6-2*radius);
set("x max",-Lc/2);

addrect;
set("y span",Wwg2);
set("y",wg_bottom_input_y2);
set("x min",Lc/2);
set("x max",Lc/2+3e-6+2*radius);

V=matrix(2,4);
V(1,1)=-Lc/2;
V(1,2)=-Lc/2;
V(1,3)=Lc/2;
V(1,4)=Lc/2;
V(2,1)=wg_bottom_input_y1-Wwg1/2;
V(2,2)=wg_bottom_input_y1+Wwg1/2;
V(2,3)=wg_bottom_input_y2+Wwg2/2;
V(2,4)=wg_bottom_input_y2-Wwg2/2;
addpoly;
set("vertices",V);

selectall;
set("material",Material.Si);
set("z min",0);
set("z max",Thickness.Si);

groupscope("::model");
addgroup; set("name","input"); groupscope("input");

addrect;
set("y span",w1);
set("y",wg_top_input_y+2*radius);
set("x min",-Lc/2-3e-6-2*radius);
set("x max",get("x min") + 3e-6);

copy;
set("x max",Lc/2+3e-6+2*radius);
set("x min",get("x max") - 3e-6);

addring;
set("inner radius",radius-w1/2);
set("outer radius",radius+w1/2);
set("theta start", 90);
set("theta stop", 180);
set("x",Lc/2+2*radius);
set("y", wg_top_input_y + radius);

copy;
set("theta start", 0);
set("theta stop", 90);
set("x",-get("x"));

```

```

copy;
set("theta start", -90);
set("theta stop", 0);
set("x", Lc/2);
set("y", wg_top_input_y+radius);

copy;
set("theta start", -180);
set("theta stop", -90);
set("x", -get("x"));

addrect;
set("y span", w1);
set("y", wg_top_input_y);
set("x", 0);
set("x span", Lc);

selectall;
set("material", Material_Si);
set("z min", 0);
set("z max", Thickness_Si);

groupscope("::model");

maxvxWAFER = Lc/2+radius+2e-6;
minvxWAFER = -maxvxWAFER;
minvy=-gap/2-3*Wwg;
maxvy=radius+gap/2+3*w1;

# buried Oxide
addrect; set("name", "Oxide");
set("x min", minvxWAFER-radius); set("y min", minvy-radius);
set("x max", maxvxWAFER+radius); set("y max", maxvy+radius);
set("z min", -2e-6); set("z max", 0);
set("material", Material_Ox);
set("alpha", 0.3);

# Cladding oxide
addrect; set("name", "Cladding");
set("x min", minvxWAFER-radius); set("y min", minvy-radius);
set("x max", maxvxWAFER+radius); set("y max", maxvy+radius);
set("z min", 0); set("z max", 2.3e-6);
set("material", Material_Ox);
set("alpha", 0.1);
set("override mesh order from material database", 1);
set("mesh order", 4); # make the cladding the background, i.e., "send to back".

MonSpanY=2e-6+Wwg;
MonSpanX=2e-6+Wwg;
yInputPort=maxvy-Wwg/2;
SimMarginX=0.5e-6;
SimMarginY=0.5e-6;
SourceMarginX=2e-6;
MonMarginX=2e-6;

addfdtd;
set("x min", minvxWAFER-SimMarginX); set("x max", maxvxWAFER+SimMarginX);
set("y min", minvy); set("y max", maxvy);
set("z min", minvzWAFER); set("z max", maxvzWAFER);
set("mesh accuracy", MESH);
set("z min bc", BC); set("z max bc", BC); set("y min bc", BC); set("y max bc", BC);
set("auto shutoff min", Autoshtutoff);
set("simulation time", Lc/c*4.5+500e-15); # distance/v_g + pulse_time

```

```

addmode; set("name", "simulation-port");
set("injection axis", "y-axis");
set("Direction", "Backward");
set("x", -Lc/2-radius); # simulate up to the layout port (wg_width x wg_width box)
set("x span", MonSpanX);
set("y", wg_top_input_y + radius);
set("z min", minvzWAFER); set("z max", maxvzWAFER);

addpower; set("name", "Coupled");
set('monitor type', '2d X-Normal');
set('y', wg_bottom_input_y2);
set("y span", MonSpanY);
set('z', Thickness.Si/2);
set('x', maxvxWAFER -1e-6);
set('override global monitor settings', 1);
set("frequency points",FREQ_PTS);

addmodeexpansion; set("name", "Coupled TE00");
set('monitor type', '2d X-Normal');
set('y', wg_bottom_input_y2);
set("y span", MonSpanY);
set('z', Thickness.Si/2);
set('x', maxvxWAFER -1e-6);
set('override global monitor settings', 1);
set("auto update before analysis",0);
set("frequency points",FREQ_PTS);

addmodeexpansion; set("name", "Coupled TE01");
set('monitor type', '2d X-Normal');
set('y', wg_bottom_input_y2);
set("y span", MonSpanY);
set('z', Thickness.Si/2);
set('x', maxvxWAFER -1e-6);
set('override global monitor settings', 1);
set("auto update before analysis",0);
set("frequency points",FREQ_PTS);

addpower; set("name", "Through");
set('monitor type', '2d Y-Normal');
set('y', wg_top_input_y+radius);
set("x span", MonSpanX);
set('z', Thickness.Si/2);
set('x', Lc/2+radius);
set('override global monitor settings', 1);
set("frequency points",FREQ_PTS);

addmodeexpansion; set("name", "Through TE00");
set('monitor type', '2d Y-Normal');
set('y', wg_top_input_y+radius);
set("x span", MonSpanX);
set('z', Thickness.Si/2);
set('x', Lc/2+radius);
set('override global monitor settings', 1);
set("frequency points",FREQ_PTS);

addpower; set("name", "Cut");
set('monitor type', '2D Z-normal');
set("x min", minvxWAFER-SimMarginX); set("x max", maxvxWAFER+SimMarginX);
set("y min", minvy); set("y max", maxvy);
set("z min", minvzWAFER); set("z max", maxvzWAFER);
set('z', Thickness.Si/2);
set('override global monitor settings', 1);
set("frequency points",FREQ_PTS);

```

```
select("FDID"); setview("extent");
```

Appendix C

Test Structure Design

Here the images of the mask layout for the multiplexer and demultiplexer test structure, and the Bezier curve test structure are presented, in Fig. 32 and Fig. 33, respectively. In Fig. 32 the loss of the device is measured from the TE_{01} output after an input is multiplexed and demultiplexed onto the TE_{01} mode. A separate structure with an input to the TE_{00} mode was used to find the TE_{01} crosstalk level. In Fig. 33 crosstalk in either Bezier or semi-circular curves is found. The fundamental mode is input into the curve. The output from the curve is then demultiplexed and transmission at the TE_{01} output is measured as crosstalk. These results are presented in Chapter 5.

Figure 32: The multiplexer and demultiplexer test structure. The input is multiplexed to the TE₀₁ mode of the bus waveguide, which is then demultiplexed after a short distance. This allows for the calculation of (de)multiplexer loss.

Figure 33: The Bezier curve test structure. The TE_{00} mode propagates through 2, 90° Bezier curves. After the curves the output is demultiplexed to determine the magnitude of the TE_{01} mode that was produced during propagation. This allows for the calculation of curve crosstalk.